FM THRESHOLD PERFORMANCE

OF

THE PHASE-LOCKED OSCILLATOR

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By

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SCOPE AND CONTENTS:

This thesis is principally concerned with the performance of the phase-locked FM demodulator under conditions of interference in comparison to the conventional FM demodulator. The linear no interference performance of the phase-locked oscillator is well known, however this aspect is included in the interests of completeness and reference.

Mechanisms for threshold effects in the phase-locked and conventional FM demodulator are discussed and compared. It is shown theoretically and experimentally that the noise threshold is reduced in the phase-locked FM demodulator by virtue of the limits of $\Psi_i(t)$ being restricted by the noise bandwidth of the feedback loop. Fall off in baseband signal level in the presence of noise was seen to be a function of $\theta_s(t)$ and $(S/N)_{IF}$.

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LIST OF PRINCIPAL SYMBOLS

e_s	=	instantaneous signal voltage at output of first IF
e_i	=	instantaneous noise voltage at output of first IF
e_l	=	instantaneous local oscillator voltage
E_s	=	peak value of sinusoidal carrier voltage
ΔE_i	=	peak value of sinusoidal incremental noise voltage
E_l	=	peak value of sinusoidal local oscillator
w_k	=	angular frequency of the carrier
$w_k + w_n$	=	angular frequency of the noise components
w_l	=	angular frequency of the local oscillator
$\theta_s(t)$	=	angle modulation of the carrier
$\theta_l(t)$	=	angle modulation of the local oscillator
c_1	=	phase constant associated with the carrier
c_2	=	phase constant associated with the local oscillator
θ_s	=	$\theta_s(t) + c_1$
θ_l	=	$\theta_l(t) + c_2$
ϕ_n	=	the random phase of an individual noise component
IF	=	intermediate frequency
BF	=	baseband frequency
S_{IF}	=	$(E_s)^2/2$, the mean IF signal power
N_{IF}	=	$\sum(\Delta E_i)^2/2$, the mean IF noise power
$2B_1$	=	the IF noise bandwidth
ρ	=	$N_{IF}/2B_1$, mean IF noise power density
η	=	rectification efficiency associated with diode detector

K_1 = sensitivity of the balanced phase detector, volts/radian

K_2 = voltage gain of the dc amplifier in the feedback loop

K_3 = sensitivity of the local oscillator, radian/sec-volt

K = $K_1 K_2 K_3$, loop gain

p = d/dt, the differential operator

$F(p)$ = low pass filter in the feedback loop

e_{BF} = output baseband frequency voltage

$\Delta w_1(t)$ = the frequency modulation of the local oscillator

$T(p)$ = the closed loop transfer function

$\Delta \theta_e(t)$ = phase error associated with phase detector operating point

$|T(jw)|$ = steady state amplitude characteristic

$B(w)$ = steady state phase shift characteristic

$2B_2$ = noise bandwidth of the feedback loop

w_o = natural resonant frequency of the loop

δ = damping factor

p_1, p_1^* = complex conjugate poles of T(p)

$\mathcal{L}$ = Laplace transform operator

$\mathcal{L}^{-1}$ = Inverse Laplace transform operator

M_i = envelope associated with $e_s + e_i$

$\Psi_i(t)$ = angle modulation associated with $e_s + e_i$

$E_p(t)$ = random in phase component of equivalent noise vector

$E_q(t)$ = random quadrature component of equivalent noise vector

$R(t)$ = random envelope of equivalent white noise carrier

Φ = random angle modulation of equivalent white noise carrier

σ^2 = the variance, or mean noise power

$P(R)$	=	the Rayleigh probability distribution
$R'(t)$	=	envelope associated with noise within the band $2B_2$ at the IF frequency
$(S/N)_{IF}$	=	mean IF signal to noise power ratio
$(S/N)_{BF}$	=	mean BF signal to noise power ratio

SECTION I

INTRODUCTION

A. Bandwidth Considerations and Conflict Between the FM Threshold and Baseband Signal to Noise Ratio:

The noise bandwidths of the Intermediate Radio Frequency (IF) and Baseband Frequency (BF) circuits in a receiver for a particular type of modulation are dictated by the characteristics of the modulated wave.

For an Amplitude Modulated (AM) wave the minimum noise bandwidths of the IF and BF circuits without distortion are both set by the highest modulation frequencies of the modulating information spectrum.

This is in contrast with a Frequency Modulated (FM) wave where the minimum BF noise bandwidth without distortion is set by the highest modulating frequency, while the minimum IF noise bandwidth without distortion is set by both the peak angular frequency deviation and the highest modulating frequency associated with the modulated wave.

It is well known that an FM system can be designed such that above a "threshold of FM improvement" the output baseband mean signal to noise power ratio $\left[S/N\right]_{BF}$ will be greater than that attainable from an AM system (either a single sideband or double sideband system) designed for the spectrum of modulating information.

Increasing the S/N_{BF} above the FM improvement threshold by increasing the modulation index must, for the case of a conventional FM receiver designed for the minimum IF noise bandwidth, be associated with a widening of the minimum IF noise bandwidth to permit undistorted passage of the modulated wave.

This automatically implies that a higher noise power is passed by the predetection, or IF noise bandwidth, for a given IF noise power density and consequently, a received signal of higher strength would be required to preserve the FM improvement threshold. Conversely, if the received signal strength is the same in both cases, then broadening the IF noise bandwidth to accomodate the FM wave of higher modulation index will for a given IF noise power density, result in a degradation of the FM improvement threshold. Thus, in the conventional FM receiver to increase the $\left[S/N\right]_{BF}$ and at the same time preserve the threshold, would require an increase in the transmitted power.

Similarly, for the conventional FM receiver, it is impossible to reduce the $\left[S/N\right]_{IF}$ at which threshold occurs without degrading the $\left[S/N\right]_{BF}$ above "threshold" for a given received signal strength and IF noise power density. Hence, for the conventional FM receiver, there is an uncompromising conflict between decreasing the $\left[S/N\right]_{IF}$ at which "threshold" occurs, and not degrading the $\left[S/N\right]_{BF}$ above "threshold", or conversely, between increasing the $\left[S/N\right]_{BF}$ above "threshold", without degrading the threshold.

For the conventional FM receiver incorporating an amplitude

Limiter, certain definite features associated with operation in the FM threshold region become noticeable when the noise amplitude peaks begin to exceed the signal amplitude peaks in the IF noise bandwidth prior to the limiter. (Refer to Section V regarding this subject.)

On an instantaneous basis the noise envelope of Rayleigh probability distribution instantaneously controls the zero crossings of the composite waveform at the limiter input with a frequency of occurrence given by the statistical nature of the problem. This is associated with spikes of baseband voltage at the output of the receiver with an increasingly significant frequency of occurrence as the $\left[S/N\right]_{IF}$ is lowered in the threshold region.

On a root mean square voltage, or a mean power basis, the characteristic constant RMS noise voltage density or constant mean noise power density above "threshold", associated with the output frequency spectrum of the limiter around the first spectral zone, becomes modified.

In this connection as the FM threshold is approached from above "threshold", an additional triangular noise voltage density, or parabolic noise power density contribution, is symmetrically superimposed upon the output IF constant noise density characteristic above "threshold". Further filtering after the limiter cannot alter the shape of this spectrum and hence the "threshold" cannot be reduced by post limiting filtering.

Corresponding to the change in shape of the IF noise density characteristic in the "threshold" region the normal triangular distribution of RMS baseband noise voltage begins to have additional contributions superimposed about the baseband zero frequency. This filling in of the baseband triangular distribution of RMS noise voltage, is more noticeable at the lower baseband frequencies than the higher baseband frequencies. For a baseband noise bandwidth several octaves wide the change in RMS noise voltage in the highest channels is virtually imperceptible.

B. Implications of FM Threshold Improvement Without Degradation of Baseband Signal to Noise Ratio:

The realization of threshold improvement for an FM receiver without degradation of the $\left[S/N\right]_{BF}$ above "threshold" associated with a wideband FM wave in a conventional FM receiver has certain logical implications, among which are the following: The first IF amplifier of a given minimum noise bandwidth must be followed by a network of lower noise bandwidth in such a way that the latter is essentially predominant in determing the FM improvement threshold. The fulfillment of the first criterion necessarily implies that the original FM wave be translated and modified in such a way as to allow undistorted transmission of the modified wave through the second IF at lower noise bandwidth. At the same time demodulation of the modified wave must yield the same $\left[S/N\right]_{BF}$ above "threshold" as would a conventional receiver designed for the original wideband FM wave.

The realization of the above requirements through operations upon the original wideband FM wave, from the very nature of the problem, implies the use of correlation techniques.

C. Noise Reduction in FM Systems:

Since the initial recognition of the noise reduction properties of the Frequency Modulation System by Armstrong[1] the performance of FM systems under conditions of interference have been extensively examined.

As the performance of an FM system, both above the FM improvement threshold and in the threshold region, is determined by the bandwidths required for transmission of the modulated wave, investigators have necessarily concerned themselves with these factors.

The general baseband noise characteristics of an FM demodulator were early appreciated from experimental results[2]. The experimental noise performance above threshold was in agreement with that predicted from a straightforward theoretical analysis. Although the basic mechanism causing the FM threshold effect was correctly interpreted, a general quantitative and qualitative analysis of threshold performance was not attempted.

The complexity of such an analysis has been the basic reason that most authors have continued to avoid the subject, with notable exceptions[3,4].

The threshold effect of the FM receiver would not represent a limitation if the receiver were always operated above threshold.

This is however, where the operation must be confined to for intelligible demodulation of the original modulating information. Consequently departures into the threshold region, or "capture" of the desired signal by noise, would be associated with fading of the received signal strength from the nominal "above threshold" design value, assuming the receiver IF noise power to be of constant power density. Thus the assurance of above threshold performance of the FM receiver is *contingent upon the existence of a safety factor in the magnitude of the received signal strength to guard against the worst expected fading conditions of the signal.*

The advent of FM communication systems having particularly severe statistical fading of the signal strength such as an FM Tropospheric Scatter Communications System, has caused an examination of the problem of FM threshold reduction without degradation of the baseband signal to noise ratio associated with a received wideband FM wave. In this connection it was shown[5] that compression of the frequency deviation of a received wideband FM wave would result in a reduction in threshold without degradation of the $\left[S/N\right]_{BF}$ associated with the uncompressed wideband wave.

Within the past few years another type of FM demodulator has received widespread use and attention, with particular application in the tracking of satellites. This demodulator has been variously defined as the "phase-locked oscillator" or "phase-lock tracking filter".

Basically this demodulator is an crosscorrelation detector. It can be designed to yield a reduction in the FM threshold over a realizable conventional FM demodulator for the same wideband FM wave. The degree of FM threshold reduction is determined by the extent to which the closed loop noise bandwidth can be made less than the noise bandwidth of the first IF. As a demodulator of narrowband FM waves, the performance of the phase-locked auto-correlation detector would be identical to that of an ideal conventional narrowband FM receiver of the same IF noise bandwidth, if the latter receiver could be attained physically. However, if the IF noise bandwidths required could not be attained physically at the IF frequency, due to the extremely high Q's of the circuits involved and from stability considerations, then since the realization of such noise bandwidths can be readily achieved at baseband frequencies, one could say that the phase-locked tracking filter in this case yielded improved performance over that available from existing conventional FM demodulators, due only to problems of physical realizability.

In addition, the phase-locked crosscorrelation detector is capable of yielding information regarding the phase constant associated with the input signal through use of an auxiliary phase comparator by virtue of the quadrature relationship between the mean phase of the signal and local oscillator.

The phase-locked crosscorrelation detector operates as an FM demodulator and as is the case of the conventional FM receiver, the performance is determined by the above threshold $\left[S/N\right]_{BF}$, the noise bandwidth of the system and the FM improvement threshold.

A complete study of the phase-locked detector must necessarily consider the above variables and the corresponding relations in a conventional FM receiver designed to demodulate the same FM wave. However, the general noise characteristics of the phase-locked detector, particularly the question of FM threshold and performance of the system in the threshold region, have not been investigated in the literature.

As the performance of the phase-locked crosscorrelation detector in any particular application, regardless of the bandwidth involved, is ultimately limited by the FM threshold effect, then it is of utmost importance that the noise performance of the system be thoroughly examined and compared with that of the conventional FM demodulator.

This thesis comprises the results of such an investigation, together with the experimental noise performance of a phase-locked crosscorrelation detector as measured in the laboratory.

The linear no interference performance of the phase-locked demodulator has been extensively examined using standard linear servo techniques. This aspect of performance is considered within the body of the present thesis in the interests of completeness and reference.

SECTION II

GENERAL PERFORMANCE OF THE PHASE-LOCKED OSCILLATOR

A. Block Diagram and Possible Applications of the System:

The system may be represented by the block diagram of Figure ~~2~~ 1, wherein negative feedback is applied around the phase detector, in such a way that the voltage controlled local oscillator follows the instantaneous phase of the input signal, by virtue of a feedback voltage proportional to instantaneous phase error between signal and local oscillator being applied so as to frequency modulate the local oscillator.

Such a system is capable of operating as an FM demodulator yielding a range of significant improvement in the FM threshold without degradation of the $\left[S/N\right]_{BF}$ associated with a conventional FM receiver designed to demodulate the same wideband FM wave.

By the addition of an integration network to operate on the baseband voltage e_{BF}, the system is capable of operating as a phase detector of linearity far exceeding that of a conventional phase detector without feedback.

The phase-locked oscillator is perhaps more adequately described as being a cross correlation detector, and use of the latter designation immediately implies certain features of general performance as well as possible applications of the device.

In addition to the above mentioned applications of the auto-correlation detector, "phase-locking" techniques may be applied so as to permit the removal of random phase variations from a number of RF carriers. This is particularly useful if it is desired to combine coherently at IF, the received signals available in a diversity communications system.

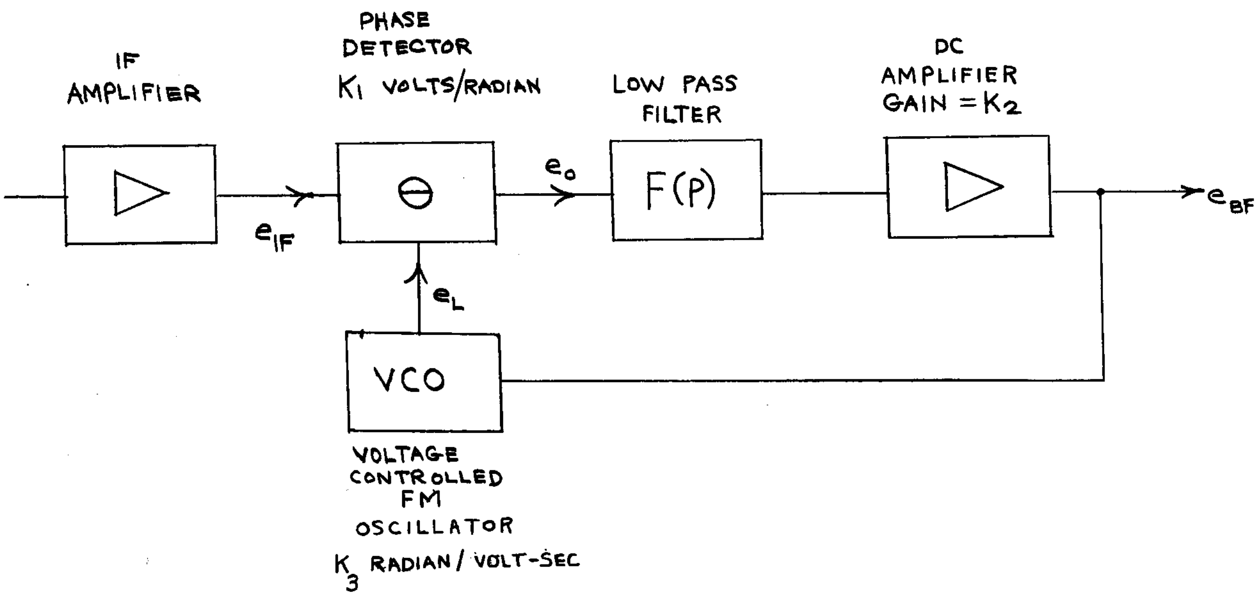


Figure 1

B.- Equations of General System Performance:

The phase detector performance is given from Equation (3.32) as

$$e_o = K_1 \left\{ \sin\left[\theta_s(t) - \theta_1(t)\right] + \frac{1}{E_s} \sum_{-B_1}^{B_1} \Delta E_i \sin\left[w_n t + \phi_n - \theta_1(t)\right] \right\} \quad \ldots . (2.1)$$

where $K_1 = \eta E_s$, and the mean phase of the local oscillator is in quadrature with the mean phase of the signal with the phase detector sensitivity assumed controlled. (See Section III, Part C.)

But $$e_{BF} = K_2 F(p)\, e_o \quad \ldots . (2.2)$$

and $$\Delta w_1 \cancel{(t)} = K_3 e_{BF} \quad \ldots . (2.3)$$

by inspection of Figure 1, where K_1, K_2 and K_3 are defined as the phase detector sensitivity (volts/ radian), amplifier gain (dimensionless) and the VCO sensitivity (radian/ sec-volt) respectively.

Hence by substitution

$$\Delta w_1(t) = K_2 K_3\, F(p)\, e_o \quad \ldots . (2.4)$$

$$= K_1 K_2 K_3\, F(p) \left\{ \sin\left[\theta_s(t) - \theta_1(t)\right] \right.$$
$$\left. \frac{1}{E_s} \sum_{-B_1}^{B_1} \Delta E_i \sin\left[w_n t + \phi_n - \theta_1(t)\right] \right\} \quad \ldots . (2.5)$$

But $p\theta_1(t) = \Delta w_1(t)$

by definition of the differential operator $p = \frac{d}{dt}$.

So

$$p\theta_1(t) = KF(p)\left\{ \sin\left[\theta_s(t) - \theta_1(t)\right] + \frac{1}{E_s}\sum_{-B_1}^{B_1}\Delta E_i \sin\left[w_n t + \phi_n - \theta_1(t)\right]\right\} \quad \ldots\ldots (2.6)$$

with $K = K_1\ K_2\ K_3$, and having dimensions of $\frac{1}{sec}$.

The system performance for general conditions of signal modulation and noise interference in the transient or steady state is defined by equation (2.6).

In subsequent Sections the no interference and interference performance will be examined.

SECTION III

PHASE DETECTOR OPERATION

A. Performance of the Ideal Multiplier:

Let the output of the first IF amplifier stage of noise bandwidth $2B_1$ consist of a wideband FM wave e_s and white noise interference e_i, where

$$e_s = E_s \cos \left[w_k t + \theta_s \right] \qquad \text{.... (3.1)}$$

$$e_i = \sum_{-B_1}^{B_1} \Delta E_i \cos \left[(w_k + w_n)t + \phi_n \right] \qquad \text{.... (3.2)}$$

E_s = peak amplitude of the FM wave.

ΔE_i = peak amplitude of each noise component within the noise bandwidth $2B_1$ containing a total mean noise power of N_{IF} watts, or

$$\Delta E_i = \sqrt{\frac{2N_{IF}}{2B_1} dw} \qquad \text{.... (3.3)}$$

$w_n = n\Delta w$, as $\Delta w \rightarrow 0$, w_n becomes continuous

w_k = angular frequency of the modulated carrier.

$w_k + w_n$ = angular frequencies of the noise spectrum.

$\theta_s = \theta_s(t) + c_1$ with $\theta_s(t)$ being the time varying angle modulation and c_1 an arbitrary constant.

ϕ_n = the random phase associated with each component of the noise spectrum.

Suppose the signal plus noise is operated upon by the local oscillator voltage e_1 in an ideal multiplier to yield an output voltage e_o, where

$$e_1 = E_1 \cos\left[w_1 t + \theta_1\right] \qquad \ldots\ldots (3.4)$$

$$e_o = 2g\,(e_i + e_s)e_1 \qquad \ldots\ldots (3.5)$$

E_1 = the peak amplitude of the local oscillator voltage.

w_1 = the angular frequency of the local oscillator.

$\theta_1 = \theta_1(t) + c_2$ with $\theta_1(t)$ being an angle modulation and c_2 an arbitrary constant.

$2g$ = the conversion constant of the multiplier.

Then

$$e_o = g\,F_s E_1 \cos\left[(w_k \pm w_1)t + \theta_s \pm \theta_1\right] \qquad \ldots\ldots (3.6)$$

$$g\,E_1 \sum_{-B_1}^{+B_1} \Delta E_i \cos\left[(w_k \pm w_1 + w_n)t + \phi_n \pm \theta_1\right]$$

by definition of the double angle expansion of cos $(A \pm B)$.

Selecting that frequency spectrum centered about $w_k - w_1$ it is seen that the minimum bandwidth requirements of the necessary filter to pass that spectrum is in general determined by w_{max} the highest modulation frequency associated with $\theta_s(t)$ and also by the modulation index associated with $\theta_s - \theta_1$.

For $w_k \neq w_1$ and $\theta_1(t) = 0$ corresponding to no baseband feedback, the minimum noise bandwidth requirements of the second IF are identical to that of the first IF. In the limit for

$\theta_1(t) \neq 0$ the noise bandwidth cannot be made less that $2w_{max}$. In the former case no reduction in the FM improvement threshold would ensue while in the latter case the "threshold" would be that associated with a narrowband FM wave. Between these limits lies a range of "threshold" reduction. It may be readily shown that the $[S/N]_{BF}$ under feedback conditions is that associated with the original wideband wave and hence, that this represents one solution to the problem of "threshold"[5.] reduction without degradation of $[S/N]_{BF}$.

