53

A SYSTEMS APPROACH TO FISSION-FUSION SYMBIOSIS

by

CHARLES WILLIAM GORDON, B.A.Sc.

PART B: MCMASTER (ON-CAMPUS) PROJECT*

A Report Submitted to the School of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Engineering

Department of Engineering Physics
McMaster University
Hamilton, Ontario, Canada.

April 1975

* One of two project reports: The other part is designated PART A: OFF-CAMPUS PROJECT.

MASTER OF ENGINEERING (1975)
Department of Engineering Physics

MCMASTER UNIVERSITY
Hamilton, Ontario.

TITLE: A Systems Approach to Fission-Fusion Symbiosis

AUTHOR: Charles William Gordon, B.A.Sc. (Toronto)

SUPERVISOR: Dr. A.A. Harms

NUMBER OF PAGES: vii, 71

ABSTRACT

Three symbiotic systems are considered. These include the possibility of coupling the tritium production in a fission reactor with the fertile conversion in a fusion blanket. Equations for the fuel dynamics, power output, efficiency and costs of a symbiotic, self-contained power station are developed and evaluated for a specific, 1500 MWe fission reactor operating on a thorium cycle and some fusion parameters. It is concluded that a system using the tritium produced in a fission reactor has lower costs and increased power output when compared to an alternate system.

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Dr. A.A. Harms of the Department of Engineering Physics for his guidance and encouragement in the preparation of this project.

TABLE OF CONTENTS

Page No.

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

1.1 Motivation

Several methods of coupling fission and fusion reactors have been proposed as a method of producing energy[1]. Basically, these can be divided into two categories: hybrid and symbiotic. A hybrid can be defined as a reactor in which a fusion core is surrounded by a blanket containing fertile and/or fissile material which is fissioned by fusion neutrons to increase the energy output over that of a pure fusion reactor. Fission-fusion symbiosis is characterized by separate fission and fusion reactors with interconnected fuel and energy circulations. The key distinction is that a hybrid is a single reactor and symbiosis involves two distinct reactors.

There are many reasons for investigating the coupling of fusion and fission reactors rather than pure fission or fusion. Fission reactors are inherently "power rich" but "neutron poor" while fusion reactors are "neutron rich" but "power poor"[2]. Coupling of these reactors may have many advantages such as:

(i) the early introduction of fusion reactors since the plasma characteristics necessary to achieve breakeven may be much less than those for pure fusion[1],

(ii) breeding of fissile materials for fission reactors without compromising the costs or safety of the fission reactor[3],

(iii) introduction of alternative fuel cycles by eliminating the ^{238}U-^{239}Pu cycle with its high toxicity and weapons grade plutonium in favour of the ^{232}Th-^{233}U cycle[2],

(iv) attainment of a self-contained electrical plant[1],

(v) the ability to adjust the power split between the fusion and fission components to meet fuel production, safety and environmental constraints[2].

These advantages seem to make a coupled fission-fusion system useful during the transition between the growing fission economy and the future fusion economy.

1.2 Lidsky's System

At the Culham Fusion Reactor Conference in 1969, L.M. Lidsky introduced the concept of fusion-fission symbiosis[3]. He considered two reactors coupled by the production of fuel for the fission reactor in a D-T fusion reactor blanket, as in Fig. 1(a). The reactors were also coupled by a common electrical generating facility, some of the output of this being used to heat the plasma for the fusion reactor, as in Fig. 1(b). Lidsky investigated the fuel dynamics, power balance, efficiency and capital cost of such a system. As a specific example, he investigated a MSR (Molten Salt Reactor) system with a salt of $LiF:BeF_2:ThF_4$ and a graphite moderator. This salt was also used in his fusion blanket which had a tritium breeding ratio of 1.126 and a Th-U conversion of 0.325 per incident fusion neutron.

A 1500 MWe plant was proposed by Lidsky with a system period of 10 years[1]. This corresponds to a fuel doubling time of 6.93 years. With a specific tritium inventory of 2g/MWt[1], the fusion-fission reaction rate ratio, R_1/R_2, was 0.585. The resultant design was a 295 MWt fusion reactor which provided fuel for a 4450 MWt MSCR while consuming 89 MWe. The net station output was 1690 MWe, assuming a thermodynamic efficiency of 40%, and the net station efficiency was 35.6%. Lidsky specifically excluded the possible tritium production in the fission component in his system.

2. SYSTEMS DESCRIPTION

2.1 Introduction and Terminology

As stated in the last chapter, Lidsky specifically excluded the production of tritium in the fission component of a symbiotic system. Tritium production rates from 3Ci/MWt-d for light water reactors up to 6000 Ci/MWt-a for heavy water reactors have been reported[4]. There are two methods of utilizing this tritium: (i) using the tritium produced by the fission reactor as the only source of tritium for the fusion reactor, and (ii) using the tritium produced in the fission reactor to supplement the tritium production in the fusion blanket. In this paper three conceptual symbiotic systems will be investigated.

In biology there are three types of symbiotic relationships[5]: parasitic, where one organism lives on the body of the host from whom it derives nourishment and does some degree of damage; commensalistic, where one organism consumes the unused food of another; and mutualistic, a relationship in which both partners benefit. All three of our systems may be termed mutualistic since both reactor partners derive some benefits from their relationship; the fission reactor receiving fuel and the fusion reactor receiving fuel and/or energy. An analogy to the biological symbiosis can be found, however, by examining the fuel circulation of the three proposed systems.

The fuel circulations for the three systems considered are shown in Figs. 2, 3 and 4. Consider the system shown in Fig. 2. The fusion blanket has been constructed to produce fissile fuel to be consumed by the fission reactor while, in terms of fuel, the fission reactor provides nothing for the fusion host. This system will be referred to as parasitic. In Fig. 4, both reactors produce fuel for themselves as well as for the other reactor partner. This fully interrelated system will be referred to as mutualistic. The system shown in Fig. 3, however, is not strictly commensalistic; the fusion reactor is consuming tritium produced as a by-product of the fission reactor operation and so this system will be referred to as commensalistic.

The proposed systems are assumed to be completely self-contained central power stations. With this constraint, the fuel processing for the fusion and fission reactors is considered part of the overall system; hence, fissile fabrication and tritium processing plant are included as components of the systems. To simplify the analysis, these plants are considered to be perfectly efficient although the coupling coefficients defined below could be altered to account for losses occurring during processing.

2.2 Parasitic System

The parasitic system shown in Fig. 2 is essentially the same as the system proposed by Lidsky. Tritium is produced in a blanket surrounding the fusion core by the $Li^6(n,\alpha)T$ and the $Li^7(n,n'\alpha)T$ reactions and the fertile nuclei, ^{232}Th or ^{238}U, are converted to fissile nuclei, ^{233}U or ^{239}Pu, by (n,γ) type reactions. Additional neutrons may be provided by $(n,2n)$ reactions with niobium or molybdeum.

The tritium to operate the fusion reactor is produced solely in the fusion blanket and the fissile fuel for the fission reactor is produced by conversion within the fission core as well as conversion within the fusion blanket. The fusion component need not be a net power producer; the excess energy for operation of the plasma, if needed, comes from the fission reactor.

2.3 Commensalistic System

In the commensalistic system, Fig. 3, the tritium to operate the fusion reactor is produced solely within the fission core. This tritium may be produced by $D(n,\gamma)T$ reactions, by boron-neutron interactions[4], in absorber rods, and in poisons, possibly supplemented by $Li^6(n,\alpha)T$ reactions. The fusion blanket is now used to convert fertile nuclei and to shield out radiation. As with the parasitic system, there is self-conversion within the fission component and any energy deficit of the fusion component is made up by the fission reactor.

2.4 Mutualistic System

In this system, Fig. 4, there is full interrelation of the fusion and fission components. Tritium for the fusion reactor is produced in both the fusion blanket and supplemented by production within the fission core. Fertile material is converted to fissile material within the fission core and supplemented by production within the fusion blanket. As in the other two systems, any power needed to drive the fusion component is provided by the fission reactor.

3. SYSTEMS THEORY

3.1 Fuel Dynamics for the Parasitic System

The distinctive characteristic of the three systems under consideration is the fuel circulation. It is expected that the major differences among the systems will be found in the fuel dynamics results.

The following notation will be used:

1 = subscript referring to the fusion component.

2 = subscript referring to the fission component.

R_i = total instantaneous reaction rate for the i'th reactor, fusions or fissions per second.

N_i = total fuel inventory of the i'th reactor, number of tritium nuclei or fissile nuclei.

C_{ij} = coupling coefficient relating the number of fissile or fusile nuclei for the j'th reactor produced by the i'th reactor for each reaction in the i'th reactor.

The following definitions will be used later:

n_i = the specific inventory of the i'th reactor

$$n_i = N_i/R_i \ , \qquad (3.1\text{-}1)$$

with the dimensions of time;

τ = the system period defined by

$$\tau = \frac{d}{dt}\ln N_1 = \frac{d}{dt}\ln N_2 \ . \qquad (3.1\text{-}2)$$

The system period also has the dimension of time and is related to the fuel doubling time, t_2, by

$$t_2 = \tau \ln 2 \ . \tag{3.1-3}$$

The time dependence of the inventory can be described by

$$N_i = N_{io} \exp(t/\tau) \ , \tag{3.1-4}$$

where N_{io} is the total fuel inventory at time $t = 0$. Thus, τ is the "e" folding time of the system.

The fuel dynamics of the three symbiotic systems can be found from simple rate balance equations,

$$\frac{d}{dt} N_i = \binom{\text{rate of}}{\text{production}} - \binom{\text{rate of}}{\text{consumption}} \ . \tag{3.1-5}$$

In the case of the parasitic system this is

$$\frac{d}{dt} N_1 = R_1 (C_{11} - 1) \ , \tag{3.1-6}$$

and

$$\frac{d}{dt} N_2 = R_2 (C_{22} - 1) + R_1 C_{12} \ . \tag{3.1-7}$$

These were solved for the following conditions:

Case A: steady state,

$$\frac{d}{dt} N_1 = \frac{d}{dt} N_2 = 0 \ , \tag{3.1-8}$$

which corresponds to constant fuel inventory.

Case B: constant fuel inventory ratio,

$$\frac{N_1}{N_2} = \text{constant} \ , \tag{3.1-9}$$

or

$$\frac{d}{dt}\left(\frac{N_1}{N_2}\right) = 0 \ , \tag{3.1-10}$$

$$\frac{1}{N_2}\frac{d}{dt}N_1 - \frac{N_1}{N_2}\frac{d}{dt}N_2 = 0 \ , \tag{3.1-11}$$

$$\frac{1}{N_1}\frac{d}{dt}N_1 = \frac{1}{N_2}\frac{d}{dt}N_2 \ , \tag{3.1-12}$$

from which is obtained,

$$\frac{d}{dt}(\ln N_1) = \frac{d}{dt}(\ln N_2) = \frac{1}{\tau} \ . \tag{3.1-13}$$

When the conditions for steady state are applied to Eq. (3.1-6) and Eq. (3.1-7), the following are found,

$$C_{11} = 1 \ , \tag{3.1-14}$$

and

$$\frac{R_1}{R_2} = \frac{1 - C_{22}}{C_{12}} \ . \tag{3.1-15}$$

For steady state, the fusion blanket must produce at least one triton per fusion and the reaction rate ratio is determined by the fissile fuel production. If $(1 - C_{22})$ is defined as the self-conversion deficit, then the fusion-fission reaction rate ratio at steady state must equal the ratio of self-conversion deficit to the fusion production of fissile nuclei.

For constant fuel inventory ratio, Eq. (3.1-13) is substituted into Eq. (3.1-6) and Eq. (3.1-7) to obtain

$$\frac{d}{dt}N_1 = \frac{N_1}{\tau} = R_1(C_{11} - 1) \ , \tag{3.1-16}$$

and

$$\frac{d}{dt} N_2 = \frac{N_2}{\tau} = R_2(C_{22} - 1) + R_1 C_{12} . \tag{3.1-17}$$

Using the definitions of n_1 and n_2, the requirements for constant fuel inventory ratio are

$$C_{11} = 1 + \frac{n_1}{\tau} , \tag{3.1-18}$$

and

$$\frac{R_1}{R_2} = \frac{1}{C_{12}} (1 + \frac{n_2}{\tau} - C_{22}) . \tag{3.1-19}$$

Note that as $\tau \to \infty$, the system approaches steady state and Eq. (3.1-18) and Eq. (3.1-19) approach Eq. (3.1-14) and Eq. (3.1-15), which are the steady state requirements. The ratio n_i/τ can be considered a measure of the fuel increase required for a system period, τ. The amount of tritium produced within a fusion blanket, C_{11}, must meet the fusion reactor consumption plus the amount needed for increasing the inventory. The amount of conversion within the blanket, $R_1 C_{12}$, must equal the deficiency of the fission reactor conversion plus the amount needed to increase the inventory.

3.2 Fuel Dynamics for the Commensalistic System

The fuel dynamics equations for the commensalistic system are similar to those for the parasitic system:

$$\frac{dN_1}{dt} = -R_1 + R_2 C_{21} , \tag{3.2-1}$$

and

$$\frac{dN_2}{dt} = R_2(C_{22} - 1) + R_1C_{12} \, . \tag{3.2-2}$$

These are again solved for steady state and constant fuel ratio. For steady state, the requirements are

$$C_{12} = \frac{1 - C_{22}}{C_{21}} \, , \tag{3.2-3}$$

and

$$\frac{R_1}{R_2} = C_{21} \, . \tag{3.2-4}$$

The steady state reactions rate ratio is completely determined by the tritium production in the fission reactor. The D-T fusion reaction,

$$D + T \rightarrow \alpha + n \, , \tag{3.2-5}$$

shows that the reaction rate is proportional to the amount of tritium, all of which, for the commensalistic system, is produced by the fission component. If Eq. (3.2-4) is substituted into Eq. (3.2-3), it is clear that the amount of fertile conversion within the fusion blanket, R_1C_{12}, must equal the self-conversion deficiency, $R_2(1 - C_{22})$.

Substituting the condition of constant inventory ratio, Eq. (3.1-13),

$$\frac{N_1}{\tau} = -R_1 + R_2C_{21} \, , \tag{3.2-6}$$

and

$$\frac{N_2}{\tau} = R_2(C_{22} - 1) + R_1C_{12} \, , \tag{3.2-7}$$

are obtained.

Rearrangement and the definitions of n_1 and n_2 yield,

$$C_{12} = \left[\frac{1 + \frac{n_1}{\tau}}{C_{21}}\right]\left[1 + \frac{n_2}{\tau} - C_{22}\right] , \tag{3.2-8}$$

and

$$\frac{R_1}{R_2} = \frac{C_{21}}{1 + \frac{n_1}{\tau}} . \tag{3.2-9}$$

As with the parasitic system, the steady state equations are modified for constant fuel inventory ratio to account for the increase in inventory.

3.3 Fuel Dynamics for the Mutualistic System

The fuel dynamics for the mutualistic system are similarly described:

$$\frac{dN_1}{dt} = R_1(C_{11} - 1) + R_2C_{21} , \tag{3.3-1}$$

and

$$\frac{dN_2}{dt} = R_2(C_{22} - 1) + R_1C_{12} . \tag{3.3-2}$$

Note that the fuel dynamics equations for the parasitic system, Eq. (3.1-6) and Eq. (3.1-7), and the commensalistic system, Eq. (3.2-1) and Eq. (3.2-2), are special cases of the mutualistic system equations where the appropriate coupling coefficients are set to zero, that is $C_{21} = 0$ for the parasitic system and $C_{11} = 0$ for the commensalistic system.

For steady state, the equations can be written:

$$0 = R_1(C_{11} - 1) + R_2C_{21} , \tag{3.3-3}$$

and

$$0 = R_2(C_{22} - 1) + R_1C_{12} \ . \tag{3.3-4}$$

By eliminating R_1/R_2, the steady state coupling coefficients must satisfy

$$(C_{11} - 1)(C_{22} - 1) - C_{21}C_{12} = 0 \ , \tag{3.3-5}$$

or

$$C_{11} = 1 - \frac{C_{21}C_{12}}{1 - C_{22}} \ . \tag{3.3-6}$$

The steady state reaction ratio is given by

$$\frac{R_1}{R_2} = \frac{1 - C_{22}}{C_{12}} \ . \tag{3.3-7}$$

Substituting Eq. (3.3-7) into Eq. (3.3-6) and rearranging, the requirements can be rewritten in the form

$$R_1C_{12} = R_2(1 - C_{22}) \ , \tag{3.3-8}$$

and

$$R_2C_{21} = R_1(1 - C_{11}) \ . \tag{3.3-9}$$

This shows the type of balance which the system must have to be in steady state and is an alternate method of arriving at the coupling coefficient and reaction rate ratio requirements.

