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AN OXYGEN MODEL FOR LAKE ONTARIO

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A Project Report
Submitted to the School of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree
Master of Engineering

McMaster University
October, 1977

MASTER OF ENGINEERING (1977)
(Chemical Engineering)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE:	An Oxygen Model for Lake Ontario
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NUMBER OF PAGES:	x, 184

ABSTRACT

A dissolved oxygen model for stratified lakes is developed and is verified for Lake Ontario. The processes affecting the dissolved oxygen budget of a lake including hydraulic inflow and outflow, production and decomposition, atmospheric reaeration, vertical transport across the thermocline, and sediment oxygen demand are analyzed and quantified. The production and decomposition oxygen fluxes are provided by a phosphorus model developed by others. These phosphorus-oxygen inter-relationships are quantified using stoichiometry developed for oceans. Nine years of temperature data for Lake Ontario are used to estimate the annual variations of epilimnetic temperatures, the rate of deepening of the epilimnion (i.e. thermocline depth vs time) and the vertical exchange coefficient. Dissolved oxygen data over a similar period are used to estimate lake-wide concentrations of DO. The vertical exchange and decomposition hypolimnetic oxygen fluxes are estimated from observed data. Model predictions of concentrations and fluxes compare favourably to the observed data. This provides some verification for the predictions of the oxygen model, the oxygen-phosphorus stoichiometry and the decomposition flux predicted by the phosphorus model.

ACKNOWLEDGEMENTS

This writer gratefully acknowledges the support of my advisor and friend, Bill Snodgrass, whose endless patience and guidance were a constant source of inspiration in the shaping of my chosen career. In addition, this understanding and advice have influenced my development as a person. For these things, I can never repay him.

I am deeply indebted to Professors P.M. Huck and M.H.I. Baird for giving of their valuable time and talents to serve on my committee.

The advice and assistance always freely and cheerfully given by Professors K.L. Murphy and A. Benedek during my association with the Water Research Group is gratefully acknowledged.

The many students, faculty and staff whom I came to know and respect during my stay at McMaster were a constant source of good company and fellowship. Their friendship is gratefully appreciated and will never be forgotten.

The invaluable assistance of Sara Gallo in typing this manuscript and the financial support of the National Research Council of Canada and McMaster University are gratefully acknowledged.

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CHAPTER 1

INTRODUCTION AND REVIEW OF PREVIOUS WATER QUALITY MODELLING STUDIES

1.1 INTRODUCTION

Water quality modelling has recently developed into an important tool in the management of both natural and man-made surface bodies of water. Much of this development occurred as a result of the investigation of one of the most serious problems facing our surface water resources, eutrophication. The magnitude of effort expended in the understanding of this natural process has been considerable. The study of the causes and effects of the eutrophication problem requires appraisal of existing and future trends in parameters such as nutrients, biomass and dissolved oxygen (DO). Models facilitate this appraisal and can be used to predict the response of a natural system to various strategies for water quality management.

The adjective "eutrophic" means well-nourished and therefore eutrophication can be defined as the process of nutrient addition, natural or artificial, occurring in a body of water (Rohlich, 1970). Eutrophication is part of the aging process and is normally applied only to lakes and other similar bodies of water since streams and rivers do not age in the same sense as lakes. Although it occurs naturally, eutrophication can be influenced by man's activities. Domestic and industrial wastes have contributed large quantities of nutrients phosphorus and nitrogen, to receiving waters, thus accelerating the eutrophication process. Significant changes in the plant and animal life of a lake occur,the most common of which is excessive growth of algae and larger aquatic plants. Effects of this growth may include loss of potable water quality, choking of open waters and reduction in aesthetics, Decomposition leads to foul odours and deficits in deep water dissolved oxygen. In a stratified lake, the bulk of decomposition occurs in the hypolimnion or deeper layers as dead biomass sinks from the upper or euphotic zone. The stratification isolates the hypolimnion from many sources of DO and thus oxygen depletion can be complete if sufficient biomass decomposes. An anoxic, or oxygen-free, hypolimnion leads to anaerobic decay and the production of objectionable reduced chemical species (Fe^{2+}, Mn^{2+}, S^{2-}) (Lorenzen, 1973) and methane (CH_4).

The loss of oxygen from the hypolimnion can have serious effects on resident fisheries in a lake. Cold water species such as trout are very sensitive to DO levels. Hypolimnetic oxygen depletion would result in a stress situation for these fish since they would be forced into warmer waters to obtain the required DO levels. Temperature sensitivity of many species would then increase the stress on body functions. The combined stress can be sufficient to result in migration (if possible) or death.

This research addresses the nature of many of the factors affecting the DO reserves in a stratified lake and attempts to develop a model capable of predicting DO fluxes and concentrations over time. The eutrophication aspect of DO depletion is included by utilizing a DO model whose decomposition/production oxygen fluxes are provided by a phosphorus model verified for total annual phosphorus concentration for lakes whose hypolimnia remain oxic.

1.2 REVIEW OF PREVIOUS ATTEMPTS AT NATURAL SYSTEMS CHEMISTRY MODELLING

Modelling of the chemisty of natural systems by the mathematical representation of mass balances was introduced in 1925 by Streeter and Phelps. Further work in modelling was very modest in the next thirty to thirty-five years and centered mainly on rivers and streams and the effects of man-introduced loadings of biochemical oxygen demand (BOD). In this sense, the stream was treated as an allochthonous system, a term meaning "introduced from outside". The source of energy or driving force for decomposition was thus from an external source. This approach was adequate for the modelling of rivers and streams but tended to ignore photosynthetic effects which can be quite significant, especially in lakes. The contrasting approach and that used in the majority of models of non-conservative substances in lakes is to treat the lake (or stream) as an autochthonous system in which in situ fixation of energy is the driving force for decomposition. The difference between the two concepts will be more clear after the following review of natural systems models.

The classical relationship for the mathematical modelling of the dissolved oxygen balance in rivers and streams was proposed by Streeter and Phelps (1925) in their study of the pollution and purification of the Ohio River. The model in differential form is

$$\frac{dD}{dt} = K_1L - K_2D \qquad (1.1)$$

where $D = (C_s - C)$ = dissolved oxygen deficit,
C = DO concentration,
C_s = DO saturation concentration,
L = carbonaceous biochemical oxygen demand,
K_1 = deoxygenation coefficient,
K_2 = reaeration coefficient, and
t = time of travel downstream.

This relationship assumes plug flow and complete vertical mixing in the stream. In addition, only two source/sink processes are considered, namely atmospheric reaeration and bacterial decomposition of carbonaceous organic matter. Hence, the Streeter-Phelps model describes a system in which the primary source of available energy is in the form of carbonaceous matter. With this as a food source, bacteria replicate, simultaneously requiring oxygen as the electron acceptor.

Since the introduction of this important contribution to water quality modelling, work has centred basically on two areas:

1) the extension of the model to include other source/sink processes which may be dominant in rivers and streams but were not studied by Streeter and Phelps, and

2) better and more up-to-date methods of estimating reaction coefficients for the source/sink terms both for specific (i.e., - a particular stream) and general application.

The sophistications introduced by subsequent works include both mass flow terms which are direct add-ons to the right hand side of equation (1.1) and rate coefficients which must be incorporated into a mass flow expression before being introduced to the model.

Some of the first extensions to the model were in regard to the definition or make-up of the coefficients. The Streeter-Phelps coefficient K_1 represented the removal of BOD by oxidative processes only. The value of K_1 was determined by laboratory BOD analysis. Thomas (1948) suggested that non-oxidative BOD removal and addition mechanisms could be important to the overall BOD and DO balance in the stream. He added the rate coefficient K_3 which can be positive or negative, indicating BOD removal or addition respectively. This coefficient, with the proper sign, can be added to K_1 to give Kr, the total or net BOD removal rate. Velz and Gannon (1962) investigated the direct absorption of oxygen by attached biological slimes and defined a coefficient B to account for this occurrence. As for K_3, the value of B can be added to K_1. The sum of K_1 and B is represented by K_d, a total deoxygenation rate. The preceding relationships are illustrated in equations 1.2 and 1.3 below.

$$\underset{\text{(overall BOD removal coefficient)}}{K_r} = \underset{\text{(deoxygenation coefficient of free-flowing water, Streeter and Phelps)}}{K_1} + \underset{\text{(coefficient describing BOD removal/addition mechanisms not exerting an oxygen demand, Thomas)}}{K_3} \quad (1.2)$$

$$\underset{\text{(overall deoxygenation coefficient)}}{K_d} = \underset{\text{(Streeter-Phelps)}}{K_1} + \underset{\text{(absorption by slimes coefficient, Velz and Gannon)}}{B} \quad (1.3)$$

These new coefficients K_d and K_r can be fit into a more sophisticated model as will be summarized presently in equations 1.4 and 1.5. Note that if B, the slime uptake coefficient, can be considered insignificant, K_1 can be considered a good estimate of K_d and K_r would be equal to the sum of K_d and K_3.

The effects of longitudinal dispersion were studied by O'Connor (1961) in his research into the reaeration process. Basically, the coefficient of longitudinal dispersion, D_L, accounts for deviations from ideal plug flow caused by turbulence and variation in velocity across a river transect. He further demonstrated the importance of this term in slow-moving, longitudinally mixed streams such as estuaries.

Dobbins (1964) summarized several relevant considerations not accounted for by Streeter-Phelps. They included:

(i) removal of BOD by sedimentation,

(ii) addition of BOD along a stretch (bottom scour, local runoff),

(iii) benthal or sediment oxygen demands,

(iv) photosynthesis and respiration (oxygen source/ sink) by plankton and fixed plants, and

(v) the effects of longitudinal dispersion.

The work of Thomas (1948) addresses considerations (i) and (ii) while that of O'Connor addresses consideration (v). Dobbins (1964) addressed himself to the benthal or sediment oxygen sink and the photosynthesis/respiration oxygen source/ sink terms. He introduced a term D_B, to account for both oxygen uptake by the sediments and respiration of green plants and oxygen release by photosynthesis of green plants. The term D_B is a mass flow rather than a coefficient and is a separate term added to the right-hand side of equation 1.1. D_B can be either positive or negative, depending on whether oxygen uptake (positive) or oxygen release (negative) was dominant. Later these two processes, sediment demands and photosynthesis/ respiration, were treated separately.

O'Connell and Thomas (1965) added the expression (P-R), a mass flow, to the right side of Equation 1.1 to explain the diurnal effects of photosynthesis (P) and respiration (R). The expression is positive during daylight hours as P greatly exceeds R and thus oxygen is added to the stream. In the absence of sunlight, P is zero, the expression is negative and thus oxygen is depleted. The (P-R) expression can be measured by two procedures:

1) consideration of the maximum variation in an overall oxygen balance equation on a daily basis and solving for (P-R) by difference (indirect) and

2) measurement of the opposing processes in an algal chamber apparatus (direct).

Novotny and Krenkel (1975) introduced a term S_B in their model to account for only benthal or sediment demands as well as a nitrogenous BOD term $K_n L^n$. These too are mass flows and are added as separate terms to the right hand side of equation 1.1.

Several investigators, including Kittrell and Kochtitzky (1967) have shown the inapplicability of the Streeter-Phelps model to specific sites. Other DO source or sink mechanisms (e.g., absorption of DO by attached slimes) legitimately assumed negligible by Streeter and Phelps in a stream the size of the Ohio River, take on a new significance in smaller streams.

Equations 1.4 and 1.5 summarize the modifications to equation 1.1 from the previously discussed investigations and studies in a model form applicable to a wider range of stream conditions with respect to the factors affecting the DO balance. It neglects the effect of longitudinal dispersion.

The term S_B is that used by Novotny and Krenkel (1975) and thus represents a true sediment oxygen demand.

$$\frac{dD}{dt} = K_d L - K_2 D - (P-R) + K_n L^n + S_B \qquad (1.4)$$

$$\text{and } L = L_o e^{-K_r t} \qquad (1.5)$$

where (P-R) = photosynthesis/respiration effects (O'Connell and Thomas, 1965),

$K_n L^n$ = nitrogenous BOD depletion term (Novotny and Krenkel, 1975), and

S_B = sediment oxygen demand (Novotny and Krenkel, 1975).

A time-based approach to time series analysis, developed by Box and Jenkins, was used to develop water quality models by Huck and Farquhar (1974). These models are stochastic in character (i.e., no input/output of mass or energy) as opposed to all the previously discussed models which are deterministic types. They succeeded in modelling hourly water quality data recorded in the St. Clair River for both a conservative (chloride) and non-conservative (dissolved oxygen) substance.

In addition to extending the applicability of the basic model form, much work has been conducted in the evaluation of the parameters of the processes affecting the DO balance in natural systems. Since the model is most often applied to streams along the course of which organic pollutants are introduced as point-source additions, most of the effort has centered on the BOD reaction and deoxygenation coefficients, K_1, K_d, K_r, K_3, and K_2. Increased emphasis is presently being placed on the sediment demand, photosynthesis/respiration and nitrification terms.

O'Connor (1962) stated that K_r and K_d may often be considered equal if BOD is removed by oxidation only. If this assumption cannot be made, the deoxygenation coefficient K_d can often be approximated by the value of K_1 which is determined from laboratory measurements of the BOD of stream samples. The specific values of these coefficients are determined by the nature of the waste entering the stream and the conditions in the stream itself. The parameters will often continuously decrease in downstream sections of a stream since the most easily oxidizable portions of the organic matter occur in the upper reaches (Kittrell and Kochtitzky, 1967).

Several investigators have studied the nature and evaluation of the reaeration coefficient, K_2, in great detail. Riddle (1970) presents a literature review of the determination of water reaeration constants. He traces initial work by Lewis and Whitman (1924) on the two-film theory of mass transfer to Higbie's (1935) proposed penetration theory. Subsequent work by Dobbins (1956) and O'Connor and Dobbins (1958) refined the previous work to include turbulent effects and surface renewal. Churchill et al. (1962) developed numerous equations relating observed reaeration rates to the hydraulic properties of stream channels, U, the velocity of the stream, S, the slope of the energy gradient and H, the average depth. Two simple estimates of the reaeration coefficient are:

$$K_2 = \frac{(D_m U)^{\frac{1}{2}}}{H^{3/2}} \quad \text{(O'Connor and Dobbins, 1956)} \qquad (1.5)$$

where D_m = molecular diffusivity of oxygen in water

and

$$K_2 = 11.56 \frac{U}{H^{2/3}} \quad \text{(Churchill et al., 1962)} \qquad (1.6)$$

Parkhurst and Pomeroy (1972) proposed a relationship that included the effects of turbulence, velocity, impurities in the water, stream geometry, temperature and slope of the energy gradient on the value of the reaeration coefficient.

A tracer method using radioactive tritium and its stable daughter product ^{3}He has recently been used to measure reaeration coefficients in bodies of water (Torgersen et al., 1977). The molecule ^{3}He has diffusion properties very similar to oxygen. This provides an important independent method for measuring reaeration.

In summary, the previous work cited above is presented as an historical background to dissolved oxygen modelling. To date, most of the research has been conducted on stream applications rather than on lakes. Although the processes and mechanisms affecting the DO balance in streams do not differ from those in lakes, the conversion to a mathematical form requires consideration of the physical differences between the two types of natural systems.

The models previously described are basically steady-state in nature. DO estimates vary with time of travel or distance downstream due to the plug-flow nature of a stream. In a lake, plug flow does not occur frequently and complete vertical mixing cannot always be assumed. The seasonal stratification of many lakes serves to reduce vertical mixing across the interface between the resulting layers. Horizontal variations of DO are often insignificant compared to vertical variations and those variations that occur with time. Thus, many models of lakes or impoundments are one-dimensional and include variations with depth and time.

There are two major types of kinetic models that can be applied to lakes (Snodgrass, 1974). The first, a compartmental or box model divides the lake into as few well-mixed boxes as possible and neglects horizontal variations completely and vertical variations except in the region of the interface between two adjoining boxes. The second type, a dispersed-plug flow (DPF) model, divides the lake into an infinite number of boxes, usually in the vertical direction. In this way concentration changes are described as a continuous function. Several examples of each in the literature are discussed below.

Wright (1961) used a vertical, one-dimensional dispersed-plug flow model to estimate biological rates of production and consumption by adding the rates of turbulent transport of oxygen to the observed rate of change in concentration at successive depths. Vertical eddy diffusivity coefficients were assumed to be equal to coefficients of vertical eddy conductivity and advection was ignored. Bella (1970 a,b) used a one-dimensional DPF model to simulate the effects of sinking and vertical mixing on the dynamics of algal populations and to compare the relative effects of reaeration,photosynthetic oxygenation, vertical mixing and oxygen uptake on the hypolimnetic DO level in stratified lakes. Bella, like many others (Wright, Rumer and Melfi) used a heat balance to calculate vertical transport coefficients. He found that vertical dispersion and hypolimnetic respiration have a greater influence on the hypolimnetic DO change during stratification than euphotic zone oxygenation or reaeration. Markofsky and Harleman (1973) take a different approach. They use a one-dimensional vertical advection model, neglecting eddy diffusivity, to predict DO variations in stratified reservoirs. Bottom demands, BOD sedimentation and algal decomposition demands on the DO budget are neglected. The shear effect of horizontal advective velocities due to river inflows and reservoir outflows is used to predict vertical advective velocities. Their predictions of vertical transport are verified from earlier work by Markofsky and Harleman (1971); their oxygen model is not verified due to insufficient field data.

The DO model developed by O'Connell and Thomas (1965) is one of several box models. It takes into consideration variations in time and horizontal distance and thus can be easily be applied to the euphotic zone of a lake with the assumption that horizontal variations are insignificant. Sedimentation of BOD and vertical transport of oxygen are ignored. Other models of this general type have been proposed by Rainey (1967), Sweers (1969b) and O'Connor and Mueller (1970) for application to conservative substances (i.e., chlorides) in the Great Lakes. Only Sweers (1969) considered the effects of stratification but did not allow transport between boxes. Other compartmental models used for the non-conservative substance phosphorus include those proposed by Vollenweider (1969) in which sedimentation is considered, O'Melia (1972) in which transport between boxes (i.e., epilimnion and hypolimnion) occurs via eddy diffusion and sedimentation and Imboden (1973) in which algal growth and decomposition are added to vertical exchange and sedimentation.

The model of Snodgrass (1974) is similar to that of Imboden (1973). Vertical exchange, sedimentation and algal growth/decomposition are modelled with respect to soluble and particulate phosphorus. Snodgrass (1974) also predicts total phosphorus concentration in a lake as a function of mean depth, hydraulic loading and areal phosphorus loading. In these phosphorus models in situ fixation of energy by the standing crop of living biomass is the driving force for decomposition. This condition thus considers the lake to be an autochthonous system.

An extension of these types of box models to dissolved oxygen in lakes was made by Rumer and Melfi (1973). They proposed a two box model for summer stratification to examine BOD loading effects on a lake's minimum DO. This model, in contrast to the phosphorus model discussed above, considers the lake to be an allochthonous system since externally applied BOD rather than in situ fixation of energy is the driving force for decomposition. Rumer and Melfi (1973) base their model on the prediction of time-varying, depth averaged DO values for the epilimnion and hypolimnion of a lake. The basic material balance for DO is expressed for each layer as

$$\begin{array}{lcccccc} \text{time rate} & & \text{mass} & & \text{mass} & & \text{mass change due} \\ \text{of change} & = & \text{in} & - & \text{out} & + & \text{to biochemical} \\ \text{of mass} & & & & & & \text{reactions} \end{array}$$

Volumetric inflow and outflow from or to points beyond the system boundary is permitted only in the epilimnion. Vertical exchange across the thermocline is permitted but sedimentation of BOD is ignored. The vertical exchange coefficient is determined from a heat balance over the hypolimnetic volume. This assumes that all heat is transported to the hypolimnion by vertical mixing across the thermocline (Sweers, 1970). All model parameters are kept constant over the stratification period. BOD concentrations were assumed to take on steady-state values and thus decomposition fluxes affecting the DO balance were constant.

Bhagat et al. (1972) also use a compartment type model to predict oxygen deficit in Lake Vancouver, Washington, but do not consider vertical transport due to the application of the model to an unstratified lake in which complete mixing is assumed to occur.

Other model forms used to predict DO behaviour include two models relating the outflow DO of a reservoir to temperature and retention time by Churchill and Nicholas (1967) and Wunderlich and Elder (1968). The former model includes reservoir operation considerations while the latter approximates all source/ sink terms as a single depletion factor. The applicability of these models to lakes is somewhat limited.

Of the various model forms available, this researcher selected the box or compartmental approach to model oxygen in Lake Ontario on a temporal scale.

The development of the DO model in this research was based on the concepts of the Rumer and Melfi (1973) model with respect to DO. However, the lake in this research is treated as an autochthonous system. The phosphorus model of Snodgrass (1974) was therefore used to predict the fluxes of soluble and particulate phosphorus which were in turn used as the driving forces for production and decomposition fluxes of dissolved oxygen required by this researcher's model. A conversion factor relating production or decomposition of particulate phosphorous to liberation or depletion of oxygen was also required. The quantification of the production/decomposition reaction developed by Redfield (Stumm and Morgan, 1970) in which a chemical formula is assigned to algal biomass was used in this research to obtain an estimate of this conversion factor. This concept is discussed in more detail in section 2.4.2. This approach, referred to as "Redfield Stoichiometry" was also used by Imboden (1973) to predict hypolimnetic oxygen depletion resulting from particulate phosphorus introduced from epilimnetic production.

CHAPTER 2

THE MODEL DEVELOPMENT

2.1 THE MODELLING PROCESS

A model is an approximation of a real system. In water quality models, certain physical, chemical or biochemical processes and their interactions are quantified in an attempt to estimate the response of the system to changes in the status quo.

The degree of complexity of a particular model is related to the available knowledge of the system under study, the processes taking place within the system and the objectives of the model.

Snodgrass (1974) identifies the modelling process as consisting of six steps:

(1) delineation of model objectives,
(2) delineation of system boundaries and discretization into accumulators and compartments,
(3) construction of the model,
(4) model calibration,
(5) model verification, and
(6) model prediction.

These might be preceded by the initial realization that a problem or potential for a problem exists.

Delineation of model objectives, the logical first step, is a statement of purpose for application of the model to the real system. The form of a particular model or the selection between competing models is strongly influenced by this summary of required information.

Definition of system and accumulator boundaries requires a thorough understanding of the system's physical characteristics and the relevant mass or energy inputs and outputs. The internal as well as external structures of the system are thus established and should be compatible with the model objectives. An example of an accumulator in the model is the hypolimnion of a stratified lake. For the purposes of this model, transfer of mass or energy between inter-connected accumulators is provided for, but there is essentially no spatial variation of mass or energy within a particular accumulator allowed. An accumulator can be further broken down into compartments containing a particular form of mass or energy (e.g., dissolved oxygen).

Construction of the model is a process involving four basic steps:

(1) identification of the physical, chemical or biochemical processes involved in the transfer of mass or energy between accumulators or between compartments within accumulators (e.g. atmospheric reaeration),

(2) definition of assumptions made to reduce the complexity of the real system to a state that can be described within the available realm of knowledge,

(3) construction of a system of equations based normally on the conservation of mass or energy, that describe the behaviour of the system with respect to particular parameters,

(4) identification or selection of appropriate boundary conditions used to solve the system of equations.

Model calibration begins with an initial selection of numerical values for the coefficients established in model construction. The values can be selected on the basis of field data and/or literature values. If model predictions do not match the actual results closely, modifications to one or more of the coefficients are made to improve the fit. This tuning procedure is facilitated by the determination of the coefficients to which the model predictions are most sensitive. This "sensitivity analysis" provides the basis for selection of coefficients to be modified to obtain the desired fit.

The calibrated model is then used to predict the system behaviour under a set of conditions independent of the conditions used for calibration. This is model verification. The independent conditions could be another year of data for the same lake or data obtained from another lake, similar in structure or biochemistry. For another year's data in the same lake, some judgement is required as to the time period between calibration and verification years. Detention time in the lake would be a reasonable separation period to apply for a chemical species which does not exchange with the atmosphere. For a species such as dissolved oxygen, this exchange occurs and therefore annual or more frequent renewal tends to guarantee independence since steady state conditions with the atmosphere are attained annually. If the verification is good, the model can then be used for predictive purposes for that particular system.

If the model verification is not good, the preceding steps in the modelling process must be reevaluated. A modification of the system structure or model construction is usually required.

It is very important to remember that a model can be used for predicting only those objectives for which it has been verified. If model predictions are required for a condition outside the realm of the model objectives, care must be taken to ensure a reliable result.

2.2 MODEL PERSPECTIVE FOR EXAMINING THE O_2 REGIME

This section summarizes the first two steps of the modelling process as described in the previous section: delineation of model objectives and delineation of the system boundaries (including internal compartments and accumulators).

2.2.1 Objectives of this Research

Dissolved oxygen (DO) is an important indicator of the overall water quality in a lake (Bhagat, et al., 1972, IJC, 1969). Dissolved oxygen in natural waters is essential for the respiration of fish and other aquatic life, the bacterial oxidation of organic material and the stabilization of the mud-water interface. Cold water species of fish show discomfort at DO levels below 7 mg/l while warm water species can tolerate DO levels as low as 5 mg/l (Coker, 1954). The amount of free oxygen needed to prevent nuisances is said to be about 2 mg/l (Eldridge, 1942).

The overall focus of this research has been to develop an understanding of the factors affecting the oxygen budget of a lake undergoing a period of thermal stratification during its yearly cycle. To this end, the specific objectives of the work are:

(1) to determine both the exogenous and endogenous variables which most seriously affect the DO concentrations in a lake,

(2) to construct and develop a quantitative dissolved oxygen model capable of predicting trends in dissolved oxygen in a stratified lake both from season to season within a year and from year to year,

(3) to test the use of a phosphorus model to estimate the production and decomposition fluxes of dissolved oxygen in a lake, and

(4) to test the use of Redfield stoichiometry to relate the conversion between phosphorus compartments to an equivalent increase or decrease in dissolved oxygen.

2.2.2 System Description-Boundaries, Accumulators and Compartments

The physical system of concern in this research consists of a lake or similar body of water (e.g., an impoundment), the overlying atmosphere and the underlying sediments. Mass exchange of oxygen occurs between the water and the atmosphere while the sediments act as an oxygen sink. Hydraulic inputs to and outputs from the lake occur at a constant rate. The type of lake considered is one that has a period of stratification during the yearly cycle. The following is a brief description of the yearly heating and resultant circulation cycle typical of a temperate lake (after Forel, 1901) or more specifically a dimictic* lake (Hutchinson and Loffler, 1956).

During the cold winter period, most lakes in the temperate zone can be considered virtually isothermal at or slightly below 4°C, the temperature of maximum density of water. A slight temperature gradient may be in evidence with cooler waters near the surface. With the advent of spring, cool surface waters below 4°C are heated by incoming solar radiation. Due to the resulting increase in density,

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*A dimictic lake is one that has two periods of complete mixing, i.e., overturns, during its annual cycle (Wetzel, 1975).

these waters sink and are replaced by cooler subsurface waters, thus inducing mixing by density gradient. Wind generated currents set the entire water mass in motion with complete intermixing due to the zero stability of the lake under isothermal conditions. Together these processes result in the period of spring turnover during which nutrient rich bottom waters mix thoroughly with nutrient poor euphotic waters.

The warming process continues throughout the spring and early summer due to warmer temperatures, higher incident solar radiation and the longer days. Aided by wind mixing, heat diffuses from the surface down through the water column. Mixing between surface and bottom waters by density gradient is prevented because the warmer surface waters are less dense than the underlying waters. At some depth below the surface there is a region or layer of water at which the temperature gradient is a maximum. This layer, called the thermocline, separates the upper zone or epilimnion from the lower zone or hypolimnion. The thermocline depth increases with time starting at the surface of the lake and reaching a maximum depth at the end of the summer heating period. Yearly variation of the thermocline depth in a lake is due to dissimilar meteorological conditions from one year to the next. During this stratification period, mixing occurs in the epilimnion as a result of wind induced currents. Mixing with the hypolimnion is prevented by the density gradient across the thermocline but circulation energy is transferred to the hypolimnion, resulting in a state of reduced internal mixing in this deeper, cooler zone.

As fall approaches, the cooler air temperatures cause a loss of heat from the surface waters. This leads to a decrease in the stability of the lake's stratification. These waters become more dense as their temperature decreases and mixing by density gradient occurs once again. Winds can overcome the resistance to mixing offered by the thermocline and cause convective entrainment of the epilimnion into the hypolimnion. Eventually a homothermal condition (uniform temperature) exists in the water mass and again, wind-induced circulation of the entire mass occurs. This is called the period of fall overturn.

During the winter, surface waters are cooled to below $4^{o}C$ thus creating a slight gradient from surface to bottom. The stability of the lake is low at this time, and again wind-generated circulation occurs. This results in a homothermal condition in the lake at a temperature at or slightly below $4^{o}C$ and complete internal mixing. This is called the winter circulation period. The complete circulation can be interrupted by the formation of ice which prevents wind induced currents or by a cooling of surface waters below $4^{o}C$ which causes a density gradient preventing mixing.

As previously mentioned, the formation of the thermocline during stratification results in the formation of two distinct bodies of water within the lake, the epilimnion overlying the hypolimnion. The thermocline represents a semi-porous layer. It is a density barrier to the free mixing between the two layers. However, transfer of mass and energy via diffusion or vertical exchange transport mechanisms across the thermocline can occur.

