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PREDICTIVE STOCHASTIC FEEDFORWARD-FEEDBACK CONTROL OF A HEAT EXCHANGER-STIRRED TANK SYSTEM

PREDICTIVE STOCHASTIC FEEDFORWARD-FEEDBACK CONTROL OF A HEAT EXCHANGER-STIRRED TANK SYSTEM

by

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ABSTRACT

An optimal stochastic feedforward-feedback control scheme is implemented on a heat exchanger-stirred tank system using an on-line minicomputer. Because variations in the measured disturbance variable have an effect on the output controlled variable before compensating action can become effective, the feedforward action must be predictive in nature. Statistical time series models are used to model both the measured disturbance and the unobserved disturbances in the system. These stochastic disturbance models and the transfer function models for the process are identified, fitted and checked using statistical model building procedures on a set of data collected on-line using the minicomputer. The predictive feedforward-feedback controller is derived from these models. The performance of the control scheme is compared with that of a pure feedback control scheme and the actual performances are shown to conform well to the theory.

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To Eileen

CHAPTER 1

INTRODUCTION

1.1 Feedback Control

Traditional feedback control is often enough to maintain a process variable close to a desired value (Fig. 1). The "controlled" variable is measured and compared to the desired value or "setpoint". The difference is used to calculate a deviation in another "manipulated" variable which through the process will affect the controlled variable and may bring it closer to the setpoint. The calculation may be simple or complex. In proportional control action, the deviaation in manipulated variable is a constant times the deviation in controlled variable. Better control can usually be obtained by adding a constant multiple of the rate of change of deviation and/or a constant multiple of the integral of deviation with respect to time. This is called Proportional-Integral-Derivative (PID) action. PID control can be executed continuously using an analog controller, which relies on physical components to measure the value, the integral and the derivative of the deviation, and to add together predetermined constant multiples of the three actions to obtain a continuously changing manipulated variable. The constants used are unique for each process, and methods of determining their best values have been closely studied.

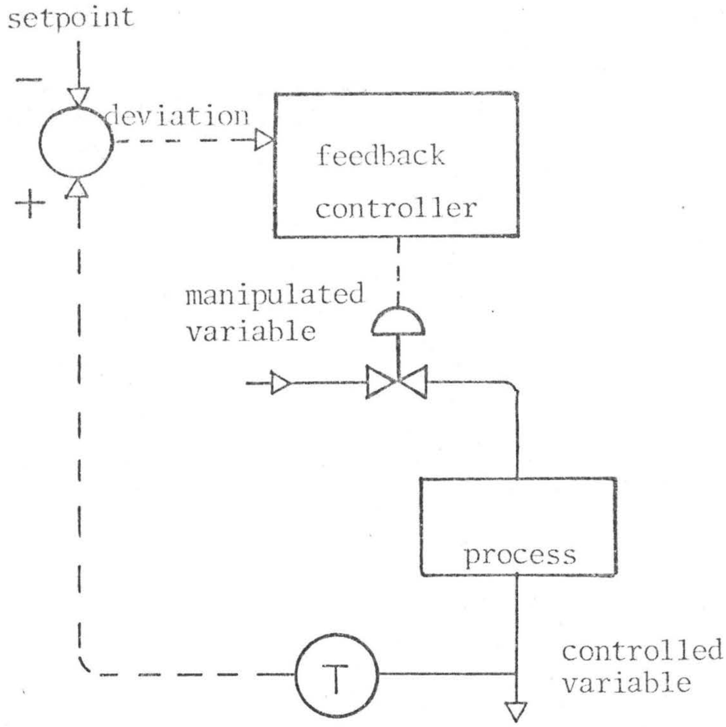


Fig. 1 Feedback Control

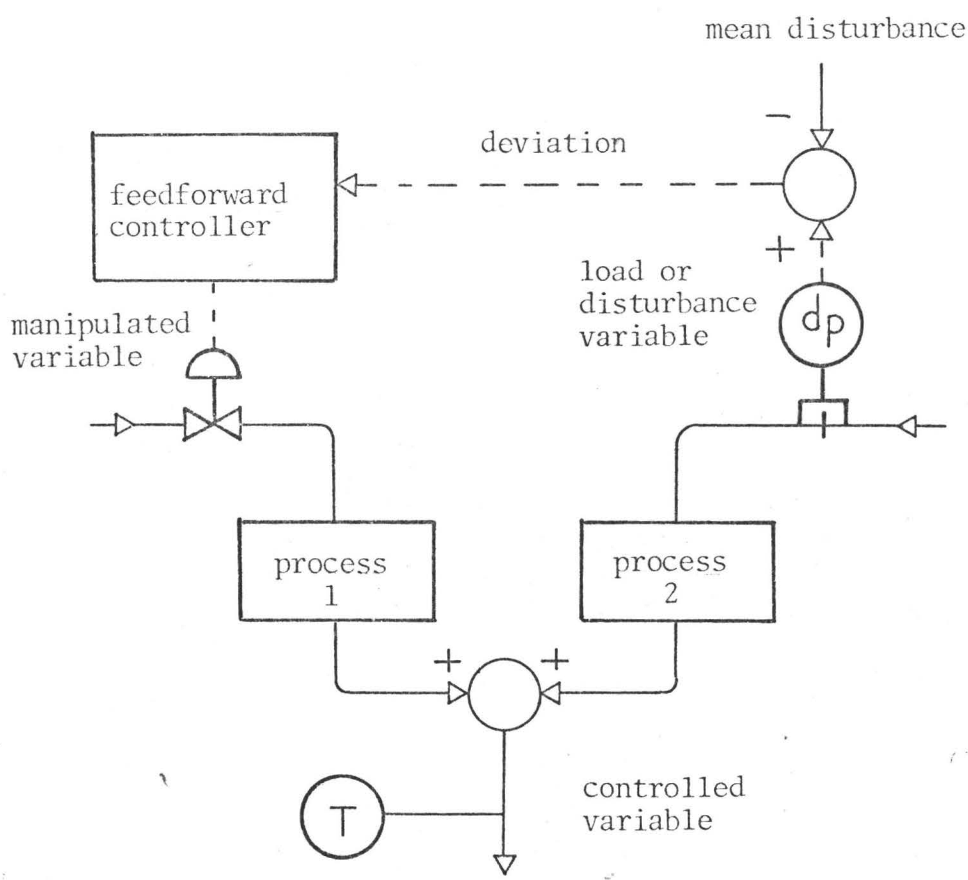


Fig. 2 Feedforward Control

Sometimes, a digital computer is preferable to an analog controller, for example, if

(1) the controlled variable has to be measured at discrete time intervals, or the manipulated variable is changeable only at discrete time intervals; or

(2) a time delay is necessary before changing the manipulated variable; or

(3) the calculation of deviation in manipulated variable includes mathematics that cannot easily be reproduced by physical elements.

1.2 Feedforward Control

In the process, there may be a "load" or "disturbance" variable whose variations are known to cause undesirable variations in the controlled variable (Fig. 2).

The best way to prevent this is to control the disturbance variable close to a constant value, so that variations in the controlled variable are negligible. However, this is not always possible, e.g., when the previous process leading to the disturbance variable is too complex or when interference is not permissible. Instead, feedforward control may be used. This relies on a knowledge of the effect of the disturbance variable on the controlled variable, and of the effect of the manipulated variable on the controlled variable. The disturbance is measured and compared to its mean value, and a deviation in manipulated variable is calculated such that its effect on the controlled variable exactly cancels out the effect of the disturbance on the controlled variable.

Feedforward control may or may not require forecasting. If the effect of the manipulated variable is seen in the controlled variable before the effect of the disturbance is seen in the controlled variable, then action can be calculated which will exactly cancel the effect of the present or previous values of the disturbance. However, if the effect of the disturbance will reach the controlled variable first, then when action is taken, it has to cancel the forecasted effects on the controlled variable of future values of the disturbance variable.

Simple feedforward control without forecasting can be accomplished with analog equipment as in ratio control, and simple forecasting can be done by analog filters. For more complex schemes, a digital computer may be required.

1.3 Feedforward-Feedback Control

If the disturbance which is compensated for by feedforward control is the only disturbance to the process, then theoretically the controlled variable should remain constant at the desired value. However, this almost never is the case: there are always other unknown or poorly identified disturbances which will cause deviation of the controlled variable. This can be corrected by feedback control, and so usually feedforward control is used in conjunction with feedback control (Fig. 3).

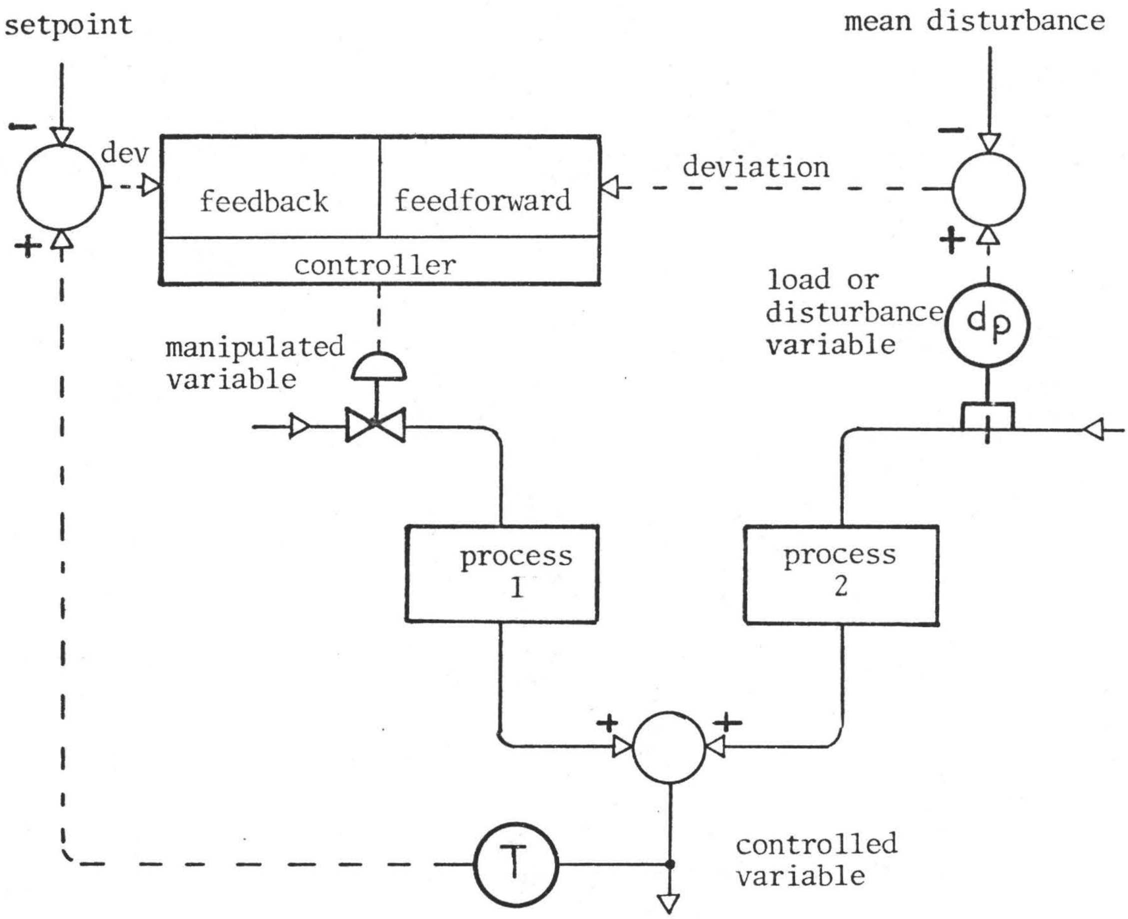


Fig. 3 Feedforward-Feedback Control

1.4 Process Models

It is not necesary to have models describing the process when using analog equipment: empirical methods have been derived to obtain the best possible values of controller parameters and ratios for a particular process; this is called "tuning". However, it is often helpful, and in other schemes, essential, to have process models. These may be derived theoretically from equations which describe physical or chemical changes in the process, or experimentally from observations of process output when it is subjected to different inputs.

Experimental observations may be analysed by the methods of frequency response analysis, based on classical control theory, or time series analysis based on statistical theory. If the experimental data is good and truly representative of a process whose characteristics will not change, then the model obtained is final, and a controller which is designed from it needs no further tuning.

1.5 Time Series Analysis

Box and Jenkins [1] have developed a method of obtaining simple models to describe single series of observations spaced at equal time intervals, and also to relate such a series to one or more other series.

For single series, the main tool is the autocorrelation between any observation and another taken a certain number of intervals later. The pattern of autocorrelations at different lags, called the autocorrelation function, often falls into one of a small number of characteristic patterns, each of which can be represented mathematically by a simple model. To relate one series to another, the cross correlation between an observation in one series and an observation in the other series taken a certain number of intervals later is calculated. The pattern of crosscorrelations at different lags again often resembles a common pattern which can be represented mathematically by a simple model. Regression analysis is used to find the best model parameters to fit the given data.

To obtain good estimates of parameters, a fairly large number of observations are often needed (e.g., one or two hundred). For processes having long time constants (> 1 minute) analog equipment such as high speed recording charts, or even readings taken manually, can be used, but for time constants of the order of seconds, a digital computer becomes preferable. If it is desired to use an artificial input to the process, then the computer can be readily programmed to calculate a sequence for input.

Box and Jenkins have extended their methods to the design of controllers. Once models have been chosen and the best parameter estimates obtained to describe the process, these models can be used to relate the controlled, manipulated and disturbance variables in a control algorithm. A feedback controller can be designed from two models: one to describe the effect of the manipulated variable on the controlled variable, and the other to describe the behaviour of the "noise"

n_t in the controlled variable. This was done in practice by Huynh [2] who compared conventional PID and Time Series feedback control algorithms experimentally on two stirred tanks in series. A simple feedforward-feedback controller needs three models: one to relate the manipulated variable to the controlled variable; one to relate the disturbance variable to the controlled variable; and one to describe the behaviour of the noise. In feedforward-feedback control, if forecasting of future values of the disturbance is required, then a fourth model is necessary to describe the behaviour of the disturbance itself. The theory of feedforward-feedback control with forecasting of the disturbance has been derived by MacGregor [3], but has not previously been tested experimentally.

1.6 Objectives

No new theory is presented in this work, which is an application study of the Box-Jenkins-MacGregor theory. The aims of the study are:

(1) Using a digital minicomputer to collect data and identify models to relate controlled, manipulated and disturbance variables in a physical process;

(2) with these models to derive feedback and feedforward-feedback control algorithms, using the Box-Jenkins-MacGregor theory;

(3) to implement the algorithms by minicomputer to control the process.

1.7 Experimental

The apparatus used was a constant volume stirred tank with two incoming water streams at different temperatures and an overflow to drain (Fig. 6). Tank temperature was the controlled variable. One input stream was cold water whose flowrate varied and could be measured by an orifice meter: this reading was the disturbance variable. The other stream was a constant flowrate of hot water of variable temperature, produced by passing cold water through 2 small heat exchangers in series. The steam supply was in parallel to both exchangers from a single steam control valve, whose setting was the manipulated variable. The scheme was to control the tank temperature by compensating for the effect of disturbances in cold water flowrate by manipulation of the steam control valve and hence hot water temperature. Readings and control action were taken regularly by minicomputer through an interface.

A disturbance in cold water flowrate affected the tank temperature quickly (detectable in less than 10 seconds). A change in steam valve position would appear much later; the dynamics of the valve, steam pressure build-up, establishment of new temperature profiles in the exchangers, a hold-up between exchangers, and pipe length from the second exchanger to the stirred tank all contributed to a total of approximately 40 seconds lag. Thus in feedforward control a manipulation of the steam valve had to be calculated to compensate for the effect of a disturbance in cold water flowrate that would occur 30-40 seconds later. To predict it so far ahead, the disturbance had to be describable by a mathematical model, and the accuracy of control would depend on the accuracy of this model.

Since disturbances in the cold water supply available in the laboratory were insignificant, it was necessary to simulate the disturbance by means of a control valve in the cold water line. A simple model was designed and programmed into the minicomputer for transmission of a signal to the water control valve every sampling interval. Knowledge of this model was not used in the overall scheme, except for the assumption of its constancy. The disturbance variable was the reading of the in-line orifice meter.

A feedback control scheme was also designed to compare the effectiveness of its control action when the water control valve was subjected to the same disturbance signals but when no use was made in the control algorithm of orifice meter readings.

CHAPTER 2

THEORY: A SUMMARY

2.1 Single Series

A time series is a series of observations z_t taken at a fixed time interval. It is possible to describe a single observation z_t in terms of a "white noise sequence" or a finite or infinite series of random "shocks" a_t where a_t is a random number drawn from a normal distribution.

$$z_t = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} \ldots\ldots = (1 + \psi_1 B + \psi_2 B^2 \ldots\ldots)a_t = \psi(B) a_t$$

where B is the backward shift operator $(Ba_t = a_{t-1})$. a_t is said to be transformed to z_t by the linear filter $\psi(B)$ which is a basic model for the series. However, it is a cumbersome model and it is more convenient to express the series in one of two basic forms:

Autoregressive Process:

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} \ldots\ldots + \phi_p z_{t-p} + a_t; \ \phi(B) z_t = a_t$$

Moving Average Process:

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots\ldots - \theta_q a_{t-q}; \ z_t = \theta(B) a_t$$

or a combination of the two forms, where p and q are small numbers. Sometimes it is desirable to take the first difference of z_t and express the new variable

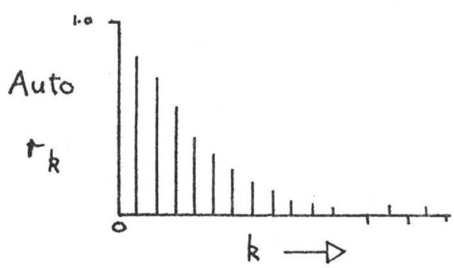
$$(z_t - z_{t-1}) = \nabla z_t = w_t$$

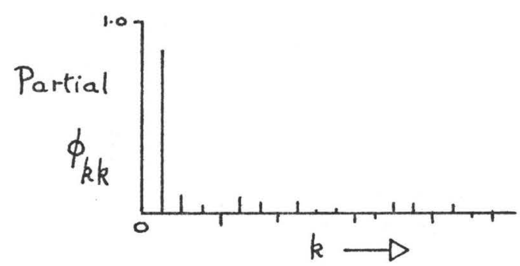
in one or another of the above forms. Again sometimes it is necessary to take the second difference of z_t: $\nabla\nabla z_t = w_t$.

To find which form to use the autocorrelation function and partial autocorrelation function are needed. The autocorrelation coefficient at lag k is between any observation and the observation made k intervals previously, and is estimated by

$$r_k = \frac{c_k}{c_o} \quad \text{where } c_k = \frac{1}{N}\sum_{t=1}^{N-k}(w_t - \bar{w})(w_{t+k} - \bar{w}) \quad k=0,1,2\ldots$$

The pattern of autocorrelation coefficients is called the autocorrelation function. The partial autocorrelation coefficient at lag k is the residual autocorrelation at lag k after the effect of the autocorrelations up to lag (k-1) has been removed. The following autocorrelation and partial autocorrelation functions would be expected from an AR(1) process:





By merely looking closely at the forms of the auto-correlation functions and partial autocorrelation functions of the series and of its first and second differences, it is possible to identify a model together with rough estimates of its parameters in one of the basic forms above or a combination called generally an Auto Regressive Integrated Moving Average (ARIMA) model.

$$\phi(B)\, w_t = \theta(B)\, a_t$$

The best estimates of the parameters of the tentatively identified model can be obtained by a nonlinear least squares procedure. The residuals from such a fit should appear as a white noise sequence and diagnostic checking procedures are based on testing for this. On the assumption of an adequate model the calculated residual autocorrelations and partial autocorrelations can be compared with 95% probability limits and any inadequacies pinpointed.