For a constant fuel inventory ratio, Eq. (3.3-1) and Eq. (3.3-2) become

$$\frac{N_1}{\tau} = R_1(C_{11} - 1) + R_2C_{21} \ , \tag{3.3-10}$$

and

$$\frac{N_2}{\tau} = R_2(C_{22} - 1) + R_1C_{12} \ . \tag{3.3-11}$$

These can be rearranged to give

$$C_{11} = 1 + \frac{n_1}{\tau} - \frac{C_{12}C_{21}}{1 + \frac{n_2}{\tau} - C_{22}} \ , \tag{3.3-12}$$

and

$$\frac{R_1}{R_2} = \frac{1 + \frac{n_2}{\tau} - C_{22}}{C_{12}} \ . \tag{3.3-13}$$

As with the other two systems, the steady state equations are modified to account for the increase in inventory.

The above results are strictly valid only when the fuels are continuously transferred from one reactor to the other; the assumption made is that the buildup of fissile precursors and the decay of tritium can be neglected(6). This is an acceptable approximation when the fabrication time is less than the system period, when the precursors decay before fuel processing and when the tritium is processed before much decay occurs. Some work has been done to account for tritium decay and residence time in a processing unit for a pure fusion system(7).

3.4 Power Balance and Efficiency

As seen in Figs. 2, 3 and 4, there are no differences in the energy circulation among the systems. Because of this, a common energy circulation will be used to determine efficiency and net output of the systems.

We define the following quantities:

U_i = energy removed per event in reactor i, MeV;

η_i = the electrical conversion efficiency for the energy removed from reactor i;

W_n = net station output, MW;

W_p = power needed to heat the fusion plasma assuming perfect heating efficiency, MW;

W_t = power needed to extract the tritium, MW;

W_f = power needed to process the fissile fuel, MW;

W_c = power requirements for station control, MW.

It is seen from Fig. 5 that the net station output, W_n, is given by

$$W_n = \eta_1(R_1U_1 + W_p) + \eta_2R_2U_2 - (W_p + W_f + W_t + W_c). \qquad (3.4\text{-}1)$$

This equation can be rewritten as

$$W_n = \eta_2R_2U_2\{1 + \frac{R_1}{R_2}\left(\frac{\eta_1U_1}{\eta_2U_2}\right)(1 - \frac{1}{Q}) - \frac{W_t + W_f + W_c}{\eta_2R_2U_2}\} , \qquad (3.4\text{-}2)$$

where

$$Q = \eta_1 \frac{(R_1U_1 + W_p)}{W_p} , \qquad (3.4\text{-}3)$$

$$= \frac{\eta_1}{1 - \eta_1}\left(\frac{R_1U_1}{W_p}\right) . \qquad (3.4\text{-}4)$$

The parameter, Q, defined in Eq. (3.4-4) is the ratio of electrical fusion output power to the power needed to heat the plasma. A value of unity for Q would correspond to a breakeven plasma. The net station output power, as given by Eq. (3.4-2), is primarily the fission power modified by correction terms for the fusion output or consumption, and the power consumption of the other components.

The net thermal efficiency, η_{net}, of the station can be defined as

$$\eta_{net} = \frac{\text{(net station output)}}{\text{(total reactors output)}} \quad . \tag{3.4-5}$$

Substituting for W_n, defined by Eq. (3.4-2), to obtain

$$\eta_{net} = \eta_2 \frac{\{1 + \frac{R_1}{R_2}\left(\frac{\eta_1 U_1}{\eta_2 U_2}\right)\left(1 - \frac{1}{Q}\right) - \frac{W_t + W_f + W_c}{\eta_2 R_2 U_2}\}}{\left(1 + \frac{R_1 U_1}{R_2 U_2}\right)} \quad . \tag{3.4-6}$$

The net thermal efficiency is essentially the fission efficiency modified by terms accounting for the fusion reactor and power consuming units.

3.5 Cost Assessment

There are two costs associated with an electrical generating station: the capital required for construction of the station, and the operation costs.

The total capital cost of the system is taken to be the sum of the individual components plus the cost of construction and installation,

$$D = D_1 + D_2 + D_t + D_f + D_e \ , \tag{3.5-1}$$

where

D = total capital invested,

D_1 = capital cost of the fusion component,

D_2 = capital cost of the fission component,

D_t = capital cost of the tritium processing unit,

D_f = capital cost of the fissile processing unit,

D_e = capital cost of the generator facility.

Usually the cost of a generating station is given as a specific cost, that is cost per installed power output, \$/kW(net). If "d" is used to designate the total specific capital cost and

d_1 = specific cost per kWe of the fusion core output,

d_2 = specific capital cost per kWe of the fission core output,

and

d_t, d_f, d_e = specific capital cost of the individual components per kWe of the station output,

then

$$D = dW_n = d_1\eta_1(R_1U_1 + W_p) + d_2\eta_2R_2U_2 + (d_t + d_f + d_e)W_n, \quad (3.5\text{-}2)$$

or

$$d = d_1 \frac{\eta_1(R_1U_1 + W_p)}{W_n} + d_2 \frac{\eta_2R_2U_2}{W_n} + d_t + d_f + d_e . \quad (3.5\text{-}3)$$

Where the other parameters are defined as above. If Eq. (3.4-2) is substituted into Eq. (3.5-3) then

$$d = \frac{(d_1(\frac{R_1}{R_2})(\frac{\eta_1U_1}{\eta_2U_2})(1 + \frac{1}{(1 - \eta_1)Q}) + d_2)}{(1 + \frac{R_1}{R_2}(\frac{\eta_1U_1}{\eta_2U_2})(1 - \frac{1}{Q}) + \frac{W_t + W_f + W_c}{\eta_2R_2U_2})} + d_t + d_f + d_c. \quad (3.5\text{-}4)$$

The operating costs of a station include upkeep, maintenance, fuel costs, and interest on the capital required to construct the plant.

If "E" is used to designate the total operating costs and "e", the specific cost per unit energy, then

$$E = E_1 + E_2 + E_t + E_f + E_{cap} , \quad (3.5\text{-}5)$$

where the subscript "cap" refers to the capital charge and the other subscripts are as defined above. The values of E_1 and E_2 include the cost of unprocessed fuel for each reactor since our system includes processing plants.

On a specific cost basis,

$$E = eW_n = e_1\eta_1(R_1U_1 + W_p) + e_2\eta_2R_2U_2 + (e_t + e_f + e_{cap})W_n, \quad (3.5\text{-}6)$$

$$e = e_1 \frac{\eta_1(R_1U_1 + W_p)}{W_n} + e_2 \frac{\eta_2R_2U_2}{W_n} + e_t + e_f + e_{cap} . \quad (3.5\text{-}7)$$

Substituting Eq. (3.4-2) into Eq. (3.5-7),

$$e = \frac{(e_1(\frac{R_1}{R_2})(\frac{\eta_1U_1}{\eta_2U_2})(1 + \frac{1}{(1 - \eta_1)Q}) + e_2)}{(1 + \frac{R_1}{R_2}(\frac{\eta_1U_1}{\eta_2U_2})(1 - \frac{1}{Q}) + \frac{W_f + W_t + W_c}{\eta_2R_2U_2})} + e_t + e_f + e_{cap}, \quad (3.5\text{-}8)$$

is obtained. The value of e will be the unit energy cost for the system.

4. AN EXAMPLE

4.1 Systems Models

It is useful to evaluate the results of the systems theory for some hypothetical model. The modelling of the systems is limited by the fact that no fusion reactors are operating to date. To overcome this, a specific fusion core will not be described but only some values will be used for the parameters, Q, defined by Eq. (3.4-4); the thermal-electrical efficiency, η_1; the specific inventory, n_1, defined by Eq. (3.1-1); and upper and lower estimates of costs. The values used are listed in Table 1. The values of Q used are for three cases: (i) an energy consuming plasma, $Q < 1$; (ii) a breakeven plasma, $Q = 1$; and (iii) an energy producing plasma, $Q > 1$. The specific inventory values used are 2 g/MWt[1,3], 6 g/MWt[8], and 20 g/MWt for comparison. The thermal-electrical efficiency of 40% was used by Lidsky[3] and seems reasonable. The costs assumptions are discussed in Section 4.6.

Fission reactors have been in operation for some time now and more accurate values for the parameters involved are known. For this reason, a more specific fission reactor will be considered, a 1500 MWe CANDU reactor operating on a ThO_2 fuel cycle as proposed by Lewis et al[9]. This reactor has an organic coolant, heavy water moderator, 61 element fuel bundle with a linear thermal power of 3.5-4.5 MWt/m. The core contains 290 channels on a 280 mm square pitch and is 5 m long, assuming an overall average-maximum power ratio of 0.65 and a thermodynamic efficiency of 40%. The parameters used to evaluate the fission core are summarized in Table 2. There are two kinds of fuel elements, a driver fuel of ThO_2 enriched to 2-3% ^{233}U and a power fuel of ThO_2. After an irradiation of 2-4 n/kb, both driver and power fuel elements would have approximately the same enrichment of 75% of the initial driver fuel elements[6]. The costs of the fission component are discussed in Section 4.6.

4.2 Tritium Production in the Fission Core

In the mutualistic and commensalistic systems, tritium produced within the fission reactor is used in the fusion reactor. It is necessary to determine the tritium production within the fission reactor under consideration in order to determine the coupling coefficient, C_{21}.

One source of tritium in a heavy water moderated core is the $D(n,\gamma)T$ reaction. The rate of tritium production in the moderator, R_T^M, is given by

$$R_T^M = N_D^M \sigma \bar{\phi}_m V_m \ . \tag{4.2-1}$$

The thermal cross section, σ, for the $D(n,\gamma)T$ reaction is 0.5 mb and all other symbols have their usual meaning.

The proposed fission core has a 5.38 m diameter and a volume of 114 m^3. In order to carry out an analysis, a 500 MWe Pickering type core[11] is considered. The Pickering core is 5.94 m long with a diameter of 6.37 m and a volume of 189 m^3. The maximum thermal flux in the Pickering reactor is 0.91×10^{14} n/cm^2-sec and the average flux in the moderator, 0.8×10^{14} n/cm^2-sec. The thermal power of the Pickering reactor is 1744 MW.

The flux in the proposed core is taken to be proportional to the thermal power; hence, average flux in the moderator of the proposed reactor, Φ_m, will be 1.72×10^{14} n/cm^2-sec. In the Pickering core the moderator temperature is 60°C. The proposed core also has a cool heavy water moderator, with a density of 1.09 g/cm^3 or 7.28×10^{22} deuterium nuclei/cm^3. The ratio of moderator to core volume in the Pickering reactor (including the reflector) is 240/189. If this ratio is preserved in the proposed reactor, the moderator volume will be 145 m^3.

The tritium production in the proposed reactor moderator can now be evaluated using Eq. (4.2-1),

$$R_T^M = 9.07 \times 10^{17} \text{ tritons/sec} .$$

The coupling coefficient, C_{21}, for production within the moderator can also be evaluated. The thermal power of the proposed reactor is 3750 MW giving a fission rate of 1.17×10^{20} fissions/sec, assuming that 200 MeV is the recoverable energy per fission. This gives $C_{21} = 0.0077$. This is a low value so alternate tritium sources must be sought.

In a Pickering type reactor there are 18 adjuster rods used for flux shaping. These use cobalt as the absorber and the Co-60 produced is used for research. For cobalt, Σ_a is 3.327 cm^{-1} and for lithium, 3.266 cm^{-1} so the cobalt in the adjuster rods could be replaced by lithium without significantly altering the control properties. Lithium can be used to produce tritium by the $Li^6(n,\alpha)T$ reaction. To a first approximation, the volumes of the Pickering core and the proposed core are equal hence, the volumes of the adjuster rods are assumed to be equal, about 10^4 cm^3.

Using the above and the following lithium data:

$\rho_{Li} = 0.53$ g/cm^3, $\quad Li^6/Li^7 = 0.08$

$\sigma_a = 71$ b , $\quad \sigma_a(Li^6) = 945$ b (thermal)

$A_{Li} = 6.939$ $\quad \sigma_a(Li^7) = 0.033$ b (thermal),

the tritium production in the adjuster rods with lithium will be

$$R_T^A = (\rho V)_{Li} \frac{N_o}{A_{Li}} \sigma_a \bar{\phi} \ ,$$

$$= 5.61 \times 10^{18} \text{ tritons/sec} \ , \qquad (4.2\text{-}2)$$

for a flux of 1.72×10^{14} n/cm^2-sec. The total tritium production is now

$$R_T^M + R_T^A = 6.517 \times 10^{18} \text{ tritons/sec} \ .$$

Using this value, the coupling coefficient, C_{21}, is 0.0553. In the evaluation of the example three values of C_{21} will be used anticipating possible increases in the tritium production. The values used are: 0.0553, 0.1106, and 0.1659, that is they vary by a factor of 1, 2, and 3 from the estimated coupling coefficient with lithium adjuster rods.

4.3 Fusion Blanket

The purpose of the fusion blanket in a symbiotic system is threefold: (1) to remove the energy deposited by the fusion neutrons; (2) to convert fertile to fissile material; and (3) in two of the three systems considered, to produce tritium for the D-T fusion reaction. In this case the fertile nuclei are ^{233}Th and this is converted to ^{233}U for use in the driver rods of the fission reactor. Several neutronic calculations have been done for various fusion blankets and Table 3 contains some of the results. The tritium production is accomplished by $Li^6(n,\alpha)T$ and $Li^7(n,n'\alpha)T$ reactions. The average value of $C_{11} + C_{12}$ is 1.454. In the analysis of fuel dynamics the constraint that $C_{11} + C_{12} \leqslant 1.454$ was used. Fig. 7 shows a possible fusion blanket configuration.

4.4 Evaluation of the Fuel Dynamics

The evaluation of the fuel dynamics is based on the data in Tables 1 and 2. The steady state results are presented in Table 5. In evaluating the mutualistic results, Eq. (3.3-6), Eq. (3.3-7), Eq. (3.3-12) and Eq. (3.3-13), an additional equation is required. The constraint adopted was

$$C_{11} + C_{12} = 1.45 \ , \tag{4.4-1}$$

then Eq. (3.3-6) becomes

$$C_{11} = \frac{1 - C_{22} - 1.45\ C_{21}}{1 - C_{22} - C_{21}} \quad . \tag{4.4-2}$$

Solving for the condition

$$C_{11} = 0 \ , \tag{4.4-3}$$

requires a value for C_{21} of 0.138. At this point the system degenerates into a commensalistic system since the fission production of tritium is sufficient to supply the fusion reactor. Imposing Eq. (4.4-1) on the results for constant inventory ratio, the result is

$$C_{11} = \frac{(1 + \frac{n_1}{\tau})(1 - C_{22} + \frac{n_2}{\tau}) - 1.45\, C_{21}}{(1 - C_{22} + \frac{n_2}{\tau}) - C_{21}} \; . \qquad (4.4\text{-}4)$$

The results of the fuel dynamics are plotted in Fig. 8 to Fig. 15.

4.5 Evaluation of the Power Balance and Efficiency

The power balance and efficiency for the systems were evaluated using the data from Tables 1, 2 and 3. The results are plotted in Fig. 16 and Fig. 17. A value for Q of unity gives a breakeven plasma and the net power output is independent of the reaction rate ratios. Altering the power consumption of the components will raise or lower the intercept, $R_1/R_2 = 0$, and altering Q will alter the slope of the line. In this case the power consumption of the components was assumed to be 21% of the fission electrical output. The values for the two processing plants are arbitrary since the consumption by these components could easily involve steam from the reactors and recycling of the waste heat rather than electrical power.

4.6 Evaluation of Costs

In evaluating the costs of the system, upper and lower costs of the components were used. The costs of the fission reactor and the generator facilities were assumed to be well known and only one value of costs was used. The data used for evaluation are given in Tables 1, 2 and 3. The range of fusion reactor values lies within most of the estimates given in the literature[3,12,13,14]. The capital costs for the fissile production plant was evaluated using the equations developed by Salmon et al[14] using the data for LWR-Pu fuel at 113 kg per day. The value obtained was halved for a lower cost and doubled for an upper cost. The tritium processing plant costs were arbitrarily chosen to be 1 million dollars and 5 million dollars. The results are plotted in Fig. 18.

The unit energy costs for the system were also evaluated for upper and lower costs. The results are plotted in Fig. 19. The fuel costs of the two reactors have little effect especially since the costs are for "raw" material and not fabricated fuel. The operation and maintenance costs of the fusion and fission reactors influence the rate of increase of the costs as a function of R_1/R_2. The tritium processing plant costs are arbitrary and the fissile production plant costs were evaluated using the data from Reference 14 and again these were halved and doubled. The major component of the unit energy costs is the fixed capital charge. The values used were 5% of the lower capital cost, 15% of the upper capital cost and 10% of both the upper and lower capital costs.

5. DISCUSSION OF THE RESULTS

5.1 Fuel Cycle

Some general trends are shown in the plots of the fuel cycle results for the three systems. The system period, the fusion reactor specific inventory, and the assumed tritium production within the fission core all effect the coupling coefficients and the reaction rate ratios.

