CREEP OF PLAIN CONCRETE AND PREDICTION OF CREEP BEHAVIOUR UNDER VARIATION OF STRESS

By

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Chapter 1

INTRODUCTION AND THEORY OF CREEP

1.1. Forward:

The limit state of excess deflection forms one of the three main criteria of serviceability of structures (the others being collapse and local damage).

In order to control excessive deflection, the design engineer must first be able to reasonably predict the short-time and long time deformation behaviour of a given structure.

In general, all of these efforts can be divided into two categories:

(1) Attempts to understand the basic mechanism and causes of creep and shrinkage of concrete.

(2) Attempts to develop empirical design methods for predicting creep behaviour that will enable the designer to control the deflection of structures, within some limit of uncertainty.

It should be pointed out, that the exact nature of creep and shrinkage of plain concrete is one of the major unsolved problems facing the Civil Engineer.

1.2. Effects of Creep

Creep of concrete effects strains and deflections and often also stress distribution, but the effects vary with the type of structure.

Creep of plain concrete does not by itself effect the strength although under very high stresses creep hastens the approach of the limiting strains at which failure takes place, this applies only when the sustained load is above approximately 85 percent of the short term ultimate load capacity[20].

Under a low sustained stress the volume of concrete decreases and this would be expected to increase the strength of concrete. However, adequate experimental data are still lacking and perhaps other factors such as microcracking nullify this anticipated effect.

The influence of creep on the ultimate strength of a simply supported reinforced concrete beam subjected to a sustained load is not significant, but the deflection increases considerably and may in many cases be a critical consideration in design.

According to Thomas [21] there are two distinct neutral surfaces in a beam subjected to sustained loading: one of zero stress, the other of zero strain. This arises from the fact that an increase in the strain in concrete leads to an increased stress in the steel and a consequent lowering of the neutral axis when an increasing depth of concrete is brought into compression.

As a result the elastic strain distribution changes, but the creep strain is not cancelled out, so that at the level of the new stress-neutral-axis a residual tensile strain will remain.

At some level above this axis there is a fibre of zero strain at any time although there is a stress acting. This is an interesting example of the influence of the stress history on strain at any time.

In reinforced concrete compression members, creep results in a gradual transfer of load from the concrete to the reinforcement. Once the steel yields, any increase in load is taken by the concrete, so that the full strength of both the steel and the concrete is developed before failure takes place - a fact recognized by the design formulae.

In statistically indeterminate structures creep may relieve stress concentrations induced by shrinkage,temperature changes, or movement of supports. In some concrete structures creep reduces internal stresses due to non-uniform shrinkage so that there is a reduction in cracking.

The loss of prestress due to creep is well known and, indeed, accounts for the failure of all early attempts at prestressing. It was only the introduction of high tensile steel, whose elongation is several times the contraction of concrete due to creep and shrinkage, that made prestressing a successful proposition.

The effects of creep may thus be harmful but, on the whole, creep, unlike shrinkage, is beneficial in relieving stress concentrations and has partly contributed to the success of concrete as a structural material.

1.3. Definitions

When a concrete specimen is subjected to a sustained compressive stress two kinds of deformation occurs.

The first is the instantaneous deformation, which represents the elastic strain corresponding to the applied stress and to the modulus of elasticity at the time of applying the load.

The second is time-dependent deformation which begins at once and continues for years.

Creep is an increase with time in the strain of concrete subjected to stress; it is conveniently expressed at a constant stress. This definition is not adequate because concrete exhibits a change in strain with time, when no external stress is acting - when drying (or swelling) takes place. This is of course drying shrinkage.

How are shrinkage and creep analyzed when they occur simultaneously? The common practise is to consider the two phenomena to be additive.

The over-all increase in strain of a stressed and drying member is assumed to consist of shrinkage (equal in magnitude to that of a similar unstressed member) and of a change in strain due to stress (creep).

After around 70 years of investigation it is generally agreed that creep can be divided into two components.

One component is associated with moisture exchange with the surroundings and a second component is considered to take place without any moisture exchange.
For example, basic creep, and drying creep, as defined by Kesler, Ali[1]

-Basic creep: strain caused by load that takes place without moisture exchange with the atmosphere.

-Drying creep: strain caused by load, in addition to shrinkage or swelling, that is associated with moisture exchange with the surroundings.

Drying creep is assumed to be the total time dependent strain measured in a loaded drying specimen minus shrinkage and minus basic creep.

The approach which consider the two phenomena creep and shrinkage to be additive has the merit of simplicity but not of accuracy. Creep and shrinkage are not independent phenomena to which the principle of superposition can be applied, and in fact the effect of shrinkage influence the magnitude of creep. The effect of shrinkage was found by many authors to increase the magnitude of creep. Meyers[2] reported that specimens sealed immediately after the curing period, but loaded some time after curing, exhibited more creep than unsealed specimens loaded at the same time.

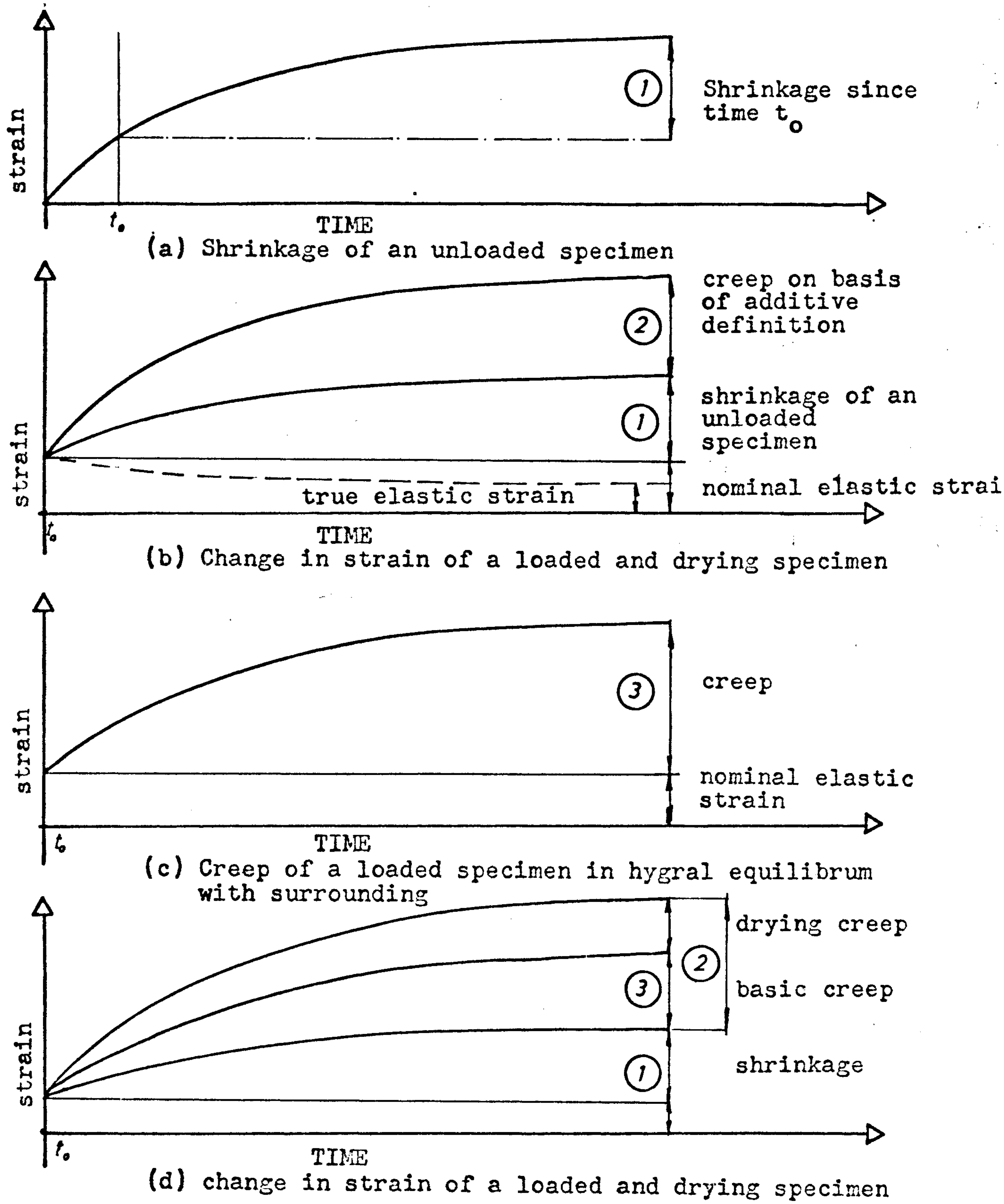


Fig. 1.1. Definitions of terms.

In the case of many actual structures, however, creep and shrinkage occur simultaneously and the treatment of the two together is from the practical standpoint often convenient. For this reason, and also because all data on creep in this research were obtained on the assumption of the additive properties of creep and shrinkage, the discussion in this thesis will consider creep as a deformation in excess of shrinkage.

It should be noted that since the modulus of elasticity of concrete increases with time, the elastic strain decreases with time. Thus, strictly speaking, creep should be reckoned as strain in excess of the elastic strain at the time considered and not in excess of a fixed value of elastic strain.

The terms and definitions involved are illustrated in Figure 1.1.

Specific creep is defined as creep strain at a given time per unit of applied stress.

Level of stress is defined as the ratio of the applied stress to the strength at the time of loading.

1.4. Theories of Creep

As mentioned in section 1.1. it was thought to be important for prediction of creep behaviour to attempt to understand the basic mechanism and causes of creep and shrinkage of concrete, based on the best available scientific information.

A great many theories of creep have been prepared, most of them being hypotheses which fit some known facts and are in disagreement with others.

1.4.1. Theory Proposed by McHenry(3)

It is well known that creep and creep recovery are related phenomena. McHenry regards recovery of creep as an elastic phenomenom and the mechanism of the creep and of the creep recovery to be the same, where the principle of superposition of strains implies that creep is a delayed elastic phenomenom in which full recovery is generally impeded by the progressive hydration of cement.

The "principle of superposition of strains" states that the strains produced in concrete at any time t by a stress increment applied at any time t_o are independent of the effect of any stress applied either earlier or later than t_o. The stress increment is understood to mean either a compressive stress increment or a tensile stress increment which could result in a relief of load. It follows then that, if the compressive stress on a specimen is removed at age t_1, the resulting creep recovery will be the same as the creep of a similar specimen subjected to the same compressive stress at the age t_1.

Creep recovery is represented by the difference between the actual inelastic strain at any time and the inelastic strain that would exist at the same time had the specimen continued to be subjected to the original compressive stress.

Although the principle of superposition of strains is a convenient working assumption and when concrete is stressed to low intensity up to 20 percent as seen from Ross[4] the theory shows very good results, the theory fails for stresses with moderate or high intensity, where the so called "specific" creep cannot be applied. The actual creep recovery is in all cases less than expected. (See Chapter 5)

Thus the principle of superposition does not fully explain the phenomena of creep and creep recovery.

1.4.2. Plastic Theory of Creep

This theory explain very little about creep. The theory holds that creep of concrete is due to crystalline flow. That is, creep is due to slipping along certain preferred planes in the crystal lattice, and to local rupture of the cement paste.

It may be noted that, while in metals undergoing plastic deformation, the volume change experienced is fairly slight, a fairly large decrease in volume occurs as concrete creeps. In addition, if this theory were completely true, creep of concrete would be wholly irrecoverable. This is not the case in any experiment made by the author.

1.4.3. The Seepage Theory of Creep

One of the most widely accepted explanations of creep is the seepage theory put forward by Lynam[5]. From this time on the theory has been developed and improved. The theory states that the application of external pressure causes diffusion or expulsion of colloidal water from the cement gel. To better understand this theory one must understand the structure of concrete and the structure of cement paste as well. The theory identifies three areas influencing creep of concrete.

-The effects of concrete constituents

-The effects of environment

-The effects of stress

1.4.3.I. The Effects of Concrete Constituents

The structure of cement paste can be described in terms of the following constituents: unhydrated cement; solid products of hydration including crystals, crystalites; and void space containing both strongly adsorbed and capillary water and air[6],[7],[8].

The porous hydrate known as the gel comprises the solid products of hydration.

Although dimension changes occur mainly in the gel, secondary volume changes which accompany continuing hydration of the cement cannot be ignored in some cases.

Crystals are defined as those particles, which are large enough to be seen with an optical microscope, while crystallites may be defined as smaller particles which size are beyond the resolution of the optical microscope. In recent years scanning electron microscope have been used to produced images of assemblies of primary particles or crystallites.

Mills[6] documented by photographs taken in a transmission electron microscope the fibrous material comprising the "petals" which form the flower-like structure of the gel. This photograph was identified as $3CaO \times SiO_2$ [C_3S].

It has been suggested[1] that the strengths of the bonds between primary particles in the "petals" are orders of magnitudes greater than those between "petals" and that the strength of cement paste is determined by the weaker bonds. It is clear that displacements of these structures contribute significant components of strain.

1.4.3.II. Water

Free water is defined as water located at distances outside the range of surface forces. It is assumed that such water may be evaporated at 98 percent relative humidity.

Capillary or pore water is defined as water between 1 mm and 30A from gel surfaces. It is assumed that all such water may be evaporated at 40 to 50 percent relative humidiy.

Adsorbed water is water that is strongly attracted to the gel at distances between 0 and 30A from gel surfaces. It may be evaporated over the range 40 to 0 percent relative humidity.

It is this type of water that has been described as "load bearing water" by Powers[8] and as "active water" by Mills[6]. It appears that the active nature of such water could influence mainly the dilation and collapse of spaces within the "petal" structure.

Water of hydration is the part of the total water which cannot be evaporated at 0 percent relative humidity. It can be regarded as part of the solid phase in the gel.

Air, or more correctly water vapour is in the remaining void space. Even in sealed specimens the volume of vapor filled space increases as hydration proceeds and the accompanying meniscus forces may cause shrinkage.

It was found[7],[8], that the specific volume of capillary water is 1.0 cc/gm and 0.9 cc/gm for adsorbed water. The statements which assign reduced specific volume to adsorbed water imply that such water is in compression.

Thus strain energy would be stored in the solids and this would result in the sides being drawn together upon removal of adsorbed water.

The time-dependent deformation of concrete is in part a result of the response of the above components to stimuli which alter the pressure in, and the hygral equilibrium of the cement paste. Deformations of primary particles in the gel structure are insignificant by comparison with deformations of the gross structure and since over dry concretes creep very little at normal stress levels, it is clear that the primary particles themselves almost do not creep. It is clear that the bulk of the time-dependent deformation is related to changes in the interaction of the water and the cement hydrate.

Free water and part of the capillary water is relatively mobile and responds rapidly to pressure changes. These changes do not significantly contribute to long-term creep.

Water that is bonded to the hydrophillies solids comprising the gel (adsorbed and the remaining capillary water) responds slowly to mechanical and thermodynamic stimuli. Because of the hydrophilic nature of the gel, adsorbed water will tend to return to its original position upon removal of the mechanical or thermodynamics stimuli causing motion. Such movement may be inhibited by permanent changes in the solid.

1.4.3.III. Aggregate

The usual normal weight aggregates used in concrete are not liable to creep to an appreciable extent, so that it is reasonable to assume that the main component of creep is in the cement paste as mentioned in the previous section, but this does not mean that the aggregate does not influence the creep of concrete.

The influence is, in fact, two-fold related to:

i) aggregate content

ii) physical properties of aggregate

Since aggregates can be assumed to be volumetrically stable, it is to be expected that creep in concrete, subjected to working stress level reduces with an increase in the volume proportions of aggregate.

Creep is thus a function of the volumetric content of cement paste in concrete, but the relationship is not linear. Neville[10] indicates that creep of concrete c and the volumetric content of aggregate g are related by:

$$\log c_p/c = A \log 1/1-g \qquad \text{(Equation 1.1.)}$$

Where c_p is creep of neat cement paste of the same quality as used in the concrete. And A is a function of Poisson's ratio and the moduli of elasticity of the aggregate and paste.

Figure 1.2. illustrates the relation between creep of concrete and its aggregate content.

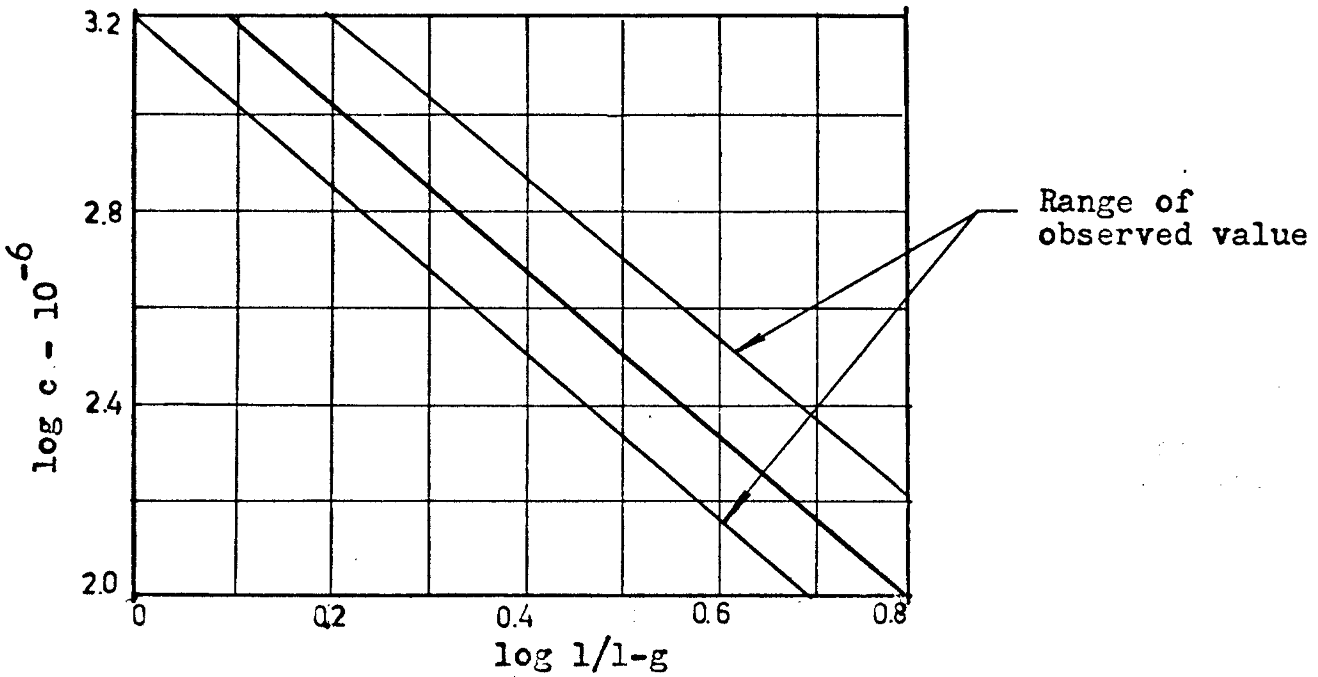


Fig. 1.2. Relation between creep after 28 days under load and content of aggregate g for wet-stored specimens loaded at age of 14 days to a stress-strength ratio of 50 percent.

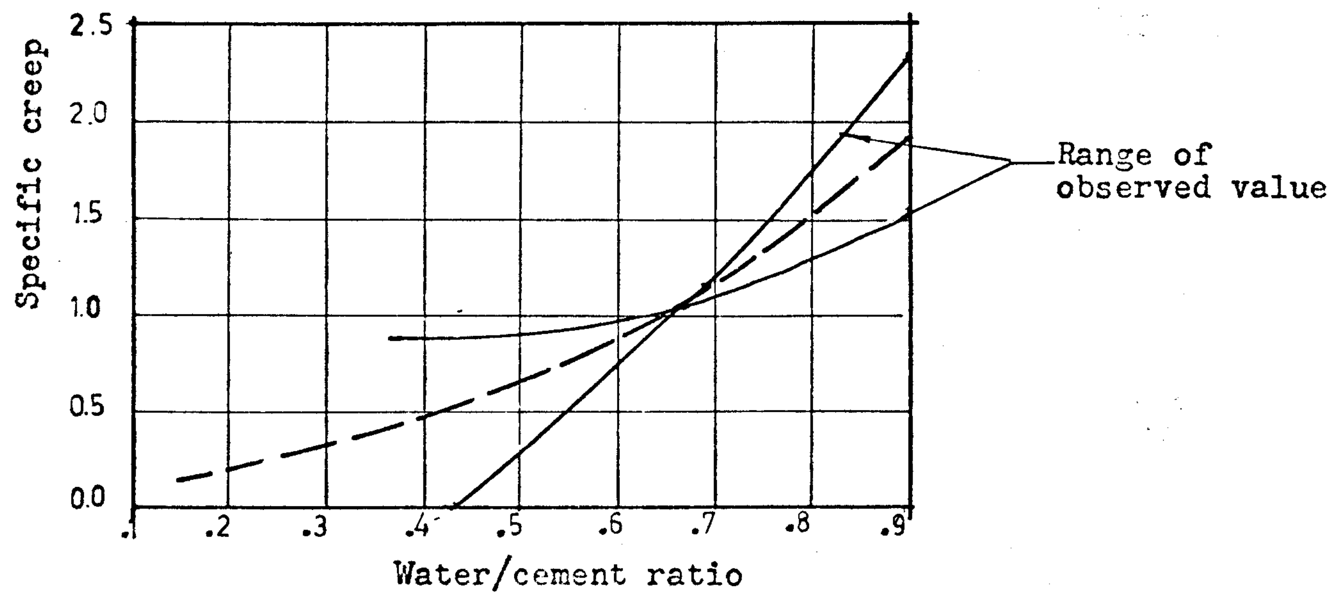


Fig. 1.3. Relation between creep and water-cement ratio, expressed in terms of creep at a water-cement ratio of 0.65.

It may be noted that in the majority of the usual mixes, the variation in the agregate content is small and therefore the variation in creep is small.

Many investigators have shown that concretes made with different aggregates exhibit widely varying creep strains.

Tests by Troxell[18] suggest that variations associated with different petrological type of aggregate arise because of differences in elastic moduli of the aggregate.

The higher the modulus, the greater is the restraint offered by the aggregate to the potential creep of the cement paste. This property influences the factor A in Equation 1.1.

Porosity of aggregate has also been found to influence the creep of concrete but since an aggregate with a higher porosity generally has a lower modulus of elasticity it is possible that porosity is not an independent factor in creep.

On the other hand the porosity of aggregate, and even more its aborption plays a direct role in the transfer of moisture within concrete, this transfer may be associated with creep.

Because of the great variation in aggregate within any minerological or petrological type, it is not possible to make a general statement about the magnitude of creep of concrete made with aggregates of different types.

1.4.3.IV. Cement

The usual portland cements differ from one another primarily in the fate of hydration but not in the ultimate strength. Any comparison of creep behaviour must therefore take into account the degree of hydration at the time of application of the load.

In conventional concrete design, the permissible stress forms a fixed proportion of the concrete strength at the time of application of the load or, more commonly, at some arbitrary age such as 28 days. For this reason, it is logical to compare concretes made with different cements under a load where the level of stress is the same in all cases under these condition, the type of cement (i.e., its composition or fineness) does not, in the first approximation, influence creep(12).

The composition is meant to include the major cement compounds, C_3S, C_2S, C_3A, C_4AF, and also the alkalies whose presence in cement tends to increase the creep and also to lower the gain in strength(12).

The amount of gypsum in the cement may effect creep in concrete. This was observed by Neville(12).

The statement that creep is not influenced by the type of cement is believed to be of importance, but should be considered because of the change in strength of concrete while under load. Therefore, concretes made with different cements and loaded at the same age at a constant stress-strength ratio should be considered.

The increase in strength beyond this age will be different for different cements, being least for Type III cement and greatest for Type IV cement.

It has been suggested by Neville[12] that the decrease with time in the rate of creep is a function of the increase in strength; the decrease in the rate being greater, the greater the increase in strength.

A partly different view held by the author, is discussed further in section 3.4. It is suggested that, in the case of very low humidity, there will be a decrease in the rate of creep with time, while strength can remain constant or even decrease.

It was thought that much better approach can be reached instead of using strength use modulus of elasticity or strain.

1.4.3.V. Mix Proportions

The quality of the cement paste has a direct influence on creep, and this can be expressed approximately by saying that for a constant cement paste content, and the same applied stress, creep is inversely proportional to the strength of concrete[11]. Thus concrete strength is, for a laboratory environment, a convenient, but approximate measure of the state of the cement paste in terms of its composition and degree of hydration.

Creep is, as is suggested by Lorman[13], approximately proportional to the square of the water - cement ratio, but the relation between creep and the water content of the mix is not basic. What happens depends on the influence of the water - cement ratio and aggregate - cement ratio, as these two factors control the water content of the mix.

Viewing creep as a function of water - cement ratio and aggregate - cement ratio gives a correct general picture of the influence of mix proportions on creep. If both the aggregate content and water - cement ratio are varied, the net effect on creep would depend on the relative magnitude of the effects of variation in the paste content and its quality.

For these reasons, and also because the strength of structural concrete is a practical quantity to measure a method for prediction of creep behaviour by relating creep to strength was thought by the author to be both convenient and fairly reliable.

An illustration of this concept is apparent from results by Klieger[14] shown in Table 1.1. where creep of concrete is expressed in terms of strength.

Table 1.1.

Strength at time of application of load, psi	Ultimate specific creep (10^6 per psi)	Ultimate creep at a stress-strength ratio of 30% (10^6)
2000	1.40	933
4000	0.80	1067
6000	0.55	1100
8000	0.40	1067

A good illustration of the general situation is given by Wagner(15).

Figure 1.3. shows the relation between specific creep and water cement ratio. The ordinate of this figure represents the ratio of the actual creep to the creep of a mix with a water - cement ratio of 0.65. Such a relation exists for both long term and short term creep.

1.4.3.VI. Effect of Environment

The effects of ambient temperature and humidity on shrinkage and creep before, during and after loading are important insofar as these factors influence the volume proportions of hydrated, adsorbed and capillary water, and influence the rate at which moisture diffuses in the concrete and evaporates to the atmosphere.

Curing Period Before Loading.

Both temperature and humidity influence the rate of hydration. The extent of hydration in saturated concrete is approximately proportional to the integral of temperature versus time, referred to a datum temperature of about 11^{o}F at which point the rate of hydration is negligible[17].

Ambient Temperature and Humidity After Loading.

An increase in thermal activity of water, accompanying temperature rise, increases the reaction rate of a diffusion controlled processes such as creep and shrinkage.

Numerous tests have shown that creep increases with a decrease in the relative humidity of the surrounding medium. For instance, at a relative humidity of 50 percent creep may be 2 to 3 times greater than at 100 percent relative humidity. Figure 1.5. shows this trend as observed by Troxell[18]. However, careful qualification is necessary because a statement, that creep is higher for lower relative humidity in the ambient medium, may be misleading.

The ambient humidity affects creep if drying takes place while the specimen is under load. But if the concrete has reached hygral equilibrium prior to loading, the magnitude of creep is independent of the relative humidity of the surrounding medium. This was found by Kesler's[16] tests on mature concretes. It appears thus that it is not the ambient humidity that is a factor in creep but the process of drying while the concrete is subject to creep.

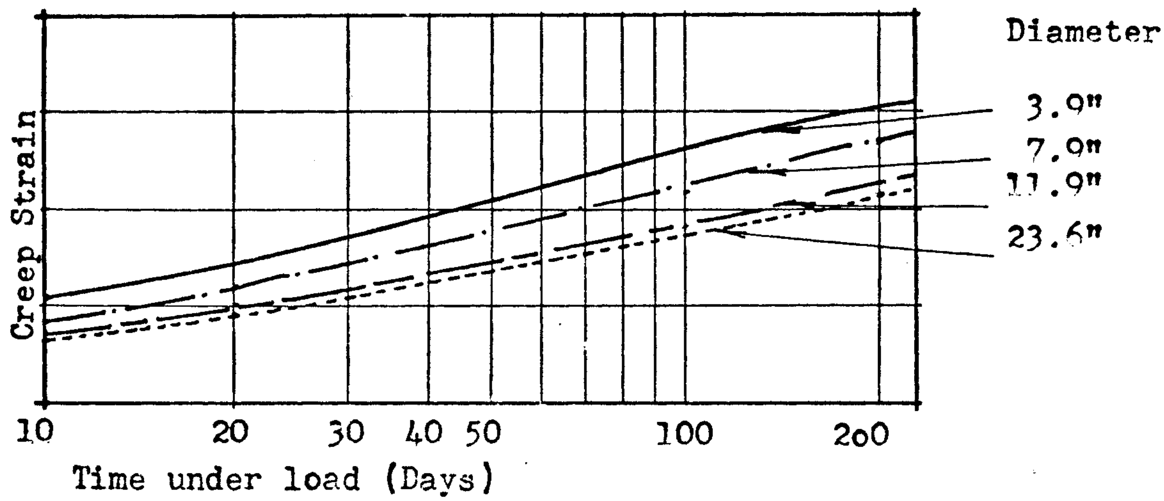


Fig. 1.4. Influence of specimen size on creep.

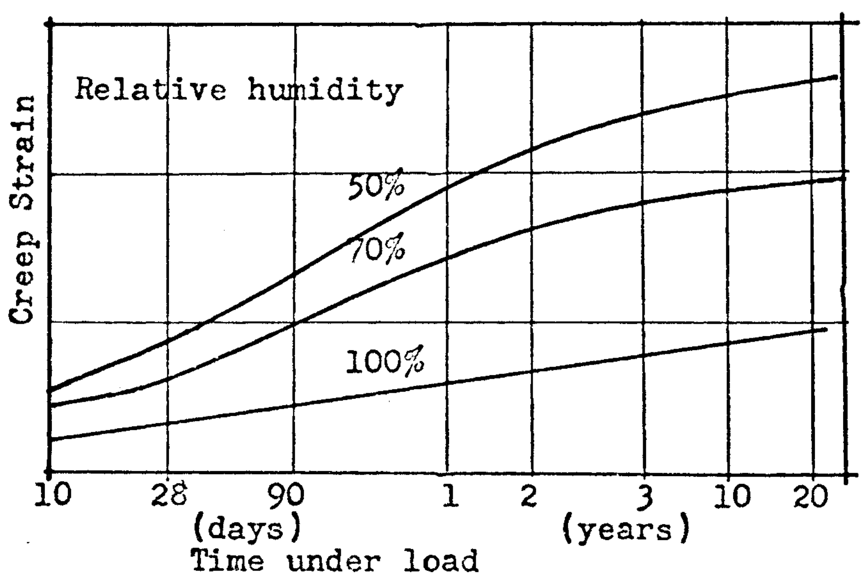


Fig. 1.5. Creep of concrete cured in fog for 28 days, then loaded and stored at different relative humidities.

This is confirmed, for instance by the fact that at later ages the rate becomes sensibly independent of the ambient relative humidity. (In Figure 1.5. after about 2 years.)

It should also be noted that the ambient humidity affects the rate of hydration and of the gain of strength.

The influence of temperature on creep plays an important role mainly for higher temperature range, where the increased thermal activity of water has a significant effect. Even within the normal range (say, up to 100°F) the effect should not be neglected.

1.4.3.VI. Effects of Stress[(1),(2),(8)]

Disturbance of the energy balance resulting from sustained mechanical load results in movement of water from areas of higher pressure to areas of lower pressure and/or relative humidity.

In the early stages of loading the rapid responses of free water results in minor volume change which is often regarded as a stress induced shrinkage. This component of volume change may be recovered upon removal of the sustained load and re-saturation of the concrete.

The water more strongly attracted to the crystal and crystallite surfaces (capillary and adsorbed water) diffused from its original position very slowly. The gel particles are brought closer together resulting in the major components of the time-dependent deformation of concrete (creep). This deformation takes place rather slowly and probably never completely reduces the adsorbed water layers to zero. Providing that there is no change in internal gel structure and the condition of the water, the associated deformation should be recoverable. However, measurements have shown that a large percentage of this deformation is not recoverable under ordinary circumstances. The origin of this irrecoverable components has been examined in relation to evidence of a permanent change of structure.