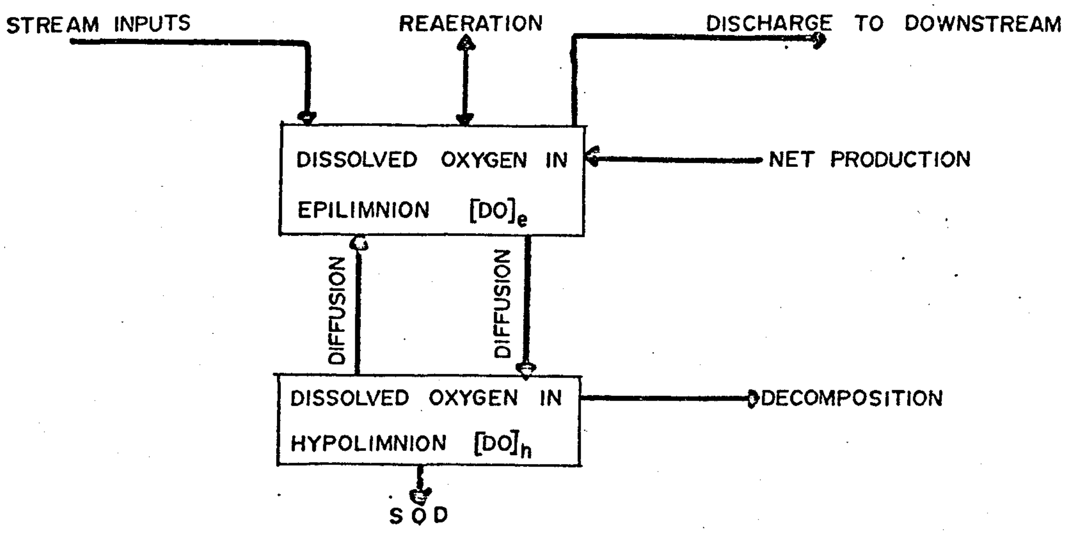
Thus, for this research, a physical discretization of winter circulation and summer stratification was selected. A single well-mixed accumulator or "box" is used to model the lake during the circulation period. Two smaller boxes representing the epilimnion overlying the hypolimnion are used to model the lake during the stratification period. The thermocline prevents hydraulic inter-mixing between the two boxes but allows for exchange of mass and energy via vertical transport mechanisms. The depth of the thermocline increases with time over stratification. The transition between winter and summer conditions is assumed to be instantaneous for the model.

Each box is subdivided into three biochemical compartments; two forms of phosphorus and dissolved oxygen. The phosphorus forms are orthophosphorus (OP) and particulate phosphorus (PP). Orthophosphorus includes all soluble forms. Particulate phosphorus includes the algal and bacterial biomass both viable and non-viable. Mass transfer between compartments is permitted via the processes of production and decomposition/respiration.

Figures 2-1 and 2-2 respectively depict the dissolved oxygen and phosphorus transport mechanisms considered in the

FIGURE 2-1
OXYGEN TRANSPORT MECHANISMS

SUMMER STRATIFICATION



WINTER CIRCULATION

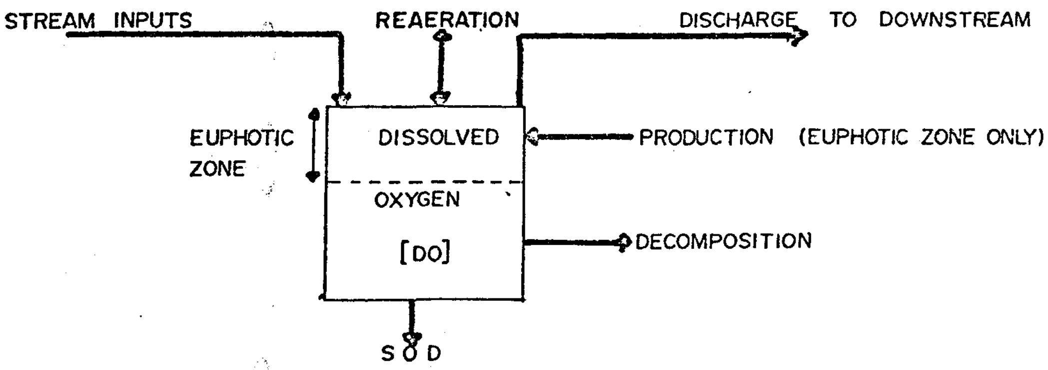
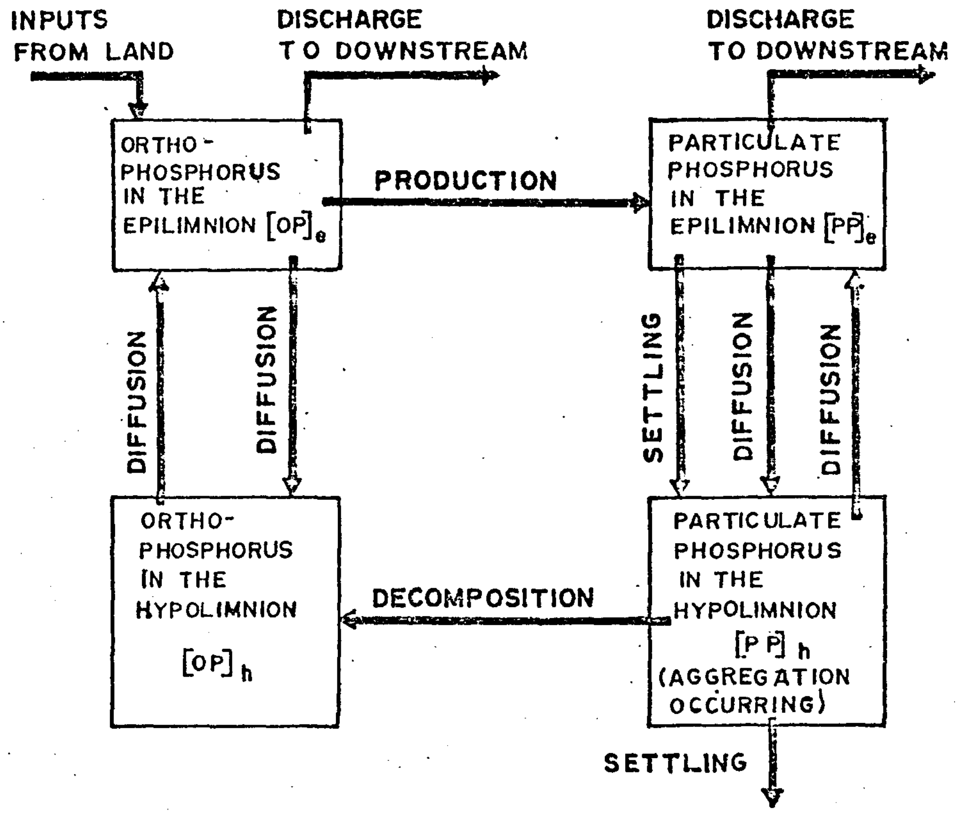
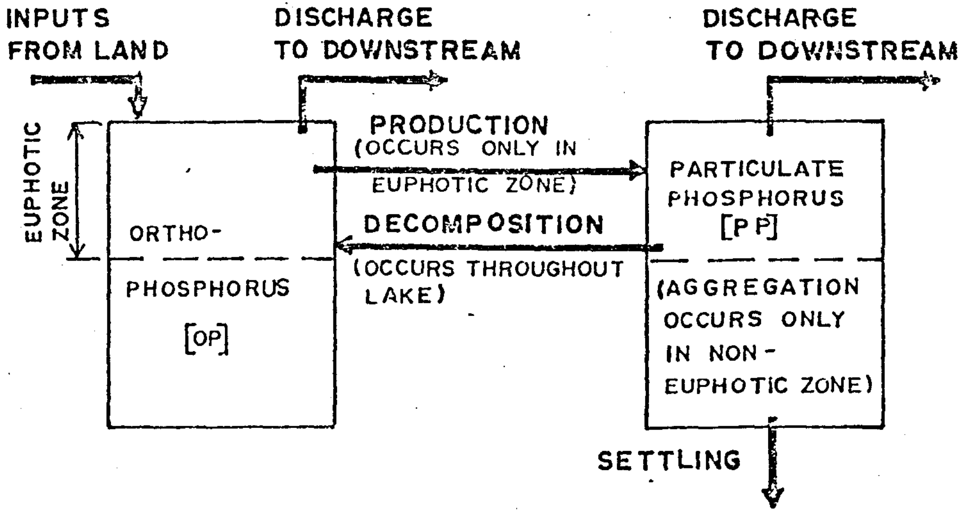


FIGURE 2-2
PHOSPHORUS TRANSPORT MECHANISMS

SUMMER STRATIFICATION (A)



WINTER CIRCULATION (B)



model for the summer stratification and winter circulation periods. These transport mechanisms include production, decomposition, atmospheric reaeration, vertical exchange, sediment demands, settling and hydraulic inputs and outputs. It should be noted that production is confined to the epilimnion during summer stratification and to the euphotic zone (to a depth of 10 meters) in the winter circulation period.

2.3 THE PHOSPHORUS MODEL

As stated previously, the development of the dissolved oxygen model in this research is to be based on a phosphorus model providing the estimates of decomposition and production related oxygen fluxes. The model selected for this purpose is the predictive model for phosphorus developed by Snodgrass and O'Melia (1975). The model has been successful in predicting annual total phosphorus concentrations. A brief description of the model including modified basic assumptions follows.

2.3.1 Definitions and Assumptions

The following definitions are made:

(1) The phosphorus species in the lake are represented by two compartments, soluble orthophosphate (OP) and particulate phosphorus (PP), and

(2) Reaction coefficients used in the model to describe physical, chemical, and biological

processes are held constant over the time period of interest (summer stratification or winter circulation). The value of each coefficient selected by Snodgrass and O'Melia does not vary from year to year. However, in this research, the vertical exchange coefficient can vary on a year to year basis.

The following basic assumptions are made:

(1) Phosphorus is the nutrient controlling the rate of growth and size of the standing crop (e.g., of particulate P). A dynamic equilibrium exists between the amount of the influx of phosphorus and the growth of the organisms.

(2) The total phosphorus concentration (TP) is an appropriate parameter for describing the trophic status of a lake.

(3) The bottom sediments act solely as a phosphorus sink via settling. Inputs from the sediments in the form of orthophosphorus are negligible in the presence of an oxic hypolimnion.

(4) Phosphorus variations in the horizontal direction are negligible due to the assumption of each box being a continuous stirred tank reactor (CSTR). Vertical variations are significant only in the region of the thermocline.

(5) The lake is homothermal and ice-free during the winter circulation period.

(6) Groundwater flows are negligible. Evaporation from the surface is equalized by precipitation on an annual basis.

(7) External hydraulic (i.e., tributary) flows and phosphorus loadings occur into and out of the epilimnion during the stratification period. All external phosphorus loadings are biochemically active and enter the lake as orthophosphorus. These loadings are considered to be constant over time.

(8) The euphotic zone is equivalent to the epilimnion during stratification but assumed to reach to a depth of 10 meters in the winter.

(9) Mass transfer coefficients are assumed to be equivalent to heat transfer coefficients in their application to the vertical transport of mass across the thermocline.

2.3.2 The Reactions and Mass Balances

The reactions used in the model are net production in the euphotic zone, vertical diffusion of OP and PP between the epilimnion and the hypolimnion (direction-dependent on the gradient), decomposition in the hypolimnion or in the entire lake, settling of particulate phosphorus and change in the settling characteristics of particulate phosphorus by natural flocculation. Inputs and outputs are land-based phosphorus inputs from all sources, hydraulic discharge of OP and PP to downstream and settling of PP to the sediments.

Mass balances are formulated for each compartment of each box to yield four interdependent differential equations for the stratification period and two for the winter circulation period. These are shown below:

SUMMER STRATIFICATION

i) Epilimnion

$$[OP_e]: \quad \underset{\text{net rate of change}}{\frac{V_e d[OP_e]}{dt}} = \underset{\text{input: loading from land-based sources}}{\Sigma Q_j [TP]_j} - \underset{\text{output: hydraulic discharge}}{Q[OP_e]} - \underset{\text{net production}}{p_e V_e [OP_e]}$$

$$+ \underset{\text{vertical exchange from hypolimnion}}{\hat{k}_{th} A_{th} [OP_h]} - \underset{\text{vertical exchange to hypolimnion}}{\hat{k}_{th} A_{th} [OP_e]} \qquad (2.3\text{-}1)$$

$$[PP_e]: \quad \underset{\text{net rate of change}}{\frac{V_e d[PP_e]}{dt}} = - \underset{\text{output: hydraulic discharge}}{Q[PP_e]} + \underset{\text{net production}}{p_e V_e [OP_e]} - \underset{\text{settling to hypolimnion}}{g_e A_{th} [PP_e]}$$

$$+ \underset{\text{vertical exchange from hypolimnion}}{\hat{k}_{th} A_{th} [PP_h]} - \underset{\text{vertical exchange to hypolimnion}}{\hat{k}_{th} A_{th} [PP_e]} \qquad (2.3\text{-}2)$$

ii) Hypolimnion

$$[OP_h]: \quad \underset{\text{net rate of change}}{\frac{V_h d[OP_h]}{dt}} = \underset{\text{decomposition}}{d_h V_h [PP_h]} + \underset{\text{vertical exchange from epilimnion}}{\hat{k}_{th} A_{th} [OP_e]} - \underset{\text{vertical exchange to epilimnion}}{\hat{k}_{th} A_{th} [OP_h]} \qquad (2.3\text{-}2)$$

$$[PP_h]:\ \underbrace{\frac{V_h d[PP_h]}{dt}}_{\substack{\text{net rate}\\ \text{of change}}} = \underbrace{g_e A_{th}[PP_e]}_{\substack{\text{input: settling}\\ \text{from epilimnion}}} - \underbrace{g_h A_s [PP_h]}_{\substack{\text{output: settling}\\ \text{to sediments}}} - \underbrace{d_h V_h [PP_h]}_{\substack{\text{decomp-}\\ \text{osition}}}$$

$$+ \underbrace{\hat{k}_{th} A_{th} [PP_e]}_{\substack{\text{vertical exchange}\\ \text{from epilimnion}}} - \underbrace{\hat{k}_{th} A_{th} [PP_h]}_{\substack{\text{vertical exchange}\\ \text{to epilimnion}}} \qquad (2.3\text{-}4)$$

<u>Subscripts:</u>

e = epilimnion,
eu = euphotic,
h = hypolimnion,
th = thermocline region, and
s = sedimentation-water interface.

Parameters:

[OP] = concentration of orthophosphate,
[PP] = concentration of particulate phosphorus,
p = net production rate coefficient,
d = decomposition rate coefficient,
$\hat{k}$ = vertical exchange coefficient which includes the effects of molecular and turbulent diffusion, internal waves, erosion of the hypolimnion and other fluid processes on the transfer of heat and materials across the thermocline,
g = effective settling velocity,
$\bar{Z}$ = mean depth,
V = volume,
A = interfacial area,
Q_j = land based volumetric rate of inflow of water, and
Q = volumetric rate of lake discharge.

WINTER CIRCULATION

$$[OP]: \underset{\substack{\text{net rate}\\ \text{of change}}}{\frac{V[OP]}{dt}} = \underset{\substack{\text{input: loading}\\ \text{from land-based}\\ \text{sources}}}{\Sigma Q_j[TP]_j} - \underset{\substack{\text{output:}\\ \text{hydraulic}\\ \text{discharge}}}{Q[OP]} - \underset{\substack{\text{production in}\\ \text{euphotic zone}}}{p_{eu}V_{eu}[OP]} + \underset{\text{decomposition}}{dV[PP]} \qquad (2.3\text{-}5)$$

$$[PP]: \underset{\substack{\text{net rate}\\ \text{of change}}}{\frac{V[PP]}{dt}} = \underset{\substack{\text{output:}\\ \text{hydraulic}\\ \text{discharge}}}{-Q[PP]} + \underset{\substack{\text{production in}\\ \text{euphotic zone}}}{p_{eu}V_{eu}[OP]} - \underset{\text{decomposition}}{dV[PP]} - \underset{\substack{\text{settling to}\\ \text{sediments}}}{gA_s\ [PP]} \qquad (2.3\text{-}6)$$

Subscripts: eu = euphotic zone

The models for the stratification and circulation periods are coupled using appropriate boundary conditions at the autumnal and vernal overturns.

2.4 THE DISSOLVED OXYGEN MODEL

As for the preceding phosphorus model, basic definitions and assumptions were established to the behaviour of dissolved oxygen in lakes. Since the two models are coupled by common production/decomposition terms, the same definitions and assumptions made for the phosphorus model apply equally to the combined DO-P model. Additional points specific to the dissolved oxygen part of the model are listed below.

2.4.1 Definitions and Assumptions

The following definition is made:

1. Symbols, subscripts, etc. common to both the phosphorus model and the dissolved oxygen model have exactly the same meaning. Only new symbols are defined in this section:
   e.g. - "A" is an interfacial area in both models.

The following assumptions are made:

1. Since inputs of DO from streams are unknown, the dissolved oxygen concentration of the tributary hydraulic inputs are at saturation levels throughout the year.

2. The oxygen demand of the lake water has been assumed to be completely due to the decomposition and respiration of the non-viable and viable portions respectively of the standing crop of algal and bacterial biomass.

3. The surface area of the water sediment interface is equal to the surface area of the lake. This is justified when one considers the lake as a box with the following dimensions: 290 km long x 70 km wide x 89 m deep. The true wetted area of such a box would be 20,326 km^2 while the surface area is 20,300 km^2. The difference is less than one per cent.

4. Redfield stoichiometry (Stumm and Morgan, 1970) can be used to relate dissolved oxygen fluxes to the production/decomposition fluxes of particulate phosphorus within the lake.

5. The sediments act as a sink for dissolved oxygen at a constant rate throughout the year over the entire sediment surface.

2.4.2 The Reactions and Mass Balances

The following processes and transport mechanisms are considered significant with respect to the dissolved oxygen budget of a lake.

1) Production of oxygen by photosynthesis,
2) Depletion of oxygen by decomposition including cell respiration,
3) Atmospheric reaeration,
4) Sediment oxygen demand,
5) Hydraulic input and output (to and from entire lake during circulation; to and from epilimnion only during stratification), and
6) Vertical transport across the thermocline during stratification.

A closer look at these sources and sinks provides the basis for the mathematical form of these processes in the model.

Primary production is the photosynthetic conversion of orthophosphate, carbon dioxide, nitrate and water to particulate phosphorus. An important by-product of this reaction is oxygen. Simultaneously, cells are respiring or decomposing, converting some particulate P to soluble P. Photosynthesis requires light and thus occurs only in the euphotic zone in a lake during daylight hours. At night, photosynthesis ceases and only respiration takes place. Since vertical transport between the epilimnion and hypolimnion is minimal during stratification, this diurnal cycle in the epilimnion results in the concept of net production. Decomposition only occurs in the non-euphotic hypolimnion. Normally, the photosynthesis reaction occurs at a greater rate than the respiration reaction and thus there is a net increase in particulate P due to these opposing processes. The reactions describing production are shown below:

Net Production of Biomass $= p_e V_e [OP_e]$ in the epilimnion during stratification, and

Gross Production of Biomass $= p_{eu} V_{eu} [OP]$ in the euphotic zone during circulation.

For the euphotic zone during winter circulation, a gross production coefficient is used since complete mixing occurs throughout the lake,

The production of particulate phosphorus predicted by the above equations is accompanied by production of oxygen.

The quantitative relationship between production of cells and oxygen is not exactly known due to many complicating factors, not the least of which is the lack of a universally acceptable chemical formula for biomass to use in the production reaction equation. Of several representations of organic matter (Eckenfelder and O'Connor, 1961, Stumm and Morgan, 1970, and Wetzel, 1975) only Redfield stoichiometry contains both nitrogen and phosphorus and for this reason was selected for application in this model.

Redfield Stoichiometry

$$106\ CO_2+16\ NO_3^-+HPO_4^{-2}+122\ H_2O+18H^+ \ \underset{\xleftarrow[D]{\text{Trace Elements}}}{\xrightarrow[\text{+ Energy +}]{P}}$$

$$C_{106}H_{263}O_{110}N_{16}P+138\ O_2$$

Stoichiometrically, the above equation predicts that the production or decomposition of 1 μg/l of particulate phosphorus respectively is equivalent to a production or consumption of 0.142 mg/l of dissolved oxygen. Thus the following equations are used to model photosynthetic production of oxygen:

$$\text{Production Rate of Oxygen} = \text{FAC} \times p_e V_e [OP_e] \quad \text{in the epilimnion during stratification}$$

$$= \text{FAC} \times p_{eu} V_{eu} [OP] \quad \text{in the euphotic zone during circulation}$$

$$\text{where FAC} = \frac{0.142\ \text{mg DO/l}}{1\ \mu\text{g P/l}}$$

Dissolved oxygen is consumed by the process of decomposition including cell respiration. The rate of depletion is a direct function of the concentration of particulate phosphorus which is used to estimate the organic biomass. Due to the concept of net production in the epilimnion, this oxygen sink was allowed to occur only in the hypolimnion during stratification. The process occurs over the entire lake during the circulation period. A similar development to that for production was used to establish equations to predict particulate phosphorus and oxygen consumption due to decomposition. The results are:

Consumption of Particulate Phosphorus	$= d_h V_h [PP_h]$	in the hypolimnion during stratification
	$= dV [PP]$	in the entire lake during circulation
Consumption of Oxygen	$= FAC \times d_h V_h [PP_h]$	in the hypolimnion during stratification
	$= FAC \times dV[PP]$	in the entire lake during circulation

In this model, the biochemical oxygen demand or BOD of the lake water is assumed to be totally due to the decomposition of particulate phosphorus.

Atmospheric reaeration is the process whereby oxygen is transferred from the surrounding atmosphere to a body of water. The driving force is the concentration deficit between the air (at saturation for any given temperature) and the lake surface waters. The reaeration reaction has been formulated by Eckenfelder and O'Connor (1961) as:

$$\frac{d}{dt}[DO] = K_2(DO_{sat}-[DO]) \qquad (2.4\text{-}1)$$

This was originally developed by Streeter and Phelps (1925) as the oxygen source in their stream DO sag model. For a lake,

$$V = A_{surf}\,\bar{z} \quad \text{- entire lake} \qquad (2.4\text{-}2)$$

$$V_e = A_{surf}\,\bar{z}_e \quad \text{- epilimnion} \qquad (2.4\text{-}3)$$

where A_{surf} = surface area of the lake,
$\bar{z}$ = mean depth of the lake, and
$\bar{z}_e$ = mean depth of the epilimnion

Converting from a concentration to a mass flow basis,

$$\text{Atmospheric Reaeration (mass/time)} = k_a\,A_{surf}(DO_{sat}-[DO_e]) \quad \text{- epilimnion during stratification}$$

$$= k_a\,A_{surf}(DO_{sat}-[DO]) \quad \text{- entire lake during circulation}$$

$$\text{where } k_a = K_2\bar{z} \qquad (2.4\text{-}4)$$

A similar form of the reaeration process involving the product of a velocity coefficient, k_a, and the surface area, A_{surf}, as the overall transfer coefficient was used by Busch (1972).

Vertical transport across the thermocline during stratification applies equally to dissolved oxygen and to the phosphorus compartments. The driving force is the concentration gradient between the epilimnion and the hypolimnion. The reaction is formulated as:

$$\text{Vertical Transport} = \hat{k}_{th}A_{th}[DO_e] \quad \text{from epilimnion to hypolimnion}$$

$$\text{Vertical Transport} = \hat{k}_{th}A_{th}[DO_h] \quad \text{from hypolimnion to epilimnion}$$

The sediment oxygen demand represents the chemical and biological consumption of oxygen by the sediments. For a lake with an oxic condition at the water-sediment interface, the demand is largely due to the benthic or bottom dwelling organisms. The reaction is formulated as the zero order reaction shown below:

Sediment Oxygen Demand = k_sA_s in the hypolimnion during stratification

= k_sA_s in the whole lake during circulation

where k_s = areal sediment oxygen demand, and

A_s = area of the sediments.

The area of sediments in the epilimnion is considered negligible in comparison to hypolimnetic sediment area and thus this oxygen sink occurs only in the hypolimnion during stratification.

The hydraulic inputs and outputs of dissolved oxygen are listed below. The tributaries are assumed to be at saturation DO concentration when they enter the lake.

Input = $[DO_{sat}]Q_T$ during stratification and circulation

Output = $[DO_e]Q$ during stratification

Output = $[DO]Q$ during circulation

where Q_T = total tributary inflow.

Construction of mass balance equations for dissolved oxygen yields two linear interdependent differential equations for the summer stratification period and one linear differential equation for the winter circulation period.

SUMMER STRATIFICATION

i) Epilimnion

$$[DO_e]:\ \underset{\substack{\text{net rate}\\ \text{of change}}}{\frac{V_e d[DO_e]}{dt}} = \underset{\substack{\text{input: from}\\ \text{tributaries}}}{DO_{sat}Q_T} - \underset{\substack{\text{output:}\\ \text{hydraulic}\\ \text{discharge}}}{Q[DO_e]} + \underset{\substack{\text{net}\\ \text{production}}}{FACp_eV_e[OP_e]}$$

$$+ \underset{\substack{\text{atmospheric}\\ \text{reaeration}}}{k_aA_{surf}(DO_{sat}-[DO_e])} + \underset{\substack{\text{vertical exchange}\\ \text{from hypolimnion}}}{\hat{k}_{th}A_{th}[DO_h]}$$

$$- \underset{\substack{\text{vertical exchange}\\ \text{to hypolimnion}}}{\hat{k}_{th}A_{th}[DO_e]} \tag{2.4-6}$$

ii) Hypolimnion

$$[DO_h]:\ \underset{\substack{\text{net rate}\\ \text{of change}}}{\frac{V_h d[DO_h]}{dt}} = \underset{\text{decomposition}}{- FACd_hV_h[PP_h]} - \underset{\substack{\text{sediment}\\ \text{demand}}}{k_sA_s} + \underset{\substack{\text{vertical exchange}\\ \text{from epilimnion}}}{\hat{k}_{th}A_{th}[DO_e]}$$

$$- \underset{\substack{\text{vertical exchange}\\ \text{to epilimnion}}}{\hat{k}_{th}A_{th}[DO_h]} \tag{2.4-7}$$

WINTER CIRCULATION

$$[DO]:\ \underset{\substack{\text{net rate}\\ \text{of change}}}{\frac{Vd[DO]}{dt}} = \underset{\substack{\text{input: from}\\ \text{tributaries}}}{DO_{sat}Q_T} - \underset{\substack{\text{output:}\\ \text{hydraulic}\\ \text{discharge}}}{Q[DO]} + \underset{\substack{\text{gross}\\ \text{production}}}{FACp_{eu}V_{eu}[OP]} \underset{\text{decomposition}}{-FACdV[PP]}$$

$$- \underset{\substack{\text{sediment}\\ \text{demand}}}{k_sA_s} + \underset{\text{atmospheric reaeration}}{k_aA_{surf}(DO_{satn}-[DO])} \qquad (2.4\text{-}8)$$

In the following section, the phosphorus and dissolved oxygen mass balance equations are combined and are represented in the final matrix form.

