# LINEAR POLARIZATION MEASUREMENTS ON $^{22}Na$

# LINEAR POLARIZATION MEASUREMENTS ON $^{22}$Na

By

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A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Science

McMaster University

December 1969

MASTER OF SCIENCE (1970)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Linear Polarization Measurements on $^{22}Na$

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NUMBER OF PAGES:

SCOPE AND CONTENTS: This thesis comprises linear polarization measurements on gamma rays emitted from the previously observed 1.984, 2.572, 2.969 and 3.059 MeV levels of $^{22}Na$ using a Ge(Li) Compton polarimeter. Consistency with previous measurements on parameters characterizing these levels was first checked before assigning $J^{\pi} = 2^{+}$ for the 1.984 MeV level and determining that both the 2.969 MeV and 3.059 MeV levels have positive parity. Investigation of the 2.572 MeV level produced inconsistency with some previous work which had indicated a $2^{-}$ assignment. However, except for some pickup reaction work, the polarization measurement is consistent with all former measurements and indicates a $2^{+}$ assignment.

ACKNOWLEDGEMENTS

I should like to thank my supervisor, Professor J. A. Kuehner, for his help and understanding. Special thanks are also extended to Dr. A. Baxter for his assistance and interest in the experimental work.

I also gratefully thank the National Research Council for its financial assistance.

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# CHAPTER I

# INTRODUCTION

The final objective in the study of a nucleus in experimental nuclear physics is to determine all possible parameters, which describe different states of the nucleus, by observation of emitted radiations or employing other techniques. With information about the nucleus - spins, lifetimes, transition rates, parities, etc. - different theories and nuclear models can be investigated to gain a better understanding about the structure of a particular nucleus, with the ultimate goal of a more comprehensive description of nuclear structure. In the low mass region ($A<40$) both the collective and shell models have been successfully applied to the measured parameters of the low-lying energy levels.

Measurement of nuclear properties involves the application of numerous experimental techniques. For example, angular correlation and angular distribution measurements of gamma rays from excited states provide information about spins and multipole mixing ratios of gamma rays, but do not give direct information about the parity of the nuclear levels. A widely used method to determine parities is observation of the angular distribution of emitted particles from stripping and pick-up reactions. These angular distribution patterns are characteristic of transferred orbital angular momentum and can yield parities when compared to theoretical calculations. Another experimental method is to observe the linear polarization of gamma radiation which contains similiar information to the angular distribution, but in certain circumstances indicates whether a parity change is involved in a transition. In addition, linear polarization measurements can also yield information on gamma ray multipole mixing ratios, or further limit a spin choice where the parities of the involved energy levels are known. It is with such a goal in mind that linear polarization measurements on gamma rays of $^{22}$Na have been made and reported in this thesis.

To discuss the principle of a linear polarization measurement it is convenient to use the density and efficiency matrix formalism of Coester and Jauch (1953) and Devans and Golfarb (1957) respectively. This formalism describes the intensity of radiation as $W = Tr\ (\rho\varepsilon)$ .... (1). The density matrix, $\rho$ provides all information about the nucleus as it describes an ensemble of nuclei in particular nuclear states while the efficiency matrix, $\varepsilon$ contains all information concerned with detection of the emitted radiation. In the case of a nuclear reaction A(a, b)B (A and B initial and final nuclei and a and b incident and outgoing particles respectively) where the outgoing particle is not observed, the nuclear states of the final nucleus with sharp spin J will have axial symmetry about the axis of quantization, which is the direction of the incident beam. This reaction also populates unequally the magnetic substates, m of the nuclear state of spin J. Since the formed nuclear states have good parity, they will also have reflectional symmetry in the plane perpendicular to the quantization axis. Under these conditions the states are said to be aligned. In general the gamma radiation from aligned states is linearly polarized, i.e. its electric vector has a preferred direction. Compton scattering of a gamma ray depends on its linear polarization (Chapter 2.4) as the gamma ray is scattered preferentially into a plane normal to the direction of its electric vector. Thus using a detection system (Compton Polarimeter, Chapter 2.4) sensitive to the Compton scattering as a function of azimuthal angle relative to the gamma ray direction, a measurement of the linear polarization can be obtained.

# CHAPTER 2

# THEORY

## 2.1 General

In the radiative decay of excited nuclear states, the gamma rays are represented quantum mechanically by a multipole expansion of the electromagnetic field, Blatt and Weisskopf (1952). The ratio of the amplitudes of the higher order multipole to lower order multipole is called the mixing ratio, $\delta$. This quantity has been defined several ways, differing primarily in the phase factor. In this thesis the convention of Ferguson (1965) is used, which is opposite in sign to that of McCallum (1961), for the case of E2/M1 radiation.

If nuclear substates of spin J and projection $m$ are denoted $|J,m\rangle$, then elements of the density matrix $\rho$ as defined in Ferguson (1965) can be written $\langle Jm|\rho|J'm'\rangle$. The statistical tensor relative to the beam axis, which has useful rotation properties, can be introduced using Clebsch-Gordan coefficients.

Ferguson (1965) has the form

$$\rho_{k\chi'}(JJ') = \Sigma_{mm'}(-)^{J-m}\,(J,m,J'\;-m'/k\,\chi')\,\langle Jm|\rho|J'm'\rangle \ldots\ldots \quad (2)$$

For a reaction A(a,b)B* leading to a state of good parity and in which the outgoing particles are undetected, the density matrix describing the levels B* relative to the beam axis is diagonal (expressing axial symmetry) and reflectionally invarient (expressing good parity), thus $m = m'$. Consideration of an isolated level involves only terms $\rho_{k\kappa}^{(JJ)}$. Since $\kappa' = m-m'$ in the Clebsch-Gordan co-efficient, $\kappa' = 0$

and the statistical tensor becomes

$$\rho_{k0} = \rho_{k0}(JJ) = \sum_m (-)^{J-m} P(m) (JmJ-m/k0) \quad .... (3)$$

It is convenient to denote the probability of populating a magnetic substate m of the nuclear state of spin J as P(m). These population parameters, P(m) are the diagonal elements of the density matrix. Since P(m) = P(-m) and due to triangle properties of the Clebsch-Gordan co-efficients, $\rho_{k0} = 0$ unless k is even. Physically this results from good parity.

## 2.2 Angular Distributions for Mixed Multipoles

When a nuclear state of spin J is de-excited to a state of spin I, the angular distribution intensity of the emitted gamma rays of mixed multipolarity $(L,L')$ and mixing ratio $\delta$ can be expressed (Ferguson (1965))

$$W(\theta) = \sum_{kLL'} (-)^{I-J} \rho_{k0} \; \delta^{r} \; \bar{Z}_1 (LJL'JIk) \; P_k (\cos \theta)$$

$$= \sum_k (-)^{I-J} \rho_{k0} \; P_k (\cos \theta) \left[ \bar{Z}_1 (LJLJIk) \right.$$

$$\left. - 2\delta \bar{Z}_1 (LJL'JIk) + \delta^2 \bar{Z}_1 (L'JL'JIk) \right] \quad .... (4)$$

where the $\bar{Z}_1$ co-efficients, (Ferguson (1965)) satisfy $|L-L'| \leq k \leq |L+L'|$ and $0 \leq k \leq 2J$. The Legendre polynominals express the rotation of the co-ordinate system such that the z-axis lies along the axis of the emitted radiation. It is convenient to write this as

$$W(\theta) = A_0 P_0 (\cos \theta) + A_2 P_2 (\cos \theta) + A_4 P_4 (\cos \theta) + .... \quad .... (5)$$

where

$$A_0 = (-)^{I-J} \rho_{00} \Big[ \bar{Z}_1(LJLJI0) - 2\delta \bar{Z}_1(LJL'JI0) + \delta^2 \bar{Z}_1(L'JL'JI0) \Big]$$

$$= (2J+1)^{\frac{1}{2}} (1+\delta^2) \rho_{00}$$

since $\bar{Z}_1(LJL'JI0) = (-)^{I-J} (2J+1)^{\frac{1}{2}} \delta_{L,L'}$

$$\therefore \ A_2/A_0 = (-)^{I-J} \rho_{20}/\rho_{00} \Big[ \bar{Z}_1(LJLJI2) - 2\delta \bar{Z}_1(LJL'JI2) + \delta^2 \bar{Z}_1(L'JL'JI2) \Big] \Big[ (2J+1)^{\frac{1}{2}} (1+\delta^2) \Big]^{-1} \quad \ldots. (6)$$

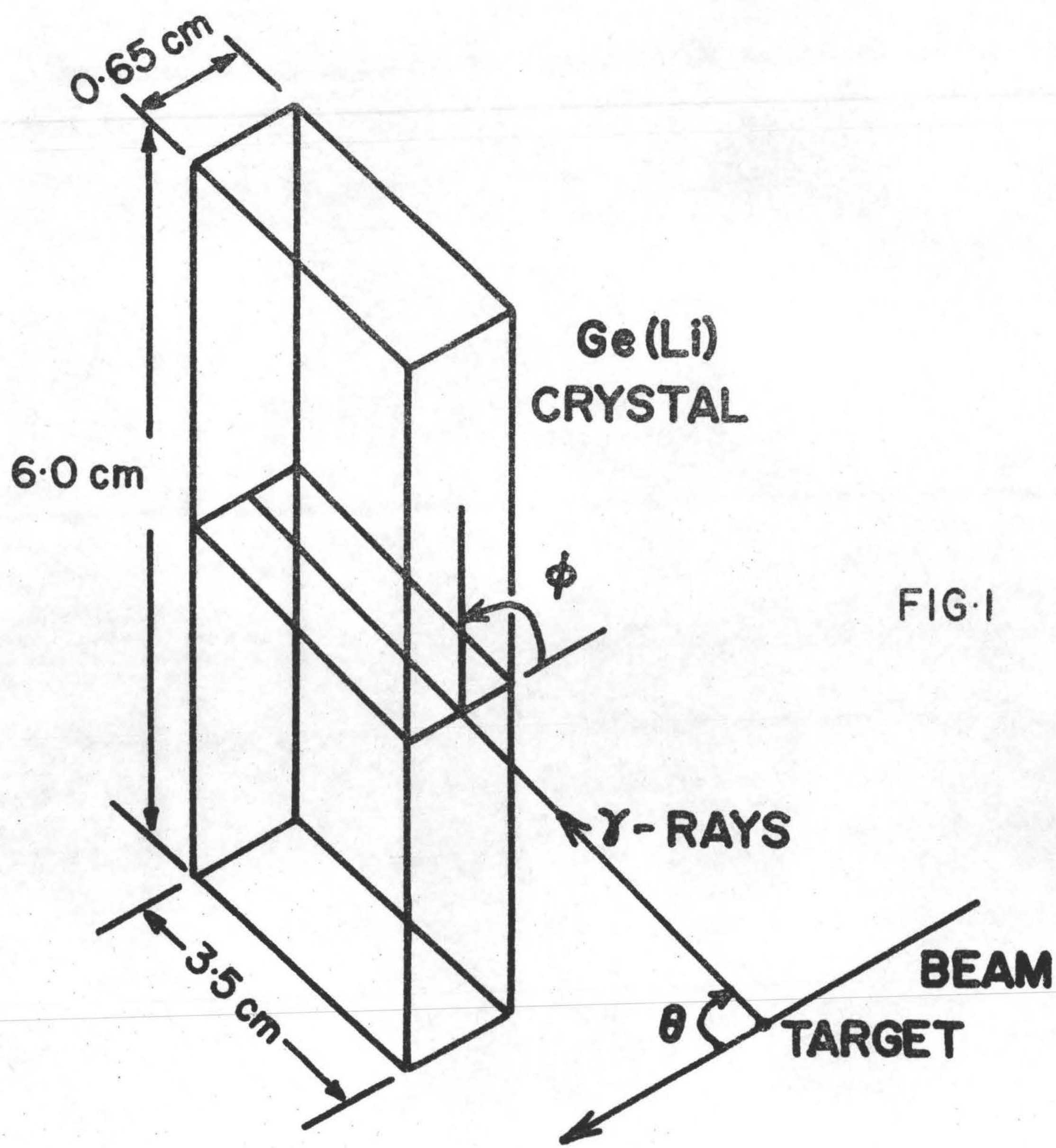
and

$$A_4/A_0 = (-)^{I-J} \rho_{40}/\rho_{00} \Big[ \bar{Z}_1(LJLJI4) - 2\delta \bar{Z}_1(LJL'JI4) + \delta^2 \bar{Z}_1(L'JL'JI4) \Big] \Big[ (2J+1)^{\frac{1}{2}} (1+\delta^2) \Big]^{-1} \quad \ldots. (7)$$

