ON THE DYNAMIC RESPONSE OF

RAILWAY VEHICLES

ON THE EFFECT OF TRACK IRREGULARITIES ON THE DYNAMIC RESPONSE OF RAILWAY VEHICLES

by

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SCOPE AND CONTENTS:

The steady state response for models of an actual six-axle locomotive running on a sinusoidally irregular track is investigated. Three mathematical models are set up, a simplified model which assumed no springing or damping of trucks or motors, and no creep forces, a full model for the "stationary" vehicle in which creep forces are assumed negligible and a full model for the "moving" vehicle, in which creep forces, gravity stiffness effects and wheel tread profiles are considered.

The steady state response of the vehicle components to varying input frequencies is calculated and the response curves are computer plotted in each case. The natural frequencies for the simplified and the full model are also calculated. For the "moving" vehicle responses for the cases of new and worn wheels are obtained. Effect of creep and wheel tread profiles is studied.

The accuracy with which each of the devised models describe the performance of the real railway vehicle is compared. A discussion of the conclusions drawn from the analysis, including the applications to the design of high speed railway vehicles is given. Much attention was devoted to the development and testing of five computer programs for which simplified flow charts are given in Appendix V.

ACKNOWLEDGEMENTS

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NOMENCLATURE

The page numbers given below refer to the page on which the symbol is first defined, and the figure numbers, when given, refer to the first figure in which the symbol appears.

A	- Inertia (or mass) matrix; p.14
AD	- Inertia (or mass) matrix after elimination of internal reactions, p.22
B	- Stiffness matrix, p.14
BD	- Stiffness matrix after elimination of internal reactions; p.22
C	- Damping matrix; p.14
CD	- Damping matrix after elimination of internal reactions, p.22
$C_1,\ldots,C_{12}$	- Damping coefficients for rubber springs (corresponding to $K_1,\ldots,K_{12}$), Figures 1a & 1b
$C_R, C_V, C_{13}\ldots C_{18}$	- Damping coefficients for shock absorbers, Figures 1a and 2
D	- Transformation matrix; p. 22
f_1	- Coefficient relating longitudinal creep force to long. creep; p.30
f_2	- Coefficient relating lateral creep force to lateral creep; p.30
f_3	- Coefficient relating creep torque to rotational creep; p.30
f_{23}	- Coupling coefficient relating creep torque to lateral creep and lateral force to rotational creep; p.30
G	- Matrix whose eigen-values are the natural frequencies; p. 64
$I_{a\alpha}, I_{a\beta}, I_{a\gamma}$	- Moments of inertia for the locomotive body about an axis passing through its c.g. and parallel to x, y and z axis respectively, pp. 145, 146, 147 respectively

$I_{b\alpha}, I_{b\beta}, I_{b\gamma}$ - Moments of inertia for a frame about an axis passing through its c.g. and parallel to x, y and z axis respectively; pp. 153, 155, 157 respectively.

$I_{c\alpha}, I_{c\beta}, I_{c\gamma}$ - Moments of inertia for a motor about an axis passing through its c.g. and parallel to x, y and z axis respectively; pp. 172, 170, 170 respectively

$I_{d\alpha}, I_{d\beta}, I_{d\gamma}$ - Moments of inertia for a wheelset about an axis passing through its c.g. and parallel to x, y and z axis respectively; pp. 172, 173, 173 respectively

$K_1, \ldots, K_{12}$ - Stiffness coefficients for rubber springs; Figures 1a, 1b and 2

$\ell_1, \ldots, \ell_{35}$ - Lengths, Figures 1a, 1b and 2.

m_a - Locomotive body mass, p.141

m_b - Mass of a frame; p.150

m_c - Mass of a motor; p.170

m_d - Mass of a wheelset; p.172

$N_{\ell 1}, \ldots N_{\ell 6}$ - Reaction between rail and left wheels, No. 1,2,...,6 respectively; p.32

$N_{r1}, \ldots N_{r6}$ - Reaction between rail and right wheels, No. 1,2,...,6 respectively; p.32

u_a - Longitudinal displacement for the c.g. of the locomotive body, Figures 1a and 1b

u_{bf} - Longitudinal displacement for the c.g. of the front frame, Figures 1a and 1b.

u_{br} - Longitudinal displacement for the c.g. of the rear frame, Figures 1a and 1b.

$u_{c1}, \ldots, u_{c6}$ - Longitudinal displacement for the c.g. for motors 1,...,6; Figures 1a and 1b.

$u_{d1}, \ldots, u_{d6}$ - Longitudinal displacement for the c.g. for wheelsets 1,...,6; Figures 1a and 1b.

$U_{1r}, \ldots, U_{6r}$ - Creep force in the longitudinal direction for the right wheels, 1,...,6 respectively; p.30

$U_{1\ell}, \ldots, U_{6\ell}$ - Creep force in the longitudinal direction for the left wheels, 1,...,6, respectively; p.30

r_r, r_ℓ - Radii of tread circles, wheelset displaced laterally; p.216

$RA_1,\ldots,RA_6$ - Internal reaction between frame and wheelsets, 1,...,6, respectively in the u direction; pp.16, 17.

$RB_1,\ldots,RB_6$ - Internal reaction between frame and wheelsets, 1,...,6, respectively in the v direction; pp. 16,17.

$RM_1,\ldots,RM_6$ - Internal reaction between frame and wheelsets, 1,...,6, respectively in the γ direction, pp. 16, 17.

$RU_1,\ldots,RU_6$ - Internal reactions between motors and wheelsets, 1,...,6, respectively in the u direction, pp.16,17.

$RV_1,\ldots,RV_6$ - Internal reactions between motors and wheelsets, 1,...,6 respectively in the v direction, pp. 16, 17.

$RW_1,\ldots,RW_6$ - Internal reactions between motors and wheelsets, 1,...,6, respectively in the w direction, pp. 16, 17.

$R\alpha_1,\ldots,R\alpha_6$ - Internal reactions between motors and wheelsets, 1,...,6, respectively in the α direction, pp. 16, 17.

$R\gamma_1,\ldots,R\gamma_6$ - Internal reactions between motors and wheelsets, 1,...,6, respectively in the γ direction, pp. 16, 17.

$RX_{1\ell},\ldots,RX_{6\ell}$ - Reaction between rail and left wheel, 1,...,6, respectively in the x direction, pp. 16, 17.

$RX_{1r},\ldots,RX_{6r}$ - Reaction between rail and right wheel, 1,...,6, respectively in the x direction, pp. 16, 17.

$RY_{1\ell},\ldots,RY_{6\ell}$ - Reaction between rail and left wheel, 1,...,6, respectively in the y direction, pp. 16, 17.

$RY_{1r},\ldots,RY_{6r}$ - Reaction between rail and right wheel, 1,...,6, respectively in the y direction, pp. 16, 17.

$RZ_{1\ell},\ldots,RZ_{6\ell}$ - Reaction between rail and left wheel, 1,...,6, respectively in the z direction, pp. 16, 17.

$RZ_{1r},\ldots,RZ_{6r}$ - Reaction between rail and right wheel, 1,...,6, respectively in the z direction, pp. 16, 17.

S - Forward speed of the locomotive; p. 217

S_i - Roots of the matrix G (eigen-values); p. 65

T - Transformation matrix; p. 59.

$T_{1\ell}, \ldots, T_{6\ell}$ - Creep torque (about the vertical axis) for the left wheels, 1,...,6, respectively, p. 30.

$T_{1r}, \ldots, T_{6r}$ - Creep torque (about the vertical axis) for the right wheels, 1,...,6, respectively, p. 30.

v_a - Lateral displacement for the c.g. of the locomotive body, Figures 1a and 1b.

v_{bf} - Lateral displacement for the c.g. of the front frame, Figures 1a and 1b.

v_{br} - Lateral displacement for the c.g. of the rear frame, Figures 1a and 1b.

$v_{c1}, \ldots, v_{c6}$ - Lateral displacement for the c.g. of the motors, 1,...,6 respectively, Figures 1a and 1b.

$V_{1\ell}, \ldots, V_{6\ell}$ - Creep force in the lateral direction for left wheels, 1,...,6, respectively, p.30.

$V_{1r}, \ldots, V_{6r}$ - Creep force in the lateral direction for right wheels, 1,...,6, respectively, p.30.

W - Gravity force per wheelset, p. 214.

w_a - Vertical displacement for the c.g. of the locomotive body, Figures 1a and 1b.

w_{bf} - Vertical displacement for the c.g. of the front frame, Figures 1a and 1b.

w_{br} - Vertical displacement for the c.g. of the rear frame, Figures 1a and 1b.

$w_{c1}, \ldots, w_{c6}$ - Vertical displacement for the c.g. for motors, 1,...,6, respectively, Figures 1a and 1b.

$w_{d1}, \ldots, w_{d6}$ - Vertical displacement for the c.g. for wheelsets, 1,...,6, respectively, Figures 1a and 1b.

x - Displacement solution vector, p. 14

y - Displacement excitations vector, p. 14.

α_a - Angle of rotation of the body about x axis passing through c.g.; Figures 1a and 1b.

α_{bf} - Angle of rotation of the front frame about x axis passing through c.g.; Figures 1a and 1b.

α_{br} - Angle of rotation of the rear frame about x axis passing through c.g.; Figures 1a and 1b.

$\alpha_{c1},\ldots,\alpha_{c6}$ - Angle of rotation of the motors, 1,...,6, respectively about x axis passing through c.g.; Figures 1a and 1b.

$\alpha_{d1},\ldots,\alpha_{d6}$ - Angle of rotation of the wheelsets, 1,...,6, respectively about x axis passing through c.g.; Figures 1a and 1b.

β_a - Angle of rotation of the body about y axis passing through c.g.; Figures 1a and 1b.

β_{bf} - Angle of rotation of the front frame about y axis passing through c.g.; Figures 1a and 1b.

β_{br} - Angle of rotation of the rear frame about y axis passing through c.g.; Figures 1a and 1b.

$\beta_{c1},\ldots,\beta_{c6}$ - Angle of rotation of the motors, 1,...,6, respectively about y axis passing through c.g.; Figures 1a and 1b.

$\beta_{d1},\ldots,\beta_{d6}$ - Angle of rotation of the wheelsets, 1,...,6, respectively about y axis passing through c.g.; Figures 1a and 1b.

γ_a - Angle of rotation of the body about z axis passing through c.g.; Figures 1a and 1b.

γ_{bf} - Angle of rotation of the front frame about z axis passing through c.g.; Figures 1a and 1b.

γ_{br} - Angle of rotation of the rear frame about z axis passing through c.g.; Figures 1a and 1b.

$\gamma_{c1},\ldots,\gamma_{c6}$ - Angle of rotation of the motors, 1,...,6, respectively about z axis passing through c.g.; Figures 1a and 1b.

$\gamma_{d1},\ldots,\gamma_{d6}$ - Angle of rotation of the wheelsets, 1,...,6, respectively about z axis passing through c.g.; Figures 1a and 1b.

λ - Effective conicity of wheel tread defined as the rate of change of rolling radius with lateral displacement of wheelset; p. 41.

ε - Rate of change of contact plane slope with lateral displacement of wheelset; p. 215.

θ_o - Angle between contact plane and horizontal, wheelset in central position; p.32.

θ_r, θ_ℓ - Angles between contact planes and horizontal, wheelset displaced laterally; Figure (III.1).

η - Rate of change of distance between wheelset centre-line and contact points with the lateral displacement of wheelset; p. 216.

CHAPTER 1

INTRODUCTION

1.1 Background

The dynamics of railway vehicles have been of interest for many decades. This interest is motivated by a desire to improve riding qualities and to reduce wear and damage to vehicles and track. One of the greatest problems in this area is that railway vehicles, under certain conditions, experience sustained oscillations in a horizontal plane; this behaviour is commonly referred to as hunting and is the main obstacle to the increase of train speed. Hunting is accompanied by large dynamic loads between the vehicle and rails since unstable oscillations are limited only by flange contact of the wheels; under certain conditions derailment of the vehicle will occur. Extensive analytical and experimental studies that have attempted to explain this phenomenon have been reported in the literature using simplified models.

Another important problem is the dynamic response of railway vehicles to rail irregularities. This problem arises from the fact that some track "shapes" are more structurally damaging or more operationally dangerous than others. A six-axle locomotive of the type commonly used in North America experienced extensive yaw oscillations when it was used in another country.

Clearly the "shape" of the surface over which a vehicle runs plays an important role in the movements of the vehicle in space.

The tracks may possess a variety of horizontal and vertical irregularities which affect the vehicle performance in various ways.

1.2 Literature Survey

Ever since trains have been running, there have been studies on the lateral oscillations of railway vehicles. These motions are due to lateral rail irregularities, coned wheel profiles, and the parameters of the contact condition between wheels and rails.

Carter [1,2]* was one of the first to recognize the importance of the contact mechanism between the wheel and the rail and he indicated the existence of critical velocities above which hunting takes place. He treated the two dimensional case of two cylinders with parallel axes rolling together with creep in the direction of rolling. Poritsky's work [3] is similar to Carter's.

An approximate theory for the three dimensional problem with elliptical contact is given by Johnson and Vermuellen [4]. Another theoretical treatment is given by Haines and Ollerton [5] for the three dimensional case of elliptical contact but for creep in the direction of rolling only. More work including spin has been done by Johnson [6], dePater [7] and Kalker [8].

To study the stability of a four-wheeled railway vehicle, dePater [9,10,11] derived the non-linear differential equations of motion and solved them using the method of Krylov and Bogoljubov.

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* *Numbers in square brackets designate references in the Bibliography*

Later Van Bommel [12,13] was able to numerically integrate these equations.

The trend towards higher speed trains demanded a better understanding of the behaviour of this complex system with many degrees of freedom. Two approaches can be distinguished in the further development of the theory of lateral motions of railway vehicles:

1. *By Linearization of the Equations and using the Well-Known Matrix Theory:*

The investigations of Bishop [14], Brann and Bishop [15], Wickens [16,17,18], Van Bommel [19] and Shaghaghi [20] have given a considerable amount of information about the stability of the systems with different parameters such as the mass, the position of the mass centre, the moments of inertia, the total load, the ratio of wheel gauge and wheelbase and the types of profiles of wheel and rail.

The influence of lateral and longitudinal stiffness has been studied by Wickens [17,21,22], Van Bommel [19] and Matsudaira [23]. The first two investigators also introduced the theory of rolling contact including spin according to Kalker [24]. Many of the investigators have, with considerable success made great use of digital computers.

2. *Considering the Non-Linear Problem:*

Non-linearities are essentially due to the non-linear profiles of wheel and rail, the presence of the flanges, the clearance between the wheelsets and the bogie, and the non-linear creep forces. dePater

[9,10,11] and Van Bommel [12,13] studied the effects of each of these parameters. The influence of non-linear features of the suspension and the forces arising between rail and wheel were described by Gilchrist, Hobbs, King and Wasby [25].

Van Bommel [13] was the first to consider track irregularities in studying the performance of railway vehicles. He considered the dynamic response of a four-wheeled vehicle running on a sinusoidal shaped track.

Birmann's [26] theoretical and experimental work showed the effect of the vertical and horizontal track elasticity on the response of railway vehicles.

Nakamura [27,28] applied covariance functions to describe the vertical irregularities of the track and hence determine the response of the system.

Research work describing the lateral irregularities of rails in a statistical sense was done by Stassen [30, 31]. In his doctoral thesis, Stassen [31] studied the dynamic response of a simplified model of a bogie with two degrees of freedom having the lateral deviations as input, and the generalized co-ordinates which describes the movements of the bogies as output; the problem is non-linear with random inputs.

1.3 Scope of the Study

In this study three models for a six-axle locomotive are considered:

a - a simplified model, assuming no springing or damping of trucks or motors, and no creep forces.

b - a full model for the "stationary" vehicle, assuming no creep forces.

c - a full model for the "moving" vehicle, including slip and corresponding creep forces.

The steady state response of the body to varying input frequencies is calculated in each case. A one inch amplitude lateral sinusoidal track irregularity is considered. For the "moving" vehicle the input frequency is a function of track sinusoidal wave length and vehicle speed.

In practice the variation in the position of the rail is random, but in many cases it could be considered sinusoidal. Moreover the response to a sinusoidal alignment function is worth considering, as mentioned by Hobbs [32], because power spectra and autocorrelation functions obtained from field measurements indicate the existence of periodic functions.

In this study the assumption is made that guidance is achieved by the creep forces acting on the wheel treads. The design should include this assumption as a goal thus avoiding flange contact in normal running conditions. With this assumption being made, it is appropriate to consider a linearized analysis.

Transient response considerations are outside the scope of this study and discussion will be limited to the steady state response and natural frequencies.

The direct application of such a study would be in the field of the design of railway vehicles, the main problem being: how to design a guided vehicle which is stable and has optimum damping such that the response to economically attractive track imperfections is satisfactory up to very high vehicle speeds?

CHAPTER 2

EQUATIONS OF MOTION

2.1 Introduction

In this chapter the three mathematical models are described:

a - the simplified model,

b - the full model - "stationary" vehicle,

c - the full model - "moving" vehicle.

The differential equations of motion of each are derived using Newton's method.

2.2 Description of the Full Model

The first step in the analysis is setting up a mathematical model. A complete mathematical model for an actual six-axle locomotive is set up with the following simplifying assumptions:

1. The vehicle components are regarded as perfectly rigid and their elasticity is lumped, in the suspension elements.
2. The axles are assumed to run freely in the journal bearings, without bearing friction, at constant speed.
3. Lateral play (due to clearance) between the wheel-sets and the frame is neglected.
4. All displacements are small.
5. All springs have linear characteristics

The accuracy of the results obtained directly depend on the accuracy with which the mathematical model approximates the actual railway vehicle. Figures 1a, 1b and 2 illustrate the full mathematical model, which is composed of fifteen rigid bodies:

- the locomotive body,
- two frames (bogies)
- six motors
- six wheelsets.

The primary suspension, which is the suspension between the wheelsets and the frames consists of coil springs and shock absorbers. Wheelsets are restrained in both the longitudinal and lateral directions.

A traction motor is supported on the axle from one side and on a rubber nose support on the other side. A rubber nose support is used to prevent motor nose lug failure due to sudden torque impulses. This assembly is such that rotation of the motor about the axle center line is the only allowable relative motion between the motor and the wheelset.

The locomotive body is supported on rubber side bearers fixed to the front and rear frames. A resiliently mounted centre pivot is used in conjunction with the side bearers, thus avoiding metallic contact between the body and the trucks. Lateral and rotational motion of the body is resisted by a set of lateral and rotational shock absorbers.

Because the body is supported freely it can oscillate in all six coordinates, i.e. along the three principal axes through their centres of gravity.

i - longitudinal or "fore and aft" oscillation (in the u direction).

ii - lateral oscillation (in the v direction).

iii - vertical or "bouncing" oscillation (in the w direction).

and rotate about these three axes:

i - about the longitudinal u-axis or "rolling" (in the α direction).

ii - about the lateral v-axis or "pitching" (in the β direction).

iii - about the vertical w-axis or "yawing" (in the γ direction).

For the fifteen rigid bodies, the total number of degrees of freedom is ninety. This number can be reduced to seventy-eight degrees of freedom and hence seventy-eight equations of motion only thirty-six of which are independent. This reduction in the number of degrees of freedom can be effected if one combines the equations of motion for the wheelset and the corresponding motor in the u direction and in the α direction.

The relation between all the variables and the independent ones will be given by the transformation matrix D which is based on the equations of constraints.

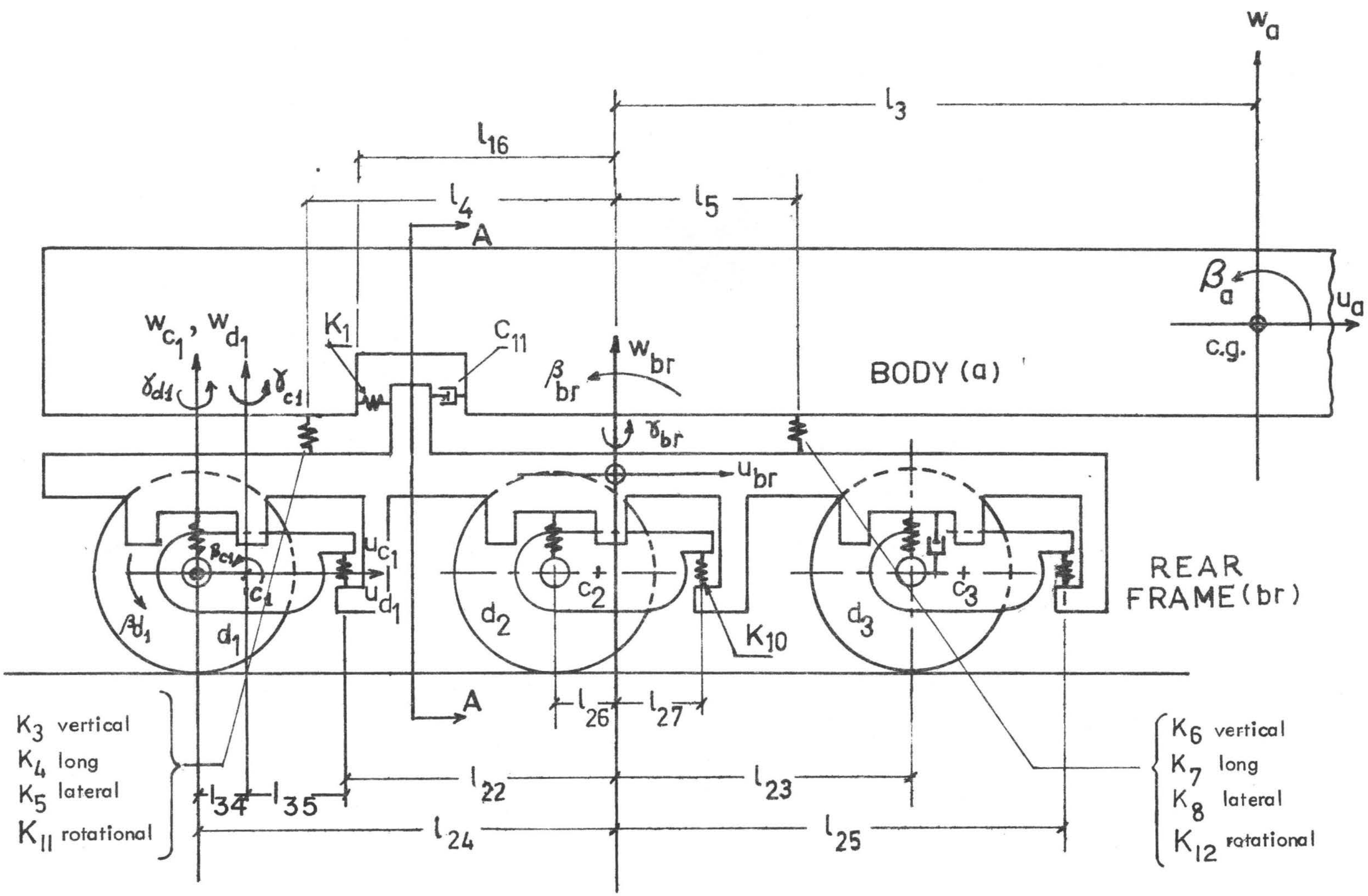

FIGURE 1a: MATHEMATICAL MODEL FOR THE LOCOMOTIVE
(Side View)

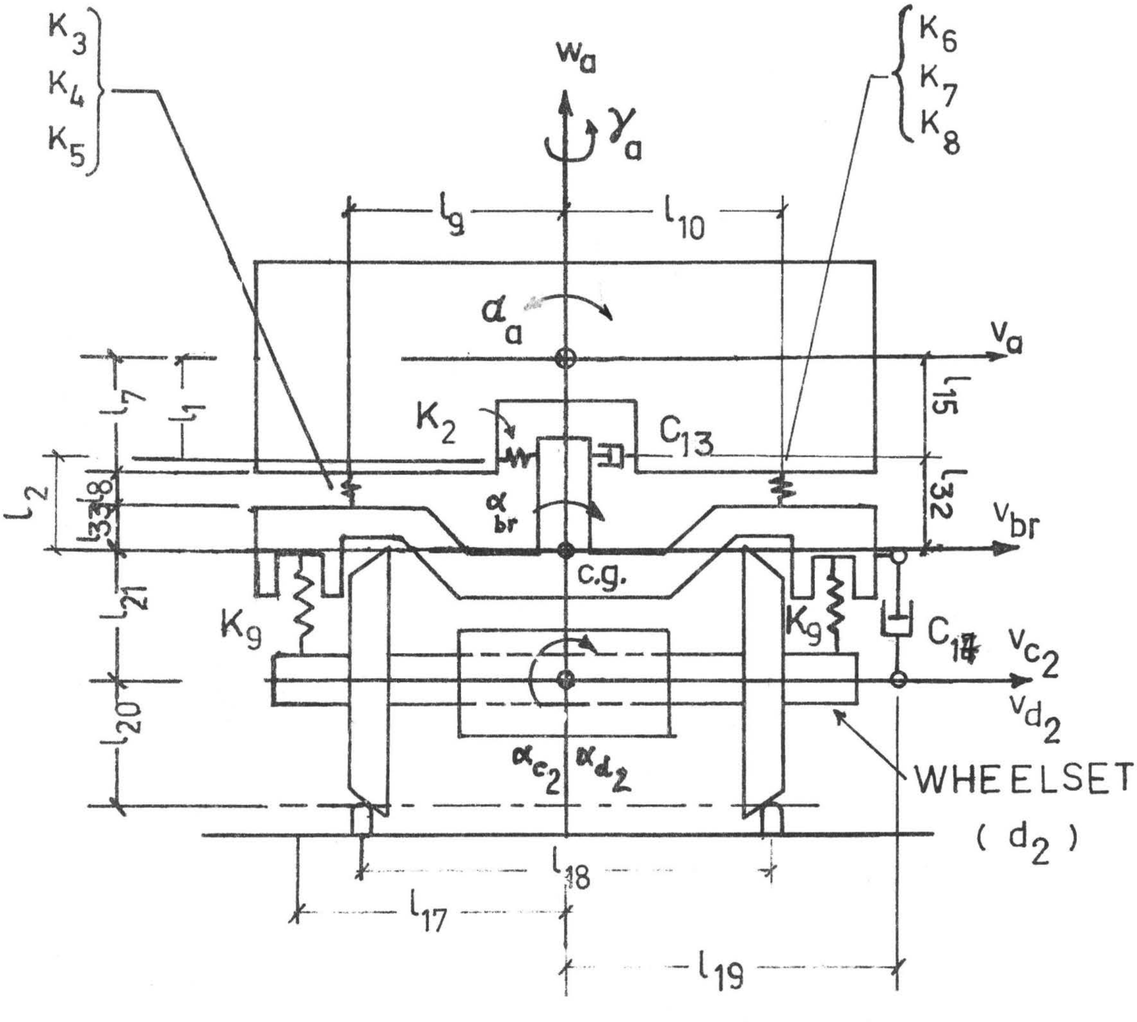

FIGURE 1b: VIEW FROM A-A

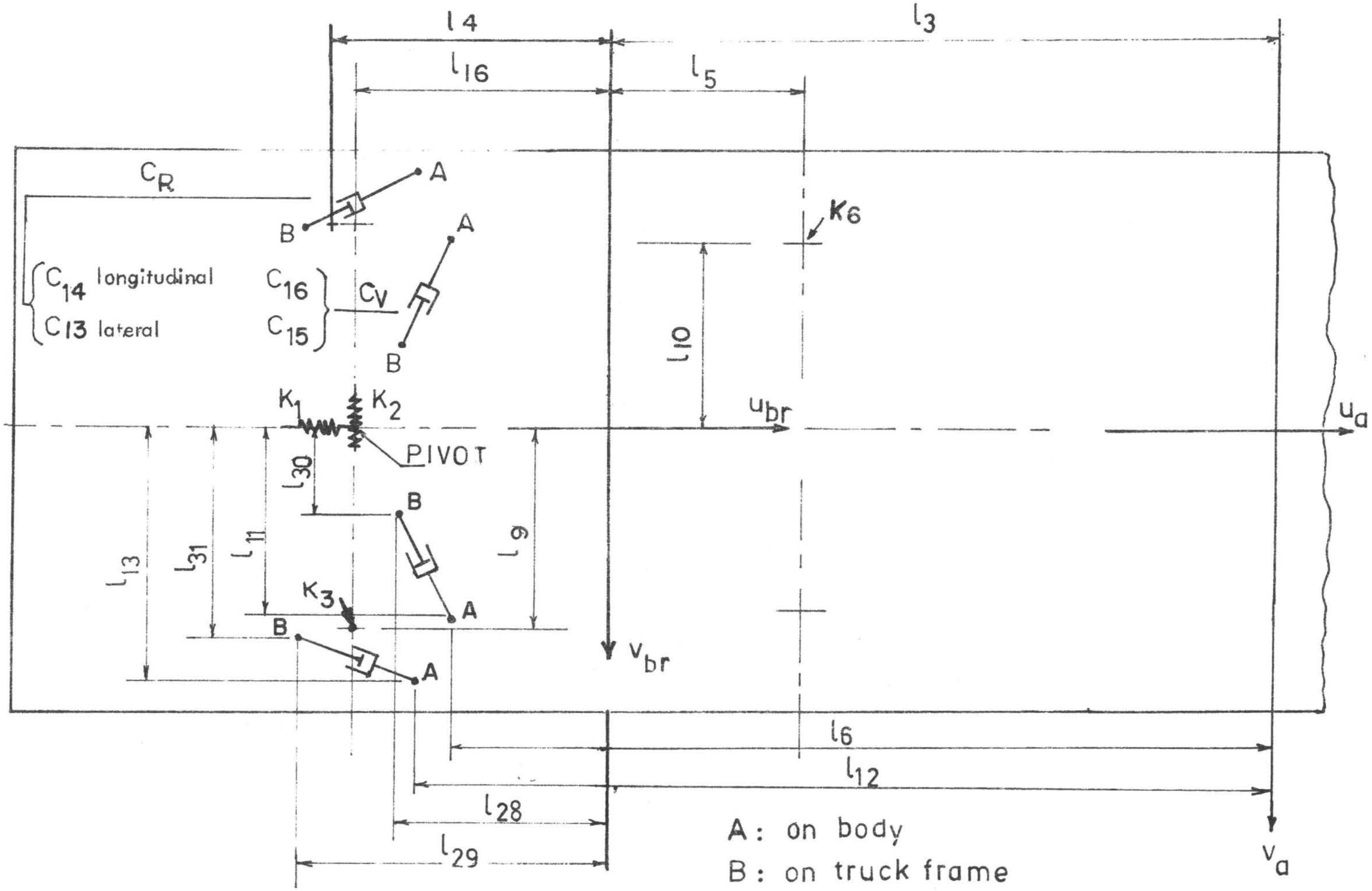

FIGURE 2: PLAN VIEW FOR SECONDARY SUSPENSION

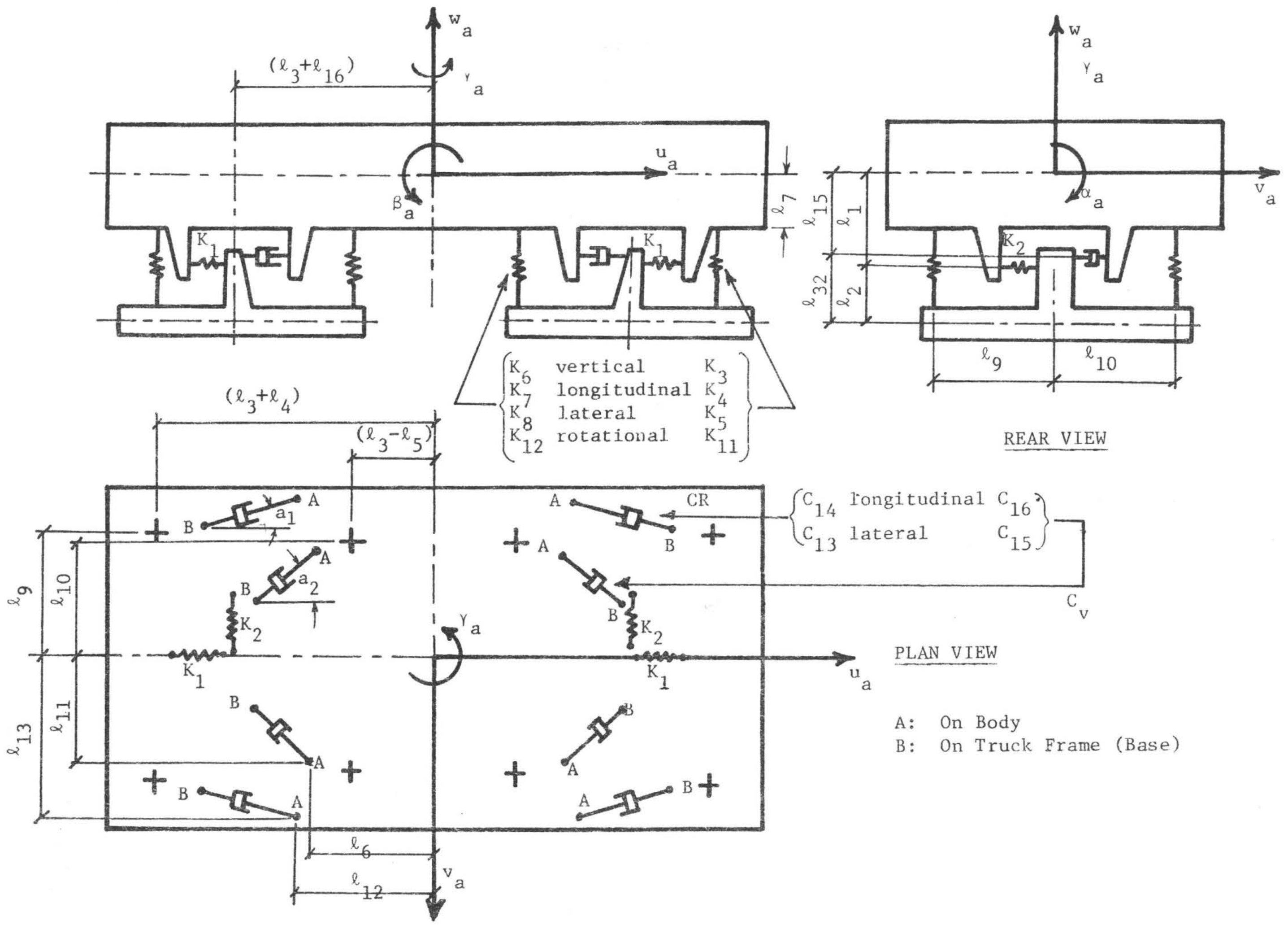

FIGURE 3: THE SIMPLIFIED MODEL

2.3 Description of the Simplified Model

Figure 3 illustrates the simplified model. The wheelsets and motors are assumed to be rigidly connected to the trucks. The locomotive body is, as described for the full model, supported freely and can oscillate in all six modes.

In this study we will consider free vibration and the steady state response of the body to varying input frequencies.

2.4 Derivation of the Equations of Motion for the Simplified Model

The equations of motion for the simplified model are given in detail in Appendix II. The six equations have the general form:

$$[A]\ \{\ddot{X}\} + [C]\ \{\dot{X}\} + [B]\ \{X\} = [C_{bf}]\{\dot{y}_{bf}\} + [K_{bf}]\{y_{bf}\} + [C_{br}]\{\dot{y}_{br}\} + [K_{br}]\{y_{br}\} \qquad (2.4.1)$$

where

- A is the inertia matrix (diagonal) (6x6)
- C is the damping matrix (symmetric) (6x6)
- B is the stiffness matrix (symmetric) (6x6)
- y_{bf} is the displacement excitation vector at the front base (6x1)
- y_{br} is the displacement excitation vector at the rear base (6x1)
- x is the displacement vector (6x1) where elements are

$x(1) = u_a$ $\quad x(2) = v_a$ $\quad x(3) = w_a$

$x(4) = \alpha_a$ $\quad x(5) = \beta_a$ $\quad x(6) = \gamma_a$

For the free vibration problem, the right hand side of equation (2.4.1) will simply be a column of zeros.

2.5 Derivation of the Equations of Motion for the Full Model - "Stationary" Vehicle

The equations of motion for the "stationary" vehicle are given in detail in Appendix I.

The system of linear differential equations is written in the form:

$$\underset{78x78}{[A]}\ \underset{78x1}{\{\ddot{x}\}} + \underset{78x78}{[C]}\ \underset{78x1}{\{\dot{x}\}} + \underset{78x78}{[B]}\ \underset{78x1}{\{x\}} + \underset{78x1}{\{R\}} = 0 \qquad (2.5.1)$$

where A - is the inertia matrix,

B - is the stiffness matrix,

C - is the damping matrix,

R - is the vector of internal reactions.

Eliminating the internal reactions by the method of substitution we get a system of 36 equations in 78 unknowns. Thirty-six of the unknowns are independent.

2.5.1 Elimination of Internal Reactions

The vector of internal reactions is given by the following column matrix (78x1):

VAR.	EQN.NO.	REACTION
u_a	1	0
v_a	2	0
w_a	3	0
α_a	4	0
β_a	5	0
γ_a	6	0
u_{br}	7	$-(RA_1 + RA_2 + RA_3)$
v_{br}	8	$-(RB_1 + RB_2 + RB_3)$
w_{br}	9	0
α_{br}	10	$\ell_{21}(RB_1 + RB_2 + RB_3)$
β_{br}	11	$-\ell_{21}(RA_1 + RA_2 + RA_3)$
γ_{br}	12	$-\ell_{24}(RB_1) - \ell_{26}(RB_2) + \ell_{23}(RB_3) - (RM_1 + RM_2 + RM_3)$
u_{bf}	13	$-(RA_4 + RA_5 + RA_6)$
v_{bf}	14	$-(RB_4 + RB_5 + RB_6)$
w_{bf}	15	0
α_{bf}	16	$+\ell_{21}(RB_4 + RB_5 + RB_6)$
β_{bf}	17	$-\ell_{21}(RA_4 + RA_5 + RA_6)$
γ_{bf}	18	$-\ell_{23}(RB_4) + \ell_{26}(RB_5) + \ell_{24}(RB_6) - (RM_4 + RM_5 + RM_6)$
v_{cl}	19	$-(RV_1)$
w_{cl}	20	$+(RW_1)$
β_{cl}	21	$-\ell_{34}(RW_1)$
γ_{cl}	22	$-\ell_{34}(RV_1) - (R\gamma_1)$
u_{dl}	23	$(RA_1) - (RX_{1r} + RX_{1\ell})$
v_{dl}	24	$(RB_1) + (RV_1)$

VAR.	EQN.NO.	REACTION
w_{d1}	25	$-(RW_1)$
α_{d1}	26	0
β_{d1}	27	$-\ell_{20}(RX_{1r} + RX_{1\ell})$
γ_{d1}	28	$(RM_1) + (R\gamma_1) - (\ell_{18}/2)(RX_{1r} - RX_{1\ell})$
v_{c2}	29	$-(RV_2)$
w_{c2}	30	$+(RW_2)$
β_{c2}	31	$-\ell_{34}(RW_2)$
γ_{c2}	32	$-\ell_{34}(RV_2) - (R\gamma_2)$
u_{d2}	33	$(RA_2) - (RX_{2r} + RX_{2\ell})$
v_{d2}	34	$(RB_2) + (RV_2)$
w_{d2}	35	$-(RW_2)$
α_{d2}	36	0
β_{d2}	37	$-\ell_{20}(RX_{2r} + RX_{2\ell})$
γ_{d2}	38	$(RM_2) + (R\gamma_2) -(\ell_{18}/2)(RX_{2r} - RX_{2\ell})$
v_{c3}	39	$-(RV_3)$
w_{c3}	40	$+(RW_3)$
β_{c3}	41	$-\ell_{34}(RW_3)$
γ_{c3}	42	$-\ell_{34}(RV_3) - (R\gamma_3)$
u_{d3}	43	$(RA_3) - (RX_{3r} + RX_{3\ell})$
v_{d3}	44	$(RB_3) + (RV_3)$
w_{d3}	45	$-(RW_3)$
α_{d3}	46	0
β_{d3}	47	$-\ell_{20}(RX_{3r} + RX_{3\ell})$

VAR.	EQN.NO.	REACTION
γ_{d3}	48	$(RM_3) + (R\gamma_3) - (\ell_{18}/2)(RX_{3r} - RX_{3\ell})$
v_{c4}	49	$-(RV_4)$
w_{c4}	50	$+(RW_4)$
β_{c4}	51	$+\ell_{34}(RW_4)$
γ_{c4}	52	$+\ell_{34}(RV_4) - (R\gamma_4)$
u_{d4}	53	$(RA_4) - (RX_{4r} + RX_{4\ell})$
v_{d4}	54	$(RB_4) + (RV_4)$
w_{d4}	55	$-(RW_4)$
α_{d4}	56	0
β_{d4}	57	$-\ell_{20}(RX_{4r} + RX_{4\ell})$
γ_{d4}	58	$(RM_4) + (R\gamma_4) - (\ell_{18}/2)(RX_{4r} - RX_{4\ell})$
v_{c5}	59	$-(RV_5)$
w_{c5}	60	$+(RW_5)$
β_{c5}	61	$+\ell_{34}(RW_5)$
γ_{c5}	62	$+\ell_{34}(RV_5) - (R\gamma_5)$
u_{d5}	63	$(RA_5) - (RX_{5r} + RX_{5\ell})$
v_{d5}	64	$(RB_5) + (RV_5)$
w_{d5}	65	$-(RW_5)$
α_{d5}	66	0
β_{d5}	67	$-\ell_{20}(RX_{5r} + RX_{5\ell})$
γ_{d5}	68	$(RM_5) + (R\gamma_5) - (\ell_{18}/2)(RX_{5r} - RX_{5\ell})$

VAR.	EQN.NO.	REACTION
v_{c6}	69	$-(RV_6)$
w_{c6}	70	$+(RW_6)$
β_{c6}	71	$+\ell_{34}(RW_6)$
γ_{c6}	72	$+\ell_{34}(RV_6) - (R\gamma_6)$
u_{d6}	73	$(RA_6) - (RX_{6r} + RX_{6\ell})$
v_{d6}	74	$(RB_6) + (RV_6)$
w_{d6}	75	$-(RW_6)$
α_{d6}	76	0
β_{d6}	77	$-\ell_{20}(RX_{6r} + RX_{6\ell})$
γ_{d6}	78	$(RM_6) + (R\gamma_6) - (\ell_{18}/2)(RX_{6r} - RX_{6\ell})$

To eliminate the vector R the procedure described below is adopted where numbers refer to equations in Appendix I:

1 = 1

2 = 2

3 = 3

4 = 4

5 = 5

6 = 6

$$7 = 7 + (23 - \frac{27}{\ell_{20}}) + (33 - \frac{37}{\ell_{20}}) + (43 - \frac{47}{\ell_{20}})$$

8 = 8 + (19+24) + (29+34) + (39+44)

9 = 9

$$10 = 10 - \ell_{21}[(19+24) + (29+34) + (39+44)]$$

$$11 = 11 + \ell_{21}[(23 - \frac{27}{\ell_{20}}) + (33 - \frac{37}{\ell_{20}}) + (43 - \frac{47}{\ell_{20}})]$$

$$12 = 12 + [\ell_{24}(19+24) + \ell_{26}(29+34) - \ell_{23}(39+44)] +$$

$$[-\ell_{34}(19+29+39) + (22+32+42) + (28+38+48)]$$

$$13 = 13 + [(53 - \frac{57}{\ell_{20}}) + (63 - \frac{57}{\ell_{20}}) + (73 - \frac{77}{\ell_{20}})]$$

$$14 = 14 + (49+54) + (59+64) + (69+74)$$

$$15 = 15$$

$$16 = 16 - \ell_{21}[(49+54) + (59+64) + (69+74)]$$

$$17 = 17 + \ell_{21}[(53 - \frac{57}{\ell_{20}}) + (63 - \frac{67}{\ell_{20}}) + (73 - \frac{77}{\ell_{20}})]$$

$$18 = 18 + [\ell_{23}(49+54) - \ell_{26}(59+64) - \ell_{24}(69+74)] + [\ell_{34}(49+59+69) +$$

$$(52+62+72) + (58+68+78)]$$

$$19 = 21 + \ell_{34}(20)$$

$$20 = 25 + 20$$

$$21 = 26$$

$$22 = 31 + \ell_{34}(30)$$

$$23 = 35 + 30$$

$$24 = 36$$

$$25 = 41 + \ell_{34}(40)$$

$$26 = 45 + (40)$$

$$27 = 46$$

$$28 = 51 - \ell_{34}(50)$$

29 = 55 + (50)

30 = 56

31 = 61 - ℓ_{34}(60)

32 = 65 + 60

33 = 66

34 = 71 - ℓ_{34}(70)

35 = 75 + (70)

36 = 76

In the computer program, this procedure is applied to the mass, stiffness and damping matrices successively, giving:

$$[A]\ \{\ddot{Z}\} + [C]\ \{\dot{Z}\} + [B]\ \{Z\} = 0 \qquad (2.5.2)$$

where [A], [C], [B] are now 36x78 each.

To eliminate the dependent variables, we use the transformation matrix [D]. [D] is a 78x36 matrix which relates all the variables and the independent variables and which is based on the equations of constraints: i.e.,

$$\underset{78x1}{\{Z\}} = \underset{78x36}{[D]}\ \underset{36x1}{\{X\}}$$

where X are the independent variables. Therefore (2.5.2) becomes

$$\underset{36x78}{[A]}\ \underset{78x36}{[D]}\ \underset{36x1}{\{\ddot{X}\}} + \underset{36x78}{[C]}\ \underset{78x36}{[D]}\ \underset{36x1}{\{\dot{X}\}} + \underset{36x78}{[B]}\ \underset{78x36}{[D]}\ \underset{36x1}{\{X\}} = 0$$

which after carrying the matrix multiplication gives the following system of 36 equations in 36 independent variables.

$$\underset{36\text{x}36}{[AD]}\ \underset{36\text{x}1}{\{\ddot{X}\}} + \underset{36\text{x}36}{[CD]}\ \underset{36\text{x}1}{\{\dot{X}\}} + \underset{36\text{x}36}{[BD]}\ \underset{36\text{x}1}{\{X\}} = 0 \tag{2.5.3}$$

It should be noted that the original mass stiffness and damping matrices (A, B and C) are symmetric and positive definite as well as the reduced mass stiffness and damping matrices AD, BD and CD respectively. In the computer program a check on the symmetry of the resulting reduced matrices is made.

In the following sections the equations of constraints and the definition of the transformation matrix [D] are given.

2.5.2 Equations of Constraints

(A) Constraints Between Wheelsets and Frames

$$u_{d1} = u_{br} + \ell_{21}\ \beta_{br} \tag{2.5.2.1}$$

$$u_{d2} = u_{br} + \ell_{21}\ \beta_{br} \tag{2.5.2.2}$$

$$u_{d3} = u_{br} + \ell_{21}\ \beta_{br} \tag{2.5.2.3}$$

$$u_{d4} = u_{bf} + \ell_{21}\ \beta_{bf} \tag{2.5.2.4}$$

$$u_{d5} = u_{bf} + \ell_{21}\ \beta_{bf} \tag{2.5.2.5}$$

$$u_{d6} = u_{bf} + \ell_{21}\ \beta_{bf} \tag{2.5.2.6}$$

$$v_{d1} = v_{br} - \ell_{21}\ \alpha_{br} + \ell_{24}\ \gamma_{br} \tag{2.5.2.7}$$

$$v_{d2} = v_{br} - \ell_{21}\ \alpha_{br} + \ell_{26}\ \gamma_{br} \tag{2.5.2.8}$$

$$v_{d3} = v_{br} - \ell_{21}\ \alpha_{br} + \ell_{23}\ \gamma_{br} \tag{2.5.2.9}$$

$$v_{d4} = v_{bf} - \ell_{21}\,\alpha_{bf} + \ell_{23}\,\gamma_{bf} \tag{2.5.2.10}$$

$$v_{d5} = v_{bf} - \ell_{21}\,\alpha_{bf} - \ell_{26}\,\gamma_{bf} \tag{2.5.2.11}$$

$$v_{d6} = v_{bf} - \ell_{21}\,\alpha_{bf} - \ell_{24}\,\gamma_{bf} \tag{2.5.2.12}$$

$$\gamma_{d1} = \gamma_{br} \tag{2.5.2.13}$$

$$\gamma_{d2} = \gamma_{br} \tag{2.5.2.14}$$

$$\gamma_{d3} = \gamma_{br} \tag{2.5.2.15}$$

$$\gamma_{d4} = \gamma_{bf} \tag{2.5.2.16}$$

$$\gamma_{d5} = \gamma_{bf} \tag{2.5.2.17}$$

$$\gamma_{d6} = \gamma_{bf} \tag{2.5.2.18}$$

(B) <u>Constraints Between Motors and Wheelsets</u>

$u_{d1} = u_{c1}$ $\qquad$ $u_{d4} = u_{c4}$

$u_{d2} = u_{c2}$ $\qquad$ $u_{d5} = u_{c5}$

$u_{d3} = u_{c3}$ $\qquad$ $u_{d6} = u_{c6}$

$\alpha_{d1} = \alpha_{c1}$ $\qquad$ $\alpha_{d4} = \alpha_{c4}$

$\alpha_{d2} = \alpha_{c2}$ $\qquad$ $\alpha_{d5} = \alpha_{c5}$

$\alpha_{d3} = \alpha_{c3}$ $\qquad$ $\alpha_{d6} = \alpha_{c6}$

These relations are already satisfied by combining the wheelset and motor equations.

$$v_{c1} = v_{br} - \ell_{21}\alpha_{br} + \ell_{24}\gamma_{br} - \ell_{34}\gamma_{c1} \quad (2.5.2.19)$$

$$v_{c2} = v_{br} - \ell_{21}\alpha_{br} + \ell_{26}\gamma_{br} - \ell_{34}\gamma_{c2} \quad (2.5.2.20)$$

$$v_{c3} = v_{br} - \ell_{21}\alpha_{br} - \ell_{23}\gamma_{br} - \ell_{34}\gamma_{c3} \quad (2.5.2.21)$$

$$v_{c4} = v_{bf} - \ell_{21}\alpha_{bf} + \ell_{23}\gamma_{bf} + \ell_{34}\gamma_{c4} \quad (2.5.2.22)$$

$$v_{c5} = v_{bf} - \ell_{21}\alpha_{bf} - \ell_{26}\gamma_{bf} + \ell_{34}\gamma_{c5} \quad (2.5.2.23)$$

$$v_{c6} = v_{bf} - \ell_{21}\alpha_{bf} - \ell_{24}\gamma_{bf} + \ell_{34}\gamma_{c6} \quad (2.5.2.24)$$

$$w_{c1} = w_{d1} + \ell_{34}\beta_{c1} \quad (2.5.2.25)$$

$$w_{c2} = w_{d2} + \ell_{34}\beta_{c2} \quad (2.5.2.26)$$

$$w_{c3} = w_{d3} + \ell_{34}\beta_{c3} \quad (2.5.2.27)$$

$$w_{c4} = w_{d4} - \ell_{34}\beta_{c4} \quad (2.5.2.28)$$

$$w_{c5} = w_{d5} - \ell_{34}\beta_{c5} \quad (2.5.2.29)$$

$$w_{c6} = w_{d6} - \ell_{34}\beta_{c6} \quad (2.5.2.30)$$

$$\gamma_{d1} = \gamma_{c1} \quad (2.5.2.31)$$

$$\gamma_{d2} = \gamma_{c2} \quad (2.5.2.32)$$

$$\gamma_{d3} = \gamma_{c3} \quad (2.5.2.33)$$

$$\gamma_{d4} = \gamma_{c4} \quad (2.5.2.34)$$

$$\gamma_{d5} = \gamma_{c5} \quad (2.5.2.35)$$

$$\gamma_{d6} = \gamma_{c6} \quad (2.5.2.36)$$

(C) Additional Constraints

The assumption of pure rolling yields the following constraints:

$$\beta_{d1} = - \frac{1}{\ell_{20}} u_{d1} \quad (2.5.2.37)$$

$$\beta_{d2} = - \frac{1}{\ell_{20}} u_{d2} \quad (2.5.2.38)$$

$$\beta_{d3} = - \frac{1}{\ell_{20}} u_{d3} \quad (2.5.2.39)$$

$$\beta_{d4} = - \frac{1}{\ell_{20}} u_{d4} \quad (2.5.2.40)$$

$$\beta_{d5} = - \frac{1}{\ell_{20}} u_{d5} \quad (2.5.2.41)$$

$$\beta_{d6} = - \frac{1}{\ell_{20}} u_{d6} \quad (2.5.2.42)$$

2.5.3 The Transformation Matrix [D]

In this section we define the transformation matrix D which gives the relation between all the variables and the independent variables based on the equations of constraints. The matrix D is defined in the following pages where:

$\rho_1 = \ell_{24} - \ell_{34}$ $\qquad$ $\rho_2 = \ell_{26} - \ell_{34}$

$\rho_3 = -(\ell_{23} + \ell_{34})$ $\qquad$ $\rho_4 = \ell_{23} + \ell_{34}$

$\rho_5 = -\ell_{26} + \ell_{34}$ $\qquad$ $\rho_6 = -\ell_{24} + \ell_{34}$

$\rho_7 = -\dfrac{1}{\ell_{20}}$ $\qquad$ $\rho_8 = -\dfrac{\ell_{21}}{\ell_{20}}$

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
		u_a	v_a	w_a	α_a	β_a	γ_a	u_{br}	v_{br}	w_{br}	α_{br}	β_{br}	γ_{br}	u_{bf}	v_{bf}	w_{bf}	α_{bf}	β_{bf}	γ_{bf}	β_{c1}	w_{d1}	α_{d1}	β_{c2}	w_{d2}	α_{d2}	β_{c3}	w_{d3}	α_{d3}	β_{c4}	w_{d4}	α_{d4}	β_{c5}	w_{d5}	α_{d5}	β_{c6}	w_{d6}	α_{d6}
1	u_a	1																																			
2	v_a		1																																		
3	w_a			1																																	
4	α_a				1																																
5	β_a					1																															
6	γ_a						1																														
7	u_{br}							1																													
8	v_{br}								1																												
9	w_{br}									1																											
10	α_{br}										1																										
11	β_{br}											1																									
12	γ_{br}												1																								
13	u_{bf}													1																							
14	v_{bf}														1																						
15	w_{bf}															1																					
16	α_{bf}																1																				
17	β_{bf}																	1																			
18	γ_{bf}																		1																		
19	v_{c1}								1		$-\ell_{21}$		ρ_1																								
20	w_{c1}																			ℓ_{34}	1																
21	β_{c1}																			1																	
22	γ_{c1}												1																								
23	u_{d1}							1				ℓ_{21}																									
24	v_{d1}								1		$-\ell_{21}$		ℓ_{24}																								
25	w_{d1}																				1																
26	α_{d1}																					1															
27	β_{d1}							ρ_7				ρ_8																									
28	γ_{d1}												1																								
29	v_{c2}								1		$-\ell_{21}$		ρ_2																								
30	w_{c2}																						ℓ_{34}	I													
31	β_{c2}																						1														
32	γ_{c2}												1																								
33	u_{d2}							1				ℓ_{21}																									
34	v_{d2}								1		$-\ell_{21}$		ℓ_{26}																								
35	w_{d2}																							1													
36	α_{d2}																								1												
37	β_{d2}							ρ_7				ρ_8																									
38	γ_{d2}												1																								

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
		u_a	v_a	w_a	α_a	β_a	γ_a	u_{br}	v_{br}	w_{br}	α_{br}	β_{br}	γ_{br}	u_{bf}	v_{bf}	w_{bf}	α_{bf}	β_{bf}	γ_{bf}	β_{c1}	w_{d1}	α_{d1}	β_{c2}	w_{d2}	α_{d2}	β_{c3}	w_{d3}	α_{d3}	β_{c4}	w_{d4}	α_{d4}	β_{c5}	w_{d5}	α_{d5}	β_{c6}	w_{d6}	α_{d6}
39	v_{c3}								1		$-\ell_{21}$		ρ_3																								
40	w_{c3}																									ℓ_{34}	1										
41	β_{c3}																									1											
42	γ_{c3}												1																								
43	u_{d3}							1				ℓ_{21}																									
44	v_{d3}								1		$-\ell_{21}$		$-\ell_{23}$																								
45	w_{d3}																										1										
46	α_{d3}																											1									
47	β_{d3}							ρ_7				ρ_8																									
48	γ_{d3}												1																								
49	v_{c4}														1		$-\ell_{21}$		ρ_4																		
50	w_{c4}																												$-\ell_{34}$	1							
51	β_{c4}																												1								
52	γ_{c4}																		1																		
53	u_{d4}													1				ℓ_{21}																			
54	v_{d4}														1		$-\ell_{21}$		ℓ_{23}																		
55	w_{d4}																													1							
56	α_{d4}																														1						
57	β_{d4}													ρ_7				ρ_8																			
58	γ_{d4}																		1																		
59	v_{c5}														1		$-\ell_{21}$		ρ_5																		
60	w_{c5}																															$-\ell_{34}$	1				
61	β_{c5}																															1					
62	γ_{c5}																		1																		
63	u_{d5}													1				ℓ_{21}																			
64	v_{d5}														1		$-\ell_{21}$		$-\ell_{26}$																		
65	w_{d5}																																1				
66	α_{d5}																																	1			
67	β_{d5}													ρ_7				ρ_8																			
68	γ_{d5}																		1																		
69	v_{c6}														1		$-\ell_{21}$		ρ_6																		
70	w_{c6}																																		$-\ell_{34}$	1	
71	β_{c6}																																				
72	γ_{c6}																		1																		
73	u_{d6}													1				ℓ_{21}																			
74	v_{d6}														1		$-\ell_{21}$		$-\ell_{24}$																		
75	w_{d6}																																			1	
76	α_{d6}																																				1
77	β_{d6}													ρ_7				ρ_8																			
78	γ_{d6}																		1																		

2.6 Derivation of the Equations of Motion for the "Moving" Vehicle

In making the transition from the "stationary" vehicle to the "moving" vehicle, two major effects have to be considered:

a) creep and the corresponding creep forces between wheels and rails

b) conicity of wheel treads.

In railway vehicles non-conservative forces arise from the phenomenon of creep. When a wheel exerts a tractive force, the distance travelled by the wheel is less than the pure rolling displacement. This effect is known as longitudinal creep-creep is the state between pure rolling and pure sliding. Also, if a wheel is rolling and a lateral force is applied, a lateral displacement of the wheel occurs which is proportional to the distance travelled.

The equations of motion are set up in a similar manner as the previous section with the introduction of these creep effects and the "gravitational stiffness" effect. This effect arises because the lateral component of the change in the normal reactions between wheel and rail as a wheelset is displaced, is proportional to the lateral displacement.

Evaluation of the parameters of the "moving" vehicle (creep coefficients, conicity of wheel treads,...etc) is given in detail in Appendix IV. The following notation is used for the creep coefficients and creep forces:

f_1 - coefficient relating longitudinal creep force to longitudinal creep,

f_2 - coefficient relating lateral creep force to lateral creep,

f_3 - coefficient relating creep torque to rotational creep,

f_{23} - coupling coefficient relating creep torque to lateral creep and lateral force to rotational creep,

U_{ir} - creep force in the longitudinal direction for the right wheel number i,

$U_{i\ell}$ - creep force in the longitudinal direction for the left wheel number i,

V_{ir} - creep force in the lateral direction for the right wheel number i,

$V_{i\ell}$ - creep force in the lateral direction for the left wheel number i,

T_{ir} - creep torque (about the vertical axis) for the right wheel number i,

$T_{i\ell}$ - creep torque (about the vertical axis) for the left wheel number i

where i = 1, 2, 3, 4, 5, 6 from rear to front.

It should be noted that the longitudinal creep forces do not include the steady state propelling force, they are variations from this force.

The equations of motion for the "moving" vehicle are given in detail in Appendix III. Notation used is the same as for the "stationary" vehicle.

The system of differential equations is written in the following form:

$$\underset{78x78}{[A]}\ \underset{78x1}{\{\ddot{Z}\}} + \underset{78x78}{[C]}\ \underset{78x1}{\{\dot{Z}\}} + \underset{78x78}{[B]}\ \underset{78x1}{\{Z\}} + \underset{78x1}{\{F\}} = 0 \qquad (2.6.1)$$

where A,B, and C are the inertia, the stiffness and the damping matrices, respectively,

and F is a column vector of reactions.

Eliminating the reactions by the method of substitution we get a system of 36 equations in 78 unknowns (only 36 of the unknowns are independent).

2.6.1 Elimination of Reactions

The column matrix of reactions is given by:

VAR.	EQN.NO.	REACTION
u_a	1	0
v_a	2	0
w_a	3	0
α_a	4	0
β_a	5	0
γ_a	6	0
u_{br}	7	$-(RA_1 + RA_2 + RA_3)$
v_{br}	8	$-(RB_1 + RB_2 + RB_3)$
w_{br}	9	0
α_{br}	10	$\ell_{21}(RB_1 + RB_2 + RB_3)$

VAR.	EQN.NO.	REACTION
β_{br}	11	$-\ell_{21}(RA_1 + RA_2 + RA_3)$
γ_{br}	12	$-\ell_{24}(RB_1) - \ell_{26}(RB_2) + \ell_{23}(RB_3) - (RM_1 + RM_2 + RM_3)$
u_{bf}	13	$-(RA_4 + RA_5 + RA_6)$
v_{bf}	14	$-(RB_4 + RB_5 + RB_6)$
w_{bf}	15	0
α_{bf}	16	$+\ell_{21}(RB_4 + RB_5 + RB_6)$
β_{bf}	17	$-\ell_{21}(RA_4 + RA_5 + RA_6)$
γ_{bf}	18	$-\ell_{23}(RB_4) + \ell_{26}(RB_5) + \ell_{24}(RB_6) - (RM_4 + RM_5 + RM_6)$
v_{c1}	19	$-(RV_1)$
w_{c1}	20	$+(RW_1)$
β_{c1}	21	$-\ell_{34}(RW_1)$
γ_{c1}	22	$-\ell_{34}(RV_1) - (R\gamma_1)$
u_{d1}	23	(RA_1)
v_{d1}	24	$(RB_1) + (RV_1) + \theta_o(N_{r1} - N_{\ell 1})$
w_{d1}	25	$-(RW_1)$
α_{d1}	26	$(-\frac{\ell_{18}}{2} - \ell_{20}\,\theta_o)(N_{r1} - N_{\ell 1})$
β_{d1}	27	0
γ_{d1}	28	$(RM_1) + (R\gamma_1)$
v_{c2}	29	$-(RV_2)$

VAR.	EQN.NO.	REACTION
w_{c2}	30	$+(RW_2)$
β_{c2}	31	$-\ell_{34}(RW_2)$
γ_{c2}	32	$-\ell_{34}(RV_2) - (R\gamma_2)$
u_{d2}	33	(RA_2)
v_{d2}	34	$(RB_2) + (RV_2) + \theta_o(N_{r2} - N_{\ell 2})$
w_{d2}	35	$-(RW_2)$
α_{d2}	36	$(\frac{\ell_{18}}{2} - \ell_{20}\,\theta_o)(N_{r2} - N_{\ell 2})$
β_{d2}	37	0
γ_{d2}	38	$(RM_2) + (R\gamma_2)$
v_{c3}	39	$-(RV_3)$
w_{c3}	40	$+(RW_3)$
β_{c3}	41	$-\ell_{34}(RW_3)$
γ_{c3}	42	$-\ell_{34}(RV_3) - (R\gamma_3)$
u_{d3}	43	(RA_3)
v_{d3}	44	$(RB_3) + (RV_3) + \theta_o(N_{r3} - N_{\ell 3})$
w_{d3}	45	$-(RW_3)$
α_{d3}	46	$(\frac{\ell_{18}}{2} - \ell_{20}\theta_o)(N_{r3} - N_{\ell 3})$
β_{d3}	47	0
γ_{d3}	48	$(RM_3) + (R\gamma_3)$

VAR.	EQN.NO.	REACTION
v_{c4}	49	$-(RV_4)$
w_{c4}	50	$+(RW_4)$
β_{c4}	51	$+\ell_{34}(RW_4)$
γ_{c4}	52	$+\ell_{34}(RV_4) - (R\gamma_4)$
u_{d4}	53	(RA_4)
v_{d4}	54	$(RB_4) + (RV_4) + \theta_o(N_{r4} - N_{\ell 4})$
w_{d4}	55	$-(RW_4)$
α_{d4}	56	$(\frac{\ell_{18}}{2} - \ell_{20}\theta_o)(N_{r3} - N_{\ell 3})$
β_{d4}	57	0
γ_{d4}	58	$(RM_4) + (R\gamma_4)$
v_{c5}	59	$-(RV_5)$
w_{c5}	60	$+(RW_5)$
β_{c5}	61	$+\ell_{34}(RW_5)$
γ_{c5}	62	$+\ell_{34}(RV_5) - (R\gamma_5)$
u_{d5}	63	(RA_5)
v_{d5}	64	$(RB_5) + (RV_5) + \theta_o(N_{r5} - N_{\ell 5})$
w_{d5}	65	$-(RW_5)$
α_{d5}	66	$(\frac{\ell_{18}}{2} - \ell_{20}\theta_o)(N_{r5} - N_{\ell 5})$
β_{d5}	67	0
γ_{d5}	68	$(RM_5) + R\gamma_5)$
v_{c6}	69	$-(RV_6)$
w_{c6}	70	$+(RW_6)$

VAR.	EQN.NO.	REACTION
β_{c6}	71	$+\ell_{34}(RW_6)$
γ_{c6}	72	$+\ell_{34}(RV_6) - (R\gamma_6)$
u_{d6}	73	(RA_6)
v_{d6}	74	$(RB_6) + (RV_6) + \theta_o(N_{r6} - N_{\ell 6})$
w_{d6}	75	$-(RW_6)$
α_{d6}	76	$(\frac{\ell_{18}}{2} - \ell_{20}\theta_o)(N_{r6} - N_{\ell 6})$
β_{d6}	77	0
γ_{d6}	78	$(RM_6) + (R\gamma_6)$

Elimination of the reactions is done in two steps:

(A) <u>Elimination of $(N_{ri}-N_{\ell i})(i = 1,...,6)$</u>

To do this, the following procedure, which is implemented in the computer program, is followed:

Equation 24 = Equation 24 + ρ · Equation 26

Equation 34 = Equation 34 + ρ · Equation 36

Equation 44 = Equation 44 + ρ · Equation 46

Equation 54 = Equation 54 + ρ · Equation 56

Equation 64 = Equation 64 + ρ · Equation 66

Equation 74 = Equation 74 + ρ · Equation 76

Where $\rho = \dfrac{-\theta_o}{\frac{\ell_{18}}{2} - \ell_{20}\theta_o}$

(B) Elimination of the Internal Reactions

To eliminate the internal reactions by substitution the procedure described below is used; for simplicity we designate the equations by their corresponding number as given in Appendix III.

$1 = 1$

$2 = 2$

$3 = 3$

$4 = 4$

$5 = 5$

$6 = 6$

$7 = 7 + 23 + 33 + 43$

$8 = 8 + (19+24) + (29+34) + (39+44)$

$9 = 9$

$10 = 10 - \ell_{21}[(19+24) + (29+34) + (39+44)$

$11 = 11 + \ell_{21}(23 + 33 + 43)$

$$12 = 12 + \ell_{24}[(19+24) + \ell_{26}(29+34) - \ell_{23}(39+44)] - \ell_{34}[(19+29+39) + (22+32+42) + (28+38+48)]$$

$13 = 13 + (53 + 63 + 73)$

$14 = 14 + (49+54) + (59+64) + (69+74)$

$15 = 15$

$16 = 16 - \ell_{21}[(49+54) + (59+64) + (69+74)]$

$17 = 17 + \ell_{21}(53 + 63 + 73)$

$$18 = 18 + [\ell_{23}(49+54) - \ell_{26}(59+64) - \ell_{24}(69+74)] + \ell_{34}[(49+59+69) + (52+62+72) + (58+68+78)]$$

$$19 = 21 + \ell_{34}(20)$$

$$20 = 25 + 20$$

$$21 = 27$$

$$22 = 31 + \ell_{34}(30)$$

$$23 = 35 + 30$$

$$24 = 37$$

$$25 = 41 + \ell_{34}(40)$$

$$26 = 45 + 40$$

$$27 = 47$$

$$28 = 51 - \ell_{34}(50)$$

$$29 = 55 + 50$$

$$30 = 57$$

$$31 = 61 - \ell_{34}(60)$$

$$32 = 65 + 60$$

$$33 = 67$$

$$34 = 71 - \ell_{34}(70)$$

$$35 = 75 + 70$$

$$36 = 77$$

In the computer program, this procedure is applied to the mass, the stiffness and the damping matrices successively and the expression (2.6.1) becomes

$$\underset{36\text{x}78}{[A]}\ \underset{78\text{x}1}{\{\ddot{Z}\}} + \underset{36\text{x}78}{[C]}\ \underset{78\text{x}1}{\{\dot{Z}\}}\ \underset{36\text{x}78}{[B]}\ \underset{78\text{x}1}{\{Z\}} = 0 \tag{2.6.2}$$

To eliminate the dependent variables, we use the transformation matrix [D] which gives the relation between all the variables and the independent variables. [D] is based on the equations of constraints for the "moving" vehicle. Therefore (2.6.2) becomes

$$\underset{36\text{x}78}{[A]}\ \underset{78\text{x}36}{[D]}\ \underset{36\text{x}1}{\{\ddot{X}\}} + \underset{36\text{x}78}{[C]}\ [D]\ \underset{36\text{x}1}{\{\dot{X}\}} + \underset{36\text{x}78}{[B]}\ [D]\ \underset{36\text{x}1}{\{X\}} = 0$$

Which after carrying the matrix multiplication gives the following system of thirty-six equations in thirty-six independent variables.

$$\underset{36\text{x}36}{[AD]}\ \underset{36\text{x}1}{\{\ddot{X}\}} + \underset{36\text{x}36}{[CD]}\ \underset{36\text{x}1}{\{\dot{X}\}} + \underset{36\text{x}36}{[BD]}\ \underset{36\text{x}1}{\{X\}} = 0 \tag{2.6.3}$$

It is worth noting here that by introducing the non-conservative forces which are due to the phenomenon of creep, the inertia, damping and stiffness matrices are no longer symmetric and there is thus no reason for the reduced matrices (AD, BD, CD) to be symmetric.

