DYNAMIC RESPONSE OF INELASTIC
MULTI-STOREY BUILDING FRAMES
BEHAVIOUR OF INELASTIC MULTI-STOREY BUILDING FRAMES SUBJECTED TO STRONG GROUND MOTIONS

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SCOPE AND CONTENTS:

The theoretical and experimental investigations presented in this thesis are primarily related to the dynamic response of inelastic multi-storey building frames subjected to strong ground motions. The main purpose is to investigate, both analytically and experimentally, those aspects of the dynamic response characteristics which are of importance in aseismic design. In the first part of the thesis, the various parameters pertaining to the structural system are varied in a systematic manner and an assessment is made of the influence of this variation on the maximum response characteristics of the dynamic system. The second part of the thesis consists of an experimental investigation into the inelastic dynamic response of multi-storey frames. The comparison of experi-
mentally obtained inelastic response and that predicted theoretically indicated a good agreement between the two.
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NOTATIONS


{B} Column matrix of constant coefficients for any phase of deformation.

[C] Damping Matrix

C_{ij} Coefficient of damping matrix.

C, C_c Coefficients of damping and critical damping respectively.

E Modulus of elasticity.

F Multiplying parameter for intensity of acceleration of accelerogram records.

{\bar{F}(t)} Column matrix of applied dynamic loads.

I Moment of inertia of the member.

K Multiplying parameter for stiffness of girders.

L Length of a member of the frame.

M_P Plastic moment of a member of the frame.

[M] Diagonal mass matrix.

P Horizontal Load acting at the floor level.

Q Vertical load on the girder.

Q_{DL}, Q_{LL} Vertical dead and live load on the girder respectively.

{X} Floor displacement column matrix.

g Acceleration due to gravity.
Indices

\( m_i \)
Mass lumped at ith floor level.

\( n \)
Number of storeys of the frame.

\( t \)
Time

\( x \)
Floor displacement.

\( \bar{x}_i, \bar{x}_j \)
Strain response amplitudes at ith and jth cycles under free vibration.

\( \alpha \)
Multiplying parameter for inclusion of live load as a pure load on the girder.

\( \beta \)
Multiplying parameter for inclusion of live load mass to floor mass due to dead load.

\( \gamma \)
Parameter defining the position of live load on the girder.

\( \delta_{ij} \)
Kronecker delta, \( = 1 \) if \( i = j \), \( = 0 \) if \( i \neq j \).

\( \bar{\delta} \)
Logarithmic decrement.

\( \zeta \)
Damping factor.

\( \mu \)
Ductility factor (dynamic).

\( \mu_s \)
Static ductility factor

\( \phi^y \)
Yield deformation.

\( \phi^p \)
Total plastic hinge rotation.
PART I

PARAMETRIC STUDY OF THE INELASTIC RESPONSE OF MULTI-STOREY FRAMES SUBJECTED TO STRONG MOTION EARTHQUAKES
CHAPTER I
INTRODUCTION

1.1 General

The dynamic response of structures in the inelastic region has been the subject of a significant amount of investigation, particularly during the last decade. Due to the nonconservative nature of inelastic structural systems, comprised of a large number of components with different individual characteristics, the behaviour of such systems under the action of strong dynamic forces becomes difficult to predict. A number of techniques have been devised which, based on a variety of simplifying assumptions, permit the calculation of the dynamic response of such systems. At the present time, the basic need is not for the development of additional similar computational techniques, but is for more knowledge of the actual behaviour of real structural systems. Having such knowledge available, one can further explore the field in order to answer the relevant questions and then make suitable recommendations leading ultimately toward the revision and improvement of the design criteria.

The development of knowledge of dynamic behaviour characteristics of real structural systems can take various forms. Two complementary approaches are used in the present
investigation of the behaviour characteristics of multi-
storey inelastic framed structures. First, it is of much practical interest to assess the influence of various system parameters, particularly with regard to the establishment of consistent behaviour patterns. Second, it is necessary to verify, by experimental observation, the validity of simplifying assumptions regarding the inelastic material properties, as used in the computational mathematical model. The purpose of the present investigation is to apply these complementary approaches toward a better understanding of the dynamic behaviour of inelastic framed structures subjected to strong ground motions.

1.2 Previous Work

A rational basis for the design of earthquake resistant multi-storey frames has been the subject of considerable study for some time. The advent of high speed digital computers has boosted the efforts of many researchers to develop various methods for the dynamic analysis of inelastic multi-storey frames subjected to strong ground motions.

DiMaggio\textsuperscript{1}\footnote{Numbers refer to the bibliography listing.} developed a normal mode approach for the dynamic analysis of elastic-plastic frames. This method was an extension of a normal mode approach for beams developed by Bleich and Salvadori\textsuperscript{2}. In this method, the motion during the successive elastic and elasto-plastic stages is expressed in terms of the normal modes of the structure. It
is necessary to recompute the normal modes of the structure each time when a plastic hinge transition occurs during the dynamic motion of the structure. Though the method is theoretically sound, it should be realized that it will require large amounts of computer time and storage for the analysis of a multi-storey frame in which numerous plastic hinge transitions may occur during a very short interval of time. These requirements may limit the practical applicability of the method.

Berg and DaDeppo\(^3\) presented a method for the dynamic analysis of elasto-plastic structures. In this method, the dynamic response is computed by numerical integration of the equation of motion for an elastic system. If the bending moments, as computed after each time interval, exceed the plastic moments, linear corrections are applied. These corrector solutions consist of frames with actual hinges and moment constraints at those points at which a plastic hinge occurs; these are superimposed in such a way so that none of the moments exceeds the plastic moment at any point of the frame. This enables the assumed idealized elastic-plastic moment curvature relationship to be satisfied at each section at which a plastic hinge forms. This procedure may not be too difficult for the dynamic analysis of small frames, but it should be noted that both the pre-calculations of basic corrector solutions for all points and the actual computation during the analysis are likely to be very time-consuming for the analysis of a large multi-
Penzien developed a method for the analysis of multi-storey frames in which girders are assumed to be infinitely rigid. In his approach, all the floors are assumed to remain parallel so that there is only relative horizontal displacement between floors. An idealized elastic-plastic force-deformation relationship is assumed to govern the inter-floor shear resistances. The equations of motion are expressed in terms of inter-floor shear resistances and are numerically integrated by the mid-acceleration method. Though the assumptions greatly simplify the analysis of multi-storey structures, they severely limit the applicability of the method for the analysis of multi-storey frames of normal proportions in which girder ductility plays an important role. The provision for the inelastic deformation of the girders is quite a significant source of energy dissipation to damp out the response of a multi-storey structure subjected to a strong earthquake motion. In the same investigation, Penzien studied the influence of the variation of natural period, yield strength and damping on the maximum dynamic response of such highly idealized shear buildings. The conclusions drawn from this study are limited to the particular class of buildings, and may not be applicable to frames with inelastically deforming girders. Since such deformation was not included in this study, no conclusions could be drawn regarding the relative ductility requirements imposed on the individual girders and columns.
Heidebrecht developed a method for the analysis of inelastic framed structures in which the inelastic behaviour of both columns and girders is considered. In this approach, the horizontal resistance to motion at each floor level is explicitly expressed in terms of the horizontal displacement at floor levels for any state of elastic-plastic behaviour, utilizing the conjugate frame method developed by Lee. The differential equations of motion are solved by using the single step forward numerical integration procedure. Though the method is versatile and includes the significant factors affecting the structural behaviour, it is not applicable to multi-storey frames because it requires a particularly large amount of computer storage capacity. This particular investigation made no attempt to study the influence of any parameters on the behaviour of a given structural system.

A computer program was developed by Benuska to analyze the dynamic response of high-rise buildings to nuclear blast loading. In this approach it was assumed that each member of the frame possesses a prescribed special bilinear moment rotation property. In doing so, any member is assumed to consist of two components in parallel. The first component is a basic elasto-plastic beam which develops a plastic hinge at either end when the respective end moment exceeds the yield moment while the second component remains fully elastic. The elasto-plastic
beam component is assumed to possess a rigid plastic moment rotation property. In this approach, the simplifying assumptions prescribing the special bilinear moment curvature relationship makes the computation relatively simple. The assumptions made obviously neglect the penetration of the plastic zone towards the centre of a member possessing usual bilinear moment curvature relationship.

In the same investigation, Clough and others\textsuperscript{9, 10} studied the ductility requirements of the various members of a typical high-rise building subjected to the ground motion recorded in the El Centro Earthquake of May 18, 1940, N-S Component. Since the study considered a particular structure subjected to a particular base excitation, the authors presented their conclusions with a caution against their general applicability to other structures. With this caution in mind, their study showed that the ductility factors developed during a severe earthquake may be of the order of four to six while the columns may remain elastic except in the top few stories. The period and height of a building do not appear to be the important factors in determining the amount and distribution of ductile deformations. The lateral displacements developed during the nonlinear earthquake response of a tall building appear to be similar in magnitude to the elastic displacement response.

Saul\textsuperscript{11} presented a method of dynamic analysis of structures assuming a piecewise bilinear moment curvature
and stress-strain relationship. The applicability of the method was demonstrated only for a shear building of four storeys. The penetration of the plastic zone towards the centre of the column was considered. Since the floors are considered to be infinitely rigid, the method of analysis is limited to shear buildings. The practical application of this method is probably limited to small structures because in the case of large multi-storey structures, the iterative procedure required to solve the differential equations of motion will be very time consuming, even with high speed digital computers.

Goel and Berg\(^{12}\) developed a procedure for evaluating the response of a multi-storey steel frame in which the girders are permitted to deform inelastically and the columns are restricted to elastic behaviour. The inelastic behaviour of the girders is represented by a Ramberg-Osgood\(^{13}\) type moment-curvature function. These assumptions make the method relatively simple since only the girders are to be considered capable of inelastic deformations. In order for this method to be used, it would be necessary to proportion the columns to remain elastic. In this way, only the girders would dissipate the energy by deforming inelastically. The method thus simplified is limited to only those particular cases in which yielding in columns is to be avoided altogether.

The above investigation considered the influence of three system parameters on the response of the structure,
namely the height and stiffness of the frame, and the earthquake characteristics. Damping other than hysteretic was not considered. The authors concluded that the inelastic deformation of girders alone can provide an important source of energy dissipation to damp out the response of a multi-storey structure subjected to a strong earthquake motion. Also, it was concluded that the elastic fundamental period of a multi-storey structure has a marked influence on inelastic response and that the general shape of the elastic velocity response spectrum of an earthquake motion can provide significant information about the expected inelastic response of a multi-storey structure. It was further concluded that the column and girder response tends to be more uniformly distributed as the height of the structure increases. It was noted that the conclusions drawn should be viewed with caution as these are based on the analysis of a limited number of situations.

The author developed a general method for evaluating the dynamic response of inelastic multi-storey building frames. The method of analysis is based on the matrix displacement method of structural analysis. In this approach, columns and girders follow an idealized moment curvature relationship. A plastic hinge is allowed to form at any section where the moments attain extremum values. The method of analysis allows a piece-wise linear behaviour of the structural system between successive plastic hinge transitions. These features make this method particularly
suitable for dynamic analysis of multi-storey steel building frames subjected to any kind of dynamic loading. This method is utilized in the investigation described in this thesis and is discussed further in Chapter II.

The foregoing review of research work depicts the state of art of the computation of inelastic response of framed structures. The methods of computation described above include a variety of approaches, each of which is characterized by assumptions which are made in order to simplify the formulation of the problem and overcome computational difficulties. Apart from the complexity of formulation, the computational difficulty of the requirement of large computer storage and computing time, have been some of the causes which have compelled various researchers to either adopt an oversimplified mathematical model or to limit the effort only to the development of the method of analysis. There have been limited efforts to consider the influence of the variation of various system parameters on the inelastic response of structures. The attempts which have been made have usually accompanied the development of the method of analysis. The parameters which have been studied include: natural period, yield strength, damping, height of building, relative strengths of girders and columns, and characteristics of ground motions. None of the studies have included all the parameters in conjunction with the same mathematical model and the same method of analysis. Consequently, whatever limited
conclusions are available cannot be interpreted in a consistent manner. Moreover some of the important system parameters such as inclusion of live load mass and live load forces, position of live load, various levels of damping, various earthquake intensities, etc. have not been considered at all. Also the effect of variation of system parameters has been considered only on the member ductility requirements and floor displacements. No attention has been given to the maximum floor accelerations which develop during the earthquake.

Compared to the large number of analytical investigations in the field of structural dynamics, there have been very few experimental investigations in the area of inelastic response of multi-degree of freedom structural systems.

Hanson\textsuperscript{15} investigated experimentally the frequency response characteristics of a one degree of freedom mild steel structure vibrating in the inelastic range. The object of his investigation was to compare the static, dynamic and theoretical behaviour of the yielding structure. The dynamic horizontal force applied to the structure was sinusoidal in nature and was generated by a shaking machine placed at the top of the structure. It was found that the dynamic and static hysteretic force deflection curves were generally in good agreement. It was also concluded that on the basis of the static virgin force deflection curve, the resonant vibrational amplitude of the structure can be
predicted within 20% and the resonant natural frequency within 2 1/2%. For large plastic deflections at low frequencies, it was found that the difference between the dynamic and static hysteresis loops was less than the changes in the static loops resulting from deterioration caused by repeated cycles of loading.

Impact tests on small model steel frames were reported by Rawlings\textsuperscript{16}. These frames were square portals with fixed bases, whose side dimensions were either 6 in. or 10 in. The frames were oxy-cut from 1/4 in. thick mild steel plate and all the members were 1/4 in. square in section. In all cases, good agreement was found between experimentally recorded transient forces and deflections and those predicted theoretically, taking into account the strain rate sensitivity of the material and strain hardening.

Heidebrecht, Zelman and Ward\textsuperscript{17} reported an exploratory experimental investigation of the dynamic behaviour of small-scale framed structures subjected to blast loading. Aluminum "I" sections were used for beams and columns and the floors were reinforced concrete slabs. The structures were tested dynamically using an air blast caused by detonations of an explosive charge comprised of approximately twenty tons of T.N.T. The permanent deflections of the floors of a six storey frame were recorded and compared with those obtained using various mathematical models; the mathematical models included an
elasto-plastic framed structure, elasto-plastic shear building, and also a structure with rigid members and flexible joints.

No definite conclusions were drawn except that the method of testing was not suitable and that the effect of joint deformations should be included in the mathematical model.

The above experimental investigations reported by various authors indicate the attempts to compare the experimental results with those obtained from theoretical considerations or based on material properties determined experimentally under static conditions. The conclusions drawn by Rawlings cannot be used in any practical manner for application to multi-storey frames as the structure used for investigation consists of members which are not representative of practical structural sections. Moreover, the mathematical model used is also too complicated for any practical application for multi-storey frames. Hanson's work is useful as it answers some of the basic questions for the verification of basic assumptions used in analytical prediction. The investigation was directed to investigate post-elastic dynamic response under steady state vibration, though in most cases the strong dynamic loading forces the structures to go through a transient type of vibration in the inelastic range. Only the yielding of columns has been considered. The author himself has indicated a need for further investigation of
the behaviour of multi-degree of freedom system.

1.3 Scope of Investigation

The analytical and experimental investigation presented in the two parts of this dissertation are primarily related to the dynamic response of inelastic multi-storey building frames subjected to strong dynamic ground motion. The main purpose is to investigate, both analytically and experimentally, those aspects of the dynamic response characteristics which are of importance in aseismic design. The knowledge of these characteristics can then lead to verification or modification of existing design criteria.

In Chapter II, the method of dynamic analysis of inelastic multi-storey frames is briefly outlined. The mathematical model consists of a lumped mass system with all masses lumped at the floor levels. The members making up the structural frame are assumed to have an idealized elastic-plastic moment curvature relationship. Plastic hinges are permitted to form at all possible locations of maximum moments. Damping, other than hysteretic damping, is assumed to be of a viscous type. A description of the differential equation of motion used for dynamic analysis of inelastic multi-storey frames, together with brief comments on the numerical integration procedure adopted to solve it, is also presented in Chapter II.
The first part of the thesis consists of the parametric study of the inelastic response of multi-storey building frames subjected to strong motion earthquakes. Various parameters pertaining to the structural system are varied in a systematic manner and an assessment is made of the influence of this variation on the maximum response characteristics of the dynamic system.

Chapter III contains the basic features of the parametric study. The procedure adopted to accomplish the parametric study is outlined. The characteristics of the basic multi-storey structure and the standard earthquake are described. The seven system parameters and three response parameters which are evaluated for variation of each of the system parameters are defined.

Results obtained from the parametric study and the related discussion are presented in Chapter IV.

Chapter V contains the conclusions drawn from the parametric study. The conclusions pertaining to the variation of each system parameter are presented in separate sections.

The second part of the thesis consists of an experimental investigation into the inelastic dynamic response of multi-storey frames. The object of this investigation is to compare the experimentally obtained inelastic response with that obtained analytically. The inelastic and elastic responses of the system, based on material properties determined under static conditions,
were computed analytically and are compared with the
dynamic response which was recorded experimentally.
Similarly the natural frequencies of the system predicted
analytically from 1) individual member properties and 2)
properties of the assembled structural system (both deter-
mined experimentally under static conditions) are compared
with those observed experimentally.

The description of the experimental structure and
the joint test procedure from which the moment-curvature
relationships are obtained are given in Chapter VI. This
chapter also describes the experimental procedure for the
determination of flexibility influence coefficients.

Chapter VII contains the analytical predictions
of elastic response to pulses and earthquake excitation
together with the inelastic response of the experimental
structure.