2.2 Transfer Functions

Given two series of observations, taken simultaneously at equal time intervals, statistical methods may be used to determine the relationship, or transfer function, between them. First, an ARIMA model is found to describe one series (the input). Then both series are "prewhitened" or multiplied by the inverse of this model to obtain two new series: one is a white noise sequence, and the second is the transformed output. Let the relationship before prewhitening be

$$y_t = v_o x_t + v_1 x_{t-1} + v_2 x_{t-2} \cdots + n_t$$

where n_t is a component of y_t that is not correlated with x_t called the noise component. Then after prewhitening it will be

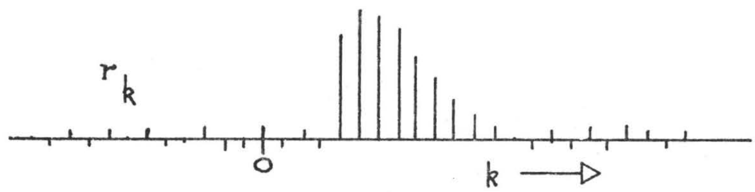
$$\beta_t = v_o \alpha_t + v_1 \alpha_{t-1} + v_2 \alpha_{t-2} \cdots + \gamma_t$$

$$= v(B)\, \alpha_t + \gamma_t$$

Estimates of the v-weights can then be obtained by calculating the cross-correlations r_k between the α_t and β_t series. Specifically [Ref.1 p.380]:

$$\hat{v}_k = r_k \frac{s_\beta}{s_\alpha}$$

where s_α and s_β are the standard deviations of the series. The crosscorrelation pattern for $-20 < k < 20$ might look thus:



It is possible to represent a transfer function using these v-weights but if there are many it is cumbersome. Box and Jenkins pointed out that a general model of the form

$$y_t - \delta_1 y_{t-1} - \delta_2 y_{t-2} \ldots \delta_r y_{t-r} = \omega_o x_{t-b} - \omega_1 x_{t-b-1}$$

$$- \omega_2 x_{t-b-2} \ldots - \omega_s x_{t-b-s}$$

where r, s are small numbers,

$$\text{or} \quad y_t = \frac{(\omega_o - \omega_1 B - \omega_2 B^2 \ldots)}{1 - \delta_1 B - \delta_2 B^2 \ldots} x_{t-b} = \frac{\omega(B)}{\delta(B)} B^b x_t$$

is more parsimonious of parameters. They have drawn the cross-correlation patterns for a number of models of the above form for different values of r and s. So having calculated the experimental crosscorrelation pattern, the model giving the closest pattern would be identified as the best to describe the physical process. Box and Jenkins [p.347] explain how to calculate rough estimates of the model parameters from the calculated v-weights.

At this stage noise is considered: there will usually be a component of the output series that is not correlated with the input, but may be autocorrelated: this is referred to as Noise. This component is calculated from

$$n_t = y_t - v(B)\, x_t$$

and is then analysed as described above for a single series. Auto and partial autocorrelations at different values of k are calculated for the n_t sequence, and the resulting patterns (functions) are viewed to identify the best ARIMA model to describe the noise together with rough estimates of its parameters. Again it may be necessary to analyse the series derived from the first or second difference of n_t.

Having obtained the forms of the transfer function model and noise model and rough estimates of their parameters, a regression routine is used to estimate all the parameters together by minimising the sum of squares of the residuals a_t which may be calculated recursively:

$$n_t = y_t - \frac{\omega(B)}{\delta(B)}\, x_{t-b}$$

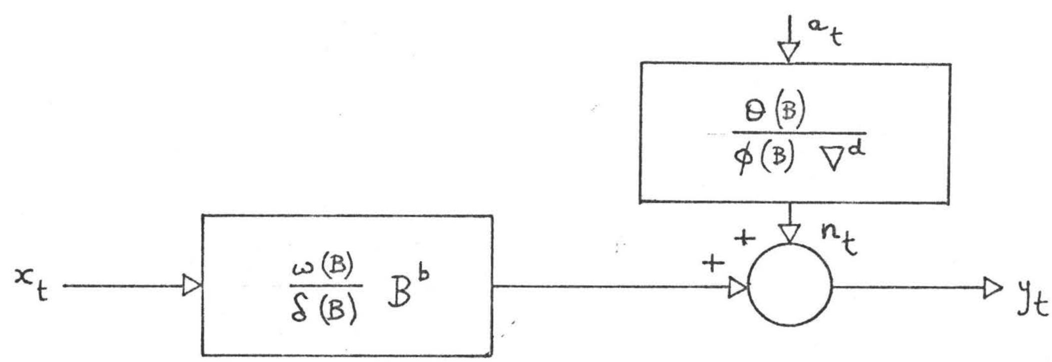
$$a_t = \frac{\phi(B)}{\theta(B)}\, \nabla^d\, n_t$$

The residuals a_t should form a white noise sequence, and should be uncorrelated with the input series, and diagnostic checks are based on this. The estimated autocorrelations and partial autocorrelations, and crosscorrelations with the input series, can be compared with their 95% confidence limits calculated on the assumption of white noise and overall chi-squared tests may also be performed. These checks will reveal any inadequacy in either of the models.

Finally

$$y_t = \frac{\omega(B)}{\delta(B)} x_{t-b} + n_t$$

and $$\nabla^d n_t = \frac{\theta(B)}{\phi(B)} a_t$$



2.3 <u>Two Input Series</u>

If there are two uncorrelated input series and one output series of observations taken simultaneously at equal time intervals, each input may be treated separately. While one input and the output are being considered, the other input is ignored. For each input an ARIMA model is found, both input and output are prewhitened with the inverse of this model, and the crosscorrelation pattern of the resulting two series at various lags is calculated. From this pattern a transfer function model is tentatively identified and rough parameter estimates calculated. Having done this for both inputs, the noise is calculated by subtracting from the output the product of each input and its corresponding v-weights:

$$y_t = v_1 (B) x_{1t} + v_2 (B) x_{2t} + n_t$$

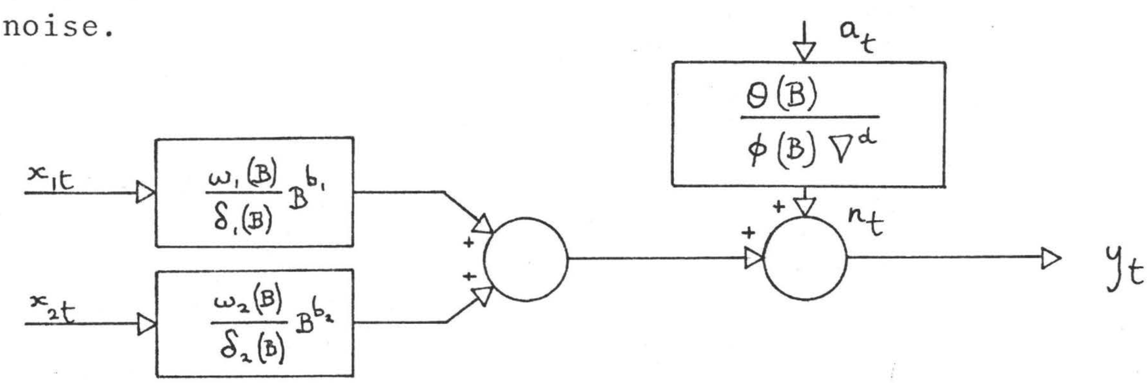
$$n_t = y_t - v_1 (B) x_{1t} - v_2 (B) x_{2t}$$

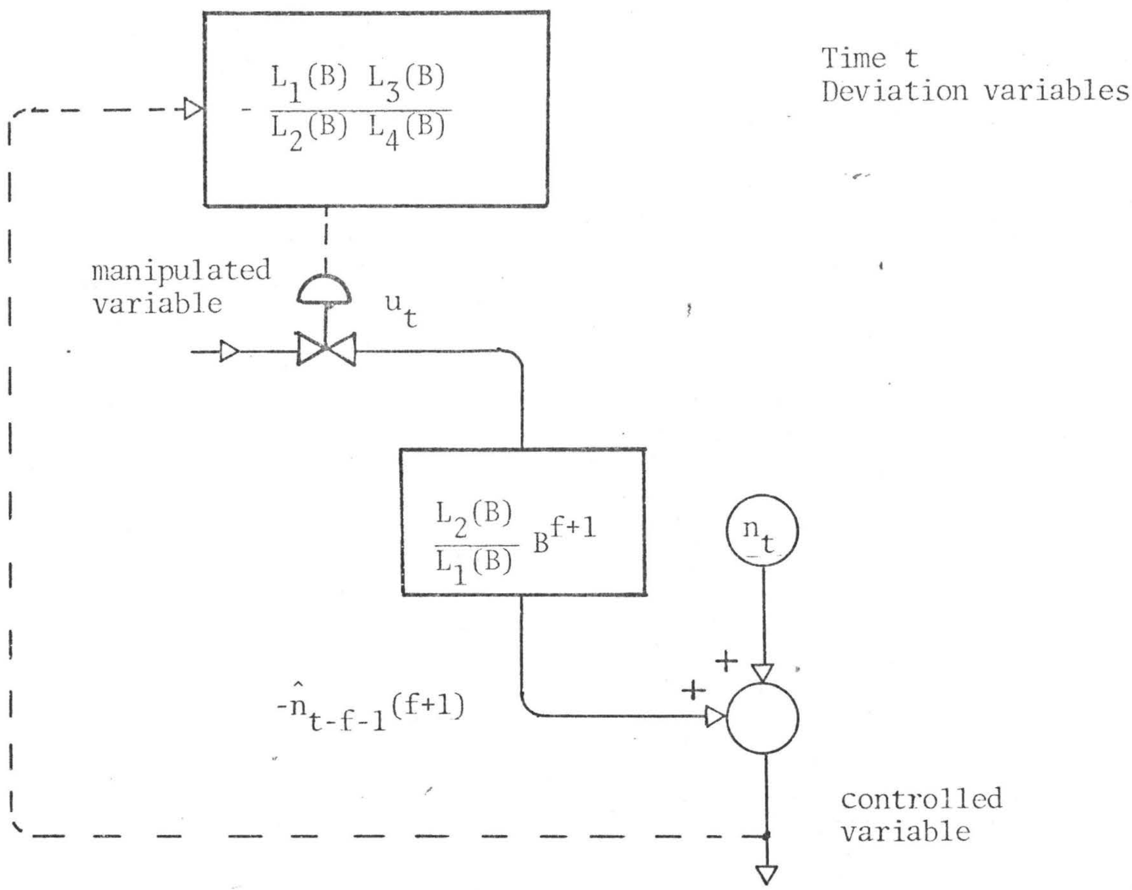
From the autocorrelation function of n_t an ARIMA model is identified and rough parameter estimates calculated. A regression routine is used to obtain best estimates of all the parameters together by minimising the sum of squares of residuals a_t from

$$n_t = y_t - \frac{\omega_1(B)}{\delta_1(B)} B^{b_1} x_{1t} - \frac{\omega_2(B)}{\delta_2(B)} B^{b_2} x_{2t}$$

$$a_t = \frac{\phi(B)}{\theta(B)} \nabla^d n_t$$

The residuals should again be white noise and hence should be neither autocorrelated nor correlated with either input. To check the models, the autocorrelation function for a_t and the crosscorrelation pattern between a_t and each prewhitened input separately may be calculated and compared with 95% confidence limits calculated under the assumption of white noise.





$$\Sigma_t = e_{t-f-1}(f+1)$$

Fig. 4 Feedback Controller

2.4 Minimum Variance Feedback Controller

Fig. 4 represents at time t a minimum variance feedback control scheme derived below, where predicted deviations in the controlled variable are cancelled out by the effect of calculated deviations in the manipulated variable. A model to describe the behaviour of noise in the controlled variable due to unknown disturbances, and a transfer function model to describe the effect of the manipulated variable on the controlled variable, are required.

Let the transfer function model be

$$y_t = u'_{t-f-1} = \frac{L_2(B)}{L_1(B)} B^{f+1} u_t$$

where (f+1) is the number of lags between a manipulation and its effect being observed in the controlled variable. When taking action u_t it should be designed to cancel a deviation in the controlled variable that will occur (f+1) time intervals later. Thus we need an expression for the (f+1) step ahead forecast of n_{t+f+1}, denoted $\hat{n}_t$ (f+1)

$$n_{t+f+1} = [a_{t+f+1} + \psi_1 a_{t+f} + \ldots + \psi_f a_{t+1}] + [\psi_{f+1} a_t + \psi_{f+2} a_{t-1} \ldots]$$

$$= L_4(B)\, a_{t+f+1} \qquad + L_3(B)\, a_t$$

$$= e_t(f+1) \qquad + \hat{n}_t(f+1)$$

The ψ-weights are obtained by expanding the ARIMA Noise model

$$\phi(B)\, \nabla^d\, n_t = \theta(B)\, a_t$$

When the scheme is implemented, $\hat{n}_t$ (f+1) should be cancelled out, and so deviations in controlled variable Σ_t will be equal to the error in forecasting n_t:

$$\Sigma_t = e_{t-f-1}(f+1)$$

The a_t's are not known directly, so $\hat{n}_t(f+1)$ is calculated from Σ_t

$$\hat{n}_t(f+1) = L_3(B)\, a_t$$

$$e_{t-f-1}(f+1) = L_4(B)\, a_t$$

$$\therefore \quad \hat{n}_t(f+1) = \frac{L_3(B)}{L_4(B)}\, e_{t-f-1}(f+1) = \frac{L_3(B)}{L_4(B)}\, \Sigma_t$$

The effect of u_t on the controlled variable is u'_t and should cancel $\hat{n}_t(f+1)$. So the action by the manipulated variable u_t should be such that:

$$\frac{L_2(B)}{L_1(B)}\, u_t = -\frac{L_3(B)}{L_4(B)}\, \Sigma_t$$

or

$$\underline{u_t = -\frac{L_1(B)}{L_2(B)} \cdot \frac{L_3(B)}{L_4(B)}\, \Sigma_t}$$

where Σ_t is the deviation of controlled variable from set-point.

2.5 Minimum Variance Feedforward-Feedback Controller

(when the delay in the effect of the manipulated variable exceeds the delay in the effect of the measured disturbance.)

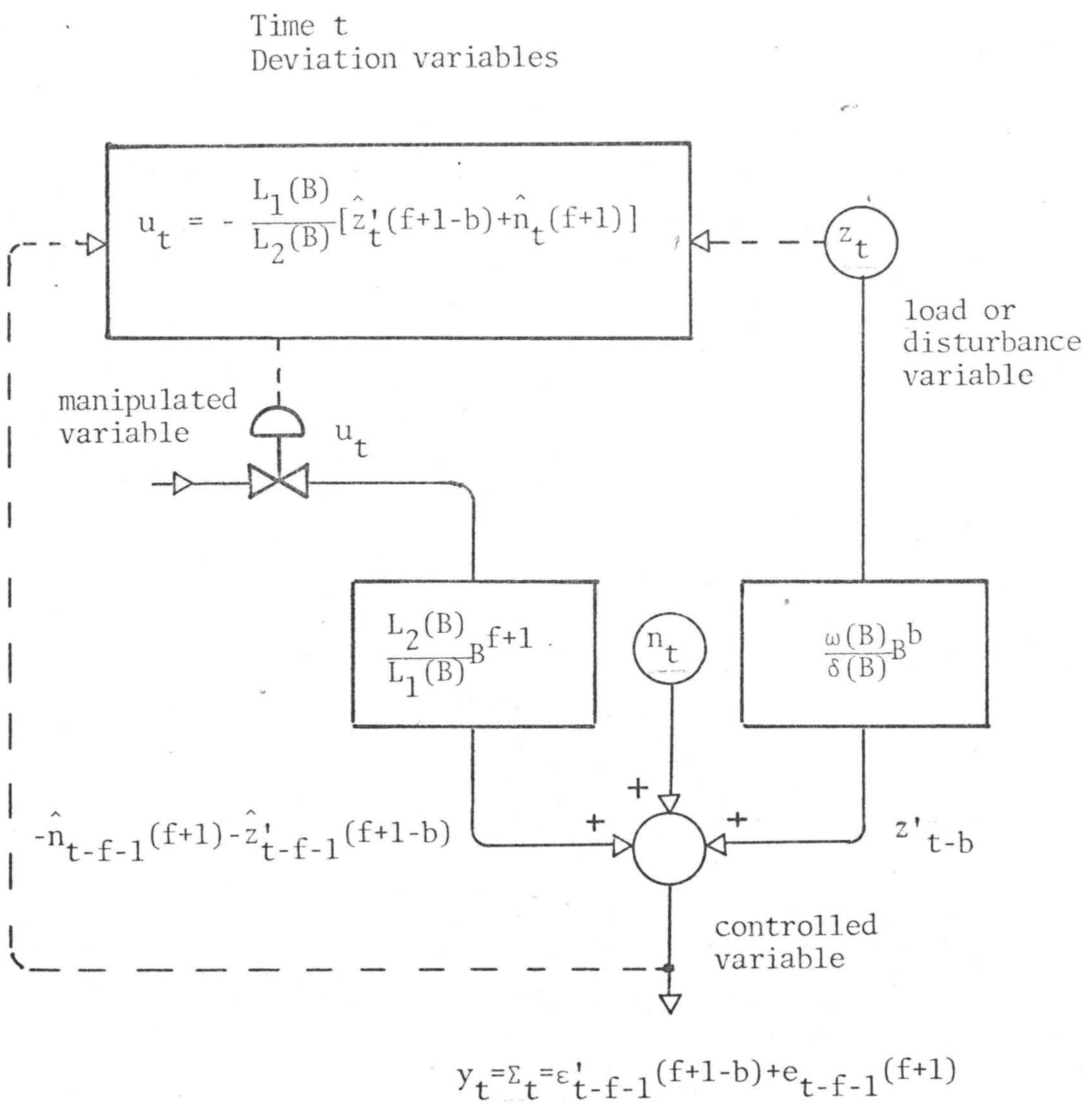


Fig. 5 Feedforward-Feedback Controller

In the minimum variance feedforward-feedback scheme shown in Fig. 5 and derived below, the predicted effect on the controlled variable due to the measured disturbance is cancelled by feedforward control action, and the deviations in controlled variable which are not correlated with the measured disturbance variable, but which are nevertheless predictable, are cancelled by feedback control action.

The notation u'_t means the effect on the controlled variable of action u_t, and z'_t means the effect on the controlled variable of disturbance z_t.

Four models are required:

1) to describe behaviour of the disturbance.
2) to relate the disturbance to the controlled variable.
3) to describe behaviour of noise (deviations in controlled variable that are not correlated with measured disturbances).
4) to relate the manipulated variable to the controlled variable.