Mills[6] has described as "collapse of structure" the mechanism whereby the surfaces of the solids originally containing load-bearing adsorbed water are drawn close enough together to form inter-molecular bonds which can be numerous and strong enough to resist penetration of water upon re-saturation and/or removal of load. By a comparison of photographs of shrinkage and creep specimens taken with scanning electronic microscope it was suggests that a sustained load results in a permanent sintering of the crystallites in the "flower" gel structure, while shrinkage does not.

The concept of "collapse of structure" suggest methods of alleviating creep.

It is well known that high pressure steam curing reduces the surface area, shrinkage, and creep of the cement gel. Conversion of colloidal to microcrystalline material, which occurs during high pressure steam curing, can be considered collapse of structure of one type.

From the above discussion it can be seen that the main variables associated with cement paste, that effect the creep of concrete, are those variables that depends ultimately on the quantity and equilibrum state of the active water in the system.

The quantity of active water depends on the volume concentration of hydrated cement and this depends on the water-cement ratio and degree of hydration. Strength is a measure of both water-cement ratio and degree of hydration. It is therefore not surprising that as an initial engineering approximation this component of creep can be evaluated in terms of the water-cement ratio, the age at time of loading, and the degree of hydration.

Figure 1.3. illustrates the influence of water-cement ratio. Table 1.1. shows the relationship between strength and specific creep (creep strain per psi). The effect of the age of loading is illustrated in Table 3.3. where the data from three identically cured prisms loaded at various times is presented.

1.5. Age of Concrete

The age of concrete at the time of loading acts in the same way as the main variables associated with behaviour of cement paste, which in turn effects the creep of concrete. It is those variables that depends on moisture movement.

Under conditions such that no sensible variation in the degree of hydration occurs, where the active water will be in state of hygral equilibrum, the age of loading ceases to influence creep.

1.6. Size of Member

As indicated from seepage theory, the size of a member or perhaps more accurately the surface - volume ratio influences the water movement.

Several investigations have indicated an influence of the size of the specimen on creep. The measured creep decreases with an increase in the size of the specimen, but when the specimen thickness exceeds about 3 ft., the size effect is no longer noticeable. Characteristic results are shown in Figure 1.4.

This apparent influence of the size of the concrete member on creep complicates an evaluation of the tests results of different investigators but the importance of the phenomenon lies in its causes.

Why does size affect creep? The first observation which should be made is that the influence of size on creep is greatest during the initial period after the application of the load.

Beyond, say, several weeks the rate of creep is nearly the same in specimens of all sizes.

The original explanation of the size effect in terms of the loss of water to the ambient medium (which would be relatively greater in a smaller specimen) is not correct for it is now known that loss of water does not play a significant role in creep. However, in actual tests, creep and shrinkage operate simultaneously. As shown earlier these two phenomena are not independent and concurrent shrinkage enchances creep. Thus in a small specimen a greater part of the concrete is subjected to creep while drying takes place, and a greater creep is therefore recorded. The converse is true in a larger specimen, and even if with time the drying effect reaches the core, the concrete there will have changed substantially from the state which existed when load was first applied. A greater degree of hydration will have taken place and a higher strength will have been developed in the core so that the creep response to the creep - while - drying condition will be small.

The size effect applies not to true creep or basic creep but to the increase in creep due to drying.

1.7. End Value of Creep

The definitions of creep refers to the increase in strain with time but says nothing about the presence or absence of a terminal value.

All creep-time curves (with the exception of those for extremely high level of stress which lead in time to failure) show a progressive decrease in the rate of creep with time.

This is particularly noticeable in the early stages after the application of the load but it is not certain that the decrease continues indefinitely. The problem is of fundamental importance for, if a continual decrease in the rate of creep is the case, creep approches asymtotically a limiting value. If on the other hand, the rate becomes stabilized at some value, creep increases indefinitely. There is no doubt, however, that even in the latter case the rate of increase of creep would be so low that with a practical limit of duration of load, say 100 years, the long term increase of creep would not bring about a large or dangerous deformation.

The practical significance of the problem is thus small.

The longest period for which creep data are available is less than 30 years and here a small but measurable rate of creep was observed. It is not possible to say whether this rate will vanish to zero.

Troxell[18], found that in general one-quarter of the 20 year creep took place in the first two weeks of loading, about one-half in the first two to three months and about three-fourths in the first year.

For average values of increase in creep compared with one-year creep see Table 1.2.[18]

Creep after	1 year	1.00
	2 years	1.14
	5 years	1.20
	10 years	1.26
	20 years	1.33

Table 1.2. Average increase in creep compared with the one-year creep.

1.8. Creep Expressions

It is convenient to express the creep-time relation in the form of an equation so that values of creep may be predicted without performing long term tests.

About a dozen equations have been suggested, most of these of a hyperbolic or an exponential type. In some cases creep is expressed by a "standard" curve, which is modified by a number of factors to allow for properties of a particular mix and storage conditions.

Among the equations which make creep tend toward a finite limit are those of Lorman[19] and Ross.

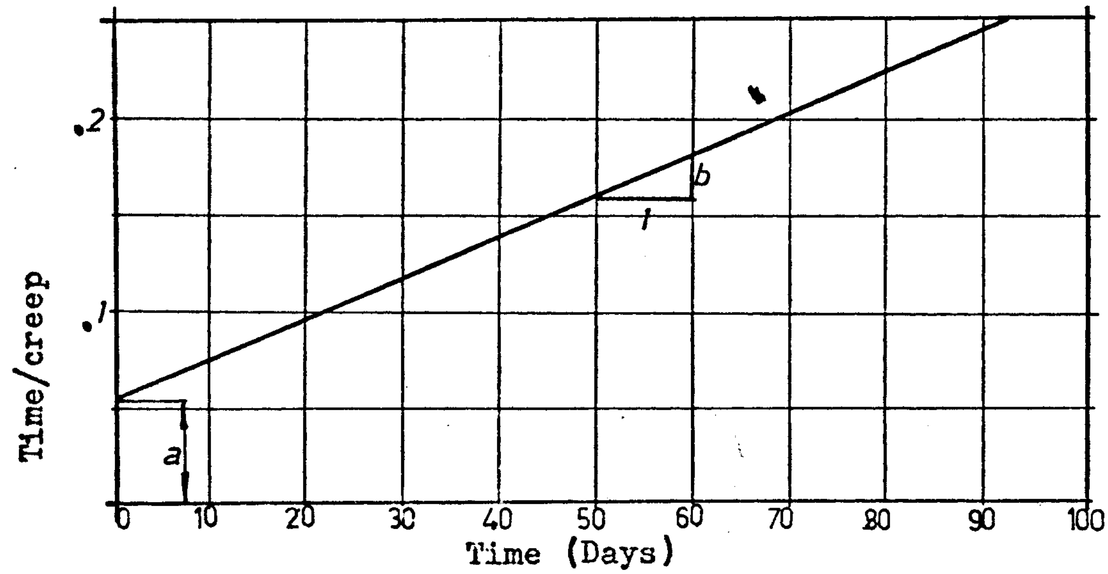


Fig. 1.6. Illustration of Ross formula for creep prediction.

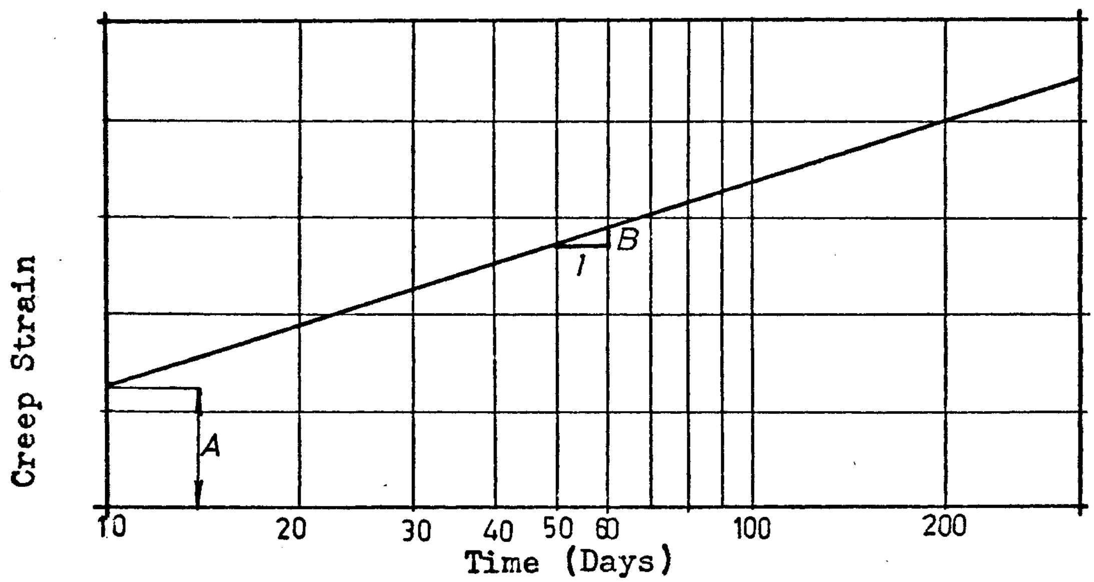


Fig. 1.7. Illustration of semi-logarithm formula for creep prediction.

Lorman proposed a hyperbolic expression

$$c = \frac{mt}{n+t} S \qquad \text{Equation 1.3.}$$

where t is time since the application of load

S is applied stress

m and n are constants

The ultimate creep, C is creep when $t = \infty$ from Equation 1.3.

$$C_\infty = mS \qquad \text{Equation 1.4.}$$

Ross suggested a similar equation

$$C = \frac{t}{a+bt} \qquad \text{Equation 1.5.}$$

Where a and b are constant.

This expression is used throughout this research. A plot of t/c against t is a straight line and the constants can be easily evaluated from Figure 1.6.

The ultimate creep is $C_\infty = \frac{1}{b}$

The logarithm method is an attempt to apply an empirical formula of the type that represented the logarithm of the deformation as a linear function of the logarithm of time and the logarithm of the stress causing the deformation.

That is, the deformation could be expressed by an Equation 1.6.

$$C = aS^m t^n \qquad \text{Equation 1.6.}$$

where a,m and n being constant

S is the stress

C is the creep strain.

This relationship means that for constant stress the logarithm of the deformation plotted against the logarithm of time should give a straight line.

This formula gives good agreement with the measured data for concrete under lower stresses, but not too good agreement at higher stresses.

The semi-logarithm method is in the form of Equation 1.7.

$$C = A + B \times \text{logarithm of Time} \qquad \text{Equation 1.7.}$$

where c is creep

A and B are constant

A is of strain units and B is of strain over time units. Thus a plot of creep strain (to a natural scale) against time (logarithm scale) is a straight line. The values of A and B can be easily obtained from a such plot as shown in Figure 1.7.

The creep constant are both functions of the level of the applied load.

From creep definition follow that for zero applied stress is creep strain equal fòr any time zero, as can be seen from Equation 1.8.

C = A + B x logarithm of time = 0 Equation 1.8.

Since A and B are not themselves function of time this implies that both A and B are zero for zero stress. The semi-log formula, however, is not a theoretical law. It is an empirical formula devised to fit experimental data. It does not necessary reflect observed creep behaviour at all times.
Figure 1.5.(18) indicates that creep is linearly proportional to the logarithm of time within a period from around two weeks to around one year for the range of humidity used in this research.

This procedure is used throughout this research.

In all the equations mentioned the emperical constant have to be determined experimentaly, therefore limited time creep tests must be made using the actual mix and storage conditions. The longer the time over which creep is measured, the better the prediction.

Summary

This chapter was dealing mainly with the mechanism and causes of creep of concrete, and various factors influencing creep.

The author made an effort with a nonsuccesful result to observe the "collapse of structure" by using scanning electronic microscope at McMaster University.

In the next chapter a description of numerous laboratory tests are given which form the basis for numerical analysis of creep.

Chapter 2

Material properties, tests specimen, and equipment and instrumentation.

2.1. Introduction

This chapter provides information about the material used for the experimental test program. The properties of the concrete include the stress-strain relationship, increased strength with age and shrinkage of the specimens.

Details of the specimen dimensions, tolerances, and fabrication methods are given. In addition, the measuring devices, the instrumentation, and the test equipment are described.

Most of the data in this chapter provided necessary information for the numerical analysis.

In order to obtain more information for numerical analysis, two series of test were performed.

The prisms in the first four tests were loaded to a maximum of 0.45 fc'_{28} and kept under load for 170 days. The prism in second pour were loaded to a maximum 0.54 fc'_{28} and kept under load for 121 days. For developing of creep analysis, data from first test series was mainly used.

Some estimates of the possible dimensional variations in fabricating and estimates of instrument reading accuracy are also included.

The effects of these types of experimental error are discussed in Chapter 6.

2.2. Concrete Properties.

2.2.1. Materials

Concrete consisted of Type I, portland cement and common sand and stone supplied by local, commercial dealers. Washed sand of fineness modules equal to 2.74 was used. The aggregate used was 3/8 inch maximum size crushed limestone.

2.2.2. The Concrete Mix

For the first series of tests, twelve 22 inch x 6 inch x 6 inch prisms and fifteen 12 inch x 6 inch diameter cylinders were cast.

In the second series of test ten 22 inch x 6 inch x 6 inch prisms and eighteen 12 inch x 6 inch diameter cylinders were cast.

Table 2.1. lists the proportions by weight of the concrete mix as well as the weights required for one batch.

Table 2.1. Concrete Mix Constituents

Components	Weight in Pounds per Batch	Percent by Weight
Portland cement Type I	88.1 lb.	14.0
Water	57.1 lb. ± correction factor	9.1
Sand	293.0 lb.	46.6
Stone	190.4 lb.	30.3
Total	628.6 lb.	100.0 percent

Volume per batch = 4.15 cubic feet (approximate)

The weight of the aggregate was for the air-dried condition.

Adjustment were made for excess moisture. Although the normal way for making the moisture adjustment would be to find the moisture content in the sand and the stone and make the correction to the proportion of water, this was not used in this work because of very high variations of moisture in the aggregate which was stored in plastic bags. Also the accuracy of the scale for measuring the weight of water was not very high. The correction for water content was based on satisfying slump condition. The design slump for a standard 12 inch high Slump Cone was $2\frac{1}{2}$ inches. In the first pour the measured slumps were 2 5/8 inches and 2 3/4 inches. In the second pour the measured slumps were $2\frac{1}{2}$ inches and 2 5/8 inch.

2.2.3. The Mixing and Pouring Procedure

In the first pour two batches were made in a 9 cubic foot capacity horizontal-drum mixer. The prism forms and cylinders were filled in three equal layers. The first two layers were composed of the concrete from the first batch and the third layer from the second batch.

Each layer was placed and vibrated as soon as it was removed from the mixer. The concrete was internally vibrated using a $1\frac{1}{4}$ inch diameter poker type vibrator. The poker was not allowed to penetrate any further than just into the surface of the preceding layer.

Since it was considered desirable to make the concrete in the cylinders as much like that in the prisms as possible, the cylinders from both pours were also mechanicaly vibrated in three layers as specified in ASTM C192.

For the top layer, the forms were filled so that the concrete was about $\frac{1}{4}$ of inch to $\frac{1}{2}$ of inch over the top after vibrating. The excess was then trowelled off the upper face. A smoothly trowelled surface was obtained without addition of water or excessive working which would cause migration of water to the upper surface.

The prism in the first series were cast in carefully cleaned and oiled steel forms. The prism in the second series were cast in wooden form which were lined with polyethylene plastic sheet in order to give the concrete a smooth finish.

The dimensions of the forms were carefully checked for accuracy, and all dimensions were correct to within 1/32 of an inch for the steel forms and 1/16 of an inch for the wooden forms.

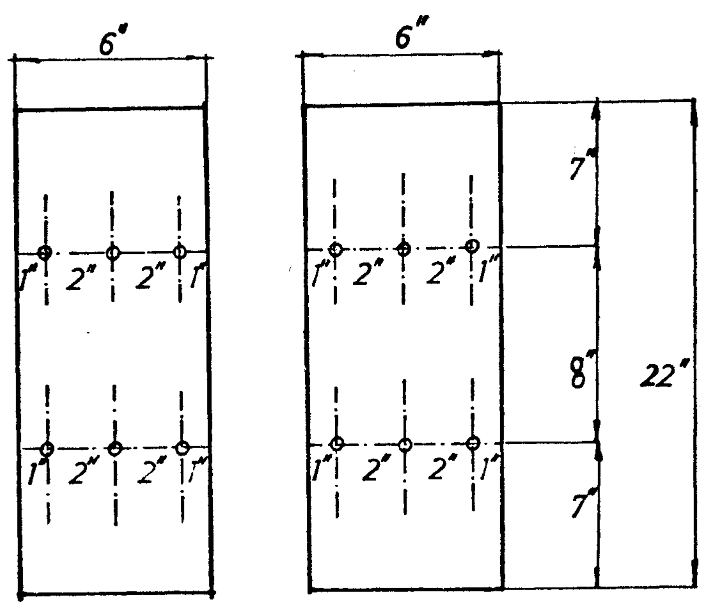
2.2.4. Curing of the Concrete and Preparations of the Test Prisms.

At the age of one day, all the prisms and cylinders were removed from their forms or molds. This procedure was adhered to for both pours. The specimens from the first pour were not covered with wet burlap for the first two weeks due to absence of the investigator. Then they were cured for seventeen days.

The prisms and cylinders from the second pour were covered with wet burlap and kept moist for fourteen days after the specimens were removed from their forms or molds.

For observation of creep and shrinkage strains, one quarter-inch diameter brass gauge points, each with a number 60 drill hole in its centre, were set in the face of each side of all prisms.

The arrangements of gauge points on the creep and the shrinkage prisms are shown in Figure 2.3.



2.3 The arrangement of gage points.

The gauge points were also affixed to several of the cylinders in order that the concrete stress-strain curve could be obtained during tests for compressive strength.

All points were glued to the concrete using a two-part epoxy glue.

After attaching the gauge points the prisms and cylinders from the first pour were moved to a humidity and temperature controlled tent. The prisms and cylinders from the second pour were kept under wet burlap until the beginning of loading.

The humidity and temperature controlled tent is described later in this chapter.

2.3. Concrete Stress-Strain Relationship and Strength.

As has been mentioned previously, a number of the concrete cylinders were fitted with gauge points in order that the concrete stress-strain curve obtained from the Tinius Olsen testing device could be checked.

For checking the Tinius Olsen plotting device just one set of points was used because of lack of space and in order that could be taken continously without stopping at various levels of stress.

The cylinders were capped with a molten sulphur compound called Vitrobond and tested in the above mentioned 300 kip capacity hydraulic Tinius Olsen testing machine.

The test procedure was standard, except that loading was stopped at a stress of about .80 fc' in order that the stress-strain plotting device could be removed to prevent damage upon failure of the concrete cylinder.

The rate of loading was slow but adhered to ASTM specifications.

The average of the strains from three cylinders tests were used for plotting the stress-strain curve.

At any time at least three cylinders were tested according to ASTM specification.

The average stress-strain data at ages 28 days, 76 days, and 135 days after pouring of the first pour, and 28 days of the second pour are included in Appendix 1.

A least-squares fit (LESQ) subroutine, available at McMaster University, was used to derive the stress-strain curve of concrete expressed as a polynomial equation. A third degree polynomical equation provided a sufficiently accurate fit of the experimented data.

Table 2.2.

Concrete Strength at Different Ages

Age [days]	Pour No. 1			Pour No. 2		
	Individual cylinder strength [psi]	Average cylinder strength [psi]	Mean deviation [psi]	Individual cylinder strength (psi)	Average cylinder strength (psi)	Mean deviation (psi)
14				3749 3501 3608	3619	87
21	3165 3056 2920	3047	85			
28	3100 3325 3442	3289	126	4209 4103 4191	4167	43
76	3952 3824 3735	3837	77			
135	4256 3993 4360	4203	140			

Average Strength $fc' = \frac{\Sigma \bar{f}c'}{n}$

Mean Deviation $= \frac{\Sigma|\bar{f}c' - fc'|}{n}$

Figure 2.1. illustrated the stress-strain curves at different ages after pouring and shows the form of the polynomial equation from both test series.

To guard against a large deviation of points obtained by measurements (input points) compared to those given by the least-squares fit plots (output points), the subroutines PLOTPT and OUTPLT have been used. These subrutines plotted input and output points. A carefull check showed no unusual differences and verified the accuracy of the concrete stress-strain expression.

From tests it appears that the strain at ultimate strength for concrete from the first pour was around .0026 in/in after 28 days and around .0027 in/in after 135 days. However this aspect was not important for the purposes of this investigation. Table 2.2. contains the results of the cylinder tests which were performed at various ages of the concrete.

As indicated in the table the average strength 28 days after pouring the first test series was 3289 psi. The gain in strength with time is evident from Table 2.2. and is illustrated in Figure 2.2.

The LESQ subroutine was used for deriving an expression for the variation of strength with time. Equation 2.5. shown in Figure 2.2. according to the author's experience should valid over the time period of one month to one year.

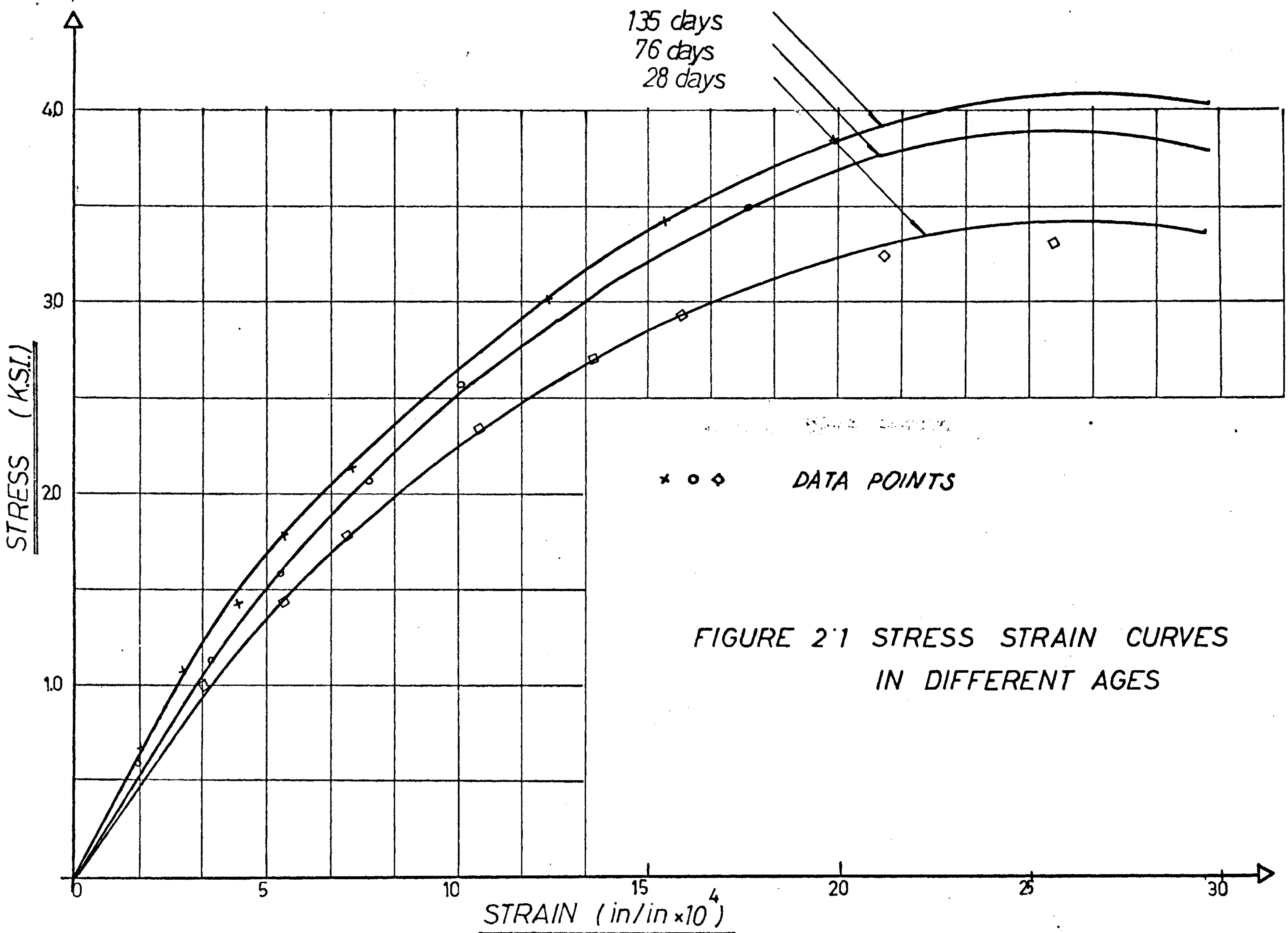


FIGURE 2.1 STRESS STRAIN CURVES IN DIFFERENT AGES

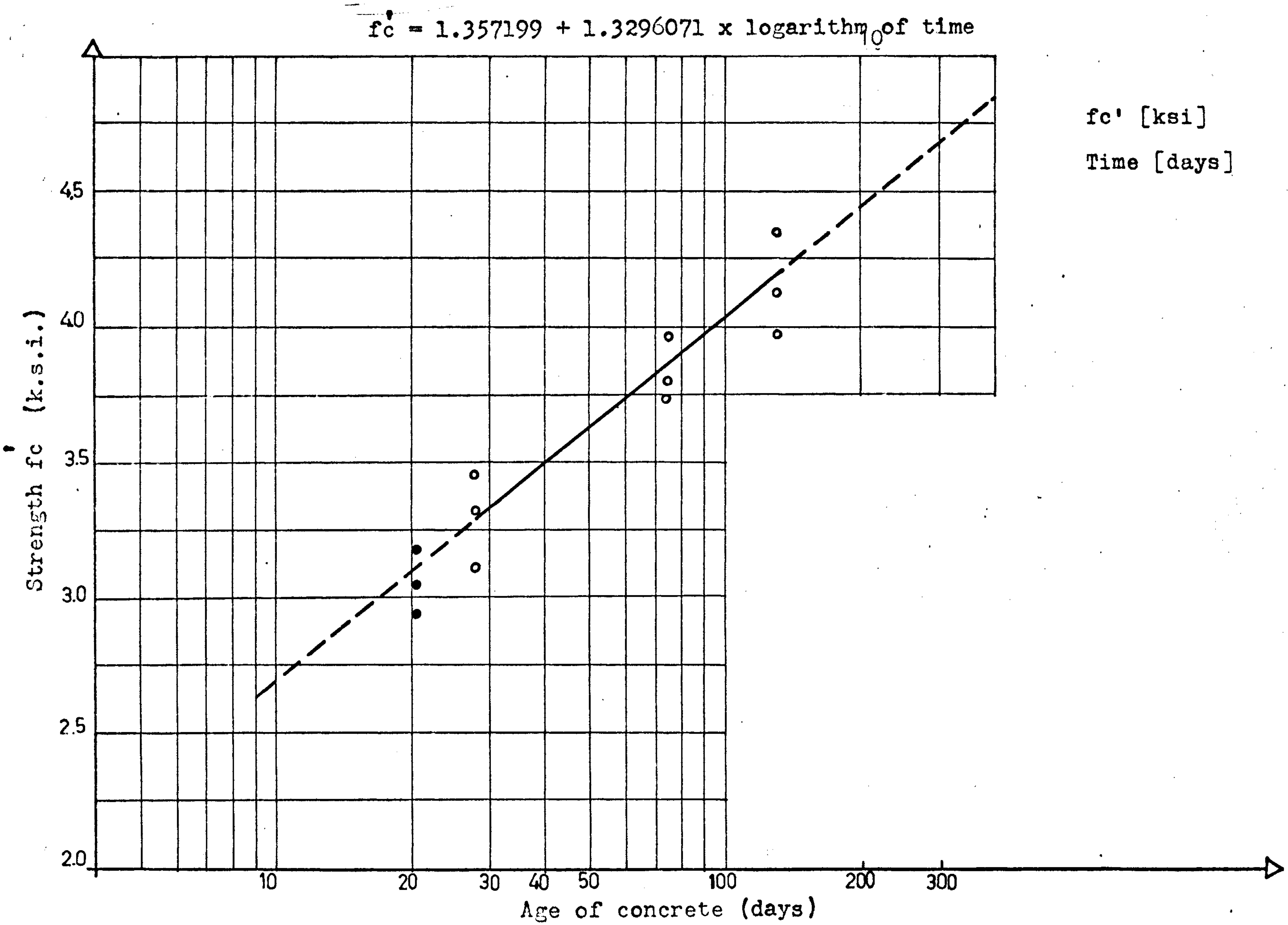


Fig. 2.2. Gain of Strength in Concrete

STRESS - STRAIN POLYNOMS

Pour 1

$$\sigma_{28}\ [K.S.I.] = .0237507 + .3112647 \times \mathcal{E} - .00995637 \times \mathcal{E}^2 + .00010715 \times \mathcal{E}^3 \quad \text{Equation 2.1.}$$

$$\sigma_{76}\ [K.S.I.] = .04914122 + .30252684 \times \mathcal{E} - .00766656 \times \mathcal{E}^2 + .00003815 \times \mathcal{E}^3 \quad \text{Equation 2.2.}$$

$$\sigma_{135}\ [K.S.I.] = .07998607 + .35092104 \times \mathcal{E} - .01280083 \times \mathcal{E}^2 + .00020176 \times \mathcal{E}^3 \quad \text{Equation 2.3.}$$

where $\mathcal{E}$ = strain (in/in x 10^4)

Pour 2

$$\sigma_{28}\ [P.S.I.] = 7.23420647 + 4.13704641 \times \mathcal{E} - .00151185 \times \mathcal{E}^2 + .00000022 \times \mathcal{E}^3 \quad \text{Equation 2.4.}$$

where $\mathcal{E}$ = strain (in/in x 10^6)

Some references indicated an appropriate general equation in the form

$$(fc')_{time} = \frac{t}{a+bt} fc'_{28}$$

would predict compressive strength at any time $(fc')_{time}$ and where a and b are constant. It was thought by the author that expression 2.5. gives a better fit for above mentioned period of time because it does not differ from any measured value at that time by more than 3 percent, and from any average value measured at that time by more than 2 percent.

2.4. Concrete Shrinkage

One twenty two inch long plain concrete prism with 6 inch by 6 inch cross-section from the first pour and two plain concrete prisms of the same dimension from the second pour were cast for the purpose of keeping a record of the length changes for unloading prisms for the duration of the test period.

One day after casting, three pairs of Demec gauge points were glued on each side of the prisms. The shrinkage prisms were stored in the same environment as the creep prisms and strain reading were taken the same time as for the creep prisms.

Shrinkage reading served as correction to the creep strain readings to account for non-load-induced strain.

No record of shrinkage was made before loading creep prisms. The arrangement of gauge points on shrinkage prism is shown in Figure 2.2.

Appendix 2. contains shrinkage data which is the average of 12 shrinkage measurements for each prism. The concrete shrinkage results are plotted in Figure 2.3.

