TURBULENT FLOW OF IRON ORE-WATER SUSPENSIONS
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BY

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SCOPE AND CONTENTS:

This thesis describes the behaviour of iron ore-water suspensions under turbulent flow conditions.

This work is divided into two parts. Part I deals with the regimes of transport under steady state flow conditions in circular and horizontal ducts. The heterogeneous flow regime is extensively analyzed; a sequential discrimination of models with an oriented design of experiments have permitted the determination of the best model to correlate hydraulic gradients for these suspensions. A critical discussion on the limit deposit conditions is also included.

Part II describes the behaviour of clear water under oscillatory flow conditions. The study demonstrates that the quasi-steady state hypothesis, i.e., fully developed flow assumption, applied to pulsatile turbulent flow under the conditions studied. Observations on the behaviour of iron ore-water suspensions under pulsatile flow are also included. The experiments were carried out using a new air-pulsing technique.
ACKNOWLEDGEMENTS

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PART I  STEADY STATE FLOW STUDIES
1.1 INTRODUCTION

Many industrial processes and natural phenomena involve some form of solid-liquid interaction. The understanding of these interactions is basic to the control of these systems.

Solids movement through pipelines is now a commercial reality and has some advantages over other forms of overland transportation. These are: continuous operation, immunity to adverse weather conditions and relatively low capital and operating costs per unit mass transported. The first part of this thesis deals with one of the main factors which must be considered in the optimization of a solids pipeline design, i.e., the energy requirements for slurry flow under steady state conditions. The main emphasis is focused on the study of the heterogeneous flow regime, because this regime is normally identified with economical operation, that is to say, the amount of material transported per unit power consumption is at a maximum. Due to its importance, great research effort has been concentrated in this regime, but unfortunately no generally accepted criterion to describe head loss under various flow conditions within this regime has yet been established as the following analysis of this thesis will show. For this reason, the author has carried out a statistical discrimination between the most-used models which describe this regime using the Bayes theorem, and a design of experiments using the Roth criterion for the optimal choice of experimental conditions, on the basis that this is the best
strategy for efficient experimentation.

All tests were carried out with aqueous slurries of hematite (size passing 30 mesh and specific gravity 5.17) in concentrations of solid up to 25% by volume.
1.2 BASIC ASPECTS

Two phase solid-liquid flow has been analyzed theoretically and experimentally by many investigators, and most of these investigations are concerned with the pressure drop of the two phase flow in a pipe using the same methods as in ordinary hydraulic research, that is, of one-dimensional treatment with simple assumptions regarding the flow pattern. However, general conclusions on this subject have not been obtained, because the nature of this flow depends on many complicated factors such as particle size, particle form, concentration, pipe diameter, density of solid and fluid, flow velocity and so on. Nardi (1959) has indicated eight physical characteristics of the solids, ten physical characteristics of the slurry and about fourteen factors in the design data, all of which should be considered in the design of a slurry pipeline.

The theoretical study of the behaviour of a particle in turbulent flow poses immense difficulties. Ayukawa (1968) et.al. indicate that the motion of a solid particle in a pipe is governed by a drag force caused by the difference of velocity between fluid and the particle, a friction force at the pipe wall, a gravitational force, a force resulting from collisions of particles or between a particle and the wall, and a lift force caused by asymmetry of pressure distribution on the surface of the particle. Because the ratio of the density of solid to that of water is in general comparable with unity, these forces have about equal
significance in determining the motion of a particle. On the other hand, the physico-chemical forces acting between pairs of particles also depend on particle size and shape but, more important, they arise because of the electro-chemical nature of the particle surface, the chemical environment in the suspending liquid and the physical interaction arising out of collisions of two or more particles. It is outside the scope of the present thesis to detail the exact nature of the inter-particle forces, except to mention that they vary from material to material and are very sensitive to small changes in ionic and surfactant concentrations in the liquid surrounding the particles. While it is convenient to distinguish between the factors considered above, in practice, further interactions between them are possible. Thus, a large particle may be supported by the mass of smaller particles held together by the inter-particle forces as if it were a homogeneous flow. Or the aggregates of particles, flocculated by the interparticle forces, will immobilize the suspending liquid within the flocs and the electrical double layer outside, and behave as larger particles. And again, when the relative velocities between the liquid flow and particle movement are large, the drag and lift experienced by the particles may overcome the gravitational pull and suspend the particles which would otherwise settle.

Therefore, each particle moves along very complicated trajectory with mutual interferences, and the pressure drop may be affected by these circumstances. Only a small number of significant contributions to this theoretical problem appear to have been made since the
pioneering studies of Tchen (1947). Whether the solution has been via analytical, numerical or stochastic methods it has generally been necessary to make numerous simplifying assumptions, which may be unrealistic in many practical situations involving the transport of solids.

In view of the immense difficulties, it is customary to resort to alternative semi-empirical theories in order to model slurry behaviour. This aspect is discussed in the following sections of this thesis.
1.3 REGIMES OF MOTION OF PARTICLES IN HYDRAULIC CONVEYING

From the point of view of behaviour during flow, solid-liquid mixtures may be divided into two groups. Some mixtures can be transported successfully even in laminar flow without appreciable deposition of solid on the pipe bottom. On the other hand, there are mixtures from which solid particles separate rapidly unless the flow is highly turbulent. It is convenient to refer to the mixtures in these two groups as "non-settling" and "settling" mixtures or slurries.

According to Brebner (1962), mixtures with low settling velocities of the order of roughly 0.005 feet per second behave as "non-settling" pseudo homogeneous fluids at almost all velocities, whereas mixtures with settling velocities greater than the mentioned value behave as "settling" mixtures. Williams (1953) and other authors have noted that "non-settling" mixtures with particle diameters less than 10 microns exhibit a clear non-Newtonian behaviour. The behaviour of these "non-settling" mixtures will not be considered further here: information concerning them is given by Gay (1969).

The present thesis deals with the flow in horizontal pipes of "settling" mixtures. Because of the interaction between the tendency of the solid particles in a "settling" mixture to settle out and the drag and lift exerted by the flowing liquid on them, four different flow patterns are possible in horizontal pipes. These are shown in Figure (1.1), where a plot of the hydraulic gradient versus the mean flow velocity,
Fig. 1.1 Typical head loss curve. Modified from Brebner (1962).

Fig. 1.2 Flow regimes for a given system after Durand. Modified from Brebner (1962).
on double logarithmic paper, yields a straight line, in pure water for a pipe with the same relative roughness in the turbulent flow regime. When the hydraulic gradient obtained with a liquid-solid mixture of a given concentration is plotted on Figure (1.1), the divergence decreases as the transportation velocity decreases. Or, in other words, for a given particle size and concentration, a certain flow velocity of the mixture is necessary to assure flow. But the flow of the mixture takes place in various forms, as shown in Figure (1.1). At very high velocities the solid particles can be more or less uniformly mixed with the carrier liquid and this results in a "homogeneous" flow pattern; thus, the vertical distribution of solid particles is nearly uniform. At somewhat lower velocities the vertical particle concentration gradient is not uniform, but all solids are in suspension. This flow pattern is termed "heterogeneous" suspension and this is probably the most important regime of hydraulic conveying because it is normally identified with economical operation. At still lower velocities, some of the solid particles move as a sliding bed on the bottom of the pipe. This flow pattern is termed "sliding bed with heterogeneous suspension". At still lower velocities, part of the pipe area is occupied by solid particles which do not move. Above these, there may be particles which slide, although most of the solid movement is by saltation. This flow pattern is termed "stationary bed with saltation" and will not be discussed in this thesis because it is essentially a rigid boundary problem not pertinent to solid transportation in pipes. The relative extent of each of these flow regimes in any pipeline flow situation depends on many factors such as particle size, particle
form, concentration, pipe diameter, density of fluid and solid, flow velocity, configuration of pipes and so on; however, the solid-fluid flow interactive mechanisms are sufficiently different from regime to regime that this classification is really justified.

Durand (1952) proposed a classification of solid-liquid mixtures based on the particle size. Mixtures with particles less than 25 microns were considered intermediate; and particles greater than 50 microns, heterogeneous. The variation of flow characteristics with particle size and mean velocity are shown diagrammatically in Figure (1.2).

1.3.1 THE HOMOGENEOUS FLOW REGIME

Zandi (1968) has indicated that this type of flow occurs when solid particles are fine and light, or the mean velocity of the flow is high enough to keep the particles uniformly in suspension throughout the pipe cross-section. The mixture usually, but not necessarily, exhibits a shear stress-shear strain relationship different from that of the conveying liquid. When water is used as the conveying fluid, the behaviour of the slurry may become non-Newtonian, as indicated by Clarke (1967). This non-depositing, non-stratified flow is encountered in the transport of many materials as indicated by Metzner (1964), Newitt (1962), Durand (1953), etc.

However, the homogeneous regime comes into being as the result of the combination of particle size, solid and liquid densities, and the mean velocity of the flow. The only criterion is the uniformity of solids' distributions in the cross-section of pipe. Newitt (1955)
proposed the following relationship for this regime,

\[
\frac{1800 g D w}{U^3} > 1
\]  

(1.1)

where \( g \) is the acceleration of gravity in ft./sec.\(^2 \), \( D \) is the diameter of the pipe in feet, \( w \) is the free settling velocity of particles in ft./sec., and \( U \) is the mean velocity of suspension in ft./sec.

Unfortunately, this criterion has not been tested extensively against experimental work, therefore it should be used only as a guideline rather than a hard and fast rule.

The homogeneous flow may be further subdivided into those regimes in which the particle-liquid interactions do not alter the rheological properties of the conveying liquid, and those in which it does. It is becoming more and more apparent that water carrying a small amount of fine particulate matter produces less pressure-gradient than when water alone is flowing, all other conditions being the same. Under certain conditions the suppression of head loss may be considerable and increased efficiency of pumping may be realized. Publications pointing to this unexpected phenomenon are now many. A recent paper by Zandi (1967) indicates that this phenomenon is observed when low concentrations of either fine coal, fine charcoal, or fine ash is added to the flowing water, however, this effect cannot be predicted quantitatively in the present state of knowledge.

When the solid-liquid system is such that because of the combination of concentration, particle size and mean velocity the flow
is still homogeneous but not of the type of decreasing head loss and when the slurry does not exhibit strong non-Newtonian characteristics, the energy loss can be computed with the following formula which is due to Newitt (1955):

\[ \phi = s - 1 \]  

(1.2)

where

\[ \phi = \frac{h_m - h_w}{h_w C_v} \]  

(1.3)

\( h_m \) is the hydraulic gradient due to the slurry in feet of water per foot of pipe, \( h_w \) is the hydraulic gradient due to pure water, \( s \) is the specific gravity of solids and \( C_v \) is the volumetric concentration of solids.

It should be emphasized that there is no way to differentiate a priori between those suspensions which may exhibit pressure gradient suppressing action and those which would not. Those homogeneous suspensions which exhibit non-Newtonian behaviour will not be considered further here.

1.3.2 HETEROGENEOUS FLOW REGIME

This type of flow occurs when the concentration of solids varies from a minimum at the top of the pipe to a maximum at the bottom due to the tendency of the particles to settle at a velocity proportional to their fall velocity, or in other words, there is a concentration distribution across the pipe cross-section. It is, however, important
to note that as defined by most of the investigators in this regime there is no deposit on the bottom of the pipe and the particles do not interact chemically with the conveying fluid. Examples of heterogeneous flow can be found in the transport of sand (Bonnington, 1958), nickel (Round, 1963), coal (Durand, 1954). This non-depositing flow regime has wide application in industry. Therefore, it has been the subject of many experimental, theoretical and review studies. Despite all these investigations, however, the main useful body of available knowledge is still empirical and in many aspects not internally consistent. The theoretical work, because of the complexity of the phenomenon, has been mainly directed toward very dilute suspensions, which are not of great industrial significance. On the other hand, the flow of concentrated suspensions, which is of industrial significance, is treated experimentally for the most part, and mainly aimed at developing some kind of correlation for the prediction of pressure gradients. In addition, most available data are for slurries consisting of uniform particles. Figure 1.1 shows a typical variation of pressure gradient with velocity when the concentration of solids is constant. From this figure it is apparent that as the mean velocity decreases, the head loss of the suspension decreases and then increases, passing through a minimum point. Through observation it is established that this minimum coincides with the appearance of a bed load on the bottom of the pipe indicating a change of the regime from heterogeneous to saltation. The velocity associated with the minimum point is defined as critical velocity or minimum deposit velocity.
Many attempts have been made to develop a relationship for the prediction of the head loss when the velocity is above the critical point. In general, the investigators have assumed that the pressure gradient required to maintain the flow of a slurry is composed of two parts. First, the pressure gradient required to maintain turbulent flow of conveying liquid, and second, the pressure gradient required to keep particles in suspension. Consequently the total pressure gradient is:

$$ h_m = h_w + h_s $$

(1.4)

where $h_m$ is the pressure gradient for suspension, $h_w$ is the head loss for pure water flowing with the average velocity of suspension, and $h_s$ is the excessive amount of energy loss as a result of suspended matters, all in feet of water per foot of pipe.

Several equations have been proposed to correlate pressure gradients in the heterogeneous regime. By far the most extensive experimental work has been carried-out in France by Durand (1952, 1953) and his co-workers. They used particles up to 1 inch in diameter in concentrations up to 22% by volume in horizontal and vertical pipes varying in diameter from 1½ to 22 inches. They found that all their results could be correlated with reasonable accuracy by the formula,

$$ \frac{h_m - h_w}{C_v h_w} = \theta \left[ \frac{g D(s-1)}{U^2} x \frac{w}{\sqrt{g d(s-1)}} \right]^{1.5} $$

(1.5)

where $U$ is the mean velocity of the mixture, $C_v$ is the volumetric concentration of solids, $s$ is the specific gravity of solids, $w$ is the
terminal falling velocity of a particle, \( g \) is the acceleration due to gravity, \( d \) is the diameter of the particle, \( D \) is the pipe diameter and \( \theta_1 \) is a constant evaluated by experiments. The value of this constant reported by Durand, op. cit., in his original paper is 121. However, values of \( \theta_1 = 60, 120, 150, 180 \) and 380 have been reported in the literature. With a little algebra equation (1.4) can be expressed in terms of friction factors,

\[
h_m = (f + \lambda_1) \frac{u^2}{2gD}
\]  

(1.6)

where \( f \) is the friction factor of pure water and

\[
\lambda_1 = f \cdot C \cdot \left( \frac{5}{1.5} \right)^{0.75} \times \left( \frac{\theta_1}{\sqrt{F_r}} \right)^{1.5}
\]

(1.7)

and \( F_r \) is the Froude Number defined as

\[
F_r = \frac{u^2}{gD}
\]

(1.8)

Charles (1969) indicated that Durand's equation tends to underestimate the hydraulic gradient due to slurry \( h_m \) in the homogeneous flow regime. In a plot of the parameter \( \phi \) versus \( U \) or the Froude Number, in double logarithmic paper for a given value of \( \theta_1 \) and \( s \), for both equations (1.5) and (1.2), the line representing equation (1.5) has a negative slope, while the line representing equation (1.2) is horizontal with the value of \( \phi \) directly dependent on \( s \). The intersection of
equations (1.2) and 1.5) suggest that there is a sudden transition from a regime in which equation (1.5) applies to one in which equation (1.2) applies. In fact, this sharp transition is physically unrealistic and is not supported by experimental data. (See figure 1.13 or 1.14)

According to Charles, op. cit., a much improved relationship is obtained by combining equations (1.2) and (1.5) to give,

\[
\frac{h_m - h_w}{C_v h_w} = \theta_2 \left[ \frac{gD(s-1)}{U^2} x \frac{w}{\sqrt{gd(s-1)}} \right]^{1.5} + (s-1) \tag{1.9}
\]

and thereby providing for a smooth relationship between \( \phi \) and \( U \) throughout the heterogeneous and homogeneous regimes. Charles, op. cit., has summarized about 500 individual tests in this way and has also indicated that equation (1.9) should be valid for concentrations up to 25% by volume. Equation (1.9) is known as the Charles equation while equation (1.6) is known as the Durand-Condolios equation. \( \theta_2 \) should be identical to \( \theta_1 \) according to the original paper of Charles.