If $w_k = w_1$ then the frequency spectrum has been translated about zero frequency, and if $c_1 - c_2 = \frac{\pi}{2}$ then

$$e_o = gE_1 \left\{ E_s \cos\left[(\theta_s(t) - \theta_1(t)\right] + \sum_{-B_1}^{B_1} \Delta E_i \cos\left[w_n t + \phi_n - \theta_1(t)\right]\right\}$$

$$= gE_1 \left\{ E_s \sin\left[\theta_s(t) - \theta_1(t)\right] + \sum_{-B_1}^{B_1} \Delta E_i \sin\left[w_n t + \phi_n - \phi_1(t)\right] \right\} \quad .... (3.8)$$

and for

$$\frac{\left|\theta_s(t) - \theta_1(t)\right|^3}{3!} << \left|\theta_s(t) - \theta_1(t)\right| \quad .... (3.9)$$

it follows that

$$e_o = gE_1 \left\{ E_s \left[\theta_s(t) - \theta_1(t)\right] + \sum_{-B_1}^{B_1} \Delta E_i \sin\left[w_n t + \phi_n - \theta_1(t)\right]\right\} \quad .... (3.10)$$

hence for $w_k = w_1$ the ideal multiplier acts as an unbalanced phase detector. In the absence of baseband feedback, $\theta_1(t) = 0$ and the range of linear operation is severely limited to waves of very low modulation index. For this case the minimum noise bandwidth is determined only by the highest modulating frequency w_{max}.

The application of negative feedback around the phase detector results in an extension of the range of linearity to waves of higher modulation index. The minimum noise bandwidth of the feedback loop is now determined by both w_{max} and the peak angular frequency deviation for linear operation $\left.\Delta w(t)\right]_{\substack{peak\\linear}}$.

It will be shown that this represents another possible solution to the problem of FM threshold resuction without degradation of $\left[S/N\right]_{BF}$.

B. Performance of the Balanced Phase Detector:

Let the output signal plus noise voltage $e_i + e_s$ and the local oscillator voltage e_1 be applied to the balanced phase detector shown in Figure 1 where e_1 and $e_{IF} = e_s + e_i$ are defined in Section A, with $w_k = w_1$.

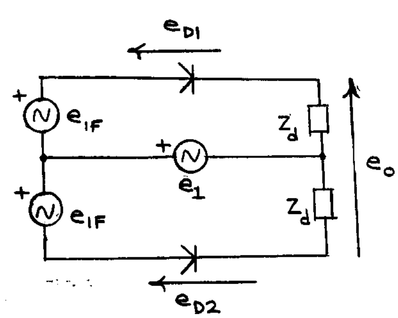


Figure 2.

The output voltage e_o may be evaluated as follows:
The voltage applied to diode D1 by the sign convention in Figure 2 is

$$e_{D1} = (Re)E_1 e^{j[w_1 t + \theta_1]}$$

$$\cdot\left\{1 + ae^{j(\theta_s - \theta_1)} + \sum_{-B_1}^{B_1} \Delta b e^{j[w_n t + \phi_n - \theta_1]}\right\} \quad \text{.... (3.11)}$$

where

$$a = \frac{E_s}{E_1} \quad \text{and } \Delta b = \frac{\Delta E_i}{E_1} \quad \text{.... (3.12)}$$

but

$$\left\{1 + ae^{j(\theta_s - \theta_1)} + \sum_{-B_1}^{B_1} \Delta b e^{j(w_n t + \phi - \theta_1)}\right\} = M_1 e^{j(\Psi_1)} \quad \text{.... (3.13)}$$

since the quantity on the left hand side is a complex number, by definition it follows that

$$\Psi_1 = \arctan \frac{a \sin(\theta_s - \theta_1) + \sum \Delta b \sin(w_n t + \phi_n - \theta_1)}{1 + a\cos(\theta_s - \theta_1) + \sum \Delta b \cos(w_n t + \phi_n - \theta_1)} \quad \text{.... (3.14)}$$

and

$$[M_1]^2 = \left[1 + a\cos(\theta_s - \theta_1) + \sum \Delta b \cos(w_n t + \phi_n - \theta_1)\right]^2$$
$$+ \left[a \sin(\theta_s - \theta_1) + \sum \Delta b \sin(w_n t + \phi_n - \theta_1)\right]^2 \quad \text{.... (3.15)}$$

or

$$[M_1]^2 = 1 + a^2 + \sum (\Delta b)^2 + 2a\cos(\theta_s - \theta_1)$$
$$+ \sum 2\Delta b \cos(w_n t + \phi_n - \theta_1) + \sum 2a\Delta b \cos(\theta_s - w_n t - \phi_n) \quad \text{.... (3.16)}$$

from an expansion and collection of terms in (3.15).

Similarly the voltage e_{D2} applied to diode D2 is

$$e_{D2} = (Re)E_1 e^{j(w_1 t + \theta_1)} \quad M_2 e^{j(\Psi_2)} \qquad \ldots. (3.17)$$

where

$$\Psi_2 = \text{arc tan} \; \frac{-a \sin(\theta_s - \theta_1) - \sum \Delta b \sin(w_n t + \phi_n - \theta_1)}{1 - a \cos(\theta_s - \theta_1) - \sum \Delta b \cos(w_n t + \phi_n - \theta_1)} \qquad \ldots. (3.18)$$

and

$$\left[M_2\right]^2 = 1 + a^2 + \sum (b)^2 - 2a \cos(\theta_s - \theta_1) - \sum 2\Delta b \cos(w_n t + \phi_n - \theta_1) + \sum 2a\Delta b \cos(\theta_s - w_n t - \phi_n) \qquad \ldots. (3.19)$$

which follows by definition and by replacing "a" and "b" in Equation (2.16) by "-a" and "-b" respectively.

Now for $a \ll 1$ and $b \ll 1$, $|M_1|$ and $|M_2|$ are of the form

$$\left[1 \pm 2\, a \cos(\theta_s - \theta_1) \pm \sum 2\Delta b \cos(w_n t + \phi_n - \theta_1)\right]^{\frac{1}{2}} \qquad \ldots. (3.20)$$

But this term is of the form $(1 \pm z)^{\frac{1}{2}}$ .... (3.21)

where

$$z = 2 \left[a \cos(\theta_s - \theta_1) + \sum \Delta b \cos(w_n t + \phi_n - \theta_1)\right] \qquad \ldots. (3.22)$$

and the expansion of

$$(1 \pm z)^n = 1 \pm nz + n(n-1)\frac{z^2}{2!} \pm n(n-1)(n-2)\frac{z^3}{3!} \qquad \ldots. (3.23)$$

Now application of e_{D1} and e_{D2} to D1 and D2 develops a baseband voltage e_o across the output terminals of the phase detector given by

$$e_o = (e_{D1} - e_{D2})\eta \qquad \text{.... (3.24)}$$

where η is the rectification efficiency of the diodes.

By by definition

$$e_{D1} - e_{D2} = E_1 \left[|M_1| - |M_2| \right] \qquad \text{.... (3.25)}$$

$$= E_1 \left[(1+z)^{\frac{1}{2}} - (1-z)^{\frac{1}{2}} \right] \qquad \text{.... (3.26)}$$

$$= E_1 \, 2 \left[\frac{z}{2} + \frac{1}{1} \frac{\cdot 3}{\cdot 2 \cdot 4 \cdot 6} \cdot z^3 \; .. \right] \qquad \text{.... (3.27)}$$

$$= E_1 \, z \qquad \text{.... (3.28)}$$

since $\frac{E_s}{E_1} \ll 1$ and $\frac{1}{E_1} \sum \Delta E_i \ll 1$.

Hence

$$e_o = \eta E_1 z = E_1 \eta \left[\frac{E_s}{E_1} \cos(\theta_s - \theta_1) + \frac{1}{E_1} \sum_{-B_1}^{B_1} \Delta E_i \cos(w_n t + \phi_n - \theta_1) \right] \qquad \text{.... (3.29)}$$

$$= \eta E_s \left[\cos(\theta_s - \theta_1) + \frac{1}{E_s} \cdot \sum_{-B_1}^{B_1} \Delta E_i \cos(w_n t \quad \phi_n - \theta_1) \right] \qquad \text{.... (3.30)}$$

and for $\theta_1 = \theta_1(t) + c_2$

and $\quad \theta_s = \theta_s(t) + c_1$

with $\quad c_1 - c_2 = \frac{\pi}{2}$ .... (3.31)

then $e_o = E_s \eta \left\{ \sin\left[\theta_s(t) - \theta_1(t)\right] \right.$

$$+ \quad \frac{1}{E_s} \sum_{-\beta_1}^{\beta_1} \Delta E_i \sin \left[w_n t + \phi_n - \theta_1(t) \right] \Bigg\} \qquad \text{.... (3.32)}$$

which is identical in form to the equation developed for the unbalanced phase detector with the same considerations discussed in that connection, applying as well in this case.

C. <u>Control of Phase Detector Sensitivity:</u>

From an inspection of the general equations of output voltage e_o for the balanced or unbalanced phase detector, it is evident that the output of the phase detector is sensitive to amplitude variations of the applied voltages e_s, e_1, or e_i. It is consequently, necessary to control the sensitivity of the phase detector by an appropriate technique. There is of course, no problem regarding e_1 as it is ideally a pure angle modulated wave, being a locally generated voltage of constant peak amplitude.

There are basically two techniques to control the amplitude variations associated with $e_s + e_i$, one maintaining the peak signal amplitude constant to the phase detector without regard to noise, and one maintaining the peak amplitude of signal and noise constant to the phase detector. The former would correspond to use of an AGC with a long time constant, while the latter would correspond to use of an amplitude limiter.

An ideal FM demodulator operating well above the FM improvement threshold has two essential features. The demodulated mean signal power associated with the desired angle modulation of the carrier is independent of $[S/N]_{IF}$, while the demodulated mean noise power associated with the undesired angle modulation of the carrier varies linearly with the $[S/N]_{IF}$.

The performance of the phase-locked crosscorrelation detector in this region is identical to that of an ideal FM demodulator. (Refer to Section VI). This is true regardless of whether an AGC or amplitude limiter is used to control phase detector sensitivity, due to the fact that the input power to the phase detector would be essentially a constant signal power in both cases.

If however, the minimum IF noise bandwidth prior to the limiter is much larger than the minimum noise bandwidth of the feedback loop, then the FM threshold of the system will be that associated with a conventional FM receiver of the same IF noise bandwidth. This would not be the result if an AGC circuit were used in the above situation. In the above case, if it were desired to use a limiter and at the same time obtain a reduction in the FM threshold it would be necessary to apply feedback around the IF stage so that the noise bandwidth prior to the limiter could be reduced.

SECTION IV

NO INTERFERENCE PERFORMANCE OF THE PHASE-LOCKED DEMODULATOR

A. Linear Equivalent Circuit and Criterion for Linear System Operation:

Under conditions of no interference, the IF noise power density is zero, hence from equation (3.6) it follows that

$$p\theta_1(t) = KF(p) \sin\left[\theta_s(t) - \theta_1(t)\right] \qquad .... (4.1)$$

Let the phase error $\Delta\theta_e(t)$ be defined as

$$\Delta\theta_e(t) = \theta_s(t) - \theta_1(t) \qquad .... (4.2)$$

hence the sufficient condition for linear operation is that

$$\left|\frac{\Delta\theta_e(t)}{3!}\right|^3 << \left|\Delta\theta_e(t)\right| \qquad .... (4.3)$$

Thus under the defined conditions of linearity

$$p\theta_1(t) = KF(p)\left[\theta_s(t) - \theta_1(t)\right] \qquad .... (4.4)$$

Rearranging (4.4) gives

$$\theta_1(t)\left[p + KF(p)\right] = KF(p).\theta_s(t) \qquad .... (4.5)$$

or

$$\theta_1(t) = T(p).\theta_s(t) \qquad .... (4.6)$$

Where the closed loop transfer function is defined from (4.5) as

$$T(p) = \frac{KF(p)}{p + KF(p)} \qquad .... (4.7)$$

The output baseband voltage e_{BF} is by definition (See Figure ~~2~~ 1)

$$e_{BF} = \frac{1}{K_3} \; p\theta_1(t) \qquad \ldots. \; (4.8)$$

or

$$e_{BF} = \frac{1}{K_3} \; p \; T(p).\theta_s(t) \qquad \ldots. \; (4.9)$$

from substitution of (4.6) into (4.8)

It is desired to represent the system by its linear equivalent circuit. It is apparent from inspection that the linear equivalent circuit shown in Figure 3 represents the actual system under linear operation since the equivalent circuit implies Equation (4.4)

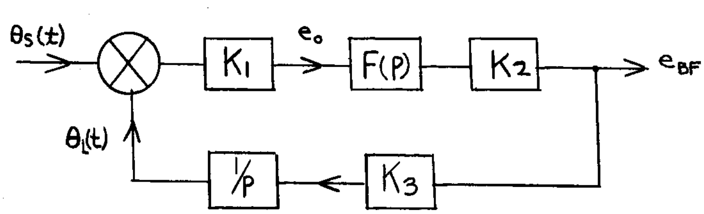


Figure 3.

Equation (4.4) can equally well be represented by the linear equivalent circuit of Figure 4, in which case it is seen that the input from the signal path is considered to be angular frequency deviation $p\theta_s(t)$ as opposed to phase deviation $\theta_s(t)$ in the representation of Figure 3. Due to the change in dimensions of the input for Figure 4, the $\frac{1}{p}$ operator must necessarily be included in the forward loop. Forms of both circuits will be found in the literature[6,7].

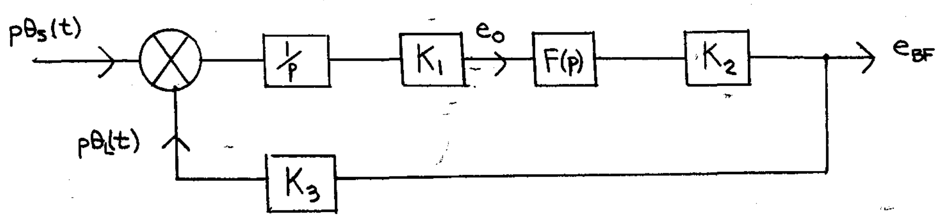


Figure 4.

Whether one considers $\theta_s(t)$ or $p\theta_s(t)$ to be the equivalent input to the loop is immaterial, as the VCO is physically following both the instantaneous phase deviation and the instantaneous frequency deviation of the input FM wave.

The important point however, is that the crosscorrelation detector must be capable of following both the highest baseband frequencies associated with $\theta_s(t)$ and the peak angular frequency deviations. Hence the minimum noise bandwidth associated with the closed loop transfer function T(p) is determined by both the above variables.

This is not at all surprising, since in a conventional FM receiver the minimum IF noise bandwidth is in general determined by both the highest modulating frequencies w_{max} and the peak angular frequency deviation $\Delta w(t)_{peak}$. However there exists a range of wideband FM waves which for demodulation in the phase-locked detector, require a smaller noise bandwidth than the IF noise bandwidth of a conventional FM receiver. This is due to the fact that the noise bandwidth of the loop is essentially set by the larger of the two variables w_{max} or $\Delta w(t)_{peak}$.

The phase error $\Delta\theta_e(t)$ between e_1 and e_s from the no modulation quadrature relation is completely specified for linear steady state or transient operation from the closed loop transfer function T(p), for a given input angle modulation.

B. Limits of Linear Operation:

In the absence of a phase error $\Delta\theta_e(t)$ between the signal and local oscillator voltage, the feedback loop is not capable of developing any baseband voltage e_{BF} since from Equations (4.1), (4.2), and (4.8)

$$p\theta_1(t) = \Delta w_1(t) = K_3 . e_{BF} = K\,F(p).\sin\left[\Delta\theta_e(t)\right] = 0 \qquad (4.10)$$

for $\Delta\theta_e(t) = 0$

Thus a certain amount of phase error is necessary to follow the rates of change of the angular frequency deviation of the modulated input signal.

The peak angular frequency deviation capable of being followed by the loop under linear operation is by definition

$$\left[\Delta w_1(t)\right]_{\substack{peak\\ linear}} = K\,F(p)\left[\Delta\theta_e(t)\right]_{\substack{peak\\ linear}} \qquad \ldots.\ (4.11)$$

where linearity implies $\dfrac{\left|\Delta\theta_e(t)\right|^3}{3!} << \left|\Delta\theta_e(t)\right|$

Now the limits of linear performance are determined by the percentage departure of $[\theta - \sin\theta]$ from linearity as a function of θ.

With reference to Figure 5, showing $\frac{\theta - \sin\theta}{\theta} \times 100$ as a function of θ, it will be observed that for $\theta = 0.1$, 0.3, 0.5, 0.7 and 0.9 the percentage departures of $\sin\theta$ from linearity are .16, 1.5, 4.1, 8.0 and 12.7, respectively.

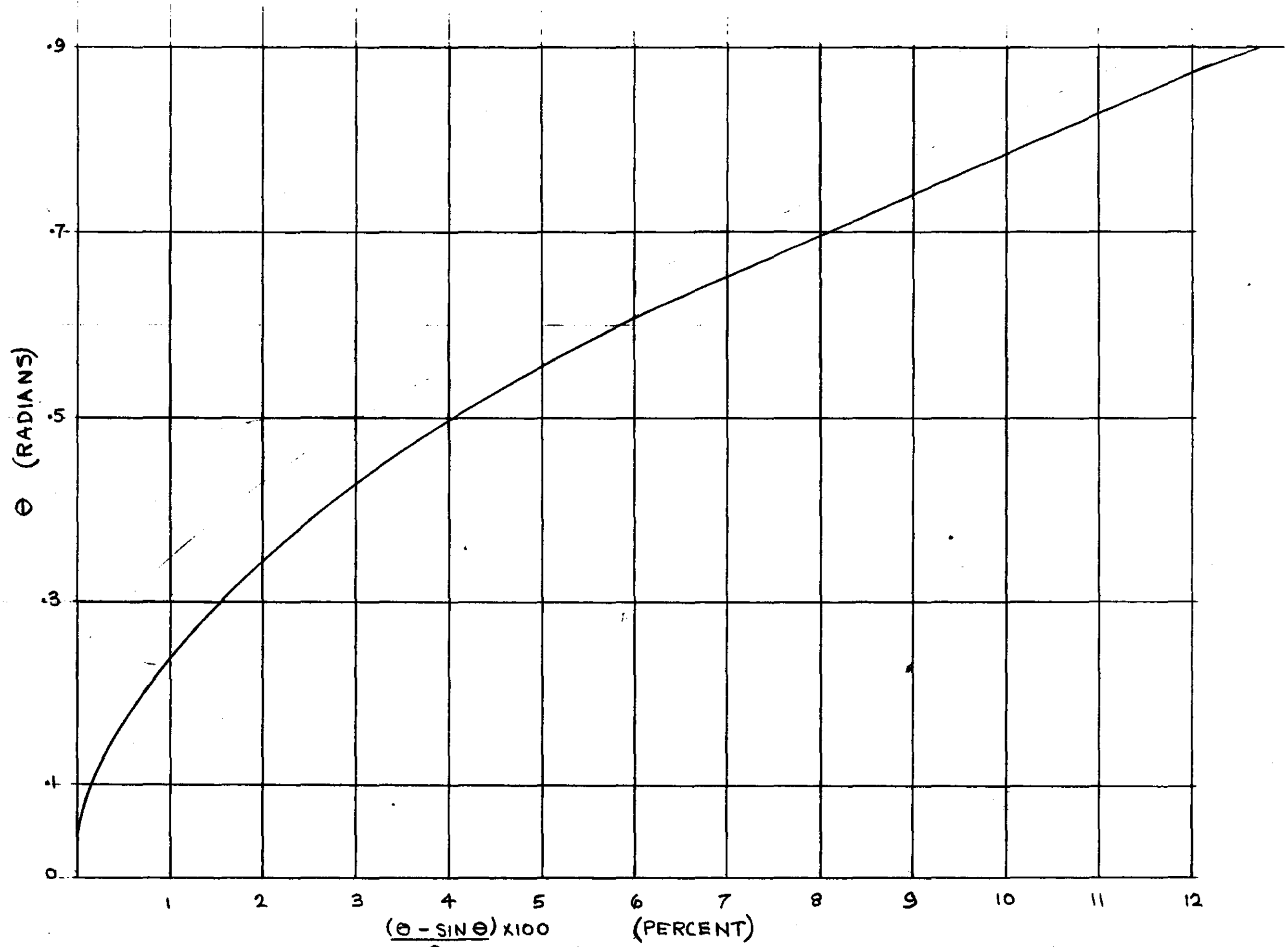


Figure 5.

The limits of linear performance will be considerably below a $\Delta\Theta_e(t)$ equal to one radian, from the above considerations. As linearity is a relative measure of the degree of non-linearity, then the permissible upper limit of $\Delta\Theta_e(t)$ in a specific application would be determined by the maximum allowable intermodulation distortion of the system.

Hence the peak angular frequency deviation for linear operation is given by

$$\left[\Delta w_1(t)\right]_{\substack{\text{peak}\\\text{linear}}} < K\, F(p)\left|\sin(1)\right| \qquad \ldots. \ (4.12)$$

since $\Delta\Theta_e(t) < 1$ radian for linear operation.