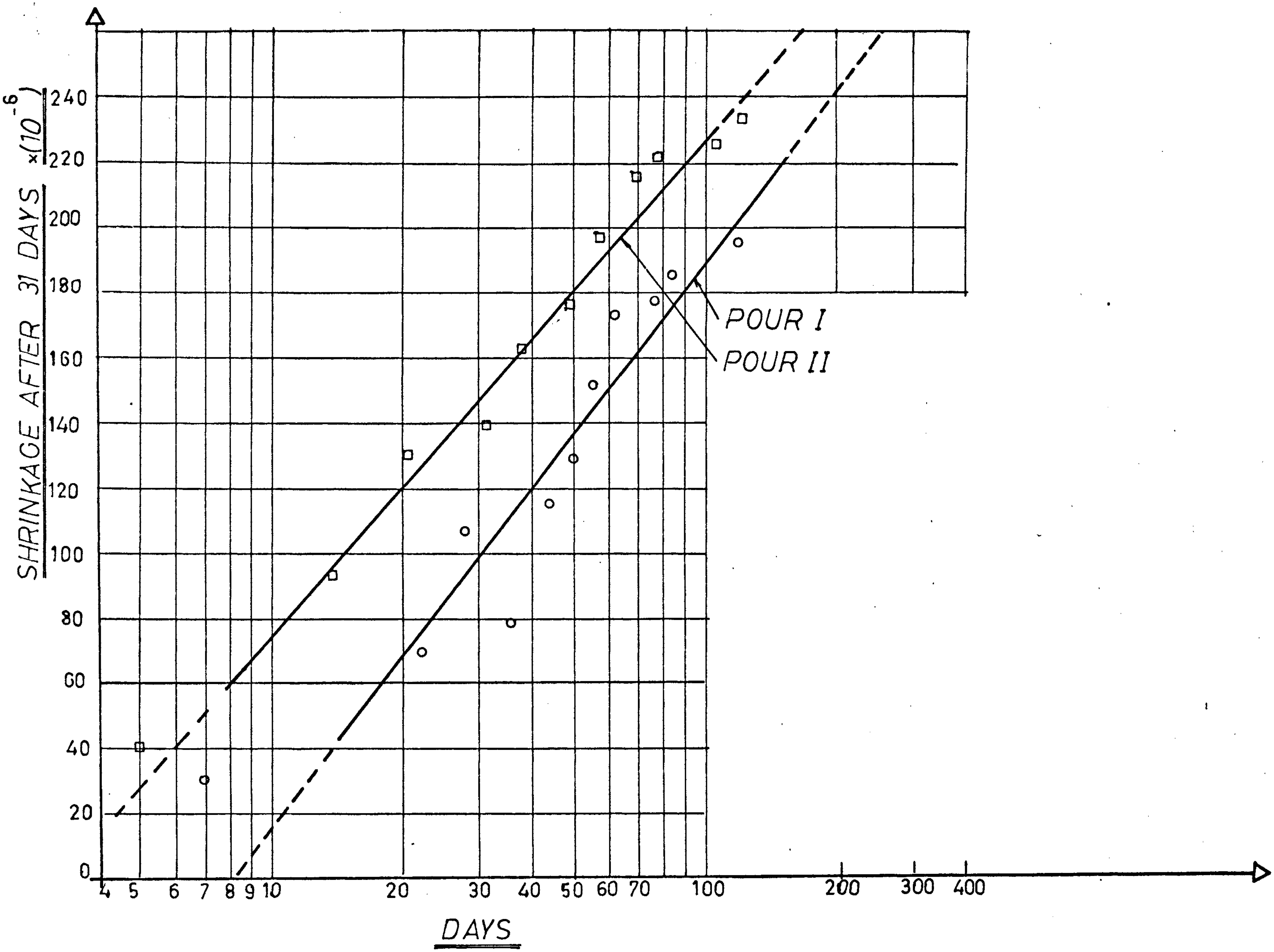


FIG. 2.3. CONCRETE SHRINKAGE

2.5. The Loading Frame

Eight identical load frames were used in the test program. The features of a load frame are shown in Figure 2.4. and Figure 2.5.

It is basically an assembly of four identical steel plates, four steel rods, and four steel springs.

The creep prism is loaded by jacking against plate (1) down unto plate (2).

The four rods are thus placed in tension, while the springs and the creep specimen are placed in compression.

When the compressive load on the concrete, as indicated by the load cell, has reached the desired level the nuts above plate (2) are screwed down into contact with plate (2). The jack force can then be released and the jack removed.

The load would decrease quite drastically due to the concrete's deformations if the springs were not incorporated in the apparatus. The use of springs restricts the loss of load in a given time to approximately 4 x (deformation of concrete) x (spring constant).

The spring characteristics are as follows:

Free length = 9"	Spring constant = 135 k/in.
Solid length = 6"	Weight = 50.5 lbs.
No. active turns = 1.69	Rod diameter = 1 5/8"
Outside diameter = 9"	Inside diameter = 5 3/4"

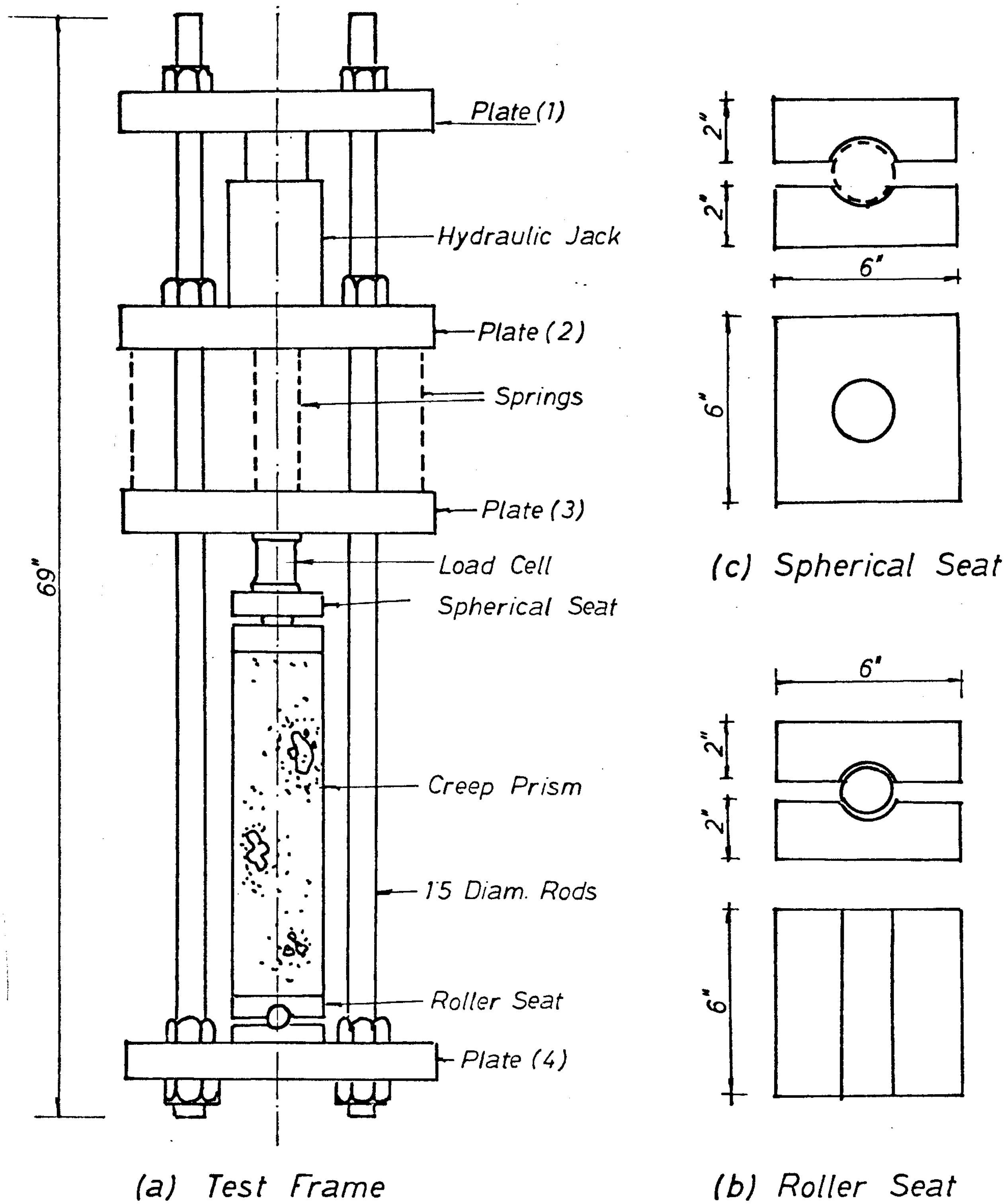


FIGURE 2.4 CONCRETE CREEP TEST EQUIPMENT

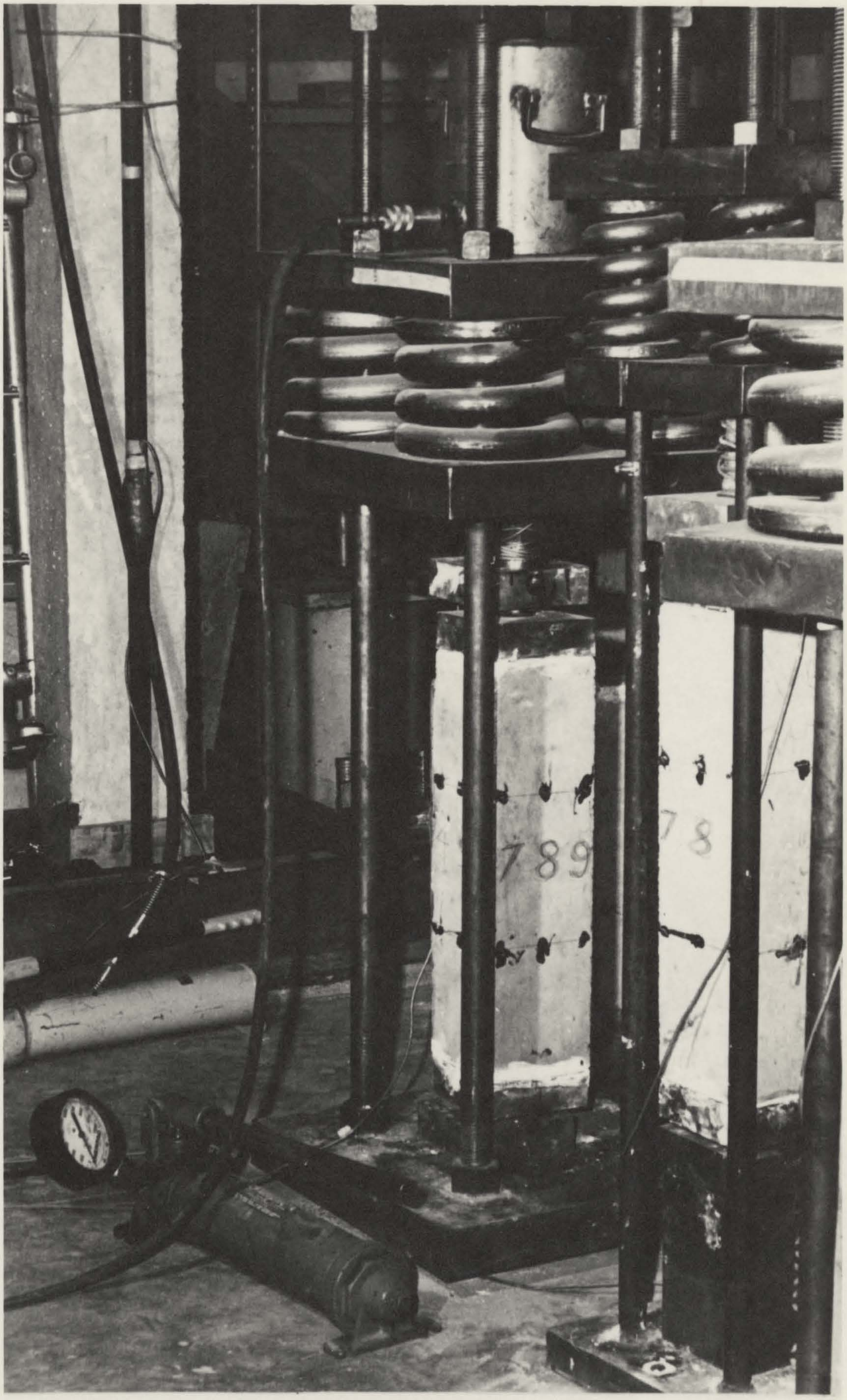


FIGURE 2.5 LOADING FRAME

The concentric load was applied to the concrete prism through a pair of roller and spherical load seats, which are also shown in Figure 2.4.b, and 2.4.c. The load seat at the top of the prisms, used an arrangement of a ball set between two plates, while the lower seat used a roller bearing.

It was felt that a ball seat was neccessary at one end in order to reduce the possibility of applying the load with an eccentricity in a direction at right angles to that intended.

The load seat plates were attached to the prisms using plaster. When the applied load is fairly low (below 50 kips, say) it is possible to adjust the load by moving the nuts above plate (2) with a wrench. At high loads, the friction between plate (2) and the nuts is too great to permit such a method of adjustment. To change the load in such cases, the jack must be used.

A 160 ton capacity hydraulic "Simplex" jack was used to apply the loads.

The load was checked, and adjusted at least once a week and more often if the load had dropped off by more than the permissible value of approximately $1\frac{1}{2}\%$.

2.6. The Load Cells

The loads were placed on prisms and adjusted during the sustained load period by using load cells which were inserted between the bottom loading platform and the spherical seat as shown in Figure 2.4.

The dimension of load cells appear in Figure 2.6.

The steel used was "Ultamo 6", which is a high yield steel with good creep characteristics.

Four Budd CG - 181 - B "Metafilm" electrical resistance strain gauges were attached to each cell using Ga - 1 heat-cured epoxy cement. The arrangement of two gauges positioned vertically and two horizontally constituted a full bridge.

The full bridge was used to compensate for temperature changes and bending of the load cells.

The load cells were calibrated in the 120 kip capacity. Tinius Olsen testing machine. For the calibration, the strain readings indicated by the gauges on the cells at various loads were recorded. The calibration procedure involved loading and unloading cycles for each cell. This process was carried out at least three times. If the calibration curve and zero load reading continued to vary after several cycles of loading, the strain gauges were replaced. The calibration curves (graphs of strain versus load) were plotted for all the cells.

For the creep tests, all the load cells were connected to a Budd SB - 1 portable switch and balance unit. This unit was connected in turn to a Budd portable strain indicator.

After the creep tests had been completed, the load cells were removed from the load frames and re-calibrated as a check on their accuracy over the test period. The recalibration procedure was as follows:

(i) The load was released and the strain reading for zero load was noted.

(ii) Each load cell was loaded until the indicated strains was that which had originally represented the applied load for its specimen minus strain indicated in point (i).

The load for this strain reading was than noted as the load at the end of the loading period.

It was found that only in few cases had the load-strain calibration altered significantly. Because the creep was represented by means of two prisms in almost all cases, the possible errors from using load cells was greatly reduced.

The record of the load variations is contained in section 2.9.

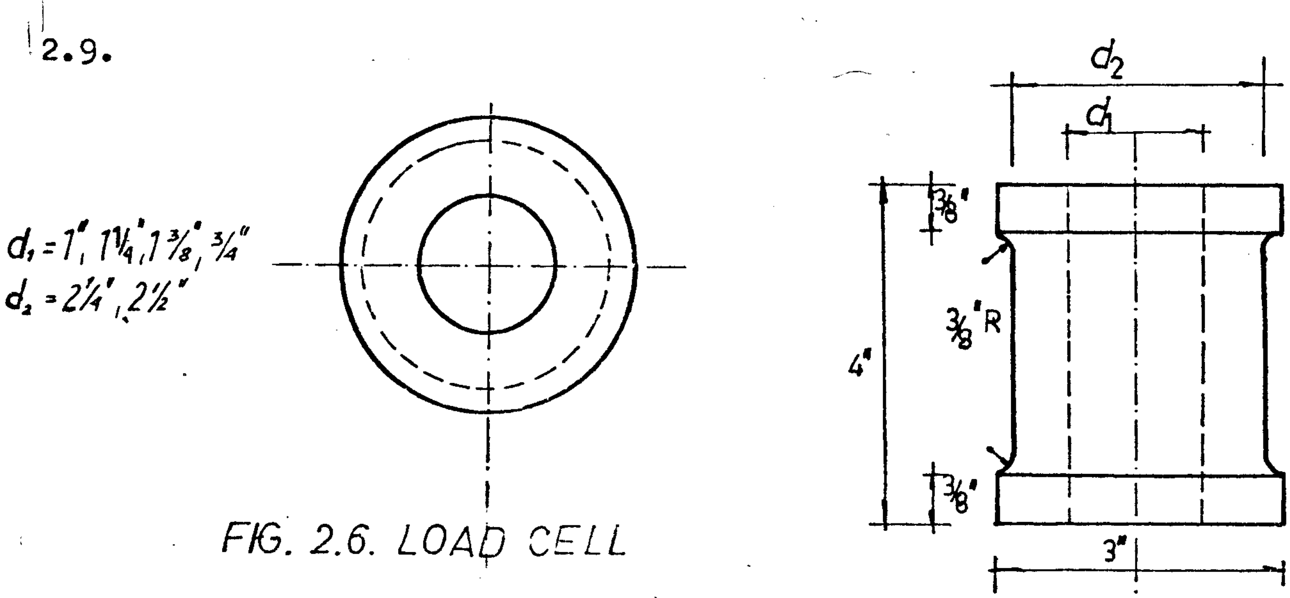


FIG. 2.6. LOAD CELL

2.7. The Demec Strain Gauge

The "Demec" gauge was used to all concrete strain measurements. It is a demountable mechanical strain gauge. The smallest division on the scale represents a strain of 10 microinches per inch. The gauge has an 8-inch gauge length. It was found possible to repeat readings to one half of a division.

Two reading were always taken, one from a standard invar bar, and one from the points glued to the concrete's surface. Any change in the difference between two following readings indicated a change in the strain of the concrete.

The errors involved in using the gauge are discussed in Chapter 6.

2.8. The Temperature and Humidity Controlled Tent

As was described in Chapter 1, temperature and humidity are factors which influence creep in concrete. The temperature in the testing laboratory was maintained near 75°F. but the relative humidity varied between extremes of 80 percent and 20 percent in the summer and winter respectively.

To avoid having to try to take into account the effects of varying temperature and humidity, it was decided to enclose the sustained loading prism test frames in a more nearly constant environment.

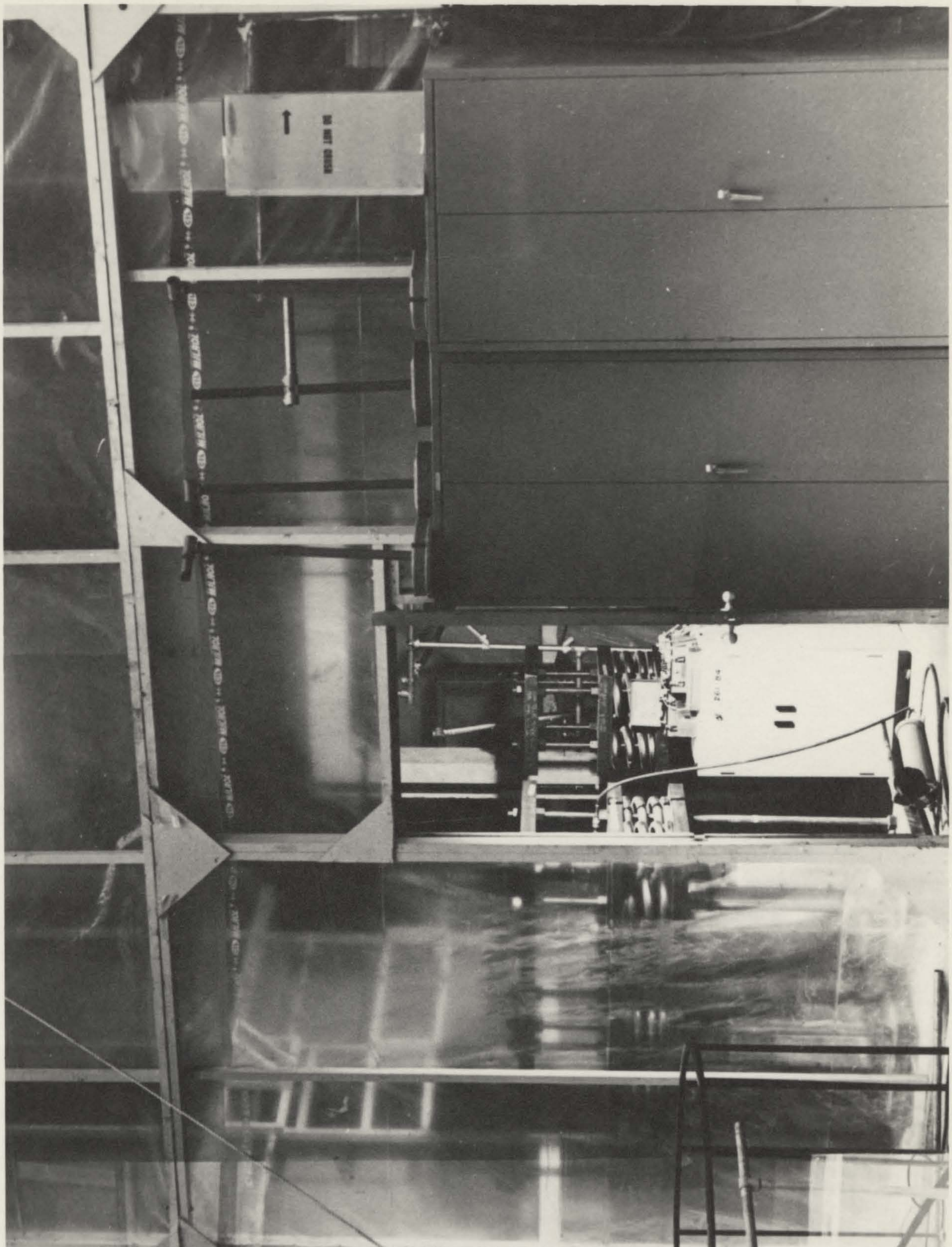


FIGURE 2.7. TEMPERATURE AND HUMIDITY CONTROLLED TENT

A frame covered with polyethelene was constructed around the prism test frames. Entrance was provided by a door at one end of the tent.

A temperature of 75°F was chosen as the temperature to be maintaned in the tent. Since the laboratory temperature was below 75°F most of the time excluding a short period during summer, no cooling unit was used. The additional heat required was provided by two heaters. Each heater was controlled by a thermostat. To be able to control the humidity it was necessary to employ both a humidifier and a dehumidifier.

A relative humidity of 50 percent was chosen as the target value, but poor control increase the average value to 54 percent.

Two oscillating desk type fans were mounted on each end of the tent. They were aimed so that the air was forced to circulate in a vertical loop. This circulation almost entirely eliminated any air stratification.

A recorder was used to check the temperature and humidity of various location around the prisms.

A photograph of the temperature and humidity controlled polyethelene tent is shown in Figure 2.7.

2.9. Experimental Procedure

A 2,000 lb. capacity workshop crane, served for lifting and moving the load frames.

Plate (3) shown in Figure 2.4. was lifted up by a small 3 kips jack and held up initially by means of temporary clips.

The polished ball and roller bearings in the load seats were well coated with grease.

Initial readings were then taken with the "Demec" strain indicator.

The creep specimens with their load seats and load cells were then aligned in the frames, and held vertically by means of short lengths of wood. Next, the temporary clips restraining plate (3) were released, thereby allowing the weight of plates (2) and (3), plus that of the springs, to bear on the specimen and hold it in place. The specimens were loaded by means of jack as was described before. Immediately after loading the measurements of elastic strains were taken. Further measurements of strain were made the next day, two days after loading and later in the week.

The load on each specimen was checked every day until some range of load level variations was established. Later load was checked every few days, and a record was kept of the fluctuations. The loads on all specimens tended to diminish with time, and in each case, the load was brought back up to a desired level after a period of time when it was necessary to prevent the load cell reading from dropping more than 30 microinches near the beginning of testing and more than 20 microinches in advanced ages of the test.

This meant that the load varied a maximum of 1 to 1.5% from the nominal value.

Eleven prisms in the first pour have been loaded as shown in Figure 2.8.

The creep specimens were given identification letters.
A, B, C, D, E, F, G, I, J, K, L.
Prism H served for shrinkage reading.

For higher accuracy of results prisms B, C, and D were kept under supposedly identical stress history until 87 days with prisms B and D continuing until 135 days.
Similarly prisms E, F, G were kept under supposedly identical stress history until 87 days and prisms F and G continuing until 135 days.

Prisms I, and J as well as K, and L had suposedly identical stress history for the duration of the test.

In the second pour, the creep specimens were given the same identification letters and number 2.
A-2, B-2, C-2, D-2, E-2, F-2, I-2, J-2.
The prisms G-2 and H-2 served for shrinkage reading.

The loading scheme is illustrated in Figure 2.9.

As was pointed out before, during the test damage to the strain indicator box occured. It was necessary to change the strain indicator box and recalibrate the load cells. The recalibration was done 135 days after pouring and it is estimated that the strain indicator box broke down at 120 days after pouring.

Hence for two weeks the tests were without proper record of load. It can be assumed that the load would drop down in some cases below the desired level. It was decided by the author that the short 3 hour period of unloading for recalibration would not influence the long term strain record.

Figure 2.8. illustrates loading error due to variations of the load cells calibration. For prism A which was loaded to 81 kips or 2250 p.s.i. the load had dropped down 3.1 percent at the end of the test.

By taking average creep strain of prism B and D, where due to load cell in case of B the load was maintaned above required value and in the case of prism D under required value there occurs up to 1% error. There was made a creep data correction of creep prism C and E. This prisms creep data were thought most important for creep analysis. It was assumed that the error in load cells was distributed linearly with time. A correction in creep reading used the assumption of specific creep (creep per p.s.i.)

The computation was as follows

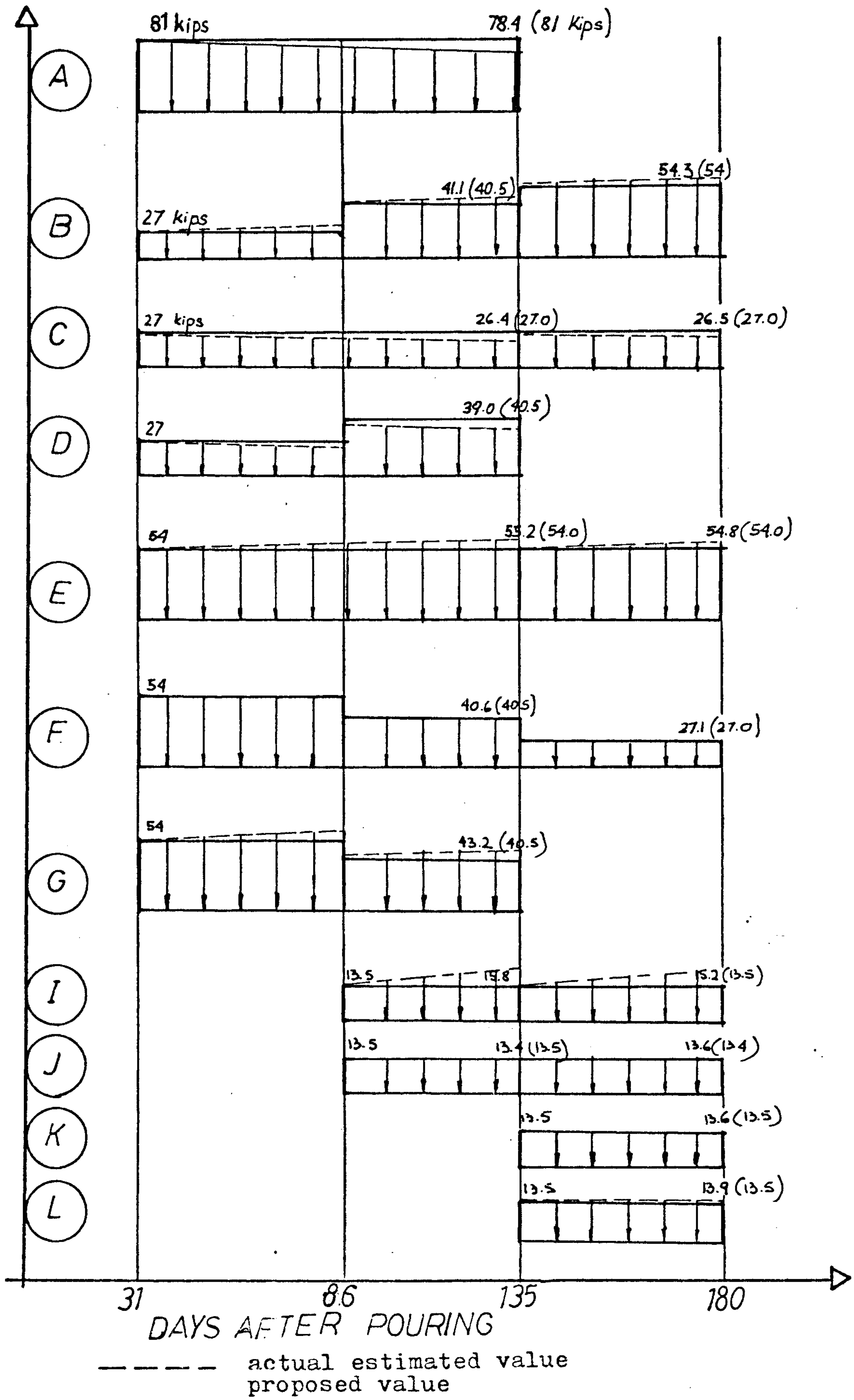
27 kip	26.9	26.8	26.7	26.6	26.5	26.4 kips

If creep because of stress 26.9 kips is x, the specific creep is $\frac{x}{26.9} = \frac{y}{27.0}$

Where x are creep value taken by Demec strain indicator box and y are new computed creep value.

Load cell I behaved very poorly and the data were not used.

FIG. 28. LOADING SCHEME POUR I



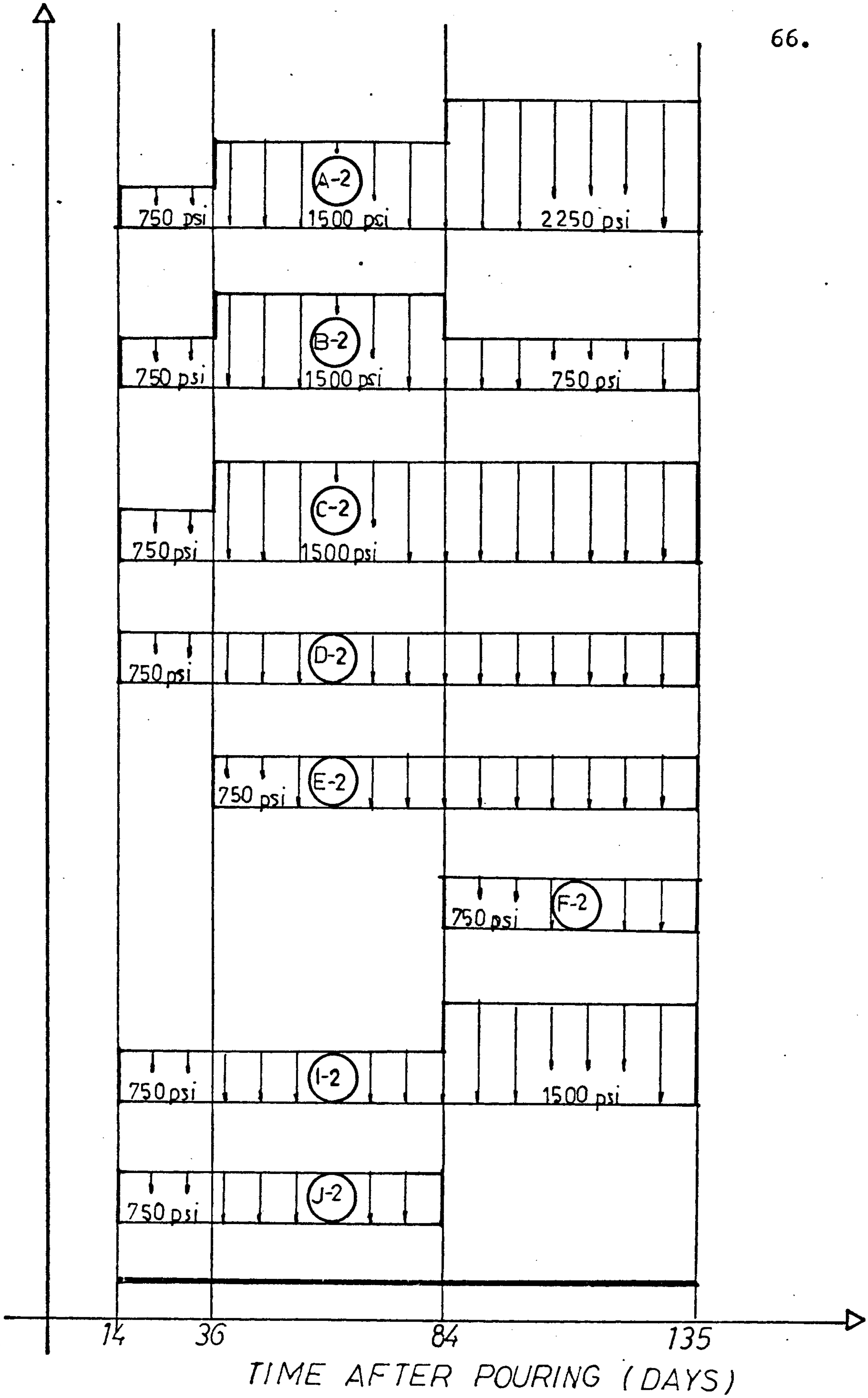


FIG. 2.9. LOADING SCHEME POUR II.

