A STUDY OF EROSIONAL SCALLOPS
THE GENERATION OF SMALL-SCALE RELIEF FEATURES
OF ERODED LIMESTONE:
A STUDY OF EROSIONAL SCALLOPS

by

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SCOPE AND CONTENTS: Proposed theories concerning the nature and mode of formation of limestone scallops are examined. Some progress is made toward a purely theoretical understanding. Scallop formation is simulated by generation on blocks of plaster of Paris in a laboratory flume under known and controlled conditions and the relationships between the resulting features, the generating conditions and the base material examined. Field evidence both confirms these relationships and reveals other unsuspected factors. The similarity between these features and others found on ablating snow surfaces is investigated and the same laws found to apply.
PREFACE

In karst studies, the underground realm has been comparatively neglected. Yet all karst surface forms, with the possible exception of the very smallest, are intimately related to a subterranean drainage system. This system is rarely accessible; only after the hydrologic pattern which generated the rooms and passages has long been drastically revised are caves open to the human explorer and investigator. In order to understand these fossil relics some information must be gathered on the quantity, velocity and direction of flow of the ancient stream pattern. To this end, speleologists have long been intrigued by complex, small-scale relief patterns which are found on the walls of many dry cave passages and appear to be related to the movement of water over the limestone at the time of formation of the cave.

It was with these considerations in mind, and with the recent flurry of interest in this question in the speleological literature, that this project was undertaken.
ACKNOWLEDGEMENTS

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CHAPTER ONE

INTRODUCTION AND REVIEW OF THE LITERATURE

1. Terminology and Morphometry

A wealth of terms has been used at various times to describe the phenomenon discussed in these pages. Besides the mundane 'scallop', the vernacular of the cave explorer, writers have favoured the terms 'flute', 'cochleark', 'dirt polygon', 'ripple', and even the magnificent 'negatively mammillate surface' (Leighly, 1933). The ambiguous terms 'flute' and 'ripple' are reserved in this work for other related forms. The terms 'dirt polygon' and 'ablation polygon' will be used synonymously with 'scallop' when dealing with the phenomena developed on snow surfaces, while the other terms will be ignored.

Eroded limestone surfaces in caverns frequently show a shallow patterned relief akin to the ripples found in sediments on the beds of flowing streams or lakes. The depressions and ridges are commonly less elongated, resembling closely a mosaic of inlaid scallop shells orientated in a single direction. Since the features are frequently isolated rather than closely packed, it is convenient to consider them as 1
depressions defined by ridges rather than vice versa. A deeply scalloped boulder from a dry passage of the Makimu Caves, Glacier National Park, British Columbia, appears in Plate 1.1.

In a general plane normal to the surface, the features have an asymmetrical cross-section. The greatest variation in curvature of this cross-section is found in a direction perpendicular to the unique symmetrical cross-section. In short, scallops are steeper on one side than on the other. The line defined by the maximum asymmetry is parallel for all features on a flat surface, and thus a unique direction can be found. Bretz (1942) was probably the first to note that this line is always parallel to the direction of water flow over the surface in cases where limestone forms the bed of a stream and that scallops are always steeper on the upstream side. Since cave passages are formed by the action of flowing water, Bretz proposed that scallop patterns in dry passages may be used to demonstrate the past existence of moving water and its direction of flow.

Three dimensions will be defined for individual scallop features. The length is defined as the maximum extent of the feature parallel to the water flow direction. In the case of closely packed features, the length is defined between adjacent ridges; the width is defined as the
maximum extent perpendicular to the flow direction and in the plane of the surface. The depth is defined as the relief of the surface.

2. Limestone Scallops

Davis (1930) observed scalloped surfaces in the caverns he investigated for his monumental work on speleo-
ogenesis. Considering a scallop pattern as a steady state, propagated into the limestone bedrock as the wall is eroded by flowing water, he investigated the implications of their form. The pattern can only be maintained if it is propagated in a direction defined by the slopes on either side of the ridges (Figure 1.1). On the basis of his obser-
vations, therefore, Davis believed that the pattern was constrained to migrate upstream. He further drew compar-
sions between these features and the better known sedimentary ripples, assuming the two to be genetically similar.

In addition to identifying scallop patterns as indicators of water flow direction, Bretz considered the phenomenon to be due to corrosive rather than abrasional erosion. He suggested the fluid vortices and the imperfect limestone wall interacted in such a manner that depressions evolved, trapping vortices and producing a stable form. This theme has been favoured by many subsequent authors as a genetic theory of the phenomenon. Bretz pointed to frail fossil mantlets and insoluble material structures left in
Figure 1.1. Parameters of the Scallop Profile
relief by the eroding water as evidence of a solutional process. He noted that ridge lines were more continuous across the flow than parallel to it.

Coleman (1949) introduced the American findings in an English journal and appropriately suggested a change in terminology from 'flute' to 'scallops'. He moved away from Bretz in maintaining that "current load caught up in vertical movements is a major tool in the excavation of scallops", but notes that insoluble material such as chert commonly remains un-scalloped.

Observed distributions of scallop lengths have stimulated many authors to speculate on controlling factors. Lengths of neighbouring scallops on a large flat wall are observed to be distributed about a mean with a standard deviation of perhaps 20 per cent of the mean value. Yet the mean length is found to vary between sites, ranging from a few millimetres (Wahhu Caves, B.C.) to several metres (Mammoth Cave, Kentucky). Glennie (1963), following a suggestion of Ashwell (1962), proposed that a small mean length is the result of fast, turbulent flow. A large mean length results from an extended period of steady flow. If allowed to remain undisturbed, such a steady flow would eventually produce an infinitely long set of scallops. R. A. Davies (1963) reported a conversation with Yeh, in which the latter suggested the size variation is a matter
of age, the length of a scallop increasing with time, apparently indefinitely. On a surface eroding into a uniform mass of limestone, insoluble fragments and inhomogeneities are exposed at a uniform rate. There is therefore a constant probability through time of the initiation of a scallop feature. Thus, although individuals may grow with time as Yeh suggested, the properties of the length distribution must remain constant. Davies' argument does not explain the observed variation in mean length between sites, but rather the deviation in individual lengths at a single site.

T. D. Ford (1964) proposed, on the basis of observation, that large features are produced by turbulent, sediment-laden water corrading the limestone, while small features are produced by solution due to slowly moving water. Eyre (1954), also on the basis of observation, suggested the opposite relationship between stream turbulence and scallop length. He noted that scallops in a passage of Caping Chyll Caverns, Northwest Yorkshire, England, which had formed on a bulge outwards from the cave wall, were smaller on the upstream side of the bulge than on the downstream side. This inverse velocity-size relationship was also the theme of Curl (1952).

Scalloped limestone surfaces have long been noted in surface streams. Gilbert (1875) described scallop-like
features on boulders in the Colorado River. Lugeon (1915)
reported similar phenomena while Maxson (1935, 1940)
attempted a more systematic analysis. He noted that the
'flute' features in the Grand Canyon were found predominant-
ly, though not exclusively, on limestone boulders. It is
clear from his descriptions and plates that only a small
subset of the features analysed are morphologically similar
to cavern scallops. From the size of the sediment load
carried in suspension by the river, Maxson proposed that
the dominant erosion mechanism is abrasion. He compared
these forms with the faceting of desert pebbles by wind-
blown sand and in the later paper attempted a quantitative
analysis of both sets of features. Defining a Reynolds
Number based on the height of the top surface of a boulder
above the surrounding stream bed or desert floor, Maxson noted
that the value obtained was much higher for stream action than
for desert wind action. He concluded that the size of trapped
vortices responsible for the features was an increasing func-
tion of the Reynolds Number. Elongated features, grooves
parallel to the flow direction, were believed to form when
vortices were not stable and migrated downstream (as in the
case of separated boundary layers). Maxson notes (page 739)
that the features were observed on wide flat surfaces, which
places the value of his Reynolds Number in some doubt.

Geze (1965) makes a radically different suggestion:
"...des 'coupes de rodage', des 'coupes de gouge', des 'vagues d'érosion' dont les dimensions peuvent aller de quelques centimètres à plus d'un mètre soit des formes résultant de l'action d'un courant violent sur les voutes, les parois ou le sol des galeries de grottes. Leur allongement et leur dissymétrie indiquent le sens de ce courant. Leur genèse exacte est d'ailleurs très discutée. Certains auteurs estiment qu'elles ne peuvent se produire que dans des galeries temporairement remplies de cailloutis, galets et blocs que le courant agiterait, tandis que pour l'autres, plus que de l'érosion proprement dite, elles resulteraient de la 'cavitation' phénomène se produisant en conduite forcée par dégagement de gaz entre le paroi rocheuse et une tranche d'eau décollée de celle-ci à la suite d'un déficit de pression."

The concept of cavitation has never been seriously proposed in the English literature, in part because the required conditions are seldom if ever achieved in the natural environment.

3. Ablation or Dirt Polygons

Ball (1954) was perhaps the first to note the similarity of form between scallops formed on limestone, and dirt polygons, features known mainly to glaciologists. During ablation, snow patches frequently develop a pattern of closely packed hollows (Plate 1.2). Dirt is often observed to accumulate on the ridges between the hollows, forming a polygonal pattern. Dirt polygons frequently show the cross-sectional asymmetry which characterises a scallop pattern, and if present the asymmetry similarly defines a unique direction.
Plate 1.2. Dirt Polygons on a Snowfield, Mt. Castleguard, Banff National Park, Alberta, in August 1968.
Sharp (1947) observed snow scallops on the Wolf Creek Glaciers, Yukon Territory, and speculated on the factors responsible for their origin. He noted that "elongation of flutes on slopes would seem to indicate that meltwater is a significant factor". Leighly (1948) commented on scallop-like forms on the roof of a snow tunnel and suggested that convection cells set up by thermal, and therefore density differences in the air are responsible for the form of the surface. Asymmetry is produced by general fluid movement, although this is not necessary to the mechanism. Further, the dirt on the ridges is washed there by meltwater. Leighly fails to explain accumulations of dirt on upward-facing snow surfaces.

Lundqvist (1948) and Klebelsberg (1948) accepted the importance of wind in the formation of snow scallops. "Die schalenformige Abtragung is das Werk der Schmelzung und Verdunstung durch bewegte Luft". Sjolin (1957) recognized the relationship between wind direction and asymmetry, and further noted that on the surfaces he observed, widths were always greater than scallop lengths. He utilized these observations to plot daily maps of wind directions on melting snow, noting the rapid adjustment of the patterns to changes in wind conditions.

The dirt accumulations associated with scallop patterns on snow have also aroused considerable interest. Ball (1954) offered a theory of "normal trajectory" to account for the
phenomenon. (Figure 1.2). As dirt particles melt out of the snow matrix, their path is normal to the surface. Hence accumulations occur on peaks and ridges. He offered no mechanism to explain this anomalous dirt movement.

Richardson (1954) tried to apply Leighly's ideas of natural convection to open snow surfaces by postulating convection cells within the snow, generated by heat fluxes from the ground. But scallop patterns are common on snow patches several tens of feet thick, and on glacier surfaces. Sharp illustrates patterns on vertical faces, further denying the possibility of such a mechanism. In a later paper Richardson and Harper (1957) note that dirt accumulations are not always present and therefore constitute a secondary effect. They note that heavier material such as gravel does not accumulate. The paper finally advances a theory of boundary layer separation and turbulence in the lee of surrounding obstacles.

The work of Ashwell and Hannell (1966) assumes an intimate relationship between dirt and polygons. An area of snow was levelled and lines of reindeer droppings laid out on the snow both parallel to and perpendicular to the prevailing wind. In cloudy weather, depressions developed on the upwind side of those lines of dirt laid transverse to the wind. In sunny weather the dirt enhanced the ablation rate due to direct solar heating by presenting an anomalously high absorption coefficient.
Figure 1.2. Theory of Normal Trajectory. Patterns of Dirt Movement on a Receding Surface of Dirt Polygons as Proposed by Ball (1954).
The authors present a theory of dirt polygon formation which incorporated both sets of climatic conditions. On cloudy windy days depressions develop on the upwind side of dirt accumulations. At this point it should be noted that in levelling the experimental plot, the authors in fact produced a shallow pit, so that the effects which are described as upwind of the dirt lines can also be described as downwind of the walls of the experimental plot, a more acceptable description.

Under calm and sunny conditions, the authors contend, ablation is controlled by the absorption of the surface and is highest under thin layers of dirt. Following such climatic conditions, dirt accumulations are observed in the bottoms of the depressions which they have generated. Further cloud and wind will either blow this dirt back onto the ridges or modify the pattern by generating depressions on the windward sides of accumulations so that dirt polygons result once more.

After noting the presence of dirt-free polygons on the under surface of a snow tunnel roof, Ashwell and Hannellend by observing that dirt accumulations are not essential to the formation of ablation polygons.

Jahn and Klapa (1968) support Ball's theory of "normal trajectory" outlined above, and offer a mechanism to explain the phenomenon. They distinguish between mineral and organic dirt, maintaining from a small number of observations that the
latter accumulated more readily. Their observations were based on the composition of polygon ridge dirt, without comparison with the composition of dirt available before accumulation. Organic dirt "sticks to a wet surface as though it were glued". Thus when movement of the dirt takes place due to recession of the supporting snow surface, such movement is normal to the surface rather than vertical. This results in cleaning of the depressions of the ablation polygons, and dirt accumulations as proposed by Ball.

Jahn and Klapa note that there is a tendency for elongated particles such as pine needles to rotate during movement in such a manner as to eventually lie with their long axes parallel to the dirt ridges. They demonstrated this phenomenon by distributing match sticks on a snow surface and observing changes in their distribution and orientation through time.

4. The Work of Curl

Curl (1966) unified the limestone scallop and ablation polygon forms in one theory. On the basis of evidence cited by Bretz and other workers, he considered limestone scallops to be the result of a wholly solutional process. The rate of erosion at any point is therefore controlled by the mass transfer rates of fresh aggressive water from the main body of the stream. Thus in principle the process is described by the diffusion equation and the hydrodynamic Navier Stokes equation.
A complete solution is of course infeasible.