K F(p) is a maximum when F(p) = 1 corresponding to either no low pass filter or to modulating frequencies low in comparison to the 3db frequencies of F(p). The maximum possible value of $\Delta w(t)$ is given from Equation (4.1) as

$$\left[\Delta w_1(t)\right]_{\max} = K\, F(p) \qquad \ldots. \ (4.13)$$

corresponding to $\left|\sin \frac{\pi}{2}\right|$. Let the maximum possible angular frequency deviation F , be donoted by w_d.

Clearly under these conditions the performance is highly non-linear, as the higher order terms in the power series expansion are not negligible in comparison to $\Delta\Theta_e(t)$. Furthermore, if the angular frequency deviation of the carrier were increased beyond $\left[\Delta w(t)\right]_{\max}$, the loop is incapable of developing the increased baseband voltage e_{BF}, as sin $\Delta\Theta_e(t)$ is a decreasing function for

$\frac{\pi}{2} < \left|\Delta\theta_e(t)\right| < \pi$. Consequently the loop will fall out of step for that period of time in which $\left[\Delta w(t)\right]_{max}$ is less than the instantaneous angular frequency deviation of the modulated carrier.

C. Steady State Linear Performance:

(1) Phase Error as a Function of Loop Parameters:

The steady state linear performance of the phase-locked demodulator is completely specified from the closed loop transfer function T(p) defined in Equation (4.7). For steady state performance

$$T(p) = T(jw) \qquad \text{.... (4.14)}$$

and T(jw), being a complex number, will in general be given by

$$T(jw) = \left|T(jw)\right| \left[\cos B(w) + j \sin B(w)\right] \qquad \text{.... (4.15)}$$

$$= \left|T(jw)\right| \; e^{jB(w)} \qquad \text{.... (4.16)}$$

where T(jw) and B(w) are the amplitude and phase shift characteristics respectively, of the closed loop transfer function.

With regard to system performance, one is interested in reproducing the variations of $\theta_s(t)$ with a minimum degree of frequency and phase shift distortion within the bandwidth containing the frequency spectrum of the desired angle modulation, and at the same time insuring the minimum possible noise bandwidth for the closed loop transfer function T(jw). In addition, linear operation of the loop restricts the phase error $\Delta\theta_e(t)$, which is necessary for the tracking operation, to fractions of a radian.

The peak angular frequency deviation capable of being followed by the loop for linear operation is given from Equation (4.11) as

$$\left|\Delta w_1(t)\right|_{\substack{\text{peak}\\\text{linear}}} = K\,F(p)\,\left|\Delta\theta_e(t)\right|_{\substack{\text{peak}\\\text{linear}}} .$$

where $\dfrac{\left|\Delta\theta_e(t)\right|^3}{3!} << \left|\Delta\theta_e(t)\right|$ for linear operation.

Now for linear steady state operation $\Delta\theta_e(t)$ is given as

$$\Delta\theta_e(t) = \theta_s(t)\left[1 - T(jw)\right] \qquad \text{.... (4.17)}$$

since $\theta_1(t) = T(jw)\,\theta_s(t)$ under steady state operation by Equation (4.6)

The dependence of $\Delta\theta_e(t)$ on the amplitude and phase shift characteristics may be seen by substituting Equation (4.15) into (4.17) to yield

$$\Delta\theta_e(t) = \theta_s(t)\left\{1 - \left|T(jw)\right|\left[\cos B(w) + j\sin B(w)\right]\right\} \qquad (4.18)$$

or

$$\Delta\theta_e(t) = \theta_s\!\left(t+\frac{\gamma}{\omega}\right)\left[1 - 2\left|T(jw)\right|\cdot\cos B(w) + \left|T(jw)\right|^2\right]^{\frac{1}{2}}$$

$$\text{.... (4.19)}$$

where

$$\gamma = \arctan\frac{-\left|T(jw)\right|\cdot\sin B(w)}{1-\left|T(jw)\right|\cdot\cos B(w)} \qquad \text{.... (4.20)}$$

An expression for $\Delta\theta_e(t)$ could equally well be expressed in the form

$$\Delta\Theta_e(t) = \Theta_s(t)\left[1 - \frac{K\,F(jw)}{jw + K\,F(jw)}\right] \qquad \ldots.\ (4.21)$$

by definition of T(jw) from Equation (4.7)

$$\Delta\Theta_e(t) = \Theta_s(t)\left[\frac{jw}{jw + K\,F(jw)}\right] = \Theta_s(t)\left[\frac{1}{1 + \frac{K\,F(jw)}{jw}}\right] \qquad (4.22)$$

where the term $\frac{K\,F(jw)}{jw}$ is a dimensionless function of frequency.

If $\Theta_s(t)$ is a sinusoidal function of time defined by

$$\Delta\Theta_s(t) = M_a \sin w_a t \qquad \ldots.\ (4.23)$$

then from Equation (4.19) or (4.22) it is seen that the error voltage $\Delta\Theta_e(t)$ is a sinusoidal wave having the same frequency as $\Theta_s(t)$, but having an amplitude and phase shift which are a function of frequency and the closed loop parameters. Suppose that the angular frequency deviation of the modulated input carrier was adjusted such that the loop performance was close to the point of being perceptively non-linear, at a modulating frequency, such that in Equation (4.22) $\left|\frac{K\,F(jw)}{jw}\right| \gg 1$

Now if the angular frequency deviation were held constant while the modulating frequency w_a were varied towards the 3db frequency of the closed loop transfer function, then the performance of the loop would become increasingly nonlinear and in the limit the loop would lose synchronism, for that period of time during which the frequency deviation of the carrier exceeds K F(jw), as discussed in connection with Equation (4.13).

(2) Amplitude, Phase-shift and Phase Error Characteristics:

It will be instructive to examine the closed loop transfer function $T(p)$ for $F(p)$, defined by the one pole low pass filter shown in Figure 6.

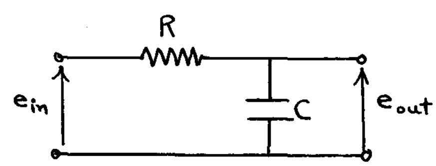


Figure 6.

By inspection

$$F(p) = \frac{e_{out}}{e_{in}} = \frac{1}{1 + pT} \qquad \text{.... (4.24)}$$

where $T = RC$ .... (4.25)

but $T(p) = \dfrac{K\,F(p)}{p + K\,F(p)}$ from Equation (4.7)

Substitution of (4.24) into (4.7) gives

$$T(p) = \frac{K}{p(1 + pT) + K} = \frac{K/T}{p^2 + \frac{p}{T} + \frac{K}{T}} \qquad \text{.... (4.26)}$$

or expressed in the standard form

$$T(p) = \frac{w_o^{\,2}}{p^2 + 2\alpha p + w_o^{\,2}} \qquad \text{.... (4.27)}$$

in terms of the poles of the function

$$T(p) = \frac{w_o^{\,2}}{(p - p_1)\,(p - p_1^{*})} \qquad \text{.... (4.28)}$$

where the following definitions are involved

$w_o = \frac{K}{T}^{\frac{1}{2}}$ the closed loop natural resonant frequency

$\alpha = \frac{1}{2T}$ the real component of the complex poles

$\delta = \frac{\alpha}{w_o}$ the damping factor

$p_1 = -\alpha + j\, w_o \left(1 - \delta^2\right)^{\frac{1}{2}}$ Roots of denominator

$p_1^{*} = -\alpha - j\, w_o \left(1 - \delta^2\right)^{\frac{1}{2}}$

The steady state transfer function is given from Equation (4.27) for $p = jw$ as

$$T(jw) = \frac{w_o^{\,2}}{w_o^{\,2}\left[-\frac{w^2}{w_o^{\,2}} + \frac{2\delta jw}{w_o^{\,2}} + 1\right]} \qquad \text{.... (4.30)}$$

or

$$T(jw) = \frac{1}{1 - \left(\frac{w}{w_o}\right)^2 + 2j\,\delta\,\frac{w}{w_o}} \qquad \text{.... (4.31)}$$

since $\delta = \frac{\alpha}{w_o}$ by definition.

Now $T(jw) = |T(jw)| \, e^{j\,B(w)}$ from Equation (4.16)

thus for the particular F(p) considered

$$|T(jw)| = \frac{1}{\left\{\left[1 - \left(\frac{w}{w_o}\right)^2\right]^2 + 4\delta^2 \left(\frac{w}{w_o}\right)^2\right\}^{\frac{1}{2}}} \qquad \ldots. \ (4.32)$$

and

$$-B(w) = \text{arc tan } \frac{2\,\delta\,\frac{w}{w_o}}{\left[1 - \left(\frac{w}{w_o}\right)^2\right]} \qquad \ldots. \ (4.33)$$

which follows by definition from an examination of (4.3)

The amplitude and phase shift characteristics $|T(jw)|$ and $B(w)$ are presented graphically as a function of $\frac{w}{w_o}$ and δ in Figures 7 and 8 respectively.

The linear steady state error $\Delta\theta_e(t)$ is given from Equation (4.17) as

$$\Delta\theta_e(t) = \theta_s(t)\left(1 - T(jw)\right)$$

The variation of $\Delta\theta_e(t)$ as a function of frequency is determined by the vector quantity $1 - T(jw)$. This quantity can be readily constructed as a polar plot from the amplitude and phase shift characteristics; this plot is presented in Figure 9.

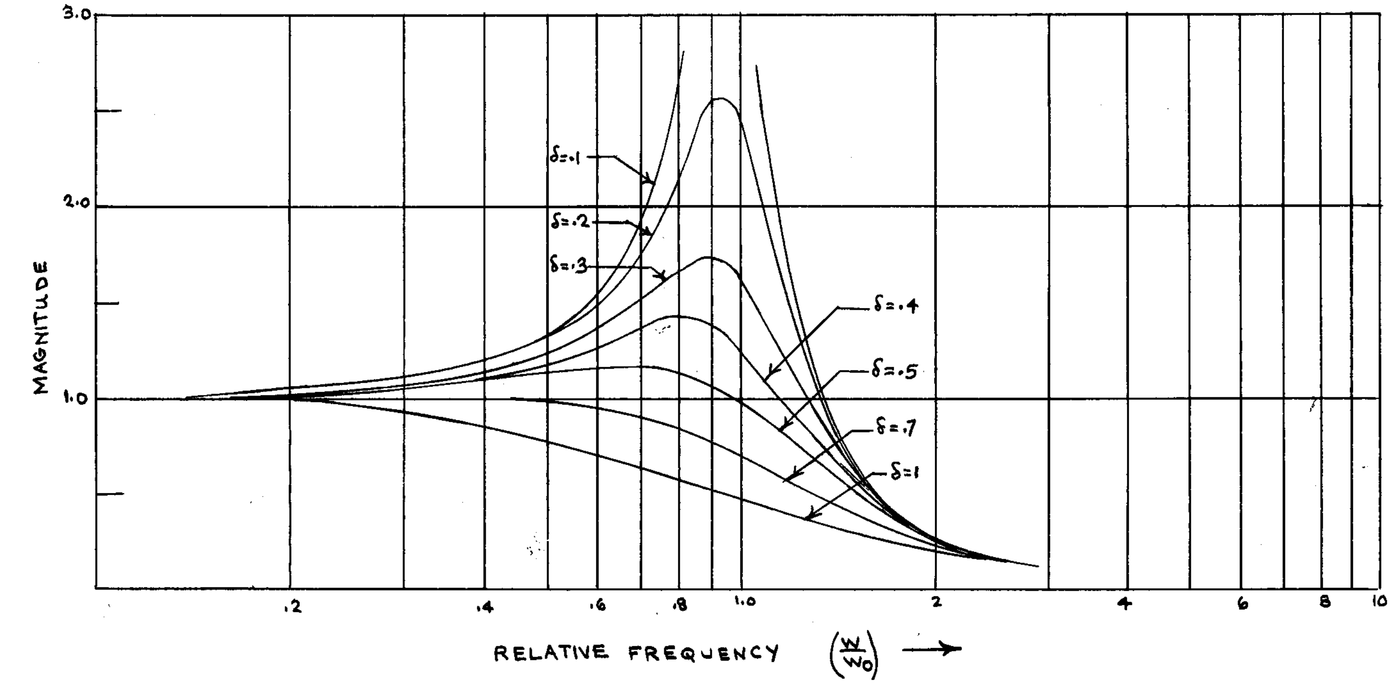


FIGURE 7

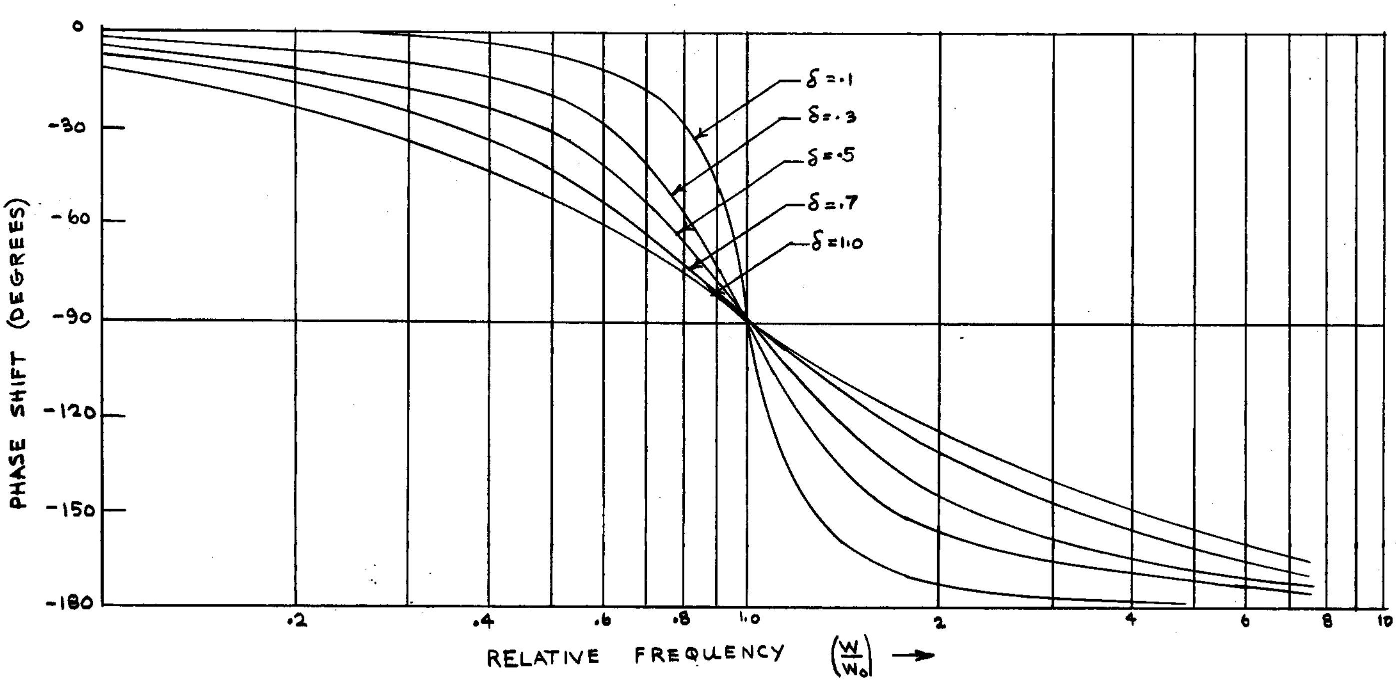


FIGURE 8

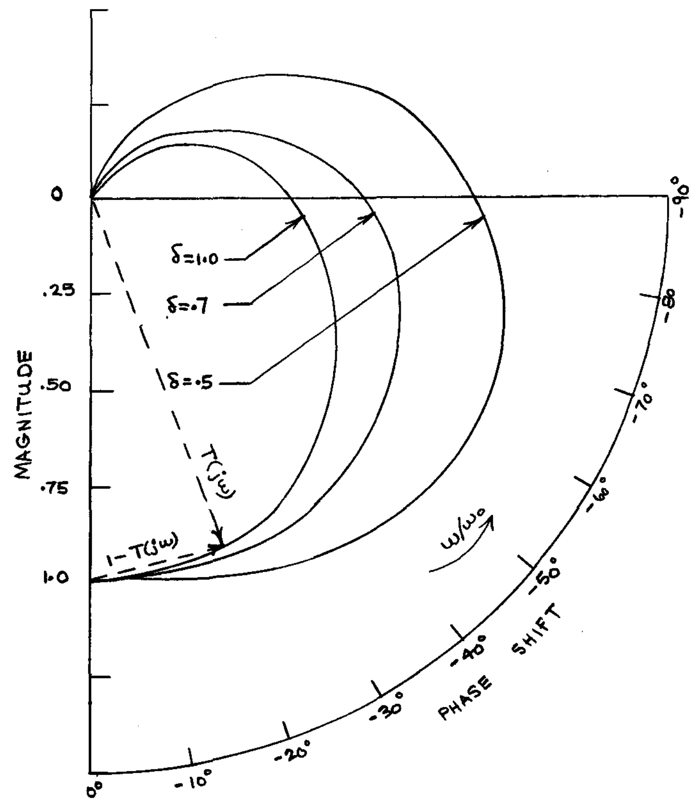


FIGURE 9

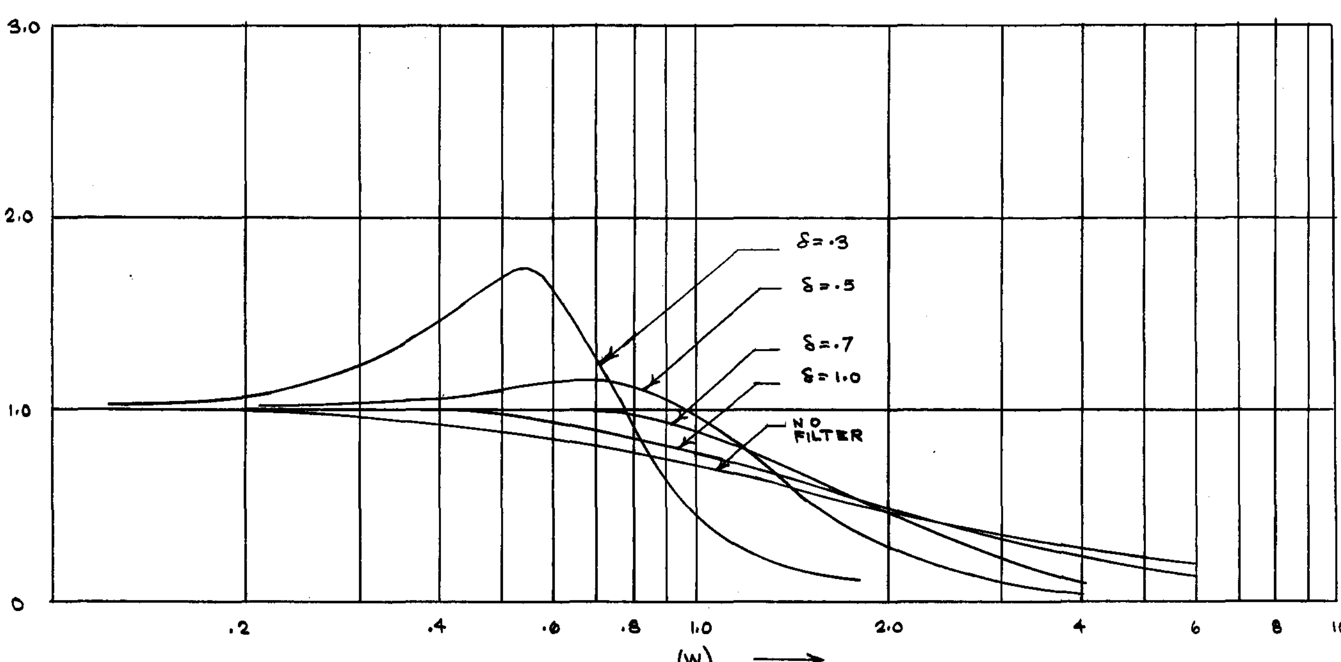


FIGURE 10

(3) Comparison of Performance With and Without the Low Pass Filter:

It is of interest to compare the performance of the feedback demodulation with and without a low pass filter $F(p)$. If $F(p) = 1 + j\,0$ corresponding to no low pass filter, then from Equation (4.26)

$$T(jw) = \frac{K}{p + K} = \frac{1}{1 + j\dfrac{w}{w_d}} \qquad \text{.... } (4.34)$$

where $w_d = K$ by definition associated with (4.13).

The transient response of the feedback demodulator is sluggish for $F(p) = 1 + j\,0$ since the closed loop transfer function contains a simple pole located on the negative real axis. The addition of the low pass filter $F(p)$ defined by Equation (4.24), can provide improved transient performance, however, the 3db frequencies are greater than that for the loop without a filter, by an amount determined by the damping factor.

It would be useful to compare the curves of $T(jw)$ for both cases as a function of frequency to observe the relation between the 3db frequencies of the two transfer functions. This can be done by determining the relation between w_o and w_d, and plotting Equations (4.31) and (4.34) as a function of $\frac{w}{w_d}$.