From creep reading K and L average values were taken because of in spite of increase of load on prism L the creep readings were slightly lower than the creep readings of prism K. For observation of creep recovery, strain reading were taken immediately after the removal of the prism from the load frames and for a period 56 days after conclusion of the last test.

FIGURE 2.10 STRAIN MEASUREMENTS

Chapter 3

Prediction of the Creep of Concrete Under Constant Sustained Load.

3.1. Introduction

The aim of this research is to study the prediction of creep strain of concrete with load varying with time from tests conducted under constant stress. An important practical consideration is the limited number of specimens required and, the limited time available for running the tests. This chapter provides creep information from concrete prisms subjected to various constant levels of sustained stress.

The Ross formula and the semi-logarithm formula were used for expressing creep. By means of these formula creep strain can be found at any time within the time span applicable to the formula.

This section describes the methods used to interpolate between the creep data results in order to calculate creep for intermediate stress levels.

Creep data expressed in the form of creep constants from semi-logarithm constant stress creep equations were plotted against the elastic strain. Polynomial equations were found to best fit the above plot. From these polynomial equations the creep constants and hence the creep can be computed for the elastic strain corresponding to any intermediate level of sustained stress.

The second method for interpolation of creep data to predict creep at intermediate stress levels is from the straight line proportionality between level of stress and the logarithm of the ratio of the rate of creep to the rate of creep of standard specimen with known level of stress.

For other ages of concrete at the beginning of loading, creep can be evaluated from the method described above because as creep tends to decrease with later ages of application of load the elastic strain for a given stress tends also to decrease with increased age of concrete.

Creep tests have been conducted under conditions of sustained compressive concentric loads on prisms resulting in uniformly distributed compressive stresses.

No attempt has been made to study creep for tensile, biaxial or triaxial stresses.

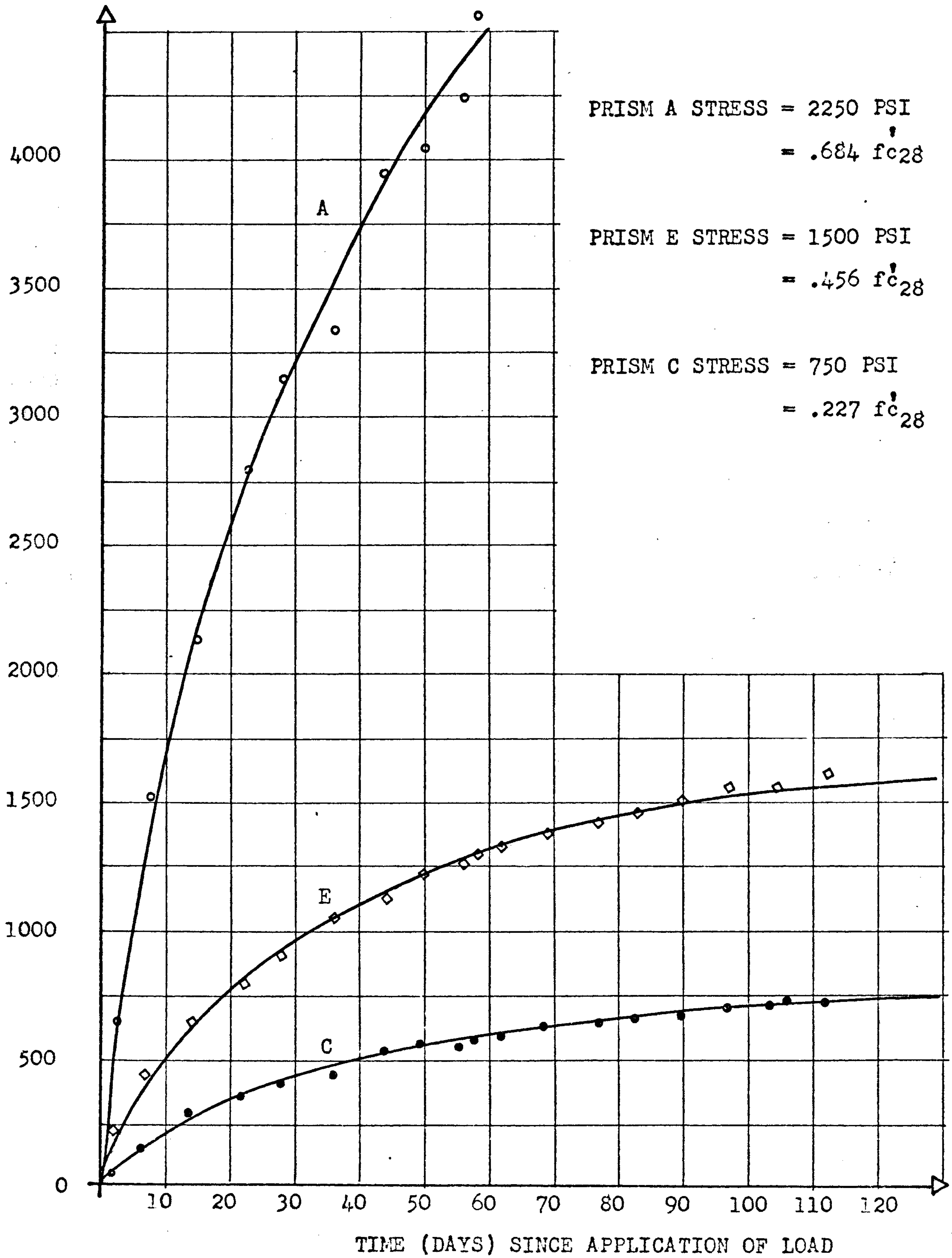
3.2. Presentation of Creep Data and Their Mathematical Expression

The deformation of concrete at any instant are defined as follows from Equation 3.1.

$$\text{Total Strain} = \text{elastic strain} + \text{creep strain} + \text{shrinkage strain} \qquad \text{Equation 3.1.}$$

Comparisons with other published data shows that the creep strain values found in this investigation are high. High levels of stress, low relative humidity, and a low aggregate to cement ratio are the main causes of this difference.

FIGURE 3.1 CREEP STRAIN PLOTTED AGAINST TIME



The creep strain data plotted to a natural scale against time are illustrated in Figure 3.1. and shown in Appendix 3.

The prisms from the first pour were stressed at the age of 31 days after pouring to stresses of 750 p.s.i., 1500 p.s.i. and 2250 p.s.i.

Observation of the curves in Figure 3.1. show that the curves rise with time, hence;

$$\frac{dc}{dt} > 0$$

and the slope decreases with time, so that;

$$\frac{d^2c}{dt^2} < 0$$

The experimental creep strain at any given time was calculated by taking the average creep strain from twelve sets of gauge points fixed to each prisms. It was thought that this practise would minimise the effects of any unintentional eccentricity of the applied load.

The creep strain data from tests were expressed in the following forms:

Ross formula

Semi-log formula

3.2.1. Ross Expression of Creep Data

It will be recalled from Chapter 1 that Ross suggested Equation 1.5. for expressing creep versus time.

$$C = \frac{t}{a+bt} \qquad \text{Equation 1.5.}$$

where c = creep strain

t = time since application of load

a and b are constants.

Any factor which effect the creep observed at a given time, will also effect the values of constants a and b.

Thus both constant are function of the magnitude of the applied load. That is, a and b may be expressed as functions of stress or "elastic" strain both of which are measures of the magnitude of the applied load.

The above relationship may be rearranged to give

$$\frac{t}{c} = a + bt \qquad \text{Equation 3.2.}$$

where a plot of $\frac{t}{c}$ against t is a straight line.

In the appendix 3 the average creep data from prisms to C, E, and A loaded to 750 p.s.i., 1500 p.s.i. and 2250 p.s.i. are included along with the computation of time/creep.

A computer subroutine, LESQ was used to give a least-squares fit of the data for the various straight line plots and thereby derive the coefficients of equation 3.2., which appear in Figure 3.2.

In order to interpolate the creep data and apply it to intermediate stress levels, it was necessary to express the creep constants a and b from the formula as function of the applied load. To this end, these constants were plotted against inital "elastic" strain, this being a convenient measure of the level of the applied load.

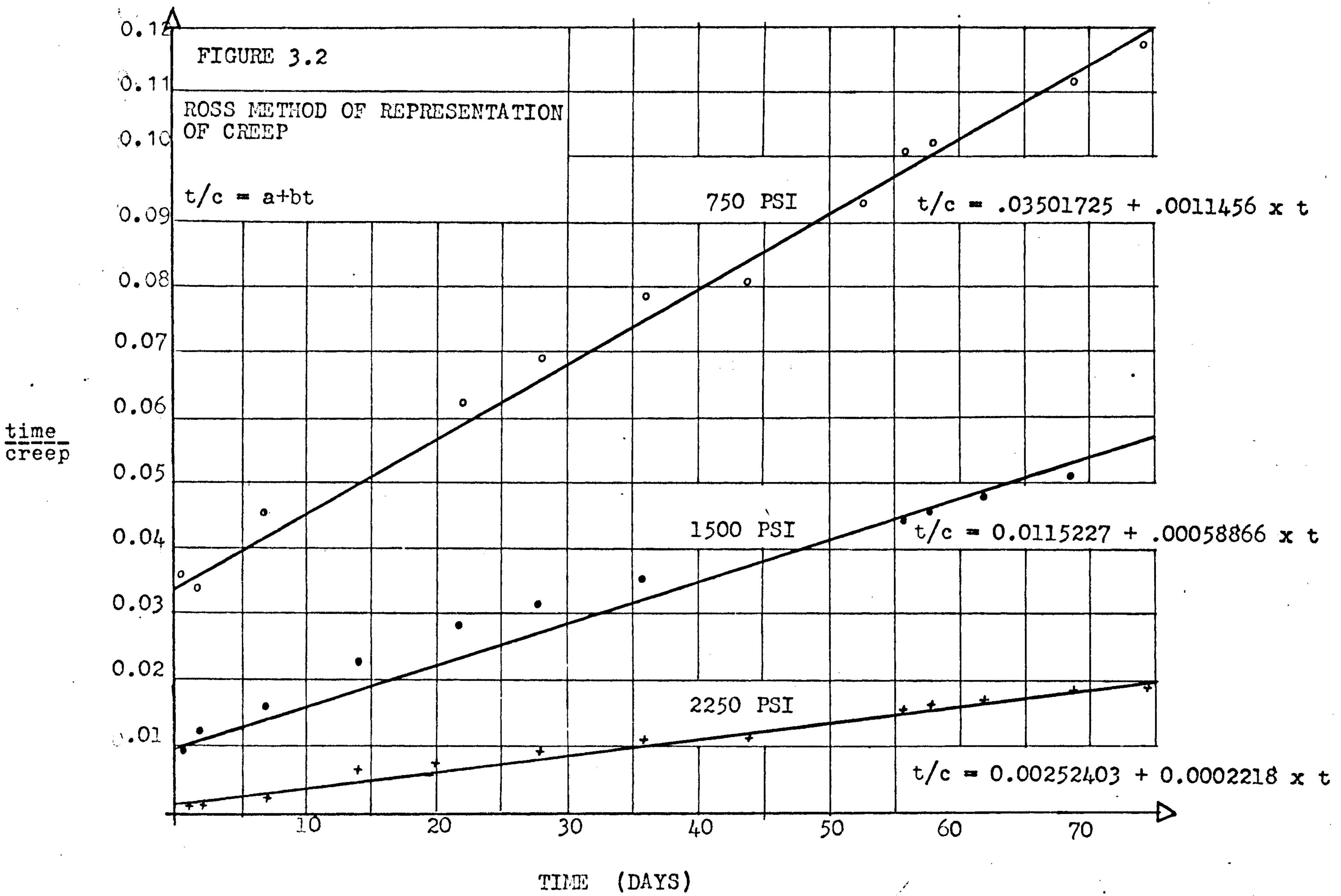


FIGURE 3.2
ROSS METHOD OF REPRESENTATION OF CREEP
t/c = a+bt
750 PSI
t/c = .03501725 + .0011456 x t
1500 PSI
t/c = 0.0115227 + .00058866 x t
2250 PSI
t/c = 0.00252403 + 0.0002218 x t
time / creep
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
.01
10
20
30
40
50
60
70
TIME (DAYS)

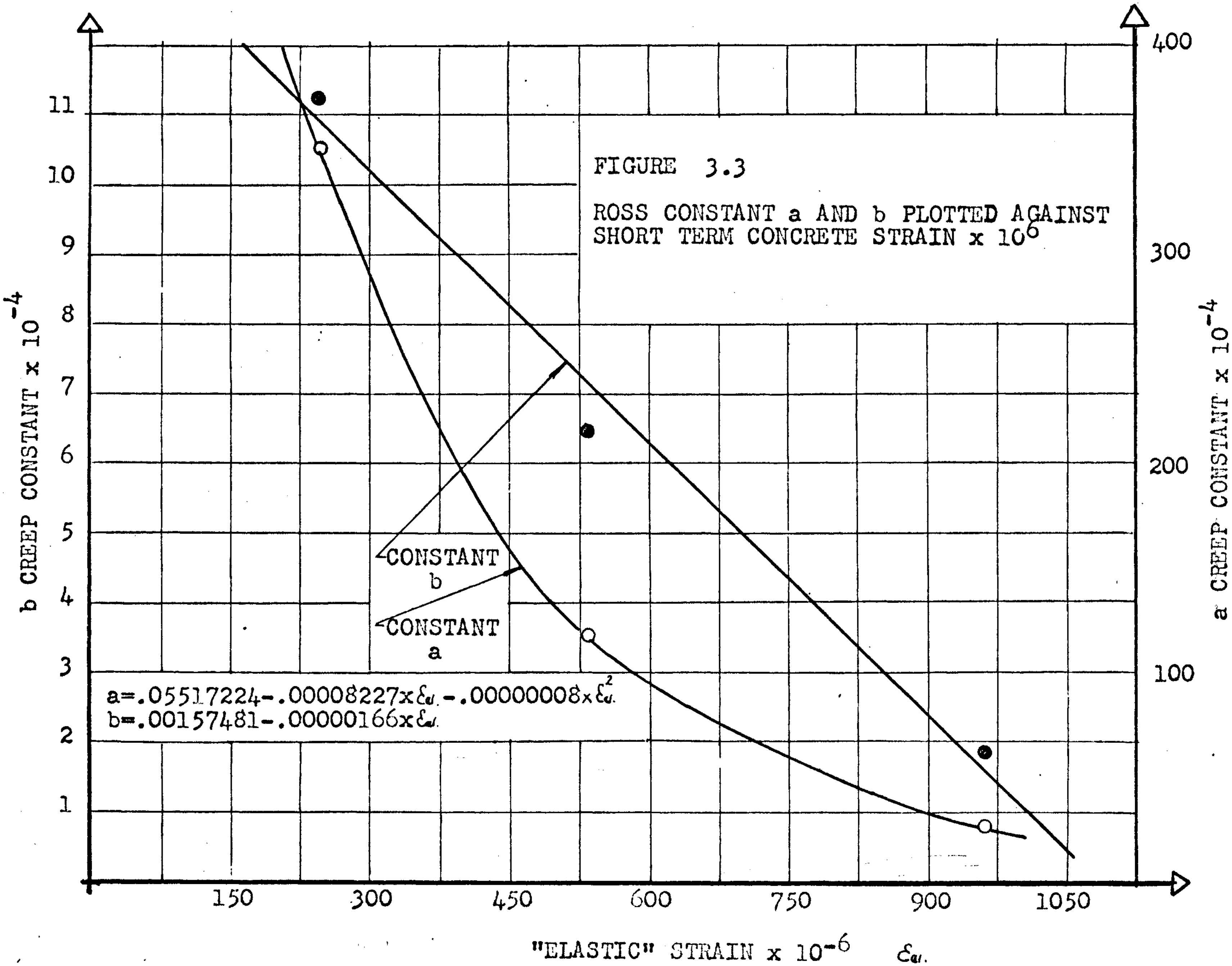


FIGURE 3.3

ROSS CONSTANT a AND b PLOTTED AGAINST SHORT TERM CONCRETE STRAIN x 10^6

Figure 3.3. illustrates Ross constants a and b plotted against "elastic" strain.

A polynomial of first degree provided a good least squares fit for b and similarly a second degree polynomial was used to represent constant a.

The elastic strains represent in Figure 3.3. an immediately strain observed on prism specimens after loading.

Because of $\frac{c}{t}$ does not represent creep very conveniently it is not used much in this analysis.

3.2.2. The Semi-Logarithm Expression of Creep Data.

As it will be re-called from Chapter 1 the semi-logarithm creep expression is in the form of Equation 1.7.

C = A + B x logarithm of time Equation 1.7.

where C is creep strain

A and B are constant

The semi-logarithmic formula is an empirical formula devised to fit experimental data. As was mentioned in section 1.8. it should fit creep data over the period 1 week to one year.

As is well known, creep strain is a function of the applied stress, therefore creep constants A and B from Equation 1.7. are functions of applied stress.

Table 3.1. illustrates how the use of creep data for 1, 2, and 7 days after loading can influence coefficients A and B from Equation 1.7. and therefore the physical interpretation of these coefficients.

Datas where input points in ages 1, 2, and 7 days were excluded are used for computation.

Table 3.1.

Applied Stress P.S.I.	For computation used input points in age 1, 2, and 7 days		Input points in ages 1, 2, and 7 days are excluded	
	A	B	A	B
750	- 44.4428	337.7045	- 300.3024	494.6035
1500	- 30.0636	710.5197	- 660.9955	1106.1546
2250	-169.4407	2549.1247	-3152.0824	4396.0412

Because creep is a function of stress and elastic strain is also a function of stress, it follows that creep is a function of "elastic" strain. More precisely in this analysis the "elastic" strain refers to the immediately strain of prism specimen after loading.

From Figure 3.5. where creep constant A and B are plotted against "elastic" strain, the creep constants for any elastic strain can be easily obtained and hence creep can be calculated from Equation 1.7. for any defined time. Figure 3.5. also illustrates the linear proportionality between creep and elastic strain up to a limit of approximately 45 percent of the concrete strength.

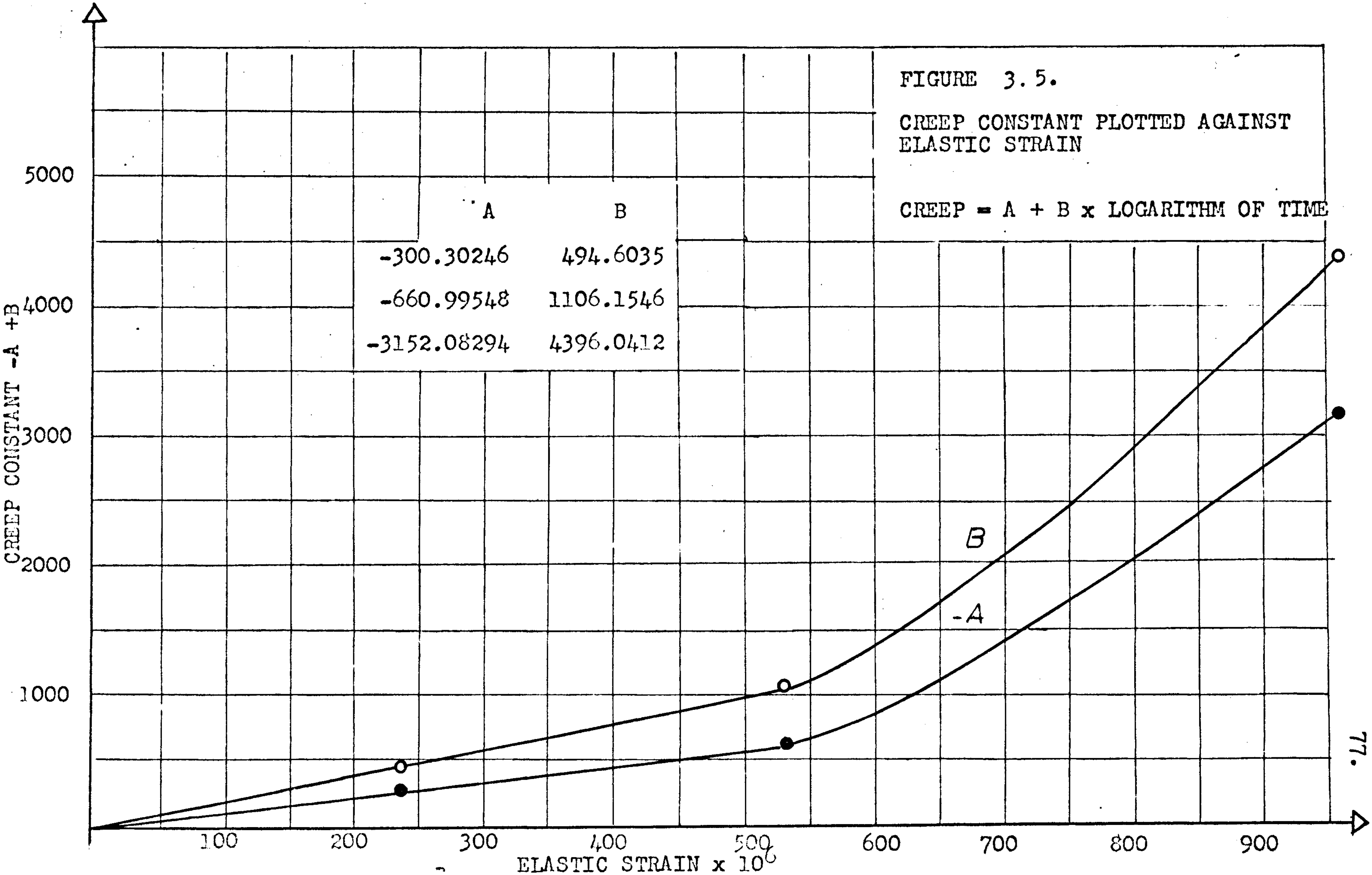


FIGURE 3.5.

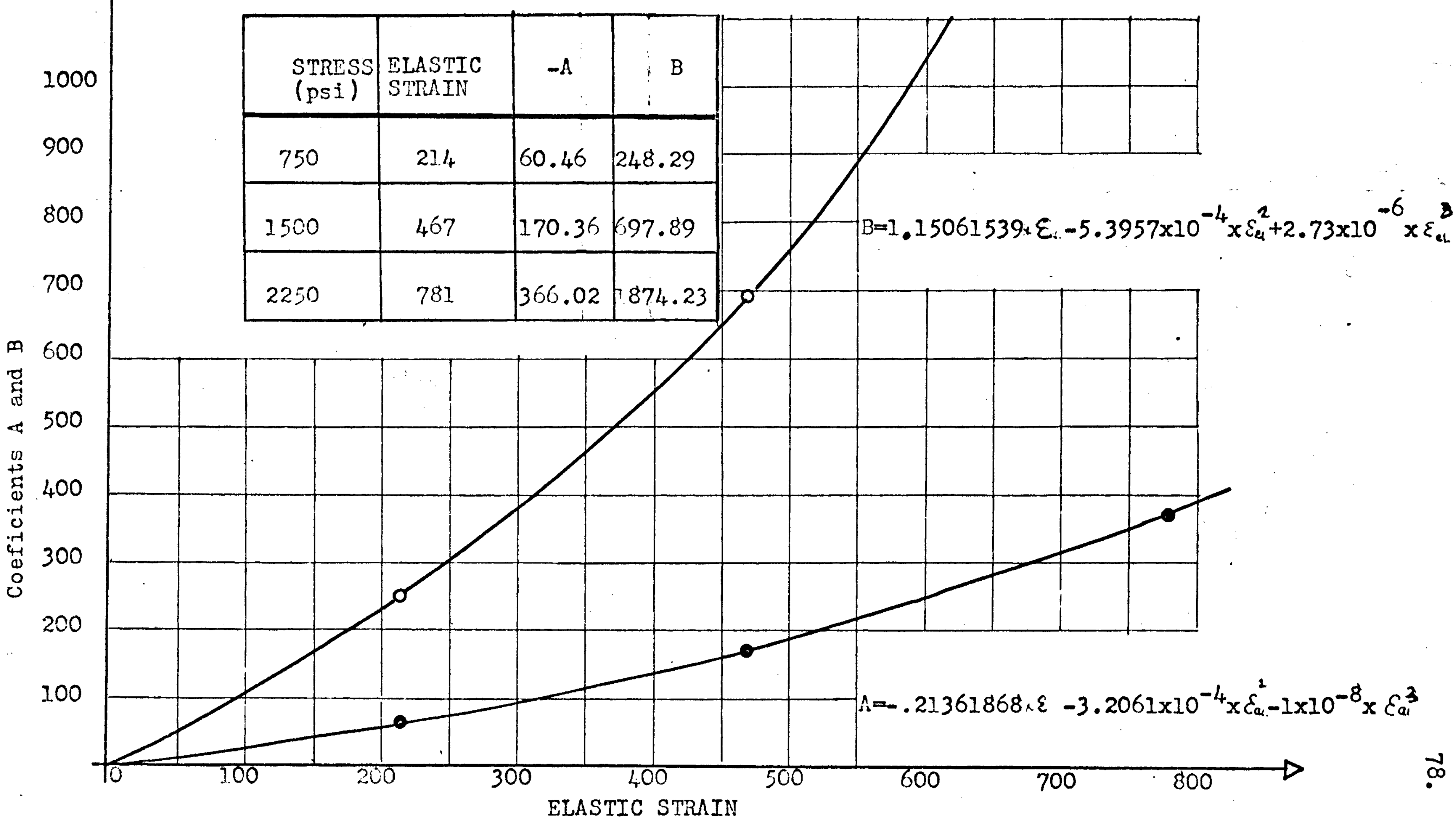
CREEP CONSTANT PLOTTED AGAINST ELASTIC STRAIN

CREEP = A + B x LOGARITHM OF TIME

A	B
-300.30246	494.6035
-660.99548	1106.1546
-3152.08294	4396.0412

FIGURE 3.6 LOGARITHM CONSTANT A AND B PLOTTED AGAINST ELASTIC STRAIN - POUR II

STRESS (psi)	ELASTIC STRAIN	-A	B
750	214	60.46	248.29
1500	467	170.36	697.89
2250	781	366.02	1874.23



A similar relationship as was developed in the first pour and shown in Figure 3.5. had to be made for the second pour. Because at the time of the first application of load only one prism was subjected to a sustained load stress of 750 psi and other data were missing was developed from information in next section.

For the computation showing how from creep data of prism D-2 loaded to 750 psi was developed for hypothatic x-2 and y-2 which would be loaded to 1500 psi and 2250 psi see Apendix 5.

Figure 3.6. illustrates the creep constants A and B from the second pour plotted against elastic strain. The development of these constants required the utilization of the data developed in Appendix 5.

3.3. Proportionality Between Creep and Applied Stress

In section 3.2.2. a logical method was described for interpolating the creep data for stress levels up to 0.75 of the concrete strength.

There is a theoreticaly requirement of a three points to define polynomials as shown in Figure 3.6. In practise to get polynomials with better accuracy there should be several more tests performed to define the shape of the curves.

Because of three sets of data points were available from the first test series and only one set of points from the second test series, it was thought that some more efficiency method should be developed to use all the available information.

For low stresses creep of concrete tends to be a linear function of stress up to stress levels of approximately 25 percent. The following investigators suggested limits to this linear proportionality: Roll[20] - 23 percent, Glanville[25] - 22½ percent, Davis[26] - 26½ percent, Mamillan[27] - 30 percent.

With regard to the above references, Ross[4] in his prediction of creep under varying stress, because the maximum load was 0.20 fc' could with very good results use creep per unit stress called "specific creep".

The "specific creep" assumption could not be used for prediction of creep under varying stress in this research where the load varies in first program between values of 0.23 fc' and 0.46 fc' and in the second program between values of 0.20 fc' and 0.62 fc'. Where fc' is the concrete strength at the time of the first loading.

For developing the analysis test results from the following references were used; Drysdale[22], Gray[28], Smith[32] and the results of the first pour. In all the above mentioned information, creep was expressed with semi-logarithm expression as is Equation 1.7. In this equation, A and B are constant, where A represents creep and B represents rate of creep per logarithmic unit of time, as illustrated in Figure 1.7.

It is shown later in this section, that there had been found experimentally a linear proportionality between the level of stress and rate of creep for range 0.2 to 0.75 fc'. The relationship is of interest in this research.

There is no sense in bothering with stresses higher than 0.80 fc', because many investigators have found that creep under this stress will cause failure after a few days.

Table 3.2. and Figure 3.7 with regard to the above mentioned references indicate that there is a straight line proportionality between level of stress and logarithm of the ratio of rate of creep per logarithmic unit of time over rate of creep per logarithmic unit of time of a standard specimen.

As is known from geometry two points define a straight line. Therefore knowing two single creep/time curve for sustained loads at defined levels of stress, these two relationship can be used to derive a curve similar to those shown in Figure 3.7. For any defined level of stress, the rate of creep per logarithmic unit of time can be interpolated using the above straight line relationship.

The slope of straight line from Figure 3.7. depends on the concrete mix, size of specimen, and likely the other factors which affect creep.

Equation 3.3. is the equation of this straight line.

$$\frac{fc}{fc'} = .25113971 + .46143835 \log \left(\frac{B_i}{B_1}\right) \qquad \text{Equation 3.3.}$$

where $\frac{fc}{fc'}$ is level of stress at time of loading B_1 is rate of creep per logarithmic unit of time for a known stress level B_i is computed rate of creep per logarithmic unit of time per stress level $\frac{fc}{fc'}$.

The above mentioned analytical method cannot be extrqpolated for very low stresses. Where for creep at low stresses the "specific creep" theory may be applied.