Consider a snow surface being eroded by warm air rather than by direct solar heating. The rate of erosion is controlled by the rate at which heat can be transferred from the warm fluid to the cold surface and, to some extent, by the rate at which water vapour can diffuse back into the air flow. The latter factor is more important in the case of sublimation at temperatures below 0°C. Thus the erosion process is described by the equation of heat transfer, and the Navier Stokes equation. Since heat transfer and mass transfer are analogous processes, we expect analogy between a dissolving limestone surface and an abating snow patch.

In order to treat the scallop phenomenon quantitatively, Curl first reduced the system to two dimensions, by introducing symmetry perpendicular to the flow direction. He invoked the name 'flute' to describe a scallop infinitely extended perpendicular to the flow direction. Features which satisfy this condition approximately may be found in some caves and in snow tunnels.

Using the technique of dimensional analysis, Curl first lists the parameters likely to affect the length (as defined above) of such flutes. These are

\[ \nu \text{ a characteristic velocity of the flow} \]
\[ H \text{ a characteristic channel dimension} \]
\[ \eta \text{ the fluid viscosity} \]
\[ \rho \text{ the fluid density} \]

\[ D \text{ the diffusivity of the solute ions} \]

Denoting units of length by \( L \), of time by \( T \) and of mass by \( M \), the above quantities have the following dimensions.

Flute length \( \lambda \)

\[ \lambda \quad L -1 \quad \frac{L}{L T} \quad \frac{L}{L^3} \quad \frac{M}{L^1 T^3} \quad \frac{M}{L T} \]

and may be combined into the dimensionless numbers

\[ \frac{\lambda v \rho}{\eta}, \quad \frac{\lambda}{H}, \quad \frac{\eta}{D \rho} \]

The number \( \lambda/H \) will only become important, argues Curl, when the dimensions of the channel are such that the flow about a flute is influenced by other surfaces in the channel. Therefore if we assume \( \lambda/H \) to be small, we can write

\[ \frac{\lambda v \rho}{\eta} \text{ is some function of } \frac{\eta}{D \rho} \]

Curl argues further that the dependence on the Schmidt Number \( \frac{\eta}{D \rho} \) is likely to be extremely weak since the rate of erosion is very slow. Thus we may write \[ \frac{\lambda v \rho}{\eta} = N_f, \] a dimensionless "Flute Reynolds Number".

Curl measured two sets of flutes in order to evaluate
the numerical value of $N_f$. One set, measured in a limestone-water system, gave a numerical value of 23500. The other, in an ice-air system, gave 21600. In view of the methods used to estimate the velocity parameter $v$, and the possibility that the velocities measured were not the velocities responsible for the flute formation, the agreement can be regarded as either fortuitous or convincing.

The theory predicts the formation of asymmetrical scallop-like features in all cases where a surface is eroded by a transfer process with a moving hydrodynamic fluid. It is encouraging to note, then, that scallops have been reported from caves developed by water in Gypsum deposits (Mogragor et al, 1962) and that similar features have been reported on meteorites ablated during passage through the atmosphere (Williams, 1963).

For symmetry to develop perpendicular to the flow direction, the conditions must be stable. Fluctuation in velocity in a periodic or random fashion will cause the breakdown of this transverse symmetry, and a scallop pattern will then result. Although a scallop pattern will change in detail with time, then, statistical descriptions of scallop lengths should remain constant and bear some, unknown, relationship to the flute periodic length.

Writing the profile of a fluted surface $y$ as a function of the distance $x$ parallel to the flow direction from a flute
crest, and making the same assumptions as before, Curl obtains the result:

$$\frac{y}{\lambda} = \frac{1}{H_f} F(x/\lambda)$$

and concludes that all flutes have the same profile $y(x)$. At every point on the profile the rate of solution is dependent on the mass transfer rate at that point, so that for a stable profile propagating into the surface the mass transfer rates must also be a function of $x/\lambda$. Curl then derives constraints concerning the possible directions of propagation of the profile.

Let the slope of the surface at any point $x$ be denoted by $\phi$, and the direction of the propagation of the constant profile by $\Theta$ (Figure 1.1):

$$\tan \phi = \frac{dy}{dx}$$

the velocity of propagation normal to the surface at any point is denoted by $v$. This is related to the overall profile velocity $V$ in the direction $\Theta$ by

$$\frac{v}{V} = \sin (\Theta - \phi) = \sin \Theta \cos \phi - \cos \Theta \sin \phi$$

Between any pair of crests there are two inflection points, with $\phi_2$ and $\phi_1$, which limit the possible range of $\Theta$ since solution rates must be everywhere positive. Thus

$$\Theta \geq \phi_2 \quad \text{or} \quad \Theta \leq \phi_1$$

It can be shown that if $\Theta$ lies in the interval
\[ \frac{\pi}{2} - |\phi_2| < \Theta < \phi_1 + \frac{\pi}{2} \]

then there are two equal solution rate maxima in each profile. Curl argues that this is too fortuitous, and that therefore \( \Theta \) must lie outside this range. He favours a downstream propagation.

Curl discounts the possibility that an ideal flute profile can have a cuspat e ridge, but his argument appears unfounded in this respect. Neither does he consider the possibility of a cuspat e ridge with one inflexion point in the profile.

5. **Morphologically Similar Forms**

Scalloped surfaces bear a striking resemblance to other forms. Allen (1968) has studied erosional forms on mud, also described as flutes. It is intriguing to speculate that these forms may be the momentum-transfer equivalents of the scallops described here. Much systematic work remains to be done.

Sediment ripples and dunes are morphologically similar, with the same asymmetrical cross section. Genetically, there is no analogy between these forms and scallop and flute features since sediment ripples are depositional forms, resulting from the integration of individual sand particle movements. The hydrodynamics are similar, however, and studies of sediment ripple mechanics will therefore be examined in the ensuing chapter.
CHAPTER TWO

HYDRODYNAMICS OF SCALLOP FORMATION

The flow of a viscous fluid is described by the Navier-Stokes equations, which relate the parameters of a small unit of the fluid.

\[
\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \eta \nabla^2 v_x
\]

\[
\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \eta \nabla^2 v_y
\]

\[
\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \eta \nabla^2 v_z
\]

where \(v_x, v_y\) and \(v_z\) denote the component velocities in the \(x, y\) and \(z\) directions respectively. The other terms are

- \(\eta\) the fluid viscosity
- \(\rho\) the fluid density
- \(g\) the acceleration due to gravity
- \(h\) height above datum
- \(p\) hydrostatic pressure

\(\nabla^2\) denotes the operator \(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\)

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The process of scalloped formation has been described by Curl and others as follows. As the limestone wall recedes in response to solution, inhomogeneities are revealed in the rock. These may be in the form of voids, or insoluble fragments. In the case of snow, density variations are important also.

The sharp break in the smooth surface produces a separation of the fluid boundary layer downstream, and an associated eddy or set of eddies, as shown in Figure 2.1. Williams (1963) has shown experimentally that the boundary layer between the reversed flow of the vortex and the surface is thinner than in the undisturbed flow.

The inhomogeneity is of initial importance only. Once a pit has been created it forces a boundary layer separation by its own geometry and is permanently associated with a "bound" vortex.

Several experimental studies (Arie and Rouse, 1956) have been concerned with boundary layer separation downstream of a sharp break, or hydrodynamic "wall". But they have little relevance to this study, as they are not concerned with the relationships between vortex chains produced by unstable free boundary layers. Such instability has been studied theoretically by Michalke (1965) who provides a review of earlier theoretical and experimental work. We expect intuitively an analogy between the roll-up of a free boundary layer into a
Figure 2.1. Pattern of Fluid Flow Downstream of a Wall.

Point of Boundary Layer Separation

Point of Reattachment
vortex sheet and the chain of bound vortices of a scallop pattern. Thus Sato (1960) has shown that boundary layer roll-up can be excited by pressure variations from an acoustical source. The wavelengths of resulting vortex sheets show an inverse dependence on velocity, as Curl predicted for the flute pattern.

The presence of one vortex or scallop feature on a surface leads to the generation of others downstream. This phenomenon can be observed on surfaces eroded for a short period of time and not completely covered with scallops, as in the case of a roof collapse into a stream in a limestone cavern. Gaster (1965) has shown on theoretical grounds that periodic disturbances of a boundary layer lead to spatially growing waves downstream of the disturbance. Much experimental work has been concentrated on the same topic, following the studies of Schubauer and Skramstad (1948).

Approximate numerical solutions of the Navier Stokes equations are feasible with mechanical computation. Through the use of an advanced computer system, a team at Los Alamos has made considerable progress with the time-dependent equations, producing solutions to such problems as dripping faucets and breaking waves, (Marlow et al, 1965). Although the nature of the scallop problem requires the use of a much finer grid in a numerical solution, and hence far more computer time, it is possible in principle to obtain a numerical solution to the
problem. Harlow et al., using a grid of 1500 points in two dimensions, required twenty to sixty minutes to produce a single solution for one time interval.

To solve the scallop problem, the Navier Stokes equations must be combined with the diffusion equation to determine the rates of erosion at points on the surface, and with equations describing the kinetics of limestone solution. (See for example Weyl (1959), Kay (1957) and Curl (1968)).

It seems reasonable to expect anomalous behavior of macroscopic parameters of the fluid flow, such as the wall friction factor, at the appropriate value of the flute number $N_f$. There is some evidence to this effect, (Schlichting (1936), Wiederhold (1949), Seiferth and Kruger (1950)) although values of $N_f$ are lower than that expected from the Curl model.

Curl (1966) measured mass transfer rates directly on a simulated flute profile. Experiments were conducted in a narrow flume so as to approximate a two-dimensional system. A flute profile was formed in perspex from a cross-section copied from a cave wall, and nickel electrodes embedded in the profile. Alternating current measurements from these electrodes to a distant common anode in an electrolyte of moving dilute alkaline potassium ferrocyanide gave a measure of the mass transfer rates, (see Reiss and Hanratty, 1962).

The experiments revealed considerable variation in
instantaneous mass transfer rates, especially at high frequencies. Curl found that the mass transfer rates were strongly dependent upon the geometry of the flute profile, especially along the lee slope. It appears that values of \( N_f \) cannot be determined by this technique because the stable flute profile cannot be known in advance to sufficient accuracy. Experiments must be made on modifiable surfaces.

Rudnicki (1966) produced artificial scallop patterns using a rapidly modified material. Blocks of Plaster of Paris \((\text{CaSO}_4 \cdot \text{H}_2\text{O})\) were cast and immersed in streams flowing at different rates. Rudnicki does not expand on the validity of the analogy between plaster and limestone surfaces, though in terms of the Curl model quantitative comparison should be valid, since both processes are solutional. The analysis of the results was qualitative, showing a decrease in scallop size with increasing velocity. "Wielkosc form jest odwrotnie proporcjonalna do prędkosci przepływu". It is possible, however, to measure scallop sizes from the published photographs. These results and their analysis appear in a later chapter.

Kennedy has published a series of papers (1963, 1964) on the analysis of fluvial bedforms using an inviscid, irrotational approach. We may follow his analysis to the point where the mechanics of sediment movement are introduced, and substitute instead the laws of mass transfer.

We consider two-dimensional flow in the \( x-y \) plane,
with the x direction parallel to the surface and the y direction perpendicular. We first define an equation for a stationary wavy bed,

\[ h(x,t) = a(t) \sin kx \]

By the Fourier theorem this may be generalised to describe any waveform, as

\[ h(x,t) = \sum_{n=1}^{\infty} a_n(t) \sin nkx \]

where \( h(x,t) \) denotes the elevation of the bed at time \( t \) and at point \( x \).

In the case of inviscid, irrotational flow, the hydrodynamic equations may be written as

\[ \nabla^2 \phi = 0 \]

the function \( \phi \) being defined by \( \frac{\partial \phi}{\partial x} = U + u \), the velocity fluctuation \( u \) at any point in the x-direction superimposed on the main stream velocity \( U \), and \( \frac{\partial \phi}{\partial y} = v \), the point velocity in the y direction. To solve the problem, then, the function \( \phi \) must first be found.

Let us first define the boundary conditions. We require \( v = 0 \) at \( y = d \), a streamline remote from the wavy surface. Along this streamline \( u = 0 \) also.

If we consider the wavy bed as a streamline, we can write that at the mean level of the bed, \( y = 0 \),

\[ v = U \frac{\partial h}{\partial x} = U k \cos kx \]
provided that the fluctuation of ",a" with time is slow so that
\[
\frac{da}{dt} \ll U a k
\]
and provided that the bed is impermeable.

The velocity potential function \( \phi \) satisfying these boundary conditions is
\[
\phi = U x - U a \cos k (y - d) \cos k x
\]
which describes the velocities in the system.

By Fick's law, we have the relationships between mass transfer rates in the \( x \) and \( y \) directions and the solute density \( p \) thus:
\[
\eta_x = -D \frac{\partial p}{\partial x} + p (u + U)
\]
\[
\eta_y = -D \frac{\partial p}{\partial y} + p v
\]
where \( D \) denotes the solute diffusivity,
and \( \eta_x, \eta_y \) the mass transfer rates.

The requirement that solute be conserved in the system gives
\[
\frac{\partial \eta_x}{\partial x} + \frac{\partial \eta_y}{\partial y} = 0
\]
Combining these relationships and the hydrodynamic considerations gives
\[
-D \nabla^2 p + (u + U) \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} = 0
\]
If the periodic length \( 1/k \) is much greater than \( a \), we can ignore variations of \( p \) with \( x \) and write
\[
D \frac{\partial^2 p}{\partial y^2} = v \frac{\partial p}{\partial y}
\]
which is the one-dimensional diffusion equation.
Writing \( \gamma \) for \( d\psi /dy \) we have
\[
D \frac{d\psi}{dy} = -\gamma \frac{k u a \cos Kx \sinh K(x-y-d)}{\sinh Kd}
\]
which may be integrated thus
\[
D \log \psi = -\frac{u a \cos Kx}{\sinh Kd} \left( \cosh Kd -1 \right) + D \log A
\]
where \( A \) is a constant of integration.