2.5 SUMMARY OF MODEL

The model is summarized in matrix form in Figures 2.3 and 2.4. The model is solved mathematically using a second order Runge-Kutta method as described in section 5.1.

$$\begin{bmatrix} \frac{d[PP_e]}{dt} \\ \frac{d[PP_h]}{dt} \\ \frac{d[OP_e]}{dt} \\ \frac{D[OP_h]}{dt} \\ \frac{d[DO_e]}{dt} \\ \frac{d[DO_h]}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{Q}{V_e} - \frac{K_{th}A_{th}}{\bar{z}_{th}V_e} - \frac{g_e A_{th}}{V_e} & \frac{K_{th}A_{th}}{V_e \bar{z}_{th}} & p_e & 0 & 0 & 0 \\ \frac{K_{th}A_{th}}{\bar{z}_{th}V_h} + \frac{g_e A_{th}}{V_h} & -\frac{K_{th}A_{th}}{\bar{z}_{th}V_h} - \frac{g_h A_s}{V_h} - d_h & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{Q}{V_e} - p_e - \frac{K_{th}A_{th}}{\bar{z}_{th}V_e} & \frac{K_{th}A_{th}}{\bar{z}_{th} V_e} & 0 & 0 \\ 0 & d_h & \frac{K_{th}A_{th}}{\bar{z}_{th}V_h} & -\frac{K_{th}A_{th}}{\bar{z}_{th}V_h} & 0 & 0 \\ 0 & 0 & p_e \times FAC & 0 & -\frac{K_a A_{surf}}{V_e} - \frac{K_{th}A_{th}}{\bar{z}_{th}V_e} - \frac{Q}{V_e} & -\frac{K_{th}A_{th}}{\bar{z}_{th}V_e} \\ 0 & -d_h \times FAC & 0 & 0 & -\frac{K_{th}A_{th}}{\bar{z}_{th}V_h} & -\frac{K_{th}A_{th}}{\bar{z}_{th}V_h} \end{bmatrix} \times \begin{bmatrix} [PP_e] \\ [PP_h] \\ [OP_e] \\ [OP_h] \\ [DO_e] \\ [DO_h] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{PHOSLD}{V_e} \\ 0 \\ \frac{k_a A_{surf} DO_{sat}}{V_e} + \frac{DO_{sat} Q_T}{V_e} \\ -\frac{K_s A_s}{V_h} \end{bmatrix}$$

FIGURE 2-3

MODEL IN MATRIX FORM SUMMER STRATIFICATION

$$\begin{bmatrix} \frac{d[PP]}{dt} \\ \frac{D[OP]}{dt} \\ \frac{d[DO]}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{Q}{V} - d - \frac{g\ A_{sed}}{V} & p_{eu}\frac{V_{eu}}{V} & 0 \\ d & -\frac{Q}{V} - p_{eu}\frac{V_{eu}}{V} & 0 \\ -d \times FAC & p_{eu} \times FAC \times \frac{V_{EU}}{V} & -\frac{k_a A_{surf}}{V} - \frac{Q}{V} \end{bmatrix} \times \begin{bmatrix} {[PP]} \\ {[OP]} \\ {[DO]} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{PHOSLD}{V} \\ \frac{k_a A_{surf} DO_{sat}}{V} + \frac{DO_{sat} Q_T}{V} - \frac{k_s A_s}{V} \end{bmatrix}$$

FIGURE 2-4

MODEL IN MATRIX FORM - WINTER CIRCULATION

CHAPTER 3

PHYSICAL AND LIMNOLOGICAL PARAMETERS OF LAKE ONTARIO

The basic model form was developed with the view of general application to any stratified lake. Simply by changing those physical parameters and/or reaction coefficients characteristic of a particular lake, the model is made to be specific for that lake. The model structure need not be changed. This implies that a basic source of data should be readily available to the researcher not only for model calibration and verification but for the development and calculation of certain parameters such as the rate of deepening of the epilimnion and the vertical exchange coefficient. Due to previous in-depth physical studies, numerous water quality cruises and successful application of the phosphorus model used in this research, Lake Ontario was selected for initial study.

This chapter presents and discusses the selection of values for morphometric and model parameters.

3.1 BASIC PHYSICAL DATA

The physical data for Lake Ontario are summarized in Table 3-1. The data in part(a) were obtained from a report to the International Joint Commission (1969). The

TABLE 3-1

PHYSICAL DATA - LAKE ONTARIO

LAKE PARAMETER	SYMBOL (for this research)	VALUE
a) IJC Report		
Volume	V	1.63×10^{12} m^3
Surface Area	A	1.83×10^{10} m^2
Mean Depth	$\bar{Z}$	89 m
Lake Discharge	Q	5.68×10^{8} m^3/day
Niagara River Flow	NRFLOW	4.77×10^{8} m^3/day
Triburary Flow	TRFLOW	0.746×10^{8} m^3/day
Phosphorus Loading	PHOSLD	34.1×10^{9} mg/day
b) Estimated		
Volume of Euphotic Zone	V_{eu}	1.83×10^{11} m^3
Depth of Euphotic Zone	Z_{eu}	10 m
Surface Area of Sediments	A_s	1.83×10^{10} m^2

data in part (b) are estimates.

During the summer stratification period, the depth of the epilimnion and its volume increases with time at the same rate as the deepening of the thermocline. The depth of the thermocline was calculated as a function of time for each year (see Section 3.3.1).

For a given depth, the interfacial area between the epilimnion and the hypolimnion was estimated using the hypsometric chart shown in Figure 3-1. For depths up to 34 meters, the following relations were determined using a linear approximation:

$$A_{th} = (-126\ \bar{Z}_e + 18300) \times 10^6\ m^2 \qquad (3.1\text{-}1)$$

$$V_e = (-63\ \bar{Z}_e^2 + 18300\ \bar{Z}_e) \times 10^6\ m^3 \qquad (3.1\text{-}2)$$

$$V_h = V - V_e \qquad (3.1\text{-}3)$$

$$\bar{Z}_h = V_h/A_{th} \qquad (3.1\text{-}4)$$

3.2 BIOLOGICAL, CHEMICAL AND PHYSICAL PROCESS COEFFICIENTS

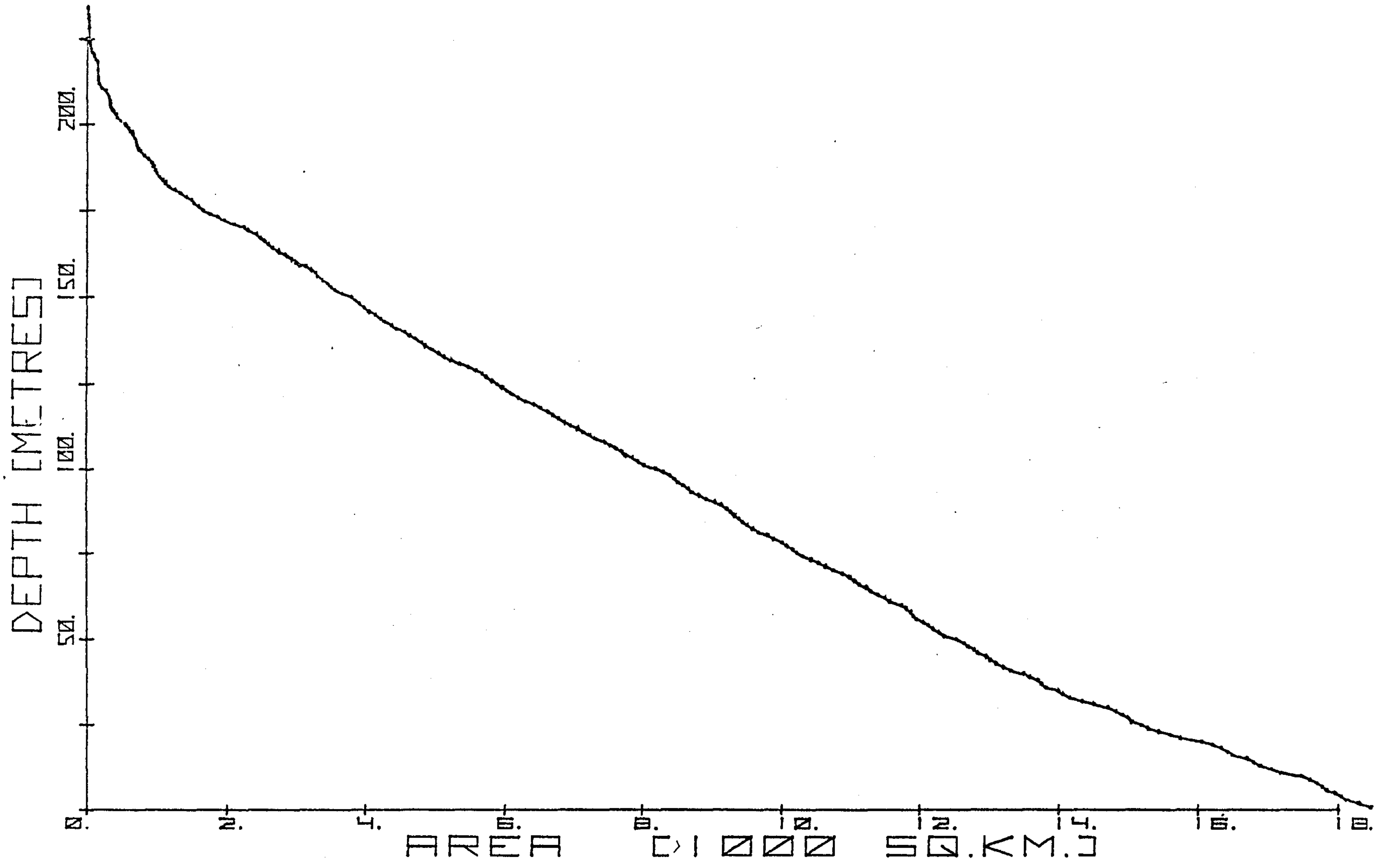
The coefficients selected by Snodgrass (1974) were used as the phosphorus model parameters with only two exceptions, the net production coefficient and the coefficient of vertical exchange across the thermocline. These are summarized in Table 3-2.

TABLE 3-2

PROCESS RATE COEFFICIENTS

COEFFICIENT	SYMBOL	VALUE (for this Research)	SNODGRASS (1974)
Net Production			
-epilimnion	p_e	0.2 day^{-1}	2.0 day^{-1}
-euphotic zone	p_{eu}	0.06 day^{-1}	0.06 day^{-1}
Decomposition			
-hypolimnion	d_h	0.03 day^{-1}	0.03 day^{-1}
-entire lake	d	0.03 day^{-1}	0.03 day^{-1}
Sedimentation			
-epilimnion	g_e	0.1 m/day	0.1 m/day
-absence of flocculation	g_o	0.05 m/day	0.05 m/day
-hypolimnion	g_h	$g_o(1+f\ \bar{Z}_h)$	$g_o(1+\ f\ \bar{Z}_h)$
-winter lake	g	$g_o(1+f(\bar{Z}-\bar{Z}_{eu}))$	$g_o(1+f(\bar{Z}-\bar{Z}_{eu}))$
Flocculation	f	0.05 m^{-1}	0.05 m^{-1}
Vertical exchange across the thermocline	$\hat{k}_{th}$	Variable year to year as calculated	0.005 $\bar{Z}$

FIGURE 3-1
HYPSOMETRIC CHART LAKE ONTARIO



Most literature sources report a reaeration coefficient, K_2, with the units of inverse time, according to the reaction equation 2.4-1. The reaeration coefficient used in this model, k_a, has units of distance/time. It can be seen from equation 2.4-4 that k_a is the product of the Streeter-Phelps reaeration coefficient K_2 and the depth of the box into which aeration occurs.

Numerical values of the reaeration coefficient K_2 for lakes and other similar bodies of water have not been investigated to the extent of those for rivers and streams. Busch (1972) converted values of K_2 (day^{-1}) established by Churchill et al. (1962) and Juliano (1969) for several rivers to equivalent k_a values (m/day). This was done by multiplying K_2 by the average depth of the river over which the coefficient was determined. Metcalf and Eddy (1972) report values of K_2 for small ponds, large lakes and sluggish streams while O'Connor and Dobbins (1958) measured K_2 in San Diego Bay at various depths. Torgersen et al. (1977) measured "piston velocities" or the rate of gas exchange at the surface for Lake Ontario using decay of radioactive helium. Table 3-3 summarizes the results of the above studies.

For the initial model run, a value of 7.5 m/day was selected for k_a. This value was used by Rumer and Melfi (1973) in their modelling of Lake Ontario and was selected from published data by Odum (1956). No distinction was made between summer and winter conditions initially but model calibration (see section 5.3) indicated the need for such a

TABLE 3-3

LITERATURE VALUES OF THE REAERATION COEFFICIENT

WATER BODY	RANGE	TEMP.	SOURCE
San Diego Bay			
at 12 ft	0.048 (day^{-1})	20°C	O'Connor and Dobbins (1958)
at 32 ft	0.018 (day^{-1})		
at 37 ft	0.026 (day^{-1})		
Rivers (5)	0.58 - 2.77 (m/day)		Churchill *et al.* (1962)
Rivers (4)	1.6 - 36.9 (m/day)		Juliano (1969)
Small Ponds and Backwaters	0.05 - 0.10 (day^{-1})	20°C	Metcalf and Eddy (1972)
Sluggish Streams and Large Lakes	0.10 - 0.15 (day^{-1})	20°C	Metcalf and Eddy (1972)
Lake Ontario	2-4 m/day		Torgersen (1977)

distinction.

Direct sediment oxygen demand (SOD) was estimated to be very low for Lake Ontario, on an average areal basis, due to large areas of post glacial muds in off-shore areas consisting of 2 to 6 per cent organic matter. Indeed even the nearshore sediments, consisting of sand, gravel and bedrock, are on the whole low in organic content (less than 2 per cent). In some areas, notably around cities and other large point sources of organic pollution, the SOD might be expected to be higher. Since most of the water/sediment interface is at an oxic condition, SOD would be due largely to oxygen uptake by benthic organisms with a minor contribution from the decomposition of organic material.

Sediment oxygen demand has been studied by several researchers. McKeown _et al._ (1968) measured SOD's in a laboratory apparatus with bottom sludge of paper mill origin. At low flow, near stagnant conditions, the measured SOD ranged from 0.2 - 0.8 g O_2/m^2-day at 20°C. They also found that SOD was a function of the flow condition over the sediment surface. Edwards and Rolley (1965) measured the SOD of river muds in a laboratory apparatus. The overlying water was maintained in a mixed state to prevent DO gradients at the water-mud interface. The measured SOD's were clustered near 2.4 g O_2/m^2-day at 20°C. Recent studies of the oxygen consumption of lake sediments have been reported by Sorizogni _et al._ (1977) for "in situ" measurements and by Brewer _et al._ (1977) for laboratory measurements. The in situ values ranged from 0.12 to 0.22

g O_2/m^2-day and averaged 0.17 g O_2/m^2-day at an unreported temperature, while the laboratory measurements ranged from 0.31 to 0.98 g O_2/m^2-day and averaged 0.68 g O_2/m^2-day at 18°C.

The sediments in Lake Ontario are assumed to be largely in the hypolimnion during stratification. The sediments will also be at or near 4°C all year round and should be less biologically active than river muds. In addition, the currents in the overlying water are expected to be of low velocity. Based on all the above considerations, a value of 0.1 g O_2/m^2-day was selected for the model SOD coefficient.

The coefficients presented in Tables 3-2 and 3-4 are considered to be constant on a seasonal basis (i.e., constant for winter circulation or summer stratification). Still to be determined are the vertical exchange coefficient $\hat{k}_{th}$ and the depth of the epilimnion Z_e throughout stratification. These require analysis of temperature data.

TABLE 3-4

SUMMARY OF DO MODEL PARAMETERS

COEFFICIENT	SYMBOL	VALUE
Reaeration	k_a	7.5 m/day
Sediment Oxygen Demand	k_s	0.1 g/m^2-day
DO/Phosphorus Factor	FAC	0.142 $\frac{mg\ O_2/l}{\mu g\ P/L}$

3.3 ASSIMILATION OF FIELD TEMPERATURE DATA

The remaining input model parameters required for the solution of the model equations are the vertical exchange coefficient $\hat{k}_{th}$ for each year, the relationship between thermocline depth and time for each year and the relationship between the saturation dissolved oxygen concentration and time throughout the year. To obtain this data, the temperature data from nine years of cruises on Lake Ontario was obtained from the Canada Centre for Inland Waters (CCIW) in Burlington, Ontario. This data was analyzed and the results are presented below.

3.3.1 Rate of Change of the Depth of the Thermocline

The available temperature data was collected by the CCIW personnel in the following form for each cruise. At each station or sampling point, the water temperature was measured at various depths thus providing a profile from surface to bottom. All measurements at a particular station and subsequently all stations for each cruise were recorded on a computer data tape. These tapes were made available to this researcher for the years 1966 to 1974 inclusive.

To obtain an overall lake temperature profile for each cruise, depth ranges were selected as follows:

0 m - 1 m
1 - 2
2 - 4
4 - 6
6 - 8
8 - 10
10 - 12
12 - 16
16 m - 20 m
20 - 25
25 - 40
40 - 60
60 - 100
100 - 150
150 - 200
>200

All individual temperature and associated depth measurements within each depth range were averaged to give a mean temperature and depth for each slice. These mean values were then plotted for each cruise and a smooth curve drawn through the points to produce a temperature depth profile. The cruise profiles for the years 1966, 1967, 1968, 1969, 1970,and 1972 are shown in Figures A-1 to A-6 in Appendix A. Due to unavailability of data, curves could not be produced for the stratification period for the years 1971, 1973, and 1974.

From these curves, the thermocline depth was determined for the cruises when the lake could be considered stratified. This depth was determined by estimating for each cruise profile the point at which the maximum gradient occurred. These depths were paired with the appropriate date, in months, of the cruise. A least squares line was fitted to the data, thus producing the rate of epilimnetic deepening (the slope) and an estimate of actual thermocline depth as a function of time for each year. Table 3-5 presents the raw data obtained from Figure 3-2 for the year 1966. The data from all years was used to provide least squares estimates for the three years for which no data was available. The overall results are shown in Table 3-6 while raw data and calculations are presented in Appendix A.

The figures in Table 3-6 indicate that the rate of increase of depth of the thermocline can vary widely from year to year. Factors affecting the rate are primarily meteorological in nature and include incoming solar radiation, air, temperature, strength and frequency of winds and

TABLE 3-5

THERMOCLINE DEPTH AT VARIOUS TIMES DURING STRATIFICATION - 1966

DATE OF CRUISE	TIME (months)	ESTIMATED* DEPTH OF THERMOCLINE (meters)
June 25	5.83	6.5
June 30	6.0	8.8
July 10	6.33	12.4
July 24	6.80	13.0
August 7	7.23	17.4
August 19	7.63	16.4
September 2	8.07	22.0
September 16	8.53	21.5

Least Squares Linear Regression

$$\bar{Z}_e = 5.50\ t - 24.0$$

---

[*] from Figure 3-2

FIGURE 3-2a

TEMPERATURE PROFILES LAKE ONTARIO 1966

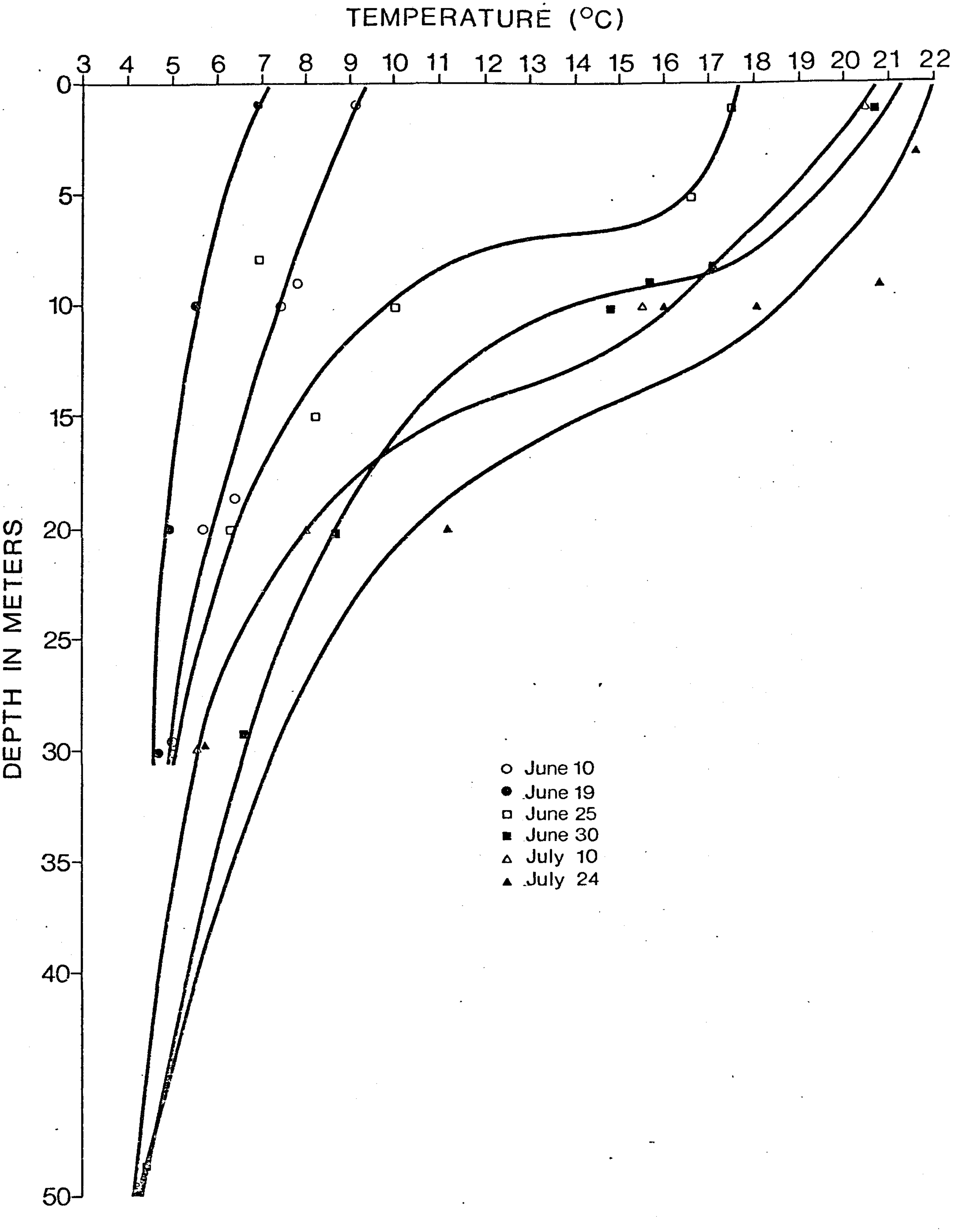


FIGURE 3-2b

TEMPERATURE PROFILES LAKE ONTARIO 1966

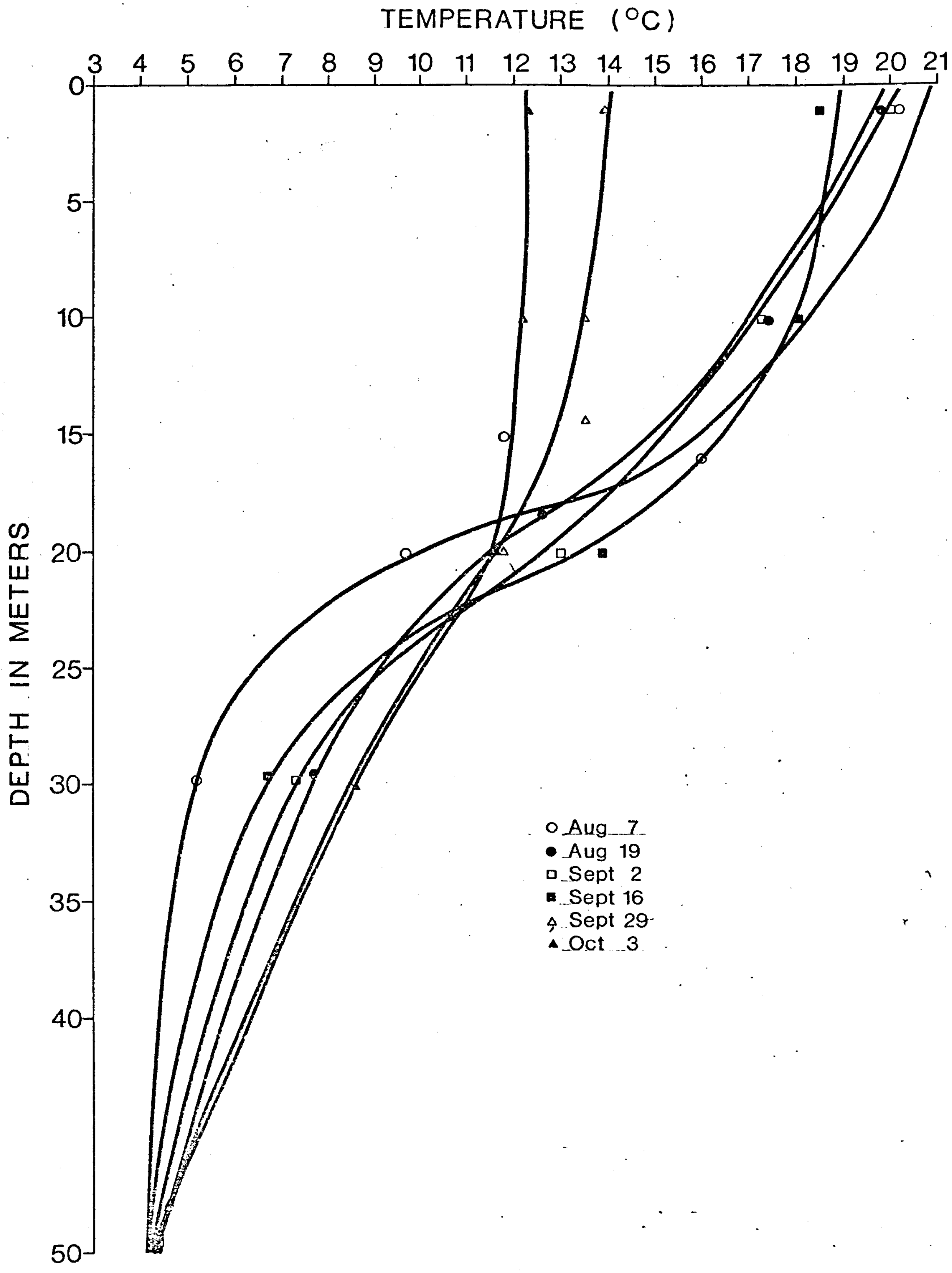


TABLE 3-6

ESTIMATES OF RATE OF DEEPENING OF THERMOCLINE AND THERMOCLINE DEPTH AS A FUNCTION OF TIME

YEAR	ESTIMATED* RATE OF INCREASE OF $\bar{Z}_e$ (meters/month)	$\bar{Z}_e$ vs t t in months $\bar{Z}_e$ in meters
1966	5.50	5.50 t - 24.0
1967	0.84	0.84 t + 3.18
1968	1.10	1.10 t + 7.95
1969	2.84	2.84 t - 5.14
1970	3.66	3.66 t - 17.14
1971	3.62**	3.62 t - 13.08**
1972	5.76	5.76 t - 28.24
1973	3.62**	3.62 t - 13.08**
1974	3.62**	3.62 t - 13.08**

---

* least squares estimate

** least squares approximate of all data 1966-1970, 1972 occurrence of storm events. Internal lake circulation is affected by all of the above and it is this circulation (convective heat transfer) along with diffusion (conductive heat transfer) and radiation that determine the thermal pattern of the lake.

These estimates were incorporated into the model to allow the epilimnion volume to increase throughout the stratification period. A similar treatment of temperature data was performed by Sweers (1969a) for the years 1966 and 1967. Thermocline intensity is discussed by Sweers using his temperature-depth profiles but he does not attempt to estimate the rate of deepening of the epilimnion.

3.3.2 The Vertical Exchange Coefficient

The transport of mass across the thermocline is due to processes such as eddy diffusivity, molecular diffusivity, internal circulation causing entrainment and leakage. This transport process can be described by:

$$\text{Mass Flow} = k_{th} A_{th} \left(\frac{d[DO]}{dZ}\right)_{th} \qquad (3.3\text{-}1)$$

The transport equation consists of the product of an effective vertical transport coefficient, k_{th}, the interfacial area at the thermocline, A_{th}, and a driving force in the form of a concentration gradient between epilimnetic and hypolimnetic waters in the region of the thermocline. A total derivative is used due to the assumption of negligible horizontal gradients.

The maximum gradient is assumed to exist at the thermocline and the thickness of the thermocline is used as an approximation to the depth parameter. Thus the equation becomes:

$$\text{Mass Flow} = k_{th} \, A_{th} \left(\frac{[DO_h] - [DO_e]}{\Delta Z_{th}} \right) \qquad (3.3\text{-}2)$$

The vertical transport coefficient, k_{th}, and the thermocline thickness, ΔZ_{th}, are combined to give the vertical exchange coefficient, $\hat{k}_{th}$, which is analagous to a velocity.

$$\hat{k}_{th} = \frac{k_{th}}{\Delta Z_{th}} \qquad (3.3\text{-}3)$$

This parameter not only changes from year to year due to different circulation patterns but also during the stratification period of any particular year. The value steadily decreases with time throughout the stratification period. This has been shown to be the case of the year 1966 as shown by the calculations presented in Appendix B. However, for the purpose of application to this model, it was decided to use an average value for the entire stratification period for each year.

The vertical exchange coefficient, $\hat{k}_{th}$, can be estimated using the temperature profiles constructed (Figures A-1 to A-6) and Fick's 1st Law. This method was used by Rumer and Melfi (1973) and is based on a heat balance over the hypolimnetic volume. It is assumed that all heat is transported to the hypolimnion by vertical mixing across the thermocline region.

The method is illustrated in Table 3-7, with sample calculation for 1966 following in Table 3-8. The time period, Δt, for this research was selected such that an average value of $\hat{k}_{th}$ over the stratification period was obtained. The raw data and sample calculations for all other years are shown in Appendix C. The results for the nine years of data are shown in Table 3-9. These values were used in the model solutions for all years.

3.3.3 Saturation Dissolved Oxygen Level Throughout the Year

The driving force for atmospheric reaeration is the dissolved oxygen gradient between the surface waters of the lake and the overlying atmosphere. Due to the assumption of complete mixing in the lake, the rate controlling step is the transfer through the gas/liquid interface. The concentration of dissolved oxygen in the gas film is assumed to be at the saturation limit at the temperature of the liquid (water) film with which it is in contract. Therefore, the surface water temperature of the lake was examined.