By definition from Equation (4.29)

$$w_o = \frac{K}{T}$$

$$\delta = \frac{\alpha}{w_o} = \frac{1}{2T\left[\frac{K}{T}\right]^{\frac{1}{2}}}$$

Let the time constant T of the filter F(p) in Equation (4.24) be defined as $\frac{1}{w_F}$. The product KT must be a dimensionless number n_1, given by

$$KT = \frac{w_d}{w_F} = n_1 \qquad \text{.... (4.35)}$$

Substituting Equation (4.34) into w_o and δ gives

$$w_o = \frac{w_d}{\sqrt{n_1}} \qquad \text{.... (4.36)}$$

$$\delta = \frac{1}{2\sqrt{n_1}} \qquad \text{.... (4.37)}$$

Now

$$\frac{w}{w_d} = \frac{w}{w_o} \cdot \frac{w_o}{w_d} = \frac{w}{w_o} \cdot \frac{1}{\sqrt{n_1}} \qquad \text{.... (4.38)}$$

Thus the frequency scale of Figure 6, when multiplied by $\frac{1}{\sqrt{n_1}}$ for each damping factor, gives rise to the desired comparison curve shown in Figure 10. (PAGE 35)

The low pass filter, although improving the transient performance is seen to result in an increase in the noise bandwidth in comparison to the case with no filter. At the same time for those frequencies $w < w_d$, the factor $1 - T(jw)$ is less than that for the case with no filter, resulting in a reduction in the deviation capable of being followed ~~by~~ in the frequency range $w < w_d$.

The parameters of the loop are determined by the deviation and modulating frequency of the input FM wave. For a narrowband FM wave the bandwidth requirements are essentially determined by the highest modulating frequency w_{max}, while for the wideband FM case the bandwidth is essentially determined by the peak angular frequency deviation. However, for a given desired damping factor, this implies that the loop is capable of following a larger deviation than actually required. This is due to the fact that the deviation capable of being followed and the 3db frequency are intimately related for a given damping factor.

Suppose for example, that the highest modulating frequency w_{max}, to be followed is w_{max} = 50 kc/s, and that the peak angular frequency deviation of the FM wave is in one case 10 kc/s and in another 100 kc/s. Since from Equation (4.1)

$$\left|\Delta w(t)\right|_{\substack{\text{peak}\\ \text{linear}}} = K\,F(p)\,\left|\Delta\theta_e(t)\right|_{\substack{\text{peak}\\ \text{linear}}}$$

where $\left|\Delta\theta_e(t)\right|_{\substack{\text{peak}\\ \text{linear}}}$ depends on the permissible distortion, then for the angular frequency deviations of the FM waves to be followed linearly

$$\left|\Delta w(t)\right|_{\substack{\text{peak}\\ \text{linear}}} = 10 \text{ kc/s} \qquad \text{in the first case}$$

and $$\left|\Delta w(t)\right|_{\substack{\text{peak}\\ \text{linear}}} = 100 \text{ kc/s} \qquad \text{in the second case}$$

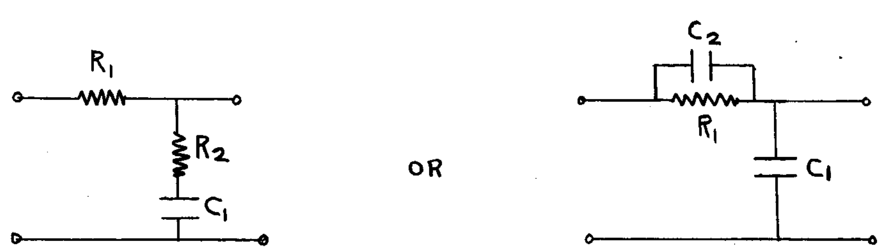
If for example, $\left|\Delta\theta_e(t)\right|_{\substack{\text{peak}\\ \text{linear}}}$ is taken as $\frac{1}{3}$ this implies that

$K_{min} = 20$ kc/s , for case one

and $K_{min} = 200$ kc/s , for the second case

These values of K_{min} are based on the requirements of angular frequency deviation, and have not been related to the actual value of K, associated with a particular dampening factor and 3db bandwidth in Figure 7. When this is done it will be seen that in general either K or the 3db frequency of the loop might be larger than K_{min} and w_{max} for the narrowband and wideband cases, respectively.

For the simple low pass filter an attempt to reduce the bandwidth of the closed loop response may result in an adverse effect on the transient response and the angular frequency deviation capable of being followed by the loop. A modified low pass filter, such as shown in Figure 11, will allow a certain degree of freedom in this respect[8]. It will be recognized that the limit cases of F(p) as defined below, correspond to no filter or the simple low pass filter.



$$F(p) = \frac{p\,c_1 R_2 + 1}{p\,c_1(R_1 + R_2) + 1}$$

$$F(p) = \frac{p\,R_1 c_2 + 1}{p\,R_1(c_1 + c_2) + 1}$$

Figure 11.

D. Transient Linear Performance:

The closed loop transfer function under linear operation for F(p) defined as in Equation (4.24) is given by

$$\frac{\theta_1(t)}{\theta_s(t)} = T(p) = \frac{w_o^2}{(p - p_1)(p - p_1^*)} \qquad \text{from (4.28)}$$

or

$$T(p) = \frac{w_o^2}{\left\{p + \alpha - jw_o\left[1 - \delta^2\right]^{\frac{1}{2}}\right\}\left\{p + \alpha + jw_o\left[1 - \delta^2\right]^{\frac{1}{2}}\right\}} \qquad \text{.... (4.39)}$$

from substitution of p_1 and p_1 as defined in (4.29).

It will be instructive to examine the response of the feedback system to a "step" of the input phase $\theta_s(t)$.

Hence by definition let

$$\begin{aligned} \theta_s(t) &= m \qquad \text{for } t > o \\ \theta_s(t) &= o \qquad \text{for } t < o \end{aligned} \qquad \text{.... (4.40)}$$

Taking the Laplace Transform of the function $\theta_s(t)$ and substituting in Equation (4.39), where the differential operator p is replaced by the complex frequency variable s, yields

$$\theta_1(s) = \frac{m}{S\left[s + \alpha - jw_o\sqrt{1 - \delta^2}\right]\left[s + \alpha - jw_o\sqrt{1 - \delta^2}\right]} \qquad \text{.... (4.41)}$$

since

$$\mathcal{L}\,\theta_s(t) = \frac{m}{s}$$

$\theta_1(s)$ may be expanded as

$$\frac{\theta_1(s)}{m} = \frac{a_1}{s} + \frac{a_2}{\left[s + \alpha - jw_o\sqrt{1-\delta^2}\right]}$$

$$\frac{a_3}{\left[s + \alpha + jw_o\sqrt{1-\delta^2}\right]} \qquad \text{.... (4.42)}$$

Evaluating the coefficients gives

$$a_1 = \left\{\left[\theta_1(s)\right] \cdot s\right\}_{s=0} \qquad \text{.... (4.43)}$$

by definition of Equations (4.41) and (4.42)

or by substitution

$$a_1 = \frac{w_o^2}{(\alpha - jw_o\sqrt{1-\delta^2})(\alpha + jw_o\sqrt{1-\delta^2})}$$

$$= \frac{w_o^2}{\alpha^2 + w_o^2(1-\delta^2)} = 1 \qquad \text{.... (4.44)}$$

similarly, by definition

$$a_2 = \left\{\left[\theta_1(s)\right] \cdot \left[s + \alpha - jw_o\sqrt{1-\delta^2}\right]\right\}_{s = \left[-\alpha + jw_o\sqrt{1-\delta^2}\right]}$$

$$\text{.... (4.45)}$$

or by substitution

$$a_2 = \frac{w_o^2}{\left[-\alpha + jw_o\sqrt{1-\delta^2}\right]\left[2\,jw_o\sqrt{1-\delta^2}\right]}$$

$$= \frac{w_o^2}{2\left[-w_o^2(1-\delta^2) - jw_o\sqrt{1-\delta^2}\right]}$$

$$= \frac{w_o^2}{-2w_o^2(1-\delta^2)\left[1+\frac{j\,\delta}{\sqrt{1-\delta^2}}\right]} = \frac{-1}{2(1-\delta^2)\left[1+\frac{\delta^2}{1-\delta^2}\right]^{\frac{1}{2}} e^{j\Gamma}}$$

.... (4.46)

$$= -\tfrac{1}{2}\sqrt{1-\delta^2}\cdot e^{-j\Gamma}$$

where $\Gamma = \arc\tan \frac{\delta}{\sqrt{1-\delta^2}}$

also, by definition

$$a_3 = \left(\theta_1(s)\cdot\left[s+\alpha+jw_o\sqrt{1-\delta^2}\right]\right)_{s=-\alpha-jw_o\cdot\sqrt{1-\delta^2}}$$

.... (4.47)

or by substitution

$$a_3 = \frac{w_o^2}{\left[-\alpha-jw_o\sqrt{1-\delta^2}\right]\left[-2\,jw_o\sqrt{1-\delta^2}\right]}$$

$$= \frac{w_o^2}{2\left[-w_o^2(1-\delta^2)+j\alpha w_o\sqrt{1-\delta^2}\right]}$$

$$= \frac{w_o^2}{-2w_o^2(1-\delta^2).\left[1-j\frac{\delta}{\sqrt{1-\delta^2}}\right]}$$

$$= \frac{1}{-2(1-\delta^2)\cdot\left[1+\frac{\delta^2}{1-\delta^2}\right]^{\frac{1}{2}} e^{-j\Gamma}} = -\tfrac{1}{2}\sqrt{\frac{1}{1-\delta^2}}\,.e^{j\Gamma}$$

.... (4.48)

Thus taking the inverse Laplace Transforms

$$\mathcal{L}^{-1} \frac{a_1}{s} = 1$$

$$\mathcal{L}^{-1}\left[\frac{a_2}{s+\alpha - jw_o\sqrt{1-\delta^2}}\right] = -\frac{1}{2}\frac{1}{\sqrt{1-\delta^2}} \cdot e^{-j\Gamma}\left[e^{-\alpha t + jw_o\sqrt{1-\delta^2}\,t}\right]$$

$$\mathcal{L}^{-1} \frac{a_3}{s+\alpha+jw_o\sqrt{1-\delta^2}} = -\frac{1}{2}\sqrt{1-\delta^2}\; e^{j\Gamma} \cdot \left[e^{-\alpha t - jw_o\sqrt{1-\delta^2}\,t}\right] \qquad \text{.... (4.49)}$$

The summation of the inverse transforms yields the desired time function, namely $\theta_1(t)$, so

$$\frac{\theta_1(t)}{m} = 1 - \frac{1}{2}\cdot\frac{1}{\sqrt{1-\delta^2}} \cdot e^{-\alpha t}$$

$$\cdot\; e^{j\left[w_o\sqrt{1-\delta^2}\;t-\Gamma\right]} + e^{-j\left[w_o\sqrt{1-\delta^2}\;t-\Gamma\right]} \qquad \text{.... (4.50)}$$

or, by definition of the cosine function

$$\frac{\theta_1(t)}{m} = 1 - \frac{e^{-\alpha t}}{\sqrt{1-\delta^2}} \cdot \cos\left[w_o\sqrt{1-\delta^2}\;t-\Gamma\right] \qquad \text{.... (4.51)}$$

$\theta_1(t)$ could also be expressed in terms of a sine function since, by definition of the right angle triangle defining

$$180^\circ = 90^\circ + \Gamma + \mu$$

Then

$$\Gamma = 90^{\circ} - \mu$$

so

$$\theta_1(t) = 1 + \frac{e^{-\alpha t}}{\sqrt{1 - \delta^2}} \cdot \sin\left[w_o \sqrt{1 - \delta^2}\ t + \mu\right] \quad \ldots. (4.52)$$

$\dfrac{\theta_1(t)}{m}$ is plotted as a function of δ and t in Figure 12

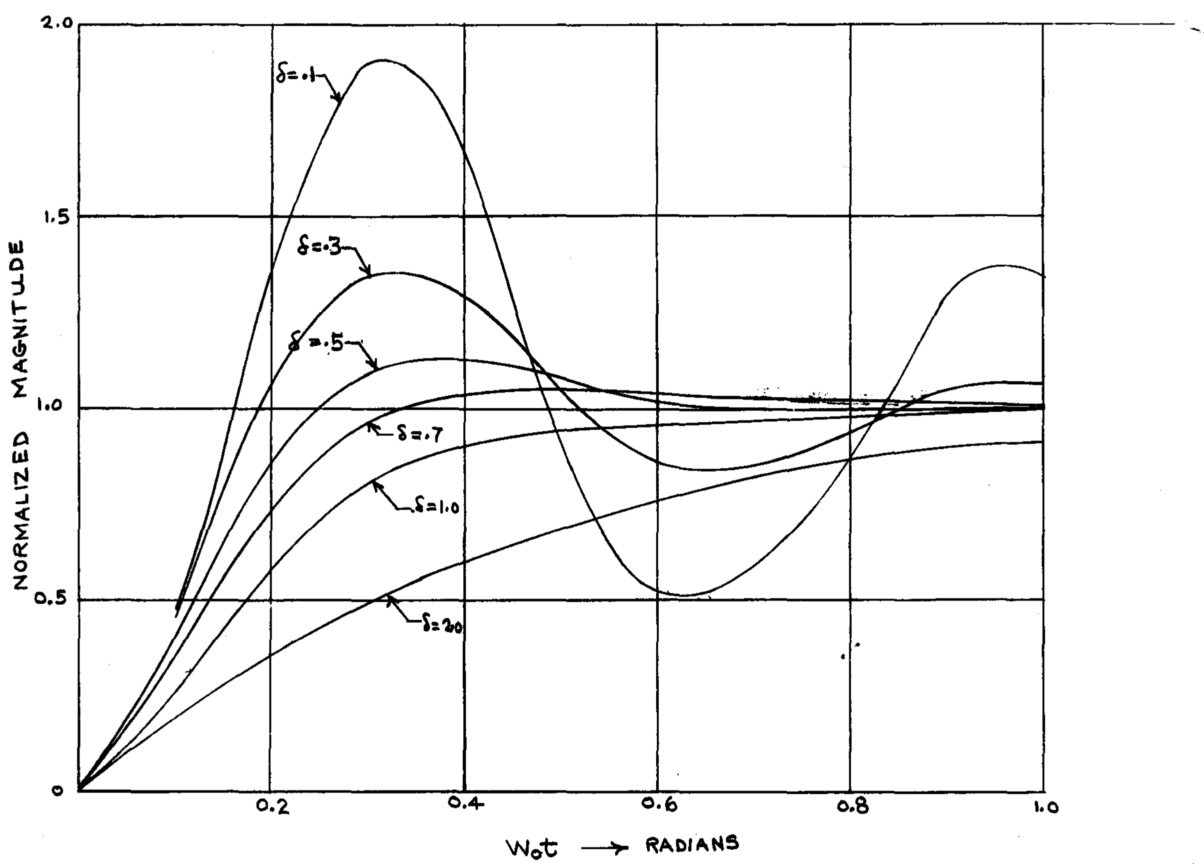


Figure 12

It will be seen that both the sluggish and oscillatory responses result in an appreciable squared phase error. The phase error $\Delta\theta_e(t)$ is given as

$$\Delta\theta_e(t) = \theta_s(t) - \theta_1(t)$$

Hence for $\theta_s(t)$ a step function, defined as in Equation (4.40), the integrated squared error is

$$\int_0^{\infty} \Delta\theta_e(t)^{\,2}\, dt = m^2 \int_0^{\infty} \left[\frac{e^{-\alpha t}}{\sqrt{1-\delta^2}} \cdot \sin\left[w_o \sqrt{1-\delta^2}\; t + \mu\right]\right]^2 dt \quad \text{.... (4.53)}$$

It can be shown that the function is a minimum for $\delta = 0.5$. The significance of the term m, in defining the step function, is that the value of the step for linear operation of the loop requires

$$\frac{\left|\Delta\theta_e(t)\right|^3}{3!} << \left|\Delta\theta_e(t)\right|$$

which implies that $\frac{m^3}{3!} << m$

It will be recognized that this value of m for linear operation will be several times less than the maximum value of $\Delta\theta_e(t)$ for linear operation under steady state conditions with sinusoidal excitation, since from Equation (4.17)

$$\Delta\theta_e(t) = \theta_s(t)\ \left[1 - T(jw)\right]$$

$$= m_a \sin w_a t \left[1 - T(jw) \right]$$

if the signal modulation is assumed to be a single sinusoid. The reason for the great difference in phase errors under step function and sinusoidal excitation is due to the fact that there is initially no feedback for a phase step, hence the phase error at that instant is just the magnitude of the phase step.

On the other hand for a ramp phase function corresponding to a frequency step function,

$$\left.\begin{aligned} \theta_s(t) &= 0 \qquad \text{for } t < 0 \\ \theta_s(t) &= \Delta w . t \qquad \text{for } t > 0 \end{aligned}\right\} \qquad \ldots. \ (4.54)$$

Since, by Equation (4.7)

$$\theta_1(t) = \theta_s(t) \quad T(p)$$

then

$$p\,\theta_1(t) = p\,\theta_s(t) \quad T(p) \qquad \ldots. \ (4.55)$$

But for $\theta_s(t)$ defined as in Equation (4.54), the transient performance to a frequency step Δw, is given by substitution of Equation (4.55) as

$$\Delta w_1(t) = p\,\theta_1(t) = \Delta w . T(p) \qquad \ldots. \ (4.56)$$

Taking the Laplace Transform of Equation (4.56) yields

$$\Delta w_1(s) = \Delta w \cdot \frac{T(s)}{s} \qquad \text{.... (4.57)}$$

But the function $\frac{T(s)}{s}$ has already been evaluated for a phase step, thus the inverse transform of $w_1(s)$ can immediately be written as

$$\Delta w_1(t) = \mathcal{L}^{-1} \Delta w_1(s) = \Delta w \left\{ 1 + e^{-\delta w_0 t} \sin\left[w_0 \sqrt{1-\delta^2}\, t + \mu \right] \right\} \qquad \text{.... (4.58)}$$

by comparison with Equation (4.52).

In contrast to the permissible phase step for transient linear operation, the magnitude of the frequency step for linear operation is essentially the same as the peak angular frequency deviation for steady state operation. This is due to the fact that the phase input to the feedback loop does not contain a discontinuity at t = 0. The expression of the phase response for the frequency step can be obtained upon integration of Equation (4.58).

Thus

$$\begin{aligned} \theta_1(t) &= \int_0^t \Delta w_1(t)\, dt \\ &= \Delta w \int_0^t \left\{ 1 + e^{-\delta w_0 t} \cdot \sin\left[w_0 \sqrt{1-\delta^2}\, t + \mu \right] \right\} dt \end{aligned} \qquad \text{.... (4.59)}$$

The considerations for optimum damping factor apply as well in this case. If the transient is assumed to exist for time t_1 with proper damping, then

$$\begin{aligned} \theta_1(t) = \; & \Delta w \int_0^{t_1} \left\{ 1 + e^{-\delta w_0 t} \sin\left[w_0 \sqrt{1-\delta^2}\, t + \mu \right] \right\} dt \\ & \Delta w \int_{t_1}^{t} dt \end{aligned} \qquad \text{.... (4.60)}$$

$$= \Delta w \; t + \underline{\underline{\Delta w}} \int_{0}^{t_1} \left\{ e^{-\delta w_o t} \; \sin \left[w_o \sqrt{1 - \delta^2} \; t + \mu \right] \right\} dt \qquad \text{.... (4.61)}$$

$\cong \Delta w \; t \qquad$ for $t \gg t_1$

SECTION V

NOISE PERFORMANCE OF THE CONVENTIONAL FM RECEIVER

A. Composite Waveform Prior to the Amplitude Limiter:

Let e_s and e_i as defined in Equations (3.1) and (3.2) represent the desired and interfering signals within the IF noise bandwidth prior to the limiter of a conventional FM receiver.

The resultant instantaneous voltage e_r, due to the desired and interfering signal is thus

$$e_r = e_s + e_i = (\mathrm{Re}) \left\{ E_s \cdot e^{j\left[w_k t + \theta_s(t)\right]} \right\} \cdot \left\{ 1 + \sum_{w_n=-B_1}^{+B_1} \frac{\Delta E_i}{E_s} \, e^{j\left[w_n t + \phi_n - \theta_s(t)\right]} \right\} \qquad \ldots. (5.1)$$

or

$$e_r = (\mathrm{Re}) \;\; M_i \, E_s \;\; e^{j\left[w_k t + \theta_s(t) + \Psi_i(t)\right]} \qquad \ldots. (5.2)$$

where M_i and $\Psi_i(t)$ must necessarily be defined as below for Equations (5.1) and (5.2) to be consistent.

$$\left[M_i\right]^2 = 1 + 2\sum_{-B_1}^{+B_1} \Delta x \cos\left[w_n t + \phi_n - \theta_s(t)\right] + \left\{ \sum_{-B_1}^{+B_1} \Delta x \cos\left[w_n t + \phi_n - \theta_s(t)\right] \right\}^2 + \left\{ \sum_{-B_1}^{B_1} \Delta x \sin\left[w_n t + \phi_n - \theta_s(t)\right] \right\}^2 \qquad \ldots. (5.3)$$

and

$$\Psi_i(t) = \arc\tan \frac{\sum \Delta x \sin[w_n t + \phi_n - \theta_s(t)]}{1 + \sum \Delta x \cos[w_n t + \phi_n - \theta_s(t)]} \quad \ldots. (5.4)$$

where Δx is defined as $\Delta E_i / E_s$.