2.6.2 Equations of Constraints

(A) Constraints between Wheelsets and Frames

$$u_{d1} = u_{br} + \ell_{21}\beta_{br} \tag{2.6.2.1}$$

$$u_{d2} = u_{br} + \ell_{21}\beta_{br} \tag{2.6.2.2}$$

$$u_{d3} = u_{br} + \ell_{21}\beta_{br} \tag{2.6.2.3}$$

$$u_{d4} = u_{bf} + \ell_{21}\beta_{bf} \qquad (2.6.2.4)$$

$$u_{d5} = u_{bf} + \ell_{21}\beta_{bf} \qquad (2.6.2.5)$$

$$u_{d6} = u_{bf} + \ell_{21}\beta_{bf} \qquad (2.6.2.6)$$

$$v_{d1} = v_{br} - \ell_{21}\alpha_{br} + \ell_{24}\gamma_{br} \qquad (2.6.2.7)$$

$$v_{d2} = v_{br} - \ell_{21}\alpha_{br} + \ell_{26}\gamma_{br} \qquad (2.6.2.8)$$

$$v_{d3} = v_{br} - \ell_{21}\alpha_{br} - \ell_{23}\gamma_{br} \qquad (2.6.2.9)$$

$$v_{d4} = v_{bf} - \ell_{21}\alpha_{bf} + \ell_{23}\gamma_{bf} \qquad (2.6.2.10)$$

$$v_{d5} = v_{bf} - \ell_{21}\alpha_{bf} - \ell_{26}\gamma_{bf} \qquad (2.6.2.11)$$

$$v_{d6} = v_{bf} - \ell_{21}\alpha_{bf} - \ell_{24}\gamma_{bf} \qquad (2.6.2.12)$$

$$\gamma_{d1} = \gamma_{br} \qquad (2.6.2.13)$$

$$\gamma_{d2} = \gamma_{br} \qquad (2.6.2.14)$$

$$\gamma_{d3} = \gamma_{br} \qquad (2.6.2.15)$$

$$\gamma_{d4} = \gamma_{bf} \qquad (2.6.2.16)$$

$$\gamma_{d5} = \gamma_{bf} \qquad (2.6.2.17)$$

$$\gamma_{d6} = \gamma_{bf} \qquad (2.6.2.18)$$

(B) Constraints Between Motors and Wheelsets

$u_{d1} = u_{c1}$ $u_{d4} = u_{c4}$

$u_{d2} = u_{c2}$ $u_{d5} = u_{c5}$

$u_{d3} = u_{c3}$ $u_{d6} = u_{c6}$

$\alpha_{d1} = \alpha_{c1}$ $\alpha_{d4} = \alpha_{c4}$

$\alpha_{d2} = \alpha_{c2}$ $\alpha_{d5} = \alpha_{c5}$

$\alpha_{d3} = \alpha_{c3}$ $\alpha_{d6} = \alpha_{c6}$

These relations are already satisfied by combining the wheelset and motor equations

$$v_{c1} = v_{br} - \ell_{21}\alpha_{br} + \ell_{24}\gamma_{br} - \ell_{34}\gamma_{c1} \quad (2.6.2.19)$$

$$v_{c2} = v_{br} - \ell_{21}\alpha_{br} + \ell_{26}\gamma_{br} - \ell_{34}\gamma_{c2} \quad (2.6.2.20)$$

$$v_{c3} = v_{br} - \ell_{21}\alpha_{br} - \ell_{23}\gamma_{br} - \ell_{34}\gamma_{c3} \quad (2.6.2.21)$$

$$v_{c4} = v_{bf} - \ell_{21}\alpha_{bf} + \ell_{23}\gamma_{bf} + \ell_{34}\gamma_{c4} \quad (2.6.2.22)$$

$$v_{c5} = v_{bf} - \ell_{21}\alpha_{bf} - \ell_{26}\gamma_{bf} + \ell_{34}\gamma_{c5} \quad (2.6.2.23)$$

$$v_{c6} = v_{bf} - \ell_{21}\alpha_{bf} - \ell_{24}\gamma_{bf} + \ell_{34}\gamma_{c6} \quad (2.6.2.24)$$

$$w_{c1} = w_{d1} + \ell_{34}\beta_{c1} \quad (2.6.2.25)$$

$$w_{c2} = w_{d2} + \ell_{34}\beta_{c2} \quad (2.6.2.26)$$

$$w_{c3} = w_{d3} + \ell_{34}\beta_{c3} \quad (2.6.2.27)$$

$$w_{c4} = w_{d4} - \ell_{34}\beta_{c4} \qquad (2.6.2.28)$$

$$w_{c5} = w_{d5} - \ell_{34}\beta_{c5} \qquad (2.6.2.29)$$

$$w_{c6} = w_{d6} - \ell_{34}\beta_{c6} \qquad (2.6.2.30)$$

$$\gamma_{d1} = \gamma_{c1} \qquad (2.6.2.31)$$

$$\gamma_{d2} = \gamma_{c2} \qquad (2.6.2.32)$$

$$\gamma_{d3} = \gamma_{c3} \qquad (2.6.2.33)$$

$$\gamma_{d4} = \gamma_{c4} \qquad (2.6.2.34)$$

$$\gamma_{d5} = \gamma_{c5} \qquad (2.6.2.35)$$

$$\gamma_{d6} = \gamma_{c6} \qquad (2.6.2.36)$$

(C) Additional Constraints

The constraint between the wheelsets and rails (due to the conicity of the wheels) yields the following constraints:

$$\alpha_{d1} = -\frac{2\lambda}{\ell_{18}} v_{d1} \qquad (2.6.2.37)$$

$$\alpha_{d2} = -\frac{2\lambda}{\ell_{18}} v_{d2} \qquad (2.6.2.38)$$

$$\alpha_{d3} = -\frac{2\lambda}{\ell_{18}} v_{d3} \qquad (2.6.2.39)$$

$$\alpha_{d4} = - \frac{2\lambda}{\ell_{18}} v_{d4} \qquad (2.6.2.40)$$

$$\alpha_{d5} = - \frac{2\lambda}{\ell_{18}} v_{d5} \qquad (2.6.2.41)$$

$$\alpha_{d6} = - \frac{2\lambda}{\ell_{18}} v_{d6} \qquad (2.6.2.42)$$

Where v_{d1} (i = 1,...,6) is the relative lateral displacement between the wheelsets and the rail. Contact must be maintained between the wheels and rails, requiring that $w_{di} = 0$ (i = 1, ...,6).

2.6.3 The Transformation Matrix [D]

This matrix gives the relation between all the variables and the independent variables, based on the equations of constraints for the "moving" vehicle. The matrix [D] is defined in the following pages where

$$\rho_1 = - \frac{2\lambda}{\ell_{18}} \qquad \rho_2 = \frac{2\lambda\ \ell_{24}}{\ell_{18}}$$

$$\rho_3 = - \frac{2\lambda\ \ell_{24}}{\ell_{18}} \qquad \rho_4 = \ell_{24} - \ell_{34}$$

$$\rho_5 = \ell_{26} - \ell_{34} \qquad \rho_6 = -(\ell_{23} + \ell_{34})$$

$$\rho_7 = \ell_{23} + \ell_{24} \qquad \rho_8 = -\ell_{26} + \ell_{33}$$

$$\rho_9 = -\ell_{24} + \ell_{34}$$

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
		u_a	v_a	w_a	α_a	β_a	γ_a	u_{br}	v_{br}	w_{br}	α_{br}	β_{br}	γ_{br}	u_{bf}	v_{bf}	w_{bf}	α_{bf}	β_{bf}	γ_{bf}	β_{c1}	w_{d1}	β_{d1}	β_{c2}	w_{d2}	β_{d2}	β_{c3}	w_{d3}	β_{d3}	β_{c4}	w_{d4}	β_{d4}	β_{c5}	w_{d5}	β_{d5}	β_{c6}	w_{d6}	β_{d6}
1	u_a	1																																			
2	v_a		1																																		
3	w_a			1																																	
4	α_a				1																																
5	β_a					1																															
6	γ_a						1																														
7	u_{br}							1																													
8	v_{br}								1																												
9	w_{br}									1																											
10	α_{br}										1																										
11	β_{br}											1																									
12	γ_{br}												1																								
13	u_{bf}													1																							
14	v_{bf}														1																						
15	w_{bf}															1																					
16	α_{bf}																1																				
17	β_{bf}																	1																			
18	γ_{bf}																		1																		
19	v_{c1}								1		$-\ell_{21}$		ρ_4																								
20	w_{c1}																			ℓ_{34}	1																
21	β_{c1}																			1																	
22	γ_{c1}												1																								
23	u_{d1}							1				ℓ_{21}																									
24	v_{d1}								1		$-\ell_{21}$		ℓ_{24}																								
25	w_{d1}																				1																
26	α_{d1}								ρ_1		ρ_2		ρ_3																								
27	β_{d1}																					1															
28	γ_{d1}												1																								
29	v_{c2}								1		$-\ell_{21}$		ρ_5																								
30	w_{c2}																						ℓ_{34}	1													
31	β_{c2}																						1														
32	γ_{c2}												1																								
33	u_{d2}							1				ℓ_{21}																									
34	v_{d2}								1		$-\ell_{21}$		ℓ_{26}																								
35	w_{d2}																							1													
36	α_{d2}								ρ_1		ρ_2		ρ_3																								
37	β_{d2}																								1												
38	γ_{d2}												1																								

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
		u_a	v_a	w_a	α_a	β_a	γ_a	u_{br}	v_{br}	w_{br}	α_{br}	β_{br}	γ_{br}	u_{bf}	v_{bf}	w_{bf}	α_{bf}	β_{bf}	γ_{bf}	β_{c1}	w_{d1}	β_{d1}	β_{c2}	w_{d2}	β_{d2}	β_{c3}	w_{d3}	β_{d3}	β_{c4}	w_{d4}	β_{d4}	β_{c5}	w_{d5}	β_{d5}	β_{c6}	w_{d6}	β_{d6}
39	v_{c3}								1		$-\ell_{21}$		ρ_6																								
40	w_{c3}																									ℓ_{34}	1										
41	β_{c3}																									1											
42	γ_{c3}												1																								
43	u_{d3}							1				ℓ_{21}																									
44	v_{d3}								1		$-\ell_{21}$		$-\ell_{23}$																								
45	w_{d3}																										1										
46	α_{d3}								ρ_1		ρ_2		ρ_3																								
47	β_{d3}																											1									
48	γ_{d3}												1																								
49	v_{c4}														1		$-\ell_{21}$		ρ_7																		
50	w_{c4}																												$-\ell_{34}$	1							
51	β_{c4}																												1								
52	γ_{c4}																		1																		
53	u_{d4}													1				ℓ_{21}																			
54	v_{d4}														1		$-\ell_{21}$		ℓ_{23}																		
55	w_{d4}																													1							
56	α_{d4}														ρ_1		ρ_2		ρ_3																		
57	β_{d4}																														1						
58	γ_{d4}																		1																		
59	v_{c5}														1		$-\ell_{21}$		ρ_8																		
60	w_{c5}																															$-\ell_{34}$	1				
61	β_{c5}																															1					
62	γ_{c5}																		1																		
63	u_{d5}													1				ℓ_{21}																			
64	v_{d5}														1		$-\ell_{21}$		$-\ell_{26}$																		
65	w_{d5}																																1				
66	α_{d5}														ρ_1		ρ_2		ρ_3																		
67	β_{d5}																																	1			
68	γ_{d5}																		1																		
69	v_{c6}														1		$-\ell_{21}$		ρ_9																		
70	w_{c6}																																		$-\ell_{34}$	1	
71	β_{c6}																																		1		
72	γ_{c6}																		1																		
73	u_{d6}													1				ℓ_{21}																			
74	v_{d6}														1		$-\ell_{21}$		$-\ell_{24}$																		
75	w_{d6}																																			1	
76	α_{d6}														ρ_1		ρ_2		ρ_3																		
77	β_{d6}																																				1
78	γ_{d6}																		1																		

CHAPTER 3

STEADY STATE RESPONSE TO RAIL IRREGULARITIES

3.1 Steady State Response of a General Multidegree of Freedom System Using the Method of Complex Algebra

The use of complex algebra simplifies the procedure for solving the system of differential equations. For forced vibration where the impressed force is harmonic, the steady state solution is also harmonic with the same frequency. In the presence of damping in the system, phase-differences between the resulting motions and the input excitation exist.

Let us first consider a single degree of freedom system with viscous damping. The impressed force and the resulting displacement are the vectors F_o and X, the latter lagging the former by the angle ϕ, as shown in Figure 4, and the two rotating together with a common angular speed ω.

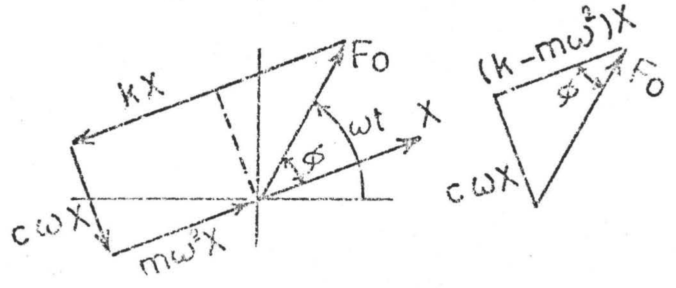

Figure 4: Vector Representation of Forced Vibration with Viscous Damping

If such vectors are represented by the exponential function, we can write

$$F = F_o\, e^{i\omega t} \tag{3.1.1}$$

$$x = X\, e^{i(\omega t-\phi)} = X\, e^{-i\phi}\, e^{i\omega t} \tag{3.1.2}$$

where F_o and X are absolute values equal to the length of the vectors.

It is also possible to write equation (3.1.2) as

$$x = \bar{X}\, e^{i\omega t} \tag{3.1.3}$$

where $\bar{X} = X\, e^{-i\phi}$ is the complex amplitude designating its angular position with respect to F_o.

Applying this type of complex notation to the forced vibration of a viscously damped spring-mass system excited by a harmonic force $F_o \sin \omega t$, we can write the differential equation of motion in the form

$$m\ddot{x} + c\dot{x} + kx = F_o\, e^{i\omega t} \tag{3.1.4}$$

Letting $x = \bar{X}\, e^{i\omega t}$, the preceding equation becomes

$$(-m\omega^2 + i\omega c + k)\, \bar{X}\, e^{i\omega t} = F_o\, e^{i\omega t} \tag{3.1.5}$$

The complex amplitude is then determined as

$$\bar{X} = \frac{F_o}{(k-m\omega^2) + i\omega c} = \frac{F_o\, e^{-i\phi}}{\sqrt{(k-m\omega^2)^2 + (\omega c)^2}} \tag{3.1.6}$$

From which the amplitude and phase are found to be:

$$X = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (\omega c)^2}} \tag{3.1.7}$$

$$\phi = \tan^{-1} \frac{\omega c}{k - m\omega^2} \tag{3.1.8}$$

The quantity $F_o\ e^{i\omega t}$ could be called a "complex force" and the quantity x a "complex displacement" related as in equation (3.1.5).

$$x = \frac{1}{(k - m\omega^2) + i\omega c} F_o\ e^{i\omega t} \tag{3.1.9}$$

which states that

Complex Displacement = α x *complex force,*

where the expression

$$\alpha = \frac{1}{(k - m\omega^2) + i\omega c},$$

relates the complex displacement and force.

We shall now consider a system having a number of degrees of freedom and find that the results obtained for the system having one degree of freedom may be extended quite naturally. It will be shown that a convenient way of allowing for this is to use matrices having complex elements. There is no reason, in matrix theory, why the elements should not be complex, provided that the mathematical rules for manipulating complex numbers are observed. Hence for a multi-degree of freedom system the equations of motions could be written in the form:

$$[M]\ \{\ddot{X}\} + [C]\ \{\dot{X}\} + [K]\ \{X\} = \{F\} \qquad (3.1.10)$$

where F and X are the vectors representing the complex forces and complex displacements respectively.

Equation (3.1.10) can be solved by means of the trial solution:

$$\{X\} = \{\bar{X}\}\ e^{i\omega t} \qquad (3.1.11)$$

where column matrix $\bar{X}$ represents the complex amplitudes. Knowing that

$$\{F\} = \{F_o\}\ e^{i\omega t} \qquad (3.1.12)$$

leads us to the equation

$$[(K - \omega^2 M) + i\omega C]\ \{\bar{X}\} = \{F_o\} \qquad (3.1.13)$$

The quantity in square brackets is a square matrix whose elements are complex. Let us denote this square matrix by Z. Provided that Z is non-singular it will have an inverse Z^{-1} and the solution of (3.1.13) will be:

$$\{\bar{X}\} = [Z^{-1}]\ \{F_o\} \qquad (3.1.14)$$

It should be noted that the derivation was done for a multi-degree of freedom system related to an arbitrary set of periodic forces all having the same frequency. The use of complex quantitites allows us to relax the latter restriction. If the system is acted upon by forces of different frequencies then the resultant displacement vector will be the sum of the displacement vectors due to the forces at each frequency taking one frequency at a time. In this study, however, the excitation forces have the same frequency.

The method outlined above provides the simplest way of finding the displacements due to an applied set of forces with or without damping.

For a system having a large number of degrees of freedom the method is ideally suitable for solution on an electronic digital computer which has the important capability of automatically observing the rules for manipulating complex numbers.

Although the result given by equation (3.1.14) is simple in principle, it should be pointed out that even for a small number of degrees of freedom the inversion of a complex matrix is a very lengthy and tedious process requiring the use of a digital computer.

3.2 Steady State Response for the Simplified Model

The simplified model was intended to give an approximate response to track irregularities. Irregularity of the track is assumed to be sinusoidal and the track width assumed constant.

As described before, for the purpose of this simplified investigation, the following assumptions are made:

1. Wheelsets and motors are rigidly connected to the frames,

2. The primary suspension has negligible effect on the dynamic behaviour of the system,

3. Creep forces have negligible effect on the steady state response of the system.

The steady state response for the body is found in the case of two out of phase displacements excitation at the two bases (the trucks). The forcing vector (the R.H.S. in equation 3.2.1) is derived for the case of input excitation in both the lateral and the vertical direction. To study the steady state response due to lateral track irregularities only, the elements of the input displacements vector will simply be replaced by zeros except for lateral displacements.

Equations of motion are given in detail in Appendix II. The system of equations has the form:

$$[A]\{\ddot{X}\} + [C]\{\dot{X}\} + [B]\{X\} = [C_{bf}]\{\dot{y}_{bf}\} + [K_{bf}]\{y_{bf}\} + [C_{br}]\{\dot{y}_{br}\} + [K_{br}]\{y_{br}\} \qquad (3.2.1)$$

where:

X - complex vector of unknown displacements (6x1)

y_{bf} - excitation displacement vector at the front base (6x1 complex)

y_{br} - excitation displacement vector at the rear base (6x1 complex)

M - inertia matrix (6x6 real diagonal matrix)

K - stiffness matrix (6x6 real symmetric matrix)

C - damping matrix (6x6 real symmetric matrix)

K_{bf} - stiffness matrix for the front frame (6x2)

C_{bf} - damping matrix for the front frame (6x2)

K_{br} - stiffness matrix for the rear frame (6x2)

C_{br} - damping matrix for the rear frame (6x2)

Using the method described in section 3.1 for solving (3.2.1) we get:

$$[(K - \omega^2 M) + i\omega C] \{\bar{X}\} = [K_{bf} + i\omega\ C_{bf}]\ \{y_{bf}\} + [K_{br} + i\omega\ C_{br}]\{\bar{y}_{br}\}$$

i.e.:

$$[Z]\ \{\bar{X}\} = \{F_o\} \tag{3.2.2}$$

in which the quantities between brackets are complex square matrices, this could be rewritten as

$$\{\bar{X}\} = [Z^{-1}]\{F_o\} \tag{3.2.3}$$

in which F_o, Z^{-1} and $\bar{X}$ are complex.

To get the steady state response equation (3.2.3) is solved for increasing values of the input frequency (ω), knowing the amplitudes and phase shifts of the elements of the forcing vector.

3.3 Steady State Response for the Full Model - "Stationary" Vehicle

The steady state response for the full model - "stationary" vehicle is obtained due to a lateral displacement input of the wheels. This excitation is equivalent to the input displacement that we would obtain if the vehicle was moving on an irregular track without relative displacements between the wheels and the rails. In other words, for the "stationary" vehicle certain generalized coordinates are "locked". These coordinates relate to motions of the wheelsets.

For the full "stationary" model the equations of motion are given by equation (2.5.3):

$$\underset{36x36}{[A]}\ \underset{36x1}{\{\ddot{x}\}} + \underset{36x36}{[C]}\ \underset{36x1}{\{\dot{x}\}} + \underset{36x36}{[B]}\ \underset{36x1}{\{x\}} = 0$$

For the "stationary" vehicle, the variables defining

a) the lateral displacements of the frames (v_{bf} and v_{br}),

b) the yaw displacementof the frames (γ_{bf} and γ_{br}),

c) the vertical displacements of the wheelsets (w_{d1} to w_{d6}) and

d) the rolling of the wheelsets (α_{d1} to α_{d6}),

are known since the wheels are assumed to follow the track (without relative displacement). Consequently these input displacements should be taken to the R.H.S. of the equations and equation (2.5.3) becomes:

$$\underset{20x20}{[A]}\ \underset{20x1}{\{\ddot{X}\}} + \underset{20x20}{[C]}\ \underset{20x1}{\{\dot{X}\}} + \underset{20x20}{[B]}\ \underset{20x1}{\{X\}} = \underset{20x16}{[AR]}\ \underset{16x1}{\{\ddot{y}\}} + \underset{20x16}{[CR]}\ \underset{16x1}{\{\dot{y}\}} + \underset{20x16}{[BR]}\ \underset{16x1}{\{y\}} \tag{3.3.1}$$

where

$\{y\}$ - is the input excitation displacement (16x1 complex)

and $\{X\}$ - is the solution vector (20x1, complex)

A - reduced inertia matrix (20x20, real symmetric)

B - reduced stiffness matrix (20x20, real symmetric)

C - reduced damping matrix (20x20, real symmetric)

AR - excitation inertia matrix (20x16, real)

BR - excitation stiffness matrix (20x16, real)

CR - excitation damping matrix (20x16, real)

Using the method described in section 3.1 for solving we get

$$[(B - \omega^2 A) + i\omega \; C]\{X\} = [(BR - \omega^2 AR) + i\omega \; C]\{y\} \qquad (3.3.2)$$

this could be rewritten as

$$[Z] \{\bar{X}\} = \{F_o\}$$

or

$$\{\bar{X}\} = [Z^{-1}]\{F_o\} \qquad (3.3.3)$$

in which all matrices are complex.

To get the steady state response for the full model, "stationary" vehicle equation (3.3.3) is solved for increasing values of the excitation frequency.

Formation of the R.H.S. inertia, stiffness and damping matrices from the original (36x36) inertia, stiffness and damping matrices respectively is done by the following procedure:

i - first, neglect all rows in the original inertia (stiffness or damping) matrix A (B or C) corresponding to the unknown displacements (row numbers: 8, 12, 14, 18, 20, 21, 23, 24, 26, 27, 29, 30, 32, 33, 35, 36).

ii - the elements left in the columns corresponding to the known displacements constitute in fact the columns of the R.H.S. matrices respectively.

The reduced mass matrix is obtained by contraction of the elements left in the original mass matrix after formation of the R.H.S. inertia matrix. Similarly for the stiffness and the damping matrices. It is important to note that the procedure described above is only valid because the original inertia stiffness and mass matrices are symmetric and that the reduced inertia, stiffness and mass matrices should remain symmetric.

3.4 The Steady State Response for the "Moving" Vehicle

When the vehicle is not stationary on the track, it is no longer true that certain of the coordinates are "locked", but it remains true that certain relationships exist between them. These relationships are those governed by the laws of "creepage". And the simultaneous equations may be expressed in the following form, representing a 36x36 matrix equation.

$$[A]\{\ddot{X}\} + [C]\{\dot{X}\} + [B]\{X\} = F(X,\dot{X},\ddot{X}) + \psi(y,\dot{y},\ddot{y}) \qquad (3.4.1)$$

where

$F(X,\dot{X},\ddot{X})$ represents the constraint imposed by the tracks (due to creep forces, conicity of the wheels and gravity stiffness effect).

$\psi(y,\dot{y},\ddot{y})$ is a column matrix of forces, due to rail irregularities, for a perfectly straight track ψ would simply be a column of noughts.

The force F depends on the instantaneously prevailing values of X, $\dot{X}$ and $\ddot{X}$. In the linear case, the function form $F(X,\dot{X},\ddot{X})$ may be expressed as a sum of column matrices of the type

$$-([a]\{\ddot{X}\} + [c]\{\dot{X}\} + [b]\{X\})$$

where: the square matrices [a], [b] and [c] are determined by the process of creepage and by the speed "S" of the railway vehicle.

These square matrices are not symmetric, and the system is non-conservative. Equation (3.4.1) is rewritten in the form:

$$[\bar{A}]\{\ddot{X}\} + [\bar{C}]\{\dot{X}\} + [\bar{B}]\{X\} = \psi(y,\dot{y},\ddot{y}) \qquad (3.4.2)$$

where $[\bar{A}]$, $[\bar{B}]$ and $[\bar{C}]$ are no longer of the original positive definite type, and are no longer symmetric.

Since we are assuming that the wheels are following the rails in the vertical direction (no creepage in this direction) it follows that $w_{di} = 0$ $(i = 1,\dots,6)$, and rows and columns corresponding to these variables should be deleted and the system of equations (3.4.2) becomes:

$$\underset{30x30}{[\bar{A}]}\ \underset{30x1}{\{\ddot{X}\}} + \underset{30x30}{[\bar{C}]}\ \underset{30x1}{\{\dot{X}\}} + \underset{30x30}{[\bar{B}]}\ \underset{30x1}{\{X\}} = \psi(y,\dot{y},\ddot{y}) \qquad (3.4.3)$$

The rolling motion of the wheelsets $[\alpha_{di}$ $(i = 1,\dots,6)]$ are not independent variables, but they do exist implicitly in the equations due to the lateral displacement of the wheelsets and the conicity of the wheels (see equations of constraints (2.6.2.37) to (2.6.2.42)).

<u>Determination of the Forcing Function $\psi(y,\dot{y},\ddot{y})$</u>

In the linear case the forcing function $\psi(y,\dot{y},\ddot{y})$ which is the R.H.S. in equation (3.4.3) has the form

$$-[AR]\{\ddot{y}\} - [CR]\{\dot{y}\} - [BR]\{y\} \tag{3.4.4}$$

where $\{y\}$ is the vector of known input displacements due to rail irregularities.

To determine the forcing function we have to go back to the original equations of motion (2.6.1) up to the point where creep forces are substituted for reactions. These are valid for any displacements whether free or induced. Let us assume that all displacements have the general form

$$x = x_r + x_i$$

x_r = displacement relative to the track due to creep

x_i = induced displacement due to track irregularities.

All x_i are zeros except for the variables describing the motion of the wheelsets, and for the frames in the longitudinal, lateral and yaw directions. For convenience, we rewrite equation (2.6.1) in the following form

$$\underset{78\times78}{[A]}\ \underset{78\times1}{\{\ddot{x}_r\}} + [C]\{\dot{x}_r\} + [B]\{x_r\} + \{F\}$$

$$+ \underset{78\times78}{[AR]}\ \underset{78\times1}{\{\ddot{x}_i\}} + [CR]\{\dot{x}_i\} + [BR]\{x_i\} = 0 \tag{3.4.5}$$

In the equations of motion for the wheelsets, the terms due to creep forces and the terms including $(N_r - N_\ell)$ and W are functions only of the x_r components; and the induced motion matrices AR, CR and BR have the same form as the "relative" motion matrices A, B, and C respectively except that the following elements are zeros:

$$\left.\begin{array}{l} BR(24,24) = 0 \\ BR(24,28) = 0 \\ BR(26,24) = 0 \\ BR(26,28) = 0 \\ BR(28,24) = 0 \\ BR(28,28) = 0 \end{array}\right\} \text{for Wheelset No. 1} \qquad \left\{\begin{array}{l} CR(23,23) = 0 \\ CR(23,27) = 0 \\ CR(24,24) = 0 \\ CR(24,26) = 0 \\ CR(24,28) = 0 \\ CR(26,24) = 0 \\ CR(26,26) = \ell_{19}(C_{17}+C_{18}) + 0 \\ CR(26,28) = 0 \\ CR(27,23) = 0 \\ CR(27,27) = 0 \\ CR(28,24) = 0 \\ CR(28,26) = 0 \\ CR(28,28) = 0 \end{array}\right.$$

etc. for other wheelsets.

We now apply the process of eliminating reactions as given in section (2.6.1) operating on AR, BR and CR as it was previously done for A, B and C. Equation (3.4.5) becomes

$$\underset{36x78}{[A']}\ \underset{78x1}{\{\ddot{x}_r\}} + [C']\{\dot{x}_r\} + [B']\{x_r\} =$$

$$- \underset{36x78}{[AR']}\ \underset{78x1}{\{\ddot{x}_i\}} - [CR']\{\dot{x}_i\} - [BR']\{x_i\} \qquad (3.4.6)$$

where the forcing function ψ is the R.H.S. of (3.4.6).

To obtain the right hand side as a function of the independent displacements, the transformation matrix T (78x42) given later in this section is used. This matrix T is based on similar constraints as those used to get the transformation matrix D (in section 2.6.2) except that the relationship between v_d and α_d does not apply.

To eliminate dependent displacements we have the relationship

$$\underset{78x1}{\{x_i\}} = \underset{78x42}{[T]} \underset{42x1}{\{y_i\}} \tag{3.4.7}$$

Substituting (3.4.7) in the R.H.S. of (3.4.6) we get

$$\psi = - \underset{36x78}{[AR]} \underset{78x42}{[T]} \underset{42x1}{\{\ddot{y}_i\}} - [CR][T]\{\dot{y}_i\} - [BR][T]\{y_i\}$$

$$\psi = - \underset{36x42}{[ART]} \underset{42x1}{\{\ddot{y}_i\}} - [CRT]\{\dot{y}_i\} - [BRT]\{y_i\} \tag{3.4.8}$$

The only non-zero elements in $\{y_i\}$ are those describing the induced motion of:

a) lateral displacement of rear frame (v_{br}) = y(8)

b) yaw motion of rear frame (γ_{br}) = y(12)

c) lateral displacement of front frame v_{bf} = y(14)

d) yaw motion of front frame γ_{bf} = y(18)

e) vertical displacements of wheelsets (w_{d1} to w_{d6}) = y(20), y(24), y(28), y(32), y(36), y(40)

f) rolling motion of wheelsets (α_{d1} to α_{d6}) = y(21), y(25), y(29), y(33), y(37), y(41)

The transformation matrix [T] is given on the following pages.

The Transformation Matrix [T]

This matrix gives the relation between all the variables and the independent variables which describe the portion of the generalized displacements due to rail irregularities (the R.H.S.).

The transformation matrix T (78x42) is defined in the following pages, where:

$\rho_1 = \ell_{24} - \ell_{34}$ $\rho_2 = \ell_{26} - \ell_{34}$

$\rho_3 = -(\ell_{23} + \ell_{34})$ $\rho_4 = \ell_{23} + \ell_{34}$

$\rho_5 = -\ell_{26} + \ell_{34}$ $\rho_6 = -\ell_{24} + \ell_{34}$

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
		u_a	v_a	w_a	α_a	β_a	γ_a	u_{br}	v_{br}	w_{br}	α_{br}	β_{br}	γ_{br}	u_{bf}	v_{bf}	w_{bf}	α_{bf}	β_{bf}	γ_{bf}	β_{c1}	w_{d1}	α_{d1}	β_{d1}	β_{c2}	w_{d2}	α_{d2}	β_{d2}	β_{c3}	w_{d3}	α_{d3}	β_{d3}	β_{c4}	w_{d4}	α_{d4}	β_{d4}	β_{c5}	w_{d5}	α_{d5}	β_{d5}	β_{c6}	w_{d6}	α_{d6}	β_{d6}
1	u_a	1																																									
2	v_a		1																																								
3	w_a			1																																							
4	α_a				1																																						
5	β_a					1																																					
6	γ_a						1																																				
7	u_{br}							1																																			
8	v_{br}								1																																		
9	w_{br}									1																																	
10	α_{br}										1																																
11	β_{br}											1																															
12	γ_{br}												1																														
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15	w_{bf}															1																											
16	α_{bf}																1																										
17	β_{bf}																	1																									
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19	v_{c1}								1		$-\ell_{21}$		ρ_1																														
20	w_{c1}																			ℓ_{34}	1																						
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22	γ_{c1}												1																														
23	u_{d1}							1				ℓ_{21}																															
24	v_{d1}								1		$-\ell_{21}$		ℓ_{24}																														
25	w_{d1}																				1																						
26	α_{d1}																					1																					
27	β_{d1}																						1																				
28	γ_{d1}												1																														
29	v_{c2}								1		$-\ell_{21}$		ρ_2																														
30	w_{c2}																							ℓ_{34}	1																		
31	β_{c2}																							1																			
32	γ_{c2}												1																														
33	u_{d2}							1				ℓ_{21}																															
34	v_{d2}								1		$-\ell_{21}$		ℓ_{26}																														
35	w_{d2}																								1																		
36	α_{d2}																									1																	
37	β_{d2}																										1																
38	γ_{d2}												1																														

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
		u_a	v_a	w_a	α_a	β_a	γ_a	u_{br}	v_{br}	w_{br}	α_{br}	β_{br}	γ_{br}	u_{bf}	v_{bf}	w_{bf}	α_{bf}	β_{bf}	γ_{bf}	β_{c1}	w_{d1}	α_{d1}	β_{d1}	β_{c2}	w_{d2}	α_{d2}	β_{d2}	β_{d3}	w_{d3}	α_{d3}	β_{d3}	β_{c4}	w_{d4}	α_{d4}	β_{d4}	β_{c5}	w_{d5}	α_{d5}	β_{d5}	β_{c6}	w_{d6}	α_{d6}	β_{d6}
39	v_{c3}								1		$-\ell_{21}$		ρ_3																														
40	w_{c3}																											ℓ_{34}	1														
41	β_{c3}																											1															
42	γ_{c3}												1																														
43	u_{d3}							1				ℓ_{21}																															
44	v_{d3}								1		$-\ell_{21}$		$-\ell_{23}$																														
45	w_{d3}																												1														
46	α_{d3}																													1													
47	β_{d3}																														1												
48	γ_{d3}												1																														
49	v_{c4}														1		$-\ell_{21}$		ρ_4																								
50	w_{c4}																															$-\ell_{34}$	1										
51	β_{c4}																															1											
52	γ_{c4}																		1																								
53	u_{d4}													1				ℓ_{21}																									
54	v_{d4}														1		$-\ell_{21}$		ℓ_{23}																								
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56	α_{d4}																																	1									
57	β_{d4}																																		1								
58	γ_{d4}																		1																								
59	v_{c5}														1		$-\ell_{21}$		ρ_5																								
60	w_{c5}																																			$-\ell_{34}$	1						
61	β_{c5}																																			1							
62	γ_{c5}																		1																								
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64	v_{d5}														1		$-\ell_{21}$		$-\ell_{26}$																								
65	w_{d5}																																				1						
66	α_{d5}																																					1					
67	β_{d5}																																						1				
68	γ_{d5}																		1																								
69	v_{c6}														1		$-\ell_{21}$		ρ_6																								
70	w_{c6}																																							$-\ell_{34}$	1		
71	β_{c6}																																							1			
72	γ_{c6}																		1																								
73	u_{d6}													1				ℓ_{21}																									
74	v_{d6}														1		$-\ell_{21}$		$-\ell_{24}$																								
75	w_{d6}																																								1		
76	α_{d6}																																									1	
77	β_{d6}																																										1
78	γ_{d6}																		1																								

Steady State Response

Substituting from (3.4.8) after eliminating rows corresponding to the relative vertical displacement of the wheelsets in the vertical (w_d) and rolling directions (α_d) in the system of equations (3.4.3) it becomes:

$$\underset{30\times30}{[\bar{A}]}\ \underset{30\times1}{\{\ddot{X}\}} + [\bar{C}]\{\dot{X}\} + [\bar{B}]\{X\} = - \underset{30\times42}{[ART]}\ \underset{42\times1}{\{\ddot{y}_i\}} - [CRT]\{\dot{y}_i\} - [BRT]\{y_i\} \quad (3.4.9)$$

where:

$\bar{A}$ - inertia matrix (including creep terms)

$\bar{B}$ - stiffness matrix (including creep terms)

$\bar{C}$ - damping matrix (including creep terms)

-[ART] - "induced motion" inertia matrix (no creep terms)

-[BRT] - "induced motion" stiffness matrix (no creep terms)

-[CRT] - "induced motion" damping matrix (no creep terms)

y - induced displacements vector (42x1, complex)

X - is the solution vector (30x1, complex)

It should be noted that $\bar{A}$, $\bar{B}$ and $\bar{C}$ vary with the forward speed of the vehicle.

To solve for a certain value of forward speed, i.e., a certain value of excitation frequency, the following procedure is adopted:

Using the method described in section 3.1, we let

$$y = e^{i(\omega t-\psi)}$$

getting $x = X\,e^{i(\omega t-\phi)} = X\,e^{-i\phi}\,e^{i\omega t}$

$$\dot{x} = i\omega\,X\,e^{-i\phi}\,e^{i\omega t}$$

$$\ddot{x} = -\,\omega^2\,X\,e^{-i\phi}\,e^{i\omega t}$$

Substituting in the system of equations (3.4.9) we get:

$$(-[\bar{A}]\omega^2 + i\omega\,[\bar{C}] + \bar{B})\,\{X\,e^{-i\phi}\} = (-[AR]\omega^2 + i\omega\,[CR] + [BR])\{y\,e^{-i\psi}\}$$

OR

$$\{X\,e^{-i\phi}\} = ([\bar{B}] - [\bar{A}]\omega^2 + i\omega\,[\bar{C}])^{-1}\,([BR] - [AR]\omega^2 + i\omega\,[C])\{y\,e^{-i\psi}\}$$

3.5 Damped Natural Frequencies

For free vibrations the R.H.S. of the equations of motions is simply a column of zeros. Hence the system becomes:

$$[A]\,\{\ddot{X}\} + [C]\,\{\dot{X}\} + [B]\,\{X\} = 0 \qquad (3.5.1)$$

where

A - is the inertia (or mass) matrix

C - is the damping matrix

B - is the stiffness matrix

Equation (3.5.1) could be rewritten as:

$$D \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -A^{-1}B & -A^{-1}C \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} \tag{3.5.2}$$

where all the above quantities are submatrices, except D which is the differential operator. Now we let

$$\{y\} = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$$

and

$$G = \begin{bmatrix} 0 & I \\ -A^{-1}B & -A^{-1}C \end{bmatrix}$$

Equation (3.5.2) becomes

$$D\ \{y\} = [G]\ \{y\} \tag{3.5.3}$$

We assume a solution in the form

$$\{y\} = \{Y\ e^{st}\} \tag{3.5.4}$$

and we substitute in (3.5.3) getting

$$S\{Y\} = [G]\ \{Y\}$$

or

$$(S[I] - [G])\ \{Y\} = 0 \tag{3.5.5}$$

The roots of [G] (the eigenvalues) may be obtained using a library subroutine. The solutions will be complex in the form:

$$S_i = \mu_i \pm i\omega_i \quad \text{(for the } i^{th} \text{ root)}$$

where μ_i represents damping

ω_i is the damped natural frequency.

Equation (3.5.4) becomes:

$$y_i = e^{\mu_i t} A \cos (\omega_i t + \psi) \tag{3.5.6}$$

If any μ_i is positive, the displacement y_i becomes unstable.

The natural frequencies for the simplified model and the full "stationary" model were found.

Hunting for the moving vehicle could also be investigated by determining the eigenvalues of the matrix [G]. The procedure is to solve for all eigenvalues for increasing values of forward speed until a μ_i becomes positive, indicating instability and the critical speed of the car. It is obvious that the state of the track plays an important role in the analysis. For the case of sinusoidal track having amplitudes of 2-3 mm it was shown by Van Bommel [13] that the critical speed is already appreciably reduced.

Stability analysis for the "moving" vehicle on an irregular track is more complicated than the same study on a straight one, and is beyond the scope of this research.

CHAPTER 4

RESULTS AND CONCLUSIONS

4.1 Introduction

In this chapter, the results obtained for the three models considered are discussed. For the "moving" vehicle responses for the cases of new wheels and worn wheels are compared. This is followed by a discussion of the conclusions that can be drawn from the analysis, including the applications to the design of high speed railway vehicles. Finally, possible extensions of this analysis are given, and suggestions for further research in related areas are made.

The results given in this chapter are for the case of lateral track irregularities only. It should be noted however that the method, as well as the computer programs, are general enough to allow also for vertical track irregularities. The sinusoidal irregularities of the two rails can be in-phase or out-of-phase, but are of the same maximum amplitude.

The steady state responses of the vehicle components to varying input frequencies is computed. The input frequency is increased from zero to 3 cycles per second. For the "moving" vehicle the input frequency is a function of the wave length and the vehicle forward speed and is given in terms of the vehicle speed.

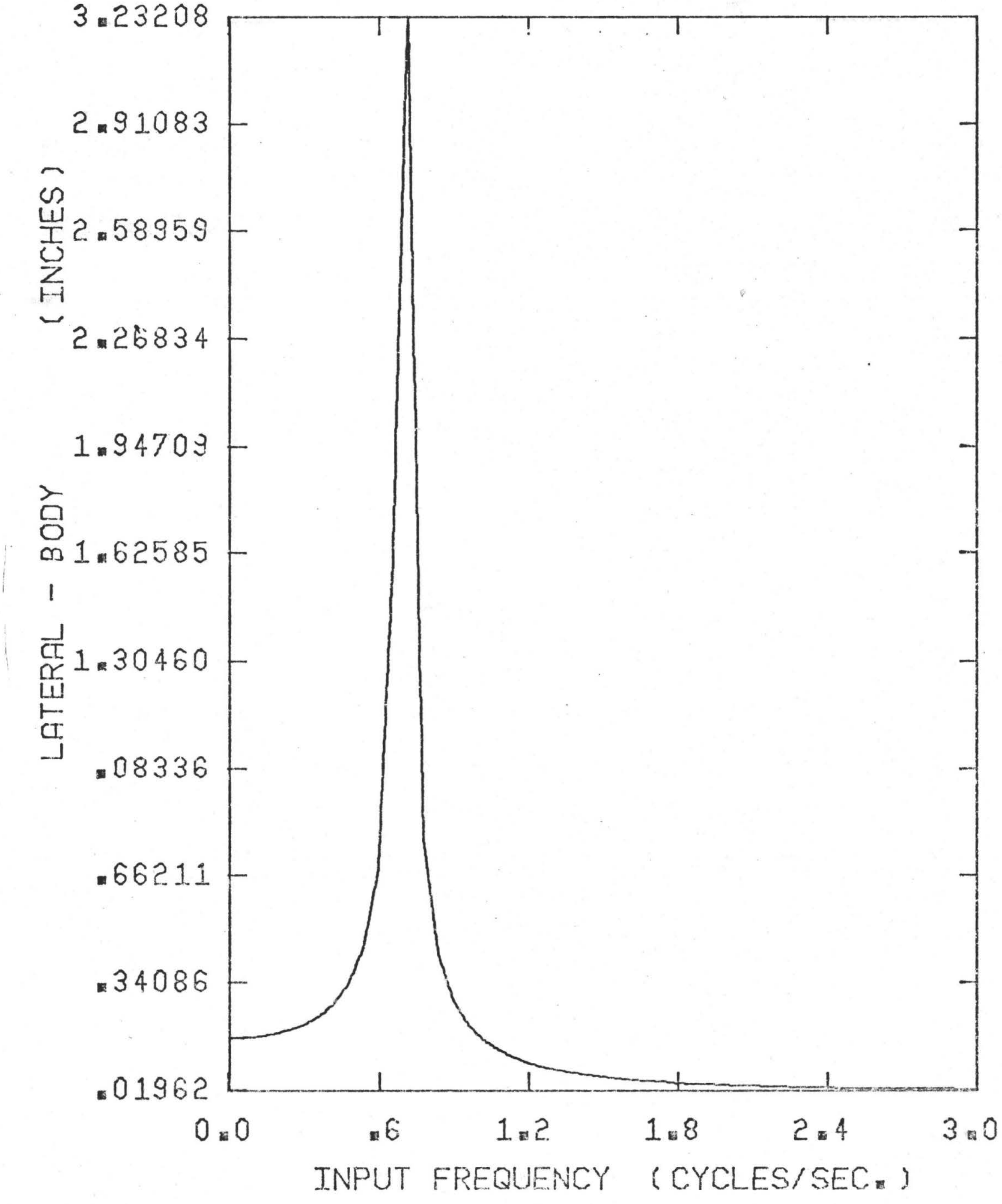
The curves are computer plotted from discrete data using a crude smoothing technique. This together with the contracted horizontal scale and the amount of damping accounts for the sharp peaks that occur. These peaks are really rounded.

4.2 Results for the Simplified Model

Figures 5 to 7 show the response curves obtained for the body in the lateral, roll and yaw directions respectively using the simplified model.

The horizontal axis represents the input frequency. The responses in the other three modes, i.e. longitudinal, vertical and pitch are coupled together but are uncoupled from the responses in the lateral, roll and yaw directions. Since the input is in the lateral direction only it follows that the steady state responses in the longitudinal, vertical and pitch directions are zeros.

Natural frequencies for the simplified model are given on the page following Figure 7.



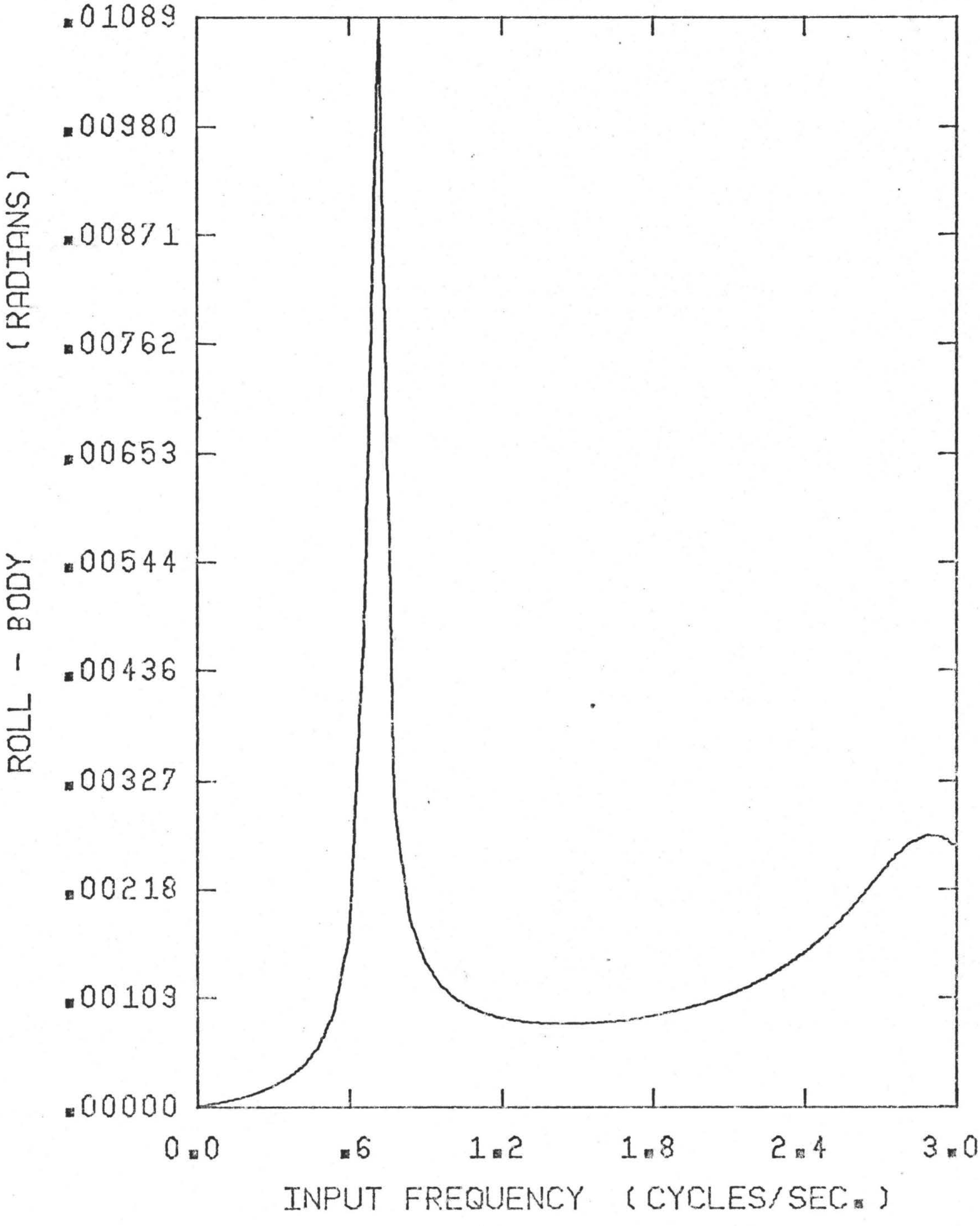
FOR THE CASE OF THE SIMPLIFIED MODEL ...

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 5: LONGITUDINAL BODY DISPLACEMENT(u_a) - SIMPLIFIED MODEL



FOR THE CASE OF THE SIMPLIFIED MODEL ...

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 6: ROLL BODY DISPLACEMENT (α_a) - SIMPLIFIED MODEL

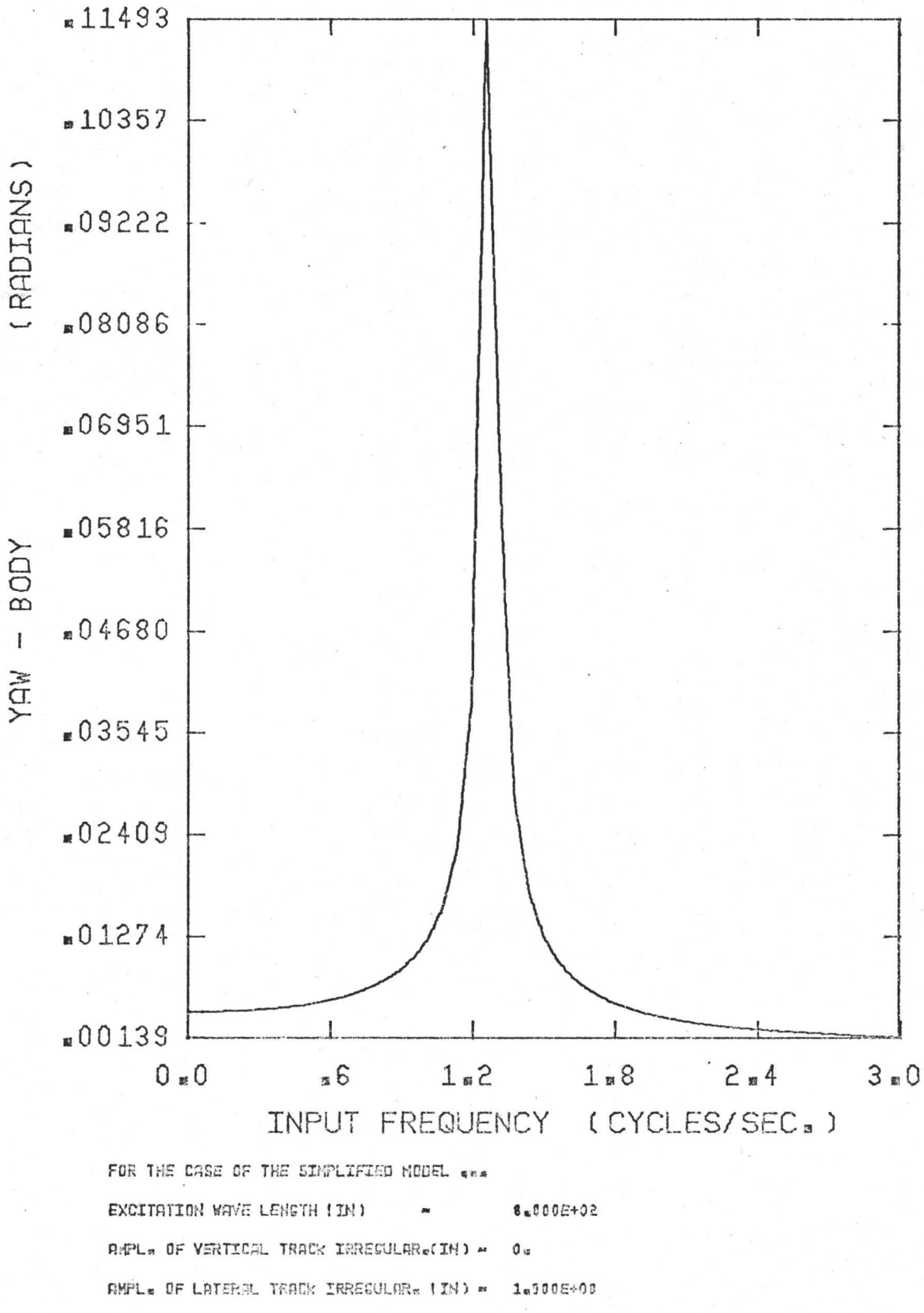


FIGURE 7: YAW BODY DISPLACEMENT (γ_a) - SIMPLIFIED MODEL

NATURAL FREQUENCIES FOR THE SIMPLIFIED MODEL

(IN CYCLES/SECOND)

OMEGA 1 =	.701
OMEGA 2 =	1.277
OMEGA 3 =	2.922
OMEGA 4 =	3.148
OMEGA 5 =	4.416
OMEGA 6 =	6.438

4.3 Results for the Full Model - "Stationary" Vehicle

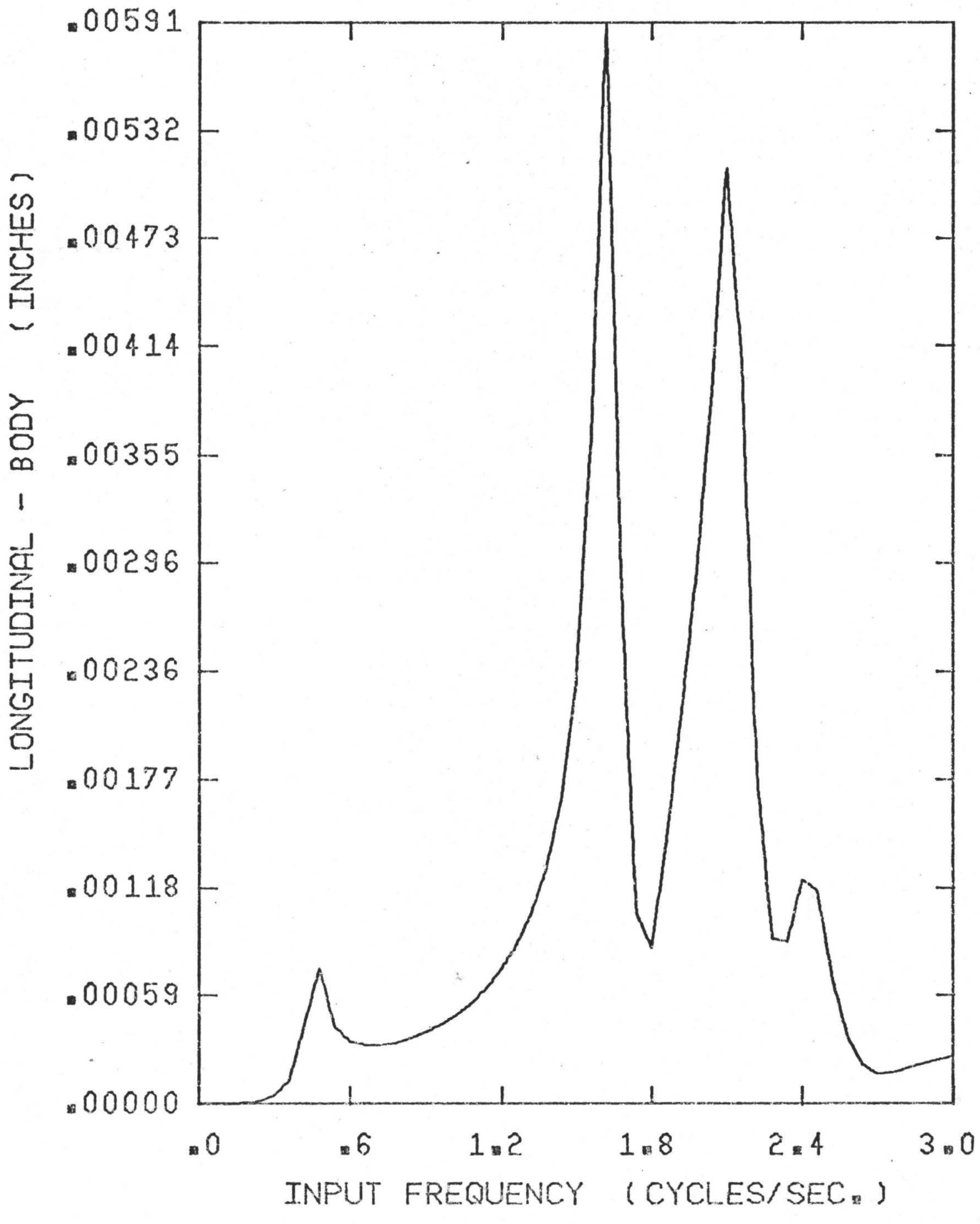
For the full model the primary suspension is not symmetric and the response curves are obtained for the body in all the six modes of oscillations.

Figures 8 to 13 shows the response curves obtained for the body in the longitudinal, lateral, vertical, roll, pitch and yaw directions respectively.

Figures 14 to 17 show the response curves for the front frame in the longitudinal, vertical, roll and pitch directions respectively. No steady state responses in the lateral or yaw directions were obtained since no slip was assumed between the wheels and rails.

Figure 18 gives the pitch for motor No. 5 displacement.

Natural frequencies for the full model are given on the page following Figure 18. Since there is no constraint or reaction preventing longitudinal motion of the system, the system is free-free in this coordinate. It is known that one of the modes of vibration must be at zero frequency with the whole system moving at constant velocity in the longitudinal direction. This automatically comes out of the solution of the full system. However this knowledge may be used to reduce the system of equations by one equation before solving.



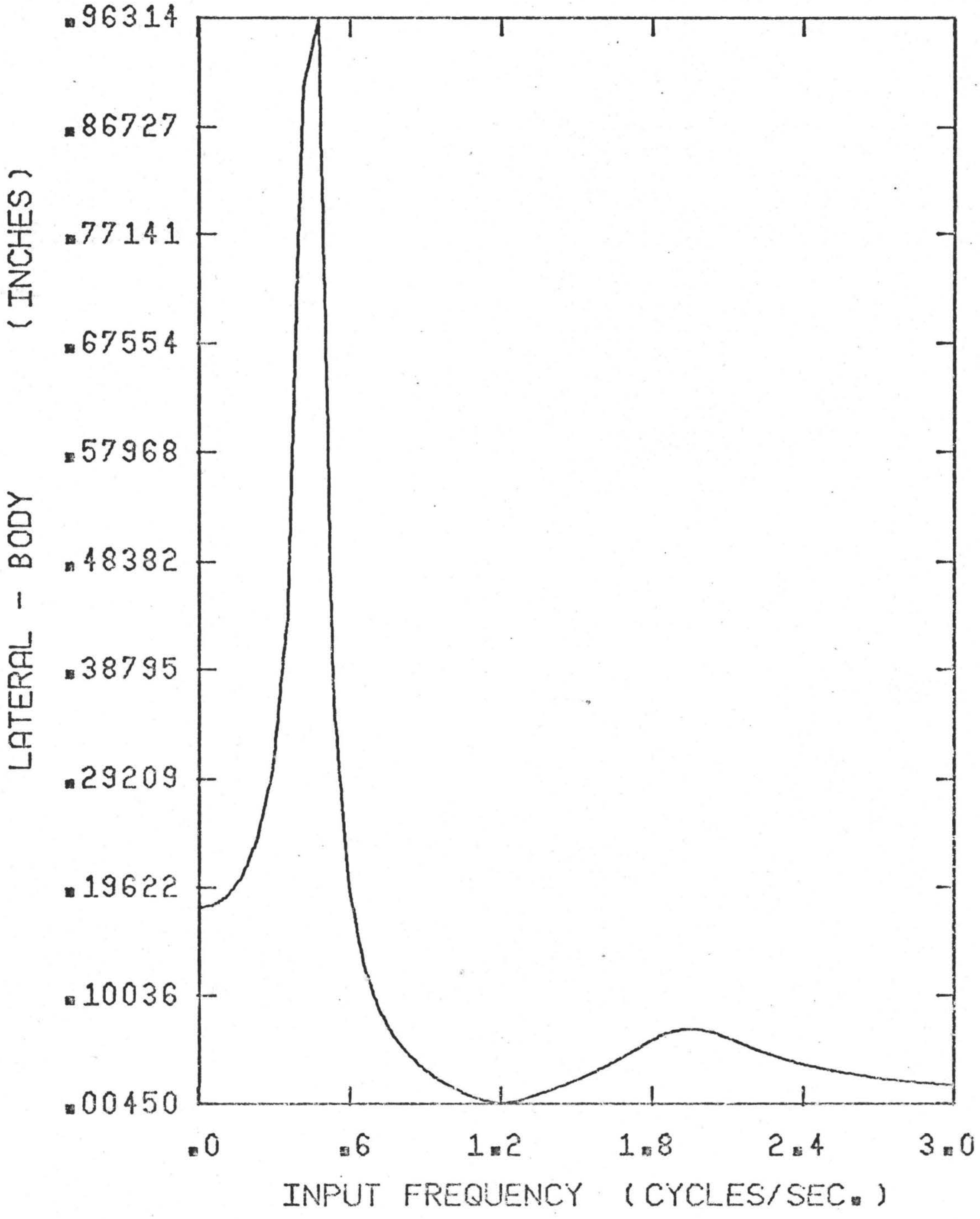
FOR THE FULL MODEL-STATIONARY VEHICLE...

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 8: LONGITUDINAL BODY DISPLACEMENT (u_a) - "STATIONARY" VEHICLE



FOR THE FULL MODEL-STATIONARY VEHICLE...

EXCITATION WAVE LENGTH (IN) = 6.000E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 9: LATERAL BODY DISPLACEMENT (v_a) -- "STATIONARY" VEHICLE

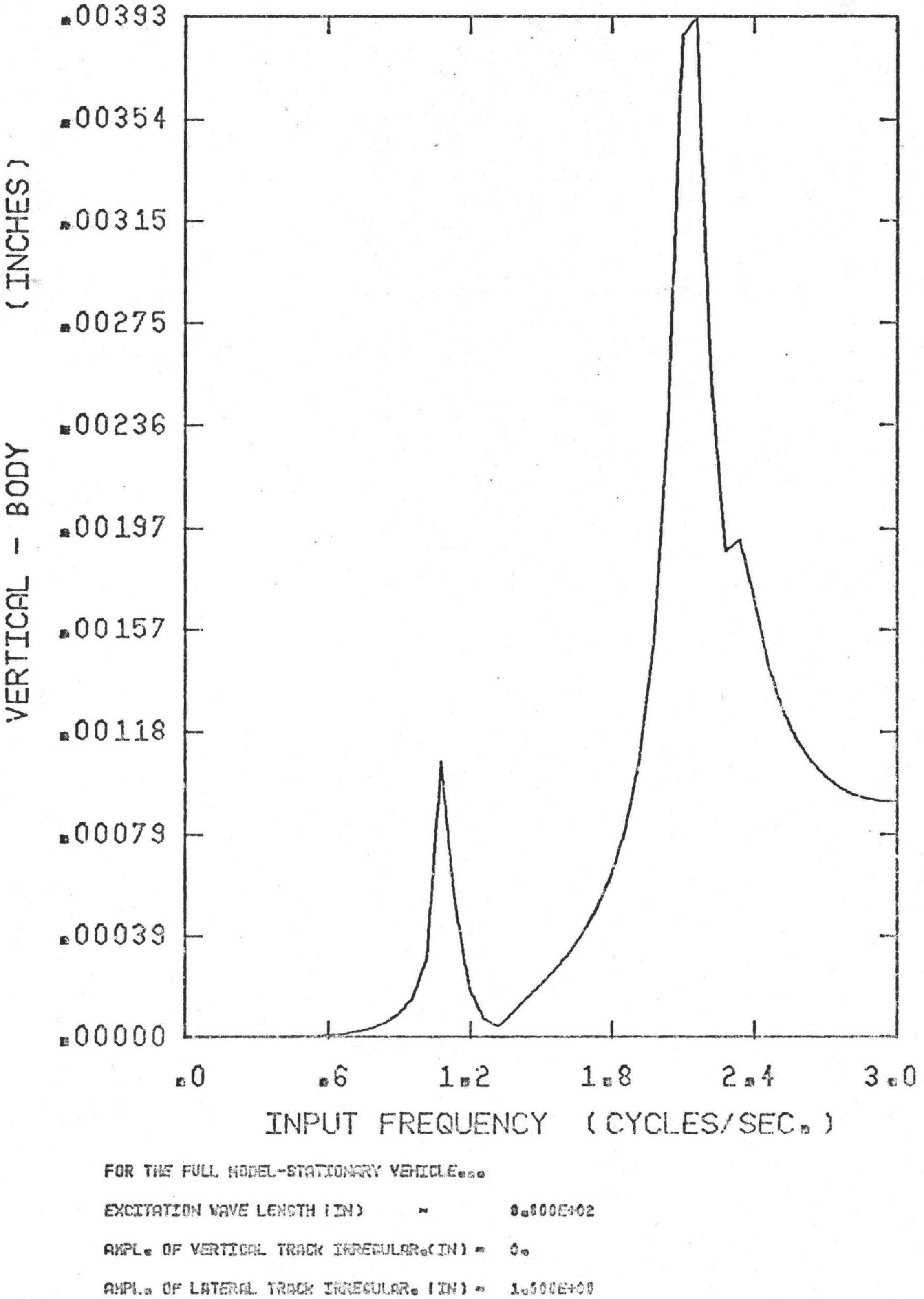


FIGURE 10: VERTICAL BODY DISPLACEMENT (w_a) - "STATIONARY" VEHICLE

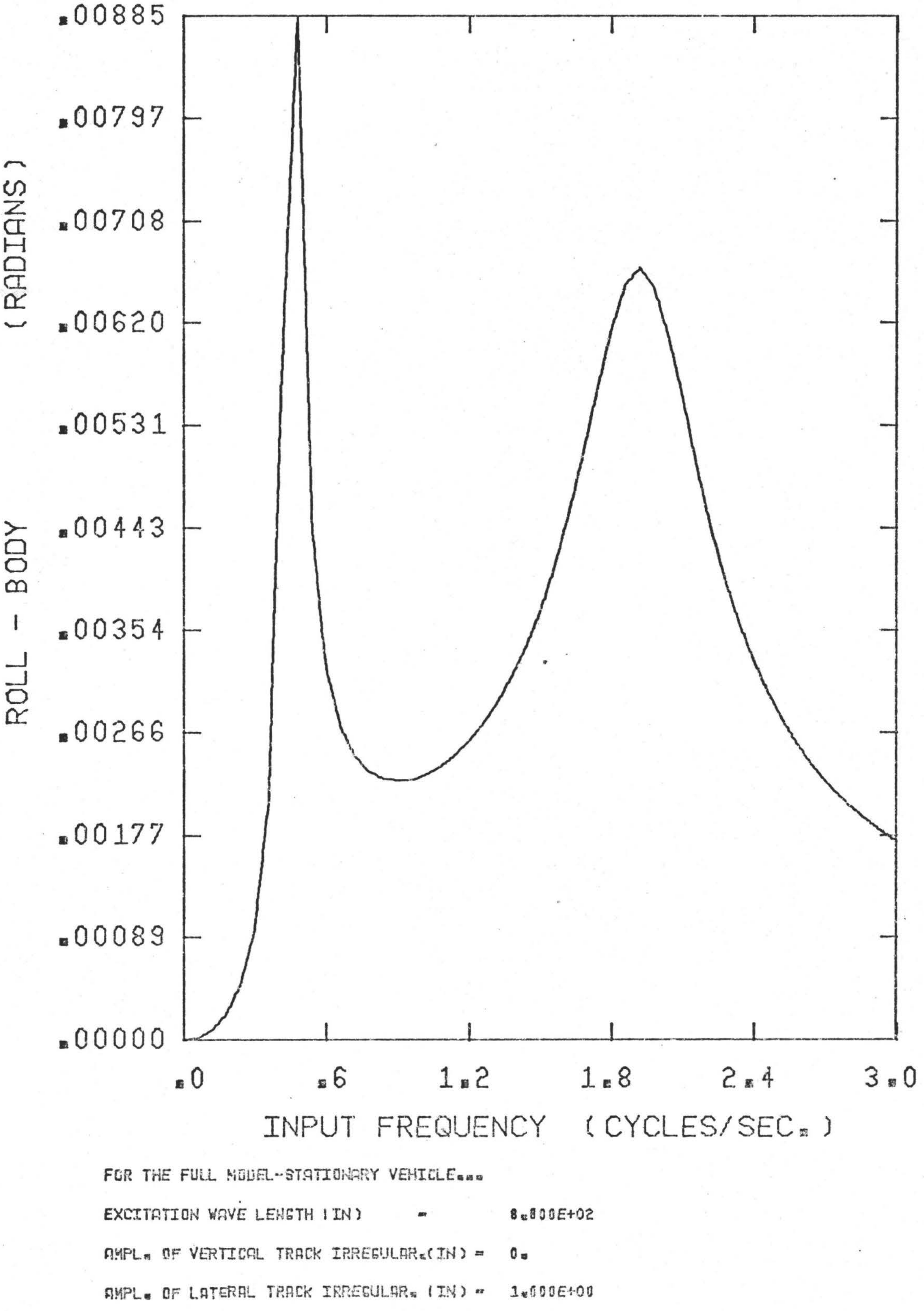


FIGURE 11: ROLL BODY DISPLACEMENT (α_a) - "STATIONARY" VEHICLE

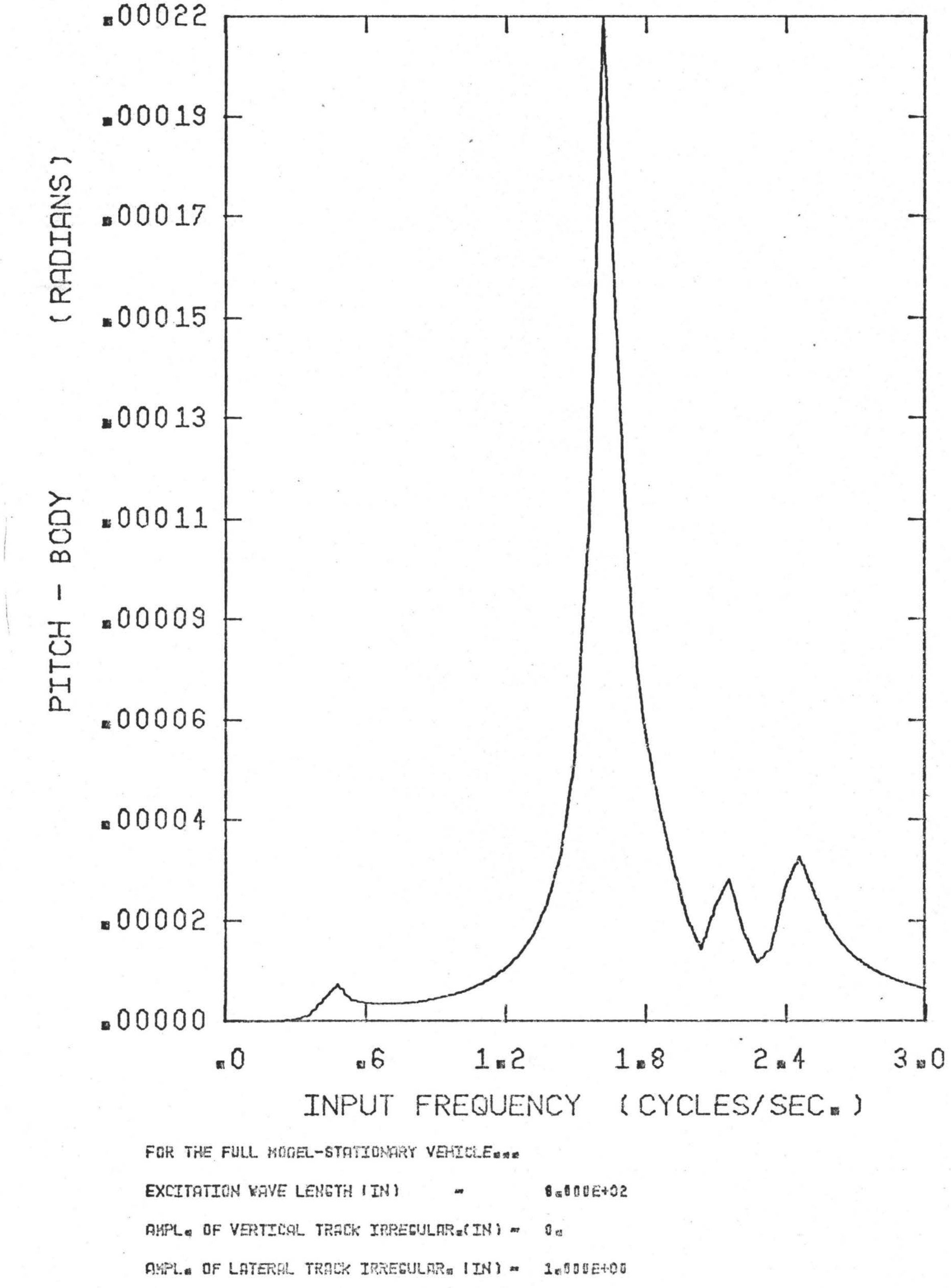
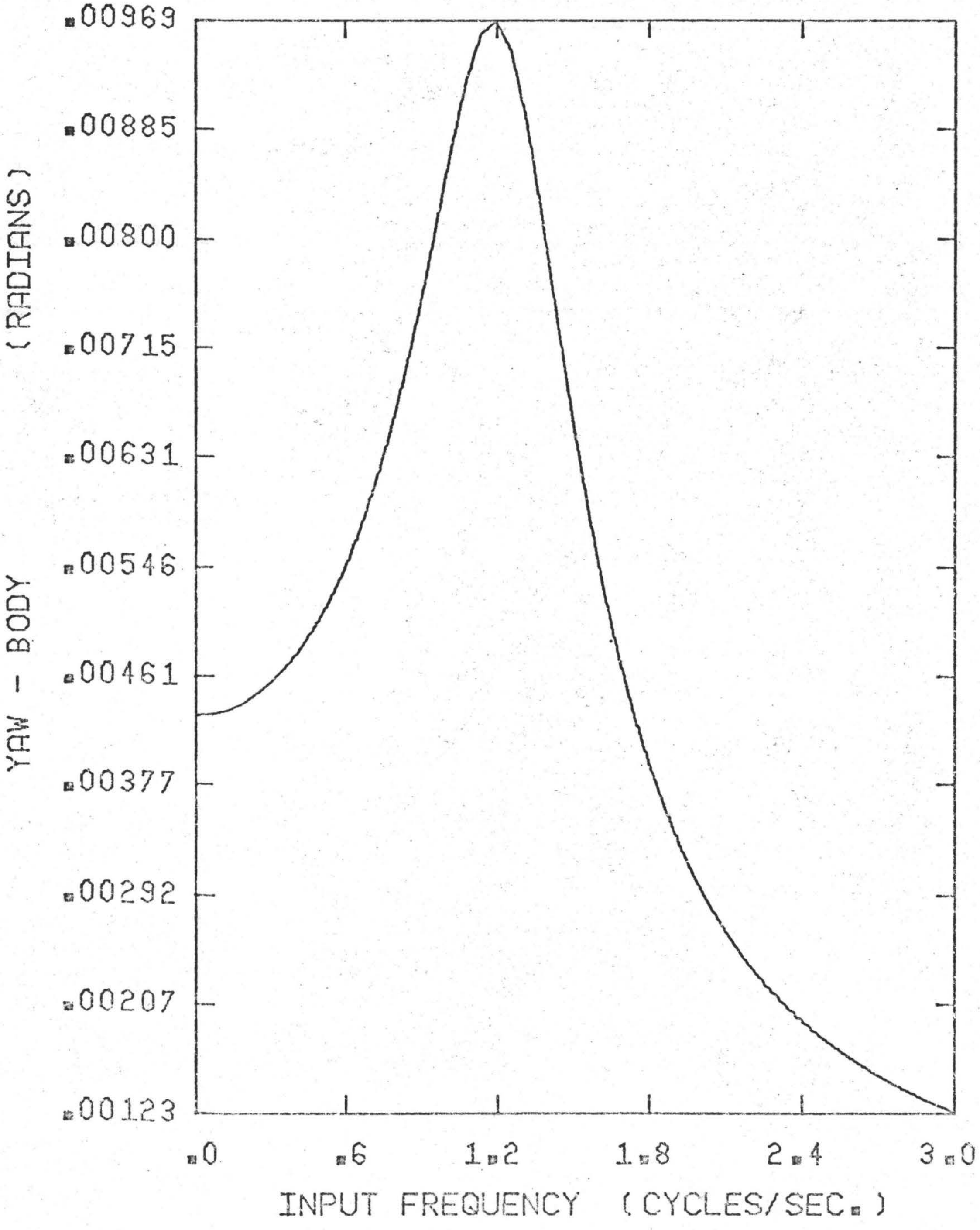


FIGURE 12: PITCH BODY DISPLACEMENT (β_a) - "STATIONARY" VEHICLE



FOR THE FULL MODEL-STATIONARY VEHICLE...