The temperature data from the nine year period 1966-1974 was analyzed. Lake temperature at a depth of 1.0 meter or less was plotted versus time of year in months. All data points were plotted on a single graph to represent a "typical" yearly cycle. The result is shown in Figure 3-3. A least squares polynomial curve fitting technique was used to obtain the mathematical representation of the relationship. The

TABLE 3-7

METHOD OF CALCULATION OF $\hat{k}_{th}$ FROM TEMPERATURE DATA (Rumer and Melfi (1973))

Heat Balance over Hypolimnion:

$$\frac{d}{dt}\{C_p \rho_h \theta_h\} = C_p \hat{k}_{th} A_i (\rho_e \bar{\theta}_e - \rho_h \bar{\theta}_h) \qquad (3.3\text{-}4)$$

where θ = temperature
C_p = heat capacity (constant)

Neglect small variations in density ($\rho_e = \rho_h$)

Averaging over time yields:

$$\hat{k}_{th} = \left[\frac{V_h}{A_i} \; \frac{\theta_h(t) - \theta_h(o)}{\Delta t(\bar{\theta}_e - \bar{\theta}_h)} \right] \qquad (3.3\text{-}5)$$

where: $\hat{k}_{th}$ = coefficient of vertical exchange across the thermocline (m/day)
V_h = avg. volume of hypolimnion over the time period Δt (m^3)
A_i = avg. interfacial area between hypolimnion and epilimnion over Δt (m^2)
$\theta_h(t)$ = temperature of hypolimnion at time t(^{o}C)
$\theta_h(o)$ = temperature of hypolimnion at time o(^{o}C)
Δt = time period (days)
$\bar{\theta}_e$ = avg.temperature of epilimnion over Δt (^{o}C)
$\bar{\theta}_h$ = avg. temperature of hypolimnion over Δt (^{o}C)

TABLE 3-8

CALCULATION OF OVERALL $\hat{k}_{th}$ FOR SUMMER OF 1966

t(o): 8 June, 1966
t(t): 14 Sept.,1966
Δt = 98 days

Depth of Thermocline: 8 June - 1.0 m
14 Sept.- 19.7 m
Avg.: 10.35

For t=0 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP.
0 - 0.3	.003	12.5	.0375
0.3 - 0.7	.005	11.0	.0550
0.7 - 1.85	.012	9.6	.1152
1.85 - 3.40	.018	8.6	.1548
2.40 - 5.90	.027	7.5	.2025
5.9 - 10.25	.048	6.5	.3120
	.113		.8770
10.25- 17.1	.069	5.5	.3795
17.1 - 32.1	.138	4.5	.6210
22.1 - bottom	.680	4.0	2.7200
	.887		3.7205

$\bar{\theta}_e = 7.76$

$\bar{\theta}_h = 4.19$

For t=98 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP.
0 - 1.3	.015	20.1	.3015
1.3 - 6.8	.060	19.5	1.1700
68 - 10.3	.039	18.65	.7274
	.114		2.1989
10.3 - 12.4	.022	18.15	.3993
12.4 - 17.25	.053	16.6	.0798
17.75- 18.7	.009	14.6	.1314
18.7 - 19.7	.011	13.0	.1430
19.7 - 20.25	.006	11.0	.0660
20.25- 21.9	.015	9.0	.1350
21.0 - 23.1	.010	2.5	.0750
23.1 - 28.5	.048	6.0	.2880
28.5 - 50.0	.186	4.5	.8370
50.0 - bottom	.526	4	2.1040
	.886		5.0585

$\bar{\theta}_e = 19.29$

$\bar{\theta}_h = 5.71$

$$\hat{k}_{th} = \frac{13.662}{.1736}\left(\frac{5.71 - 4.19}{98(13.52 - 4.95)}\right) = 0.1424$$

TABLE 3-9

VERTICAL EXCHANGE COEFFICIENT $\hat{k}_{th}$

YEAR	$\hat{k}_{th}$ (m/day)
1966	0.142
1967	0.116
1968	0.074
1969	0.121
1970	0.128
1971	0.122*
1972	0.153
1973	0.122*
1974	0.122*

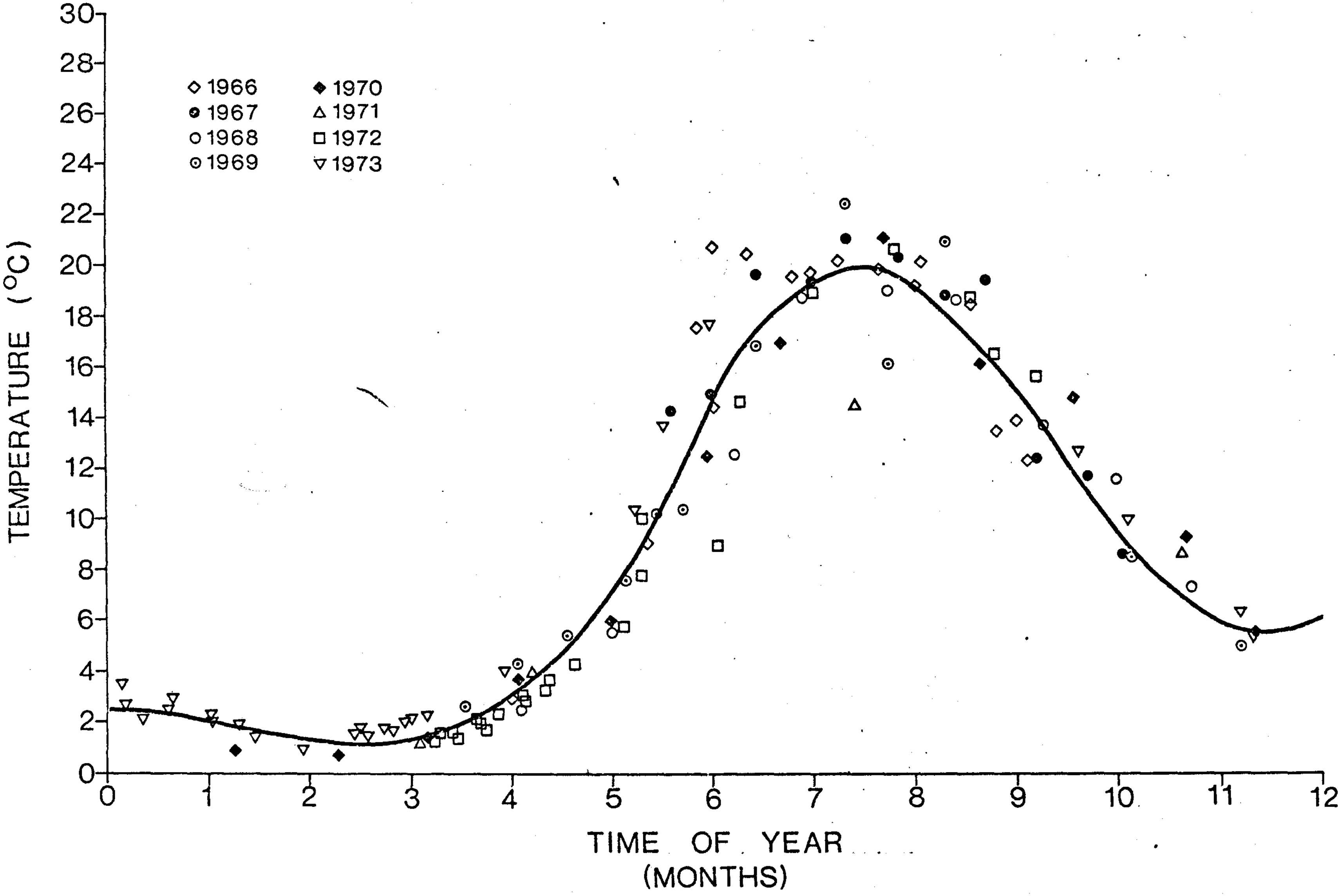
---

[*] average of $\hat{k}_{th}$ values for 1966-1970 and 1972

Note: Values of $\hat{k}_{th}$ are constant for the entire stratification period

FIGURE 3-3

SURFACE WATER TEMPERATURE LAKE ONTARIO



routine is called Cubic Splines and is a package program available on the CDC 6400 Computer used in this research. As the name implies, a series of third-order polynomials are used to fit the data. The number of polynomials used is determined by the number of node points, selected by the programmer after viewing the graphical form of the data. The routine begins with the initial locations of the nodes (provided by the programmer) and searches for the "best" location for these points.

The number of nodes selected for the data shown in Figure 3-3 was two. Thus the routine provides cubic estimates for three sections of data which together comprise the entire set. The results are shown below.

For $0.1 \leq t \leq 5.31$, $D = t$

and $T = ((0.1584*D-0.5972)*D-0.0716)*D+2.6058$ (3.3-6)

For $5.31 \leq t \leq 6.61$, $D = t - 5.31$

and $T = ((-1.4288*D+1.9278)*D+6.9978)*D+9.1264$ (3.3-7)

For $6.61 \leq t \leq 12.00$, $D = t - 6.61$

and $T = ((0.4278*D-3.6162)*D+4.8140)*D+18.3106$ (3.3-8)

where t = time (months)

T = surface temperature (°C, and

D = dummy variable.

Thus, the surface water temperature can be estimated at any time of the year. The saturation dissolved oxygen concentration for water at that temperature is determined using a relationship developed by Markofsky and Harleman (1971) shown below:

$$\underset{(\mathrm{mg/l})}{\text{Saturated DO}} = 14.48 - 0.36T + 0.0043\,T^2 \qquad (3.3\text{-}9)$$

where T = surface temperature (°C).

The change of surface temperature and thus saturated dissolved oxygen with time of year was incorporated into the model. The calculated saturation DO at any time is used as an estimate of the DO concentration of tributary inputs to the lake.

CHAPTER 4

SYNTHESIS OF DATA BASE FOR MODEL COMPARISON

The data base obtained from CCIW included measurements of the concentrations of several chemical species in the lake. These were presented in much the same format as the temperature data previously discussed. Among these species were dissolved oxygen and three forms of phosphorus = total P, soluble reactive P and filtered P. Many more dissolved oxygen measurements were made than were measurements of phosphorus. For phosphorus, most measurements are as total P and soluble reactive P. Model predictions can only be compared to these data if it is assumed that stable orthophosphorus (OP) corresponds to soluble reactive P. Hence, any comparisons between actual and predicted phosphorus concentrations in the lake will be restricted to total phosphorus.

The available oxygen and phosphorus data was transformed into lakewide averages for the winter circulation period. Epilimnetic and hypolimnetic averages were calculated for the summer stratification period. This observed data was then analyzed for significant fluxes of dissolved oxygen.

4.1 CALCULATION OF LAKEWIDE AVERAGES

The raw DO and P data supplied by CCIW was in the form of a measurement and an associated depth for a number of sampling locations in the lake during each cruise. As for the temperature data, the DO and P data were arranged into specific depth slices. An average concentration and depth was calculated for each slice. With the aid of a hyposometric chart, which provided the portion of the total lake volume contained in each slice, a volumetric average over the entire lake was calculated for each species. The procedure is illustrated in Figure 4-1. For the stratification period, epilimnetic and hypolimnetic averages were determined using the same technique. The equations of epilimnetic depth versus time developed by this researcher were used to establish the division between the two boxes. A four month stratification period was selected, starting in mid-June and running to mid-October.

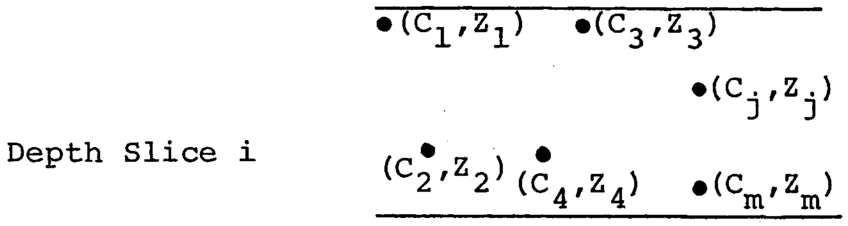
For each survey year (1966-1974), lakewide averages of dissolved oxygen and phosphorus were established. These were used for direct comparison of concentrations (observed vs model predictions) and also to calculate actual fluxes occurring in the lake for comparison with those predicted by the model.

These lakewide averages, henceforth referred to as observed data, are presented in Tables D-1 to D-9 contained

FIGURE 4-1

SYNTHESIS OF DATA INTO LAKEWIDE AVERAGES

i) FOR EACH INDIVIDUAL SLICE



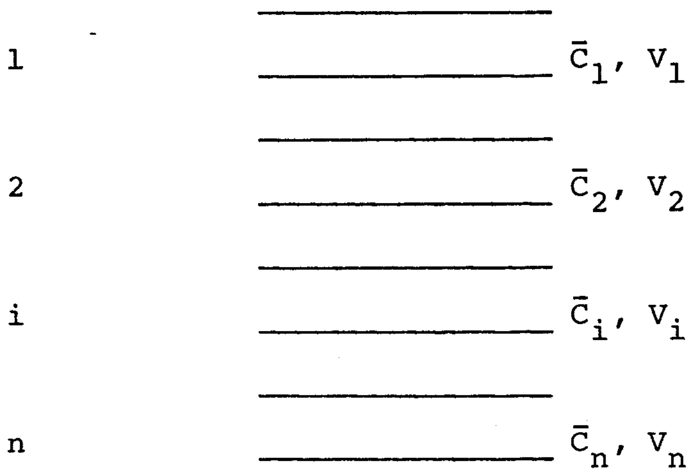
m = individual pairs of C and Z within a slice

$$\bar{C}_i = \frac{\Sigma C_j}{m}$$

$$\bar{Z}_i = \frac{\Sigma Z_j}{m}$$

ii) FOR ENTIRE LAKE, EPILIMNION, HYPOLIMNION

n Depth Slices



$$\left[\begin{array}{l}\text{Lake Wide}\\ \text{Concentration}\end{array}\right] = \frac{\Sigma \bar{C}_i V_i}{\Sigma V_i}$$

in Appendix D. The equations of lake volume versus depth developed from the hyposometric chart are shown below with those for interfacial area:

For $0 < Z \leq 34$

$$A = (-126\ Z + 18300) \times 10^6 \qquad (4.1\text{-}1)$$

$$V = (-63\ Z^2 + 18300\ Z) \times 10^6 \qquad (4.1\text{-}2)$$

For $34 < Z \leq 182$

$$A = (-86.4Z + 16920) \times 10^6 \qquad (4.1\text{-}3)$$

$$V = (-43.2\ Z^2 + 16920\ Z) \times 10^6 \qquad (4.1\text{-}4)$$

For $Z > 182$

$$A = (-41.4\ Z + 9100) \times 10^6 \qquad (4.1\text{-}5)$$

$$V = (-20.7\ Z^2 + 9100\ Z) \times 10^6 \qquad (4.1\text{-}6)$$

Area equations are obtained directly from the hyposometric chart (Figure 3-1). Volume equations are integrations of the associated area equation with respect to depth.

4.2 DISSOLVED OXYGEN FLUXES CALCULATED FROM OBSERVED DATA

The nature of dissolved oxygen fluxes in Lake Ontario was examined by analyzing the observed data developed in section 4.1. The vertical exchange flux between the hypolimnion and epilimnion, the decomposition flux of particulate phosphorus in the hypolimnion and the flux of oxygen between the atmosphere and the epilimnion are examined. Due to paucity of DO data and also $\hat{k}_{th}$ values for the years 1971, 1973 and

1974, the analysis of fluxes was restricted to the years 1966-1970 and 1972.

4.2.1 Total Oxygen Depletion in the Hypolimnion

The total mass of oxygen consumed in the hypolimnion during a period of time approximating stratification was determined from the observed data. Estimates were then made of the rate of depletion or flux of dissolved oxygen from the hypolimnion in the following manner. Plots of hypolimnetic dissolved oxygen concentration versus time of the year (during stratification) were made from the observed data. Linear estimates were drawn through the points. A time period, Δt, over which depletion occurs was selected such that extrapolation would be kept to a minimum and such that it would be close to the time period selected earlier for the calculation of $\hat{k}_{th}$ from temperature data (see Section 3.3.2).

The total depletion over the time period was used to calculate estimates of the flux of dissolved oxygen.

$$\text{Flux of DO} = \frac{[DO_h(o) - DO_h(t) \times \bar{V}_h]}{\bar{A}_{th} \times \Delta t} \qquad (4.2\text{-}1)$$

$$\frac{\bar{V}_h}{\bar{A}_{th}} = \bar{Z}_h = \text{average depth of the hypolimnion} \qquad (4.2\text{-}2)$$

Thus equation 4.2-1 can be simplified to:

$$\text{DO Flux} = [DO_h(o) - DO_h(t)] \times \frac{\bar{Z}_h}{\Delta t} \qquad (4.2\text{-}3)$$

The average depth of the hypolimnion, $\bar{Z}_h$, is calculated at the midpoint of the time period.

Yearly plots of hypolimnetic dissolved oxygen versus time and calculations are presented in Appendix E. The results, for the years 1966-1970 and 1972 are presented in Table 4-1.

4.2.2 Vertical Exchange Across the Thermocline Flux

On the same plots of hypolimnetic DO versus time used in Section 4.2.1 (Appendix E, Figures E1 to E6), epilimnetic DO versus time is plotted so that estimates of the flux of oxygen across the thermocline could be determined. This vertical exchange flux is calculated over the same time period as the total flux. To simplify calculations, linear estimates were made for epilimnetic DO versus time. Observation of the data points however shows that two linear estimates are required to approximate the data over the entire time period. This results in the formation of two smaller time periods which together make up the total time.

Calculation of the vertical exchange flux requires that the driving force of oxygen concentrations between the hypolimnion and epilimnion be determined. Thus average DO_h and DO_e were estimated for each of the smaller time segments by selecting the concentration at the midpoint of each segment. The flux was then determined as follows:

$$\Delta t = \Delta t_1 + \Delta t_2 \qquad (4.2\text{-}5)$$

For Δt_1:

$$\text{Vertical Exchange Flux} = \hat{k}_{th}(\overline{DO}_{h1} - \overline{DO}_{e1}) \qquad (4.2\text{-}6)$$

$$\text{or } VEF_1 = \hat{k}_{th}(\overline{DO}_{h1} - \overline{DO}_{e1}) \qquad (4.2\text{-}6)$$

TABLE 4-1

SUMMARY OF DISSOLVED OXYGEN FLUXES IN THE HYPOLIMNION ESTIMATED FROM OBSERVED DATA

YEAR	TOTAL FLUX	VERTICAL EXCHANGE FLUX	SEDIMENT DEMAND FLUX	DECOMPOSITION FLUX
1966	0.53	0.29	0.10	0.14
1967	1.37	0.18	0.10	1.09
1968	1.10	0.12	0.10	0.88
1969	1.50	0.16	0.10	1.24
1970	1.51	0.19	0.10	1.22
1972	1.69	0.18	0.10	1.41

Note: Sediment Demand Flux assumed constant at 0.1.
Units in all cases: g O_2/m^2-day.

Similarly for Δt_2: $VEF_2 = \hat{k}_{th}(\overline{DO}_{h2} - \overline{DO}_{e2})$ (4.2-7)

Over the total time period Δt : $\text{Vertical Exchange} = VEF_1\left(\frac{\Delta t_1}{\Delta t}\right) + VEF_2\left(\frac{\Delta t_2}{\Delta t}\right)$ (4.2-8)

The observed vertical exchange fluxes calculated in the above manner are presented in Table 4-1.

4.2.3 The Sediment Oxygen Demand Flux

The exact value of the sediment oxygen demand flux cannot be estimated from the observed data. Hence the value 0.1 g/m^2-day, used in the model, will also be used to estimate the observed flux. It is assumed that this is a constant term.

4.2.4 The Decomposition Flux

The flux of dissolved oxygen from the hypolimnion due to the decomposition reaction can be estimated using the following relationship:

$$\begin{matrix}\text{Total Flux}\\ \text{of DO}\end{matrix} = \begin{matrix}\text{Vertical Exchange}\\ \text{Flux}\end{matrix} + \begin{matrix}\text{Sediment Demand}\\ \text{Flux}\end{matrix} + \begin{matrix}\text{Decomposition}\\ \text{Flux}\end{matrix} \quad (4.2\text{-}9)$$

Decomposition fluxes are estimated by difference. Since the sediment demand flux has not been calculated from observed data but is merely estimated from the literature, the decomposition fluxes are not to be considered accurate values but simply "best estimates". These are also presented in Table 4-1.

4.2.5 The Reaeration Flux

The variation of saturation dissolved oxygen with time for the stratified period as determined in Section 3.3.3 is presented in each of Figures E1 to E6. The saturation DO is less than the epilimnetic DO for approximately the first three months of stratification in all years. This implies that there is a net flux of oxygen from the epilimnion to the atmosphere during this period of time.

4.3 SUMMARY OF OBSERVED HYPOLIMNETIC DO FLUXES

Table 4-1 is a summary of the hypolimnetic DO fluxes calculated from observed data. Most noteworthy are the results for 1966. The total flux of 0.53 g O_2/m^2-day is more than 50 per cent less than the next lowest value (1968). The vertical exchange flux of 0.29 g O_2/m^2-day is more than 50 per cent higher than the next highest value (1970). And finally, the estimated decomposition flux is less than one-sixth of the next lowest value (1968).

CHAPTER 5

SOLUTION, CALIBRATION AND VERIFICATION OF THE MODEL

The final model form as developed in Chapter 2, including selected model parameters is solved numerically using computer techniques. A sensitivity analysis is performed to determine the major factors affecting model predictions. This analysis is used to calibrate the model to summer stratification conditions in 1966 and winter circulation conditions in 1969. The changes made are summarized and then the model is used to predict conditions in the other years of the data base. In addition, dissolved oxygen fluxes in the hypolimnion predicted by the model are compared to those observed by analysis of the actual data (see Chapter 4) for further model verification.

5.1 SOLUTION OF MODEL EQUATIONS

The model is in the form of six simultaneous, interdependent, linear differential equations for the stratification period. The number reduces to three during the winter circulation period. Section 2.5 presents the model in matrix form. This form particularly lends itself to a clear comprehension of the numerical solution technique performed by the computer.

The numerical integration technique used to solve the equations is that used by Snodgrass (1974) for his summer model solution. This method was originally developed by Di Toro *et al.* (1972) for the solution of a phytoplankton model. Numerical integration involves starting from an initial or boundary condition, dividing the time scale into time step sizes, Δ, estimating the slope of the true temporal concentration curve over the step size by linear extrapolation and calculating the concentration at the end of the step size. The above procedure is repeated for every time step. This method employed herein utilizes a second order two step Runge method involving an Euler predictor step and a half-step corrector. The method is outlined below:

Half Step Corrector $DO^1 = DO_o + \frac{\Delta}{2} * f\ (t_o, DO_o)$

Solution After Δ Time Step $DO_1 = DO_o + \Delta * f(t_o + \frac{\Delta}{2}, DO^1)$

where DO_o = initial concentration vector at t_o

DO^1 = half step concentration vector

DO_1 = concentration vector after $t_o + \Delta$

The combined winter/summer model was programmed in Fortran IV and solved on a CDC 6400 system computer. The numerical formulation is shown in the computer program in Appendix F.

The initial conditions used were selected by observing actual field data for Lake Ontario and are summarized below:

DO_e = 13.1 mg/l (g/m^3)
DO_h = 13.1 mg/l
PP_e = 4 μg/l (mg/m^3)
PP_h = 4 μg/l
OP_e = 20 μg/l
OP_h = 20 μg/l

The modelling procedure was commenced at the beginning of stratification for 1966.

5.2 SENSITIVITY ANALYSIS

A sensitivity analysis of the model parameters can provide valuable information for subsequent calibration procedures. This analysis establishes the response of the model predictions to changes in one or more parameters. For this DO model, the parameters of interest are the physical, chemical and biological reaction coefficients and, in addition, the external phosphorus loading. Snodgrass and O'Melia (1975) performed an extensive analysis of the sensitivity of the phosphorus portion of the model. The result of their analysis will be summarized. Sensitivity in this work centered on the dissolved oxygen portion of the model. Model sensitivity is quantitatively measured by the per cent increase or decrease in concentration from that at standard conditions caused by a change in the value of a parameter. Concentrations are compared after 2, 60, and 120 days of stratification.

The initial selection of model parameters is summarized below. This set of conditions represents the "base" case against which the results of changes in these conditions will be compared.

Parameter	Value
$\hat{k}_{th}$	$0.142 \frac{m}{day}$ (1966)
p_e	$0.2 \quad day^{-1}$
p_{eu}	$0.06 \quad day^{-1}$
d_h	$0.03 \quad day^{-1}$
FAC	$0.142 \frac{mg\ O_2/l}{\mu g\ P/l}$
k_s	$0.1 \quad \frac{g\ O_2}{m^2\text{-}day}$
k_a	$7.5 \quad \frac{m}{day}$ (all year)
g_e	$0.1 \quad \frac{m}{day}$
g_o	$0.05 \quad \frac{m}{day}$
f	$0.05 \quad m^{-1}$
PHOSLD (Phosphorus Loading)	$3.41 \times 10^{10} \frac{mg}{day}$

The boundary conditions of DO, PP and OP are summarized at the end of section 5.1.

There are several lake and system characteristics that are allowed to change with time during the solution of the model equations. Since Z_e, V_e, Z_h, V_h, and saturation DO change with time, the first sensitivity test conducted was to check whether the frequency of re-evaluation would affect the results of the mathematical solution. Thus, these values were changed every day in one run and every two days in a second run and the results compared. The comparison is shown in Table 5-1. The value of Δ, the integration time step was kept constant at 0.25 days.

The figures in Table 5-1 indicate negligible effect on concentrations by changing the re-evaluation frequency.

Snodgrass and O'Melia (1975) found that the phosphorus model predictions of total phosphorus concentrations in the spring are sensitive to the following parameters in order of decreasing importance.

Order	Symbol	Parameter
1	L_a	land based areal phosphorus loading
2	g(or g_h)	sedimentation coefficient
3	Q_a	areal rate of total water input or discharge
4	r(or r_h)	decomposition coefficient
5	p_{eu}	production coefficient, euphotic zone
6	$\bar{z}_{eu}$	depth of euphotic zone
7	$\hat{k}_{th}$	vertical exchange coefficient

In addition, they demonstrated that the net production coefficient in the epilimnion, p_e, and the sedimentation coefficient in the epilimnion, g_e, have negligible effects on

TABLE 5-1

COMPARISON OF MODEL CONCENTRATION PREDICTIONS WITH VARIABLE RE-EVALUATION FREQUENCY DURING STRATIFICATION

TIME OF YEAR	RE-EVALUATION FREQUENCY (days)	CONCENTRATION OF SPECIES					
		PPE (μg/l)	PPH (μg/l)	OPE (μg/l)	OPH (μg/l)	DOE (mg/l)	DOH (mg/l)
5.57	1.0	10.38	3.77	13.9	20.22	11.65	13.06
	2.0	10.37	3.77	13.9	20.22	11.69	13.06
6.50	1.0	23.26	2.51	2.14	21.94	9.66	12.56
	2.0	23.25	2.50	2.16	21.94	9.69	12.56
7.50	1.0	24.53	2.18	1.47	22.96	9.13	12.07
	2.0	24.53	2.18	1.48	22.96	9.13	12.08
8.50	1.0	25.43	2.15	1.13	23.74	9.55	11.62
	2.0	25.43	2.15	1.13	23.74	9.54	11.62
9.50	1.0	26.20	2.22	0.92	24.47	10.62	11.22
	2.0	26.20	2.21	0.92	24.47	10.59	11.23

total phosphorus concentrations when changed over several orders of magnitude. The net production coefficient, p_e, does however affect the balance between the two phosphorus compartments, OP and PP in the epilimnion during the initial stages of stratification. At the beginning of stratification, the concentration of OP is much higher than that of PP. As production proceeds, this balance shifts to the point where the PP fraction represents virtually all the phosphorus in the epilimnion. The value of p_e regulates the time over which this transition takes place. Doubling p_e from 0.2 day^{-1} to 0.4 day^{-1} had the effect of halving the time required for PP in the epilimnion to reach its maximum concentration. As might be expected, the sedimentation coefficient, g_e, has its greatest effect towards the end of stratification when PP_e is very high. A 50 per cent decrease and a 100 per cent increase in the value of g_e increased PP_e by 17% and decreased PP_e by 24% respectively at levels of approximately 20 μg/l at the standard value of g_e. The simultaneous effects on PP_h produced by these changes were less than 10 per cent at levels of less than 2 μg/l at the standard g_e.

The sensitivity of the dissolved oxygen model was analyzed by increasing and then decreasing a particular parameter by a factor of two (i.e., 100 per cent increase and 50% decrease) while keeping all others at the standard condition. Dissolved oxygen concentrations in both the epilimnion and hypolimnion were compared with those existing under the standard conditions at periods of 2, 60, and 120 days after the beginning of stratification. The results are shown in Tables 5-2, 5-3 and 5-4 respectively. None of the changes made caused any significant change in the dissolved oxygen concentration in the lake at the end of the winter circulation period.

Based on the results presented in Tables 5-2 to 5-4, the following statements can be made concerning the sensitivity of the dissolved oxygen model:

1. At the base values selected for the parameters, and initial DO conditions, the model dissolved oxygen predictions are relatively insensitive (less than 10% change) to a doubling or halving of the model parameters throughout stratification.

2. Model predictions of hypolimnetic oxygen concentrations <u>at the end of stratification</u> are sensitive, in decreasing order of significance to the following parameters:

1.	FAC	DO/P factor
2.	$\hat{k}_{th}$	Vertical exchange coefficient
3.	d_h	Decomposition coefficient for hypolimnion
4.	k_s	Sediment oxygen demand

   The model is insensitive to the other parameters.