Details of the dynamic experimental investigation are
given in Chapter VIII. These details contain a brief
description of the experimental system and the experimental
procedures adopted to determine damping factor, dynamic
properties and transient elastic and inelastic response.

In Chapter IX, comparisons of the analytical and
experimental results are presented and discussed.

Chapter X contains general conclusions drawn from
both the parametric study and the experimental investiga-
tions. Recommendations based on the conclusions are also
presented in this chapter.
CHAPTER II
DYNAMIC ANALYSIS

2.1 Introduction

As mentioned in Chapter I, a number of methods of analysis for computing the dynamic response of inelastic framed structures have been reported in the literature. The method used in this investigation was developed by the author and has features which make it particularly suitable for computing the response of multi-storey frames. It is used in Part I of this thesis to compute the inelastic response, with varying system parameters, of a ten storey building frame subjected to strong motion earthquakes. It is also used in Part II to give the analytical predictions of the elastic and inelastic response of the experimental test frames subjected to varying forms of base excitation.

2.2 Assumptions

The various assumptions made for the dynamic analysis of inelastic multi-storey building frames used for this study are the following:

1) The girders and columns of the frame are of a ductile material which has an idealized elasto-plastic stress
strain relationship. Structural steel used in multi-storey building frames is quite ductile, with ductility factors varying from eight to fifteen for various grades of steel, as shown by Beedle. For wide flange and "I" sections which are usually used in multi-storey building frames, the moment-curvature relationship of flexural members, i.e. girders and columns, can reasonably be assumed to be of the idealized form shown in Fig. 2.1, as the shape factor for these shapes is approximately 1.15. Various authors in this field have confirmed this assumption of idealized moment-curvature relationship to be practically the same as that obtained experimentally.

2) The joints connecting the members are assumed to be infinitely rigid. This is the usual assumption made in the analysis of moment resisting frames. Though it is realized that no real joint is perfectly rigid, consideration of such factors might complicate the whole concept of the simplicity of elasto-plastic behaviour of the frame. Recently a method has been formulated to consider the non-rigidity of the connections.

3) Effect of shear deformation is neglected. This assumption is justified as the columns of a normally proportioned frame are approximately ten times larger than their depth. Shear deformation can become appreciable only when the length to depth ratio of members is small. All authors except one in this area have disregarded
FIG. 2.1 MOMENT–CURVATURE RELATIONSHIP
the effect of shear deformation in their analysis. It is considered that the effect of shear on the overall deflections is negligible.

4) Effect of axial strain is neglected. This has been done for the sake of simplicity by all the authors in this area. On the basis of experience, it has been reported in the literature\(^2^3\) that if the height to width ratio in the frame is no greater than five, axial strains in columns may be neglected without appreciably affecting the dynamic response of the structure. On the basis of this criterion, the effect of axial deformation would be very small in the frame adopted for the parametric study, since its aspect ratio is about 5.7.

5) The destabilizing effect of gravity under large lateral floor deflections has been neglected for the sake of simplicity. This has been done by all the previous authors. It is only recently that the destabilizing effect of gravity has been considered for an oversimplified single storey frame in order to explain the formation of a collapse mechanism in single storey structures damaged by severe earthquakes\(^2^4\).

The mathematical model based on the above assumptions consists of a lumped mass system with all the masses lumped at the floor levels. These masses may include a percentage of the mass due to live load on the floors, in addition to the mass due to dead load of the floor and structural system. The frame is assumed to have only
planar motion. Plastic hinges are permitted to form at any point of maximum moment, including girder points where the resultant of the vertical dead and live load may act. Thus the stiffnesses of both columns and girders contribute to the stiffness of the frame which is characterized by a piecewise linear function between successive transitions of the plastic hinges in accordance with the elastic perfectly plastic moment-curvature law. Any damping is assumed to be of viscous type and is expressed as a percentage of critical damping for the fundamental elastic mode. Two terms are used in subsequent chapters for damping. The term "close coupled" refers to that damping matrix whose off diagonal elements are zero while the term "far coupled" refers to the damping matrix which is fully populated.

2.3 Differential Equation of Motion

The differential equation of motion for a viscously damped multi-degree of freedom system is given by

\[ [M]\{\ddot{X}\} + [C]\{\dot{X}\} + \{R(X)\} = \{F(t)\} \tag{2.1} \]

where

\( \{X\} \) is the column matrix of displacements,

\( \{\dot{X}\} \) is the column matrix of velocities,

\( \{\ddot{X}\} \) is the column matrix of accelerations,

\( [M] \) is a diagonal matrix consisting of masses,

\( [C] \) is the matrix of damping coefficients,
\{R(x)\} is the column matrix of structural resistance forces, and
\{F(t)\} is the column matrix of applied dynamic loads.

If "n" is the number of storeys in the structure, each of the matrices in Eq. 2.1 is of "n" order and the column matrices contain quantities evaluated at each floor level. The matrix \{R(x)\} is given by

\[
\{R(x)\} = [A]\{x\} + \{B\}
\]

in which \([A]\) and \(\{B\}\) are matrices of constant coefficients for any phase of deformation. The term "phase" refers to a particular state of elastic-plastic deformation resulting from the formation or release of one or more plastic hinges at a certain instant of time. The procedure of evaluation of \([A]\) and \(\{B\}\) due to change of phase at any instant is based on the method described by the author.

Eq. 2.1 is solved using a single step forward numerical integration procedure. In this method of solution, the deflection-velocity and velocity-acceleration relationships are assumed to be linear over each small time interval. The computations used in this thesis were done at the McMaster University Data Processing and Computing Centre on an IBM 7040 computer. Output data for the parametric study in Part I were stored on magnetic tape and the graphical representation of the results were drawn by the Benson Lehner plotter.
CHAPTER III
FEATURES OF THE INVESTIGATION

3.1 Introduction

As indicated in Chapter I, significant efforts have been made to develop computational techniques for the calculation of the dynamic response of structures characterized by particular mathematical models. However, little work has been done to study the significance of the various factors which are incorporated in the mathematical model. The purpose of this particular study is to evaluate the effects of so-called "system parameters" on the response characteristics of inelastic multi-storey structures subjected to strong motion earthquakes.

The "system parameters" are properties of the structure and its loading system and are defined in detail later in this chapter. The response characteristics are identified by a small group of so-called "response parameters" also described later in this chapter. The procedure used in this analysis is to vary one particular system parameter over a realistic range with all other parameters constant, and to evaluate the response parameters during the same range of variation. The behaviour of response parameters is then examined in detail in order to evaluate the significance of the particular system parameter. The analysis
concludes with a discussion of the design implications of such results.

3.2 Characteristics of Basic Structure and Standard Earthquake Basic Structure

For the purpose of this investigation, the single bay ten storey framed structure shown in Fig. 3.1 is used as the basic structure. The storey height is 12.0 ft. and the bay width is 21.0 ft. Based on an assumed frame spacing of 28.0 ft. and a dead load of 100 lb. per square foot acting on each floor of the finished structure, an equivalent concentrated dead load of 59 kips is assumed to act at the mid-span of each girder. The relative stiffnesses of the members are shown in Table 3.1. In addition to ensuring the general adequacy of the performance of the basic structure under lateral loads, this structure was designed so that the tenth floor load deflection curve for a monotonically increasing static horizontal load (proportional to floor mass) showed a reasonable ductility of the structure. The load deflection curve has a nearly flat plateau in the region prior to collapse, as shown in Fig. 3.2, which gives the non-dimensional load-deflection curve for a constant value of vertical girder load Q and a monotonically increasing horizontal load parameter P. First-hinge formation occurs when $P = 23$ kips at a tenth floor deflection of 8.02 in. At collapse the maximum deflection is 4.77 times
FIG. 3.1 THE BASIC STRUCTURE
<table>
<thead>
<tr>
<th>Stories</th>
<th>Columns</th>
<th>Girders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size</td>
<td>Relative Stiffness</td>
</tr>
<tr>
<td>10</td>
<td>14 WF 95</td>
<td>1.00</td>
</tr>
<tr>
<td>9</td>
<td>&quot; 103</td>
<td>1.10</td>
</tr>
<tr>
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<td>&quot; 184</td>
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<td>&quot; 246</td>
<td>3.03</td>
</tr>
</tbody>
</table>

**TABLE 3.1** RELATIVE STIFFNESSES OF THE MEMBERS OF BASIC TEN STOREY SINGLE BAY FRAME
FIG. 3.2 LOAD DEFLECTION CURVE FOR BASIC STRUCTURE.

TOTAL DEFLECTION OF 10th FLOOR

DEFLECTION OF 10th FLOOR AT FIRST HINGE FORMATION

μs

Q = 59 K

Q1

Q2

Q3

Q4

Q5

Q6

Q7

Q8

Q9

Q10

P

P

P

P

P

P

P

P

P

P

21'
the value at first-hinge formation; this ratio is defined as the static ductility factor $\mu_s$.

**Standard Earthquake**

The basic structure is subjected to the N-S Component of the May 18, 1940 El Centro California earthquake with the acceleration multiplied by a constant factor "F" so that the peak acceleration is 0.5 g. In the subsequent text, this intensified earthquake is referred to as the "standard earthquake". After studying nine different cases in which the full duration of earthquake excitation was used, it was observed that the maximum response occurred within 12 sec. duration of the earthquake. Hence for the investigation of further cases, a duration of first 14 sec. of the above mentioned "standard earthquake" was considered to be appropriate. This was done to save computing time.

### 3.3 System Parameters

The system parameters used in this investigation are the properties of the structure itself and/or the loading system (static or dynamic) applied to the structure. The following paragraphs define the system parameters and their ranges of variation.

**Girder Live Load**

This parameter is the percentage of live load included as vertical load on the girder and is denoted by the symbol "$a$". The concentrated vertical load $Q$ acting at the centre of each girder is then given by
\[ Q_{\text{TOTAL}} = Q_{\text{DL}} + \alpha \times Q_{\text{LL}} \]

For a uniformly distributed live load of 75 lbs. per square ft., \( Q_{\text{LL}} \) is 44.25 kips. The value of the parameter \( \alpha \) is varied from 0.0 to 0.8. It is necessary to consider this parameter because the value of vertical load on the girder influences the inelastic response of the structural system. The constant stresses due to dead load, if superimposed on those due to live load and seismic forces generated by the earthquake excitation, may result in either early or delayed yielding of the members.

**Live Load Position**

The parameter \( \gamma \), as shown in the diagram on top of Table 4.2, defines the position of live load on the girder and is given by

\[
\gamma = \frac{\text{distance of live load from mid span of girder}}{\text{half girder span}}
\]

The value of parameter \( \gamma \) is varied from 0.0 to 1.0. The consideration of the variation of this parameter is also important for the same reasons as those given for the parameter \( \alpha \). However, this may have a far worse effect on the response than the previous parameter because of the initial sway of the structure.

**Live Load Contribution to Mass**

This parameter is the percentage of live load mass
added to the lumped mass at floor level due to the dead weight of the structural system, and is denoted by "β". The lumped mass at each floor level is then given by

\[ m_i = \left( Q_{DL} + \beta \times Q_{LL} \right)/g. \]

While adding the live load mass to floor mass, the corresponding live load is also added to the vertical load on the girder. Therefore, in this case, the concentrated vertical load \( Q \) acting at the centre of each girder is given by

\[ Q_{TOTAL} = Q_{DL} + \beta \times Q_{LL} \]

The value of parameter \( \beta \) is varied from 0.0 to 0.8.

It is of much importance to investigate the effect of variation of this parameter on the response parameters because the contribution of live load mass to the mass of each floor results in a different dynamical system which will have different response characteristics.

**Stiffness Distribution**

In this case the girder stiffnesses are proportionately increased, while the column stiffnesses are the same as those of basic structure. The multiplying parameter is denoted by "K", so that the stiffness of each girder is then given by

Girder Stiffness = \( K \times \) Girder Stiffness of the Basic Structure. The value of the parameter \( K \) is varied from 1.0 to 10.0.
In the lower storeys of a multi-storey building, the stiffness of the column governs the design while in the uppermost few storeys, the stiffnesses of the girders govern. The stiffness of the girders is increased by the composite action of the floor and girder system. Thus it is of interest to know the effect of increasing the stiffness of the girders on the response parameters of the new dynamical system resulting from the change of girder stiffnesses. Such investigation is also important with regard to the aseismic design of structures insofar as the evaluation of the relative contribution of different floor systems to girder stiffnesses is concerned.

**Damping**

This parameter is the percentage of critical damping denoted by the symbol "\( \zeta \)" and is given by

\[
\zeta = \frac{c}{C_c}
\]

in which \( c \) and \( C_c \) are respectively the coefficients of damping and critical damping. The value of the parameter is varied from 0.0 to 0.10 for close coupled damping of a viscous type. A case of far coupled damping with the parameter \( \zeta \) equal to 0.02 is also studied and comparisons of close and far coupled cases for this value of \( \zeta \) are made in Chapter IV.

The dynamic response of elastic systems is also computed and compared with that of the corresponding inelastic system. This has been done to evaluate whether
or not the usual concept of approximately equal maximum displacements for elasto-plastic and elastic single degree of freedom system also holds true in multi-degree of freedom systems.

**Earthquake Intensity**

This parameter is a factor "F" by which the amplitude of the actual accelerogram of N-S component of the May 18, 1940 El Centro California earthquake is multiplied in order to vary the intensity of the excitation of ground motion. Therefore, the amplitude of the resulting intensified accelerogram is given by

\[
\text{Amplitude of Intensified Accelerogram} = \text{Amplitude of Actual Accelerogram} \times F.
\]

The value of the parameter F is varied from 1.0 to 1.567 thereby resulting in the variation of maximum base acceleration from 0.32 g to 0.5 g. The effect of the variation of this factor may provide information about the probable effect of intensity of base acceleration on the response of the inelastic structure.

**Earthquake Characteristics**

The effect of this parameter has been assessed by using six different earthquake accelerogram records. In each case the amplitude of acceleration of the particular earthquake was multiplied by a constant factor (for that particular earthquake) so that the maximum base acceleration for each earthquake has an "intensified" value of
0.5 g. The six earthquakes together with the respective multiplying constant factors are given in Table 4.7. This study may provide information as to the reliability of using the maximum base acceleration as the primary measure of probable damage to structures, even with earthquakes having vastly different characteristics.

3.4 Response Parameters

The response of an inelastic multi-storey building frame under the action of earthquake motions may be studied with the help of the following so-called "Response Parameters".

**Ductility Factor**

An important parameter defining the behaviour of an inelastic framed structural system is the maximum inelastic flexural deformation in each member of the frame during the time history of the earthquake. In order to estimate the magnitude of maximum inelastic flexural deformation, it is compared with the maximum elastic rotation angle \( \phi^Y \) which the member may develop. This is also referred to as yield deformation which is given by

\[
\phi^Y = \frac{M_p L}{6EI}
\]  

(3.1)

where

\( M_p \) = plastic moment,
\( L \) = length of the member,
\( E \) = modulus of elasticity, and
\[ I = \text{moment of inertia of the member.} \]

Note that in Eq. 3.1, the fully plastic moment \( M_p \) is used to determine the yield deformation, instead of yield moment \( M_y \). This is perfectly legitimate in accordance with the basic assumption of an idealized moment-curvature relationship valid for wide flange and "I" sections of ductile material possessing an idealized elasto-plastic stress strain relationship as indicated in section 2.2. Thus the member ductility factor, which is defined as the ratio of the total flexural deformation (including the inelastic portion) to the flexural yield deformation, is represented by

\[
\mu = \frac{\phi_y + \phi_p}{\phi_{y,\text{max}}} \tag{3.2}
\]

in which

\[ \mu = \text{member ductility factor, and} \]

\[ \phi_p = \text{total plastic hinge rotation.} \]

The ductility factor is particularly important since it is an index of energy absorption capacity of the multi-storey building in the inelastic range. It is necessary to design a building to have sufficient ductility in order to absorb energy inelastically without failure when subjected to strong earthquakes. Moreover, from the point of view of aseismic design, it is of interest to determine which parameters influence the maximum ductility and its distribution throughout the system.
**Displacements**

Evaluation of maximum displacement of floors during the time history of response is of interest and importance because of the following reasons:

1) The maximum displacements control the design of the multi-storey building. Masonry and plaster become sensitive to cracking when the average building drift is of the order of \( \frac{1}{1000} \) to \( \frac{1}{250} \).

2) Adjacent buildings may sway out of phase during an earthquake. This can result in hammering of buildings against one another unless predictions of probable maximum displacements have been made at the time of plan layout and the required building separation has been provided. Hammering may cause considerable local damage at the points of contact resulting in serious damage to the building and may endanger people in the street below.

3) During large displacements, the destabilizing effect of gravity may assist in causing the collapse of the structure.

4) It is worthwhile to investigate whether or not the displacement response spectrum is valid for the prediction of maximum displacements of an inelastic multi-degree of freedom system. A detailed discussion on the results of such investigation is presented in section 4.5.

**Accelerations**

During the course of an earthquake, different floors of the multi-storey structure are subjected to accelerations which may be magnified considerably above
the maximum ground acceleration. It is important to evaluate the maximum acceleration developed at any floor level for the following reasons:

1) The maximum acceleration indicates the amount of force experienced by the corresponding floor. This can be utilized in evaluating the maximum probable shears at different levels of the building.

2) The maximum accelerations may be used to evaluate the so-called seismic coefficient \( c \) for the design of the most vulnerable parts of the building, i.e. parapet walls, interior decorations, etc.