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 13: YAW BODY DISPLACEMENT (γ_a) - "STATIONARY" VEHICLE

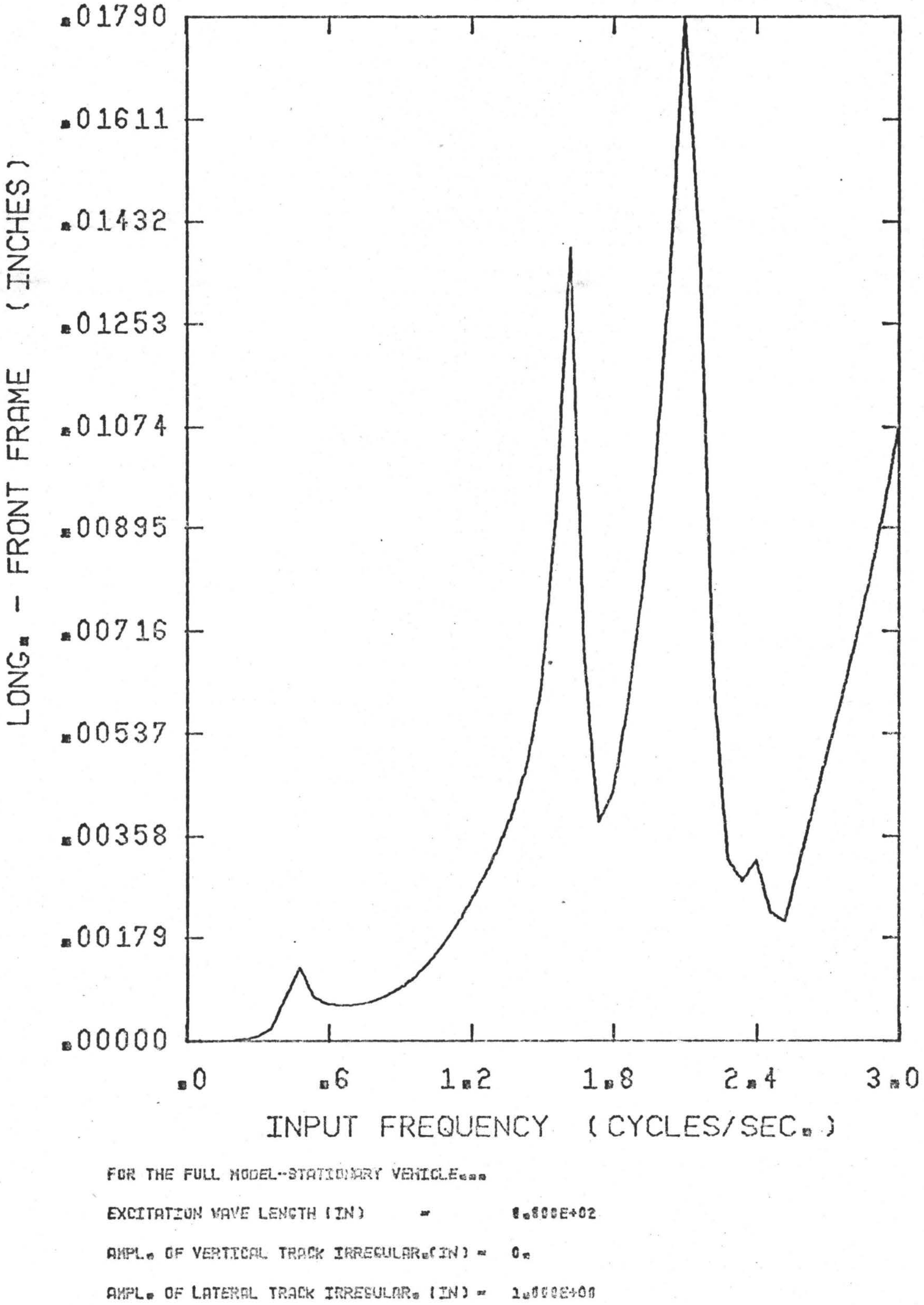
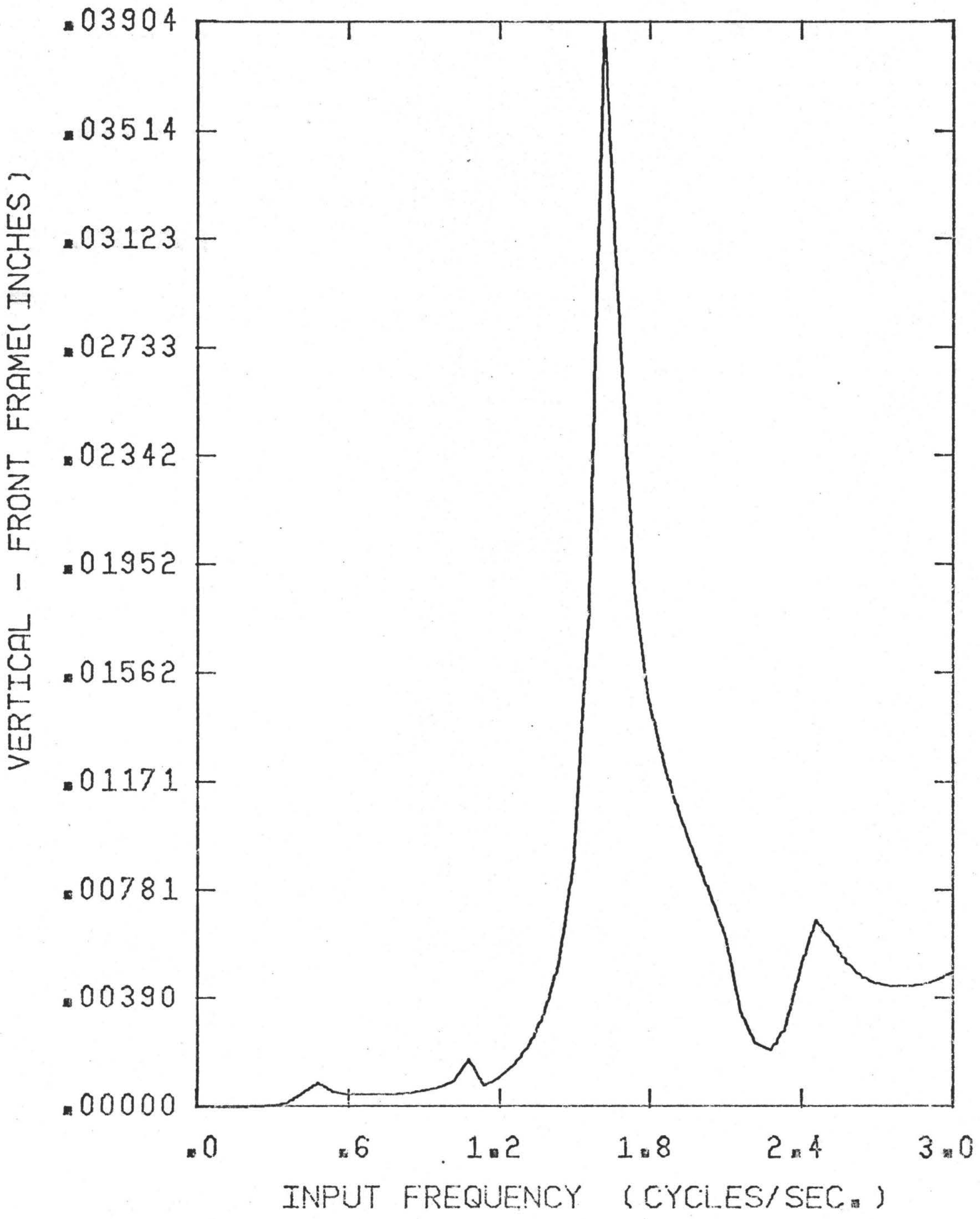


FIGURE 14: LONGITUDINAL FRONT FRAME DISPLACEMENT (u_{bf}) - "STATIONARY" VEHICLE



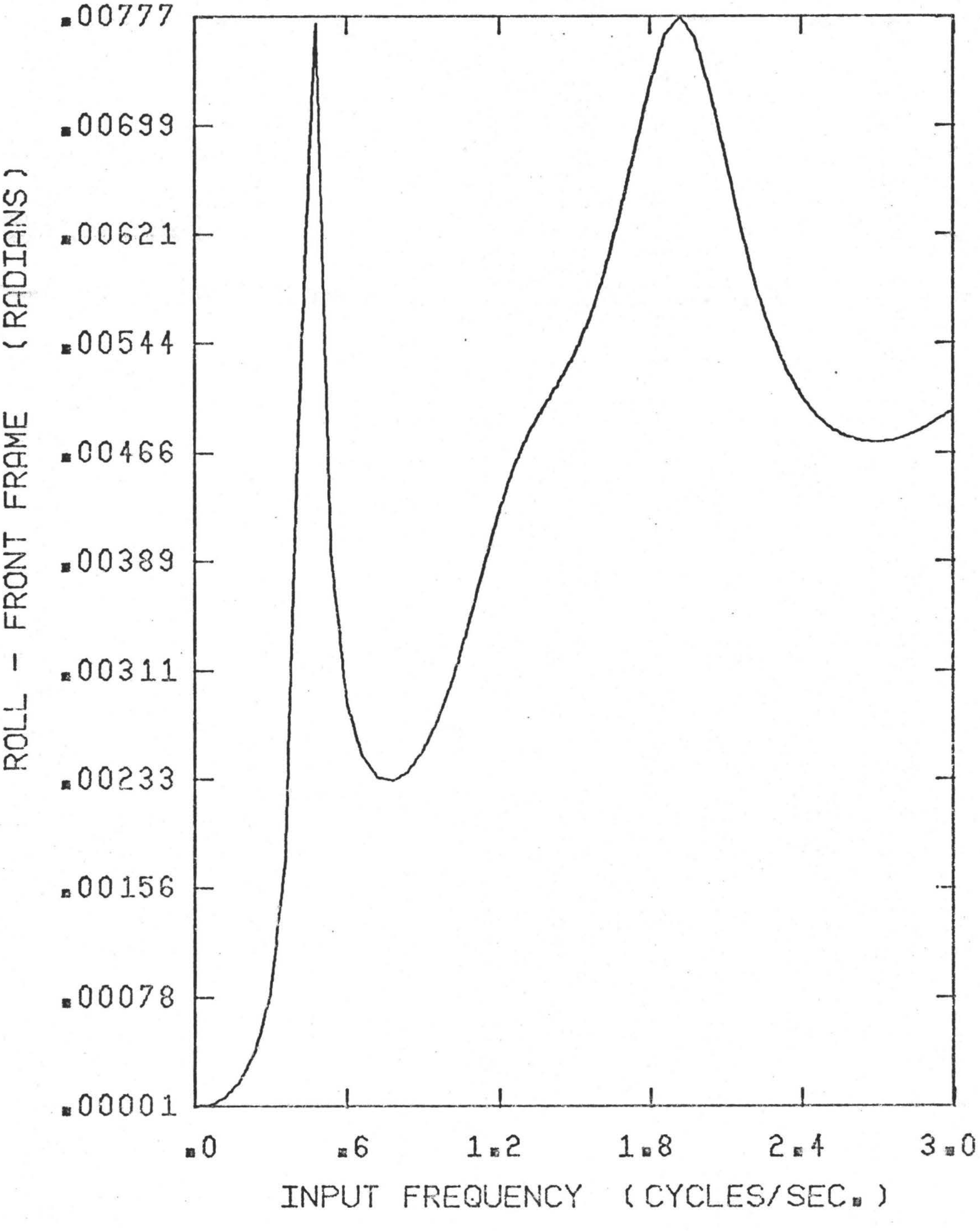
FOR THE FULL MODEL-STATIONARY VEHICLE...

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 15: VERTICAL FRONT FRAME DISPLACEMENT (w_{bf}) - "STATIONARY" VEHICLE



FOR THE FULL MODEL-STATIONARY VEHICLE...

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 16: ROLL FRONT FRAME DISPLACEMENT (α_{bf}) - "STATIONARY" VEHICLE

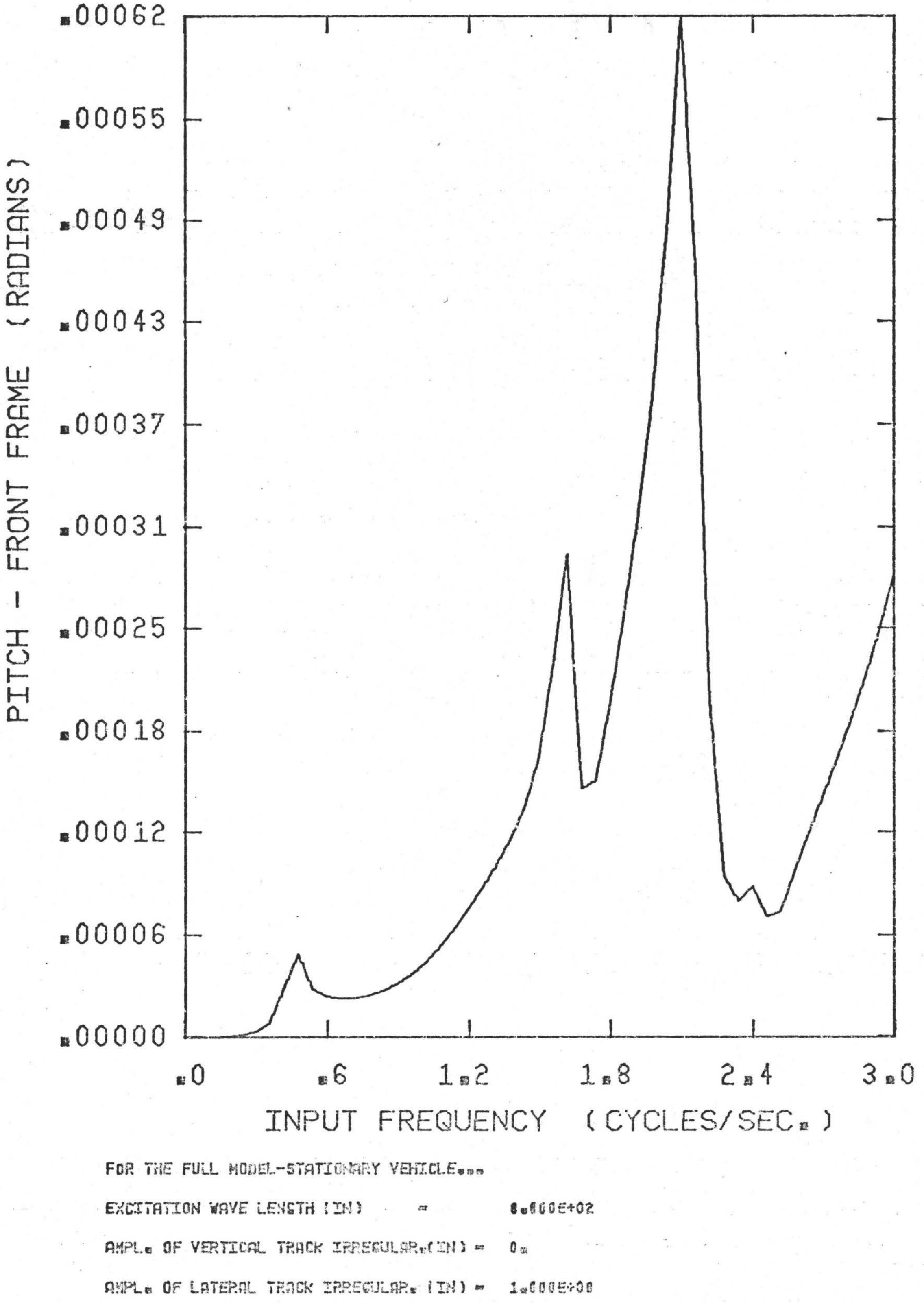
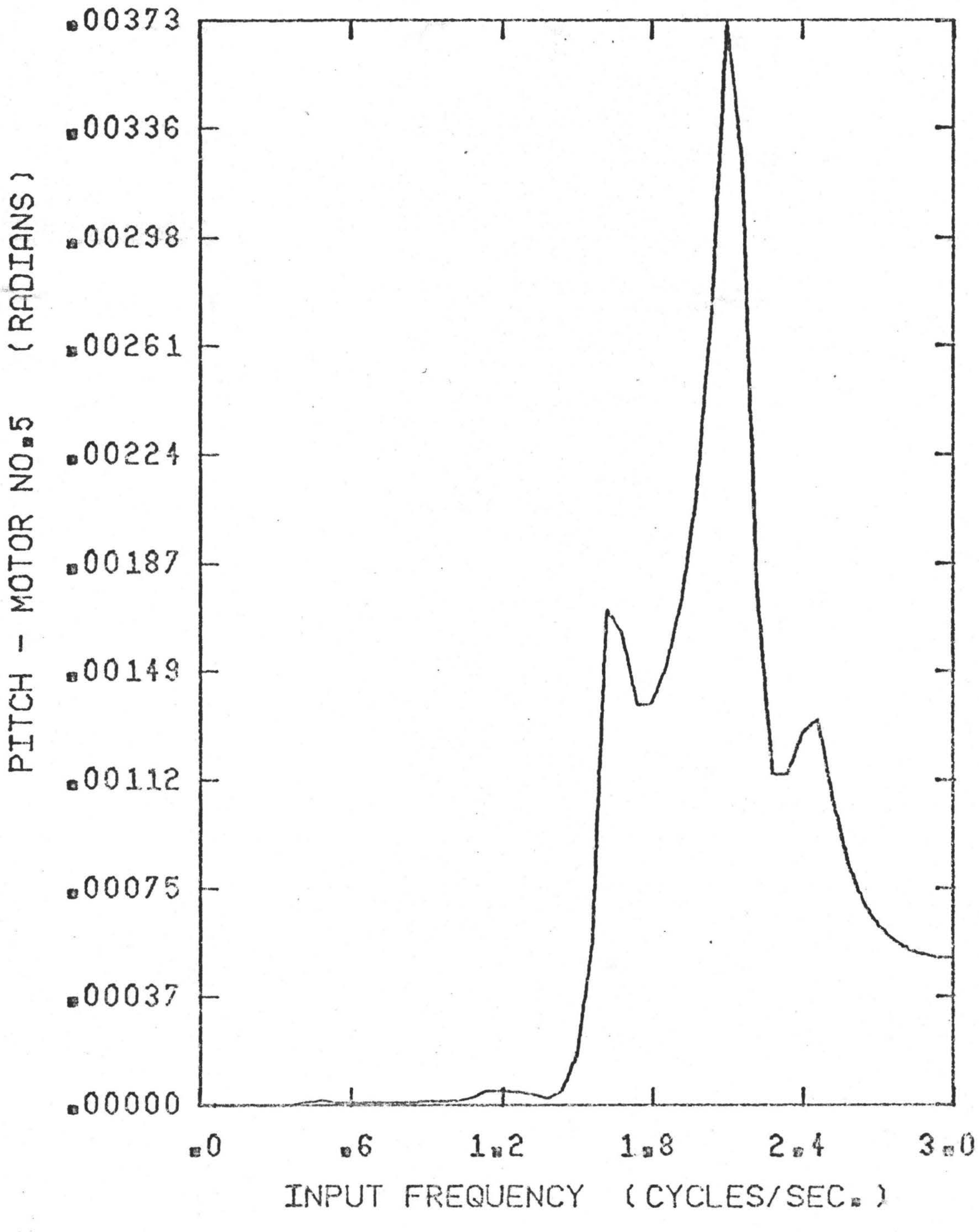


FIGURE 17: PITCH FRONT FRAME DISPLACEMENT (β_{bf}) - "STATIONARY" VEHICLE



FOR THE FULL MODEL-STATIONARY VEHICLE...

EXCITATION WAVE LENGTH (IN) = [illegible]

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = [illegible]

FIGURE 18: PITCH MOTOR NO. 5 DISPLACEMENT (u_{c5}) - "STATIONARY" VEHICLE

NATURAL FREQUENCIES FOR THE STATIONARY VEHICLE

(IN CYCLES/SECOND)

OMEGA 1 = .000

OMEGA 2 = .455

OMEGA 3 = 1.096

OMEGA 4 = 1.255

OMEGA 5 = 1.626

OMEGA 6 = 1.936

OMEGA 7 = 2.130

OMEGA 8 = 2.134

OMEGA 9 = 2.298

OMEGA 10 = 2.298

OMEGA 11 = 2.323

OMEGA 12 = 2.435

OMEGA 13 = 3.677

OMEGA 14 = 4.404

OMEGA 15 = 9.470

OMEGA 16 = 9.780

OMEGA 17 = 10.477

OMEGA 18 = 11.470

OMEGA 19 = 18.155

OMEGA 20 = 18.707

4.4 Results for the Full Model - "Moving" Vehicle

For the "moving" vehicle response curves are plotted against the forward speed of the vehicle. The input frequency is a function of the wave length and the speed of the vehicle.

Because creep is considered, response curves for the body and frames were calculated in all six modes, and in the pitch direction for motors and wheelsets. Steady state responses for the two cases of new wheels and worn wheels are given.

1. For the case of new wheels:
   - a) Figures 19 to 24 give the displacements for the body,
   - b) Figures 25 to 30 give the displacements for the front frame,
   - c) Figure 31 gives the pitch displacement for motor No. 5,
   - d) Figure 32 gives the pitch displacement for wheelset No. 5.

2. For the case of worn wheels:
   - a) Figures 33 to 38 give the displacements for the body,
   - b) Figures 39 to 44 give the displacements for the front frame,
   - c) Figure 45 gives the pitch displacement for motor No. 5,
   - d) Figure 46 gives the pitch displacement for wheelset No. 5.

Comparison of the response curves for the cases of new wheels and worn wheels shows that the condition of the wheel does not affect the shape of the response curves in general except for peak responses.

Comparison of the response curves for the "moving" vehicle and the "stationary" vehicle indicates that creep forces have a significant effect on peak responses.

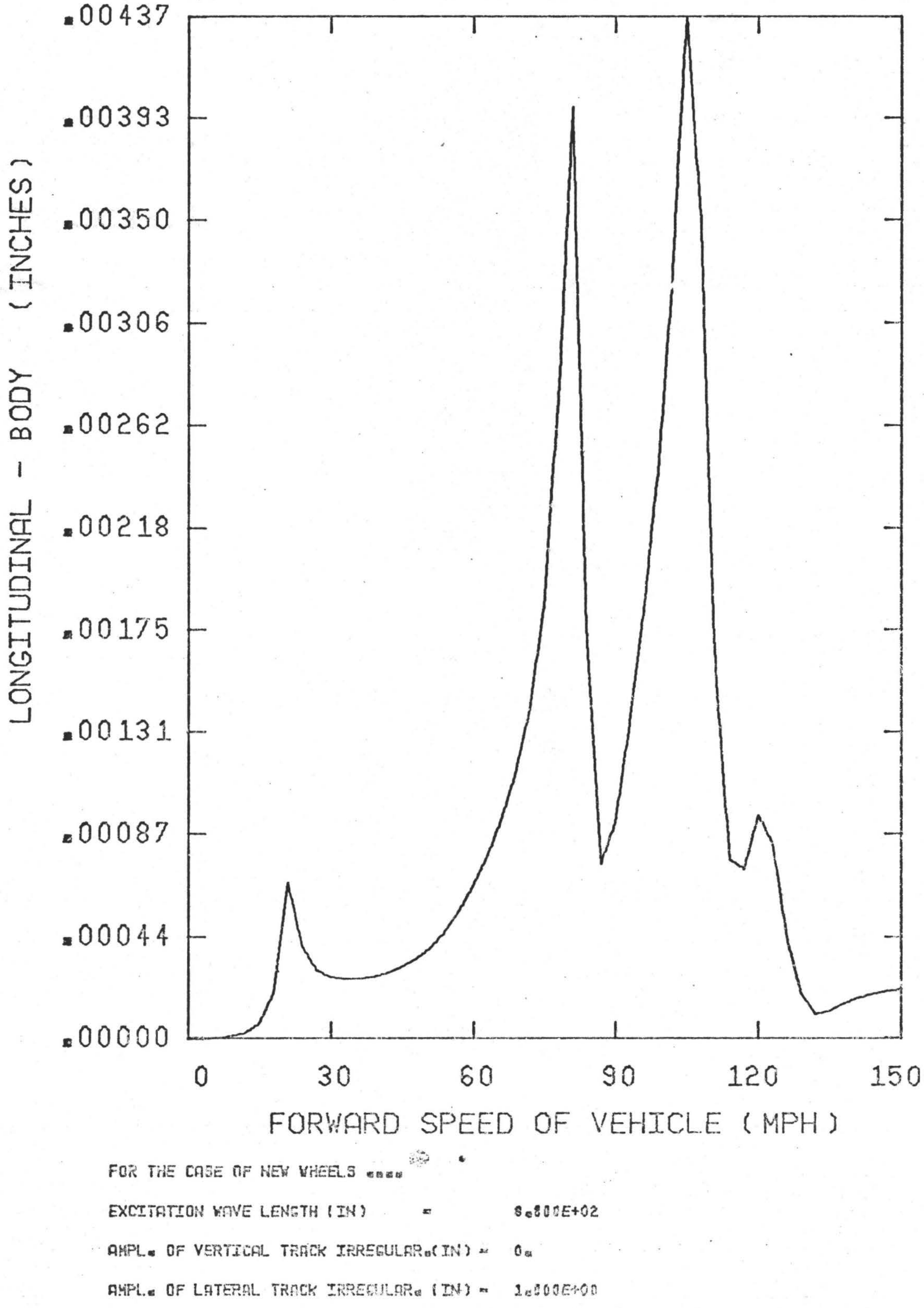
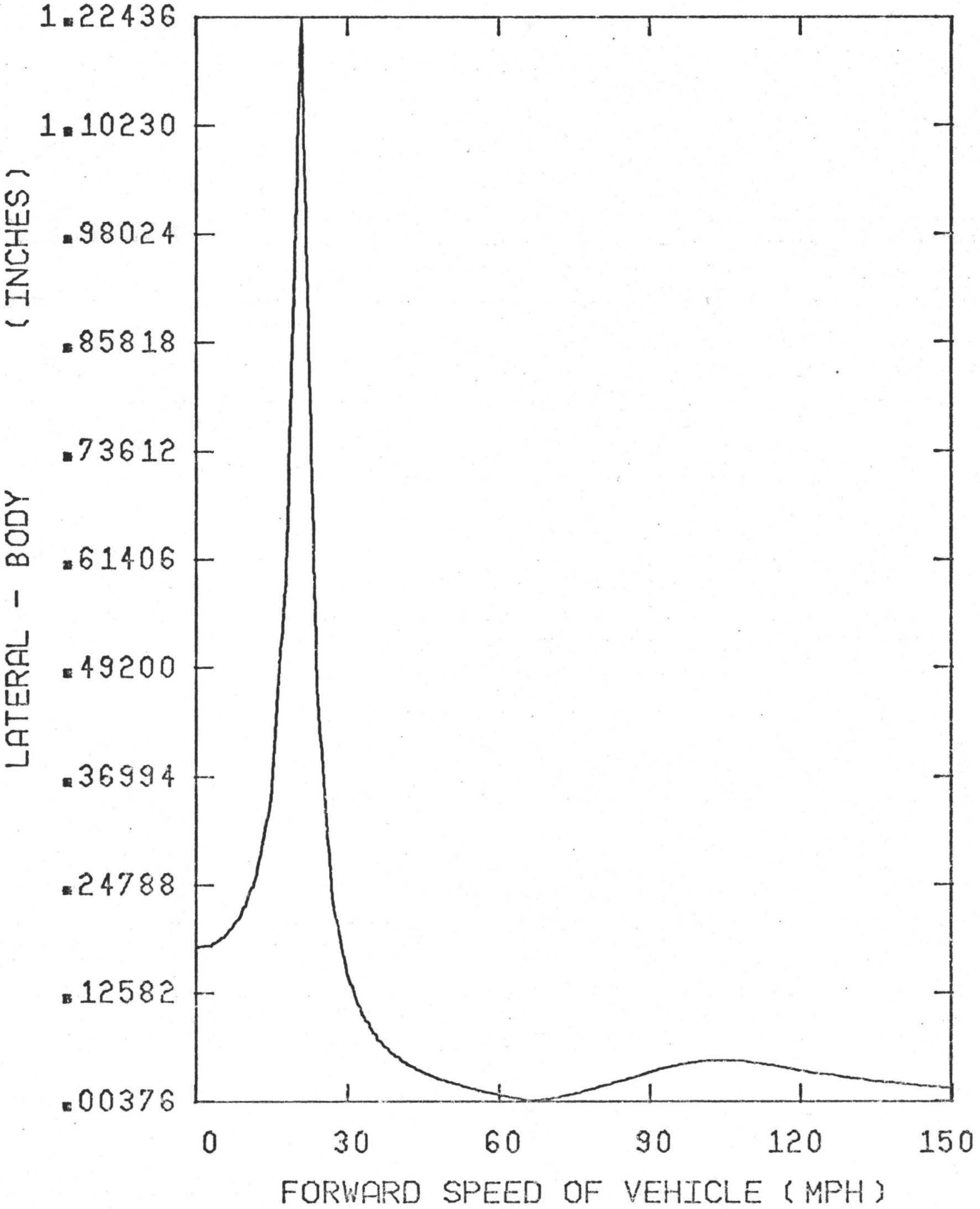


FIGURE 19: LONGITUDINAL BODY DISPLACEMENT (u_a) -- NEW WHEELS



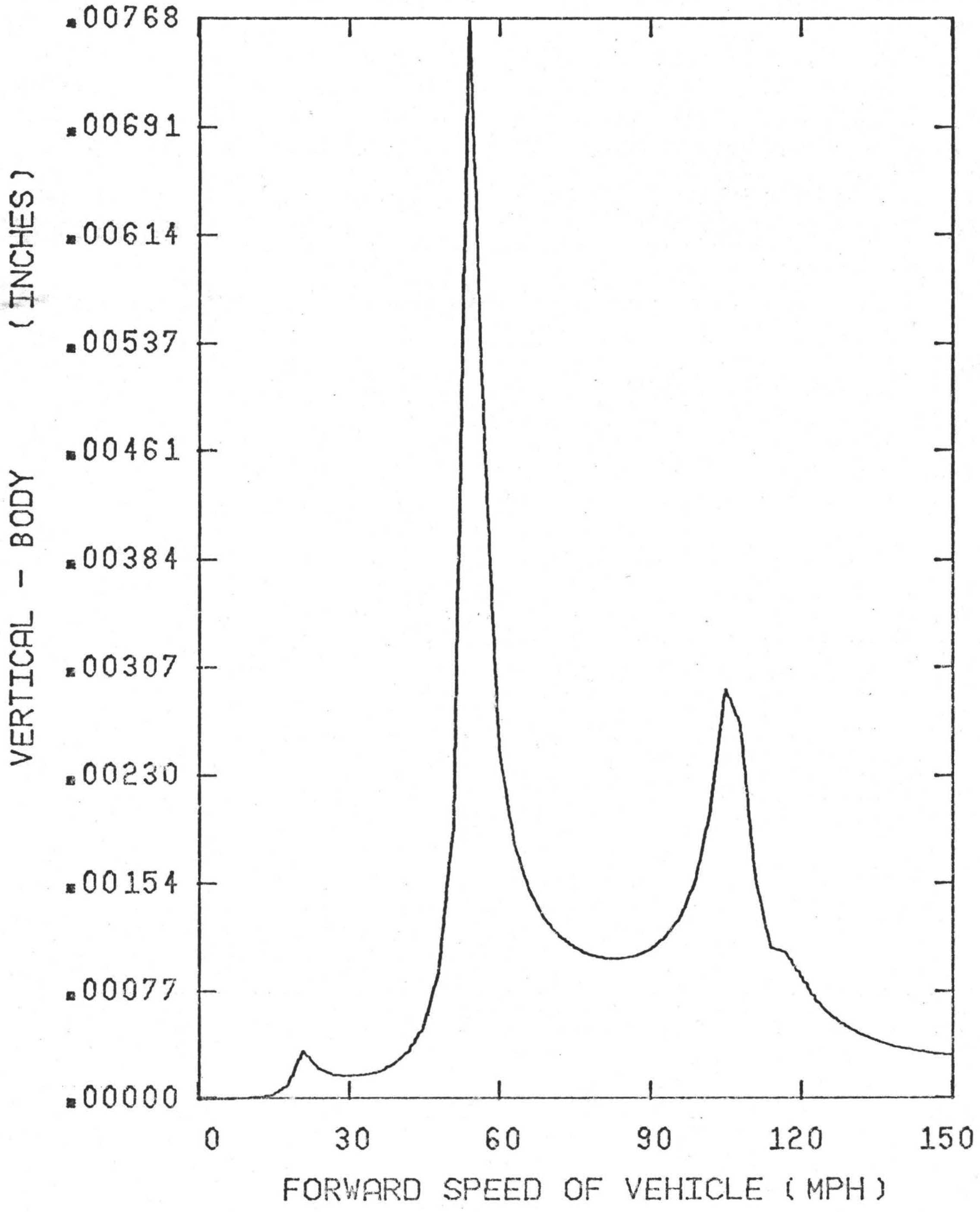
FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.000E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 20: LATERAL BODY DISPLACEMENT (v_a) - NEW WHEELS



FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 6.000E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 21: VERTICAL BODY DISPLACEMENT (w_a) - NEW WHEELS

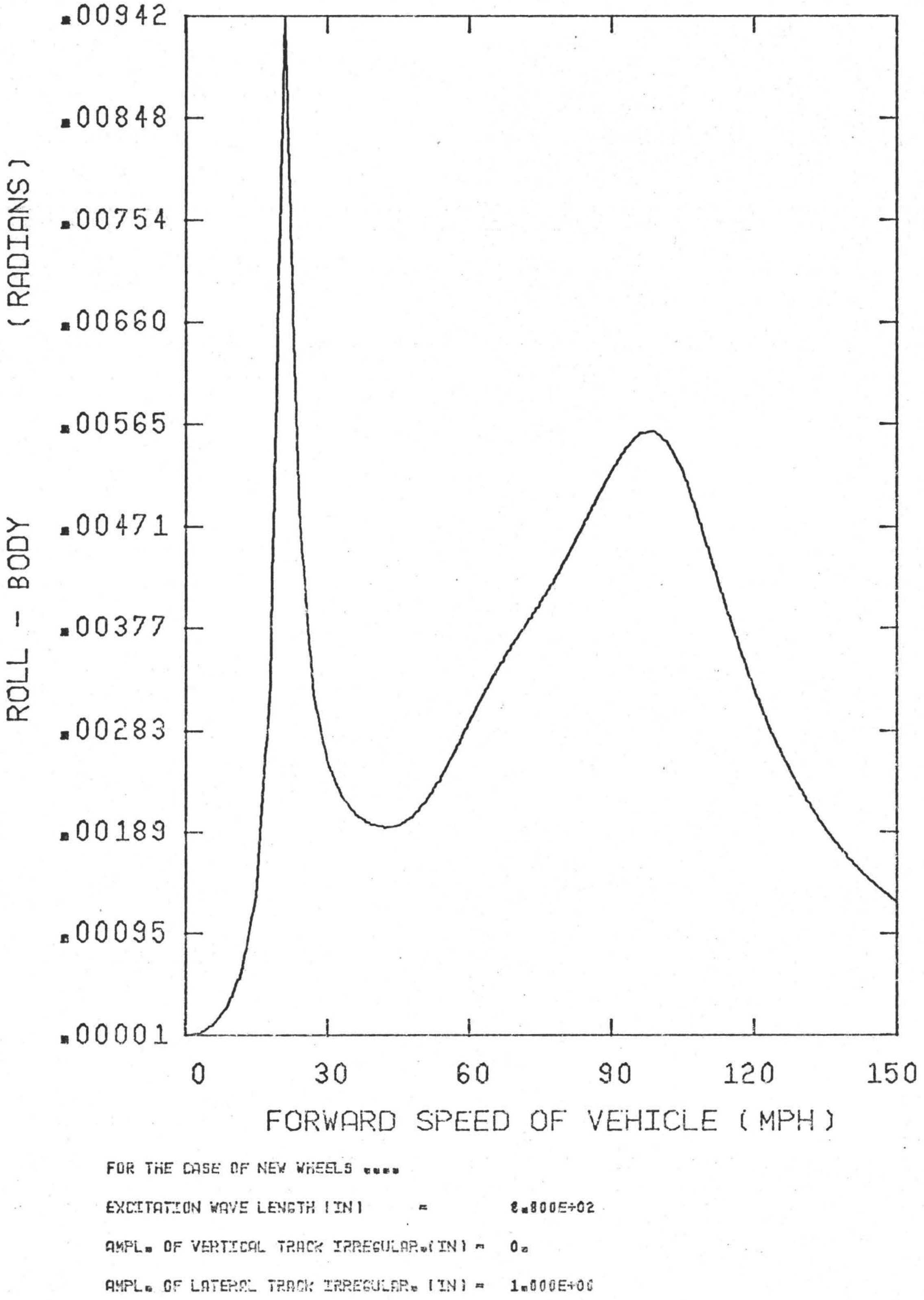
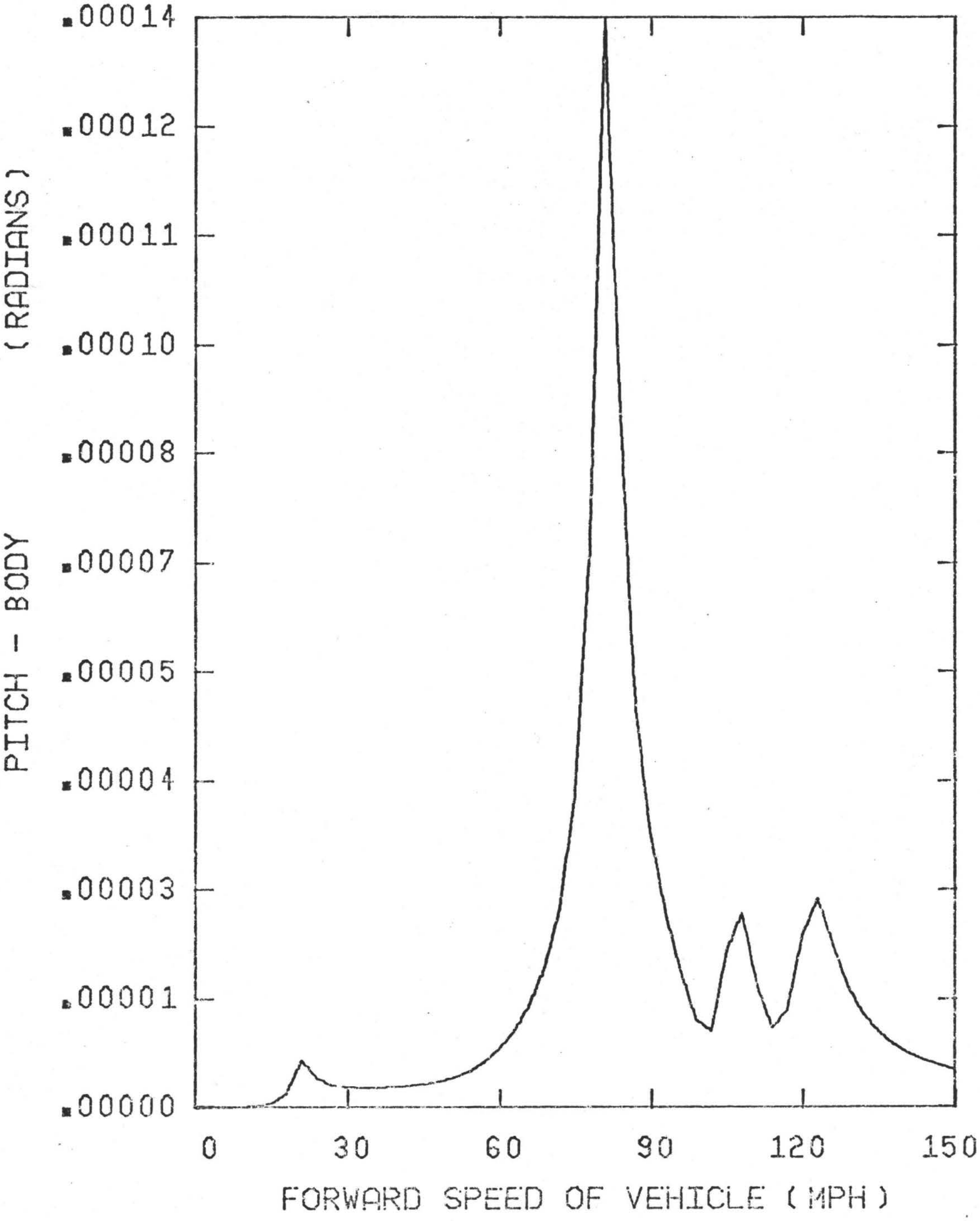


FIGURE 22: ROLL BODY DISPLACEMENT (α_a) - NEW WHEELS



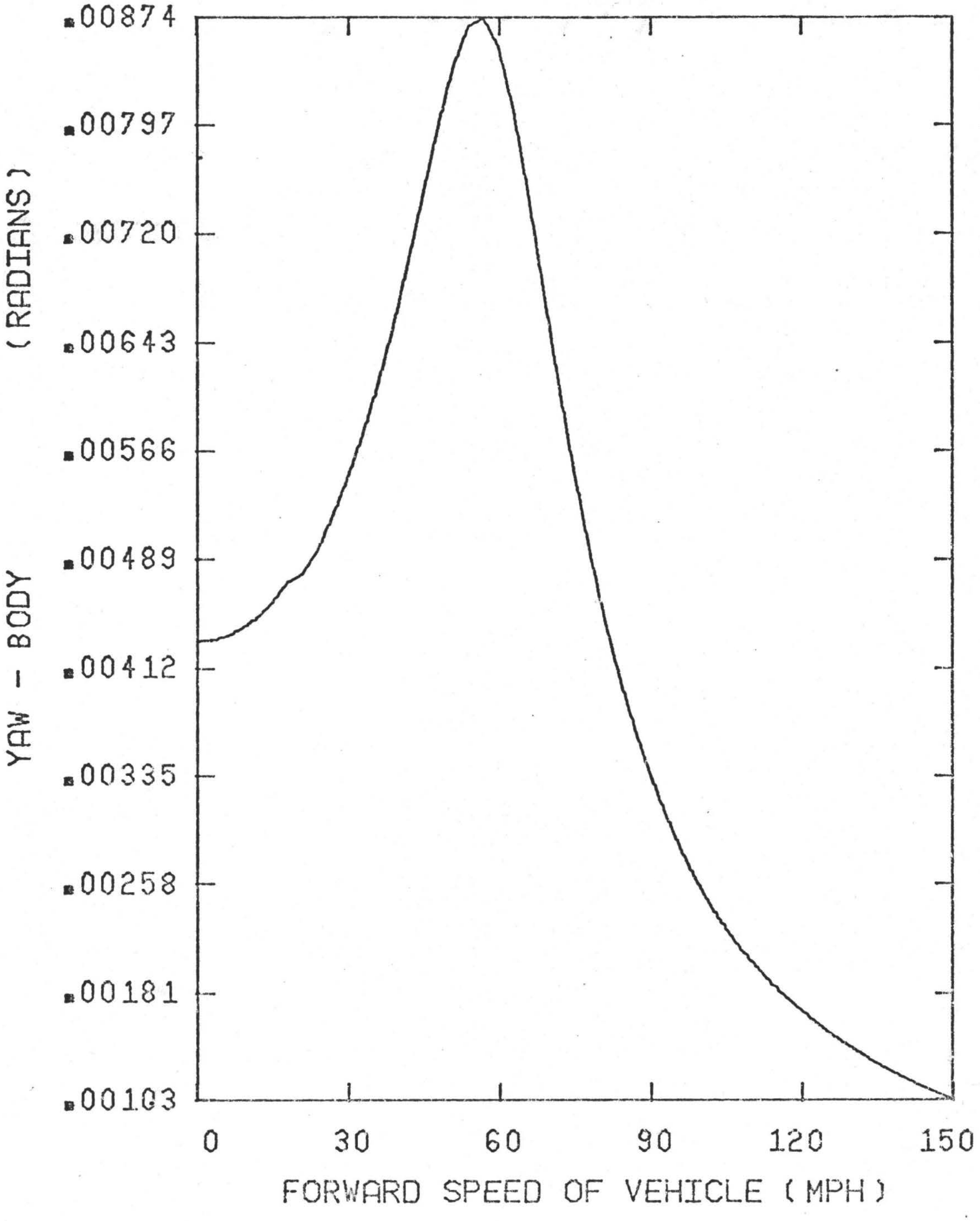
FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 23: PITCH BODY DISPLACEMENT (β_a) - NEW WHEELS



FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.600E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 24: YAW BODY DISPLACEMENT (γ_a) - NEW WHEELS

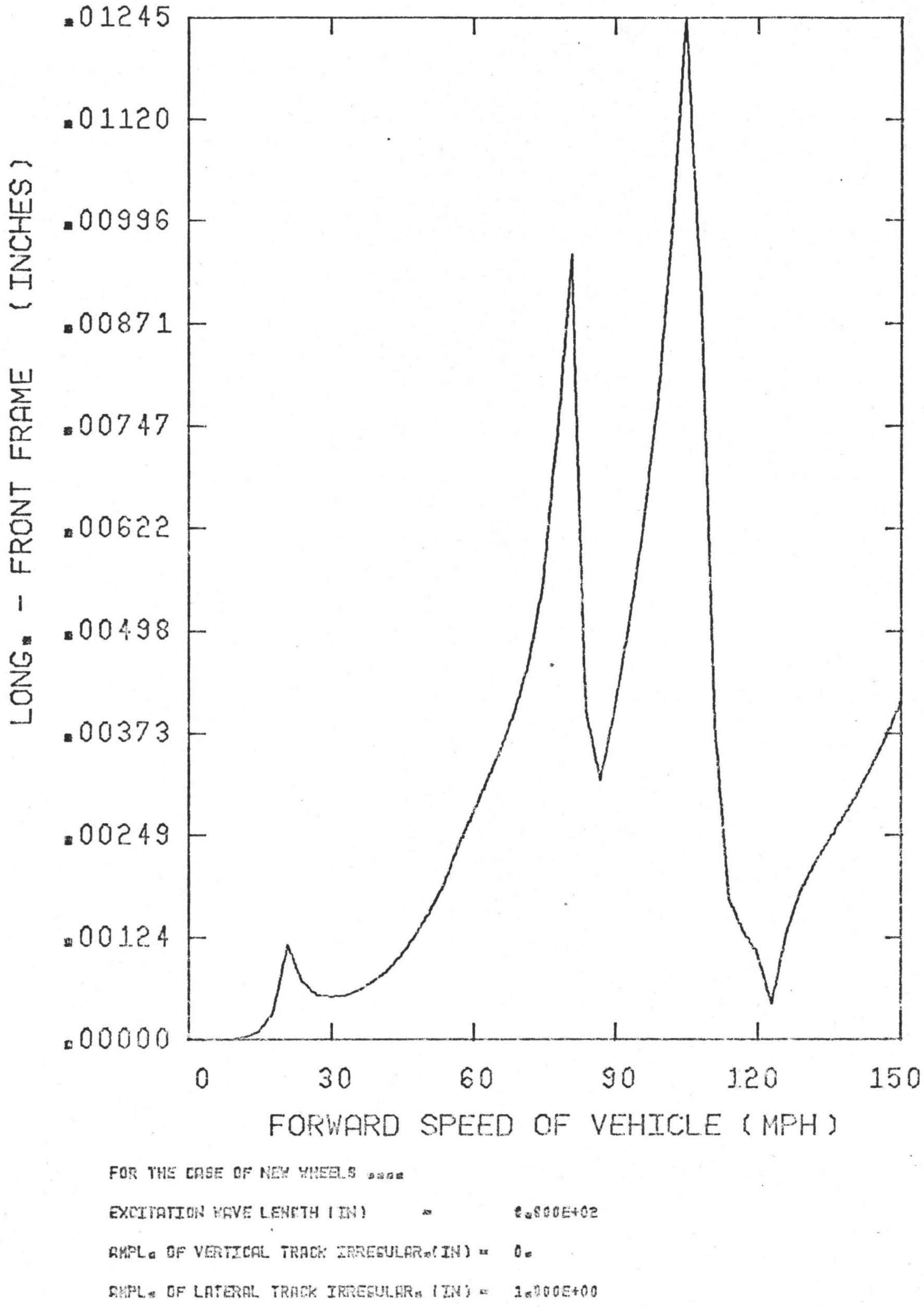


FIGURE 25: LONGITUDINAL FRONT FRAME DISPLACEMENT (u_{bf}) - NEW WHEELS

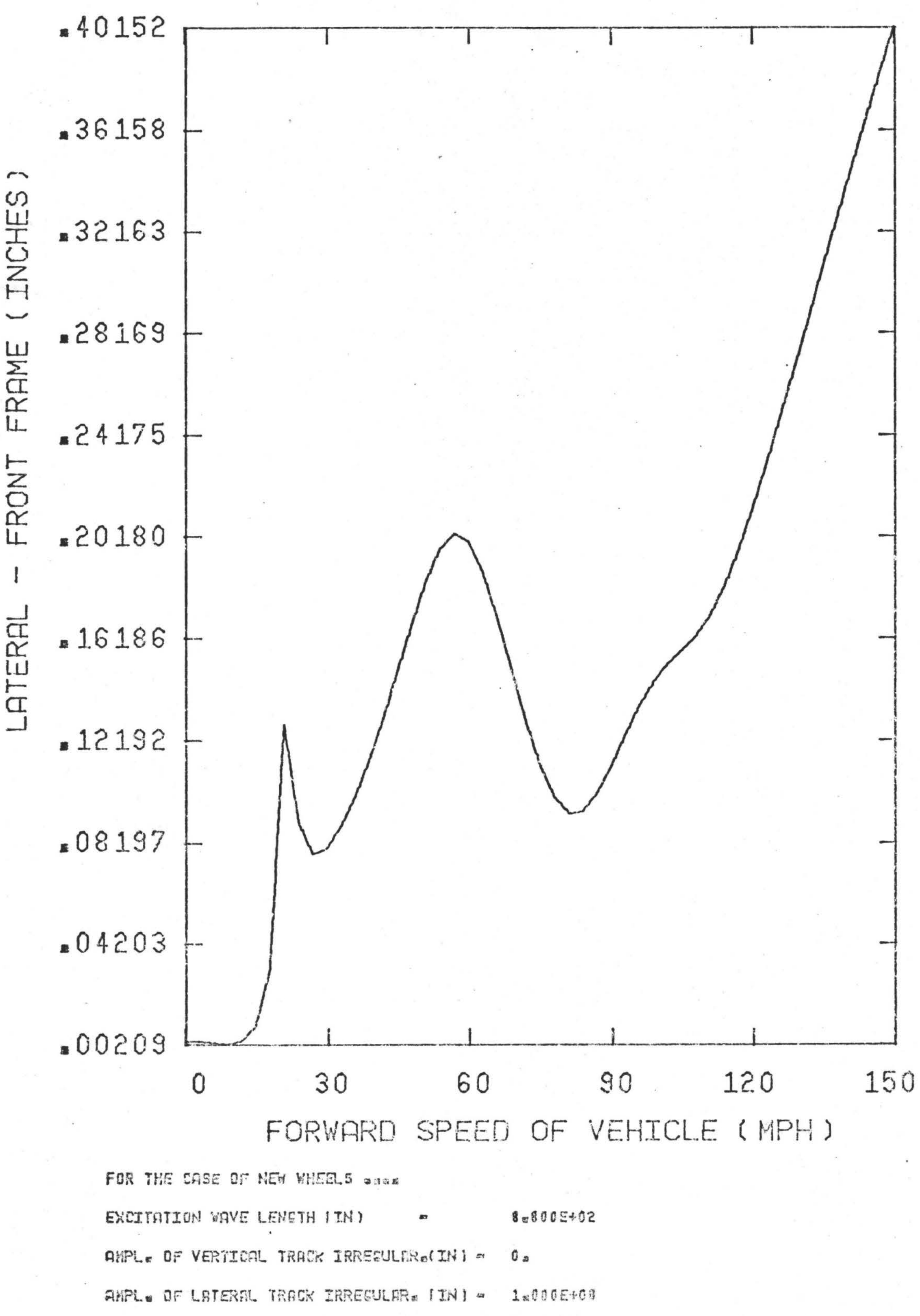
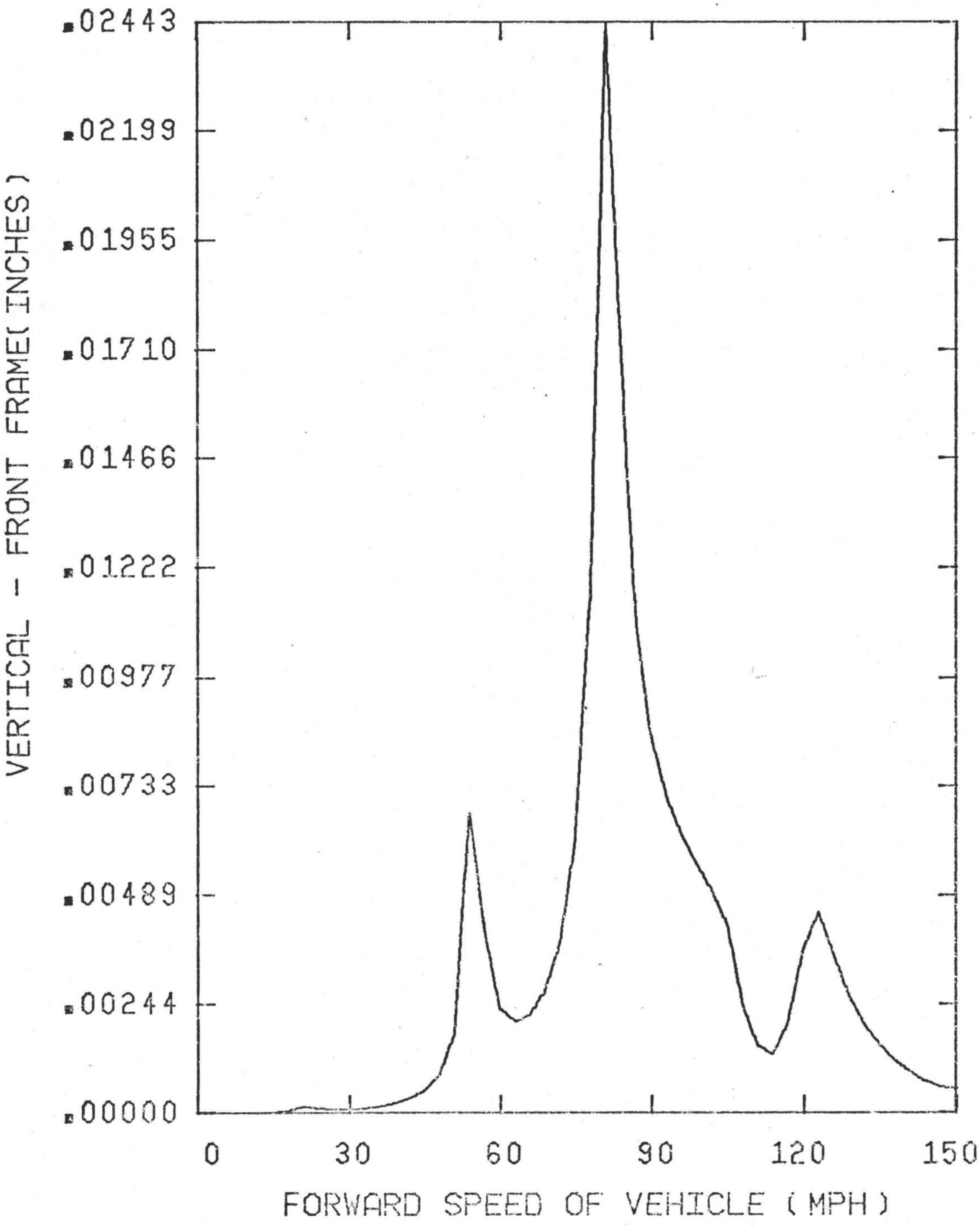


FIGURE 26: LATERAL FRONT FRAME DISPLACEMENT (v_{bf}) - NEW WHEELS



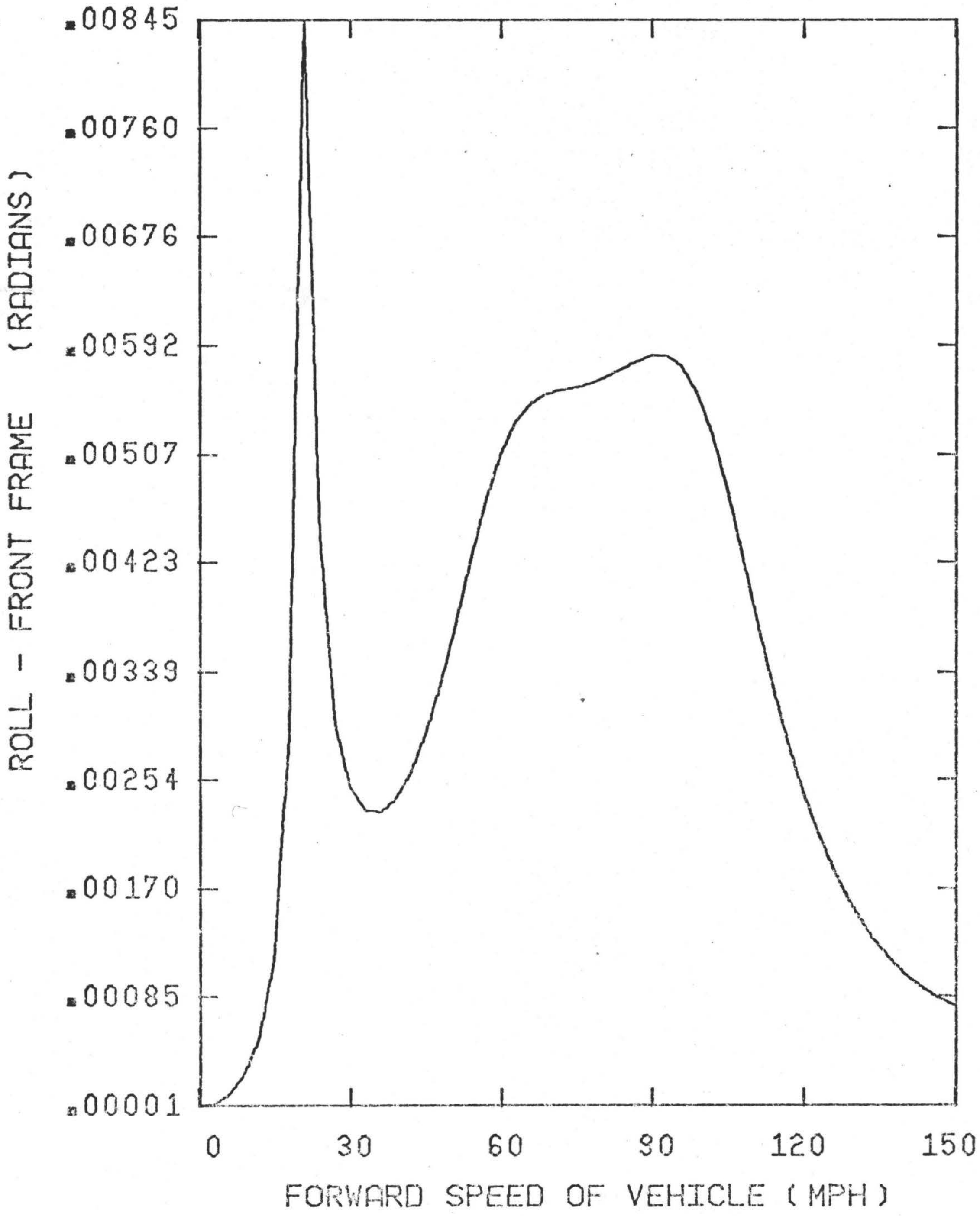
FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 27: VERTICAL FRONT FRAME DISPLACEMENT (w_{bf}) - NEW WHEELS



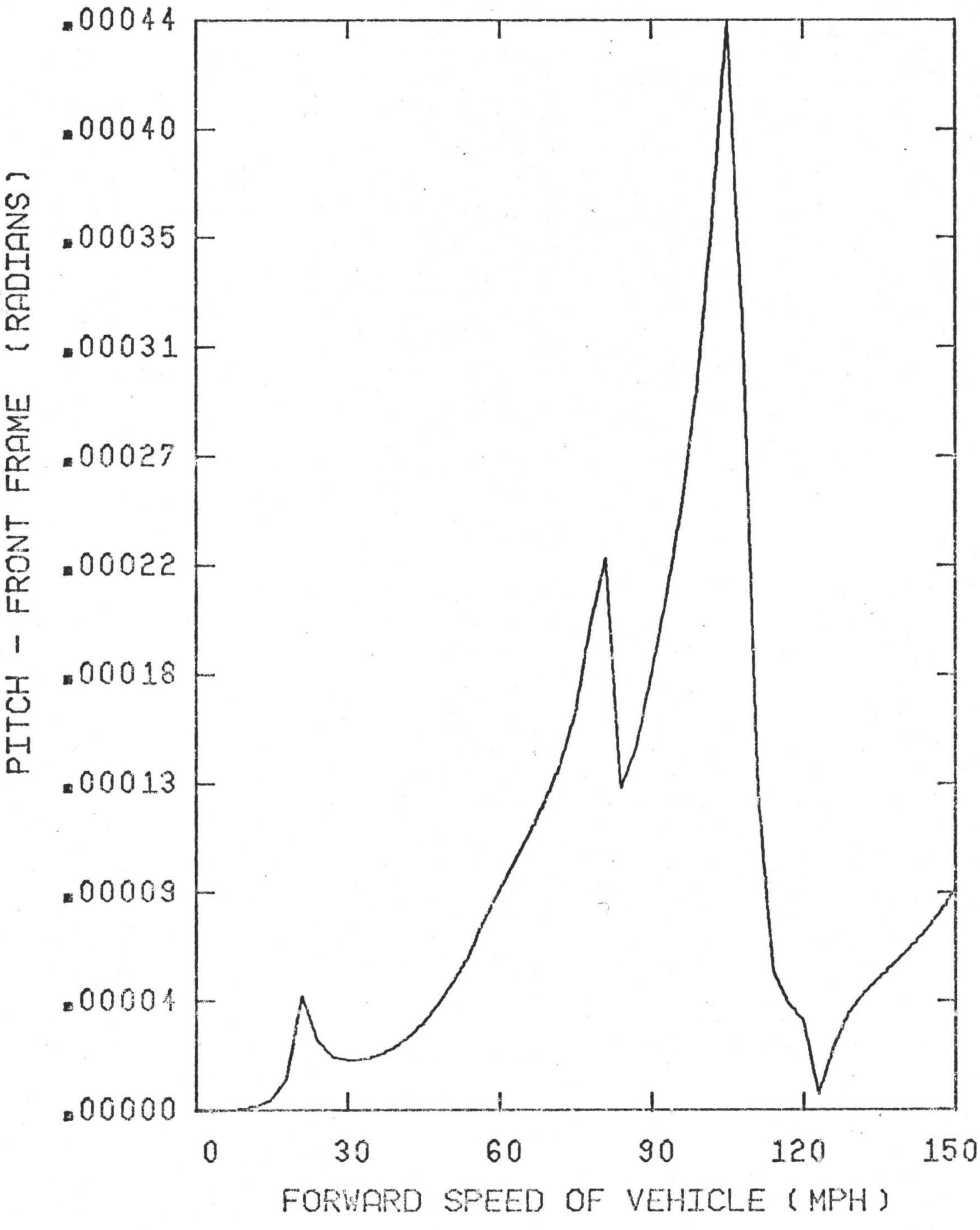
FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.600E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 28: ROLL FRONT FRAME DISPLACEMENT (α_{bf}) - NEW WHEELS