3. Epilimnion dissolved oxygen concentrations are insensitive to all the parameters <u>at the end of stratification</u>.

4. Hypolimnetic dissolved oxygen concentrations are insensitive to all the parameters <u>at the beginning of the stratification period</u>.

TABLE 5-2

SENSITIVITY ANALYSIS OF MODEL AFTER 2 DAYS OF STRATIFICATION

SPECIES	% CHANGE IN CONCENTRATION FROM STANDARD CONDITIONS											
	$\hat{k}_{th}$		p_e		d_h		FAC		k_s		k_a	
	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%
EPILIMNETIC DO	-.1	+.2	-1	+2	0	0	-1	+3	0	0	+5	-3
HYPOLIMNETIC DO	0	0	0	0	+.2	-.2	+.2	-.2	0	0	0	0
EPILIMNETIC PP	+7	-1	-28	+42	+.1	0	0	0	0	0	0	0
HYPOLIMNETIC PP	0	+.3	-.3	+1	+3	-6	0	0	0	0	0	0
EPILIMNETIC OP	-5	+.9	+21	-32	-.1	0	0	0	0	0	0	0
HYPOLIMNETIC OP	+.1	+.1	+.1	-.1	-1	+1	0	0	0	0	0	0

SPECIES	CSEP		CSAF		CFLOC		PHOSLD	
	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%
EPILIMNETIC DO	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.2
HYPOLIMNETIC DO	0	0	0	0	0	0	0	0
EPILIMNETIC PP	+1.1	-20	0	0	0	0	-0.5	+1.0
HYPOLIMNETIC PP	-0.3	+0.5	+0.3	-0.5	+0.3	-0.5	0	0
EPILIMNETIC OP	0	0	0	0	0	0	-1.7	+3.4
HYPOLIMNETIC OP	0	0	0	0	0	0	0	0

TABLE 5-3

SENSITIVITY ANALYSIS OF MODEL AFTER 60 DAYS OF STRATIFICATION

SPECIES	% CHANGE IN CONCENTRATION FROM STANDARD CONDITION											
	k_{th}		p_e		d_h		FAC		k_s		k_a	
	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%
EPILIMNETIC DO	-1	+.1	0	0	0	0	-.4	1.1	0	0	+1.5	.8
HYPOLIMNETIC DO	+1	-3	+.3	-.2	+2	-1	+2	-5	+.3	-.6	+.2	-.1
EPILIMNETIC PP	-1	-.1	-4	+2	+.1	+.1	0	0	0	0	0	0
HYPOLIMNETIC PP	-19	+36	-6	+3	+65	-51	0	0	0	0	0	0
EPILIMNETIC OP	-28	+51	+102	-50	-3	+3	0	0	0	0	0	0
HYPOLIMNETIC OP	+2	-4	+.2	-.1	-6	+5	0	0	0	0	0	0

SPECIES	CSEP		CSAF		CFLOC		PHOSLD	
	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%
EPILIMNETIC DO	0	0	0	0	0	0	-0.2	+0.3
HYPOLIMNETIC DO	+0.2	-0.4	-0.2	+0.2	-0.2	+0.2	+0.1	-0.2
EPILIMNETIC PP	+14.5	-21.7	+0.3	-0.3	+0.3	-0.2	-13.3	+26.9
HYPOLIMNETIC PP	-8.2	+11.3	+5.2	8.8	+4.1	-7.2	-7.7	+15.5
EPILIMNETIC OP	-0.7	+0.7	0	-0.7	0	-0.7	-20.4	+39.4
HYPOLIMNETIC OP	-1.0	+1.4	+0.5	-1.0	+0.4	-0.9	-0.7	1.3

TABLE 5-4

SENSITIVITY ANALYSIS OF MODEL
AFTER 120 DAYS OF STRATIFICATION

SPECIES	% CHANGE IN CONCENTRATION FROM STANDARD CONDITION											
	$\hat{k}_{th}$		pe		d_h		FAC		k_s		k_a	
	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%
EPILIMNETIC DO	-.3	+.4	0	0	0	0	-.4	-.7	0	0	-.3	+.2
HYPOLIMNETIC DO	+3	-6	+.4	-.2	+2	-1	+4.2	+4.3	+.7	-1.3	+.3	-.1
EPILIMNETIC PP	-3	+3	-2	+.7	-.3	+.3	0	0	0	0	0	0
HYPOLIMNETIC PP	-28	+51	-3	+1	+74	-54	0	0	0	0	0	0
EPILIMNETIC OP	-28	+50	+101	-50	-4	+3	0	0	0	0	0	0
HYPOLIMNETIC OP	+3	-1.5	+.1	-.1	-7	+5	0	0	0	0	0	0

SPECIES	CSEP		CSAF		CFLOC		PHOSLD	
	-50%	+100%	-50%	+100%	-50%	+100%	-50%	+100%
EPILIMNETIC DO	+0.3	+0.3	+0.3	+0.3	+0.3	+0.2	+.1	+.6
HYPOLIMNETIC DO	+.6	-.8	-.4	+0.6	-.3	+.5	+.6	-1.2
EPILIMNETIC PP	+16.8	-23.6	+.5	-.8	+.4	-.7	-16.9	+33.8
HYPOLIMNETIC PP	-7.8	+8.4	+5.6	-9.0	+4.5	-6.7	-14.6	+30.3
EPILIMNETIC OP	-2.2	+1.1	0	-2.2	0	-2.2	-22.0	+41.8
HYPOLIMNETIC OP	-1.9	+2.6	+1.2	-2.1	+.9	-1.6	-2.2	+4.5

5. Epilimnetic dissolved oxygen concentrations are sensitive to the following parameters <u>at the beginning of stratification</u> (in decreasing order of significance):

1.	k_a	Reaeration coefficient
2.	FAC	DO/P factor
3.	p_e	Net Production coefficient

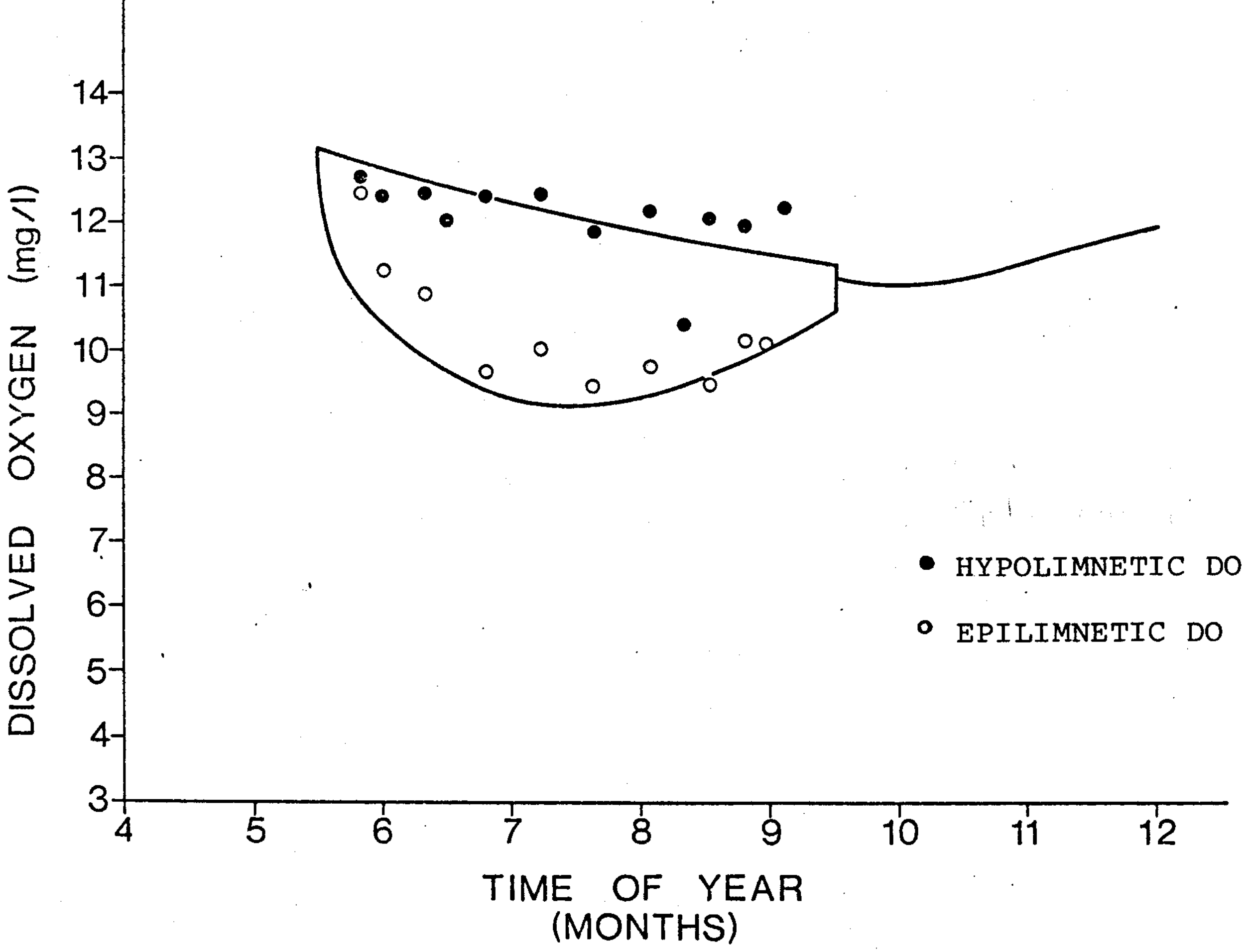
5.3 MODEL CALIBRATION

The model was calibrated with respect to dissolved oxygen concentrations observed during the 1966 stratification period and the 1969 circulation period. The lack of winter circulation data for 1966 precluded the use of this year for the entire model calibration. The year 1969 was selected for calibration during the winter period due to the quantity of data available.

The initial run at standard or "base" conditions of the stratification parameters indicated that the predicted dissolved oxygen was much lower than the observed epilimnetic dissolved oxygen during most of the stratification period. This is illustrated in Figure 5-1. In addition, it was noted that the rate of epilimnetic oxygen depletion in the initial stages of stratification was higher than the trend of observed data. As discussed in section 4.2.5, there is a net transport of oxygen from the epilimnion to the atmosphere during this period since $DO_e >$ saturation DO. If k_a, the reaeration-coefficient, is too high for this period, dissolved oxygen

FIGURE 5-1

INITIAL CALIBRATION RUN—1966 STRATIFICATION
DISSOLVED OXYGEN VS TIME



is lost too fast from the epilimnion. However, model predictions show good agreement with the observed results for the 1969 circulation period. These results imply that k_a = 7.5 m/day is appropriate for the circulation period as the calibration value but must be reduced during stratification.

The development of the reaeration coefficient in section 2.4.2 justifies the reduction of k_a during stratification since the mean depth of the epilimnion is much smaller than the mean depth of the lake. The mean depth of the lake is 89 meters while the mean depth of the epilimnion is approximately 17 meters over the stratification period (IJC, 1969). Since k_a is the product of the Streeter-Phelps coefficient K_2 and the mean depth of the box, the value of k_a should be reduced during stratification by approximately the factor of 17/89. However, the average surface temperature of the lake is approximately 20°C during stratification and 10°C during the circulation period. This temperature difference results in a higher rate of reaeration in the summer. The temperature dependence is normally formulated as

$$\frac{K_T}{K_{20}} = \theta^{T-20} \qquad (5.3\text{-}1)$$

where 20 - the reference temperature (°C)

T = any other temperature (°C)

θ = the temperature factor, and

K = any rate coefficient.

For the reaeration process, a common value of the correction factor θ is 1.025 (Rich, 1973). Application of the above relationship would increase k_a by approximately 25 per cent in the summer. Taking into consideration the above factors, a value for k_a of 2.1 m/day can be calculated. Based upon several runs, a value of 2.0 m/day was found to give the best fit for calibration.

The values selected for k_a can be converted to equivalent K_2 values at 20°C. For the summer and winter periods, respectively, the results are 0.12 and 0.11 day^{-1}. These values are within the range of literature values of K_2 for lakes and similar bodies of water presented in Table 3-3.

The above argument for the change in k_a is based on the physical change in the lake from a stratified to a non-stratified condition. The change in the biochemical reaction rates within the lake from summer to winter may however also affect the flux of oxygen.

However, since the reaeration coefficient is purely a physical quantity, its numerical value will be independent of concentration, and therefore will not be affected by the change in the production and decomposition rates.

The hypolimnetic dissolved oxygen concentrations predicted by the model exhibited close agreement with those observed in 1966. However the decomposition flux predicted by the model was far in excess of that estimated from observed data as indicated in Table 5-5. Since model predictions of hypolimnetic oxygen are insensitive to the reaeration

TABLE 5-5

SUMMARY OF PARAMETERS OF CALIBRATED MODEL

PARAMETER	VALUE
$\hat{k}_{th}$	$0.142\ \frac{m}{day}$ (1966)
p_e	$0.2\ day^{-1}$
p_{eu}	$0.06\ day^{-1}$
d_h	$0.03\ day^{-1}$
FAC	$0.142\ \frac{mg\ O_2/l}{\mu g\ P/l}$
k_s	$0.1\ \frac{g\ O_2}{m^2\text{-}day}$
k_a	$7.5\ \frac{m}{day}$ (circulation)
	$2.0\ \frac{m}{day}$ (stratification)
g_e	$0.1\ \frac{m}{day}$
g_o	$0.05\ \frac{m}{day}$
f	$0.05\ m^{-1}$
PHOSLD (Phosphorus Loading)	$3.41 \times 10^{10}\ \frac{mg}{day}$

coefficient, the difference between model predictions and observations for 1966 is not due to the change in reaeration coefficient. Since the observed decomposition flux in 1966 is much lower than for other years and since the predicted decomposition flux is in the same order of magnitude as those observed in other years, the decision was made to leave d_h unchanged for model verification.

The parameters of the calibrated model, summer and winter, are summarized in Table 5-5. The comparisons of the calibrated model predicted dissolved oxygen concentrations and actual data for the calibration periods are presented graphically in Figures 5-2 and 5-5.

5.4 MODEL VERIFICATION

Verification was conducted by obtaining model predictions of DO and P concentrations and DO fluxes from the beginning of stratification in 1966 through to the end of 1974 and comparing these predictions with the observed data. The vertical exchange coefficient, $\hat{k}_{th}$, and the epilimnion depth/time relationship calculated from temperature data are specific to each year and thus were changed annually. Otherwise, all calibrated model parameters are summarized at the end of section 5.3 were used and remained unchanged.

The phosphorus loading, PHOSLD, was also left unchanged. This may result in an over-prediction of total phosphorus due to the phosphorus controls imposed in Ontario at municipal plants releasing treated sewage to the Great Lakes. However, this writer decided to make predictions at this constant level of phosphorus loading as estimates of the temporal change of P loading over time are imprecise.

The emphasis of model verification was placed on concentrations and fluxes of DO due to two factors:

1. The phosphorus model had been previously verified for total phosphorus (Snodgrass, 1974),
2. There is a serious lack of data for the two compartments, ortho-phosphorus and particulate phosphorus used in the model.

Total phosphorus predictions are however compared to the available observed data for time scale of less than a year. The dissolved oxygen concentrations, predicted and observed, are also presented along with a discussion of predicted versus observed fluxes.

5.4.1 Verification of Dissolved Oxygen Model

The results of the model runs for each year are presented graphically in Figures 5-2 to 5-10. In these figures, the solid line represents the model prediction while the symbols "O", "X" and"⊗" represent actual observed data. The "O" represents epilimnetic average dissolved oxygen while the "X" represents hypolimnetic average dissolved oxygen in the stratification period. The "⊗" represents lakewide average dissolved oxygen in the circulation period.

FIGURE 5-2

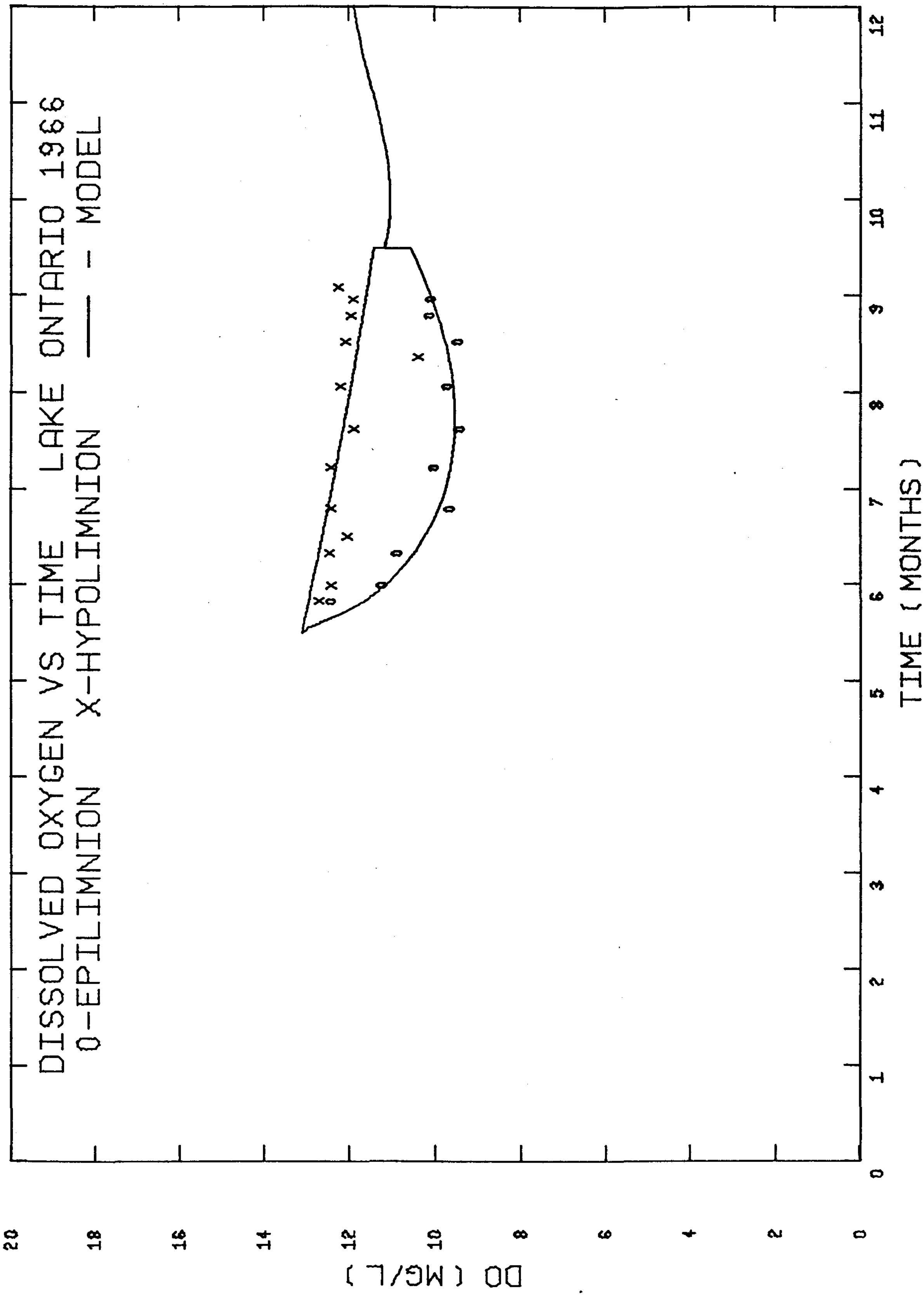

DISSOLVED OXYGEN VS TIME   LAKE ONTARIO 1966
0-EPILIMNION   X-HYPOLIMNION   —— - MODEL
DO (MG/L)
0
2
4
6
8
10
12
14
16
18
20
TIME (MONTHS)
0
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE 5-3

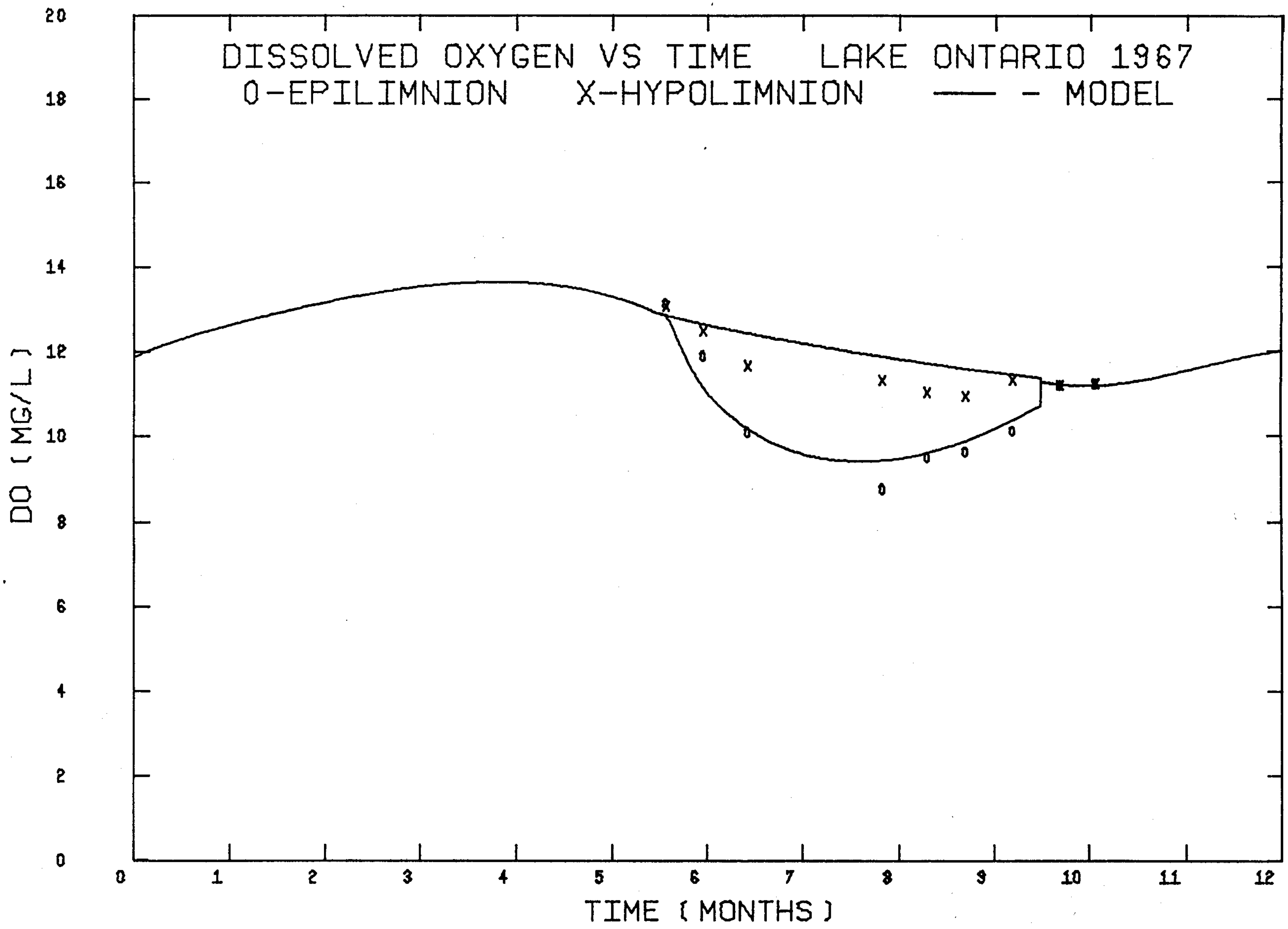

DISSOLVED OXYGEN VS TIME LAKE ONTARIO 1967
0-EPILIMNION X-HYPOLIMNION —— - MODEL
DO (MG/L)
0
2
4
6
8
10
12
14
16
18
20
TIME (MONTHS)
0
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE 5-4

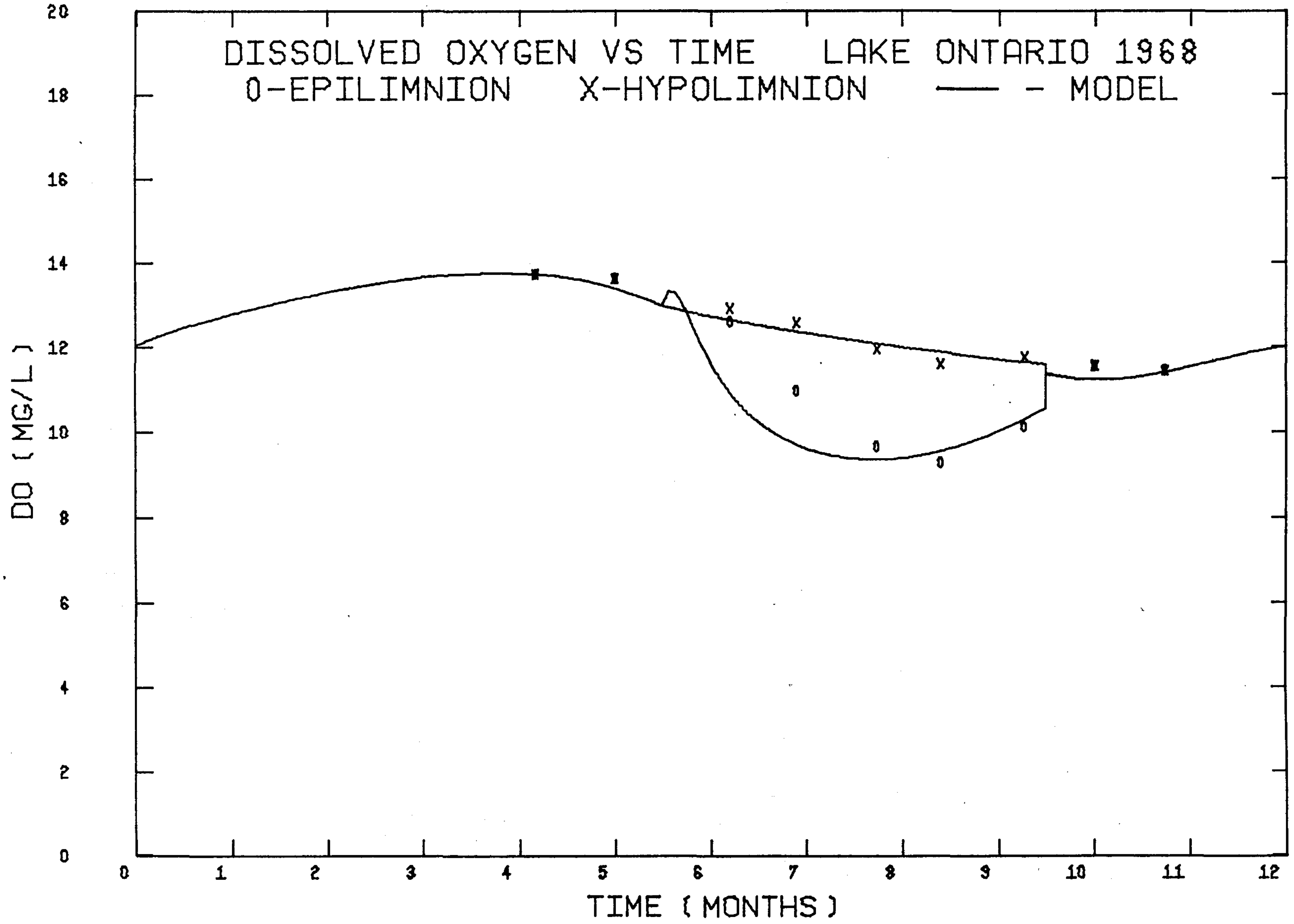

DISSOLVED OXYGEN VS TIME LAKE ONTARIO 1968
0-EPILIMNION X-HYPOLIMNION —— - MODEL
DO (MG/L)
TIME (MONTHS)
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE 5-5