Newitt et al. (1955) using an energy approach developed the following expression to describe the head loss for heterogeneous flow,

\[
\frac{h_m - h_w}{C_v h_w} = \theta_3 (s-1)gD x \frac{w}{U^3} \tag{1.10}
\]

where \( \theta_3 \) is a constant evaluated by experiments. Newitt, op. cit., conducted tests with sediment diameters ranging from 0.005 to 0.235 inches and sediment concentrations ranging from 2% to 35% by volume, specific gravity of solids ranging from 1.18 to 4.60, but all his
experiments were carried out in a 1 inch diameter pipe. The value of \(\theta_3\) reported by Newitt, op. cit., is 1100. It should be noted that both equation (1.10) and Durand's equation (1.6) indicate that the parameter \(\psi\), as defined in equation (1.3), is inversely proportional to the cube of the mean velocity \(U\). With a little algebra, equation (1.10) can be expressed in terms of friction factors,

\[
h_m = (f + \lambda_\psi) \frac{u^2}{2gD} \tag{1.11}
\]

where

\[
\lambda_\psi = f C_v \theta_\psi (s-1)(F_c)^{-1.5} \times \frac{w}{gD} \tag{1.12}
\]

Kriegel and Brauer (1966) have studied theoretically and experimentally the hydraulic transport of solids in horizontal pipes for some suspensions of coke, coal and ore granulates. They used particles from 0.115 to 1.67 mm in diameter in concentrations up to 42\% by volume, and specific gravity of solids ranging from 1.38 to 4.62. This investigation was especially concerned with the conveying mechanism in the turbulent fluid. By means of a semi-theoretical model for the turbulent mixing of mass the friction factor caused by mixing of materials could be derived and was experimentally confirmed up to volumetric concentrations of 25%.

\[
h_m = (f + \lambda_\psi) \frac{u^2}{2gD} \tag{1.13}
\]
where

\[ \lambda_4 = \frac{6 \, c \, (s-1)}{g \, v} \left( \frac{v^3}{g v} \right)^{1/3} (F_r)^{-4/3} \]  

(1.14)

\( v \) is the kinematic viscosity of the carrier fluid and all other symbols have the same meaning as explained before. The value of \( \theta_4 \) reported by Kriegel and Brauer, op. cit., is 0.282 and the settling velocity \( u \) should be multiplied by a form factor, which depend on particle size and kind of material, whose value ranges from 0.5 up to 1.0. For concentrations higher than 25\% by volume, equation (1.14) was extended empirically. According to Kriegel and Brauer equation (1.14) would be also valid for multi-particle size suspensions if a mean diameter of the grains is used. The effect of kinematic viscosity was not checked since all experiments were carried out with clear water as carrier fluid.

Ayukawa and Ochi (1968) derived a formula for the pressure drop in a flow with a sliding bed of particles through a horizontal straight pipe by equating the dissipation of energy of solid particles caused by sliding on a pipe wall to the work done by the additional pressure drop due to the conveying of solid particles. However, the range of velocities covered by these investigators indicates clearly that this equation should be valid for the entire heterogeneous regime, not only for velocities near the limit deposit velocity. The total pressure drop is expressed as

\[ h_m = (f + \lambda_5) \frac{v^2}{2gD} \]  

(1.15)
where $\lambda_5$ is the additional friction factor caused by mixing of materials, which is given by

$$\lambda_5 = \eta \theta_5 \frac{2.0g \ (s^{-1}) \ D \ C_v}{w^2} \quad (1.16)$$

where $\theta_5$ represents a coefficient of friction due to a friction between a pipe wall and particles, which could eventually be determined using a plate of the same quality as the pipe. $\eta$ is the modification factor which is determined as a function of parameters obtained from the similarity conditions for particle motions and is a compensating factor to the effect of the existence of floating parts of solid particles caused by their complicated motions. According to the experiments carried out by Ayukawa, op. cit., this modification factor is given by,

$$\eta = 0.90 \left( \frac{d}{D} \right)^{-0.707} \left( \frac{F_d}{w} \right)^{-2.72} U_{c}^{2} \quad (1.17)$$

where $F_d$ is a modified version of the particle Froude Number defined as,

$$F_d = \frac{U}{\sqrt{gd(s-1)}} \quad (1.18)$$

The critical velocity $U_c$ or the limit deposit velocity for solid liquid-mixture is the velocity below which solid particles settle out and form a stationary bed (not a sliding bed). It is important to note that some authors appear to be confused with this definition and indicate that the limit deposit velocity is that at which solids begin to settle to the bottom of the pipe forming a moving bed. However, Durand (1952),
Carstens (1969) and most other authors consider that this velocity is that at which the solids form a stationary bed, and they hydraulic gradient due to slurry can be fairly well correlated by any of the equations indicated in this section at velocities greater than the limit deposit velocity. From the practical point of view, this velocity is not precisely defined and usually corresponds to a "region" whose boundaries can be determined experimentally.

There have been many attempts to present a generalized correlation of limit deposit velocities. Perhaps the best known is that due to Durand (1952), which can be expressed as follows:

\[ U_c = F_L \sqrt{2.0 \, gD(s-1)} \]  \hspace{1cm} (1.19)

where \( F_L \) is a function of particle size and slurry concentration.

Spells (1955) determined the following relationship from literature data;

\[ U_c = 0.075 \left[ (s-1)gd \right]^{0.816} \left[ 1 + \frac{C_v (s-1)^D}{\mu} \right]^{0.633} \]  \hspace{1cm} (1.20)

where \( \mu \) is the viscosity of the fluid. Recently, Charles (1970) recommended the following expression as an estimate of critical velocity,

\[ U_c = \frac{4.80}{C_D^{1/4}} \left[ \frac{C_v(s-1)}{\left[ C_v(s-1) + 1 \right]^{1/3}} \right]^{1/3} \]  \hspace{1cm} (1.21)

where \( C_D \) is the drag coefficient for the largest particle present.

Finally, the simplest relationship is the one proposed by Newitt (1955),
\[ U_c = 17 w \]  

(1.22)

Unfortunately, this equation has not been verified extensively.

1.3.3 STATIONARY BED WITH SALTATION

Generally, head losses in transportation pipelines with a stationary bed are greater than those associated with the limit deposit velocity. The scatter of data in this regime is considerable. According to Condolios and Chapus (1963) Durand's equation can predict the head loss in this regime if \( D \) is replaced by the hydraulic diameter. This equation appears to be the most suitable one but it could be used only as a guideline. Since in all applications under steady state flow conditions the systems are designed to prevent occurrence of this regime, this topic is outside the scope of this thesis and will not be discussed further here.
1.4 THE DISCRIMINATION OF MODELS AND THE DESIGN OF EXPERIMENTS

The most important problem in designing a hydraulic haulage system to transport solid materials is the prediction of the head loss and subsequent power consumption. For settling slurries, heterogeneous flow is the most important mode of solid conveying because it is always the most economical regime in which to operate, i.e., it gives the maximum amount of solid transported per unit energy expended. The models introduced in the previous section of this thesis, represent the most widely used to correlate pressure gradients in the heterogeneous regime, but there are many others such as the ones proposed by Wilson (1942), Worster (1954), Wayment (1962), Toda (1969), etc. However, the scatter between the predictions of the models is well known and indeed was so from the very beginning. This fact has not been overlooked by most of the previous authors. Now the obvious question is: how should a researcher who has to analyse many alternatives, determine beforehand the "best" model for a determined system? The question has no answer, a priori. The solution can be found only using an adequate strategy for efficient experimentation, with special emphasis on the analysis of those levels of the independent variables which show up differences between the models. On the other hand, it should be noted that for a given model the agreement between the values of the coefficients determined experimentally and those given by that model, does not necessarily mean that it is the best functional model. The mechanism for determining the "best" model requires a sequential discrimination between rival models. For these and other reasons, the author of this
thesis has been strongly urged to perform a statistical design of experiments. The Bayesian Approach has been selected for the discrimination of models because of its proved capability. The Roth Criterion was chosen for the optimal selection of experimental coordinates to improve ulterior discrimination by Bayesian Analysis. A lucid description of these techniques is given by Reilly (1970) and only a brief summary is presented here. The models to be discriminated are presented in Figure 1.5

The Bayesian Approach

The models can be represented in the form

\[ y_i = f_k(\theta_k, x_i) + e_i \quad i = 1, 2 \ldots i \text{ observations} \quad (1.23) \]
\[ k = 1, 2 \ldots k \text{ models} \]
\[ j = 1, 2 \ldots j \text{ parameters} \]

where \( y_i \) is the response or the value of the dependent variable at the \( i \)th measurement, \( x_i \) is the vector of the independent variables at the \( i \)th trial, \( \theta_k \) is the vector of the parameters for model \( k \), \( e_i \) is the error associated with the \( i \)th measurement, and \( f_k(\theta_k, x_i) \) represents the "true" value of the dependent variable at \( \theta_k \) and \( x_i \).

It is possible to linearize around \( \alpha_j \) for the \( k \)-th model (Box et.al., 1967) as follows:

\[ \mu = X(\theta_k - \alpha_j) + e \quad (1.24) \]
where $a_j$ is a prior estimate of $\theta_k$, the vector of the unknown parameters, and $\mu$ is a vector whose $i$th element is

$$\mu = y_i - f_k(a_j, x_i)$$  \hspace{1cm} (1.25)

$X$ is a matrix defined as

$$X = \begin{bmatrix} \frac{\partial f_k(\theta_k, x_i)}{\partial \theta_k} \end{bmatrix}_{0_k} = x_j$$  \hspace{1cm} (1.26)

and $e_i$ is the error vector, assumed to have the multivariate normal distribution with mean 0 and covariance matrix $\Sigma = \sigma^2$, where $0$ and $I$ are the null and identity matrices respectively. The application of Bayes theorem requires however some prior information, such as the estimated variance of the experimental errors $\sigma^2$, the parameters covariance matrix $\Sigma_j$ considering the possible range of the parameter values reported in the literature and, also, an initial estimate of the parameters. The initial model probability could be taken as $1/k$, where $\sum Pr(M_k) = 1$, showing no initial preference for any model.

Bayes' Theorem states:

\[
\{\text{Posterior model probability}\} \propto \{\text{Prior model probability}\} \times D_f(\mu/M_k)
\]

or

$$Pr(M_k/\mu) \propto Pr(M_k) \times D_f(\mu/M_k)$$  \hspace{1cm} (1.27)

where $Pr(M_k)$ is the known prior probability of model $k$ and $D_f(\mu/M_k)$ is the likelihood density function for $\mu$ given the model $k$, which can be evaluated as follows:
MODEL 1

\[ \phi_1 = \theta_1 \left( \frac{gD(s-1)}{U^2} x \frac{w}{\sqrt{g(d(s-1))}} \right)^{1.5} \]

DURAND (1952)

MODEL 2

\[ \phi_2 = \theta_2 \left( \frac{gD(s-1)}{U^2} x \frac{w}{\sqrt{g(d(s-1))}} \right)^{1.5} + (s-1) \]

CHARLES (1968)

MODEL 3

\[ \phi_3 = \theta_3 x \frac{0.282}{f} (s-1) \left( \frac{w^3}{g\sqrt{V}} \right)^{1/3} x \frac{1}{Fr}^{4/3} \]

KRIEGEL

BRAUER (1966)

MODEL 4

\[ \phi_4 = \theta_4 x \frac{1.80}{f} g(s-1) D \left( \frac{U}{gd(s-1)} \right)^{-2.72} x \left( \frac{d}{D} \right)^{-0.707} x \frac{1}{w^2} \]

AYUKAWA-OCHI (1968)
where \( n \) is the number of experimental points and \( \Sigma(\mu) \) is the covariance matrix given by

\[
\Sigma(\mu) = X_j^T U_j X_j + V
\]

Note that the superscript \( T \) indicates the transpose of \( X_j \) and \( |\Sigma(\mu)| \) is the determinant of \( \Sigma(\mu) \). The posterior model probability is then calculated with equation (1.27).

After the initial set of experiments, the known posterior model probabilities become the prior model probabilities for the next experiment. Also, the posterior parameter distribution becomes the prior parameter distribution for the next run. The posterior estimate of \( \theta_j \) is given by

\[
(X_j^T V^{-1} X_j + U_j^{-1})^{-1} (X_j^T V^{-1} B_j + U_j^{-1} \alpha_j)
\]

where \( B_j \) is the least squares estimate of \( \theta_j \) and \( \alpha_j \) is the prior estimate of \( \theta_j \). Similarly

\[
(X_j^T V^{-1} X_j + U_j^{-1})^{-1}
\]

becomes the new prior estimate of \( U_j \). The sequential application of these equations allow to determine posterior model probabilities and to compute new estimates of the parameters.
The Roth Criterion

After the initial set of experiments the problem is how to select the new set of experimental conditions, i.e., the values of the independent variables, to achieve maximum discrimination. This objective can be attained using the Roth Criterion (Roth, 1965), which gives a weighted average of the total separation among the models, the weights being the Bayesian posterior model probabilities. That is, once the point defined by the independent variables \( (x_1, x_2) \) is chosen, the amount of separation \( Z \) is computed as follows:

\[
Z(x_1, x_2) = \sum_{k=1}^{k} \Pr(M_k) \cdot C_k
\]

where \( C_k = \prod_{j=1}^{k} \left| y_j(x) - y_k(x) \right| \)

and \( y_j(x) \) and \( y_k(x) \) are the predicted responses of model \( j \) and \( k \) under the experimental conditions \( (x_1, x_2) \) using the current best least squares estimates of the parameters. A grid search is defined for which sets of \( (x_1, x_2) \) are taken and the corresponding \( Z \) values are calculated. The set \( (x_1, x_2) \) that defines a maximum value for \( Z(x_1, x_2) \) is selected as the new experimental condition for the next run.

For the present case the two independent variables are obviously the volumetric concentration of solids and the mean velocity of the mixture.
1.5 EXPERIMENTAL

1.5.1 APPARATUS

A schematic diagram of the actual experimental apparatus is presented in Figure 1.3.

