FACULTY OF BUSINESS

RESEARCH AND WORKING PAPER SERIES

COST-VOLUME-PROFIT ANALYSIS AND THE VALUE OF INFORMATION: AN EVALUATION FOR THE NORMAL AND LOGNORMAL DISTRIBUTIONS

by

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Working Paper No. 273

February, 1987

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The financial support of the Accounting Research and Education Centre of the Faculty of Business, McMaster University, is gratefully acknowledged.

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ABSTRACT

In contrast to most papers dealing with cost-volume-profit analysis which focus on the decision to accept or reject a project, this paper examines the decision whether or not to seek improved information about the inputs to the decision model. The maximum benefit from this information-seeking process is measured by the expected value of perfect information. Expressions for the expected value of perfect information for the lognormal distribution, an asymmetric distribution which is generally more realistic in such decisions, are presented. The results from these expressions are compared to those obtained from the expression for the normal distribution over a range of parameters. This comparison shows that the results for the normal distribution approximate those for the lognormal distribution only over a narrow range of parameters.

Cost-volume-profit (CVP) analysis has been widely taught and used as a tool for short run business decision making. The basic CVP model, which assumes a single product, a single time period and certain knowledge of the variables to be used, is useful but has obvious limitations. Research has been done to extend the basic model in regard to these assumptions, e.g. multiple time periods [Manes, 1966], multiple products [Johnson and Simik, 1971], and uncertainty of the variables [Jaedicke and Robichek, 1964]. The work of Jaedicke and Robichek has been extended in a number of research studies. Some studies have examined alternative specifications for the statistical distributions of the inputs to the CVP model [Buzby, 1974; Hilliard and Leitch, 1975; and Liao, 1975] while other studies have examined the effects of related statistical assumptions such as the correlation of input parameters [Ferrara, Hayya and Nachman, 1972]. Other research studies have examined the stochastic CVP model when allowance is made for the possible inequality of production and demand with different assumptions about fixed and variable costs and various penalties for over- or under-production [Ismail and Louderback, 1979; Lau, 1980; and Shih, 1979].

The primary decision criterion in the various CVP models is the maximization of profit. Recognition that performance is frequently evaluated by use of return on investment for both external and internal purposes has led to research in which the decision criterion of maximization of expected return on investment has been applied to the case of stochastic demand CVP analysis [Thakkar, Finley and Liao, 1984].

All the research discussed above can be characterized as focusing on the accept/reject decision for the project, assuming the availability of acceptable data on the required inputs for the particular model. Refinements in the basic model have involved an improved decision criterion (e.g. maximize expected return on investment rather than profit) or an additional decision criterion (e.g. evaluate risk of the project via a probability statement as well as expected profit). These refinements differ in their practical applicability because of the need for information on details of the distributions of input parameters and their correlation, which may not be either available or known with any accuracy.

It has been suggested that another decision relating to the uncertainty of knowledge of the input data for CVP analysis should be considered [Richardson and Wesolowsky, 1977; Kaplan, 1982]. The particular suggestion is to use cost-benefit analysis in the decision whether to seek more information about the input variables before making an accept/reject decision on the project. The present paper extends the work in this area. In particular, a measure of the benefit from seeking additional information is presented for an asymmetric (lognormal) distribution rather than the usual symmetric (normal) distribution. In addition, the extent to which the normal distribution may be used as an approximation for the lognormal distribution is examined.

The plan of this paper is as follows. First, the basic issue of the approach to decision making in CVP analysis is examined. Then, the relevant previous research is reviewed. Next, measures of benefit for the normal and lognormal distributions to use in the cost-benefit decision described above are presented, and then compared and evaluated. Finally, some conclusions are drawn.

DECISION APPROACH IN CVP ANALYSIS

In general, the approach to the decision process in CVP analysis can be characterized as illustrated in Figure 1. That is, the information on the inputs appropriate to the particular CVP model being used is assumed to be available and the issue is whether an accept or reject decision should be made based on the specified decision criteria, normally related to return and risk.

A decision process that is more realistic in general is presented in Figure 2. This model recognizes that an initial decision whether to make the accept/reject decision with the available information or to seek additional information must be made first. The focus of the current research is on the decision whether to seek additional information. The results of previous research on the accept/reject decision are taken as given.

In order to decide whether to seek further information, the relevant benefits and costs must be estimated. For a particular situation, the cost may be estimated more or less easily. However, it is generally much more difficult to estimate the benefit which would result from seeking further information. An upper limit on the benefit, which is very useful in many situations, is provided by the expected value of perfect information (EVPI). Expressions for the EVPI for a normal distribution are well known. However, it is generally recognized that the distribution of the key variables relevant for CVP analysis are unlikely to be symmetric, let alone normal. Therefore, an expression for the EVPI of an asymmetric distribution--specifically the lognormal--should be more useful. The conditions under which the EVPI for the normal distribution is a good approximation to the EVPI for the lognormal distribution are also of interest.