Let the rate of loss of solute across the remote streamline \( y = d \) be constant. Thus at \( \gamma = a, \psi = B \). The mass transfer rate at the surface \( y = 0 \) is given by \( S \) where
\[
D \log S = -\frac{u a \cos Kx}{\sinh Kd} \left( \cosh Kd -1 \right) + D \log B
\]
\[
S = B \exp \left[ -\frac{u a \cos Kx}{D \sinh Kd} \left( \cosh Kd -1 \right) \right]
\]
To determine the stationary points of this function, we differentiate with respect to \( x \)
\[
\frac{dS}{dx} = B \exp \left[ -\frac{u a \cos Kx}{D \sinh Kd} \left( \cosh Kd -1 \right) \right] \frac{k u a \left( \cosh Kd -1 \right) \sin Kx}{D \sinh Kd} = 0
\]
Stationary points thus occur for \( \sin Kx = 0 \).

To determine the nature of the stationary points, we differentiate again
\[
\frac{d^2S}{dx^2} = \kappa \sin^2 Kx - \beta \cos Kx
\]
and note that for \( x = 0 \), \( S \) is maximum; for \( x = \pi \), \( S \) is minimum.

For a stable surface, we must be able to define a direction \( w \) in the plane \( x, y \) such that the profile is propagated through time without change in this direction. We require then an angle \( \omega \) for which
\[
S = \frac{S_{\text{max}} \cos \omega}{\cos (\omega + \omega)}
\]
where \( S_{\text{max}} \) is the maximum value of \( S \), and where

\[
\tan \alpha = \frac{dk}{d\xi} \quad S = S_{\text{max}} / (\cos \omega - \tan \alpha \sin \omega)
\]

Clearly the simple sinusoidal bedform is not stable.

Consider, however, the general bedform

\[
h(x, t) = \sum_{n=1}^{\infty} A_n \sin \lambda k x
\]

The potential function which satisfies the boundary conditions is now

\[
\phi = U k - \sum_{n=1}^{\infty} \left[ U a_n \frac{\cosh \lambda k (y_d - d) \cos \lambda k x}{\sin \lambda k \lambda k d} \right]
\]

The solution for \( S \) now reads

\[
S = B \exp \left[ -\frac{U}{d} \sum_{n=1}^{\infty} a_n \cos \lambda k x \frac{(\cosh \lambda k d - 1)}{\cosh \lambda k d} \right]
\]

For a stable flute profile we require a constant \( \lambda \) such that

\[
S_{\text{max}} / (\cos \omega - \tan \alpha \sin \omega) = S
\]

This equation is satisfied by a certain set of \( a_n \)'s, and we conclude that there is a stable profile in this model.

Further, its direction of propagation is, intuitively, downstream.

This inviscid, irrotational model above is of course unrealistic. No account is taken of separation effects or the effects of the fluid boundary layer. It is however useful to note the conclusions. In any real system an inviscid, irrotational model represents a first approximation whose
validity can only be judged by comparison of its conclusions with those of observation.
CHAPTER THREE

FLUME SIMULATION AND ANALYSIS OF SCALLOP LENGTHS

1. Flume Design

Embodied in this chapter are design details and operating techniques used in a laboratory flume in which the formation of scallops was simulated under controlled conditions. The Plaster of Paris - water system was chosen for this simulation for various reasons. Plaster of Paris is soluble in water, requiring no addition of acid or any other substance to the fluid. The solution rate is such that a scalloped surface can be produced in a few hours at all but the slowest operating speeds. Its greatest advantage over other substances is that it can be cast in situ. It proved essential to provide a very tight fit between cast blocks and the flume walls because the high hydraulic gradients present rapidly enlarged any alternative paths for the water. Plaster of Paris is cheap and available in a high degree of purity.

Water was used as a fluid because the anticipated relationship between velocity and scallop size indicated that
for similar velocities a liquid would give scallops of more reasonable size than a gas. Gases can be used in heat transfer systems, but close control is required of gas and solid temperature and consequently more complex equipment is necessary.

Flume cross section, velocity range and anticipated scallop sizes are all interrelated. The independent parameter considered was therefore the capacity of the pump in a recirculating flume. Only a recirculating device could continue to operate for sufficient time periods and with sufficient stability of water velocity during those periods. A pump capacity of 4 cfs was considered the most reasonable in view of cost, since such a volume could be delivered by a propeller set in the flume and powered by a small electric motor.

With this projected capacity, it was anticipated that a flume of a square cross section of one square foot with velocities up to 4 ft/sec would produce scallops of a size much less than the width of the flume, but of sufficient size to be easily measurable. Thus Curl's assumption that the conduit dimensions be much greater than the flute wavelength was satisfied. A motor of one quarter horse power could produce such velocities, lower speeds being achieved by gear reduction.

The scallop phenomenon is a function primarily of the
fluid boundary. The presence of secondary currents and
turbulence in the main fluid stream is not directly rele-
vant and has no serious effect on the resulting pattern.
Thus the flume need only be designed with sufficient length
to ensure an adequate working area, and the efficient opera-
tion of the propeller drive. Figure 3.1 shows a plan of
the McMaster flume with dimensions. The working area was
constructed of one quarter inch perspex sheet, and the re-
mainder of plywood. The entire inside surface was coated
with epoxy resin to prevent leakage.

Figure 3.2 shows elevations of the flume at various
points. Plates 3.1 and 3.2 show the completed structure.

The drive was obtained through the lower section of
an outboard motor. This was driven from an electric motor
by a belt, velocity variation being achieved by the use of
a selection of drive wheels on the outboard-drive shaft.
The drive system is shown in Plate 3.3.

Because of the low solubility of Calcium Sulphate in
water (0.2 gm/100 ml), continuous interchange of the flume
water is necessary. Input of fresh water was continuous
via the input orifice. Water was removed from the flume
intermittently and at a faster rate, controlled by the level
control. Water was pumped from the output orifice, located
in the bottom of the flume to permit draining after each
experiment. The pump used was a 1/4 hp flexible liner pump
Figure 3.1. Plan of the Flume to Scale 1:24
Plywood Section. Raised 1" from Floor with Wood Blocks.

Perspex Section.

Corners Bolted with 1" Angle Iron

Figure 3.2. Flume Cross Sections. Scale 1:12.
Plate 3.1. The Experimental Flume.
Plate 3.2. Working Section of the Flume.
Plate 3.3. Flume Drive System.
which had the advantage of being unaffected by air locks. However, flooding did occur on occasion when the pump was accidentally shut off.

Flume level was controlled by a float in a tank linked to the flume itself by 9 ft of 1/4 inch hose, thus ensuring full damping of minor short term level fluctuations caused by the drive. A microswitch activated by the float controlled the pump directly. A second microswitch turned on the drive automatically during filling as soon as the water level reached sufficient depth. (Figure 3.3)

Problems were experienced during operation of the flume with this drive system. The propeller operates less efficiently when partially out of water. Long waves can thus build up in the flume as soon as any level fluctuation develops. The periodic length of these waves is equal to the complete circuit made by the water. Only a small range of water levels will produce long waves. Deeper water keeps the propeller completely immersed at all times. More shallow water prevents any complete immersion.

Blocks were cast in the flume using a wooden mould, which was removed as soon as the material had set. This procedure ensured a complete fit between the plaster and the flume walls. The plaster was cast initially in the form of a rectangular block with a short taper on the leading edge. (Figure 3.4) This was found to give excessive hydraulic
Figure 3.3. Details of the Level Control. Scale about 1:2
First Block Geometry Used

High Gradient Block Geometry

Low Gradient Block Geometry

Figure 3.4. Profiles of the Various Experimental Blocks
gradients leading to the development of subsidiary channels between the block and the flume walls. In the experiments this was avoided by the use of a wedge shaped block as shown. The resultant velocity variation was taken into account in the analysis as described below.

The plaster used initially was of a grade used in interior decorating, with a high sand content. It was found that a flat surface of this material is totally stable, generating no scallops when eroded by running water. Grooves were cut in the surface transverse to the water flow in an effort to initiate scallop formation but these were rapidly eradicated unless greater than 1/8 inch in depth, in which case scallops developed but no downstream chains were produced.

Due to the high sand content of the plaster, accumulations of sand form in any pits on the surface, reducing the rate of solution there to such an extent that the pit eventually disappears. Thus the initial stage of scallop formation is destroyed, and no stable features can form. Only pits of depth greater than 1/4 inches can remain clear of sand and form scallops.

The substitution of analytic grade Plaster of Paris, Fisher Scientific Company Catalog C-138, removed this problem entirely and produced excellent scallop patterns. But since even the analytic grade plaster used has a small insoluble
residue, traps were installed upstream of the plaster block to remove as much as possible from the flume bedload. These consisted of small wooden wedges fixed transverse to the flow, behind which the sand accumulated. Accumulations also formed behind the experimental block.

In addition to fluid velocity, fluid temperature was varied as an independent parameter in the experiments. The viscosity of water, which appears in the formulation of Curl's theory, varies rapidly with temperature, decreasing at a rate of 10 per cent for every 5°C rise in temperature in the range 0 – 30°C.

Flume temperature was varied by altering the temperature of the inlet water. A thermostat was installed in the flume, and used to control the input of either hot or cold water, depending on whether the flume temperature was below or above that desired, respectively. This was achieved through the operation of a pair of solenoid-operated faucets, adapted from washing machine inlet valves. The complete control system is shown in Figure 3.5. Water temperature was measured independently using a thermocouple embedded in the wall of the flume at a point shown in Figure 3.1. This was coupled to a recording potentiometer and a junction maintained at thermostatically controlled temperature. The charts showed that the flume temperature varied by 0.2°C during operation, due to the operation of the thermostat.
Figure 3.5. Flume Control System.
Local velocities were measured using a Pitot tube constructed as shown in Figure 3.6. The pressure difference between the two orifices of a Pitot tube, one aligned perpendicular to and one parallel to the flow, gives a measure of fluid velocity. At 100 percent efficiency, in an inviscid system, the pressure difference is given by the Bernoulli equation

\[ p = \frac{\rho v^2}{2} \]

Pressure difference can be measured by a manometer, in which case the difference in height \( h \) of the fluid on the two sides of the U-tube is given by

\[ h \rho g = p \]

where \( \rho \) is in this case the density of manometer fluid.

Because of the low velocities being measured and hence the small pressure differences, Carbon Tetrachloride was used in the manometer, the rest of the system being water-filled. This gave an effective density of about 0.1 gm/cc, and hence a high sensitivity to the device.

The Pitot tube was calibrated in the flame, in the absence of an experimental block, against a Gurley flow meter of the rotating cup type, (Plate 3.2), fixed at the point shown in Figure 3.1. The calibration curve is shown in Figure 3.7 for each operating temperature.

Once calibrated, the Pitot tube was used to measure velocity profiles over the experimental block. One such pro-
Figure 3.6. Pitot Tube Construction.
Figure 3.7. Pitot Tube Calibration Curves for Separate Operating Temperatures.

Velocity, fps

Manometer Level Difference, cms
file is shown in Figure 3.8. Velocities used in the analysis of results were obtained separately for each experiment, at 3 cms above the plaster surface.

Formation of scallops was observed by time lapse photography. A control unit was constructed to run from a 12 volt DC power source and expose one frame of 8 mm film every 5 minutes via a cable release. The control unit is shown in Figure 3.9. The camera used was an Elmo C-200 using 8 mm Fujichrome Single-8 colour film. The camera work indicated that Plaster of Paris scallops migrate slowly downstream, a result anticipated by Curl but conflicting with the predictions of other workers.

The operation of the control unit is as follows.
The minute hand of a 12 volt DC electric clock wipes contacts every 5 minutes. This contact switches RL1, discharging the 100 microfarad capacitor through the coil of RL2. The time constant of the RL2 capacitor circuit is such that RL2 is switched on for 1/4 second. This completes the motor circuit for sufficient time to allow the wiper arm illustrated to complete a full revolution, pushing a cable release connected to the camera.

2. Analysis of Flume Results

Because of the variation in velocity over the experimental block, a complex method of analysis was used. The length of each scallop was measured, together with its dis-
Figure 3.8. Velocity Contours at 3.15 fps over the High Gradient Block

(Pitot Tube Readings in Centimetres)
Figure 3.9. Time Lapse Camera Control System.
tance from the downstream end of the block. The length was defined as before as the maximum distance between ridges, parallel to the fluid flow. In cases where scallops were formed as isolated individuals, referred to as an immature surface, the distance measured was that which gave the scallop a similar cross section to a scallop on a mature surface. This method is admittedly approximate: measurement of immature scallops was avoided wherever possible, but was necessary in the case of surfaces formed at low temperatures and low velocities, which would otherwise have taken an unreasonable length of time to reach maturity.

Velocity profiles over the experimental blocks showed that acceleration was uniform (Figure 3.10). Accordingly a line was fitted by regression to the scallop length observations with respect to distances from the downstream end of the block, and the intersection of this line with the axis corresponding to zero distance used as the characteristic mean scallop length in the analysis. The velocity used was the extrapolated velocity at this point.

This procedure assumes only that variation in scallop lengths over the block surface is a linear function of the distance from the end of the block. It does not imply the assumption of a linear relationship between size and velocity, nor does it preclude an absence of systematic variation over the block.
Figure 3.10. Velocity Profiles at the Four Operating Speeds over the High Gradient Block

Velocity, fps

Drive Ratio 1

Drive Ratio 2

Drive Ratio 3

Drive Ratio 4

Distance from Downstream End of Block  cms

10  20  30  40  50  60  70
Experiments were made at four different velocities as shown in Table 3.1. The combination of four velocities and three temperatures, approximately 10, 20 and 30°C, gave twelve sets of results. The temperatures and corresponding viscosities are shown in Table 3.1. In addition to the mean scallop length, the 10th, 20th, 30th through 100th percentiles of the corrected scallop length distributions were calculated after each scallop length had been adjusted according to its distance from the end of the block. Figure 3.11 shows a sample histogram before and after application of this procedure.

