ON THE DYNAMIC ANALYSIS

OF

A STANDARD AND SELF-STEERING SEMITRAILERS

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by

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SCOPE AND CONTENTS:

This thesis describes an analytical study of the dynamics of a tractor-semitrailer vehicle.

Two mathematical models; an articulated vehicle with self steering semitrailer and an articulated vehicle with a standard semitrailer, are developed to describe the longitudinal, lateral, vertical, pitching, rolling and yawing motions of the vehicle on a rough road surface.

The natural frequencies and the damped eigenvalues for both models are calculated.

The steady state response of the vehicle components to a sinusoidal input profile of varying frequencies is calculated and

the response curves are computer plotted in each case. For the self-steering semitrailer, the effect of varying the spring stiffness at the fifth wheel is studied. The dynamic loads imparted to the pavement due to the dynamic action of the vehicle in response to road irregularities, are also calculated. A discussion of the conclusions drawn from the analysis is given.

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LIST OF SYMBOLS

Notes:

(1) Symbols which are underscored represent vectors.

(2) Subscripts which denote locations may take on values from 1 to 12, denoting locations, as follows:

1 - tractor, left front
2 - tractor, right front
3 - tractor, left rear
4 - tractor, right rear
5 - bogie, left front
6 - bogie, right front
7 - bogie, left rear
8 - bogie, right rear
9 - tractor, fifth-wheel
10 - semitrailer, fifth-wheel
11 - semitrailer, vertical torque shaft
12 - bogie, frame journal

(3) k may take the values 1, 3, 5, 7 for tractor front axle, tractor rear axle, bogie front axle and bogie rear axle respectively.

a_j vehicle dimensions

A mass matrix

b_j vehicle dimensions

B stiffness matrix

C damping matrix

C_1, C_3, C_5, C_7 suspension vertical damping constant, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

C_2, C_4, C_6, C_8 suspension lateral damping constant, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

$C_{t1}, C_{t2}, C_{t3}, C_{t4}$ tire vertical damping constant, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

$C_{t2}, C_{t4}, C_{t6}, C_{t8}$ tire lateral damping constant, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

C_h fifth-wheel viscous damping

C_p vertical shaft viscous damping

$C_{r1}, C_{r3}, C_{r5}, C_{r7}$ roll damping constants, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

d_j vehicle dimensions

D transformation matrix

$\underline{f}_1$ force on tractor at suspension

$\underline{f}_2$ force on tractor at fifth-wheel

$\underline{f}_3$ force on semitrailer at fifth-wheel

$\underline{f}_4$ force on semitrailer at vertical torque shaft

$\underline{f}_5$ force on bogie at vertical torque shaft

$\underline{f}_6$ force on bogie at suspension

$\underline{f}_7$ force on tractor front axle at suspension

$\underline{f}_8$ force on tractor front axle from the road

F force vector

F_{ij} force at location i in direction j

G matrix whose eigenvalues are the natural frequencies.

G_i displacement function of the road contour in vertical direction applied at location i

h_j vehicle dimensions

I unit matrix

$\underline{I}_b$ bogie inertia tensor

$\underline{I}_{jk}$ product of moment of inertia with reference to jk axes

$\underline{I}_K$ axle inertia tensor

$\underline{I}_s$ semitrailer inertia tensor

$\underline{I}_t$ tractor inertia tensor

K stiffness matrix

K_1, K_3, K_5, K_7 suspension vertical stiffness, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

K_2, K_4, K_6, K_8 suspension lateral stiffness, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

$K_{t1}, K_{t3}, K_{t5}, K_{t7}$ tire vertical stiffness, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

$K_{t2}, K_{t4}, K_{t6}, K_{t8}$ tire lateral stiffness, locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

K_h fifth-wheel spring constant

$K_{r1}, K_{r3}, K_{r5}, K_{r7}$ roll stiffness for locations 1 and 2, 3 and 4, 5 and 6, 7 and 8 respectively.

ℓ_j vehicle dimensions

m_b bogie mass

m_k axle mass

m_s semitrailer sprung mass

m_t tractor sprung mass

$\underline{m}_1$ moment of $\underline{f}_1$ about tractor centre of mass

$\underline{m}_2$ couples on tractor at suspension

$\underline{m}_3$ moment of $\underline{f}_2$ about tractor centre of mass

$\underline{m}_4$ couples on tractor at fifth-wheel

$\underline{m}_5$ moment of $\underline{f}_3$ about semitrailer centre of mass

$\underline{m}_6$ couples on semitrailer at fifth-wheel

$\underline{m}_7$ moment of $\underline{f}_4$ about semitrailer centre of mass

$\underline{m}_8$ couples on semitrailer at vertical torque shaft

$\underline{m}_9$ couples on bogie at vertical torque shaft

$\underline{m}_{10}$ moment of $\underline{f}_6$ about bogie centre of mass

$\underline{m}_{11}$ couples on the bogie at suspension

$\underline{m}_{12}$ moment due to suspension forces about tractor front axle centre of mass

$\underline{m}_{13}$ couples on tractor front axle at suspension

$\underline{m}_{14}$ moment due to road forces about tractor front axle centre of mass

M mass matrix

M_{ij} couple at location i in direction j

q displacement vector

Q force vector

r_i tire rolling radius

$\underline{r}_j$ position vector

t time

u_b longitudinal displacement for bogie mass centre in x direction

u_k longitudinal displacement for axle mass centre in x direction

u_s longitudinal displacement for semitrailer mass centre in x direction

u_t longitudinal displacement for tractor mass centre in x direction

v_b lateral displacement for bogie mass centre in y direction

v_k lateral displacement for axle mass centre in y direction

v_s lateral displacement for semitrailer mass centre in y direction

v_t lateral displacement for tractor mass centre in y direction

$\underline{V}_b$ lineal velocity of bogie centre of mass

$\underline{V}_k$ lineal velocity of axle centre of mass

$\underline{V}_s$ lineal velocity of semitrailer centre of mass

$\underline{V}_t$ lineal velocity of tractor centre of mass

w_b vertical displacement for bogie centre of mass

w_k vertical displacement for axle centre of mass

w_s vertical displacement for semitrailer centre of mass

w_t vertical displacement for tractor centre of mass

X displacement solution vector

Y displacement excitation vector

Y_i dynamic lateral reaction of road on tire, location i

Z_i	dynamic vertical reaction of road on tire, location i
α_b	angle of rotation of bogie about x axis
α_k	angle of rotation of axle about x axis
α_s	angle of rotation of semitrailer about x axis
α_t	angle of rotation of tractor about x axis
β_b	angle of rotation of bogie about y axis
β_k	angle of rotation of axle about y axis
β_s	angle of rotation of semitrailer about y axis
β_t	angle of rotation of tractor about y axis
γ_b	angle of rotation of bogie about z axis
γ_k	angle of rotation of axle about z axis
γ_s	angle of rotation of semitrailer about z axis
γ_t	angle of rotation of tractor about z axis
λ	classical eigenvalue
λ_i	displacement function of the road contour in lateral direction applied at location i
ω	forced frequency
$\underline{\omega}_b$	angular velocity of bogie centre of mass
$\underline{\omega}_k$	angular velocity of axle centre of mass
$\underline{\omega}_s$	angular velocity of semitrailer centre of mass
$\underline{\omega}_t$	angular velocity of tractor centre of mass

CHAPTER 1

INTRODUCTION

When a tractor-semitrailer vehicle, travels along a road the irregularities of the road-surface impose continuously varying displacements at the points of contact of tires and road, and so give rise to responses in the form of stresses or accelerations in the various components of the vehicle.

To determine the response characteristics of the vehicle to road surface undulation requires a complete dynamical description of the vehicle and this is only possible if sufficient degrees of freedom are considered.

Axle dynamics can be expected to affect the behaviour of the vehicle significantly when the vehicle is operating on bumpy roads.

1.1 Background

A brief historical discussion of research on the dynamic behaviour of vehicles is desirable in developing a basis for this work.

The previous investigators have shown the effects of applying mathematical techniques and the computer to increasingly sophisticated and more complete linear and non-linear models.

Slibar and Pasly [1]* studied the lateral behaviour of a semitrailer having a constant forward velocity. The king pin was assumed to have a small periodic lateral velocity and displacement, and the frequency of the lateral oscillation was proportional to the forward velocity. The results of this analysis showed that the damping of the lateral oscillations was inversely proportional to the forward velocity and that self-excited motion never occurred.

The same investigators [2] considered the motion of a tractor when a sinusoidal force and moment excitation acted at the fifth wheel. The heading angle and the steering angle required to keep the velocity of the centre of mass constant and without a lateral component, were determined.

Janeway [3] has researched driver comfort and has found that the fore-aft motions of the tractor are of utmost importance to driver comfort.

Clark and Huang [4] simplified the model by completely ignoring the bounce of each axle and the fore-aft motions of the tractor and trailer.

Ellis [5] studied the riding qualities of an articulated vehicle and concluded that fore and aft shake is a significant factor in determining driver and load ride conditions. This shake is due to the relative positions of the centres of gravity of the two units and the height of the fifth wheel. First approximations to the various

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*Number in square brackets designate reference in the Bibliography natural frequencies were obtained and a comment on the probable effects of spring friction was given.

LeFevre [6] discussed qualitative aspects of vehicle ride and also considered the effects of axle vibration on vehicle ride.

The paper of Walther, Gossard and Fensel [7] has taken a quantitative approach to the problem. Parameters studied included spring and damping rates for different axles of the combination, laden mass of the trailer, vehicle speed, coupler position on the tractor and cab mounting. The vehicle was treated as a linear dynamic system with seven degrees of freedom. These degrees of freedom are: vertical and rotational displacements of both tractor and trailer, and vertical displacements of three axle-suspension assemblies. The fifth wheel was considered as a high rate spring.

Van Deusen [8] described techniques that predict and analyze dynamic response of a vehicle traversing random rough surfaces.

Chiesa and Rinonapoli [9] developed a mathematical model of seven nonlinear differential equations, three of which involve lateral, yaw and roll movements of the car and four of which involve the relative movements between the tread of the tires and the wheel due to lateral flexibility of the tires. Lateral and vertical stiffnesses of tires were investigated. Calculations showed the damping effect of lateral stiffness and the need for increasing both cornering and lateral stiffnesses.

Walker and Potts [10] developed and discussed a computer solution of the linear vibration of a 3-axle semi-trailer truck. An adjunct program which provides a spectral density analysis of the output of each coordinate given random road surface input was described. The mathematical model used is of six independent coordinates; vertical displacements of the tractor and the three axles, and pitching of both tractor and semitrailer.

In 1968, Mikulcik [11] presented a more detailed and general nonlinear mathematical model of a tractor-semitrailer vehicle. The model included eight degrees of freedom. Both the tractor and the semitrailer were free to yaw, pitch, roll, and translate in the forward, lateral, and vertical directions, except as constrained by the fifth wheel. The model also had the capability to include stabilizing elastic and damping moments at the fifth wheel. The truck model was used to study the effects of steering, braking, and fifth wheel devices on the behaviour of the vehicle.

McHenry and Deleys [12] presented an eleven degree of freedom computer model of a passenger vehicle. In this model one degree of freedom was included for each of the two unsprung masses and two degrees of freedom were included for the rear unsprung mass, a beam axle. The degrees of freedom for the unsprung masses were included so that simulations of violent vehicle maneuvers could be performed.

Potts and Walker [13] investigated the nonlinear vibratory motions of a three-axle semitrailer truck. They also described an experimental vibration study, performed on a model truck. The analysis allows any shape of suspension force-deflection curve (including wheel hop, suspension stops, and dry friction damping) and a similar liberality of truck absorber force-velocity characteristics.

1.2 Objective of the Research Program

The objective of this research is to investigate the dynamics of the tractor-semitrailer vehicle as it is affected by the road profile. To accomplish this work two mathematical models, tractor-standard semitrailer and tractor-self-steering semitrailer describing the longitudinal, lateral, vertical, pitching, rolling and yawing motions of the vehicle are developed.

The three dimensional study of the mathematical model would consider the entire vehicle as a vibrating system and analyze the problem as a multiple input-output system.

The steady state response of the vehicle components to a sinusoidal input profile of varying frequencies is calculated. The input frequency is made dependent on vehicle speed and expansion joint spacing of the road.

Results obtained in the computer study are discussed and presented graphically. Finally, some suggestions for future research are presented.

CHAPTER 2

THE PHYSICAL CHARACTERISTICS OF THE MODEL

In this chapter the self-steering bogie for the semitrailer, the characteristics of the tractor-semitrailer mathematical model are described and the simplifying assumptions are presented.

2.1 Self-Steering Bogie For Semitrailer

The aim of providing, at the rear of a semitrailer pulled by a road tractor, a bogie that has a steerable front axle is to improve the stability of the semitrailer when rounding a curve and to reduce the tendency for the tractor and the semitrailer to "jackknife" relative to one another.

A self-steering bogie for a semitrailer, Figure 2.1, has a bogie frame (6) with at least one fixed rear axle (5) and with a front axle (1) having, at its ends, steerable wheel-carrying stub axles (7) connected to the front axle by king pins (8). Above the frame is an intermediate slide (4) that can be adjusted longitudinally along the underside of the semitrailer body. Fixed to the slide is a first swivel, consisting of a vertical shaft (3) on which said bogie frame is jounalled. This swivel is connected to the stub axles by a suitable links(13, 12, 11, 2, 10, 9) so as to steer the wheel-carrying stub axles

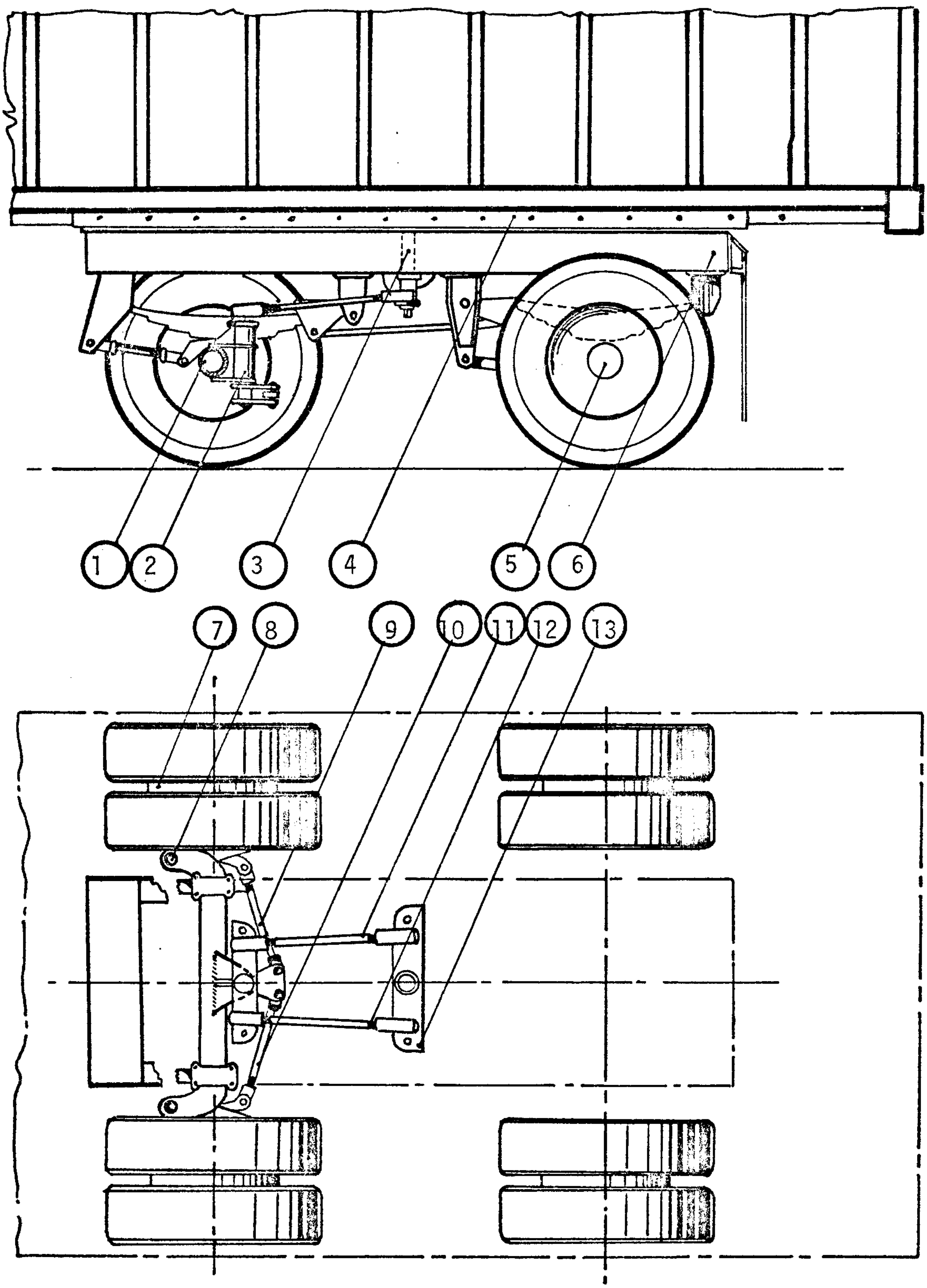

FIGURE 2.1 SELF-STEERING BOGIE FOR SEMITRAILER

in accordance with the relative swivelling of the semitrailer body and bogie frame.

2.2. General Description

a) The Tractor-Semitrailer

The tractor-semitrailer is considered to be operating on a rough road. Both units are allowed to translate in the forward, vertical and lateral directions, and roll, pitch and yaw --- except as constrained by the fifth wheel.

b) The Self-Steering Bogie

The self-steering bogie is capable of turning relative to the semitrailer body about a swivel axis in the vertical direction, but roll and pitch with the semitrailer.

c) Vehicle Dimensions

Overall vehicle dimensions are shown in Figures 2.2, 2.3 and 2.4.

d) Fifth Wheel

The motions of the tractor and the semitrailer are related by the constraints imposed by the fifth wheel, Figure 2.5. It allows the semitrailer to rotate with respect to the tractor about two mutually perpendicular axes. Fifth wheel stabilizing elements, such as torsional springs, torsional dampers can exert moments about the axis of this pin.

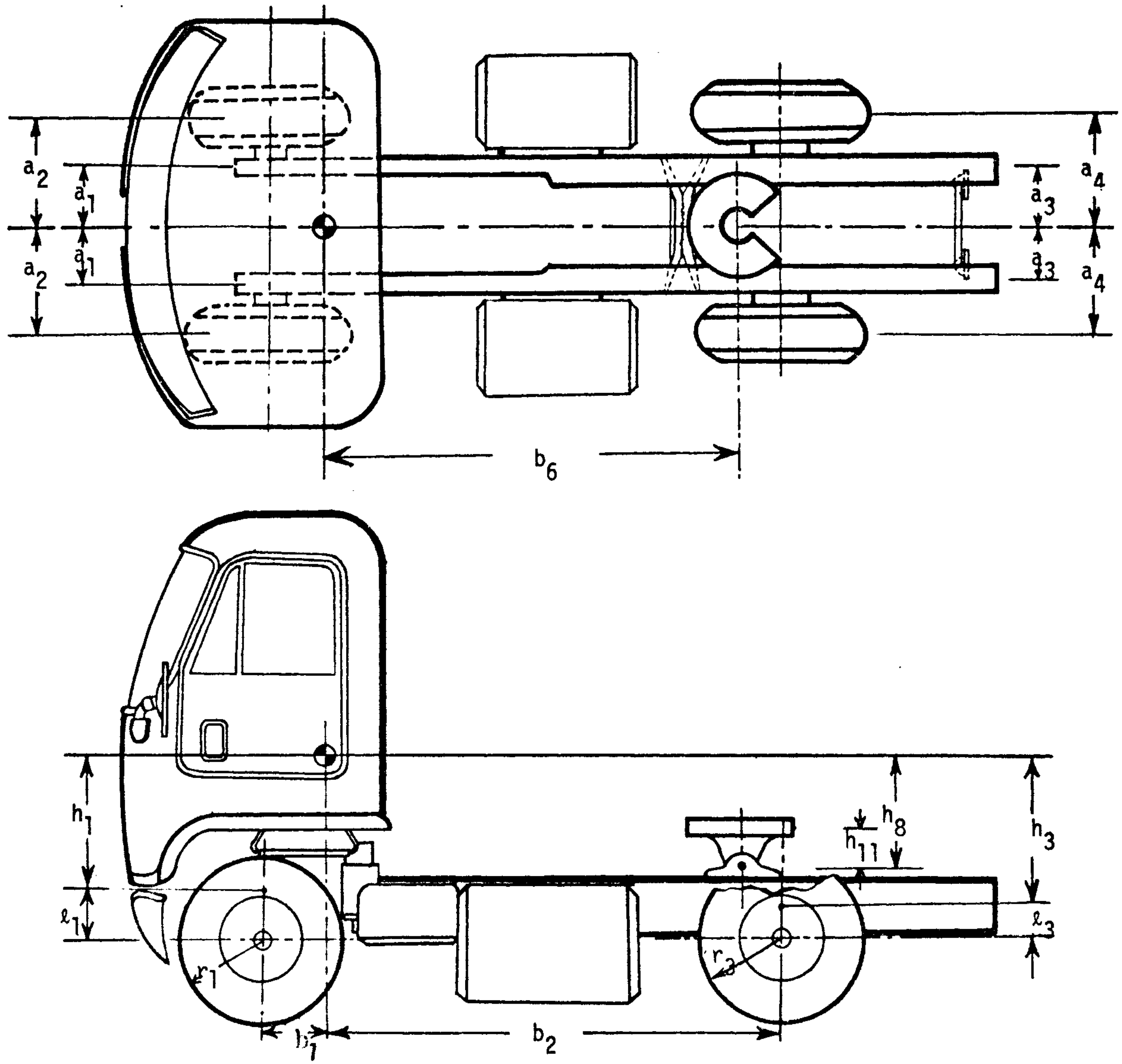

FIGURE 2.2 TRACTOR DIMENSIONS

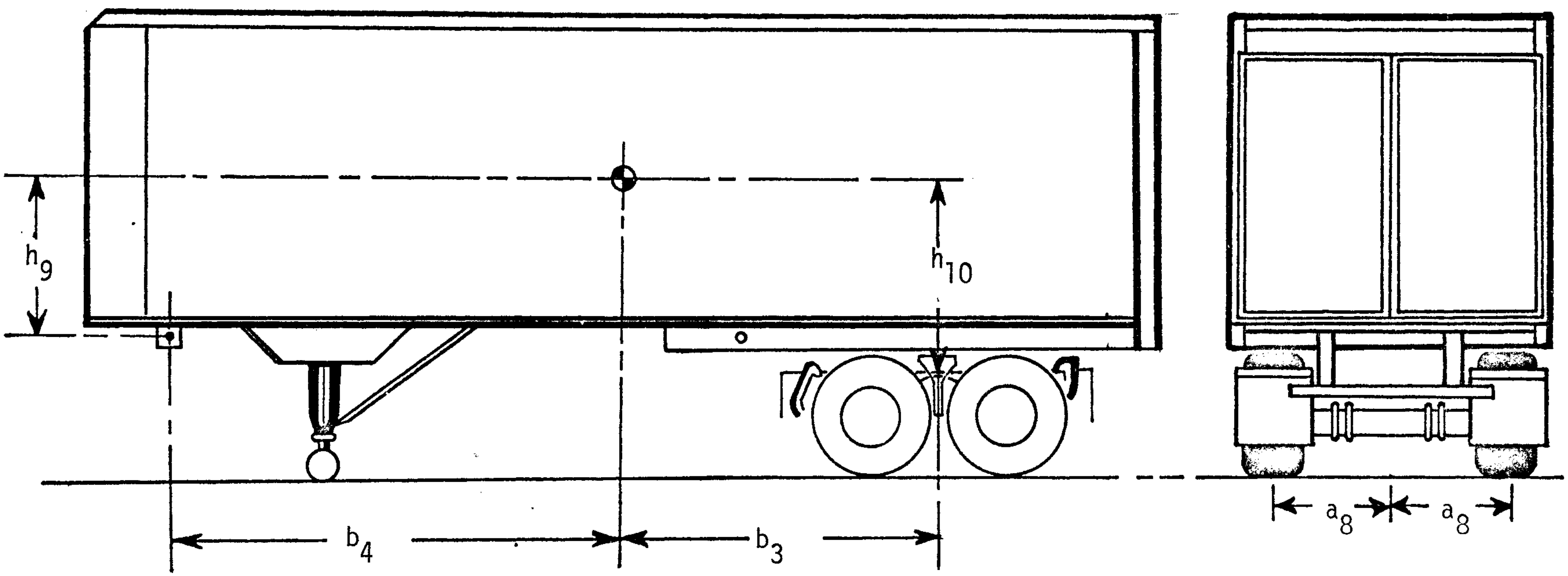

FIGURE 2.3 SEMITRAILER DIMENSIONS

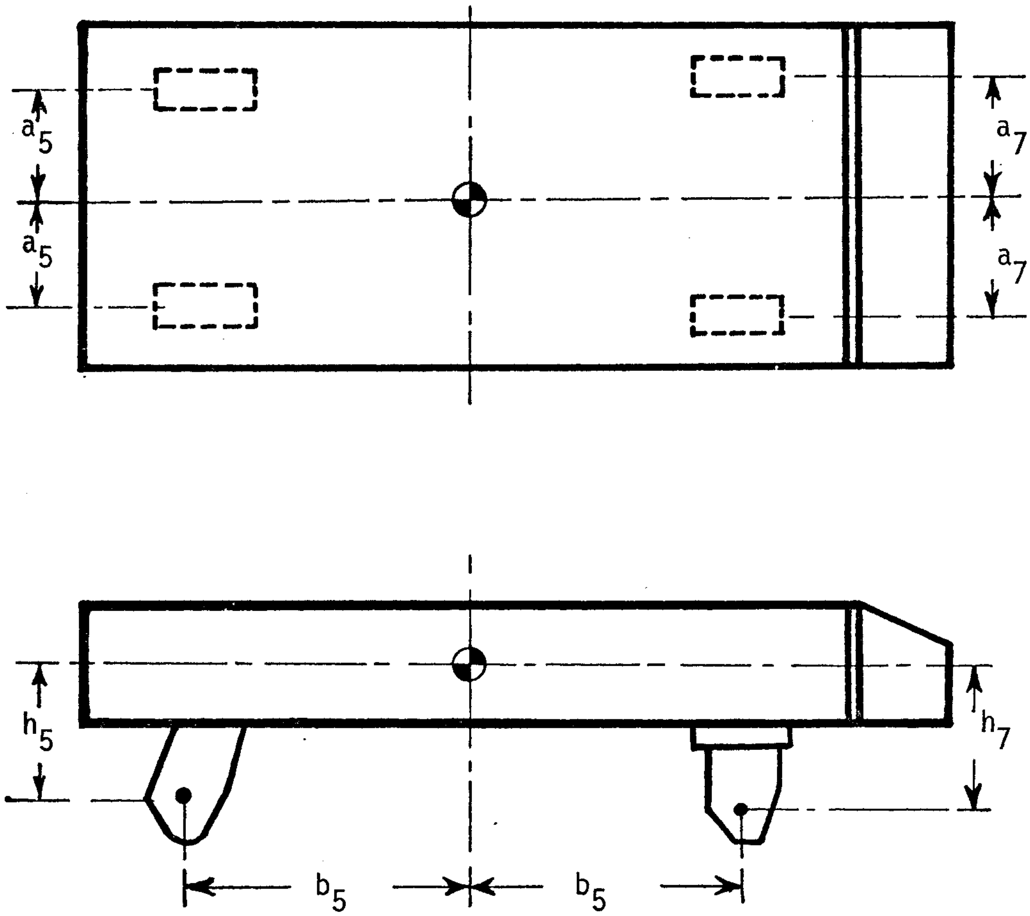

FIGURE 2.4 BOGIE DIMENSIONS

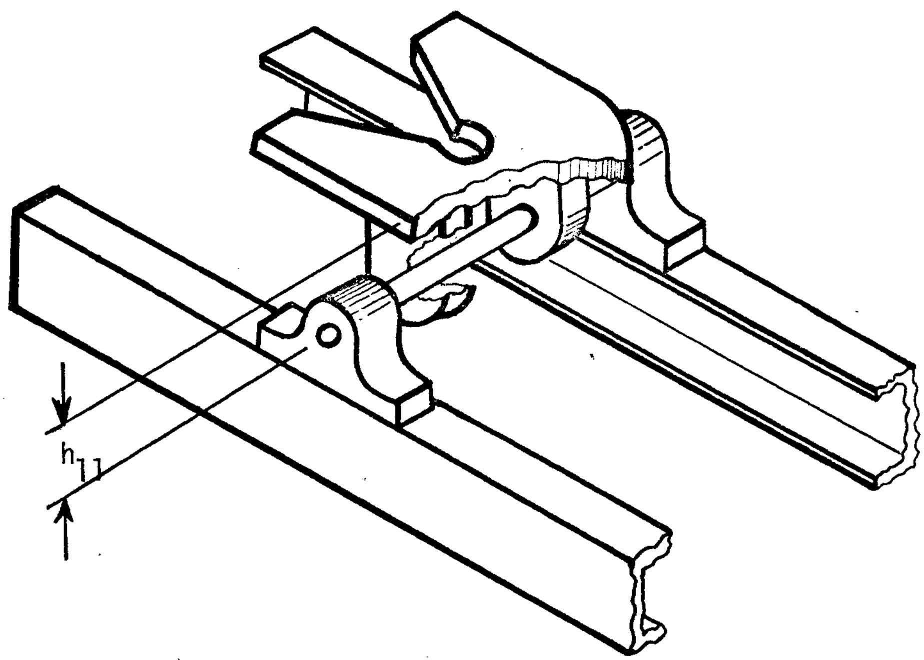

FIGURE 2.5 FIFTH WHEEL

e) Suspensions and Tires

A mass beam axle suspension, Figure 2.6, is used for the four axles of the vehicle. Each axle assembly is connected to two spring-shock absorbers in both vertical and lateral directions; these units are attached to the sprung mass at two points.

In practice, the suspension, fifth wheel and the vertical torque shaft are attached to the chassis at many points. However, the reactions associated with the suspension, fifth wheel and the vertical torque shaft can be represented by force and moment combinations and are taken to act at single points on the chassis. Each point is denoted by a subscript used in reference forces and moments when written in component form as shown in Figure 2.7.

The subscripts are defined as follows:

1 tractor, left front
2 tractor, right front
3 tractor, left rear
4 tractor, right rear
5 bogie, left front
6 bogie, right front
7 bogie, left rear
8 bogie, right rear
9 tractor, fifth wheel
10 semitrailer, fifth wheel
11 semitrailer, vertical torque shaft
12 bogie, vertical torque shaft

For Axle 1, i = 1, j = 2
For Axle 3, i = 3, j = 4
For Axle 5, i = 5, j = 6
For Axle 7, i = 7, j = 8

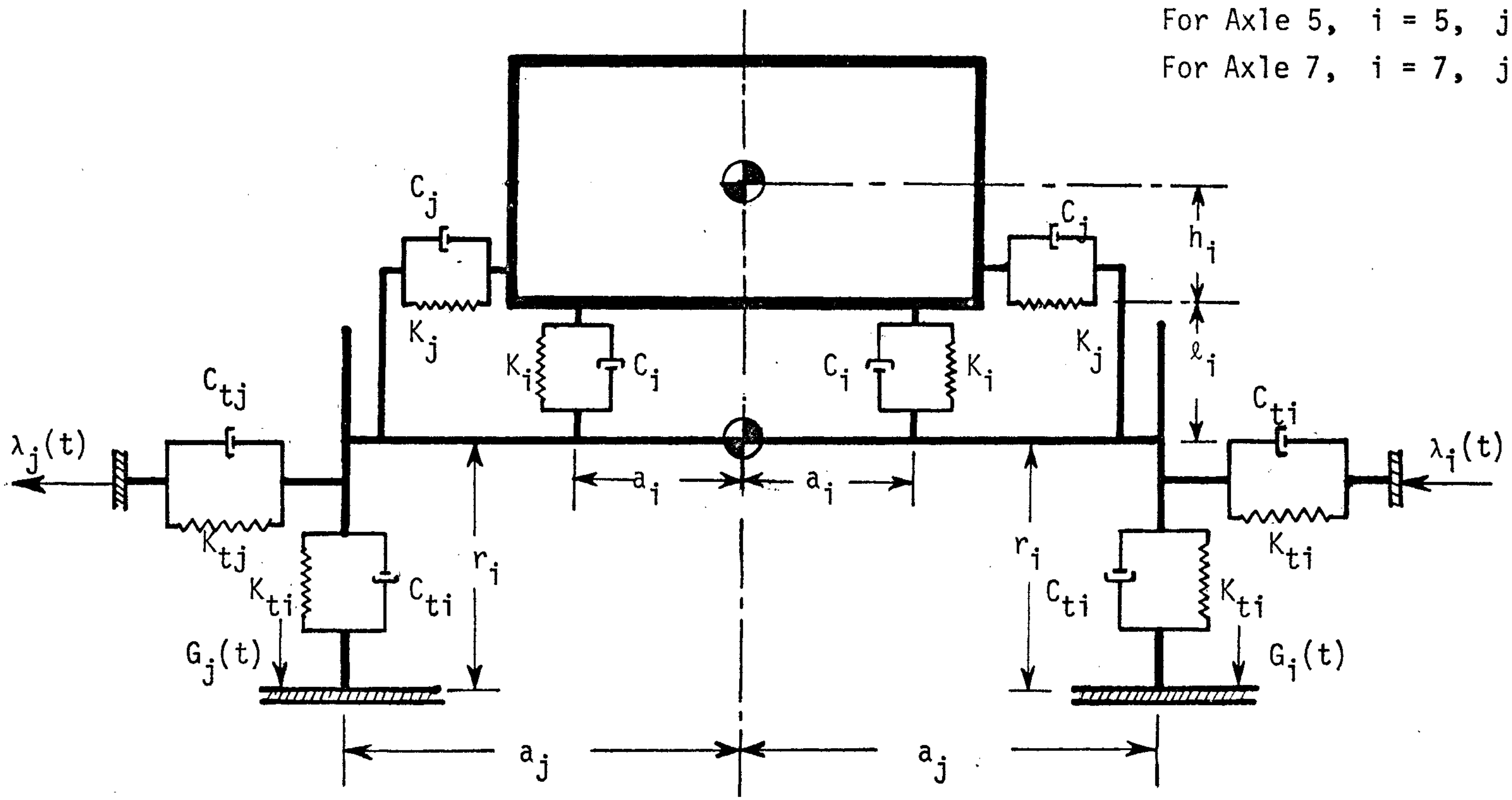

FIGURE 2.6 AXLE ATTACHMENT GEOMETRY, DYNAMIC ANALYSIS

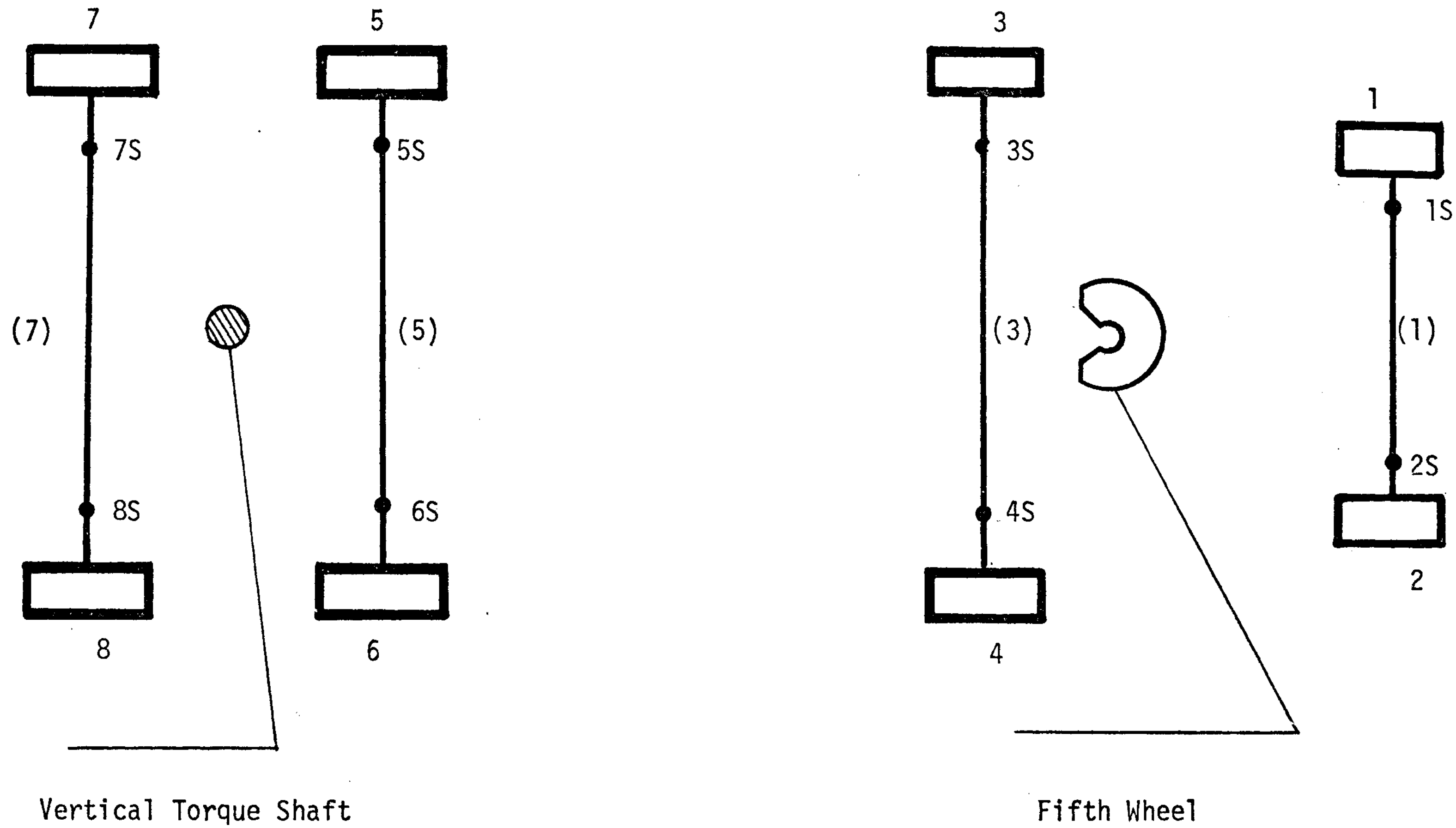

FIGURE 2.7 AXLE AND WHEEL LABELING CONVENTION

(1) Tractor front axle

(3) Tractor rear axle

(5) Bogie front axle

(7) Bogie rear axle

The tires are considered as a system having vertical and lateral springing and damping characteristics.

The axle can translate relative to the sprung mass in the vertical and lateral directions and can rotate about the longitudinal axis.

f) The Input

The input to the mathematical vehicle model is the perturbations of the road surface.

2.3 The Assumptions

The simplifying assumptions used in order to keep the mathematics tractable, are the following:

1) The vehicle is moving in a straight line with a constant forward speed.

2) All the displacements are small.

3) The sprung masses of the tractor, semitrailer and the bogie are assumed to be rigid bodies.

4) Forces and couples are transmitted to the sprung masses through the suspensions, fifth wheel and the vertical torque shaft.

5) Springs and dampers are considered to be described by linear functions of displacement and velocity, respectively. Also, the springing and damping of the tires have linear characteristics.

6) The input displacement function of the road is considered to be applied to a point at the centre of the tire contact patch.

7) The wheels must remain in contact with the road surface at all times.

8) Camber effect is neglected.

CHAPTER 3

EQUATIONS OF MOTION

3.1 Coordinate Systems

The coordinates as shown in Figures 3.1 and 3.2 are chosen to describe the vibrational motion of the articulated vehicle. The components of the vehicle (tractor, semitrailer, bogie and four axles) are three dimensional objects and are allowed to translate (vertical, longitudinal and lateral) pitch, yaw and roll. Then each component has six degrees of freedom, but the constraints of the fifth wheel, and the constraints between the semitrailer and bogie, and between the axles and the sprung masses reduce the total number of degrees of freedom.

x, y, z: this coordinate system is fixed to the body (tractor, semitrailer, bogie and each axle), the origin of the coordinate system is at the centre of mass of the body.

The x axis points forward, the y axis points to the right (from the point of view of the driver) and the z axis points directly downward.

u is the linear displacement in the x direction.

v is the linear displacement in the y direction.

w is the linear displacement in the z direction.

α is the angular rotation about the x axis.

β is the angular rotation about the y axis.

γ is the angular rotation about the z axis.

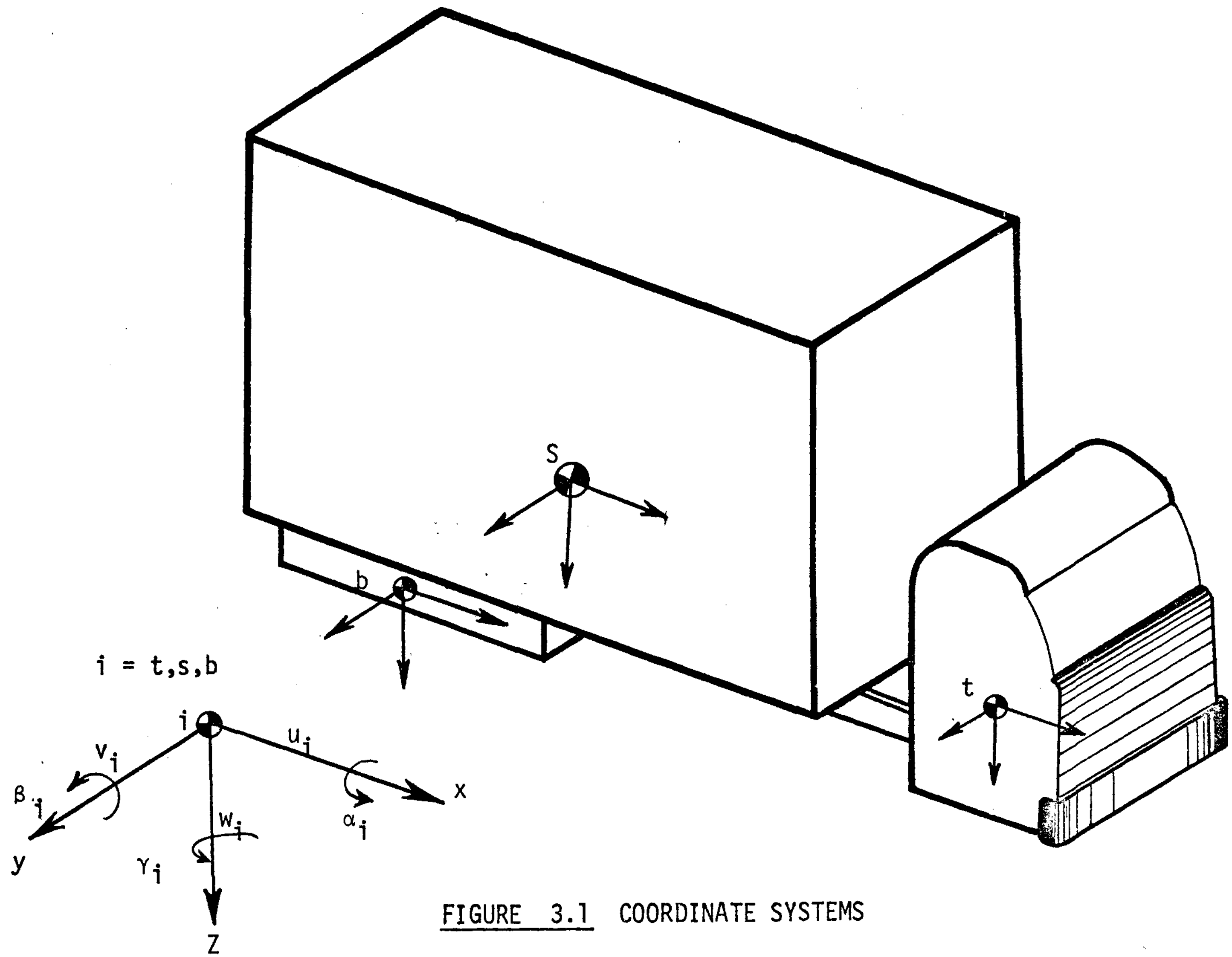

FIGURE 3.1 COORDINATE SYSTEMS

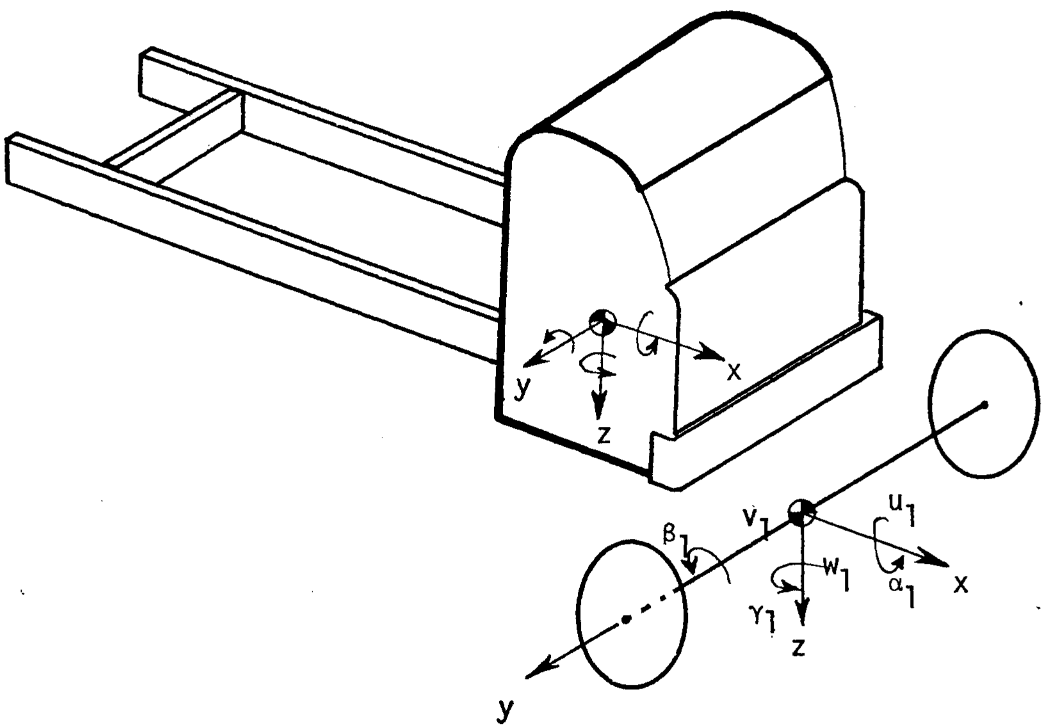

FIGURE 3.2 TRACTOR FRONT AXLE COORDINATE SYSTEM

Subscripts are as follows:

- t - for the tractor
- s - for the semitrailer
- b - for the bogie
- $i = 1, 3, 5, 7$ for the axles

The equations of motion are derived for the tractor-self-steering semitrailer, and then modified to obtain the equation of motion for the tractor-standard semitrailer.