## 2.3 Linear Polarization for Mixed Multipoles

For a gamma ray of multipolarity $(L,L')$ from a state of spin J to a state of spin I, the linear polarization may be described as a function of the angle $\theta$ between the beam direction and the emitted gamma ray, and the azimuthal angle $\phi$ about the gamma ray direction. However to simplify the analysis of experimental results, the linear polarization of a gamma ray is expressed as the ratio of intensities of gamma rays whose electric vectors are perpendicular ($\phi = 90^\circ$) to the reaction plane to those whose electric vectors lie in the reaction plane ($\phi = 0^\circ$) (see

Ge(Li) Polarimeter Crystal




FIG·1

Figure 1). Since a gamma ray Compton scatters preferentially into a plane normal to its electric vector, a polarimeter based on this effect (see 2.4) may be used to determine the linear polarization of gamma rays. Ferguson (1965) expresses the linear polarization as

$$P(\theta) = \Big[ \sum_{kLL'} (-)^{I-J} \rho_{k0} \bar{Z}_1(LJL'JIk) \; \delta^r \left(P_k(\cos\theta) + (-)^{\pi} K_k(LL') P_k^2(\cos\theta)\right)\Big] \Big[ \sum_{kLL'} (-)^{I-J} \rho_{k0} \bar{Z}_1(LJL'JIk) \; \delta^r \left(P_k(\cos\theta) - (-)^{\pi} K_k(LL')\right)\Big]^{-1} \quad \ldots. (8)$$

Note that the intensity in either plane differs from the angular distribution expression by the second term in the bracket, which is included because the linear polarization of the gamma ray is observed. The term $K_k(L,L')$ contains information about the multipolarity of the gamma ray while $P_k^2(\cos\theta)$ expresses the angular dependence of the polarization due to the rotation of the co-ordinate system such that the z-axis lies along the direction of radiation. The factor $(-)^{\pi}$ distinguishes between electric and magnetic radiation, if $\pi$ is even the radiation is electric and odd if it is magnetic. When $\theta = 90^\circ$ for an E2/M1 transition, the linear polarization can be expressed (Ferguson (1965))

$$P(90) = \Big\{(-)^{I-J} (1+\delta^2)(2J+1)^{\frac{1}{2}} + \rho_{20}/\rho_{00}\left[\bar{Z}_1(1J1JI2) - 2\delta\bar{Z}_1(1J2JI2) + \delta^2\bar{Z}_1(2J2JI2)\right] + \rho_{40}/\rho_{00}\,\delta^2\bar{Z}_1(2J2JI4)\Big\} \Big\{(-)^{I-J}(1+\delta^2)(2J+1)^{\frac{1}{2}}$$

$$- 2 \rho_{20}/\rho_{00} \left[\bar{Z}_1(L J1 L J I2) + \delta^2 \bar{Z}_1(2J2JI2)\right]$$

$$- \tfrac{1}{4} \delta^2 \rho_{40}/\rho_{00} \bar{Z}_1(2J2JI4)\Big\}^{-1}$$

and for M3/E2 radiation it is (Appendix I)

$$P(90) = \Big\{(-)^{I-J}(1 + \delta^2)(2J + 1)^{\frac{1}{2}} + \rho_{20}/\rho_{00}\left[\bar{Z}_1(2J2JI2)\right.$$

$$\left. - \tfrac{1}{2}\delta \bar{Z}_1(2J3JI2) - 3/2\ \delta^2 \bar{Z}_1(3J3JI2)\right]$$

$$+ \rho_{40}/\rho_{00}\left[\bar{Z}_1(2J2JI4) - \tfrac{1}{2}\delta \bar{Z}_1(2J3JI4)\right.$$

$$\left. + 23/8\ \delta^2 \bar{Z}_1(3J3JI4)\right]\Big\}\Big\{(-)^{I-J}(1 + \delta^2)(2J + 1)^{\frac{1}{2}}$$

$$- \rho_{20}/\rho_{00}\left[2\bar{Z}_1(2J2JI2) - 5/2\ \delta \bar{Z}_1(2J3JI2)\right.$$

$$\left. - \tfrac{1}{2}\delta^2 \bar{Z}_1(3J3JI2)\right] - \rho_{40}/\rho_{00}\left[\tfrac{1}{4}\bar{Z}_1(2J2JI4)\right.$$

$$\left. + \delta \bar{Z}_1(2J3JI4) + 17/8\ \delta^2 \bar{Z}_1(3J3JI4)\right]\Big\}^{-1}$$

It is useful to note that the expression for linear polarization as defined, is such that the expression for $M_L'/E_L$ is the reciprocal for that of $E_L'/M_L$.

## 2.4 Ge(Li) Compton Polarimeter

The Compton effect is one of many possible interactions of a gamma ray with matter. As the Compton scattering process scatters the gamma ray preferentially into a plane normal to its electric vector (Fagg and Hanna (1959)) the resulting azimuthal asymmetry distribution relative to the gamma ray direction may be measured using a rectangular Ge(Li) crystal shown in Figure 1. This Ge(Li) crystal is able to detect this distribution since gamma rays Compton scattered into the plane of the Ge(Li) crystal will be detected with a much higher probability than those scattered out of the detector plane and hence out of the crystal. Measurement of the total absorption peak as a function of the azimuthal angle about the gamma ray direction will contain the relative azimuthal asymmetry distribution due the gamma ray's linear polarization. It is convenient to define an asymmetry ratio, N as the intensity of totally absorbed radiation detected by the Ge(Li) crystal in the reaction plane ($\phi = 0^\circ$) to that in the corresponding perpendicular plane ($\phi = 90^\circ$). For a gamma ray completely polarized in the reaction plane, the asymmetry ratio is called the sensitivity and denoted $R$. Then the linear polarization defined in Section III can be expressed as (Ferguson (1965))

$$P = (1-NR)/(N-R) \quad .... \quad (11)$$

$$\text{or} \quad N = (1+PR)/(P-R) \quad .... \quad (12)$$

Thus by measuring the asymmetry ratio N where the sensitivity R is known, the linear polarization of the gamma ray may be determined.

## 2.5 Physical Considerations

Observation of the linear polarization of a gamma ray from a nuclear state may determine whether or not a parity change has occurred. It may also establish the value of the multipole mixing ratio, $\delta$. From this information and the value of $A_2/A_0$ from the angular distribution data, calculation of the statistical tensor $\rho_{20}/\rho_{00}$ from equation (6) is possible. One must check, if this result is consistent with the physical situation, expressed in equation (3) is the dependence of the statistical tensor on the population parameters. Within the allowed values of P(m), a range of values can be placed on $\rho_{20}/\rho_{00}$, outside this range requires $P(m) < 0$. Using the symmetry relation

$$(JJ\ m{-}m|20) = \frac{(-)^{J-m}\ 5^{\frac{1}{2}}\ (3m^2 - J(J+1))}{(2J+1)^{\frac{1}{2}}\ (J(J+1)\ (2J+1)\ (2J+3))^{\frac{1}{2}}}$$

equation (3) becomes,

$$\rho_{20}/\rho_{00} = \frac{5^{\frac{1}{2}} \sum_m P(m)\ (3m^2 - J(J+1)}{(J(J+1)\ (2J+1)\ (2J+3))^{\frac{1}{2}}}$$

which puts limits on $\rho_{20}/\rho_{00}$ and the corresponding value calculated from the experimental data may be compared and checked for consistency. A similar check can be made with the population parameters using equation (3) to get limits on $\rho_{40}/\rho_{00}$. However the calculated values of $\rho_{40}/\rho_{00}$ use the experimental value of $A_4/A_0$ which in general is difficult to determine accurately, while $\rho_{20}/\rho_{00}$ is a function of $A_2/A_0$ which is generally easy to determine with the required accuracy. Hence the calculated value of $\frac{\rho_{20}}{\rho_{00}}$ is a far more reliable indication of the validity of the experimental results.

# CHAPTER 3

# EXPERIMENTAL METHOD AND REDUCTION OF DATA

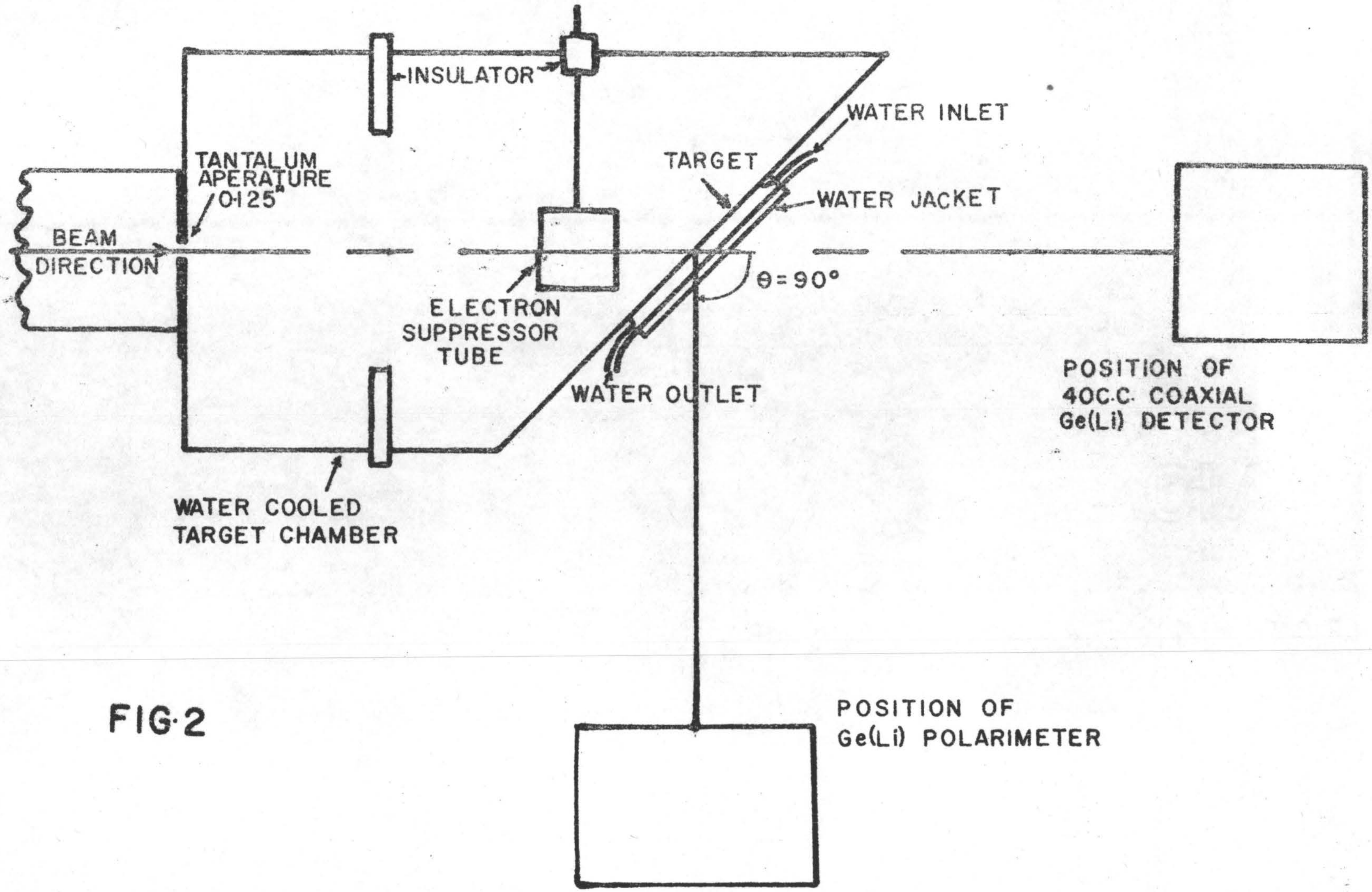
## 3.1 Experimental Method

Low-lying excited states of $^{22}Na$ were investigated using the $^{19}F(\alpha, n)^{22}Na$ reaction. The helium ion beam was obtained from the McMaster University 7.5 Mega-Volt F.N. Tandem Accelerator and collimated to about a three millimeter spot on target using tantalum apertures.

The target - $PbF_2$ evaporated on a tantalum backing - was held in a water-cooled target chamber (see Figure 2) and aligned optically in the center of the Lotus Goniometer, (an array of counter holders capable of accurately positioning detectors with respect to the target and beam axis). An insulated circular aperture was placed in the chamber in front of the target to aid in the focussing of a circular beam spot centered on the target. The target was electrically insulated along with the chamber. By maximizing the target current and simultaneously minimizing the aperture current, the beam was positioned on the target. For typical beam energies of 6.13 and 7.00 MeV, target currents from 200 to 550 nA were obtained. It was found necessary to bias the electron suppressor tube by -300 volts to suppress secondary electrons emitted from the target.