The system period is related to the fuel doubling time by Eq. (3.1-3). The rapid increase of the coupling coefficient C_{11} for the parasitic and mutualistic systems and of the coefficient C_{12} for the commensalistic system as the system period is shortened reflects this. Note, that since the constraint $C_{11} + C_{12} \leq 1.45$ is imposed, as the tritium production must be increased to satisfy the doubling time imposed on the system, the fertile conversion in the blanket must decrease. The reason that the fertile production in the commensalistic system increases with shorter system periods is that there is no tritium production in the blanket. As with the coupling coefficients, the fusion-fission reaction rate ratio increases as the doubling time or system period is shortened; the exception is in the commensalistic system where the opposite is true. As the system period is shortened, the rate of increase of the inventory is increased so more tritium must be produced in the blanket and less fissile material consumed. Increasing C_{11} will increase the tritium production but so will increasing the fusion reaction rate, thus increasing the fusion-fission reaction rate ratio. For the commensalistic system, tritium is produced within the fission core; hence, to increase the tritium production rate the fission reaction rate must be increased, for a fixed value of C_{21}, and this will decrease the reaction ratio.

The effect of the specific inventory of the fusion core, n_1, can easily be seen in the fuel cycle results. As the specific fusion core inventory is increased for a given system period, there is an increased demand for tritium. This increase must be satisfied by increasing C_{11} and R_1/R_2 for the parasitic and mutualistic systems, and decreasing R_1/R_2 for the commensalistic system which simultaneously reduces fusion consumption and increases the fission tritium production rate. Increasing the fission rate in the commensalistic system will increase the fissile consumption rate and require an increase in C_{12}.

The assumed tritium production in the fission core will effect only the commensalistic and mutualistic systems. For the commensalistic system, as the value of C_{21} is increased, a larger fusion reaction rate can be supported for the same for the same fission reaction rate. This will mean more neutrons are available for fertile conversion but, with the same demand, C_{12} may be decreased. For the mutualistic system, increasing the tritium supply from the fission reactor allows for less emphasis on tritium production in the blanket so C_{11} decreases and more fertile conversion is possible; hence, C_{12} increases. As the amount of fission produced tritium increases in a mutualistic system, the fusion-fission reaction rate ratio decreases since, by increasing C_{21},

C_{12} is increased; thus, for a given fusion reaction rate, more fertile conversion occurs allowing for a higher fission reaction rate.

It is interesting to note the characteristics of the curves for the fusion coupling coefficients for the mutualistic system as a function of system period. For each coefficient there are three asymptotes as $\tau \to \infty$ corresponding to the three values assumed for C_{21} and, also, there are three asymptotes as $\tau \to 0$ corresponding to the three assumed fusion specific inventories. In the equation,

$$C_{11} = \frac{(1 + \frac{n_1}{\tau})(1 - C_{22} + \frac{n_2}{\tau}) - 1.45\ C_{21}}{(1 - C_{22} + \frac{n_2}{\tau}) - C_{21}} , \tag{4.4-4}$$

as $\tau \to \infty$, the steady state results are approached,

$$C_{11} \to \frac{1 - C_{22} - 1.45\ C_{21}}{1 - C_{22} - C_{21}} \quad \text{as} \quad \tau \to \infty , \tag{5.1-1}$$

which clearly shows the effect of C_{21} on the asymptote. As $\tau \to 0$, since the terms all involve $1/\tau$ and these tend to infinity as τ tends to zero, and thus Eq. (4.4-4) tends to

$$C_{11} \to \frac{n_1}{\tau} \quad \text{as} \quad \tau \to 0 . \tag{5.1-2}$$

The effect of increasing n_1 will be to shift the asymptote to the right as seen in Fig. 12.

Figures 14 and 15 show a comparison of the three systems. For the fusion coupling coefficients, the commensalistic system requires such a large value of C_{12} that it was more than twice the limit of 1.45 for the value of C_{21} assumed. There is a 12% reduction of C_{11} of the mutualistic system over the parasitic system for a system period of 20 years. The reaction rate ratios are plotted in Fig. 15. The effect of introducing the fission produced tritium is clearly seen. This would mean a smaller fusion reactor for a given fission reactor is required. The very low value for R_1/R_2 for the commensalistic system is due to the very small amount of tritium which is produced in a fission reactor relative to the fusion consumption. This makes this system unviable except for very long system periods and large values of C_{21}.

5.2 Power Balance and Efficiency

The net station output of the symbiotic system is a linear function of the reaction rate ratios. For a breakeven fusion component, the fusion reactor can just run itself, thus it neither adds to the net output nor consumes the fission component output. The intersection at $R_1/R_2 = 0$ is determined by the fission core output and the fuel production facilities consumption. The slope of the lines for a fusion power consumer or producer is determined by the value of Q; thus, if a fusion core originally constructed with $Q \leqslant 1$ was improved during its lifetime to be a power producer, the net station output may increase dramatically. Note that for a fusion power consumer, if the size of the fusion component is increased, there is a point where it will consume all of the fission component output. Such a station would be a zero power system possibly useful for research.

The net station efficiency is also a linear function of R_1/R_2. For a breakeven fusion core, $Q = 1$, the station efficiency drops as the fusion reaction rate increases with a constant value of R_2 since, although the fusion core does not consume power, its output is included in the station efficiency, Eq. (3.4-5). For a power consuming fusion core, the negative slope of the line is even greater. In the example it turns out that the net efficiency is nearly independent of the reaction rate ratios for Q = 4. The intercept at $R_1/R_2 = 0$ is determined by the thermodynamic efficiencies of the power units and the consumption of the fuel processing units.

5.3 Costs

The actual values of the costs in the example may not be meaningful but the trends are. The specific capital cost of a symbiotic system with a breakeven fusion core is a linear function of R_1/R_2, the slope being determined by the specific cost of the fusion component. For a power consuming fusion core, the cost increases sharply as the fusion component size is increased since the capital outplay increases and the net station output falls. For a power producing core, the capital cost can decrease as the fusion core size is increased. This occurs when the specific capital cost of the fusion core is less than that of the fission core. In this situation a pure fusion system may be more economically attractive.

The operating costs of the symbiotic system, and in fact for pure fusion or fission, are strongly dependent on the capital charge or interest. For the symbiotic system where the fuelling costs are extremely low, the capital charge makes up almost all of the unit energy cost except for the operation and maintenance charge. As with the capital costs, the operating cost of the symbiotic system with a power consuming fusion component is strongly dependent on the fusion core size. With a power producing fusion component, the effect is not as great and the specific operating cost may actually decrease since the capital costs decrease and the capital charge represents the major component of operating cost. The slopes of the lines are determined by the fusion specific capital cost and the intercept, by the fission and fuel processing units capital costs.

5.4 Steady State Results

Some useful comparisons of the three systems can be made by studying the steady state results. A steady state symbiotic system would be one which does not increase its inventory. Note that the commensalistic system does not satisfy the constraint, $C_{11} + C_{12} \leq 1.45$, unless the value of C_{21} is large.

Comparing the reaction rate ratios, there is a reduction for the viable commensalistic and all three mutualistic systems over the value for the parasitic system. A mutualistic system with the estimated tritium production of 0.0553 per fission has a fusion-fission reaction rate ratio which is 28.4% lower than the parasitic system. By increasing the tritium production in the fission component, even lower values can be achieved. There is also an increase in net power output for the mutualistic and commensalistic systems over that of the parasitic system. For the same mutualistic system as before, and a Q of 0.25, there is a 6.49% increase in power output over the parasitic system. Note that for the mutualistic system with a power producing fusion reactor, as the tritium production in the fission component is increased, the net output of the system decreases. There is an increase in the net station efficiency of the commensalistic and mutualistic systems over the parasitic system in all cases.

The capital costs for the commensalistic system and the mutualistic systems are less; 12.3% less for the above mutualistic system compared to the parasitic system. This reduction of capital cost is reflected in the unit energy costs.

6. CONCLUSIONS

From the discussion of the previous section, some conclusions about the symbiotic systems can be made. If the tritium production in the fission core is included in the fuel cycle of a symbiotic system, there is improvement in the power output and efficiency as well as a reduction in both the capital costs and the unit energy costs of the system. A symbiotic system, in which the fusion core is operated solely on fission produced tritium, is not viable neutronically in the fusion blanket unless the tritium output of the fission reactor is very high. The best system appears to be the mutualistic system.

There are disadvantages to the symbiotic system compared to the pure fusion or pure fission systems. There is the added ecological and biological danger of increased tritium handling since tritium is extracted both from the fission and fusion cores. For a power consuming symbiotic fusion reactor, better power output and unit energy costs could be achieved by pure fission. The increase in handling of irradiated fissile material is a danger especially when compared to a pure fusion system where the only activated materials would be from the blanket components which must be replaced and the tritium fuel.

There are advantages in introducing symbiotic systems. The introduction of fusion reactors can be advanced by heating of the fusion plasma with the energy produced in the fission reactor. If the resources for fission power are considered, the breeding of fissile material in the fusion blanket becomes important. Not only can fertile material be converted for consumption in a fission reactor, but this can be accomplished without endangering the safety requirements of the fission core requiring special reactor designs. Thus, the fission core can be designed for optimal power output and for usage of different fuel cycles.

Two points have been introduced in studying the systems. The first is a symbiotic research system which would be a non-power consuming system to study the operation of fusion. The other point is the incorporation of fuel processing plants on site thus eliminating the dangers and costs of transportation and reducing costs of fabrication by the availability of cheap power and steam from the reactors on site. For large nuclear installations, this cooperation of utilities and industry might prove economically interesting. It is interesting to note that in the Bruce Generating and Gentilly Generating Stations, heavy water production plants are being built on site.

Perhaps the greatest use of a symbiotic system would be in the transition stages, between the technological demonstration of fusion power and its economic improvement over fission reactors. At this time there will probably be a large economy based on fission reactors. Utilities with fission reactors possibly less than half way through their estimated lifetime would be reluctant to invest in fusion to the extent of scrapping their fission reactors. The conversion of some fission reactor sites to symbiotic systems might prove valuable by lowering fuel costs for the other fission reactors. An added advantage would be in the training of personnel in fusion core operations and plant administrations when the future conversion to fusion power is started.

Thus, symbiotic systems appear to be a useful method of power production in the future especially in the transition from a fission power economy to a fusion power economy. Further, the utilization of the tritium produced in the fission partner of such systems will improve the performance of these systems and possibly lower the costs.

APPENDIX A

FUEL DYNAMICS IN MATRIX FORM

The fuel dynamics equations for the symbiotic systems can be written in matrix form,

$$\dot{\underline{N}} = \underline{\underline{C}}\underline{R} , \tag{A-1}$$

where,

$$\dot{\underline{N}} = \begin{pmatrix} \frac{dN_1}{dt} \\ \frac{dN_2}{dt} \end{pmatrix} , \tag{A-2}$$

$$\underline{\underline{C}} = \begin{pmatrix} C_{11} - 1 & C_{21} \\ C_{12} & C_{22} - 1 \end{pmatrix} , \tag{A-3}$$

and

$$\underline{R} = \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} . \tag{A-4}$$

This form of the equations holds for all three systems; $C_{21} = 0$ for the parasitic system and $C_{11} = 0$ for the commensalistic system.

These fuel dynamics equations are solved for two conditions:

(i) steady state,

$$\dot{\underline{N}} = \underline{0} , \tag{A-5}$$

(ii) constant fuel inventory ratio,

$$N_1/N_2 = \text{a constant} , \tag{A-6}$$

or

$$\dot{\underline{N}} = (1/\tau)\underline{N} \ . \tag{A-7}$$

For a steady state,

$$\underline{\underline{C}}\underline{R} = \underline{0} \ , \tag{A-8}$$

or

$$\begin{pmatrix} C_{11} - 1 & C_{21} \\ C_{12} & C_{22} - 1 \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \ . \tag{A-9}$$

For a non-trivial solution,

$$\det \underline{\underline{C}} = 0 \ , \tag{A-10}$$

or

$$(C_{11} - 1)(C_{22} - 1) - C_{21}C_{12} = 0 \ . \tag{A-11}$$

The conditions which must be satisfied are the following: for the parasitic system, where $C_{21} = 0$ and $C_{22} < 1$, then

$$C_{11} = 1 \ ; \tag{A-12}$$

for the commensalistic system, where $C_{11} = 0$, then

$$C_{12} = (1 - C_{22})/C_{21} \ ; \tag{A-13}$$

and for the mutualistic system,

$$C_{11} = 1 - \frac{C_{12}C_{21}}{1 - C_{22}} \ . \tag{A-14}$$

To obtain information about the steady state reaction rate ratios, Eq. (A-9) can be rewritten as

$$\begin{pmatrix} C_{11} - 1 & 1 \\ C_{12} & 1 \end{pmatrix} \begin{pmatrix} R_1/R_2 \\ 0 \end{pmatrix} = - \begin{pmatrix} C_{21} \\ C_{22} - 1 \end{pmatrix} . \tag{A-15}$$

This can be solved using Cramer's Rule,

$$\frac{R_1}{R_2} = \frac{(C_{22} - 1) - C_{21}}{(C_{11} - 1) - C_{12}} . \tag{A-16}$$

From Eq. (A-11),

$$C_{21} = \frac{(C_{11} - 1)}{C_{21}} (C_{22} - 1) , \tag{A-17}$$

is obtained. This is substituted into Eq. (A-16) to obtain,

$$\frac{R_1}{R_2} = \frac{(1 - C_{22})}{C_{12}} . \tag{A-18}$$

Eq. (A-18) holds in this form for both the parasitic and mutualistic systems but can be simplified for the commensalistic system by using Eq. (A-13); hence,

$$\frac{R_1}{R_2} = C_{21} . \tag{A-19}$$

Solving for constant fuel inventory ratio, the fuel dynamics equations can be written in matrix form as

$$\frac{1}{\tau} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} C_{11} - 1 & C_{21} \\ C_{12} & C_{22} - 1 \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} . \tag{A-20}$$

From the definition of specific inventory, this can be written as

$$\begin{pmatrix} \frac{n_1}{\tau} & 0 \\ 0 & \frac{n_2}{\tau} \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} C_{11} - 1 & C_{21} \\ C_{12} & C_{22} - 1 \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} , \tag{A-21}$$

or

$$\begin{pmatrix} C_{11} - 1 - \frac{n_1}{\tau} & C_{21} \\ C_{12} & C_{22} - 1 - \frac{n_2}{\tau} \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} , \qquad \text{(A-22)}$$

or

$$\underline{\underline{C}}'\underline{R} = \underline{0} . \qquad \text{(A-23)}$$

For a non-trivial solution,

$$\det \underline{\underline{C}}' = 0 , \qquad \text{(A-24)}$$

or

$$(C_{11} - 1 - \frac{n_1}{\tau})(C_{22} - 1 - \frac{n_2}{\tau}) - C_{12}C_{21} = 0 . \qquad \text{(A-25)}$$

For the condition of constant fuel inventory ratio, the conditions which must be satisfied are: for the parasitic system, where $C_{21} = 0$ and $C_{22} < 1$,

$$C_{11} = 1 + \frac{n_1}{\tau} \; ; \qquad \text{(A-26)}$$

for the commensalistic system, where $C_{11} = 0$,

$$C_{12} = \frac{(1 + \frac{n_1}{\tau})}{C_{21}} (1 + \frac{n_2}{\tau} - C_{22}) \; ; \qquad \text{(A-27)}$$

and for the mutualistic system,

$$C_{11} = 1 + \frac{n_1}{\tau} - \frac{C_{12}C_{21}}{1 + \frac{n_2}{\tau} - C_{22}} . \qquad \text{(A-28)}$$

To obtain information about the reaction rate ratios for constant fuel inventory ratio, Eq. (A-22) can be rewritten as

$$\begin{pmatrix} C_{11} - 1 - \frac{n_1}{\tau} & 1 \\ C_{12} & 1 \end{pmatrix} \begin{pmatrix} R_1/R_2 \\ 0 \end{pmatrix} = - \begin{pmatrix} C_{21} \\ C_{22} - 1 - \frac{n_2}{\tau} \end{pmatrix} , \tag{A-29}$$

which can be solved using Cramer's Rule to obtain

$$\frac{R_1}{R_2} = \frac{-C_{21} + (C_{22} - 1 - \frac{n_2}{\tau})}{(C_{11} - 1 - \frac{n_1}{\tau}) - C_{12}} . \tag{A-30}$$

From Eq. (A-25),

$$C_{21} = \frac{(C_{11} - 1 - \frac{n_1}{\tau})}{C_{12}} (C_{22} - 1 - \frac{n_2}{\tau}) , \tag{A-31}$$

can be substituted into Eq. (A-30) to obtain

$$\frac{R_1}{R_2} = \frac{1 + \frac{n_2}{\tau} - C_{22}}{C_{12}} . \tag{A-32}$$

This form for the reaction rate ratios holds for the parasitic and mutualistic systems but, using Eq. (A-27), for the commensalistic systems this can be simplified to

$$\frac{R_1}{R_2} = \frac{C_{21}}{1 + \frac{n_1}{\tau}} . \tag{A-33}$$

The matrix form of the equations yields the same results as manipulating the balance equations. The matrix form, Eq. (A-1), may be more useful if the systems are to be solved for conditions other than steady state or constant fuel inventory ratio such as optimal power output, optimum tritium production or optium fissile production. Matrix forms are also more suitable for systems involving more than two reactors such as one fusion core coupled with two or more fission reactors.

