EXPERIMENTAL STUDIES OF
COMBINED RELIABILITY

MASTER OF ENGINEERING
(Mechanical Engineering)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Experimental Studies of Combined Reliability

AUTHOR: Richard Dumala, B.A.Sc.(University of Toronto)

SUPERVISOR: Professor J. N. Siddall

NUMBER OF PAGES: vi, 99

SCOPE AND CONTENTS:

In the field of Reliability, a new concept is introduced. The Combined Dependability theory is put forth in this thesis.

Attempts are made to prove this theory experimentally by use of accelerated failure tests of GE-47 miniature lamps. The lamps are tested individually and ten in series. The individual lamp test results are then used for the prediction of the reliability of the ten lamps in series.

The ten lamps in series simulate a machine with ten components. A failure of one of the components will produce a failure of the machine. The reliability of the machine can be found if the reliability of each part is known. The single part when tested individually must be subjected to the same stresses and conditions that it would encounter when operating in the machine. If this is not accomplished, the machine's calculation of reliability is invalid.

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ACKNOWLEDGEMENTS

It gives the author pleasure to express his thanks to his supervisor, Professor J. N. Siddall, for his guidance and for instilling in the author the drive and desire to continue.

This project was supported in part by a grant from the Ford Foundation.

ABSTRACT

In today's technology, the science of reliability is increasingly becoming a large factor in the design of machines and their components. Yet, the science of reliability is not progressing at the same pace as the other sciences.

This thesis is an attempt to further the knowledge of reliability by introducing a new theorem in probability and with the aid of experimental results to verify this theorem.

When predicting the reliability of a device, the standard method is to use the product of the reliabilities of each component. A new theory displaces this product rule for certain cases. When a large number of components are used, the reliability of a device calculated by the product rule can be erroneous.

A theoretical section included in this thesis explains the theory of combined dependability and compares the results of this theory with the product rule. The experimental section attempts to verify one of these theorems conclusively.

INTRODUCTION

The object of this paper is to propose a new approach in reliability studies. The product rule is compared with the newly introduced combined dependability theorem. Both are valid in their predictions of the reliability of a device with "n" components, but each must make different basic assumptions. If, in fact, each component in a device is independent of the others while operating in the device, the product rule applies. If the components are dependent upon one another for each others operation life, then the combined dependability theorem is the proper procedure for predicting the reliability of the device.

The combined dependability theorem is described and proven mathematically. Along with the theoretical description, the combined dependability theorem is compared with the product rule for varying number of components in a device and for different component probabilities. It is shown that the combined dependability theorem predicts a higher probability of survival for a given time period than the product rule. By considering the components of a device to be dependent upon one another for their own operational life, then the life of the device is theoretically longer than if this assumption were not considered.

The probability of survival of a machine for a low time is basically the same when using either approach. At the high time mark is where the combined dependability theorem shows a definite increase in probability over the product rule. An identical divergence between the two methods occurs when the number of components in the machine is increased.

To verify this theorem experimentally, a mock machine was constructed of miniature lamps placed in series. It was hoped to simulate a device with "n" number of components. For this experiment "n" equalled ten. By placing the lamps in series electrically, the intention was to make each lamp independent in operation but its life dependent upon the other lamps.

The lamps were tested singularly under the same operating conditions or stress as they would experience when in the simulated machine. A sufficient number of lamps were tested to establish a satisfactory frequency distribution of lamps versus lamp life.

Because of the number of lamps expected to be tested, an accelerated test technique was adopted. Various methods were considered. The first attempt at an accelerated test technique, subjected the lamps to a current of 0.230 amperes along with cycling. The lamp circuit was opened and closed at a frequency of 175 cpm. This method proved to be time consuming and inconsistent in operation.

Another method considered was the use of a high constant voltage. This method was satisfactory for single lamp tests where voltage could be controlled without difficulty but became an immense problem when ten lamps were tested in series. The operator could not efficiently handle the voltage adjustment required on ten lamps simultaneously.

The third method, which was officially adopted, consisted of a constant filament current of 0.240 amperes. Only one potentiometer was required in the circuit when either one lamp or ten lamps were operated.

Every effort was made to keep all controllable variables constant when performing the tests. It was not realized until after the tests that the filament temperature had affected the lamp lives. Because of the accelerated test technique of using a current higher than rated, the filament was operating at a temperature near the melting point of tungsten. The lamps were on the verge of instability and only slight increases in current or heat would lead to an unexpected short lamp life.

The lamps, when tested individually, produced a consistent frequency distribution. When tested in series, it was concluded that each lamp had in fact affected the other. The lamps in series were not considered to be under the same stresses as when operated singularly.

This is considered to be the main reason for the discrepancy in the results between the predicted device life with ten components and the experimental device life.

Recommendations for future testing are put forth to aid in the next attempt in proving the combined dependability theorem experimentally.

It is hoped that by bringing forth a new theorem in reliability, and although a first attempt at proving it experimentally has not succeeded, that enough interest has been generated to initiate further efforts in the combined dependability theorem.

THEORETICAL ANALYSIS

To establish the reliability of a device with "n" number of items, it is usually customary to use reliability data of each individual item based on tests performed on each item in isolation. At this time, this seems to be the best method of approximating the reliability of a device. To illustrate the product rule theory, consider a machine M composed of "n" components, such that if one component failed the machine itself would fail.

When "n" equals 1, the machine has only one component, A. The probability of M operating failure free for X hours is the direct probability of A reaching X hours without failure.

Now consider a machine M with two components, A and B. The probability of A and B reaching X hours separately as independent events is $P_A(X) = .8$ and $P_B(X) = .6$, respectively. It must be assumed that A and B are independent while operating in M and that M is dependent on both of them. If A has survived X hours of service, then B has a probability of .6 of surviving the same X hours. In other words, M's probability of surviving X hours is .6 of .8 which is .48. This is known as the "Both and" rule of probability or the "Product Rule".

The same logic applies for many components in M. If there are "n" components, the probability of each component reaching X hours as independent events is

$P_1(X) = .8$, $P_2(X) = .9$,......, $P_{n-1}(X) = .6$, $P_n(X) = .7$. The probability of M reaching X hours is $P_1(X)$ of $P_2(X)$ of ...... of $P_{n-1}(X)$ of $P_n(X)$. That is, the probability of M reaching X hours is the product of the individual probabilities of the components as independent events. But, failure of a component in isolation cannot be considered to be the same situation as a failure of the same component while operating in the device. Siddall [1], in his paper, has stated that if the i^{th} component fails in the device, and this failure constitutes a failure of the device, then the resulting reliability of the device due to this failure is R_d but, the reliability of the part is R_i. Since these two reliabilities were defined in different context, (e.g. in isolation and in a device) then they cannot be directly related by a probability theorem.

It is, therefore, necessary to redefine the probability of failure of the item in the device.

The probability of the complement of the event of failure of the i^{th} component has already been established as R_i. A_i will be defined as the event that failure of the i^{th} component occurs while operating in the device. Therefore, $\bar{A}_i$ is the event of non-failure of the i^{th} component while operating in the device. Now it is obvious that

$$P(\bar{A}_i) \neq R_i \qquad (1)$$

If all A_i's, where $i = 1,2,\ldots,n$, are mutually exclusive and dependent events, the equation can be written in full as

$$P(\bar{A}_i | \bar{A}_1\bar{A}_2 \ldots . \bar{A}_{i-1}\bar{A}_{i+1} \ldots . \bar{A}_n) = R_i \qquad (2)$$

For n components, there are n expressions of this type, and 2^n-n-1 expressions representing the mutually exclusive property of the A_i's.

$$P(A_iA_j \ldots . A_m) = 0 \qquad (3)$$

The reliability of the device can then be written as a combined reliability

$$R_d = P(\bar{A}_1\bar{A}_2 \ldots . \bar{A}_n) \qquad (4)$$

In reviewing the situation, we see that

$$\begin{aligned} R_1 &= P(\bar{A}_1 | \bar{A}_2\bar{A}_3 \ldots . \bar{A}_n) \\ R_2 &= P(\bar{A}_2 | \bar{A}_1\bar{A}_3 \ldots . \bar{A}_n) \\ &\vdots \\ R_n &= P(\bar{A}_n | \bar{A}_1\bar{A}_2 \ldots . \bar{A}_{n-1}) \end{aligned} \qquad (5)$$

Each conditional probability has different conditioning events. Because of this they cannot be used in the probability theorem.

By converting the problem into basic probabilities the reliability of the device, R_d, can be solved. Consider a simple case when $n = 2$. Using the set of equations (5)

$$R_1 = P(\bar{A}_1 | \bar{A}_2)$$

$$R_2 = P(\bar{A}_2 | \bar{A}_1) \qquad (6)$$

equation (3) gives

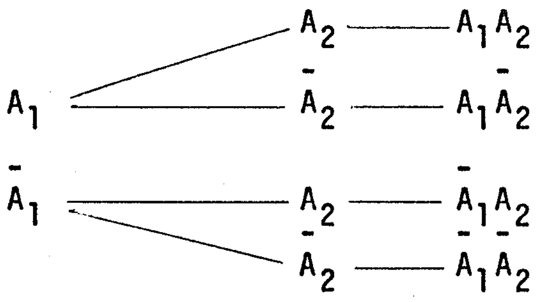
$$P(A_1 A_2) = 0$$

considering all basic probabilities for the device we get

$$P(A_1 A_2) + P(A_1 \bar{A}_2) + P(\bar{A}_1 A_2) + P(\bar{A}_1 \bar{A}_2) = 1 \qquad (7)$$

where one of these four probabilities will and must occur.

Equation (7) was derived from an event network which is represented as



This event network can be interpreted as such; either A_1 occurs or not, that is A_1 or $\bar{A}_1$. For either occurrences, there will be an accompanying occurrence of A_2, either A_2 or $\bar{A}_2$. Breaking this network down into its basic form, we get four possible occurrences for a two component device.

We must now analyze the probability of each individual occurrence. That is, for A_1 to occur, we must have the occurrence of either A_1A_2 or $A_1\bar{A}_2$, as seen from the event network.

$$P(A_1) = P(A_1A_2 + A_1\bar{A}_2)$$

This equation can be rewritten as

$$P(A_1) = P(A_1A_2) + P(A_1\bar{A}_2)$$

Similarly, using the event network, the probability of the remaining three occurrences can be written

$$P(\bar{A}_1) = P(\bar{A}_1A_2) + P(\bar{A}_1\bar{A}_2)$$

$$P(A_2) = P(A_2A_1) + P(A_2\bar{A}_1)$$

$$P(\bar{A}_2) = P(\bar{A}_2A_1) + P(\bar{A}_2\bar{A}_1)$$

Returning to the individual reliability of equation (6)

$$R_1 = P(\bar{A}_1|\bar{A}_2)$$

This conditional probability can be rewritten in basic form as

$$R_1 = P(\bar{A}_1|\bar{A}_2) = P(\bar{A}_1\bar{A}_2)/P(\bar{A}_2)$$

substituting in for $P(\bar{A}_2)$

$$R_1 = P(\bar{A}_1\bar{A}_2)/(P(\bar{A}_2A_1) + P(\bar{A}_2\bar{A}_1))$$

similarly R_2 can be written as

$$R_2 = P(\bar{A}_1\bar{A}_2)/(P(\bar{A}_1A_2) + P(\bar{A}_1\bar{A}_2))$$

by rearranging the above equations we get

$$P(\bar{A}_2A_1) = P(\bar{A}_1\bar{A}_2)\ \frac{[1 - R_1]}{R_1}$$

and

$$P(\bar{A}_1 A_2) = P(\bar{A}_1 \bar{A}_2) \frac{[1 - R_2]}{R_2}$$

which are two of four basic probabilities.

Since

$$P(A_1 A_2) = 0$$

the basic probability equation becomes

$$P(A_1 \bar{A}_2) + P(\bar{A}_1 A_2) + P(\bar{A}_1 \bar{A}_2) = 1$$

substituting into the above equation for the basic probabilities we get

$$P(\bar{A}_1 \bar{A}_2) \frac{[1 - R_1]}{R_1} + P(\bar{A}_1 \bar{A}_2) \frac{[1 - R_2]}{R_2}$$

$$+ P(\bar{A}_1 \bar{A}_2) = 1$$

$$P(\bar{A}_1 \bar{A}_2)[\frac{(1 - R_1)}{R_1} + \frac{(1 - R_2)}{R_2} + 1] = 1$$

$$P(\bar{A}_1 \bar{A}_2) = \frac{R_1 R_2}{R_1 + R_2 - R_1 R_2} \tag{8}$$

$$= R_d \text{ as defined by equation (4)}$$

Since

$$P(\bar{A}_1) = \frac{P(\bar{A}_1 \bar{A}_2)}{P(\bar{A}_2/\bar{A}_1)} \quad \text{and} \quad R(\bar{A}_2/\bar{A}_1) = R_2$$

Therefore, the equation for the probability of the non-occurrence of A_1 for the case n = 2, is

$$P(\bar{A}_1) = \frac{R_1}{R_1 + R_2 - R_1 R_2}$$

and $P(\bar{A}_2)$ can be written as

$$P(\bar{A}_2) = \frac{R_2}{R_1 + R_2 - R_1R_2}$$

Returning to general statements, equation (5) is

$$R_i = P(\bar{A}_i | \bar{A}_1\bar{A}_2 \ldots \bar{A}_{i-1}\bar{A}_{i+1} \ldots \bar{A}_n)$$

rewritten as

$$P(\bar{A}_1\bar{A}_2 \ldots . \bar{A}_n) - R_i P(\bar{A}_1\bar{A}_2 \ldots . \bar{A}_{i-1}\bar{A}_{i+1} \ldots \bar{A}_n) = 0$$

for i = 1,2,3,....,n.

This gives in terms of basic probabilities

$$(1 - R_i)P(\bar{A}_1\bar{A}_2 \ldots . \bar{A}_n) - R_i P(\bar{A}_1\bar{A}_2 \ldots \bar{A}_{i-1}\bar{A}_i\bar{A}_{i+1} \ldots . \bar{A}_n) = 0 \quad (9)$$

also

$$\Sigma \text{ basic probabilities} = 1$$

There are $2^n - n - 1$ expressions representing the mutually exclusive property of the A_i's and these expressions are equal to zero. The first two are

$$P(A_1A_2 \ldots A_n) = 0 \quad (10)$$

$$P(A_1A_2 \ldots A_{n-1}) = 0 \quad (11)$$

In terms of basic probabilities equation (11) becomes

$$P(A_1A_2 \ldots . A_n) + P(A_1A_2 \ldots . A_{n-1}\bar{A}_n) = 0$$

Since the first term is zero as seen in equation (10), the second must be zero also. It can also be shown that all basic probabilities except those occurring in equation (9) are zero. Rewritting equation (9), we get

$$P(\bar{A}_1\bar{A}_2\ldots\bar{A}_{i-1}\bar{A}_i\bar{A}_{i+1}\ldots.\bar{A}_n) = (\frac{1 - R_i}{R_i})\ R_d$$

substituting into basic probability equation

$$R_d = \frac{1}{1 + \sum_{i=1}^{n} (\frac{1 - R_i}{R_i})}$$

if all R_i's are equal to R, then

$$R_d = \frac{R}{n - R(n - 1)} \qquad (12)$$

To verify equation (12), we have two consistency checks. We can apply the theory of decomposition. There are n components and $\frac{n}{2}$ of them can be joined in a "subassembly". We have two "subassemblies",each of which has its' own reliability, R_s. The combination of the R_s's should produce the reliability of the device, R_d, if equation (12) is correct. To prove equation (12) using the decomposition theory, we first write the expression for R_s in terms of R.

$$R_s = \frac{R}{\frac{n}{2} - R(\frac{n}{2} - 1)} \qquad (13)$$

for n = 2

$$R_d = \frac{R_s}{2 - R_s(2 - 1)} = \frac{R_s}{2 - R_s} \qquad (14)$$

substituting for R_s from equation (13) into equation (14) we get

$$R_d = \frac{R}{n - nR + R} = \frac{R}{n - R(n + 1)}$$

which is equation (12).

Another consistency check is the use of a simple problem where the probabilities are known. The problem consists of predicting the probability of drawing a white marble from a container in which there are 5 white marbles and 20 black marbles.

Let A, be defined as the event of drawing out the white marble, labelled number 1. Therefore $\bar{A}_1$ is the event of not drawing out the white marble, number 1. $\bar{A}_2$ is the event of not drawing out the number 2 white marble, and so on until $\bar{A}_5$, which is the event of not drawing out white marble, number 5. From basic probabilities we get

$$P(\bar{A}_1) = \frac{24}{25}$$

$$P(\bar{A}_1|\bar{A}_2) = \frac{23}{24}$$

$$P(\bar{A}_1|\bar{A}_2\bar{A}_3) = \frac{22}{23}$$

$$P(\bar{A}_1|\bar{A}_2\bar{A}_3\bar{A}_4) = \frac{21}{22}$$

$$P(\bar{A}_1|\bar{A}_2\bar{A}_3\bar{A}_4\bar{A}_5) = \frac{20}{21} \tag{15}$$

The last equation in the group of equations (15) is read as the probability of not drawing out number 1 white marble having already drawn out 4 marbles which have not been white.

The probability of drawing a white marble, using the conventional product rule would be $[1 - P(\bar{A}_1\bar{A}_2 \ldots A_5)]$

$$P = [1 - (\tfrac{20}{21})^5] = 0.2385$$

But by applying equation (12) we get, for n = 5

$$P = 1 - R_d = 1 - \frac{\frac{20}{21}}{5 - \frac{20}{21}(5 - 1)}$$

$= 1/5$ which is the correct solution.

As seen from the above example, the product rule has an inherent error in it when circumvention exists. To illustrate this further, values of probability have been calculated using the product rule and the theory of combined dependabilities. These values are listed in Table 1.

It is obvious that the product rule is more pessimistic of the final reliability than the combined dependability theory regardless of the number of components in the device.

DESCRIPTION OF TEST TECHNIQUE

LAMP THEORY

The experimental model used for testing a series reliability combination was ten miniature "GE-47" incandescent lamps. To estimate the reliability of a single component, the lamps were operated in isolation until failure. The major problem was to ensure that each lamp was subject to the same environment or "stress" both alone and in combination.

The manufacture rates this lamp for a design voltage of 6.3 volts and a design current of 0.15 amperes, resulting in a mean life of 3000 hours. A diagram of this lamp showing filament shape is shown in Figure 1.

This long life of the lamp made it imperative to use an accelerated test technique. The easiest method to decrease the life of a lamp is to increase the applied voltage.

A change in the applied voltage in a tungsten filament lamp from the rated voltage will increase or decrease lamp life, depending in which direction the voltage was changed. Lamp life varies inversely as the 12th power of the ratio of the applied voltage versus the rated voltage.

Figure 2 illustrates this effect. Also shown in Figure 2 is the effect applied voltage has on candlepower and current. Candlepower is directly proportional to the

3.5 power of the ratio of the applied voltage versus the rated voltage. Current consumption is approximately directly proportional to the 0.55 power of the ratio of applied voltage versus the rated voltage, for small ratios of voltage not exceeding approximately 2.0.

With this guideline, the applied voltage was increased until a favourable mean life occurred. A mean life of approximately 200 seconds was considered reasonable. A mean life of 200 seconds occurred when the applied voltage was approximately 21 volts. This voltage was verified experimentally and empirically. The resulting current would be 0.24 amperes as determined experimentally.

