NEUTRON STAR MATTER

by

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SCOPE AND CONTENTS:

An expression is obtained for the energy per particle in neutron star matter. The energy per particle is expressed as a function of, y, the ratio of protons to the total number of nucleons in the system. Minimizing the energy with respect to y gives the optimum proton ratio at a given density. Using an effective nuclear force, the results were extrapolated to a density of $\rho = 6\rho_{NM}$. The proton ratio is rather sensitive to the force used, but all forces used indicated a peak in the proton concentration at $\rho \approx 2\rho_{NM}$. The expression for the energy as a function of y was also used to interpolate the energy per particle between the nuclear matter and neutron gas limits. The form of this interpolation is important in determining the stability of neutron-rich nuclei.

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TABLE OF CONTENTS

CHAPTER 1 INTRODUCTION

In the last few years considerable work has been done in attempting to extend the results of nuclear matter calculations to the neutron gas limit. More recently this work has been stimulated by the discovery of pulsars late in 1967 and their subsequent interpretation by Gold (1968) as rotating neutron stars. The work presented here will deal with matter near the neutron gas limit. It was motivated by the fact that the Nemeth-Sprung paper (1968) appears to be wrong, and by a desire to extend the calculation to a higher density region. A recent paper by Baym, Bethe and Pethick (1971) indicates that neutron stars have a lattice structure up to the density of normal nuclear matter. Only then do the nuclei really dissolve and uniform neutron star matter come into existence. The calculation presented here is thus of interest mainly in the region greater than that of ordinary nuclear matter, although results will be given for lower densities where the calculation gives an upper bound on the actual energy.

An expression for the energy of the neutron star matter as a function of the fractional concentration of protons, y, will be obtained. The main result will be to obtain the fraction of protons which minimizes the energy of the system at a given density. The energy expression will also be used, along with results from elsewhere, for the nuclear matter region, to obtain an expression which interpolates the energy per particle between symmetric nuclear matter and a pure neutron gas.

Chapter 2 begins with a discussion of the model for neutron star matter. It discusses the terms which will be included in the energy expression and obtains an expression for the kinetic and rest mass energies of the neutrons, protons, electrons and muons. In Chapter 3 the method of obtaining the expression for the nuclear potential energy is discussed. Chapter 4 treats some of the details of the calculation and presents the results obtained from the energy minimization. Chapter 5 deals with the problem of interpolating between nuclear matter and neutron gas limits.

CHAPTER 2 KINETIC AND REST MASS ENERGIES

Neutron stars were first postulated by Landau in 1932, as products of a supernova explosion. The supernova remnant theory is presently backed up by the evidence that the pulsar with the shortest period, and therefore presumably one of the youngest, is in the Crab Nebula - which was formed by the supernova explosion of 1054 A.D. The theory proposes that a star of several solar masses and density $\sim 10^7$ to 10^8 g/cm^3 gradually loses energy and contracts, eventually becoming unstable against collapse. A supernova explosion follows and under certain conditions leaves behind a superdense core. This core is rich in neutrons because the electrons at this density are highly relativistic, and it is energetically favourable for inverse β decay to occur; so that an electron and a proton form a neutron (and a neutrino which escapes). Some of the electrons at the top of the Fermi sea may have sufficient energy to form muons. Since the star before collapse has net charge of zero, and these interactions conserve charge, the resulting neutron star is also electrically neutral. Since neutron stars are believed to be at a temperature $\sim 10^8$°K after the initial cooling and the energy per particle near nuclear matter densities is several MeV, corresponding to $\sim 10^{11}$°K, we can use the zero temperature limit in our calculations without introducing any error.

Our model for a neutron star is therefore quite a simple one. It consists of a degenerate Fermi gas of neutrons with a small fraction of protons and enough electrons and muons to keep the system electrically neutral. In the following discussion the term neutron star matter will refer to the mixture of neutrons, protons, electrons and muons. If the system is entirely neutrons, it will be called a neutron gas. When it consists of an equal number of protons and neutrons with no electrons, it will be referred to as nuclear matter. The Coulomb force is assumed not to be present when we discuss nuclear matter.

Having decided on a model to use, the next step is to develop an expression for the energy of the system. Included in the energy calculation are the rest mass energies and a two body interaction between the nucleons based on the Reid potential. Because the particles are assumed to form a uniform gas, the net charge in any volume element is zero; there is, therefore, no Coulomb contribution to the energy of the system.

Consider first the kinetic energy of the neutrons and protons. The kinetic energy of a particle in state i of a Fermi gas is

$$E_i = \frac{1}{2m} p_i^{\,2} \qquad (1)$$

and the total energy of n particles,

$$E = \sum_{i=1}^{n} E_i \tag{2}$$

For a degenerate gas (i.e., in the T = 0°K limit) all the available states below the Fermi momentum $p_f = \hbar k_f$ are occupied, and all the states above p_f, unoccupied. Transforming to the continuum:

$$E \rightarrow 2 \frac{\Omega}{h^3} \int E(p) d^3p \tag{3}$$

where the factor 2 arises from the two spin states of a nucleon, and $\frac{\Omega}{h^3}$ is a normalization factor. The total kinetic energy of the system is therefore

$$\begin{aligned} E &= 2 \frac{\Omega}{h^3} 4\pi \int_0^{p_f} \frac{p^2}{2m} p^2 \, dp \\ &= 2 \frac{\Omega}{h^3} 4\pi \frac{1}{2m} \frac{p_f^{\,5}}{5} \end{aligned} \tag{4}$$

The number of particles in the system is $A = \Sigma g_i$, where g_i is the occupancy of state i. Since $g_i = 2$ for fermions,

$$\begin{aligned} A &\rightarrow 2 \frac{\Omega}{h^3} \int d^3p \\ &= 2 \frac{\Omega}{h^3} 4\pi \frac{p_f^{\,3}}{3} \end{aligned} \tag{5}$$

The energy per particle is then

$$E/A = \frac{3}{5}\frac{p_f^{\,2}}{2m} = \frac{3}{5}\frac{\hbar^2}{2m}k_f^{\,2} \tag{6}$$

The number of particles per unit volume, the number density, is obtained from equation (5)

$$\rho = \frac{A}{\Omega} = \frac{1}{3\pi^2}k_f^{\,3} \tag{7}$$

For a system of neutrons and protons the total density is simply the sum of the neutron and proton densities

$$\rho = \rho_n + \rho_p \tag{8}$$

$$= \frac{1}{3\pi^2}(k_n^{\,3} + k_p^{\,3}) = \frac{2}{3\pi^2}k_F^{\,3} \tag{9}$$

where

$$2k_F^{\,3} = k_n^{\,3} + k_p^{\,3} \tag{10}$$

In symmetric nuclear matter $k_n = k_p = k_F$.

Consider a system of A nucleons composed of N neutrons and Z protons. If the fraction of protons is $y = \frac{Z}{A}$, then

$$\rho_p = y\,\rho \qquad \rho_n = (1-y)\rho \tag{11}$$

$$k_p^{\,3} = 2y\,k_F^{\,3} \qquad \text{and} \qquad k_n^{\,3} = 2(1-y)k_F^{\,3} \tag{12}$$

or $$k_p = (2y)^{1/3}\, k_F \qquad\qquad k_n = 2^{1/3}(1-y)^{1/3}\, k_F \tag{13}$$

Therefore, for the kinetic energy of the system

$$\begin{aligned} T_k &= N\left(\frac{3}{5}\frac{\hbar^2}{2m_n} k_n^{\,2}\right) + Z\left(\frac{3}{5}\frac{\hbar^2}{2m_p} k_p^{\,2}\right) \\ &= A(1-y)\left[\frac{3}{5}\frac{\hbar^2}{2m_n}\, 2^{2/3}(1-y)^{2/3}\, k_F^{\,2}\right] + Ay\left[\frac{3}{5}\frac{\hbar^2}{2m_p}\, y^{2/3}\, k_F^{\,2}\right] \\ &= A\,\frac{2^{2/3}\cdot 3\hbar^2\, k_F^{\,2}}{10}\left[\frac{1}{m_n}(1-y)^{5/3} + \frac{1}{m_p}\, y^{5/3}\right] \end{aligned} \tag{14}$$

The rest mass energy of the system is

$$\begin{aligned} E_M &= N\, m_n c^2 + Z\, m_p c^2 \\ &= A\, c^2[m_n + y(m_p - m_n)] \end{aligned} \tag{15}$$

The electrons and muons, because of their small mass, must be treated relativistically so instead of equation (1) we have to use

$$E_i = \sqrt{p_i^{\,2}c^2 + m^2c^4} \tag{16}$$

where m is the rest mass.