Angular distributions of the observed gamma rays were measured using a 40 cc co-axial Ge(Li) detector, about 9 cms from the target. The detector pulses were sent through a conventional pre-

Water Cooled Target Chamber


INSULATOR
WATER INLET
TANTALUM APERATURE 0·125"
TARGET
WATER JACKET
BEAM DIRECTION
θ=90°
ELECTRON SUPPRESSOR TUBE
WATER OUTLET
POSITION OF 40C.C. COAXIAL Ge(Li) DETECTOR
WATER COOLED TARGET CHAMBER
POSITION OF Ge(Li) POLARIMETER

FIG·2

amplifier* - amplifier** - baseline restorer*** assembly to an analog-to-digital converter (A.D.C.). This configuration obtained an overall resolution of 4.2 KeV (F.W.H.M.) for the 1.275 MeV gamma ray in $^{22}Ne$. Spectra at 0, 30, 45, 60 and 90 degrees were recorded in the 3300 Nuclear Data Analyzer and stored on magnetic tape.

The Ge(Li) polarimeter - on loan from G. T. Ewan of the Chalk River Nuclear Laboratories - was positioned 11.5 cms from the target at 90 degrees to the beam direction. Two spectra were recorded with the polarimeter (see Figure 1) in each of two positions: horizontal ($\phi = 0^\circ$) and vertical ($\phi = 90^\circ$). This was achieved for 6.13 MeV beam energy on two independent runs (identified (a) & (b)) months apart, while all data for 7.00 MeV were collected during one day. The electronic set-up for the polarimeter was similar to that of the 40 cc detector except a Simtec pre-amplifier and a Tennelec TC 203 amplifier were used. An overall resolution of 6.8 KeV (F.W.H.M.) at 1.275 MeV was obtained. The spectrum from each detector was put into an A.D.C. of the 3300 Analyzer which was operated in its digiplex mode. This mode enables the A.D.C.'s to share the analyzer memory with about a 40 micro-sec. dead time for memory storage. This reaction however, had a low counting rate permitting almost every detected event to be analyzed.

* Ortec Model C1415

** Canberra Industries Spectroscopy Amplifier Model 1417

*** Tennelec TC 203

The sensitivity of a Ge(Li) polarimeter (using a single planar crystal) to the linear polarization of a gamma ray is an order of magnitude smaller than for a NaI polarimeter (consisting of a NaI scattering crystal and a NaI crystal to detect the scattered gamma ray). This imples that the asymmetry ratio N (Chapter 2.4) to be observed will be smaller in its difference from unity for a Ge(Li) polarimeter than a NaI polarimeter. However, the resolution of a Ge(Li) crystal is vastly superior to the resolution of a NaI crystal and because of this, a Ge(Li) polarimeter was used to investigate the complex level structure of $^{22}$Na. From equation (12) taken in the limit as $P \rightarrow \infty$, $N = R$ while for $P = 0$, $N = 1/R$. Considering a 1.0 MeV gamma ray where $R = 0.81$ (see Figure 8), the value of N may vary from 0.81 to 1.24. Thus for a 1.0 MeV gamma ray the maximum observable asymmetry will be about 20%. Furthermore since the value of R increases towards unity with increasing gamma ray energy (see Figure 8) the size of the observable asymmetry decreases, stressing that the linear polarization becomes increasingly more difficult to observe as the gamma ray's energy increases.

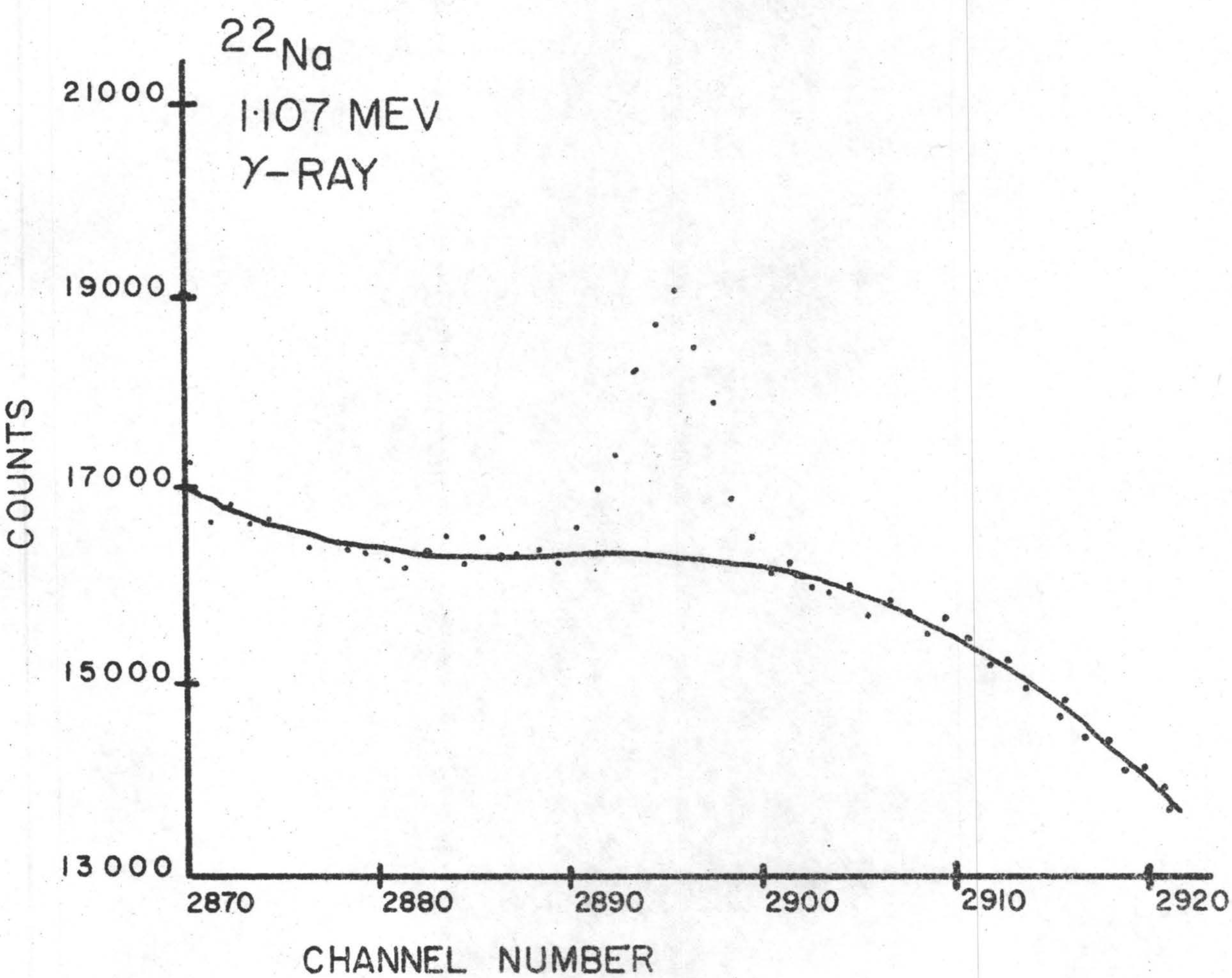
## 3.2 Reduction of Data

The Compton edge associated with the detection of a gamma ray using a Ge(Li) crystal adds non-linearity to the observed background. This creates a problem in background evaluation. Here also poor statistics can be a factor. The intensities of weak gamma rays that are not much stronger than the background must be determined consistently if meaningful comparisons to other intensity measurements are to be obtained. With this motivation a third, fourth and fifth order polynominal expansion was used to obtain a least-squares fit to the observed background about the peak. By choosing a region of between fifteen and fifty channels on either side of the peak, statistical fluctuations in the observed background can be averaged out, enabling a high degree of consistency to be obtained among different spectra for the intensity of a given peak. Both the observed gamma ray peak and the calculated background were simultaneously plotted using a program written for the McMaster University C.D.C. 6400 computer (see Figure 3). This facilitates comparison of the background fit of a given gamma ray for different spectra. By consistently choosing in each spectrum similar amounts of background, the intensity of the peak and an estimate of the error involved can be obtained. In addition to the statistical error in the number of events of a particular gamma peak, the variation in the relative amount of the peak lying outside the chosen peak limits for each spectrum was inspected. This was necessary due to both the Dopplar shift and broading which affected some gamma rays. Additional error was added to include this variation. Next the quality of the calculated background fit was estimated. On this basis an error was assigned by inspecting for consistency all the fits made to the background under a particular gamma ray peak. A gamma ray emitted from the target must pass through the targets tantalum backing and the target chamber's water jacket. Due to the geometry of the target chamber and the Ge(Li) 40 cc detector, gamma ray absorption will vary with the angle of detection. The angular distribution measurements were corrected for this absorption effect and an additional error was included to take into account the uncertainty in this process.

A particularly troublesome case was the following. The Compton

Fourth Order Polynomial Expansion to the Background

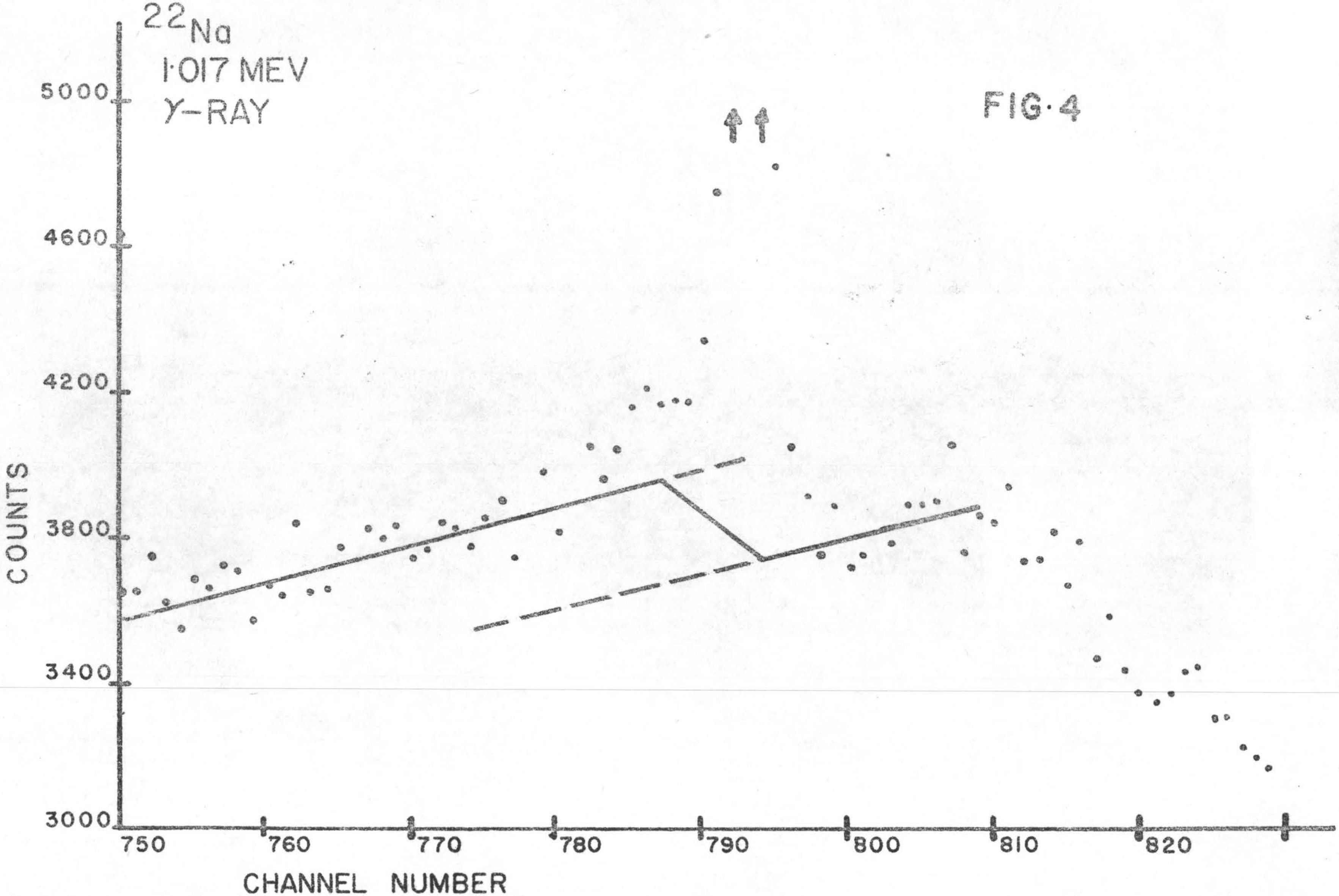
FIG·3


22Na
1·107 MEV
γ-RAY
21000
19000
17000
15000
13 000
COUNTS
2870
2880
2890
2900
2910
2920
CHANNEL NUMBER

edge of the 1.235 MeV gamma ray arising from the 1.345 MeV to 0.110 MeV transition in $^{19}F$ was found to be directly beneath the 1.017 MeV full absorption peak from the 2.969 MeV to 1.952 MeV transition in $^{22}Na$. It was necessary to obtain the detailed line-shape of this Compton edge if an accurate determination of the intensity of the 1.017 MeV gamma ray was to be made. As the position and shape of the Compton edge corresponding to the 1.275 MeV transition from the 1.275 MeV to the ground state in $^{22}Ne$ was known, the position and shape for the Compton edge of the 1.235 MeV gamma ray could also be determined by assuming a similar structure. Thus the Compton edge under the 1.017 MeV peak was constructed (see Figure 4), representing the background in each spectrum. An estimate of the error, similar to the calculated background fits, was made also in this case.

