COMPARISON BETWEEN "PQR" AND DIRECT ELIMINATION METHODS OF FORMULATING POWER SYSTEM COEFFICIENT MATRICES

COMPARISON BETWEEN "PQR" AND DIRECT ELIMINATION METHODS OF FORMULATING POWER SYSTEM COEFFICIENT MATRICES

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ABSTRACT

In power systems, dynamic stability analysis is an important field of interest for both design and operation studies. This stability analysis requires the formulation of the linearized power system equations in the state-space form.

In this thesis, the state-space matrices of multi-machine systems are constructed by implementing two matrix formulation techniques, the "PQR" and the direct elimination "ELIM" methods. Two computer programs have been devised to apply these formulation techniques. The programs are capable of handling systems up to a maximum order of 70, with available central memory of about 49,000 words (decimal). Another feature of these programs is their capability of accommodating generating units with different degrees of complexity, by allowing a variety of models for the sub-system components. Both programs have been applied to two test examples to illustrate their validity.

The two formulation technique programs were compared from the point of view of computational time, storage requirements and eigenvalue sensitivity evaluation.

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LIST OF PRINCIPAL SYMBOLS

STATE SPACE MODEL FOR THE OVERALL SYSTEM

$\underset{\sim}{f}, \underset{\sim}{g}, \underset{\sim}{h}, \underset{\sim}{k}$	Vector functions
A, B, C, D	Matrices for the state-space description
P, Q, R, S, E	Matrices associated with the "PQR" method
H, Z	Matrices associated with the "Elimination" method
$\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{u}$	Vectors of states, algebraic variables and inputs
PA, PB, QA, QB, RA, RB, PX, PS, PT, PI, PN	Submatrices associated with the "PQR" partitioning method
I, O	Identity and null matrices

GENERATING UNIT

$\underset{\sim}{X}_g$	States associated with generator
v_d, v_q	Direct and quadrature axis components of machine terminal voltage
v_t	Magnitude of machine terminal voltage
i_d, i_q	Direct and quadrature axis stator currents of machine
i_{fd}, i_{kd}, i_{kq}	Rotor circuit currents of synchronous machine
ψ_d, ψ_q	Direct and quadrature axis armature flux linkages
ψ_{md}, ψ_{mq}	Direct and quadrature axis mutual flux linkages
ψ_{fd}, ψ_{kd}, ψ_{kq}	Field, direct axis amortisseur and quadrature axis amortisseur flux linkages
r_s	Stator resistance of synchronous machine

r_{fd}, r_{kd}, r_{kq}	Rotor circuit resistances of synchronous machine
$L_{s\ell}$	Stator leakage inductance
L_{md}, L_{mq}	Direct and quadrature axis mutual inductances
$L_{f\ell}$, L_{kd}, L_{kq}	Field, direct axis amortisseur and quadrature axis amortisseur leakage inductances
PX	Machine internal reactance matrix
T_e	Machine electrical torque
P_o	Machine output power
δ	Rotor angle
ω_o, ω	Synchronous and instantaneous angular frequencies

MECHANICAL SHAFT SYSTEM

$\underset{\sim}{X}_s$	States associated with mechanical shaft
H	Inertia constant
D	Damping coefficient
M	Inertia of the lumped mass element; $M = 2H/\omega_o$
S_{ij}	Stiffness between i^{th} and j^{th} lumped mass

TURBINE-GOVERNOR

$\underset{\sim}{X}_t$	States associated with turbine-governor
P_m	Output mechanical power
P_{ref}	Reference (control) power

Steam Unit

K_g	Speed sensor gain
τ_3	Speed sensor time constant
τ_{ch}	Turbine time constant

Hydro Unit

K	Speed relay gain
τ_1	Speed relay time constant
τ_3	Servomotor time constant
τ_5, τ_w	Time constants associated with turbine

EXCITATION SYSTEM

$\underset{\sim}{X}_e$	States associated with exciter-stabilizer
V_{fd}	Equivalent field voltage
V_{ref}	Reference voltage
V_v	Voltage sensor output
τ_v	Voltage sensor time constant
k_e, k_Q	Exciter and stabilizer gain
τ_e, τ_Q, τ_a, τ_x	Time constants associated with exciter, washout and lead lag circuit
V_a, V_b	Velocity and acceleration components of stabilizing signal
V_s	Stabilizing signal

NETWORK SYSTEM

R_t, X_t	Resistance and inductive reactance of a transmission line
PN, Y_{NN}	Nodal admittance matrix
V_D, V_Q	Direct and quadrature axis components of nodal voltage referred to the network reference frame
i_D, i_Q	Direct and quadrature axis components of nodal current referred to the network reference frame
n	Number of generating units in the system
T	Network transformation matrix

i_{DI}, i_{QI}	Direct and quadrature axis components of an infinite bus referred to the network reference

EIGENVALUES AND EIGENVALUE SENSITIVITIES

λ	System eigenvalue
$\hat{\lambda}$	Estimated eigenvalues
$\dot{\lambda}$	First-order eigenvalue sensitivity
$\ddot{\lambda}$	Second-order eigenvalue sensitivity
ξ	System parameters
V, W	Eigenvectors of the [A] matrix and its transpose

MISCELLANEOUS

Δ	Prescript denoting incremental change
$\cdot$	Subscript denoting differentiation with respect to time
$\sim$	Subscript denoting vector quantity
t	Subscript denoting matrix or vector transpose
-1	Subscript denoting matrix inverse
o	Subscript denoting equilibrium value
S	Laplace operator
Units:	All time constants in seconds, all angles in radians and other quantities are in per unit (p.u.)

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CHAPTER 1

INTRODUCTION

1.1 Power System Stability

One of the important considerations in the design and operation of a power system is the stability of the system. The stability problem is concerned with the behaviour of synchronous machines after they have been perturbed. If the perturbation does not involve any net change in power, the machines should return to their original state. If an unbalance between the supply and demand is created as a result of a change in load, in generation or in network conditions, a new operating condition should be achieved through different controllers of the system. In any case, all interconnected synchronous machines should remain in synchronism if the system is stable; (i.e., they should all remain operating in parallel and at the same speed).

Perturbations may be observed in different forms; one is a major disturbance such as the loss of a generator, a fault or the loss of a line, or a combination of these disturbances. A second form is a small disturbance such as a random or small load change occurring under normal operating conditions. Stability depends strongly upon the magnitude and location of the disturbance, and to a lesser extent on the initial operating condition of the system.

There are two main categories of power system stability. These are transient and dynamic stability. Transient stability is concerned with the behaviour of the system following a "major" or large disturbance which can arise as a result of an abnormal condition. Immediate loss of synchronism is generally of major concern affecting stability. The differential equations describing the dynamic performance of the system are nonlinear due mainly to the sinusoidal nature of the torque-load angle relationships. Saturation in the exciter, prime mover response and magnetic saturation are also significant factors in the nonlinearities [19]. The system behaviour after a "major" disturbance is a function of the nature of the fault and the system properties.

Dynamic stability is concerned with the behaviour of the system following a "small" disturbance around a steady-state operating condition. For a sufficiently small disturbance, linearized differential equations may be used to describe the system dynamics. These equations are derived by perturbing the nonlinear equations of the system around the equilibrium operating point. This admits the use of modern control theory concepts.

The dominant theme of this thesis is dynamic stability analysis.

1.2 Dynamic Stability Evaluation

A power system is a nonlinear system. However, for the purpose of investigating the "small signal" behaviour, the system equations may be linearized and analyzed by any of several methods applicable for linear systems. Methods used in conventional linear control theory, such as a Routh or Nyquist criteria for evaluating dynamic stability of power systems are restricted to the analysis of small systems such as a single machine infinite bus system. They are also of limited value in the analysis of systems having a wide range in frequency of oscillations [19]. However, the techniques of modern control theory have now removed this difficulty, subject to the requirement that the system can be described by a set of differential equations in the state space form.

The differential and algebraic equations describing the performance of a power system are basically nonlinear. System performance can be described by a set of first-order differential equations and associated algebraic relationship [7], [11], as shown in (1.1).

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{f}\ (\tilde{x}) + \tilde{g}\ (\tilde{u}) \\ \tilde{y} &= \tilde{h}\ (\tilde{x}) + \tilde{k}\ (\tilde{u}) \end{aligned} \tag{1.1}$$

where, the $\tilde{x}$, $\tilde{u}$ and $\tilde{y}$ are vectors of state, input and algebraic variables of order n, m and r, respectively; the f, g, h and k are vector functions [24], i.e.,

$$f(x) = \begin{bmatrix} f_1\ (x_1, x_2, \ldots, x_n) \\ \cdot \\ \cdot \\ \cdot \\ f_n\ (x_1, x_2, \ldots, x_n) \end{bmatrix}$$

When dealing with small disturbance stability of a system, equation (1.1) can be expressed in terms of deviations from the equilibrium point. If the disturbance is small enough, second-order and higher-order terms are negligible in a Taylor series expansion. Therefore, the equations will be described by the following linear form:

$$\begin{aligned} \dot{\underset{\sim}{\Delta x}} &= [A]\, \underset{\sim}{\Delta x} + [B]\, \underset{\sim}{\Delta u} \\ \underset{\sim}{\Delta y} &= [C]\, \underset{\sim}{\Delta x} + [D]\, \underset{\sim}{\Delta u} \end{aligned} \tag{1.2}$$

which is the standard state-space equation representation. [A], [B], [C] and [D] are real constant matrices with appropriate dimensions. The entries of these matrices depend on the system parameter values and also on the steady-state operating conditions.

The matrix [A] is called the coefficient or the state matrix and its elements a_{ij} are given by equation (1.3) which is evaluated at the equilibrium condition prior to the disturbance.

$$A = \frac{\partial f}{\partial x}\bigg|_{\underset{\sim}{x}_o} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{1.3}$$

The stability of the system is determined by computing the eigenvalues of the state matrix. The eigenvalues correspond to the natural modes of the system response and may be either complex or real [19]. A real eigenvalue is associated with a non-oscillatory mode. Complex eigenvalues always occur as conjugate pairs and each pair is associated with an oscillatory mode. The imaginary part of each pair represents the natural angular frequency of oscillation and the real part represents the amount of damping associated with the mode. A negative real part of a complex pair is an indication of a damped oscillatory mode, whereas a positive real part indicates instability through oscillations of increasing amplitude.

1.3 Formulation Approaches

For dynamic stability studies, the differential and algebraic equation sets of the system are manipulated into the state-space form (1.2). Then the state matrix [A] may be examined for stability using eigenvalue analysis. Different methods have been proposed for forming the [A] matrix.

Laughton [4] proposed a method of forming the [A] matrix for a multimachine power system by using a "direct elimination" technique to extract [A] from the complete differential and algebraic equations of the whole system, this name will be used for this general method.

Undrill [7] extended Laughton's method with more accurate generator, governor and exciter representation. His approach depends on building up the matrix [A] of the multi-machine power system from submatrices representing system segments and thus large blocks of null elements can be avoided.

Anderson [11], [14] extended the approach of Enns et al [2] and represented the differential and algebraic equations of the system in a linearized form, as shown in (1.4) and termed by him the "PQR" method.

$$[P] \begin{bmatrix} \Delta\dot{\underset{\sim}{x}} \\ \\ \Delta\underset{\sim}{y} \end{bmatrix} = [Q]\ \Delta\underset{\sim}{x} + [R]\ \Delta\underset{\sim}{u} \tag{1.4}$$

where, $\Delta\underset{\sim}{x}$, $\Delta\underset{\sim}{y}$ and $\Delta\underset{\sim}{u}$ are the state, algebraic and input vectors of perturbations from the steady-state equilibrium point and are of the same dimensions as those of equation (1.2). [P], [Q] and [R] are real constant matrices of compatible dimensions with $\Delta\underset{\sim}{x}$, $\Delta\underset{\sim}{y}$ and $\Delta\underset{\sim}{u}$ vectors. The state-space form could be obtained from equation (1.4) using a matrix inversion routine, it will be discussed in Chapter 3.

Alden and Zein El-Din [28] combined the simplicity of the PQR technique with the efficiency of the submatrix build-up technique while retaining the identity between submatrices and system components.

1.4 Arrangement of the Material

In Chapter 2, a representation of the nonlinear equations and the linearized state-space equations of one model for each subsystem will be introduced as an example.

In Chapter 3, the details of a computer program are presented. It is used in building up the coefficient matrices A, B and C for multimachine dynamic stability studies taking into account several models for each subsystem. This program is based on the basic idea of using the combined PQR and submatrix build up technique.

A second computer program is presented in Chapter 4 to form the matrix [A], based on the "direct elimination" technique.

In Chapter 5, both of these computer programs have been applied to two test examples to illustrate their validity. Detailed comparison between these two programs and extensive analysis of both algorithms have been done from the point of view of computational time, storage requirements and eigenvalue sensitivity evaluation. In Chapter 6, the main conclusions of the thesis are summarized.

CHAPTER 2

SUBSYSTEM MODELLING

2.1 Introduction

The power system, shown in Figure 2.1, consists of two major subsystems. The first one is the electric network which can be divided into two sections: the transmission or bulk power system, and the distribution system. The second major subsystem comprises mutually uncoupled generating units. The input to this subsystem is the vector of stator voltages, and the output is the vector of stator currents. Each generating unit consists of four elements, as shown in Figure 2.1: generator, mechanical shaft, turbine-governor and exciter-stabilizer. By appropriate choice of generating unit subsystems, a wide variety of model types and complexities may be considered. Table 2.1 shows the different subsystem models used in this thesis. In this chapter, one model for each element will be discussed and the other models are presented in Appendix A.

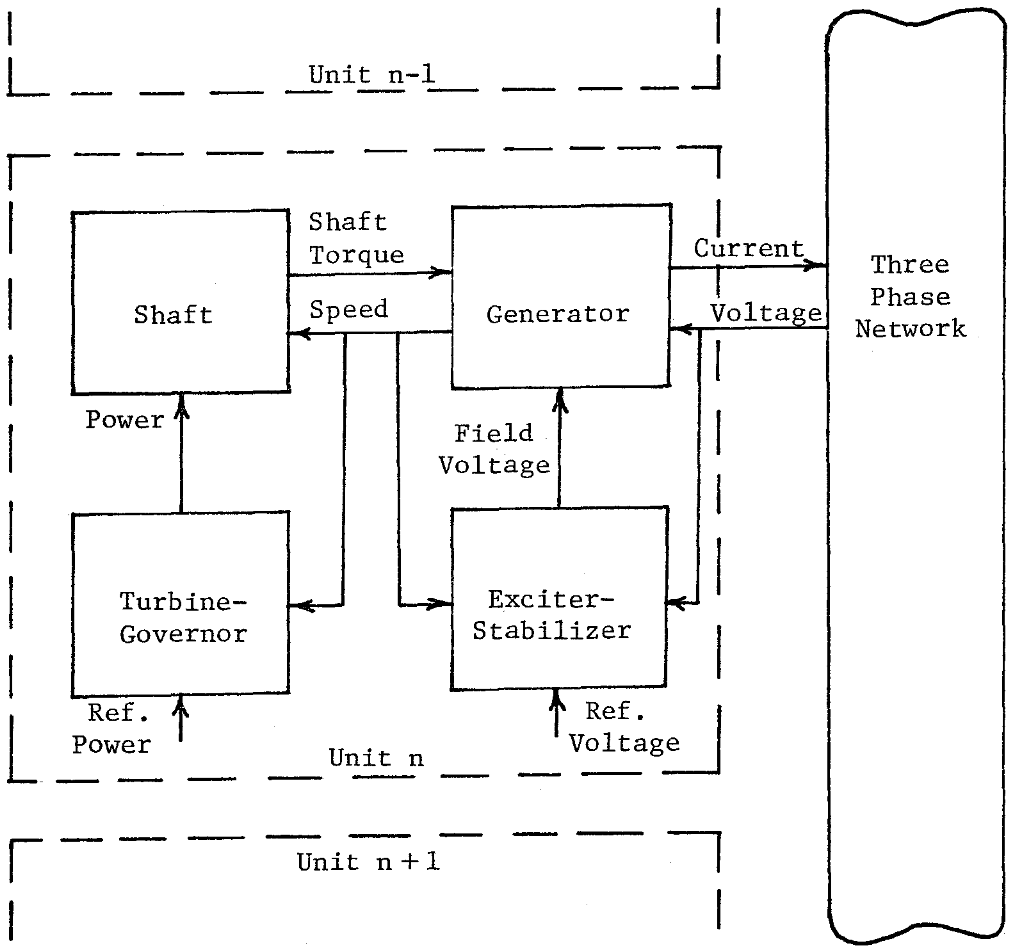


Figure 2.1 System Structure

Table 2.1 Subsystem Models

Element	Type	Classification	Order
Generator	G_o	classical model	0
	G_1	one rotor circuit (no damper windings)	1
	G_2	three rotor circuits (two damper windings)	3
	G_3	three rotor circuits (two damper windings + stator transient included)	5
System Shaft	S_1	lumped mass	2
	S_2	two mass model	4
	S_3	five mass model	10
Turbine-Governor	T_o	no turbine (constant mechanical power)	0
	T_1	steam turbine	2
	T_2	hydraulic turbine	3
System Exciter	E_o	no exciter (constant field voltage)	0
	E_1	simple exciter	1
	E_2	static exciter	2
	E_3	static exciter with speed-stabilizer	4

2.2 Synchronous Machine Model

Nonlinear Model Including Stator Transients

This model is used in the case studies where the effects of the d.c. offset in the stator circuits are important, hence, the stator transients must be retained [19]. The equation set no. (2.1) are based on Park's transformations describing the nonlinear performance of a synchronous machine in a reference frame rotating with the rotor. The equivalent circuit representing this model is shown in Figure (2.2a).

Stator Equations:

$$\dot{\psi}_d = \omega_o \left(V_d + \frac{\omega}{\omega_o}\,\psi_q + r_s\, i_d\right)$$

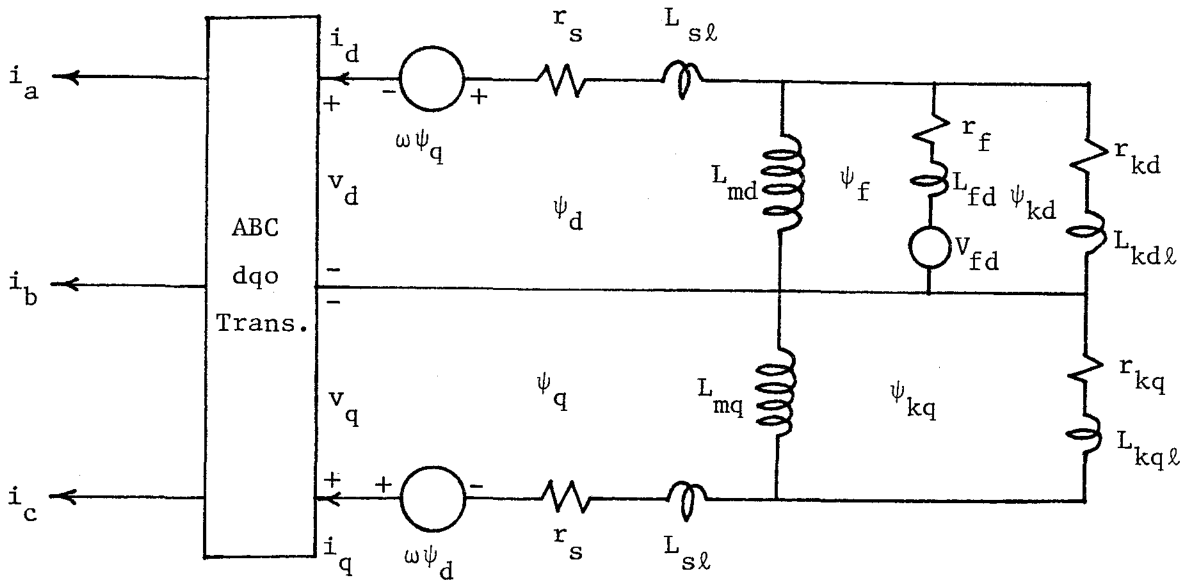
$$\dot{\psi}_q = \omega_o \left(V_q - \frac{\omega}{\omega_o}\,\psi_d + r_s\, i_q\right) \tag{2.1a}$$

Rotor Equations:

$$\dot{\psi}_{fd} = \omega_o \left(V_{fd} - r_{fd}\, i_{fd}\right)$$

$$\dot{\psi}_{kd} = -\,\omega_o \left(r_{kd}\, i_{kd}\right) \tag{2.1b}$$

$$\dot{\psi}_{kq} = -\,\omega_o \left(r_{kq}\, i_{kq}\right)$$



(a) Equivalent Circuit

$$\begin{bmatrix} \Delta\dot{\psi}_{fd} \\ \Delta\psi_d \\ \Delta\psi_{kd} \\ \Delta\psi_q \\ \Delta\psi_{kq} \end{bmatrix} = \begin{bmatrix} \omega_o r_f & & & & \\ & -\omega_o r_s & & & \\ & & \omega_o r_{kd} & & \\ & & & -\omega_o r_s & \\ & & & & \omega_o r_{kq} \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_d \\ \Delta i_{kd} \\ \Delta i_q \\ \Delta i_{kg} \end{bmatrix} + \begin{bmatrix} & \\ \omega_o & \\ & \\ & \omega_o \\ & \end{bmatrix} \begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} +$$

$$\begin{bmatrix} & & & & \\ & & & \omega_o & \\ & & & & \\ & -\omega_o & & & \\ & & & & \end{bmatrix} \begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_d \\ \Delta\psi_{kd} \\ \Delta\psi_{kq} \\ \Delta\psi_{kg} \end{bmatrix} + \begin{bmatrix} & \omega_o \\ \psi_{qo} & \\ & \\ -\psi_{do} & \\ & \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta V_{fd} \end{bmatrix}$$

(b) State-Space Equations

Figure 2.2 Synchronous Machine "G_3" Model

The stator and rotor current components used in equation (2.1) are presented in equation set no. (2.2) in terms of the flux linkages.

$$i_d = \frac{\psi_{md} - \psi_d}{L_{s\ell}} \qquad i_q = \frac{\psi_{mq} - \psi_q}{L_{s\ell}}$$

$$i_{fd} = \frac{\psi_{fd} - \psi_{md}}{L_{f\ell}} \qquad i_{kd} = \frac{\psi_{kd} - \psi_{md}}{L_{kd}} \tag{2.2}$$

$$i_{kq} = \frac{\psi_{kq} - \psi_{mq}}{L_{kq}}$$

The air-gap flux linkages are given by:

$$\psi_{md} = k_1 \left(\frac{\psi_d}{L_{s\ell}} + \frac{\psi_{fd}}{L_{f\ell}} + \frac{\psi_{kd}}{L_{kd\ell}}\right)$$

$$\psi_{mq} = k_2 \left(\frac{\psi_q}{L_{s\ell}} + \frac{\psi_{kq}}{L_{kq\ell}}\right) \tag{2.3}$$

where:

$$\psi_d = X_{md}\, i_f - X_d\, i_d + X_{md}\, i_{kd}$$

$$\psi_q = -X_q\, i_q + X_{mq}\, i_{kq}$$

$$\psi_{fd} = X_f\, i_{fd} - X_{md}\, i_d + X_{md}\, i_{kd} \tag{2.4}$$

$$\psi_{kd} = X_{md}\, i_f - X_{md}\, i_d + X_{kd}\, i_{kd}$$

$$\psi_{kq} = -X_{mq}\, i_q + X_{kq}\, i_{kq}$$

$$\frac{1}{k_1} = \frac{1}{L_{s\ell}} + \frac{1}{L_{md}} + \frac{1}{L_{f\ell}} + \frac{1}{L_{kd\ell}}$$

$$\frac{1}{k_2} = \frac{1}{L_{s\ell}} + \frac{1}{L_{mq}} + \frac{1}{L_{kq\ell}}$$

The state variables in equations (2.1 - 2.4) are the flux linkages. The flux linkage state-space model is convenient for studying the effect of magnetic saturation causing nonlinearity, as all the terms of the model equations are linear except for the magnetizing flux linkages ψ_{md} and ψ_{mq}. These flux linkages are affected by saturation of the mutual inductances L_{md} and L_{mq} and only these terms need to be corrected for saturation. This can be done by computing a saturation function to adjust (2.3) at all times to reflect the state of the mutual inductances [31]. Practically, the q-axis inductance L_{mq} seldom saturates, so it is usually necessary to adjust only L_{md} for saturation using the saturation curve shown in Figure (2.3).

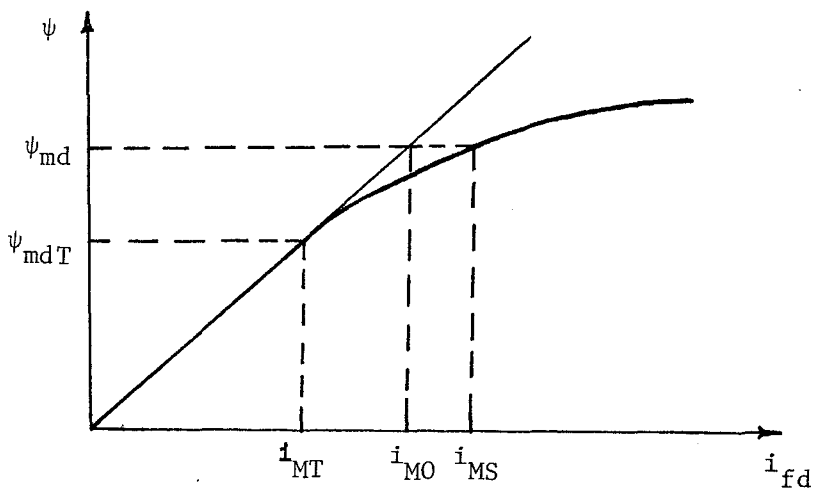


Figure 2.3 Saturation Curve for ψ_{md}

The procedure for including the magnetic circuit saturation for salient pole machines is as follows:

(a) From the saturation curve, the threshold flux linkage (before saturation), ψ_{mdT}, which corresponds to a magnetizing current i_{MT} is determined.

(b) For a given ψ_{md}, the unsaturated magnetizing current i_{Mo}, corresponding to L_{mdo} is determined.

(c) For a flux linkage greater than ψ_{mdT}, the current increment $i_{M\Delta}$ is calculated.

$$i_{M\Delta} = A_s \exp [B_s (\psi_{md} - \psi_{mdT})]$$

where, A_s and B_s are constants to be determined from the generator saturation curve.

(d) The saturated current i_{Ms} can be evaluated as follows:

$$i_{Ms} = i_{Mo} + i_{M\Delta}$$

(e) The saturated value of the magnetizing inductance L_{md} will be:

$$L_{md} = K_s L_{mdo}$$

where:

$$K_s \text{ (saturation function)} = \frac{i_{Mo}}{i_{Ms}}$$

Other forms of nonlinearities beside the magnetic saturation are the product nonlinearities and trigonometric functions [31].

Product Nonlinearities

Considering the state variables X_i and X_j having the initial values X_{io} and X_{jo}, and $X_{i\Delta}$, $X_{j\Delta}$ are the small changes of these variables.

The new value of their product will be:

$$(X_{io} + X_{i\Delta})\ (X_{jo} + X_{j\Delta}) = X_{io}\ X_{jo} + X_{io}\ X_{j\Delta} + X_{jo}\ X_{i\Delta} + X_{i\Delta}\ X_{j\Delta}$$

It is seen from this equation that the last term, $X_{i\Delta}\ X_{j\Delta}$, causes nonlinearity. Since this term is very small, so it could be neglected. Thus, for a first-order approximation, the change in the product $(X_i X_j)$ is given by:

$$(X_{io} + X_{j\Delta})\ (X_{jo} + X_{j\Delta}) - X_{io}\ X_{jo} = X_{io}\ X_{j\Delta} + X_{jo}\ X_{i\Delta}$$

where, X_{io} and X_{jo} are known quantities and treated as coefficients, while $X_{i\Delta}$ and $X_{j\Delta}$ are "increment" variables.