A visual inspection of the lamps revealed that each lamp had variations in filament length and diameter. These variations have very large effects, proportionally, on the voltage drop across each lamp which in turn effects the current.

DISCUSSION OF TEST PROCEDURE

In the reliability tests, it is necessary for each lamp to be subjected to the same stress. One possibility is to use constant voltage, in which a potentiometer would be placed in parallel with the lamp in order to adjust each individual voltage to the same fixed amount. The disadvantage of this arrangement is that the voltage across the lamp must be monitored constantly and the potentiometer would, in turn, be adjusted as the lamp filament deteriorated. This lamp circuitry is shown in Figure 3.

When considering ten lamps in series, the constant voltage situation makes the monitoring of ten lamps difficult. Monitoring ten voltages simultaneoulsy cannot be done with the same accuracy as monitoring one voltage; therefore, there would be errors in potentiometer settings. Some potentiometers would not be attended to as regularly as required because of the heavy work load put on the experimenter.

The other disadvantage of the "constant voltage stressing" occurs at the commencement of the test. As stated earlier, each lamp varies in filament characteristics, that is, filament diameter, resulting in variations in resistance of the lamp. Because each lamp has a slightly different "hot" resistance than another lamp, the voltage drop across it will also vary. A more detailed explanation will be presented later.

Each lamp would have to be adjusted to the proper voltage at the start of the test. This initial adjustment would require approximately 5 seconds to complete. During this adjustment phase, the lamp would not be subjected to the voltage of the test. But, this adjustment time would only constitute about 2 percent of the total test time, and the voltage variation initially is usually only plus or minus 1.5 volts. This variation in the relatively short time was not considered a threat to the accuracy of the test results.

The major problem of the constant voltage occurs when ten lamps are operated in series. Since each lamp must be adjusted initially, then the 10th lamp would not be adjusted until the preceeding nine lamps were corrected for proper voltage. Using the above figures of a 2 percent adjustment time and ±1.5 volts variation, the extreme case would show that the 10th lamp could be operating at 22.5 volts (21 volts standard) for approximately 20 percent of the test before being corrected to 21 volts. This higher voltage over this period of time would considerably shorten the mean life of the lamp, which would invalidate the test because the prime purpose of the test is to determine the reliability of the lamps for a specific condition of stress.

The above arguments favoured the use of "constant current stressing" over "constant voltage stressing". The new circuitry for constant current is illustrated in Figure 4. The current is easier to control than the voltage. Since each lamp has a different resistance, the voltage drop in the lamps will vary. The test results using constant current would be more valid than the results using constant voltage.

THEORY OF CONSTANT CURRENT

The use of constant current eliminates the variable, current decay. If a lamp is subjected to an initial current and then allowed to operate without any adjustment to the current, the filament will degenerate. This degeneration of the tungsten filament will increase the lamp resistance and there will be a resulting current decrease.

Because of the slight variation in initial resistance and filament structure, which effects the voltage drops across the lamp, the current will also vary due to these imperfections in construction of the lamp.

Figure 5 illustrates a typical plot of current versus time. The current is plotted from time zero, which occurs when the lamp circuit is closed, to burn-out time, when the filament fails and opens the lamp circuit. The curve can be segmented into three major regions.

The first region of the current plot is due to current surge, also known as inrush current. This high inrush current occurs in a condition of "cold" resistance of the filament. Filament resistance is a function of filament temperature. When voltage is applied to a "cold" lamp, the resistance is very low and a high surge current develops. Once the filament is stabilized the resistance is high and current will be at its' rated value. The steady state value of current is reached when the rate of electrical energy input is exactly balanced by the equivalent rate of thermal energy output. The ratio of initial inrush current to rated current at which the lamp stabilizes is a function of the increase in resistance from room temperature to the final operating temperature. The Chicago Miniature Lamp Works quote the inrush current at about 11 times rated current for vacuum lamps.[2]

Region 2 is the largest interval. This is the normal operating region where the filament is decaying at reasonably steady rate. In this region normal wear-out is occurring.

The third region is the interval in which rapid failure begins and is completed. The rate of decay increases in this interval until complete failure has occurred. This interval occurs in approximately the last five per cent of the total time. In other words, the remaining five per cent of the life of the lamp begins when an increase in current decay is noticed until the filament fails and opens the lamp circuitry.

The objective was to maintain a constant current and thereby eliminate the problem of current decay in each lamp. Each lamp, if left unattended, would develop its own decay rate. Although the initial current would be the same in each case, the resulting decays and failure times would not be kept constant. Thus, the lamps were not allowed to "run wild" after initial set-up. The current was to be controlled, in order that each lamp would be subjected to a constant, continuous stress which would be common to all lamps.

The resulting current-time plot was to be that of Figure 6. The current was monitored and as drop in current was noticed, the potentiometer(which was in series with the lamp) was adjusted to maintain the original current of 0.24 amperes. The initial current surge could not be eliminated. Since it occurred for every lamp, it was not considered a determining factor in lamp life.

In practice, the actual constant current curve resembled that of Figure 7. Considering the scale on the ammeter and the thickness of the pointer, the best current tolerance possible on 0.24 amperes, was plus or minus 0.002 amperes. This accounts for the cycling curve about the theoretical constant current line.

When ten lamps are operated in series, the reason for using a "constant current stressing" is even more obvious. If all ten lamps were allowed to "run wild" the actual current in the circuit, as seen in the ammeter, would be an average of the currents of each lamp due to their individual current decays. In this case, each lamp in the series would not be allowed to develop its own current decay, as it was when tested individually. Because of this average current, some lamps would be experiencing a greater current than they would normally if alone. This would decrease their expected life because they would not be operating at the same stress as when tested individually.

Figure 8 illustrates this point. Only five lamps are used for this example in order to keep the plot simple. As seen from the curves, lamp #2 has the largest rate of current decay and lamp #3 has the smallest rate of decay. When all five lamps are connected in series and subjected to an initial common current, the resulting average current would not represent any one of the five lamps. Thus lamp #2 would be experiencing a current greater than expected which would decrease its life considerably. Lamp #3 would experience a lower current than expected which would increase its life. No lamp of the five would be operating according to the test conditions which prevailed in the testing procedure of the lamps individually. The resulting reliability curve for the single lamps therefore, could not apply to this set of lamps when they are tested in series.

CYCLIC TESTING

In the initial stages of adopting an accelerated test method, a cycling system was conceived along with an applied current higher than rated.

It is known that a lamp will fail at an earlier time when it is constantly turned off and on. This cycling produces large temperature gradients in the filament which is accompanied with very high current surges when the lamp circuit is closed. These temperature gradients will also produce large thermal stresses in the filament structure itself. The conditions produced by cycling deteriorate the filament at a faster rate than constant current would.

Following this line of thought, an apparatus was constructed which would open and close the lamp circuit at any frequency rate desired. The system consisted of a universal AC-DC motor driving a rotary switch through a v-belt and pulleys. In the motor circuit, a rheostat was installed to control the motor voltage drop and thus the motor speed. The cam in the rotary switch was adjusted to allow the lamp circuit to be closed for 90 degrees of the revolution.

The test procedure for this apparatus consisted of first establishing a lamp cycling frequency of 175 cpm by adjusting the rheostat while operating a strobescope on the rotary switch pulley. Having adjusted the motor speed to the planned switching rate, the rheostat was left at this setting.

The lamp was then installed in the socket and the circuit closed for five to ten seconds for the purpose of adjusting the filament steady state current to 0.230 amperes. The motor was then turned on and a digital counter and photo-cell arrangement recorded the number of cycles until lamp failure. The digital counter and photo-cell used in this test method are the identical pieces of apparatus as used in the accepted accelerated test technique, which has been described previous to this section.

Once under way, the entire apparatus was left unattended, but it soon became apparent that, with time, the motor began to slow down. The constant operation caused a temperature increase of approximately 30°F in the motor. This temperature increase caused a resistance change in the motor-rheostat circuit, leading to a voltage change across the motor and consequently to a speed change. After two hours of operation, it required approximately a seven volt increase across the motor to maintain the original speed.

It became necessary to constantly monitor the rotary switch speed with a strobescope and adjust the rheostat accordingly. This produced a heavy work load on the experimenter which was a definite disadvantage because it required total attention to the apparatus. The other problem encountered in this method was the motor control itself. The best possible speed control with the rheostat was plus or minus ten rpm.

The test conditions could not be kept consistent for each lamp which invalidated the results. This method was discarded for a more feasible technique.

TEST APPARATUS

Having decided upon the configuration of the lamp circuitry, the next step was to devise a system of accurately measuring the time-to-failure of the lamps.

Installing a relay in the lamp circuit to trigger a timer was considered. However, this was considered not to be practical because the relay might draw too much voltage or current and this might present a problem when the lamps are tested in series.

The optimum method would be to allow the lamp circuit to be independent of all other circuitry in the experiment. This meant that an external sensing device was necessary to measure the time-to-failure of the lamps. In order to attain maximum independence of the lamp circuitry, the lamps were operated by direct current and all other measuring systems used alternating current. This insured that if, for some reason, one of the measuring units created a power surge or possibly a short circuit the lamps would continue to operate and the test could possibly still be salvaged instead of being discarded.

The external sensing device consisted of a photo-cell circuit which triggered a digital counter. A detailed description of the test apparatus is given in the appendix. At this point, only a brief explanation will be given of the function of the photo-cell system.

The photo-cell principle is that the resistance of the photo-cell will vary depending upon the intensity of light to which it is exposed. That is, when there is an absence of light, the photo-cell will take on the characteristics of a very high resistance. In some cases this can almost be considered an open circuit. At the other extreme, when there is a large intensity of light upon it, the photo-cell will have a very low resistance, causing a large current flow and a small voltage drop across the cell.

By using this principle, the photo-cell was able to turn a digital counter on and off. In the "rest" condition, when the lamp was off, the photo-cell had a high resistance and a large voltage drop across it. This meant that the other resistances in the photo-cell circuit had very low voltage drops. The counter was connected across one of these resistances. Only when the resistance had a large voltage drop, 6 to 9 volts, across it would the counter operate. The counter would continue to operate until the voltage across the resistance dropped to less than 1.0 volts, which meant that the lamp had failed and the photo-cell's resistance had increased.

The only difficulty encountered in this arrangement was the amount of light on the photo-cell. Although the light output of the lamp was quite significant due to the three-fold increase in voltage, the light intensity did not remain constant which was due to darkening of the lamp glass.

During the test, the tungsten filament evaporated slowly and was deposited on the interior surface of the lamp glass. This deposit darkened the glass and produced a significant decrease in light output. This decrease in light was sufficient to increase the photo-cell resistance and trigger the counter to the off position. The photo-cell sensed the decrease in illumination as sufficient to describe lamp failure. To overcome the sensitivity of the photo-cell system, a paper tube was placed around the lamp and aimed at the photo-cell so that most of the light would be concentrated in a small area, the area which is occupied by the photo-cell. The darkening of the glass would only produce a relatively small decrease in illumination with respect to the total light concentration. This proved to be sufficient, in that the photo-cell was not triggered until lamp failure.

When ten lamps were tested in series, the general illumination produced was adequate to control the photo-cell without the aid of a tube. No noticeable difference in life was perceived when the lamps were tested with and without the paper tube.

RESISTANCE MEASUREMENT

A secondary experiment was also being performed at this time. It was hoped to correlate the initial cold resistance of a lamp with its time-to-failure and also to predict the voltage drop across the lamp, knowing its cold resistance and the applied current.

A high accuracy ohmmeter is required for this, capable of maintaining a reasonable accuracy over a range from 4 ohms to 150 ohms. The low end of the range would be used most often.

The normal ohmmeter does not possess an accurate low ohm scale and an accurate ohmmeter was unavailable for this experiment. The solution was to rig up an ohmmeter using a strain gauge meter reading out into a digital voltmeter. The reader is referred to the Appendix for a complete description of the apparatus.

A strain gauge meter measures change in resistance. The initial resistance of a strain gauge varies from 50 ohms to 2000 ohms depending on the make of the gauge. Since the resistances of the lamps were to be below the strain gauge meter minimum of 50 ohms, two resistances of 1000 ohms each were substituted for the strain gauges, to bring the overall resistance into the center portion of the strain gauge meter capabilities. A lamp socket for the lamps was placed in series with one of the resistances, as shown in Figure 9.

In this way, the bridge could be balanced using these two resistance to increase the resistance of the circuit to an acceptable level. When the lamp was placed in the socket, in series with the resistances, the bridge would register a change in resistance in one of the two circuits. The other circuit was used as a reference. This change in resistance was recorded as a change in voltage. The read out dial on the strain gauge meter was not capable of distinguishing the very small changes in voltage. To retain the accuracy of this bridge, this voltage was read into a digital voltmeter.

The lamp could be placed in the socket and the change in voltage in the loop would be shown on the digital voltmeter. This technique could thus be used to measure the initial cold resistance of the lamp before it was tested.

The calibration of this apparatus involved the use of two sliding resistors and a Wheatstone Bridge. The sliding resistor was placed in the psuedo strain gauge circuit in place of the lamp socket. The calibration involved varying the sliding resistor from 0 ohms to 50 ohms in regular intervals. The voltage shown on the digital voltmeter was recorded and then, using the Wheatstone Bridge, the resistance of the sliding resistors was accurately found. The resulting data produced a linear calibration curve of the form:

$$V = 0.1403\ R - 3.495$$

or

$$R = 7.276\ V + 24.93$$

where V is volts and R is ohms.

INTERPRETATION OF DATA

The transformation of raw data into useful results for the resistance measurements was straight forward. The calibration curve was linear and no correction factors were necessary.

The calculations involved in the analysis of the time-to-failure of the lamps and their reliability was slightly more complicated, though a long and tedious operation. A Fortran computer program was written to perform these calculations. The failure times were read in the program in the order in which they occurred. The program then proceeded to arrange the times into numerically ascending order and calculated their corresponding rank-order numbers.

A visual check of the data showed the times to range from 100 seconds to 300 seconds. This 200 second interval was divided into 20 intervals of 10 seconds each. The failure times were then categorized into one of these 20 intervals. This produced a frequency histogram. The cumulative frequency and then both curves were normalized. A reliability curve was then calculated from the cumulative frequency curve.

These results were yet unsmoothed. A Weibull function was fitted to the pointsof the frequency histogram. The actual process consisted of plotting the normalized cumulative frequency curve versus the time. The two variables were plotted on Weibull co-ordinates and the resulting points were fitted by the method of least squares. To check the validity of the calculated curve, a correlation factor was calculated between the fitted curve and the data points. The fitted Weibull curves are shown later with the Frequency Distribution Histograms for the single lamp tests.

All calculations to this point were common for both the single lamp tests and the test of the lamps in series. The calculations for the lamps in series was terminated here. An additional calculation, for the single lamps, included predicting the reliability curve for the ten lamps in series using the Product Rule and also predicting the reliability curve for the ten lamps in series using the Combined Reliability Theory.

SIMULATION

A Fortran computer program was written for the purpose of simulating the failure distribution of ten lamps in series, knowing the failure frequency of the lamps when operated individually. The program was intended to simulate a failure in one of the lamps of the ten lamps in series. Everything remaining constant and common among the ten lamps, the lamp with the lowest reliability, of the ten would fail.

Monte Carlo simulation was incorporated into the program to generate a reliability curve for the lamps in series. The program read in points from the normalized frequency histogram of the single lamp failures. This frequency histogram was determined from the experimental results. Random numbers were then generated two at a time. The first random number was located on the abscissa or time axis. The frequency corresponding to this time was found from the input frequency histogram.

Since the program was working with histograms, the first random number of the generated pair was located in an interval and the frequency of this interval was noted. No form of interpolation was used between frequency points. All curves generated in this program remained as histograms.

The second random number generated was compared with the frequency which corresponded with the first random number. If the second random number was less than or equal to this frequency, then the first random number was considered to be a valid failure time and it was then allocated to a temporary storage location. If the second random number was greater than this frequency, both random numbers were rejected and two new random numbers were generated. This process was repeated until ten valid points were generated. These ten points represented ten lamps to be tested in series.

If these ten lamps are tested in series, then the lamp with the lowest life will fail, if all conditions and stresses are common to the ten lamps. The ten generated points were arranged in numerically ascending order and the lowest numerical point was considered to be the time at which the ten lamps in series failed. This time-to-failure is placed into a storage matrix and the entire procedure is repeated. That is, random numbers are generated in pairs until ten valid points are found and the lowest time of the ten is the time-to-failure of the ten.

The simulation program repeats itself until 500 failure times were generated. This represented 500 tests of ten lamps in series. These 500 times were categorized into one of the 20 intervals which subdivide the time axis. A frequency histogram is formed which becomes the basis for the cumulative frequency curve. The cumulative frequency curve is then normalized and the reliability curve is calculated.

This reliability curve is the simulated reliability curve of the ten lamps in series when the lamp with the lowest life fails. The resulting curve is plotted in Figure 10. The Fortran simulation program is listed in the appendix.

This simulated curve agreed very closely with the reliability curve produced by the Product Rule from the single lamp results. In fact, the theory used by the simulation program is actually the Product Rule only stated in a different manner. It was then concluded that this simulation program could not be used for the prediction of reliability results for the Combined Dependability theory.

RESULTS AND DISCUSSION

Four separate tests were performed for the single lamps to establish a reliability curve for the GE-47 miniature lamps at 0.24 amperes. The raw data for the single lamp tests is listed in Tables 2a, b, c, and d. The frequency distribution histograms are shown in Figures 11a, b, c, and d. In all, four separate single lamp tests were run. Each test was performed at a separate time from the other tests. There was approximately a one week time difference between the tests so that time would be available to check and verify the validity of each test. If one test produced inconsistent results, the entire test program would not have to be repeated, only a small section would require repetition. A Weibull curve was fitted to each histogram and the correlation factor was calculated.

A bias of the Weibull curves on the ends of the frequency histogram was discovered. The initial point on the Weibull curve was always greater than zero and the final point reached zero before the histogram itself became zero.

The overall correlation between the experimental points and the curve was approximately 0.98 or better. Although the correlation seemed good, the ends of the Weibull curve, from two standard deviations outward, did not seem to follow the same pattern as the experimental points. The Weibull curve produced an early increase at zero time and a premature decrease at the final time interval.

The effect could be partially explained by the steep slope of the Weibull curve. Generally, the Weibull formula is valid when the slope of the curve lies between 0.5 and 6.0. The curves produced by the experimental data had slopes greater than 6.0 but less than 8.0.

The inaccuracy of the end points caused by the Weibull fit could not be tolerated, and its use was abandoned. The raw data points were considered to produce a sufficiently smooth frequency curve so that no curve need be fitted to them.

The four frequency histograms were summed into one histogram which could be considered as a more accurate representation of the distribution of the times-to-failure of the GE-47 miniature lamps at 0.24 amperes current. The resulting normalized frequency distribution histogram is shown in Figure 12. The corresponding cumulative frequency and reliability curves are depicted in Figures 13 and 14. From these curves it is seen that the unfitted curves themselves are relatively smooth and that no curve fitting need be applied. All calculations performed on the single lamp data used the experimental results rather than the fitted curve points.

Having determined the reliability curve for the single lamps for a given condition of stress, calculations were then made to predict the reliability curve for ten lamps in series by the methods of Combined Dependability and the Product Rule.