But $\Psi_i(t)$ is also given by

$$\Psi_i(t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \quad (\mathrm{Im}) \left\{ \sum_{w_n=-B_1}^{B_1} \Delta x \quad e^{j[w_n t + \phi_n - \theta_s(t)]} \right\}^n \quad \ldots. (5.5)$$

valid for $\sum_{w_n=-B_1}^{B_1} (\Delta x)\, e^{j[w_n t + \phi_n - \theta_s(t)]} < 1$

Equation (5.5) is readily shown by taking the logarithm of the vector $M e^{j\Psi_i(t)}$ defined in (5.1) and selecting the "imaginary" part.

For the simple case of $\sum_{-B_1}^{B_1} (\Delta x) e^{j[w_n t + \phi_n - \theta_s(t)]} \ll 1$, the first term only of the series defined by Equation (5.5) is important, thus for this case

$$\Psi_i(t) = \sum_{w_n=-B_1}^{B_1} \Delta x \sin[w_n t + \phi_n - \theta_s(t)] \quad \ldots. (5.6)$$

which of course agrees with the result from Equations (5.4) for the same conditions

B. Statistical Properties of the Noise Interference:

The mean and RMS values of e_i are given by a direct inspection of Equation (3.2). However, because of the random phase relations ϕ_n, between noise components, the peak value of e_i must necessarily be of a statirtical nature.

It would be convenient to express the interference e_i in a form which lends itself to statistical evaluation as follows, by definition of Equation (3.2)

$$e_i = (Re) \sum_{-B_1}^{B_1} \Delta E_i \quad e^{j\left[(w_k + w_n)t + \phi_n\right]}$$

Rearranging gives

$$e_i = (Re)\left\{(\cos w_k t + j \sin w_k t)\right\}$$
$$\cdot \left\{\sum_{-B_1}^{B_1} \Delta E_i \cos (w_n t + \phi_n) + j \sum_{-B_1}^{B_1} \Delta E_i \sin (w_n t + \phi_n)\right\} \quad \ldots\ldots (5.7)$$

or

$$e_i = \left\{\sum_{-B_1}^{B_1} \Delta E_i \cos (w_n t + \phi_n)\right\} \cdot \cos w_k t$$
$$- \left\{\sum_{-B_1}^{B_1} \Delta E_i \sin (w_n t + \phi_n)\right\} \cdot \sin w_k t \quad \ldots\ldots (5.8)$$

or

$$e_i = E_p(t) \cos w_k t \quad - E_q(t) \sin w_k t$$
$$\ldots\ldots (5.9)$$
$$= E_p(t) \cos w_k t \quad + \quad E_q(t) \cos(w_k t + \tfrac{\pi}{2})$$

Where $E_p(t)$ and $E_q(t)$ are defined by Equation (5.8) and may be interpreted as the in phase and quadrature component of noise of an equivalent noise carrier centered at w_k. Due to the random nature of the problem the variables $E_p(t)$ and $E_q(t)$ may be individually described by the quassian probability distribution as a consequence of the central limit theorem[9].

e_i could also be expressed in the form

$$e_i = \sqrt{\left[E_p(t)\right]^2 + \left[E_q(t)\right]^2} \cdot \cos\left[w_k t + \Phi\right]$$

.... (5.10)

$$= R(t) \cos (w_k t + \Phi)$$

with R(t) defining the envelope of the wave where

$$\left[R(t)\right]^2 = \left[E_p(t)\right]^2 + \left[E_q(t)\right]^2$$

and $\Phi = \arc \tan \dfrac{E_q(t)}{E_p(t)}$

Now the statistical distribution in time of $E_p(t)$ and $E_q(t)$ is given by

$$P(E_p) = \frac{e^{-E_p^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \qquad \text{.... (5.11)}$$

where σ^2 is the mean square noise voltage within the IF noise bandwidth.

$$P(E_q) = \frac{e^{-E_q^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \qquad \text{.... (5.12)}$$

by definition of the gaussian probability distribution, where E_p and E_q are particular values of $E_p(t)$ and $E_q(t)$.

Thus the joint probability distribution of E_p and E_q is

$$P(E_p, E_q) = \frac{e^{-(E_p^2 + E_q^2)/2\sigma^2}}{2\pi\sigma^2} \qquad \text{.... (5.13)}$$

and after a suitable change of variables[10],

$$P(R, \Phi) = \frac{R\, e^{-R^2/2\sigma^2}}{2\pi\sigma^2} \qquad \text{.... (5.14)}$$

thus

$$P(R) = \frac{R\, e^{-R^2/2\sigma^2}}{\sigma^2} \qquad \text{.... (5.15)}$$

by integration over all values of Φ,

since

$$\int_0^{2\pi} P(R, \Phi)\, d\Phi = P(R) \qquad \text{.... (5.16)}$$

and

$$\int_0^{\infty} P(R, \Phi)\, dr = P(\Phi) \qquad \text{.... (5.17)}$$

Equation (5.15) is known as the Rayleigh Probability Distribution.

From the definitions of $E_p(t)$, $E_q(t)$, $R(t)$ and $\cos \Phi$ it follows that the $\Psi_i(t)$ associated with the resultant signal plus noise could be expressed in the form below which will prove useful if a statistical evaluation of the noise effects are desired.

$$\Psi_i(t) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (Im) \left[\frac{R(t)}{E_s} e^{j\left[\Phi - \theta_s(t)\right]} \right]^n \qquad \text{.... (5.18)}$$

valid for $\left|\Psi_i(t)\right| < 1$

If the first three terms of the power series expansion of $\Psi_i(t)$ are written then

$$\Psi_i(t) = \frac{R(t)}{E_s} \sin\left[\Phi - \theta_s(t)\right] - \frac{R^2(t)}{2E_s^2} \sin 2\left[\Phi - \theta_s(t)\right]$$

$$+ \frac{R^3(t)}{3E_s^3} \sin 3\left[\Phi - \theta_s(t)\right] \qquad \text{.... (5.19)}$$

C. IF and Baseband Noise Spectrum Above Threshold After the Limiter:

After passage of the waveform defined by Equation (5.2) through an ideal limiter which removes the amplitude variations M_i of the waveform without modifying the zero crossings, one obtains after filtering and prior to demodulation, the voltage e_{1m} where

$$e_{1m} = E_{1m} \cos\left[w_k t + \theta_s(t) + \Psi_i(t)\right] \qquad \text{.... (5.20)}$$

where E_{1m} is the peak amplitude of the angle modulated sinusoid e_{1m}, resulting from the limiting and filtering operation.

The application of e_{1m} to an ideal frequency demodulator yields a demodulated baseband voltage e_{BF}, given by

$$e_{BF} = k_d \frac{d}{dt}\left[\theta_s(t) + \Psi_i(t)\right] \; T(jw) \qquad \text{.... (5.21)}$$

where k_d is the sensitivity of the demodulator in $\frac{\text{volts-sec}}{\text{radian}}$ and $T(jw)$ is an equivalent transfer function to baseband frequencies, and is a dimensionless operator.

For $\Psi_i(t) << 1$ (well above the FM improvement threshold) only the first order term in the expansion of $\Psi_i(t)$ is significant. Thus

$$\Psi_i(t) = \sum_{-B_1}^{B_1} \Delta x \sin\left[w_n t + \phi_n - \theta_s(t)\right] \qquad \text{.... (5.22)}$$

The characteristic performance at intermediate and baseband frequencies of the FM demodulator well above the improvement threshold can readily be deduced from Equation (5.22) for $\Psi_i(t)$ defined by (5.24).

It is of interest to note that the mean signal/noise power ratio at the input to the demodulator is enhanced above that ratio in the noise bandwidth of the IF prior to limiting. Since this result is independent of $\theta_s(t)$, the latter may be taken as zero for convenience to demonstrate the point.

Now since for the assumed conditions,

$$e_{1m} = E_{1m} \cos\left[w_k t + \frac{R(t)}{E_s} . \sin \Phi \right] \quad \text{.... (5.23)}$$

$$= (\mathrm{Re})\ E_{1m}\, e^{j\left[\frac{R(t)}{E_s} . \sin \Phi\right]}\ e^{j\omega_k t} \quad \text{.... (5.24)}$$

then

$$e_{1m} = (\mathrm{Re})\ E_{1m}\, e^{j w_k t} \sum_{-\infty}^{\infty} J_n \left[\frac{R(t)}{E_s}\right] e^{j n \Phi} \quad \text{.... (5.25)}$$

by definition of the Bessel Series expansion.

But since $\frac{R(t)}{E_s} \ll 1$ .then

$$J_o \left[\frac{R(t)}{E_s}\right] = 1$$

and

$$J_1 \left[\frac{R(t)}{E_s}\right] = \frac{R(t)}{2E_s}$$

and the higher order terms are zero, hence

$$e_{1m} = E_{1m} \left\{ 1 \cos w_k t + \frac{R(t)}{2E_s} . \cos (w_k t + \Phi) - \frac{R(t)}{2E_s} . \cos (w_k t - \Phi) \right\} \quad \text{.... (5.26)}$$

The mean power is given by

$$\overline{(e_{1m})^2} = E_{1m}^{\ 2} \left\{ \frac{1}{2} + \frac{\overline{R^2(t)}}{8E_s^{\ 2}} + \frac{\overline{R^2(t)}}{8E_s^{\ 2}} \right\} \quad \text{.... (5.27)}$$

The ratio of the mean signal power to mean noise power is

$$\frac{\frac{1}{2}}{\frac{\overline{R^2(t)}}{4E_s^2}} = \frac{2E_s^2}{\overline{R^2(t)}} \qquad \text{.... (5.28)}$$

for $\frac{\overline{R^2(t)}}{E_s^2} << 1$

But the mean signal to noise power ratio at the limiter input is

$$\frac{E_s^2}{\overline{R^2(t)}} \qquad \text{.... (5.29)}$$

and it is seen that mean signal noise ratio is enhanced by a factor of 2 or 3db powerwise, for the high signal/noise case.

In a similar manner it can be shown that the signal noise power ratio will be degraded by a maximum factor of $\frac{\pi}{4}$ for $(E_s)^2 << \overline{R^2(t)}$.

The actual IF or baseband frequency spectrum for conditions of interference and $\Psi_i(t) << 1$ is given as follows,

$$e_{1m} = E_{1m} \cos\left\{ w_k t + \theta_s(t) + \sum_{-B_1}^{B_1} \Delta x \sin\left[w_n t + \phi_n - \theta_s(t)\right]\right\} \qquad \text{.... (5.30)}$$

if $\theta_s(t) = 0$

then

$$e_{BF} = k_d\, T(jw) \sum_{-B_1}^{B_1} w_n\, \Delta x \cos(w_n t + \phi_n) \qquad \text{.... (5.31)}$$

and

$$\sqrt{\overline{e_{BF}}^2} = \sqrt{k_d^2\, T(jw)^2 \sum_{-B_1}^{B_1} w_n^2 \frac{(\Delta x)^2}{2}} \qquad \text{.... (5.32)}$$

which exhibits ~~is~~ the characteristic triangular spectrum of RMS noise voltage for an FM receiver well above the FM improvement threshold. Under these conditions the frequency spectrum at IF prior to demodulation will consist of the components

$$w_k \mp w_n \qquad \text{.... (5.33)}$$

as is apparent since

$$e_{1m} = (\mathrm{Re})\, E_{1m}\, e^{j\, w_k t} \cdot e^{j \sum_{-B_1}^{B_1} \Delta x \sin (w_n t + \phi_n)} \qquad \text{.... (5.34)}$$

$$= (\mathrm{Re})\, E_{1m}\, e^{j\, w_k t} \prod \Big\{ J_o(\Delta x) + J_1(\Delta x) \left[e^{j(w_n t + \phi_n)} - e^{-j(w_n t + \phi_n)} \right] \Big\} \qquad \text{.... (5.35)}$$

$$e_{1m} = (\mathrm{Re})\, E_{1m}\, e^{j\, w_k t} \Big\{ J_o(\Delta x)^{N'} + J_o(\Delta x)^{N'-1} \cdot J_1(\Delta x) \sum_{-B_1}^{B_1} e^{j(w_n t + \phi_n)} - \sum_{-B_1}^{B_1} e^{-j(w_n t + \phi_n)} \Big\} \qquad \text{.... (5.36)}$$

where N' is the number of spectral components

Similarly if

$$\theta_s(t) \neq 0$$

but

$$\theta_s(t) = m_a \sin w_a t$$

then the IF frequency spectrum prior to the demodulation contains those components

$$w_k \mp r\, w_a \mp w_n \qquad \ldots. (5.37)$$

The baseband frequency spectrum for $\theta_s(t) = m_a \sin w_a t$ and $\Psi_i(t) \ll 1$ is given as follows

$$e_{BF} = k_d \frac{d}{dt} T(jw) \left\{ m_a \sin w_a t + \sum_{-B_1}^{B_1} \Delta x \sin\left[w_n t + \phi_n - m_a \sin w_a t\right] \right\} \qquad \ldots. (5.38)$$

but

$$\sum_{-B_1}^{B_1} \Delta x \sin\left[w_n t + \phi_n - m_a \sin w_a t\right]$$

$$= \sum_{-B_1}^{B_1} \Delta x (\mathrm{Im}) e^{j\left[w_n t + \phi_n - m_a \sin w_a t\right]}$$

$$= \sum_{-B_1}^{B_1} \Delta x (\mathrm{Im}) e^{-j\, m_a \sin w_a t} \cdot e^{j(w_n t + \phi_n)} \qquad \ldots. (5.39)$$

$$= \sum_{-B_1}^{B_1} \Delta x (\mathrm{Im}) \cdot \sum_{-\infty}^{\infty} (-1)^r J_r(m_a)\, e^{j\, r\, w_a t}\, e^{j(w_n t + \phi_n)}$$

$$= \sum_{-B_1}^{B_1} \Delta x \sum_{-\infty}^{\infty} (-1)^r J_r(m_a) \sin\left[(r\, w_a + w_n)t + \phi_n\right] \qquad \ldots. (5.40)$$

hence

$$e_{BF} = k_d T(jw) \left\{ m_a w_a \cos w_a t + \sum_{B_1}^{B_1} \Delta x \sum_{-\infty}^{\infty} (-1)^r J_r(m_a) \cdot (r\, w_a + w_n) \cdot \cos\left[r\, w_a t + w_n t + \phi_n\right] \right\} \qquad \ldots. (5.41)$$

In the presence of signal modulation it is seen that well above threshold, the noise spectrum at baseband is modified from the no modulation case, being composed of a number of triangular spectra centered at $\pm rw_a$, for $r = 0, \pm 1, \pm 2$ etc., with the upper limit of r being determined by the modulation index.

The mean square voltage $\overline{(e_{BF})^2}$ above threshold is given by performing the implied operation on Equation (5.41).

$$\text{For } \sum_{-B_1}^{B_1} \frac{(\Delta x)^2}{2} << 1 \quad \text{and} \quad \frac{(m_a)^2}{2} >> \sum_{-B_1}^{B_1} \frac{(\Delta x)^2}{2}$$

the $\left[S/N\right]_{BF}$ is essentially S_{BF} for $\Delta x = 0$, divided by N_{BF}, for $m_a = 0$.

$$\left.\overline{(e_{BF})^2}\right]_{\Delta x = 0} = \left[k_d \; T(jw)\right]^2 \frac{(\Delta w)^2}{2} \qquad \text{where } \Delta w = m_a \, w_a \tag{5.42}$$

$$\left.\overline{(e_{BF})^2}\right]_{m_a = 0} = \left[k_d \; T(jw)\right]^2 \sum_{-B_1}^{B_1} (w_n)^2 \frac{(\Delta x)^2}{2}$$

$$= \left[k_d \; T(jw)\right]^2 \sum_{-B_1}^{B_1} (w_n)^2 \rho \frac{dw}{(E_s)^2} = \left[k_d \; T(jw)\right]^2 \frac{2}{(E_s)^2} \rho \left[\frac{w_n^3}{3}\right]_0^{B_2}$$

$$= \left[k_d \; T(jw)\right]^2 \frac{1}{S_{IF}} \rho \frac{(B_2)^3}{3} \qquad \ldots\ldots \tag{5.43}$$

Hence

$$\frac{S_{BF}}{N_{BF}} = \frac{(\Delta w)^2 \; S_{IF} \; 3}{2 \; \rho \quad (B_2)^3} = \frac{(\Delta w)^2}{(B_2)^2} \; \frac{S_{IF} \; 3}{2 \rho (B_2)} \qquad \ldots\ldots \tag{5.44}$$

So

$$\left[\frac{S_{BF}}{N_{BF}}\right]_{FM} = (F)^2 \left[\frac{S_{IF}}{N_{IF}}\right]_{FM} \frac{3\ B_1}{B_2} = (F)^2\ 3 \left[\frac{S_{IF}}{N_{IF}}\right]_{AM} \quad .... (5.45)$$

where $2 \rho B_2$ is the IF noise power in a double sideband AM system, and ρB_2 is the noise power in a single sideband AM system.

D. Frequency Spectrum in the Threshold Region:

Thus far we have considered the performance of the conventional FM receiver well above the improvement threshold. Under these conditions the in-phase component of noise voltage $E_p(t) << E_s$ and the quadrature component $E_q(t) << E_s$. Hence the demodulated noise voltage of an ideal FM demodulator is that due to the quadrature component of noise within the noise bandwidth of the receiver regardless of whether a limiter were used or not.

As the noise power N_{IF} is increased relative to $S_{IF} = \frac{E_s^2}{2}$, eventually the angle modulation due to the interference, $\Psi_i(t)$ no longer bears a linear relation with the signal noise ratio in the noise bandwidth of the receiver.

This is due to the second order term of the $\Psi_i(t)$ series expansion, and is seen to be the product of noise beating with noise, since

$$\Psi_i(t) = \frac{R(t)}{E_s} \sin \Phi - \frac{R^2(t)}{2E_s^2} \sin 2\Phi \quad .... (5.46)$$

or

$$\Psi_i(t) = \sum_{-B_1}^{B_1} \Delta x \sin[w_n t + \phi_n] - \tfrac{1}{2} \operatorname{Im} \left[\sum_{-2B_1}^{2B_1} \Delta x \, e^{j(w_n t + \phi_n)} \right]^2 \qquad \ldots\ldots (5.47)$$

where Equations (5.46) and (5.47) follow from (5.18), and (5.5), respectively taking the first two terms of the series and letting $\theta_s(t) = 0$.

In the "threshold of FM improvement" region the baseband output is characterized by two related features, one associated with the statistical nature of the noise envelope and the other associated with the mean signal/noise power ratio within the noise bandwidth of the receiver.

The first feature characterizing the baseband voltage and due to the statistical nature of noise, is that spikes of baseband voltage begin to occur in a particular region with a frequency of repetition which increases sharply as the IF signal/noise ratio is decreased.

The angle modulation $\Psi_i(t)$ of the resultant carrier plus noise $\Psi_i(t)$ is given as

$$\Psi_i(t) = \frac{R(t)/E_s \sin \Phi}{1 + R(t)/E_s \cos \Phi} \qquad \ldots\ldots (5.48)$$

for $R(t)/E_s < 1$

or

$$\Psi_i(t) = \frac{-E_s/R(t)\ \sin \Phi}{1 + E_s/R(t)\ \cos \Phi} \qquad \text{.... (5.49)}$$

for $E_s/R(t) < 1$

where $e_r = e_s + e_i$ are defined by Equations (3.1) and (3.2)

and

$$e_r = (\mathrm{Re})\ E_s\ e^{j\, w_k t} \left[1 + \frac{R(t) e^{j\Phi}}{E_s}\right] \qquad \text{.... (5.50)}$$

for $R(t)/E_s < 1$.

$$e_r = (\mathrm{Re})\ R(t)\ e^{j[w_k t + \Phi]} \left[1 + E_s/R(t)\ e^{-j\Phi}\right] \qquad \text{.... (5.51)}$$

for $E_s/R(t) < 1$.

For a given mean white noise power density, denoted by σ^2, the probable values of R(t) are given by the Rayleigh probability distribution. An increase in noise power is associated with an increasing probability of a particular noise voltage. The random phase orientation Φ, of the equivalent noise vector R(t) cos Φ, is however uniformly distributed and independent of σ^2.

As long as R(t) has a negligible probability of exceeding the carrier for a given $[S/N]_{IF}$, then the angle modulation $\Psi_i(t)$ will be confined to the region

$$0 < \left|\Psi_i(t)\right| < \frac{\pi}{2}$$

For an increasing noise power relative to the carrier power it becomes more probable that the angle modulation $\Psi_i(t)$ will be given by

$$0 < \left|\Psi_i(t)\right| < \pi$$

and that the instantaneous zero crossings of the composite waveform be controlled by noise.

As the occurrence of baseband spikes can be correlated with the probability of the noise envelope exceeding the carrier it would be instructive to plot this probability for a range of $\left[S/N\right]_{IF}$.

The probability of occurrence of a particular noise voltage is given from the Rayleigh distribution in Equation (5.15) as

$$P(R) = \frac{R\, e^{-R^2/2\sigma^2}}{\sigma^2}$$

hence the probability that R lies between 0 to R is just

$$\int_0^R P(R)\;dR = \left[\frac{-e^{-R^2/2\sigma^2}}{\sigma^2}\right]_0^R \qquad \ldots\ldots (5.52)$$

$$= \left[1 - \frac{e^{-R^2/2\sigma^2}}{\sigma^2}\right]$$

Thus the probability that R lies outside this range is

$$1 - \int_0^R P(R)\;dR = \frac{e^{-R^2/2\sigma^2}}{\sigma^2} \qquad \ldots\ldots (5.53)$$

By definition $\left[S/N\right]_{IF} = \frac{E_s^2}{2\sigma^2}$, thus the probability that R exceeds E_s for a given $\left[S/N\right]_{IF}$ is

$$P(R)\Big]_{R>E_s} = e^{-(S/N)_{IF}} \qquad \text{.... (5.54)}$$

This function is plotted in Figure 13.