APPENDIX B

CALCULATION OF SPECIFIC INVENTORY

In evaluating the fuel cycles of the symbiotic systems with constant fuel inventory ratio, the parameters n_1, the fusion core specific inventory, and n_2, the fission core specific inventory , are important. Two units for this parameter are used in the evaluation: g/MWt and years. It is not obvious that these units are compatible.

The energy recovered per fusion is assumed to be 22.4 MeV[3] where this includes the energy of fusion neutrons and the energy from reactions occuring within the fusion blanket. To convert n_1 (g/MWt) to n_1 (years), the following conversion is used:

$$n_1 \left(\frac{g}{MWt}\right) 10^{-6} \left(\frac{MW\text{-}sec}{J}\right) 1.602 \times 10^{-13} \left(\frac{J}{MeV}\right) 22.4 \left(\frac{MeV}{fusion}\right)$$

$$\times\ 1 \left(\frac{fusion}{triton}\right) \frac{2 \times 6.02 \times 10^{23}}{6} \left(\frac{tritons}{g}\right)$$

$$\times\ \frac{1}{3.156 \times 10^{7}} \left(\frac{years}{sec}\right) = n_1 \text{ (years)} \ , \qquad \text{(B-1)}$$

or

$$n_1 \text{ (years)} = 2.28 \times 10^{-2} n_1 \text{ (g/MWt)} \ . \qquad \text{(B-2)}$$

Similarly, the recoverable energy per fission is assumed to be 200 MeV. Thus, using a conversion scheme of the same form as Eq. (B-1),

$$n_2 \text{ (years)} = 2.605 \times 10^{-3} n_2 \text{(g/MWt)} \ . \qquad \text{(B-3)}$$

To evaluate the fission core specific inventory, it is necessary to evaluate the linear density of ^{233}U. The fuel pellet assembly used in the proposed fission reactor is shown in Fig. 6. The pellet is 1.29 cm in diameter, 0.645 cm long, and has a volume of 0.81 cm^3. The volume of the graphite disk is 0.0845 cm^3. The length of the pellet and graphite disk together is 0.71 cm; thus, in a 5 m channel there will be 705 pellets. If the ThO_2 density is 9.7 g/cm^3, there is 11.1 gm of ThO_2 per cm of fuel element. The enrichment after irradiation is assumed to be 1.5% ^{233}U hence there will be 0.166 g of ^{233}U per cm of fuel element. With 61 elements in each bundle, there will be 10.0 g of ^{233}U per cm of channel. The linear thermal power is between 3.5 and 4.5 MW/m, thus for a value of 4.2 MW/m the specific inventory of ^{233} is (linear density of ^{233}U)/(linear thermal power) or 238 g/MWt.

APPENDIX C

TRITIUM PRODUCTION IN THE FISSION CORE

W. Köhler and J. Voss[4] report tritium production rates in heavy water reactors up to 6000 Ci/MWt.a. To compare the values used in the example to this it is necessary to convert the coupling coefficients from tritons/fission to Ci/MWt.a. This requires a conversion factor,

$$R_T\left(\frac{\text{Ci}}{\text{MWt.a}}\right) = C_{21}\left(\frac{\text{tritons}}{\text{fission}}\right) \frac{1}{3.7 \times 10^{10}} \left(\frac{\text{Ci-sec}}{\text{triton}}\right) \frac{\ln 2}{t_1/2} \left(\frac{1}{\text{years}}\right)$$

$$\times \frac{1}{200} \left(\frac{\text{fissions}}{\text{MeV}}\right) 6.24 \times 10^{12} \left(\frac{\text{MeV}}{\text{J}}\right) 10^6 \left(\frac{\text{J}}{\text{MW-sec}}\right) , \qquad \text{(C-1)}$$

or

$$R_T\left(\frac{\text{Ci}}{\text{MWT.a}}\right) = 4.75 \times 10^4 \, C_{21} \left(\frac{\text{tritons}}{\text{fission}}\right) . \qquad \text{(C-2)}$$

There are four values for C_{21} given in the report; the value for tritium produced in the moderator, 0.0077; the value for the tritium produced in the moderator and in lithium adjuster rods, 0.0553; and the last value doubled and tripled, 0.1106 and 0.1659. These values correspond to 366, 2620, 5240, and 7860 Ci/MWt.a, respectively. These values are comparable to those reported by Köhler and Voss.

TABLE 1

FUSION CORE AND FUSION BLANKET DATA

FUSION CORE	
SPECIFIC INVENTORY	
n_1	2, 6, 20 g/MW(th)
	0.0456, 0.137, 0.456 years
THERMODYNAMIC EFFICIENCY	
η_1	40%
Q	0.25, 1.0, 4.0
CAPITAL COST	
d_1	50, 200 $/kW(e)
OPERATING COSTS	
Operation and Maintenance	0.38, 1.52 m$/kWh
* Fuel	0.0001835, 0.000367 m$/kWh
Total, e_1	0.38, 1.52 m$/kWh
FUEL COST	
Deuterium	20, 40 m$/g
Lithium	2, 40 m$/g
FUSION BLANKET	
C_{11}	
C_{12}	$C_{11} + C_{12} \leq 1.45$

TABLE 2

FISSION REACTOR DATA (PROPOSED 1500 MWe)

CORE	
Bundle Diameter	129.9 mm
Calandria Tube Diameter	160.62 mm
Lattice Spacing	280 mm
Number of Channels	290
Fuel Elements/Bundle	61
Average-Maximum Power Ratio	0.65
$\int_{\text{Surface}}^{\text{Maximum}} \lambda d\theta$	45 W/cm
Linear Thermal Power	3.5-4.5 MW/m
Burnup	35 MW.d/kgTh
Specific Inventory	238 g/MW(th) 0.62 years
Thermal Power	3750 MW
Thermodynamic Efficiency	40%
CAPITAL COSTS	
d_2	225 $/kWe
OPERATING COSTS	
Capacity Factor	80%
Operation and Maintenance	0.38 m$/kWh
Heavy Water Upkeep	0.13 m$/kWh
Fuel Cost (Raw)	0.0356, 0.0712 m$/kWh
Cost of ThO_2	10, 20 $/kg
C_{21}	0.0553, 0.1106, 0.1659
C_{22}	0.80

TABLE 3

PROCESSING PLANTS DATA

POWER CONSUMPTION	
Tritium, $W_t/\eta_2 R_2 U_2$	0.1
Fissile, $W_f/\eta_2 R_2 U_2$	0.05
Control, $W_c/\eta_2 R_2 U_2$	0.06
CAPITAL COSTS	
Tritium, D_t	1, 5 M$
Fissile, D_f	4.27, 17.06 M$
Generators, d_e	60 $/kWe
OPERATING COSTS	
Tritium, e_t	0.13, 0.52 m$/kWh
Fissile, E_f	1.99, 7.96 M$/year

TABLE 4

SUMMARY OF FUSION BLANKET NEUTRONIC CALCULATIONS

SOURCE		C_{11}	C_{12}	$C_{11} + C_{12}$
LIDSKY[3]		1.126	0.325	1.451
		1.005	0.40	1.405
* LA VERGNE ET AL[6]	--	1.454	0	1.454
Thorium added to Li (% vol)	0.5	1.436	0.0174	1.4534
	1	1.399	0.0345	1.4335
	2	1.385	0.0677	1.4527
	4	1.323	0.131	1.454
	6	1.263	0.191	1.454
	8	1.198	0.247	1.445
	10	1.154	0.301	1.455
Thin layers of Th: at front surface		1.41	0.11	1.52
		1.28	0.22	1.50
		1.16	0.33	1.49
At 230 cm		1.37	0.09	1.46
		1.30	0.17	1.47
		1.24	0.24	1.48
With 2% U^{233}		1.24	0.19	1.43
At Graphite surface		1.35	0.13	1.48
		1.30	0.19	1.49
		1.28	0.22	1.50
With % U^{233}		1.27	0.19	1.46
Thin layers of ThO_2 at front surface		1.35	0.08	1.43
		1.17	0.17	1.34
		1.02	0.27	1.29
At 230 cm		1.37	0.07	1.44
		1.30	0.14	1.44
		1.24	0.21	1.45
At Graphite surface		1.36	0.11	1.47
		1.31	0.17	1.48
		1.28	0.22	1.50
Thin layers of ThC at Graphite surface		1.35	0.12	1.47
		1.30	0.18	1.48
		1.28	0.22	1.50
Average		1.281	0.174	1.455

* Results for blanket shown in Fig. 7.

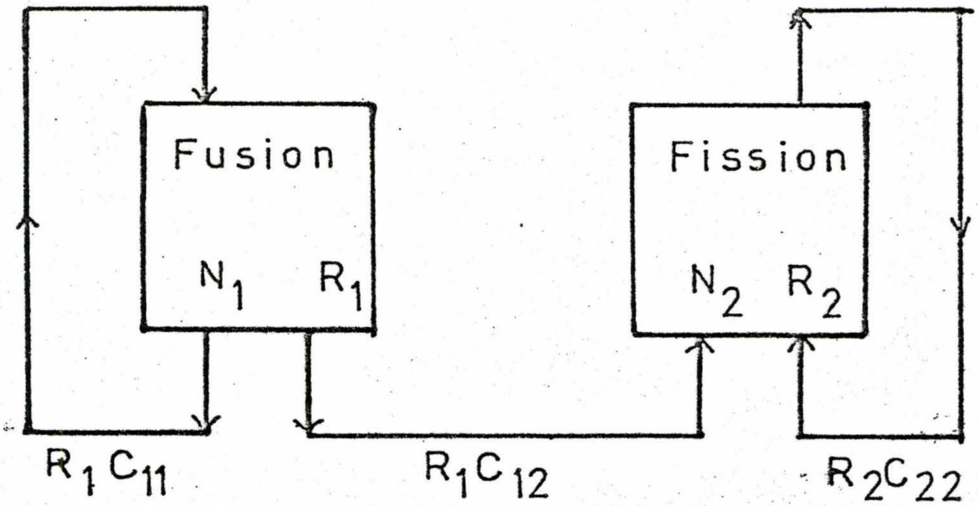
TABLE 5

STEADY STATE SYSTEMS RESULTS

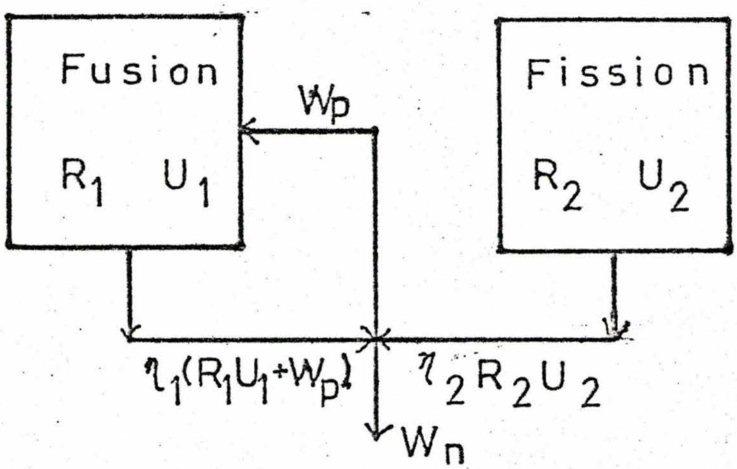
	C_{11}	C_{12}	C_{21}	C_{22}	$\frac{R_1}{R_2}$	W_n (MW)			η_{net}			d (\$/kWe) *			e (m\$/kWh) †		
						Q 0.25	Q 1.0	Q 4.0	Q 0.25	Q 1.0	Q 4.0	Q 0.25	Q 1.0	Q 4.0	Q 0.25	Q 1.0	Q 4.0
Parasitic	1.0	0.45	--	0.80	0.445	958	1185	1218	0.24	0.30	0.318	542	395	365	11.25	7.50	7.30
Commensal-istic	--	3.62	0.0553	0.80	0.0553	1175	1185	1188	0.316	0.3175	0.318	365	365	363.5	7.30	7.20	7.20
	--	1.82	0.1106	0.80	0.1106	1125	1185	1200	0.295	0.311	0.318	397	370	364	8.05	7.25	7.25
	--	1.205	0.1659	0.80	0.1659	1095	1185	1208	0.283	0.309	0.318	420	375	364	8.30	7.28	7.25
Mutual-istic	0.824	0.626	0.0553	0.80	0.319	1020	1185	1228	0.261	0.304	0.318	475	387	364	9.85	7.40	7.28
	0.445	1.005	0.1106	0.80	0.20	1085	1185	1220	0.28	0.308	0.318	433	377	364	8.80	7.30	7.25
	0.0	1.45	0.1659	0.80	0.138	1112	1185	1203	0.291	0.31	0.318	408	370	364	8.25	7.27	7.25

* upper cost d+.

† upper cost with a fixed capital charge, 10%.