One of the problems frequently encountered in gamma ray intensity measurements is the necessity of normalizing in order to compare two or more measurements. Factors leading to uncertainties in the absolute intensities include target deterioration, movement of the beam on the target, inaccurate current-integration, etc. In this reaction however, the decay of the first excited state of $^{22}Na$ has a lifetime, $\tau$ = 351 n-sec. (Sunyar et al (1966)) which is sufficiently long to cause the state to lose all its alignment before emission of the gamma ray. Thus the emitted gamma ray has no polarization and an isotropic angular distribution. Each spectrum was thereby normalized using this internal monitor which compensates for all the previously mentioned problems associated with normalization.

## Background Evaluation Where the Compton Edge of the 1.235 MeV Gamma Ray Was Under the 1.017 MeV Full Energy Peak


22Na
1·017 MEV
γ-RAY
FIG·4
COUNTS
5000
4600
4200
3800
3400
3000
750
760
770
780
790
800
810
820
CHANNEL NUMBER

The angular distribution measured using the 40 cc Ge(Li) co-axial detector was fitted to a Legendre polynominal expansion (equation (5)) using a standard least squares technique (Ferguson (1965)) on the CDC 6400 computer. From this, values of $A_2/A_0$ and $A_4/A_0$ with their corresponding, correlated errors were calculated. These values were then corrected for solid angle effects, as calculated in Appendix II.

# CHAPTER 4

# RESULTS

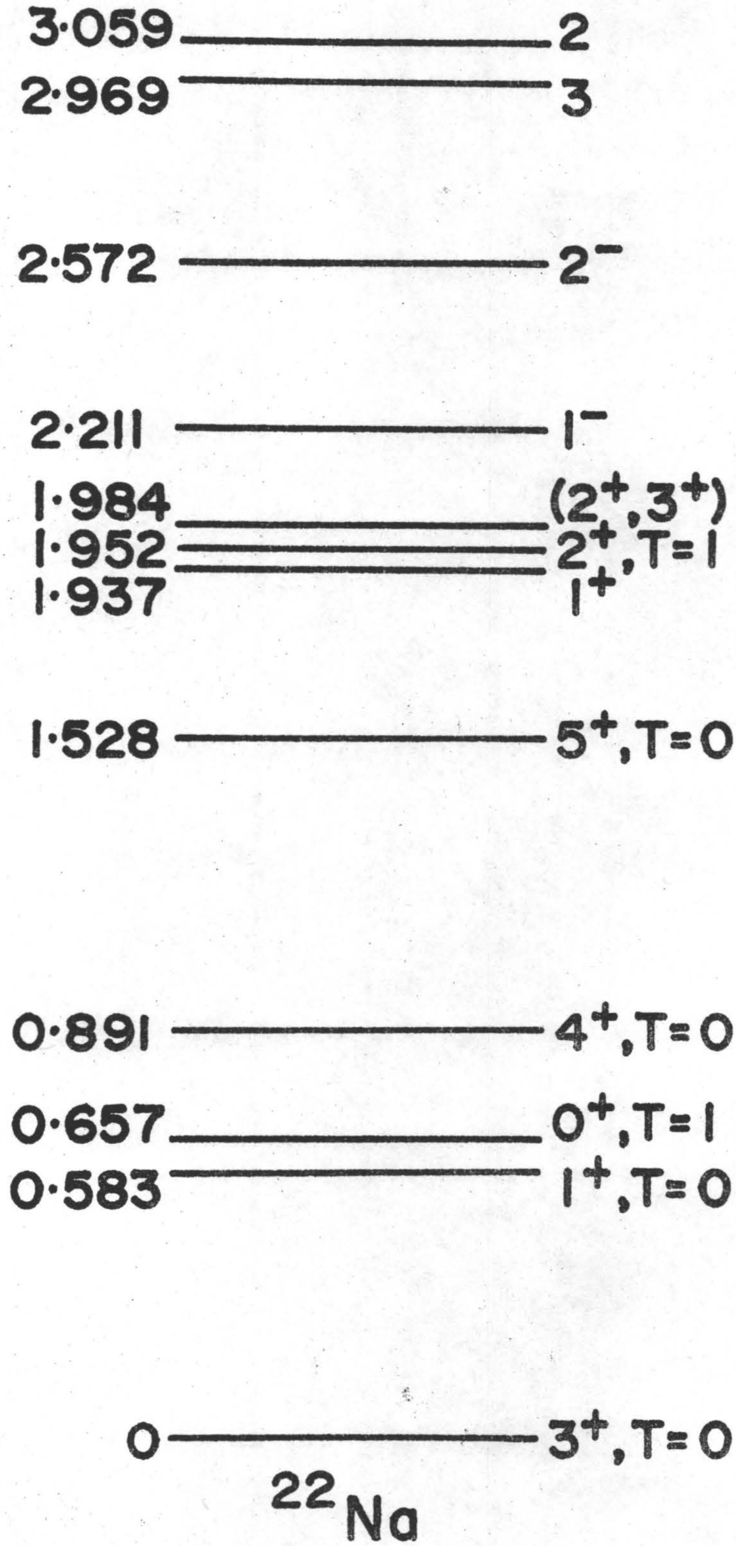
## 4.1 Introduction

Low-lying energy levels of $^{22}Na$ have been in general carefully studied. The known parameters characterizing these levels, before the linear polarization measurements in this thesis were carried out, were for the most part measured by a Brookhaven Group (Warburton et al (1967), Poletti et al (1967, a), Poletti et al (1967, b), Warburton et al (1968), and Paul et al (1968)). Their assignments are illustrated in Figure 5. As $^{22}Na$ lies in the s-d shell and the rotational model has had considerable success in this region, it is expected that some aspects of the level structure can be explained by a number of rotational bands. Two such bands are evident from Figure 5: firstly the K = 3, T = 0 ground state band has its first three members at 0 MeV ($3^+$), 0.891 MeV ($4^+$), 1.528 MeV ($5^+$); and secondly the first two members of the K=0, T = 1 band: $0^+$, 0.657 MeV and $2^+$, 1.952 MeV levels. From the angular correlation measurements of Warburton et al (1968) the spins of the 2.969 MeV and 3.059 MeV levels have been assigned 3 and 2 respectively. Furthermore, both were proposed to have positive parity on the basis of their possible membership in rotational bands. This of course is highly speculative since their parities have not in fact been experimentally determined. With the idea of expanding the amount of existing information, linear polarization measurements were made.

Preliminary investigation showed that most low-lying levels

$^{22}$Na Level Scheme

FIG·5


3·059
2
2·969
3
2·572
2⁻
2·211
1⁻
1·984
(2⁺,3⁺)
1·952
2⁺, T=1
1·937
1⁺
1·528
5⁺, T=0
0·891
4⁺, T=0
0·657
0⁺, T=1
0·583
1⁺, T=0
0
3⁺, T=0
$^{22}$Na

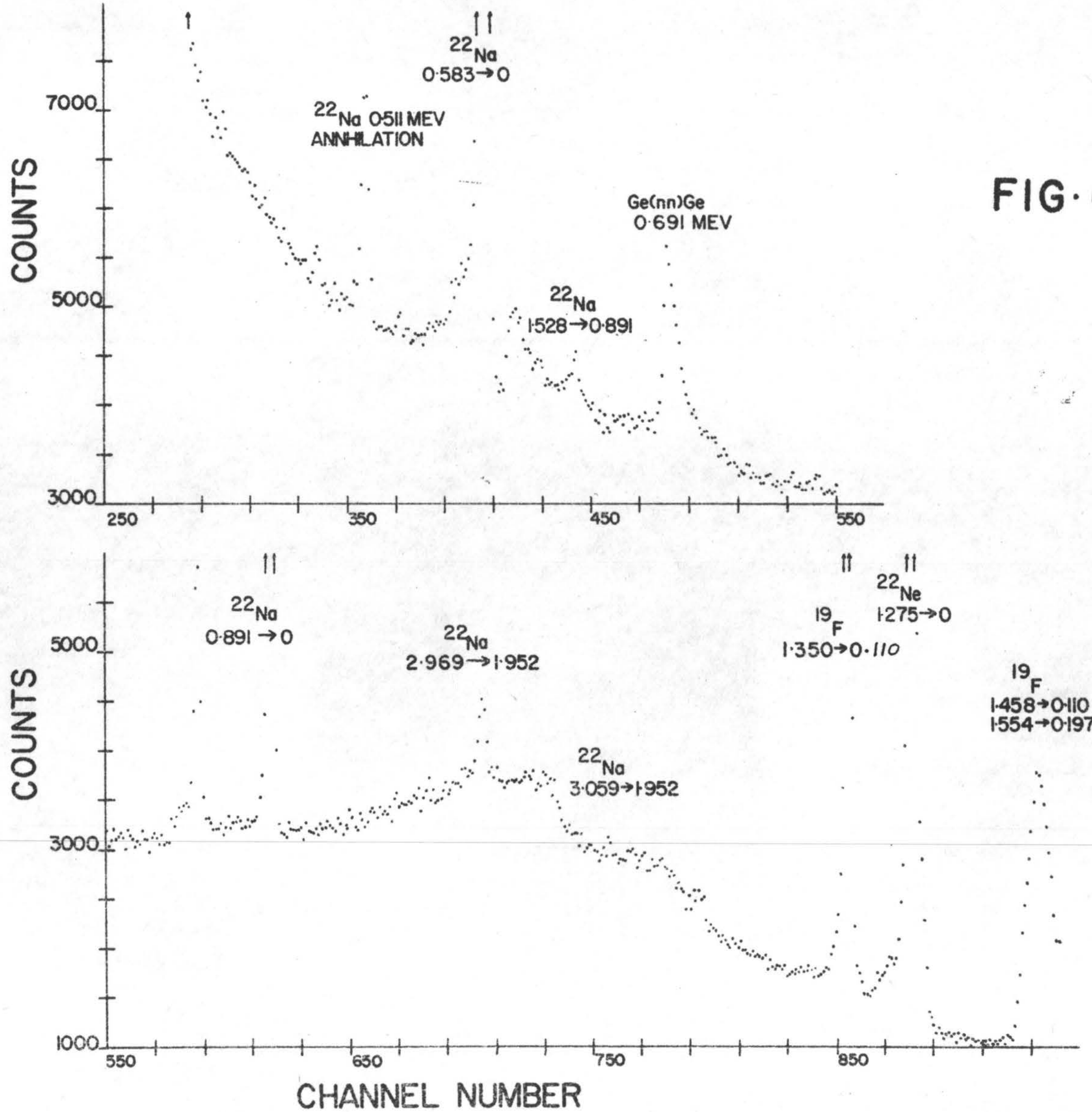
were strongly populated with a 6.13 MeV bombarding energy in the reaction $^{19}F(\alpha,n)^{22}Na$. A complete spectrum recorded by the 40 cc co-axial detector at $0^{\circ}$ is shown in Figures 6 and 7. A linear polarization measurement was made at this energy and denoted in the results as $E_{\alpha}$ = 6.13 MeV (a). A month later the apparatus was again set up and the measurement repeated. This independent measurement is identified by $E_{\alpha}$ = 6.13 MeV (b). Each of the polarization measurements were made with the Ge(Li) polarimeter crystal first in a vertical position, then in a horizontal position. For a 6.13 MeV bombarding energy the 2.969 and 3.059 MeV levels were weakly populated. However, for a 7.00 MeV beam energy they were strong enough to permit a polarization measurement to be made. For this incident beam energy, their full absorption gamma peaks were observed during runs with the Ge(Li) polarimeter crystal vertical, then horizontal, repeating the horizontal run and finally in the vertical position.

