ON THE DEFINITION OF INTRAURBAN MOBILITY CONCEPTS

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by

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ABSTRACT

Several concepts have been developed recently in the intraurban mobility literature. Among them the concepts of aspirations, place utility and stress are of particular importance. Nevertheless, their definition appears to be not very clear.

This paper aims to clarify them by employing concepts from the consumer choice theory as they are used in equilibrium models in Geography. With this purpose in mind the world of a specific equilibrium model is used. The distinction between the hypothetical world of this model and the real world is continuously emphasized.

In order to clarify further the concepts mentioned above a particular example is given. In this example a Cobb-Douglas is used as a utility function.

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SECTION I

INTRODUCTION

In recent years, intraurban mobility has attracted the interest of geographers and psychologists. As a result of this, a literature has been developed, reflecting several different views and approaches. A large portion of this literature is devoted to analysing the mechanisms that lead the household to the decision to seek a new residence. This kind of analysis involves the introduction of several new concepts and ideas. It is worthwhile to mention that the household is studied in its actual environment, and the concepts and ideas are considered within the real world.

Equilibrium models, in contrast, are built in a hypothetical world where the theoretical basis is consumer choice theory. The household is viewed as a rational decision maker with the ability to choose from a set of different alternatives. It has certain preferences, whereby the different alternatives are set in an order according to these preferences. The household chooses the most prefered alternative, even if it only provides slightly more satisfaction than what it already has. Economic factors such as the budget constraint of the household, are taken into account in its decision making process.

The objective of the present paper is twofold. First, to relate concepts of intraurban mobility to those of consumer choice theory and second, to provide a model of residential mobility based on a particular hypothetical world. In this case the hypothetical world refered to is that used by Papageorgiou (1976). The ultimate purpose of the paper is an attempt to understand the household as a decision maker in the real world through the hypothetical world of the model.

To this end, the paper is divided into five sections. In the second section, the most basic concepts in residential mobility literature are reviewed. Most important among them are the concepts of aspirations, place utility and stress.

The third section carries the heaviest load. The hypothetical world of the above mentioned model is described in the first subsection. Concepts from consumer choice theory are reviewed and some of them are slightly modified. Each of the concepts occupy a particular subsection. The concepts of the consumption set, preferences, utility and budget constraint are discussed in the next four subsections. In the last subsection a definition of stress is given. Here, the distinction between the real and the hypothetical world is emphasized. Stress is, in a way, the link between the two worlds.

The maximum satisfaction the household can attain with its income in the hypothetical world is considered identical to its expectations in reality. The difference between its expectations at a location and the satisfaction imposed by the limitations of the household's environment at the same location are defined as the stress of the household at this particular location. The price of land plays an important role in determining the expected level of satisfaction. The household is willing to pay a certain rent at each location in order to achieve the level of satisfaction it expects.

A particular case, where the Cobb-Douglas function is used to express the preferences of the household, is treated in the fourth section. The expectations of a household are determined as a function of its income and location.

The last section is devoted to a brief recapitulation of the previous three sections and conclusion. It must remain explicit that all the discussion refers to an individual household, hence we are not faced with any aggregation problem. Also, although equilibrium models are frequently refered to, the notion of equilibrium is not used.

The notation used is of the most common form. A vector is denoted by an underline, e.g., $\underset{\sim}{x}$, $\underset{\sim}{z}$. Functions are indicated by square brackets, e.g., $f[\underset{\sim}{x}]$. All vectors are considered as column vectors. A transpose of a vector is denoted by a prime. Having two vectors $\underset{\sim}{x}$ and $\underset{\sim}{y}$, the inner product of them is denoted by $\underset{\sim}{x}'\cdot\underset{\sim}{y}$. The sets are denoted by capital letters or by chain brackets, e.g., $\{1,2\}$. Also, $\underset{\sim}{x} >> \underset{\sim}{y}$ means $x_i > y_i$ for all i. Membership in a set is denoted by ε. $A\times B$ is the cartesian product of the sets A and B. Theorems, propositions, lemmas, corollaries and definitions are numbered consecutively within each subsection. The n-dimensional real space is denoted by $\mathbb{R}^n$. I_n is the set $\{1,2,\ldots n\}$.

SECTION II

INTRAURBAN MOBILITY CONCEPTS

Several writers have attempted to analyse intraurban mobility. The field is dominated by the ideas of Rossi (1955), Wolpert (1965), Brown and Moore (1970) and others. The purpose of this section is to provide a brief review of the concepts developed in these studies[1]. Most important among these are the concepts of aspirations, place utility and stress.

In early studies it has been clarified that mobility is a response to the discrepancy between the needs and wants of the household and the offerings of the environment (Rossi 1955, Leslie and Richardson 1961). More recently, Wolpert (1965) has emphasized that the household does not respond to the environment itself but to its perception and evaluation of that environment. In this context, the household may be viewed as an intendedly rational decision maker, i.e., the household is assumed to have imperfect information but a considerably high evaluational ability. The household subjectively determines a threshold of net utility or an attainable aspiration level which is influenced by past experience, needs and desires. This aspiration level functions as a standard from which differences may be evaluated.

Brown and Moore (1970) provided the intendedly rational household with an aspiration region. This may be thought of as a multidimensional space in which the dimensions correspond to the attributes of a residence. Members of the aspiration region then are vectors consisting of the particular attributes of each residence. Vectors corresponding to residences in the action space, i.e., the area over which the household has information, may or may not fall within the aspiration region. The limits of the aspiration region are a lower aspiration vector and an upper aspiration vector. Those are defined by the household according to its needs and are sensitive in time. In this context the term aspiration is unfortunate because aspirations and needs, with their usual meanings, are not consistent. Recently Kennedy (1975) has used the term expectations to describe the objectives of a household that are constrained by its ability to attain a particular type of residence. The concept of constrained expectations is essentially the same as attainable aspirations in the sense that Wolpert defines. It is obvious that attainable aspirations fall within the aspiration region.

The concept of place utility is defined by Wolpert (1965) as the relative value attached to a particular residence by a household. It is relative in the sense that its value is proportional to the difference of the particular residence from the aspiration level. Moore (1972) provides a different conceptualization. According to him a household is assigned a set of basic values. Each residence site in the action space, then, is associated with a place utility, which is a measure of the satisfaction the household obtains (or would obtain) from that particular residence. A preference ordering of the residences in the action space takes place.

Definitions of place utility given by Simmons (1968) and Brown and Longbrake (1970) are not essentially different from the one given by Wolpert.

Concerning stress, Brown and Moore (1970) define that as a measure of a set of stressors. Stressors refer to the disparity between the household's needs and its environmental offerings. Nevertheless, this definition almost coincides with the definition of place utility given by Wolpert (1965). Clark and Cadwallader (1973) and Clark (1975) proposed a different conceptualization. According to them a household has a level of satisfaction in its present place and it believes it may attain another level, naturally higher, elsewhere. The difference between those two levels is defined by them as stress.

SECTION III

EQUILIBRIUM MODELS AND INTRAURBAN MOBILITY

III.1 A Model

The framework of neoclassical consumer choice theory has served as a prototype for equilibrium models in Geography. One could mention, without exhausting the list, work by Alonso (1964), Beckmann (1969), Long (1971), Casetti (1971), Solow (1973) and Papageorgiou (1976) as representative, in this context[2].

For our discussion we are going to adopt the world described by Papageorgiou (1976) as more general than the others. It is more general in the sense that according to it households live in a multicentre environment and are distinguished by their income. A brief description is given by the following summary.

Everything takes place in an urban region. The centres are connected subsets of it. All the goods and services are provided by the centres. Goods and services are classified into n groups. Prices change with location due to transportation cost. All the goods and services whose prices change proportionally belong to the same group. Every centre is associated with a set of different groups. A centre is said to be of order i if the set associated with it contains i groups. The highest order in the system is obviously n. Christaller's (1933) assumption is adopted:

> "If a centre is of order i, then it provides all the groups a centre of order (i-1) provides."

Clearly, then, for any two centres of the same order the associated sets are such that the one is a subset of the other.

Every location s in the city, if it is not on the boundary of a Dirichlet region, has one and only one centre of order i closest to it. Each such location has one and only one corresponding n-dimensional vector $\underset{\sim}{s}$ representing distances to its closest centres of all the orders. The vector $\underset{\sim}{s}$ then represents the location of a household in relation to the spatial economy. We shall refer to the physical location of a household as s. Our discussion will be limited to locations that are not on the boundaries of the Dirichlet regions. All the centres offering the same goods, are charging the same price for them, regardless of their order. This means that economies of scale are not taken into account.

In the remainder of this section the hypothetical world described above will be enriched with some concepts and assumptions from the theory of consumer choice. However, the distinction between this hypothetical world and the world reviewed in Section I will remain explicit. Our incentive will be to analyse intraurban mobility through the model described above.

III.2 The Consumption Set

The basic concept in consumer choice theory is that of the consumption set Z. Generally:

$$Z \subseteq \mathbb{R}^n .$$

For a $\underset{\sim}{z}' = (z_1, z_2, \ldots z_n) \in Z$, z_i represents the amount of the ith group of goods in the particular consumption $\underset{\sim}{z}$. Furthermore, Z is traditionally taken to be the nonnegative orthant of $\mathbb{R}^n$. As Takayama (1974) points out, this convention implicitly contains the assumptions that:

(a) All the individual households have the same starvation point or minimum subsistence consumption which is the origin.

(b) A household can consume any amount of commodities.

(c) The set Z is convex.

Regarding (a) we may imagine that every household has its own subsistence consumption. The mean value, say, of all consumptions in the city may be accepted as the subsistence consumption of every household. For our discussion we shall adopt $\hat{\underset{\sim}{z}}$ as this consumption. Some of the elements of $\hat{\underset{\sim}{z}}$, however, may be zero. Certainly, substitution and trade offs do not make sense below this level.

In an analogous manner we may accept that every household has its own upper limit in the consumption of certain goods. A household e.g. is unable to consume more than 30 Kgr of bread per week. For a good like bread, we may accept the highest upper limit in the city as a limit for all the households. It is recognized, however, that there is not an upper limit for every z_i in a consumption $\underset{\sim}{z}$. A household may get satisfaction from owning commodities

Therefore, if $\check{\underset{\sim}{z}}$ is an upper bound, some of its elements are equal to infinity.

