ELECTROMAGNETIC STRUCTURE OF A BOUND NUCLEON

ELECTROMAGNETIC STRUCTURE OF A BOUND NUCLEON

By

ROLF VOLKER GERSTENBERGER, B.SC.

A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Science

McMaster University

July 1973

MASTER OF SCIENCE (Physics)

McMASTER UNIVERSITY Hamilton, Ontario.

TITLE: Electromagnetic Structure of a Bound Nucleon

AUTHOR: Rolf Volker Gerstenberger

B.Sc. (Saint Lawrence University)

SUPERVISOR: Professor Y. Nogami

NUMBER OF PAGES: (viii), 123

SCOPE AND CONTENTS:

Effects of binding on the electromagnetic structure of a nucleon in a nucleus is examined for the two-pion part of the nucleon form factor. The nucleon is assumed to be bound in a harmonic oscillator potential and also coupled to the pion field through the Chew-Low type interaction. In the tight-binding limit, the nucleon structure approaches that given in the static Chew-Low theory, as expected, while the loose-binding limit gives the free nucleon case with nucleon recoil corrections. Implications of this binding on the magnetic moments of the tri-nucleon systems are investigated, and it is found that for a realistic strength of the harmonic oscillator potential the binding effect is too small to be very significant. Effects of the Pauli Principle are also discussed.

ACKNOWLEDGEMENTS

I am deeply indebted to Dr. Y. Nogami for his constant help and supervision during the progress of this work. I would like to thank the National Research Council of Canada for supporting a year of my study and McMaster University for making my stay here worthwhile. And finally, I would like to thank Miss Erie Long for typing the manuscript.

TABLE OF CONTENTS

TABLE OF CONTENTS - continued

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES - continued

CHAPTER I

INTRODUCTION

The experimental values of the magnetic moments of ^3He and ^3H are known very accurately. However, a clear discrepancy has long existed between the experimental and theoretical values of the magnetic moments of the tri-nucleon systems. In this work we attempted to explain this discrepancy.

Up to the time that this work was begun in early 1972, the theoretical value of the isovector magnetic moment of the tri-nucleon systems was from 6 to 8% below the experimental value. The theoretical value is composed of several factors. One is the expectation value of the one-body magnetic moment operator. However, from the expression for this contribution it becomes apparent that this expression cannot account for the experimental value. In fact, this was the first clear evidence that exchange current effects give an important contribution to the magnetic moment calculations. The exchange current contribution had been investigated in considerable detail, but a discrepancy still remained.

We thought that the electromagnetic structure of a bound nucleon might be appreciably different from that of a free nucleon. To examine this effect of binding, we investigated the two pion process shown in Fig. 8 (p. 68), which gives the two pion contribution to the long range part of the charge and magnetic moment distributions of a nucleon and is believed to be the most important contribution. First we calculated the charge and magnetic moment distributions for a free nucleon, and then we considered two different cases. We took the nucleon as bound in the intermediate state by an effective potential V(S), and calculated the corrections to the charge and magnetic moment densities of the free nucleon. However, this analysis, where we took the static approximation and failed to consider the initial nucleon wave function renormalization led, we now believe, to an incorrect result. We, therefore, proceed to consider the case where the nucleon is bound by a harmonic oscillator potential, and we take account of nucleon recoil. We find that the effect of binding on the electromagnetic structure of a nucleon is very small. For example, the magnetic moment of a proton becomes only 1∿2% larger when it is bound in the triton.

The form of this work is as follows: In Chapter II we start by reviewing what is meant by the electromagnetic structure of a nucleon and how we can experimentally determine if a particle has such a structure. In Chapter III we describe the pion field, the nucleon field and their interactions, all of which are required to explain the structure of a nucleon. Chapter IV gives a summary of the situation in regard to the magnetic moments of the tri-nucleon systems, up to the time when our work began and up to the present time. In fact, most of the outstanding discrepancy between the experimental and theoretical values of the magnetic moments of the tri-nucleon systems has now been explained, not by the effect which we have examined, but by taking into account the S to D state electromagnetic transition probability, which had been neglected until recently. In Chapters V and VI we present our calculations, results and conclusion.

CHAPTER II

DEFINITION OF ELECTROMAGNETIC STRUCTURE

Before beginning an investigation into the electromagnetic structure of nucleons, it is, of course, necessary to define precisely what is meant by the electromagnetic structure of a nucleon. This is most easily accomplished by considering electron-nucleon scattering and in particular, electron-proton scattering, from which most of the experimental information has been obtained.

Using the notation of Appendix I, we calculate the scattering matrix for the scattering of an electron from a proton, where the proton is treated initially as a free structureless Dirac particle (Fig. 1). The electron and nucleon are Dirac particles, i.e., they have spin 1/2 and satisfy the free Dirac equation

$$(i\nabla\!\!\!/ - m)\psi = 0 \qquad [1]$$

where $\nabla\!\!\!/ = \gamma^{\mu}\,\partial/\partial x^{\mu}$, $p_{\mu} = +i\,\partial/\partial x^{\mu}$, m is the mass of the particle, $\hbar = c = 1$, and ψ is the solution of the Dirac equation, of the form

$$\psi^{r}(x) = w^{r}(\underline{p})\,e^{-i\varepsilon_{r}(p_{\mu}x^{\mu})} \qquad [2]$$

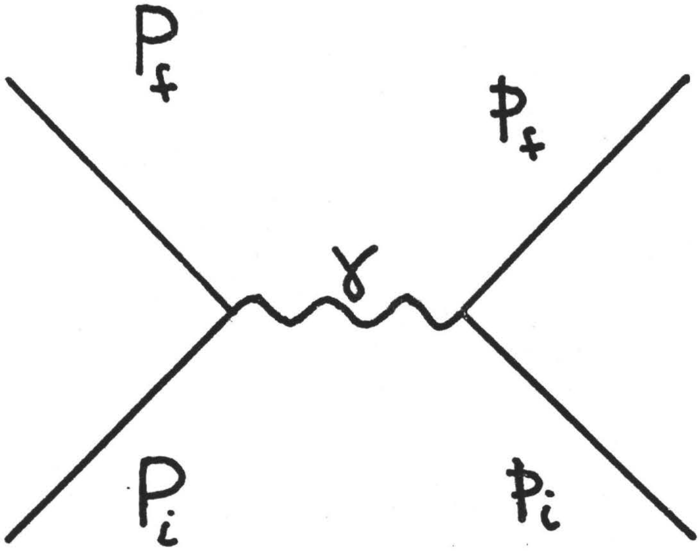


Fig. 1: Electron-proton scattering

where $\varepsilon_r = +1$ for $r = 1, 2$ and $\varepsilon_r = -1$ for $r = 3, 4$; which are the positive and negative energy solutions respectively, $p_\mu x^\mu = Et - \underline{p}\cdot\underline{x}$, and where $w^r(p)$ are the four spinors listed in the Appendix.

If a spin 1/2 particle has a charge $-e$ and interacts with an external field specified by a four-vector potential, $A_\mu(x) = (\phi,\underline{A})$, where ϕ is the scalar potential, and $\underline{A}$ is the (three-component) vector potential, which in the Lorentz gauge satisfies the gauge condition $\partial^\mu A_\mu(x) = 0$, then the Dirac equation describing its motion is obtained by the gauge invariant replacement

$$p_\mu \to p_\mu + eA_\mu \quad . \qquad [3]$$

The Dirac equation then becomes

$$(\not{p} - m)\psi = -eA\psi \quad . \qquad [4]$$

The current for the electron is assumed to be (1)

$$j^e_\mu(x) = -e\overline{\psi}_{ef}(x)\gamma_\mu\psi_{ei}(x) \qquad [5]$$

where e is the charge of the electron and $\overline{\psi}_{ef}$ and ψ_{ei} represent the final and initial plane wave solutions of the Dirac equations for the electron. The corresponding current for the proton is given by

$$j_\mu^p(x) = e\overline{\psi}_{pf}(x)\gamma_\mu\psi_{pi}(x) \quad , \qquad [6]$$

where e is the charge of the proton, $\overline{\psi}_{pf}$ and ψ_{pi} represent the final and initial plane wave solutions of the Dirac equation for the free proton, and j_μ^p describes the current of the Dirac proton, which satisfies the field equation

$$\Box A_\mu(x) = j_\mu^p(x) \qquad [7]$$

where $A_\mu(x)$ describes the electromagnetic field produced by the proton, with the Lorentz gauge.

We can calculate the scattering matrix [2] (S-matrix) for the process shown in Fig. 1 by using perturbation theory, and it is found that

$$S_{fi} = -i\frac{e^2}{V^2}(2\pi)^4\,\delta^4(P_f^\mu - P_i^\mu + p_f^\mu - p_i^\mu)\sqrt{\frac{m^2}{E_f E_i}}\sqrt{\frac{M^2}{\varepsilon_f\varepsilon_i}}$$

$$\times\;[\overline{u}(p_f,s_f)\gamma_\mu u(p_i,s_i)]\,\frac{1}{q^2+i\varepsilon}\,[\overline{u}(P_f,s_f)\gamma^\mu u(P_i,s_i)] \qquad [8]$$

where p_i and p_f are the initial and final four momenta of the electron, respectively, P_i and P_f are the corresponding four momenta for the proton, m and M are the masses of the electron and proton, respectively, E_f, E_i and ε_f, ε_i are the energies of the electron and proton, the u's are the four component spinors for the spin 1/2 particles, and $1/q^2$ is the propagator for the virtual photon which is exchanged, where $q = p_f - p_i = P_i - P_f$.

The basic assumption involved in deducing equation [8] is that the coupling of the proton to the virtual photon is described simply by quantum electrodynamics, where the proton is a point particle of unit charge, with the usual Dirac magnetic moment. However, it has long been known that this simple picture of the proton does not hold since the proton has a magnetic moment of 2.79274 nuclear magnetons, while the neutron has a magnetic moment of -1.91314 nuclear magnetons (nm). The expected values using the above theory are 1 nm and 0 for the proton and neutron, respectively.

The proton current in the scattering matrix is

$$\langle p_f | \int j_\mu^P(x) e^{-iq_\mu \cdot x^\mu} d^4x | p_i \rangle$$

$$= e\sqrt{\frac{M^2}{\varepsilon_i \varepsilon_f}} [\bar{u}(P_f, s_f)\gamma^\mu u(p_i, s_i)] \int d^4x e^{i(p_f - p_i - q)_\mu \cdot x^\mu} \tag{9}$$

where we assume that the photon is absorbed by a point proton. We can, therefore, replace

$$\int d^4x e^{i(p_f - p_i)_\mu \cdot x^\mu} e^{-iq_\mu \cdot x^\mu} \delta(x'-x)$$

$$= \int d^4x' \, d^4x e^{i(p_f - p_i)_\mu \cdot x^\mu} F(x'-x) e^{-iq_\mu \cdot x^{\mu\prime}} \tag{10}$$

which can be rewritten as

$$\int d^4x e^{i(p_f - p_i - q)_\mu \cdot x^\mu} F(q^2) \qquad [11]$$

where

$$F(q^2) = \int d^4y e^{-iq_\mu \cdot y^\mu} F(y) \qquad [12]$$

and (3)

$$F(y) = \int \frac{d^4q}{(2\pi)^4} e^{iq_\mu \cdot y^\mu} F(q^2) \quad . \qquad [13]$$

Here we consider the proton not as a point, but a particle with some spatial extent due to processes that will be considered later. The structure is described by the function $F(x'-x)$, which describes the scattering of a photon from a particle at x' which originated at x.

In addition to the change incorporated into Eq. [11], the correction for the anomalous magnetic moment means that in addition to the $\gamma_\mu A^\mu$ in Eqs. [4] and [8], we must include a Pauli term of the form $\sigma_{\mu\nu}F^{\mu\nu}$, where

$$\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

and [14]

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu = i(q_\nu A_\mu - q_\mu A_\nu) \quad .$$

Combining this with Eq. [13], we obtain

$$\langle P_f | j_\mu^p(x) e^{-iq_\mu \cdot x^\mu} | P_i \rangle$$

$$= e \sqrt{\frac{M^2}{\varepsilon_i \varepsilon_f}} \{ (\bar{u}(P_f, s_f) | \gamma_\mu F_1(q^2)$$

$$+ i \frac{\sigma_{\mu\nu} q^\mu \kappa}{2M} F_2(q^2) | u(P_i, s_i)) \} \qquad [15]$$

where κ is the anomalous magnetic moment of the nucleon in nm. Note that a different spatial extent is associated with the original convection current coupling $\gamma_\mu A^\mu$ and with the subsequently introduced Pauli term, $\sigma_{\mu\nu} F^{\mu\nu}$. This form is the most general form which is allowed for the coupling of a photon to a physical proton by the requirements of gauge invariance and Lorentz invariance (4); it can be shown from the fact that j_μ^p is a Hermitian operator that both $F_1(q^2)$ and $F_2(q^2)$ are real.

The final form of the scattering matrix to lowest order in the electric charge is

$$S_{fi} = - i \frac{e^2}{V^2} (2\pi)^4 (\delta^4 (P_f - P_i + p_f - p_i) \sqrt{\frac{m^2 M^2}{\varepsilon_f \varepsilon_i E_f E_i}}$$

$$\times (\bar{u}(P_f, s_f) | \gamma_\mu F_1(q^2) + i \frac{\sigma_{\mu\nu}}{2M} q^\nu \kappa F_2(q^2) | \bar{u}(P_i, s_i))$$

$$\times \frac{1}{q^2 + i\varepsilon} (\bar{u}(p_f, s_f) \gamma^\mu u(p_i, s_i)) \quad , \qquad [16]$$

while the differential cross section that is obtained from this scattering matrix (commonly called the Rosenbluth cross section) [5] is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2[(F_1^2 - \frac{\kappa^2 q^2}{4m^2} F_2^2)\cos^2\frac{\theta}{2} - \frac{q^2}{4M^2}(F_1 + \kappa F_2^2)\sin^2\frac{\theta}{2}]}{4E^2[1 + (\frac{2E}{M})\sin^2\frac{\theta}{2}]\sin^4\frac{\theta}{2}} \qquad [17]$$

where E denotes the incident energy, θ is the scattering angle of the electron in the laboratory system, and α is the electromagnetic coupling constant, $\alpha = e^2/4\pi$. Individual determinations of F_1 and F_2 can be obtained by comparing measurements taken at different scattering angles and energies, but for the same q^2.

A point proton of charge e and total magnetic moment $(1 + \kappa)$nm is a particle for which $F_1(q^2) = 1$ and $F_2(q^2) = 1$ for all values of q^2. Therefore, a particle has an electromagnetic structure if and only if the function $F_1(q^2)$ and/or $F_2(q^2)$ are not constant. The functions F_1 and F_2 are called the charge and magnetic form factor, respectively, although the form factors

$$G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2 \quad ; \quad G_M = F_1 + \kappa F_2 \ , \qquad [18]$$

which have a more geometrical interpretation, are in wider use at the present time [6].

Of course, the above analysis also holds for the scattering of an electron off of a neutron, which lets us apply the isotopic spin formalism, such that isoscalar and isovector components are:

$$F_1^S = \frac{1}{2}(F_1^p + F_1^n) \qquad (= \frac{1}{2} \quad \text{for } q = 0)$$

$$F_1^V = \frac{1}{2}(F_1^p - F_1^n) \qquad (= \frac{1}{2} \quad \text{for } q = 0)$$

$$F_2^S = \frac{1}{2}(F_2^p + F_2^n) \qquad (= -\ .06 \text{ for } q = 0) \tag{19}$$

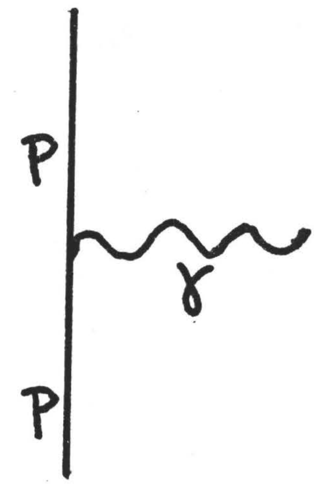
$$F_2^V = \frac{1}{2}(F_2^p - F_2^n) \qquad (= +\ 1.85 \text{ for } q = 0) \quad .$$

CHAPTER III

PION-NUCLEON FIELD THEORY

Now that we have seen what is meant by the electromagnetic structure of nucleons, it is necessary to describe the factors that contribute to this structure. For instance, the anomalous magnetic moment of the nucleon is due to a pion cloud around the nucleon; these virtual pions have an effective current which contributes to the observed anomalous magnetic moment and it also makes it appear that the charge of the proton is distributed in space. Equation [15] implies that if the spatial extent of the proton is due to a virtual meson cloud, the photon could be absorbed at x' by a virtual meson emitted from the proton at x (Fig. 2), which justifies the substitution in Eq. [10].

Similarly, Eq. [16] includes all the effects due to clouds of virtual particles around the proton, and due to any contribution to the proton vertex which does not violate Lorentz and gauge invariance (Fig. 3). To understand how this virtual cloud of mesons affects the charge and magnetic moment distributions, it is necessary to describe the nuclear field, the pion field, the electromagnetic field and their interactions.



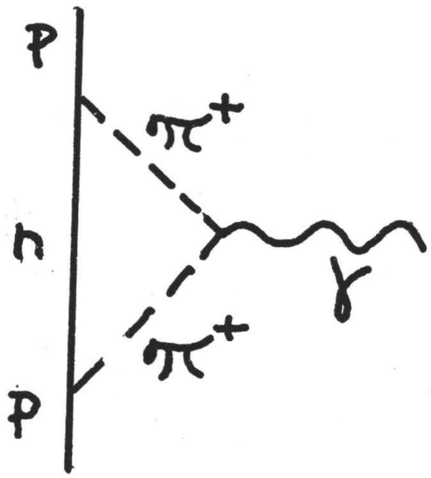


Fig. 2: Electromagnetic field interacting a) with the bare proton and b) with the π^+ surrounding the nucleon

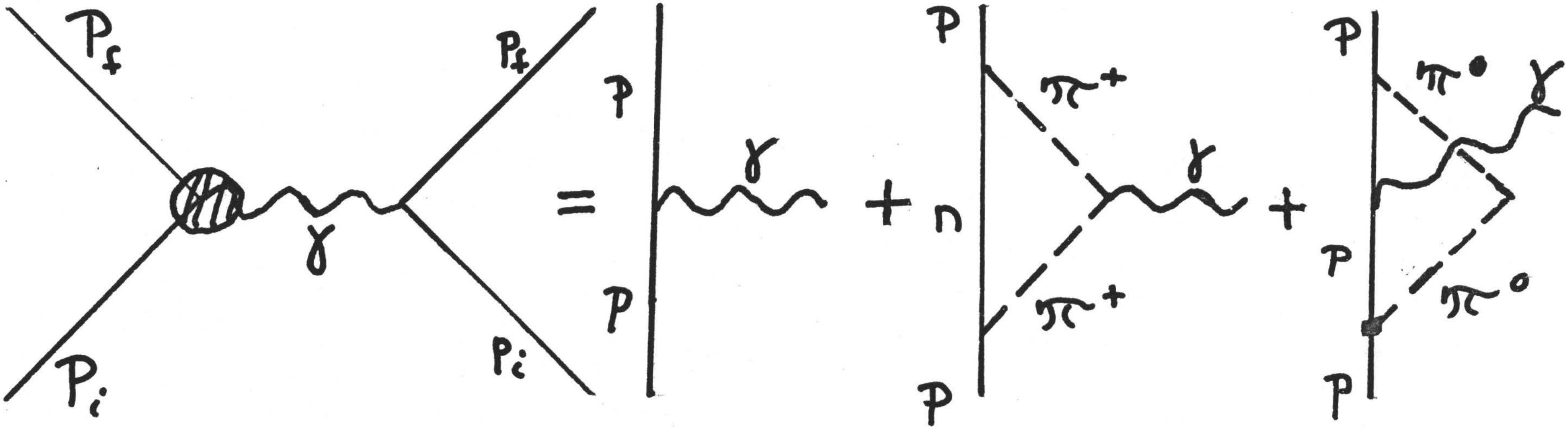


Fig. 3: Various types of electron-proton scattering diagrams, all of which contribute to the total electromagnetic structure of a proton

3.1 The Pion Field

Much of the pion-nucleon interaction is developed in analogy to the electromagnetic interaction. Just as the electron charges are the source of the electromagnetic field, so the nucleons are assumed to be the source of the pion field. The basic rules for describing this field is that in the interaction describing the Hamiltonian of the system, the source term must contain the wave function of the source and it must have the same tensor properties of the field. To determine these properties of the field, it is necessary to investigate the experimental characteristics of the pions.

Accurate values of the masses of the charged pions are obtained from the measurements of the $\pi \to \mu$ decay, and the result is that [7]

$$M(\pi^{\pm}) = (139.59 \pm .05) \text{ MeV} \qquad [20]$$

while the mean life time of the charged pion is

$$\tau(\pi^{\pm}) \sim 2.6 \times 10^{-8} \text{ sec} \qquad [21]$$

which is measured directly from the decay of the π^+ at rest [8]. The equality of the mean lives of the free particles of opposite sign is required under the assumption of invariance under charge conjugation.