3.2 Equations of Motion in General Form

In this section, the dynamic equations of motion for the tractor, semitrailer, bogie, and the front tractor axle are obtained in general form. The sets of equations for the seven bodies (tractor, trailer, bogie, and four axles) are, of course, not independent, and the constraint equations establishing their dependence will be developed in Section 3.3.

The second order differential equations which govern the motion of the masses are written by applying Newton's second law.

3.2.a Equations of Motion for the Tractor

Referring to Figure 3.3 and equating the rate of change of lineal momentum of the sprung mass to the external forces gives:

$$m_t \frac{d}{dt} (\underline{V}_t) = \underline{f}_1 + \underline{f}_2 \qquad (3.2.1)$$

where

m_t = the mass of the tractor

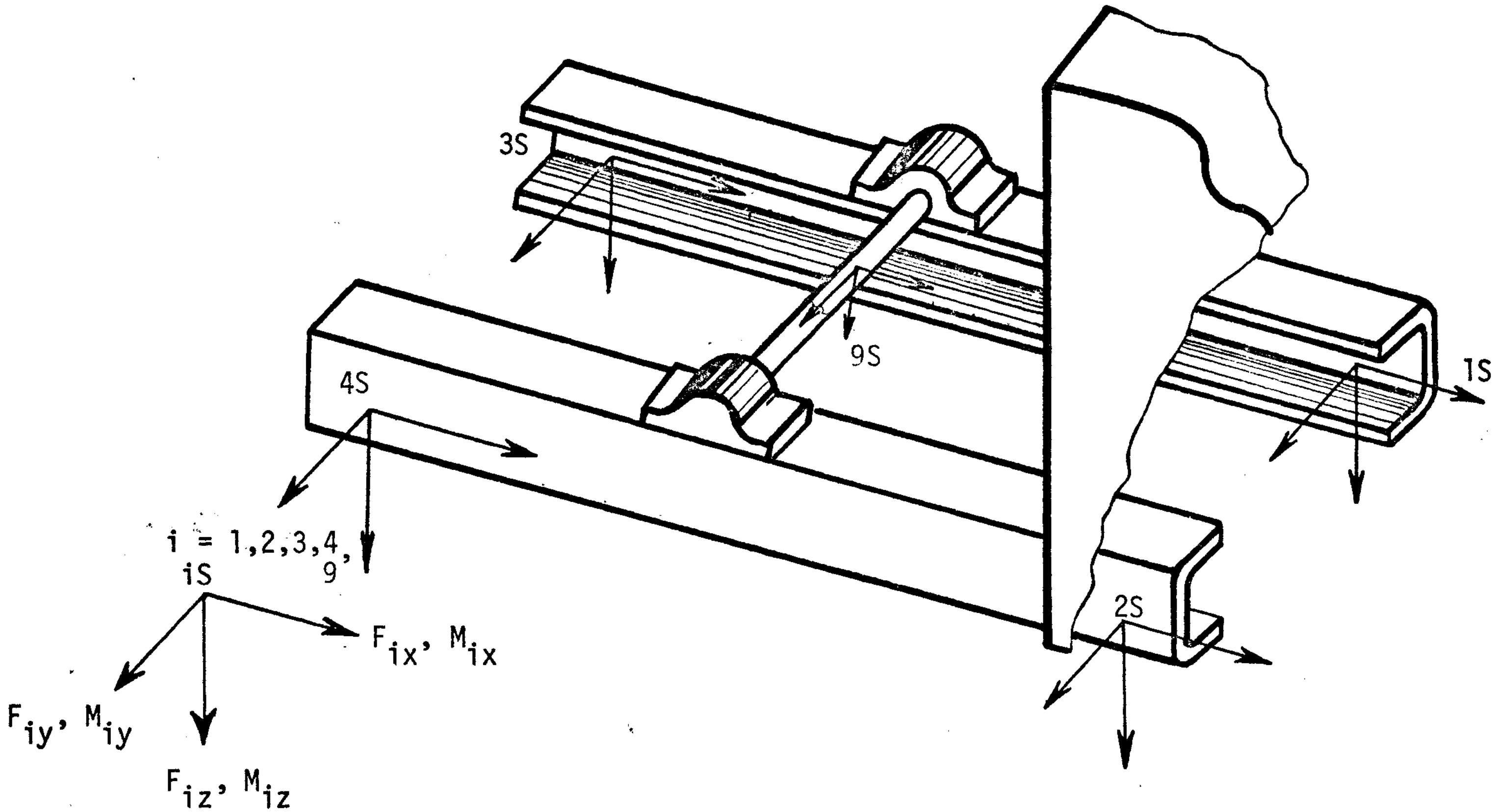

FIGURE 3.3 GENERAL FORCES AND COUPLES ON TRACTOR SPRUNG MASS

$$\underline{V}_t = \begin{bmatrix} \dot{u}_t \\ \dot{v}_t \\ \dot{w}_t \end{bmatrix}$$ the velocity of the tractor mass centre

$\underline{f}_1$ = the sum of the dynamic forces of the suspensions on the tractor sprung mass.

$$\underline{f}_1 = \begin{bmatrix} F_{1x} + F_{2x} + F_{3x} + F_{4x} \\ F_{1y} + F_{2y} + F_{3y} + F_{4y} \\ F_{1z} + F_{2z} + F_{3z} + F_{4z} \end{bmatrix}$$

$\underline{f}_2$ = the forces which act on the tractor at the fifth-wheel.

$$\underline{f}_2 = \begin{bmatrix} F_{9x} \\ F_{9y} \\ F_{9z} \end{bmatrix}$$

Equating the rate of change of the angular momentum of the mass to the external moments gives:

$$\frac{d}{dt}(\underline{I}_t \, \underline{\omega}_t) = \underline{m}_1 + \underline{m}_2 + \underline{m}_3 + \underline{m}_4 \qquad (3.2.2)$$

where

$\underline{I}_t$ = the inertia tensor of the tractor

$$\underline{I}_t = \begin{bmatrix} I_{txx} & -I_{txy} & -I_{txz} \\ -I_{txy} & I_{tyy} & -I_{tyz} \\ -I_{txz} & -I_{tyz} & I_{tzz} \end{bmatrix}$$

If the tractor is symmetrical about the xz plane, then

$$\underline{I}_t = \begin{bmatrix} I_{txx} & 0 & -I_{txz} \\ 0 & I_{tyy} & 0 \\ -I_{txz} & 0 & I_{tzz} \end{bmatrix}$$

$$\underline{\omega}_t = \begin{bmatrix} \dot{\alpha}_t \\ \dot{\beta}_t \\ \dot{\gamma}_t \end{bmatrix}$$ the angular velocity of the tractor

$\underline{m}_1$ = the moments of $\underline{f}_1$ about the centre of mass

$$\underline{m}_1 = \begin{bmatrix} b_1 \\ -a_1 \\ h_1 \end{bmatrix} \times \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{1z} \end{bmatrix} + \begin{bmatrix} b_1 \\ a_1 \\ h_1 \end{bmatrix} \times \begin{bmatrix} F_{2x} \\ F_{2y} \\ F_{2z} \end{bmatrix}$$

$$+ \begin{bmatrix} -b_2 \\ -a_3 \\ h_3 \end{bmatrix} \times \begin{bmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{bmatrix} + \begin{bmatrix} -b_2 \\ a_3 \\ h_3 \end{bmatrix} \times \begin{bmatrix} F_{4x} \\ F_{4y} \\ F_{4z} \end{bmatrix}$$

$$= \begin{bmatrix} -h_1(F_{1y}+F_{2y}) - a_1(F_{1z}-F_{2z}) - h_3(F_{3y}+F_{4y}) - a_3(F_{3z}-F_{4z}) \\ h_1(F_{1x}+F_{2x}) - b_1(F_{1z}+F_{2z}) + h_3(F_{3x}+F_{4x}) + b_2(F_{3z}+F_{4z}) \\ a_1(F_{1x}-F_{2x}) + b_1(F_{1y}+F_{2y}) + a_3(F_{3x}-F_{4x}) - b_2(F_{3y}+F_{4y}) \end{bmatrix}$$

The moment $\underline{m}_2$ is the sum of the dynamic moments, transmitted to the tractor sprung mass at the suspension attachment points,

$$\underline{m}_2 = \begin{bmatrix} M_{1x} + M_{2x} + M_{3x} + M_{4x} \\ M_{1y} + M_{2y} + M_{3y} + M_{4y} \\ M_{1z} + M_{2z} + M_{3z} + M_{4z} \end{bmatrix}$$

The moment $\underline{m}_3$ is the sum of the moments of the forces which are acting at the fifth wheel about the centre of mass of the tractor,

$$\underline{m}_3 = \begin{bmatrix} -b_6 \\ 0 \\ h_8 \end{bmatrix} \times \begin{bmatrix} F_{9x} \\ F_{9y} \\ F_{9z} \end{bmatrix} = \begin{bmatrix} 0 & -h_8 & 0 \\ h_8 & 0 & b_6 \\ 0 & -b_6 & 0 \end{bmatrix} \begin{bmatrix} F_{9x} \\ F_{9y} \\ F_{9z} \end{bmatrix}$$

$\underline{m}_4$ = the couples act on the tractor at the fifth wheel

$$\underline{m}_4 = \begin{bmatrix} M_{9x} \\ M_{9y} \\ M_{9z} \end{bmatrix}$$

3.2.b Equations of Motion for the Semitrailer

Referring to Figure 3.4 and equating the time rate of change of the lineal momentum of the semitrailer sprung mass to the external forces gives:

$$m_s \frac{d}{dt} (\underline{V}_s) = \underline{f}_3 + \underline{f}_4 \tag{3.2.3}$$

where

m_s = the mass of the semitrailer

$$\underline{V}_s = \begin{bmatrix} \dot{u}_s \\ \dot{v}_s \\ \dot{w}_s \end{bmatrix} \quad \text{the velocity of the semitrailer}$$

$\underline{f}_3$ = the forces which act on the semitrailer at the fifth wheel

$$\underline{f}_3 = \begin{bmatrix} F_{10x} \\ F_{10y} \\ F_{10z} \end{bmatrix} = - \underline{f}_2$$

$\underline{f}_4$ = the forces which act on the semitrailer at the vertical shaft from the bogie

$$\underline{f}_4 = \begin{bmatrix} F_{11x} \\ F_{11y} \\ F11z \end{bmatrix}$$

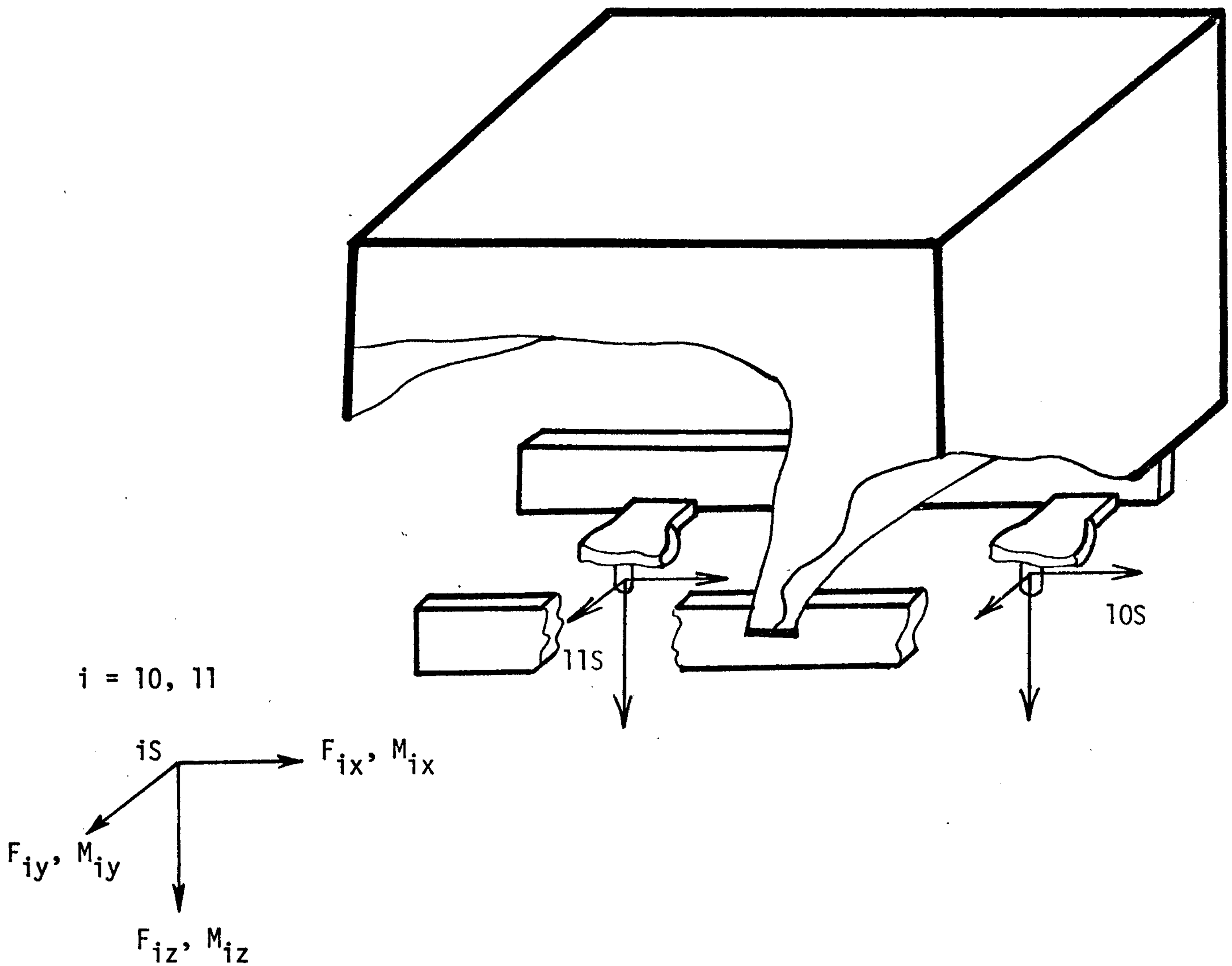

FIGURE 3.4 GENERAL FORCES AND COUPLES ON SEMITRAILER SPRUNG MASS

Equating the rate of change of angular momentum of the semitrailer sprung mass about its centre of mass to the external moment gives:

$$\frac{d}{dt}(\underline{I}_s\ \underline{\omega}_s) = \underline{m}_5 + \underline{m}_6 + \underline{m}_7 + \underline{m}_8 \qquad (3.2.4)$$

where

$\underline{I}_s$ = the inertia tensor of the semitrailer

$$\underline{I}_s = \begin{bmatrix} I_{sxx} & -I_{sxy} & -I_{sxz} \\ -I_{sxy} & I_{syy} & -I_{syz} \\ -I_{sxz} & -I_{syz} & I_{szz} \end{bmatrix}$$

$\underline{\omega}_s = \begin{bmatrix} \dot{\alpha}_s \\ \dot{\beta}_s \\ \dot{\gamma}_s \end{bmatrix}$ the angular velocity of the semitrailer centre of mass

$\underline{m}_5$ = the sum of the moments of the forces at the fifth wheel about the centre of mass of the semitrailer

$$\underline{m}_5 = \begin{bmatrix} b_4 \\ 0 \\ h_9 \end{bmatrix} \times \begin{bmatrix} F_{10x} \\ F_{10y} \\ F_{10z} \end{bmatrix} = \begin{bmatrix} 0 & -h_9 & 0 \\ h_9 & 0 & -b_4 \\ 0 & b_4 & 0 \end{bmatrix} \begin{bmatrix} F_{10x} \\ F_{10y} \\ F_{10z} \end{bmatrix}$$

$\underset{\sim}{m}_6$ = the couples act on the semitrailer at the fifth wheel

$$\underset{\sim}{m}_6 = \begin{bmatrix} M_{10x} \\ M_{10y} \\ M_{10z} \end{bmatrix}$$

$\underset{\sim}{m}_7$ = the moments about the centre of mass of the forces transmitted to the semitrailer at the vertical shaft

$$\underset{\sim}{m}_7 = \begin{bmatrix} -b_3 \\ 0 \\ h_{10} \end{bmatrix} \times \begin{bmatrix} F_{11x} \\ F_{11y} \\ F_{11z} \end{bmatrix} = \begin{bmatrix} 0 & -h_{10} & 0 \\ h_{10} & 0 & b_3 \\ 0 & -b_3 & 0 \end{bmatrix} \begin{bmatrix} F_{11x} \\ F_{11y} \\ F_{11z} \end{bmatrix}$$

$\underset{\sim}{m}_8$ = the couples act on the semitrailer at the vertical shaft

$$\underset{\sim}{m}_8 = \begin{bmatrix} M_{11x} \\ M_{11y} \\ M_{11z} \end{bmatrix}$$

3.2.c Equations of Motion for the Bogie

Similarly, the equations of motion for the bogie can be obtained. From Figure 3.5 and by applying Newton's second law:

$$m_b \frac{d}{dt} (\underset{\sim}{V}_b) = \underset{\sim}{f}_5 + \underset{\sim}{f}_6 \qquad (3.2.5)$$

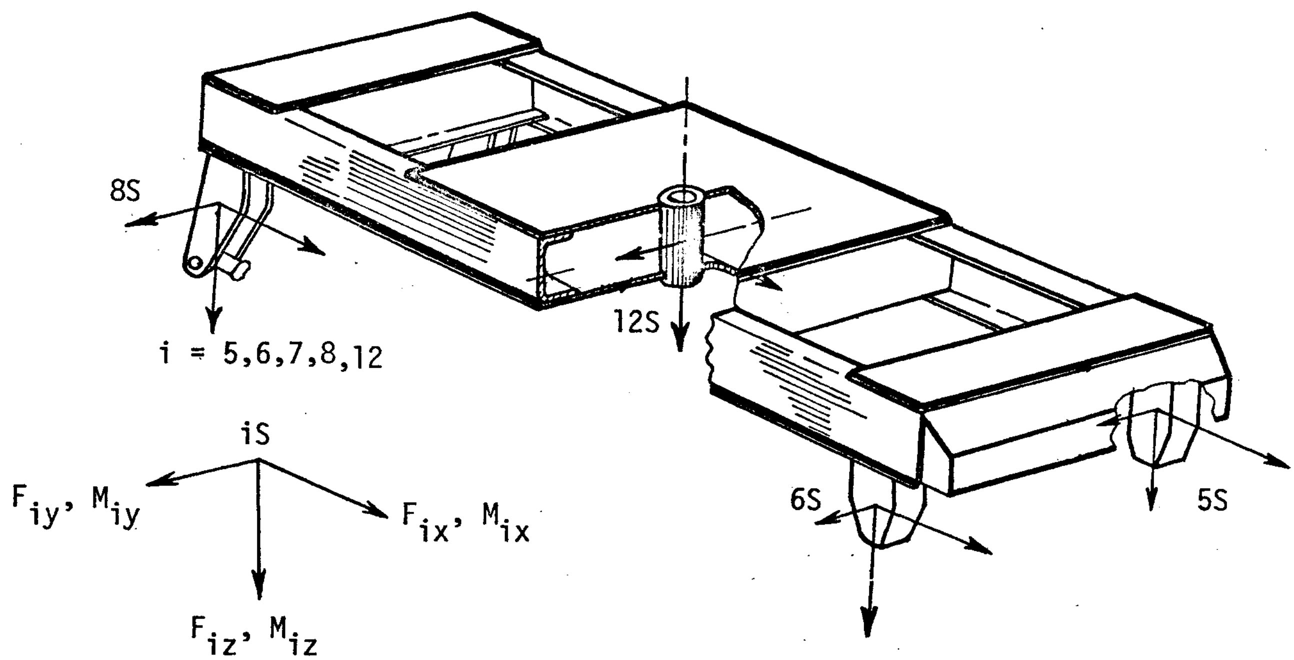

FIGURE 3.5 GENERAL FORCES AND COUPLES ON BOGIE SPRUNG MASS

where

m_b = the mass of the bogie

$$\underline{V}_b = \begin{bmatrix} \dot{u}_b \\ \dot{v}_b \\ \dot{w}_b \end{bmatrix} \quad \text{the velocity of the bogie centre of mass}$$

$\underline{f}_5$ = the forces transmitted from the semitrailer to the bogie at the vertical shaft

$$\underline{f}_5 = \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{12z} \end{bmatrix} = - \underline{f}_4$$

$\underline{f}_6$ = the sum of the forces which act on the bogie at the suspension points

$$\underline{f}_6 = \begin{bmatrix} F_{5x} + F_{6x} + F_{7x} + F_{8x} \\ F_{5Y} \quad F_{6y} + F_{7y} + F_{8y} \\ F_{5z} \quad F_{6z} + F_{7z} + F_{8z} \end{bmatrix}$$

and

$$\frac{d}{dt} (\underline{I}_b \, \underline{\omega}_b) = \underline{m}_9 + \underline{m}_{10} + \underline{m}_{11} \qquad (3.2.6)$$

where

$\underline{I}_b$ is the inertia tensor of the bogie, and it is diagonal since the axes xyz are regarded as being principal axes for the bogie.

This inertia tensor can be written as:

$$\underline{I}_b = \begin{bmatrix} I_{bxx} & 0 & 0 \\ 0 & I_{byy} & 0 \\ 0 & 0 & I_{bzz} \end{bmatrix}$$

$$\underline{\omega}_b = \begin{bmatrix} \dot{\alpha}_b \\ \dot{\beta}_b \\ \dot{\gamma}_b \end{bmatrix}$$ the angular velocity of the bogie mass centre

$\underline{m}_9$ = the couples act on the bogie from the semitrailer at the vertical shaft.

$$\underline{m}_9 = \begin{bmatrix} M_{12x} \\ M_{12y} \\ M_{12z} \end{bmatrix} = - \underline{m}_8$$

The moment $\underline{m}_{10}$ is the moment of $\underline{f}_6$ about the centre of mass of the bogie.

$$\underline{m}_{10} = \begin{bmatrix} b_5 \\ -a_5 \\ h_5 \end{bmatrix} \times \begin{bmatrix} F_{5x} \\ F_{5y} \\ F_{5z} \end{bmatrix} + \begin{bmatrix} b_5 \\ a_5 \\ h_5 \end{bmatrix} \times \begin{bmatrix} F_{6x} \\ F_{6y} \\ F_{6z} \end{bmatrix}$$

$$+ \begin{bmatrix} -b_5 \\ -a_7 \\ h_7 \end{bmatrix} \times \begin{bmatrix} F_{7x} \\ F_{7y} \\ F_{7z} \end{bmatrix} + \begin{bmatrix} -b_5 \\ a_7 \\ h_7 \end{bmatrix} \times \begin{bmatrix} F_{8x} \\ F_{8y} \\ F_{8z} \end{bmatrix}$$

$$= \begin{bmatrix} -h_5(F_{5y}+F_{6y}) - h_7(F_{7y}+F_{8y}) - a_5(F_{5z}-F_{6z}) - a_7(F_{7z}-F_{8z}) \\ h_5(F_{5x}+F_{6x}) + h_7(F_{7x}+F_{8y}) - b_5(F_{5z}+F_{6z}-F_{7z}-F_{8z}) \\ a_5(F_{5x}-F_{6x}) + a_7(F_{7x}-F_{8x}) + b_5(F_{5y}+F_{6y}+F_{7y}+F_{8y}) \end{bmatrix}$$

$\underline{m}_{11}$ = the sum of the dynamic moments, transmitted to the bogie at the suspension attachment points

$$\underline{m}_{11} = \begin{bmatrix} M_{5x} + M_{6x} + M_{7x} + M_{8x} \\ M_{5Y} + M_{6y} + M_{7y} + M_{8y} \\ M_{5z} + M_{6z} + M_{7z} + M_{8z} \end{bmatrix}$$

3.2.d Equations of Motion for Tractor Front Axle in General Form

The equations of motion for tractor front axle can be written in vector form in terms of the forces and moments which act on the axle.

Since all components of the suspension between the axle and sprung mass are assumed massless, the forces which act at the suspension - sprung mass attachment point can be regarded as acting at the associated suspension - axle attachment point.

Figure 3.6 shows axle 1 and the forces and couples which act on it. Definition for Z_1, Z_2, Y_1 and Y_2 are given in Appendix I.

$$Z_1 = + K_{t1} (w_1 - a_2\alpha_1 - G_1(t)) + C_{t1}(\dot{w}_1 - a_2\dot{\alpha}_1 - \dot{G}_1(t))$$

$$Z_2 = + K_{t1} (w_1 + a_2\alpha_1 - G_2(t)) + C_{t1}(\dot{w}_1 + a_2\dot{\alpha}_1 - \dot{G}_2(t))$$

$$Y_1 = K_{t2} (v_1 - r_1\alpha_1 - \lambda_1(t)) + C_{t2}(\dot{v}_1 - r_1\dot{\alpha}_1 - \dot{\lambda}_1(t))$$

$$Y_2 = K_{t2} (v_1 - r_1\alpha_1 - \lambda_2(t)) + C_{t2}(\dot{v}_1 - r_1\dot{\alpha}_1 - \dot{\lambda}_2(t))$$

The moments M_{x1} and M_{y1} are added to the body free diagram of Figure 3.6 in order to maintain equilibrium about the x and y axes.

$$M_x = -(F_{1y} + F_{2y})\, d_1$$

$$M_y = (F_{1x} + F_{2x})\, \ell_1$$

where d_1 is the distance from the centre of mass of axle 1 to the acting point of the dynamic forces of the horizontal suspension stiffness-damping system.

By applying Newton's second law:

$$m_1 \frac{d}{dt} (\underline{V}_1) = \underline{f}_7 + \underline{f}_8 \qquad (3.2.7)$$

where

m_1 = the mass of the tractor front axle

For Axle 1 $i = 1, j = 2$
For Axle 3 $i = 3, j = 4$
For Axle 5 $i = 5, j = 6$
For Axle 7 $i = 7, j = 8$

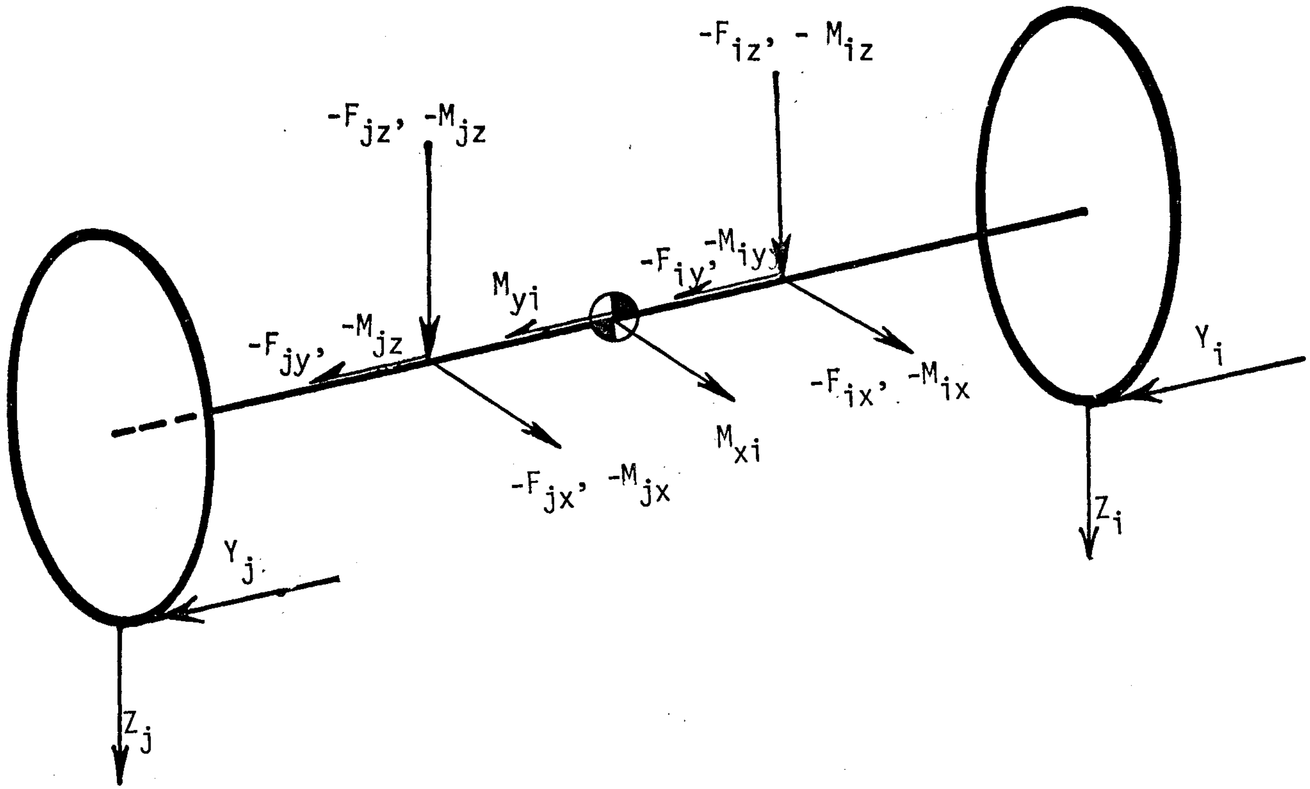

FIGURE 3.6 GENERAL FORCES AND COUPLES ON THE AXLE

$$\underline{V}_1 = \begin{bmatrix} \dot{u}_1 \\ \dot{v}_1 \\ \dot{w}_1 \end{bmatrix}$$ the velocity of the centre of mass of the tractor front axle.

$\underline{f}_7$ = the sum of the suspension forces which act on the axle

$$\underline{f}_7 = \begin{bmatrix} F_{1x} + F_{2x} \\ F_{1y} + F_{2y} \\ F_{1z} + F_{2z} \end{bmatrix}$$

$\underline{f}_8$ = the sum of the road input forces

$$\underline{f}_8 = \begin{bmatrix} 0 \\ Y_1 + Y_2 \\ Z_1 + Z_2 \end{bmatrix}$$

$$\frac{d}{dt}(\underline{I}_1\ \underline{\omega}_1) = \underline{m}_{12} + \underline{m}_{13} + \underline{m}_{14} \tag{3.2.8}$$

where $\underline{I}_1$ is the inertia tensor for tractor front axle. It is assumed diagonal since the axes xyz of the axle are regarded as being the principal axes for the axle.

$$\underline{I}_1 = \begin{bmatrix} I_{1xx} & 0 & 0 \\ 0 & I_{1yy} & 0 \\ 0 & 0 & I_{1zz} \end{bmatrix}$$

$$\underline{\omega}_1 = \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\beta}_1 \\ \dot{\gamma}_1 \end{bmatrix}$$ the angular velocity of the axle

$\underline{m}_{12}$ = the sum of the moments due to suspension forces

$$\underline{m}_{12} = \begin{bmatrix} a_1(F_{1z} - F_{2z}) - d_1(F_{1y} + F_{2y}) \\ \ell_1(F_{1x} + F_{2x}) \\ a_1(F_{2x} + F_{1x}) \end{bmatrix}$$

$\underline{m}_{13}$ = the suspension couples which act on the axle

$$\underline{m}_{13} = \begin{bmatrix} M_{1x} + M_{2x} \\ M_{1y} + M_{2y} \\ M_{1z} + M_{2z} \end{bmatrix}$$

$\underline{m}_{14}$ = the sum of the moments due to road forces

$$\underline{m}_{14} = \begin{bmatrix} a_2(Z_2 - Z_1) - r_1(Y_1 + Y_2) \\ 0 \\ 0 \end{bmatrix}$$

A complete derivation of the equations of motion of the vehicle are given in detail in Appendix I.

3.3 Equations of Constraints

3.3.1.a Constraints Between the Tractor and Semitrailer

The position of the fifth wheel king pin on the tractor must be coincident with its position on the semitrailer.

Let

$$\underline{r}_1 = \begin{bmatrix} -b_6 \\ 0 \\ h_8 \end{bmatrix}$$ the position vector from the centre of mass of the tractor to the fifth wheel king pin

and

$$\underline{r}_2 = \begin{bmatrix} b_4 \\ 0 \\ h_9 + h_{11} \end{bmatrix}$$ the position vector from the semitrailer mass centre to the fifth wheel king pin

then

$$\begin{bmatrix} u_s \\ v_s \\ w_s \end{bmatrix} + \begin{bmatrix} \alpha_s \\ \beta_s \\ \gamma_s \end{bmatrix} \times \begin{bmatrix} b_4 \\ 0 \\ h_9 + h_{11} \end{bmatrix} = \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} + \begin{bmatrix} \alpha_t \\ \beta_t \\ \gamma_t \end{bmatrix} \times \begin{bmatrix} -b_6 \\ 0 \\ h_8 \end{bmatrix}$$

i.e.

$$\begin{bmatrix} u_s \\ v_s \\ w_s \end{bmatrix} = \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} + \begin{bmatrix} h_8\beta_t \\ -h_8\alpha_t - b_6\gamma_t \\ b_6\beta_t \end{bmatrix} - \begin{bmatrix} (h_9+h_{11})\beta_s \\ -(h_9+h_{11})\alpha_s + b_4\gamma_s \\ -b_4\beta_s \end{bmatrix} \qquad (3.3.1)$$

$$\alpha_s = \alpha_t \qquad (3.3.2)$$

3.3.1.b Constraints Between Semitrailer and Bogie

Let

$$\underline{r}_3 = \begin{bmatrix} -b_3 \\ 0 \\ h_{10} \end{bmatrix}$$ the position vector from the semitrailer mass centre to the vertical shaft connecting the semitrailer to the bogie.

then

$$\begin{bmatrix} u_b \\ v_b \\ w_b \end{bmatrix} = \begin{bmatrix} u_s \\ v_s \\ w_s \end{bmatrix} + \begin{bmatrix} \alpha_s \\ \beta_s \\ \gamma_s \end{bmatrix} \times \begin{bmatrix} -b_3 \\ 0 \\ h_{10} \end{bmatrix}$$

$$= \begin{bmatrix} u_s \\ v_s \\ w_s \end{bmatrix} + \begin{bmatrix} h_{10}\beta_s \\ -(h_{10}\alpha_s + b_3\gamma_s) \\ b_3\beta_s \end{bmatrix} \qquad (3.3.3)$$

Substituting from equation (3.3.1) into equation (3.3.3) we obtain:

$$\begin{bmatrix} u_b \\ v_b \\ w_b \end{bmatrix} = \begin{bmatrix} u_t \\ v_t \\ w_t \end{bmatrix} + \begin{bmatrix} h_8\beta_t - (h_9 + h_{11} - h_{10})\beta_s \\ (h_9+h_{11}-h_8-h_{10})\alpha_t - b_6\,\gamma_t - (b_3+b_4)\gamma_s \\ b_6\beta_t + (b_3 + b_4)\beta_s \end{bmatrix}$$

(3.3.4)

The bogie must roll and pitch with the semitrailer:

$$\alpha_b = \alpha_s = \alpha_t \tag{3.3.5}$$

$$\beta_b = \beta_s \tag{3.3.6}$$

3.3.1.c Constraints Between the Sprung Masses and the Axles

All the axles can translate in the vertical and lateral directions and can rotate about the longitudinal axis. The axle must yaw and pitch with the sprung mass.

(a) Tractor Front Axle

$$u_1 = u_t + (h_1 + \ell_1)\beta_t \tag{3.3.7}$$

$$\beta_1 = \beta_t \tag{3.3.8}$$

$$\gamma_1 = \gamma_t \tag{3.3.9}$$

(b) Tractor Rear Axle

$$u_3 = u_t + (h_3 + \ell_3)\beta_t \tag{3.3.10}$$

$$\beta_3 = \beta_t \tag{3.3.11}$$

$$\gamma_3 = \gamma_t \tag{3.3.12}$$

(c) Bogie Front Axle

$$u_5 = u_b + (h_5 + \ell_5)\beta_b$$

$$= u_t + h_8\beta_t + (h_{10}+h_5+\ell_5-h_9-h_{11})\beta_s \qquad (3.3.13)$$

$$\beta_5 = \beta_b = \beta_s \qquad (3.3.14)$$

$$\gamma_5 = \gamma_b \qquad (3.3.15)$$

(d) Bogie Rear Axle

$$u_7 = u_b + (h_7 + \ell_7)\beta_b$$

$$= u_t + h_8\beta_t + (h_7+h_{10}+\ell_7-h_9-h_{11})\beta_s \qquad (3.3.16)$$

$$\beta_7 = \beta_b = \beta_s \qquad (3.3.17)$$

$$\gamma_7 = \gamma_b \qquad (3.3.18)$$

Since there is no stiffnesses or dampings in the longitudinal direction, the vehicle moves as a rigid body in the longitudinal direction and u_t is an ignorable coordinate and may be substituted out of the differential equations of motion thus reducing the number of degrees of freedom by one.

By adding the equations of motion of all the components of the vehicle in the x direction we obtain:

$$m_t\ddot{u}_t+m_s\ddot{u}_s+m_b\ddot{u}_b+m_1\ddot{u}_1+m_3\ddot{u}_3+m_5\ddot{u}_5+m_7\ddot{u}_7 = 0 \qquad (3.3.19)$$

Integrating twice with respect to time, we obtain:

$$m_t u_t + m_s u_s + m_b u_b + m_1 u_1 + m_3 u_3 + m_5 u_5 + m_7 u_7 = E_1 t + E_2 \qquad (3.3.20)$$

where E_1 and E_2 are constants to be determined from the initial conditions.

For the velocity:

$$\left.(m_t \dot{u}_t + m_s \dot{u}_s + m_b \dot{u}_b + m_1 \dot{u}_1 + m_3 \dot{u}_3 + m_5 \dot{u}_5 + m_7 \dot{u}_7)\right|_{t=0} = E_1 \qquad (3.3.21)$$

For the displacement:

$$\left.(m_t u_t + m_s u_s + m_b u_b + m_1 u_1 + m_3 u_3 + m_5 u_5 + m_7 u_7)\right|_{t=0} = E_2 \qquad (3.3.22)$$

But u_t describes only the oscillatory motions of the tractor centre of gravity. Thus E_1 must be equal to zero, otherwise u_t would grow with time. E_2 may also be equal to zero by assuming that u_t is to be at the centre of gravity of the tractor when all of the independent coordinates are in their neutral positions.

By substituting from the constraint equations, we obtain:

$$\begin{aligned} & m_t u_t + m_s(u_t + h_8\beta_t - (h_9 + h_{11})\beta_s) \\ & \quad + m_b(u_t + h_8\beta_t - (h_9 + h_{11} - h_{10})\beta_s) \\ & \quad + m_1(u_t + (h_1 + \ell_1)\beta_t) \\ & \quad + m_3(u_t + (h_3 + \ell_3)\beta_t) \\ & \quad + m_5(u_t + h_8\beta_t + (h_{10} + h_5 + \ell_5 - h_9 - h_{11})\beta_s) \\ & \quad + m_7(u_t + h_8\beta_t + (h_{10} + h_7 + \ell_7 - h_9 - h_{11})\beta_s) \\ & \quad = 0 \end{aligned} \qquad (3.3.23)$$

thus,

$$u_t = \rho_1 \beta_t - \rho_2 \beta_s \tag{3.3.24}$$

where:

$$\rho_1 = [(m_s+m_b+m_5+m_7)h_8+m_1(h_1+\ell_1) + m_3(h_3+\ell_3)]/M$$

$$\rho_2 = [(m_s(h_9+h_{11})+m_b(h_9+h_{11}-h_{10}) + m_5(h_9+h_{11}-h_{10}-h_5-\ell_5) + m_7(h_9+h_{11}-h_{10}-h_7-\ell_7)]/M$$

and

$$M = m_t+m_s+m_b+m_1+m_3+m_5+m_7$$

3.3.2 The Transformation Matrix [D]

In this section the transformation matrix D which gives the relation between all the variables and the independent variables based on the equations of constraints is defined in Table 3.1.

Where

$$\rho_3 = \rho_1 + h_8$$

$$\rho_4 = \rho_2 - (h_9 + h_{11})$$

$$\rho_5 = h_9 + h_{11} - h_8$$

$$\rho_6 = \rho_2 - (h_9 + h_{11} - h_{10})$$

$$\rho_7 = h_9 + h_{11} - h_{10} - h_8$$

TABLE 3.1

THE TRANSFORMATION MATRIX $[D]_{42\times20}$

(Articulated Vehicle With Self-Steering Semitrailer)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		v_t	w_t	α_t	β_t	γ_t	β_s	γ_s	γ_b	v_1	w_1	α_1	v_3	w_3	α_3	v_5	w_5	α_5	v_7	w_7	α_7
1	u_t				ρ_1		ρ_2														
2	v_t	1																			
3	w_t		1																		
4	α_t			1																	
5	β_t				1																
6	γ_t					1															
7	u_s				ρ_3		ρ_4														
8	v_s	1		ρ_5		$-b_6$		$-b_4$													
9	w_s		1		b_6		b_4														
10	α_s			1																	
11	β_s						1														
12	γ_s							1													
13	u_b				ρ_3		ρ_6														
14	v_b	1		ρ_7		$-b_6$		$-\rho_8$													
15	w_b		1		b_6		ρ_6														
16	α_b			1																	
17	β_b						1														
18	γ_b								1												
19	u_1				ρ_9		ρ_2														
20	v_1									1											
21	w_1										1										
22	α_1											1									
23	β_1				1																
24	γ_1					1															
25	u_3				ρ_{10}		ρ_2														
26	v_3												1								
27	w_3													1							
28	α_3														1						
29	β_3				1																
30	γ_3					1															
31	u_5				ρ_3		ρ_{11}														
32	v_5															1					
33	w_5																1				
34	α_5																	1			
35	β_5						1														
36	γ_5								1												
37	u_7				ρ_3		ρ_{12}														
38	v_7																		1		
39	w_7																			1	
40	α_7																				1
41	β_7						1														
42	γ_7								1												

$$\rho_8 = b_3 + b_4$$

$$\rho_9 = \rho_1 + (h_1 + \ell_1)$$

$$\rho_{10} = \rho_1 + (h_3 + \ell_3)$$

$$\rho_{11} = \rho_2 + (h_{10} + h_5 + \ell_5 - h_9 - h_{11})$$

$$\rho_{12} = \rho_2 + (h_{10} + h_7 + \ell_7 - h_9 - h_{11})$$

3.4 Manipulation of the Equations

3.4.1 Tractor-Self-Steering Semitrailer

The system of second order linear differential equations is written in the form:

$$\underset{42\times42}{[A]}\ \underset{42\times1}{\{\ddot{Z}\}} + \underset{42\times42}{[C]}\ \underset{42\times1}{\{\dot{Z}\}} + \underset{42\times42}{[B]}\ \underset{42\times1}{\{Z\}} + \underset{42\times1}{\{R\}} = \underset{42\times1}{\{Q\}} \tag{3.4.1}$$

where

A = Inertia matrix

C = Damping matrix

B = Stiffness matrix

R = Vector of the internal reactions

Q = Vector representing the input to the system

Z = Vector representing the coordinates of the various degrees of freedom.

We eliminate the internal reactions by the method of substitution and thus we get a system of 20 equations in 42 variables; twenty of these variables are independent.