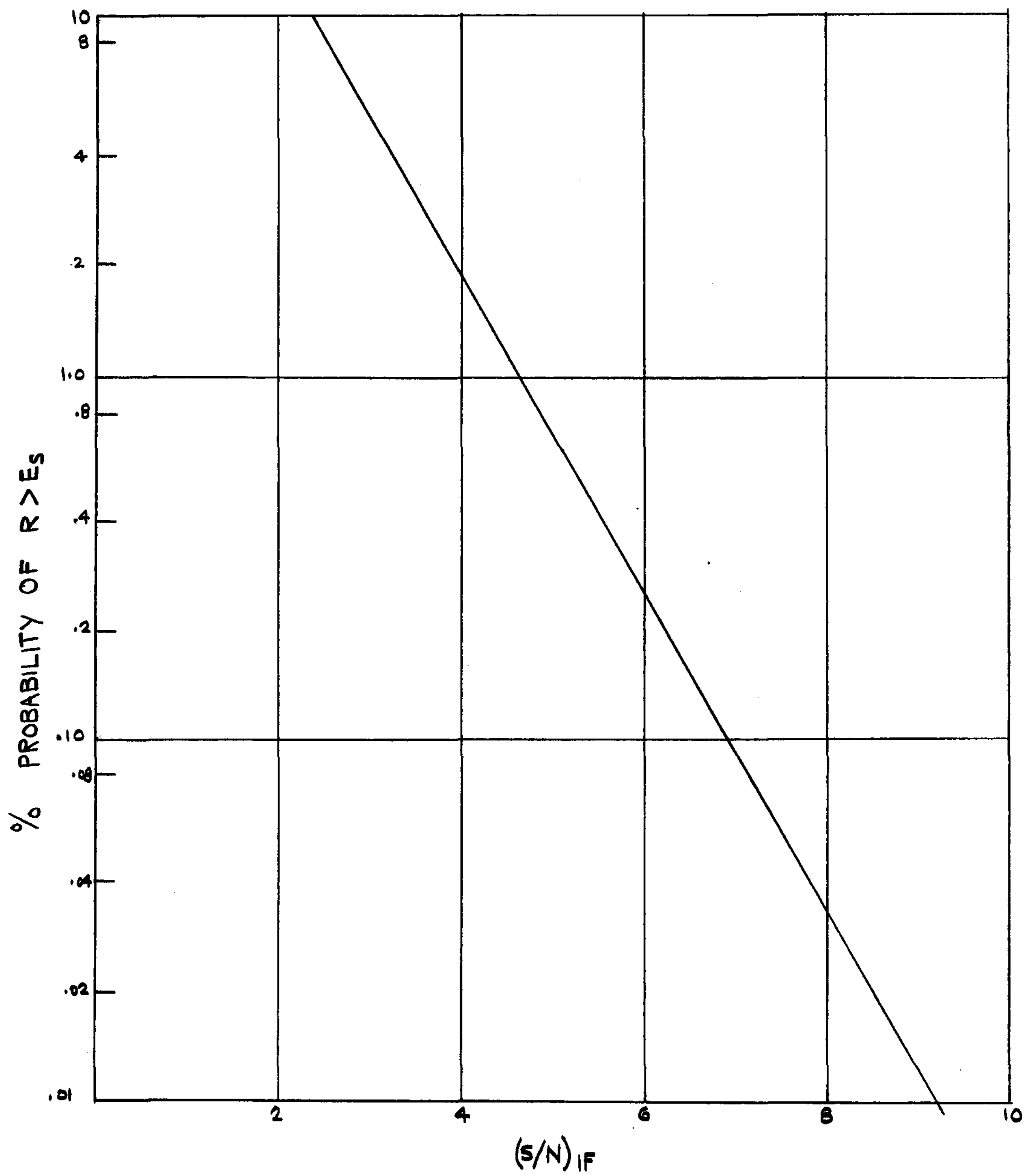


Figure 13.

The second feature associated with the threshold performance of an FM receiver is that the curve of demodulated RMS noise voltage (or mean power) versus IF signal noise ratio departs from linearity as that ratio is decreased in the threshold region with the effect showing up more sharply at the lower baseband frequencies than the higher.

Now in this region the frequency spectrum of noise voltage may be obtained from Equation (5.5).

It will be realized that the second term representing the squaring operation on N^{1} noise components must contain $(N^{1})^2$ components of the form

$$\mathrm{Im}\sum\sum \Delta\, x^2\, e^{j[(w_n + w_n)t + \phi_n + \phi_m]}$$

$$= \sum\sum \Delta x^2 \sin\left[(w_n + w_m)t + \phi_n + \phi_m\right]$$

The number of spectral components at a given frequency $w_q = w_n + w_m$ is clearly given by solution of the linear algebretic equation $q = n+m$ for all n and m, where n is a member of one set of N^1 components and m is a member of the other set of N^1 components, (with half of the components upper sideband and the remaining half lower sidebands).

Although the commutative law holds for addition of frequencies this is clearly not the case for phase angles because of their random nature, hence $\phi_n + \phi_m \neq \phi_{n+m}$.

At the center of the noise bandwidth the number of spectral components is given by $0 = n+m$, which to be satisfied implies $m = -n$ for all n and m, consequently there will be N^1 components at w_o.

Similarly at $w_q = +2B_1$ the number of components is given by $\pm q = n+m = \pm N^1$

thus $+q = N^1 = n+m$ can only be satisfied by one value of n and m, namely $n = m = +N^1/2$,

since this is the maximum value of n and m, and

$-q = -N^1 = n+m$ similarly implies $n = m = -N^1/2$.

Thus a graph of the number of randomly phased contributions in the second order term as a function of frequency may be immediately drawn since the equation defining the curve has been shown to be that of a straight line and the end points have been defined.

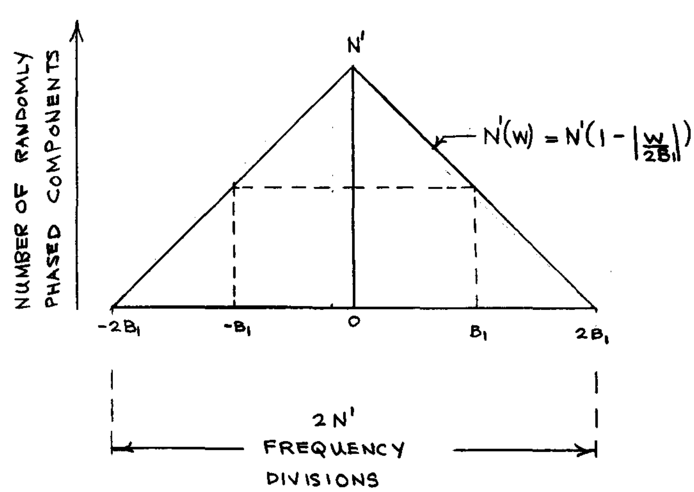


Figure 14.

With reference to Figure 14 it is seen that since the frequency interval $4B_1$ is assumed divided into $2N^1$ components, then the total number of spectral components in the second order term is just the area of the triangle. This area is by inspection $\frac{1}{2}(2N^1)\cdot N^1 = (N^1)^2$, which provides a check on the model.

Since $\Psi_i(t)$ for the threshold region is given from Equation (5.45) then

$$\overline{\left[\Psi_i(t)\right]^2} = \sum_{-B_1}^{B_1} \frac{(\Delta X)^2}{2} + \frac{1}{4}\sum_{-2B_1}^{2B_1} \frac{(\Delta X)^4}{2} N^1\left(1 - \frac{|w|}{2B_1}\right) \quad \ldots . (5.55)$$

where $N^1\left(1 - \frac{|w|}{2B_1}\right)$ is the number of spectral components within the band $0 < |w| < 2B_1$ from inspection of Figure 14.

But by definition

$$(\Delta X)^2 = (N/S)_{IF} \frac{dw}{2B_1} = (N/S)_{IF} \frac{1}{N^1}$$

and substitution for $(\Delta X)^2$ into (5.52) yields

$$\overline{\left[\Psi_i(t)\right]^2} = 2\int_0^{B_1} \frac{(N/S)_{IF}}{2} \frac{dw}{2B_1} + \frac{2}{4}\int_0^{2B_1} \frac{(N/S)^2_{IF}}{2} \frac{dw}{2B_1} \frac{N^1}{N^1}\left(1 - \frac{w}{2B_1}\right)$$

$$= \tfrac{1}{2}(N/S)_{IF} + 1/8 \quad (N/S)^2_{IF} \frac{dw}{2B_1}$$

$$= \tfrac{1}{2}(N/S)_{IF} + 1/8 \quad (N/S)^2_{IF} \quad \ldots . (5.56)$$

If the interference were a single sine wave component of the same mean noise power $N_{IF} = \sigma^2$ then $\Psi_i(t)$ is given by

$$\Psi_i(t) = \sqrt{\frac{2\sigma^2}{(E_s)^2}} \sin(w_n t + \phi_n) - \frac{1}{2}\frac{2\sigma^2}{(E_s)^2} \sin 2(w_n t + \phi_n)$$

The mean square value of $\Psi_i(t)$ is then

$$\overline{\left[\Psi_i(t)\right]} = \frac{2\sigma^2}{2E_s^2} + \frac{\sigma^4}{2E_s^4}$$

$$= \tfrac{1}{2}(N/S)_{IF} + 1/8\,(N/S)^2_{IF}$$

which is the same magnitude as for the case of white noise interference.

The demodulated baseband voltage e_{BF} in this region is given by

$$e_{BF} = K_d T(jw) \frac{d}{dt} \Psi_i(t)$$

$$= K_d T(jw) \left\{ \sum_{-B_1}^{B_1} w_n \Delta X \cos\left[w_n t + \phi_n\right] - \sum_{-2B_1}^{2B_1} (w_n + w_m)\frac{\Delta X^2}{2} \cos\left[(w_n + w_m)t + \phi_n + \phi_m\right] \right\} \quad \ldots. (5.57)$$

The mean squared baseband noise voltage is then

$$\overline{(e_{BF})^2} = \left[K_d T(jw)\right]^2 \left\{ \sum_{-B_1}^{B_1} (w_n)^2 \frac{(\Delta X)^2}{2} + \sum_{-2B_1}^{2B_1} (2w_n)^2 \frac{(\Delta X)^4}{8} N^1 \left(1 - \frac{|w|}{2B_1}\right) \right\}$$

or

$$\overline{(e_{BF})^2} = \left[K_d T(jw)\right]^2 \left\{ \int_0^{B_1} \frac{w_n^2 \, (N/S)_{IF}}{2B_1} \, dw + \int_0^{2B_1} w_n^2 \, (N/S)_{IF}^2 \left(1 - \frac{|w|}{2B_1}\right) dw \right\}$$

.... (5.58)

For the conventional FM demodulator the bandwidth of $T(jw)$ is determined by the highest modulating frequency, hence only noise within these limits will contribute to e_{BF}.

The departure of $\overline{(e_{BF})^2}$ due to noise versus $(N/S)_{IF}$ is seen from Equation (5.58).

SECTION VI

NOISE PERFORMANCE OF THE PHASE-LOCKED FM DEMODULATOR

A. Above Threshold Noise Performance:

(1) Comparison With the Conventional FM Demodulator

The general performance of phase-locked feedback demodulator incorporating a balanced phase detector is given from Equation (3.6) as

$$p\theta_1(t) = KF(p)\left\{\sin\left[\theta_s(t) - \theta_1(t)\right] + \frac{1}{E_s}\sum_{-B_1}^{B_1}\Delta E_i \sin\left[w_n t + \phi_n - \theta_1(t)\right]\right\}$$

For the purposes of analysis it will be convenient to express the signal plus noise in terms of a composite waveform as follows:

$$p\theta_1(t) = KF(p)\ (\mathrm{Im})e^{j\left[\theta_s(t) - \theta_1(t)\right]}\left\{1 + \frac{1}{E_s}\sum_{-B_1}^{B_1}\Delta E_i e^{j\left[w_n t + \phi_n - \theta_s(t)\right]}\right\} \quad \text{.... (6.1)}$$

$$= KF(p)M_i\ (\mathrm{Im})e^{j\left[\theta_s(t) - \theta_1(t) + \Psi_i(t)\right]} \quad \text{.... (6.2)}$$

$$= KF(p)\ M_i\ \sin\left[\theta_s(t) - \theta_1(t) + \Psi_i(t)\right]$$

It will be noted that M_i and $\Psi_i(t)$ have been previously defined in Equations (5.3) and (5.4) respectively. It will be shown that for the phase locked demodulator, the limits of M_i and $\Psi_i(t)$ are essentially set by the closed loop noise bandwidth of the feedback loop.

Let the phase error $\Delta\theta_e(t)$ under general conditions of signal modulation and interference be defined as

$$\Delta\theta_e(t) = \left[\theta_s(t) + \Psi_i(t) - \theta_1(t)\right] \qquad \text{.... (6.3)}$$

Linear operation of the phase detector implies

$$\frac{\left|\Delta\theta_e(t)\right|^3}{3!} << \left|\Delta\theta_e(t)\right|$$

Thus under these conditions

$$p\theta_1(t) = KF(p)\, M_i \left[\theta_s(t) + \Psi_i(t) - \theta_1(t)\right] \qquad \text{.... (6.4)}$$

Rearranging gives

$$\theta_1(t) + \frac{KF(p)\, M_i\, \theta_1(t)}{p} = \left[\theta_s(t) + \Psi_i(t)\right]\frac{KF(p) M_i}{p} \qquad \text{.... (6.5)}$$

Under conditions of no interference it is seen that Equation (6.4) reduces to Equation (4.4). The linear no interference performance has been previously examined in Section IV. M_i may be regarded as a time varying coefficient and well above the FM threshold, M_i equals unity.

Under conditions of interference and linearity the baseband voltage e_{BF} is given by

$$e_{BF} = \frac{p\theta_1(t)}{K_3} = \frac{T(p)}{K_3} \; p\left[\theta_s(t) + \psi_i(t)\right] \qquad \ldots. (6.6)$$

which is identical in form to that for the conventional FM receiver above threshold.

For $\theta_s(t) = 0$,

$$e_{BF} = \frac{T(p)}{K_3} \sum_{-B_1}^{B_1} w_n \,\Delta x \cos(w_n t + \phi_n) \qquad \ldots. (6.7)$$

which indicates the triangular distribution of RMS noise voltage versus frequency common to an ideal FM demodulator. For $\theta_s(t) = m_a \sin w_a t$, e_{BF} is given by equation (5.41) for the corresponding conditions.

It is apparent that for this region of operation the phase-locked FM demodulator and the conventional FM demodulator exhibit identical performance. For this region of operation the magnitude of M_i is essentially unity and hence the loop parameters are unmodified from their no interference value.

(2) Noise Modulation of the VCO

The mean square noise modulation of the VCO under conditions of zero signal modulation and above threshold is given from Equation (6.4) as

$$\overline{\left[\theta_1(t)\right]}^{\,2} = \overline{\left|T(jw)\right|^2 \left[\Psi_i(t)\right]^2} \qquad \text{.... (6.8)}$$

But $\Psi_i(t)$ is defined in Equation (5.6) as

$$\Psi_i(t) = \sum_{-B_1}^{B_1} \Delta x \sin(w_n t + \phi_n) \qquad \text{, hence}$$

$$\overline{\left[\theta_1(t)\right]}^{\,2} = \sum_{-B_1}^{B_1} \overline{\left|T(jw)\right|^2 \frac{(\Delta x)^2}{2}}$$

$$= \sum_{-B_1}^{B_1} \overline{\left|T(jw)\right|^2 \frac{\Delta x^2}{2}} \qquad \text{.... (6.9)}$$

By definition $(\Delta x)^2 = \dfrac{2\varrho dw}{\overline{(E_s)^2}} = (N/S)_{IF} \dfrac{dw}{2B_1}$

Substitution into (6.10) yields

$$\overline{\left[\theta_1(t)\right]}^{\,2} = \frac{1}{2} (N/S)_{IF} \int_{-B_1}^{B_1} \frac{\left|T(jw)\right|^2}{2B_1} dw \qquad \text{.... (6.10)}$$

The noise bandwidth $2B_2$ associated with the closed loop transfer function $T(jw)$ is defined as

$$2B_2 = \int_{-\infty}^{\infty} \left|T(jw)\right|^2 dw \qquad \text{.... (6.11)}$$

where integration is from $-\infty$ to ∞ because the noise spectrum applied to the phase detector is symmetrical about the IF frequency.

If $2B_2 << 2B_1$ then for this case integration over the limits

$-B_1$ to B_1 would yield the same result as integration from $-\infty$ to ∞, that is to say,

$$\int_{-\infty}^{\infty} \left|T(jw)\right|^2 \, dw = \int_{-B_1}^{B_1} \left|T(jw)\right|^2 \, dw$$

for $B_2 << B_1$

Hence under these conditions

$$\overline{\left[\theta_1(t)\right]^2} = \tfrac{1}{2}\,(N/S)_{IF}\,\frac{B_2}{B_1} \qquad \text{.... (6.12)}$$

Alternately, if $B_2 >> B_1$, then

$$\overline{\left[\theta_1(t)\right]^2} = \tfrac{1}{2}\,(N/S)_{IF} \qquad \text{.... (6.13)}$$

The bandwidth requirements of the feedback demodulator are determined by the highest modulating frequency and peak deviation of the FM wave to be demodulated. These considerations have been dealt with in Section IV. The FM threshold in the conventional FM demodulator is determined by the IF bandwidth $2B_1$ prior to the limiter as discussed in Section V. As the relative noise immunity of the phase-locked demodulator in comparison to the conventional FM demodulator is determined by the ratio B_2/B_1, the case where B_2 is appreciably less than B_1 will be examined in detail. For the phase-locked demodulator the phase of a reference or local oscillator voltage is correlated to the phase of the composite input wave by virtue of the negative feedback action of the closed loop. The essential spectral contributions to the VCO phase lie within the closed loop noise bandwidth $2B_2$ of the feedback loop, consequently it is these components which determine the extent to which the demodulated signal is corrupted by noise.

The mean square angle modulation of the VCO by noise is defined by Equation (6.10) for any arbitrary noise bandwidth $2B_2$. Suppose that the closed loop transfer function of noise bandwidth $2B_2$ is defined by

$$\begin{aligned} |T(jw)| &= 1 \quad \text{for} \quad 0 < |w| < B_2 \\ |T(jw)| &= 0 \quad \text{for} \quad |w| > B_2 \end{aligned} \qquad \ldots\ldots (6.14)$$

and that the phase characteristic B(w) associated with T(jw) is suitable for following the desired signal modulation within the noise bandwidth. Under these assumptions the phase-locked demodulator would be immune to noise outside the noise bandwidth $2B_2$. Under these conditions the mean square angle modulation is given by

$$\overline{\left[\theta_1(t)\right]}^{\,2} = \tfrac{1}{2}\,(N/S)_{IF} \int_{-B_2}^{B_2} \frac{dw}{2B_1} = \tfrac{1}{2}\,(N/S)\,\frac{B_2}{B_1} \qquad \ldots\ldots (6.15)$$

The magnitude of $\overline{\left[\theta_1(t)\right]}^{\,2}$ for this case is identical to that in Equation (6.13). For the case defined by Equation (6.15) it follows from Equation (6.1) that

$$KF(p)\sum_{-B_1}^{B_1} \Delta x \sin\left[(w_n t + \phi_n) - \theta_1(t)\right] = KF(p)\sum_{-B_2}^{B_2} \Delta x \sin\left[(w_n t + \phi_n) - \theta_e(t)\right]$$

since $KF(p)\ 2 \sum_{B_2}^{B_1} \Delta x \sin[(w_n t + \phi_n) - \theta_1(t)]$ must be zero in order for components in this band not to modulate the VCO. Hence the limits of $\psi_i(t)$ need only be taken over the limits $-B_2 < w < B_2$ in Equation (6.4).

For the closed loop transfer function T(jw) departing from the idealized case, the difference between the actual and idealized transfer function is implicit in Equation (6.10) since

$$\overline{\left[\theta_1(t)\right]^2} = \tfrac{1}{2}\,(N/S)_{IF} \left\{ 2 \int_0^{B_2} \left|\frac{T(jw)}{2B_1}\right|^2 dw + 2 \int_{B_2}^{B_1} \left|\frac{T(jw)}{2B_1}\right|^2 dw \right\} \qquad \ldots\ (6.16)$$

The extent to which noise components outside the bandwidth $2B_2$ are effective in modulating the VCO is determined by the relative magnitude of the two terms in the above equation. A typical spectrum of mean square angle modulation of the VCO by noise is depicted in Figure 15

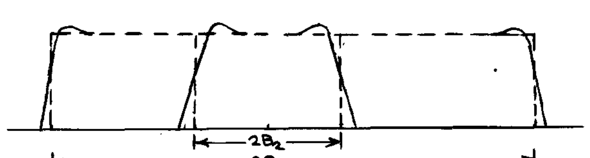


Figure 15.