The spin of the pions has been obtained from the detailed balance comparison of the reaction [9]

$$p + p \rightarrow D + \pi^+ \qquad [22]$$

and its inverse

$$\pi^+ + D \rightarrow p + p \qquad [23]$$

which requires that the spin of the pion be zero.

Since like nucleons cannot exchange charged pions, the observed charged independence of nuclear forces requires the existence of a neutral pion, π^0. The π^0 has a mass of 135.00 ± .05 MeV and its spin is assumed to be zero, while its mean life is extremely short, approximately 10^{-17} sec [10].

If the reaction [11]

$$\pi^- + D \rightarrow n + n \qquad [24]$$

is considered, and both the balance of spin and parity are studied, it can be concluded that the π^- has odd intrinsic parity in relation to the nucleons. If we arbitrarily assign an even parity to the nucleons, i.e., they are $1/2^+$, then the pions are 0^-. The pion wave function is a state of 0 angular momentum and is invariant under a rotation of the axis but changes sign under a reflection of the axis through the origin. The parity of the π^+ is assumed to be odd because of invariance under charge conjugation. The $\pi^0 \to 2\gamma$ decay determines that the parity of the π^0 is odd. This supports the theory of charge independence [12].

It is necessary to describe this system of pions in a complete field theory description. This is best accomplished by first considering a system of neutral, spin zero particles of mass μ (i.e., the π^0 particle). A real scalar field, $\phi(x)$, which describes this system, satisfies the Klein-Gordon equation

$$(\Box + \mu^2)\phi(x) = 0 \qquad [25]$$

where $\hbar = c = 1$ and the metric and notation are given in Appendix I. The Lagrangian density which gives this equation is

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} - \mu^2\phi^2\right) \quad , \qquad [26]$$

where Eq. [25] is obtained from Eq. [26] using standard field theory techniques, with the conjugate momentum defined as

$$\pi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \dot{\phi} \quad . \qquad [27]$$

Using the well-known canonical quantization procedure, π and ϕ become Hermitian operators satisfying equal-time commutator relations

$$[\phi(\underline{x},t),\phi(\underline{x}',t)] = [\pi(\underline{x},t),\pi(\underline{x}',t)] = 0$$
$$[\pi(x,t),\phi(x',t)] = -i\delta^3(x-x') \quad . \tag{28}$$

The Hamiltonian for this field, obtained from Eqs. [26] and [27] is

$$H = \int d^3x \mathcal{L}(\pi,\phi) \tag{29}$$

where

$$\mathcal{H}(\pi,\phi) = \pi\dot{\phi} - \mathcal{L} = \frac{1}{2}[(\pi(\underline{x},t))^2 + |\nabla\phi(\underline{x},t)|^2 + \mu^2\phi^2(\underline{x},t)] \tag{30}$$

while the momentum operator is

$$P = -\int \pi\nabla\phi d^3x \quad . \tag{31}$$

We have to construct a complete set of state vectors Φ by forming eigenvectors of momentum and energy if we want to describe the properties of the quantized Klein-Gordon field. This can be accomplished by expanding an arbitrary solution of [25] as a Fourier integral over elementary plane wave solutions, such that

$$\phi(\underline{x},t) = \int \frac{d^3k}{\sqrt{(2\pi)^3\ 2w_k}} [a(k)e^{(ik\cdot x - iw_k t)} + a^+(k)e^{-ik\cdot x + iw_k t}] \tag{32}$$

where $w_k = \sqrt{\underline{k}^2 + m^2}$. The amplitudes $a(k)$ and $a^+(k)$ become operators with $a^+(k)$ the Hermitian conjugate of $a(k)$, where $a(k)$ and $a^+(k)$ satisfy the following commutation relations:

$$\begin{aligned} [a(k),a^+(k')] &= \delta^3(k-k') \\ [a(k),a(k')] &= [a^+(k),a^+(k')] = 0 \ . \end{aligned} \tag{33}$$

From Eqs. [29] - [33], we can obtain expressions for the total energy and momentum for the free Klein-Gordon field,

$$\begin{aligned} H &= \frac{1}{2}\int d^3k\ w_k[a^+(k)a(k) + a(k)a^+(k)] \\ \underline{P} &= \frac{1}{2}\int d^3k\ \underline{k}[a^+(k)a(k) + a(k)a^+(k)] \ . \end{aligned} \tag{34}$$

From Eq. [29] we see that H is a continuous sum of terms

$$H_k = \frac{1}{2} w_k [a^+(k)a(k) + a(k)a^+(k)] \qquad [35]$$

which is the expression for a Hamiltonian for a simple harmonic oscillator of frequency w_k. The $a^+(k)$ and $a(k)$ are the rising and lowering operators. To go to discrete notation,

$$\int d^3k \to \sum_k \Delta V_k \quad \text{and} \quad \delta^3(\underline{k}-\underline{k}') = \frac{\delta_{kk'}}{\Delta V_k} \qquad [36]$$

so that

$$H = \sum_k H_k = \sum_k \frac{1}{2} w_k [a_k^+ a_k + a_k a_k^+] \quad , \quad a_k = \sqrt{\Delta V_k}\, a(k)$$

$$[a_k, a_{k'}^+] = \delta_{kk'} \quad , \quad [a_k, a_{k'}] = [a_k^+, a_{k'}^+] = 0 \quad . \qquad [37]$$

Since H is a sum of mutually commuting terms, H_k for each wave number $\underline{k}$ and frequency $w_k = (\underline{k}^2 + m^2)^{1/2}$, the energy eigenfunctions will be products of eigenfunctions ϕ_k of each H_k. General state vectors Φ can be built from a superposition of such products over all $\underline{k}$ values.

The solution to the oscillator eigenvalue problem for each $\underline{k}$ may be characterized by an integer $n_k = 0, 1, \ldots$ in terms of which the energy eigenfunction and eigenvalue are

$$H_k \phi_k(n_k) = w_k(n_k + \frac{1}{2}) \Phi_k(n_k) \qquad [38]$$

$$\Phi_k(n_k) = \frac{1}{\sqrt{n_k!}} (a_k^+)^{n_k} \Phi_k(0) \qquad [39]$$

where $\Phi_k(0)$ is the ground state, defined by

$$a_k \Phi_k(0) = 0 \qquad [40]$$

and the states are normalized to

$$(\Phi_k(n_k), \Phi_k(n_k')) = \delta_{n_k, n_k'} \quad . \qquad [41]$$

The momentum operator may be decomposed as

$$\underline{P} = \sum_k \underline{P}_k = \sum_k \frac{1}{2} \underline{k}(a_k^+ a_k + a_k a_k^+) \qquad [42]$$

with

$$\underline{P}_k \phi_k(n) = \underline{k}(n_k + \frac{1}{2}) \Phi_k(n_k) \qquad n_k = 0, 1, 2, \ldots$$

The energy momentum eigenfunctions Φ are products of the ϕ_k for each momentum cell, and are characterized by integers n_k for each $\underline{k}$:

$$\Phi(n_{k_1} \ldots n_{k_\alpha} \ldots) = \prod_k \phi_k(n_k)$$
$$P^\mu \Phi(\ldots n_k\ ) = \sum_k k^\mu (n_k + \tfrac{1}{2})\ (\ldots n_{k_\alpha} \ldots) \quad . \qquad [43]$$

The ground state is the state of lowest energy, i.e., the state with all $n_{k_\alpha} = 0$; such that

$$\Phi_0 = \prod_k \phi_k(0) \qquad [44]$$

with an energy

$$E_0 = \sum_k \frac{1}{2} w_k \quad , \qquad [45]$$

which is infinite, but this divergence is easily removed by subtracting an infinite constant from H to cancel E_0, so that the energy-momentum operator becomes

$$P'_\mu = P_\mu - (\phi_0, P_\mu \phi_0) = \sum k_\mu a_k^+ a_k \quad , \qquad [46]$$

or

$$P'_\mu = \int d^3k\ k_\mu\ a^+(k)a(k) \qquad [47]$$

for the continuum, where $P_0 = w_k$, and $P_1 = k_1$, etc. such that $\underline{P} = \underline{k}$. From Eqs. [43] and [46] we find that the eigenvalues of P'_μ are

$$P'_\mu \Phi(\ldots\, n_{k_\alpha}\, \ldots) = \sum_k n_k\, k_\mu \Phi(\ldots\, n_{k_\alpha}\, \ldots) \quad ,$$
$$n_k = 0,\ 1,\ 2,\ \ldots \qquad [48]$$

The different eigenstates for each normal mode k carry four momenta corresponding to n_k quanta, each with four momentum k^μ and mass μ according to the Einstein relation

$$k_\mu k^\mu = E^2 - k^2 = \mu^2 \quad , \qquad [49]$$

where a particle picture of the field emerges, because n_k is called the occupation number of the k'th momentum state, and by specifying the numbers of quanta n_{k_α}, we get a complete description of the eigenstate $\Phi(\ldots\, n_{k_\alpha}\, \ldots)$.

To further facilitate the presentation of this field theory approach to meson theory, we introduce a number operator

$$N = a_k^+ a_k \qquad [50]$$

with integer eigenvalues

$$N_k \Phi(\ldots\, n_k\, \ldots) = n_k \Phi(\ldots\, n_k\, \ldots) \quad , \quad n_k = 0,\ 1,\ 2,\ \ldots \qquad [51]$$

which gives

$$P^{\mu} = \sum_{k} k^{\mu} N_k \quad , \tag{52}$$

where N_k satisfies the commutation relations:

$$[N_k, a^{+}_{k'}] = \delta_{kk'}\, a^{+}_{k'} \quad , \quad [N_k, a_{k'}] = -\,\delta_{kk'}\, a_{k'} \quad . \tag{53}$$

This relation, along with Eq. [52] implies that a^{+}_{k} is a creation operator for a quantum of momentum k^{μ} because it produces a state with n_{k+1} quanta of this momentum from a state with n_k such quanta; e.g.,

$$\begin{aligned} P_{\mu} a^{+}_{k} \phi(\ldots\, n_k \,\ldots) &= a^{+}_{k} [P_{\mu} + k_{\mu}] \Phi(\ldots\, n_k \,\ldots) \\ &= [\sum_{k'} n_{k'} k'_{\mu} + k_{\mu}\,] a^{+}_{k} \Phi(\ldots\, n_k \,\ldots) \; . \end{aligned} \tag{54}$$

Similarly, a_k destroys a quantum with k_{μ}, and if it acts on a state with zero quanta, $a_k \phi_k(0) = 0$. From harmonic oscillator theory, we know that a_k and a^{+}_{k} connect states that differ by one quanta,

$$\begin{aligned} (\Phi_k(n'_k), a_k \Phi_k(n_k)) &\equiv \langle n'_k | a_k | n_k \rangle = \sqrt{n_k}\, \delta_{n_{k'}, n_{k-1}} \\ \langle n'_k | a^{+}_{k} | n_k \rangle &= \sqrt{n_{k+1}}\, \delta_{n_{k'}, n_{k+1}} \end{aligned} \tag{55}$$

Up to this point, we have been concerned only with the description of a neutral pion field. We now have to extend this to a description of a charged scalar field, i.e., a π^+, π^- system. A charged particle is described in terms of a complex wave function

$$\phi(x) = \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x)) \qquad [56]$$

with ϕ_1 and ϕ_2 real. First consider two identical non-interacting real fields which satisfy the Klein-Gordon field equations

$$(\Box + \mu_1^2)\phi_1(x) = 0 \quad , \quad (\Box + \mu_2^2)\phi_2(x) = 0 \qquad [57]$$

which follow from the Langrangian density

$$\mathcal{L} = \frac{1}{2} (\frac{\partial\phi_1}{\partial x_\mu} \frac{\partial\phi_1}{\partial x^\mu} - \mu_1^2\phi_1^2 + \frac{\partial\phi_2}{\partial x_\mu} \frac{\partial\phi_2}{\partial x^\mu} - m_2^2\phi_2^2) \qquad [58]$$

where the canonical momenta are

$$\pi_1 = \dot{\phi}_1 \quad , \quad \pi_2 = \dot{\phi}_2 \quad , \qquad [59]$$

and both ϕ and π satisfy the same canonical commutation relations as the neutral scalar case. The numbers of particles 1 and 2 are separately conserved in the absence of interaction terms, and we can label states by the eigenvalues of the number operators:

$$N_1(k) = a_1^+(k)a_1(k) \quad , \quad N_2(k) = a_2^+(k)a_2(k) \quad . \qquad [60]$$

As a special case where $\mu_1 = \mu_2 = \mu$ (which holds for the charged pions), we may replace ϕ_1 and ϕ_2 by

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \quad , \quad \phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \qquad [61]$$

where ϕ and ϕ^* satisfy the Klein-Gordon equation, and the Lagrangian density becomes

$$\mathcal{L} = \frac{\partial\phi^*}{\partial x_\mu}\frac{\partial\phi}{\partial x^\mu} - \mu^2\phi^*\phi \quad , \qquad [62]$$

and the canonical momenta for these coordinates are

$$\pi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \dot{\phi}^* = \frac{\dot{\phi}_1 - i\dot{\phi}_2}{\sqrt{2}}$$
$$\pi^* = \frac{\partial\mathcal{L}}{\partial\dot{\phi}^*} = \dot{\phi} = \frac{\dot{\phi}_1 + i\dot{\phi}_2}{\sqrt{2}} \qquad [63]$$

The Hamiltonian density is found to be

$$\mathcal{H} = \pi\dot{\phi} + \pi^*\dot{\phi}^* - \mathcal{L} = \pi^*\pi + (\underline{\nabla}\phi^*)\cdot(\underline{\nabla}\phi) + m^2\phi^*\phi \quad . \qquad [64]$$

The commutation relations at equal times become

$$[\pi(\underline{x},t),\phi(\underline{x}',t)] = [\pi^*(\underline{x},t),\phi^*(\underline{x}',t)]$$

$$= -\, i\delta^3(\underline{x}-\underline{x}') \quad . \qquad [65]$$

Following the form of Eq. [32], we see that the Fourier transform of the solutions to the Klein-Gordon equation in k space are

$$\phi(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3\ 2w_k}} [a_+(k)e^{-ik\cdot x} + a_-^+(k)e^{ik\cdot x}]$$

$$\phi^*(x) = \int \frac{d^3k}{(2\pi)^{3/2}\ (2w_k)^{1/2}} [a_+^+(k)e^{ik\cdot x} + a_-(k)e^{-ik\cdot x}] \qquad [66]$$

where

$$a_+(k) = \frac{1}{\sqrt{2}} (a_1(k) + ia_2(k))$$

$$a_+^+(k) = \frac{1}{\sqrt{2}} (a_1^+(k) - ia_2^+(k))$$

$$a_-(k) = \frac{1}{\sqrt{2}} (a_1(k) - ia_2(k)) \qquad [67]$$

$$a_-^+(k) = \frac{1}{\sqrt{2}} (a_1^+(k) + ia_2^+(k)) \quad .$$

The commutation relations for the $a_{\pm}(k)$ are readily constructed to be

$$[a_+(k), a_+^+(k')] = [a_-(k), a_-^+(k')] = \delta^3(k-k')$$

$$[a_+(k), a_-^+(k')] = [a_-(k), a_+^+(k')] = 0 \qquad [68]$$

$$[a_{\pm}(k), a_{\pm}(k')] = [a_{\pm}^+(k), a_{\pm}^+(k')] = 0$$

where new number operators can be formed,

$$N_k^+ = a_{+,k}^+ \, a_{+,k} \quad , \qquad N_k^- = a_{-,k}^+ \, a_{-,k} \qquad [69]$$

such that

$$P_\mu = \sum_k k_\mu (N_k^+ + N_k^-) \quad . \qquad [70]$$

The operators $a_{\pm,k}$ are destruction-operators for the + and - quanta of momentum k, respectively, and $a_{\pm,k}^+$ the corresponding creation operators.

Solutions to the Klein-Gordon equation satisfy a continuity equation

$$\frac{\partial j_\mu(x)}{\partial x^\mu} = \frac{\partial}{\partial x^\mu} \left(i\phi^* \frac{\partial \phi}{\partial x_\mu} - i\phi \frac{\partial \phi^*}{\partial x_\mu}\right) = 0 \qquad [71]$$

which gives, after applying the divergence theorem,

$$Q = \int d^3x\, j_0(x) = i \int d^3x(\phi^*\dot{\phi} - \phi\dot{\phi}^*) = \text{constant}\,, \quad [72]$$

i.e., the charge is conserved. In the theory presented here,

$$Q = \int d^3k[a_+^+(k)a_+(k) - a_-^+(k)a_-(k)] \quad , \quad [72]$$

or if stated in the discrete notation,

$$Q = \sum_k (N_k^+ - N_k^-) \quad , \quad [74]$$

and from [45] and [47],

$$[Q, P_\mu] = 0 \quad . \quad [75]$$

From Eq. [51], it is seen that the + and - quanta carry +1 and -1 units of charge Q, respectively. Since $[P_\mu, a_+^+(k)] = + k_\mu a_+^+(k)$ and $[Q, a_+^+(k)] = + a_+^+$, a_+^+ is an operator which increases the energy of the system by k_μ and the charge by +1, while a_+ annihilates such a quantum. Similarly, a_-^+ creates a particle of energy k_μ and charge -1, while a_- annihilates it. For our case, a_+^+ creates a π^+ from the vacuum and a_-^+ creates a π^- pion. (The formulation of pion field theory can be found in many books on Quantum

Field Theory [13]; (in particular, Bjorken and Drell, Relativistic Quantum Fields, Chapter 12, McGraw-Hill, Inc., 1968)).

Much of the theoretical work with this pion field theory assumes that $\mu_{\pi^+} = \mu_{\pi^-} = \mu_{\pi^\pm}$ and then considers the field $\phi_0 = \phi_3$, ϕ_1 and ϕ_2 as the components of a vector $\underline{\phi}$ in some isotopic charge space, where the Lagrangian from Eqs. [26] and [58] becomes

$$\mathcal{L} = -\frac{1}{2}\sum_{j=1}^{3}[\mu_j^2\phi_j\phi_j - \phi_{j\mu}\phi_j^{\mu}] \quad . \tag{76}$$

If we let T_3 be the operator which generates infinitesimal rotations about the "3 axis",

$$\begin{aligned} e^{-i\varepsilon T_3}\phi_1 e^{+i\varepsilon T_3} &= \phi_1 + \varepsilon\phi_2 \\ e^{-i\varepsilon T_3}\phi_2 e^{+i\varepsilon T_3} &= -\varepsilon\phi_1 + \phi_2 \\ e^{-i\varepsilon T_3}\phi_3 e^{+i\varepsilon T_3} &= \phi_3 \quad , \end{aligned} \tag{77}$$

since for infinitesimal ε

$$e^{-i\varepsilon T_3}\phi_i(x)e^{+i\varepsilon T_3} \simeq \phi_i(x) - i\varepsilon[T_3,\phi_i(x)] \quad . \tag{78}$$

We see then, that

$$i[T_3,\phi_1] = -\phi_2$$

$$i[T_3,\phi_2] = \phi_1 \qquad [79]$$

$$i[T_3,\phi_3] = 0 \quad ,$$

so a possible choice for T_3 is

$$T_3 = -\int \{\pi_1(x)\phi_2(x) - \pi_2(x)\phi_1(x)\}d^3x \quad , \qquad [80]$$

which equals, from Eq. [72]

$$T_3 = \frac{1}{e}Q = \frac{1}{e}\int d^3x\rho(x) \quad . \qquad [81]$$

If we assume that the masses of the three pions are equal, then $\mathcal{L}$ is invariant under rotation in isotopic spin space, and the components T_i, i = 1, 2, 3 are the components of a vector $\underline{T}$ with

$$\underline{T} = -\int d^3x(\underline{\pi}(x) \times \underline{\phi}(x)) \quad , \qquad [82]$$

in analogy with Eq. [80]. The commutation rules of the operators T_i are deduced from Eq. [28], such that

$$[T_\ell , T_j] = \int d^3x \sum_{r,s,m,n=1}^{3} \varepsilon_{\ell rs}\varepsilon_{jmn}[\pi_r(x)\phi_s(x), \pi_m(x')\phi_n(x')]$$

$$= i\varepsilon_{\ell jm}\, T_m \quad . \qquad [83]$$

There is a great similarity between this formalism and that of the angular momentum, and in fact, the operators T_i satisfy the same commutation relationship as the angular momentum operators, such that $\underline{T}^2$ and T_3 can be chosen diagonal. One speaks of the π meson system as having total isotopic spin +1, with the three different charge states given by $t_3 = 0, \pm 1$, i.e., if the eigenvalue of $\underline{T}^2$ is denoted by $t(t+1)$, then $t = 1$, and

$$T_3|\phi_0\rangle = 0$$

$$T_3|\pi^+\rangle = T_3(\phi_1 + i\phi_2)|\phi_0\rangle = +\,1|\pi^+\rangle \qquad [84]$$

$$T_3|\pi^-\rangle = T_3(\phi_1 - i\phi_2)|\phi_0\rangle = -\,1|\pi^-\rangle$$

where $|\phi_0\rangle$ is the state of one neutral pion, $|\pi^+\rangle$ is the state of one π^+ particle, etc.