Let the transfer function models be (for the disturbance)

$$z'_{t-b} = \frac{\omega\ (B)}{\delta\ (B)}\ B^b\ z_t$$

and (for the manipulated variable)

$$u'_{t-f-1} = \frac{L_2\ (B)}{L_1\ (B)}\ B^{f+1}\ u_t$$

When all contributions are added

$$y_t = u'_{t-f-1} + z'_{t-b} + n_t$$

or, rewriting at the time when action is taken:

$$\Sigma_{t+f+1} = y_{t+f+1} = u'_t + z'_{t+f+1-b} + n_{t+f+1}$$

When control action is being taken, the deviation in the controlled variable is $\Sigma_t = y_t$. If u'_t could cancel out $(z'_{t+f+1-b} + n_{t+f+1})$, the error Σ_t would be zero. However at time t, neither of them are known, and they have to be forecasted:

The disturbance model is $z_t = \frac{\theta_\ell (B)}{\phi_\ell (B)} \alpha_t$

$$z'_{t+f+1-b} = \frac{\omega (B)}{\delta (B)} z_{t+f+1-b} = \frac{\omega (B)}{\delta (B)} \cdot \frac{\theta_\ell (B)}{\phi_\ell (B)} \alpha_{t+f+1-b}$$

which can be expanded in terms of α_t:

$$z'_{t+f+1-b} = [\xi_o\alpha_{t+f+1-b}+\xi_1\alpha_{t+f-b}\ldots+\xi_{f-b}\alpha_{t+1}]+[\xi_{f-b+1}\alpha_t + \xi_{f-b+2}\alpha_{t-1}\ldots]$$

$$= L_6' (B) \alpha_{t+f+1-b} + L_5' (B) \alpha_t$$

Substitute for α_t:

$$z'_{t+f+1-b} = L_6' (B) \frac{\phi_\ell (B)}{\theta_\ell (B)} z_{t+f+1-b} + L_5' (B) \frac{\phi_\ell (B)}{\theta_\ell (B)} z_t$$

$$= L_6 (B) z_{t+f+1-b} + L_5 (B) z_t$$

$$= \varepsilon'_t (f+1-b) + \hat{z}'_t (f+1-b)$$

N.B. Both L_5 (B) and L_3 (B) below can be put in the form of parsimonious expressions.

The noise model is: $n_t = \frac{\theta_n (B)}{\phi_n (B)\nabla^d} a_t$

and may be expanded as in the pure feedback model:

$$n_{t+f+1} = [a_{t+f+1} + \psi_1 a_{t+f} \cdots + \psi_f a_{t+1}] + [\psi_{f+1} a_t + \psi_{f+2} a_{t-1} \cdots]$$

$$= L_4 (B)\, a_{t+f+1} \qquad + L_3 (B)\, a_t$$

$$= e_t (f+1) \qquad + \hat{n}_t (f+1)$$

Rewriting the equation for Σ_t:

$$\Sigma_{t+f+1} = u'_t + \hat{z}'_t(f+1-b) + \varepsilon'_t (f+1-b) + \hat{n}_t (f+1) + e_t (f+1)$$

A minimum variance controller is designed to minimise Var (Σ_t)

$$\mathrm{Var}(\Sigma_{t+f+1}) = \mathrm{Var}[u'_t + \hat{z}'_t(f+1-b) + \hat{n}_t(f+1)] + \mathrm{Var}[\varepsilon'_t(f+1-b)] + \mathrm{Var}[e_t(f+1)]$$

All covariances are zero.

The first term can be set equal to zero by the proper choice of control action; the others are fixed positive values.

By setting $-u'_t = \hat{z}'_t (f+1-b) + \hat{n}_t (f+1)$

the error in the controlled variable becomes:

$$\Sigma_{t+f+1} = \varepsilon'_t (f+1-b) + e_t (f+1)$$

and $\Sigma_t = \varepsilon'_{t-f-1} (f+1-b) + e_{t-f-1} (f+1)$

The noise forecast is obtained as in the pure feedback controller:

$$\hat{n}_t(f+1) = \frac{L_3(B)}{L_4(B)} e_{t-f-1}(f+1)$$

The forecasting error $e_{t-f-1}(f+1)$ in the noise n_t can be obtained from the above expression for the deviation in controlled variable Σ_t:

$$\hat{n}_t(f+1) = \frac{L_3(B)}{L_4(B)} [\Sigma_t - \varepsilon'_{t-f-1}(f+1-b)]$$

Thus the feedforward-feedback controller becomes:

$$\underline{u_t = -\frac{L_1(B)}{L_2(B)} \{L_5(B)\, z_t + \frac{L_3(B)}{L_4(B)}[\Sigma_t - L_6(B)\, z_{t-b}]\}}$$

CHAPTER 3

APPARATUS AND DATA COLLECTION

3.1 Apparatus

A diagram of the apparatus used in this work is shown in Fig. 6. The variable to be controlled was the temperature of water in an open-top constant volume stirred tank. There were four vertical side baffles, stirring was vigorous, and mixing was top to bottom with little swirl. Water level was usually 2/3 of the way up the 3" pipe overflow to drain, and the tank volume was 7.3 U.S. gallons.

There were two water flows entering the tank. Cold water from the main entered through 1" pipe. The flow could be varied between 0-6 USGPM by a 1/2" pneumatic control valve, and this was used to generate a disturbance. An orifice plate (diameter 0.437 inch) was used to measure the disturbance variable and was connected via a dp-cell and transmitter to the computer interface.

The other flow entering the tank was hot water produced by a fairly complex arrangement. Mains cold water was maintained at constant flowrate by a manual valve and rotameter: initially 4.8 USGPM was used, but for the final identification run and control runs, 7.2 USGPM was used. The water passed down through a small heat exchanger, a hold-up volume and a second heat exchanger, all close together in series. Then up through 12 feet of 1" pipe full of water,

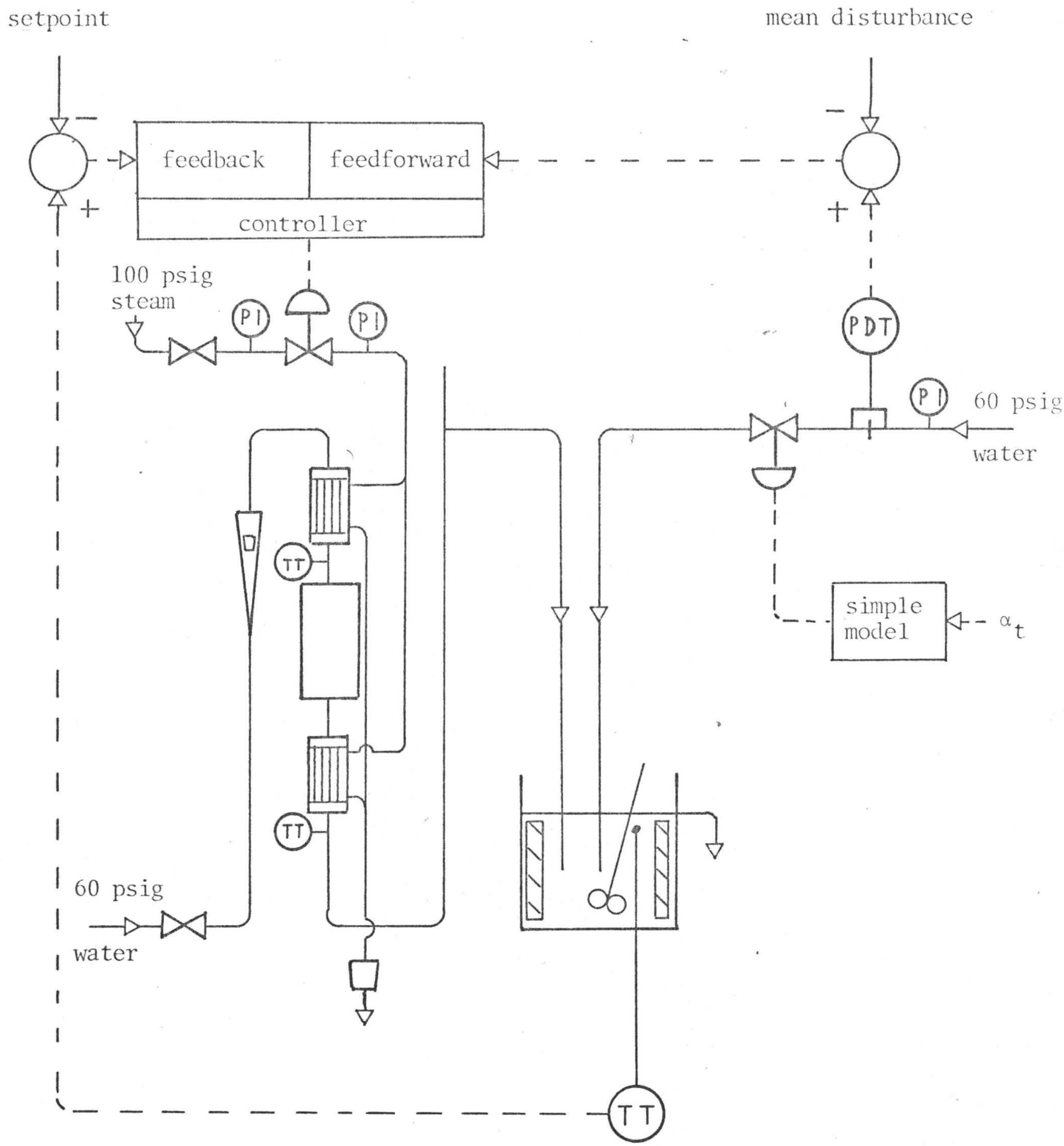


Fig. 6 Apparatus

past an atmospheric vent and down through 8 feet of 2" pipe partly filled, before entry into the tank. Each heat exchanger shell was approximately 8" long and 2" diameter. The intermediate hold-up volume was approximately 2.4 U.S. gal. A copper constantan thermocouple was used to measure the water temperature at the exit from each heat exchanger: each was connected via a transmitter to the computer interface. The tank thermocouple was connected similarly. Only the latter was used for control; the two heat exchanger thermocouples were used only in checking results.

The steam supply was in parallel to each heat exchanger, and was controlled by a single 1/2" pneumatic control valve, whose setting was our manipulated variable. Both this valve and the water control valve received signals via their transducers from the computer interface. Pressure gauges indicated pressure up and down stream of the control valve, and upstream of the first gauge was a manual valve used for throttling (see below). Steam supply pressure was nominally 100 psig (a constant 110 psig as measured by the first gauge). Condensate from the heat exchangers passed through the same continuously-operating bucket-type steam trap to drain.

The water flowrate through the heat exchangers as indicated by the rotameter was held reasonably constant by manual adjustments, which had to be made during runs only on one or two occasions. Based on observations of the rotameter, the standard deviation in the hot water rate would be approximately 0.1 USGPM. These small variations contributed to the noise n_t observed in the output.

The water control valve was purchased to linear specification and upon checking was found to be approximately linear in the range 2-8 volt in signal to the transducer. (The full signal range was 0-10 volts.) In any case, this was not important since disturbances were measured by orifice meter. The valve had a very short time constant and did not stick.

Linearity of the steam control valve was checked by observations of the valve setting and of the steady-state temperature rise caused at a constant water flowrate. It was found that above a 6 volt signal to the transducer, steam flow increased more rapidly. To counteract this, the manual steam valve upstream of the control valve was throttled, which had the effect of reducing supply pressure at wider control valve openings. By trying different manual valve settings it was not difficult to obtain a water temperature rise that was linear with steam control valve opening, and also to obtain a maximum hot water temperature with the steam control valve fully open that was high enough on discharge (50-60^{o}C) without incidence of boiling in the second heat exchanger.

Having determined a manual valve setting it had to be repeatable, so before each run, after a half-hour apparatus warm-up with the water rate set at 7.2 USGPM, the manual valve was adjusted until at certain control valve settings (7 and 10 volts) the pressure between manual and control valves was always the same (72 and 45 psig respectively).

The steam control valve steam packing was oiled before each run, before application of steam, to prevent sticking.

All thermocouple and dp-cell transmitters were found to be linear. The temperature conversion for the transmitters used was

$$T = .278 * (\text{Value in A/D units}) - 30.4 \quad (^{o}C)$$

$$3.60 \text{ A/D units} = 1 \text{ Centigrade degree}$$

Differential pressure across the orifice meter was proportional to the square of flowrate and so the flowrate conversion was

$$F = .566 * \sqrt{(\text{Value in A/D units}) - 105} \quad (\text{USGPM})$$

These calibrations were done before any runs were made, and were not rechecked after the runs. While it is true that transmitter characteristics change with time, it is a reasonable assumption that the changes in spans (which are important in this work) were insignificant. A/D and D/A units have been used in this study: no conversions were made to engineering units.

The interface between the Data General NOVA 2 minicomputer and the apparatus is shown in Fig. 7.

3.2 Procedure

In feedforward-feedback control of tank temperature, simultaneous measurements are needed of tank temperature and cold water flowrate at constant sampling interval. Combined feedback and feedforward control action is calculated by the computer in a few microseconds and immediately output to the steam control valve. As mentioned in the introduction,

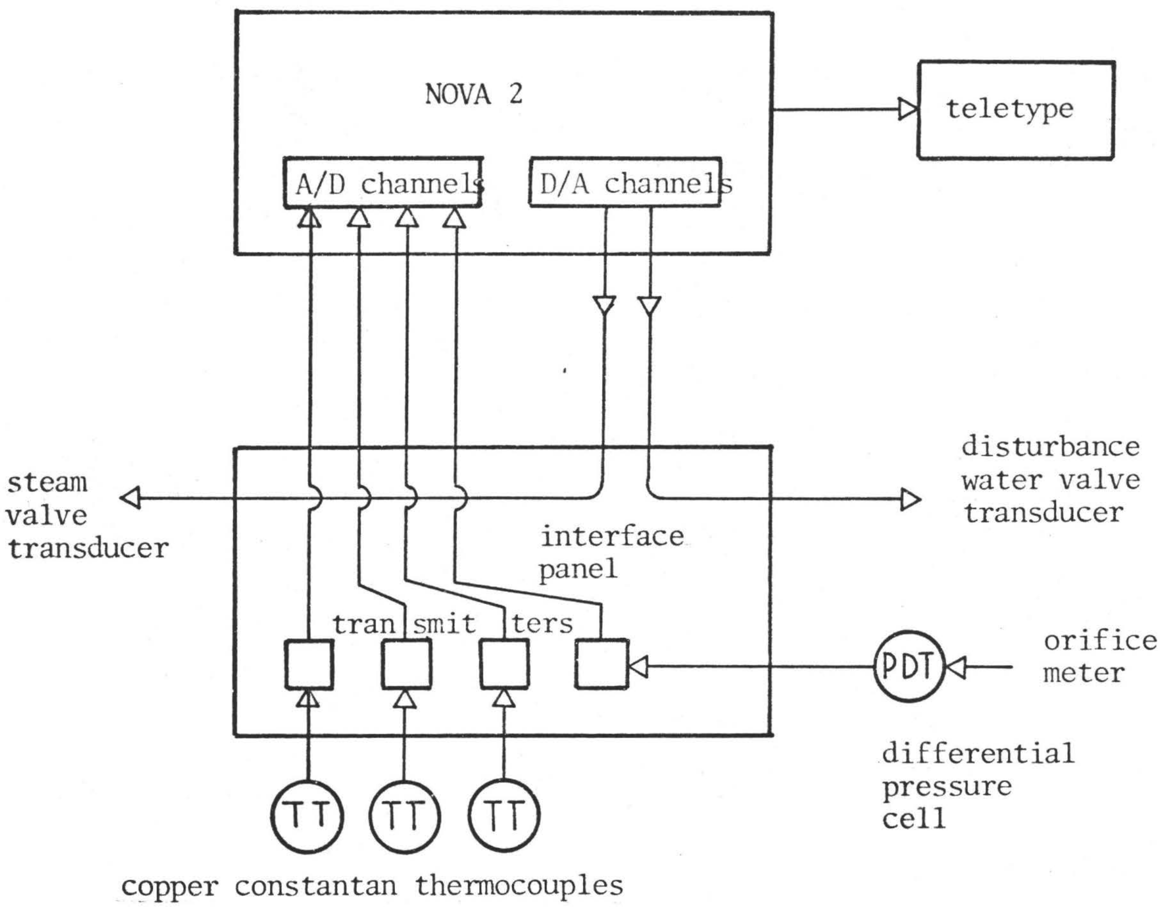


Fig. 7 Minicomputer Interface

the process has no significant inherent disturbance, and so in order to test a control scheme a simulated disturbance is needed. To achieve this a signal is output by the computer to the water control valve during each sampling interval.

Experimental runs were in two groups: identification, and control.

Identification

Using random numbers and two ARIMA models, the computer calculated two stochastic inputs, one to each control valve, to be transmitted in between sets of readings of tank temperature and orifice meter at a regular sampling interval. The length of interval, and the times within the interval of transmission of each valve signal, were the same as in control runs. For feedforward-feedback control the data was used to derive four models: an ARIMA model for the orifice meter readings, transfer functions relating the steam valve setting and orifice meter to the tank temperature, and a noise model. For feedback control the same data was used to derive two models only for the transfer function and noise models relating steam valve setting to tank temperature.

The models were then used to derive feedforward-feedback and feedback control algorithms respectively.

Control

At every sampling interval, simultaneous readings of tank temperature and orifice meter were used in the feed-forward-feedback algorithm to calculate control action which was immediately output to the steam control valve. Using random numbers and the same ARIMA model as used in identification, a stochastic input signal was calculated and sent to the disturbance water control valve 7 seconds after each reading of tank temperature and orifice meter.

When testing the pure feedback algorithm, the same stochastic input signal was sent to the disturbance water control valve, but no use was made of the orifice meter readings.

3.3 Choice of Values for Input Parameters

Previous studies on the heat exchangers alone had shown that a good sampling interval was 5 seconds. The stirred tanks added extra capacity to the system, and for this work a 10 second interval was found to be suitable.

A value was chosen for the variance of the stochastic input series to each control valve u_t based on linearity. Since both valves had been shown to be approximately linear in the range 2-8 volts, σ_u was chosen at 1.16 volts (59.5 A/D units), such that 99% of observations would fall within this range (2.58 σ_u).

An AR (1) model was chosen to calculate each stochastic input series; different ϕ-values were chosen to reflect the apparently different time constants. Then

$$(1 - \phi B)\, u_t = a_t$$

where a_t is a white noise sequence, and the desired variance of a_t is calculated from

$$\sigma_a^2 = (1 - \phi^2)\ \sigma_u^2 \qquad \text{[Ref.1 p.58]}$$

Box and Jenkins explain that in designing an input for a process having slow dynamics (a large δ) a similarly large ϕ should be chosen to produce a slowly drifting input. Thus $\phi = 0.9$ was chosen for both inputs, which gave fairly good results. For the final identification run, $\phi_s = 0.8$ was used for the steam valve stochastic input, and $\phi_w = 0.5$ for the water valve, thinking that its effect on tank temperature had a shorter time constant; the latter value was used for all control runs too. When results were analysed it turned out that the disturbance - tank temperature transfer function actually had the same δ value as the steam valve - tank temperature.

3.4 Minicomputer Software

Two routines in assembler language were used for identification: ACQUI (the executive) and subroutine RANDO which calculated valve signals from random numbers. The flow charts in Appendix 1 show the sequence of operations. RANDO calls a subroutine RAND in the NOVA program library, which calculates a "random" number x in the uniform band range 0 to 2^{16}-1, using the algorithm

$$x_{n+1} = (x_n * A + C) \bmod 2**16$$

where $A = (2**11 + 2**2 + 1)$

and $C = 33031$ octal

The variance of a random variable x from such a uniform distribution is theoretically

$$\mathrm{Var}(x) = \frac{(\mathrm{Range})^2}{12}$$

where $\mathrm{Range} = 2^{16} - 1$

RANDO obtains 16 numbers from RAND and calculates the average, which is multiplied by

$$\frac{\text{desired standard deviation of } a_t \ (\sigma_a)}{\text{standard deviation of this average}}$$

to obtain a_t in A/D units.