The Trigonometric Nonlinearities

This type of nonlinearity is treated in a form where the expansion of the function is used, as follows:

$$\cos\ (\delta_o + \delta_\Delta) = \cos\ \delta_o\ \cos\ \delta_\Delta - \sin\ \delta_o\ \sin\ \delta_\Delta$$

with $\cos\ \delta_\Delta \simeq 1$ and $\sin\ \delta_\Delta \simeq \delta_\Delta$. Therefore,

$$\cos\ (\delta_o + \delta_\Delta) - \cos\ \delta_o \simeq (-\sin\ \delta_o)\ \delta_\Delta$$

The increment change in cos δ is then $(-\sin\delta_o)\delta_\Delta$, the incremental variable is δ_Δ and its coefficient is $(-\sin\delta_o)$. Similarly, for the incremental change in the term $(\sin\delta)$ is given by:

$$\sin(\delta_o + \delta_\Delta) = \sin\delta_o \cos\delta_\Delta + \cos\delta_o \sin\delta_\Delta$$

or,
$$\sin(\delta_o + \delta_\Delta) - \sin\delta_o = (\cos\delta_o)\,\delta_\Delta$$

Linearized Equations

The different techniques adopted for developing the linearized state-space equations in all approaches are basically similar. The nonlinear differential and algebraic equations of each subsystem model are linearized around an operating condition, then the overall system equations are formulated. The steady-state equilibrium condition of the overall system is usually obtained using a load flow program [35]. The linearized equations of the differential equations (2.1) and the algebraic equations (2.4) are:

Stator Equations

$$\begin{aligned} \Delta\dot{\psi}_d &= \omega_o\Delta V_d + \omega_o\Delta r_s \cdot \Delta i_d + \psi_{qo} \cdot \Delta\omega + \omega_o \cdot \Delta\psi_q \\ \Delta\dot{\psi}_q &= \omega_o\Delta V_q + \omega_o\Delta r_s \cdot \Delta i_q - \psi_{do} \cdot \Delta\omega - \omega_o \cdot \Delta\psi_d \end{aligned} \tag{2.5}$$

Rotor Equations

$$\begin{aligned} \Delta\dot{\psi}_{fd} &= \omega_o \cdot \Delta V_{fd} - \omega_o r_{fd} \cdot \Delta i_{fd} \\ \Delta\dot{\psi}_{kd} &= -\omega_o r_{kd} \cdot \Delta i_{kd} \\ \Delta\dot{\psi}_{kq} &= -\omega_o r_{kq} \cdot \Delta i_{kq} \end{aligned} \tag{2.6}$$

Algebraic Equations

$$
\begin{aligned}
\Delta\psi_d &= X_{md}\,\Delta i_{fd} - X_d\Delta i_d + X_{md}\Delta i_{kd} \\
\Delta\psi_q &= -X_q\Delta i_q + X_{mq}\Delta i_{kq} \\
\Delta\psi_{fd} &= X_{fd}\Delta i_{fd} - X_{md}\Delta i_d + X_{md}\Delta i_{kd} \qquad (2.7) \\
\Delta\psi_{kd} &= X_{md}\Delta i_{fd} - X_{md}\Delta i_d + X_{kd}\Delta i_{kd} \\
\Delta\psi_{kq} &= -X_{mq}\Delta i_q + X_{kq}\Delta i_{kq}
\end{aligned}
$$

The linearized state-space form is represented in Figure (2.2b).

2.3 Mechanical System Model

Nonlinear Model Including the Effects of Torsional Vibrations of the Mechanical System

The effects of turbine-generator torsional vibration effects are considered in power system analyses and design. One of the possibilities to have torsional oscillations is when a feed back of rotor speed to the excitation system is used for damping power angle oscillations. Another possibility causing the occurrence of such a problem is when series capacitors are used to compensate long-distance high-voltage transmission lines; this could introduce potential modes of dynamic instability and may include interactions with turbo-alternator shaft oscillations. Analytical methods used to predict torsional instability require the modelling of the mechanical system dynamics. The following equation set no. (2.8) describes the dynamic performance of the turbo-alternator mechanical system which considers the shaft system as a five-mass system [18], [19], as shown in Figure (2.4a).

Generator:

$$\dot{\delta}_1 = \omega_1 - \omega_o$$

$$\dot{\omega}_1 = \frac{S_{12}}{M_1}\delta_2 + \frac{(-S_{12})}{M_1}\delta_1 - \frac{D_1}{M_1}\omega_1 - \frac{1}{M_1}T_e$$

LP_B:

$$\dot{\delta}_2 = \omega_2 - \omega_o$$

$$\dot{\omega}_2 = \frac{S_{23}}{M_2}\delta_3 - \frac{(S_{23}+S_{12})}{M_2}\delta_2 + \frac{S_{12}}{M_2}\delta_1 - \frac{D_2}{M_2}\omega_2 + \frac{1}{M_2}P_{LP}$$

LP_A:

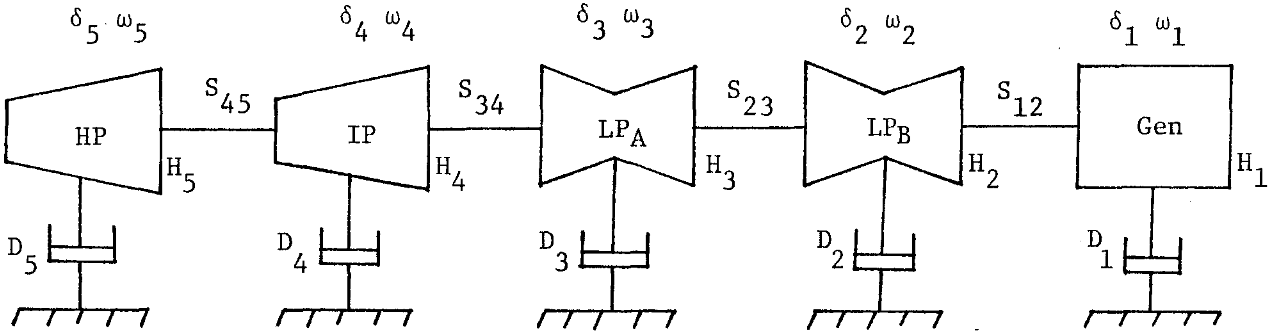
$$\dot{\delta}_3 = \omega_3 - \omega_o \tag{2.8}$$

$$\dot{\omega}_3 = \frac{S_{34}}{M_3}\delta_4 - \frac{(S_{34}+S_{23})}{M_3}\delta_3 + \frac{S_{23}}{M_3}\delta_2 - \frac{D_3}{M_3}\omega_3 + \frac{1}{M_3}P_{LP}$$

IP:

$$\dot{\delta}_4 = \omega_4 - \omega_o$$

$$\dot{\omega}_4 = \frac{S_{45}}{M_4}\delta_5 - \frac{(S_{45}+S_{34})}{M_4}\delta_4 + \frac{S_{34}}{M4}\delta_3 - \frac{D_4}{M_4}\omega_4 + \frac{1}{M_4}P_{IP}$$



HP = High Pressure Stage
IP = Intermediate Pressure stage
LP = Low Pressure Stages

H = Inertia Constant (sec)
D = Damping Factor (P.U. Torque/Radian/Sec)
S = Shaft Stiffness Factor (P.U. Torque/Radian)

(a) Equivalent Mechanical System

Figure 2.4 Mechanical Shaft System "S_3" Model

H.P.:

$$\dot{\delta}_5 = \omega_5 - \omega_o$$

$$\dot{\omega}_5 = \frac{S_{45}}{M_5} \delta_4 - \frac{S_{45}}{M_5} \delta_5 - \frac{D_5}{M_5} \omega_5 + \frac{1}{M_5} P_{HP}$$

where,

$$T_e = \psi_d i_q - \psi_q i_d$$

The linearized state-space equations of this model are shown in Figure (2.4b).

The above model is used for thermal generating units where the generator rotor and each turbine stage is represented by one equivalent rotating mass [18]. Such representation is sufficiently accurate for the prediction of the lower shaft natural frequencies (below 60 hertz) at which torsional sub-synchronous resonance occurs. For hydraulic turbines (shown in Appendix A), a two-mass equivalent system is considered adequate, one mass corresponding to the rotor inertia and the other representing the turbine inertia.

A single-mass equivalent may be employed in applications involving many stability predictions in which the stability of a specific shaft mode is required to be analyzed. This model is shown in Appendix A.

$$
\left[\begin{array}{c}
\Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \hline
\Delta\omega_1 \\ \Delta\omega_2 \\ \Delta\omega_3 \\ \Delta\omega_4 \\ \Delta\omega_5
\end{array}\right]
=
\left[\begin{array}{ccccc|ccccc}
 & & & & & \omega_o & & & & \\
 & & & & & & \omega_o & & & \\
 & & & & & & & \omega_o & & \\
 & & & & & & & & \omega_o & \\
 & & & & & & & & & \omega o \\ \hline
\frac{-S_{12}}{M_1} & \frac{S_{12}}{M_1} & & & & \frac{-D_1}{M_1} & & & & \\
\frac{S_{12}}{M_2} & \frac{-(S_{23}+S_{12})}{M_2} & \frac{S_{23}}{M_2} & & & & \frac{-D_2}{M_2} & & & \\
 & \frac{S_{23}}{M_3} & \frac{-(S_{34}+S_{23})}{M_3} & \frac{S_{34}}{M_3} & & & & \frac{-D_3}{M_3} & & \\
 & & \frac{S_{34}}{M_4} & \frac{-(S_{45}+S_{34})}{M_4} & \frac{S_{45}}{M_4} & & & & \frac{-D_4}{M_4} & \\
 & & & \frac{S_{45}}{M_5} & \frac{-S_{45}}{M_5} & & & & & \frac{-D_5}{M_5}
\end{array}\right]
\left[\begin{array}{c}
\Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \hline
\Delta\omega_1 \\ \Delta\omega_2 \\ \Delta\omega_3 \\ \Delta\omega_4 \\ \Delta\omega_5
\end{array}\right]
+
\left[\begin{array}{c}
 \\ \\ \\ \\ \\ \hline
\frac{-1}{M_1} \\ \\ \\ \\ \\
\end{array}\right]
\Delta T_e
+
\left[\begin{array}{ccc}
 & & \\ & & \\ & & \\ & & \\ & & \\ \hline
 & & \\
 & & \frac{1}{M_2} \\
 & & \frac{1}{M_3} \\
 & \frac{1}{M_4} & \\
\frac{1}{M_5} & &
\end{array}\right]
\left[\begin{array}{c}
P_{HP} \\ P_{IP} \\ P_{LP}
\end{array}\right]
$$

$$\Delta T_e = \psi_{do}\,\Delta i_q - \psi_{qo}\,\Delta i_d + i_{qo}\,\Delta\psi_d - i_{do}\,\Delta\psi_q$$

(b) State-Space Equations

Figure 2.4 Mechanical Shaft System "S_3" Model

2.4 Turbine/Governor Model

Nonlinear Model for Hydraulic Systems

In this section, a simplified model for a hydro turbine-governor subsystem is described [18]. Another model which is shown in Appendix A represents a simplified model for steam units [18]. The turbine-governor model for the steam unit is recommended also for nuclear units.

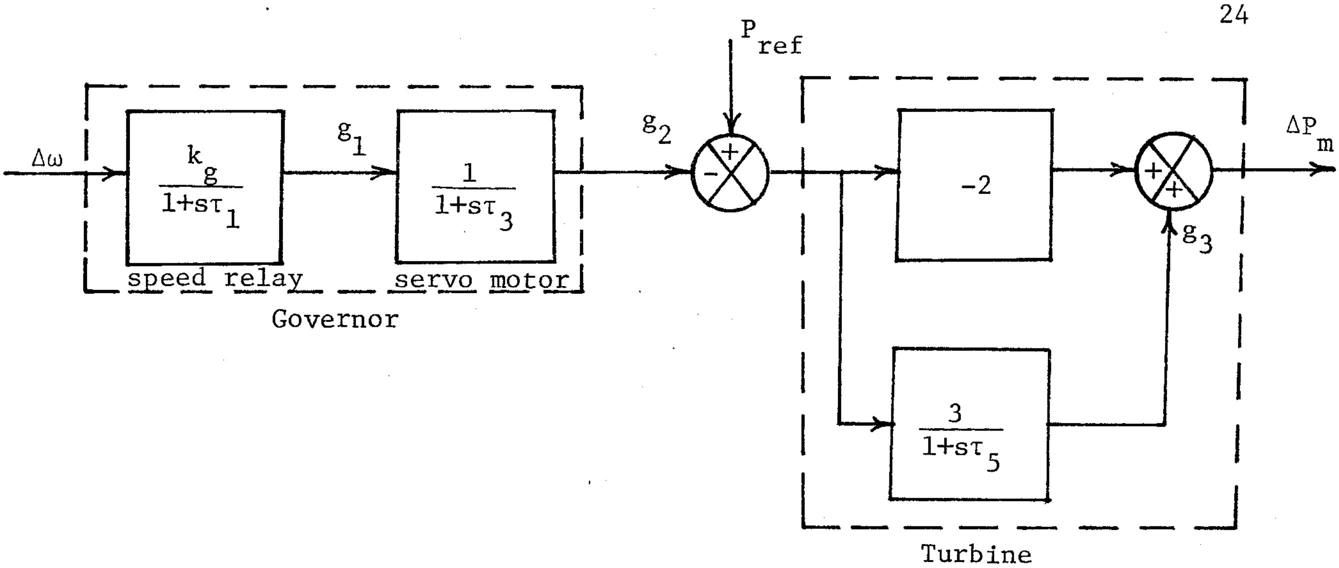
The dynamic model for a hydro turbine-governor subsystem is shown in Figure (2.5a). The turbine representation in Figure (2.5a) is an equivalent for the block diagram description in Figure (2.5b). The governor model includes two different transfer functions representing the speed relay and servo motor. The nonlinear differential and algebraic equations representing this model are as follows:

Differential Equations

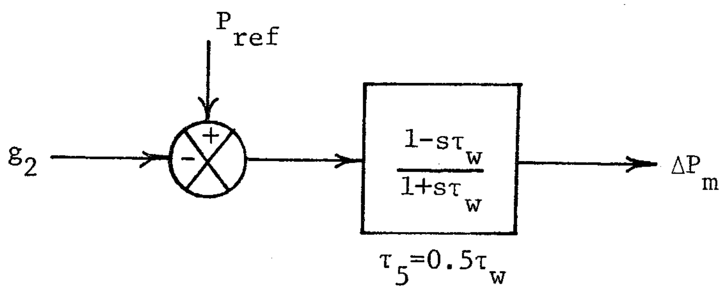
$$
\begin{aligned}
\dot{g}_1 &= \frac{-1}{\tau_1} g_1 + \frac{k_g}{\tau_1} \omega \\
\dot{g}_2 &= \frac{-1}{\tau_3} g_2 + \frac{1}{\tau_3} g_1 \\
\dot{g}_3 &= \frac{-1}{\tau_5} g_3 - \frac{3}{\tau_5} g_2 + \frac{3}{\tau_5} P_{ref}
\end{aligned}
\qquad (2.9)
$$

Algebraic Equation:

$$P_m = g_3 - 2\,(P_{ref} - g_2) \qquad (2.10)$$



(a) Block Diagram



(b) Turbine Transfer Function

$$\overset{\bullet}{\begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}} \begin{bmatrix} \frac{-1}{\tau_1} & & \\ \frac{1}{\tau_3} & \frac{-1}{\tau_3} & \\ & \frac{-3}{\tau_5} & \frac{-1}{\tau_5} \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} + \begin{bmatrix} \frac{k_g}{\tau_1} \\ \\ \\ \end{bmatrix} \Delta\omega + \begin{bmatrix} \\ \\ \frac{3}{\tau_5} \end{bmatrix} P_{ref}$$

$$\Delta P_m = \Delta g_3 - 2\ (P_{ref} - \Delta g_2)$$

(c) Linearized State-Space Equations

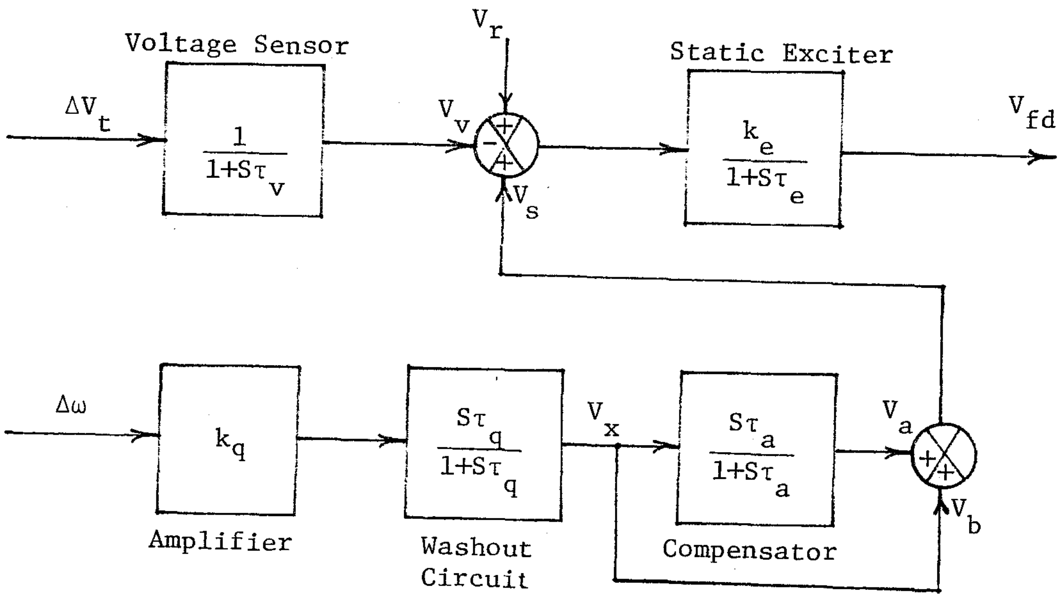
Figure 2.5 Turbine-Governor "T_2" Model

2.5 Excitation System Model

Static Exciter with Speed Stabilizer Model

In this section, the static exciter with speed stabilizer model is presented. Another model which is a simple exciter is presented in Appendix A. The static exciter has a very powerful capability which can be used very effectively to control power system swings [16]. It has also the advantage that there is no exciter saturation as in rotating exciters. This saturation is a result of the nonlinear relation between the exciter field voltage and the exciter field current in rotating exciters. To correct for nonlinearity in rotating exciters, a similar approach to that used in overcoming the magnetic saturation nonlinearities in a generator is used. A device for producing a signal proportional to small changes in generator speed has been developed by Ontario Hydro. It has been found that a stabilizing signal based on direct measurement of shaft speed has the advantage of being virtually independent of system configuration and operating procedures [15]. This will provide satisfactory damping of the generator oscillations.

In Figure (2.6a), a block diagram description for the static exciter-speed stabilizer, using a signal derived from machine rotor speed, is shown. This model is used by Ontario Hydro [15]. In this model, the exciter is represented by a single time constant transfer function, the inputs are the stabilizing signal (v_s) and the difference (v_r - v_v) between the reference voltage (v_r) and the signal corres-



(a) Block Diagram

$$
\begin{bmatrix} \dot{V}_v \\ V_{fd} \\ V_x \\ V_y \end{bmatrix}
=
\begin{bmatrix}
\frac{-1}{\tau_v} & & & \\
\frac{-k_e}{\tau_e} & \frac{-1}{\tau_e} & \frac{k_e T_c}{\tau_e} & \frac{k_e}{\tau_e} \\
 & & \frac{-1}{\tau_q} & \\
 & & \frac{-\tau_a}{\tau_x^2} & \frac{-1}{\tau_x}
\end{bmatrix}
\begin{bmatrix} V_v \\ V_{fd} \\ V_x \\ V_y \end{bmatrix}
+
\begin{bmatrix} \frac{1}{\tau_v} \\ \\ \\ \\ \end{bmatrix} \Delta v_t
+
\begin{bmatrix} \\ \frac{k_e k_q T_c}{\tau_e} \\ \frac{-k_q}{\tau_q} \\ \frac{-\tau_a k_q}{\tau_x^2} \end{bmatrix} \Delta\omega
+
\begin{bmatrix} \\ \frac{k_e}{\tau_e} \\ \\ \\ \end{bmatrix} v_r
$$

$$
T_c = \frac{\tau_a}{\tau_x} + 1
$$

(b) Linearized State-Space Equations

Figure 2.6 Static Exciter-Speed Stabilizer "E_3" Model ponding to the machine terminal voltage (v_v). The washout circuit eliminates any steady-state offset of the speed signal into the exciter input. The compensator is used for cancellation of phase lag contributed by the machine and exciter. The differential equations describing the performance of this model are shown in (2.11).

$$\begin{aligned}
\dot{v}_v &= \frac{1}{\tau_v} v_t - \frac{1}{\tau_v} v_v \\
\dot{v}_{fd} &= \frac{K_e K_q}{\omega_o \tau_e} \left(\frac{\tau_a}{\tau_x} + 1\right) \omega - \frac{K_e}{\tau_e} v_v - \frac{1}{\tau_e} v_{fd} + \frac{K_e}{\tau_e} \left(\frac{\tau_e}{\tau_x} + 1\right) v_x + \\
&\quad \frac{K_e}{\tau_e} v_y + \frac{K_e}{\tau_e} v_r \\
\dot{v}_x &= \frac{-K_q}{\omega_o \tau_q} \omega - \frac{1}{\tau_q} v_x \\
\dot{v}_y &= \frac{-\tau_a K_q}{\omega_o \tau_x^2} \omega - \frac{\tau_a}{\tau_x^2} v_x - \frac{1}{\tau_x} v_y
\end{aligned} \qquad (2.11)$$

where,

$$v_y = v_a - \frac{\tau_a}{\tau_x} v_b$$

The linearized state-space equations are presented in Figure (2.6b)

CHAPTER 3

"PQR" FORMULATION

3.1 Matrix Formulation

In dynamic stability (small signal) studies of power systems, it is useful to manipulate the linearized differential and algebraic equation sets describing the performance of the system into the state-space linear form, equation (3.1).

$$\begin{aligned} \Delta\dot{\underset{\sim}{x}} &= [A]\,\Delta\underset{\sim}{x} + [B]\,\Delta\underset{\sim}{u} \\ \Delta\underset{\sim}{y} &= [C]\,\Delta\underset{\sim}{x} + [D]\,\Delta\underset{\sim}{u} \end{aligned} \tag{3.1}$$

where Δx, Δu, and Δy are vectors of state, input, and algebraic variables of order n, m, and r, respectively. These vectors are considered the vectors of perturbation from steady-state equilibrium point. The matrices, A, B, and C are real constant matrices with appropriate dimensions. The entries of these matrices are functions of all the system parameters. The state-space form, equation (3.1), is convenient for the applications of modern control theory.

For a small problem such as a single machine-infinite bus, the number of the differential and algebraic equations describing the system performance is relatively small. The reduction of these equations into state-space form is simple. However, for interconnected systems, it is difficult and a systematic reduction technique should be used.

Enns et al, [2], suggested a systematic formulation technique which has been extended by Anderson [11] and [14], where the method was termed by him the PQR method. The linearized differential and algebraic equations of the system are formulated in the following form:

$$[P] \begin{bmatrix} \Delta \dot{\underset{\sim}{x}} \\ \\ \Delta \underset{\sim}{y} \end{bmatrix} = [Q] \; \Delta \underset{\sim}{x} + [R] \; \Delta \underset{\sim}{u} \tag{3.2}$$

The matrices P, Q, and R are real constant matrices of compatible dimensions with the vectors $\Delta \underset{\sim}{x}$, $\Delta \underset{\sim}{u}$, and $\Delta \underset{\sim}{y}$. These matrices are functions of the system structure and the steady-state operating conditions. These matrices are formed within the digital computer. Equation (3.2) is then premultiplied by the inverse of the matrix [P] to give:

$$\begin{bmatrix} \Delta \dot{\underset{\sim}{x}} \\ \\ \Delta \underset{\sim}{y} \end{bmatrix} = [P]^{-1} \; [Q] \; \Delta \underset{\sim}{x} + [P]^{-1} \; [R] \; \Delta \underset{\sim}{u} \tag{3.3}$$

If the matrices $[P^{-1} \; Q]$ and $[P^{-1} \; R]$ in equation (3.3) are conformably partitioned [28], then the system equations are obtained in the state-space form (3.1). For a large system, the inversion time is relatively long and has to be performed every time a parameter setting is changed.

Undrill [7], recommended a procedure for computing a multi-machine model which represented each generator with order 5 and required a matrix inversion of order 11n, where n is the number of machines. On the other hand, the formulation recommended by Anderson [14] required the inversion of n matrices of order 15 to produce the same model. He used a modular approach which replaces the inversion of one large matrix by the inversion of a number of a lower order.

The approach suggested by Alden and Zein El-Din [28], avoided the inversion of a large matrix by ordering the state, algebraic and output variables of each individual machine in such a way as to set up the [P] matrix in a quasi-block diagonal form. The procedure developed requires the inversion of n machine reactance matrices of order 5 and the second one is the real network admittance matrix, of order 2n. If the network impedance matrix is developed instead of the admittance matrix, no inversion is needed for the last matrix.

In the last approach [28] equation (3.2) is rewritten after partitioning the matrices P, Q and R, as follows:

$$\left[\begin{array}{c|c} I & PA \\ \hline 0 & PB \end{array}\right] \left[\begin{array}{c} \Delta\dot{\underset{\sim}{x}} \\ \hline \Delta\underset{\sim}{y} \end{array}\right] = \left[\begin{array}{c} QA \\ \hline QB \end{array}\right] \Delta\underset{\sim}{x} + \left[\begin{array}{c} RA \\ \hline RB \end{array}\right] \Delta\underset{\sim}{u} \qquad (3.4)$$

The state-space form (3.1) can be obtained by matrix manipulation described in section (3.5) using inversion by parts to invert the matrix [PB].

A computer program which is a part of the interactive McMaster University Multi-Machine Analysis System (MUMMAS) package has been produced for building up the coefficient matrices A, B and C for multi-machine dynamic stability studies taking into account several models for each subsystem. This program is based on the basic idea of using the combined PQR and sub-matrix build-up technique [28].

3.2 Network Formulation

The network can be described by the nodal admittance matrix equation [1]:

$$
\left[\begin{array}{c}
i_{D1} \\ i_{Q1} \\ \hline
i_{D2} \\ i_{Q2} \\ \hline
\vdots \\ \hline
i_{Dn} \\ i_{Qn}
\end{array}\right]
=
\left[\begin{array}{cc|cc|c|cc}
g_{11} & -b_{11} & g_{12} & -b_{12} & \cdots & g_{1n} & -b_{1n} \\
b_{11} & g_{11} & b_{12} & g_{12} & \cdots & b_{m} & g_{m} \\ \hline
g_{12} & -b_{12} & g_{22} & -b_{22} & \cdots & g_{2n} & -b_{2n} \\
b_{21} & g_{21} & b_{22} & g_{22} & \cdots & b_{2n} & g_{2n} \\ \hline
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \hline
g_{n1} & -b_{n1} & g_{n2} & -b_{n2} & \cdots & g_{nn} & -b_{nn} \\
b_{n1} & g_{n1} & b_{n2} & g_{n2} & \cdots & b_{nn} & g_{nn}
\end{array}\right]
\left[\begin{array}{c}
v_{D1} \\ v_{Q1} \\ \hline
v_{D2} \\ v_{Q2} \\ \hline
\vdots \\ \hline
v_{Dn} \\ v_{Qn}
\end{array}\right]
\qquad (3.5)
$$

Equation (3.5) is the result of expanding a set of n simultaneous complex equations into a set of 2n real equations.

This equation can be written symbolically as:

$$
\underset{\sim}{i}_N = [Y_{NN}]\, \underset{\sim}{v}_N \qquad (3.6)
$$

Each load is represented in this approach as a linear static load. Hence, they are combined in the bus admittance matrix as a constant admittance. This is achieved [35] by eliminating all non-generator buses which are connected only to a linear static load.

The components of the terminal voltage of a synchronous machine with respect to its direct and quadrature (d, q) reference axes (which rotate in synchronism with the machine rotor) are related to the components in the D, Q reference frame of the network (which rotates at the angular frequency of the steady-state network current), as shown in Figure (3.1).