It was hypothesised that in addition to velocity and viscosity, acceleration behaved as an independent variable affecting the resulting scallop pattern. Accordingly the experiments were repeated with two different block geometries as shown in Figure 3.4, giving 24 experimental runs in all. This was reduced to 23 because the generation of scallops at the lowest temperature and velocity on a low gradient block proved excessively slow. The duration of each run was between 18 hours and 4 days, erosion proceeding more slowly at low velocities and low temperatures.

Sample size varied according to the conditions. Most samples were of the order of 100 individuals, but sample size was as low as 7 for the low velocity, low temperature run. The full experimental data is included in Appendix 1.
### Table 3.1. Flume Operating Conditions

<table>
<thead>
<tr>
<th>Drive Ratio</th>
<th>High Gradient Block</th>
<th>Low Gradient Block</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.08</td>
<td>3.54</td>
</tr>
<tr>
<td>2</td>
<td>3.60</td>
<td>2.88</td>
</tr>
<tr>
<td>3</td>
<td>3.13</td>
<td>2.53</td>
</tr>
<tr>
<td>4</td>
<td>2.52</td>
<td>2.04</td>
</tr>
</tbody>
</table>

### Flume Temperatures and Corresponding Viscosities

<table>
<thead>
<tr>
<th>Temp. °F</th>
<th>Temp. °C</th>
<th>Viscosity (cgs units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0</td>
<td>10.0</td>
<td>0.01310</td>
</tr>
<tr>
<td>70.0</td>
<td>21.1</td>
<td>0.00988</td>
</tr>
<tr>
<td>90.0</td>
<td>32.2</td>
<td>0.00761</td>
</tr>
</tbody>
</table>
Figure 3.11. Histograms of a Scallop Sample Before and After Correction for Position on Block.
The eleven length parameters derived from each experiment, that is, the arithmetic mean length and the percentiles 10 through 100 of the length distribution, were compared to the variables velocity, acceleration and viscosity. In particular, the model

\[(\text{length parameter})^a (\text{velocity})^b \text{ is a function of} (\text{viscosity})^c\]

determined that scallops are the result of a breakdown of an ideal flute pattern due to fluctuation in controlling parameters: and that the distribution of scallop lengths is related to the flute length through one or more of its properties.

The model was evaluated by multiple regression, using a logarithmic transformation to linearize the equation

\[
\log y = a_0 + a_1 \log x_1 + a_2 \log x_2 + a_3 \log x_3
\]

Acceleration was quantified on a nominal scale by a value of 1.0 for the first, high gradient block geometry, and 2.0 for the low gradient. Eleven models were run, using the different parameters of length in turn.

The relevant parameters of the regression analysis are given in Table 3.2. The particular multiple regression technique used inserts variables in order of the ratio of variance before and after insertion.
Table 3.2 Regression Analysis of the Flume Data

<table>
<thead>
<tr>
<th>Length Parameter</th>
<th>Pure Velocity Constant $a_0$</th>
<th>Velocity Coefficient $a_1$</th>
<th>Viscosity Coefficient $a_2$</th>
<th>Standard Error of Estimate</th>
<th>Multiple Correlation Coefficient</th>
<th>Fraction of Variability Accounted for</th>
<th>Order of Insertion</th>
<th>Constant of Constrained Model $y = ax_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.149</td>
<td>-0.9740</td>
<td>1.2997</td>
<td>0.0537</td>
<td>0.9353</td>
<td>0.8747</td>
<td>2 1 3</td>
<td>11476</td>
</tr>
<tr>
<td>10th Percentile</td>
<td>3.464</td>
<td>-1.1981</td>
<td>1.4824</td>
<td>0.0811</td>
<td>0.9270</td>
<td>0.8594</td>
<td>2 1 3</td>
<td>8187</td>
</tr>
<tr>
<td>20th Percentile</td>
<td>3.540</td>
<td>-1.1692</td>
<td>1.4967</td>
<td>0.0699</td>
<td>0.9343</td>
<td>0.8729</td>
<td>2 1 3</td>
<td>9457</td>
</tr>
<tr>
<td>30th Percentile</td>
<td>3.550</td>
<td>-1.1999</td>
<td>1.4753</td>
<td>0.0712</td>
<td>0.9203</td>
<td>0.8469</td>
<td>2 1 3</td>
<td>10404</td>
</tr>
<tr>
<td>40th Percentile</td>
<td>3.526</td>
<td>-1.1075</td>
<td>1.4713</td>
<td>0.0649</td>
<td>0.9275</td>
<td>0.8602</td>
<td>2 1 3</td>
<td>11026</td>
</tr>
<tr>
<td>50th Percentile</td>
<td>3.313</td>
<td>-1.0319</td>
<td>1.3701</td>
<td>0.0575</td>
<td>0.9331</td>
<td>0.8707</td>
<td>2 1 3</td>
<td>11508</td>
</tr>
<tr>
<td>60th Percentile</td>
<td>3.213</td>
<td>-0.9669</td>
<td>1.3233</td>
<td>0.0614</td>
<td>0.9199</td>
<td>0.8461</td>
<td>2 1 3</td>
<td>12148</td>
</tr>
<tr>
<td>70th Percentile</td>
<td>3.005</td>
<td>-0.8949</td>
<td>1.2250</td>
<td>0.0589</td>
<td>0.9157</td>
<td>0.8385</td>
<td>2 1 3</td>
<td>12631</td>
</tr>
<tr>
<td>80th Percentile</td>
<td>2.983</td>
<td>-0.8366</td>
<td>1.2008</td>
<td>0.0572</td>
<td>0.9154</td>
<td>0.8380</td>
<td>2 1 3</td>
<td>13226</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>2.836</td>
<td>-0.8346</td>
<td>1.1264</td>
<td>0.0606</td>
<td>0.9004</td>
<td>0.8107</td>
<td>2 1 3</td>
<td>14192</td>
</tr>
<tr>
<td>100th Percentile</td>
<td>2.519</td>
<td>-0.4872</td>
<td>1.0075</td>
<td>0.0651</td>
<td>0.8672</td>
<td>0.7522</td>
<td>2 1 3</td>
<td>16947</td>
</tr>
</tbody>
</table>
The weight attached to the acceleration term is very low. This may be due to either of two factors. First, acceleration may play no part as an independent term in the formation of scallops. Secondly, the actual value of the corresponding coefficient may be so low that differences in scallop measurements due to this factor are masked by sampling error and the probabilistic nature of the distributions themselves.

The form of variation of multiple correlation coefficients between the different models is as anticipated. Mean length is a significantly better variable with which to explain variations in scallop populations under varying conditions.

Figures 3.12 and 3.13 show two sections of the data. It is graphically impossible to illustrate the entire model at one time. The line corresponding to Curl's Flute Reynolds Number is also shown for comparison. By substituting mean scallop length for the Flute Reynolds Number, the value of the constant in the Curl model is much reduced. Since the values of the set of coefficients "a" were not constrained to those appropriate to a linear dependence on the variables, the precise values of these constants were not calculated under this assumption. The actual values of $a_1$ and $a_2$ selected by the multiple regression procedure are shown in Table 3.2. A value of 1.0 is expected in the case of linear
Figure 3.12.
Mean Scallop Lengths at 70 F,
High Gradient Block
Figure 3.13.

90th Percentile of the Corrected Length Distribution at 50 F, Low Gradient Block.
dependence on the variable. The values determined differ from 1.0 less in the case of velocity, based on four different values, than that of viscosity, which was only based on three.

A second model of the form \( y = ax_2/x_1 \) was investigated. This model is less critical of the theory, but enables estimates of the constant "a" to be made. The constant computed was that which gave the least square deviation from the observed scallop length parameters, according to the formula:

\[
\alpha = \frac{\sum (y_{x_2}/x_1)}{\sum (x_2/x_1)}
\]

The derivation is given in Appendix 2.

The values of the constant thus computed are given in Table 3.2. It would be useful to compare these values with an accurate determination of Curl's Flute Reynolds Number. Much light would then be cast on the relationships between scallops and flutes. Conceivably the maximum length in the scallop population corresponds to the flute periodic length. Although comparison suggests that no scallop in the population approaches the flute period, the determination of the Flute Reynolds Number could certainly be improved.
CHAPTER FOUR

COMPLETE DESCRIPTION OF

A SCALLOP SURFACE

Discussion has been concerned principally with the length dimension of an individual scallop, as defined above. The nature of the relationship between this parameter and the Flute Reynolds Number has been investigated. However, the distribution of scallop lengths represents only a very small part of the information contained in a scallop pattern.

The length of a scallop on a mature surface is in a sense the distance over which the scallop profile repeats itself. A flute pattern exhibits perfect periodicity parallel to the flow direction, in that the flute profile is identical for every feature. Transverse to the flow direction there is no repetition. A scallop pattern is far more complex. There is repetition in both directions, and the repetition is not perfect. Chains of repetition exist over short distances, broken by shorter or longer features.

Two functions are commonly used to describe such patterns in any number of dimensions. Wave patterns, stock market fluctuations and many other phenomena have been
investigated in this manner. Both functions describe the amount of repetition of different periods or wavelengths in the phenomenon.

The Autocorrelation function is defined as

\[ C_1(d) = \int_{a} F(x) F(x + d) \, da \]

where \( da \) is an infinitesimal portion of \( n \)-dimensional space displaced from the origin by \( x \). \( G(d) \) describes the amount of repetition over a distance \( d \) found in the function \( F \). \( d \) can be in any direction, and \( G \) has clearly the same dimensions as \( F \).

The Spectral Density function is defined as the Fourier transform

\[ \xi(k) = \int_{a} F(x) e^{-i \frac{k}{\lambda} x} \, da \]

This function is given in terms of the vectorial wave number \( k \) which is related to the previously defined \( d \) by

\[ d = \frac{2\pi}{k} \]

The Autocorrelation function of a perfect wave is itself a perfect wave. The Spectral Density function is a series of sharp peaks at integral multiples of \( \frac{2\pi}{d} \).

Both functions may be evaluated by optical analogs.
A diagram of the autocorrelator constructed is given in Figure 4.1. Two identical photographic negatives of the scallop surface are placed in omni-directional white light. Consider a beam of parallel rays as shown in Figure 4.1. These will be diminished in passing through the first negative by an amount proportional to the light reflected from the surface during photography, which we will call $A(x)$. All the parallel rays will have suffered the same displacement upon reaching the second plate, so that the intensity after passing through the second plate will be $A(x) A(x+\delta)$. These parallel rays are brought to a focus by the convex lens, so that the total light reaching the focus is

$$
\int_{-\infty}^{\infty} A(x) A(x+\delta) \, d\lambda
$$

Now for every value of $\delta$ there corresponds a point on the final photographic plate, so that upon developing this plate the Autocorrelation function is displayed.

A pattern of equally spaced stripes was constructed for alignment purposes. It was found necessary to use high-contrast film, and to control exposure very accurately. Plates 4.1 and 4.2 show the autocorrelograms of this test surface on HP3 and high-contrast Kodalith Ortho Type 3 respectively, using an exposure of one second in each case.

Both functions are evaluated for an isotropic surface. Velocity variations over the experimental blocks could
Figure 4.1. Construction of the Autocorrelator.

4"x5" Photographic Plate in Focal Plane

Convex Lens
not be taken into account in the autocorrelator. Accordingly the flow pattern was modified by closing the surface of water in the flume in the manner shown in Figure 4.2 to give a uniform flow rate. This modification severely limited the velocity range of the flume since operation at high velocity produced a back-up upstream of the pinched conduit and severe flooding of the immediate area. Several runs were made however at different temperatures and velocities.

Surface 1001, produced at 35°C and about 3.5 feet per second is shown in Plate 4.3. The Autocorrelogram is reproduced in Plate 4.4. The central peak, corresponding to \( d = 0 \) and thus to

\[
\int_a A(x)^2 \, dx
\]

is broadened. The brightest areas on the negative correspond to the shadows on the photograph, whose finite area is responsible for this broadened central peak. The two nulls on either side indicate that there is no repetition of this length parallel to the flow direction. The scallop lengths measured correspond to a point beyond the nulls in the grey area.

Surface 1003 was produced at 20°C at about 2.0 feet per second, and photographed at the same distance and with identical illumination (Plate 4.5). This surface is less mature. The Autocorrelogram is shown in Plate 4.6 and
Plate 4.1. Autocorrelogram of a Test Pattern of Parallel Stripes Recorded on Ilford HP3 Film
Plate 4.2. The Test Autocorrelogram Recorded on Kodalith Ortho Type J.
Figure 4.2. Experimental Design to Eliminate Acceleration over Block
Plate 4.3. Scallop Surface 1001.
4.4. Autocorrelogram of Scallop Surface 1001.
4.5. Scallop Surface 1003.
4.6. Autocorrelogram of Scallop Surface 1003.
appears to lack the nulls above and below the central peak. The peak is much enlarged and reduced in amplitude. The Autocorrellogram indicates that scallop features are isolated, the degree of spatial structuring being low.

The Spectral Density function can also be produced optically, and is more sensitive since periodicity is indicated by absolute rather than relative values of the function. The Fraunhofer diffraction pattern of an image is the Fourier transform of the image, and hence the Spectral Density function.

Scallop pattern negatives were illuminated by coherent light from a Helium-Neon laser as shown in Figure 4.3. Because of the scale of the surface relative to the wavelength of red light, patterns were extremely difficult to photograph. The Fraunhofer diffraction pattern of surface 1001 is shown in detail in Figure 4.4 as observed on a ground glass screen. The central peak represents the mean

\[ \int A(\chi) \, d\chi \]

The nulls extending outwards across the photograph indicate no repetition perpendicular to the flow direction. Parallel to the flow there is a distribution of repetition lengths until a cut-off point corresponding to the nulls of the Autocorrellogram. Geometrically the two functions are shown in a reciprocal fashion, since points further from the origin
Figure 4.3. Diffraction System to Produce Spectral Density Functions.
Figure 4.4. Spectral Density Function of Scallop Pattern

Mean

Heavy Repetition of Pattern Parallel to Flow Direction

Little Repetition Perpendicular to Flow

Flow Direction
of a spectral density function density function correspond to smaller distances.