We shall accept the multidimensional set

$$Z = \{\underset{\sim}{z} \varepsilon \mathbb{R}^n | \hat{\underset{\sim}{z}} \leq \underset{\sim}{z} \leq \check{\underset{\sim}{z}} \quad ; \quad \hat{\underset{\sim}{z}} > 0 \quad , \quad \check{\underset{\sim}{z}} >> \hat{\underset{\sim}{z}}\}$$

as our consumption set. Since some of the elements of $\check{\underset{\sim}{z}}$ are equal to infinity, this set is not bounded.

However, assumption (c) is still valid, but it is quite strong. It assumes that all commodities are perfectly divisible. Certainly, this is not the case in reality but as we shall see later in this section, it helps us to gain insights to the household's behaviour.

The multidimensional set Z corresponds to the aspiration region of the Brown and Moore (1970) framework. The limits $\hat{\underset{\sim}{z}}$ and $\check{\underset{\sim}{z}}$ correspond to the lower and upper aspiration vectors, respectively. Some of the elements of a particular $\underset{\sim}{z} \varepsilon Z$ represent the attributes of the residence corresponding to $\underset{\sim}{z}$. Therefore, $\underset{\sim}{z} \varepsilon Z$ is a more general vector than a member of a Brown and Moore (1970) aspiration region. Also, a vector corresponding to a residence in the action space may or may not fall within the aspiration region whereas any possible consumption $\underset{\sim}{z}$ by a household has to be a member of Z.

The difference between Z and the actual consumption set of a household is that the first is convex whereas the second is not. To understand this difference better, let us imagine the action space of a household as the set of locations $\{s^1, s^2, \ldots s^\ell\}$. Each location is associated with a particular residence. Every residence has its own attributes. A household in a particular residence has to choose goods and services other than housing. Therefore, with each residence in the action space is associated a number of different consumptions. Each one of those consumptions represents an n-dimensional point in Z. The set of all those points is clearly a subset of Z. This is the actual consumption set of the household. Figure 1 illustrates the case when there are only two goods z_1 and z_2. The actual consumption set is the set of the dots.

III.3 Preferences

The concept of preferences came into consumption theory from a more general branch called choice theory. Strictly speaking, the notion of preference is a primitive one. Given two consumptions $\underset{\sim}{z}^1$ and $\underset{\sim}{z}^2$ in Z, one of the following three alternatives is assumed to hold for a household:

(a) $\underset{\sim}{z}^1$ is prefered to $\underset{\sim}{z}^2$;

(b) $\underset{\sim}{z}^2$ is prefered to $\underset{\sim}{z}^1$;

(c) $\underset{\sim}{z}^1$ is indifferent to $\underset{\sim}{z}^2$.

Following Debreu (1959) we provide this particular household with the binary relation "is not prefered to". Given $\underset{\sim}{z}^1$ and $\underset{\sim}{z}^2$, it is denoted by $\underset{\sim}{z}^1 \precsim \underset{\sim}{z}^2$ and is read "$\underset{\sim}{z}^1$ is not prefered to $\underset{\sim}{z}^2$". An equivalent way of denoted the same thing is $\underset{\sim}{z}^2 \succsim \underset{\sim}{z}^1$. This relation is proven to be preordering. Taking into account the assumption that for any two members of Z exactly one of the above three alternatives holds, this

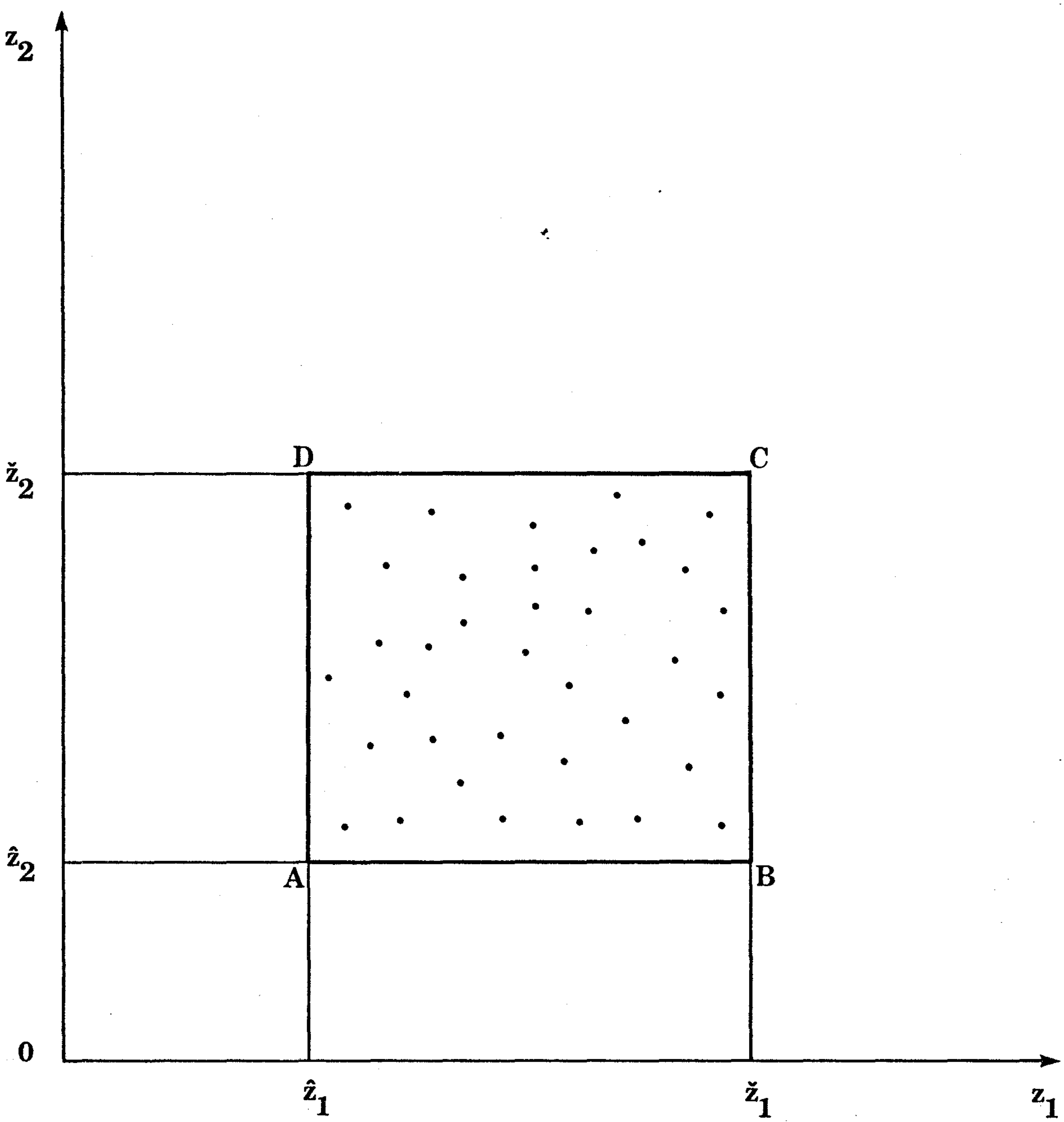

FIGURE 1

The Consumption Set.

Z is the orthogonal ABCD, whereas the actual consumption set is the set of the dots.

is a complete preordering[3].

We may define the indifference relation now as follows:

$$(\underset{\sim}{z}^1 \precsim \underset{\sim}{z}^2 \quad \text{and} \quad \underset{\sim}{z}^2 \precsim \underset{\sim}{z}^1) \iff \underset{\sim}{z}^1 \sim \underset{\sim}{z}^2 .$$

$\underset{\sim}{z}^1 \sim \underset{\sim}{z}^2$ is read "$\underset{\sim}{z}^1$ is indifferent to $\underset{\sim}{z}^2$". This relation is easily proven to be an equivalence[4]. We may also define the following relation:

$$(\underset{\sim}{z}^1 \precsim \underset{\sim}{z}^2 \quad \text{and not} \quad \underset{\sim}{z}^2 \precsim \underset{\sim}{z}^1) \iff \underset{\sim}{z}^1 \prec \underset{\sim}{z}^2 .$$

"Not $\underset{\sim}{z}^2 \precsim \underset{\sim}{z}^1$" is the denial of the statement "$\underset{\sim}{z}^2 \precsim \underset{\sim}{z}^1$". "$\underset{\sim}{z}^1 \prec \underset{\sim}{z}^2$" may be read as "$\underset{\sim}{z}^2$ is prefered to $\underset{\sim}{z}^1$". This is written alternatively as "$\underset{\sim}{z}^2 \succ \underset{\sim}{z}^1$". For some $\underset{\sim}{z}^1 \in Z$ we define the set:

$$\{\underset{\sim}{z} \in Z \mid \underset{\sim}{z} \sim \underset{\sim}{z}^1\}$$

and we call it the indifference class corresponding to $\underset{\sim}{z}^1 \in Z$. It is easy to see that each member of Z belongs to one and only one indifference class. Alternatively we may say that the set of all the indifference classes forms a partition of Z. It must remain explicit that each particular household is associated with its own partition of Z.

The consumption $\underset{\sim}{z} \in Z$ will be called the satiation consumption of a particular household if no other consumption in Z is prefered to it. We shall adopt Debreu's insatiability assumption:

"No satiation consumption exists for any household in Z."

This assumption is consistent with the definition of Z. As it has been defined, it is not bounded. Also, we have accepted satiation points for certain goods in $\underset{\sim}{z}$ but not for the $\underset{\sim}{z}$ itself.

Since Z is convex and the preordering relation is complete, we need an assumption on the continuity of preferences. Thus:

"For every $\underset{\sim}{z}^1 \varepsilon Z$, the sets $\{\underset{\sim}{z} \varepsilon Z | \underset{\sim}{z} \precsim \underset{\sim}{z}^1\}$ and $\{\underset{\sim}{z} \varepsilon Z | \underset{\sim}{z}^1 \precsim \underset{\sim}{z}\}$ are closed in Z."

Each indifference class may be represented as a curve within Z. Traditionally those curves are taken to be convex towards the origin. To guarantee this shape we shall assume that the preference ordering is a strictly convex one[5].