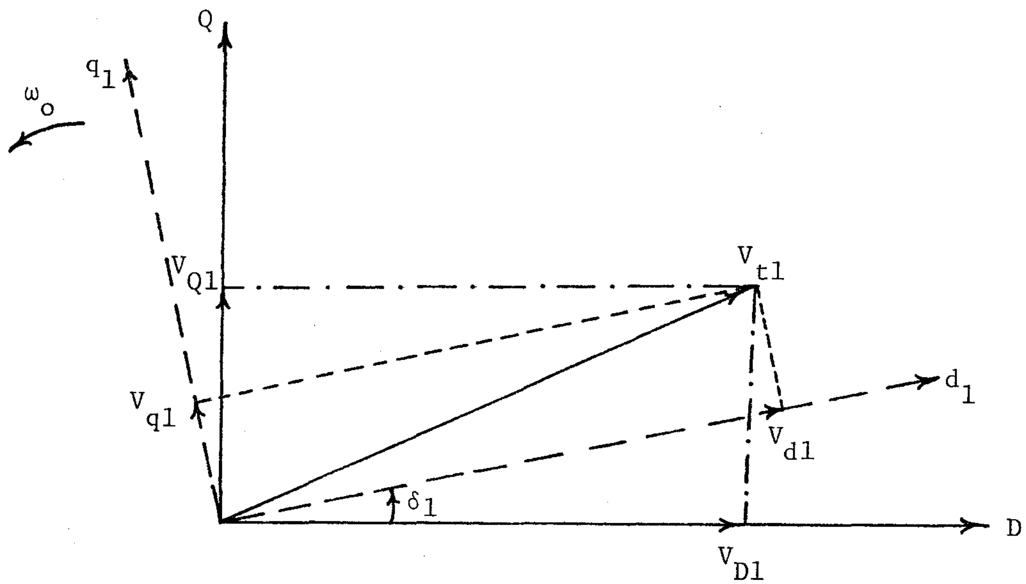


Figure 3.1 Angular Relationships Between Network and Synchronous Machine Reference Axes

This relationship can be expressed by the following equation:

$$\begin{bmatrix} v_{di} \\ v_{qi} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & \sin\delta_i \\ -\sin\delta_i & \cos\delta_i \end{bmatrix} \begin{bmatrix} v_{Di} \\ v_{Qi} \end{bmatrix} \qquad (3.7)$$

or, $\underset{\sim}{v}_m = [T]\,\underset{\sim}{v}_N$

Considering n generating units are connected to the network, equation (3.7) will be as follows:

$$\underset{\sim}{v}_m = \begin{bmatrix} T_{11} & 0 & \cdots & 0 \\ 0 & T_{22} & & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & T_{nn} \end{bmatrix} \underset{\sim}{v}_N \tag{3.8}$$

For small disturbances in the system, equation (3.8) can be linearized around the operating condition. This yields:

$$\Delta v_m = [T]_o \, \Delta v_N + \begin{bmatrix} v_{q1} & 0 & \cdots & 0 \\ -v_{d1} & 0 & \cdots & 0 \\ 0 & v_{q2} & & \\ 0 & -v_{d2} & & \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & v_{qn} \\ 0 & 0 & \cdots & -v_{dn} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \vdots \\ \Delta\delta_n \end{bmatrix} \tag{3.9}$$

Similarly, the machine and network current vectors are related by:

$$[T]^t \, \underset{\sim}{i}_m = \underset{\sim}{i}_N \tag{3.10}$$

where $[T]^t$ is the transpose of the transformation matrix $[T]$. From equations (3.6) and (3.10), it can be proved that:

$$[T]^t \, \underset{\sim}{i}_m = [Y_{NN}] \, \underset{\sim}{v}_N \tag{3.11}$$

For small perturbations, equation (3.11) can be linearized as follows:

$$[T]_o^t \, \Delta \underset{\sim}{i}_m = [Y_{NN}] \, \Delta \underset{\sim}{v}_N + \begin{bmatrix} i_{Q1} & 0 & \cdots\cdots & 0 & \Delta\delta_1 \\ -i_{D1} & 0 & \cdots\cdots & 0 & \Delta\delta_2 \\ 0 & i_{Q2} & & 0 & \cdot \\ 0 & -i_{D2} & & 0 & \cdot \\ \vdots & & & \vdots & \vdots \\ 0 & & & i_{Qn} & \cdot \\ 0 & & & -i_{Dn} & \Delta\delta_n \end{bmatrix} \tag{3.12}$$

3.3 Inclusion of an Infinite Bus

An infinite bus is considered to be rated at constant frequency and voltage (both in magnitude and angle). A very large capacity bus compared to the rating of the machine and connected in the power system is considered as an infinite bus. In the case of existence of an infinite bus, equation (3.6) is re-formulated as follows:

$$\begin{bmatrix} \underset{\sim}{i}_I \\ \hline \underset{\sim}{i}_N \end{bmatrix} = \left[\begin{array}{c|c} Y_{II} & Y_{IN} \\ \hline Y_{NI} & Y_{NN} \end{array}\right] \begin{bmatrix} \underset{\sim}{v}_I \\ \hline \underset{\sim}{v}_N \end{bmatrix} \tag{3.13}$$

where, v_I and i_I are voltage and current vectors of the infinite bus. If small perturbation occurs in the system, equation (3.13) can be linearized. This yields:

$$\Delta \underset{\sim}{i}_I = [Y_{II}] \Delta \underset{\sim}{v}_I + [Y_{IN}] \Delta \underset{\sim}{v}_N$$
$$\Delta \underset{\sim}{i}_N = [Y_{NI}] \Delta \underset{\sim}{v}_I + [Y_{NN}] \Delta \underset{\sim}{v}_N \qquad (3.14)$$

Since the infinite bus voltage is constant, hence:

$$\Delta \underset{\sim}{v}_I = \underset{\sim}{0} \qquad (3.15)$$

As a result, the linearized equation (3.13) will be:

$$\begin{bmatrix} \Delta \underset{\sim}{i}_I \\ \hline \Delta \underset{\sim}{i}_N \end{bmatrix} = \begin{bmatrix} Y_{NI} \\ \hline Y_{NN} \end{bmatrix} \Delta \underset{\sim}{v}_N \qquad (3.16)$$

Using the same procedure adopted in the last section (3.3) to refer the individual machine currents to the general reference frame, equation (3.16) can be replaced by the following form:

$$\begin{bmatrix} I & 0 \\ 0 & [T_o]^t \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{i}_I \\ \Delta \underset{\sim}{i}_m \end{bmatrix} = \begin{bmatrix} Y_{NI} \\ Y_{NN} \end{bmatrix} \Delta \underset{\sim}{v}_N + \begin{bmatrix} 0 & 0 & \cdots & 0 & \Delta\delta_1 \\ 0 & 0 & & 0 & \Delta\delta_2 \\ i_{Q1} & 0 & & 0 & \cdot \\ -i_{D1} & 0 & & 0 & \cdot \\ 0 & i_{Q2} & & 0 & \cdot \\ 0 & -i_{D2} & & 0 & \cdot \\ \cdot & 0 & & \cdot & \cdot \\ \vdots & 0 & & \vdots & \cdot \\ \cdot & 0 & & \cdot & \cdot \\ 0 & 0 & & i_{Qn} & \cdot \\ 0 & 0 & & -i_{Dn} & \Delta\delta_n \end{bmatrix} \qquad (3.17)$$

If there is no infinite bus included in the system, it is assumed [7] that the network frequency is always identical to that of one arbitrarily chosen machine so that the axes (D, Q) rotate in synchronism with the axes (d, q) of that machine. This implies that the rotor angle deviation, $\Delta\delta$, of the chosen machine is always zero.

As a result, one angle, and hence one state, is eliminated, and equation (3.12) replaces equation (3.17).

3.4 Ordering of the System Vectors

There are two approaches for grouping the state variables:

(a) Type Grouping

In which all the state variables associated with the same process in each machine are grouped together, e.g., the grouping of rotor angles of all machines, rotor speeds of all machines, etc.

$$\underset{\sim}{X} = [\delta_1, \delta_2, \ldots \delta_n; \omega_1, \omega_2, \ldots \omega_n; \ldots .]$$

(b) Generator Grouping

In which all the state variables associated with a particular generating unit are grouped together.

$$\underset{\sim}{X} = [\delta_1, \omega_1, \ldots; \delta_2, \omega_2, \ldots; \delta_n, \omega_n, \ldots]$$

The first approach has been adopted in [4], [7] and [22]; whereas the second approach has been used in [14], [25] and [28]. The second approach is simpler than the first, especially for the general case with different degrees of subsystem modelling, and also for system updating. The generator approach has been used in the work presented in this thesis.

The state variable vector, $\Delta \underset{\sim}{X}_i$, for each individual machine, is constructed from the perturbed values of the rotor angle, rotor speed, internal flux linkages, governor and exciter state variables. The choice of the flux linkages as state variables instead of the machine currents is preferred as it is considered more convenient for studying the effect of a magnetic saturation in the synchronous machine, as mentioned in section (2.2). The state variable vector for the i^{th} generating unit is constructed as follows:

$$\Delta \underset{\sim}{X}_i = [\Delta\delta_i, \Delta\omega_i, \Delta\underset{\sim}{\psi}_i^t, \underset{\sim}{X}_{gi}^t, \underset{\sim}{X}_{ei}^t]^t \tag{3.18}$$

The state variable vector $\Delta \underset{\sim}{X}$ of the whole system is then constructed from all the individual vectors $\Delta \underset{\sim}{X}_i^t$ of each machine, as follows:

$$\Delta \underset{\sim}{X} = [\Delta \underset{\sim}{X}_1^t,\ \Delta \underset{\sim}{X}_2^t,\ \ldots\ \Delta \underset{\sim}{X}_n^t]^t \tag{3.19}$$

The algebraic vector of the whole system is constructed from the algebraic variables of each individual machine, as shown in equation (3.20).

To avoid inversion of a large matrix, the algebraic vector of the whole system is constructed from the algebraic variables of each individual machine, each group of variables being placed alternately, as shown in equation (3.20).

$$\begin{aligned}\Delta \underset{\sim}{y} \;=\; \Delta[&\underset{\sim}{i}_{m1}^t,\ \underset{\sim}{i}_{m2}^t,\ \ldots,\ \underset{\sim}{i}_{mn}^t,\ \underset{\sim}{v}_{m1}^t,\ \underset{\sim}{v}_{m2}^t,\ \ldots,\ \underset{\sim}{v}_{mn}^t,\ v_{t1},\ v_{t2},\ \ldots,\ v_{tn},\\ &T_{e1},\ T_{e2},\ \ldots,\ T_{en},\ P_{o1},\ P_{o2},\ \ldots\ P_{on},\ i_{DI},\ i_{QI},\ \underset{\sim}{v}_N^t]^t\end{aligned} \tag{3.20}$$

The input vector is constructed from the input vectors of each machine, as follows:

$$\Delta \underset{\sim}{u} = [\Delta \underset{\sim}{u}_1^t,\ \Delta \underset{\sim}{u}_2^t,\ \ldots,\ \Delta \underset{\sim}{u}_n^t] \tag{3.21}$$

3.5 State-Space Formulation

In this section, the [P], [Q] and [R] matrices of equation (3.2) will be constructed in detail based on the formulation proposed in reference [14]. The [P] matrix is partitioned, as follows:

$$[P] = \left[\begin{array}{c|c} I & PA \\ \hline 0 & PB \end{array}\right] \begin{array}{l} ns \\ nv \end{array} \tag{3.22}$$

where ns and nv are the total number of state and algebraic variables, respectively. [I] is an identity matrix, [O] is a null matrix. Since we want to reformulate equation (3.2) to be in the state-space form, equation (3.1), the matrix [P] should be inverted, as shown in equation (3.23):

$$[P]^{-1} = \left[\begin{array}{c|c} I & -[PA]\,[PB]^{-1} \\ \hline 0 & [PB]^{-1} \end{array}\right] \tag{3.23}$$

The [PA] and [PB] matrices are partitioned as shown in equation (3.24):

$$[PA] = \begin{array}{c} nv \\ \left[\begin{array}{c} PA_1 \\ \hline PA_2 \\ \hline \cdots \\ \hline PA_n \end{array}\right] \end{array} \begin{array}{c} \\ ns_1 \\ ns_2 \\ \\ ns_n \end{array} \quad \text{and} \quad [PB] = \begin{array}{cc} nx_m & nv-nx_m \\ \end{array} \left[\begin{array}{c|c} PX & 0 \\ \hline PC & PD \end{array}\right] \begin{array}{c} nx_m \\ nv-nx_m \end{array} \tag{3.24}$$

where ns_1, ns_2, ... ns_n are the number of state variables of machine 1, 2, ..., n, respectively and nx_m is the total number of state variables associated with the synchronous generator (flux linkages), i.e.,
$nx_m = \sum_{i=1}^{n} nx_i$.

The matrix [PB] is of particular interest since it has to be inverted, using inversion by parts as:

$$[PB]^{-1} = \left[\begin{array}{c|c} [PX]^{-1} & 0 \\ \hline -[PD]^{-1}\,[PC]\,[PX]^{-1} & [PD]^{-1} \end{array}\right] \tag{3.25}$$

From equation (3.25), it is noticed that the matrices [PX] and [PD] should be inverted. Hence, further partitioning is done to reduce the inversion computation time, as follows:

$$
[PX] = \left[\begin{array}{cccc} PX_1 & & & 0 \\ & PX_2 & & \\ & & \ddots & \\ 0 & & & PX_n \end{array}\right] \quad \text{and} \quad [PD] = \begin{array}{c} \left[\begin{array}{c|c|c|c} I & 0 & 0 & -[PT] \\ \hline [PS] & I & 0 & 0 \\ \hline 0 & 0 & I & -[PI] \\ \hline 0 & 0 & 0 & -[PN] \end{array}\right] \end{array} \begin{array}{l} nv_1 \\ nv_2 \\ nv_3 \\ nv_4 \end{array} \tag{3.26}
$$

where, nv_1 and nv_4 equals 2xn, nv_2 equals 3xn and nv_3 equals 2 if the system includes an infinite bus, and equals zero if it is not included. The matrix [PX] is a block diagonal and includes all the reactance matrices, one block per machine and the matrix [PD] can be inverted using inversion by parts as follows:

$$
[PD]^{-1} = \left[\begin{array}{c|c|c|c} I & 0 & 0 & [PT]\,[PN]^{-1} \\ \hline -[PS] & I & 0 & -[PS]\,[PT]\,[PN]^{-1} \\ \hline 0 & 0 & I & -[PI]\,[PN]^{-1} \\ \hline 0 & 0 & 0 & -[PN]^{-1} \end{array}\right] \tag{3.27}
$$

where, the matrix [PN] is the real network admittance matrix $[Y_{nn}]$ and the matrix [PI] is the real infinite bus admittance matrix $[Y_{NI}]$. The form of one block of the matrix [PX] considering the 5th order generation model will be as shown in Figure (3.2).

$$
[PX_i] = \left[\begin{array}{ccc|cc} X_{fi} & -X_{mdi} & X_{mdi} & & \\ X_{mdi} & -X_{di} & X_{mdi} & & 0 \\ X_{mdi} & -X_{mdi} & X_{kdi} & & \\ \hline & 0 & & -X_{qi} & X_{mqi} \\ & & & -X_{mqi} & X_{kqi} \end{array}\right] \qquad \begin{array}{l} i = 1, \ldots, n \\ n = \text{number of machines} \end{array}
$$

Figure (3.2) [PX] Matrix for One Machine (Order 5)

The matrices [PS], [PT], [PC], $[PA_i]$, $[QA_i]$, [QB] and $[RA_i]$ are shown in Figure (3.3) - Figure (3.9) in full detail representing a three machine system, the order of each machine is thirteen. The subsystem models are chosen, as follows:

a) Mechanical shaft - Type 1 (order 2)

b) Synchronous generator - Type 3 (order 5)

c) Turbine/governor - Type 1 (order 2)

d) Exciter/stabilizer - Type 3 (order 4)

where, nx_s, nx_m, nx_g and nx_e are the number of the states associated with mechanical shaft, generator, turbine/governor and exciter/stabilizer, respectively.

To obtain the coefficient matrices A, B, C and D, the following procedure may be followed:

Recalling equation (3.4),

$$\left[\begin{array}{c|c} I & PA \\ \hline O & PB \end{array}\right]\left[\begin{array}{c} \Delta\dot{\underset{\sim}{x}} \\ \hline \Delta\underset{\sim}{y} \end{array}\right] = \left[\begin{array}{c} QA \\ \hline QB \end{array}\right] \Delta\underset{\sim}{x} + \left[\begin{array}{c} RA \\ \hline RB \end{array}\right] \Delta\underset{\sim}{u} \tag{3.28}$$

$$\text{or,} \quad \Delta\dot{\underset{\sim}{x}} + [PA]\,\Delta\underset{\sim}{y} = [QA]\,\Delta\underset{\sim}{x} + [RA]\,\Delta\underset{\sim}{u} \tag{3.29}$$

$$[PB]\,\Delta\underset{\sim}{y} = [QB]\,\Delta\underset{\sim}{x} + [RB]\,\Delta\underset{\sim}{u} \tag{3.30}$$

Equation (3.30) can be re-written as follows:

$$\Delta\underset{\sim}{y} = [PB]^{-1}\,[QB]\,\Delta\underset{\sim}{x} + [PB]^{-1}\,[RB]\,\Delta\underset{\sim}{u} \tag{3.31}$$

Substituting equation (3.31) into equation (3.29), leads to:

$$\dot{\underset{\sim}{\Delta x}} = \{[QA] - [PA]\,[C]\}\,\Delta x + [RA]\,\underset{\sim}{\Delta u} \tag{3.32}$$

Comparing the equation set (3.1) with equations (3.31) and (3.32), we can conclude:

$$\begin{aligned} [A] &= [QA] - [PA]\,[C] \\ [B] &= [RA] \\ [C] &= [PB]^{-1}\,[QB] \\ [D] &= [PB]^{-1}\,[RB] \end{aligned} \tag{3.33}$$

In power system configurations, [RB] and consequently [D] are zero matrices. The program package description, user guide and program listing have been presented in detail in a McMaster internal report [34]. The flow chart of this program is presented in Figure (3.10). The program is capable of representing a synchronous machine, either in detail or by a classical model (fixed voltage behind transient reactance). Detailed model representation is used for machines close to the point of interest (study system) and the less detailed model representation is used for the rest of the machines (external system). Using different subsystem models is very important, as it facilitates representing the system dynamics in different degrees of complexity. The different types of mechanical shaft, governor/turbine and exciters shown in Table (2.1), have been utilized in this program. The program is capable of handling systems up to seventy states in an interactive mode.

$$
\begin{array}{cc}
 & \begin{array}{cccccc} V_{d1} & V_{q1} & V_{d2} & V_{q2} & V_{d3} & V_{q3} \end{array} \\
\begin{array}{c} V_{t1} \\ V_{t2} \\ V_{t3} \\ T_{e1} \\ T_{e2} \\ T_{e3} \\ P_{01} \\ P_{02} \\ P_{03} \end{array} &
\left[\begin{array}{cc|cc|cc}
\frac{-V_{d1}}{V_{t1}} & \frac{-V_{q1}}{V_{t1}} & & & & \\
 & & \frac{-V_{d2}}{V_{t2}} & \frac{-V_{q2}}{V_{t2}} & & \\
 & & & & \frac{-V_{d3}}{V_{t3}} & \frac{-V_{d3}}{V_{t3}} \\
\hline
 & & & & & \\
 & & & & & \\
 & & & & & \\
\hline
-i_{d1} & -i_{q1} & & & & \\
 & & -i_{d2} & -i_{q2} & & \\
 & & & & -i_{d3} & -i_{q3}
\end{array}\right]
\end{array}
$$

Figure 3.3 [PS] Matrix

$$
\begin{array}{c|cc|cc|cc|}
 & V_{D1} & V_{Q1} & V_{D2} & V_{Q2} & V_{D3} & V_{Q3} \\
\hline
V_{d1} & \cos\delta_1 & \sin\delta_1 & & & & \\
V_{q1} & -\sin\delta_1 & \cos\delta_1 & & & & \\
\hline
V_{d2} & & & \cos\delta_2 & \sin\delta_2 & & \\
V_{q2} & & & -\sin\delta_2 & \cos\delta_2 & & \\
\hline
V_{d3} & & & & & \cos\delta_3 & \sin\delta_3 \\
V_{q3} & & & & & -\sin\delta_3 & \cos\delta_3 \\
\hline
\end{array}
$$

Figure 3.4 [PT] Matrix

$$
\begin{array}{c|ccccc|ccccc|ccccc|}
 & i_{f1} & i_{d1} & i_{kd1} & i_{q1} & i_{kq1} & i_{f2} & i_{d2} & i_{kd2} & i_{q2} & i_{kq2} & i_{f3} & i_{d3} & i_{kd3} & i_{q3} & i_{kq3} \\
\hline
V_{d1} &&&&&&&&&&&&&&& \\
V_{q1} &&&&&&&&&&&&&&& \\
V_{d2} &&&&&&&&&&&&&&& \\
V_{q2} &&&&&&&&&&&&&&& \\
V_{d3} &&&&&&&&&&&&&&& \\
V_{q3} &&&&&&&&&&&&&&& \\
V_{t1} &&&&&&&&&&&&&&& \\
V_{t2} &&&&&&&&&&&&&&& \\
V_{t3} &&&&&&&&&&&&&&& \\
\hline
T_{e1} && \psi_{q1} && -\psi_{d1} &&&&&&&&&&& \\
T_{e2} &&&&&&& \psi_{q2} && -\psi_{d2} &&&&&& \\
T_{e3} &&&&&&&&&&&& \psi_{q3} && -\psi_{d3} & \\
\hline
P_{01} && -V_{d1} && -V_{q1} &&&&&&&&&&& \\
P_{02} &&&&&&& -V_{d2} && -V_{q2} &&&&&& \\
P_{03} &&&&&&&&&&&& -V_{d3} && -V_{q3} & \\
\hline
i_{DI} &&&&&&&&&&&&&&& \\
i_{QI} &&&&&&&&&&&&&&& \\
\hline
V_{D1} && \cos\delta_1 && -\sin\delta_1 &&&&&&&&&&& \\
V_{Q1} && \sin\delta_1 && \cos\delta_1 &&&&&&&&&&& \\
V_{D2} &&&&&&& \cos\delta_2 && -\sin\delta_2 &&&&&& \\
V_{Q2} &&&&&&& \sin\delta_2 && \cos\delta_2 &&&&&& \\
V_{D3} &&&&&&&&&&&& \cos\delta_3 && -\sin\delta_3 & \\
V_{Q3} &&&&&&&&&&&& \sin\delta_3 && \cos\delta_3 & \\
\hline
\end{array}
$$

Figure 3.5 [PC] Matrix

$$
\begin{array}{cccccccccccccccccccccccccccccccccccccc|c}
i_{f1} & i_{d1} & i_{kd1} & i_{q1} & i_{kq1} & i_{f2} & i_{d2} & i_{kd2} & i_{q2} & i_{kq2} & i_{f3} & i_{d3} & i_{kd3} & i_{q3} & i_{kq3} & -V_{d1} & V_{q1} & V_{d2} & V_{q2} & V_{d3} & V_{q3} & V_{t1} & V_{t2} & V_{t3} & T_{e1} & T_{e2} & T_{e3} & P_{01} & P_{02} & P_{03} & i_{DI} & i_{QI} & V_{D1} & V_{Q1} & V_{D2} & V_{Q2} & V_{D3} & V_{Q3} & \\
\hline
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& \delta_1 \\
&&&&&&&&&&&&&&&&&&&&&&&& \frac{\omega_o}{2H_1} &&&&&&&&&&&&&& \omega_1 \\
\omega_o r_{f1} &&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& \psi_{f1} \\
& -\omega_o r_{d1} &&&&&&&&&&&&&& \omega_o &&&&&&&&&&&&&&&&&&&&&&& \psi_{d1} \\
&& \omega_o r_{q1} &&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& \psi_{kd1} \\
&&& -\omega_o r_{kq1} &&&&&&&&&&&&& -\omega_o &&&&&&&&&&&&&&&&&&&&&& \psi_{q1} \\
&&&& \omega_o r_{kq1} &&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& \psi_{kq1} \\
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& P_{m1} \\
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& g_1 \\
&&&&&&&&&&&&&&&&&&&&& \frac{-1}{\tau v_1} &&&&&&&&&&&&&&&&& V_{v1} \\
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& V_{f1} \\
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& V_{x1} \\
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&& V_{y1}
\end{array}
$$

Figure 3.6 $[PA_1]$ Matrix

$$
\begin{array}{c}
\begin{array}{ccccccccccccc}
\delta_1 & \omega_1 & \psi_{f1} & \psi_{d1} & \psi k_{d1} & \psi_{q1} & \psi_{kq1} & P_{m1} & g_1 & V_{v1} & V_{f1} & V_{x1} & V_{y1}
\end{array}
\\
\left[
\begin{array}{cc|ccccc|cc|cccc}
 & 1 & & & & & & & & & & & \\
 & -k_{md1} & & & & & & \frac{\omega_o}{2H_1} & & & & & \\
\hline
 & & & & & & & & & & \omega_o & & \\
 & \psi_{q1} & & & & \omega_o & & & & & & & \\
 & -\psi_{d1} & & -\omega_o & & & & & & & & & \\
\hline
 & & & & & & & \frac{-1}{\tau_{ch1}} & \frac{-1}{\tau_{ch1}} & & & & \\
 & \frac{kg1}{\omega_o\ \tau_{31}} & & & & & & & \frac{-1}{\tau_{31}} & & & & \\
\hline
 & & & & & & & & & \frac{-1}{\tau_{V1}} & & & \\
 & \frac{k_e^*\ k_Q\ \tau_{ax}}{\omega_o\ \tau_e} & & & & & & & & \frac{-k_e^*}{\tau_e} & \frac{-1}{\tau_e} & \frac{k_e^*\ \tau_{ax}}{\tau_e} & \frac{k_e^*}{\tau_e} \\
 & \frac{-k_Q}{\omega_o\ \tau_Q} & & & & & & & & & & \frac{-1}{\tau_Q} & \\
 & \frac{-k_Q\ \tau_a}{\omega_o\ \tau_x^2} & & & & & & & & & & \frac{-\tau_a}{\tau_x^2} & \frac{-1}{\tau_x}
\end{array}
\right]
\end{array}
\begin{array}{c}
nXs \\ \\ nXm \\ \\ nXg \\ \\ nXe
\end{array}
\qquad
\begin{array}{c}
\begin{array}{cc} V_r & P_r \end{array}
\\
\left[
\begin{array}{cc}
 & \\
 & \\
\hline
 & \\
 & \\
 & \\
\hline
 & \frac{1}{\tau_{ch}} \\
 & \\
\hline
\frac{k_e^*}{\tau_e} & \\
 & \\
 & \\
 &
\end{array}
\right]
\end{array}
$$