Then $u_t = \phi\, u_{t-1} + a_t$

u_t is added to the mean valve setting to obtain the valve action, which is stored for use by ACQUI.

RANDO performs this operation twice, once for each control valve, in less than one millisecond.

ACQUI records the time, reads the A/D channels, calls RANDO to calculate the stochastic inputs to the two valves, immediately outputs the steam valve signal via a D/A channel, waits 7 seconds and then outputs the disturbance water valve signal, waits another 3 seconds and returns to record the time again. A subsidiary TASK outputs data from ACQUI and RANDO to the teletype and to punched tape for future analysis.

A12 was the final identification run, and was used to determine models for all feedback and feedforward-feedback controllers.

CHAPTER 4

MODEL IDENTIFICATION AND ESTIMATION

4.1 Single Series

After a run of 300 readings, the data on tape was converted to cards, and the mean value of each data column calculated, to enable subsequent calculation of data about mean zero. The orifice meter series and the steam valve series were identified using Fortran program IDENT from the time series programs in the Department of Chemical Engineering which calculated the autocorrelation and partial autocorrelation coefficients at various lags. Both series showed, as expected, patterns clearly identifiable as AR(1) processes. Program TSHAUS, a least squares regression routine, was used to estimate ϕ for each process. In the final identification run, ϕ_S for the steam valve included .80 in its 95% confidence limits, and since .80 had been used in calculating its input series, this value was used subsequently. ϕ was .54 for the orifice meter reading and was .55 for the series formed from the square root of its reading. The confidence limits of both the latter series included .50 which had been used for the stochastic input to the disturbance water control valve, but this fact was not used, since the water valve series was presumed unknown.

Having determined a value of ϕ, it was used with an

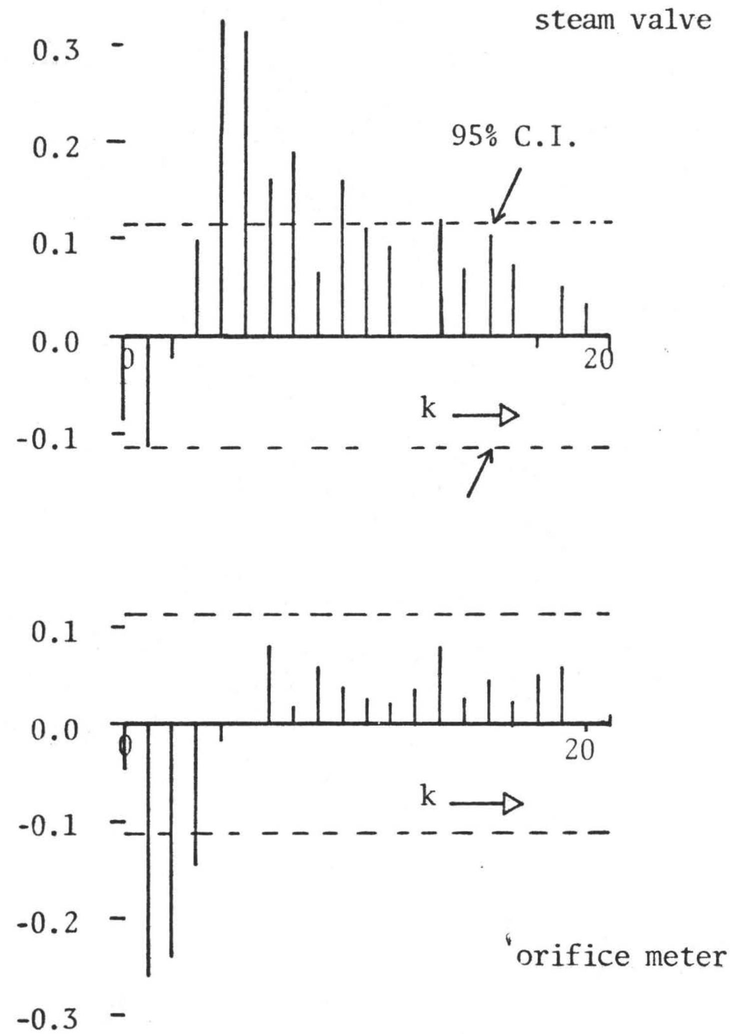


Fig. 8 Crosscorrelations of the Prewhitened Steam Valve Settings and of the Prewhitened Square Root of Orifice Meter Reading with the Transformed Tank Temperature for the Final Identification run A12. These Patterns were Used in Deriving All Controllers

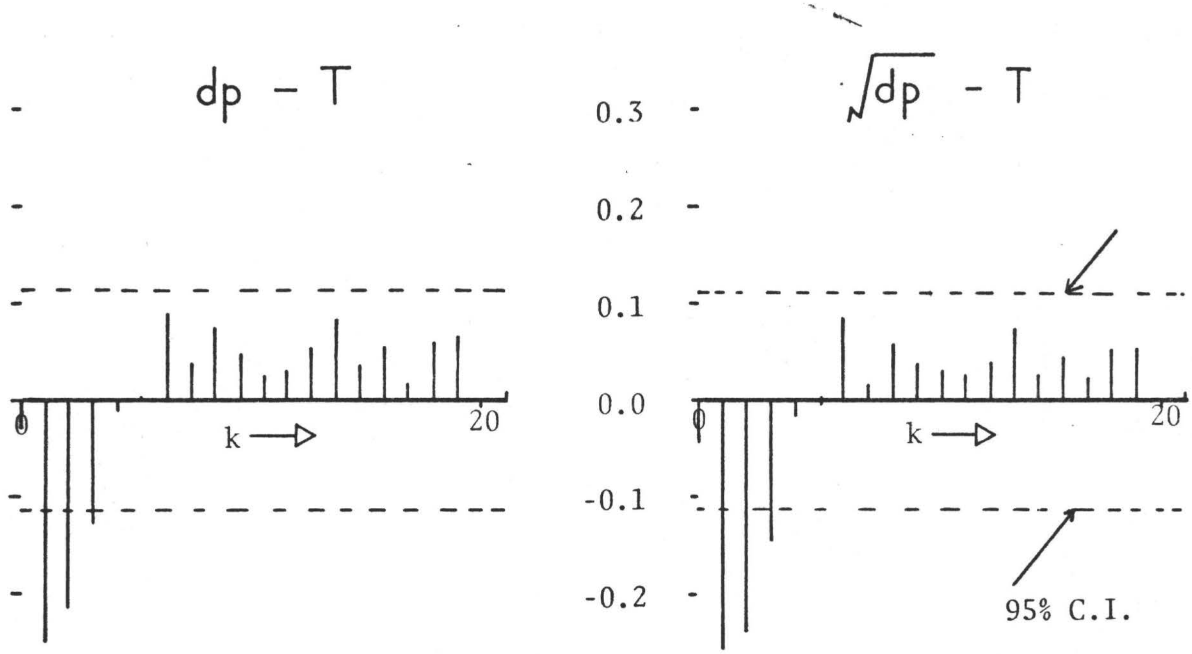


Fig. 9 Crosscorrelations of the Prewhitened Orifice Meter Reading and of the Prewhitened Square Root of the Reading with the Transformed Tank Temperature (Final Identification Run A12)

AR(1) model to prewhiten input and output series before calculating crosscorrelations at various lags between the series. This was done for each input using its respective ϕ-value, giving a crosscorrelation pattern for each input, shown in Fig. 8 for the final identification run A12.

4.2 Choice of Disturbance Variable

Since the orifice meter reading was proportional to the square of the cold water flowrate, the reading and its square root were separately cross-correlated with the tank temperature, and it was found that the square root gave slightly more significant crosscorrelations (Fig. 9). Therefore the square root was chosen as the disturbance variable.

4.3 Transfer Function and Noise Model Identification and Estimation

Firstly the transfer function v-weights were obtained by multiplying the significant crosscorrelations by the ratio of standard deviations of output and input series, as described in Chapter 2.

For feedforward-feedback control, by subtracting the product of each input and its respective v-weights from the tank temperature, and analysing the resulting series by its auto-and partial-autocorrelations, the noise model was identified best as an AR model, but possibly as an IMA model (Fig. 10).

For feedback control, only the product of the steam valve setting and its v-weights was subtracted from the tank temperature to obtain the noise series, which was identified

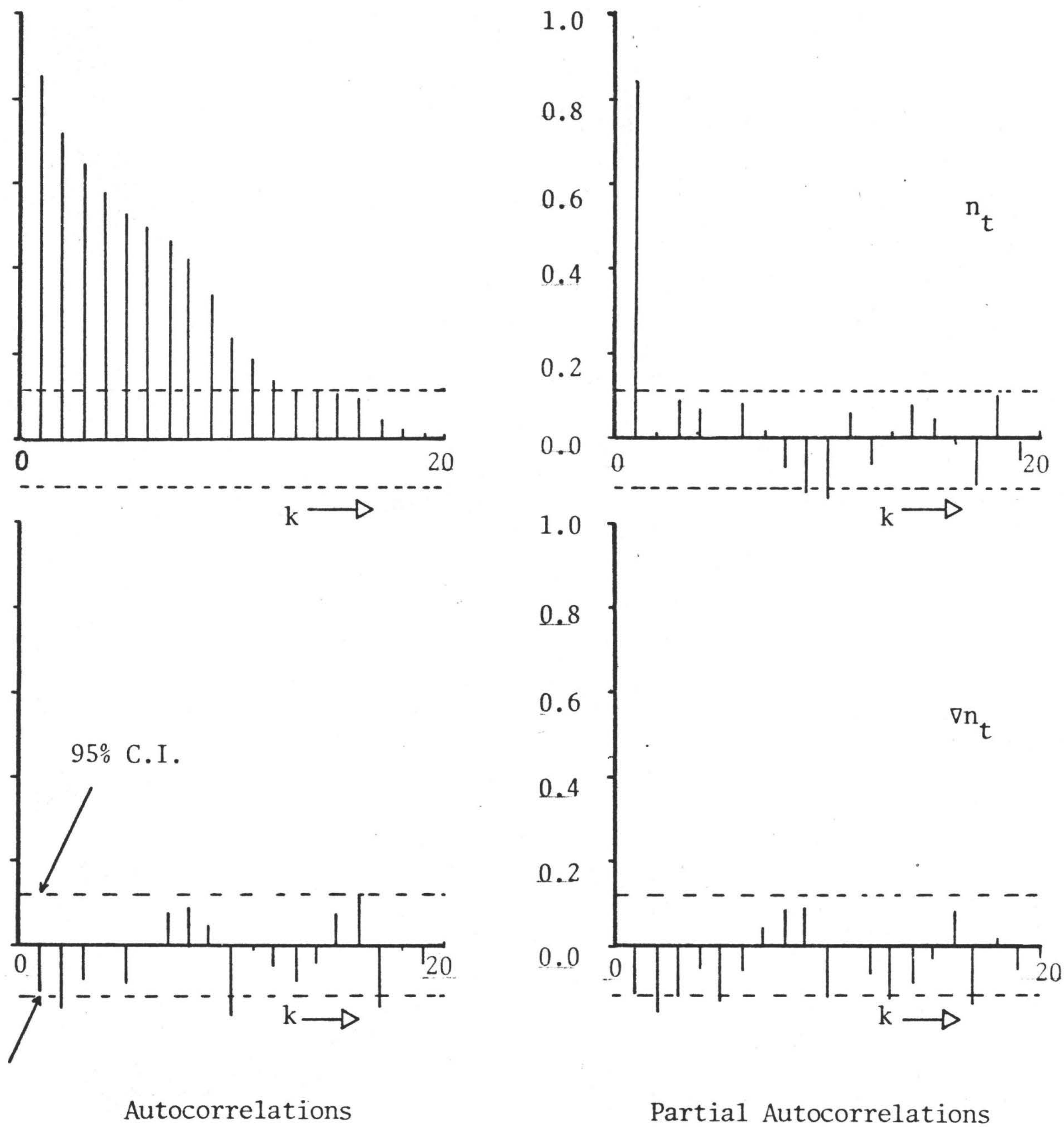


Fig. 10 Noise Identification for Feedforward-Feedback Controller

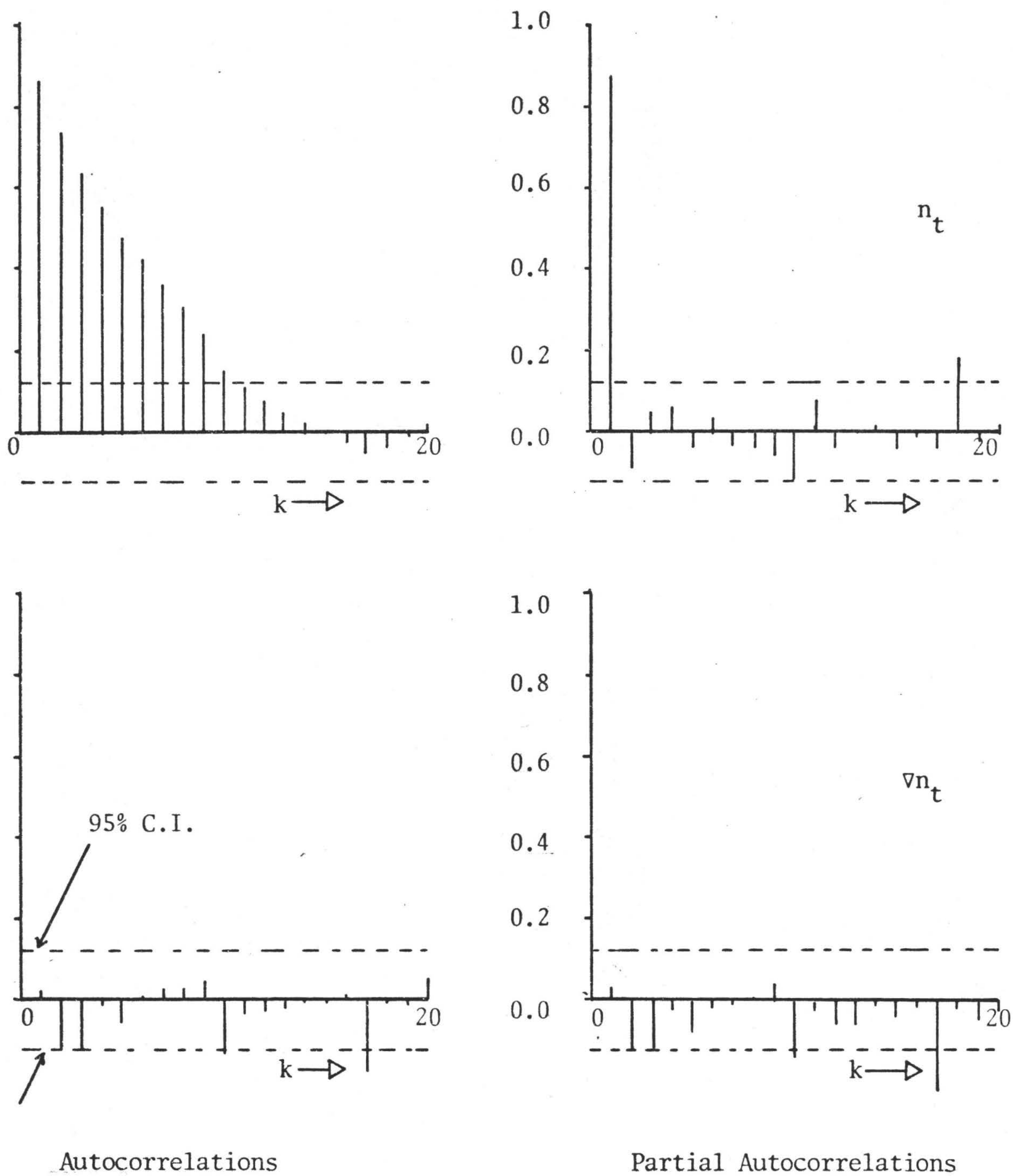


Fig. 11 Noise Identification, Ignoring Measured Disturbance in Water Flow, for Feedback Controller

again as an AR or possibly an IMA (013) model (Fig. 11).

The crosscorrelation patterns were studied and suitable transfer function models chosen. The orifice meter reading (square root) pattern was clear and a good model was chosen immediately, but the steam valve pattern was not clear and many models were attempted. For the noise an AR(1) model was a clear choice and worked well; an IMA noise model was also fitted to permit design of a controller to include integral action. The same models were used for feedback and feedforward-feedback control, but with different parameter values.

Table 1 shows the results of all regression programs to estimate the best parameter values.

4.4 Regression Using Both Orifice Meter and Steam Valve Inputs

The orifice-tank crosscorrelations had the pattern of a first order decay and the first significant crosscorrelation at lag 1 was also the biggest. The model

$$y_t = \frac{\omega_o}{1-\delta_1 B} z_{t-1}$$

was chosen for the transfer function. From noise model identification, an AR(1) model was the obvious choice. These two models were used in testing out different steam valve-tank transfer function models.

The first significant crosscorrelation between steam valve and tank temperature was at lag 4, but since lag 3 was

Model No.	Square Root of Orifice Meter Reading Transfer Function			Steam Valve Transfer Function			Noise	Residuals Variance σ_a^2	Square Root of Orifice Meter Reading Transfer Function			Steam Valve Transfer Function			Noise				Controller Type
	r	s	b	r	s	b		(A/D Units)^2	δ_ℓ	ω_ℓ	b	δ_m	ω_m	f+1	ϕ_n	θ_1	θ_2	θ_3	
1	δ_1	ω_o	1	δ_1	ω_1	3	AR(1)	2.40											
2		"		δ_2	ω_1	3	"	2.40											
3		"		δ_2	ω_o	4	"	2.62											
4		"		δ_1	ω_o	4	"	2.62	.83	-.625	1	.82	.0314	4	.505				AF
5		"		δ_1	ω_o	4	IMA(013)	2.93	.79	-.648	1	.82	.0298	4		.297	.258	.178	IF
6		-		δ_1	ω_o	4	AR(1)	3.75		-		.84	.0272	4	.76				AB
7		-			"		AR(2)	3.70											
8		-			"		IMA(013)	3.98		-		.85	.0262	4		.056	.219	.243	IB
9		-			"		IMA(012)	4.20											
10		-			"		IMA(010)	4.26											

TABLE 1. Estimation of parameters for different transfer function and noise models from data of final identification run A12.

δ_2 implies two δ-coefficients, δ_1 and δ_2

ω_1 implies two ω-coefficients, ω_0 and ω_1

b refers to the initial lag

[Ref. 1]

almost significant, a model with b = 3 was tried first. Because the most significant crosscorrelation was at lag 4, ω_o and ω_1 were essential. After lag 4, the pattern approximated an exponential decay, and so a first order δ_1 was chosen. Thus the model

$$y_t = \frac{\omega_o - \omega_1 B}{1 - \delta_1 B} \; u_{t-3}$$

was chosen this time. Then, using these three models, parameters were estimated by regression routine, minimizing the variance of the residuals (σ_a in Table 1). The final σ_a (Model 1) was the lowest ever obtained and the autocorrelations of residuals and the crosscorrelations of residuals with inputs, showed only one or two peaks that were just significant.