At this point of the experiment, the resistance measurements were terminated due to a lack of meaningful results.

The lamps were then tested in series. One test was conducted with the ten lamps in series. The resulting data from the tests is given in Table 3. To ensure that the lamps were failing randomly in all of the ten lamp sockets, the frequency of failure in each socket was noted. A histogram showing the number of failures versus socket number, for all lamps tested in series, is given in Figure 15.

A CHI-SQUARE Goodness-of-fit test was performed on the lamp failures in the 10 sockets. The probability of a lamp failure in anyone of the 10 sockets should be 0.10. For a level of significance of 0.01, the maximum absolute difference between sample and population functions is 0.490. The actual test results produce a chi-square of 0.484, which is below the maximum allowable difference. Therefore, the failures in the sockets can be considered as being random failures.

The frequency distribution histogram for ten lamps in series is shown in Figure 16. This histogram is formed from the combined results of the two tests. The reliability curve for the ten lamps in series along with the predicted reliability curves by the Combined Dependability method and the Product Rule is illustrated together in Figure 17, for the purpose of comparison of the curves.

Another attempt was made to fit the experimental data to a Weibull curve, but the slope of the Weibull curve was extraordinarily high, approximately 11 to 12. This large slope produced a lack of confidence in the Weibull fit, and the fitted Weibull curve was discarded.

The reliability curves produced by the various methods showed large deviations from each other. The curve produced from the theory of Combined Dependability is the most optimistic of the reliability curves. This curve diverges from the Product Rule as the failure times increase. At 100 seconds, the two curves are almost identical. but at zero reliability for both curves, the Combined Dependability curve has a failure time approximately 30% greater than the Product Rule. The experimental reliability curve is very pessimistic with respect to the other reliability curves. A possible explanation for this variation of the experimental curve from the theoretical curve is given in the next section.

The results of the resistance measurements did not prove to be of any aid in predicting failure times of the GE-47 miniature lamps. A plot of time versus "cold" resistance for the lamps of test number 1 and 2 are given in Figure 18. No obvious correlation could be found to explain the random pattern produced. No attempt was made to fit any curve through these points.

The discrepancy between the experimental results and the theoretical results can be partially explained by the operation characteristics of the lamp itself.

A filament lamp fails when current cannot flow through the filament and thus not being able to produce illumination. This can be caused either by an open circuit in the lamp (a filament break) or the filament is shorted out. The most frequent failure is the destruction of the filament due to deterioration and evaporation of the tungsten climaxing in a break in the tungsten filament.

There are many variables to be considered when performing destructive tests on miniature lamps. The most obvious of these are current, voltage and temperature. Current and voltage can be controlled directly while temperature is controlled indirectly through current and voltage. Physical characteristics and properties of the lamp effect the lamp life and cause erratic behaviour of the lamps. Aside from the physical characteristics, a higher applied voltage than rated voltage brings about unforeseen complications resulting in life variations.

Since voltage and current effects are known, let us consider the physical characteristics of the lamp when being operated until failure.

One cause for a deviation from normal life, or a shortening of life, can be attributed to filament shorting. This occurs to some degree in most types of coil filament lamps. Shorting is caused by filament movement which can result from inconsistencies in the normal expansion and contraction of the coiled filament. Temperature and time produce stress relaxation in the filament. The stresses which are removed are stresses which were induced in filament fabrication, inherent stresses resulting from the drawn fibrous nature of the filament wire and stresses induced in filament mounting. This stress relaxation can cause change in the filament form and/or touching of adjacent filament coils. The degree of stress relaxation varies according to the design and the extent to which their effects have been minimized through processing. If a filament twists and shorts out causing the effective length of the filament to become less, the filament temperature increases and results in a greater rate of darkening due to tungsten evaporation.

When operating lamps with a D.C. power supply, as in this case, lamp life is affected by electromigration in the tungsten filament which results in a more pronounced filament grain growth. The Chicago Miniature Lamp Works state that they believe that electromigration and the Soret Effect both contribute to the development of hot spots and the eventual failure of lamp filaments.[3]

Electromigration is the tendency for a grain of tungsten to take on a sawtooth form when a direct current potential is applied. The directional orientation is dependent upon the polarity. Each lamp will be affected to some extent by this phenomenon.

Filament grain growth is the tendency for smaller crystals of tungsten to combine and form larger crystals when heated above the recrystalization temperature.

Another factor which affects lamp life is vacuum deterioration. This is the result of either leakage or outgassing of some component within the lamp envelope. Out gassing is the tendency of a liquid or solid material to turn to gas as temperature increases and pressure decreases.

During accelerated testing, too much wattage will cause outgassing of the glass envelope, premature darkening and failure.

The ability of the lamp to dissipate wattage contributes to the life of a lamp. The wattage dissipated within a lamp controls the rate at which the water cycle occurs. The larger the lamp, the greater the wattage that can be safely dissipated. The water cycle is a deterioration phenomena of water vapour outgassing from the interior glass surface of a lamp caused by temperature of the filament, wattage dissipated by the lamp and/or the thermal ambient in which the lamp operates.

A brief description of the water cycle can be given here. Evolved gases are brought about by chemical decomposition and release of absorbed gases which are principally water vapour, carbon dioxide and nitrogen which are detrimental to the lamp. Molecules of water vapour decompose when heated by the filament. Free hydrogen passes-off, but oxygen combines with tungsten to form tungsten oxide which is transmitted to the glass walls of the envelope. There free hydrogen reduces the oxide, leaving metallic tungsten on the glass walls and combines with free oxygen to form water molecules. The molecules then return to the filament and a new cycle is begun, continually removing tungsten from the filament to be deposited on the walls.[3]

Temperature of the filament and lamp and the rate of dissipation of this temperature is an important property of a lamp when considering and predicting its life. A lamp filament must have the ability to dissipate the thermal energy generated at a rate fast enough to prevent any temperature build-up on any spot on the glass envelope, so that there will be no release of gaseous impurities into an otherwise good vacuum surrounding the filament.

In a lamp, the dominant characteristic that determines life performance is the maximum temperature attained by the filament in a zone midway between the filament terminals, on low voltage lamps without anchor wires.[4]

The filament temperature will control the rate at which evaporation causes darkening. The amount of glass surface area and proximity of the filament to the glass determines how quickly the lamp will darken. The smaller the surface area the sooner the lamp will darken.

The results of two individual lamps tested by Chicago Miniature Lamps Works are listed in Table 4. Of particular interest is the rapid rate of temperature increase as the voltage increases. The temperature of the filament is approaching the melting point of tungsten, which is 3655^{o}K.

It is normally recommended that higher volts than 150% of design volts is not advisable from a control stand point, mostly from overheating of the lamp[5]. The actual voltage used in the test was in the neighbourhood of 350% of design voltage. At this applied voltage, too many factors controlled the lamp life.

The governing factor in the operation of an incandescent filament lamp is the large changes in the characteristics of a lamp which are produced by a small change in filament temperature.

CONCLUSIONS

Lamps under voltage that is higher than normal tend to be less stable in terms of brightness and current and exhibit more erratic life performance than when operated under lower voltages.

The life of a miniature lamp is affected by several conditions. These are:

(1) The filament colour temperature will determine the filament evaporation rate and can be directly related to lamp life.

(2) The electrical conditions under which the lamp is operated also determine lamp life.

(3) The particular processing techniques used in the manufacture of the lamp affects life due to such things as filament shorting and not a perfect vacuum.

Of the above three conditions, the filament temperature was a factor least considered and about which little was known. As always, this factor became the one of most concern and importance. The lack of control of filament temperature, among other things that were mentioned in the text, possibly produced the unexpected results.

Although current was maintained at a constant amperage, it is not unlikely that the filament temperature was not constant for all lamps.

When the lamps were tested singularly, they were not affected as much by filament temperature as when tested in series. The physical proximity of the ten lamps in series with each other (centre line distance = 2.0") was a possible contributing factor to the life of the lamps.

Because the lamps were tested at voltages much higher than the rated voltage, at a point where the filament temperature was quite near the melting point of tungsten, the lamp was approaching instability and the slightest bit of extra energy could cause premature failure. Being close to one another, the radiant heat generated by one lamp would almost certainly affect its neighbour. This extra energy, slight as it is, could cause the glass envelope to raise in temperature and cause the lamp to increase its rate of outgassing and thus produce vacuum deterioration. Which is to say that there is doubt that the lamps in series were exposed to the same stresses that were used in the single lamp tests.

It is unfortunate that the Combined Dependability theory could not be verified in this experiment, but the experiment itself has helped to form a better basis and understanding of the application of stresses to lamps.

This lack of verification of the Combined Dependability theory should not discourage any further attempts to prove it. The problems encountered in this experiment have proven to be a great asset in establishing a better approach to the problem of testing lamps for the purpose of performing reliability studies.

A section of recommendations has been prepared to guide future experimentation in lamp life testing. It is hoped that these recommendations will prove fruitful in their attempt to prove the Combined Dependability theorem.

At this time, consideration must be given to the immense difficulty and inherent complications that are involved in the prediction of the reliability of even a simple device consisting of a few basic components.

The life of a component and ultimately the life of the device in which the component exists and the degree of component interaction is extremely difficult to predict using the reliability and probability theories of today. This fact has been shown in this thesis.

Every possible variable, whether it be direct or indirect, influences the life of a part or a machine. It is no easy task to estimate the operating life of a component. Extreme caution must be exercised in first, recognizing all stresses affecting the component and second, determining the influence of each on this component. A slight miscalculation of a single stress factor will produce an erroneous expected component life which in turn will adversely influence the reliability of the parent device.

The interaction between components is difficult to predict since each component is tested as an individual unit. Component interaction is difficult to measure and more difficult to simulate. Caution must be exercised when performing life tests on components to duplicate actual operating conditions of the component as if it were operating in the device.

In general, there can be no perfect simulation of operational stresses. Crude simulations are used which produce under-rated lives for components. This can be considered adequate for most applications but it is not utilizing the ultimate possible performance of a component and, thus, the design of the device can not be considered optimum.

Again, it must be said that performing life tests on components and then predicting their reliability and the reliability of the parent device is a delicate project requiring extreme care in simulating actual operational conditions.

RECOMMENDATIONS

A more realistic approach to life determination of lamps and from there a reliability study of them is to use a modified accelerated life test. This form of testing would use low voltages, much lower than the voltages used in the previous tests, in order to avoid high filament temperatures. Chicago Miniature Lamp Works have established a test procedure for lamp life[5] and reliability where not one voltage but several different voltages are applied to different batches of lamps. This will integrate the variables that affect life performance for each single lamp; so that the average value to burnout of each test lot will follow and satisfy the power law equation:

$$T'\left(\frac{V'}{V'}\right)^{x} = T''\left(\frac{V''}{V'}\right)^{x} = T'''\left(\frac{V'''}{V'}\right)^{x}$$

$$= T'^{v}\left(\frac{V'^{v}}{V'}\right) = \text{etc}$$

V', V'', V''', V'^{v},etc. are different higher than design voltage values and x is the common exponent that will satisfy the v' as a common denominator with equal life for each one of the accelerated life tests. T', T'', T''', T'^{v},etc. are burnout times for each voltage V', V'', V''', V'^{v},etc.

To further illustrate this technique, an example is listed of a fictituous test using three batches of lamps with each lamp batch being subjected to a voltage, higher than rated, and different from the other batch voltages. An assumption is made that the number of lamps in each lot is sufficient to produce an accurate estimate of average life.

Lamp Lot	Voltage	Assumed Average Life
1	$V' = 7$	850 hr.
2	$V'' = 8$	175 hr.
3	$V''' = 9$	40 hr.

Because of the different voltages each lot of lamps will vary with respect to time-to-failure. Since the rated voltage of these lamps is $v' = 6.3$, then the voltage values can be substituted into the above equation to solve for the exponent x.

$$\left(\frac{7}{6.3}\right)^x = (1.111)^x = \text{antilog}(x \log 1.111)\text{test } 1$$

$$\left(\frac{8}{6.3}\right)^x = (1.270)^x = \text{antilog}(x \log 1.270)\text{test } 2$$

$$\left(\frac{9}{6.3}\right)^x = (1.428)^x = \text{antilog}(x \log 1.428)\text{test } 3$$

By equating any two of the test conducted, x can be found:

eg. test 1 = test 2

$$\text{antilog}[x \log 1.111]\ 850 = \text{antilog}[x \log 1.270]\ 175$$

$$x = 12.58$$

Equate test 2 to test 3, and test 1 to test 3. The x exponent should coume out the same or nearly so for each of the three equations. This is true provided that the quality of the lamps are uniform in workmanship, particularly in regard to a stable vacuum in each lamp throughout the respective test lots to burnout time.

As stated previously, the control of the R.M.S. value of volts when applied on many lamps during a test is critical. This statement is reiterated in the formula $T'(\frac{V'}{V})^x$, where the value of x is large. A slight discrepancy in V' will result in a relatively large error in x.

The reliability factor of these lamps should rather be based on the consistency with which the x exponent in the previous equations remains approximately constant for three or more accelerated life tests, even though the normalized average life for this lamp at 6.3 volts rated, is in excess or lower than the rated life of 3000 hours.

Theoretically, the results should produce the rated life of 3000 hours but due to many characteristics that are present in all incandescent lamps, the life of each lamp will vary from the norm.

If the exponent x is valid and consistent for various test voltages, the lamps in the different tests could be considered to be under the same stresses.

Considering the previous example of the three test voltages of seven, eight and nine volts, if upon solving for x it was found that the x for a fourth test was 13.6 as compared with 12.58 for the other three tests, then it is logical to assume that test four had other factors influencing the lamps in that test that were not present in tests one, two and three.

The results of test four could not then be used in the reliability study for the lamps under a given condition. The different voltages do not affect the test results because they are all normalized to one given voltage, e.g. rated voltage.

A filament temperature change causing a shift in the frequency distribution curve could be discovered when the x exponent is calculated for that particular test batch of lamps. The advantage of this type of multi-voltage testing is that one can detect any discrepancies that might arise in the individual test batches that might not be realized in the normal course of testing.

APPENDIX I

PHOTO CELL CIRCUIT DESIGN

The photo cell chosen to monitor the lamps was capable of a maximum voltage drop across it of 30 volts, with a corresponding maximum current of 3 milliamps. Its "dark current" was less than 15 microamps. The photo cell was incorporated into a DELTA configuration as shown in Figure 19.

The photo cell served as a switching device in this circuit. When the miniature lamp was off, the resistance of the photo cell could be assumed to be infinitely large, thereby resulting in an almost open circuit. Very little voltage drop would be experienced across the three potentiometers. When the miniature lamp was on, the resistance of the photo cell would decrease appreciably allowing for current to flow and allowing a significant voltage drop across the potentiometers.

The potentiometers were arranged so that the 0.1 megohm resistor would draw the most current through its own branch thus reducing the current in the other branch to a small amount. To illustrate this point, consider the extreme cases when resistor A is (1) 0 ohms and (2) 0.1 megohms.

<u>Case (1)</u>: When resistor A is 0 ohms a short circuit can be considered to exist across the resistor and the only

limitation to current in the circuit would be the photo cell resistance. In essence, the current through the other loop, which contains resistors B and C, can be considered to be near zero.

Case (2): When resistor A is 0.1 megohms, the maximum limit of this resistor, and assuming the voltage drop across it is 10 volts, then the current would be $0.1 \times 10^6 I_A = 10.$ $I_A = 100 \times 10^{-6}$ amperes.

Considering the other loop as having the maximum resistance which it is capable of, the current would be $I_{BC}[5 \times 10^6 + 0.5 \times 10^6] = 10.$ $I_{BC} = 1.82 \times 10^{-6}$ amperes.

It can now be seen that the purpose of resistance A is to maintain the current, in the loop containing resistances B and C, at a reasonable level. Resistor A would be considered a coarse adjustment of the voltage drop across resistor C. Resistor B would then be considered as a medium adjustment. The final resistor C, whose voltage drop was used to trigger the counter, was used as a fine adjustment of the voltage.

The final adjustment gave resistor C approximately a 6 volt drop across it when using a 10 volt D.C. power source and with the photo cell at 1 to 2 inches from a miniature lamp consuming close to 21 volts.

INSTRUMENTATION

The digital counter used in the experiment was a Hewlett Packard counter, model 5223L. The controls were set to enable the counter to initiate counting when subjected to a 6 volt input. It would stop counting when there was more than a 2 volt decrease from the input voltage. The scale on the counter was adjusted to give a three decimal place accuracy. When the time was recorded, it was rounded off to two decimal figures. This accuracy was more than adequate considering the errors produced by voltage and current adjustments during the tests.

The time lag from the closing of the lamp circuit till the counter began to count was in the micro second range. This was not critical considering the time-to-failure of the lamps. Also, this time lag was common for every test.

All calibration for the Hewlett Packard digital counter was internal. Spot checks throughout the entire experiment were performed to check the accuracy of the count.

The instrumentation used in the resistance test consisted of an Ellis Associates BAM-1 strain gauge meter and an Honeywell 630S Digital Voltmeter.

The cold resistance of a miniature GE-47 lamp is approximately 5 ohms. At time of testing, there was no ohmmeter available which was accurate in the low ohm range. A strain gauge meter was used, in its place, because of its sensitivity. Since the scale on the strain gauge meter was small and the resistances of the lamps would only produce a fractional movement of the indicator, the volt meter on the strain gauge meter was bypassed and the Honeywell digital voltmeter replaced it.

The resistances of strain gauges vary from 500 to 2000 ohms. The strain gauge meter is manufactured to operate in this range. It was necessary to bring the resistances of the lamps into this range. This was accomplished by installing two 1000 ohm ± 10% resistors in series with the lamp as shown in Figure 9.

The calibration of this apparatus involved the use of a pair of sliding resistors, a 5.2 ohm resistor and a 100 ohm resistor, and a Honeywell Wheatstone Bridge, model number 1071. The sliding resistors replaced the lamp in the strain gauge circuit. The resistors were set at regular physical intervals. The exact resistance produced was found through the Wheatstone Bridge and the voltage shown on the digital voltmeter was noted. The calibration was involved in the range from 0 ohms to 50 ohms. The resulting calibration curve was linear and of the form

$$R = 7.276\ V + 24.93$$

This curve gave every indication that it would continue to be linear for much higher resistances.

APPENDIX 2

INCANDESCENT LAMPS

Electric lamps can be categorized into two groups, the filament or incandescent types and the discharge types. The first category of lamp is the main interest in this section.

A filament lamp can be described as containing a tungsten filament, manufactured by drawing and coiling a tungsten wire. This formed filament is placed in a glass envelope containing either a vacuum or an inert gas.

The purpose of a lamp is to produce light. To produce light, something must be caused to give out radiant energy of sufficient intensity and within proper wavelength limits (0.38 μ to 0.78 μ) to affect the eye.[6] To cause any material to radiate, energy must be applied to it to cause some of its electrons to be accelerated.

Any radiator, when in a definite physical condition and excited in a definite manner, emits the same type of radiation; that is, it gives radiation having a definite distribution with wavelengths. This holds true for the radiation from the atoms of a gas, such as hydrogen and also for the radiation from a solid radiator, such as tungsten filament.