## 4.2 Method of Analysis

### (i) Ge(Li) Polarimeter Calibration

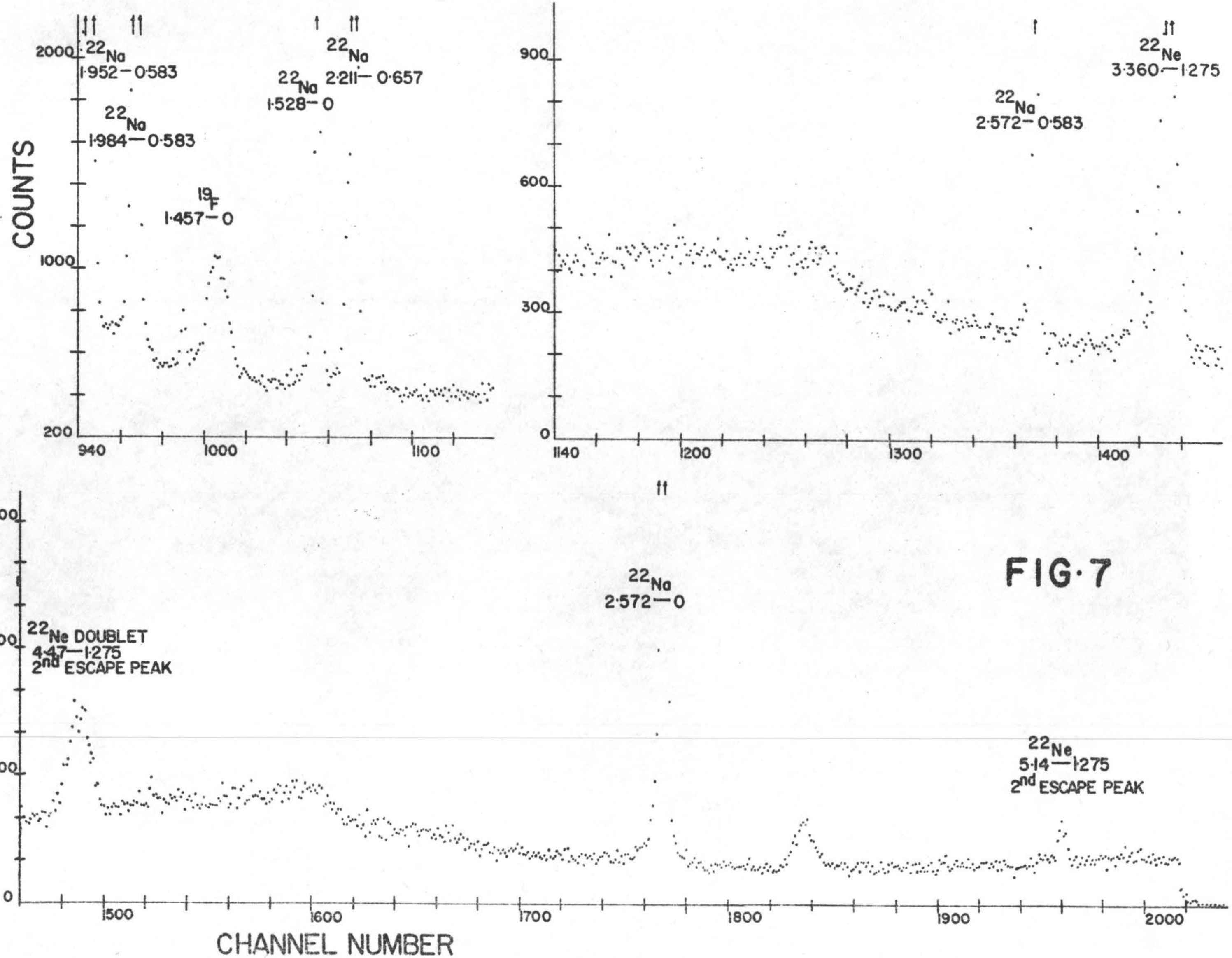
Calculation of a gamma-ray's linear polarization utilizing the polarimeter requires knowledge of the polarimeter's polarization sensitivity at that particular gamma energy. Measurement and substitution of the asymmetry ratio N in equation (11) will only relate the sensitivity to the linear polarization for the observed gamma ray. If however the angular distribution co-efficients of a gamma ray, whose mixing ratio is known, are measured, the linear polarization can be independently determined from equation (9). Pure E2 gamma rays are particularly useful for calibration purposes

$^{22}Na$ Gamma Spectrum from $^{19}F(\alpha,n)^{22}Na$



COUNTS
7000
5000
3000
250
350
450
550
22Na 0·583→0
22Na 0·511 MEV
ANNHILATION
Ge(nn)Ge
0·691 MEV
22Na
1·528→0·891
COUNTS
5000
3000
1000
550
650
750
850
22Na
0·891 →0
22Na
2·969→1·952
22Na
3·059→1·952
19F
1·350→0·110
22Ne
1·275→0
19F
1·458→0·110
1·554→0·197
CHANNEL NUMBER


FIG·6



COUNTS
CHANNEL NUMBER
22Na 1·952—0·583
22Na 1·984—0·583
19F 1·457—0
22Na 1·528—0
22Na 2·211—0·657
22Na 2·572—0·583
22Ne 3·360—1·275
22Na 2·572—0
22Ne DOUBLET 4·47—1·275 2nd ESCAPE PEAK
22Ne 5·14—1·275 2nd ESCAPE PEAK


FIG·7

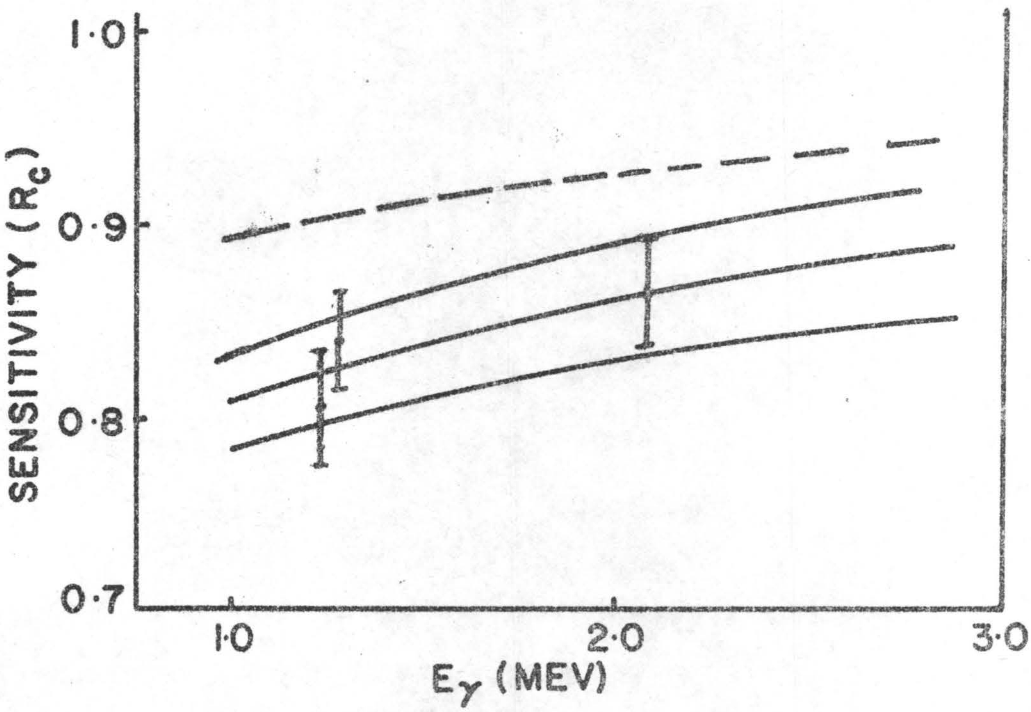
as both equations (9) and (10) reduce to

$$P = \frac{1 \quad + A_2/A_0 \quad + A_4/A_0}{1 \quad - 2\,A_2/A_0 - \tfrac{3}{4}A_4/A_0} \;.... \quad (14)$$

The sensitivity in equation (11) is not only dependent on the gamma-ray energy but also on the gamma-ray's angular distribution and the different angles subtended with respect to the target by the polarimeter in its horizontal and vertical positions. Theoretical calculations of the sensitivity (dotted line in Figure 8) for the polarimeter were done on the CDC 6400 computer, using a program provided by T. Lam, University of Toronto. This program also calculated the effect of the gamma-ray's angular distribution on the sensitivity. Using the 1.235 MeV gamma ray ($^{19}F$: 1.345 MeV $\rightarrow$ 0.110 MeV) whose lifetime, $\tau = 8$ p-sec. (Warburton et al (1967)) requires $\delta\,|(M3/E2)| \leq 10^{-3}$; the 1.275 MeV gamma ray ($^{22}Ne$: 1.275 MeV $(2^+) \rightarrow 0$ MeV $(0^+)$) which is pure E2 and the 2.082 MeV gamma ray ($^{22}Ne$: 3.359 MeV $\rightarrow$ 1.275 MeV) whose lifetime $\tau = 0.40 \pm 0.11$ p-sec. (Warburton et al (1967)) requires $\delta\,|(M3/E2)| \leq 10^{-4}$. The radiation characterizing each gamma ray was considered pure E2 and the polarization was calculated from equation (14). The above restrictions on $\delta\,|(M3/E2)|$ were imposed assuming an M3 transition strength of 1 W.u. and using the single particle estimate tabulated in Skorka et al (1966), which assumes the nuclear radius parameter to be $r_0 = 1.2 \times 10^{-13}$ cms. Using equation (11) the sensitivity was then calculated and corrected to exclude angular distribution effects (see Table I). These values were plotted as a band of possible values to reflect the experimental error in the measured parameters (see Figure 8). With the calibration established, it is then possible to determine from the graph the sensitivity for a particular

Sensitivity Calibration Plot of the Polarimeter

FIG·8




WEIGHTED MEAN VALUES

| $E_\gamma$ | $R_c$ | $\Delta R_c$ |
|---|---|---|
| 1·235 Mev | 0·805 | 0·030 |
| 1·275 Mev | 0·839 | 0·025 |
| 2·082 Mev | 0·860 | 0·030 |

| $E_\gamma$ | $E_\alpha$ | $A_2/A_0$ | $A_4/A_0$ | P | N | $R_c$ |
|---|---|---|---|---|---|---|
| 1.235 Mev | 6.1 Mev(a) | 0.295±.009 | -0.285±.010 | 2.11±.10 | 0.960±.011 | 0.856±.030 |
| | 6.1 Mev(b) | 0.295±.009 | -0.321±.010 | 1.99±.10 | 0.929±.011 | 0.768±.030 |
| | 7.0 Mev | 0.037±.011 | 0.079±.011 | 1.23±.05 | 0.966±.009 | 0.719±.079 |
| 1.275 Mev | 6.1 Mev(a) | 0.209±.008 | -0.086±.007 | 1.86±.06 | 0.951±.013 | 0.833±.039 |
| | 6.1 Mev(b) | 0.203±.009 | -0.093±.009 | 1.80±.07 | 0.958±.015 | 0.848±.048 |
| | 7.0 Mev | 0.240±.015 | -0.172±.016 | 1.89±.13 | 0.955±.014 | 0.841±.043 |
| 2.085 Mev | 6.1 Mev(a) | 0.422±.034 | -0.135±.036 | 6.8 ± 2.7 | 0.885±.016 | 0.823±.026 |
| | 6.1 Mev(b) | 0.400±.049 | -0.144±.055 | 5.3± 2.5 | 0.866±.022 | 0.786±.044 |
| | 7.0 Mev | 0.393±.034 | -0.167±.011 | 4.8± 1.5 | 0.968±.016 | 0.922±.026 |

TABLE I Calibration Of Polarimeter Sensitivity

$R_c$ = Sensitivity Excluding Angular Distribution Effects gamma ray. After adjusting the value of the sensitivity to include angular distribution effects due to this gamma ray the value can be used to calculate the linear polarization of the gamma ray expressed in equation (11) upon determination of the asymmetry ratio N using the polarimeter.

(ii) Evaluation of Polarimeter Measurements

For an observed gamma ray, the linear polarization was measured using the polarimeter. At the same time the independently obtained angular distribution co-efficients enable calculation of the polarization as a function of $\delta$ for either M2/E1 or E2/M1 multipole mixing. Plotting the experimental values of polarization as a crosshatched area on the same graph as a plot of polarization against arctan $\delta$ (shown in Figure 10) results in overlaping areas which restrict the ranges of $\delta$ (M2/E1) or $\delta$ (E2/M1) for the transition. Other experimental work may have also set restrictions on the mixing ratio, or a lifetime measurement may eliminate values of mixing ratio where they imply highly improbable transition strengths. The angular distribution co-efficients and linear polarization measurements experimentally obtained are summarized in Table II.