Figure 3.7 [QA_1] Matrix

Figure 3.8 [RA_1] Matrix

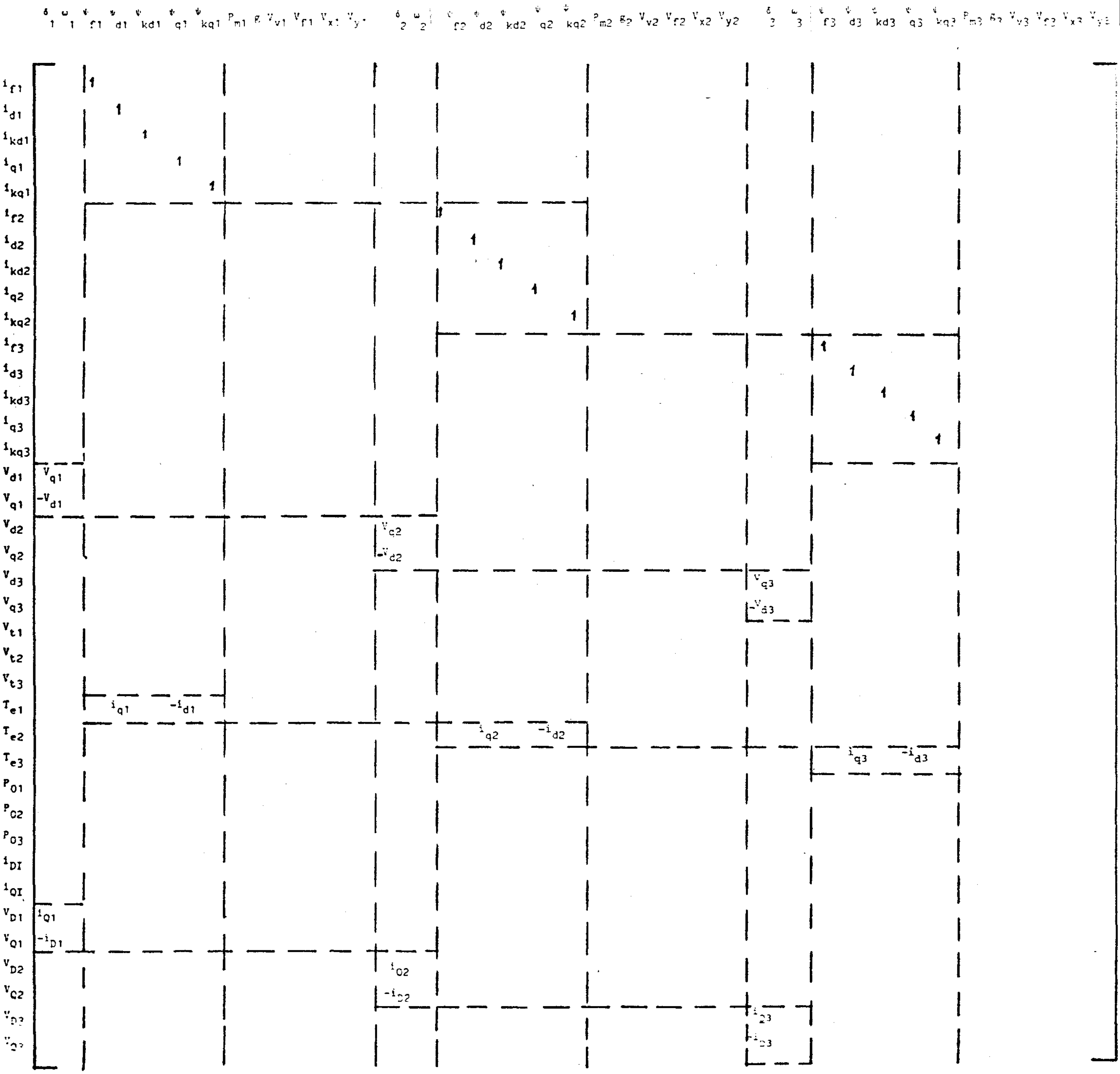
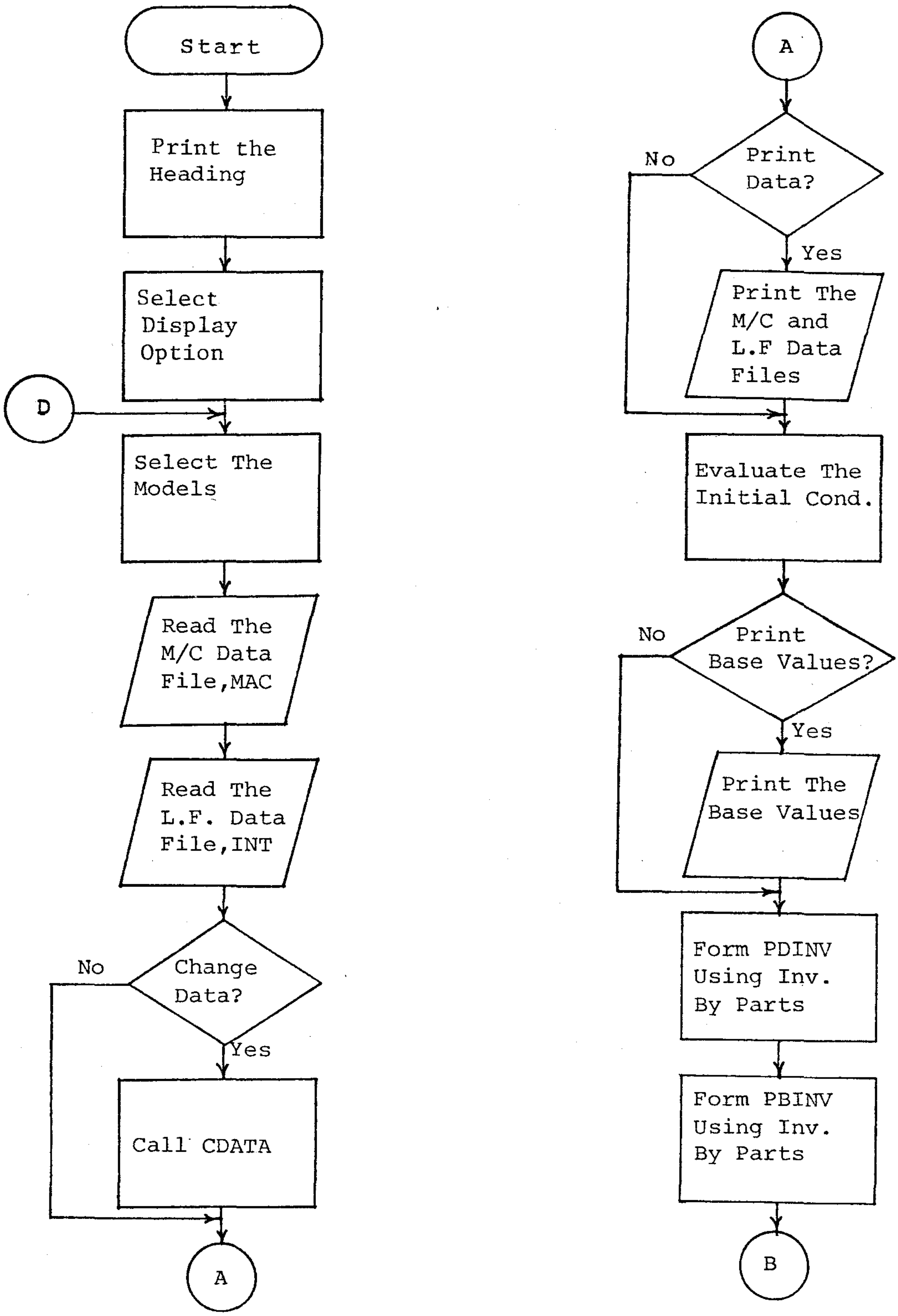
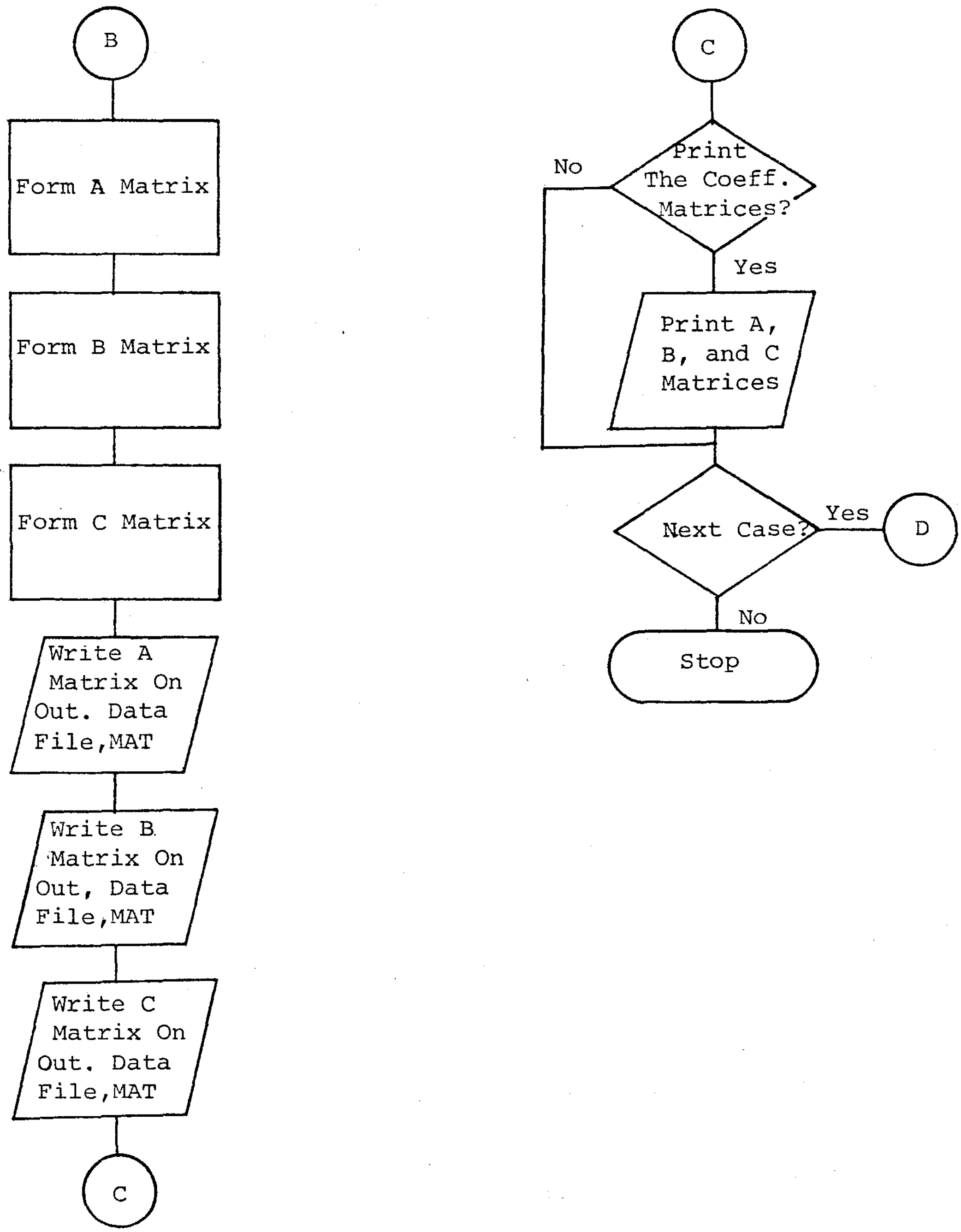


Figure 3.9 [QB] Matrix

Figure 3.10 " PQR "

PROGRAM FLOW CHART




B
Form A Matrix
Form B Matrix
Form C Matrix
Write A Matrix On Out. Data File,MAT
Write B Matrix On Out, Data File,MAT
Write C Matrix On Out. Data File,MAT
C
C
Print The Coeff. Matrices?
No
Yes
Print A, B, and C Matrices
Next Case?
Yes
D
No
Stop

3.6 Eigenvalue Sensitivity

The eigenvalues of the system state matrix [A] indicate system dynamic stability. These eigenvalues are, in general, functions of all control and design parameters in the system. A change in any of these parameters affects the system performance, and a shift in the whole eigenvalue pattern may occur.

If a change $\Delta\xi$ in a certain parameter ξ occurs, an estimate $\hat{\lambda}_i$, can be obtained using Taylor series expansion around a base value λ_{io}, as follows [32],

$$\hat{\lambda}_i = \lambda_{io} + \frac{\partial \lambda_i}{\partial \xi}\bigg|_{\xi_o} (\Delta\xi) + \frac{1}{2}\frac{\partial^2 \lambda_i}{\partial \xi^2}\bigg|_{\xi_o} (\Delta\xi)^2 + \ldots \tag{3.34}$$

In equation (3.34), the term:

$$\frac{\partial \lambda_i}{\partial \xi}\bigg|_{\xi_o}$$

is defined as the first-order sensitivity coefficient of the eigenvalue λ_i with respect to the parameter at the original parameter value ξ_o. If only the first term of the Taylor series expansion is taken into consideration, the estimation is called a first-order eigenvalue sensitivity. The second-order partial derivative in equation (3.34)

$$\frac{\partial^2 \lambda_i}{\partial \xi^2}\bigg|_{\xi_o}$$

is called the second-order sensitivity coefficient of the eigenvalue λ_i with respect to the system parameter ξ. Eigenvalue first and second-order sensitivity analysis has been applied in references [3], [26] and [27]. Higher order eigenvalue sensitivities were computed in reference [33] for determining the changes in the eigenvalues for a large change in system parameters to obtain a more accurate estimate of the new eigenvalue location. The expressions for first and second-order sensitivity coefficients with respect to different control parameters, are given in equations (3.35) and (3.36). These expressions are taken directly from reference [32].

$$\frac{\partial \lambda_i}{\partial \xi} = \frac{\{\frac{\partial [A]}{\partial \xi} \underset{\sim}{V}_i\} \underset{\sim}{W}_i}{(\underset{\sim}{V}_i \underset{\sim}{W}_i^t)} \tag{3.35}$$

$$\frac{\partial^2 \lambda_i}{\partial \xi^2} = \frac{[\frac{\partial^2 [A]}{\partial \xi^2} \underset{\sim}{V}_i \underset{\sim}{W}_i] + 2\, [(\frac{\partial [A]}{\partial \xi} \sum_{\substack{j=1 \\ j \neq i}}^{ns} \alpha_{ij} \underset{\sim}{V}_j) \underset{\sim}{W}_i]}{(\underset{\sim}{V}_i \underset{\sim}{W}_i)} \tag{3.36}$$

where:

λ_i	:	ith eigenvalue
$\underset{\sim}{V}_i$	:	eigenvector of [A] corresponding to ith eigenvalue
$\underset{\sim}{W}_i$	:	eigenvector of $[A]^t$ corresponding to ith eigenvalue
ξ	:	system parameter of interest
$\frac{\partial [A]}{\partial \xi}$	:	state matrix partial derivatives with respect to parameter ξ
α_{ij}	:	$(\frac{\partial [A]}{\partial \xi} \underset{\sim}{V}_i \underset{\sim}{W}_j) / (\underset{\sim}{V}_i \underset{\sim}{W}_j)\ (\lambda_i - \lambda_j)$

From equations (3.35) and (3.36), it is seen that to find the sensitivity of the eigenvalues to a system parameter, it is necessary to compute:

1. The partial derivative of the state matrix [A] with respect to that parameter.
2. The system eigenvalues, the normal and transpose eigenvectors of the matrix [A].

Nolan et al, [21] and [27], proved that the state matrix first-order derivatives with respect to a variable parameter, ξ, using the "PQR" matrix formulation, is as follows:

$$\frac{\partial[A]}{\partial\xi} = [I \ 0] \frac{\partial[S]}{\partial\xi} \tag{3.37}$$

or,

$$\frac{\partial[A]}{\partial\xi} = [I \ 0] [P]^{-1} [\frac{\partial[Q]}{\partial\xi} - \{\frac{\partial[P]}{\partial\xi}\}S] \tag{3.38}$$

where

$$[S] = [P]^{-1} [Q] = \left[\frac{A}{C}\right], \tag{3.39}$$

and [I] is the unit matrix of order ns.

The "PQR" technique, described in the reference [32], formulates the [A] matrix from the addition of two matrices. One of them, [QA], contains most of the control and design parameters in the system as simple explicit functions. Consequently, for most variable parameters and specifically all control parameters (control gains, time constants, etc.), $\frac{\partial[P]}{\partial\xi} = [0]$, hence,

$$\frac{\partial[A]}{\partial\xi} = [I \;\; 0] \frac{\partial[Q]}{\partial\xi} = \frac{\partial[QA]}{\partial\xi} \tag{3.40}$$

and the second-order derivatives of the system state matrix [A] are given by:

$$\frac{\partial^2[A]}{\partial\xi^2} = \frac{\partial^2[QA]}{\partial\xi^2} \tag{3.41}$$

On the other hand, if it is required to compute eigenvalue sensitivities with respect to parameters that appear in the matrix [P], $\frac{\partial[Q]}{\partial\xi} = [0]$, and hence,

$$\frac{\partial[A]}{\partial\xi} = -[I \;\; 0] \, [P]^{-1} \frac{\partial[P]}{\partial\xi} [S] \tag{3.42}$$

The approach used by Nolan et al for calculating matrix first-order derivatives is extended by Zein El-Din, [32], to calculate the second-order derivatives as follows:

$$\frac{\partial^2[A]}{\partial\xi^2} = -[I \;\; 0] \, [P]^{-1} \, \{\frac{\partial^2[P]}{\partial\xi^2} [S] + 2 \, \frac{\partial[P]}{\partial\xi} \frac{\partial[S]}{\partial\xi}\} \tag{3.43}$$

CHAPTER 4

"DIRECT ELIMINATION" FORMULATION

4.1 Introduction

The dynamic system is represented by the state matrix [A] which is based directly on the algebraic and first-order differential equations, or indirectly on the equations and block diagrams representing the system performance.

Laughton [4] used the "direct elimination" technique to obtain the state matrix [A] from the complete algebraic and differential equations of the whole system. He formulated the general nonlinear equations describing the performance of a single machine without associated excitation or prime-mover control, when connected to an equivalent transmission system and linearized these equations by considering the first terms only of a Taylor series expansion of the equations around any operating point. He initially constructed the operating matrix equation which summarizes the relationships between all machine and system variables, keeping the variables of particular interest. The system input variables (ΔV_{fd} and ΔT_m), the controlled machine variables ($\Delta\delta$, ΔV_t and ΔI), and all time-derivative quantities, are in the first equations. The variables not of interest may be eliminated by matrix reduction. This method is of great significance, because it is a practical method for obtaining the required differential equations.

But, the reduction of the operating matrix by hand manipulation of the system equations, may lead to errors in the calculations.

Van Ness [3] and Muir [30] used the general block diagram technique instead of direct representation of the controller equations. For each block, the name of the output variable, the parameters (gains and time constants), and the input variables must be provided on the input cards. The input variable may include a plus or minus sign to indicate the sign of the input. The system equations are formulated in the program according to these informations. This facilitates addition of voltage regulators, different types of governors, and other control equipment to study their effect on systems dynamics. The interactions between blocks are identified by integers, certain integers being reserved to connect the controller to the controlled device. Each block is given a name that is used to refer to a state, input or output variable. The method described by Muir [30] used the elimination method described by Laughton [4]. He formed the state matrix [A] by forming and storing the network equations first, then the equations of one machine with its exciter/governor were formed and reduced until only the differential equations of that machine and the algebraic equations of the network were left. Then the two network equations for that node could be eliminated. The same procedure is repeated for each machine until the full state matrix of the system is formed. This method is more flexible as it allows for future modifications such as the addition of control equipment. But, the application of this method is restricted to high standard users because any mistake in entering the data (the parameters, input and output variable names) of each block diagram may give misleading results, as the input data are supplied in a transfer function form.

This chapter describes another computer program package which forms the state matrix and computes the eigenvalues for determining the stability of the system. This program is based on both ideas, direct elimination [4], and reduction of the machine shaft, governor and exciter equations to state format, machine by machine, starting with the interconnection between the machine and the network at the beginning [30]. The matrices are constructed directly from the linearized equations representing the different subsystems. The user communicates with the program through a series of questions, which enables him to select different models for each subsystem.

4.2 Matrix Formulation

The linearized differential and algebraic equations describing the performance of a single synchronous machine without associated excitation or prime-mover control, when connected to an equivalent transmission system, are summarized in reference [4]. These equations can be written in matrix form as shown in equation (4.1):

$$\left[\begin{array}{c} \Delta T_m \\ \Delta V_{fd} \\ 0 \\ 0 \\ \hline 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{array}\right] = \left[\begin{array}{c:c} & \\ & \\ Z_1 & Z_2 \\ & \\ \hdashline & \\ & \\ & \\ & \\ Z_3 & Z_4 \\ & \\ & \end{array}\right] \left[\begin{array}{c} \Delta\delta \\ \Delta\psi_{fd} \\ \Delta V_t \\ \Delta I \\ \hline \Delta V_d \\ \Delta V_q \\ \Delta i_{fd} \\ \Delta i_d \\ \Delta i_q \\ \Delta\psi_d \\ \Delta\psi_q \end{array}\right] \begin{array}{l} \\ \text{variables of} \\ \text{interest} \\ \\ \\ \\ \\ \text{variables not} \\ \text{of interest} \\ \\ \\ \end{array} \tag{4.1}$$

Equation (4.1) can be written in a symbolic form as follows:

$$\left[\begin{array}{c} \underset{\sim}{\Delta u} \\ \hline \underset{\sim}{0} \end{array}\right] = [Z_p] \left[\begin{array}{c} \underset{\sim}{\Delta x} \\ \hline \underset{\sim}{\Delta y} \end{array}\right] \tag{4.2}$$

where, $[Z_p]$ is an operational matrix.

This equation describes one machine connected to an infinite bus and summarizes the relationships between all machine and system variables, where the variables of particular interest (the forcing functions ΔV_{fd} and ΔT_m representing system inputs through the excitation system or prime mover, the controlled machine variables $\Delta\delta$, ΔV_t and ΔI, and all time derivative quantities) are in the first equations. The variables not of interest (ΔV_d to $\Delta\psi_q$, equation (4.1)) may be eliminated by matrix reduction. This leads to the following equation:

$$\begin{bmatrix} \Delta T_m \\ \Delta V_{fd} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_1 - Z_2 Z_4^{-1} Z_3 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\psi_{fd} \\ \Delta V_t \\ \Delta I \end{bmatrix} \tag{4.3}$$

The system represented by equation (4.2) can be formulated in the following state-space equations (4.4) and (4.5).

$$\Delta \dot{\tilde{x}} = [A]\, \Delta \tilde{x} + [B]\, \Delta \tilde{u} \tag{4.4}$$

$$\Delta \tilde{y} = [C]\, \Delta \tilde{x} \tag{4.5}$$

This can be done through a few substitutions [4]. Assuming that: $x_1 = \Delta\delta$, $x_2 = \dot{x}_1 = \Delta\omega$, $x_3 = \Delta\psi_{fd}$, and for the control input variables, $u_1 = \Delta V_{fd}$, $u_2 = \Delta T_m$. Thus, equation (4.4) becomes:

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \\ \Delta\dot{\psi} \end{bmatrix} = [A] \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta\psi_{fd} \end{bmatrix} + [B] \begin{bmatrix} \Delta V_{fd} \\ \Delta T_m \end{bmatrix} \tag{4.6}$$

The output variables may be represented also in matrix form as a function of the system variables, which in the case of equation (4.5) can be expanded by substituting: $y_1 = \Delta V_t$, $y_2 = \Delta I$; this yields,

$$\begin{bmatrix} \Delta V_t \\ \Delta I \end{bmatrix} = [C] \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta\psi_{fd} \end{bmatrix} \tag{4.7}$$

Applying this approach [4] for the multimachine dynamic stability problem, the matrix equation (4.2) may be rewritten as follows:

$$\begin{array}{c} ns \\ nv \end{array} \begin{bmatrix} \Delta \underset{\sim}{u}_1 \\ \hline \underset{\sim}{0} \end{bmatrix} = [Z_{d1}] \quad \Delta \dot{\underset{\sim}{x}} + \begin{bmatrix} & & \\ & Z_1 & \\ & & \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{x}_1 \\ \hline \Delta \underset{\sim}{y}_1 \end{bmatrix} \begin{array}{c} ns \\ nv \end{array} \tag{4.8}$$

where, ns and nv are the total number of the state variables and algebraic variables of the first machine $[Z_{d1}]$ and $[Z_1]$ are constant real matrices, $[Z_{d1}]$ is a diagonal matrix and $[Z_1]$ can be partitioned, as follows:

$$[Z_1] = \begin{array}{cc} \begin{array}{cc} ns & nv \end{array} & \\ \left[\begin{array}{c|c} Z_{11} & Z_{21} \\ \hline Z_{31} & Z_{41} \end{array}\right] & \begin{array}{c} ns \\ nv \end{array} \end{array} \tag{4.9}$$

The differential and algebraic equations representing one machine of order six are shown in equations (4.10) and (4.11).

The chosen subsystem models are as follows:

a) Mechanical shaft - Type 1 (2nd order)

b) Synchronous machine - Type 2 (3rd order)

c) Governor/Turbine - Type 0 (constant mech. power)

d) Exciter/Stabilizer - Type 1 (1st order)

The procedure used is based on the idea in reference [30]. The steps of this procedure can be summarized as follows:

1) The 2n real admittance matrix, $[Y_{NN}]$, which is formed in the load-flow program, is stored first.

2) The equations of the first machine with its exciter and governor are formed according to the user choice for the subsystem models.

Nonlinear Differential Equations

$$
\begin{aligned}
0 &= \dot{\delta} - \omega + \omega_o \\
T_m &= \frac{2H}{\omega_o} \dot{\omega} + \frac{D}{\omega_o} \omega + T_e \\
0 &= \dot{\psi}_{fd} - \omega_o V_{fd} + \omega_o r_{fd} i_{fd} \\
0 &= \dot{\psi}_{kd} + \omega_o r_{kd} i_{kd} \\
0 &= \dot{\psi}_{kq} + \omega_o r_{kq} i_{kq} \\
V_{ref} &= -\frac{\tau_e}{k_e} \dot{V}_{fd} + V_t - \frac{1}{k_e} V_{fd}
\end{aligned}
\tag{4.10}
$$

Nonlinear Algebraic Equations

$$
\begin{aligned}
0 &= \psi_{fd} - X_{fd}\, i_{fd} + X_{md}\, i_d - X_{md}\, i_{kd} \\
0 &= \psi_{kd} - X_{md}\, i_{fd} + X_{md}\, i_d - X_{kd}\, i_{kd} \\
0 &= \psi_{kq} + X_{mq}\, i_q - X_{kq}\, i_{kq} \\
0 &= r_s\, i_q + V_q - \omega\psi_d \\
0 &= r_s\, i_d + V_d + \omega\psi_q \\
0 &= V_t^2 - V_d^2 - V_q^2 \\
0 &= P_o - V_d\, i_d - V_q\, i_q \\
0 &= T_e - \psi_d\, i_q + \psi_q\, i_d \\
0 &= V_d - \cos\delta_1\, V_D - \sin\delta_1\, V_Q \\
0 &= V_q + \sin\delta_1\, V_D - \cos\delta_1\, V_Q
\end{aligned}
\tag{4.11}
$$

3) The matrix $[Z_n]$, which includes all the network algebraic equations, the differential and algebraic equations of the first machine, is reduced until only the differential equations of that machine and the algebraic equations of the network are left.

4) The two real network equations for that node connected to the first machine are eliminated.

5) The next machine then is added and reduced and so on until the full state matrix of the whole system is formed.

4.3 Formulation of Network Equations

The formulation of the network equations is similar to that in Chapter 3. The linearized equation (3.7) of the machine voltage referred to the general reference frame for one machine can be formulated as follows:

$$\Delta \underset{\sim}{v}_{mi} - [T_{ii}]_o \, \Delta \underset{\sim}{v}_{Ni} - \begin{bmatrix} v_{qi} \\ -v_{di} \end{bmatrix} \Delta\delta_i = \underset{\sim}{0} \qquad (4.12)$$

$i = 1, 2, \ldots, \quad n$: number of machines

Also, the linearized equation (3.11) of the machine currents referred to the general reference frame for one machine will be shown in equation (4.13):

$$[T_{ii}]_o^t \, \Delta \underset{\sim}{i}_{mi} - [Y_{NN}] \, \Delta v_N - \begin{bmatrix} i_{Qi} \\ -i_{Di} \end{bmatrix} \Delta\delta_i = \underset{\sim}{0} \qquad (4.13)$$

where,

$$[T_{ii}]_o = \begin{bmatrix} \cos\delta_{io} & \sin\delta_{io} \\ -\sin\delta_{io} & \cos\delta_{io} \end{bmatrix} \qquad (4.14)$$

δ_{io} is the rotor angle of machine i referred to the network reference frame at steady-state, $\Delta \underset{\sim}{v}_{mi}$ and $\Delta \underset{\sim}{i}_{mi}$ are the voltage and current vectors of machine i, i_{Di} and i_{Qi} are the components of the nodal current which can be represented in terms of the components of the machine currents by the following relation:

$$\begin{bmatrix} i_{Di} \\ i_{Qi} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{bmatrix} \begin{bmatrix} i_{di} \\ i_{qi} \end{bmatrix} \tag{4.15}$$

and the matrix $[Y_{NN}]$ is the real nodal admittance matrix.

4.4 Inclusion of an Infinite Bus

The infinite bus absence assumption is similar to what was mentioned in the previous chapter by selecting a machine having a reference axes (d_r, q_r) rotating in synchronism with the network reference frame axes (D, Q), i.e. rotate with synchronous angular frequency as shown in Figure (4.1).

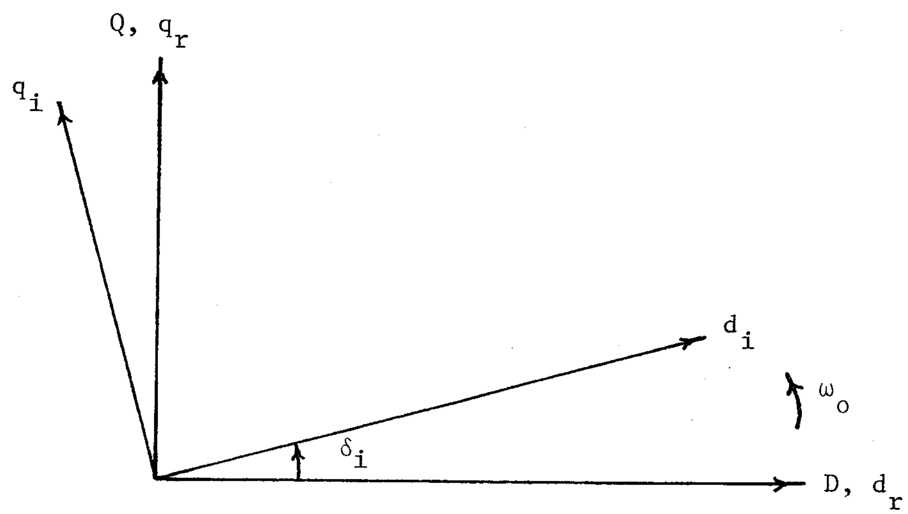


Figure 4.1 Absence of an Infinite Bus

This means that the rotor angle of that chosen machine and consequently the corresponding state should be eliminated.