REVIEW OF RELEVANT RESEARCH

The simplest CVP model is the single period, single product case where all input variables are assumed to be deterministic. The key decision results for this case are given by the following two equations.

$$Z = Q\ (P - V) - F = Q\ M - F \qquad (1)$$

$$Q_b = F\ /\ (P - V) = F\ /\ M \qquad (2)$$

where

Z = profit

Q = quantity to be produced and sold

P = unit selling price

V = unit variable cost (total)

$M = P - V$ = contribution margin per unit

F = fixed costs (total)

Q_b = breakeven quantity

and all variables apply for some specified time period. Given that values for P, V and F, and possibly Q, can be determined, a decision can be made on the basis of Z and Q_b calculated from these equations.

Because of the obvious weakness of assuming that the input variables to equations (1) and (2) are deterministic, Jaedicke and Robichek [1964] incorporated uncertainty explicitly by making the following assumptions:

(i) Q, P, V and F are normally distributed random variables.

(ii) Q, P, V and F are statistically independent.

(iii) Z is a normally distributed random variable.

The mean and standard deviation for Z can then be calculated straightforwardly from the means and standard deviations of Q, P, V and F. This allows the computation of probability statements about profit. For example, the probability of profit exceeding breakeven is given by

$$Pr(\ Z>=0\ ) = \int_{0}^{\infty} f_N(Z)\ dZ \qquad (3)$$

where $f_N(Z)$ is the probability density function of profit Z which is assumed to be normally distributed.

$$f_N(Z) = \frac{1}{\sqrt{2\pi}\ \sigma_Z} \exp\left[-\frac{1}{2}\left(\frac{Z-\mu_Z}{\sigma_Z}\right)^2\right] \qquad (4)$$

Probability statements like equation (3) provide information related to the risk of the project's which may be used with the information related to the project's return from equations (1) and (2) (calculated using expected values) in the decision to accept or reject a project.

Subsequent research has examined the limitations of assumptions (i) [Buzby, 1974; Hilliard and Leitch, 1975; and Liao, 1975] and (ii) and (iii) [Ferrara, Hayya and Nachman, 1972] and has produced interesting and useful results.

THE BENEFIT OF ADDITIONAL INFORMATION

As illustrated in Figure 2, the decision whether to seek additional information about the input variables, before making a decision about the project itself, is being examined. In many situations, it is generally possible to estimate the cost of obtaining additional information within reasonable bounds. However, estimating the benefit that would result from that information is typically much more difficut because of the need for more detailed specifications and more complex calculations than are generally feasible. A useful upper limit to the benefit to be obtained from obtaining additional information, and so to the cost to incur in any information gathering process, is the expected value of perfect information (EVPI).

It is well known that, for a normal distribution, the EVPI can be calculated from the equation

$$EVPI = \sigma_Z L_N(|R_3|) \tag{5}$$

where

$$R_3 = \frac{\mu_Z}{\sigma_Z} \tag{6}$$

and $L_N(|R_3|)$ can be obtained from standard normal loss function tables [Winkler, 1972] or calculated from the equation

$$L_N(|R_3|) = -R_3 F_N(R_3) + f_N(R_3) \tag{7}$$

where $f_N(R_3)$ is obtained using equation (4) and

$$F_N(a) = \int_a^\infty f_n(x)dx \tag{8}$$

THE LOGNORMAL DISTRIBUTION AND EVPI

The use of the normal distribution in CVP analysis is unrealistic in some, if not all, cases because it is symmetric and allows negative values of the independent variable. The lognormal distribution has been suggested for use in this situation because, among other useful properties, it can be made to have a specified lower limit and a specified degree of skewness [Hilliard and Leitch, 1975]. For F fixed at a value μ_F, the lognormal probability density function of profit Z, $f_L(Z)$, is given by the equation

$$f_L(Z) = \frac{1}{(Z + \mu_F) \sqrt{2 \pi} \sigma_*} \exp \left[-\frac{1}{2} \left(\frac{\log (Z+\mu_F) - \mu_*}{\sigma_*} \right)^2 \right] \tag{9}$$

where

$$\mu_* = \log \left[\frac{\mu_Q^2}{\sqrt{\sigma_Q^2 + \mu_Q^2}}\right] + \log \left[\frac{\mu_M^2}{\sqrt{\sigma_M^2 + \mu_M^2}}\right] \quad (10)$$

$$\sigma_*^2 = \log\left[\left(\frac{\sigma_Q}{\mu_Q}\right)^2 + 1\right] + \log \left[\left(\frac{\sigma_M}{\mu_M}\right)^2 + 1\right]$$

$$+ \log \left[\frac{\sigma_q(\rho_{QP}\sigma_P - \rho_{QV}\,\sigma_V)}{\mu_Q\mu_M} + 1\right] \quad (11)$$