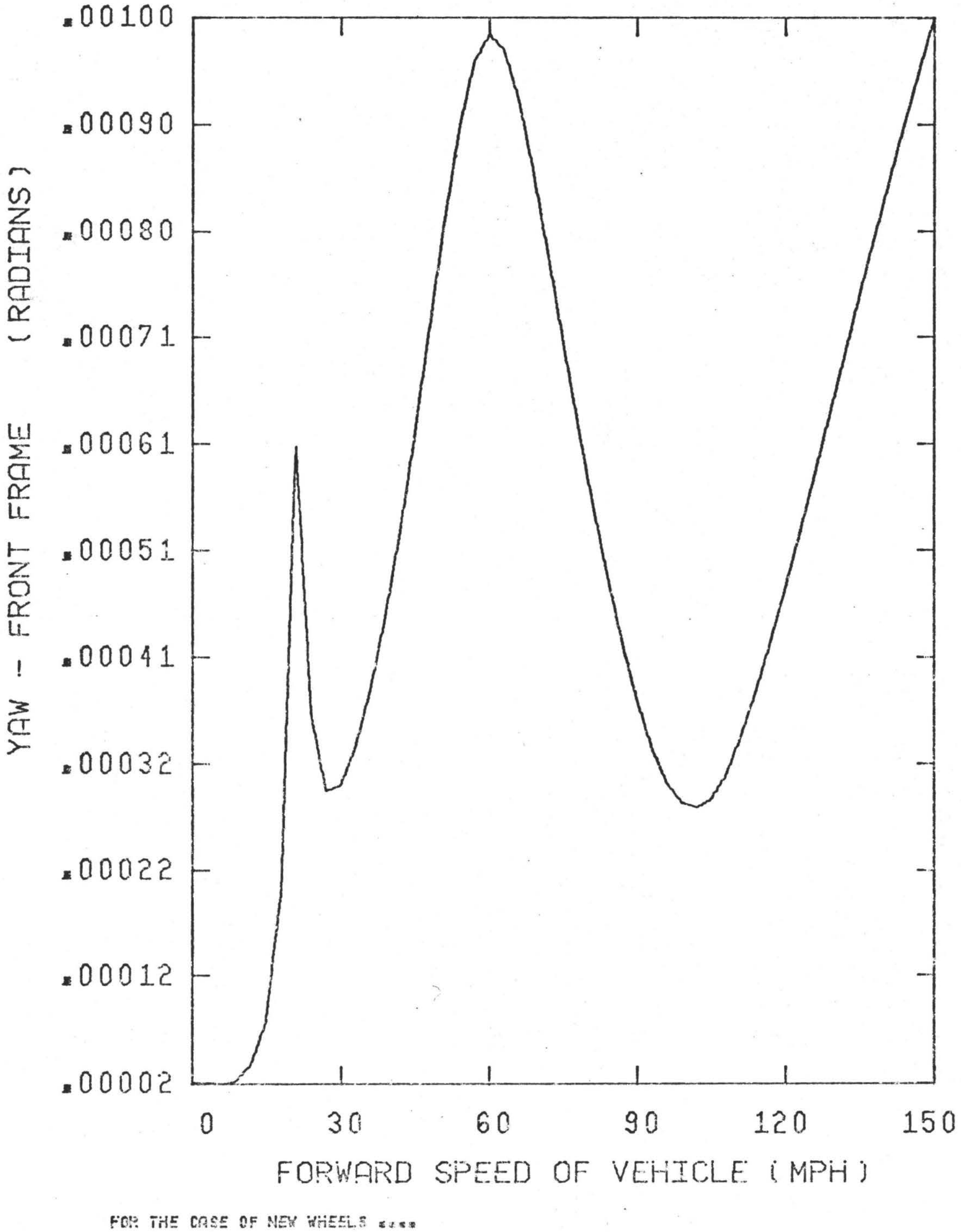
FOR THE CASE OF NEW WHEELS

EXCITATION WAVE LENGTH (IN) = 6.600E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 29: PITCH FRONT FRAME DISPLACEMENT (β_{bf}) - NEW WHEELS



FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 30: YAW FRONT FRAME DISPLACEMENT (γ_{bf}) - NEW WHEELS

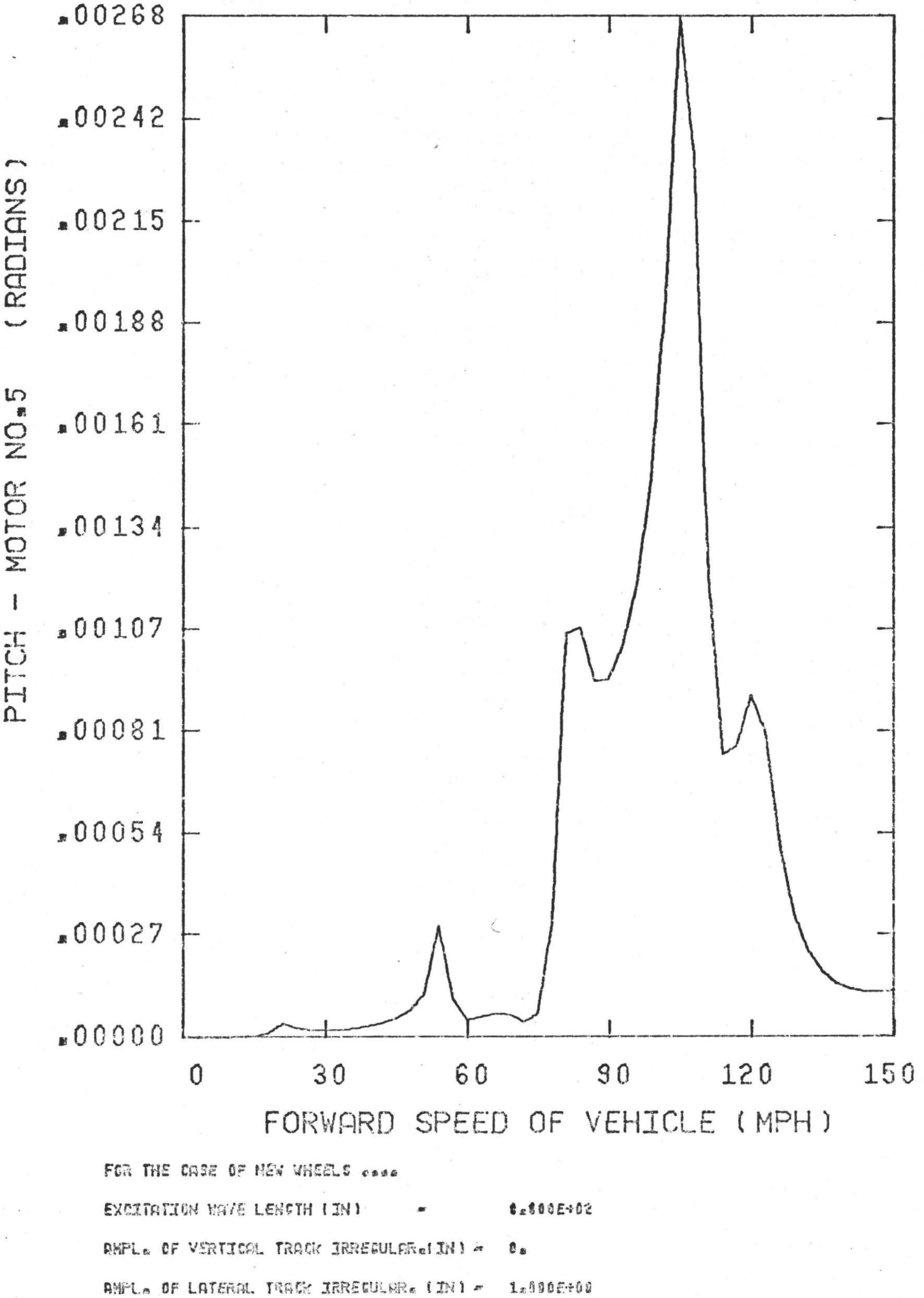
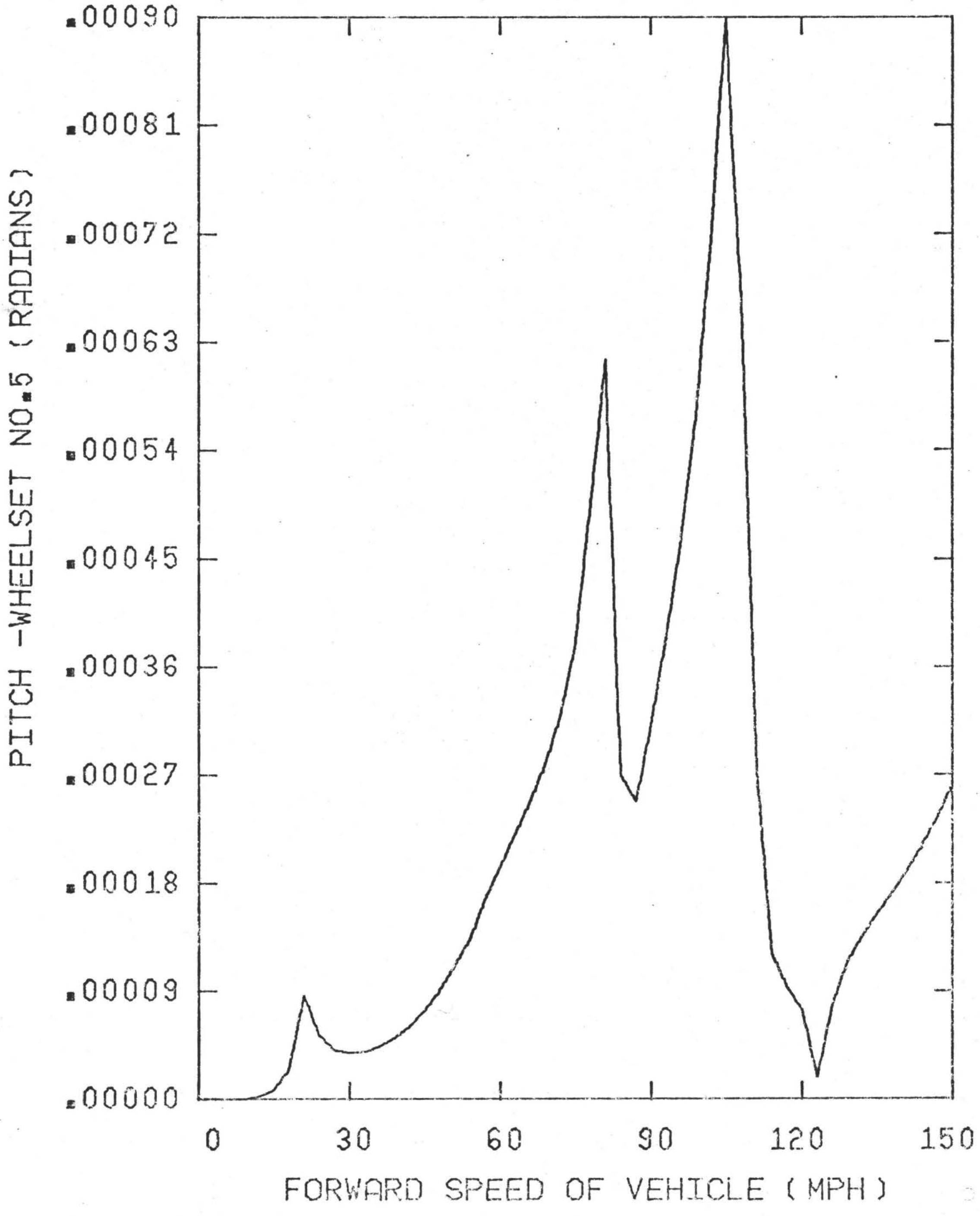


FIGURE 31 - PITCH MOTOR NO. 5 DISPLACEMENT (u_{c5}) - NEW WHEELS



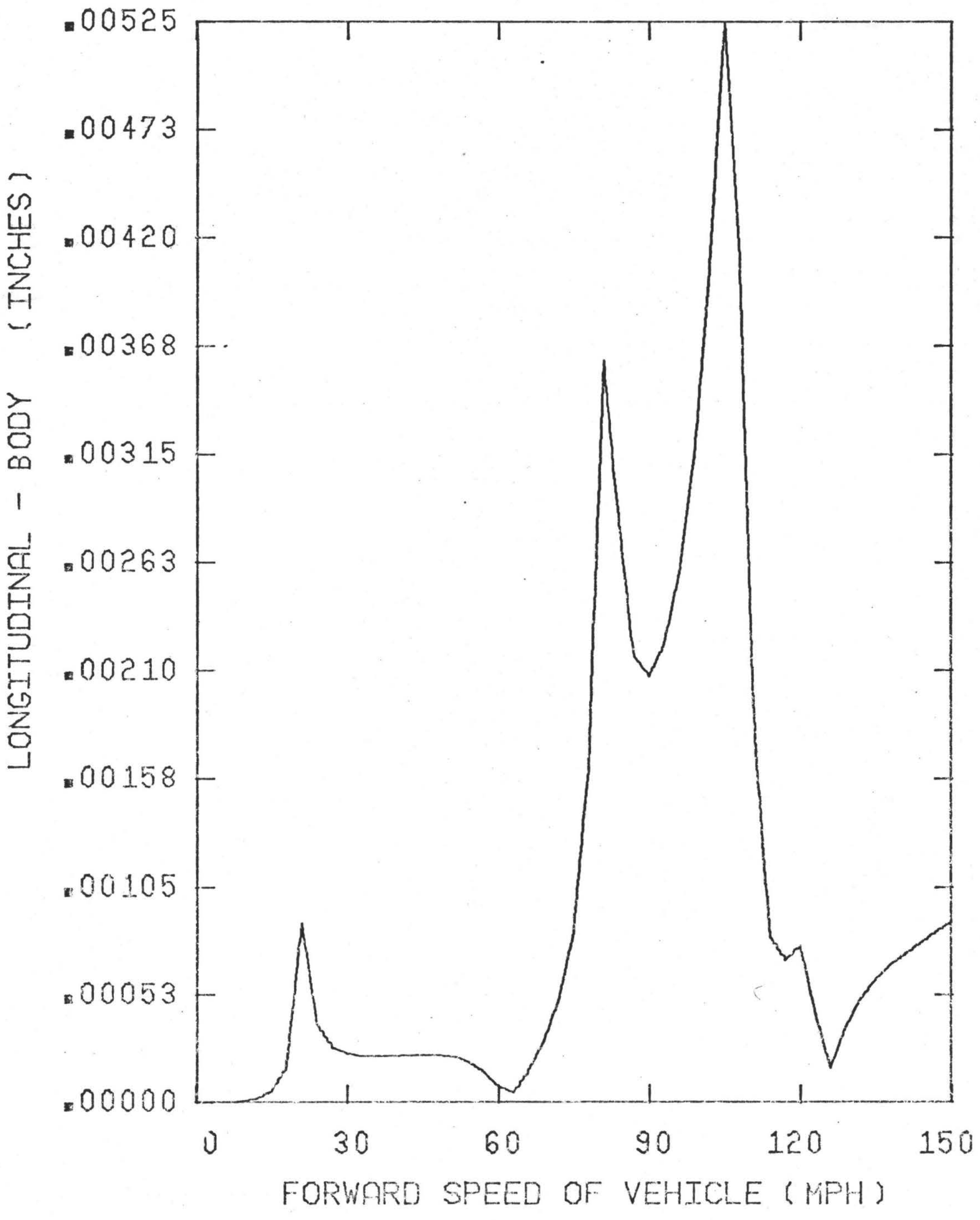
FOR THE CASE OF NEW WHEELS ....

EXCITATION WAVE LENGTH (IN) = 0.500E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 32: PITCH WHEELSET NO. 5 DISPLACEMENT (u_{d5}) - NEW WHEELS



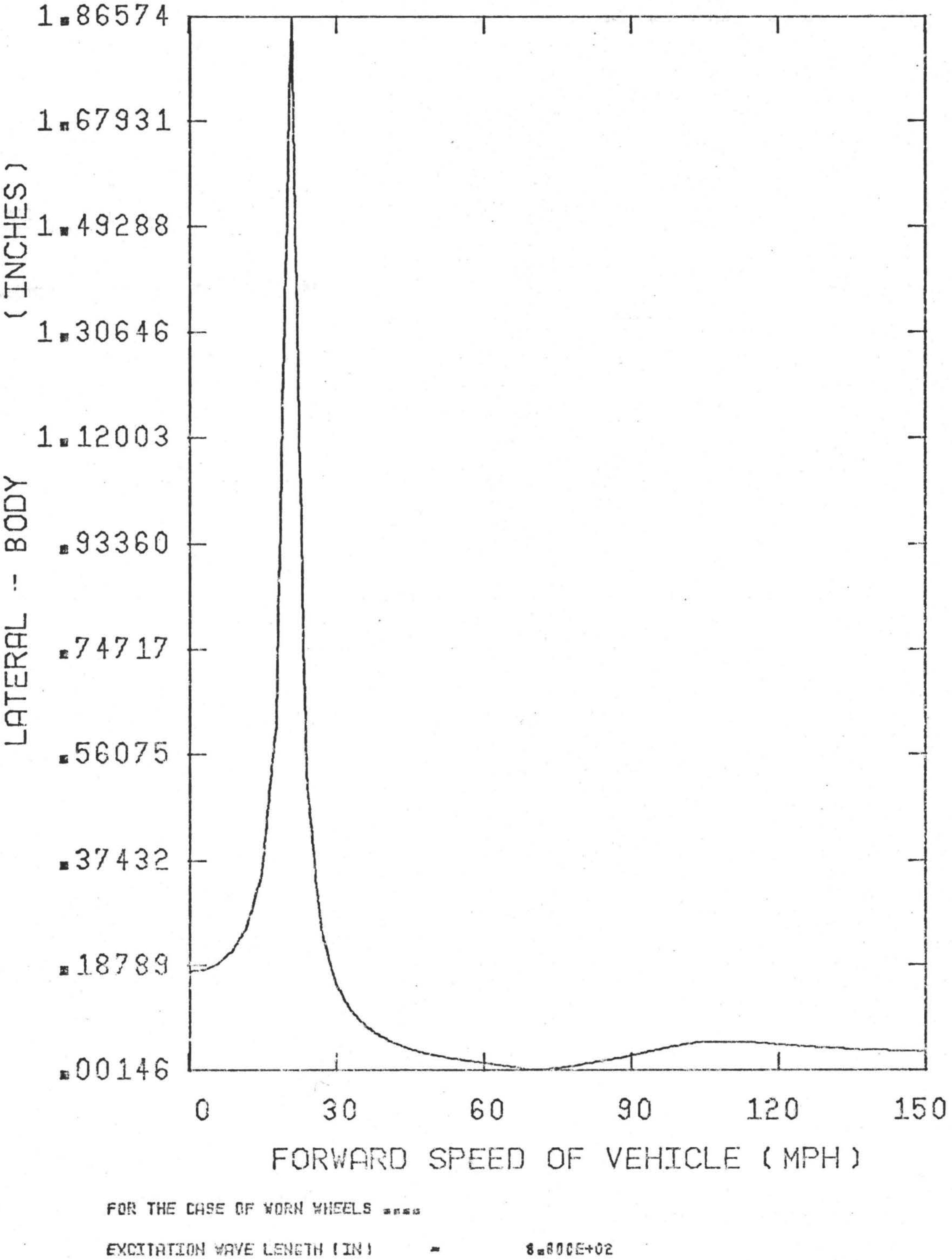
FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.600E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 33: LONGITUDINAL BODY DISPLACEMENT (u_a) - WORN WHEELS



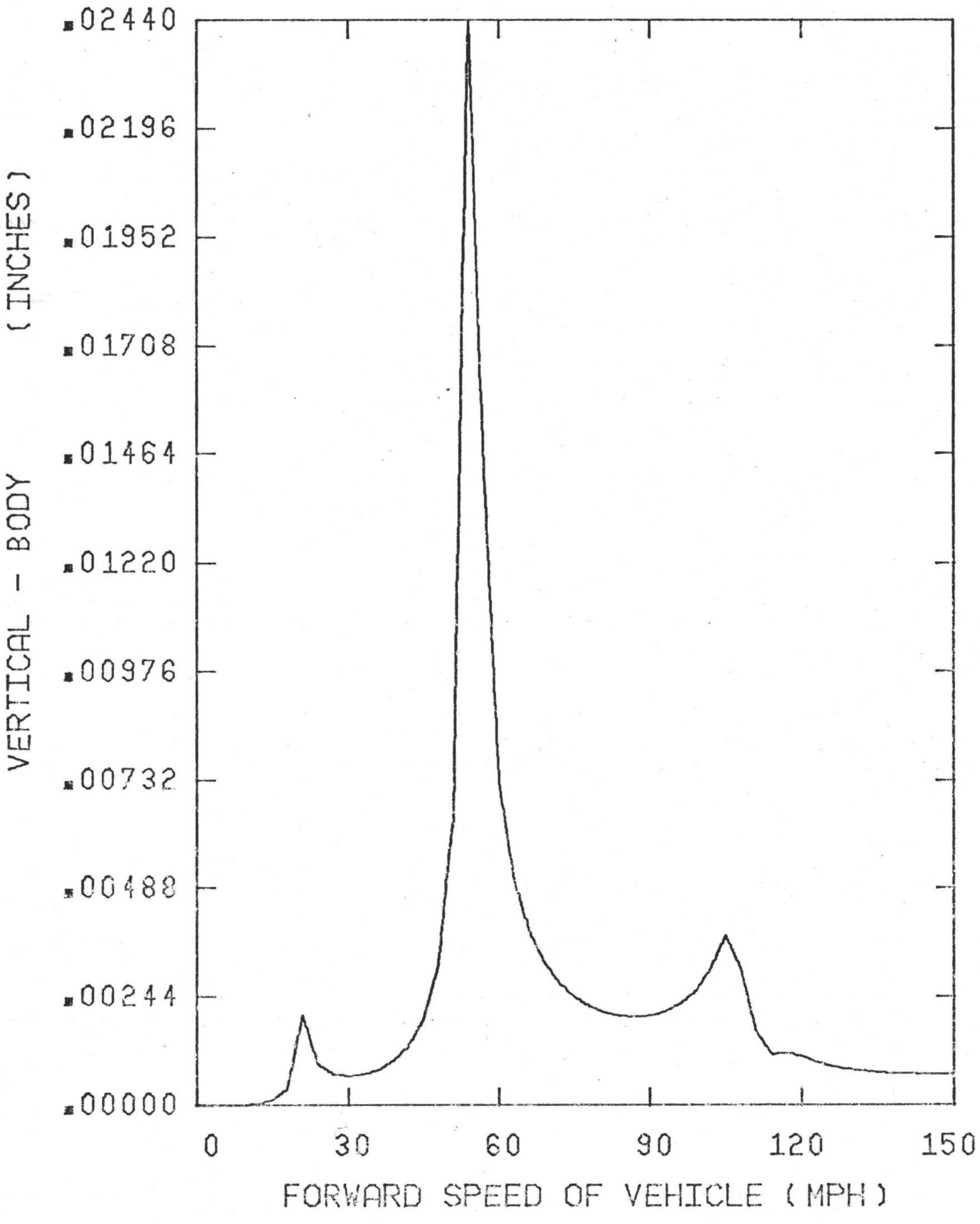
FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 34: LATERAL BODY DISPLACEMENT (v_a) - WORN WHEELS



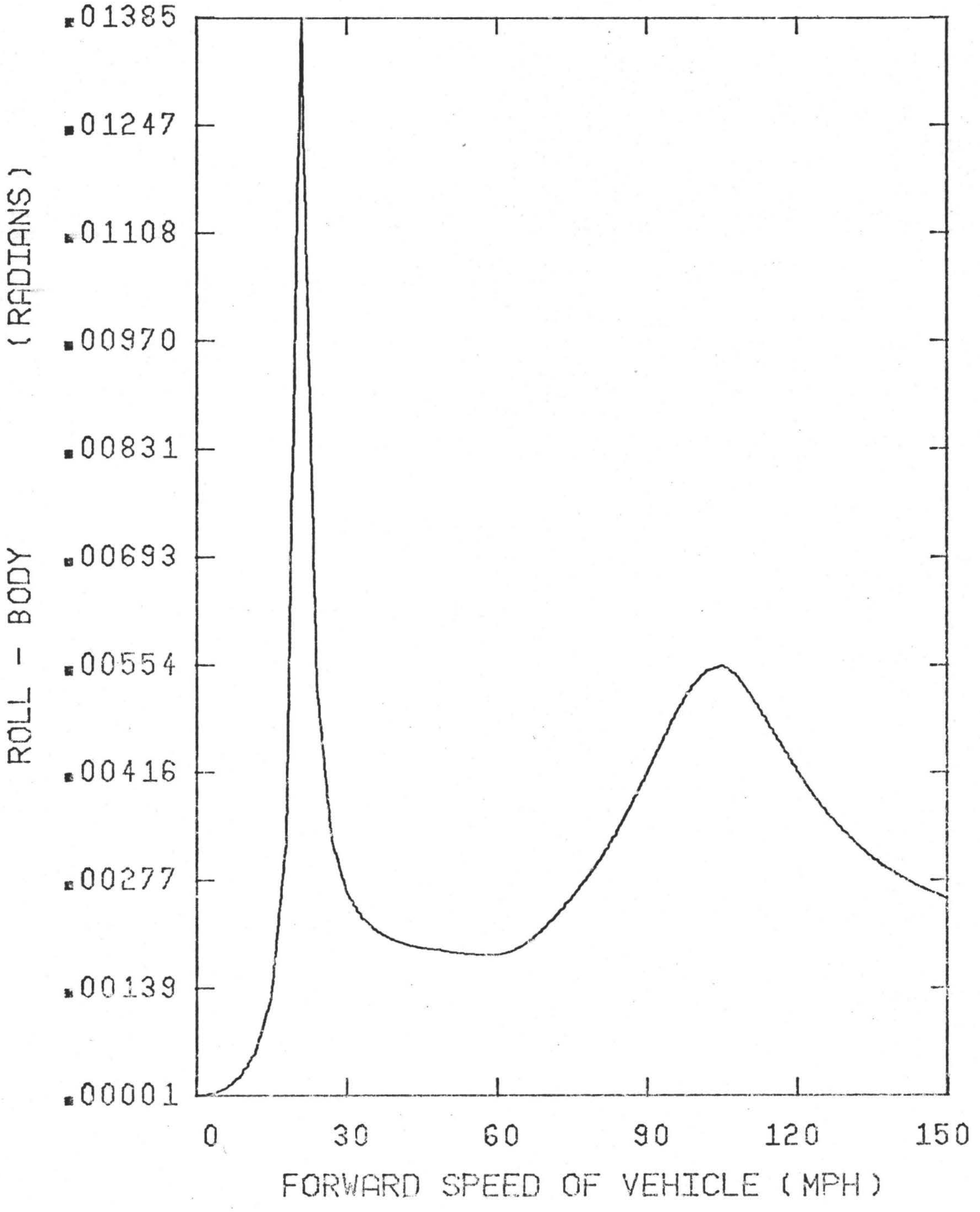
FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 35: VERTICAL BODY DISPLACEMENT (w_a) -- WORN WHEELS



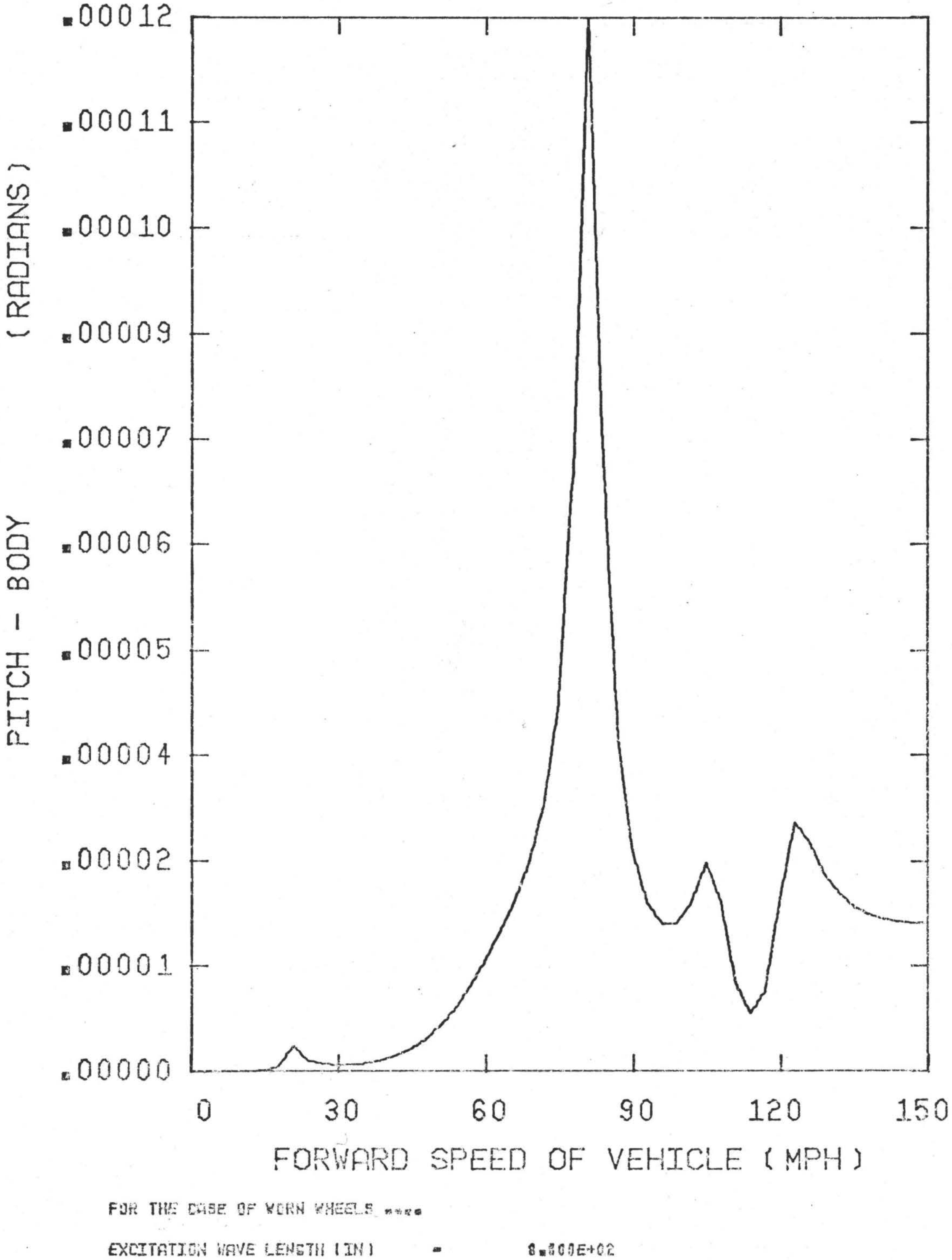
FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.600E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 36: ROLL BODY DISPLACEMENT (α_a) - WORN WHEELS



FOR THE CASE OF WORN WHEELS ****

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 37: PITCH BODY DISPLACEMENT (β_a) - WORN WHEELS

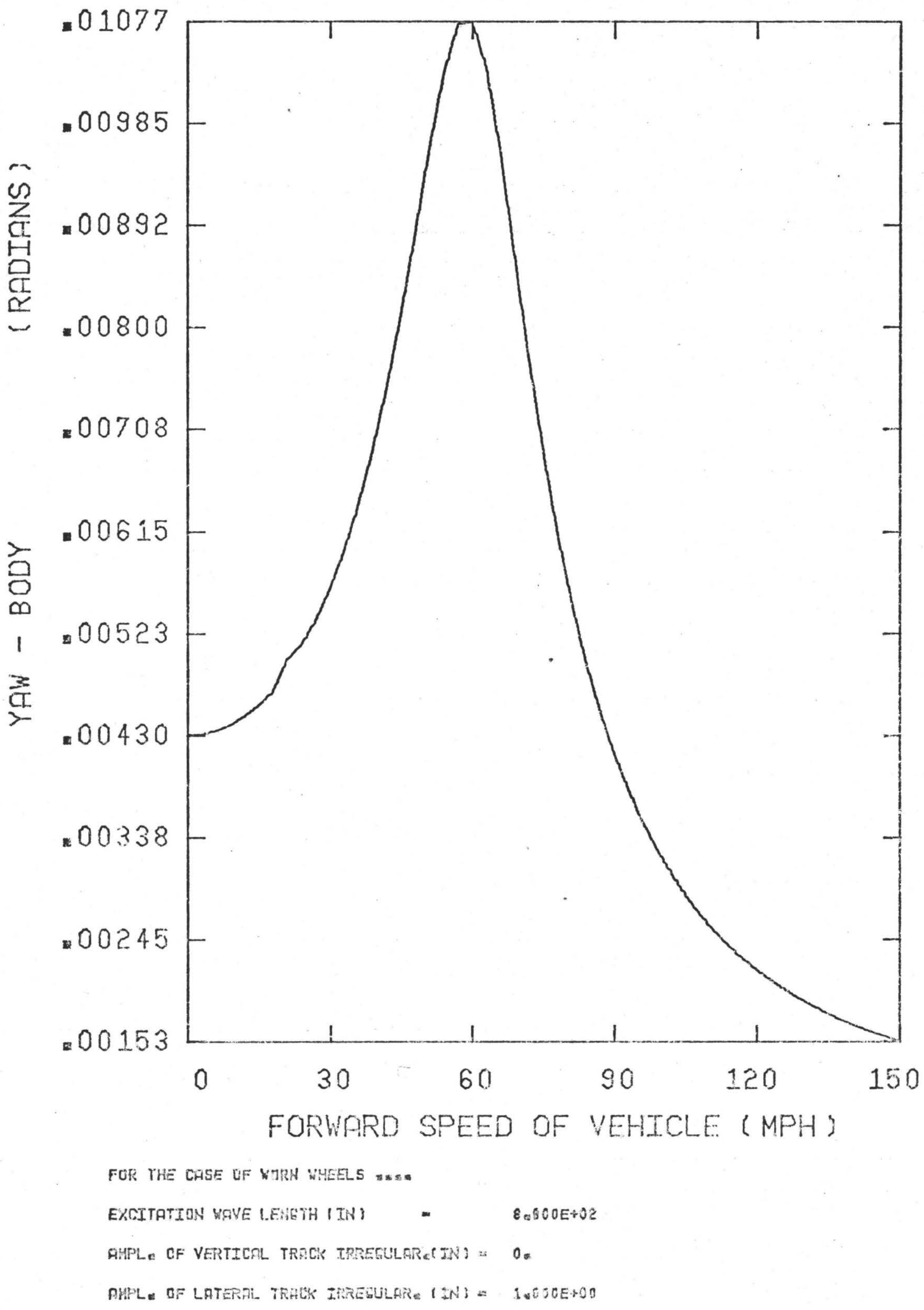


FIGURE 38: YAW BODY DISPLACEMENT (γ_a) - WORN WHEELS

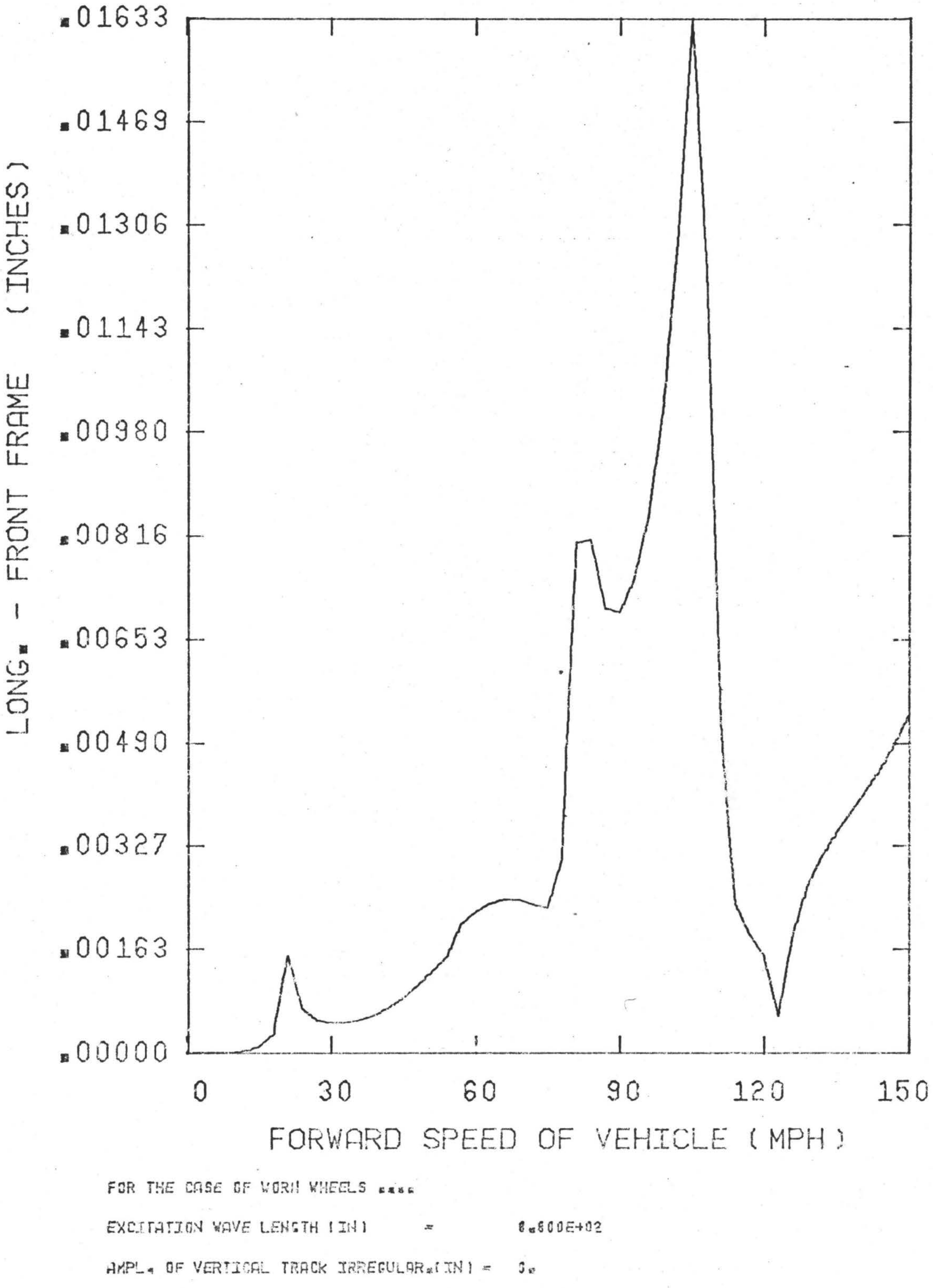


FIGURE 39: LONGITUDINAL FRONT FRAME DISPLACEMENT (u_{bf}) - WORN WHEELS

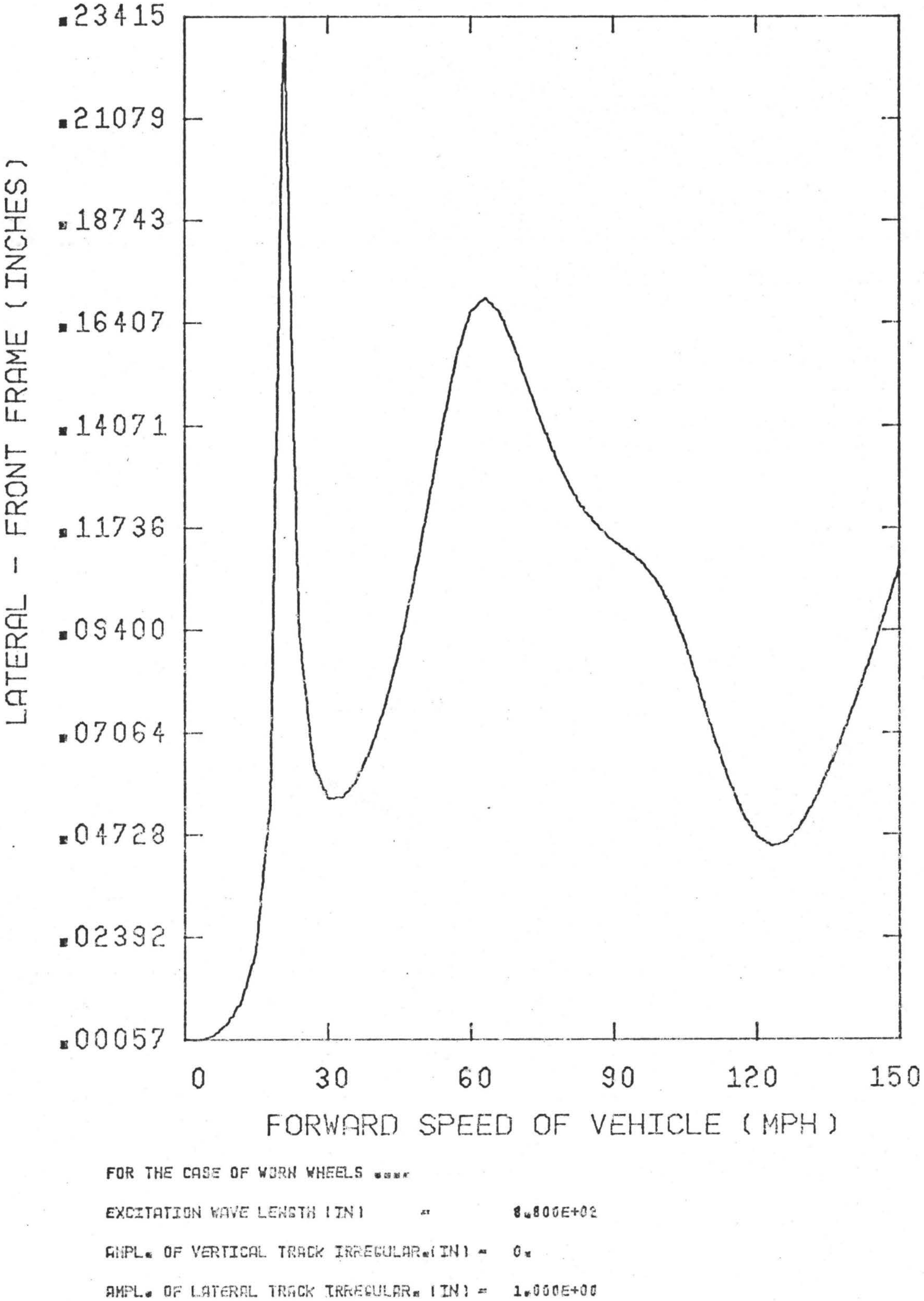
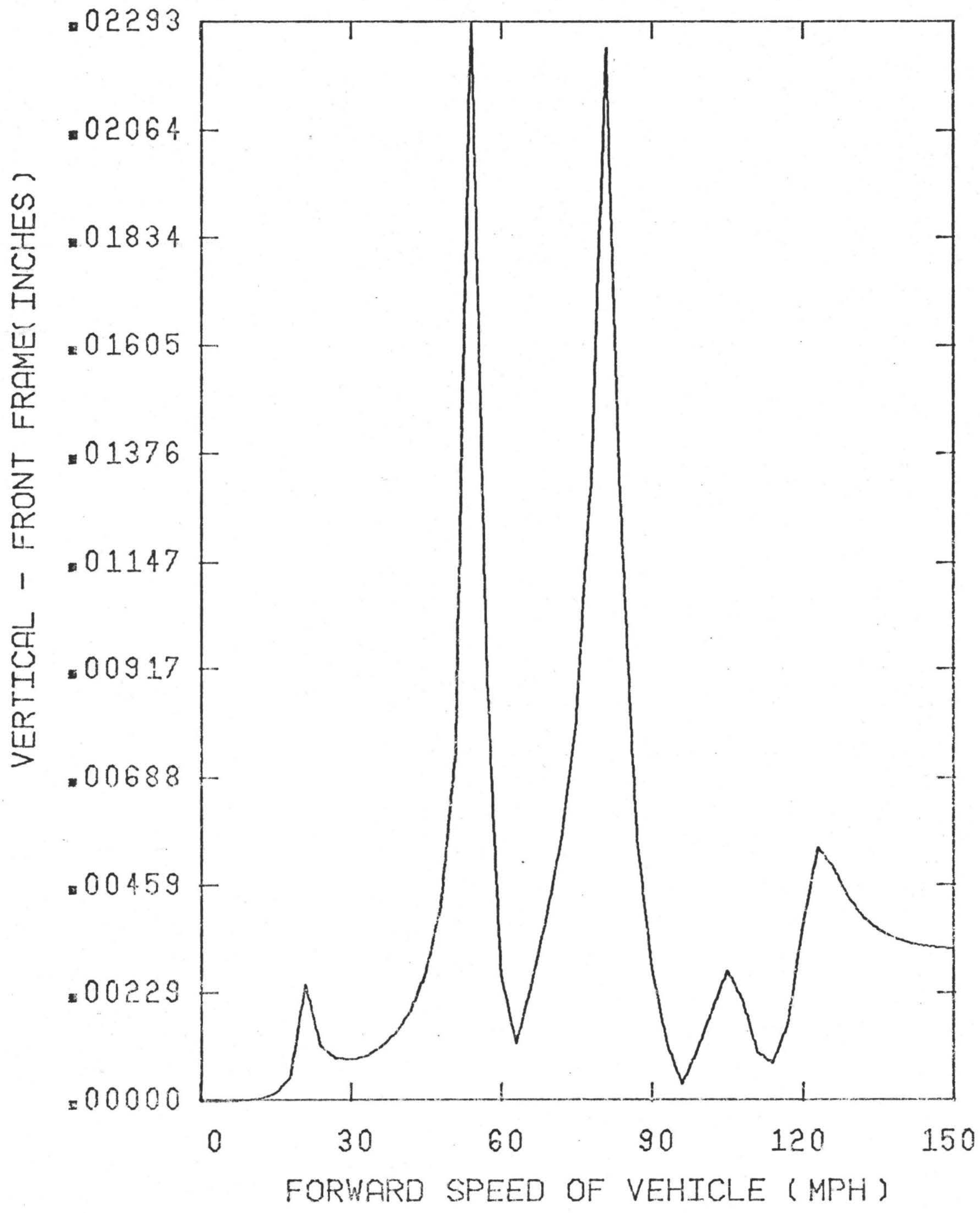


FIGURE 40: LATERAL FRONT FRAME DISPLACEMENT (v_{bf}) - WORN WHEELS



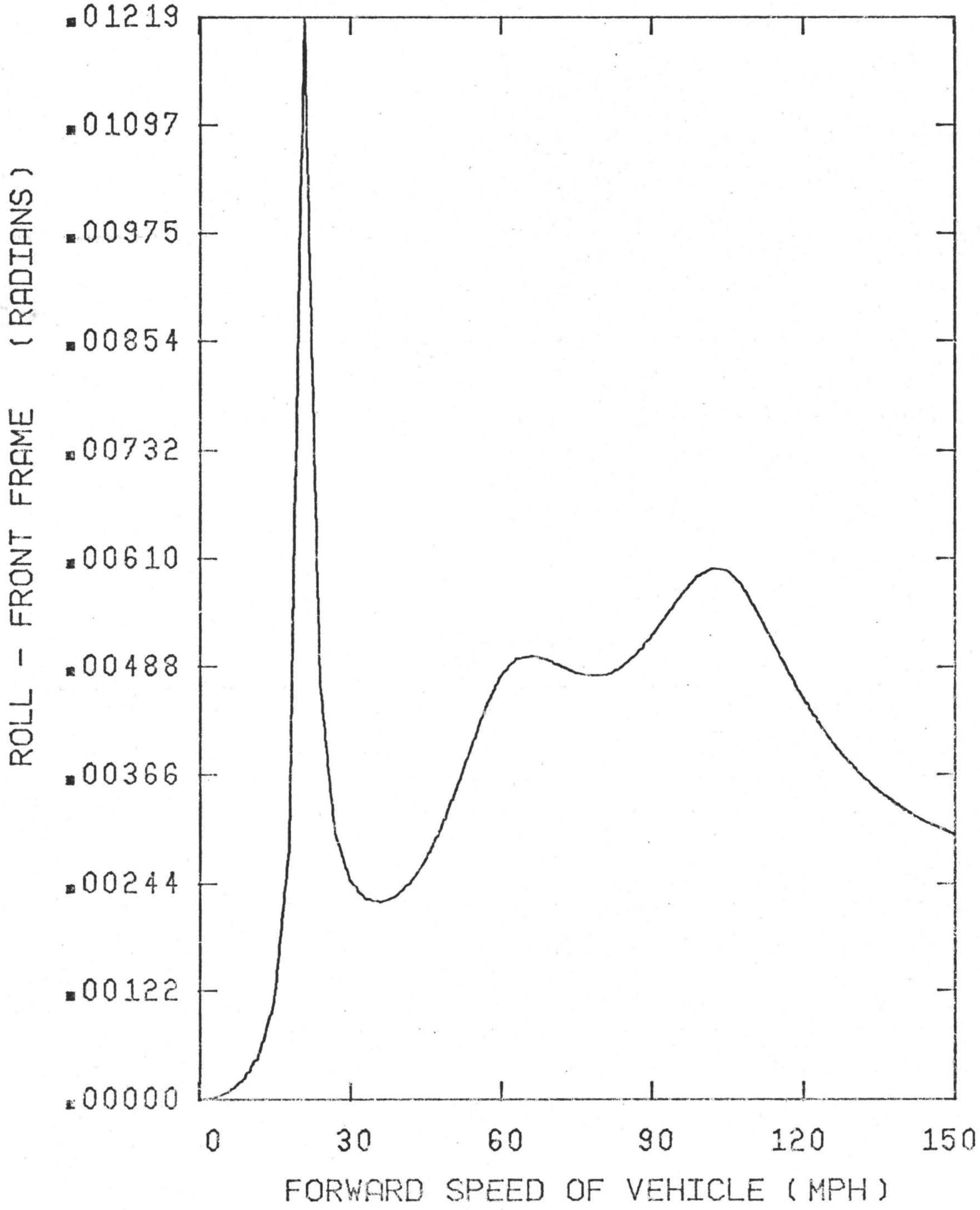
FOR THE CASE OF WORN WHEELS ****

EXCITATION WAVE LENGTH (IN) = 6.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 41: VERTICAL FRONT FRAME DISPLACEMENT (w_{bf}) - WORN WHEELS



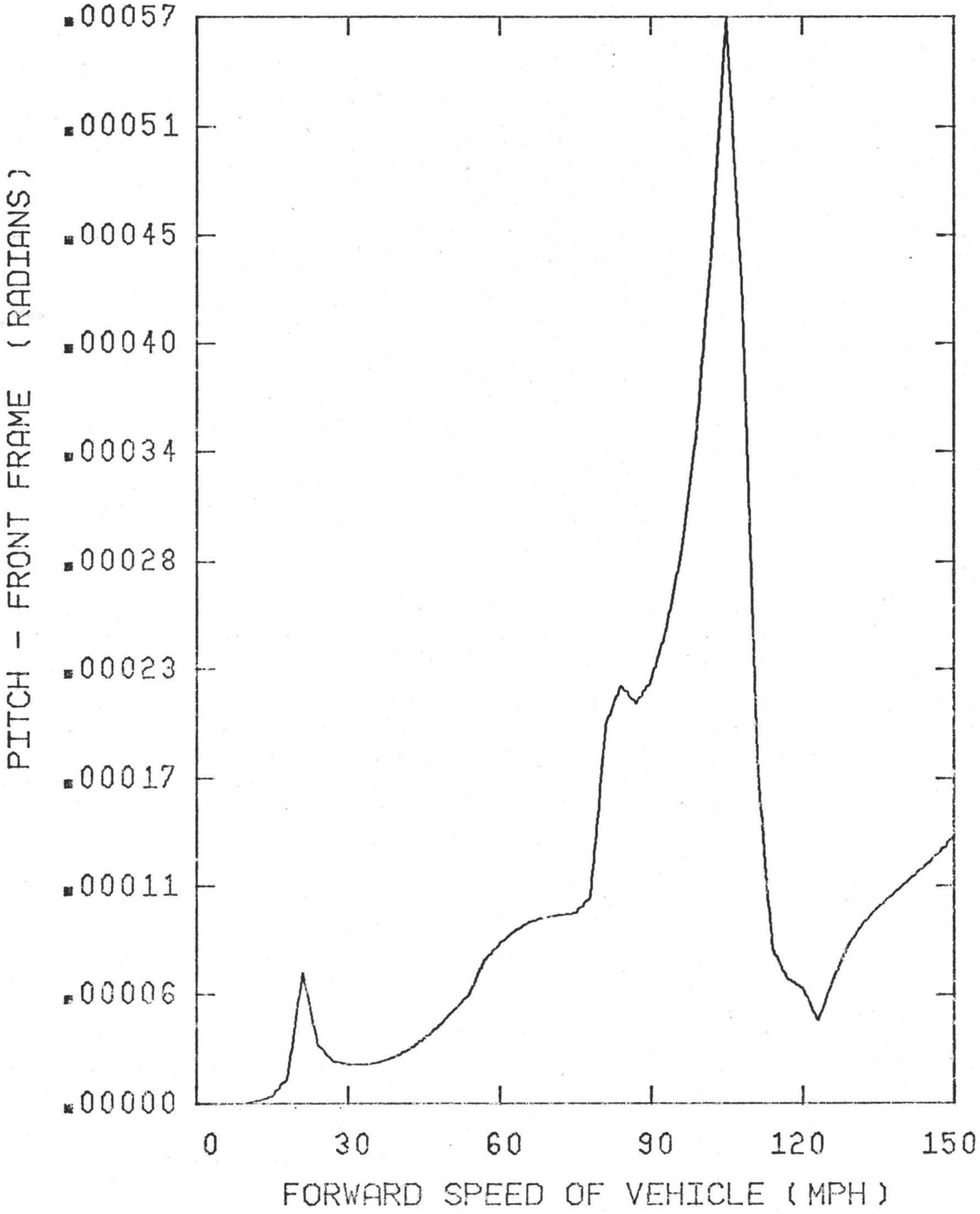
FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 6.600E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 42: ROLL FRONT FRAME DISPLACEMENT (α_{bf}) --WORN WHEELS



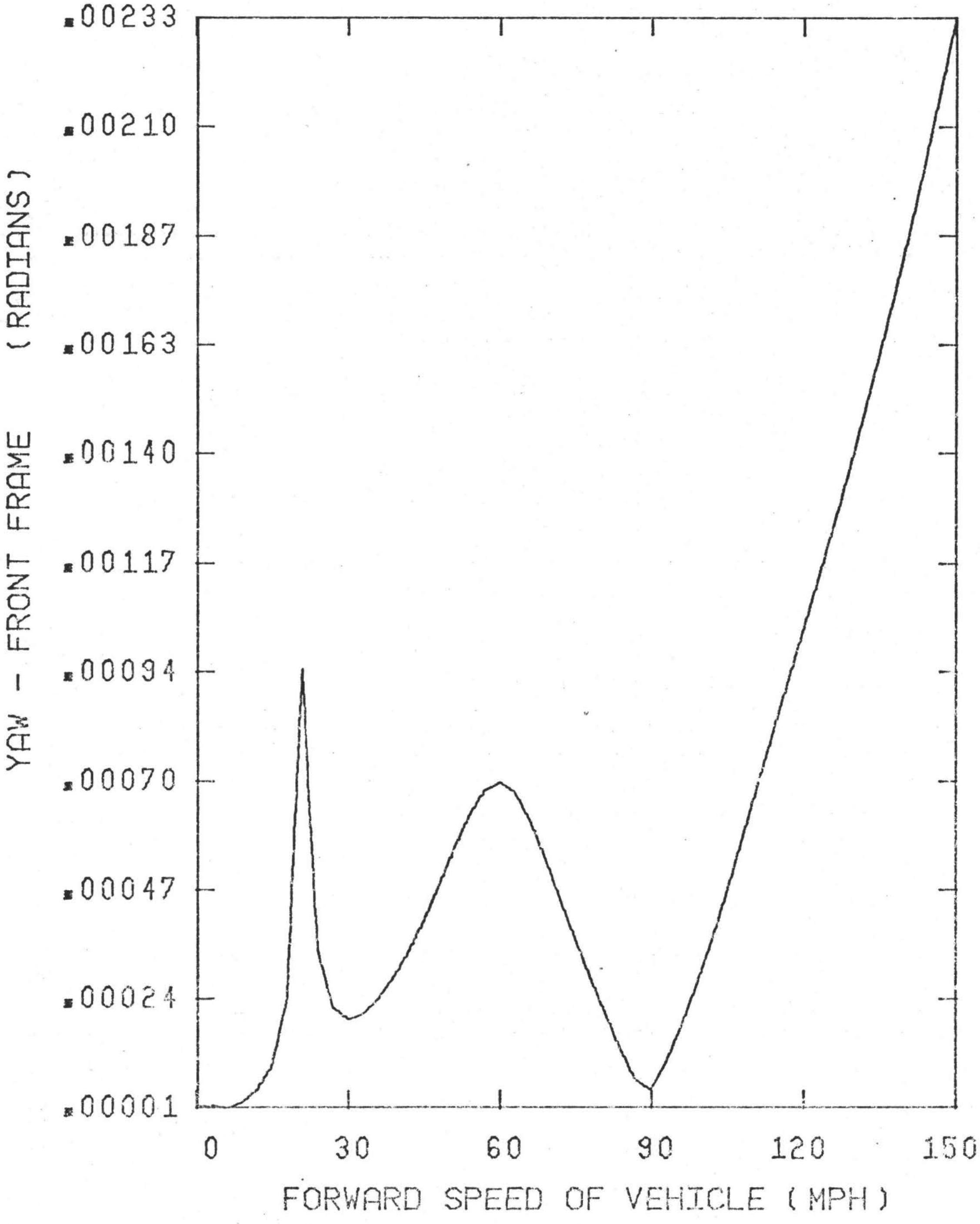
FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.000E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 43: PITCH FRONT FRAME DISPLACEMENT (β_{bf}) - WORN WHEELS



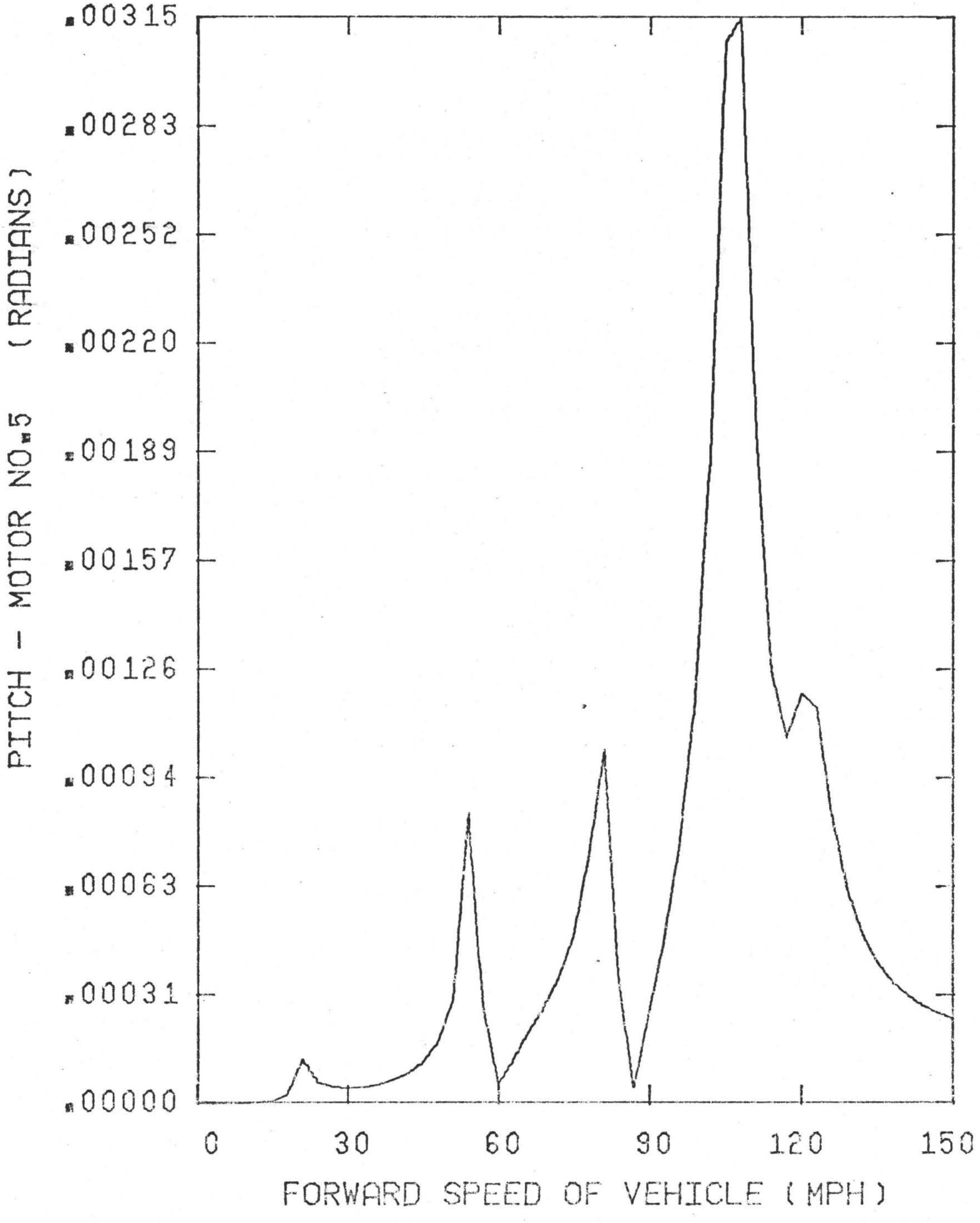
FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 0.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 44: YAW FRONT FRAME DISPLACEMENT (γ_{bf}) - WORN WHEELS



FOR THE CASE OF WORN WHEELS ....

EXCITATION WAVE LENGTH (IN) = 8.800E+02

AMPL. OF VERTICAL TRACK IRREGULAR.(IN) = 0.

AMPL. OF LATERAL TRACK IRREGULAR. (IN) = 1.000E+00

FIGURE 45: PITCH MOTOR NO. 5 DISPLACEMENT (β_{c5}) - WORN WHEELS

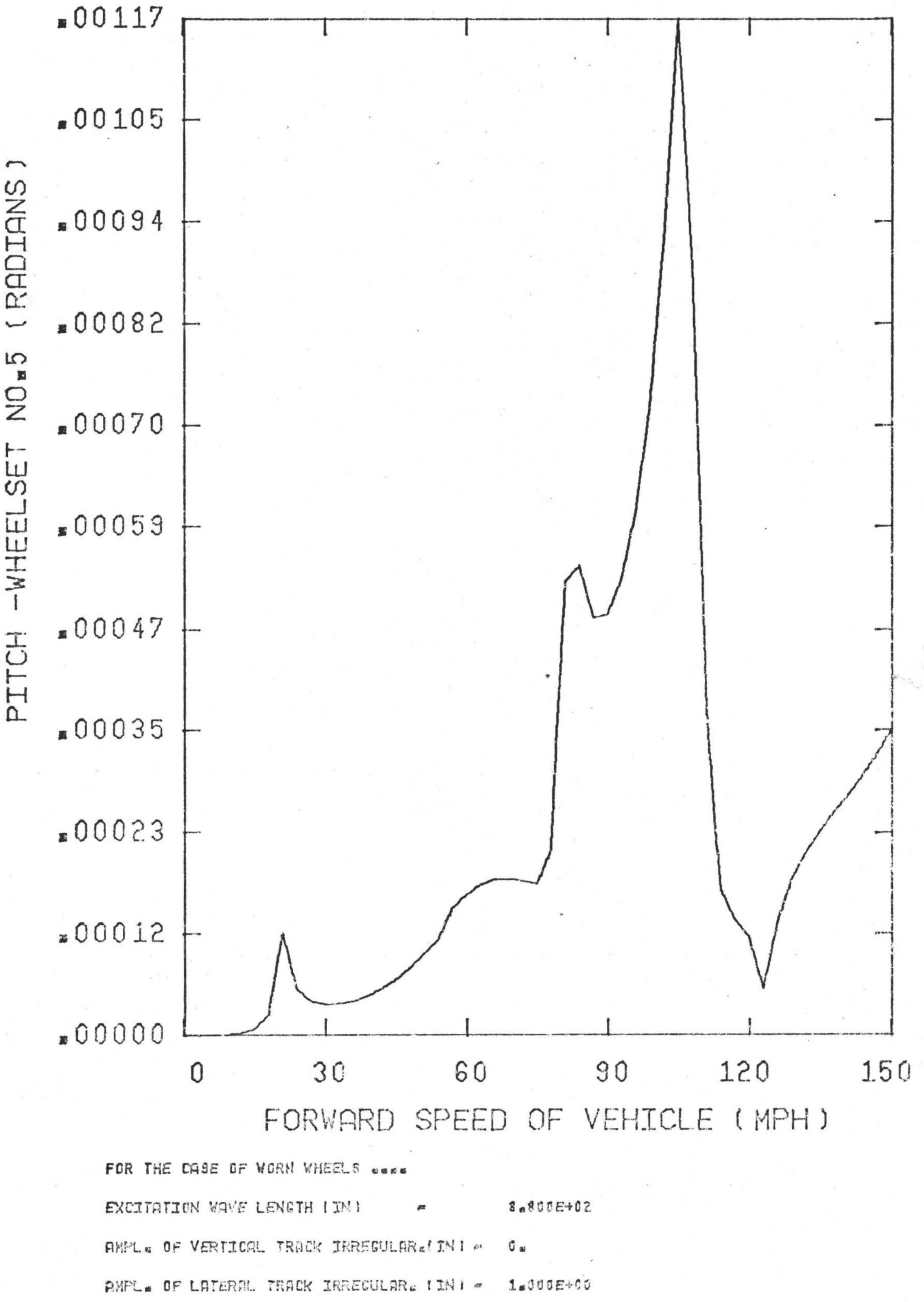


FIGURE 46: PITCH WHEELSET NO. 5 DISPLACEMENT (β_{d5}) - WORN WHEELS

4.5 Conclusions

Three models were investigated: a simplified model which assumed no springing or damping of trucks or motors, and no creep forces, a full model for the "stationary" vehicle, in which creep forces are assumed negligible and a full model for the "moving" vehicle, in which creep forces, gravity stiffness effects and tread profiles are considered.

Several conclusions can be drawn from the analysis presented in this thesis. The accuracy with which the devised models describe the performance of the real railway vehicle is compared.

The analysis and results show that:

1. The simplified model does not adequately approximate the response of the railway vehicle. Since all vehicles have finite primary suspension stiffness, this conclusion means that the simplified model is inadequate for simulating the performance of an actual railway vehicle.

2. The comparison of the steady state responses for the "stationary" and the "moving" vehicles indicates that creep forces have a significant effect on peak responses.

3. Comparison of the responses in the cases of worn wheels and new wheels shows that tread wear have a significant effect on peak responses but generally not on the shape of the response curves.

4. It is recommended that slip and corresponding creep forces, wheel tread and rail profiles, and gravity stiffness effect be included in the steady state response analysis of railway vehicle to track irregularities.

The study also shows that modern engineering techniques and use of a digital computer can contribute significantly to the analysis and design of a complex vehicle suspension. The direct application of the analysis outlined in this thesis is in the field of design of railway vehicles. By successive computer trials for different system parameters as input data it is possible to determine the effect of these parameters on the performance of the railway vehicle and in particular on the steady state response to rail irregularities. It is then possible to:

1. Determine the optimum suspension to meet passenger comfort requirements and provide sufficient damping to control the car body motion.

2. Optimize the suspension stiffness in order to maximize the critical velocity.

3. Choose an optimum tread profile.

4. Determine the response of the railway vehicle to different track "shapes".

4.6 Suggestions for Further Research

There are many problems in this area of high speed railway dynamics that need attention. Theoretical work such as presented in this thesis should be extended as described below:

1. Refine the mathematical model to include the effect of clearance between wheelsets and frames and include the nonlinear constraints.

2. Expand the analysis to a model for a complete train and study the effect of rail irregularities on the train stability.

3. Introduce statistical properties of the track irregularities which results in random fluctuations in the response.

4. Introduce driving and braking traction into the analysis and its effect on the amount of creep in order to determine its influence on stability and performance.

5. Consider the elastic deformation of the track when a train is passing and its effect on the adhesion mechanism.

The experimental adhesion study is important. At present, there is still a large discrepancy between the results measured in the laboratory and those experienced in railway operation.

Experiments are needed in order to find quantitative values and conduct comparative studies between theoretical and experimental results. Experiments could be performed in the field by running real railway vehicles or in laboratories by testing prototype models.

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A P P E N D I C E S

APPENDIX I

EQUATIONS OF MOTION FOR THE FULL MODEL ("STATIONARY" VEHICLE)

I.1 INTRODUCTION

In this Appendix equations of motion for the full model are derived in detail. The following notations for displacements are used:

u - linear displacement in the x direction,

v - linear displacement in the y direction,

w - linear displacement in the z direction,

α - angular displacement about the x direction,

β - angular displacement about the y direction,

γ - angular displacement about the z direction.

Subscripts are as follows:

a - for the body,

bf - for the front frame,

br - for the rear frame,

c1,...,c6 - motors numbered from rear to front,

d1,...,d6 - wheelsets numbered from rear to front.

For the "deformed" configuration the following assumptions are made:

1. $u_a > u_{bj} > u_{ci} > u_{di}$

$v_a > v_{bj} > v_{ci} > v_{di}$ $\quad (j \equiv f \text{ or } r)$ $\quad (i = 1, 2, \ldots, 6)$

$w_a > w_{bj} > w_{ci} > w_{di}$

2. All α, β, γ, rotations are positive (anticlockwise).

In each equation the terms are written in the following order:

Front - Right

Front - Left

Rear - Right

Rear - Left

For the internal reactions the following notations are used:

1. Internal reactions between wheelsets and frame

i - In the u direction RA_i

ii - In the v direction RB_i

iii - In the γ direction RM_i

2. Internal reactions between motor and wheelsets

i - In the u direction RU_i

ii - In the v direction RV_i

iii - In the w direction RW_i

iv - In the α direction $R\alpha_i$

v - In the γ direction $R\gamma_i$

Where i = 1, 2, 3, 4, 5, 6 from rear to front.