Then, the total energy is

$$E = \Sigma E_i \rightarrow 2\left(\frac{\Omega}{h^3}\right) 4\pi \int_0^{p_f} \sqrt{p^2c^2 + m^2c^4}\; p^2 dp \tag{17}$$

The total number of particles is the same as in the non-degenerate case, which is given by equation (5). The energy per particle is therefore

$$T/A = \frac{3}{p_f^{\,3}} \int_0^{p_f} p^2 \sqrt{p^2c^2 + m^2c^4}\, dp \tag{18}$$

Letting $x = \frac{p}{mc} = \frac{\hbar k}{mc}$, equation (18) becomes

$$\begin{aligned} T/A &= \frac{3}{(mcx_f)^3} (mc)^3 mc^2 \int_0^{x_f} x^2 \sqrt{x^2 + 1}\, dx \\ &= \frac{3mc^2}{8x_f^{\,3}} [x_f(x_f^{\,2} + 1)^{1/2}(2x_f^{\,2} + 1) - \sinh^{-1} x_f] \\ &= \frac{3m^4c^5}{8\hbar^3 k_f^{\,3}} [x_f(x_f^{\,2} + 1)^{1/2}(2x_f^{\,2} + 1) - \sinh^{-1} x_f] \end{aligned} \tag{19}$$

If there are y_1A electrons and y_2A muons then their energy contribution is

$$\begin{aligned} T_e + T_\mu = A\frac{3}{8} \frac{c^5}{\hbar^3} [&\frac{m_e^{\,4} y_1}{k_e^{\,3}} \{x_1(x_1^{\,2}+1)^{1/2}(2x_1+1) - \sinh^{-1} x_1\} \\ &+ \frac{m_\mu^{\,4} y_2}{k_\mu^{\,3}} \{x_2(x_2^{\,2}+1)^{1/2}(2x_2+1) - \sinh^{-1} x_2\}] \end{aligned} \tag{20}$$

where x_1 refers to the electrons and x_2 to the muons. We know that

$$x_1 = \frac{\hbar k_e}{m_e c} \qquad \text{and} \qquad \rho_e = \frac{1}{3\pi^2} k_e^{\,3} \tag{21}$$

Also $$\rho_e = y_1\rho = \frac{2y_1}{3\pi^2} k_F^3 \tag{22}$$

$$\therefore \qquad k_e = (3\pi^2\rho_e)^{1/3} = (2y_1)^{1/3} k_F \tag{23}$$

Similar equations apply for the muons. Then equation (20) becomes

$$T_e + T_\mu = A \frac{3}{16} \frac{c^5}{\hbar^3 k_F^3} [m_e^4\{x_1(x_1^2+1)^{1/2}(2x_1+1)-\sinh^{-1}x_1\}$$
$$m_\mu^4\{x_2(x_2^2+1)^{1/2}(2x_2+1)-\sinh^{-1}x_1\}] \tag{24}$$

where $x_1 = 2^{1/3} \frac{\hbar k_F}{m_e c} y_1^{1/3}$, and $x_2 = 2^{1/3} \frac{\hbar k_F}{m_\mu c} y_2^{1/3}$ (25)

There are two constraints on y_1 and y_2 that must be applied in this problem:

(i) to preserve charge neutrality $y_1 + y_2 = y$, where y is the proton density; and

(ii) no muons will be created unless the relativistic mass of the last electron at the electron Fermi surface is greater than the rest mass of the muon, i.e.,

$$c^2 p_e^2 + m_e^2 c^4 \geq m_\mu^2 c^4 \tag{26}$$

Since $p_e = m_e c x_1$ this condition requires x_2 (and y_2) to be zero unless

$$x_1 > \sqrt{\left(\frac{m_\mu}{m_e}\right)^2 - 1} \approx 207 \tag{27}$$

The next chapter will deal with the interaction potential of the nucleons. This will be written as an expansion in y since we are interested in small proton concentrations. The energy per particle will be written in the form

$$U = A(c_0 + c_1 y + c_2 y^2) \tag{28}$$

where the coefficients c_i will be determined in Chapter 3. The total energy of neutron star matter is, therefore,

$$E = U + E_{Mass} + T_{Kin} + T_e + T_\mu \tag{29}$$

and the energy per particle obtained from equations (14), (15), (24), (28) and (29):

$$\begin{aligned} E/A = {} & c_0 + c_1 y + c_2 y^2 \\ & + m_n c^2 + (m_p - m_n) y c^2 \\ & + \frac{3}{10}\, 2^{2/3}\, \hbar^2 k_F{}^2 \left[\frac{1}{m_n}(1-y)^{5/3} + \frac{1}{m_p} y^{5/3}\right] \\ & + \frac{3}{16}\, \frac{c^5}{\hbar^3 k_F{}^3} \left[m_e{}^4 \{x_1 (x_1{}^2+1)^{1/2} (2x_1{}^2+1) - \sinh^{-1} x_1\}\right. \\ & \left. + m_\mu{}^4 \{x_2 (x_2{}^2+1)^{1/2} (2x_2{}^2+1) - \sinh^{-1} x_2\}\right] \end{aligned} \tag{30}$$

CHAPTER 3 THE NUCLEAR POTENTIAL ENERGY

In this chapter the method of determining the potential energy due to the internucleon forces will be discussed. The energy was determined using the reaction matrix G of the Brueckner-Bethe-Goldstone theory of nuclear matter, taken from a previous calculation. Before describing the procedure that was followed, a few general comments on nuclear matter theory will be presented.

The Brueckner theory is the generally accepted method of dealing with nuclear matter. It provides a way of calculating the way in which the interaction between two nucleons is modified by the presence of the other nucleons. In an infinite Fermi gas for which a static two-body potential v is assumed, the interaction can be described in terms of the reaction matrix $G(\underset{\sim}{k}_1,\underset{\sim}{k}_2,\underset{\sim}{k}_1',\underset{\sim}{k}_2')$, where $\underset{\sim}{k}_1$ and $\underset{\sim}{k}_2$ are the initial Fermi momenta of the particles, and k_1' and k_2' the final momenta. (Actually k is the wave number and $\hbar\underset{\sim}{k}$ the momentum, but $\underset{\sim}{k}$ is commonly referred to as the Fermi momentum.) The G matrix is a generalization of the T matrix of scattering theory and satisfies a similar equation, the Brueckner-Goldstone equation

$$G = v - v\,\frac{Q}{e}\,G \qquad (1)$$

where v is a realistic two-body interaction, Q is the Pauli operator preventing scattering into already occupied states, and e is the energy difference between the intermediate and initial states. The G matrix is essentially an effective interaction in which all two-body clusters are treated exactly and higher order clusters allowed for in an average way.

Instead of writing the reaction matrix in terms of the individual momenta it can be described in terms of the center of mass and relative momenta $\underset{\sim}{P}$, $\underset{\sim}{k}$, $\underset{\sim}{P}'$ and $\underset{\sim}{k}'$. A number of simplifications follow immediately. From the conservation of momentum, $\underset{\sim}{P} = \underset{\sim}{P}' = \hbar\underset{\sim}{K}$. In determining the energy only the diagonal elements are needed, so we restrict our attention to $\underset{\sim}{k} = \underset{\sim}{k}'$. Thus the reaction matrix elements which we require can be written as $G(\underset{\sim}{k},\underset{\sim}{K})$. In practice $G(\underset{\sim}{k},\underset{\sim}{K})$ is evaluated at an average K for each k, so that G is a function of k only. It will, however, be a function of the density and depend on both k_n and k_p, the neutron and proton Fermi momenta, when $N \neq Z$. So G is finally written as $G(k_n,k_p;k)$.

The G matrix elements used in the calculations were obtained for nuclear matter and neutron gas from the work of Banerjee at McMaster University and by Sprung at Orsay; both used the soft core Reid potential. An effective force of Sprung and Banerjee (1971) was used in order to extrapolate matrix elements into the high density region where actual nuclear matter matrix elements were unavailable.

Several approximations were used in evaluating $G(k_n,k_p;k)$. We can think of $G(k_n,k_p;k)$ as being comprised of three parts: $G_{nn}(k_n,k_p;k)$, $G_{pp}(k_n,k_p;k)$ and $G_{np}(k_n,k_p;k)$ which describe the neutron-neutron, proton-proton, and neutron-proton interactions respectively. The Brueckner-Dabrowski approximation (1964) allows us to write the unsymmetric nuclear matter G in terms of symmetric nuclear matter G for which we have G matrix elements available. For a small difference in the number of protons and neutrons the Brueckner-Dabrowski approximation states that

$$G_{nn}(k_n,k_p;k) = G_{nn}(k_n,k_n;k) \qquad 2(a)$$

$$G_{pp}(k_n,k_p;k) = G_{pp}(k_p,k_p;k) \qquad 2(b)$$

$$G_{np}(k_n,k_p;k) = G_{np}(k_A,k_A;k) \qquad 2(c)$$

where

$$k_A{}^2 = \tfrac{1}{2}(k_n{}^2 + k_p{}^2) \qquad 2(d)$$

The rationale of this hypothesis is that for nn (or pp) interactions, the Pauli principle will be the dominant effect and the value of k_p (or k_n) will be unimportant. For n-p interactions this cannot be so, but some average value k_A for the Fermi momentum may describe suitably the interaction.

We are using this approximation in the limit of large neutron excess. In the actual calculation really only 2(c) was used, as the G_{nn} was replaced by matrix elements obtained from a pure neutron gas calculation, and G_{pp} was considered small enough to be neglected especially in view of the small number of proton-proton pairs. Discussions with Dr. Sprung revealed that equation 2(d) might be replaced by $k_A{}^3 = \frac{1}{2}(k_n{}^3 + k_p{}^3)$ as an alternative formulation of the Brueckner-Dabrowski hypothesis. This was also considered and will be further discussed in Chapter 4; however, 2(d) was used in most of the calculations and will be used in further derivations in this chapter.

A second approximation used is to make G linear in k as done by Nemeth et al. (1968). This approximation is good for $k_F < 2.0\ \text{fm}^{-1}$ and leads to a considerable reduction in the complexity of determining the nuclear potential energy. A sample of the curves and their linear fits is given in Figure 1.