Because the lognormal distribution is not symmetric, the calculation of the EVPI is more complex than for the case of a normal distribution. In particular, the computation of the EVPI is different for the cases of $\mu_Z > 0$ and < 0. For $\mu_Z > 0$, the EVPI is related to the possibility of Z actually turning out to be negative and is given by the expression

$$EVPI_1 = -\int_{\mu_F}^{o} Z\, f_L(Z)\, dZ$$

$$= -\exp\left[\frac{\sigma_*^2 + 2\mu_*}{2}\right] G_N \left[\frac{\log \mu_F - \mu_*}{\sigma_*} - \sigma_*\right]$$

$$+ \mu_F\, G_N \left[\frac{\log \mu_F - \mu_*}{\sigma_*}\right] \quad (12)$$

where

$$G_N(a) = 1 - F_N(a) \quad (13)$$

For $\mu_Z < 0$, the EVPI is related to the possibility of Z turning out to be positive and is given by the expression

$$\text{EVPI}_2 = \int_0^\infty Z\, f_L(Z)\, dZ \tag{14}$$

The definition of μ_Z leads to the following result.

$$\begin{aligned} \mu_Z &= \int_{-\mu_F}^\infty Z\, f_L(Z) dZ \\ &= \int_{-\mu_F}^0 Z\, f_L(Z) dZ + \int_0^\infty Z\, f_L(Z) dZ \\ &= -\text{EVPI}_1 + \text{EVPI}_2 \end{aligned} \tag{15}$$

Therefore,

$$\text{EVPI}_2 = \text{EVPI}_1 + \mu_Z \tag{16}$$

or the EVPI for the lognormal distribution can be written in the more general form

$$\text{EVPI}_L = \text{EVPI}_1 + \min[\mu_Z, 0] \tag{17}$$

Examination of the preceding shows that, although the computations are somewhat more involved, the EVPI for the lognormal distribution requires the same information about the distributions of Q, P, V and F as for the normal distribution.

COMPARISON OF THE EVPI FOR THE NORMAL AND LOGNORMAL DISTRIBUTIONS

The reason for calculating the EVPI is to establish an upper limit for the benefit from, and therefore the cost to be incurred in, seeking additional information before making a final decision using the CVP model. Given the familiarity of the normal distribution, it is of interest to determine whether the EVPI calculated for it is a good approximation to the EVPI calculated using the more realistic lognormal distribution. In order to make this evaluation more tractable, a stochastic demand CVP model will be used; that is, it will be assumed that only Q, the quantity to be produced and sold, is uncertain. This is not an unreasonable assumption because in most short run decisions, for which CVP analysis is an appropriate tool, the value of Q is the most uncertain variable. Research using this assumption in a different approach to CVP analysis has recently been reported in the literature [Thakkar, Finley and Liao, 1984].

The EVPI for the normal distribution is given by equation 5 which depends primarily on the variable R_3. With the assumption that Q is the only uncertain variable, it may be shown that

$$R_3 = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_Q M - F}{\sigma_Q M}$$

$$= (1 - R_1) / R_2 \qquad (18)$$

where R_1 relates the breakeven quantity to the expected quantity

$$R_1 = Q_b / \mu_Q \qquad (19)$$

and R_2 is the coefficient of variation which relates to the width of the normal distribution

$$R_2 = \sigma_Q / \mu_Q \qquad (20)$$

Equation (5) can now be rewritten in a standardized form as

$$\frac{EVPI_N}{\sigma_Z} = L_N[|(1 - R_1) / R_2 |] \qquad (21)$$

That is, the EVPI for a normal distribution, in units of the standard deviation of profit σ_Z, is a function of the two variables R_1 and R_2.

With the assumption above that Q is the only uncertain variable, equations (10) and (11) for the lognormal distribution can be simplified to the following.

$$\mu_* = \log \left[\frac{\mu_Q M}{(R_2^2 + 1)^{1/2}}\right] \qquad (22)$$

$$\sigma_*^2 = \log\,[R_2^2 + 1] \qquad (23)$$

In addition, equation (17) for the EVPI of the lognormal distribution can be expressed in a standardized form as

$$\frac{EVPI_L}{\sigma_Z} = \frac{R_1}{R_2} G_N \left[\frac{\log R_1 + 1/2 \log (R_2^2+1)}{[\log (R_2^2+1)]^{1/2}}\right] - \frac{1}{R_2} G_N \left[\frac{\log R_1 - 1/2 \log (R_2^2+1)}{[\log (R_2^2+1)]^{1/2}}\right] + \min[\,(1-R_1)/R_2,\ 0\,] \qquad (24)$$

Note that, for the lognormal distribution, R_1 still relates the breakeven and expected quantities, but the coefficient of variation R_2 is a measure of the skewness of the distribution.