3.4.1.a Elimination of Internal Reactions

VARIABLE	EQUATION NUMBER	REACTION
u_t	1	$-(F_{1x}+F_{2x}+F_{3x}+F_{4x})-F_{5x}$
v_t	2	$-F_{9y}$
w_t	3	$-F_{9z}$
α_t	4	$h_8F_{9y}-M_{9x}$
β_t	5	$-h_1(F_{1x}+F_{2x})-h_3(F_{3x}+F_{4x})$
		$-h_8F_{9x}-b_6F_{9z}-(M_{1y}+M_{2y}+M_{3y}+M_{4y})$
γ_t	6	$-a_1(F_{1x}-F_{2x})-a_3(F_{3x}-F_{4x})+b_6F_{9y}$
u_s	7	$F_{9x}-F_{11x}$
v_s	8	$F_{9y}-F_{11y}$
w_s	9	$F_{9z}-F_{11z}$
α_s	10	$-(h_9+h_{11})F_{9y}+M_{9x}+h_{10}F_{11y}-M_{11x}$
β_s	11	$(h_9+h_{11})F_{9x}-b_4F_{9z}-h_{10}F_{11x}-b_3F_{11z}$
		$-M_{11y}$
γ_s	12	$b_4F_{9y}+b_3F_{11y}$
u_b	13	$F_{11x}-(F_{5x}+F_{6x}+F_{7x}+F_{8x})$
v_b	14	F_{11y}

VARIABLE	EQUATION NUMBER	REACTION
w_b	15	F_{11z}
α_b	16	M_{11x}
β_b	17	$-h_5(F_{5x}+F_{6x})-h_7(F_{7x}+F_{8x})$
		$+M_{11y}-(M_{5y}+M_{6y}+M_{7y}+M_{8y})$
γ_b	18	$-a_5(F_{5x}-F_{6x})-a_7(F_{7x}-F_{8x})$
u_1	19	$F_{1x}+F_{2x}$
v_1	20	0
w_1	21	0
α_1	22	0
β_1	23	$-\ell_1(F_{1x}+F_{2x})+(M_{1y}+M_{2y})$
γ_1	24	$a_1(F_{1x}-F_{2x})$
u_3	25	$F_{3x}+F_{4x}$
v_3	26	0
w_3	27	0
α_3	28	0
β_3	29	$-\ell_3(F_{3x}+F_{4x})+(M_{3y}+M_{4y})$
γ_3	30	$a_3(F_{3x}-F_{4x})$
u_5	31	$F_{5x}+F_{6x}$

VARIABLE	EQUATION NUMBER	REACTION
v_5	32	0
w_5	33	0
α_5	34	0
β_5	35	$-\ell_5(F_{5x}+F_{6x})+(M_{5y}+M_{6y})$
γ_5	36	$a_5(F_{5x}-F_{6x})$
u_7	37	$F_{7x}+F_{8x}$
v_7	38	0
w_7	39	0
α_7	40	0
β_7	41	$-\ell_7(F_{7x}+F_{8x})+(M_{7y}+M_{8y})$
γ_7	42	$a_7(F_{7x} - F_{8x})$

To eliminate the vector R the procedure described below is adopted where numbers refer to equations in Appendix I:*

$$1 = 2 + 8 + 14$$

$$2 = 3 + 9 + 15$$

$$3 = 4 + 10 + 16 + (h_9+h_{11}-h_8)\ (8)$$
$$+ (h_9+h_{11}-h_{10}-h_8)\ (14)$$

*For simplicity Equations 1, 2, ------, 42, refer to Equations (I.104), (I.105), ------, (I.145).

$$4 = 5 + b_6(9+15)+h_8(7+13+31+37)$$
$$+(h_1+\ell_1)\ (19) + (h_3+\ell_3)\ (25)$$
$$+ 23 + 29$$

$$5 = 6 + 24 + 30 - b_6\ (8 + 14)$$

$$6 = 11 + 17 + 35 + 41 + b_4\ (9) + (b_3+b_4)\ (15)$$
$$+ (h_{10}-h_9-h_{11})\ (13) - (h_9+h_{11})\ (7)$$
$$+ (h_{10}+h_5+\ell_5-h_9-h_{11})\ (31)$$
$$+ (h_{10}+h_7+\ell_7-h_9-h_{11})\ (37)$$

$$7 = 12 - b_4\ (8) - (b_3+b_4)\ (14)$$

$$8 = 18 + 36 + 42$$

$$9 = 20$$

$$10 = 21$$
$$11 = 22$$
$$12 = 26$$
$$13 = 27$$
$$14 = 28$$
$$15 = 32$$
$$16 = 33$$
$$17 = 34$$
$$18 = 38$$
$$19 = 39$$
$$20 = 40$$

This procedure is applied to the mass, stiffness and damping matrices successively, and it gives:

$$\underset{20\times42}{[A]}\;\underset{42\times1}{\{\ddot{Z}\}} + \underset{20\times42}{[C]}\;\underset{42\times1}{\{Z\}} + \underset{20\times42}{[B]}\;\underset{42\times1}{\{Z\}} = \underset{20\times1}{\{F\}} \tag{3.4.2}$$

To eliminate the dependent variables, we use the transformation matrix [D] which gives the relation between all the variables and the independent variables and which is based on the equations of constraints.

Let

$$\underset{42\times1}{\{Z\}} = \underset{42\times20}{[D]}\;\underset{20\times1}{\{X\}}$$

Therefore, equation 3.4.2 becomes:

$$\underset{20\times42}{[A]}\;\underset{42\times20}{[D]}\;\underset{20\times1}{\{\ddot{X}\}} + \underset{20\times42}{[C]}\;\underset{42\times20}{[D]}\;\underset{20\times1}{\{\dot{X}\}} + \underset{20\times42}{[B]}\;\underset{42\times20}{[D]}\;\underset{20\times1}{\{X\}} = \underset{20\times1}{\{F\}} \tag{3.4.3}$$

i.e.

$$\underset{20\times20}{[AD]}\;\underset{20\times1}{\{\ddot{X}\}} + \underset{20\times20}{[CD]}\;\underset{20\times1}{\{\dot{X}\}} + \underset{20\times20}{[BD]}\;\underset{20\times1}{\{X\}} = \underset{20\times1}{\{F\}} \tag{3.4.4}$$

The above matrix equation gives twenty simultaneous second order differential equations in twenty unknowns.

3.4.2 Tractor-Standard Semitrailer

If the bogie is locked to the semitrailer we obtain the standard semitrailer. In this case, there is an extra unknown reaction M_{12z} between the semitrailer and the bogie. To eliminate the reactions we use the method of substitution as described below where the numbers refer to equations in Appendix I.*

$$1 = 2 + 8 + 14$$

$$2 = 3 + 9 + 15$$

$$\begin{aligned} 3 = {} & 4 + (h_9 + h_{11} - h_8)\ (8) + 10 \\ & + (h_9 + h_{11} - h_{10} - h_8)\ (14) + 16 \end{aligned}$$

$$\begin{aligned} 4 = {} & 5 + h_8\ (7 + 13 + 31 + 37) + b_6\ (9 + 15) \\ & + (h_1 + \ell_1)\ (19) + 23 + (h_3 + \ell_3)\ (25) \\ & + 29 \end{aligned}$$

$$5 = 6 - b_6(8 + 14) + 24 + 30$$

$$\begin{aligned} 6 = {} & 11 - (h_9 + h_{11})\ (7) + (h_{10} - h_9 - h_{11})\ (13) \\ & + b_4\ (9) + (b_3 + b_4)\ (15) + 17 \\ & + (h_{10} + h_5 + \ell_5 - h_9 - h_{11})\ (31) + 35 \\ & + (h_{10} + h_7 + \ell_7 - h_9 - h_{11})\ (37) + 41 \end{aligned}$$

*For simplicity Equations 1, 2, ..., 42 refer to Equations (I.104), (I.105), ..., (I.145).

$7 = 12 + 18 + 36 + 42 - b_4(8) - (b_3 + b_4)\ (14)$

$8 = 20$

$9 = 21$

$10 = 22$

$11 = 26$

$12 = 27$

$13 = 28$

$14 = 32$

$15 = 33$

$16 = 34$

$17 = 38$

$18 = 39$

$19 = 40$

In the computer program this procedure is applied to the mass, stiffness and damping matrices successively, giving:

$$\underset{19\times42}{[A]}\ \underset{42\times1}{\{\ddot{Z}\}} + \underset{19\times42}{[C]}\ \underset{42\times1}{\{\dot{Z}\}} + \underset{19\times42}{[B]}\ \underset{42\times1}{\{Z\}} = \underset{19\times1}{\{F\}} \qquad (3.4.2.1)$$

Using the transformation matrix [D] which is defined in Table 3.2 to eliminate the dependent variables, we obtain a system of nineteen second order linear differential equations in nineteen variables.

Let

$$\underset{42\times1}{\{Z\}} = \underset{42\times19}{[D]} \; \underset{19\times1}{\{X\}}$$

Equation (3.4.2.1) becomes

$$\underset{19\times42}{[A]} \; \underset{42\times19}{[D]} \; \underset{19\times1}{\{\ddot{X}\}} + \underset{19\times42}{[C]} \; \underset{42\times19}{[D]} \; \underset{19\times1}{\{\dot{X}\}} + \underset{19\times42}{[B]} \; \underset{42\times19}{[D]} \; \underset{19\times1}{\{X\}} = \underset{19\times1}{\{F\}} \qquad (3.4.2.2)$$

or

$$\underset{19\times19}{[AD]} \; \underset{19\times1}{\{\ddot{X}\}} + \underset{19\times19}{[CD]} \; \underset{19\times1}{\{\dot{X}\}} + \underset{19\times19}{[BD]} \; \underset{19\times1}{\{X\}} = \underset{19\times1}{\{F\}} \qquad (3.4.2.3)$$

Note that the equations involving the thirteen variables v_t, α_t, γ_t, γ_s, γ_b, v_1, α_1, v_3, α_3, v_5, α_5, v_7, α_7 in case of self-steering semitrailer and the twelve variables v_t, α_t, γ_t, γ_s, v_1, α_1, v_3, α_3, v_5, α_5, v_7, α_7 in case of standard semitrailer, are coupled to the equations involving w_t, β_t, β_s, w_1, w_3, w_5, w_7 only be the products of inertia, I_{sxy}, I_{syz}. The lateral and vertical motions are, hence, uncoupled for situations in which the semitrailer and its load are symmetric in the xy and yz planes.

TABLE 3.2

THE TRANSFORMATION MATRIX $[D]_{42x19}$

(Articulated Vehicle With a Standard Semitrailer)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		v_t	w_t	α_t	β_t	γ_t	β_s	γ_s	v_1	w_1	α_1	v_3	w_3	α_3	v_5	w_5	α_5	v_7	w_7	α_7
1	u_t				ρ_1		ρ_2													
2	v_t	1																		
3	w_t		1																	
4	α_t			1																
5	β_t				1															
6	γ_t					1														
7	u_s				ρ_3		ρ_4													
8	v_s	1		ρ_5		$-b_6$		$-b_4$												
9	w_s		1		b_6		b_4													
10	α_s			1																
11	β_s						1													
12	γ_s							1												
13	u_b				ρ_3		ρ_6													
14	v_b	1		ρ_7		$-b_6$		$-\rho_8$												
15	w_b		1		b_6		ρ_6													
16	α_b			1																
17	β_b						1													
18	γ_b							1												
19	u_1				ρ_3		ρ_2													
20	v_2								1											
21	w_1									1										
22	α_1										1									
23	β_1				1															
24	γ_1					1														
25	u_3				ρ_{10}		ρ_2													
26	v_3											1								
27	w_3												1							
28	α_3													1						
29	β_3				1															
30	γ_3					1														
31	u_5				ρ_3		ρ_{11}													
32	v_5														1					
33	w_5															1				
34	α_5																1			
35	β_5						1													
36	γ_5							1												
37	u_7				ρ_3		ρ_{12}													
38	v_7																	1		
39	w_7																		1	
40	α_7																			1
41	β_7						1													
42	γ_7							1												

CHAPTER 4

THEORY OF SOLUTIONS OF THE DIFFERENTIAL EQUATIONS OF MOTION

For an n-degree of freedom system the equations of motion could be written in the matrix form:

$$[M]\ \{\ddot{X}\} + [C]\ \{\dot{X}\} + [K]\ \{X\} = \{F\} \tag{4.1}$$

where M, C and K are the mass, damping and stiffness matrices respectively. F and X are the vectors representing the complex forces and complex displacements respectively.

The general solution of Equation (4.1) is composed of a complementary function and a particular solution.

a) Complementary Function

The homogeneous matrix equation

$$[M]\ \{\ddot{X}\} + [C]\ \{\dot{X}\} + [K]\ \{X\} = \{0\} \tag{4.2}$$

could be rearranged into the form

$$\left[\begin{array}{c|c} [M] & [0] \\ \hline [C] & [M] \end{array}\right] \left\{\begin{array}{c} \{\dot{X}\} \\ \hline \{\ddot{X}\} \end{array}\right\} + \left[\begin{array}{c|c} [0] & -[M] \\ \hline [K] & [0] \end{array}\right] \left\{\begin{array}{c} \{X\} \\ \hline \{\dot{X}\} \end{array}\right\} = \left\{\begin{array}{c} \{0\} \\ \hline \{0\} \end{array}\right\} \tag{4.3}$$

This equation, often referred to as the "reduced" form of (4.2) can be written as:

$$[A]\,\{\dot{q}\} + [B]\,\{q\} = \{0\} \tag{4.4}$$

where

$$[A] = \left[\begin{array}{c|c} [M] & [0] \\ \hline [C] & [M] \end{array}\right]$$

$$[B] = \left[\begin{array}{c|c} [0] & -[M] \\ \hline [K] & [0] \end{array}\right]$$

$$\{q\} = \left\{\begin{array}{c} \{X\} \\ \hline \{\dot{X}\} \end{array}\right\}$$

The column matrix $\{q\}$ is of order 2n.

In general, [A] is a nonsingular matrix, and its inverse is:

$$[A]^{-1} = \begin{bmatrix} [M]^{-1} & [0] \\ -[M]^{-1}[C][M]^{-1} & [M]^{-1} \end{bmatrix} \tag{4.5}$$

Premultiplying (4.4) by $[A]^{-1}$ yields

$$\{\dot{q}\} + [A]^{-1}\,[B]\,\{q\} = \{0\} \tag{4.6}$$

Let

$$[G] = -[A]^{-1}[B]$$

$$= \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \tag{4.7}$$

where [G] is a 2nth-order matrix.

Equation (4.6) can be written as:

$$\{\dot{q}\} - [G]\{q\} = \{0\} \tag{4.8}$$

We assume a solution in the form

$$\{q\} = e^{\lambda t}\{\phi\} \tag{4.9}$$

where $\{\phi\}$ is a column matrix of 2n constants and λ is a parameter. When {q} is substituted into (4.8) the exponential factors cancel out, and a set of homogeneous equations in ϕ_i is obtained:

$$(-\lambda[I] + [G])\{\phi\} = \{0\} \tag{4.10}$$

Equation (4.10) can be put in the form

$$[G]\{\phi\} = \lambda\{\phi\} \tag{4.11}$$

which is a familiar form for the statement of the eigenvalue problem of the matrix [G]. Here λ is the eigenvalue of the matrix [G]. Equation (4.10) is a system of homogeneous algebraic equations which have non-trivial solutions if and only if the matrix $(-\lambda[I] + [G])$ is singular. Therefore, we have

$$|-\lambda[I] + [G]| = 0 \tag{4.12}$$

which is the characteristic equation of the matrix [G]. The eigenvalues

λ_i of the matrix [G] are the roots of the characteristic equation, which for stable system are real negative or complex with negative real parts. The complex roots occur in a pair of conjugates.

If the roots are complex, the eigenvalues λ_i will be in the form

$$\lambda_i = \mu_i \pm i\omega_i \qquad \text{(for the } i^{th} \text{ root)} \tag{4.13}$$

where:

μ_i represents the damping rate for the particular mode arising for that eigenvalue

ω_i represents the damped natural frequency for the same mode.

For each root or eigenvalue λ_i a set of values for ϕ can be calculated from the system of equations (4.10). $\{\phi\}_i$ is called the modal column or the eigenvector of the matrix [G]. Each modal column satisfies

$$[G]\ \{\phi\}_i = \lambda_i\ \{\phi\}_i \tag{4.14}$$

Since complex roots occur in conjugate pairs, the modal columns associated with the complex roots must also form conjugate pairs in order that the motions $\{X\}$ be real. It is evident that by combining these complex functions, the motions can be expressed as sine and cosine functions with amplitude diminishing exponentially. To each eigenvalue λ_i corresponds a principal mode of frequency ω_i, whose relative amplitudes of oscillation in each coordinate are given by the elements of $\{\phi\}_i$. The reason that $\{\phi\}_i$ gives the relative magnitude instead of the absolute magnitude is that

(4.10) is a system of homogeneous equations in ϕ_i.

Let

$$\{X\} = \{\psi\} \; e^{\lambda t} \tag{4.15}$$

then

$$\{\dot{X}\} = \lambda\{\psi\} \; e^{\lambda t} \tag{4.16}$$

and we have

$$\{q\} = \left\{\begin{array}{c}\{X\}\\ \hline \{\dot{X}\}\end{array}\right\} = \left\{\begin{array}{c}\{\psi\}\\ \hline \lambda\{\psi\}\end{array}\right\} e^{\lambda t} \tag{4.17}$$

Consequently

$$\{\phi\}_i = \left\{\begin{array}{c}\{\psi\}_i\\ \hline \lambda_i\{\psi\}_i\end{array}\right\} \tag{4.18}$$

Hence, the elements of $\{\psi\}$ can be computed from those of $\{\phi\}$.

Alternatively, by combining the modal columns $\{\psi\}$ to obtain the modal matrix $[\psi]$, the complete transient solution to the general problem is given by:

$$\underset{n\times 1}{\{X\}} = \underset{n\times 2n}{[\psi]} \; \underset{2n\times 2n}{[e^{\lambda t}]} \; \underset{2n\times 1}{\{C\}} \tag{4.19}$$

where

$$[e^{\lambda t}] = \begin{bmatrix} e^{\lambda_1 t} & 0 & --- & 0 \\ 0 & e^{\lambda_2 t} & --- & 0 \\ \hline 0 & 0 & ---- & e^{\lambda_{2n} t} \end{bmatrix} \tag{4.20}$$

The constant $\{C\}$ may be determined if the initial conditions on displacements and velocities for all the coordinates are known, i.e. $\{X\}$ and $\{\dot{X}\}$ must be known for time $t = 0$.

b) Particular Solution

The matrix formulation for the particular (steady state) solution is:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F(t)\} = \{F\}e^{i\omega t} \tag{4.21}$$

A solution for $\{X\}$ is assumed as some function of $F(t)$

Let

$$\{X\} = \{\bar{X}\}e^{i\omega t} \tag{4.22}$$

where the column matrix $\{\bar{X}\}$ is allowed to be complex - that is, to contain complex elements. This leads to the equation:

$$[(K - \omega^2 M) + i\omega C]\ \{\bar{X}\} = \{F\} \tag{4.23}$$

in which the quantity in square brackets is a square matrix whose elements are complex. We denote this square matrix by Z. Provided that Z is non-singular it will have an inverse Z^{-1}, and the solution of (4.21) will be:

$$\{\bar{X}\} = [Z]^{-1}\ \{F\} \tag{4.24}$$

We may call Z^{-1} the "complex receptance" matrix of the system.

All the theory for systems having n degrees of freedom has so far related to an arbitrary set of forces, all having the same frequency. If the system is acted upon by forces of different frequencies then the resultant displacement matrix will be the sum of the displacement matrices due to the forces of each frequency taken one frequency at a time.

Solution of equation (4.24) is obtained by using the digital computer.

For the problem in hand, the irregularity of the track is assumed to be sinusoidal, and the equation of motion takes the form:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = [CF]\{\dot{Y}\} + [KF]\{Y\} \qquad (4.25)$$

where

M - inertia matrix

C - damping matrix

K - stiffness matrix

X - complex vector of unknown displacements

Y - excitation displacement vector

CF - Forced damping matrix

KF - Forced stiffness matrix

Now

$$\{Y\} = \{\bar{Y}\}e^{i\omega t}$$

then

$$\{X\} = \{\bar{X}\}e^{i\omega t}$$

and the equation (2.25) will be

$$[(K - \omega^2 M) + i\omega C]\{\bar{X}\} = [KF + i\omega CF]\{\bar{Y}\} \qquad (4.26)$$

i.e.

$$[Z]\{\bar{X}\} = \{F\} \qquad (4.27)$$

or

$$\{\bar{X}\} = [Z]^{-1}\{F\} \qquad (4.28)$$

Thus, the solution for a particular value of frequency ω results in a complex solution vector $\{X\}$, whose individual elements represent the steady state frequency response of the various coordinates of the various degrees of freedom. Each individual complex part can then be written as a magnitude and as a phase angle.

CHAPTER 5

RESULTS AND CONCLUSIONS

5.1 Input to the System

The road profile that is presented to the vehicle tires is a function of the constructional details employed in building the highway. The full joint and the saw cut in the concrete highway construction allow differential settling along the length of the slab, resulting in a small but regularly spaced wave forms.

For this study, a sinusoidal input profile of a one inch amplitude peak to peak in the vertical direction and 0.2 inch amplitude peak to peak in lateral direction is considered to be applied to the vehicle from the road through the tire contact point. The phase angles of road displacement application due to the tractor rear axle and the bogie axles offset from the front tractor axle, are taken into account.

The input frequency of the forcing function is made dependent on the vehicle forward speed and expansion joint spacing. For the left and right track, the sinusoidal irregularities can be in-phase or out-of-phase and the input displacement function may be assumed unequal.

5.2 Results

The results of this section were obtained with the vehicle dimensions and parameters listed in Table 5.1.

The digital computer was used for computational analysis of the derived equations and for plotting the steady state frequency response curves.

5.2.1 Results for Tractor-Self-Steering Semitrailer

In general, the response of multidegree of freedom system is quite complex. However, two characteristics of the dynamic systems which give some indication of the overall response of the vibratory system are described in the following paragraphs, along with the major analytical ideas behind each description. The effect of adding a spring at the fifth wheel is also discussed.

1.a The results of undamped natural frequencies of the system, which are the frequencies at which the system could oscillate when displaced from its equilibrium position, are presented in Table 5.2.

1.b The damped eigenvalues are next presented in Table 5.3. They are generally complex numbers arising in conjugate pairs and are roots of the damped fortieth degree characteristic equation. The real part of a complex eigenvalue represents the damping rate associated with the decay of the amplitude, and the imaginary part is the damped frequency of oscillation.

TABLE 5.1

VEHICLE PARAMETERS

I. General

Tractor weight	14200 lb.
Trailer weight	33500 lb.
Bogie weight	2000 lb.
Tractor front axle weight	800 lb.
Tractor rear axle weight	3200 lb.
Bogie front axle weight	1600 lb.
Bogie rear axle weight	1400 lb.
Tractor inertia tensor (inertias in in.lb./sec^2)	$\begin{bmatrix} 9600 & 0 & 2400 \\ 0 & 30000 & 0 \\ 2400 & 0 & 90000 \end{bmatrix}$
Trailer Inertia tensor (inertias in in.lb./sec^2)	$\begin{bmatrix} 100800 & 0 & 25200 \\ 0 & 1344000 & 0 \\ 25200 & 0 & 1440000 \end{bmatrix}$
Bogie inertia tensor (inertias in in.lb./sec^2)	$\begin{bmatrix} 650 & 0 & 0 \\ 0 & 6500 & 0 \\ 0 & 0 & 8130 \end{bmatrix}$

Table 5.1 (continued)

Tractor Front axle inertia tensor (inertias in in.lb./sec^2)	$\begin{bmatrix} 1728 & 0 & 0 \\ 0 & 189 & 0 \\ 0 & 0 & 1728 \end{bmatrix}$
Tractor rear axle inertia tensor (inertias in in.lb./sec^2)	$\begin{bmatrix} 6480 & 0 & 0 \\ 0 & 470.5 & 0 \\ 0 & 0 & 6480 \end{bmatrix}$
Bogie front axle inertia tensor (inertias in in.lb./sec^2)	$\begin{bmatrix} 3240 & 0 & 0 \\ 0 & 228.3 & 0 \\ 0 & 0 & 3240 \end{bmatrix}$
Bogie rear axle inertia tensor (inertias in in.lb./sec^2)	$\begin{bmatrix} 2835 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 2835 \end{bmatrix}$

II. Dimensions

a_1	19.00 in.	h_3	0.36 in.
a_2	36.50 in.	h_5	9.50 in.
a_3	20.50 in.	h_7	9.50 in.
a_4	36.00 in.	h_8	-12.00 in.

Table 5.1 (continued)

a_5	17.00 in.	h_9	40.00 in.
a_6	37.00 in.	h_{10}	45.00 in.
a_7	17.00 in.	h_{11}	3.00 in.
a_8	37.00 in.	ℓ_1	4.00 in.
b_1	50.00 in.	ℓ_3	11.00 in.
b_2	80.00 in.	ℓ_5	11.00 in.
b_3	156.00 in.	ℓ_7	11.00 in.
b_4	180.00 in.	r_1	20.50 in.
b_5	36.00 in.	r_3	19.70 in.
b_6	55.00 in.	r_5	19.70 in.
h_1	8.00 in.	r_7	19.70 in.

III. Suspension Characteristics

Suspension Spring Constants (lb/in)

K_1	1100.00	K_2	22000.00
K_3	1800.00	K_4	36000.00
K_5	900.00	K_6	18000.00
K_7	900.00	K_8	18000.00

Table 5.1 (continued)

Suspension Damping Constants (lb.sec./in.)			
C_1	33.50	C_2	8.5
C_3	84.00	C_4	21.0
C_5	42.00	C_6	10.5
C_7	42.00	C_8	10.5
Roll Stiffness at Axles 1, 3, 5, 7 (lb.in./rad.)			
K_{r1}	794200.00		
K_{r3}	1512900.00		
K_{r5}	520200.00		
K_{r7}	520200.00		
Roll Damping at Axles 1, 3, 5, 7 (lb.in.sec./rad.)			
C_{r1}	24187.00		
C_{r3}	70602.00		
C_{r5}	24276.00		
C_{r7}	24276.00		

Table 5.1 (continued)

IV. Tire Characteristics

Tire Spring Constants (lb./in.)

K_{t1}	4500.00	K_{t2}	2750.00
K_{t3}	20000.00	K_{t4}	7000.00
K_{t5}	10000.00	K_{t6}	4000.00
K_{t7}	10000.00	K_{t8}	4000.00

Tire Damping Constants (lb.sec./in.)

C_{t1}	5.00	C_{t2}	2.00
C_{t3}	20.00	C_{t4}	4.00
C_{t5}	10.00	C_{t6}	2.00
C_{t7}	10.00	C_{t8}	2.00

V. Fifth Wheel Parameters

Fifth wheel spring constants	0.0 lb.in./rad.
Fifth wheel damping constant	0.0 lb.in./rad.
Vertical torque shaft damping constant	0.0 lb.in.sec./rad.

TABLE 5.2

NATURAL FREQUENCIES OF THE VEHICLE

(SELF-STEERING SEMITRAILER)

(IN CYCLES/SECONDS)

OMEGA	1	=	0.449
OMEGA	2	=	1.292
OMEGA	3	=	1.493
OMEGA	4	=	2.130
OMEGA	5	=	2.539
OMEGA	6	=	2.925
OMEGA	7	=	3.493
OMEGA	8	=	4.171
OMEGA	9	=	11.392
OMEGA	10	=	11.522
OMEGA	11	=	11.603
OMEGA	12	=	12.295
OMEGA	13	=	14.989
OMEGA	14	=	15.303
OMEGA	15	=	15.758
OMEGA	16	=	16.847
OMEGA	17	=	18.242
OMEGA	18	=	19.207
OMEGA	19	=	21.085
OMEGA	20	=	25.678

TABLE 5.3

RESULTS OF DAMPED EIGENVALUE ANALYSIS

(SELF-STEERING SEMITRAILER)

THE EIGENVALUE NO.	1	=	-.5460366E+01	,	.1612448E+03
THE EIGENVALUE NO.	2	=	-.5460366E+01	,	-.1612448E+03
THE EIGENVALUE NO.	3	=	-.5267394E+01	,	.1323754E+03
THE EIGENVALUE NO.	4	=	-.5267394E+01	,	-.1323754E+03
THE EIGENVALUE NO.	5	=	-.4336981E+01	,	.1206303E+03
THE EIGENVALUE NO.	6	=	-.4336981E+01	,	-.1206303E+03
THE EIGENVALUE NO.	7	=	-.4140326E+01	,	.1145433E+03
THE EIGENVALUE NO.	8	=	-.4140326E+01	,	-.1145433E+03
THE EIGENVALUE NO.	9	=	-.9162576E+01	,	.1054570E+03
THE EIGENVALUE NO.	10	=	-.9162576E+01	,	-.1054570E+03
THE EIGENVALUE NO.	11	=	-.1131710E+02	,	.9349694E+02
THE EIGENVALUE NO.	12	=	-.1131710E+02	,	-.9349694E+02
THE EIGENVALUE NO.	13	=	-.9415099E+01	,	.9568641E+02
THE EIGENVALUE NO.	14	=	-.9415099E+01	,	-.9568641E+02
THE EIGENVALUE NO.	15	=	-.8033413E+01	,	.9868260E+02
THE EIGENVALUE NO.	16	=	-.8033413E+01	,	-.9868260E+02
THE EIGENVALUE NO.	17	=	-.1949418E+02	,	.7024962E+02
THE EIGENVALUE NO.	18	=	-.1949418E+02	,	-.7024962E+02
THE EIGENVALUE NO.	19	=	-.1452400E+02	,	.7587608E+02
THE EIGENVALUE NO.	20	=	-.1452400E+02	,	-.7587608E+02
THE EIGENVALUE NO.	21	=	-.1320513E+02	,	.7034772E+02
THE EIGENVALUE NO.	22	=	-.1320513E+02	,	-.7034772E+02
THE EIGENVALUE NO.	23	=	-.1269140E+02	,	.7127042E+02
THE EIGENVALUE NO.	24	=	-.1269140E+02	,	-.7127042E+02
THE EIGENVALUE NO.	25	=	-.2398468E+00	,	.2620851E+02
THE EIGENVALUE NO.	26	=	-.2398468E+00	,	-.2620851E+02
THE EIGENVALUE NO.	27	=	-.5120711E+00	,	.2194260E+02

TABLE 5.3 (continued)

THE EIGENVALUE NO. 28 =	-.5120711E+00	,	-.2194260E+02
THE EIGENVALUE NO. 29 =	-.1380315E+00	,	.1837458E+02
THE EIGENVALUE NO. 30 =	-.1380315E+00	,	-.1837458E+02
THE EIGENVALUE NO. 31 =	-.1140446E+00	,	.1338206E+02
THE EIGENVALUE NO. 32 =	-.1140446E+00	,	-.1338206E+02
THE EIGENVALUE NO. 33 =	-.1608061E+00	,	.2815148E+01
THE EIGENVALUE NO. 34 =	-.1608061E+00	,	-.2815148E+01
THE EIGENVALUE NO. 35 =	-.4558223E+01	,	.1528696E+02
THE EIGENVALUE NO. 36 =	-.4558223E+01	,	-.1528696E+02
THE EIGENVALUE NO. 37 =	-.1221146E+01	,	.8028223E+01
THE EIGENVALUE NO. 38 =	-.1221146E+01	,	-.8028223E+01
THE EIGENVALUE NO. 39 =	-.1611329E+01	,	.9241422E+01
THE EIGENVALUE NO. 40 =	-.1611329E+01	,	-.9241422E+01

2. The frequency responses of the system, which are the amplitude responses of the system when it is forced to oscillate at various excitation frequencies, are next considered. With a constant wave length of 30 feet and varying speed of the vehicle, the following graphs are obtained.

   Figures 5.1 to 5.6 show the response curves for the tractor in the longitudinal, lateral, vertical, roll, pitch and yaw coordinates respectively.

   Figure 5.7 to 5.11 show the response curves for the semitrailer in the longitudinal, lateral, vertical, pitch and yaw coordinates respectively.

   Figure 5.12 gives the response curve for the bogie in yaw coordinate.

   The frequency response curves in the lateral, vertical and roll coordinates for the tractor front axle, tractor rear axle, bogie front axle and bogie rear axle are given in Figures 5.13 to 5.24 respectively.

3. Spring at the Fifth Wheel

   A modification has been made to the articulated vehicle by incorporating a torsional spring into the fifth wheel mechanism.

   Figures 5.25 to 5.27 show the effect of varying spring stiffness on the yaw angle difference of tractor and effect on the vehicle peak response in the yaw mode, since part of the available moment has been used to deflect the torsional spring. The spring stiffness has a very little effect on all other modes.

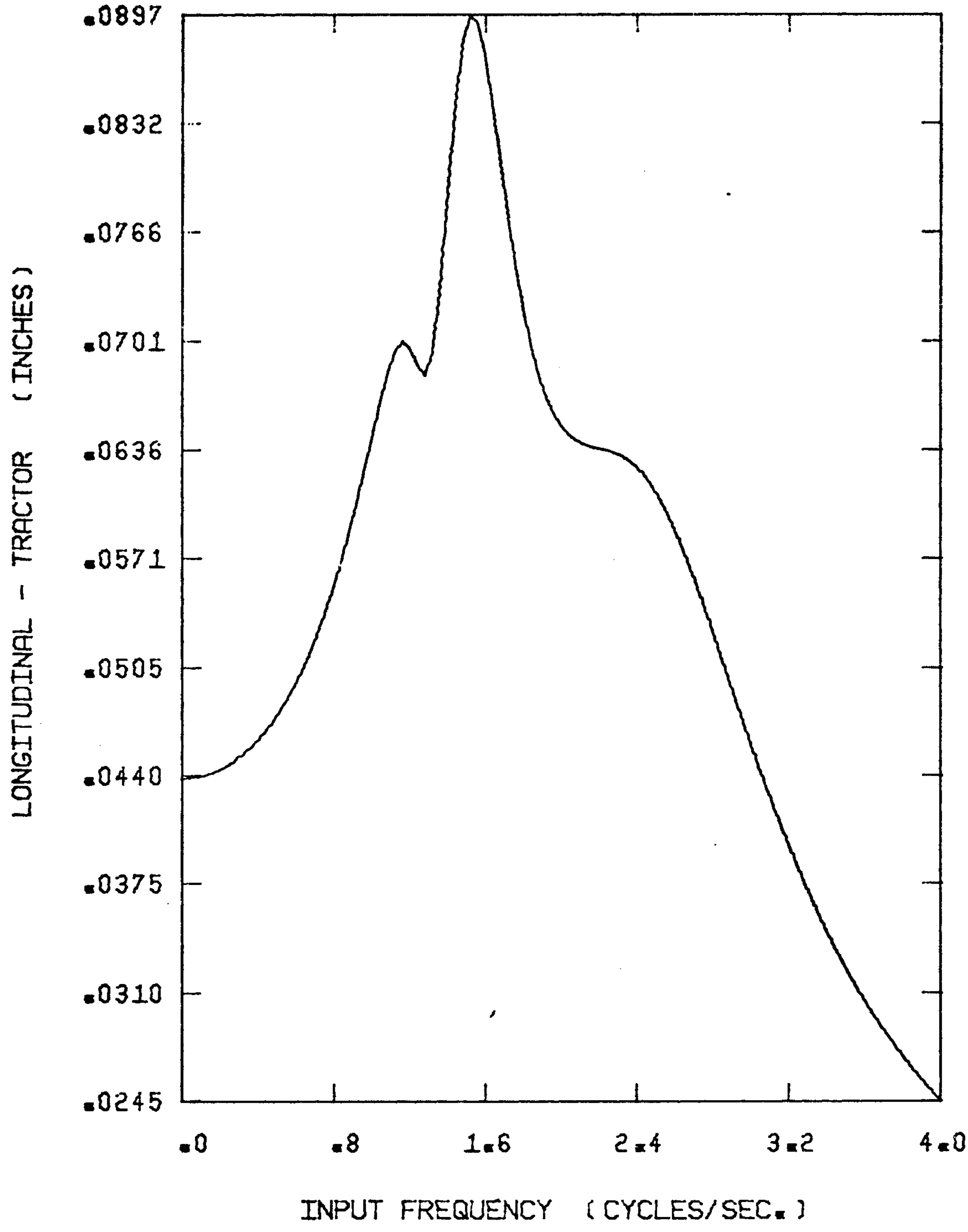

FIGURE 5.1: LONGITUDINAL TRACTOR DISPLACEMENT (u_t)
(SELF-STEERING SEMITRAILER)

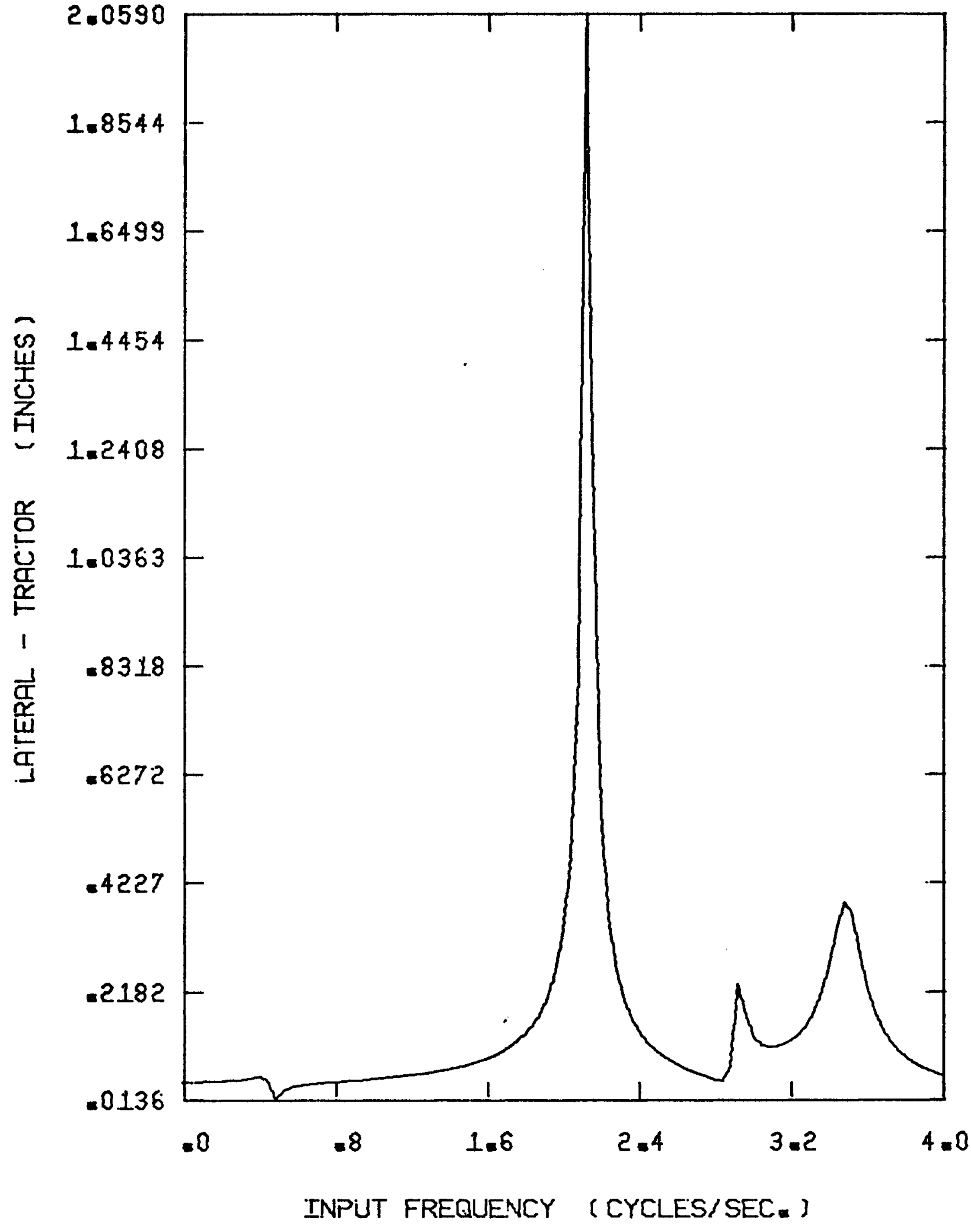

FIGURE 5.2: LATERAL TRACTOR DISPLACEMENT (v_t)
(SELF-STEERING SEMITRAILER)

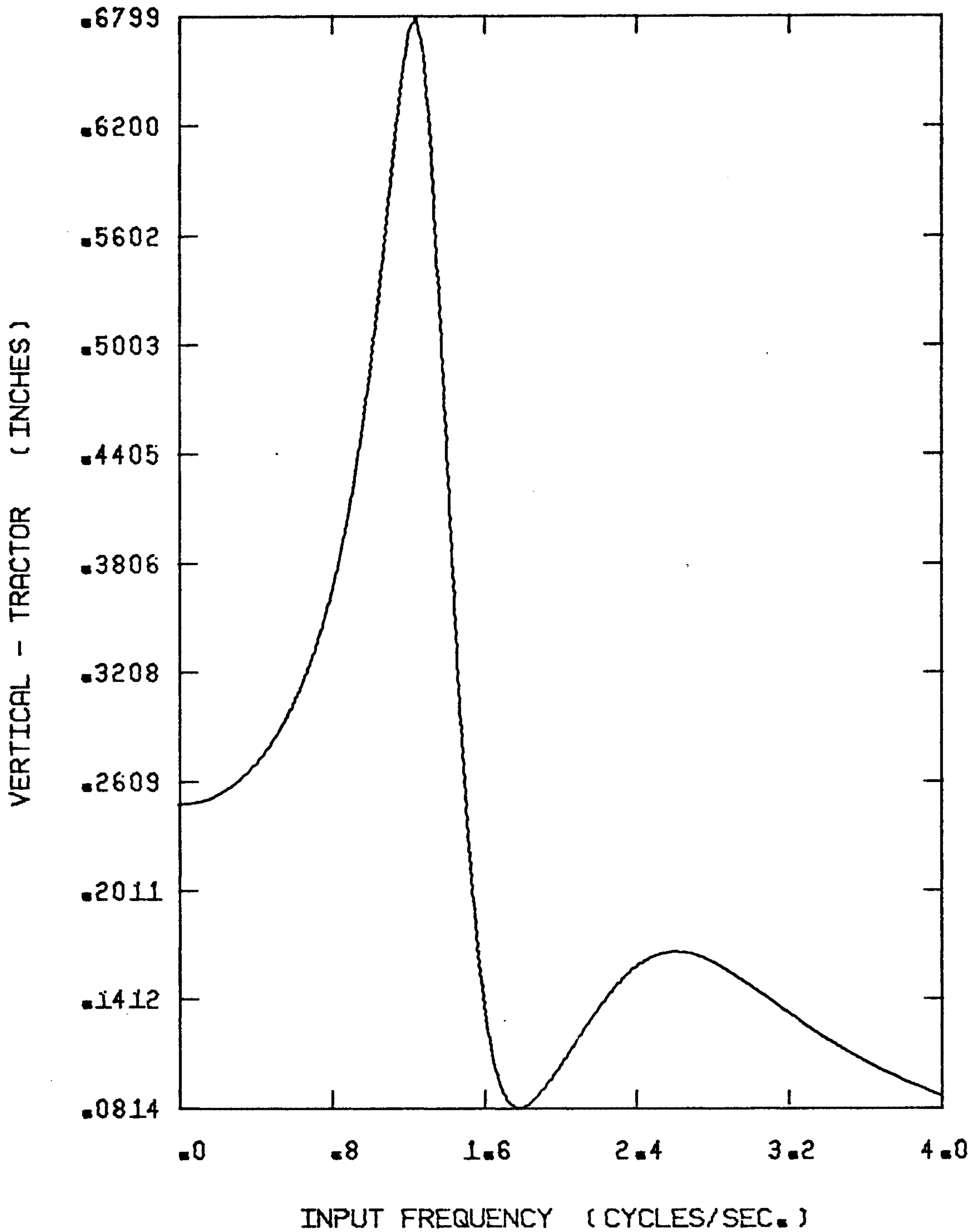

FIGURE 5.3: VERTICAL TRACTOR DISPLACEMENT (w_t)
(SELF-STEERING SEMITRAILER)

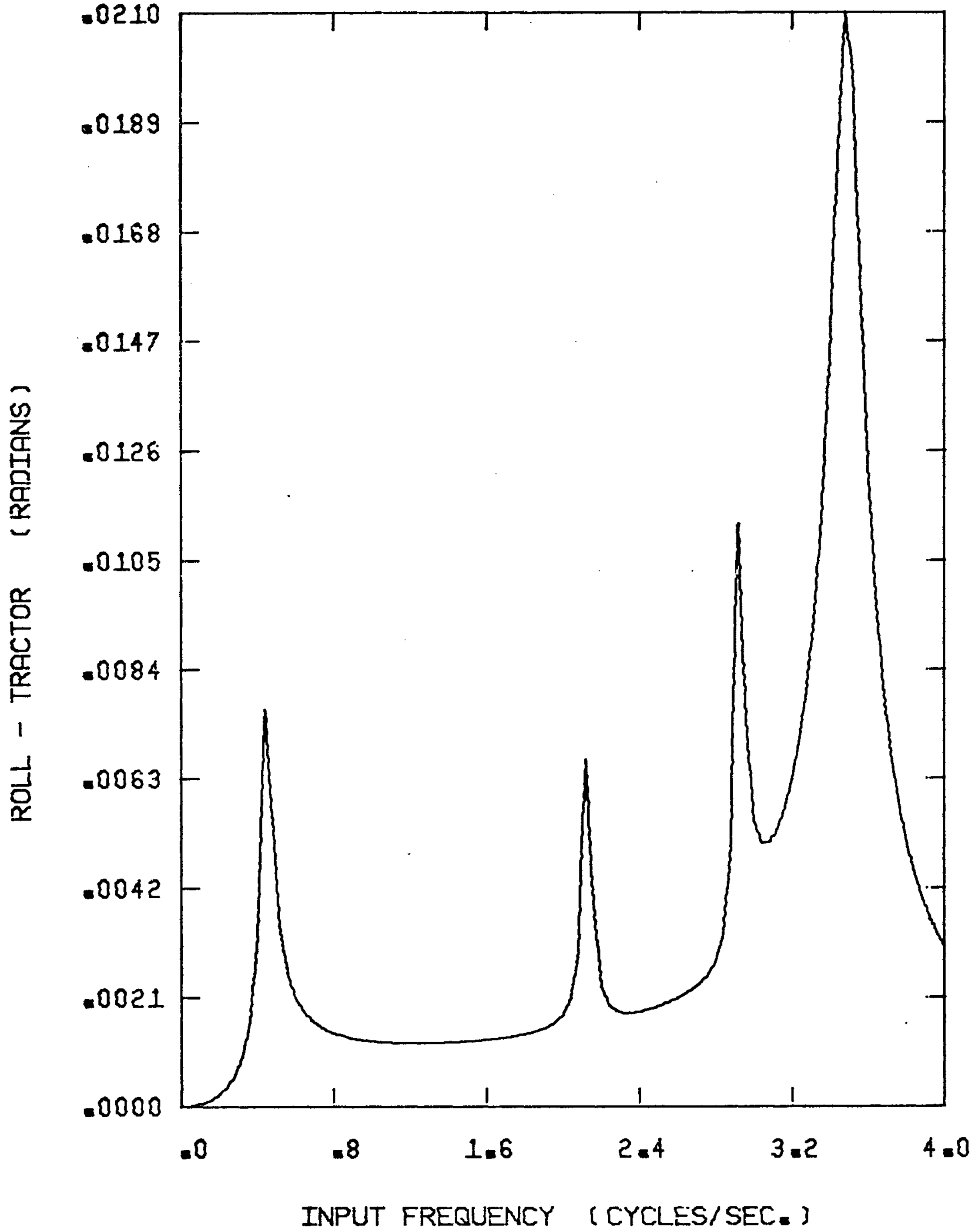

FIGURE 5.4: ROLL TRACTOR ANGLE (α_t)
(SELF-STEERING SEMITRAILER)