The fluid was circulated from the reservoir R through the pipeline system, and then back to the reservoir by a rotary, positive displacement, Moyno pump type CDR, serial S-56275, equipped with a Lovejoy #3225 variable speed Pulley Drive, with a speed range from 227 to 687 r.p.m. The pump capacity vs. 75 PSI is 50 USGPM (min.) and 150 USGPM (max.). A Brooks Mag Electromagnetic flowmeter model 7300 was used to measure the flow rate. It consists of two basic and separate components; a flowhead which develops an electrical signal proportional to flow rate, and a signal converter which amplifies and converts the ac output signal into a dc signal. This dc signal is used to drive a meter on the signal converter that indicates percent of maximum flow. Also, this signal is used with a recorder for registering the instantaneous mean flow. The flow rate was doubly checked for the experiments with slurries by counting the revolutions of the pump wheel with a Strobocat stroboscope type 6310B1, serial 29213. Calibration charts are presented in appendix 1.10.2

The pipeline system consists of a 80 foot loop of 2 inch internal diameter steel pipe. The U section with a radius of curvature 1.33 feet is, however, only 1½ inch diameter in order to prevent the partial blockage of the system. The test section was situated in the upper part of the loop and for the investigation with slurries it was 20 feet long
FIG. 1.3 Experimental setup.
but for the experiments with clear water it was 31.66 feet long. The instantaneous pressure drop in the test section was measured with a Pace Transducer model P7D. This transducer operates with a Pace Carrier-Demodulator model CD10 whose output signal is registered by the recorder. The recorder is a Rikadenki model B-241 with two independent input and pen systems and was used for registering simultaneously the flow rate and the pressure drop in the test section. This recorder uses the principle of a null balancing servo potentiometer and the limit of error is less than ± 0.3% of full scale. The pen speed is 1 sec. travel full scale and the chart speed used in the experiments was 400 mm/min.

The reservoir (see Figure 1.3.a) is provided with a cooling jacket to keep a uniform slurry temperature. It also contains a sampling system for determining the delivered volumetric concentration of solids of the upper section of the loop (test-section). This is done by rotating an adjustable section of the pipeline in such a manner that a sample of the flowing mixture is collected in a calibrated vessel through a connecting line fixed to the reservoir. Two high-power stirrers were located in the reservoir in order to maintain a uniform distribution of solids and to prevent the partial deposition of particles on the bottom at low flow rates.

1.5.2 PROCEDURE

The initial set of experiments under steady state flow conditions was carried out with tap water. The pressure drop in the test section was measured at different water velocities, from 7.0 ft./sec. up to 15.4 ft./sec. The purpose of these initial experiments was to ascertain the reliability and accuracy of the pressure measuring equipment and consequently to establish a correlation between friction
factor and Reynolds number which represents the "true" behaviour of water flowing in the test section. The experimental data is presented in Table I.1.

The second part of the experimental work was concerned with the pressure drop under steady flow conditions with hematite slurries of different volumetric concentrations at different flow rates. The slurries were prepared by adding a determined weight of hematite mineral to water circulating in the system and allowing it to circulate for approximately half and hour in order to achieve constant distribution of solids and constant temperature. Visual observations were made in a 2 ft. long glass tube of 2 in. internal diameter, which formed part of the test section. This made it possible to determine whether solid particles were fully suspended or in the bed transport regime. When the system achieved stability, the pressure drop and the mean mixture velocity were both recorded using the electronic device mentioned in the previous section. Almost simultaneously, samples of slurry were collected in a calibrated vessel to determine the true delivered volumetric concentration of solids. The flow rate was doubly checked by counting the speed of the pump using a stroboscope. See table 1.12.

The procedure indicated above was repeated for different flow rates with a constant overall concentration of solids in the system. When the complete range of velocities was covered, the total concentration of solids was increased by adding hematite to the system and the entire process repeated.

The above experiments with slurries permitted the determination
of the modes or regimes of transport for hematite-water mixtures at different experimental conditions. The limit deposit velocity was estimated for five different concentrations of solids. The plane of the independent variables $U$ and $C_v$ was divided in a rectangular grid pattern having equally spaced length. The range of velocities was limited to 10.5-16. ft./sec. to make certain that the design of experiments covers only the non-deposition transport regime. The range of concentrations was 0.-0.25 on a volumetric basis. The procedure in the Bayesian analysis was started-up using the subjective prior information that the probability of each of the proposed models was 0.25. The initial parameter covariance matrix was estimated considering values of parameters reported in the literature. The variance associated with experimental errors was selected on the basis of previous measurements. Four experimental points were chosen initially and the procedure for discrimination of models and design of experiments was started-up. The Roth criterion selected the vector of experimental conditions for the following run. The procedure indicated above was then repeated for 5 experimental points. The sequential procedure continued until one model probability reached a value that caused acceptance of the model by a logic criterion.
1.5.3 Estimation of experimental errors.

The estimated errors of the principal variables or their uncertainties are:

- **Pipe diameter.** The nominal diameter of the pipe is 2.0 inches. Taking into account the deviations from roundness, caliper error, etc., the uncertainty in the pipe diameter is about ± 0.04 inch.

- **Water density.** The estimated uncertainty in the water temperature is ± 6°F corresponding to a density variation of less than 0.2%.

- **Length of the test section.** ± 0.5 inch.

- **Frequency.** The frequency was determined simply by measuring the time required for ten oscillations. This reproduced to within less than 1.2%.

- **Amplitude.** ± 0.25 inches.

- **Suspension throughput.** Approximately 1.02 % (flow in U.S. Gallon per minute). The determination of the pump speed using the stroboscope-light gives a reproduction to within less than 1.2%.

- **Particle settling velocity.** ± 0.0113 feet/sec.

- **Delivered volumetric concentration of solids.** ± 3%. However, for very low concentration of solids, say 2-5% by volume, the error could be considerably large.

- **Pressure drop.** Replication of experiments indicated that pressure drop is reproduced within 0.018 feet of water/foot of pipe (over the test section of 20 feet) using the electronic device mentioned in section 1.5.1. This corresponds to an error of approximately 5%. The relationship between the pressure drop and the signal
in the recorder chart was of the type

\[ y = 2.267 x^{0.9539} \]  

(1.33.1)

where \( y \) is the pressure drop in inches of mercury and \( x \) is the reading in recording units. This type of equation changes slightly with the span and/or the diaphragm of the transducer, however, in all cases a slight non-linearity was noted in this relationship.
1.6 DISCUSSION OF RESULTS

Pressure drop with clear water

Pressure drop was measured for different fluid velocities under steady-state conditions. The experimental data is presented in table 1.1. The friction factor was calculated from the data of table 1.1 using the well-known Darcy-Weisbach equation,

\[ \Delta P = f \left( \frac{L}{D} \right) \left( \frac{\rho U^2}{2g_c} \right) \]  

(1.34)

Friction factors and Reynolds numbers were fitted according to the Blasius model and the Prandtl model (Streeter, 1966) using a least squares technique. The estimated regression equations were:

Blasius model

\[ f = 1.5233 \ (Re)^{-0.3844} \]  

(1.35)

Prandtl model

\[ \frac{1}{\sqrt{f}} = 4.021 \ \log(Re \ \sqrt{f}) - 9.195 \]  

(1.36)

In table 1.2 are tabulated the set of friction factors calculated from equation 1.34 and those predicted by equations 1.35 and 1.36 along with the residuals associated with these models, as well as the corresponding
FIG. 1.4 Friction factors under steady state conditions.
Reynolds number. Figure 1.4 shows the experimental friction factor as a function of Reynolds number on double logarithmic paper. The scatter of the data points is consistent with the experimental accuracy of the individual determinations of $f$. A slight difference is observed between the numerical values of Prandtl and Blasius parameters given by standard references and those obtained with the present experimental data. However, this fact is not really significant because very similar responses are given by these models when using the present parameters or those given by the standard references.

**Particle Size Distribution and Settling Velocity of Hematite**

The mineral used in the experiments with slurries was hematite whose specific gravity is 5.17. From previous experiments it was noted that the effect of particle attrition, due to prolonged recirculation of the slurry, tended to proceed towards a quasi-equilibrium size distribution after a few hours of recirculation. The mineral used in the pressure drop studies had at least ten hours of recirculation and the results of the screen analysis of this material is presented in table 1.3. The cumulative particle size distribution curve is shown in figure 1.6. The characteristic particle shape is shown in figure 1.7 and it can be appreciated that the solid particles are not nearly spherical. However, as is customary in this type of work, the "mean diameter" of particles found on any screen is expressed as a mean length between the openings in the screen above and that on which the particles rest. The "equivalent diameter" of the mixture was calculated
FIG. 1.6 CUMULATIVE PARTICLE SIZE DISTRIBUTION FOR HEMATITE
according to the Sauter mean diameter definition

\[ d = \frac{\sum \omega_i}{\sum \omega_i d_i} \] (1.37)

where \( \omega_i \) is the fraction by weight of particles with diameter \( d_i \), and \( d \) is the equivalent diameter of the mixture. The terminal settling velocities in still water were measured in a five-foot-long, six inch diameter, vertical glass tube, by timing the descent of particles selected at random from each screen. These results are presented in table 1.4. On figure 1.8 the terminal settling velocity has been plotted against the mean diameter on double logarithmic scale. The shape of the curve is similar to that of spheres of similar diameters. This curve has been taken from Perry (1969). The functional relationship between settling velocity and mean diameter was determined using the IBM Share-Program SD3094 for non-linear least squares curve fitting. It was found that data is represented well by the equation

\[ w = 6.8916 (d)^{0.7037} - 0.0552 \] (1.38)

where \( w \) is the settling velocity in feet/sec and \( d \) is the mean particle diameter in inches. This equation can be used for hematite particles within the range 0.002-0.06 inches with an standard deviation of 0.0113.
FIG. 1.7 Photograph of hematite particles. Courtesy of Dr. K. Chan, McMaster University.
Delivered Concentration of Solids

The delivered volumetric concentration of solids $C_v$, defined as the ratio of volume of solids to the volume of slurry, was determining by weighing the collected samples of slurry in a calibrated vessel. It was noted that the delivered concentration of solids was a function of mixture velocity and it was not equal to the overall concentration of solids in the mixture loaded into the system. While the difference is quite small at high flow rates, it is appreciable at low flow rates especially those below the limit deposit velocity, at which a substantial portion of the solids is held-up in the pipe and particularly in the reservoir. This aspect can be observed on figure 1.9, where the delivered volumetric concentration of solids has been plotted against the average velocity of the mixture. There is a systematic decrease in $C_v$ as the average mixture velocity decreases, especially when the total concentration of solids in the system is higher than 10%. However, the error in the determination of $C_v$ is relatively high when the mean velocity is in the neighborhood of the limit deposit velocity, mainly due to the unstable characteristics of the flow pattern in this region.

Pressure Drop with Slurries

Three flow patterns were observed for the flow of hematite-water mixtures in the range 5.0-16.0 ft./sec.

(a) Continuous and discontinuous stationary bed with saltation.
(b) Intermediate flow regime.
(c) Non-depositing transport regime or heterogeneous flow regime.
FIG. 1.8 SETTLING VELOCITY OF HEMATITE PARTICLES AT 22°C

\[ W = 6.8916 \times (d)^{0.7037} - 0.0552 \]

- EXPERIMENTAL
- SPHERICAL PARTICLES
These flow regimes are indicated in figure 1.10, where the hydraulic gradient due to slurry $h_m$ (feet of water per foot of pipe) is plotted versus the mean mixture velocity $U$ (the total rate of discharge per cross-sectional area of the pipe, ft./sec.) on linear paper. The points indicated on this figure are, of course, experimental values and the coordinates of most of these are presented in tables 1.5. The clear-water-line has been also included on figure 1.10 for reference. The lines of constant delivered concentration (figure 1.10) have been drawn as the better representations of the experimental coordinates and consequently do not represent the responses of any specific model. The tendency of these curves is similar to those given by most of the authors in the sense that for a given concentration of solids, as the mean mixture velocity decreases from a rather high value, first the hydraulic gradient decreases and then increases, passing through a minimum point. At high velocities the hydraulic gradient lines are approximately parallel to the clear-water-line and the deviation from the slope of this curve increases with concentration. The heterogeneous flow regime is characterized by the full suspension of the particles and it occurs at any velocity greater than 10.5 ft./sec. when the concentration of solids is not greater than 25% by volume.

The transition flow regime is characterized by the tendency for bed formation and the flow in this zone is unstable mainly due to incipient formation of a stationary bed. When deposition of particles occurs there is a reduction in the free area available for flow, and the velocity in the free area is therefore greater than the apparent
FIG. 1.9 Variation of delivered concentration of solids with mean velocity.
mean velocity; consequently, the shear forces exerted by the flowing water are greater than the frictional forces at the pipe wall and the bed slides along the bottom of the pipe. Associated with this unstable regime is the limit deposit velocity or the critical velocity \( U_c \). As a consequence of this, the pressure behaves erratically and data is not reliable. The velocity \( U_c \) does not correspond to a unique number, it is essentially a "region" whose boundaries can eventually be determined experimentally. In the present case, the limit deposit velocity shows a sensitivity to the volumetric concentration of solids, which is not always the case. For example, Round (1963) in his study on nickel (specific gravity 8.9) slurries observed no dependency between \( U_c \) and \( C_v \).

Several correlations have been proposed to determine the limit deposit velocity and some of these models were discussed in section 1.3.2. Figure 1.11 represents the critical Froude number versus the volumetric concentration of solids on double logarithmic scale, on which the predicted values of \( U_c \) due to different models are presented along with the estimated experimental values of \( U_c \). The scatter among the different responses is very great indeed; however, all models with exception of the one proposed by Newitt show a dependency of the limit deposit velocity with the concentration of solids. Durand's equation appears to correlate best the experimental data, even if the shape of the curve predicted by Charles' model seems to be very similar to the one projected by the experimental data. About this, Charles (1970) recommended using his correlation considering the drag coefficient of the
FIG. 1.10 Relationship between $h_m-U-C_v$ for hematite-water mixtures in a 2-inch pipe.
largest particle present. In the present case, a mean diameter of 0.05 inches with an equivalent drag coefficient 0.55 was considered in using equation 1.21, but it seems better to use the drag coefficient of a particle with the equivalent diameter of the mixture rather than the original proposal. The equation proposed by Spells, which considers most of the characteristics of the system, including the viscosity of the carrier fluid, predicts values of $U_c$ considerably in error, and no definite reason for this effect can be advanced. Aside from the possibility of differences in the experimental data used to develop these empirical correlations, the wide deviations in their predictions suggest that the correlations themselves are in error. At the present time no comprehensive theory of the limiting deposit condition, in circular pipes, exists. Shen (1970) has developed an interesting approach, using the Shield's criterion for incipient sediment motion, which is only valid for rectangular closed conduit turbulent flow.

There is no consistent data on hematite-water slurries published in the literature, with exception of some incomplete information given by Castro (1963). He determined the pressure gradients for slurries only 57% by weight of hematite and concluded that Durand-Condolios equation correlated the data well if the value of the constant is taken as 120. No attempt was made to determine the general characteristic of the system over a whole range of concentration. Some results on iron ore have been reported by Watanabe (1958), Thomas (1961), Sinclair (1960) and Linford (1969). However, the physical and chemical properties of the solids studied are entirely different from author to author, consequently the present data cannot be compared with the information
FIG. 1.11  Comparison between experimental and predicted critical Froude Number according to different authors for hematite-water suspensions.
already published.

**Discrimination of Models**

The purpose of this work was to determine in a scientific manner the best correlation for the heterogeneous flow of hematite-water slurries, as well as to obtain the best estimation of the parameters of each model. The regions of maximum separation between models and experimental data have been determined along with a critical discussion of the alternatives available at different levels of concentration and mixture velocity.

The entire process of sequential discrimination of models with design of experiments was performed using a computer program (see Appendix 1.1) written in single precision Fortran IV language. The process is summarized as follows:

1. Four experimental points were chosen initially. These are indicated in table 1.5 from sequence 1 to 4 and they represent the wide spectrum of possibilities for the heterogeneous flow regime, i.e., low and high concentrations along with maximum and minimum velocities permitted by the constraints of this flow regime. Also, the initial probability for each of the 4 models was taken as 0.25, showing no specific preference for any model. The parameter covariance matrix was selected considering values of the parameters reported in the literature. The variance of the experimental errors was evaluated on the basis of previous experiments.
2. A first estimation of the parameters is carried out using a least squares technique. Subroutine LEASQ repeats this operation for each model, considering initially only the 4 experimental points.