Tungsten is the metal used most for converting electrical energy into light and into neighbouring infrared and ultraviolet radiation. This metal has the highest melting point of any of the metallic elements. The reason for using tungsten is to secure satisfactory lamp life; a filament that will remain solid, neither melting nor evaporating at an unduly high rate.

Tungsten becomes incandescence when excited electrically. Incandescence refers to radiation that is due to the temperature of the source; its intensity increases rapidly with the temperature of the source, and the wavelength of the maximum of the intensity shifts towards shorter wavelengths as the temperature is increased.

Getting the energy into the light-producing part of any type of lamp results in a loss in efficiency. Conduction of energy away from the filaments of an incandescent lamp by the leads and filament supports result in such a loss. For the 120 volt tungsten-filament lamp these losses in efficiency amount ot 3% to 4% but can be in the range of 30% to 50% for miniature lamps with a very short filament[6].

To produce a gain in efficiency the filaments are coiled. A coiled filament coil does not operate at a higher filament temperature, as this would shorten lamp life, the increased efficiency is due to the increased area of incandescent surface, which results from the longer filament which can be employed.

The melting point of tungsten is 3655°K and at this temperature the efficiency of an incandescent lamp is about 53 lumens/watt. A coiled tungsten filament operates at 2800°K with an efficiency of 7 to 20 lumens/watt.[7]

The chief limiting factor in the improvement of tungsten filament lamps is not the melting point of the tungsten but the temperature at which it disintegrates or volatilises. The volatilisation of the tungsten soon blackens the bulb and also increases the resistance of the filament itself.

LAMP UNDER TEST

The lamp used for these tests is a miniature based lamp whose prime function is to illuminate radio and instrument panels. The reason for this choice is obvious. The lamp is very common and inexpensive.

The lamp number is 47 but this lamp is produced by almost every lamp manufacturer, which accounts for the code letters preceeding the number, e.g. CGE-47, produced by Canadian General Electric and CM-47, produced by Chicago Miniature Lamp Works.

Regardless of the manufacturer, the number 47 lamp has the same specifications and properties.

The recognized rating for this lamp is 6.3 volts at

0.15 amps which produces 0.50 nominal mean spherical candlepower. It is rated for an average life of 3,000 hours to burnout. This lamp is so designed that it can be made on high speed automatic equipment, and the glass envelope is large enough to easily dissipate 0.95 watt energy without undue rise in its temperature.[3]

The filament mass is equal to 0.00021 grams between terminals and the wire length is 2.5 cm. with a diameter of 0.00235 cm.[3]

REFERENCES

1. J.N. Siddall - "The Prediction of Combined Dependabilities"
McMaster University.

2. H.G. Chassie - "Inrush Current"
Chicago Miniature Lamp Works, Technical
Bulletin No. 121, May 28, 1963.

3. Chicago Miniature Lamp Works, Catalog No. CMT-3.

4. O.J. Forsberg - "Pulsing and Lyching Effects"
Chicago Miniature Lamp Works, Information
Sheet No. 130, April 15, 1965.

5. O.J. Forsberg - "In a Communication with O.J. Forsberg,
Chief Design Engineer, Chicago Miniature
Lamp Works, December 22, 1969.

6. E.S. Steeb & W.E. Forsythe - "Incandescent Lamps"
Handbook of Physics, McGraw Hill Book
Company, 1967.

7. General Electric Company, Research Laboratory, London
"Incandescent Lamps", Dictionary of
Applied Physics, Macmillan Company, Ltd.

TABLE 1

TABLE COMPARING PRODUCT RULE TO EQUATION (12)

Component Dependability	Number of Components	Combined Dependability	Product of Dependability
0.3000	2	0.17647	0.09000
	3	0.12500	0.02700
	5	0.07895	0.00243
	10	0.04110	10^{-5}
	100	0.00427	-----
0.50000	2	0.3333	0.25000
	3	0.25000	0.12500
	5	0.16667	0.03125
	10	0.09091	0.00098
	100	0.00938	-----
0.70000	2	0.53846	0.48000
	3	0.43750	0.34300
	5	0.31818	0.16807
	10	0.18919	0.02825
	100	0.02280	-----
0.90000	2	0.81818	0.81000
	3	0.75000	0.72900
	5	0.64286	0.59049
	10	0.47368	0.34868
	100	0.08257	0.00003
0.99000	2	0.98020	0.98010
	3	0.97059	0.97030
	5	0.95192	0.95099
	10	0.90826	0.90438
	100	0.49749	0.36603

TABLE 1 cont'd

Component Dependability	Number of Components	Combined Dependability	Product of Dependability
0.99900	2	0.998002	0.998001
	3	0.997006	0.997003
	5	0.99502	0.99501
	10	0.99009	0.99004
	100	0.90901	0.90479
0.99990	2	0.999800	0.999800
	3	0.999700	0.999700
	5	0.999500	0.999500
	10	0.999001	0.999000
	100	0.990098	0.990049
0.99999	2	0.999980	0.999980
	3	0.999970	0.999970
	5	0.9999500	0.999950
	10	0.9999000	0.9999000
	100	0.99900099	0.99900049

TABLE 2(a)

SINGLE LAMP TEST - TEST NO. 1

Δ Voltage "volt"	Resistance "ohm"	Time "sec"	Δ Voltage "volt"	Resistance "ohm"	Time "sec"
-2.733	5.045	266.9	-2.739	5.001	214.0
-2.736	5.023	246.9	-2.755	4.885	213.0
-2.757	4.870	156.5	-2.740	4.994	177.3
-2.736	5.023	147.1	-2.666	5.532	173.7
-2.739	5.001	248.0	-2.744	4.965	208.4
-2.693	5.299	261.8	-2.763	4.826	160.5
-2.729	5.074	249.2	-2.737	5.016	160.6
-2.747	4.943	165.4	-2.735	5.030	204.8
-2.723	5.117	130.0	-2.673	5.481	196.6
-2.747	4.843	154.5	-2.753	4.899	194.3
-2.753	4.899	220.2	-2.756	4.877	177.7
-2.756	4.877	166.8	-2.757	4.870	235.4
-2.746	4.950	156.3	-2.750	4.921	188.4
-2.723	5.117	209.3	-2.750	4.921	174.2
-2.731	5.059	134.1	-2.746	4.950	184.9
-2.754	4.892	207.3	-2.760	4.848	180.2
-2.704	5.256	236.7	-2.703	5.263	201.3
-2.719	5.147	201.8	-2.743	4.972	151.0
-2.695	5.321	240.7	-2.738	5.008	180.7
-2.763	5.212	189.7	-2.706	5.241	177.4
-2.757	4.870	254.8	-2.729	4.994	201.3
-2.740	4.994	261.3	-2.736	5.023	188.2
-2.744	4.965	248.8	-2.733	5.045	210.6
-2.234	5.110	222.8	-2.726	5.096	220.1
-2.718	5.154	172.4	-2.738	5.008	162.2
-2.748	4.936	167.6	-2.752	4.906	272.0
-2.726	5.096	205.7	-2.755	4.885	202.6

TABLE 2(a) cont'd

SINGLE LAMP TEST - TEST NO. 1

Δ Voltage "volt"	Resistance "ohm"	Time "sec"	Δ Voltage "volt"	Resistance "ohm"	Time "sec"
-2.702	5.270	196.8	-2.705	5.248	234.7
-2.746	4.950	224.3	-2.745	4.957	204.0
-2.720	5.139	180.7	-2.742	4.979	185.1
-2.761	4.841	246.0	-2.759	4.856	178.5
-2.685	5.394	184.7	-2.745	4.957	174.2
-2.747	4.943	193.7	-2.747	4.943	184.2
-2.722	5.125	216.1	-2.734	5.037	183.5
-2.721	5.132	179.3	-2.731	5.059	134.0
-2.720	5.139	175.7	-2.724	5.110	153.3
-2.736	5.023	204.1	-2.730	5.067	187.1
-2.741	4.986	194.1	-2.716	5.168	209.4
-2.732	5.052	220.9	-2.721	5.132	284.4
-2.726	5.096	203.7	-2.711	5.205	169.2
-2.821	5.132	208.3	-2.725	5.103	196.5
-2.740	5.074	201.3	-2.697	5.307	208.4
-2.727	5.088	243.2	-2.731	5.059	260.9
-2.731	5.059	249.4	-2.713	5.190	254.3
-2.644	5.692	289.1	-2.741	4.986	185.4
-2.737	5.016	189.6	-2.732	5.052	147.5
-2.726	5.096	190.1	-2.746	4.950	205.0
-2.724	5.110	155.9	-2.718	5.154	204.2
-2.737	5.016	171.4	-2.717	5.161	215.9
-2.716	5.168	226.9	-2.724	5.110	186.4

TABLE 2(b)

SINGLE LAMP TEST - TEST NO. 2

Δ Voltage "volt"	Resistance "ohm"	Time "sec"	Δ Voltage "volt"	Resistance "ohm"	Time "sec"
-2.699	5.292	204.0	-2.737	5.016	200.6
-2.699	5.292	205.0	-2.725	5.103	233.6
-2.707	5.234	123.4	-2.732	5.052	184.1
-2.697	5.307	222.6	-2.729	5.074	223.4
-2.679	5.438	169.1	-2.717	5.161	164.2
-2.709	5.219	207.6	-2.711	5.205	225.2
-2.711	5.205	285.1	-2.672	5.489	219.3
-2.705	5.248	216.6	-2.697	5.307	224.1
-2.721	5.132	257.2	-2.695	5.321	154.8
-2.728	5.081	250.9	-2.708	5.227	226.3
-2.732	5.052	220.9	-2.687	5.379	207.6
-2.727	5.088	209.4	-2.686	5.387	183.4
-2.805	4.521	181.1	-2.720	5.139	231.1
-2.742	4.979	271.3	-2.713	5.190	163.8
-2.724	5.110	182.5	-2.706	5.241	245.2
-2.744	4.965	183.9	-2.666	5.532	196.9
-2.755	4.885	192.5	-2.718	5.154	249.1
-2.747	4.943	201.6	-2.726	5.096	215.1
-2.720	5.139	170.4	-2.733	5.045	211.0
-2.720	5.139	221.5	-2.716	5.168	186.7
-2.740	4.994	200.4	-2.719	5.147	252.5
-2.702	5.270	196.8	-2.714	5.183	252.3
-2.731	5.059	103.6	-2.715	5.176	208.7
-2.741	4.986	223.7	-2.726	5.096	257.2
-2.723	5.117	244.3	-2.680	5.430	183.4
-2.722	5.125	230.3	-2.663	5.554	186.6
-2.723	5.117	243.9	-2.684	5.401	192.2
-2.707	5.234	219.4	-2.693	5.336	205.5

TABLE 2(b) cont'd

SINGLE LAMP TEST - TEST NO. 2

Δ Voltage "volt"	Resistance "ohm"	Time "sec"	Δ Voltage "volt"	Resistance "ohm"	Time "sec"
-2.736	5.023	260.3	-2.724	5.110	243.5
-2.731	5.059	232.8	-2.720	5.139	203.2
-2.701	5.278	231.5	-2.709	5.219	201.1
-2.709	5.210	201.2	-2.683	5.408	168.9
-2.721	5.132	262.2	-2.690	5.358	184.8
-2.718	5.154	223.9	-2.705	5.248	210.8
-2.696	5.314	196.4	-2.705	5.248	173.3
-2.708	5.227	199.4	-2. 713	5.190	179.2
-2.706	5.241	204.1	-2.720	5.139	267.9
-2.700	5.285	199.2	-2. 700	5.285	183.4
-2.696	5.314	200.8	-2.717	5.161	231.2
-2.726	5.096	208.7	-2.687	5.379	125.1
-2.692	5.343	194.7	-2.708	5.227	192.6
-2.736	5.023	278.6	-2.705	5.248	236.3
-2.717	5.161	179.4	-2.724	5.110	190.9
-2.692	5.343	237.9	-2.707	5.234	196.7
-2.685	5.394	157.0	-2.725	5.103	211.1
-2.803	5.263	246.9	-2.687	5.379	201.9
-2.710	4.826	189.7	-2.703	5.263	191.1
-2.716	5.168	224.8	-2.727	5.088	192.7

TABLE 2(c)

SINGLE LAMP TEST - TEST NO. 3

Time "sec"	Time "sec"	Time "sec"	Time "sec"	Time "sec"	Time "sec"
146.2	265.7	188.2	238.5	232.8	222.2
145.2	201.5	173.0	172.0	253.3	171.9
176.2	208.1	171.4	232.3	198.2	203.3
182.3	203.7	226.2	180.9	197.9	270.7
154.5	188.7	245.6	188.7	204.7	179.9
145.2	271.9	225.3	277.3	159.2	191.4
161.4	259.0	187.3	188.9	195.5	186.3
167.2	193.6	167.4	216.1	183.4	176.5
126.8	230.0	200.2	168.2	207.3	198.0
191.2	172.0	181.9	215.2	174.3	227.2
197.3	218.6	160.2	196.6	178.5	140.1
208.5	197.8	256.5	235.1	189.3	219.5
187.7	245.6	232.1	186.1	195.8	143.7
270.0	177.6	204.6	232.3	188.8	154.2
187.0	214.7	177.1	175.6	150.5	240.4
164.2	230.9	163.6	203.9	244.0	239.8
178.4	201.2	216.6	255.1	194.7	150.7
134.4	270.8	189.9	217.9	267.6	175.3
200.2	161.3	188.9	188.4	186.5	208.8
126.4	249.5	211.9	185.9	237.1	262.5
95.1	176.1	181.3	171.8	175.4	261.8
113.0	195.3	269.0	205.1	169.1	227.4
138.4	218.3	236.3	199.8	173.4	229.2
118.9	187.7	240.8	237.3	190.4	147.1
135.6	198.4	218.1	229.5	101.4	215.1
103.3	165.6	162.9	167.8	236.6	263.2
89.2	260.1	172.2	243.9	188.0	218.9
282.6	173.7	241.5	174.2	163.6	204.3

TABLE 2(c)

SINGLE LAMP TEST - TEST NO. 3

Time "sec"	Time "sec"	Time "sec"	Time "sec"	Time "sec"	Time "sec"
253.6	236.3	188.5	216.0	214.2	224.6
260.4	215.5	199.4	218.6	164.9	190.9
146.0	201.3	207.3	254.1	185.8	252.5
179.0	173.6	196.1	192.8	230.4	246.3
261.2	216.1	153.0	225.2	188.7	202.6
274.9	249.4	200.4	202.2	175.4	268.2

TABLE 2(d)

SINGLE LAMP TEST - TEST NO. 4

Time "sec"	Time "sec"	Time "sec"	Time "sec"
209.5	127.1	201.5	170.0
180.2	178.0	218.7	241.7
200.1	168.9	205.2	224.5
221.7	181.4	217.8	201.2
194.0	221.7	266.1	255.3
222.7	243.3	197.5	209.5
226.2	255.1	170.9	163.6
217.9	261.4	198.8	204.7
200.8	248.1	173.0	244.7
184.9	246.7	206.1	178.7
207.8	200.1	220.6	171.0
189.9	239.7	200.2	216.7
192.1	260.6	290.1	179.6
162.1	160.3	171.8	178.0
193.2	186.3	170.9	185.9
161.0	180.9	221.7	207.1
206.8	212.9	163.3	245.9
163.0	195.6	224.2	
180.7	184.7	228.3	
209.5	202.3	219.6	
143.5	255.7	205.9	
145.9	176.4	171.2	
200.7	243.9	260.9	
228.3	180.2	207.5	
231.1	176.7	165.8	
158.4	217.4	257.2	
209.5	209.6	206.3	
182.5	200.7	242.2	
132.5	173.9	208.0	
165.6	200.0	258.8	
265.9	207.2	191.6	

TABLE 3

TEN LAMPS IN SERIES - TEST NO. 1

Time "sec"	Failed Lamp	Time "sec"	Failed Lamp	Time "sec"	Failed Lamp	Time "sec"	Failed Lamp
131.2	1	115.4	9	117.8	7	134.8	7
126.2	3	122.1	10	134.6	2	150.7	7
131.9	1	123.3	4	125.3	5	127.1	10
103.5	9	135.8	7	137.6	9	124.4	8
117.8	7	156.3	6	133.6	10	117.5	10
117.6	3	136.2	9	136.4	3	137.1	5
116.6	6	124.4	2	143.3	1	140.1	7
135.7	10	129.6	2	150.2	6	143.9	10
135.2	10	124.2	4	135.8	9	142.6	3
123.8	9	107.5	4	125.5	7	135.1	9
128.3	1	108.0	10	138.9	4	125.4	6
138.0	7	134.9	1	135.8	9	143.9	5
126.1	8	173.1	4	131.4	5	161.0	8
133.1	8	120.9	3	155.9	10	151.6	2
133.6	1	142.3	9	122.6	1	146.1	7
123.8	9	104.9	10	144.0	8	150.1	10
130.2	9	141.9	8	138.4	5	141.3	1
145.5	4	148.9	6	134.2	8	159.1	4
124.6	9	150.5	5	149.0	3	165.2	2
108.9	5	124.2	1	132.5	9	142.6	7
131.6	10	138.5	3	123.8	1	141.1	2
142.8	10	133.1	5	136.4	3	136.4	8
128.0	3	147.0	1	123.0	6	141.3	1
140.9	2	138.7	4	157.2	6	124.9	1
133.5	8	135.6	6	126.0	6	115.1	8
107.1	7	138.6	5	145.7	4	148.7	3
110.2	1	150.6	7	140.1	2	135.2	1
123.5	4	136.2	6	146.6	7	141.2	10
127.8	5	123.5	9	163.6	10	137.6	6
111.1	4	140.3	10	140.9	7	128.1	10

TABLE 3 cont'd

TEN LAMPS IN SERIES - TEST NO. 1

Time "sec"	Failed Lamp	Time "sec"	Failed Lamp
132.4	3	138.9	1
167.6	1	129.9	6
118.0	1	150.5	4
145.5	2	140.7	4
145.3	10	131.8	5
107.8	7	115.0	2
128.6	8	122.2	10
134.4	2	111.4	3
135.3	5	145.3	9
133.6	2	141.9	8
112.4	9	123.5	9
143.2	4	153.4	3
133.6	9	141.7	2
118.7	8	127.1	6
140.5	2	142.3	5
132.0	2	136.8	6
119.2	8	139.5	4
135.6	8	139.6	9
111.3	7	130.4	1
120.9	7	138.5	4
135.9	10	136.3	4
136.1	2	146.9	8
125.5	2	142.9	1
137.7	9	140.7	9
137.0	8	117.4	2
129.7	8	145.6	5
141.8	7	136.9	6
145.5	4	129.2	7
135.5	4	137.8	1
119.7	4	152.9	7
136.3	2	128.0	2
127.5	9	118.4	7

TABLE 4

EFFECT OF VOLTAGE ON CURRENT, CANDLE POWER AND TEMPERATURE OF A CM-47 MINIATURE LAMP

	Volts	Amps	M.S.C.P.*	Temp ^{o}K
Lamp # 1	1.0	0.0530	---	--
	2.0	0.0795	0.001	1240
	3.0	0.102	0.022	1835
	4.0	0.120	0.080	2000
	5.0	0.137	0.200	2150
	5.5	0.144	0.295	2235
	6.0	0.152	0.409	2290
	6.3	0.156	0.493	2330
	7.0	0.166	0.724	2400
	7.5	0.173	0.926	2490
Lamp # 2	1.0	0.0523	---	--
	2.0	0.0785	0.002	1580
	3.0	0.100	0.021	1830
	4.0	0.118	0.079	1995
	5.0	0.136	0.197	2140
	5.5	0.144	0.289	2230
	6.0	0.150	0.403	2260
	6.3	0.155	0.485	2325
	7.0	0.165	0.708	2390
	7.5	0.172	0.910	2480

Equipment used: Weston Photocell and Inductronic Amplifier

Temperature at filament center was measured with an Optical Pyrometer.