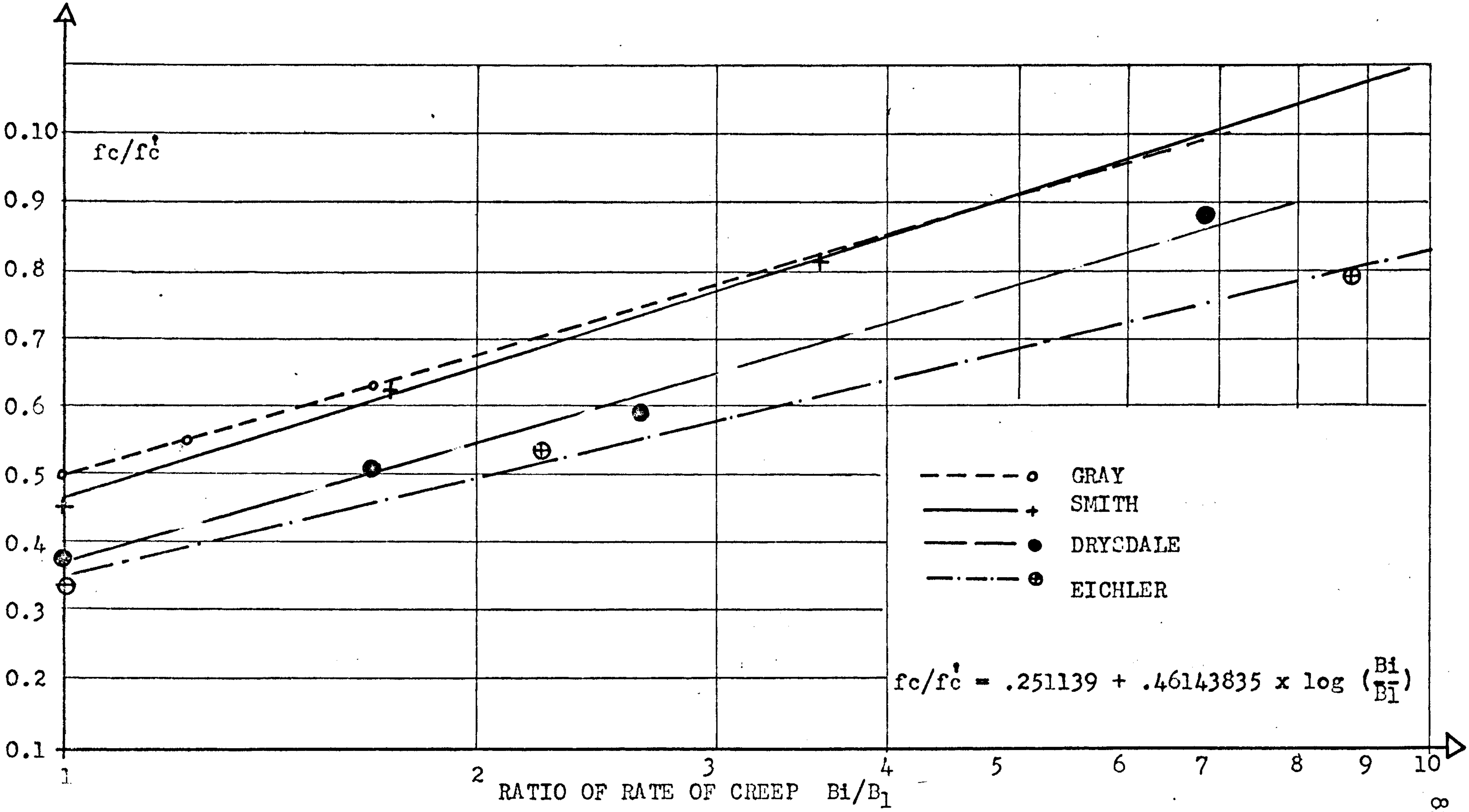
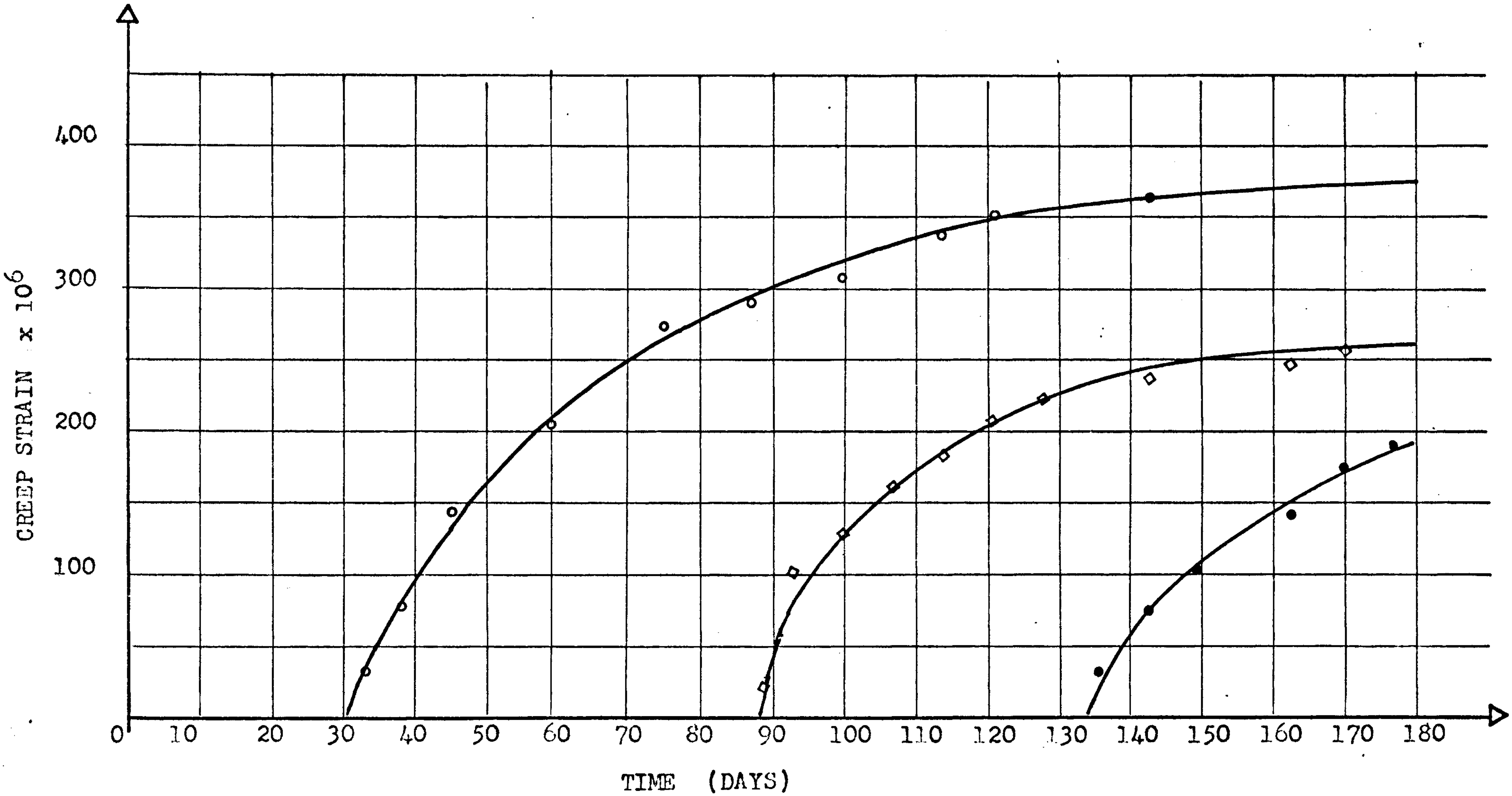


FIGURE 3.7 LEVEL OF STRESS IN TIME OF LOADING PLOTTED AGAINST RATIO OF RATE OF CREEP

Table 3.2.

$\frac{fc}{fc'}$ in time of loading	$B_{(i)}$ i = 0,1,2,3,4,5	$\frac{B_i}{B_1}$	Remark
0.16	B_0 = 258	0.80	Test made by Drysdale[22]
0.28	B_1 = 321	1.00	loaded at 28 days
0.41	B_2 = 544	1.69	fc' = 3850 psi
0.49	B_3 = 858	2.67	
0.78	B_4 = 2215	6.90	
0.177	B_0 = 96.5	0.569	Test made by Gray[28]
0.354	B_1 = 169.6	1.00	loaded at 36 days,
0.531	B_2 = 291.9	1.72	waxed.
0.709	B_3 = 607.5	3.58	fc' = 4230 psi
0.149	B_0 = 22.3	0.76	Test made by Smith[32]
0.297	B_1 = 29.2	1.00	loaded at 28 days
0.446	B_2 = 36.1	1.23	fc' = 4200 psi
0.595	B_3 = 49.9	1.71	
0.227	B_1 = 529.9	1.00	Test from Pour 1
0.450	B_2 = 1095.0	2.23	loaded at 14 days
0.675	B_3 = 4396.0	8.89	fc' = 3620 psi

FIGURE 3.8 CREEP STRAIN OF LOAD 375 PSI APPLIED AT DIFFERENT AGES



3.4. Effect of Concrete Age at Time of Loading on Creep

It is a well established fact, that the time of application of load influences the magnitude of creep.

For observing how time of loading effects the magnitude of creep the following prisms were poured and loaded at; Prism C - 31 days, Prism I and Prism J - 87 days, Prism K and Prism L - 135 days.

Figure 3.8. and Appendix 4 illustrate rate of creep for the above mentioned prisms loaded to a stress of 375 psi at these different ages. An exception was that Prism C was loaded to a sustained stress of 750 psi, and creep strain of 375 psi have been recalculated by dividing creep strain by 2.

Creep has been expressed by the semi-log formula, Creep = A + B x $\log_{10}$ of time, and used the LESQ computer subroutine for the best least square fit.

Creep data for the prisms loaded at ages 87 and 135 days after pouring represents the mean reading from two prisms.

Equations 3.4, 3.5, and 3.6, express creep for a stress of 375 psi for loading at different ages.

$$\text{Creep}_{(31)} = -158.01 + 252.74 \times \log_{10} \text{ of time} \qquad \text{Eq.3.4}$$

$$\text{Creep}_{(87)} = -138.07 + 208.73 \times \log_{10} \text{ f time} \qquad \text{Eq.3.5}$$

$$\text{Creep}_{(135)} = -127.41 + 185.90 \times \log_{0} \text{ of time} \qquad \text{Eq.3.6}$$

where 31,87, and 135 are the ages at the time of loading.

The strength of concrete cylinders at different ages can be computed from Equation 2.4.

$fc'_{(time)} = 1357.19 + 1329.60 \times \text{logarithm}_{10}$ of time Equation 2.4.

$fc'_{31} = 3325$ psi

$fc'_{87} = 3932$ psi

$fc'_{135} = 4197$ psi

The "elastic" strain of the concrete prisms at different ages was found experimentally for stress 375 psi.

at age 31 days = 114 in/in x $10^6 = \mathcal{E}_{31}$

87 days = 96 in/in x $10^6 = \mathcal{E}_{87}$

135 days = 85 in/in x $10^6 = \mathcal{E}_{135}$

A proportionality between decreasing rate of creep per log.unit of time with age and between decreasing "elastic" strain with time has been found.

Table 3.3.

Decrease of Creep with Time of Loading	Decrease of Elastic Strain with Time of Loading	Increase of Strength with Time of Loading
$\frac{B_{31}}{B_{87}} = \frac{252.7}{208.7} = 1.21$	$\frac{\mathcal{E}_{31}}{\mathcal{E}_{87}} = \frac{114}{96} = 1.19$	$\frac{fc'_{87}}{fc'_{31}} = 1.18$
$\frac{B_{31}}{B_{135}} = \frac{252.7}{185.9} = 1.36$	$\frac{\mathcal{E}_{31}}{\mathcal{E}_{135}} = \frac{114}{85} = 1.34$	$\frac{fc'_{135}}{fc'_{31}} = 1.28$

From Table 3.3. it is evident that from the age 31 to 87 days after pouring in the rate of creep per logarithmic unit of time decreases 21 percent whereas the "elastic" strain decreases 19 percent, and gain in concrete strength is 18 percent.

Over the period from age of loading at 31 days to 135 days after pouring is decrease in rate of creep 36 percent, decrease in elastic strain 34 percent and its increase in strength is 28 percent.

Although this is a limited source of data it follows that increasing time since application of load influences the rate of creep in the same manner as "elastic" strain is influenced. The gain of strength is not influenced as much as "elastic" strain.

From the above observations it follows, that from a plot of "elastic" strain versus creep (expressed in the form of creep constants) can be employing change of "elastic" strain with time easily used to evaluate creep.

Two empirically developed methods are described in the next chapter for prediction of creep. By increasing the load on a creep specimen, the new creep strain consist of two parts.

Whether we take as a basis, the change of strength with time or the change of elastic strain with time, this aspect is to be recognized in one of those creep parts.

Chapter 4

Prediction of the Effect of Creep Under Increasing Load Increments.

4.1. Introduction

If the stress in concrete structures would remain constant, estimation of the effects of creep would present no problem because this is precisely the condition under which most creep data have been obtained experimentally. Real concrete structure are however, subjected to changing stresses. The stresses may be gradually declining, such as occurs due to creep and shrinkage losses in prestressed concrete members, or they may consist of increasing step functions due to construction stages. They may, of course, be a combination of the two. The basic problem then is predicting the creep under varying stress from the results of tests under constant stress.

As previously indicated application of additional stress of the same sense causes additional creep. The first part of this chapter is used to acquaint the reader with some previous methods devised to account for variable sustained stress.

Two modified superposition method are introduced.

Modified superposition method I is based on the change of elastic strain at the time of applying additional stress. Modified superposition method II is base on the change of level of stress in time of applying additional stress.

Examples are used to show the modified superposition methods for predicting creep behavior and compared with observed values.

It is indicated that the creep problem in concrete design of real structure subjected to changing stresses may be solved by creep tests conducted to constant sustained loading conditions.

4.2. Discussion of Previous Investigations.

To determine the creep strain of concrete due to variation of load with time three main methods with slight variations on each method have been previously used. These are;

- -Effective modulus method
- -Rate of creep method
- -Superposition method

These method are briefly described here.

(a) Effective Modules Method

This is the simplest and least accurate of the creep prediction methods, and takes no account of stress history, or of age at first loading. It is based on a single creep strain versus time curve for a specimen under constant stress, such as that shown for Specimen (2) in Figure 4.1.

The effective modulus of elasticity E'_c is defined by Equation 4.1.

$$E'_c = \frac{\text{stress}}{\text{creep strain + elastic strain}} \qquad \text{Equation 4.1.}$$

Thus for specimen (2) in the Figure 4.1.

$$E'_c = fo/(\mathcal{E}_c + \mathcal{E}_{EL}) \qquad \text{Equation 4.2.}$$

and the ordinates of the curve are proportional to $1/E'_c$.
At time $t = t_o$, $E_c' = E_{co}$ and for $t>t_o$, $E_c' < E_{co}$ due to creep.

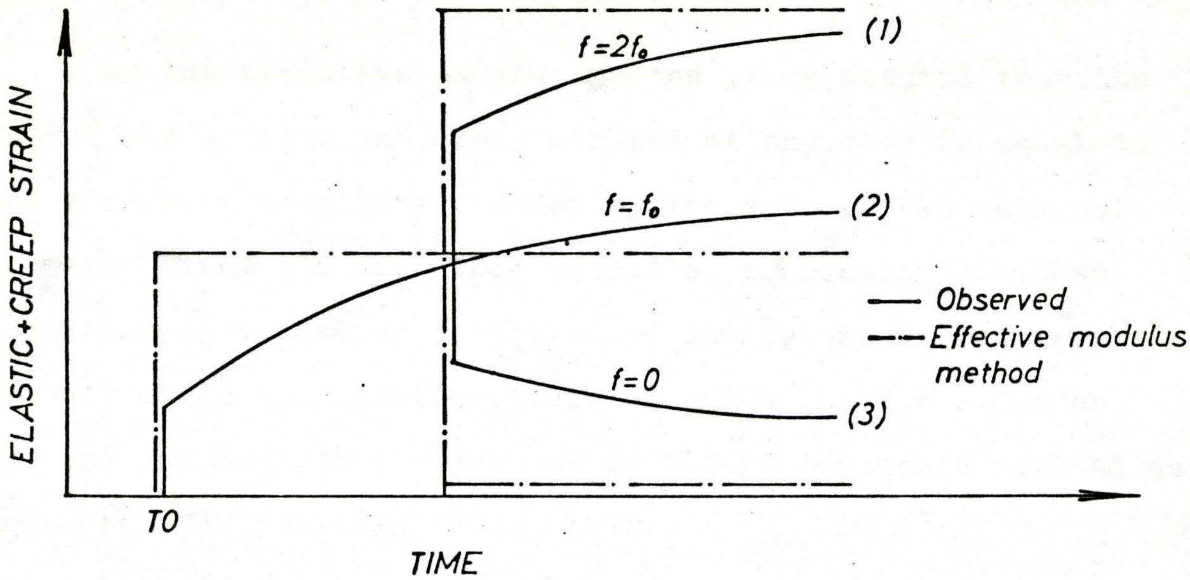


FIGURE 4.1 ILLUSTRATION OF EFFECTIVE MODULUS METHOD

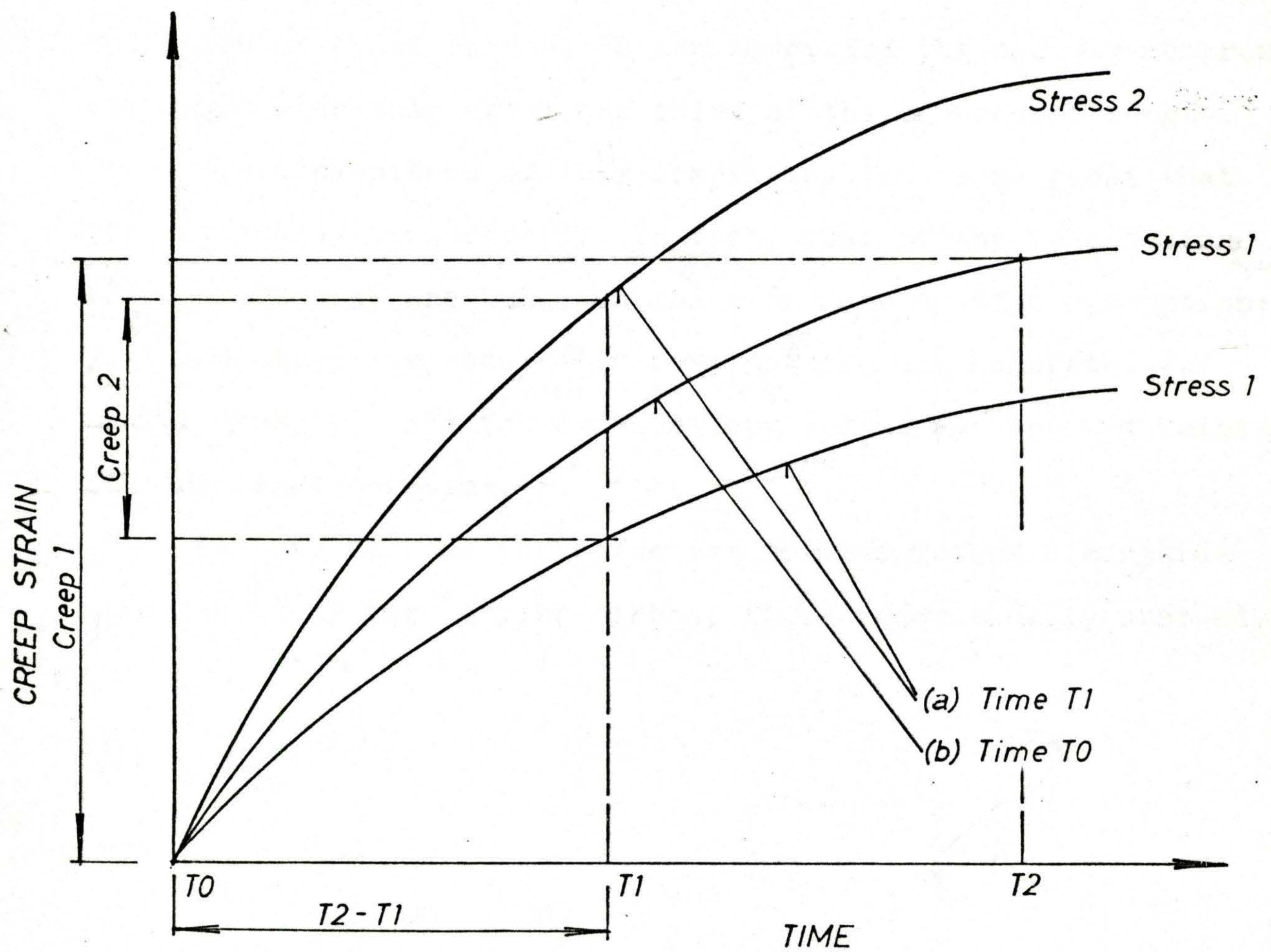


FIGURE 4.2 STANDARD SUPERPOSITION METHOD FOR CALCULATING CONCRETE CREEP

In the effective modulus method it is assumed that the sum of the elastic and creep strains at any time is equal to the stress at that time divided by the appropriate value of the effective modulus. This method of prediction as shown in Figure 4.1. assumes complete and instantaneous recovery of both creep and elastic strain on unloading for specimen (3) and overestimates creep due to later increments of load as indicated for specimen (1).

The effective modulus method makes it possible to use in the analysis of concrete structures all the well-known theories and folmulae based on Hooke's law, provided the maximum compressive stress is less than about one-third of the concrete strength.

The convenience of this simplification is so great that its inaccuracy is accepted. In fact, most of the calculations made in practise are based on the even more drastic assumption; that the effective modulus is constant for all concretes (of normal density) and for stresses applied at any age and maintained for any length of time.

However because the effective modulus method disregards stress history for varying stress, it is theoretically unsound.

(b) Rate of Creep Method

This method is based on the use of a single specific-creep versus time curve for the concrete. From this curve the rate of creep, dc_1/dt, at any time t is known and if, at this time, the stress in a structural member is f, the increment of creep in the element of time dt is assumed to be fdc_1. Creep under a variable stress after a time t is therefore:

$$\text{Creep} = \int_0^t f \frac{dc_1}{dt} \, dt \qquad \text{Equation 4.3.}$$

This method attempts to avoid some weaknesses of the effective modulus method by integrating all elementary increments of creep, each of which is estimated from the unit - stress creep curve and the particular stress acting during the relevant element of time. In this sense it takes into account variations of stress, but it does not reflect any influence of previous stress history and therefore cannot be considered theoretically sound because of this.

If after a loaded period the stress is removed, then $f\,dc_1/dt$ is zero which means, that the method predicts no time - dependent creep recovery. Concrete however, definitely exhibits some recovery of creep. Again, in another sense, the rate of creep method disregards stress history because it assumes that concrete will creep at the rate of $f\,dc_1/dt$ regardless of whether it was stressed severely or negligibly in its earlier history.

The method also assumes creep to be proportional to stress which has been found to be true only for very low stresses.

Hence this method can be approximately used for very low stresses where large variations in stress are not encountered.

(c) Superposition Method

This method simply involves the addition of creep results for a number of intervals of time. During each time interval the stress was assumed to remain constant. Most present day superposition methods are similar to that outlined by Ross[4].

To extend the superposition method to include the non-linear portion of the stress - creep relationship as influenced by age of loading the principle of superposition was illustrated as follows:

(see Figure 4.2.)

Concrete specimen was stressed to stress 1 from a time t_o to time t_1, then the stress was changed to stress 2 and maintained for the time t_1, to time t_2. The creep, as calculated by the superposition method, was required for the interval of time t_o to t_2.

Using creep curves for initial loading at age t_1, one portion of the creep was found by taking the creep due to stress 2 for the time $t_2 - t_1$ minus the creep due to stress 1 over the same time interval. The second part of the creep was that which would have occured at stress 1 during the time t_o to t_2 when loaded at time t_o. The addition of these two parts gave the total creep occuring during the interval of t_o to t_2.

After studying the existing creep prediction methods, it was thought that none accurately compensated for previous stress history.

The Effective modulus method completely disregards stress history.

The Rate of creep method takes into account variation in stress assuming creep directly proportional to stress ignores stress history. The method of superposition fully recognizes stress history, but for higher stresses it is theoreticaly unsound, because similar as rate of creep method, it assumes creep directly linear proportional to stress.

In a significant series of tests Ross[(4)] conducted tests on axially loaded cylinders subjected to:

decreasing increments of stress,

increasing increments of stress,

decreasing and increasing increments of stress.

He then compared three method of evaluating time-dependent strains due to variable stresses.

It was found that the use of an effective modulus of elasticity gave poor results by underestimating strains when stresses were decreasing, and vice versa. It also predicted complete recovery when stress was removed.

The rate of creep method described in the paper, gave a fair agreement between theoretical and experimental results, but it could not predict creep recovery following a reduction of stress. It tended to over - estimate creep under a declining stress. The superposition method predicted creep strains fairly well.

However tests were conducted at stresses under 0.2 x concrete strength. At low stresses the earlier mentioned principle of "specific creep" is valid. For high stresses the superposition method underestimates creep under increasing load increment.

All of existing creep prediction methods assume that creep and creep recovery are identical phenomena. This assumption is challenged by the author, hence prediction of creep effect under decreasing load is described in Chapter 5.

4.3. Modified Superposition Method 1 for Creep Prediction

The modified superposition method 1 in this form was presented by Drysdale[22] and used by him in a study of sustained loading effect on long slender column.

In this method creep is related directly to the elastic portion of the total strain. The equations for the experimental creep results for the various levels of constant stress were obtained by a least squares fit of the data.

The relationship used for creep at a constant stress was: Creep = -A + B x logarithm of Time. Variable A and B were dependent on the stress and hence also on the "elastic" strain as shown in Figures 3.5 and 3.6. Next, equations for A and B, were expressed as a functions of strain. Polynomials of third order yielded the best agreement for both curves, although for the working stress - level range up to .45fc', the relationship was found to be linearly proportional.

The modified superposition method computed the creep due to varying stress in two parts.

The first part of the creep calculation was similar to the rate of creep method. That is the creep for an increment of time due to a new "elastic" strain was calculated as though these conditions had prevailed since the beginning of loading. For the second part of the calculation, the creep which would occur for a stress equal to the difference between the previous stress and the present stress was found.

This was done using the increment of time under consideration but assuming that it commenced at the beginning of loading. However, account was taken of the increase and age of the concrete because the "elastic" portion of the strain was experimentaly found to change in the same manner as creep with age of loading of concrete. (Table 3.3.)

Figure 4.3. graphically portrays the following example of modified superposition method 1.

The creep, Creep 1, which would occur for an initial "elastic" strain $\mathcal{E}_{EL.1}$, during the time t_o to t_1, was taken directly from the experimentally found data. Then an increased stress resulting in "elastic" strain $\mathcal{E}_{EL.2}$ has in turn maintaned for the period t_1 to t_2. Now the effect of stress history became a factor. The first part of Creep 2, Creep 2' was the strain which would have occurred for $\mathcal{E}_{EL.2}$ over time t_1 to t_2 if the element had been loaded at the same level for time t_o to t_1. The second part, Creep 2", was the creep which would result from the "elastic" strain $\mathcal{E}_{EL.2}-\mathcal{E}_{EL.1}$ for an element loaded to that strain at the time t_1 for the length of time $t_2 - t_1$. The total creep up to time t_2 was then: Creep 1 + Creep 2' + Creep 2". Each additional change for increments of time was treated in the same manner.

TOTAL CREEP IN TIME T2 = CREEP 1 + CREEP 2' + CREEP 2"

FOR CHANGES IN STRESS CORRESPONDING TO $\varepsilon_{cel.2}$ AND $\varepsilon_{cel.1}$

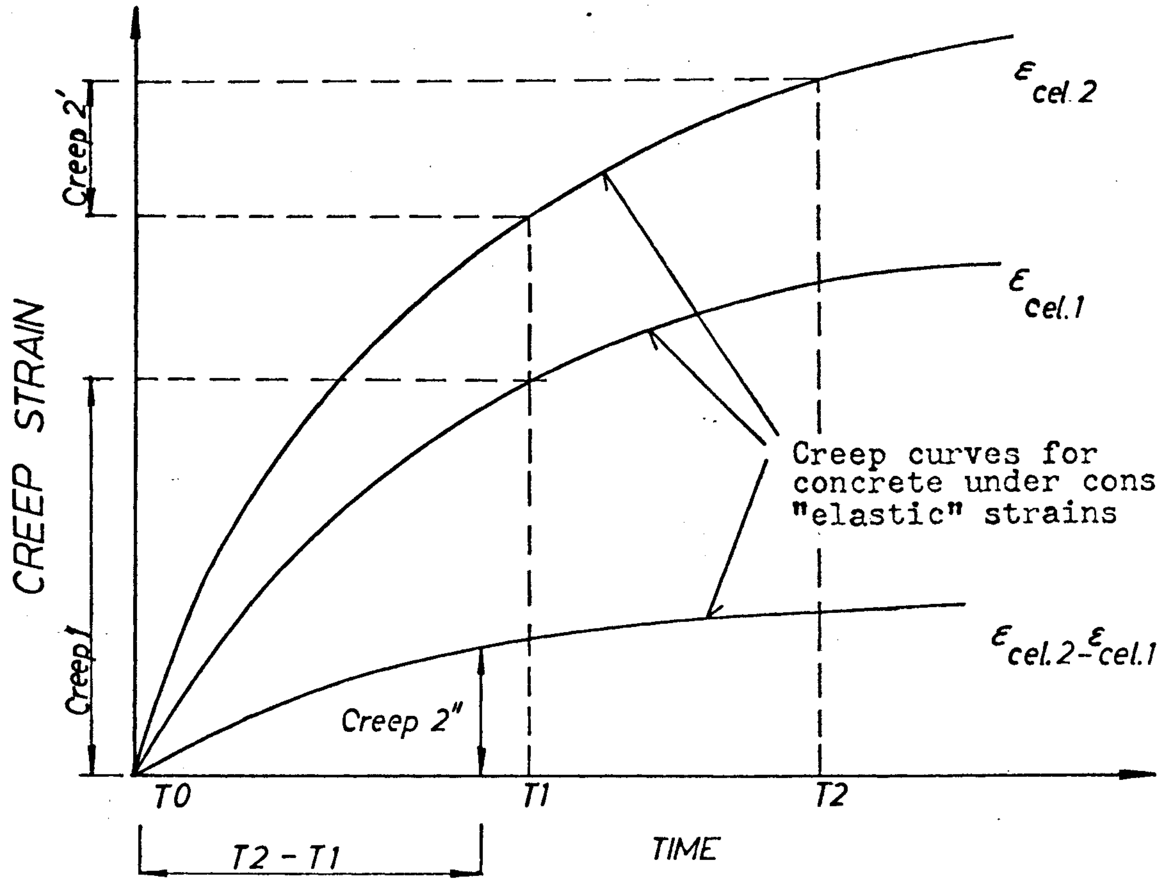
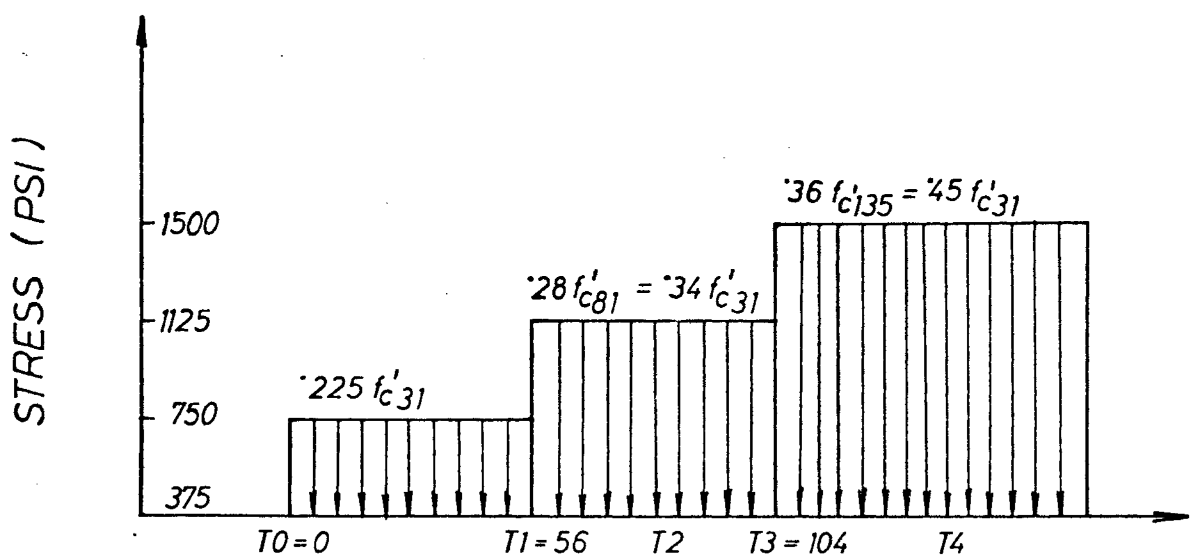


FIGURE 4.3 MODIFIED SUPERPOSITION METHOD 1 FOR CALCULATING CREEP



TIME SINCE FIRST APPLICATION OF LOAD

FIGURE 4.4 LOADING SCHEME OF PRISM B

4.3.1. Observation and Prediction of Creep for Variable Stress on Specimen B

Prism B was loaded as illustrated in Figure 4.4. At age 31 days, the prism was stressed to 750 psi, and at age 87 days was additional loaded to 1125 psi and finally at age 135 days the prism was additionally loaded to a stress of 1500 psi.

"Elastic" strains (more accurately short term cylinder strain) were computed from polynomial equations 2.1, 2.2, and 2.3. These equations were derived from cylinders tested at ages 28 days, 76 days and 135 days after pouring. The "elastic" strain at ages 31 days and 87 days hence were computed assuming straight line change of elastic strain with time compared to the 28 day and 76 day results.

Table 4.1.

Age since first applying of load	Age since pouring (days)	Elastic Strains for Stresses		
		750 [psi]	1125 [psi]	1500 [psi]
	28	253.0	403.6	572.8
0	31	252.6		
45	76	246.0	393.8	524.1
56	87		383.2	
104	135	206.3	337.0	483.4

"Elastic" strain (in/in x 10^6)

Hence plot of creep constants A and B against "elastic" strain is illustrated in Figure 3.5. where for "elastic" strains up to an "elastic" strain corresponding to a stress of 1500 psi the relationships are straight lines. These values of creep constants A and B can be obtained from Equations 4.4. and 4.5.