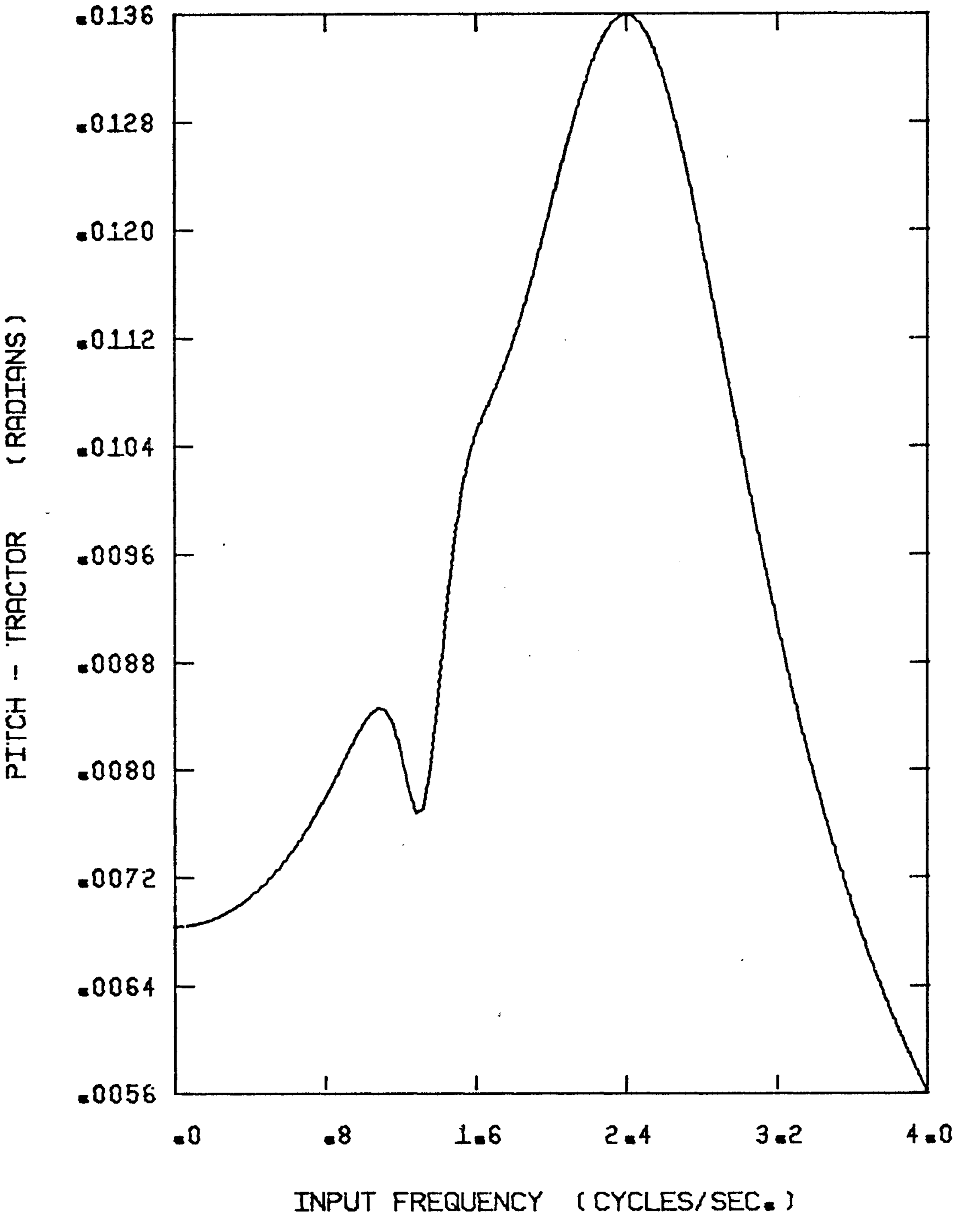

FIGURE 5.5: PITCH TRACTOR ANGLE (β_t)
(SELF-STEERING SEMITRAILER)

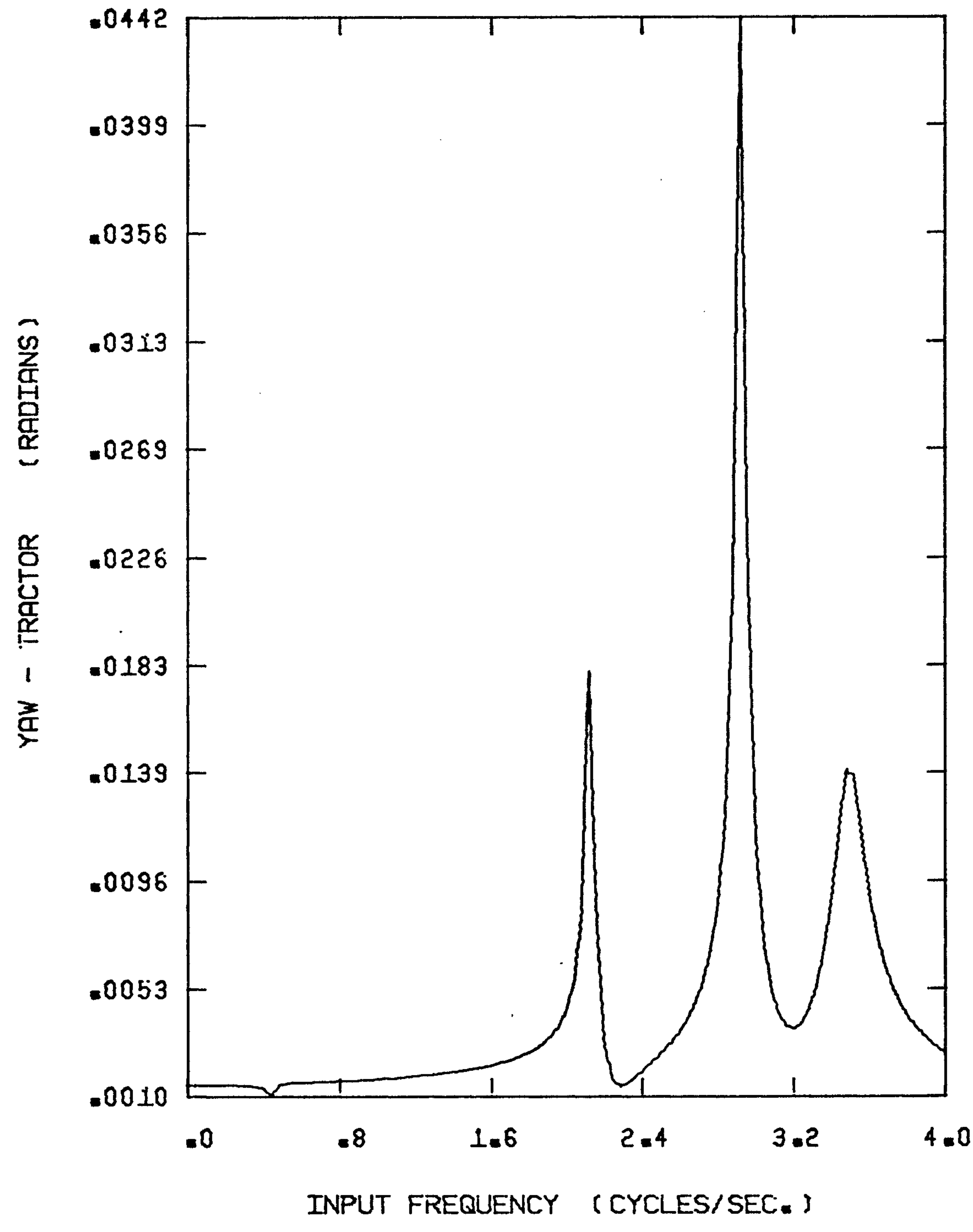

FIGURE 5.6: YAW TRACTOR ANGLE (γ_t)
(SELF-STEERING SEMITRAILER)

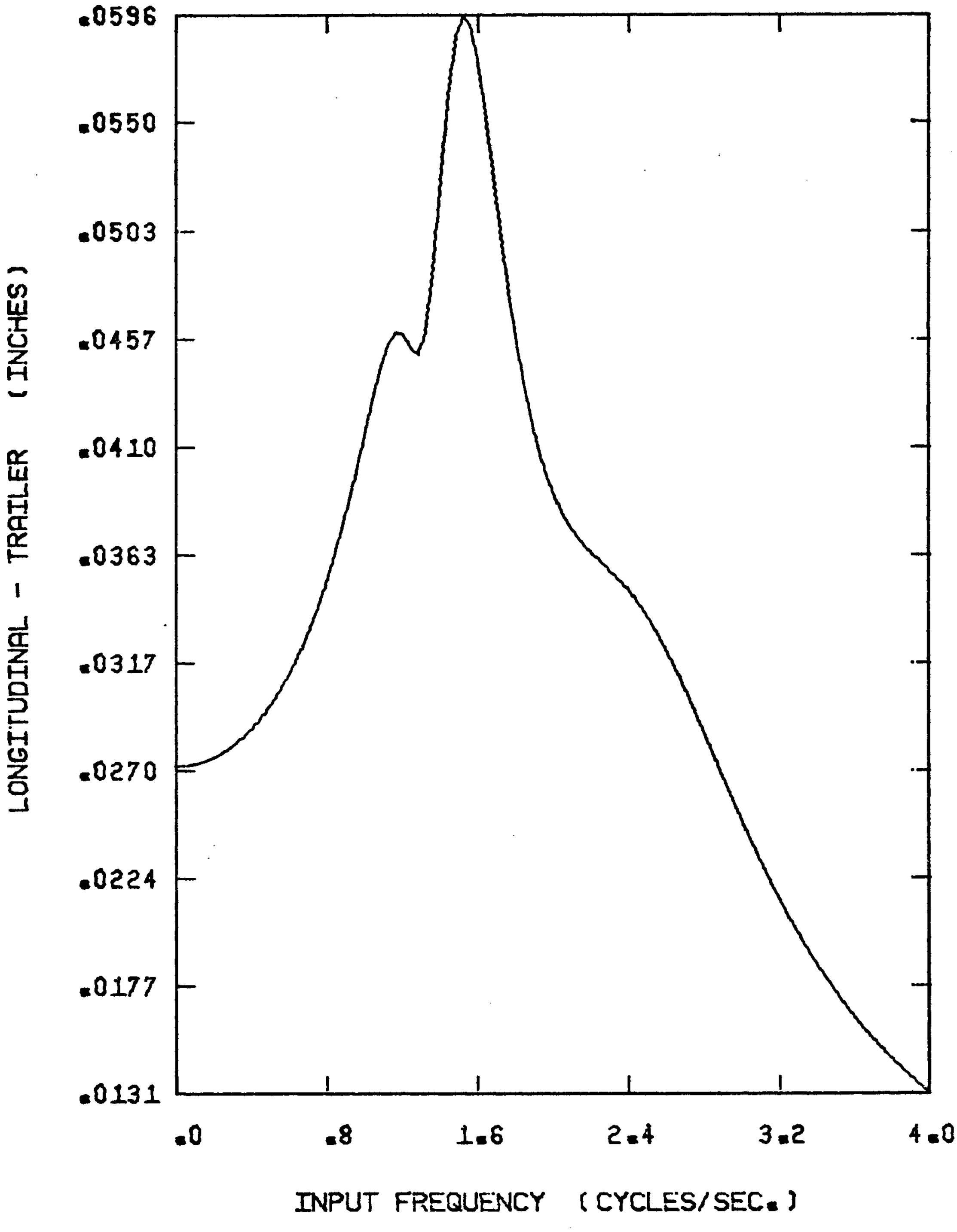

FIGURE 5.7: LONGITUDINAL SEMITRAILER DISPLACEMENT (u_s)
(SELF-STEERING SEMITRAILER)

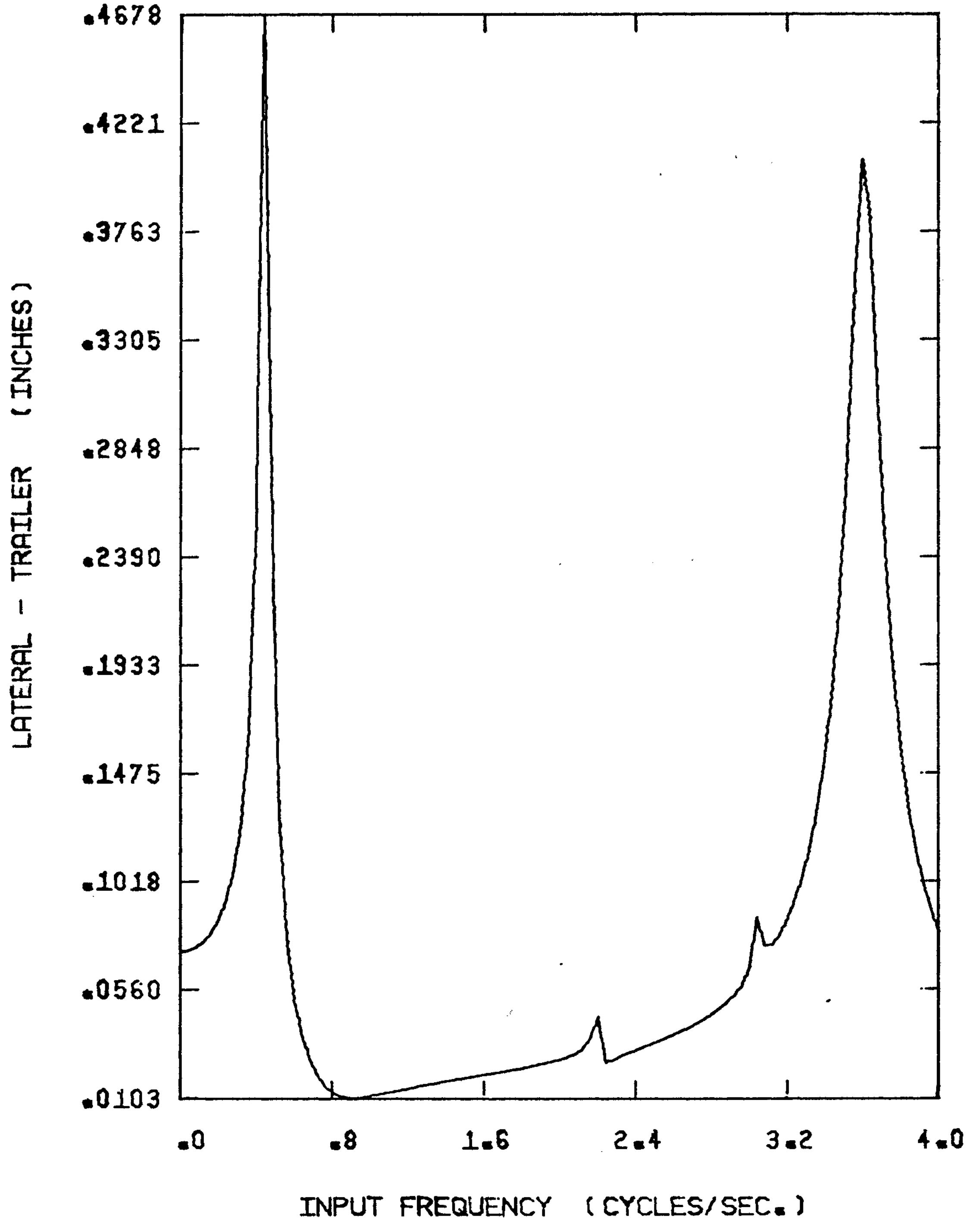

FIGURE 5.8: LATERAL SEMITRAILER DISPLACEMENT (y_s)
(SELF-STEERING SEMITRAILER)

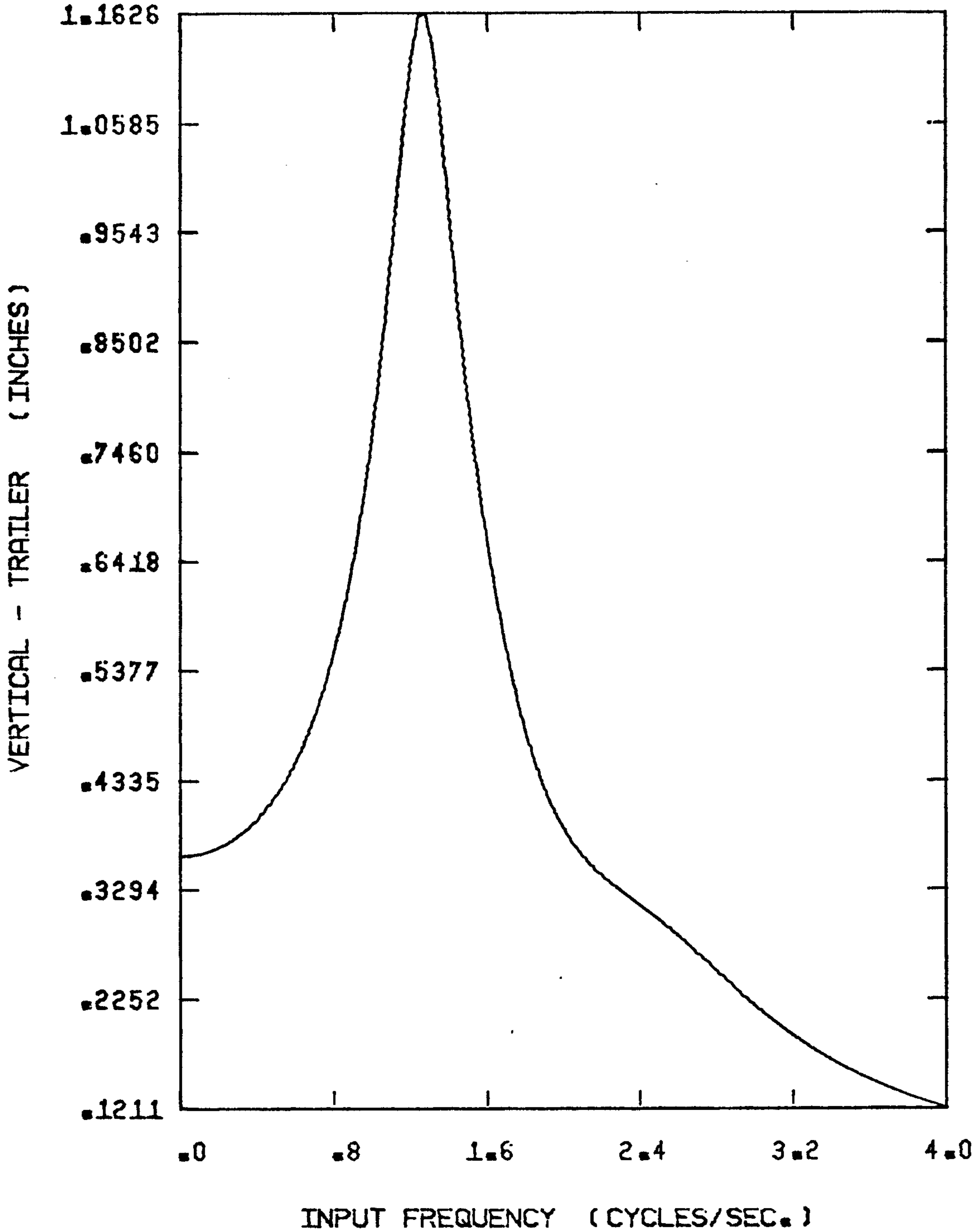

FIGURE 5.9: VERTICAL SEMITRAILER DISPLACEMENT (w_s)
(SELF-STEERING SEMITRAILER)

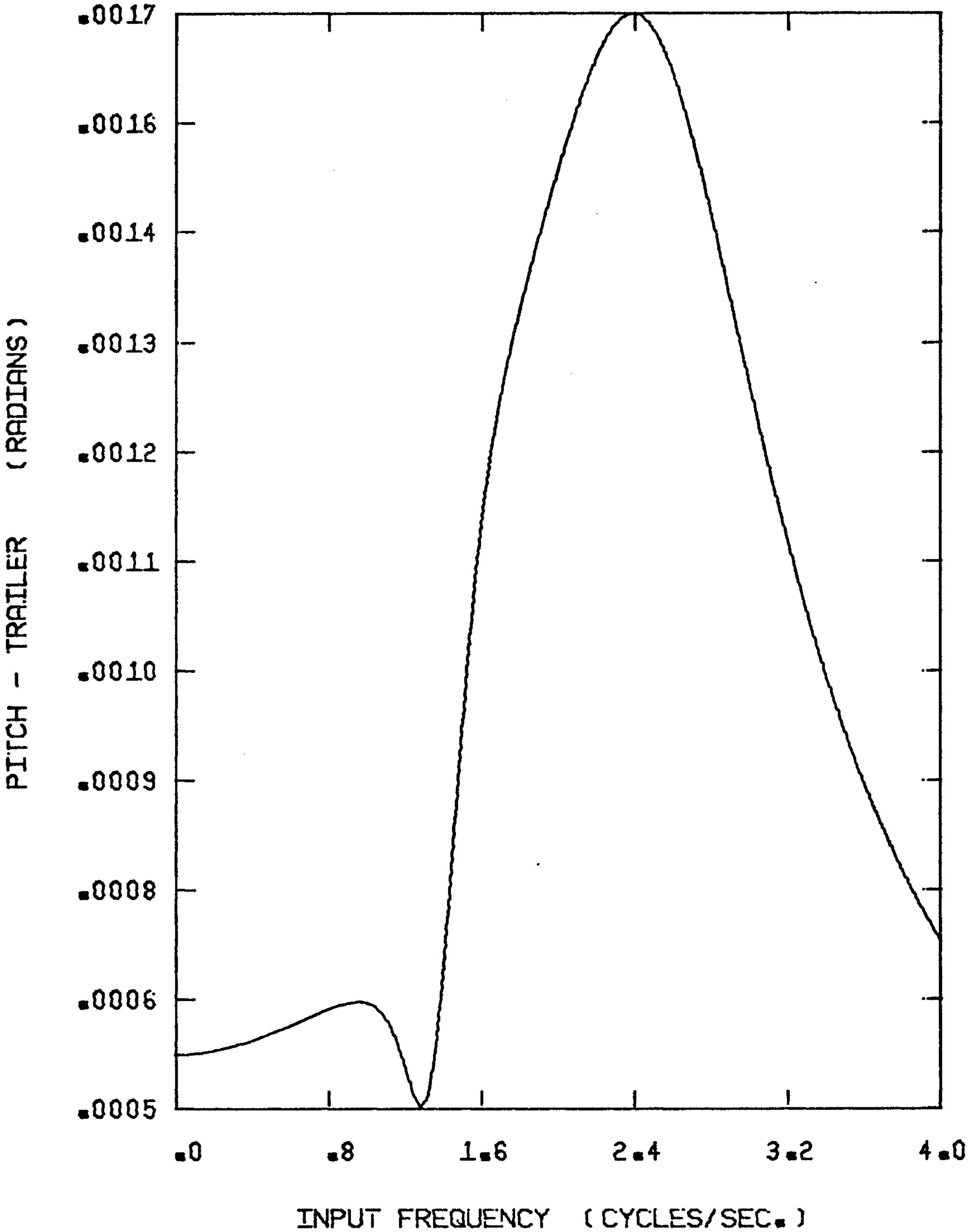

FIGURE 5.10: PITCH SEMITRAILER ANGLE (β_S)
(SELF-STEERING SEMITRAILER)

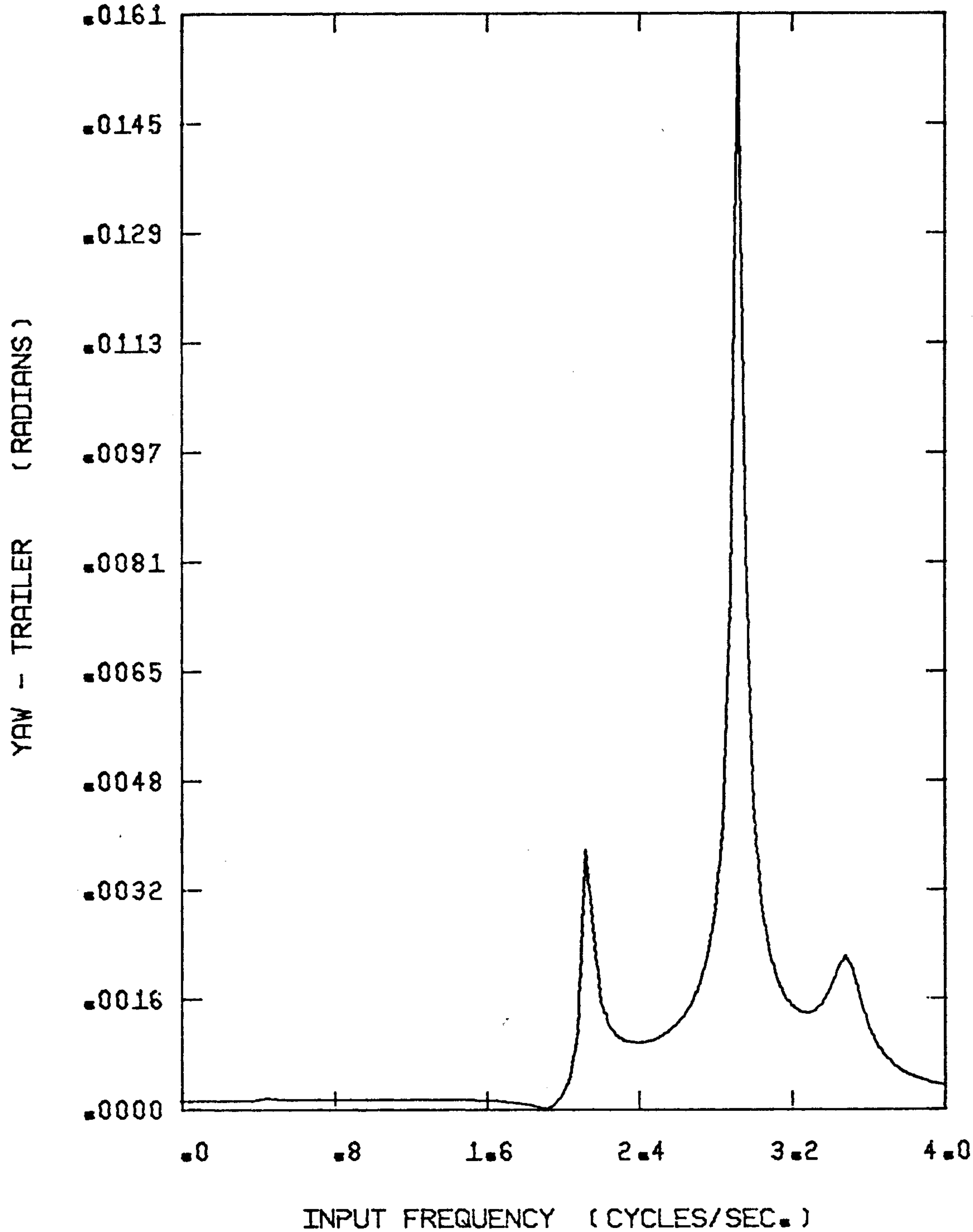

FIGURE 5.11: YAW SEMITRAILER ANGLE (γ_s)
(SELF-STEERING SEMITRAILER)

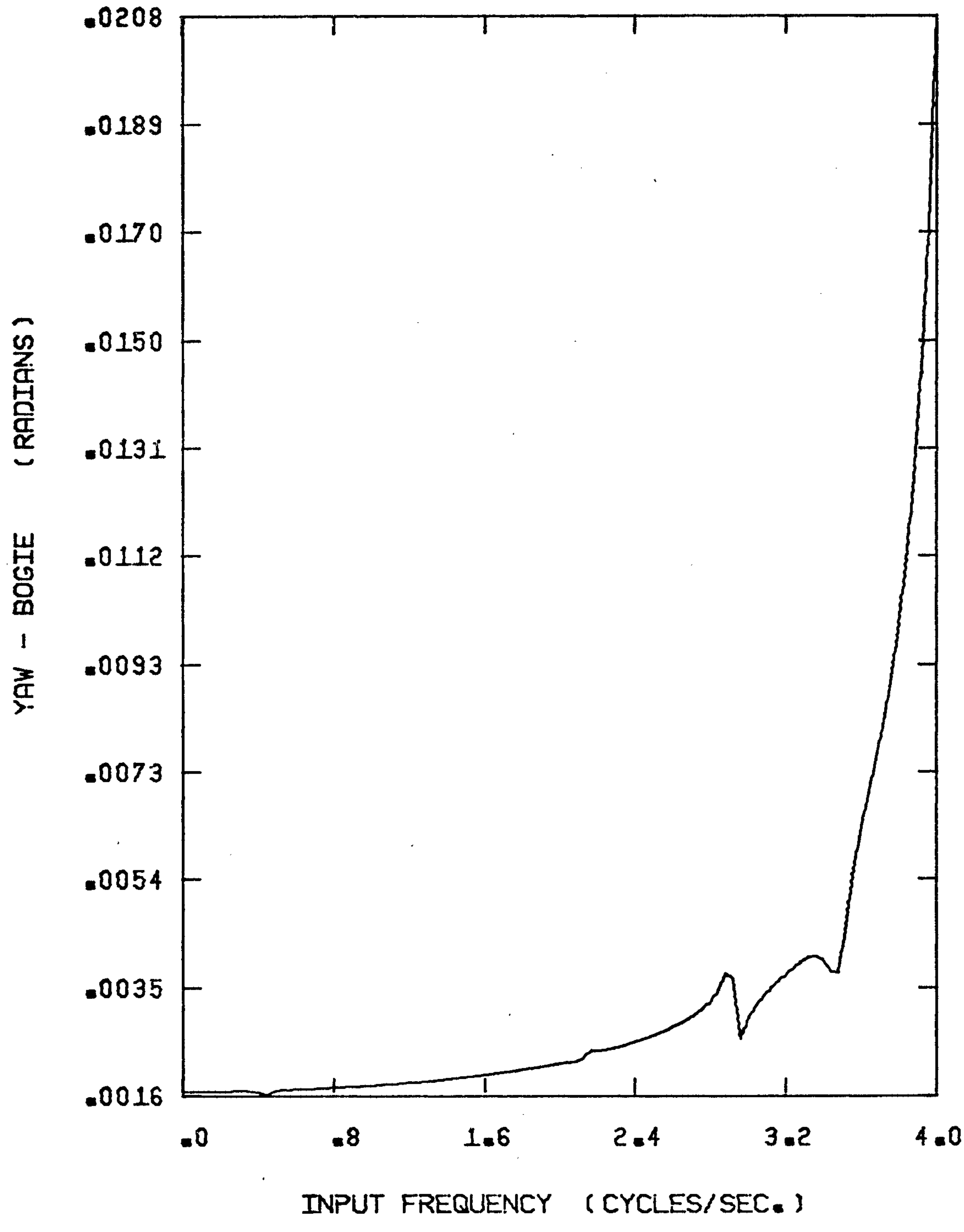

FIGURE 5.12: YAW BOGIE ANGLE (γ_b)
(SELF-STEERING SEMITRAILER)

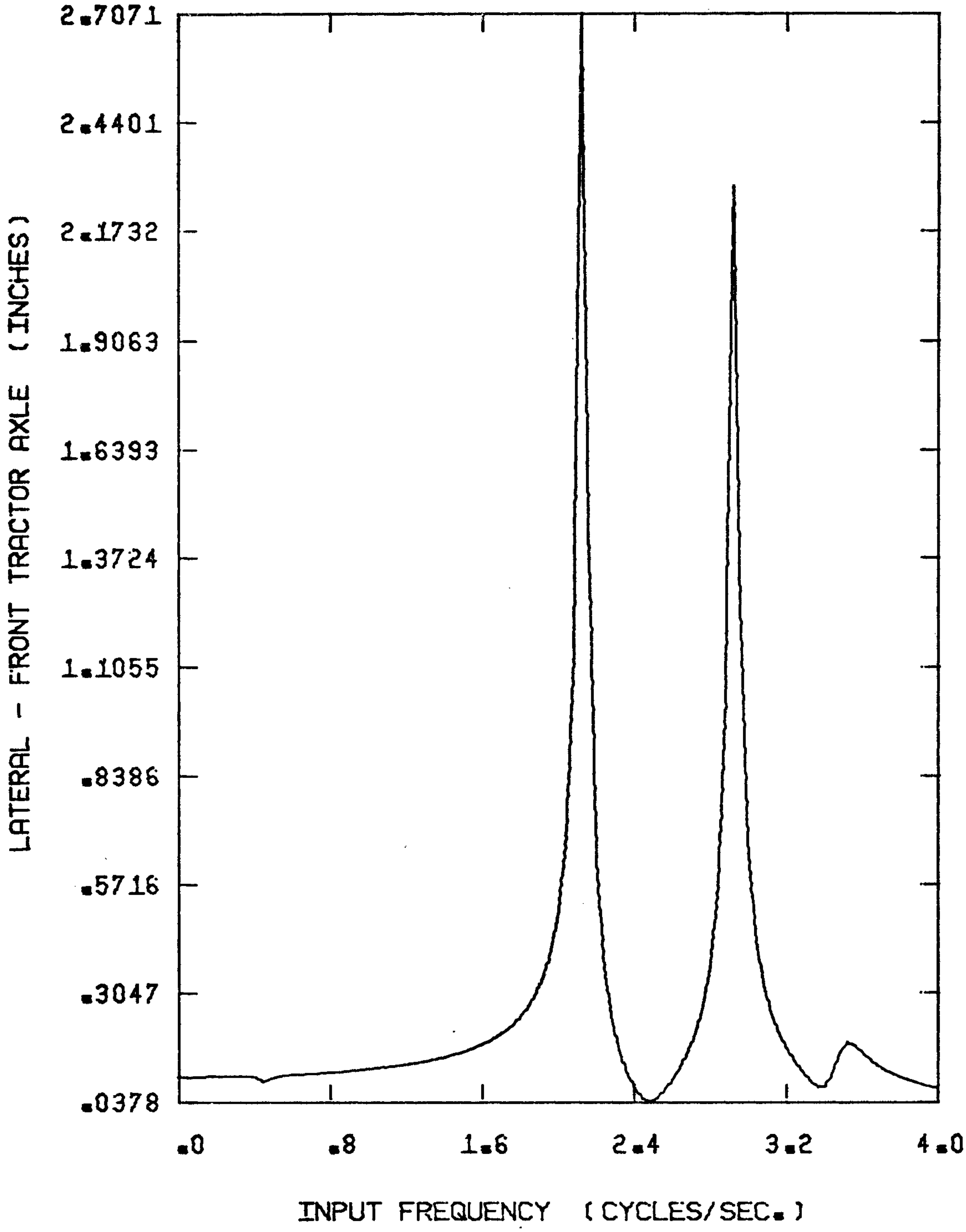

FIGURE 5.13: LATERAL TRACTOR FRONT AXLE DISPLACEMENT (v_1)
(SELF-STEERING SEMITRAILER)

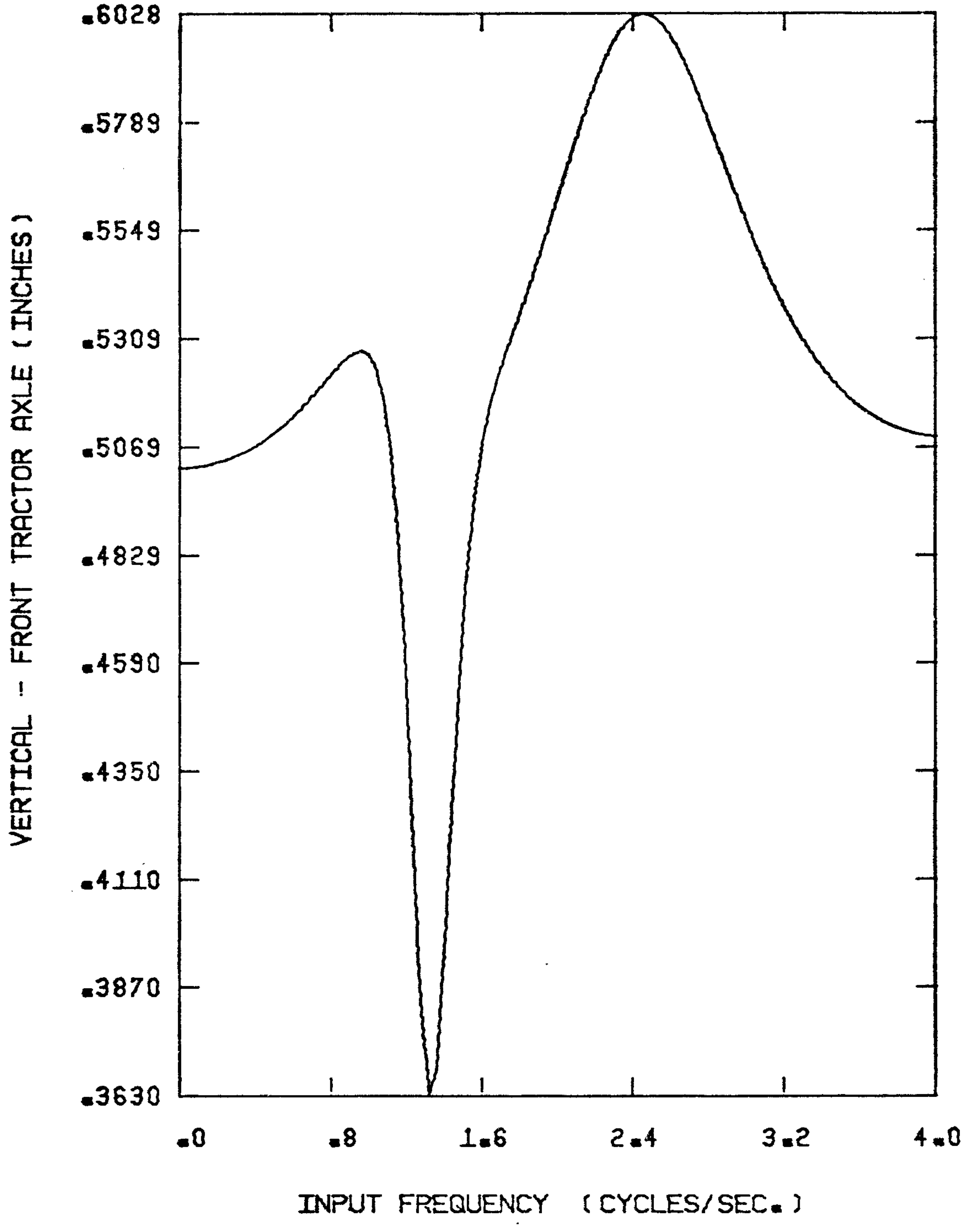

FIGURE 5.14: VERTICAL TRACTOR FRONT AXLE DISPLACEMENT (w_1)
(SELF-STEERING SEMITRAILER)

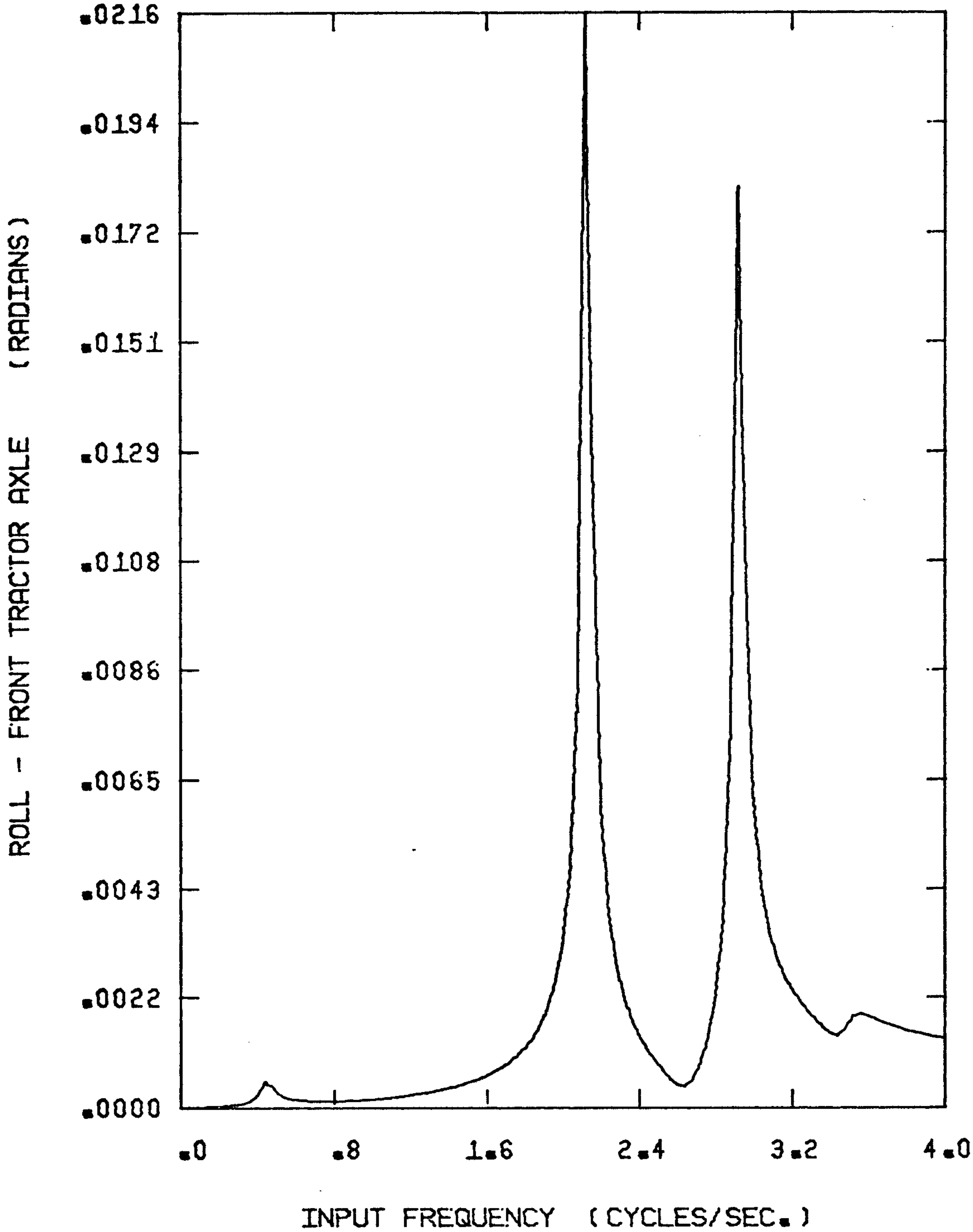

FIGURE 5.15: ROLL TRACTOR FRONT AXLE ANGLE (α_1)
(SELF-STEERING SEMITRAILER)