"If $\underset{\sim}{z}^1 \sim \underset{\sim}{z}^2$ then $t\underset{\sim}{z}^1 + (1-t)\underset{\sim}{z}^2 \succ \underset{\sim}{z}^1$, $0 < t < 1$."

According to Chipman (1960) the interpretation of this assumption is that:

"... people desire to consume a variety of products rather than limit their consumption to any one commodity alone."

The household, according to its preferences, associates each residence in its action space with the best consumption of goods other than housing. Therefore, the group of consumptions associated with s^i in the action space is represented by the most prefered consumption of the group, say, $\underset{\sim}{z}^i$. In case another consumption of the group is in the same indifference curve with $\underset{\sim}{z}^i$, one of them is chosen. In the simple case of two goods (Figure 2), the dots represent such consumptions. Each dot is assumed to belong to some indifference curve. The case of having two of them on the same indifference curve is not excluded.

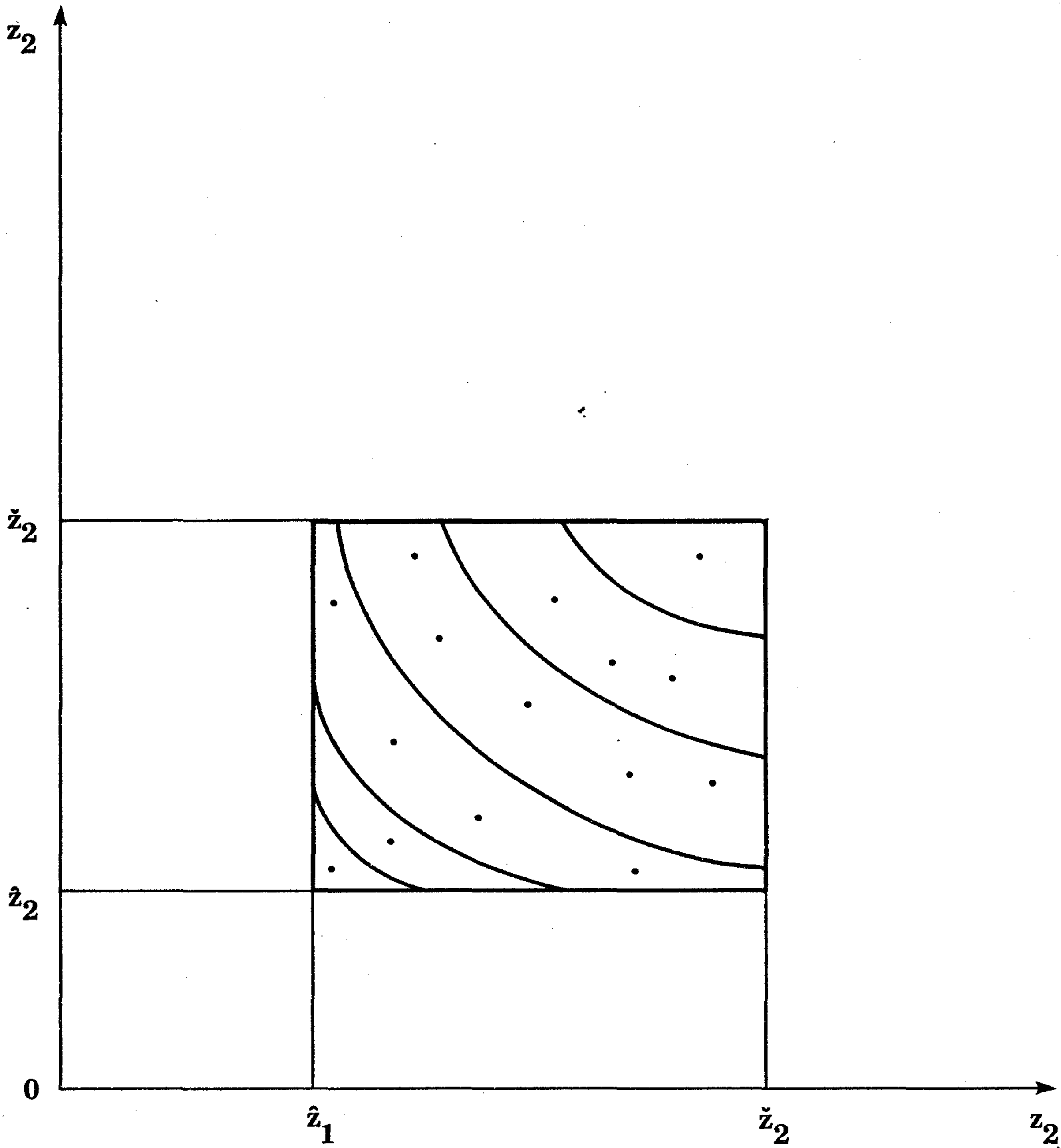

FIGURE 2

Indifference Curves.

Finally, the notion of preference is similar to what Moore (1972) has called basic set of values. Both of the notions are related to a particular individual.

III.4 Utility

The task of this subsection is to answer the question: Is it possible to associate with each indifference curve a real number such that if one indifference curve is prefered to another, then the number associated with the first is greater than the number associated with the second? What is actually required is to establish a continuous, real valued, increasing function u with:

$$u : \quad Z \to \mathbb{R}$$

$$\underset{\sim}{z}^1 \precsim \underset{\sim}{z}^2 \quad \Leftrightarrow \quad u(\underset{\sim}{z}^1) \leq u(\underset{\sim}{z}^2)$$

$$\underset{\sim}{z}^1 \sim \underset{\sim}{z}^2 \quad \Leftrightarrow \quad u(\underset{\sim}{z}^1) = u(\underset{\sim}{z}^2) \ .$$

This function will be called a utility function. The answer as to the existence of such a function is given by a theorem, proven by Debreu. This is stated as:

> "If Z is a connected subset of $\mathbb{R}^n$ and there exist a complete preordering in it which fulfills the assumption of continuity, then there exist a continuous utility function on Z."

Z as it has been specified in subsection III.2 is convex and, therefore, connected. All the requirements of the theorem are satisfied and the existence of a continuous utility function in Z is guaranteed.

It has been pointed out that the consumption set is not connected in reality. We may see this clearly in Figure 1, where the set of dots is not connected. We may easily accept the existence of a complete preordering in the actual consumption set. However, the assumption of preference continuity is highly unrealistic. It is impossible to have continuity of preferences in a non-connected set. The actual consumption set is a subset of Z. Every member of the actual consumption set belongs to exactly one indifference curve of Z. Thus, despite the fact that the real consumption set does not fulfill the assumptions of the Debreu theorem, a real number is associated with each member of it such that: if one consumption is prefered to another, the number associated with the first is greater than the number associated with the second.

This kind of utility is similar to the place utility defined by Moore (1972). Certainly, his concept of the basic set of values, with which every household is assigned, corresponds to the preferences of the household.

III.5 The Budget Constraint

In the consumer choice theory it is assumed that the household's choice on consumption is constrained by its income. This is expressed by:

$$\underset{\sim}{p}'\cdot\underset{\sim}{z} \leq y \tag{1}$$

where $\underset{\sim}{p}'$ represents the transpose of the vector of prices. The inner product $\underset{\sim}{p}'\cdot\underset{\sim}{z}$ is the total money spent on the consumption $\underset{\sim}{z}$ and this amount must not exceed the income of the household y.

In our discussion we shall assume that prices change due to transportation cost. This implies that the prices are functions of the vector of distances $\underset{\sim}{s}$. The equivalent of (1) in our case will be:

$$\underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z} \leq y \ . \tag{2}$$

A household with income y at the particular location s has to choose from the set $\{\underset{\sim}{z} \in Z | \underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z} \leq y\}$, where $\underset{\sim}{s}$ corresponds to s.

Certainly, there are pairs of $(\underset{\sim}{s},y)$ such that this set is empty. We may consider the subset S×Y of $\mathbb{R}^{n+\ell}$ that contains the pairs $(\underset{\sim}{s},y)$ and only those that correspond to a nonempty $\{\underset{\sim}{z} \in Z | \underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z} \leq y\}$.

Then, we may establish a correspondence γ from S×Y to the set of subsets of Z.

$$S\times Y \ni (\underset{\sim}{s},y) \rightarrow \gamma[\underset{\sim}{s},y] = \{\underset{\sim}{z} \in Z | p'[\underset{\sim}{s}]\cdot\underset{\sim}{z} \leq y\} \ .$$

For a pair $(\underset{\sim}{s},y) \in S\times Y$ the equation $\underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z} = y$ represents a hyperplane. Due to the shape of the indifference curves there exist one and only one tangent to this hyperplane. Therefore, the most prefered consumption of a household with income y at the particular location s is given by the tangent point $\underset{\sim}{z}^*$. The indifference curve of this point reflects the rational expectations of the household with income y. This is similar to the attainable aspirations of Wolpert (1965) and the expectations of Kennedy (1975).

The best consumption available at s is $\underset{\sim}{z}$. The consumption $\underset{\sim}{z}$ may or may not belong to the indifference curve of $\underset{\sim}{z}^*$. This depends on environmental constraints at s. Figure 3 indicates the two dimensional case.

III.6 Stress and the Household Maximization Problem

One particular good, land, and its price, rent, will play an important role in the discussion of this subsection. For convenience we shall abstract the quantity of land q from the consumption $\underset{\sim}{z}$. A consumption will be represented by the pair $(\underset{\sim}{z},q)$. Yet $\underset{\sim}{z}$ will be considered as an n-dimensional vector.