In the reaction matrix theory the potential energy of a system of nucleons, neglecting higher order cluster energies, is

$$U = \sum_{\text{all pairs}} \sum \langle mn|G|mn\rangle \tag{3}$$

Transforming to the continuum

$$U \to \left[\frac{\Omega}{(2\pi)^3}\right]^2 g^2 \int d^3\underset{\sim}{m} \int d^3\underset{\sim}{n}\ \frac{1}{\Omega}\, G(k_n, k_p; k) \tag{4}$$

For the neutron-proton interaction the limits on the integrals over m and n are different, so that transforming to center of mass and relative coordinates is awkward, however, the Brueckner-Dabrowski approximation introduced in equation (2) allows us to set the limits on the integrals equal. Then with $k = \frac{m-n}{2}$, the integral can be shown to reduce to

$$U = g^2 \frac{\Omega}{(2\pi)^6} \frac{2^6\pi^2 k_f^{\ 6}}{3} \int_0^1 k^2(1-k)^2(2+k)G(k_f,k_f;k)dk \tag{5}$$

In nuclear matter $k_F^{\ 3} = 1.5\ \pi^2\rho$, and A, the total number of particles is

$$A = g \frac{\Omega}{h^3} \frac{4\pi}{3} \hbar^3 k_F^{\ 3} = \frac{g\Omega}{6\pi^2} k_F^{\ 3} \tag{6}$$

The energy per particle is

$$U/A = \frac{g6\pi^2}{2^6\pi^6} \frac{2^6\pi^2 k_F^{\ 3}}{3} \int_0^1 k^2(1-k)^2(2+k)G(k_F,k_F;k)dk$$

$$= \frac{2g}{\pi^2} \frac{3}{2} \pi^2\rho \int_0^1 k^2(1-k)^2(2+k)G(k_F,k_F;k)dk$$

which gives, (with g = 4 since there are two spin and two isospin states),

$$U/A = 12\rho \int_0^1 k^2(1-k)^2(2+k)G(k_F,k_F;k) \tag{7}$$

When $G(k_F,k_F;k)$ has the simple functional form of $a + bk$ the integral is easily evaluated to give

$$U/A = \rho(a + \frac{18}{35}b) \tag{8}$$

For a neutron gas $\rho_n = \frac{1}{3\pi^2} k_n^{\ 3}$ and $g = 2$ so that equation (5) leads to equation (8) with A replaced by N, and ρ by ρ_n. We can also define an average G matrix element $\overline{G}(k_F) = a + \frac{18}{35}b$ so that equation (8) becomes

$$U/A = \rho\overline{G}(k_F) \tag{9}$$

For nuclear matter the G matrix elements are calculated separately for T = 0 and T = 1 and defined in such a way that

$$U/A = \rho(\overline{G}_0 + \overline{G}_1) \tag{10}$$

which in relation to equation (8) means that $\overline{G}_0 = a_0 + \frac{18}{35}b_0$, $\overline{G}_1 = a_1 + \frac{18}{35}b_1$ with $a = a_0 + a_1$ and $b = b_0 + b_1$. If we consider only the n-p interactions, this means that the $T_3 = 1$ and $T_3 = -1$ parts of the T = 1 interaction are turned off, leaving only the $T_3 = 0$ contribution. Assuming that the three components contribute equally, the contribution to the energy from neutron-proton pairs is given by

$$U_{np} = \rho A(\overline{G}_0 + \frac{1}{3}\,\overline{G}_1) \tag{11}$$

There are $NZ = \frac{A^2}{4}$ n-p pairs so that the energy per pair is

$$U_{np} \quad \frac{4}{A^2} = \frac{4\rho}{A}(\overline{G}_0 + \tfrac{1}{3}\overline{G}_1) = \frac{4}{\Omega}(\overline{G}_0 + \tfrac{1}{3}\overline{G}_1) = \frac{4}{\Omega}\,\overline{G}_{np} \qquad (12)$$

The n-n interaction will be the same as in a neutron gas of density ρ_n, so the energy contribution is

$$U_{nn} = \rho_n\, N\, \overline{G}_{nn} \qquad (13)$$

where $\overline{G}_{nn}$ is evaluated at k_n. There are $\frac{N^2}{2}$ pairs so the energy per pair is given by

$$U_{nn} \quad \frac{2}{N^2} = \frac{2\rho_n}{N}\overline{G}_{nn} = \frac{2}{\Omega}\overline{G}_{nn} \qquad (14)$$

The proton-proton energy contribution is given by an equation identical to equation (14) with nn replaced by pp.

If there are $N = A(1-y)$ neutrons and $Z = Ay$ protons, the energy will be

$$U = \frac{A^2(1-y)^2}{2}\left(\frac{2}{\Omega}\overline{G}_{nn}\right) + A^2y(1-y)\left(\frac{4}{\Omega}\overline{G}_{np}\right) + \frac{A^2y}{2}\left(\frac{2}{\Omega}\overline{G}_{pp}\right)$$

which gives, since $\rho = A/\Omega$

$$U = \rho A[(1-y)^2\overline{G}_{nn} + 4y(1-y)\overline{G}_{np} + y^2\overline{G}_{pp}] \qquad (15)$$

where the various G's are to be evaluated at k_n, k_A and k_p respectively. In the neutron star limit, because of the small number of proton-proton pairs, the last term in equation (15) is negligible and is omitted.

For the purpose of minimizing the energy we want an expression which is a simple function of y. However, k_n, k_A and k_p depend on y so we make some further simplifications. Neutron star matter is composed of almost entirely neutrons so that k_n has almost the same value as for a pure neutron gas, in which the fermi momentum is $q_F = 2^{1/3}k_F$. Expanding as a Taylor series about q_F

$$G_{nn}(k_n,0;k) = G_{nn}(q_F,0;k) + (k_n-q_F)\frac{\partial G_{nn}(q_F,0;k)}{\partial q_F} + \frac{1}{2!}(k_n-q_F)^2 \frac{\partial^2 G_{nn}(q_F,0;k)}{\partial q_F^{\,2}} \tag{16}$$

but

$$k_n - q_F = 2^{1/3}(1-y)^{1/3}k_F - q_F$$

$$= q_F(1-\tfrac{1}{3}y-\tfrac{1}{9}y^2+\ldots-1)$$

$$\simeq -(\tfrac{1}{3}y+\tfrac{1}{9}y^2)q_F$$

and

$$(k_n - q_F)^2 \simeq \frac{1}{9}\, y^2\, q_F^{\,2}$$

Equation (16) becomes

$$G_{nn}(k_n,0;k) = G_{nn}(q_F,0;k) - (\tfrac{1}{3}y + \tfrac{1}{9}y^2)q_F \frac{\partial G_{nn}}{\partial q_F} + \frac{y^2}{18}q_F{}^2 \frac{\partial^2 G_{nn}}{\partial q_F{}^2} \tag{17}$$

We could write $k_A{}^2 = \frac{1}{2}(k_n{}^2 + k_p{}^2)$ in terms of y as

$$k_A{}^2 = 2^{-1/3}k_F{}^2[(1-y)^{2/3} + y^{2/3}]$$

but this would involve non-integral powers of y. Instead we introduce the parameter $\alpha = \frac{N-Z}{A} = 1-2y$, so that we can write

$$k_A{}^2 = \tfrac{1}{2}k_F{}^2[(1+\alpha)^{2/3} + (1-\alpha)^{2/3}]$$

$$\simeq k_F{}^2(1 - \tfrac{1}{9}\alpha^2)$$

We then re-define k_A by

$$k_A = k_F(1 - \frac{1}{18}\alpha^2)$$

$$= k_F[1 - \tfrac{1}{18}(1-2y)^2]$$

$$= k_F[\tfrac{17}{18} + \tfrac{4}{18}y - \tfrac{4}{18}y^2]$$

$$= k_{A_0}(1 + \tfrac{4}{17}y - \tfrac{4}{17}y^2) \tag{18}$$

where we have introduced a new quantity $k_{A_0} = \frac{17}{18}k_F$. The expression for G_{np} becomes

$$G_{np}(k_A,k_A;k) = G_{np}(k_{A_0},k_{A_0};k) + (k_A-k_{A_0})\frac{\partial G_{np}(k_{A_0},k_{A_0};k)}{\partial k_{A_0}}$$

$$+ \frac{1}{2!}(k_A-k_{A_0})^2 \frac{\partial^2 G_{np}(k_{A_0},k_{A_0};k)}{\partial k_{A_0}^2}$$

$$= G_{np}(k_{A_0},k_{A_0};k) - \frac{4}{17}\, y(1-y)k_{A_0} \frac{\partial G_{np}(k_{A_0},k_{A_0};k)}{\partial k_{A_0}}$$

$$+ \frac{8}{289}y^2\, k_{A_0}^{\ 2}\frac{\partial^2 G_{np}(k_{A_0},k_{A_0};k)}{\partial k_{A_0}^2} \tag{19}$$

To obtain the average G's needed for equation (15) the derivation of equation (8) still applies. All that is needed, therefore, is to replace the G's in equations (16) and (20) by their average values evaluated at q_F and k_{A_0} respectively.

CHAPTER 4 CALCULATIONS AND RESULTS

The input data for this calculation were G matrix elements, given for seven values of k between 0 and 1 (in units of k_F), taken at a number of values of k_F. A linear fit of the seven points was made, weighting the central points more strongly since $\bar{G} = a + \frac{18}{35} b \simeq G(0.5)$. This fit defined an intercept, a, and a slope, b, for each k_F. The quality of the fits can be seen from Fig. 1 which shows several typical examples of G as a function of k and the corresponding linear fit. For the G matrix elements obtained from the nuclear matter programs of Sprung and Banerjee the data could be fitted with the functions

$$a(k_F) = a_1/k_F + a_2 + a_3 k_F + a_4 k_F^2 \qquad 1(a)$$

$$b(k_F) = b_1 + b_2 \sqrt{k_F} + b_3 k_F \qquad 1(b)$$

as suggested by Nemeth and Sprung (1968). The values of the coefficients are listed in Table 1 for the neutron gas and nuclear matter T=0 and T=1 parts. The effective force of Sprung and Banerjee (1971) was used to obtain G matrix elements over a larger range of k_F. The functions of equation (1) above failed to give satisfactory fits so they were replaced by the polynomials

$$a(k_F) = a_1 + a_2 k_F + a_3 k_F^2 + a_4 k_F^3 \quad 2(a)$$

$$b(k_F) = b_1 k_F + b_2 k_F^2 + b_3 k_F^3 + b_4 k_F^4 \quad 2(b)$$

Note that b will be slightly in error near $k_F=0$ but we are mainly interested in what happens for large k_F. Figures 2(a) and 2(b) show the parameters a and b as functions of k_F for the two forces used. A description of these forces is given in Sprung and Banerjee (1971). For our purposes it is sufficient to say that force G-0 has a density dependence proportional to $\sqrt{k_F}$; force G-1 has a dependence proportional to k_F. Throughout most of the calculation only $\overline{G} = a + \frac{18}{35} b$ was needed, instead of a and b separately. Moreover $\overline{G}$ is a smoother function than either a or b and can, therefore, be fitted more accurately. Figure 2(c) shows $\overline{G}$ as a function of k_F. The reasons for considering a and b separately were, partly inertia because Mrs. Nemeth had done so, and partly that it is useful to have these available when we come to renormalize the force to give the proper nuclear matter binding energy and saturation density. This will be discussed below.