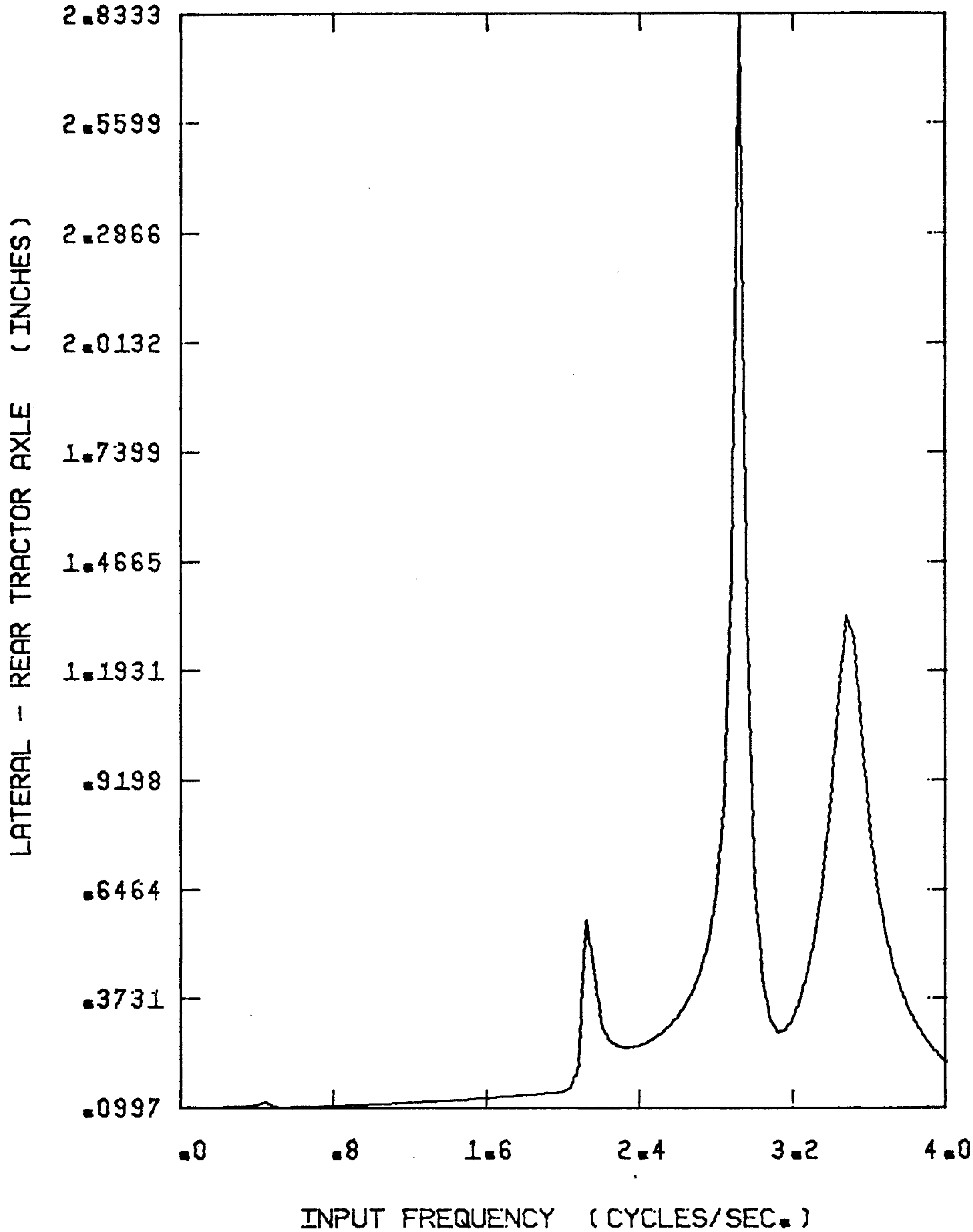

FIGURE 5.16: LATERAL TRACTOR REAR AXLE DISPLACEMENT (v_3)
(SELF-STEERING SEMITRAILER)

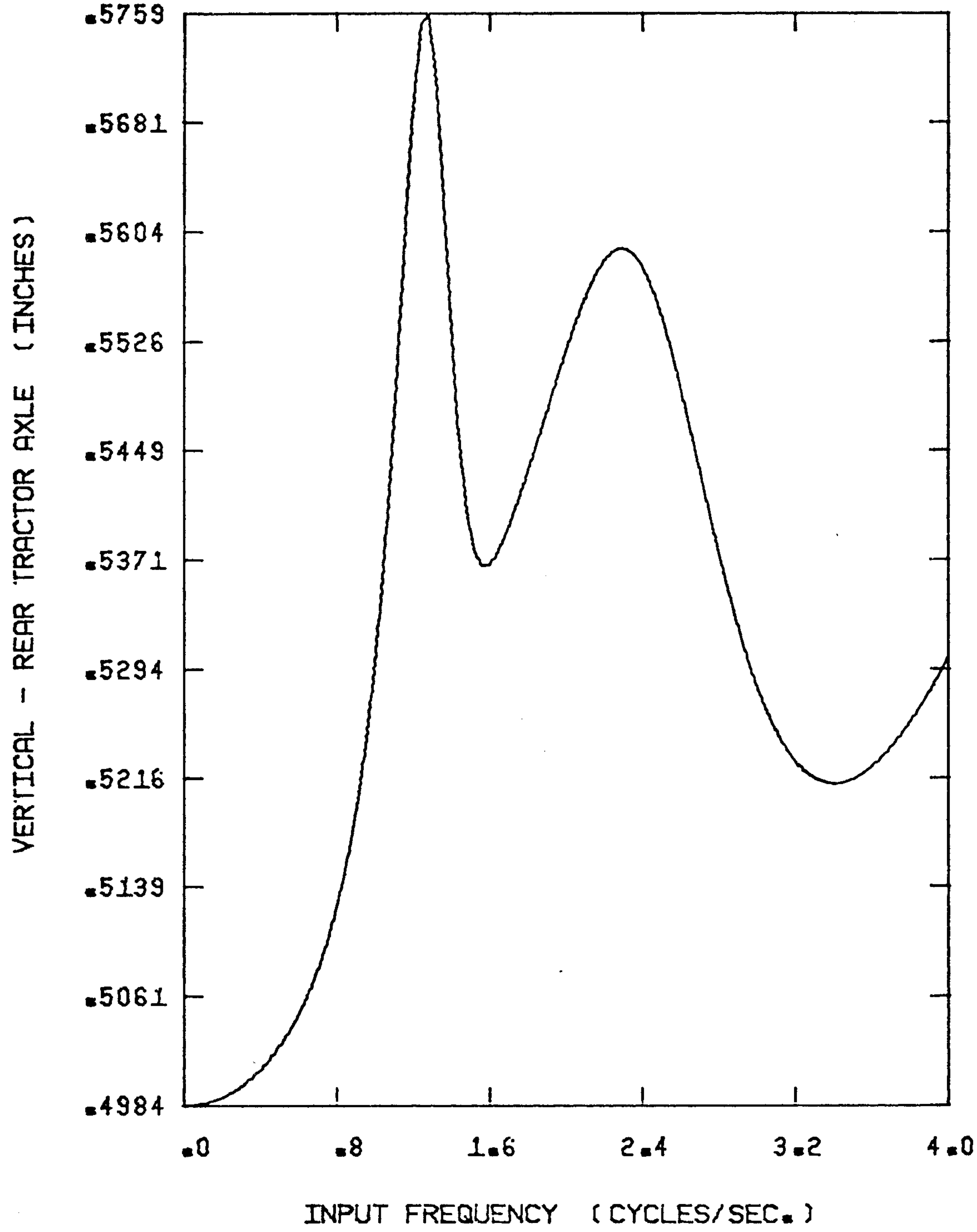

FIGURE 5.17: VERTICAL TRACTOR REAR AXLE DISPLACEMENT (w_3)
(SELF-STEERING SEMITRAILER)

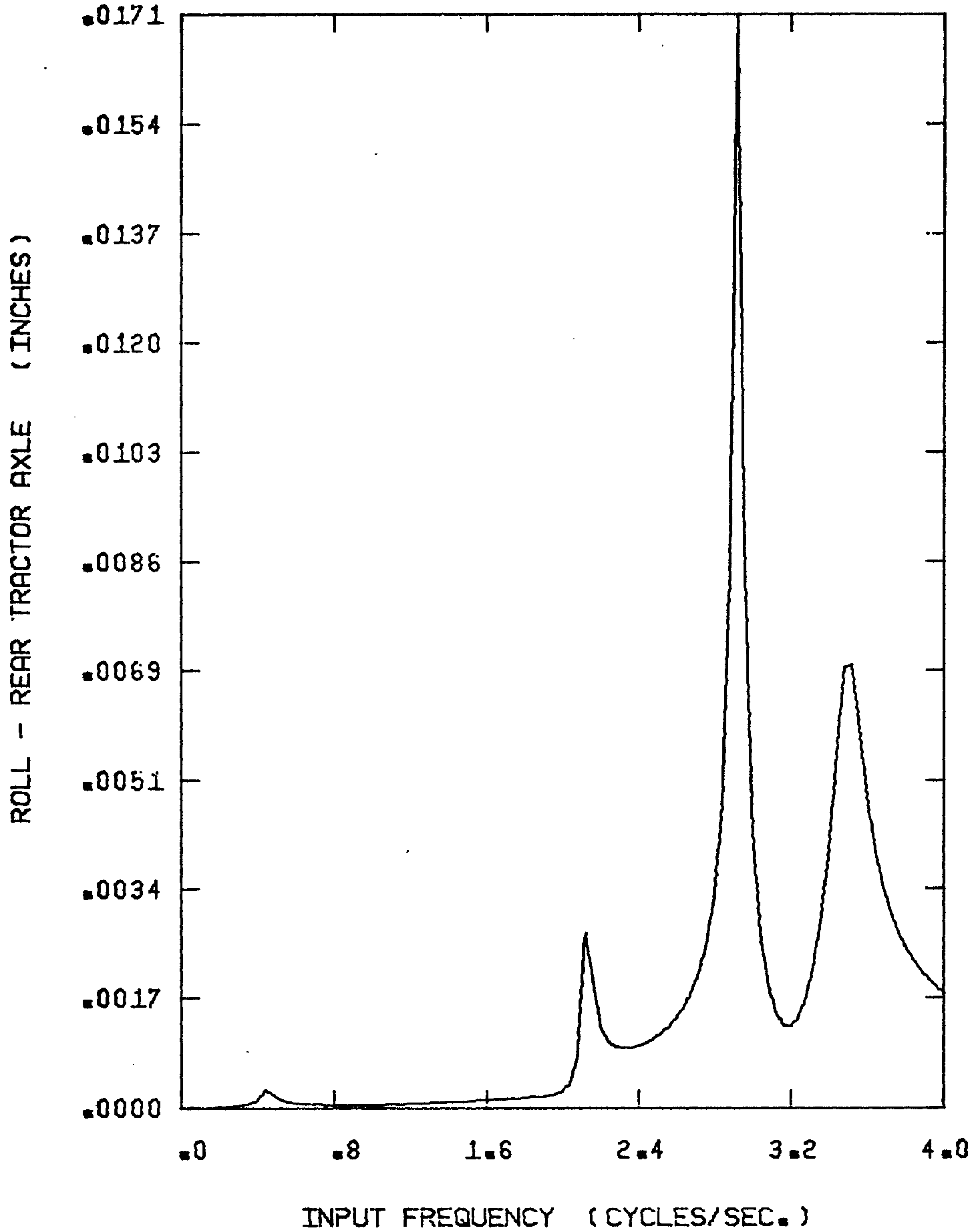

FIGURE 5.18: ROLL TRACTOR REAR AXLE ANGLE (α_3)
(SELF-STEERING SEMITRAILER)

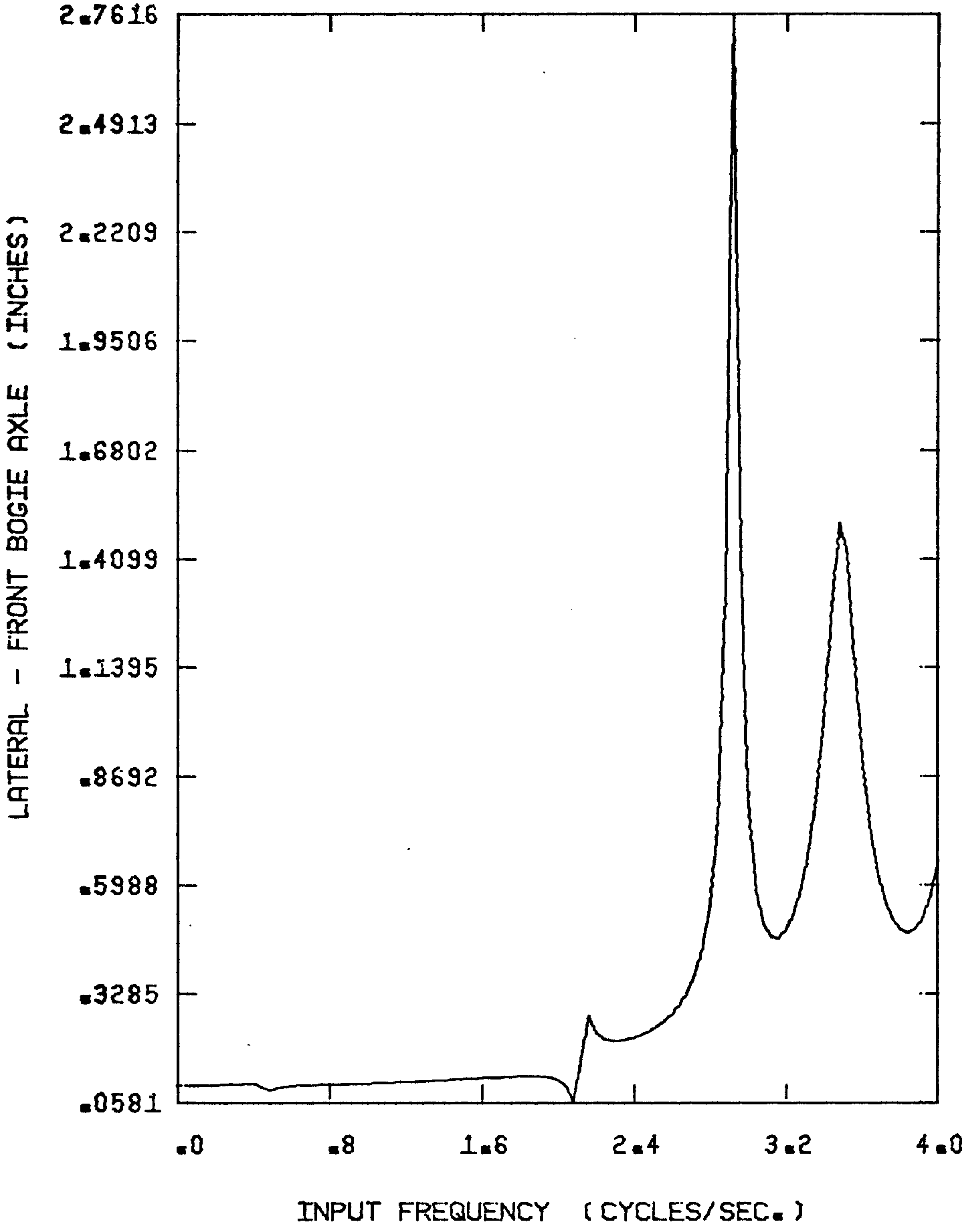

FIGURE 5.19: LATERAL BOGIE FRONT AXLE DISPLACEMENT (v_5)
(SELF-STEERING SEMITRAILER)

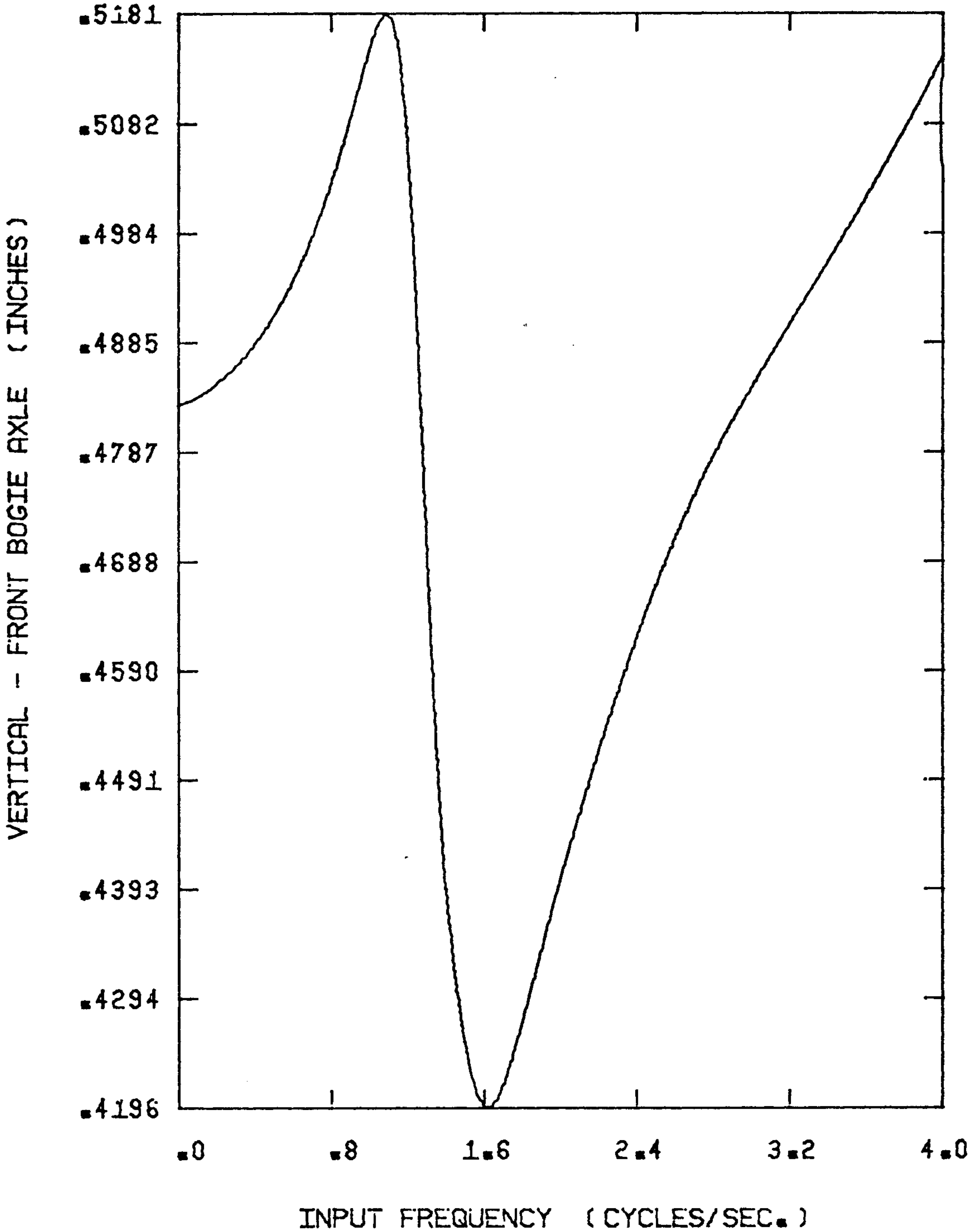

FIGURE 5.20: VERTICAL BOGIE FRONT AXLE DISPLACEMENT (w_5)
(SELF-STEERING SEMITRAILER)

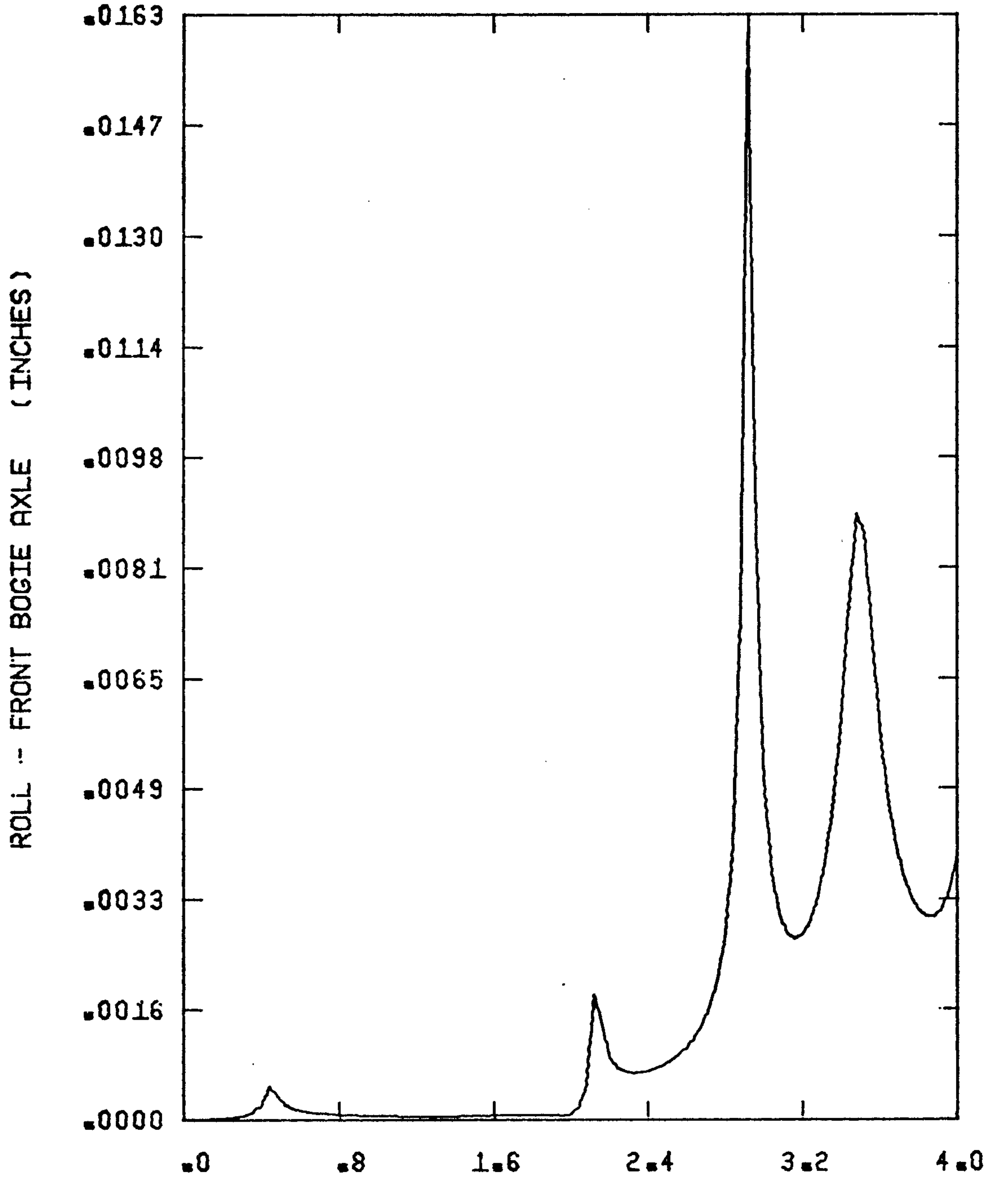

FIGURE 5.21: ROLL BOGIE FRONT AXLE ANGLE (α_5)
(SELF-STEERING SEMITRAILER)

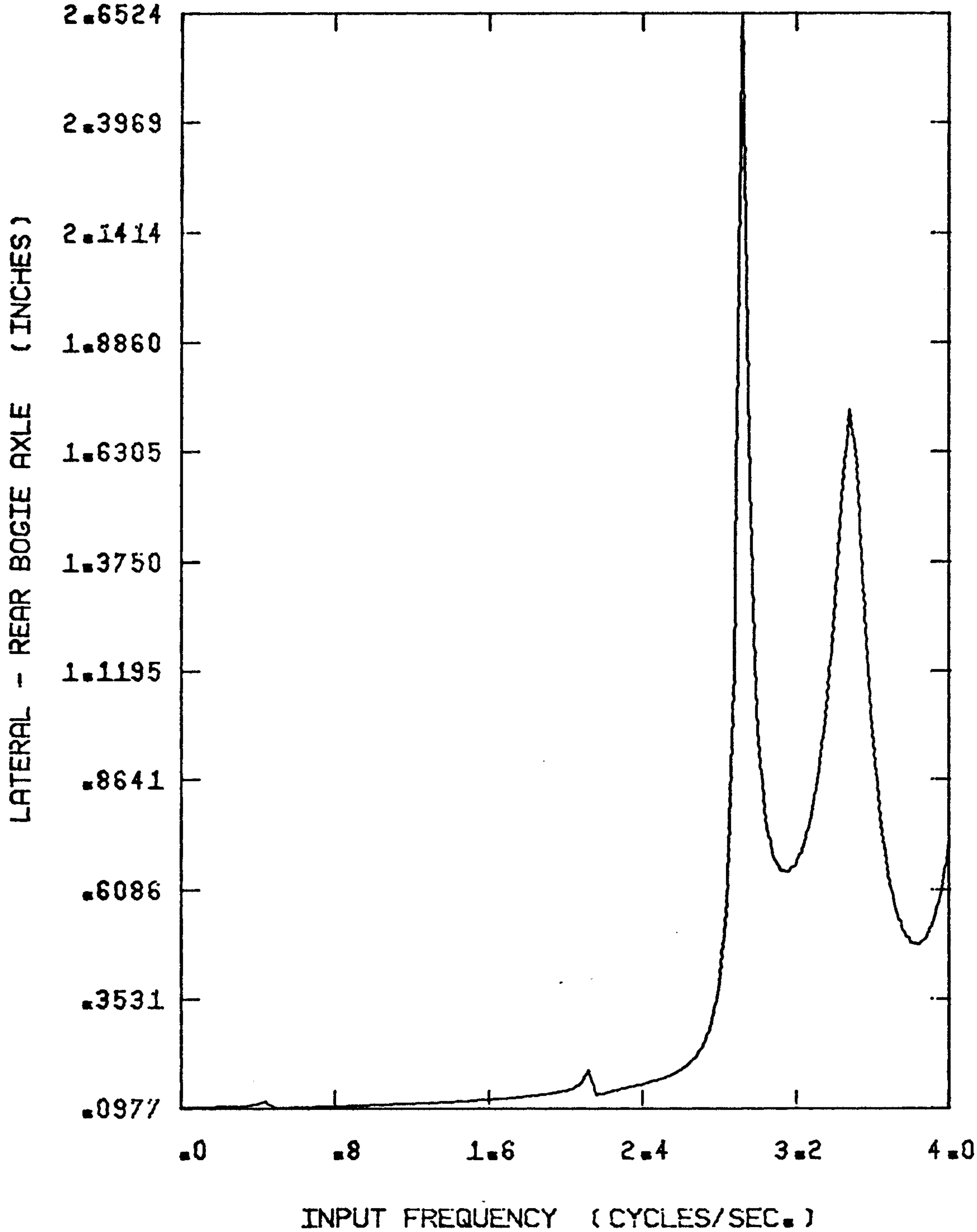

FIGURE 5,22; LATERAL BOGIE REAR AXLE DISPLACEMENT (v_7)
(SELF-STEERING SEMITRAILER)

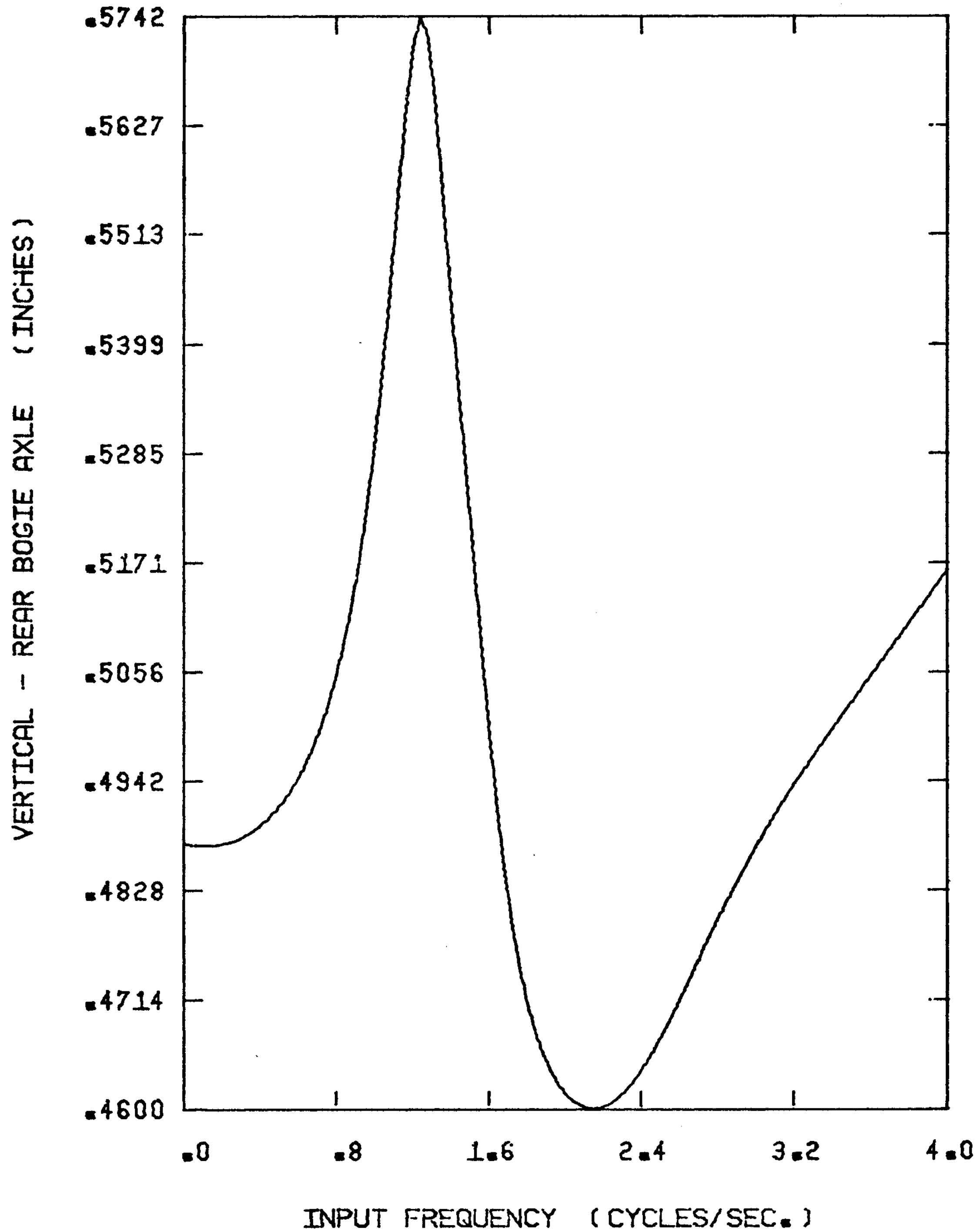

FIGURE 5.23: VERTICAL BOGIE REAR AXLE DISPLACEMENT (w_7)
(SELF-STEERING SEMITRAILER)

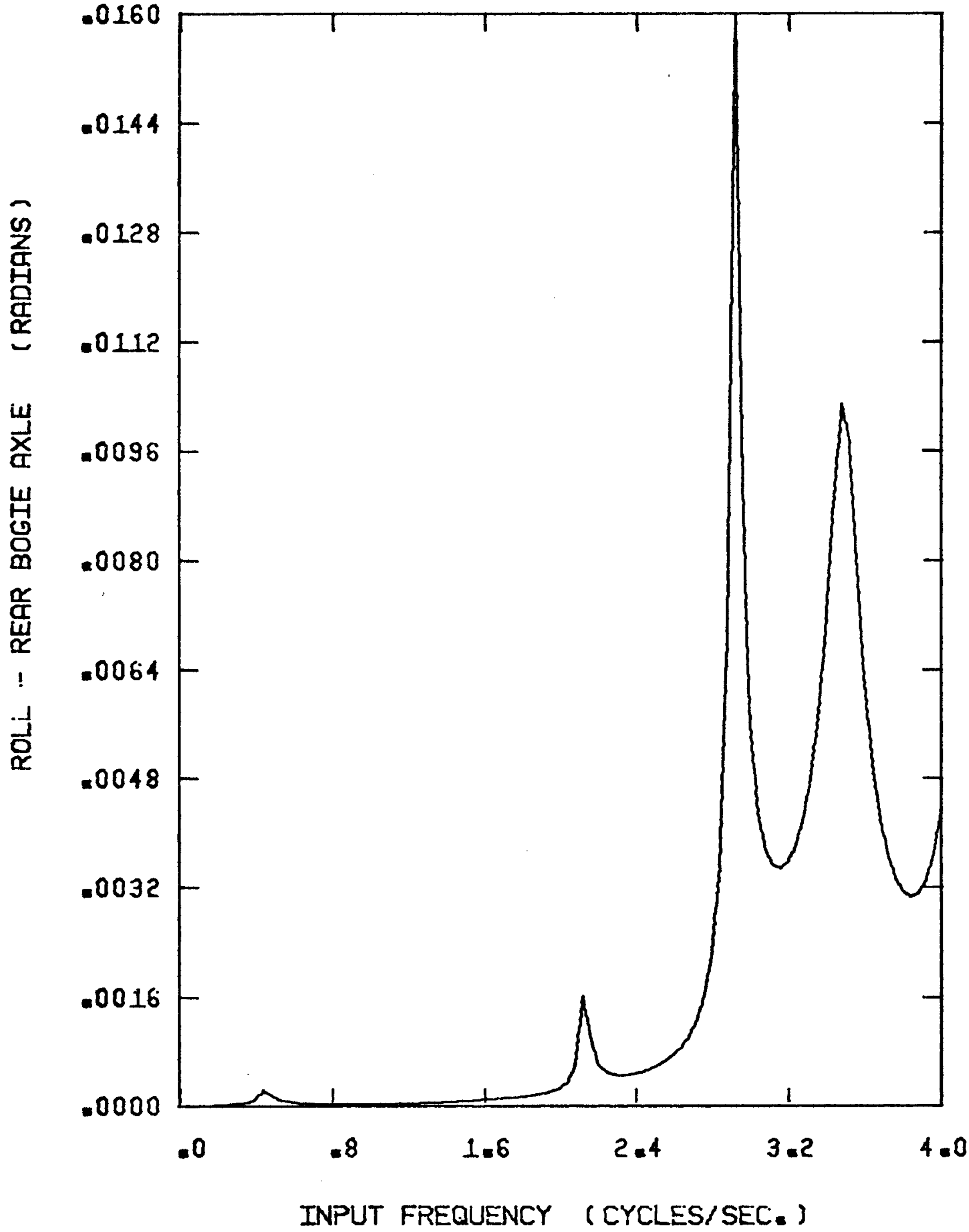

FIGURE 5.24: ROLL BOGIE REAR AXLE ANGLE (α_7)
(SELF-STEERING SEMITRAILER)

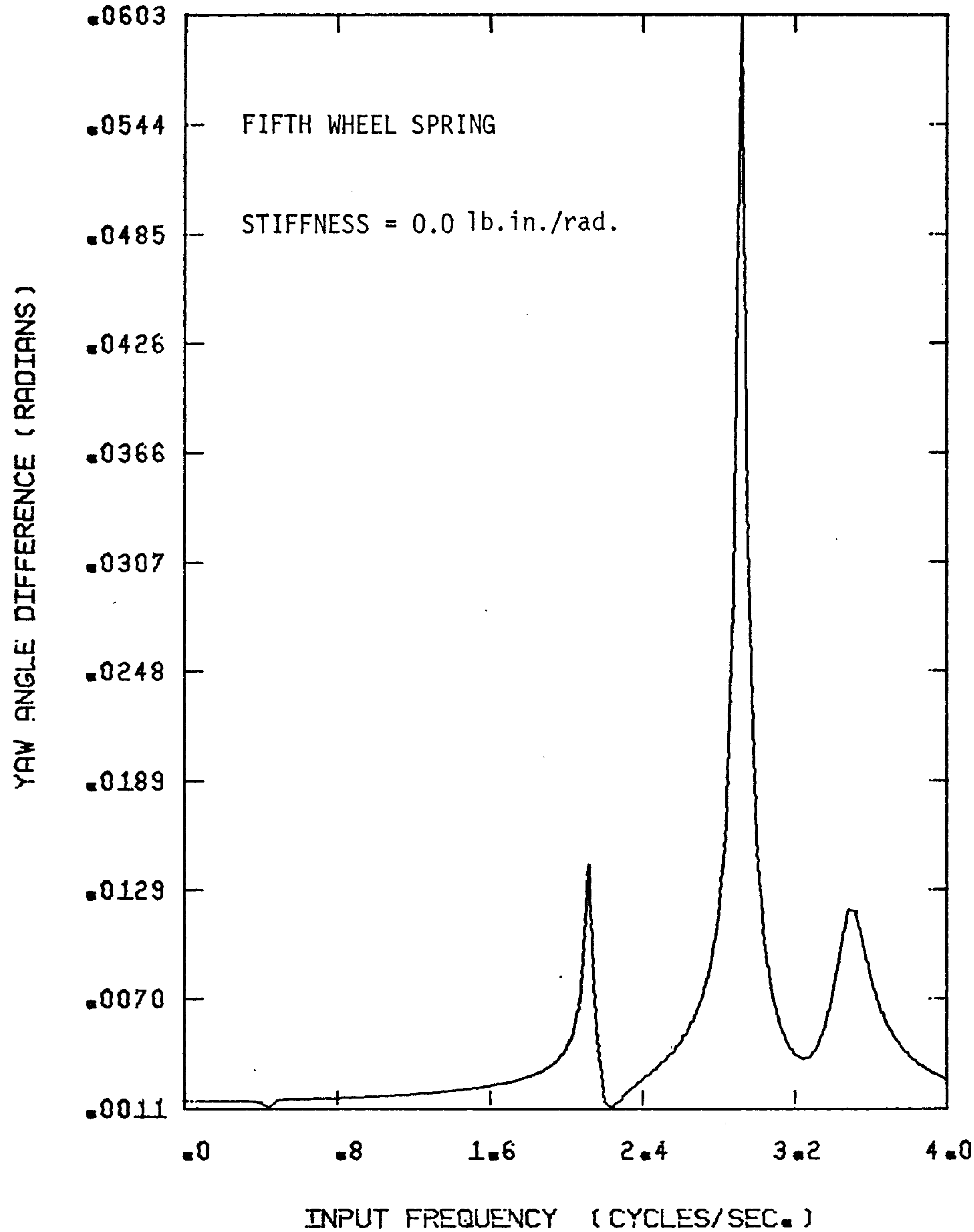

FIGURE 5.25: YAW ANGLE DIFFERENCE OF TRACTOR AND SEMITRAILER FOR K_h = 0.0 lb.in./rad.
(SELF-STEERING SEMITRAILER)

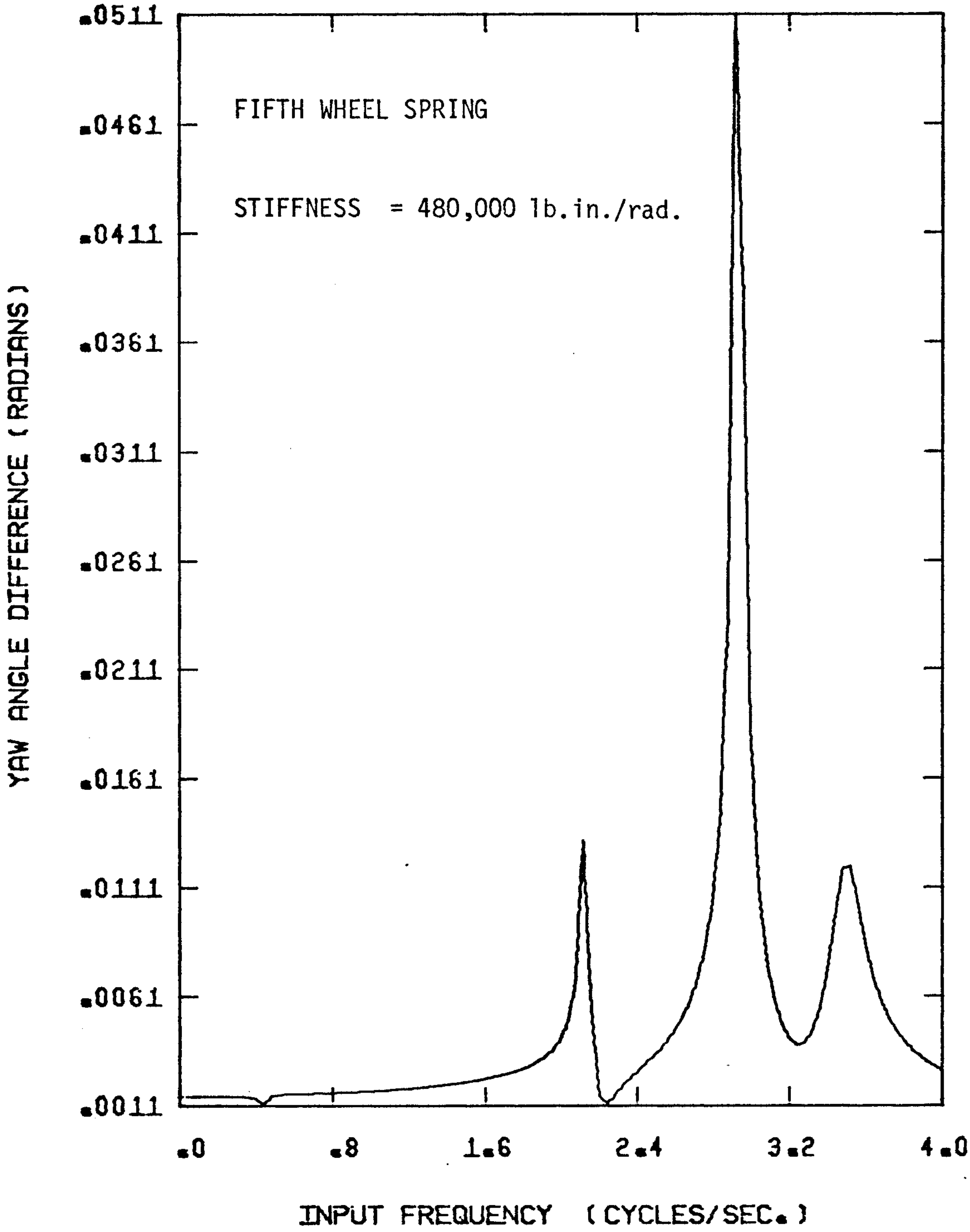

FIGURE 5.26: YAW ANGLE DIFFERENCE OF TRACTOR AND SEMITRAILER FOR K_h = 480,000 lb.in./rad.
(SELF-STEERING SEMITRAILER)

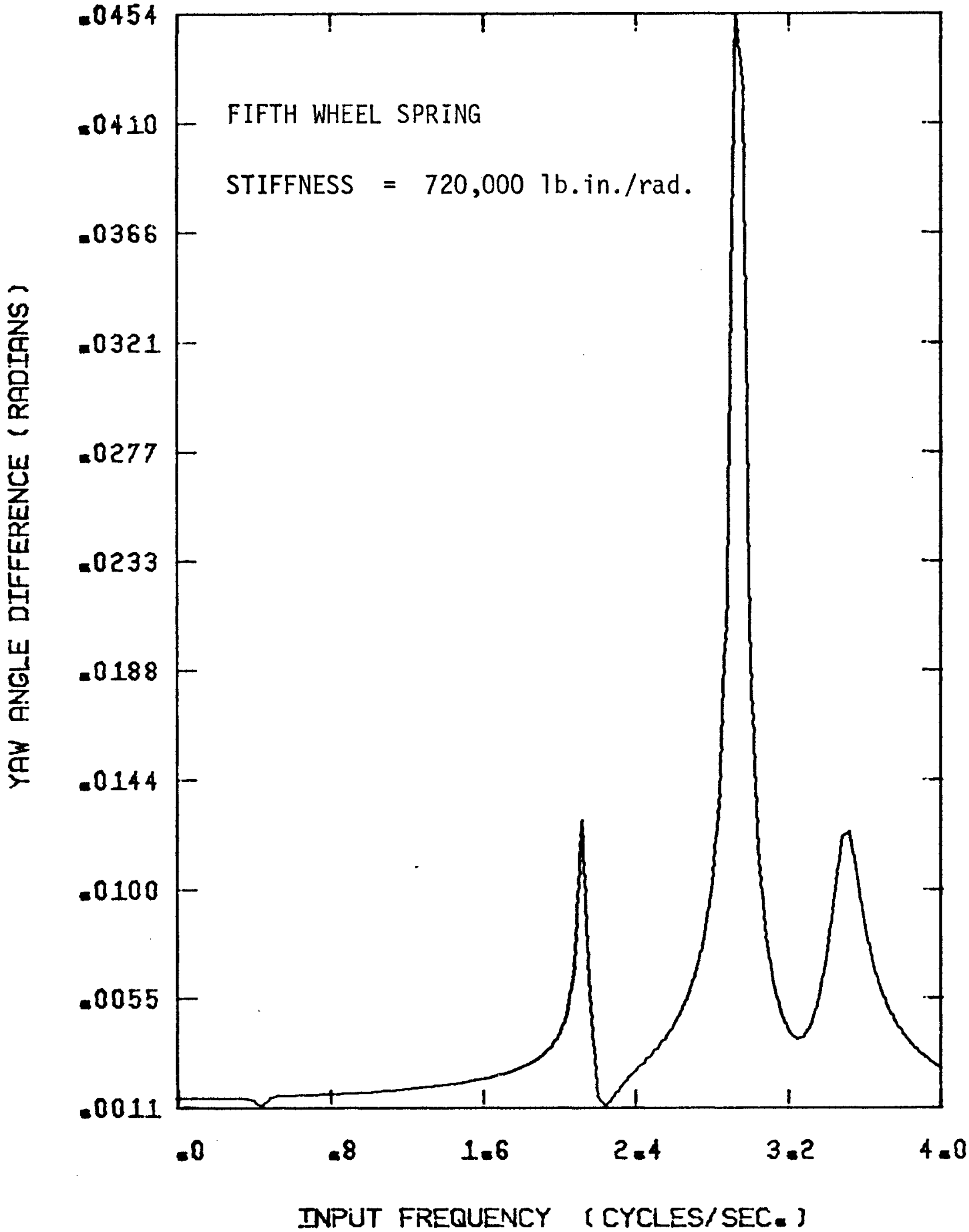

FIGURE 5.27: YAW ANGLE DIFFERENCE OF TRACTOR AND SEMITRAILER FOR K_h = 720,000 lb.in./rad.
(SELF-STEERING SEMITRAILER)

5.2.2 Results for Tractor-Standard Semitrailer

Since the semitrailer is assumed to be symmetric in the xy and yz planes, then the three modes, longitudinal, vertical and pitch are coupled together and they are decoupled to the other three modes, lateral, roll and yaw. Thus, the vertical motion for the articulated vehicle with a standard semitrailer is the same as the one with self-steering semitrailer.

The undamped eigenvalues are presented in Table 5.4 while the damped eigenvalues are presented in Table 5.5.

The steady state response curves are drawn for each generalized coordinate concerning the lateral motion of the vehicle.

Figures 5.28 to 5.30 show the response curves for the tractor in the lateral, roll and yaw coordinates respectively.

Figure 5.31 gives the response curve for the semitrailer in yaw coordinate.