3. The Bayesian Analysis procedure is started. Subroutine Bayes computes the posterior model probabilities and the maximum likelihood estimates of the parameters. However, it should be noted that for the first iteration, in which 4 points are being considered, the values of the maximum likelihood estimates are identical to the least squares values as a condition imposed by the technique. Also, the parameters covariance matrix is updated for the next iteration.

4. The new set of experimental conditions, i.e., the values of the volumetric concentration of solids and the mean velocity of the mixture, is determined by Subroutine Roth. The process was then repeated at step 2 with 5 experimental points. The sequential procedure was continued until sequence 16, where the probability of Ayukawa-Ochi model reached the value 70%. The posterior model probabilities for each sequence are presented in table 1.17 and figure 1.12 is a graphical representation of this data. It is important to note that the Roth Criterion gave always the coordinates of experimental conditions at the maximum value of concentration (25 % by volume) and at the minimum or the maximum velocity imposed by the boundaries of this flow regime. However, it was not always possible to run an experiment at
FIG. 1.12 Model discrimination by sequential design.
Fig. 1.12.a. Posterior probabilities for Ayukawa-Ochi model considering different values for the variance of errors.
the conditions indicated by the Roth Criterion, and the nearest experimental conditions were taken (for some of the cases) from experimental data that had been previously collected when the boundaries and the general operation curves of this regime were determined. This fact is frequently encountered in the practical design of experiments but the final result is not altered, only a few more iterations are necessary. The design of experiments was terminated at sequence 16 because of the evidence that one of the models (Ayukawa-Ochi) gave the best correlation of the data. This fact can be readily appreciated by figure 1.12 where the tendency of the probability curve for model 1 is steadily increasing and significantly better than the curves of other models. Figure 1.12.a shows the posterior probabilities of Ayukawa-Ouchi model for different values of the error variance. As expected, when the error variance increases the posterior probabilities decrease. However, the initial estimate of the error variance appears to be correct; the ratio of the error variance to the residual variance (for Ayukawa-Ochi model) is approximately 1.4. The experimental points indicated by sequence 17 to 27 were randomly chosen (not considered in the design of experiments) and they are representative of those level of concentration not considered of significant importance in the general design of experiments. See table 1.5.

The equation proposed by Ayukawa-Ochi correlates the experimental data well and for high concentrations of solid and low velocities it can predict the hydraulic gradient with an error no greater than 6% (using the parameters here estimated) while Durand's equation predicts the data with an error no lower than 10% for these conditions. The region around 13 ft./sec. is correlated well by any of the models with exception of the one proposed by Charles. The separation in the responses of the models increases again when velocity is high, 15-16 ft./sec. This fact points out the importance of the design of experiments because if measurements had been carried out over equally spaced intervals, including high and low levels of concentrations, the result would have been significantly different.
The coefficient $\phi$ has been plotted versus the Froude number in figure 1.13, using a double logarithmic scale. The responses of different models are also shown, with exception of the one due to Kriegel and Brauer in order to reduce congestion. Experimental points tend to be ordered in two straight lines, one following the other, with a slight inflexion point around $\phi = s-1 = 4.17$. The model proposed by Ayukawa-Ochi gives a dependency of $\phi$ on the Froude number and the friction factor, the same for Kriegel and Brauer, while Durand's model does not consider the effect of change of the friction factor with the velocity. For these reasons, Durand's model gives a straight line on this plane and consequently cannot fit those points at high velocities. The response of the Ayukawa-Ochi model, and the same for Kriegel-Brauer, is slightly curved, with slope directly dependent on the value of the friction factor for clear water at a given Froude number. However, the effect of the friction factor may not be significant for large pipes, when the Reynolds number is large and the friction factor is practically constant. Figure 1.13 also indicates that differences in the responses between Ayukawa-Ochi and Durand equations are significant at high or low Froude numbers, while at intermediate flow rates, say 13 ft./sec., both models correlates the data well. Since the presentation of data on the $\phi-F_r$ plane is presented here for first time, it is not possible to make comparisons with other references.

The equations due to Durand and Charles are always represented on the $\phi-\psi$ plane (figure 1.14). Durand's equation appears to fit the experimental data better on the $\phi-\psi$ plane than in the $\phi-F_r$ one,
FIG. 1.13 Comparison of different models on $\phi$-$F_R$ plane.
This has been noted by Babcock (1970), who has shown, using Durand's original data, how the experimental data looks when it is plotted in different coordinate systems. However, the $\phi-F_r$ plane is completely general while the $\phi-\psi$ one is only valid for Durand or Durand-Charles equations. From figure 1.13 it would seem that the Durand-Charles model in no way represents the experimental data. Charles (1970) indicated that the intersection of Durand's equation and equation 1.2 suggests that there is a sudden transition from a regime in which Durand's equation applies to one in which equation 1.2 applies. He also pointed-out that this sharp transition is physically unrealistic and is not supported by experimental data. However, this statement of Professor Charles does not appear entirely correct. Indeed, the data presented by Round (1963), Newitt (1955), Nora Blatch (1902), Durand (1952), Babcock (1970), Hayden (1970) etc., do not show the tendency indicated by Charles. On the other hand, some data (Babcock, 1970) indicates that when specific gravity of particles is not large, say 2, then the systems appear to behave as stated by Charles. Positively, the modification of Durand's equation proposed by Charles is not a good alternative for the present data.

The fact that the Ayukawa-Ochi equation does not fit the lower concentration data as well as it does the higher concentration data could be interpreted in the following manner: the parameter in this model represents the friction factor due to a friction between particles and the pipe wall; it is correct to assume that this effect of friction becomes significant when the volume occupied by particles also becomes
FIG. 1.14  Comparison of Durand-Condolios and Charles equations on $\phi$-$\psi$ plane.
large, i.e., when concentration of solids is large and/or the velocity of the mixture is low.

Durand's equation has been widely used since it was proposed in 1952 and apparently the most representative value of its parameter is 121. The Ayukawa-Ochi model, proposed in 1968, has been used only by its authors and its parameter depends directly on the kind of pipe and material to be transported. However, this equation takes into account the factors that govern the mechanism of transport in a more detailed form. While Durand's equation is valid at any velocity greater than the limit deposit velocity, the Ayukawa-Ochi model does not predict the pressure gradient at high velocities well, as indicated by its authors, who have suggested as the upper limiting velocity:

\[ U \approx 2.9 \sqrt{gD(s-1)} \]  

(1.39)

For the present system this velocity is approximately 14 ft./sec. but the correlation was extended up to 15.5 ft./sec. without detriment to the model. However, this fact is not important because from the practical point of view it would not be economic to transport hematite at 14 ft./sec.

The equation proposed by Kriegel-Brauer represents a good alternative when its parameter is fitted by experimental data, otherwise it is advisable to use Durand's equation.
FIG. 1.15 Experimental $\phi$ vs. predicted $\phi$
1.7 CONCLUSIONS

- The studies on the turbulent flow of hematite-water suspensions indicated that this mixture behaves as "settling slurries".
- The hematite mineral can be slurried and conveyed in a 2 inch-diameter and horizontal duct. Three flow regimes were observed:
  (a) stationary bed with saltation at velocities below 7.0 ft./sec., depending on concentration of solids.
  (b) intermediate flow regime with unstable tendency of bed formation. (7. - 10. ft./sec.)
  (c) heterogeneous flow regime with full suspension of particles at velocities greater than 10.5 ft./sec. when the delivered concentration of solids is lower than 25% by volume.

The limit deposit velocity shows dependency on the delivered concentration of solids.

- A sequential discrimination of model using the Bayesian Approach and a design of experiments using the Roth Criterion indicated that the best model to correlate hydraulic gradients for these suspensions in the heterogeneous flow regime is the one proposed by Ayukawa-Ochi (1968). These experiments also showed that the difference between experimental and predicted data increase with the increasing concentration of solids and are significant at velocities near the limit deposit velocity.
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1.9 SYMBOLS.

A Posterior mean estimate of $\theta$.
B Least square estimate of parameter $\theta$.
CD Drag coefficient.
Cv Delivered volumetric concentration of solids.
d Particle diameter
D Pipe diameter.
Df likelihood density function.
f Friction factor.
Fr Froude Number.
g Acceleration of gravity.
hm Hydraulic gradient due to suspension.
hw Hydraulic gradient for clear water.
Re Reynolds Number.
s specific gravity.
P Pr Probability.
U Mean velocity.
Uc Limit deposit velocity.
Uj Covariance parameter matrix.
w particle settling velocity.
v Kinematic viscosity.
x Independent variable.
X differential matrix.
y Dependent variable.
Z Auxiliar variable (Roth Criterion).
\( \alpha \) Prior estimate of parameter \( \theta \).

\( \mu \) Difference between experimental and predicted value of the dependent variable.

\( \sigma^2 \) Variance.

\( \rho \) Density of clear water.

\( \omega \) Fraction by weight.

\( \Theta \) Parameter to be determined by experiments.
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PREDICTED PRESSURE GRADIENTS USING LEAST SQ. ESTIMATES.*

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* The models are presented in figure 1.5
**TABLE 1.7**

**PREDICTED PRESSURE GRADIENTS USING POSTERIOR MEAN ESTIMATES.**

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* The models are presented in figure 1.5
### TABLE 1.8

**ESTIMATED VALUES OF PARAMETERS AFTER EACH ITERATION.**

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A Posterior mean estimate.

B Least Squares estimate.

* The models are presented in figure 1.5
TABLE 1.9

PREDICTED COEFFICIENTS USING THE POSTERIOR MEAN ESTIMATES,*

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* The models are presented in figure 1.5
### TABLE 1.10

PREDICTED COEFFICIENTS \( \phi \) USING LEAST SQ. ESTIMATES

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### Table 1.11

**POSTERIOR MODEL PROBABILITIES AFTER EACH ITERATION.**

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* The models are presented in figure 1.5
APPENDIX 1.10.1

COMPUTER PROGRAM FOR DISCRIMINATION OF MODELS.
COMMON/UNO/RHO, VIS, DIA, D, S, W
COMMON/DOS/HM(30), U(30), CV(30), FR(30), FI(30), PSI(30), HW(30)
COMMON/CUATRO/UX(5,30), UX(5,30), V(5,5,5), Y(5,30)
COMMON/CINCO/NMODEL, ALPHA(5), BETA(5), PRPRE(5), PRPOST(5), SU(5),
PHI1, PHI3, PHI4
COMMON/OCHO/PHI(4,30), SSQ(5), RATIO(4,30), R(30)

READ(5,10) RHO, VIS, DIA, D, S, W
READ(5,20) (SU(M), M=1,5)
READ(5,30) SIGMA
READ(5,35) (PRPRE(M), M=1,5)

DO 40 M=1,4
DO 40 J=1,4
DO 40 I=1,4
VCM,J,J) = 0.0
DO 50 M=1,4
DO 50 J=1,4
V(M,J,I) = SIGMA

NORMALIZATION OF AUXILIAR CONSTANTS
D = D/17.
PHI1 = (W/SQRT(32.174*D))^1.5
PHI3 = W
PHI4 = (D/DIA)^(-0.707)*(SQRT(32.174*D*(S-1.)))*(2.72/W)^2

READ(5,70) NEXP, NINIT, NTOTAL, NCOUNT, NMODEL
IFR = 1
IF(NTOTAL.NE.NINIT) ITER=NTOTAL
READ(5,80) (CV(I), U(I), HM(I), I=ITER, NTOTAL)
CONTINUE
IF(NTOTAL.GT.30) GO TO 150

SUBROUTINE LEASQ RETURNS THE BEST ESTIMATES OF THE PARAMETERS.
CALL LEASQ(NINIT, NEXP, NTOTAL)

SUBROUTINE GRAD RETURNS VECTORS X AND UU.
CALL GRAD(NTOTAL, NEXP, NINIT)

WRITE(6,100)
1*M5*,,/
WRITE(6,110) (BETA(M), M=1, NMODEL)
WRITE(6,111) (ALPHA(M), M=1, NMODEL).
WRITE(6,114) (SU(M), M=1, NMODEL)
WRITE(6,112) (PRPRE(M), M=1, NMODEL)

SUBROUTINE BAYES PERFORMS BAYESIAN ANALYSIS.
CALL BAYES(NTOTAL, NEXP, NINIT, SIGMA)

WRITE(6,113) (PRPOST(M), M=1, NMODEL)
WRITE(6,120)
WRITE(6,130) (CV(I), U(I), HM(I), (Y(M,I), UU(M,I), M=1, NMODEL),
I=1, NTOTAL)

SUBROUTINE ROTH RETURNS THE NEW EXPERIMENTAL COORD NATES.
CALL ROTH(NINIT, NEXP, CVEXP, UEXP)
WRITE(6,140) CVEXP, UEXP
IF(INCOUNT.EQ.1)GO TO 60

CONTINUE

10 FORMAT(6F10.0)
20 FORMAT(5F10.0)
30 FORMAT(F10.0)
35 FORMAT(5F10.0)
70 FORMAT(5I5)
80 FORMAT(3F10.0)
110 FORMAT(/,X,*PFTA*,36X, 5F15.5)
111 FORMAT(/,X,*ALPHA*,35X,5E15.5)
114 FORMAT(/,X,*SU*,28X, 5E15.5)
112 FORMAT(/,X,*PRPRE*, 35X, 5E15.5)
113 FORMAT(/,X,*PRPOST*, 35X, 5E15.5)
120 FORMAT(//,2X,*NEW EXPERIMENTAL COORDINATES*, 10X, *CV*, F10.4,
130 FORMAT(/,2X,*CV*, F10.4)
STOP
END

SUBROUTINE GRAD(NTOTAL,NEXP,NINIT)
COMMON/UNO/RHO,VIS,DIA,D,S,W
COMMON/DOS/HM(30),U(30),CV(30),FR(30),FI(30),PSI(30),HW(30)
COMMON/CUATRO/UU(5,30),X(5,30),V(5,5,5),YMODEL(5,30)
COMMON/CINCO/ NMODEL,ALPHA(5),BETA(5),PRPRE(5),PRPOST(5),SU(5),
1PHI1,PHI3,PHI4
COMMON/OCHO/PHI(4,30),SSQ(5),RATIO(4,30),R(30)
DIMENSION F(30)

IF(NTOTAL.GT.NINIT) GO TO 15
DO 10 M=1,NMODEL
ALPHA(M) = BETA(M)
SSQ(M) = 0.
10 CONTINUE

ITER = NTOTAL
IF(NEXP.EQ.NINIT) ITER = 1
DO 20 I=ITER,NTOTAL
YMODEL(1,I) = HW(I) + ALPHA(1)*CV(I)*HW(I)*((S-1.0)**0.75)*PHI1/
1FR(I)**1.5
UU(I,1) = HM(I) - YMODEL(1,I)
X(I,1) = CV(I)*HW(I)*((S-1.0)**0.75)*PHI1/FR(I)**1.5
PHI(I,1) = ALPHA(1)*((S-1.0)**0.75)*PHI1/FR(I)**1.5
SSQ(1) = SSQ(1) + UU(I,1)*UU(I,1)
YMODEL(2,I) = HW(I) + ALPHA(2)*CV(I)*HW(I)*((S-1.0)**0.75)*PHI1/
20 CONTINUE
END
1FR(I)**1.5 + CV(I)*HW(I)*(S-1.0)
UU(2,I) = HM(I) - YMODEL(2,I)
X(2,I) = X(1,I)
PHI(2,I) = ALPHA(2)*PHI(1,I)/ALPHA(1) + S-1.
SSQ(2) = SSQ(2) + UU(2,I)**2