3) An acceleration greater than 15% of gravitational acceleration is considered to be unbearable from the point of view of occupants' comfort. It is expected that in tall buildings, accelerations developed during a strong earthquake may be several times greater than 15% of gravitational acceleration. Hence it is of interest to evaluate the maximum accelerations so as to be aware of the possible risk to human comfort and safety.
CHAPTER IV
RESULTS AND DISCUSSION

4.1 Effect of the Amount of Live Load Included as Vertical Load on the Girders

Figures 4.1 through 4.5 show the response histories of the horizontal floor displacements for values of $\alpha$ varying between 0.0 and 0.8. As the contribution of live load as a pure load on the girder increases, the response characteristics of the system do not change but the amplitude of displacements is mildly affected. The displacements tend to decrease as the value of $\alpha$ increases, with the maximum displacement for $\alpha = 0.8$ approximately 10 percent less than that for $\alpha = 0.0$.

Fig. 4.6 shows the effect of variation of $\alpha$ on the maximum ductility factors of columns and girders in each storey. For all variations of $\alpha$, the columns in the lower two-thirds of the structure, except for those in the lowest storey, behave elastically while yielding occurs in the columns in the upper third of the structure. However, girder yielding occurs in all storeys, with the maximum ductility occurring at two-thirds of the height of the structure above the base.

There is insignificant variation in the column ductility factors in the lower two-thirds of the structure.
FIG. 4.1 INELASTIC RESPONSE HISTORY OF BASIC STRUCTURE,
EL CENTRO MAY 18, 1940, N-S X 1567
FIG. 4.2 INELASTIC RESPONSE HISTORY FOR $\alpha = 0.2$
FIG. 4.3 INELASTIC RESPONSE HISTORY FOR $\alpha = 0.4$
FIG. 4.4 INELASTIC RESPONSE HISTORY FOR $\alpha = 0.6$
FIG. 4.5 INELASTIC RESPONSE HISTORY FOR $\alpha = 0.8$, $\gamma = 0.0$
FIG. 4.6 EFFECT OF LIVE LOAD CONTRIBUTION AS A PURE LOAD ON THE GIRDER

DUCTILITY FACTORS
and only slight variation in the upper stories. No significance can be attached to the nature of variation of column ductility factors within any given storey.

The effect of the variation of \( \alpha \) on the girder ductility factors is more significant. The amount of change of ductility factor varies from storey to storey but it is a general observation that the girder ductility factor increases with an increase in \( \alpha \). The maximum relative change is approximately 15% but the change at the maximum value is negligible.

The increase in load on the girders affects the maximum moments on the ends of girders more directly than those on the columns. This results in an early onset of yielding of the girders. Consequently, the girder ductility factors are increased. With the increase in girder ductility factors, there must be more consumption of energy which results in reduced ductility factors of the columns. This is indicated by the decrease in column ductility factors associated with corresponding increase in girder ductility factors due to increase in \( \alpha \).

Fig. 4.7 shows the effect of variation of \( \alpha \) on the maximum acceleration of each floor. It is seen from this figure that the maximum acceleration does not increase continuously from the lower to upper storeys. Though there is a general increase with height, the pattern of such change is erratic. This is due to the various degrees
FIG. 4.7 EFFECT OF LIVE LOAD CONTRIBUTION AS A PURE LOAD ON THE GIRDERs
of yielding of the different sections. Yielding of the girders inhibits increase in acceleration. Also the non-uniform distribution of ductility in different storeys results in a non-uniform pattern of variation of acceleration from storey to storey. The variation of maximum acceleration does not have a constant pattern from storey to storey which is consistent with the observation of similar variation of girder ductility factors. This phenomenon is to be expected because the ductility factors play two important roles. First, a ductility factor of more than one indicates some loss of stiffness of the structure, the degree of which depends on the location of the plastic hinges and the number of such hinges. A plastic hinge in a column will result in more loss of stiffness compared with a plastic hinge occurring in a girder. Second, a larger ductility factor indicates more consumption of energy due to plastic work at the hinges. Both these factors interact to affect the response of the structure. It is expected that the formation of a hinge in the column in the lower storey will tend to increase the maximum displacement in the storeys above.

The maximum horizontal floor displacements resulting from the variation of $\alpha$ are shown in Fig. 4.8. There is very little variation of maximum floor displacements in the lower two-thirds of the structure. However, there is a decrease in maximum floor displacements in the upper one-third of the structure with an increase in the
FIG. 4.8 EFFECT OF LIVE LOAD CONTRIBUTION AS A PURE LOAD ON THE GIRDERs
value of $\alpha$. As discussed in an earlier paragraph, the decrease in maximum displacements is consistent with the decrease in column ductility factors which in turn results from the increase in girder ductility as a consequence of the increase in $\alpha$. Thus the changes in the maximum displacements are not directly related to the variation of $\alpha$, but are a consequence of change in column ductility factors in particular. Consequently, no significance could be attached to the decrease in maximum displacements with increase in $\alpha$.

Table 4.1 lists the maximum displacement of the tenth floor, its time of occurrence, the overall maximum acceleration and the overall maximum column and girder ductility factors. It is clear that the time of occurrence of maximum displacement of the tenth floor remains essentially constant as $\alpha$ varies from 0.0 to 0.6. When $\alpha = 0.8$, the time of maximum displacement is only slightly later than that for other values of $\alpha$. No significance can be attached to this difference.

The maximum tenth floor displacement and overall maximum accelerations are shown in Fig. 4.9. It is seen that the maximum tenth floor displacement decreases linearly with $\alpha$. Little significance can be attached to this observation because the range of variation of the maximum displacement is only 10% and it is probable that the linearity observed here may change for excitations by other earthquakes having different characteristics.
<table>
<thead>
<tr>
<th>System Parameter $\alpha$</th>
<th>Maximum Displacement of 10th floor in inches</th>
<th>Time for Maximum Displacements in seconds</th>
<th>Maximum Acceleration $/g$</th>
<th>Ductility Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Columns</td>
</tr>
<tr>
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<td>2.190 ($X$)</td>
<td>1.31 ($X$)</td>
</tr>
<tr>
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<td>2.155 ($X$)</td>
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</tr>
<tr>
<td>0.4</td>
<td>9.51</td>
<td>6.070</td>
<td>2.170 ($X$)</td>
<td>1.33 ($X$)</td>
</tr>
<tr>
<td>0.6</td>
<td>9.25</td>
<td>6.075</td>
<td>2.160 ($X$)</td>
<td>1.44 ($X$)</td>
</tr>
<tr>
<td>0.8</td>
<td>8.98</td>
<td>6.750</td>
<td>2.150 ($X$)</td>
<td>1.46 ($X$)</td>
</tr>
</tbody>
</table>

*Roman numeral enclosed by parentheses refers to storey number of the frame.*

**TABLE 4.1 EFFECT OF VARIATION OF LIVE LOAD AS A PURE LOAD ON THE GIRDER**
FIG. 4.9 EFFECT OF VARIATION OF LIVE LOAD AS A PURE LOAD ON THE GIRDER.
The above figure indicates that the maximum acceleration is nearly constant for all values of $\alpha$. This is probably due to the small excursions of the columns into the plastic region and also due to the small variations of girder ductility factors, because both of these factors interact to affect the accelerations.

4.2 Effect of Different Positions of the Live Loads

Figures 4.5, 4.10 and 4.11 show the response histories of the horizontal floor displacements for values of $\gamma$ equal to 0.0, 0.5 and 1.0 respectively. As the position of live load moves from the centre of the girder towards the end of the girder, there is no change in the displacement response characteristics except that the magnitude of displacement is increased by approximately 15%. Obviously there is initial sway of the frame due to the eccentric position of the live load (when $\gamma = 0.5$ or 1.0); this can be seen from the displacements at zero time in Figures 4.10 and 4.11.

The effect of variation of $\gamma$ on maximum column and girder ductility factors in each storey is shown in Fig. 4.12. There is very little change in the maximum column ductility factors as $\gamma$ varies from 0.0 to 1.0. This constancy of column ductility factor is consistent with the observations made for the variation of live load magnitude.

The maximum girder ductility factors increase significantly with the eccentricity of the live load posi-
FIG. 4.10 INELASTIC RESPONSE HISTORY FOR $\gamma = 0.5$, $\alpha = 0.8$
FIG. 4.11 INELASTIC RESPONSE HISTORY FOR $\gamma = 1.0$, $\alpha = 0.8$
DUCTILITY FACTORS

FIG. 4.12 EFFECT OF THE CHANGE OF LIVE LOAD POSITION
tion. This would be anticipated since a position of resultant of dead and live load closer to the column would induce earlier yielding, thereby tending to increase the maximum girder ductility factor.

As seen in Fig. 4.12, the general shape of the curves for maximum ductility factors remain essentially the same. The shifts in the curves indicate the effects of differing degrees of inelastic activity.

The amount of change of girder ductility factor varies from storey to storey but in general it can be seen that the maximum girder ductility factors tend to increase with \( \gamma \) at points of relative maxima (located near the upper and lower part of the structure). In contrast with this, they tend to decrease at points of relative minima (located in the middle of the structure). Though the maximum relative change is about 45\%, the change in the overall maximum value is about 30\%.

Fig. 4.13 indicates the effect of variation of \( \gamma \) on the maximum acceleration of each floor. Referring to Fig. 4.12, it appears that increase in maximum accelerations at a certain portion of the structure (middle portion in this case) corresponds to a decrease in ductility factors of the same portion and vice versa (upper and lower portion). This observation indicates that increase in ductility factors at some portion of the structure tend to decrease the maximum acceleration and vice versa. Though such interactions of two parameters are not expli-
Fig. 4.13 Effect of the change of live load position

ACCELERATION

ACCELERATION OF GRAVITY

\( y = \text{DISTANCE OF LIVE LOAD FROM MID-SPAN} \)

\( \text{HALF GIRDER LENGTH} \)
citly related to each other, the above observation seems to be consistent with the fact that increase in ductility will absorb energy and result in reduced accelerations and vice versa. Little significance should be attached to such drastic changes of accelerations in terms of design implications of the overall structures. Of course, it may be noted that such an increase in acceleration is possible anywhere in the structure, and considerations of designing nonstructural elements and their connections to the main structure for a larger horizontal force (as stipulated in the SEAOC code) are reasonable and desirable.

The maximum horizontal floor displacements resulting from the variation of $\gamma$ are shown in Fig. 4.14. As observed in the previous case, here again the inelastic activity in the columns in the upper third of the structure affects the maximum displacements of the floors above. The displacement curves have also departed from the original curve ($\gamma = 0.0$) because of initial sway. It is seen in this case that the maximum displacement of the tenth floor tends to increase with increase in $\gamma$. This observation cannot be generalized since it is not directly related to $\gamma$, rather to a greater extent on the column ductility factors which are themselves related in turn to $\gamma$ and other system and response parameters. However, it is worth noting that the maximum change in maximum displacement is only about 15% for extreme changes in live load position.
FIG. 4.14 EFFECT OF THE CHANGE OF LIVE LOAD POSITION
Table 4.2 lists the maximum displacement of the tenth floor, its time of occurrence, the overall maximum acceleration and the overall maximum column and girder ductility factors. Again, as in Table 4.1, the time at which the maximum displacement occurs varies very slightly and has no apparent significance.

Fig. 4.15 shows the variation of maximum displacement and maximum acceleration with changes in the parameter $\gamma$. For reasons mentioned earlier in this section, the variation of tenth floor displacement and maximum acceleration with variation in $\gamma$ cannot be generalized. Of course, for this particular case, the maximum displacement varies more or less in a linear fashion. It may also be noted from Fig. 4.13 that for extreme values of $\gamma$ there is slight change in girder ductility. This in turn results in constant maximum acceleration for the same range of variation of $\gamma$ as shown in Fig. 4.15.

4.3 Effect of Contribution of Live Load to Floor Mass

Figures 4.1 and 4.16 through 4.19 show the response histories of the horizontal floor displacements for values of $\beta$ varying between 0.0 and 0.8.

As the contribution of mass due to live load is added to the dead load floor mass, the dynamic system changes and hence the response characteristics will also change. This can be seen from the displacement response histories in the figures referred to above.
### TABLE 4.2 EFFECT OF THE CHANGE OF LIVE LOAD POSITION

<table>
<thead>
<tr>
<th>System Parameter (γ)</th>
<th>Maximum Displacement of 10th floor in inches</th>
<th>Time for Maximum Displacements in seconds</th>
<th>Maximum Acceleration /g</th>
<th>Maximum Ductility Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Columns</td>
</tr>
<tr>
<td>1.0</td>
<td>10.26</td>
<td>6.070</td>
<td>3.810 (IV)</td>
<td>1.36 (X)</td>
</tr>
<tr>
<td>0.5</td>
<td>9.44</td>
<td>6.072</td>
<td>3.840 (IV)</td>
<td>1.44 (X)</td>
</tr>
<tr>
<td>0.0</td>
<td>8.98</td>
<td>6.750</td>
<td>2.150 (X)</td>
<td>1.46 (X)</td>
</tr>
</tbody>
</table>
FIG 4.15 EFFECT OF THE CHANGE OF LIVE LOAD POSITION.
FIG. 4.16 INELASTIC RESPONSE HISTORY FOR $\beta = 0.2$
FIG. 4.17 INELASTIC RESPONSE HISTORY FOR $\beta = 0.4$
FIG. 4.18 INELASTIC RESPONSE HISTORY FOR $\beta = 0.6$
FIG. 4.19 INELASTIC RESPONSE HISTORY FOR $\beta = 0.8$
Fig. 4.20 shows the effect of variation of $\beta$ on the maximum ductility factors of the columns and girders in each storey. The maximum column ductility factors are very nearly constant in the lower half of the structure, but considerable variation is observed in the upper storeys. It can be seen that there is a lack of systematic change of maximum column ductility factors in the upper half of the structure. Such an irregular behaviour should be expected as a consequence of the complete change of dynamical system.

The girder ductility factors are severely affected. The variations of girder ductility curves more or less retain the original shape corresponding to $\beta = 0.0$. This is partly because the statical properties of the system have not changed. It is interesting to note that for this particular problem, the overall maximum girder and column ductility factors increase with increase in $\beta$.

Fig. 4.21 shows the maximum floor acceleration for various values of $\beta$. For this particular situation, the increase in $\beta$ results in an overall tendency for a reduction of maximum accelerations although the individual storey by storey variation is not systematic. This can be attributed to both inelastic activity and also to the changed dynamic system.

The general increase of girder ductility curves for $\beta$ greater than 0.0 (Fig. 4.20), results in a general decrease of acceleration values as seen in Fig. 4.21. Thus
DUCTILITY FACTORS

FIG. 4.20 EFFECT OF LIVE LOAD CONTRIBUTION TO FLOOR MASS
FIG. 4.21 EFFECT OF LIVE LOAD CONTRIBUTION TO FLOOR MASS

ACCELERATION
ACCELERATION OF GRAVITY
the observation made in the previous sections that the increase in girder ductility factors at a certain location results in reduced maximum acceleration at the same section is seen to be consistent here as well.

The maximum horizontal floor displacements resulting from the variation of \( \beta \) are shown in Fig. 4.22. There are significant changes in the displacement curves in the upper half of the structure for values of \( \beta \) greater than 0.0 compared to the curve for \( \beta = 0.0 \). A similar change of column ductility curves at about two-thirds of the height of the structure is seen in Fig. 4.20. Also there is a lack of systematic change of displacements with changes in \( \beta \). The same lack of systematic change of column ductility curves in the upper one-third of the structure can be seen from Fig. 4.20. Therefore, the observations and the related reasoning stated in earlier sections further confirm that the maximum floor displacements are heavily dependent on maximum column ductility factors.

Table 4.3 shows the effect of variations of \( \beta \) on the overall maxima of various response parameters along with the periods of vibration of the undamped system. As expected, the period of vibration increases with the increase in the value of \( \beta \), i.e. the mass of the system. It may be noted that the time of occurrence of maximum displacement is very different because each case corresponds to a different dynamic system possessing different response characteristics.
Fig. 4.22 Effect of live load contribution to floor mass
<table>
<thead>
<tr>
<th>System Parameter $\beta$</th>
<th>Maximum Displacement of 10th floor in inches</th>
<th>Time for Maximum Displacements in seconds</th>
<th>Maximum Acceleration $/g$</th>
<th>Maximum Ductility Factors</th>
<th>Period in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Columns</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>10.00</td>
<td>6.070</td>
<td>2.190 (X)</td>
<td>1.31 (X)</td>
<td>1.2812</td>
</tr>
<tr>
<td>0.2</td>
<td>12.57</td>
<td>12.430</td>
<td>1.720 (X)</td>
<td>1.76 (VII)</td>
<td>1.3739</td>
</tr>
<tr>
<td>0.4</td>
<td>12.90</td>
<td>12.070</td>
<td>1.700 (V)</td>
<td>1.83 (VIII)</td>
<td>1.4608</td>
</tr>
<tr>
<td>0.6</td>
<td>12.35</td>
<td>8.730</td>
<td>1.960 (II)</td>
<td>1.97 (VII)</td>
<td>1.5282</td>
</tr>
<tr>
<td>0.8</td>
<td>13.26</td>
<td>5.640</td>
<td>1.460 (VII)</td>
<td>2.15 (IX)</td>
<td>1.6207</td>
</tr>
</tbody>
</table>

|                          |                                             |                                           |                           | Girders                   |                  |
|                          |                                             |                                           |                           |                          |                  |
|                          |                                             |                                           |                           | 3.30 (VII)               |                  |
|                          |                                             |                                           |                           | 3.56 (VIII)              |                  |
|                          |                                             |                                           |                           | 4.39 (VII)               |                  |
|                          |                                             |                                           |                           | 4.16 (VII)               |                  |
|                          |                                             |                                           |                           | 4.70 (VIII)              |                  |

**TABLE 4.3** EFFECT OF LIVE LOAD CONTRIBUTION TO FLOOR MASS
Fig. 4.23 shows the effect of variation of $\beta$ on tenth floor displacements and overall maximum accelerations. The apparent increase in maximum displacement with $\beta$ is probably due to the fact that the higher period gives a larger spectral displacement. The changes in maximum displacements and overall maximum accelerations cannot be directly related to changes in $\beta$, for reasons discussed earlier.