Also, the price of land r will be abstracted from $\underset{\sim}{p}[\underset{\sim}{s}]$. A price system will be denoted by $(\underset{\sim}{p}[\underset{\sim}{s}],r)$. A household with income y expects a definite level of satisfaction $\bar{\bar{u}}[y]$. This level depends on its income and not on any particular location. The answer to the question as to how this level is determined will be given within the hypothetical world. We shall assume that the particular

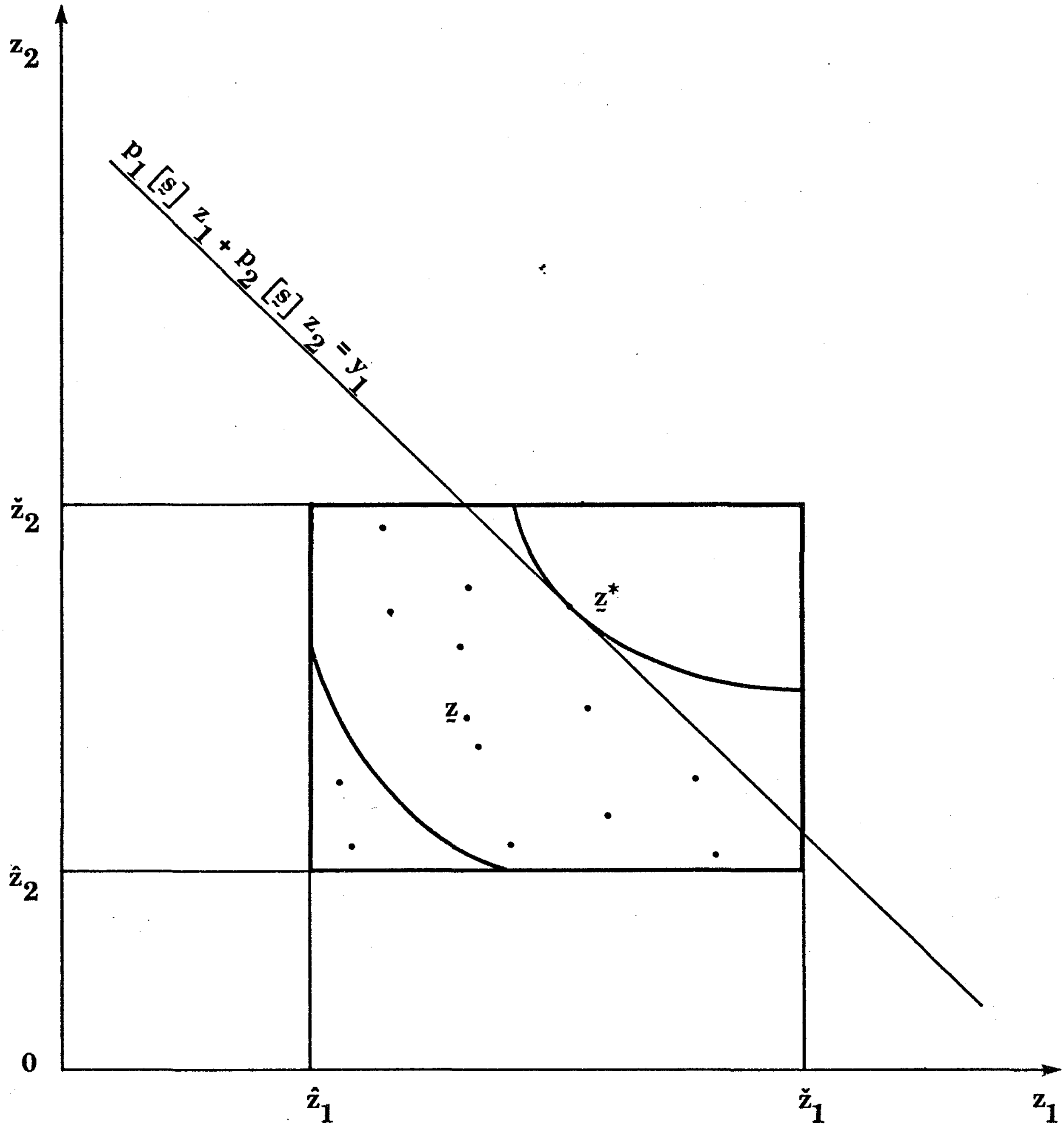

FIGURE 3

The Budget Constraint.

household has the ability to calculate $(\underset{\sim}{z}^*,q^*)$, the counterpart of $\underset{\sim}{z}^*$ in Section III.5. This calculation takes place for a particular location s. In other words, it is assumed that a household with income y at a location s has the ability to solve the problem:

$$\max_{\underset{\sim}{z},q} u[\underset{\sim}{z},q] \qquad \text{(III.6.1)}$$

with $\quad (\underset{\sim}{z},q) \varepsilon \{(\underset{\sim}{z},q) \varepsilon Z \mid p'[\underset{\sim}{s}]\cdot\underset{\sim}{z} + rq \leq y\}$.

The consumption satisfying problem (III.6.1) is $(\underset{\sim}{z}^*,q^*)$. The indifference curve, to which this consumption belongs, reflects the rational expectations of the household. The utility level corresponding to this indifference curve is denoted by $\bar{\bar{u}}[y]$.

It is important to note that the household is prepared to pay the particular rent r at the particular location s in order to achieve $(\underset{\sim}{z}^*,q^*)$. In another location s^1 the household is prepared to pay another rent r^1 in order to achieve a consumption in the same indifference curve as $(\underset{\sim}{z}^*,q^*)$.

To go back to reality now, the rational expectations of a household with income y are normally such that:

$$\bar{\bar{u}}[y] \geq u[\underset{\sim}{z}^i,q^i] \ .$$

The consumption $(\underset{\sim}{z}^i,q^i)$ corresponds to the location s^i of the action space. We may define now as stress of a household with income y at s^i the following:

$$S[s^i] = \bar{\bar{u}}[y] - u[\underset{\sim}{z}^i, q^i]$$

with $S[s^i] \geq 0$.

When stress at the present residence of a household exceeds a certain level, the household starts searching for a new residence. This certain level depends on psychological factors. If the place associated with the least stress is its present residence, the household will not move. Assume that the present residence of the household is at s^i and the residence associated with the least stress at s^j, then:

$$S[s^j] \leq S[s^i] \ .$$

The household will move as long as:

$$S[s^i] - S[s^j] > a$$

where a may be interpreted as a certain level of stress associated with moving expenses. The stress $S[s^j]$ may or may not be zero. It will be zero if and only if:

$$\bar{\bar{u}}[y] = u[\underset{\sim}{z}^j, q^j]$$

where $(\underset{\sim}{z}^j, q^j)$ is the consumption associated with the residence at s^j.

It is possible during the search, a residence with $(\underset{\sim}{z}, q)$ to be found such that:

$$u[\underset{\sim}{z},q] > \bar{\bar{u}}[y] \ .$$

This residence will be considered an exceptional case as it is beyond the expectations of the household. In the simple two dimensional case (Figure 4), the one of the two goods is the amount of land q. The other may be any good z. An upper limit for the consumption of land does not exist. The indifference curve to which all the straight lines are tangent represents the rational expectations of the household with income y. At each location, s^k, the household is prepared to pay a rent r^k (k = 1, 2, 3, 4) to be able to have a consumption consistent with its expectations. Let (z^1,q^1) represent consumption at the present residence of the household. The household will move to s^4 as long as:

$$S[s^1] - S[s^4] > a \ .$$

The consumption (z^5,q^5) represents an exceptional case as that described above.

The remainder of this paper is devoted to the discussion of problem (III.6.1) with the additional requirement:

$$u[\underset{\sim}{z},q] = \bar{\bar{u}}[y] \ .$$

u must be a strictly increasing real valued function of $\underset{\sim}{z}$ and q. As

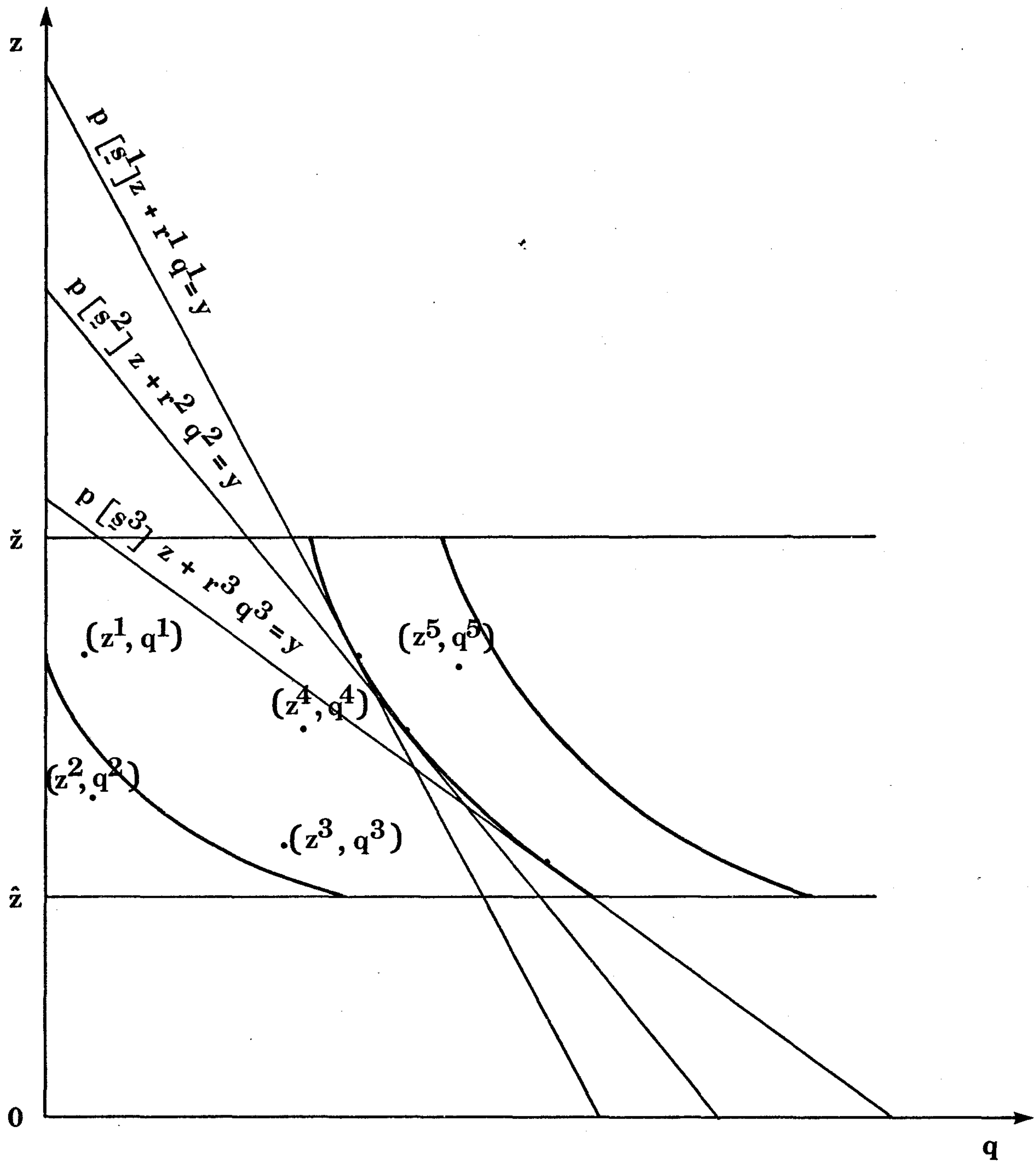

FIGURE 4

Expected Utility Level and the Budget Constraint

at Different Locations.

such the well known Cobb-Douglas function will be used.