(a) Fuel Circulation



(b) Energy Circulation

FIG.1 LIDSKY'S SYMBIOTIC POWER PLANT

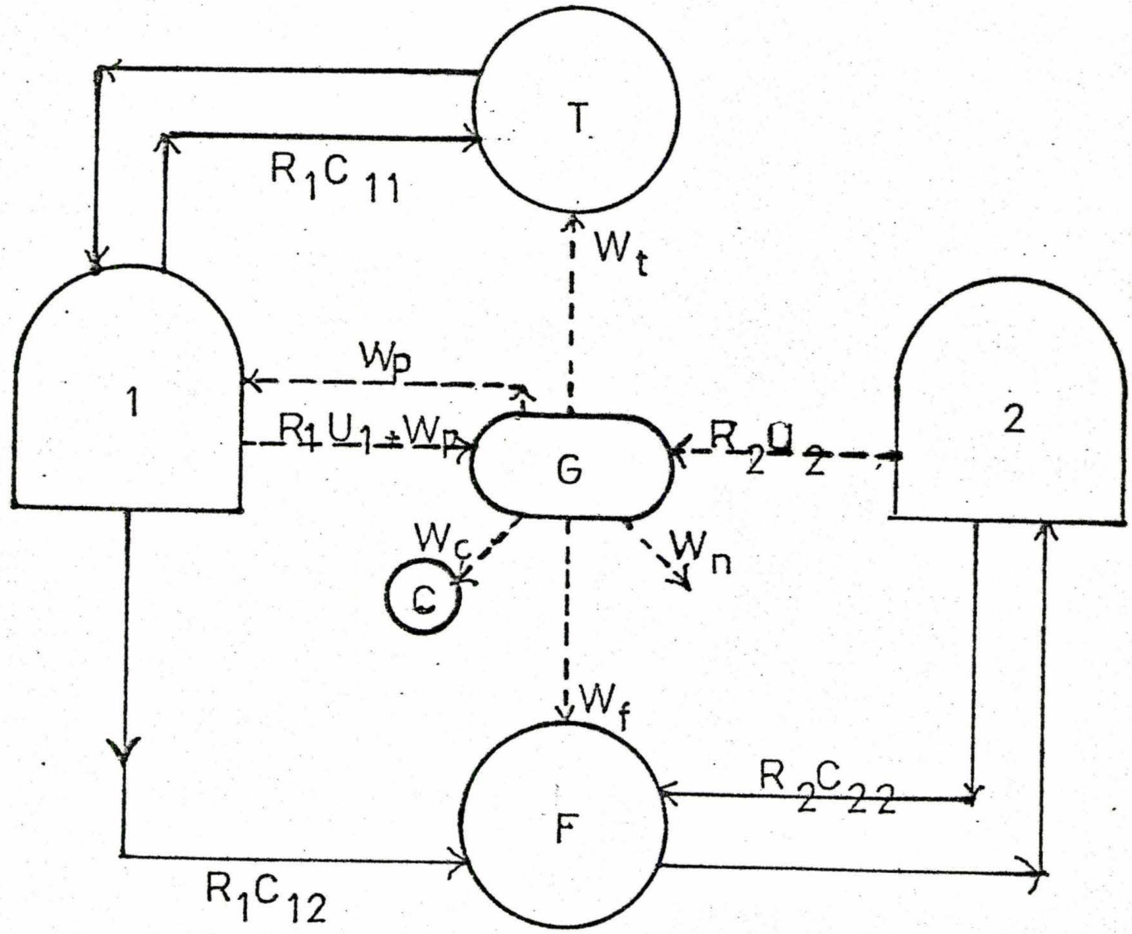


FIG. 2 PARASITIC SYSTEM

1 Fusion Reactor

2 Fission Reactor

T Tritium Processing

F Fissile Processing

G Generators

C Station Control

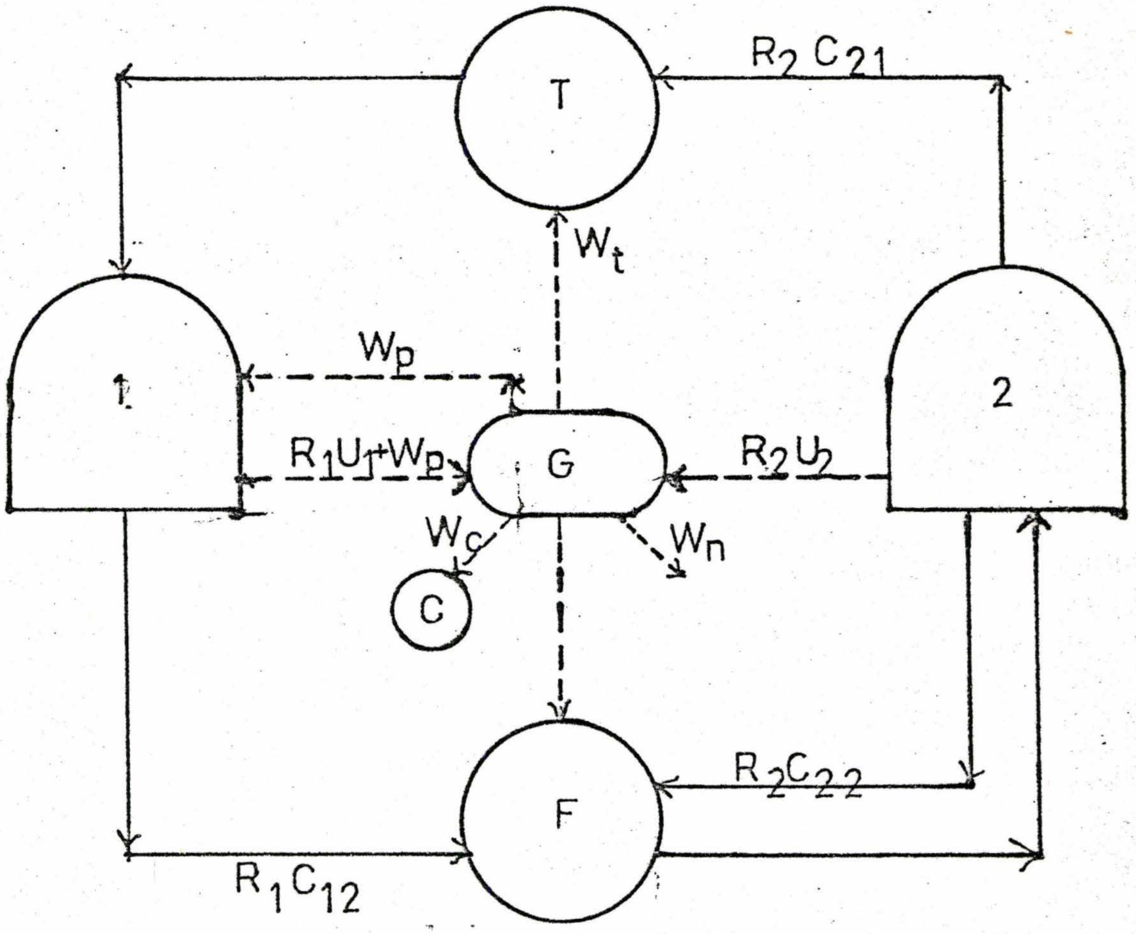


FIG.3 COMMENSALISTIC SYSTEM

1 Fusion Reactor

2 Fission Reactor

T Tritium Processing

F Fissile Processing

G Generators

C Station Control

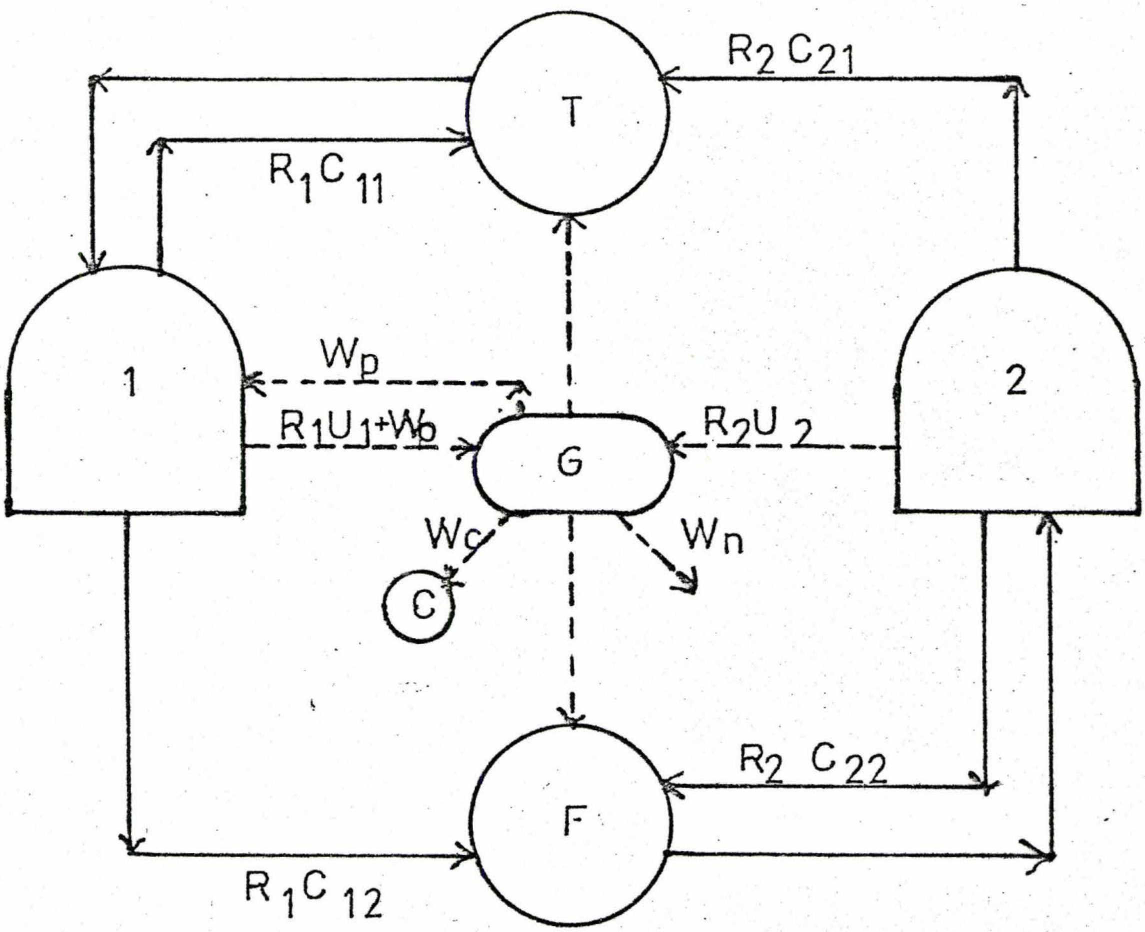


FIG. 4 MUTUALISTIC SYSTEM

1 Fusion Reactor

2 Fission Reactor

T Tritium Processing

F Fissile Processing

G Generators

C Station Control

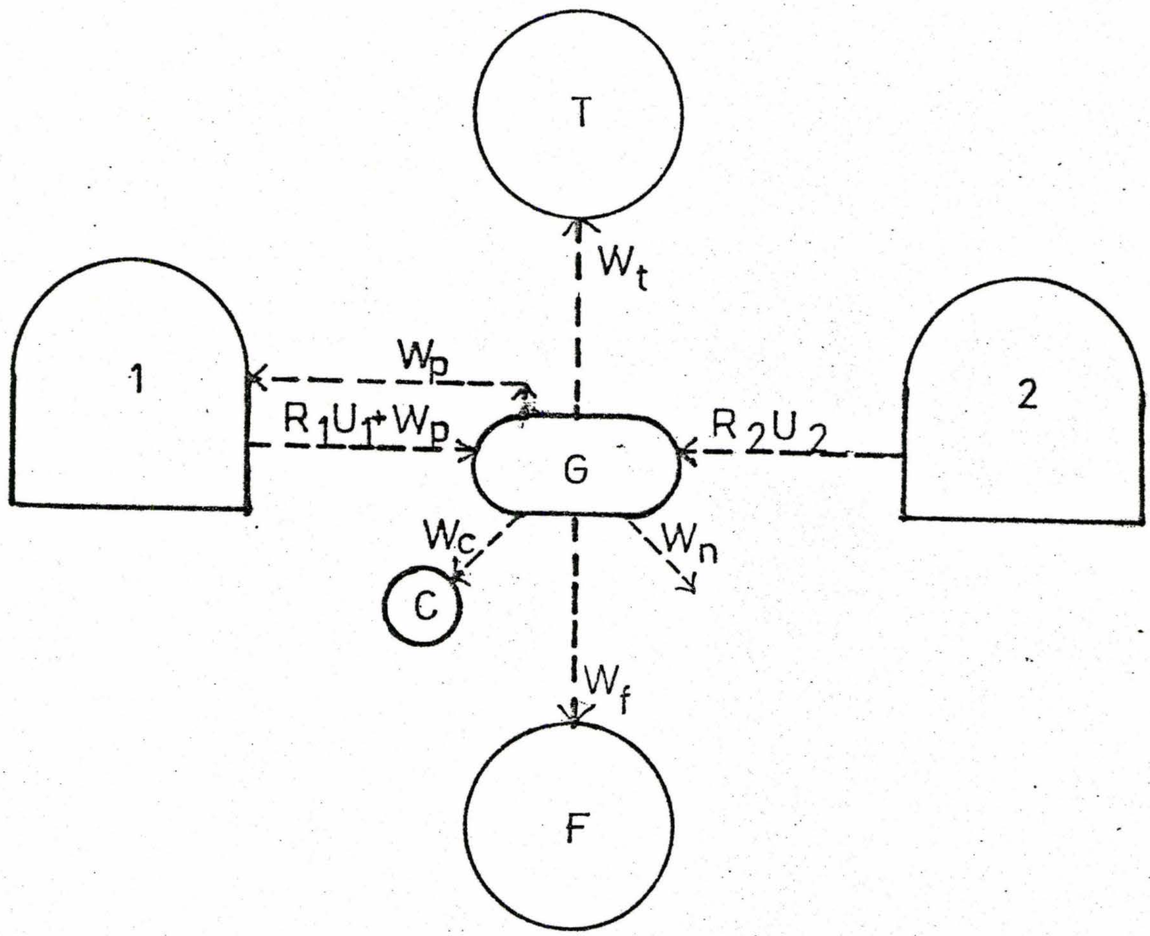


FIG. 5 ENERGY CIRCULATION

1 Fusion Reactor

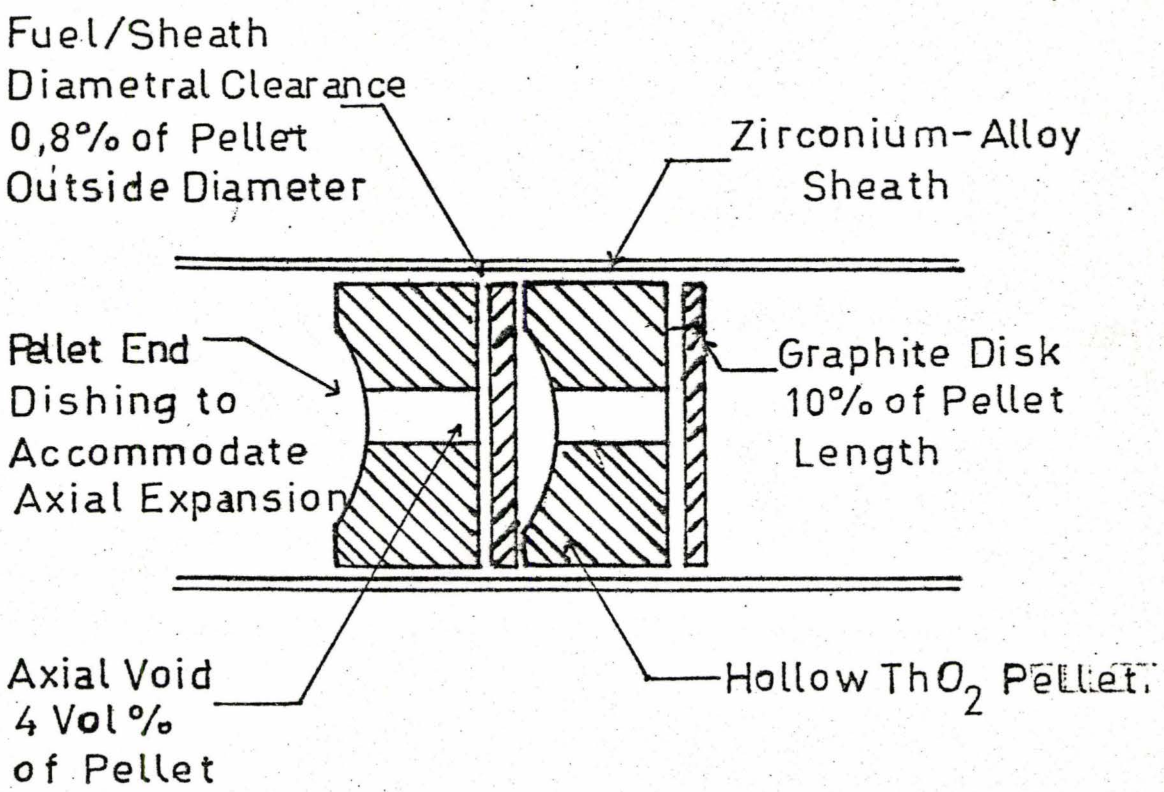
2 Fission Reactor

T Tritium Processing

F Fissile Processing

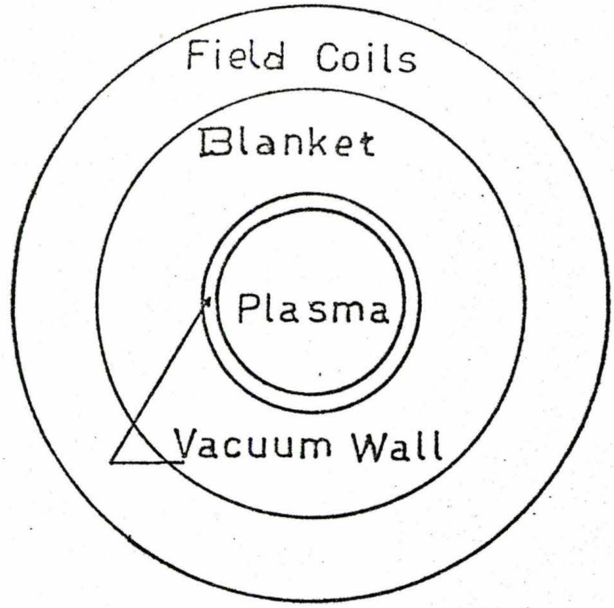
G Generators

C Station Control

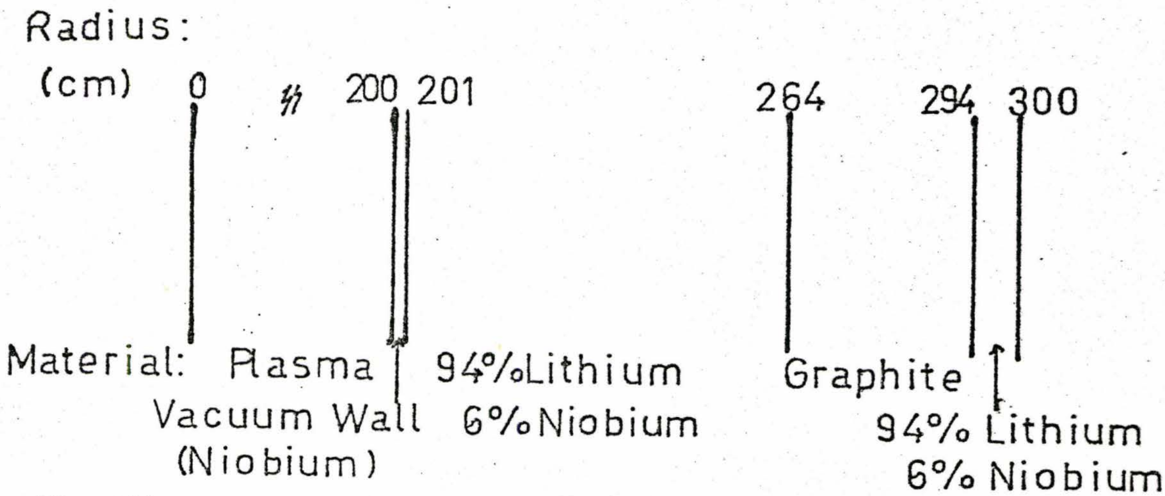


Fuel Sheath Diameter - 13.88 mm
Fuel Sheath Thickness - 0.38 mm
Pellet l/d - 1/2
Oxide density - 9.7 g/cm^3