3.2 The Nucleon Field

A similar analysis can hold for the nucleon system, the neutron and the proton. There is much evidence that to the approximation that electromagnetic and weak interactions can be ignored, protons and neutrons have identical properties. They both have spin 1/2, they have the same space parity, taken to be +1 and very nearly the same mass. So, for this system, the isotopic spin is similar to an angular momentum 1/2 system, that is, a 2 component system. The eigenvalues for T_3 are, therefore, ±1/2 such that

$$\begin{aligned} T_3|p\rangle &= \frac{1}{2}\,|p\rangle \\ T_3|n\rangle &= -\,\frac{1}{2}\,|n\rangle \\ T_3|\bar{p}\rangle &= -\,\frac{1}{2}\,|\bar{p}\rangle \\ T_3|\bar{n}\rangle &= \frac{1}{2}\,|\bar{n}\rangle \end{aligned} \qquad [85]$$

where $|\bar{p}\rangle$ and $|\bar{n}\rangle$ are the proton and neutron antiparticle states.

A quantized view of the pion field has been presented, where the field is composed of pions of various charges, momenta and energies which can be created or destroyed, and which satisfy Bose statistics, i.e., their creation and annihilation operators satisfy commutation rules. It will be beneficial to apply this same formalism to the spin 1/2 particles, and in particular, to the nucleon system, composed of protons and neutrons. The state of the nucleon field, or n particle system, is described by the number of quanta in each single particle state. But for fermions, or spin 1/2 particles, the wave functions are anti-symmetric, and the occupation numbers for each state can only be 0 or 1, as opposed to the pion case, where there was no restriction on the number of particles in a state. We will, therefore, introduce creation and annihilation operators for the nucleon field, but they will satisfy anti-commutation rules rather than the commutation rules of the Boson field.

Another difference between the nucleon field and the scalar pion field is that the Dirac equation allows for negative energy solutions, which, if extended to the quantized field theory point of view, implies that there exist negative energy particles, or anti-particles, which have been observed experimentally.

The solution to the free Dirac equation (Eq. [1]) can be written as

$$\psi(\underline{x},t) = \sum_{\pm s} \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{E_p}} \, [b(p,s)u(p,s)e^{-ip_\mu \cdot x^\mu} + d^+(p,s)u(p,s)e^{ip_\mu \cdot x^\mu}]$$

$$\psi^+(\underline{x},t) = \sum_{\pm s} \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{E_p}} \, [b^+(p,s)u(p,s)e^{-ip_\mu \cdot x^\mu} + d(p,s)u(p,s)e^{ip_\mu \cdot x^\mu}] \qquad [86]$$

where $E_p = p_0 = \sqrt{|\underline{p}|^2 + m^2}$ and $u(p,s)$ and $v(p,s)$ satisfy the relations given in Appendix I , while ψ and ψ^+ satisfy the following anti-commutation relation:

$$[\psi_\alpha(\underline{x},t),\ \psi^+_\beta(\underline{x}',t')]_+ = \delta^3(\underline{x}-\underline{x}')\delta_{\alpha\beta}$$

$$[\psi(\underline{x},t), \psi(\underline{x}',t)]_+ = [\psi^+(\underline{x},t),\ \psi^+(x',t)]_+ = 0 \quad . \qquad [87]$$

The energy and momentum of the system then become

$$H = \int \psi^+(i\not{\partial} - m)\psi d^3x$$

$$= \sum_{\pm s} \int d^3pE_p[b^+(p,s)b(p,s) - d(p,s)d^+(p,s)] \qquad [88]$$

and

$$\underline{P} = \int \psi^{+}(-i\underline{\nabla})\psi d^3x$$

$$= \sum_{\pm s} \int d^3p\underline{p}[b^{+}(p,s)b(p,s) - d(p,s)d^{+}(p,s)] \quad , \qquad [89]$$

from which it is obvious that d(p,s) creates a negative energy particle with $(-E_p,-\underline{p})$ and thus $b^{+}(p,s)$ creates a positive energy particle with $(E_p,\underline{p})$, with $d^{+}(p,s)$ and b(p,s) the corresponding annihilation operators; or in terms of the hole theory often applied to the Dirac equation, $d^{+}(p,s)$ creates an anti-particle, while d(p,s) destroys an anti-particle. The vacuum state in this theory is defined as the state where all the negative energy states are filled and the positive energy states are empty.

We can now define the number operator for positive energy particles as

$$N^{+}(p,s) = b^{+}(p,s)b(p,s) \quad , \qquad [90]$$

such that $N^{+}(p,s)d^3p$ tells us how many particles of spin s are in the momentum interval d^3p, while

$$N^{-}(p,s) = d^{+}(p,s)d(p,s) \qquad [91]$$

is the number operator for anti-particles of positive energy. Therefore, the energy-momentum four vector becomes

$$P^{\mu} = \sum_{\pm s} \int d^3p\,p^{\mu}[N^{+}(p,s) + N^{-}(p,s)] \quad . \qquad [92]$$

Also, since $Q = \int d^3x\psi^{+}\psi$, by inserting Eq. [53] we see that the conserved charge becomes

$$Q = \sum_{\pm s} \int d^3p[N^{+}(p,s) - N^{-}(p,s)] \quad . \qquad [93]$$

3.3 The Pion-Nucleon Interaction

We have now seen that to every particle (and its anti-particle) we can associate a quantized field. Each such particle, when moving freely in space, is characterized by a mass, a spin, an electric charge and possibly some other quantum number, such as its nucleonic charge (i.e., a nucleon is either a proton or a neutron). Now, however, we must extend this formalism to take into account the interaction between these fields. The interaction between the fields is introduced by adding to the Lagrangian of the free uncoupled fields, $\mathcal{L}_0$, an interaction term, $\mathcal{L}_I$, which must satisfy the quantum mechanical condition of hermiticity, relativistic invariance, and for simplicity, it cannot have space-time derivatives of the field derivatives higher than the first, so that the corresponding field equations are at most of second order. The strength of the interaction term in the Lagrangian is measured by the magnitude of a multiplicative factor called the coupling constant.

First of all, we have to distinguish between local and nonlocal couplings. In local coupling the interaction term is built up from field quantities, referring to the same space-time point, e.g.,

$$\mathcal{L}_I = G\overline{\psi}(x)\psi(x)\phi(x) \qquad [94]$$

where $\psi(x)$ is a spinor field, $\phi(x)$ a scalar field, and G the coupling constant. For nonlocal coupling, this is not the case, for

$$\mathcal{L}_I = G\int \overline{\psi}(x)\psi(x)F(x-x')\phi(x')d^4x' \quad ,$$

where $F(x-x')$ is a prescribed scalar function which characterizes the space-time "region" over which the interaction takes place. We also define direct coupling as a coupling where no derivatives of field quantities exist, as opposed to derivative coupling, where derivatives of field quantities appear in $\mathcal{L}_I$.

The simplest way to present a local, relativistically invariant interaction between a scalar field ϕ and a spinor field ψ is to couple the invariant quantity $\overline{\psi}\psi$ with ϕ and to write, classically,

$$\mathcal{L}_I = G \int d^4x\overline{\psi}(x)\psi(x)\phi(x) \qquad [96]$$

where G is the coupling constant. To determine the dimensionality of the coupling constant, we note that the field quantities are normalized in terms of certain free field expressions, as in the free scalar boson field, (Eq. [30]),

$$H = \frac{1}{2}\int d^3x\{\mu^2\phi^2 + \frac{1}{c^2}\pi^2 + (\nabla\phi)^2\} \qquad [97]$$

where μ is the inverse compton wave length of the particle.

In natarual units it equals the mass of the particle. H has the dimension of energy, so that ϕ^2 has the dimensions of $(\hbar c/V\mu)$ where V is a volume. We are including $\hbar$ and c in this case in order to determine the dimensionality of G. Similarly, for the Dirac Field,

$$H = \int d^3x\overline{\psi}(-i\hbar c\gamma^\mu\cdot\partial_\mu + mc^2)\psi \qquad [98]$$

so that $\overline{\psi}\psi V$ is dimensionless. Therefore, since the quantity $G\int d^3x\overline{\psi}\psi\phi$ has dimension of energy, it can be shown that $G^2/\hbar c$ is dimensionless.

For the pion field, which is a pseudoscalar field, ϕ, the coupling which guarantees the invariance of $\mathcal{L}_I$ under proper Lorentz transformations as well as under spatial inversions is of the form

$$\mathcal{L}_I(x) = G\bar{\psi}\gamma_5\psi\phi(x) \tag{99}$$

resulting from combining the pseudoscalar $\bar{\psi}\gamma_5\psi$ with ϕ, so that $\bar{\psi}\gamma_5\psi\phi$ is invariant under inversion. If we wish to describe the interaction of charged as well as neutral pions with nucleons in such a way that the prediction of the theory will be charge independent, then the interaction must be invariant under rotations in isotopic spin space. The simplest such interaction is one which couples the nucleonic isotopic vector

$$\bar{\psi}\ \underline{\tau}\ \gamma_5\psi \tag{100}$$

with the meson isotopic vector $\underline{\phi}$, with ϕ_3 describing the neutral meson and $1/\sqrt{2}(\phi_1 \pm i\phi_2)$ the corresponding charged pions. The interaction terms in the Lagrangian would then be of the form

$$\mathcal{L}_I = G\bar{\psi}\gamma_5\underline{\tau}\psi\cdot\underline{\phi} = \sum_{i=1}^{3} G\bar{\psi}\gamma_5\ \tau_i\psi\phi_i \quad . \tag{101}$$

A more formal field theory description of this interaction Lagrangian is developed as follows. In accordance with the assignment of an isotopic spin 1 to the pion system, it can be described by a field operator $\underline{\phi}$ which transforms like a vector in isotopic spin space, while the nucleon field can be described by an eight component spinor operator

$$\psi(x) = \begin{pmatrix} \psi_p(x) \\ \psi_n(x) \end{pmatrix} \qquad [102]$$

which transforms like a two component spinor in isotopic spin space. The possible interaction term between these fields which are invariant under rotations in isotopic spin space, and that conserves charge is of the form

$$\mathcal{L}_I = g_1 \, \overline{\psi}_n \, O_1 \cdot \psi_p \phi + g_2 \, \overline{\psi}_p \, O_2 \psi_n \phi^* \qquad [103]$$

where g_1 and g_2 are coupling constants, O_1 and O_2 are operators which are to be determined, and the other quantities have been defined previously. Note that ϕ creates a charge e, while $\overline{\psi}_n \psi_p$ destroys a charge e, and similarly for the second term, so that charge is conserved. We also require that the interaction Lagrangian be charge symmetric, i.e., invariant under the transformations:

$$\psi_n \to \psi_p \quad , \quad \phi \to \phi^*$$
$$\psi_p \to \psi_n \quad , \quad \phi^* \to \phi \quad , \qquad [104]$$

which leads to the new Lagrangian

$$\mathcal{L} \to g_1 \, \overline{\psi}_p \, O_1 \cdot \psi_n \, \phi^* + g_2 \, \overline{\psi}_n \, O_2 \cdot \psi_p \, \phi \quad , \qquad [105]$$

which, therefore, implies that

$$g_1 O_1 = g_2 O_2 \quad . \qquad [106]$$

The fact that the Lagrangian is Hermitian, i.e., $\mathcal{L}_I = \mathcal{L}_I^*$, leads to

$$\overline{g}_1 \gamma_0 O_1^+ \gamma_0 = g_2 O_2$$
$$\overline{g}_2 \gamma_0 O_2^* \gamma_0 = g_1 O_1 \qquad [107]$$

which when combined with [106] gives a charge symmetric $\mathcal{L}_\pm$

$$\mathcal{L}_I = g \; \overline{\psi}_n \; O \psi_p \phi^* + \overline{g} \; \overline{\psi}_p \gamma_0 O^* \gamma_0 \psi_n \phi$$

with

$$gO = \overline{g} \; \gamma_0 \; O^* \; \gamma_0 \quad .$$

This then leads to the following possible interactions for the pion-nucleon system:

$$
\begin{aligned}
&(a) \quad g_s(\overline{\psi}_p \cdot \psi_n \phi^* + \overline{\psi}_n \cdot \psi_p \phi) \\
&(b) \quad g_{ps}(\overline{\psi}_p \gamma_5 \cdot \psi_n \phi^* + \overline{\psi}_n \gamma_5 \cdot \psi_p \phi) \\
&(c) \quad g_v(\overline{\psi}_p \gamma_\mu \cdot \psi_n \partial^\mu \phi^* + \overline{\psi}_n \gamma_\mu \cdot \psi_p \partial^\mu \phi) \\
&(d) \quad ig_{pv}(\overline{\psi}_p \gamma_5 \gamma_\mu \cdot \psi_n \partial^\mu \phi^* + \overline{\psi}_n \gamma_5 \gamma_\mu \psi_p \partial^\mu \phi) \quad ,
\end{aligned}
\qquad [108]
$$

where $g_s \equiv$ scalar coupling, $g_{ps} \equiv$ pseudoscalar coupling, $g_v \equiv$ vector coupling and $g_{pv} \equiv$ pseudovector coupling. If we also require a charge independent pion-nucleon interaction (i.e., an interaction invariant under rotations in isotopic spin space), we require that

$$[\mathcal{L}_I, T_i] = 0 \quad , \quad i = 1, 2, 3 \qquad [109]$$

where T_i is the i^{th} component of the total isotopic spin. This will clearly be the case if we form an isotopic scalar quantity from the isotopic vector $\overline{\psi}\ O\ \underline{\tau} \cdot \psi$ with the iso-vector $\underline{\phi}$ to get Eq. [101] which can also be written in the form

$$\mathcal{L}_I = \sqrt{2}\, g[\overline{\psi}\tau_- O\cdot\psi(\phi_1+i\phi_2) + \overline{\psi}\tau_+ O\cdot\psi(\phi_1-i\phi_2)] + g\overline{\psi}\tau_3 O\cdot\psi\phi_3$$

$$= \sqrt{2}\, g[\overline{\psi}_n O\psi_p\phi + \overline{\psi}_p O\cdot\psi_n\phi^*] + g[\overline{\psi}_p O\cdot\psi_p - \overline{\psi}_n O\psi_n]\phi_3 \qquad [110]$$

where $\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$. Therefore, in a charge independent theory with Yukawa coupling, i.e., of the form $\overline{\psi}O\psi\phi$, the coupling of the nucleon field to the neutral meson field is $\sqrt{2}$ times weaker than to the charged meson field, and, the coupling constants measuring the interaction of the neutron and proton fields with the neutral pion field are equal in magnitude but opposite in sign.

If we now restrict ourselves to nonderivative Yukawa type couplings, the most general coupling of the nucleon field to the pion field which satisfies the requirements of Lorentz invariance, charge conjugation and parity conservation [(14)] gives

$$\mathcal{L}_I = g[\overline{\psi}_p\gamma_5\cdot\psi_n\phi^* + \overline{\psi}_n\gamma_5\cdot\psi_p\phi] + g_3'\overline{\psi}_p\gamma_5\cdot\psi_p\phi_3 + g_3''\overline{\psi}_n\gamma_5\cdot\psi_n\phi_3$$

$$= g[\overline{\psi}\tau_-\gamma_5\cdot\psi\phi + \overline{\psi}\tau_+\gamma_5\cdot\psi\phi^*] + g_3'\overline{\psi}_p\gamma_5\cdot\psi_p\phi_3 + g_3''\overline{\psi}_n'\gamma_5\cdot\psi_n\phi_3 \qquad [111]$$

where g, g_3' and g_3'' are real constants. If we impose the requirements of charge independence, it is clear that $g_3' = -\, g_3'' = 1/\sqrt{2}\, g$. The interaction term, therefore, reduces to

$$\mathcal{L}_{\underline{I}} = \frac{1}{2}\, g[\overline{\psi}\gamma_5\underline{\tau},\psi]\cdot\underline{\phi} \quad . \qquad [112]$$

The interacting meson-nucleon system can thus be described by the following Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\,\overline{\psi}(-i\gamma^{\mu}\partial_{\mu} + M)\psi - \frac{1}{2}\,(i\partial_{\mu}\overline{\psi}\gamma^{\mu} + M\overline{\psi})\cdot\psi$$

$$-\frac{1}{2}\,(\mu^2\underline{\phi}\cdot\underline{\phi} - \partial_{\mu}\underline{\phi}\cdot\partial^{\mu}\underline{\phi}) + \frac{1}{2}\,G\sum_{j=1}^{3}\,[\overline{\psi}\gamma_5\tau_j,\psi]\phi_j \qquad [113]$$

where M is the mass of the bare nucleon, and μ is the mass of the bare pion, ψ is an eight component nucleon field operator, and the corresponding Hamiltonian for the system is

$$H = \int d^3x\;\overline{\psi}(-i\gamma^{\mu}\partial_{\mu} + M)\psi + \frac{1}{2}\int \pi(x)\cdot\pi(x)$$

$$+\frac{1}{2}\int \pi(x)\cdot\pi(x) + \nabla\phi\cdot\nabla\phi(x) + \mu_0^2\phi\cdot\phi(x)$$

$$+\frac{G}{2}\int d^3x\sum_{j=1}^{3}\,[\overline{\psi}\gamma_5\tau_j,\psi]\phi_j \quad . \qquad [114]$$

The Hamiltonian in Eq. [114] can be divided into 3 parts,

$$H = H_N + H_M + H_I \quad , \qquad [115]$$

where

$$H_N = \int d^3x\ \overline{\psi}(-i\gamma^{\mu}\partial_{\mu} + M)\psi$$

$$H_M = \frac{1}{2}\int \{\pi^2 + (\underline{\nabla}\phi)^2 + \mu^2\phi^2\}d^3x \qquad [116]$$

$$H_I = G\sum_j \int d^3x\ \overline{\psi}\gamma_5\tau_j\psi\phi_j \quad .$$

If we apply a Foldy-Wouthuysen transformation to the nucleon field (Appendix II), we remove odd γ matrices which connect positive and negative energy states, and H_N becomes

$$H_N = \int d^3x\ \overline{\psi}\beta(\underline{p}^2 + M^2)^{1/2}\ \psi \quad , \qquad [117]$$

which in the non-relativistic limit finally becomes

$$H_N = \int d^3x\ \overline{\psi}\beta(\frac{\underline{p}^2}{2M} + M)\psi \quad . \qquad [118]$$

Applying a canonical transformation to H which changes the pseudoscalar coupling term to a pseudovector coupling (Appendix II), and keeping terms only to order 1/M which contribute to the p-wave pion nucleon interaction we get,

$$H_I = (\frac{G}{2M})\int d^3x \sum_{j=1}^{3} \overline{\psi}(x)\tau_j\ \underline{\sigma}\psi(x)\cdot\underline{\nabla}\phi_j(x) \qquad [119]$$

which is equivalent to

$$H_I = \frac{g}{\mu}\int d^3x\rho(\underline{x})\overline{\psi}\underline{\sigma}\cdot\underline{\nabla}\ \underline{\tau}\cdot\underline{\phi}(x)\psi \quad . \qquad [120]$$

where $g/\mu = G/2M$ is called the pseudovector coupling constant, ψ is a four component spinor describing a nucleon which transforms like a two component spinor in both isotopic spin space and ordinary spin space. We have defined a source function $\rho(x)$, which describes the extent of the meson-nucleon interaction region, which is assumed to be spherically symmetric. It will be normalized such that

$$\int \rho(x) d^3x = 1 \quad , \tag{121}$$

and if we introduce the Fourier transform of $\rho(x)$, we can define a cutoff function $v(k)$,

$$v(k) = \int e^{+i\underline{k}\cdot\underline{x}} \rho(x) d^3\underline{x} \quad . \tag{122}$$

$v(k)$ (or $\rho(x)$) is introduced to account for a convergent theory. From the meson field operator (Eq.[66]), we can see that it implies that the value of ϕ or $\nabla\phi$ is taken exactly at the position of the nucleon, i.e., $\rho(x)$ is a delta function $\delta(\underline{x})$ if the nucleon is at the origin. But this assumption that the nucleon is pointlike leads to $v(k) = 1$ for all values of k, which leads to divergences in the interaction. This makes it necessary to attribute a finite extension to the nucleon in order to get a convergent theory. This cutoff function can also, in some manner, take account of anti-nucleon effects, direct meson-meson interactions and kaon and hyperon effects, all of which are ignored in our approximation and are believed to be small. If we take R_0 as the radius of the interaction region, i.e., where $\rho(x)$ is appreciably different from zero, then if $|\underline{k}|R_0 >> 1$, $v(k)$ falls rapidly to zero. The radius R_0 will be less than $1/\mu$, and it is believed to be of the order $1/M$, where M is the nucleon mass.