The reactions between wheels and rails are designated by:

i - In the x direction RX_{ij}

ii - In the v direction RY_{ij}

iii - In the w direction RZ_{ij}

Cont'd.

where i = 1, 2, 3, 4, 5, 6 from rear to front

j = r for the right wheel

j = ℓ for the left wheel

The equations of motion for the wheelsets are first written in terms of horizontal reactions between wheels and rails. These are later rewritten with these reactions transformed to creep forces.

I.2 CALCULATIONS OF THE FORCES IN SPRINGS: (See Fig. 3)

For Spring K_1:

i) Front

$$F_{1f} = K_1\{(u_a - u_{bf}) + \ell_1\beta_a + \ell_2\,\beta_{bf}\} \tag{I.1}$$

ii) Rear

$$F_{1r} = K_1\{(u_a - u_{br}) + \ell_1\beta_a + \ell_2\,\beta_{br}\} \tag{I.2}$$

For Spring K_2:

i) Front

$$F_{2f} = K_2\{(v_a - v_{bf}) - \ell_1\alpha_a - \ell_2\alpha_{bf} - (\ell_3+\ell_{16})\gamma_a + \ell_{16}\gamma_{bf}\} \tag{I.3}$$

ii) Rear

$$F_{2r} = K_2\{(v_a - v_{br}) - \ell_1\alpha_a - \ell_2\alpha_{br} + (\ell_3+\ell_{16})\gamma_a - \ell_{16}\gamma_{br}\} \tag{I.4}$$

For Spring K_3:

i) Front Right

$$F_{3fr} = K_3\{(w_a - w_{bf}) - \ell_9\alpha_a + \ell_9\alpha_{bf} + (\ell_3+\ell_4)\beta_a - \ell_4\beta_{bf}\} \tag{I.5}$$

ii) Front Left

$$F_{3f\ell} = K_3\{(w_a - w_{bf}) + \ell_9\alpha_a - \ell_9\alpha_{bf} + (\ell_3+\ell_4)\beta_a - \ell_4\beta_{bf}\} \qquad (I.6)$$

iii) Rear Right

$$F_{3rr} = K_3\{(w_a - w_{br}) - \ell_9\alpha_a + \ell_9\alpha_{br} - (\ell_3+\ell_4)\beta_a + \ell_4\beta_{br}\} \qquad (I.7)$$

iv) Rear Left

$$F_{3r\ell} = K_3\{(w_a - w_{br}) + \ell_9\alpha_a - \ell_9\alpha_{br} - (\ell_3+\ell_4)\beta_a + \ell_4\beta_{br}\} \qquad (I.8)$$

For Spring K_6:

i) Front Right

$$F_{6fr} = K_6\{(w_a - w_{bf}) - \ell_{10}\alpha_a + \ell_{10}\alpha_{bf} + (\ell_3-\ell_5)\beta_a + \ell_5\beta_{bf}\} \qquad (I.9)$$

ii) Front Left

$$F_{6f\ell} = K_6\{(w_a - w_{bf}) + \ell_{10}\alpha_a - \ell_{10}\alpha_{bf} + (\ell_3-\ell_5)\beta_a + \ell_5\beta_{bf}\} \qquad (I.10)$$

iii) Rear Right

$$F_{6rr} = K_6\{(w_a - w_{br}) - \ell_{10}\alpha_a + \ell_{10}\alpha_{br} - (\ell_3-\ell_5)\beta_a - \ell_5\beta_{br}\} \qquad (I.11)$$

iv) Rear Left

$$F_{6r\ell} = K_6\{(w_a - w_{br}) + \ell_{10}\alpha_a - \ell_{10}\alpha_{br} - (\ell_3-\ell_5)\beta_a - \ell_5\beta_{br}\} \qquad (I.12)$$

For Spring K_4:

i) Front Right

$$F_{4fr} = K_4\{(u_a - u_{bf}) + \ell_7\beta_a + \ell_{33}\beta_{bf} + \ell_9\gamma_a - \ell_9\gamma_{bf}\} \qquad (I.13)$$

ii) <u>Front Left</u>

$$F_{4f\ell} = K_4\{(u_a - u_{bf}) + \ell_7\beta_a + \ell_{33}\beta_{bf} - \ell_9\gamma_a + \ell_9\gamma_{bf}\} \quad (I.14)$$

iii) <u>Rear Right</u>

$$F_{4rr} = K_4\{(u_a - u_{br}) + \ell_7\beta_a + \ell_{33}\beta_{br} + \ell_9\gamma_a - \ell_9\gamma_{br}\} \quad (I.15)$$

iv) <u>Rear Left</u>

$$F_{4r\ell} = K_4\{(u_a - u_{br}) + \ell_7\beta_a + \ell_{33}\beta_{br} - \ell_9\gamma_a + \ell_9\gamma_{br}\} \quad (I.16)$$

<u>For Spring K_7:</u>

i) <u>Front Right</u>

$$F_{7fr} = K_7\{(u_a - u_{bf}) + \ell_7\beta_a + \ell_{33}\beta_{bf} + \ell_{10}\gamma_a - \ell_{10}\gamma_{bf}\} \quad (I.17)$$

ii) <u>Front Left</u>

$$F_{7f\ell} = K_7\{(u_a - u_{bf}) + \ell_7\beta_a + \ell_{33}\beta_{bf} - \ell_{10}\gamma_a + \ell_{10}\gamma_{bf}\} \quad (I.18)$$

iii) <u>Rear Right</u>

$$F_{7rr} = K_7\{(u_a - u_{br}) + \ell_7\beta_a + \ell_{33}\beta_{br} + \ell_{10}\gamma_a - \ell_{10}\gamma_{bf}\} \quad (I.19)$$

iv) <u>Rear Left</u>

$$F_{7r\ell} = K_7\{(u_a - u_{br}) + \ell_7\beta_a + \ell_{33}\beta_{br} - \ell_{10}\gamma_a + \ell_{10}\gamma_{br}\} \quad (I.20)$$

For Spring K_5:

i) Front Right

$$F_{5fr} = K_5\{(v_a - v_{bf}) - \ell_7\alpha_a - \ell_{33}\alpha_{bf} - (\ell_3+\ell_4)\gamma_a + \ell_4\gamma_{bf}\} \quad (I.21)$$

ii) Front Left

$$F_{5f\ell} = K_5\{(v_a - v_{bf}) - \ell_7\alpha_a - \ell_{33}\alpha_{bf} - (\ell_3+\ell_4)\gamma_a + \ell_4\gamma_{bf}\} \quad (I.22)$$

iii) Rear Right

$$F_{5rr} = K_5\{(v_a - v_{br}) - \ell_7\alpha_a - \ell_{33}\alpha_{br} + (\ell_3+\ell_4)\gamma_a - \ell_4\gamma_{br}\} \quad (I.23)$$

iv) Rear Left

$$F_{5r\ell} = K_5\{(v_a - v_{br}) - \ell_7\alpha_a - \ell_{33}\alpha_{br} + (\ell_3+\ell_4)\gamma_a - \ell_4\gamma_{br}\} \quad (I.24)$$

For Spring K_8:

i) Front Right

$$F_{8fr} = K_8\{(v_a - v_{bf}) - \ell_7\alpha_a - \ell_{33}\alpha_{bf} - (\ell_3-\ell_5)\gamma_a - \ell_5\gamma_{bf}\} \quad (I.25)$$

ii) Front Left

$$F_{8f\ell} = K_8\{(v_a - v_{bf}) - \ell_7\alpha_a - \ell_{33}\alpha_{bf} - (\ell_3-\ell_5)\gamma_a - \ell_5\gamma_{bf}\} \quad (I.26)$$

iii) Rear Right

$$F_{8rr} = K_8\{(v_a - v_{br}) - \ell_7\alpha_a - \ell_{33}\alpha_{br} + (\ell_3-\ell_5)\gamma_a + \ell_5\gamma_{bf}\} \quad (I.27)$$

iv) Rear Left

$$F_{8r\ell} = K_8\{(v_a - v_{br}) - \ell_7\alpha_a - \ell_{33}\alpha_{br} + (\ell_3-\ell_5)\gamma_a + \ell_5\gamma_{br}\} \quad (I.28)$$

For Spring K_{11}:

i) Front Right

$$F_{11fr} = K_{11}(\gamma_a - \gamma_{bf}) \quad (I.29)$$

ii) Front Left

$$F_{11f\ell} = K_{11}(\gamma_a - \gamma_{bf}) \quad (I.30)$$

iii) Rear Right

$$F_{11rr} = K_{11}(\gamma_a - \gamma_{br}) \quad (I.31)$$

iv) Rear Left

$$F_{11r\ell} = K_{11}(\gamma_a - \gamma_{br}) \quad (I.32)$$

For Spring K_{12}:

i) Front Right

$$F_{12fr} = K_{12}(\gamma_a - \gamma_{bf}) \quad (I.33)$$

ii) Front Left

$$F_{12f\ell} = K_{12}(\gamma_a - \gamma_{bf}) \quad (I.34)$$

iii) Rear Right

$$F_{12rr} = K_{12}(\gamma_a - \gamma_{br}) \quad (I.35)$$

iv) Rear Left

$$F_{12r\ell} = K_{12}(\gamma_a - \gamma_{br}) \quad (I.36)$$

For Spring K_9:

(From rear to front 1, 2, 3,...,6)

i) 1 Right

$$F_{9,1r} = K_9\{(w_{br} - w_{d1}) - \ell_{17}\alpha_{br} + \ell_{17}\alpha_{d1} - \ell_{24}\beta_{br}\} \quad (I.37)$$

ii) 1 Left

$$F_{9,1\ell} = K_9\{(w_{br} - w_{d1}) + \ell_{17}\alpha_{br} - \ell_{17}\alpha_{d1} - \ell_{24}\beta_{br}\} \quad (I.38)$$

iii) 2 Right

$$F_{9,2r} = K_9\{(w_{br} - w_{d2}) - \ell_{17}\alpha_{br} + \ell_{17}\alpha_{d2} - \ell_{26}\beta_{br}\} \quad (I.39)$$

iv) 2 Left

$$F_{9,2\ell} = K_9\{(w_{br} - w_{d2}) + \ell_{17}\alpha_{br} - \ell_{17}\alpha_{d2} - \ell_{26}\beta_{br}\} \quad (I.40)$$

v) 3 Right

$$F_{9,3r} = K_9\{(w_{br} - w_{d3}) - \ell_{17}\alpha_{br} + \ell_{17}\alpha_{d3} + \ell_{23}\beta_{br}\} \quad (I.41)$$

vi) 3 Left

$$F_{9,3\ell} = K_9\{(w_{br} - w_{d3}) + \ell_{17}\alpha_{br} - \ell_{17}\alpha_{d3} + \ell_{23}\beta_{br}\} \quad (I.42)$$

vii) 4 Right

$$F_{9,4r} = K_9\{(w_{bf} - w_{d4}) - \ell_{17}\alpha_{bf} + \ell_{17}\alpha_{d4} - \ell_{23}\beta_{bf}\} \quad (I.43)$$

viii) 4 Left

$$F_{9,4\ell} = K_9\{(w_{bf} - w_{d4}) + \ell_{17}\alpha_{bf} - \ell_{17}\alpha_{d4} - \ell_{23}\beta_{bf}\} \quad (I.44)$$

ix) 5 Right

$$F_{9,5r} = K_9\{(w_{bf} - w_{d5}) - \ell_{17}\alpha_{bf} + \ell_{17}\alpha_{d5} + \ell_{26}\beta_{bf}\} \quad (I.45)$$

x) 5 Left

$$F_{9,5\ell} = K_9\{(w_{bf} - w_{d5}) + \ell_{17}\alpha_{bf} - \ell_{17}\alpha_{d5} + \ell_{26}\beta_{bf}\} \quad (I.46)$$

xi) 6 Right

$$F_{9,6r} = K_9\{(w_{bf} - w_{d6}) - \ell_{17}\alpha_{bf} + \ell_{17}\alpha_{d6} + \ell_{24}\beta_{bf}\} \quad (I.47)$$

xii) 6 Left

$$F_{9,6\ell} = K_9\{(w_{bf} - w_{d6}) + \ell_{17}\alpha_{bf} - \ell_{17}\alpha_{d6} + \ell_{24}\beta_{bf}\} \quad (I.48)$$

For Spring K_{10}:

(from rear to front 1, 2, ...,6)

$$F_{10,1} = K_{10}\{(w_{br} - w_{c1}) - \ell_{22}\beta_{br} - \ell_{35}\beta_{c1}\} \quad (I.49)$$

$$F_{10,2} = K_{10}\{(w_{br} - w_{c2}) + \ell_{27}\beta_{br} - \ell_{35}\beta_{c2}\} \quad (I.50)$$

$$F_{10,3} = K_{10}\{(w_{br} - w_{c3}) + \ell_{25}\beta_{br} - \ell_{35}\beta_{c3}\} \quad (I.51)$$

$$F_{10,4} = K_{10}\{(w_{bf} - w_{c4}) - \ell_{25}\beta_{bf} + \ell_{35}\beta_{c4}\} \quad (I.52)$$

$$F_{10,5} = K_{10}\{(w_{bf} - w_{c5}) - \ell_{27}\beta_{bf} + \ell_{35}\beta_{c5}\} \quad (I.53)$$

$$F_{10,6} = K_{10}\{(w_{bf} - w_{c6}) + \ell_{22}\beta_{bf} + \ell_{35}\beta_{c6}\} \quad (I.54)$$

I.3 FORCES DUE TO DAMPING

These forces are due to:

A - Rubber springs (in parallel with stiffness)

(C_1, C_2, ..., C_8, C_{11}, and C_{12})

B - Shock absorbers

(C_{13}, C_{14}, C_{15}, C_{16}, C_{17} and C_{18})

(A) Damping in the Rubber Springs

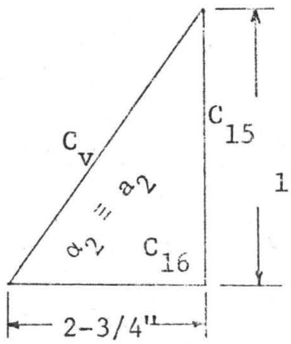
Equations (I.55) to (I.90) giving the forces P due to damping, could be obtained from equations (I.1) to (I.36) by replacing K_n by C_n and the variables by their first derivatives.

(B) Damping Due to Shock Absorbers (for C_R and C_V)

$\alpha_2 = \tan^{-1}(17.75/2.75)$

$C_{15} = C_V \sin \alpha_2$

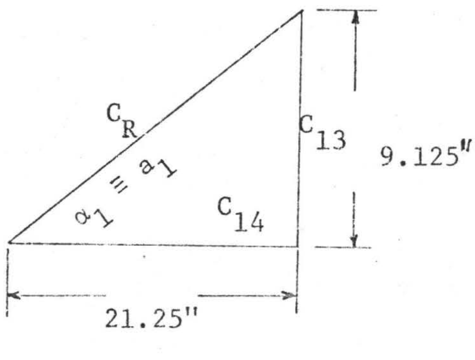
$C_{16} = C_V \cos \alpha_2$

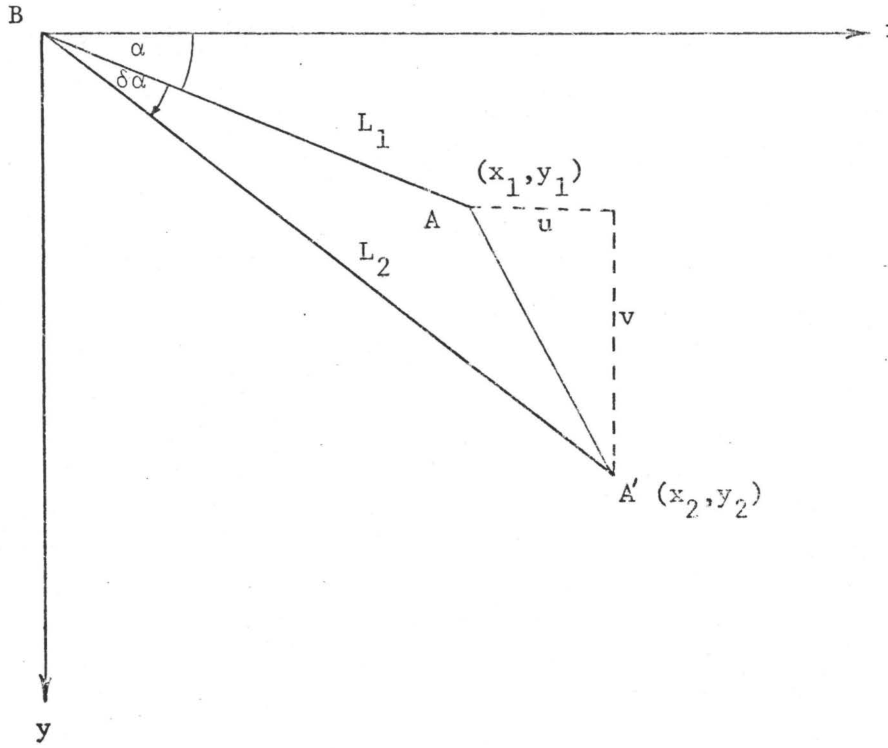


$\alpha_1 = \tan^{-1}(9.125/21.25)$

$C_{13} = C_R \sin \alpha_1$

$C_{14} = C_R \cos \alpha_1$





Initial Length $L_1 = \overline{BA}$, $L_1^2 = x_1^2 + y_1^2$

Final Length $L_2 = \overline{BA'}$, $L_2^2 = x_2^2 + y_2^2$

Assuming small u and v,

$$(L_2 - L_1)(L_2 + L_1) \simeq 2(ux_1 + vy_1)$$

or, taking $L_2 + L_1 \simeq 2L_1$

$$L_2 - L_1 \simeq \frac{ux_1 + vy_1}{L_1}$$

Therefore:

$L_2 - L_1 = u \cos \alpha + v \sin \alpha$

Hence:

x component of displacement = $(u.\cos \alpha + v \sin \alpha)\cos \alpha$

y component of displacement = $(u.\cos \alpha + v \sin \alpha)\sin \alpha$

for small $\delta\alpha$.

It should be noted that:

1. u and v are measured in the positive directions of x and y respectively.

2. The angle α is measured positive from the x-axis in a clockwise direction.

3. u_A = Displacement of A in the positive direction,

   u_B = Displacement of B in the positive direction,

   v_A = Displacement of A in the positive direction,

   v_B = Displacement of B in the positive direction.

SUMMARY

For the rotational and the lateral shock absorbers, components at points A and B due to displacement of the center of gravity, and rotation about it are:

POSITION	POINT	u_A and u_B	v_A and v_B
Front Right	A	$(u \cos\alpha_1^* - v \sin\alpha_1)\cos\alpha_1$	$(-u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$
Front Right	B	$(u \cos\alpha_1 - v \sin\alpha_1)\cos\alpha_1$	$(-u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$
Front Left	A	$(u \cos\alpha_1 + v \sin\alpha_1)\cos\alpha_1$	$(u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$
Front Left	B	$(u \cos\alpha_1 + v \sin\alpha_1)\cos\alpha_1$	$(u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$
Rear Right	A	$(u \cos\alpha_1 + v \sin\alpha_1)\cos\alpha_1$	$(u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$
Rear Right	B	$(u \cos\alpha_1 + v \sin\alpha_1)\cos\alpha_1$	$(u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$
Rear Left	A	$(u \cos\alpha_1 - v \sin\alpha_1)\cos\alpha_1$	$(-u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$
Rear Left	B	$(u \cos\alpha_1 - v \sin\alpha_1)\cos\alpha_1$	$(-u \cos\alpha_1 + v \sin\alpha_1)\sin\alpha_1$

* α_1 for C_R and α_2 for C_V

For the rotational shock absorber (C_R), we get the following:

Displacements in the positive u and v directions

POSITION	Direc-tion	Displacements of Points A	Displacements of Points B
Front Right	u	$u_a + \ell_{15}\beta_a + \ell_{13}\gamma_a$	$u_{bf} - \ell_{32}\beta_{bf} + \ell_{31}\gamma_{bf}$
	v	$v_a - \ell_{15}\alpha_a - \ell_{12}\gamma_a$	$v_{bf} + \ell_{32}\alpha_{bf} - \ell_{29}\gamma_{bf}$
Front Left	u	$u_a + \ell_{15}\beta_a - \ell_{13}\gamma_a$	$u_{bf} - \ell_{32}\beta_{bf} - \ell_{31}\gamma_{bf}$
	v	$v_a - \ell_{15}\alpha_a - \ell_{12}\gamma_a$	$v_{bf} + \ell_{32}\alpha_{bf} - \ell_{29}\gamma_{bf}$
Rear Right	u	$u_a + \ell_{15}\beta_a + \ell_{13}\gamma_a$	$u_{br} - \ell_{32}\beta_{br} + \ell_{29}\gamma_{br}$
	v	$v_a - \ell_{15}\alpha_a + \ell_{12}\gamma_a$	$v_{br} + \ell_{32}\alpha_{br} + \ell_{29}\gamma_{br}$
Rear Left	u	$u_a + \ell_{15}\beta_a - \ell_{13}\gamma_a$	$u_{br} - \ell_{32}\beta_{br} - \ell_{31}\gamma_{br}$
	v	$v_a - \ell_{15}\alpha_a + \ell_{12}\gamma_a$	$v_{br} + \ell_{32}\alpha_{br} + \ell_{29}\gamma_{br}$

Relative Displacements of A w.r.t. B

POSITION	Relative Displacement (+ve Directions)	
Front Right	u	$(u_a - u_{bf}) + \ell_{15}\beta_a + \ell_{32}\beta_{bf} + \ell_{13}\gamma_a - \ell_{31}\gamma_{bf}$
	v	$(v_a - v_{bf}) - \ell_{15}\alpha_a - \ell_{32}\alpha_{bf} - \ell_{12}\gamma_a + \ell_{29}\gamma_{bf}$
Front Left	u	$(u_a - u_{bf}) + \ell_{15}\beta_a + \ell_{32}\beta_{bf} - \ell_{13}\gamma_a + \ell_{31}\gamma_{bf}$
	v	$(v_a - v_{bf}) - \ell_{15}\alpha_a - \ell_{32}\alpha_{bf} + \ell_{12}\gamma_a + \ell_{29}\gamma_{bf}$
Rear Right	u	$(u_a - u_{br}) + \ell_{15}\beta_a + \ell_{32}\beta_{br} + \ell_{13}\gamma_a - \ell_{31}\gamma_{br}$
	v	$(v_a - v_{br}) - \ell_{15}\alpha_a - \ell_{32}\alpha_{br} + \ell_{12}\gamma_a - \ell_{29}\gamma_{br}$
Rear Left	u	$(u_a - u_{br}) + \ell_{15}\beta_a + \ell_{32}\beta_{br} - \ell_{13}\gamma_a + \ell_{31}\gamma_{br}$
	v	$(v_a - v_{br}) - \ell_{15}\alpha_a - \ell_{32}\alpha_{br} + \ell_{12}\gamma_a - \ell_{29}\gamma_{br}$

Forces in the Longitudinal Direction due to C_R:

i) Front Right

$$P_{14,1} = C_{14}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{bf} + \ell_{13}\dot{\gamma}_a - \ell_{31}\dot{\gamma}_{bf}]\cos\alpha_1$$

$$+[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_{12}\dot{\gamma}_a + \ell_{29}\dot{\gamma}_{bf}]\sin\alpha_1\} \quad (I.91)$$

ii) Front Left

$$P_{14,2} = C_{14}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{bf} - \ell_{13}\dot{\gamma}_a + \ell_{31}\dot{\gamma}_{bf}]\cos\alpha_1$$

$$-[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_{12}\dot{\gamma}_a + \ell_{29}\dot{\gamma}_{bf}]\sin\alpha_1\} \quad (I.92)$$

iii) Rear Right

$$P_{14,3} = C_{14}\{[(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} + \ell_{13}\dot{\gamma}_a - \ell_{31}\dot{\gamma}_{br}]\cos\alpha_1$$

$$+[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_{12}\dot{\gamma}_a - \ell_{29}\dot{\gamma}_{br}]\sin\alpha_1\} \quad (I.93)$$

iv) Rear Left

$$P_{14,4} = C_{14}\{[(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} - \ell_{13}\dot{\gamma}_a + \ell_{31}\dot{\gamma}_{br}]\cos\alpha_1$$

$$-[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_{12}\dot{\gamma}_a - \ell_{29}\dot{\gamma}_{br}]\sin\alpha_1\} \quad (I.94)$$

Forces in the Lateral Direction Due to C_R:

i) Front Right

$$P_{13,1} = C_{13}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{bf} + \ell_{13}\dot{\gamma}_a - \ell_{31}\dot{\gamma}_{bf}]\cos\alpha_1$$

$$+[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_{12}\dot{\gamma}_a + \ell_{29}\dot{\gamma}_{bf}]\sin\alpha_1\} \quad (I.95)$$

ii) Front Left

$$P_{13,2} = C_{13}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{bf} - \ell_{13}\dot{\gamma}_a + \ell_{31}\dot{\gamma}_{bf}]\cos\alpha_1$$

$$-[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_{12}\dot{\gamma}_a + \ell_{29}\dot{\gamma}_{bf}]\sin\alpha_1\} \quad (I.96)$$

iii) Rear Right

$$P_{13,3} = C_{13}\{+[(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} + \ell_{13}\dot{\gamma}_a - \ell_{31}\dot{\gamma}_{br}]\cos\alpha_1$$

$$+[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_{12}\dot{\gamma}_a - \ell_{29}\dot{\gamma}_{br}]\sin\alpha_1\} \quad (I.97)$$

iv) Rear Left

$$P_{13,4} = C_{13}\{+[(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} - \ell_{13}\dot{\gamma}_a + \ell_{31}\dot{\gamma}_{br}]\cos\alpha_1$$

$$-[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_{12}\dot{\gamma}_a - \ell_{29}\dot{\gamma}_{br}]\sin\alpha_1\} \quad (I.98)$$

Similarly we will get the forces in the longitudinal and lateral directions due to C_V.

Forces in the Longitudinal Direction Due to C_V:

i) Front Right

$$P_{16,1} = C_{16}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{bf} + \ell_{11}\dot{\gamma}_a - \ell_{30}\dot{\gamma}_{bf}]\cos\alpha_2$$

$$+[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_6\dot{\gamma}_a + \ell_{28}\dot{\gamma}_{bf}]\sin\alpha_2\} \quad (I.99)$$

ii) Front Left

$$P_{16,2} = C_{16}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{bf} - \ell_{11}\dot{\gamma}_a + \ell_{30}\dot{\gamma}_{bf}]\cos\alpha_2$$

$$-[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_6\dot{\gamma}_a + \ell_{28}\dot{\gamma}_{bf}]\sin\alpha_2\} \quad (I.100)$$

iii) Rear Right

$$P_{16,3} = C_{16}\{[(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} + \ell_{11}\dot{\gamma}_a - \ell_{30}\dot{\gamma}_{br}]\cos\alpha_2$$

$$+[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_6\dot{\gamma}_a - \ell_{28}\dot{\gamma}_{br}]\sin\alpha_2\} \quad (I.101)$$

iv) Rear Left

$$P_{16,4} = C_{16}\{[(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} - \ell_{11}\dot{\gamma}_a + \ell_{30}\dot{\gamma}_{br}]\cos\alpha_2$$

$$-[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_6\dot{\gamma}_a - \ell_{28}\dot{\gamma}_{br}]\sin\alpha_2\} \quad (I.102)$$

Forces in the Lateral Direction Due to C_V:

i) Front Right

$$P_{15,1} = C_{15}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{bf} + \ell_{11}\dot{\gamma}_a - \ell_{30}\dot{\gamma}_{bf}]\cos\alpha_2$$

$$+[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_6\dot{\gamma}_a + \ell_{28}\dot{\gamma}_{bf}]\sin\alpha_2\} \quad (I.103)$$

ii) Front Left

$$P_{15,2} = C_{15}\{-[(\dot{u}_a-\dot{u}_{bf}) + \ell_{15}\dot{\beta}_a + \ell_{32}\ddot{\beta}_{bf} - \ell_{11}\dot{\gamma}_a + \ell_{30}\dot{\gamma}_{bf}]\cos\alpha_2$$

$$-[(\dot{v}_a-\dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{bf} - \ell_6\dot{\gamma}_a + \ell_{28}\dot{\gamma}_{bf}]\sin\alpha_2\} \quad (I.104)$$

iii) Rear Right

$$P_{15,3} = C_{15}\{[(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} + \ell_{11}\dot{\gamma}_a + \ell_{30}\dot{\gamma}_{br}]\cos\alpha_2$$

$$+[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_6\dot{\gamma}_a - \ell_{28}\dot{\gamma}_{br}]\sin\alpha_2\} \quad (I.105)$$

iv) Rear Left

$$P_{15,4} = C_{15}\{ [(\dot{u}_a-\dot{u}_{br}) + \ell_{15}\dot{\beta}_a + \ell_{32}\dot{\beta}_{br} - \ell_{11}\dot{\gamma}_a - \ell_{30}\dot{\gamma}_{br}]\cos\alpha_2$$

$$-[(\dot{v}_a-\dot{v}_{br}) - \ell_{15}\dot{\alpha}_a - \ell_{32}\dot{\alpha}_{br} + \ell_6\dot{\gamma}_a - \ell_{28}\dot{\gamma}_{br}]\sin\alpha_2\} \qquad (I.106)$$

Forces Due to Shock Absorber C_{17}:

$$P_{17,1} = C_{17}\{(\dot{w}_{br} - \dot{w}_{d1}) + \ell_{19}\dot{\alpha}_{br} - \ell_{19}\dot{\alpha}_{d1} - \ell_{24}\dot{\beta}_{br}\} \qquad (I.107)$$

$$P_{17,2} = C_{17}\{(\dot{w}_{br} - \dot{w}_{d3}) - \ell_{19}\dot{\alpha}_{br} + \ell_{19}\dot{\alpha}_{d3} + \ell_{23}\dot{\beta}_{br}\} \qquad (I.108)$$

$$P_{17,3} = C_{17}\{(\dot{w}_{bf} - \dot{w}_{d4}) + \ell_{19}\dot{\alpha}_{bf} - \ell_{19}\dot{\alpha}_{d4} - \ell_{23}\dot{\beta}_{bf}\} \qquad (I.109)$$

$$P_{17,4} = C_{17}\{(\dot{w}_{bf} - \dot{w}_{d6}) - \ell_{19}\dot{\alpha}_{bf} + \ell_{19}\dot{\alpha}_{d6} + \ell_{24}\dot{\beta}_{bf}\} \qquad (I.110)$$

Forces Due to Shock Absorber C_{18}:

$$P_{18,1r} = C_{18}\{(\dot{w}_{br} - \dot{w}_{d1}) - \ell_{19}\dot{\alpha}_{br} + \ell_{19}\dot{\alpha}_{d1} - \ell_{24}\dot{\beta}_{br}\} \qquad (I.111)$$

$$P_{18,2r} = C_{18}\{(\dot{w}_{br} - \dot{w}_{d2}) - \ell_{19}\dot{\alpha}_{br} + \ell_{19}\dot{\alpha}_{d2} - \ell_{26}\dot{\beta}_{br}\} \qquad (I.112)$$

$$P_{18,2\ell} = C_{18}\{(\dot{w}_{br} - \dot{w}_{d2}) + \ell_{19}\dot{\alpha}_{br} - \ell_{19}\dot{\alpha}_{d2} - \ell_{26}\dot{\beta}_{br}\} \qquad (I.113)$$

$$P_{18,3\ell} = C_{18}\{(\dot{w}_{br} - \dot{w}_{d3}) + \ell_{19}\dot{\alpha}_{br} - \ell_{19}\dot{\alpha}_{d3} + \ell_{23}\dot{\beta}_{br}\} \qquad (I.114)$$

$$P_{18,4r} = C_{18}\{(\dot{w}_{bf} - \dot{w}_{d4}) - \ell_{19}\dot{\alpha}_{bf} + \ell_{19}\dot{\alpha}_{d4} - \ell_{23}\dot{\beta}_{bf}\} \qquad (I.115)$$

$$P_{18,5r} = C_{18}\{(\dot{w}_{bf} - \dot{w}_{d5}) - \ell_{19}\dot{\alpha}_{bf} + \ell_{19}\dot{\alpha}_{d5} + \ell_{26}\dot{\beta}_{bf}\} \qquad (I.116)$$

$$P_{18,5\ell} = C_{18}\{(\dot{w}_{bf} - \dot{w}_{d5}) + \ell_{19}\dot{\alpha}_{bf} - \ell_{19}\dot{\alpha}_{d5} + \ell_{26}\dot{\beta}_{bf}\} \qquad (I.117)$$

$$P_{18,6\ell} = C_{18}\{(\dot{w}_{bf} - \dot{w}_{d6}) + \ell_{19}\dot{\alpha}_{bf} - \ell_{19}\dot{\alpha}_{d6} + \ell_{24}\dot{\beta}_{bf}\} \qquad (I.118)$$

I.4 EQUATIONS OF MOTION

(A) For the Body (See Fig. I.1)

$\Sigma F_x = 0$

$$m_a\ddot{u}_a + F_{1f} + F_{1r} + P_{1f} + P_{1r}$$
$$+ F_{4fr} + F_{4f\ell} + F_{4rr} + F_{4r\ell} + F_{7fr} + F_{7f\ell} + F_{7rr} + F_{7r\ell}$$
$$+ P_{4fr} + P_{4f\ell} + P_{4rr} + P_{4r\ell} + P_{7fr} + P_{7f\ell} + P7rr + P_{7r\ell}$$
$$- P_{14fr} - P_{14f\ell} + P_{14rr} + P_{14r\ell}$$
$$- P_{16fr} - P_{16f\ell} + P_{16rr} + P_{16r\ell} = 0$$

$$m_a\ddot{u}_a + (2K_1 + 4K_4 + 4K_7)u_a + (-K_1 - 2K_4 - 2K_7)u_{bf}$$
$$+ (-K_1 - 2K_4 - 2K_7)u_{br} + (2\ell_1 K_1 + 4\ell_7 K_4 + 4\ell_7 K_7)\beta_a$$
$$+ (\ell_2 K_1 + 2\ell_{33}K_4 + 2\ell_{33}K_7)\beta_{bf} + (\ell_2 K_1 + 2\ell_{33}K_4 + 2\ell_{33}K_7)\beta_{br}$$
$$+ (-C_1 - 2C_4 - 2C_7 - 2C_{14}\cos\alpha_1 - 2C_{16}\cos\alpha_2)\dot{u}_{bf}$$
$$+ (-C_1 - 2C_4 - 2C_7 - 2C_{14}\cos\alpha_1 - 2C_{16}\cos\alpha_2)\dot{u}_{br}$$
$$+ (2\ell_1 C_1 + 4\ell_7 C_4 + 4\ell_7 C_7 + 4\ell_{15}C_{14}\cos\alpha_1 + 4\ell_{15}C_{16}\cos\alpha_2)\dot{\beta}_a$$
$$+ (\ell_2 C_1 + 2\ell_{33}C_4 + 2\ell_{33}C_7 + 2\ell_{32}C_{14}\cos\alpha_1 + 2\ell_{32}C_{16}\cos\alpha_2)\dot{\beta}_{bf}$$
$$+ (\ell_2 C_1 + 2\ell_{33}C_4 + 2\ell_{33}C_7 + 2\ell_{32}C_{14}\cos\alpha_1 + 2\ell_{32}C_{16}\cos\alpha_2)\dot{\beta}_{br}$$
$$+ (2C_1 + 4C_4 + 4C_7 + 4C_{14}\cos\alpha_1 + 4C_{16}\cos\alpha_2)\dot{u}_a$$
$$= 0 \qquad (1)$$

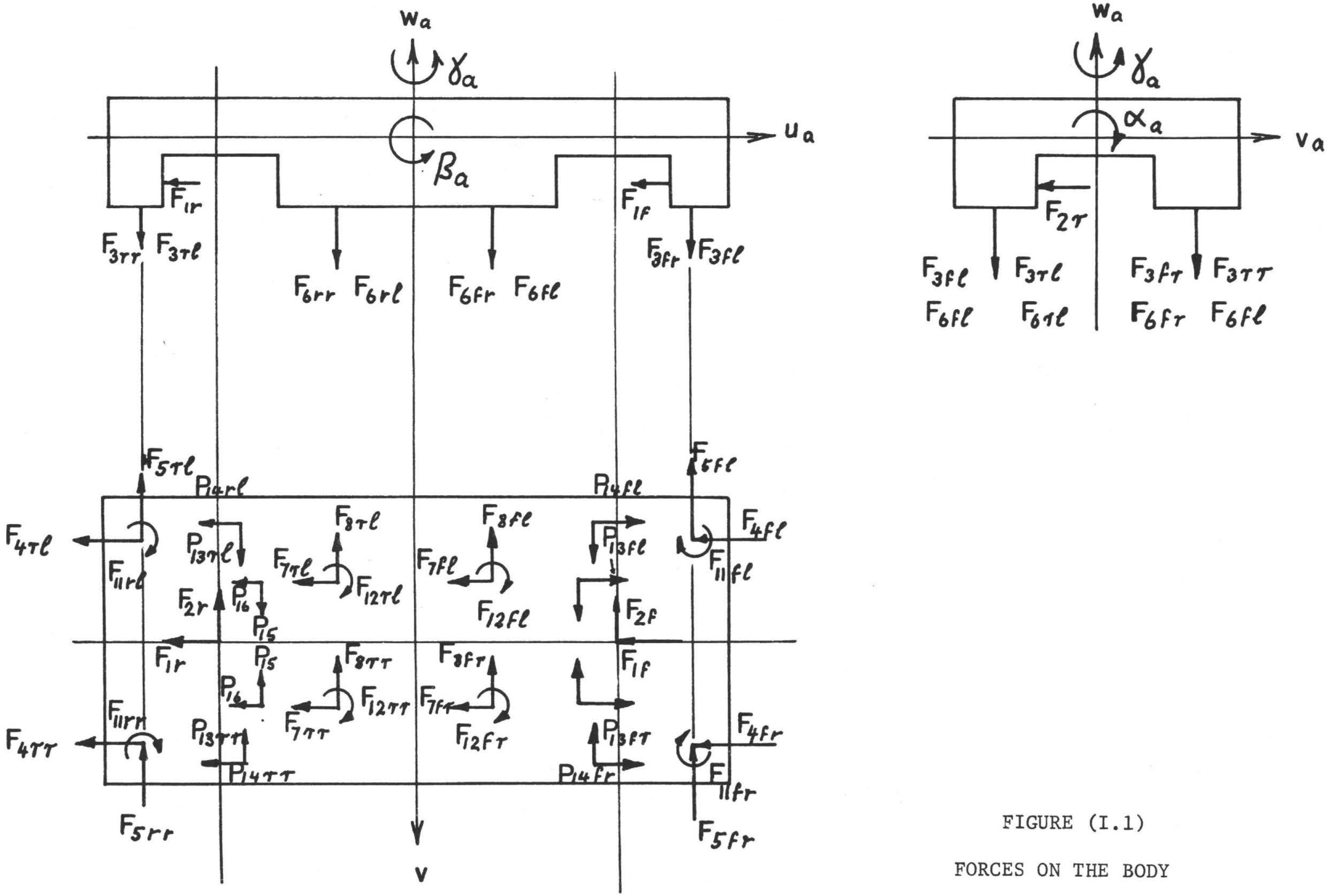


FIGURE (I.1)

FORCES ON THE BODY

$\Sigma F_y = 0$

$$m_a \ddot{v}_a + F_{2f} + F_{2r} + F_{5fr} + F_{5f\ell} + F_{5rr} + F_{5r\ell} + P_{5fr} + P_{5f\ell}$$
$$+ P_{5rr} + P_{5r\ell} + F_{8fr} + F_{8f\ell} + F_{8rr} + F_{8r\ell} + P_{8fr} + P_{8f\ell}$$
$$+ P_{8rr} + P_{8r\ell} + P_{13fr} - P_{13f\ell} + P_{13rr} - P_{13r\ell} + P_{15fr} - P_{15f\ell}$$
$$+ P_{15rr} - P_{15r\ell}$$
$$= 0$$

$$m_a \ddot{v}_a + (2K_2 + 4K_5 + 4K_8)v_a$$
$$+ (-K_2 - 2K_5 - 2K_8)v_{bf} + (-K_2 - 2K_5 - 2K_8)v_{br}$$
$$+ (-2\ell_1 K_2 - 4\ell_7 K_5 - 4\ell_7 K_8)\alpha_a$$
$$+ (-\ell_2 K_2 - 2\ell_{33} K_5 - 2\ell_{33} K_8)\alpha_{bf} + (-\ell_2 K_2 - 2\ell_{33} K_5 - 2\ell_{33} K_8)\alpha_{br}$$
$$+ (\ell_{16} K_2 + 2\ell_4 K_5 - 2\ell_5 K_8)\gamma_{bf} + (-\ell_{16} K_2 - 2\ell_4 K_5 + 2\ell_5 K_8)\gamma_{br}$$
$$+ (2C_2 + 4C_5 + 4C_8 + 4C_{13} \sin\alpha_1 + 4C_{15} \sin\alpha_2)\dot{v}_a$$
$$+ (-C_2 - 2C_5 - 2C_8 - 2C_{13} \sin\alpha_1 - 2C_{15} \sin\alpha_2)\dot{v}_{bf}$$
$$+ (-C_2 - 2C_5 - 2C_8 - 2C_{13} \sin\alpha_1 - 2C_{15} \sin\alpha_2)\dot{v}_{br}$$
$$+ (-2\ell_1 C_2 - 4\ell_7 C_5 - 4\ell_7 C_8 - 4\ell_{15} C_{13} \sin\alpha_1 - 4\ell_{15} C_{15} \sin\alpha_2)\dot{\alpha}_a$$
$$+ (-\ell_2 C_2 - 2\ell_{33} C_5 - 2\ell_{33} C_8 - 2\ell_{32} C_{13} \sin\alpha_1 - 2\ell_{32} C_{15} \sin\alpha_2)\dot{\alpha}_{bf}$$
$$+ (-\ell_2 C_2 - 2\ell_{33} C_5 - 2\ell_{33} C_8 - 2\ell_{32} C_{13} \sin\alpha_1 - 2\ell_{32} C_{15} \sin\alpha_2)\dot{\alpha}_{br}$$
$$+ [\ell_{16} C_2 + 2\ell_4 C_5 - 2\ell_5 C_8 + 2C_{13}(\ell_{31} \cos\alpha_1 + \ell_{29} \sin\alpha_1)$$
$$+2C_{15}(\ell_{30} \cos\alpha_2 + \ell_{28} \sin\alpha_2)]\dot{\gamma}_{bf}$$
$$+ [(-\ell_{16} C_2 - 2\ell_4 C_5 + 2\ell_5 C_8 - 2C_{13}(\ell_{31} \cos\alpha_1 + \ell_{29} \sin\alpha_1)$$
$$- 2C_{15}(\ell_{30} \cos\alpha_2 + \ell_{28} \sin\alpha_2)]\dot{\gamma}_{br} = 0 \quad (2)$$

$\underline{\Sigma F_z = 0}$

$$m_a\ddot{w}_a + F_{3fr} + F_{3f\ell} + F_{3rr} + F_{3r\ell} + F_{6fr} + F_{6f\ell} + F_{6rr} + F_{6r\ell}$$

$$+ P_{3fr} + P_{3f\ell} + P_{3rr} + P_{3r\ell} + P_{6fr} + P_{6f\ell} + P_{6rr} + P_{6r\ell}$$

$$= 0$$

$$m_a\ddot{w}_a + (4K_3 + 4K_6)w_a$$

$$+ (-2K_3 - 2K_6)w_{bf} + (-2K_3 - 2K_6)w_{br}$$

$$+ (-2\ell_4K_3 + 2\ell_5K_6)\beta_{bf} + (2\ell_4K_3 - 2\ell_5K_6)\beta_{br}$$

$$+ (4C_3 + 4C_6)\dot{w}_a$$

$$+ (-2C_3 - 2C_6)\dot{w}_{bf} + (-2C_3 - 2C_6)\dot{w}_{br}$$

$$+ (-2\ell_4C_3 + 2\ell_5C_6)\dot{\beta}_{bf} + (2\ell_4C_3 - 2\ell_5C_6)\dot{\beta}_{br}$$

$$= 0 \qquad (3)$$

$\underline{\Sigma M_\alpha = 0}$

$$I_{a\alpha}\ddot{\alpha}_a - \ell_1(F_{2f}+F_{2r}) - \ell_7(F_{5fr}+F_{5f\ell}+F_{5rr}+F_{5r\ell}+F_{8fr}+F_{8f\ell}+F_{8rr}+F_{8r\ell})$$

$$+ \ell_9(-F_{3fr}+F_{3f\ell}-F_{3rr}+F_{3r\ell}) + \ell_{10}(-F_{6fr}+F_{6f\ell}-F_{6rr}+F_{6r\ell})$$

$$- \ell_1(P_{2f}+P_{2r}) - \ell_7(P_{5fr}+P_{5f\ell}+P_{5rr}+P_{5r\ell}+P_{8fr}+P_{8f\ell}+P_{8rr}+P_{8r\ell})$$

$$+ \ell_9(-P_{3fr}+P_{3f\ell}-P_{3rr}+P_{3r\ell}) + \ell_{10}(-P_{6fr}+P_{6f\ell}-P_{6rr}+P_{6r\ell})$$

$$- \ell_{15}(P_{13fr}-P_{13f\ell}+P_{13rr}-P_{13r\ell}) - \ell_{15}(P_{15fr}-P_{15f\ell}+P_{15rr}-P_{15r\ell})$$

$$= 0$$

$$
\begin{aligned}
& I_{a\alpha}\ddot{\alpha}_a + (-2\ell_1 K_2 - 4\ell_7 K_5 - 4\ell_7 K_8)v_a \\
& \quad + (\ell_1 K_2 + 2\ell_7 K_5 + 2\ell_7 K_8)v_{bf} + (\ell_1 K_2 + 2\ell_7 K_5 + 2\ell_7 K_8)v_{br} \\
& \quad + (2\ell_1^2 K_2 + 4\ell_7^2 K_5 + 4\ell_7^2 K_8 + 4\ell_9^2 K_3 + 4\ell_{10}^2 K_6)\alpha_a \\
& \quad + (\ell_1\ell_2 K_2 + 2\ell_7\ell_{33}K_5 + 2\ell_7\ell_{33}K_8 - 2\ell_9^2 K_3 - 2\ell_{10}^2 K_6)\alpha_{bf} \\
& \quad + (\ell_1\ell_2 K_2 + 2\ell_7\ell_{33}K_5 + 2\ell_7\ell_{33}K_8 - 2\ell_9^2 K_3 - 2\ell_{10}^2 K_6)\alpha_{br} \\
& \quad + (-\ell_1\ell_{16}K_2 - 2\ell_7\ell_4 K_5 + 2\ell_7\ell_5 K_8)\gamma_{bf} + (\ell_1\ell_{16}K_2 + 2\ell_7\ell_5 K_5 - 2\ell_7\ell_5 K_8)\gamma_{br} \\
& \quad + (\ 2\ell_1 C_2 - 4\ell_7 C_5 - 4\ell_7 C_8 - 4\ell_{15}C_{13}\sin a_1 - 4\ell_{15}C_{15}\sin a_2)\dot{v}_a \\
& \quad + (\ell_1 C_2 + 2\ell_7 C_5 + 2\ell_7 C_8 + 2\ell_{15}C_{13}\sin a_1 + 2\ell_{15}C_{15}\sin a_2)\dot{v}_{bf} \\
& \quad + (\ell_1 C_2 + 2\ell_7 C_5 + 2\ell_7 C_8 + 2\ell_{15}C_{13}\sin a_1 + 2\ell_{15}C_{15}\sin a_2)\dot{v}_{br} \\
& \quad + (2\ell_1^2 C_2 + 4\ell_7^2 C_5 + 4\ell_7^2 C_8 + 4\ell_9^2 C_3 + 4\ell_{10}^2 C_6 + 4\ell_{15}^2 C_{13}\sin a_1 + 4\ell_{15}^2 C_{15}\sin a_2)\dot{\alpha}_a \\
& \quad + (\ell_1\ell_2 C_2 + 2\ell_7\ell_{33}C_5 + 2\ell_7\ell_{33}C_8 - 2\ell_9^2 C_3 - 2\ell_{10}^2 C_6 + 2\ell_{15}\ell_{32}C_{13}\sin a_1 + \\
& \quad\quad 2\ell_{15}\ell_{32}C_{15}\sin a_2)\dot{\alpha}_{bf} \\
& \quad + (\ell_1\ell_2 C_2 + 2\ell_7\ell_{33}C_5 + 2\ell_7\ell_{33}C_8 - 2\ell_9^2 C_3 - 2\ell_{10}^2 C_6 + 2\ell_{15}\ell_{32}C_{13}\sin a_1 \\
& \quad\quad + 2\ell_{15}\ell_{32}C_{15}\sin a_2)\dot{\alpha}_{br} \\
& \quad + (-\ell_1\ell_{16}C_2 - 2\ell_7\ell_4 C_5 + 2\ell_7\ell_5 C_8 - 2\ell_{15}C_{13}(\ell_{31}\cos a_1 + \ell_{29}\sin a_1) \\
& \quad\quad - 2\ell_{15}C_{15}(\ell_{30}\cos a_2 + \ell_{28}\sin a_2))\dot{\gamma}_{bf} \\
& \quad + (\ell_1\ell_{16}C_2 + 2\ell_7\ell_4 C_5 - 2\ell_7\ell_5 C_8 + 2\ell_{15}C_{13}(\ell_{31}\cos a_1 + \ell_{29}\sin a_1) \\
& \quad\quad + 2\ell_{15}C_{17}(\ell_{30}\cos a_2 + \ell_{28}\sin a_2))\dot{\gamma}_{br} \\
& \quad = 0 \qquad (4)
\end{aligned}
$$

$\underline{\Sigma M_\beta = 0}$

$$I_{a\beta}\ddot{\beta}_a + \ell_1(F_{1f} + F_{1r})$$

$$+ (\ell_3+\ell_4)(F_{3fr}+F_{3f\ell}-F_{3rr}-F_{3r\ell}) + (\ell_3-\ell_5)(F_{6fr}+F_{6f\ell}-F_{6rr}-F_{6r\ell})$$

$$+ \ell_7(F_{4fr}+F_{4f\ell}+F_{4rr}+F_{4r\ell}) + \ell_7(F_{7fr}+F_{7f\ell}+F_{7rr}+F_{7r\ell})$$

$$+ \ell_1(P_{1f} + P_{1r})$$

$$+ (\ell_3+\ell_4)(P_{3fr}+P_{3f\ell}-P_{3rr}-P_{3r\ell}) + (\ell_3-\ell_5)(P_{6fr}+P_{6f\ell}-P_{6rr}-P_{6r\ell})$$

$$+ \ell_7(P_{4fr}+P_{4f\ell}+P_{4rr}+P_{4r\ell}+P_{7f\ell}+P_{7fr}+P_{7rr}+P_{7r\ell})$$

$$+ \ell_{15}(-P_{14fr}-P_{14f\ell}+P_{14rr}+P_{14r\ell}-P_{16fr}-P_{16f\ell}+P_{16rr}+P_{16r\ell})$$

$$= 0$$

$$I_{a\beta}\ddot{\beta}_a + (2\ell_1K_1 + 4\ell_7K_4 + 4\ell_7K_7)u_a$$

$$+ (-\ell_1K_1 - 2\ell_7K_4 - 2\ell_7K_7)u_{bf}$$

$$+ (-\ell_1K_1 - 2\ell_7K_4 - 2\ell_7K_7)u_{br}$$

$$+ (-2[\ell_3+\ell_4]K_3 - 2[\ell_3-\ell_5]K_6)w_{bf}$$

$$+ (+2[\ell_3+\ell_4]K_3 + 2[\ell_3-\ell_5]K_6)w_{br}$$

$$+ (2\ell_1^2K_1 + 4(\ell_3+\ell_4)^2K_3 + 4(\ell_3-\ell_5)^2K_6 + 4\ell_7^2K_4 + 4\ell_7^2K_7)\beta_a$$

$$+ (\ell_1\ell_2K_1-2\ell_4(\ell_3+\ell_4)K_3 + 2\ell_5(\ell_3-\ell_5)K_6 + 2\ell_7\ell_{33}K_4+2\ell_7\ell_{33}K_7)\beta_{bf}$$

$$+ (\ell_1\ell_2K_1-2\ell_4(\ell_3+\ell_4)K_3 + 2\ell_5(\ell_3-\ell_5)K_6 + 2\ell_7\ell_{33}K_4+2\ell_7\ell_{33}K_7)\beta_{br}$$

$$+ (2\ell_1C_1+4\ell_7C_4+4\ell_7C_7+4\ell_{15}C_{14}\cos a_1 + 4\ell_{15}C_{16}\cos a_2)\dot{u}_a$$

(Cont'd)

$$+ (-\ell_1C_1-2\ell_7C_4-2\ell_7C_7-2\ell_{15}C_{14} \cos a_1 - 2\ell_{15}C_{16} \cos a_2)\dot{u}_{bf}$$

$$+ (-\ell_1C_1-2\ell_7C_4-2\ell_7C_7-2\ell_{15}C_{14} \cos a_1 - 2\ell_{15}C_{16} \cos a_2)\dot{u}_{br}$$

$$+ (-2(\ell_3+\ell_4)C_3 - 2(\ell_3-\ell_5)C_6)\dot{w}_{bf}$$

$$+ (2(\ell_3+\ell_4)C_3 + 2(\ell_3-\ell_5)C_6)\dot{w}_{br}$$

$$+ [\ell_1\ell_2C_1-2\ell_4(\ell_3+\ell_4)C_3 + 2\ell_5(\ell_3-\ell_5)C_6 + 2\ell_7\ell_{33}C_4+$$

$$2\ell_7\ell_{33}C_7 + 2\ell_{15}\ell_{32}C_{14} \cos a_1 + 2\ell_{15}\ell_{32}C_{16} \cos a_2]\dot{\beta}_{bf}$$

$$+ [\ell_1\ell_2C_1-2\ell_4(\ell_3+\ell_4)C_3 + 2\ell_5(\ell_3-\ell_5)C_6 + 2\ell_7\ell_{33}C_4 +$$

$$2\ell_7\ell_{33}C_7 + 2\ell_{15}\ell_{32}C_{14} \cos a_1 + 2\ell_{15}\ell_{32}C_{16} \cos a_2]\dot{\beta}_{br}$$

$$+ [2\ell_1^2C_1 + 4(\ell_3+\ell_4)^2C_3 + 4(\ell_3-\ell_5)^2C_6 + 4\ell_7^2C_4 + 4\ell_7^2C_7 +$$

$$4\ell_{15}^2C_{14} \cos a_1 + 4\ell_{15}^2C_{16} \cos a_2]\dot{\beta}_a$$

$$= 0 \qquad (5)$$

$\underline{\Sigma M_\gamma = 0}$

$$I_{a\gamma}\ddot{\gamma}_a + \ell_{16}(-F_{2f} + F_{2r}) + \ell_9(F_{4fr} - F_{4f\ell} + F_{4rr} - F_{4r\ell})$$

$$+ \ell_{10}(F_{7fr} - F_{7f\ell} + F_{7rr} - F_{7r\ell})$$

$$+ (\ell_3+\ell_4)(-F_{5fr} - F_{5f\ell} + F_{5rr} + F_{5r\ell})$$

$$+ (\ell_3-\ell_5)(-F_{8fr} - F_{8f\ell} + F_{8rr} + F_{8r\ell})$$

$$+ (F_{11fr} + F_{11f\ell} + F_{11rr} + F_{11r\ell}) + (F_{12fr} + F_{12f\ell} + F_{12rr} + F_{12r\ell})$$

+ Damping Terms

(Cont'd)

$$+ \ell_{13}(-P_{14fr} + P_{14f\ell} + P_{14rr} - P_{14r\ell})$$

$$+ \ell_{11}(-P_{16fr} + P_{16f\ell} + P_{16rr} - P_{16r\ell})$$

$$+ \ell_{12}(-P_{13fr} + P_{13f\ell} + P_{13rr} - P_{13r\ell})$$

$$+ \ell_{6}(-P_{15fr} + P_{15f\ell} + P_{15rr} - P_{15r\ell})$$

$$= 0$$

$$I_{a\gamma}\ddot{\gamma}_a + [(\ell_3+\ell_{16})K_2 + 2(\ell_3+\ell_4)K_5 + 2(\ell_3-\ell_5)K_8]v_{bf}$$

$$+ [-(\ell_3+\ell_{16})K_2 - 2(\ell_3+\ell_4)K_5 - 2(\ell_3-\ell_5)K_8]v_{br}$$

$$+ [\ (\ell_3+\ell_{16})\ell_2K_2 + 2(\ell_3+\ell_4)\ell_{33}K_5 + 2(\ell_3-\ell_5)\ell_{33}K_8]\alpha_{bf}$$

$$+ [-(\ell_3+\ell_{16})\ell_2K_2 - 2(\ell_3+\ell_4)\ell_{33}K_5 - 2(\ell_3-\ell_5)\ell_{33}K_8]\alpha_{br}$$

$$+ [2(\ell_3+\ell_{16})^2K_2 + 4\ell_9^2K_4 + 4\ell_{10}^2K_7 + 4(\ell_3+\ell_4)^2K_5 +$$
$$4(\ell_3-\ell_5)^2K_8 + 4K_{11} + 4K_{12}]\gamma_a$$

$$+ [-\ell_{16}(\ell_3+\ell_{16})K_2 - 2\ell_9^2K_4 - 2\ell_{10}^2K_7 - 2(\ell_3+\ell_4)\ell_4K_5 +$$
$$2(\ell_3-\ell_5)\ell_5K_8 - 2K_{11} - 2K_{12}]\gamma_{bf}$$

$$+ [-\ell_{16}(\ell_3+\ell_{16})K_2 - 2\ell_9^2K_4 - 2\ell_{10}^2K_7 - 2(\ell_3+\ell_4)\ell_4K_5 +$$
$$2(\ell_3-\ell_5)\ell_5K_8 - 2K_{11} - 2K_{12}]\gamma_{br}$$

$$+ [(\ell_3+\ell_{16})C_2 + 2(\ell_3+\ell_4)C_5 + 2(\ell_3-\ell_5)C_8 + 2(\ell_{12}C_{13}+\ell_{13}C_{14}) \sin a_1$$
$$+ 2(\ell_6C_{15} + \ell_{11}C_{16}) \sin a_2]\mathring{v}_{bf}$$

(Cont'd.)

$$+ [-(\ell_3+\ell_{16})C_2 - 2(\ell_3+\ell_4)C_5 - 2(\ell_3-\ell_5)C_8 - 2(\ell_{12}C_{13} + \ell_{13}C_{14})$$

$$\sin a_1 - 2(\ell_6C_{15} + \ell_{11}C_{16}) \sin a_2]\dot{v}_{br}$$

$$+ [(\ell_3+\ell_{16})\ell_2C_2 + 2(\ell_3+\ell_4)\ell_{33}C_5 + 2(\ell_3-\ell_5)\ell_{33}C_8 +$$

$$2\ell_{32}(\ell_{12}C_{13} + \ell_{13}C_{14}) \sin a_1 + 2\ell_{32}(\ell_6C_{15}+\ell_{11}C_{16}) \sin a_2]\dot{\alpha}_{bf}$$

$$+ [-(\ell_3+\ell_{16})\ell_2C_2 - 2(\ell_3+\ell_4)\ell_{33}C_5 - 2(\ell_3-\ell_5)\ell_{33}C_8 +$$

$$- 2\ell_{32}(\ell_{12}C_{13}+\ell_{13}C_{14}) \sin a_1 - 2\ell_{32}(\ell_6C_{15}+\ell_{11}C_{16} \sin a_2]\dot{\alpha}_{br}$$

$$+ [2(\ell_3+\ell_{16})^2C_2 + 4\ell_9^2C_4 + 4\ell_{10}^2C_7 + 4(\ell_3+\ell_4)^2C_5 + 4(\ell_3-\ell_5)^2C_8 +$$

$$4C_{11} + 4C_{12} + 4(\ell_{13} \cos a_1 + \ell_{12} \sin a_1)(\ell_{12}C_{13}+\ell_{13}C_{14}) +$$

$$4(\ell_{11} \cos a_2 + \ell_6 \sin a_2)(\ell_6C_{15} + \ell_{11}C_{16})]\dot{\gamma}_a$$

$$+ [-\ell_{16}(\ell_3+\ell_{16})C_2 - 2\ell_9^2C_4 - 2\ell_{10}^2C_7 - 2(\ell_3+\ell_4)\ell_4C_5 +$$

$$2(\ell_3-\ell_5)\ell_5C_8 - 2C_{11} - 2C_{12} - 2(\ell_{31} \cos a_1 +\ell_{29} \sin a_1)$$

$$(\ell_{12}C_{13} + \ell_{13}C_{14}) - 2(\ell_{30} \cos a_2 + \ell_{28} \sin a_2)$$

$$(\ell_6C_{15} + \ell_{11}C_{16})]\dot{\gamma}_{bf}$$

$$+ [-\ell_{16}(\ell_3+\ell_{16})C_2 - 2\ell_9^2C_4 - 2\ell_{10}^2C_7 - 2(\ell_3+\ell_4)\ell_4C_5 +$$

$$2(\ell_3-\ell_5)\ell_5C_8 - 2C_{11} - 2C_{12} - 2(\ell_{31} \cos a_1 + \ell_{29} \sin a_1)$$

$$(\ell_{12}C_{13} + \ell_{13}C_{14}) - 2(\ell_{30} \cos a_2 + \ell_{28} \sin a_2)$$

$$(\ell_6C_{15} + \ell_{11}C_{16})]\dot{\gamma}_{br}$$

$$= 0 \qquad (6)$$

(B) <u>For The Rear Frame (See Fig. I.2)</u>

$\Sigma F_x = 0$

$$m_b \ddot{u}_{br} - (F_{1r}) - (F_{4rr} + F_{4r\ell}) - (F_{7rr} + F_{7r\ell})$$
$$- (P_{1r}) - (P_{4rr} + P_{4r\ell}) - (P_{7rr} + P_{7r\ell})$$
$$- (P_{14rr} + P_{14r\ell}) - (P_{16rr} + P_{16r\ell}) - (RA_1 + RA_2 + RA_3)$$
$$= 0$$

$$m_b \ddot{u}_{br} + (-K_1 - 2K_4 - 2K_7)u_a + (K_1 + 2K_4 + 2K_7)u_{br}$$
$$+ (-\ell_1 K_1 - 2\ell_7 K_4 - 2\ell_7 K_7)\beta_a + (-\ell_2 K_1 - 2\ell_{33} K_4 - 2\ell_{33} K_7)\beta_{br}$$
$$+ (-C_1 - 2C_4 - 2C_7 - 2C_{14} \cos a_1 - 2C_{16} \cos a_2)(\dot{u}_a - \dot{u}_{br})$$
$$+ (-\ell_1 C_1 - 2\ell_7 C_4 - 2\ell_7 C_7 - 2\ell_{15} C_{14} \cos a_1 - 2\ell_{15} C_{16} \cos a_2)\dot{\beta}_a$$
$$+ (-\ell_2 C_1 - 2\ell_{33} C_4 - 2\ell_{33} C_7 - 2\ell_{32} C_{14} \cos a_1 - 2\ell_{32} C_{16} \cos a_2)\dot{\beta}_{br}$$
$$- (RA_1 + RA_2 + RA_3)$$
$$= 0 \qquad (7)$$

$\Sigma F_y = 0$

$$m_b \ddot{v}_{br} - (F_{2r}) - (F_{5rr} + F_{5r\ell}) - (F_{8rr} + F_{8r\ell})$$
$$- (P_{2r}) - (P_{5rr} + P_{5r\ell}) - (P_{8rr} + P_{8r\ell})$$
$$- P_{13rr} + P_{13r\ell} - P_{15rr} + P_{15r\ell} - (RB_1 + RB_2 + RB_3)$$
$$= 0$$

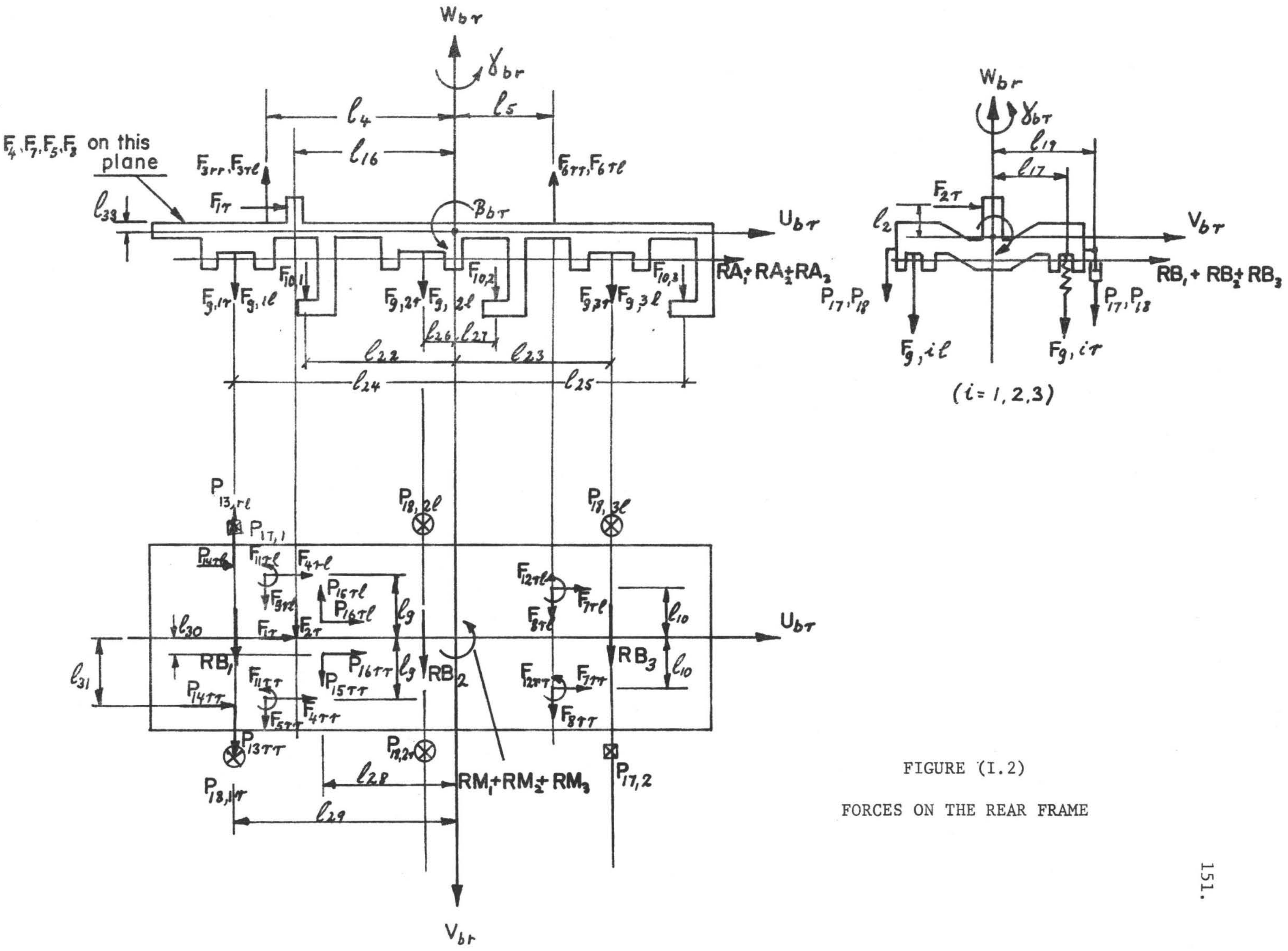


FIGURE (I.2)