FIG.6 SINTERED ThO_2 FUEL PELLET



(a) Cross Section of Cylindrical Fusion Reactor[15]



Fertile Material at Various Positions in Blanket

(b) Structure of Fusion Blanket

FIG.7 BLANKET CONFIGURATIONS

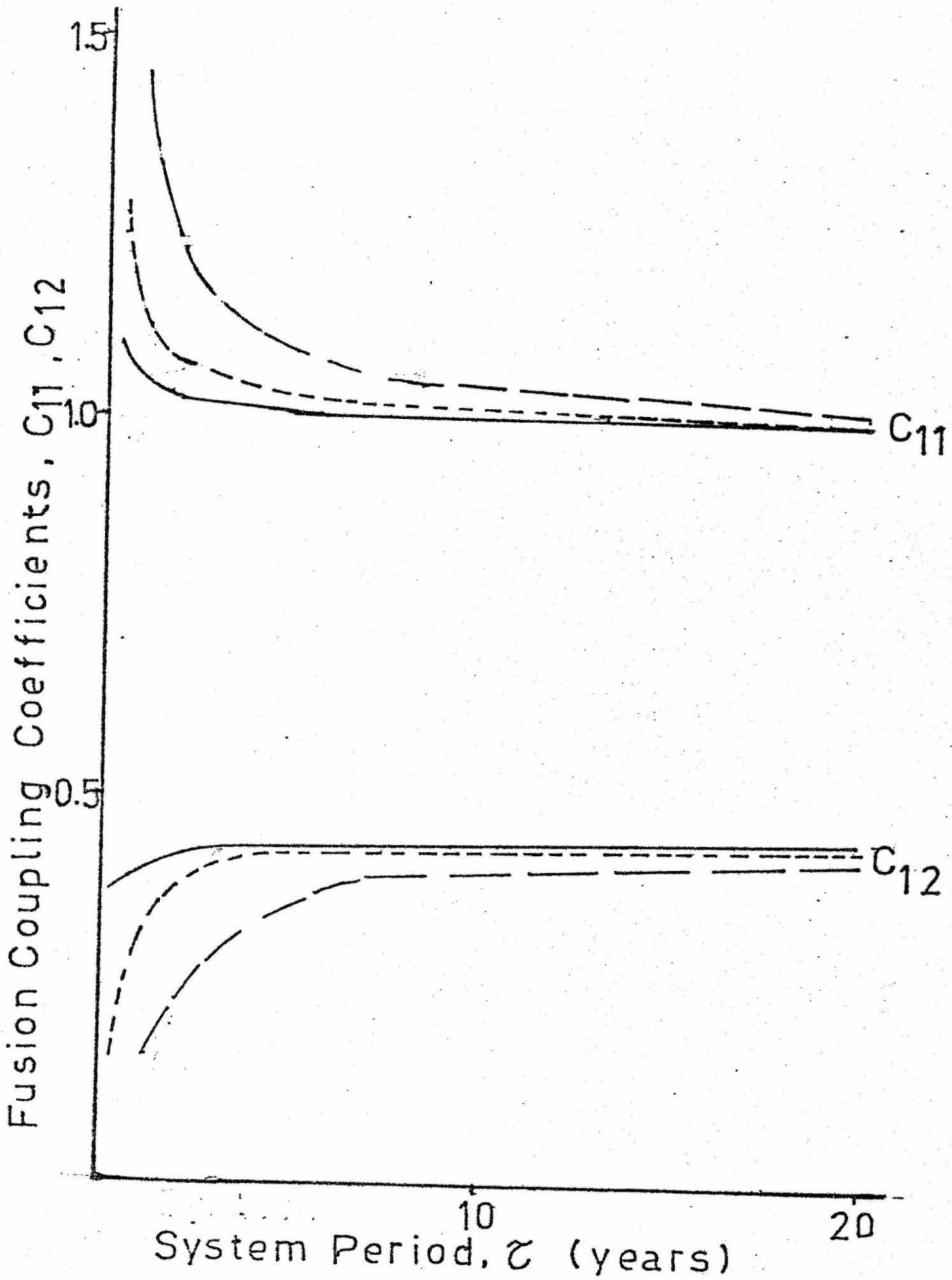


FIG.8 FUSION COUPLING COEFFICIENTS (PARASITIC)
(——) $n_1 = 2g/MWt$, (----) $n_1 = 6g/MWt$,
(— —) $n_1 = 20g/MWt$

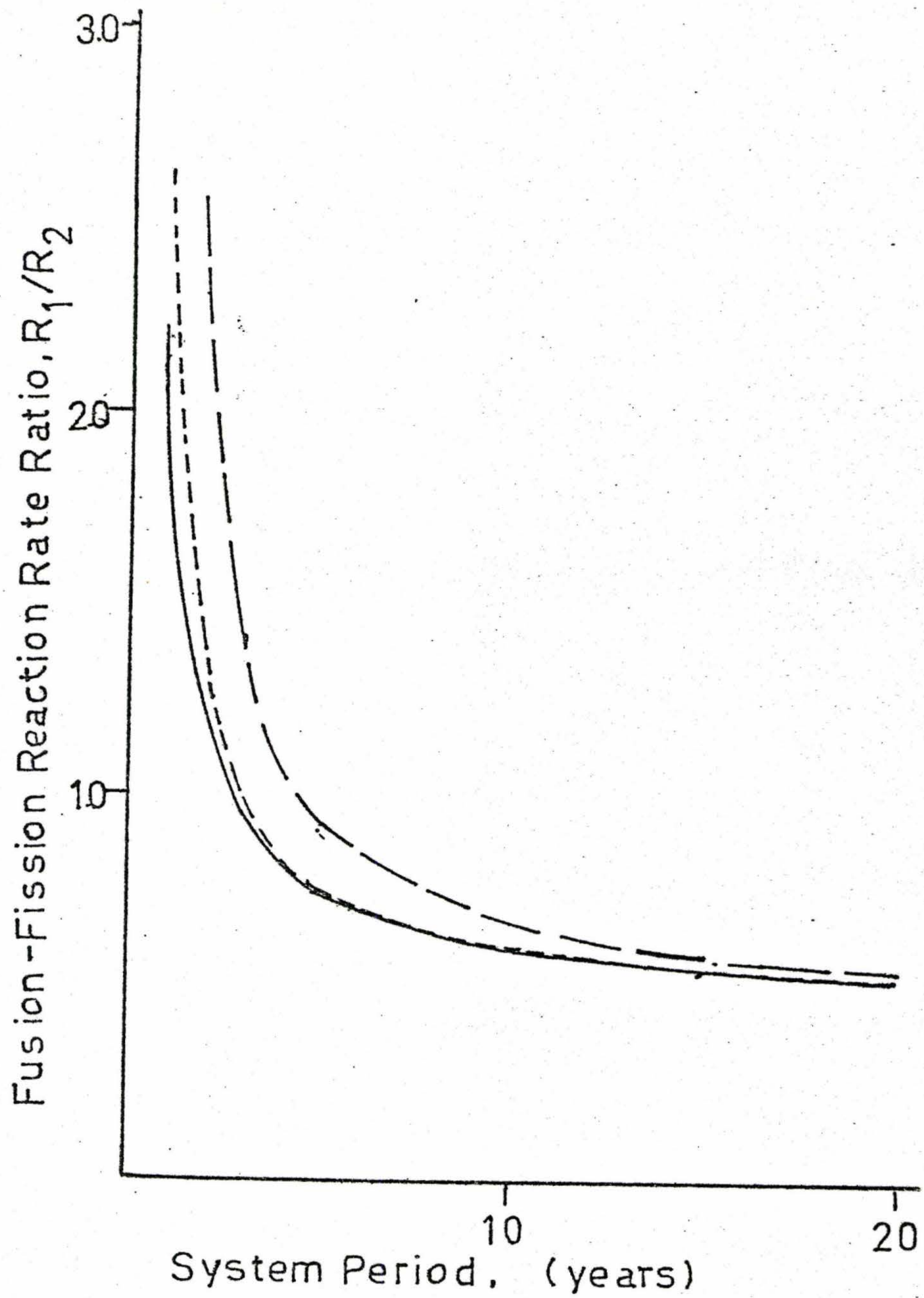


FIG.9 REACTION RATE RATIO
(PARASITIC)

(———) $n_1 = 2$ g/MWt, (------) $n_1 = 6$ g/MWt,
(— —) $n_1 = 20$ g/MWt

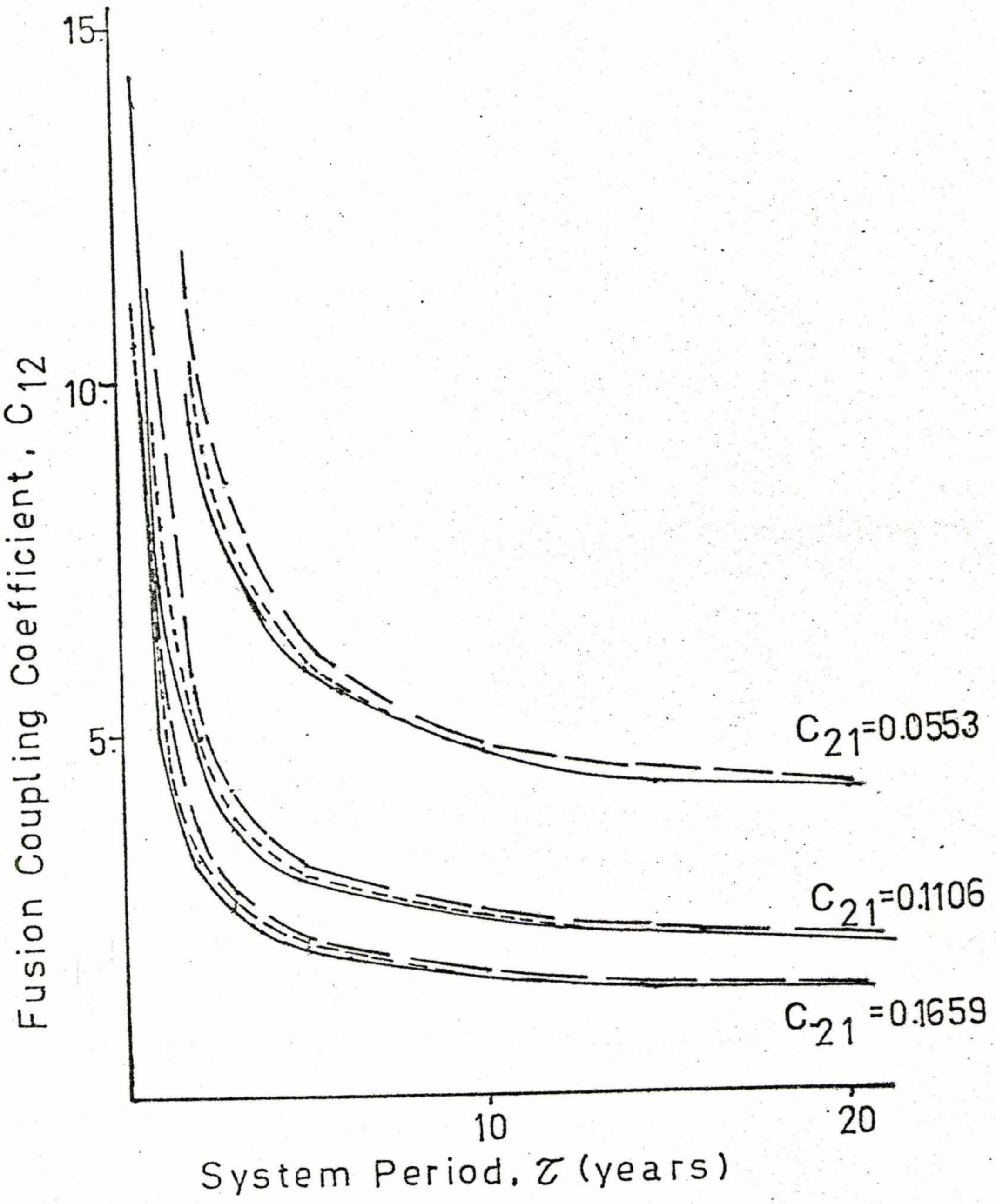


FIG.10 FUSION COUPLING COEFFICIENTS (COMMENSALISTIC)

(——) n_1=2 g/MWt, (----) n_1= 6g/MWt,
(— —) n_1=20 g/MWt

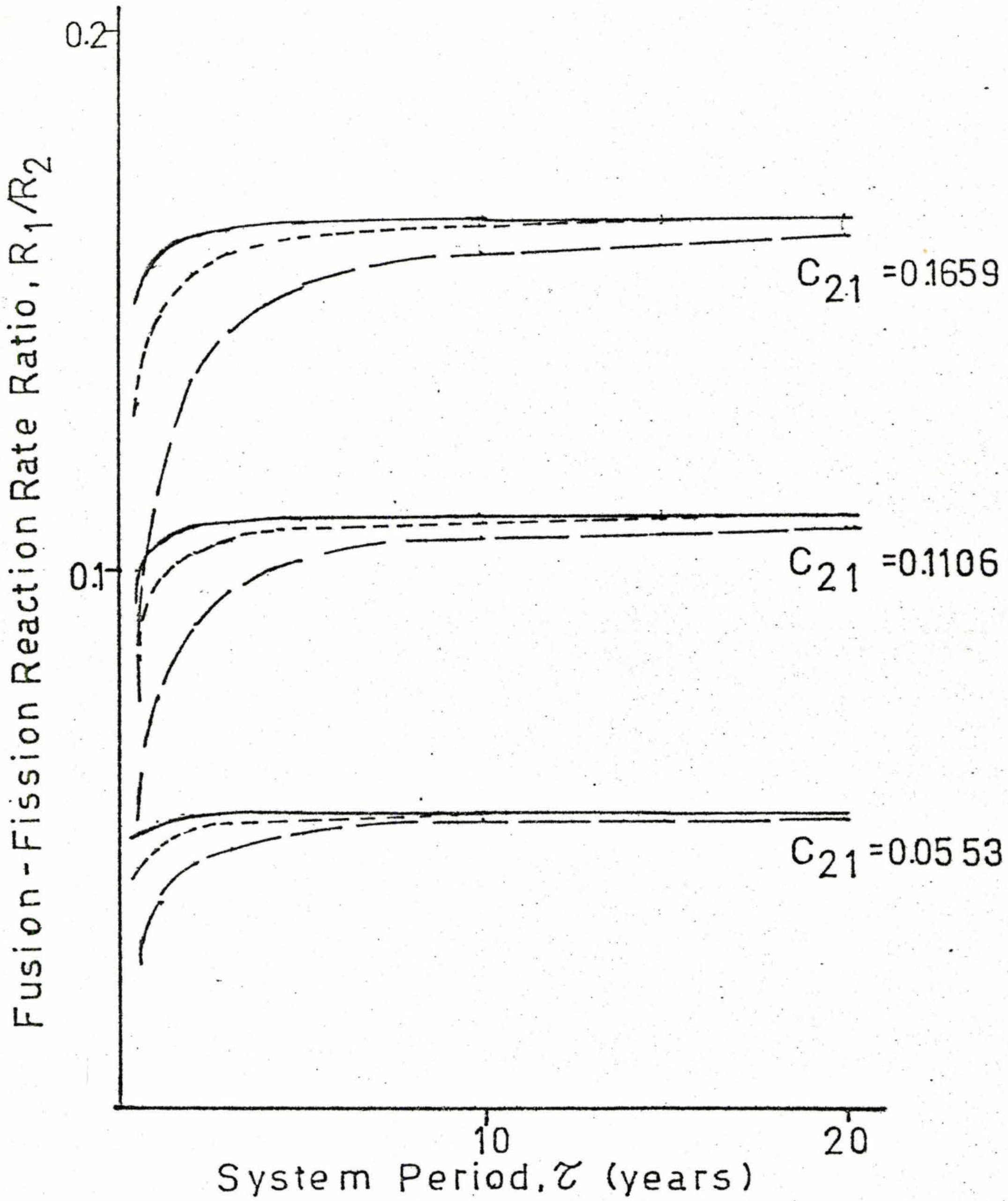


FIG. 11 REACTION RATE RATIO (COMMENSALISTIC)