And finally, we must take into account the possible interaction of our π-N system with an electromagnetic field specified by the four-vector potential $A_\mu(x) = (\phi, \underline{A})$. The Dirac field has already been considered in this respect. We apply the same formula as in Chapter II, and recall that the gauge invariant introduction of electromagnetic interactions requires that the operator $\underline{\nabla}\underline{\phi}$ in H_I be replaced by $(\underline{\nabla} - et_3\underline{A})\underline{\phi}$, e.g., the $\underline{\nabla}$ operator be replaced by $\underline{\nabla} + iet_3\underline{A}$ when acting on the π^+ operator. Therefore, in the presence of electromagnetic effects, an additional term of the form

$$\frac{eg}{\mu}\psi[\underline{\sigma}\cdot\underline{A}\phi\tau_- + \underline{\sigma}\cdot\underline{A}\phi^*\tau_+]\psi \qquad [123]$$

occurs in the interaction.

We now have expressions which describe the nucleon field , pion field , pion-nucleon interaction and electromagnetic interactions, and we are, therefore, in a position to evaluate the scattering amplitudes for many processes. Hopefully, we have not lost sight of our initial objective, which is to try to explain how the electromagnetic structure of a nucleon is altered when it is bound in a nucleus.

CHAPTER IV

PRESENT STATUS OF TRI-NUCLEON MAGNETIC MOMENT CALCULATIONS

In the preceding section we developed equations which describe the nucleon field, the pion field, and interactions between these two fields. It is believed that the electromagnetic structure of a nucleon is due to the strong coupling of a pion field and heavy meson fields to a nucleon field. We wish to show how the structure of a single nucleon is altered by the presence of other nucleons. First we investigate the two nucleon system, since many of the techniques used to solve the magnetic moment problem of the deuteron are applicable to the tri-nucleon system.

4.1 The Deuteron Magnetic Moment

The simplest system that can be studied is the two nucleon system, and in particular, the deuteron, a system composed of one proton and one neutron, with a corresponding isospin T=0. In the absence of any interaction currents, the magnetic moment operator is

$$\underline{M} = \sum_{i=1}^{2} \left[\frac{1}{2} [(1-\tau_z(i)] \mu_n \underline{\sigma}(i)) + \frac{1}{2} [1+\tau_z(i)] [\mu_p \underline{\sigma}(i) + \underline{L}(i)] \right] \qquad [124]$$

where μ_n and μ_p are the free neutron and proton magnetic moments, respectively, and where $\underline{\sigma}(i)$ and $\underline{L}(i)$ are the spin and angular momentum operators for the i^{th} particle, respectively. The observed magnetic moment of the deuteron is close to the sum of the magnetic moments of the neutron and proton; i.e., the experimental value is $\mu_D^{exp} = .857$ nm while $\mu_p + \mu_n = .879$ nm. To a first approximation the two nucleons are in a $^3S_1$ state, where the magnetic moment is almost totally contributed by the magnetic moments associated with the parallel spins of the constituent nucleons. However, it is also found that the quadrupole moment of the deuteron is not zero, which implies that the deuteron ground state wave function is not a pure S-state, but rather, it is a superposition of $^3S_1$ and $^3D_1$ states. It can be shown [16] that the expectation value of operator $\underline{M}$ is given by

$$\mu_D = (\mu_p + \mu_n) - \frac{3}{2}(\mu_p + \mu_n - \frac{1}{2})P_D \qquad [125]$$

where P_D is the D-state probability. In order for the theoretical magnetic moment to equal the experimental value, we need a D-state probability of 3.9 percent. On the other hand, realistic nucleon-nucleon potentials, such as the Hamada-Johnston and Reid soft core potential, lead to a D-state probability of about 7 percent [17]. Several experimental results, such as the coherent photoproduction of π° from deuterium [18] also suggest a D-state probability of 7 percent. This leaves a discrepancy of about 1.7×10^{-2} nm for the magnetic moment.

Adler and Drell [19] showed that this discrepancy could be lifted if one considers the exchange current contribution. The simplest exchange current would be the one shown in Fig. 4, where the electromagnetic field interacts with the π^+ intermediate charged pion. Since the deuteron has isotopic spin $T = 0$, only the isospin scalar part of the electromagnetic current contributes to the electromagnetic structure of the deuteron; that is, only electromagnetic processes that correspond to the emission of photons without a change in isospin can take place, since the coupling between the electromagnetic and nuclear fields must be an isoscalar. We introduce the concept of G parity, which is the rotation by angle π of a system about the second axis in isospin space, followed by charge conjugation. The photon is odd under charge conjugation, while the isoscalar part of the electromagnetic interaction is even under rotation by π about the second axis in isospin space. The isovector part of the electromagnetic interaction is odd under this rotation. A system of n pions has G parity $(-1)^n$; if we assume invariance of the strong interactions under G parity, we find that all diagrams with an even number of pions contribute to the iso-vector coupling to the electromagnetic field, while all

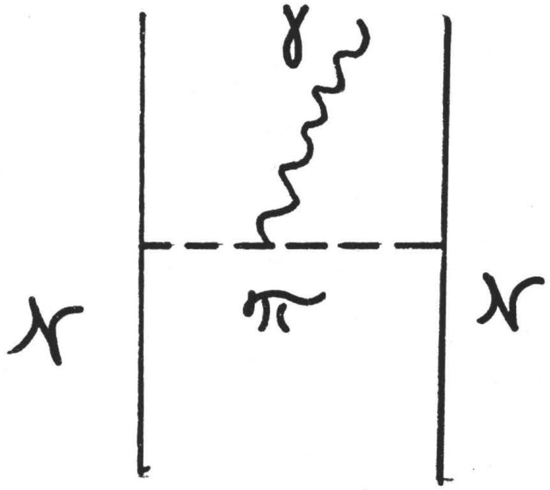


Fig. 4: Simplest exchange current diagram. The intermediate charged pion interacts with the electromagnetic field

diagrams with an odd number of pions contribute to the isoscalar coupling. Since only the isoscalar part of the electromagnetic interaction acts on the deuteron, all even pion states are not present. Therefore, the least massive state is that state composed of three pions (Fig. 5a). Adler and Drell investigated the exchange current contribution due to this process, in the approximation that the 3π state may be approximated by a two particle (ρ,π) system (Fig. 5b), with the ρ (a 2π resonance) and π landing on different nucleons and thus constituting an exchange current. They calculated the contribution of this process to the deuteron magnetic moment and obtained a value of $\Delta\mu \approx (1-2) \times 10^{-2}$ nm, which is comparable in magnitude with the existing discrepancy of 1.7×10^{-2} nm. The major part of the discrepancy seems to be explained by this exchange current calculation.

It should be mentioned, however, that in dealing with apparent discrepancies of this small magnitude a comparable correction may be present when we consider relativistic effects not included in a treatment of the deuteron as a bound state of two Pauli particles interacting via an instantaneous potential. This problem will also be present when we consider the three-body problem.

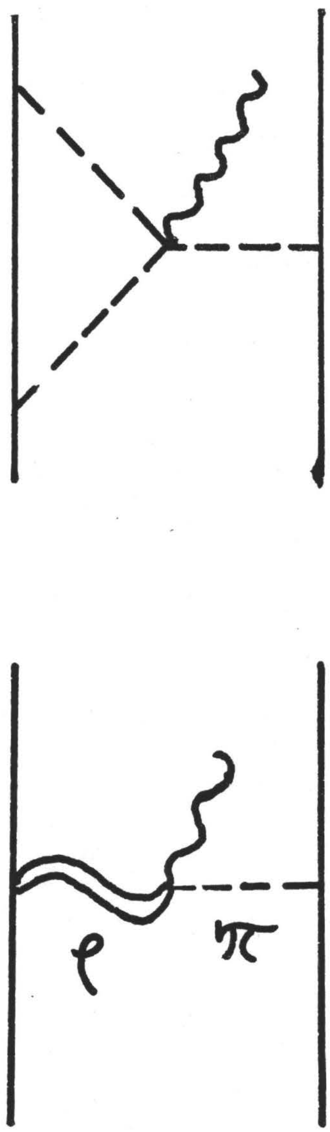


Fig. 5: Exchange current contributions due to the 3π state where a) we consider 3 pions, b) we approximate a 3π state by a two particle (ρ,π) system

4.2 Magnetic Moment of He^3 and H^3

We now consider the three nucleon system, He^3 and H^3. He^3 is composed of two protons and one neutron with a magnetic moment of -2.1274 nm with a spin 1/2, while H^3 is composed of one proton and two neutrons with a magnetic moment of 2.9788 nm and a spin 1/2. If we consider the three nucleon system in an S-state where there is no orbital contribution to the magnetic moment, the magnetic moment contribution would come from the unpaired nucleon. However, this is certainly not the case. If we evaluate Eq. [124] for the three-body system, and determine the isovector and isoscalar magnetic moments for the two three-body systems, we find [20]

$$\begin{aligned}
\mu_s &= \frac{1}{2}\,[\mu(He^3) + \mu(H^3)] \\
&= \frac{1}{2}\,(\mu_p + \mu_n)[P(S) + P(S') - P(D)] + \frac{1}{2}\,P(D) \\
\mu_v &= \frac{1}{2}\,[\mu(He^3) - \mu(H^3)] \\
&= \frac{1}{2}\,(\mu_p + \mu_n)[P(S) - \frac{1}{3}\,P(S') + \frac{1}{3}\,P(D)] - \frac{1}{6}\,P(D)
\end{aligned} \qquad [126]$$

where P(S) is the principal S-state probability, P(D) is the D-state probability, P(S') is the probability for the mixed symmetry state, where $P(S) + P(S') + P(D) \approx 1$.

The classification of allowed states of the tri-nucleon system with ($J^P = \frac{1}{2}^+$, $T = \frac{1}{2}$) gives 10 distinct states [21] corresponding to the spectroscopic terms: three $^2S_{1/2}$, $^2P_{1/2}$ and $^4D_{1/2}$ and one $^4D_{1/2}$ states. According to the work of Gibson [22] only the S-state (fully space symmetric), S' state (mixed space symmetry), D and $T = \frac{3}{2}$ (mixed space symmetry) states are appreciable.

To find the best values for P(S), P(S') and P(D), various authors have made extensive calculations for the tri-nucleon bound states using realistic nucleon-nucleon interactions, such as the Hamada-Johnston potential [23] and the Reid soft core potential [24] (Table I). Substituting these values into Eq. [126], we see from Table I that there is a large discrepancy between the experimental and theoretical values for the isovector magnetic moment ($\sim$ 15 percent), while the discrepancy between the isoscalar values is much smaller ($\sim$ 5%). Historically, the anomaly in the tri-nucleon magnetic moment was the first piece of clear evidence of meson-exchange effects [25], since if we consider Eq. [126], we see that no values of P(S), P(S') and P(D) will fit μ_s and μ_v to the experimental values.

A rather thorough study of the meson exchange effects in He^3 and H^3 was done by Chemtob and Rho [26]. They consider contributions to the isoscalar and isovector magnetic moments of the He^3 and H^3 system due to the

TABLE I

Expectation values of the one-body magnetic moment operators and their deviation from corresponding experimental values. The experimental values are $\mu_v^{exp} = 2.553$ nm and $\mu_s^{exp} = .426$ nm. Case A is for the Hamada-Johnston potential (Ref. 23) and case B is for the Reid soft-core potential (Ref. 24).

Case	P(S) (%)	P(S') (%)	P(D) (%)	$\mu_v^{(1)}$ (nm)	$\mu_v^{exp}-\mu_v^{(1)}$ (nm)	$\mu_s^{(1)}$ (nm)	$\mu_s^{exp}-\mu_s^{(1)}$ (nm)
A	89	2	9	2.134	.419 (16%)	.406	.020 (5%)
B	90.56	.52	8.92	2.182	.371 (15%)	.406	.020 (5%)

exchange of one pion and of vector mesons. In their calculations, they used the Gaussian form for the wave function of the S-state, and considered only S-state to S-state transitions. They calculated the two-body exchange operators due to the one-pion exchange (OPE) and the heavy meson exchange (HME) contributions. For the OPE terms (Fig. 6), they used the familiar low-energy theorem information and supplemented these calculations by using the Chew-Low model. For the HME case they considered the ρ (2π resonance) and ω (three π resonance) exchange graphs (Fig. 7).

In both the OPE and HME terms, three types of currents were considered: the pionic current (Fig. 6a), the pair excitation current (Figs. 6b and 6c), and the nucleon recoil current (Figs. 6d and 6e). The matrix elements of the exchange current operators depend on the radial wave function. As stated previously, Chemtob and Rho used the simplest analytical form, the Gaussian, and since some of the pion exchange and heavy meson exchange terms are very sensitive to the part of the wave function where $r_{ij} = |r_i - r_j|$ (where r_i is the distance from the origin to the i^{th} nucleon) is small, they employed a Jastrow type correlation function of the form

$$\prod_{i<j} (1 - e^{-\gamma^2 r_{ij}^2})^{1/2}$$

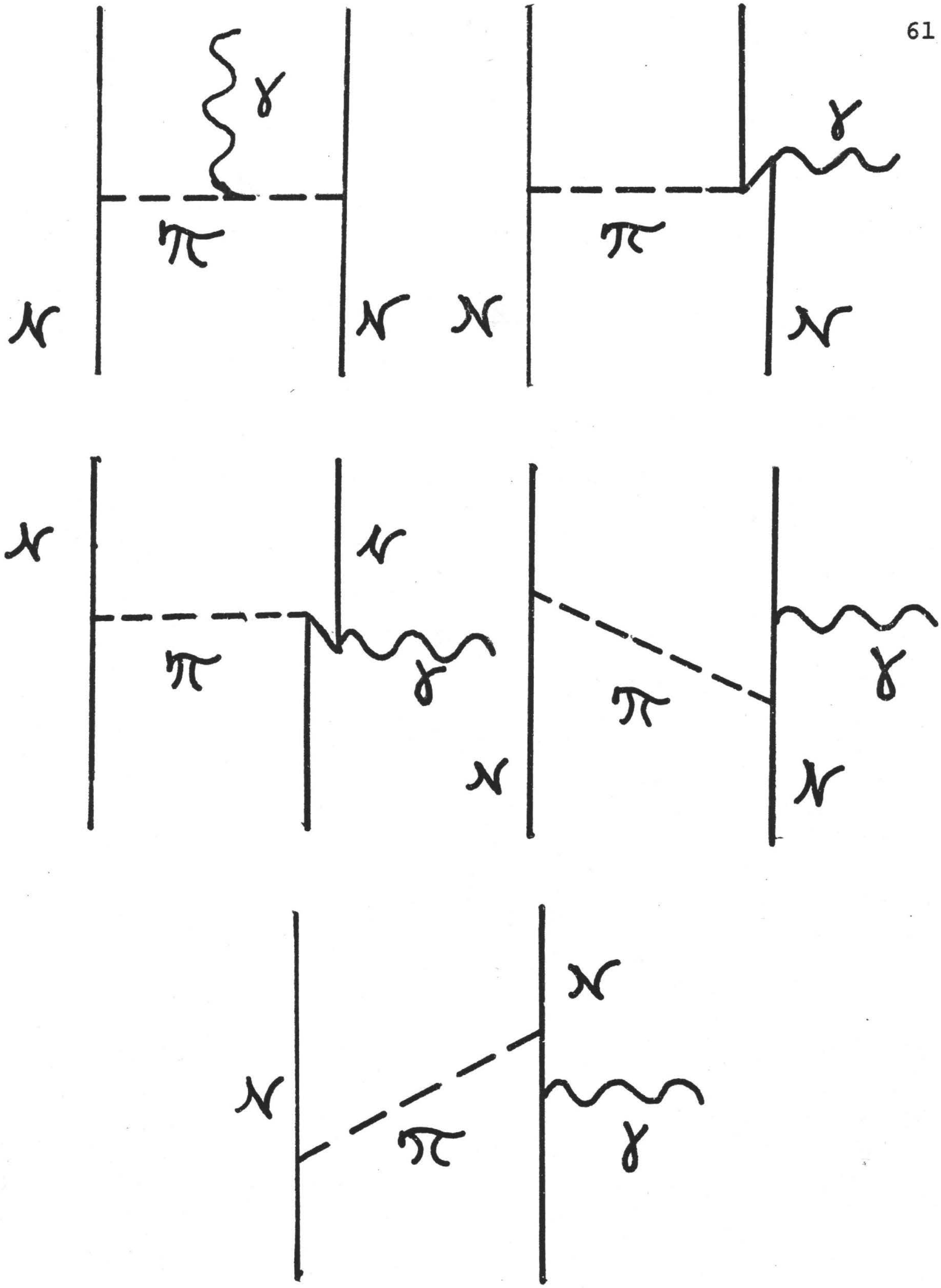


Fig. 6: The diagrams for the OPE process where a) gives the pionic current, b) and c) give the pair excitation exchange current, d) and e) give the nucleon-recoil exchange current

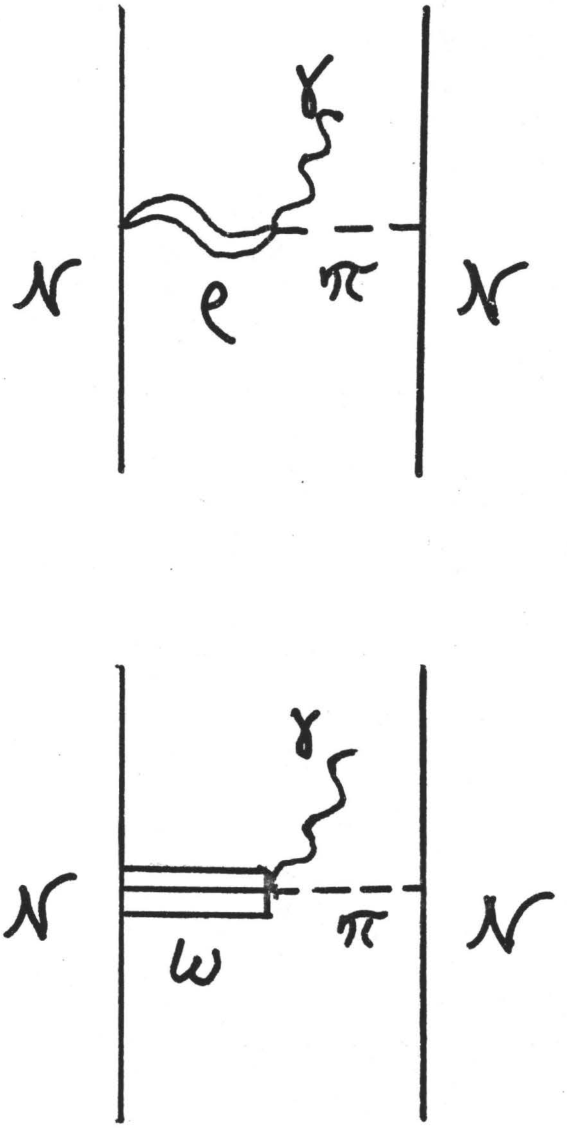


Fig. 7: One pion exchange diagram representing the vertex corrections due to the vector meson intermediate states

to simulate short distance behaviour, where the parameter γ is related to the repulsive core radius in nucleon-nucleon potential theory. The results they obtain for the isovector and isoscalar magnetic moments are

$$\mu_v^2 = .193 \ (\pm\ .041) \ \text{nm}$$

$$\mu_s^2 = .0093 \ \left(\begin{smallmatrix}+ & .0077 \\ - & .0053\end{smallmatrix}\right) \ \text{nm} \quad . \qquad [127]$$

This reduces the discrepancy between μ^{exp} and $\mu^{(1)}$ in Table I as follows:

$$\mu_v^{exp} - (\mu_v^{(1)} + \mu_v^{(2)}) = \begin{matrix} .226 \ (9\%) \ \text{for case A} \\ .178 \ (7\%) \ \text{for case B} \end{matrix}$$

$$\mu_s^{exp} - (\mu_s^{(1)} + \mu_s^{(2)}) = .011 \ (3\%) \ \text{for cases A and B.} \qquad [128]$$

Chemtob and Rho find that if we require $\mu^{exp} = \mu^{(1)} + \mu^{(2)}$, we need $P(S') \approx 0$, $P(D) \approx 6$ percent, which is not very realistic.

Riska and Brown [27], in trying to explain a 10% discrepancy between the theoretical and experimental values of the cross section of the reaction $n + p \rightarrow D + \gamma$, found that the exchange current contributions could explain this discrepancy if the $^1S_0$ to $^3D_1$ transition is considered, which had been overlooked until then. Harper et al. [28]

applied this idea to the calculations of Chemtob and Rho. They used tri-nucleon wave functions derived from an exact solution of the Faddeev equation [29] for a nucleon-nucleon interaction (effective in the $^1S_0$ and $^3S_1$ - $^3D_1$ states) given by the Reid soft core potential [30]. The tri-nucleon binding energy they obtained, in the absence of Coulomb effects, was 6.7 MeV, and the percentages of P(S), P(S') and P(D) were 89.7%, 1.7% and 8.6% respectively. They obtained the values given in Table II. It can be seen that the SD transition is extremely important here. However, using Harper et al.'s values for P(S), P(S') and P(D), we find that

$$\delta\mu_s = \mu_s^{exp} - \mu_s^{(1)} = .017 \text{ nm}$$

$$\delta\mu_v = \mu_v^{exp} - \mu_v^{(1)} = .401 \text{ nm} \quad .$$

The isovector magnetic discrepancy is now only 2% of the experimental value, which is encouraging. However, the isovector magnetic moment is overcompensated, which implies that the above analysis must be corrected to some extent.