Equations (21) and (24) allow the EVPI, in units of σ_Z, for the normal and lognormal distributions to be compared as a function of R_1 and R_2. A comparison in this form is particularly useful because of the meanings of the two variables for the normal and lognormal distributions. Table 1 provides values of the EVPI, in units of σ_Z, for the normal and lognormal distributions as a function of the two variables R_1 and R_2. The table also provides the difference between these two values as a percentage of the EVPI for both the normal and lognormal distributions.

Examination of Table 1 leads to several conclusions. For a given value of the coefficient of variation $R_2 = \sigma_Q / \mu_Q$, which reflects the skewness of the lognormal distribution,

(i) The two values are closest for $R_1 = 1$, i.e. for $Q_b = \mu_Q$.

(ii) The value of $EVPI_N$ gets increasingly larger (smaller) than the value of $EVPI_L$ as R_1 moves farther below (above) 1, i.e. as Q_b gets increasingly smaller (larger) than μ_Q.

For a given value of R_1, the difference between $EVPI_N$ and $EVPI_L$ varies smoothly, with the particular relationship depending primarily upon whether R_1 is less or greater than 1.

The most striking observation is that the difference between $EVPI_N$ and $EVPI_L$ increases rapidly as R_1 moves away from 1, i.e. as the difference between the breakeven and expected quantities increases, and as R_2 increases, i.e. as the skewness of the lognormal distribution increases. These general observations result from the difference in the nature of the distributions. It is interesting to see how rapidly the EVPI calculated for the normal distribution becomes significantly different from the EVPI calculated for the lognormal distribution.

CONCLUSIONS

In this paper, the decision whether to seek additional information about the input variables of the CVP decision model, a decision that is generally not explicitly dealt with in the accounting literature, has been investigated. A cost-benefit approach to this decision has been employed, with EVPI, the expected value of perfect information, used as the upper limit to the benefit which would result from seeking additional information. Expressions for the EVPI of the lognormal distribution, an asymmetric distribution which should be useful in the CVP decision, have been derived and presented. The values of the EVPI calculated for the normal and lognormal distributions have been compared. This comparison shows that the EVPI for a normal distribution approximates that of a lognormal distribution only under relatively restricted conditions.

In summary, the paper has presented an explicit approach to the decision whether to seek additional information before making an accept or reject decision using CVP analysis. It has shown that the computations for the lognormal distribution, which will better approximate reality in most situations, should be used rather than the more restricted normal distribution.

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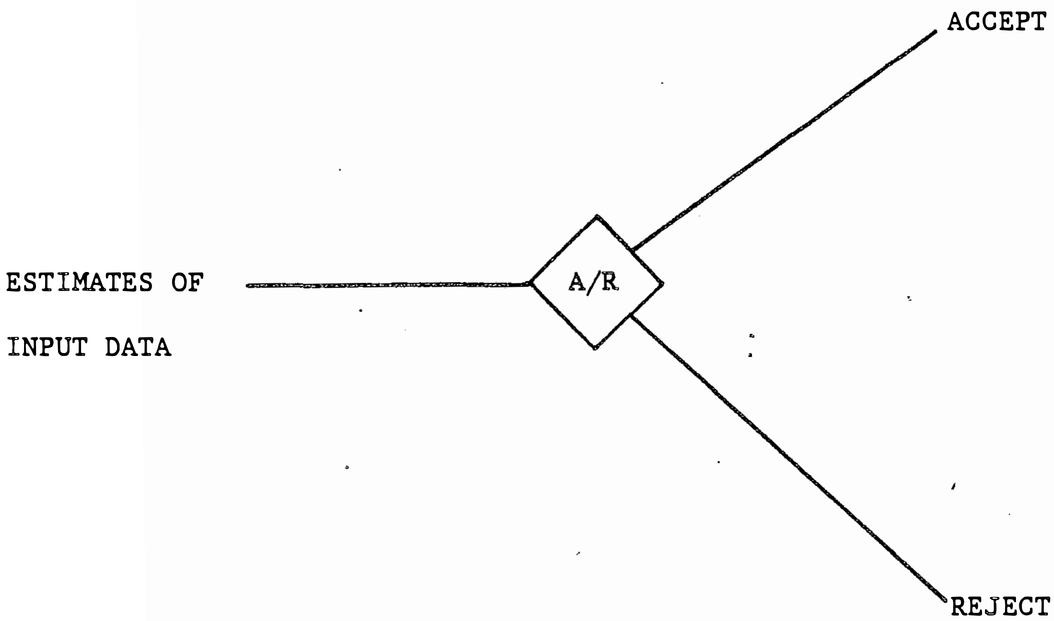
Figure 1