4.4 Effect of Stiffness Distribution Between Girders and Columns

The horizontal displacement response histories of floors for various values of $K$ are shown in Fig. 4.1 and Figures 4.24 through 4.27.

As in the previous section, in this case also the dynamical system completely changes due to changes of the stiffness of girders. Accordingly, the response characteristics also change completely.

The effect of the variation of $K$ on maximum column and girder ductility factors is shown in Fig. 4.28. Increase in girder stiffness drastically reduces the girder ductility factors, forcing the columns to undergo large inelastic deformations, so that the column ductility factors increase significantly. When the girder stiffnesses are doubled, the maximum girder ductility factors are reduced by two thirds and are always less than 1.0, indicating entirely elastic girder behaviour. At the same time,
FIG. 4.23 EFFECT OF LIVE LOAD CONTRIBUTION TO FLOOR MASS.
FIG. 4.24 INELASTIC RESPONSE HISTORY FOR K = 2.0
FIG. 4.25 INELASTIC RESPONSE HISTORY FOR $K = 3.0$
FIG. 4.26 INELASTIC RESPONSE HISTORY FOR $k = 6.0$
FIG. 4.27 INELASTIC RESPONSE HISTORY FOR K = 10.0
DUCTILITY FACTORS

FIG. 4.28 EFFECT OF VARIATION OF RELATIVE STIFFNESS BETWEEN GIRDERS AND COLUMNS
column ductility factors have more than doubled; all columns have yielded during the response. Larger values of $K$ indicate a continuation of this trend, but the storey by storey variations of column ductility factor is not systematic. The reason for this is that both the dynamic and static structural system properties change when $K$ is changed. It is reasonable that by increasing the girder stiffness, almost all of the inelastic deformation is found to take place in the columns. As noted in previous sections, the displacements are directly related to the column ductility factors so that one would anticipate significantly larger displacements for $K$ greater than 1. Since the column ductility factors are not systematically larger in every storey with increasing $K$, there may not be a direct relationship between maximum displacement and the value of $K$. Fig. 4.30 shows this to be the case; the largest maximum displacements occur for $K = 3.0$ and not for $K = 10.0$. This is because a sway mechanism has developed in the seventh storey, as can be seen in Fig. 4.28. It should again be emphasized that the columns yield due to increase in $K$, but that the amounts of yielding and the locations of hinge formation depend completely upon the inelastic properties of the differing dynamical systems.

Girder ductility factors do decrease consistently with increasing $K$, but since the girder remains essentially elastic for $K$ greater than 1.0, this has little direct significance.
Fig. 4.29 shows the maximum floor accelerations for various values of K. Here again the variation of the maximum acceleration parameter cannot be directly related to variation of K, for the reasons already stated. However, it may be noted from Table 4.4 that overall maxima of accelerations are of the same order of magnitude. This is because the yielding of columns in nearly every storey consumes energy and inhibits development of larger accelerations.

The maximum floor displacements are shown in Fig. 4.30. The maximum floor displacements vary considerably for different values of K. As indicated earlier, the maximum values are not consistently changing with corresponding changes in K, but depend largely on the column ductility factor variations. Since each case has a varying stiffness, it is not significant to compare actual values of maximum storey displacement for different values of K.

Table 4.4 shows that the time of occurrence of maximum displacement varies considerably with changes in K. As noted in the previous section, this is expected because the dynamic characteristics affecting the detailed response history change when K changes.

Fig. 4.31 shows the variation of tenth floor displacement and overall maximum acceleration for various values of K. As expected due to changes of the dynamical system as well as the static structural system, there is a lack of general relationship between the K and maximum
FIG. 4.29 EFFECT OF VARIATION OF RELATIVE STIFFNESS BETWEEN GIRDERS AND COLUMNS
<table>
<thead>
<tr>
<th>System Parameter $K$</th>
<th>Maximum Displacement of 10th floor in inches</th>
<th>Time for Maximum Displacements in seconds</th>
<th>Maximum Acceleration $/g$</th>
<th>Maximum Ductility Factors</th>
<th>Period in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10.00</td>
<td>6.070</td>
<td>2.190 (X)</td>
<td>1.31 (X)</td>
<td>1.2812</td>
</tr>
<tr>
<td>2.0</td>
<td>11.99</td>
<td>4.420</td>
<td>2.330 (VI)</td>
<td>2.97 (I)</td>
<td>1.0295</td>
</tr>
<tr>
<td>3.0</td>
<td>15.29</td>
<td>9.315</td>
<td>2.230 (X)</td>
<td>7.18 (VII)</td>
<td>0.9277</td>
</tr>
<tr>
<td>6.0</td>
<td>8.17</td>
<td>11.020</td>
<td>1.810 (IX)</td>
<td>1.69 (VII)</td>
<td>0.8110</td>
</tr>
<tr>
<td>10.0</td>
<td>6.99</td>
<td>11.900</td>
<td>2.550 (X)</td>
<td>2.07 (VII)</td>
<td>0.7590</td>
</tr>
</tbody>
</table>

**TABLE 4.4**  
**EFFECT OF RELATIVE STIFFNESS BETWEEN GIRDERS AND COLUMNS**
FIG. 4.30  EFFECT OF VARIATION OF RELATIVE
STIFFNESS BETWEEN GIRDERS AND COLUMNS

MAXIMUM DEFLECTIONS IN INCHES
FIG. 4.31  EFFECT OF RELATIVE STIFFNESS BETWEEN GIRDER AND COLUMNS.

MAXIMUM 10th FLOOR DISPLACEMENT IN INCHES

MAXIMUM ACCELERATION IN g
displacement and maximum accelerations. Such lack of general relationship should also be expected for other values of \( K \).

4.5 Effect of Various Levels and Kinds of Damping

**Inelastic Case**

Figures 4.1 and 4.32 through 4.35 show the floor displacement response histories for various values of \( \zeta \) for close coupled damping. It is seen that the general response characteristics remain similar, except for decreasing amplitudes of displacement for larger values of \( \zeta \).

Fig. 4.36 shows the maximum ductility factors in each storey for various values of \( \zeta \), including one case of far coupled damping. There is insignificant variation in the column ductility factors in the lower two-thirds of the structure and only slight variation in the upper storeys. Moreover, there is a lack of systematic change in the column ductility factors which indicates that inelastic deformation and viscous type of damping cannot be related in a simple way. No significance can be attached to the nature of variation of column ductility factors within any given storey.

The effect of the variation of \( \zeta \) on the girder ductility factors is more significant. The amount of change of girder ductility factors varies from storey to storey and again in this case also there is a lack of
FIG. 4.32 INELASTIC RESPONSE HISTORY FOR $\xi = 0.0$
FIG. 4.33 INELASTIC RESPONSE HISTORY FOR $\xi = 0.01$
FIG. 4.34 INELASTIC RESPONSE HISTORY FOR $\zeta = 0.05$
FIG. 4.35 INELASTIC RESPONSE HISTORY FOR $\zeta = 0.10$
FIG. 4.36 EFFECT OF DAMPING
systematic change of girder ductility factors with corresponding change in $\zeta$. Though the maximum variation in girder ductility factor is of the order of 50% (in the bottom storey in this case), the variation at the point of occurrence of overall maximum is of the order of 10 to 15%.

From the above observations on the variation of maximum ductility factors, it appears that for the range of $\zeta$ considered, i.e. 0.0 to 0.10, the effect of viscous damping is of minor significance compared with the hysteretic damping resulting from the inelastic deformation. Even though this may be true for the structure and earthquake considered for this problem, it may have a far different effect for other cases. Even in this case the range of variation of maximum ductility factors by 50% of the maximum indicate a strong interaction between viscous and hysteretic type of damping. It is quite conceivable that for a certain optimum value of $\zeta$, viscous type of damping will tend to decrease the inelastic deformation by dissipating a part of the energy. This may result in slight changes in ductility factors with the increase in $\zeta$ beyond the level referred to above. In contrast to this, if the viscous damping is below the above mentioned (so-called optimum) level, the system will tend to have early excursions in the inelastic region which again will result in increased consumption of energy and therefore will tend to limit the changes in ductility factors.
This self-adjusting phenomenon of the viscously and hysteretically damped system is perhaps the reason why the changes in ductility factors are, in general, confined within a certain range.

Fig. 4.37 shows the effect of variation of $\zeta$ on maximum floor accelerations which is quite significant. The variation of this parameter is more or less consistent with the previous observations made and the reasoning described in sections 4.1 and 4.2. The maximum accelerations occur when the girder ductility factors are relatively small. Again there is a lack of systematic change of maximum accelerations due to lack of systematic change of girder ductility factors.

Fig. 4.38 shows the maximum floor displacements for various values of $\zeta$. In general, the amplitude of maximum displacement decreases with increase in damping as can be seen from the family of curves in which there is little overlapping. In previous sections it was observed that the maximum displacements largely depend on column ductility factors in the lower storeys. In this case also, the pattern and amount of change is related to a similar change in column ductility factors in the upper one-third of the structure.

Fig. 4.39 shows the variation of tenth floor displacement and overall maximum acceleration as a function of the variation of $\zeta$. The tenth floor displacement decreases very slowly as $\zeta$ increases. This is consistent with the
ACCELERATION

ACCELERATION OF GRAVITY

FIG. 4.37 EFFECT OF DAMPING
FIG. 4.38 EFFECT OF DAMPING

\( \zeta = \text{FRACTION OF CRITICAL DAMPING} \)

CLOSED COUPLED IF NOT MENTIONED
F.C. FAR COUPLED

MAXIMUM DEFLECTIONS IN INCHES
FIG. 4.39 EFFECT OF DAMPING.
observed variation of column ductility factors and also the reasoning given earlier in this section regarding the interaction between viscous and hysteretic type of damping.

From Table 4.5 and Fig. 4.39, it is apparent that deflections are reduced if damping is changed from close coupled to far coupled though the reduction is quite small. There is a significant change in maximum acceleration between the close coupled and the far coupled damping case for this particular structure. Of course, too much significance should not be attached to this observation of a single case.

Elastic Case

It has been reported by other authors\textsuperscript{29,30} that, for a single degree of freedom system, the maximum displacements of an elastic and an elasto-plastic system, with or without damping, are of the same order of magnitude. It is quite meaningful to investigate whether the same observation holds true in case of a multi-degree of freedom system. Some previous observations\textsuperscript{9} have indicated that in an undamped case, the maximum displacement of an inelastic system is greater than the corresponding elastic one, and the same authors\textsuperscript{9} report that the situation will be reversed if the damping is included in addition to the inelastic behaviour.

In order to further investigate this specific problem area, the elastic responses of the basic structure were computed both with and without damping. The maximum
<table>
<thead>
<tr>
<th>Kind of Damping</th>
<th>System Parameter $\zeta$</th>
<th>Maximum Displacement of 10th floor in inches</th>
<th>Time for Maximum Displacements in seconds</th>
<th>Maximum Acceleration /g</th>
<th>Maximum Ductility Factors</th>
<th>System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Close Coupled</td>
<td>0.0</td>
<td>10.08</td>
<td>6.070</td>
<td>4.120 (IV)</td>
<td>1.28 (X) 3.33 (VII)</td>
<td>Inelastic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.76</td>
<td>7.375</td>
<td>3.279 (X)</td>
<td>2.05 (IX) 2.30 (VIII)</td>
<td>Elastic</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>9.96</td>
<td>6.070</td>
<td>3.940 (III)</td>
<td>1.29 (X) 3.25 (VII)</td>
<td>Inelastic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.00</td>
<td>6.070</td>
<td>2.190 (X)</td>
<td>1.31 (X) 3.30 (VII)</td>
<td>Inelastic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.05</td>
<td>7.375</td>
<td>3.064 (X)</td>
<td>1.86 (IX) 2.13 (VIII)</td>
<td>Elastic</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>9.96</td>
<td>6.067</td>
<td>3.620 (V)</td>
<td>1.16 (VII) 3.10 (VII)</td>
<td>Inelastic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.04</td>
<td>7.375</td>
<td>2.784 (X)</td>
<td>1.72 (IX) 2.03 (VII)</td>
<td>Elastic</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>9.86</td>
<td>6.055</td>
<td>3.260 (V)</td>
<td>1.15 (IX) 2.91 (VII)</td>
<td>Inelastic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.44</td>
<td>6.038</td>
<td>2.100 (X)</td>
<td>1.45 (VII) 1.90 (VII)</td>
<td>Elastic</td>
</tr>
<tr>
<td>Far Coupled</td>
<td>0.10</td>
<td>9.86</td>
<td>6.055</td>
<td>3.260 (V)</td>
<td>1.15 (IX) 2.91 (VII)</td>
<td>Inelastic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12.44</td>
<td>6.038</td>
<td>2.100 (X)</td>
<td>1.45 (VII) 1.90 (VII)</td>
<td>Elastic</td>
</tr>
<tr>
<td></td>
<td>0.22</td>
<td>9.70</td>
<td>6.06</td>
<td>4.370 (IV)</td>
<td>1.37 (X) 3.12 (VII)</td>
<td>Inelastic</td>
</tr>
</tbody>
</table>

**TABLE 4.5 EFFECT OF DAMPING**
response values are given in Table 4.5 along with the inelastic results.

Contrary to the previous observations, the results shown in Table 4.5 indicate that even for the undamped case, the maximum tenth floor displacements in an elastic system are significantly larger than those of the inelastic system. In this case the ductility factors given are purely a measure of maximum elastic deformation as compared to the yield deformation in the corresponding inelastic case. However, the overall maximum girder ductility factors obtained for a given elastic system are smaller than those of the corresponding inelastic system. The same observations have been noted by Clough, Benuska and Wilson. This situation is reversed in the case of overall maximum column ductility factors, for which the inelastic values are smaller than the corresponding elastic values.

Referring to Fig. 4.38, it is seen that the family of curves for the inelastic cases indicate smaller displacements than the comparable family of curves in the elastic case. Also it may be noted that the curves for inelastic case consistently give smaller displacements. This occurs because the system possesses inelastic capability to permit girders to deform more and the columns to deform less. As the displacements largely depend on column ductility factors (according to corresponding observations in the previous sections), the small variations of displacements
of the inelastic case are consistent with corresponding small variations of column ductility factors. In contrast a larger and consistent variation of displacements is seen for the elastic case. This is because in the elastic system girders cannot deform inelastically so as to have more relative deformation than that of columns. Of course, as noted above, this happens in the inelastic case, in which relatively larger inelastic deformations of girders dissipate energy and consequently inhibit larger displacements.

As far as maximum accelerations are concerned, these all occur at tenth floor in the elastic systems, while those for the inelastic system occur at different positions for different values of $\zeta$. As would be expected, the maximum elastic accelerations decrease consistently with an increase in $\zeta$.

4.6 Effect of Relative Earthquake Intensities

Figures 4.1 and 4.40 through 4.43 show the response history of the horizontal floor displacements of the basic structure for various values of $F$. From these figures it is quite clear that the variation of $F$ does not change the general characteristics of the response except for the increase in amplitude of maximum displacement with increase in intensity. Column 3 of Table 4.6 gives the maximum tenth floor displacements for various values of $F$. As $F$ increases, the maximum displacements increase in a consist-
FIG. 4.40 INELASTIC RESPONSE HISTORY FOR $F = 1.1$
FIG. 4.1 INELASTIC RESPONSE HISTORY FOR F = 1.2
FIG. 4.42 INELASTIC RESPONSE HISTORY FOR F=1.3
Fig. 4.43 Inelastic response history for $F = 1.4$
<table>
<thead>
<tr>
<th>Maximum Base Acceleration /g</th>
<th>System Parameter F</th>
<th>Maximum Displacement of 10th floor in inches</th>
<th>Time for Maximum Displacements in seconds</th>
<th>Maximum Acceleration /g</th>
<th>Maximum Ductility Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.319</td>
<td>1.000</td>
<td>8.53</td>
<td>7.390</td>
<td>2.580 (IV)</td>
<td>1.08 (IX) 2.60 (VII)</td>
</tr>
<tr>
<td>0.352</td>
<td>1.100</td>
<td>9.11</td>
<td>6.050</td>
<td>2.820 (IV)</td>
<td>1.13 (IX) 2.90 (VII)</td>
</tr>
<tr>
<td>0.383</td>
<td>1.200</td>
<td>9.36</td>
<td>6.050</td>
<td>3.010 (IV)</td>
<td>1.16 (IX) 2.89 (VII)</td>
</tr>
<tr>
<td>0.415</td>
<td>1.300</td>
<td>9.56</td>
<td>6.060</td>
<td>3.240 (V)</td>
<td>1.18 (IX) 3.17 (VII)</td>
</tr>
<tr>
<td>0.447</td>
<td>1.400</td>
<td>9.64</td>
<td>6.060</td>
<td>3.340 (V)</td>
<td>1.27 (X) 3.12 (VII)</td>
</tr>
<tr>
<td>0.500</td>
<td>1.567</td>
<td>10.00</td>
<td>6.070</td>
<td>2.190 (X)</td>
<td>1.31 (X) 3.30 (VII)</td>
</tr>
</tbody>
</table>

TABLE 4.6 EFFECT OF EARTHQUAKE INTENSITY
ent manner though the increase is not linear. This can be seen from Fig. 4.47. The maximum value of $F = 1.567$ corresponds to a peak ground acceleration of $0.5 \, g$.