Miniature Lamps in test were number CM-47.

* Mean Spherical Candle Power.

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C
C     SIMULATION PROGRAM TO ESTABLISH RELIABILITY HISTOGRAM OF THE LOWEST
C     FAILURE IN SETS OF TEN FAILURES
C
      DIMENSION F(500),RN(500),IFF(500),ICF(500),CF(500),R(500),RT(500),
     1RF(500), FR(500)
      READ (5,1) IKOUNT,N
    1 FORMAT (2I5)
C
C     READ IN FREQUENCY CURVE OF 'ONE LAMP' TEST
C
      READ (5,4) ( FR(I), I=1,N)
    4 FORMAT (5F10.0)
      ICOUNT=0
      J=3
C
C     ZERO COUNTER
C
      DO 20 I=1,N
   20 IFF(I)=0
      WRITE (6,5)
    5 FORMAT (1X,14H RANDOM NUMBER)
   80 K=0
C
C     GENERATE RANDOM NUMBERS, TWO AT A TIME
C
   30 CALL FRANDN (RN,2,J)
      J=0
C
C     LOCATE FIRST RANDOM NUMBER ON TIME SCALE
C
      B=0.0
      DO 10 I=1,N
      B=B+0.05
      A=B-0.05
   10 IF (RN(1).LT.B.AND.RN(1).GE.A) L=I
C
C     CHECK SECOND RANDOM NUMBER TO BE LESS THAN INPUT FREQUENCY CURVE
C
      IF (RN(2).GT.FR(L)) GO TO 30
      K=K+1
      RT(K)=RN(1)
C
C     GENERATE TEN 'GOOD' RANDOM NUMBERS
C
      IF (K.LT.10) GO TO 30
C
C     ARRANGE IN NUMERICAL ASCENDING ORDER
C
      CALL ASCEND (RT,10)
      ICOUNT=ICOUNT+1
```

```
C
C     RELOCATE LOWEST RANDOM NUMBER INTO LARGER MATRIX
C
      R(ICOUNT)=RT(1)
      WRITE (6,3)         R(ICOUNT), ICOUNT
    3 FORMAT            (5X,F10.5,5X,I5)
C
C     LOCATE THE RANDOM NUMBER ON THE 'TIME' AXIS AND ADD 1 TO THE
C     FREQUENCY HISTOGRAM
C
      B=0.0
      DO 40 I=1,N
      B=B+0.05
      C=B-0.05
   40 IF (R(ICOUNT).LT.B.AND.R(ICOUNT).GE.C) IFF(I)=IFF(I)+1
      IF (ICOUNT.LT.IKOUNT) GO TO 80
C
C     CALCULATE CUMULATIVE FREQUENCY
C
      ICF(1)=IFF(1)
      DO 50 L=2,N
   50 ICF(L)=ICF(L-1)+IFF(L)
C
C     CHANGE TO REAL NUMBERS
C
      DO 60 I=1,N
      F(I)=FLOAT(IFF(I))
   60 CF(I)=FLOAT(ICF(I))
C
C     NORMALIZE CUMULATIVE FREQUENCY
C
      C=CF(N)
      DO 70 I=1,N
   70 CF(I)=CF(I)/C
C
C     CALCULATE RELIABILITY HISTOGRAM
C
      DO 75 I=1,N
   75 R(I)=1.0-CF(I)
      WRITE (6,6)
    6 FORMAT (5X,10H FREQUENCY,3X,12H RELIABILITY,3X,9H INTERVAL)
      WRITE (6,2) (F(I),R(I),I,I=1,N)
    2 FORMAT (5X,F10.3,5X,F10.3,5X,I3)
      STOP
      END

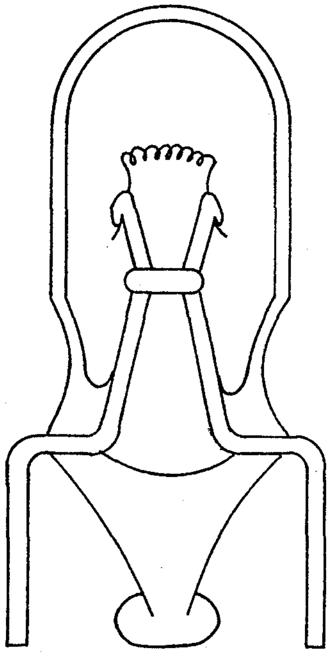
                                   CD TOT    0109
```

```
      SUBROUTINE ASCEND (X,N)
C
C     SUBPROGRAM TO ARRANGE DATA INTO NUMERICALLY ASCENDING ORDER
C
      DIMENSION X(1)
      M=N-1
      DO 10 I=1,M
      K=I+1
      DO 10 L=K,N
      IF (X(I)-X(L)) 10,10,20
   20 TEMP=X(I)
      X(I)=X(L)
      X(L)=TEMP
   10 CONTINUE
      RETURN
      END
```