If the system includes an infinite bus, equation (4.16) replaces equation (4.13).

$$\left[\begin{array}{c|c} [T_{ii}]_o^t & 0 \\ \hline 0 & I \end{array}\right] \left[\begin{array}{c} \Delta i_{mi} \\ \hline \Delta i_{ni} \end{array}\right] - \left[\begin{array}{c} Y_{NI} \\ \hline Y_{NN} \end{array}\right] \Delta \underset{\sim}{v}_N - \left[\begin{array}{c} i_{Q1} \\ -i_{D1} \\ 0 \\ 0 \end{array}\right] \Delta\delta_i = \underset{\sim}{0} \qquad (4.16)$$

4.5 State-Space Formation

To clarify how to get the state matrix from equation (4.8) the matrix $[Z_1]$ can be constructed from the 2n real network equations, the differential and algebraic equations of the first machine and partitioning of this matrix, as shown in equations (4.17) and (4.18). Figure (4.2) shows the matrix $[Z_1]$ in detail after adding the first machine to the network, where,

$$k_1 = -(i_{d1} \sin \delta_1 + i_{q1} \cos \delta_1)$$

$$k_2 = (i_{d1} \cos \delta_1 - i_{q1} \sin \delta_1)$$

This example is based on the system equations (4.10) and (4.11).

$$\begin{array}{r} \\ \text{network equations} \\ \text{inf. bus equations} \\ \text{Differential equations of machine 1} \\ \text{Algebraic equations of machine 1} \end{array} \begin{array}{c} \begin{array}{cccc} 2n & 2 & ns_1 & nv_1 \end{array} \\ \left[\begin{array}{c|c|c|c} Y_N & & h_1 & h_2 \\ \hline & & & \\ \hline 0 & & H_{11} & H_{21} \\ \hline h_3 & & H_{31} & H_{41} \end{array}\right] \end{array} \begin{array}{l} \\ 2n \\ 2 \\ ns_1 \\ nv_1 \end{array} \qquad (4.17)$$

ΔV_{D1}	ΔV_{Q1}	ΔV_{D2}	ΔV_{Q2}	ΔV_{D3}	ΔV_{Q3}	Δi_{DI}	Δi_{QI}	$\Delta\delta_1$	$\Delta\omega_1$	$\Delta\psi_{f1}$	$\Delta\psi_{kd1}$	$\Delta\psi_{kq1}$	ΔV_{f1}	Δi_{f1}	Δi_{d1}	Δi_{kd1}	Δi_{q1}	Δi_{kq1}	ΔV_{d1}	ΔV_{q1}	ΔV_{t1}	ΔT_{e1}	ΔP_{01}	$\Delta\psi_{d1}$	$\Delta\psi_{q1}$
$-g_{11}$	b_{11}	$-g_{12}$	b_{12}	$-g_{13}$	b_{13}			K_1							$\cos\delta_1$		$-\sin\delta_1$								
$-b_{11}$	$-g_{11}$	$-b_{12}$	$-g_{12}$	$-b_{13}$	$-g_{13}$			$K2$							$\sin\delta_1$		$\cos\delta_1$								
$-g_{21}$	b_{21}	$-g_{22}$	b_{22}	$-g_{23}$	b_{23}																				
$-b_{21}$	$-g_{21}$	$-b_{22}$	g_{22}	$-b_{23}$	$-g_{23}$																				
$-g_{31}$	b_{31}	$-g_{32}$	b_{32}	$-g_{33}$	b_{33}																				
$-b_{31}$	$-g_{31}$	$-b_{32}$	$-g_{32}$	$-b_{33}$	$-g_{33}$																				
$-g_{I1}$	b_{I1}	$-g_{I2}$	b_{I2}	$-g_{I3}$	b_{I3}	1																			
$-b_{I1}$	$-g_{I1}$	$-b_{I2}$	$-g_{I2}$	$-b_{I3}$	$-g_{I3}$		1																		
									-1																
									$\frac{D_1+P_m}{\omega_o}$													1			
													$-\omega_o$	$\omega_o r_{f1}$											
																$\omega_o r_{kd1}$									
																		$\omega_o r_{kq1}$							
													$\frac{1}{k_e^*}$								1				
										1				$-X_{f1}$	X_{md1}	$-X_{md1}$									
														$-X_{md1}$	X_{d1}	$-X_{md1}$								1	
											1			$-X_{md1}$	X_{md1}	$-X_{kd1}$									
																	X_{q1}	$-X_{mq1}$							1
												1					X_{mq1}	$-X_{kq1}$							
$-\cos\delta_1$	$-\sin\delta_1$							$-V_{q1}$											1						
$\sin\delta_1$	$-\cos\delta_1$							V_{d1}												1					
																			$\frac{-V_{d1}}{V_{t1}}$	$\frac{-V_{q1}}{V_{t1}}$	1				
															ψ_{q1}		$-\psi_{d1}$					1		$-i_{q1}$	i_{d1}
															$-V_{d1}$		$-V_{q1}$		$-i_{d1}$	$-i_{q1}$			1		
									$-\psi_{d1}$								$\omega_o r_{s1}$			ω_o				$-\omega_o$	
									ψ_{q1}						$\omega_o r_{s1}$				ω_o						

Figure 4.2 [Z_1] Matrix

The matrix $[Z_1]$ is partitioned as shown in equation (4.18) to eliminate the algebraic equations of the first machine.

$$\begin{array}{cc} \left[\begin{array}{c|c} Z_{11} & Z_{21} \\ \hline Z_{31} & Z_{41} \end{array}\right] & \begin{array}{c} na_1 \\ nv_1 \end{array} \end{array} \qquad (4.18)$$

where, $na_1 = 2n + 2 + ns_1$

ns_1 = number of state variables of machine 1

nv_1 = number of algebraic variables of machine 1

Using matrix reduction for equation (4.18) to keep the algebraic network equations and the differential equations of machine 1, leads to:

$$[M_{R1}] = [Z_{11} - Z_{21}\, Z_{41}^{-1}\, Z_{31}] \qquad (4.19)$$

The reduced matrix $[M_{R1}]$ is partitioned to eliminate the two real network equations of the node connected to machine 1, as follows:

$$[M_{R1}] = \begin{array}{cc} \begin{array}{cc} 2 & (na_1-2) \end{array} & \\ \left[\begin{array}{c|c} Z_{R41} & Z_{R31} \\ \hline Z_{R21} & Z_{R11} \end{array}\right] & \begin{array}{c} 2 \\ (na_1-2) \end{array} \end{array} \qquad (4.20)$$

Using matrix reduction for equation (4.20) leads to:

$$[Z_{R1}] = [Z_{R11} - Z_{R21}\, Z_{R41}^{-1}\, Z_{R31}] \tag{4.21}$$

Then the differential and algebraic equations of the second machine will be added to the matrix $[Z_{R1}]$ as shown in equation (4.22):

$$\begin{array}{l c}
 & \begin{array}{ccc} \quad na_1-2 & \quad ns_2 & \quad nv_2 \end{array} \\
\begin{array}{l} \text{2n network equations +} \\ \text{diff. equations of machine 1} \\[1ex] \text{Diff. equations of machine 2} \\[1ex] \text{Algebraic equations of} \\ \text{machine 2} \end{array} &
\left[\begin{array}{c|c:c} Z_{R1} & h_3 & h_4 \\ \hline 0 & H_{12} & H_{22} \\ \hdashline h_5 & H_{32} & H_{42} \end{array}\right]
\begin{array}{l} na_1-2 \\ ns_2 \\ nv_2 \end{array}
\end{array} \tag{4.22}$$

then the whole matrix will be partitioned, as shown in equation (4.23):

$$\begin{array}{l c}
 & \begin{array}{cc} \quad na_2 & \qquad nv_2 \end{array} \\
\begin{array}{l} \text{2n network equations + diff.} \\ \text{equations of machine 1 + diff.} \\ \text{equations of machine 2} \\[2ex] \text{Algebraic equations of} \\ \text{machine 2} \end{array} &
\left[\begin{array}{c:c} Z_{12} & Z_{22} \\ \hdashline Z_{32} & Z_{42} \end{array}\right]
\begin{array}{l} na_2 \\ nv_2 \end{array}
\end{array} \tag{4.23}$$

where, $na_2 = na_1-2 + ns_2$

Using matrix elimination for equation (4.23) to retain the remaining algebraic network equations and the differential equations of machines 1 and 2 leads to:

$$[M_{R2}] = [Z_{12} - Z_{22}\, Z_{42}^{-1}\, Z_{32}] \tag{4.24}$$

The reduced matrix $[M_{R2}]$ is partitioned to eliminate the two real network equations of the node connected to machine 2, as follows:

$$
[M_{R2}] = \begin{array}{c} \begin{array}{cc} 2 & (na_2-2) \end{array} \\ \left[\begin{array}{c|c} Z_{R42} & Z_{R32} \\ \hline Z_{R22} & Z_{R12} \end{array}\right] \end{array} \begin{array}{c} \\ 2 \\ (na_2-2) \end{array} \tag{4.25}
$$

Using matrix reduction for equation (4.25) leads to:

$$
[M_{R2}] = [Z_{R12} - Z_{R22}\, Z_{R42}^{-1}\, Z_{R32}] \tag{4.26}
$$

Then the differential and algebraic equations of the third machine will be added to the matrix $[Z_{R2}]$ and the same steps will be repeated. Finally, the last reduced matrix $[Z_{Rn}]$, which is shown in equation (4.27), will be formed after the elimination of the two real network equations of the node connected to the last machine and the two real equations of the network connected to an infinite bus.

$$
[Z_{Rn}] = \left[\begin{array}{c|c|c|c} H_1 & f_{12} & \cdots & f_{1n} \\ \hline f_{21} & H_2 & \cdots & f_{2n} \\ \hline \cdots & \cdots & \cdots & \cdots \\ \hline f_{n1} & f_{n2} & \cdots & H_n \end{array}\right] \begin{array}{c} ns_1 \\ ns_2 \\ \vdots \\ ns_n \end{array} \tag{4.27}
$$

It is noted that at each step a machine is introduced to the matrix formulation, two submatrices should be inverted. The first matrix is $[H_{4i}]$, where i is the machine number, this matrix may be inverted by parts to reduce the computation time. This can be done by partitioning the matrix, as follows:

$$[H_{4i}] = \left[\begin{array}{c|c} Z_{xi} & 0 \\ \hline Z_{ci} & Z_{Di} \end{array}\right] \tag{4.28}$$

and hence the inversion of this matrix will be:

$$[H_{4i}]^{-1} = \left[\begin{array}{c|c} Z_{xi}^{-1} & 0 \\ \hline -Z_{Di}^{-1} \cdot Z_{ci} \cdot Z_{xi}^{-1} & Z_{Di}^{-1} \end{array}\right] \tag{4.29}$$

and the second matrix which should be inverted is $[Z_{R4i}]$ matrix.

The coefficient matrices [A], [B] and [C] can be obtained from equations (4.8) and (4.9) as follows:

$$[\Delta \underset{\sim}{u}] = [Z_{dn}]\, \Delta \underset{\sim}{\dot{x}} + [Z_1]\, \Delta \underset{\sim}{x} + [Z_2]\, \Delta y \tag{4.30}$$

$$[\underset{\sim}{0}] = [Z_3]\, \Delta \underset{\sim}{x} + [Z_4]\, \Delta y \tag{4.31}$$

From equation (4.31),

$$[\Delta \underset{\sim}{y}] = -[Z_4]^{-1} [Z_3] \tag{4.32}$$

By substitution from equation (4.32) into equation (4.30), we obtain,

$$[\Delta \underset{\sim}{u}] = [Z_{dn}]\, \Delta \underset{\sim}{\dot{x}} + \{[Z_1] - [Z_2]\, [Z_4]^{-1} [Z_3]\}\, \Delta \underset{\sim}{x} \tag{4.33}$$

or, $$[\Delta \underset{\sim}{\dot{x}}] = -[Z_{dn}]^{-1} \{[Z_1] - [Z_2]\, [Z_4]^{-1} [Z_3]\}\, \Delta \underset{\sim}{x} + [Z_{dn}]^{-1}\, \Delta \underset{\sim}{u} \tag{4.34}$$

Comparing equations (4.4) and (4.34), we get:

$$[A] = -[Z_{dn}]^{-1} \{[Z_1] - [Z_2]\, [Z_4]^{-1} [Z_3]\} \tag{4.35}$$

or, $$[A] = -[Z_{dn}]^{-1}\,[Z_{Rn}] \tag{4.36}$$

$$[B] = [Z_{dn}]^{-1} \tag{4.37}$$

where, the matrix $[Z_{dn}]$ is a block diagonal matrix, one block per machine, and each block is a diagonal matrix. Equation (4.38) shows the matrix $[Z_{dn}]$ and the first block $[Z_{d11}]$ of the first machine based on the equations (4.10) and (4.11).

$$[Z_{dn}] = \left[\begin{array}{c|c|c|c} Z_{d11} & & & \\ \hline & Z_{d22} & & \\ \hline & & \cdots & \\ \hline & & & Z_{dnn} \end{array}\right] \tag{4.38a}$$

$$[Z_{d11}] = \left[\begin{array}{c|c|c|c|c} 1 & & & & \\ \hline & \frac{2H_1}{\omega_o} & & & \\ \hline & & 1 & & \\ \hline & & & 1 & \\ \hline & & & & \frac{\tau_{el}}{k^*_{el}} \end{array}\right] \tag{4.38b}$$

Since $[Z_{dn}]$ is a diagonal matrix, there is no need for using matrix inversion routine and the inversion can be done direclty by storing the inverse of the non-unity entries.

Since the algebraic variable equations are eliminated in sequence, the output matrix, [C], could be formed as a diagonal matrix, one block per machine, as follows:

$$[C] = \left[\begin{array}{c|c|c|c} C_1 & & & \\ \hline & C_2 & & \\ \hline & & \cdots & \\ \hline & & & C_n \end{array}\right] \tag{4.39}$$

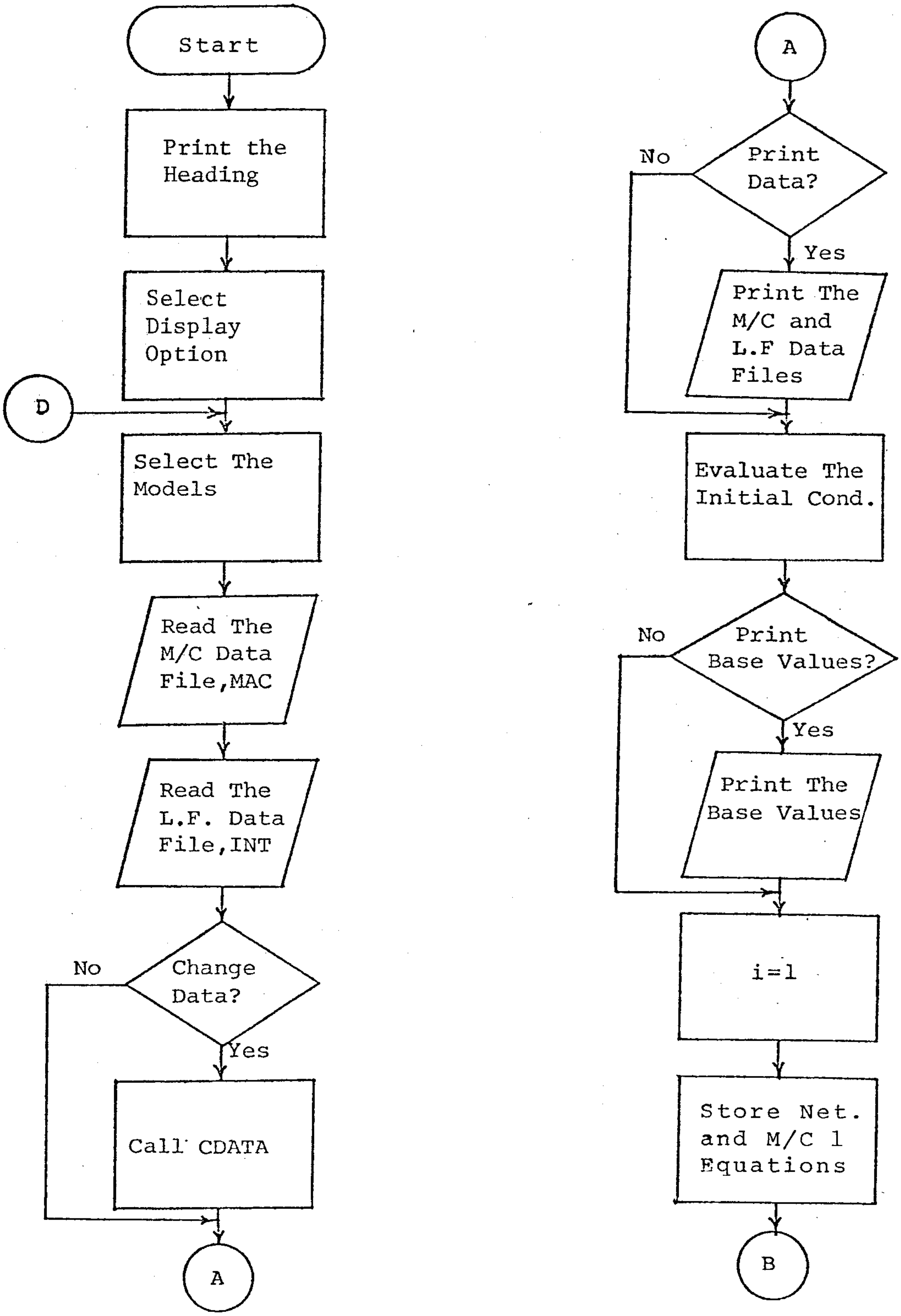
where, $C_i = -[H_{4i}]^{-1}\ [H_{3i}]$, and

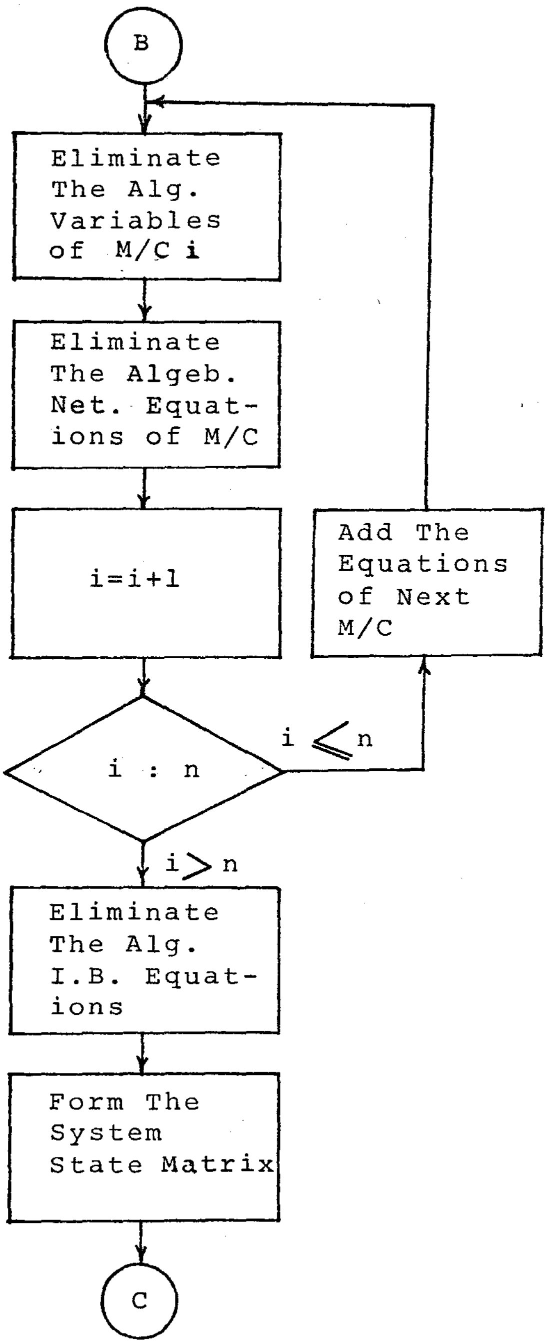
i is the machine number

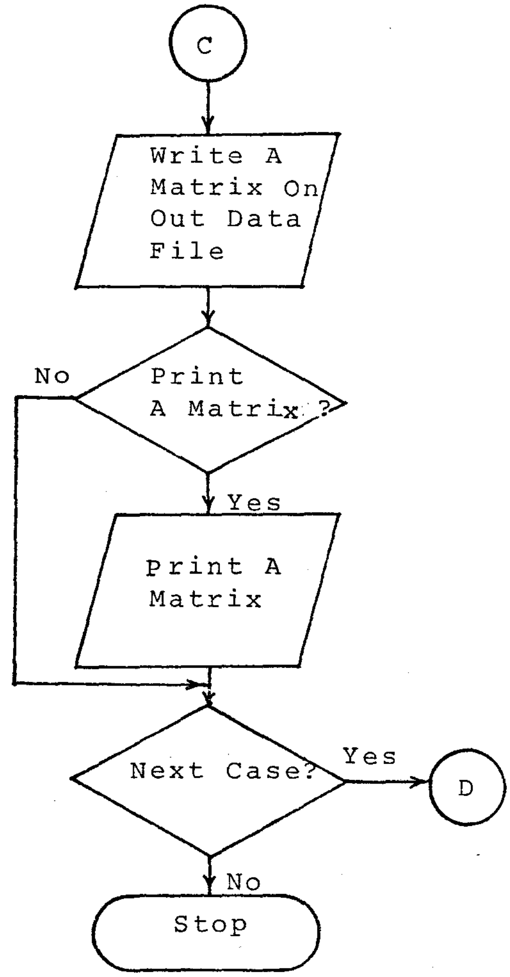
The program package description, user guide and program listing have been presented in detail in another internal report [36]. The flow chart of this program is presented in Figure (4.3). Some of subsystem models, which are shown in Table (2.1), have been utilized in this program. The models used are: G_2, G_3, S_1, T_o, T_1, E_o, E_1 and E_4.

Figure 4.3 "ELIM"

PROGRAM FLOW CHART




B
Eliminate The Alg. Variables of M/C i
Eliminate The Algeb. Net. Equat-ions of M/C
i=i+1
Add The Equations of Next M/C
i : n
i ≤ n
i > n
Eliminate The Alg. I.B. Equat-ions
Form The System State Matrix
C


C
Write A Matrix On Out Data File
Print A Matrix ?
No
Yes
Print A Matrix
Next Case?
Yes
D
No
Stop

4.6 Eigenvalue Sensitivity

The sensitivity of the system eigenvalues with respect to control parameters can be expressed in terms of the derivatives of the system state matrix [A] with respect to any of these parameters and the normal and the transposed eigenvectors. Following the same expressions used in Chapter 3, for first and second-order sensitivities,

$$\frac{\partial \lambda_i}{\partial \xi} = \frac{\{\frac{\partial [A]}{\partial \xi} \underset{\sim}{V}_i\} \underset{\sim}{W}_i}{(\underset{\sim}{V}_i \underset{\sim}{W}_i^t)}$$

$$\frac{\partial^2 \lambda_i}{\partial \xi^2} = \frac{[\frac{\partial^2 [A]}{\partial \xi^2} \underset{\sim}{V}_i \underset{\sim}{W}_i] + 2\,[(\frac{\partial [A]}{\partial \xi} \sum_{j=1}^{ns} \alpha_{ij} \underset{\sim}{V}_j) \underset{\sim}{W}_i]}{(\underset{\sim}{V}_i \underset{\sim}{W}_j)}$$

An expression has been derived for the [A] matrix first-order derivatives with respect to a system control parameter, ξ, using the "DIRECT ELIMINATION" technique, as follows:

$$\frac{\partial [A]}{\partial \xi} = -[Z_{dn}]^{-1} \frac{\partial [Z_{Rn}]}{\partial \xi} - \{\frac{\partial [Z_{dn}]^{-1}}{\partial \xi}\} [Z_{Rn}] \qquad (4.39)$$

A detailed derivation of this expression is given in Appendix B.

The system control parameters τ_{ch} (steam turbine chest time constant) and τ_e (exciter time constant) exist in the matrix $[Z_{dn}]$, hence $\frac{\partial [Z_{Rn}]}{\partial \xi} = 0$ and for these parameters,

$$\frac{\partial [A]}{\partial \xi} = -\{\frac{\partial [Z_{dn}]^{-1}}{\partial \xi}\} [Z_{Rn}] \qquad (4.40)$$

The exciter gain, k_e, exists in both the matrices $[Z_{dn}]$ and $[Z_{Rn}]$. Consequently for this control parameter, the expression in equation (4.39) is used.

The other control parameters, like stabilizer gain, stabilizer time constant, voltage sensor time constant, etc., exist in the $[Z_{Rn}]$. matrix only. Hence, $\frac{\partial [Z_{dn}]^{-1}}{\partial \xi} = 0$ and for these parameters,

$$\frac{\partial [A]}{\partial \xi} = -[Z_{dn}]^{-1} \{\frac{\partial [Z_{Rn}]}{\partial \xi}\} \tag{4.41}$$

where,

the matrix $[Z_{dn}]$ is a block diagonal matrix, each block is a diagonal matrix, most of its entries are unity, and the matrix $[Z_{Rn}]$ is as shown in equation (4.42),

$$[Z_{Rn}] = \begin{array}{c} \left[\begin{array}{c|c|c|c} H_1 & f_{12} & --- & f_{1n} \\ \hline f_{21} & H_2 & --- & f_{2n} \\ \hline --- & --- & --- & --- \\ \hline f_{n1} & f_{n2} & --- & H_n \end{array}\right] \end{array} \begin{array}{l} n_{s1} \\ n_{s2} \\ \vdots \\ n_{sn} \end{array} \tag{4.42}$$

The off-diagonal matrices, $[f_{ij}]$, do not include the system control parameters. The diagonal matrices, $[H_i] = [H_{1i}(\xi) - \overline{h}_{ii}]$, which means that the system control parameters exist only in the matrices $[H_{1i}(\xi)]$, hence,

$$\frac{\partial [f_{ij}]}{\partial \xi} = 0 \qquad i,\ j = 1,\ 2,\ \ldots,\ n \tag{4.43}$$

and,

$$\frac{\partial[H_i]}{\partial\xi} = \frac{\partial[H_{1i}(\xi)]}{\partial\xi} \qquad i = 1, 2, \ldots, n \tag{4.44}$$

From equations [(4.42) - (4.44)], the partial derivatives of the matrix $[Z_{Rn}]$ with respect to system control parameter, ξ, could be obtained as follows:

$$\frac{\partial[Z_{Rn}]}{\partial\xi} = \text{diag}\ \{\frac{\partial[H_{1i}(\xi)]}{\partial\xi}\} \tag{4.45}$$

Substituting from equation (4.45) into equation (4.39), yields,

$$\frac{\partial[A]}{\partial\xi} = -[Z_{dn}]^{-1}\ \text{diag}\ \{\frac{\partial[H_{1i}(\xi)]}{\partial\xi}\} - \{\frac{\partial[Z_{dn}]^{-1}}{\partial\xi}\}[Z_{Rn}] \tag{4.46}$$

CHAPTER 5

VALIDATION AND COMPARISON

5.1 Program Validation

The computer programs developed have been successfully tested. The stability of a synchronous machine connected to an infinite bus through a transmission line has been chosen as the test problem to illustrate the validity of these programs. Two specific examples have been considered, namely, a simplified second-order system (classical generator model) and a seventh-order system (detailed generator model). The results obtained in both examples are presented and compared with other results in the literature.

5.1.1 The Simplified Second-Order System Example

In this example, a 2-axis machine representation is considered with the field circuit in the direct-axis. The damper effect is neglected. Both the flux linkages and the input mechanical power are assumed constant, i.e., no excitation and governor controls are represented. Hence, the system can be easily described by a constant voltage behind transient reactance (classical model) as shown in Fig. (5.1). The parameters and the operating point are listed in Table (5.1).

The system equations (5.1), also given in Appendix A, are linearized around a steady-state operating condition, and have been developed in the state-space form using the techniques described in Chapters 3 and 4. These techniques were programmed on McMaster University CYBER 170/730 computer.

$$\begin{aligned} \Delta\dot{\delta}_i &= \Delta\omega_i \\ \Delta\dot{\omega}_i &= \frac{-D'}{2H_i}\Delta\omega_i - \frac{\omega_o}{2H_i} k_{1ij}\Delta\delta_i \end{aligned} \tag{5.1}$$

where

$$k_{1ij} = \sum_{\substack{j=1 \\ j\neq i}}^{n} E_{i0}\, E_{j0}\, Y_{ij} \sin\,(\theta_{ij0} - \delta_{i0} + \delta_{j0})$$

$Y_{ij}\underline{\big|\theta_{ij}}$ = negative of the transfer admittance between nodes i + j

The computed eigenvalues of this system using the developed computer programs are:

$$\lambda_{1,2} = -0.0714 \pm j\ 12.3326 \tag{5.2}$$

The stability of the torque-angle loop of this system, i.e., the behaviour of the rotor angle and speed, following a small disturbance has been analysed by deMello and Concordia [12]. They have shown that the characteristic equation of this system is as follows:

$$S^2 + (D/2H)\ S + (\omega_0 k_1/2H) = 0 \tag{5.3}$$

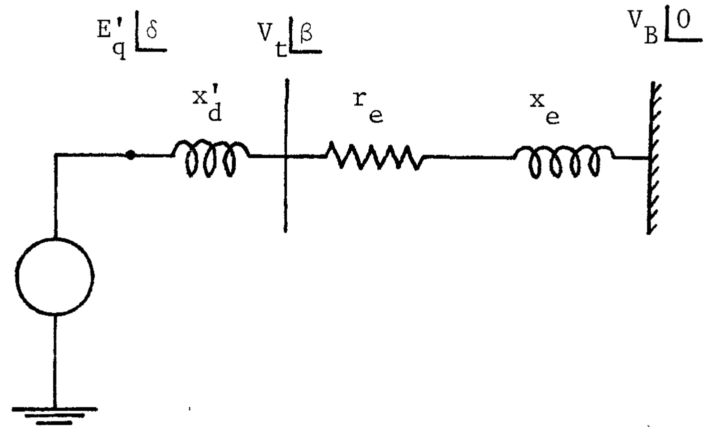


Fig. (5.1): Equivalent Circuit of One Machine Connected to an Infinite Bus Through a Transmission Line

$H = 3.5$ sec.	$D = 1.0$	
$X'_d = 0.235$	$r_e = 0.005$	$X_e = 0.133$
$P_B = 0.5$	$Q_B = 0.1$ (lag)	

Table (5.1): Data for Classical Model System

The roots of this equation are:

$$\lambda_{1,2} = \frac{-D}{4H} \pm j \left[\frac{\omega_0 k_1}{2H} - \frac{D^2}{16H^2}\right]^{1/2} \qquad (5.4)$$

where D is the damping coefficient, M is the inertia coefficient, H is the inertia constant and ω_0 is the synchronous speed. k_1 is the synchronizing power coefficient which is computed as follows:

$$k_1 = \frac{E'_{q0} V_B}{A} [r_e \sin \delta_0 + (X_e + X'_d) \cos \delta_0] + \frac{i_{q0} V_B}{A} [(X_q - X'_d)(X_e + X_q) \sin \delta_0 = r_e (X_q - X'_d) \cos \delta_0] \qquad (5.5)$$

where

$$A = [r_e^2 + (X_e + X'_d)(X_q + X_e)] ,$$

E'_q is the voltage proportional to the direct-axis flux linkages, V_B is the infinite bus voltage, r_e is the transmission line resistance, X_e is the transmission line reactance, X'_d is the direct-axis transient reactance, X_d is the direct-axis synchronous reactance, X_q is the quadrature-axis synchronous reactance and δ is the angle between quadrature axis and the finite bus.