Fig. 4.44 shows the maximum column and girder ductility factors developed in each storey for various values of $F$. The columns remain elastic in the lower two-thirds of the structure. The inelastic deformations in the columns occur in the upper storeys and their extent increases as the value of $F$ increases. There is a consistent increase in maximum column ductility factors with increase in $F$. However, the increase in column ductility factors is non-linear; an increase of the order of 50% in the maximum ground acceleration results in an increase in column ductility factors of about 20%. As would be expected, the consistent increase in maximum ductility factors should result in a consistent increase in maximum floor displacements. Fig. 4.46 shows this to be the case.

The girder ductility factors also increase consistently with an increase in $F$. The relative increases in girder ductility are larger compared with the relative increases in column ductility factors. For this problem an increase of the order of 25% in overall maximum girder ductility factors can be seen from Figs. 4.44 and Table 4.6. Relatively large increases in girder ductility factors combine with relatively small increases in column ductility factors. This explains the non-linear relationship between maximum displacements and $F$ as pointed out earlier.
DUCTILITY FACTORS

FIG. 4.44 EFFECT OF VARIATION OF EARTHQUAKE INTENSITY

F = AMPLITUDE OF INTENSIFIED ACCELEROGRAM
   AMPLITUDE OF ACTUAL ACCELEROGRAM
FIG. 4.45  EFFECT OF VARIATION OF
EARTHQUAKE INTENSITY
FIG. 4.46 EFFECT OF VARIATION OF EARTHQUAKE INTENSITY

MAXIMUM DEFLECTIONS IN INCHES

F = AMPLITUDE OF INTENSIFIED ACCELEROGRAM
AMPLITUDE OF ACTUAL ACCELEROGRAM
FIG. 4.47 EFFECT OF EARTHQUAKE INTENSITY
Fig. 4.45 shows the maximum floor accelerations for various values of F. As observed in sections 4.1 and 4.2, the maximum accelerations tend to be of relatively higher magnitude at locations which develop relatively smaller ductility factors and vice versa. This can be seen from the above figure. The maximum acceleration curves consistently have large peaks at about mid-height of the structure, as long as the girders remain elastic at the same location. Once the girders yield significantly, as is this case for the largest value of F, the maximum acceleration curve has much lower values at mid-height.

4.7 Effect of Earthquake Characteristics

Figures 4.1 and 4.48 through 4.52 show the response history of horizontal floor displacement for various intensified earthquakes having the same maximum acceleration of 0.5 g. It can be observed that not only the response characteristics are different for the different earthquakes, but also that the maximum floor displacements and their time of occurrence are different.

Fig. 4.53 shows the maximum column and girder ductility factors developed in each storey during the different earthquakes. In all these cases as well, the columns remain elastic in the lower two-thirds of the structure, except for some yielding in the lowest storey in several cases. As would be expected, the girder and column ductility factors have vastly different behaviour.
FIG. 4.48 INELASTIC RESPONSE HISTORY, EL CENTRO
DEC 30, 1934, N-S x 1.96
FIG. 4.49 INELASTIC RESPONSE HISTORY, OLYMPIA
APR 13, 1949, N80E x 1.815
FIG. 4.50 INELASTIC RESPONSE HISTORY, EL CENTRO
DEC 30, 1934, E-W x 2.838
FIG. 4.51  INELASTIC RESPONSE HISTORY, EL CENTRO
MAY 18, 1940, E-W x 2.200
FIG. 452  INELASTIC RESPONSE HISTORY, OLYMPIA APR 13, 1949
N1OW x 2.660
FIG. 4.53 EFFECT OF DIFFERENT EARTHQUAKE CHARACTERISTICS

DUCTILITY FACTORS

(1) El Centro Dec. 30, 1934, N-S x 1.960
(2) El Centro May 18, 1940, N-S x 1.567
(3) Olympia Apr. 13, 1949, N80E x 1.815
(4) El Centro Dec. 30, 1934, E-W x 2.838
(5) El Centro May 18, 1940, E-W x 2.200
(6) Olympia Apr. 13, 1949, N10W x 2.66

STOREY

COLUMNS

1 2 3 4

GIRDERS

1 2 3 4
characteristics for the different earthquakes. The girders absorb the bulk of the inelastic deformations.

Fig. 4.54 shows the variation of maximum floor accelerations for each earthquake. Similar to the ductility factor variation, there are different amounts of magnification of accelerations for the different earthquakes although the peak ground acceleration of each of these is 0.5 g.

Fig. 4.55 shows the maximum floor displacements for each earthquake. There is a wide range of variation of maximum displacements. Such a variation is not unexpected because the characteristics of each of the earthquakes used are different although the peak acceleration of all the earthquakes is the same.

Fig. 4.56 shows the maximum tenth floor displacement and overall maximum accelerations for each earthquake. No significance can be attached to the variations of these parameters except to indicate that the variation of overall maximum accelerations fall within a reasonably narrow range of 0.5 g for this group of earthquakes.

Table 4.7 lists the maximum displacement of the tenth floor, its time of occurrence, overall maximum acceleration and overall maximum column and girder ductility factors for the different earthquake excitations. Though the number of earthquakes used to study the effect of earthquake characteristics on the inelastic structural response is too small for making any significant statistical comparisons or observations, an attempt is made below to
ACCELERATION OF GRAVITY

FIG. 4.54 EFFECT OF DIFFERENT EARTHQUAKE CHARACTERISTICS

(1) El Centro Dec. 30, 1934, N-Sx 1.980
(2) El Centro May 18, 1940, N-Sx 1.567
(3) Olympia Apr. 13, 1949, N80E x1.815
(4) El Centro Dec. 30, 1934, E-Wx 2.838
(5) El Centro May 18, 1940, E-Wx 2.200
(6) Olympia Apr. 13, 1949, N10Wx 2.600
MAXIMUM DEFLECTIONS IN INCHES

FIG. 4.55 EFFECT OF DIFFERENT EARTHQUAKE CHARACTERISTICS

(1) El Centro Dec. 30, 1934, N-Sx 1.960
(2) El Centro May 18, 1940, N-Sx 1.567
(3) Olympia Apr. 13, 1949, N80Lx1.815
(4) El Centro Dec. 30, 1934, E-Wx 2.838
(5) El Centro May 18, 1940, E-Wx 2.200
(6) Olympia Apr. 13, 1949, N10Wx2.660
OLYMPIA APR. 13, 1949, N10W x 2.660

EL CENTRO MAY 18, 1940, E-W x 2200

EL CENTRO DEC. 30, 1934 E-W x 2.838

OLYMPIA APR. 13, 1949, N80E x 1815

EL CENTRO MAY 18, 1940, N-S x 1.567

EL CENTRO DEC. 30, 1934, N-S x 1.960

MAXIMUM 10TH FLOOR DISPLACEMENT IN INCHES

MAXIMUM ACCELERATION IN g

FIG. 4.56 EFFECT OF EARTHQUAKE CHARACTERISTICS
### TABLE 4.7 EFFECT OF EARTHQUAKE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Different Earthquakes</th>
<th>Maximum Displacement of 10th floor in inches</th>
<th>Time for Maximum Displacements in seconds</th>
<th>Maximum Acceleration $/g$</th>
<th>Maximum Ductility Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro Dec. 30, 1934, N-S x 1.960</td>
<td>10.88</td>
<td>11.26</td>
<td>1.81 (X)</td>
<td>1.1 (IX) 2.4 (VII)</td>
</tr>
<tr>
<td>El Centro May 18, 1940, N-S x 1.567</td>
<td>10.00</td>
<td>6.07</td>
<td>2.19 (X)</td>
<td>1.3 (X) 3.3 (VII)</td>
</tr>
<tr>
<td>Olympia Apr. 13, 1949, N80Ex1.815</td>
<td>11.61</td>
<td>21.50</td>
<td>2.36 (X)</td>
<td>2.4 (IX) 3.6 (VIII)</td>
</tr>
<tr>
<td>El Centro Dec. 30, 1934, E-W x 2.838</td>
<td>12.24</td>
<td>18.13</td>
<td>2.16 (VIII)</td>
<td>1.8 (IX) 3.3 (VIII)</td>
</tr>
<tr>
<td>El Centro May 18, 1940, E-W x 2.200</td>
<td>14.19</td>
<td>12.66</td>
<td>2.11 (X)</td>
<td>1.3 (IX) 3.4 (V)</td>
</tr>
<tr>
<td>Olympia Apr. 13, 1949, N10Wx2.660</td>
<td>14.61</td>
<td>16.04</td>
<td>2.07 (X)</td>
<td>1.8 (IX) 3.7 (VII)</td>
</tr>
</tbody>
</table>
compare the various responses arbitrarily. For this purpose, the response due to the El Centro Dec. 30, 1934 E-W \times 2.838 earthquake is considered to be an approximate mean. Compared to the maximum displacement in this "mean" case, the maximum displacements due to other accelerograms vary within ±25%. Similarly the overall maximum column ductility factors and girder ductility factors for other accelerograms considered vary within ±40 and ±30% respectively. The range of variation of overall maximum accelerations fall within ±20% of the so-called mean.
CHAPTER V
CONCLUSIONS

The following sections give the conclusions drawn from the results and related discussion presented in Chapter IV for the variation of each system parameter.

5.1 Effect of the Amount of Live Load Included as Vertical Load on the Girders

The observations of this effect indicate that the response parameter variations corresponding to varying percentage of live load are insignificant in terms of implications on design. Some consistency was observed in the variation of girder ductility factors and the storey displacements were found to be highly dependent upon the column ductility factors. However, neither variation was of such a magnitude so as to warrant additional investigation of this parameter. One can therefore conclude that it is not important, in terms of pure girder load, to specify a realistic percentage of live load.

5.2 Effect of Different Positions of Live Loads

The observations of this section indicate that the maximum displacement (which is one of the most important
response parameters) increases slightly with increase in eccentricity of the position of live load. In contrast, as observed in section 4.1, the maximum displacements decrease with increase in percentage of live load. Thus the eccentrically positioned live load counteracts the effect of the additional pure girder load. Similar to the observations of section 4.1, the storey displacements were found to be essentially dependent on the column ductility factors. It was also observed that there is a significant change in the girder ductility factors and maximum floor accelerations for extreme position of live load. Of course, such changes correspond to the increase in these response values in some portions of the structure and decrease of the same in other portions. In view of such compensating changes for variation of live load position and the more or less counteracting effects of both, the position of and the contribution of live load as pure load on the girder, it can be concluded that all live load acting as a pure girder load can be neglected without significantly affecting the reliability of the response data from the point of view of its implications on the design of the structure.

5.3 Effect of Live Load Contribution to Floor Mass

The observations of this effect clearly show that the response parameter variations corresponding to varying
percentage contributions of live load mass to floor mass are quite significant in terms of their implications on dynamic analysis and design. The increase in the masses of the system as a result of the inclusion of a percentage of live load mass results in an entirely different dynamical system with altogether different response characteristics. Thus it was found that there is a lack of systematic change of response parameters with changes in contribution of live load mass to the mass of the system. Therefore, it is concluded that it is very important to make a proper estimation of the mass of the system preceding the analysis and design. The mass thus estimated should include a reasonable contribution of mass due to live load. From the observation, it may be seen that neither maximum nor minimum live load mass contributions may necessarily yield the most severe response characteristics.

5.4 Effect of Stiffness Distribution Between Girders and Columns

The observations of this effect indicates that the response parameter variations corresponding to increasing the relative girder stiffnesses are quite significant from the point of view of its implications on the dynamic analysis of the system and its design. The increase in relative girder stiffnesses results in a system whose dynamic and static structural properties are entirely
different. As a result of such changes in the static and dynamic characteristics of the system, it was found that there is a lack of systematic change of the response parameters with corresponding increase in relative girder stiffnesses.

It was observed that the burden of inelastic deformations falls on the columns while it is relieved from the girders when the relative stiffnesses of the girder increase. This observation of attraction of inelastic activity by weaker sections and repulsion of it by relatively stiffer members is in agreement with similar observations made by Clough and Benuska 10.

It is concluded that the changes of girder stiffnesses should be viewed as changes in dynamic and static characteristics of the system. As a stiffer floor system may force all the inelastic deformations into the columns, which in turn may lead to larger displacements, serious damage and instability, it is very important to give due consideration to the assessment of the contribution of different floor systems to girder stiffness, before doing the dynamic analysis and the aseismic design of the structure.

It was observed in Chapter IV that the maximum floor displacements largely depend upon column ductility factors in the lower storeys. It was also observed in section 4.4 that if girders are made stronger than columns, the column ductility factors increase; this will result in
large floor displacements. In tall buildings larger floor displacements are likely to involve the danger of total collapse or irreparable damage due to the destabilizing effect of the gravity loads. To avoid such a dangerous situation, it seems logical to avoid inelastic deformation of columns. Therefore, on the basis of above stated observations and critical assessment of the situation, it can be concluded that for tall buildings it is quite important to design relatively stronger columns and weaker girders. Such a design will provide sufficient energy dissipation capacity in the structure by allowing the girders to deform inelastically. Also, completely elastic behaviour of columns will inhibit large displacements of the floors. Thus the probability of collapse of the structure will be reduced to a minimum.

However, it may be noted as a caution, that the design of columns in the above approach must be given due consideration from the point of view of static stability during the occurrence of strong earthquakes. In this event, when all girder ends in different storeys meeting at a particular column develop plastic hinges simultaneously, it is very likely that local instability of a part or full length of such a column may become the starting point of overall instability of the structure. Because of the above consideration, it is logical to recommend that the inelastic deformation capacity of the girders should be staggered in alternate storeys. In such an approach, the
unsupported column length will never exceed two times the storey height, even when all the weaker girders form plastic hinges simultaneously. In such an approach, the caution against the design of columns, as stated in the beginning of this paragraph, may be removed.

5.5 Effect of Various Levels and Kinds of Damping

It was observed in section 4.5 that the effect of viscous damping (close coupled) on the column ductility factors was non-systematic and nearly insignificant. The same was also observed for maximum floor displacements since, according to previous observations, the maximum displacements largely depend upon the column ductility factors. In the case of girder ductility factors, the effect of damping* was observed to be relatively more significant than that on column ductility factors. Of course, in this case also, there was no systematic decrease in girder ductility factors with the corresponding increase in damping. Moreover, the amounts of change in girder ductility factors varied from storey to storey. The same erratic effect was observed on maximum floor accelerations because these also depend on maximum girder ductility factors. Therefore, on the basis of the above observations, it can be concluded that the effect of viscous type of damping* was observed to be relatively more significant than that on column ductility factors.

*For further discussions, the term "damping" will refer to close coupled viscous type of damping unless otherwise stated.
damping on the response of the structure is relatively small compared with the effect of hysteretic type of damping resulting from the inelastic deformation of the members of the structural system.

Indeed, the non-systematic changes in girder ductility factors are a result of the strong interaction between the viscous type and the hysteretic type of damping. It may be noted that a low level of viscous type of damping will tend to allow the girders to start yielding earlier. This results in more energy dissipation in yielding rather than in viscous type of damping. Thus the maximum displacements are not significantly affected. In the other situation, when the level of viscous type of damping is high, there will be more dissipation of energy due to viscous damping. Consequently, the yielding of members will be delayed and the amount and distribution of deformations will also be different from those corresponding to low level of damping. Here again the net result will be to inhibit larger displacements. On the basis of the above argument, it can be further concluded that viscous type of damping plays a significant role in conjunction with the hysteretic type of damping to affect the inelastic response of the structural system. Certainly from the point of view of aseismic design of structures, it is important to assess the damping characteristics of both the structural and non-structural elements of the building in evaluating the relative value of damping and
the strength of the coupling.

The comparison of a case of far coupled and close coupled damping indicates that the effect of far coupled damping on the response parameters is relatively more than that of close coupled damping. (No general conclusions can be drawn from this as only one case was investigated.)

Comparison of Elastic and Inelastic Cases

For the structure used in this investigation, the maximum displacements of the inelastic system were found to be considerably less than those of the comparable elastic system. Therefore, it can be concluded that the usual concept, which is more or less correct for single degree of freedom systems that the maximum displacement of the elastic and elasto-plastic system are of the same order of magnitude, does not hold true for either damped or undamped multi-degree of freedom systems.

5.6 Effect of Relative Earthquake Intensities

The observations of this effect indicate that the response parameter variations corresponding to varying relative earthquake intensities are significant in terms of their implications on design and analysis. The increase in relative earthquake intensity results in a consistent increase in maximum displacements and column and girder ductility factors, but such increases are nonlinear with respect to intensity level. As the intensity
increases, the relative increase in displacements and column ductility factors decrease. This is because the additional increase in girder ductility factors consumes increasingly more energy to damp out the system response thereby inhibiting larger increases in column ductility factors which in turn control the maximum displacements.

It is also interesting to note that the increase in earthquake intensity may not necessarily amplify the maximum accelerations in a consistent manner. A higher ground acceleration intensity may induce larger inelastic deformations and thus may in turn reduce the magnification of accelerations.

It is therefore concluded that the relationship between the intensity and damage to the structure and its response is non-proportional.

5.7 Effect of Earthquake Characteristics

The observation of the effect of various earthquake excitations representing different characteristics but having the same peak acceleration of 0.5 g, indicates that the maximum intensity of ground acceleration is not the only basis for assessing the probable damage to the structure, but that the characteristics of the earthquakes are also important.

As noted in section 5.6, the effect of varying the maximum base acceleration level for a given earthquake type also indicates that the relationship between damage
and maximum base acceleration is non-linear. On the basis of this, it can be concluded that both the intensity of acceleration and frequency response spectra for the earthquake type interact in a non-linear manner in influencing the dynamic response of the structure. Both of these factors are of significant importance, and therefore must be given due consideration in designing the structure.