$-A = 1.175 \times$ "Elastic" strain Equation 4.4.

$B = 1.949 \times$ "Elastic" strain Equation 4.5.

For a stress increment at time $T_1 = 56$ days since first application of load creep strain any time T_2 after 56 days will be computed from Figure 4.5. where;

Total Creep = Creep 1 + Creep 2' + Creep 2"

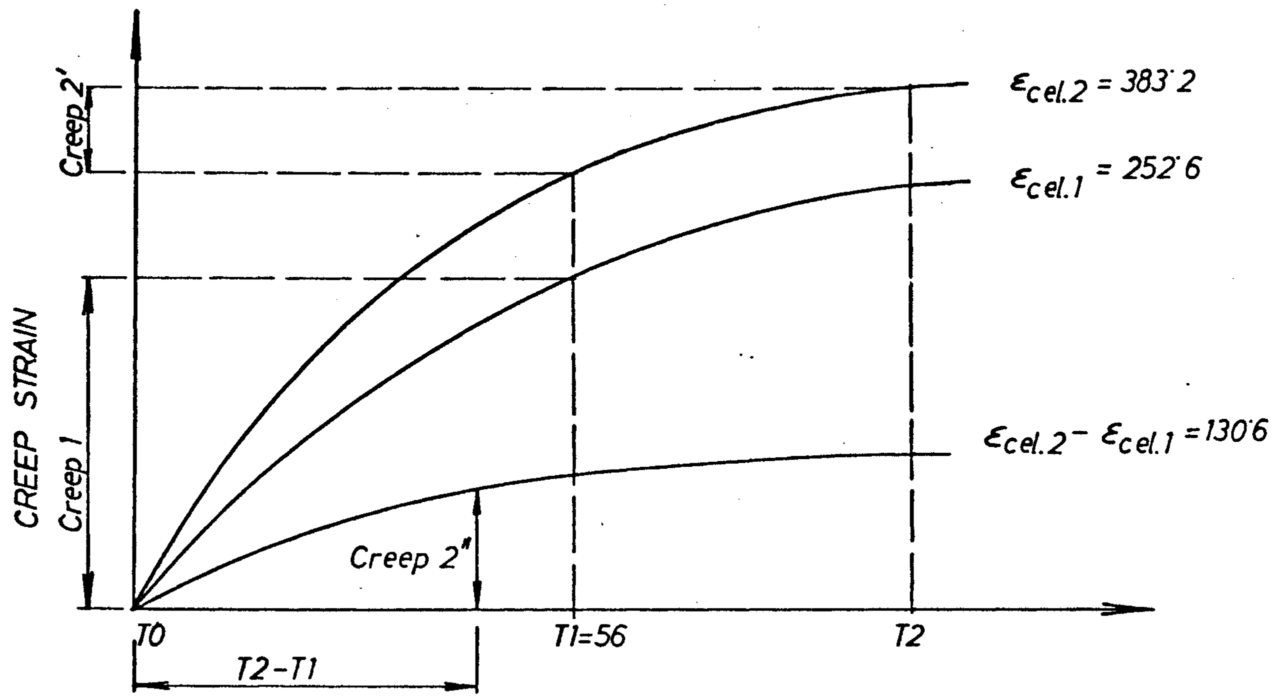
Hence the solution requires creep versus time curves for "elastic" strains $\mathcal{E}_{CEL.2} = 383.2 \times 10^{-6}$ and $\mathcal{E}_{CEL.2} - \mathcal{E}_{CEL.1} = 130.6 \times 10^{-6}$. The creep strain corresponding to $\mathcal{E}_{CEL.1} = 252.6 \times 10^{-6}$ has been found experimentally.

Creep strain is expressed in form of Equation 1.7.

Creep = A + B x $\log_{10}$ of Time Equation 1.7.

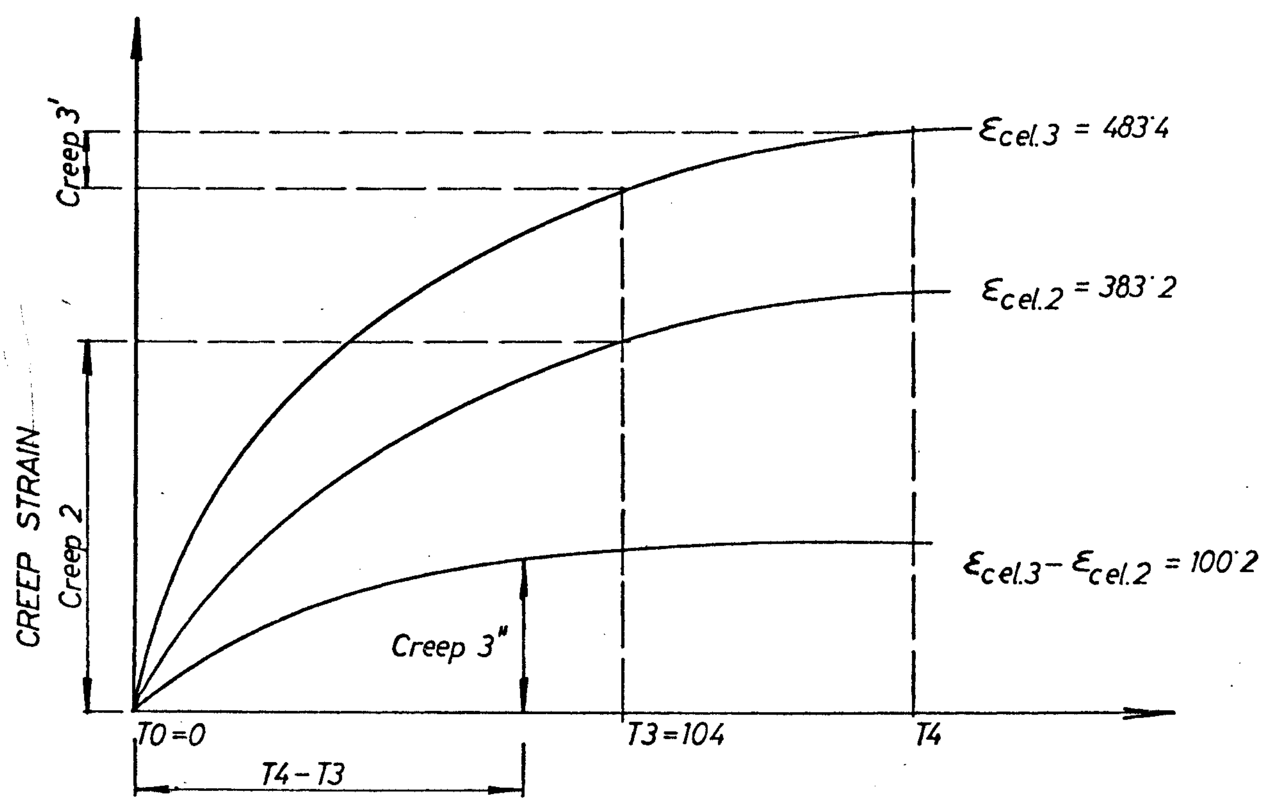
For any "elastic" strains, the creep constants in this example are found from Equations 4.4. and 4.5.

PREDICTION OF CREEP STRAIN OF PRISM WITH B STRESS HISTORY BY MODIFIED SUPERPOSITION METHOD 1



TIME SINCE FIRST APPLICATION OF LOAD

FIGURE 4.5 FIRST INCREMENT OF LOAD AT AGE 56 DAYS



TIME SINCE FIRST APPLICATION OF LOAD

FIGURE 4.6 SECOND INCREMENT OF LOAD AT AGE 104 DAYS

Hence

$\text{Creep}_{\varepsilon_{cEL.2}} = -450.26 + 746.85 \times \log_{10}$ of Time   Equation 4.6.

$\text{Creep}_{\varepsilon_{cEL.2}-\varepsilon_{cEL.1}} = -153.45 + 254.54 \times \log_{10}$ of Time   Equation 4.7.

The computation of Creep 2' and Creep 2" at time T_2 after 56 days are contained in Appendix 6.

Table 4.2. shows the observed and predicted values of creep strain (in/in x 10^6) for Prism B.

Prediction is by modified superposition method I.

For the second stress increment to a stress of 1500 psi at age T_3 = 104 days there is a change of "elastic" strain from $\varepsilon_{cEL.2}$ = 383.2 in/in x 10^6 to $\varepsilon_{cEL.3}$ = 483.4 in/in x 10^6. Figure 4.6. similarly to the previous "elastic" strain increment illustrates the creep computation.

Total creep after time T_3 = 104 days since first loading is

Creep = Creep 2 + Creep 3' + Creep 3"

where Creep 2 = Creep 1 + Creep 2' + Creep 2"

The computation of Creep 3' and Creep 3" is shown in Appendix 6a.

The predicted and observed values for this phase of the creep of Prism B are illustrated in Table 4.2. and Figure 4.7.

Table 4.2.

Observed and Predicted Values of Creep Strain [in/in x 10^6] for Prism B.

Time since pouring	Time since first loading	Observed creep strain	Creep 1	Creep 2'	Creep 2"	Computed creep strain
87	56	554	554	0	0	554
100	69	688	554	67	130	751
114	83	799	554	128	211	893
135	104	857	554	201	274	1029
			Creep 2	Creep 3'	Creep 3"	
135	104	857	1029	0	0	1029
143	112	919	1029	30	59	1118
163	132	1090	1029	98	165	1292
170	139	1138	1029	119	192	1340

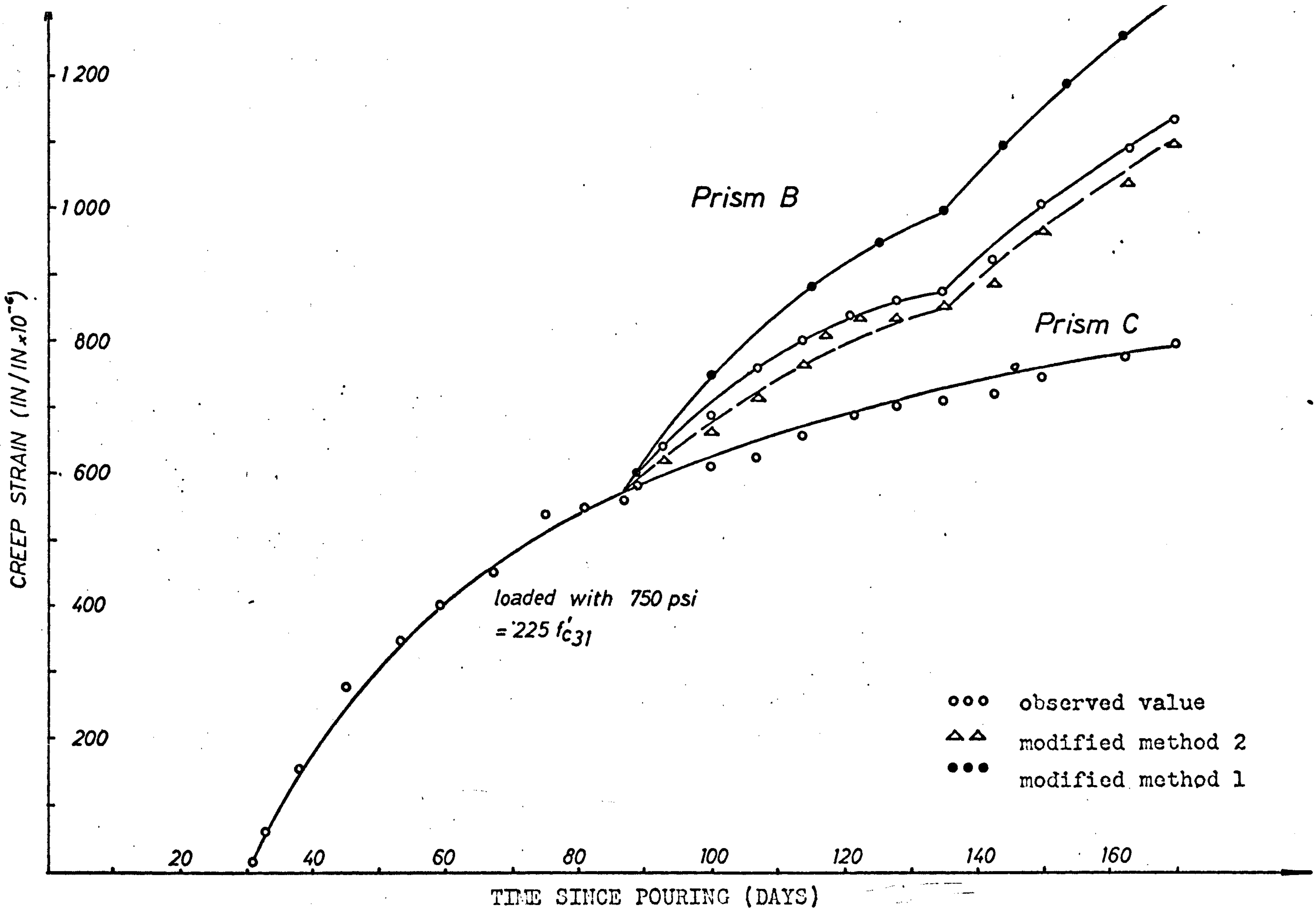


FIGURE 4.7 OBSERVATION AND PREDICTION OF CREEP BEHAVIOUR OF PRISM B

4.3.2. Observation and Prediction of Creep Behaviour of Prism A-2

The loading schedule of prism A-2 is illustrated in Figure 4.14.

Creep is expressed in form of Equation 1.7.

Creep = A + B x $\log_{10}$ of Time.

Creep constants A and B are evaluated for any "elastic" strain from Equations 4.8. and 4.9. where;

$$A = -.21361868 \times \varepsilon_{CEL.} - 3.2061 \times 10^{-4} \times \varepsilon_{CEL.}^{2} - 1 \times 10^{-8} \times \varepsilon_{CEL.}^{3}$$
Equation 4.8.

$$B = 1.15061539 \times \varepsilon_{CEL.} - 5.3957 \times 10^{-4} \times \varepsilon_{CEL.}^{2} + 2.73 \times 10^{6} \times \varepsilon_{CEL.}^{3}$$
Equation 4.9.

For evaluation of polynomials in the forms of Equations 4.8. and 4.9. more observed data was required. As well some of the stress strain data in Table 4.3. were not found experimentally. Appendix 5. explains the method used for evaluation of missing observed data.

For an "elastic" strain increment at age 22 days since first application of load

Total creep = Creep 1 + Creep 2' + Creep 2" as illustrated in Figure 4.8.

$$\text{Creep}_{\varepsilon_{CEL.2}} = -142.6492 + 570.1219 \times \log_{10} \text{ of Time} \qquad \text{Equation 4.10.}$$

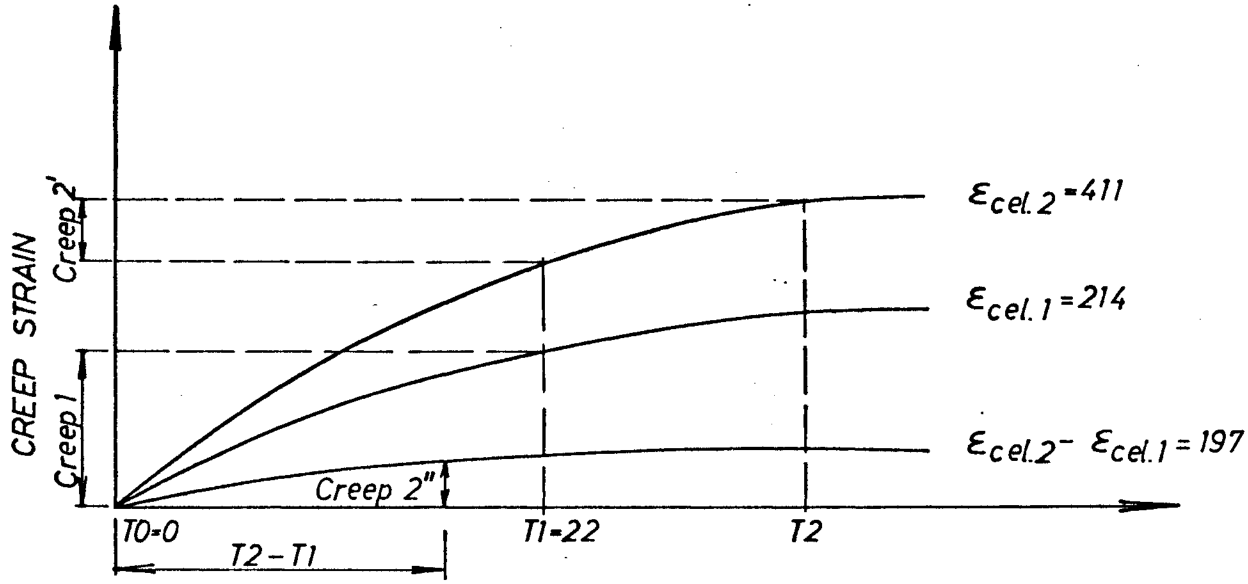
$$\text{Creep}_{\varepsilon_{CEL.2} - \varepsilon_{CEL.1}} = -54.58 + 226.599 \times \log_{10} \text{ of Time} \qquad \text{Equation 4.11.}$$

where $\varepsilon_{CEL.1} = 411$ in/in x 10^{6}

$\varepsilon_{CEL.2} - \varepsilon_{CEL.1} = 197$ in/in x 10^{6}

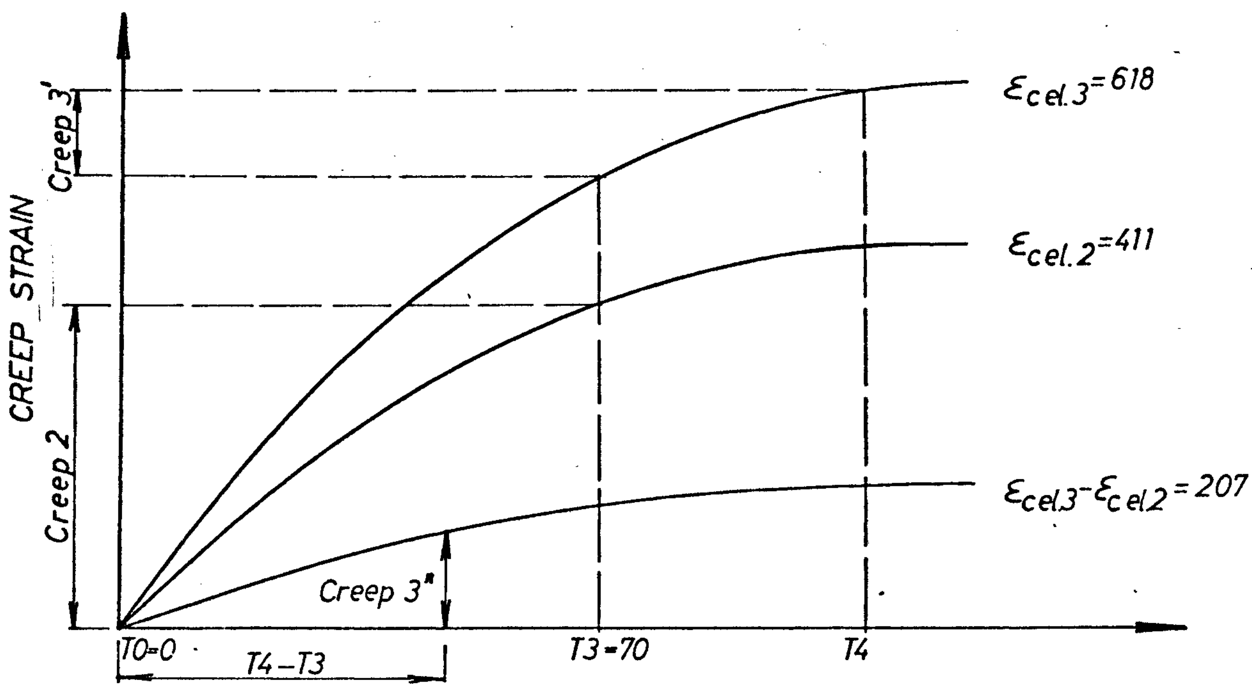
The computations of A and B constants for Equations 4.10. and 4.11. are shown in Appendix 7.

PREDICTION OF CREEP STRAIN OF PRISM WITH A-2 STRESS HISTORY BY MODIFIED SUPERPOSITION METHOD 1



TIME SINCE FIRST APPLICATION OF LOAD (DAYS)

FIGURE 4.8 FIRST INCREMENT ON-LOAD AT AGE 22 DAYS



TIME SINCE FIRST APPLICATION OF LOAD (DAYS)

FIGURE 4.9 SECOND INCREMENT OF LOAD AT AGE 70 DAYS

The next step is the computation of Creep 2' and Creep 2".
The computation of Creep 2' and Creep 2" is in Appendix 7.

For second elastic strain increment in age 70 days.
Total creep in time T_4 = 70 days will be computed as illustrated in Figure 4.9.

Total creep = Creep 2 + Creep 3' + Creep 3"

Where Creep 2 = Creep 1 + Creep 2' + Creep 2"

The creep strains corresponding to $\mathcal{E}_{CEL.3}$ = 618 in/in x 10^6 and $\mathcal{E}_{CEL.3}-\mathcal{E}_{CEL.2}$ = 207 in/in x 10^6 by using Equations 4.8. and 4.9. are in the form.

$\text{Creep}_{\mathcal{E}_{CEL.3}} = 256.7 + 1149.0 \times \log_{10}$ of Time Equation 4.12.

$\text{Creep}_{\mathcal{E}_{CEL.3}-\mathcal{E}_{CEL.2}} = -55.9 + 239.3 \times \log_{10}$ of Time Equation 4.13.

The next step is the computation of Creep 3' and Creep 3". These computations are shown in Appendix 7a.

Figure 4.10. and Table 4.4. illustrate the observed and computed creep strains for prism A-2.

Table 4.3.

Stress Versus Strain value Required for Computation of Creep Constants for Prism A-2.

Time Since Pouring	Time Since First Loading	Elastic Strain for Stresses		
		750 [psi]	1125 [psi]	1500 [psi]
14	0	214		
36	22		411	
84	70			618

Table 4.4.

Observation and Prediction of Prism A-2.

Time since pouring	Time since first loading	Creep 1	Creep 2'	Creep 2"	Computed creep strain	Observed creep strain
36	22	276	0	0	276	276
43	29	276	68	137	481	521
62	48	276	193	266	735	741
72	58	276	240	298	814	790
84	70	276	286	326	888	891
		Creep 2	Creep 3'	Creep 3"		
84	70	888	0	0	888	891
92	78	888	54	160	1102	1132
128	114	888	243	337	1468	1467

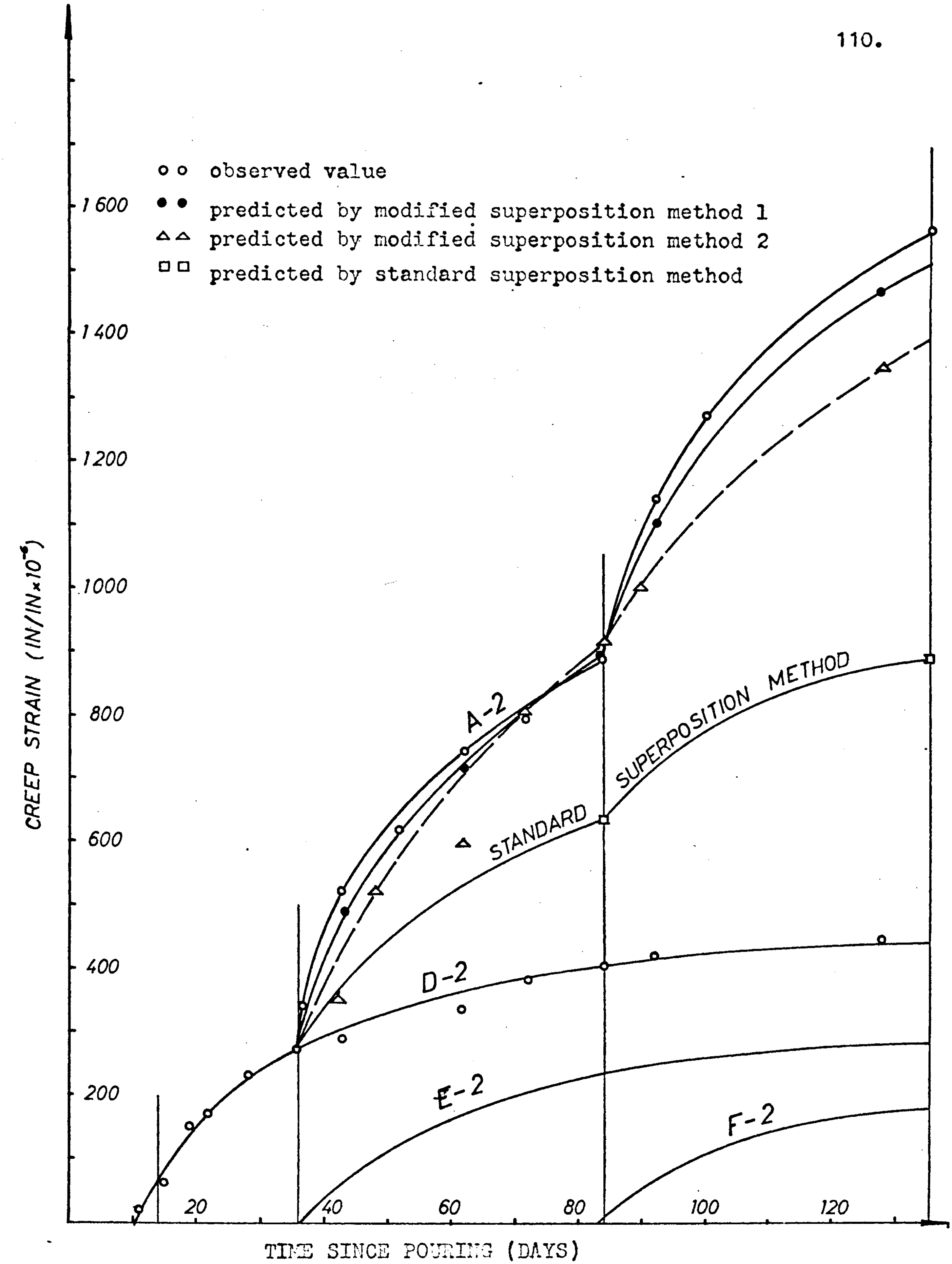


FIGURE 4.10 OBSERVATION AND PREDICTION OF CREEP STRAIN OF PRISM A-2

4.4. Modified Superposition Method 2

Modified superposition method 2, serves a similar purpose as the method described in the previous section for prediction of creep under increasing stresses.

The method uses calculation of the single creep versus time curve, relationship between level of stress versus rate of creep to describe the creep versus time relationship. Also change of strength with time.

The modified superposition method 2 computes the creep due to varying stress in two parts. The first part of the creep calculation for an increased stress over a time interval is similar to the rate of creep method. That is, the creep for an increment of time due to a new stress level is calculated as thought these condition has prevailed since the beginning of loading.

For the second part of the creep calculation, the creep which would occur for a stress equal to the difference between the previous stress level and present stress level was found.

In section 3.4. it was found that the change in concrete strength with age of loading is smaller than the change in the rate of creep with age of loading or the change in "elastic" strain with the age of loading.

MODIFIED SUPERPOSITION METHOD 2 FOR CALCULATING CONCRETE CREEP STRAIN

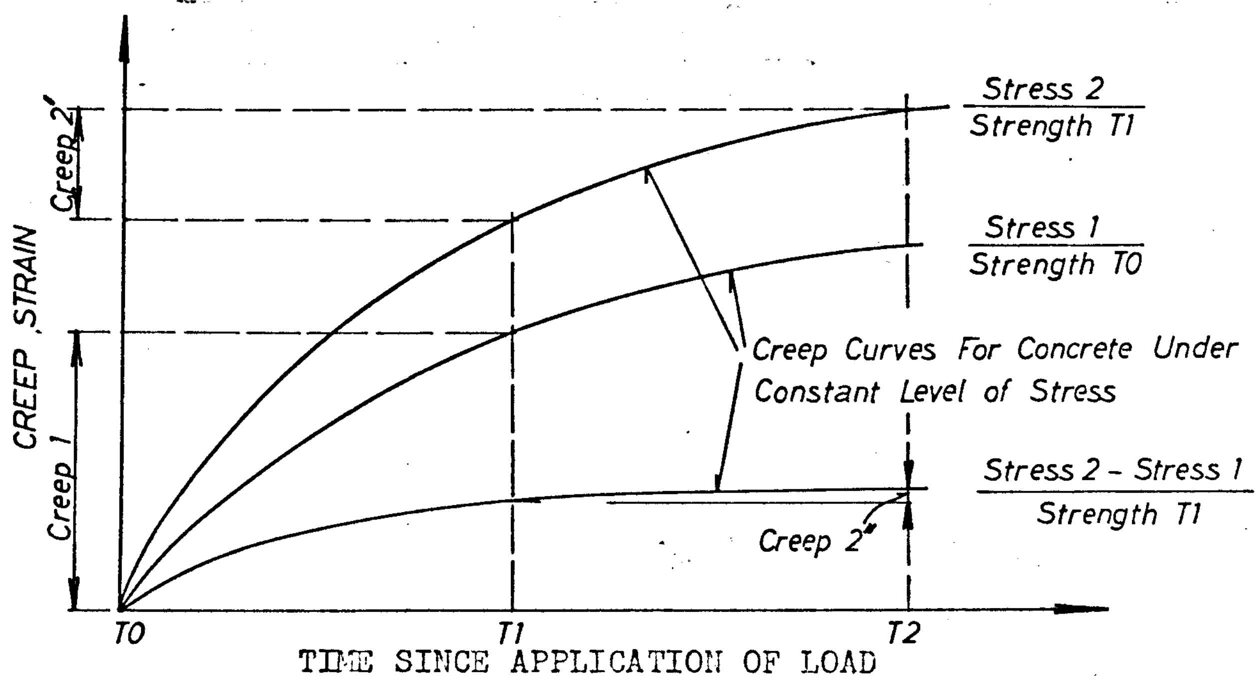


FIGURE 4.11.A

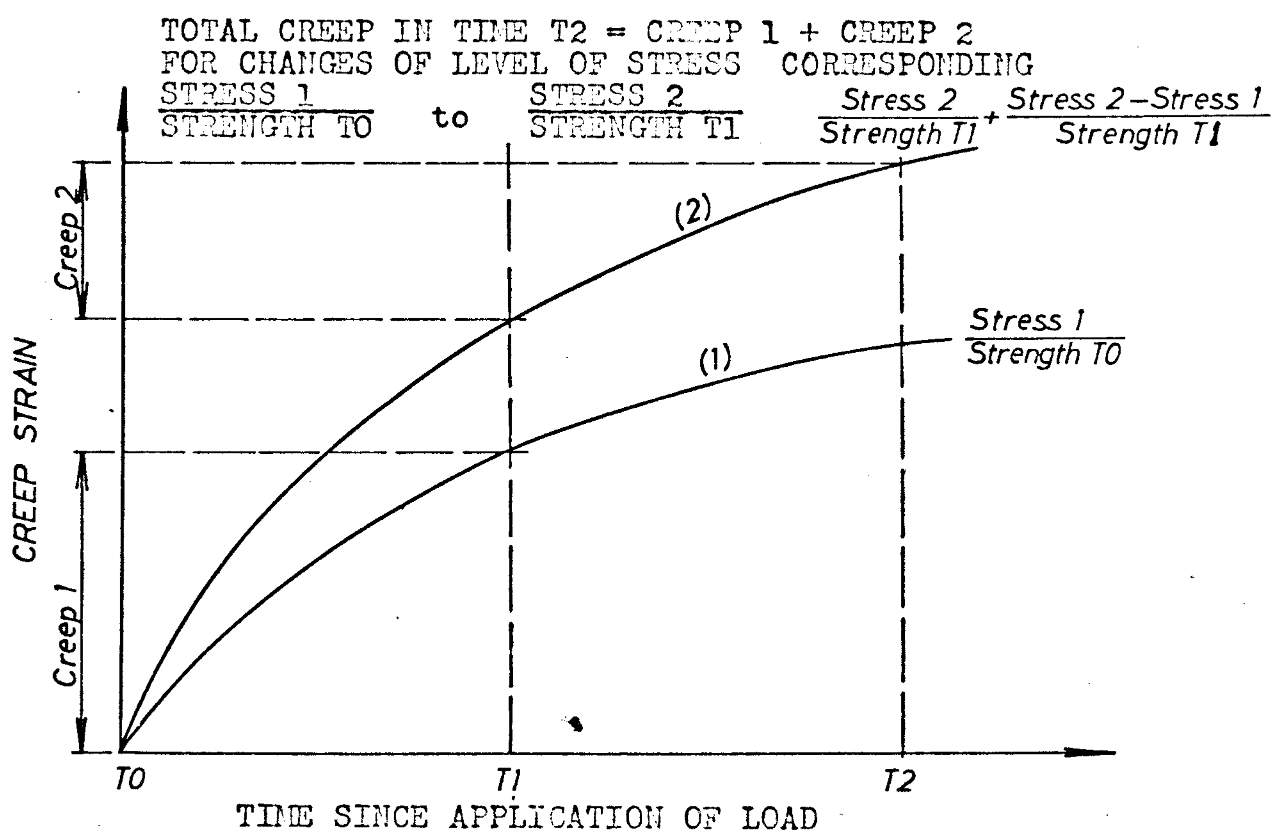


FIGURE 4.11. B

Hence this influence creep 2".