(———) n_1 = 2g/MWt, (------) n_1 = 6g/MWt,
(— —) n_1 = 20 g/MWt

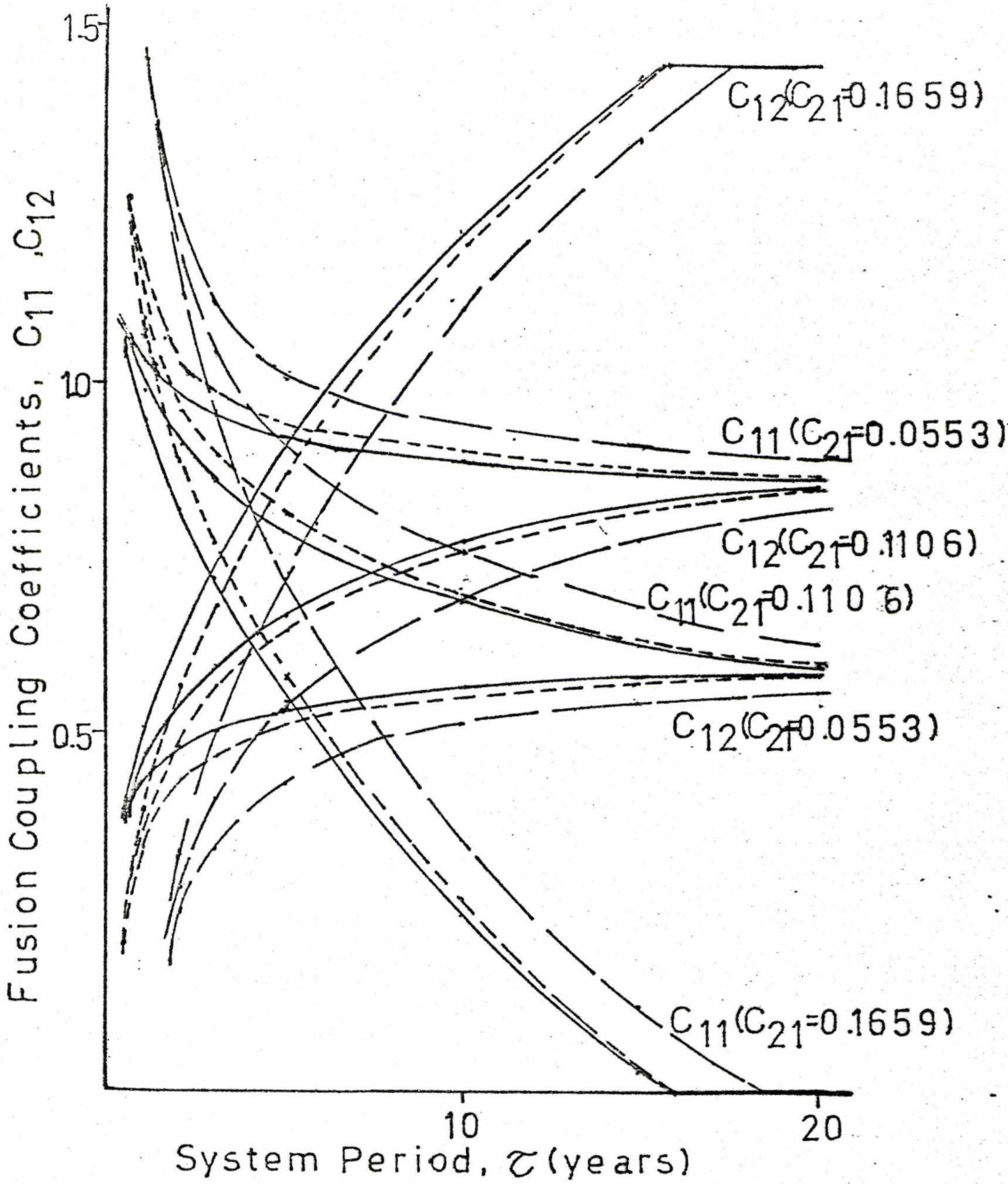


FIG.12 FUSION COUPLING COEFFICIENTS (MUTUALISTIC)

(——) $n_1 = 2g/MWt$, (------) $n_1 = 6g/MWt$, (— —) $n_1 = 20g/MWt$

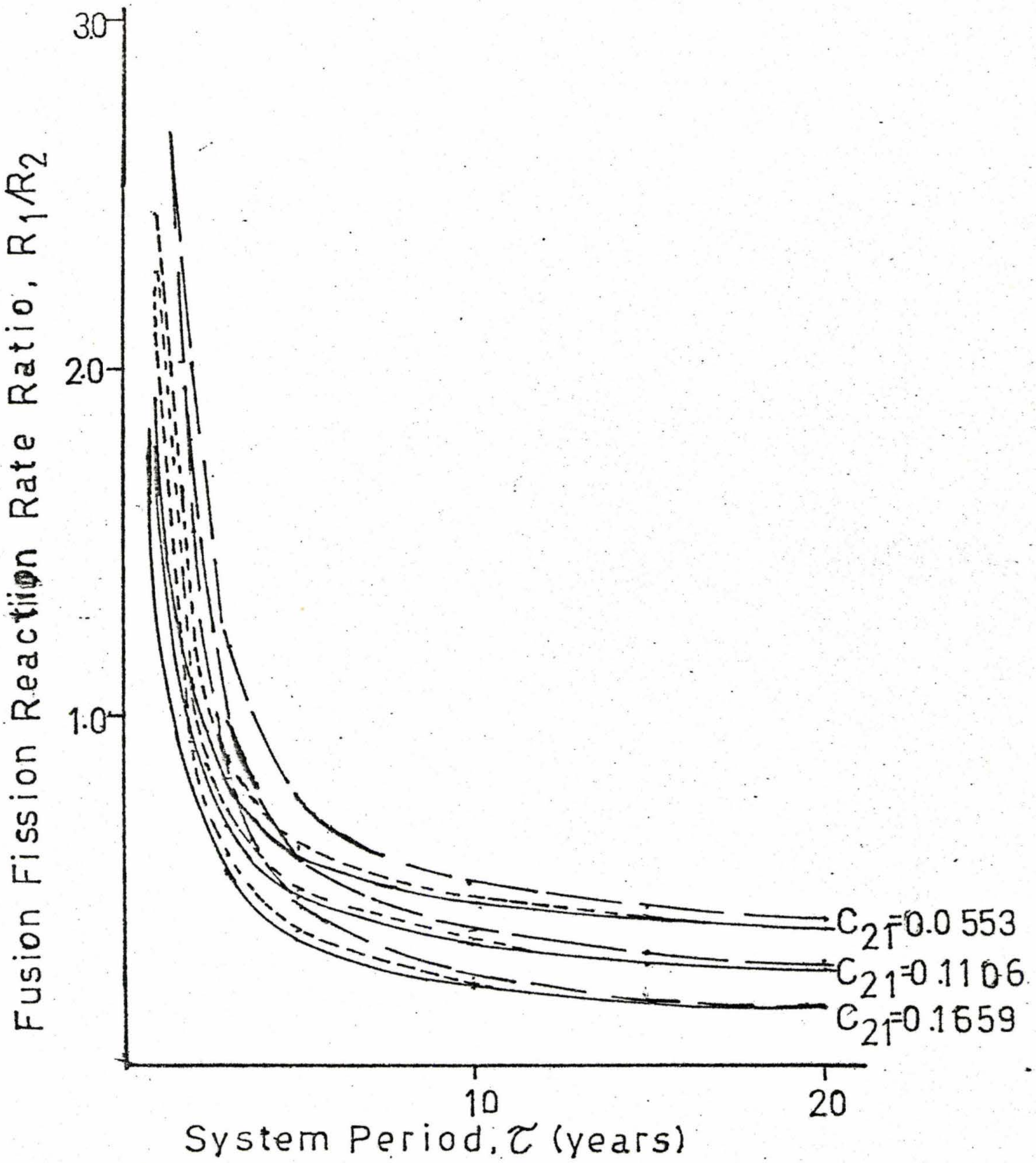


FIG.13 REACTION RATE RATIO (MUTUALISTIC)