FORCES ON THE REAR FRAME

$$m_b\ddot{v}_{br} + (-K_2 - 2K_5 - 2K_8)v_a + (K_2 + 2K_5 + 2K_8)v_{br}$$

$$+ (\ell_1K_2 + 2\ell_7K_5 + 2\ell_7K_8)\alpha_a + (\ell_2K_2 + 2\ell_{33}K_5 + 2\ell_{33}K_8)\alpha_{br}$$

$$+ [-(\ell_3+\ell_{16})K_2 - 2(\ell_3+\ell_4)K_5 - 2(\ell_3-\ell_5)K_8]\gamma_a$$

$$+ (\ell_{16}K_2 + 2\ell_4K_5 - 2\ell_5K_8)\gamma_{br}$$

$$+ (-C_2 - 2C_5 - 2C_8 - 2C_{13} \sin a_1 - 2C_{15} \sin a_2)\dot{v}_a$$

$$+ (C_2 + 2C_5 + 2C_8 + 2C_{13} \sin a_1 + 2C_{15} \sin a_2)\dot{v}_{br}$$

$$+ (\ell_1C_2 + 2\ell_7C_5 + 2\ell_7C_8 + 2\ell_{15}C_{13} \sin a_1 + 2\ell_{15}C_{15} \sin a_2)\dot{\alpha}_a$$

$$+ (\ell_2C_2 + 2\ell_{33}C_5 + 2\ell_{33}C_8 + 2\ell_{32}C_{13} \sin a_1 + 2\ell_{32}C_{15} \sin a_2)\dot{\alpha}_{br}$$

$$+ [-(\ell_3+\ell_{16})C_2 - 2(\ell_3+\ell_4)C_5 - 2(\ell_3-\ell_5)C_8 - 2\ell_{13}C_{13} \cos a_1 -$$

$$2\ell_{12}C_{13} \sin a_1 - 2\ell_{11}C_{15} \cos a_2 - 2\ell_6C_{15} \sin a_2]\dot{\gamma}_a$$

$$+ (\ell_{16}C_2 + 2\ell_4C_5 - 2\ell_5C_8 + 2\ell_{31}C_{13} \cos a_1 + 2\ell_{29}C_{13} \sin a_1 +$$

$$2\ell_{30}C_{15} \cos a_2 + 2\ell_{28}C_{15} \sin a_2)\dot{\gamma}_{br}$$

$$- (RB_1 + RB_2 + RB_3)$$

$$= 0 \qquad (8)$$

<u>$\Sigma F_z = 0$</u>

$$m_b\ddot{w}_{br} - (F_{3rr} + F_{3r\ell}) - (F_{6rr} + F_{6r\ell}) + (F_{9,1r}+F_{9,1\ell}+F_{9,2r} +F_{9,2\ell}$$

$$+F_{9,3r}+F_{9,3\ell})$$

$$+ (F_{10,1} + F_{10,2} + F_{10,3})$$

$$- (P_{3rr} + P_{3r\ell}) - (P_{6rr} + P_{6r\ell}) + (P_{10,1}+P_{10,2}+P_{10,3})$$

$$+ (P_{17,1}+P_{17,2}) + (P_{18,1r}+P_{18,2r}+P_{18,2\ell}+P_{18,3\ell}) = 0$$

$$m_b\ddot{w}_{br} + (-2K_3-2K_6)w_a + (2K_3+2K_6+6K_9+3K_{10})w_{br}$$

$$+ (-K_{10})w_{c1} + (-K_{10})w_{c2} + (-K_{10})w_{c3} + (-2K_9)w_{d1} + (-2K_9)w_{d2} +$$

$$[2(\ell_3+\ell_4)K_3 + 2(\ell_3-\ell_5)K_6]\beta_a + (-2K_9)w_{d3}$$

$$+ [-2\ell_4K_3+2\ell_5K_6-2(-\ell_{23}+\ell_{26}+\ell_{24})K_9 - (-\ell_{25}-\ell_{27}+\ell_{22})K_{10}]\beta_{br}$$

$$+ [-\ell_{35}\ K_{10}]\beta_{c1} + [-\ell_{35}\ K_{10}]\beta_{c2} + [-\ell_{35}\ K_{10}]\beta_{c3}$$

$$+ (-2C_3-2C_6)\dot{w}_a + (2C_3+2C_6+2C_{17}+4C_{18}+3C_{10})\dot{w}_{br}$$

$$+ (-C_{17}-C_{18})\dot{w}_{d1} + (-2C_{18})\dot{w}_{d2} + (-C_{17}-C_{18})\dot{w}_{d3} + [-\ell_{19}(C_{17}-C_{18})]\dot{\alpha}_{d1}$$

$$- (C_{10})\dot{w}_{c1} + (-C_{10})\dot{w}_{c2} + (-C_{10})\dot{w}_{c3} + (+\ell_{19}(C_{17}-C_{18}))\dot{\alpha}_{d3}$$

$$+ [2(\ell_3+\ell_4)C_3 + 2(\ell_3-\ell_5)C_6]\dot{\beta}_a$$

$$+ [-2\ell_4C_3+2\ell_5C_6+(-\ell_{24}+\ell_{23})C_{17} + (-\ell_{24}-2\ell_{26}+\ell_{23})C_{18} +$$

$$(\ell_{25} + \ell_{27} - \ell_{22})C_{10}]\dot{\beta}_{br}$$

$$+ (-\ell_{35}C_{10})\dot{\beta}_{c1} + (-\ell_{35}C_{10})\dot{\beta}_{c2} + (-\ell_{35}C_{10})\dot{\beta}_{c3}$$

$$= 0 \qquad (9)$$

$\Sigma M_\alpha = 0$

$$I_{b\alpha}\ddot{\alpha}_{br} - \ell_2(F_{2r}) - \ell_{33}(F_{5rr} + F_{5r\ell}) - \ell_{33}(F_{8rr} + F_{8r\ell})$$

$$+ \ell_9(F_{3rr} - F_{3r\ell}) + \ell_{10}(F_{6rr} - F_{6r\ell})$$

$$+ \ell_{17}(-F_{9,1r} + F_{9,1\ell} - F_{9,2r} + F_{9,2\ell} - F_{9,3r} + F_{9,3\ell})$$

$$- \ell_2(P_{2r}) - \ell_{33}(P_{5rr} + P_{5r\ell}) - \ell_{33}(P_{8rr} + P_{8r\ell})$$

$$+ \ell_9(P_{3rr} - P_{3r\ell}) + \ell_{10}(P_{6rr} - P_{6r\ell})$$

$$+ \ell_{32}(-P_{13rr} + P_{13r\ell}) + \ell_{32}(-P_{15rr} + P_{15r\ell})$$

$$+ \ell_{19}(P_{17,1}-P_{17,2}) + \ell_{19}(-P_{18,1r}-P_{18,2r}+P_{18,2\ell}+P_{18,3\ell})$$

$$+ \ell_{21}(RB_1 + RB_2 + RB_3)$$

$$= 0$$

$$I_{b\alpha}\ddot{\alpha}_{br} + (-\ell_2K_2-2\ell_{33}K_5-2\ell_{33}K_8)v_a + (\ell_2K_2+2\ell_{33}K_5+2\ell_{33}K_8)v_{br}$$

$$+ (\ell_2\ell_1K_2+2\ell_{33}\ell_7K_5+2\ell_{33}\ell_7K_8-2\ell_9^2K_3-2\ell_{10}^2K_6)\alpha_a$$

$$+ (\ell_2^2K_2+2\ell_{33}^2K_5+2\ell_{33}^2K_8+2\ell_9^2K_3+2\ell_{10}^2K_6+6\ell_{17}^2K_9)\alpha_{br}$$

$$+ (-2\ell_{17}^2K_9)\alpha_{d1} + (-2\ell_{17}^2K_9)\alpha_{d2} + (-2\ell_{17}^2K_9)\alpha_{d3}$$

$$+ [-\ell_2(\ell_3+\ell_{16})K_2 - 2\ell_{33}(\ell_3+\ell_4)K_5 - 2\ell_{33}(\ell_3-\ell_5)K_8]\gamma_a$$

$$+ (\ell_2\ell_{16}K_2 + 2\ell_{33}\ell_4K_5 - 2\ell_{33}\ell_5K_8)\gamma_{br}$$

$$+ (-\ell_2C_2 - 2\ell_{33}C_5 - 2\ell_{33}C_8 - 2\ell_{32}C_{13} \sin a_1 - 2\ell_{32}C_{15} \sin a_2)\dot{v}_a$$

$$+ (\ell_2C_2 + 2\ell_{33}C_5 + 2\ell_{33}C_8 + 2\ell_{32}C_{13} \sin a_1 + 2\ell_{32}C_{15} \sin a_2)\dot{v}_{br}$$

$$+ (\ell_2\ell_1C_2 + 2\ell_{33}\ell_7C_5 + 2\ell_{33}\ell_7C_8 - 2\ell_9^2C_3 - 2\ell_{10}^2C_6 + 2\ell_{32}\ell_{15}C_{13}$$

$$\sin a_1 + 2\ell_{32}\ell_{15}C_{15} \sin a_2)\dot{\alpha}_a$$

$$+ [\ell_{19}(-C_{17} + C_{18})]\dot{w}_{d1} + [\ell_{19}(C_{17} - C_{18})]\dot{w}_{d3}$$

$$+ (\ell_2^2 C_2 + 2\ell_{33}^2 C_5 + 2\ell_{33}^2 C_8 + 2\ell_9^2 C_3 + 2\ell_{10}^2 C_6 + 2\ell_{32}^2 C_{13}$$

$$\sin a_1 + 2\ell_{32}^2 C_{15} \sin a_2 + 2\ell_{19}^2 C_{17} + 4\ell_{19}^2 C_{18})\dot{\alpha}_{br}$$

$$+ (-\ell_{19}^2 C_{17} - \ell_{19}^2 C_{18})\dot{\alpha}_{d1} + (-2\ell_{19}^2 C_{18})\dot{\alpha}_{d2} + (-\ell_{19}^2 C_{17} - \ell_{19}^2 C_{18})\dot{\alpha}_{d3}$$

$$+ [\ell_{19}(-\ell_{24} - \ell_{23})C_{17} + \ell_{19}(\ell_{24} + \ell_{23})C_{18}]\dot{\beta}_{br}$$

$$+ [-\ell_2(\ell_3 + \ell_{16})C_2 - 2\ell_{33}(\ell_3 + \ell_4)C_5 - 2\ell_{33}(\ell_3 - \ell_5)C_8 -$$

$$2\ell_{32}\ell_{13}C_{13} \cos a_1 - 2\ell_{32}\ell_{12}C_{13} \sin a_1 - 2\ell_{32}\ell_{11}C_{15}$$

$$\cos a_2 - 2\ell_{32}\ell_6 C_{15} \sin a_2]\dot{\gamma}_a$$

$$+ [\ell_2\ell_{16}C_2 + 2\ell_{33}\ell_4 C_5 - 2\ell_{33}\ell_5 C_8 + 2\ell_{32}\ell_{31}C_{13} \cos a_1 +$$

$$2\ell_{32}\ell_{29}C_{13} \sin a_1 + 2\ell_{32}\ell_{30}C_{15} \cos a_2 + 2\ell_{32}\ell_{28}C_{15} \sin a_2]\dot{\gamma}_{br}$$

$$+ \ell_{21}(RB_1 + RB_2 + RB_3) = 0 \qquad (10)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{b\beta}\ddot{\beta}_{br} + \ell_2(F_{2r}) + \ell_{31}(F_{4rr} + F_{4r\ell}) + \ell_{31}(F_{7rr} + F_{7r\ell}) + \ell_2(P_{2r}) +$$

$$\ell_{31}(P_{4rr} + P_{4r\ell})$$

$$+ \ell_{31}(P_{7rr} + P_{7r\ell}) + \ell_4(F_{3rr} + F_{3r\ell}) - \ell_5(F_{6rr} + F_{6r\ell}) +$$

$$\ell_4(P_{3rr} + P_{3r\ell}) - \ell_5(P_{6rr} + P_{6r\ell})$$

$$- \ell_{24}(F_{9,1r} + F_{9,1\ell}) - \ell_{26}(F_{9,2r} + F_{9,2\ell}) + \ell_{23}(F_{9,3r} + F_{9,3\ell})$$

$$- \ell_{22}(F_{10,1}) + \ell_{27}(F_{10,2}) + \ell_{25}(F_{10,3}) - \ell_{22}(P_{10,1}) +$$

$$\ell_{27}(P_{10,2}) + \ell_{25}(P_{10,3})$$

$$+ \ell_{32}(P_{14rr} + P_{14r\ell}) + \ell_{32}(P_{16rr} + P_{16r\ell})$$

$$- \ell_{24}(P_{17,1}+P_{18,1r}) - \ell_{26}(P_{18,2r}+P_{18,2\ell}) + \ell_{23}(P_{17,2}+P_{18,3\ell})$$

$$- \ell_{21}(RA_1 + RA_2 + RA_3) = 0$$

$$I_{b\beta}\ddot{\beta}_{br} + (\ell_2K_1+2\ell_{33}K_4+2\ell_{33}K_7)u_a + (-\ell_2K_1-2\ell_{33}K_4-2\ell_{33}K_7)u_{br}$$

$$+ [2\ell_4K_3-2\ell_5K_6]w_a + (\ell_{22}K_{10})w_{c1} + (-\ell_{27}K_{10})w_{c2} + (-\ell_{25}K_{10})w_{c3}$$

$$+ (2\ell_{24}K_9)w_{d1} + (2\ell_{26}K_9)w_{d2} + (-2\ell_{33}K_9)w_{d3}$$

$$+ [-2\ell_4K_3+2\ell_5K_6+2(-\ell_{24}-\ell_{26}+\ell_{23})K_9 + (-\ell_{22}+\ell_{27}+\ell_{25})K_{10}]w_{br}$$

$$+ [\ell_2\ell_1K_1+2\ell_{33}\ell_7K_4+2\ell_{33}\ell_7K_7-2\ell_4(\ell_3+\ell_4)K_3+2\ell_5(\ell_3-\ell_5)K_6]\beta_a$$

$$+ [\ell_2^2K_1+2\ell_{33}^2K_4+2\ell_{33}^2K_7+2\ell_4^2K_3+2\ell_5^2K_6+2(\ell_{24}^2+\ell_{26}^2+\ell_{23}^2)K_9 +$$
$$(\ell_{22}^2+\ell_{27}^2+\ell_{25}^2)K_{10}]\beta_{br}$$

$$+ [\ell_{22}\ell_{35}K_{10}]\beta_{c1} + [-\ell_{27}\ell_{35}K_{10}]\beta_{c2} + [-\ell_{25}\ell_{35}K_{10}]\beta_{c3}$$

$$+ (\ell_2C_1+2\ell_{33}C_4+2\ell_{33}C_7+2\ell_{32}C_{14}\cos a_1 + 2\ell_{32}C_{16}\cos a_2)\dot{u}_a$$

$$+ (-\ell_2C_1-2\ell_{33}C_4-2\ell_{33}C_7-2\ell_{32}C_{14}\cos a_1 - 2\ell_{32}C_{16}\cos a_2)\dot{u}_{br}$$

$$+ (2\ell_4C_3-2\ell_5C_6)\dot{w}_a + [-2\ell_4C_3+2\ell_5C_6-\ell_{24}(C_{17}+C_{18}) -2\ell_{26}C_{18} +$$
$$(-\ell_{22}+\ell_{27}+\ell_{25})C_{10} + \ell_{23}(C_{17}+C_{18})]\dot{w}_{br}$$

$$+ \ell_{24}(C_{17}+C_{18})\dot{w}_{d1} + (2\ell_{26}C_{18})\dot{w}_{d2} + (-\ell_{23}(C_{17}+C_{18}))\dot{w}_{d3}$$

$$+ (\ell_{22}C_{10})\dot{w}_{c1} + (-\ell_{27}C_{10})\dot{w}_{c2} + (-\ell_{25}C_{10})\dot{w}_{c3}$$

$$+ (-\ell_{24}\ell_{19}C_{17} + \ell_{24}\ell_{19}C_{18} - \ell_{23}\ell_{19}C_{17} + \ell_{23}\ell_{19}C_{18})\dot{\alpha}_{br}$$

$$+ (\ell_{24}\ell_{19}C_{17}-\ell_{24}\ell_{19}C_{18})\alpha_{d1} + (\ell_{23}\ell_{19}C_{17}-\ell_{23}\ell_{19}C_{18})\dot{\alpha}_{d3}$$

$$+ (\ell_{2}\ell_{1}C_{1}+2\ell_{33}\ell_{7}C_{4}+2\ell_{33}\ell_{7}C_{7}-2\ell_{4}(\ell_{3}+\ell_{4})C_{3} + 2\ell_{5}(\ell_{3}-\ell_{5})C_{6} +$$

$$2\ell_{32}\ell_{15}C_{14} \cos a_{1} + 2\ell_{32}\ell_{15}C_{16} \cos a_{2})\dot{\beta}_{a}$$

$$+ [\ell_{2}^{2}C_{1}+2\ell_{33}^{2}C_{4}+2\ell_{33}^{2}C_{7}+2\ell_{4}^{2}C_{3}+2\ell_{5}^{2}C_{6}+2\ell_{32}^{2}C_{14} \cos a_{1} +$$

$$2\ell_{32}^{2}C_{16} \cos a_{2} +\ell_{24}^{2}C_{17}+\ell_{24}^{2}C_{18}+2\ell_{26}^{2}C_{18}+\ell_{23}^{2}C_{17}+\ell_{23}^{2}C_{18} +$$

$$(\ell_{23}^{2}+\ell_{27}^{2}+\ell_{25}^{2})C_{10}]\dot{\beta}_{br}$$

$$+(\ell_{22}\ell_{35}C_{10})\dot{\beta}_{c1} + (-\ell_{35}\ell_{27}C_{10})\dot{\beta}_{c2} + (-\ell_{25}\ell_{35}C_{10})\dot{\beta}_{c3} -$$

$$\ell_{21}(RA_{1} + RA_{2} + RA_{3}) = 0 \qquad (11)$$

$\Sigma M_{\gamma} = 0$

$$I_{b\gamma}\ddot{\gamma}_{br} + \ell_{9}(-F_{4rr}+F_{4r\ell}) + \ell_{10}(-F_{7rr}+F_{7r\ell}) + \ell_{9}(-P_{4rr}+P_{4r\ell}) +$$

$$\ell_{10}(-P_{7rr}+P_{7r\ell})$$

$$+ \ell_{16}(-F_{2r}) + \ell_{16}(-P_{2r})$$

$$+ \ell_{4}(-F_{5rr}-F_{5r\ell}) + \ell_{5}(F_{8rr}+F_{8r\ell}) + \ell_{4}(-P_{5rr}-P_{5r\ell}) +$$

$$\ell_{5}(P_{8rr}+P_{8r\ell})$$

$$+ (-F_{11rr}-F_{11r\ell}) + (-F_{12rr}-F_{12r\ell}) + (-P_{11rr}-P_{11r\ell}) + (-P_{12rr}-P_{12r\ell})$$

$$- \ell_{31}(P_{14rr}-P_{14r\ell}) - \ell_{30}(P_{16rr}-P_{16r\ell})$$

$$- \ell_{29}(P_{13rr}-P_{13r\ell}) - \ell_{28}(P_{15rr}-P_{15r\ell})$$

$$- \ell_{24}(RB_1) - \ell_{26}(RB_2) + \ell_{23}(RB_3)$$

$$- (RM_1 + RM_2 + RM_3)$$

$$= 0$$

$$I_{b\gamma}\ddot{\gamma}_{br} + (-\ell_{16}K_2 - 2\ell_4K_5 + 2\ell_5K_8)v_a$$

$$+ (\ell_{16}K_2 + 2\ell_4K_5 - 2\ell_5K_8)v_{br}$$

$$+ (\ell_{16}\ell_1K_2 + 2\ell_4\ell_7K_5 - 2\ell_5\ell_7K_8)\alpha_a$$

$$+ (\ell_{16}\ell_2K_2 + 2\ell_4\ell_{33}K_5 - 2\ell_5\ell_{33}K_8)\alpha_{br}$$

$$+ [-2\ell_9^2K_4 - 2\ell_{10}^2K_7 - \ell_{16}(\ell_3+\ell_{16})K_2 - 2\ell_4(\ell_3+\ell_4)K_5 +$$

$$2\ell_5(\ell_3-\ell_5)K_8 - 2K_{11} - 2K_{12}]\gamma_a$$

$$+ [2\ell_9^2K_4 + 2\ell_{10}^2K_7 + \ell_{16}^2K_2 + 2\ell_4^2K_5 + 2\ell_5^2K_8 + 2K_{11} + 2K_{12}]\gamma_{br}$$

$$+ [-\ell_{16}C_2 - 2\ell_4C_5 + 2\ell_5C_8 - 2\ell_{31}C_{14}\sin a_1 - 2\ell_{29}C_{13}$$

$$\sin a_1 - 2\ell_{30}C_{16}\sin a_2 - 2\ell_{28}C_{15}\sin a_2]\dot{v}_a$$

$$+ [\ell_{16}C_2 + 2\ell_4C_5 - 2\ell_5C_8 + 2\ell_{31}C_{14}\sin a_1 + 2\ell_{29}C_{13}\sin a_1 +$$

$$2\ell_{30}C_{16}\sin a_2 + 2\ell_{28}C_{15}\sin a_2]\dot{v}_{br}$$

$$+ [\ell_{16}\ell_1 C_2 + 2\ell_4\ell_7 C_5 - 2\ell_5\ell_7 C_8 + 2\ell_{31}\ell_{15}C_{14} \sin a_1 +$$

$$2\ell_{29}\ell_{15}C_{13} \sin a_1 + 2\ell_{30}\ell_{15}C_{16} \sin a_2 + 2\ell_{28}\ell_{15}C_{15} \sin a_2]\dot{\alpha}_a$$

$$+ [\ell_{16}\ell_2 C_2 + 2\ell_4\ell_{33}C_5 - 2\ell_5\ell_{33}C_8 + 2\ell_{31}\ell_{32}C_{14} \sin a_1 +$$

$$2\ell_{29}\ell_{32}C_{13} \sin a_1 + 2\ell_{30}\ell_{32}C_{16} \sin a_2 + 2\ell_{28}\ell_{32}C_{15} \sin a_2]\dot{\alpha}_{br}$$

$$+ [-2\ell_9^2 C_4 - 2\ell_{10}^2 C_7 - \ell_{16}(\ell_3+\ell_{16})C_2 - 2\ell_4(\ell_3+\ell_4)C_5 + 2\ell_5(\ell_3-\ell_5)C_8 -$$

$$2C_{11} - 2C_{12} + (-2\ell_{31}C_{14} - 2\ell_{29}C_{13})(\ell_{13} \cos a_1 + \ell_{12} \sin a_1) +$$

$$(-2\ell_{30}C_{16} - 2\ell_{28}C_{15})(\ell_{11} \cos a_2 + \ell_6 \sin a_2)]\dot{\gamma}_a$$

$$+ [\ 2\ell_9^2 C_4 + 2\ell_{10}^2 C_7 + \ell_{16}^2 C_2 + 2\ell_4^2 C_5 + 2\ell_5^2 C_8 + 2C_{11} + 2C_{12} +$$

$$(2\ell_{31}C_{14} + 2\ell_{29}C_{13})(\ell_{31} \cos a_1 + \ell_{29} \sin a_1) +$$

$$(2\ell_{30}C_{16} + 2\ell_{28}C_{15})(\ell_{30} \cos a_2 + \ell_{28} \sin a_2)]\dot{\gamma}_{br}$$

$$- \ell_{24}(RB_1) - \ell_{26}(RB_2) + \ell_{23}(RB_3) - (RM_1 + RM_2 + RM_3) = 0 \qquad (12)$$

(C) <u>For The Front Frame (See Fig. I.3)</u>

<u>$\Sigma F_x = 0$</u>

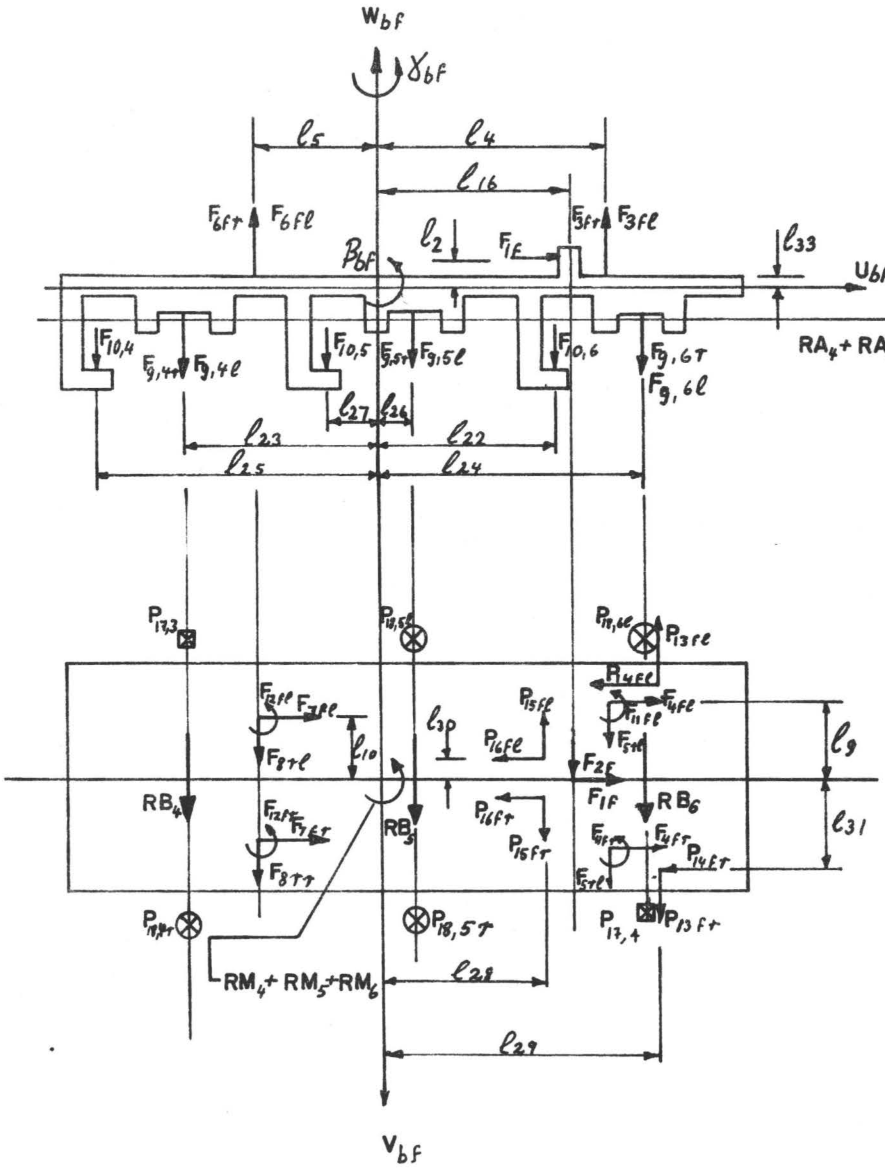
$$m_b\ddot{u}_{bf} - (F_{1f}) - (F_{4fr} + F_{4f\ell}) - (F_{7fr} + F_{7f\ell})$$

$$- (P_{1f}) - (P_{4fr} + P_{4f\ell}) - (P_{7fr} + P_{7f\ell})$$

$$+ (P_{14fr} + P_{14f\ell}) + (P_{16fr} + P_{16f\ell})$$

$$- (RA_4 + RA_5 + RA_6)$$

$$= 0$$



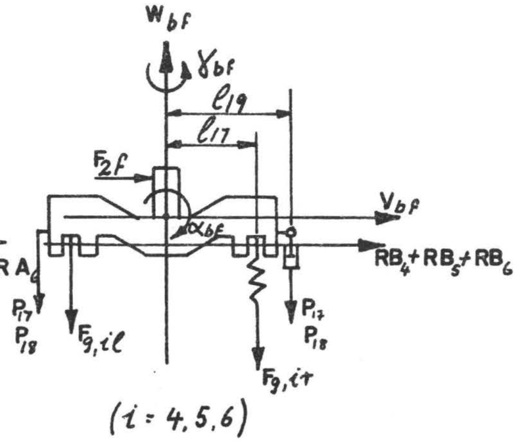


FIGURE (I.3)

FORCES ON THE FRONT FRAME

$$m_b\ddot{u}_{bf} + (-K_1 - 2K_4 - 2K_7)u_a + (K_1 + 2K_4 + 2K_7)u_{bf}$$

$$+ (-\ell_1K_1 - 2\ell_7K_4 - 2\ell_7K_7)\beta_a + (-\ell_2K_1 - 2\ell_{33}K_4 - 2\ell_{33}K_7)\beta_{bf}$$

$$+ (-C_1 - 2C_4 - 2C_7 - 2C_{14}\cos a_1 - 2C_{16}\cos a_2)\dot{u}_a$$

$$+ (C_1 + 2C_4 + 2C_7 + 2C_{14}\cos a_1 + 2C_{16}\cos a_2)\dot{u}_{bf}$$

$$+ (-\ell_1C_1 - 2\ell_7C_4 - 2\ell_7C_7 - 2\ell_{15}C_{14}\cos a_1 - 2\ell_{15}C_{16}\cos a_2)\dot{\beta}_a$$

$$+ (-\ell_2C_1 - 2\ell_{33}C_4 - 2\ell_{33}C_7 - 2\ell_{32}C_{14}\cos a_1 - 2\ell_{32}C_{16}\cos a_2)\dot{\beta}_{bf}$$

$$- (RA_4 + RA_5 + RA_6) = 0 \qquad (13)$$

$\underline{\Sigma F_y = 0}$

$$m_b\ddot{v}_{bf} - (F_{2f}) - (F_{5fr} + F_{5f\ell}) - (F_{8fr} + F_{8f\ell})$$

$$- (P_{2f}) - (P_{5fr} + P_{5f\ell}) - (P_{8fr} + P_{8f\ell})$$

$$- P_{13fr} + P_{13f\ell} - P_{15fr} + P_{15f\ell} - (RB_4 + RB_5 + RB_6)$$

$$= 0$$

$$m_b\ddot{v}_{bf} + (-K_2 - 2K_5 - 2K_8)v_a + (K_2 + 2K_5 + 2K_8)v_{bf}$$

$$+ (\ell_1K_2 + 2\ell_7K_5 + 2\ell_7K_8)\alpha_a + (\ell_2K_2 + 2\ell_{33}K_5 + 2\ell_{33}K_8)\alpha_{bf}$$

$$+ [(\ell_3+\ell_{16})K_2 + 2(\ell_3+\ell_4)K_5 + 2(\ell_3-\ell_5)K_8]\gamma_a$$

$$+ (-\ell_{16}K_2 - 2\ell_4K_5 + 2\ell_5K_8)\gamma_{bf}$$

$$+ (-C_2 - 2C_5 - 2C_8 - 2C_{13}\sin a_1 - 2C_{15}\sin a_2)\dot{v}_a$$

$$+ (C_2 + 2C_5 + 2C_8 + 2C_{13} \sin a_1 + 2C_{15} \sin a_2)\dot{v}_{bf}$$

$$+ (\ell_1 C_2 + 2\ell_7 C_5 + 2\ell_7 C_8 + 2\ell_{15} C_{13} \sin a_1 + 2\ell_{15} C_{15} \sin a_2)\dot{\alpha}_a$$

$$+ (\ell_2 C_2 + 2\ell_{33} C_5 + 2\ell_{33} C_8 + 2\ell_{32} C_{13} \sin a_1 + 2\ell_{32} C_{15} \sin a_2)\dot{\alpha}_{bf}$$

$$+ [(\ell_3 + \ell_{16})C_2 + 2(\ell_3 + \ell_4)C_5 + 2(\ell_3 - \ell_5)C_8 + 2\ell_{13} C_{13} \cos a_1 +$$

$$2\ell_{12} C_{13} \sin a_1 + 2\ell_{11} C_{15} \cos a_2 + 2\ell_6 C_{15} \sin a_2]\dot{\gamma}_a$$

$$+ [-\ell_{16} C_2 - 2\ell_4 C_5 + 2\ell_5 C_8 - 2\ell_{31} C_{13} \cos a_1 - 2\ell_{29} C_{13} \sin a_1 -$$

$$2\ell_{30} C_{15} \cos a_2 - 2\ell_{28} C_{15} \sin a_2]\dot{\gamma}_{bf}$$

$$- (RB_4 + RB_5 + RB_6)$$

$$= 0 \qquad (14)$$

$\Sigma F_z = 0$

$$m_b \ddot{w}_{bf} - (F_{3fr} + F_{3f\ell}) - (F_{6fr} + F_{6f\ell})$$

$$+ (F_{9,4r} + F_{9,4\ell} + F_{9,5r} + F_{9,5\ell} + F_{9,6r} + F_{9,6\ell})$$

$$+ (F_{10,4} + F_{10,5} + F_{10,6})$$

$$- (P_{3fr} + P_{3f\ell}) - (P_{6fr} + P_{6f\ell})$$

$$+ (P_{10,4} + P_{10,5} + P_{10,6})$$

$$+ (P_{17,3} + P_{17,4})$$

$$+ (P_{18,4r} + P_{18,5r} + P_{18,5\ell} + P_{18,6\ell})$$

$$= 0$$

$$m_b\ddot{w}_{bf} + (-2K_3 - 2K_6)w_a + (2K_3 + 2K_6 + 6K_9 + 3K_{10})w_{bf}$$

$$+ (-K_{10})w_{c4} + (-K_{10})w_{c5} + (-K_{10})w_{c6}$$

$$+ (-2K_9)w_{d4} + (-2K_9)w_{d5} + (-2K_9)w_{d6}$$

$$+ [-2(\ell_3+\ell_4)K_3 - 2(\ell_3-\ell_5)K_6]\beta_a$$

$$+ [2\ell_4K_3 - 2\ell_5K_6 + 2(-\ell_{23}+\ell_{26}+\ell_{24})K_9 + (-\ell_{25}-\ell_{27}+\ell_{22})K_{10}]\beta_{bf}$$

$$+ (\ell_{35}\ K_{10})\beta_{c4} + (\ell_{35}\ K_{10})\beta_{c5} + (\ell_{35}\ K_{10})\beta_{c6}$$

$$+ (-2C_3-2C_6)\dot{w}_a + (2C_3+2C_6+2C_{17}+4C_{18})\dot{w}_{bf}$$

$$+ (-C_{17}-C_{18})\dot{w}_{d4} + (-2C_{18})\dot{w}_{d5} + (-C_{18}-C_{17})\dot{w}_{d6}$$

$$+ (-C_{10})\dot{w}_{c4} + (-C_{10})\dot{w}_{c5} + (-C_{10})\dot{w}_{c6}$$

$$+ (-\ell_{19}C_{17} + \ell_{19}C_{18})\dot{\alpha}_{d4} + (\ell_{19}C_{17} - \ell_{19}C_{18})\dot{\alpha}_{d6}$$

$$+ [-2(\ell_3+\ell_4)C_3 - 2(\ell_3-\ell_5)C_6]\dot{\beta}_a$$

$$+ [2\ell_4C_3 - 2\ell_5C_6 + (-\ell_{23}+\ell_{24})C_{17} + (-\ell_{23} + 2\ell_{26} + \ell_{24})C_{18} +$$

$$(-\ell_{25} - \ell_{27} + \ell_{22})C_{10}]\dot{\beta}_{bf}$$

$$+ \ell_{35}C_{10}\dot{\beta}_{c4} + \ell_{35}C_{10}\dot{\beta}_{c5} + \ell_{35}C_{10}\dot{\beta}_{c6}$$

$$= 0 \qquad (15)$$

$\underline{\Sigma M_{\alpha} = 0}$

$$I_{b\alpha}\ddot{\alpha}_{bf} - \ell_2(F_{2f}) - \ell_{33}(F_{5fr}+F_{5f\ell}) - \ell_{33}(F_{8fr}+F_{8f\ell})$$

$$- \ell_2(P_{2f}) - \ell_{33}(P_{5fr}+P_{5f\ell}) - \ell_{33}(P_{8fr}+P_{8f\ell})$$

$$+ \ell_{32}(-P_{13fr}+P_{13f\ell}) + \ell_{32}(-P_{15fr}+P_{15f\ell})$$

$$+ \ell_9(F_{3fr}-F_{3f\ell}) + \ell_{10}(F_{6fr}-F_{6f\ell}) + \ell_9(P_{3fr}-P_{3f\ell}) + \ell_{10}(P_{6fr}-P_{6f\ell})$$

$$+ \ell_{17}(-F_{9,4r} + F_{9,4\ell} - F_{9,5r} + F_{9,5\ell} - F_{9,6r} + F_{9,6\ell})$$

$$+ \ell_{19}(P_{17,3}-P_{17,4}) + \ell_{19}(-P_{18,4r} - P_{18,5r} + P_{18,5\ell} + P_{18,6\ell})$$

$$+ \ell_{21}(RB_4 + RB_5 + RB_6)$$

$$= 0$$

$$I_{b\alpha}\ddot{\alpha}_{bf} + (-\ell_2K_2-2\ell_{33}K_5-2\ell_{33}K_8)v_a + (\ell_2K_2+2\ell_{33}K_5+2\ell_{33}K_8)v_{bf}$$

$$+ (\ell_2\ell_1K_2 + 2\ell_{33}\ell_7K_5 + 2\ell_{33}\ell_7K_8 - 2\ell_9^2K_3 - 2\ell_{10}^2K_6)\alpha_a$$

$$+ (\ell_2^2K_2 + 2\ell_{33}^2K_5 + 2\ell_{33}^2K_8 + 2\ell_9^2K_3 + 2\ell_{10}^2K_6 + 6\ell_{17}^2K_9)\alpha_{bf}$$

$$+ (-2\ell_{17}^2K_9)\alpha_{d4} + (-2\ell_{17}^2K_9)\alpha_{d5} + (-2\ell_{17}^2K_9)\alpha_{d6}$$

$$+ [\ell_2(\ell_3+\ell_{16})K_2 + 2\ell_{33}(\ell_3+\ell_4)K_5 + 2\ell_{33}(\ell_3-\ell_5)K_8]\gamma_a$$

$$+ (-\ell_2\ell_{16}K_2 - 2\ell_{33}\ell_4K_5 + 2\ell_{33}\ell_5K_8)\gamma_{bf}$$

$$+ (-\ell_2C_2 - 2\ell_{33}C_5 - 2\ell_{33}C_8 - 2\ell_{32}C_{13}\sin a_1 - 2\ell_{32}C_{15}\sin a_2)\dot{v}_a$$

$$+ (\ell_2C_2 + 2\ell_{33}C_5 + 2\ell_{33}C_8 + 2\ell_{32}C_{13}\sin a_1 + 2\ell_{32}C_{15}\sin a_2)\dot{v}_{bf}$$

$$+ [\ell_{19}(-C_{17}+C_{18})]\dot{w}_{d4} + [\ell_{19}(C_{17}-C_{18})]\dot{w}_{d6}$$

$$+ [\ell_2\ell_1C_2 + 2\ell_{33}\ell_7C_5 + 2\ell_{33}\ell_7C_8 - 2\ell_9^2C_3 + 2\ell_{10}^2C_6 +$$

$$2\ell_{32}\ell_{15}C_{13} \sin a_1 + 2\ell_{32}\ell_{15}C_{15} \sin a_2]\dot{\alpha}_a$$

$$+ [\ell_2^2C_2 + 2\ell_{33}^2C_5 + 2\ell_{33}^2C_8 + 2\ell_9^2C_3 + 2\ell_{10}^2C_6 + 2\ell_{32}^2C_{13} \sin a_1 +$$

$$2\ell_{32}^2C_{15} \sin a_2 + 2\ell_{19}^2C_{17} + 4\ell_{19}^2C_{18}]\dot{\alpha}_{bf}$$

$$+ (-\ell_{19}^2C_{17}-\ell_{19}^2C_{18})\dot{\alpha}_{d4} + (-2\ell_{19}^2C_{18})\dot{\alpha}_{d5} + (-\ell_{19}^2C_{17}-\ell_{19}^2C_{18})\dot{\alpha}_{d6}$$

$$+ [\ell_{19}(-\ell_{23}-\ell_{24})C_{17} + \ell_{19}(\ell_{23}+\ell_{24})C_{18}]\dot{\beta}_{bf}$$

$$+ [\ell_2(\ell_3+\ell_{16})C_2 + 2\ell_{33}(\ell_3+\ell_4)C_5 + 2\ell_{33}(\ell_3-\ell_5)C_8 +$$

$$2\ell_{32}\ell_{13}C_{13} \cos a_1 + 2\ell_{32}\ell_{12}C_{13} \sin a_1 + 2\ell_{32}\ell_{11}C_{15}$$

$$\cos a_2 + 2\ell_{32}\ell_6C_{15} \sin a_2]\dot{\gamma}_a$$

$$+ [-\ell_2\ell_{16}C_2 - 2\ell_{33}\ell_4C_5 + 2\ell_{33}\ell_5C_8 - 2\ell_{32}\ell_{31}C_{13} \cos a_1 -$$

$$2\ell_{32}\ell_{29}C_{13} \sin a_1 - 2\ell_{32}\ell_{30}C_{15} \cos a_2 - 2\ell_{32}\ell_{28}C_{15} \sin a_2]\dot{\gamma}_{bf}$$

$$+ \ell_{21}(RB_4 + RB_5 + RB_6) = 0 \qquad (16)$$

$\Sigma M_\beta = 0$

$$I_{b\beta}\ddot{\beta}_{bf} + \ell_2F_{1f} + \ell_{33}(F_{4fr} + F_{4f\ell}) + \ell_{33}(F_{7f\ell} + F_{7fr})$$

$$+ \ell_2(P_{1f}) + \ell_{33}(P_{4fr}+P_{4f\ell}) + \ell_{33}(P_{7fr}+P_{7f\ell})$$

$$- \ell_{32}(P_{14fr} + P_{14f\ell}) - \ell_{32}(P_{16fr} + P_{16f\ell})$$

$$- \ell_4(F_{3fr}+F_{3f\ell}) + \ell_5(F_{6fr}+F_{6f\ell}) - \ell_4(P_{3fr}+P_{3f\ell}) + \ell_5(P_{6fr}+P_{6f\ell})$$

$$- \ell_{23}(F_{9,4r}+F_{9,4\ell}) + \ell_{26}(F_{9,5r}+F_{9,5\ell}) + \ell_{24}(F_{9,6r}+F_{9,6\ell})$$

$$- \ell_{25}(F_{10,4}) - \ell_{27}(F_{10,5}) + \ell_{22}(F_{10,6}) - \ell_{25}(P_{10,4}) -$$

$$\ell_{27}(P_{10,5}) + \ell_{22}(P_{10,6})$$

$$- \ell_{23}(P_{17,3}+P_{18,4r}) + \ell_{26}(P_{18,5r}+P_{18,5\ell}) + \ell_{24}(P_{17,4}+P_{18,6\ell})$$

$$- \ell_{21}(RA_4 + RA_5 + RA_6)$$

$$= 0$$

$$I_{b\beta}\ddot{\beta}_{bf} + (\ell_2K_1 + 2\ell_{33}K_4 + 2\ell_{33}K_7)u_a + (-\ell_2K_1 - 2\ell_{33}K_4 - 2\ell_{33}K_7)u_{bf}$$

$$+ [-2\ell_4K_3 + 2\ell_5K_6]w_a + [2\ell_4K_3 - 2\ell_5K_6 + 2(-\ell_{23}+\ell_{26}+\ell_{24})K_9 +$$

$$(-\ell_{25}-\ell_{27}+\ell_{22})K_{10}]w_{bf}$$

$$+ (\ell_{25}K_{10})w_{c4} + (\ell_{27}K_{10})w_{c5} + (-\ell_{22}K_{10})w_{c6}$$

$$+ (2\ell_{23}K_9)w_{d4} +(-2\ell_{26}K_9)w_{d5} + (-2\ell_{24}K_9)w_{d6}$$

$$+ [\ell_2\ell_1K_1 + 2\ell_{33}\ell_7K_4 + 2\ell_{33}\ell_7K_7 - 2\ell_4(\ell_3+\ell_4)K_3 + 2\ell_5(\ell_3-\ell_5)K_6]\beta_a$$

$$+ [\ell_2^2K_1 + 2\ell_{33}^2K_4 + 2\ell_{33}^2K_7 + 2\ell_4^2K_4 + 2\ell_5^2K_6 + 2(\ell_{23}^2+\ell_{26}^2+\ell_{24}^2)K_9 +$$

$$(\ell_{25}^2+\ell_{27}^2+\ell_{22}^2)K_{10}]\beta_{bf}$$

$$+ [-\ell_{25}\ell_{35}K_{10}]\beta_{c4} + [-\ell_{27}\ell_{35}K_{10}]\beta_{c5} + [\ell_{22}\ell_{35}K_{10}]\beta_{c6}$$

$$+ [\ell_2C_1 + 2\ell_{33}C_4 + 2\ell_{33}C_7 + 2\ell_{32}C_{14}\cos a_1 + 2\ell_{32}C_{16}\cos a_2]\dot{u}_a$$

$$+ (-\ell_2 C_1 - 2\ell_{33} C_4 - 2\ell_{33} C_7 - 2\ell_{32} C_{14} \cos a_1 - 2\ell_{32} C_{16} \cos a_2)\dot{u}_{bf}$$

$$+ (-2\ell_4 C_3 + 2\ell_5 C_6)\dot{w}_a + [2\ell_4 C_3 - 2\ell_5 C_6 - \ell_{23}(C_{17}+C_{18}) +$$

$$2\ell_{26} C_{18} + \ell_{24}(C_{17}+C_{18}) + (-\ell_{25}-\ell_{27}+\ell_{22})C_{10}]\dot{w}_{bf}$$

$$+ [\ell_{23}(C_{17}+C_{18})]\dot{w}_{d4} + (-2\ell_{26} C_{18})\dot{w}_{d5} + [-\ell_{24}(C_{17}+C_{18})]\dot{w}_{d6}$$

$$+ (\ell_{25} C_{10})\dot{w}_{c4} + (\ell_{27} C_{10})\dot{w}_{c5} + (-\ell_{22} C_{10})\dot{w}_{c6}$$

$$+ (-\ell_{23}\ell_{19} C_{17} + \ell_{23}\ell_{19} C_{18} - \ell_{24}\ell_{19} C_{17} + \ell_{24}\ell_{19} C_{18})\dot{\alpha}_{bf}$$

$$+ (\ell_{23}\ell_{19} C_{17} - \ell_{23}\ell_{19} C_{18})\dot{\alpha}_{d4} + (\ell_{24}\ell_{19} C_{17} - \ell_{24}\ell_{19} C_{18})\dot{\alpha}_{d6}$$

$$+ [\ell_2 \ell_1 C_1 + 2\ell_{33}\ell_7 C_4 + 2\ell_{33}\ell_7 C_7 - 2\ell_4(\ell_3+\ell_4)C_3 + 2\ell_5(\ell_3-\ell_5)C_6 +$$

$$2\ell_{32}\ell_{15} C_{14} \cos a_1 + 2\ell_{32}\ell_{15} C_{16} \cos a_2]\dot{\beta}_a$$

$$+ (-\ell_{35}\ell_{25} C_{10})\dot{\beta}_{c4} + (-\ell_{35}\ell_{27} C_{10})\dot{\beta}_{c5} + (\ell_{35}\ell_{22} C_{10})\dot{\beta}_{c6}$$

$$+ [\ell_2^2 C_1 + 2\ell_{33}^2 C_4 + 2\ell_{33}^2 C_7 + 2\ell_4^2 C_3 + 2\ell_5^2 C_6 + 2\ell_{32}^2 C_{14} \cos a_1 +$$

$$\ell_{25}^2 C_{10} + \ell_{27}^2 C_{10} + \ell_{22}^2 C_{10} + 2\ell_{32}^2 C_{16} \cos a_2 + \ell_{23}^2 C_{17} + \ell_{23}^2 C_{18} +$$

$$2\ell_{26}^2 C_{18} + \ell_{24}^2 C_{17} + \ell_{24}^2 C_{18}]\dot{\beta}_{bf}$$

$$- \ell_{21}(RA_4 + RA_5 + RA_6)$$

$$= 0 \qquad (17)$$

$\Sigma M_\gamma = 0$

$$I_{b\gamma}\ddot{\gamma}_{bf} + \ell_9(-F_{4fr}+F_{4f\ell}) + \ell_{10}(-F_{7fr}+F_{7f\ell}) + \ell_9(-P_{4fr}+P_{4f\ell}) + \ell_{10}(-P_{7fr}+P_{7f\ell}$$

$$+ \ell_{16}(F_{2f}) + \ell_{16}(P_{2f}) + \ell_5(-P_{8fr} - P_{8f\ell})$$

$$+ \ell_4(F_{5fr} + F_{5f\ell}) + \ell_5(-F_{8fr} - F_{8f\ell}) + \ell_4(P_{5fr} + P_{5f\ell})$$

$$+ (-F_{11fr}-F_{11f\ell}) + (-F_{12fr}-F_{12f\ell}) + (-P_{11fr}-P_{11f\ell}) + (-P_{12fr}-P_{12f\ell})$$

$$+ \ell_{31}(P_{14fr} - P_{14f\ell}) + \ell_{30}(P_{16fr} - P_{16f\ell})$$

$$+ \ell_{29}(P_{13fr} - P_{13f\ell}) + \ell_{28}(P_{15fr} - P_{15f\ell})$$

$$- \ell_{23}(RB_4) + \ell_{26}(RB_5) + \ell_{24}(RB_6) - (RM_4 + RM_5 + RM_6)$$

$$= 0$$

$$I_{b\gamma}\ddot{\gamma}_{bf} + (\ell_{16}K_2 + 2\ell_4K_5 - 2\ell_5K_8)v_a + (-\ell_{16}K_2 - 2\ell_4K_5 + 2\ell_5K_8)v_{bf}$$

$$+ (-\ell_{16}\ell_1K_2 - 2\ell_4\ell_7K_5 + 2\ell_5\ell_7K_8)\alpha_a + (-\ell_{16}\ell_2K_2 -$$

$$2\ell_4\ell_{33}K_5 + 2\ell_5\ell_{33}K_8)\alpha_{bf}$$

$$+ [-2\ell_9^2K_4 - 2\ell_{10}^2K_7 - \ell_{16}(\ell_3+\ell_{16})K_2 - 2\ell_4(\ell_3+\ell_4)K_5 +$$

$$2\ell_5(\ell_3-\ell_5)K_8 - 2K_{11} - 2K_{12}]\gamma_a$$

$$+ [2\ell_9^2K_4 + 2\ell_{10}^2K_7 + \ell_{16}^2K_2 + 2\ell_4^2K_5 + 2\ell_5^2K_8 + 2K_{11} + 2K_{12}]\gamma_{bf}$$

$$+ (\ell_{16}C_2 + 2\ell_4C_5 - 2\ell_5C_8 + 2\ell_{31}C_{14} \sin a_1 + 2\ell_{29}C_{13} \sin a_1 +$$

$$2\ell_{30}C_{16} \sin a_2 + 2\ell_{28}C_{15} \sin a_2)\dot{v}_a$$

$$+ (-\ell_{16}C_2 - 2\ell_4C_5 + 2\ell_5C_8 - 2\ell_{31}C_{14} \sin a_1 - 2\ell_{29}C_{13} \sin a_1 -$$

$$2\ell_{30}C_{16} \sin a_2 - 2\ell_{28}C_{15} \sin a_2)\dot{v}_{bf}$$

$$+ (-\ell_{16}\ell_1C_2 - 2\ell_4\ell_7C_5 + 2\ell_5\ell_7C_8 - 2\ell_{31}\ell_{15}C_{14} \sin a_1 -$$

$$2\ell_{29}\ell_{15}C_{13} \sin a_1 - 2\ell_{30}\ell_{15}C_{16} \sin a_2 - 2\ell_{28}\ell_{15}C_{15} \sin a_2)\dot{\alpha}_a$$

$$+ (-\ell_{16}\ell_2C_2 - 2\ell_4\ell_{33}C_5 + 2\ell_5\ell_{33}C_8 - 2\ell_{31}\ell_{32}C_{14} \sin a_1 -$$

$$2\ell_{29}\ell_{32}C_{13} \sin a_1 - 2\ell_{30}\ell_{32}C_{16} \sin a_2 - 2\ell_{28}\ell_{32}C_{15} \sin a_2)\dot{\alpha}_{bf}$$

$$+ [-2\ell_9^2C_4 - 2\ell_{10}^2C_7 - \ell_{16}(\ell_3+\ell_{16})C_2 - 2\ell_4(\ell_3+\ell_4)C_5 +$$

$$2\ell_5(\ell_3-\ell_5)C_8 - 2C_{11} - 2C_{12} + (2\ell_{31}C_{14} + 2\ell_{29}C_{13})$$

$$(-\ell_{13} \cos a_1 - \ell_{12} \sin a_1) + (2\ell_{30}C_{16} + 2\ell_{28}C_{15})$$

$$(-\ell_{11} \cos a_2 - \ell_6 \sin a_2)]\dot{\gamma}_a$$

$$+ [2\ell_9^2C_4 + 2\ell_{10}^2C_7 + \ell_{16}^2C_2 + 2\ell_4^2C_5 + 2\ell_5^2C_8 + 2C_{11} + 2C_{12} +$$

$$(2\ell_{31}C_{14} + 2\ell_{29}C_{13})(\ell_{31} \cos a_1 + \ell_{29} \sin a_1) +$$

$$(2\ell_{30}C_{16} + 2\ell_{28}C_{15})(\ell_{30} \cos a_2 + \ell_{28} \sin a_2)]\dot{\gamma}_{bf}$$

$$- \ell_{23}(RB_4) + \ell_{26}(RB_5) + \ell_{24}(RB_6)$$

$$- (RM_4 + RM_5 + RM_6)$$

$$= 0 \qquad (18)$$

(D) For Motors and Wheelsets (1,2,3) (See Fig. I.4)

For Motor and Wheelset No. 1

$$v_{c1} = v_{d1} - \ell_{34}\, \gamma_{c1}$$

$$w_{c1} = w_{d1} + \ell_{34}\, \beta_{c1}$$

$\Sigma F_y = 0$

$$m_c \ddot{v}_{c1} - (RV_1) = 0 \qquad (19)$$

$\Sigma F_z = 0$

$$m_c \ddot{w}_{c1} + (-K_{10}) w_{br} + (K_{10}) w_{c1} + (\ell_{22} K_{10}) \beta_{br} + (\ell_{35} K_{10}) \beta_{c1}$$
$$+ (-C_{10}) \dot{w}_{br} + (C_{10}) \dot{w}_{c1} + (\ell_{22} C_{10}) \dot{\beta}_{br} + (\ell_{35} C_{10}) \dot{\beta}_{c1}$$
$$+ (RW_1)$$
$$= 0 \qquad (20)$$

$\Sigma M_\beta = 0$

$$I_{c\beta} \ddot{\beta}_{c1} + (-\ell_{35} K_{10}) w_{br} + (\ell_{35} K_{10}) w_{c1} + (\ell_{22} \ell_{35} K_{10}) \beta_{br} + (\ell_{35}^2 K_{10}) \beta_{c1}$$
$$+ (-\ell_{35} C_{10}) \dot{w}_{br} + (\ell_{35} C_{10}) \dot{w}_{c1} + (\ell_{22} \ell_{35} C_{10}) \dot{\beta}_{br} + (\ell_{35}^2 C_{10}) \dot{\beta}_{c1}$$
$$- \ell_{34} (RW_1)$$
$$= 0 \qquad (21)$$

$\Sigma M_\gamma = 0$

$$I_{c\gamma} \ddot{\gamma}_{c1} - \ell_{34} (RV_1) - (R\gamma_1) = 0 \qquad (22)$$

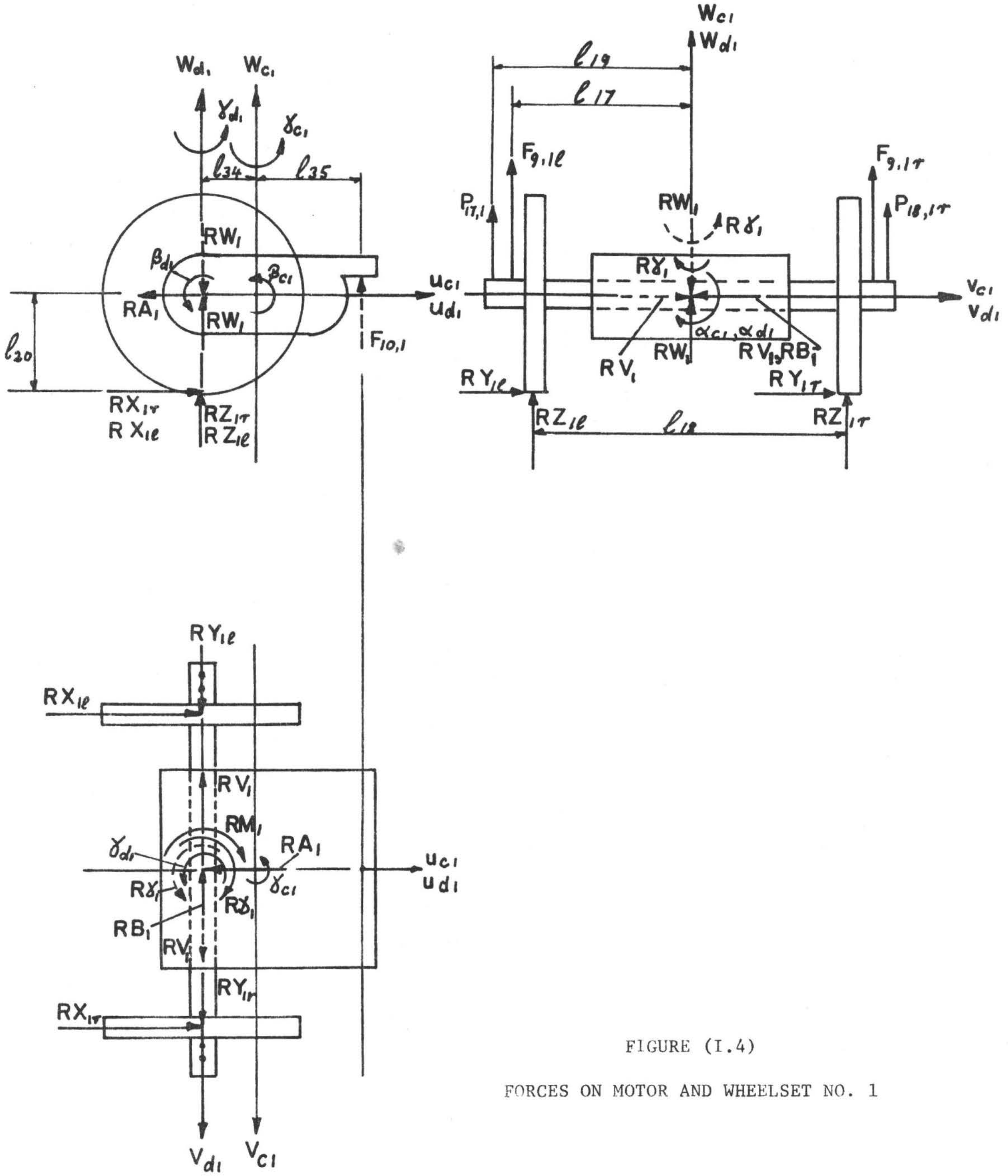


FIGURE (I.4)

FORCES ON MOTOR AND WHEELSET NO. 1

$\underline{\Sigma F_x = 0}$

$$(m_c + m_d)\ddot{u}_{d1} + (RA_1) - (RX_{1r} + RX_{1\ell}) = 0 \quad (23)$$

$\underline{\Sigma F_y = 0}$

$$m_d\ddot{v}_{d1} + (RB_1) + (RV_1) - (RY_{1r} + RY_{1\ell}) = 0 \quad (24)$$

$\underline{\Sigma F_z = 0}$

$$m_d\ddot{w}_{d1} + (-2K_9)w_{br} + (2K_9)w_{d1} + 2(\ell_{24}K_9)\beta_{br}$$

$$+ (-C_{17}-C_{18})\dot{w}_{br} + (C_{17}+C_{18})\dot{w}_{d1} + \ell_{19}(-C_{17}+C_{18})\dot{\alpha}_{br}$$

$$+ \ell_{19}(C_{17}-C_{18})\dot{\alpha}_{d1} + [\ell_{24}(C_{17}+C_{18})]\dot{\beta}_{br}$$

$$- (RW_1) - (RZ_{1r} + (RZ_{1\ell}) = 0 \quad (25)$$

$\underline{\Sigma M_\alpha = 0}$

$$(I_{c\alpha}+I_{d\alpha})\ddot{\alpha}_{d1} + (-2\ell_{17}^2K_9)\alpha_{br} + (2\ell_{17}^2K_9)\alpha_{d1}$$

$$+ [\ell_{19}(C_{18}-C_{17})]\dot{w}_{br} + [\ell_{19}(-C_{18}+C_{17})]\dot{w}_{d1} + [\ell_{19}^2(-C_{18}-C_{17})]\dot{\alpha}_{br}$$

$$+ [\ell_{19}^2(C_{18}+C_{17})]\dot{\alpha}_{d1} + [\ell_{19}\ell_{24}(-C_{18}-C_{17})]\dot{\beta}_{br}$$

$$+ \ell_{20}(RY_{1r} + RY_{1\ell}) + (\ell_{18}/2)\ (RZ_{1r} + RZ_{1\ell})$$

$$= 0 \quad (26)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{d\beta}\ddot{\beta}_{d1} - \ell_{20}(RX_{1r} + RX_{1\ell}) = 0 \quad (27)$$

$\underline{\Sigma M_\gamma = 0}$

$$I_{d\gamma}\ddot{\gamma}_{d1} - (\ell_{18}/2)\ (RX_{1r} - RX_{1\ell}) + (RM_1) + (R\gamma_1) = 0 \quad (28)$$

<u>For Motor and Wheelset No. 2</u>

$\underline{\Sigma F_y = 0}$

$$m_c\ddot{v}_{c2} - (RV_2) = 0 \quad (29)$$

$\underline{\Sigma F_z = 0}$

$$m_c\ddot{w}_{c2} + (-K_{10})w_{br} + (K_{10})w_{c2} + (-\ell_{27}K_{10})\beta_{br} + (\ell_{35}K_{10})\beta_{c2}$$

$$+ (-C_{10})\dot{w}_{br} + (C_{10})\dot{w}_{c2} + (-\ell_{27}C_{10})\dot{\beta}_{br} + (\ell_{35}C_{10})\dot{\beta}_{c2}$$

$$+ (RW_2) = 0 \quad (30)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{c\beta}\ddot{\beta}_{c2} + (-\ell_{35}K_{10})w_{br} + (\ell_{35}K_{10})w_{c2} + (-\ell_{35}\ell_{27}K_{10})\beta_{br} + (\ell_{35}^2K_{10})\beta_{c2}$$

$$+ (-\ell_{35}C_{10})\dot{w}_{br} + (\ell_{35}C_{10})\dot{w}_{c2} + (-\ell_{35}\ell_{27}C_{10})\dot{\beta}_{br} + (\ell_{35}^2C_{10})\dot{\beta}_{c2}$$

$$- \ell_{34}(RW_2) = 0 \quad (31)$$

$\underline{\Sigma M_{\gamma} = 0}$

$$I_{c\gamma}\ddot{\gamma}_{c2} - \ell_{34}(RV_2) - (R\gamma_2) = 0 \qquad (32)$$

$\underline{\Sigma F_x = 0}$

$$(m_c + m_d)\ddot{u}_{d2} + (RA_2) - (RX_{2r} + RX_{2\ell}) = 0 \qquad (33)$$

$\underline{\Sigma F_y = 0}$

$$m_d\ddot{v}_{d2} + (RB_2) + (RV_2) - (RY_{2r} + RY_{2\ell}) = 0 \qquad (34)$$

$\underline{\Sigma F_z = 0}$

$$m_d\ddot{w}_{d2} + (-2K_9)w_{br} + (2K_9)w_{d2} + (2\ell_{26}K_9)\beta_{br}$$

$$+ (-2C_{18})\dot{w}_{br} + (2C_{18})\dot{w}_{d2} + (2\ell_{26}C_{18})\dot{\beta}_{br}$$

$$- (RW_2) - (RZ_{2r} + RZ_{2\ell}) = 0 \qquad (35)$$

$\underline{\Sigma M_{\alpha} = 0}$

$$(I_{c\alpha} + I_{d\alpha})\ddot{\alpha}_{d2} + (-2\ell_{17}^2K_9)\alpha_{br} + (2\ell_{17}^2K_9)\alpha_{d2}$$

$$+ (-2\ell_{19}^2C_{18})\dot{\alpha}_{br} + (2\ell_{19}^2C_{18})\dot{\alpha}_{d2}$$

$$+ \ell_{20}(RY_{2r} + RY_{2\ell}) + (\ell_{18}/2)(RZ_{2r} - RZ_{2\ell})$$

$$= 0 \qquad (36)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{d\beta}\ddot{\beta}_{d2} - \ell_{20}(RX_{2r} + RX_{2\ell}) = 0 \quad (37)$$

$\underline{\Sigma M_\gamma = 0}$

$$I_{d\gamma}\ddot{\gamma}_{d2} - (\ell_{18}/2)(RX_{2r} - RX_{2\ell}) + (RM_2) + (R\gamma_2) = 0 \quad (38)$$

For Motor and Wheelset No. 3

$\underline{\Sigma F_x = 0}$

$$m_c\ddot{v}_{c3} - (RV_3) = 0 \quad (39)$$

$\underline{\Sigma F_z = 0}$

$$m_c\ddot{w}_{c3} + (-K_{10})w_{br} + (K_{10})w_{c3} + (-\ell_{25}K_{10})\beta_{br} + (\ell_{35}K_{10})\beta_{c3}$$

$$+ (-C_{10})\dot{w}_{br} + (C_{10})\dot{w}_{c3} + (-\ell_{25}C_{10})\dot{\beta}_{br} + (\ell_{35}C_{10})\dot{\beta}_{c3}$$

$$+ (RW_3) = 0 \quad (40)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{c\beta}\ddot{\beta}_{c3} + (-\ell_{35}K_{10})w_{br} + (\ell_{35}K_{10})w_{c3} + (-\ell_{35}\ell_{25}K_{10})\beta_{br} + (\ell_{35}^2K_{10})\beta_{c3}$$

$$+ (-\ell_{35}C_{10})\dot{w}_{br} + (\ell_{35}C_{10})\dot{w}_{c3} + (-\ell_{35}\ell_{25}C_{10})\dot{\beta}_{br} + (\ell_{35}^2C_{10})\dot{\beta}_{c3}$$

$$- \ell_{34}(RW_3) = 0 \quad (41)$$

$\underline{\Sigma M_{\gamma} = 0}$

$$I_{c\gamma}\ddot{\gamma}_{c3} - \ell_{34}(RV_3) - (R\gamma_3) = 0 \qquad (42)$$

$\underline{\Sigma F_{x} = 0}$

$$(m_c+m_d)\ddot{u}_{d3} + (RA_3) - (RX_{3r} + RX_{3\ell}) = 0 \qquad (43)$$

$\underline{\Sigma F_{y} = 0}$

$$m_d\ddot{v}_{d3} + (RB_3) + (RV_3) -(RY_{3r} + RY_{3\ell}) = 0 \qquad (44)$$

$\underline{\Sigma F_{z} = 0}$

$$m_d\ddot{w}_{d3} + (-2K_9)w_{br} + (2K_9)w_{d3} + (-2\ell_{23}K_9)\beta_{br}$$
$$+ (-C_{17}-C_{18})\dot{w}_{br} + (C_{17}+C_{18})\dot{w}_{d3} + \ell_{19}(C_{17}-C_{18})\dot{\alpha}_{br} + \ell_{19}(-C_{17}+C_{18})\dot{\alpha}_{d3}$$
$$+ [-\ell_{23}(C_{17}+C_{18})]\dot{\beta}_{br} - (RW_3) - (RZ_{3r} + RZ_{3\ell})$$
$$= 0 \qquad (45)$$

$\underline{\Sigma M_{\alpha} = 0}$

$$(I_{c\alpha}+I_{d\alpha})\ddot{\alpha}_{d3} + (-2\ell_{17}^2K_9)\alpha_{br} + (2\ell_{17}^2K_9)\alpha_{d3}$$
$$+ [\ell_{19}(C_{17}-C_{18})]\dot{w}_{br} + [\ell_{19}(-C_{17}+C_{18})]\dot{w}_{d3} + [\ell_{19}^2(-C_{17}-C_{18})]\dot{\alpha}_{br}$$
$$+ [\ell_{19}^2(C_{17}+C_{18})]\dot{\alpha}_{d3} + [\ell_{19}\ell_{23}(C_{17}-C_{18})]\dot{\beta}_{br}$$
$$+ \ell_{20}(RY_{3r} + RY_{3\ell}) + (\ell_{18}/2)(RZ_{3r} - RZ_{3\ell})$$
$$= 0 \qquad (46)$$

$\Sigma M_\beta = 0$

$$I_{d\beta}\ddot{\beta}_{d3} - \ell_{20}(RX_{3r} + RX_{3\ell}) = 0 \qquad (47)$$

$\Sigma M_\gamma = 0$

$$I_{d\gamma}\ddot{\gamma}_{d3} - (\ell_{18}/2)(RX_{3r} - RX_{3\ell}) + (RM_3) + (R\gamma_3) = 0 \qquad (48)$$

(E) For Motors and Wheelsets (4,5,6) (See Fig. I.5)

For Motor and Wheelset No. 4

$$v_{c4} = v_{d4} + \ell_{34}\gamma_{c4}$$

$$w_{c4} = w_{d4} - \ell_{34}\beta_{c4}$$

$\Sigma F_y = 0$

$$m_c\ddot{v}_{c4} - (RV_4) = 0 \qquad (49)$$

$\Sigma F_z = 0$

$$m_c\ddot{w}_{c4} + (-K_{10})w_{bf} + (K_{10})w_{c4} + (\ell_{25}K_{10})\beta_{bf} + (-\ell_{35}K_{10})\beta_{c4}$$
$$+ (-C_{10})\dot{w}_{bf} + (C_{10})\dot{w}_{c4} + (\ell_{25}C_{10})\dot{\beta}_{bf} + (-\ell_{35}C_{10})\dot{\beta}_{c4}$$
$$+ (RW_4) = 0 \qquad (50)$$

$\Sigma M_\beta = 0$

$$I_{c\beta}\ddot{\beta}_{c4} + (\ell_{35}K_{10})w_{bf} + (-\ell_{35}K_{10})w_{c4} + (-\ell_{35}\ell_{25}K_{10})\beta_{bf} + (\ell_{35}^2K_{10})\beta_{c4}$$
$$+ (\ell_{35}C_{10})\dot{w}_{bf} + (-\ell_{35}C_{10})\dot{w}_{c4} + (-\ell_{35}\ell_{25}C_{10})\dot{\beta}_{bf} + (\ell_{35}^2C_{10})\dot{\beta}_{c4}$$
$$+ \ell_{34}(RW_4) = 0 \qquad (51)$$