Creep 2" is part of the creep which occurs after time t_1. Figure 4.11.a. and 4.11.b. illustrate the above method. The creep 1 from Figures 4.11. which would occur for stress level stress 1/strength T_o during time t_o to t_1 was observed. Creep 2 consist of two parts. Together these parts give the creep strain which corresponds to a stress level.

$$\frac{\text{Stress 2} + \text{Stress 2} - \text{Stress 1}}{\text{Strength } T_1}$$

The creep value evaluated from the relationship shown in Figure 3.7. Hence the total creep after time t_2 is Creep 1 + Creep 2 as shown in Figure 4.11.b. The next part of this section explains the prediction of the creep of prisms with stress histories similar to Prism B and Prism A-2.

4.4.1. Observation and Prediction of Creep for Variable Stress in Prism B by Modified Superposition Method 2.

Prism B was loaded as illustrated in Figure 4.4. For prediction of creep behaviour only the single creep versus time curve from the sustained loading of Prism C to the stress level of $fc = .225\, fc'_{31}$ and Equation 3.3. which was experimentally derived and explained in section 3.3. were used.

For a change in stress from 750 psi to 1125 at time t_1 = 56 days since the first application of load, creep strain at any time t_2 is described by.

Total creep = Creep 1 + Creep 2.

Figure 4.12. demonstrates this calculation. The concrete strength is computed from Equation 2.4.

fc' = 1357.2 + 1329.6 x logarithm of Time   Equation 2.4.

and hence at;

Time 31 fc'_{T0} = 3325 psi,

Time 87 fc'_{T1} = 3932 psi,

Time 135 fc'_{T3} = 4197 psi.

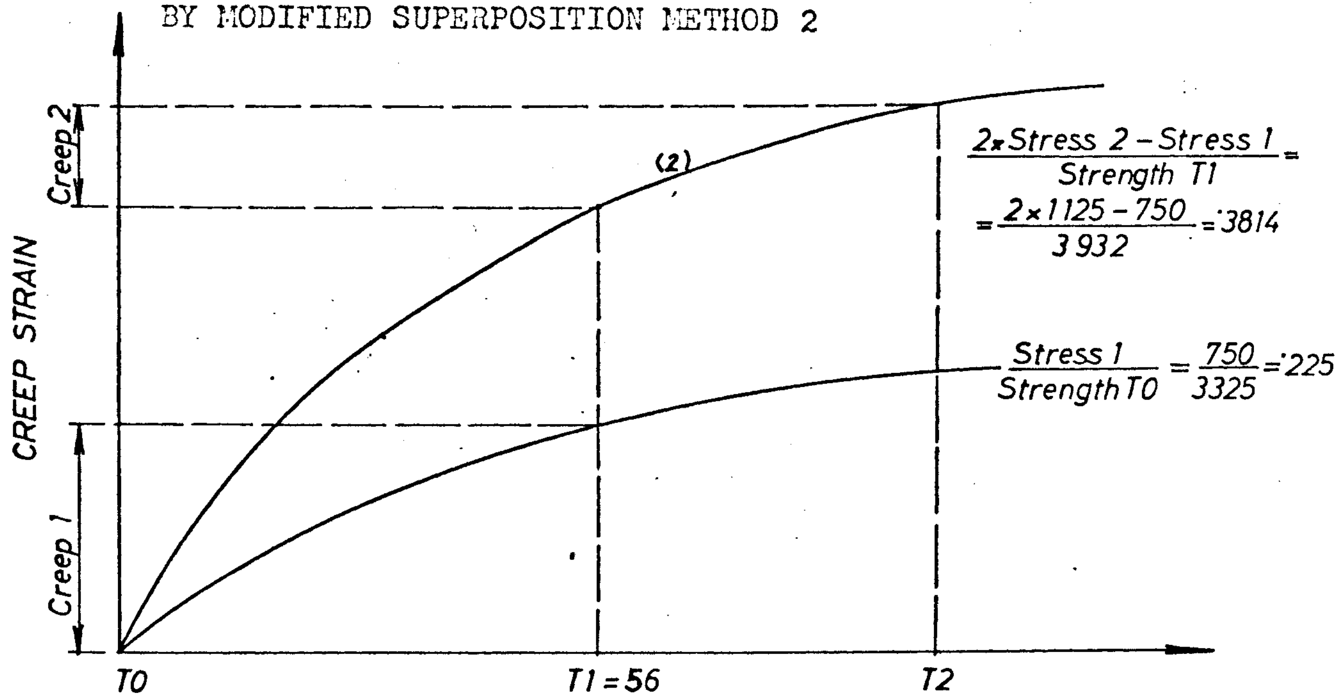
The stress level for curve (2) in Figure 4.12. is 0.3814 and the rate of creep is computed from Equation 3.3.

0.3814 = 0.2511 + 0.4614 x logarithm (B_2/B_c) Equation 3.3.

The equation is satisfied for B_2/B_c = 1.916

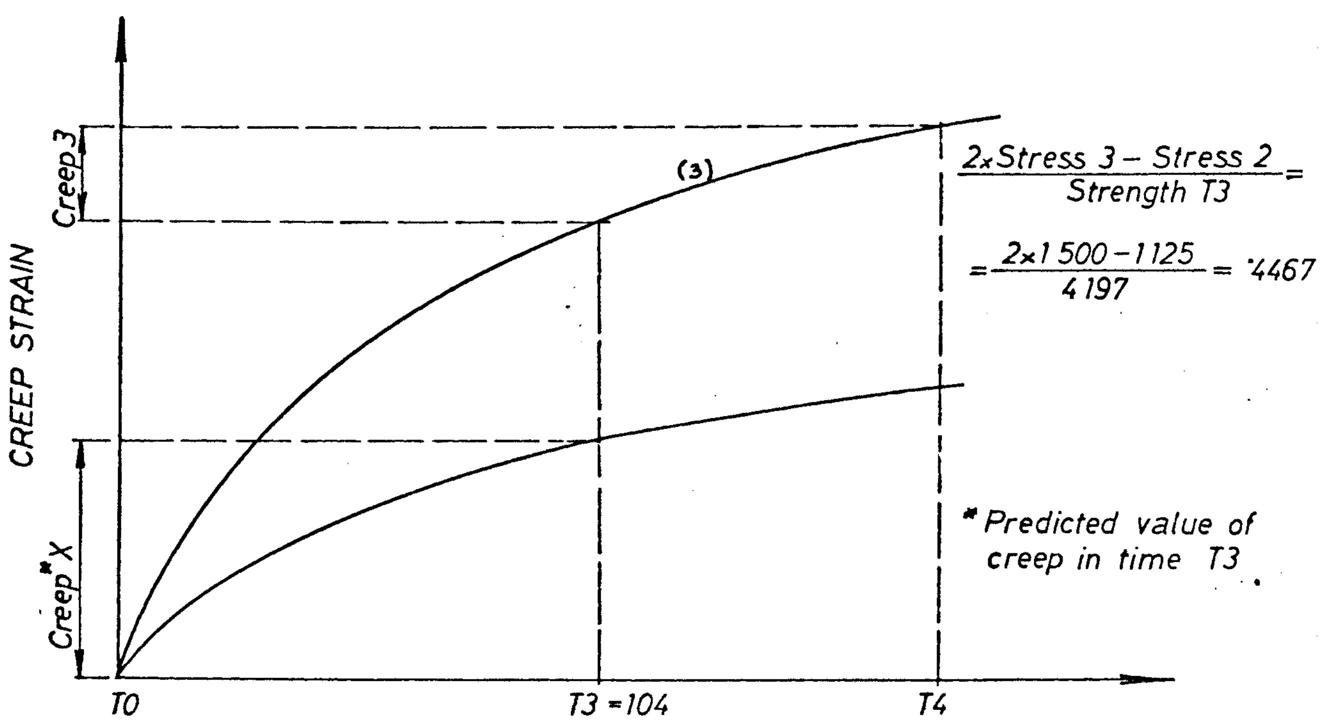
PREDICTION OF CREEP STRAIN OF PRISM WITH B STRESS HISTORY

BY MODIFIED SUPERPOSITION METHOD 2



TIME SINCE FIRST APPLICATION OF LOAD (DAYS)

FIGURE 4.12 FIRST INCREMENT OF LOAD IN AGE 56 DAYS



TIME SINCE FIRST APPLICATION OF LOAD (DAYS)

FIGURE 4.13 SECOND INCREMENT OF LOAD IN AGE 104 DAYS

This means that curve 2 will have creep strain of 1.916 x creep strain of prism C.

The computation of curve 2 and hence creep 2 are illustrated in Table 4.5.

For a new increment of load to a stress of 1500 psi at age t_3 = 104 days since first loading, there is an increment in stress level as follows from Figure 4.13. The creep due to the increment in load is the creep due to a stress level of 0.4467. The rate of creep of prism 3 for the level of stress equal to 0.4467 is computed from Equation 3.3.

$$0.4467 = .2511 + .4614 \times \text{logarithm } (B_3/B_c) \qquad \text{Equation 3.3.}$$

This equation is satisfied for B_c/B_c = 2.654 and hence the creep strain of curve 3 is 2.654 x creep strain of prism C. For computation look into Table 4.5. and predicted value of creep are illustrated in Figure 4.7.

Table 4.5.

Observation of Prism B and Prediction by Modified Superposition Method 2

Time since first loading	Time since pouring	Observed Creep Strain		Computed Creep Strain		
		Prism C	Prism B	of curve 2 (3)	increment during time interval	total predicted value of Prism B
1		28	28			
2	33	59	59			
7	38	153	153			
14	45	280	280			
22	53	350	350			
28	59	403	403			
36	67	457	457			
44	75	543	543			
50	81	545	545		Creep 2	
56	87	554	554	1061		554
58	89	567	599	1086	25	579
62	93	590	647	1130	69	623
69	100	609	688	1167	100	660
77	107	635	759	1217	156	710
83	114	661	799	1266	205	759
90	121	692	836	1326	265	819
97	128	702	855	1345	284	838
104	135	711	857	1362	301	854
				1887	Creep 3	
106	137	720		1911	24	878
112	143	720	919	1911	24	878
119	150	753	1012	1998	111	965
132	163	780	1090	2070	183	1037
139	170	803	1138	2131	244	1098

4.4.2. Observation and Prediction of Creep for Variable Stress in Prism A-2 by Modified Superposition Method2.

The loading schedule of prism A-2 is shown in Figure 4.14. For prediction of creep behaviour creep strain data from prism D-2 was used.

Some data representing strength at ages 36 days and 84 days since pouring was missing. With regard to references (22, 28, 32) concrete strength can be found at any time within 28 to 100 days after pouring from Equation 4.14.

$$fc'_T = fc'_{28} \, (1 + 2 \, (T-28)/1000) \qquad \text{Equation 4.14.}$$

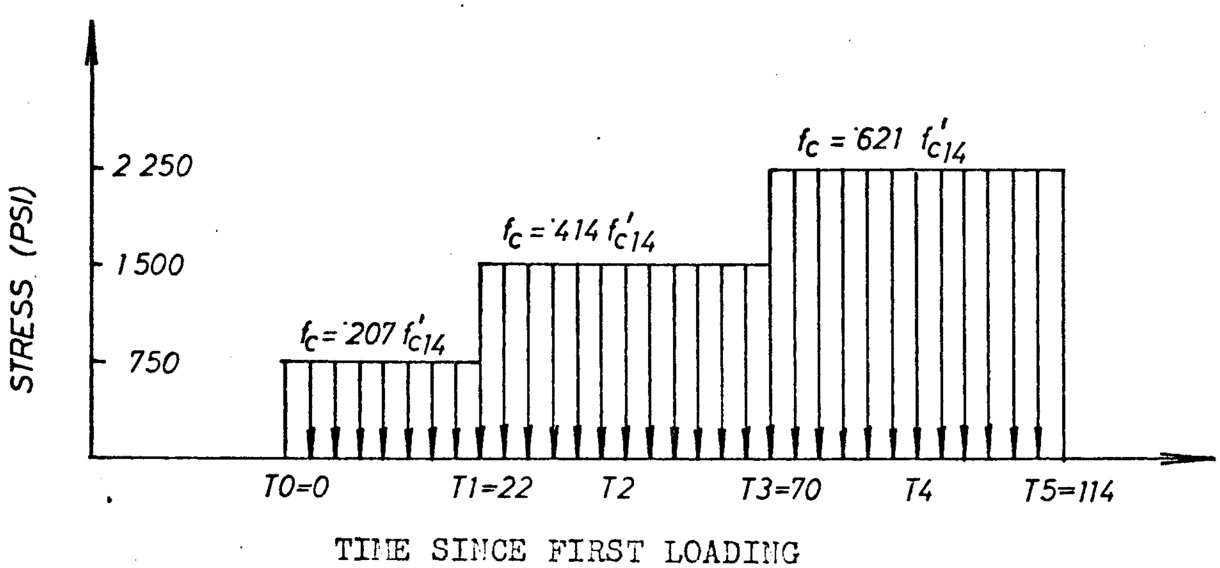


FIGURE 4.14 LOADING SCHEDULE OF PRISM A-2

The strength at time 14 days and 28 days after pouring was found experimentally, and listen in Table 2.2. where;

Time 0 $\quad fc'_{14} = 3619$ psi

$\quad\quad\quad fc'_{28} = 4167$ psi

and at ages 36 days and 84 days strengths were computed from Equation 4.14.

Time 1 $\quad fc'_{36} = 4230$ psi

Time 3 $\quad fc'_{84} = 4620$ psi

In this example insufficient data was available for developing the proportionality between ratio of rate of creep and level of stress. However because the concrete for this test was made with the same mix proportion as the concrete in Pour 1 and because the prisms were in the same environment, it was assumed that Equation 3.3. can be re-used.

It can be mathematically proved, that the first number of Equation 3.3. is just represents a constant point where the straight line intersects the vertical axis. The slope of this straight line is important for the computation and serves for computation of the rate of creep of known level of stress.

Equation 3.3. is used in the form of Equation 4.15.

$$\frac{fc}{fc'} = .2070 + .4614 \times \text{logarithm } (B_c/B_{D-2}) \qquad \text{Equation 4.15.}$$

For a load increment at time T_1 = 22 days since first application of load, the total creep strain will be creep 1 + creep 2.

Creep 1 is creep at age T_1 due to

$$\frac{\text{stress 1}}{\text{strength } T_o} = \frac{750 \text{ psi}}{3619 \text{ psi}}$$

and creep 2 is due to the change of the level of stress to 1500 psi so that this part of the creep corresponds to a stress ratio of

$$\frac{1500 + 1500 - 750}{4230} = 0.5319$$

From Equation 4.15. it follows that

$$0.5319 = 0.2070 + 0.4614 \text{ x logarithm } (B_2/B_{D-2}).$$

This equation is satisfied for $B_2/B_{D-2} = 5.06$

This means that the rate of creep of curve 2 is 5.06 x rate of creep of prism D-2.

The computation of the increment of creep during T_1 and T_2 and prediction of creep of prism with A-2 stress history see Table 4.6. and Figure 4.10.

For a new stress increment to a stress of 2250 at time T_3 = 70 days since first loading, the creep strain at time T_4 will be the predicted creep strain in time T_3 + Creep 3.

Creep 3 is due to level of stress

$$\frac{2250 + 2250 - 1500}{4620} = 0.6493$$

In time interval $T_4 - T_3$

from Equation 4.15. follow

$$0.6493 = 0.2070 + 0.4614 \times \text{logarithm } (B_3/B_{D-2})$$

the solution is for $B_3/B_{D-2} = 9.09$
this mean that the rate of creep of curve 3 is 9.09 x rate of creep of prism D-2.
For computation of curve 3, creep 3 and prediction of creep of prism with A-2 stress history see Table 4.6. and Figure 4.10.

Table 4.6.

Observed and Predicted Value of Creep Strain of Prism A-2

Prediction with Modified Superposition Method 2

Time since pouring	Time since first loading	Observed Creep Strain		Computated Creep Strain		
		Prism A-2	Prism D-2	of curve 2 (3)	Increment during time interval	Predicted creep strain A-2
15	1	67	67	339		67
19	5	154	154	779		154
22	8	165	165	835		165
28	14	233	233	1179		233
					Creep 2	
36	22	276	276	1397		276
37	23	343				
43	29	521	289	1462	65	341
52	38	624				
62	48	741	337	1705	308	584
72	58	790	379	1918	521	797
84	70	891	403	2039	642	918
				Prism 3	Creep 3	
84	70	891	403	3663		918
92	78	1132	423	3845	182	1100
100	86	1275				
128	114	1467	450	4090	427	1345
135	121	1570				

Time (days)

Creep (in/in x 10^6)

4.5. Discussion

Two methods were presented for prediction of creep under increasing load increments in the medium to high stress range.

It seems that modified superposition method 1 overestimates creep for medium stresses or for small stress increments. For higher stresses modified superposition method 1 as shown in Figure 4.10. provides an excellent prediction. It should be remembered of course that some data which should have been observed were missing and were computed from a relationship valid for the first pour. It is quite possible that some errors could occurs.

The modified superposition method 2 predicted creep strain for medium stresses with good accuracy but slightly underestimates creep.

For high stresses this method underestimates creep strain by a wider margin.

In Figure 4.10. the modified superposition methods are compared with the best published method "Standard Superposition Method". The modified superposition methods show superior prediction. These results are referred to again in Chapter 7.

Chapter 5

Creep Under Decreasing Stresses

5.1. Introduction

When a concrete specimen which has been subjected to a sustained compressive stress is subsequently unloaded, the recovery of strain is of two types.

The first strain recovery is the instantaneous recovery, which represents the elastic strain corresponding to the stress removed and to the modulus of elasticity at the time of the removal of the load. This immediate recovery is followed by an additional gradual recovery which is often called creep recovery by analogy to the creep under load.

Comparatively little is known about creep recovery, although it is of great importance in elucidating the mechanism of creep of concrete.

A frequently accepted explanation of creep recovery is based on the principle of superposition of strains as described in Section 1.4.1. Assuming superposition to be valid it follows that if the compressive stress on a specimen is removed at the age t_1, the resulting creep recovery will be the same as the creep of a similar specimen subjected to the same compressive stress at the age t_1. This theory thus considers the mechanism of creep and of creep recovery to be the same and the principle of superposition of strains implies that creep is a delayed elastic phenomenon in which full recovery is generally impeeded by the progressive hydration of cement.

A basically different view is taken by Dutron[33], who does not regard creep as an elastic phenomenon. According to Dutron the creep recovery is probably caused by a slight swelling of the cement paste when released from load as the concrete adjusts to a state of hygral equilibrium with the unchanged surrounding medium.

Mills[6] has described "collapse of structure" the mechanism whereby the surfaces of the solids originally containing load-bearing adsorbed water are drawn close enough together to form inter-molecular bonds which can be strong enough to resist penetration of water upon removal of load.

More searching tests of the principle of superposition of strains made by Ross[4], Davies[26], and McHenry[3], indicated that the principle of superposition of strain which represents a convenient working assumption overestimates the creep recovery.

A typical creep - recovery - time curve is shown in Figure 5.1. It can be seen that this shape differs from the usual creep - time curve in two respects.

Firstly, the recovery is much steeper during the first few days after removal of the load. Secondly, the full recovery is completed within a short period of time often two to three weeks as opposed to the slow continuation of creep over many months or years. These differences are of interest with regard to the theories which assume both creep and creep recovery to be governed by the same principles.

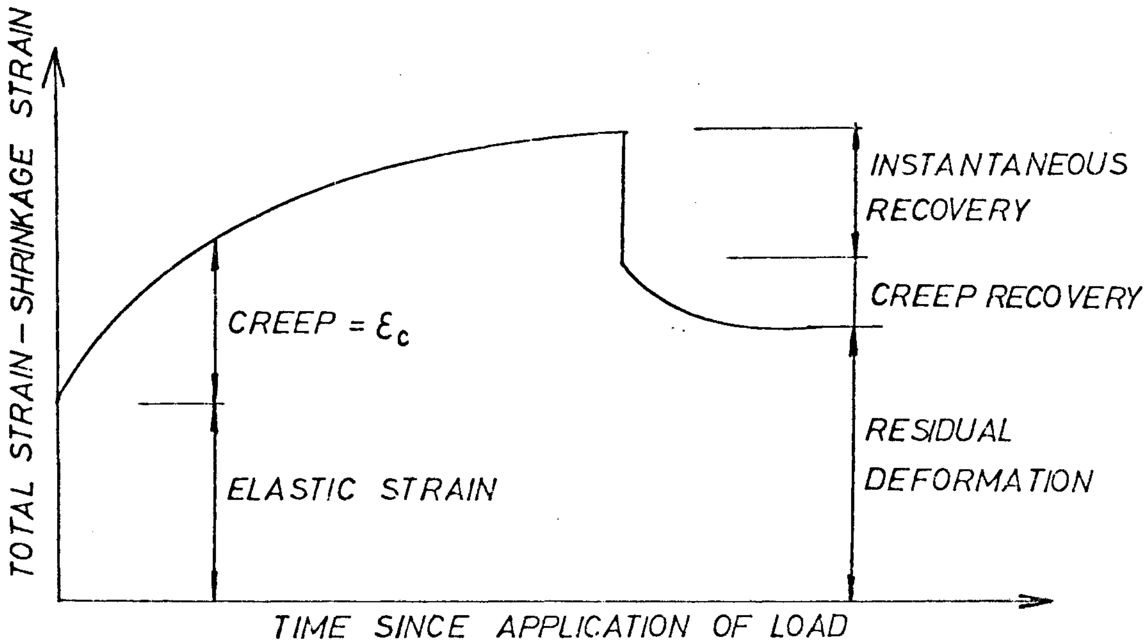


FIGURE 5.1 TYPICAL CREEP RECOVERY - TIME CURVE

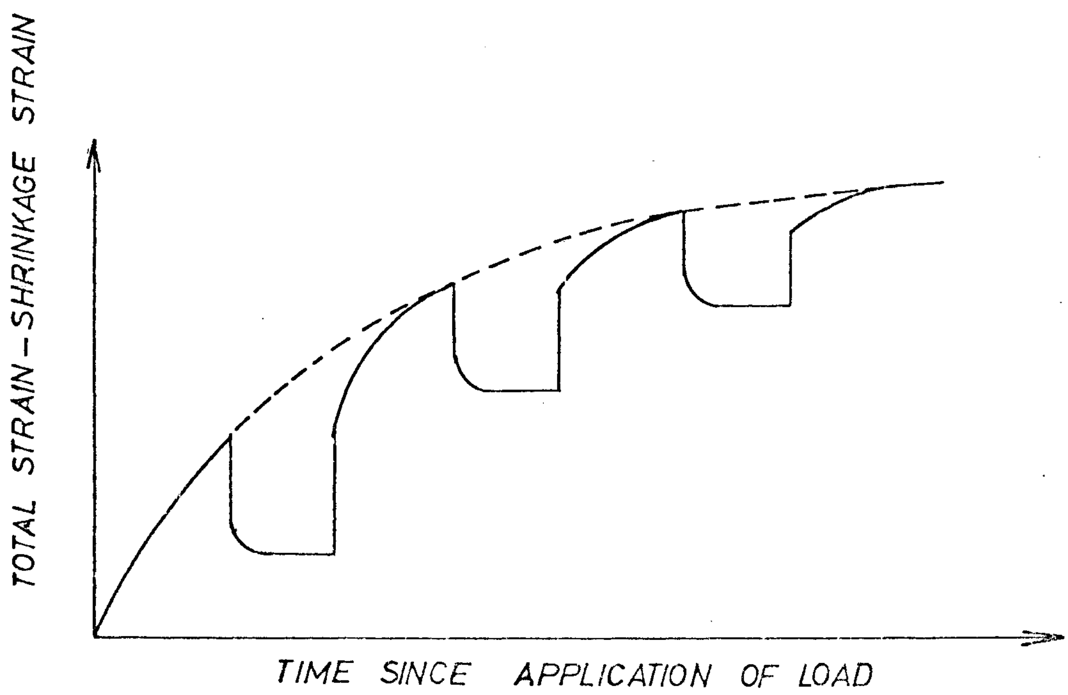


FIGURE 5.2 OBSERVATION OF SPECIMEN PERIODICALLY UNLOADED

Meyers[34] indicated, that for specimens loaded at the same time, the creep recovery decreases with increasing time under load.

There are not available informations to describe how the age of beginning of loading influences creep recovery. The tests describe later in this chapter, where the creep recovery data of prism, J and K are shown, indicated that higher portions of creep are recoverable with increasing age of application of load.

An interesting series of tests run by Roll[20], where cylindrical specimens were stressed to 31.5 and 63 percent of 28-day compressive strength and periodically unloaded is illustrated in Figure 5.2. The creep recovery appears to be complete within a few weeks after removal of stress, and appears to be a linear function of previously sustained stress, regardless of magnitude of that stress level. Also the number of proceeding load cycles does not seem to have any effect upon the creep - recovery, or on the shape of the creep curve for continued sustained loaded.

5.2. Test Program

Some of the prisms from the constant sustained load tests served the purpose to study creep under decreasing streses.

The prisms were kept in the same controlled environment.

Prisms C, E, I, J, K, and L which had been subjected to sustained loaded where unloaded at end of loading test and strain reading of creep recovery were made using the Demec strain indicator.

Prism F was loaded at time $t_o = 0$ to a stress of 1500 psi which is equal to $.45fc'_{31}$. The load was maintained until time $t_1 = 56$ days and then decreased to 1125 psi. The next decrease of load to 750 psi was made at time $t_3 = 104$ days. Prism F served for the study of creep under decreasing stresses. Table 5.1. and Figure 5.3. show data for creep and creep recovery of above mentioned prisms. Creep recovery may be defined as the ratio $= \frac{\text{creep recovery}}{\text{creep up to time of unloading}}$

for prism E $= \frac{124}{1672} = .074$

prism C $= \frac{73}{803} = .091$

prism I, J $= \frac{38}{253} = 0.15$

prism K, L $= \frac{52}{169} = 0.34$

The prisms subjected to decreasing load increments or to complete removal of load indicated the following behaviour:

i.) Upon complete removal of load from Prisms G and D there were no observed change of strains after 33 days of unloading. Observations indicated a permanent set or strain for this environment. Hence only part of the creep strain is recovered.

ii.) Meyers(34) indicated that for specimens loaded at the same age, the creep recovery decreases with increased time under load due to increased stability of the gel and water at the time of load release.

iii.) If a specimen is loaded soon after pouring, creep recovery will be small compared with the creep strain. This is due to fact that physical changes are taking place rapidly; the gel structure is less stable than that of a specimen loaded when hydration is closer to completion. Thus more microcracking and collapse of the structure will take place. On the other hand, if a mature specimen is loaded when the gel and water content are relatively stable, creep will be a greater percentage of the creep strain. As can be shown by comparison of prisms I and J with K and L in Figure 5.3.

iv.) If a specimens is loaded to higher stresses the creep recovery ratio is smaller, as indicated prisms C and E.

Table 5.1. Creep and Creep Recovery Data

Time since first loading (days)	Creep and Creep Recovery of Prisms						
	C	X	E	F	G	I,J	K,L
1	28	44	110	110			
2	59	92	229	229			
7	153	240	434	434			
14	278	436	642	642			
22	350	549	786	786			
28	403	633	895	895			
36	457	717	1033	1037			
44	543	852	1190	1190			
50	545	856	1220	1220			
56	554	870	1254	1254			
58	567	890	1286	1229		18	
62	590	926	1324	1256		100	
69	609	956	1370	1274		124	
77	635	997	1418	1313		154	
83	661	1038	1454	1341		178	
90	692	1086	1501	1373		206	
97	702	1102	1551	1396		218	
104	711	1116	1559	1378		227	
106	720	1130		1340	1290		45
112	720	1130	1603	1347	1278	232	71
119	753	1182	1622	1364	1267	239	104
132	780	1224	1656	1360			133
139	803	1261	1672	1381	1265	253	169
Unload (Creep Recovery)							
153	781		1605			228	131
167	730		1556			217	115
181	730		1548		1266	215	117

Creep Strain [in/in x 10^{6}]

CREEP RECOVERY

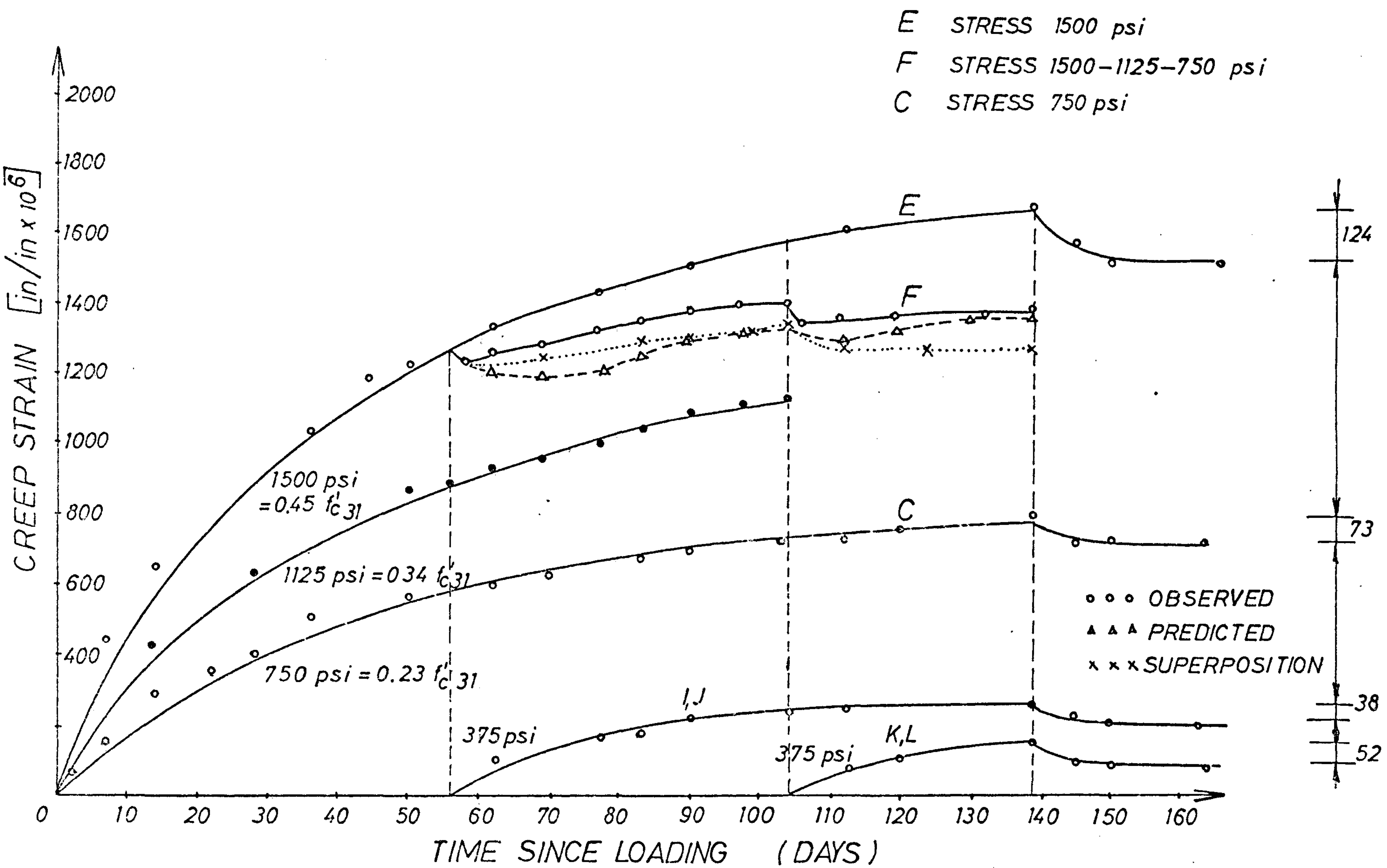


FIGURE 5.3. OBSERVATION OF CREEP RECOVERY

5.3. Prediction of Creep Under Decreasing Increments of Stress

It was thought by author that much more searching tests should be done to better understand creep recovery and hence the prediction of creep under decreasing load with time. One very simple approximate method is described which takes into consideration all four of the remarks noted in the previous section.

