DETERMINATION OF THE COMPOSITION
OF
METAMORPHIC ROCKS
BY USE OF THE
POINT COUNTER

By

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Scope and contents of this thesis:

A short history of micrometric analyses of rocks; the use of the point counter to determine the effect of orientation on the bulk composition of metamorphic rocks, the variations and limitations of the method with comparison of calculated analyses from the modes with an ordinary chemical analysis. The discrepancies lie within the range of experimental error. A quantitative interpretation by the use of statistics is offered.

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HISTORY OF EXPERIMENTAL WORK

Since the days of Sorby when thin-sections were first used, qualitative microscopic methods have steadily developed. But quantitative methods have found little favour among petrographers probably due to the inaccuracy of results, the tedious processes, and the time involved. All these methods involve the measurement of areas either directly or by weighing dissected rock patterns, or of intercepts of the certain minerals along certain selected lines.

The first attempts of Delesse (1848) were purely macroscopic. This method involved the tracing of mineral grains from a polished and oiled rock slab. The grain outlines were characteristically tinted in order that the various mineral species could be easily distinguished. Tinfoil was pasted to the transparent sheet and subsequently cut up along grain boundaries. The fragments were then grouped according to colours, the paper washed away and the tinfoil weighed. To quote C.E. Thomson (1930):

> "Delesse was the first to enunciate the principal that the volumetric might be related to areal proportions provided enough area was included in the determinations to take care of the third dimension".

A great contribution to quantitative petrography was furnished by A. Rosiwal (1898). This is the simplest and one of the most accurate geometric methods of rock analyses. As stated by F.C. Lincoln and H.L. Rietz (1913):

> "The Rosiwal method consists of measuring the intercepted lengths of grains along a line or series of lines and calculating percentage by volume by dividing the total distance measured into the sum of the intercepts for each component".

The cover glass of the thin-section is ruled with thin lines and measurements along these lines are made with an eyepiece micrometer.

Further detail may be obtained by referring to the excellent work of F.E. Lincoln and H.L. Rietz (ob. cit. 1913).

The accuracy of the Rosiwal method is directly proportional to the length of traverse measured and inversely proportional to the grain size of the rock. To obtain a measurement accurate to 1 per cent, the total length of traverse must be at least 100 times the grain size.

Not all petrographers agreed with the accuracy of Rosiwal's method. The fundamental premise of Rosiwal that linear measurements are directly proportional to the volumes of the minerals measured was challenged by Julien (1903), (from Lincoln and Rietz, 1913) and Williams (1905).

Julien used photomicrographs - cutting them up and weighing the fragments.

Williams measured grains directly by means of an eyepiece micrometer cross-ruled into small squares. Squares which came within the boundaries of each particle were counted, and from the measurements obtained the relative volumes of the various minerals were computed. Williams states that:

> "Any one of the direct measurements with the microscope does not appear to fulfill even approximately the necessary conditions for a statement of complete quantitative composition of an igneous rock such as a granite".

A.A. Johannsen and E.A. Stephenson (1919) defended Rosiwal's method satisfactorily. They analyzed carefully the determinations carried out by Williams and found his methods to be unjust in their application.

Later in the same year Johannsen (1919) published a paper in which a scheme for quantitative measurement of a thin-section by planimeter is outlined. This method is based on surface measurements and is shown to be proportional to the volume in any uniform, non-banded rock.

The Rosiwal method as originally set forth required a day for the analysis of one thin-section. It was thus a time-consuming process requiring a great amount of patience. The linear method of Rosiwal has been simplified by the introduction of a recording micrometer for the automatic integration of the measurements. Each successive device has had certain favourable features to commend it over the earlier ones.

The first recording micrometer was devised by Shand (1916) employing the Rosiwal principle. This mechanical stage not only makes the measurements but also performs the addition of them. The method measures linear intercepts from which an estimation of percentage volumes and percentage weights are obtained. Shand (op. cit. pp. 399) states that:

> "In the case of a rock with "parallel" structure it is necessary, and in most cases desirable to take measurements both along and across the section".

One important defect of the Shand micrometer is that it can be used to measure only two constituents at a time.

In order to eliminate the separate traverse for each mineral, Wentworth (1923) constructed a device which permits the measurement of as many as five minerals at one time. The Wentworth-type counter has the advantage in that only one hand is used in its operation and the other is left free.

The most recent mechanical aid to the Rosiwal method of rock analysis is a microscope stage designed by Dollar (1937). This instrument permits the measurement of six different mineral constituents at one time. It is so constructed as to permit the stage to rotate through 360°.

C.E. Thomson (1930) has attacked the problems of accuracy of measurements from a more practical standpoint. His experiments have shown that the linear method of Rosiwal is round and easily capable of an accuracy within 1 or 2 percent for each constituent and that areal methods are slightly less accurate. These three methods include the areal, lineal, and lantern method. These are nothing new but removed from the confines of the microscope stage and magnified.

A form of integrating stage requiring counting by area estimation or by diameters was brought out by Thackwell (1933). Relative areas of the various minerals are determined by direct area estimation by grain count using a travelling stage and a net-ruled eyepiece.

An electric counter based on the Shand micrometer is described by C.S. Hurlbut (1939). The electric counter differs from all the previous devices for micrometric measurements in that the recording unit is entirely separate from the microscope stage.

A new basis for thin-section analysis was introduced by Glagolev (1934, in Chayes 1949). In Glagolev's procedure the regular spacing between traverses is retained, but the traverse itself is broken into a series of equally spaced points. The operator identifies the material under the cross-hair intersection and depresses an appropriate key of a tabulating device, thereby triggering a mechanism which translates the stage a fixed distance along the line of traverse. The sample is thus a bilaterally symmetrical grid of points.

F. Chayes (1949) adapted a standard Bausch and Lomb mechanical stage for point counting. A detailed description may be had from Chayes original paper (op. cit. 1949). The precision of the instrument is somewhat better than that of the Wentworth-Hunt and Hurlbut integrators. The average operating speed is about four times that of the Wentworth-Hunt and twice that of the Hurlbut machine.

The point counter used by the writer is a Bausch and Lomb adaptation and is quite similar in principle to the counter employed by Chayes. The Clay-Adams tabulator was fitted with five keys which will record only five constituents. Although this may be convenient for rocks of simple mineralogy it is hardly adequate for tabulating metamorphic rocks with as many as eleven and twelve recognizable mineral species. In the present work as many as six additional minerals were tallied separately.

The thin-section analysis consisted of a series of parallel traverses .25 mm. apart across the face of the section in the course of which the frequences of points underlain by the minerals of different species are summed for each species. The distance between points is .03 mm.

Area-Volume Relationship

All mechanical stages are similar in that they give an estimate of the area of a thin-section. It is fundamental in the practice of integrating methods that this area be an unbiased estimate of the volume of the rock. This problem has plagued geologists for a century.

Both Delesse and Rosiwal used simple calculus but believed only homogeneous rocks were suitable for analysis. However, it must be remembered that no mineral can be equally developed in all directions.

Delesse outlined a proof by calculus (1848) that the sums of the areas of the components in a plane surface of rock are proportional to the volumes of the components in the rock. Rosiwal extended the form of the proof to show that intercepts on lines are also proportional to volumes. Both Delesse and Rosiwal worked on oriented rocks. Delesse believed that sections normal to the plane of foliation would be more accurate and used a cylindrical-rod model to show the proportionality of area to volume. The earlier workers thought more of single measurements rather than averages. Our main interest however, is in the relative volume of the rock occupied by grains of each species. Chayes in a recent paper (1954) observes that:

> "No variation in size, shape, or orientation introduces any bias into the fundamental relationship connecting area and volume; and the way to improve the precision of an unbiased estimate is to expand the sample on which it is based".

Niggli (1954) in his discussion on grain size in coherent and noncoherent rocks considers a random section through a homogeneous aggregate of equal-sized spheres in contact with one another. (p. 223, Fig. 98). Sections through the centre of the sphere will give a true picture of the diameters while others lead to smaller circles. Any average grain diameter would be smaller than the true value:

> "In practice, the grains are, of course, of different sizes and shapes, and, if the texture is directed, planes of certain onentations may not be considered "random" sections". (Niggli pp. 224)

Nigglie noted that the ratio $\frac{\text{surface area}}{\text{volume}}$ of a particle is dependent upon the grain-shape of the particle.