```
 DATA


  500   20
   0.037     0.025     0.062      0.099      0.136
   0.198     0.420     0.667     0.778      0.617
   1.00      0.444     0.432     0.333      0.395
   0.235     0.272     0.111      0.049     0.012
                                 CD TOT    0030
```

FIGURE 1

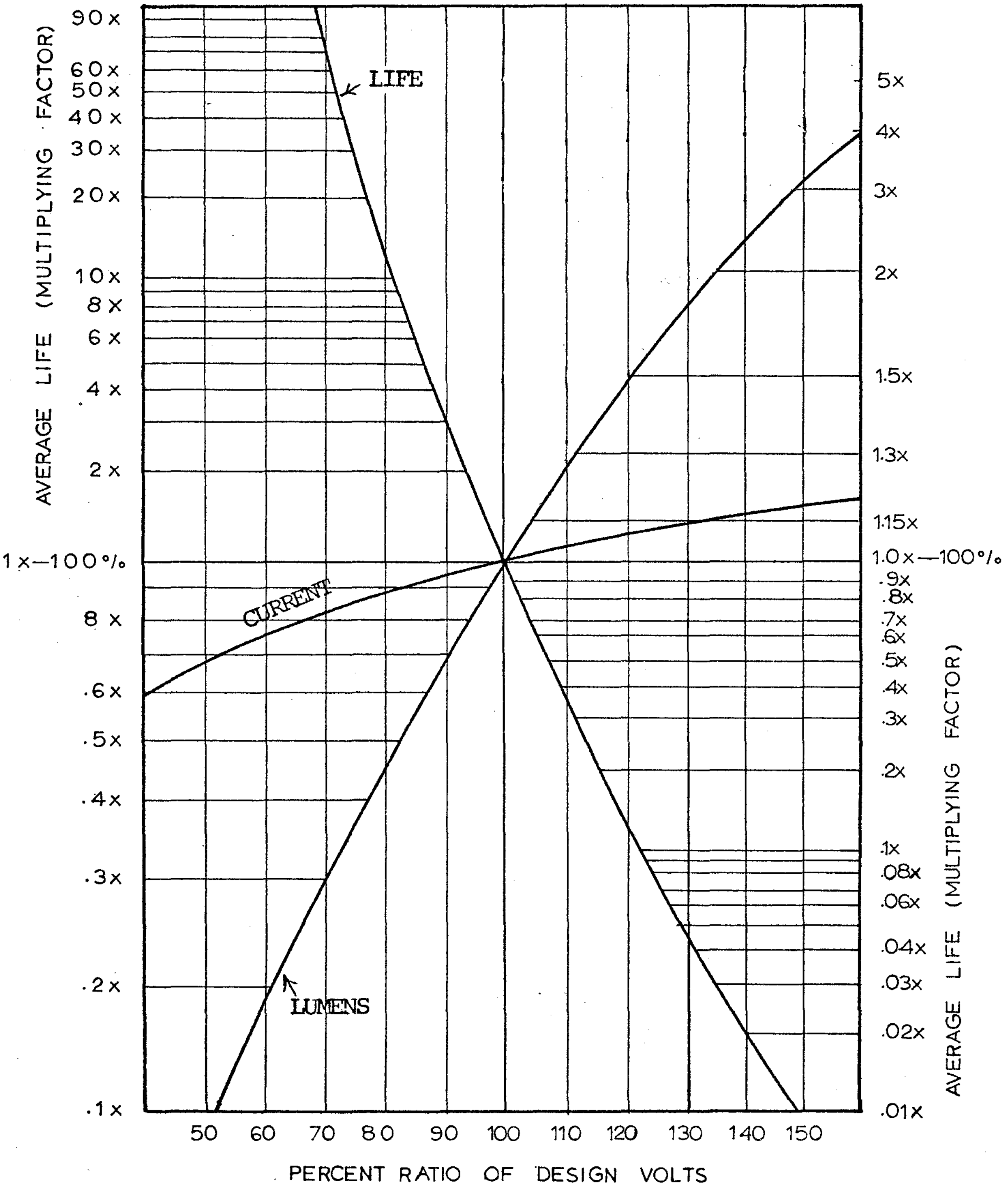


BUTT SEALED NO. 47 LAMP

Nominal Voltage - 6.3 volts

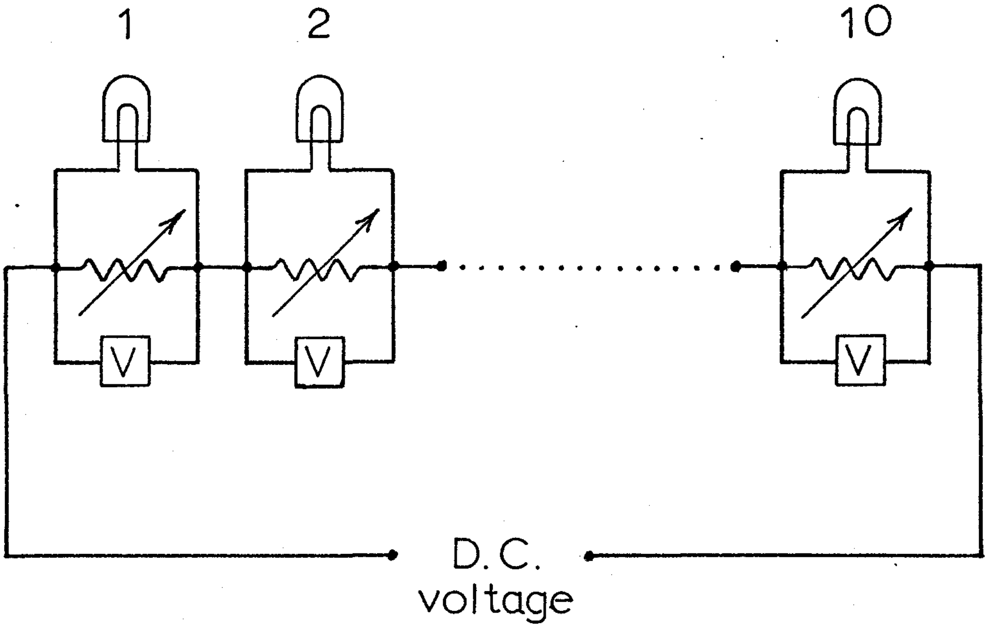
Current - 0.15 amperes

FIGURE 2



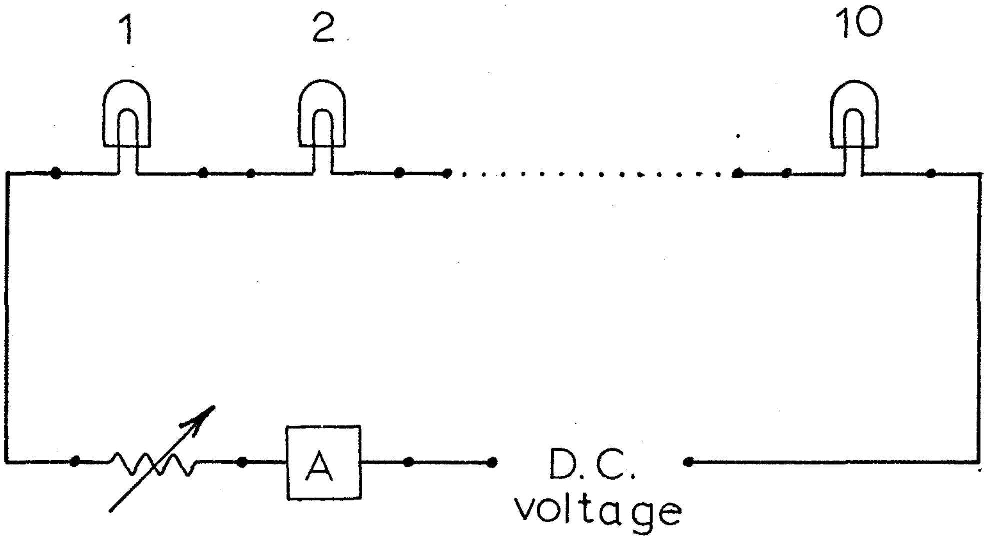
PERFORMANCE OF LAMP LIFE, CURRENT AND CANDLEPOWER FOR VARIOUS LAMP VOLTAGES.

FIGURE 3



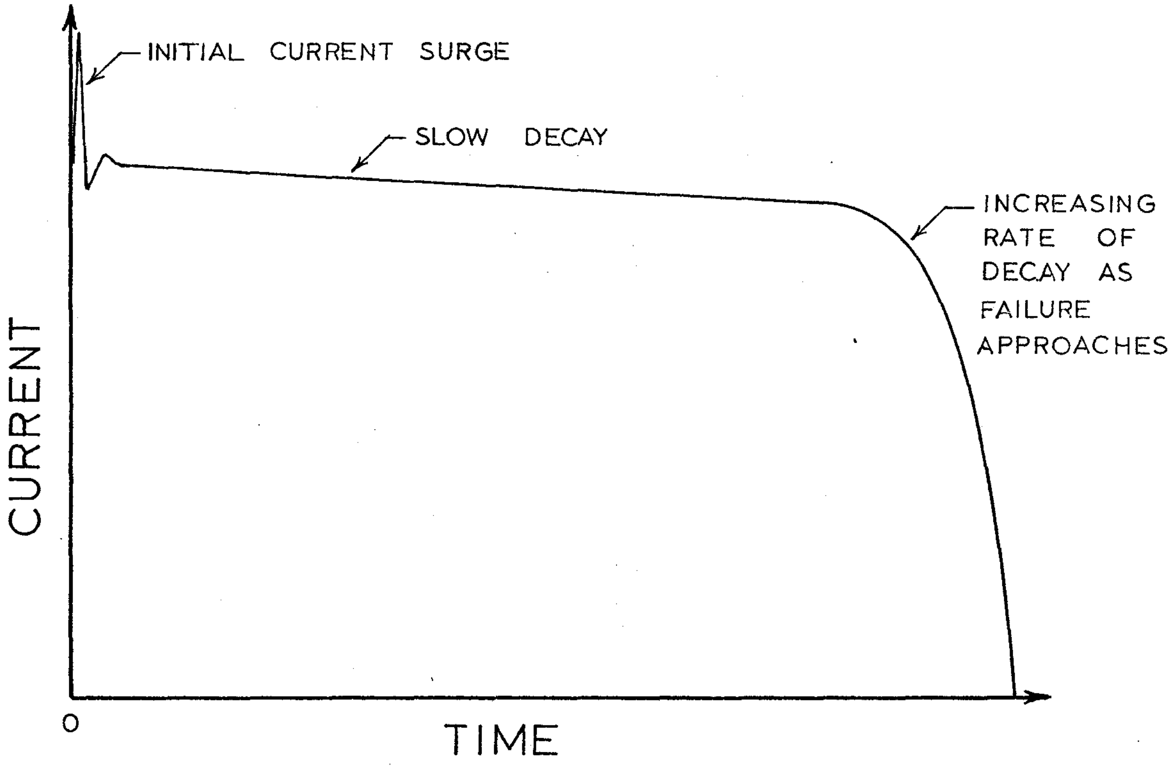
CONSTANT VOLTAGE CIRCUIT

FIGURE 4



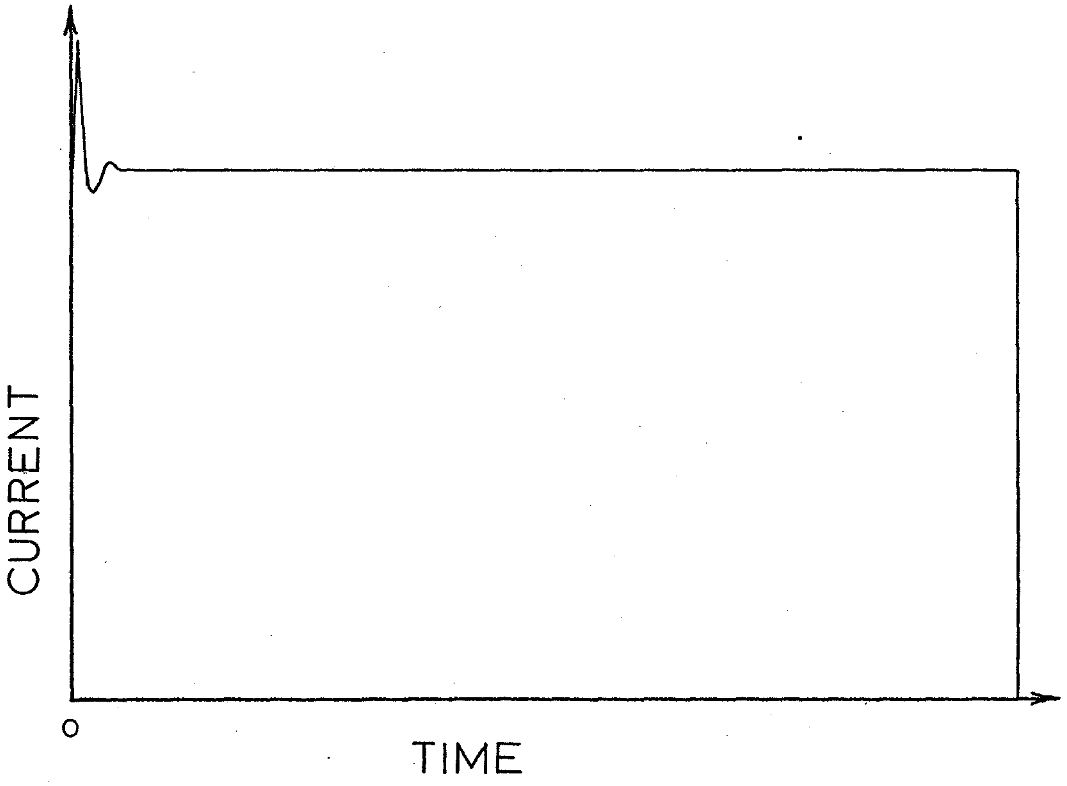
CONSTANT CURRENT CIRCUIT

FIGURE 5



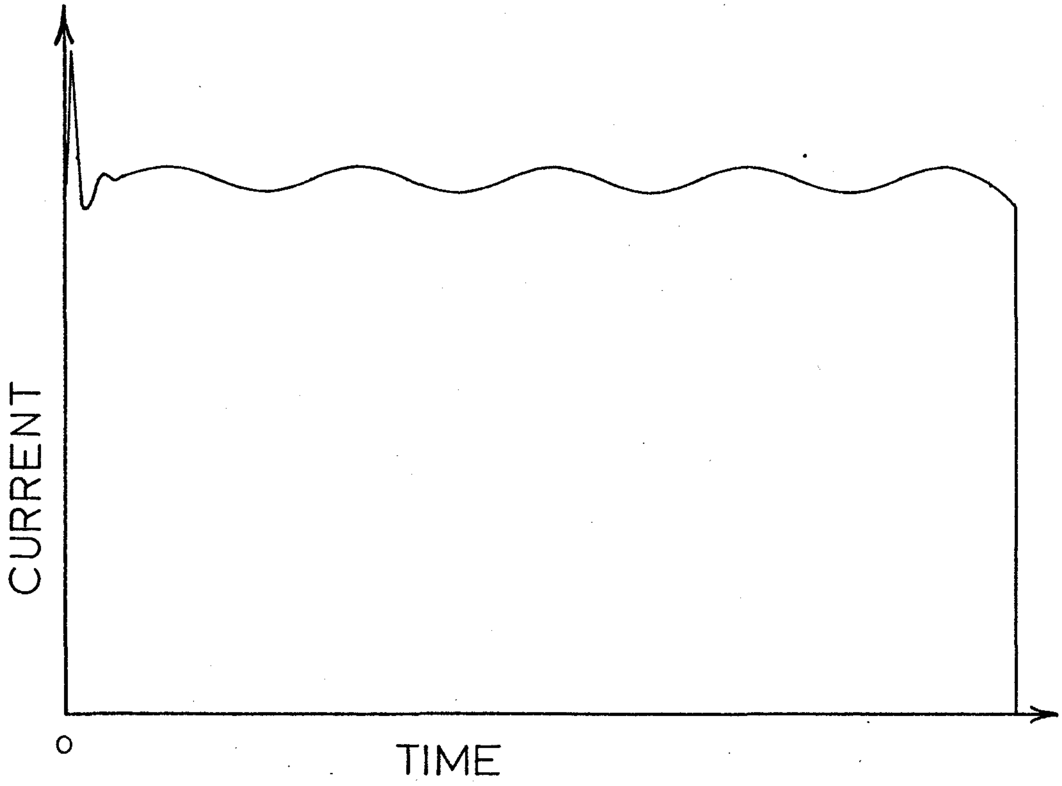
CURRENT DECAY

FIGURE 6



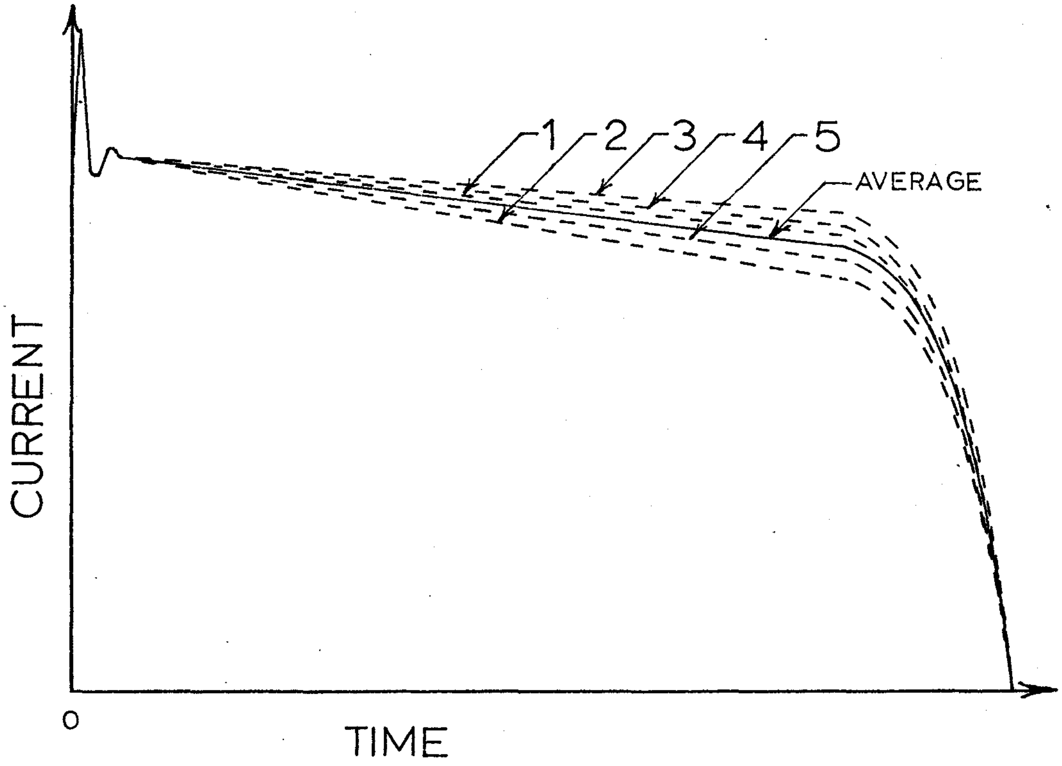
THEORETICAL CONSTANT CURRENT

FIGURE 7