Representation of Decision Process in CVP Analysis

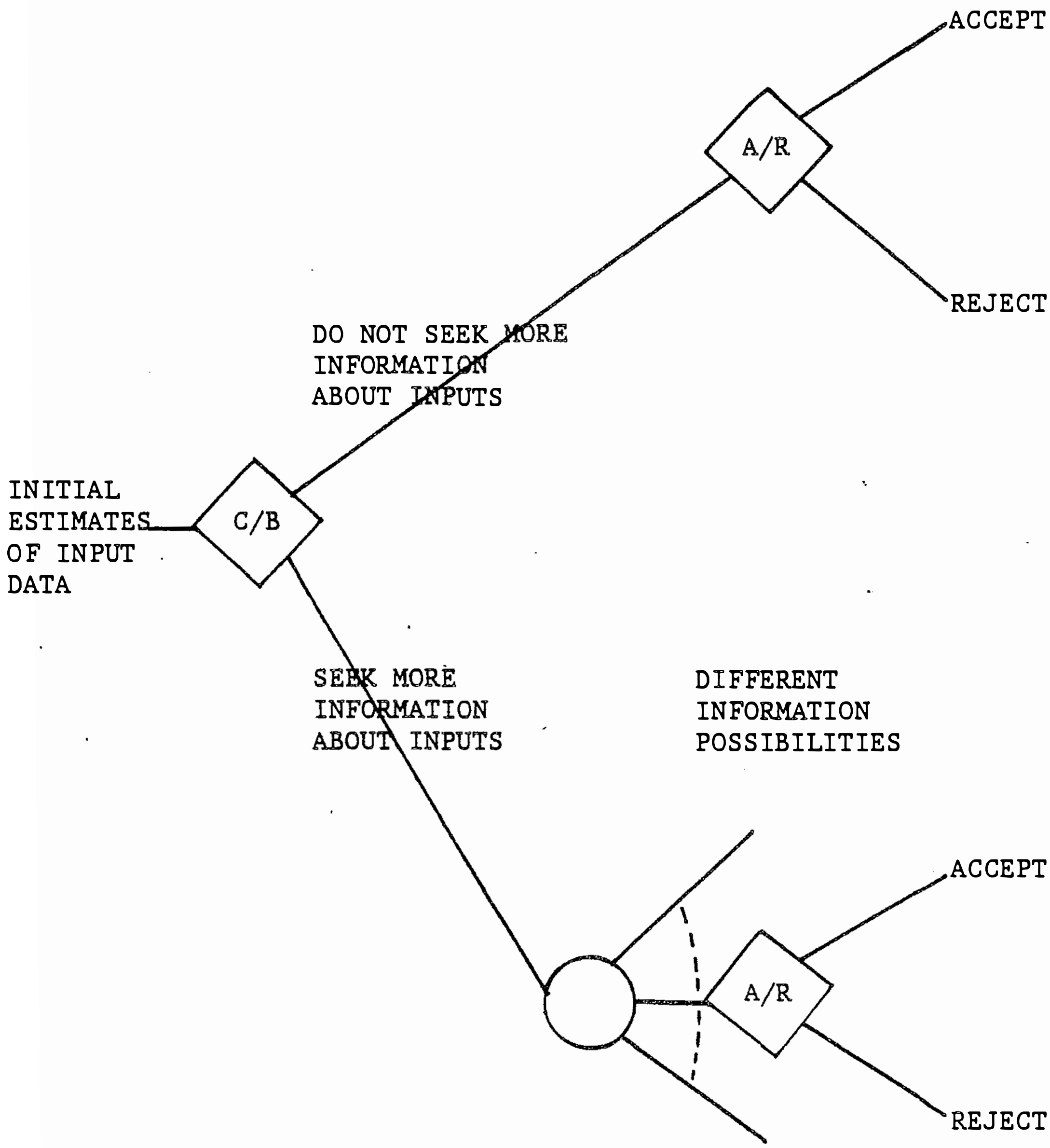
as Usually Assumed



A/R: CVP ACCEPT/REJECT DECISION

Figure 2

Improved Representation of Decision Process in CVP Analysis



C/B: COST/BENEFIT DECISION
A/R: CVP ACCEPT/REJECT DECISION

Table 1

Comparison of the EVPI Calculated for the Normal and Lognormal Distributions(1)