Both functions analyse the geometric relationship between the light and dark areas of the negatives. These are related by a cosine function to the slope of the surface itself, with the added complexity of shadow areas. The slope is related in turn to the relief.

These methods reveal little that is unexpected. The precise values of the functions are controlled too much by inherent variation in the photographic materials and in the illumination of the surface to be of quantitative use. These problems can of course be overcome by digital analysis using complex and expensive machinery to record the relief of the surface, and extensive computer use to perform the analyses, (Nordin and Algert, 1966).
CHAPTER FIVE

FIELD STUDIES OF SCALLOPED LIMESTONES
AND GYPSUMS

1. Relationships to Environmental Parameters

Two lines of investigation are embodied in this section: firstly, a demonstration that the use of Plaster of Paris as a medium with which to simulate the formation of scallops on carbonate rocks is valid, and secondly that the laws demonstrated on the plaster apply equally to limestone. Philosophically there is little distinction between these objectives: the difference is more one of experimental design.

The presence of scallop-like features on Gypsum and Alabaster formations has been described in the literature. Bretz (1952) has published a geomorphic analysis of Alabaster Caverns, Woodward County, Oklahoma, which are developed in the Blaine Formation of Permian Age. He notes the presence of current flutes on the walls and floor of a small trench carved through the caverns by a contemporary vadose stream. In classifying current flutes or scallops as characteris-
tically vadose features Bretz is mistaken, since these features are frequently found on cave roofs. Bretz himself earlier noted (1942) that "Flutes cannot be used to prove that vadose conditions existed at the time of formation." But there appears to be some variance between Bretz's understanding of the terms vadose and phreatic, and the current definitions involving the water table. These are simply that vadose processes of cavern development take place above, and phreatic processes below, the water table which may be defined as that surface below which water exists at a pressure greater than one atmosphere. The surface need be neither continuous nor unique at every location.

Bretz describes the cutting of the ceiling channel or inverted trench in Alabaster Caverns by a stream flowing on top of sediment against the cave ceiling as a vadose process. Yet in his classic paper on speleogenesis (Bretz, 1942) a vadose stream is defined as having a free surface. It appears more likely that in Alabaster Caverns Bretz was using the term "vadose" to indicate a rapid stream flow in a conduit, while reserving the term "phreatic" for slow ground-water circulation in a disconnected network of capillaries and cavities.

Phreatic pockets are found frequently in the main passage of Alabaster Caverns, where the intersection of a
joint or bedding plane with the cave passage has permitted the removal of an exceptional amount of material. These pockets are asymmetrical, demonstrating a flow of water through the cave during their formation. It is interesting to note that in July 1968 the Superintendent of Alabaster Caverns State Park referred to these features as "flutes" and remarked on the relationship between asymmetry and fluid flow, whilst ignoring the much smaller and more frequent features noted by Bretz.

McGregor et al (1962) describe gypsum caves in North Central Texas in the same formation. They note that fluting of the walls, ceiling and floor is present in many of these caves. Flutes are assigned a vadose significance, with reference to Bretz (1942) despite the quotation from that paper above.

Plates 5.1 and 5.2 show scallops formed on crystalline gypsum beds and on alabaster beds in Pothooks Cave, located on the Sam Vest ranch, 25 miles north of Childress, Texas. The scallop surface is developed independently of the coarse rock structure, which is on a somewhat larger scale. Scallops on gypsum obey the asymmetry rule of limestone scallops. Where a wall protrudes into the inferred stream flow, scallops are shorter on the upstream side of the protrusion and longer on the downstream side.

In view of the time scale of limestone erosion it
Plate 5.1. Scallops Formed on Gypsum Beds in Pothooks Cave, Childress County, Texas.
Plate 5.2. Scallops Formed on Gypsum Beds in Pothook's Cave, Childress County, Texas.
is virtually impossible to produce limestone scallops under controlled conditions. Under natural conditions, the stream velocity and viscosity are observable only at one point in time, and therefore may be entirely atypical of the conditions which led to the generating of a limestone scallop surface.

However, the rate of erosion of a limestone surface increases very rapidly with stream velocity at the velocities typical of alpine streams; such that nearly all limestone erosion is done under summer flood conditions. With this in mind, a block of Plaster of Paris was placed in a vadose stream in an effluent cave known as Raspberry Rising, in Glacier National Park, British Columbia, in August 1966. Scallops are found on the walls of this cave both above and below water level. After two days the block was removed and the scallop features on the block surface compared with those on the cave wall below water level, (Plate 5.3). In view of the strong current, (of the order of 6 ft/sec), the block had to be cast in a wooden frame to provide an adequate anchor. The disruption of flow around the edges of the block meant that only features toward the centre of the surface could be regarded as typical of the stream flow conditions. A sample of 52 such features had a mean length of 2.26 cms with variance 0.243 cms$^2$, while a sample of 33 limestone scallops on the cave wall had a mean length of
Plate 5.3. Scallops Generated on Plaster of Paris for Comparison with Those on Limestone in the Same System.
2.43 cms with variance 0.159 cms$^2$.

Comparison of the width measurements was not so encouraging. The mean width of the plaster surface was 2.28 cms while that of the limestone was 3.22 cms. However, this discrepancy is to be expected in terms of the Curl model, which sees limited width as the result of increased turbulence and instability of the flow downstream of the edge of the plaster block.

The work of Rudnicki has been referred to above, (Rudnicki, 1960). Rudnicki analysed the patterns in terms of the number of features per square decimeter, concluding that features became smaller as velocity increased. Since this parameter depends both on scallop lengths and scallop widths, measurements were taken directly from the published photographs. These appear in Table 5.1. The constants were computed using the mean length and the viscosity of water at 70°F.

Very little importance should be attached to these results since the measurements were made from poorly reproduced photographs which tended to obscure the smaller features.

Since no information is available on water flow conditions during the generation of limestone scallop surfaces, tests of the relationships between these must be by inference only. In general, only the ratios of water velocities bet-
Table 5.1. Data from Rudnicki's Plaster Experiments

<table>
<thead>
<tr>
<th></th>
<th>Velocity (cms/sec)</th>
<th>Mean Length (cms)</th>
<th>Sample Size</th>
<th>Scallopc constant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75-85</td>
<td>1.31</td>
<td>10</td>
<td>14480</td>
</tr>
<tr>
<td></td>
<td>250-300</td>
<td>0.79</td>
<td>25</td>
<td>19750</td>
</tr>
</tbody>
</table>
ween different sites can be inferred, so that direct numerical tests are impossible. We can only test the forms of functional relationships.

In a long section of cave conduit of constant cross section, friction factor, stream volume and slope or gradient, the velocity of a stream is such that the gravitational forces balance the frictional forces. Change in any of the controlling parameters will result in a change in stream velocity through acceleration or deceleration of the stream. Thus if we consider long straight sections of conduit of constant slope, geometry and friction characteristics and then compare the velocities of the same stream in sections of differing slope, we will find that the velocity is a function of slope alone.

In the Nakimu Caves, Glacier National Park, British Columbia, a complex of conduits has been carved since the last glaciation by the subterranean Cougar Creek. This complex represents successive captures of the same stream and has been the result of predominantly vadose development, (Ford et al, 1966). Scallops were measured in straight sections of these conduits of constant cross section and slope. The conduits are formed along bedding planes, connected by joint-controlled chutes, so that the assumption of constant friction factor is reasonable. Scallops were measured near the low point of the passage cross section, since these
features are most typical of the currently observed passage geometry. All sample sizes were greater than 25. Slopes of passages were measured over distances of 25 feet around the sample sites, using an Abney level.

The mean length was calculated for each of the 18 samples and correlated with the sine of the slope angle by least-squares regression analysis. The data is shown in Figure 5.1 plotted on a linear scale. The regression parameters are as shown in Table 5.2. The model giving the highest per cent explanation of the data is the last listed, with regression equation

\[ \log_{10} \text{Length} = -0.7343 \log_{10} \text{Slope} + 0.4240 \]

Since we expect that velocity is a monotonically increasing function of slope, we can deduce that length is a monotonically decreasing function of velocity. Alternatively, if we assume Curl’s law that \( \frac{\lambda v f}{\gamma} = \text{constant} \), we can deduce that since the regression equation indicates a power law relationship between length and slope,

\[ \text{Velocity} = B \text{ (Slope)}^{c} \text{ where } c = 0.7434 \]

By assuming a value for Curl’s constant, the hydraulic channel law could be deduced.

In a phreatic passage there is no such relationship between velocity and slope. Velocities may be positive even with negative slopes. However, velocities may be inferred...
Figure 5.1. Nakimu Vadose-Flow Data. Mean Scallop Length Against Conduit Slope
Table 5.2  Regression Analysis of Nakimu Data. Variation of Mean Scallop Length with Passage Slope Analysed by Various Models.

<table>
<thead>
<tr>
<th>X Variable</th>
<th>Y Variable</th>
<th>Correlation Coefficient</th>
<th>Per Cent Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>Length</td>
<td>0.6781</td>
<td>46.0</td>
</tr>
<tr>
<td>Slope</td>
<td>Log Length</td>
<td>0.7755</td>
<td>60.1</td>
</tr>
<tr>
<td>Log Slope</td>
<td>Length</td>
<td>0.7206</td>
<td>51.9</td>
</tr>
<tr>
<td>Log Slope</td>
<td>Log Length</td>
<td>0.7968</td>
<td>63.2</td>
</tr>
</tbody>
</table>
from cross sections by means of the continuity condition
\[ Q = vA \]
where \( Q \) is the mean stream discharge, \( v \) is the mean stream velocity and \( A \) is the cross-sectional area.

Two experiments were performed using this relationship. In the first case, in Glen Park Labyrinths, Water- town, New York, flows were inferred in the links of a hydrologic network by scallop and cross section measurements and then the continuity condition was applied at each junction. Figure 5.2 shows a schematic plan of the system.

This cave system is a fine example of passage control by a system of rectangular joints. Plate 5.4 shows a link in the network, developed from the intersection of a strongly defined bedding plane at floor level and the vertical joint. Development downwards has been restricted by the sediment on the passage floor.

The geometry of the cave indicates that the network was determined under conditions of slow flow of phreatic water captured from the nearby chute of the Black River. However, the contemporary walls carry scallops indicative, on the basis of previous work, of high flow rates. These flow rates can only have been achieved during the last stages of the cave's development since the overall plan of the cave has not been modified appreciably by them.

At least 25 scallops were measured at each of the
Figure 5.2. Glen Park Labyrinths Schematic Plan

△ Sample Site

→ Inferred Direction of Flow
Plate 5.4. Measuring the Cross Section of a Passage in Glen Park Labyrinths, Watertown, New York.
sample points shown on the diagram. The volumes of water input to and output from each junction are shown in Table 5.3. The units are an arbitrary volume per unit time scale. The correlation coefficient between input and output is 0.6478.

A second hypothesis was tested: that volumes could be estimated by assuming a constant velocity, and computing discharges from the cross section alone. In this case, the correlation coefficient between input and output was found to be marginally better, at 0.6697. Both correlations are significant at the 95 per cent level in terms of the non-parametric rank correlation coefficient.

Three lines of argument are possible. First, we may regard the difference in correlation coefficients as insignificant and fortuitous, in which case we conclude that neither method has effectively estimated discharges in the cave passages. Second, we may argue that Curl's law is not applicable in this case: that there is no relationship between stream velocities and scallop lengths. Third, we may argue that the velocities estimated by this method are not reliable estimates of the mean stream velocities in the network links. This last is reasonable in terms of the short links and high flow rates, and consequent high secondary flows.
Table 5.3. Input and Output Flows at Junctions in Glen Park Labyrinths, Watertown, New York, as Interpreted from Scallop Patterns and Cross Section Measurements

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.8</td>
<td>8.7</td>
</tr>
<tr>
<td>29.0</td>
<td>16.4</td>
</tr>
<tr>
<td>11.4</td>
<td>18.4</td>
</tr>
<tr>
<td>23.0</td>
<td>22.7</td>
</tr>
<tr>
<td>6.8</td>
<td>6.7</td>
</tr>
<tr>
<td>10.1</td>
<td>5.3</td>
</tr>
<tr>
<td>12.9</td>
<td>18.3</td>
</tr>
<tr>
<td>10.4</td>
<td>7.7</td>
</tr>
</tbody>
</table>
A second experiment was conducted in a single phreatic conduit of varying cross section in the Bonnechere Caves, Eganville, Ontario. This cavern system has been described in detail by D.C. Ford (1961), Ongley (1965) and Marshall (1966). The caves are in the Chauvont limestone of the Black River Group in the Mohawkian Series of the Ordovician (Kay 1942). The beds are nearly horizontal and have been eroded by water captured from the Bonnechere River into a series of passages. Directions of flow can be deduced from the scallop patterns in the passages. The water exit is higher than much of the system and it is thus evident that the passages were full of water when formed. They have subsequently drained as the river cut into its bed, removing the supply of water. This conclusion is supported by the geometry of the passage cross sections, which are consistent with phreatic or water-filled development.

Five quite massive, heavily scalloped beds have been identified throughout the cave system and numbered 0 through 4, from uppermost downwards. Between these beds lie groups of thin, platy limestones and intercalated shales, bearing no scallops. The thicknesses of the scalloped beds are given in Table 5.4.

A length of passage of particular geometry was studied. The intersection of two joint-controlled lengths of the same conduit on different bearings has here resulted in a widen-
Table 5.4. Analysis of Lithology in Bonnechere Caves, Renfrew County, Ontario, in Relation to Mean Scallop Length on the Various Beds.

<table>
<thead>
<tr>
<th>Bed</th>
<th>Thickness</th>
<th>Mean Ratio</th>
<th>% CaCO₃</th>
<th>% MgCO₃</th>
<th>% insoluble</th>
<th>Rank of Inhomogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>1.95</td>
<td>95.1</td>
<td>0.1</td>
<td>4.8</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3.4</td>
<td>1.60</td>
<td>93.9</td>
<td>2.5</td>
<td>3.6</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>1.53</td>
<td>91.9</td>
<td>5.0</td>
<td>3.1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2.0</td>
<td>1.90</td>
<td>95.6</td>
<td>1.3</td>
<td>3.1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1.1</td>
<td>1.00</td>
<td>93.4</td>
<td>1.7</td>
<td>4.9</td>
<td>5</td>
</tr>
</tbody>
</table>
ing of the passage. Figure 5.3 shows an idealised plan of the area. Under phreatic conditions the continuity equation above must hold for any cross section. Thus, since discharge is constant, the mean velocity in any cross section can be related to the area of the cross section. The absence of any large-scale eddies is indicated by the consistency of flow direction inferred from the scallop patterns.