Hadjimichael et al. [31] calculated similar meson-exchange corrections to the magnetic moments of H^3 and He^3 using a method by means of which the two-body short range correlations in the nucleon-nucleon interaction are taken

TABLE II

Contribution to the magnetic moment expectation values obtained by Harper et al. (Ref. 28) due to the processes shown in Figs. 6 and 7. The notation (X,SS) implies that this is the contribution for the X type of interaction for an S to S state transition.

X	$\mu_v^{(2)}$ (X,SS)	$\mu_v^{(2)}$ (X,S'S')	$\mu_v^{(2)}$ (X,SD)	Total (nm)
N*	.002	-.000	.164	.166
ω	.000	-.000	.012	.012
Pair & pionic	.138	-.002	.105	.241
Total	.140	-.002	.281	.419

accurately into account in three-body bound state calculations. If we assume a 100% S-state probability from Eq. [126], they find $\mu_V^{(sb)} = 2.353$ and $\mu_S^{(sb)} = .440$, while if they take P(S) = 91%, P(S') = 1% and P(D) = 8%, they obtain $\mu_V^{(sb)} = 2.213$ and $\mu_S^{(sb)} = .410$. As we know, the bulk of the difference between the single-body results and the experimental values is expected to come from meson-exchange corrections to the magnetic operator. The corrections found by Hadjimichael using a S-state of 92%, are μ_V(S-S) = .183 nm and μ_S(S-S) = $.775 \times 10^{-2}$ nm for a S- to S-state transition. They also applied the theory of Riska and Brown that the S-D matrix elements of the two-body operators are comparable to the S-S matrix elements, and taking a 8% D-state probability, they find that $\mu_V^{(tb)}$(S-D) = .170. They note, however, that their expression for the D-state wave function does not have the correct asymptotic behaviour and that $\mu_V^{(tb)}$(S-D) is probably overestimated. Therefore, for a S-state probability of 92%, P(S') = 0, and P(D) = 0, they find that

$$\mu_V = 2.217 + .183 + .170 = 2.57 \text{ nm}$$

which agrees favourably with the experimental value, but again is larger than the experimental value.

Our work on the anomalous magnetic moment of the tri-nucleon system began after the work of Chemtob and Rho but before the work of Riska and Brown, Harper et al., and Hadjimichael was published. We considered the contribution due to the process in Fig. 8, which we believe was not considered up to that time, and whose effect is still not taken into account.

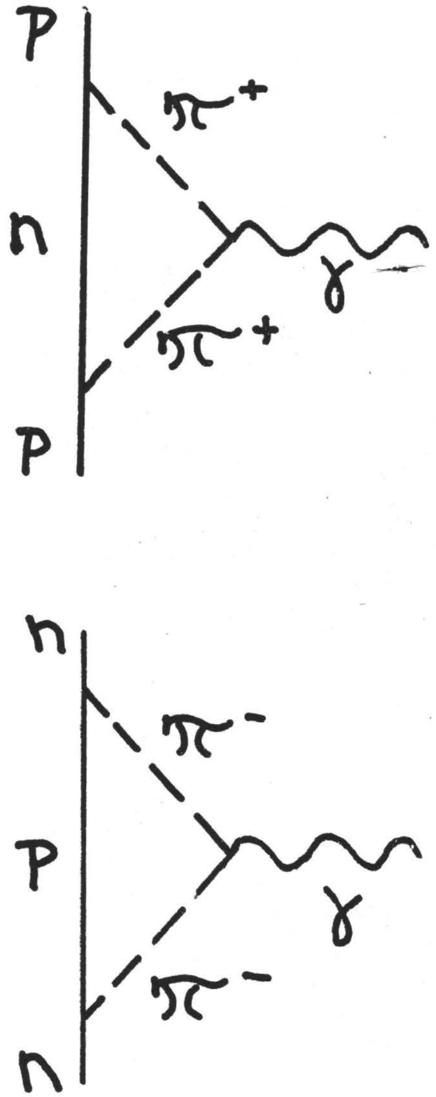


Fig. 8: Two pion electromagnetic process that contributes to the long range part of the charge and magnetic form factors of a nucleon

CHAPTER V

CALCULATIONS OF THE ELECTROMAGNETIC STRUCTURE OF NUCLEONS

The basic problem that is dealt with is calculating the effect that a nucleon has on the electromagnetic structure of another nearby nucleon. We are concerned mainly with the charge and magnetic moment densities due to the process shown in Fig. 8, which is known as the "two pion contribution" to the long range part of the charge and magnetic moment distributions of a nucleon. As we have shown in Section 4.1, this two pion process will contribute to the isovector magnetic moment of the system. This process is believed to give the largest contribution to the anomalous magnetic moment of the nucleon. Basically it describes a state where a "bare" nucleon spends part of its time as a pion-nucleon system. We will discuss the contributions to the magnetic moment and charge densities of a nucleon obtained from these diagrams in various approximations, and try to show how these contributions are altered when the nucleons are bound, or when they interact with other nucleons.

5.1 The Free Nucleon Case

Many authors have calculated the effect shown in Fig. 8 by the Chew-Low theory in the static approximation, i.e., in the limit as the nucleon mass goes to infinity. Using the well known Feynman propagator approach, the rules for constructing the scattering matrix are [33,34]

1) for each internal line include a factor

$$\frac{i\,\delta_{ij}}{(2\pi)^4}\,\frac{1}{k_0^2 - w_k^2 + i\varepsilon} = \frac{i\,\delta_{ij}}{(2\pi)^4}\,\frac{1}{k_\mu \cdot k^\mu - \mu^2 + i\varepsilon} \qquad [129]$$

where k_μ is the four momentum of the pion, $w_k^2 = k^2 + \mu^2$ where μ is the mass of the pion, and k_0 is the energy variable in the pion propagator;

2) for each internal nucleon line include a factor

$$(2\pi)^4\,\frac{1}{\not{k} - M + i\varepsilon} \rightarrow \frac{i}{2\pi}\,\frac{1}{k_0 - \frac{k^2}{2M} + i\varepsilon} \xrightarrow[M\to\infty]{} \frac{i}{2\pi}\,\frac{1}{k_0 + i\varepsilon} \qquad [130]$$

where the first term is the relativistic propagator for a nucleon of mass M and 4-momentum k, the second term is the nonrelativistic limit, and the third term is the static approximation;

3) at each πNN vertex include the factor

$$(2\pi i)\delta(\Sigma k_0)\ \frac{f}{\mu}\ \tau_i \underline{\sigma}\cdot\underline{k}\ e^{\pm i\underline{k}\cdot\underline{r}_N} \qquad [131]$$

which gives the pion nucleon coupling, where $\underline{r}_N$ is the position of the nucleon, which will be taken to be zero in the static approximation, and $\frac{f}{\mu} = \frac{g}{\mu}$.

4) and at the $\gamma\pi\pi$ vertex

$$2\pi\delta(\Sigma k_0)(-i)(\delta_{1i}\delta_{2j} - \delta_{2i}\delta_{1j})(k_\mu+k'_\mu)e^{i(\underline{k}'-\underline{k})\cdot\underline{r}_\pi}\cdot eA^\mu \qquad [132]$$

which describes the interaction of the charged pion with the electromagnetic field described by the field potential A_μ. We also multiply by $(-i)^N$, where N is the order of the diagram and integrate over all internal momenta. For the charge density, Eq. [132] is

$$(2\pi)\delta(\Sigma k_0)(-i)(\delta_{1i}\delta_{2j} - \delta_{2i}\delta_{1j})(k_0+k'_0)e^{i(\underline{k}'-\underline{k})\cdot\underline{r}_\pi}\cdot eA_0 \qquad [133]$$

while for the magnetic moment density, it is

$$(2\pi)\delta(\Sigma k_0)(-i)(\delta_{1i}\delta_{2j} - \delta_{2i}\delta_{1j})(\underline{k}+\underline{k}')e^{(i\underline{k}'-\underline{k})\cdot\underline{r}_\pi}\cdot e\underline{A}\ , \qquad [134]$$

where A_0 is the external scalar potential and $\underline{A}$ is the vector potential. Applying these rules, we find that the relevant S-matrix is

$$S = (-i)^3\delta(0)\int_{-\infty}^{+\infty} dt\int \underline{dk}\int \underline{dk} \; (\frac{i}{(2\pi)^4})^2 \, \frac{i}{2\pi} \, e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}$$

$$\times \; (2\pi i)^2 (\frac{f}{\mu})^2 (-2\tau_3) \quad \frac{(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k})(2\pi)(k_\mu+k'_\mu)eA^\mu v(k)v(k')}{(t^2-w_k^2+i\varepsilon)(t^2-w_{k'}^2+i\varepsilon)(-t+i\varepsilon)} \qquad [135]$$

where $t \equiv k_0 \equiv t'$, r is the pion coordinate, and v(k) is the cutoff function, while in terms of the charge and current densities $\rho(r)$ and $\underline{j}(r)$

$$S = -(2\pi i)\delta(0)\rho_c(r)eA_0(r) + 2\pi i\delta(0)\underline{j}(r)\cdot e\underline{A}(r)$$

$$= -2\pi i\delta(0)\cdot e\{\rho_c(r)A_0(r) - \underline{j}(r)\cdot\underline{A}(r)\} \qquad [136]$$

since the interaction Hamiltonian is

$$H_I = ej_\mu A^\mu = e\rho_c A_0 - e\underline{j}\cdot\underline{A} \quad . \qquad [137]$$

From Eqs. [135] and [136] we see that the charge density is

$$\rho_c(r) = \frac{4}{(2\pi)^6} \, (\frac{f}{\mu})^2 \, \frac{\tau_3}{(2\pi i)} \int_{-\infty}^{+\infty} dt\int \underline{dk}\int \underline{dk}'$$

$$\times \; \frac{(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k})e^{i(\underline{k}'-\underline{k})\cdot\underline{r}} \; v(k)v(k')}{(t^2-w_k^2+i\varepsilon)(t^2-w_{k'}^2+i\varepsilon)}$$

continued...

$$= \frac{2\tau_3}{(2\pi)^6} \left(\frac{f}{\mu}\right)^2 \int \underline{dk}\,\underline{dk}'\ \underline{k}\cdot\underline{k}'\ \frac{e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}\, v(k)v(k')}{ww'(w+w')} \qquad [138]$$

where τ_3 = +1 for the proton and -1 for the neutron, f is the renormalized coupling constant for this process such that $f^2/4\pi = .08$, the t (or energy) integration has been done, and there is no spin flip such that

$$(\underline{\sigma}\cdot\mathrm{k}')(\underline{\sigma}\cdot\underline{k}) = \underline{k}'\cdot\underline{k} + i\underline{\sigma}\cdot\underline{k}'\times\underline{k} = \underline{k}'\cdot\underline{k} \quad .$$

A similar calculation for the current density gives

$$\underline{j}(\underline{r}) = \frac{\tau_3}{(2\pi)^6} \frac{f^2}{\mu^2} \int \underline{dk} \int \underline{dk}' \times \frac{(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k})(\underline{k}'+\underline{k})e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}\, v(k)v(k')}{w^2\, w'^2}$$

$$= \frac{i\tau_3}{(2\pi)^6} \frac{f^2}{\mu^2} \int \underline{dk} \int \underline{dk}' \times \frac{(\underline{\sigma}\cdot\underline{k}'\times\underline{k})(\underline{k}'+\underline{k})e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}\, v(k)v(k')}{w^2\, w'^2} \qquad [139]$$

where the $\underline{k}'\cdot\underline{k}$ term of the spinor product gives a zero contribution, which becomes obvious if $\underline{k}$ is replaced by $-\underline{k}$ and then integrated.

If we define $\rho_m(r)$ as the magnetic moment density distribution of the pion-nucleon system, the current density is defined as

$$\underline{j}(\underline{r}) = (\underline{\nabla}\times\underline{\sigma})\rho_m(r) = -(\underline{\sigma}\times\underline{r})\frac{1}{r}\frac{\partial}{\partial r}\rho_m(r) \qquad [140]$$

where $\underline{\sigma}$ is the spin of the nucleon. The magnetic moment density is

$$\underline{M}(\underline{r}) = \frac{e}{2M}\frac{1}{2}(\underline{r}\times\underline{j}(\underline{r})) = \frac{e}{2M}\mu(r)\underline{\sigma} \qquad [141]$$

where

$$\mu(r) = -\frac{r}{3}\frac{\partial}{\partial r}\rho_m(r) \quad . \qquad [142]$$

After some manipulation (see Appendix III), Eq. [139] becomes

$$\begin{aligned} j(\underline{r}) &= (\underline{\sigma}\times\underline{r})\frac{2\tau_3}{(2\pi)^6}\left(\frac{f}{\mu}\right)^2\frac{1}{r^2} \\ &\times\int\frac{\underline{dk}\underline{dk}'\ v(k)v(k')\underline{k}\cdot\underline{k}'\ e^{i(\underline{k}-\underline{k}')\cdot\underline{r}}}{w^2\ w'^2} \quad . \end{aligned} \qquad [143]$$

Therefore, from Eqs. [140] and [143]

$$\begin{aligned} \frac{\partial}{\partial r}\rho_m(r) &= -\frac{2\tau_3}{(2\pi)^6}\left(\frac{f}{\mu}\right)^2\frac{1}{r}\int\underline{dk}\underline{dk}' \\ &\times\frac{v(k)v(k')\underline{k}\cdot\underline{k}'\ e^{i(\underline{k}-\underline{k}')\cdot\underline{r}}}{w^2\ w'^2} \end{aligned} \qquad [144]$$

and from [144] and [142], the magnetic moment density

$$\mu(r) = \frac{2\tau_3}{3(2\pi)^6} \left(\frac{f}{\mu}\right)^2 \int d\underline{k} \int d\underline{k}' \times \frac{v(k)\,v(k')\,\underline{k}\cdot\underline{k}'\; e^{i(\underline{k}-\underline{k}')\cdot\underline{r}}}{w^2\, w'^2} \qquad [145]$$

and the magnetic moment is

$$\mu_m = \int \mu(r)\,d\underline{r} = \frac{2\tau_3}{2(2\pi)^3} \left(\frac{f}{\mu}\right)^2 \int d\underline{k}\; \frac{v^2(k)\,k^2}{w^4}$$

$$= \tau_3\, \frac{4}{3\pi} \left(\frac{f^2}{4\pi}\right) \frac{1}{\mu^2} \int_0^\infty dk\, \frac{k^4}{w^4}\, v^2(k) \qquad [146]$$

while the total charge (from Eq. [138]) is

$$Q = \int \rho_c(r)\,d\underline{r} = \frac{2\tau_3}{\pi} \left(\frac{f^2}{4\pi}\right) \frac{1}{\mu^2} \int_0^\infty dk\, \frac{k^4}{w^3}\, v^2(k) \quad . \qquad [147]$$

5.2 Nucleon Bound by an Effective Potential

We now consider how the charge and magnetic moment densities are altered by the presence of another nucleon, as shown in Fig. 9. We assume that the two nucleons are interacting through an effective potential V(S), where S is the distance between the two nucleons. We also assume that

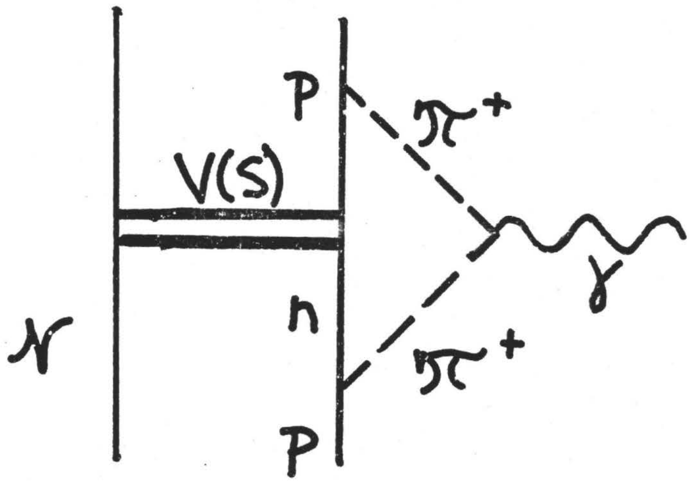


Fig. 9: A process in which the neutron in the intermediate state interacts with another nucleon through an effective interaction V(S)

V(S) is independent of spin and isospin. If we treat V(S) in lowest order perturbation theory $\Delta\rho_c$, the correction to $\rho_c(r)$ due to this process, is simply proportional to V(S), and similarly for $\Delta\underline{j}(r)$. In Fig. 9 there is an additional internal nucleon line, so a factor

$$-2\pi i\ V(S)\ \frac{i}{2\pi}\ \frac{1}{-k_0 + i\varepsilon} = -\ \frac{V(S)}{t-i\varepsilon} \qquad [148]$$

must be included in the scattering matrix calculation in Eq. [137]. In the charge density calculation, the t or energy integration (Appendix III) replaces $1/ww'(w+w')$ by $1/w^2\ w'^2$ so that

$$\Delta\rho_c(r) = \frac{2\tau_3}{(2\pi)^6}\left(\frac{f}{\mu}\right)^2 \int \underline{dk}\underline{dk}'\ \underline{k}\cdot\underline{k}'$$

$$\times\ \frac{e^{i(k-k')\cdot r}\ v(k)\,v(k')\,\langle V(S)\rangle}{w^2\ w'^2} \qquad [149]$$

where $\langle V(S)\rangle$ is the expectation value of the potential V(S), while the new current density is obtained by replacing $1/w^2\ w'^2$ by

$$\frac{w^2 + ww' + w'^2}{(ww')^3(w+w')}$$

so that the additional current density is

$$\Delta \mathrm{j}(r) = \frac{i\tau_3 \langle -V \rangle}{(2\pi)^6} \left(\frac{f^2}{\mu^2}\right) \int \underline{dk} \int \underline{dk}' (\underline{\sigma}\cdot\underline{k}'\times\underline{k})(\underline{k}'+\underline{k})$$

$$\times \frac{e^{i(\underline{k}-\underline{k}')\cdot\underline{r}}\, v(k)v(k')(w^2 + ww' + w'^2)}{(ww')^3(w + w')} \quad . \quad [150]$$

The magnetic moment density correction becomes

$$\Delta\mu(r) = \frac{2\tau_3 \langle -V \rangle}{3(2\pi)^6} \left(\frac{f}{\mu}\right)^2 \int \underline{dk} \int \underline{dk}'$$

$$\times \frac{v(k)v(k')(w^2 + ww' + w'^2)\underline{k}\cdot\underline{k}'\, e^{i(\underline{k}-\underline{k}')\cdot\underline{r}}}{(ww')^3(w + w')}$$

[151]

which finally gives

$$\Delta\mu = \int \Delta\mu(r)\underline{dr} = \frac{2}{\pi} \langle -V(S) \rangle \frac{f^2}{4\pi} \frac{1}{\mu^2} \int_0^\infty dk\, \frac{k^4}{w^5} v^2(k) \quad ,$$

[152]

while the correction to the charge is obtained from Eq. [149]

$$\Delta Q = \int \Delta\rho_c(r)\underline{dr} = \tau_3 \langle -V(S) \rangle \frac{4}{\pi} \frac{f^2}{4\pi} \frac{1}{\mu^2} \int_0^\infty dk\, \frac{k^4}{w^4} v^2(k) \quad .$$

[153]

We now have expressions that enable us to calculate the two pion contribution to the charge and magnetic moments, Eqs. [147] and [146] respectively, and the correction to the charge and magnetic moments, Eqs. [153] and [152] respectively. To obtain some numerical results, we must take appropriate values for $\langle -V(S) \rangle$ and for the nucleon cutoff, $v(k)$.

Our main interest is in the three nucleon systems, so for the expectation value of $-V(S)$ we take 2/3 of the total potential energy E_p, because two out of three bonds are contributing. According to a variational calculation by Ohmura et al. [35], E_p is in the range of 55 - 80 MeV. The potential with a small hard core gave 55 MeV, while the one with a hard core radius of .6 F gave 80 MeV. Our effective interaction should be regarded as a K matrix which shows no singular behaviour like the realistic nucleon-nucleon potential, so that E_p should be obtained from a potential without a hard core. Also Law and Bhaduri [36] showed that, for the binding energy calculation of the triton, it is a good approximation to take only the long range part of the nucleon-nucleon potential, which gave them a value of $E_p = 37$ MeV. If we note that Law and Bhaduri underestimated the triton binding energy while Ohmura et al. overestimated, we take $E_p \simeq 45$ MeV, so that

$$-\langle V(S)\rangle = \frac{2}{3} E_p \approx 30 \text{ MeV} \quad . \qquad [154]$$

For our cutoff function, we take $v(k) = (\Lambda^2-\mu^2)/(k^2+\Lambda^2)$, where μ, the pion mass is taken to be 139.6 MeV, the cutoff parameter Λ is taken for two cases, $\Lambda = 5\mu$ and $\Lambda = 6\mu$. The renormalized coupling constant is such that $f^2/4\pi = .08$, and all quantities are taken in terms of the pion mass. The results for Q, ΔQ, μ_m, $\Delta\mu$ are shown in Table III. It is obvious that $\Delta Q/Q \sim 7 \sim 9\%$, and $\Delta\mu/\mu_m = 9 \sim 10\%$, which shows that the corrections calculated here are quite large. However, the approximations made in obtaining these corrections are quite suspect.