## 4.3 Analysis

(i) 2.969 Mev Level

The 2.969 Mev level, whose lifetime is 0.06 $\pm$ 0.013 p-sec. (Paul et al (1968)), decays 100% to the 1.952 Mev level. While the parity of the 2.969 Mev level is unknown its spin has been experimentally de-

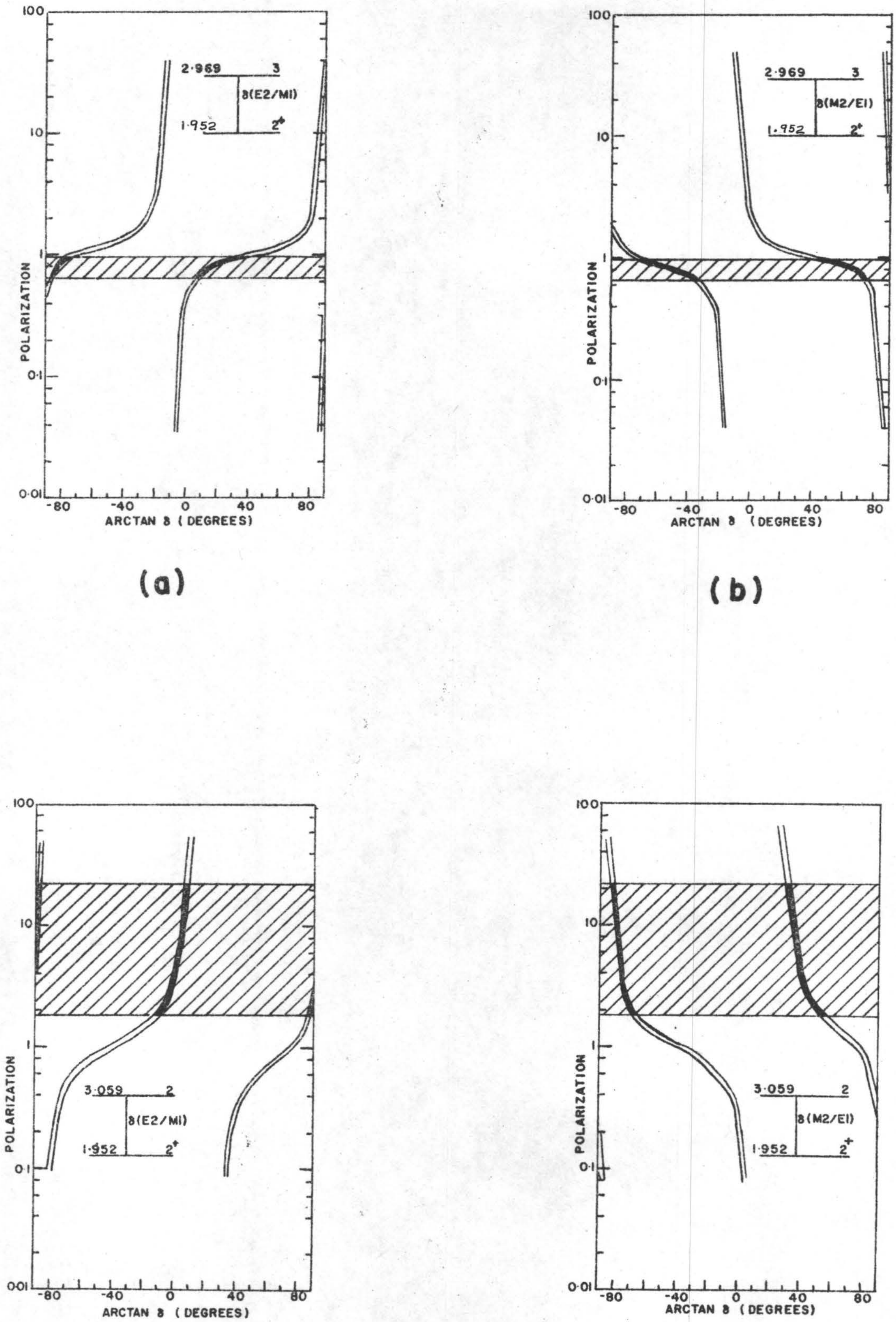
| $E_\gamma$ | $E_\alpha$ | $A_2/A_0$ | $A_4/A_0$ | N | $R_e$ | P |
|---|---|---|---|---|---|---|
| 1.017 Mev | 7.0 Mev | -0.256±.019 | -0.094±.022 | 1.024±.020 | 0.810±.026 | 0.80±0.15 |
| 1.107 Mev | 7.0 Mev | 0.336±.038 | -0.001±.044 | 0.894±.054 | 0.826±.027 | $3.8^{+18.}_{-2.1}$ |
| 1.401 Mev | 6.1 Mev(a) | 0.394±.013 | -0.056±.015 | 0.990±.018 | 0.848±.028 | 1.13±0.25 |
| | 6.1 Mev(b) | 0.365±.013 | -0.043±.015 | 0.984±.018 | 0.848±.028 | 1.22±0.28 |
| | 7.0 Mec | 0.396±.013 | -0.216±.064 | 0.924±.016 | 0.848±.028 | $3.4^{+1.6}_{-0.9}$ |
| 1.989 Mev | 6.1 Mev(a) | -0.369±.027 | 0.00 ±.029 | 1.024±.032 | 0.849±.031 | $0.75^{+0.36}_{-0.25}$ |
| | 6.1 Mev(b) | -0.366±.028 | 0.023±.033 | 1.029±.036 | 0.849±.031 | $0.70^{+0.36}_{-0.28}$ |
| 2.572 Mev | 6.1 Mev(a) | -0.106±.015 | 0.031±.016 | 1.051±.019 | 0.874±.031 | 0.46±0.17 |
| | 6.1 Mev(b) | -0.091±.015 | 0.058±.017 | 1.059±.021 | 0.874±.031 | 0.41±0.18 |

TABLE II Observed Polarization

$R_e$ = Sensitivity including angular distribution effects termined to be 3 and the mixing ratio restricted to $-0.017 \leqslant \delta \leqslant 0.051$ (Warburton et al (1968)). Since the spin and parity of the 1.952 level are known to be $2^+$, the gamma radiation will be a quadrupole/dipole mixture. Because the lower level has positive parity, the radiation will be M2/E1 if the upper level has negative parity or E2/M1 if it has positive parity. Polarimeter measurements determined the polarization (Table II) to be $0.80 \pm 0.15$. This value and values of polarization for $-\infty \leqslant \delta \leqslant \infty$ using the angular distribution data (Table II) are plotted together for both the $\delta$(E2/M1) and $\delta$(M2/E1) cases (see Figure 9). Intersection of the experimental value - cross hatched area - with the calculated band of values is shaded. This shaded area leads to limitations on the mixing ratios for each case. If the radiation is an M2/E1 mixture then the mixing ratio would be limited to the ranges $-2.9 \leqslant \delta \leqslant -0.6$ and $1.0 \leqslant \delta \leqslant 4.7$ while for E2/M1 radiation it would be $-14.5 \leqslant \delta \leqslant -3.3$ and $0.11 \leqslant \delta \leqslant 0.75$. No M2 strength greater than 1 W.u. has yet been observed in s-d shell nuclei (Skorka et al (1966)). Assuming that the M2 strength in this case does not exceed 10 W.u. the measured lifetime implies that $|\delta(M2/E1)| \leqslant 0.01$. Applying this restriction to the results of this work leads to the conclusion that the radiation is not M2/E1. Thus the 2.969 MeV level has positive parity and the decay to the 1.952 MeV level is E2/M1. Comparing the values of $\delta$(E2/M1) obtained in this work with those of Warburton et al (1968) indicates that only the range $0.11 \leqslant \delta \leqslant 0.75$ is applicable and the corresponding minimum E2 strength is 41 W.u. While agreement of the range of values for mixing ratio with the previously mentioned work of Warburton et al (1968) is poor, consistency can be attained if the errors used are extended to two standard deviations. If $\delta = 0.05$ the

Polarization Plots of the 1.017 MeV

and the 1.107 MeV Gamma Rays


100
10
1
0·1
0·01
POLARIZATION
-80
-40
0
40
80
ARCTAN δ (DEGREES)
2·969
3
δ(E2/M1)
1·952
2+
(a)
δ(M2/E1)
(b)
3·059
2
δ(E2/M1)
1·952
2+
δ(M2/E1)

(c) (d)

FIG·9

strength would still be quite large at 9 W.u. From equation (13) the physical limits placed on the statistical tensor, $\rho_{20}/\rho_{00}$ by requiring positive population parameters for the 2.969 MeV level are $4.0 \leq \rho_{20}/\rho_{00}$ 1.7. To check if the mixing ratio for E2/M1 radiation satisfies this condition, equation (6) is used. For the experimentally determined range of $\delta$ (E2/M1), it was found that $0.22 \leq \rho_{20}/\rho_{00} \leq 0.48$. Although the value of $|\delta(E2/M1)|$ implies a rather large E2 strength for a $\Delta T = 1$ transition, the results nevertheless indicate that the assignment of positive parity to the 2.969 MeV level is definite.

(ii) 3.059 MeV Level

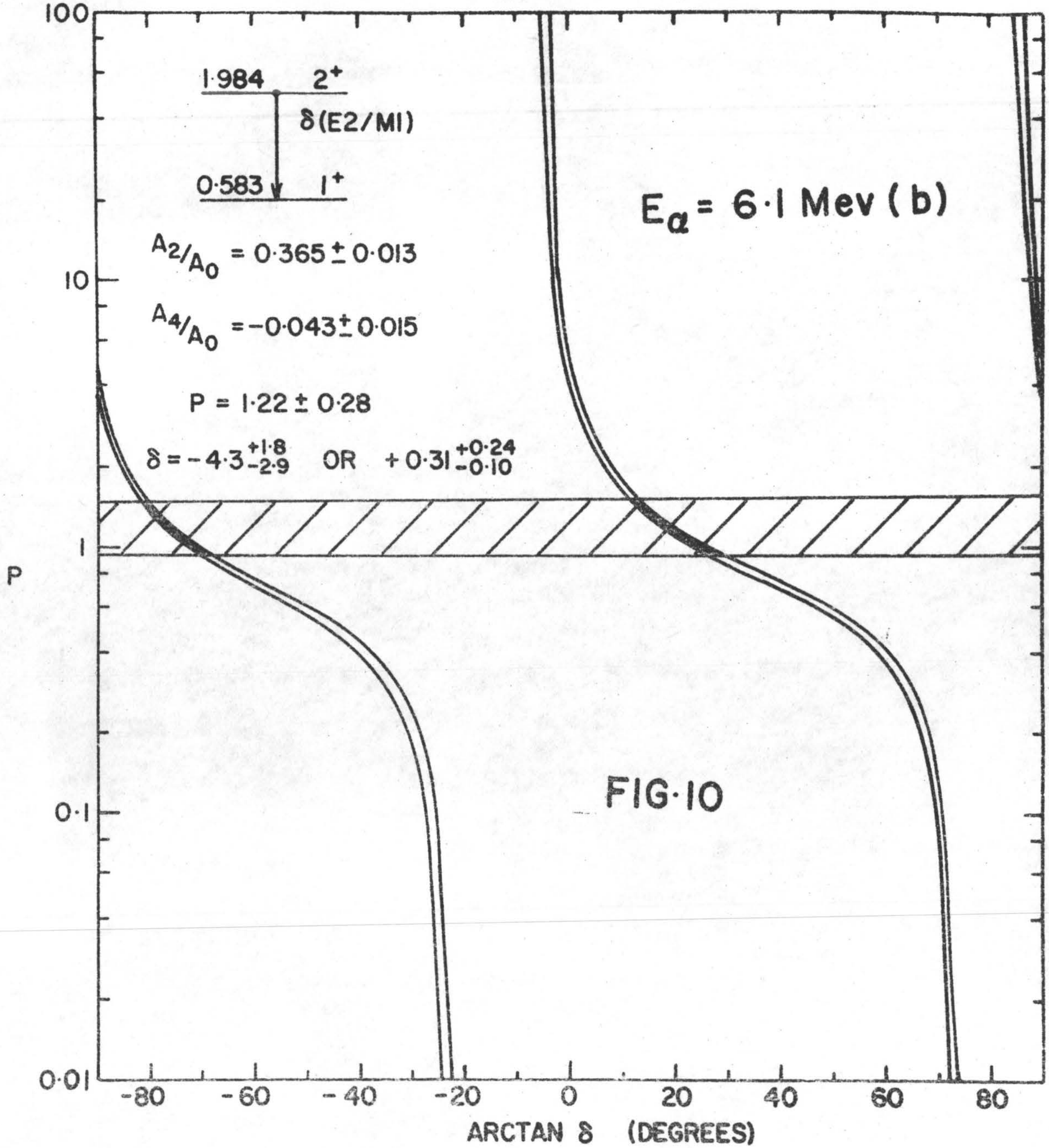
The state at 3.059 MeV de-excites 97% to the state at 1.952 MeV and the remainder goes to the first excited state. While the parity of the level is unknown, the lifetime, $\tau = 0.04 \pm 0.01$ p-sec. (Paul et al (1968)) and the spin J = 2 from angular correlation measurements (Warburton et al (1968)) have been measured. The angular correlation measurements, which determined the spin, also limited the mixing ratio to $-0.20 \leq \delta \leq +0.10$ for the transition to the 1.952 MeV level. Since spin and lifetime of the upper level and spin and parity of the lower level for the 97% branch are unknown, the choice of radiation is limited to a quadrupole/dipole mixture. If the multipole mixing is (M2/E1), for a transition strength of $\leq$ 10 W.u., the lifetime measurement would allow $|\delta(M2/E1)| \leq 0.01$ while an E2 strength $\leq$ 10 W.u. for a E2/M1 mixture restricts $|\delta(E2/M1)| \leq$ 0.06. Comparison of the measured polarization $P = 3.8^{+18.}_{-2.1}$ (Table II) to the band of calculated polarization values, using equation (9), restricts the range of the mixing ratio in the case of $\delta$ (M2/E1) to $-5.2 \leqslant \delta \leqslant -1.9$ and $0.62 \leqslant \delta \leqslant 1.06$ and for E2/M1 ratio to $\delta < -30$, $-0.30 \leqslant \delta \leqslant 0.10$ and $\delta > 11$. To be consistent with the lifetime measurement the only possible value is $-0.30 \leqslant \delta \leqslant 0.10$ (Skorka et al (1966)). To satisfy the physical conditions implied in equation (13), the statistical tensor is limited to the range $-1.2 \leqslant \rho_{20}/\rho_{00} \leqslant 1.2$. If the determined value of $\delta$ (E2/M1) is to be physically allowed, calculation of $\rho_{20}/\rho_{00}$ in equation (6) should fall within this limit. From equation (6) for the above values of $\delta$ (E2/M1), $0.62 \leqslant \rho_{20}/\rho_{00} \leqslant 1.10$ which is within the allowed range and agrees with the angular correlation work of Warburton et al (1968). Since the radiation is E2/M1 the parity of the 2.969 MeV level is positive.