(———) n_1=2g/MWt, (- - - -) n_1=6g/MWt,
(— —) n_1=20g/MWt

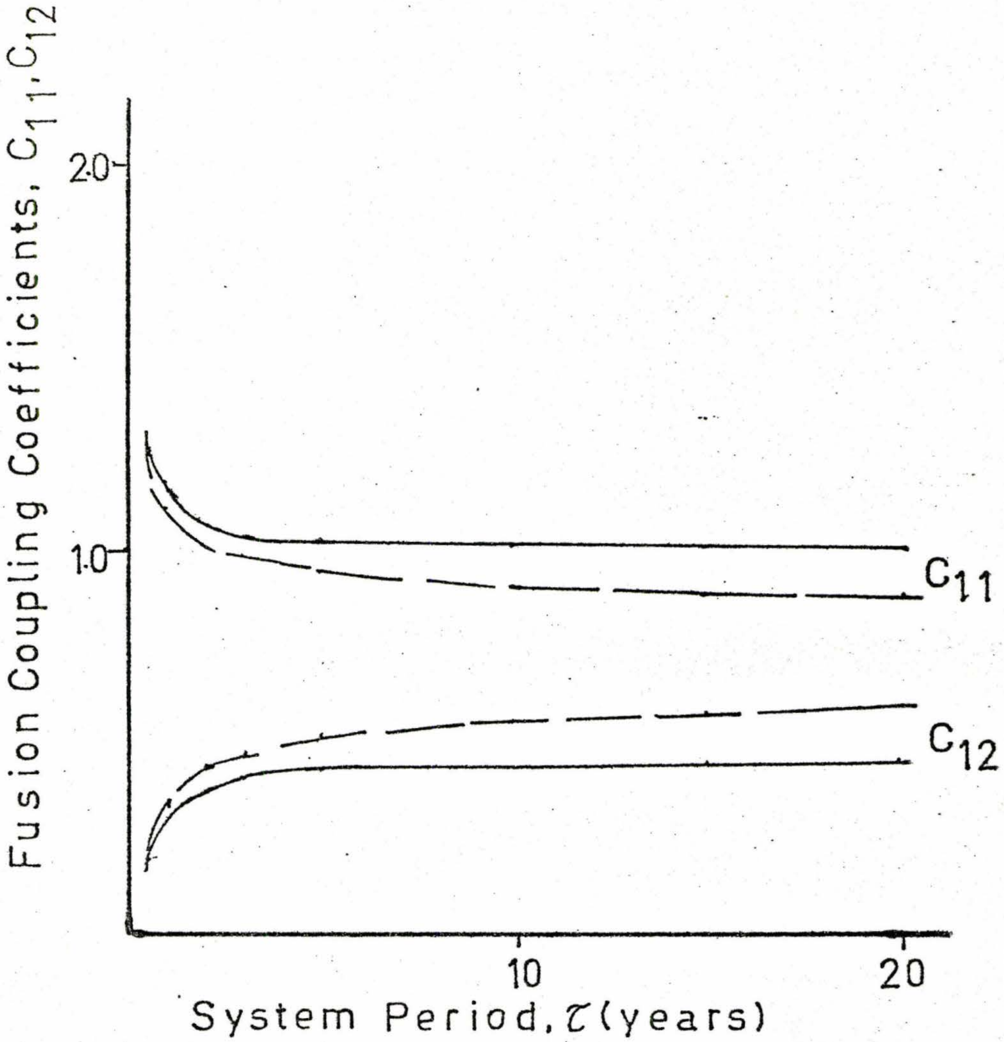


FIG. 14 FUSION COUPLING COEFFICIENTS

$n_1 = 6g/MWt$, $C_{21} = 0.0553$

Note: for Commensalistic System $C_{11}=0, C_{12}>4$

(———) Parasitic, (— —) Mutualistic

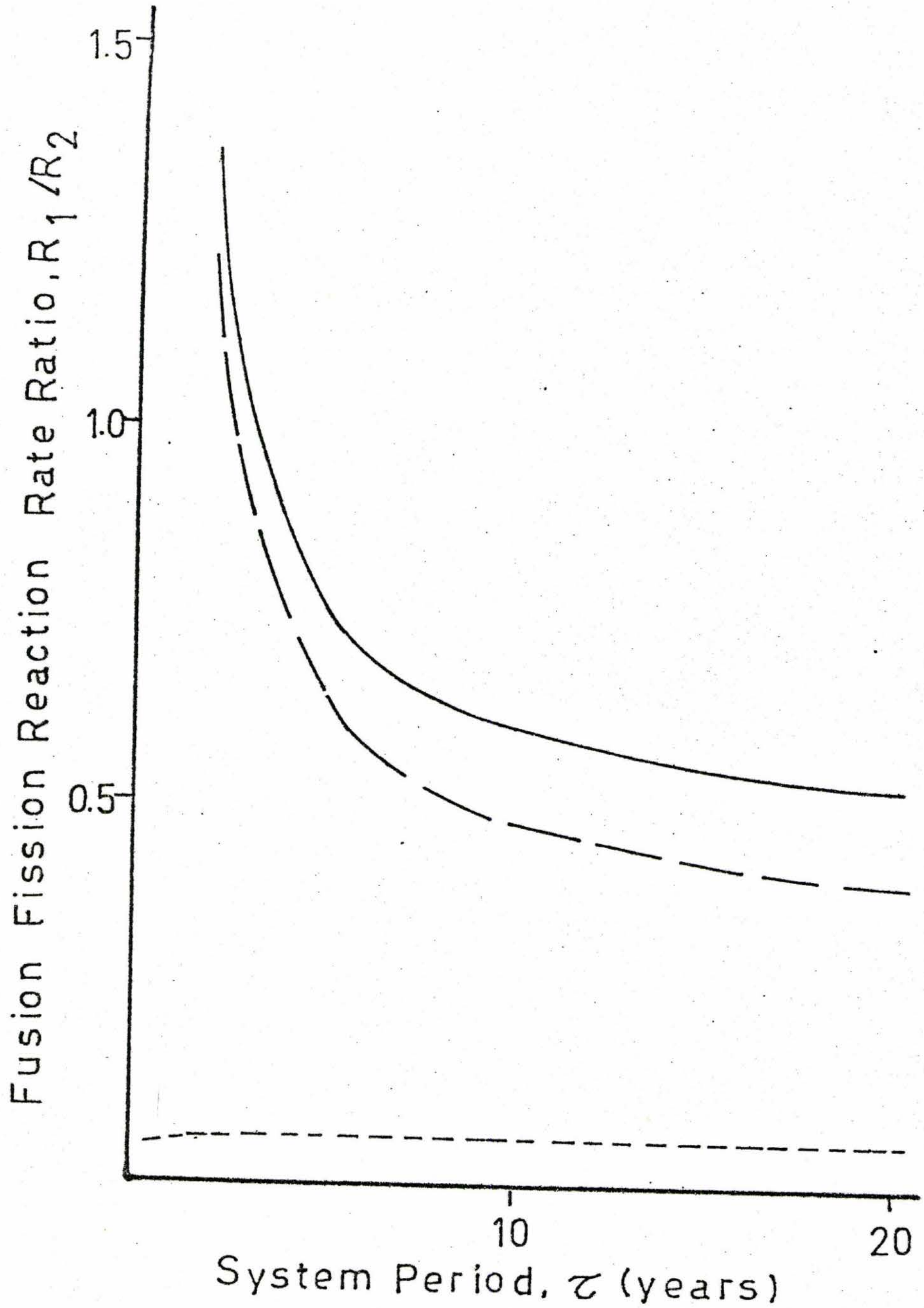


FIG. 15 REACTION RATE RATIOS

$n_1 = 6 g/MWt$, $C_{21} = 0.0553$

(———) Parasitic, (------) Commensalistic

(— — —) Mutualistic

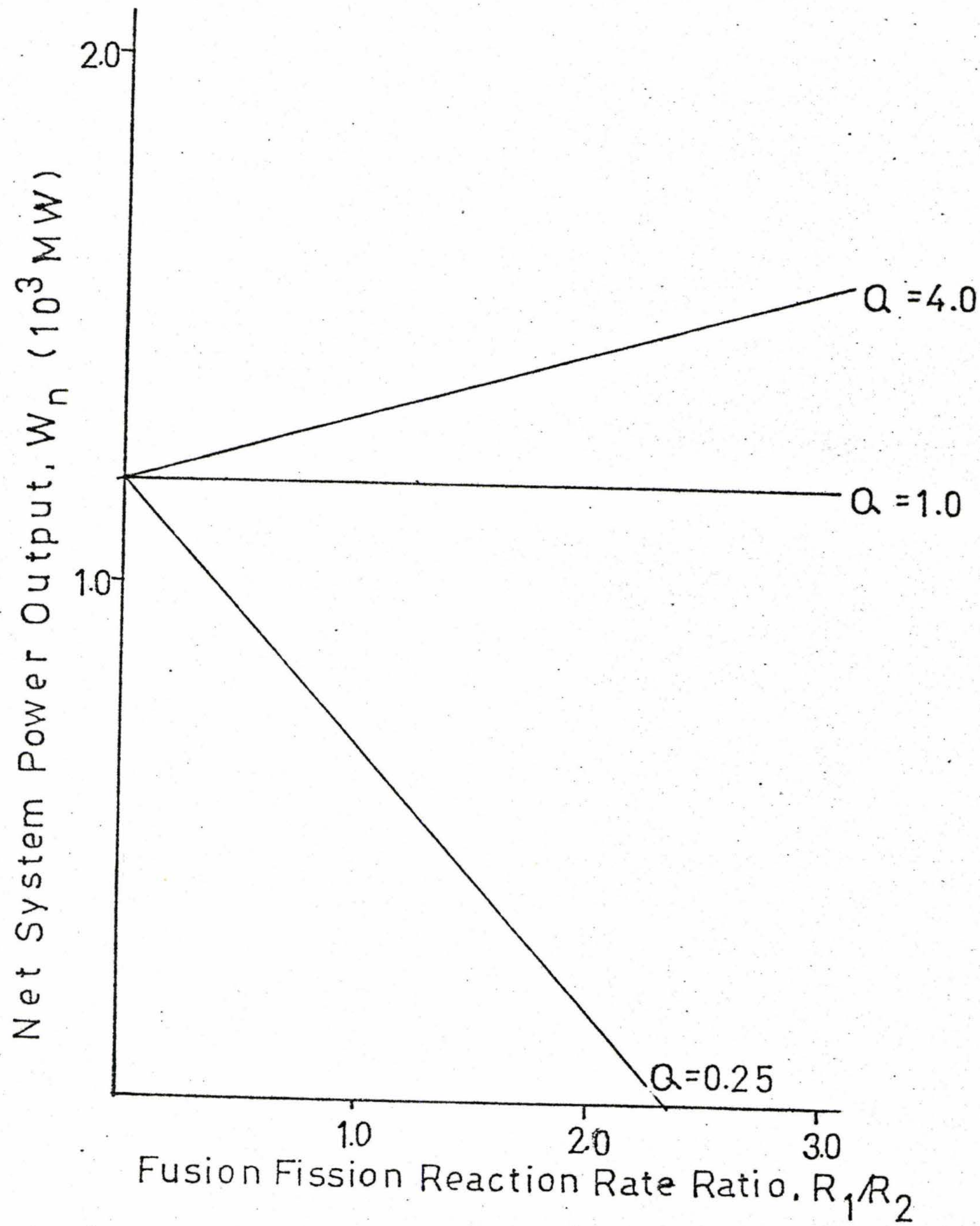


FIG.16 NET SYSTEM POWER OUTPUT

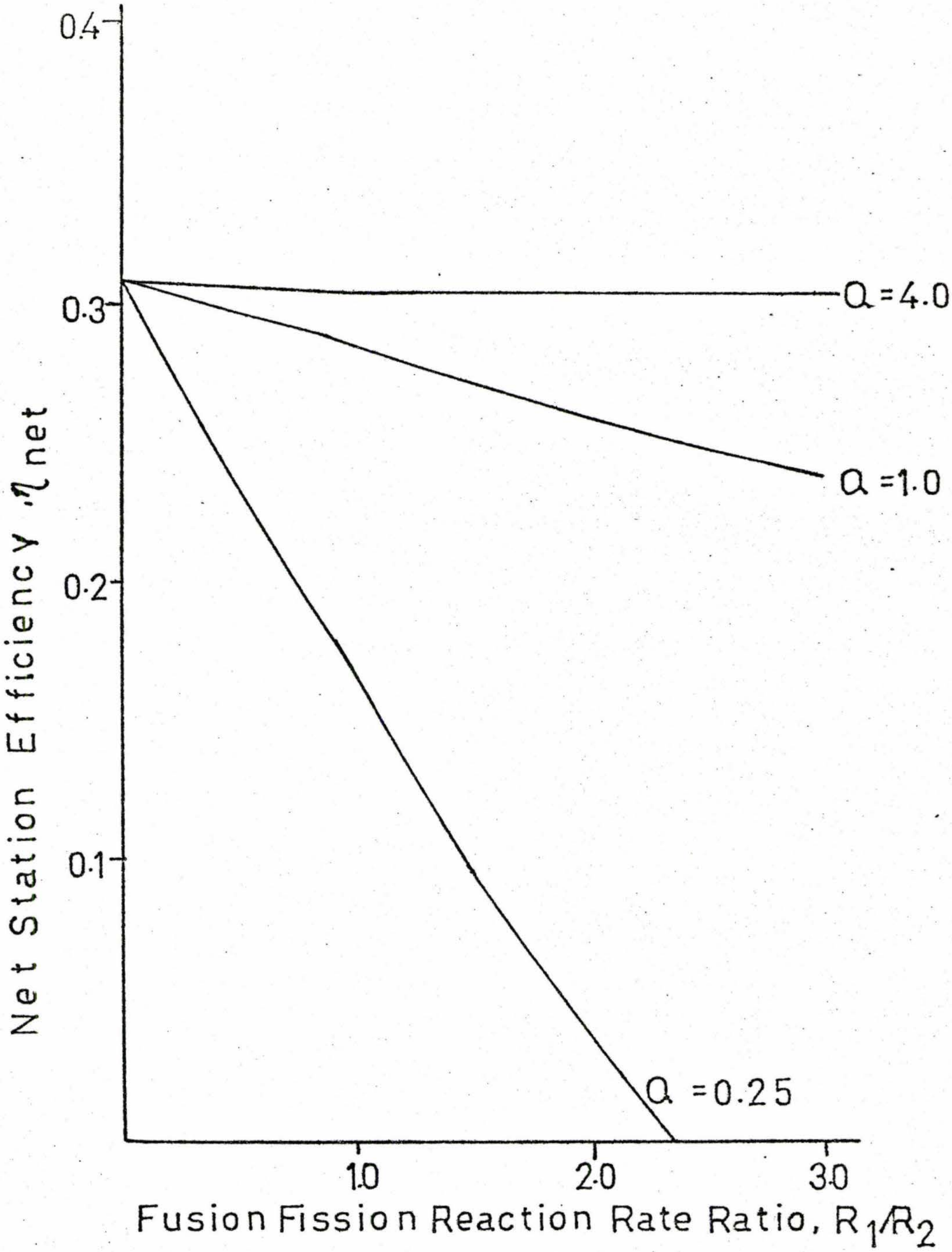


FIG. 17 NET STATION EFFICIENCY

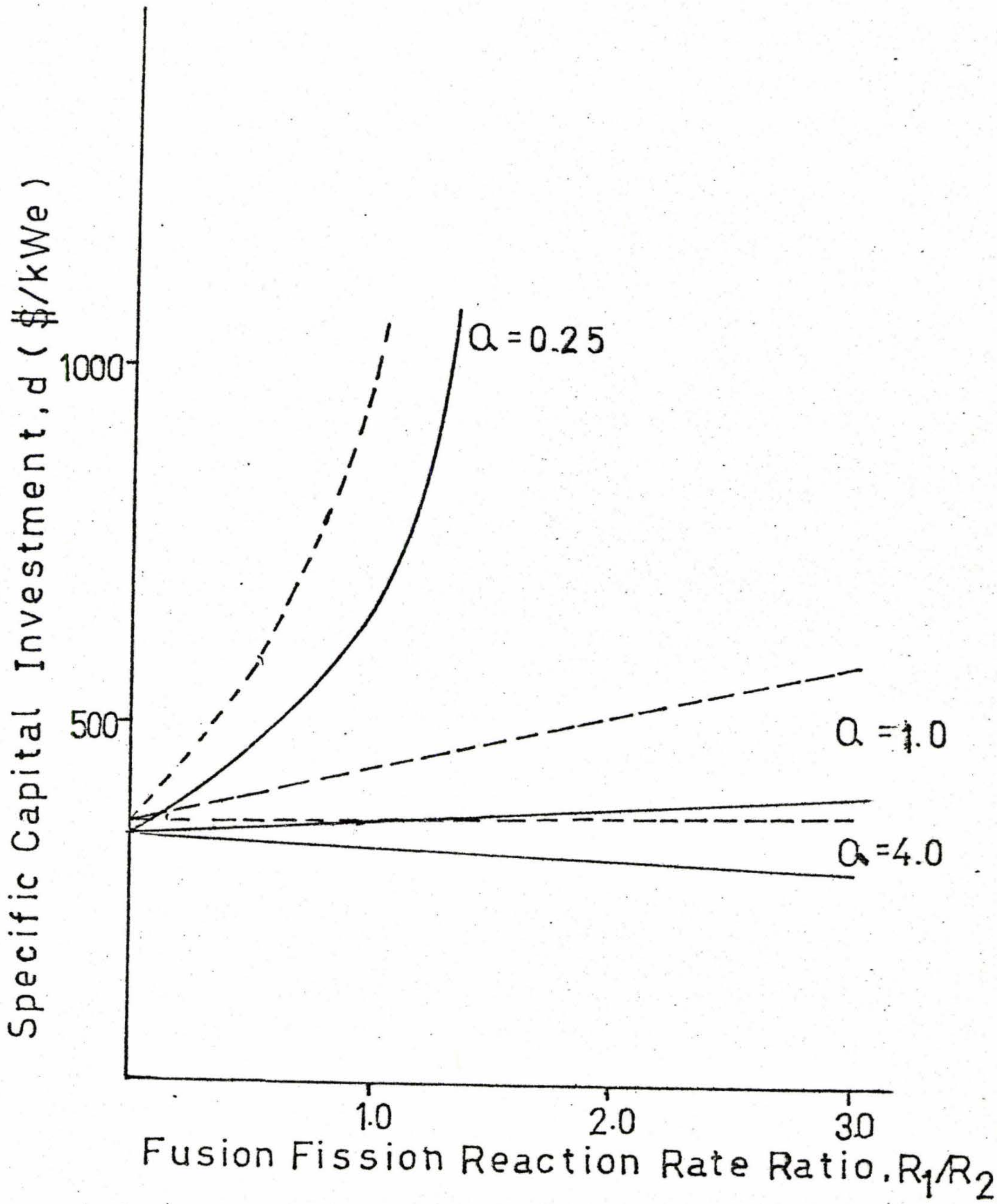


FIG. 18 SPECIFIC CAPITAL INVESTMENT

(———) d_-, (-----) d_+

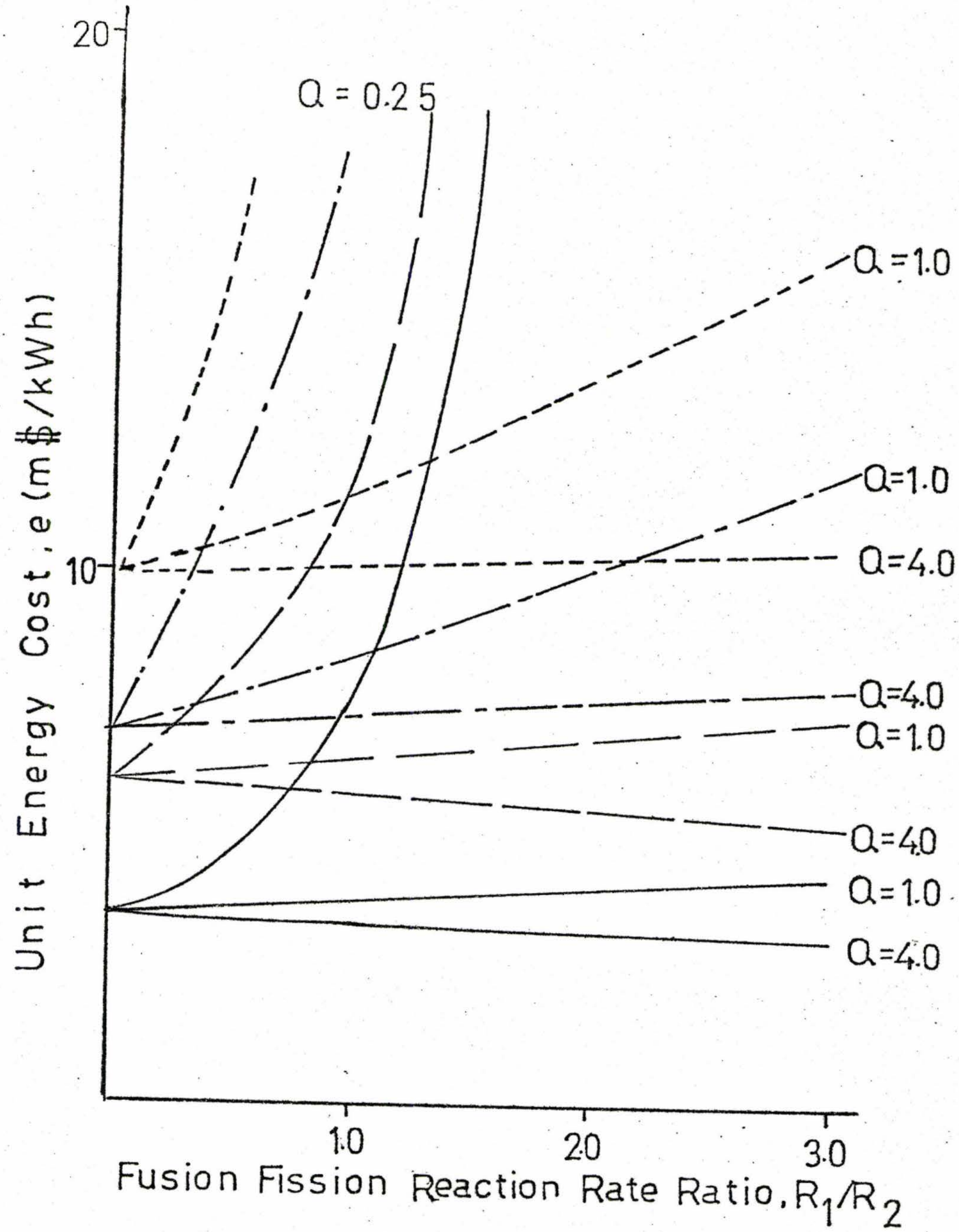


FIG. 19 UNIT ENERGY COST

(———) 5% d_-, (— — —) 10% d_-,
(—·—) 10% d_+, (- - - - -) 15% d_+

REFERENCES

1. B.R. Leonard, Jr., "A Review of Fusion-Fission (Hybrid) Concepts", Nuclear Technology, 20, 161 (1973).
2. E.L. Draper, Jr. and S.J. Gage, "The Fusion-Fission Breeder: Its Potential in a Fuel-Starved Thermal Reactor Economy", Texas Symposium on the Technology of Controlled Thermonuclear Fusion Experiments and the Engineering Aspects of Fusion Reactors, Austin, Texas, Nov. 20-22 (1972).
3. L.M. Lidsky, "Fission-Fusion Symbiosis: General Considerations and a Specific Example", B.N.E.S. Nuclear Fusion Conference, Culham Laboratory, U.K., Sept. (1969).
4. W.H. Köhler and J. Voss, "Tritium Demand for Fusion Reactor Development", Proceedings of the First Topical Meeting on the Technology of Controlled Nuclear Fusion, San Diego, California, April 16-18 (1974).
5. J.W. Kimball, Biology, Addison-Wesley Publishing Co. Inc., Reading, Mass. (1965).
6. G. LaVergne, J.E. Robinson and J.G. Martel, "On the Matching of Fusion Breeders to Heavy Water Reactors", Nuclear Technology, (in press, 1975).
7. W.F. Vogelsang, "Breeding Ratio, Inventory, and Doubling Time in a D-T Fusion Reactor", Nuclear Technology, 15, 470 (1972).

8. D. Steiner and A.P. Fraas, "Preliminary Observations on the Radiological Implications of Fusion Power", Nuclear Safety, 13, 353, (1972).

9. W.B. Lewis, M.F. Duret, D.S. Graig, J.I. Veeder, and A.S. Bain, "Large-Scale Nuclear Energy from the Thorium Cycle", Proc. 4th Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, 9, 239 (1971).

10. Design Description, Pickering Generating Station, prepared by A.E.C.L. for the Hydro Electric Power Commission of Ontario, 1969.

11. J.D. Lee, "Tritium Breeding and Direct Energy Conversion", Proc. of a Symposium on the Role of Chemistry in the Development of Controlled Fusion, Boston, April (1972).

12. D.J. Rose, "Engineering Feasibility of Controlled Fusion", Nuclear Fusion, 9, 183 (1969).

13. R.G. Mills, "Some Engineering Problems of Thermonuclear Reactors", Nuclear Fusion, 7, 223 (1964).

14. R. Salmon, J.T. Roberts, A.L. Lotts, T.N. Washurn, and W.H. McVey, "Price Forecasting and Resource Utilization for the Fuel-Cycle Industry of the United States of America", Proc. of the 4th Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, 4, 255 (1971).

15. D.H. Rose and M. Clark, Jr., Plasmas and Controlled Fusion, M.I.T. Press, Cambridge, Mass. (1961).

16. D.L. Bate, P.F. Mayes and W.S. Philip, "Costing of Canadian Nuclear Power Plants", Proc. of the 4th Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, 2, 443 (1971).

17. R.G. Hart, L.R. Haywood and G.A. Pon, "The CANDU Nuclear Power System: Competative for the Foreseeable Future", Proc. of the 4th Int. Conf. on Peaceful Uses of Atomic Energy, Geneva, 5, 239 (1971).