Since the effective force was available for nuclear matter only, a problem arose in deciding what to use for the neutron gas matrix elements in the higher density region. They were obtained from the nuclear matter values by the following reasoning. Consider nuclear matter at density ρ

and corresponding Fermi momentum k_F. We can obtain a neutron gas at the same density by doubling the density of the original system, thus increasing k_F by a factor of $2^{1/3}$. We also must turn off the T=0 and T=1, T_3=0 parts of the nuclear interaction (between unlike particles). This means that the T=1 part of the interaction now has a statistical weight of 2 instead of the usual 3. We have thus created a neutron and a proton gas which do not interact with each other and both of which are at the original density ρ. The energy of the system is, therefore, twice the neutron gas energy since the charge symmetry of the nuclear force makes the two gases equivalent. The energy per particle is the same as for the neutron gas alone, since there are twice as many particles. What this means is that

$$U_{NG}(k_F) = \frac{2}{3} U_{NM}^{T=1}(2^{1/3}k_F) \tag{3}$$

or

$$\rho G_{NG}(k_F,0;k) = \frac{2}{3}(2\rho)\ G_{NM}^{T=1}(2^{1/3}k_F,2^{1/3}k_F;k)$$

$$\therefore \quad G_{NG}(k_F,0;k) = \frac{4}{3} G_{NM}^{T=1}(2^{1/3}k_F,2^{1/3}k_F;k) \tag{4}$$

The neutron gas matrix elements calculated from this expression agree quite well with a correct calculation using the actual nuclear programs in the lower density region where both sets were available. The agreement is close enough to assume that the disagreement is due to the approximate nature of the G-matrix calculation. We, therefore, used the relation equation (4) to extrapolate the neutron gas matrix elements to higher density.

With the G matrix elements known and fitted as functions of k_F the values of $\overline{G}_{nn}$ and $\overline{G}_{np}$ and any required derivatives can be calculated. The coefficients of y in equation (2-13) and (2-14) are now known so the coefficients c_1, c_2, and c_3 of equation (3-16) are determined. Hence, for any values of k_F and y the average energy per particle can be calculated from equation (2-30).

The next step is to minimize equation (2-30) with respect to y_1 and y_2 for a series of values of k_F. The conditions for a minimum are

$$\frac{\partial(E/A)}{\partial y_1} = 0 \qquad , \qquad \frac{\partial(E/A)}{\partial y_2} = 0 \tag{5}$$

$$\frac{\partial(E/A)}{\partial y_1} = c_2 + 2c_3 + (m_p - m_n)c^2$$
$$+2^{-1/3}\hbar^2 k_F{}^2 [\frac{1}{m_p} y^{2/3} - \frac{1}{m_n}(1-y)^{2/3}] \frac{\partial y}{\partial y_1}$$
$$+ \frac{3}{16}\frac{m_e{}^4 c^5}{\hbar^3 k_F{}^3} [(x_1{}^2+1)^{1/2}(6x_1{}^2+1) - \frac{x_1{}^2(2x_1{}^2+1)}{(x_1{}^2+1)^{1/2}} - \frac{1}{(x_1{}^2+1)^{1/2}}] \frac{dx_1}{dy_1}$$

(6)

Since $y = y_1 + y_2$, then $\frac{\partial y}{\partial y_1} = 1$. From (2- 5)

$y_1 = \frac{m_e{}^3 c^3}{2h^3 k_F{}^3} x_1{}^3$; thus

$$\frac{\partial y_1}{\partial x_1} = \frac{3m_e{}^3 c^3}{2h^3 k_F{}^3} x_1{}^2 \tag{7}$$

Equation (6) thus reduces to

$$\begin{aligned} 0 = {} & c_2 + 2c_3 + (m_p - m_n)c^2 \\ & + 2^{-1/3}\hbar^2 k_F{}^2 \left[\frac{1}{m_p} y^{2/3} - \frac{1}{m_n}(1-y)^{2/3}\right] \\ & + m_e c^2 \sqrt{x_1{}^2+1} \end{aligned} \tag{8}$$

Similarly the condition $\frac{\partial (E/A)}{\partial y_2} = 0$ gives the equation

$$\begin{aligned} 0 = {} & c_2 + 2c_3 + (m_p - m_n)c^2 \\ & + 2^{-1/3}\hbar^2 k_F{}^2 \left[\frac{1}{m_p} y^{2/3} - \frac{1}{m_n}(1-y)^{2/3}\right] \\ & + m_\mu c^2 \sqrt{x_2{}^2+1} \end{aligned} \tag{9}$$

Subtracting equations (8) and (9) gives

$$m_e c^2 \sqrt{x_1{}^2+1} = m_\mu c^2 \sqrt{x_2{}^2+1} \tag{10}$$

It should be recalled that $x_2=0$ unless $x_1 > \sqrt{\frac{m_\mu^2}{m_e^2} - 1}$.

The nuclear matter programs and effective forces used produce G matrix elements which give binding energies for nuclear matter of about -12 MeV. These were renormalized to give -16.5 MeV binding at $\rho = \rho_{NM}$. We believe that a renormalization is necessary because the "observed" nuclear binding energy is about -16 MeV. Moreover, the G matrix formalism represents the effect of the two body clusters and neglects the effects of three-body and higher body clusters, which make the nuclear potential more attractive by 2 or 3 MeV. Several forms of renormalization were used. One method was to add a constant to all of the G matrix elements to make them more attractive; this corresponds to adding a zero range force. The second method was to multiply the G matrix elements by a factor; this corresponds to increasing the strength of the nuclear force. These adjustments were done in two ways: (1) all the correction was put into the T=0 part of the interaction; (2) the correction was divided equally between the T=0 and T=1 parts of the interaction.

The first method is believed to be better because of the uncertainty in the tensor force which effects the T=0 part of the interaction. Therefore, four forms of renormalization have been used:

1) T=0 plus a constant

2) T=0 and T=1 plus a constant

3) T=0 multiplied by a factor

4) T=0 and T=1 multiplied by a factor.

In the graphs the curves will be identified as 1, 2, 3 or 4.

The results of the minimization are shown in Figures 3 and 4 for forces G-0 and G-1. The contributions of the various terms to the energy are listed in Table 3. One interesting thing to note is the peak in the proton concentration near $k_F = 1.7$ ($\rho=2\rho_{NM}$).

The above calculations have been performed using the Brueckner-Dabrowski approximation of equation (3-2) with $k_A{}^2 = \frac{1}{2}(k_n{}^2+k_p{}^2)$. When we use the alternative approximation $k_A{}^3 = \frac{1}{2}(k_n{}^3+k_p{}^3) = k_F{}^3$, G_{np} is no longer dependent on k_n and k_p but only on k_F. Equation (3-16) simplifies to

$$G_{np}(k_A,k_A;k) = G_{np}(k_F,k_F;k)$$

This change is easily incorporated into the computer program by making $k_{A_0} = k_F$ and setting $\frac{\partial G_{np}}{\partial k_{A_0}}$ and $\frac{\partial^2 G_{np}}{\partial k_{A_0}^2}$ equal to 0 in equation (3-16). The results of this calculation are given in Figures 5 and 6.

Comparison of the graphs shows that the method of renormalizing the force has far more effect on the equilibrium proton concentration than does the force used or the method of approximating the n-p interaction.

CHAPTER 5 INTERPOLATION BETWEEN NUCLEAR MATTER AND NEUTRON GAS

Another use for our expression for the nuclear potential energy is to obtain a method for interpolating between the neutron gas and nuclear matter limits. The variation of the binding energy per particle as a function of the neutron excess is important in determining the stability of neutron-rich nuclei. The semi-empirical mass formula, valid for small neutron excess uses

$$E(k_F,\alpha) = E_0(k_F,0) + \alpha^2 E_S \tag{1}$$

where E_0 is the energy of symmetric nuclear matter, about -16 MeV and E_S is the symmetry energy, about 32 MeV. Since we are interested in large neutron excess, we shall write

$$E(k_F,\alpha) = E_0(k_F,0) + E_S\ \alpha^2 f(\alpha) \tag{2}$$

and investigate the nature of $f(\alpha)$. If $f(\alpha) \equiv 1$, equation (1) would be valid for all neutron excesses. However, from data from atomic bomb tests, Cameron and Elkin (1965) suggested that neutron-rich nuclei were formed more easily than implied by the semi-empirical mass formula, and proposed

$$E(k_F,\alpha) = a \exp(-\alpha^2 \frac{b}{a}) \qquad (3)$$

with a = -17.2928 and b = -26.587. Since the neutron gas is unbound by about 16 MeV, this formula is valid only for small α. We, therefore, write it in the expanded form

$$E(k_F,\alpha) = a(1 - \frac{b}{a}\alpha^2 + \frac{1}{2}(\frac{b}{a})^2 \alpha^4)$$

$$= a - b\alpha^2(1 - \frac{b}{2a}\alpha^2) \qquad (4)$$

with $E_S = -b$ and $f(\alpha) = 1 - \lambda\alpha^2$, where $\lambda = \frac{b}{2a} = .77$.

Brueckner, Coon and Dabrowski (1968) using nuclear matter calculations at α = 0, .2, and .4 suggested the same form for $f(\alpha)$ and obtained E_S = 28.0 and λ = .67, which fitted their points at α = 0, .2 and .4 exactly. Siemens (1970) used the same method to arrive at E_S = 31.0 MeV and λ = 1.7, but there seems to a mistake in his calculations and his figures should be revised to E_S = 29.3 and λ = .05. The only thing consistent about these three calculations is that $f(\alpha) < 1$ for small α.