Smith and Guttman (1953) have shown from a study of geometric probabilities that the average number of intercepts per unit length of a random line drawn through a three-dimensional structure is exactly half of the true ratio of surface to volume. They conclude:

> "It is not possible to derive the volume of the average cell from measurements in two-dimensional sections except on the assumption of a constant shape and size. Formulae derived for identical spheres are tabulated and may be of some use".

F. Chayes (1953) believes that the passage from area to volume has nothing to do with the rock-type or the size and shape of the grains. The fundamental formula relating area to volume, viz.,

$$V = \int_a^b A\ (h)\ d\ h,$$

where V is the volume, and where A, taken perpendicular to an axis along which h is measured, is the area and is expressed as a function of h.

> "It follows that if the distances of "a" (or h) from the plane of the thin-section are equally likely, the proportion of the area of any section occupied by the grain is an unbiased estimate of its relative volume, for the sum of an infinite number of such infinitely thin areas would be its volume".

INTRODUCTION

Location

The Littleton formation is one of the more important stratigraphic units in New Hampshire. These rocks are intruded by at least four magna series, the most extensive being the Oliverian, the New Hampshire (Middle or Upper Devonian), and the White Mountain (Mississippian). The discovery of fossils definitely establishes the age of the Littleton formation as Lower Devonian.

Sampling

The four samples came from the White Mountains. Oriented hand-specimens of sillimanite garnet gneiss and pseudo-andalusite schist were taken for sectioning. Representative chip samples were also collected for chemical analyses. These high-grade rocks reveal evidences of stress in the form of schistosity and lineation.

Method of Cutting and Labelling the Samples

The hand-specimens collected were sectioned in planes parallel to the schistosity and perpendicular to the schistosity. One of the planes normal to the schistosity includes a lineation.

Nine thin-sections were cut from three hand-specimens of the sillimanite garnet gneiss (L29), three parallel to the schistosity, three normal to schistosity and any linear features in the rock; and three normal to the schistosity but including a weak lineation.

Nine sections were also taken from one hand-specimen of the pseudo-andalusite schist (L49); two sections including the schistosity, two sections normal to any parallelism, and two sections perpendicular to the plane of schistosity but containing lineation. Three sections were also cut at oblique angles to any possible structural features within the hand-specimen.

Table 1
L29

Hand-Specimen 1	Hand-Specimen 2	Hand-Specimen 3
A_1 - $\perp$ to schistosity and including lineation	A_2	A_3
B_1 - $\parallel$ to plane of schistosity	B_2	B_3
C_1 - $\perp$ to A_1 and B_1	C_2	C_3

Table 2
L49

A_1 - $\perp$ to schistosity and including lineation	A_2	$\parallel$ to A_1	R_1 oblique
B_1 - $\parallel$ to plane of schistosity	B_2	$\parallel$ to B_1	R_2 oblique
C_1 - $\perp$ to A_1 and B_1	C_2	$\parallel$ to C_1	R_3 oblique

LABORATORY WORK

Petrography

The Littleton series of rocks display a considerable range in mineral composition because of the differences in the grades of metamorphism and in original composition. The rocks analysed by the writer belong to a high-grade zone of metamorphism.

Sillimanite garnet gneiss (L29)

This rock is grey, fine to medium-grained and reveals strong schistosity and possible lineation. It consists of muscovite, sillimanite, quartz and biotite. Accessories include garnet, ilmenite, magnetite, chlorite, tourmaline and plagioclase.

Muscovite flakes are well developed with their longer axes parallel to the lineation. Slender sillimanite needles are intergrown with the muscovite and occur as inclusions in quartz. It also occurs in bands as wide as 15 mm. and as larger prismatic sections.

Quartz shows undulose extinction and is elongate parallel to lineation. In a fine groundmass of quartz, sillimanite, and muscovite, the quartz is difficult to distinguish from untwinned plagioclase.

The Biotite has a preferred orientation which contributes to the schistosity and lineation of the rock. In places it alters to a pale green chlorite.

Porphyroblasts of garnet averaging 1.5 mm in diameter are pale pink and relatively free of inclusions.

Pseudo-andalusite Schist (L49)

This rock is dark grey, medium-grained, and consists essentially of interlocking grains of quartz, biotite, muscovite, sillimanite, and staurolite with an xenoblastic structure. Accessories include magnetite, ilmenite garnet, plagioclase, apatite, chlorite, and tourmaline.

Quartz is present as fine grains with a granular texture surrounding garnet porphyroblasts and often carrying needles of sillimanite.

Garent, staurolite, and muscovite porphyroblasts exhibit a poikiloblastic. Structure Staurolite cyrstals have maximum dimensions of 10 mm.by 3 mm. and fractures in the crystals are filled with quartz.

Method of Integration

Sillimanite garnet gneiss (L29)

Six microtraverses were run on each of the nine sections from the sillimanite garnet gneiss. Approximately 2000 points were tallied for each analysis. The same section was then rotated 180° with reference to the mechanical stage and another analysis of about 2000 was completed. The results are given in Table 5.

The area of the rock section in each slide is as follows:

Table 3.

A_1 - 19 mm. x 8 mm.	A_2 - 14 mm. x 8 mm.	A_3 - 15 mm. x 5 mm.
B_1 - 16 mm. x 10 mm.	B_2 - 7 mm. x 6 mm.	B_3 - 15 mm. x 12 mm.
C_1 - 22 mm. x 16 mm.	C_2 - 14 mm. x 7 mm.	C_3 - 22 mm. x 9 mm.

Six replicate analyses were made for each of A_1, B_1, and C_1, but only two replicates were made for each of the remaining six slides.

Pseudo-andalusite Schist

From one hand-specimen nine thin-sections were cut as explained on page 12. Appendix II is a list of the results.

The area of the rock sample in each slide is given below:

Table 4

A_1	25 mm. x 32 mm.	A_2	14 mm. x 8 mm.	R_1	13 mm. x 13 mm.
B_1	32 mm. x 19 mm.	B_2	11 mm. x 8 mm.	R_2	13 mm. x 13 mm.
C_1	13 mm. x 16 mm.	C_2	19 mm. x 19 mm.	R_3	19 mm. x 10 mm.

The large variation between parallel sections cut .03 mm. apart may be seen from A_1 and A_2 of Appendix II. Each section was analysed twice.

Limitations of the Micrometric Analysis

Holes in the Slide

When the cross-hair intersections fall upon small openings in the slide these were left uncounted. However, if the holes are large and numerous another thin-section must be used. If only a small percentage of grains appear in the section an addition or subtraction of one or two grains of say garnet, will have an appreciable effect upon the results. The tearing out of grains in the grinding of the thin-section is still more serious, because it leads to under-estimation of the harder minerals. At times it is possible to estimate from the shape of the holes the character of the grains thus lost and to allow for them accordingly.

Mineral Identification

This is probably one of the larger sources of error in micrometric analyses. If care is not taken basal-sections of muscovite with low interference colours may be readily mistaken for quartz, especially if the operator is working at high speed. Untwinned plagioclase is present in the sillimanite garnet gneiss and may be abundant. Only twinned plagioclase was tallied as such; the remainder assumed to be quartz since the indices were nearly equal.

Sillimanite is the most difficult mineral to estimate since it occurs in quartz, in muscovite, as independent slender prisms, and in black almost opaque aggregates. Whenever the cross-hair intersection encountered a definite sillimanite needle it was tallied.

Partial alternation products such as sericitization of the plagioclase was estimated as original plagioclase. Where the altered masses could be definitely identified as a particular mineral species such as bands of chlorite in biotite it was totalled as such.

Grain Size

Errors in the measurement of a single thin-section increase with decrease in the size of the grains. On the other hand, an increase in grain size limits the number of grains counted. Garnet and Staurolite porphyroblasts in the sillimanite garnet gneiss and the pseudo-andalusite schist create a different problem. A possible solution to this problem would be the use of a grid on a flat, polished rock specimen.

Results of Micrometric Analyses

Variability is a familiar idea in the science of geology. Every rock and every mineral is a little different from every other, even though the two occur side by side and were formed at the same time and by the same processes.

All mechanical stages are similar in that they give an estimate of the area of a thin-section. It is fundamental to the practice of integrating methods that this area be an unbiased estimate of the volume of the rock.