YMODEL(3,I) = HW(I) + ALPHA(3)*0.282*(S-1.0)*CV(I)*((PHI3**3/(32.174*VIS))**(1./3)*1(1+1.0)*FR(I)**(-4./3.))*U(I)**2/(2.0*32.174*2DIA)
UU(3,I) = HM(I) - YMODEL(3,I)
X(3,I) = 0.282*(S-1.0)*CV(I)*((PHI3**3/(32.174*VIS))**(1./3)*1(1+1.0)*FR(I)**(-4./3.))*PHI(I)/ALPHA(3)*U(I)**2/(2.0*32.174*2DIA)

SUBROUTINE BAYES(NTOTAL,NEXP,NINIT,SIGMA)
COMMON/CUATRO/ UU(5,30),X(5,30),V(5,5,5),YMODEL(5,30)
COMMON/CINCO/ NMODEL,ALPHA(5), BETA(5), PRPRE(5), PRPOST(5),SU(5)
DIMENSION AUX(5,5), AUX1(5,5,5), VAR(5,5,5), AUX2(5,5,5),
1AUX3(5,5,5), DETVAR(5), DETAUX(5), DF(5), AUX4(5,30), AUX5(5),
2VAR1(5,30)

IF(NEXP.NE.NINIT) GO TO 65
DO 10 M=1,NMODEL
DO 10 I=1,NINIT
AUX(M,I) = SU(M)*X(M,I)
10 CONTINUE
DO 20 M=1,NMODEL
DO 20 I=1,NINIT
DO 20 J=1,NINIT
AUX1(M,I,J) = AUX(M,I)*X(M,J)
20 CONTINUE
DO 30 M = 1,NMODEL
DO 30 I=1,NINIT
DO 30 J=1,NINIT
VAR(M,I,J) = VM(M,I,J) + AUX1(M,I,J)
30 CONTINUE

CONTINUE
DO 40 M=1,NMODEL
DO 40 I=1,NINIT
DO 40 J=1,NINIT
AUX2(M,I,J) = UU(M,1)*UU(M,J)
40 CONTINUE

DO 50 M=1,NMODEL
DO 50 I=1,NINIT
DO 50 J=1,NFEXP
AUX3(M,I,J) = VAR(M,I,J) + AUX2(M,I,J)
50 CONTINUE

SUBROUTINE DETER RETURNS THE DETERMINANTS OF VAR AND AUX3
DO 60 M=1,NMODEL
CALL DETER(NINIT,DFTAUX(M),AUX3,M)
CALL DETER(NINIT,DETFVAR(M),VAR,M)
60 CONTINUE
GO TO 66
65 CONTINUE
DO 66 M=1,NMODEL
VAR1(M,NTOTAL) = X(M,NTOTAL)*SU(M)*X(M,NTOTAL) + SIGMA
DETVAR(M) = VAR1(M,NTOTAL)
DETAUX(M) = VAR1(M,NTOTAL) + UU(M,NTOTAL)*UU(M,NTOTAL)
66 CONTINUE

CALCULATION OF THE LIKELIHOOD FUNCTIONS.
DO 70 M=1,NMODEL
Z = -0.5*(DETAUX(M)/DETVAR(M) - 1.0)
DF(M) = EXP(Z)/SQRT(DETVAR(M))
70 CONTINUE

EVALUATION OF UNNORMALIZED POSTERIOR MODEL PROBABILITIES.
ANORM = 0.
DO 80 M=1,NMODEL
PRPOST(M) = PRPRE(M)*DF(M)
ANORM = ANORM + PRPOST(M)
80 CONTINUE

NORMALIZED POSTERIOR MODEL PROBABILITIES
DO 90 M=1,NMODEL
PRPOST(M) = PRPOST(M)/ANORM
PRPRE(M) = PRPOST(M)
90 CONTINUE

POSTERIOR PARAMETERS DISTRIBUTION.
DO 100 M=1,NMODEL
DO 100 I=1,NTOTAL
AUX4(M,I) = X(M,I)/SIGMA
100 CONTINUE

DO 110 M=1,NMODEL
AUX5(M) = 0.0
DO 110 I=1,NTOTAL
AUX5(M) = AUX5(M) + X(M,I)*AUX4(M,I)
110 CONTINUE

DO 120 M=1,NMODEL
Z = 1.0/(AUX5(M) + 1.0/SU(M))
ALPHA(M) = Z*(AUX5(M)*BETA(M) + ALPHA(M)/SU(M))
SU(M) = Z
120 CONTINUE
RETURN
FND

SUBROUTINE ROTH(NINIT,NEXP,CVEXP,UEXP)

COMMON/UNO/PHO,VT~,nTA,n,~,w
COMMON/CINCO/NMODEL,ALPHA(5),BETAC5),PRPREC5>,PRPOST(5),SU(5),  
PHI1,PHI3,PHI4
DIMENSION GRADW(30),F(30),YMODEL(5,30,30),CV(30),U(30),C(5),  
Z(30,30),ZMAX(30,30)
DIMENSION FR(30)

IF(NEXP.NE.NINIT) GO TO 20
CVMIN = 0.03
CVMAX = 0.25
UMIN = 10.
UMAX = 16.0
IDELEN = IFIX(100*(CVMAX-CVMIN))
JDELTA = 2*IFIX(UMAX-UMIN)
DO 10 I=1,IDELEN
CV(I) = CVMIN + FLOAT(I)/100.0
10 CONTINUE
DO 20 I=1,JDELTA
U(I) = UMIN + 0.5*FLOAT(I)
CALL WATER(U(I),GRADW(I))
FR(I) = U(I)**2/(2.0*32.174*DIA)
20 CONTINUE
DO 25 L=1,IDELEN
DO 30 K=1,JDELTA
YMODEL(1,K,L) = GRADW(K) + BETA(1)*CV(L)*GRADW(K)*((S-1.)***0.75)*  
1PHI1/FR(K)**1.5
YMODEL(2,K,L) = GRADW(K) + BETA(2)*CV(L)*GRADW(K)*((S-1.)***0.75)*  
2PHI1/FR(K)**1.5 + CV(L)*GRADW(K)*((S-1.)***0.75)
YMODEL(3,K,L) = GRADW(K) + BETA(3)*0.282*(S-1.)*CV(L)*((PHI3**3/  
3(32.174*VIS))**((1./3.))*FR(K)**(-4./3.))*U(K)**2/(2.0*32.174*  
4DIA)
YMODEL(4,K,L) = GRADW(K) + BETA(4)*1.80*32.174*(S-1.)*DIA*CV(L)*  
5(U(K)**(-2.72))*PHI4*U(K)**2/(2.0*32.174*DIA)
DO 25 I=1,NMODEL
C(I) = 1.
DO 25 J=1,NMODFL
IF(J.EQ.I) GO TO 25
C(J) = C(J)*NMODEL(I,K,L)*C(J)
25 CONTINUE
SUBROUTINE DETER(N,DET,A,M)

THIS SUBROUTINE CALCULATES THE DETERMINANT OF A SQUARE MATRIX OF ORDER N BY THE METHOD OF PIVOTAL CONDENSATION.

DIMENSION A(5,5,5)

K = 2
L = 1

DO 10 I=K,N
RATIO = A(M,I,L)/A(M,L,L)
DO 10 J=K,N
A(M,I,J) = A(M,I,J) - A(M,L,J)*RATIO
IF(K-N)15,20,20
15 L = K + 1
GO TO 5
20 DET = 1.0
DO 25 L=1,N
DET = DET*A(M,L,L)
25 CONTINUE
RETURN
END

SUBROUTINE FRIC(F,U)
THIS SUBROUTINE WILL COMPUTE THE FRICTION FACTOR F FOR A PURE FLUID UNDER LAMINAR OR TURBULENT REGIMES. FOR TURBULENT LOW FRICTION FACTOR IS EVALUATED ACCORDING WITH VON-KARMAN-PRANDTL EQUATION AND THE NEWTON-RAPHSON ITERATION METHOD IS USED FOR FINDING THE ROOT OF THIS EQUATION.

B. CARNAHAN ET. AL. APPLIED NUMERICAL METHODS. CHAPTR 3. JOHN WILEY AND SONS INC. (197).

GLOSSARY OF PRINCIPAL SYMBOLS:
A,B = EMPIRICAL COEFFICIENTS IN VON-KARMAN-PRANDTL EQUATION.
C,D = EMPIRICAL COEFFICIENTS IN BLAUSIUS EQUATION.
VIS = FLUID VISCOSITY, LBM/FT•SEC.
DVIS = DYNAMIC FLUID VISCOSITY, LB•SEC/CU•FT.
U = MEAN FLUID VELOCITY, FEET/SEC.
F = FRICTION FACTOR, DIMENSIONLESS.
DIA = PIPELINE INTERNAL DIAMETER, FEET.
RHO = FLUID DENSITY, LBM/CU•FT.
RE = REYNOLDS NUMBER, DIMENSIONLESS.

DATA DIA,RHO, DVIS/0.1667, 62.4, 0.0000300/
DATA A,B,C,D/-9.1947749, 4.0212147, 1.523312, -0.3843671/

TRANSFORMATION OF DYNAMIC VISCOSITY (LB•SEC/CU•FT.) INTO THE ENGLISH ENGINEERING UNIT SYSTEM (LB/FT•SEC)
VIS = DVIS*37.174

F EVALUATION OF REYNOLDS NUMBER.
RE = DIA*U*RHO/VIS

CHECK FOR TURBULENT FLOW
IF(RE•GT•2000.0) GO TO 10
F = 64.0/RE
RETURN

THE BLAUSIUS EQUATION IS USED TO GET A FIRST ESTIMATE OF F.

10 F = C•RE**D

BEGIN NEWTON-RAPHSON ITERATION.
DO 20 I=1,50
PRO = RE•SQRT(F)
FNEW = F - (1.0/SQRT(F) - B•ALOG10(PRO) - A)*(2.0•F•SQRT(F))/
1(-1.0 - 0.4342944819•B•SQRT(F))

CHECK FOR CONVERGENCE.
IF(ABS(F-FNEW)•LT•1.0E-6) GO TO 40
F = FNEW
CONTINUE
SUBROUTINE WATER(U, GRADW)

THIS SUBROUTINE WILL CALCULATE THE HYDRAULIC GRADIENT FOR PURE WATER ACCORDING WITH DARCY-WEISBACH EQUATION.


GLOSSARY OF PRINCIPAL SYMBOLS.

DIA = PIPELINE DIAMETER, FEET.
F = FRICTION FACTOR, DIMENSIONLESS.
GRADW = HYDRAULIC GRADIENT FOR PURE WATER, FEET OF WATER PER FT. OF PIPE.
U = AVERAGE FLUID VELOCITY, FEET/SEC.

DATA DIA,RHO,DVIS/0.1667, 62.4, 0.0000167/

SUBROUTINE FRICTION RETURNS THE FRICTION FACTOR.
CALL FRIC(F,U)
GRADW = (F/DIA)*U**2/(12.0*32.174)
RETURN
FND
APPENDIX 1.10.2  

Calibration of Electromagnetic Flowmeter.

Figure 1.16 represents the calibration curve for the electromagnetic flowmeter (see 1.5.1). The electronic calibration given by the maker (also indicated on figure 1.16) certifies an error no greater than 1.05% on the flow rate. The water displacement of the pump (as percent of maximum flow) as a function of the pump speed is given in table 1.12.

**TABLE 1.12**

<table>
<thead>
<tr>
<th>Percent of maximum flow</th>
<th>Pump speed (r.p.m.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>188</td>
</tr>
<tr>
<td>36</td>
<td>228</td>
</tr>
<tr>
<td>41</td>
<td>256</td>
</tr>
<tr>
<td>45</td>
<td>280</td>
</tr>
<tr>
<td>51</td>
<td>314</td>
</tr>
<tr>
<td>56</td>
<td>352</td>
</tr>
<tr>
<td>61</td>
<td>386</td>
</tr>
<tr>
<td>67</td>
<td>423</td>
</tr>
<tr>
<td>73</td>
<td>458</td>
</tr>
<tr>
<td>78</td>
<td>480</td>
</tr>
<tr>
<td>84</td>
<td>515</td>
</tr>
<tr>
<td>90</td>
<td>549</td>
</tr>
</tbody>
</table>
FIG. 1.16 CALIBRATION CURVE FOR ELECTROMAGNETIC FLOWMETER.
PART II OSCILLATORY FLOW STUDIES
2.1 INTRODUCTION

Part II of this thesis deals with the behaviour of clear water under pulsed turbulent flow. Some preliminary observations on the behaviour of settling mixtures under oscillatory flow conditions are also included.

The behaviour of particles in pulsating flows is of interest in such diverse fields as fluidized-bed technology, atmospheric dispersion, transport of solids, etc. When pulsations were applied to solid-fluid systems, increases of up to 13-fold were reported in the mass-transfer coefficients (Lemlich, 1961) and up to 4-fold in the heat transfer coefficients (Hogg, 1966). But the enhancement of heat and mass transfer are not the only advantages gained by the application of pulsations to particulate systems. It was reported that pulsations enhance the stability of fluidized beds (Massimilla, 1964), decrease the viscosity of dispersed systems (Mikhailov, 1964) and increase the settling rate of flocculated suspensions (Obiakor, 1965). It has also been discovered that under the influence of pulsations it is possible to force bubbles to move downwards against the net buoyant force (Buchanan, 1962, Jameson, 1966) and to suspend particles against gravity in liquids (Houghton, 1963).

Pulsations are not necessarily externally superimposed on the system. Many solid-fluid systems will themselves generate macroscopic pulsations which otherwise will not exist in the fluid alone. This was reported during the hydraulic transport of solids (Lamb, 1932) and

2.2 THE BEHAVIOUR OF CLEAR WATER UNDER PULSED TURBULENT FLOW

Theory, results and comments on pressure drop, air consumption, fluid velocity and power dissipation for clear water under pulsatile flow are presented in appendix 2.5.1. Some complementary results are presented in appendix 2.5.2. The computer program listing for determining Fourier coefficients by numerical integration is included in appendix 2.5.3.

2.3 COMMENTS ON THE BEHAVIOUR OF SETTLING MIXTURES UNDER PULSATILE FLOW

When a solid-liquid mixture flows in a horizontal pipeline under steady state conditions at velocities below the minimum deposit velocity, partial or total deposition of solids will occur, depending on concentration of solids and mean velocity of the mixture. The objective of these observations was to estimate the effect of including an oscillatory component in the liquid flow when partial or total deposition of solids exists.

The apparatus used was the same indicated in appendix 2.5.1 and the slurry was prepared by adding a determined amount of hematite to the system, obtaining an overall concentration of solids of about 10% by volume.

Three flow regimes were visually observed when pulsations are applied to the system above mentioned:
1. At low amplitudes a stationary bed exists on the bottom of the pipe at all times.

2. At intermediate amplitudes a transition region exists, in which a stationary bed of solids exists for part of the time only.

3. At large amplitudes all particles are moving in water at all times.

Figure 2.1 gives some indication of the effect of pulsations on the flow regimes involved when the mean velocity of the mixture is changed. It would appear that the pulsatile component of velocity is as effective as the mean flow velocity in keeping particles in suspension. Thus, in pulsed flow a high solid concentration can be transported by a relatively small flow of water.
SYSTEM: 10% BY VOLUME SLURRY OF HEMATITE (30 MESH) IN 2 IN. DIAM. 80 FT. TEST LOOP

PARTICLES TOTALLY SUSPENDED IN WATER AT ALL TIMES.