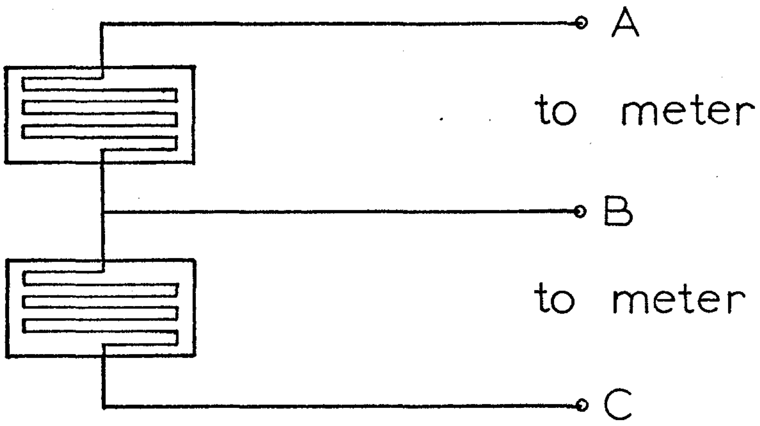
ACTUAL CONSTANT CURRENT

FIGURE 8



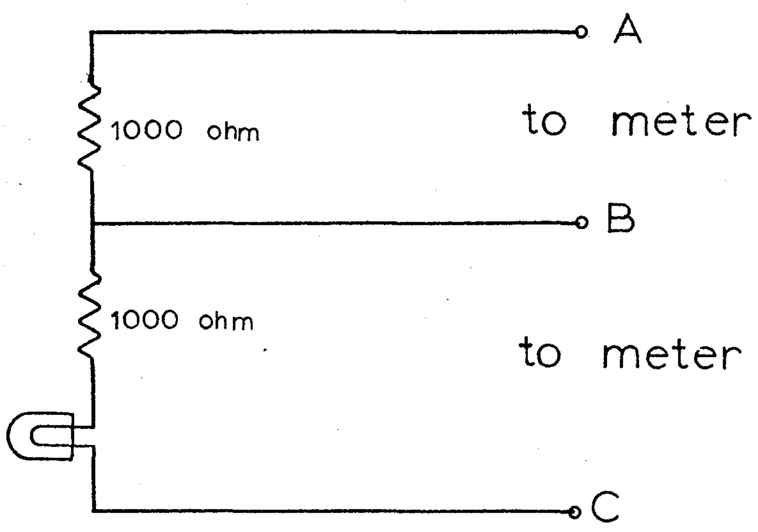
AVERAGE CURRENT DECAY

FIGURE 9(a)



STRAIN GAUGE CONNECTION FOR BAM-1 METER

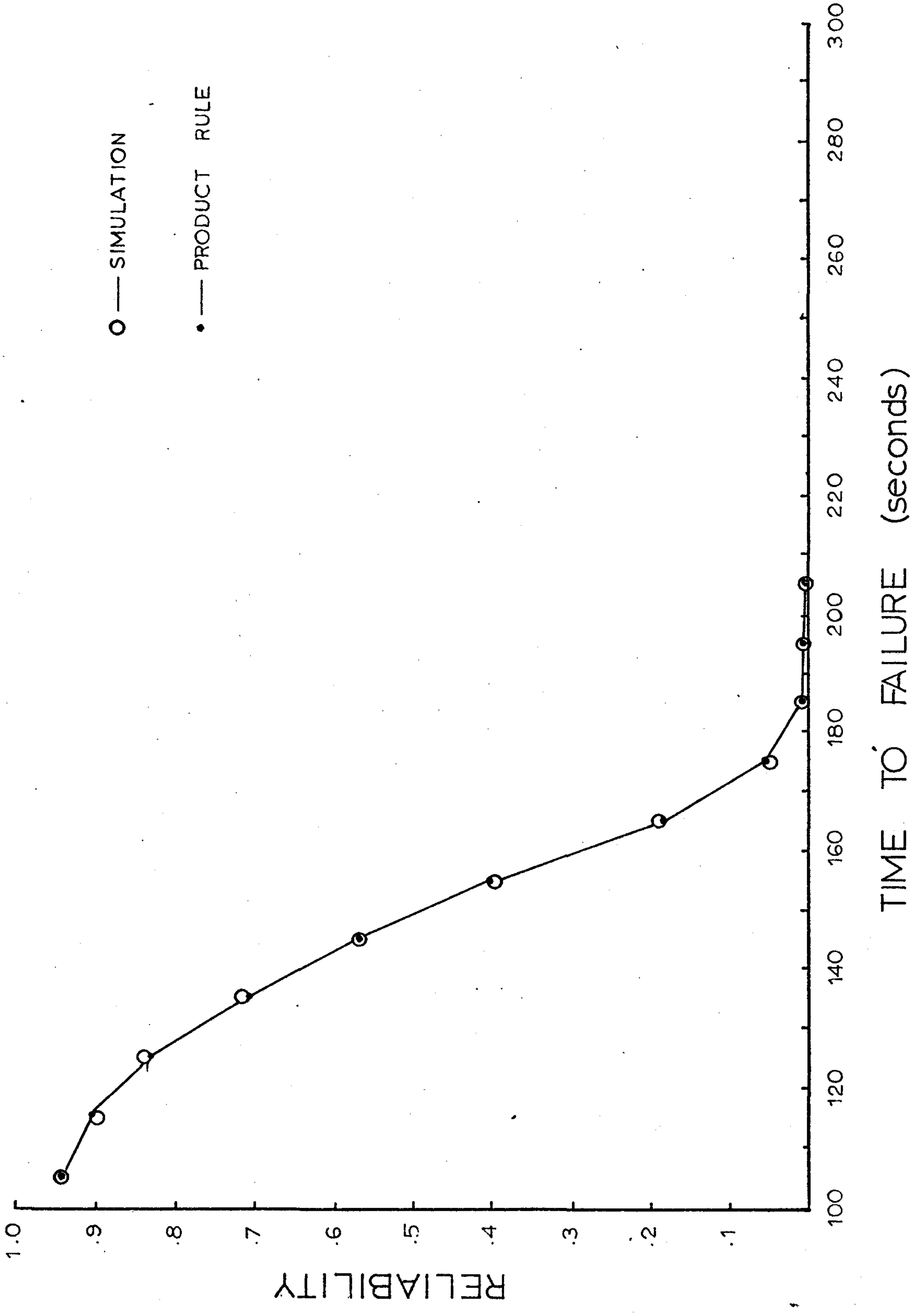
FIGURE 9(b)



SUBSTITUTE CONNECTION FOR BAM-1 METER

COMPARISON OF PRODUCT RULE WITH COMPUTER SIMULATION FOR TEN LAMPS IN SERIES

FIGURE 10



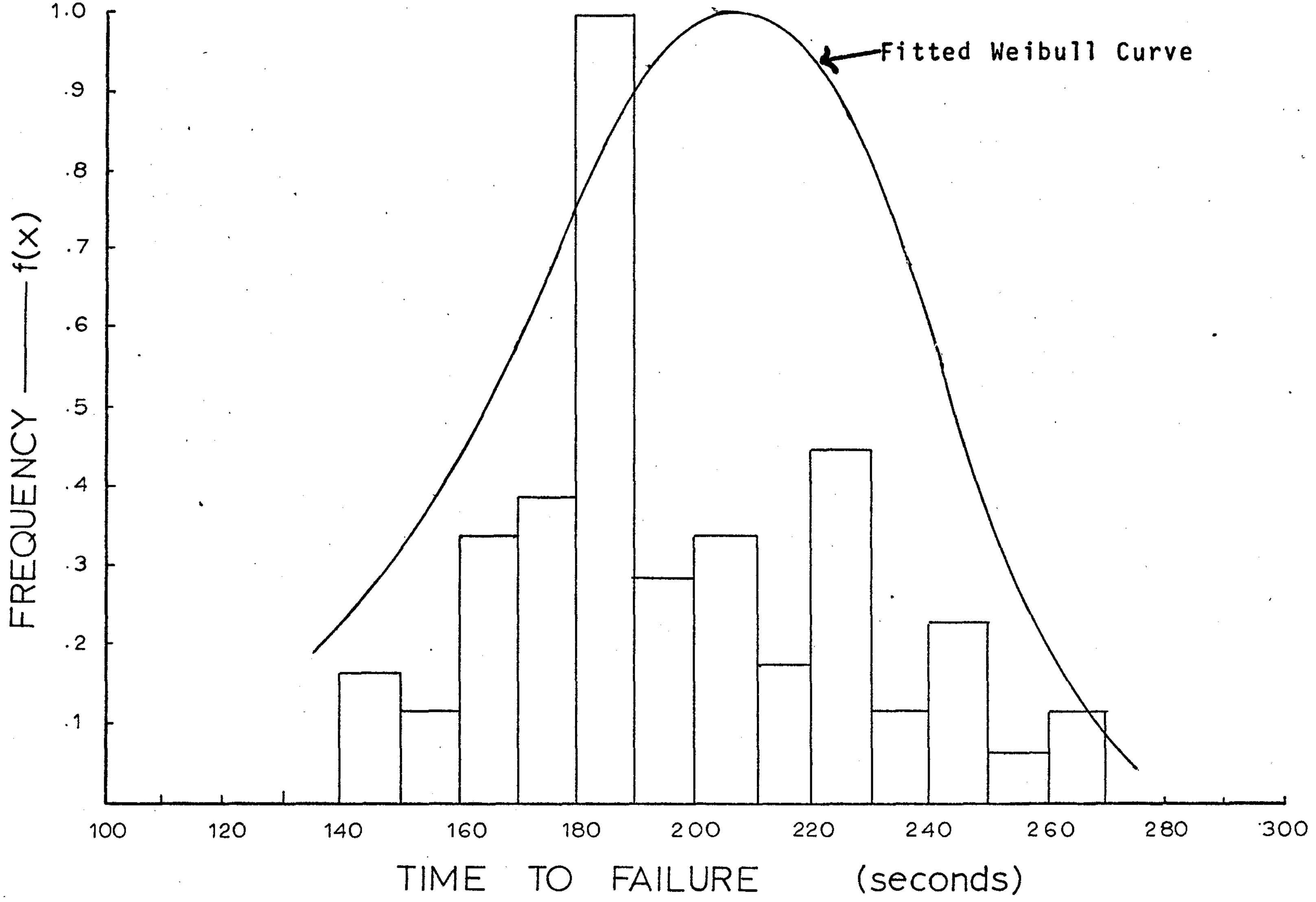
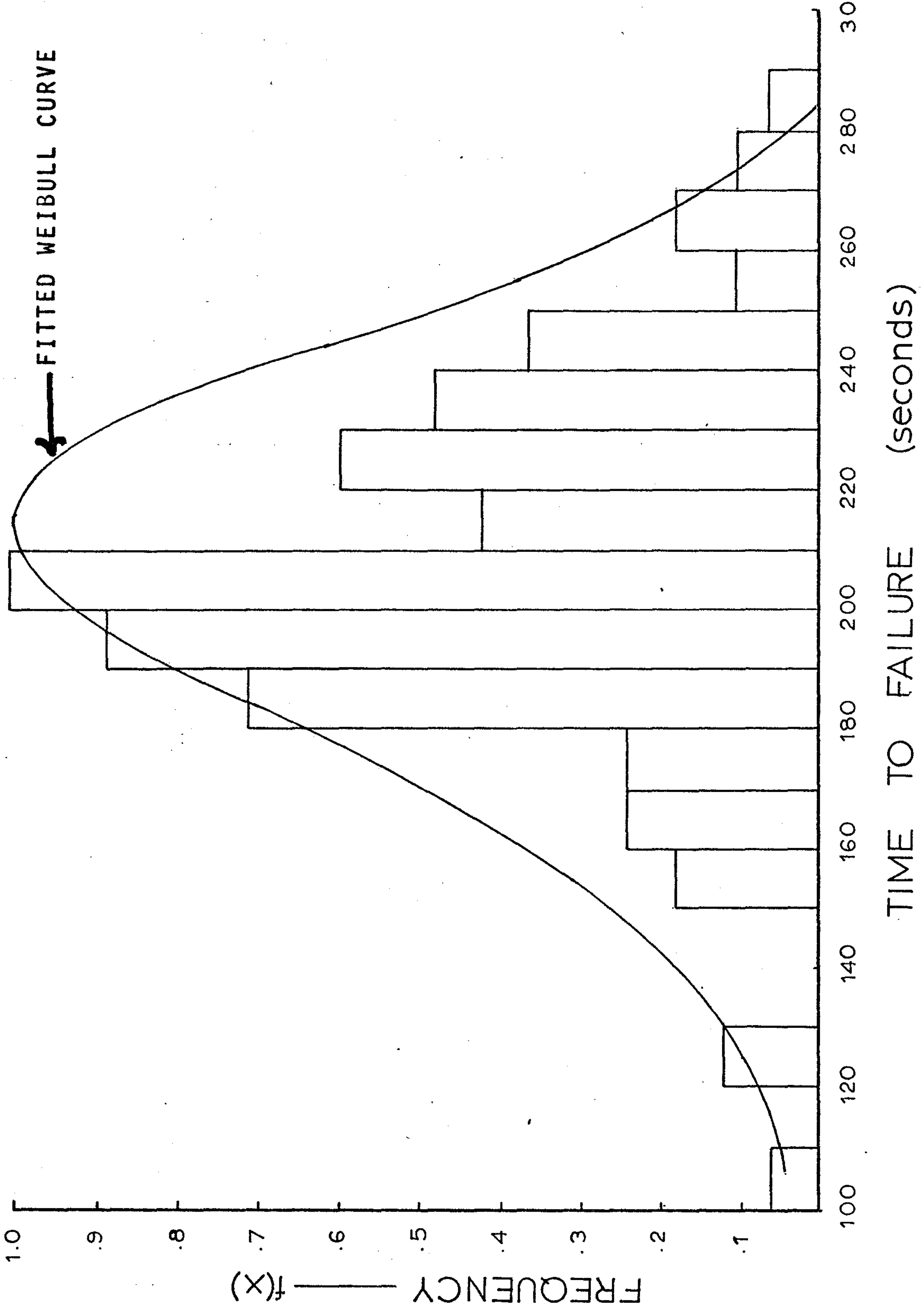


FIGURE 11(a)

FREQUENCY HISTOGRAM FOR TEST NO.1 OF SINGLE LAMP TESTS

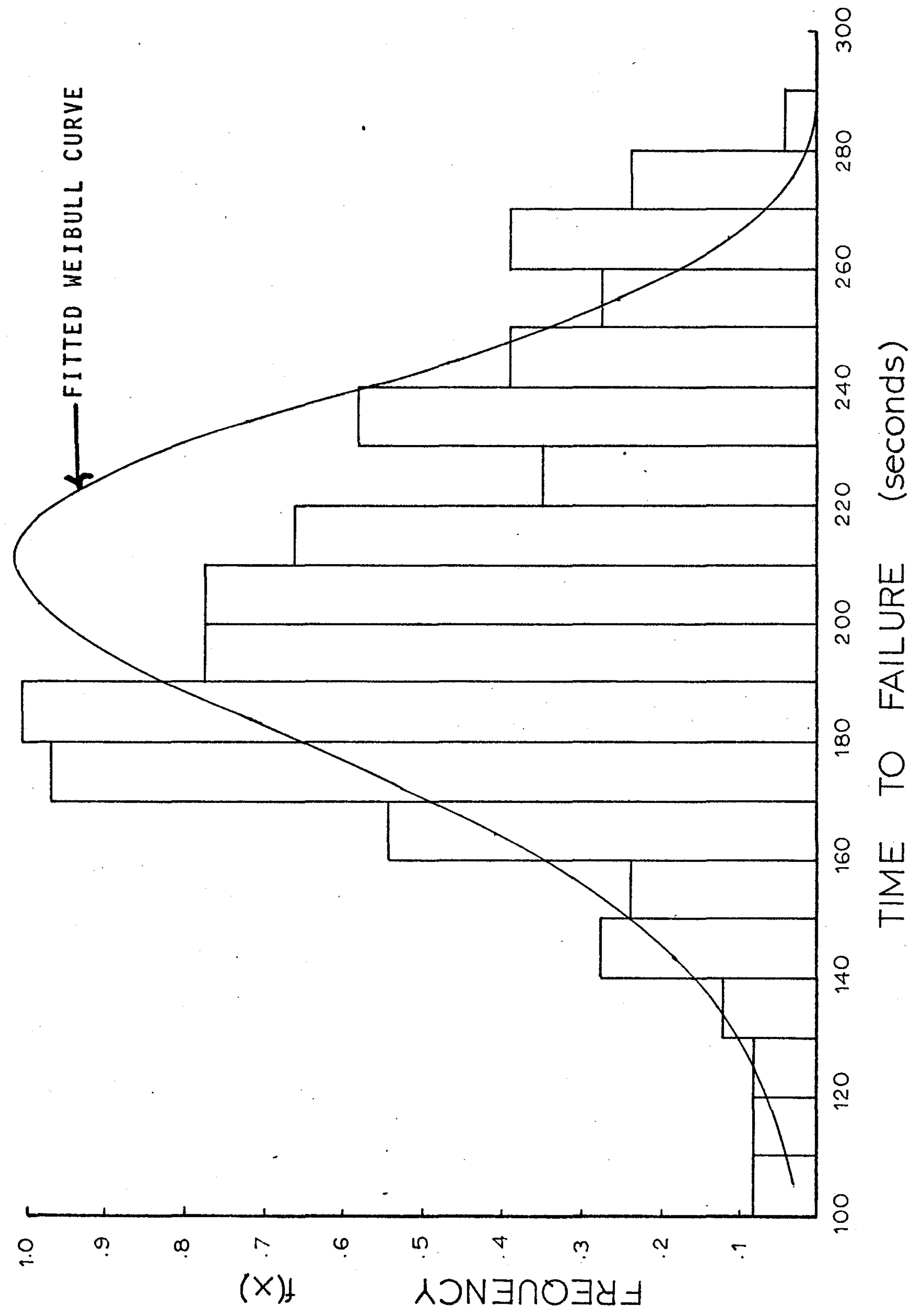
FREQUENCY DISTRIBUTION HISTOGRAM FOR TEST NO.2 OF SINGLE LAMP TESTS

FIGURE 11(b)



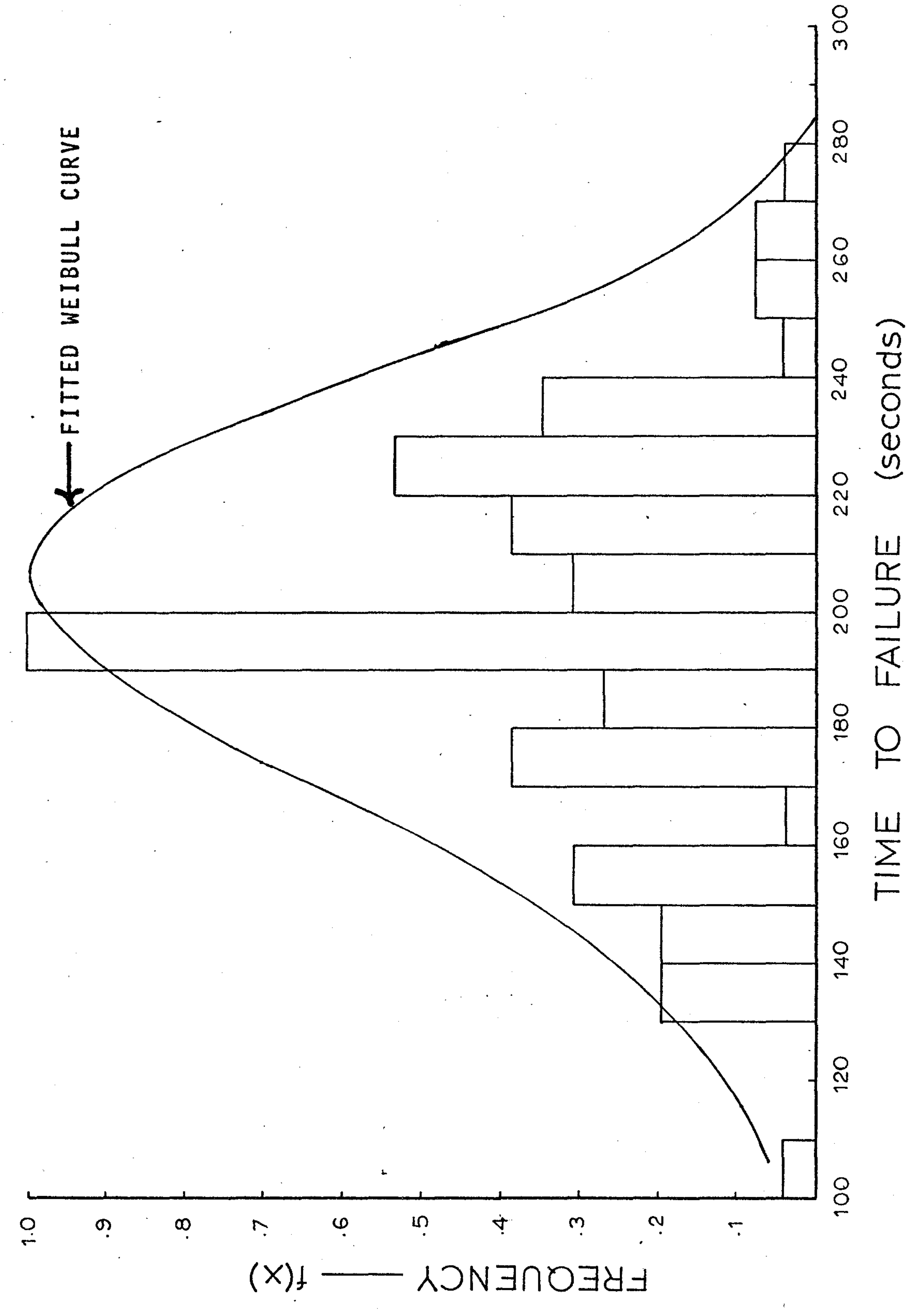
FREQUENCY DISTRIBUTION HISTOGRAM FOR TEST NO. 3 OF SINGLE LAMP TESTS

FIGURE 11(c)



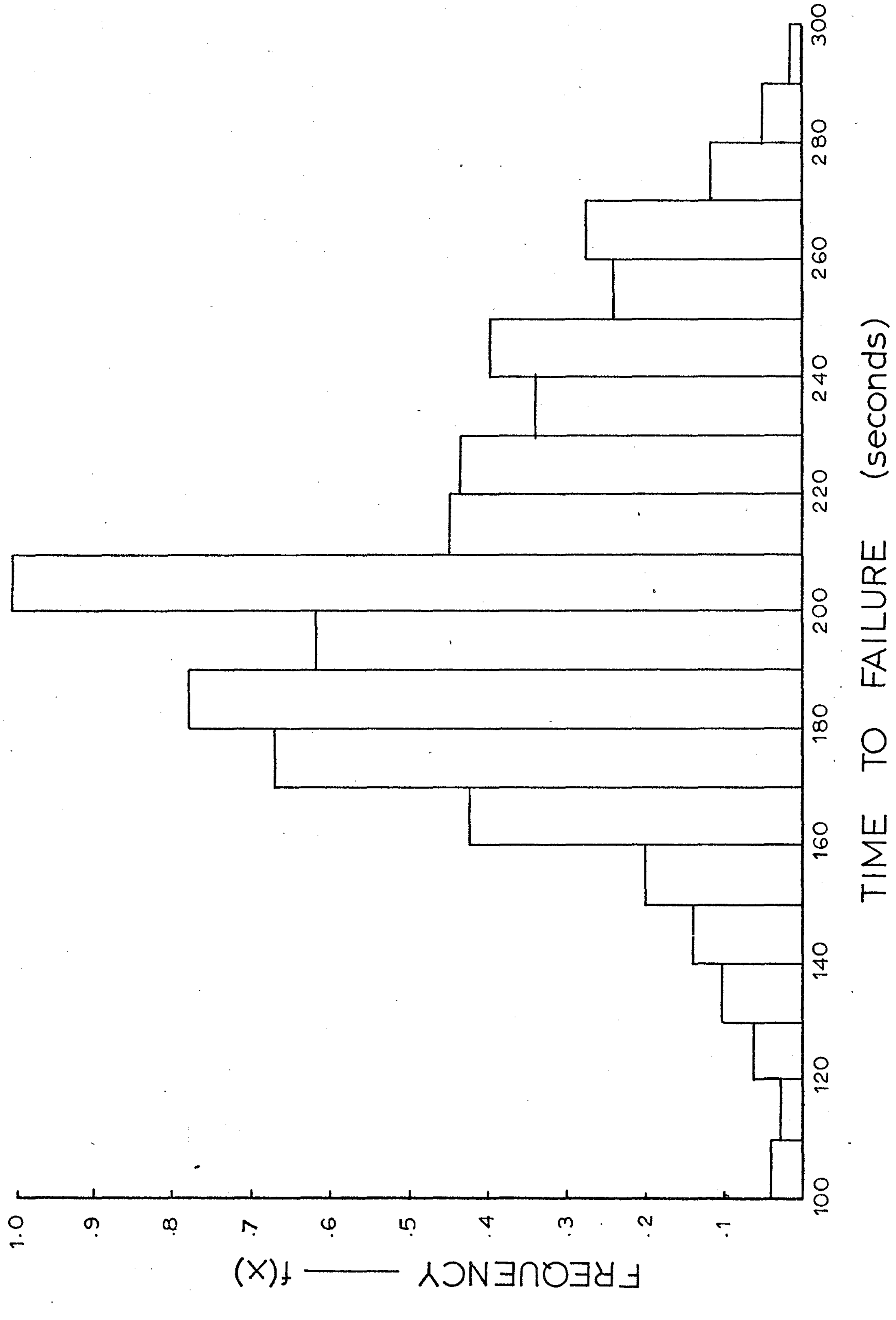
FREQUENCY DISTRIBUTION HISTOGRAM FOR TEST NO.4 FOR SINGLE LAMP TESTS