Variation in One Section

It is apparent from the examination of Table 5 that the results of six replicate analyses on a given slide are in fairly close correspondence. The proportions of minerals such as biotite, garnet, and tourmaline which are not easily misidentified in thin-section do not deviate very greatly from the mean. However, muscovite variation is caused by confusion with sillimanite in small grains.

In marked contrast to the above results are the discrepancies between the figures for the two halves of one section. For example Table 6 gives the results of such an analysis. The greatest variation is seen in the muscovite and garnet percentages. The only garnet porphyroblast lies in the upper half of the thin-section.

Another striking example is in the pseudo-andalusite schist shown in Table 6 ($I49A_1$). The staurolite varies as much as 10.9 per cent of the total in the two halves while garnet, the other mineral forming porphyroblasts differs as much as 5.2 per cent of the total.

These discrepancies in a single thin-section have been noted by Larsen and Miller (1935) in granodiorite and gabbro from the San Luis Rey Quadrangle, California. The plagioclase differs as much as 8.4 per cent of the total in the upper and lower halves of a granodiorite, and the pyroxene as much as 21.9 per cent of the total for gabbro, even although these rocks appear to be quite uniform.

Larsen and Miller believe that the traverses should be transverse to and fluidal banding or linear structure which the rock may contain. In Table 6 ($I49C_1$) which is normal to the schistosity and the lineation there is very little discrepancy in the result in contrast with the analyses in planes parallel to the schistosity and including lineation.

As Chayes (1950) says:

> "We never analyse more than a very small part of a rock and in what sense may we suppose that the material used in a single analysis represents the rock? The material analysed is part of the rock and to this extent represents it. The amount of a given rock is finite with a fixed composition. The average of any two determinations should give a better result than either single result".

A single thin-section has a definite composition but there will no doubt be errors in any estimate of it as may be seen from any of the Tables.

This important problem of how well a single thin-section represents the parent outcrop or hand-specimen is even more striking in metamorphic rocks than in the more homogeneous igneous rocks.

Variation in Oriented Sections from One Hand-Specimen

The next source of variation to be considered is that between thin-sections cut from the same hand-specimen. Grain-size, experimental error, and orientation result in the largest discrepancies. Differences might reflect compositional gradients in the hand-specimen. Even if all the sources of variation were evaluated for a single hand-specimen there would still be no assurance that other hand-specimens from the outcrop were similar. As seen from Table 5 and Table 6 there is no close correspondence of values from the three slides from a single hand-specimen.

Variation in Different Hand-Specimens

Still more serious in the problem of sampling any considerable body of rock is local variations in the proportions of minerals, or even in the assemblage of minerals present. In the sillimanite garnet gneiss three hand-specimens were selected at random and the analyses show the variation that might be expected within hand-specimens. Compare the results in Table 7A. The analyses in the nine oriented slides in three hand-specimens are grouped to give the average composition of each hand-specimen.

Orientation

The sillimanite garnet gneiss is schistose and lineated while the pseudo-andalusite schist is more massive and coarser-grained. Lineation is not pronounced but in a section containing the lineation the muscovite and sillimanite grains are elongated in a single direction and equant normal to this direction. Muscovite and sillimanite occur in bands 1.6 mm. in width which continue across the length of the section.

Even when the muscovite and sillimanite percentages are counted together there is a significant difference between orientation.

In Table 7B are the results of traversing normal to lineation. The results may be compared to these in appendix I (129, A_3), which were run parallel to lineation on the same slide. There is a noticeable difference in the quartz percentages.

The effect of schistosity on the modes is apparent from Table 5. More will be said about the effect of orientation on the final result in a later section.

Effect of Varying the Point Count

As seen from Table 8A, increasing the point count affects the final result. Analyses (1) and (2) are continuations covering the complete slide. In (3) the slide has been rotated 180° with respect to the counter, and there is a noticeable increase in the quartz content. The number of points measured is directly proportional to the area measured on the face of the thin-section and the larger the area the closer the correspondence to the bulk composition of the rock. In many of the analyses the point count is similar. Some of the sections are small and in some cases less than 2000 points were tallied.

Operator Variance

There are, as would be expected, differences between observers and in some instances these differences are quite large. Operator bias is a consistent and personal characteristic. To quote J.C. Griffiths and M.A. Rosenfeld (1954) -

> "One must respect not only differences between different operators but also inconsistent differences between different operators at different times and even within the same operator at different times".

Identification of minerals should theoretically be objective but may become subjective after a number of analyses. The large variation is the percentages of sillimanite and muscovite analysed by different unskilled operators and may be seen in Table 8B. The traverse interval in analyses in Table 8B is 0.5 mm. and can be compared with the results of one operator with a traverse interval of 0.25 mm. in Table 5. Variation in the results are due to the varying skill of the operators uncertain mineral identification, and the number of points tallied.

Before an analysis is attempted the thin-section should be studied in detail in order to classify aggregates and alternation products. If biotite has partly altered to chlorite some operators may estimate the chlorite as biotite and others may attempt to separate them. If a grain is unknown while attempting an analysis the classification of such a grain becomes subjective. The possibility of errors in identification can never be entirely excluded but can be held to a minimum in this type of work.

TABLE 5

Modes of Six Replicate Analyses on Three Oriented Slides From One-Hand Specimens of L29

Mineral	A_1						B_1						C_1					
Muscovite	39.52	39.79	44.59	44.14	44.33	41.84	29.55	33.18	34.12	30.38	33.12	31.21	38.81	38.47	35.97	37.18	37.59	37.20
Sillimanite	19.32	19.84	15.99	17.87	18.01	17.11	8.14	7.36	5.68	8.52	4.68	7.33	10.65	10.42	10.87	10.97	11.10	11.18
Biotite	15.57	14.92	14.51	14.86	14.34	15.28	23.50	23.18	21.95	23.62	22.60	23.83	18.96	18.05	19.41	18.58	18.77	19.17
Quartz	17.27	17.33	16.09	14.25	15.64	16.75	32.05	29.55	31.56	30.48	34.12	30.29	23.08	24.79	24.58	24.39	23.85	24.88
Garnet	5.29	5.40	5.33	6.04	5.48	5.48	3.23	3.64	3.54	3.43	2.60	3.33	4.98	5.32	5.21	5.00	5.03	4.59
Opaques	1.49	.96	1.49	1.21	.90	1.67	1.27	1.41	1.49	1.81	1.24	1.83	1.39	1.16	1.58	1.20	1.27	1.13
Chlorite	.93	.96	1.33	.97	.90	1.04	1.36	.91	.93	.71	.96	1.29	1.49	.95	1.69	1.11	1.81	1.29
Tourmaline	.51	.43	.31	.42	.22	.52	.18	.09	.14	.19	.16	.29	.35	.63	.37	.39	.29	.05
Plagioclase	.10	.37	.36	.24	.17	.31	.73	.68	.59	.86	.52	.58	.30	.21	.32	.19	.29	.52
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.01	100.00	100.00	100.00	100.00	99.98	100.01	100.00	100.00	100.00	100.00	100.01
Points	1946	1870	1951	1656	1771	1917	2200	2200	3558	2100	2500	2400	2010	1900	1896	2079	2046	1941

T A B L E 6

Variations in Single Thin-Sections

Mineral	$L29B_3$		$L49A_1$		$L29A_1$		$L49A_1$		$L49B_1$		$L49C_1$	
Muscovite	21.50	18.60	5.15	9.25	39.79	35.01	5.15	9.25	6.84	4.29	27.38	28.90
Sillimanite	3.35	3.41	25.50	10.85	19.84	12.10	25.50	10.85	14.59	22.55	6.00	9.71
Biotite	11.75	22.45	28.60	32.05	14.92	10.80	28.60	32.05	28.95	14.42	22.21	18.00
Quartz	52.90	51.34	35.15	20.75	17.33	39.29	35.15	20.75	44.38	38.04	34.49	32.14
Garnet	5.20		.25	6.40	5.40		.25	6.40			1.39	.43
Staurolite			3.40	14.25			3.40	14.25	2.92	15.73	6.40	8.10
Opaques	1.50	1.09	1.15	3.90	.96	1.50	1.15	3.90	1.95	4.14	1.34	1.38
Chlorite	.85	.20	.35	2.25	.96	.48	.35	2.25		.39	.06	.19
Tourmaline	2.20	2.82	.05	.05	.43	.07	.05	.05	.03		.06	.05
Apatite	.15		.40	.25			.40	.25	.22		.31	.43
Plagioclase	.60	.10			.37	.75			.14	.44	.37	.67
Total	100.00	100.01	100.00	100.00	100.00	100.00	100.00	100.00	100.03	100.00	100.01	100.00
Points	2000	2022	2000	2000	1871	1471	2000	2000	3700	2053	3517	2100