The whole problem then is summarized as:

$$
\begin{aligned}
&\underset{\underset{\sim}{z},q}{\text{Max}}\ u[\underset{\sim}{z},q] = \prod_i z_i^{\alpha_i} q^{\beta} \qquad 0 < \alpha_i < 1,\ i \varepsilon I_n \\
&\qquad\qquad\qquad\qquad\qquad\qquad\ 0 < \beta < 1 \\
&\underset{\sim}{p}'[\underset{\sim}{s}] \cdot \underset{\sim}{z} + rq \leq y \\
&\quad \hat{\underset{\sim}{z}} \leq \underset{\sim}{z} \\
&\quad \hat{q} \leq q \\
&\quad z_h \leq \check{z}_h \qquad h \varepsilon I_m \subset I_n \\
&u[\underset{\sim}{z},q] = \bar{\bar{u}}[y] \ .
\end{aligned}
\qquad \text{(III.6.2)}
$$

Problem (III.6.2) has been treated by Papageorgiou and Mullally (1976). Nevertheless their discussion does not include constraints such as $z_h \leq \check{z}_h$, $h \varepsilon I_m \subset I_n$.

CHAPTER IV

THE HOUSEHOLD PROBLEM

In subsection III.6 we assumed that a household with income y at s has the ability to solve problem (III.6.1). The present section is devoted to the discussion of this problem, as it was summarized in (III.6.2).

Some mathematical preliminaries are given in the first subsection. These enable us, in the second subsection, to identify the necessary and sufficient conditions for the consumption $(\underset{\sim}{z},q)$ to be a solution to problem (III.6.1).

Different types of consumption will be considered as possible solutions to the problem. In the general case, the expected utility level $\bar{\bar{u}}[y]$, for a household with income y, is assumed to be given exogenously. Despite this fact we can determine the function $\bar{\bar{u}}[y]$ for the particular case examined in this section.

The rent that a household with income y is prepared to pay at s in order to achieve utility level $\bar{\bar{u}}[y]$ will be determined for different types of consumption. This rent will be expressed as a function of the pair $(y,\underset{\sim}{s})$. For consistency with the previous literature, the rent function will be denoted by $\bar{\bar{r}}[y,\underset{\sim}{s}]$.

Also, a household willing to pay rent $\bar{\bar{r}}[y,\underset{\sim}{s}]$, will consume a quantity of land. This quantity will be expressed as a function of the pair $(y,\underset{\sim}{s})$ and will be denoted by $\bar{\bar{q}}[y,\underset{\sim}{s}]$.

IV.1 Mathematical Preliminaries

The Kuhn-Tucker conditions are usually given in the economic literature after taking into account non-negativity constraints. Our purpose in this subsection will be to supply the theoretical basis which will enable us to derive those conditions for the problem (III.6.2).

Two theorems are stated first without proof[6].

Theorem IV.1.1 (necessary condition)

Given: $f : \mathbb{R}^n \to \mathbb{R}$ and

$g_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, 2, \ldots h$ differentiable functions.

Consider the nonlinear programming problem:

$$\max_{\tilde{x}} f[\tilde{x}]$$

Subject to: $g_i[\tilde{x}] \geq 0 \quad i = 1, 2, \ldots h.$

Let $\tilde{x}^*$ be an optimal solution and assume the constraint qualification holds. Then the following three conditions also hold:

(1) $\tilde{x}^*$ is feasible.

(2) There exist multipliers $\lambda_i \geq 0$, $i = 1, 2, \ldots h$ such that

$$\lambda_i g_i[\tilde{x}^*] = 0 \quad i = 1, 2, \ldots h.$$

(3) $\nabla f[\tilde{x}^*] + \sum_{i=1}^{m} \lambda_i \nabla g_i[\tilde{x}^*] = 0 .$

Theorem IV.1.2 (sufficient condition)

In the nonlinear programming problem:

$$\underset{\tilde{x}}{\text{MAX}}\ f[\tilde{x}]$$

$$\text{s.t} \quad g_i[\tilde{x}] \geq 0 \quad i = 1, 2, \dots h.$$

Let f be differentiable and pseudoconcave and g_i, $i = 1, 2, \dots h$ differentiable and quasiconcave[7]. If an $\tilde{x}^*$ satisfies the conditions (1), (2), (3) of Theorem (IV.1.1), then it is an optimal solution for the problem.

Corollary IV.1.1

Having to solve the problem of Theorem (IV.1.1), the conditions (1), (2), (3) are equivalent to:

(1)' $\tilde{x}^*$ is feasible.

(2)' There exist multipliers $\lambda_i \geq 0$, $i = 1, 2, \dots h$ such that:

$$\lambda_i \left.\frac{\partial L}{\partial \lambda_i}\right|_{\tilde{x}^*} = 0 \quad i = 1, 2, \dots h$$

for $L = f[\tilde{x}] + \sum_{i=1}^{m} \lambda_i g_i[\tilde{x}]$.

(3)' $$\left.\frac{\partial L}{\partial x_j}\right|_{\tilde{x}^*} = 0 \quad j = 1, 2, \dots n.$$

Proof: $\frac{\partial L}{\partial \lambda_i} = g_i[\tilde{x}] \quad \forall\, i\varepsilon I_h$. Therefore, (2)' is equivalent to (2).

Condition (3) of Theorem IV.1.1 may be written:

$$\left.\frac{\partial f}{\partial x_j}\right|_{\tilde{x}^*} + \sum_{i=1}^{m} \lambda_i \left.\frac{\partial g_i}{\partial x_j}\right|_{\tilde{x}^*} = 0 \quad \forall\, j\varepsilon I_n.$$

Or, from the definition of L:

$$\left.\frac{\partial L}{\partial x_j}\right|_{\underset{\sim}{x}^*} = 0 \quad \forall\, j \varepsilon I_n. \;\|$$

An application of Theorem IV.1.1 and Corollary IV.1.1 is:

Corollary IV.1.2

Given: $f : \mathbb{R}^n \to \mathbb{R}$ and

$g_i : \mathbb{R}^n \to \mathbb{R}$, $i \varepsilon I_h$ differentiable functions

and the real numbers $\hat{x}_j$, $j \varepsilon I_n$ and $\check{x}_k$, $k \varepsilon I_m \subseteq I_n$ with $\hat{x}_k < \check{x}_k$, $k \varepsilon I_m$.

Consider the problem:

$$\text{Max } f[\underset{\sim}{x}]$$

$$\text{s.t} \quad g_i[\underset{\sim}{x}] \geq 0 \quad \forall\, i \varepsilon I_h$$

$$x_j \geq \hat{x}_j \quad \forall\, j \varepsilon I_n$$

$$\check{x}_k \geq x_k \quad \forall\, k \varepsilon I_m \subseteq I_n.$$

If x* is a solution to this problem and the constraint qualification holds, then:

(1) x* is feasible.

(2) There exist multipliers $\lambda_i \geq 0$ $i = 1, 2, \ldots h$ such that:

$$\lambda_i \left.\frac{\partial L}{\partial \lambda_i}\right|_{x^*} = 0 \quad \forall\, i \varepsilon I_h$$

for $L = f[\underset{\sim}{x}] + \sum_{i=1}^{h} \lambda_i g_i[\underset{\sim}{x}]$.

(3) $\left.\frac{\partial L}{\partial x_\ell}\right|_{\underset{\sim}{x}^*} \leq 0$ for $\ell \varepsilon A = \{j \varepsilon I_n \mid x_j^* = \hat{x}_j\}$

$$\left.\frac{\partial L}{\partial x_\ell}\right|_{\underset{\sim}{x}^*} \geq 0 \quad \text{for} \quad \ell \varepsilon B = \{k \varepsilon I_m \mid x_k^* = \check{x}_k\}$$

$$\left.\frac{\partial L}{\partial x_\ell}\right|_{\underset{\sim}{x}^*} = 0 \quad \text{for} \quad \ell \varepsilon I_n - (AUB).$$

Proof: If $\underset{\sim}{x}^*$ is a solution to the problem, then according to Theorem IV.1.1 and Corollary IV.1.1:

(1) $\underset{\sim}{x}^*$ is feasible.