Baym, Bethe and Pethick (1971) have obtained an expression in powers of α^2 for interpolating between nuclear matter and the neutron gas. Their expression for the energy is

$$E(k_F,\alpha) = E_0(k_F,0) + E_S(k)\alpha^2(1-\lambda(k_F)\alpha^2+E(k_F)\alpha^4) \qquad (5)$$

where they took $E_0 = -16.5$ MeV and $E_S = 33$ MeV at $k_F = 1.43$. At $k_F = 1.36$ where the other authors have done their calculations they have $E_S = 29.85$ MeV, $\lambda = -.24$ and $\varepsilon = -.19$. Their calculation, therefore, gives λ a different sign than the other authors. From a reading of their paper, it appears that they did not pay attention to the sign of λ in writing down their energy expression.

Our expression for the energy is given by $E = T_{kin} + U$ where

$$U = \rho[(1-y)^2\ \overline{G}_{nn} + 4y(1-y)\overline{G}_{np} + y^2\ \overline{G}_{pp}] \tag{6}$$

It should be correct in the case of symmetric nuclear matter when we use the exact values of k_n, k_p and k_A instead of expanding as a power series in terms of y. For an equal number of protons and neutrons we have $k_n = k_p = k_A = k_F$ so that the n-p interaction is given correctly. The neutron and proton gases will be equivalent and have half the density of a pure neutron gas at this density; thus the fermi momentum is reduced by a factor of $2^{1/3}$. From equation (4-4) we, therefore, have

$$G_{nn} = G_{pp} = G_{NG}(2^{1/3}k_F, 0; k) = \frac{4}{3}\, G_{NM}^{T=1}(k_F, k_F; k) \tag{7}$$

Thus substituting into equation (6) with $y = \frac{1}{2}$ correctly gives

$$U = \rho[\frac{1}{4}\ (\frac{4}{3}\ \bar{G}_{NM}^{T=1}) + 4(\frac{1}{4})(\bar{G}_{NM}^{T=0} + \frac{1}{3}\ \bar{G}_{NM}^{T=1}) + \frac{1}{4}(\frac{4}{3}\ \bar{G}_{NM}^{T=1})]$$

$$= \rho(\bar{G}_{NM}^{T=0} + \bar{G}_{NM}^{T=1}) \tag{8}$$

Evaluating the energy per particle for $\alpha = 0$, .2, and .4 allows us to evaluate $f(\alpha) = 1 - \lambda\alpha^2$. We obtain $E_S \simeq 30$ MeV and $\lambda \simeq -0.08$ which agrees in sign with Baym et al. (1971) but conflicts with the other calculations. Using

$$f(\alpha) = 1 - \lambda\alpha^2 + \varepsilon\alpha^4$$

and fitting the energy at $\alpha = .8$ and 1.0 using E_S as determined from the fit near nuclear matter we obtain $\lambda = .04$ and $\varepsilon = .14$. This gives a fair fit to our data in the region near the pure neutron gas and tends to agree with Siemens (corrected) value for λ. The numbers given here are for force G-0 renormalized by multiplying the T=0 strength by a factor which gives -16.5 MeV binding energy for symmetric nuclear matter at $k_F = 1.36$. The results are similar for the other forces and types of adjustment.

The conclusions that can be drawn from this are uncertain. The more recent calculations appear to agree that $f(\alpha) \simeq 1$ everywhere, and probably is slightly greater than one for small neutron excess. This is at variance with

Cameron and Elkin. Possibly the division into volume and surface symmetry energies is different from what they used; this could affect the energies of finite nuclei without showing up in the infinite system studied here. For large neutron excess, the value of $f(\alpha)$ is of order one, but whether it is a bit smaller or larger depends strongly on the value used for E_S. To use $f(\alpha) \equiv 1$ seems not to be in flagrant disagreement with the calculations; this appears to be the choice adopted by Brueckner, Meldner and Chirka for their Thomas-Fermi calculations.

In conclusion then, the main result of our calculations is that there is a peak in the proton concentration. Nemeth et al. (1967) found a peak at $k_F = 1.3\ fm^{-1}$ but in a later calculation (Nemeth and Sprung, 1968) no peak or even levelling off of the proton concentration was obtained. Our calculation gives a higher proton concentration and peaks at $1.7\ fm^{-1}$, just above the highest value of k_F in the Nemeth-Sprung calculation. According to Baym et al. (1971) neutron stars have a lattice structure which does not completely dissolve until $\rho \simeq 2.4 \times 10^{14}\ gm/cm^3$ ($k_F = 1.3\ fm^{-1}$), so that our calculation is applicable only for densities greater than this. Buchler and Ingber (1971) state that the validity of a non-relativistic many-body calculation ceases at $\rho \simeq 10^{15}\ gm/cm^3$ ($k_F = 2.0\ fm^{-1}$). However, it is noteworthy that in our calculation the form of the density dependence of the nuclear potential (i.e., G-0 or G-1) seems to have less effect on the proton concentration than does the method of renormalizing the nuclear force. We have not allowed for the existence of negative pions or other particles in our calculation. However, if their interaction with the nucleons is small, their effect on the calculation will not be too much different from that of the muons since they have roughly the same masses. We, therefore, believe that our calculation should give a reasonably good description of neutron star matter up to 2 or 3 times normal nuclear matter density.

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TABLE 1

Coefficients used in fitting the slopes and intercepts of the G matrix elements obtained from nuclear matter and neutron gas calculations

	a_1	a_2	a_3	a_4	b_1	b_2	b_3
Neutron gas	-5.0988	2.0748	-12.3872	3.7777	2.1496	-9.2486	15.0942
Nuclear matter T=0	-0.5107	-21.2849	15.3358	-3.6621	3.5609	-5.9450	6.8650
Nuclear matter T=1	-0.9893	- 9.2810	1.6472	-0.4319	2.9204	-5.7830	7.3488

TABLE 2

Coefficients used in fitting the slopes and intercepts of the G matrix elements obtained from the effective forces G-0 and G-1

Force G-0

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
Nuclear matter T=0	-16.2572	4.1088	1.6284	-0.3555	6.6901	1.9662	-2.7556	0.4797
Nuclear matter T=1	- 9.2139	-1.6380	2.2313	-0.3888	2.6198	4.6164	-3.2520	0.5317

Force G-1

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
Nuclear matter T=0	-13.8082	-0.1247	3.9412	-0.6478	4.7018	5.1350	-4.3868	0.7181
Nuclear matter T=1	- 9.2954	-1.8128	2.6399	-0.4729	3.5370	3.3621	-2.8928	0.5084

TABLE 3

Energy contributions for force G-0 renormalized by multiplying the T=0 part by a factor

k_F	% protons	(electrons)	E_{mass}	T_{Kin}	T_e	T_μ	U	E
1.36	5.871	(5.084)	-.076	33.324	4.776	.927	-24.840	14.110
.20	.031	(.031)	-.000	.789	.001	0.000	- .152	.637
.40	.175	(.175)	-.002	3.148	.016	0.000	- 1.066	2.096
.60	.690	(.690)	-.009	7.025	.147	0.000	- 3.190	3.973
.80	1.740	(1.740)	-.023	12.281	.673	0.000	- 6.837	6.095
1.00	3.158	(3.158)	-.041	18.769	1.861	0.000	-12.135	8.455
1.20	4.528	(4.522)	-.059	26.469	3.605	.006	-18.746	11.277
1.40	6.149	(5.152)	-.080	35.169	5.005	1.200	-26.366	14.927
1.60	6.940	(5.163)	-.090	45.407	5.735	2.351	-33.414	19.989
1.80	6.754	(4.712)	-.087	57.623	5.712	2.883	-38.937	27.195
2.00	5.769	(3.913)	-.075	72.176	4.954	2.710	-42.761	37.005
2.20	4.231	(2.882)	-.055	89.368	3.624	1.959	-45.351	49.545
2.40	2.452	(1.760)	-.032	109.301	2.049	.947	-47.572	64.693
2.60	.873	(.749)	-.011	131.520	.711	.147	-50.031	82.335

LIST OF FIGURES

Figure 4: Proton concentration vs. k_F for force G-1. The n-p interaction is given by $k_A{}^2 = \frac{1}{2}(k_n{}^2+k_p{}^2)$. The various curves are labelled as in Figure 3.

Figure 5: Proton concentration vs. k_F for force G-0 with the n-p interaction using $k_A = k_F$. The curves are labelled as in Figure 3.

Figure 6: Proton concentration vs. k_F for force G-1 with $k_A = k_F$.