FIGURE 11(d)



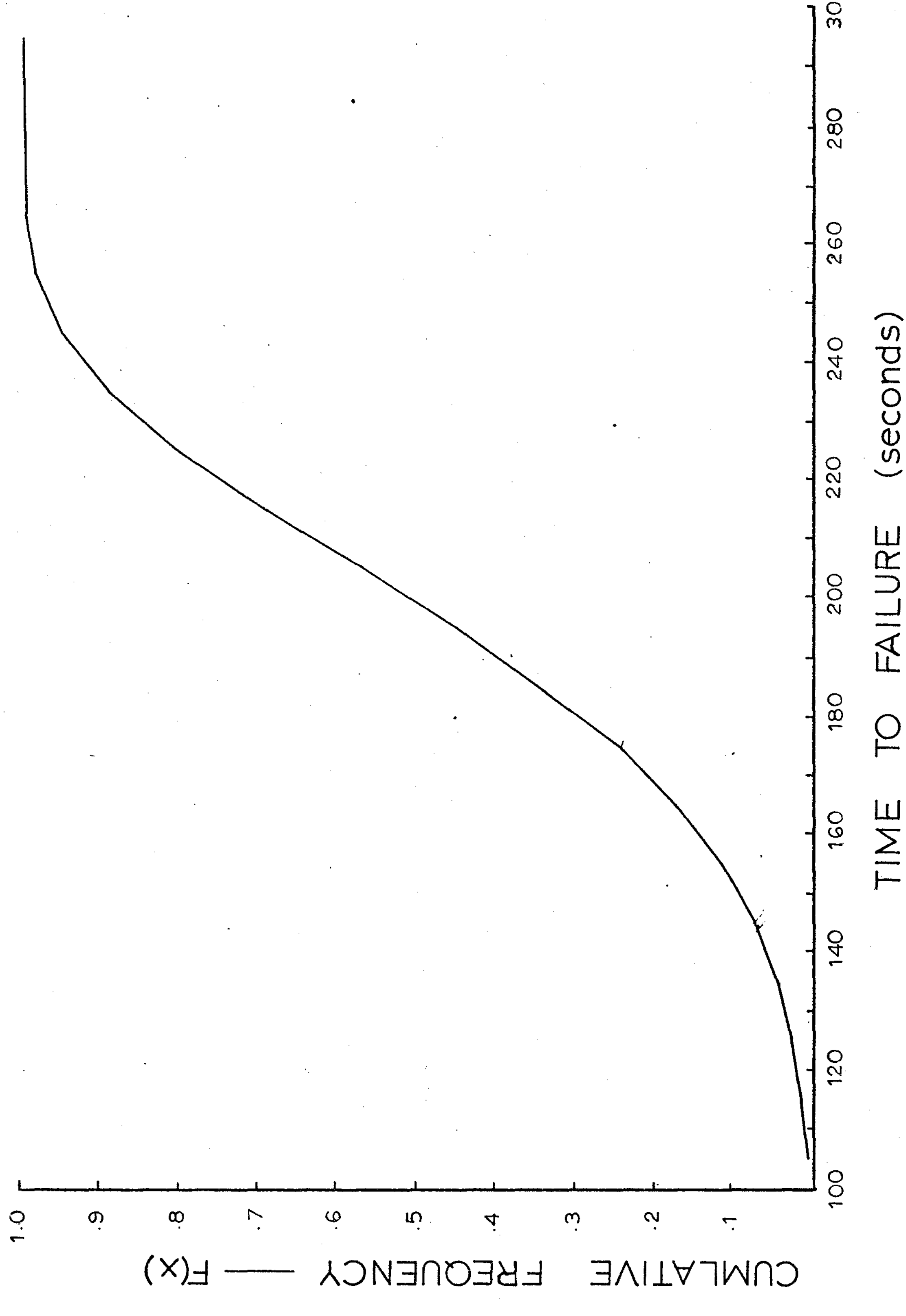
COMBINED FREQUENCY DISTRIBUTION HISTOGRAM FOR SINGLE LAMPS

FIGURE 12



CUMULATIVE FREQUENCY DISTRIBUTION CURVE FOR
SINGLE LAMPS

FIGURE 13



PROBABILITY OF SURVIVAL CURVE FOR SINGLE LAMPS

FIGURE 14

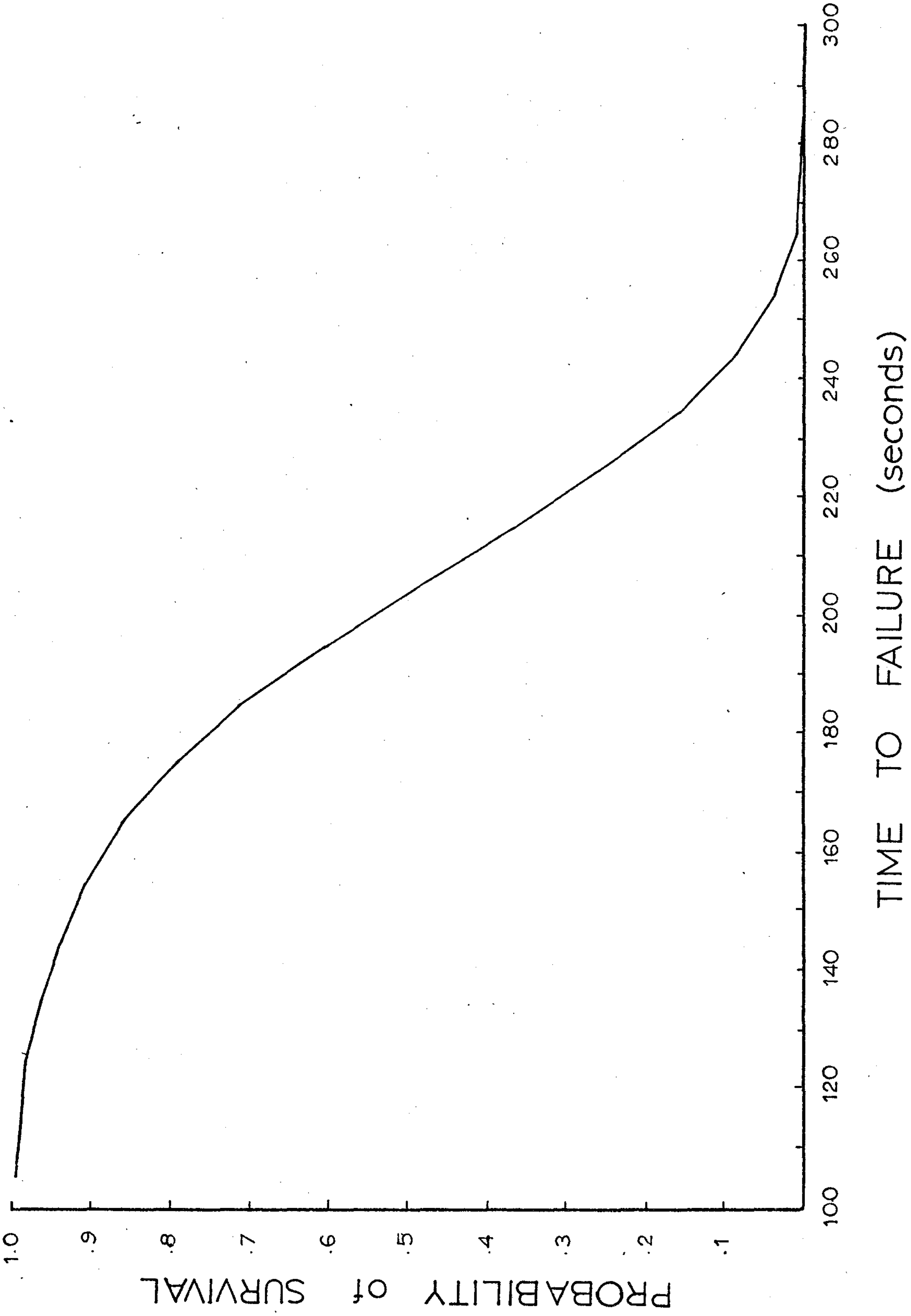
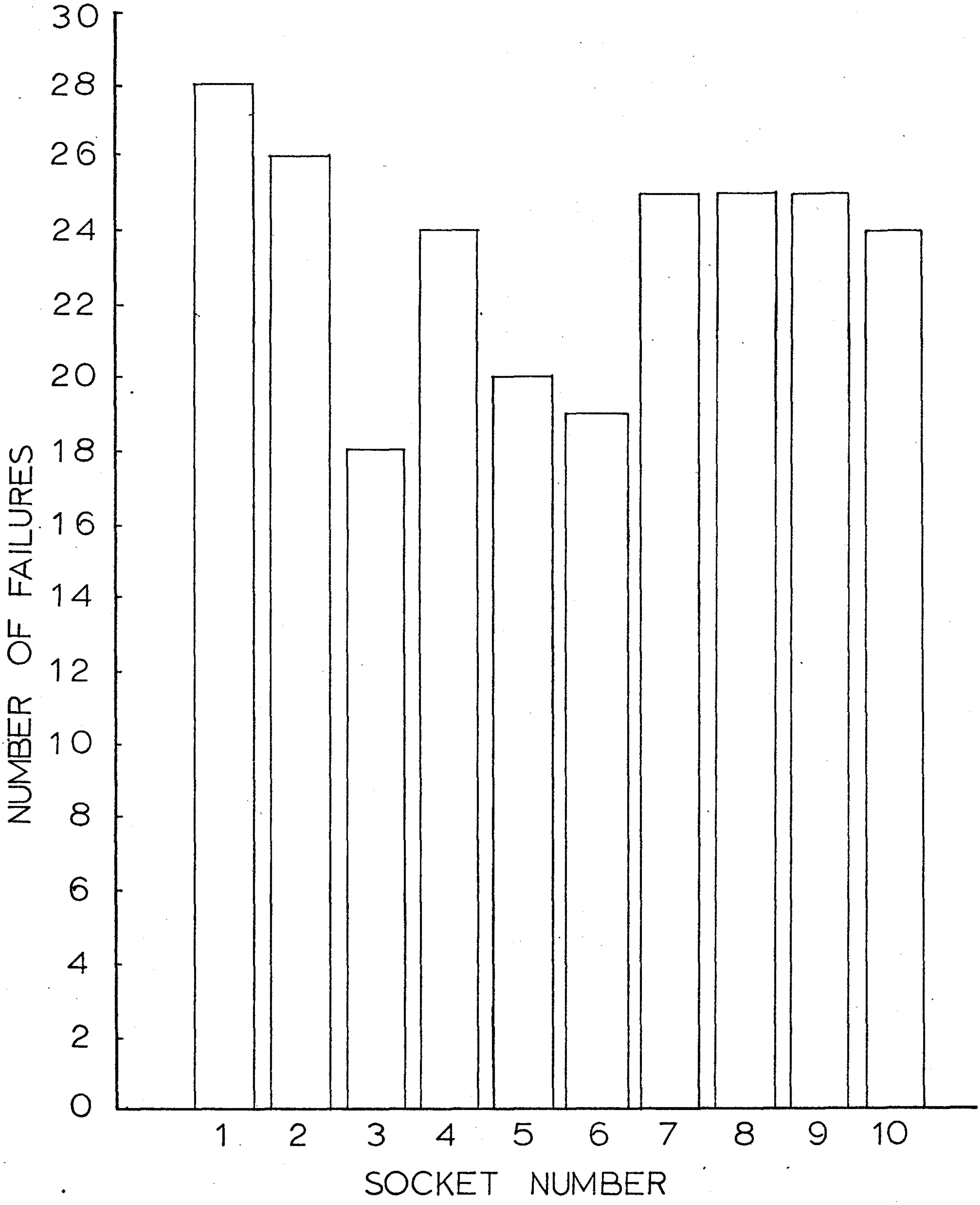
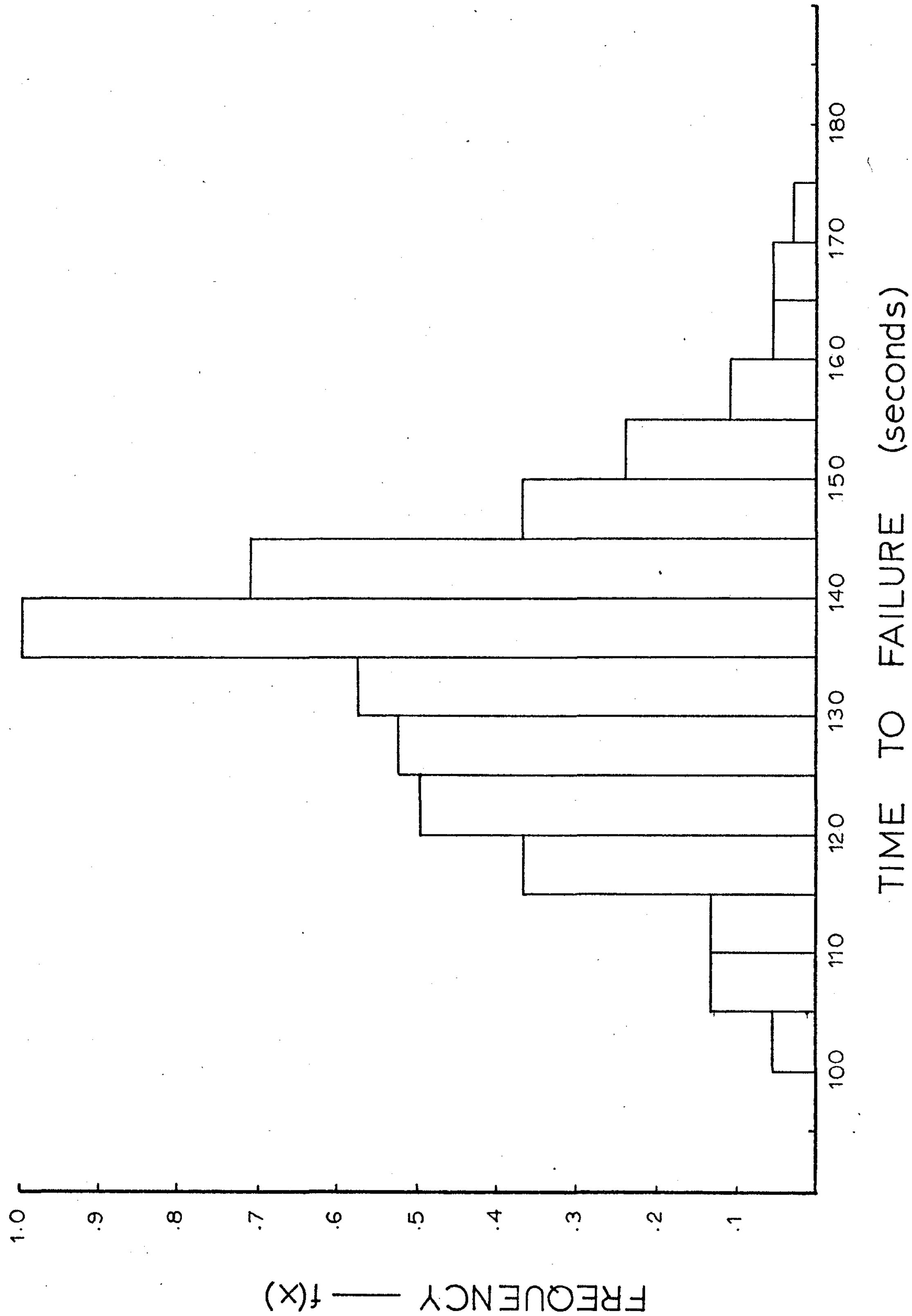


FIGURE 15


30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
NUMBER OF FAILURES
1
2
3
4
5
6
7
8
9
10
SOCKET NUMBER

FREQUENCY DISTRIBUTION HISTOGRAM FOR TEN LAMPS
IN SERIES

FIGURE 16



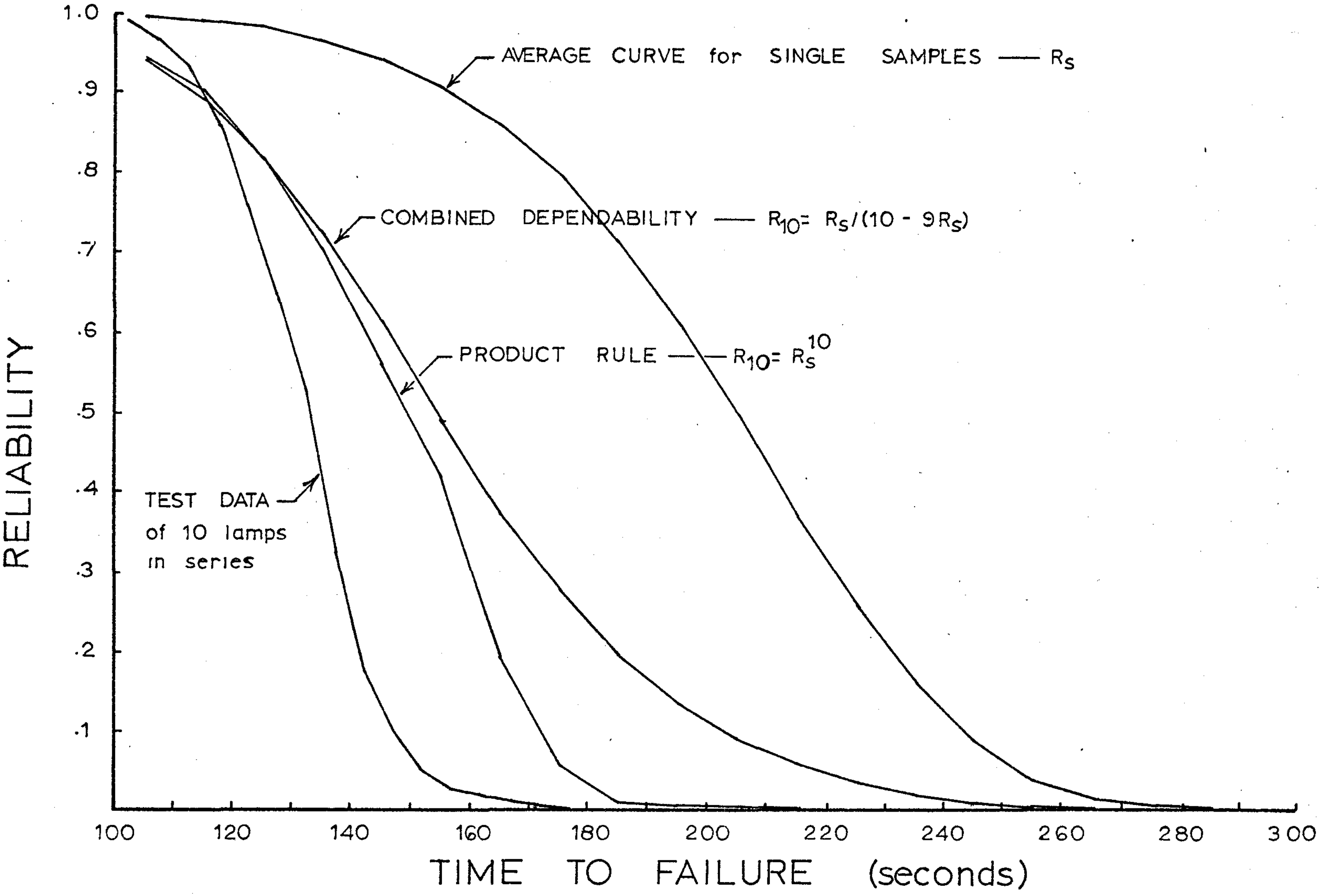
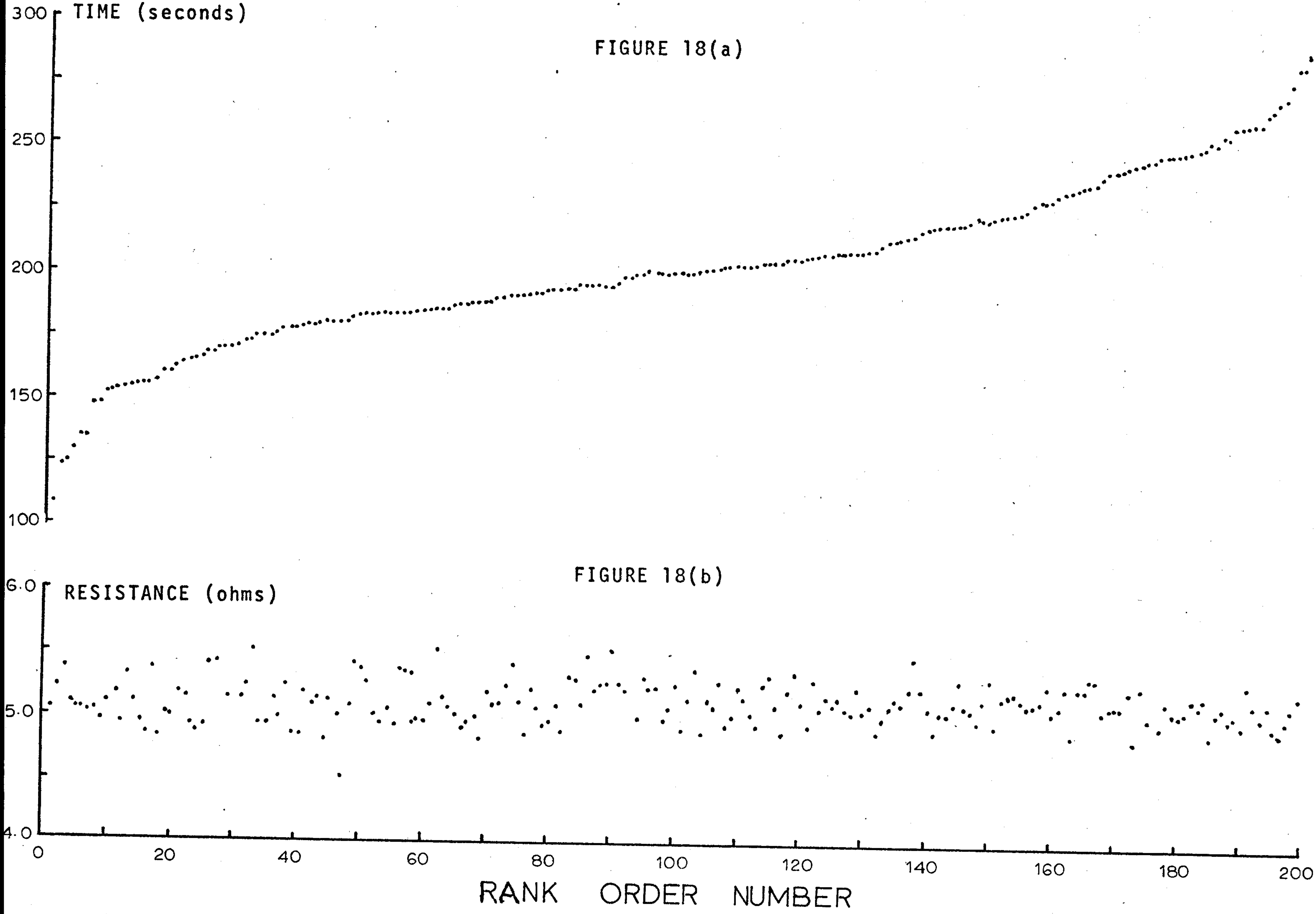


FIGURE 17

COMPARISON OF THEORETICAL AND EXPERIMENTAL RELIABILITY CURVES FOR TEN LAMPS IN SERIES



COMPARISON OF FAILURE TIME WITH COLD RESISTANCE

FIGURE 19

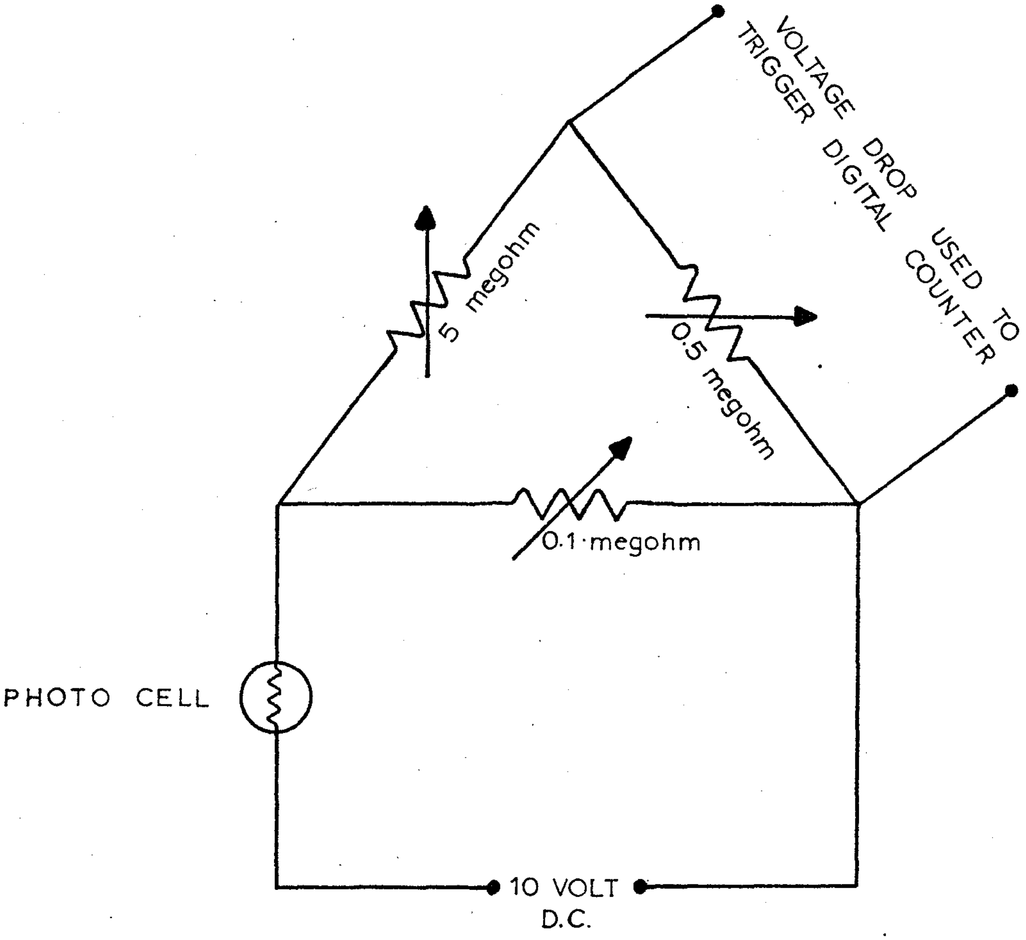


PHOTO-CELL CIRCUIT DESIGN