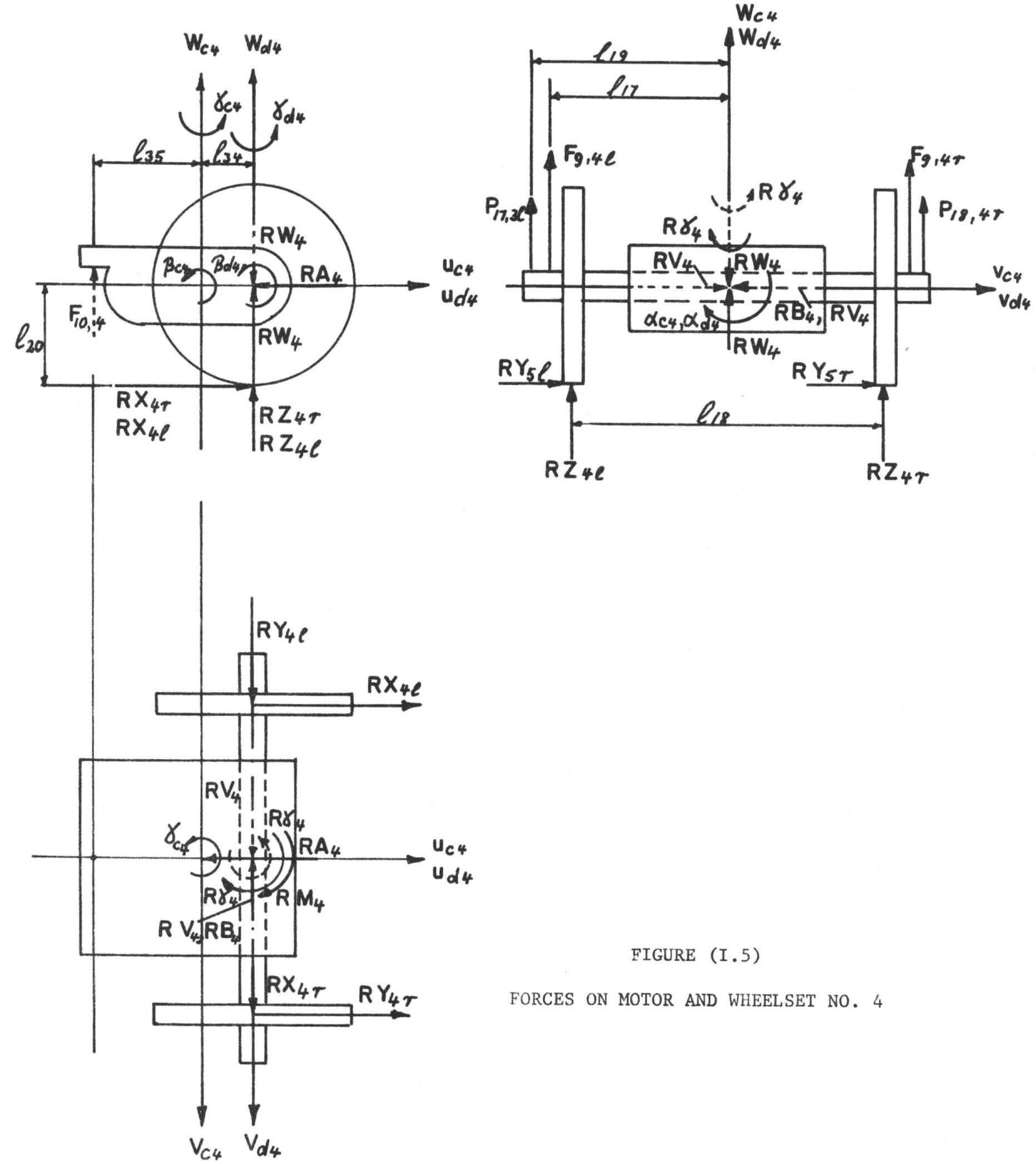


FIGURE (I.5)

FORCES ON MOTOR AND WHEELSET NO. 4

$\underline{\Sigma M_{\gamma} = 0}$

$$I_{c\gamma}\ddot{\gamma}_{c4} + \ell_{34}(RV_4) - (R\gamma_4) = 0 \qquad (52)$$

$\underline{\Sigma F_x = 0}$

$$(m_c+m_d)\ddot{u}_{d4} + (RA_4) - (RX_{4r} + RX_{4\ell}) = 0 \qquad (53)$$

$\underline{\Sigma F_y = 0}$

$$m_d\ddot{v}_{d4} + (RB_4) + (RV_4) - (RY_{4r} + RY_{4\ell}) = 0 \qquad (54)$$

$\underline{\Sigma F_z = 0}$

$$m_d\ddot{w}_{d4} + (-2K_9)w_{bf} + (2K_9)w_{d4} + (2\ell_{23}K_9)\beta_{bf}$$

$$+ (-C_{18}-C_{17})\dot{w}_{bf} + (C_{18}+C_{17})\dot{w}_{d4} + [\ell_{19}(C_{18}-C_{17})]\dot{\alpha}_{bf}$$

$$+ [\ell_{19}(-C_{18}+C_{17})]\dot{\alpha}_{d4} + [\ell_{23}(C_{18}+C_{17})]\dot{\beta}_{bf}$$

$$- (RW_4) - (RZ_{4r} + RZ_{4\ell}) = 0 \qquad (55)$$

$\underline{\Sigma M_{\alpha} = 0}$

$$(I_{c\alpha}+I_{d\alpha})\ddot{\alpha}_{d4} + (-2\ell_{17}^2K_9)\alpha_{bf} + (2\ell_{17}^2K_9)\alpha_{d4}$$

$$+ [\ell_{19}(C_{18}-C_{17})]\dot{w}_{bf} + [\ell_{19}(-C_{18}+C_{17})]\dot{w}_{d4} + [\ell_{19}^2(-C_{18}-C_{17})]\dot{\alpha}_{bf}$$

$$+ [\ell_{19}^2(C_{18}+C_{17})]\dot{\alpha}_{d4} + [\ell_{19}\ell_{23}(-C_{18}+C_{17})]\dot{\beta}_{bf}$$

$$+ \ell_{20}(RY_{4r} + RY_{4\ell}) + (\ell_{18}/2)(RZ_{4r} - RZ_{4\ell})$$

$$= 0 \qquad (56)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{d\beta}\ddot{\beta}_{d4} - \ell_{20}(RX_{4r} + RX_{4\ell}) = 0 \qquad (57)$$

$\underline{\Sigma M_\gamma = 0}$

$$I_{d\alpha}\ddot{\gamma}_{d4} + (\ell_{18}/2)(-RX_{4r} + RX_{4\ell}) + (RM_4) + (R\gamma_4) = 0 \qquad (58)$$

For Motor and Wheelset No. 5

$\underline{\Sigma F_y = 0}$

$$m_c\ddot{v}_{c5} - (RV_5) = 0 \qquad (59)$$

$\underline{\Sigma F_z = 0}$

$$m_c\ddot{w}_{c5} + (-K_{10})w_{bf} + (K_{10})w_{c5} + (\ell_{27}K_{10})\beta_{bf} + (-\ell_{35}K_{10})\beta_{c5}$$
$$+ (-C_{10})\dot{w}_{bf} + (C_{10})\dot{w}_{c5} + (\ell_{27}C_{10})\dot{\beta}_{bf} + (-\ell_{35}C_{10})\dot{\beta}_{c5}$$
$$+ (RW_5) = 0 \qquad (60)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{c\beta}\ddot{\beta}_{c5} + (\ell_{35}K_{10})w_{bf} + (-\ell_{35}K_{10})w_{c5} + (-\ell_{35}\ell_{27}K_{10})\beta_{bf} + (\ell_{35}^2K_{10})\beta_{c5}$$
$$+ (\ell_{35}C_{10})\dot{w}_{bf} + (-\ell_{35}C_{10})\dot{w}_{c5} + (-\ell_{35}\ell_{27}C_{10})\dot{\beta}_{bf} + (\ell_{35}^2C_{10})\dot{\beta}_{c5}$$
$$+ \ell_{34}(RW_5) = 0 \qquad (61)$$

$\underline{\Sigma M_\gamma = 0}$

$$I_{c\gamma}\ddot{\gamma}_{c5} + \ell_{34}(RV_5) - (R\gamma_5) = 0 \qquad (62)$$

$\Sigma F_x = 0$

$$(m_c + m_d)\ddot{u}_{d5} + (RA_5) - (RX_{5r} + RX_{5\ell}) = 0 \qquad (63)$$

$\Sigma F_y = 0$

$$m_d\ddot{v}_{d5} + (RB_5) + (RV_5) - (RY_{5r} + RY_{5\ell}) = 0 \qquad (64)$$

$\Sigma F_z = 0$

$$\begin{aligned} m_d\ddot{w}_{d5} &+ (-2K_9)w_{bf} + (2K_9)w_{d5} + (-2\ell_{26}K_9)\beta_{bf} \\ &+ (-2C_{18})\dot{w}_{bf} + (2C_{18})\dot{w}_{d5} + (-2\ell_{26}C_{18})\dot{\beta}_{bf} \\ &- (RW_5) - (RZ_{5r} + RZ_{5\ell}) \\ &= 0 \end{aligned} \qquad (65)$$

$\Sigma M_\alpha = 0$

$$\begin{aligned} (I_{c\alpha} + I_{d\alpha})\ddot{\alpha}_{d5} &+ (-2\ell_{17}^2K_9)\alpha_{bf} + (2\ell_{17}^2K_9)\alpha_{d5} \\ &+ (-2\ell_{19}^2C_{18})\dot{\alpha}_{bf} + (2\ell_{19}^2C_{18})\dot{\alpha}_{d5} \\ &+ \ell_{20}(RY_{5r} + RY_{5\ell}) + (\ell_{18}/2)(RZ_{5r} - RZ_{5\ell}) \\ &= 0 \end{aligned} \qquad (66)$$

$\Sigma M_\beta = 0$

$$I_{d\beta}\ddot{\beta}_{d5} - \ell_{20}(RX_{5r} + RX_{5\ell}) = 0 \qquad (67)$$

$\underline{\Sigma M_{\gamma} = 0}$

$$I_{d\gamma}\ddot{\gamma}_{d5} + (\ell_{18}/2)(-RX_{5r} + RX_{5\ell}) + (RM_5) + (R\gamma_5) = 0 \quad (68)$$

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$\underline{\Sigma F_{y} = 0}$

$$m_c\ddot{v}_{c6} - (RV_6) = 0 \quad (69)$$

$\underline{\Sigma F_{z} = 0}$

$$\begin{aligned} m_c\ddot{w}_{c6} &+ (-K_{10})w_{bf} + (K_{10})w_{c6} + (-\ell_{22}K_{10})\beta_{bf} + (-\ell_{35}K_{10})\beta_{c6} \\ &+ (-C_{10})\dot{w}_{bf} + (C_{10})\dot{w}_{c6} + (-\ell_{22}C_{10})\dot{\beta}_{bf} + (-\ell_{35}C_{10})\dot{\beta}_{c6} \\ &+ (RW_6) = 0 \end{aligned} \quad (70)$$

$\underline{\Sigma M_{\beta} = 0}$

$$\begin{aligned} I_{c\beta}\ddot{\beta}_{c6} &+ (\ell_{35}K_{10})w_{bf} + (-\ell_{35}K_{10})w_{c6} + (\ell_{35}\ell_{22}K_{10})\beta_{bf} + (\ell^2_{35}K_{10})\beta_{c6} \\ &+ (\ell_{35}C_{10})\dot{w}_{bf} + (-\ell_{35}C_{10})\dot{w}_{c6} + (\ell_{35}\ell_{22}C_{10})\dot{\beta}_{bf} + (\ell^2_{35}C_{10})\dot{\beta}_{c6} \\ &+ \ell_{34}(RW_6) = 0 \end{aligned} \quad (71)$$

$\underline{\Sigma M_{\gamma} = 0}$

$$I_{c\gamma}\ddot{\gamma}_{c6} + \ell_{34}(RV_6) - (R\gamma_6) = 0 \quad (72)$$

$\Sigma F_x = 0$

$$(m_c+m_d)\ddot{u}_{d6} + (RA_6) - (RX_{6r} + RX_{6\ell}) = 0 \qquad (73)$$

$\Sigma F_y = 0$

$$m_d\ddot{v}_{d6} + (RB_6) + (RV_6) - (RY_{6r} + RY_{6\ell}) = 0 \qquad (74)$$

$\Sigma F_z = 0$

$$\begin{aligned} m_d\ddot{w}_{d6} &+ (-2K_9)w_{bf} + (2K_9)w_{d6} + (-2\ell_{24}K_9)\beta_{bf} \\ &+ (-C_{17}-C_{18})\dot{w}_{bf} + (C_{17}+C_{18})\dot{w}_{d6} + [\ell_{19}(C_{17}-C_{18})]\dot{\alpha}_{bf} \\ &+ [\ell_{19}(-C_{17}+C_{18})]\dot{\alpha}_{d6} + [\ell_{24}(-C_{17}-C_{18})]\dot{\beta}_{bf} \\ &- (RW_6) - (RZ_{6r} + RZ_{6\ell}) = 0 \end{aligned} \qquad (75)$$

$\Sigma M_\alpha = 0$

$$\begin{aligned} (I_{c\alpha}+I_{d\alpha})\ddot{\alpha}_{d6} &+ (-2\ell_{17}^2K_9)\alpha_{bf} + (2\ell_{17}^2K_9)\alpha_{d6} \\ &+ [\ell_{19}(C_{17}-C_{18})]\dot{w}_{bf} + [\ell_{19}(-C_{17}+C_{18})]\dot{w}_{d6} + [\ell_{19}^2(-C_{17}-C_{18})]\dot{\alpha}_{bf} \\ &+ [\ell_{19}^2(C_{17}+C_{18})]\dot{\alpha}_{d6} + [\ell_{19}\ell_{24}(C_{17}-C_{18})]\dot{\beta}_{bf} \\ &+ \ell_{20}(RY_{6r} + RY_{6\ell}) + (\ell_{18}/2)(RZ_{6r} - RZ_{6\ell}) = 0 \end{aligned} \qquad (76)$$

$\Sigma M_\beta = 0$

$$I_{d\beta}\ddot{\beta}_{d6} - \ell_{20}(RX_{6r} + RX_{6\ell}) = 0 \qquad (77)$$

$\Sigma M_\gamma = 0$

$$I_{d\gamma}\ddot{\gamma}_{d6} + (\ell_{18}/2)(-RX_{6r} + RX_{6\ell}) + (RM_6) + (R\gamma_6) = 0 \qquad (78)$$

I.5 DEFINITION OF THE ELEMENTS OF THE MATRICES

All elements which are not defined below are zeros. All matrices are symmetric [i.e. A(J,I) = A(I,J), B(J,I) = B(I,J) and C(J,I) = C(I,J)].

(A) Elements of the Inertia Matrix [A]

$A(1,1) = m_a$ $\qquad$ $A(4,4) = I_{a\alpha}$

$A(2,2) = m_a$ $\qquad$ $A(5,5) = I_{a\beta}$

$A(3,3) = m_a$ $\qquad$ $A(6,6) = I_{a\gamma}$

$A(7,7) = A(13,13) = m_b$

$A(8,8) = A(14,14) = m_b$

$A(9,9) = A(15,15) = m_b$

$A(10,10) = A(16,16) = I_{b\alpha}$

$A(11,11) = A(17,17) = I_{b\beta}$

$A(12,12) = A(18,18) = I_{b\gamma}$

$A(19,19) = A(29,29) = A(39,39) = A(49,49) = A(59,59) = A(69,69) = m_c$

$A(20,20) = A(30,30) = A(40,40) = A(50,50) = A(60,60) = A(70,70) = m_c$

$A(21,21) = A(31,31) = A(41,41) = A(51,51) = A(61,61) = A(71,71) = I_{c\beta}$

$A(22,22) = A(32,32) = A(42,42) = A(52,52) = A(62,62) = A(72,72) = I_{c\gamma}$

$A(23,23) = A(33,33) = A(43,43) = A(53,53) = A(63,63) = A(73,73) = m_c+m_d$

$A(24,24) = A(34,34) = A(44,44) = A(54,54) = A(64,64) = A(74,74) = m_d$

$A(25,25) = A(35,35) = A(45,45) = A(55,55) = A(65,65) = A(75,75) = m_d$

$A(26,26) = A(36,36) = A(46,46) = A(56,56) = A(66,66) = A(76,76) = I_{c\alpha}+I_{d\alpha}$

$A(27,27) = A(37,37) = A(47,47) = A(57,57) = A(67,67) = A(77,77) = I_{d\beta}$

$A(28,28) = A(38,38) = A(48,48) = A(58,58) = A(68,68) = A(78,78) = I_{d\gamma}$

(B) <u>Elements of the Stiffness Matrix [B]</u>

$B(1,1) = 2K_1 + 4(K_4 + K_7)$

$B(1,5) = 2\ell_1K_1 + 4\ell_7(K_4 + K_7)$

$B(1,7) = -K_1 - 2(K_4 + K_7) = -B(1,1)/2.0$

$B(1,11) = \ell_2K_1 + 2\ell_{33}(K_4 + K_7)$

$B(1,13) = B(1,7)$

$B(1,17) = B(1,11)$

$B(2,2) = 2K_2 + 4(K_5 + K_8)$

$B(2,4) = -2\ell_1K_2 - 4\ell_7(K_5 + K_8)$

$B(2,8) = B(2,2)/2.0$

$B(2,10) = -\ell_2K_2 - 2\ell_{33}(K_5 + K_8)$

$B(2,12) = -\ell_{16}K_2 - 2\ell_4K_5 + 2\ell_5K_8$

$B(2,14) = B(2,8)$

$B(2,16) = B(2,10)$

$B(2,18) = -B(2,12)$

$B(3,3) = 4(K_3 + K_6)$

$B(3,9) = -B(3,3)/2.0$

$B(3,11) = 2\ell_4K_3 - 2\ell_5K_6$

$B(3,15) = B(3,9)$

$B(3,17) = -B(3,11)$

$B(4,4) = 2\ell_1^2K_2 + 4\ell_7^2(K_5 + K_8) + 4\ell_9^2K_3 + 4\ell_{10}^2K_6$

$B(4,8) = B(2,4)/2.0$

$B(4,10) = \ell_1\ell_2K_2 + 2\ell_7\ell_{33}(K_5 + K_8) - 2\ell_9^2K_3 - 2\ell_{10}^2K_6$

$B(4,12) = \ell_1\ell_{16}K_2 + 2\ell_7(\ell_4K_5 - \ell_5K_8)$

$B(4,14) = B(4,8)$

$B(4,16) = B(4,10)$

$B(4,18) = -B(4,12)$

$B(5,5) = 2\ell_1^2 K_1 + 4(\ell_3+\ell_4)^2 K_3 + 4(\ell_3-\ell_5)^2 K_6 + 4\ell_7^2(K_4 + K_7)$

$B(5,7) = -B(1,5)/2.0$

$B(5,9) = 2(\ell_3+\ell_4)K_3 + 2(\ell_3 -\ell_5)K_6$

$B(5,11) = \ell_1\ell_2K_1 - 2\ell_4(\ell_3+\ell_4)K_3 + 2\ell_5(\ell_3-\ell_5)K_6 + 2\ell_7\ell_{33}(K_4 + K_7)$

$B(5,13) = B(5,7)$

$B(5,15) = -B(5,9)$

$B(5,17) = B(5,11)$

$B(6,6) = 2(\ell_3+\ell_{16})^2K_2 + 4\ell_9^2K_4 + 4\ell_{10}^2K_7 + 4(\ell_3+\ell_4)^2K_5 + 4(\ell_3-\ell_5)^2K_8$

$+ 4(K_{11} + K_{12})$

$B(6,8) = -(\ell_3+\ell_{16})K_2 - 2(\ell_3+\ell_4)K_5 - 2(\ell_3-\ell_5)K_8$

$B(6,10) = -(\ell_3+\ell_{16})\ell_2K_2 - 2(\ell_3+\ell_4)\ell_{33}K_5 - 2(\ell_3-\ell_5)\ell_{33}K_8$

$B(6,12) = -\ell_{16}(\ell_3+\ell_{16})K_2 - 2\ell_9^2K_4 - 2\ell_{10}^2K_7 - 2(\ell_3+\ell_4)\ell_4K_5 +$

$2(\ell_3-\ell_5)\ell_5K_8 - 2(K_{11} + K_{12})$

$B(6,14) = -B(6,8)$

$B(6,16) = -B(6,10)$

$B(6,18) = B(6,12)$

$B(7,7) = K_1 + 2(K_4 + K_7)$

$B(7,11) = -\ell_2K_1 - 2\ell_{33}(K_4 + K_7)$

$B(8,8) = K_2 + 2(K_5 + K_8)$

$B(8,10) = \ell_2 K_2 + 2\ell_{33}(K_5 + K_8)$

$B(8,12) = \ell_{16}K_2 + 2\ell_4 K_5 - 2\ell_5 K_8$

$B(9,9) = 2(K_3 + K_6) + 6K_9 + 3K_{10}$

$B(9,11) = -2\ell_4 K_3 + 2\ell_5 K_6 - 2(-\ell_{23}+\ell_{26}+\ell_{24})K_9 - (-\ell_{25}-\ell_{27}+\ell_{22})K_{10}$

$B(9,20) = -K_{10}$

$B(9,21) = -\ell_{35}K_{10}$

$B(9,25) = -2K_9$

$B(9,30) = -K_{10}$

$B(9,31) = -\ell_{35}K_{10}$

$B(9,35) = -2K_9$

$B(9,40) = -K_{10}$

$B(9,41) = -\ell_{35}K_{10}$

$B(9,45) = -2K_9$

$B(10,10) = \ell_2^2 K_2 + 2\ell_{33}^2(K_5 + K_8) + 2\ell_9^2 K_3 + 2\ell_{10}^2 K_6 + 6\ell_{17}^2 K_9$

$B(10,12) = \ell_2 \ell_{16} K_2 + 2\ell_{33}(\ell_4 K_5 - \ell_5 K_8)$

$B(10,26) = -2\ell_{17}K_9$

$B(10,36) = B(10,26)$

$B(10,46) = B(10,26)$

$$B(11,11) = \ell_2^2K_1 + 2\ell_{33}^2(K_4 + K_7) + 2\ell_4^2K_3 + 2\ell_5^2K_6 + 2(\ell_{24}^2+\ell_{26}^2+\ell_{23}^2)K_9$$

$$+ (\ell_{22}^2+\ell_{27}^2+\ell_{25}^2)K_{10}$$

$$B(11,20) = \ell_{22}K_{10}$$

$$B(11,21) = \ell_{22}\ell_{35}K_{10}$$

$$B(11,25) = 2\ell_{24}K_9$$

$$B(11,30) = -\ell_{27}K_{10}$$

$$B(11,31) = -\ell_{27}\ell_{35}K_{10}$$

$$B(11,35) = 2\ell_{26}K_9$$

$$B(11,40) = -\ell_{25}K_{10}$$

$$B(11,41) = -\ell_{25}\ell_{35}K_{10}$$

$$B(11,45) = -2\ell_{23}K_9$$

$$B(12,12) = 2\ell_9^2K_4 + 2\ell_{10}^2K_7 + \ell_{16}^2K_2 + 2\ell_4^2K_5 + 2\ell_5^2K_8 + 2(K_{11} + K_{12})$$

$$B(13,13) = K_1 + 2(K_4 + K_7)$$

$$B(13,17) = -\ell_2K_1 - 2\ell_{33}(K_4 + K_7)$$

$$B(14,14) = K_2 + 2(K_5 + K_8)$$

$$B(14,16) = \ell_2K_2 + 2\ell_{33}(K_5 + K_8)$$

$$B(14,18) = -\ell_{16}K_2 - 2\ell_4K_5 + 2\ell_5K_8$$

$$B(15,15) = 2(K_3 + K_6) + 6K_9 + 2K_{10}$$

$$B(15,17) = 2\ell_4K_3 - 2\ell_5K_6 + 2(-\ell_{23}+\ell_{26}+\ell_{24})K_9 + (-\ell_{25}-\ell_{27}+\ell_{22})K_{10}$$

$B(15,50) = -K_{10}$

$B(15,51) = \ell_{35}K_{10}$

$B(15,55) = -2K_9$

$B(15,60) = -K_{10}$

$B(15,61) = \ell_{35}K_{10}$

$B(15,65) = -2K_9$

$B(15,70) = -K_{10}$

$B(15,71) = \ell_{35}K_{10}$

$B(15,75) = -2K_9$

$B(16,16) = \ell_2^2K_2 + 2\ell_{33}^2(K_5 + K_8) + 2\ell_9^2K_3 + 2\ell_{10}^2K_6 + 6\ell_{17}^2K_9$

$B(16,18) = -\ell_2\ell_{16}K_2 - 2\ell_{33}\ell_4K_5 + 2\ell_{33}\ell_5K_8$

$B(16,56) = -2\ell_{17}^2K_9$

$B(16,66) = -2\ell_{17}^2K_9$

$B(16,76) = -2\ell_{17}^2K_9$

$B(17,17) = B(11,11)$

$B(17,50) = \ell_{25}K_{10}$

$B(17,51) = -\ell_{25}\ell_{35}K_{10}$

$B(17,55) = 2\ell_{23}K_9$

$B(17,60) = \ell_{27}K_{10}$

$B(17,61) = -\ell_{27}\ell_{35}K_{10}$

$B(17,65) = -2\ell_{26}K_9$

$B(17,70) = -\ell_{22}K_{10}$

$B(17,71) = \ell_{22}\ell_{35}K_{10}$

$B(17,75) = -2\ell_{24}K_9$

$B(18,18) = B(12,12)$

$B(20,20) = K_{10}$

$B(20,21) = \ell_{35}K_{10}$

$B(21,21) = \ell_{35}^2K_{10}$

$B(25,25) = 2K_9$

$B(26,26) = 2\ell_{17}^2K_9$

$B(30,30) = K_{10}$

$B(30,31) = \ell_{35}K_{10}$

$B(31,31) = \ell_{35}^2K_{10}$

$B(35,35) = 2K_9$

$B(36,36) = 2\ell_{17}^2K_9$

$B(40,40) = K_{10}$

$B(40,41) = \ell_{35}K_{10}$

$B(41,41) = \ell_{35}^2K_{10}$

$B(45,45) = 2K_9$

$B(46,46) = 2\ell_{17}^2K_9$

$B(50,50) = K_{10}$

$B(50,51) = -\ell_{35}K_{10}$

$B(51,51) = \ell_{35}^2K_{10}$

$B(55,55) = 2K_9$

$B(56,56) = 2\ell_{17}^2K_9$

$B(60,60) = K_{10}$

$B(60,61) = -\ell_{35}K_{10}$

$B(61,61) = \ell_{35}^2K_{10}$

$B(65,65) = 2K_9$

$B(66,66) = 2\ell_{17}^2K_9$

$B(70,70) = K_{10}$

$B(70,71) = -\ell_{35}K_{10}$

$B(71,71) = \ell_{35}^2K_{10}$

$B(75,75) = 2K_9$

$B(76,76) = 2\ell_{17}^2K_9$

(C) Elements of the Damping Matrix [C]

The elements of the damping matrix will consist of two parts:

i - Elements due to damping in rubber springs: These are obtained from the elements of [B] by replacing the stiffnesses K's by their corresponding terms C's.

ii - Additional damping terms: These are damping due to shock absorbers and are given below.

$C(1,1) = 4(C_{14} \cos a_1 + C_{16} \cos a_2)$

$C(1,5) = 4\ell_{15}(C_{14} \cos a_1 + C_{16} \cos a_2)$

$C(1,7) = -2(C_{14} \cos a_1 + C_{16} \cos a_2)$

$C(1,11) = 2\ell_{32}(C_{14} \cos a_1 + C_{16} \cos a_2)$

$C(1,13) = C(1,7)$

$C(1,17) = C(1,11)$

$C(2,2) = 4(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(2,4) = -4\ell_{15}(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(2,8) = -2(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(2,10) = -2\ell_{32}(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(2,12) = -2C_{13}(\ell_{31} \cos a_1 + \ell_{29} \sin a_1) - 2C_{15}(\ell_{30} \cos a_2 + \ell_{28} \sin a_2)$

$C(2,14) = C(2,8)$

$C(2,16) = -C(2,10)$; $C(2,18) = - C(2,12)$

$C(4,4) = 4\ell_{15}^2(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(4,8) = 2\ell_{15}(C_{13} \sin a_1 + C_{15} \sin a_2)$

$$C(4,10) = 2\ell_{15}\ell_{32}(C_{13} \sin a_1 + C_{15} \sin a_2)$$

$$C(4,12) = 2\ell_{15}C_{13}(\ell_{31} \cos a_1 + \ell_9 \sin a_1) + 2\ell_{15}C_{15}(\ell_{30} \cos a_2 + \ell_{28} \sin a_2$$

$$C(4,14) = C(4,8)$$

$$C(4,16) = C(4,10)$$

$$C(4,18) = -C(4,12)$$

$$C(5,5) = 4\ell_{15}^2(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(5,7) = 2\ell_{15}(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(5,11) = 2\ell_{15}\ell_{32}(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(5,13) = C(5,7)$$

$$C(5,17) = C(5,11)$$

$$C(6,6) = 4(\ell_{13} \cos a_1 + \ell_{12} \sin a_1)(\ell_{12}C_{13} + \ell_{13}C_{14}) +$$

$$4(\ell_{11} \cos a_2 + \ell_6 \sin a_2)(\ell_6 C_{15} + \ell_{11}C_{16})$$

$$C(6,8) = -2(\ell_{12}C_{13} + \ell_{13}C_{14})\sin a_1 - 2(\ell_6 C_{15} + \ell_{11}C_{16}) \sin a_2$$

$$C(6,10) = -2\ell_{32}(\ell_{12}C_{13} + \ell_{13}C_{14}) \sin a_1 - 2\ell_{32}(\ell_6 C_{15} + \ell_{11}C_{16}) \sin a_2$$

$$C(6,12) = -2(\ell_{31} \cos a_1 + \ell_{29} \sin a_1)(\ell_{12}C_{13} + \ell_{13}C_{14}) -$$

$$2(\ell_{30} \cos a_2 + \ell_{28} \sin a_2)(\ell_6 C_{15} + \ell_{11}C_{16})$$

$$C(6,14) = -C(6,8)$$

$$C(6,16) = -C(6,10)$$

$$C(6,18) = C(6,12)$$

$$C(7,7) = 2(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(7,11) = -2\ell_{32}(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$C(8,8) = 2(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(8,10) = 2\ell_{32}(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(8,12) = 2(\ell_{31}C_{13} \cos a_1 + \ell_{29}C_{13} \sin a_1 + \ell_{30}C_{15} \cos a_2 +$
$\ell_{28}C_{15} \sin a_2)$

$C(9,9) = 2C_{17} + 4C_{18}$

$C(9,11) = (-\ell_{24} + \ell_{23})C_{17} + (-\ell_{24} - 2\ell_{26} + \ell_{23})C_{18}$

$C(9,25) = -C_{17} - C_{18}$

$C(9,26) = \ell_{19}(-C_{17} + C_{18})$

$C(9,35) = -2C_{18}$

$C(9,45) = -C_{17} - C_{18}$

$C(9,46) = \ell_{19}(C_{17} - C_{18})$

$C(10,10) = 2\ell_{32}^2(C_{13} \sin a_1 + C_{15} \sin a_2) + 2\ell_{19}^2 C_{17} + 4\ell_{19}^2 C_{18}$

$C(10,11) = \ell_{19}(\ell_{24} + \ell_{23})(-C_{17} + C_{18})$

$C(10,12) = 2\ell_{32}(\ell_{31}C_{13} \cos a_1 + \ell_{29}C_{13} \sin a_1 + \ell_{30}C_{15} \cos a_2 +$
$\ell_{28}C_{15} \sin a_2)$

$C(10,25) = \ell_{19}(-C_{17} + C_{18})$

$C(10,26) = -\ell_{19}^2(C_{17} + C_{18})$

$C(10,36) = -2\ell_{19}^2(C_{18})$

$C(10,45) = \ell_{19}(C_{17} - C_{18})$

$C(10,46) = C(10,26)$

$$C(11,11) = 2\ell_{32}^2(C_{14} \cos a_1 + C_{16} \cos a_2 + \ell_{24}^2(C_{17} + C_{18}) +$$

$$2\ell_{26}^2 C_{18} + \ell_{23}^2(C_{17} + C_{18})$$

$$C(11,25) = \ell_{24}(C_{17} + C_{18})$$

$$C(11,26) = \ell_{24}\ell_{19}(C_{17} - C_{18})$$

$$C(11,35) = 2\ell_{26}C_{18}$$

$$C(11,45) = -\ell_{23}(C_{17} + C_{18})$$

$$C(11,46) = \ell_{23}\ell_{19}(C_{17} - C_{18})$$

$$C(12,12) = +(2\ell_{31}C_{14} + 2\ell_{29}C_{13})(\ell_{31} \cos a_1 + \ell_{29} \sin a_1) +$$

$$(2\ell_{30}C_{16} + 2\ell_{28}C_{15})(\ell_{30} \cos a_2 + \ell_{28} \sin a_2)$$

$$C(13,13) = +2(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(13,17) = -2\ell_{32}(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(14,14) = +2(C_{13} \sin a_1 + C_{15} \sin a_2)$$

$$C(14,16) = +2\ell_{32}(C_{13} \sin a_1 + C_{15} \sin a_2)$$

$$C(14,18) = -2\ell_{31}C_{13} \cos a_1 - 2\ell_{29}C_{13} \sin a_1 - 2\ell_{30}C_{15} \cos a_2 -$$

$$2\ell_{28}C_{15} \sin a_2$$

$$C(15,15) = +2C_{17} + 4C_{18}$$

$$CL5,17) = (-\ell_{23} + \ell_{24})C_{17} + (-\ell_{23} + 2\ell_{26} + \ell_{24})C_{18}$$

$$C(15,55) = -C_{17} - C_{18}$$

$$C(15,56) = \ell_{19}(-C_{17} + C_{18})$$

$C(15,65) = -2C_{18}$

$C(15,75) = -C_{18} - C_{17}$

$C(15,76) = \ell_{19}(C_{17} - C_{18})$

$C(16,16) = C(10,10)$

$C(16,17) = C(10,11)$

$C(16,18) = -C(10,12)$

$C(16,55) = C(10,25)$

$C(16,56) = C(10,26)$

$C(16,66) = C(10,36)$

$C(16,75) = C(10,45)$

$C(16,76) = C(10,46)$

$C(17,17) = C(11,11)$

$C(17,55) = \ell_{23}(C_{17} + C_{18})$

$C(17,56) = \ell_{23}\ell_{19}(C_{17} - C_{18})$

$C(17,65) = -2\ell_{26}C_{18}$

$C(17,75) = -\ell_{24}(C_{17} + C_{18})$

$C(17,76) = \ell_{24}\ell_{19}(C_{17} - C_{18})$

$C(18,18) = C(12,12)$

$C(25,25) = C_{17} + C_{18}$

$C(25,26) = \ell_{19}(C_{17} - C_{18})$

$C(26,26) = \ell_{19}^2(C_{17} + C_{18})$

$C(35,35) = 2C_{18}$

$$C(36,36) = 2\ell_{19}^2 C_{18}$$

$$C(45,45) = C_{17} + C_{18}$$

$$C(45,46) = \ell_{19}(-C_{17} + C_{18})$$

$$C(46,46) = \ell_{19}^2(C_{17} + C_{18})$$

$$C(55,55) = C_{18} + C_{17}$$

$$C(55,56) = \ell_{19}(-C_{18} + C_{17})$$

$$C(56,56) = \ell_{19}^2(C_{17} + C_{18})$$

$$C(65,65) = 2C_{18}$$

$$C(66,66) = 2\ell_{19}^2 C_{18}$$

$$C(75,75) = (C_{17} + C_{18})$$

$$C(75,76) = \ell_{19}(-C_{17} + C_{18})$$

$$C(76,76) = \ell_{19}^2(C_{17} + C_{18})$$

APPENDIX II

EQUATIONS OF MOTION FOR THE SIMPLIFIED MODEL

II.1 INTRODUCTION

Equations of motion for the simplified model (shown in Figure 3) are derived in detail in this appendix. In the equations the following notations for displacements are used:

u_a - linear displacement of the body in the x direction,
v_a - linear displacement of the body in the y direction,
w_a - linear displacement of the body in the z direction,
α_a - angular displacement of the body about the x direction,
β_a - angular displacement of the body about the y direction,
γ_a - angular displacement of the body about the z direction,

u_{bf} - linear displacement of the front frame in the x direction,
v_{bf} - linear displacement of the front frame in the y direction,
w_{bf} - linear displacement of the front frame in the z direction,
u_{br} - linear displacement of the rear frame in the x direction,
v_{br} - linear displacement of the rear frame in the y direction,
w_{br} - linear displacement of the rear frame in the z direction.

For the deformed configuration the following assumptions are made:

1. $u_a > u_{bj}$

   $v_a > v_{bj}$ where $j \equiv f$ for the front frame, $j \equiv r$ for the rear frame.

   $w_a > w_{bj}$

2. All rotations are anti-clockwise.

3. In each equation terms are written in the following order:

i - Front - Right

ii - Front - Left

iii - Rear - Right

iv - Rear - Left

II.2 CALCULATION OF THE FORCES IN SPRINGS (See Figure 3)

For Spring (K_1)

$$F_{1f} = K_1 \{u_a + \ell_1\beta_a\} \tag{II.1}$$

$$F_{1r} = K_1 \{u_a + \ell_1\beta_a\} \tag{II.2}$$

For Springs K_2

$$F_{2f} = K_2\{(v_a - v_{bf}) - \ell_1\alpha_a - (\ell_3+\ell_{16})\gamma_a\} \tag{II.3}$$

$$F_{2r} = K_2\{(v_a - v_{br}) - \ell_1\alpha_a + (\ell_3+\ell_{16})\gamma_a\} \tag{II.4}$$

For Springs K_3

$$F_{3fr} = K_3\{(w_a - w_{bf}) - \ell_9\alpha_a + (\ell_3+\ell_4)\beta_a\} \tag{II.5}$$

$$F_{3f\ell} = K_3\{(w_a - w_{bf}) + \ell_9\alpha_a + (\ell_3+\ell_4)\beta_a\} \tag{II.6}$$

$$F_{3rr} = K_3\{(w_a - w_{br}) - \ell_9\alpha_a - (\ell_3+\ell_4)\beta_a\} \tag{II.7}$$

$$F_{3r\ell} = K_3\{(w_a - w_{br}) + \ell_9\alpha_a - (\ell_3+\ell_4)\beta_a\} \tag{II.8}$$

For Springs K_6

$$F_{6fr} = K_6\{(w_a - w_{bf}) - \ell_{10}\alpha_a + (\ell_3-\ell_5)\beta_a\} \qquad (II.9)$$

$$F_{6f\ell} = K_6\{(w_a - w_{bf}) + \ell_{10}\alpha_a + (\ell_3-\ell_5)\beta_a\} \qquad (II.10)$$

$$F_{6rr} = K_6\{(w_a - w_{br}) - \ell_{10}\alpha_a - (\ell_3-\ell_5)\beta_a\} \qquad (II.11)$$

$$F_{6r\ell} = K_6\{(w_a - w_{br}) + \ell_{10}\alpha_a - (\ell_3-\ell_5)\beta_a\} \qquad (II.12)$$

For Springs K_4

$$F_{4fr} = K_4\{u_a + \ell_7\beta_a + \ell_9\gamma_a\} \qquad (II.13)$$

$$F_{4f\ell} = K_4\{u_a + \ell_7\beta_a - \ell_9\gamma_a\} \qquad (II.14)$$

$$F_{4rr} = K_4\{u_a + \ell_7\beta_a + \ell_9\gamma_a\} \qquad (II.15)$$

$$F_{4r\ell} = K_4\{u_a + \ell_7\beta_a - \ell_9\gamma_a\} \qquad (II.16)$$

For Springs K_7

$$F_{7fr} = K_7\{u_a + \ell_7\beta_a + \ell_{10}\gamma_a\} \qquad (II.17)$$

$$F_{7f\ell} = K_7\{u_a + \ell_7\beta_a - \ell_{10}\gamma_a\} \qquad (II.18)$$

$$F_{7rr} = K_7\{u_a + \ell_7\beta_a + \ell_{10}\gamma_a\} \qquad (II.19)$$

$$F_{7r\ell} = K_7\{u_a + \ell_7\beta_a - \ell_{10}\gamma_a\} \qquad (II.20)$$

For Springs K_5

$$F_{5fr} = K_5\{(v_a - v_{bf}) - \ell_7\alpha_a - (\ell_3+\ell_4)\gamma_a\} \qquad (II.21)$$

$$F_{5f\ell} = K_5\{(v_a - v_{bf}) - \ell_7\alpha_a - (\ell_3+\ell_4)\gamma_a\} \qquad (II.22)$$

$$F_{5rr} = K_5\{(v_a - v_{br}) - \ell_7\alpha_a + (\ell_3+\ell_4)\gamma_a\} \quad (II.23)$$

$$F_{5r\ell} = K_5\{(v_a - v_{br}) - \ell_7\alpha_a + (\ell_3+\ell_4)\gamma_a\} \quad (II.24)$$

For Springs K_8

$$F_{8fr} = K_8\{(v_a - v_{bf}) - \ell_7\alpha_a - (\ell_3-\ell_5)\gamma_a\} \quad (II.25)$$

$$F_{8f\ell} = K_8\{(v_a - v_{bf}) - \ell_7\alpha_a - (\ell_3-\ell_5)\gamma_a\} \quad (II.26)$$

$$F_{8rr} = K_8\{(v_a - v_{br}) - \ell_7\alpha_a + (\ell_3-\ell_5)\gamma_a\} \quad (II.27)$$

$$F_{8r\ell} = K_8\{(v_a - v_{br}) - \ell_7\alpha_a + (\ell_3-\ell_5)\gamma_a\} \quad (II.28)$$

For Springs K_{11}

$$F_{11fr} = K_{11}\,\gamma_a \quad (II.29)$$

$$F_{11f\ell} = K_{11}\,\gamma_a \quad (II.30)$$

$$F_{11rr} = K_{11}\,\gamma_a \quad (II.31)$$

$$F_{11r\ell} = K_{11}\,\gamma_a \quad (II.32)$$

For Springs K_{12}

$$F_{12fr} = K_{12}\,\gamma_a \quad (II.33)$$

$$F_{12f\ell} = K_{12}\,\gamma_a \quad (II.34)$$

$$F_{12rr} = K_{12}\,\gamma_a \quad (II.35)$$

$$F_{12r\ell} = K_{12}\,\gamma_a \quad (II.36)$$

II.3 FORCES DUE TO DAMPING

(A) Damping in the Rubber Springs

C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_{11} and C_{12} will work in parallel with the force due to the springs: $K_1, K_2, \ldots, K_8, K_{11}$ and K_{12} respectively . Equations (II.37) to (II.72) could be obtained by replacing K_n and C_n and the variables by their first derivative w.r.t. time.

(B) Damping Due to the Shock Absorbers C_R and C_V

Details for this are the same as given in Appendix I for the full model.

Forces in C_{14}

$$P_{14,fr} = C_{14}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{13}\dot{\gamma}_a]\cos a_1 + [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.73}$$

$$P_{14,f\ell} = C_{14}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{13}\dot{\gamma}_a]\cos a_1 - [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.74}$$

$$P_{14,rr} = C_{14}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{13}\dot{\gamma}_a]\cos a_1 + [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.75}$$

$$P_{14,r\ell} = C_{14}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{13}\dot{\gamma}_a]\cos a_1 - [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.76}$$

Forces in C_{13}

$$P_{13,fr} = C_{13}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{13}\dot{\gamma}_a]\cos a_1 + [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.77}$$

$$P_{13,f\ell} = C_{13}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{13}\dot{\gamma}_a]\cos a_1 - [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.78}$$

$$P_{13,rr} = C_{13}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{13}\dot{\gamma}_a]\cos a_1 + [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.79}$$

$$P_{13,r\ell} = C_{13}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{13}\dot{\gamma}_a]\cos a_1 - [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_{12}\dot{\gamma}_a]\sin a_1\} \tag{II.80}$$

Forces in C_{16}

$$P_{16,fr} = C_{16}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{11}\dot{\gamma}_a]\cos a_2 + [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.81}$$

$$P_{16,f\ell} = C_{16}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{11}\dot{\gamma}_a]\cos a_2 - [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.82}$$

$$P_{16,rr} = C_{16}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{11}\dot{\gamma}_a]\cos a_2 + [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.83}$$

$$P_{16,r\ell} = C_{16}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{11}\dot{\gamma}_a]\cos a_2 - [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.84}$$

Forces in C_{15}

$$P_{15,fr} = C_{15}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{11}\dot{\gamma}_a]\cos a_2 + [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.85}$$

$$P_{15,f\ell} = C_{15}\{-[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{11}\dot{\gamma}_a]\cos a_2 - [(\dot{v}_a - \dot{v}_{bf}) - \ell_{15}\dot{\alpha}_a - \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.86}$$

$$P_{15,rr} = C_{15}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a + \ell_{11}\dot{\gamma}_a]\cos a_2 + [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.87}$$

$$P_{15,r\ell} = C_{15}\{[\dot{u}_a + \ell_{15}\dot{\beta}_a - \ell_{11}\dot{\gamma}_a]\cos a_2 - [(\dot{v}_a - \dot{v}_{br}) - \ell_{15}\dot{\alpha}_a + \ell_6\dot{\gamma}_a]\sin a_2\} \tag{II.88}$$

II.4 EQUATIONS OF MOTION (See Figure I.3)

$\Sigma F_x = 0$

$$\begin{aligned} & m_a\ddot{u}_a + [2K_1 + 4(K_4 + K_7)]u_a \\ & + [2\ell_1K_1 + 4\ell_7(K_4 + K_7)]\beta_a \\ & + [2C_1 + 4(C_4 + C_7) + 4(C_{14}\cos a_1 + C_{16}\cos a_2)]\dot{u}_a \\ & + [2\ell_1C_1 + 4\ell_7(C_4 + C_7) + 4\ell_{15}(C_{14}\cos a_1 + C_{16}\cos a_2)]\dot{\beta}_a \\ & = 0 \end{aligned} \tag{1}$$

$\Sigma F_y = 0$

$$\begin{aligned} & m_a\ddot{v}_a + [2K_2 + 4(K_5 + K_8)]v_a \\ & + [-2\ell_1K_2 - 4\ell_7(K_5 + K_8)]\alpha_a \\ & + [2C_2 + 4(C_5 + C_8) + 4(C_{13}\sin a_1 + C_{15}\sin a_2)]\dot{v}_a \\ & + [-2\ell_1C_2 - 4\ell_7(C_5 + C_8) - 4\ell_{15}(C_{13}\sin a_1 + C_{15}\sin a_2)]\dot{\alpha}_a \\ & = [C_2 + 2(C_5 + C_8) + 2(C_{13}\sin a_1 + C_{15}\sin a_2)]\dot{v}_{bf} \\ & + [C_2 + 2(C_5 + C_8) + 2(C_{13}\sin a_1 + C_{15}\sin a_2)]\dot{v}_{br} \\ & + [K_2 + 2(K_5 + K_8)]v_{bf} + [K_2 + 2(K_5 + K_8)]v_{br} \end{aligned} \tag{2}$$

$\underline{\Sigma F_z = 0}$

$$m_a\ddot{w}_a + [4(K_3 + K_6)]w_a + [4(C_3 + C_6)]\dot{w}_a$$

$$= [2(K_3 + K_6)]w_{bf} + [2(K_3 + K_6)]w_{br}$$

$$+ [2(C_3 + C_6)]\dot{w}_{bf} + [2(C_3 + C_6)]\dot{w}_{br} \qquad (3)$$

$\underline{\Sigma M_\alpha = 0}$

$$I_{a\alpha}\ddot{\alpha}_a + [-2\ell_1K_2 - 4\ell_7(K_5+K_8)]v_a$$

$$+ [2\ell_1^2K_2 + 4\ell_7^2(K_5+K_8) + 4\ell_9^2K_3 + 4\ell_{10}^2K_6]\alpha_a$$

$$+ [-2\ell_1C_2 - 4\ell_7(C_5+C_8) - 4\ell_{15}(C_{13} \sin a_1 + C_{15} \sin a_2)]\dot{v}_a$$

$$+ [2\ell_1^2C_2 + 4\ell_7^2(C_5+C_8) + 4\ell_9^2C_3 + 4\ell_{10}^2C_6 + 4\ell_{15}^2(C_{13} \sin a_1 + C_{15} \sin a_2)]\dot{\alpha}_a$$

$$= [-\ell_1K_2 - 2\ell_7(K_5+K_8)]v_{bf} + [-\ell_1K_2 - 2\ell_7(C_5+C_8)]v_{br}$$

$$- [\ell_1C_2 + 2\ell_7(C_5+C_8) + 2\ell_{15}(C_{13} \sin a_1 + C_{15} \sin a_2)]\dot{v}_{bf}$$

$$- [\ell_1C_2 + 2\ell_7(C_5+C_8) + 2\ell_{15}(C_{13} \sin a_1 + C_{15} \sin a_2)]\dot{v}_{br} \qquad (4)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{a\beta}\beta_a + [2\ell_1K_1 + 4\ell_7(K_4+K_7)]u_a$$

$$+ [2\ell_1^2K_1 + 4(\ell_3+\ell_4)^2K_3 + 4(\ell_3-\ell_5)^2K_6 + 4\ell_7^2(K_4+K_7)]\beta_a$$

$$+ [2\ell_1 C_1 + 4\ell_7(C_4+C_7) + 4\ell_{15}(C_{14}\cos a_1 + C_{16}\cos a_2)]\dot{u}_a$$

$$+ [2\ell_1^2 C_1 + 4(\ell_3+\ell_4)^2 C_3 + 4(\ell_3-\ell_5)^2 C_6 + 4\ell_7^2(C_4+C_7) +$$

$$4\ell_{15}^2(C_{14}\cos a_1 + C_{16}\cos a_2)]\dot{\beta}_a$$

$$= [2(\ell_3+\ell_4)K_3 + 2(\ell_3-\ell_5)K_6]w_{bf} + [-2(\ell_3+\ell_4)K_3 - 2(\ell_3-\ell_5)K_6]w_{br}$$

$$+ [2(\ell_3+\ell_4)C_3 + 2(\ell_3-\ell_5)C_6]\dot{w}_{bf} + [-2(\ell_3+\ell_4)C_3 - 2(\ell_3-\ell_5)C_6]\dot{w}_{br} \quad (5)$$

$\underline{\Sigma M_\gamma = 0}$

$$I_a\ddot{\gamma}_a + [2\ell_{16}(\ell_3+\ell_{16})^2K_2 + 4\ell_9^2K_4 + 4\ell_{10}^2K_7 + 4(\ell_3+\ell_4)^2K_5 +$$

$$4(\ell_3-\ell_5)^2K_8 + 4(K_{11} + K_{12})]\gamma_a$$

$$+ [2\ell_{16}(\ell_3+\ell_{16})^2C_2 + 4\ell_9^2C_4 + 4\ell_{10}^2C_7 + 4(\ell_3+\ell_4)^2C_5 +$$

$$4(\ell_3-\ell_5)^2C_8 + 4(C_{11} + C_{12})]\dot{\gamma}_a$$

$$= -\ell_{16}K_2 + 2(\ell_3+\ell_4)K_5 + 2(\ell_3-\ell_5)K_8]v_{bf}$$

$$[(\ell_3+\ell_{16})K_2 + 2(\ell_3+\ell_4)K_5 + 2(\ell_3-\ell_5)K_8]v_{br}$$

$$- [(\ell_3+\ell_{16})C_2 + 2(\ell_3+\ell_4)C_5 + 2(\ell_3-\ell_5)C_8 + 2(\ell_{12}C_{13} + \ell_{13}C_{14})\sin a_1$$

$$+ 2(\ell_6C_{15} + \ell_{11}C_{16})\sin a_2]\dot{v}_{bf}$$

$$+ [(\ell_3+\ell_{16})C_2 + 2(\ell_3+\ell_4)C_5 + 2(\ell_3-\ell_5)C_8 + 2(\ell_{12}C_{13} + \ell_{13}C_{14})\sin a_1$$

$$+ 2(\ell_6C_{15} + \ell_{11}C_{16})\sin a_2]\dot{v}_{br} \quad (6)$$

II.5 DEFINITION OF THE ELEMENTS OF THE MATRICES

All the elements which are not defined below are zeros.

Elements of the Mass Matrix [A]

[A] is the mass matrix (diagonal matrix)

$A(1,1) = m_a,$ $A(4,4) = I_{a\alpha}$

$A(2,2) = m_a,$ $A(5,5) = I_{a\beta}$

$A(3,3) = m_a,$ $A(6,6) = I_{a\gamma}$

Elements of the Stiffness Matrix [B]

[B] is the stiffness matrix (6x6 and symmetrical)

$$B(1,1) = 2K_1 + 4(K_4 + K_7)$$

$$B(1,5) = 2\ell_1 K_1 + 4\ell_7(K_4 + K_7)$$

$$B(2,2) = 2K_2 + 4(K_5 + K_8)$$

$$B(2,4) = -2\ell_1 K_2 - 4\ell_7(K_5 + K_8)$$

$$B(3,3) = 4(K_3 + K_6)$$

$$B(4,4) = 2\ell_1^2 K_2 + 4\ell_7^2(K_5 + K_8) + 4\ell_9^2 K_3 + 4\ell_{10}^2 K_6$$

$$B(5,5) = 2\ell_1^2 K_1 + 4(\ell_3+\ell_4)^2 K_3 + 4(\ell_3-\ell_5)^2 K_6 + 4\ell_7^2(K_4 + K_7)$$

$$B(6,6) = 2(\ell_3+\ell_{16})^2 K_2 + 4\ell_9^2 K_4 + 4\ell_{10}^2 K_7 + 4(\ell_3+\ell_4)^2 K_5 + 4(\ell_3-\ell_5)^2 K_8 + 4(K_{11} + K_{12})$$

Elements of the Damping Matrix [C]

[C] is the damping matrix (6x6, symmetric)

$$C(1,1) = 2C_1 + 4(C_4+C_7) + 4(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(1,5) = 2\ell_1 C_1 + 4\ell_7(C_4+C_7) + 4\ell_{15}(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(2,2) = 2C_2 + 4(C_5+C_8) + 4(C_{13} \sin a_1 + C_{15} \sin a_2)$$

$$C(2,4) = -2\ell_1 C_2 - 4\ell_7(C_5+C_8) - 4\ell_{15}(C_{13} \sin a_1 + C_{15} \sin a_2)$$

$$C(3,3) = 4(C_3+C_6)$$

$$C(4,4) = 2\ell_1^2 C_2 + 4\ell_7^2(C_5+C_8) + 4\ell_9^2 C_3 + 4\ell_{10}^2 C_6 + 4\ell_{15}^2(C_{13} \sin a_1 + C_{15} \sin a_2$$

$$C(5,5) = 2\ell_1^2 C_1 + 4(\ell_3+\ell_4)^2 C_3 + 4(\ell_3-\ell_5)^2 C_6 + 4\ell_7^2(C_4+C_7) + 4\ell_{15}^2(C_{14} \cos a_1 + C_{16} \cos a_2)$$

$$C(6,6) = 2(\ell_3+\ell_{16})^2 C_2 + 4\ell_9^2 C_4 + 4\ell_{10}^2 C_7 + 4(\ell_3+\ell_4)^2 C_5 + 4(\ell_3-\ell_5)^2 C_8$$

Elements of the Forcing Stiffness Matrix (at rear frame)

$[B_{br}]$ = stiffness matrix for the rear frame

$$B(2,2) = K_2 + 2(K_5+K_8)$$

$$B(3,3) = 2(K_3+K_6)$$

$$B(4,2) = -\ell_1 K_2 - 2\ell_7(K_5+K_8)$$

$$B(5,3) = -[2(\ell_3+\ell_4)K_3 + 2(\ell_3-\ell_5)K_6]$$

$$B(6,2) = \ell_{16} K_2 + 2(\ell_3+\ell_4)K_5 + 2(\ell_3-\ell_5)K_8$$

Elements of the Forcing Damping Matrix (at rear frame)

$[C_{br}]$ = Damping matrix for the rear frame

$C(2,2) = C_2 + 2(C_5+C_8) + 2(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(3,3) = 2(C_3+C_6)$

$C(4,2) = -[\ell_1 C_2 + 2\ell_7(C_5+C_8) + 2\ell_{15}(C_{13} \sin a_1 + C_{15} \sin a_2)]$

$C(5,2) = -[2(\ell_3+\ell_4)C_3 + 2(\ell_3-\ell_5)C_6]$

$C(6,2) = \ell_{16}C_2 + 2(\ell_3+\ell_4)C_5 + 2(\ell_3-\ell_5)C_8 + 2(\ell_{12}C_{13} + \ell_{13}C_{14})$

$\sin a_1 + 2(\ell_6 C_{15} + \ell_{11}C_{16}) \sin a_2$

Elements for the Forcing Stiffness Matrix (at front frame)

$[B_{bf}]$ = stiffness matrix for the front frame

$B(2,2) = K_2 + 2(K_5+K_8)$

$B(3,3) = 2(K_3+K_6)$

$B(4,2) = -\ell_1 K_2 - 2\ell_7(K_5+K_8)$

$B(5,3) = 2(\ell_3+\ell_4)K_3 + 2(\ell_3-\ell_5)K_6$

$B(6,2) = -[(\ell_3+\ell_{16})K_2 + 2(\ell_3+\ell_4)K_5 + 2(\ell_3-\ell_5)K_8]$

Elements for the Forcing Damping Matrix (at front frame)

$[C_{bf}]$ = Damping matrix for the front frame

$C(2,2) = C_2 + 2(C_5+C_8) + 2(C_{13} \sin a_1 + C_{15} \sin a_2)$

$C(3,3) = 2(C_3+C_6)$

$$C(4,2) = -[\ell_1 C_2 + 2\ell_7(C_5+C_8) + 2\ell_5(C_{13} \sin a_1 + C_{15} \sin a_2)]$$

$$C(5,3) = 2(\ell_3+\ell_4)C_3 + 2(\ell_3-\ell_5)C_6$$

$$C(6,2) = -[(\ell_3+\ell_{16})C_2 + 2(\ell_3+\ell_4)C_5 + 2(\ell_3-\ell_5)C_8 +$$

$$2(\ell_{12}C_{13} + \ell_{13}C_{14})\sin a_1 + 2(\ell_6 C_{15} + \ell_{11}C_{16})\sin a_2]$$

APPENDIX III

EQUATIONS OF MOTION FOR THE "MOVING" VEHICLE

III.1 INTRODUCTION

For the "moving" vehicle the effect of creep forces (described in detail in Appendix IV) is introduced. The effect of the "shape" of wheel treads is considered.

The equations of motion, except those for the wheelsets, are the same as those derived for the "stationary" vehicle. But the equations describing the motion of the wheelsets have to be derived taking into consideration the effects of creep forces and conicity of the wheel treads.

In this appendix we derive in detail the equations of motion for wheelset No. 1 (equations 23, 24, 26, 27 and 28). For the rest of wheelsets (No. 2 to 6) the equations will be similar.

Before proceeding with the equations of motion for the wheelsets, it is necessary to consider the effect of wheel tread profiles on the reactions.

III.2 MOVEMENTS OF WHEELSETS

In order to derive expressions for the forces acting between wheels and rails it is necessary to consider the mutual geometry of wheel and rail. We also now include gravity forces.

When the wheels are centralized, both tread circles have the same radius ℓ_{20}. When the wheelset is displaced laterally, contact occurs at new points. Figure (III.1) shows the reactions between wheels and rail for a laterally displaced wheelset, where

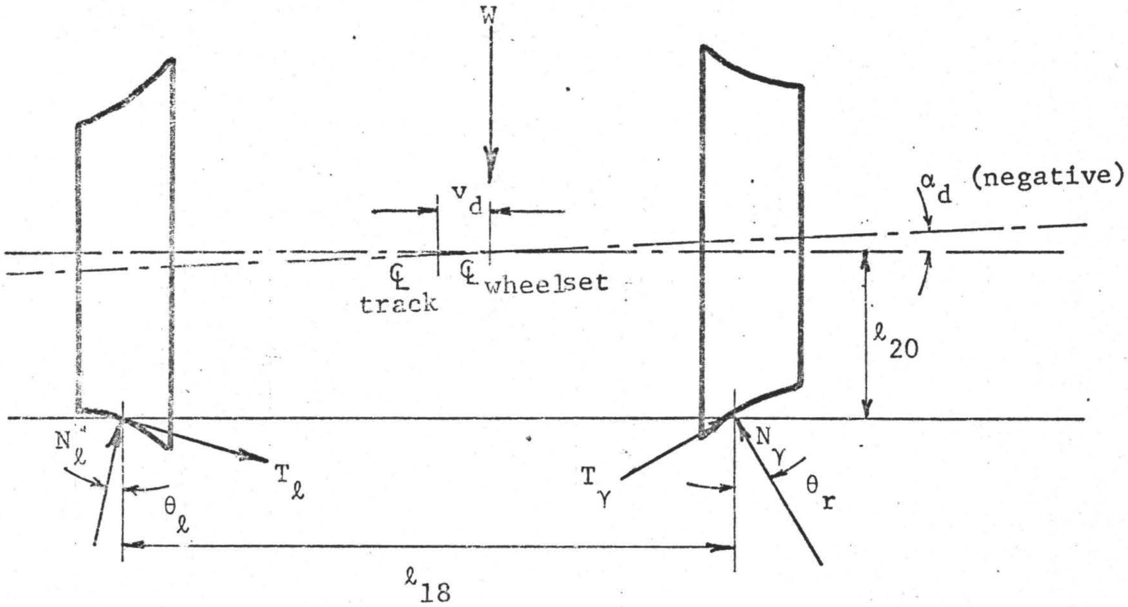


FIGURE (III.1): REACTIONS BETWEEN WHEELS AND RAIL FOR A LATERALLY DISPLACED WHEELSET

T_γ and T_ℓ - are the lateral creep forces,

N_γ and N_ℓ - are the normal reactions, including the component due to gravity force in the car,

W - is the gravity force per wheelset,

There is a complex non-linear constraint between rolling motion of the wheelset (α_d) and its lateral displacement (v_d). Following Wickens [18] this is linearized as follows:

$$\theta_r = \theta_o + \varepsilon\, v_{d1} \frac{2}{\ell_{18}} \qquad \text{(III.1)}$$

$$\theta_\ell = \theta_o - \varepsilon\, v_{d1} \frac{2}{\ell_{18}} \qquad \text{(III.2)}$$

where θ_o - is the angle between the contact plane and horizontal when wheelset is in central position.

θ_r, θ_ℓ - angles between contact planes and horizontal wheelset displaced laterally.

v_{d1} - lateral displacement of wheelset 1.

ε - rate of change of contact plane slope with lateral displacement of wheelset (for new wheels ε becomes λ).

From Equations (III.1) and (III.2) we see that θ_r and θ_ℓ are both equal to θ_o if the lateral displacement of the wheelset (v_{d1}) is zero. We also have that

$$r_r = \ell_{20} + \lambda\, v_{d1} \tag{III.3}$$

$$r_\ell = \ell_{20} - \lambda\, v_{d1} \tag{III.4}$$

where ℓ_{20} - wheel tread circle radius, wheelset in central position,

r_r, r_ℓ - radii of tread circles, wheelset displaced laterally,

λ - effective conicity which is defined as the rate of change of rolling radius with lateral displacement of wheelset.

The distance between the wheelset center line and contact points is given by:

$$\frac{\ell_{18}}{2} \pm \eta\, v_{d1} \tag{III.5}$$

where η is the correction factor. For straight coned wheels $\eta = 1$.

We should also note that θ_r and θ_ℓ are small angles so that:

$$\sin\theta_r \simeq \theta_r, \quad \sin\theta_\ell \simeq \theta_\ell$$

$$\cos\theta_r \simeq 1, \quad \cos\theta_\ell \simeq 1$$

And the equation for lateral motion of wheelset 1 is (equation 24 in Appendix I)

$$m_d \ddot{v}_{d1} + (RB_1) + (RV_1) - (RY_{1r} + RY_{1\ell}) = 0$$

where RB_1, RV_1 - are internal reactions,

$RY_{1r}, RY_{1\ell}$ - are lateral creep forces at wheels.

Substituting for the creep forces (from equation (IV.3.7) we get:

$$m_d \ddot{v}_{d1} + (RB_1) + (RV_1) + (2f_2)\gamma_{d1} + (\frac{2f_2}{S})\dot{v}_{d1} + (-\frac{2\ell_{20} f_2}{S})\dot{\alpha}_{d1} + (\frac{2f_{23}}{S})\dot{\gamma}_{d1} + N_{r1}(\theta_o + \frac{2\varepsilon\, v_{d1}}{\ell_{18}}) - N_{\ell 1}(\theta_o - \frac{2\varepsilon\, v_{d1}}{\ell_{18}}) = 0 \quad \text{(III.6)}$$

But, $N_{r1}(\theta_o + \frac{2\varepsilon\, v_{d1}}{\ell_{18}}) - N_{\ell 1}(\theta_o - \frac{2\varepsilon\, v_{d1}}{\ell_{18}})$ can be rewritten as:

$$(N_{r1} - N_{\ell 1})\theta_o + (N_{r1} + N_{\ell 1}) \frac{2\varepsilon\, v_{d1}}{\ell_{18}} .$$

However, $N_{r1} + N_{\ell 1} \simeq W$ (weight/wheelset). Hence, Equation (III.6) can be rewritten as:

$$m_d \ddot{v}_{d1} + (RB_1) + (RV_1) + (2f_2)\gamma_{d1} + (\frac{2f_2}{S})\dot{v}_{d1} + (-\frac{2\ell_{20} f_2}{S})\dot{\alpha}_{d1} + (\frac{2f_{23}}{S})\dot{\gamma}_{d1} + (\frac{2W\varepsilon}{\ell_{18}})v_{d1} + (N_{r1} - N_{\ell 1})\theta_o = 0 \quad \text{(III.7)}$$

where the quantity $\frac{2W\varepsilon}{\ell_{18}}$ is the so called "gravitational stiffness" and for new wheels $\varepsilon = \lambda = 0.2$.

Now the equation of motion of wheelset 1 in the α direction can be obtained using Equation 26 in Appendix I after substituting for the creep forces using equation (IV.3.7)

In the α direction: $\underline{\Sigma M_\alpha = 0.}$

$$(I_{c\alpha} + I_{d\alpha})\ddot{\alpha}_{d1} + (-2\ell_{17}^2 K_9)\alpha_{br} + (2\ell_{17}^2 K_9)\alpha_{d1}$$

$$+ [\ell_{19}(C_{18}-C_{17})]\dot{w}_{br} + [\ell_{19}(-C_{18}+C_{17})]\dot{w}_{d1} + [\ell_{19}^2(-C_{18}-C_{17})]\dot{\alpha}_{br}$$

$$+ [\ell_{19}^2(C_{18}+C_{17})]\dot{\alpha}_{d1} + [\ell_{19}\ell_{24}(-C_{18}+C_{17})]\dot{\beta}_{br}$$

$$+ (-2\ell_{20}f_2)\gamma_{d1} + (-\frac{2\ell_{20}f_2}{S})\dot{v}_{d1} + (\frac{2\ell_{20}^2 f_2}{S})\dot{\alpha}_{d1} + (-\frac{2\ell_{20}f_{23}}{S})\dot{\gamma}_{d1}$$

$$+ (\frac{\ell_{18}}{2} - \eta v_{d4})N_{r1} - (\frac{\ell_{18}}{2} + \eta v_{d4})N_{\ell 1}$$

$$+ \ell_{20}(\theta_o - \frac{2\varepsilon\, v_{d1}}{\ell_{18}})N_{\ell 1} - \ell_{20}(\theta_o + \frac{2\varepsilon\, v_{d1}}{\ell_{18}})N_{r1} = 0$$

Combining the terms in N_{r1} and $N_{\ell 1}$ as was done before, we get

$$(I_{c\alpha} + I_{d\alpha})\ddot{\alpha}_{d1} + (-2\ell_{17}^2 K_9)\alpha_{br} + (2\ell_{17}^2 K_9)\alpha_{d1}$$

$$+ [\ell_{19}(C_{18}-C_{17})]\dot{w}_{br} + [\ell_{19}(-C_{18}+C_{17})]\dot{w}_{d1} + [\ell_{19}^2(-C_{18}-C_{17})]\dot{\alpha}_{br}$$

$$+ [\ell_{19}^2(C_{18}+C_{17})]\dot{\alpha}_{d1} + [\ell_{19}\ell_{24}(-C_{18}+C_{17})]\dot{\beta}_{br}$$

$$+ (-2\ell_{20}f_2)\gamma_{d1} + (-\frac{2\ell_{20}f_2}{S})\dot{v}_{d1} + (\frac{2\ell_{20}^2f_2}{S})\dot{\alpha}_{d1} + (-\frac{2\ell_{20}f_{23}}{S})\dot{\gamma}_{d1}$$

$$+ (-W\eta)v_{d1} + (\frac{\ell_{18}}{2} - \ell_{20}\theta_o)(N_{r1} - N_{\ell 1}) - (\frac{2\ell_{20}\varepsilon W}{\ell_{18}})v_{d1}$$

$$= 0 \qquad \text{(III.8)}$$

To eliminate $(N_{r1} - N_{\ell 1})$ from equations (III.7) and (III.8), the following procedure (which is implemented in the computer program) is followed:

$$\text{Equation (III.7)} = \text{Equation (III.7)} + \frac{-\theta_o}{\frac{\ell_{18}}{2} - \ell_{20}\theta_o} \times \text{Equation (III.8)}$$

and we use the constraint

$$\alpha_{d1} = -\frac{2\lambda\, v_{d1}}{\ell_{18}}$$

to reduce the number of variables by one (α_{d1}).

In the γ direction, the equation of motion for wheelset (after substituting for the values of creep forces from Appendix IV) is:

$\underline{\Sigma M_\gamma = 0}$

$$I_{d\gamma}\ddot{\gamma}_{d1} + (RM_1) + (R\gamma_1) + (\frac{\ell_{18}^2 f_1}{2S})\dot{\gamma}_{d1} + (-\frac{\ell_{18}f_1}{\ell_{20}})v_{d1}$$

$$+ (\frac{2f_3}{S})\dot{\gamma}_{d1} + (2f_{32})\gamma_{d1} + (\frac{2f_{32}}{S})\dot{v}_{d1} + (-\frac{2\ell_{20}f_{32}}{S})\dot{\alpha}_{d1}$$

$$+ [-(\theta_o - \frac{2\varepsilon\, v_{d1}}{\ell_{18}}) \frac{\ell_{18}}{2} N_{\ell 1}]\gamma_{d1} + [-(\theta_o + \frac{2\varepsilon\, v_{d1}}{\ell_{18}}) \frac{\ell_{18}}{2} N_{r1}]\gamma_{d1}$$

$$= 0 \qquad \text{(III.9)}$$

Substituting for $N_{\ell 1} + N_{r1} = W$, and neglecting $(N_{\ell 1}-N_{r1})\frac{\varepsilon v_{d1}}{\ell_{18}}$

Equation (III.9) becomes:

$$I_{d\gamma}\ddot{\gamma}_{d1} + (RM_1) + (R\gamma_1)$$

$$+ (-\frac{\ell_{18} f_1 \lambda}{\ell_{20}})v_{d1} + (2f_{32} - W\frac{\ell_{18}}{2}\theta_o)\gamma_{d1}$$

$$+ (\frac{2f_{32}}{S})\dot{v}_{d1} + (-\frac{2\ell_{20} f_{32}}{S})\dot{\alpha}_{d1} + (\frac{\ell_{18}^2 f_1}{2S} + \frac{2f_3}{S})\dot{\gamma}_{d1}$$

$$= 0 \qquad \text{(III.10)}$$

Some typical values for the parameters ε, λ and θ_o are given by Wickens [18].

For the analysis the following values are used for the parameters.