This method assumes that for creep due to a load applied for time t_o to t_1, just the creep which would occur in the last third of the time period will be recovered. Figure 5.4. illustrates this method. The total creep at time t_2 = Creep 1 + Creep 2 - Creep 3 where stress 1 decreased to stress 2 at time t_1. Creep 1 is due to Stress 1 applied during time t_o to t_1. Creep 2 is due to Stress 2 applied during time t_1 to t_2 for initial loading at t_o. Creep 3 is due to the complete unloading of Stress 1 at time t_1. For each additional increment of negative load, the prediction method continues in the same manner. The author is aware that the shape of the creep recovery curve will not be similar to the observed curve and this could mean that prediction of creep recovery in short time intervals of two to three weeks may be in error.

In the following example this method underestimates creep recovery. The theoretical accuracy of this method could be improved by making the creep 3 component much steeper at early ages. Many more tests are required to find a better relationship between creep and creep recovery. Table 5.2. and Figure 5.3. show observed and predicted values of creep for Prism F.

CREEP IN ANY TIME AFTER T1 = CREEP 1 + CREEP 2 - CREEP 3

DUE TO CHANGE IN STRESS FROM STRESS 1 TO STRESS 2

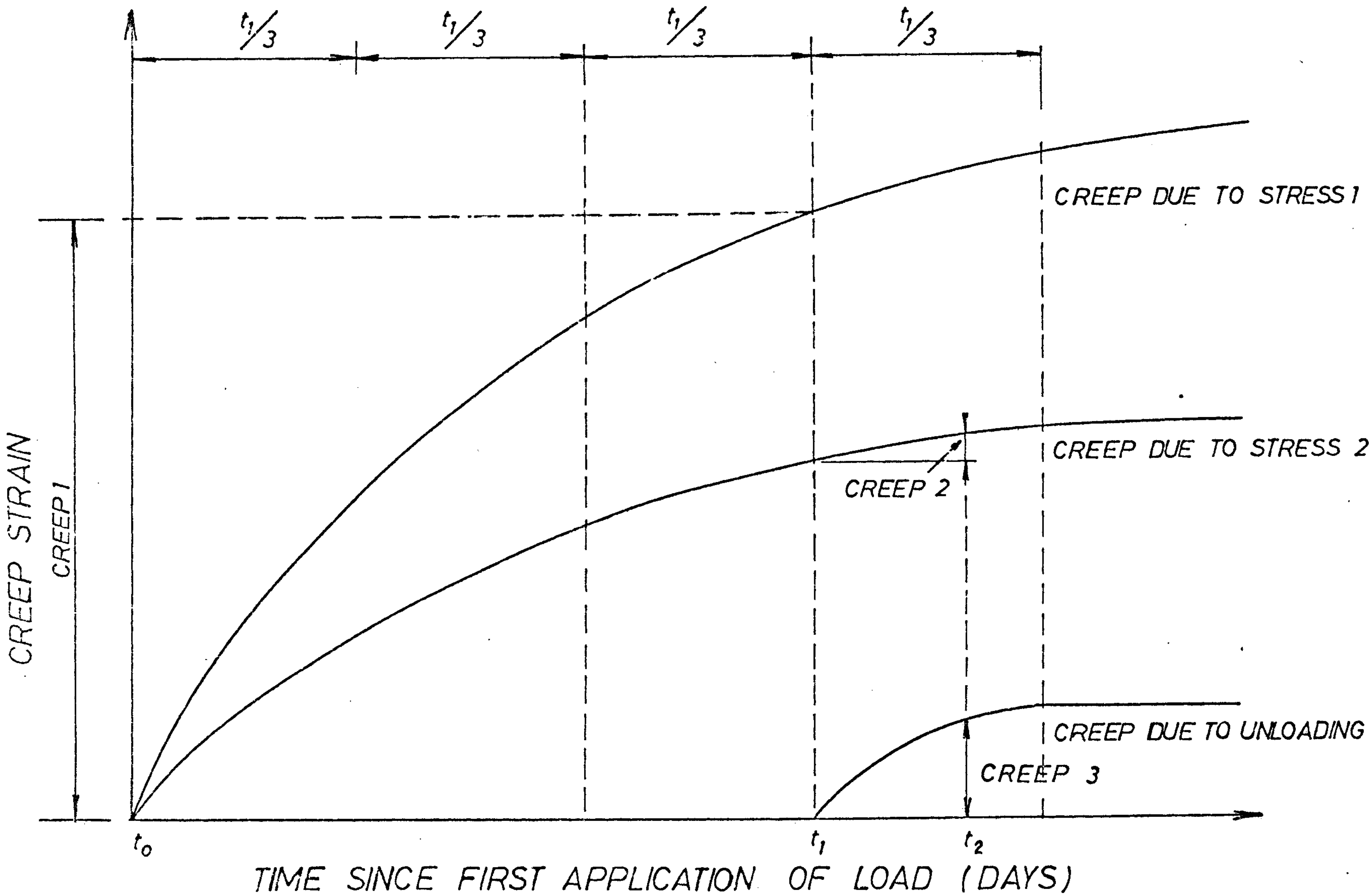


FIGURE 5.4 PREDICTION OF CREEP UNDER DECREASING LOAD INCREMENT

Table 5.2.

Observation and Prediction of Creep of Prism F

Time since loading	Observed creep strain	Creep 2 [+]	Creep 3 [-]	Predicted Creep Strain	
				Proposed method	Superposition method
2	229				
7	434				
14	642				
22	786				
28	845				
36	1037				
44	1190				
50	1220				
56	1254				
58	1229	20			
62	1256	56	120	1190	1224
69	1274	86	150	1190	1244
77	1313	127	184	1197	1264
83	1341	168	184	1238	1276
90	1373	216	184	1286	1295
97	1396	232	184	1302	1333
104	1378	246	184	1316	1332
106	1340	9			
112	1347	9	34	1297	1300
119	1364	42	34	1324	1279
132	1360	69	34	1351	
139	1381	92	34	1374	1250

For the change in stress from 1500 psi to 1125 psi at time t_1 = 56 days it was necessary to compute creep strain for a specimen loaded to a stress of 1125 psi at time t_o = 0. Equation 3.3. reused.

$$0.34 = .25 \times .46 \times \text{logarithm } (Bx/Bc) \qquad \text{Equation 5.2.}$$

Where Bc is the rate of creep of specimen with Prism C load history as observed experimentally, and
Bx is the rate of creep of a specimen loaded to $1125/fc'_{31}$.
From Equation 5.2. Bx/Bc = 1.57 and the computation of creep for specimen x is listed in Table 5.1. and the values for Creep 2 in Table 5.2.
For the next decrement of load, to a stress of 750 psi, a corresponding creep relationship for a 750 psi stress applied at time t_o = 0 is required. Creep 3 is the recoverable part of the creep or in other words is equal to the creep strain occuring between the time 69 days to 104 days. For comparison purposes the standard superposition method issued, where creep strains of Prism E were decreased by the amount of creep strain for Prism J in the time period 58 to 104 days and by the amount of creep strain for Prisms J and K after 104 days. These results were listed in Table 5.3.

The proposed prediction method shows slightly better results then the simple superposition method. To the author it seemed that although the proposed method is not fully developed, it is more theoretically sound than the simple superposition method and is possible to use this with very little effort and less data.

Chapter 6

Discussion of Errors in the Experimental Procedure and in the Numerical Analysis.

6.1. Introduction

In the preceding chapters frequent reference was made to sources of error and their possible influence on the interpretation of the theoretical and experimental results.

In this chapter an attempt is made to indicate the possible relative influence of errors due to particular considerations. No mathematical examination of probable error is attempted.

6.2. Discussion of Errors in Test Results Arising from Inaccuracies in Experimental Set-up.

There were several factors which, by nature of the experimental procedure, could influence the results obtained. It was not found to be feasible to make any numerical estimate of the discrepancies which these factors caused, since, unlike the errors involved in the computation process their effects were difficults to evaluate realistically.

Since the results of the various tests indicate consistent trends, and since the predicted creep values differ from those observed by a margin no greater than that which might reasonably occur due to computation error, it is thought that these factors which are detailed below probably had little effect on the test results. It was thought that in the later pour, as a result of the gain of experience the experimental measurements were carried out with nearly as much accuracy as the experimental equipment permitted.

6.2.1. Errors in Fabrication of Creep Prisms

The creep prisms were fabricated and prepared for testing with a great amount of care.

Each step was rechecked in an attempt to insure a high degree of accuracy. The first prisms were cast in wooden forms and later in steel forms in order to eliminate as much initial crookedness as possible. The dimensions of the prisms cross-section which were controlled by the forms were within 0.04 inches of the required size. The measured thickness to the trowelled surface were occasionally in error by as much as 0.10 inch, corresponding to 1.7% error in prism dimension.

However, since most preparations were performed by hand and checked with the naked eye, a certain amount of error has to be anticipated. In addition, some errors may have undetected or may have resulted from movement after checking.

Nonetheless small dimensional variations will have relatively minor effect.

6.2.2. Errors Due to the Positioning of Load

The spherical loading seats 6 inch by 6 inch were positioned on the prisms of cross section 6 inch by 6 inch. It was thought that the effect of any small unintentional eccentricity would be eliminated by the practise of averaging the strain readings. Considering the method of measurements, it is believed that a spherical seat was usually located within 0.02 inches, with maximum misalignment of 0.04 inches.

6.2.3. Errors Resulting from the Load Control System

The load cells used to control the load on prisms during the sustained load tests were calibrated in the "Tinius Olsen" load testing machine. This was not the same machine as for short term cylindical tests. The accuracy of the load sensing device was guaranteed to be within 0.5 percent. The small difference between strain reading on prisms and strain reading on cylinders, could be partly because different load testing machines were used. The extra load cell calibration and use stage could increase the error. In addition there occurs differences in strain readings between cylinders in supposedly identical tests. Therefore some apparent errors maybe due to the natural variability of concrete. A load cell was discarded during calibration if the calibration curve was not repeated within reading corresponding to 100 pounds for several repetition of loading.

The calibration curves and the zero load readings for a few of the load cells varied during the sustained load period. This resulted in the variation of the sustained load which were indicated in the presentation of the test results. Re-calibration of the load cells at the end of the sustained load period provided the final value for the load on the prisms. Usually, for lack of a better estimate, the load was assumed to have changed linearly over the period of sustained loading. Since wild fluctuations of the load were improbable it was thought that the above assumption is reasonable.

Fortunately most of the load cells behaved very well under sustained load and because of creep strain results represented the means of two prisms in many cases it was easy to identify and exclude error. The maximum error of 3.1% occurs on load cell of Prism A.

The loads for the sustained load tests were checked frequently and adjusted near the beginning of loading at least twice a week and more often and later at least once a week if the load dropped off by more than 500 pounds. After a load dropped off, it was increased to a value of 500 pounds higher than required. Therefore the average load was the specified load. No allowance for this fluctuation in load was made in the analysis.

There arose one additional error due to breakdown of the strain indicator box at around age 120 days after pouring in pour one. All specimens were unloaded and the load cells were recalibrated. It was felt that smaller errors would be obtained knowing exactly the characteristics of all load cells in the new strain indicator box, than those which occur by neglecting the effect of unloading for a few hours and reloading of specimen.

Although for short term experiments it is known that loading cycles increases strain. Rolls[20] experiment for sustained loading indicated that additional strain due to reloading can be negligable.

6.2.4. Errors Arising Due to Experimental Measurement of Strain

The experimentally measured strain have been found using the "Demec" Strain indicator. This device is able to read with an accuracy of .00001 inch per inch for one division. There could be a slightly increased accuracy by reading to one half of one division on the scale.

The measured strains were obtained as differences between reading taken from a standard invar bar, and a second reading taken from a gauge points fixed to the concrete. The difference between the two reading was recorded. The change between this value and one recorded of a later date indicated the magnitude of additional strain.

If one reading is say R1 and second reading say R2 and human factor is H. By taking reading to be accurate to one division on the scale 0.5 x H, then it is apparent that

$$R1 - R2 = R1 \pm 0.5H - R2 \pm 0.5H$$

Hence the maximum possible error in the quantity (R1 - R2) is $\pm$ H or one division.

It was assumed that with more readings this factor will perish. The practise of averaging a twelve sets of points and discarding of poor points decrease the probability of error.

6.3.
Errors in the Prediction of Creep Behaviour Due to the Method of Analysis.

The purpose of this section is to obtain an estimate of the probable accuracy of the values of creep strain obtained by the computation methods.

Two method have been used for prediction of creep strain under increasing load and one method for decreasing load.

6.3.1.
Errors Arising from the Scatter of Points About "Least - Squares Fit" Plots of Creep Data According to the Semi - Log. Methods.

Errors will arise from the use of Equation 1.7.

Creep = A + B x logarithm of Time

where both A and B were expressed in modified superposition method 1 as functions of initial "elastic" strain.

$A = f_1$ (EL.str) $\qquad$ $B = f_2$ (EL.str)

The erros arising from the use of this expression are derived from two sources.

(a) The original semi - log plots, obtained from creep data, give rise to an errors due to deviation of the points from the best straight line drawn through them.

It was already mentioned in Chapter 3, that the straight line expression does not fit for data in the first 7 days following application of load. Hence not usable for times less than 7 days.

(b) Errors arrising from the fact that A and B are both expressed as non linear function of elastic strain. Again there is some scatter of the values of A and B about the line given by a least squares fit. Only four data points were available for defining each of these polynomials.

6.3.2.

The Errors Arising from the Use of Semi - Log Methods in Modified Superposition Method 2.

From Expression 1.7, Creep = A + B x logarithm of Time, only constant B is used for computation. Constant B is further expressed as a function of stress level.

$B = f_3$ (stress level)

The errors arising from the use of this expression is again derived from two sources.

(a) errors similar to Section 6.3.1.a.

(b) errors arising from the fact that B is expressed as a function of level of stress. Again there is some scatter of the values of B about the line given by a least squares fit.

6.3.3.

Errors Arising Due to the Scatter of Points About the Least Squares Fit Curve Representing the Concrete Stress-Strain Relationship.

The data for the stress - strain curves have been evaluated from plots drawn by the testing machine and in some cases checked with the "Demec" strain indicator.

By comparison the values of strain readins of cylindrical specimens and prisms there was found to be a maximum difference in some cases as high as 4 percent.

6.3.4.

There Arise Some Error for the Prediction of Creep Strain of Specimen A - 2.

For prediction of creep strain with both used method is necessary to have data of at least three specimens sustained loaded at time of first application of load to different stress levels. This data are missing for prediction of prism A - 2. The above mentioned data were evaluated assuming similar behaviour as in the first pour.

Level of stress against rate of creep ratio was used with the same slope of straight line as in first pour. The slopes of straight lines in Figure 3.7. from specimens made of almost identical mix proportion by four investigators varied less than 10 percent.

Hence estimated errors due to second set of test run by author is far below 10 percent.

Chapter 7 Conclusion

7.1. Introduction

Many aspects of concrete creep have been carefully observed and described during the past 70 years. The prediction of creep behaviour under time dependent change in stress is solved in practise sometimes using very drastic methods.

An investigation was conducted into the effects of creep of concrete under time dependent change in stress. The purpose of this investigation was to device and test methods of predicting the time dependent stresses and strains by using data from specimens subjected constant sustained load.

The investigation included both experimental and theoretical work.

7.2. Experimental Investigation

In the experimental program, tests were made for prediction of creep strain under medium and high stresses. Seven prisms in first pour were subjected to constant sustained load and four prisms were loaded with varying sustained loads which were maintained at constant values over specified time intervals. Similarly four prisms were loaded under constant sustained load in second pour and four prisms were time dependent loaded. In first pour one prism and in the second pour two prisms served for shrinkage strain reading.

A record was kept of the creep strains over the whole period of that program. An additional record was kept of shrinkage strain. The concrete stress - strain - time relationship and concrete strength - time relationship was found.

7.3. Theoretical Investigation

It was intended that the strains of prisms loaded with time varying load will be compared with strains predicted by theoretical approach using constant sustained load results. Such a procedure would provide the basis for a method which could provide the same information for reinforced concrete or prestressed concrete members.

For prediction of creep strain under varying stresses with time it was found that creep and creep recovery are not identical phenomena and prediction of creep under increasing increment of load has to be made in a different manner from the meditation with decreasing increments of load. Two methods were presented for prediction of creep under increasing increment of load. Both method were modificantions of the superposition method.

The modified superposition method 1 considered creep strain through using creep constants to be proportional to applied load expressed in the form of "elastic" strain. This proportionality does not change with time of application of load because creep strain tends to change with time in the same manner as "elastic" strain. This relationship has been experimentally proved.

By applying additional "elastic" strain at later times, the total creep strain is found from creep strain corresponding to "elastic" strain due to old load, new load and the difference of "elastic" strains.

Prediction of creep by modified superposition method 1 overestimated creep when specimens are loaded to medium stresses and shows excellent prediction for high stresses.

The modified superposition method 2 considers, for time of application of load stress directly proportional to creep strain on the semi - logarithmic scale. This proportionality was not found to be true for lower stresses. This is a disadvantage of this method.

Change of creep strain with time of application of load does not change in the same manner as change of strength with time. If there occurs a situation of decreasing strength versus time, modified superposition method 2 would fail. Prediction of creep by modified superposition method 2 slightly underestimated creep in both cases.

It was thought by author that modified superposition method 1 is better theoretically sound but need more observed data than modified superposition method 2. Modified superposition method 2 need only two prisms to be sustained loaded for prediction of creep under increases stresses versus time.

One very simple method have been proposed for prediction of creep strain under declining stresses. The method assumes that creep which occurs in last third of the period of time of application of load is recoverable.

This method, although underestimating creep, showed better prediction than the standard superposition method.

As an addition a good accuracy were obtained in all prediction.

7.4. Final Conclussion and Sugestion for Further Research.

A comparison of creep prediction for low and medium stresses has not demonstrated any superior prediction of the modified superposition methods compared to the standard superposition method. For higher stresses the predictions using the modified superposition methods gave superior results.

The proposed method for prediction of creep under decreasing load increment gave better results than superposition method and could be used with very little effort.

The author recomends a few more complete tests to be run to further improve the methods described in this research.

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APPENDICES

Appendix 1

INPUT DATA FOR STRESS-STRAIN RELATIONSHIP

	POUR I						POUR II	
	28 days after pouring		76 days after pouring		135 days after pouring		28 days after pouring	
	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN
1.	0	0	0	0	0	0	0	0
2.	0.891	2.93	0.990	3.50	0.336	0.75	.371	0.91
3.	1.425	5.33	1.309	4.50	0.672	1.750	1.121	3.01
4.	1.783	7.17	1.592	5.60	1.418	4.20	1.406	3.91
5.	2.316	10.66	2.052	7.90	1.770	5.601	1.837	5.41
6.	2.674	13.59	2.494	11.35	25.47	10.10	2.143	6.61
7.	2.896	15.99	3.148	17.70	2.830	13.70	2.546	8.41
8.	3.189	21.32	3.732	21.66	3.000	14.88	2.778	9.61
9.	3.289	25.85	3.837	26.34	3.833	20.00	3.034	11.11
10.	3.250	26.65	3.732	30.00	4.103	26.66		

Stress (K.S.I.)

Strain (in/in x 10^4)

Appendix 2

SHRINKAGE STRAINS OF UNLOADED PRISMS

STRAIN (in/in x 10^6)

	POUR I			POUR II		
Age since pouring (days)	Increment of shrinkage strain	Total shrinkage strain	Storage humidity %	Age since pouring (days)	Increment of shrinkage strain	Total shrinkage strain
33	19	19	58	15	7	7
38	10	29	57	19	34	41
45	11	40	38	22	20	61
53	25	65	50	28	35	96
59	43	108	59	35	33	129
67	-30	78	59	43	17	146
75	37	115	58	52	15	161
81	21	136	50	62	14	175
87	15	151	53	72	22	197
93	23	174	51	84	17	214
100	4	178	50	92	8	222
108	4	182	54	128	8	230
114	3	185	54	134	1	231
121	-14	171	62			
128	22	193	52			
135	-14	177	62			
143	10	187	54			
150	2	189	52			

Appendix 3

CREEP DATA OF PRISMS C, E AND A AND COMPUTATION OF TIME/CREEP

	Time after loading	Average creep of prism C 750 psi	t/c	Average creep of prism E 1500 psi	t/c	Average creep of prism A 2250 psi	t/c
1.	1.0	28.2	.0354	109.7	0.0091	418.5	0.0023
2.	2.0	59.5	.0336	228.8	0.0087	779.5	0.0025
3.	7.0	152.8	0.458	433.8	0.0161	1533.0	0.0045
4.	14.0	279.7	.0500	641.7	0.0218	2134.7	0.0065
5.	22.0	350.9	.0626	786.4	0.0279	2803.3	0.0078
6.	28.0	403.0	.0694	895.4	0.0312	3142.3	0.0089
7.	36.0	457.1	.0787	1033.4	0.0348	3332.8	0.0108
8.	44.0	543.5	.0809	1190.5	0.0369	3950.3	0.0111
9.	50.0	544.8	.0917	1220.4	0.0409	4078.4	0.0122
10.	56.0	554.1	.1010	1253.6	0.0446	4239.4	0.0150
11.	58.0	567.1	.1022	1286.9	0.0450	4569.6	0.0158
12.	62.0	590.4	.1050	1324.4	0.0468	5038.3	0.0168
13.	69.0	608.8	.1133	1370.7	0.0503	5426.9	0.0182
14.	77.0	635.3	.1212	1418.0	0.0543		
15.	83.0	661.1	.1255	1454.6	0.0570		
16.	90.0	691.7	.1301	1501.9	0.0599		
17.	97.0	702.2	.1381	1559.0	0.0622		
18.	104	711.1	.1480	1552.8	0.0669		
19.	106	720.0	.1480				
20.	112	720.3	.1550	1603.0	0.0698		

Time (days)
Creep (in/in x 10^6)

Appendix 4

Creep Strain of Specimens Loaded to 375psi at Different Ages

Age since pouring	Age since loading 31 days	Creep strain of prism C/2	Age since loading at 87 days	Creep strain of prism I and J	Age since loading at 135 days	Creep of prism K and L
33	2	29.7				
38	7	76.4				
45	14	139.8				
53	22	175.4				
59	28	201.5				
67	36	228.5				
75	44	271.7				
81	50	272.4				
87	56	277.0				
89	58	283.5	2	18.0		
93	62	295.2	6	100.5		
100	69	304.4	13	126.4		
107	76	317.6	20	154.7		
114	83	330.5	27	178.8		
121	90	345.6	34	206.8		
128	97	351.1	41	218.4		
135	104	355.5	48	227.8		
143	110	360.0	56	232.3	8	71.0
150	117	376.7	63	239.3	15	104.2
163	132	390.2	76	237.3	28	133.3
170	139	401.6	83	253.3	35	169.2
177					44	189.0
182					49	197.0
189					56	198.0

Creep Strain (in/in x 10^6)

Appendix 5

For establishing creep constant equations as functions of "elastic" strain for the second pour data was recorder only for a sustained stress of 750 psi.

The creep constants due to stresses of 1500 psi and 2250 psi were computed assuming the slope of a straight line from the plot of level of stress against the ratio of rate of creep to rate of creep of standard specimen to be similar this second pour to the first pour. This straight line relationship is shown in Figure 3.7. and discussed in Section 3.3.

Equation 3.3. has been re-used in form:

level of stress = .2070 + .4614 x logarithm (B/B_{D-2})

For a stress of 1500 psi which equals 0.414 fc'_{14} it follows that

0.414 = .207 + .4614 x logarithm (B_{x2}/B_{D-2})

the solution for B_{x2}/B_{D-2} = 2.81

For a stress of 2250 psi = 0.621 fc'_{14} then

0.621 = .207 + .4614 x logarithm (B_{y2}/B_{D-2})

and the solution for B_{y2}/B_{D-2} = 7.892

The Table Appendix 5a shows the observed values for creep of prism D-2 under a sustained stress of 750 psi from the age of application of load at 14 days after pouring and the computed values of creep for prisms subjected to sustained stresses of 1500 psi and 2250 psi.

From this data shown in Appendix 5a creep versus time curves were derived using the least squares computer subroutine to establish the relationships for constants A and B.

Appendix 5a

Time since application of load	Observed creep of prism D-2	Computed creep of curve X-2 2.81 x (D-2)	Computed creep of curve Y-2 7.892 x (D-2)
1	67	188	528
5	154	432	1215
8	165	463	1302
14	233	655	1839
22	276	775	2178
29	289	812	2280
48	337	947	2660
58	379	1065	2991
70	403	1132	3180
78	423	1189	3338
114	450	1264	3351

Creep strain (in/in x 10^6)

Time (days)

Appendix 6

Computation of Creep 2'

From Equation 4.6.

$\text{Creep}_{\varepsilon_{CEL.2}} = -450.26 + 746.85 \times$ logarithm of Time

where $\varepsilon_{CEL.2} = 383.2$ in/in x 10^6

Time T_2		Creep 2'
56	Creep = -450.26 + 746.85 x 1.74819 = 855.23	0
69	Creep = -450.26 + 746.85 x 1.83885 = 922.45	67.2
83	Creep = -450.26 + 746.85 x 1.91908 = 982.96	127.7
104	Creep = -450.26 + 746.85 x 2.01703 =1056.13	200.9

Computation of Creep 2"

From Equation 4.7.

$\text{Creep}_{\varepsilon_{CEL.2} - \varepsilon_{CEL.1}} = -153.45 + 254.54 \times$ logarithm of Time

where $\varepsilon_{CEL.2} - \varepsilon_{CEL.1} = 130.6$ in/in x 10^6

Time (days)

Time (T_2)	Time (T_2-T_1)		Creep 2"
56	0	Creep = -153.45 + 254.54 x log. 0	= 0
69	13	Creep = -153.45 + 254.54 x 1. 11394	= 130.17
83	27	Creep = -153.45 + 254.54 x 1. 43136	= 210.91
104	48	Creep = -153.45 + 254.54 x 1. 68124	= 274.47

creep strain (in/in x 10^6)

Appendix 6a

Computation of Creep 3'

$\text{Creep}_{\varepsilon_{CEL.3}} = -567.99 + 942.14 \times \text{logarithm of Time}$

where $\varepsilon_{CEL.3} = 483.4 \text{ in/in} \times 10^6$

Time		Creep 3'
104	Creep = −567.99 + 942.14 x 2.01703 = 1332.3	0
112	Creep = −567.99 + 942.14 x 2.04922 = 1362.7	30.4
132	Creep = −567.99 + 942.14 x 2.12057 = 1429.9	97.6
139	Creep = −567.99 + 942.14 x 2.14301 = 1451.0	118.7

Computation of Creep 3"

$\text{Creep}_{\varepsilon_{CEL.3}-\varepsilon_{CEL.2}} = -117.7 + 195.3 \times \text{logarithm of Time}$

where $\varepsilon_{CEL.3}-\varepsilon_{CEL.2} = 100.2 \text{ in/in} \times 10^6$

Time $T_4 - T_3$		Creep 3"
0	Creep = −117.7 + 195.3 x 0 =	0
8	Creep = −117.7 + 195.3 x 0.90309 =	58.7
28	Creep = −117.7 + 195.3 x 1.44716 =	164.9
39	Creep = −117.7 + 195.3 x 1.59106 =	192.4

Appendix 7

$$A_{\varepsilon_{CEL}=411} = -.21361868 \times 411 - 3.2061 \times 10^{-4} \times 16.8921 \times 10^{4} - 1 \times 10^{-8} \times 0.6942 \times 10^{8} = -87.7972 - 54.1578 - 0.6942$$

$$= \underline{142.6492}$$

$$B_{\varepsilon_{EL.S}=411} = 1.1506 \times 411 - 5.3957 \times 10^{-4} \times 16.8921 \times 10^{4} + 2.73 \times 10^{-6} \times 69 \times 10^{6} = 472.8966 - 91.1447 + 188.3700$$

$$= \underline{570.1219}$$

$$A_{\varepsilon_{EL.S}=197} = -.2136 \times 197 - 3.2061 \times 10^{-4} \times 3.8809 \times 10^{4} - 1 \times 10^{-8} \times 0.076 \times 10^{8} = -42.07 - 12.44 - 0.07$$

$$= \underline{-54.58}$$

$$B_{\varepsilon_{EL.S}=197} = 1.1506 \times 197 - 5.3957 \times 10^{-4} \times 3.8809 \times 10^{4} + 2.73 \times 10^{-6} \times 7.6453 \times 10^{6} = 226.668 - 20.940 + 20.871$$

$$= \underline{226.599}$$

Elastic Strain (in/in x 10^6)

Computation of Creep 2'

Creep due to $\varepsilon_{CEL.2}$ = 411 in/in x 10^6

Time T_2	Creep = -142.65 + 570.12 x $\log_{10}$ of time	Creep 2'
22	Creep = -142.65 + 570.12 x 1.34242 = 622.69	0
29	Creep = -142.65 + 570.12 x 1.46240 = 691.09	68
48	Creep = -142.65 + 570.12 x 1.68124 = 815.85	193
58	Creep = -142.65 + 570.12 x 1.76343 = 862.72	240
70	Creep = -142.65 + 570.12 x 1.84510 = 909.27	286

Appendix 7a

Computation of Creep 2"

Creep Due to $\mathcal{E}_{CEL.2}-\mathcal{E}_{CEL.1}$ = 197 in/in x 10^6

Time T_2	T_2-T_1	Creep = 54.58 + 226.6 x $\log_{10}$ of Time	Creep2"
22	0	Creep = 0 0	0
29	7	Creep = -54.58 + 226.6 x 0.84510 = 136.91	137
48	26	Creep = -54.58 + 226.6 x 1.41497 = 266.05	266
58	36	Creep = -54.58 + 226.6 x 1.55630 = 298.07	298
70	48	Creep = -54.58 + 226.6 x 1.68124 = 326.38	326

Computation of Creep 3'

Creep due to $\mathcal{E}_{CEL.3}$ = 618 in/in x 10^6

Time T_4	Creep = -256.7 + 1149.0 x $\log_{10}$ of Time	Creep 3'
70	Creep = -256.7 + 1149.0 x 1.84510 = 1863.3	0
78	Creep = -256.7 + 1149.0 x 1.89209 = 1917.3	54
114	Creep = -256.7 + 1149.0 x 2.05690 = 2106.6	243

Computation of Creep 3"

Creep due to $\mathcal{E}_{CEL.3}-\mathcal{E}_{CEL.2}$ = 207 in/in x 10^6

Time T_4	T_4-T_3	Creep = -55.9 + 239.3 + $\log_{10}$ of Time	Creep 3"
70	0	Creep = 0	
78	8	Creep = -55.9 + 239.3 x 0.90309 = 160	
114	44	Creep = -55.9 + 239.3 x 1.64345 = 337	