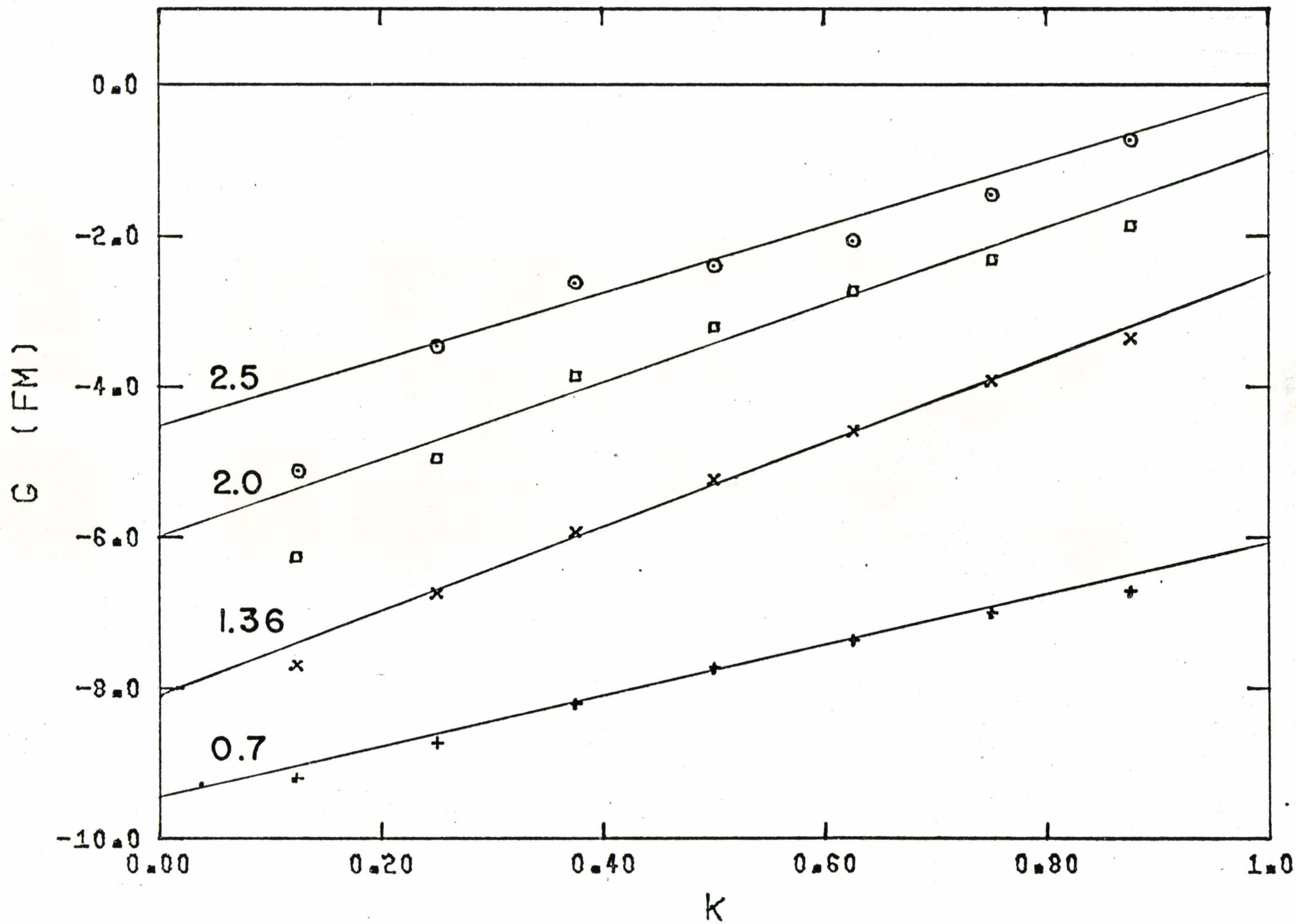
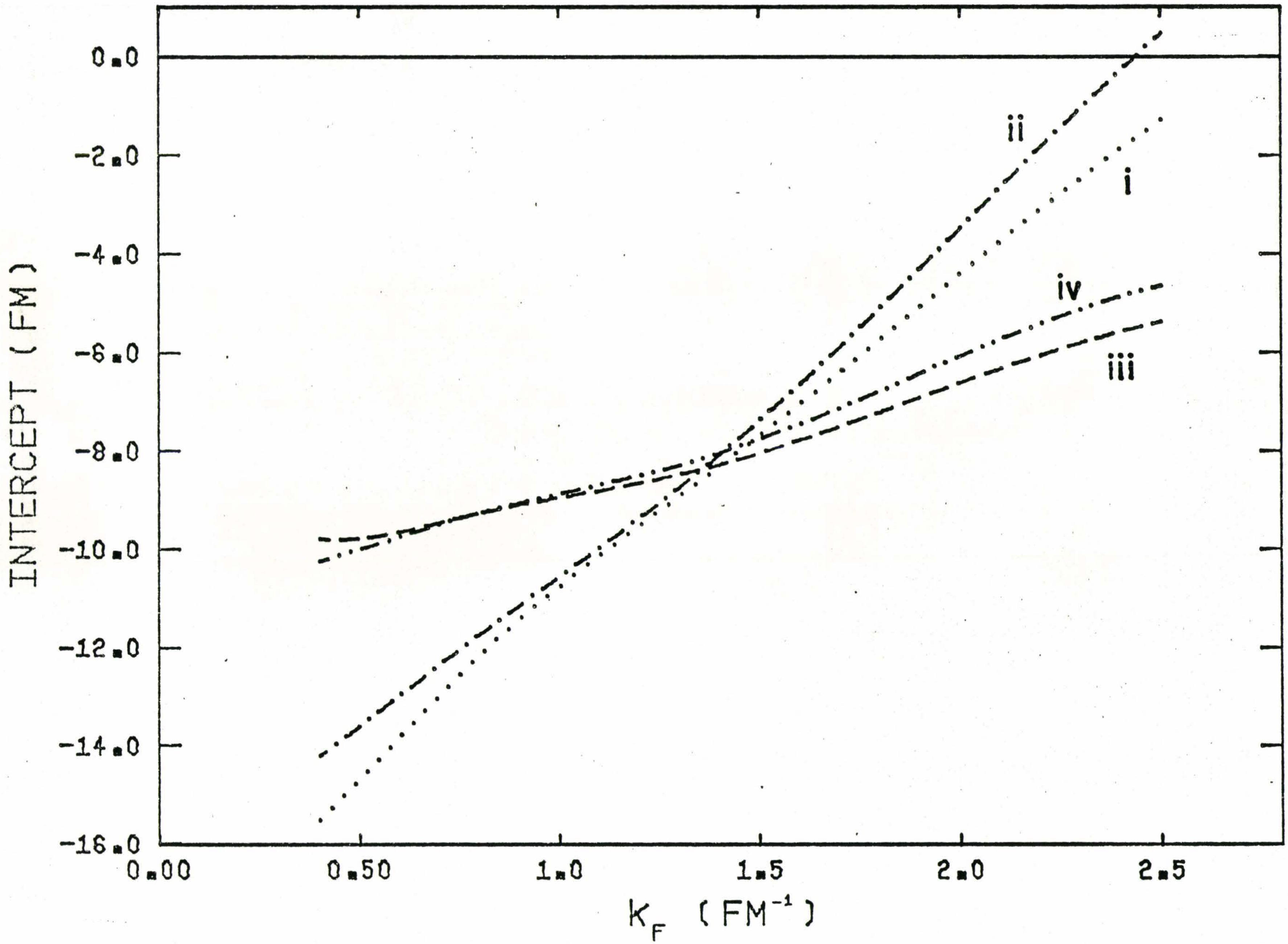


Fig. 1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-6.0
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-16.0
INTERCEPT (FM)
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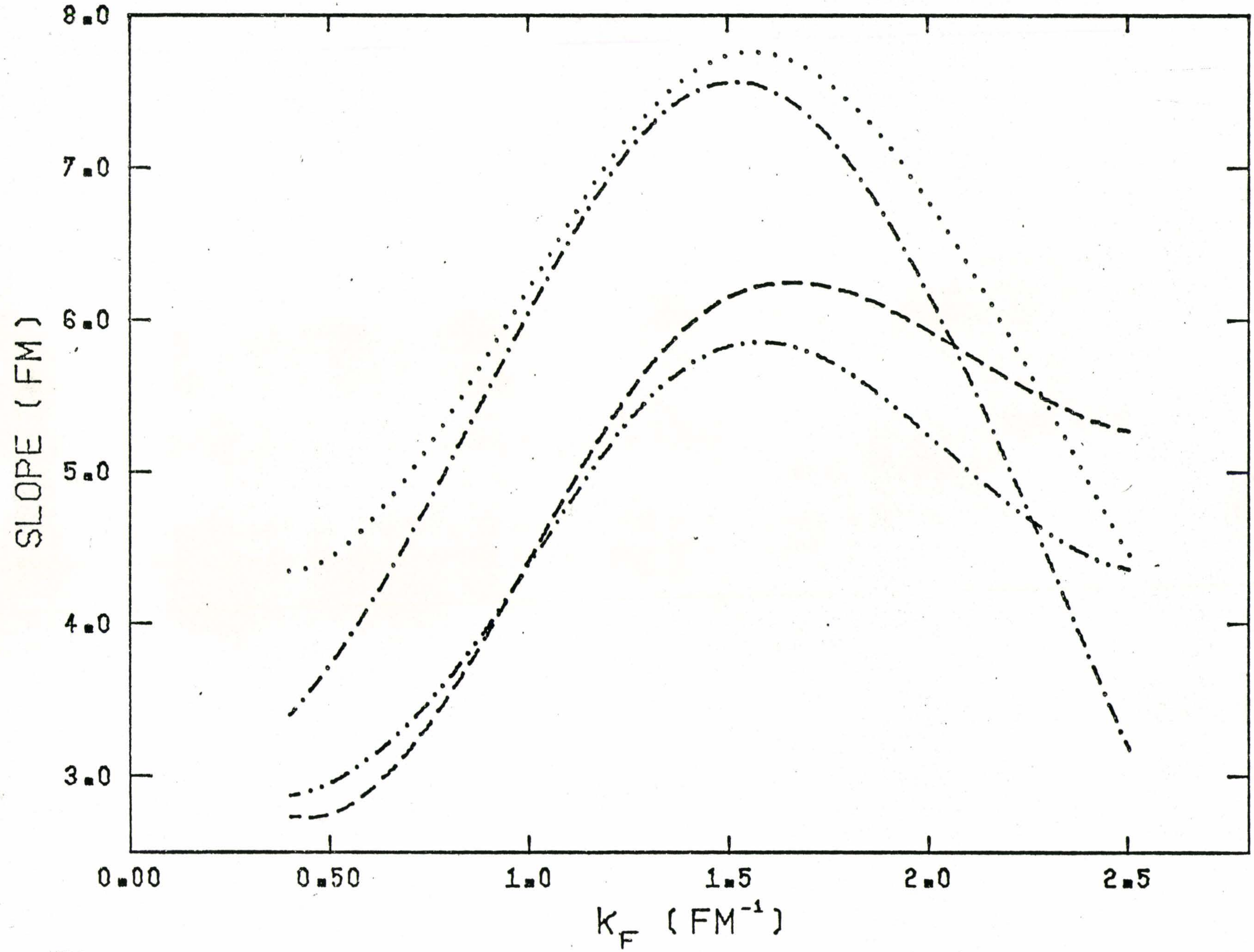
0.50
1.0
1.5
2.0
2.5
K_F (FM^-1)
i
ii
iii
iv