Beds 2 and 3 are exposed throughout the illustrated area. Twenty-five scallops were measured at each indicated location and the mean lengths were computed. Standard deviations were typically 15 per cent of the mean value. The mean values are given in Figures 5.3 and 5.4, superimposed on the plan of the area. Regression analysis yielded the following relationships between mean scallop length "y" and cross sectional area "x" for the two beds.

Bed 2 \( y = 0.0816 \times + 1.2209 \quad r = 0.49 \)

Bed 3 \( y = 0.0964 \times + 1.3255 \quad r = 0.56 \)

The rank correlation coefficients are 0.6091 and 0.5935 respectively, both significant at the 95 per cent level.

Comparison of the measurements for the two beds shows that the scallops in Bed 2 are consistently shorter than those in Bed 3. Three possible reasons for this difference were considered.

The cave passage is narrower at the level of Bed 2 than at the level of Bed 3. This would imply a difference
Figure 5.3. Bonnechere Caves Tapered Section
with Mean Scallop Lengths at
Sample Sites, Bed 3 (Lower)
Scale 1:60
Figure 5.4. Bonnechere Caves
Mean Scallop Lengths at Sample Sites,
Bed 2 (Upper)
in channel friction and thus a lower local flow velocity over Bed 2 than over Bed 3. But according to the theory this would give an increased, rather than a diminished scallop length. Alternatively, scallop lengths may depend directly on the width of the channel in which they are generated. Finally, the lithological differences between the two beds may be responsible for this variation.

Accordingly, a second set of locations was studied, where passage widths vary between ten and twenty feet. These passages were also formed under phreatic conditions. Here there is negligible variation in passage widths at the levels of the different beds. This eliminated the second hypothesis noted and gave a direct test of lithological control. Scallops were measured in the exposed beds at eight sites. Mean lengths were expressed as multiples of the mean lengths in Bed 2, and compared. The mean values of these numbers are set out in Table 5.4 for each bed.

The figures for the individual sites show an extreme variation about these average figures of 15 per cent. Plate 5.5 shows scallops developed differently in separate beds.

There are several possible explanations of this lithological control of the scallop patterns in Sommechere Caves. The lithology may affect the solution kinetics of the system or its hydrodynamics. Under given conditions of temperature, pH and CO₂ concentration, limestone solution depends prin-
Plate 5.5. Variable Scallop Development under the Same Hydrodynamic Conditions in Bonnehore Caves, Renfrew County, Ontario.
cipally on the chemical composition of the carbonate rock. For example, the solution of Dolomite proceeds at a much slower rate than the solution of Calcite. Alternatively, the physical structure of the rock, the presence of insoluble fragments and voids, may produce local variation in solution rates, altering wall friction, the boundary layer thicknesses and hence the mass transfer rates. Again, irregularities developed by non-uniform solution may cause a larger number of initial vortices leading to development of a larger number of scallops with consequent increase in the competition between scallops for the available space.

Samples of the beds were analysed by EDTA titration to determine the CaCO$_3$, MgCO$_3$ and insoluble contents. No clear correlation with scallop sizes is shown; apparently, variations in the mean lengths cannot be explained simply by a variation in the rate of solution.

Thin polished slices cut at random orientations to the geological structure showed varying physical characteristics. All beds are fine-grained crystalline limestones. But on a scale comparable to the scallop length, i.e. 2 mm to 2 cm, the slices showed a clear ranking of inhomogeneity, shown in the table in increasing sense. The slices are shown in Plate 5.6. The inhomogeneities consist largely of fragments of fossil material. Apparently this material has been responsible for an increase in spatial competition
Plate 5.6. Polished Sections of Five Limestone Beds in Bonnechere Caves, Renfrew County, Ontario, illustrating variable physical structure clockwise from top beds 0, 4, 2, 1, 3.
between the scallops in the manner observed mean scallop length.

2. **Statistical Properties of Scallop Measurements**

After sampling populations in the field in connection with the experiments outlined above, a set of statistical parameters was calculated on each sample. For each scallop, length and width were measured, as defined in Chapter One. From these fifteen parameters were calculated as shown in Table 5.5. The normalised parameters, which express moments as proportions of the mean, were calculated to enable direct comparison between samples of scallops of very different linear dimensions.

These parameters were calculated for 62 scallop samples measured in the Nakimu Caves, Glacier National Park, British Columbia, in the summer of 1966. Many properties already noted with reference to MIL, the mean length, were confirmed. The observation of Eyre (1963), that scallops on the upstream side of a bulge in a cave wall are shorter than those on the downstream side, was confirmed quantitatively.

An experiment was conducted in a straight conduit of circular cross section formed under phreatic conditions by water flowing quite steeply uphill. Scallops were measured on the walls and roof in an effort to check exactly the hypothesis that there is no significant difference between the scallop populations so formed.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Length</td>
<td>M1L</td>
<td>$\sum l_i / n$</td>
</tr>
<tr>
<td>Mean Width</td>
<td>M1W</td>
<td>$\sum w_i / n$</td>
</tr>
<tr>
<td>Mean length/width</td>
<td>M1R</td>
<td>$\sum (l_i / w_i) / n$</td>
</tr>
<tr>
<td>Length Variance</td>
<td>N2L</td>
<td>$\sum (l_i - M1L)^2 / n$</td>
</tr>
<tr>
<td>Normalized Variance</td>
<td>N2L</td>
<td>$M2L / M1L^2$</td>
</tr>
<tr>
<td>Third Moment</td>
<td>N3L</td>
<td>$\sum (l_i - M1L)^3 / n$</td>
</tr>
<tr>
<td>Normalized Third Moment</td>
<td>N3L</td>
<td>$M3L / M1L^3$</td>
</tr>
</tbody>
</table>

Two sites were selected, and at each site 25 scallops were measured on the left wall, right wall and roof, making six scallop samples in all. The length and width of each individual were measured. Results are shown in Table 5.6.

The hypothesis that the three sets of observations at one site are derived from the same statistical population cannot be accepted or rejected on the basis of this data. All the samples have appreciable third moments, so that tests assuming normality are invalid. Accordingly, a test was made of the hypothesis by comparing each sample to the median of the combined samples at each site. Contingency tables for lengths and widths at each site were prepared as shown in Table 5.7, and the tables compared to the contingencies expected if the hypothesis were true. \( \chi^2 \) was then calculated for each of the four cases as shown in the table.

None of these values of \( \chi^2 \), when evaluated for two degrees of freedom, is sufficient to cause rejection of the hypothesis at the 95 per cent level of significance. We conclude, therefore, that there are no significant differences between scallops formed on the walls and roof of a straight phreatic conduit.

The third moment of a distribution is a measure of the asymmetry or skewness of the distribution. A positive third moment indicates that the distribution is more drawn
<table>
<thead>
<tr>
<th>Site 1</th>
<th>Wall Type</th>
<th>M1L</th>
<th>M1W</th>
<th>M2L</th>
<th>M2W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>Wall</td>
<td>2.34</td>
<td>2.75</td>
<td>0.0248</td>
<td>0.0206</td>
</tr>
<tr>
<td>Roof</td>
<td></td>
<td>2.50</td>
<td>2.90</td>
<td>0.0371</td>
<td>0.0991</td>
</tr>
<tr>
<td>Right</td>
<td>Wall</td>
<td>2.52</td>
<td>2.51</td>
<td>0.0434</td>
<td>0.0804</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site 2</th>
<th>Wall Type</th>
<th>M1L</th>
<th>M1W</th>
<th>M2L</th>
<th>M2W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>Wall</td>
<td>1.14</td>
<td>1.51</td>
<td>0.0439</td>
<td>0.0540</td>
</tr>
<tr>
<td>Roof</td>
<td></td>
<td>1.21</td>
<td>1.51</td>
<td>0.0437</td>
<td>0.0587</td>
</tr>
<tr>
<td>Right</td>
<td>Wall</td>
<td>1.27</td>
<td>1.38</td>
<td>0.0441</td>
<td>0.0631</td>
</tr>
</tbody>
</table>
Table 5.7. Chi Square Analysis of Medians of Scallop Distributions as Measured on the Two Walls and Roof of a Passage in Nakimu Caves

<table>
<thead>
<tr>
<th>Contingency Table for Median Test</th>
<th>Left Wall</th>
<th>Right Wall</th>
<th>Roof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above Median</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below Median</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values of Chi Square

<table>
<thead>
<tr>
<th></th>
<th>Left Wall</th>
<th>Right Wall</th>
<th>Roof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lengths at Site 1</td>
<td>2.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Widths at Site 1</td>
<td>3.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lengths at Site 2</td>
<td>2.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Widths at Site</td>
<td>2.72</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
out to the right of the mean than to the left. Of the 62 samples measured in Nakimu Caves, 14 had negative third moments of the length distribution, and 13 had negative third moments of width distribution. Of these, four had both third moments negative. If negative third moments are assigned on a random basis so that each sample has an equal chance of receiving one, and if the probability of assigning a negative third moment is \( \frac{14}{62} \), the expected number of samples receiving two negatives is approximately three. Most scallop samples have positive third moments and are skewed to the left. It is hypothesised, though a test is difficult to devise, that the negative third moments observed are chance combinations of sampling errors.

Table 5.8 shows the correlation coefficients between the fifteen parameters for the 62 samples from Nakimu Caves. For a sample of 60 normally distributed pairs of observations the percentiles of the distribution of the correlation coefficient are as shown in Table 5.9.

The correlation between ML and MLW is very strong and indicates a relationship between mean scallop length and mean width. This relationship is also discernable on the individual level. In only 7 out of the 62 samples is the variance M25 greater than both M2L and M2W. This was also the finding of Marshall (1966).

The first, second and third moments of each variable
Table 5.8. Correlation Coefficients for the Fifteen Distribution Parameters of 62 Samples measured in the Naimu Caves

|   | N2L | N2R | N3L | N3R | N2L | N2R | N3L | N3R | N2L | N2R | N3L | N3R | N2L | N2R | N3L | N3R | N2L | N2R | N3L | N3R |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| M1L | 94  | 80  | 97  | 92  | 70  | 13  | 13  | 33  | 25  | 24  | 40  | 49  | 32  | 30  |     |     |     |     |     |     |
| M2L | 94  | 91  | 93  | 64  | 15  | 9   | 35  | 23  | 27  | 40  | 43  | 33  | 32  |     |     |     |     |     |     |     |
| M3L | 77  | 79  | 40  | 13  | 9   | 37  | 22  | 29  | 34  | 26  | 32  | 34  |     |     |     |     |     |     |     |     |
| N1L | 93  | 72  | 15  | 6   | 27  | 26  | 33  | 48  | 52  | 39  | 35  |     |     |     |     |     |     |     |     |     |
| N2L | 86  | 12  | 7   | 23  | 21  | 25  | 45  | 59  | 34  | 31  |     |     |     |     |     |     |     |     |     |     |
| N3L | 9   | 5   | 12  | 9   | 7   | 35  | 67  | 22  | 13  |     |     |     |     |     |     |     |     |     |     |     |
| M1R | 86  | 78  | 28  | 5   | 19  | 6   | 47  | 8   |     |     |     |     |     |     |     |     |     |     |     |     |
| M2R | 90  | 14  | 21  | 0   | 2   | 42  | 8   |     |     |     |     |     |     |     |     |     |     |     |     |     |
| M3R | 24  | 3   | 17  | 7   | 65  | 45  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| N2L | 66  | 54  | 22  | 39  | 30  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| N3L |     | 42  | 13  | 27  | 36  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| N2L |     |     | 73  | 56  | 46  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| N2R |     |     |     | 22  | 19  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|     |     |     |     | 80  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
Table 5.9. Percentiles of the Correlation Coefficient Between Two Independent Normally Distributed Parameters Sampled 60 Times.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.0</td>
<td>0.214</td>
</tr>
<tr>
<td>97.5</td>
<td>0.254</td>
</tr>
<tr>
<td>99.0</td>
<td>0.300</td>
</tr>
<tr>
<td>99.5</td>
<td>0.330</td>
</tr>
<tr>
<td>99.95</td>
<td>0.414</td>
</tr>
</tbody>
</table>
form groupings as expected, because they are highly interrelated. The length and width factor groups are themselves interrelated as noted above. The normalised moments of the same variables are also interrelated. However, there are no significant correlations between normalised and unnormalised moments, which indicates that scallop size distributions are basically similar and scaled with the mean.

An evaluation of the error inherent in the measurement technique was made on several occasions by repeating observations using different observers. The measurements in Table 5.10 are the means of samples at stations in Bonnechere Caves repeated by different observers. The samples are of 25 randomly selected features.

Measurements of the variation in scallop size through the enlarged section of Bonnechere Caves examined above were made by two observers, working in different beds. The measurements were repeated on a different occasion by a second group. Since the precise sites were not the same, the measurements cannot be compared directly. The regression parameters are given in Table 5.11 for first and second observers.

Except in the case of Bonnechere Caves, however, all the measurements noted herein were obtained by one observer, so that the validity of MLL as an estimate of the hypothetical population mean must be evaluated from the sample variance.
Table 5.10. Estimates of the Mean Length of the Same Scallop Population by Different Observers in the Bonnechere Caves, Renfrew County, Ontario, based on Samples of 25 Scallops.