TABLE 7

	A Variations in Modes of Three Hand-Specimens of L29			B Variations in the Modes of a Single Section Analyzed and to Mineral Alignment of L29A$_3$	
Mineral	First Hand-Specimens	Second Hand-Specimens	Third Hand-Specimens	Parallel	Perpendicular
Muscovite	37.28	28.68	17.89	11.10	9.45
Sillimanite	11.95	14.79	5.86	11.00	22.18
Biotite	19.00	24.95	16.06	16.45	17.82
Quartz	23.95	22.54	56.81	60.35	49.18
Garnet	4.61	4.40	.87		
Opaques	1.36	3.12	1.14	1.00	1.18
Chlorite	1.15	1.14	.19		
Tourmaline	.31	.33	1.02	.05	.09
Apatite				.05	
Plagioclase	.41	.08	.16		.09
Total	100.01	100.03	100.04	100.00	99.99
Points				2,000	1100

TABLE 8

	A			B				
Mineral	Variations in Point Count - L29C_3			Five Unskilled Operators' Results on a Single Section of L29C_1				
	(1)	(2)	(3)	D	E	F	G	H
Muscovite	45.55	21.41	21.70	47.59	46.50	53.61	52.20	52.10
Sillimanite	7.03	4.95	4.25					
Biotite	25.27	15.16	13.55	19.94	18.20	16.18	13.80	17.20
Quartz	18.33	56.72	59.00	23.81	27.80	21.26	25.73	23.30
Garnet				5.35	5.30	5.05	4.50	5.10
Opaques	2.58	1.15	.95					
Chlorite			.05	3.39	2.10	2.99	2.64	2.30
Tourmaline	1.07	.52	.40					
Plagioclase	.18	.14	.10					
Total	100.01	100.01	100.00	100.08	98.80	99.09	98.87	93.00
Points	1124	3484	2000	1159	1117	1039	1022	1127

CALCULATIONS OF GRAND AVERAGES FROM MICROMETRIC ANALYSES

Sillimanite Garnet Gneiss

From the first hand-specimen a mean was calculated for each mineral from the eighteen replicate analyses on A_1, B_1 and C_1 (Table 9). Similarily with the other two-hand-specimens. The average of these three means gave the grand mean $\bar{\bar{X}}$. (Table 10). Each thin-section is seen to be weighted equally. The scheme of averaging is given below.

Pseudo-Andalusite Schist

Eighteen analyses were completed on the schist and a grand mean calculated (Table 11). Table 11 also gives the individaul means.

Even although the nine sections are from the same hand-specimen it was decided to group the results for comparison.

TABLE 9

Mineral	Average Mode of 6 Analyses L29A$_1$	Average Mode of 2 Analyses L29A$_2$	Average Mode of 2 Analyses L29A$_3$	Average Mode of 10 Analyses L29A	Average Mode of 6 Analyses L29B$_1$	Average Mode of 2 Analyses L29B$_2$	Average Mode of 2 Analyses L29B$_3$	Average Mode of 10 Analyses
Muscovite	42.37	23.23	12.05	25.88	31.93	32.50	20.05	28.16
Sillimanite	18.02	11.69	9.60	13.10	6.95	14.99	3.38	8.44
Biotite	14.91	27.18	16.73	19.61	23.11	24.02	17.10	21.41
Quartz	16.22	29.73	60.45	35.47	31.34	15.40	52.12	32.95
Garnet	5.50	3.36		2.95	3.30	6.86	2.60	4.25
Opaques	1.29	3.34	1.08	1.90	1.51	4.08	1.30	2.30
Chlorite	1.02	1.37		.80	1.03	1.30	.53	.95
Tourmaline	.40	.03	.08	.17	.18	.78	2.51	1.16
Apatite			.03	.01			.08	.03
Plagioclase	.26	.10		.12	.66	.08	.35	.36
Total	99.99	100.00	100.02	100.01	100.01	100.01	100.02	100.01

TABLE 10

	Average Mode of Six Analyses L29C$_1$	Average Mode of Two Analyses L29C$_2$	Average Mode of Two Analyses L29C$_3$	Average Mode of Ten Analyses L29C	Average Mode of First Hand-Specimen	Average Mode of Second Hand-Specimen	Average Mode of Third Hand-Specimen	Mode of L29, Grand Average $\bar{\bar{X}}$
Mineral								
Muscovite	37.54	30.32	21.55	29.80	37.28	28.68	17.89	27.95
Sillimanite	10.87	17.70	4.60	11.06	11.95	14.79	5.86	10.86
Biotite	18.99	23.65	14.35	19.00	19.00	24.95	16.06	20.01
Quartz	24.26	22.48	57.86	34.87	23.94	22.54	56.81	34.43
Garnet	5.02	2.97		2.66	4.61	4.40	.87	3.29
Opaques	1.29	1.93	1.05	1.42	1.36	3.12	1.14	1.87
Chlorite	1.39	.74	.03	.72	1.15	1.14	.19	.83
Tourmaline	.35	.17	.46	.33	.31	.33	1.02	.55
Apatite							.04	.01
Plagioclase	.31	.06	.12	.16	.41	.08	.16	.22
Total	100.02	100.02	100.04	100.02	100.01	100.03	100.04	100.02

TABLE 11

	Average Mode of Two Analyses $L49A_1$	Average Mode of Two Analyses $L49A_2$	Average Mode of Two Analyses $L49B_1$	Average Mode of Two Analyses $L49B_2$	Average Mode of Two Analyses $L49C_1$	Average Mode of Two Analyses $L49C_2$	Average Mode of Two Analyses $L49R_1$	Average Mode of Two Analyses $L49R_2$	Average Mode of Two Analyses $L49R_3$
Mineral									
Quartz	27.95	40.28	41.21	7.96	33.32	38.23	18.23	33.85	45.03
Muscovite	7.20	3.03	5.57	44.14	28.14	8.30	33.90	22.75	16.08
Biotite	30.33	35.85	21.69	31.78	20.11	38.75	25.45	29.89	26.08
Staurolite	8.83	4.00	9.33	5.24	7.25	3.50	3.18	4.10	1.95
Sillimanite	18.18	14.94	18.58	9.57	7.86	9.70	5.65	8.10	9.00
Garnet	3.33				.91		7.50		
Opaques	2.53	1.54	3.05	1.05	1.36	1.08	2.40	1.05	1.63
Chlorite	1.30	.03	.20	.24	.13	.15	3.70	.16	
Tourmaline	.05	.03	.02	.04	.06	.03		.09	.05
Apatite	.33	.15	.11		.37	.28			.05
Plagioclase		.09	.29		.52			.05	.15
Total	100.03	99.99	100.05	100.02	100.03	100.02	100.01	100.04	100.02

TABLE 12

Average Modes of L49 Grouped According to Orientation

Mineral	$A_1B_1C_1$	$A_2B_2C_2$	$R_1R_2R_3$	Mode of L49 Grand Average
Quartz	34.16	28.82	32.37	31.78
Moscovite	13.64	18.49	24.24	18.79
Biotite	24.04	35.46	27.14	28.88
Staurolite	8.47	4.25	3.08	5.26
Sillimanite	14.87	11.42	7.58	11.29
Garnet	1.41		2.50	1.30
Opaques	2.31	1.22	1.69	1.74
Chlorite	.54	.14	1.29,	.66
Tourmaline	.04	.03	.05	.04
Apatite	.27	.14	.02	.14
Plagioclase	.27	.03	.07	.12
Total	100.02	100.00	100.03	100.00

CHEMICAL COMPOSITION

Introduction

Studies of variation in composition are usually based on chemical analyses. Few petrographers have placed much weight on micrometric analyses. There are several advantages of microscopic as opposed to chemican analysis.