No definite conclusions can be derived quantitatively except that the bounds for various response parameters could be established after more extensive study in this area. Roughly, an upper bound of 140% of that due to intensified El Centro December 30, 1934, E-W x 2.838 Earthquake could be said to be reasonably conservative for the group of earthquakes considered.
PART II

EXPERIMENTAL INVESTIGATION
OF THE DYNAMIC RESPONSE
OF INELASTIC MULTI-STOREY
BUILDING FRAMES
6.1 Introduction

The main criteria for the design of experimental structure could be outlined as follows. First of all, the experimental structure should be such that the basic objectives of the investigations should be fulfilled from the observations made during the testing of such a structure. As the main purpose of the experimental investigation is to compare the observed gross behaviour of the experimental model during laboratory testing with that predicted theoretically using the numerical approach described in Chapter II, the experimental structure must meet the basic assumptions as closely as practically possible under laboratory conditions. Secondly, the experimental structure designed should be such that it would be possible to test the structure under laboratory conditions. These laboratory conditions include the availability of the equipment to apply the necessary dynamic force to deform the structure in the inelastic region. The above consideration puts a restriction on the stiffness of the structure which in itself is a function of the size of the members and the mechanical properties
of the material out of which the members have been made. Lastly, the experimental structure should have realistic geometrical proportions so that it will be reasonably representative of the actual structure. It is this aspect which is most difficult to realize in the laboratory. There are two ways in which this difficulty can be overcome. In the first approach, a small size prototype structure representing a frame may be designed according to the availability of dynamic force to deform it in the inelastic region. Of course, in this case, the relative depth, length, and width ratios should be such so that the basic assumptions of the mathematical model are realized. In the second approach, a small model could be scaled down from an actual structure. In a dynamical system in which material is expected to deform in the inelastic region, it is rather difficult to design such a small scale model which will represent geometry, stiffness, mass and period of vibration to a certain scale. It will also be difficult to interpret the experimental results in terms of evaluating the gross behaviour of such a system when compared to the predictions made theoretically.

In the present investigation, a small size prototype structure was designed. Low yield strength aluminum members were used for fabricating the experimental structure. This was done because of two reasons. Firstly, suitable wide flange sections for columns and girders rolled from this material were available from a stock
maintained by the Canada Emergency Measures Organization. Secondly, the sections available were rolled from the same batch of material thus ensuring a reasonable degree of homogeneity of the material.

The structure designed was a three storey single bay frame. The details of the structure are given in section 6.2. The framed structure was designed to behave as a planar frame so that the experimental results obtained from the test of such structure could be compared with those predicted theoretically for a similar frame.

6.2 Description of the Structure

Fig. 6.1 shows the front elevation of the three storey planar frame proportioned for dynamic testing. Three such planar frames were connected together at girder column joint level by two structural steel channels at each of the six joints. The plan view of the three planar frames thus connected is shown in Fig. 6.2. It may be noted that structural steel channels connecting the three frames were also utilized for girder column connections as shown in Fig. 6.3. Fig. 6.4 shows the assembled structure consisting of three frames mounted on the dynamic simulation table acting as a base of the frame. The column to base connections are shown in Fig. 6.5.

As shown in the above figures, the columns and girders are small, wide flange aluminum sections. The reasons for using aluminum for the girders and columns are
3 WF 2.159 (AL)

2 PLATES 3\frac{1}{2} x 1" x \frac{1}{8} (SS)

2" x 1" x \frac{3}{16} (SS)

3" x 1\frac{5}{8} x \frac{1}{4} (SS)

3 WF 2.159 (AL)

BASE PLATE (AL)

4' - 8\frac{1}{2}

SS STRUCTURAL STEEL
AL ALUMINUM
A, B, D, - STRAIN GAGES

FRONT ELEVATION

FIG. 6.1 THREE STOREY EXPERIMENTAL FRAME.
FIG. 6.2 PLAN OF THE THREE STOREY EXPERIMENTAL FRAME.
FIG. 63 DETAILS OF GIRDER COLUMN JOINT

2 PLATES 3\(\frac{3}{8}\) x 1\" x \(\frac{3}{16}\) (SS)

FRONT ELEVATION

CIRDER
3 WF 2.159 (AL)

COLUMN, 2\(\frac{3}{2}\) WF 0.927 (AL)

SIDE ELEVATION

PLAN

AL. ALUMINUM
SS, STRUCTURAL STEEL
ALL BOLTS ARE OF \(\frac{3}{16}\) DIA. (SS)

FIG. 63 DETAILS OF GIRDER COLUMN JOINT
FIG. 6.4 EXPERIMENTAL STRUCTURE
FIG. 6.5 DETAILS OF COLUMN BASE JOINT

BASE PLATE (AL)

L 3"x3"x\(\frac{1}{2}\)" (SS)

COLUMNS, 2\(\frac{1}{2}\)" TF 0.927 (AL)

1" DIA BOLT

FRONT ELEVATION

SIDE ELEVATION

AL ALUMINUM
SS STRUCTURAL STEEL
ALL BOLTS ARE OF \(\frac{3}{16}\)" DIA (SS)
UNLESS OTHERWISE MENTIONED
the following. First, such sections were easily available in enough quantity for the fabrication of small size structures as described above. Secondly, these sections were rolled from the same batch of aluminum alloy thus ensuring homogenity of the material for all the members and both the frames tested. Thirdly, as will be shown in this chapter, the alloy is of sufficiently low yield strength; this property was helpful in enabling the structure to be deformed inelastically with the available dynamic load capacity of the dynamic simulator.

It may also be noted that the column to girder and column to base connections were fabricated by using structural steel sections. This was done to ensure that both the above connections behave as rigid joints without any appreciable deformation compared to the deformation of the members meeting at these joints.

All the connectors except those on the flanges of the columns were of structural steel. The connectors on the flanges of the columns are of high tensile steel. The reason for using high tensile steel connectors on the flanges of the columns was to eliminate the failure of the connectors in shear as it was practically impossible to put more than eight connectors on the flanges of the columns due to the small width of the column flange. The use of the above structural steel and high tensile steel connectors made the joints near to perfectly rigid joints relative to the aluminum members. Thus one of the basic
assumptions specifying perfectly rigid joints in a moment resisting frame was approximately realized in fabricating the experimental structure.

Fig. 6.6 shows the stress strain curve of aluminum used for columns and girders of the frame. This stress strain curve was obtained by testing coupons from the web of sections used for both columns and girders. It may be noted that the material used is quite ductile and has low yield strength which enabled the dynamic testing of the structure in the inelastic region using the available dynamic force capacity of the dynamic simulator.

Here it is worth mentioning that, although the experimental structure was designed as a small prototype structure, its relative geometrical dimensions reasonably represent the similar proportions normally existing in a real full size structure. For example, the ratios of depth to length of the columns and girders were approximately 1:11 and 1:19. Similarly the length ratio of column and girder was approximately 1:2.

The assumption of lumped mass system was approximately realized by concentrating the maximum mass of steel channels connecting the girders and columns at the floor level. The ratio of floor mass to mass of columns in between floors was approximately 15:1.

6.3 Computed Structural Properties

The following assumptions were made to compute
FIG. 6.6  ALUMINUM STRESS—STRAIN CURVE
theoretically the properties of the three storey framed structure:

(a) the structure remains elastic
(b) the amplitudes of lateral vibrations are small
(c) the damping is absent.

The natural frequencies and corresponding mode shapes were computed using the standard procedures available in structural dynamics. However, a reduced stiffness matrix was used as it gives a greater accuracy in the prediction of natural frequencies and corresponding relative modal deflections.

Table 6.1 shows the computed structural properties of the system. Columns 2 through 4 list the flexibility influence coefficients computed from the geometrical and elastic properties of the system. The masses lumped at each floor level are shown in column 5. The natural frequencies and corresponding relative modal deflections as given in columns 6 through 10 were computed using the above flexibility influence coefficients and masses lumped at each floor level. As will be described later on in section 6.5, the flexibility influence coefficients of the structure determined experimentally will be compared with those computed above to ascertain whether or not the experimental structure possesses the same static and dynamic properties as those computed theoretically from the assumed mathematical model.
<table>
<thead>
<tr>
<th>Floor Level</th>
<th>Flexibility Influence Coefficient x 10^6 in/lb</th>
<th>Mass at Floor Levels x 10^3 lb x sec^2/in</th>
<th>Mode No.</th>
<th>Natural Frequency Cycles per Second</th>
<th>Relative Modal Displacements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flexibility Influence Coefficient x 10^6 in/lb</td>
<td>Mass at Floor Levels x 10^3 lb x sec^2/in</td>
<td>Mode No.</td>
<td>Natural Frequency Cycles per Second</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>53 64 65 474</td>
<td>1 12.03</td>
<td></td>
<td>0.45 0.97 1.23</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>64 141 155 474</td>
<td>2 37.21</td>
<td></td>
<td>0.40 -0.25 -0.33</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>65 155 236 459</td>
<td>3 60.97</td>
<td></td>
<td>0.17 -0.18 0.07</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 6.1** Computed Properties of Structural System
6.4 Joint Tests to Determine the Moment Curvature Properties

The prediction of the elastic as well as inelastic response of a moment resisting frame requires the knowledge of moment-curvature relationship of the members meeting at a joint. By plotting this relationship, one can find out the moment at which yielding of the member starts and also which inelastic deformations are caused by additional increments of moments. As the deformations in a dynamic process are likely to be cyclic, one has to determine the moment-curvature relationship by a process of loading and unloading. The loading should be carried out up to a level which will give inelastic deformation of at least the order of magnitude to be expected during dynamic loading.

It is realized that the determination of moment-curvature relationship should be done by the application of dynamic loading. However, it is also of interest to investigate whether or not the moment-curvature relationship determined under static conditions and subsequently used to predict the dynamic behaviour of the frame gives a satisfactory correlation with the results observed experimentally. This aspect of investigation is of special interest as the moment-curvature relationship of the members of an actual frame can more easily be determined under static loading rather than under dynamic loading.
Fig. 6.7 shows the experimental set-up for the girder-column joint test. The joint consists of about half column lengths projecting on either side of the joint and oriented horizontally between simple supports. The vertical member is about half the girder length. The load was applied through a load cell and acting at the top of girder end. Two strain gages were mounted on the outer faces of the flanges of both columns and the girder. Similarly the displacement gages were mounted to monitor the horizontal and vertical deflection of the joint.

The moment-curvature relationship obtained from the above mentioned joint test is shown in Fig. 6.8. To interpret the magnitude of inelastic deformation and the magnitude of moment, the above quantities are shown in non-dimensionalized form. The value of plastic moment determined from these tests was 5.91 kip-in. Rotation corresponding to a ductility factor of 1.0 was determined to be $3.39 \times 10^{-3}$ radians.

Fig. 6.9 shows the experimental set-up for base-column joint test. In this case the two base column joint test pieces were bolted together. The column lengths projecting on each side of the joint shown were of length equal to half the storey height. The ends of these columns are simply supported and the load is shown to be applied at the middle of the joint. Two such joints were tested to determine the moment-curvature characteristics
FIG. 6.7 EXPERIMENTAL SET-UP FOR GIRDER-COLUMN JOINT TEST
FIG. 6.8  MOMENT CURVATURE RELATIONSHIP OF COLUMNS
FROM GIRDER COLUMN JOINT TEST
FIG. 6.9 EXPERIMENTAL SET-UP FOR COLUMN BASE JOINT TEST
of the columns. Two strain gages were mounted on each of the column lengths. These gages were located as near to the joint as was practically possible.

The moment-curvature characteristic determined from the column-base joint test is shown in Fig. 6.10. In this case also, both the quantities are shown in non-dimensionalized units. The plastic moment determined from the tests was 5.87 kip-in. The rotation corresponding to a ductility factor of 1.0 was determined to be \(3.42 \times 10^{-3}\) radians.

It may be noted that the above moment-curvature relationship obtained experimentally are very similar to those reported by Panlilio\(^{34}\). Neal\(^{35}\) has also justified the use of plastic hinge hypothesis for such ductile metals as used in the present investigation.

### 6.5 Experimental Determination of Flexibility Influence Coefficients

Fig. 6.11 shows the experimental set-up for the determination of the flexibility influence coefficients for the experimental structure. As shown on the right hand side, displacement gages were mounted to record the horizontal displacement of each floor. The displacement of any floor was monitored by three displacement gages each of which was located at the centre of the girder-column joint. A displacement gage was also mounted against the shaking table to monitor any displacement of the base.
FIG. 6.10  MOMENT CURVATURE RELATIONSHIP OF COLUMNS FROM COLUMN-BASE JOINT TESTS
FIG. 6.11 EXPERIMENTAL SET-UP FOR THE DETERMINATION OF FLEXIBILITY INFLUENCE COEFFICIENTS
The base of the frame was strongly restricted for any movement in the direction of applied load or perpendicular to it.

For determining the flexibility influence coefficients, a load of 100 lbs. was applied at any floor level and corresponding displacements at each floor level were recorded from the displacement gages mounted at each floor level. The displacement of the base was also recorded. The load was applied through a Dynamometer. The level of load was attained by adjusting the tension in the cables attached on either side of the Dynamometer through Turn-Buckles.

Table 6.2 gives the flexibility influence coefficients thus recorded. In arriving at the influence coefficients at any floor level, the average of the displacements recorded by the three displacement gages located at the same floor level was taken. The maximum variation in the three values of displacements thus recorded was found to be 2.8% of the average value at that level. The flexibility influence coefficients matrix thus obtained was further adjusted to make it symmetric. The maximum adjustment required to do this was of the order of 1.5% of the value adjusted.

Table 6.3 shows the comparison of both computed and experimentally determined flexibility influence coefficients. It may be noted that the maximum difference between the experimental and computed values is about 6%
<table>
<thead>
<tr>
<th>Frame No.</th>
<th>Floor Level</th>
<th>Experimental Flexibility Influence Coefficient x $10^6$ in/lb</th>
<th>Adjusted Flexibility Influence Coefficients x $10^6$ in/lb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Floor</td>
<td>Floor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1   2   3</td>
<td>1   2   3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>55  66  68</td>
<td>55  67  69</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>67  148 161</td>
<td>67  148 162</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>69  163 245</td>
<td>69  162 245</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>50  61  63</td>
<td>50  62  64</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>62  139 152</td>
<td>62  139 152</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>64  151 234</td>
<td>64  152 234</td>
</tr>
</tbody>
</table>

**TABLE 6.2** Experimentally Determined Flexibility Influence Coefficients
<table>
<thead>
<tr>
<th>Frame No.</th>
<th>Floor Level</th>
<th>Flexibility Influence Coefficient x 10^6 in/lb</th>
<th>Remarks</th>
<th>Percentage Difference of the Experimental Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Floor</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>67</td>
<td>148</td>
<td>162</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>69</td>
<td>162</td>
<td>245</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>50</td>
<td>62</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>62</td>
<td>139</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>64</td>
<td>152</td>
<td>234</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*E = Experimental  
T = Theoretical  

**TABLE 6.3** Comparison of Flexibility Matrix
of the experimental value. The range of variation expressed as percentage of the experimental values is between 0.9 to 6.0%.

The frequencies computed from the above experimental flexibility influence coefficients are given in Table 6.4 along with the frequencies which were computed theoretically as discussed earlier in section 6.3. The table also shows the frequencies actually observed during a cycling of sinusoidal base excitation through a range of frequencies. (A detailed description of the experimental techniques will be given in Chapter VIII.) The maximum difference in the values of frequencies predicted from the experimental flexibility influence coefficients and the actual observed frequencies, expressed as a percentage of the latter, are also given in this table. It may be noted that the maximum difference is of the order of 1.6% for the first natural frequency. The maximum difference for the remaining two higher frequencies is of the order of 2.5%. For all practical purposes the above mentioned differences show an excellent agreement between frequencies determined from experimental flexibility influence coefficients and frequencies actually observed.

At this point, it is pertinent to compare the experimentally observed frequencies and the frequencies computed from material and member properties. This is also shown as a percentage difference in Table 6.4. The maximum percentage difference of the frequencies is of
<table>
<thead>
<tr>
<th>Description</th>
<th>Frame No.</th>
<th>Natural Frequencies Cycles per second</th>
<th>% Difference from Actual Frequencies</th>
<th>Mode No.</th>
<th>Relative Modal Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Frame No.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Predicted from material and member properties</td>
<td>1 &amp; 2</td>
<td>12.03</td>
<td>3.7</td>
<td>1.4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>37.21</td>
<td>1.6</td>
<td>1.0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60.97</td>
<td>3.3</td>
<td>3.2</td>
<td>3</td>
</tr>
<tr>
<td>Predicted from experimental flexibility influence coefficient matrix</td>
<td>1</td>
<td>11.78</td>
<td>1.6</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>36.85</td>
<td>2.5</td>
<td>0.8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>58.53</td>
<td>0.8</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>12.13</td>
<td>0.6</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>37.67</td>
<td>0.2</td>
<td>1.8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60.16</td>
<td>1.8</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Observed experimentally during a cycling of sinusoidal base</td>
<td>1</td>
<td>11.60</td>
<td>1.7</td>
<td>0.0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>37.80</td>
<td>0.0</td>
<td>2.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>59.00</td>
<td>2.1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>12.20</td>
<td>0.0</td>
<td>1.8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>37.60</td>
<td>1.8</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>59.10</td>
<td>2.0</td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

First and second modes tally roughly by visual observation. Third mode could not be observed.