A BED EXISTS ON BOTTOM OF PIPE AT ALL TIMES.

FIGURE 2.1
2.4 REFERENCES*


7. Lamb, H., Hydrodynamics, Dover, New York (1932)


*SEE ALSO APPENDIX 2.5.1
APPENDIX 2.5.1

THE BEHAVIOUR OF CLEAR WATER UNDER PULSED TURBULENT FLOW.
Friction Factors in Pulsed Turbulent Flow

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Friction Factors in Pulsed Turbulent Flow

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An apparatus for investigating pulsed turbulent liquid flow in a 2 in. diameter, 80 ft. pipeline is described. The pulsation unit was powered by compressed air with a consumption of up to 2.7 std. cu.ft./min. at 35 lb./in² gauge. The pressure drop for water flowing at mean velocities of 7.66 to 12.28 ft./sec. has been measured, both for steady flow and for pulsed flow, at frequencies between 0.48 and 0.82 Hz. The experimentally measured pressure versus time curves for pulsed flow can be matched fairly closely by a solution of Euler's equation employing the friction factors measured under steady flow conditions.

Pulsating flow of fluids occurs widely, both in nature and industry. One area that has interested chemical engineers for many years is the improvement of processes by the deliberate application of pulsed flow¹. The present authors have begun a program of research on pipeline conveying of slurries in a pulsed flow of water, using an air pulsation technique². Such a technique has an advantage over pistons or flow interrupters which would be adversely affected by suspended solids.

The equipment, which is described in detail following, has first been operated with pure water alone. The objective of this work has been to obtain data on instantaneous pressure drops and friction losses in pulsed turbulent flow. The operating characteristics of the pulsation unit are also given.

On décrit un appareil pour étudier un courant liquide turbulent et pulsatoire dans un pipeline de 80 pieds de longueur et 2 pouces de diamètre. On a actionné le dispositif de pulsation avec de l'air comprimé à raison de 2.7 std. pieds cubes à la minute à une pression de 35 livres au pouce carré. On a mesuré la chute de pression de l'eau qui coule à des vitesses moyennes de 7.66 à 12.28 pieds à la seconde et ce dans les cas d'un courant stable et d'un courant pulsatoire et à des fréquences variant entre 0.48 et 0.82 Hz. Les graphiques reproduisant les mesures expérimentales de la pression vers le temps, dans le cas d'un écoulement pulsatoire, s'harmonisent assez bien avec la solution d'une équation d'Euler où l'on emploie les facteurs de frottement mesurés dans des conditions correspondant à celles d'un écoulement stable.

Pulsed laminar flow in pipes has been quite thoroughly investigated⁵,⁷ and it has been found that above a certain limiting frequency friction factors are greater than the values for steady flow. Consequently the energy losses are greater than would be expected using a quasi-steady model⁸. In the case of turbulent flow, Schultz-Grunow⁹ found that a quasi-steady state model was satisfactory, i.e., the instantaneous frictional pressure drop could be predicted from the instantaneous velocity using the steady-flow friction factor values. This early work⁸ was at frequencies up to only 0.25 Hz, but more recently Streeter and Wylie⁹ have successfully used the quasi-steady model in analyzing the hydraulic transients from a reciprocating pump at frequencies in the order of 10 Hz.

¹Department of Mechanical Engineering.
²Department of Chemical Engineering.
However this analysis was valid for flows with only a relatively small oscillatory component. Recently, Brown and others studied the response of turbulent flows to small-amplitude oscillations at acoustic frequencies (50 - 3000 Hz). At these high frequencies, the observed attenuation agreed with calculation based on "constant turbulence", i.e. the flow pattern and turbulence did not have time to adjust to the rapid fluctuations in velocity. The present investigation concerns pulsed turbulent flows at frequencies of 0.48-0.82 Hz with flow fluctuations of up to ±50% of the mean flow.

**Apparatus**

The circulation loop used is shown schematically in Figure 1. Water from the reservoir was pumped at a steady rate to the 80-ft. loop of 2-in. internal diameter steel pipe by a rotary, positive displacement pump (Moyno, type CDR). The pump speed could be varied to give a flow range from 6.7 - 20-cu.ft./min.

Shortly downstream of the pump, the water line was connected to the pulsation unit, the purpose of which was to impose an oscillatory component on the flow. The pulsation unit, which is shown in detail in Figure 2, operated on the self-triggering principle. This principle has been useful in pulsing gas absorbers, extraction columns and hydraulic test tanks. The present investigation deals with a new application of the principle to a continuous turbulent flow system. The water rising into the vertical section of 2-in. bore glass tubing activated a conductivity probe which operated a solenoid valve, supplying compressed air (normally 35 psi). As the water level receded past the probe, the solenoid switched to the "exhaust" position and the air space was connected to atmosphere. In this way the cycle repeated itself, with the water level oscillating about the probe. Previous investigations have shown that such a pulsator tends to operate at the natural frequency of the system, giving a smooth waveform and a relatively efficient use of compressed air. The pulsation frequency could be altered by adjusting the probe vertically, and the amplitude could be adjusted by varying the air supply. An air shut-off valve was also provided to permit unpulsed operation of the loop.

The water flow continued along the lower part of the loop, then via a U-section with a radius of curvature 1.33-ft. to the upper part of the loop in which the pressure-drop test section was situated. The test section began 3-ft. from the U-Section. The pressure-drop between two points 31.66 ft. apart was measured by a diaphragm transducer (Pace Engineering Co., type P7D) and transmitted to a high-speed recorder. Downstream of the test section, an electromagnetic flowmeter (Brooks Instruments, model 7300) measured the fluid velocity which was recorded on the same chart as the pressure drop signal.

The measurements taken in a typical pulsed-flow test included the pressure drop and velocity as functions of time, the frequency of pulsation (by stopwatch timing of ten cycles), the stroke (amplitude) of the water level in the pulsation unit, and the air consumption. This latter measurement was made by connecting the exhaust air line to an inverted water-filled measuring cylinder for a known number of cycles. The air consumption was obtained as the volume collected per cycle multiplied by the frequency.
Friction in steady flow

The friction factor in steady flow was calculated from the experimental pressure and velocity data using the well-known Darcy-Weisbach equation:

\[ \Delta P = f \cdot \left( \frac{L}{D} \right) \cdot \left( \frac{\rho}{2g_e} \right) \cdot \left( \frac{U^2}{2D} \right) \]  

(1)

The experimental values of \( f \) are shown in Figure 3. Also shown are the Blasius equation and the von Karman-Nikuradse equation for the friction factor in smooth pipes:

Blasius:

\[ f = 0.316 \cdot Re^{-0.25} \]  

(2)

Von Karman-Nikuradse:

\[ 1/\sqrt{f} = 0.86 \ln [Re\sqrt{f}] - 0.8 \]  

(3)

The scatter of the data points on Figure 3 is consistent with the experimental accuracy of the individual determinations of \( f \). Although the measured values of \( f \) are in reasonable agreement with Equations (2) and (3), the effect of Reynolds number appears to be slightly greater than expected; no definite reason for this effect can be advanced.

In interpreting pulsed-flow data, it was necessary to consider a Reynolds number range greater than that over which data (Figure 3) could be obtained in steady flow. The Blasius-type equation for \( f \), though simple, should not be applied at Reynolds numbers greater than 10^5. Accordingly it was decided to use a form of Equation (3) with coefficients adjusted to fit the data on Figure 3.

\[ 1/\sqrt{f} = 1.746 \ln [Re\sqrt{f}] - 9.195 \]  

(4)

Pulsed flow operation

Figure 4 shows the effect of air flow rate upon frequency and amplitude, at three different liquid flow velocities and a single position of the probe. A volumetric efficiency may be defined as the ratio of the volume of liquid displaced per cycle, divided by the volume of air (standard conditions) supplied per cycle:

\[ \eta = \frac{2\pi A w S}{Q} \]  

(5)

The values of \( n \) obtained in the present work are between 0.25 and 0.4, a range somewhat lower than that obtained in pulsing systems with no net flow. The friction due to turbulent flow undoubtedly contributes heavily to damping effects.

The negative effect of air consumption upon frequency, shown in Figure 4, is also characteristic of heavily damped systems. The curves resemble the curve obtained by Baird and Garstang for the pulsing of water through a bed of Raschig rings.

Quasi steady state model

If the velocity \( u \) as a function of time is known, and assuming the dependence of \( f \) upon \( u \) given by Equation (4), the pressure gradient variation may be calculated.

The equation of motion may be written in terms of the instantaneous flow velocity \( u \) (averaged across the pipe section), taking a length of pipe as the control volume. Incompressible flow and rigid pipe walls are assumed.

\[ \omega R^2/v \approx 0.025 \cdot Re \]

\[ \omega \approx 0.1 \cdot U/D \]  

(8)
Power dissipation

Table 1

<table>
<thead>
<tr>
<th>Velocity U, ft/s</th>
<th>Amplitude A, inches</th>
<th>Frequency H, Hz</th>
<th>Power dissipation J, ft lb/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(observed)</td>
<td>(calculated)</td>
<td></td>
</tr>
<tr>
<td>7.67</td>
<td>2.5</td>
<td>0.621</td>
<td>12.52</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>0.650</td>
<td>12.77</td>
</tr>
<tr>
<td></td>
<td>7.95</td>
<td>0.580</td>
<td>13.35</td>
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<tr>
<td></td>
<td>10.35</td>
<td>0.541</td>
<td>14.08</td>
</tr>
<tr>
<td></td>
<td>12.8</td>
<td>0.482</td>
<td>14.29</td>
</tr>
<tr>
<td>9.82</td>
<td>2.8</td>
<td>0.701</td>
<td>23.49</td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td>0.741</td>
<td>24.47</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.662</td>
<td>25.33</td>
</tr>
<tr>
<td></td>
<td>9.55</td>
<td>0.629</td>
<td>25.92</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td></td>
<td>9.55</td>
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<td>46.23</td>
</tr>
</tbody>
</table>

Thus, for the present conditions, the transition frequency would be in the order of 6 radians/sec or 1 Hz.

Although the frequencies used in this work were of the order of 1 Hz, the response lag was no more than may be expected from the magnetic flowmeter, so the transition frequency must be somewhat greater than that given by Equation (8).

Further support for the contention of a higher transition frequency is given by the success of the quasi-steady model for frequencies up to 10 Hz in a 3-inch pipe.

Power dissipation

The power dissipation averaged over a cycle is an important factor in the design of pulsed flow equipment. It is given by:

\[ J = \frac{\omega}{2\pi} \int_0^{\pi/4} D^2 u \Delta P \, dt \]  

Experimental values of this quantity are obtained from the values of \( u \) and \( \Delta P \) on the chart record. The value of \( J \) may also be estimated from the experimental measurement of \( u \) and a value of \( \Delta P \) calculated from \( u \) using Equation (6) in conjunction with the steady flow friction relationship of Equation (4). In both these calculations of \( J \), due attention is paid in the integration to that period in which \( \Delta P \) has a negative sign, in which case there is a net recovery of energy from the water as it decelerates.

The results of the tests are summarised in Table 1, and it will be seen that the observed and calculated values of \( J \) agree within 1%. The small error due to velocity measurement lag, apparent on Figure 5, is largely cancelled out when integration over the complete oscillation cycle is performed.

Conclusions

This investigation has confirmed that the quasi-steady state hypothesis (i.e., fully developed flow assumption) applied to pulsatile turbulent flow in the conditions studied. The air-pulsing principle\(^9\) can be applied for turbulent water flow in a 2-inch pipeline.

Acknowledgments

We are grateful to the Department of Energy, Mines and Resources (Mines Branch) and the National Research Council of Canada for financial support. One of us (J. N. C.) acknowledges the support of an Inter-American Development Bank scholarship.

Nomenclature

- \( A \): amplitude (stroke) measured at pulsation unit
- \( \Delta P \): Fourier coefficients
- \( D \): pipe diameter
- \( D_i \): friction factor
- \( D_{ik} \): gravitational constant
- \( f \): pressure
data
- \( f \): frequency (average over 1 cycle)
- \( L \): length of test section
- \( p \): volume of air supplied per cycle
- \( R \): radius of pipe
- \( S \): cross-sectional area of pipe
- \( u \): velocity (instantaneous)
- \( U \): velocity (average over 1 cycle)
- \( x \): axial distance
- \( \omega \): angular frequency
- \( \eta \): volumetric efficiency
- \( v \): kinematic viscosity
- \( Re \): Reynolds number = \( UD/v \)

References


* * *


RESULTS OF RUN NUMBER 271

EXPERIMENTAL CONDITIONS

AMPLITUDE (FEET) = 0.2786
FREQUENCY (1/SEC) = 1.76666
AVERAGE VELOCITY (FEET/SEC) = 13.21000
NUMBER OF EXPERIMENTAL POINTS = 11

FACTORX = 1.00000 FACTORY = .40000 FACTORZ = .35664

INDEPENDENT COEFFICIENT

AO = 13.1245006

COEFFICIENTS CORRESPONDING TO SINE SERIES AND VELOCITIES CALCULATED WITH THEM

A1 = B(1,1) = 1.0695911 A2 = B(1,2) = .0301216
A3 = B(1,3) = -.0086353 A4 = B(1,4) = .2091010

TIME UEXP U(1,1,1) U(1,2,1) U(1,3,1) U(1,4,1) U(1,4,1)
1.050000 11.89983 12.10719 .20777 12.15429 -.25435 12.14921 -.24975 11.77916 .13279
1.200000 11.77907 12.10719 .32813 12.08010 -.28102 12.05502 -.27596 11.49204 .17101

SUM ABS VALUE OF DIFFERENCE = 2.82585 2.82585 2.82585 1.12777

MEAN DEVIATION = .25690 .25690 .25690 .1207

STANDARD DEVIATION = .31002 .30376 .30324 .25451

315. PROCESSING AND COMPUTER CENTRE
COEFFICIENTS CORRESPONDING TO COSINE SERIES AND VELOCITIES CALCULATED WITH THEM

\[
B_1 = B(2,1) = -2.9951035 \\
B_2 = B(2,2) = 0.3564604 \\
B_3 = B(2,3) = -1.473822 \\
B_4 = B(2,4) = 0.1317646
\]

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<th>UEXP</th>
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<th>D(2,1)</th>
<th>L(2,2)</th>
<th>D(2,2)</th>
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<th>D(2,3)</th>
<th>U(2,4)</th>
<th>D(2,4)</th>
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SUM ABS VALUE OF DIFFERENCE = 7.59795
MEAN DEVIATION = .69872
STANDARD DEVIATION = .76197
RESULTS OF RUN NUMBER 272

EXPERIMENTAL CONDITIONS

AMPLITUDE (FEET) = .42708
FREQUENCY (1/SEC) = .71000
AVERAGE VELOCITY (FEET/SEC) = 13.21000
NUMBER OF EXPERIMENTAL POINTS = 11
FACTOY = 1.00000 FACTOY = .38350 FACTOZ = .35609

INDEPENDENT COEFFICIENT

A0 = 13.1952125

COEFFICIENTS CORRESPONDING TO SINE SERIES AND VELOCITIES CALCULATED WITH THEM

<table>
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<th>U(1,2)</th>
<th>D(1,2)</th>
<th>U(1,3)</th>
<th>D(1,3)</th>
<th>U(1,4)</th>
<th>D(1,4)</th>
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</table>