First of all, chemical analyses of silicate rocks are expensive. Secondly, the time taken for a microscopic analysis is much less than that taken for a complete chemical analysis. A real advantage of micrometric analysis is that it estimates directly the amounts of minerals actually present. When the petrographer calculates the chemical analysis back to the percentages of the various minerals present in the rock the answers are variable.

However, micrometric analysis cannot completely replace the chemist. Many rocks cannot be handled adequately by present techniques. Fine-grained volcanic rocks cannot be analysed with any mechanical stage. If the rocks are very coarse-grained fragment analysis may have to be employed. Micrometric analysis cannot accurately estimate the amount of H_2O, P_2O_5, CO_2 and the rare elements within a rock.

Errors in Calculation to Chemical Composition

Once the mode of the rock is determined it is then necessary to determine specifically the composition of each mineral. The exact composition of the minerals is not always easily ascertained, unless, of course, they have been separately analysed. In the case of sillimanite, tourmaline, apatite, and staurolite, the theoretical composition was assumed with little risk of serious error, but for the micas, chlorite, and garnet (minerals of variable composition) the safest method is to assume that they have approx-

TABLE 13

Chemical Analyses of Minerals in High-Grade Rocks of Dutchess County, N.Y. (Barth, 1936)

	Garnet	Muscovite	Biotite	Chlorite	
SiO_2	37.94	45.49	37.39	24.7	
TiO_2	1.04	.89	2.48	1.7	
Al_2O_3	22.87	37.51	20.17	17.4	
Fe_2O_3	-	-	2.08	5.2	
FeO	32.45	-	14.32	-	
MgO	1.83	.20	9.80	4.3	33.0
MnO	1.77	-	-		
CaO	2.04	.50	.51	4.4	
Na_2O	-	1.72	.87	-	
K_2O	-	8.86	8.56	-	
H_2O^+	-	4.92	3.03	9.3	
P_2O_5	-	-	-	-	
Cr_2O_3	-	-	.02	-	
F	-	-	1.67	-	
Total	99.94	100.09	99.93	100.00	

imately the composition of optically similar types which have been analysed. Thus the literature was searched for analyses of similar varieties of the highly variable minerals. Suitable analyses are given by Barth (1936) from rocks in Dutchess County, New York, and are tabulated in Table 13.

The plagioclase content is very small and the theoretical composition of oliogoclase is assumed. (Untwinned plagioclase may be present, masquerading as quartz). The next step in the calculation is to determine the specific gravity of each of the main components. A 10 ml. picnometer was used in these determinations. Two to three gravity determinations were completed on each of the minerals in Table 13 and the average taken. Each of the mineral fractions was washed in acetone to ensure "wetting" of the minerals.

TABLE 14

Specific Gravities of Some Minerals in L29 and L49

	L29	L49
Biotite	3.20	2.91
Muscovite	2.71	2.79
Staurolite	-	3.63

From the volume percentages of the grand means $\overline{X}$ and $\overline{Y}$, and the specific gravities of the individual components, the weight percentage of each mineral was determined.

The opaque minerals in the rocks are magnetite and/or ilmenite. Results are evaluated on two assumptions; firstly that there are equal percentages of each in the rock, and secondly that only magnetite is present.

Critical Examination of the Results

Variability of chemical results makes the geologist wary of using these results in the interpretation of geological data. A very high degree of accuracy for ordinary rock or mineral analyses is very seldom attained. Results reported by thirty-four chemists analyzing the same rocks (Schlecht and Stevens, 1951) show what variability may be expected from different laboratories.

Both the sillimanite garnet gneiss and the pseudo-andalusite schist have been chemically analyzed in laboratories at the University of Minnesota.

E.S. Larsen (Jr. (1938) has presented the results of our chemical analyses of an amphibole. In comparing the results SiO_2, TiO_2, and Al_2O_3 show differences as great as 2.5 per cent, Fe_2O_3 as much as 1.8, FeO up to 3.2, MgO up to 2.5 and CaO up to 4.0 per cent of the rock.

In the calculated composition of the pseudo-andalusite schist for example, the values used for sections R_{1-3} give results which are in fairly good correspondence to the chemical analysis. However, SiO_2 varies as much as 2.0 per cent, TiO_2 up to .58, and Al_2O_3 up to .07 per cent of the rock. The last is negligible.

Comparing the variation of the results discussed by Larsen on amphiboles analyzed in four different laboratories with the variation in the writers calculated results, it is found that the calculated results could very well be within the range of experimental error incurred in the chemical laboratories.

Sillimanite garnet gneiss

In Table 15 are given the calculated compositions for the gneiss. Analyses (1) and (2) differ only in the TiO_2, Fe_2O_3 and FeO values since in (2) the opaque mineral is taken as magnetite alone. Analyses (3) - (5) give an idea of the variability among hand-specimens. The most conspicuous variation is in the SiO_2, Al_2O_3, TiO_2, and the FeO/Fe_2O_3. The third hand-specimen, (5) is silica rich which in part accounts for the extreme variation from the chemical analysis of the rock (6).

The TiO_2 content is variable with respect to the chemical analysis due to the variability within the complex micas, garnet, and chlorite. Analysis (2) is in good agreement with the TiO_2 content of the chemical analysis.

The Al_2O_3 content presents a problem. On the whole it is higher in the calculated results. If untwinned plagioclase was determined there would be even more alumina to account for. One answer is the possibility that one of the analyses used in Table 21 for the micas is too rich in Al_2O_3. There is also the over-estimation of the sillimanite-muscovite ratio.

The excess of FeO and Fe_2O_3 in the calculated results may be due to the figure for biotite being too high. Also the presence of sulphur in the chemical analysis indicates the possibility of sulphides as an accessory. This would lower the percentage of magnetite and ilmenite and the final FeO, Fe_2O_3 result.

In the calculated results the MgO and MnO content is higher which could be due to any of the factors discussed in the preceeding paragraphs.

The average calculated lime content (6) is quite close to the chemical analysis. Na_2O for the calculated results is low and correspondingly the K_2O is too high. Substitution of K^+ for Na^+ in the micas is quite common and it is surprising that the estimates are as close as they are. The total $Na_2O + K_2O$ content for the chemical analysis is 5.1 per cent of the rock while that of the average calculated result (6) is 4.8 per cent of the rock.

Barths' analysis for the biotite contains 1.67 per cent fluorine which introduces a high figure for the fluorine content in the calculated compositions.

Pseudo-Andalusite Schist

The calculated composition (4) is on the whole close to the chemical analysis. (Table 17)

As in the gneiss the SiO_2 percentages are low and the TiO_2 high. Due to the garnet porphyroblasts MnO is high especially in (4).

Also like the gneiss the calculated Na_2O is low while K_2O is correspondingly high due to substitution. But the $Na_2O + K_2O$ content is 4.8 per cent of the rock in the calculated and chemical analyses.

Sillimanite in Quartz

In order to determine the amount of sillimanite needles in quartz one operator analyzed three sections of the sillimanite garnet gneiss. The results are given in Table 16.

TABLE 16

Percentage of Sillimanite in Quartz

Mineral	A_1	B_1	C_1
Sillimanite in Quartz	2.02	1.52	1.39
Quartz	27.01	26.37	22.08
Remainder	70.86	72.22	76.53
Total	99.89	100.11	100.00
Points	1235	1639	1010

TABLE 15

Calculated Compositions of the Sillimanite Garnet Gneiss

	Opaque Calculated as Magnetite Ilmenite (1)	Opaque Calculated as Magnetite (2)	First Hand-Specimen (3)	Second Hand-Specimen (4)	Third Hand-Specimen (5)	Chemical Analyses Rock Analysis Laboratory University of Minnesota (Doris Thaemlitz) (6)	Average of (2)(3)(4)(5) (7)
S_1O_2	57.69	57.69	52.72	49.75	71.32	62.51	57.69
T_1O_2	1.67	.83	1.53	2.37	.62	1.06	1.39
Al_iO_3	23.05	23.05	27.13	26.56	15.04	18.91	22.95
Fe_2O_3	1.59	2.68	1.30	2.37	1.78	.92	1.94
FeO	5.98	5.72	6.12	7.89	3.67	6.08	5.88
MgO	2.37	2.37	2.30	2.82	1.98	1.66	2.37
MnO	.21	.21	.29	.28	.05	.10	.21
CaO	.38	. 38	.48	.43	.27	.33	.39
Na_2O	.67	.67	.82	.69	.47	1.13	. 66
K_2O	4.16	4.16	4.79	4.52	3.08	3.94	4.14
$H_2\ O$	2.02	2.02	2.42	2.17	1.41	2.58	2.01
H_2^-O						.12	
P_2O_5					.01	.10	.01
PO_2						.02	
B_2O_3	.06	.06	.03	.03	,11		.06
F	.36	.36	.34	.44	.30		.36
S						.03	
Cr_2O_3				.01			. 01
C						.33	
BaO						.07	
Total	100.21	100.20	100.27	100.27	100.11	99.89	100.07

TABLE 17

Calculated Compositions of the Pseudo-Andalusite Schist.