(c) 1.984 MeV Level

The 1.984 MeV level decays 100% to the 0.583 MeV first excited state. Angular correlation measurements on the 1.401 MeV gamma ray have shown the level to be either $2^+$ or $3^+$ (Warburton et al (1967)). The parity assignment was based on a lifetime measurement of $1.74 \pm 0.34$ p-sec. and a limitation of the mixing ratio to $\delta < -0.2$ or $\delta > 5$ for the $2^+$ case.

If $J^{\pi} = 3^+$, the radiation will be essentially pure E2, enabling calculation of the polarization directly from the angular distribution co-efficients in equation (14). This value can then be compared with the polarimeter measurement and they are both tabulated for three independent measurements in Figures 10 and 11. Since for the two runs at 6.13 MeV observed and calculated values of polarization do not agree

Polarization Plot of the 1.401 MeV Gamma Ray



100
10
1
0·1
0·01
P
1·984 2+
δ(E2/M1)
0·583 1+
A2/A0 = 0·365 ± 0·013
A4/A0 = −0·043 ± 0·015
P = 1·22 ± 0·28
δ = −4·3 +1·8 −2·9 OR +0·31 +0·24 −0·10
Eα = 6·1 Mev (b)
-80 -60 -40 -20 0 20 40 60 80
ARCTAN δ (DEGREES)


FIG·10

Summary of Data on 1.401 MeV Gamma Ray

# FIG·II

$^{22}Na$

1.984 MeV ($3^+, 2^+$) 0.583 MeV (1+)

If J = 3, transition is essentially pure E2

POLARIZATION

| | Calculated from a.d. | Measured |
|---|---|---|
| (a) | 5.92 ± 0.69 | 1.13 ± 0.25 |
| (b) | 4.71 ± 0.50 | 1.22 ± 0.28 |
| (c) | 4.5 ± 2.1 | $3.40^{+1.56}_{-0.91}$ |

If J = 2

| Alpha Energy | $A_2/A_0$ | Measured Polarization | $\delta$(E2/M1) | $\rho_{20}/\rho_{00}$ |
|---|---|---|---|---|
| 6.1 MeV (a) | 0.394 ± 0.013 | 1.13 ± 0.25 | $-(3.8^{+1.9}_{-1.4})$ | $-(0.55^{+0.12}_{-0.09})$ |
| 6.1 MeV (b) | 0.365 ± 0.013 | 1.22 ± 0.28 | $-(4.3^{+2.9}_{-1.8})$ | $-(0.52^{+0.16}_{-0.09})$ |
| 7.0 MeV | 0.396 ± 0.053 | $3.4^{+1.56}_{-0.91}$ | -18 → -1000 | -1.0 → -1.3 |

it is concluded the level does not have $J^{\pi} = 3^+$.

For the other possible choice, $J^{\pi} = 2^+$ the emitted radiation from the 1.984 MeV level would be E2/M1. Comparison of the measured polarization to the polarization calculated from the angular distribution co-efficients as a function of mixing ratio (see Figure 10) restricts the range of $\delta$ for each of the three measurements. Comparison with the previously measured values of mixing ratio (Warburton et al (1967)), further restricts the value in each case and the assigned values are tabulated in Figure 11. Again the statistical tensor $\rho_{20}/\rho_{00}$ is restricted to the range $-1.2 \leq \rho_{20}/\rho_{00} \leq 1.2$ (equation (13)) and this is satisfied for the ranges of $\delta$ obtained in each measurement. However it should be noted that the range of $\delta$ obtained at 7.0 MeV $\alpha$ energy does not agree with those obtained at 6.13 MeV and it is necessary to extend the error in the measurement of polarization to two standard deviations to obtain overlap. The weighted mean value of the mixing ratio is $-(4.0^{+1.7}_{-1.2})$, which corresponds to an E2 transition strength of 22 W.u.. While this is a large strength, it is however not unacceptable. It is concluded that the 1.984 MeV level has $J^{\pi} = 2^+$.

(iii) 2.572 MeV Level

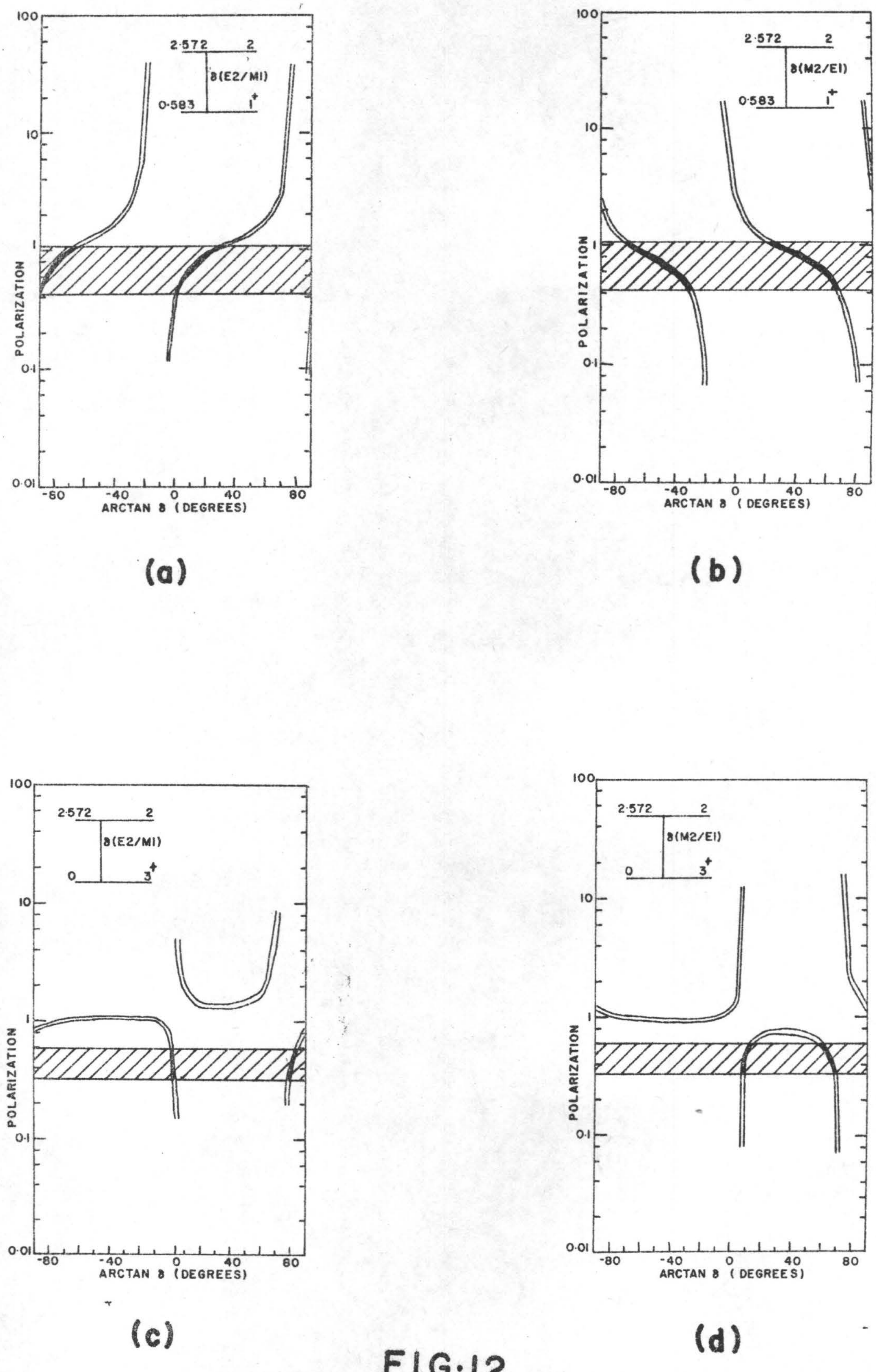
Angular correlation measurements of Poletti et al (1967, a) on the 2.572 MeV level using the $^{20}Ne(^3He,p)^{22}Na$ reaction has narrowed the choice of spins to J = 1 or 2 while at the same time restricting the values of $\delta$ to those tabulated below. Paul et al (1968) measured the life time as $\tau = 5.7 \pm 0.9$ p-sec.

| Initial Level (MeV) | Final Level (MeV) | $J_i$ | $J_f$ | Restrictions on $\delta$ |
|---|---|---|---|---|
| 2.572 | 0 | 2 | $3^+$ | $0.13 \pm 0.10$ OR $3.5 \pm 1.0$ |
| | 0.583 | 2 | $1^+$ | $0.03 \pm 0.15$ OR $2.8 \pm 0.7$ |
| | 0 | 1 | $3^+$ | No restriction |
| | 0.583 | 1 | $1^+$ | 4 OR $0.14 \pm 0.11$ |

The state decays $81 \pm 3\%$ to the ground state and 19% to the first excited state. In their work it was decided the state was not $1^-$ as this would require a large M2 strength ($7.1 \pm 1.1$ W.u.) for the ground state transition, which is expected to be reduced by the isospin selection rule (Preston (1965)). The pickup reactions $^{22}Na(P,d)^{22}Na$ and $^{23}Na(d,t)^{22}Na$ forming this level (Wei et al (1968) and Haight (1969)) determined the level to have negative parity, since D.W.B.A. analysis indicates transfer of a p-wave nucleon in populating this level. This result eliminates all choices other than $J^{\pi} = 2^-$, for which angular distribution measurements establish the only possible values of the mixing ratio for the two decays to be the smaller values in the above table.