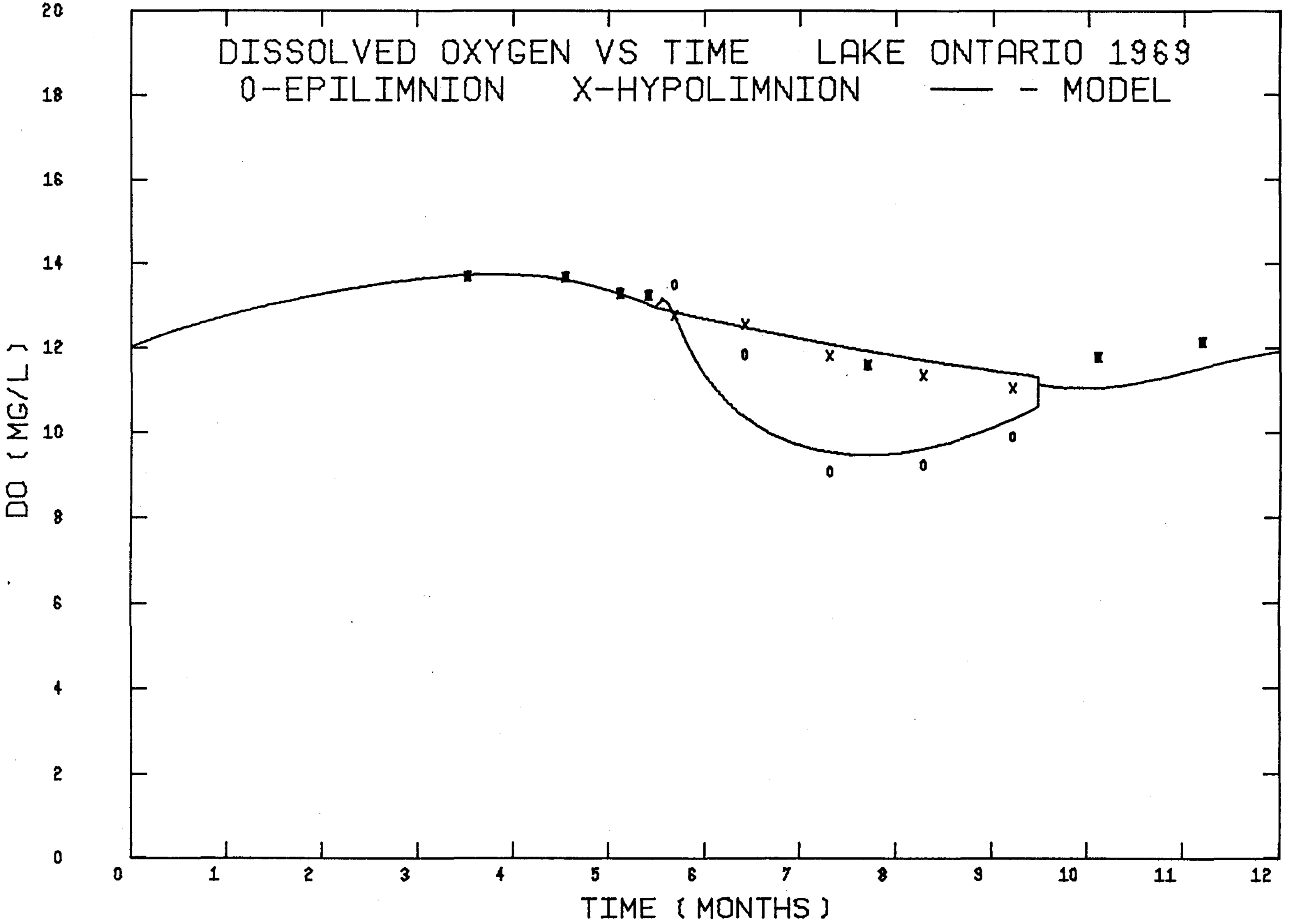

DISSOLVED OXYGEN VS TIME LAKE ONTARIO 1969
0-EPILIMNION X-HYPOLIMNION —— - MODEL
DO (MG/L)
0
2
4
6
8
10
12
14
16
18
20
TIME (MONTHS)
0
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE 5-6

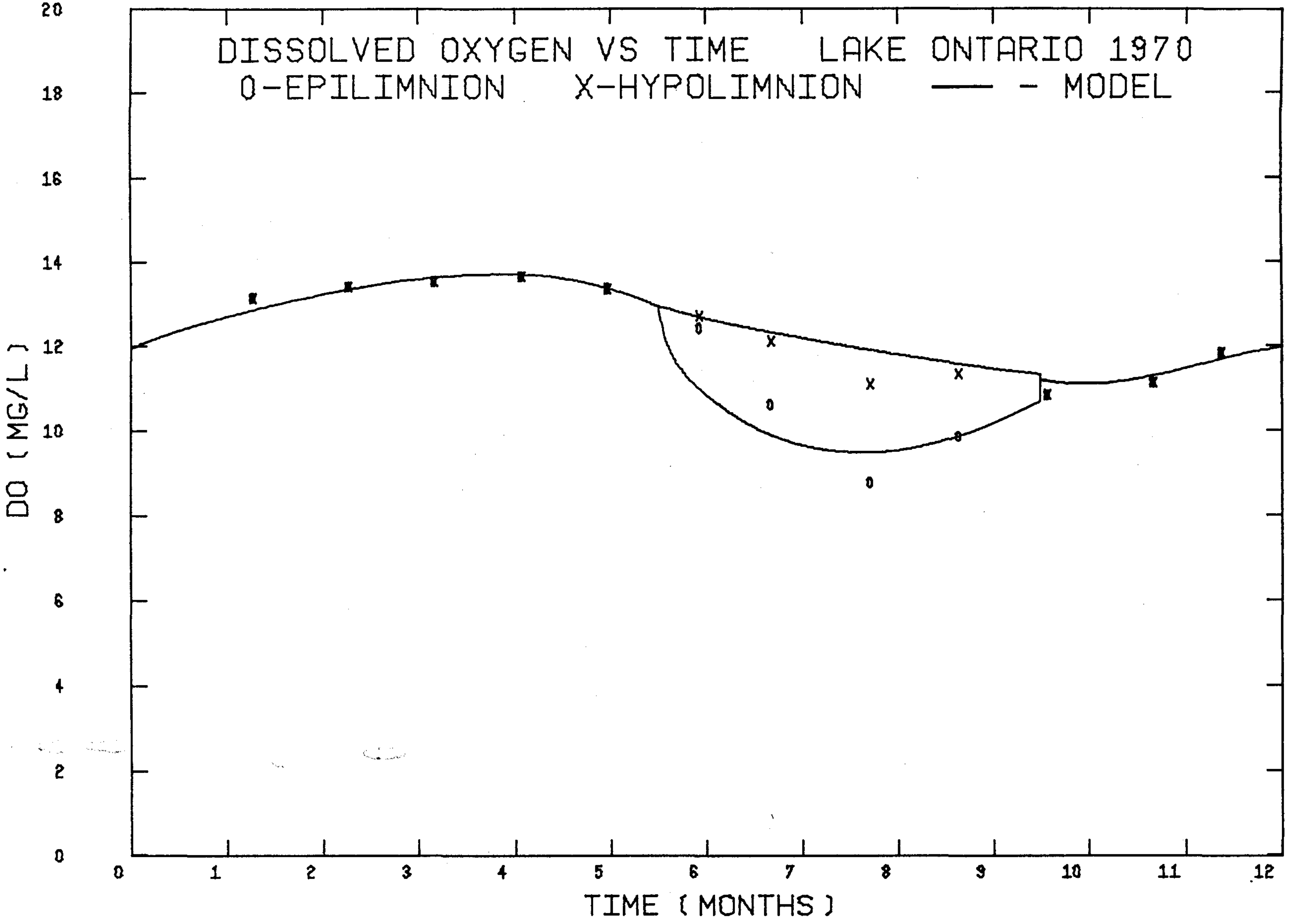

DISSOLVED OXYGEN VS TIME LAKE ONTARIO 1970
0-EPILIMNION X-HYPOLIMNION —— - MODEL
DO (MG/L)
0
2
4
6
8
10
12
14
16
18
20
TIME (MONTHS)
0
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE 5-7

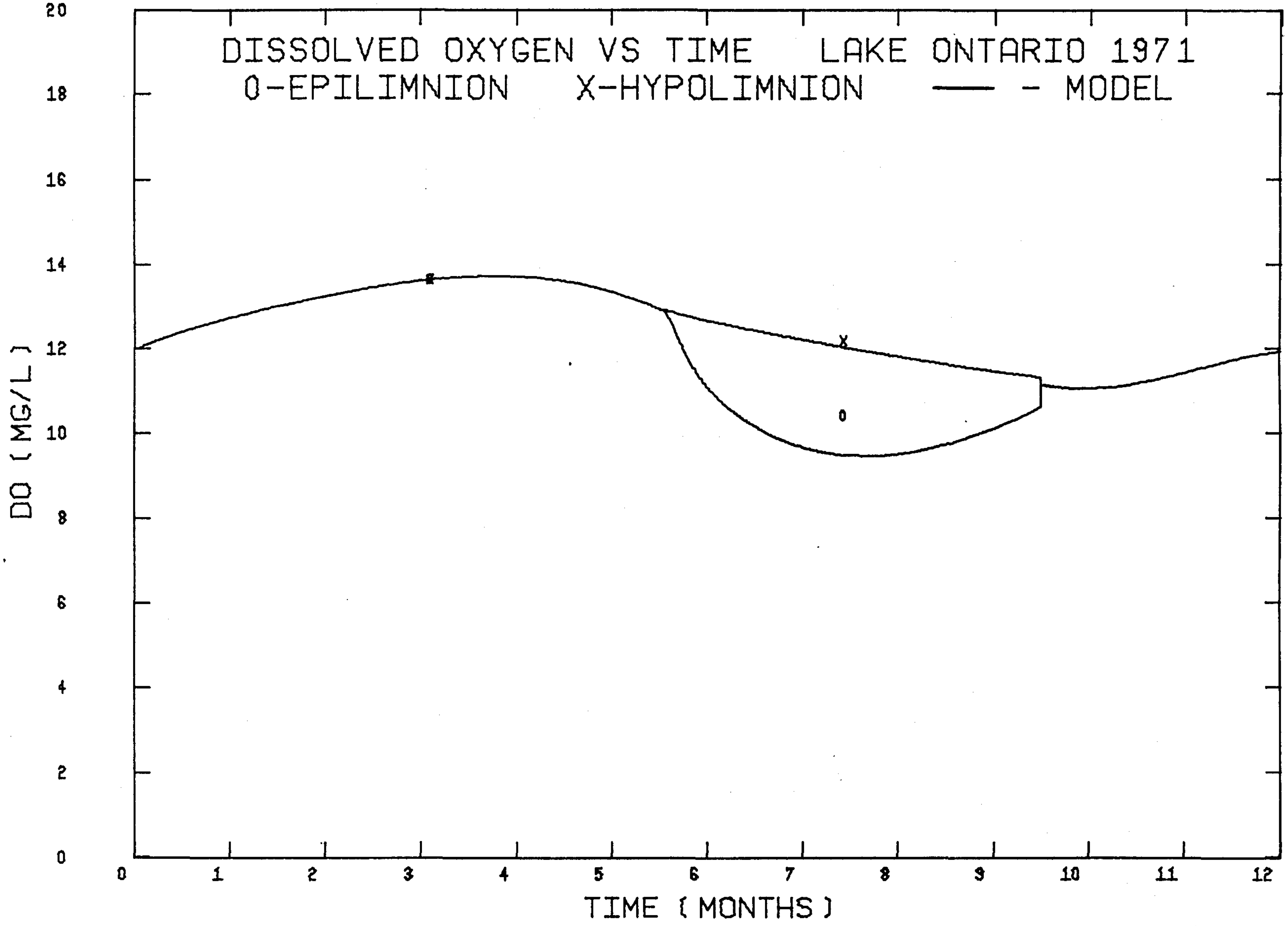

DISSOLVED OXYGEN VS TIME   LAKE ONTARIO 1971
0-EPILIMNION   X-HYPOLIMNION   —— - MODEL
DO (MG/L)
20
18
16
14
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
9
10
11
12
TIME (MONTHS)
X
0

FIGURE 5-8

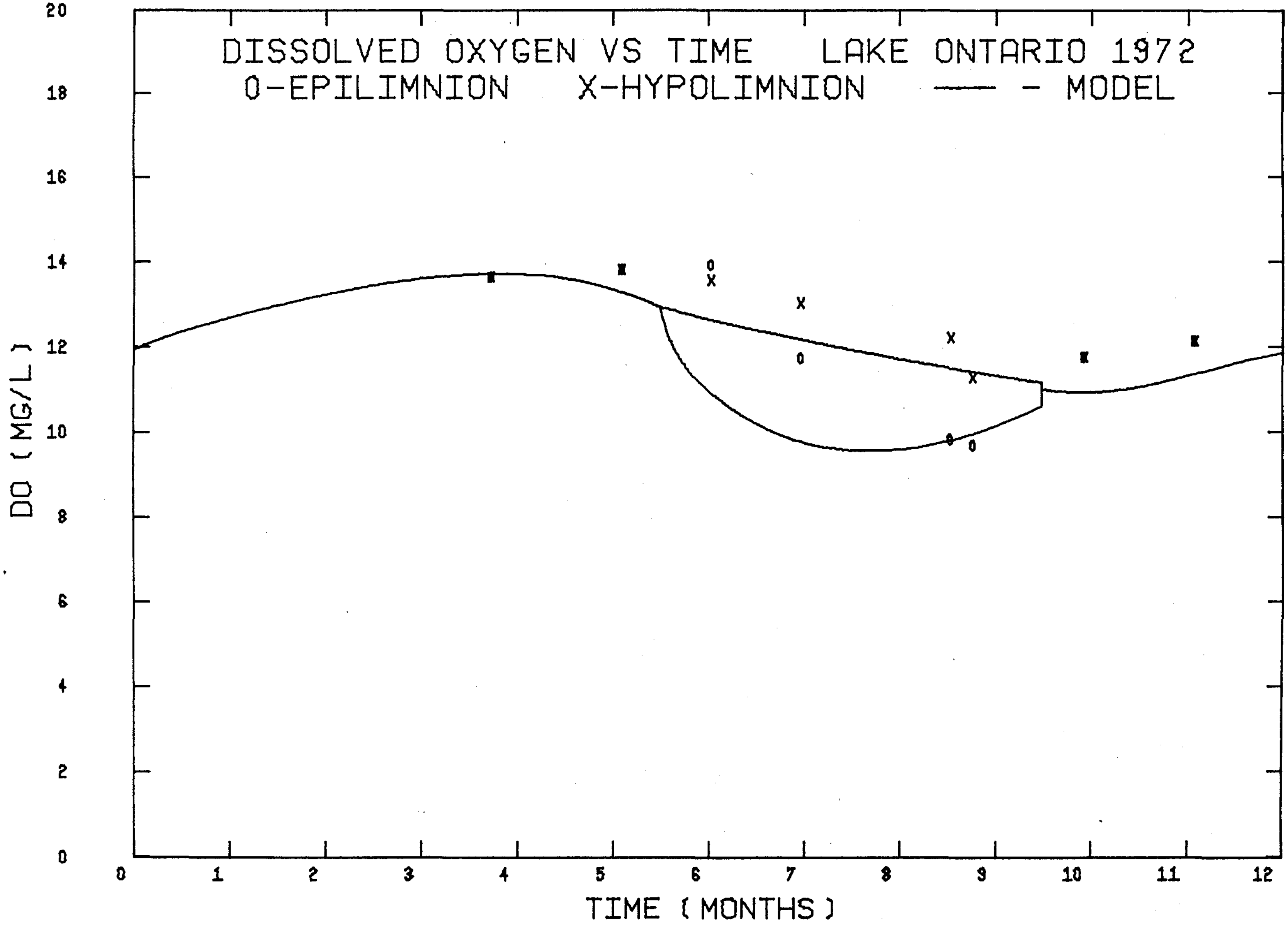

DISSOLVED OXYGEN VS TIME LAKE ONTARIO 1972
0-EPILIMNION X-HYPOLIMNION —— - MODEL
DO (MG/L)
TIME (MONTHS)
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE 5-9

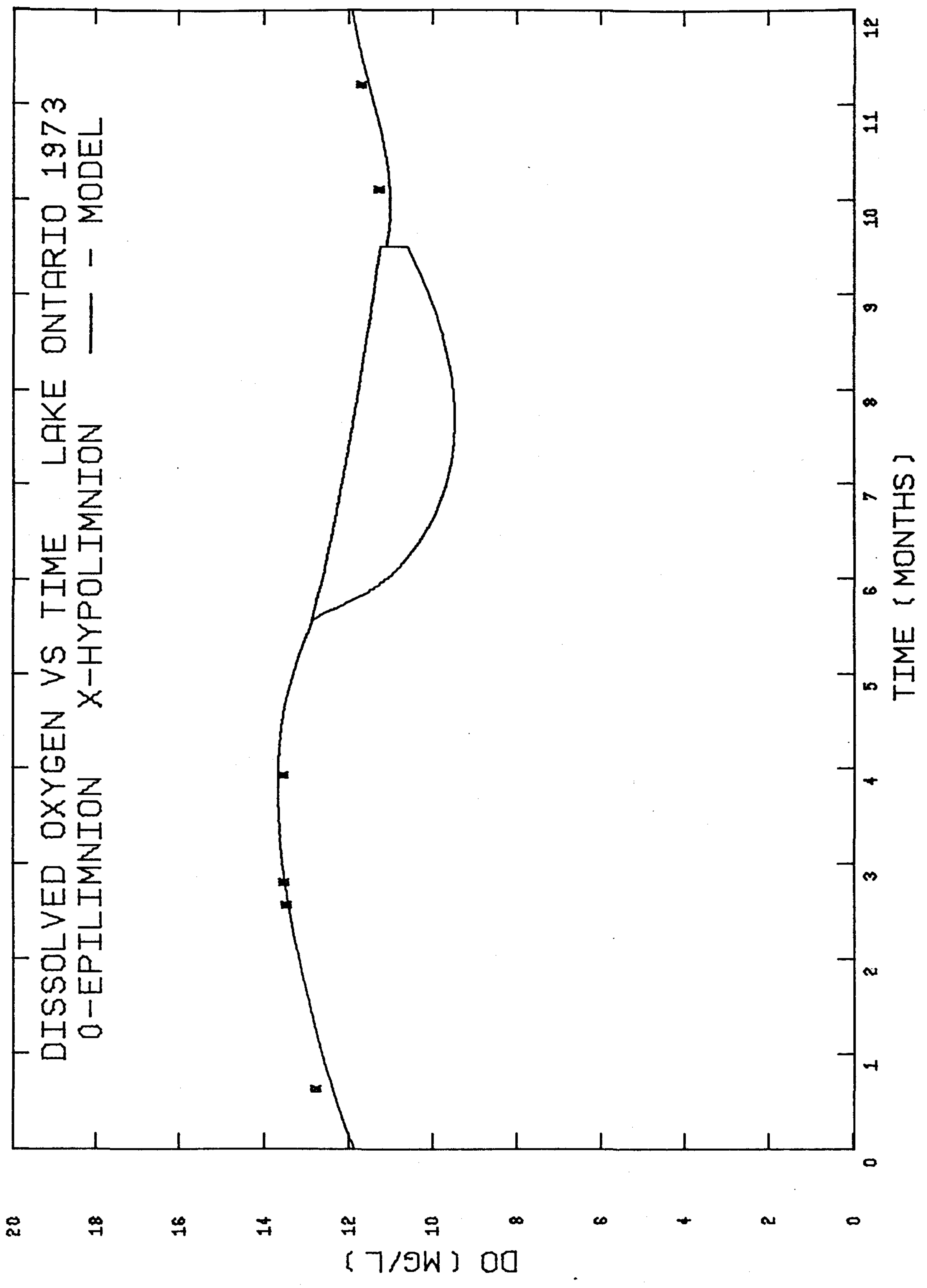
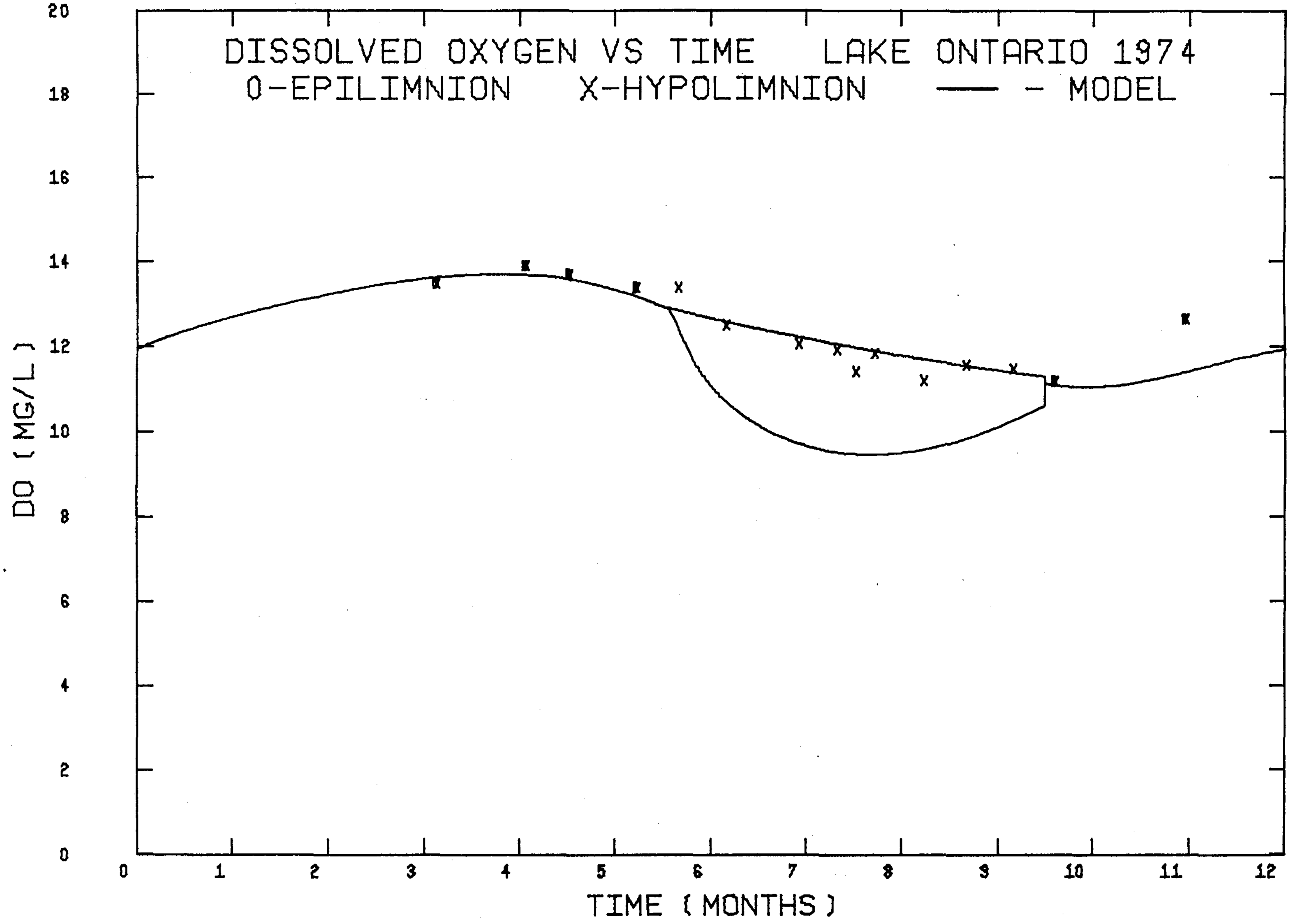

DISSOLVED OXYGEN VS TIME LAKE ONTARIO 1973
0-EPILIMNION X-HYPOLIMNION —— - MODEL
DO (MG/L)
0
2
4
6
8
10
12
14
16
18
20
TIME (MONTHS)
0
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE 5-10


DISSOLVED OXYGEN VS TIME LAKE ONTARIO 1974
O-EPILIMNION X-HYPOLIMNION —— - MODEL
DO (MG/L)
TIME (MONTHS)
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7
8
9
10
11
12

The model shows very good agreement with the observed data when one considers the entire dissolved oxygen data base. Taking each year on its own, agreement is very good in all years except 1968 and 1972. In 1968, it would appear that the model initiates stratification (mid-June) about one-half a month too early. It may be that in 1968, due to unusual conditions, the on set of thermal stratification was delayed. The temperature data obtained for 1968 does not rule out this possibility but is inconclusive due to a lack of data in the period of interest, the month of June. However, it is apparent that a one-half month shift in the model prediction in Figure 5-4 would result in much better agreement. In 1972, the results predicted for the period beginning June 1 and ending December 31 are not generally in good agreement with observed data as is seen in Figure 5-8. This year was a particularly unusual year in terms of the condition of the lake due to some severe meteorological disturbances in late spring and in early summer.

The fluxes affecting the hypolimnion oxygen budget as predicted by the model were compared to those calculated from actual data (see Table 4-1). The results are shown in Table 5-6. Since in both cases, the sediment demand flux was estimated from the literature, only the vertical exchange flux and the decomposition flux are tabulated in Table 5-6.

The predicted model vertical exchange flux was calculated using the following relationship:

$$\text{Vertical Exchange Flux} = \hat{k}_{th}(DO_h - DO_e) \qquad (5.4\text{-}1)$$

TABLE 5-6

HYPOLIMNETIC DISSOLVED OXYGEN BUDGET MODEL PREDICTIONS VS OBSERVED FLUXES

YEAR	VERT. EX. FLUX ($g/m^2/day$)		DECOMP. FLUX (g/m^2-day)	
	Observed	Model	Observed	Model
1966	0.29	0.23	0.14	0.83
1967	0.18	0.17	1.09	0.82
1968	0.12	0.12	0.88	0.75
1969	0.16	.17	1.24	0.86
1970	0.19	0.20	1.22	0.87
1972	0.18	0.23	1.41	0.93

This was evaluated after 2 days, 30 days, 60 days, 90 days and 120 days of stratification. The five values were then averaged to give a mean flux for the stratification period. In general, the agreement between observed and predicted vertical exchange fluxes is very good. Only in 1966 and 1972 is there a significant deviation between the two.

There is less agreement with respect to the decomposition fluxes. The model fluxes were calculated using the following relationship:

$$\text{Decomposition Flux} = d_h[PP_h]\ \bar{Z}_h \times \text{FAC} \qquad (5.4\text{-}2)$$

This was evaluated after 2 days, 30 days, 60 days, 90 days and 120 days of stratification. These five values were averaged to give a mean flux for the stratification period. The observed fluxes were calculated by a method of difference. The total flux of oxygen and the vertical exchange component were calculated from actual data. A constant value for the sediment demand flux (that used in the model) was added to the vertical exchange flux and the sum subtracted from the total flux. The result is an estimate of the decomposition flux that is significantly influenced by the sediment demand flux selected.

The model fluxes, with the exception of 1966, were consistently lower than those observed, in many cases by significant amounts (15 to 34 per cent). This consistent underestimation suggests the possibility of a bias in calculation of the model flux. The model flux is a function of d_h, the hypolimnetic decomposition coefficient. Raising the selected value from 0.03 to 0.04 day^{-1} would increase the predicted flux and improve the agreement. Conversely, the observed flux may have been overestimated. The sediment demand flux, selected from literature values, may be too low due to the method of calculation (by difference). This latter possibility is more likely because it is difficult to estimate the sediment oxygen demand of a lake the size of Lake Ontario due to varying bottom conditions and lack of concrete field or laboratory data.

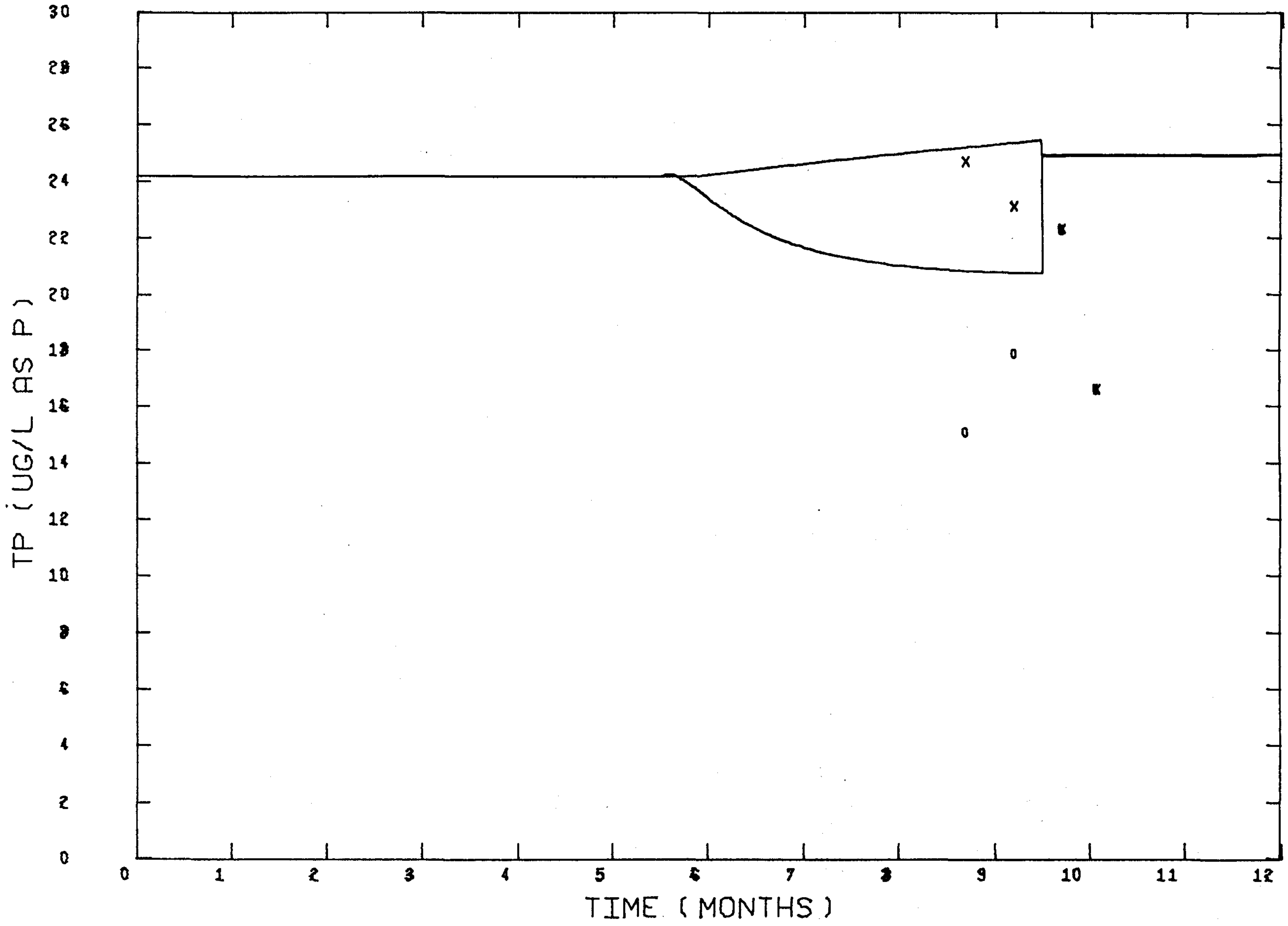
5.4.2 Verification of Total Phosphorus Predictions

The model predictions of total phosphorus are compared to the observed data in Figures 5-11 to 5-18. There was no observed data in 1966. The agreement is not as good as with dissolved oxygen concentrations but nonetheless is acceptable. There is insufficient data to form any significant conclusions concerning the ability of the model to predict total phosphorus. The phosphorus model of Snodgrass (1974) used in this research, is not verified to predict phosphorus concentration on a time scale of days or weeks. In addition, the phosphorus load W was kept constant during the nine years of prediction. In actuality, this load changed significantly with the introduction of phosphorus control measures in 1972.