<table>
<thead>
<tr>
<th>Bed</th>
<th>Site</th>
<th>1st Observer</th>
<th>2nd Observer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>I</td>
<td>1.98</td>
<td>1.94</td>
</tr>
<tr>
<td>5</td>
<td>I</td>
<td>2.39</td>
<td>2.44</td>
</tr>
<tr>
<td>1</td>
<td>O</td>
<td>3.40</td>
<td>3.05</td>
</tr>
<tr>
<td>2</td>
<td>O</td>
<td>2.39</td>
<td>2.43</td>
</tr>
<tr>
<td>3</td>
<td>O</td>
<td>3.07</td>
<td>2.92</td>
</tr>
</tbody>
</table>
Table 5.11. Comparison of Regression Parameters Between Scallop Mean Length and Cross Section in the Tapered Section of Bonnechere Caves as Determined by Two Sets of Observers

<table>
<thead>
<tr>
<th>Observer 1 Bed 2</th>
<th>Coefficient a</th>
<th>Correlation r</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observer 2 Bed 2</td>
<td>0.0816</td>
<td>0.49</td>
<td>11</td>
</tr>
<tr>
<td>Observer 1 Bed 3</td>
<td>0.0802</td>
<td>0.45</td>
<td>7</td>
</tr>
<tr>
<td>Observer 2 Bed 3</td>
<td>0.0964</td>
<td>0.56</td>
<td>13</td>
</tr>
<tr>
<td>Observer 2 Bed 3</td>
<td>0.1581</td>
<td>0.70</td>
<td>7</td>
</tr>
</tbody>
</table>
1. **Closed Conduits**

Periodic spatial non-uniformity of snow surfaces is widely observed, and covers a variety of forms. Surfaces will first be divided into tunnel and open types since the conditions in the former are much simpler and more readily understood.

Consider a body of snow or ice resting on a hill slope. During early summer streams develop under the snow or ice to carry away the melt. Air currents are set up in the tunnels either by convection or by the movement of water. Figure 6.1 shows typical forms. The advection of warm air into the cool tunnel enlarges the tunnel by ablation and produces scalloped or fluted surface analogous to the limestone surfaces considered in the previous chapter. Scallops formed in a tunnel of type (a) in Glacier National Park in the summer of 1956 are shown in Plate 6.1.

Also shown in the plate are transverse lines representing different stages in the deposition of the snow bank, in this case probably successive avalanches. It is clear
Figure 6.1. Snow Tunnels with Associated Air Convection Currents

(a) Snow Mass

(b) Air Current

Water
Plate 6.1. Snow Scallops in a Snow Tunnel in Glacier National Park, British Columbia.
that such inhomogeneities in the snow do not affect the resulting pattern.

Wind currents in snow tunnels can exist in only two directions, and tend to be stable because of the damping effect of the long, closed conduit. Melting occurs when the wind is down-slope, since up-slope winds are generated by lower, below freezing, outside temperatures unless there are very strong macroscopic winds in this direction. Thus the scallops are asymmetrical and resemble closely those formed on limestone in cave conduits. Two samples of 75 snow scallops taken from opposite walls of the tunnel of Plate 6.1 had the properties shown in Table 6.1.

Conditions in snow tunnels are complicated by the effect of meltwater. Some ablation proceeds by evaporation, or even direct sublimation, but in many cases meltwater is able to collect under the influence of gravity. The meltwater continues to evaporate, thus cooling the surfaces upon which it collects. As a result, pendants develop near the arch of the tunnel roof, (Plate 6.2). These pendants are enhancements of positive relief features which already exist as parts of a scallop pattern.

Partially closed conduits, as shown in Figure 6.1(b), generate air currents, and hence scallop patterns, by convection. A through current of air is not necessary, though this has been regarded as a problem by some workers. However, a
Table 6.1. Comparison Between Scallop Patterns Developed on Opposite Walls of a Snow Tunnel in Glacier National Park

<table>
<thead>
<tr>
<th></th>
<th>M1L</th>
<th>M2L</th>
<th>M1W</th>
<th>M2W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left Wall</td>
<td>32.3</td>
<td>122.9</td>
<td>27.4</td>
<td>129.2</td>
</tr>
<tr>
<td>Right Wall</td>
<td>31.7</td>
<td>82.5</td>
<td>29.6</td>
<td>173.7</td>
</tr>
</tbody>
</table>
Plate 6.2. Development of Pendants in the Roof of a Snow Tunnel.
source of heat being necessary to the melting of any solid, it is impossible that an entirely closed cavity wholly within a snow or ice mass could have scalloped walls.

Plates 6.3 and 6.4 show scalloping of ice masses within limestone caves. In the first case, from Moose Mountain cave in the Elbow River district of Bow Provincial Forest, Alberta, scallops have been produced by air currents flowing down through the cave during the summer months. The asymmetry is not clear, however, suggesting that ablation may also occur during the winter months in the opposite direction. The presence of hoar frost on the walls of this cave indicates that ablation is mostly by sublimation, and suggests that the ice deposits are of some considerable age. There is a horizontal banding present in the ice deposits of about four inches vertical separation and unknown origin.

A similar phenomenon occurs in Coulthard Cave in Crownest Provincial Forest, some ten miles south of Coleman, Alberta. This cave, developed under conditions of deep phreatic flow, but now situated some 2000 feet above sea level and 3000 feet above the valley floor in a steep mountain face, has a temperature of 27°F in mid-August. At some time in the past Coulthard Cave was filled with water, which then froze. There are no laminations or inhomogeneities in the ice to indicate an extended period of deposition. In all probability the event occurred during the late stages of ice
Plate 6.3. Scalloped Ice Deposits, Moose Mountain Cave, Bow Provincial Forest, Alberta.
Plate 6.4. Cuspat e Ice Deposits, Coulthard Cave, Coleman, Alberta.
wastage in the area, at a time when all available exits for
the water were frozen shut.

Since that time the ice has been gradually removed
by the retreat of ice down each passage away from the main
heat source at the cave entrance. At the present time four
cave passages are all blocked by walls of solid ice at about
500 feet from the cave entrance. These ice walls are cuspate,
as illustrated in Plate 6.4. In two passages there are con-
siderable deposits of dirt on the cusps, the dirt being a
red material derived from small, highly coloured drips in
the roof near the ice walls. The local nature of these dirt
deposits would appear to indicate that they have not melted
out of the ice, but have been deposited from the air as a
fine dust during the melting process. The concentration on
the ridges, typical of cuspate snow surfaces, is due to
preferential deposition, and the movement of the dirt par-
ticles over the surface after deposition.

2. Dirt Concentrations

In Chapter One, reference is made to a paper by Ball
(1954) in which the pattern of dirt accumulation on a scalloped
surface is related to a theory of "Normal Trajectory". Ball
suggested no mechanism by which the hypothetical dirt paths
could be related to known physical laws. In an undisturbed
environment, particles of dirt which melt out of a snow mass
are influenced only by gravity, and therefore follow a ver-
tical path. If the surface is not perfectly flat, trajectories may deviate somewhat from the vertical, but on average will be down the scallop slopes toward the depressions rather than towards the ridges.

John and Klapa (1968) propose a cohesion between dirt particles and the snow surface, which produces trajectories between the vertical and normal. Clearly any such mechanism, however weak, will eventually produce concentrations on the ridges, unless the ridges themselves migrate more rapidly due to very rapid ablation. Plate 6.5 illustrates a snow field in Banff National Park which shows a patterned relief and a dirt pattern precisely out of phase, due to rapid movement of the patterned relief under highly ablative conditions.

If the dirt distribution in the original unmelted snow mass is known to be uniform, observation of the subsequent distribution of dirt particles provides a direct test of those hypotheses. John and Klapa scattered conifer needles and match sticks on a snow patch and note that after two days the scattered material assumed "a concentric arrangement".

During the early part of an alpine summer the melting of ice frozen into cracks in bedrock outcrops releases blocks, which may then roll down hill. Frequently this process results in lines of roughly uniform dirt deposits along the
Plate 6.5. Accumulations of dirt out of step with the relief pattern on a snow field, Mt. Castle-guard, Banff National Park, Alberta in August 1963.
line of steepest slope of snow patches. Later in the summer the distribution of this dirt illustrates the pattern of dirt movement on the snow patch.

Plate 6.6 illustrates the observed dirt distribution in late summer. The dirt pattern is still basically uniform. Dirt has certainly not moved horizontally away from the original line. If dirt has in fact moved parallel to the line, the movement has not been sufficient to clear the bases of scallop depressions. However, the rest of the snow patch has a pattern of dirt concentrations which follow scallop ridges and appear to cut directly across the dirt lines.

In some cases the dirt lines have resulted in a modification of the relief, characterised by ablation pits where the dirt is thickest. These are the result of direct solar heating, and are not linked to the scallop process.

There are two possible sources of ridge dirt concentrations. Dirt may originate in the snow matrix, in which case concentration implies a dirt movement: or it may originate in the air. No single parameter controls the movement of sediment in a fluid. Fluid velocity parallel to the surface and fluid shear at the surface are the two parameters most likely to influence the initiation or termination of the movement of a dust particle in the atmosphere. In terms of the scallop profile illustrated in Figure 6.2, both parameters are minimised between points A and B, and maximised at point
Plate 6.6  Dirt Stripes Attributed to Rolling Debris, Illustrating Apparent Lack of Movement and Concentration on the Snow Surface.
Figure 6.2. Patterns of Dirt Movement, Velocity and Shear Stress on a Scalloped Snow Surface

A, B  Velocity and Shear Stress Minimised
C  Velocity and Shear Stress Maximised

→ →  Dirt Movement Pattern
C. Thus wind-blown dirt is most likely to concentrate between points A and B. This tendency will also apply to dirt particles which melt out of the matrix and are of sufficient size to be influenced by wind patterns in their subsequent movement. The proposed pattern of dirt movement is as shown in Figure 6.2, and differs from the "Normal Trajectory" hypothesis.

The observed pattern of dirt concentration is further influenced by the overall migration of the patterned relief as melting proceeds. Studies using time-lapse photography indicate that snow scallops migrate downstream in the same manner as limestone scallops. Plate 6.5 illustrates the result of a migration of the relief pattern independent of the dirt concentrations due either to excessively rapid melting or the inability of this dirt to migrate once deposited.

The combined evidence of this work and that of John and Klapa suggests the following conclusions: Dirt from the atmosphere is preferentially deposited on scallop ridges. Dirt melting from the snow matrix migrates to the scallop ridges at varying rates due to the air flow pattern and to the cohesion between dirt particles and the surface which results in a trend towards a "Normal Trajectory". The scallop pattern itself migrates at a rate which may exceed that of the surficial dirt.
3. Open Snow Surfaces

A wide range of scallop-like forms are found on open snow surfaces. Wind patterns are very complex, being omnidirectional and of widely varying strength. The simplest forms, defined as those which closely resemble forms found in snow tunnels or limestone conduits, have been generated by air currents in a single direction and therefore show clear asymmetry and an elliptical plan in which the wind direction forms either the major or the minor principal axis.

Features observed on snow fields on the flanks of Mt. Castleguard in late August 1968, were of this form. Wind patterns are restricted by the presence of Thompson Pass to the West. The wind direction was defined on each snow field by the direction of maximum asymmetry. Also measured on each of thirty snow areas were the strike direction, dip of the snow field, and average scallop length.

The wind directions indicated by scallop patterns were in all cases compatible with those intuitively expected from the local topography, (Figure 6.3). In addition, scallop lengths were related to wind patterns. This was demonstrated using multiple regression analysis by fitting a model in which the dependent variable was the average scallop length for each snow field. It was hypothesized that the scallop length depended on the velocity of air over the snow field.
Figure 6.3. Directions of Maximum Asymmetry on Scalloped Snow Fields, Mt. Castleguard, Banff National Park, August, 1968. Arrows Indicate Implied Flow Direction.
which in turn depended on the geometry of the snow patch with respect to the regional wind direction. On the basis of exposed snow patches on top of the hill, the regional wind direction was taken to be 220° Magnetic. Independent variables were defined as $x_1$, the absolute differences between regional wind direction and strike of the snow patch; $x_2$, the difference between local wind direction, interpreted from the scallop pattern, and the strike of the snow patch; $x_3$, the dip of the snow patch. The model $y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$ had a correlation coefficient of 0.6451 and explained 42 per cent of the observed variation in mean scallop lengths. We conclude that slower velocities result in larger scallops with a fair degree of regularity.

Snow scallops are the direct result of ablation by warm moving air. Direct solar heating under calm conditions will result in the destruction of the pattern, as dirt concentrations on ridges enhance the rate of melting there and so destroy the relief. Records obtained with time-lapse photography indicate that scallop patterns develop rapidly as soon as the correct climatic conditions are realised. In such cases observed scallop lengths can be correlated with the wind speeds at the time of scallop conception, and the exact form of the relationship between velocity and size deduced.

Equipment was set up on a variety of snow fields
during the summers of 1967 and 1968. The time-lapse equipment was described fully in Chapter Three. Wind speeds were recorded by a RIMCO "Summer" Mk 2 long period recorder type 2/WS-D, which records wind speed and direction every 6 minutes. The wind speed is recorded as the number of signals accumulated from the anemometer during the six-minute period. The anemometer, which is mounted independently on a five-foot high pole, has five-inch cups and activates a mercury switch. The complete equipment, set up at Maligne Lake, Jasper National Park, Alberta, in May 1968, is shown in Plate 6.7.

The chances are heavily against obtaining the correct conditions during a run with the equipment. Ablation proceeds rapidly during the summer months and snow patches may disappear before the correct conditions are reached. However, several evaluations of scallop size were made under known wind conditions. The variation in wind speed on an open snow surface reduces the accuracy of these evaluations, which at best are poor. The results obtained are given in Table 6.2. Other results were obtained from the snow tunnel in Glacier National Park and from Mt. Castleguard, but in neither case was the conception of the scallop surfaces observed, so that the velocity term cannot be evaluated. The constant was calculated from the formula \( \frac{\text{HLL} \cdot v}{\rho} \) using the appropriate values of density and viscosity for air at the prevailing air temperature. The velocities are mean velocities over a
Table 6.2. Scallop Mean Lengths and Wind Speeds Over Snow Patches at the Moment of Initiation of the Scallop Pattern

<table>
<thead>
<tr>
<th>Wind Speed (ft/sec)</th>
<th>MLL (cms)</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athabasca Glacier, Jasper National Park, August 1967</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>Maligne Lake, Jasper National Park, May 1968</td>
<td>6</td>
<td>15</td>
</tr>
</tbody>
</table>
period of several hours while the scallop surface was developing, as observed by the camera.