The system eigenvalues are computed by substituting the parameter values of Table (5.1) in equations (5.4 - 5.5), as shown in (5.6),

$$\lambda_{1,2} = -0.0714 \pm j\ 12.3329 \qquad (5.6)$$

Comparing the eigenvalues in (5.2) and (5.6), it could be seen that they are the same which consequently proves the validity of the devised programs.

5.1.2 The Seventh-Order System Example

In this 7th order example, a detailed generator model (5th order), is chosen where the synchronous machine is represented with one field winding and one damper winding in the direct-axis, and one damper winding in the quadrature-axis. The stator transient is included and the mechanical shaft system is represented by one rotating mass, (2nd order model), corresponding to the generator rotor.

A single line diagram of the generator connected to an infinite bus through an external reactance is shown in Fig. (5.2). The system parameter values and the machine working point are given in Table (5.2), this data is taken directly from reference [37]. The system state variables are: $\Delta\delta$, $\Delta\omega$, $\Delta\psi_f$, $\Delta\psi_d$, $\Delta\psi_{kd}$, $\Delta\psi_q$ and $\Delta\psi_{kq}$.

The system initial conditions are as follows:

$v_{to} = 1.0$	$v_{do} = 0.652$	$v_{qo} = 0.758$
$i_o = 1.0$	$i_{do} = 0.917$	$i_{qo} = 0.398$
$\psi_{do} = 0.7590$	$\psi_{qo} = -0.630$	$P_{mo} = 0.901$
$\delta = 64.25$ (deg.)	$v_{fd} = 0.001$	$i_{fo} = 1.496$

Assuming that the generator is working under both constant field voltage and constant mechanical power imput, the system computed eigenvalues are shown in (5.7),

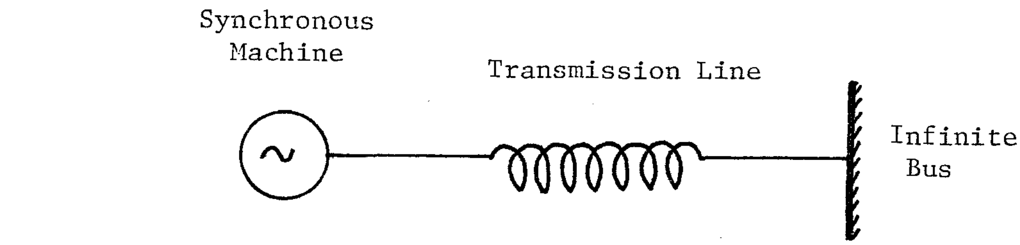


Fig. (5.2) System Line Diagram

$X_d = 1.7$	$X_f = 1.65$	$X_{kd} = 1.605$
$X_{md} = 1.55$	$X_q = 1.64$	$X_{mq} = 1.49$
$X_{kq} = 1.526$	$r_s = 0.0011$	$r_f = 0.00074$
$r_{kd} = 0.0131$	$r_{kq} = 0.054$	$X_e = 0.4$
$F_B = 50$ Hz	$H = 2.37$	$D = 0.0$
$P_{go} = 0.9$	$Q_{go} = 0.436$ (lag)	$v_{to} = 1.0$

Table (5.2) Data for Single Machine Infinite Bus System (7th order)

$$
\begin{aligned}
\lambda_1 &= -34.7922 + j\ 992.0015 \\
\lambda_2 &= -34.7922 - j\ 992.0015 \\
\lambda_3 &= -0.4814 + j\ 8.7725 \\
\lambda_4 &= -0.4814 - j\ 8.7725 \\
\lambda_5 &= -38.4784 \\
\lambda_6 &= -31.2552 \\
\lambda_7 &= -0.1638
\end{aligned} \tag{5.7}
$$

The evaluation of the 7th-order system eigenvalues using the method described in reference [37], under the same initial conditions, had led to the following values (5.8)

$$
\begin{aligned}
\lambda_1 &= -29.50 + j\ 314.2 \\
\lambda_2 &= -29.50 - j\ 314.2 \\
\lambda_3 &= -0.4464 + j\ 8.777 \\
\lambda_4 &= -0.4464 - j\ 8.777 \\
\lambda_5 &= -39.71 \\
\lambda_6 &= -31.91 \\
\lambda_7 &= -0.1639
\end{aligned}
$$

Comparing the obtained eigenvalues in (5.7 - 5.8) showed that the first two eigenvalues, which are corresponding to the stator transient mode, are different and that is due to the absence of the network transients in the studied techniques, "PQR" and "ELIM". Comparing the other five eigenvalues showed that they are close to each other within an average tolerance of (about 3.2%).

5.2 Comparison between the "PQR" and "ELIM" Techniques

In this section, a comparison between the "PQR" and the direct elimination "ELIM" matrix formulation techniques is presented from the point of view of required core storage and computation time and the effect upon eigenvalue sensitivity computation.

5.2.1 Matrix Formulation Comparison

A. "PQR" Matrix Formulation

The differential and algebraic equations of all machines in the system are formed and stored once at the beginning of the PQR method, as shown in equation (5.9). P, Q and R are constant real matrices associated with the state variable vector $\Delta \underset{\sim}{x}$, the algebraic variable vector $\Delta \underset{\sim}{y}$, and the control variable vector $\Delta \underset{\sim}{u}$.

$$[P]\begin{bmatrix} \Delta \dot{\underset{\sim}{x}} \\ \hline \Delta \underset{\sim}{y} \end{bmatrix} = [Q]\,\Delta \underset{\sim}{x} + [R]\;\;\Delta \underset{\sim}{u} \tag{5.9}$$

To avoid inversion of the whole matrix [P], which may be large, the P, Q and R matrices are partitioned (equation 5.10) as illustrated in Chapter 3.

$$\begin{array}{cc} & \begin{array}{cc} ns & nv \end{array} \\ \begin{array}{c} ns \\ nv \end{array} & \left[\begin{array}{c|c} I & PA \\ \hline 0 & PB \end{array}\right] \end{array} \begin{bmatrix} \Delta \dot{\underset{\sim}{x}} \\ \hline \Delta \underset{\sim}{y} \end{bmatrix} = \begin{array}{c} ns \\ \begin{bmatrix} QA \\ \hline QB \end{bmatrix} \end{array} \Delta \underset{\sim}{x} + \begin{bmatrix} RA \\ \hline 0 \end{bmatrix} \;\; \Delta \underset{\sim}{u} \tag{5.10}$$

As a result of this partitioning, only matrix [PB], of order nv, is inverted, where nv is the total number of the algebraic variables in the system. For further simplification, the inversion of the whole matrix [PB] is avoided by arranging the system algebraic variables in a certain manner. The approach of Zein El-Din shown in equation (5.11),

$$\Delta \underset{\sim}{y} = \Delta[\underset{\sim}{i}_{m1}^{t}, \underset{\sim}{i}_{m2}^{t}, \ldots, \underset{\sim}{i}_{mn}^{t}, \underset{\sim}{v}_{m1}^{t}, \underset{\sim}{v}_{m2}^{t}, \ldots, \underset{\sim}{v}_{mn}^{t}, v_{t1}, v_{t21}, \ldots,$$
$$v_{tn1}, T_{e1}, T_{e2}, \ldots, T_{en}, P_{o1}, P_{o2}, \ldots, P_{on}, i_{DI}, i_{QI}, \underset{\sim}{v}_{N}^{t}]^{t}$$

(5.11)

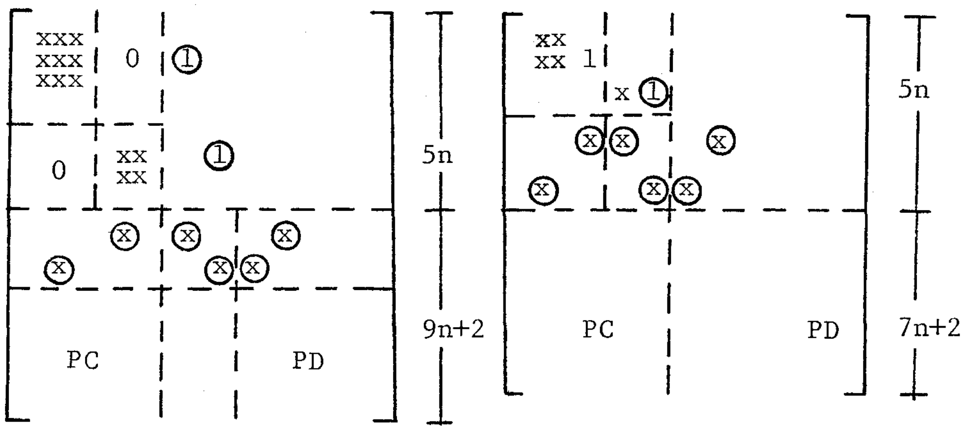
Matrix [PB] has the form shown in equation (5.12) for a 5th order generator model. The partitioning of the matrix [PB] in this case shows that we have to deal only with the inversion of two sub-matrices [PX] and [PN]. Sub-matrix [PX] is a block diagonal which includes all the machine reactances, two blocks per machine, the first one of order 3 and the second of order 2.

$$[PB] = \begin{array}{c} \begin{array}{cc} 5n & 7n+2 \end{array} \\ \left[\begin{array}{c|c} PX & PE \\ \hline PC & PD \end{array}\right] \end{array} = \left[\begin{array}{c|cccc} \begin{array}{c|c} \begin{matrix} xxx \\ xxx \\ xxx \end{matrix} & 0 \\ \hline 0 & \begin{matrix} xx \\ xx \end{matrix} \end{array} & & & 0 & \\ \hline & I & & & PT \\ PC & PS & I & & \\ & & & I & PI \\ & & & & PN \end{array}\right] \begin{array}{c} 5n \\ 2n \\ 3n \\ 2 \\ 2n \end{array}$$

(5.12)

The second sub-matrix [PN] to be inverted represents the reduced real network admittance of order 2n, where n is the number of generators in the system. Sub-matrix [PI], of order 2, only exists if there is an infinite bus.

When applying this partitioning to 3rd-order and/or 1st-order generator models, the matrix [PE] which was originally null in the 5th order case is no longer null, as shown in Figure (5.3). This will affect the efficient procedure used for finding the matrix inverse, which is shown in (5.13),



(a) 3rd-order generator model (b) 1st-order generator model

Fig. (5.3) Matrix [PB]

$$[PB]^{-1} = \left[\begin{array}{c|c} (PX-PE\cdot PD^{-1}\cdot PC)^{-1} & -PX^{-1}\cdot PE(PD-PC\cdot PX^{-1}\cdot PE)^{-1} \\ \hline -PD^{-1}\cdot PC(PX-PE\cdot PD^{-1}\cdot PC)^{-1} & (PD-PC\cdot PX^{-1}\cdot PE)^{-1} \end{array}\right] \quad (5.13)$$

It is clear that this partitioning is very efficient only when applied to a system where all the machines are represented by a 5th-order generator model.

The partitioning approach developed in this thesis which ensures the existence of a null matrix irrespective of the generator model order used is shown in equation (5.15). This has been achieved after reordering the system algebraic variables, as shown in equation (5.14),

$$\Delta \underset{\sim}{y} = \Delta[\underset{\sim}{i}_{m1}{}^{t}, \underset{\sim}{i}_{m2}{}^{t}, \ldots, \underset{\sim}{i}_{mn}{}^{t}, \underset{\sim}{v}_{m1}{}^{t}, \underset{\sim}{v}_{m2}{}^{t}, \ldots, \underset{\sim}{v}_{mn}{}^{t}, \underset{\sim}{v}_{N}{}^{t}, v_{t1}, v_{t2}, \ldots,$$

$$v_{tn}, T_{e1}, T_{e2}, \ldots, T_{en}, P_{o1}, P_{o2}, P_{on}, i_{DI}, i_{QI}]^{t} \qquad (5.14)$$

$$[PB] = \left[\begin{array}{c|c} PY & 0 \\ \hline PZ & I \end{array}\right] \begin{array}{c} \ell \\ 3n+2 \end{array} \qquad (5.15)$$

This partitioning requires the inversion of only one matrix, [PY] of order ℓ which is equal to (5m + 7k + 4n), where m is the number of generators represented by a 1st or 5th-order model and k is the number of generators represented by a 3rd order model. The order of the matrix [PY] is larger than in the previous case, but this partitioning method is applicable to all generator models.

B. "ELIM" Matrix Formulation

In the "ELIM" matrix formulation technique, the linearized 2n equations relating the machine currents and network nodal voltages are formed and stored first, then the differential and algebraic equations of the first machine are constructed and stored in the matrix form shown in equations (5.16) and (5.17), where most entries of the input vector (Δu_1) are zero.

$$\begin{array}{c} 2n \\ ns_1 \\ nv_1 \end{array} \begin{bmatrix} \underset{\sim}{0} \\ \hline \Delta \underset{\sim}{u}_1 \\ \hline \underset{\sim}{0} \end{bmatrix} = [Z_{d1}] \begin{bmatrix} \underset{\sim}{0} \\ \hline \Delta \dot{\underset{\sim}{x}}_1 \\ \hline \underset{\sim}{0} \end{bmatrix} + [Z_1] \begin{bmatrix} \Delta \underset{\sim}{v}_N \\ \hline \Delta \underset{\sim}{x}_1 \\ \hline \Delta \underset{\sim}{y}_1 \end{bmatrix} \tag{5.16}$$

or,

$$\begin{bmatrix} \underset{\sim}{0} \\ \hline \Delta \underset{\sim}{u}_1 \\ \hline \underset{\sim}{0} \end{bmatrix} = \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & Z_{d11} & 0 \\ \hline 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \underset{\sim}{0} \\ \hline \Delta \dot{\underset{\sim}{x}}_1 \\ \hline \underset{\sim}{0} \end{bmatrix} + \left[\begin{array}{c|c|c} y_N & h_1 & h_2 \\ \hline 0 & H_{11} & H_{12} \\ \hline h_3 & H_{31} & H_{41} \end{array}\right] \begin{bmatrix} \Delta \underset{\sim}{v}_N \\ \hline \Delta \underset{\sim}{x}_1 \\ \hline \Delta \underset{\sim}{y}_1 \end{bmatrix} \tag{5.17}$$

where $[Y_N]$ is the real network admittance matrix and $[Z_{d11}]$ is a diagonal matrix with most of its entries equal to one. After the addition of machine i, two successive matrix eliminations are done as illustrated in Chapter 4. The first elimination is to remove the algebraic equations of the added machine which requires the inversion of the matrix $[H_{4i}]$ of order n_{vi} (number of machine i algebraic variables). The matrix $[H_{4i}]$ is partitioned as shown in equation (5.18) to facilitate a quick inversion for different generator model orders.

$$[H_{4i}] = \left[\begin{array}{c|c} H_x & 0 \\ \hline H_c & I \end{array}\right] \begin{array}{c} r \\ 3 \end{array} \tag{5.18}$$

r is equal to 7 when the generator is represented by a 1st or 5th order model and is equal to 9 when the generator is represented by a 3rd order model. It is clear from equation (5.18) that only the matrix $[H_x]$ of order r has to be inverted each time a generating unit is added.

This matrix formulation approach compared with the "PQR" approach is more economical in the computer storage because the matrices are stored for each machine separately, in sequence, to build up the system coefficient matrices. For example, for n, 5th order machines, the storage capacity of the matrix [PB] using the "PQR" technique, (which includes the machine algebraic equations and the network equations), requires the storage of a matrix of order 12n (number of machine algebraic variables are 12) which means that $144n^2$ entries are stored, while using the "ELIM" technique, the corresponding matrices to be stored are $[H_4]$ of order 10 and $[Y_N]$ of order 2n which means that $(4n^2 + 100)$ entries are stored. Another advantage of the "ELIM" Matrix formulation approach is the reduction of the matrix inversion time, that is because the elimination of the algebraic network equations are not performed once as in the "PQR" technique, but it is performed successively two by two. Both the storage and time requirements of the two methods will now be studied in detail.

5.2.2 Computation Time Comparison

For both the "PQR" and "ELIM" techniques, the state matrix and eigenvalue computation times were obtained for different cases, where the system had different orders ranging from 2 up to 39, as shown in Table (5.3). Both techniques were implemented on the CYBER 170/730 McMaster University Computer.

For both the "PQR" and "ELIM" programs, the eigenvalue computation time is the same, as the same eigenvalue evaluation subroutine was used and the two techniques produce identical state matrices. On the other hand, the state matrix computation time is different. It was found that this time using the "ELIM" approach is less than that of the "PQR". The average ratio between the "PQR" and "ELIM" times was found to be around 2.2 for the range of system orders studied, (Table 5.3).

Since the computation time is a function of the number of arithmetic operations (multiplication, addition and subtraction), an estimate of these operations has been done in terms of number of machines (n), number of state variables (ns) and number of algebraic variables (nv), as follows:

For the "PQR" Method:

The number of multiplication operations = $24n^3 + 162n^2 + [2 \times ns \times (nv)^2]$

The number of addition operations = $243n^3 + 35n^2 - 18n + [2 \times ns \times (nv^2 - nv)]$

The number of subtraction operations = $(ns)^2$

For the "ELIM" Method:

The number of multiplication operations = $[(NT + ns_1) \times nv_1]^2 \times nv_1$

$$+ \sum_{i=2}^{n} [\{[NT - 2(i-1)] + \sum_{j=1}^{i} ns_j\}^2 \times nv_i]$$

$$+ \sum_{k=1}^{n} \{[NT - 2xk) + \sum_{j=1}^{k} ns_j]^2 \times 2\}$$

The number of addition operations $= [(NT + ns_1) \times nv_1]^2 \times (nv_1 - 1)$

$$+ \sum_{i=2}^{n} \{[NT - 2(i-1)] + \sum_{j=1}^{i} ns_j\}^2 \times (nv_i - 1)$$

$$+ \sum_{k=1}^{n} [(NT - 2xk) + \sum_{j=1}^{k} ns_j]^2$$

where NT is the order of the real network admittance matrix $= 2n + 2$ (if there is an infinite bus).

To compare between the number of arithmetic operations for both the "PQR" and "ELIM" techniques, a 2-machine infinite bus system example is presented in this section, where the selected number of state variables is 13 and the algebraic variables are 12 for each machine, i.e.,

$ns_{(tot)} = 2 \times 13 = 26$ state variables

& $nv_{(tot)} = (2 \times 12) + 2 = 26$ algebraic variables

The total number of arithmetic operations are calculated for both techniques and listed in Table (5.4). From this table, it can be seen that the number of multiplication operations using the "PQR" approach is almost twice that using the "ELIM" approach and the number of addition and subtraction operations using the "PQR" is about double that using the "ELIM" method. This is because the formulation of the state matrices requires constructing the

TABLE (5.3) State Matrix and Eigenvalue Computation Time Comparison

System Order	Eigenvalue Computation Time (sec)	State Matrix Computation Time			
		"PQR"		"ELIM"	
		Observed	Estimated	Observed	Estimated
2	0.30	0.46	0.41	---	---
7	0.44	0.72	0.71	0.33	0.28
10	0.65	0.81	0.91	0.36	0.38
13	0.96	0.94	1.16	0.42	0.52
18	1.40	1.67	1.7	0.77	0.78
24	3.20	2.58	2.46	1.18	1.14
29	5.00	3.28	3.15	1.52	1.46
34	8.00	3.89	3.87	1.78	1.79
39	11.00	4.58	4.7	2.14	2.16
140	---	---	36.7	---	15.8

Note: Computation time includes any necessary inversion.

Table (5.4) Number of Arithmetic Operations for both "PQR" and "ELIM"

Method	PQR	ELIM
No. of multiplication operations	37,744	20,288
No. of addition and subtraction operations	36,924	17,500

P, Q and R matrices. Since these matrices are large, successive operations on them require a longer time. In the "ELIM" formulation, the state matrices are formulated by the elimination of variables taken one machine at a time.

From the observations of computation times for both techniques, an estimation equation was derived using the Least Mean Square method. It was found that the optimum equations to predict time as a function of system order are as follows:

$$T_{(PQR)} = 0.2 + 0.05N + 0.0015(N)^2 \tag{5.19}$$

$$T_{(ELIM)} = -0.04 + 0.04N + 0.0005(N)^2 \tag{5.20}$$

The estimated results are shown in Table (5.3) beside the observed values and in Figure (5.4) to demonstrate that the error is very small. Additionally, the estimation equations are used to predict the computation time required for a system of order 140. This larger system is discussed in section (5.2.4) where storage requirements are computed. To place the matrix computation time in perspective, the eigenvalue computation times are also listed in Table (5.3).

5.2.3 Matrix Inversion Time Comparison

Since the matrix inversion time is relatively long for a large power system and has to be performed every time as a parameter setting is changed, an analytical comparison has been done between the two methods to choose the most economical method for dynamic stability analysis of large power system. The matrix inversion time is a function of the number

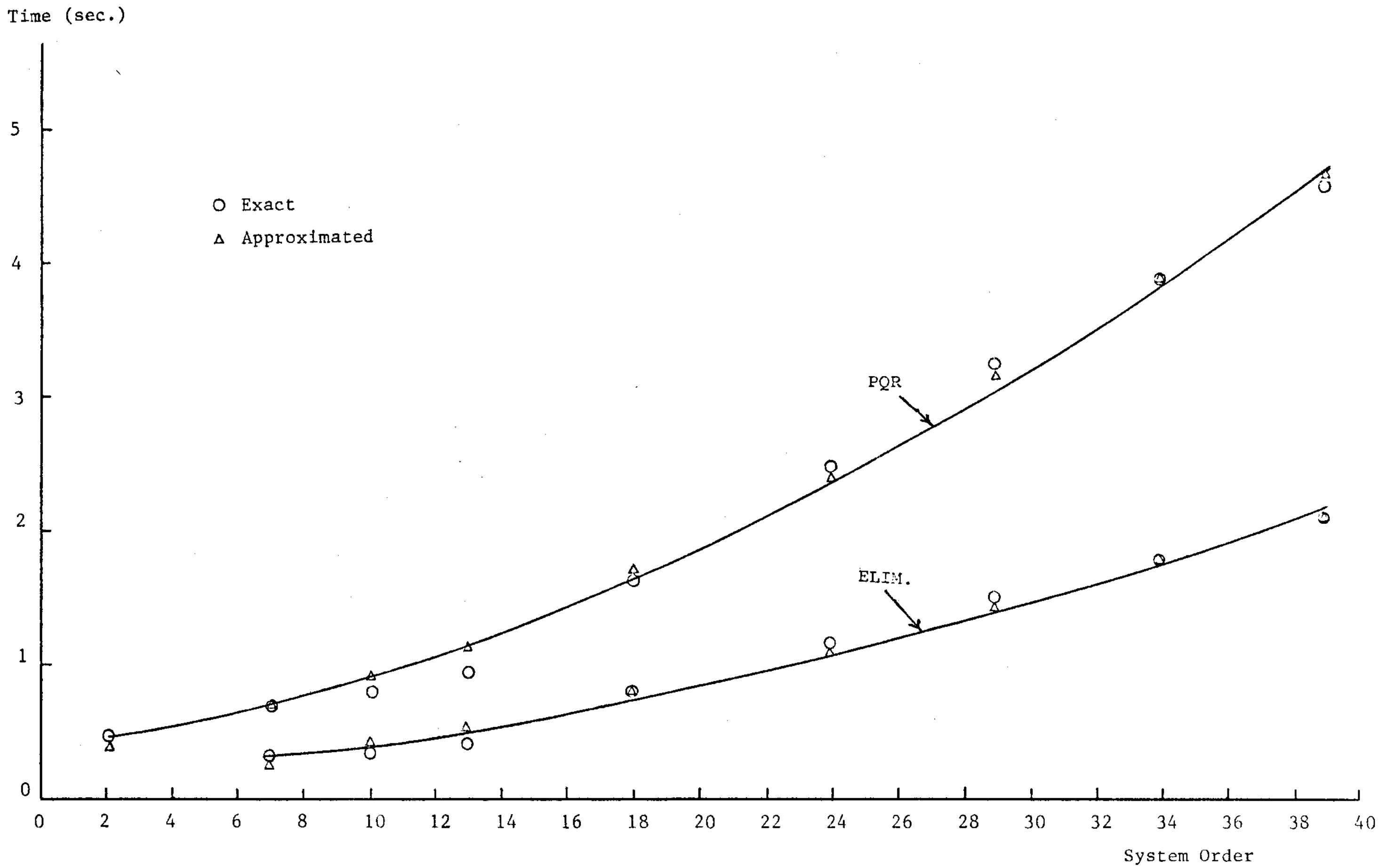


Figure (5.4) State Matrix Computation Time Comparison of arithmetic operations (division, multiplication and addition), which are necessary to carry out a solution of ℓ equations.

For a Gauss-elimination method, which is used for matrix inversion in both programs, it is possible to estimate the numbers of these operations as a function of the matrix order. It is found [10] that the number of arithmetic operations of an (ℓ x ℓ) matrix are as follows:

- The number of division operations = ℓ
- The number of multiplication operations = $\ell^3 - 1$ (5.21)
- The number of addition operations = $\ell^3 - 2\ell^2 + \ell$

Hence,

$$\begin{aligned} \text{Inversion time} = {} & \text{time of one division} \times \ell \; + \\ & \text{time of one multiplication} \times (\ell^3 - 1) \; + \\ & \text{time of one addition} \times (\ell^3 - 2\ell^2 + \ell) \end{aligned} \tag{5.22}$$

For a CYBER 170/730 computer, it is known [20] that:

- Time of one division = 5.6 μs
- Time of one multiplication = 1.0 μs (5.23)
- Time of one addition = 0.3 μs

In Chapter 3, it is shown that, to produce the state matrix, matrix manipulation using the "PQR" technique requires the inversion of n machine reactance matrices of order 5 (stator transients are included), and the real network matrix of order 2n. So, the time required for inversion as a function of machine number could be formulated using equations (5.21), (5.22) and (5.23), as follows:

$$T_{inv} = 5.6\ell + (\ell^3 - 1) + 0.3\ (\ell^3 - 2\ell^2 + \ell)$$

$$= 1.3\ell^3 - 0.6\ell^2 + 5.9\ell - 1 \tag{5.24}$$

Applying equation (5.24) to the "PQR" approach, the inversion time, T_1, will be:

$$T_1 = 1.3\ (2n)^3 - 0.6\ (2n)^2 + 5.9\ (2n) - 1 + n[1.3\ (5)^3 - 0.6\ (5)^2 + 5.9\ (5) - 1 \tag{5.25}$$

$$= 10.4\ n^3 - 2.4\ n^2 + 187.8\ n$$

From Chapter 4, it can be found that matrix manipulation to produce the state matrix, using the "ELIM" technique, for the same machine model, requires the inversion of n matrices of order 10 and n matrices of order 2. Applying equation (5.24) to the "ELIM" approach, the inversion time, T_2, could be formulated as follows:

$$T_2 = n\ [1.3\ (2)^3 - 0.6\ (2)^2 + 5.9\ (2) - 1] + n\ [1.3\ (10)^3 - 0.6\ (10)^2 + 5.9\ (10) - 1]$$

$$= 1316.8\ n \tag{5.26}$$

Equating equations (5.25) and (5.26), we get:

$$n \simeq 11$$

From Figure (5.5), we can see that when the number of machines in the system equals fifty, the matrix inversion time using the "PQR" technique will be about twenty times that using the "ELIM" technique.