R1=	2	4	6	8	1
R2= .02	VARIANCE= .0004 AND LOGNORMAL SKEWNESS= 8.00016E-06				
	40	30	20	10	0
	0	0	0	7.694608E-23	.3989423
	0	0	0	0	.3988636
	9999	9999	9999	9999	-1.978536E-02
	9999	9999	9999	-99.99999	-1.978144E-02
R2= .04	VARIANCE= .0016 AND LOGNORMAL SKEWNESS= 3.20256E-03				
	20	15	10	5	0
	0	0	7.694608E-23	5.119461E-08	.3989423
	0	0	0	[illegible].791889E-09	.398756
	9999	9999	9999	-2757.019	-4.723481E-02
	9999	9999	-99.99999	-96.49985	-4.722518E-02
R2= .06	VARIANCE= .0036 AND LOGNORMAL SKEWNESS= 7.212959E-03				
	13.33333	10	6.666667	3.333333	0
	0	7.694608E-23	1.409522E-12	1.115906E-04	.3989423
	0	0	0	1.104625E-05	.3985203
	9999	9999	9999	-411.9618	-.1255919
	9999	-99.99999	-100	-81.19036	-.1257739
R2= .08	VARIANCE= .0064 AND LOGNORMAL SKEWNESS= 1.284096E-02				
	10	7.5	5	2.5	0
	7.694608E-23	2.1667E-15	5.119461E-08	2.004103E-03	.3989423
		0	7.110279E-12	6.933341E-06	.3982006
	9999	9999	-561843.3	-139.353	-.1862605
	-99.99999	-100	-99.9822	-55.40627	-.1859162
R2= .1	VARIANCE= .01 AND LOGNORMAL SKEWNESS= .0201				
	8	6	4	2	0
	5.626968E-17	1.351521E-10	7.086083E-06	3.408648E-03	.3989423
	0	0	2.125241E-08	3.918122E-03	.3977967
	9999	9999	-33226.81	-126.7071	-.2905137
	-100	-100	-99.69994	-53.35477	-.2396772
R2= .3	VARIANCE= 9.000001E-02 AND LOGNORMAL SKEWNESS= .1881				
	2.666667	2	1.333333	.6666666	0
	1.181521E-03	5.408648E-03	4.239504E-02	.1511197	[illegible]
	1.567334E-09	1.515458E-06	1.461266E-02	.1121027	[illegible]
	-7.635862E+07	-5502.827	-240.2148	-34.30467	[illegible]
	-99.99988	-98.21518	-70.60681	-25.31859	[illegible]
R2= .5	VARIANCE= .25 AND LOGNORMAL SKEWNESS= .5625				
	1.6	1.2	.8	.4	0
	2.326197E-02	5.610239E-02	.1202073	.2304389	.3989423
	3.470775E-05	5.825721E-03	.0511553	.1720375	.3736301
	-56866.78	-883.012	-134.9851	-33.94688	-6.83186
	-99.85066	-89.61591	-57.44611	-25.36356	-6.396969
R2= .7	VARIANCE= .49 AND LOGNORMAL SKEWNESS= 1.2201				
	1.142857	.8571429	.5714286	.2857143	0
	6.300212E-02	.108567	.1766456	.2722588	.3989423
	5.562794E-06	.0180258	3.028648E-02	.1946897	.3540019
	-3355.811	-502.2565	-120.0188	-39.86235	-12.59433
	-98.94265	-83.39661	-56.54936	-28.4909	-11.26619
R2= .9	VARIANCE= .81 AND LOGNORMAL SKEWNESS= 2.2761				
	.8888889	.6666667	[illegible]	.2222222	0
	.1024928	.1511197	.2156858	.2976613	.3989423
	2.403097E-03	2.951964E-02	.0967203	.2005827	.3331836
	-4165.029	-411.9296	-122.7927	-48.3883	-19.73668
	-97.65535	-80.46605	-55.11523	-32.50924	-16.48326
R2= 1.1	VARIANCE= 1.21 AND LOGNORMAL SKEWNESS= 3.8841				
	.7272727	.5454545	.3636364	.1818182	0
	.1363958	.186133	.2432136	.3146092	.3989423
	[illegible]04115E-02	3.798263E-02	.1047498	.1986112	.3125912
	-2799.5	-386.3646	-132.1853	-58.40655	-27.9202
	-96.55112	-79.4266	-56.93096	-36.8705	-21.9425
R2= 1.3	VARIANCE= 1.69 AND LOGNORMAL SKEWNESS= 6.236099				
	.6153846	.4615385	.3076923	.1538462	0
	.1644928	.2099254	.2638334	.3267311	[illegible]
	5.358548E-03	4.332936E-02	.1077723	.1929514	[illegible]
	-2261.896	-386.4877	-164.8066	-69.33339	-35.9942
	-95.75969	-79.35966	-59.1514	-40.9449	-26.5215
R2= 1.5	VARIANCE= 2.25 AND LOGNORMAL SKEWNESS= 9.5625				
	.5333334	.4	.2666667	.1333333	0
	.1877065	.2304389	.2797131	.3358165	.3989423
	8.881293E-03	4.560664E-02	.1078978	.1856812	[illegible]
	-2013.506	-394.4336	-159.2362	-80.5565	[illegible]38109
	-95.26852	-79.77483	-61.42515	[illegible]	[illegible]
R2= 1.7	VARIANCE= 2.89 AND LOGNORMAL SKEWNESS= 14.1321				
	.4705882	.3529412	.2352941	.1176471	0
	.2070242	.2470645	.292288	.3428756	[illegible]
	1.340796E-02	4.338261E-02	.1063783	.1773624	[illegible]
	-1889.095	[illegible]	[illegible]	[illegible]	[illegible]
	-94.77258	-80.41701	-63.50498	-48.1256	[illegible]
R2= 1.9	VARIANCE= 3.61 AND LOGNORMAL SKEWNESS= 20.2521				
	.4210527	.3157895	.2105263	.1052632	0
	.223266	.2607757	.3024874	.3485189	.3989423
	1.156958E-02	4.914469E-02	.1039518	.1700383	.2439451
	-1829.767	-430.5875	-190.9882	-104.9649	-63.53773
	-94.81802	-81.15297	-65.63435	-51.21117	-38.85203
R2= 2.1	VARIANCE= 4.41 AND LOGNORMAL SKEWNESS= 28.26809				
	.3809524	.2857143	.1904762	.0952381	0
	.2370692	.2722586	.3109195	.3531312	.3989423
	1.242624E-02	4.924357E-02	[illegible]	.1624809	.2305192
	-1807.911	-452.8816	-207.6725	-117.3371	-73.0625
	[illegible]5839	-81.91295	-67.49791	-53.98853	[illegible]