$\lambda = 0.05$, $\varepsilon = 0.05$, $\theta_o = 0.05$ and $\eta=1$ (for new wheels)

$\lambda = 0.2$, $\varepsilon = 25.4$, $\theta_o = 0.04$ and $\eta=1$ (for worn wheels)

III.3 EQUATIONS OF MOTION FOR WHEELSETS

For Wheelset No. 1:

$\underline{\Sigma F_x = 0}$

$$(m_c + m_d)\ddot{u}_{d1} + (RA_1) + (\frac{2f_1}{S})\dot{u}_{d1} + (\frac{2\ell_{20} f_1}{S})\dot{\beta}_{d1} = 0$$

23

$\underline{\Sigma F_y = 0}$

$$m_d\ddot{v}_{d1} + (RB_1) + (RV_1) + (2f_2)\gamma_{d1} + (\frac{2f_2}{S})\dot{v}_{d1}$$

$$+ (-\frac{2\ell_{20}f_2}{S})\dot{\alpha}_{d1} + (\frac{2f_{23}}{S})\dot{\gamma}_{d1} + (\frac{2W\varepsilon}{\ell_{18}})v_{d1} + (N_{r1}-N_{\ell 1})\theta_o$$

$$= 0 \qquad (24)$$

$\underline{\Sigma M_\alpha = 0}$

$$(I_{c\alpha} + I_{d\alpha})\ddot{\alpha}_{d1} + (-2\ell_{17}^2K_9)\alpha_{br} + (2\ell_{17}^2K_9)\alpha_{d1}$$

$$+ [\ell_{19}(C_{18}-C_{17})]\dot{w}_{br} + [\ell_{19}(-C_{18}+C_{17})]\dot{w}_{d1} + [\ell_{19}^2(-C_{18}-C_{17})]\dot{\alpha}_{br}$$

$$+ [\ell_{19}^2(C_{18}+C_{17})]\dot{\alpha}_{d1} + [\ell_{19}\ell_{24}(-C_{18}+C_{17})]\dot{\beta}_{br}$$

$$+ (-2\ell_{20}f_2)\gamma_{d1} + (-\frac{2\ell_{20}f_2}{S})\dot{v}_{d1} + (\frac{2\ell_{20}^2f_2}{S})\dot{\alpha}_{d1} + (-\frac{2\ell_{20}f_{23}}{S})\dot{\gamma}_{d1}$$

$$+ (-W\eta + \frac{2\ell_{20}\,\varepsilon W}{\ell_{18}})v_{d1} + (\frac{\ell_{18}}{2} - \ell_{20}\theta_o)(N_{r1} - N_{\ell 1})$$

$$= 0 \qquad (26)$$

$\underline{\Sigma M_\beta = 0}$

$$I_{d\beta}\ddot{\beta}_{d1} + (\frac{2\ell_{20}^2f_1}{S})\dot{\beta}_{d1} + (\frac{2\ell_{20}f_1}{S})\dot{u}_{d1} = 0 \qquad (27)$$

$\underline{\Sigma M_{\gamma} = 0}$

$$I_{d\gamma}\ddot{\gamma}_{d1} + (RM_1) + (R\gamma_1)$$

$$+ (\frac{2f_{32}}{S})v_{d1} + (-\frac{2\ell_{20}f_{32}}{S})\alpha_{d1} + (\frac{\ell_{18}^2 f_1}{2S} + \frac{2f_3}{S})\dot{\gamma}_{d1}$$

$$= 0 \qquad (28)$$

For other wheelsets (2 to 6) we will have similar equations with the proper indices (2 to 6) respectively.

III.4 ADDITIONAL ELEMENTS FOR MATRICES

In addition to the elements defined in Section (I.5) elements of the stiffness and damping matrices for the moving vehicle are defined below (for wheelset No. 1).

(A) For the Stiffness Matrix

$$B(24,24) = \frac{2W\varepsilon}{\ell_{18}}$$

$$B(24,28) = 2f_2$$

$$B(26,24) = \frac{-2\ell_{20}\ \varepsilon W}{\ell_{18}} - W\eta$$

$$B(26,28) = -2\ell_{20}f_2$$

$$B(28,24) = -\frac{\ell_{18}f_1\lambda}{\ell_{20}}$$

$$B(28,28) = 2f_{32} - \frac{W\ell_{18}}{2} \theta_o$$

We have similar terms for wheelsets 2,3,4,5 and 6 (easy to obtain successively by incrementing the indices by 10 respectively). e.g.,

$B(34,34) = B(24,24)$

$B(34,38) = B(24,28)$

$B(36,34) = B(26,24)$

$B(36,38) = B(26,28)$

$B(38,34) = B(28,24)$

$B(38,38) = B(28,28)$

etc.

(B) <u>For the Damping Matrix</u>

$$C(23,23) = \frac{2f_1}{S}$$

$$C(23,27) = \frac{2\ell_{20}f_1}{S}$$

$$C(24,24) = \frac{2f_2}{S}$$

$$C(24,26) = \frac{-2\ell_{20}f_2}{S}$$

$$C(24,28) = \frac{2f_{23}}{S}$$

$$C(26,24) = \frac{-2\ell_{20}f_2}{S}$$

$$C(26,26) = \frac{2\ell_{20}^2 f_2}{S} + C(26,26)$$

$$C(26,28) = (-\frac{2\ell_{20} f_{23}}{S})$$

$$C(27,23) = \frac{2\ell_{20} f_1}{S}$$

$$C(27,27) = \frac{2\ell_{20}^2 f_1}{S}$$

$$C(28,24) = \frac{2f_{32}}{S}$$

$$C(28,26) = \frac{-2\ell_{20} f_{32}}{S}$$

$$C(28,28) = \frac{\ell_{18}^2 f_1}{2S} + \frac{2f_3}{S}$$

We have similar terms for wheelsets 2, 3, 4, 5 and 6. These terms can be obtained in a similar way as described in part (A) for the elements of the stiffness matrix.

APPENDIX IV

DETERMINATION OF CREEP FORCES AND EVALUATION OF CREEP COEFFICIENTS

IV.1 INTRODUCTION

The phenomenon of creep between wheel and rail is of fundamental importance in the study of the lateral dynamics of railway vehicles. Complete slip of the wheel on the rail, which is simply the limiting case of creep, is important for studies of traction and braking.

If the contact area between a wheel and a rail has insufficient contact pressure to maintain friction adhesion, an effective forward slip or creep will be present.

Longitudinal or transverse creep velocities are generated between the two bodies rolling together when any tangential force is transmitted, either in the direction of rolling or transversely. If, in addition, there exists a relative angular velocity, or spin, between the two bodies about the normal to the contact plane, then a transverse creep velocity is produced.

Many theoretical and experimental studies have been carried out to explain this phenomenon. In this appendix we will review briefly the progress of theoretical work up to the present time.

IV.2 SURVEY OF CREEP THEORY

The problem of creep was first treated by Carter [35] who recognized its importance in the railway field. He treated the two-dimensional case of two cylinders with parallel axes rolling together with creep in the direction of rolling.

An approximate theory for the three dimensional problem with elliptical contact was given by Johnson and Vermuellen [4], who approximated the area of adhesion by an ellipse similar to the contact ellipse. The theory treats the case of longitudinal and, transverse, or lateral, creep for all values of creepage. According to Kalker [24], the error due to the approximation is never more than 25%. Johnson [6] provided a solution to the creep problem which for the first time includes spin; however, the theory is only valid for vanishing creep and spin and for circular contact areas.

Similar but more recent work has been published by dePater [7] and Kalker [8], dePater's solution being for vanishing Poisson's ratio, and Kalker's being extended later to cover elliptical contact.

The most recent advance has been made by Kalker [38] who has given a numerical method for the calculation of the three dimensional case with any value of creep and spin.

IV.3 CREEP FORCES

Carter [35] originally proposed the following relationship to predict creep forces

$$\textit{Tangential Creep Forces} = -f\left(\frac{\textit{Creep or Relative Slip Velocity In Direction of Force}}{\textit{Horizontal Velocity of Wheel Axis}}\right) \quad \text{(IV.3.1)}$$

where the coefficient f is defined empirically. The selection of its value will be discussed below.

Similarly for rotational creep we have:

$$\frac{\textit{Creep Torque About An Axis}}{\textit{Normal to Creep Surface}} = -f\left(\frac{\textit{Rotational Creep in Direction of Force}}{\textit{Horizontal Velocity of Wheel Axis}}\right) \qquad \text{(IV.3.2)}$$

In addition there may be coupling between lateral creep and spin [36]. The relation between creep and spin was first investigated by Johnson. As previously defined:

f_1 - coefficient relating longitudinal creep force to longitudinal creep,

f_2 - coefficient relating lateral creep force to lateral creep,

f_3 - coefficient relating creep torque to rotational creep,

f_{23} - coupling coefficient relating creep torque to lateral creep and lateral force to rotational creep,

U_{ir} - creep force in the longitudinal direction for the right wheel (i),

$U_{i\ell}$ - creep force in the longitudinal direction for the left wheel (i),

V_{ir} - creep force in the lateral direction for the right wheel (i),

$V_{i\ell}$ - creep force in the lateral direction for the left wheel (i),

T_{ir} - creep torque in the γ direction for the right wheel (i),

$T_{i\ell}$ - creep torque in the γ direction for the left wheel (i).

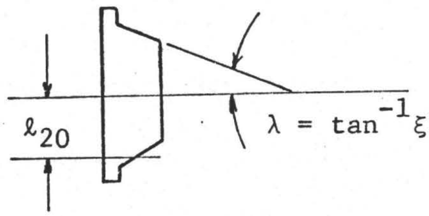
where i = 1, 2, 3, 4, 5, 6 from rear to front.

Tangential forces U_{ir} $(i=1,\ldots,6)$ do not include the steady state propelling force. They are variations about this force. Correspondingly we omit the steady state creep force in the longitudinal direction due to the propelling force.

i) Longitudinal Creep - Right Rear Wheel

We let S = forward speed

and λ = coning ratio of the wheel.



Longitudinal creep was defined by Carter [35] as:

$$CR_{\ell} = \frac{\textit{Actual Forward Displacement - Pure Rolling Forward Displacement}}{\textit{Forward Displacement Attributable to Rolling}}$$

In a small interval of time dt, the actual forward displacement at the wheel tread is:

$$S\,dt + d\,u_{d1} + \frac{\ell_{18}}{2}\,d\gamma_{d1}$$

The pure rolling forward displacement is:

$$\frac{(\ell_{20} + \lambda v_{d1})}{\ell_{20}}\,S\,dt - (\ell_{20} + \lambda v_{d1})\,d\beta_{d1}$$

Hence the forward creep displacement in time dt =

$$(\ell_{20} + \lambda v_{d1})\,d\beta_{d1} + du_{d1} + \frac{\ell_{18}}{2}\,d\gamma_{d1} - \lambda v_{d1}\,\frac{S\,dt}{\ell_{20}} \qquad \text{(IV.3.3)}$$

The first two terms represent slip induced by the difference between the axle longitudinal displacement and the displacement corresponding to the angular rotation of the axle. The third term is slip induced by angular displacement and the fourth term is slip induced by the forward steady state displacement S dt due to the rolling radius deviating from the mean ℓ_{20}.

To get the creep velocity we divide the terms in equations (IV.3.3) by dt and get the limit as dt approaches zero.

$$\textit{Creep Velocity} = (\ell_{20} + \lambda v_{d1})\dot{\beta}_{d1} + \dot{u}_{d1} + \frac{\ell_{18}}{2}\dot{\gamma}_{d1} - \lambda v_{d1}\frac{S}{\ell_{20}} \qquad \text{(IV.3.4)}$$

The nonlinear term is assumed negligible and equation (IV.3.4) becomes:

$$\textit{Creep Velocity} = \ell_{20}\dot{\beta}_{d1} + \dot{u}_{d1} + \frac{\ell_{18}}{2}\dot{\gamma}_{d1} - \lambda v_{d1}\frac{S}{\ell_{20}} \qquad \text{(IV.3.5)}$$

It should be noted that References [1], [36] and [39] do not have the first two terms, but reference [40] has them. Using equations (IV.3.1) and (IV.3.4) we get for the creep force.

$$U_{ir} = - f_1(\ell_{20}\dot{\beta}_{d1} + \dot{u}_{d1} + \frac{\ell_{18}}{2}\dot{\gamma}_{d1} - \lambda v_{d1}\frac{S}{\ell_{20}})/\frac{S}{\ell_{20}}(\ell_{20}+\lambda v_{d1})$$

The denominator is the local velocity of roll. However $S\lambda v_{d1}/\ell_{20}$ is considered second order small and will be neglected. Hence

$$U_{ir} = -\frac{f_1}{S}(\ell_{20}\dot{\beta}_{d1} + \dot{u}_{d1} + \frac{\ell_{18}}{2}\dot{\gamma}_{d1} - \lambda v_{d1}\frac{S}{\ell_{20}}) \qquad \text{(IV.3.6)}$$

ii) Lateral Creep - Right Rear Wheel

The actual lateral displacement in time dt is given by

$$d\, v_{d1} - \ell_{20}\, d\alpha_{d1}$$

The pure rolling lateral displacement (which is the lateral component of the forward speed) is -S dt γ_{d1}. Finally the rotational creep at the contact point is $\dot{\gamma}_{d1}$. Hence the lateral creep force at the right rear wheel is given by

$$V_{1r} = \{-f_2[\dot{v}_{d1} + S\gamma_{d1} - \ell_{20}\dot{\alpha}_{d1}] - \ell_{23}\dot{\gamma}_{d1}\}/S \qquad \text{(IV.3.7)}$$

iii) Rotational Creep - Right Rear Wheel

The rotational creep torque T_{1r} is given by:

$$T_{1r} = \{-f_{32}[\dot{v}_{d1} + S\gamma_{d1} - \ell_{20}\dot{\alpha}_{d1}] - f_3\dot{\gamma}_{d1}\}/S \qquad \text{(IV.3.8)}$$

In a similar manner we get the creep forces at the left wheel.

iv) Longitudinal Creep - Left Rear Wheel

$$U_{1\ell} = -f_1(\ell_{20}\dot{\beta}_{d1} + \dot{u}_{d1} - \frac{\ell_{18}}{2}\dot{\gamma}_{d1} + \lambda\, v_{d1}\frac{S}{\ell_{20}})/S \qquad \text{(IV.3.9)}$$

v) Lateral Creep - Left Rear Wheel

$$V_{1\ell} = \{-f_2]\dot{v}_{d1} + S\gamma_{d1} - \ell_{20}\dot{\alpha}_{d1}] - f_{23}\dot{\gamma}_{d1}\ /S \qquad \text{(IV.3.10)}$$

vi) Rotational Creep - Left Front Wheel

$$T_{1\ell} = \{-f_{32}[\dot{v}_{d1} + S\gamma_{d1} - \ell_{20}\dot{\alpha}_{d1}] - f_3\,\dot{\gamma}_{d1}\}/S \qquad \text{(IV.3.11)}$$

The other wheelsets will have similar expressions.

Some writers [36] include a "gravity stiffness" due to rising of the wheelset c.g., when there is lateral displacement. Its amount will depend on the shape of the wheel tread and the track profile. Gilchrist et.al. [25] indicate that the effect is negligible for unworn straight coned wheels. Clark and Law [39] neglect gravity stiffness completely.

IV.4 EVALUATION OF CREEP COEFFICIENTS

i) Longitudinal and Lateral Creep Coefficients

The creep coefficient f was defined by Carter [35] in an empirical equation

$$f = 3500(2\ell_{20}\, W_a)^{\frac{1}{2}} \text{ lb./wheel} \qquad \text{(IV.4.1)}$$

where W_a = axle loading in lb.

$= g(m_a + 2m_b + 6m_c + 6m_d)/6$

ℓ_{20} = wheel radius in inches.

This assumes $f_1 = f_2$ and neglects f_3 and f_{23}.

Clark and Law [39] used this evaluation. However Wickens [36] and Gilchrist et.al. [25] do not. They appear to use the work of

Johnson and Vermuellen [4] and Kalker [24] for longitudinal and lateral coefficients and of Johnson [6] for the rotational coefficient. A similar practice was adopted by Dokainish and Siddall [40]. The same is followed here and specific values are given below.

Let a and b be the semiaxes of the elliptical area of contact between the two surfaces. The relations between tangential contact forces and the creep velocities are given by the following expressions (reference 4, equations 21 and 23):

$$\xi_x = \frac{3\ \mu N}{G\pi ab} \quad [1 - (1 - \frac{T_x}{\mu N})^{1/3}] \qquad \text{(IV.4.2)}$$

$$\xi y = \frac{3\ \mu N}{G\pi ab} \quad \psi_1[1 - (1 - \frac{T_y}{\mu N})^{1/3}] \qquad \text{(IV.4.3)}$$

where T_x - is the tangential force in the direction of rolling,

T_y - is the transverse tangential force,

N - is the normal force between the two surfaces,

G - is the modulus of rigidity,

μ - is the coefficient of friction,

ξ_x and ξ_y - are creep ratios,

$$\xi_x = \frac{\Delta U}{S} = \frac{\text{Creep Velocity in the Direction of Rolling}}{\text{Steady Rolling Velocity}}$$

$$\xi_y = \frac{\Delta V}{S} = \frac{\text{Creep Velocity in the Transverse Direction}}{\text{Steady Rolling Velocity}}$$

ϕ and ψ_1 - functions of the ratio $\frac{a}{b}$ and ν (Poisson's Ratio)

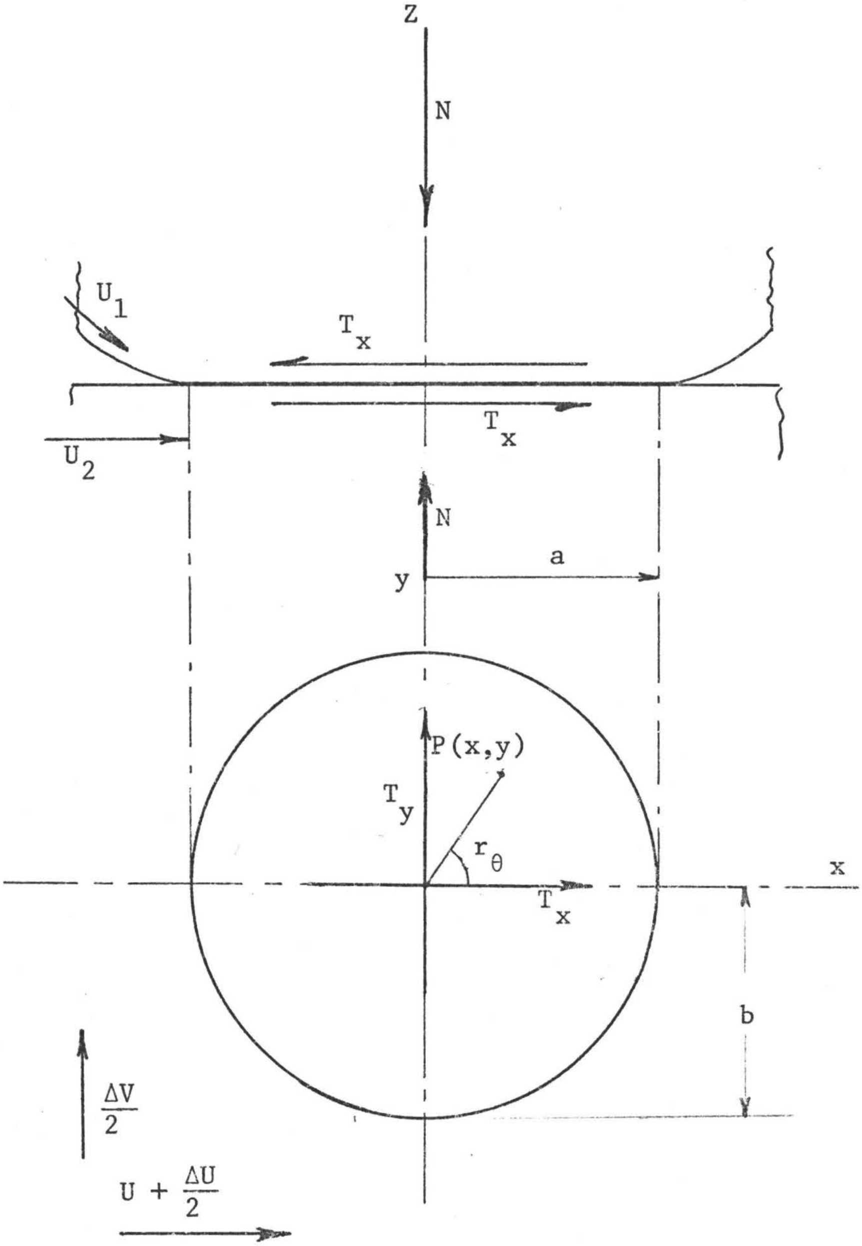


FIGURE (IV.1) - Area of Contact Showing Tangential Force Components T_x and T_y. Creep Velocities are Denoted by ΔU and ΔV.

Equation (IV.4.2) can also be rewritten in the following form:

$$\xi_x = - \frac{3\ \mu N}{G\pi ab} \phi\{1-[1 - \frac{T_x}{3\mu N} - \frac{1}{9} (\frac{T_x}{\mu N})^2 - \frac{5}{81} (\frac{T_x}{\mu N})^3 \ldots]\} \qquad \text{(IV.4.4)}$$

If the tangential force is small compared with the limiting friction force (i.e. for small values of $\frac{T_x}{\mu N}$ say 0.5), equaticn (IV.4.4) can be expressed as

$$\xi_x = - \frac{3\ \mu N}{G\pi ab} \phi \cdot \frac{T_x}{3\mu N}$$

$$\simeq - T_x \frac{\phi}{G\pi ab} \qquad \text{(IV.4.5)}$$

To get the longitudinal creep coefficient f_1 we use equation (IV.4.5). We know that

$$\textit{Longitudinal Creep Force} = - f_1 \frac{\Delta U}{S} \qquad \text{(IV.4.6)}$$

we get:

$$f_1 = T_x \frac{G\pi ab}{T_x \phi} = \frac{G\pi ab}{\phi} \qquad \text{(IV.4.7)}$$

Similarly for small values of $\frac{T_y}{\mu N}$ we can get the lateral creep coefficient f_2

$$f_2 = \frac{G\pi ab}{\psi_1} \qquad \text{(IV.4.8)}$$

As tangential force approaches the limiting frictional force the creep coefficient can be obtained by substituting $\frac{T_x}{\mu N} = 1$ in equation (IV.4.2) and $\frac{T_y}{\mu N} = 1$ in equation (IV.4.3). The limiting values for the creep ratios will be

$$\xi_x = - \frac{3\ \mu N}{G\pi ab}\ \phi = - \frac{3\ T_x}{G\pi ab}\ \phi \qquad \text{(IV.4.9)}$$

and the longitudinal creep coefficient will be given by

$$f_1 = \frac{G\pi ab}{3\phi} \qquad \text{(IV.4.10)}$$

Similarly the lateral creep coefficient

$$f_2 = \frac{G\pi ab}{3\ \psi_1} \qquad \text{(IV.4.11)}$$

These values are one third of the values obtained for small $(\frac{T_x}{\mu N})$ and $(\frac{T_y}{\mu N})$. Experience of some of the researchers working in this area indicates that the values of creep coefficients determined for small values of $(\frac{T_x}{\mu N}$ and $\frac{T_y}{\mu N})$ should be multiplied by a correction factor. The value for the correction factor η has been suggested as 0.5.

ii) Creep Coefficients Relating Spin with Transverse Tangential Force and Moment About the Spin Axis

Johnson [6] considered the effect of spin upon the rolling motion of an Elastic Sphere on a plane. Equations 26 and 27 express the following relations:

$$\xi_y = \frac{2(2 - \nu)}{3(3 - 2\nu)} \frac{\dot{\gamma}C}{S} \qquad \text{(IV.4.12)}$$

and

$$M_z = - \frac{32(2 - \nu)}{9(3 - 2\nu)} G\, C^3\, p \qquad \text{(IV.4.13)}$$

where $\dot{\gamma}$ - Angular velocity due to spin,

S - Velocity in the rolling direction,

p - A non-dimensional spin factor $= \frac{\dot{\gamma}C}{S}$,

ν - Poisson's ratio,

C - Radius of the circular area of contact,

ξ_y - Transverse creep ratio,

M_z - Moment about the spin axis.

The expressions given by equations (IV.4.12) and (IV.4.13) have been obtained for a circular area of contact. For an elliptical area of contact, we can have the following approximation:

$$C = \sqrt{ab} \qquad \text{(IV.4.14)}$$

where a and b are the two semiaxes for the ellipse. Moreover for the area of contact between wheel and rail, the ratio of the two axes is approximately one. Therefore equations (IV.4.12) and (IV.4.13) are considered to be quite satisfactory. Equation (IV.4.13) can be rewritten as:

$$M_z = - \frac{32(2 - \nu)}{9(3 - 2\nu)} G C^4 \frac{\dot{\gamma}}{S}$$

Substituting for $\nu = 0.265$ (the value of Poisson's ratio for cast steel) we get:

$$M_z = - \frac{32(2 - 0.265)}{9(3 - 0.530)} 11.3 C^4 \frac{\dot{\gamma}}{S} \times 10^6 \text{ lb.in.}$$

$$= 28.2 C^4 (\frac{\dot{\gamma}}{S}) \times 10^6 \text{ lb.in.}$$

where $\frac{\dot{\gamma}}{S}$ in the creep ratio in inch^{-1}. But, the creep coefficient, f_3, is defined by:

$$M_z = - f_3 \frac{\dot{\gamma}}{S} \quad \text{(IV.4.15)}$$

Therefore

$$f_3 = 28.2 \times 10^6 C^4 \text{ lb.in}^2 \quad \text{(IV.4.16)}$$

and equation (IV.4.12) gives us

$$\xi_y = \frac{2(2 - \nu)}{3(3 - 2\nu)} \frac{\dot{\gamma}}{S} C = 0.469 C \frac{\dot{\gamma}}{S} \quad \text{(IV.4.17)}$$

The coupled creep coefficient (f_{23} or f_{32}) is defined by:

$$\textit{Lateral Force} = - f_{23} \frac{\dot{\gamma}}{S}$$

OR

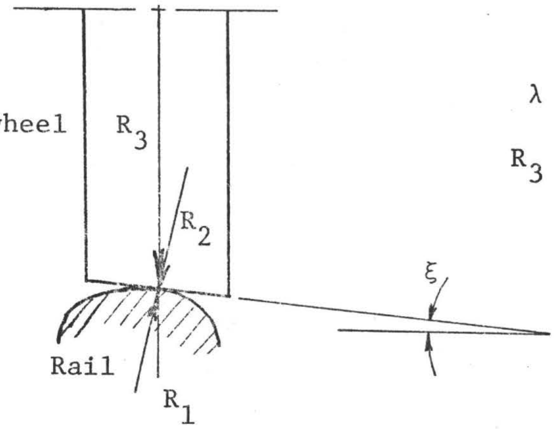
$$M_z = - f_{32} \frac{\Delta V}{S}$$

But, the lateral force is $= - f_2 \xi_y$

Therefore $\qquad f_{23} = f_2 (0.469)\ C$ lb.in. (IV.4.18)

From Equations (IV.4.7), (IV.4.8), (IV.4.16) and (IV.4.18) we note that to determine numerical values for the creep coefficients, the semiaxes of the elliptical area of contact must be determined. To do this, the method described in Timoshenko and Goodier [41] is used.

iii) Determination of the Semiaxes of the Elliptical Area of Contact



$\lambda = \tan^{-1} \xi$

$R_3 = \ell_{20}$ = radius of wheel

R_1 and R_2 are the principal radii of curvature at the point of contact for the rail and the wheel respectively. For the present case, the other two principal radii of curvature for the two surfaces are infinite. Therefore equation (d) on page 378 of Timoshenko and Goodier [41] gives us

$$A + B = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \text{(IV.4.19)}$$

where A and B are constants depending on the magnitude of the principal curvatures of the surfaces in contact and on the angle between the planes of principal curvatures of the two surfaces.

The relation between R_2, R_3 and λ (where R_3 is the radius of the wheel) is given in [40]:

$$\begin{aligned} R_2 &= R_3 \sqrt{1 + \lambda^2} \\ &= 20[1 + (\frac{1}{10})^2]^{1/2} \\ &\simeq 20 \end{aligned} \qquad \text{(IV.4.20)}$$

Therefore: $$A + B = \frac{1}{2}[\frac{1}{10} + \frac{1}{20}] \simeq \frac{1}{13}$$

Equation (d) on page 378 of reference [41] also gives us the following relation (for R_1' and $R_2' = 0$):

$$B - A = \frac{1}{2}[\frac{1}{R_1^2} + \frac{1}{R_2^2} + \frac{2}{R_1 R_2} \cos 2\psi]^{1/2} \qquad \text{(IV.4.21)}$$

where ψ is the angle between the planes containing the curvatures $\frac{1}{R_1}$ and $\frac{1}{R_2}$. For the present case $\psi = \frac{\pi}{2}$, hence

$$\begin{aligned} B - A &= \frac{1}{2}[\frac{1}{R_1^2} + \frac{1}{R_2^2} - \frac{2}{R_1 R_2}]^{1/2} \\ &= \frac{1}{2}\sqrt{(\frac{1}{R_1} - \frac{1}{R_2})^2} = \frac{1}{2}(\frac{1}{R_1} - \frac{1}{R_2}) \\ &= \frac{1}{2}(\frac{1}{10} - \frac{1}{20}) = \frac{1}{40} \end{aligned}$$

And using equation (h) on pages 379 of reference [41] we obtain:

$$\text{Cos } \theta = \frac{B - A}{B + A} \simeq \frac{13}{40} = .325 \qquad \text{(IV.4.22)}$$

and $\theta \simeq 72°$

From the table given on pages 379 reference [41] we have:

$$m = 1.202 + \frac{6}{10}(1.284 - 1.202)$$

$$= 1.25$$

And

$$n = 0.846 - \frac{6}{10}(0.846 - 0.802)$$

$$= 0.82$$

Semiaxes of the ellipse of contact are given by equations (224) on page 379 of reference [41]:

$$[\frac{a}{m}]^3 = [\frac{b}{n}]^3 = \frac{3(1-\nu)N}{2(\frac{1}{R_1} + \frac{1}{R_2})G} \qquad \text{(IV.4.23)}$$

For cast steel we have

$$\nu = 0.265$$

And $G = 11.3 \times 10^6$ psi.

Substituting for these values and values of m and n found above in Equation (IV.4.23) we find the semiaxes length of the ellipse in terms of the normal force between the wheel and the rail N.

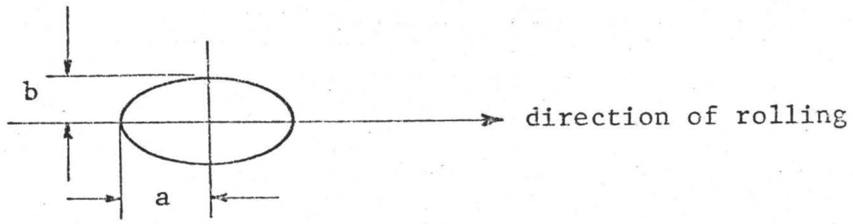
$$\left.\begin{aligned} a &\simeq 0.0095\ (N)^{1/3} \\ \text{And}\qquad b &\simeq 0.0075\ (N)^{1/3} \end{aligned}\right\} \qquad \text{(IV.4.24)}$$

Note that the vertical component of N is the weight of the vehicle per wheel. Since the conicity of wheels is small, it is possible to show that N is almost equal to the half weight per axle (see reference [41] page A.21). Thus in our case

$$N = \frac{\text{Total Weight}}{\text{No. of Wheels}} = \frac{348,100}{12} = 41295 \text{ lbs.}$$

iv) Numerical Values for the Creep Coefficient

The contact area is elliptical as we mentioned, and the ratio of two semiaxes is given from equation (IV.4.24),



$$\frac{b}{a} = \frac{0.0075\ N^{1/3}}{0.0096\ N^{1/3}} = 0.79$$

And from Figure 2 of reference [4] we get the values of ϕ and ψ_1 corresponding to the ratio $\frac{b}{a}$ getting

$$\phi = 0.59$$

$$\psi_1 = 0.78$$

Using equations (IV.4.7) and (IV.4.8), we can determine the values of longitudinal and lateral creep coefficients respectively:

$$f_1 = \frac{G\pi ab}{\phi} = \frac{11.3 \times 10^6 \times \pi \times 0.0095 \times 0.0075\ N^{2/3}}{0.59} \text{ lbs.}$$

$$\simeq 4243\ N^{2/3} \text{ lbs.} \qquad \text{(IV.4.25)}$$

and
$$f_2 = \frac{G\pi ab}{\psi_1} = \frac{11.3 \times 10^6 \times \pi \times 0.0095 \times 0.0075\ N^{2/3}}{0.78} \text{ lbs.}$$

$$\simeq 3175\ N^{2/3} \text{ lbs.} \qquad \text{(IV.4.26)}$$

and from equation (IV.4.16) we get

$$f_3 = 28.2 \times 10^6 \times (0.0075 \times 0.0095\ N^{2/3})^2 \text{ lb.in.}^2$$

$$\simeq 0.14\ N^{4/3} \text{ lb.in.}^2 \qquad \text{(IV.4.27)}$$

Finally we get f_{32} from Equation (IV.4.18)

$$f_{32} = 0.469\ C\ f_2$$

$$= 0.469 \times (0.0075 \times 0.0095\ N^{2/3})^{1/2}\ f_2$$

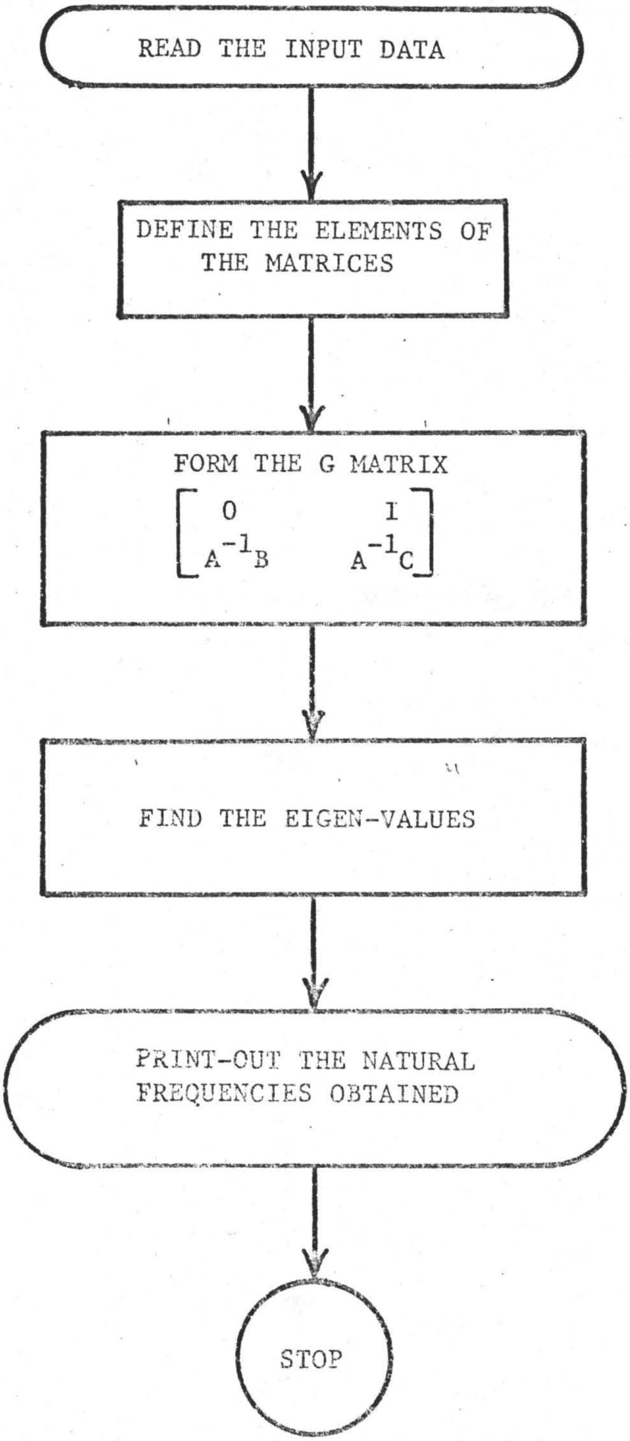
$$\simeq 1270\ N \text{ lb.in.} \qquad \text{(IV.4.28)}$$

APPENDIX V

FLOW CHARTS FOR THE COMPUTER PROGRAMS

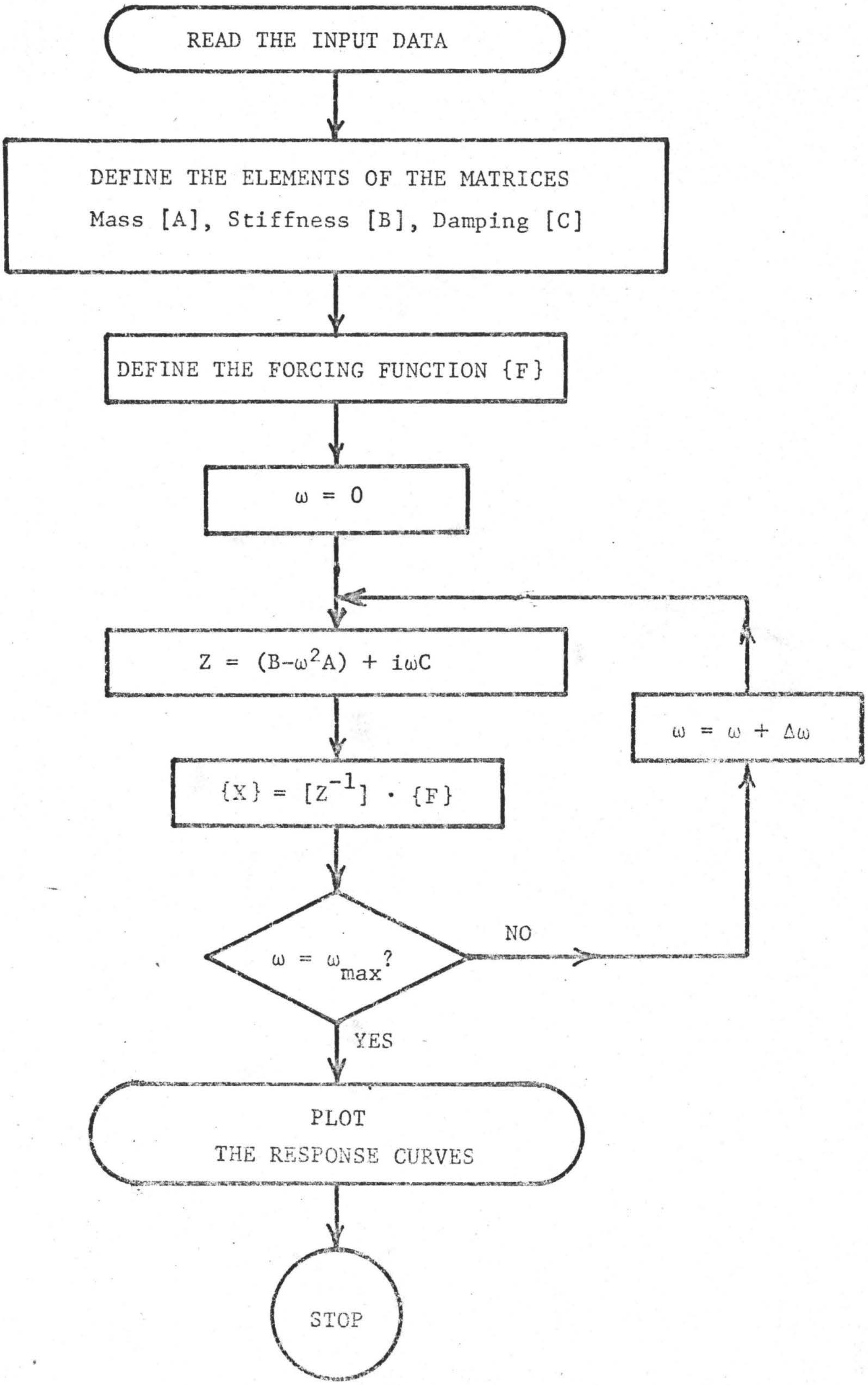
FLOW CHART 1:

NATURAL FREQUENCIES FOR THE SIMPLIFIED MODEL



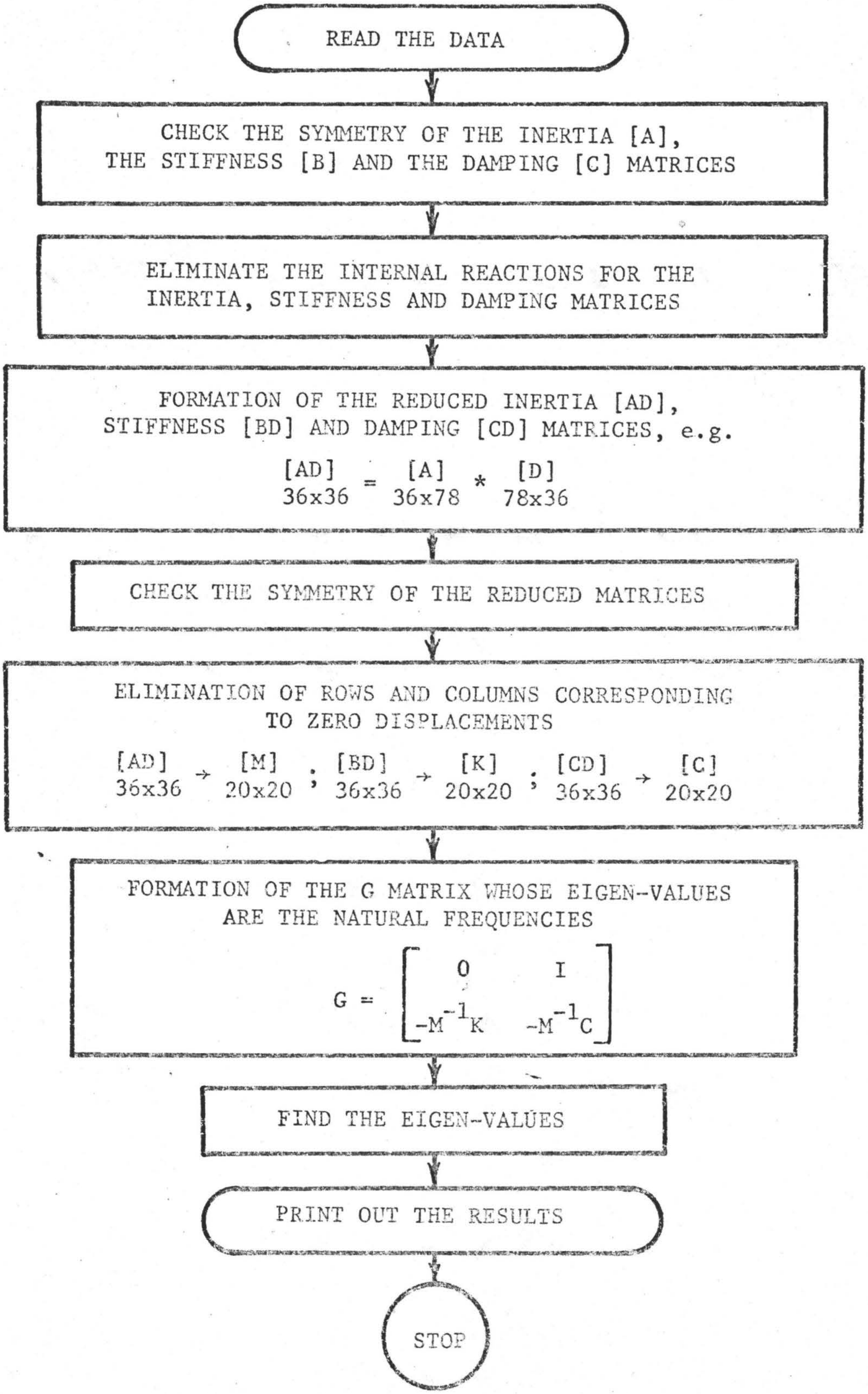
FLOW CHART 2:

STEADY STATE RESPONSE FOR THE SIMPLIFIED MODEL



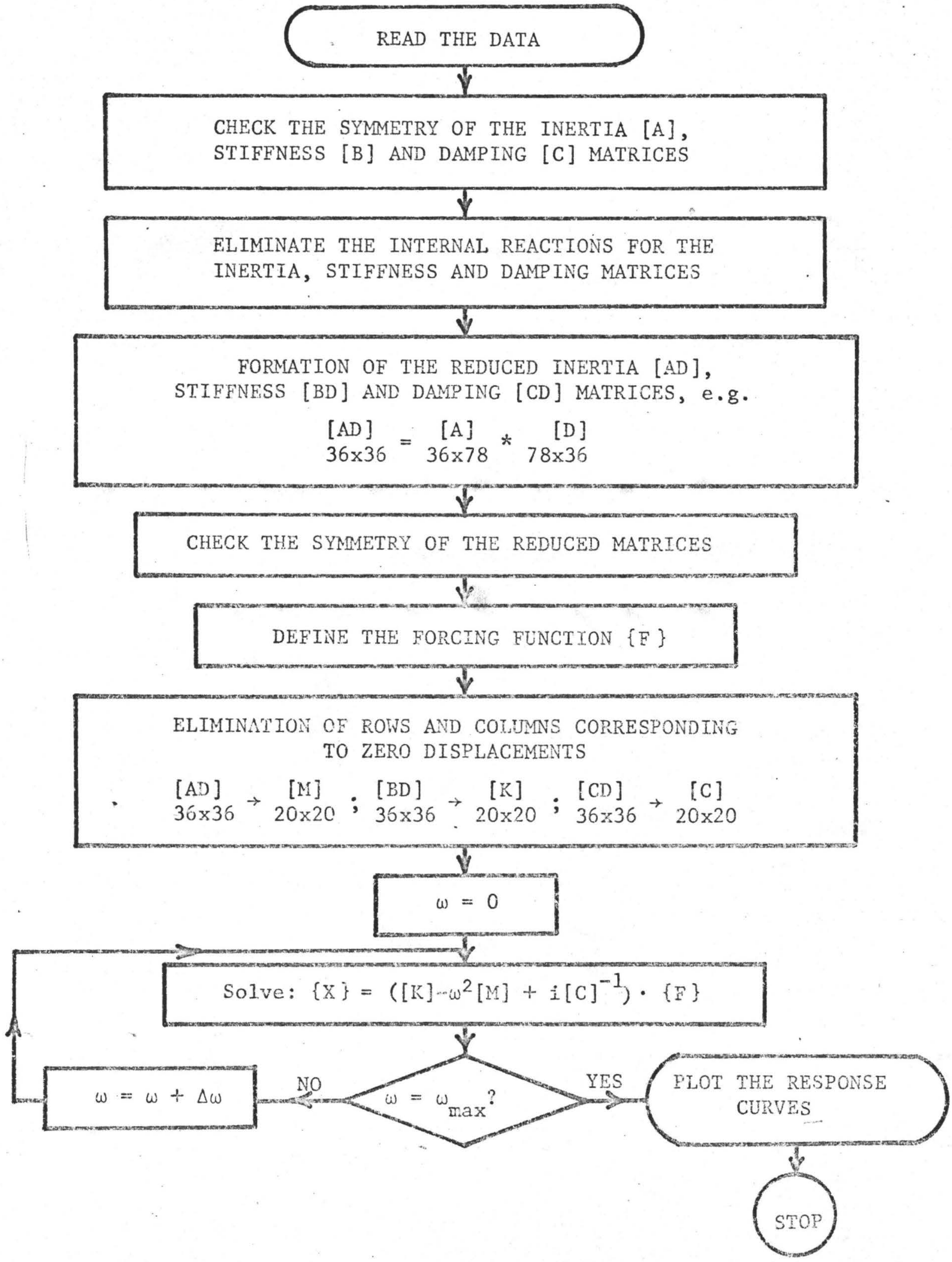
FLOW CHART 3:

NATURAL FREQUENCIES FOR THE FULL MODEL - "STATIONARY" VEHICLE



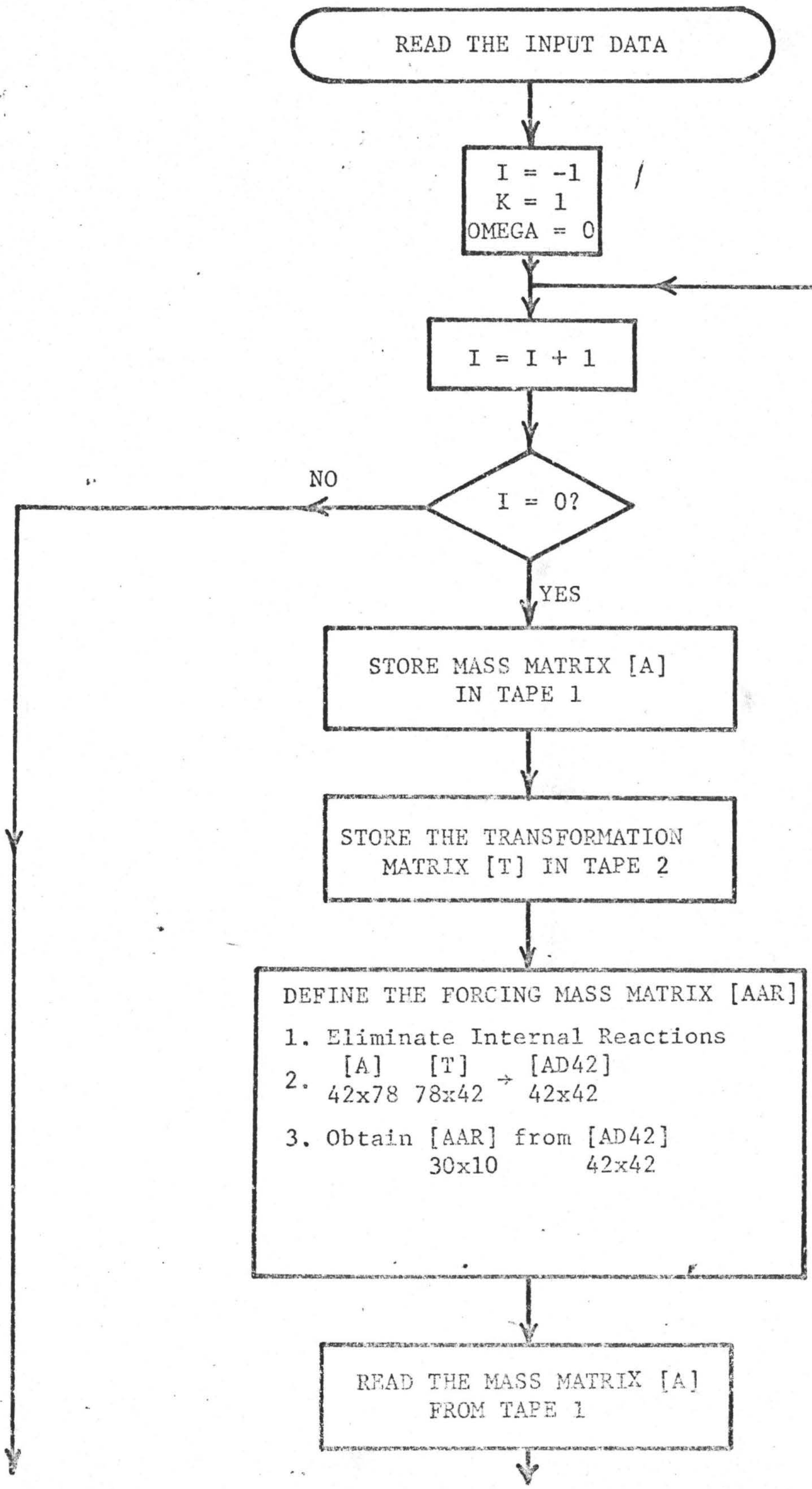
FLOW CHART 4:

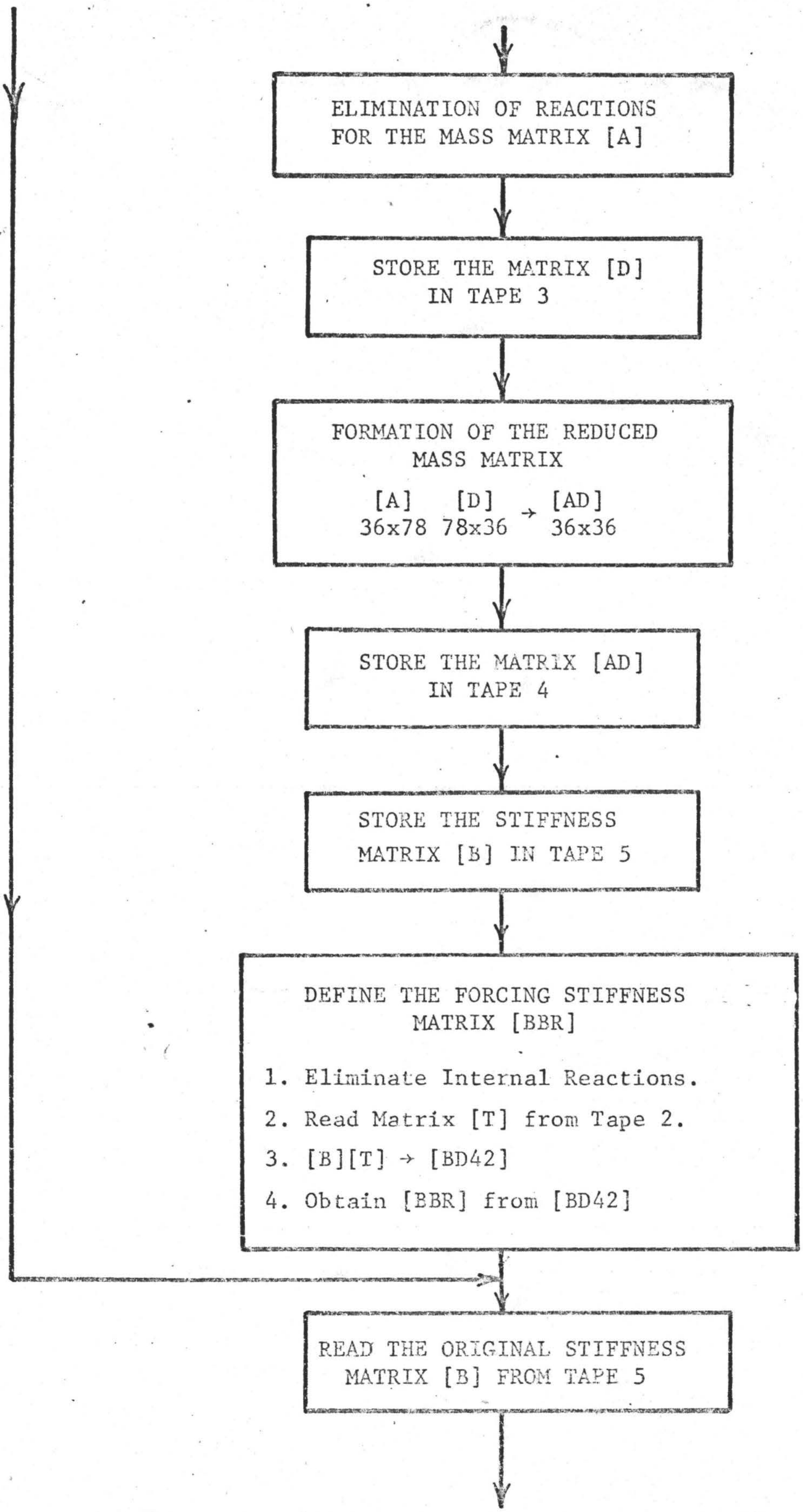
STEADY STATE RESPONSE FOR THE FULL MODEL - "STATIONARY" VEHICLE



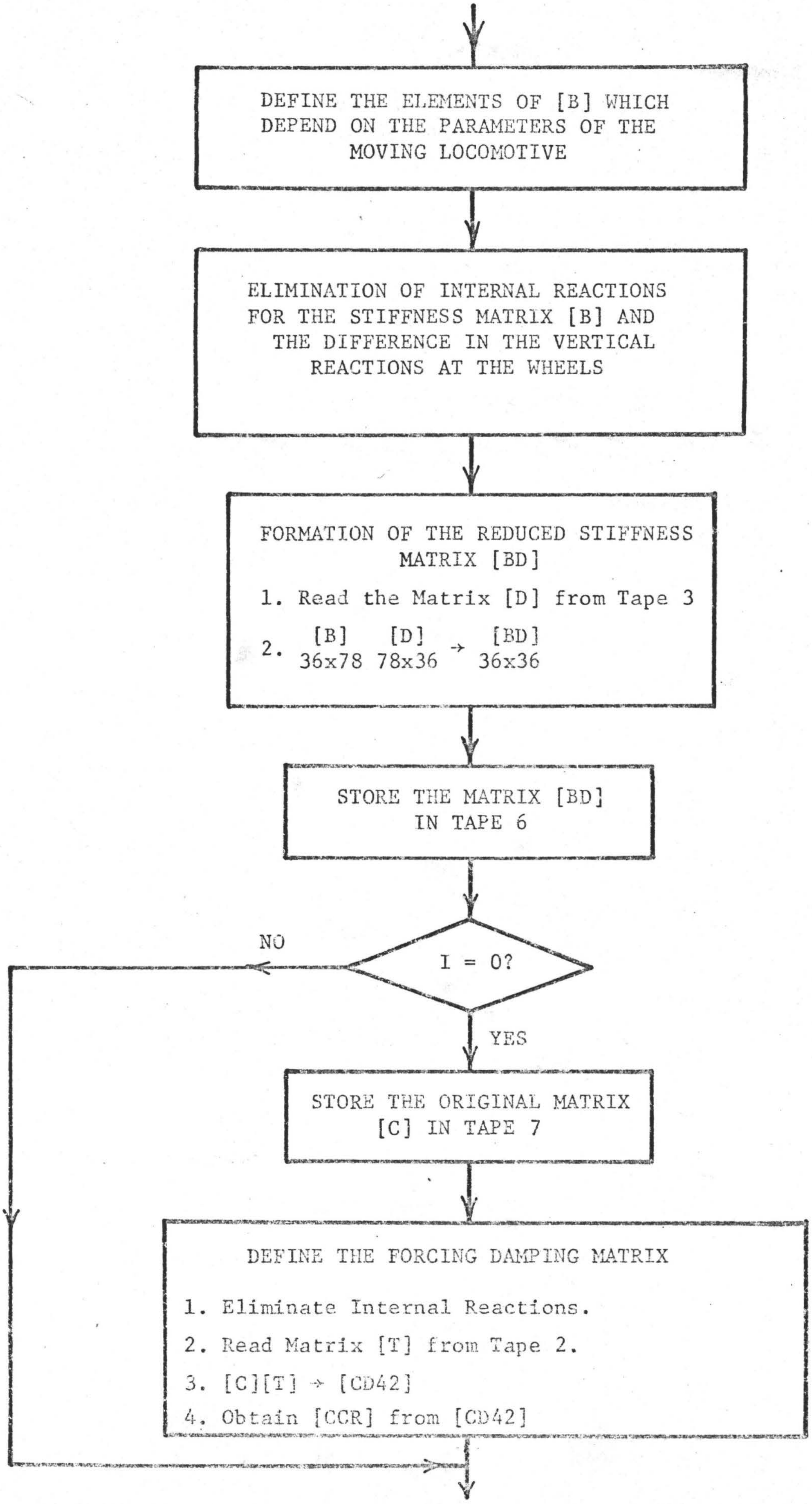
FLOW CHART 5:

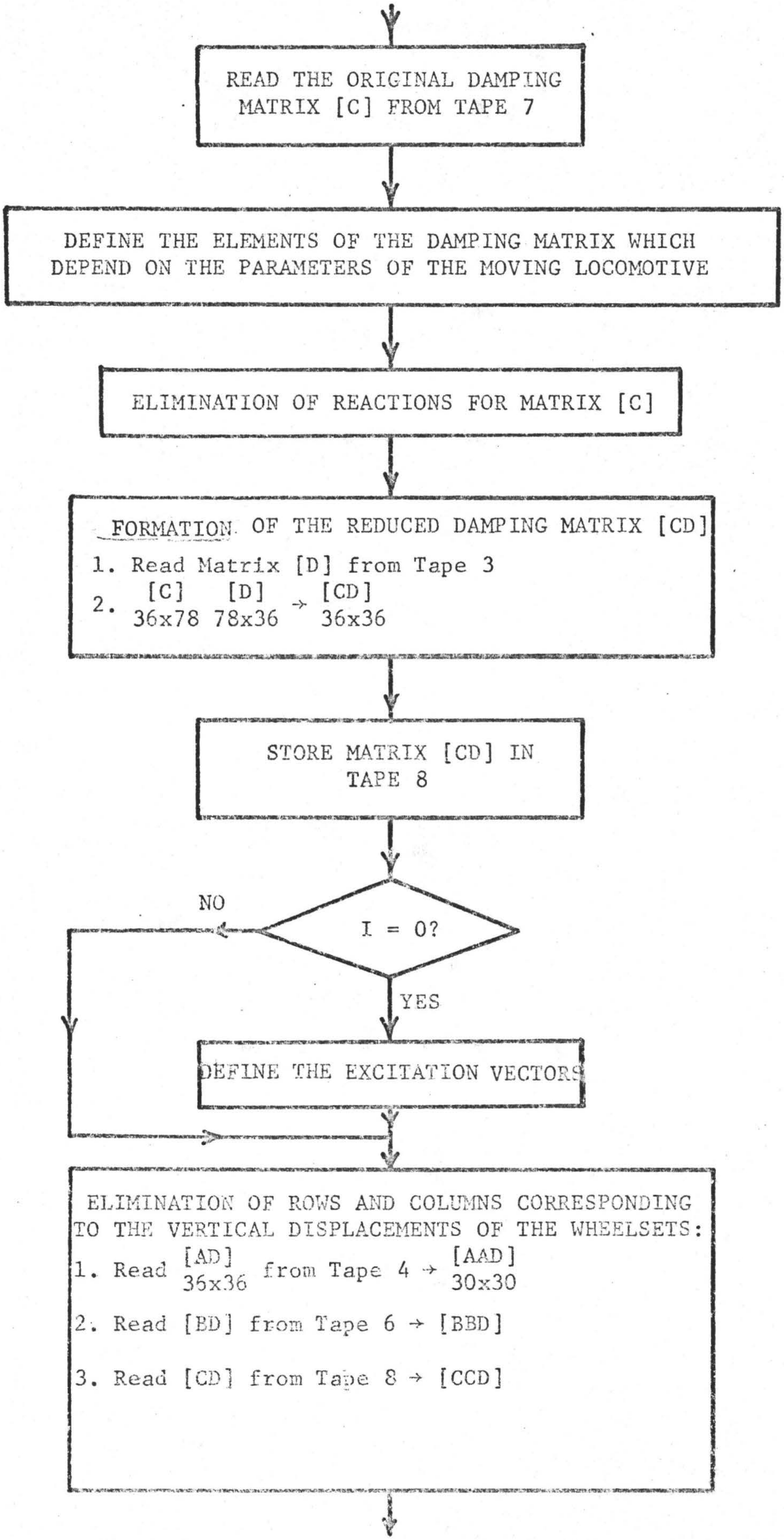
STEADY STATE RESPONSE FOR THE FULL MODEL - "MOVING" VEHICLE

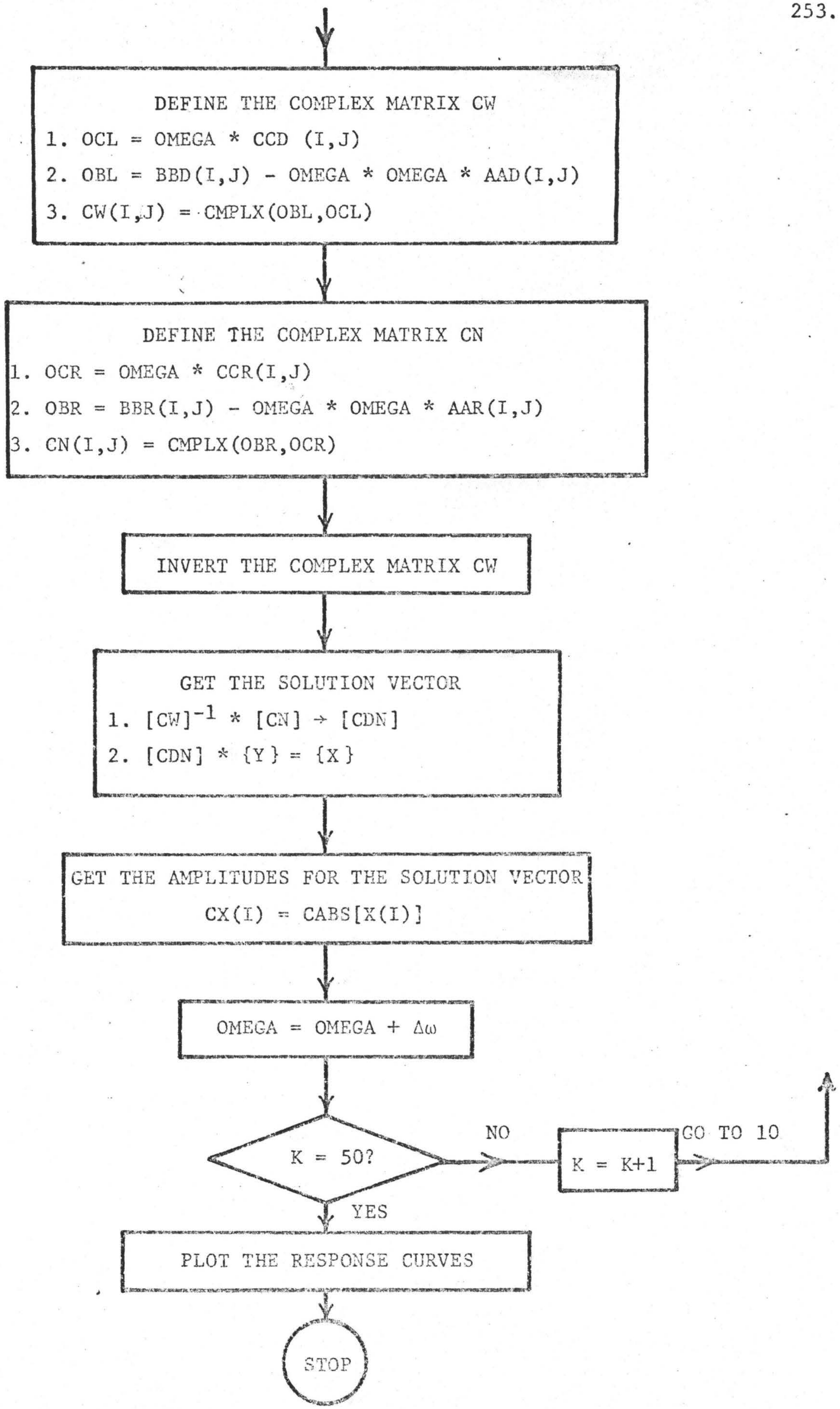




ELIMINATION OF REACTIONS FOR THE MASS MATRIX [A]
STORE THE MATRIX [D] IN TAPE 3
FORMATION OF THE REDUCED MASS MATRIX
[A] [D] → [AD]
36x78 78x36 36x36
STORE THE MATRIX [AD] IN TAPE 4
STORE THE STIFFNESS MATRIX [B] IN TAPE 5
DEFINE THE FORCING STIFFNESS MATRIX [BBR]
1. Eliminate Internal Reactions.
2. Read Matrix [T] from Tape 2.
3. [B][T] → [BD42]
4. Obtain [BBR] from [BD42]
READ THE ORIGINAL STIFFNESS MATRIX [B] FROM TAPE 5


DEFINE THE ELEMENTS OF [B] WHICH DEPEND ON THE PARAMETERS OF THE MOVING LOCOMOTIVE
ELIMINATION OF INTERNAL REACTIONS FOR THE STIFFNESS MATRIX [B] AND THE DIFFERENCE IN THE VERTICAL REACTIONS AT THE WHEELS
FORMATION OF THE REDUCED STIFFNESS MATRIX [BD]
1. Read the Matrix [D] from Tape 3
2. [B] 36x78 [D] 78x36 → [BD] 36x36
STORE THE MATRIX [BD] IN TAPE 6
NO
I = 0?
YES
STORE THE ORIGINAL MATRIX [C] IN TAPE 7
DEFINE THE FORCING DAMPING MATRIX
1. Eliminate Internal Reactions.
2. Read Matrix [T] from Tape 2.
3. [C][T] → [CD42]
4. Obtain [CCR] from [CD42]


READ THE ORIGINAL DAMPING MATRIX [C] FROM TAPE 7
DEFINE THE ELEMENTS OF THE DAMPING MATRIX WHICH DEPEND ON THE PARAMETERS OF THE MOVING LOCOMOTIVE
ELIMINATION OF REACTIONS FOR MATRIX [C]
FORMATION OF THE REDUCED DAMPING MATRIX [CD]
1. Read Matrix [D] from Tape 3
2. [C] 36x78 [D] 78x36 → [CD] 36x36
STORE MATRIX [CD] IN TAPE 8
NO
I = 0?
YES
DEFINE THE EXCITATION VECTORS
ELIMINATION OF ROWS AND COLUMNS CORRESPONDING TO THE VERTICAL DISPLACEMENTS OF THE WHEELSETS:
1. Read [AD] 36x36 from Tape 4 → [AAD] 30x30
2. Read [BD] from Tape 6 → [BBD]
3. Read [CD] from Tape 8 → [CCD]


DEFINE THE COMPLEX MATRIX CW
1. OCL = OMEGA * CCD (I,J)
2. OBL = BBD(I,J) - OMEGA * OMEGA * AAD(I,J)
3. CW(I,J) = CMPLX(OBL,OCL)
DEFINE THE COMPLEX MATRIX CN
1. OCR = OMEGA * CCR(I,J)
2. OBR = BBR(I,J) - OMEGA * OMEGA * AAR(I,J)
3. CN(I,J) = CMPLX(OBR,OCR)
INVERT THE COMPLEX MATRIX CW
GET THE SOLUTION VECTOR
1. [CW]$^{-1}$ * [CN] → [CDN]
2. [CDN] * {Y} = {X}
GET THE AMPLITUDES FOR THE SOLUTION VECTOR
CX(I) = CABS[X(I)]
OMEGA = OMEGA + Δω
K = 50?
NO
K = K+1
GO TO 10
YES
PLOT THE RESPONSE CURVES
STOP