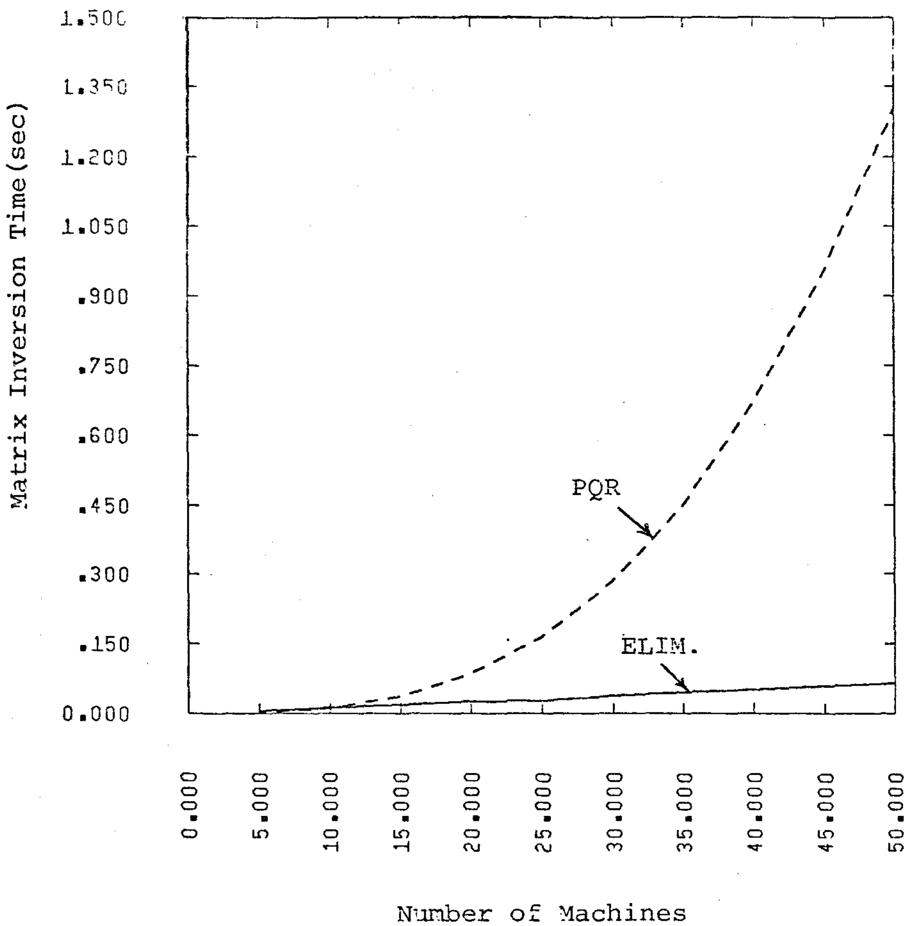


Figure 5.5 Comparison of Matrix Inversion Time

From the previous analysis, we could conclude that the matrix inversion time using the "PQR" approach is less than that using the "ELIM" approach for a system including number of machines less than eleven, but for a larger system (greater than eleven machines), the matrix inversion time using the "ELIM" technique will be less, and consequently, this matrix formulation method will be more economical than the "PQR" matrix formulation method.

5.2.4 Storage Requirement Comparison

Two examples are presented in this section to compare between the required core storage for both the "PQR" and "ELIM" techniques.

The first example is a 3 machine infinite bus system (System I) where the selected order of each machine is 13, as shown in Table (5.5). The second example is a 20 machine infinite bus system (System II), where the first five machines are represented by a 13-order model and the rest of the machines are represented by a 5-order model (less detailed model). For each machine in both examples, the algebraic variables are 12, so the total system algebraic variables, taking into account the infinite bus, will be $(12n + 2)$.

SYSTEM I							SYSTEM II						
No. of Machines	Subsystem Model Order				Machine Order	Total Order	No. of Machines	Subsystem Model Order				Machine Order	Total Order
	G	S	E	T				G	S	E	T		
3	5	2	4	2	13	39	5	5	2	4	2	13	65
–	–	–	–	–	--	--	15	1	2	2	–	5	75
SYSTEM ORDER = 39							SYSTEM ORDER = 140						

TABLE (5.5) System Orders

A. "PQR" Core Storage

Using the matrix formulation approach illustrated in equation (5.9), where the whole matrices P, Q and R are stored completely, it is found that for the 3 machine infinite bus system a (12,397) entries need to be stored and for the 20 machine infinite bus system, the number of entries to be stored are (268,164).

By using the matrix formulation approach illustrated in equation (5.10) where the matrices P, Q and R are partitioned, it is found that for the 3 machine infinite bus system, the number of entries to be stored are (4,524) entries and for the 20 machine infinite bus system, we need to store (126,464) entries.

It could be seen that the saving in the required core storage using the partitioned "PQR" formulation is about 63% for System I and 53% for System II.

B. "ELIM" Core Storage

Using the "ELIM" matrix formulation approach, which is shown in equation (5.16), it is found that for the 3-machine infinite bus system the total number of entries to be stored are (3,922) and for the 20-machines infinite bus system, the number of stored entries are (51,760).

Now, by comparing the required storage for both the partitioned "PQR" and "ELIM" techniques, it can be seen that there will be quite a saving in the required storage when using the second technique. This saving is about 13% for System I (3 machines), while for System II (20 machines), which is larger, this saving has increased to about 59%.

The "PQR" and "ELIM" programs are capable of handling systems with up to about 70 state variables with available central memory of 49.2 k in time sharing mode. Computer memory requirements for both programs are shown in Table (5.6). K denotes thousands of words.

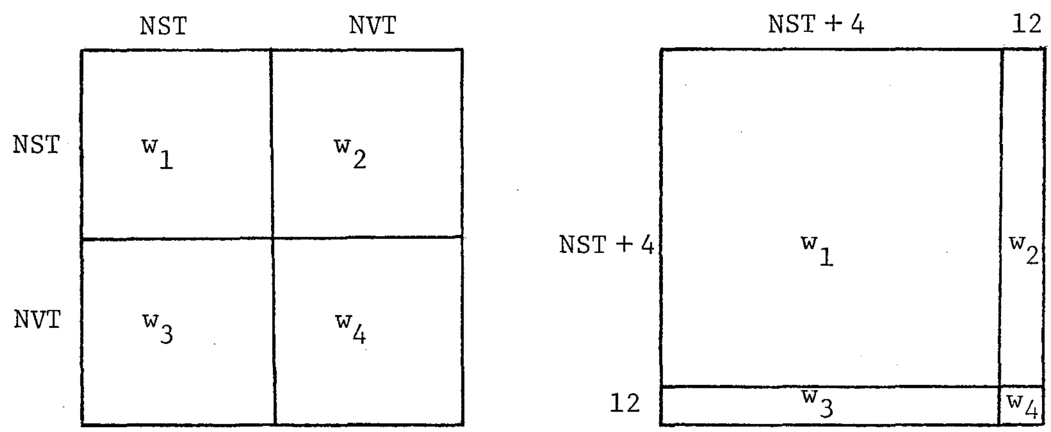
Table (5.6) CDC Memory Requirements

Function	PQR	ELIM
Data Storage	26.1 k	24.3 k
Programming	15.9 k	11.2 k*
System Executive	6.8 k	6.8 k
TOTAL	48.8 k	42.3 k
Available Space	49.2 k	49.2 k
Unused Space	0.4 k	6.9 k

* "ELIM" program not as comprehensive as "PQR"

The criterion of the "PQR" formulation requires the storage of the whole system state and algebraic variables all at once. The number of the system variables (NST) is flexible according to subsystem models required complexity, while the number of the system algebraic variables (NVT) does not have a wide range of choices, it depends on the generator model only which could be represented by 7 or 12 or 14 algebraic variables. This could be followed from Figure 5.6(a) where four working arrays are used (W1, W2, W3 and W4) the size of which is (70 x 70) which allows to a general utilizing process. In the case of low-order machine models, the NST value could be small, hence in the special case this allows us to increase the number of the algebraic variables (NVT), which requires a change in the dimensioning of the working arrays of the main program. Hence, in the "PQR" method, the maximum number of machines to be considered is governed by both the maximum number of algebraic variables (this number is governed by the available computer storage) and the number of algebraic variables associated with each machine.

An appropriate manipulation of the relative sizes of the working arrays as shown in Figure 5.6(b) will improve the storage efficiency of the "ELIM" formulation method. It is shown from Figure 5.6(b) that the total number of state variables (NST) is equal to 124, which is greater than that of the "PQR" formulation method, that is because in the "ELIM" method one machine is considered at a time. As a result, the number of algebraic variables to be stored is equal to 12 which represents the largest number of algebraic variables for one machine. Hence, in the "ELIM" program the maximum number of machines to be considered is governed by the maximum number of state variables and the number of state variables associated with each machine. This indicates that the "ELIM" program has a higher degree of freedom regarding the maximum number of machines to be chosen.



(a) Partitioning for the "PQR (b) Partitioning for the "ELIM"

Figure 5.6 Dimension of Working Arrays

5.2.5 Storage Comparison Using Sparse Technique

To apply both methods, the "PQR" and "ELIM" to a large power system, the implementation of sparse matrix techniques is needed. The matrices involved in both methods are quite sparse. Sparse matrix techniques enable us to store only the non-zero elements. So, a saving in memory and computation is usually achieved.

In both methods, the inversion of a given matrix is required. Instead of finding the matrix inverse which is usually full, we find a factorization of the matrix inverse (Bi-Factorization), [38]. Before performing the factorization process, a re-ordering of the matrix is needed. This re-ordering minimizes the number of newly generated elements (fill-ins) during the factorization process. A simulation of the needed computations (matrix inversion, matrix multiplication and row/column elimination) gives an accurate estimation of the needed storage.

A. "PQR" Core Storage

Utilizing the sparse techniques in the "PQR" matrix formulation of the 20 machine system (System II) requires the storage of (21342) non-zero entries. This number is obtained after a complete simulation of the required computations. On the other hand, the required full storage of all matrices involved in the "PQR" formulation is (126464) entries as calculated in section (5.2.4). It can be seen that the saving in the required core storage using sparse technique is about 83%.

B. "ELIM" Core Storage

Utilizing the sparse techniques in the "ELIM" matrix formulation of the same system (System II), requires the storage of (4390) non-zero entries. In section (5.2.4) the required full storage of all matrices involved in the "ELIM" formulation was found to be (51760) entries. This shows that utilizing sparse techniques results in a core storage saving of about 91%.

5.2.6 Eigenvalue Sensitivity Computation Comparison

From section (3.6), it is seen that to compute the eigenvalue sensitivity with respect to a system parameter, ξ, the state matrix derivatives with respect to that parameter are computed.

A. State Matrix Derivatives Using "PQR" Technique

Using the "PQR" matrix formulation approach [32], the [A] matrix is formulated from the addition of two matrices,

$$[A] = [QA] - [PA]\,[PB]^{-1}\,[QB] \tag{5.27}$$

One of these matrices, [QA], contains most of the control and design parameters (control gains, time constants, damping coefficient and inertia constant). The other matrix is the product of the three matrices: [PA], $[PB]^{-1}$ and [QB] as shown in equation (5.27). The matrix [PA] includes resistances of all the machines. The matrix [PB] includes: machine reactances, bus admittance matrix and other parameters depending on the operating condition as machine currents, voltages, flux linkages and rotor angles. The matrix [QB] contains machine currents, voltages and rotor angles.

All the system parameters exist in the matrices [P] and [Q] as simple explicit functions. This facilitates the direct calculation of the [A] matrix derivatives, and a general expression has been developed [27], as follows:

$$\frac{\partial[A]}{\partial\xi} = [I \,\vdots\, O]\,[P]^{-1}\left[\frac{\partial\,[Q]}{\partial\xi} - \left\{\frac{\partial\,[P]}{\partial\xi}\right\}[S]\right] \tag{5.28}$$

where,

$$[S] = [P]^{-1}\ [Q] = [\frac{A}{C}]$$

or,

$$[S] = \left[\begin{array}{c|c} I & -[PA]\ [PB]^{-1} \\ \hline 0 & [PB]^{-1} \end{array}\right] \left[\begin{array}{c} QA \\ \hline QB \end{array}\right]$$

For all control parameters, $\frac{\partial\,[P]}{\partial\xi} = [0]$, also the matrix [QB] does not contain any control parameter, hence, $\frac{\partial[QB]}{\partial\xi} = [0]$. Based on the previous two considerations, equation (5.28) will be as follows:

$$\frac{\partial[A]}{\partial\xi} = \frac{\partial[QA]}{\partial\xi} \tag{5.29}$$

Computing the state matrix derivatives w.r.t. matrix [P] parameters only hence, $\frac{\partial[Q]}{\partial\xi}, = [0]$, and the state matrix derivatives will be as follows:

$$\frac{\partial[A]}{\partial\xi} = -\left[\frac{\partial[PA]}{\partial\xi} - \{[PA]\ [PB]^{-1}\}\ \frac{\partial[PB]}{\partial\xi}\right][C] \tag{5.30}$$

Computing the state matrix derivatives w.r.t. the machine resistances only hence, $\frac{\partial[PB]}{\partial\xi} = [0]$ and the state matrix derivatives will be simpler than that of equation (5.16) as shown below:

$$\frac{\partial[A]}{\partial\xi} = -\ \{\frac{\partial[PA]}{\partial\xi}\}\ [C] \tag{5.31}$$

Computing the state matrix derivatives w.r.t. the machine reactances only hence, $\frac{\partial[PA]}{\partial\xi} = [0]$, and equation (5.16) will have the following form:

$$\frac{\partial[A]}{\partial\xi} = \left[[PA]\ [PB]^{-1}\ \{\frac{\partial[PB]}{\partial\xi}\}\right][C] \tag{5.32}$$

B. State Matrix Derivatives Using "ELIM" Technique

Using the "ELIM" matrix formulation approach, [4] and [30], the system state matrix is formulated from the multiplication of two matrices,

$$[A] = - [Z_{dn}]^{-1} [Z_{Rn}] \tag{5.33}$$

The first matrix, $[Z_{dn}]$ of order ns, is a block diagonal matrix where each block is a diagonal matrix and includes some of the control parameters as: steam turbine chest time constant (τ_{ch}), exciter time constant (τ_e) and exciter gain (k_e). The second matrix, $[Z_{Rn}]$ of order ns, includes machine damping coefficient (D) and inertia constant (H), control gains and time constants of voltage sensor, static exciter (except τ_e), speed stabilizer and governor. The other parameters as machine resistances and reactances and the parameters depending on the operating condition do not exist explicitly as in the "PQR" matrix formulation approach due to the successive matrix elimination operations which are carried out whenever a machine is added to the system.

A general expression for the [A] matrix derivatives has been derived in section (4.6), as follows:

$$\frac{\partial[A]}{\partial\xi} = - [Z_{dn}]^{-1} \{\frac{\partial[Z_{Rn}]}{\partial\xi}\} - \{\frac{\partial[Z_{dn}]^{-1}}{\partial\xi}\} [Z_{Rn}] \tag{5.34}$$

If it is required to compute the state matrix derivatives with respect to the control parameters τ_{ch} or τ_e which exist only in the matrix $[Z_{dn}]$, hence $\frac{\partial[Z_{Rn}]}{\partial\xi} = [0]$, and equation (5.34) is rewritten as follows:

$$\frac{\partial[A]}{\partial\xi} = - [Z_{Rn}] \{\frac{\partial[Z_{dn}]^{-1}}{\partial\xi}\} \tag{5.35}$$

and for the exciter gain (k_e) which exists in both the two matrixes: $[Z_{dn}]$ and $[Z_{Rn}]$, the general expression in equation (5.34) is used.

For the other control parameters, machine damping coefficient and inertia constant, which exist only in the matrix $[Z_{Rn}]$, the [A] matrix derivatives are computed by considering $\frac{\partial[Z_{dn}]^{-1}}{\partial\xi} = [0]$, hence equation (5.34) is rewritten in this case as follows:

$$\frac{\partial[A]}{\partial\xi} = - [Z_{dn}]^{-1} \{\frac{\partial[Z_{Rn}]}{\partial\xi}\} \tag{5.36}$$

Since the control parameters exist only in the block diagonal matrices $[H_i]$, of the matrix $[Z_{Rn}]$, as shown in equation (B.18). So, the derivatives of the off-diagonal matrices, $[f_{ij}]$, with respect to these parameters equal zero, where, i, j=1, 2, ..., n, and hence,

$$\frac{\partial[Z_{Rn}]}{\partial\xi} = \text{diag } \{\frac{\partial[H_i]}{\partial\xi}\} \tag{5.37}$$

where,

$$[H_i] = [H_{1i}\ (\xi)] - [\bar{h}_{ii}]$$

Since the control parameters exist specifically in the matrices $[H_{1i}(\xi)]$, hence equation (5.37) is rewritten as follows:

$$\frac{\partial[Z_{Rn}]}{\partial\xi} = \text{diag } \{\frac{\partial[H_{1i}(\xi)]}{\partial\xi}\} \tag{5.38}$$

From what have been mentioned above, it is concluded that all system parameters exist explicitly in the "PQR" matrix formulation approach which is an advantage, and this facilitates the direct calculation of the system state matrix derivatives w.r.t. any system parameter.

On the other hand, when using the "ELIM" matrix formulation approach, it is found from the derivation in Appendix B, that only the control parameters, damping coefficient and inertia constant exist explicitly in the matrices $[H_{1i}(\xi)]$ and $[Z_{dn}]$, while the other system parameters do not appear due to the successive matrix eliminations to build up the system state matrix.

The state matrix derivatives w.r.t. the control parameters using the "PQR" technique requires computing the derivatives of the ns-top rows of the [Q] matrix w.r.t. these parameters, as shown in equation (5.29), while using the "ELIM" technique requires computing the derivatives of the two matrices $[Z_{dn}]$ and $[Z_{Rn}]$, of order ns, as shown in equation (5.34).

Hence, we could conclude that from the eigenvalue sensitivity computation point of view, both the "PQR" and "ELIM" formulation techniques are at the same level of adequacy as the most system control parameters are generally available in both formulation methods.

CHAPTER 6

CONCLUSIONS

The dynamic stability analysis of power systems requires the formulation of the linearized power system equations in the state-space form. In this thesis, two matrix formulation techniques have been implemented by constructing two computer programs, which have been verified and applied to a test system of two different orders. The two programs were documented and compared.

The first matrix formulation technique is the "PQR" method, which is based on grouping the states of each individual machine together and ordering the system algebraic variables in a certain manner to reduce the matrix inversion time. A computer program has been developed to construct the power system state matrices using the "PQR" method for multi-machine systems. A variety of models are included for the system components; the synchronous machine, exciter and governor control systems; this facilitates the representation of generating units with different degrees of complexity. The system algebraic variables are re-ordered to adapt the different generator models.

The second matrix formulation approach is the direct elimination "ELIM" method which is based on the addition of one complete generating unit equations to the network at a time and reduction until only the differential equations of that unit and the algebraic equations of the network are left. Then, the two network equations for that unit node are eliminated. The next generating unit is then added and reduced and so on until the full system state matrix is formed. A computer program has been produced to construct the state matrix for multi-machine systems. Both of these computer programs are capable of handling systems up to about 70 state variables with the available central memory of about 49,000 words (decimal) on the CDC CYBER 170/730 McMaster University computer. The two programs are generalized by using variable dimensions.

To illustrate the validity of the computer programs, an example of a synchronous machine connected to an infinite bus through a transmission line is chosen as a test problem. Two specific examples, second and seventh-order systems, have been considered. When applying the two programs to these examples, the achieved results were found to be in agreement with the corresponding results in the literature.

A comparison between the two computer program computation times has been done in Chapter 5 for different system orders and it was found that the computation time of the "PQR" program is higher than that of "ELIM" program. Also based on these computation times, a prediction equation has been developed to predict the computation time for a large power system (20 machine system, 140 state variables). To verify and extend the computation time comparison, analytical expressions have been constructed. These analytical expressions were based on the number of arithmetic operations performed in terms of the number of machines, number of state variables and number of algebraic variables. Based on the above comparison, it was found that the computation time using the "PQR" program is almost twice that of the "ELIM" program.

Since a matrix inversion is required for both methods and it is a relatively long process for a large power system and has to be performed whenever a parameter setting changes, an analytical inversion time comparison has been done between the two approaches. It is found that the matrix inversion time using the "PQR" approach is less than that using the "ELIM" approach for a system including number of machines less than eleven, while by increasing the number of machines in the system (greater than eleven) the matrix inversion time using the "ELIM" approach will be less than that when using the "PQR" approach. This shows that the "ELIM" technique is more economical than the "PQR" technique for larger systems.

The storage requirement for both the "PQR" and "ELIM" programs is compared in Chapter 5 and it was found that the storage required for the "ELIM" program is less than that required for the "PQR" program. As documented in Chapter 5, appropriate manipulation of the relative sizes of the working arrays will increase the advantage of the "ELIM" method, in addition to improving the storage efficiency of both methods. Also, an estimate of the storage requirements for both matrix formulations utilizing sparse techniques (only non-zero elements to be stored) has been done for a 20 machine system (140 state variables). The analytical comparison proved that there is quite a saving using sparse techniques for both formulation methods and the saving is larger for the "ELIM" formulation method.

Eigenvalue sensitivity evaluation with respect to a system parameter requires the computation of the state matrix derivatives with respect to that parameter. A comparison between state matrix derivative computation using the "PQR" and "ELIM" matrix formulation approaches has also been done in Chapter 5. It was found that the system control parameters in both formulation methods are generally available. The formulation of specific derivative expressions with respect to a system control parameter was developed for the "PQR" method by Zein El-Din, while in this thesis these derivatives have been developed for the "ELIM" method. The applicability of utilizing a similar eigenvalue sensivitity approach for the "ELIM" method has been proven in Chapter 5 and Appendix B, this adds to the advantages and flexibility of using the "ELIM" formulation technique.

Finally, the specific contributions of the study in this thesis are summarized as follows:

(1) Two state matrix formulation programs have been developed and implemented on the University CDC CYBER computer. Detailed documentation has been presented in two internal reports. The validity of these programs has been established by comparing with other published material.

(2) Detailed comparison between these two programs and extensive analysis of both algorithms has led to the conclusion that the direct elimination method requires considerably less storage and less running time than the "PQR" method when full advantage is taken of the implicit data structure in both methods.

(3) The advantage of direct eigenvalue sensitivity computation in the "PQR" method has been extended in this thesis to the direct elimination method.

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APPENDIX A

SUBSYSTEM MODELS

In this Appendix, the nonlinear equations and the linearized state-space equations which describe the performance of each subsystem model will be presented. The models in this Appendix and in Chapter 2 are taken directly from the appropriate references.

A.1 Synchronous Machines

The modelling of a synchronous machine in state-space form has been considered in many references. Two different approaches have been adopted in choosing the states of the model. The stator and rotor currents (referred to the machine rotor frame) were used as states in Reference 11. Alternatively, the stator and rotor fluxes (referred to the machine rotor frame) were used as states in References 7 and 32. The choice of this second approach is followed in this thesis. Four models are used for the synchronous machine as shown in Table (2.1), Chapter 2: The classical model (G_o), one rotor circuit (no damper windings), model (G_1), three rotor circuits (two damper windings), model (G_2) and three rotor circuits (two damper windings + stator transient included), model (G_3). The last model, G_3, has been discussed in Chapter 2, and the other models will be discussed in this section.

<u>Nonlinear Model (G_1)</u>

In this model, the stator transients and damper winding effects are eliminated which means that only the field circuit effect is included. The nonlinear differential and algebraic equation are shown in (A.1a) and (A.1b). The equations of a model based on a linear approximation around an appropriate operating condition for a synchronous machine are presented in matrix form in Figure (A.1). This model is of first order.

$$\dot{\psi}_{fd} = \omega_o (v_{fd} - r_{fd} i_{fd}) \tag{A.1a}$$

$$\begin{aligned} \psi_{fd} &= X_{fd} i_{fd} - X_{md} i_d \\ 0 &= r_s i_d + v_d + \omega\psi_q \\ 0 &= r_s i_q + v_q - \omega\psi_d \end{aligned} \tag{A.1b}$$

$$[\Delta\dot{\psi}_{fd}] = -[\omega_o r_{fd}] \Delta i_{fd} + [\omega_o] \Delta v_{fd}$$

$$\begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_d \\ \Delta\psi_q \end{bmatrix} = \begin{bmatrix} X_{fd} & -X_{md} & \\ & & r_s \\ & -r_s & \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_d \\ \Delta i_q \end{bmatrix} - \begin{bmatrix} & \\ & 1 \\ -1 & \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} - \begin{bmatrix} \\ \frac{\psi_{do}}{\omega_o} \\ \frac{\psi_{qo}}{\omega_o} \end{bmatrix} [\Delta v]$$

Figure (A.1): The Linearized State and Algebraic Equations for a Synchronous Machine (Model G_1)

Nonlinear Model (G_2)

In this model, the stator transients are eliminated, while the damper winding and field winding effects are included. The nonlinear differential and algebraic equations are shown in (A.2a) and (A.2b). The linearized state and algebraic equations in matrix form are shown in Figure (A.2). This model is of third order.

$$\begin{aligned}
\dot{\psi}_{fd} &= \omega_o (v_{fd} - r_{fd}\, i_{fd}) \\
\dot{\psi}_{kd} &= -\omega_o (r_{kd}\, i_{kd}) \\
\dot{\psi}_{kq} &= -\omega_o (r_{kq}\, i_{kq})
\end{aligned} \tag{A.2a}$$

$$\begin{aligned}
\psi_{fd} &= X_{fd}\, i_{fd} - X_{md}\, i_d + X_{md}\, i_{kd} \\
\psi_{kd} &= X_{md}\, i_{fd} - X_{md}\, i_d + X_{kd}\, i_{kd} \\
\psi_{kq} &= -X_{mq}\, i_q + X_{kq}\, i_{kq} \\
0 &= r_s\, i_d + v_d + \omega\psi_q \\
0 &= r_s\, i_q + v_q - \omega\psi_d
\end{aligned} \tag{A.2b}$$

$$
\begin{bmatrix} \Delta\dot{\psi}_{fd} \\ \Delta\dot{\psi}_{kd} \\ \Delta\dot{\psi}_{kq} \end{bmatrix} = \begin{bmatrix} -\omega_o r_f & & \\ & -\omega_o r_{kd} & \\ & & -\omega_o r_{kq} \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_{kd} \\ \Delta i_{kq} \end{bmatrix} + [\omega_o] \; \Delta\omega_{fd}
$$

$$
\begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_{kd} \\ \Delta\psi_{kq} \\ \Delta\psi_{d} \\ \Delta\psi_{q} \end{bmatrix} = \begin{bmatrix} X_{fd} & -X_{md} & X_{md} & & \\ X_{md} & -X_{md} & X_{kd} & & \\ & & & -X_{mq} & X_{kq} \\ & & & & r_s \\ & & & -r_s & \end{bmatrix} \begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_{kd} \\ \Delta\psi_{kq} \\ \Delta\psi_{d} \\ \Delta\psi_{q} \end{bmatrix} + \begin{bmatrix} & \\ & \\ & \\ & 1 \\ -1 & \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} - \begin{bmatrix} \\ \\ \\ \frac{\psi_{do}}{\omega_o} \\ \frac{\psi_{qo}}{\omega_o} \end{bmatrix} [\Delta\omega]
$$

Figure (A.2): The Linearized State and Algebraic Equations for a Synchronous Machine (Model G_2)

Classical Model (G_o)

In this model, the generating unit is represented by a classical model which means that the generator is described by a constant voltage behind transient reactance as shown in Figure (A.3). The nonlinear motion equations are shown in (A.3a) and the linearized equations in (A.3b).

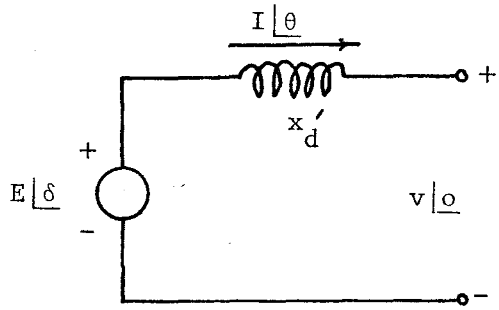

Figure (A.3): Classical Model Representation

$$\dot{\delta}_i = \omega_i - \omega_o$$

$$\frac{2H_i}{\omega_o}\dot{\omega}_i + \frac{D_i}{\omega_o}\omega_i = P_{mi} - [E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j)] \tag{A.3a}$$

$$\Delta\dot{\delta}_i = \Delta\omega_i$$

$$\Delta\dot{\omega}_i = -\frac{D_i}{2H_i}\Delta\omega_i - \frac{\omega_o}{2H_i} k_{1ij}\Delta\delta_i \tag{A.3b}$$

where:

$$k_{1ij} = \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_{jo} Y_{ijo} \sin(\theta_{ijo} - \delta_{io} + \delta_{jo})$$

A.2 Mechanical Shaft Systems

For the analysis of shaft torsional effects in power system stability studies, the shaft system is represented by a number of concentrated rotating masses connected by weightless springs [25]. In Chapter 2, the shaft is represented by five equivalent rotating masses, one equivalent rotating mass corresponding to each turbine stage and one equivalent mass representing the generator rotor. In this section, the shaft system will be represented by two additional models: a single equivalent mass model (S_1) and two equivalent mass models (S_2).