R1=	1.2	1.4	1.6	1.8	2
R2= .02	VARIANCE= .0004 AND LOGNORMAL SKEWNESS= 8.00016E-06				
	-10	-20	-30	-40	-50
	7.694608E-23	0	0	0	0
	2.533197E-07	7.450581E-07	4.470348E-08	1.490116E-06	-1.117587E-06
	100	100	100	100	100
	1.292276E+17	9999	9999	9999	9999
R2= .04	VARIANCE= .0016 AND LOGNORMAL SKEWNESS= 3.20255E-03				
	-5.000002	-10	-15	-20	-25
	5.119608E-08	7.694608E-23	0	0	0
	5.03697E-07	3.725291E-07	2.235172E-08	7.450581E-07	-5.587936E-07
	91.51709	100	100	100	100
	1078.841	4.861613E+17	9999	9999	9999
R2= .06	VARIANCE= .0036 AND LOGNORMAL SKEWNESS= 7.212959E-03				
	-3.333334	-6.666667	-10	-13.33333	-16.66667
	1.118901E-06	1.409522E-12	7.694526E-23	0	0
	3.507323E-06	8.940698E-08	3.327928E-07	1.78814E-07	8.990367E-07
	58.78231	99.99861	100	100	100
	222.3987	5342973	6.325057E+17	9999	9999
R2= .08	VARIANCE= .0064 AND LOGNORMAL SKEWNESS= 1.284096E-02				
	-2.500001	-5	-7.500001	-10	-12.5
	2.004096E-03	5.119461E-08	2.166039E-15	7.694608E-23	4.695201E-35
	4.20982E-03	3.047288E-06	1.117588E-08	3.72529E-07	-2.793968E-07
	52.39471	98.32	99.99999	100	100
	110.0607	5852.361	5.159383E+08	6.861613E+17	-5.950689E+29
R2= .1	VARIANCE= .01 AND LOGNORMAL SKEWNESS= .0201				
	-2.000001	-4	-6	-8	-10
	8.490834E-03	7.086083E-06	1.351521E-10	5.628348E-17	7.694608E-23
	1.452807E-02	1.133084E-04	3.278256E-07	1.192093E-07	1.490116E-07
	41.55585	93.7462	99.95877	100	100
	71.10295	1499.028	242460.7	2.115016E+11	1.936372E+17
R2= .3	VARIANCE= 9.000001E-02 AND LOGNORMAL SKEWNESS= .1881				
	-.6666668	-1.333333	-2	-2.666667	-3.333333
	.1511197	4.239504E-02	5.490864E-03	1.18152E-03	[illegible]E-04
	.1736699	7.396366E-02	2.847028E-02	1.098596E-02	4.19169E-03
	12.98655	41.07173	70.17646	89.24518	97.33066
	14.92212	59.69783	235.3056	929.8137	3646.238
R2= .5	VARIANCE= .25 AND LOGNORMAL SKEWNESS= .5625				
	-.4000001	-.8	-1.2	-1.6	-2
	.2304388	.1202073	5.610238E-02	2.326197E-02	8.490864E-03
	.2603368	.1539646	9.876322E-02	6.366527E-02	4.132939E-02
	6.117593	21.92525	43.19507	63.49349	79.65566
	6.29662	28.08239	76.04106	173.9237	388.7526
R2= .7	VARIANCE= .49 AND LOGNORMAL SKEWNESS= 1.2201				
	-.2857144	-.5714286	-.8571429	-1.142857	-1.428572
	.2722586	.1766456	.1085669	6.300213E-02	3.442105E-02
	.2620835	.1954109	.1468903	.1113576	8.513523E-02
	-3.882406	9.603118	26.08978	43.42351	59.56897
	-3.737307	10.62328	35.29929	76.75185	147.3348
R2= .9	VARIANCE= .81 AND LOGNORMAL SKEWNESS= 2.2761				
	-.2222223	[illegible]	-.6666667	-.8888888	-1.111111
	.2976612	.2156858	.1511197	.1024928	6.712546E-02
	.2650811	.2129121	.1725649	.1410517	.116201
	-12.25308	-1.20878	12.42733	27.33676	42.23329