TABLE 6.4 Natural Frequencies and Relative Modal Displacements
the order of 3.7% which shows an excellent agreement between the computed and experimentally observed frequencies of the system.
CHAPTER VII
ANALYTICAL PREDICTIONS

7.1 Introduction

The prediction of the response of the structural system used for the experimental investigation was done by using the numerical procedure described in Chapter II. The modulus of elasticity required for this prediction was determined from the slope of the moment-curvature relationships obtained experimentally by joint tests as described earlier in Chapter VI. The value of plastic moment was also obtained from these tests. The geometrical property, i.e. second moment of area, was taken from the properties of Alcan Extruded Shapes\textsuperscript{36}.

The forcing functions used for the predictions of the response were those monitored by the accelerometer mounted on the base of the structure.

The various quantities predicted for comparison with those obtained experimentally (details follow in Chapter VIII) are the following:

(a) Maximum acceleration at each floor level of the frame;

(b) Maximum ductility factor in each storey;

(c) Maximum horizontal displacements of each floor.
7.2 Elastic Response

The elastic response of the structure was predicted by using the numerical approach described in Chapter II. The following base excitations were used to predict the response:

(a) A single sinusoidal pulse of peak amplitude of 0.5 g at 12 cycles per second;
(b) A single sinusoidal pulse of peak amplitude of 1.0 g at 12 cycles per second;
(c) Base acceleration recorded due to an input of the first nine seconds of accelerogram of El Centro California Earthquake of May 1940, N-S Component x 2.0.

In all these predictions, a damping value of 0.5% of the critical viscous type damping was used. As will be described later in Chapter VIII, this value of damping was determined from the decay of amplitude under free vibration of the system following an impulse. The damping used was assumed to be of close coupled nature.

Table 7.1 shows the maximum ductility factors developed in each storey and the maximum accelerations obtained from numerical computation at each floor level. These predicted values are given in rows 1, 4 and 7 of the table.

7.3 Inelastic Response

As indicated in the introduction, the inelastic response was predicted by using the numerical approach
<table>
<thead>
<tr>
<th>Description of Acceleration Excitation of the Base</th>
<th>Maximum Column Ductility Factors</th>
<th>Maximum Acceleration of Gravity</th>
<th>Remarks</th>
<th>Row No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Storeys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Single sinusoidal pulse of peak amplitude of 0.5 g at 12 cycles/sec.</td>
<td>0.33</td>
<td>0.25</td>
<td>0.16</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>0.31</td>
<td>0.30</td>
<td>0.12</td>
<td>Not Recorded</td>
</tr>
<tr>
<td></td>
<td>6.5</td>
<td>16.7</td>
<td>33.3</td>
<td>6.3</td>
</tr>
<tr>
<td>Single sinusoidal pulse of peak amplitude of 1.0 g at 12 cycles/sec.</td>
<td>0.65</td>
<td>0.49</td>
<td>0.32</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>0.63</td>
<td>0.57</td>
<td>0.37</td>
<td>Not Recorded</td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td>14.0</td>
<td>13.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Accelerogram of El Centro California Earthquake of May 18, 1940, N-S Component x 2.0</td>
<td>0.67</td>
<td>0.49</td>
<td>0.30</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>0.65</td>
<td>0.45</td>
<td>0.30</td>
<td>Not Recorded</td>
</tr>
<tr>
<td></td>
<td>3.1</td>
<td>8.9</td>
<td>0.0</td>
<td>16.2</td>
</tr>
</tbody>
</table>

TABLE 7.1 Elastic Response of the Frame Subjected to Low Amplitude Excitation
already described in Chapter II. As the moment-curvature relationship of the members of the system is of a bi-linear nature with a mild slope in the linear strain hardening range (Figures 6.8 and 6.10), it was necessary to simplify the above relationship to an elasto-plastic moment-curvature relationship. Fig. 7.1 shows such simplifications. In the process of this simplification, the plastic moment is raised to a value so that the area of M-\(\phi\) diagram under the actual curve and the idealized curve is the same. Such a simplification is justifiable if the excursions are limited to a ductility factor approximately equal to four. This is because in such a case the inaccuracy involved is very little. This can be seen from Fig. 7.1(a). If a positive excursion is followed by a reversal of stresses and a subsequent opposite excursion of about half the magnitude of that of the previous one, the lowering of fully plastic moment on the opposite side (shown on negative side of Fig. 7.1(a)) is very little. In this case, the possible adjustment is shown by a chain line. If the excursions in the inelastic region are of greater magnitude, the fully plastic moment of the idealized curve is to be raised to a higher value to satisfy the criteria of equal area under the two curves. Such a situation is shown in Fig. 7.1(b) in which an excursion corresponding to a ductility factor of 10 is considered. In this case if the excursion of one kind is followed by an excursion of another kind, and of the same
(a) $\mu = 4$

(b) $\mu = 10$

**FIG. 7.1 IDEALIZATION OF MOMENT CURVATURE RELATIONSHIP**
order of magnitude as the former one, it is evident from the M-\(\phi\) curve on the negative side (Fig. 7.1(b)) that larger inaccuracies are likely to be involved. On the negative side, the area under the experimental M-\(\phi\) curve is less than the area under idealized curve. In such a situation, the magnitude of fully plastic moment on the opposite side (negative side in this case) is to be reduced to make the two areas equal. This is shown by a chain line. Such adjustments can reasonably be justified in situations as in the present case in which the material of experimental model is of bi-linear nature compared to elasto-plastic steel used in actual buildings.

Using the above simplifications, and base acceleration record monitored experimentally (Fig. 8.6), the inelastic response of the first frame was predicted numerically. This is given in Table 7.2.

As will be described later in Chapter VIII, the second frame was inelastically deformed by a sudden application of base acceleration resulting from accidental power failure. This unexpected dynamic load was taken into account to predict the inelastic response of second frame which is given in Table 7.3.
<table>
<thead>
<tr>
<th>Row No.</th>
<th>Description</th>
<th>Maximum Ductility Factor</th>
<th>Maximum Acceleration of Gravity</th>
<th>Maximum Deflections in inches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Storeys</td>
<td>Storeys</td>
<td>Storeys</td>
</tr>
<tr>
<td>1</td>
<td>Predicted</td>
<td>1</td>
<td>9.50</td>
<td>17.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>2.87</td>
<td>14.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>2.12</td>
<td>14.89</td>
</tr>
<tr>
<td>2</td>
<td>Experimental</td>
<td>1</td>
<td>Not Recorded</td>
<td>14.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td>15.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td>1.26*</td>
</tr>
<tr>
<td>3</td>
<td>Difference</td>
<td>1</td>
<td>2.0</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td>between the above two as a percentage of the experimental value</td>
<td>2</td>
<td>2.0</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>23.2</td>
<td>8.7</td>
</tr>
</tbody>
</table>

* Estimated approximately from high speed photographic record.

**TABLE 7.2** Inelastic Response of Frame No. 1
<table>
<thead>
<tr>
<th>Row No.</th>
<th>Description</th>
<th>Maximum Ductility Factor</th>
<th>Maximum Acceleration of Gravity</th>
<th>Maximum Deflections in inches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Storeys</td>
<td>Storeys</td>
<td>Storeys</td>
</tr>
<tr>
<td>1</td>
<td>Predicted</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Experimental</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Difference between the above two as a percentage of the experimental value</td>
<td>1.7</td>
<td>5.7</td>
<td>8.5</td>
</tr>
</tbody>
</table>

**TABLE 7.3** Inelastic Response of Frame No. 2
CHAPTER VIII
DYNAMIC EXPERIMENTAL INVESTIGATION

8.1 Introduction

The main purpose of the dynamic experimental investigation was to determine experimentally the dynamic characteristics of the experimental structure and its transient response in the elastic as well as inelastic region. The determination of dynamic characteristics includes determination of damping factor and the natural frequencies of the system. Once the dynamic properties and the response to various forcing functions are recorded experimentally, the experimental results can be compared with those predicted earlier on the basis of an assumed mathematical model and using the numerical procedure. Such a comparison may lead to the important conclusions as to whether or not the assumed mathematical model and the employed computational procedure predict the response reasonably accurately. Conclusions drawn in this way will have significant implications in terms of the validity of the approach used for prediction and its subsequent use in predicting the dynamic response of inelastic multi-storey framed structures.
8.2 Experimental System

The experimental system consists of a shaking table 6.5 ft. wide and 7 ft. long in plan dimensions, and has a live weight capacity (during dynamic motion) of 3000 lbs. It is excited by a servo-controlled actuator which can apply base accelerations of 1 g to the shaking table (with maximum live weight attached) at frequencies which may exceed 100 cycles per second.

The excitation of the shaking table was done by using the 903.34 Structures Loading System supplied by MTS Systems Corporation. This system has features which allow the application of any type of function to the shaking table, although the motion of the shaking table is controlled by corresponding displacement. For example, if it is desired that the shaking table should follow a prescribed acceleration function, the prescribed acceleration is integrated twice by the integrating unit of the system to obtain corresponding displacement which governs the motion of the table. In case of prescribed velocity function, the function is integrated once to obtain corresponding displacement.

One of the important features of the system is the Stata Trak Model 5124. This contains a drum on which any type of displacement, velocity or acceleration function, etched on a special silver plated program chart, can be mounted. The etching of the desired function is done by a hot-stylus etcher. For example, an earthquake accelerato-
gram record could be etched on the program chart to simulate an earthquake excitation of the shake table. The Stata Trak drum is capable of being rotated with a wide range of speeds so that the time scale of the prescribed function could be changed. As will be indicated later, the Stata Trak was used to apply an earthquake excitation to the base of the experimental structure.

The motion of the shaking table is controlled by a servo-controlled loading feedback system. The principle behind this system is the comparison of the desired condition of the shaking table displacement with the actual displacement of the shaking table at any instant of time. If any difference is detected between these two, a correction signal is sent to the servovalve which adjusts the flow of hydraulic fluid into the actuator to eliminate the detected difference of the desired and existing displacement. This process of comparison and correction or precisely command and feedback takes place 10,000 times per second. Such high frequency of command and feedback system enables the displacement of the table to continuously follow the desired displacement very closely.

The instrumentation used for the experimental investigation is as follows:

Strain Gages

The strain gages were mounted at all critical sections of the frame as shown in Fig. 6.1. These gages were of High Elongation Type HE-141-B as manufactured by
the Budd Company, Phoenixville, PA. The resistance and gage factor of the gages were 120 ± 0.2 ohms and 2.05 ± 1/2\% respectively. The gages were capable of tolerating up to 15\% elongation of their grid length of 1/4 inch. These were specified to be suitable for use up to a temperature of +200°F.

The strain gages denoted by letter 'B' in Fig. 6.1 were connected to a direct writing type R Dynograph (Beckman Instruments, Inc., Offener Division, Schiller Park, Illinois). The other sets of strain gages denoted by 'A' were connected through Bridge Amplifiers (Model No. BA-4, Ellis Associates) to an Ampex, CP-100 magnetic tape recorder. After the test was over, the output of the strain gages recorded on the Ampex recorder was reproduced for graphical representation on the direct writing Dynograph.

The strain gages denoted by letter 'D' in Fig. 6.1 were connected to a Digital Strain Indicator (Budd Model TC-22) to monitor the residual strains, if any, at these locations after the test was completed.

Accelerometer

As shown in Fig. 6.1, three accelerometers, one at each floor level, were mounted to monitor the accelerations. One accelerometer was also mounted on the shaking table to monitor the base acceleration applied to the experimental structure.

The accelerometers mounted on the second and third
floor level were Endevco Series 2200 Accelerometers. These accelerometers are of a Piezoelectric type. The operating frequency range varies from 2 cps to 12 kc. The acceleration range varies from 0.001 g to more than 10,000 g's. The accelerometers used on the second and third floor level were models 2221c serial ED60 and 2221c serial EC54 respectively.

A Kistler Quartz Accelerometer was used at the first floor level. Unfortunately, no output was recorded from it. It appeared that there was something wrong either in the accelerometer or in the amplifying system connected to it. Attempts to trace the cause of malfunctioning were unsuccessful.

The output from the Endevco Accelerometer mounted at the second floor level was amplified by a High Impedance Laboratory Amplifier Model 2616B. The resulting output was fed to Model CEC 1-165 D.C. Amplifier. The output signal thus obtained was fed to the Direct Writing Oscillograph recorder type RG 32.12/15. This recorder employs an ultra-violet light source, using pencil type mirror galvanometers focussed on to ultra-violet sensitive recording paper.

The Endevco Accelerometer mounted at the third floor level was connected to the Dynograph Direct Writing Recorder.

The accelerometer mounted on the shaking table was a Universal Servo Accelerometer, Model 305A, S/N 2477.
Its sensitivity is 0.2 ma/g, 0.100 v/g. The output from this accelerometer was recorded on the type R Dynograph Direct Writing Recorder after amplifying the output through Servo Amplifier Model 515 S/N 168.

**Accuracy of Output**

All the accelerometers were directly calibrated at a frequency of 12 cycles per second. This eliminated the inertia effect of the pen while interpreting the recorded results except for those results recorded due to an earthquake excitation. As the frequency content in this situation is of a random nature, there seems to be no direct way to take into account the inertia effect of the galvanometer writing pen. Thus there may be an unknown degree of inaccuracy involved in the results shown for earthquake response in section 8.5. The inertia effect of the galvanometer writing pen is shown in Fig. 8.1.

The strain gage calibration was done by the calibration pulses of known strain available in the Bridge Amplifiers (BA-4). The linearity of this equipment varies from 1/2% to 2%. Similarly, the linearity of Dynograph is also 1/2% for the curvilinear recording. It is estimated that inaccuracy in reading the recorded results might be of the order of 1%. Thus the overall accuracy, except for the earthquake excited situation, should be well within 3%.

**High Speed Camera**

A High Speed Model 1-B Recording Camera (manufactured by Photo-Sonics, Inc., Burbank, California) was
FIG. 8.1 FREQUENCY RESPONSE OF DIRECT WRITING DYNOGRAPH GALVANOMETER.
used to photograph the response of the experimental structure during its vibration. This camera has a capacity for taking full frame motion pictures at rates between 12 and 1000 frames per second.

The first frame tested was photographed at 1000 frames per second. The quality of the pictures taken was not very good due to lack of appropriate illumination at such high speed. The response of the second frame was photographed at 400 frames per second and with more illumination than that of the previous one. In this case, the quality of the picture improved considerably.

8.3 Damping Factor Determination

For determining the damping factor, a low amplitude impulse was applied to the base of the structure. The strain responses from the strain gages mounted at critical sections of the frame were monitored. A typical strain response to impulse is shown in Fig. 8.2. Other strain response records were of similar nature. Fig. 8.3 shows the plot of the logarithm of amplitude versus the number of cycles under free vibration conditions. It may be noted that plot is a straight line which shows that the damping is constant and is of viscous type.

The value of logarithmic decrement \( \delta \) is given by

\[
\delta = \ln \left( \frac{x_i}{x_j} \right) / (j-i)
\]

in which \( x_i \) and \( x_j \) are the maximum amplitudes of free
FIG. 8.2 TYPICAL STRAIN GAUGE RESPONSE TO IMPULSE
FIG. 8.3 LOGARITHMIC DECREMENT
vibrations in \( i \)th and \( j \)th cycles \((j>i)\). From the plot shown in Fig. 8.3, \( \ln(\bar{x}_i/\bar{x}_j) = 1.0 \) for \( j-i = 31.5 \).

Thus the logarithmic decrement \( \bar{\delta} \) is given by

\[
\bar{\delta} = \frac{1}{31.5} = 0.0317
\]

Therefore the damping factor \( \zeta \) is given by

\[
\zeta = \frac{\bar{\delta}}{2\pi} = \frac{0.0317}{2\pi} = 0.00505
\]

8.4 Dynamic Properties

The natural frequencies of the system were determined by giving the base of the structure a sinusoidal displacement and varying the frequency. For convenience in observing the magnification of the response, the signal from a strain gage mounted on the bottom end of the lowermost storey column was fed to an Oscilloscope. In order to keep the response well within the elastic region and also the amplitudes of displacements to a minimum to avoid the possible damage due to inelastic deformation, the amplitude of base displacement was kept at ±0.0025 inches. At such small amplitude, the attempts to plot the frequency versus amplitude relationship were quite disappointing. At frequencies slightly lower than the natural frequency, the magnification was not enough so that it could be observed and recorded. As soon as the natural frequency was reached while increasing the base frequency, a spontaneous amplification of amplitude of about five times or more was observed. As the damping
determined prior to this indicated a very low value, it was not thought proper to keep the structure in resonance condition for a longer time than absolutely necessary. To avoid any possibilities of damage resulting from resonance condition, base amplitude was immediately brought to zero value. The procedure was repeated several times to verify whether or not the frequency thus obtained remains stationary. It was observed that this remains stationary within the readable accuracy of the system.

The determination of natural frequencies was done during a cycling of sinusoidal base excitation both while increasing the frequencies and decreasing the frequencies. It was observed that while increasing the base frequencies, the magnified amplitude died out immediately once the natural frequency was passed. Of course, this was not so while decreasing the frequency beginning from a frequency higher than the natural frequency. In this case, a further magnification was observed if the decrease of excitation frequency was continued after striking the natural frequency. The reason for this may be due to overhang of amplitude frequency plot on the decreasing side of the frequency. In this case also, no risk was taken to magnify the displacement of the top floor such that inelastic deformations might occur. Thus the amplitude of base excitation was immediately reduced to zero once the natural frequency was reached.

The natural frequencies thus observed are given
in Table 6.4. A comparison of these frequencies with those computed from material and member properties and also from experimentally determined influence coefficients has already been made in Chapter VI.