	Average Composition (1)	$A_1B_1C_1$ (2)	$A_2B_2C_2$ (3)	$R_1R_2R_3$ (4)	Chemical Analyses University of Minnesota Doris Thaemlitz, analyst (5)
S_1O_2	55.86	54.97	53.93	56.20	58.33
T_1O_2	1.31	1.74	1.59	1.68	1.17
al_2O_3	24.51	25.40	24.50	22.48	22.50
Fe_2O_3	1.16	1.86	1.47	1.63	2.09
FeO	6.61	7.39	6.85	6.35	5.84
MgO	2.93	2.38	3.52	2.78	1.87
MnO	.13	.11	.02	.26	.06
CaO	.40	.40	.36	.38	.25
Na_2O	.57	.44	.62	.64	.98
K_2O	4.10	3.14	4.61	4.39	3.62
H_2O	2.12	1.84	2.19	2.24	2.44
H_2O					.13
P_2O_5	.19	.37	.19	.02	.18
CO_2					.02
B_2O_3	.49	.39	.59	.01	
F					

STATISTICAL ANALYSIS

Introduction

Statistical analysis is a specialized and powerful tool for certain quantitative investigations and being specialized it is not appliable under all circumstances.

During recent years geologists have used statistical methods mainly as an aid in summarizing large numbers of observations in petrology and other fields. In addition, statistics provide methods for analysis of data; and quantitative interpretation of geological data are possible by the use of modern techniques of statistical analysis.

One of the most applicable techniques in statistics is analysis of variance.

Analysis of Variance

The main object of variance analysis is not to describe a sample but to reach inferences about the parent population from which the sample is drawn. The first step in the investigation is to define the characteristic quality or qualities to be sampled. Any particular quality will be called a "population". A hand-specimen may represent a population from which the thin-sections are cut.

The actual test of variances involves the assumption that:

> "the parent variation of each source be normally or "sufficiently normally" distributed". (Chayes, 1950).

Normal distribution is discussed by Dixon and Massey (1951, pp. 47-66). Briefly, the normal distribution is represented as a frequency - distribution diagram by a symmetrical, bell-shaped curve.

> "One way in which a close approach to the normal curve may be obtained is to compile a large number of "precision" measurements" (Strahler, 1954).

The next step is to define some of the statistical measures used in the analysis. The arithmetic means of a set of observations is the quotient obtained when the sum of the set of observations is divided by the number of observations:

$$(1) \qquad \bar{X} = \frac{\sum (X)}{N}$$

A statistics that indicates the extent to which the individual readings fail to coincide with the mean is the standard deviation:

$$(2) \qquad \sigma = \frac{\sqrt{(x - \bar{x})^2}}{N}$$

where is the standard deviation, $\bar{x}$ the arithmetic mean, x the reading, and N the total number of readings.

As defined by Krumbein and Miller (1953) residual variation is

> "the random and uncontrolled variance within the population plus some other factors in the study which were not included in the experimental design".

Degrees of freedom is simply the number of independent measurements, and is one less than the sample size.

Analysis of variance requires that data be collected and analyzed according to specific models or designs in order to observe the maximum information from the method. One problem is to recognize the appropriateness of the model to the study. A large number of mathematical models have been designed by statisticians. Discussion of these, with examples, are given by Dixon and Massey (1951, chap. X). The model used by the writer is given in the following tables and is based on the method given in Snedecor (1946, p. 283). Each of the nine "cells" in the variance model in Table 18 contains observations from a particular thin-section.

The following variance analyses are for the sillimanite garnet gneiss; the pseudo-andalusite schist is not included.

TABLE 18

Variance Analysis of Muscovite

Muscovite Values

Hand-Specimen	Orientation A	 B	 C
1.	39.51	30.38	38.80
	39.78	34.12	38.47
	44.59	33.12	35.97
	44.14	31.20	37.18
	44.32	29.54	37.58
	41.83	33.18	37.19
2.	22.45	33.53	32.24
	24.00	31.46	28.40
3.	13.00	21.50	21.41
	11.10	18.60	21.70

TABLE 19

Sums of A, B, C of Muscovite

	A	B	C	T
1.	254.17	191.54	225.19	670.90
2.	46.45	64.99	60.64	172.08
3.	24.10	40.10	43.11	107.31
T.	324.72	296.63	328.94	950.29

The Method

The various steps in the method will be given for the variance analysis of the muscovite and only the results of the remaining minerals will be presented.

(1) $C = \frac{T_{..}^2}{n} = \frac{(950.29)^2}{30} = 30101.7028$

(2) Total: $\sum X^2 - C = 2350.0193$

(3) Subclasses: $\sum\left(\frac{T_{ij}^2}{w_{ij}}\right) - C = 2284.6155$

(4) Within subclasses: (2) - (3) = 65.4038

(5) Orientations: $\frac{T_{\cdot A} + T_{\cdot B} + T_{\cdot C}}{10} - C = 61.6931$

(6) Rocks: $\frac{T_1^2}{18} + \frac{T_2^2}{6} + \frac{T_3^2}{6} - C = 1758.7249$

(7) Interaction: (3) - (5) - (6) = 464.1975

TABLE 20

Variance Analysis of Muscovite

Source of Variation	Degree of Freedom	Sum of Squares	Mean Square
Orientations	2	61.6931	30.8466
Rocks	2	1758.7249	879.3625
Interactions	4	464.1975	116.0494
Residual	21	65.4038	3.1145
Total	29	2350.0193	

The experimental error: $= \sqrt{3.1145} = 1.7648$. The next step is to calculate the F ratios with reference to the tables in Dixon and Massey (1951, p. 310).

The ratio of the Interaction/Residual is given by:

$$F = \frac{116.0494}{3.1145} = 37.26$$

$$F_{.99} \ (4,21) = 4.37$$

Inasmuch as the observed value is greater than the $F_{.99}$ there is a significant variation at the 1 per cent level.

The ratio Orientations/Interaction is:

$$F = \frac{30.8466}{116.0494} = .27$$

$$F_{.95} \ 2 \ (2,4) = 6.94$$

Since F is less than 1 there is no significant variation among orientations.

For the ratio Rocks/Interactions

$$F = \frac{879.3625}{116.0494} = 7.58$$

$$F_{.95}\ (2,4) = 6.94$$

$$F_{.99}\ (2,4) = 18.00$$

This shows no significant variation on the 5 per cent level of significance but the variation is significant on the 1 per cent level of significance.

TABLE 21

Variance Analysis of Sillimanite

Sillimanite Values			
Hand Specimen	Orientation		
	A	B	C
1.	19.32	8.14	10.65
	19.84	7.36	10.42
	15.99	5.68	10.87
	17.87	8.52	10.97
	18.01	4.68	11.10
	17.11	7.33	11.18
2.	11.57	14.80	16.84
	11.80	15.15	18.55
3.	8.20	3.35	4.95
	11.00	3.41	4.25

TABLE 22

Sums of A, B, C of Sillimanite

	A	B	C	Ti.	
1.	108.14	41.71	65.19	215.04	
2.	23.37	29.98	35.39	88.74	
3.	19.20	6.76	9.20	35.16	
$T._j$	150.71	78.45	109.78	338.94	T..

TABLE 23

Variance Analysis of Sillimanite

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Orientations	2	262.6114	131.3057
Rocks	2	257.1693	128.5847
Interactions	4	196.3554	49.0889
Residual	21	27.0704	1.2891
Total	29	743.2065	

The experimental error: $= \sqrt{1.2891} = 1.136$

Interactions/Residual

$F = 38.08$

$F_{.99}$ (4,21) = 4.37

Significant

Orientations/Interaction

$F = 2.68$

$F_{.95}$ (2,4) = 6.94

Not Significant

Rocks/Interaction

$F = 2.62$

$F_{.95}$ = (2,4) = 6.94

Not Significant

Hence, for the sillimanite values there is a significant variation only in the ratio Interactions/Residual.