(3) Phase Error Above Threshold

The phase error $\Delta\theta_e(t)$ for linear operation is given by definition as

$$\Delta\theta_e(t) = \left[\theta_s(t) + \psi_i(t)\right]\left[1 - T(jw)\right] \qquad \ldots\ (6.17)$$

The mean square phase error $\overline{\left[\Delta\theta_e(t)\right]}^2$ is given as

$$\overline{\left[\Delta\theta_e(t)\right]}^2 = \left\{\overline{\left[\theta_s(t)\right]}^2 + \overline{\left[\psi_i(t)\right]}^2 + 2\,\overline{\theta_s(t)\,\psi_i(t)}\right\}\frac{1}{1+\frac{KF(p)}{p}}$$

$$= \left\{\overline{\left[\theta_s(t)\right]}^2 + \overline{\left[\psi_i(t)\right]}^2\right\}\frac{1}{\left[1+\frac{KF(p)}{p}\right]} \qquad \ldots\ (6.18)$$

for zero correlation between $\theta_s(t)$ and $\psi_i(t)$.

It is seen that an increase in the mean square phase error must be associated with an increase in either the mean square signal modulation or the $(S/N)_{IF}$. Thus for a higher Rms modulation index associated with $\theta_s(t)$, (with $\theta_s(t)$ represented as a single sinusoid or a summation of randomly phased sinusoids), departure from linearity in loop performance will occur at a higher $(S/N)_{IF}$ then would be the case for a lower modulation index.

For T(jw) defined by the idealized transfer function, the mean square phase error associated with noise is due to those components lying within the noise bandwidth $2B_2$, which have been translated from their symmetrical position at the IF frequency to a symmetrical position centered about zero frequency. For the non-idealized transfer function, the mean square angle modulation of the VCO by noise is essentially due to these components lying within the noise bandwidth $2B_2$.

B. Threshold Effects

(1). General Considerations

The general equation for system performance is given by Equation (6.2) as

$$p\theta_1(t) = KF(p)M_i \quad \sin\left[\theta_s(t) - \theta_1(t) + \psi_i(t)\right]$$

The definition of the FM threshold of the phase-locked FM demodulator must necessarily be based on those characteristics observed for the conventional FM demodulator under threshold conditions. This implies such features as the onset of a non-linear relation between $(S/N)_{BF}$ as a function of $(S/N)_{IF}$, or the frequency of occurrence of baseband "spikes" associated with the impulsive frequency modulation of the carrier by noise.

There is, however, associated with the phase-locked FM demodulator certain features not encountered in the conventional FM demodulator which contribute to performance characteristic of threshold operation. The imperfection of the phase detector requires that $\frac{\left|\Delta\theta_e(t)\right|^3}{3!} \ll \left|\Delta\theta_e(t)\right|$, to prevent intermodulation of the signal and noise. At the same time a certain $\Delta\theta_e(t)$ is required for the maximum frequency deviations of the modulated carrier to be followed. It is apparent that one could have through improper design, noise and signal crossproducts occurring at a $(S/N)_{IF}$ above the minimum $(S/N)_{IF}$ at which these effects would be observed. This is because the phase error $\Delta\theta_e(t)$ is due to both $\theta_s(t)$ and $\psi_i(t)$. The time varying coefficient M_i is seen to result in a modulation of the loop gain K from the no interference value.

Well above the FM threshold the effect of M_i on the loop performance is negligable. With M_i departing from unity as the threshold region is approached, the presence of M_i results in a modulation of the damping factor and bandwidth of the feedback loop.

In addition to these factors peculiar to the phase-locked FM demodulator, the expansion of $\psi_i(t)$ contains cross products which become important in the threshold region as is the case for the conventional FM receiver. In the threshold region of a conventional FM demodulator additional filtering after the limiter cannot alter the FM threshold. This is due to the fact that the spectrum of noise mapped onto the phase of the pure angle modulated wave at the limiter output, is set by conditions existing at the input terminals to the limiter. This is in contrast to the phase-locked demodulator in which a narrow band circuit can be inserted after the wideband IF with the threshold being set by the noise bandwidth of the feedback demodulator. This is due to the fact that no non-linear operation is performed upon the composite waveform in the IF prior to the demodulator.

(2) Threshold Limits

Two extreme cases of system operation are apparent from Equation (6.1). Firstly, with no interference $\Delta E_1 = o$, hence

$$p\theta_1(t) = KF(p) \sin\left[\theta_s(t) - \theta_1(t)\right]$$

Alternately with $E_s = o$ and $\Delta E_i \not< o$

$$p\theta_1(t) = \eta K_2 K_3 F(p) \sum \Delta E_i \sin\left[\pm w_n t + \phi_n - \theta_1(t)\right]$$

since $K_1 = \eta E_s$ and $\Delta x = \Delta E_i / E_s$ by definition. For this latter case the balanced multiplier operates as a frequency changer with the IF noise spectrum being translated about zero frequency. The feedback $\theta_1(t)$ results in an angle modulation of each noise component which has no useful effect. Between these extreme cases lies the normal operating conditions encountered in a practical system.

Under conditions of linear operation of the loop the mean square angle modulation of the VCO by noise for no signal modulation is given from Equation (6.12) for B_2 appreciably less than B_1 as

$$\overline{\left[\theta_1(t)\right]^2} = \frac{1}{2} (N/S)_{IF} \frac{B_2}{B_1}$$

It is desired to examine the minimum $(S/N)_{IF}$ at which threshold effects, such as noise cross products and noise spikes, occur for the phase-locked demodulator. Let the noise interference be represented by a single interfering sinusoid of angular frequency $w_k + w_n$ within the noise bandwidth $2B_2$ centered at the IF frequency and producing the same mean square angle modulation of the VCO. Thus the necessary interfering vector is given by

$$e_i = \sqrt{2N_{IF} \frac{B_2}{B_1}} \cos\left[(w_k + w_n)t + \phi_n\right] \qquad \ldots. (6.19)$$

and the angle modulation $\psi_i(t)$ associated with the composite waveform of signal plus noise at the phase detector input is

$$\psi_i(t) = \sqrt{(N/S)_{IF} \frac{B_2}{B_1}} \sin(w_n t + \phi_n) - \tfrac{1}{2} (N/S)_{IF} \frac{B_2}{B_1} \sin\left[2(w_n t + \phi_n)\right] \quad \text{.... (6.20)}$$

from the power series expansion of $\psi_i(t)$ defined in Equation (5.5). Well above the FM threshold, corresponding to the second order term being negligable it is seen that the angle modulation of the VCO is

$$\overline{\left[\theta_1(t)\right]^2} = \tfrac{1}{2} (N/S)_{IF} \frac{B_2}{B_1}$$

which is the same result as for the interference constituted by white noise. The mean square value of $\psi_i(t)$ including the second order term is by inspection of Equation (6.20)

$$\overline{\left[\psi_i(t)\right]^2} = \tfrac{1}{2} (N/S)_{IF} \frac{B_2}{B_1} + \frac{1}{8}\left[(N/S)_{IF} \frac{B_2}{B_1}\right]^2 \quad \text{.... (6.21)}$$

The corresponding mean square baseband voltage is from examination of Equation (6.20)

$$\overline{(e_{BF})^2} = \left[\frac{1}{k_3}\right]^2 \cdot \tfrac{1}{2}\left\{\left[(N/S)_{IF} \frac{B_2}{B_1} (w_n)^2 + \tfrac{1}{4}\left[(N/S)_{IF} \frac{B_2}{B_1}\right]^2 (2w_n)^2\right]\right\} \quad \text{.(6.22)}$$

Although the representation of the interference as a single sinusoidal waveform does not correspond to the physical reality experienced in practice, it does provide a picture of general affects of interference upon the operation of the phase-locked oscillator. It will prove instructive to examine the experimental performance of the phase-locked oscillator for both single sine wave interference and white noise interference. This is done in Part C of the present section.

Representation of the interference effects by a single sinusoidal interfering vector fails to take into account the random nature of the commonly encountered type of interference. If white noise centered at the IF and within the noise bandwidth $2B_2$ is represented by an interfering vector having random in phase and quadrature components, then from Equation (5.10)

$$e_i = R^1(t) \cos \left[(w_k + w_n)t + \phi_n \right] \qquad \text{.... (6.23)}$$

with $\overline{(e_i)^2} = (N_{IF}) \dfrac{B_2}{B_1}$ by definition , where $R'(t)$ is the envelope of noise WITHIN THE BANDWIDTH $2B_2$

The angle modulation $\Psi_i(t)$ associated with the composite signal plus noise waveform within the noise bandwidth $2B_2$ is in the absence of signal modulation

$$\Psi_i(t) = \frac{(R^1(t)/E_s) \sin \Phi}{1 + (R^1(t)/E_s) \cos \Phi} \qquad \text{.... (6.24)}$$

which is valid for $(R^1(t)/E_s) < 1$, corresponding to the zero crossings of the waveform being controlled mainly by the carrier. The probable values of $R^1(t)$ are defined by the Rayleigh probability distribution function of Equation (5.15).

It is evident that there will be particularly severe impulsive response when $R^1(t) = E_s$ for the appropriate values of Φ. For $R^1(t) < E_s$, and $R^1(t) > E_s$, the zero crossings of the composite waveform are instantaneously controlled mainly the signal and noise respectively. The probability that the crest value of white noise exceeds 4 is given from the Rayleigh distribution as 0.034 %. If $R_1(t)$ does not exceed E_s for more than 0.034 % of the time then

$$(N/S)_{IF} \frac{B_2}{B_1} = (\sqrt{2}/4)^2 \qquad \text{.... (6.25)}$$

since the crest value of the sine wave carrier equals $\sqrt{2}$. A measurement could be performed to determine the exact relation between the impulses of instantaneous frequency deviation and the $(S/N)_{IF}$ for a particular FM receiver.

The representation of the white noise interference by a summation of randomly phased sinusoids of the same mean incremental power is capable of yielding information on the statistical properties, as well as the actual frequency spectrum of interference. By definition the equivalent input angle modulation to the phase-locked FM demodulator due to components within the noise bandwidth $2B_2$ is for $\theta_s(t) = 0$,

$$\psi_i(t) = \sum_{-B_2}^{B_2} \Delta x \sin(w_n t + \phi_n) - \tfrac{1}{2} \cdot \sum\sum_{-2B_2}^{2B_2} (\Delta x)^2 \sin\left[(w_n + w_m)t + \phi_n + \phi_m\right] \qquad \text{.... (6.26)}$$

from Equation (5.47) with the limits being taken over the noise bandwidth $2B_2$.

Above the FM threshold only the first order terms of $\Psi_i(t)$ are important. The extent to which the second order terms are important as the threshold region is approached may be determined by evaluating $\overline{[\Psi_i(t)]^2}$ associated with both the first and second order terms. Following the corresponding development for the conventional FM demodulator in Equation (5.56), it can be shown that for phase-locked FM demodulator the equivalent input mean square angle modulation due to noise components within the noise bandwidth $2B_2$ is

$$\overline{[\Psi_i(t)]^2} = \frac{1}{2}\left[(N/S)_{IF}\,\frac{B_2}{B_1}\right] + \frac{1}{8}\left[(N/S)_{IF}\,\frac{B_2}{B_1}\right]^2 \qquad \ldots. \text{ (6.27)}$$

which is seen to correspond to that result for the single interfering vector in Equation (6.21).

For the conventional FM demodulator in the threshold region $\overline{[\Psi_i(t)]^2}$ is given from Equation (5.56) as

$$\overline{[\Psi_i(t)]^2} = \frac{1}{2}\left[(N/S)_{IF}\right] + \frac{1}{8}\left[(N/S)_{IF}\right]^2$$

It is seen that for a given $(N/S)_{IF}$ measured in the IF noise bandwidth $2B_1$, the percent contribution of the second order term in $\overline{[\Psi_i(t)]^2}$ is less for the phase-locked FM demodulator by an amount proportional to $\frac{B_2}{B_1}$. Hence for the phase-locked FM demodulator one would expect freedom from threshold effects for a higher range of

$(N/S)_{IF}$ then for the conventional FM demodulator by an amount proportional to $\frac{B_2}{B_1}$.

C. Experimental Noise Performance:

(1) Experimental Apparatus and Method of Measurement

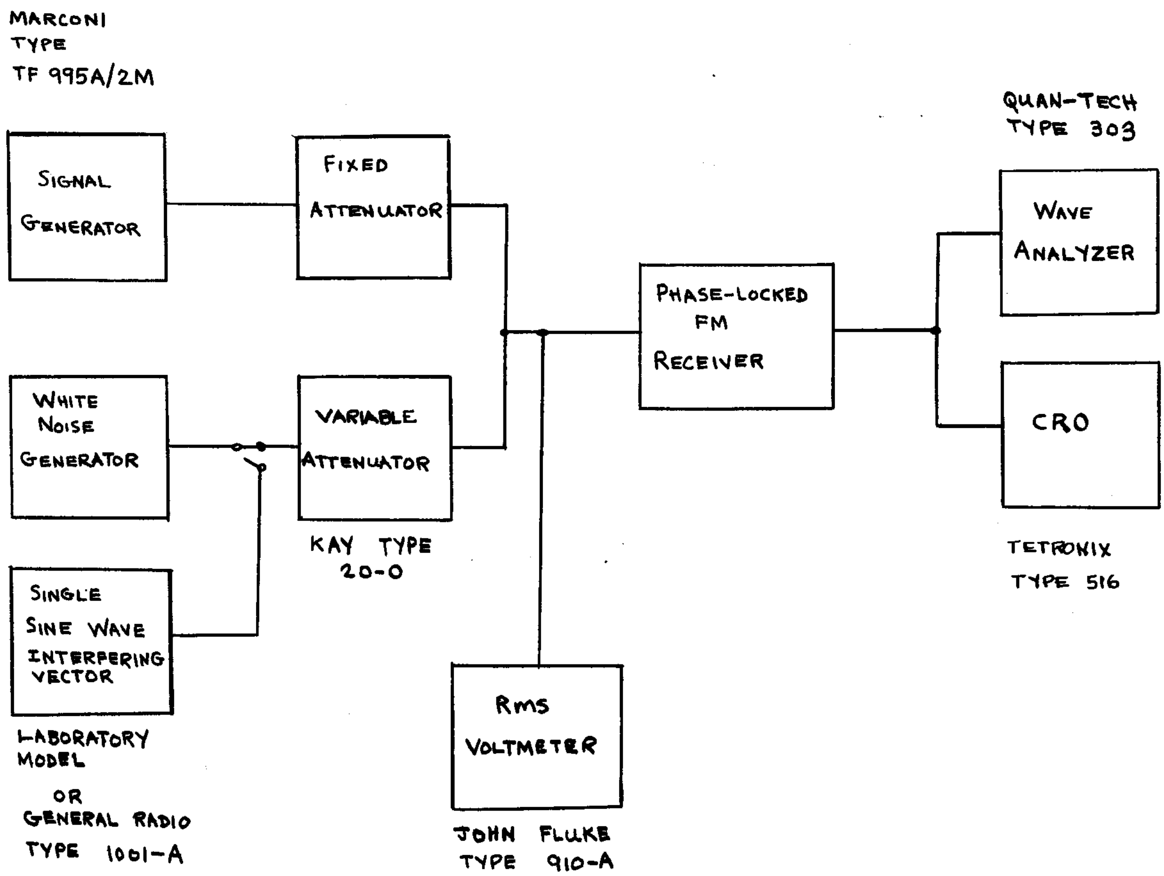
The interference performance of the phase-locked FM demodulator was investigated using the experimental arrangement shown in Figure 16. The apparatus not designated by a manufacturers type number was of laboratory construction. The majority of the remaining apparatus was purchased with the aid of a grant from the Defense Research Board.

The noise generator comprises a back biased semiconductor noise diode followed by a high gain broad band two stage amplifier in an arrangement as indicated in the block diagram of Figure 17.

The experimental phase-locked FM demodulator is represented by the block diagram of Figure 18. The IF amplifier comprises two stages of staggered single tuned circuits plus the double tuned magnetically coupled interstage network associated with the balanced phase detector. The VCO comprises an LC phase shift oscillator and a reactance tube in which the quadrature component is derived from the phase shift section. The AC equivalent circuit of the VCO is similar to a frequency modulator of this type described in a reference text book[11].

The signal generator was adjusted to the nominal carrier amplitude and frequency. Fixed amounts of interference were added by adjusting the variable attenuator and the corresponding Rms signal and interference voltages monitored on the Rms voltmeter. The base-band Rms interference voltage in the absence of signal modulation was measured in a 1 kc/s slot centered at various baseband frequencies as the $(S/N)_{IF}$ was varied over the range of operation of the demodulator. For the same $(S/N)_{IF}$, the corresponding baseband Rms signal voltage was measured in a 10 c/s bandwidth for a sinusoidal test tone modulation of the carrier.

This procedure was followed for broad band white noise interference and for a single sinusoidal interfering vector lying within the narrow band noise bandwidth $2B_2$ of the phase-locked demodulator. To further demonstrate the noise rejection properties of the demodulator the closed loop noise bandwidth was reduced from the above case and for the same IF noise power densities, the corresponding noise performance was observed.



EXPERIMENTAL ARRANGEMENT OF APPARATUS

FIGURE 16

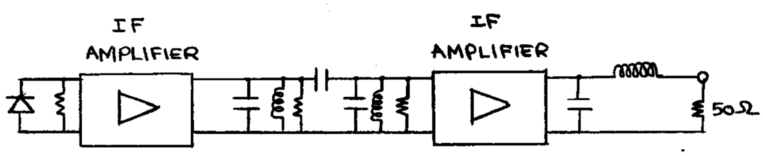


FIGURE 17

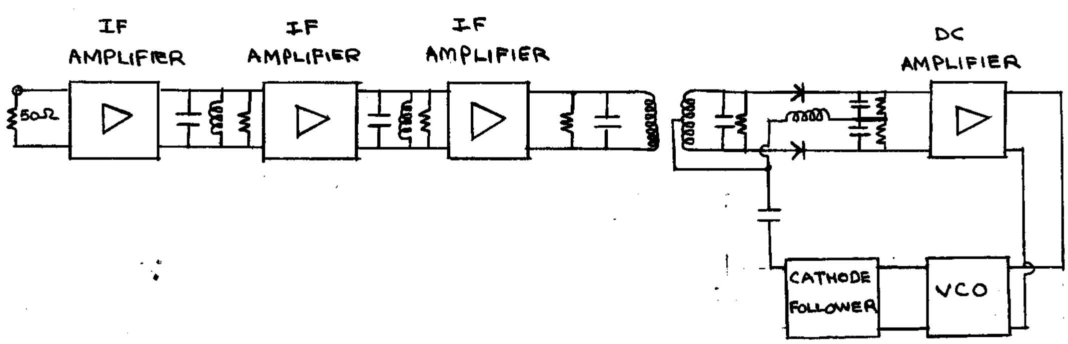


FIGURE 18

(2) Experimental Results and Interpretation

a. Single Interfering Vector

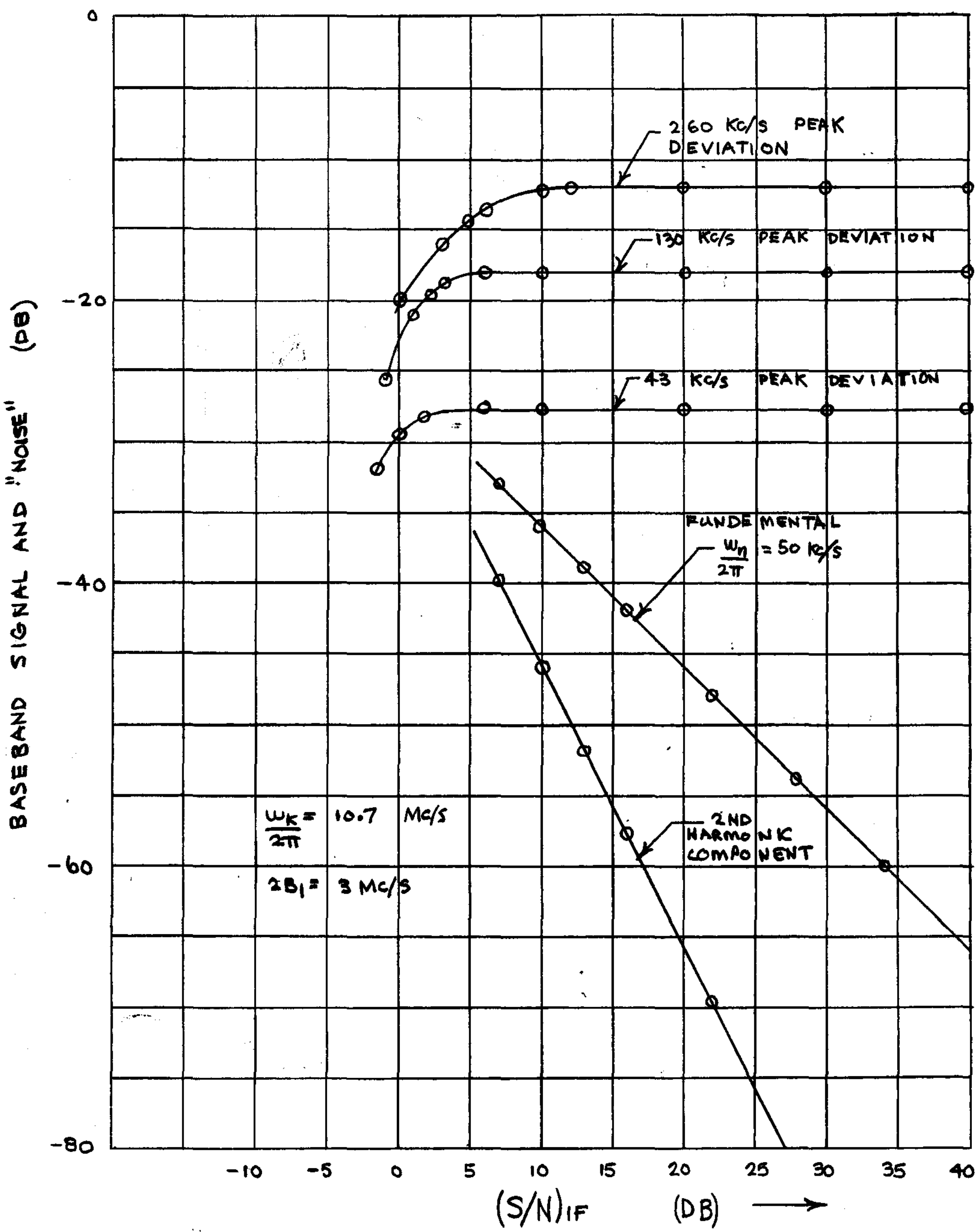
The experimental performance of the phase-locked oscillator for a single interfering vector is shown in Figure 19. The baseband voltage was measured in the absence of signal modulation, due to an interfering vector at $w_n/2\pi = 50$ kc/s from the carrier. It is seen that for $(S/N)_{IF} >> 1$ within the noise bandwidth $2B_2$, the baseband interference output is linearly related to the $(S/N)_{IF}$. Under these conditions the baseband interference voltage as viewed on the cathode ray oscilloscope is a pure sine wave at the difference frequency w_n. As the ratio of the IF interference to carrier amplitude increases, the waveform departs from a sine wave due principally to the second harmonic component associated with the second order term in the power series expansion of $\Psi_i(t)$. The equivalent input angle modulation to the feedback loop $\Psi_i(t)$ under these conditions is given from Equation (6.20) as

$$\Psi_i(t) = x \sin(w_n t + \phi_n) - \frac{x^2}{2} \sin 2(w_n t + \phi_n),$$

where $x = \sqrt{(N/S)_{IF}(B_2/B_1)}$. The corresponding baseband frequency voltage is given as

$$e_{BF} = \frac{1}{K_3} \; T(jw) \left\{ \left[x\, w_n \cos(w_n t + \phi_n) - \frac{x^2}{2}\, 2\, w_n \cos 2(w_n t + \phi_n) \right] \right\}$$

The experimental results for this region of operation are in good agreement with the predicted value.