However, the values of $(\omega_o - \omega_1 B)$ were $(.142 + .170\ B)$. Looking ahead, the controller design would contain the terms

$$(.142 + .170\ B)\ u_t = \ldots.$$

or

$$u_t = - \frac{.170}{.142}\, u_{t-1} + \ldots.$$

$$= - 1.2\ u_{t-1} + \ldots.$$

This controller design is unstable, and when tested it proved to be so. Therefore a different steam valve-tank transfer function model had to be used.

To avoid using two ω-values, the almost significant crosscorrelation coefficient at lag 3 was ignored; then lag 4 had the first and most significant crosscorrelation, after

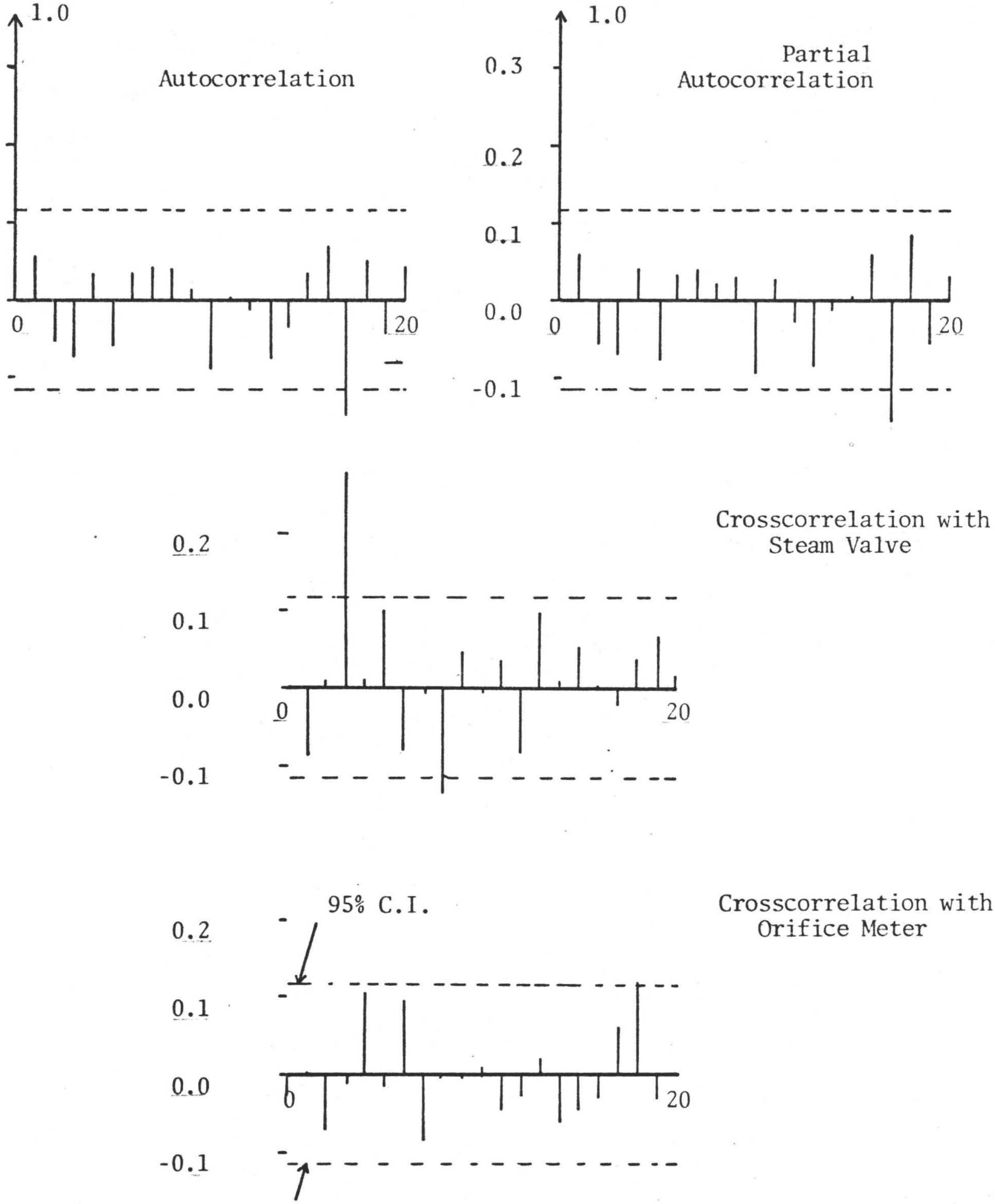


Fig. 12 Residual Autocorrelations and Partial Autocorrelations and Crosscorrelations with Prewhitened Steam Valve Settings and with Prewhitened Orifice Meter (Square Root) for Model 4

which the pattern approximated to exponential decay. Thus the model

$$y_t = \frac{\omega_o}{1-\delta_1 B} u_{t-4}$$

was chosen and used with the previously mentioned models for orifice transfer function and for the noise to estimate parameters (Model 4). Residual σ_a was not much worse than Model 1 and the residual autocorrelation and cross-correlation checks in Fig. 12 showed no very significant peaks except one at lag 3 in the steam valve-residuals pattern; r_3 = .278, which was undesirable, but as we have seen, not easily removable. An additional paramter δ_2 for a second order steam valve-tank model (Model 3) did not reduce the residual variance significantly. The parameters estimated here were used in designing the feedforward-feedback controller using an AR(1) noise model, (AF controller).

Using the models derived above, a controller analogous to a classical proportional controller was obtained, which meant that there might be an offset in controlled variable. If an IMA noise model were used, the resulting controller would have integral action, which would eliminate offset.

Although not as good as an AR(1), it was possible to fit an IMA noise model, and IMA (013) was chosen arbitrarily because that was the best model derived for pure feedback control (see below). Regression in Model 5 left a residual σ_a larger than Model 4, but residual autocorrelation and cross-correlation checks were almost as good. The parameters estimated

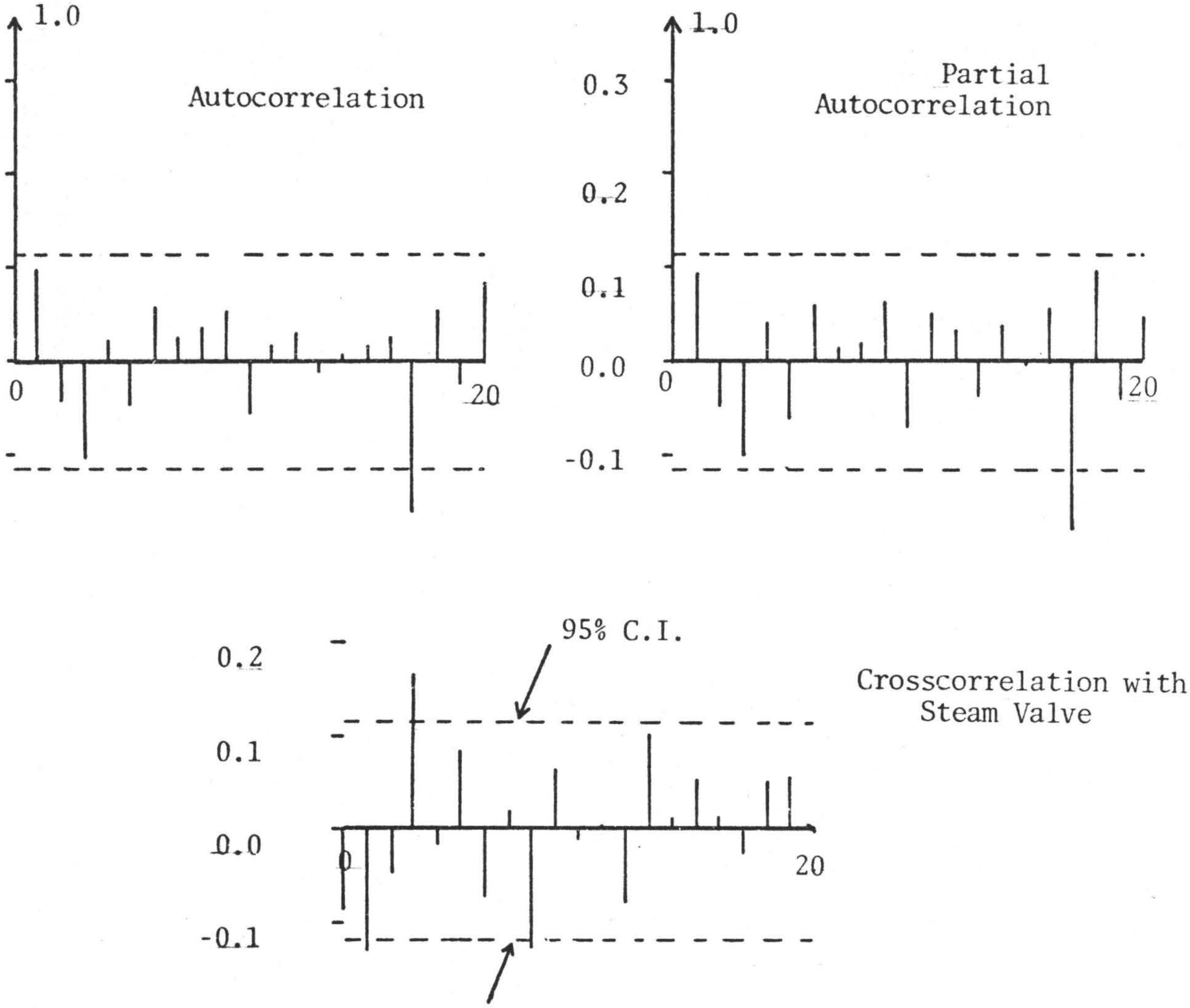


Fig. 13 Residual Autocorrelation and Partial Autocorrelations and Crosscorrelations with Prewhitened Steam Valve Settings for Model 6

here were used in designing the feedforward-feedback controller using IMA (013) noise model, (IF controller).

4.5 Regression Using Knowledge of the Steam Valve Input Only

The orifice meter input was ignored. The noise was identified previously as an AR(1) model, possible AR(2), possible IMA(013). For the same reasons as above, the model for the steam valve-tank transfer function was:

$$y_t = \frac{\omega_0}{1 - \delta_1 B} u_{t-4}$$

Using this and an AR(1) noise model to estimate parameters in Model 6, the residual σ_a was considerably higher than before, as might have been expected since we had ignored the orifice meter input. The residual autocorrelation at lag 17 and crosscorrelation with input at lags 1, 3, 8 were all slightly significant (Fig. 13).

AR(2) was tried instead for the noise, but the decrease in σ_a was not much, and so the above two models and their estimated parameters were used in designing a pure feedback controller with AR(1) noise model, (AB controller).

To include integral action in the controller, an IMA (013) noise model was tried with the above steam valve-tank model in Model 8. The residual σ_a and correlation checks were worse than Model 6, but significantly better than IMA (012) and so these models were used to design a pure feedback controller with IMA noise model, (IB controller).

For both pure feedback and feedforward-feedback controllers, the IMA noise model gave a greater residual σ_a than the AR model, and so it might be expected that the variance of the tank temperature about the mean value using a controller based on an IMA model would be greater than on an AR model. However, because it has integral action, the IMA model would be expected to cause the tank temperature to vary about a fixed set-point, while the AR model might cause an offset.

Parameter values from the four useful regression programs are in Table 1. A striking feature of the parameter table is that the δ values for orifice-tank and steam valve-tank transfer functions are very close. Their confidence limits all include a common value, and this fact enabled the expression $(1-\delta B)$ to be cancelled, thus making the design of controllers much simpler.

The physical interpretation is that the time constants of the two separate "processes" were the same.

CHAPTER 5

DERIVATION OF CONTROLLERS

Here minimum variance feedforward-feedback and pure feedback controllers of the position-type are derived, both using AR(1) noise models. Velocity-type controllers having integral action are derived from IMA noise models in Appendix 2. The model for the disturbance variable (square root of the orifice meter reading):

$$(1-\phi_\ell B)\ z_t = \alpha_t \qquad \text{where } \phi_\ell = 0.55$$

is common to all control schemes.

5.1 Feedforward-Feedback Controller Using an AR(1) Noise Model (Controller AF)

Parameter values were estimated in Model 4 (see Table 1), and the four required models are:

(1) Disturbance variable model $(1-\phi_\ell B)\ z_t = \alpha_t$

(2) Disturbance transfer function model

$$z'_{t-1} = \frac{\omega_\ell}{1-\delta_\ell B}\ z_{t-1}$$

(3) Manipulated variable transfer function model:

$$u'_{t-4} = \frac{\omega_m}{1-\delta_m B}\ u_{t-4} = \frac{L_2(B)}{L_1(B)}\ u_{t-4}$$

(4) Noise model $(1-\phi_n B)\ n_t = a_t$

Now $\qquad y_t = z'_{t-1} + u'_{t-4} + n_t$ or at the time of

taking action $\qquad y_{t+4} = z'_{t+3} + u'_t + n_{t+4}$

To minimise the variance of y, we should set

$$-u'_t = z'_{t+3} + n_{t+4}$$

but these are future unknown values, and so we can only use their estimates.

Set $\qquad -u'_t = \hat{z}'_t(3) + \hat{n}_t(4)$

Then $\qquad y_{t+4} = u'_t + \hat{z}'_t(3) + \varepsilon'_t(3) + \hat{n}_t(4) + e_t(4)$

and the error or deviation in controlled variable equals the sum of the errors in the two estimates: $\Sigma_{t+4} = \varepsilon'_t(3) + e_t(4)$ or at time t: $\Sigma_t = \varepsilon'_{t-4}(3) + e_{t-4}(4)$

To obtain $\hat{z}'_t(3)$, expand z'_{t+3} in terms of α_t:

$$z'_{t+3} = \frac{\omega_\ell}{1-\delta_\ell B} z_{t+3} = \frac{\omega}{1-\delta B} \cdot \frac{\alpha_{t+3}}{1-\phi B} \qquad \omega = \omega_\ell,\ \delta = \delta_\ell,\ \phi = \phi_\ell$$

$$= \omega\,\alpha_{t+3} + \omega\frac{[(\delta+\phi)-\delta\phi B]}{(1-\delta B)(1-\phi B)}\alpha_{t+2}$$

$$= \omega\,\alpha_{t+3} + \omega(\delta+\phi)\,\alpha_{t+2} + \omega\frac{[(\delta+\phi)(\delta+\phi-\delta\phi B)-\delta\phi]}{(1-\delta B)(1-\phi B)}\alpha_{t+1}$$

$$= \omega\,\alpha_{t+3} + \omega(\delta+\phi)\,\alpha_{t+2} + \omega[(\delta+\phi)^2-\delta\phi]\,\alpha_{t+1} + \omega\frac{\{[(\delta+\phi)^2-\delta\phi][\delta+\phi-\delta\phi B]-(\delta+\phi)\delta\phi\}}{(1-\delta B)(1-\phi B)}\alpha_t$$

$$= \omega\{1+(\delta+\phi)B + [(\delta+\phi)^2-\delta\phi]B^2\}\alpha_{t+3} + \omega\frac{\{f(B)\}}{(1-\delta B)(1-\phi B)}\alpha_t$$

backsubstituting for α_t:

$$= \omega\{1+(\delta+\phi)B+[(\delta+\phi)^2-\delta\phi]B^2\}\{1-\phi B\}\ z_{t+3} + \omega\frac{\{f(B)\}}{(1-\delta B)}\ z_t$$

$$= L_6(B)\ z_{t+3}\ \text{(unknown at t)} + L_5(B)\ z_t\ \text{(known at t)}$$

$$= \varepsilon'_t(3) + \hat{z}'_t(3)$$

Thus $\hat{z}'_t(3) = L_5(B)\ z_t$ and $\varepsilon'_{t-4}(3) = L_6(B)\ z_{t-1}$

Similarly expand n_{t+4} to obtain $\hat{n}_t(4)$:

$$n_{t+4} = \frac{a_{t+4}}{1-\phi_n B} = a_{t+4} + \frac{\phi}{1-\phi B}\ a_{t+3} \qquad \phi = \phi_n$$

$$= a_{t+4} + \phi\ a_{t+3} + \frac{\phi^2}{1-\phi B}\ a_{t+2}$$

$$= a_{t+4} + \phi\ a_{t+3} + \phi^2\ a_{t+2} + \frac{\phi^3}{1-\phi B}\ a_{t+1}$$

$$= a_{t+4} + \phi\ a_{t+3} + \phi^2\ a_{t+2} + \phi^3\ a_{t+1} + \frac{\phi^4}{1-\phi B}\ a_t$$

$$= (1 + \phi B + \phi^2 B^2 + \phi^3 B^3)\ a_{t+4} + \frac{\phi^4}{1-\phi B}\ a_t$$

$$= L_4(B)\ a_{t+4}\ \text{(unknown at t)} + L_3(B)\ a_t\ \text{(known at t)}$$

$$= e_t(4) + \hat{n}_t(4)$$

Now $\hat{n}_t(4) = L_3(B)\ a_t$ and $e_{t-4}(4) = L_4(B)\ a_t$

Rearranging:

$$\hat{n}_t(4) = \frac{L_3(B)}{L_4(B)} e_{t-4}(4)$$

$$= \frac{L_3(B)}{L_4(B)} (\Sigma_t - \varepsilon'_{t-4}(3))$$

$$= \frac{L_3(B)}{L_4(B)} (\Sigma_t - L_6(B)\ z_{t-1})$$

The control action is described by:

$$-u'_t = -\frac{L_2(B)}{L_1(B)} u_t = L_5(B)\ z_t + \frac{L_3(B)}{L_4(B)} (\Sigma_t - L_6(B)\ z_{t-1})$$

Dropping the subscript ℓ, such that:

$\omega\ \delta\ \phi$ are $\omega_\ell\ \delta_\ell\ \phi_\ell$, referring to the disturbance variable;

$\omega_m\ \delta_m$ refer to the manipulated variable;

ϕ_n refers to the noise;

Substituting in the control equation:

$$-\frac{\omega_m}{1-\delta_m B} u_t = \omega \frac{\{[(\delta+\phi)^2 - \delta\phi][\delta+\phi-\delta\phi B] - (\delta+\phi)\delta\phi\}}{(1-\delta B)} z_t + \frac{\phi_n^4}{1-\phi_n^4 B^4}\{\Sigma_t - \omega[1+(\delta+\phi)B+((\delta+\phi)^2-\delta\phi)B^2][1-\phi B]\ z_{t-1}\}$$

Since $\delta_m \triangleq \delta_\ell = \delta$, $(1-\delta_m B)$ and $(1-\delta B)$ may be considered equal.

Multiply through by $(1-\phi_n^4 B^4)(1-\delta B)/\omega$:

$$-\frac{\omega_m}{\omega}(1-\phi_n^4 B^4)u_t = \{1-\phi_n^4 B^4\}\{[(\delta+\phi)^2-\delta\phi][\delta+\phi-\delta\phi B] - (\delta+\phi)\delta\phi\}\ z_t + \phi_n^4\ \{1-\delta B\}\{\frac{\Sigma_t}{\omega} - [1+(\delta+\phi)B+((\delta+\phi)^2-\delta\phi)B^2][1-\phi B]z_{t-1}\}$$

Some terms will cancel out, but at this stage it is simpler to substitute values:

$$= \{1-.065B^4\}\{[1.38^2-.457][1.38-.457B] - 1.38 \times .457\} z_t +$$

$$.065(1-.83B)\{-1.6\ \Sigma_t - [1+1.38B + (1.38^2-.457)B^2][1-.55B]z_{t-1}\}$$

$$= (-.104 + .0865B)\ \Sigma_t + (1.36-.725B)\ z_t$$

$$\therefore\ (1-.065B^4)\ u_t = +\ 19.9\ [(-.104+.0865B)\ \Sigma_t + (1.36-.725B)\ z_t]$$

$$\therefore\ \underline{u_t = .065\ u_{t-4} - 2.07\ \Sigma_t + 1.71\ \Sigma_{t-1} + 27.2\ z_t - 14.4\ z_{t-1}}$$

(Controller AF)

This controller and the next one are both position-type controllers, having no integral action, and are not convertible to velocity algorithms. $u\ \Sigma\ z$ are deviation variables and for implementation have to be added to the known steady-state physical values of mean steam valve setting, mean tank temperature, and mean disturbance.