JMJ ABS VALUE OF DIFFERENCE = 3.22009 3.22009 3.22009 4.42429
MEAN DEVIATION = .29274 .29274 .29274 .42221
STANDARD DEVIATION = .35713 .35404 .35437 .51027
### COEFFICIENTS CORRESPONDING TO COSINE SERIES AND VELOCITIES CALCULATED WITH THEM

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<tr>
<th>TIME</th>
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<th>U(2,2,1)</th>
<th>D(2,2,1)</th>
<th>U(2,3,1)</th>
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<td>13.11214</td>
<td>0.17032</td>
</tr>
</tbody>
</table>

**SUM ABS VALUE OF DIFFERENCE** = 11.15972

**MEAN DEVIATION** = 1.01452

**STANDARD DEVIATION** = 1.13635

**SUM VALUE OF DIFFERENCE** = 10.12244

**MEAN DEVIATION** = 0.92822

**STANDARD DEVIATION** = 1.10724
RESULTS OF RUN NUMBER 273

Experimental Conditions

Amplitude (Feet) = 0.63541
Frequency (1/sec) = 70.000
Average velocity (Feet/sec) = 13.21900
Number of experimental points = 11

Factor = 1.00000
Factor = 0.37710
Factor = 0.35609

Independent Coefficient

\[ A_0 = 13.153938 \]

Coefficients corresponding to sine, cosine and velocities calculated with them

\[
\begin{align*}
A_1 &= B(1,1) = 2.2831967 \\
A_2 &= B(1,2) = 0.2153013 \\
A_3 &= B(1,3) = 0.1551513 \\
A_4 &= B(1,4) = 0.7737807
\end{align*}
\]

<table>
<thead>
<tr>
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<th>D(1,4)</th>
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Sum Abs Value of Difference = 4.20984
Mean Deviation = 0.36271
Standard Deviation = 0.46249

-- A PROFESSING AND COMPUTER CENTRE --

13
COEFFICIENTS CORRESPONDING TO COSINE SERIES AND VELOCITIES CALCULATED WITH THEM

<table>
<thead>
<tr>
<th>B1 = b(2, 1) = -0.5735996</th>
<th>B2 = b(2, 2) = 0.2697001</th>
<th>B3 = b(2, 3) = 0.392804</th>
<th>B4 = b(2, 4) = 0.2755242</th>
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</table>

<table>
<thead>
<tr>
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SUM ABS VALUE OF DIFFERENCE = 15.63640
MEAN DEVIATION = 1.42149
STANDARD DEVIATION = 1.57360

104 A PROCESSING AND COMPUTER CENTRE
RESULTS OF RUN NUMBER 274

Experimental Conditions

Amplitude (Feet) = .8320
Frequency (1/Sec) = .6666
Average Velocity (Feet/Sec) = 13.2100
Number of Experimental Points = 11

Factor X = 1.0000
Factor Y = .3835
Factor Z = .3564

Independent Coefficient

\[ A_0 = 13.11340 \]

Coefficients Corresponding to Sine Series and Velocities Calculated with Them

\[ A_1 = B(1,1) = 2.9095313 \]
\[ A_2 = B(1,2) = .0429549 \]
\[ A_3 = B(1,3) = .0011266 \]
\[ A_4 = B(1,4) = .9497416 \]

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<th>U(1,3,1)</th>
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Sum Abs Value of Difference = 2.06426 2.06427 2.06427 6.33043

Mean Deviation = .18766 .18766 .18766 .57549

Standard Deviation = .22862 .22568 .22568 .49315
COEFFICIENTS CORRESPONDING TO COSINE SERIES AND VELOCITIES CALCULATED WITH THEM

\[
\begin{align*}
B_1 &= B(2,1) = -1.163762 \\
B_2 &= B(2,2) = .1219837 \\
B_3 &= B(2,3) = .1186688 \\
B_4 &= B(2,4) = .0164770
\end{align*}
\]

<table>
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<th>(U_{(2,2)})</th>
<th>(D_{(2,2)})</th>
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</table>

**SUM ABS VALUE OF DIFFERENCE** =

\[
\begin{align*}
18.52641 &
18.40443 &
18.04842 &
18.53195
\end{align*}
\]

**MEAN DEVIATION** =

\[
\begin{align*}
1.68422 &
1.67313 &
1.64077 &
1.63927
\end{align*}
\]

**STANDARD DEVIATION** =

\[
\begin{align*}
1.98676 &
1.97212 &
1.96828 &
1.95843
\end{align*}
\]
APPENDIX 2.5.3
PROGRAM R-10

PIPELINE CONVEYING PROJECT, McMaster University, Hamilton, Canada.

SUPERVISORS DR. Z. SARD, CH. ENG., AND DR. G. ROUND, MECH. ENG.

PROGRAMMER GEORGE CARDENAS, CHEMICAL ENGINEERING DEPARTMENT.

THIS PROGRAM WILL EVALUATE IN PART ONE THE FOURIERS COEFFICIENTS AND VELOCITIES FROM AN EXPERIMENTAL SET OF VELOCITY AND TIME VALUES. IN PART TWO WILL CALCULATE VELOCITIES ACCORDING WITH THE SINUSOIDAL MODEL. IN PART THREE WILL COMPUTE PRESSURE DROPS ACCORDING WITH DIFFERENT MODELS. IN PART FOUR WILL PLOT THE EXPERIMENTAL DATA. EVENTUALLY A FIFTH PART CAN BE ADDED IN ORDER TO PUNCH THE DATA CALCULATED.

EACH EXPERIMENTAL POINT IS EXPRESSED IN TERMS OF VELOCITY, TIME AND PRESSURE DROP RESPECTIVELY. THE NUMBER OF EXPERIMENTAL POINTS OF EACH SET OF DATA MUST CORRESPOND TO A COMPLETE PERIOD AND MUST ALWAYS BE AN ODD NUMBER (THE MAXIMUM NUMBER CAN BE 91). THE TIME INTERVAL BETWEEN SUCCESSIVE TIME VALUES MUST BE CONSTANT.

THE ARRANGEMENT OF EACH SET OF DATA IS AS FOLLOWS

1. ONE CARD WITH THE CONDITIONS OF THE RUN
2. THE SET OF EXPERIMENTAL POINTS
3. THE IDENTIFICATION CARDS OF THE GRAPHS
4. A CONTROL CARD. IF IT IS EQUALS ONE ANOTHER COMPLETE SET OF DATA MUST FOLLOW. IF IT IS NOT EQUALS ONE THE PROGRAM WILL EXIT.

GLOSSARY OF PRINCIPAL SYMBOLS:

AREA = TRANSVERSAL SECTION OF THE PIPE, SQ. FEET.
LENGTH = LENGTH OF THE PIPE, FEET.
AMPL = AMPLITUDE, FEET.
DP = PRESSURE DROP, PSI.
DIA = DIAMETER OF PIPE, FEET.
DPEXP = EXPERIMENTAL PRESSURE DROP, PSI.
DVIS = DYNAMIC VISCOSITY OF THE FLUID, POUND/SEC/SQ. FEET.
DERIV = DERIVATIVE OF VELOCITY RESPECT TO TIME.
FACTOY, FACTOX, FACTOZ = CONVERSION FACTORS FOR VELOCITY, TIME AND PRESSURE DROP FROM EXPERIMENTAL UNITS TO FEET/SEC, SEC AND PSI.
FRE = FREQUENCY, 1/SEC.
F = FRCTION FACTOR.
N = NUMBER OF EXPERIMENTAL POINTS.
RHO = DENSITY OF FLUID, POUND/CU. FEET.
RN = NUMBER OF THE EXPERIMENTAL RUN.
T = TIME, SEC.
UAVE = AVERAGE VELOCITY, FEET/SEC.
UEXP = EXPERIMENTAL VELOCITY, FEET/SEC.
USIN = VELOCITY CALCULATED ACCORDING WITH SINUSOIDAL MODEL, FEET/SEC.
U(L, M, 1) = VELOCITY CALCULATED ACCORDING WITH FOURIERS MODEL, FEET/SEC.
XI, YI, ZI = EXPERIMENTAL COORDINATES FOR TIME, VELOCITY AND PRESSURE DROP.

DIMENSION XI(51), YI(51), ZI(51), T(51), UEXP(51), DPEXP(51),
IHX(51), FT(51), B(5, 5, 5), SUX(5), DIFSS(5), AMEXAN(5, 5),
1STAND(5, 5), U(5, 5, 5, 1), D(5, 5, 5, 1)
DIMENSION USIN(21), DIFSR(21)
DIMENSION SDIFF(10), SDIFFS(5), SUX(10, 21), DERIV(100), DP(10, 21)
1, DIFFR(10, 21), ZMEAN(10), ZSTAND(10)
DIMENSION TT(100), UA(100), UPR(100)
READ EXPERIMENTAL DATA. CONVERSION TO CONVENTIONAL UNITS.
1 READ(U4, I1, R6, FRE, AMP, UAVE, FACTO, FACTY, FACTZ
10 FORMAT(15, F5.0, U5.0)
   K = N - 1
   DO 30 I = 1, N
      READ(5,20) XI(I), YI(I), ZI(I)
   20 FORMAT(3F15.6)
      T(I) = FACTO*(1.0/FRE*FLOAT(K))/XI(I)
      UEXP(I) = FACTO*(0.70679/2.12/YI(I)+UAVE
      OEXP(I) = FACTZ*ZI(I)
   30 CONTINUE
WRITE(6,40) RN
WRITE(6,50)
WRITE(6,60)
WRITE(6,70)
WRITE(6,80) AMP
WRITE(6,90) FRE
WRITE(6,95) UAVE
WRITE(6,100) N
WRITE(6,110) FACTO, FACTY, FACTZ
WRITE(6,120)
40 FORMAT(1H1, 35X, * RESULTS OF RUN NUMBER*; 1X, F4.0)
50 FORMAT(1H1, 34X, '*EXPERIMENTAL CONDITIONS')
60 FORMAT(1H1, 34X, '///\')
70 FORMAT(1H1, 34X, '///')
80 FORMAT(1H1, 31X, 'AMPLITUDE (FEET) = *, F7.5)
90 FORMAT(1H1, 31X, 'FREQUENCY (1/SEC) = *, F7.5)
95 FORMAT(1H1, 31X, 'AVG VELOCITY (FEET/SEC) = *, F9.5)
100 FORMAT(1H1, 31X, 'HUMOT OF EXPERIMENTAL Points: = *, I, )
110 FORMAT(1H1, 14X, 9HFACTO = *, F9.5, 6X, 9HFACTY = *, F9.5, 6X, 9HFACTZ = *, F9.5)
120 FORMAT(1H1, '///, 9X)

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PART ONE.- EVALUATION OF FOURIERS COEFFICIENTS FOR THE FOLLOWING MODEL

\[ U = A_0 + A_1 \sin(x) + A_2 \sin(2x) + A_3 \sin(3x) + A_4 \sin(4x) + H_1 \cos(x) + H_2 \cos(2x) + H_3 \cos(3x) + H_4 \cos(4x) \]

EVALUATION OF THE INDEPENDENT COEFFICIENT \( A_0 \). THE SIMPSON'S RULE IS USED
IN ORDER TO EVALUATE THE CORRESPONDING INTEGRAL:

\[ \text{SUM4} = 0.0 \]
\[ \text{SUM2} = 0.0 \]
\[ H = T(N) - T(1)/FLOAT(K) \]
\[ \text{DO 200 I = 2, J, 2} \]
\[ \text{SUM4 = SUM4 + UEXP(I)} \]
\[ \text{SUM2 = SUM2 + UEXP(I+1)} \]
\[ \text{200 CONTINUE} \]
\[ A_0 = \text{FRE} \times \{(H/3.0) \times \text{UEXP(1)} + 4.0 \times \text{SUM4} + \text{UEXP(K)} + 2.0 \times \text{SUM2} + 1.0 \times \text{UEXP(N)}\) \]

EVALUATION OF COEFFICIENTS CORRESPONDING TO SINE AND COSINE SERIES.
\( M \) IS THE SUBSCRIPT OF THE FOURIERS COEFFICIENT.
\[ L = 1 \]
\[ 210 N = 1 \]
220 SUM4 = 0.0
      SUM2 = 0.0
      DO 240 I = 1,N
      XX(I) = FLOAT(I)*2.0E3*141502*FREST(I)
      IF(L.EQ.2) GO TO 230
      F(I) = UEXP(I)*SIN(XX(I))
      GO TO 240
230   F(I) = UEXP(I)*COS(XX(I))
240 CONTINUE
      DO 250 I = 2,J,2
      SUM4 = SUM4 + F(I)
      SUM2 = SUM2 + F(I+1)
250 CONTINUE
      SL(I,*) = 2.0*FREST((I/2.0)*(F(I) + 4.0*(SUM4 + F(I)) + 2.0*SUM2
       + F(I+1)))