INTERFERENCE PERFORMANCE
FOR SINGLE
INTERFERING VECTOR

Figure 19.

In the absence of interference, linear operation of the phase detector requires that the modulation index and angular frequency deviation lie within certain limits as discussed in Section IV. If these conditions are satisfied, then non-linear operation of the phase detector under conditions of interference and signal modulation, is due to the presence of the interference. Non-linear operation of the phase detector is associated with the cubic term of the power series expansion of sin $[\Delta\theta_e(t)]$ not being negligible in comparison to the first. This type of non-linearity should be distinguished from the inherent non-linearity associated with the power series expansion of $\Psi_i(t)$. That the first and second order terms associated with $\Psi_i(t)$, for $\theta_s(t) = o$, are demodulated as predicted by the linear feedback loop analysis, for the particular w_n, serves to demonstrate that the phase detector is operated in the linear mode under these conditions. In the presence of signal modulation and interference, and under conditions of linearity of the phase detector, the baseband voltage is given from Equation (6.6) as

$$e_{BF} = \frac{T(p)}{K_3} \left[p\, \theta_s(t) + p\, \Psi_i(t)\right] \qquad \text{for } p = d/dt$$

or for $\theta_s(t) = m_a \sin w_a t$,

$$e_{BF} = \frac{T(jw)}{K_3} \left\{ w_a\, m_a \cos w_a t + x \sum_{-\infty}^{\infty} Jr(m_a)(-1)^r (rw_a + w_n) \cos \left[w_n t + \phi_n + rw_a \right] \right.$$

$$- \frac{x^2}{2} \sum_{-\infty}^{\infty} Jr(2m_a)(-1)^r(rw_a + 2w_n) \cos 2 \left[w_n t + \phi_n + \frac{r}{2} w_a\right] \Bigg\} \Bigg]$$

which is in the form of Equation (5.43) and where the values x and w_n are such that operation of the phase detector is linear. In the presence of signal modulation the baseband voltage measured at w_n and $2w_n$ will be less by virtue of the energy being spread over a spectral range determined by the Bessel coefficients $J_r(m_a)$ and $J_r(2m_a)$. From Equation (6.18) the mean square phase error under linear operation is

$$\overline{\left[\Delta\theta_e(t)\right]}^2 = \left\{\overline{\left[\theta_s(t)\right]}^2 + \overline{\left[\Psi_i(t)\right]}^2\right\} \left[\frac{1}{1 + \frac{KF(p)}{p}}\right]$$

Now fall off of the test tone level in a given slot implies either non-linear operation of the phase detector or a reduction in the equivalent modulation index while the phase detector is still under linear operation. With regard to the latter case it can be seen that for the case of the interfering vector at the carrier frequency $w_n = o$, by inspection

$$e_{BF} = \frac{T(jw)}{K_3} \left\{ w_a m_a \cos w_a t + x \sum_{-\infty}^{\infty} J_r(m_a) (-1)^r rm_a \cos\left[rw_a t + \phi_n\right] - \frac{(x)^2}{2} \sum_{-\infty}^{\infty} J_r(2m_a)(-1)^r rm_a \cos\left[rw_a t + 2\phi_n\right] \right\}$$

hence the conditions for fall off of the test tone level at the angular frequency w_a could be satisfied under linear operation. On the other hand for the observations in question $\frac{w_n}{2\pi} = 50$ kc/s, consequently a loss in test tone level under the conditions of interference is due to non-linear operation of the phase detector. As indicated by Equation (6.18), this would be expected to be a function of the signal modulation and the $(S/N)_{IF}$. This is seen to be the case from Figure 19 where loss in test tone level for a higher angular frequency deviation of the signal occurs at a higher $(S/N)_{IF}$ than for a lower deviation.

Although the single sinusoidal interfering vector approach represents an approximation to the actual interference problem, it does provide an insight into the performance of the phase-locked FM demodulator.

b. White Noise Interference

Following the procedure previously outlined, the experimental performance of the phase-locked FM demodulator was observed under conditions of white noise interference. This performance is shown in Figure 20 for a closed loop noise bandwidth of $2B_2$ = 900 kc/s. The baseband noise output in a 1 kc/s slot was measured in the absence of signal modulation for baseband center frequencies of 10,20,40 and 80 kc/s as a function of the $(S/N)_{IF}$ in the wideband IF noise bandwidth of $2B_1$ = 3 Mc/s. The corresponding signal to noise ratio within the noise bandwidth $2B_2$ and centered at the IF frequency is given by $(S/N)_{IF}$ (B_2/B_1). The results of Figure 20 plus those of Figure 21 for a narrower noise bandwidth of the feedback loop, demonstrate that it is the signal to noise ratio within the closed loop noise bandwidth $2B_2$ which determines the FM threshold. The baseband noise characteristics of Figure 20 are similar to that of a conventional FM demodulator for operation above the FM threshold. For $(S/N)_{IF}\ B_2/B_1 \gg 1$, the baseband noise output is linearly related to $(S/N)_{IF}\ B_2/B_1$. As this ratio is reduced the baseband noise characteristics depart from linearity with the effect being earlier noticed at the lowest baseband frequencies. Well above the threshold, the theoretical baseband noise power measured in a slot of noise bandwidth Bm centered at ω_n is given from Equation (6.7) as

$$\Delta N_{BF} = \frac{1}{(K_3)^2}\ 2 \sum_{\omega_n}^{\omega_n+B_m} \frac{(\Delta x)^2}{2} (w_n)^2 = \frac{1}{(K_3)^2} \sum_{\omega_n}^{\omega_n+B_m} (N/S)_{IF}\ \frac{dw}{2B_1} (w_n)^2$$

hence the incremental noise power in the bandwidth $\Delta w = B_m$ for $B_m \ll w_n$ is

$$\Delta N_{BF} = \frac{1}{(K_3)^2} \frac{(N/S)_{IF}\ (w_n)^2}{2B_1} B_m$$

The corresponding signal power in a given slot due to a test tone modulation of $\theta_s(t) = m_a \sin w_a t$ is

$$S_{BF} = \frac{1}{(K_3)^2} \frac{[m_a\ w_a]^2}{2} = \frac{1}{(K_3)^2} \frac{[\Delta w_a]^2}{2}$$

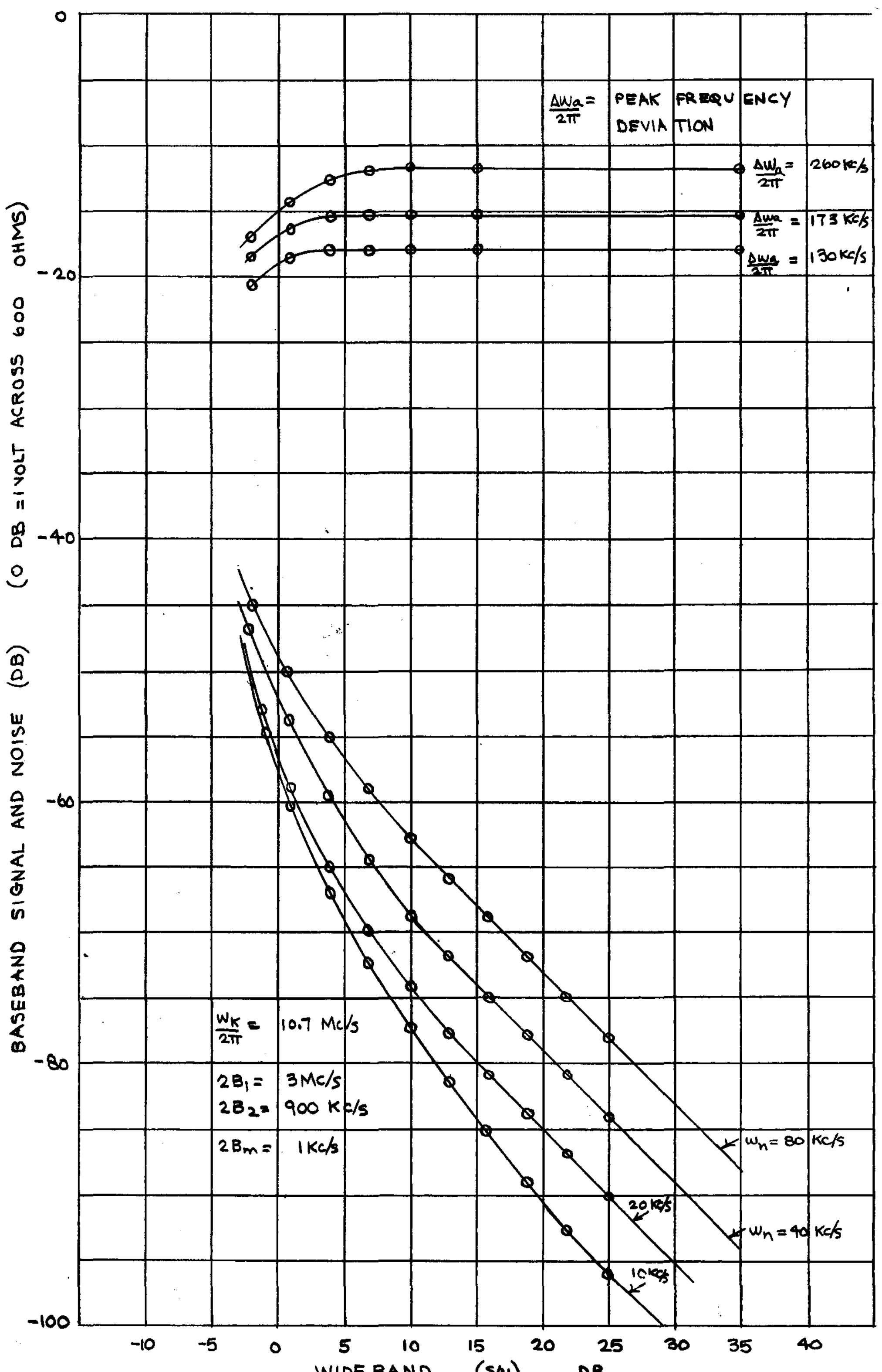
and the ratio of the two powers is

$$(F)^2\ (S/N)_{IF}\ B_1/B_m \qquad , \text{ where } F = \frac{\Delta w_a}{w_n}$$

The experimental baseband signal to noise ratios for this range of operation are in close agreement with the predicted value. The fall off of the baseband signal level is seen to be a function of the angular frequency deviation of the FM wave being demodulated and the IF signal to noise ratio. This may be attributed to non-linear operation of the phase detector and the dependence of the phase error $\Delta\theta_e(t)$ upon $\theta_s(t)$ and $\psi_1(t)$ is given by Equation (6.18) for linear operation. When fall off of the signal level first becomes noticeable it was observed that the peak to peak value of the output noise voltage in the absence of signal modulation was close to that peak to peak voltage associated with the maximum sinusoidal frequency deviation capable of being followed by the feedback demodulator in the absence of interference.

For $2B_1 = 3$ Mc/s and $2B_2 = 900$ Kc/s , unity signal to noise ratio in the wideband IF corresponds to a signal to noise ratio within the noise bandwidth of the feedback demodulator of $10 \log \frac{B_1}{B_2}$ or 5.2 db. If the signal to noise ratio at which threshold effects become noticeable, in the wideband IF of a conventional FM receiver, is taken as 10 to 12 db, corresponding to the intersection of the "below threshold" and "above threshold" asymptotes then it is evident from Figure 20 that the phase-locked FM demodulator provides a degree of FM improvement. One might expect a threshold improvement using the phase-locked demodulator of $10 \log \frac{B_1}{B_2}$ or approximately 5 db. for this case. This would appear to be borne out from the observations.

To further demonstrate the noise rejection properties of the phase-locked loop the noise bandwidth $2B_2$ was reduced from 900 Kc/s to 300 Kc/s. This was accomplished by decreasing the bandwidth of the low pass filter and reducing the signal level E_s at the same time to maintain essentially the same shape of the closed loop transfer function as before. The noise performance of the phase-locked FM demodulator was then examined using the experimental technique previously outlined. The results are shown in Figure 21. A $(S/N)_{IF}$ equal to zero db in the wideband IF of 3 Mc/s would correspond to a signal to noise ratio of 10 log 3/.9 = 5.2 db and 10 log 3/.3 = 10 db within the closed loop noise bandwidths centered at the IF of 900 Kc/s and 300 Kc/s respectively. This represents an enhancement of 4.8 db for the 300 Kc/s filter in comparison to the 900 Kc/s filter. The experimental results demonstrate that the range of linearity between baseband noise power and the wideband $(S/N)_{IF}$ is extended for the lower closed loop noise bandwidth and that it is the noise bandwidth of the feedback loop which determines the noise rejection ability and hence the FM threshold in the Phase-Locked FM demodulator.



0
-20
-40
-60
-80
-100
BASEBAND SIGNAL AND NOISE (DB) (0 DB = 1 VOLT ACROSS 600 OHMS)
$\frac{\Delta W_a}{2\pi}$ = PEAK FREQUENCY DEVIATION
$\frac{\Delta W_a}{2\pi}$ = 260 Kc/s
$\frac{\Delta W_a}{2\pi}$ = 173 Kc/s
$\frac{\Delta W_a}{2\pi}$ = 130 Kc/s
$\frac{W_K}{2\pi}$ = 10.7 Mc/s
$2B_1$ = 3 Mc/s
$2B_2$ = 900 Kc/s
$2B_m$ = 1 Kc/s
w_n = 80 Kc/s
20 Kc/s
w_n = 40 Kc/s
10 Kc/s
-10
-5
0
5
10
15
20
25
30
35
40
WIDE BAND $(S/N)_{IF}$ DB

FIGURE 20

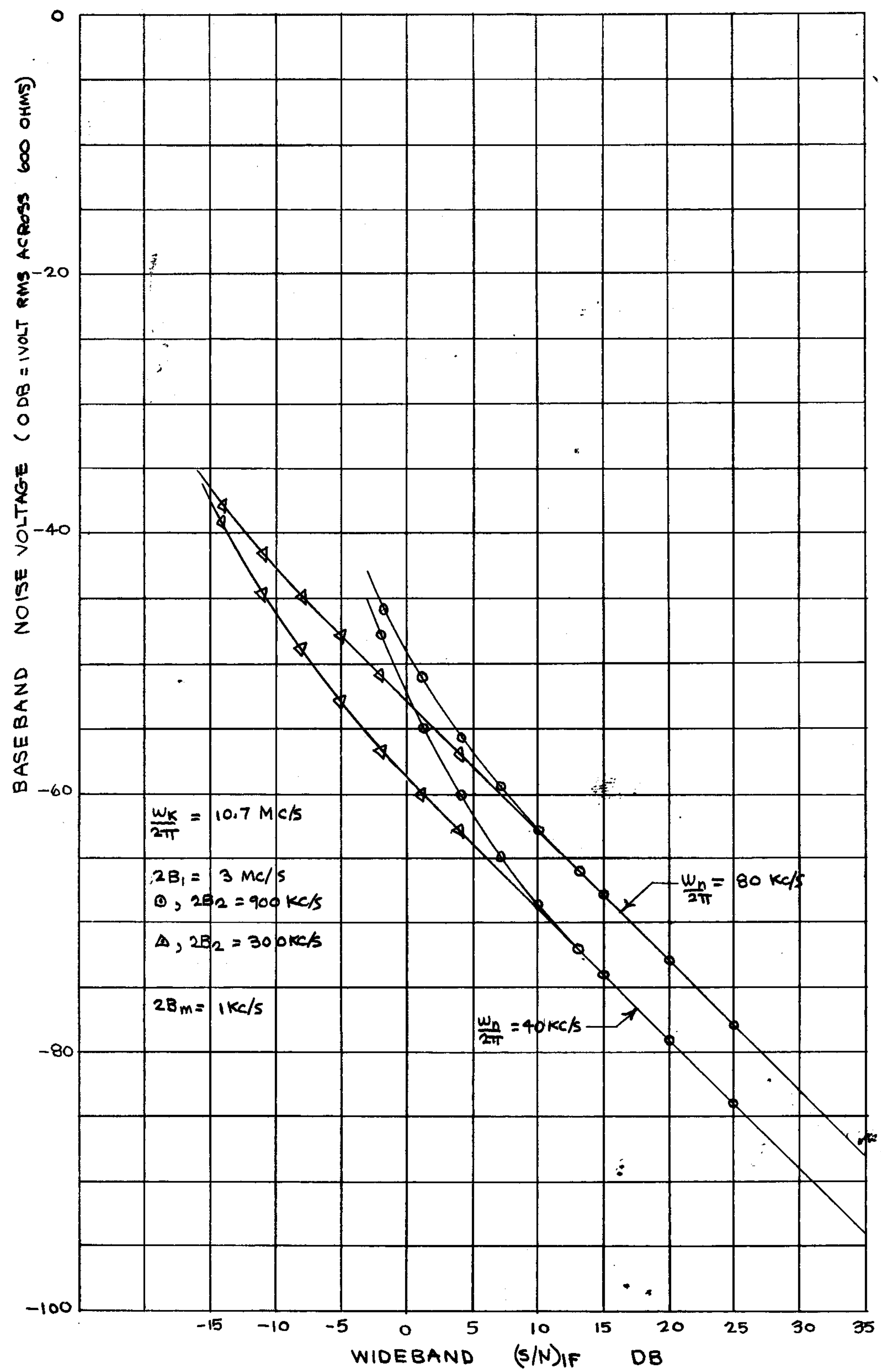

BASEBAND NOISE VOLTAGE (O DB = 1 VOLT RMS ACROSS 600 OHMS)
0
-20
-40
-60
-80
-100
-15 -10 -5 0 5 10 15 20 25 30 35
WIDEBAND (S/N)IF DB
ωK/2π = 10.7 Mc/s
2B1 = 3 Mc/s
⊙, 2B2 = 900 Kc/s
△, 2B2 = 300 Kc/s
2Bm = 1 Kc/s
ωn/2π = 80 Kc/s
ωn/2π = 40 Kc/s

Figure 21.

(3) Summary

The theoretical and experimental interference performance of the phase-locked FM demodulator have been investigated. It was shown experimentally that the noise threshold effect is determined by the closed loop noise bandwidth. This justifies the theoretical model in which the limits of the interference were taken over the closed loop noise bandwidth. The noise threshold performance of the phase-locked demodulator is in contrast to the conventional FM demodulator wherein the noise threshold is determined by the wideband IF bandwidth prior to the limiter. It was shown that fall off in level of the demodulated signal is a function of both the angular frequency deviation of the signal and the IF signal to noise ratio within the closed loop noise bandwidth. This could be explained as being due to non-linear operation of the phase detector and could be minimized by lowering the deviation of the signal.

An analysis of the performance under general conditions of signal modulation and non-linear operation of the phase detector has not been attempted at this time. This would proceed from the general equation of loop performance when higher order terms of the sine function are not negligible in comparison to the first.

SECTION VII

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