Fig. 2a



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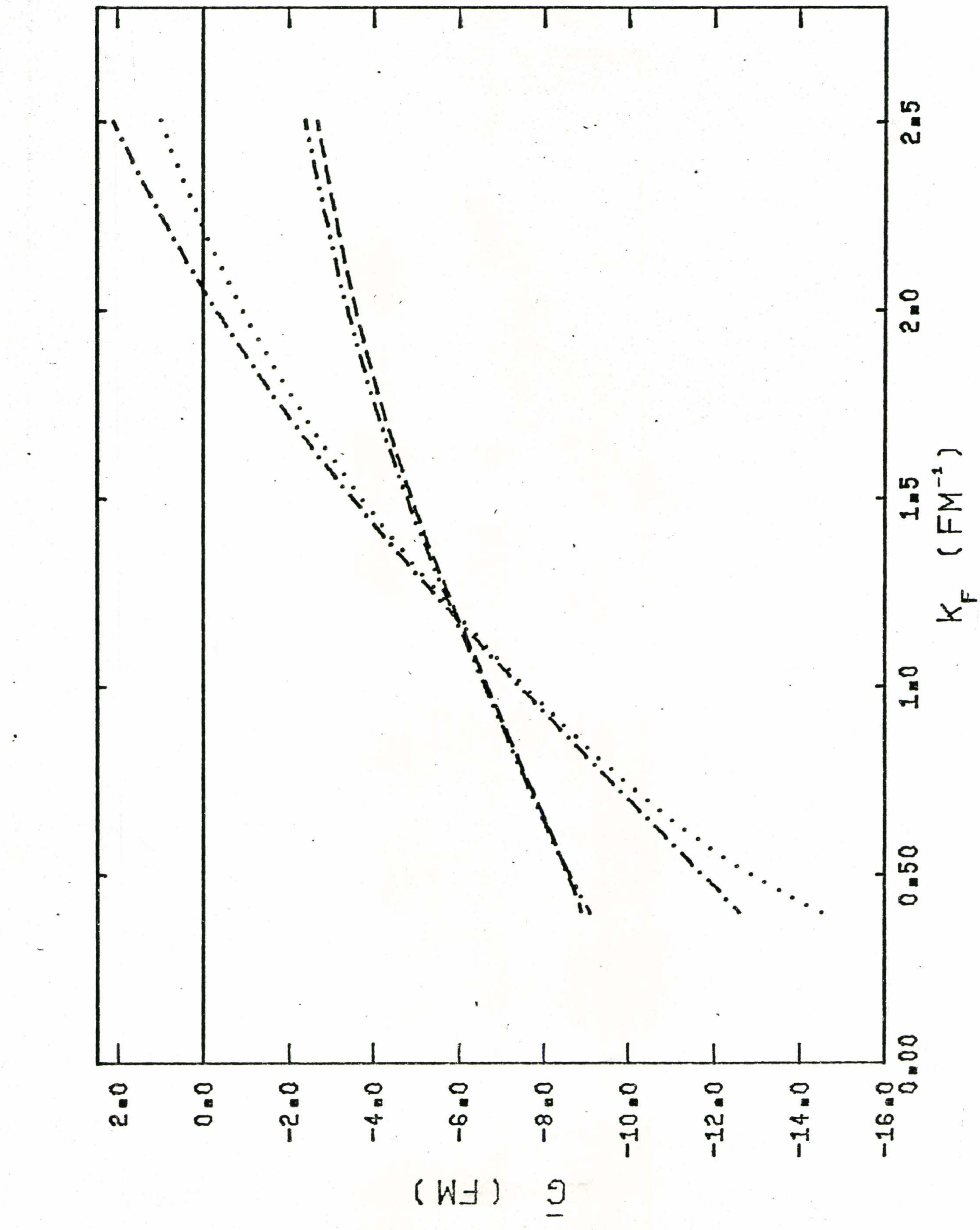

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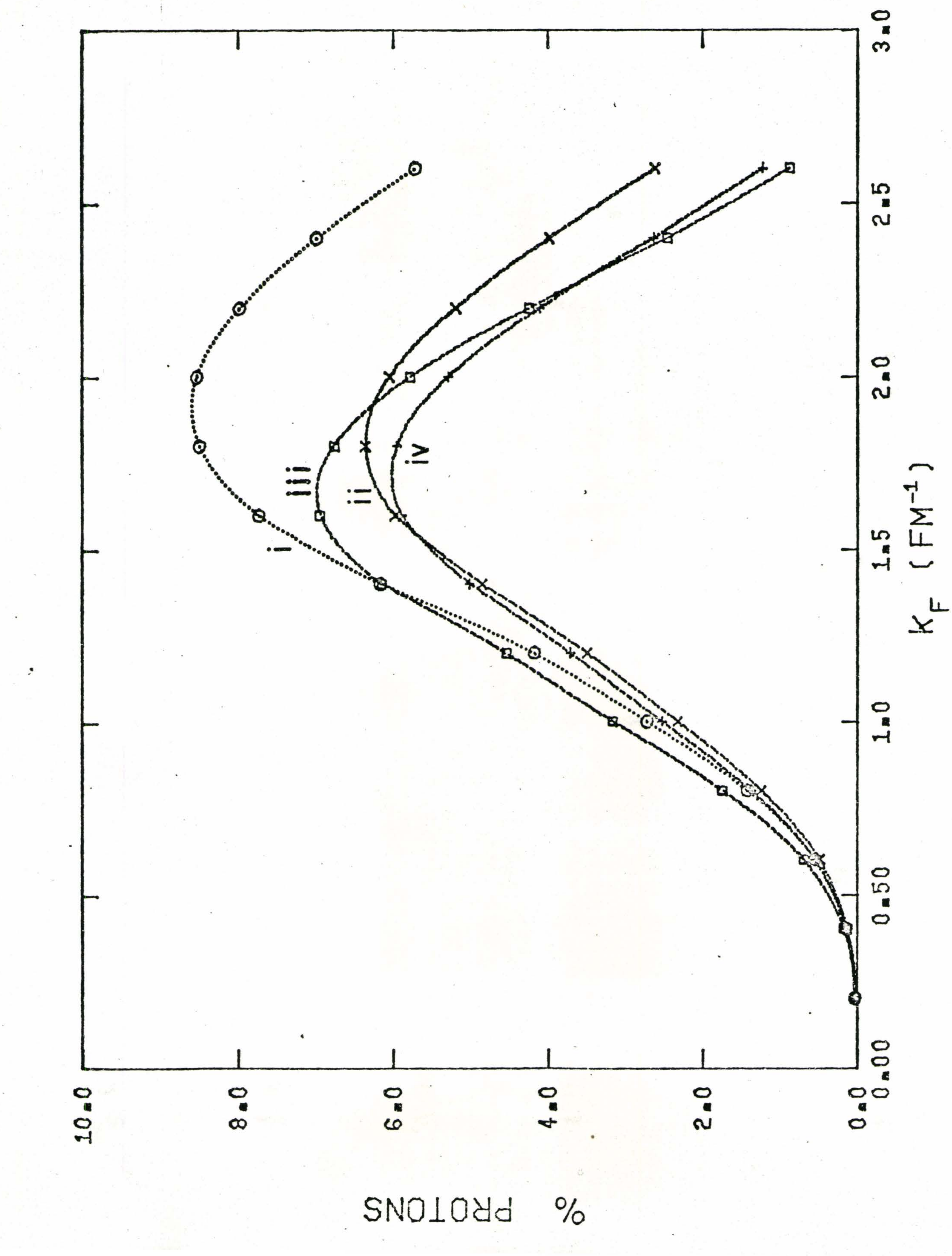