(2) There exist multipliers $\lambda_i \geq 0 \quad i\varepsilon Ih$; $\mu_j \geq 0 \quad j\varepsilon In$ $\nu_k \geq 0 \quad k\varepsilon Im$, such that:

$$\lambda_i \left.\frac{\partial L}{\partial \lambda_i}\right|_{\underset{\sim}{x}^*} = 0 \quad \forall\, i\varepsilon I_h \qquad \text{(a)}$$

$$\mu_j(x_j^* - \hat{x}_j) = 0 \quad \forall\, j\varepsilon I_n \qquad \text{(b)}$$

$$\nu_k(\check{x}_k - x_k^*) = 0 \quad \forall\, k\varepsilon I_m. \qquad \text{(c)}$$

(3) Condition (3) of Theorem IV.1.1 can be written:

$$\nabla f[\underset{\sim}{x}^*] + \sum_{i=1}^{h} \lambda_i \nabla g_i[\underset{\sim}{x}^*] + \sum_{j=1}^{n} \mu_j \nabla(x_j - \hat{x}_j)\Big|_{\underset{\sim}{x}^*} + \sum_{k=1}^{m} \nu_k \nabla(\check{x}_k - x_k)\Big|_{\underset{\sim}{x}^*} = 0$$

This equation may be written as the following system of equations:

$$\left.\frac{\partial L}{\partial x_\ell}\right|_{\underset{\sim}{x}^*} + \mu_\ell - \nu_\ell = 0 \quad \forall\, \ell\varepsilon I_m$$

$$\left.\frac{\partial L}{\partial x_\ell}\right|_{\underset{\sim}{x}^*} + \mu_\ell = 0 \qquad \forall\, \ell\varepsilon I_n - I_m .$$

The first of them in conjunction with (b) and (c) gives:

$$\left.\frac{\partial L}{\partial x_\ell}\right|_{\underset{\sim}{x}^*} = -\mu_\ell \leq 0 \quad \text{for} \quad \ell \varepsilon A$$

$$\left.\frac{\partial L}{\partial x_\ell}\right|_{\underset{\sim}{x}^*} = \nu_\ell \geq 0 \quad \text{for} \quad \ell \varepsilon B.$$

For any other index, $\ell \varepsilon I_n$, we have:

$$x_\ell^* - \hat{x}_\ell \neq 0$$

and $$\check{x}_\ell - x_\ell^* \neq 0.$$

From (b) and (c), then $\mu_\ell = \nu_\ell = 0$. Therefore,

$$\left.\frac{\partial L}{\partial x_\ell}\right|_{x^*} = 0 \quad \forall\ \ell \varepsilon I_n - (A \cup B). \|$$

Remark IV.1.1: Utilizing Theorem IV.1.2 we can say that if f is pseudoconcave and g_i, $i \varepsilon I_h$ quasiconcave, then (1), (2), and (3) of Corollary IV.1.2 are sufficient conditions also.

IV.2 The Maximization Problem

Our task in this subsection is to apply Corollary IV.1.2 to the problem (III.6.1). Using a Cobb-Douglas function, the problem is stated as:

$$\underset{(\underset{\sim}{z},q)}{\text{Max}}\ u[\underset{\sim}{z},q] = \prod_{i=1}^{n} z_i^{\alpha_i} q^{\beta} \qquad \begin{array}{l} 0 < \alpha_i < 1 \\ 0 < \beta < 1 \end{array} \tag{IV.2.1}$$

$$\text{s.t} \quad \underset{\sim}{p}'[\underset{\sim}{s}] \cdot \underset{\sim}{z} + rq \leq y$$

...continued... $\underset{\sim}{\hat{z}} \leq \underset{\sim}{z}$

$$\hat{q} \leq q \tag{IV.2.1}$$

$$z_h \leq \check{z}_h \quad \forall\, h \varepsilon I_m \subset I_n .$$

In comparison to the problem of Corollary IV.1.2, the budget constraint replaces the h constraints $g_i(x) \geq 0$, $i \varepsilon I_h$.

Definition IV.2.1: For a particular $(\underset{\sim}{z},q) \in Z$, we define the sets:

$$K' = \{k' \varepsilon I_n |\ z_{k'} = \hat{z}_{k'}\}$$

$$L' = \{\ell' \varepsilon I_m |\ z_{\ell'} = \check{z}_{\ell'}\} .$$

With the above definition a classification of the elements of a consumption $(\underset{\sim}{z},q)$ takes place. The indices of the elements of $\underset{\sim}{z}$, that are equal to the corresponding lower (upper) bound, belong to K' (L'). This classification is helpful in the following proposition, where we derive the necessary and sufficient conditions for a particular consumption $(\underset{\sim}{z}^*,q^*)$ to be a solution to problem (IV.2.1). For notational simplicity, we shall put:

$$u^* = u[\underset{\sim}{z}^*,q^*] .$$

Proposition IV.2.1: For given $(\underset{\sim}{s},y) \varepsilon S\times Y$, $(\underset{\sim}{z}^*,q^*)$ is a unique solution to problem (IV.2.1) iff: (1) $(\underset{\sim}{z}^*,q^*)$ is feasible; and (2) there exist some $\lambda \geq 0$ such that:

$$\alpha_{k'}(z^*_{k'})^{-1}u^* - \lambda p_{k'}[\underset{\sim}{s}] \leq 0 \quad \forall k' \varepsilon K'$$

$$\alpha_{\ell'}(z^*_{\ell'})^{-1}u^* - \lambda p_{\ell'}[\underset{\sim}{s}] \geq 0 \quad \forall \ell' \varepsilon L'$$

$$\alpha_{j'}(z^*_{j'})^{-1}u^* - \lambda p_{j'}[\underset{\sim}{s}] = 0 \quad \forall j' \varepsilon I_n - (K'UL')$$

$$\beta(q^*)^{-1}u^* - \lambda r \leq 0 \ , \ \hat{q} \leq q^* \ , \ (\beta(q^*)^{-1}u^* - \lambda r)(q^*-\hat{q}) = 0$$

$$y - \underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z}^* - rq^* \geq 0 \ , \ \lambda \geq 0 \ , \ (y - p'[\underset{\sim}{s}]\cdot\underset{\sim}{z}^* - rq^*)\lambda = 0.$$

Proof: (a) The "if" statement

The objective function of problem (IV.2.1) is obviously differentiable. So is the function $(y - \underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z} - rq)$ for given $(\underset{\sim}{s},y)$. All the constraint functions are linear and the constraint qualification holds[8]. According to Corollary IV.1.2, if $(\underset{\sim}{z}^*,q^*)$ is a solution to the problem (IV.2.1), then:

(1) $(\underset{\sim}{z}^*,q^*)$ is feasible.

(2) There exists a multiplier $\lambda \geq 0$, such that for:

$$L = u + \lambda(y-\underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z}-rq)$$

$$\lambda \left.\frac{\partial L}{\partial \lambda}\right|_{(\underset{\sim}{z}^*,q^*)} = 0 \quad \text{or} \quad \lambda(y - \underset{\sim}{p}'[\underset{\sim}{s}]\cdot\underset{\sim}{z}^* - rq^*) = 0.$$

$$(3)\quad \left.\frac{\partial L}{\partial z_{k'}}\right|_{(\underset{\sim}{z}^*,q^*)} = a_{k'}(z^*_{k'})^{-1}u^* - \lambda p_{k'}[\underset{\sim}{s}] \leq 0 \quad \forall\, k'\varepsilon K'$$

$$\left.\frac{\partial L}{\partial z_{\ell'}}\right|_{(\underset{\sim}{z}^*,q^*)} = a_{\ell'}(z^*_{\ell'})^{-1}u^* - \lambda p_{\ell'}[\underset{\sim}{s}] \geq 0 \quad \forall\, \ell'\varepsilon L'$$

$$\left.\frac{\partial L}{\partial z_{j'}}\right|_{(\underset{\sim}{z}^*,q^*)} = a_{j'}(z^*_{j'})^{-1}u^* - \lambda p_{j'}[\underset{\sim}{s}] = 0 \quad \forall\, j'\varepsilon I_n - (K'UL').$$

Since there is not an upper bound for q:

$$\left.\frac{\partial L}{\partial q}\right|_{(\underset{\sim}{z}^*,q^*)} = \beta(q^*)^{-1}u^* - \lambda r \leq 0 \quad \text{for} \quad \hat{q} = q^*$$

and

$$\left.\frac{\partial L}{\partial q}\right|_{(\underset{\sim}{z}^*,q^*)} = \beta(q^*)^{-1}u^* - \lambda r = 0 \quad \text{for} \quad \hat{q} < q^*.$$

The last two statements are summarized as in the proposition IV.2.1.

(b) The "only if" statement

The utility function, as it is defined, is strictly concave and therefore, pseudoconcave. Also, for a pair $(\underset{\sim}{s},y)$ the function $(y - \underset{\sim}{p}[\underset{\sim}{s}]\cdot\underset{\sim}{z} - rq)$ is linear, hence quasiconcave. According to Remark IV.1.1, if $(\underset{\sim}{z}^*,q^*)$ satisfies conditions (1) and (2), then this is an optimal solution. Due to the strict concavity of the utility function, the solution $(\underset{\sim}{z}^*,q^*)$ is unique.||

Let us consider an element $z^*_{k'}$ of $\underset{\sim}{z}^*$ with $k'\varepsilon K'$. The equivalent of this, according to Definition IV.2.1, is $z^*_{k'} = \hat{z}_{k'}$. In such a case, according to Proposition IV.2.1, the expression $a_{k'}(z^*_{k'})^{-1}u^* - \lambda p_{k'}[\underset{\sim}{s}]$

will be either zero or negative.

Suppose that we have to solve Problem (IV.2.1), but without a lower bound for the k'th element of $\underset{\sim}{z}$ and the solution is $(\underset{\sim}{z}^*, q^*)$. Then, the equality $\alpha_{k'}(z^*_{k'})^{-1}u^* - \lambda p_{k'}[\underset{\sim}{s}] = 0$ means that the k'th element of $\underset{\sim}{z}^*$ is the quantity $\hat{z}_{k'}$. Whereas, the inequality $\alpha_{k'}(z^*_{k'})^{-1}u^* - \lambda p_{k'}[\underset{\sim}{s}] < 0$ means that the k'th element of $\underset{\sim}{z}^*$ is less than $\hat{z}_{k'}$. In the case of inequality, the household will consume less from some other commodity for which the optimal consumption is well above the subsistence level in order to buy more from $z_{k'}$ and reach the subsistence level $\hat{z}_{k'}$. Analogous statements may be made for the upper bounds.

The question that arises is whether the household is capable of saving part of its income. We know that for certain goods there are no upper bounds (see in Subsection III.2, definition of Z). The more of these goods a household consumes the greater its utility. Hence, the household as a utility maximizer will spend all of its income in order to obtain as many more of these goods as possible. A formal statement about this is given by the following proposition.

<u>Proposition IV.2.2</u>: For given $(\underset{\sim}{s}, y) \varepsilon S \times Y$, the conditions of Proposition IV.2.1 reduce to:

(1) $(\underset{\sim}{z}^*, q^*)$ is feasible.