TABLE 24

Variance Analysis of Biotite

Biotite Values			
Hand Specimen	Orientation		
	A	B	C
1.	15.57	23.50	18.96
	14.92	23.18	18.05
	14.51	21.95	19.41
	14.86	23.62	19.58
	14.24	22.60	18.77
	15.28	23.83	19.17
2.	28.11	23.89	23.62
	26.25	24.15	23.68
3.	17.00	11.75	15.16
	16.45	22.45	13.55

TABLE 25

Sums of A, B, C of Biotite

	A	B	C	Ti
1.	89.48	138.68	113.94	342.10
2.	54.36	48.04	47.30	149.70
3.	33.45	34.20	28.71	96.36
$T._j$	177.29	220.92	189.95	588.16 T..

TABLE 26

Variance Analysis of Biotite

Source of Variation	Degree of Freedom	Sum of Squares	Mean Square
Orientations	2	100.7264	50.3632
Rocks	2	253.2846	126.6423
Interactions	4	124.9128	31.2282
Residual	21	65.5485	3.1214
Total	29	544.4723	

Experimental Error: $= \sqrt{3.1214} = 1.767$

Interaction/Residual

$F = 10.00$

$F_{.99}\ (4,21) = 4.37$

Significant

Orientations/Interactions

$F = 1.61$

$F_{.95}\ (2,4) = 6.94$

Not Significant

Rocks/Interactions

$F = 4.06$

$F_{.95}\ (2,4) = 6.94$

Not Significant

Again, for the biotite the only significant variation from the level of significance is in the case of the ratio Interactions/Residual.

TABLE 27

Variance Analysis of Quartz

Quartz Values			
Hand Specimen	Orientations		
	A	B	C
1.	17.27	32.05	23.08
	17.33	29.55	24.79
	16.09	31.56	24.58
	14.25	30.48	24.39
	15.64	34.12	24.85
	16.75	30.29	24.88
2.	30.41	14.36	21.79
	29.05	16.44	23.16
3.	60.55	52.90	56.72
	60.35	51.34	59.00

TABLE 28

Sums of A, B, C of Quartz

	A	B	C	$T_{i.}$
1.	97.33	188.05	145.57	430.95
2.	59.46	30.80	44.95	135.21
3.	120.90	104.24	115.72	340.86
$T_{.j}$	277.69	323.09	306.24	907.02 $T_{..}$

TABLE 29

Variance Analysis of Quartz

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Orientation	2	105.3395	52.6696
Rocks	2	5306.0324	2653.0162
Interactions	4	859.4815	214,8704
Residual	21	6301.2377	1.0128
Total	29		

Experimental Error: $= \sqrt{1.0128} = 1.007$

Interaction/Residual

$F = 212.16$

Highly Significant

$F_{.99}$ (4,21) = 4.37

Orientations/Interactions

$F = 0.2451$

Not Significant

$F_{.95}$ (2,4) = 6.94

Rocks/Interactions

$F = 12.35$

$F_{.95}$ (2,4) = 6.94

Probably Significant

$F_{.99}$ (2,4) =18.00

The observed value for the F ratio is significant in all cases except in the ratio Orientation/Interactions. However, the analysis of variance shows that there is no significant variation in any of the minerals among orientations.

Two factors may cause significance. Firstly, the unequal number of observations in the cells and secondly, a single observation in a cell may be very much larger or smaller than any of the other constituents in the analysis, thereby strongly affecting the mean. This is true especially in the analysis of the quartz where the third hand-specimen is silica rock.

In summary, it may be noted that there is definitely interaction as shown by the ratio Interactions/Residuals. Calculations for the ratio of Orientations/Interaction suggest that no matter how a metamorphic rock is sectioned and subsequently analysed, it will not affect the final result.

It is difficult to interpret the results for the ratio of Rocks/Interactions; but on the whole they suggest that there is probably no significance. In other words, the geologist is not restricted to a particular hand-specimen from an outcrop.

CONCLUSIONS

The conclusions may now be summarized on the basis of experimental results.

There is undoubtedly variation within the mode of a single hand-specimen. There is also a large variation between different hand-specimens of the same rock type. This is probably the variation noted by Delésse and Rosiwal when they believed that only homogeneous rocks could be successfully integrated. Rosiwal worked on single measurements and did not take averages into account. Now, when the averages of ten analyses are taken on each of the oriented planes the results are in close correspondence to one another. (Table 9-10). However, separate hand-specimens present a problem. In Table 10 the analyses for the three hand-specimens from the sillimanite garnet gneiss show a wide variation which will naturally affect the final results as presented in Table 15. These results suggest that neither of the three-hand-specimens themselves closely correspond to the chemical analysis, but when the average of the three hand-specimens is taken the calculated result is close to the chemical analysis. Although single results taken normal to the orientation vary greatly from the final result, the average of analyses from sections cut at high angles to mineral directions gives a final result that is quite close to the chemical analysis (Table 17).

With such variation how many thin-section-analyses must be carried out to give at least an idea of the chemical composition? The number of thin-section analyses necessary to represent a rock must be determined by a consideration of the grain-size and the uniformity of the rock. A medium-grained igneous rock would be well represented by say three analyses. If on the other hand the rock contains variable sizes of grains from fine aggregates to large porphyroblasts the number of representative analyses is a difficult question. The writer's work suggests that no less than three sections should be analysed. For example in the pseudo-andalusite schist three sections were cut at high angles to orientation and the average of these (each measured three) is close to the chemical analysis.

The fundamental question of the relationship between area and volume is still being debated. The writer agrees with F. Chayes that orientation has no effect on the final result, as long as the final results are computed from averages. With all limitations and possible sources of errors the results do not show too large a spread.

Only the most simple operations and tests of statistics have been brought into this paper and explanations of the theory behind them may be found in Snedecor and Dixon and Massey.

Statistical tests may or may not reveal significant differences and the reasons for these differences lie outside the realm of statistics. However, statistics may test the results of the petrologist. In this paper these tests on the whole confirm intuitive observations.

In conclusion, micrometic analysis can be successfully applied, with limitations, to oriented rocks. However, much work has yet to be done in this field and the author hopes that this paper will at least stimulate further interest and research in this direction.

APPENDIX-I

Modes of Second and Third Hand-Specimens of L29

Mineral	A_2		A_3		B_2		B_3		C_2		C_3	
Muscovite	22.46	24.00	13.00	11.10	33.53	31.47	21.50	18.60	32.24	28.40	21.41	21.70
Sillimanite	11.57	11.80	8.20	11.00	14.80	15.18	3.35	3.41	16.84	18.55	4.95	4.25
Biotite	28.11	26.25	17.00	16.45	23.89	24.15	11.75	22.45	23.62	23.68	15.16	13.55
Quartz	30.41	29.05	60.55	60.35	14.36	16.44	52.90	51.34	21.79	23.16	56.72	59.00
Garnet	3.12	3.60			6.59	7.13	5.20		3.00	2.94		
Opaques	8.12	3.55	1.15	1.00	4.62	3.54	1.50	1.09	1.61	2.25	1.15	.95
Chlorite	1.13	1.60			1.38	1.21	.85	.20	.72	.75		
Tourmaline		.05	.10	.05	.74	.82	2.20	2.82	.17	.17	.52	.40
Apatite				.05			.15					
Plagioclase	.09	.10			.10	.05	.60	.10		.12	.14	.10
Total	100.01	100.00	100.00	100.00	100.01	99.99	100.00	100.01	99.99	100.01	100.01	100.00
Points	2213	2000	2000	2000	2034	2062	2000	2022	1799	1736	3484	2000