Calculation of the theoretical variance of the controlled variable when the controller is in action:

$$\Sigma_t = \varepsilon'_{t-4}(3) + e_{t-4}(4)$$

$$= \omega[1+(\delta+\phi)B + ((\delta+\phi)^2 - \delta\phi)B^2]\alpha_{t-1} + (1+\phi_n B+\phi_n^2 B^2+\phi_n^3 B^3)\ a_t$$

$$= -.625[1+1.38B+1.444B^2]\alpha_{t-1}+(1+.505B+.255B^2+.129B^3)\ a_t$$

$$= \quad \xi(B)\ \alpha_{t-1} \quad + \quad \psi(B)\ a_t$$

$$\mathrm{Var}(\Sigma_t) = \mathrm{Var}\ \varepsilon'_{t-4}(3) + \mathrm{Var}\ e_{t-4}(4) = \sum(\xi^2)\ \mathrm{Var}\ \alpha_t + \sum(\psi^2)\ \mathrm{Var}\ a_t$$

Var α_t is the variance of the residuals α_t after fitting a model to the disturbance series (square root of orifice meter reading), which we did in Chapter 4 before crosscorrelating the disturbance and tank temperature. Var a_t is the variance of the residuals a_t after fitting both transfer function models and the noise model to all series in model 4 (see Table 1)

$$\text{Var}(\Sigma_t) = .39[1+1.9+2.09] \text{ Var } \alpha_t + (1+.255+.065+.0166) \text{ Var } a_t$$

$$= 1.95\text{x}1.487+1.34\text{x}2.617 = 6.4$$

$$\underline{\sigma_\Sigma = 2.53}$$

5.2 Feedback Controller Using an AR(1) Noise Model (Controller AB)

Parameter values were estimated in Model 6 (see Table 1), and the two required models are:

(1) manipulated variable transfer function model

$$u'_{t-4} = \frac{\omega_m}{1-\delta_m B} u_{t-4} = \frac{L_2(B)}{L_1(B)} u_{t-4}$$

(2) noise model:

$$(1-\phi_n B)\, n_t = a_t$$

Now $y_t = u'_{t-4} + n_t$ or, at time of taking action

$$y_{t+4} = u'_t + n_{t+4}$$

To minimise the variance of y_t, we should set $-u'_t = n_{t+4}$ Since the future noise value must be estimated, we set $-u'_t = \hat{n}_t(4)$.

Then $y_{t+4} = u'_t + \hat{n}_t(4) + e_t(4)$ and $e_t(4)$ will be equal to the error or deviation in controlled variable Σ_{t+4}.

Expand
$$n_{t+4} = \frac{a_{t+4}}{1-\phi_n B} = a_{t+4}+\phi_n a_{t+3}+\phi_n^2 a_{t+2}+\phi_n^3 a_{t+1} + \frac{\phi_n^4}{1-\phi_n B} a_t$$

$$= (1+\phi_n B+\phi_n^2 B^2+\phi_n^3 B^3)\, a_{t+4} + \frac{\phi_n^4}{1-\phi_n B} a_t$$

$$= L_4(B)\, a_{t+4} \text{ (unknown at t)} + L_3(B)\, a_t \text{ (known at t)}$$

$$= e_t(4) + \hat{n}_t(4)$$

If $\hat{n}_t(4) = L_3(B)\, a_t$ and $e_{t-4}(4) = L_4(B)\, a_t$

then $$\hat{n}_t(4) = \frac{L_3(B)}{L_4(B)}\, e_{t-4}(4) = \frac{L_3(B)}{L_4(B)}\, \Sigma_t$$

The control action is described by:

$$-u'_t = -\frac{L_2(B)}{L_1(B)}\, u_t = \frac{L_3(B)}{L_4(B)}\, \Sigma_t$$

Substituting:

$$-\frac{\omega_m}{1-\phi_m B}\, u_t = \frac{\phi_n^4}{1-\phi_n^4 B^4}\, \Sigma_t$$

$$(1-\phi_n^4 B^4)\, u_t = -\frac{(1-\delta_m B)}{\omega_m}\, \phi_n^4\, \Sigma_t$$

$$u_t - .336\, u_{t-4} = -\, 12.3\, (\Sigma_t - .84\, \Sigma_{t-1})$$

$$\underline{u_t = .336\, u_{t-4} - 12.3\, \Sigma_t + 10.4\, \Sigma_{t-1}} \quad \text{(Controller AB)}$$

Like the previous controller, this is a position-type which requires the knowledge of mean values of steam valve setting and tank temperature for implementation.

Calculation of the theoretical variance of the controlled variable when the controller is in action:

$$\Sigma_t = e_{t-4}(4) = (1 + \phi_n B + \phi_n^2\, B^2 + \phi_n^3\, B^3)\, a_t$$

$$= (1 + .76B + .578\, B^2 + .439\, B^3)\, a_t = \psi(B)\, a_t$$

$$\text{Var}\ \Sigma_t = \sum (\psi^2)\ \text{Var}\ a_t$$

Var a_t is the variance of residuals a_t after fitting transfer function and noise models to the manipulated and controlled variable series in Model 6.

$$\text{Var } \Sigma_t = (1 + .578 + .334 + .192) \text{ Var } a_t$$

$$= 2.1 \times 3.751$$

$$= 7.88$$

$$\underline{\sigma_\Sigma = 2.81}$$

Controllers using IMA noise models are derived in Appendix 2.

CHAPTER 6

IMPLEMENTATION OF CONTROLLERS

6.1 Apparatus and Procedure

The apparatus was set up as for the identification runs and experimental conditions were adjusted to be as close as possible to the conditions prevailing during identification. The steam control valve packing was well oiled and the valve actuated several times before opening up the steam supply. The constant hot water flowrate was set at 7.2 USGPM, checked by rotameter and rechecked before starting a run. The manual steam valve was set and the pressure between it and the control valve was checked at the same control valve settings as described in Chapter 3. The cold water temperature was not under our control, and if it was different from the identification run a corresponding change in setpoint was made; but this was not critical since each controller would settle the tank temperature to its own mean value, and our criterion of performance was variance about this value. The apparatus was allowed to warm up for at least 30 minutes before each run.

The disturbance signal to the water control valve was calculated from the same model and variance as for the identification run. Controller performance was measured when the "random" number sequence (which could be determined) was (PREV 1) the same as, and (PREV 2, PREV 3) different from that used in the identification run. (A different "random" number sequence is generated when a different initial number is specified.) The timing of output of the disturbance signal in control runs was the same as in the identification run: 7 seconds after reading the A/D channels. Each run had 300 observations (PREV 1), or 160 observations (PREV 2, PREV 3).

6.2 Minicomputer Software

The two programs ACQUI and RANDO used in the identification run were modified slightly for the control runs and a third program AR/IMCON was written to calculate control action. Immediately after reading the time of day and the A/D channels, ACQUI called the controller subroutine AR/IMCON to calculate control valve action based on the latest readings, whereupon the result was immediately output to the steam control valve by ACQUI. Then ACQUI called subroutine RANDO to calculate the disturbance signal from "random" numbers as in the identification run. After a 7 second wait this signal was sent to the water control valve by ACQUI. After a 3 second wait ACQUI returned to read the time of day and A/D channels again. The calculation times of RANDO, AR/IMCON were each less than one millisecond.

Subroutine ARCON used a position-type algorithm suitable for both feedforward-feedback and pure feedback controllers derived using an AR(1) noise model:

$$u_t = a_1 u_{t-1} + a_2 u_{t-2} + \ldots b_o \Sigma_t + b_1 \Sigma_{t-1} + \ldots + c_o z_t + c_1 z_{t-1} + \ldots$$

For feedback control the z-coefficients are all zero.

Subroutine IMCON used a velocity-type algorithm suitable for both feedforward-feedback and pure feedback controllers derived using an IMA noise model:

$$\nabla u_t = a_1 \nabla u_{t-1} + a_2 \nabla u_{t-2} + \dots + b_o \Sigma_t + b_1 \Sigma_{t-1} + \dots +$$

$$+ c_o \nabla z_t + c_1 \nabla z_{t-1} + \dots + d_o z_t$$

The final term is a multiple of the latest deviation in disturbance variable, which is produced in the derivation; in practice this term is not very large. Again for feedback control the ∇z- and z-coefficients are all zero.

The flow charts of ARCON and IMCON are very similar (Appendix 1). ARCON is described here; the flowchart shows where IMCON differs.

The setpoint, the orifice meter dp-cell transmitter reading under zero flow, the mean value of the square root of the net orifice meter reading, and the mean valve signal, were all calculated from identification data and specified in ARCON: (SETPT, NOFLO,ZAV16,MOUT).

When called, using A/D readings from ACQUI, ARCON calculates the latest deviations in tank temperature (Σ_t) and in the square root of the net orifice meter reading, and stores them in a variable list. The square root is calculated by a library subroutine: ISQR. ARCON has only one variable list containing 5 previous u_t values, Σ_t and 4 previous Σ_t values, z_t and 4 previous z_t values. There is one coefficient list.

Pointers are set to each list, and a counter set to the number of variables.

A cyclic procedure calculates the product of each variable and its coefficient and adds it to a double precision sum, which is then divided by 1,000* to obtain u_t. u_t is added to the mean valve signal to obtain the valve signal, which is then restricted if necessary between 0-10 volts. A back-calculation gives the actual u_t and stores it in the variable list. The program then returns to ACQUI.

6.3 Test Runs

ACQUI, RANDO, were used with ARCON to test both feed-forward-feedback, and pure feedback controllers derived in Chapter 5, and with IMCON to test both controllers derived in Appendix 2. A fifth run was done with the steam valve set at a constant 5 volts, to test the apparatus with no control. In all 5 cases the "random" number sequence used in calculating the signal to the disturbance water valve was the same as in the identification run. Results of the 5 test runs are in Table 2(a).

The criterion of performance adopted was the standard deviation of the tank temperature about its mean, which with feedforward-feedback control was smaller than with feedback control, which was smaller than with no control.

The largest standard deviation of the steam control

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* For the reason, see note on Flowchart.

Run No.	B1	B2	B3	B4	B5
Controller type	Forward-Back	Back	0	Forward-Back	Back
Noise model	AR(1)	AR(1)		IMA(013)	IMA(013)
Setpoint	230	225		240	240
Average tank temp.	229.52	224.89	226.95	239.56	240.08
Standard deviation σ_{Σ}	2.48	2.87	3.46	2.34	2.71
Valve setpoint	270	270		270	270
Average valve setting	272.65	270.11	(256)	255.31	254.32
Standard deviation σ_u	34.9	23.5	0	39.8	49.9

TABLE 2(a) Results of controller test runs on different days: 300 readings per run. (PREV 1 was used)

Temperature in A/D units: 3.60 A/D units = 1 Centigrade degree

Valve setting in D/A units: 51 D/A units = 1 Volt

Run Section	1	2	3	4	5	6
Controller type	Back	Forward-Back	0	Back	Forward-Back	0
Noise model	IMA(013)	AR(1)		AR(1)	IMA(013)	
Setpoint	235	235		235	235	
Average tank temp.	234.82	232.84	234.71	233.51	234.65	232.66
Standard deviation σ_Σ	2.70	2.16	3.11	2.62	2.27	3.21
Valve Setpoint	270	270		270	270	
Average valve setting	290.81	274.47	(270)	274.49	281.66	(270)
Standard deviation σ_u	45.2	35.0	0	27.8	41.7	0

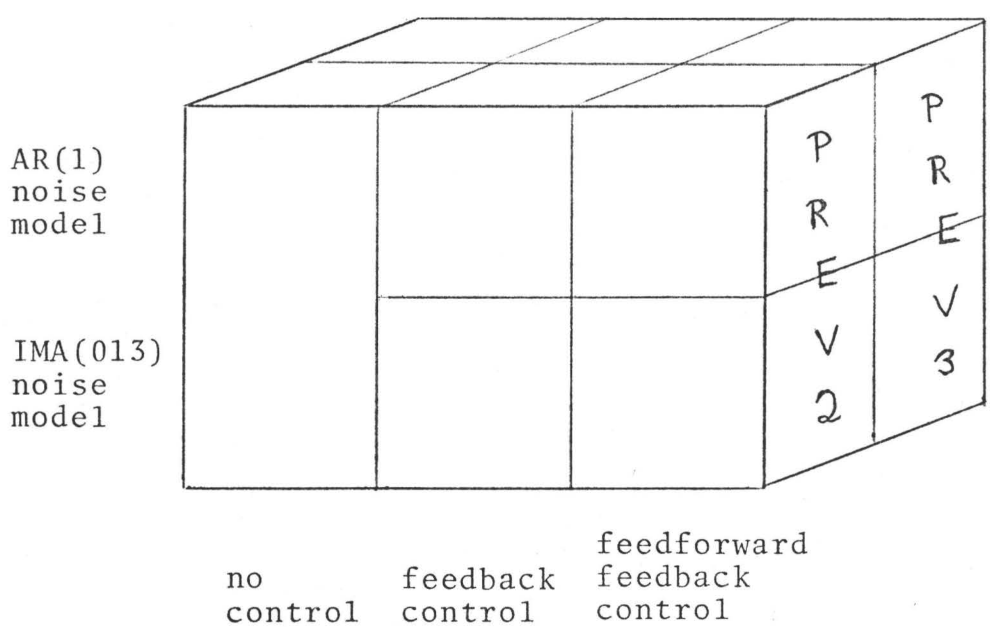
Run section	7	8	9	10*	11*	12
Controller type	Forward-Back	Forward-Back	Back	0	Back	0
Noise model	AR(1)	IMA(013)	AR(1)		IMA(013)	
Setpoint	235	235	235		235	
Average tank temp.	231.37	234.92	232.72	233.96	234.99	231.33
Standard deviation σ_Σ	2.43	2.32	2.96	3.50	3.36	3.27
Valve setpoint	270	270	270		270	
Average valve setting	275.42	288.05	276.82	(270)	300.64	(270)
Standard deviation σ_u	35.7	37.4	22.1	0	40.9	0

TABLE 2(b). Results of continuous control run B6: 160 readings per section (Sections 1-6 used PREV 2; Sections 7-12 used PREV 3).
*N.B. Sections 10 and 11 were adversely affected.

valve setting was 50 D/A units or 1 volt, which means that more than 99% of valve actions were within the linear range of the valve. Therefore it was not necessary to constrain the valve action, which would have called for much more complicated theory.

The above tests both suggested that the controllers were working properly. However since the 5 runs had been done on different days it was possible that unmeasured external disturbances (e.g., water pressure or temperature, steam pressure, or ambient temperature) could have influenced the results, and so a sixth run was planned to test all controllers again, this time using two different "random" number sequences in calculating the disturbance water valve action.

6.4 Final Run



Twelve sections were planned, as shown in the block diagram. The first 6 sections were done using PREV 2 (the second "random" number sequence), and then 6 sections using PREV 3 (the third sequence). Within each group of six the order was picked out of a hat:

	PREV 2						PREV 3					
Warm Up	IB	AF	O	AB	IF	O	AF	IF	AB	O*	IB*	O

Each block was a half hour long, containing 180 readings of which the first 20 (3 min. 20 secs.) were discarded, leaving 160 readings for analysis.

The programs ACQUI, RANDO, AR/IMCON were modified to permit a continuous run through all the above combinations. The constant hot water flow rate had to be adjusted once or twice, but otherwise all went well until the program stopped unexpectedly during section 10 (regime: no control) due to lack of storage space. It was restarted again to complete the run, but the results of sections 10 and 11 were adversely affected. Results of the other sections were good, and are tabulated in Table 2(b).

CHAPTER 7

RESULTS AND DISCUSSION

Tabulated results have been transferred to graphs (Fig. 14) which may be used to compare the following:

(1) The effect on controller performance of a different "random" number sequence in calculating the disturbance water valve signal.

(2) The standard deviation of the controlled variable under no control, pure feedback control, and feedforward-feedback control.

(3) The performance of controllers derived using an AR(1) noise model and using an IMA noise model.

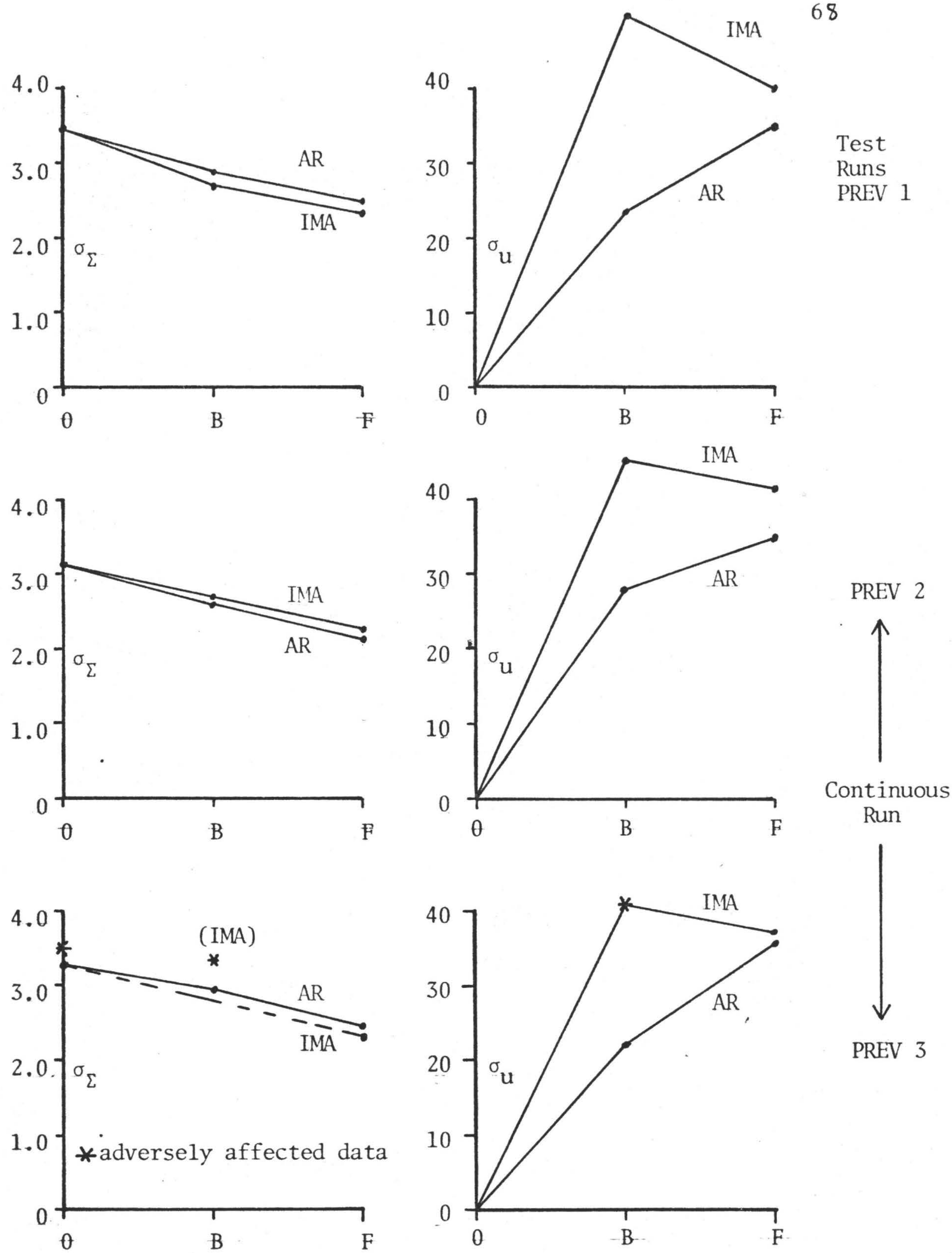
(4) The performance of controllers with their theoretical performance.