(2) $\alpha_{k'}(z^*_{k'})^{-1}u^* - \lambda p_{k'}[\underset{\sim}{s}] \leq 0 \quad \forall\, k' \varepsilon K'$

$$\alpha_{\ell'}(z^*_{\ell'})^{-1}u^* - \lambda p_{\ell'}[\underset{\sim}{s}] \geq 0 \quad \forall\, \ell' \varepsilon L'$$

$$\alpha_{j'}(z^*_{j'})^{-1}u^* - \lambda p_{j'}[\underset{\sim}{s}] = 0 \quad \forall\, j' \varepsilon I_n - (K' \cup L')$$

$$\beta(q^*)^{-1}u^* - \lambda r \leq 0 \ , \ \hat{q} \leq q^* \ , \ (\beta(q^*)^{-1}u^* - \lambda r)(q^* - \hat{q}) = 0$$

$$y - \underset{\sim}{p}'[\underset{\sim}{s}] \cdot \underset{\sim}{z}^* - rq^* = 0 \ .$$

Proof: Assume $\lambda = 0$, then $\beta q^{*-1}u^* \leq 0$ is a contradiction. Thus, $\lambda > 0$. The last condition of Proposition IV.2.1 becomes:

$$y - \underset{\sim}{p}'[\underset{\sim}{s}] \cdot \underset{\sim}{z}^* - rq^* = 0. \|$$

Clearly, another classification of the elements of a solution $(\underset{\sim}{z}^*, q^*)$ may be made as following:

Definition IV.2.2: For a solution $(\underset{\sim}{z}^*, q^*)$ to Problem (IV.2.1):

$$K = \{k \varepsilon I_n | \alpha_k (z_k^*)^{-1} u^* - \lambda p_k [\underset{\sim}{s}] < 0\}$$

$$L = \{\ell \varepsilon I_m | \alpha_\ell (z_\ell^*)^{-1} u^* - \lambda p_\ell [\underset{\sim}{s}] > 0\} \ .$$

We shall call any element $z_k^*, k \varepsilon K$ lower suboptimal and any element z_ℓ^*, $\ell \varepsilon L$ upper suboptimal. Any other element, i.e. z_j^*, $j \varepsilon I_n$ - (KUL) will be called optimal.

For notational simplicity, we shall put:

$$I_n - (KUL) = J \ .$$

The sets K and L will be called respectively, lower and upper suboptimal sets, whereas J will be called an optimal set. We shall assume, also, that q^* is always optimal. The relationship between the two kinds of classification is given by the following lemma.

Lemma IV.2.1: $K \subseteq K'$, $L \subseteq L'$.

Proof: Consider $k \in K$. Then, $\alpha_k (z_k^*)^{-1} u^* - \lambda p_k[\underset{\sim}{s}] < 0$. From condition (2) of Proposition IV.2.1, $k \notin L'$ and $k \notin I_n - (K' \cup L')$. Hence, $k \in K'$. The proof for L is analogous.||

In the most general case, where none of the sets K,L,J are empty, the solution is given by the following proposition.

Proposition IV.2.3: For given $(\underset{\sim}{s}, y) \in S \times Y$, let $(\underset{\sim}{z}^*, q^*)$ be the unique solution to Problem (IV.2.1). Then:

$$z_k^* = \hat{z}_k \qquad \forall\, k \in K$$

$$z_\ell^* = \check{z}_\ell \qquad \forall\, \ell \in L$$

$$z_j^* = ((\sum_J \alpha_j + \beta) p_j[\underset{\sim}{s}])^{-1} \alpha_j (y - \sum_K \hat{z}_k p_k[\underset{\sim}{s}] - \sum_L \check{z}_\ell p_\ell[\underset{\sim}{s}]) \quad \forall\, j \in J$$

$$q^* = ((\sum_J \alpha_j + \beta) r)^{-1} \beta (y - \sum_K \hat{z}_k p_k[\underset{\sim}{s}] - \sum_L \check{z}_\ell p_\ell[\underset{\sim}{s}]) .$$

Proof: The first and the second of the above equations are obvious by applying Lemma IV.2.1. Summing the equations $\alpha_j u^* = \lambda z_j^* p_j[\underset{\sim}{s}]$, $j \in J$ and $\beta u^* = \lambda q^* r$ by parts, we get:

$$(\sum_J \alpha_j + \beta) u^* = \lambda (\sum_J z_j^* p_j[\underset{\sim}{s}] + q^* r) .$$

Taking into account the last equation of Proposition IV.2.2, we get

$$(\sum_J \alpha_j + \beta)u^* = \lambda(y - \sum_K \hat{z}_k p_k[\underset{\sim}{s}] - \sum_L \check{z}_\ell p_\ell[\underset{\sim}{s}]) .$$

From this and each of $\alpha_j u^* = \lambda z_j^* p_j[\underset{\sim}{s}]$ $j \varepsilon J$, $\beta u^* = \lambda q^* r$, we get the last two equations of Proposition IV.2.3.||

A particular consumption is well defined by the sets (K,L,J). In this sense all the possible solutions to Problem (IV.2.1) are consumptions of the following six types:

(1) (K,L,J).

(2) (∅,L,J).

(3) (K,∅,J).

(4) (K,L,∅).

(5) (K,∅,∅).

(6) (∅,∅,J).

Types (3), (5) and (6) have been discussed by Papageorgiou and Mullally (1976) in a more general context. Specifically, they called a consumption of type (6) an optimal consumption.

IV.3 Spatial Indifference and the Rent Function

It is easy to see that the conditions of Proposition IV.2.2 in the case of a type (6) consumption are:

$$\alpha_i (z_i^*)^{-1} u^* - \lambda p_i[\underset{\sim}{s}] = 0 \quad \forall\ i \varepsilon I_n$$

$$\beta (q^*)^{-1} u^* - \lambda r = 0 \qquad \text{(IV.3.1)}$$

$$y - \underset{\sim}{p}'[\underset{\sim}{s}] \cdot \underset{\sim}{z}^* - rq^* = 0 \ .$$

The equations of Proposition IV.2.3 for the optimal consumption become:

$$z_i^* = ((\sum_{I_n} \alpha_i + \beta) p_i[\underset{\sim}{s}])^{-1} \alpha_i y \quad \forall\ i \varepsilon I_n$$

$$q^* = ((\sum_{I_n} \alpha_i + \beta) r)^{-1} \beta y \ . \qquad \text{(IV.3.2)}$$

Instead of these, for the consumption type $(\emptyset, L, J)$, we get:

$$z_\ell^* = \check{z}_\ell \qquad \forall\ \ell \varepsilon L$$

$$z_j^* = ((\sum_J \alpha_j + \beta) p_j[\underset{\sim}{s}])^{-1} \alpha_j \ (y - \sum_L \check{z}_\ell p_\ell[\underset{\sim}{s}]) \quad \forall\ j \varepsilon J.$$

$$q^* = ((\sum_J \alpha_j + \beta) r)^{-1} \beta \ (y - \sum_L \check{z}_\ell p_\ell[\underset{\sim}{s}]). \qquad \text{(IV.3.3)}$$

We may now determine the expected utility level of a household with income y:

<u>Proposition IV.3.1</u>: $\bar{\bar{u}}[y] = y^{\Sigma_{I_n} \alpha_i + \beta}$

<u>Proof</u>: Following Papageorgiou and Mullally (1976), we add by parts the equations $\alpha_i u^* = \lambda p_i[\underset{\sim}{s}] z_i^*$, $i \varepsilon In$ and $\beta u^* = \lambda r q^*$. We get:

$$\sum_{I_n} \alpha_i + \beta u^* = \lambda y \ . \tag{IV.3.4}$$

The usual interpretation of the Lagrangian multiplier λ is given by the identity[9]:

$$\frac{\partial u^*}{\partial y} = \lambda \ . \tag{IV.3.5}$$

Equations (IV.3.4) and (IV.3.5),give:

$$\frac{\partial \bar{\bar{u}}}{\partial y} y = (\sum_{I_n} \alpha_i + \beta) \bar{\bar{u}} \ .$$

Solving this differential equation, we get:

$$\bar{\bar{u}}[y] = y^{\Sigma_{I_n} \alpha_i + \beta} . || \tag{IV.3.6}$$

We want to know now,how much rent a household with income y is prepared to pay at a location s in order to achieve utility level $y^{(\Sigma_{I_n} \alpha_i + \beta)}$. The rent function is going to be determined for the most general case, the type (1) consumption. The formula for type (2) comsumption will follow as a corollary. Type (4) will not be discussed as it is unlikely to occur.

Proposition IV.3.2: For K,L,J $\neq \emptyset$

$$\bar{\bar{r}}^{\beta}[y,\underset{\sim}{s}] = (\sum_J \alpha_j+\beta)^{-(\sum_J \alpha_j+\beta)} (\prod_J \alpha_j^{\alpha_j})\beta^{\beta}(\prod_K \hat{z}_k^{\alpha_k})(\prod_L \check{z}_{\ell}^{\alpha_{\ell}})$$

$$(\prod_J p_j^{-\alpha_j}[\underset{\sim}{s}])y^{-(\sum_{I_n} \alpha_i+\beta)} (y - \sum_K \hat{z}_k p_k[\underset{\sim}{s}] - \sum_L \check{z}_{\ell} p_{\ell}[\underset{\sim}{s}])^{(\sum_J \alpha_j+\beta)} . \qquad \text{(IV.3.7)}$$

Proof: The maximum utility for a consumption of type (1) is:

$$u[\underset{\sim}{z}^*,\underset{\sim}{g}^*] = (\sum_J \alpha_j+\beta)^{-(\sum_J \alpha_j+\beta)} (\prod_J \alpha_j^{\alpha_j})\beta^{\beta}(\prod_K \hat{z}_k^{\alpha_k})(\prod_L \check{z}_{\ell}^{\alpha_{\ell}})(\prod_J p_j^{-\alpha_j}[\underset{\sim}{s}])r^{-\beta}$$

$$(y - \sum_K \hat{z}_k p_k[\underset{\sim}{s}] - \sum_L \check{z}_{\ell} p_{\ell}[\underset{\sim}{s}])^{(\sum_J \alpha_j+\beta)} .$$

From this and (IV.3.6), we get (IV.3.7).||

Corollary IV.3.1: For a type (2) consumption, we have:

$$\bar{\bar{r}}^{\beta}[y,\underset{\sim}{s}] = (\sum_J \alpha_j+\beta)^{-(\sum_j \alpha_j+\beta)} (\prod_J \alpha_j^{\alpha_j})\beta^{\beta}(\prod_L \check{z}_{\ell}^{\alpha_{\ell}})(\prod_J p_j^{-\alpha_j}[\underset{\sim}{s}])$$

$$y^{-(\sum_{I_n} \alpha_i+\beta)} (y - \sum_L p_{\ell}[\underset{\sim}{s}]\check{z}_{\ell})^{(\sum_J \alpha_j+\beta)} . \qquad \text{(IV.3.8)}$$

The proof of the above corollary is similar to the one given in Proposition (IV.3.2). Comparing the expressions of the second part of the Equations (IV.3.7) and (IV.3.8), we see that these are essentially the same. Only the part concerning K is omitted in (IV.3.8).