The frequency response curves in the lateral and roll coordinates for the tractor front axle, tractor rear axle and bogie rear axle are shown in Figures 5.32 to 5.37 respectively.

TABLE 5.4

NATURAL FREQUENCIES OF THE VEHICLE

(STANDARD SEMITRAILER)

(IN CYCLES/SECOND)

OMEGA	1	=	.449
OMEGA	2	=	1.292
OMEGA	3	=	1.493
OMEGA	4	=	2.135
OMEGA	5	=	2.539
OMEGA	6	=	2.938
OMEGA	7	=	3.495
OMEGA	8	=	11.392
OMEGA	9	=	11.521
OMEGA	10	=	11.603
OMEGA	11	=	12.295
OMEGA	12	=	14.989
OMEGA	13	=	15.303
OMEGA	14	=	15.775
OMEGA	15	=	16.764
OMEGA	16	=	16.865
OMEGA	17	=	18.401
OMEGA	18	=	19.418
OMEGA	19	=	25.678

TABLE 5.5

RESULTS OF DAMPED EIGENVALUE ANALYSIS

(STANDARD SEMITRAILER)

THE EIGENVALUE NO.	1 =	-.5460364E+01	,	.1612447E+03
THE EIGENVALUE NO.	2 =	-.5460364E+01	,	-.1612447E+03
THE EIGENVALUE NO.	3 =	-.4454441E+01	,	.1219258E+03
THE EIGENVALUE NO.	4 =	-.4454441E+01	,	-.1219258E+03
THE EIGENVALUE NO.	5 =	-.4203110E+01	,	.1155386E+03
THE EIGENVALUE NO.	6 =	-.4203110E+01	,	-.1155386E+03
THE EIGENVALUE NO.	7 =	-.3486006E+01	,	.1052755E+03
THE EIGENVALUE NO.	8 =	-.3486006E+01	,	-.1052755E+03
THE EIGENVALUE NO.	9 =	-.9178666E+01	,	.1055654E+03
THE EIGENVALUE NO.	10 =	-.9178666E+01	,	-.1055654E+03
THE EIGENVALUE NO.	11 =	-.1131709E+02	,	.9349686E+02
THE EIGENVALUE NO.	12 =	-.1131709E+02	,	-.9349686E+02
THE EIGENVALUE NO.	13 =	-.9414843E+01	,	.9568851E+02
THE EIGENVALUE NO.	14 =	-.9414843E+01	,	-.9568851E+02
THE EIGENVALUE NO.	15 =	-.7943909E+01	,	.9879906E+02
THE EIGENVALUE NO.	16 =	-.7943909E+01	,	-.9879906E+02
THE EIGENVALUE NO.	17 =	-.1949418E+02	,	.7024962E+02
THE EIGENVALUE NO.	18 =	-.1949418E+02	,	-.7024962E+02
THE EIGENVALUE NO.	19 =	-.1452400E+02	,	.7587608E+02
THE EIGENVALUE NO.	20 =	-.1452400E+02	,	-.7587608E+02
THE EIGENVALUE NO.	21 =	-.1320513E+02	,	.7034772E+02
THE EIGENVALUE NO.	22 =	-.1320513E+02	,	-.7034772E+02
THE EIGENVALUE NO.	23 =	-.1269140E+02	,	.7127042E+02
THE EIGENVALUE NO.	24 =	-.1269140E+02	,	-.7127042E+02
THE EIGENVALUE NO.	25 =	-.5129658E+00	,	.2195163E+02
THE EIGENVALUE NO.	26 =	-.5129658E+00	,	-.2195163E+02
THE EIGENVALUE NO.	27 =	-.1381706E+00	,	.1845680E+02

TABLE 5.5 (continued)

THE EIGENVALUE NO.	28 =	-.1381706E+00	,	-.1845680E+02
THE EIGENVALUE NO.	29 =	-.1150690E+00	,	.1341244E+02
THE EIGENVALUE NO.	30 =	-.1150690E+00	,	-.1341244E+02
THE EIGENVALUE NO.	31 =	-.1608068E+00	,	.2815496E+01
THE EIGENVALUE NO.	32 =	-.1608068E+00	,	-.2815496E+01
THE EIGENVALUE NO.	33 =	-.4553223E+01	,	.1528695E+02
THE EIGENVALUE NO.	34 =	-.4553223E+01	,	-.1528695E+02
THE EIGENVALUE NO.	35 =	-.1221146E+01	,	.8028223E+01
THE EIGENVALUE NO.	36 =	-.1221146E+01	,	-.8028223E+01
THE EIGENVALUE NO.	37 =	-.1611329E+01	,	.9241422E+01
THE EIGENVALUE NO.	38 =	-.1611329E+01	,	-.9241422E+01

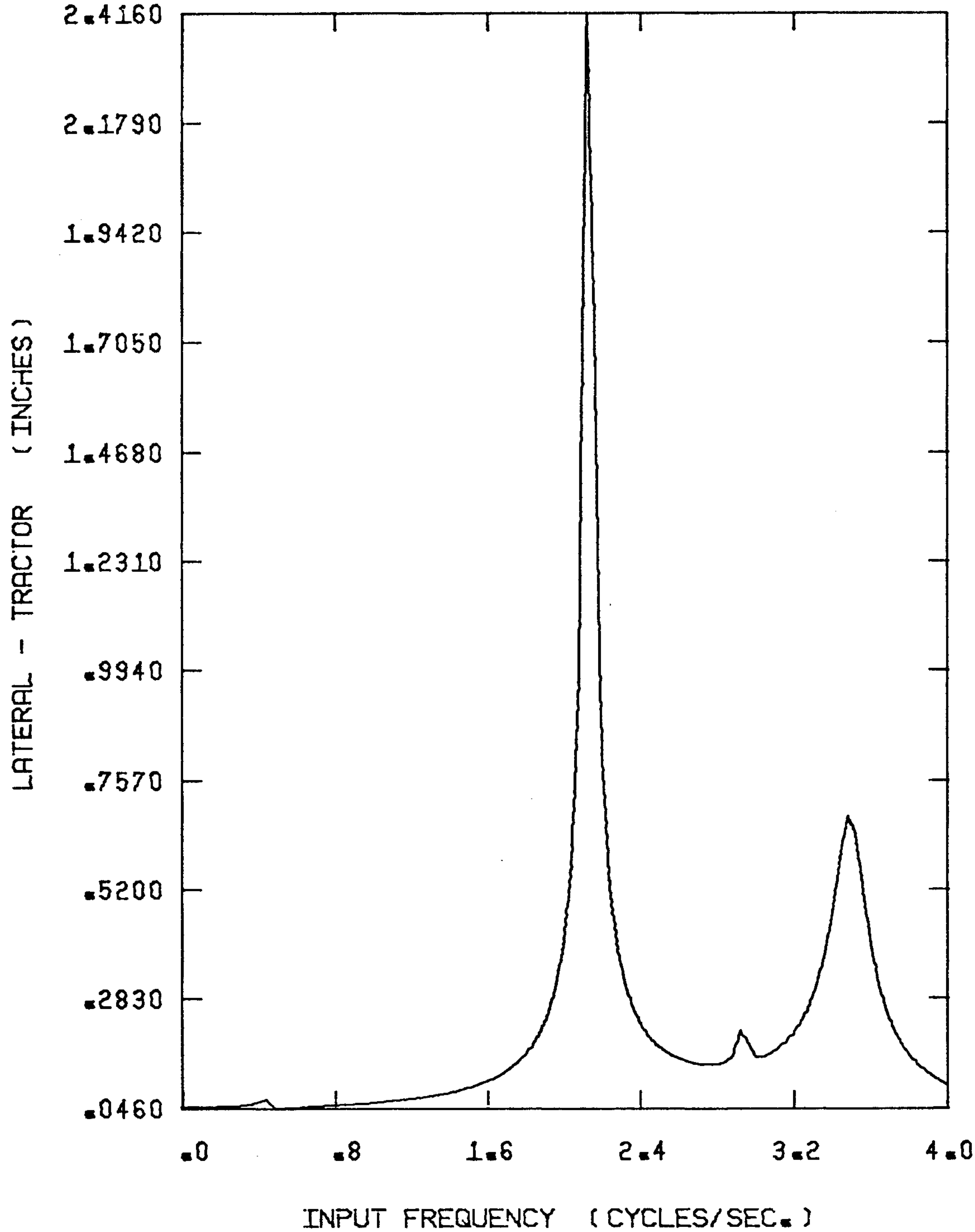

FIGURE 5.28: LATERAL TRACTOR DISPLACEMENT (v_t)
(STANDARD SEMITRAILER)

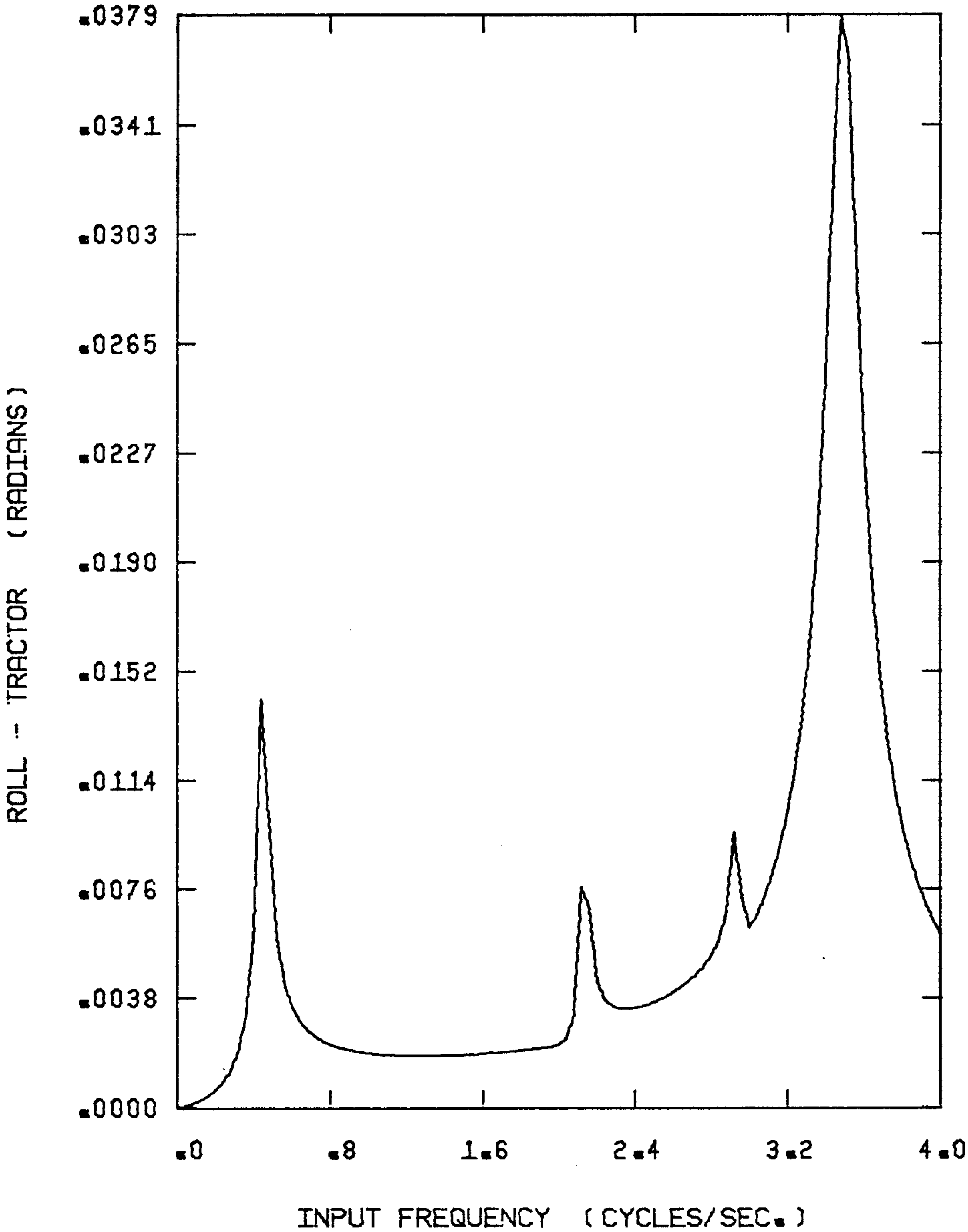

FIGURE 5.29: ROLL TRACTOR ANGLE (α_t)
(STANDARD SEMITRAILER)

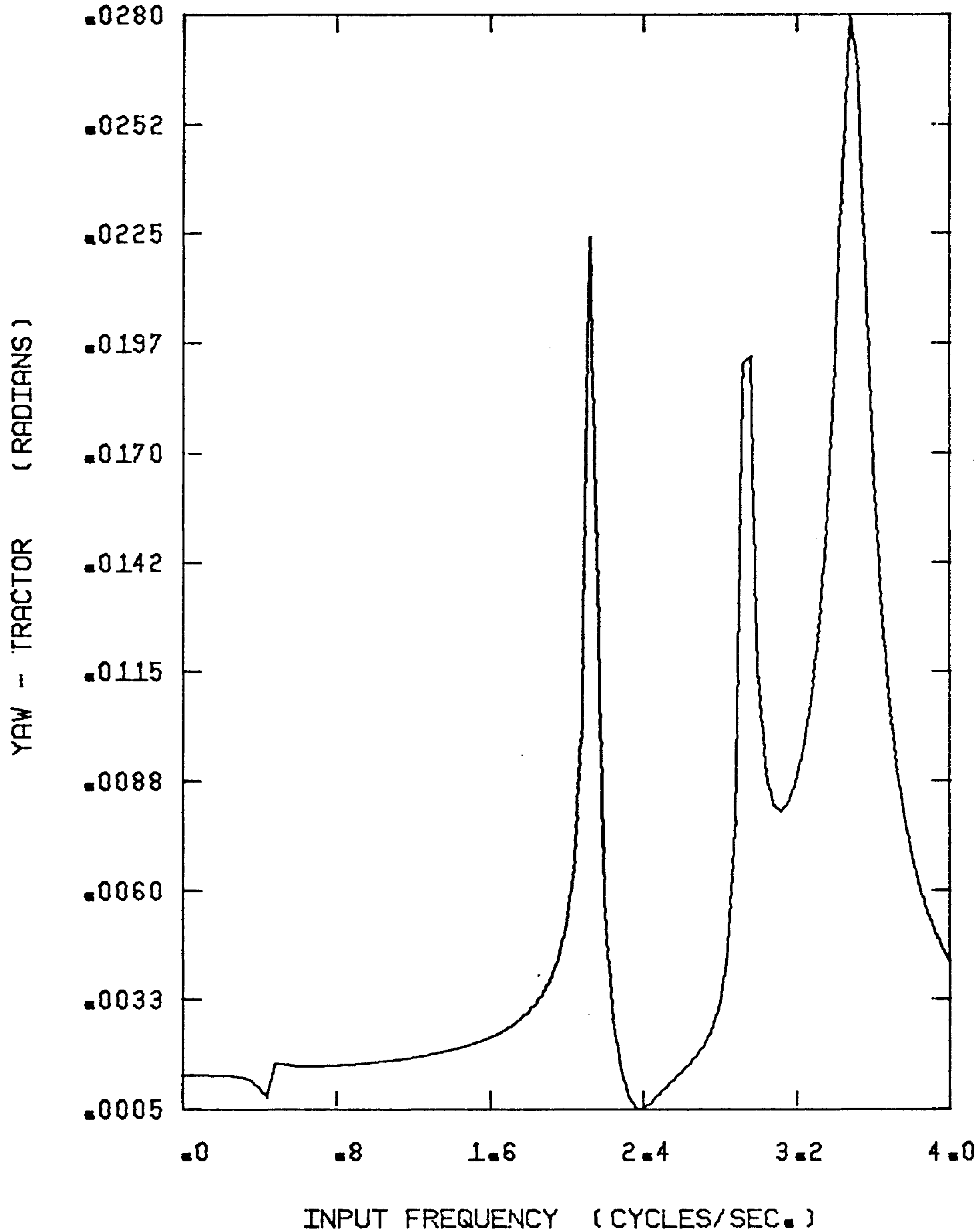

FIGURE 5.30: YAW TRACTOR ANGLE (γ_t)
(STANDARD SEMITRAILER)

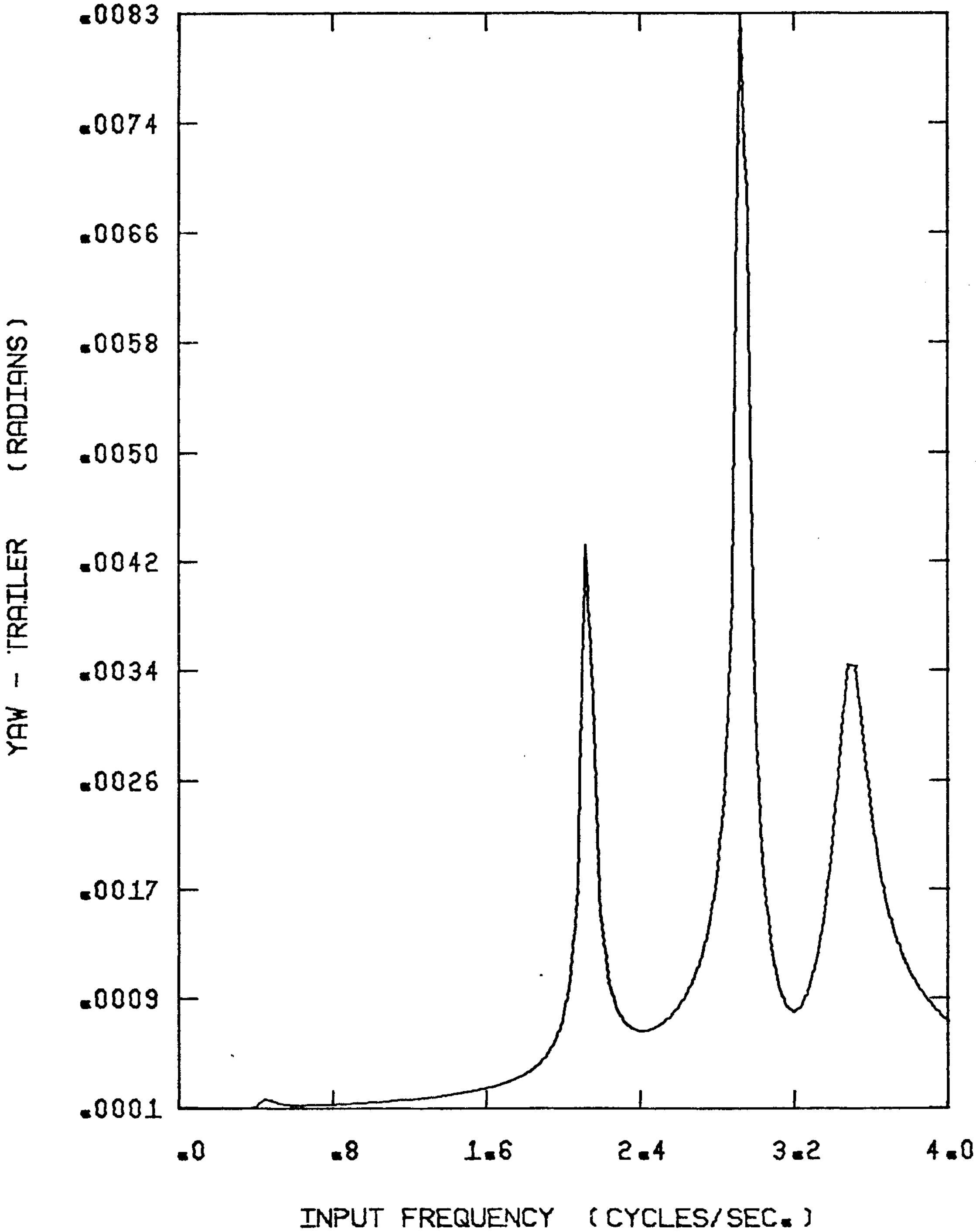

FIGURE 5.31: YAW SEMITRAILER ANGLE (γ_S)
(STANDARD SEMITRAILER)

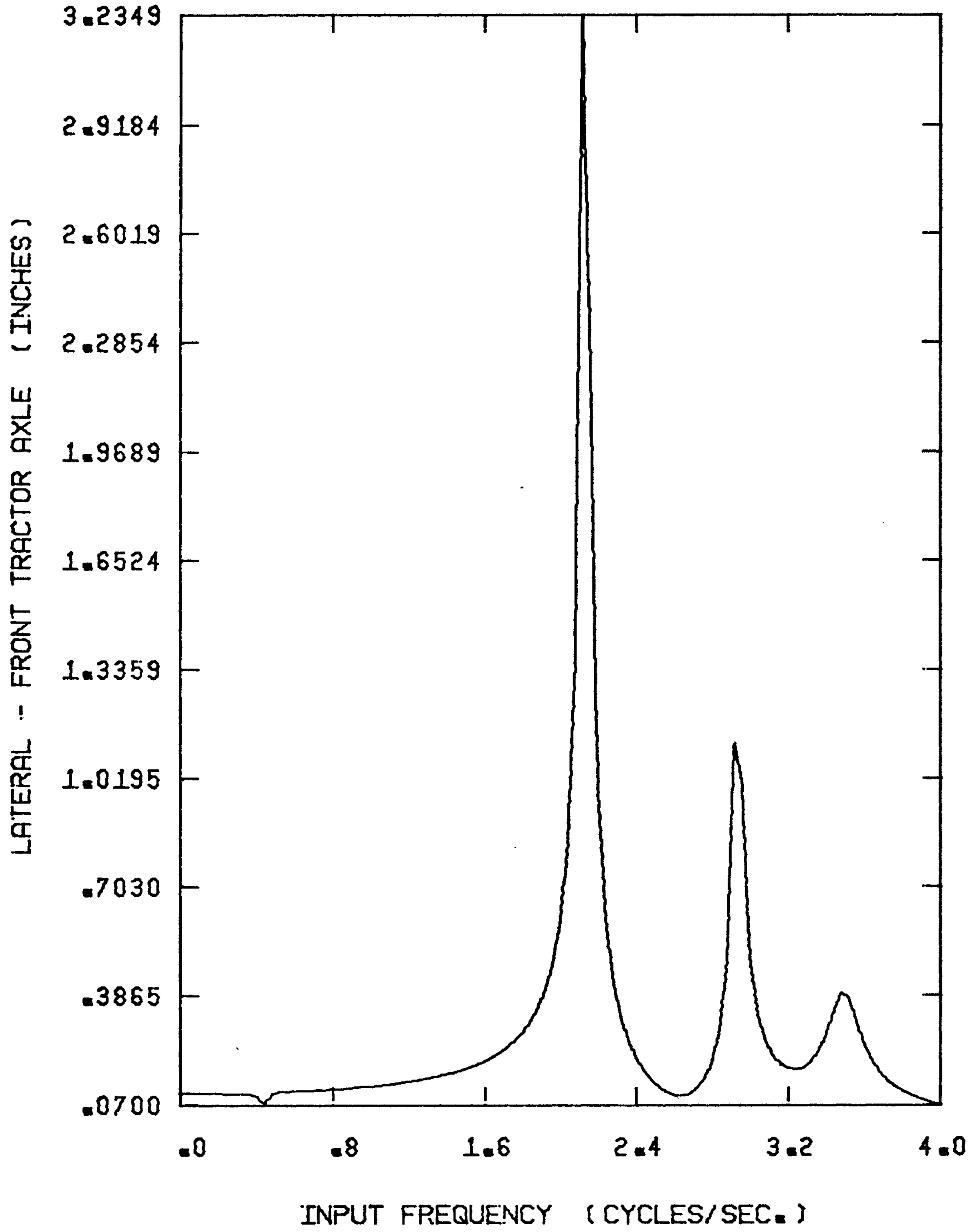

FIGURE 5.32: LATERAL TRACTOR FRONT AXLE DISPLACEMENT (v_1)
(STANDARD SEMITRAILER)

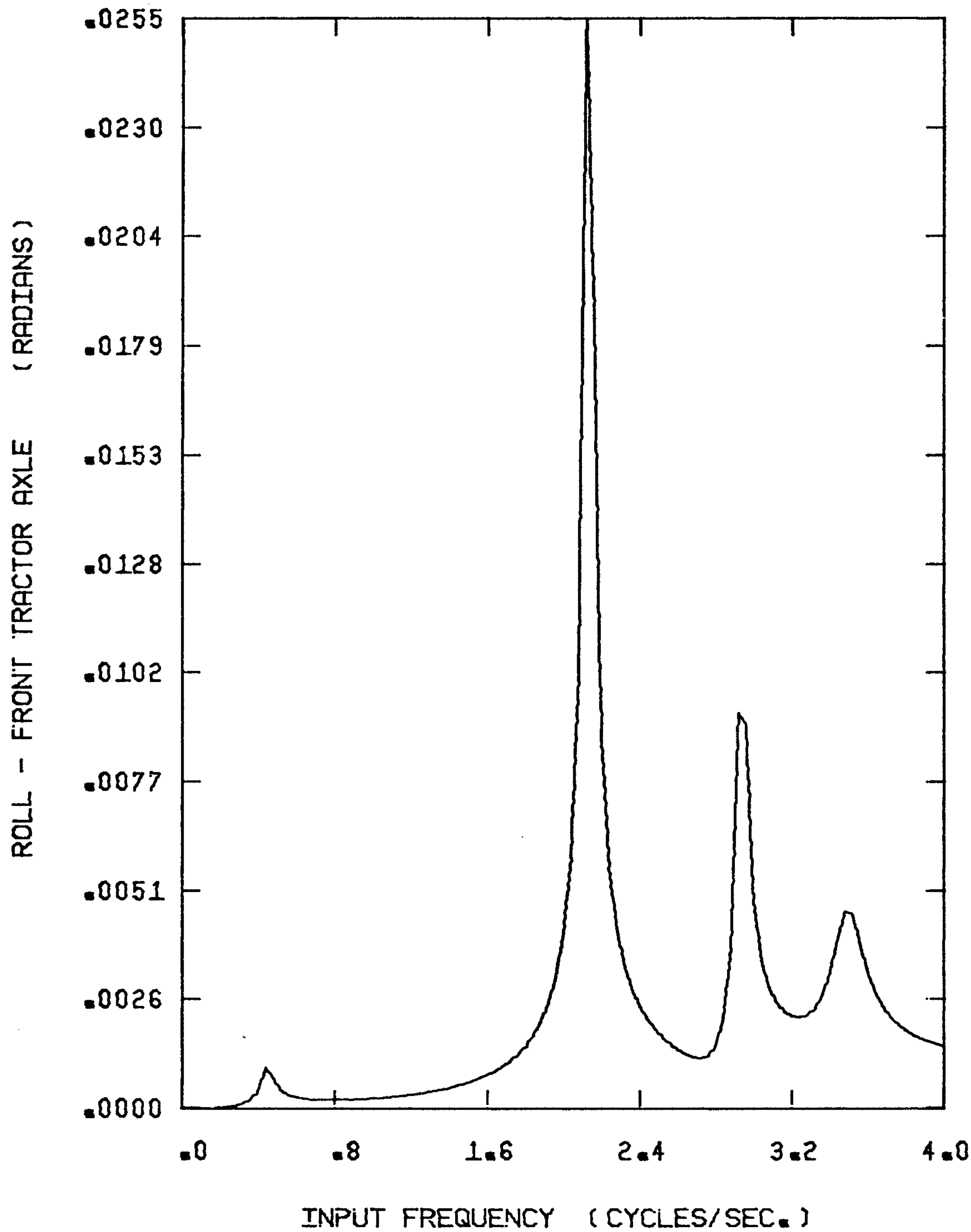

FIGURE 5.33: ROLL TRACTOR FRONT AXLE ANGLE (α_1)
(STANDARD SEMITRAILER)

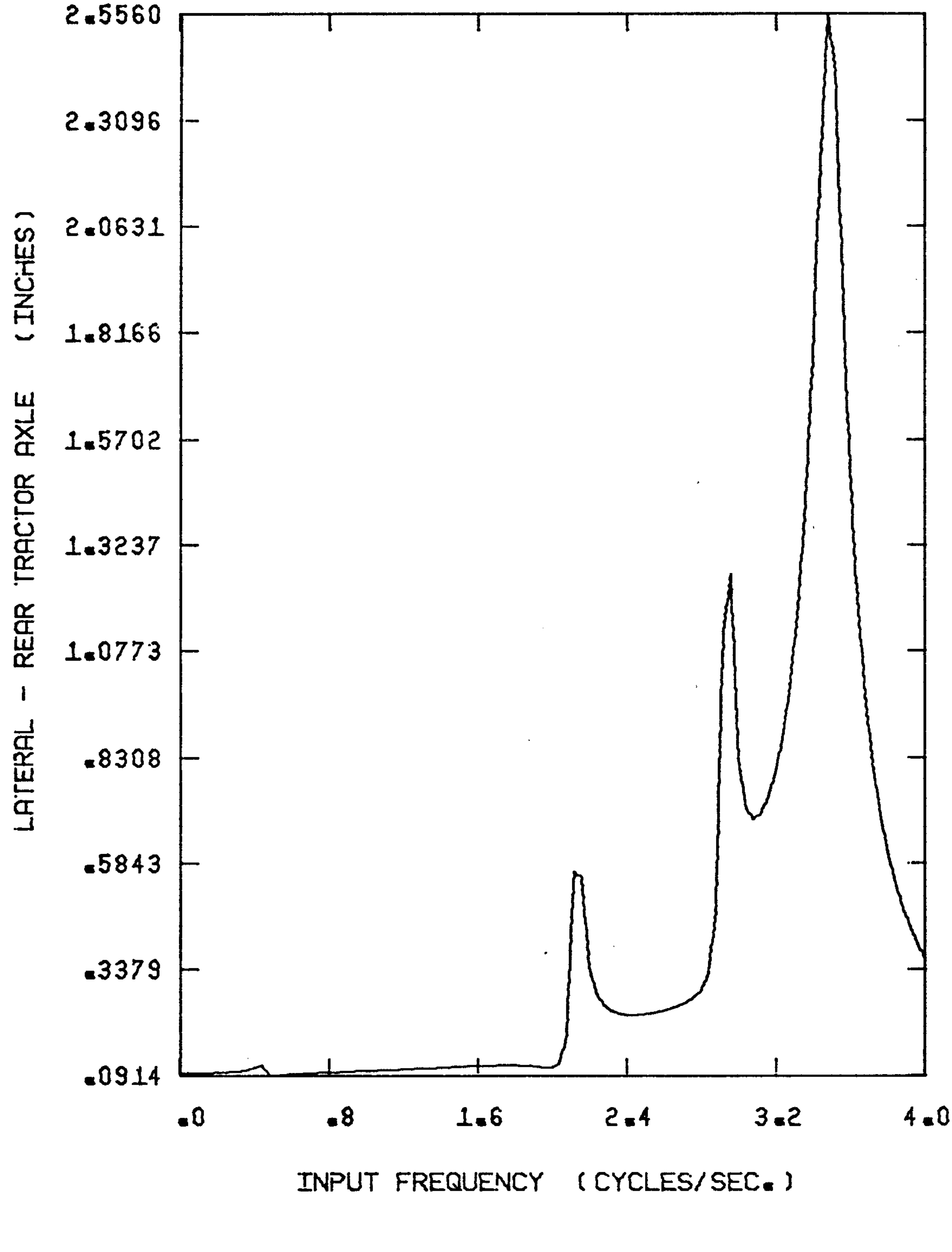

FIGURE 5.34: LATERAL TRACTOR REAR AXLE DISPLACEMENT (v_3)
(STANDARD SEMITRAILER)

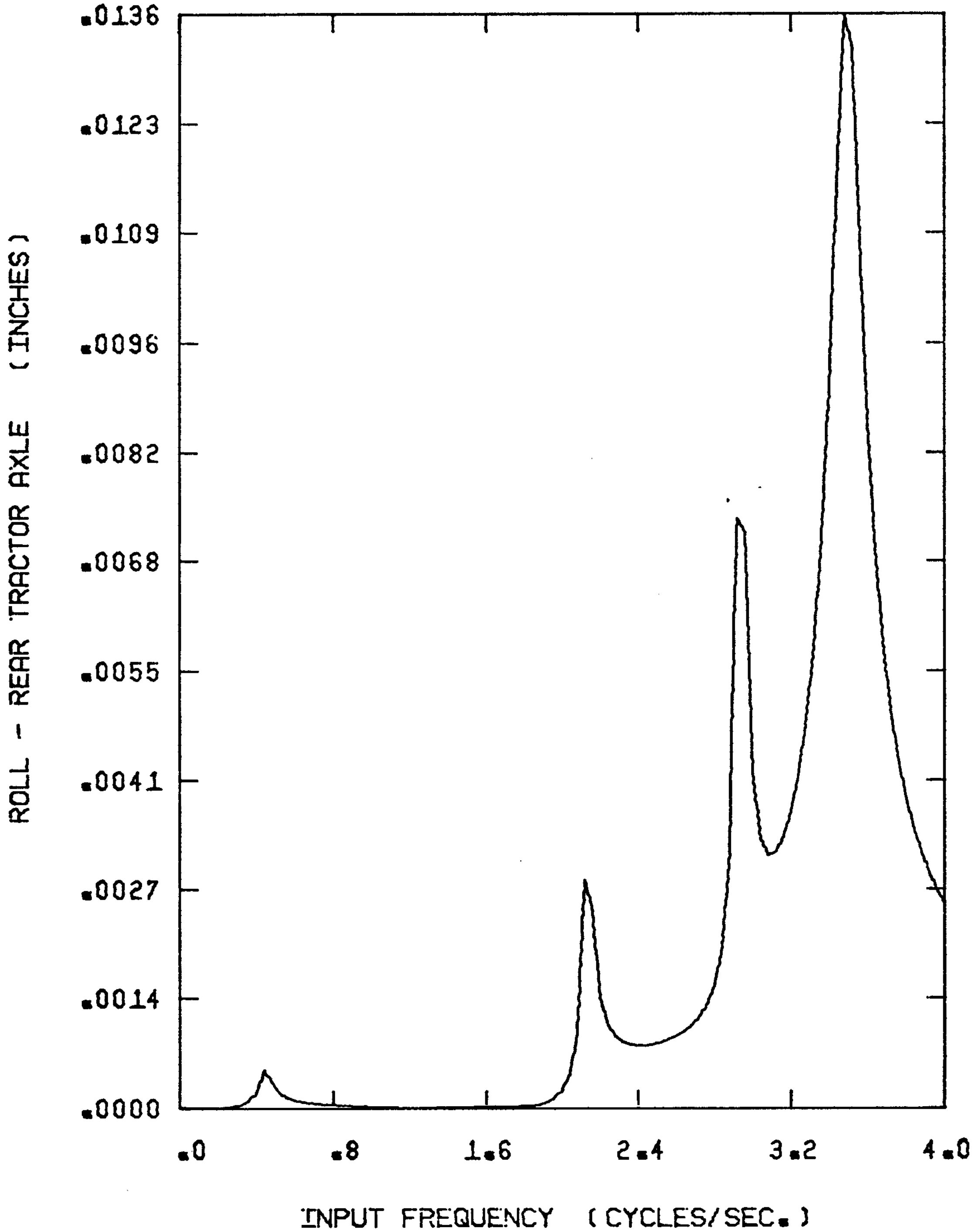

FIGURE 5.35: ROLL TRACTOR REAR AXLE ANGLE (α_3)
(STANDARD SEMITRAILER)

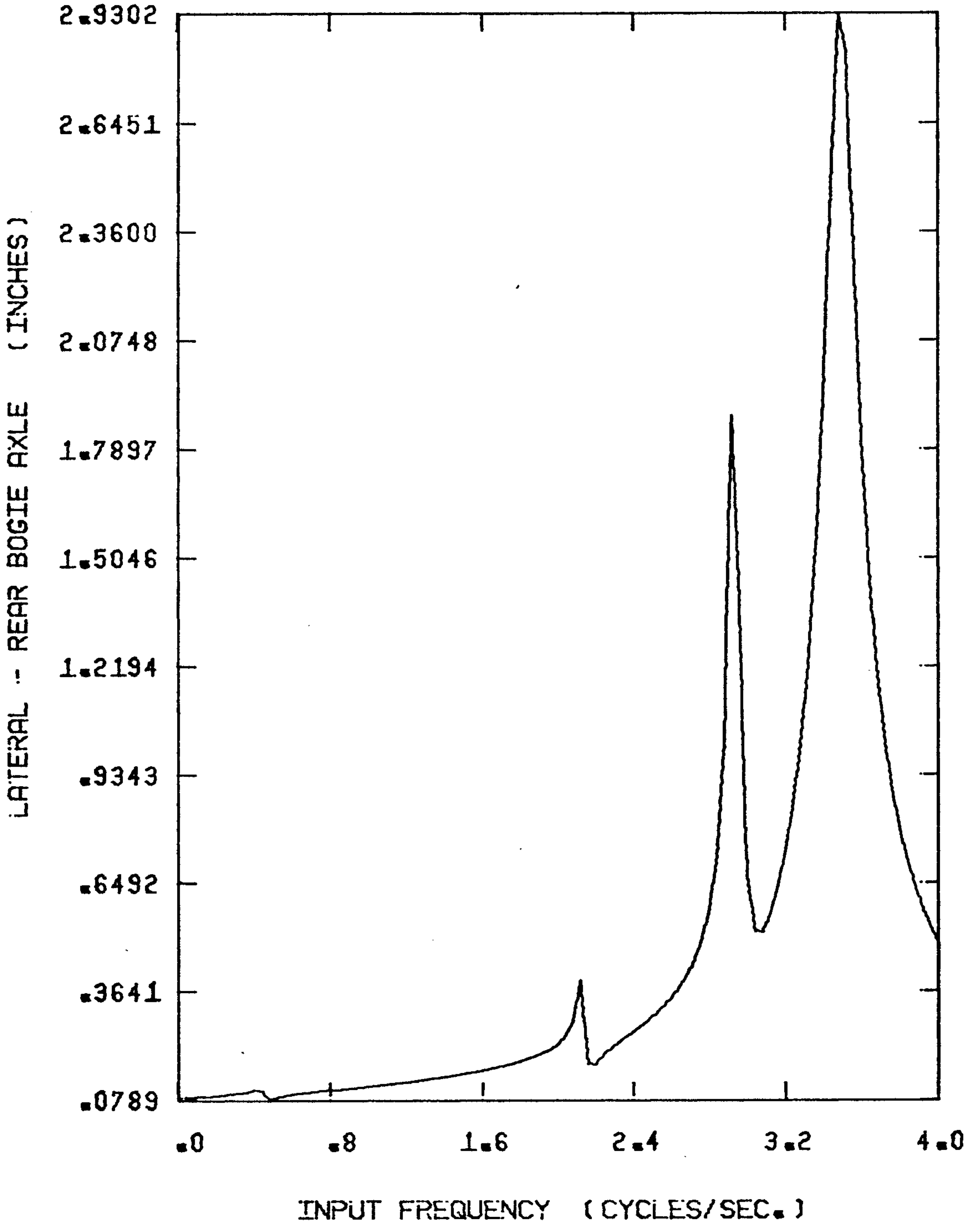

FIGURE 5.36: LATERAL BOGIE REAR AXLE DISPLACEMENT (v_7)
(STANDARD SEMITRAILER)

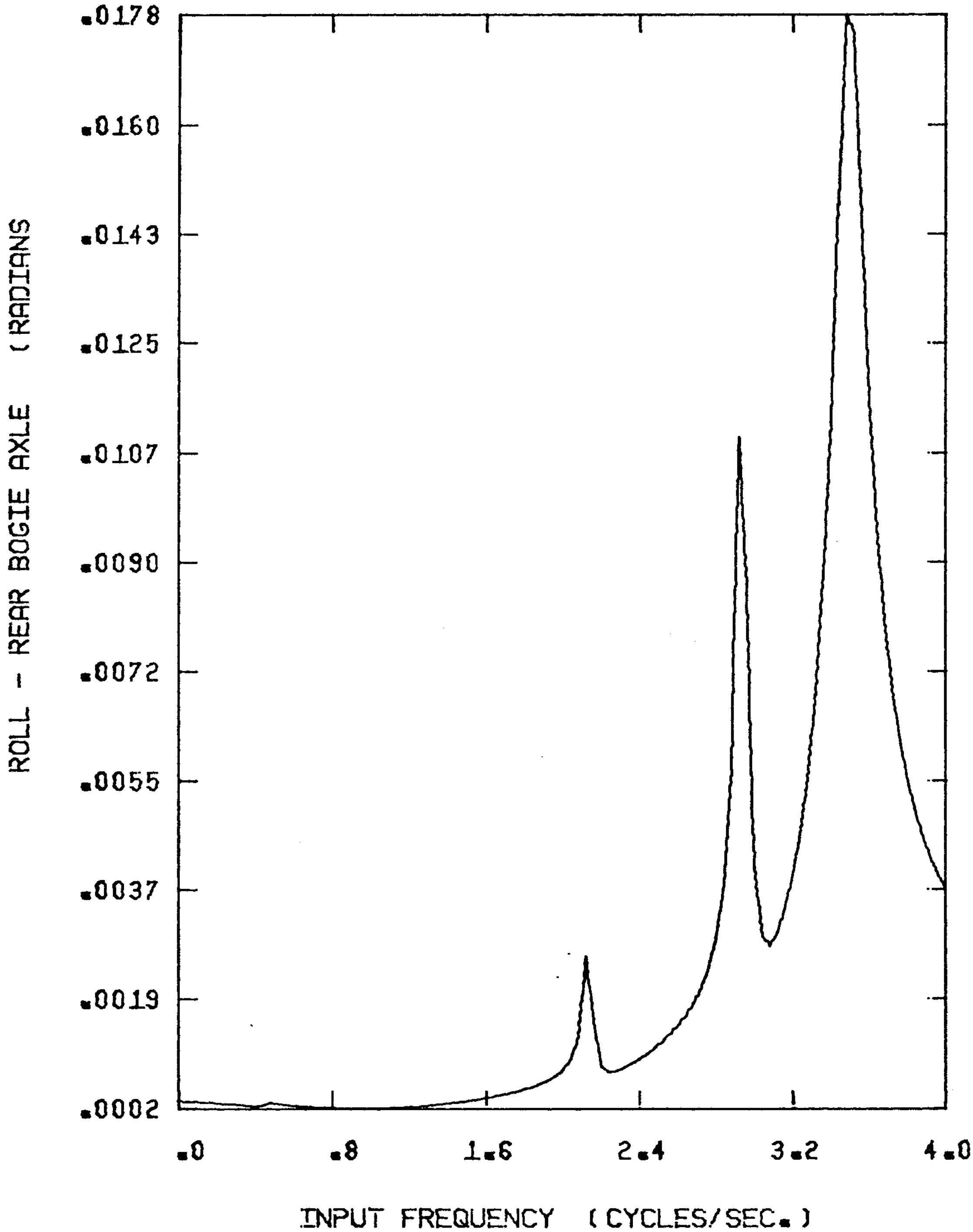

FIGURE 5.37: ROLL BOGIE REAR AXLE ANGLE (α_7)
(STANDARD SEMITRAILER)

5.2.3 An Investigation Into the Dynamic Behaviour of Articulated Vehicle and Highways

It is well known that the road loading problem is one of the important aspects of the overall objective of building adequate life into vehicular highways.

The vehicle and highway have the resonating characteristics at certain frequencies, thereby magnifying the peak dynamic road load forces exerted by the vehicle on the road, and the peak dynamic deflections and bending moments that are induced in the road by the vehicle.

The excitation of the road profile limited to that causes vertical motion of the axles and consequently pitch and bounce of the vehicle. Since the forcing frequency may vary due either to a change in the speed of the vehicle or to a change in the frequency of the road input sinusoid, a Bode surface may be defined for this problem. However, section plots of this surface, one for each axle coordinate at speed of 40.0 m.p.h. are plotted.

The ratio of the steady-state dynamic load to the static load is plotted as a function of frequency (the ratio of vehicle speed to road wave length) in Figures 5.38 to 5.41. These plots represent the dynamic loads imparted to the pavement at tractor front axle, tractor rear axle, bogie front axle and bogie rear axle respectively.