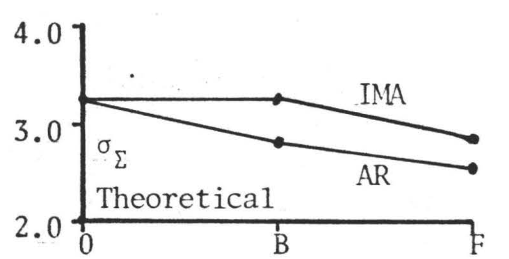
(5) The standard deviation of the manipulated variable for different controllers.

The graphs for the third "random" number sequence (PREV 3) have some starred points, which are the adversely affected results of sections 10 and 11 in the final run. The dotted line shows a trend which has been inferred from the previous graphs.

Taking the above points in order:

(1) If the model form and model parameter values are the same, the use of different random numbers in calculating the disturbance signal in different runs should cause no





σ_u = standard deviation of manipulated variable D/A units; 51 units=1 volt

σ_Σ = standard deviation of controlled variable A/D units; 3.6 units = 1 centigrade degree

Fig. 14

0 No control: open loop

B Pure feedback control

F Feedforward-feedback control

difference in the standard deviation of the controlled variable, provided that the random numbers are drawn from the same distribution. From the graphs it may be seen that the magnitudes of σ_Σ and the trend between controller-types are not very different between runs using different random number sequences, which bears out the above assertion.

(2) In all graphs, σ_Σ for the feedforward-feedback controller is smaller than for the pure feedback controller, which demonstrates the better performance of the feedforward-feedback controller. This is true for controllers derived using both AR(1) and IMA noise models.

The improvement from feedback to feedforward-feedback controller appears to be about as much as from no control to feedback controller. The amount of improvement in either step is not great, because of the lack of precision in forecasting either noise or disturbance 30-40 seconds ahead.

(3) When fitting models by regression, the fits using IMA noise models had an appreciably larger residual variance σ_a^2 than had the fits using AR(1) noise models, which suggests that the former should control with a larger σ_Σ than the latter. However this is not supported by results of runs using the first and second "random" number sequences: σ_Σ values for controllers using AR(1) or IMA noise models are very close and neither is better than the other.

(4) Theoretical standard deviations in the controlled variable have been calculated at the end of the derivation of each controller. Experimental estimates of model parameters

contain errors and therefore theoretical standard deviations calculated from these estimates will be expected to differ from the experimentally obtained standard deviations. They have been plotted in a graph under the experimental graphs. The reduction in σ_Σ from no control to feedback to feed-forward-feedback control follows a trend similar to the experimental. Theoretically σ_Σ is bigger using an IMA noise model, which is not borne out in practice. It is interesting to note that in almost all cases, the controller performance as measured by σ_Σ is better in practice than in theory, but this observation is most likely not statistically significant.

(5) The standard deviation of the manipulated variable is rather different for different controllers, as may be expected from the widely differing algorithms.

(6) The preceding discussion of results has been based entirely on σ_Σ the standard deviation in controlled variable about its <u>mean value</u>. Another basis for comparison is the difference between this mean value and the setpoint, which is tabulated for different controllers in Table 3. The differences are smaller for controllers using an IMA noise model, which demonstrates the effect of the integral action.

Noise model Controller type	AR(1) Forward-Back	Back	IMA (013) Forward-Back	Back
Runs B1-B5	-.48	-.11	-.44	+.08
B6 Sections 1-6	-2.16	-1.49	-.35	-.18
B6 Sections 7-12	-3.63	-2.28	-.08	-.01

TABLE 3 Difference between Setpoint and actual mean tank temperature (A/D Units)

3.60 A/D Units = 1 Centigrade degree

CHAPTER 8

CONCLUSIONS

The object of this study was to demonstrate the use of time series analysis in designing a feedforward-feedback control scheme to compensate for an unavoidable, but measurable, disturbance to a system. A physical process having a manipulated variable capable of affecting the controlled variable was subjected to a repeatable simulated external disturbance. Since the disturbance affected the output before the manipulated variable did, it was necessary to forecast the effect of future disturbance values. Stochastic inputs to the manipulated variable and disturbance variable produced data from which models describing the relationship of each with the controlled variable were identified. From these models was derived a control scheme which made use of the disturbance readings, and also a scheme which did not: feedforward-feedback and pure feedback schemes respectively. Implementation of the feedforward-feedback controller reduced the variance of the controlled variable about its mean value more than did the pure feedback controller, demonstrating the advantage of feedforward control, and successfully testing the Box-Jenkins-MacGregor theory.

The reduction in variance of the controlled variable was in fact not much; 15-20% reduction in σ_Σ was achieved in each of the two steps: from open loop to feedback control, and from feedback to feedforward-feedback control. This was because disturbance values had to be forecasted far ahead: had the forecast been shorter, greater improvement might have been obtained. The simple model chosen for the effect of the manipulated variable was only approximate: had the real process been less complex then its representative model might have been a closer fit. The magnitude of the effect of disturbance on the controlled variable was not great ($\sigma_{\Sigma} \simeq 1^{O}C$): a bigger effect could have been chosen and still been mostly within the linear range of the control valves.

It should be noted that the time constant for the effects of both disturbance and manipulated variable was the same.

Time did not allow the comparison of the method used with 2 other methods which would have been interesting. Pulses and step changes in input and their effect on the output could have provided data to be analysed by classical methods to yield models usable in controller design. Secondly, theoretical models could have been derived to describe the effect of each input on the output, and the data obtained in this study used with a regression routine to obtain model parameters.

In this study it turned out that the variance in manipulated variable did not exceed the linear range of the control valve, and so it was not necessary to constrain the manipulations. However, MacGregor [3] has extended the theory deriving a feedforward controller by time series analysis to include the constraint of the manipulated variable by a state variable technique.

REFERENCES

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APPENDIX I

FLOWCHARTS FOR MINICOMPUTER SOFTWARE

Program ACQUI (Executive)

Enter: Set up TASK to print out data

→ Read time of day and store it in data block

Were all A/D channels read last time? —No→ Error
Yes ↓
Reset to first A/D channel

Call A/D channel reading routine
Read 4 A/D channels and store readings
in data block and in program

Receive message saying A/D channel reading is finished

Identification Version: Call subroutine RANDO } to calculate steam
Control Version: Call subroutine AR/IMCON } valve signal

Identification Version: Send signal from RANDO } via D/A channel
Control Version: Send signal from AR/INCON } to steam valve

Call subroutine RANDO to calculate water valve signal

Wait 7 seconds

Send signal from RANDO via D/A channel to water valve

Wait 3 seconds

TASK: every 10 seconds print out a line of data on teletype:

Time of Day	4 A/D channels		RANDO or AR/IMCON Steam Valve	RANDO Water Valve
	Tank HE(2) HE(1) temperatures	dp Cell	a_t u_t Action	a_t u_t Action
Seconds	A/D Units		D/A Units	D/A Units

Print out lags 30 seconds behind readings.

Calculation times of RANDO, AR/IMCON were each less than 1 millisecond.

Program RANDO

(Calculates valve signals from random numbers.)

External: Library subroutine RAND which calculates "random" numbers.

Enter: Store return address
Set N = -2 for steam valve calculation
→ Get 5 "random" numbers by calling RAND and discard them
Set SUM = 0 and M = 16
→ Get a "random" number from RAND
Divide it by 16
Add the result to SUM
Decrement M
non zero (loop back to: Get a "random" number from RAND) ↓ Zero
From SUM subtract mean 2^{15}
Check sign; make positive if necessary; store sign
Get desired standard deviation of a_t σ_a
Multiply SUM by σ_a
Divide result by standard deviation of SUM $\sigma_r \rightarrow a_t$
Round up or down and restore sign
Store a_t in program and in data block
Get previous value of u_t
Check sign; make positive if necessary; store sign
Get value of ϕ * 100
Multiply u_{t-1} by ϕ * 100
Divide by 100
Round up or down and restore sign
$u_t = \phi * u_{t-1} + a_t$
Store u_t in program and in data block
Get mean valve setting VBAR and add it to $u_t \rightarrow$ Valve action
Store valve action in program and in data block
Increment N
N=-1 (loop back to: Get 5 "random" numbers) First cycle was for steam valve
Second cycle was for water valve
↓ N=0
Return to main program ACQUI

N.B. The above is the version used in Identification.
In the control version, both cycles are completed, but the steam valve cycle values are neither used for output nor stored in the data block nor print out.

Program ARCON/IMCON

(Calculates a steam valve signal using a position algorithm (ARCON) or a velocity algorithm (IMCON), for feedforward-feedback or feedback control.)

External: Library subroutine .ISQR for the square root of an integer

Enter:
Store return address
Get latest reading of tank temperature from ACQUI
Subtract SETPOINT → Σ_t
Store result in variable list
Get latest orifice meter reading from ACQUI
Subtract reading at zero flow (NOFLO)
Restrict result to the range 0 to + 127.
Multiply by 256.
Call library subroutine .ISQR to find latest square root; store result.
Subtract mean value of square root ZAV16 → z_t
Store result in variable list

[IMCON only:
Get latest and previous square root
Store latest square root in location for previous square root for use after next interval
Subtract previous from latest square root → ∇z_t
Store result in variable list]

Set 2 pointers to variable list and 1 pointer to coefficient list
Set sum of (variable * coefficient) to zero
Set N to the number of variables in control algorithm
→ Get a variable. Re-store it down one location for use after next interval.
Get coefficient of variable and check if it is zero
(Yes → go to "Decrement N"; ↓ No)
Multiply variable by coefficient, preserving sign
Add result to or subtract from previous sum and store new sum
→ Decrement N:- last variable?
(↓ No; Yes → go to "Divide sum")
Decrement variable and coefficient pointers (→ back to "Get a variable")
Divide sum of (variable x coefficient) by 1000, rounding up or down, preserving sign

[IMCON:
→ Calculated ∇u_t
Get previous actual value u_{t-1}
$u_t = u_{t-1} + \nabla u_t$]

[ARCON:
→ calculated u_t]

Get mean valve signal MOUT; add calculated u_t → Valve signal
Restrict signal 0-10 volts and store value in program and in data block
Subtract mean valve signal MOUT → actual u_t

[IMCON only:
Get previous actual u_{t-1} and in its location store actual u_t for use after next interval
actual ∇u_t = actual u_t - actual u_{t-1}]

Store result in variable list and in data block
Return to main program ACQUI

NB: ARCON variables are u_t Σ_t z_t

IMCON variables are ∇u_t Σ_t ∇z_t z_t

ARCON/IMCON

Notes:

(1) SETPOINT, ZAV16, and MOUT were specified after calculation from the identification data.

(2) To obtain high enough precision using integer calculations, u_t or ∇u_t, and Σ_t coefficients were multiplied by 1000 before inclusion in AR/IMCON. Hence the need to divide the sum of (variable * coefficient) by 1000.

z_t or ∇z_t coefficients were only multiplied by 62.5, because the variable z_t or ∇z_t itself contains a factor of 16 introduced to obtain maximum precision from the square root subroutine.

(3) Thick lines enclose sections where IMCON differs from ARCON.

APPENDIX 2

DERIVATION OF FEEDFORWARD-FEEDBACK AND FEEDBACK CONTROLLERS USING IMA NOISE MODELS

(For the derivation of controllers using AR(1) noise models, see Chapter 5.)

A2.1 Feedforward-Feedback Controller using an IMA Noise Model (Controller IF)

Parameter values were estimated in Model 5 (see Table 1). Three out of four required models are the same as used with the AR(1) noise model:

(1) Disturbance variable (square root of orifice meter reading) model:

$$(1-\phi_\ell B)\ z_t = \alpha_t$$

(2) Disturbance transfer function model

$$z'_{t-1} = \frac{\omega_\ell}{1-\delta_\ell B}\ z_{t-1}$$

(3) Manipulated variable transfer function model

$$u'_{t-4} = \frac{\omega_m}{1-\delta_m B}\ u_{t-4} = \frac{L_2(B)}{L_1(B)}\ u_{t-4}$$

(4) The noise model is different:

$$\nabla n_t = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)\ a_t$$

As in Chapter 5, the general expression for y_{t+4} is:

$$y_{t+4} = u'_t + \hat{z}'_t(3) + \varepsilon'_t(3) + \hat{n}_t(4) + e_t(4)$$

The control action

$$-u'_t = \hat{z}'_t(3) + \hat{n}_t(4)$$

and the error or deviation in controlled variable

$$\Sigma_t = \varepsilon'_{t-4}(3) + e_{t-4}(4)$$

When z'_{t+3} is expanded in terms of α_t, we obtain expressions for

$$\hat{z}'_t(3) = L_5(B)\ z_t \quad \text{and} \quad \varepsilon'_{t-4}(3) = L_6(B)\ z_{t-1}$$

which are identical to those in Chapter 5 (feedforward-feedback controller).

Expansion of n_{t-4} to obtain $\hat{n}_t(4)$ is different:

$$\nabla n_{t+4} = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)\ a_{t+4}$$

$$n_{t+4} = \frac{(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)}{1-B}\ a_{t+4} = a_{t+4}+\frac{(1-\theta_1-\theta_2 B-\theta_3 B^2)}{1-B}\ a_{t+3}$$

$$= a_{t+4} + (1-\theta_1)\ a_{t+3} + \frac{(1-\theta_1-\theta_2-\theta_3 B)}{1-B}\ a_{t+2}$$

$$= a_{t+4} + (1-\theta_1)\ a_{t+3} + (1-\theta_1-\theta_2) a_{t+2} + \frac{(1-\theta_1-\theta_2-\theta_3)}{1-B}\ a_{t+1}$$

$$= a_{t+4}+ (1-\theta_1) a_{t+3} + (1-\theta_1-\theta_2) a_{t+2}+(1-\theta_1-\theta_2-\theta_3)\ a_{t+1} + \frac{(1-\theta_1-\theta_2-\theta_3)}{1-B}\ a_t$$

$$= [1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3]a_{t+4}+\frac{(1-\theta_1-\theta_2-\theta_3)}{1-B}a_t$$

$$= L_4(B)\ a_{t+4}\quad \text{(unknown at t)} \quad + L_3(B)\ a_t \quad \text{(known at t)}$$

$$= e_t(4) \qquad\qquad + \hat{n}_t(4)$$

Rearranging:

$$\hat{n}_t(4) = \frac{L_3(B)}{L_4(B)}\ e_{t-4}\ (4)$$

$$= \frac{L_3(B)}{L_4(B)}\ (\Sigma_t - \varepsilon'_{t-4}(3))$$

$$= \frac{L_3(B)}{L_4(B)}\ (\Sigma_t - L_6(B)\ z_{t-1})$$

The control action is again described by

$$-u'_t = -\frac{L_2(B)}{L_1(B)}\ u_t = L_5(B)\ z_t + \frac{L_3(B)}{L_4(B)}\ (\Sigma_t - L_6(B)\ z_{t-1})$$

Substituting parameters obtained in Table 1 (Model 5), and dropping the subscript ℓ such that $\omega = \omega_\ell$ $\delta = \delta_\ell$ $\phi = \phi_\ell$

$$\frac{-\omega_m}{1-\delta_m B}\ u_t = \frac{\omega\{[(\delta+\phi)^2-\delta\phi][\delta+\phi-\delta\phi B]-(\delta+\phi)\delta\phi\}}{1-\delta B}\ z_t +$$

$$\frac{(1-\theta_1-\theta_2-\theta_3)}{[1-B][1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3]}\{\Sigma_t-\omega[1+(\delta+\phi)\ B +$$

$$((\delta+\phi)^2-\delta\phi)B^2][1-\phi B]\ z_{t-1}\}$$

Since $\delta_m \triangleq \delta_\ell = \delta$, the expressions $(1-\delta_m B)$ and $(1-\delta B)$ may be considered equal. (1-B) in the denominator of the feedback part will give integral action. We will derive a velocity algorithm.