	-10.93938	-1.194361	14.19087	37.62116	73.11009
R2= 1.1	VARIANCE= 1.21 AND LOGNORMAL SKEWNESS= 3.8841				
	-.1818182	-.3636364	-.5454546	-.7272727	-.9090909
	.3146092	.2432136	.186133	.1363958	9.876587E-02
	.259669	[illegible]	.1842367	.1571231	.1369333
	-21.16712	[illegible]58081	.0670993	13.19174	25.30123
	-17.46936	[illegible]	5.714435E-02	15.19642	36.61525
R2= 1.3	VARIANCE= 1.69 AND LOGNORMAL SKEWNESS= 6.236099				
	-.1538462	-.3076923	-.4615385	-.6153846	-.7692308
	.3267311	.2638334	.2279254	.1644928	.1259642
	.2504802	.2160473	.1878757	.1645534	[illegible]
	-30.4419	-22.11832	-11.73633	3.684689E-02	12.53449
	-23.33751	-18.1122	-10.50359	3.586047E-02	14.33078
R2= 1.5	VARIANCE= 2.25 AND LOGNORMAL SKEWNESS= 9.5625				
	-.1333336	-.2666667	[illegible]	-.5333333	-.6666667
	.3358165	.2797101	.2304388	.1877065	[illegible]
	.2398616	.2129502	.186939	.1567556	[illegible]
	-40.0063	-32.59532	-23.26952	-12.56382	[illegible]
	-28.57362	-24.58256	-13.87695	[illegible]	[illegible]
R2= 1.7	VARIANCE= 2.89 AND LOGNORMAL SKEWNESS= [illegible]				
	-.1176471	[illegible]	-.3529412	-.4705882	[illegible]
	.3428756	.292288	.2470645	.2070242	[illegible]
	.2289336	[illegible]	.1334868	.1538558	[illegible]
	-49.77116	[illegible]	[illegible]	[illegible]	[illegible]
	-37.23146	[illegible]	-25.73407	[illegible]	[illegible]
R2= 1.9	VARIANCE= 3.61 AND LOGNORMAL SKEWNESS= 20.2521				
	-.1052632	-.2105263	-.3157895	-.4210526	-.5263158
	.3485189	.3024876	.2607757	.2232659	[illegible]
	.2182598	.1968336	[illegible]	.1631502	[illegible]
	-59.68071	-53.6769	-45.93389	-36.84684	[illegible]
	-37.37503	-36.92861	-31.47582	-26.92561	-21.13107
R2= 2.1	VARIANCE= 4.41 AND LOGNORMAL SKEWNESS= 28.26809				
	-9.523812E-02	-.1904762	-.2857143	-.3809524	-.4761905
	.3531311	.3109195	.2722586	.2370692	.2052429
	.2081056	.1892892	.1732564	.1594278	.147378
	[illegible]68846	-64.25632	-57.14205	[illegible]	-39.26286
	-41.06867	-39.11954	-36.36331	-32.75051	-28.19335

(1) The variable $R_1 = Q_b/\mu_Q$ (see Equation (19)) varies horizontally and the variable $R_2 = \sigma_Q/\mu_Q$ (see Equation (20)) varies vertically.

The variance is R_2^2 and the skewness of the lognormal distribution is $R_2^2 (R_2^2+2)$.

For each R_1, R_2 combination, the information given is:

(i) $R_3 = \mu_z/\sigma_z$ (see Equation (6)).

(ii) $EVPI_N/\sigma_z$ (for the normal distribution (see Equation (21)).

(iii) $EVPI_L/\sigma_z$ for the lognormal distribution (see Equation (24)).

(iv) $100* (EVPI_L - EVPI_N)/EVPI_L$

(v) $100* (EVPI_L - EVPI_N/EVPI_N$

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