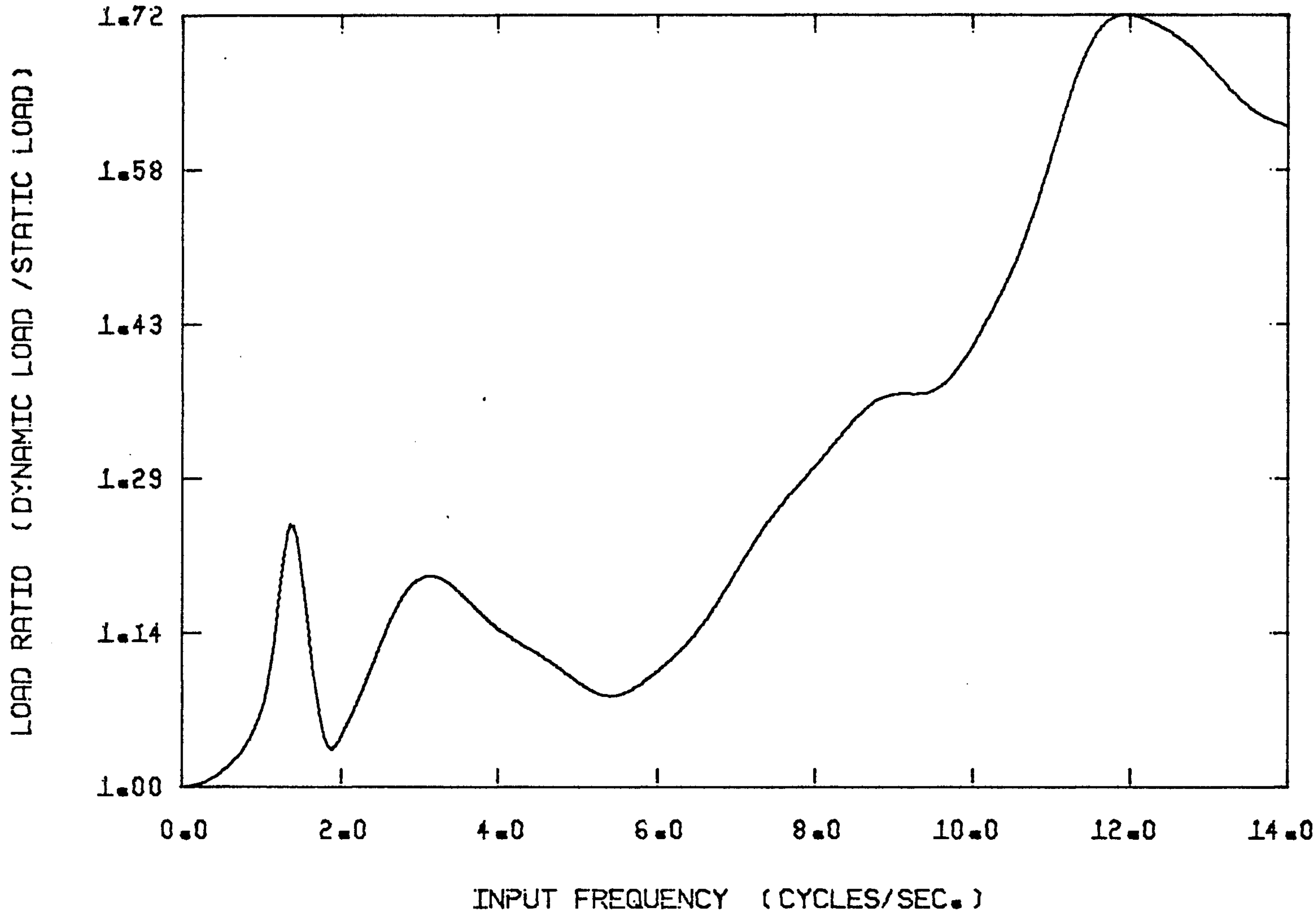

FIGURE 5.38: VERTICAL ROAD LOAD FREQUENCY RESPONSE FOR: TRACTOR FRONT AXLE

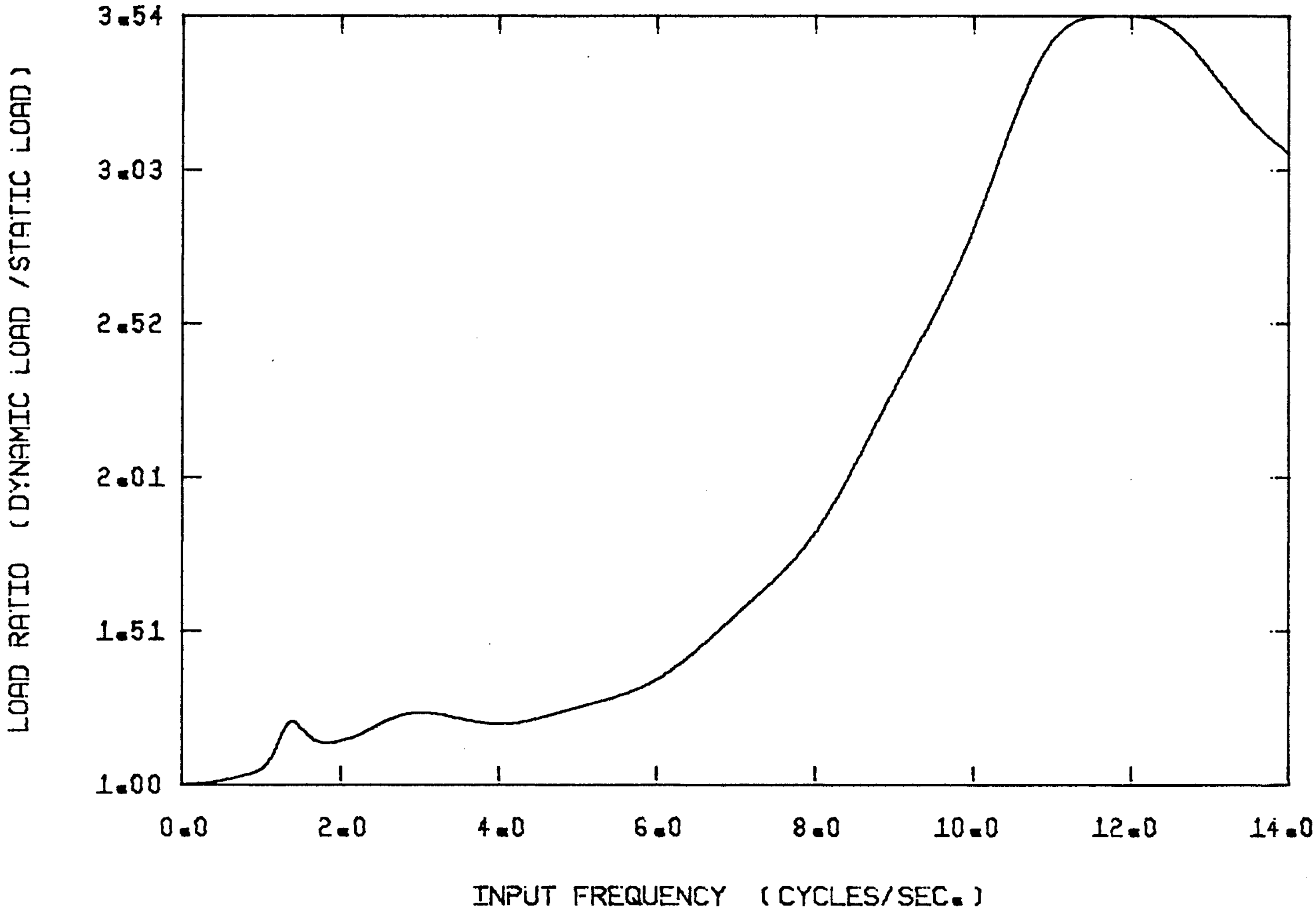

FIGURE 5.39: VERTICAL ROAD LOAD FREQUENCY RESPONSE FOR: TRACTOR REAR AXLE

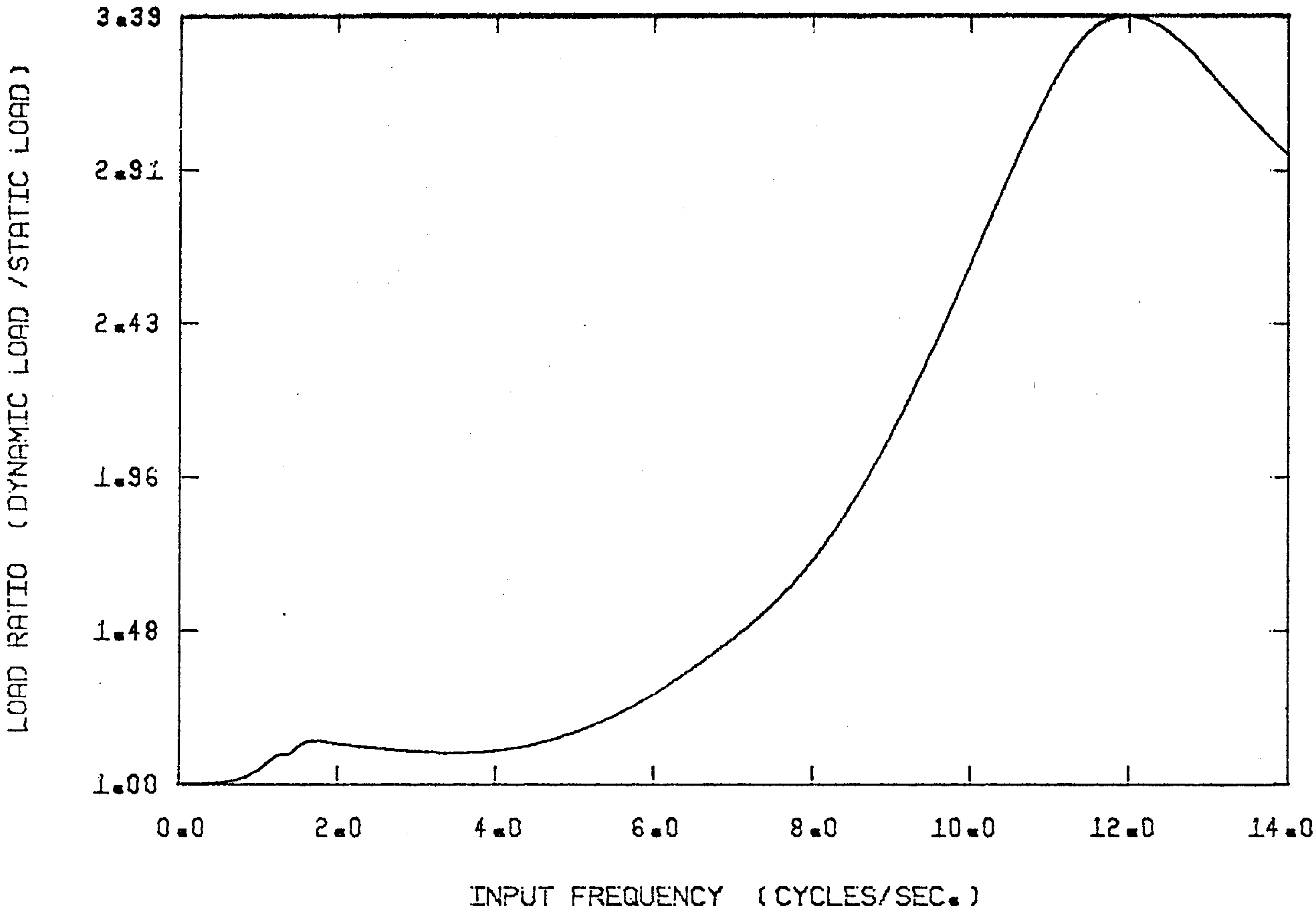

FIGURE 5.40: VERTICAL ROAD LOAD FREQUENCY RESPONSE FOR: BOGIE FRONT AXLE

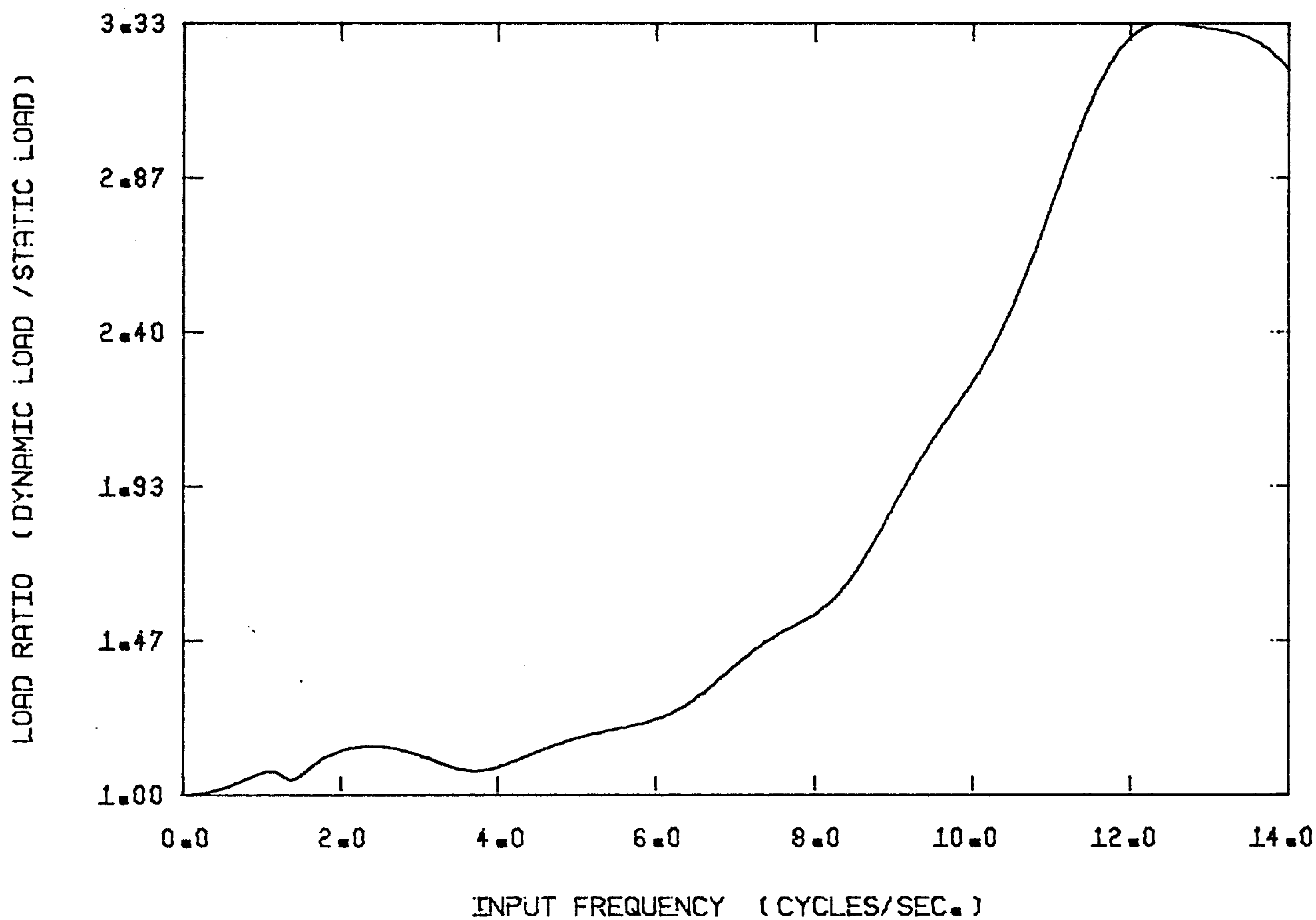

FIGURE 5.41: VERTICAL ROAD LOAD FREQUENCY RESPONSE FOR: BOGIE REAR AXLE

5.3 Conclusions

A digital computer simulation has been used to study the dynamic of a tractor-semitrailer vehicle. Two mathematical models, a tractor-standard semitrailer and a tractor self-steering semitrailer have been developed to describe the longitudinal, lateral, vertical, pitching, rolling and yawing motions of the vehicle on a rough road surface.

The results obtained in this thesis indicate that:

1) The peak responses in the lateral and roll coordinates in the case of standard semitrailer are higher, compared to the case of self-steering semitrailer, while the peak responses in yaw coordinates are lower.
   It should be mentioned that the non-conservative forces which are the side forces acting on each tire at the tire-road interface were not included in the derivation of the equations of motion, i.e. the vehicle was considered as stationary one. In other words, for the stationary vehicle certain generalized coordinates relating to the motions of the axles were locked.

2) Introducing a spring at the fifth wheel reduces the peak response in the yaw modes, but to be effective, large spring forces are needed.

3) The vehicle and highway are dynamic systems, i.e. mechanical resonance occurs at certain frequencies and magnifies the transmitted loads imparted to the pavement due to the dynamic action of the vehicle in response to road irregularities.

The program with some modifications may be applied in the following areas:

The vehicle designers could determine the optimum suspension to satisfy driver comfort, acceptable cargo ride and an acceptable level of suspension wear.

The program can be used to determine the influence of road and vehicle dimensions on the amplitude of sprung masses motions and dynamic wheel loads.

The program might be utilized to reduce the vibrational difficulties encountered in pulling small trailers (i.e. boat trailers, utility trailers).

5.4 Suggestions for Future Research

There are many vehicle characteristics, surface factors, and specified control inputs which may be investigated.

Some of the areas which should be explored are:

For Vehicle Ride

1) Random road surface should be applied to the vehicle to determine the response of the vehicle components to the undulations of the road surface over which it passes. The random vibration theory allows a set of compromise parameters in vehicle design to be developed.

2) Study the effect of fifth-wheel position on the tractor and the spring and damping rates of the different axles of the vehicle.

3) A nonlinear model needs to be developed to consider the nonlinearities of the suspension springs, the shock absorbers, the pneumatic tires, the coloumb friction in the suspension and the wheel hop. Thus, it would be able to gauge the inaccuracies present in a linear analysis.

For Vehicle Handling

A mathematical model for the self-steering semitrailer should be developed to include the vehicle speed as a parameter and to study:

1) The stability of the semitrailer when rounding a curve.

2) The response of the vehicle to steering and braking input to determine the influence of design parameters, operating conditions and the environment on the directional behavior of the vehicle.

3) The tire rolling resistance, the side force and the slip angle.

Finally, actual vehicle tests should be performed in order to vertify the analytical studies which would be performed.

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APPENDIX I

DERIVATION OF THE EQUATIONS OF MOTION OF SELF-STEERING SEMITRAILER

I.1 Introduction

In this Appendix equations of motion for the articulated vehicle with self-steering semitrailer are derived in detail. The following notations for the displacements and angular rotations are used, Figures 3.1 and 3.2:

u - linear displacement in the x direction,

v - linear displacement in the y direction,

w - linear displacement in the z direction,

α - angular rotation about the x direction,

β - angular rotation about the y direction,

γ - angular rotation about the z direction.

Subscripts are as follows:

t - for the tractor,

s - for the semitrailer,

b - for the bogie,

$i = 1, 3, 5, 7$ - for the four axles.

I.2 Calculations of the Forces in Springs

a) For Tractor Front Axle:

For Spring K_1:

i) Left

$$P_{1\ell} = K_1\{(w_t - w_1) - a_1\,\alpha_t + a_1\,\alpha_1 - b_1\,\beta_t\} \qquad (I.1)$$

ii) Right

$$P_{1r} = K_1\{(w_t - w_1) + a_1\,\alpha_t - a_1\,\alpha_1 - b_1\,\beta_t\} \tag{I.2}$$

For Spring K_2:

i) Left

$$P_{2\ell} = K_2\,\{(v_t - v_1) - h_1\,\alpha_t - d_1\,\alpha_1 + b_1\,\gamma_t\} \tag{I.3}$$

ii) Right

$$P_{2r} = K_2\,\{(v_t - v_1) - h_1\,\alpha_t - d_1\,\alpha_1 + b_1\,\gamma_t\} \tag{I.4}$$

For Spring K_{t1}:

i) Left

$$P_{11\ell} = K_{t1}\,\{w_1 - a_2\,\alpha_1 - G_1(t)\} \tag{I.5}$$

ii) Right

$$P_{11r} = K_{t1}\,\{w_1 + a_2\,\alpha_1 - G_2(t)\} \tag{I.6}$$

For Spring K_{t2}:

i) Left

$$P_{12\ell} = K_{t2}\{v_1 - r_1\,\alpha_1 - \lambda_1(t)\} \tag{I.7}$$

ii) Right

$$P_{12r} = K_{t2}\{v_1 - r_1\,\alpha_1 - \lambda_2(t)\} \tag{I.8}$$

b) For Tractor Rear Axle:

For Spring K_3:

i) Left

$$P_{3\ell} = K_3\{(w_t - w_3) - a_3\,\alpha_t + a_3\,\alpha_3 + b_2\,\beta_t\} \tag{I.9}$$

ii) Right

$$P_{3r} = K_3\{(w_t - w_3) + a_3\,\alpha_t - a_3\,\alpha_3 + b_2\,\beta_t\} \tag{I.10}$$

For Spring K_4:

i) Left

$$P_{4\ell} = K_4\{(v_t - v_3) - h_3\,\alpha_t - d_3\,\alpha_3 - b_2\,\gamma_t\} \tag{I.11}$$

ii) Right

$$P_{4r} = K_4\{(v_t - v_3) - h_3\,\alpha_t - d_3\,\alpha_3 - \gamma_t\,b_2\} \tag{I.12}$$

For Spring K_{t3}:

i) Left

$$P_{13\ell} = K_{t3}\,\{w_3 - a_4\,\alpha_3 - G_3(t)\} \tag{I.13}$$

ii) Right

$$P_{13r} = K_{t3}\,\{w_3 + a_4\,\alpha_3 - G_4(t)\} \tag{I.14}$$

For Spring K_{t4}:

i) Left

$$P_{14\ell} = K_{t4}\{v_3 - r_3\,\alpha_3 - \lambda_3(t)\} \qquad (I.15)$$

ii) Right

$$P_{14r} = K_{t4}\{v_3 - r_3\,\alpha_3 - \lambda_4(t)\} \qquad (I.16)$$

c) For Bogie Front Axle

For Spring K_5:

i) Left

$$P_{5\ell} = K_5\{(w_b - w_5) - a_5\,\alpha_b + a_5\,\alpha_5 - b_5\,\beta_b\} \qquad (I.17)$$

ii) Right

$$P_{5r} = K_5\{(w_b - w_5) + a_5\,\alpha_b - a_5\,\alpha_5 - b_5\,\beta_b\} \qquad (I.18)$$

For Spring K_6:

i) Left

$$P_{6\ell} = K_6\{(v_b - v_5) - h_6\,\alpha_b - d_5\,\alpha_5 + b_5\,\gamma_b\} \qquad (I.19)$$

ii) Right

$$P_{6r} = K_6\{(v_b - v_5) - h_6\,\alpha_b - d_5\,\alpha_5 + b_5\,\gamma_b\} \qquad (I.20)$$

For Spring K_{t3}:

i) Left

$$P_{15\ell} = K_{t5} \{w_5 - a_6 \alpha_5 - G_5(t)\} \tag{I.21}$$

ii) Right

$$P_{15r} = K_{t5} \{w_5 + a_6 \alpha_5 - G_6(t)\} \tag{I.22}$$

For Spring K_{t6}:

i) Left

$$P_{16\ell} = K_{t6} \{v_5 - r_5 \alpha_5 - \lambda_5(t)\} \tag{I.23}$$

ii) Right

$$P_{16r} = K_{t6} \{v_5 - v_5 \alpha_5 - \lambda_6(t)\} \tag{I.24}$$

d) For Bogie Rear Axle

For Spring K_7:

i) Left

$$P_{7\ell} = K_7 \{(w_b - w_7) - a_7 \alpha_b + a_7 \alpha_7 + b_5 \beta_b\} \tag{I.25}$$

ii) Right

$$P_{7r} = K_7 \{(w_b - w_7) + a_7 \alpha_b + a_7 \alpha_7 + b_5 \beta_b\} \tag{I.26}$$

For Spring K_8:

i) Left

$$P_{8\ell} = K_8\{(v_b - v_7) - h_6\,\alpha_b - d_7\,\alpha_7 - b_5\,\gamma_b\} \qquad (I.27)$$

ii) Right

$$P_{8r} = K_8\{(v_b - v_7) - h_6\,\alpha_b - d_7\,\alpha_7 - b_5\,\gamma_b\} \qquad (I.28)$$

For Spring K_{t7}:

i) Left

$$P_{17\ell} = K_{t7}\,\{w_7 - a_8\,\alpha_7 - G_7(t)\} \qquad (I.29)$$

ii) Right

$$P_{17r} = K_{t7}\,\{w_7 + a_8\,\alpha_7 - G_8(t)\} \qquad (I.30)$$

For Spring K_{t8}:

i) Left

$$P_{18\ell} = K_{t8}\,\{v_7 - r_7\,\alpha_7 - \lambda_7(t)\} \qquad (I.31)$$

ii) Right

$$P_{18r} = K_{t8}\,\{v_7 - r_7\,\alpha_7 - \lambda_8(t)\} \qquad (I.32)$$

I.3 Forces Due to Damping

Equations (I.33) to (I.64) give the damping forces, N, that could be obtained from equations (I.1) to (I.32) by replacing K_n by C_n and the variables by their first derivatives.

I.4 Dynamic Reactions of The Suspension

a) In the Vertical Direction

$$F_{1z} = K_1\{(w_t - w_1) - a_1\alpha_t + a_1\alpha_1 - b_1\beta_t\}$$
$$+ C_1\{(\dot{w}_t - \dot{w}_1) - a_1\dot{\alpha}_t + a_1\dot{\alpha}_1 - b_1\dot{\beta}_t\} \qquad (I.65)$$

$$F_{2z} = K_1\{(w_t - w_1) + a_1\alpha_t - a_1\alpha_1 - b_1\beta_t\}$$
$$+ C_1\{(\dot{w}_t - \dot{w}_1) + a_1\dot{\alpha}_t - a_1\dot{\alpha}_1 - b_1\dot{\beta}_t\} \qquad (I.66)$$

$$F_{3z} = K_3\{(w_t - w_3) - a_3\alpha_t + a_3\alpha_3 + b_2\beta_t\}$$
$$+ C_3\{(\dot{w}_t - \dot{w}_3) - a_3\dot{\alpha}_t + a_3\dot{\alpha}_3 + b_2\dot{\beta}_t\} \qquad (I.67)$$

$$F_{4z} = K_3\{(w_t - w_3) + a_3\alpha_t - a_3\alpha_3 + b_2\beta_t\}$$
$$+ C_3\{(\dot{w}_t - \dot{w}_3) + a_3\dot{\alpha}_t - a_3\dot{\alpha}_3 + b_2\dot{\beta}_t\} \qquad (I.68)$$

$$F_{5z} = K_5\{(w_b - w_5) - a_5\alpha_b + a_5\alpha_5 - b_5\beta_b\}$$
$$+ C_5\{(\dot{w}_b - \dot{w}_5) - a_5\dot{\alpha}_b + a_5\dot{\alpha}_5 - b_5\dot{\beta}_b\} \qquad (I.69)$$

$$F_{6z} = K_5\{(w_b - w_5) + a_5\alpha_b - a_5\alpha_5 - b_5\beta_b\}$$
$$C_5\{(\dot{w}_b - \dot{w}_5) + a_5\dot{\alpha}_b - a_5\dot{\alpha}_5 - b_5\dot{\beta}_b\} \qquad (I.70)$$

$$F_{7z} = K_7\{(w_b-w_7) - a_6\alpha_b + a_7\alpha_7 + b_5\beta_b\}$$

$$+ C_7\{(\dot{w}_b-\dot{w}_7) - a_7\dot{\alpha}_b + a_7\dot{\alpha}_7 + b_5\dot{\beta}_b\} \qquad (I.71)$$

$$F_{8z} = K_7\{(w_b-w_7) + a_7\alpha_b - a_7\alpha_7 + b_5\beta_b\}$$

$$+ C_7\{(\dot{w}_b-\dot{w}_7) + a_7\dot{\alpha}_b - a_7\dot{\alpha}_7 + b_5\dot{\beta}_b\} \qquad (I.72)$$

b) In the Lateral Direction

$$F_{1y} = F_{2y} = K_2\{(v_t-v_1) - h_1\alpha_t - d_1\alpha_1 + b_1\gamma_t\}$$

$$+ C_2\{(\dot{v}_t-\dot{v}_1) - h_1\dot{\alpha}_t - d_1\dot{\alpha}_1 + b_1\dot{\gamma}_t\} \qquad (I.73)$$

$$F_{3y} = F_{4y} = K_4\{(v_t-v_3) - h_3\alpha_t - d_3\alpha_3 - b_2\gamma_t\}$$

$$+ C_4\{(\dot{v}_t-\dot{v}_3) - h_3\dot{\alpha}_t - d_3\dot{\alpha}_3 - b_2\dot{\gamma}_t\} \qquad (I.74)$$

$$F_{5y} = F_{6y} = K_6\{(v_b-v_5) - h_5\alpha_b - d_5\alpha_5 + b_5\gamma_b\}$$

$$+ C_6\{(\dot{v}_b-\dot{v}_5) - h_5\dot{\alpha}_b - d_5\dot{\alpha}_5 + b_5\dot{\gamma}_b\} \qquad (I.75)$$

$$F_{7y} = F_{8y} = K_8\{(v_b-v_7) - h_5\alpha_b - d_7\alpha_7 - b_5\gamma_6\}$$

$$+ C_8\{(\dot{v}_b-\dot{v}_7) - h_5\dot{\alpha}_b - d_7\dot{\alpha}_7 - b_5\dot{\gamma}_b\} \qquad (I.76)$$

I.5 Dynamic Reactions of the Tires

a) In the Vertical Direction

$$Z_1 = K_{t1}\{w_1 - a_2\alpha_1 - G_1(t)\}$$
$$+ C_{t1}\{\dot{w}_1 - a_2\dot{\alpha}_1 - \dot{G}_1(t)\} \qquad (I.77)$$

$$Z_2 = K_{t1}\{w_1 + a_2\alpha_1 - G_2(t)\}$$
$$+ C_{t1}\{\dot{w}_1 + a_2\dot{\alpha}_1 - \dot{G}_2(t)\} \qquad (I.78)$$

$$Z_3 = K_{t3}\{w_3 - a_4\alpha_3 - G_3(t)\}$$
$$+ C_{t3}\{\dot{w}_3 - a_4\dot{\alpha}_3 - \dot{G}_3(t)\} \qquad (I.79)$$

$$Z_4 = K_{t3}\{w_3 + a_4\alpha_3 - G_4(t)\}$$
$$+ C_{t3}\{\dot{w}_3 + a_4\dot{\alpha}_3 - \dot{G}_4(t)\} \qquad (I.80)$$

$$Z_5 = K_{t5}\{w_5 - a_6\alpha_5 - G_5(t)\}$$
$$+ C_{t5}\{\dot{w}_5 - a_6\dot{\alpha}_5 - \dot{G}_5(t)\} \qquad (I.81)$$

$$Z_6 = K_{t5}\{w_5 + a_6\alpha_5 - G_6(t)\}$$
$$+ C_{t5}\{\dot{w}_5 + a_6\dot{\alpha}_5 - \dot{G}_6(t)\} \qquad (I.82)$$

$$Z_7 = K_{t7}\{w_7 - a_8\alpha_7 - G_7(t)\}$$

$$+ C_{t7}\{\dot{w}_7 - a_8\dot{\alpha}_7 - \dot{G}_7(t)\} \qquad (I.83)$$

$$Z_8 = K_{t7}\{w_7 + a_8\alpha_7 - G_8(t)\}$$

$$+ C_{t7}\{\dot{w}_7 + a_8\dot{\alpha}_7 - \dot{G}_8(t)\} \qquad (I.84)$$

b) In the Lateral Direction

$$Y_1 = K_{t2}\{v_1 - r_1\alpha_1 - \lambda_1(t)\}$$

$$+ C_{t2}\{\dot{v}_1 - r_1\dot{\alpha}_1 - \dot{\lambda}_1(t)\} \qquad (I.85)$$

$$Y_2 = K_{t2}\{v_1 - r_1\alpha_1 - \lambda_2(t)\}$$

$$+ C_{t2}\{\dot{v}_1 - r_1\dot{\alpha}_1 - \dot{\lambda}_2(t)\} \qquad (I.86)$$

$$Y_3 = K_{t4}\{v_3 - r_3\alpha_3 - \lambda_3(t)\}$$

$$+ C_{t4}\{\dot{v}_3 - r_3\dot{\alpha}_3 - \dot{\lambda}_3(t)\} \qquad (I.87)$$

$$Y_4 = K_{t4}\{v_3 - r_3\alpha_3 - \lambda_4(t)\}$$

$$+ C_{t4}\{\dot{v}_3 - r_3\dot{\alpha}_3 - \dot{\lambda}_4(t)\} \qquad (I.88)$$

$$Y_5 = K_{t6}\{v_5 - r_5\alpha_5 - \lambda_5(t)\}$$

$$+ C_{t6}\{\dot{v}_5 - r_5\dot{\alpha}_5 - \dot{\lambda}_5(t)\} \qquad (I.89)$$

$$Y_6 = K_{t6}\{v_5 - r_5\alpha_5 - \lambda_6(t)\}$$

$$+ C_{t6}\{\dot{v}_5 - r_5\dot{\alpha}_5 - \dot{\lambda}_6(t)\} \qquad (I.90)$$

$$Y_7 = K_{t8}\{v_7 - r_7\alpha_7 - \lambda_7(t)\}$$

$$+ C_{t8}\{\dot{v}_7 - r_7\dot{\alpha}_7 - \dot{\lambda}_7(t)\} \qquad (I.91)$$

$$Y_8 = K_{t8}\{v_7 - r_7\alpha_7 - \lambda_8(t)\}$$

$$+ C_{t8}\{\dot{v}_7 - r_7\dot{\alpha}_7 - \dot{\lambda}_8(t)\} \qquad (I.92)$$

I.6 Definitions and Relations of Other Reactions

a) Since both the axle and the sprung mass are rigid, then

$$M_{1z} = M_{2z} = M_{3z} = M_{4z} = M_{5z} = M_{6z} = M_{7z} = M_{8z} = 0 \qquad (I.93)$$

b) The moments at the suspension connection points in the x-direction

For the Tractor Front Axle

$$M_{1x} + M_{2x} = (K_{r1} - 2a_1^2\, K_1)\,(\alpha_t - \alpha_1)$$

$$+(C_{r1} - 2a_1^2\, C_1)\,(\dot{\alpha}_t - \dot{\alpha}_1) \qquad (I.94)$$

For the Tractor Rear Axle

$$M_{3x} + M_{4x} = (K_{r3} - 2a_3^2 K_3)(\alpha_t - \alpha_3)$$

$$+(C_{r3} - 2a_3^2 C_3)(\dot{\alpha}_t - \dot{\alpha}_3) \qquad (I.95)$$

For the Bogie Front Axle

$$M_{5x} + M_{6x} = (K_{r5} - 2a_5^2 K_5)(\alpha_b - \alpha_5)$$

$$+(C_{r5} - 2a_5^2 C_5)(\dot{\alpha}_b - \dot{\alpha}_5) \qquad (I.96)$$

For the Bogie Rear Axle

$$M_{7x} + M_{8x} = (K_{r7} - 2a_7^2 K_7)(\alpha_b - \alpha_7)$$

$$+(C_{r7} - 2a_7^2 C_7)(\dot{\alpha}_b - \dot{\alpha}_7) \qquad (I.97)$$

where

K_{r1} is the overall roll stiffness coefficient, tractor front axle

K_{r3} is the overall roll stiffness coefficient, tractor rear axle.

K_{r5} is the overall roll stiffness coefficient, bogie front axle.

K_{r7} is the overall roll stiffness coefficient, bogie rear axle.

C_{r1} is the overall roll damping coefficient, tractor front axle.

C_{r3} is the overall roll damping coefficient, tractor rear axle.

C_{r5} is the overall roll damping coefficient, bogie front axle.

C_{r7} is the overall roll damping coefficient, bogie rear axle

c) The forces and couples at the fifth wheel:

By assuming negligible weight and inertia of the fifth wheel, then from Figure I.1,

for forces

$$\begin{bmatrix} F_{10x} \\ F_{10y} \\ F_{10z} \end{bmatrix} = - \begin{bmatrix} F_{9x} \\ F_{9y} \\ F_{9z} \end{bmatrix} \tag{I.98}$$

and for couples

$$\begin{bmatrix} M_{10x} \\ M_{10y} \\ M_{10z} \end{bmatrix} = - \begin{bmatrix} M_{9x} \\ M_{9y} \\ M_{9z} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ h_{11} \end{bmatrix} \times \begin{bmatrix} F_{9x} \\ F_{9y} \\ F_{9z} \end{bmatrix} \tag{I.99}$$

$M_{9y} = 0$ since the fifth wheel is affixed to the tractor frame through bearing.

$$M_{10z} = K_h(\gamma_t - \gamma_s) + C_h(\dot{\gamma}_t - \dot{\gamma}_s) \tag{I.100}$$

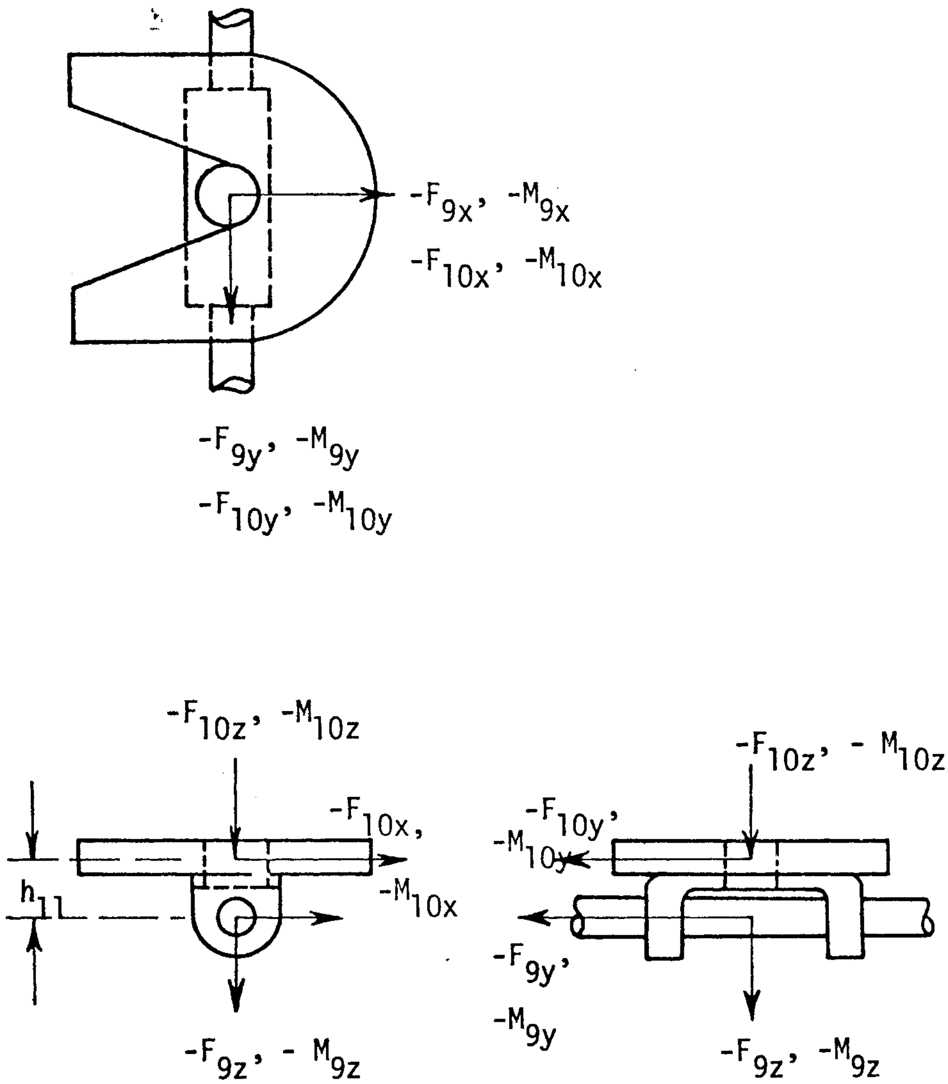


FIGURE I.1 FORCES AND COUPLES ON FIFTH WHEEL where

K_h the spring stiffness at the fifth wheel.

C_h the viscous damping at the fifth wheel.

d) The forces and couples between the semitrailer and bogie:

The balance of forces,

$$\begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{12z} \end{bmatrix} = - \begin{bmatrix} F_{11x} \\ F_{11y} \\ F_{11z} \end{bmatrix} \tag{I.101}$$

and the balance of the couples,

$$\begin{bmatrix} M_{12x} \\ M_{12y} \\ M_{12z} \end{bmatrix} = - \begin{bmatrix} M_{11x} \\ M_{11y} \\ M_{11z} \end{bmatrix} \tag{I.102}$$

M_{12z} is usually very small because the area where the bogie rotates about the vertical shaft is well lubricated with grease.

By introducing a damping at this location, then:

$$M_{12z} = C_p(\dot{\gamma}_x - \dot{\gamma}_b) \tag{I.103}$$

where

C_p is the viscous damping at the vertical shaft.