Multiply through by $[1-B][1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3][1-\delta B]/\omega$:

$$-\frac{\omega_m}{\omega}[1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3]\,\nabla u_t =$$

$$\{1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3\}\{[(\delta+\phi)^2-\delta\phi][\delta+\phi-\delta\phi B]-(\delta+\phi)\delta\phi\}\,\nabla z_t$$

$$+\{1-\delta B\}\{1-\theta_1-\theta_2-\theta_3\}\{\frac{\Sigma_t}{\omega}-[1+(\delta+\phi)B+((\delta+\phi)^2-\delta\phi)B^2][1-\phi B]\,z_{t-1}\}$$

Substituting values obtained from Table 1 (Model 5):

$$\frac{1}{21.75}[1+.703B+.445B^2+.267B^3]\nabla u_t = \{1+.703B+.445B^2+.267B^3\}\{[1.8-.435][1.34-.435B]$$

$$-1.34*.435\}\,\nabla z_t$$

$$+\{1-.79B\}\,.267\{-1.543\,\Sigma_t-[1+1.34B+(1.8-.435)B^2][1-.55B]\,z_{t-1}\}$$

When the right-hand side is multiplied out, and terms in z_t are re-expressed as terms in ∇z_t (plus a residual term in z_t), it becomes:

$$\{1.339+.108B-.038B^2-.109B^3\}\,\nabla z_t-.092\,z_t-.412\,\Sigma_t+.326\,\Sigma_{t-1}$$

Finally: $\underline{\nabla u_t = -.703\,\nabla u_{t-1}-.445\nabla u_{t-2}-.267\nabla u_{t-3}-8.96\Sigma_t+7.09\Sigma_{t-1}}$

$\underline{+\,29.2\nabla z_t+2.42\nabla z_{t-1}-.76\nabla z_{t-2}-2.26\nabla z_{t-3}-2.02z_t}$

(Controller IF)

In this velocity algorithm the required change in steam valve action is expressed in terms of previous changes in valve action, changes in disturbance variable, the latest deviation in disturbance variable, and deviations in controlled variable. The deviations in controlled variable will provide "integral" action and ensure that control is about the set-point. No knowledge of a mean valve setting is required. The mean disturbance is needed only for the term in z_t, whose coefficient is small compared with the ∇z_t coefficients, and so the mean disturbance need be only approximately known. Calculation of the theoretical variance of the controlled variable when the controller is in action:

$$\Sigma_t = \varepsilon'_{t-4}(3) + e_{t-4}(4)$$

$$= \omega[1+(\delta+\phi)B+((\delta+\phi)^2-\delta\phi)B^2]\ \alpha_{t-1}+[1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3]a_t$$

$$= -.648[1+1.34B+1.365B^2]\alpha_{t-1} + [1+.703B+.445B^2+.267B^3]\ a_t$$

$$\text{Var}\ \Sigma_t = \sum(\xi^2)\ \text{Var}\ \alpha_{t-1} + \sum(\psi^2)\ \text{Var}\ a_t$$

$$= 1.95 * 1.487 \quad + 1.763 * 2.933$$

$$= 8.07$$

$$\underline{\sigma_\Sigma = 2.84}$$

A2.2 Pure Feedback Controller using an IMA Noise Model (Controller IB)

Parameter values were estimated in Table 1 (Model 8), and the two required models are:

(1) Manipulated variable transfer function model:

$$u'_{t-4} = \frac{\omega_m}{1-\delta_m B}\ u_{t-4} = \frac{L_2(B)}{L_1(B)}\ u_{t-4}$$

(2) Noise model

$$\nabla n_t = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)\ a_t$$

As in Chapter 5, the equation for y_{t+4} when the disturbance is ignored is:

$$y_{t+4} = u'_t + \hat{n}_t(4) + e_t(4)$$

The control action

$$-\ u'_t = \hat{n}_t(4)$$

and $e_t(4)$ is the error or deviation in controlled variable Σ_t. Expanding n_{t+4} in terms of a_t:

$$n_{t+4} = \frac{(1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)}{1-B}\ a_{t+4} = a_{t+4} + \frac{(1-\theta_1-\theta_2 B-\theta_3 B^2)}{1-B}\ a_{t+3}$$

$$= a_{t+4} + (1-\theta_1)\ a_{t+3} + \frac{(1-\theta_1-\theta_2-\theta_3 B)}{1-B}\ a_{t+2}$$

$$= a_{t+4} + (1-\theta_1)\ a_{t+3} + (1-\theta_1-\theta_2)\ a_{t+2} + \frac{(1-\theta_1-\theta_2-\theta_3)}{1-B}\ a_{t+1}$$

$$= a_{t+4} + (1-\theta_1)\ a_{t+3} + (1-\theta_1-\theta_2)\ a_{t+2} + (1-\theta_1-\theta_2-\theta_3)\ a_{t+1} + \frac{(1-\theta_1-\theta_2-\theta_3)}{1-B}\ a_t$$

$$= [1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3]\ a_{t+4} + \frac{(1-\theta_1-\theta_2-\theta_3)}{1-B}\ a_t$$

$$= L_4(B)\ a_{t+4}\ \text{(unknown at t)} + L_3(B)\ a_t\ \text{(known at t)}$$

$$= \qquad e_t(4) \qquad\qquad + \hat{n}_t(4)$$

Rearranging $\hat{n}_t(4) = \dfrac{L_3(B)}{L_4(B)}\ e_{t-4}(4) = \dfrac{L_3(B)}{L_4(B)}\ \Sigma_t$

and the control action is described by:

$$-u'_t = -\frac{L_2(B)}{L_1(B)} u_t = \frac{L_3(B)}{L_4(B)} \Sigma_t$$

Substituting parameters obtained from Table 1 (Model 8)

$$\frac{-\omega_m}{1-\delta_m B} u_t = \frac{(1-\theta_1-\theta_2-\theta_3)}{[1-B][1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3]} \Sigma_t$$

(1-B) in the denominator will give "integral" action. We derive the velocity algorithm. Multiply through by

$$-[1-B][1-\delta_m B][1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3]/\omega_m:$$

$$[1+(1-\theta_1)B+(1-\theta_1-\theta_2)B^2+(1-\theta_1-\theta_2-\theta_3)B^3] \nabla u_t = -\frac{(1-\theta_1-\theta_2-\theta_3)(1-\delta_m B)}{\omega_m} \Sigma_t$$

Substituting values obtained from Table 1 (Model 8):

$$[1+.944B+.725B^2+.482B^3] \nabla u_t = -\frac{.482}{.0262} (1-.85\ B) \Sigma_t$$

Finally

$$\nabla u_t = -.944\nabla u_{t-1} - .725\ \nabla u_{t-2} - .482\nabla u_{t-3} - 18.4\Sigma_t + 15.6\Sigma_{t-1}$$

(Controller IB)

The change in manipulated variable is expressed only in terms of previous changes in manipulated variable and of the deviation in controlled variable. The latter provides "integral" action, ensuring that control is about the set-point. The mean steam valve setting is not required. Calculation of the theoretical variance of the controlled variable when the controller is in action:

$$\Sigma_t = e_{t-4}(4)$$

$$= [1+(1-\theta_1)\ B + (1-\theta_1-\theta_2)B^2 + (1-\theta_1-\theta_2-\theta_3)B^3]\ a_t$$

$$= [1+ .944B + .725\ B^2 + .482\ B^3]\ a_t$$

$$\text{Var}\ \Sigma_t = \sum(\psi^2)\ \text{Var}\ a_t$$

$$= 2.646 * 3.98$$

$$= 10.53$$

$$\underline{\sigma_\Sigma = 3.25}$$

(Position-type controllers are derived using AR(1) noise models in Chapter 5.)

APPENDIX 3

PROCESS DEVELOPMENT AIDED BY CROSS-CORRELATION

The equipment used in this work was a general purpose laboratory apparatus and some development work was done before carrying out the final identification run A12. The key diagnostic tools used in this were the two input-output crosscorrelation patterns, and their use is illustrated here in studying three practical aspects.

(1) The cross correlation pattern between steam valve and tank temperature for some of the early identification runs showed reasonable crosscorrelations at lags 40 and 50 seconds, but the most significant one was at 100 seconds.

To trace the cause, two extra diagnostic checks were done: one was to crosscorrelate the steam valve setting with the thermocouple readings at the outlets from heat exchangers (1) and (2). These are shown together with the crosscorrelations between the steam valve and tank temperature in Fig. 15.

For HE(1) there was one large crosscorrelation at a lag of 10 seconds, and very little else of significance. For HE(2) there were many crosscorrelations of the same order of significance between lags 40-200 seconds. Much of this was due to the hold-up volume between exchangers, but it was also thought that if condensate were not draining fast from

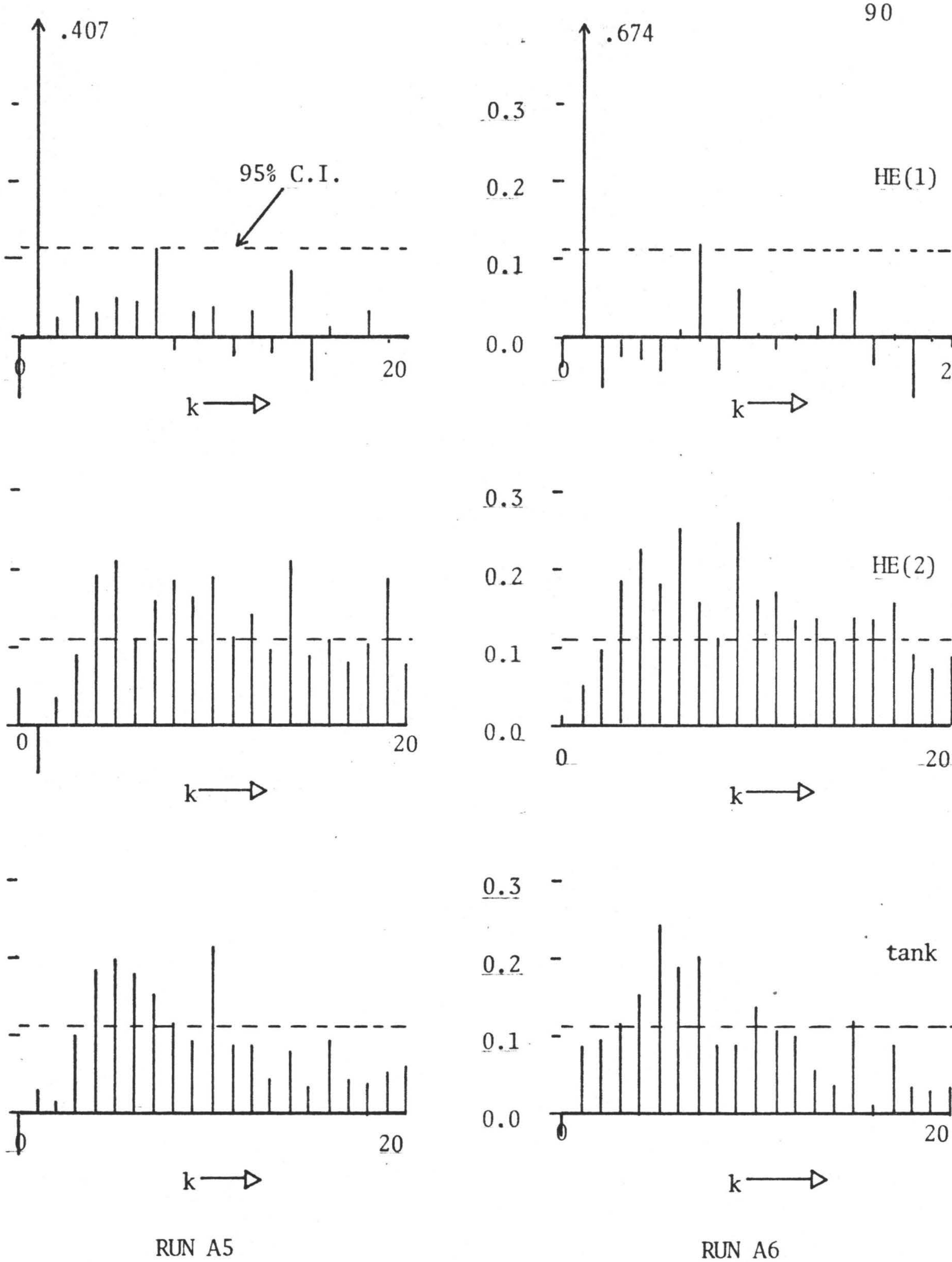

Fig. 15 Crosscorrelations of the Prewhitened Steam Valve Setting with the Transformed Heat Exchanger and Tank Temperatures Before and After Changing Steam Condensate Trap

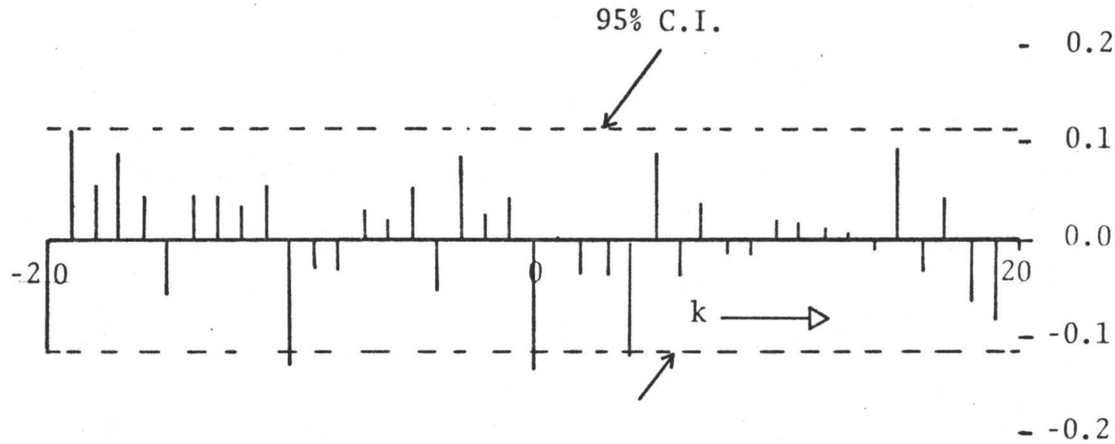

Fig. 16 Crosscorrelations between the Two "Random" Number Sequences used in Calculating Stochastic Inputs to the Two Control Valves in all Identification Runs

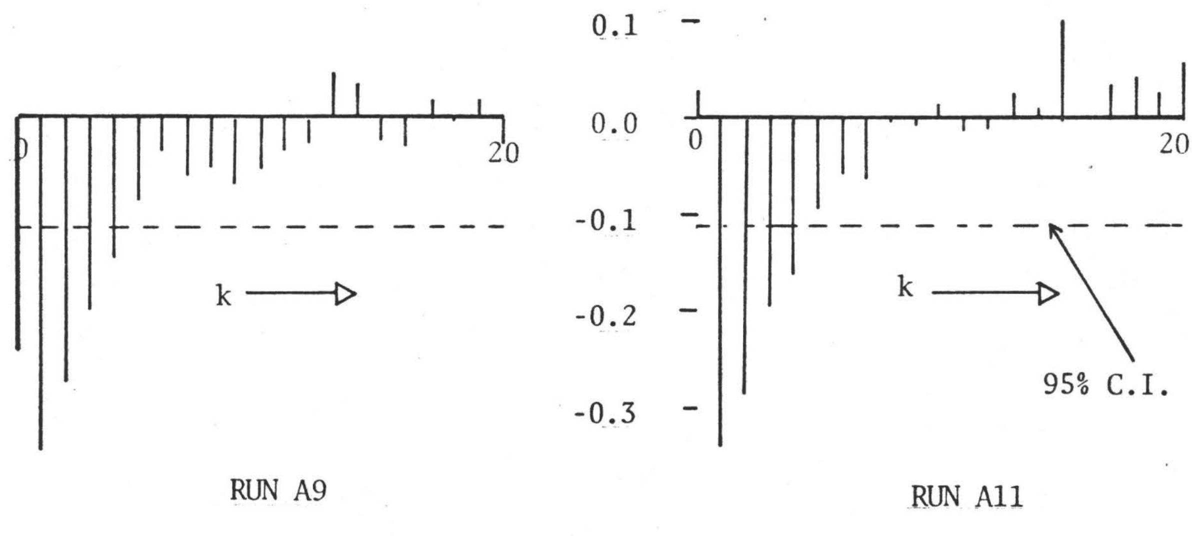

Fig. 17 Crosscorrelations of the Prewhitened Square Root of Orifice Meter Reading and the Transformed Tank Temperature Before and After Putting a 7 Second Delay Before the Water Valve Signal

the second exchanger it might be having an aggravating effect.

The second diagnostic check was to crosscorrelate the two "random" number sequences calculated from RAND as shown in Fig. 16. There were a few just-significant correlations which might have been transferred to the two stochastic input series and contributed to the undulating pattern between steam valve and tank temperature; however the method of obtaining "random" numbers was not altered since correcting condensate back-up appeared more fruitful.

Condensate was thought to be backing up into the second heat exchanger and so the thermodynamic trap was replaced by a continuously operating bucket-type trap positioned further away, which resulted in a definite change, as shown in Fig. 15. The crosscorrelation at 100 seconds lag was still present but much less significant, while others became more significant.

(2) Initially the simulated disturbance signal was output immediately after the reading of the A/D channels. When the orifice meter was read 10 seconds later, the disturbance had already caused a change in tank temperature, as shown by a crosscorrelation between orifice meter and tank temperature at lag zero (Fig. 17). The inference was that in this situation we might as well not read the orifice meter because the information is already contained in the tank temperature: i.e., feedforward control would give little advantage. So the outputting of the disturbance signal was delayed 7 seconds after reading the A/D channels. When the A/D channels were

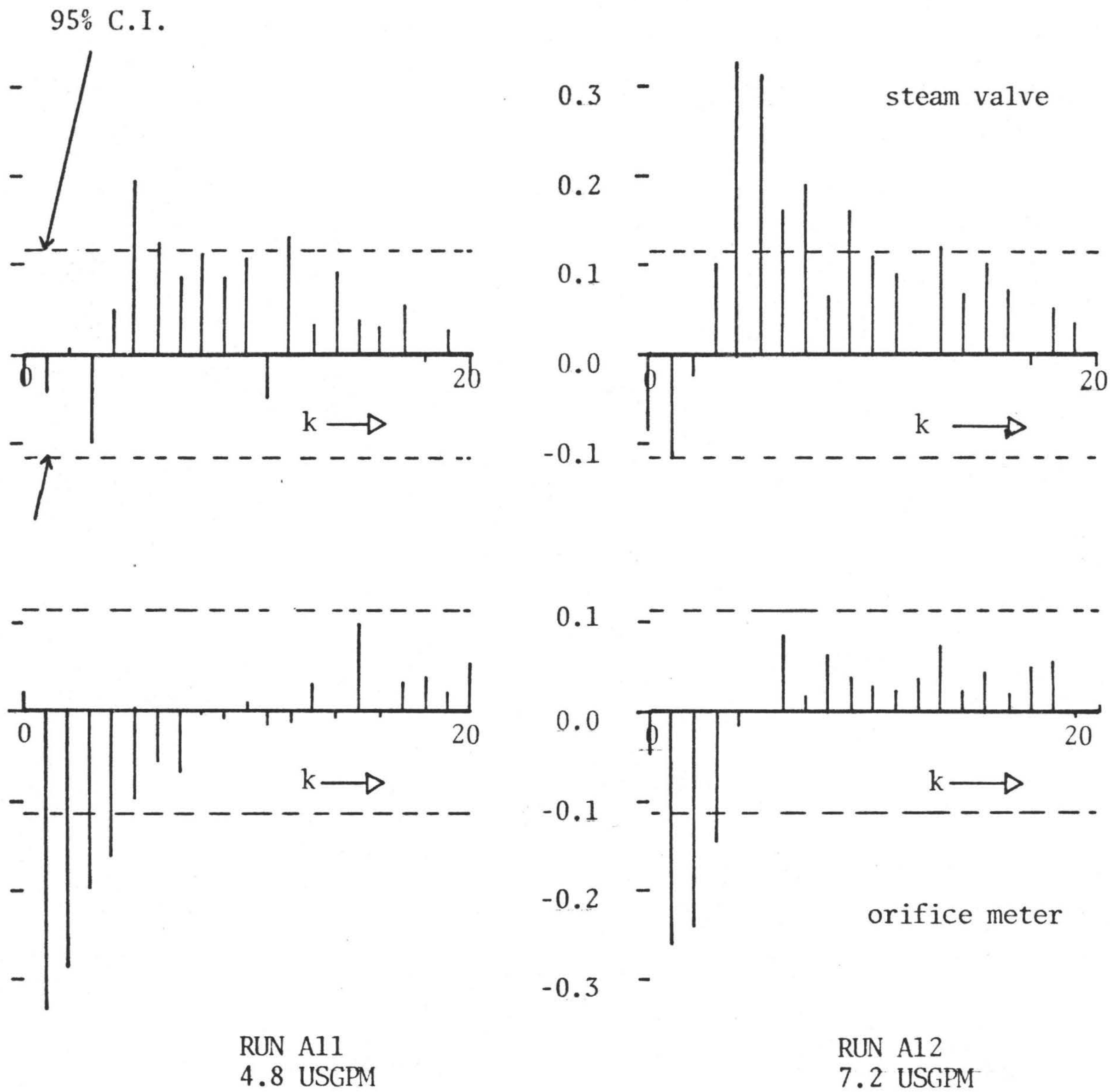

Fig. 18 Crosscorrelations of the Prewhitened Steam Valve Settings and of the Prewhitened Square Root of Orifice Meter Reading with the Transformed Tank Temperature at Different Hot Water Flowrates. RUN A12 Data was Used in the Final Identification

again read 3 seconds after the disturbance signal, the orifice meter had already reacted but the tank temperature had not, as shown in Fig. 17. The steam valve signal was still sent immediately after reading the A/D channels.

(3) Data was collected from two runs with the hot water flow rate set at 4.8 USGPM and 7.2 USGPM. (In the latter run the manual steam valve was throttled to a lesser extent; in both runs the steam control valve was linear.) Fig. 18 shows the effect of this process variable on each of the input-output cross correlation patterns.

APPENDIX 4

Study Procedure Diagram