If one uses these values of $\delta$ and looks at the polarization plot for $\delta$ (M2/E1) (Figure 12), there is not agreement between the polarization calculated from the angular distribution data and the measured polarization. It would appear that perhaps the parity of the 2.572 MeV level is not negative. If the parity were instead positive, both $J^{\pi} = 1^+$ or $2^+$ are possible (Poletti et al (1967, a)). If $J^{\pi} = 1^+$, then the radiation to the ground state would be essentially E2 as $|\delta\ (M3/E2)| \lesssim 10^{-3}$

Polarization Plot of Gamma Ray's from the 2.572 MeV Level


100
10
1
0·1
0·01
POLARIZATION
-80
-40
0
40
80
ARCTAN δ (DEGREES)
2·572 2
δ(E2/M1)
0·583 1+
(a)
2·572 2
δ(M2/E1)
0·583 1+
(b)
2·572 2
δ(E2/M1)
0 3+
60
(c)
2·572 2
δ(M2/E1)
0 3+
(d)

FIG·12

from the measured life time. Taking the mean values of the two 6.1 MeV runs, $A_2/A_0 = -0.098 \pm 0.011$ and $A_4/A_0 = 0.045 \pm 0.012$, the corresponding polarization from equation (14) is $0.80 \pm 0.03$ while the mean value of $N = 1.055 \pm 0.03$ corresponds to a measured polarization of $P = 0.43 \pm 0.15$ (equation (11)). The lack of agreement within two standard deviations, indicates the 2.572 level is not $1^+$. The only remaining possibility if the parity is positive is $J^{\pi} = 2^+$. Calculation of the polarization for the measured angular distribution co-efficients for both of the emitted gamma rays as a function of $\delta$ (E2/M1) is shown as a band of possible values to include experimental errors in Figure 12. The crosshatched areas are the measured values of polarization and the area of intersection places limits on the range of $\delta$ (E2/M1) for each gamma ray. These ranges were each checked for agreement with the physical restriction on the statistical tensor (equation (13)). A further physical check can be made by eliminating the statistical tensor $\rho_{20}/\rho_{00}$ in equation (6) for both the 1.989 MeV and the 2.572 MeV gamma rays since they originate from the same level, and checking that allowed values of $\delta$ (E2/M1) for the 1.989 MeV gamma ray corresponded to an allowed value of $\delta$ (E2/M1) for the 2.572 MeV gamma ray through this mathematical relationship. Imposition of all the previous restrictions and checks on the mixing ratio determined $5.2 \leq \delta(E2/M1) \leq 9.2$ for the 2.572 MeV gamma ray and $0.05 \leq \delta(E2/M1) \leq 0.23$ for the 1.989 MeV gamma ray. Both of these values agree with ranges of mixing ratio assigned by the angular correlation work of Poletti et al (1967, a). These mixing ratios imply E2 transition strengths of 0.0007 W.u. and 0.28 W.u. for the decay of the 2.572 MeV level to the first excited state and ground state respectively. In addition a further solution, with E2/M1 mixing ratios for both gamma rays approximately zero can be obtained if the errors in each of the polarization measurements are extended to twice the uncertainty given in Table II. This solution would also be consistent with Poletti et al (1967, a) but it can be regarded as relatively unlikely compared to the other solution given above.

In conclusion, because of the inconsistency between this work and Paul et al (1968), the previous assignment of $2^-$ to the 2.572 MeV level is regarded as doubtful. An alternative solution is $2^+$ and the polarization results are then in agreement with the angular correlation work and lifetime measurements. However the conclusion from the pickup reaction studies of Wei (1968) and Haight (1969) would then be wrong.

# CHAPTER 5

# CONCLUSION

Previous knowledge of lifetimes, parities, and restrictions on possible values of spin and mixing ratios of gamma decay branches of the low lying levels of $^{22}Na$, have enabled a partial description of the observed nuclear structure in terms of a number of rotational bands (Olness et al (1969)). They have proposed a $K\pi = 3^+$ band based on the $3^+$ ground state and including the excited states at 0.891 MeV ($4^+$), 1.528 MeV ($5^+$) and a tentatively assigned $6^+$ level at 3.71 MeV. Based on the first excited state at 0.583 MeV a $K\pi = 1^+$ band was proposed which included the levels at 1.984 MeV (tentatively assigned $3^+$) and at 4.71 MeV (tentatively assigned $5^+$). In addition they also proposed a $K\pi = 0^+$, $T = 1$ band with levels at 0.657 MeV ($0^+$), 1.952 MeV ($2^+$) and a tentatively assigned $4^+$ level at 4.077 MeV along with a $K\pi = 1^-$ band composed of the $1^-$ level at 2.211 MeV and the $2^-$ level at 2.572 MeV.

The linear polarization measurements in this thesis were made on gamma rays emitted from the low lying levels directly included in this description or associated with these bands. They helped to resolve ambiguities in spin and parity assignments as well as determining a value of the mixing ratio for each transition. These measurements shed new light on the nuclear structure; in particular, they raise problems in the proposed description.

Two tentative assignments of the Brookhaven group which are shown to be incorrect by this work are negative parity for the 2.572 MeV state and spin 3 for the 1.984 MeV state instead of positive parity and spin 2 as required by the polarization work. While these new results eliminate two of the proposed rotational bands, there is still strong evidence for collective effects in $^{22}$Na as evidenced by the number of strong E2 transitions (Warburton et al (1967)).

Additional implications arise from this work due to an enhanced E2 strength ($\sim$40 W.u.) between the $3^+$ state at 2.969 MeV and the $2^+$, T = 1 state at 1.952 MeV. Because of its energy the 2.969 MeV state is expected to have T = 0, as there is no corresponding analog state in $^{22}$Ne. This means the above mentioned E2 transition is between states of different T and is very difficult to explain in the context of the proposed bands. A similar situation exists connected with the decay of the proposed $4^+$, T = 1 state at 4.077 MeV (Olness et al (1969)). This state apparently decays nearly 100% to the 1.984 MeV state by an E2 transition in preference to decaying by an enhanced E2 within the T = 1 band to the 1.952 MeV state. Further complications arise from the measured E2 strength of 22 W.u. in the decay of the 1.984 MeV state to the 0.583 MeV state. It would seem that several strongly enhanced E2 strengths are present but that the interpretation in terms of rotational band structure is not clear. A plausible solution might be the existence of an unresolved doublet at either 1.952 MeV or 1.984 MeV.

The linear polarization measurements, while raising these problems, cannot resolve them. Thus, further investigation is required to consider questions such as the possibility of an unresolved level or additional cascading.

APPENDIX I

Derviation of the Expression for Polarization for M3/E2 Radiation at $\theta = 90^\circ$.

Using equation (8) three terms in the numerator must be evaluated. Using

$$P_0(\cos 90^\circ) = 1 \qquad P(\cos 90^\circ) = -0.5 \qquad P_4(\cos 90^\circ) = .375$$

$$P_0^2(\cos 90^\circ) = 0 \qquad P_2^2(\cos 90^\circ) = 3.0 \qquad P_4^2(\cos 90^\circ) = -7.50$$

| | | $k=2$ | $k=4$ |
|---|---|---|---|
| for $L = 2,\ L' = 2,\ (\pm)_{L'} = +$ | $\mathcal{K}_k(22)$ | $= 1/2$ | $-1/12$ |
| $L = 2,\ L' = 3,\ (\pm)_{L'} = -$ | $\mathcal{K}_k(23)$ | $= -1/4$ | $-1/60$ |
| $L = 3,\ L' = 3,\ (\pm)_{L'} = -$ | $\mathcal{K}_k(33)$ | $= 1/3$ | $1/3$ |

and $\qquad \bar{Z}_1(LJLJI0) = (-)^{I-J}(2J+1)^{\frac{1}{2}}\ \delta_{LL'}$

the three numerator terms are

(i) $$(-)^{I-J}\Big[\rho_{00}\,\bar{Z}_1(2J2JI0)\,(1+0) + \rho_{20}\,\bar{Z}_1(2J2JI2)\,(-\tfrac{1}{2}+3/2) + \rho_{40}\,\bar{Z}_1(2J2JI4)\,(0.375 + 1/12 \times 15/2)\Big]$$

$$= (-)^{I-J}\Big[\rho_{00}\,\bar{Z}_1(2J2JI0) + \rho_{20}\,\bar{Z}_1(2J2JI2) + \rho_{40}\,\bar{Z}_1(2J2JI4)\Big]$$

(ii) $$-2\,\delta\,(-)^{I-J}\Big[\rho_{00}\,\bar{Z}_1(2J3JI0)\,(1-0) + \rho_{20}\,\bar{Z}_1(2J3JI2)\,(-\tfrac{1}{2}-(-\tfrac{1}{4})^3)\ \rho_{40}\ Z\,(2J3JI4)\,(.375 - (-1/60)\,(-7.5))\Big]$$

$$= -2\,\delta\,(-)^{I-J}\Big[\rho_{00}\,\bar{Z}_1(2J3JI0) + \tfrac{1}{4}\,\rho_{20}\,\bar{Z}_1(2J3JI2) + \tfrac{1}{4}\,\rho_{40}\,\bar{Z}_1(2J3JI4)\Big]$$

(iii) $$\delta^2\,(-)^{I-J}\Big[\rho_{00}\,\bar{Z}_1(3J3JI0)\,(1-0) + \rho_{20}\,\bar{Z}_1(3J3JI2)\,(-\tfrac{1}{2}-1/3(3)) + \rho_{40}\,\bar{Z}_1(3J3JI4)\,(.375 - (1/3)\,(-7.5))\Big]$$

$$= (-)^{I-J} \delta^2 \left[\rho_{00} \bar{Z}_1(3J3JI0) - 3/2\, \rho_{20} \bar{Z}_1(3J3JI2) + 23/8\, \rho_{40} \bar{Z}_1(3J3JI4)\right]$$

Omiting the term $(-)^{I-J}$, the numerator is

$$(-)^{I-J} \rho_{00} (1 + \delta^2)(2J+1)^{\frac{1}{2}} + \rho_{20}\left[\bar{Z}_1(2J2JI2) - \tfrac{1}{2}\delta \bar{Z}_1(2J3JI2) - 3/2\, \delta^2 \bar{Z}_1(3J3JI2)\right] + \rho_{40} \left[\bar{Z}_1(2J2JI4) - \tfrac{1}{2}\delta\bar{Z}_1(2J3JI4) + 23/8\, \delta^2 \bar{Z}_1(3J3JI4)\right]$$

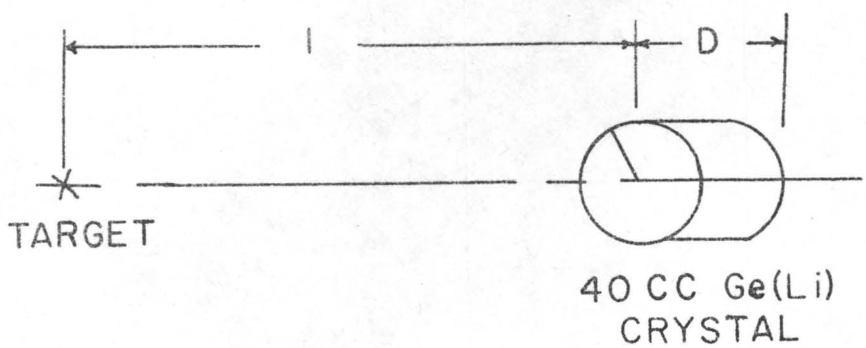
A similar expression for the denominator can be derived which leads to the linear polarization expression at $\theta = 90^\circ$ for M3/E2 radiation.

$$P(90^\circ) = \left\{(-)^{I-J} (1+\delta^2)(2J+1)^{\frac{1}{2}} + \rho_{20}/\rho_{00} \left[\bar{Z}_1(2J2JI2) - \tfrac{1}{2}\bar{Z}_1(2J3JI2) - 3/2\, \delta^2 \bar{Z}_1(3J3JI2)\right] + \rho_{40}/\rho_{00} \left[\bar{Z}_1(2J2JI4) - \tfrac{1}{2}\delta \bar{Z}_1(2J3JI4) + 23/8\,\delta^2 \bar{Z}_1(3J3JI4)\right]\right\}$$
$$\left\{(-)^{I-J} (1+\delta^2)(2J+1)^{\frac{1}{2}} - \rho_{20}/\rho_{00}\left[2\bar{Z}_1(2J2JI2) - 5/2\,\delta \bar{Z}_1(2J3JI2) - \tfrac{1}{2}\delta^2 \bar{Z}_1(3J3JI2)\right] - \rho_{40}/\rho_{00}\left[\tfrac{1}{4}\bar{Z}_1(2J2JI4) + \delta \bar{Z}_1(2J3JI4) + 17/8\,\delta^2 \bar{Z}_1(3J3JI4)\right]\right\}^{-1}$$

## APPENDIX II

### Attenuation Co-efficients of 40 c.c. Ge Detector

The attenuation co-efficients, $Q_k$ due to solid angle effects, can be calculated from the equation below Smith (1962) . This equation is applicable throughout the range of energies wherein the absorption length of the gamma ray is not small compared with the dimensions of the detector.




$$Q_k = \frac{P_{k-1}(\cos\alpha) - \cos \quad P_k(\cos\alpha)}{(k + 1) \quad (1 - \cos\alpha)}$$

where $\tan\alpha = r/(l + D/2)$

$l$ = distance of crystal to target ($10 \pm 0.5$ cms)

$r$ = radius of crystal ($1.98 \pm 0.05$ cms)

$D$ = crystal depth ($3.63 \pm 0.05$ cms.)

For the 40 c.c. GE (Li) crystal and the geometry of the experimental set-up, $Q_2 = 0.980$ & $0.005$ & $Q_4 = 0.933 \pm 0.005$ neglecting the dead region in the center of the detector (approximately 1 cm diameter).

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