I.7 Equations of Motion

a) For the Tractor

$$\Sigma F_x = 0$$

$$m_t\ddot{u}_t - (F_{1x} + F_{2x} + F_{3x} + F_{4x}) - (F_{9x}) = 0 \qquad (I.104)$$

$$\Sigma F_y = 0$$

$$m_t\ddot{v}_t - (F_{1y} + F_{2y} + F_{3y} + F_{4y}) - (F_{9y}) = 0$$

$$\begin{aligned}
m_t\ddot{v}_t &+ 2(K_2 + K_4)v_t \\
&+ 2(-K_2)v_1 + 2(-K_4)v_3 \\
&+ 2(-h_1K_2 - h_3K_4)\alpha_t \\
&+ 2(-d_1K_2)\alpha_1 + 2(-d_3K_4)\alpha_3 \\
&+ 2(b_1K_2 - b_2K_4)\gamma_t \\
&+ 2(C_2 + C_4)\dot{v}_t \\
&+ 2(-C_2)v_1 + 2(-K_4)\dot{v}_3 \\
&+ 2(-h_1C_2 - h_3C_4)\dot{\alpha}_t \\
&+ 2(-d_1C_2)\dot{\alpha}_1 + 2(-d_3C_4)\dot{\alpha}_3 \\
&+ 2(b_1C_2 - b_2C_4)\dot{\gamma}_t \\
&+ (-F_{9y}) = 0
\end{aligned} \qquad (I.105)$$

$$\Sigma F_z = 0$$

$$m_t\ddot{w}_t - (F_{1z} + F_{2z} + F_{3z} + F_{4z}) - (F_{9z}) = 0$$

$$\begin{aligned}
m_t\ddot{w}_t &+ 2(K_1 + K_3)w_t \\
&+ 2(-K_1)w_1 + 2(-K_3)w_3 \\
&+ 2(-b_1K_1 + b_2K_3)\beta_t \\
&+ 2(C_1 + C_3)\dot{w}_t \\
&+ 2(-C_1)\dot{w}_1 + 2(-C_3)\dot{w}_3 \\
&+ 2(-b_1C_1 + b_2C_3)\beta_t \\
&+ (-F_{9z}) = 0
\end{aligned} \tag{I.106}$$

$$\Sigma M_x = 0$$

$$\begin{aligned}
I_{txx}\ddot{\alpha}_t - I_{txz}\ddot{\gamma}_t &+ h_1(F_{1y} + F_{2y}) + a_1(F_{1z} - F_{2z}) \\
&+ h_3(F_{3y} + F_{4y}) + a_3(F_{3z} - F_{4z}) \\
&+ h_8(F_{9y} - (M_{1x} + M_{2x} + M_{3x} + M_{4x}) \\
&- (M_{9x}) = 0
\end{aligned}$$

$$\begin{aligned}
I_{txx}\ddot{\alpha}_t - I_{txz}\ddot{\gamma}_t &+ 2(-h_1K_2 - h_3K_4)v_t \\
&+ 2(h_1K_2)v_1 + 2(h_3K_4)v_3 \\
&+ 2(h_1^2K_2 + h_3^2K_4 + 0.5\ K_{r1} + 0.5\ K_{r3})\alpha_t \\
&+ 2(h_1d_1K_2 - 0.5\ K_{r1})\alpha_1 + 2(h_3d_3K_4 - 0.5\ K_{r3})\alpha_3
\end{aligned}$$

$$+ 2(-h_1 b_1 K_2 + h_3 b_2 K_4)\gamma_t$$

$$+ 2(-h_1 C_2 - h_3 C_4)\dot{v}_t$$

$$+ 2(h_1 C_2)\dot{v}_1 + 2(h_3 C_4)\dot{v}_3$$

$$+ 2(h_1^2 C_2 + h_3^2 C_4 + 0.5\, C_{r1} + 0.5\, C_{r3})\dot{\alpha}_t$$

$$+ 2(h_1 d_1 C_2 - 0.5\, C_{r1})\dot{\alpha}_1$$

$$+ 2(h_3 d_3 C_4 - 0.5\, C_{r3})\dot{\alpha}_3$$

$$+ 2(-h_1 b_1 C_2 + h_3 b_2 C_4)\dot{\gamma}_t$$

$$+ (h_7 F_{9y}) - M_{9y} = 0 \qquad \text{(I.107)}$$

$$\Sigma M_y = 0$$

$$I_{tyy}\,\ddot{\beta}_t - h_1(F_{1x} + F_{2x}) + b_1(F_{1z} + F_{2z})$$

$$- h_3(F_{3x} + F_{4x}) - b_2(F_{3z} + F_{4z})$$

$$-(h_8 F_{9x} + b_6 F_{9z})$$

$$-(M_{1y} + M_{2y} + M_{3y} + M_{4y}) = 0$$

$$I_{tyy}\,\ddot{\beta}_t + 2(-b_1 K_1 + b_2 K_3) w_t$$

$$+ 2(b_1 K_1) w_1 + 2(-b_2 K_3) w_3$$

$$+ 2(b_1^2 K_1 + b_2^2 K_3)\beta_t$$

$$+ 2(-b_1 C_1 + b_2 C_3)\dot{w}_t$$

$$+ 2(b_1C_1)\dot{w}_1 + 2(-b_2C_3)\dot{w}_3$$

$$+ 2(b_1^2C_1 + b_2^2C_3)\dot{\beta}_t$$

$$- h_1(F_{1x} + F_{2x}) - h_3(F_{3x} + F_{4x})$$

$$-(h_8F_{9x} + b_6F_{9z})$$

$$-(M_{1y} + M_{2y} + M_{3y} + M_{4y}) = 0 \qquad (I.108)$$

$$\Sigma M_z = 0$$

$$- I_{txz}\ddot{\alpha}_t + I_{tzz}\ddot{\gamma}_t - a_1(F_{1x} - F_{2x})$$

$$- b_1(F_{1y} + F_{2y}) - a_3(F_{3x} - F_{4x})$$

$$+ b_2(F_{3y} + F_{4y}) + b_6F_{9y} - M_{9z}$$

$$- (M_{1z} + M_{2z} + M_{3z} + M_{4z}) = 0$$

$$- I_{txz}\ddot{\alpha}_t + I_{tzz}\ddot{\gamma}_t$$

$$+ 2(b_1K_2 - b_2K_4)v_t$$

$$+ 2(-b_1K_2)v_1 + 2(b_2K_4)v_3$$

$$+ 2(-h_1b_1K_2 + h_3b_2K_4)\alpha_t$$

$$+ 2(-b_1d_1K_2)\alpha_1 + 2(b_2d_3K_4)\alpha_3$$

$$+ 2(b_1^2K_2 + b_2^2K_4 + 0.5\,K_h)\gamma_t$$

$$+ 2(-0.5\,K_h)\,\gamma_s$$

$$+ 2(b_1C_2 - b_2C_4)\dot{v}_t$$

$$+ 2(-b_1C_2)\dot{v}_1 + 2(b_2C_4)\dot{v}_3$$

$$+ 2(-h_1b_1C_2 + h_3b_2C_4)\dot{\alpha}_t$$

$$+ 2(-b_1d_1C_2)\dot{\alpha}_1 + 2(b_2d_3C_4)\dot{\alpha}_3$$

$$+ 2(b_1^2C_2 + b_2^2C_4 + 0.5\ C_h)\dot{\gamma}_t$$

$$+ 2(-0.5\ C_h)\dot{\gamma}_s$$

$$- a_1(F_{1x} - F_{2x}) - a_3(F_{3x} - F_{4x})$$

$$+ b_6\ F_{9y} = 0 \qquad \text{(I.109)}$$

b) For the Semitrailer

$$\Sigma F_x = 0$$

$$m_s\ddot{u}_s + F_{9x} - F_{11x} = 0 \qquad \text{(I.110)}$$

$$\Sigma F_y = 0$$

$$m_s\ddot{v}_s + F_{9y} - F_{11y} = 0 \qquad \text{(I.111)}$$

$$\Sigma F_z = 0$$

$$m_s\ddot{w}_s + F_{9z} - F_{11z} = 0 \qquad \text{(I.112)}$$

$$\Sigma M_x = 0$$

$$I_{sxx}\ \ddot{\alpha}_s - I_{sxy}\ \ddot{\beta}_s - I_{sxz}\ \ddot{\gamma}_s$$

$$- (h_9 + h_{11})F_{9y} + h_{10}\ F_{11y} + M_{9x} - M_{11x} = 0 \qquad \text{(I.113)}$$

$$\Sigma M_y = 0$$

$$- I_{sxy}\ \ddot{\alpha}_s + I_{syy}\ \ddot{\beta}_s - I_{syz}\ \ddot{\gamma}_s$$

$$+ (h_9 + h_{11})\ F_{9x} - b_4\ F_{9z} - h_{10}\ F_{11x}$$

$$- b_3\ F_{11z} - M_{11y} = 0 \qquad (I.114)$$

$$\Sigma M_z = 0$$

$$- I_{sxz}\ \ddot{\alpha}_s - I_{syz}\ \ddot{\beta}_s + I_{szz}\ \ddot{\gamma}_s$$

$$+ 2(-0.5\ K_h)\gamma_t + 2(0.5\ K_h)\gamma_s$$

$$+ 2(-0.5\ C_h)\dot{\gamma}_t$$

$$+ 2(0.5\ C_h + 0.5\ C_p)\dot{\gamma}_s$$

$$+ 2(-0.5\ C_p)\dot{\gamma}_b$$

$$+ b_4\ F_{9y} + b_3\ F_{11y} = 0 \qquad (I.115)$$

c) <u>For the Bogie</u>

$$\Sigma F_x = 0$$

$$m_b\ddot{u}_b + F_{11x} - (F_{5x} + F_{6x} + F_{7x} + F_{8x}) = 0 \qquad (I.116)$$

$$\Sigma F_y = 0$$

$$m_b\ddot{v}_b - F_{12y} - (F_{5y} + F_{6y} + F_{7y} + F_{8y}) = 0$$

$$m_b\ddot{v}_b + 2(K_6 + K_8)v_b$$

$$+ 2(-K_6)v_5 + 2(-K_8)v_7$$

$$+ 2(-h_5K_6 - h_7K_8)\alpha_b$$

$$+ 2(-d_5K_6)\alpha_5 + 2(-d_7K_8)\alpha_7$$

$$+ 2(b_5K_6 - b_5K_8)\gamma_b$$

$$+ 2(C_6 + C_8)\dot{v}_b$$

$$+ 2(-C_6)\dot{v}_5 + 2(-C_8)\dot{v}_7$$

$$+ 2(-h_5C_6 - h_7C_8)\dot{\alpha}_b$$

$$+ 2(-d_5C_6)\dot{\alpha}_5 + 2(-d_7C_8)\dot{\alpha}_7$$

$$+ 2(b_5C_6 - b_5C_8)\dot{\gamma}_b$$

$$+ F_{11y} = 0 \qquad \text{(I.117)}$$

$$\Sigma F_z = 0$$

$$m_b\ddot{w}_b - F_{12z} - (F_{5z} + F_{6z} + F_{7z} + F_{8z}) = 0$$

$$m_b\ddot{w}_b + 2(K_5 + K_7)w_b$$

$$+ 2(-K_5)w_5 + 2(-K_7)w_7$$

$$+ 2(-b_5K_5 + b_5K_7)\beta_b$$

$$+ 2(C_5 + C_7)\dot{w}_b$$

$$+ 2(-C_5)\dot{w}_5 + 2(-C_7)\dot{w}_7$$

$$+ 2(-b_5C_5 + b_5C_7)\dot{\beta}_b$$

$$+ F_{11z} = 0 \qquad \text{(I.118)}$$

$$\Sigma M_x = 0$$

$$I_{bxx}\ddot{\alpha}_b + h_5 (F_{5y} + F_{6y}) + h_7 (F_{7y} + F_{8y})$$
$$+ a_5 (F_{5z} - F_{6z}) + a_7 (F_{7z} - F_{8z})$$
$$- M_{12x} - (M_{5x} + M_{6x} + M_{7x} + M_{8x}) = 0$$

$$I_{bxx}\ddot{\alpha}_b + 2(-h_5K_6 - h_7K_8)v_b$$
$$+ 2(h_5K_6)v_5 + 2(h_7K_8)v_7$$
$$+ 2(h_5^2K_6 + h_7^2K_8 + 0.5\ K_{r5} + 0.5\ K_{r7})\alpha_b$$
$$+ 2(h_5d_5K_6 - 0.5\ K_{r5})\alpha_5$$
$$+ 2(h_7d_7K_8 - 0.5\ K_{r7})\alpha_7$$
$$+ 2(-h_5b_5K_6 + h_7b_5K_8)\gamma_b$$
$$+ 2(-h_5C_6 - h_7C_8)\dot{v}_b$$
$$+ 2(h_5C_6)\dot{v}_5 + 2(h_7C_8)\dot{v}_7$$
$$+ 2(h_5^2C_6 + h_7^2C_8 + 0.5\ C_{r5} + 0.5\ C_{r7})\dot{\alpha}_b$$
$$+ 2(h_5d_5C_6 - 0.5\ C_{r5})\dot{\alpha}_5$$
$$+ 2(h_7d_7C_8 - 0.5\ C_{r7})\dot{\alpha}_7$$
$$+ 2(-h_5b_5C_6 + h_7b_5C_8)\dot{\gamma}_b$$
$$+ M_{11x} = 0 \qquad \text{(I.119)}$$

$$\Sigma M_y = 0$$

$$I_{byy}\ddot{\beta}_b - h_5(F_{5x} + F_{6x}) - h_7(F_{7x} + F_{8x})$$
$$+ b_5(F_{5z} + F_{6z} - F_{7z} - F_{8z})$$
$$- M_{12y} - (M_{5y} + M_{6y} + M_{7y} + M_{8y}) = 0$$

$$I_{byy}\ddot{\beta}_b + 2(-b_5K_5 + b_5K_7)w_b$$
$$+ 2(b_5K_5)w_5 + 2(-b_5K_7)w_7$$
$$+ 2(b_5^2(K_5 + K_7))\beta_b$$
$$+ 2(-b_5C_5 + b_5C_7)\dot{w}_b$$
$$+ 2(b_5C_5)\dot{w}_5 + 2(-b_5C_7)\dot{w}_7$$
$$+ 2(b_5^2(C_5 + C_7))\dot{\beta}_b$$
$$- h_5(F_{5x} + F_{6x}) - h_7(F_{7x} + F_{8x})$$
$$+ M_{11y} - (M_{5y} + M_{6y} + M_{7y} + M_{8y}) = 0 \qquad (\text{I.120})$$

$$\Sigma M_z = 0$$

$$I_{bzz}\ddot{\gamma}_b - a_5(F_{5x} - F_{6x}) - a_7(F_{7x} - F_{8x})$$
$$- b_5(F_{5y} + F_{6y} - F_{7y} - F_{8y})$$
$$- M_{12z} = 0$$

$$
\begin{aligned}
I_{bzz}\,\ddot{\gamma}_b &+ 2(b_5(K_6-K_8))v_b \\
&+ 2(-b_5K_6)v_5 + 2(b_5K_8)v_7 \\
&+ 2(-b_5h_5K_6 + b_5h_7C_8)\alpha_b \\
&+ 2(-b_5d_5K_6)\alpha_5 \\
&+ 2(b_5d_7K_8)\alpha_7 \\
&+ 2(b_5^2(K_6 + K_8))\gamma_b \\
&+ 2(b_5(C_6 - C_8))\dot{v}_b \\
&+ 2(-b_5C_6)\dot{v}_5 + 2(b_5C_8)\dot{v}_7 \\
&+ 2(-b_5h_5C_6 + b_5h_7C_8)\dot{\alpha}_b \\
&+ 2(-b_5d_5C_6)\dot{\alpha}_5 + 2(b_5d_7C_8)\dot{\alpha}_7 \\
&+ 2(b_5^2(C_6 + C_8) + 0.5\ C_p)\dot{\gamma}_b \\
&+ 2(-0.5\ C_p)\dot{\gamma}_s \\
&- a_5(F_{5x} - F_{6x}) - a_7(F_{7x} - F_{8x}) = 0 \qquad (I.121)
\end{aligned}
$$

d) For the Tractor Front Axle

$$\Sigma F_x = 0$$

$$m_1\ddot{u}_1 + F_{1x} + F_{2x} = 0 \qquad (I.122)$$

$$\Sigma F_y = 0$$

$$m_1\ddot{v}_1 - (Y_1 + Y_2) + F_{1y} + F_{2y} = 0$$

$$m_1\ddot{v}_1 + 2(-K_2)v_t + 2(K_2 + K_{t2})v_1$$

$$+ 2(h_1K_2)\alpha_t + 2(d_1K_2 - r_1K_{t2})\alpha_1$$

$$+ 2(-b_1K_2)\gamma_t$$

$$+ 2(-C_2)\dot{v}_t + 2(C_2 + C_{t2})\dot{v}_1$$

$$+ 2(h_1C_2)\dot{\alpha}_t + 2(d_1C_2 - r_1C_{t2})\dot{\alpha}_1$$

$$+ 2(-b_1C_2)\dot{\gamma}_t$$

$$+ (-K_{t2}\,(\lambda_1(t) + \lambda_2(t)))$$

$$+ (-C_{t2}(\dot{\lambda}_1(t) + \dot{\lambda}_2(t))) = 0 \qquad \text{(I.123)}$$

$$\Sigma F_z = 0$$

$$m_1\ddot{w}_1 + (F_{1z} + F_{2z}) - (Z_1 + Z_2) = 0$$

$$m_1\ddot{w}_1 + 2(-K_1)w_t + 2(K_1 + K_{t1})w_1$$

$$+ 2(b_1K_1)\beta_t$$

$$+ 2(-C_1)\dot{w}_t + 2(C_1 + C_{t1})\dot{w}_1$$

$$+ 2(b_1C_1)\dot{\beta}_t$$

$$+ (-K_{t1}(G_1(t) + G_2(t)))$$

$$+ (-C_{t1}(\dot{G}_1(t) + \dot{G}_2(t))) = 0 \qquad \text{(I.124)}$$

$$\Sigma M_x = 0$$

$$I_{1xx}\ddot{\alpha}_1 - a_1(F_{1z} - F_{2z}) + d_1(F_{1y} + F_{2y})$$

$$+ (M_{1x} + M_{2x}) - a_2(Z_2 - Z_1)$$

$$+ r_1(Y_1 + Y_2) = 0$$

$$I_{1xx}\ddot{\alpha}_1 + 2(-d_1K_2)v_t + 2(d_1K_2 - r_1K_{t2})v_1$$

$$+ 2(d_1h_1K_2 - 0.5\ K_{r1})\alpha_t$$

$$+ 2(d_1^2K_2 + a_2^2K_{t1} + r_1^2K_{t2} + 0.5\ K_{r1})\alpha_1$$

$$+ 2(-d_1b_1K_2)\gamma_t$$

$$+ 2(-d_1C_2)\dot{v}_t + 2(d_1C_2 - r_1C_{t2})\dot{v}_1$$

$$+ 2(d_1h_1C_2 - 0.5\ C_{r1})\dot{\alpha}_t$$

$$+ 2(d_1^2C_2 + a_2^2C_{t1} + r_1^2C_{t2} + 0.5\ C_{r1})\dot{\alpha}_1$$

$$+ 2(-d_1b_1C_2)\dot{\gamma}_t$$

$$+ (a_2K_{t1}(G_1(t) - G_2(t)))$$

$$+ (a_2C_{t1}(\dot{G}_1(t) - \dot{G}_2(t)))$$

$$+ (r_1K_{t2}(\lambda_1(t) - \lambda_2(t)))$$

$$+ (r_1C_{t2}(\dot{\lambda}_1(t) - \dot{\lambda}_2(t))) = 0 \qquad \text{(I.125)}$$

$$\Sigma M_y = 0$$

$$I_{1yy}\ddot{\beta}_1 - \ell_1(F_{1x} + F_{2x}) + (M_{1y} + M_{2y}) = 0 \tag{I.126}$$

$$\Sigma M_z = 0$$

$$I_{1zz}\ddot{\gamma}_1 - a_1(F_{2x} - F_{1x}) = 0 \tag{I.127}$$

e) For the Tractor Rear Axle

$$\Sigma F_x = 0$$

$$m_3\ddot{u}_3 + F_{3x} + F_{4x} = 0 \tag{I.128}$$

$$\Sigma F_y = 0$$

$$m_3\ddot{v}_3 + (F_{3y} + F_{4y}) - (Y_1 + Y_2) = 0$$

$$m_3\ddot{v}_3 + 2(-K_4)v_t$$

$$+ 2(K_4 + K_{t4})v_3$$

$$+ 2(K_4h_3)\alpha_t + 2(-r_3K_{t4} + d_3K_4)\alpha_3$$

$$+ 2(b_2K_4)\gamma_t$$

$$+ 2(-C_4)\dot{v}_t + 2(C_4 + C_{t4})\dot{v}_3$$

$$+ 2(h_3C_4)\dot{\alpha}_t$$

$$+ 2(-r_3C_{t4} + d_3C_4)\dot{\alpha}_3$$

$$+ 2(b_2C_4)\dot{\gamma}_t$$

$$+ (-K_{t4}(\lambda_3(t) + \lambda_4(t)))$$

$$+ (-C_{t4}(\dot{\lambda}_3(t) + \dot{\lambda}_4(t))) \qquad (I.129)$$

$$\Sigma F_z = 0$$

$$m_3\ddot{w}_3 + 2(-K_3)w_t + 2(K_3 + K_{t3})w_3 + 2(-b_2K_3)\beta_t$$

$$+ 2(-C_3)\dot{w}_t + 2(C_3 + C_{t3})\dot{w}_3$$

$$+ 2(-b_2C_3)\dot{\beta}_t$$

$$+ (-K_{t3}(G_3(t) + G_4(t)))$$

$$+ (-C_{t3}(\dot{G}_3(t) + \dot{G}_4(t))) \qquad (I.130)$$

$$\Sigma M_x = 0$$

$$I_{3xx}\ \ddot{\alpha}_3 - a_3(F_{3z} - F_{4z}) + d_3\ (F_{3y} + F_{4y})$$

$$+ (M_{3x} + M_{4x}) - a_4(Z_4 - Z_3)$$

$$+ r_3(Y_3 + Y_4) = 0$$

$$I_{3xx}\ \ddot{\alpha}_3 + 2(-d_3K_4)v_t$$

$$+ 2(d_3K_4 - r_3K_{t4})v_3$$

$$+ 2(d_3h_3K_4 - 0.5\ K_{r3})\alpha_t$$

$$+ 2(d_3^2K_4 + a_4^2K_{t3} + 0.5\ K_{r3})\alpha_3$$

$$+ 2(d_3 b_2 K_4)\gamma_t$$

$$+ 2(-d_3 C_4)\dot{v}_t$$

$$+ 2(d_3 C_4 - r_3 C_{t4})\dot{v}_3$$

$$+ 2(h_3 d_3 C_4 - 0.5\ Cr_3)\dot{\alpha}_t$$

$$+ 2(d_3^2 C_4 + a_4^2 C_{t3} + 0.5\ C_{r3})\dot{\alpha}_3$$

$$+ 2(d_3 b_2 C_4)\dot{\gamma}_t$$

$$+ (a_4 K_{t3}(G_3(t) - G_4(t)))$$

$$+ (a_4 C_{t3}(\dot{G}_3(t) - \dot{G}_4(t)))$$

$$+ (r_3 K_{t4}(\lambda_3(t) + \lambda_4(t)))$$

$$+ (r_3 C_{t4}(\dot{\lambda}_3(t) + \dot{\lambda}_4(t))) = 0 \qquad (I.131)$$

$$\Sigma M_y = 0$$

$$I_{3yy}\ddot{\beta}_3 - \ell_3(F_{3x} + F_{4x}) + (M_{3y} + M_{4y}) = 0 \qquad (I.132)$$

$$\Sigma M_z = 0$$

$$I_{3zz}\ddot{\gamma}_3 + a_3(F_{3x} - F_{4x}) = 0 \qquad (I.133)$$

f) For the Bogie Front Axle

$$\Sigma F_x = 0$$

$$m_5\ddot{u}_5 + (F_{5x} + F_{6x}) = 0 \tag{I.134}$$

$$\Sigma F_y = 0$$

$$m_5\ddot{v}_5 + (F_{5y} + F_{6y}) - (Y_5 + Y_6) = 0$$

$$\begin{aligned} m_5\ddot{v}_5 &+ 2(-K_6)v_b + 2(K_6 + K_{t6})v_5 \\ &+ 2(h_5K_6)\alpha_b \\ &+ 2(-r_5K_{t6} + d_5K_6)\alpha_5 \\ &+ 2(-b_5K_6)\gamma_b \\ &+ 2(-C_6)\dot{v}_b + 2(C_6 + C_{t6})\dot{v}_5 \\ &+ 2(C_6h_5)\dot{\alpha}_b \\ &+ 2(-r_5C_{t6} + d_5C_6)\dot{\alpha}_5 \\ &+ 2(-b_5C_6)\dot{\gamma}_b \\ &+ (-K_{t6}(\lambda_5(t) + \lambda_6(t))) \\ &+ (-C_{t6}(\dot{\lambda}_5(t) + \dot{\lambda}_6(t))) = 0 \end{aligned} \tag{I.135}$$

$$\Sigma F_z = 0$$

$$m_5\ddot{w}_5 + (F_{5z} + F_{6z}) - (Z_5 + Z_6) = 0$$

$$m_5\ddot{w}_5 + 2(-K_5)w_b$$

$$+ 2(K_5 + K_{t5})w_5$$

$$+ 2(b_5K_5)\beta_b$$

$$+ 2(-C_5)\dot{w}_b$$

$$+ 2(C_5 + C_{t5})\dot{w}_5$$

$$+ 2(b_5C_5)\dot{\beta}_b$$

$$+ (-K_{t5}(G_5(t) + G_6(t)))$$

$$+ (-C_{t5}(\dot{G}_5(t) + \dot{G}_6(t))) = 0 \qquad (I.136)$$

$$\Sigma M_x = 0$$

$$I_{5xx}\ddot{\alpha}_5 - a_5(F_{5z} - F_{6z}) + d_3(F_{5y} + F_{6y})$$

$$+ (M_{5x} + M_{6x}) - a_6(Z_6 - Z_5)$$

$$+ r_5(Y_5 + Y_6) = 0$$

$$
\begin{aligned}
I_{5xx}\,\ddot{\alpha}_5 &+ 2(-d_5K_6)v_b \\
&+ 2(d_5K_6 - r_5K_{t6})v_5 \\
&+ 2(d_5h_5K_6 - 0.5\,K_{r5})\alpha_b \\
&+ 2(d_5^2K_6 + a_6^2K_{t5} + 0.5\,K_{r5})\alpha_5 \\
&+ 2(-d_5b_5K_6)\gamma_b \\
&+ 2(-d_5C_6)\dot{v}_b \\
&+ 2(d_5C_6 - r_5C_{t6})\dot{v}_5 \\
&+ 2(d_5h_5C_6 - 0.5\,C_{r5})\dot{\alpha}_b \\
&+ 2(d_5^2C_6 + a_6^2C_{t5} + 0.5\,C_{r5})\dot{\alpha}_5 \\
&+ 2(-d_5b_5C_6)\dot{\gamma}_b \\
&+ (a_6K_{t5}(G_5(t) - G_6(t))) \\
&+ (a_6C_{t5}(\dot{G}_5(t) - \dot{G}_6(t))) \\
&+ (r_5K_{t6}(\lambda_5(t) + \lambda_6(t))) \\
&+ (r_5C_{t6}(\dot{\lambda}_5(t) + \dot{\lambda}_6(t))) = 0 \qquad \text{(I.137)}
\end{aligned}
$$

$\Sigma M_y = 0$

$$I_{5yy}\,\ddot{\beta}_5 - \ell_5\,(F_{5x} + F_{6x}) + (M_{5y} + M_{6y}) = 0 \qquad \text{(I.138)}$$

$$\Sigma\, M_z = 0$$

$$I_{5zz}\,\ddot{\gamma}_5 - a_5(F_{6x} - F_{5x}) = 0 \qquad (I.139)$$

g) For the Bogie Rear Axle

$$\Sigma\, F_x = 0$$

$$m_7\ddot{u}_7 + (F_{7x} + F_{8x}) = 0 \qquad (I.140)$$

$$\Sigma\, F_y = 0$$

$$m_7\ddot{v}_7 + (F_{7y} + F_{8y}) - (Y_7 + Y_8) = 0$$

$$m_7\ddot{v}_7 + 2(-K_8)v_b$$

$$+ 2(K_8 + K_{t8})v_7$$

$$+ 2(h_5K_8)\alpha_b + 2(d_7K_8 - r_7K_{t8})\alpha_7$$

$$+ 2(b_5K_8)\gamma_b$$

$$+ 2(-C_8)\dot{v}_b$$

$$+ 2(C_8 + C_{t8})\dot{v}_7$$

$$+ 2(h_5C_8)\dot{\alpha}_b$$

$$+ 2(-r_7C_{t8} + d_7C_8)\dot{\alpha}_7$$

$$+ 2(b_5C_8)\dot{\gamma}_b$$

$$+ (-K_{t8}(\lambda_7(t) + \lambda_8(t))$$

$$+ (-C_{t8}(\dot{\lambda}_7(t) + \dot{\lambda}_8(t)) \quad = \quad 0 \qquad \text{(I.141)}$$

$$\Sigma F_z \quad = \quad 0$$

$$m_7\ddot{w}_7 + (F_{7z} + F_{8z}) - (Z_7 + Z_8) \quad = \quad 0$$

$$m_7\ddot{w}_7 + 2(-K_7)w_b + 2(K_7 + K_{t7})w_7$$

$$+ 2(-b_5K_7)\beta_b$$

$$+ 2(-C_7)\dot{w}_b$$

$$+ 2(C_7 + C_{t7})\dot{w}_7$$

$$+ 2(b_5C_7)\dot{\beta}_b$$

$$+ (-K_{t7}(G_7(t) + G_8(t))$$

$$+ (-C_{t7}(\dot{G}_7(t) + \dot{G}_8(t)) \quad = \quad 0 \qquad \text{(I.142)}$$

$$\Sigma M_x \quad = \quad 0$$

$$I_{7xx}\,\ddot{\alpha}_7 + 2(-d_7K_8)v_b + 2(d_7K_8 - r_7K_{t8})v_7$$

$$+ 2(d_7h_5K_8 - 0.5\ K_{r7})\alpha_b$$

$$+ 2(d_7^2K_8 + a_8^2K_{t7} + 0.5\ K_{r7})\alpha_7$$

$$+ 2(d_7 b_5 K_8)\gamma_b$$

$$+ 2(-d_7 C_8)\dot{v}_b$$

$$+ 2(d_7 C_8 - r_7 C_{t8})\dot{v}_7$$

$$+ 2(h_5 d_7 C_8 - 0.5\ C_{r7})\dot{\alpha}_b$$

$$+ 2(d_7^2 C_8 + a_8^2 C_{t7} + 0.5\ C_{r7})\dot{\alpha}_7$$

$$+ 2(d_7 b_5 C_8)\dot{\gamma}_b$$

$$+ (a_8 K_{t7}(G_{7(t)} - G_8(t)))$$

$$+ (a_8 C_{t7}(\dot{G}_7(t) - \dot{G}_8(t)))$$

$$+ (r_7 K_{t8}(\lambda_7(t) + \lambda_8(t)))$$

$$+ (r_7 C_{t8}(\dot{\lambda}_7(t) + \dot{\lambda}_8(t))) = 0 \qquad \text{(I.143)}$$

$$\Sigma M_y = 0$$

$$I_{7yy}\ \ddot{\beta}_7 - \ell_7\ (F_{7x} + F_{8x}) + (M_{7y} + M_{8y}) = 0 \qquad \text{(I.144)}$$

$$\Sigma M_z = 0$$

$$M_{7zz}\ \ddot{\gamma}_4 + a_7\ (F_{7x} - F_{8x}) = 0 \qquad \text{(I.145)}$$

I.8 Definition of the Elements of the Matrices

All the elements which are not defined below are zeros. All matrices are symmetric [i.e. A(J,I) = A(I,J), B(J,I) = B(I,J) and C(J,I) = C(I,J)].

a) Elements of the Inertia Matrix [A]

$A(1,1) = A(2,2) = A(3,3) = m_t$

$A(4,4) = I_{txx}$

$A(4,6) = I_{txz}$

$A(5,5) = I_{tyy}$

$A(6,6) = I_{tzz}$

$A(7,7) = A(8,8) = A(9,9) = m_s$

$A(10,10) = I_{sxx}$

$A(10,11) = -I_{sxy}$

$A(10,12) = -I_{sxz}$

$A(11,11) = I_{syy}$

$A(11,12) = -I_{syz}$

$A(12,12) = I_{szz}$

$A(13,13) = A(14,14) = A(15,15) = m_b$

$A(16,16) = I_{bxx}$

$A(17,17) = I_{byy}$

$A(18,18) = I_{bzz}$

$A(19,19) = A(20,20) = A(21,21) = m_1$

$A(22,22) = I_{1xx}$

$A(23,23) = I_{1yy}$

$A(24,24) = I_{1zz}$

$A(25,25) = A(26,26) = A(27,27) = m_3$

$A(28,28) = I_{3xx}$

$A(29,29) = I_{3yy}$

$A(30,30) = I_{3zz}$

$A(31,31) = A(32,32) = A(33,33) = m_5$

$A(34,34) = I_{5xx}$

$A(35,35) = I_{5yy}$

$A(36,36) = I_{5zz}$

$A(37,37) = A(38,38) = A(39,39) = m_7$

$A(40,40) = I_{7xx}$

$A(41,41) = I_{7yy}$

$A(42,42) = I_{7zz}$

b) Elements of the Stiffness Matrix [B]

Each element must multiply by 2.0 to obtain the exact one.

$B(2,2) = K_2 + K_4$

$B(2,4) = -h_1K_2 - h_3K_4$

$B(2,6) = b_1K_2 - b_2K_4$

$B(2,20) = -K_2$

$B(2,22) = -d_1K_2$

$B(2,26) = -K_4$

$B(2,28) = -d_3K_4$

$B(3,3) = K_1 + K_3$

$B(3,5) = -b_1K_1 + b_2K_3$

$B(3,21) = -K_1$

$B(3,27) = -K_3$

$B(4,4) = h_1^2K_2 + h_3^2K_4 + 0.5\ K_{r1} + 0.5\ K_{r3}$

$B(4,6) = -h_1b_1K_2 + h_3b_2K_4$

$B(4,20) = h_1K_2$

$B(4,22) = d_1h_1K_2 - 0.5\ K_{r1}$

$B(4,26) = h_3K_4$

$$B(4,28) = h_3 d_3 K_4 - 0.5\ K_{r3}$$

$$B(5,5) = b_1^2 K_1 + b_2^2 K_3$$

$$B(5,21) = b_1 K_1$$

$$B(5,27) = -b_2 K_3$$

$$B(6,6,) = b_1^2 K_2 + b_2^2 K_4 + 0.5\ K_h$$

$$B(6,12) = -0.5\ K_h$$

$$B(6,20) = -b_1 K_2$$

$$B(6,22) = -b_1 d_1 K_2$$

$$B(6,26) = b_2 K_4$$

$$B(6,28) = b_2 d_3 K_4$$

$$B(12,12) = 0.5\ K_h$$

$$B(14,14) = K_6 + K_8$$

$$B(14,16) = -h_5 K_6 - h_7 K_8$$

$$B(14,18) = b_5(K_6 - K_8)$$

$$B(14,32) = -K_6$$

$$B(14,34) = -d_5 K_6$$

$$B(14,38) = -K_8$$

$$B(14,40) = -d_7 K_8$$

$$B(15,15) = K_5 + K_7$$

$$B(15,17) = -b_5(K_5 - K_7)$$

$$B(15,33) = -K_5$$

$$B(15,39) = -K_7$$

$$B(16,16) = H_5^2 K_6 + H_7^2 K_8 + 0.5\ K_{r5} + 0.5\ K_{r7}$$

$$B(16,18) = -b_5 h_5 K_6 + b_5 h_7 K_8$$

$$B(16,32) = h_5 K_6$$

$$B(16,34) = h_5 d_5 K_6 - 0.5\ K_{r3}$$

$$B(16,38) = h_7 K_8$$

$$B(16,40) = h_7 d_7 K_8 - 0.5\ K_{r7}$$

$$B(17,17) = b_5^2(K_5 + K_7)$$

$$B(17,33) = b_5 K_5$$

$$B(17,39) = -b_5 K_7$$

$$B(18,18) = b_5^2(K_6 + K_8)$$

$$B(18,32) = -b_5 K_6$$

$$B(18,38) = b_5 K_8$$

$$B(18,40) = b_5 d_7 K_8$$

$$B(20,20) = K_2 + K_{t2}$$

$$B(20,22) = d_1 K_2 - r_1 K_{t2}$$

$$B(21,21) = K_1 + K_{t1}$$

$$B(22,22) = d_1^2K_2 + a_2^2K_{t1} + r_1^2K_{t2} + 0.5\ K_{r1}$$

$$B(26,26) = K_4 + K_{t4}$$

$$B(26,28) = d_3K_4 - r_3K_{t4}$$

$$B(27,27) = K_3 + K_{t3}$$

$$B(28,28) = d_3^2K_4 + a_4^2K_{t3} + r_3^2K_{t4} + 0.5\ K_{r3}$$

$$B(32,32) = K_6 + K_{t6}$$

$$B(32,34) = d_5K_6 - r_5K_{t6}$$

$$B(33,33) = K_5 + K_{t5}$$

$$B(34,34) = d_5^2K_6 + a_6^2K_{t5} + r_5^2K_{t6} + 0.5\ K_{r5}$$

$$B(38,38) = K_8 + K_{t8}$$

$$B(38,40) = d_7K_8 - r_7K_{t8}$$

$$B(39,39) = K_7 + K_{t7}$$

$$B(40,40) = d_7^2K_8 + a_8^2K_{t7} + r_7^2K_{t8} + 0.5\ K_{r7}$$

c) Elements of the Damping Matrix [C]

The elements of the damping matrix are obtained from the elements of the stiffness matrix by replacing the stiffnesses K's by their corresponding terms C's.

There are three additional terms due to the damping in the vertical torque shaft,

$$C(12,12) = 0.5\ C_p$$

$$C(12,18) = -0.5\, C_p$$

$$C(18,18) = 0.5\, C_p$$

d) The Force Vector {Q}

All the elements which are not defined below are zeros.

$$Q(20) = K_{t2}(\lambda_1(t)+\lambda_2(t)) + C_{t2}(\dot{\lambda}_1(t)+\dot{\lambda}_2(t))$$

$$Q(21) = K_{t1}(G_1(t)+G_2(t)) + C_{t1}(\dot{G}_1(t)+\dot{G}_2(t))$$

$$Q(22) = a_2K_{t1}(G_2(t)-G_1(t)) + a_2C_{t1}(\dot{G}_2(t)-\dot{G}_1(t))$$
$$-r_1K_{t2}(\lambda_1(t)+\lambda_2(t)) - r_1C_{t2}(\dot{\lambda}_1(t)+\dot{\lambda}_2(t))$$

$$Q(26) = K_{t4}(\lambda_3(t)+\lambda_4(t)) + C_{t4}(\dot{\lambda}_3(t)+\dot{\lambda}_4(t))$$

$$Q(27) = K_{t3}(G_3(t)+G_4(t)) + C_{t3}(\dot{G}_3(t)+\dot{G}_4(t))$$

$$Q(28) = a_4K_{t3}(G_4(t)-G_3(t)) + a_4C_{t3}(\dot{G}_4(t)-\dot{G}_3(t))$$
$$-r_3K_{t4}(\lambda_3(t)+\lambda_4(t)) - r_3C_{t4}(\dot{\lambda}_3(t)+\dot{\lambda}_4(t))$$

$$Q(32) = K_{t6}(\lambda_5(t)+\lambda_6(t)) + C_{t6}(\dot{\lambda}_5(t)+\dot{\lambda}_6(t))$$

$$Q(33) = K_{t5}(G_5(t)+G_6(t)) + C_{t5}(\dot{G}_5(t)+\dot{G}_6(t))$$

$$Q(34) = a_6K_{t5}(G_6(t)-G_5(t)) + a_6C_{t5}(\dot{G}_6(t)-\dot{G}_5(t))$$
$$-r_5K_{t6}(\lambda_5(t)+\lambda_6(t)) - r_5C_{t6}(\dot{\lambda}_5(t)+\dot{\lambda}_6(t))$$

$$Q(38) = K_{t8}(\lambda_7(t)+\lambda_8(t)) + C_{t8}(\dot{\lambda}_7(t)+\dot{\lambda}_8(t))$$

$$Q(39) = K_{t7}(G_7(t)+G_8(t)) + C_{t7}(\dot{G}_7(t)+\dot{G}_8(t))$$

$$Q(40) = a_8K_{t7}(G_8(t)-G_7(t)) + a_8C_{t7}(\dot{G}_8(t)-\dot{G}_7(t))$$

$$-r_7K_{t8}(\lambda_7(t)+\lambda_8(t)) - r_7C_{t8}(\dot{\lambda}_7(t)+\dot{\lambda}_8(t))$$

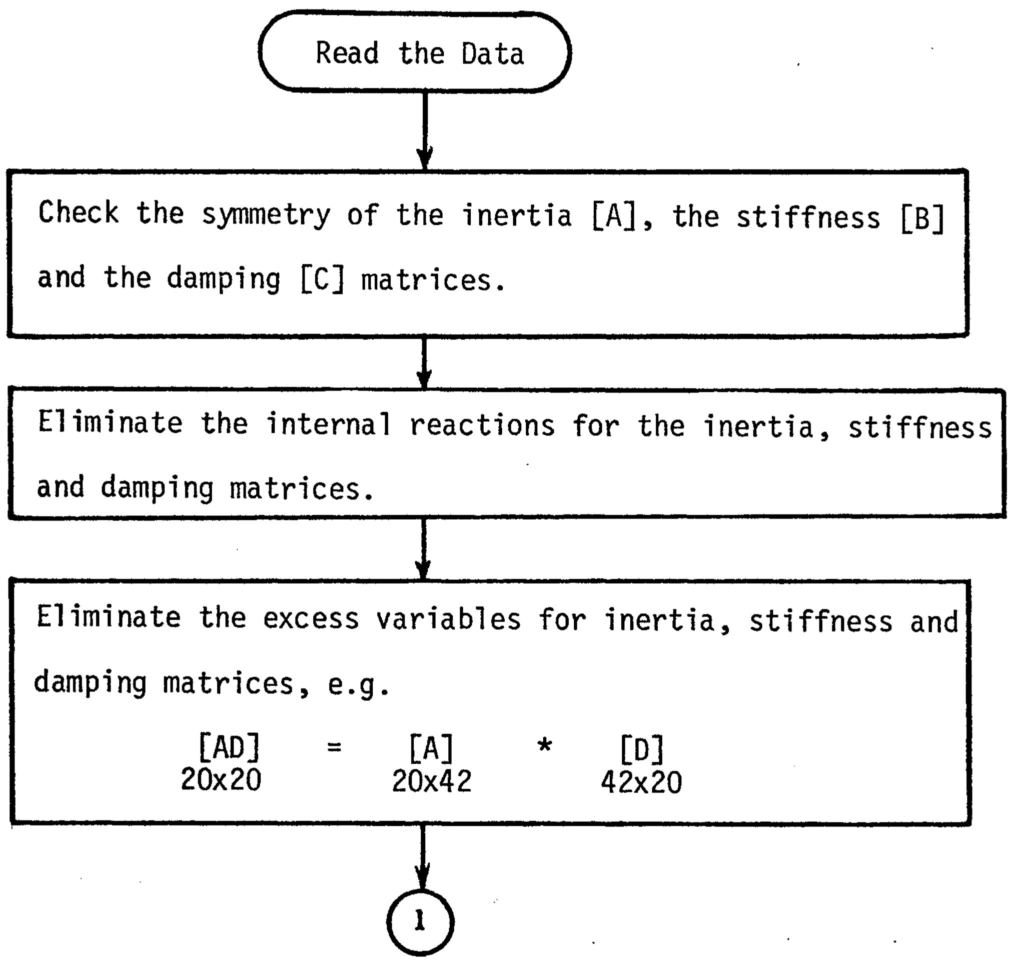
APPENDIX II

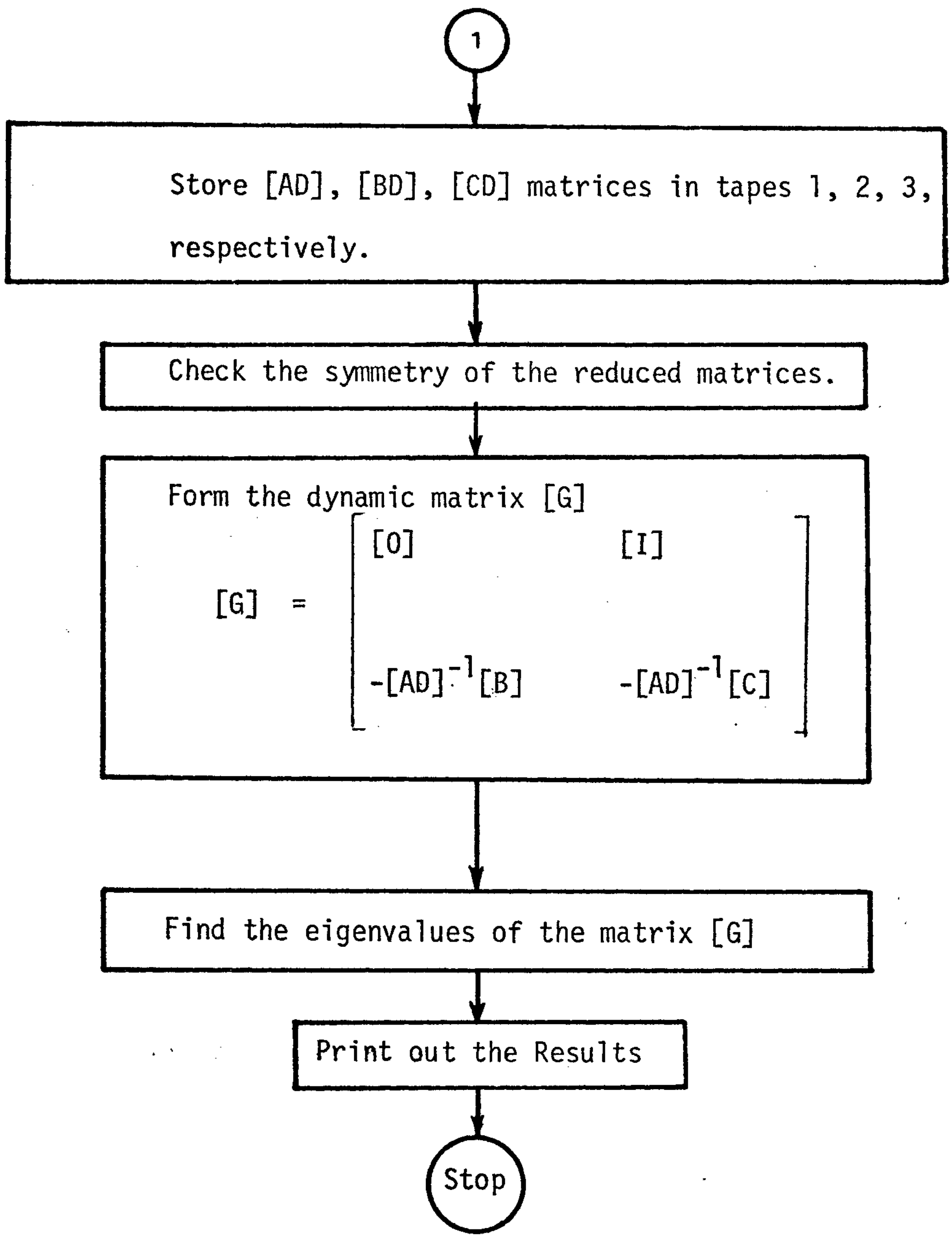
FLOW CHARTS FOR THE COMPUTER PROGRAMS

This Appendix includes flow charts of the main programs that were written to solve the linear equations of motion for the tractor-self-steering semitrailer. The main steps of the program for tractor-standard semitrailer are the same as for tractor-self-steering semitrailer.

II.1 Flow Chart 1:

Natural Frequencies for the Self-Steering Semitrailer





II.2 Flow Chart 2:

Steady State Response for the Self-Steering Semitrailer

Read The Data

Read the inertia matrix [A], stiffness matrix [B] and damping matrix [C] from tapes 1, 2, 3.

Define the forcing stiffness [FKL], [FKR] and the forcing damping [FCL], [FCR] matrices for left and right tracks respectively.

OMEGA = 0.001

Define the excitation vectors {YL} and {YR} for right and left tracks respectively.

Define the complex matrix [CD]

1. OC = OMEGA * C(I,J)
2. OB = B(I,J) - OMEGA * OMEGA * A(I,J)
3. CD(I,J) = Cmplx (OB, OC)

Define the complex matrices [CNL], [CNR]

1. OCF = OMEGA * FCL(I,J)
2. OBF = FKL(I,J)
3. CNL(I,J) = Cmplx (OBF, OCF)

1

1

4. OCF = OMEGA * FCR(I,J)

5. OBF = FKR(I,J)

6. CNR(I,J) = Cmplx (OBF, OCF)

Invert the complex matrix [CD]

Get the solution vector

For left track:

1. [CDNL] = $[CD]^{-1}$ * [CNL]

2. {XL} = [CDNL] * {YL}

For right track:

1. [CDNR] = $[CD]^{-1}$ * [CNR]

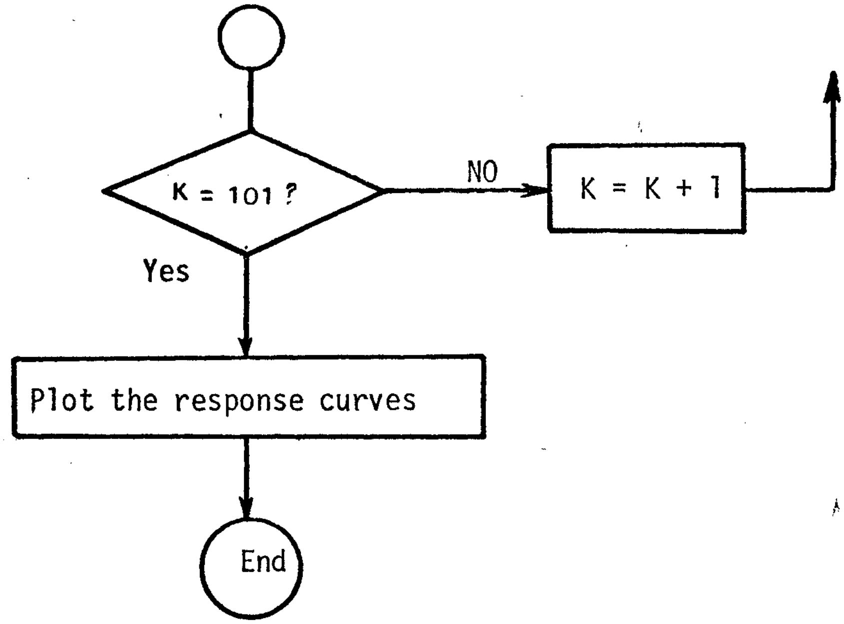
2. {XR} = [CDNR] * {YR}

Get the amplitude for the solution vector

CX(I) = CABS [XL(I) + XR(I)]

OMEGA = OMEGA + $\Delta\omega$

2


K = 101 ?
NO
K = K + 1
Yes
Plot the response curves
End