8.5 Transient Response

For the determination of transient response of the structure in the elastic and inelastic region, the following experimental procedure was followed in each case.

Elastic Response

Single sinusoidal pulses of peak amplitude of 0.5 g and 1.0 g at 12 cycles/sec were applied at different times to the base of the structure through the appropriate setting on the servo-control panel of the experimental system. The maximum response values observed from the oscillograph recording for each of the above pulses are tabulated in the 2nd and 5th rows of Table 7.1.

Elastic Earthquake Response

The response of the structure was also determined experimentally by exciting the base of the structure (shaking table) by the first 9 seconds of the accelerogram record of El Centro California Earthquake of May, 1940, N-S Component x 2.0. To achieve this, the actual accelerogram record for the first 10 seconds was plotted on a suitable scale so as to give the same time scale as that of the actual earthquake record and twice the amplitude when mounted on the Stata Trak Drum of the Servo-Control System previously described in section 8.2. This
record was then transferred and etched on silver plated paper. The paper was then mounted on the Stata Trak Drum. By operating the system in the acceleration mode, the acceleration pulses were twice integrated and the resulting displacement was applied to the shaking table acting as the base of the structure. The resulting base acceleration as monitored by the accelerometer mounted on the shaking table is shown in Fig. 8.4. It may be pointed out that the accelerogram record is slightly different than that which was drawn on the Stata Trak. This is due to the inherent characteristics of the integrator and the corresponding errors introduced in the process of double integration of the input record. Stability of the servo-control system is another factor which may be responsible for this in some way.

The maximum response obtained from the output of strain gages and accelerometers is shown in the 8th row of Table 7.1. A detailed time history of strain response from the strain gage mounted at the lower end of the lowermost storey column of the middle frame is shown in Fig. 8.5. It may be noted that, although the record drawn on the Stata Trak Drum was for the 10 seconds duration, the base excitation occurred only for a duration of 9 seconds. This happened because the drum rotation stopping device was set at about 9 seconds.

Inelastic Response

For determination of inelastic response of the
FIG. 8.4 RECORD OF BASE ACCELERATION
FIG. 8.5 ELASTIC RESPONSE DUE TO EARTHQUAKE
structure, the base of the structure was excited with a single high intensity sinusoidal pulse of the same frequency as the frequency of the structure. This frequency was 11.60 for the first frame. The base acceleration monitored by the accelerometer mounted on the base of this frame is shown in Fig. 8.6. It may be noted from this figure that, although it was intended to apply a single sinusoidal pulse, two half sine pulses of smaller amplitude and larger frequency preceded and followed the intended sine curve. This was due to the instability of the servo-control system at such high accelerations.

The maximum response values monitored by strain gages and accelerometers mounted on the first frame are given in the 2nd row of Table 7.2. A detailed time history of the strain response from the strain gage mounted at the lower end of the first storey column is shown in Fig. 8.7.

While testing the second frame, the power failed suddenly prior to the actual test. This happened due to the over-current drawn by the high wattage lights switched on for high-speed photography of the structure. The power failure resulted in the application of a base acceleration of a random nature due to sudden shut-off of the hydraulic system of the closed loop MTS system. Due to this, the frame No. 2 underwent inelastic deformations. This was indicated by some permanent deformations at the critical sections of the first and second storeys. Because of the power failure, all recording instruments were off so that
FIG. 8.6 BASE ACCELERATION
FIG. 8.7 INELASTIC STRAIN RESPONSE OF BOTTOM END OF FIRST STOREY COLUMN
no response could be monitored. It was decided to continue the test to determine the inelastic response of this frame considering the state of residual stresses and strains (to be determined later on by evaluating the base acceleration pulse resulting from the power failure) as the initial state of stress and strain. After the test was over, the base accelerations of the shaking table, under the same conditions as those present at the time of accidental power failure, were recorded several times by manually shutting off the power. The base acceleration function thus recorded is shown in Fig. 8.8. The repetitive nature of the forcing function suggested that most probably this kind of acceleration function must have been applied at the time of accidental power failure. Assuming this to be the true forcing function, the response of the structure was computed and the system was allowed to come to a stationary equilibrium position. The strains and state of residual stresses thus obtained were used as initial conditions to predict the final state of stress and deformations as pointed out in Chapter VII. Also, these strains were superimposed on those determined experimentally during tests after the accidental power failure.

The semi-experimental results thus obtained are given in Table 7.3.
FIG. 8.8 BASE ACCELERATION DUE TO POWER FAILURE
CHAPTER IX
COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS AND DISCUSSION

9.1 Natural Frequencies

The prediction of natural frequencies was based on (a) member and material properties and (b) flexibility influence coefficients obtained experimentally. The values as listed in Table 6.4 show an excellent* agreement with those obtained experimentally. The differences between the actual and predicted first natural frequency from (a) material and member properties and (b) flexibility coefficients are 3.7% and 1.6% respectively for the first frame and 1.4% and 0.6% respectively for the second frame. Such excellent agreement in case of single degree of freedom system has been reported elsewhere15.

*For purposes of comparison, the difference between experimental and predicted values expressed as a percentage of the experimental value will be categorized as excellent, very good, good, fair and poor if the percentage differences are 5, 10, 15, 20 and more than 20 respectively. However, it is realized that the above categorization is an arbitrary one and should not be considered as a generalization of the interpretation of results.
The maximum difference in case of second and third natural frequencies is 3.3%. Certainly so close an agreement can be considered as excellent.

It was difficult to record actual mode shapes because of the low amplitudes during sinusoidal cycling. However, the mode shapes observed visually roughly agree with those computed theoretically. The agreement between the mode shapes determined from experimental flexibility influence coefficients and those obtained from member and material properties is very nearly perfect.

Such close agreement can be attributed to (1) correct determination of material property of the frame and (2) almost perfect rigidity of the joints and correctness of the geometry of the frame.

9.2 Elastic Response

Response to Pulses

The maximum ductility factors and maximum acceleration obtained analytically and experimentally are shown in Table 7.1. The maximum percentage difference in ductility factors in the first, second and third storeys are 6.5, 16.7 and 33.3% respectively for the two pulses. The average percentage difference for both the pulses for the three storeys amount to 4.9, 15.4 and 23.4%. This may be categorized as excellent, fair and poor for the three storeys respectively. In the first and third storeys, the predicted response exceeds slightly the experimental
values while in the second storey it is less than the experimental. No particular significance can be attached to such fluctuation.

For the acceleration responses of second and third storeys, the predicted responses are less than those recorded. The maximum percentage difference for the second and third storeys being 9.5 and 8.8. The average differences for the above two storeys amount to 7.9 and 8.6 respectively, which is a very good agreement.

Response to Earthquake Excitation

Referring to rows 7 and 8 of Table 7.1, the differences between the predicted and experimental maximum ductility factors for the three storeys expressed as a percentage of the experimental values are 3.1, 8.9, and 0.0. This agreement can be termed excellent, very good and excellent respectively. The average of the three will work out to be an excellent agreement. Similarly, the percentage differences of maximum accelerations for the second and third storeys are 16.2 and 26.2. This can be categorized as fair and poor. The average of the two together works out to be poor. In this case, the predicted accelerations are lower and higher than experimental values for the second and third storeys respectively.

9.3 Inelastic Response

Response of Frame No. 1

The difference between the predicted and experi-
mental values expressed as a percentage of the experimental values are shown in the third row of Table 7.2. The agreement between experimental and predicted values is excellent for first and second storey maximum ductility factors while it is poor for third storey ductility factor. The average of the three percentages can be termed a very good agreement.

For categorizing the maximum accelerations of the second and third floors, the agreement between the analytical and experimental values is excellent.

In the case of maximum deflections of the first, second and third floor, the percentage difference between predicted and experimental values is 2.8, 5.7 and 8.7 which can be categorized as excellent, very good and very good respectively, and very good as an average of the three.

Response of Frame No. 2

As already stated in section 8.5, the experimental results shown in the second row of Table 7.3 are partly recorded directly and partly computed on the basis of base acceleration recorded by simulating power shut-off. Though these results cannot be relied upon as being totally correct, there seems to be good agreement between those computed and the so-called experimental. For maximum ductility factors in three storeys, the percentage differences of 1.7, 5.7 and 8.5 fall in the category of excellent, very good and very good individually, and very good on the average.
In case of maximum accelerations of the second and third floor, the classification is poor and fair individually and poor as an average of the two.

For maximum floor displacements, the percentage differences for first, second and third floors are 24.4, 18.1 and 8.3 which amount to poor, fair and very good individually, and fair as an average of the three.

9.4 Assessment of Overall Agreement Between Predicted and Experimental Results

The percentage differences for a total of fifteen response values are shown in Table 7.1. Although these percentage differences are evaluated for maximum ductility factors for different storeys and maximum accelerations at different floor levels, an attempt is made below to find out the number of these differences which show a difference of 15% or less. It can be seen that out of 15 differences, 11 differences are less than 15%. This gives an overall picture of the agreement between the predicted and experimental response values. Thus, about 70% of the comparisons of maximum elastic response values show a good agreement between the predicted and experimental maximum response values.

A similar assessment can be made for inelastic response. Tables 7.2 and 7.3 show percentage differences of 16 maximum response values. In this case, 11 differences are less than 10%. Thus in this case also, about 70% of the comparisons of maximum response values show a
very good agreement between the predicted and experimental maximum response values.

On the basis of the above assessment of overall good agreement between the predicted and experimental elastic as well as inelastic responses, it can be concluded that the theoretical prediction based on the assumed mathematical model is quite satisfactory for all practical purposes.

It may be emphasized that the evaluation of the capability of the theoretical procedure to predict the inelastic response of a multi-storey frame is of much significance from the point of view of its implication on aseismic design and dynamic analysis. The mathematical model assumed in the theoretical approach adopted to predict the response is reasonable and practically realistic. This feature is quite significant for design as it takes into account the basic mechanism involved (elasto-plastic moment-curvature relationship) in the prediction of inelastic response. The effect of other factors such as normal compressive stresses due to the weight of the structure, temperature, rate of straining, etc., is not considered because of the following two reasons. Firstly, the consideration of the above stated factors complicates the theoretical approach so much that it becomes nearly impracticable to compute the inelastic response of a multi-storey framed structure, even using a modern high speed digital computer. Thus, the formulation of such an
approach remains only of an academic significance. Secondly, the effect of such factors is quite small and therefore it may be neglected for all practical purposes.

It may be noted that the gross evaluation of the theoretical approach is based on the comparison of the predicted maximum response values with those obtained from the tests of two frames. However, it is important to note that an excellent agreement was found between the experimental flexibility influence coefficients of the experimental frames and those predicted theoretically. Similarly, the agreement between the predicted natural frequencies and those observed experimentally was also found to be excellent. Thus, on the basis of the above excellent agreement, one could conclude that the good agreement between the predicted and experimental response could be relied upon, even though the number of frames tested is small.
CHAPTER X
CONCLUSIONS AND RECOMMENDATIONS

The following sections give the general conclusions and recommendations based on the results of the theoretical and experimental investigations presented in the foregoing Parts I and II of this thesis respectively.

10.1 Parametric Study of the Inelastic Response of Multi-Storey Frames Subjected to Strong Motion Earthquakes

The results of this investigation indicate that the following general conclusions can be drawn. These conclusions are of interest and importance in dynamic analysis and aseismic design of multi-storey frames.

1. The effects of the live load as a pure girder load and the live load position are of minor importance. An eccentrically positioned live load counteracts approximately the effect of the additional pure girder load. Therefore, it can be concluded that all live load acting as a pure girder load can be neglected without significantly affecting the reliability of response data from the point of view of its implications on the design of the structure.

2. In contrast to the foregoing conclusion, the effect of live load contribution towards the mass at the
floor levels has a significant effect on the response of the structure. The increase in the masses of the system as a result of the inclusion of a percentage of live load mass results in an entirely different dynamical system with altogether different response characteristics. These resulting response characteristics can, therefore, produce effects which are not a simple function of the amount of live load mass included in the total floor mass. Therefore, it is concluded that it is very important to make a proper estimation of the mass of the system preceding the analysis and design. The mass thus estimated should include a reasonable contribution of mass due to live load. Neither maximum nor minimum live load mass contributions may necessarily yield the most severe response characteristics.

3. The effect of the change of relative stiffness distribution between girders and columns has quite a significant effect on the response parameter variations from the point of view of its implications on the dynamic analysis of the structural system and its design. The change in relative girder stiffnesses results in a system whose dynamic and static structural properties are entirely different. The observations indicate that the burden of inelastic deformations falls on the columns while it is relieved from the girders when the girders are relatively much stiffer than columns. The observations also indicate
that maximum floor displacements largely depend upon column ductility factors. As a stiffer floor system may force all the inelastic deformations into columns, which in turn may lead to larger displacements, serious damage and instability, it is recommended that due consideration must be given to the assessment of the contribution of different floor systems to girder stiffness, before doing the dynamic analysis and the aseismic design of the structure.

4. In tall buildings, larger floor displacements are likely to involve the danger of total collapse or irreparable damage due to the destabilizing effect of the gravity loads. To avoid such a dangerous situation, it seems logical to design relatively stiffer columns and weaker girders. Such a design will provide sufficient energy dissipation capacity in the structure by allowing the girders to deform inelastically. Also, completely elastic behaviour of the columns will inhibit large displacements of the floors. Thus, the probability of collapse of the structure will be reduced to a minimum.

It is also recommended that the inelastic deformation capacity of the girders should be staggered in alternate storeys of a tall building. This is necessary to avoid the situation in which all the girder ends in different storeys meeting at a particular column may develop plastic hinges which may exist simultaneously. This is very likely when the structure is subjected to
a strong earthquake. In such an event, the effective column length of the girder may become very large. Consequently, local instability of a part or full length of such a column may become the starting point of overall instability of the structure. However, if the inelastic deformation capacity of the girders is staggered in alternate storeys, the unsupported column length will never exceed two times the storey height, even when all the ends of weaker girders form plastic hinges simultaneously. Thus the chances of the initiation of local instability leading to overall collapse or irreparable damage will be reduced to a minimum.

5. The observations indicate that the effect of viscous damping on the response parameters is non-systematic and nearly insignificant. Thus it is concluded that the effect of viscous type of damping up to a maximum value of 10% of the critical damping of the first elastic mode is relatively small compared with the effect of hysteretic type of damping resulting from the inelastic deformation of the members of the structural system.

6. The non-systematic changes in the girder ductility factors (as observed and discussed in sections 4.5 and 5.5) due to a systematic change in damping value suggested that there is a strong interaction between the viscous type and hysteretic type of damping. The viscous type of damping plays a significant role in conjunction with the hysteretic type of damping to affect the in-
elastic response of the structural system. Thus from the point of view of aseismic design of structures, it is important to assess the damping characteristics of both the structural and non-structural elements of the building in evaluating the relative value of damping and the strength of the coupling.

It appears that the effect of far coupled damping on the response parameters is relatively more than that of close coupled damping. (No general conclusions can be drawn from this as only one case was investigated.)

7. The maximum displacements of the inelastic system were found to be considerably less than those of comparable elastic systems. Therefore, it is concluded that the usual concept, which is more or less correct for single degree of freedom systems, that the maximum displacement of the elastic and elasto-plastic system are of the same order of magnitude does not hold true, in general, for either damped or undamped multi-degree of freedom systems.

8. The response parameter variations corresponding to varying relative earthquake intensities are significant in terms of their implications on design and analysis. The increase in relative earthquake intensity results in a consistent but non-proportional increase in maximum displacements and column and girder ductility factors. It is therefore concluded that the relationship between the intensity and damage to the structure and its response is
9. The observations of the effect of various earthquake excitations representing different characteristics but having the same peak acceleration of 0.5 g indicate that the maximum intensity of ground acceleration is not the only basis for assessing the probable damage to the structure, but the characteristics of the earthquakes are also important. As the relationship between damage and maximum base acceleration is also non-linear, it is concluded that both the intensity of acceleration and the frequency response spectra for the earthquake type interact in a non-linear manner in influencing the dynamic response of a structure. Both of these factors are of significant importance and, therefore, must be given due consideration in designing the structures.

10. It is also concluded that a more extensive study in this area may lead to the establishment of the bounds for various response parameters. Such a study may take into consideration the response of a wide range of multi-storey structures (normally designed and constructed) subjected to a majority of strong earthquake records available so far.

10.2 Experimental Investigation of the Inelastic Dynamic Response of Multi-Storey Frames

1. The comparison of the flexibility influence
coefficients, computed from member and material properties, and those obtained experimentally, shows an excellent agreement between the two. The comparison of the natural frequencies of the experimental structure determined (a) experimentally, (b) from experimental flexibility influence coefficients, and (c) from member and material properties, also indicates an excellent agreement between the above three sets of frequencies. Therefore, it is concluded that the theoretical approach used to predict the static and dynamic properties of the experimental structure is reasonably accurate for all practical purposes.

2. The comparison of theoretically predicted elastic and inelastic response values, with those determined experimentally, indicates that about 70% of these are in good agreement. The comparison of the detailed time histories of a typical elastic strain response and a typical inelastic strain response showed a satisfactory agreement between the experimental response histories and those computed theoretically. Therefore, it is concluded that the mathematical model assumed and the numerical approach adopted for the prediction of inelastic response of multi-storey structures are appropriate for all practical purposes of analysis and design.

As the results obtained from the three storey frames are quite promising, a further investigation of frames having a larger number of storeys than those in-
vestigated here would be of interest in furthering the knowledge in this area.
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4. A. C. Heidebrecht and B. P. Guru, "Experimental Investigation of Inelastic Dynamic Response of Multi-Storey Frames", accepted for presentation at the coming ASCE meeting in Chicago this fall.