When wind patterns become more complex, scallop patterns degenerate into a variety of forms. These depend mainly on the relationships between the different wind directions. Plate 6.3 shows an example of a distinct form for which the term "furrow" is proposed. These are linear forms which run principally down-slope as if the snow surface had been ploughed.

Following Sharp (1947) it was first hypothesised that forms were influenced by the flow of melt water over the surface. However, it is easy to find cases where the furrows do not follow the line of steepest slope. Further, melt-water has never been observed in the furrows in any quantity.

There is no critical slope at which furrows appear. Although they are more common on steeply sloping snow fields, they were observed to break up into scallop patterns on slopes inclined at between five degrees and the vertical.

Although they do not precisely follow the line of steepest slope, as would be expected for a free body moving down a snow field, furrows closely approximate to such a line. Thus a process tending to follow the slope is clearly suggested. The possible role of katabatic air flow was therefore investigated.

Suppose the daily pattern of air flow over a surface
falls into two categories—the regional air flow during the entire day, the katabatic air drainage during the night. The snow pattern set up will attempt to combine the two air flows. Consider first the case in which the regional air flow is directly across the surface. In this case, the two air currents are perpendicular. Furrows of asymmetric cross section are compatible with both air currents if regarded as scallop forms. To the regional air flow they represent flutes, or scallops of extended width, and to the katabatic flow they represent long narrow scallops appropriate to the slower flow.

As the relative directions of micro and macro air flow diverge from perpendicular, furrows break down into features of limited length. The principal furrow axes are still roughly parallel to the line of steepest slope of the surface. Further divergence results in a roughly circular form, compatible with winds of roughly equal strengths from all directions. No asymmetry is apparent in the cross sections.

Plate 6.9 illustrates the complete hypothesis. In the lower steeper areas wind is channeled by the topography and blows across the slope, giving rise to furrows. On the flatter top of the snow bank the furrows degenerate first into roughly circular forms and then as the slope decreases still further and katabatic winds are largely absent, the
Mt. Castleward, August 1963.
simple uni-directional scallops are found.

A variety of snowfield forms can thus be understood as the result of interactions of air flows from various directions. All air flows lead to the same basic pattern of dirt accumulations. Finally, it must be noted that these considerations apply only to slow alpine ablation. In areas of more rapid ablation the adjustment of scallop patterns to changing wind conditions is more rapid, so that patterns do not represent a combination of the effects of previous winds on the surface.
CHAPTER SEVEN

CONCLUSIONS

The major aim of this study was to extend Curl's theory of flutes to scallop patterns; to investigate the relationships, if any, between flute periodic lengths and the assemblage of linear measurements found on a scalloped surface. Curl proposed that a scallop pattern is the result of incessant variation in velocity over a limestone surface, preventing the establishment of stable vortices extended across the flow.

Theoretical investigation is virtually impossible, though the inviscid, irrotational approach pursued in Chapter 2 yielded interesting results. Approximate solutions, though feasible in principle, are beyond the reasonable capability of present-day technology.

Scallops were therefore studied empirically, by generation in a laboratory flume on a Plaster of Paris surface. Scallop patterns were generated under varying fluid conditions, and the resulting length distributions correlated with the fluid parameters. In particular, the mean and the ten percentiles 10th through 100th were used as characteristic para-
meters of the length distributions. These parameters are themselves estimates of others, as was demonstrated in Chapter 5, Section 2. For example, the first moment, or mean, of a scallop distribution is a good estimate of the second, or variance; and the 50th percentile is a good estimate of the geometric mean.

The relationship tested between these variables was specifically

\[ y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 \]

where the set of \( a \) are constants fitted by multiple regression

- \( x_1 \) is the velocity of flow
- \( x_2 \) is the fluid viscosity
- \( x_3 \) is the acceleration term

The variable \( x_3 \), not treated in Curl's analysis, was included but found to be insignificant.

The values of \( a_1 \) were satisfactorily close to 1.0, implying a linear relationship between \( y \) and \( x_1 \), analogous to Curl's flute model. The values of \( a_2 \), however, differed considerably from 1.0. Only three values of \( x_2 \) were used in the experiments, which may explain this. Alternatively, the non-linearity of the viscosity term may be due to the presence of acceleration in the system. Wider variation of \( x_2 \) would resolve this question, but is not possible in the
present flume because of the adverse behaviour of perspex under high temperature gradients.

These considerations apply to all the length parameters "y". The model explained the means marginally better than any of the percentile measures. The degree of explanation dropped markedly for the upper percentile measures, notable the maximum value. In all cases, however, the degree of explanation of the data was quite satisfactory.

A second model was tested in which the variables were constrained to a relationship of the form

\[ y = a \frac{x_2}{x_1} \]

which is directly equivalent to Curl's flute equation. The values of "a" thus obtained were approximately one half of Curl's 22500.

Lengths measured on a scalloped surface represent only a minute fraction of the total set of linear measurements parallel to the flow direction between ridges. The Autocorrelation and Spectral Density functions of scallop patterns were therefore generated. The forms of both functions were as expected, though providing a more complete description of the surface than previously. Length distributions are more readily obtained in the field, and the aim of the study should therefore be to provide laws concerning them rather than the Autocorrelation or Spectral Density functions.

Laws concerning the generation of scallop patterns
on Plaster of Paris under laboratory conditions need not be valid for limestone scallops produced over long periods of time under fluctuating conditions. Curl's theory of scallop and flute formation predicts that the processes are analogous. A program of field investigation was therefore initiated.

The flow conditions which led to the formation of limestone scallops cannot be measured: they may only be inferred. Only relative rates of flow can be determined in conduits abandoned by water. Thus the tests of these relationships which were performed under vadose and phreatic flow conditions could only verify the form of the relationship between flow and scallop pattern.

Investigation of phreatic flow conditions in Bonnechere Caves showed that although scallop sizes are inversely proportional to flow rate, the constant of proportionality is significantly different for scallops developed on differing limestones. The observed differences between five limestones can be related to the physical structure of the rock, specifically the variation on a scale comparable with the scallop length.

The statistical properties of scallop length distributions were investigated. Fifteen moment parameters were defined from length and width measurements, and their inter-
relationships investigated. It was found that scallop lengths and widths have a positive third moment, and that the second and third moment of length, width and length/width ratio distributions scale with the respective means.

Inherent observer error was investigated by repeating measurements with different observers. Agreement was found to be satisfactory.

The analogy between limestone scallops and snow ablation polygons was confirmed quantitatively. Snow scallops obey the same laws and use of the appropriate density and viscosity gives similar values of the scallop constant.

Dirt accumulations on scalloped snow surfaces were seen to be the result of the interaction of relief and air flow. The hypothetical Normal Trajectory of Ball (1954) is found to be unnecessary to their explanation. Other snow relief forms, including furrows, are the result of interaction of air flows from several directions on a single snow field.

The process of growth of a single scallop or flute is initiated by an inhomogeneity in the base material. This inhomogeneity may be of greater or lesser resistance to erosion than the surrounding material. Thus, in the case of Plaster of Paris, a bubble or an insoluble sand grain will suffice.

The inhomogeneity causes differential solution and
formation of a surface irregularity which in turn causes separation of the fluid boundary layer, and the creation of a vortex, downstream of the sand grain or within the bubble.

The presence of a vortex enhances the transfer rates and hence the rates of ablation in the immediate area. Thus the vortex erodes a saucer-shaped depression. The growth rate of the depression depends upon its size. It is possible that the depression would continue to grow in size indefinitely; but the presence of one depression disturbs the fluid near the boundary and causes the generation of other features regularly spaced downstream. The length of each scallop feature is therefore limited by this periodicity, as can be observed by time-lapse photography.

On a mature scallop surface all space is occupied. Scallop lengths are uniformly distributed on a probabilistic basis. The distribution has one maximum and in general has a positive third moment.

While growing, scallops also move into the surface as the latter is lowered in response to erosion. In addition, a general migration takes place in the plane of the surface, which may be combined with the normal movement to give a resultant direction. This general migration can be observed by time-lapse photography, which indicates that on Plaster of Paris and snow surfaces, migration is generally downstream. In individual cases, however, the pattern may be
complicated by rapid capture.

In Curl's view a fluted surface results from generation under stable velocity conditions. Under such conditions lengths are controlled as outlined above, but widths continue to grow indefinitely. Flutes are occupied by single, cylindrical vortices, which can be terminated at each end only by walls.

Any stationary vortex in a stream must either be temporary or must terminate in a wall at both ends. Therefore, the vortices occupying and responsible for the depressions on a scalloped surface must be temporary. Thus Curl argues that a scalloped surface is the result of variation in the velocity of the fluid, preventing vortices from establishing themselves permanently and forming flutes.

This is not entirely true. Close examination of plaster surfaces inside scallop depressions gives an indication of the flow pattern, shown in Figure 7.1 (Allen, 1966). It is clear that the flow pattern is complex, involving rotation about more than the simple transverse axis. We cannot therefore dismiss the possibility of stable flows within the scallop pattern.

A second possibility presents itself. The probability of exposing an inhomogeneity, and thus initiating a scallop, is uniform across the surface and through time. On a mature surface there is still a constant probability of initiating
Figure 7.1. Pattern of Fluid Flow in an Isolated Scallop as Indicated by Flow Markings on the Plaster Surface.
new scallops.

New scallops on a mature surface must be initiated within existing features. Some will occur within normal and some in reversed flow conditions. The relative growth rates are not known. Perhaps under reversed flow conditions transfer rates are not further enhanced by incipient scallops so that no new features form in these areas, (Figure 7.2). However, it is clear that some form of competition occurs on a mature surface for the limited space available. This competition is not limited to scallop-forming conditions. New scallops can be generated within the normal flow regions of flutes.

In this context, then, flutes may only form on materials lacking scallop-forming inhomogeneities. Stability of velocity is also important, however, since the degree to which flute and flow "fit" must affect the likelihood of scallop generation, (Figure 7.2).

A scallop surface may be visualized as a set of linear measurements, lengths and widths, coupled with a set of growth rates. A mature surface represents a dynamic equilibrium, in which growth is balanced by spatial competition and capture.

The growth rates are functions of size, a small feature growing rapidly compared to a large feature. They are also functions of velocity, so that a change in velocity merely affects the rates at which features are growing in
Figure 7.2. Region in Which Scallop Initiation is Feasible

Illustrated under a) Excessively Fast and b) Normal Flow.
the statistical assemblage of linear measurements.

The relationship between the scallop distribution of lengths and the flute periodic length has been investigated. A more precise analysis will be possible when a more precise numerical value for the latter is available. Curl's dimensional analysis has been seen to apply to the arithmetic mean and to the ten percentile measures, though with different values of the constant. Other parameters, such as the logarithmic mean, are related to those investigated. Conclusions drawn from the relative values must be seen in the light of the approximate manner in which Curl arrived at his value of 22500. A precise determination of this number is clearly vital to the development of this area of study.

At present we can only repeat the finding that despite the experimental work in the laboratory flume, scallop length distributions can be widely affected by variation in the physical structure of the material on which they are formed. However, scallops developed on a single rock type have length distributions whose characteristic parameters, mean and 10th through 100th percentiles, are related to fluid viscosity, density and velocity 3 cms above the surface by the relation

\[ \frac{\lambda f}{\eta} = \text{constant} \]
For Plaster of Paris the values of the constant are given in Table 3.2.

Further uncertainty centres around the definition of the velocity parameter. Measurement of velocities further from the experimental block or the use, for example, of a mean stream velocity, would have in effect raised the calculated values of R, perhaps by as much as 20 per cent.

The rate of erosion of limestone is highly sensitive to fluid velocity. In a stream of fluctuating velocities, responding to periodic floods and droughts, the scallop pattern will respond far more rapidly to the faster flows, and thus probably represent the stream in near flood, rather than mean discharge. Even in its present stage of sophistication, then, the interpretation of a scallop pattern can provide information of an adequate accuracy on flow conditions in underground channels long abandoned by ground water.
APPENDIX ONE

FLUME DATA

The data is organised in the following manner. Each scallop sample is given a three digit serial number, which in the case of the flume data begins at 274. The first line of each sample record contains the sample serial number, the Plaster of Paris block serial number, details of the experimental conditions, and towards the far end of the line, the sample size followed by 1.0 to indicate measurements. Each line covers 20 scallops. The first two digits of the four digit scallop record are the distance in centimetres from the downstream end of the block; the last two digits are the scallop length in millimetres.

Following the data are two lines of computed parameters. These are the flume velocity 3 cms above the downstream end of the block: the fluid viscosity; the geometry, denoted by 1.0 for high gradient, 2.0 for low gradient; the mean of the distribution of corrected lengths, and the ten percentiles 10 through 100. The sample serial number is repeated at the end of each complete sample record.
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<th>Value</th>
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<td>1.535</td>
<td>1.565</td>
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</tr>
</tbody>
</table>
APPENDIX TWO

Consider the model \( y = ax \) which relates observed values of variable \( x \) to calculated values of variable \( y \).

The observed values of \( y \) are denoted by \( y_0 \).

The difference between observed and calculated values of \( y \) is

\[
y - y_0 = ax - y_0
\]

The total squared deviation for all observations is given by

\[
\sum (y - y_0)^2 = \sum (ax - y_0)^2
\]

We wish to find that value of the constant \( a \) which minimises \( \sum (y - y_0)^2 \), the total squared deviation.

\[
\sum (y - y_0)^2 = \sum (ax - y_0)^2
\]

\[
= \sum (a^2 x^2 - 2axy_0 + y_0^2)
\]

\[
D = a^2 \sum x^2 - 2a \sum xy_0 + \sum y_0^2
\]

Stationary points in the variation of \( D \) with \( a \) are given by

\[
\frac{dD}{da} = 2a \sum x^2 - 2 \sum xy_0 = 0
\]

Thus the value of \( a \) which gives the best "fit" between \( y_0 \) and \( y \) is given by

\[
a = \frac{\sum xy_0}{\sum x^2}
\]

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