In particular, the static approximation cannot be relied upon to give a useful quantitative value, as is mentioned by Hiida et al.[37]. They find that in low energy reactions such as pion-nucleon scattering, recoil effects are not essential, while in the nucleon structure problem, although the static approximation may give a useful qualitative argument, recoil effects are very important. Also, the real significance of V(S) is not apparent; we chose V(S) as an effective potential for simplicity. We failed, however, to consider the initial nucleon wave function renormalization in our calculation. Instead of treating it as an effective potential, it would be more appropriate to take some potential such as a harmonic oscillator potential, and then solve the same problem.

5.3 Nucleon Bound by Harmonic Oscillator Potential

We now consider a nucleon bound in a nucleus by some potential V(r), and we take account of nucleon recoil; therefore, in the free nucleon case we have to take account of the nucleon kinetic energy, so that in

TABLE III

The charge and magnetic moment densities and their corrections due to an interaction given by an effective potential V(S). The cut-off function is $v(k) = (\Lambda^2-m^2)/(k^2+\Lambda^2)$, while the cut-off parameter Λ is shown. The charge is in units of e and the magnetic moment in nuclear magnetons, and m is the pion mass.

	Q	ΔQ	μ_m	$\Delta\mu_m$
Λ=5m	.507	.046	.957	.095
Λ=6m	.772	.061	1.281	.116

Eq. [130], the free nucleon propagator is

$$\frac{i}{2\pi} \frac{1}{k_0 - \frac{k^2}{2M} + i\varepsilon} \qquad [155]$$

where M is the mass of the nucleon. Equation [118] becomes

$$H_N = \int d\underline{r}\; \psi^+(\underline{r}) \left(\frac{p^2}{2M} + V(r)\right) \psi(\underline{r}) \quad , \qquad [156]$$

which is the Hamiltonian for a nucleon bound in a potential $V(r)$, where $\psi(r)$ can be expanded in terms of creation and annihilation operators as in Eq. [86],

$$\psi(\underline{r}) = \sum_{\nu} C_{\nu}\; \psi_{\nu}(\underline{r}) \quad , \qquad [157]$$

where $\psi_{\nu}(r)$ is a quantum mechanical wave function such that

$$\left(\frac{p^2}{2M} + V(r)\right) \psi_{\nu}(\underline{r}) = E_{\nu}\; \psi_{\nu}(\underline{r}) \quad . \qquad [158]$$

$\psi_{\nu}(r)$ can be expanded as

$$\psi_{\nu}(r) = f_{n\ell}(r) Y_{\ell m}(\hat{r}) \quad , \quad \hat{r} = (\theta,\phi) \text{ of } \underline{r} \quad , \qquad [159]$$

where $f_{n\ell}(r)$ is the radial dependence of $\psi_{\nu}(r)$, $Y_{\ell m}(\hat{r})$ are the spherical harmonics, and C_{ν} is a destruction operator and E_{ν} is the energy eigenvalue. Similarly

$$\psi^+(\underline{r}) = \Sigma\, C_\nu^+ \, \psi_\nu^*(\underline{r}) \qquad [160]$$

such that

$$H_N = \sum_\nu E_\nu C_\nu^+ C_\nu \qquad \text{where } \nu \equiv (n, \ell, m) \quad . \qquad [161]$$

The pion field can be expanded as in Eq. [66], and since we are using the Schroedinger picture so that the quantities are time independent, we have

$$\phi_\alpha(\underline{r}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\underline{k}}{\sqrt{2w}} \left(a_k e^{i\underline{k}\cdot\underline{r}} + a_k^+ e^{-i\underline{k}\cdot\underline{r}}\right) \qquad [162]$$

where a_k and a_k^+ are destruction and creation operators, respectively, and $w = (k^2 + \mu^2)^{1/2}$. Therefore, Eq. [116] becomes

$$H_M = \int d\underline{k}\; w a_k^+ a_k \quad . \qquad [163]$$

Also, substituting Eqs. [159] and [162] into Eq. [120]

$$H_I = i\, (\frac{g}{\mu})\, (2\pi)^{-3/2} \sum_\alpha \sum_{\nu\nu'} C_{\nu'}^+ C_\nu \tau_\alpha \int d\underline{r}\; \psi_{\nu'}^* \psi_\nu(r)$$

$$\times \int \frac{d\underline{k}}{\sqrt{2w}} (\underline{\sigma}\cdot\underline{k}) \left(a_k e^{i\underline{k}\cdot\underline{r}} - a_k^+ e^{-i\underline{k}\cdot\underline{r}}\right) \qquad [164]$$

where the suffix k on a_k specifies the isospin state of the pion of momentum k, and H_I describes the interaction of a meson field with the nucleon field. We are interested in the case where a nucleon in the ground state ($\nu = (n\ell m) = (000)$) goes to an intermediate state ν' and interacts with the pion field. We first do the $\underline{r}$ integration. Expand

$$e^{i\underline{k}\cdot\underline{r}} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^{\ell} j_{\ell}(kr) Y^*_{\ell m}(\hat{k}) Y_{\ell m}(\hat{r}) \qquad [165]$$

such that

$$\int \underline{dr}\ \psi^*_{\nu}(r)\psi_0(r) e^{i\underline{k}\cdot\underline{r}}$$

$$= \int r^2\, dr\, f_{\nu}(r) f_0(r) \int d\hat{r}\ Y_{\ell m}(\hat{r}) Y_{00}(\hat{r}) e^{i\underline{k}\cdot\underline{r}}$$

$$= \sqrt{4\pi}\cdot i^{\ell} \int_0^{\infty} r^2 dr\ f_{\nu}(r) f_0(r) j_{\ell}(kr)\cdot Y^*_{\ell m}(\hat{k}) \qquad [166]$$

since $Y_{00}(\hat{r}) = 1/\sqrt{4\pi}$, and $\int Y^*_{\ell m}(\hat{r}) Y_{\ell m}(\hat{r}) d\hat{r} = 1$ and a $\sum_{\nu} = \sum_{n,\ell,m}$ is assumed. Also, since

$$e^{-i\underline{k}\cdot\underline{r}} = (4\pi) \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (-i)^{\ell} j_{\ell}(kr) Y_{\ell m}(\hat{k}) Y^*_{\ell m}(\hat{r})$$

$$= 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (-i)^{\ell} j_{\ell}(kr) Y^*_{\ell m}(\hat{k}) Y_{\ell m}(\hat{r}) \quad , \qquad [167$$

$$\int \underline{dr}\ \psi_\nu^*(\hat{r})\psi_0(\hat{r})e^{-i\underline{k}\cdot\underline{r}} = (-i)^\ell\sqrt{4\pi}\int_0^\infty r^2 dr$$

$$\times\ f_\nu(r)f_0(r)j_\ell(kr)Y_{\ell m}^*(\hat{k}) \quad . \tag{168}$$

Therefore, Eq. [164] becomes

$$H_I = i\ (\tfrac{g}{\mu})(2\pi)^{-3/2}\ i^\ell \sum_{\alpha\nu}\sum C_\nu^+ C_0\tau_\alpha \int \frac{\underline{dk}}{\sqrt{2w}} F_\nu(k)\underline{\sigma}\cdot\underline{k}$$

$$\times\ Y_{\ell m}^*(\hat{k})(a_k - (-)^\ell a_k^+) \tag{169}$$

where

$$F_\nu(k) = \sqrt{4\pi}\int_0^\infty r^2 dr\ f_\nu(r)f_0(r)j_\ell(kr)$$

and we have applied the relation

$$Y_{\ell m}^*(\theta,\phi) = (-)^m\ Y_{\ell -m}(\theta,\phi) \quad .$$

Our purpose again is to calculate the charge and magnetic moment densities for the free nucleon (without the static approximation) and the correction required if the nucleon is bound by a harmonic oscillator potential. The S matrix is calculated as before, except that the πNN vertex is altered, as shown by Eq. [169]. The πNN vertex becomes for the ground state to intermediate state transition ($0 \rightarrow \nu$)

$$0 \to \nu: \quad (2\pi i)\delta(\Sigma k_0)\ \frac{g}{\mu}\ i^{\ell+1}(-1)^{\ell+1}\ \tau_i(\underline{\sigma}\cdot\underline{k})F_\nu(k)Y^*_{\ell m}(\hat{k}) \qquad [170]$$

while the intermediate state to ground state vertex factor is

$$\nu \to 0: \quad 2\pi i\delta(\Sigma k_0)\ \frac{g}{\mu}\ i^{\ell-1}\ \tau_i(\underline{\sigma}\cdot\underline{k})F_\nu(k)Y_{\ell m}(\hat{k}) \quad . \qquad [171]$$

The nucleon propagator becomes

$$\frac{i}{2\pi}\,\frac{1}{k_0 - E_\nu + i\varepsilon} \qquad [172]$$

where $E_\nu = E_\nu - E_0$, i.e., the excitation energy.

The S matrix is then found to be

$$S = (-i)^3\delta(0)\ \sum_\nu \int_{-\infty}^{+\infty} dt \int \underline{dk} \int \underline{dk}'\ \left(\frac{i}{(2\pi)^4}\right)^2 \frac{i}{2\pi}$$
$$\times \frac{e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}}{(t^2-w_k^2+i\varepsilon)(t^2-w_{k'}^2+i\varepsilon)}\ (2\pi i)^2\left(\frac{g}{\mu}\right)^2(2\tau_3)^*(-1)$$
$$\times \frac{Y^*_{\ell m}(\hat{k})Y_{\ell m}(\hat{k}')F_\nu(k)F_\nu(k')(2\pi)(k_\mu+k'_\mu)\cdot eA^\mu}{t - E_\nu + i\varepsilon} \quad . \qquad [173]$$

Therefore, from Eqs. [136] and [173] we see that

$$\rho_c(r) = \frac{4}{(2\pi)^6}\left(\frac{g}{\mu}\right)^2 \frac{\tau_3}{2\pi i}\sum_\nu \int_{-\infty}^{+\infty} dt \int \underline{dk} \int \underline{dk}'\, F_\nu(\hat{k}) F_\nu(\hat{k}')$$

$$\times\, Y^*_{\ell m}(\hat{k}) Y_{\ell m}(\hat{k}')\, \frac{t(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k}) e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}\, v(k) v(k')}{(t^2-w^2+i\varepsilon)(t^2-w'^2+i\varepsilon)(t-E_\nu+i\varepsilon)} \;. \tag{174}$$

After doing the t integration (see Appendix III), we find that

$$\rho_c(r) = \frac{2}{(2\pi)^6}\left(\frac{g}{\mu}\right)^2 \tau_3 \int \underline{dk} \int \underline{dk}'\, F_\nu(k) F_\nu(k') Y^*_{\ell m}(\hat{k}) Y_{\ell m}(\hat{k}')$$

$$\times\, \frac{(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k}) e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}\, v(k) v(k')}{(w'+w)(w+E_\nu)(w'+E_\nu)} \tag{175}$$

while

$$Q = \int \rho_c(r)\underline{dr} = \frac{1}{(2\pi)^3}\left(\frac{g}{\mu}\right)^2 \tau_3 \sum_\nu (2\ell+1)$$

$$\times \int_0^\infty \frac{dk\, v^2(k) k^4 F_\nu^2(k)}{w(w+E_\nu)^2} \tag{176}$$

since there is no spin flip so that $(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k}) = \underline{k}'\cdot\underline{k}$, and $F_\nu(k) = \sqrt{4\pi}\int_0^\infty dr\, r^2 f_\nu(r) f_0(r) j_\ell(kr)$,

$$\sum_{m=-\ell}^{\ell} Y^*_{\ell m}(\hat{k}) Y_{\ell m}(\hat{k}) = \frac{2\ell+1}{4\pi} \;.$$

If we define $G_\nu(k) = F_\nu(k)/\sqrt{4\pi}$,

$$Q = \frac{2}{\pi}\left(\frac{g^2}{4\pi}\right)\frac{1}{\mu^2}\sum_\nu (2\ell+1)\int_0^\infty dk\, \frac{k^4\, G_\nu^2(k)}{w(w+E_\nu)^2}\, v^2(k) \quad . \qquad [177]$$

Also, from Eqs. [136] and [173] we see that

$$\underline{j}(r) = -\frac{2(4\pi)}{(2\pi)^6}\left(\frac{g}{\mu}\right)^2 \frac{\tau_3}{(2\pi i)}\sum_\nu \int_{-\infty}^{+\infty} dt \int \underline{dk} \int \underline{dk}'\, G_\nu(k) G_\nu(k')$$

$$\times\, v(k)v(k')\, \frac{(\underline{k}+\underline{k}')(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k})\, e^{i(k'-k)\cdot r}\, Y^*_{\ell m}(\hat{k})Y_{\ell m}(\hat{k}')}{(t^2-w_k^2+i\varepsilon)(t^2-w_{k'}'^2+i\varepsilon)(t+E_\nu-i\varepsilon)} \quad . \qquad [178]$$

Evaluating the t integration (Appendix III) we obtain

$$\underline{j}(r) = -\frac{2\tau_3}{(2\pi)^5}\left(\frac{g^2}{4\pi}\right)\frac{4\pi}{\mu^2}\sum_\nu \int \underline{dk}\underline{dk}'(\underline{k}+\underline{k}')\, e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}\, v(k)v(k')$$

$$\times\, \frac{(\underline{\sigma}\cdot\underline{k}')(\underline{\sigma}\cdot\underline{k})}{ww'(w+E_\nu)(w'+E_\nu)}\left\{1+\frac{E_\nu}{w+w'}\right\} Y^*_{\ell m}(\hat{k})Y_{\ell m}(\hat{k})G_\nu(k)G_\nu(k') \quad . \qquad [179]$$

Remembering that

$$\mu_m = \frac{1}{2}\int(\underline{r}\times\underline{j})\,d\underline{r} \quad \text{and} \quad \frac{1}{2}(\underline{r}\times\underline{j}) = \mu(r)\underline{\sigma} \quad , \qquad [180]$$

and noting that

$$\underline{r} \to -\frac{i}{2}(\underline{\nabla}_k' - \underline{\nabla}_k)\, e^{i(\underline{k}'-\underline{k})\cdot\underline{r}} \qquad [181]$$

we let

$$f(k',k) = -\frac{2\tau_3/(2\pi)^5(g^2/4\pi)4\pi/\mu^2}{ww'(w+E_\nu)(w'+E_\nu)}\{1+\frac{E_\nu}{w+w'}\}$$

$$\times Y^*_{\ell m}(\hat{k})Y_{\ell m}(\hat{k}')v(k)v(k')G_\nu(k)G_\nu(k') \qquad [182]$$

which is symmetric with respect to k and k'.

$$\underline{r}\times\underline{j}(r) = -\frac{i}{2}(\underline{\nabla}'_{\underline{k}}-\underline{\nabla}_{\underline{k}})e^{i(\underline{k}'-\underline{k})\cdot\underline{r}}(\underline{k}+\underline{k}')$$

$$\times(\underline{k}\cdot\underline{k}' + i\underline{\sigma}\cdot\underline{k}'\times\underline{k})f(k,k') \quad . \qquad [183]$$

The $(\underline{k}\cdot\underline{k}')$ does not contribute since it gives a symmetric contribution, while $(\nabla'_k-\nabla_k)$ is antisymmetric, and therefore, this term gives a zero contribution when integrated.

$$\mu_m = \frac{1}{2}\int\frac{1}{2}(\underline{\nabla}'_{\underline{k}}-\underline{\nabla}_{\underline{k}})e^{i\underline{k}'-\underline{k}\cdot\underline{r}}\times(\underline{k}+\underline{k}')(\underline{\sigma}\cdot\underline{k}'\times\underline{k})f(\underline{k},\underline{k}')d\underline{r}$$

which gives, after integrating by parts,

$$\mu_m = -\frac{1}{4}\int d\underline{k}\int d\underline{k}'\int e^{i\underline{k}'-\underline{k}\cdot\underline{r}}(\underline{\nabla}'_{\underline{k}}-\underline{\nabla}_{\underline{k}})\times(\underline{k}+\underline{k}')(\underline{\sigma}\cdot\underline{k}'\times\underline{k})f(k,k')d\underline{r}$$

$$[185]$$

$$\mu_m = \frac{2}{4}\int d\underline{k}\int d\underline{k}'\delta(\underline{k}-\underline{k}')f(\underline{k},\underline{k}')\underline{k}\times(\underline{\nabla}'-\underline{\nabla}_k)(\underline{\sigma}\cdot\underline{k}'\times\underline{k}) \quad ,$$

since $(\underline{k}+\underline{k}')$ and $f(k,k')$ are symmetric with respect to k and k', so that when they are operated on by $(\nabla'_k-\nabla_k)$ and integrated, they give a zero contribution. The factor

$$f(k,k')(k\times\underline{\nabla}'-\underline{\nabla})(\underline{\sigma}\cdot\underline{k}'\times\underline{k}) = -2f(k,k')(\underline{k}\times\underline{\sigma}\times\underline{k}) \qquad [186]$$

in the limit as $k \to k'$, and since

$$(\underline{k}\times\underline{\sigma}\times\underline{k}) \to k^2\underline{\sigma} - \underline{k}(\underline{\sigma}\cdot\underline{k}) = \frac{2}{3}k^2\underline{\sigma} \qquad [187]$$

we see that, when we combine Eqs. [180], [185] and [187],

$$\mu_m = \frac{2\tau_3}{(2\pi)^5}\left(\frac{g^2}{4\pi}\right)\frac{4}{\mu^2}(2\pi)^3 \sum_\nu \int \underline{dk}\; v^2(k)\,\frac{\frac{2}{3}k^2G_\nu^2(k)}{w^2(w+E_\nu)^2}$$

$$\times\, Y^*_{\ell m}(\hat{k})Y_{\ell m}(\hat{k})\{1 + \frac{E_\nu}{2w}\}$$

$$= \tau_3\left(\frac{4}{3\pi}\right)\left(\frac{g^2}{4\pi}\right)\frac{1}{\mu^2}\sum_{n,\ell}(2\ell+1)\int_0^\infty dk v^2(k)$$

$$\times\,\frac{k^4G_\nu^2(k)}{w^2(w+E_\nu)^2}\left(1 + \frac{E_\nu}{2w}\right) \qquad [188]$$

where we have used the fact that

$$\sum_{m=-\ell}^{\ell} Y^*_{\ell m}(k)Y_{\ell m}(k) = \frac{2\ell+1}{4\pi} .$$

We now solve for Q and μ_m for the case where the nucleon is bound by a harmonic oscillator potential such that Eq. [158] becomes

$$H\psi_\nu(r) = \left(\frac{p^2}{2M} + \frac{f}{2} r^2\right)\psi_{\nu(r)} = E\psi_\nu(\underline{r}) \qquad [189]$$

where f is the harmonic oscillator force constant, which can be determined in terms of the size of the nucleus, M is the nucleon mass such that $2M = 13.453\mu$, and p is the nucleon momentum. As we assumed previously, $\psi_\nu(r) = \psi_{n\ell m}(r)$ can be decomposed into a radial part and an angular part, Eq. [158], where $f_{n\ell}(r) = f_{n\ell\kappa}(r)$ is a normalized function $(\kappa = \frac{1}{2}(n-\ell))$ (38) such that

$$f_{n,\ell,\kappa}(r) = a^{3/2}\sqrt{\frac{2}{ar}}\left[\Gamma(\ell + \tfrac{3}{2})\binom{\kappa + \ell + \frac{1}{2}}{\kappa}\right]^{-1/2}$$

$$\times\, e^{-(ar)^2/2}\,(ar)^{\ell+1/2}\,L_\kappa^{\ell+1/2}(a^2r^2) \qquad [190]$$

where $\Gamma(\ell + \frac{3}{2}) = (\ell + \frac{1}{2})!\sqrt{\pi}$,

$$\binom{n}{m} = \frac{n!}{(n-m)!m!} \quad ,$$

$L_\kappa^{\ell+1/2}$ are the Laguerre polynomials; if we define the oscillator frequency $\varepsilon = (f/M)^{1/2}$, then the characteristic oscillator length is

$$b = \left(\frac{\hbar}{M\varepsilon}\right)^{1/2} = \frac{1}{(\ \varepsilon)^{1/2}} \qquad (\hbar = c = 1) \quad ,$$

so that $a^2 = 1/b^2 = M\varepsilon$.