FIGURE 5-11

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1967

0-EPILIMNION X-HYPOLIMNION —— - MODEL



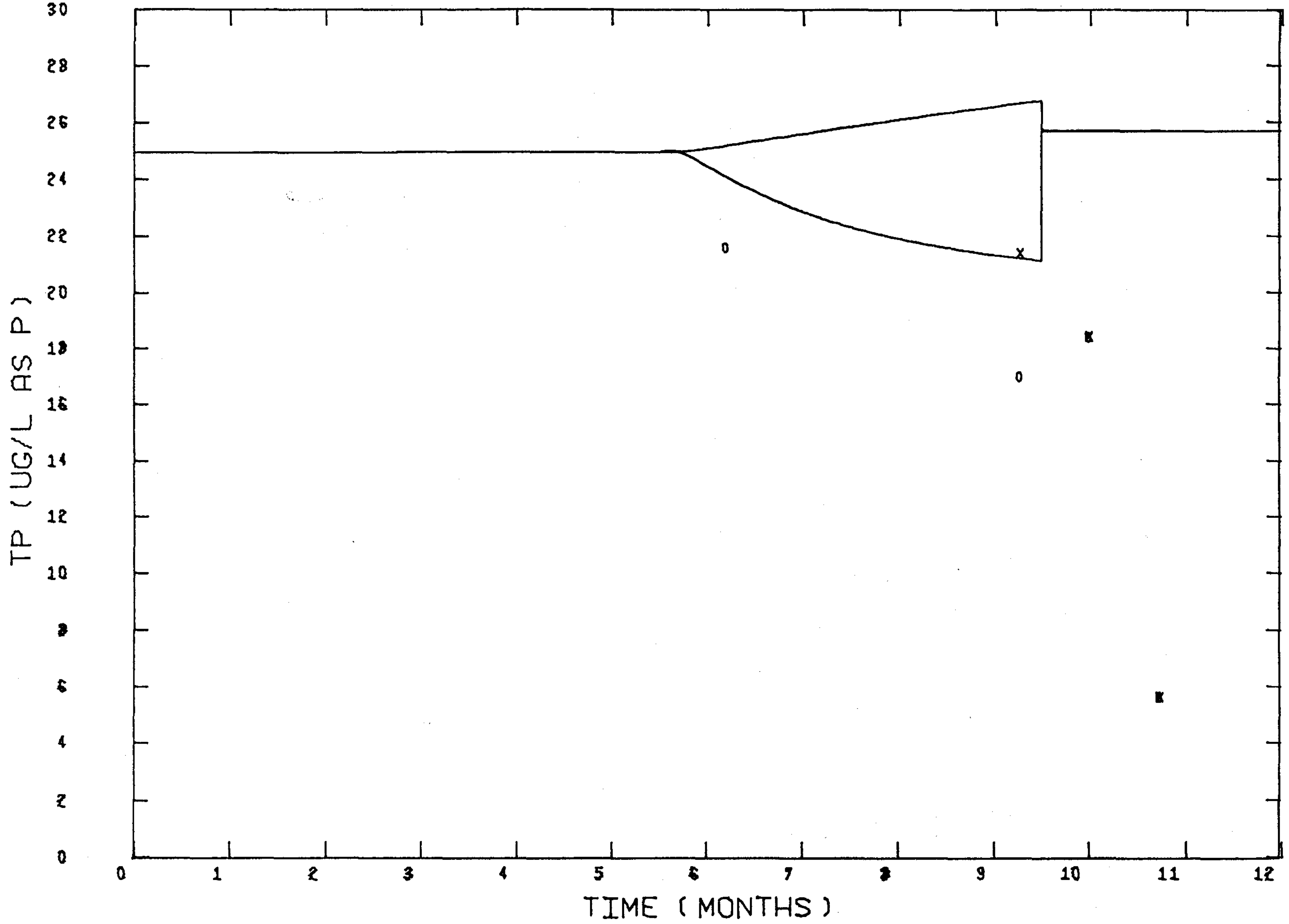


FIGURE 5-12

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1968

0-EPILIMNION X-HYPOLIMNION —— - MODEL

FIGURE 5-13

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1969

0-EPILIMNION X-HYPOLIMNION —— - MODEL

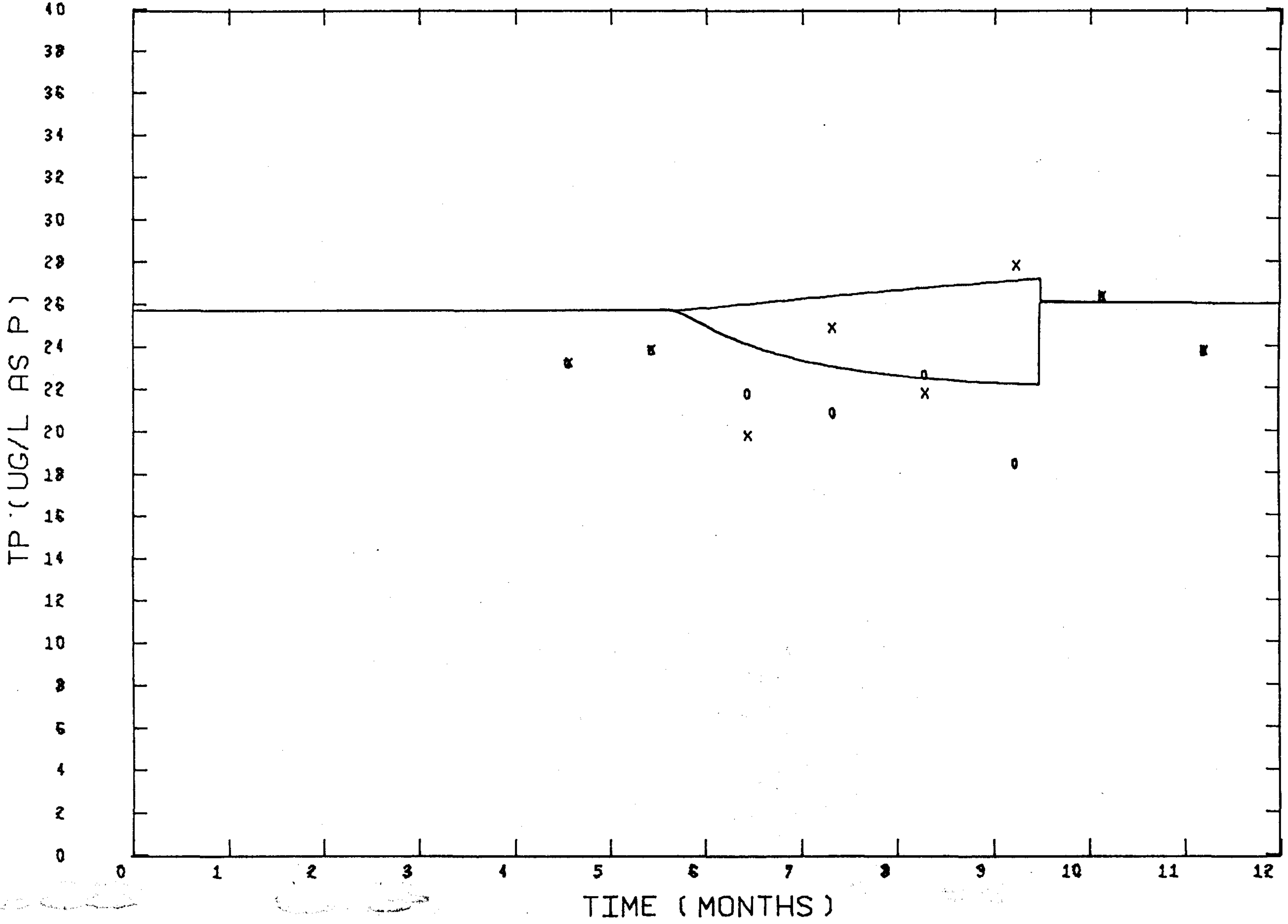


FIGURE 5-14

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1970

0-EPILIMNION X-HYPOLIMNION —— - MODEL

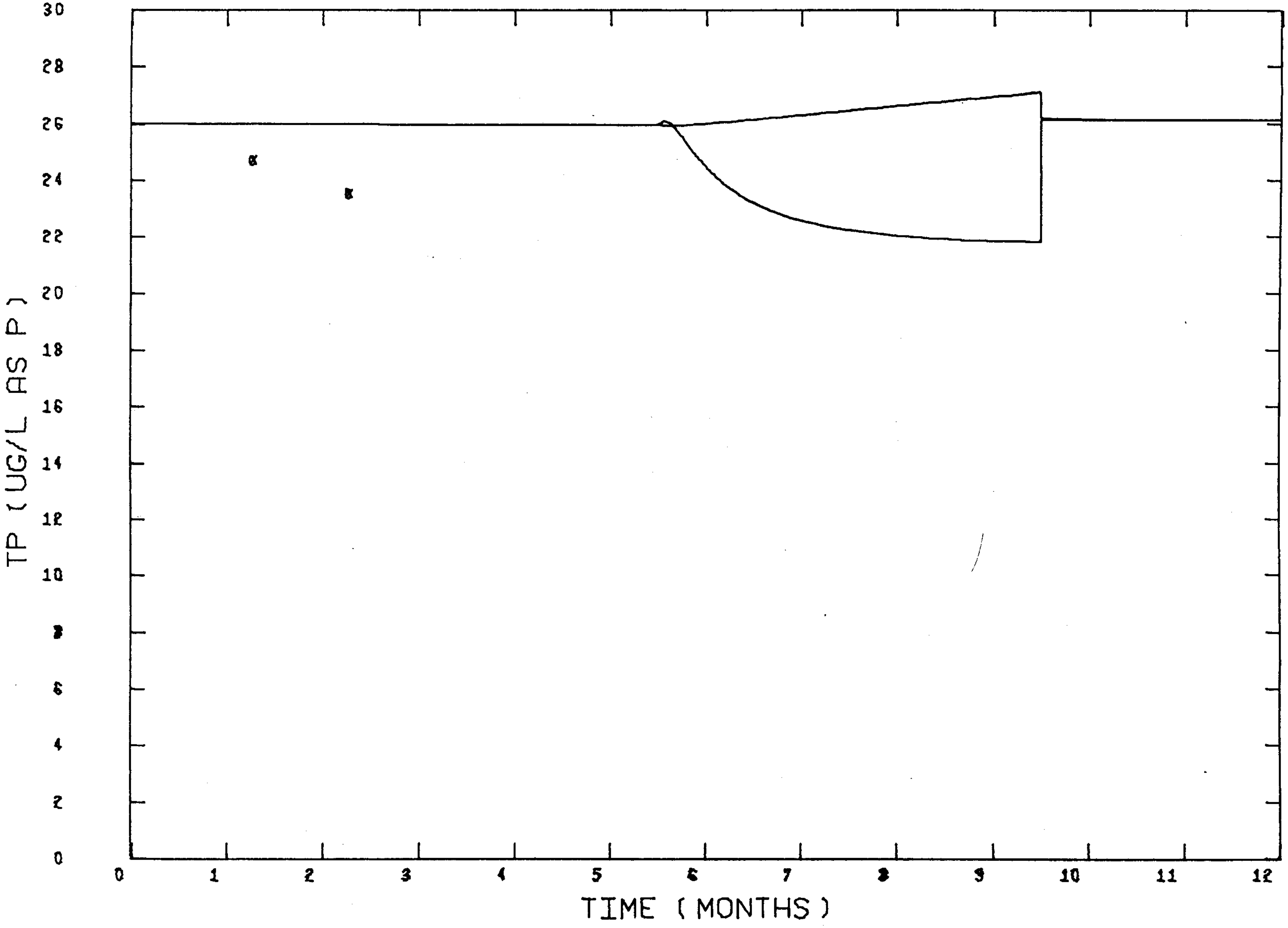


FIGURE 5-15

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1971

0-EPILIMNION X-HYPOLIMNION —— - MODEL

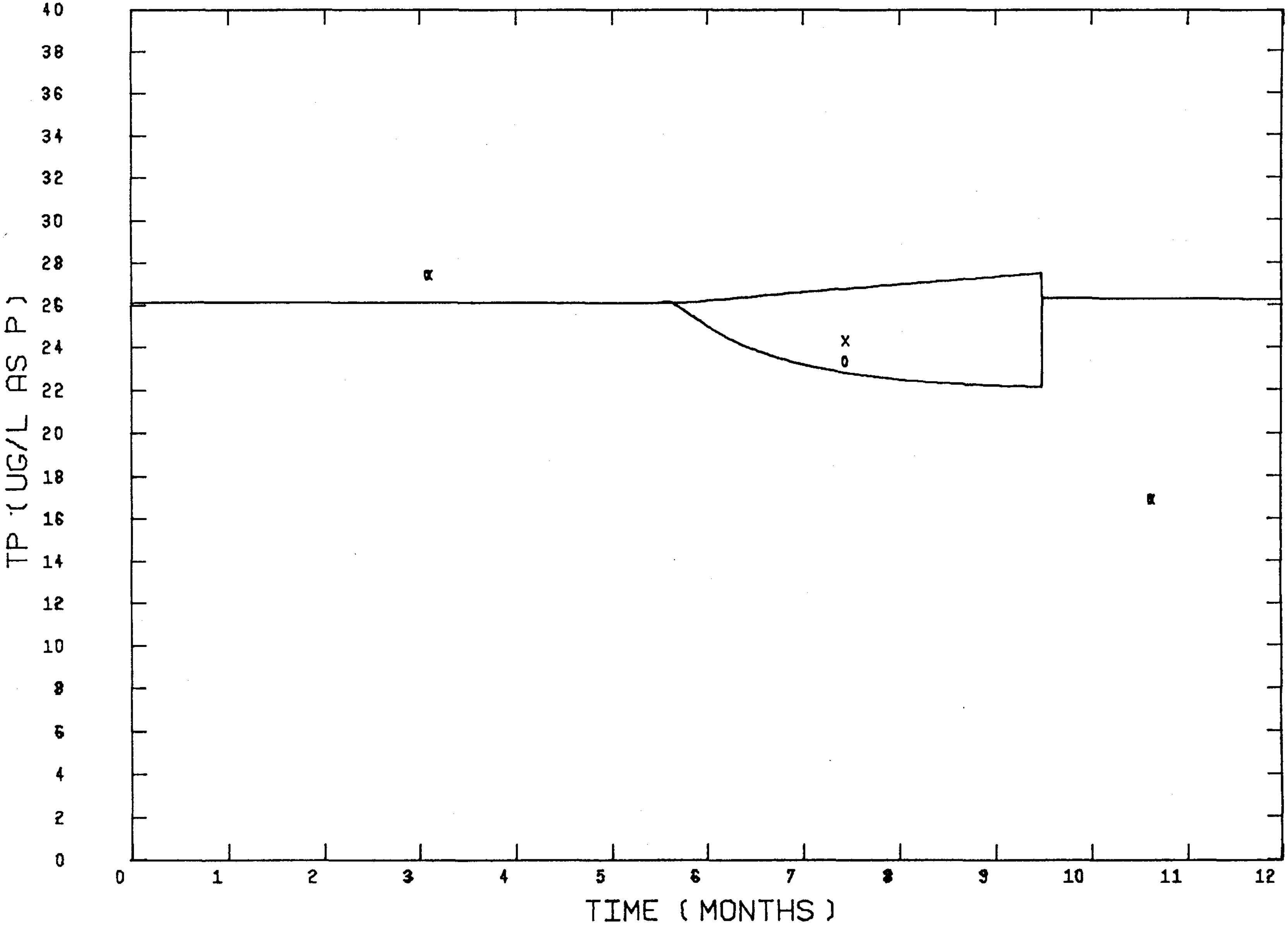


FIGURE 5-16

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1972

0-EPILIMNION X-HYPOLIMNION —— - MODEL

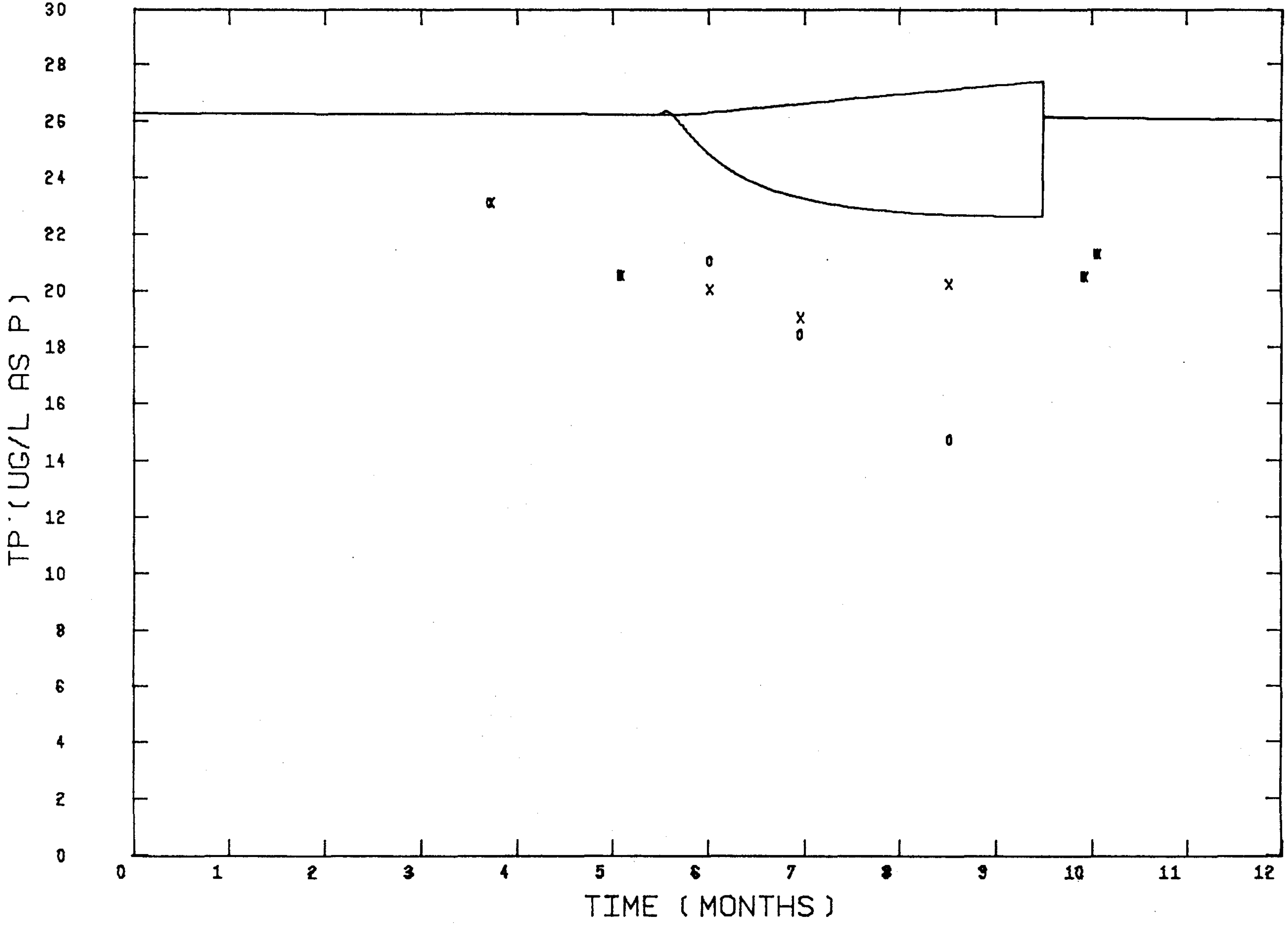


FIGURE 5-17

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1973

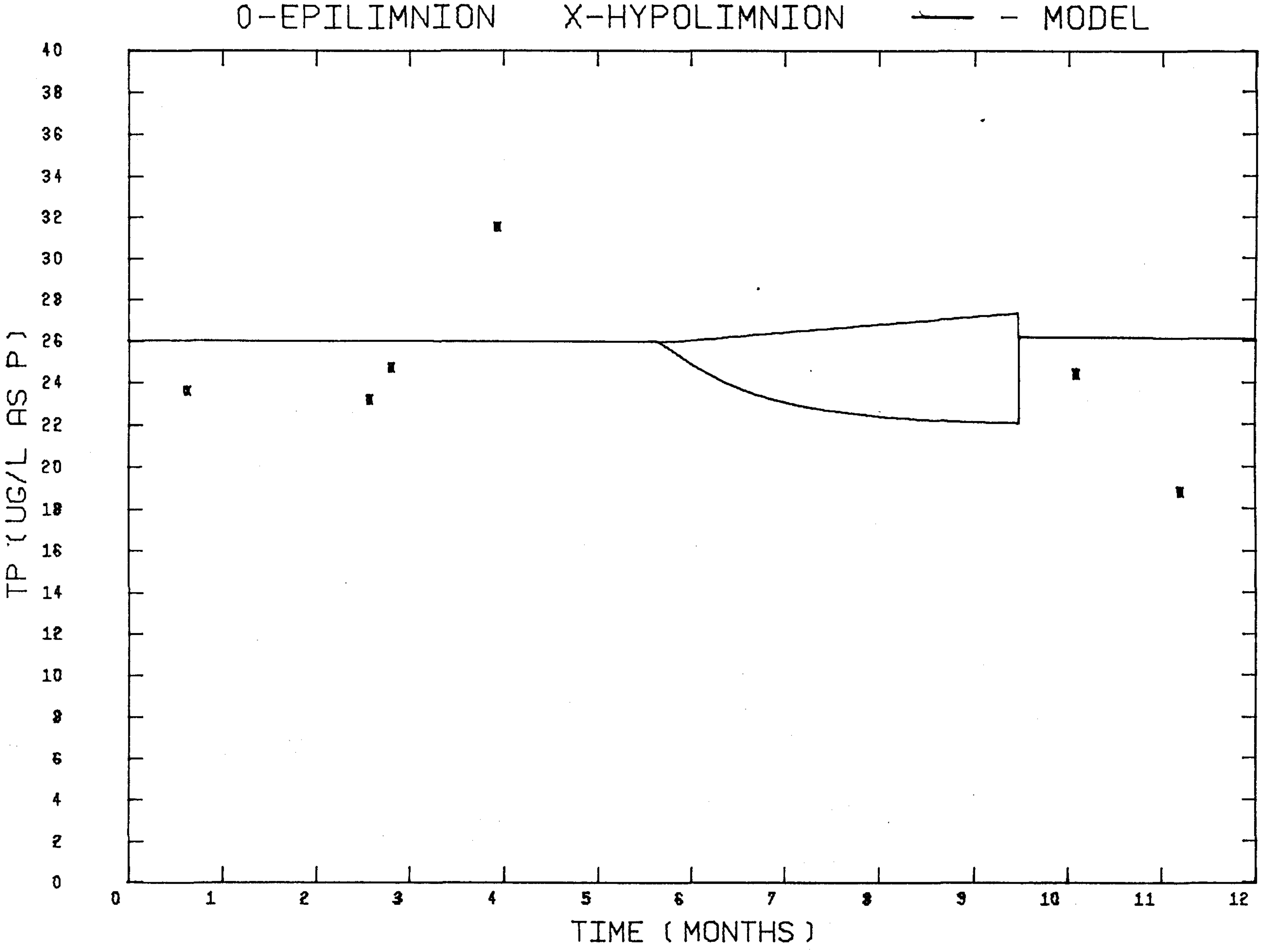
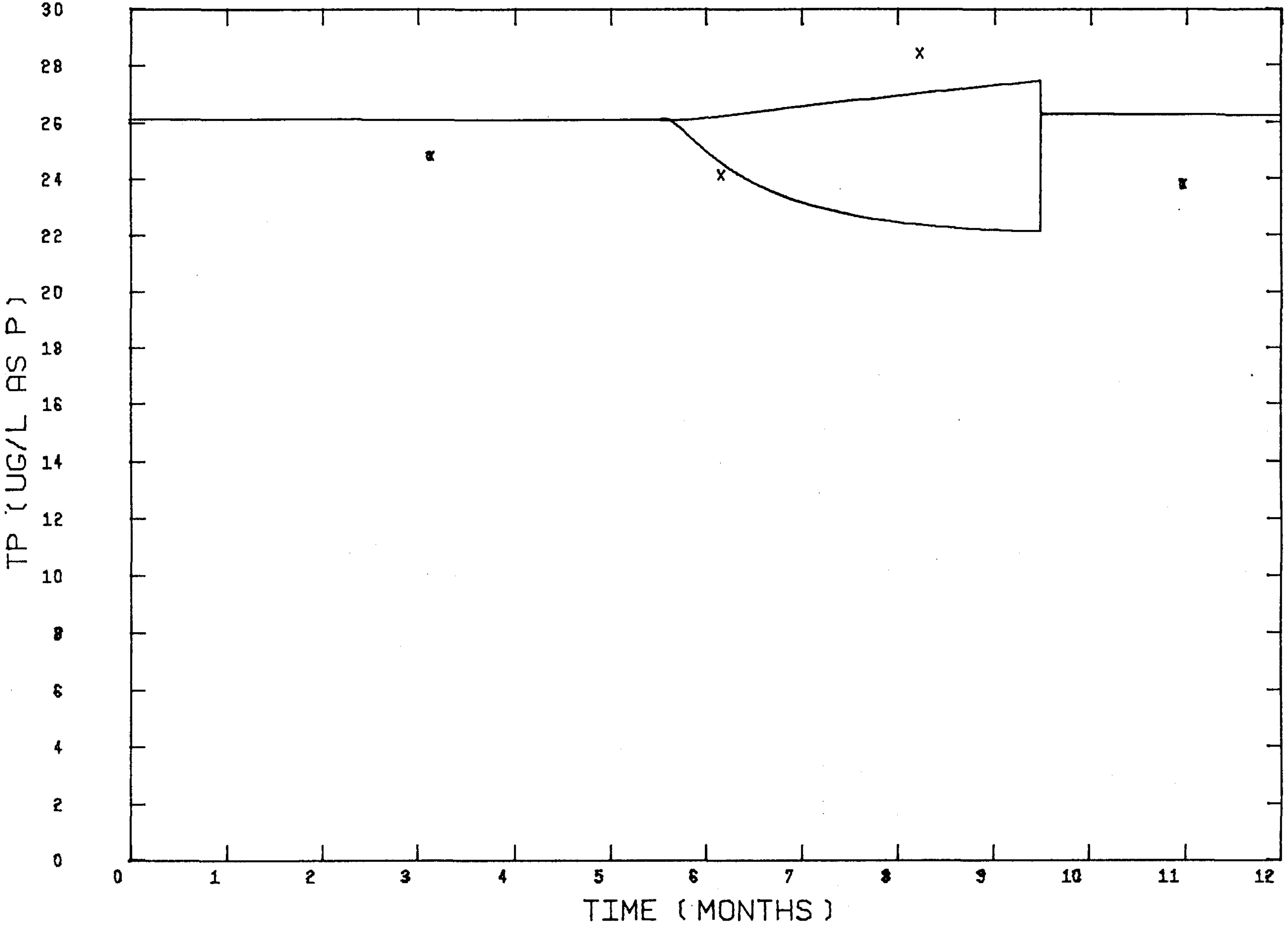


FIGURE 5-18

TOTAL PHOSPHORUS VS TIME LAKE ONTARIO 1974

O-EPILIMNION X-HYPOLIMNION —— - MODEL



CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

The following conclusions can be made based upon the research reported in preceding sections. For clarity, these conclusions are grouped under three major areas:

1) Conclusions based on the analysis of the data base,
2) Conclusions based on the model verification, and
3) Conclusions based on the model parameters.

6.1.1 Conclusions Based on Analysis of Data Base

The analysis of the dissolved oxygen, phosphorus and temperature data for the nine year period 1966-1974 for Lake Ontario yields significant information upon which the following conclusions are made concerning the behaviour of the lake.

1. The rate of increase of depth of the epilimnion is estimated to range from 0.8 m/mo to 5.8 m/mo. There is no correlation between rate of deepening and year.

2. The average vertical exchange coefficient over the stratification period is estimated to range from 0.07 m/day to 0.15 m/day. The individual values of $\hat{k}_{th}$ vary from year to year randomly.