      EVALUATION OF VELOCITIES.
      DIFFS(L,*) = 0.0
      SU1(L,*) = 0.0
      DO 300 I = 1,N
      IF(L.EQ.2) GO TO 260
      UTRANS = SL(L,*)&SIN(XX(I))
      GO TO 270
260   UTRANS = SL(L,*)&COS(XX(I))
270 IF(L.EQ.1) GO TO 280
      U(L,*,I) = U(L,*,I-1,I) + UTRANS
      GO TO 290
280   U(L,*,I) = AO + UTRANS
290   D(L,*,I) = UEXP(I) - U(L,*,I)
      SU1(L,*) = SL(L,*)&SU1(L,*) + AO*(U(L,*,I))
      DIFFS(L,*) = DIFFS(L,*) + U(L,*,I)**2
300 CONTINUE
      AMEAN(L,*) = (SU1(L,*)/FLOAT(N)
      STDDEV(L,*) = SQRT((DIFFS(L,*)/FLOAT(N))
      IF(L.EQ.4) GO TO 310
      M = M + 1
      GO TO 220
310  L = L + 1
      IF(L.EQ.3) GO TO 320
      GO TO 210
320 M = 1
330   SDI(L,M) = 0.0
      DIFFS(L,*) = 0.0
      DO 350 I = 1,N
      U(L,*,I) = U(L-2,*,I) + U(L-1,*,I) - AO
      D(L,*,I) = UEXP(I) - U(L,*,I)
      SU1(L,*) = SL(L,*)&SU1(L,*) + AO*(U(L,*,I))
      DIFFS(L,*) = DIFFS(L,*) + U(L,*,I)**2
350 CONTINUE
      AMEAN(L,*) = (SU1(L,*)/FLOAT(N)
      STDDEV(L,*) = SQRT((DIFFS(L,*)/FLOAT(N))
      IF(L.EQ.4) GO TO 360
      M = M + 1
      GO TO 330
360 WRITE(6,370) AO
      WRITE(6,380)
      WRITE(6,390)
      WRITE(6,400)
```fortran
WRITE(6,410)  B(1,1),  B(1,2)
WRITE(6,420)  B(1,3),  B(1,4)
WRITE(6,430)
DO 460  I = 1,N
WRITE(6,450)  T(I),  UEXP(I),  U(1,1,I),  U(1,1,1),  U(1,1,1),  U(1,2,1),
I,  U(1,3,1),  U(1,3,1),  U(1,4,1),  U(1,4,1)
460  CONTINUE
WRITE(6,470)  SSD(1,1),  SSD(1,2),  SSD(1,3),  SSD(1,4)
WRITE(6,480)  AAREA(1,1),  AAREA(1,2),  AAREA(1,3),  AAREA(1,4)
WRITE(6,490)  STC(1,1),  STC(1,2),  STC(1,3),  STC(1,4)
WRITE(6,500)
WRITE(6,510)  B(2,1),  B(2,2)
WRITE(6,520)  B(2,3),  B(2,4)
WRITE(6,530)
DO 540  I = 1,N
WRITE(6,450)  T(I),  UEXP(I),  U(2,1,I),  U(2,1,1),  U(2,1,1),  U(2,2,1)
I,  U(2,3,1),  U(2,3,1),  U(2,4,1),  U(2,4,1)
540  CONTINUE
WRITE(6,470)  SSD(2,1),  SSD(2,2),  SSD(2,3),  SSD(2,4)
WRITE(6,480)  AAREA(2,1),  AAREA(2,2),  AAREA(2,3),  AAREA(2,4)
WRITE(6,490)  STC(2,1),  STC(2,2),  STC(2,3),  STC(2,4)
WRITE(6,500)
WRITE(6,510)
DO 570  I = 1,N
WRITE(6,450)  T(I),  UEXP(I),  U(3,1,I),  U(3,1,1),  U(3,1,1),  U(3,2,1)
I,  U(3,3,1),  U(3,3,1),  U(3,4,1),  U(3,4,1)
570  CONTINUE
WRITE(6,470)  SSD(3,1),  SSD(3,2),  SSD(3,3),  SSD(3,4)
WRITE(6,480)  AAREA(3,1),  AAREA(3,2),  AAREA(3,3),  AAREA(3,4)
WRITE(6,490)  STC(3,1),  STC(3,2),  STC(3,3),  STC(3,4)
370 FORMAT(1n,1H, */*, X, *INDEPENDENT COEFFICIENTS
380 FORMAT(1H, *, X, /*)
390 FORMAT(1H, *, X, *COEFFICIENTS CORRESPONDING TO SINE SERIES AND VELOCITIES
400 FORMAT(1H, /*)
410 FORMAT(1H, /*, 5X, *A1 = B(1,1) = *, F15.7, 15X, *A2 = B(1,2) = *, 1F15.7)
420 FORMAT(1H, /*, 5X, *A3 = B(1,3) = *, F15.7, 15X, *A4 = B(1,4) = *, 1F15.7)
430 FORMAT(1H, /*, 7X, *TIME UEXP U(1,1,1) U(1,1,1) U(1,2,1)
I,  U(1,3,1),  U(1,3,1),  U(1,4,1),  U(1,4,1)*)
440 FORMAT(1H, /*, 6X)
450 FORMAT(1H, 10E11.5)
470 FORMAT(1H, /*, X, *SUM ABS VALUE OF DIFFERENCE = *, 2X, F11.5, 3F22.15)
480 FORMAT(1H, /*, 11X, *MEAN DEVIATION = *, 6X, F11.5, 3F22.9)
490 FORMAT(1H, /*, 7X, *STANDARD DEVIATION = *, 6X, F11.5, 3F22.3)
500 FORMAT(1H, /*, X, *COEFFICIENTS CORRESPONDING TO COSINE SERIES AND VELOCITIES
510 FORMAT(1H, /*, 5X, *B1 = B(2,1) = *, F15.7, 15X, *B2 = B(2,2) = *, 1F15.7)
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111

PART TWO.

CALCULATION OF VELOCITY ACCORDING WITH SINE AND COSINE MODELS AND EXPERIMENTAL CONDITIONS OF THE RUN.

600 \textbf{SDIFER} = 0.0
601 \textbf{SDIFS} = 0.0
602 \textbf{x} = 2.1 \times 3.14 \times 592 \times \text{FRE}
603 \textbf{DO} 610 \textbf{I} = 1, N
604 \textbf{USIN}(I) = \text{UAVER} + \text{ARMSPL} \times \sin(n \times \text{T}(1))
605 \textbf{DIFER}(I) = \text{UEXP}(I) - \text{USIN}(I)
606 \textbf{SDIFER} = \text{SDIFER} + \text{ABS}(\text{DIFER}(I))
607 \textbf{SDIFS} = \text{SDIFS} + (\text{DIFER}(I) \times 2)
608 \textbf{CONTINUE}
609 \textbf{XMEAN} = \text{SDIFER}/\text{FLOAT}(N)
610 \textbf{XSTAND} = \text{SQRT}(\text{SDIFS}/\text{FLOAT}(N))

COMPARISON BETWEEN EXPERIMENTAL FACTOR A1 AND UPRIM.

UPRIM = (*ARSPL
611 \textbf{XDIFF} = \text{UPRIM} - R(1,1)
612 \textbf{WRITE}(6,620)
613 \textbf{WRITE}(6,630)
614 \textbf{WRITE}(6,640) \text{UPRIM, XDIFF}
615 \textbf{WRITE}(6,650)
616 \textbf{WRITE}(6,660)
617 \textbf{DO} 680 \textbf{I} = 1, N
618 \textbf{WRITE}(6,670) \text{UEXP}(I), 1(I), \text{USIN}(I), \text{DIFER}(I)
619 \textbf{CONTINUE}
620 \textbf{WRITE}(6,680) \text{SDIFER}
621 \textbf{WRITE}(6,690) \text{XMEAN}
622 \textbf{WRITE}(6,700) \text{XSTAND}

623 \textbf{FORMAT}(14H, \# \text{CALCULATION OF VELOCITY ACCORDING WITH SINE AND COSINE MODELS AND EXPERIMENTAL CONDITIONS OF THE RUN}).

624 \textbf{FORMAT}(14H).

626 \textbf{FORMAT}(14H, \#USIN)
627 \textbf{FORMAT}(14H)

628 \textbf{FORMAT}(14H, 4F20.5)
629 \textbf{FORMAT}(14H, 4X, \#SDF ADD VALUE OF DIFFERENCE = *, F11.5)

7.3 \textbf{FORMAT}(14H, 5Z, \#MEAN DEVIATION = *, F11.5)
7.6 \textbf{FORMAT}(14H, 4S, \#STANDARD DEVIATION = *, F11.5)

PART THREE.

CALCULATION OF PRESSURE DROP USING EULERS EQUATION AND THE FOLLOWING VELOCITY MODELS.

A. \text{ EXPERIMENTAL VELOCITY} (\kappa = 1)
B. \text{ U} = \text{ A0} + \kappa 1 \times \sin(x) + \kappa 2 \times \sin(2x) + \kappa 3 \times \cos(x) + \kappa 4 \times \cos(2x) (\kappa = 2)
C = U = A0 + A1*SIN(X) (K=3)
D = U = UAVE + UNR1N*SIN(X) (K=4)

K = 1

810 SDIFF(K) = 3.0
SDIFFS(K) = 0.0
GO TO 850

IF(K.EQ.1) UAUX(K,1) = UEXP(1)
IF(K.EQ.2) UAUX(K,1) = U(3,2,1)
IF(K.EQ.3) UAUX(K,1) = U(1,1,1)
IF(K.EQ.4) UAUX(K,1) = US1N(1)

DIMENSION OF APPARATUS AND PROPERTIES OF THE FLUID.

DIA = 6.1667
ALEN = 31.6666
RKC = 62.4
UVIS = 0.000224

REYNOLDS NUMBER EVALUATION.

RE = ABS(UUAUX(K,1)*RKC/(UVIS*G1*174))

EVALUATION OF INSTANTANEOUS FRICTION FACTOR ACCORDING WITH EQUATION DETER-
MINED EXPERIMENTALLY (VOL. KAPITZ-KRAUTZ MODEL). THE RAPONS-NEWTON INTER-
ATION METHOD IS USED FOR SOLVE THE EQUATION.

AA = -19.626202
BB = 28276654
F = J.J20820

K IS A COUNT OF THE NUMBER OF INTERATIONS
R = 1.0

810 PRO = RINSORT(F)
FMEV = F - (1.0/50RT(F)) - RINSORT(PRO) - 940*(Z*RE*5ORT(F))/

I(1.1 - E*5ORT(F))
IF(DIFF=FLAN)*TVI.1E-6.0K*RT.50.*0) GO TO 820
F = R - 1.0
F = FMEV
GO TO 810

820 IF(R.TE.2) GO TO 830

SERIV(1) = 1(1.1)*COS(*ST(1)) + 1(1.2)*2.0.*COS(2.0.*ST(1))
SERIV(1) = B(2.1)*SIN(*ST(1)) - B(2.2)*2.0.*SIN(2.0.*ST(1))
GO TO 840

830 IF(K.EQ.1) D(1,1) = DPRT1

SERIV(1) = D(1,1)*COS(*ST(1))

840 DPK(K,1) = (SERIV(1) + *COSM(1,1)*2.0.*1.0.*5RT(K,1))

1(32.174*144.0)

DIFF(K,1) = DPKXP(1) - DPK(K,1)
DIFF(K) = SDIFF(K) + ABS(DIFF(K,1))

SDIFFS(K) = SDIFFS(K) + (DIFF(K,1)**2

850 CONTINUE

ZMEAN(K) = (DIFF(K))/FLOAT(N)
ZSTAND(K) = SORT(I(SDIFFS(K)))/FLOAT(N)
K = K + 1
IF(K.EQ.5) GO TO 860
GO TO 800

860 WRITE(6,876)
WRITE(6,880)
WRITE(6,900)
WRITE(6,910)
WRITE(6,920)
WRITE(6,930)
WRITE(6,940)
WRITE(6,950)
WRITE(6,960)
WRITE(6,970)
GO 970

112
WRITE (6, 960) T(I), DPEXP(1), DP(1,1), U1F(1,1), DP(2,1), U1F(2,1)  
 1, DP(3,1), U1F(3,1), DP(4,1), U1F(4,1)
97. CONTINUE
WRITE (6, 960) U1F(1,1), U1F(2,1), U1F(3,1), U1F(4,1)
WRITE (6, 960) ZETA(1, 1), ZETA(2, 1), ZETA(3, 1), ZETA(4, 1)
WRITE (6, 960) ZSTA(1, 1), ZSTA(2, 1), ZSTA(3, 1), ZSTA(4, 1)
870 FORMAT (1H1, 2X, *PRESSURE DROOP (DP) RESULTS USING ELLIPS EQUATION  
1 AND THE FOLLOWING VELOCITY MODELS*)
890 FORMAT (1H1, *).
900 FORMAT (1H1, * EXPERIMENTAL VELOCITY*)
910 FORMAT (1H1, '3X, * U = A0 + A1* Sin(X) + A2*Cos(X) + B/COS(X) + C/COS(X^2)')
920 FORMAT (1H1, '3X, * U = A0 + A1* Sin(X)*)
930 FORMAT (1H1, '3X, * U = DAVE* + UPRIM* Sin(X)*)
940 FORMAT (1H1, '3X, * TIME DPEXP(1), DP(1), U1F(1)
 1, DP(1), U1F(1), DP(1), U1F(1), DP(1), U1F(1)
 1(2)*)
950 FORMAT (1H1, '5X;)
960 FORMAT (1H1, '2F11.5, 8F12.5)
980 FORMAT (1H1, '3X, * SUM AND VALUE OF DIFFERENCE = *, F14.5, 12X,  
 1F12.5, 12X, F12.5, 12X, F12.5)
990 FORMAT (1H1, '14X, * MEAN DEVIATION = *, F14.5, 12X, F12.5, 12X,  
 1F12.5, 12X, F12.5)
999 FORMAT (1H1, '1X, * STANDARD DEVIATION = *, F14.5, 12X, F12.5, 12X,  
 1F12.5, 12X, F12.5)

PART FOUR - PLOTTING THE DIFFERENT PARAMETERS - ****************************************
STORING 50 POINTS FOR A CONTINUOUS CURVE USING THE FOURIERS 4 TERMS VELOCI  
MODEL. (MODEL B)
PERIOD = 1.0*FRE
ST = PERIOD/50.0
I = 1
TT(I) = 0.0

1000 X = W*TT(I)
UA(1) = A0 + A1*Sin(X) + A2*Cos(X) + B/COS(X) + C/COS(X^2)
DERIV(1) = A1*Cos(X) + A2*Sin(X) - B*Sin(X) - C*Sin(X^2)
EL = A0 + UA(1)*Rho/UVIS*32.174)
F = 0.000010

1100 PRO = R*SR(T)
FR = F - (1.0/SD(F)) - 0.5*ACU(PRO) - PA*(2.0*F*SR(T)))
M(1-0.0 - 6.0*SR(T))
IF(ASS(F - FR)) LT. 1.0 E - 6, GO TO 1020
F = FR
GO TO 1010

1120 DP6(I) = (DERIV(1) + U(AU(I)))*2/(2.0*U(I)))*(AUGT*NH)*1/32.174  
1*144.0)
IF(TT(I) < PERIOD) GO TO 1030
I = I + 1
TT(I) = TT(I-1) + DT
GO TO 1000

FIGURE ONE. VELOCITY VS. TIME
1030 DO 1040 I = 1, 50
1040
CALL PLOTPT(T(1), UA(I), 4)

1040 CONTINUE
DO 1050 I = 1,N
CALL PLOTPT(T(I), UEXP(I), 44)
CALL PLOTPT(T(I), U(I,1,I), 23)
CALL PLOTPT(T(I), USIN(I), 24)
1050 CONTINUE
CALL OUTPLT
CALL COPYCD

C C
FIGURE TWO. PRESSURE DROP VS. TIME.
DO 1060 I = 1,N
CALL PLOTPT(TT(I), DP(I), 4)
1060 CONTINUE
DO 1070 I = 1,N
CALL PLOTPT (T(I), DPLXP(I), 44)
CALL PLOTPT (T(I), DP(3*I), 23)
CALL PLOTPT (T(I), DP(4*I), 24)
1070 CONTINUE
CALL OUTPLT
CALL COPYCD

C C
FIGURE THREE. PRESSURE DROP VS. VELOCITY.
DO 1080 I = 1,N
CALL PLOTPT(U(I), DP(I), 4)
1080 CONTINUE
DO 1090 I = 1,N
CALL PLOTPT (UEXP(I), DPLXP(I), 44)
CALL PLOTPT (U(I,1,I), DP(3*I), 23)
CALL PLOTPT (USIN(I), DP(4*I), 24)
1090 CONTINUE
CALL OUTPLT
CALL COPYCD

C C
FIGURE FOUR. EXPERIMENTAL PRESSURE DROP VS. CALCULATED PRESSURE DROP.
DO 1100 I = 1,N
CALL PLOTPT (DP(2*I), UEXP(I), 4)
CALL PLOTPT (DP(3*I), UEXP(I), 23)
CALL PLOTPT (DP(4*I), UEXP(I), 24)
1100 CONTINUE
CALL OUTPLT
CALL COPYCD

C C
FIGURE FIVE. EXPERIMENTAL VELOCITY VS. CALCULATED VELOCITY.
DO 1110 I = 1,N
CALL PLOTPT (U(3*,2*I), UEXP(I), 4)
CALL PLOTPT (U(I,1,I), UEXP(I), 23)
CALL PLOTPT (USIN(I), UEXP(I), 24)
1110 CONTINUE
CALL OUTPLT
CALL COPYCD

C C
FIGURE SIX. DERIV VS. TIME.
DO 1120 I = 1,N
CALL PLOTPT (TT(I), DERIV(I), 2)
1120 CONTINUE
CALL OUTPLT
CALL COPYCD
C
END IS ' CONTROL CARD.
READ(5,1200) END
1200 FORMAT(F5.1)
IF(END.EQ.1.0) GO TO 1
END