As is shown in Appendix IV, a useful expression for $G_\nu(k)$ is

$$G_\nu(k) = \int_0^\infty r^2 dr\ f_\nu(r) f_0(r) j_\ell(kr)$$

$$= (2^{n+1/2} \sqrt{\kappa!(\kappa + \ell + 1/2)!})^{-1}\ e^{-k^2/4a^2} \cdot (\frac{k}{a})^n$$

$$= C_\nu e^{-k^2/4a^2}\ (\frac{k}{a})^n \qquad [191]$$

where n is summed from 0 to ∞, ℓ is summed from 0 to n, $\kappa = \frac{1}{2}(n-\ell)$, and

$$C_\nu = (2^{n+1/2} \sqrt{\kappa!(\kappa + \ell + 1/2)!})^{-1} \quad .$$

We see from Eqs. [188] and [176] that the charge and magnetic moment involve terms where

$$\alpha_n = \sum_\ell (2\ell+1) C_{n\ell}^2 = \frac{1}{(2^n \cdot n!)} \qquad [192]$$

which is shown in Appendix IV. The charge and magnetic moment expressions become

$$Q = \sum_n Q(n) = \tau_3 \frac{2}{\pi} (\frac{g^2}{4\pi}) \frac{1}{\mu^2} \sum_n \alpha_n \int_0^\infty \frac{k^4 dk v^2(k) e^{-k^2/2a^2}}{w(w+n\varepsilon)^2} (\frac{k}{a})^{2n} \qquad [193a]$$

$$\mu_m = \sum_n \mu(n) = \tau_3 \frac{4}{3\pi} \left(\frac{g^2}{4\pi}\right) \frac{1}{\mu^2} \sum_n \alpha_n$$

$$\times \int \frac{dkk^4 v^2(k) e^{-k^2/2a^2}}{w^2 (w+n\varepsilon)^2} \left(\frac{k}{a}\right)^{2n} \{1 + \frac{n\varepsilon}{2w}\} \qquad [193b]$$

where the energy of the intermediate state is

$$E_n = (n + \tfrac{3}{2})\varepsilon - \tfrac{3}{2}\varepsilon = n\varepsilon \quad .$$

If we let $t/\varepsilon = k^2/2a^2$ where $t = k^2/2M$, and define

$$S(\varepsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{(w+n\varepsilon)^2} \left(\frac{t}{\varepsilon}\right)^n$$

$$= -\frac{d}{dw} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{w+n\varepsilon} \left(\frac{t}{\varepsilon}\right)^n$$

$$= -\frac{d}{dw} \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^\infty d\lambda e^{-\lambda(w+n\varepsilon)} \left(\frac{t}{\varepsilon}\right)^n$$

$$= -\frac{d}{dw} \int_0^\infty d\lambda e^{-\lambda w} \cdot \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{t}{\varepsilon} e^{-\lambda\varepsilon}\right)^n \qquad [194]$$

and since $e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$,

$$S(\varepsilon) = -\frac{d}{dw} \int_0^\infty d\lambda e^{-\lambda w} \exp\left(\frac{t}{\varepsilon} e^{-\lambda\varepsilon}\right)$$

$$= \int_0^\infty d\lambda \lambda e^{-\lambda w} \exp\left(\frac{t}{\varepsilon} e^{-\lambda\varepsilon}\right) \quad . \qquad [195]$$

The terms in the integrand of Eq. [193] are of the form $S(\varepsilon)e^{-t/\varepsilon}$, so that

$$Q = \frac{2}{\pi}\left(\frac{g^2}{4\pi}\right)\frac{1}{\mu^2}\int_0^\infty dk\,\frac{k^4v^2(k)}{w}\int_0^\infty d\lambda e^{-\lambda w}\,\lambda\exp\left(\frac{t}{\varepsilon}(e^{-\lambda\varepsilon}-1)\right) \qquad [196]$$

and

$$\mu_m = \frac{4}{3\pi}\left(\frac{g^2}{4\pi}\right)\frac{1}{\mu^2}\int_0^\infty dk\,\frac{v^2(k)k^4}{w^2}\int_0^\infty d\lambda\lambda e^{-\lambda w}$$
$$\times\exp\left(\frac{t}{\varepsilon}(e^{-\lambda\varepsilon}-1)\right)\left(1+\frac{k^2e^{-\lambda\varepsilon}}{4wM}\right) \qquad [197]$$

which becomes

$$\mu_m = \frac{2}{3\pi}\,\frac{g^2}{4\pi}\,\frac{1}{\mu^2}\left[\int_0^\infty dk\,\frac{v^2(k)k^4}{w^2}\int_0^\infty d\lambda\lambda e^{-\lambda w}\exp\left(\frac{t}{\varepsilon}(e^{-\lambda\varepsilon}-1)\right)\right.$$
$$\left.+\int_0^\infty dk\,\frac{v^2(k)k^4}{w^3}\int_0^\infty d\lambda e^{-\lambda w}\exp\left(\frac{t}{\varepsilon}(e^{-\lambda\varepsilon}-1)\right)\right]. \qquad [198]$$

These are the final forms of the expressions for the magnetic moment and charge densities for a nucleon bound in a Harmonic oscillator characterized by the parameter ε.

We now investigate the two limiting cases of the charge and magnetic moment: The loose binding ($\varepsilon \to 0$) and the tight binding limit ($\varepsilon \to \infty$). Q and μ_m depend on through $S(\varepsilon)e^{-t/\varepsilon}$. In the loose binding limit we obtain

$$\lim_{\varepsilon\to 0} (S(\varepsilon)e^{-t/\varepsilon}) = \lim_{\varepsilon\to 0} [- \frac{d}{dw} \int_0^\infty d\lambda e^{-\lambda w} \exp[\frac{t}{\varepsilon} (e^{-\lambda\varepsilon}-1)]]$$

$$= - \frac{d}{dw} \int_0^\infty d\lambda e^{-\lambda w} \; e^{-\lambda t}$$

$$= - \frac{d}{dw} \frac{1}{w+t} = \frac{1}{(w+t)^2} \quad . \qquad [199]$$

Then the expressions for the magnetic moment and charge of the nucleon are

$$Q_L = \frac{2}{\pi} (\frac{g^2}{4\pi}) \frac{1}{\mu^2} \int_0^\infty dk \quad \frac{k^4 v^2(k)}{w(w + \frac{k^2}{2M})^2}$$

$$\mu_L = \frac{4}{3\pi} (\frac{g^2}{4\pi}) \frac{1}{\mu^2} \int_0^\infty dk \quad \frac{k^4 v^2(k)}{w^2(w + \frac{k^2}{2M})^2} (1 + \frac{k^2}{4wM}) \quad . \qquad [200]$$

Q_L and μ_L can be regarded as the charge and magnetic moment of a free nucleon, respectively.

Our expression for μ_L disagrees with that obtained by Goto (39) who investigated the magnetic moment of a nucleon starting with a relativistic interaction and introducing a cut-off function. This difference may be attributed to the fact that our transformed Hamiltonian for the pion nucleon system, Eq. [120], is correct only to order 1/M and terms in the expansion were neglected, while Goto considers all orders of M and all the various contributions.

In the tight binding limit, we see from Eqs. [196] and [197] that if $\varepsilon \to \infty$,

$$Q_T = \frac{2}{\pi} \left(\frac{g^2}{4\pi}\right) \frac{1}{\mu^2} \int_0^\infty dk \, \frac{k^4}{w^3} \, v^2(k)$$

$$\mu_T = \frac{4}{3\pi} \left(\frac{g^2}{4\pi}\right) \frac{1}{\mu^2} \int_0^\infty dk \, \frac{k^4}{w^4} \, v^2(k) \qquad [201]$$

which are identical to the expressions obtained in Eqs. [146] and [147] in the static approximation ($M \rightarrow \infty$).

The correct values for the charge and magnetic moment of the tri-nucleon system is obtained by taking ε between these extreme values. If we take the harmonic oscillator characteristic length

$$b = \frac{1}{(M\varepsilon)^{1/2}} = 1.4 \text{ fm} \quad ,$$

the value which gives the best binding energy for He^3 and H^3 according to the calculations of Law and Bhaduri (40), we obtain the best values for Q and μ.

CHAPTER VI

RESULTS AND CONCLUSIONS

We calculate the charge and magnetic moment both for a free nucleon and for a bound nucleon. In these calculations we take two values for the cut-off momentum, $\Lambda = 6\mu$ and $\Lambda = 7\mu$, and two forms for the cut-off function,

$$v(k) = e^{-k^2/2\Lambda^2}$$

and

$$v(k) = \frac{\Lambda^2 - \mu^2}{k^2 + \Lambda^2} \;;$$

we take the harmonic oscillator characteristic length b to be equal to 1.4 fm. The results are shown in Table IV. As can be seen, we find that the corrections to the charge and magnetic moment due to binding are quite small. The process shown in Fig. 8 accounts for over 50% of the total isovector magnetic moment. We found that the bound nucleon magnetic moment is enhanced by approximately 2%, which implies that our calculations give an enhancement of approximately 1% to the total experimental value of the magnetic moment. Our corrections also appear to be insensitive to the cut-off function and the cut-off parameter.

TABLE IV

The magnetic moment μ_m (in units of nm) and charge Q (in units of e). The bound nucleon values are primed, where Λ is the momentum cut-off parameter and v(k) is the cut-off function and μ is the pion mass. The percent differences are given in parenthesis.

		Q	Q'	μ_m (nm)	μ'_m (nm)
$v(k) = e^{-k^2/2\Lambda^2}$	$\Lambda=6\mu$	.4280	.4422 (3.3)	1.087	1.110 (2.1)
	$\Lambda=7\mu$	.5537	.5713 (3.1)	1.293	1.320 (2.0)
$v(k) = \frac{\Lambda^2-\mu^2}{k^2+\Lambda^2}$	$\Lambda=6\mu$	.3494	.3612 (3.4)	.8600	.8779 (2.1)
	$\Lambda=7\mu$	.4558	.4712 (3.4)	1.044	1.065 (2.0)

If we compare the results of Table IV with those of Table III we see that there is a rather large discrepancy between the results obtained by considering the nucleon bound in a harmonic oscillator potential and the result obtained by using the method derived in Section 5.2. Goebels [41] pointed out that a possible source of error in our treatment in Section 5.2 was that we failed to take account of the initial nucleon wavefunction renormalization, i.e., our initial work assumed that the nucleon is bound only in the intermediate state.

In Fig. 10 we have plotted the magnetic moment μ_m (Eq. [198]) as a function of b for the case where

$$v(k) = \frac{\Lambda^2 - \mu^2}{k^2 + \Lambda^2}$$

and $\Lambda = 7\mu$. We see that μ_m approaches the free limit as $b \to \infty$ and μ_m approaches the static limit as $b \to 0$.

If we examine the results of Table IV, we find that the corrections to the magnetic moments of the tri-nucleon systems are not significant. In Chapter IV we found that the corrections of Harper et al. and Hadjimichael account for the major part of the discrepancy between the experimental and theoretical values of the magnetic moments of the tri-nucleon systems; in fact, their calculations somewhat overcompensate the difference. If our small

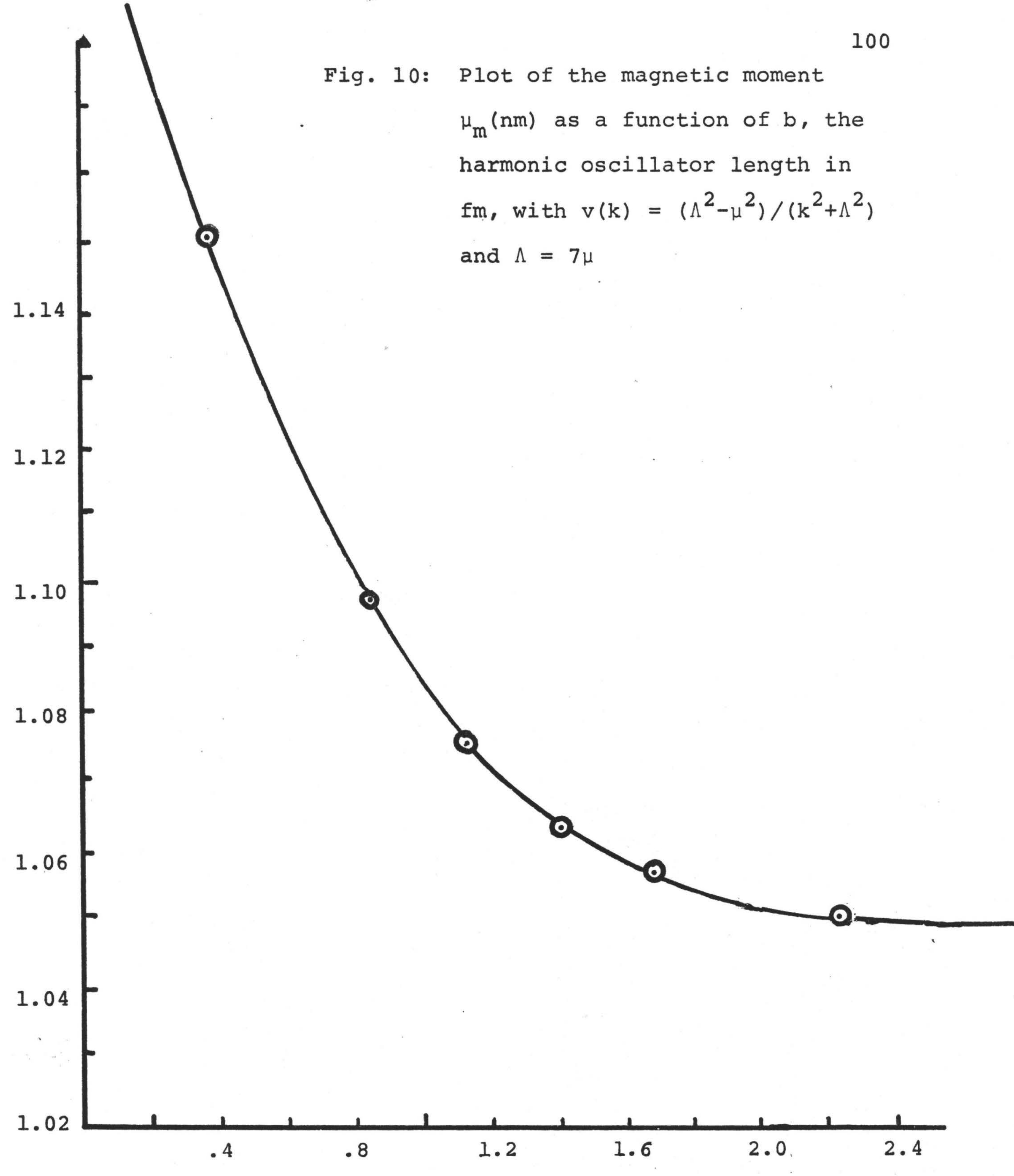


Fig. 10: Plot of the magnetic moment μ_m(nm) as a function of b, the harmonic oscillator length in fm, with $v(k) = (\Lambda^2-\mu^2)/(k^2+\Lambda^2)$ and $\Lambda = 7\mu$

contribution is taken into account, the discrepancy increases, which implies that some additional work is still necessary to completely understand this problem.

If we consider Eqs. [193a] and [193b], we see that Q and μ_m are given in terms of a sum over the intermediate states, and an interesting problem can be considered. For a closed-shell nucleus such as He^4 and O^{16}, there should be a "quenching" [(42)] of the magnetic moment due to the fact that the Pauli exclusion principle will prevent the nucleon from recoiling into intermediate states that are already filled. If the s-state is completely filled, the term with $n = 0$ (= .196 nm) in the summation of Eq. [193b] should be removed. Or more generally, if the n-shell is filled, the term with that n should be removed (Table V). The "quenching" effect seems to be quite large, and should provide further impetus to investigate this problem thoroughly.

TABLE V

Terms of the charge and magnetic moment expansions

$$Q = \sum_{n=0}^{\infty} Q(n), \quad \mu_m = \sum_{n=0}^{\infty} \mu(n),$$

Eq. [193], for the case where

$$v(k) = (\Lambda^2 - \mu^2)/(k^2+\Lambda^2) \text{ and } \Lambda = 7\mu.$$

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7
$\mu_m(n)$	.196	.158	.117	.089	.069	.055	.044	.037
$Q(n)$	.045	.044	.038	.033	.028	.024	.021	.018

APPENDIX I

A. Coordinates and Momenta

The space-time coordinates $(t,x,y,z) \equiv (t,\underline{x})$ used throughout the text are denoted by the contravariant four vector (with $\hbar = c = 1$):

$$x^{\mu} \equiv (x^0, x^1, x^2, x^3) \equiv (t,x,y,z) \quad . \qquad \text{[A1]}$$

The covariant four vector x_{μ} is obtained by changing the sign of the space components

$$x_{\mu} \equiv (x_0, x_1, x_2, x_3) \equiv (t, -x, -y, -z)$$

$$= g_{\mu\nu} x^{\nu} \qquad \text{[A2]}$$

where the metric used here is

$$g_{\mu\nu} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \quad . \qquad \text{[A3]}$$

The summation convention is used throughout, and the inner product is

$$x^2 = x_{\mu} x^{\mu} = t^2 - \underline{x}^2 \quad .$$

Momentum vectors are defined as

$$p^{\mu} = (E,\ p_x,\ p_y,\ p_z) \qquad [A4]$$

such that

$$p_1 \cdot p_2 = p_1^{\mu} p_{2\mu} = E_1 E_2 - \underline{p}_1 \cdot \underline{p}_2 \qquad [A5]$$

and $$x \cdot p = tE - \underline{x} \cdot \underline{p} \quad . \qquad [A6]$$

B. Dirac Matrices and Spinor

A Dirac spinor for a particle of momentum p_{μ} and polarization s is denoted by $u_{\alpha}(p_{\mu},s)$, while for the antiparticle it is called $v_{\alpha}(p_{\mu},s)$. In each case the energy

$$p_0 = E_p = +\sqrt{|\underline{p}|^2 + m^2}$$

is positive. Also, the vector s^{μ}, which in the rest frame for the form

$$s^{\mu} = (0,\hat{s}) \quad , \quad \hat{s} \cdot \hat{s} = 1 \qquad [A7]$$

represents the direction of spin of the physical particle in the rest frame. The γ matrices in the Dirac equation satisfy the anticommutation relation

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu} \quad , \tag{A8}$$

and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5$

where the γ matrices are 4 by 4 matrices. A familiar representation for these γ matrices is

$$\gamma^0 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \quad , \quad \{\gamma^i\} = \underset{\sim}{\gamma} = \begin{vmatrix} 0 & \underset{\sim}{\sigma} \\ -\underset{\sim}{\sigma} & 0 \end{vmatrix} \tag{A9}$$

where

$$\sigma^1 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \quad \sigma^2 = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \quad \sigma^3 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

$$1 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad . \tag{A10}$$

A frequently occurring combination is

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^{\mu}, \gamma^{\nu}] \quad .$$

The inner product of a γ matrix with an ordinary four-vector is often encountered and is denoted by

$$\gamma_\mu A^\mu = \not{A} = \gamma^0 A^0 - \underline{\gamma}\cdot\underline{A}$$

[A11]

$$p_\mu \gamma^\mu = \not{p} = E\gamma^0 - \underline{p}\cdot\underline{\gamma} \quad .$$

The spinors u and v satisfy the Dirac equation

$$(\not{p} - m)u(p,s) = 0$$

[A12]

$$(\not{p} + m)v(p,s) = 0 \quad .$$

In terms of adjoint spinor,

$$\bar{u} = u^+\gamma^0 \qquad \text{and} \qquad \bar{v} = v^+\gamma^0 \quad ,$$

$$\bar{u}(p,s)(\not{p} - m) = 0$$

[A13]

$$\bar{v}(p,s)(\not{p} + m) = 0 \quad .$$

The following normalization and completeness conditions are satisfied by the spinors:

$$\bar{u}(p,s)u(p,s) = 1$$

$$\bar{v}(p,s)v(p,s) = -1 \qquad \text{[A14]}$$

$$\sum_s [u_\alpha(p,s)\bar{u}_\beta(p,s) - v_\alpha(p,s)\bar{v}_\beta(p,s)] = \delta_{\alpha\beta} \quad .$$

Another representation of the Dirac equation that is commonly used is given in terms of $\underline{\alpha}$ and β matrices, where the Dirac equation is

$$i\hbar \frac{\partial\psi}{\partial t} = \frac{\hbar c}{i} \left(\alpha_1 \frac{\partial\psi}{\partial x_1} + \alpha_2 \frac{\partial\psi}{\partial x_2} + \alpha_3 \frac{\partial\psi}{\partial x_3}\right) + \beta mc^2\psi$$

$$\equiv H\psi \qquad \text{[A15]}$$

where the γ matrices are related to $\underset{\sim}{\alpha}$ and β by $\underset{\sim}{\gamma} = \beta\underset{\sim}{\alpha}$, $\gamma_0 = \beta$; if $\hbar = c = 1$, the Dirac equation becomes

$$H\psi = (\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta m)\psi \quad . \qquad \text{[A16]}$$

APPENDIX II

The total Hamiltonian of the pion-nucleon system is

$$H = H_N + H_M + H_I$$

$$= \int d^3x \psi^*(\underset{\sim}{x}) (\underset{\sim}{\alpha} \cdot \underset{\sim}{p} + \beta m) \psi(\underset{\sim}{x})$$

$$+ \frac{1}{2} \int d^3x \{\pi^2(\underset{\sim}{x}) + \underset{\sim}{\nabla}\phi \cdot \underset{\sim}{\nabla}\phi(\underset{\sim}{x}) + \mu^2\phi^2(\underset{\sim}{x})\}$$

$$+ G \sum_j \int d^3x \ \bar{\psi}\gamma_5\tau_j\psi\phi_j \qquad [A17]$$

where the pion-nucleon fields interact through direct pseudoscalar coupling.