We may now derive the equation giving the quantity of land that a household with income y desires to consume at each location s.

Proposition IV.3.3: For K,L,J $\neq \emptyset$

$$\bar{\bar{q}}^{\beta}[y,\underset{\sim}{s}] = (\underset{J}{\Sigma}\alpha_j+\beta)^{\underset{J}{\Sigma}\alpha_j}(\underset{J}{\Pi}\alpha_j^{-\alpha_j})(\underset{K}{\Pi}\hat{z}_k^{-\alpha_k})(\underset{L}{\Pi}\check{z}_\ell^{-\alpha_\ell})(\underset{J}{\Pi}p_j^{\alpha_j}[\underset{\sim}{s}])$$

$$y^{(\underset{I_n}{\Sigma}\alpha_i+\beta)}(y - \underset{K}{\Sigma}\hat{z}_k p_k[\underset{\sim}{s}] - \underset{L}{\Sigma}\check{z}_\ell p_\ell[\underset{\sim}{s}])^{-\underset{J}{\Sigma}\alpha_j} . \tag{IV.3.9}$$

Proof: Substituting $\bar{\bar{r}}[y,\underset{\sim}{s}]$ from (IV.3.7) into the last equation of Proposition IV.2.3, we get (IV.3.9).||

Corollary IV.3.2: For K = $\emptyset$ and L,J $\neq \emptyset$

$$\bar{\bar{q}}^{\beta}[y,\underset{\sim}{s}] = (\underset{J}{\Sigma}\alpha_j+\beta)^{\underset{J}{\Sigma}\alpha_j}(\underset{J}{\Pi}\alpha_j^{-\alpha_j})(\underset{L}{\Pi}\check{z}_\ell^{-\alpha_\ell})(\underset{J}{\Pi}p_j^{\alpha_j}[\underset{\sim}{s}])y^{(\underset{I_n}{\Sigma}\alpha_i+\beta)}$$

$$(y - \underset{L}{\Sigma}\check{z}_\ell p_\ell[\underset{\sim}{s}])^{-\Sigma\alpha_j} . \tag{IV.3.10}$$

We may easily see that (IV.3.10) is essentially the same as (IV.3.9) and only the parts involving K are omitted in (IV.3.10).

SECTION V

CONCLUSIONS

To recapitulate, the purpose of the present study is an attempt to analyse behaviour of the household in intraurban mobility. A review of the existing literature with an emphasis on the concepts of aspirations, place utility and stress was deemed necessary. The review, albeit brief, indicated the indecisiveness which is evident in this field. This indecision stems from the fact that definition of the concepts mentioned above is not always clear.

The model which followed the review aimed in providing rigorous definition of these concepts and clarification of the mechanisms that lead the household to the decision to seek a new residence. The hypothetical world described by Papageorgiou (1975) was chosen as the world of our model. Concepts from consumer choice theory were employed and adapted, where it was necessary, to fit the model. Thus, the consumption set Z was defined as a convex subset of $\mathbb{R}^n$. Subsequently the concepts of preferences, utility and the budget constraint were defined for a particular household in the consumption set Z. The assumptions behind these definitions were often explored and evaluated. At the same time the actual consumption set of a household was identified as a non-convex subset of Z. A way of ordering the members of the actual consumption set, according to the preferences defined in Z was provided. Also, the relationship, if any, of the above mentioned concepts, to those of aspirations and place utility was discussed.

The concept of stress was given special attention and was defined in a rather rigorous way. However, the definition was not essentially different from the one given by Clark and Cadwallader (1973) and Clark (1975). Thus, every location in the action space was associated with a real number called stress. This number was defined as the difference between the utility $\bar{\bar{u}}[y]$ the household with income y expected to have and the utility the particular location was capable of providing.

The rent $\bar{\bar{r}}[y,\underset{\sim}{s}]$, a household with income y was willing to pay at a location s, played the important role of a regulatory mechanism that made the household's expectations $\bar{\bar{u}}[y]$ spatially invariant.

The remainder of the study has devoted to a particular case. This case was specific in the sense that instead of a general utility function, a Cobb-Douglas was used. Also, we were restricted in determining:

First, the functions giving the quantities of each element of $\underset{\sim}{z}*$ that the household is planning to consume at each location s

Second, the rent $\bar{\bar{r}}[y,\underset{\sim}{s}]$, a household with income y is prepared to pay at each location s, in order to achieve utility level $\bar{\bar{u}}[y]$.

Third, the quantity of land $\bar{\bar{q}}[y,\underset{\sim}{s}]$, the household would like to consume.

In other words, the household with income y expects utility $\bar{\bar{u}}[y]$ by consuming $(z*,\bar{\bar{q}}[y,\underset{\sim}{s}])$ and by paying rent $\bar{\bar{r}}[y,\underset{\sim}{s}]$ at each location s. But in the housing market the situation is often other than what the household expects. Houses in the action space are usually associated with more or less than $\bar{\bar{q}}[y,\underset{\sim}{s}]$ quantity of land, and their other attributes included in some $\underset{\sim}{z}$ are, more than likely, different from those in $\underset{\sim}{z}^*$. Also, the rents are different from $\bar{\bar{r}}[y,\underset{\sim}{s}]$. Therefore, the utility associated with each location in the action space is less than $\bar{\bar{u}}[y]$. As a result, each house in the action space is associated with a stress which is the difference between $\bar{\bar{u}}[y]$ and the actual utility a house can provide. It was demonstrated in Section III.6 that the house with the least stress will be chosen.

FOOTNOTES

1. A more extensive review is given in Brummell (1976).
2. For a review on this kind of work, see Anas and Dendrinos (1975).
3. Given a set A, a binary relation R on A is defined as a subset of the Cartesian product A×A. The sets A×A and ∅ represent the universal and the empty relation on A. Membership of a $(x,y)\varepsilon A\times A$ in R is denoted either by $(x,y)\varepsilon R$ or by xRy. Subsequently, we mention some of the properties that are usually assigned to a binary relation. A binary relation R is called:

   (a) reflexive, iff xRx, for every $x\varepsilon A$.

   (b) symmetric, iff xRy implies yRx.

   (c) Transitive, iff xRy and yRz implies xRz.

   (d) Complete, iff for every $(x,y)\varepsilon A\times A$, either xRy or yRx.

   A relation R is called a preordering if it is reflexive and transitive. A preordering is called complete if it is complete. For further reading on the subject, refer to Fishburn (1972).
4. A binary relation is called an equivalence if it is reflexive, symmetric and transitive. For the indifference relation defined in Section III.3, the reflexivity is obvious. The proof for the symmetry is:

$$\underset{\sim}{z}^1 \sim \underset{\sim}{z}^2 \iff (\underset{\sim}{z}^1 \not\precsim \underset{\sim}{z}^2 \text{ and } \underset{\sim}{z}^2 \not\precsim \underset{\sim}{z}^1) \iff (\underset{\sim}{z}^2 \not\precsim \underset{\sim}{z}^1 \text{ and } \underset{\sim}{z}^1 \not\precsim \underset{\sim}{z}^2) \iff \underset{\sim}{z}^2 \sim \underset{\sim}{z}^1.$$

The transitivity is proven as follows:

$$(\underset{\sim}{z}^1 \sim \underset{\sim}{z}^2 \text{ and } \underset{\sim}{z}^2 \sim \underset{\sim}{z}^3) \iff (\underset{\sim}{z}^1 \precsim \underset{\sim}{z}^2 \text{ and } \underset{\sim}{z}^2 \precsim \underset{\sim}{z}^1) \text{ and}$$

$$(z^3 \precsim z^2 \text{ and } z^2 \precsim z^3) \iff (z^1 \precsim z^2 \text{ and } z^2 \precsim z^3) \text{ and}$$

$$(z^3 \precsim z^2 \text{ and } z^2 \precsim z^1) \iff (z^1 \precsim z^3 \text{ and } z^3 \precsim z^1)$$

$$\iff z^1 \sim z^3 \ .$$

5. Two other definitions for convexity usually exist in the literature (see, e.g., Debreu (1959), Takayama (1974), Walsh (1970)). These along with their names are:

   (a) If $\underset{\sim}{z}^1 \succsim \underset{\sim}{z}^2$ then $t\underset{\sim}{z}^1 + (1-t)\underset{\sim}{z}^2 \succsim \underset{\sim}{z}^2$; $0 < t < 1$ and $\underset{\sim}{z}^1 \neq \underset{\sim}{z}^2$ (weak convexity).

   (b) If $\underset{\sim}{z}^1 \succ \underset{\sim}{z}^2$ then $t\underset{\sim}{z}^1 + (1-t)\underset{\sim}{z}^2 \succ \underset{\sim}{z}^2$; $0 < t < 1$ and $\underset{\sim}{z}^1 \neq \underset{\sim}{z}^2$ (convexity).

6. These two theorems come from Zangwill (1969).

7. A differentiable function $f\colon \mathbb{R}^n \to \mathbb{R}$ is called pseudoconcave if

$$(\nabla f[x])' \cdot (y-x) \leq 0$$

implies $f[y] \leq f[x]$ for $x,y \in \mathbb{R}^n$.

Quasiconcavity is a more general concept than pseudoconcavity. A differentiable function $f\colon \mathbb{R}^n \to \mathbb{R}$ is quasiconcave if:

$$f[\lambda x + (1-\lambda)] \geq \min\{f[x], f[y]\}$$

for $x,y \in \mathbb{R}^n$ and $0 < \lambda < 1$.

8. This point is clarified by Chiang (1974), p. 718.

9. See e.g., Intriligator (1971), p. 60.

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