APPENDIX II

Mineral	A_1		A_2		B_1		B_2		C_1		C_2	
Quartz	35.15	20.75	42.57	37.98	44.38	38.04	9.12	6.80	34.49	32.14	39.45	37.00
Muscovite	5.15	9.25	3.16	2.90	6.84	4.29	45.85	42.42	27.38	28.90	8.50	8.10
Biotite	28.60	32.05	34.09	37.60	28.95	14.42	28.75	34.81	22.21	18.00	39.25	38.25
Staurolite	3.40	14.25	4.07	3.92	2.92	15.73	5.38	5.10	6.40	8.10	3.75	3.25
Sillimanite	25.50	10.85	14.30	15.68	14.59	22.55	9.61	9.52	6.00	9.71	7.90	11.50
Garnet	.25	6.40							1.39	.43		
Opaques	1.15	3.90	1.41	1.67	1.95	4.14	1.22	.88	1.17	1.38	.70	1.45
Chlorite	.35	2.25	.06			.39		.48	.06	.19	.20	.10
Tourmaline	.05	.05	.06	.06	.03		.08		.06	.05	.05	
Apatite	.40	.25	.17	.13	.22				.31	.43	.20	.35
Plagioclase			.11	.06	.14	.44			.37	.67		
Total	100.00	100.00	100.00	100.00	100.03	100.00	100.01	100.01	100.01	100.00	100.00	100.00
Points	2000	2000	1769	1556	3700	2053	1228	1471	3517	2100	2000	2000

APPENDIX III

Modes of Oblique Sections of L49

Mineral	R_1		R_2		R_3	
Quartz	14.95	21.50	36.45	31.24	46.85	43.20
Muscovite	33.30	34.50	21.15	24.34	16.10	16.65
Biotite	24.50	26.40	29.30	30.48	25.25	26.90
Staurolite	4.05	2.30	4.45	3.75	1.90	2.00
Sillimenite	7.20	4.10	7.50	8.69	8.40	9.60
Garnet	8.80	6.20	-	-	-	-
Opaques	2.50	2.30	.85	1.24	1.20	2.05
Chlorite	4.70	2.70	.15	.16	-	-
Tourmaline	-	-	.10	.08	.05	.05
Apatite	-	-	-	-	.10	-
Plagiocalse	-	-	.05	.04	.15	.15
Total	100.00	100.00	100.00	100.02	100.00	100.00
Points	2000	3000	2000	2510	2000	2000

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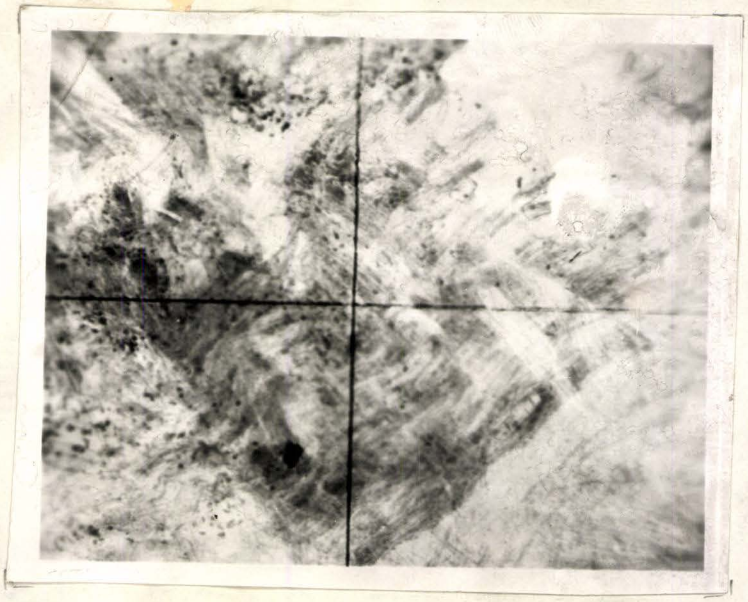
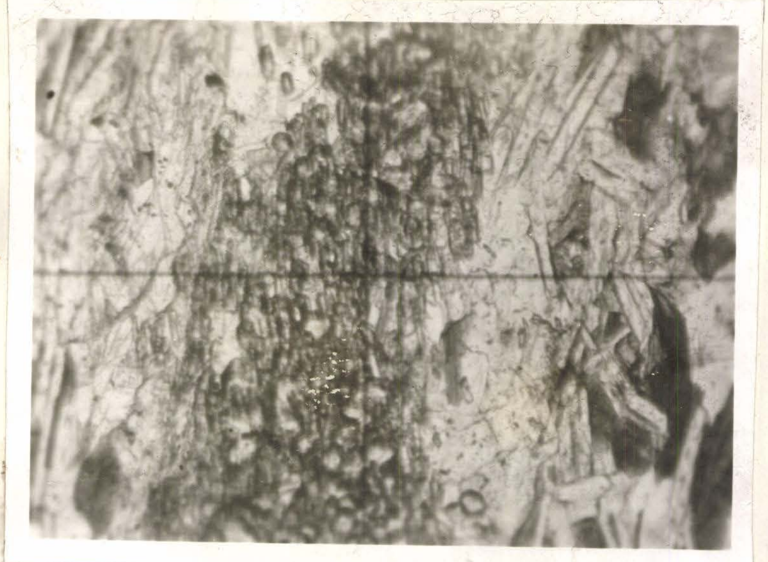
PLATE I



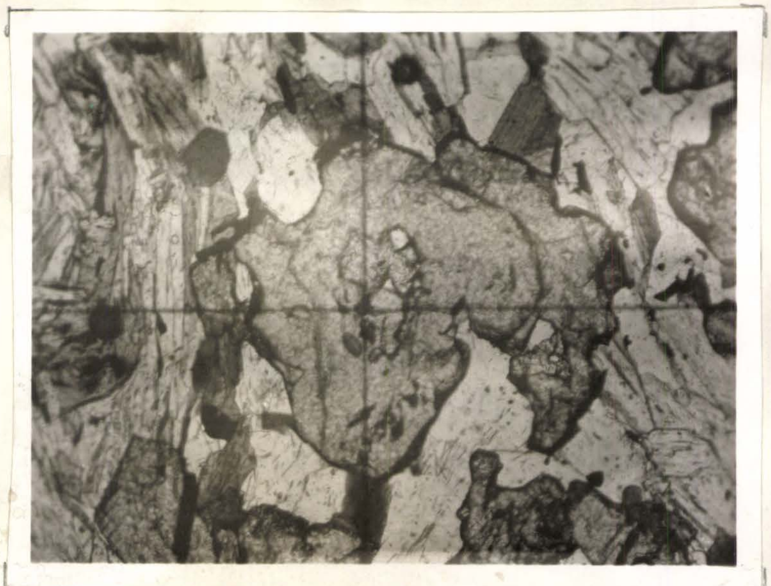
Sillimanite Aggregate in L29

PLATE II



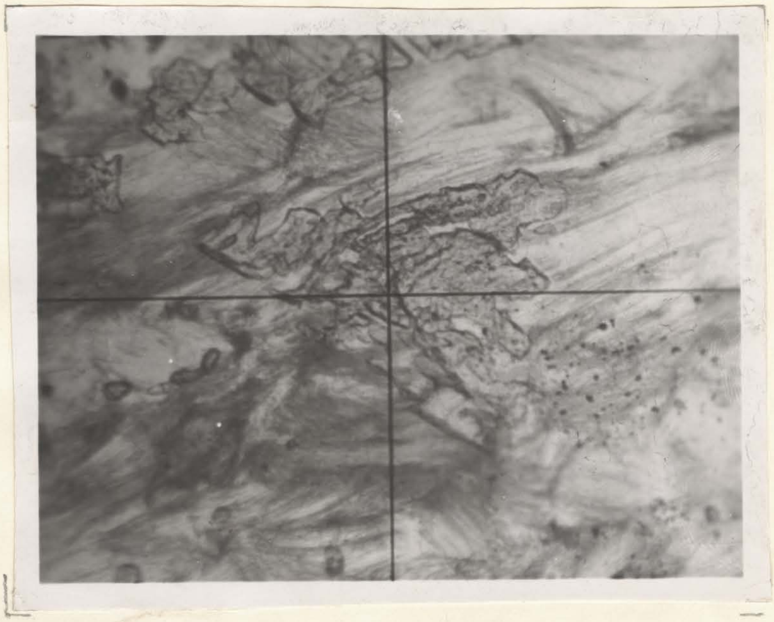
Sillimanite - Muscovite Aggregates

PLATE III



Garnet Porphyroblast in L29

PLATE IV



Staurolite Porphyroblast in Sillimanite - L49