Fig. 3

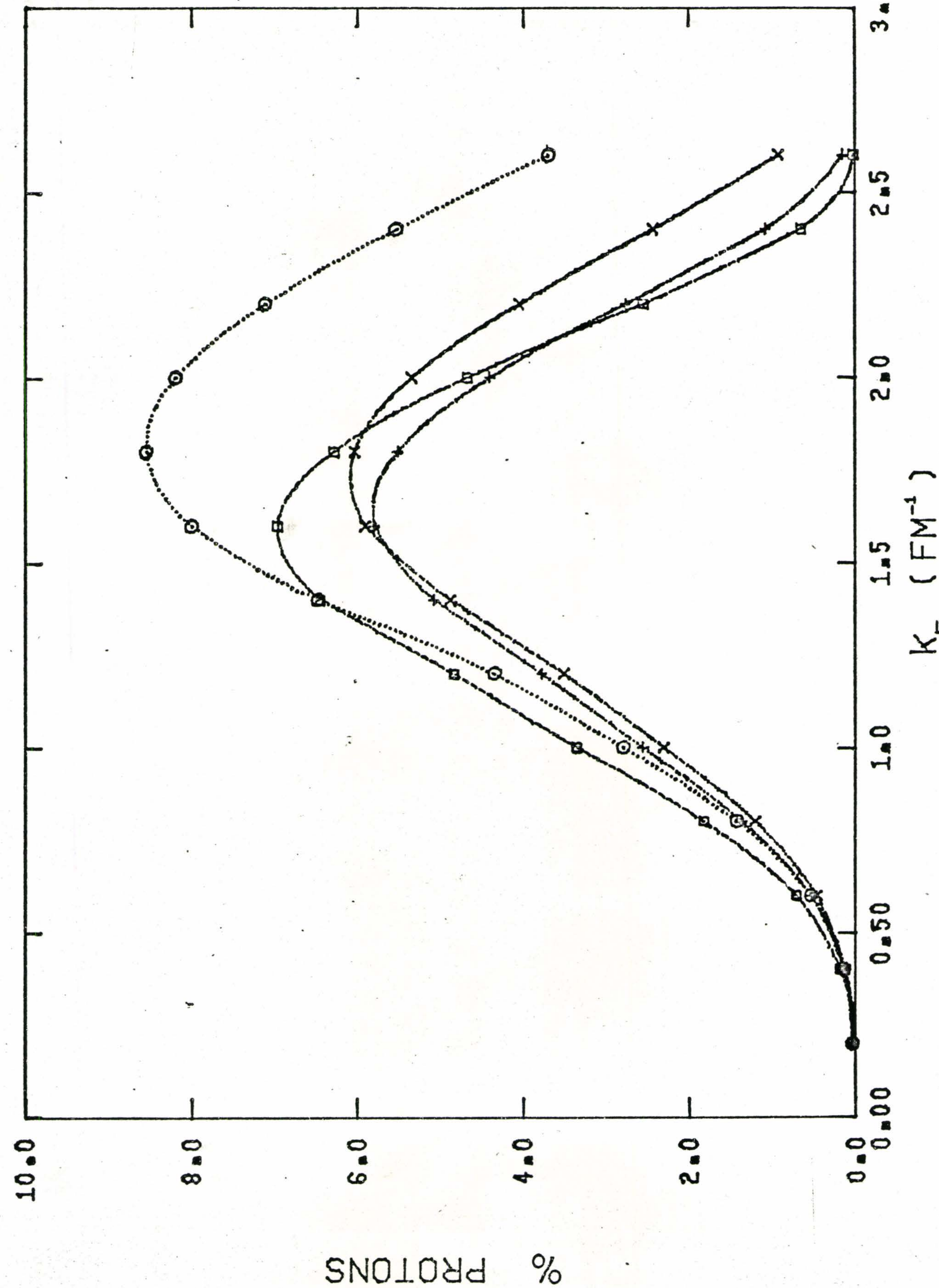

Fig. 4

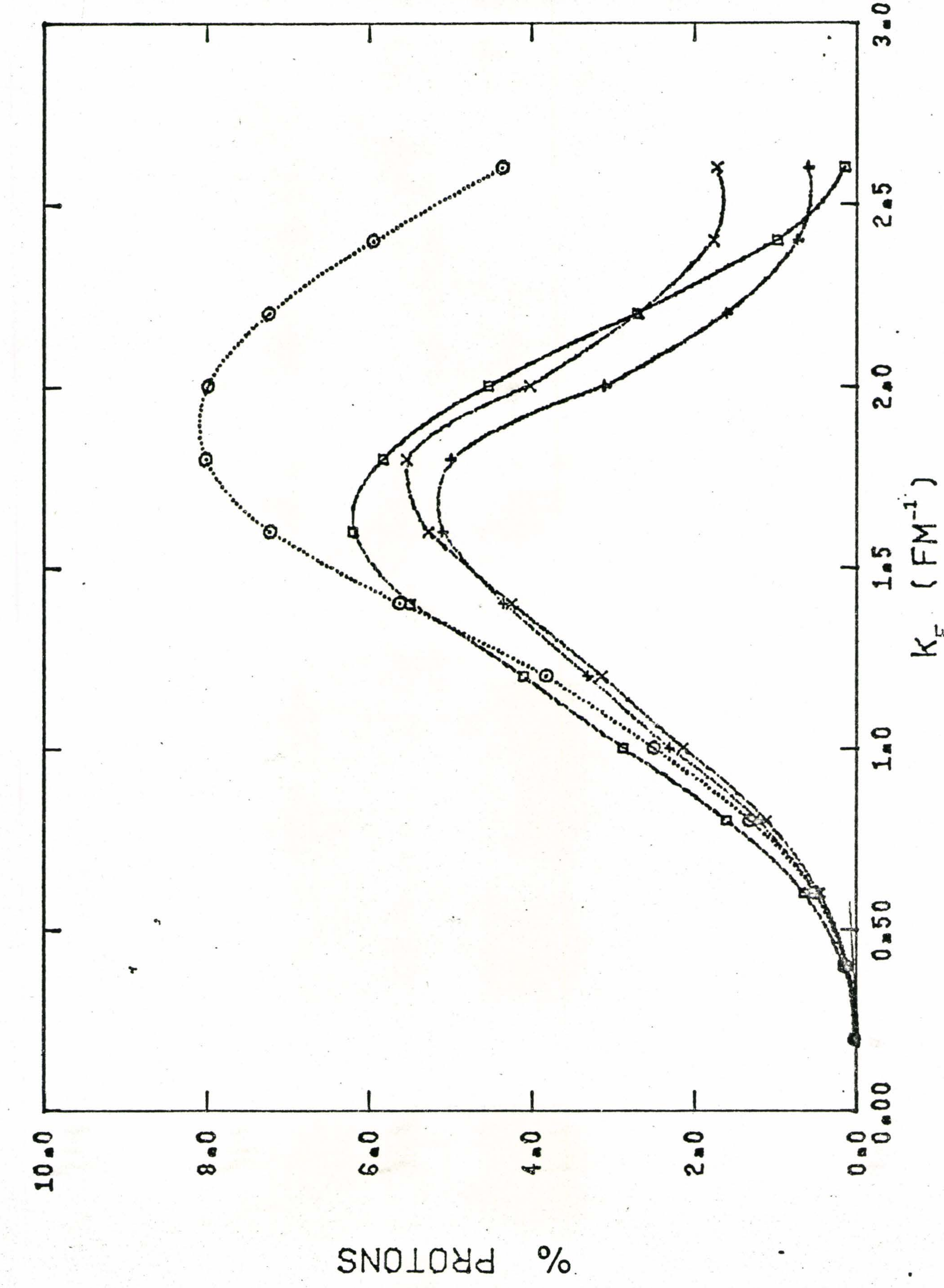

Fig. 5

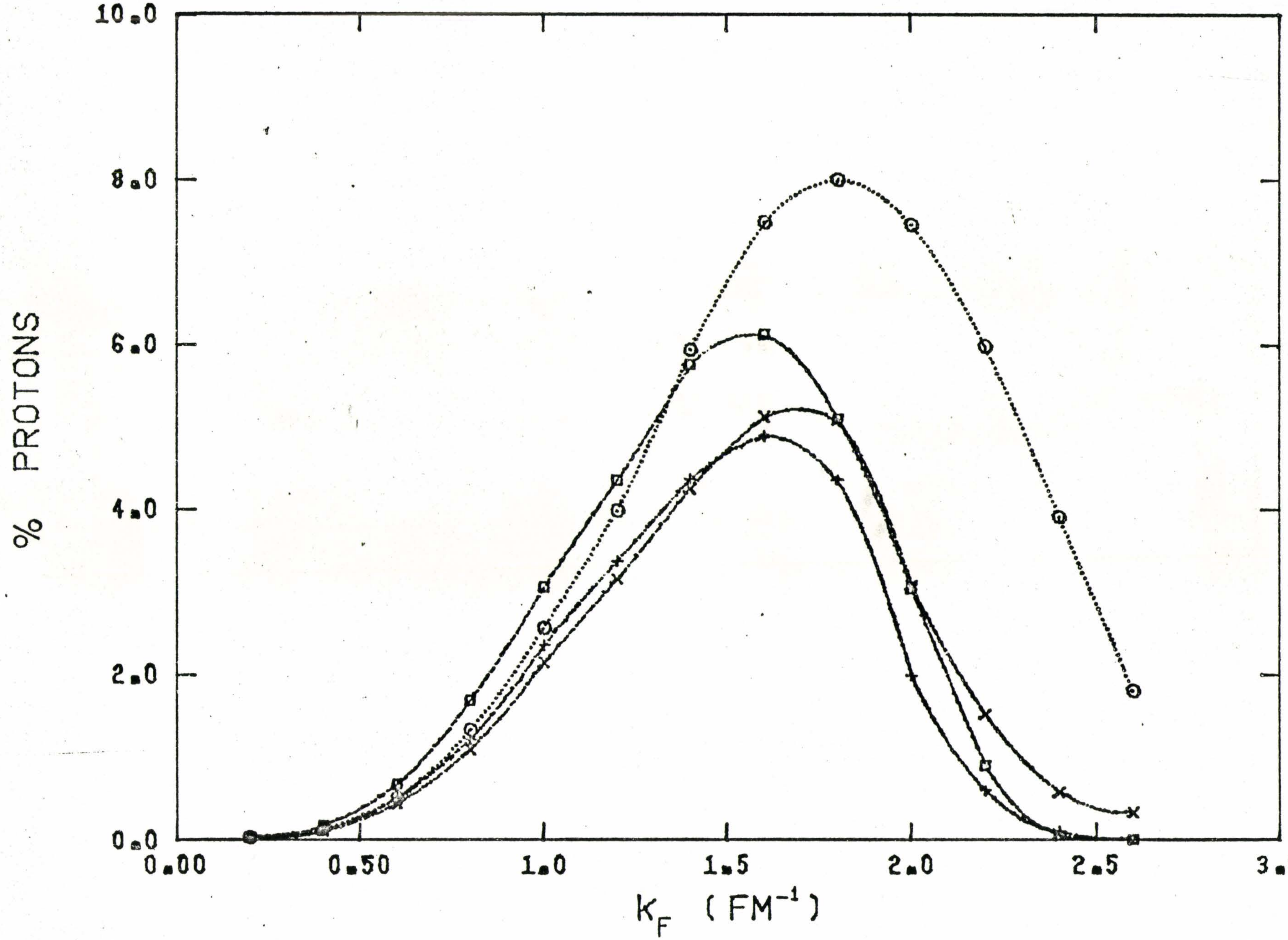

Fig. 6