Nonlinear Model (S_1)

In this model, the mechanical shaft system is represented by a single equivalent rotating mass which corresponds to the generator rotor. This model is represented by two states: rotor angle and rotor speed. Equation (A.4a) describes the dynamic performance of the mechanical shaft system and equation (A.4b) represents the linearized equation.

$$\begin{aligned} \dot{\delta} &= \omega - \omega_o \\ \dot{\omega} &= -\frac{D}{2H}\,\omega - \frac{\omega_o}{2H}\,T_{eu} + \frac{\omega_o}{2H}\,P_{mu} \end{aligned} \tag{A.4a}$$

$$\begin{aligned} \Delta\dot{\delta} &= \Delta\omega \\ \Delta\dot{\omega} &= \frac{-D}{2H}\,\Delta T_{eu} - \left(\frac{D+P_{mu}}{2H}\right)\Delta\omega + \frac{\omega_o}{2H}\,\Delta\,P_{mu} \end{aligned} \tag{A.4b}$$

<u>Nonlinear Model (S_2)</u>

In this model, the mechanical shaft system is represented by two equivalent rotating masses which correspond to the turbine and the generator rotor. This model is of fourth order. Equation (A.5) describes the dynamic performance of the turboalternator mechanical system. The linearized equations in matrix form are shown in figure (A.4).

$$\begin{aligned} \dot{\delta}_1 &= \omega_1 - \omega_o \\ \dot{\delta}_2 &= \omega_2 - \omega_o \\ \dot{\omega}_1 &= \frac{-D_1}{2H_1}\,\omega_1 - \frac{\omega_o}{2H_1}\,S_{12}\,(\delta_1 - \delta_2) - \frac{\omega_o}{2H_1}\,T_{eu} \\ \dot{\omega}_2 &= \frac{-D_2}{2H_2}\,\omega_2 - \frac{\omega_o}{2H_2}\,S_{12}\,(\delta_2 - \delta_1) + \frac{\omega_o}{2H_2}\,P_{LP} \end{aligned} \tag{A.5}$$

$$
\begin{bmatrix} \Delta\dot{\delta}_1 \\ \Delta\delta_2 \\ \hline \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix}
=
\left[\begin{array}{cc|cc}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\ \hline
\frac{-\omega_o S_{12}}{2H_1} & \frac{\omega_o S_{12}}{2H_1} & \frac{-D_1}{2H_1} & 0 \\
\frac{\omega_o S_{12}}{2H_2} & \frac{-\omega_o S_{12}}{2H_2} & 0 & \frac{-D_2}{2H_2}
\end{array}\right]
\begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \hline \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix}
-
\begin{bmatrix} 0 \\ 0 \\ \hline \frac{\omega_o}{2H_1} \\ 0 \end{bmatrix}
[\Delta T_{eu}]
+
\begin{bmatrix} 0 \\ 0 \\ \hline 0 \\ \frac{\omega_o}{2H_2} \end{bmatrix}
[\Delta P_{LP}]
$$

Figure (A.4): The Linearized State Equations for a Mechanical Shaft System (Model S_2)

A.3 <u>Excitation Systems</u>

The static exciter with speed stabilizer has been discussed in Chapter 2. In this section, two models of the exciter are presented.

<u>Nonlinear Model (E_1)</u>

The exciter in this model is represented by a single time constant transfer function. The input is the difference between the terminal voltage and the reference voltage, as shown from the block diagram, Figure (A.5). The differential equation and the linearized equation are shown in (A.6a) and (A.6b).

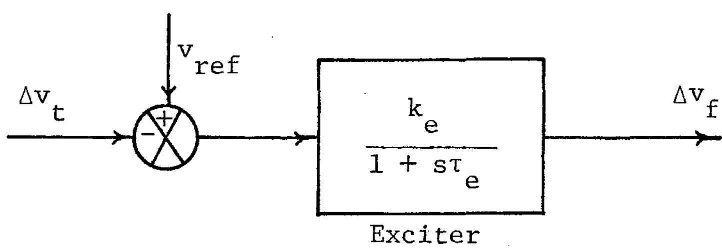

Figure (A.5): Simple Exciter Block Diagram

$$\dot{v}_{fd} = \frac{-k_e}{\tau_e} v_t - \frac{1}{\tau_e} v_{fd} + \frac{k_e}{\tau_e} v_{ref} \qquad \text{(A.6a)}$$

$$\Delta \dot{v}_{fd} = \frac{-k_e}{\tau_e} \Delta v_t - \frac{1}{\tau_e} \Delta v_{fd} + \frac{k_e}{\tau_e} v_{ref} \qquad \text{(A.6b)}$$

Nonlinear Model (E_2)

The exciter in this case is represented by two transfer functions. As shown in Figure (A.6), one represents the voltage sensor and the other represents the exciter. There is no stabilizer in this model, and it is of order 2. The differential equations are shown in (A.7) and the linearized equations are in matrix form, Figure (A.7).

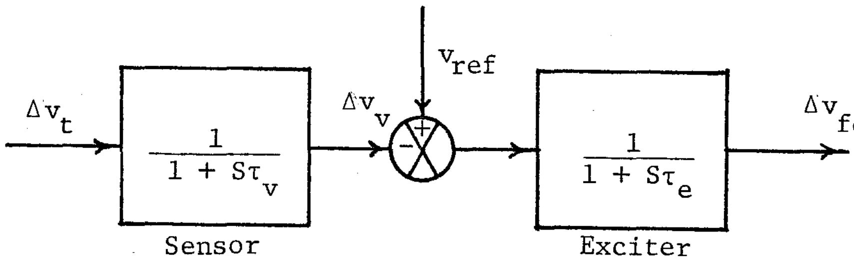

Figure (A.6): Block Diagram of Second Order Exciter

$$\begin{aligned} \dot{v}_v &= \frac{1}{\tau_v} v_t - \frac{1}{\tau_v} v_v \\ \dot{v}_{fd} &= \frac{-k_e}{\tau_e} v_v - \frac{1}{\tau_e} v_{fd} + \frac{k_e}{\tau_e} v_{ref} \end{aligned} \qquad \text{(A.7)}$$

$$\begin{bmatrix} \Delta \dot{v}_v \\ \Delta \dot{v}_{fd} \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_v} & \\ \frac{-k_e}{\tau_e} & \frac{-1}{\tau_e} \end{bmatrix} \begin{bmatrix} \Delta v_v \\ \Delta v_{fd} \end{bmatrix} + \begin{bmatrix} \frac{1}{\tau_v} \\ \\ \end{bmatrix} \Delta v_t + \begin{bmatrix} \\ \frac{k_e}{\tau_e} \end{bmatrix} v_{ref}$$

Figure (A.7): The Linearized State Equations for a Second Order Exciter

A.4 Turbine-Governor (Model T_1)

In Chapter 2, a third-order turbine-governor model for a hydro unit has been presented. In this section, a second-order turbine-governor model for steam and nuclear units will be presented. These two models are taken directly from reference [18]. The turbine-governor model for steam and nuclear units is shown in Figure (A.8). The turbine is modelled by a single-time constant transfer function. The input is the difference between the reference power (P_{ref}) and the feedback signal through the governor. The governor is also described by a single-time constant transfer function. The differential equations representing the model are given in (A.8) and the linearized equations in matrix form are given in Figure (A.9).

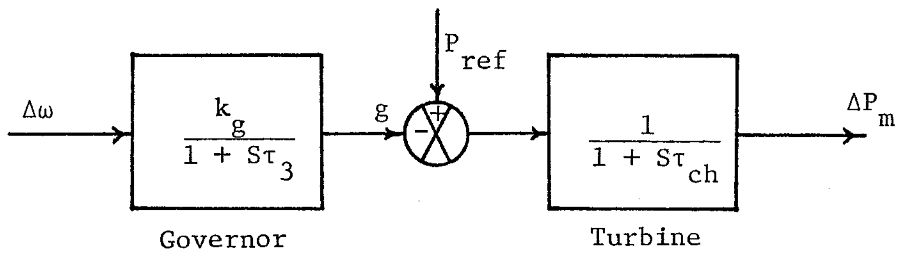

Figure (A.8): Block Diagram of Turbine-Governor Model for Steam Unit

$$\begin{aligned} \dot{P}_m &= \frac{-1}{\tau_{ch}} P_m - \frac{1}{\tau_{ch}} g + \frac{1}{\tau_{ch}} P_{ref} \\ \dot{g} &= \frac{k_g}{\omega_o \tau_3} \omega - \frac{1}{\tau_3} g \end{aligned} \tag{A.8}$$

$$\begin{bmatrix} \dot{\Delta P}_m \\ \\ \dot{g} \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_{ch}} & \frac{-1}{\tau_{ch}} \\ \\ & -\frac{1}{\tau_3} \end{bmatrix} \begin{bmatrix} \Delta P_m \\ \\ g \end{bmatrix} + \begin{bmatrix} \frac{1}{\tau_{ch}} \\ \\ \\ \end{bmatrix} P_{ref} + \begin{bmatrix} \\ \\ \frac{k_g}{\omega_o \tau_3} \end{bmatrix} \Delta\omega$$

Figure (A.9): The Linearized State Equations for Turbine-Governor (Model T_1)

APPENDIX B

STATE MATRIX PARTIAL DERIVATIVES USING THE "ELIM" APPROACH

Referring to equation (4.7) in Chapter 4, it is seen that the system differential and algebraic equations could be written in the following matrix form [4],

$$\begin{bmatrix} \Delta u \\ \hline 0 \end{bmatrix} = [Z_p] \begin{bmatrix} \Delta x \\ \hline \Delta y \end{bmatrix} \tag{B.1}$$

where $[Z_p]$ is an operational matrix.

Addition of Machine 1 to the Network

Applying the approach adopted by Muir [30], the n bus network equations are formed and stored first and then the differential and algebraic equations of one machine are formed and added to the network equations (in matrix form) as shown in equation (B.2), assuming that the number of buses equals the number of machines (non-generator buses having been previously eliminated),

$$\left[\begin{array}{c} 0 \\ \hline \Delta u_1 \\ \hline 0 \end{array}\right] = \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & Z_{d11} & 0 \\ \hline 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} 0 \\ \hline \Delta \dot{x}_1 \\ \hline 0 \end{array}\right] + \left[\begin{array}{c|c|c} Y_N & h_1 & h_2 \\ \hline 0 & H_{11}(\xi) & H_{12} \\ \hline h_3 & H_{31} & H_{41} \end{array}\right] \begin{array}{c} \left[\begin{array}{c} \Delta v_N \\ \hline \Delta x_1 \\ \hline \Delta y_1 \end{array}\right] \end{array} \begin{array}{c} 2n \\ ns_1 \\ nv_1 \end{array} \tag{B.2}$$

Equation (B.2) can be rewritten as follows:

$$\begin{bmatrix} \underset{\sim}{0} \\ \hline \Delta \underset{\sim}{u}_1 \\ \hline \underset{\sim}{0} \end{bmatrix} = \begin{bmatrix} \\ Z_{d1} \\ \\ \end{bmatrix} \begin{bmatrix} \underset{\sim}{0} \\ \hline \Delta \dot{\underset{\sim}{x}}_1 \\ \hline \underset{\sim}{0} \end{bmatrix} + \begin{bmatrix} \\ Z_1 \\ \\ \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{v}_N \\ \hline \Delta \underset{\sim}{x}_1 \\ \hline \Delta \underset{\sim}{y}_1 \end{bmatrix} \begin{array}{l} 2n \\ ns_1 \\ nv_1 \end{array} \tag{B.3}$$

where $[Z_{d1}]$ is a constant real diagonal matrix for machine 1 and the network,

$[Z_1]$ is a constant real matrix for machine 1 and the network,

$\Delta \underset{\sim}{u}_1$, $\Delta \underset{\sim}{x}_1$ are dimensioned ns_1 (the number of state variables of machine 1),

$\Delta \underset{\sim}{y}_1$ is dimensioned nv_1 (the number of algebraic variables associated with machine 1),

$\Delta \underset{\sim}{v}_N$ is dimensioned 2n (the number of algebraic variables associated with the network)

Looking at the different submatrices of the matrix $[Z_1]$,

$[Y_N]$ is the bus admittance matrix in 2n real equation form 1 .

$[h_1]$ includes the initial value of the network nodal current components (i_D, i_Q) which depends on the operating point.

$[h_2]$,$[h_3]$ include the relationship between the machine quantities and the network components of nodal voltage (v_D, v_Q) which depends on the initial condition of the rotor angle (δ).

$[H_{11}(\xi)]$ includes most of the control parameters (gains and time constants) of interest for the governor and excitation systems, inertia constant and damping coefficient

$[H_{21}]$ includes the machine resistances and the sensor voltage time constant (τ_v) in the excitation system

$[H_{31}]$ includes machine currents and voltage components which depend on the operating point

$[H_{41}]$ includes the machine reactances, currents, voltages and flux linkages which again depend on the operating point

Thus, it can be concluded that most of the significant system control parameters exist in the submatrix $[H_{11}(\xi)]$.

To eliminate the algebraic equations of the first machine from (B.2), the matrix $[Z_1]$ is partitioned as follows:

$$\begin{array}{l} \\ \text{network equations} \\ \text{differential equations of machine 1} \\ \text{algebraic equations of machine 1} \end{array} \begin{array}{c} \begin{array}{ccc} 2n & ns_1 & nv_1 \end{array} \\ \left[\begin{array}{c|c|c} Y_N & h_1 & h_2 \\ \hline 0 & H_{11}(\xi) & H_{21} \\ \hline h_3 & H_{31} & H_{41} \end{array}\right] \end{array} = \left[\begin{array}{c|c} Z_{11} & Z_{21} \\ \hline Z_{31} & Z_{41} \end{array}\right] \begin{array}{l} 2n + ns_1 \\ \\ nv_1 \end{array} \tag{B.4}$$

Using matrix elimination for equation (B.4),

$$[M_{R1}] = [Z_{11} - Z_{21}\, Z_{41}^{-1}\, Z_{31}] \tag{B.5}$$

where,

$$M_{R1} = \left[\begin{array}{c|c} Y_N & h_1 \\ \hline 0 & H_{11}(\xi) \end{array}\right] - \left[\begin{array}{c|c} h_4 & h_5 \\ \hline h_6 & h_7 \end{array}\right] = \left[\begin{array}{c|c} \bar{h}_1 & \bar{h}_2 \\ \hline \bar{h}_3 & H_{11}(\xi) - h_7 \end{array}\right] \begin{array}{l} 2n \\ ns_1 \end{array} \tag{B.6}$$

To eliminate the two real network equations of the node connected to the first machine, the matrix $[M_{R1}]$ could be partitioned as follows:

$$[M_{R1}] = \begin{array}{c} \begin{array}{ccc} 2 & 2n-2 & ns_1 \end{array} \\ \left[\begin{array}{c|c|c} t_1 & t_2 & r_1 \\ \hline t_3 & t_4 & 0 \\ \hline r_2 & 0 & H_{11}(\xi)-h_7 \end{array}\right] \end{array} = \begin{array}{cc} \begin{array}{cc} 2 & (2n-2)+ns_1 \end{array} & \\ \left[\begin{array}{c|c} Z_{R41} & Z_{R31} \\ \hline Z_{R21} & Z_{R11} \end{array}\right] & \begin{array}{c} 2 \\ (2n-2)+ns_1 \end{array} \end{array} \tag{B.7}$$

Using matrix elimination for equation (B.7),

$$[Z_{R1}] = [Z_{R11} - Z_{R21}\, Z_{R41}^{-1}\, Z_{R31}] \tag{B.8}$$

where,

$$[Z_{R1}] = \left[\begin{array}{c|c} t_4 & 0 \\ \hline 0 & H_{11}(\xi)-h_7 \end{array}\right] - \left[\begin{array}{c|c} r_3 & r_4 \\ \hline r_5 & r_6 \end{array}\right] = \begin{array}{cc} \left[\begin{array}{c|c} \bar{r}_1 & \bar{r}_2 \\ \hline \bar{r}_3 & H_{11}(\xi)-h_{11} \end{array}\right] & \begin{array}{c} 2n-2 \\ ns_1 \end{array} \end{array} \tag{B.9}$$

$$[h_{11}] = [h_7] + [r_6]$$

Substituting the reduced matrix of $[Z_1]$, $[Z_{R1}]$, in (B.3),

$$\begin{array}{c} 2n-2 \\ ns_1 \end{array} \left[\begin{array}{c} \tilde{0} \\ \hline \Delta \tilde{u}_1 \end{array}\right] = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & Z_{d11} \end{array}\right] \left[\begin{array}{c} \tilde{0} \\ \hline \Delta \tilde{x}_1 \end{array}\right] + \left[\begin{array}{c|c} \bar{r}_1 & \bar{r}_2 \\ \hline \bar{r}_3 & H_{11}(\xi)-h_{11} \end{array}\right] \left[\begin{array}{c} \Delta \tilde{v}_N \\ \hline \Delta \tilde{x}_1 \end{array}\right] \tag{B.10}$$

Addition of the Second Machine

Adding the second machine to the network and first machine differential equations,

$$
\begin{array}{c}
2n-2 \\ ns_1 \\ ns_2 \\ nv_2
\end{array}
\begin{bmatrix} \underset{\sim}{0} \\ \Delta \underset{\sim}{u}_1 \\ \Delta \underset{\sim}{u}_2 \\ \underset{\sim}{0} \end{bmatrix}
=
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & Z_{d11} & 0 & 0 \\ 0 & 0 & Z_{d22} & 0 \\ 0 & 0 & & 0 \end{bmatrix}
\begin{bmatrix} 0 \\ \Delta \dot{\underset{\sim}{x}}_1 \\ \Delta \dot{\underset{\sim}{x}}_2 \\ \underset{\sim}{0} \end{bmatrix}
+
\begin{bmatrix} \bar{r}_1 & \bar{r}_2 & s_1 & s_2 \\ r_3 & H_{11}(\xi)-h_{11} & 0 & 0 \\ 0 & 0 & H_{12}(\xi) & H_{22} \\ s_3 & 0 & H_{32} & H_{42} \end{bmatrix}
\begin{bmatrix} \Delta \underset{\sim}{v}_N \\ \Delta \underset{\sim}{x}_1 \\ \Delta \underset{\sim}{x}_2 \\ \Delta \underset{\sim}{y}_2 \end{bmatrix}
\tag{B.11}
$$

Equation (B.11) can be re-written as follows:

$$
\begin{array}{c}
2n-2 \\ ns_1 \\ ns_2 \\ nv_1
\end{array}
\begin{bmatrix} \underset{\sim}{0} \\ \Delta \underset{\sim}{u}_1 \\ \Delta \underset{\sim}{u}_2 \\ \underset{\sim}{0} \end{bmatrix}
=
\begin{bmatrix} \\ Z_{d2} \\ \\ \end{bmatrix}
\begin{bmatrix} \underset{\sim}{0} \\ \Delta \dot{\underset{\sim}{x}}_1 \\ \Delta \dot{\underset{\sim}{x}}_2 \\ \underset{\sim}{0} \end{bmatrix}
+
\begin{bmatrix} \\ Z_2 \\ \\ \end{bmatrix}
\begin{bmatrix} \Delta \underset{\sim}{v}_N \\ \Delta \underset{\sim}{x}_1 \\ \Delta \underset{\sim}{x}_2 \\ \Delta \underset{\sim}{y}_2 \end{bmatrix}
\tag{B.12}
$$

To eliminate the algebraic equations of the second machine from equation (B.12), the matrix $[Z_2]$ could be partitioned as follows:

$$
\begin{array}{c}
2n-2 \\ ns_1 \\ ns_2 \\ nv_2
\end{array}
\left[\begin{array}{cccc} \bar{r}_1 & \bar{r}_2 & s_1 & s_2 \\ \bar{r}_3 & H_{11}(\xi)-h_{11} & 0 & 0 \\ 0 & 0 & H_{12}(\xi) & H_{22} \\ \hline s_3 & 0 & H_{32} & H_{42} \end{array}\right]
=
\left[\begin{array}{c|c} Z_{12} & Z_{22} \\ \hline Z_{32} & Z_{42} \end{array}\right]
\tag{B.13}
$$

Using matrix elimination for equation (B.13),

$$
[M_{R2}] = [Z_{12} - Z_{22}\, Z_{42}^{-1}\, Z_{32}]
\tag{B.14}
$$

where,

$$
[M_{R2}] =
\begin{bmatrix} \bar{r}_1 & \bar{r}_2 & s_1 \\ \bar{r}_3 & H_{11}(\xi)-h_{11} & 0 \\ 0 & 0 & H_{12}(\xi) \end{bmatrix}
-
\begin{bmatrix} s_4 & 0 & s_5 \\ 0 & 0 & 0 \\ s_6 & 0 & s_7 \end{bmatrix}
\begin{array}{c}
2n-2 \\ ns_1 \\ ns_2
\end{array}
\tag{B.15}
$$

To eliminate the two real network equations of the node connected to the second machine, the matrix $[M_{R2}]$ could be partitioned as follows:

$$[M_{R2}] = \begin{array}{c} \left[\begin{array}{c|c|c|c} t_5 & t_6 & r_7 & r_8 \\ \hline t_7 & t_8 & 0 & 0 \\ \hline r_9 & r_{10} & H_{11}(\xi)-h_{11} & 0 \\ \hline r_{11} & r_{12} & 0 & H_{12}(\xi)-s_7 \end{array}\right] \end{array} \begin{array}{l} 2 \\ 2n-4 \\ ns_1 \\ ns_2 \end{array} - \left[\begin{array}{c|c} Z_{R42} & Z_{R32} \\ \hline Z_{R22} & Z_{R12} \end{array}\right] \tag{B.16}$$

Using the matrix elimination for equation (B.16),

$$[Z_{R2}] = [Z_{R12} - Z_{R22} \cdot Z_{R42}^{-1} \cdot Z_{R32}] \tag{B.17}$$

where,

$$[Z_{R2}] = \left[\begin{array}{c|c|c} t_8 & 0 & 0 \\ \hline r_{10} & H_{11}(\xi)-h_{11} & 0 \\ \hline r_{12} & 0 & H_{12}(\xi)-s_7 \end{array}\right] - \left[\begin{array}{c|c|c} k_1 & k_2 & k_3 \\ \hline k_4 & k_5 & k_6 \\ \hline k_7 & k_8 & k_9 \end{array}\right] = \left[\begin{array}{c|c|c} \bar{k}_1 & -k_2 & -k_3 \\ \hline \bar{k}_2 & H_{11}(\xi)-\bar{h}_{11} & -k_6 \\ \hline \bar{k}_3 & -k_8 & H_{12}(\xi)-\bar{\bar{h}}_{22} \end{array}\right] \tag{B.18}$$

$\bar{h}_{11} = h_{11} + k_5$, $\quad \bar{h}_{22} = s_7 + k_9$,

and the elements in $[k_5]$ corresponding to the control parameters in $[H_{11}(\xi)]$ are zeros, also the elements in $[k_9]$ corresponding to the control parameters in $[H_{12}(\xi)]$ are zeros. This shows that the control parameters do not change during matrix manipulation and appear explicitly in the submatrices $[H_{1i}(\xi)]$, where i is the machine number.

Substituting the reduced matrix of $[Z_2]$, $[Z_{R2}]$, in (B.11),

$$\begin{array}{c} 2n-4 \\ ns_1 \\ ns_2 \end{array} \left[\begin{array}{c} \underset{\sim}{0} \\ \hline \Delta \underset{\sim}{u}_1 \\ \hline \Delta \underset{\sim}{u}_2 \end{array}\right] = \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & Z_{d11} & 0 \\ \hline 0 & 0 & Z_{d22} \end{array}\right] \left[\begin{array}{c} \underset{\sim}{0} \\ \hline \Delta \dot{\underset{\sim}{x}}_1 \\ \hline \Delta \dot{\underset{\sim}{x}}_2 \end{array}\right] + \left[\begin{array}{c|c|c} \bar{k}_1 & -k_2 & -k_3 \\ \hline \bar{k}_2 & H_{11}(\xi)-\bar{h}_{11} & -k_6 \\ \hline \bar{k}_3 & -k_8 & H_{12}(\xi)-\bar{h}_{22} \end{array}\right] \left[\begin{array}{c} \Delta \underset{\sim}{v}_N \\ \hline \Delta \underset{\sim}{x}_1 \\ \hline \Delta \underset{\sim}{x}_2 \end{array}\right] \quad (B.19)$$

Addition of the nth Machine

The same procedure is followed when adding the next machine. After the addition of the nth machine, the following matrix form is obtained:

$$\begin{array}{c} 2 \\ ns_1 \\ \vdots \\ ns_n \\ nv_n \end{array} \left[\begin{array}{c} \underset{\sim}{0} \\ \hline \\ \Delta \underset{\sim}{u} \\ \\ \hline \underset{\sim}{0} \end{array}\right] = \left[\begin{array}{c} \\ Z_{dn} \\ \\ \\ \\ \end{array}\right] \left[\begin{array}{c} \underset{\sim}{0} \\ \hline \\ \Delta \dot{\underset{\sim}{x}} \\ \\ \hline \underset{\sim}{0} \end{array}\right] + \left[\begin{array}{c} \\ Z_n \\ \\ \\ \\ \end{array}\right] \left[\begin{array}{c} \Delta \underset{\sim}{v}_N \\ \hline \\ \Delta \underset{\sim}{x} \\ \\ \hline \Delta y_n \end{array}\right] \quad (B.20)$$

After the elimination of the nth machine algebraic equations and the left network algebraic equations, the reduced matrix of $[Z_n]$, $[Z_{Rn}]$ is obtained:

$$[Z_{Rn}] = \left[\begin{array}{c|c|c|c} H_{11}(\xi)-h_{11} & f_{12} & \cdots & f_{1n} \\ \hline f_{21} & H_{12}(\xi)-\bar{h}_{22} & \cdots & f_{2n} \\ \hline \cdots & \cdots & \cdots & \cdots \\ \hline f_{n1} & f_{n2} & \cdots & H_{1n}(\xi)-\bar{h}_{nn} \end{array}\right] \begin{array}{c} ns_1 \\ ns_2 \\ \vdots \\ ns_n \end{array} = \left[\begin{array}{c|c|c|c} H_1 & f_{12} & \cdots & f_{1n} \\ \hline f_{21} & H_2 & \cdots & f_{2n} \\ \hline \cdots & \cdots & \cdots & \cdots \\ \hline f_{n1} & f_{n2} & \cdots & H_n \end{array}\right] \quad (B.21)$$

Substituting the reduced matrix of $[Z_n]$, $[Z_{Rn}]$, in equation (B.20 leads to the following matrix form:

$$\begin{array}{c} ns_1 \\ ns_2 \\ \vdots \\ ns_n \end{array} \begin{bmatrix} \Delta \underset{\sim}{u}_1 \\ \Delta \underset{\sim}{u}_2 \\ \cdots \\ \Delta \underset{\sim}{u}_n \end{bmatrix} = \begin{bmatrix} Z_{d11} & & & \\ & Z_{d22} & & \\ & & \cdots & \\ & & & Z_{dnn} \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{\dot{x}}_1 \\ \Delta \underset{\sim}{\dot{x}}_2 \\ \cdots \\ \Delta \underset{\sim}{\dot{x}}_n \end{bmatrix} + \begin{bmatrix} H_1 & f_{12} & \cdots & f_{1n} \\ f_{21} & H_2 & \cdots & f_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ f_{n1} & f_{n2} & \cdots & H_n \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{x}_1 \\ \Delta \underset{\sim}{x}_2 \\ \cdots \\ \Delta \underset{\sim}{x}_n \end{bmatrix} \tag{B.22}$$

where, $[H_i] = [H_{1i}(\xi)] - [\bar{h}_{ii}]$ $\qquad (i = 1, 2, \ldots, n)$

or, generally,

$$\Delta \underset{\sim}{u} = [Z_{dn}]\, \Delta \underset{\sim}{\dot{x}} + [Z_{Rn}]\, \Delta \underset{\sim}{x} \tag{B.23}$$

or,

$$\Delta \underset{\sim}{\dot{x}} = -\{[Z_{dn}]^{-1} [Z_{Rn}]\} \Delta \underset{\sim}{x} + [Z_{dn}]^{-1}\, \Delta \underset{\sim}{u} \tag{B.24}$$

Comparing equation (B.24) with the state-space equation:

$$\Delta \underset{\sim}{\dot{x}} = [A]\, \Delta \underset{\sim}{x} + [B]\, \Delta \underset{\sim}{u} \tag{B.25}$$

The system state matrix, $[A]$, is formed for n machines as follows:

$$[A] = -[Z_{dn}]^{-1} [Z_{Rn}] \tag{B.26}$$

State Matrix Partial Derivatives

The general form of partial derivatives of the system state matrix with respect to system control parameters (ξ) is obtained as follows:

$$\frac{\partial [A]}{\partial \xi} = -[Z_{dn}]^{-1} \left\{\frac{\partial [Z_{Rn}]}{\partial \xi}\right\} - [Z_{Rn}]\left\{\frac{\partial [Z_{dn}]^{-1}}{\partial \xi}\right\} \tag{B.27}$$