In order to study the nonrelativistic Hamiltonian, matters are greatly simplified if we first transform H so as to eliminate the pseudoscalar coupling term for a pseudovector coupling term. We perform a unitary transformation on H such that

$$H' = e^{iS} \ H \ e^{-iS} \qquad [A18]$$

where throughout we are working in the Schrödinger picture. Berger et al. [43] take

$$S = \int d^3x\psi^* s\psi(\underset{\sim}{x}) = \int d^3x \sum_{\alpha,\beta=1}^{4} \psi_j^* s_{\alpha\beta}\psi_\beta \qquad [A19]$$

with $s(\underset{\sim}{x}) = i\gamma_5\omega(\phi(\underset{\sim}{x}))$. We note that S is not a matrix but a c-number, and therefore it commutes with all Dirac matrices. If we note that

$$\psi'(\underset{\sim}{x},\lambda) = e^{iS\lambda}\psi(\underset{\sim}{x})e^{-iS\lambda}$$

$$\frac{\partial\psi'(\underset{\sim}{x},\lambda)}{\partial\lambda} = i[S,\psi'(\underset{\sim}{x},\lambda)] \qquad [A20]$$

and remember that the ψ's anti-commute, and make use of the fact that

$$e^{iS}Qe^{-iS} = Q + \frac{i}{1!}[S,Q] + \frac{i^2}{2!}[S,[S,Q]] + \ldots \quad , \qquad [A21]$$

the expression we finally obtain for the transformed Hamiltonian is

$$H' = \int d^3x\psi^*(\underline{x})(\underline{\alpha}\cdot\underline{p} + \beta m)\psi(\underline{x})$$

$$+ \frac{1}{2}\int d^3x\{\pi^2(\underline{x}) + \nabla\phi\cdot\nabla\phi(\underline{x}) + \mu^2\phi^2(\underline{x})\}$$

$$+ \int d^3x\psi^*(\underline{x})(e^{2is}-1)\beta M\psi(\underline{x})$$

$$+ \int d^3x\psi^*(\underline{x})e^{2is}G\beta\gamma_5\phi(\underset{\sim}{x})\psi(\underset{\sim}{x}) \qquad \ldots$$

$$- \lambda \int d^3x\psi^*(\underline{x}) \underset{\sim}{\Sigma} \cdot \underset{\sim}{\nabla}\phi(\underset{\sim}{x})\psi(\underset{\sim}{x})$$

$$- i\lambda \int d^3x\psi^*(\underline{x})\gamma_5\psi(\underline{x})\pi(\underline{x})$$

$$- \frac{1}{2} \lambda^2 \left(\int d^3x\psi^*(\underline{x})\gamma_5\psi(\underline{x})\right)^2 \qquad [A22]$$

where $\Sigma_i = i\gamma_5\alpha_i$. If in this expression we expand the exponential factor and retain terms quadratic in ϕ, and if we take $-\lambda = G/2M$, then the pseudoscalar coupling will be eliminated to this order. [The equivalence of pseudoscalar and pseudovector coupling is most clearly established by Foldy's transformation (44), where s is taken to be

$$S = \frac{1}{2} \int d^3x\psi^*(\underline{x})\gamma_5\psi(\underline{x}) \tan^{-1}\left(\frac{G\phi(\underline{x})}{M}\right) . \qquad [A23]$$

If we use this canonical transformation in [A18], the pseudoscalar coupling is completely eliminated, while the pseudovector coupling term now appears with a nonlinear coefficient.] This gives

$$H' = H_N + H_M + H_I'' \qquad [A24]$$

where

$$H_I'' = \frac{G}{2M} \int d^3x\psi^*(\underline{x})(\underset{\sim}{\Sigma}\cdot\nabla\phi(\underline{x}) + i\gamma_5\pi(\underline{x}))$$

$$+ \frac{G^2}{2M} \int d^3x\psi^*(\underline{x})\beta\psi(\underline{x})\phi^2(x)$$

$$- \frac{G^2}{2M} (\int d^3x\psi^*(\underline{x})\gamma_5\psi(\underline{x}))^2 \quad . \qquad [A25]$$

The first term in Eq. [A25] is called the derivative coupling form of the meson-nucleon Hamiltonian, and if we remember that $\pi(\underline{x})$ is essentially $\partial_t\phi(\underline{x})$, this term becomes

$$\frac{F}{\mu}\,\bar{\psi}(\underline{x})\gamma_5\gamma_\mu\psi\partial^\mu\phi \qquad [A26]$$

which is the conventional way of expressing the pseudovector interaction, and where

$$\frac{F}{\mu} = \frac{G}{2M} \quad . \qquad [A27]$$

The two meson term $G^2/2M \int d^3x\psi^*(\underline{x})\beta\psi(\underline{x})\phi^2(x)$ is analogous to the quadratic $(e^2/2m)\underset{\sim}{A}^2$ in the nonrelativistic radiation theory. The last term in [A25] is called the contact term and is always present in the Hamiltonian for a derivative coupling.

Having now replaced the pseudoscalar pion-nucleon interaction term by a pseudovector interaction, we can perform successive canonical transformations on H' to remove all the odd Dirac matrices, and to order 1/M the transformed Hamiltonian becomes [45]

$$H' = \int d^3x\bar{\psi}(\underline{x})(M + \frac{\underline{p}^2}{2M})\psi(x)$$

$$+ H_M + \frac{G}{2M}\int d^3x\psi^*(x)\tau_j(\underset{\sim}{\sigma}\cdot\nabla\phi_j(x))\psi(\underline{x})$$

$$+ \mathcal{O}(\frac{1}{M^2}) \quad . \qquad [A27]$$

APPENDIX III

In Eq. [135], we are required to perform the t, or energy integrations. The charge integral is of the form

$$I_c = \int_{-\infty}^{+\infty} dt \frac{1}{(t^2 - \omega_k^2 + i\varepsilon)(t^2 - \omega_{k'}^2 + i\varepsilon)} \qquad [A28]$$

If we take the counter-clockwise path of integration in the complex plane, Fig. 10, we see that

$$I_c = 2\pi i \, \{- \frac{1}{2\omega'} \frac{1}{\omega'^2 - \omega^2} + \frac{-1}{2\omega} \frac{1}{\omega^2 - \omega'^2}\}$$

$$I_c = 2\pi i \frac{1}{2\omega\omega'(\omega + \omega')} \quad . \qquad [A29]$$

For the case of the current density, the t integral is of the form

$$I_j = \int_{-\infty}^{+\infty} \frac{dt}{(t^2 - \omega_k^2 + i\varepsilon)(t^2 - \omega_{k'}^2 + i\varepsilon)(t - i\varepsilon)} \qquad [A30]$$

taking the same path as in Fig. 11, where now there is an extra pole at $t = i\varepsilon$; we see that

$$I_j = 2\pi i \, \{\frac{1}{\omega^2\omega'^2} + \frac{1}{2\omega^2(\omega^2 - \omega'^2)} + \frac{1}{2\omega'^2(\omega'^2 - \omega^2)}\}$$

$$I_j = \frac{2\pi i}{2\omega^2\omega'^2} \quad . \qquad [A31]$$

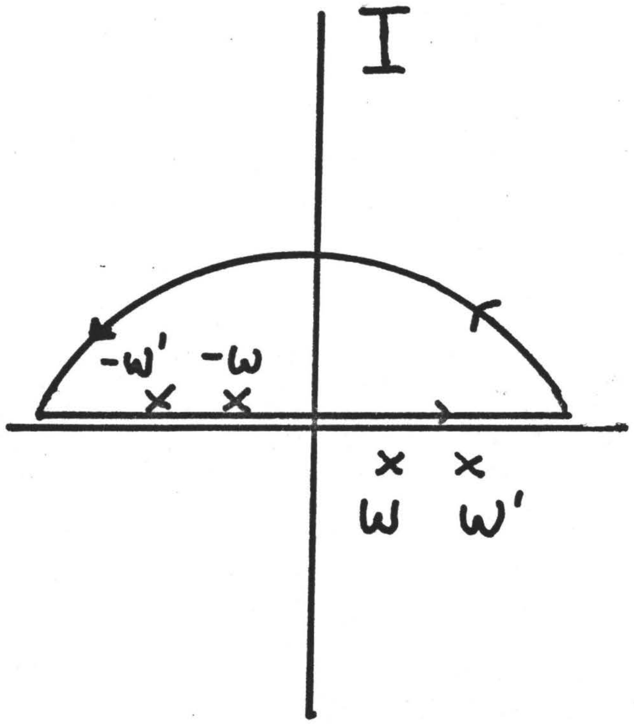


Fig. 11: Integration path for the integral of Eq. [A26] with the corresponding poles

In order to obtain Eq. [150], the energy integral is of the form

$$I = \int_{-\infty}^{+\infty} \frac{dt}{(t - i\varepsilon)^2 (t^2 - \omega_k^2 + i\varepsilon)(t^2 - \omega_{k'}^2 + i\varepsilon)} \qquad [A32]$$

$$I = \frac{2\pi i}{2} \frac{1}{\omega^2 - \omega'^2} \left(\frac{1}{\omega^3} - \frac{1}{\omega'^3}\right)$$

$$I = \frac{2\pi i}{2} \frac{\omega^2 + \omega\omega' + \omega'^2}{(\omega\omega')^3 (\omega + \omega')} \qquad [A33]$$

where we take the clockwise path of integration in the complex plane.

When we perform the t integration in Eq. [174], we obtain

$$I = \int_{-\infty}^{+\infty} dt \frac{t}{(t^2 - \omega^2 + i\varepsilon)(t^2 - \omega'^2 + i\varepsilon)(t - E_\nu + i\varepsilon)}$$

$$= 2\pi i \left\{\frac{-\omega}{-2\omega(\omega^2 - \omega'^2)(-\omega - E_\nu)} + \frac{-\omega'}{-2\omega'(\omega'^2 - \omega^2)(-\omega' - E_\nu)}\right\}$$

$$= \frac{2\pi i}{2} \left\{\frac{1}{(\omega'^2 - \omega^2)(\omega + E_\nu)} - \frac{1}{(\omega'^2 - \omega^2)(\omega' + E_\nu)}\right\}$$

$$= 2\pi i \frac{1}{2(\omega' + \omega)(\omega + E_\nu)(\omega' + E_\nu)} \qquad [A34]$$

where we take the counter-clockwise path of integration.

Finally, the t integration in Eq. [178] gives

$$I = 2\pi i \int_{-\infty}^{+\infty} dt \; \frac{1}{(t^2-\omega^2+i\varepsilon)(t^2-\omega'^2+i\varepsilon)(t+E_\nu-i\varepsilon)}$$

$$= -2\pi i \; \{\frac{1}{2\omega(\omega^2-\omega'^2)(\omega+E_\nu)} + \frac{1}{2\omega'(\omega'^2-\omega^2)(\omega'+E_\nu)}\}$$

$$= \frac{-2\pi i}{2(\omega^2-\omega'^2)} \; \{\frac{1}{\omega(\omega+E_\nu)} - \frac{1}{\omega'(\omega'+E_\nu)}\}$$

$$= 2\pi i \; \frac{}{2\omega\omega'(\omega+E_\nu)(\omega'+E_\nu)} \; \{1 + \frac{E_\nu}{\omega+\omega'}\} \quad . \qquad [A35]$$

APPENDIX IV

If we let

$$G_\nu(k) = \int_0^\infty r^2 dr f_\nu(r) f_0(r) j_\ell(kr) \qquad [A36]$$

where $\nu = (n,\ell,\kappa)$ and $\kappa = \frac{1}{2}(n-\ell)$, we have to show that

$$G_\nu(k) = C_\nu e^{-k^2/4a^2} \left(\frac{k}{a}\right)^n$$

where

$$C_\nu = \frac{1}{2^{n+1/2}(\kappa!(\kappa + \ell + \frac{1}{2})!)}$$

and

$$f_{n\ell\kappa}(r) = a^{3/2}\sqrt{\frac{2}{ar}}\,[\Gamma(\ell + \tfrac{3}{2})\begin{pmatrix}\kappa + \ell + \frac{1}{2}\\ \kappa\end{pmatrix}]^{-1/2}$$

$$\times\, e^{-(ar)^2/2}\,(ar)^{\ell+1/2}\,L_\kappa^{\ell+1/2}(a^2r^2) \quad . \qquad [A37]$$

For example,

$$f_0 = 2a^{3/2}\,\pi^{-1/4}\,e^{-(ar)^2/2} \quad ,$$

so that

$$G_\nu(k) = \frac{2a^{\ell+3}\, k^{-1/2}}{(\ell + \frac{1}{2})!\binom{\kappa + \ell + \frac{1}{2}}{\kappa}^{1/2}} \int_0^\infty drr^{\ell+3/2}\, e^{-(ar)^2}$$

$$\times\, L_\kappa^{\ell+1/2}(a^2r^2)\, J_{\ell+1/2}(kr) \qquad [A38]$$

where $L_\kappa^{\ell+1/2}(a^2r^2)$ is a Laguerre Polynomial and $J_{\ell+1/2}(kr)$ is a Bessel function.

A useful formula is

$$\int_0^\infty r^{\lambda+1}\, e^{-\beta r^2}\, L_\kappa^\lambda(\alpha r^2) J_\lambda(kr)\, dr$$

$$= (2\beta)^{-(\lambda+1)} \left(\frac{\beta-\alpha}{\beta}\right)^\kappa k^\lambda\, e^{-k^2/4\beta}\, L_\kappa^\lambda\left(\frac{\alpha k^2}{4\beta^2} \cdot \frac{\beta}{\alpha-\beta}\right) . \qquad [A39]$$

The Laguerre polynomials are defined as

$$L_\kappa^\lambda(x) = \sum_{r=0}^{\kappa} (-1)^r \binom{\kappa+\lambda}{\kappa-r} \frac{x^r}{r!} \qquad [A40]$$

so that

$$\lim_{\beta\to\alpha} \left(\frac{\beta-\alpha}{\beta}\right)^\kappa L_\kappa^\lambda\left(\frac{\alpha k^2}{4\beta^2}\, \frac{\beta}{\alpha-\beta}\right) = \frac{1}{\kappa!} \left(\frac{k^2}{4\beta}\right)^\kappa . \qquad [A41]$$

Putting $\beta = a^2$, we get

$$\int_0^\infty drr^{\lambda+1}\, e^{-(ar)^2}\, L_\kappa^\lambda(a^2r^2)J_\lambda(kr)$$

$$= \frac{k^\lambda}{(2a^2)^{\lambda+1}} \cdot \frac{1}{\kappa!}\left(\frac{k^2}{4a^2}\right)^\kappa e^{-k^2/4a^2} \qquad \text{[A42]}$$

Therefore, $G_\nu(k)$ becomes

$$G_\nu(k) = \frac{2a^{\ell+3}\, k^{-1/2}}{\left[(\ell+\frac{1}{2})!\binom{\kappa+\ell+\frac{1}{2}}{\kappa}\right]^{1/2}} \frac{k^{\ell+1/2}}{(2a^2)^{\ell+3/2}}$$

$$\times \frac{1}{\kappa!}\left(\frac{k^2}{4a^2}\right)^\kappa e^{-k^2/4a^2} \qquad \text{[A43]}$$

so that

$$G_\nu(k) = \left(2^{n+1/2}\sqrt{\kappa!(\kappa+\ell+\tfrac{1}{2})!}\right)^{-1} e^{-k^2/4a^2}\left(\frac{k}{a}\right)^n \quad . \qquad \text{[A44]}$$

Using this expression for $G_\nu(k)$, and applying the principle of Unitarity of the S-matrix, we see that

$$1 = \sum_{\nu=n,\ell} \langle 0|e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}}|\nu\rangle\langle\nu|e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}|0\rangle \qquad \text{[A45]}$$

$$= \sum_\nu (2\ell+1)G_\nu^2(k) \qquad \text{[A45]}$$

where we sum over all the intermediate states

$$1 = \sum_\nu (2\ell+1)C_\nu^2 \left(\frac{k}{a}\right)^{2n} e^{-k^2/2a^2}$$

$$1 \equiv \sum_n \alpha_n \left(\frac{k}{a}\right)^{2n} e^{-k^2/2a^2} \quad . \qquad [A46]$$

This gives

$$\sum_n \alpha_n \left(\frac{k}{a}\right)^{2n} = e^{k^2/2a^2} = \sum_n \frac{1}{n!} \left(\frac{k^2}{2a^2}\right)^n$$

$$= \sum_n \frac{1}{2^n n!} \left(\frac{k}{a}\right)^{2n} \qquad [A47]$$

which implies that

$$\alpha_n = \frac{1}{2^n n!} \quad . \qquad [A48]$$

REFERENCES

1. S. D. Drell and F. Zachariasen, Electromagnetic Structure of Nucleons, Oxford University Press, p. 2 (1961).

2. J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics, McGraw-Hill, p. 109ff (1964).

3. S. D. Drell and F. Zachariasen, op. cit., p. 5.

4. Ibid., p. 4.

5. M. N. Rosenbluth, Phys. Rev. 79, 615 (1950).

6. L. N. Hand, D. G. Miller and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

7. C. Backenstass, B. Hyams, S. Knop, T. Marin and M. Stierlin, Phys. Rev. Lett. 6, 415 (1961).

8. W. H. Barkas and A. H. Rosenfeld, "Rochester Conference Report", Interscience, N.Y., 878 (1960).

9. W. F. Cartwright, C. Richman, M. H. Whitehead and H. A. Wilcox, Phys. Rev. 81, 652 (1951).

10. A. Tollestrup, Proc. Int. Conf. High Energy Phys., Interscience, N.Y. (1960).

11. W. H. Panofsky, R. L. Aamodt and J. Hadley, Phys. Rev. 81, 565 (1951).

12. S. DeBenedetti, Nuclear Interactions, John Wiley & Son, Inc., p. 432 (1964).

13. J. D. Bjorken and S. D. Drell, op. cit., Chapter 12.

14. S. S. Schweber, An Introduction to Relativistic Quantum Field Theory, Harper and Row, p. 283 (1961).

15. S. DeBenedetti, op. cit., p. 242ff.

16. Ibid., p. 51ff.

17. C. Michael and C. Wilkin, Nucl. Phys. B11, 99 (1969).

18. F. J. Hadjioannou, Phys. Rev. 125, 1414 (1962).

19. R. J. Adler and S. D. Drell, Phys. Rev. Lett. 13, 349 (1964).

20. L. M. Delves and A. C. Phillips, Rev. Mod. Phys. 41, 375 (1969).

21. B. F. Gibson and L. I. Schiff, Phys. Rev. 138, B26 (1965).

22. B. F. Gibson, Phys. Rev. 139, B1153 (1965).

23. L. M. Delves, J. M. Blatt, C. Pask and B. Davies, Phys. Lett. 28B, 472 (1969).

24. A. D. Jackson, A. Lande and P. U. Sauer, Phys. Lett. 35B, 365 (1971).

25. F. Villars, Phys. Rev. 72, 257 (1947).

26. M. Chemtob and M. Rho, Nucl. Phys. A163, 1 (1971).

27. D. O. Riska and G. E. Brown, Phys. Lett. 38B, 193 (1972).

28. E. P. Harper, Y. E. Kim, A. Tubis and M. Tho, Phys. Lett. 40B, 533 (1972).

29. E. P. Harper, Y. E. Kim and A. Tubis, Phys. Rev. Lett. 28, 1533 (1972).

30. R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

31. E. Hadjimichael, S. N. Yang and G. E. Brown, Phys. Lett. 39B, 594 (1972).

32. S. Fubini, Nuovo Cimento 3, 1425 (1956).

G. Salzman, Phys. Rev. 99, 973 (1955); 435 (1956).

F. Zachariasen, Phys. Rev. 102, 295 (1956).

S. Treiman and R. G. Sachs, Phys. Rev. 103, 435 (1956).

33. S. S. Schweber, op. cit., p. 471.

34. H. Miyazawa, Phys. Rev. 104, 1741 (1956).

35. T. Ohmura, M. Morita and M. Yamada, Progr. Theor. Phys. 17, 326 (1957).

36. J. Law and R. K. Bhaduri, Can. J. Phys. 47, 2825 (1969).

37. K. Hiida, N. Nakanishi, Y. Nogami and M. Uehara, Progr. Theor. Phys. 22, 247 (1959).

38. J. L. Powell and B. Crasemann, Quantum Mechanics, Addison-Wesley, Chapter 7 (1961).

39. S. Goto, Progr. Theor. Phys. 12, 699 (1954).

40. J. Law and R. K. Bhaduri, op. cit.

41. C. J. Goebel, private communication.

42. E. M. Nyman, Nucl. Phys. B1, 535 (1967).

43. J. M. Berger, L. L. Foldy and R. K. Osborn, Phys. Rev. 87, 1061 (1952).

44. L. L. Foldy, Phys. Rev. 84, 1681 (1951).

45. J. D. Bjorken and S. D. Drell, op. cit., Chapter 4.