3. During the initial three months of the stratification period, there is a net flux of oxygen from the epilimnion to the atmosphere. This flux is due to the surface temperature increasing at a faster rate than the rate of DO exchange with the atmosphere.

4. Since the dissolved oxygen concentration in the hypolimnion is always higher than that of the epilimnion, the net vertical transport of dissolved oxygen is from the hypolimnion to the epilimnion during stratification. This contrasts with observations on several other lakes such as Lake Erie (IJC, 1969) and Lake Sammamish (Bella, 1970a) in which the transport takes place from the epilimnion to the hypolimnion.

5. Vertical transport of oxygen out of the hypolimnion represents 55%, 13%, 11%, 11%, 13% and 11% of the total loss of oxygen from the hypolimnion during the stratification period in the years 1966,1967,1968, 1969,1970 and 1972 respectively.

6. The hypolimnetic oxygen loss due to decomposition is estimated to be 0.14, 1.09, 0.88, 1.24, 1.22 and 1.41 g/m^2-day for the same years.

6.1.2 Conclusions Based on Model Verification

The verification of the model for several years of observed data provides the basis for the following conclusions:

1. The dissolved oxygen model developed in this research shows very good agreement between observed and predicted dissolved oxygen concentrations in Lake Ontario for a nine year data base under both winter circulation and summer stratification conditions. Of particular note is the fact that the dissolved oxygen concentration in the hypolimnion is always higher than that in the epilimnion.

2. Model predictions of vertical exchange fluxes of dissolved oxygen from the hypolimnion compare favourably with those observed.

3. Model predictions of decomposition fluxes of dissolved oxygen from the hypolimnion show acceptable agreement with but are generally lower than those observed.

4. Model verification can be achieved <u>only</u> by a data set <u>independent</u> of the set used for model calibration. If the data set used for verification is for a time period directly after that used for calibration and if the verification data set is strongly influenced by conditions of the calibration data set, then the two data sets are <u>not</u> independent. Due to the return to the same oxygen conditions annually in Lake Ontario, oxygen concentrations for the two

succeeding years are independent and two succeeding years may be used for calibration and verification.

5. It is concluded that verification has been achieved for the following elements of the model by application to Lake Ontario.

   i) the hypolimnetic mineralization flux predicted by the oxygen model and
   ii) the use of Redfield stoichiometry to relate particulate phosphorus mineralization to oxygen depletion.

   However, this model should be applied to lakes with a wide range of characteristics before it can be used with complete confidence.

6.1.3 Conclusion Based on Model Parameters

The following conclusions are made based upon the sensitivity analysis:

1. The model predictions of dissolved oxygen concentrations are relatively insensitive to doubling and halving the model parameters. Deviations from predictions for the standard conditions were less than 10% in all cases. At the early stages of stratification, model predictions of epilimnetic DO are most sensitive to the reaeration coefficient. At the end of stratification, the model predictions are most sensitive to the DO/P relationship factor

(FAC) and fairly sensitive to the vertical exchange coefficient.

2. Model predictions of epilimnetic and hypolimnetic dissolved oxygen are insensitive to the phosphorus loading into the lake as well as the initial phosphorus and dissolved oxygen values used as boundary conditions. The decomposition coefficient in the hypolimnion, d_h, has a small effect on predictions of hypolimnetic DO.

3. Due to the form of the reaeration coefficient, k_a, used in this model, its value must change from summer to winter. Reaeration oxygen transfer takes place between the atmosphere and a shallow box (epilimnion) in summer and a deep box (the entire lake) in winter. The reaeration coefficient must reflect this difference.

6.2 RECOMMENDATIONS

The following major areas of future research have been indicated by this study:

1. Apply the model to a variety of lakes for the purpose of ultimate verification. Part of this application would be to conduct temperature studies on these other lakes to determine estimates of the rate of increase of epilimnetic depth and the vertical exchange coefficient using computer techniques.

2. Sediment oxygen demand research, including development of meaningful in situ methods, on Lake Ontario and other bodies of water.

3. Application to a lake for which complete data for the two phosphorus compartments, (particulate and orthophosphorus) are available.

4. Attempt the modelling of the entire Great Lakes system with respect to dissolved oxygen using this model as a basis.

Why would
you not
model
Lake Erie

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APPENDIX A

CALCULATION OF EPILIMNETIC DEPTH (THERMOCLINE POSITION) AS A FUNCTION OF TIME OF YEAR

CALCULATION OF THERMOCLINE AS A FUNCTION OF TIME OF YEAR

$$Z_e = mt + b$$

RAW DATA - 1966

TIME OF YEAR (t) (months)	THERMOCLINE POSITION (Z_e) (meters)
5.83	6.5
6.00	8.8
6.33	12.4
6.80	13.0
7.23	17.4
7.63	16.4
8.07	22.0
8.53	21.5

m = 5.50
b = -24.00

RAW DATA - 1967

t	Z_e
6.43	8.0
7.00	9.8
7.33	9.5
7.83	9.5
8.30	10.0
8.70	10.5

m = 0.84
b = 3.18

RAW DATA - 1968

t	Z_e
6.2	15.0
6.9	15.0
7.73	17.5
8.40	16.0
9.27	18.5
10.00	19.0

m = 1.10
b = 7.95

RAW DATA - 1969

t	Z_e
5.43	10.5
6.43	13.0
7.33	16.0
8.30	17.0
9.23	22.0

m = 2.84
b = -5.14

RAW DATA - 1970

t	Z_e
5.93	4.0
6.67	8.0
7.70	11.0
8.63	14.0
9.57	18.0

m = 3.66
b = -17.14

RAW DATA - 1972

t	Z_e
5.67	5.5
5.90	6.0
6.20	7.0
6.57	8.0
7.07	15.0
7.93	17.0
8.20	18.0
8.43	18.0
8.60	21.0
8.83	22.0
9.13	22.0
9.37	31.0

m = 5.76
b = -28.24

For 1971, 1973, and 1974, use all above data: m = 3.62
b = -13.08

LAKE ONTARIO TEMPERATURE PROFILES

In Figures A-1 to A-6, there are some data points that are based on fewer than five individual temperature values within a particular depth slice. These points have been disregarded in drawing the profiles.

FIGURE A-1a

TEMPERATURE PROFILES LAKE ONTARIO 1966

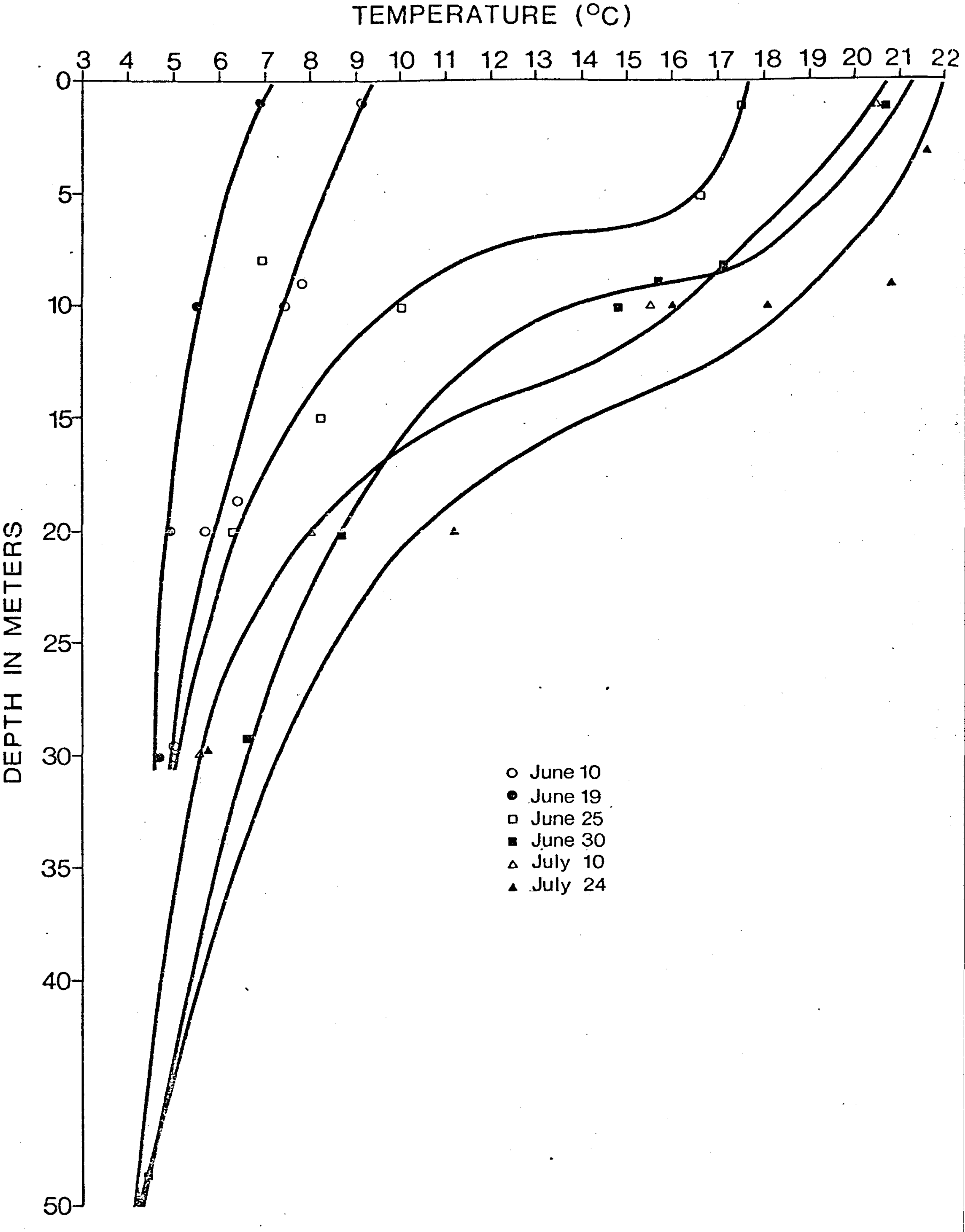


FIGURE A-1b

TEMPERATURE PROFILES LAKE ONTARIO 1966

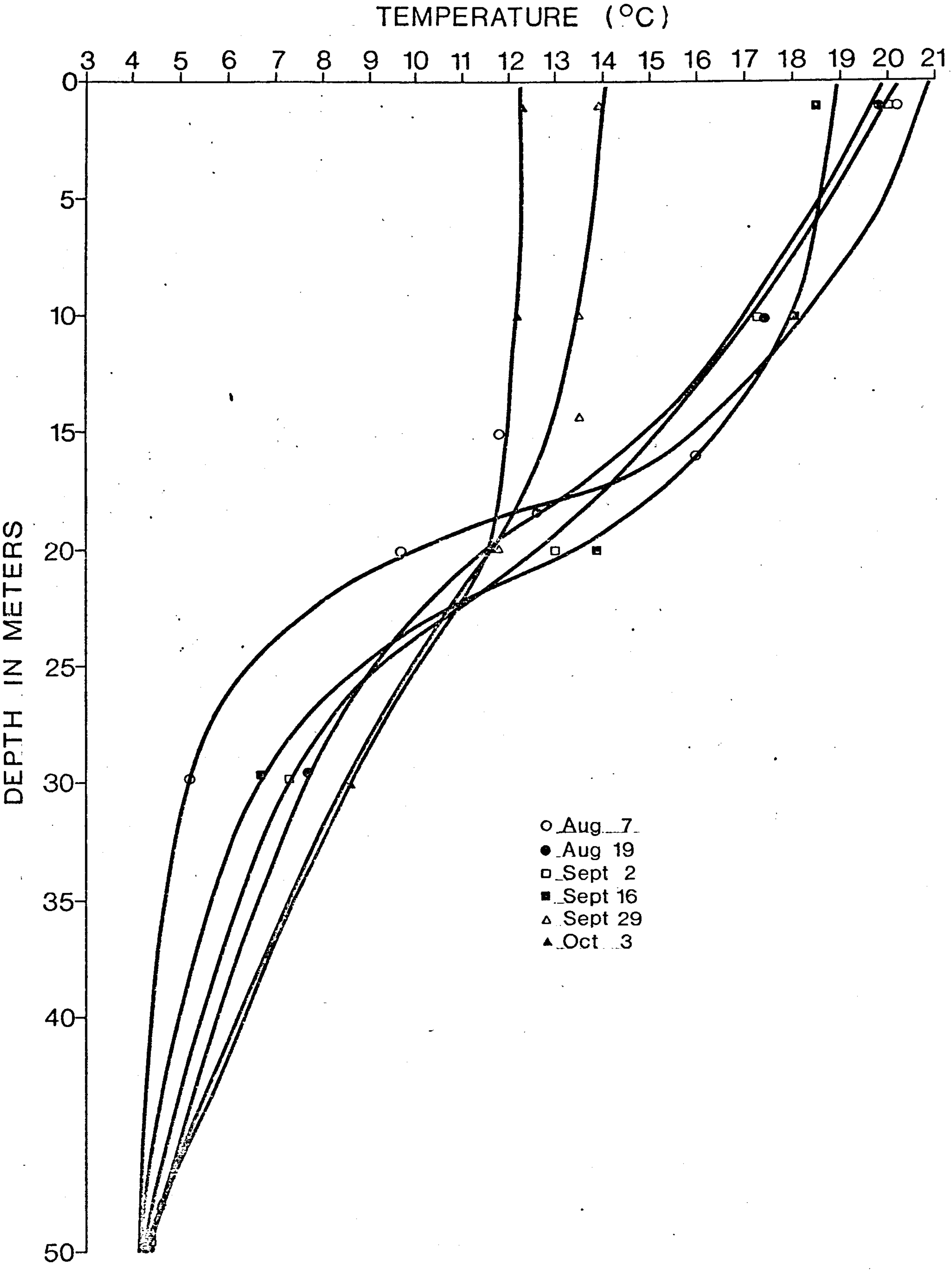


FIGURE A-2a

TEMPERATURE PROFILES LAKE ONTARIO 1967

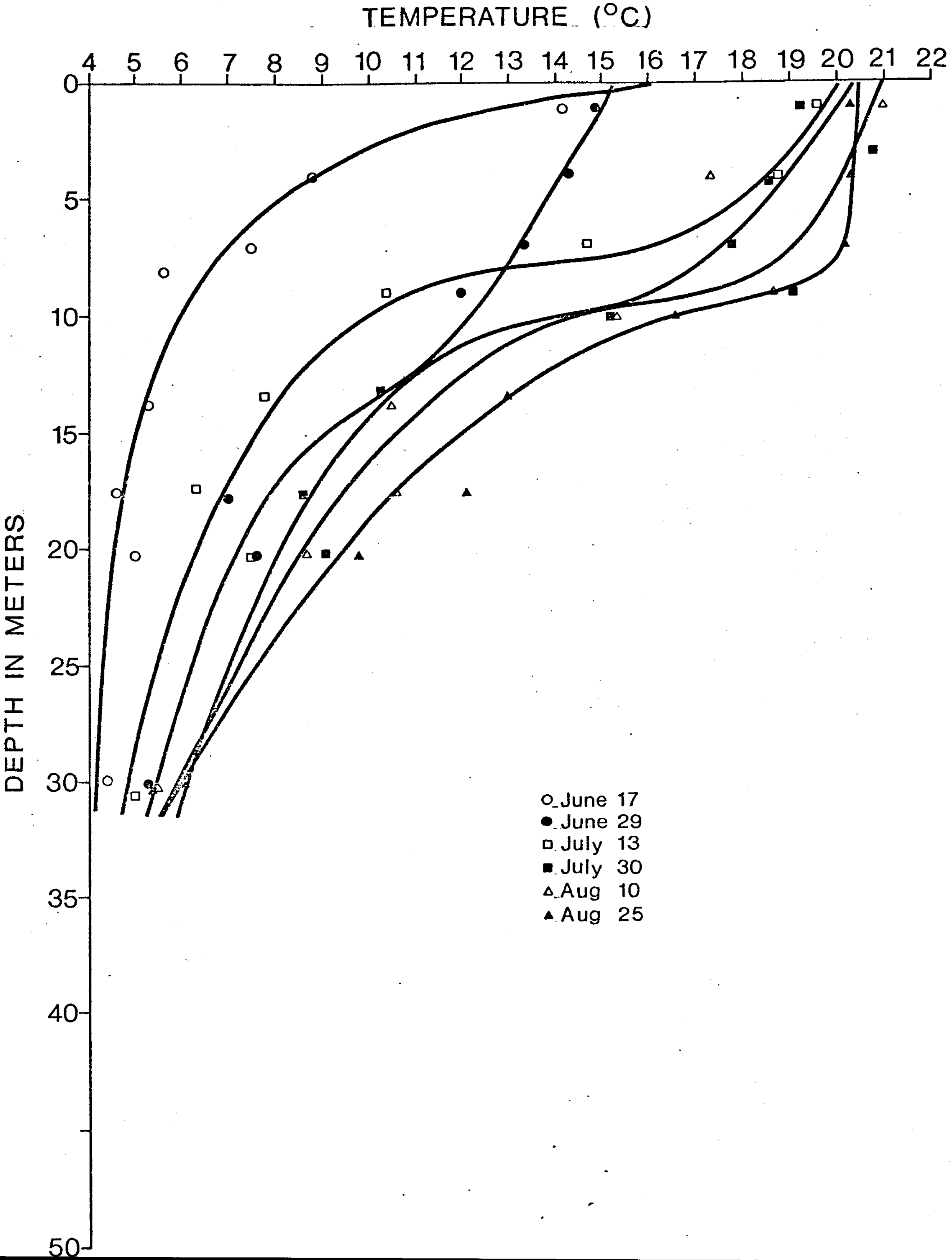


FIGURE A-2b

TEMPERATURE PROFILES LAKE ONTARIO 1967

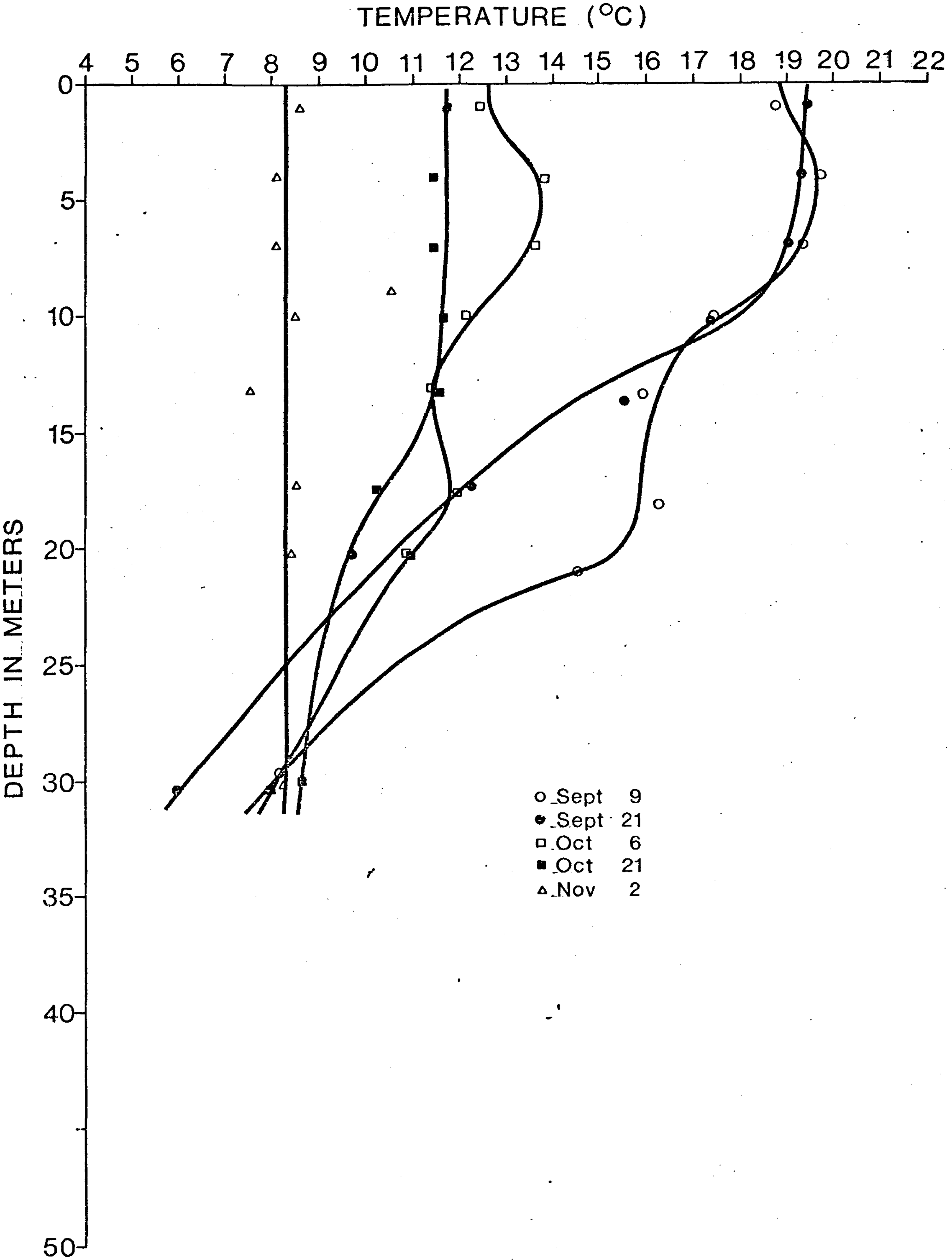


FIGURE A-3

TEMPERATURE PROFILES LAKE ONTARIO 1968

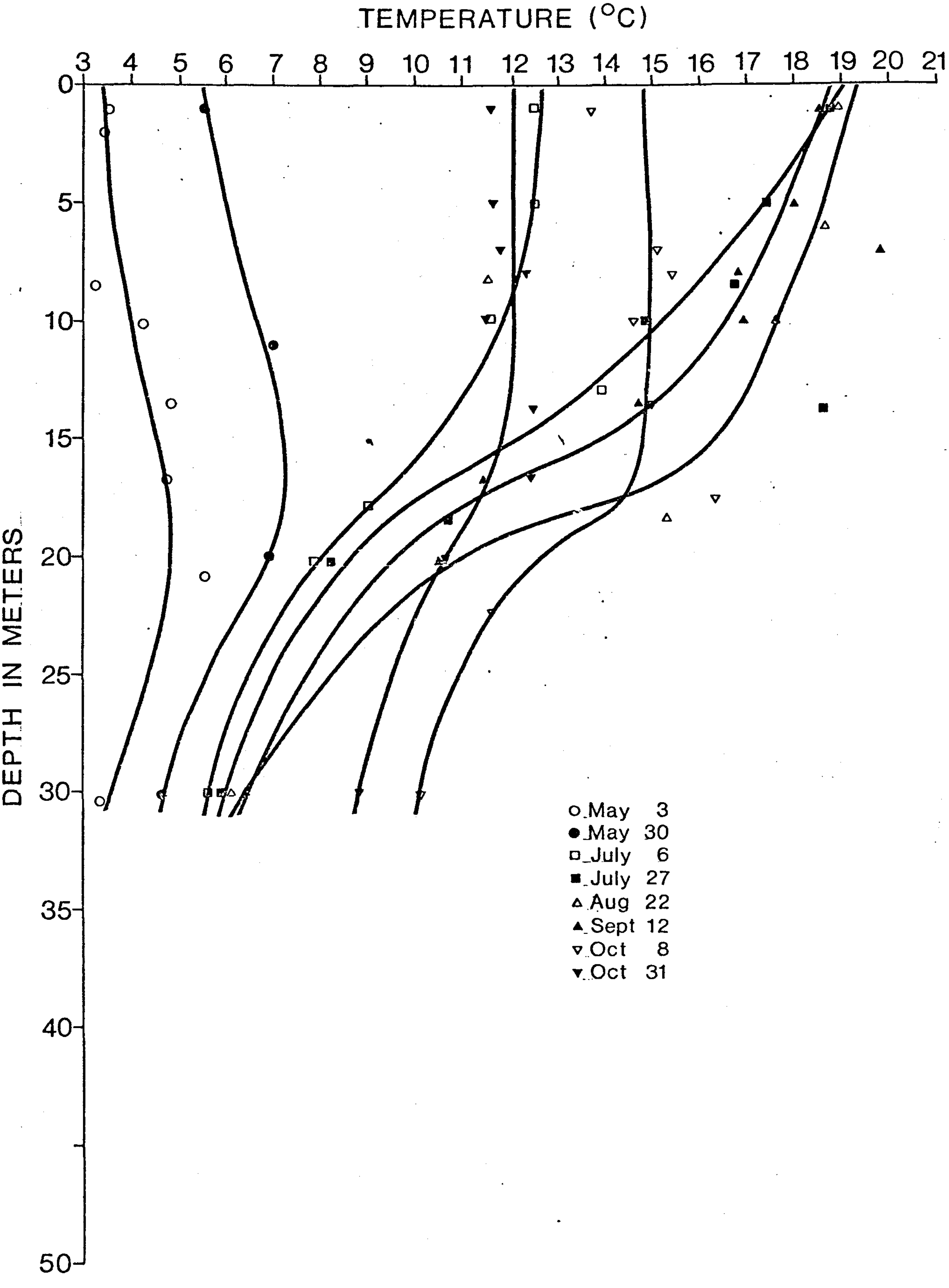


FIGURE A-4a

TEMPERATURE PROFILES LAKE ONTARIO 1969

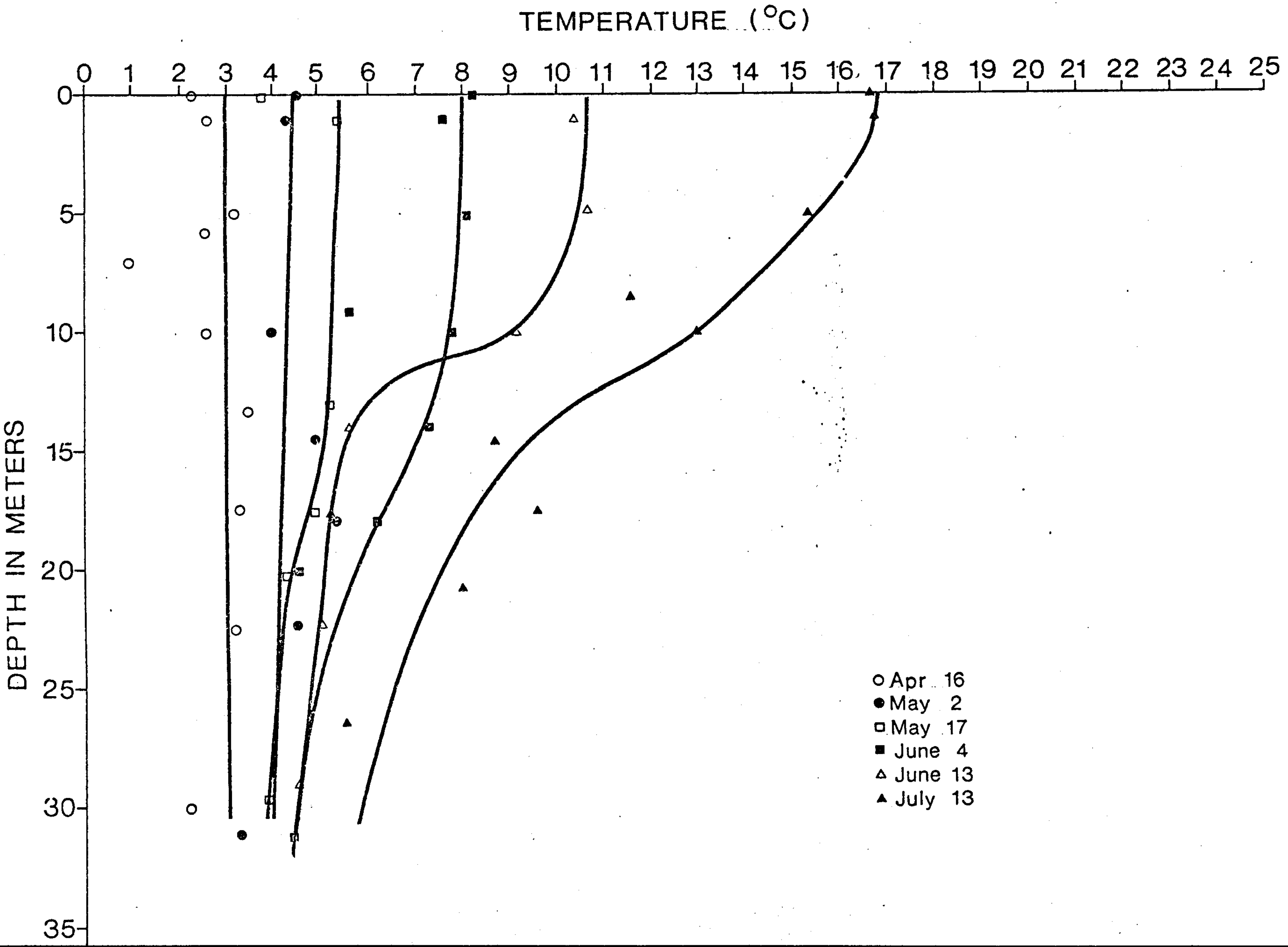


FIGURE A-4b

TEMPERATURE PROFILES LAKE ONTARIO 1969

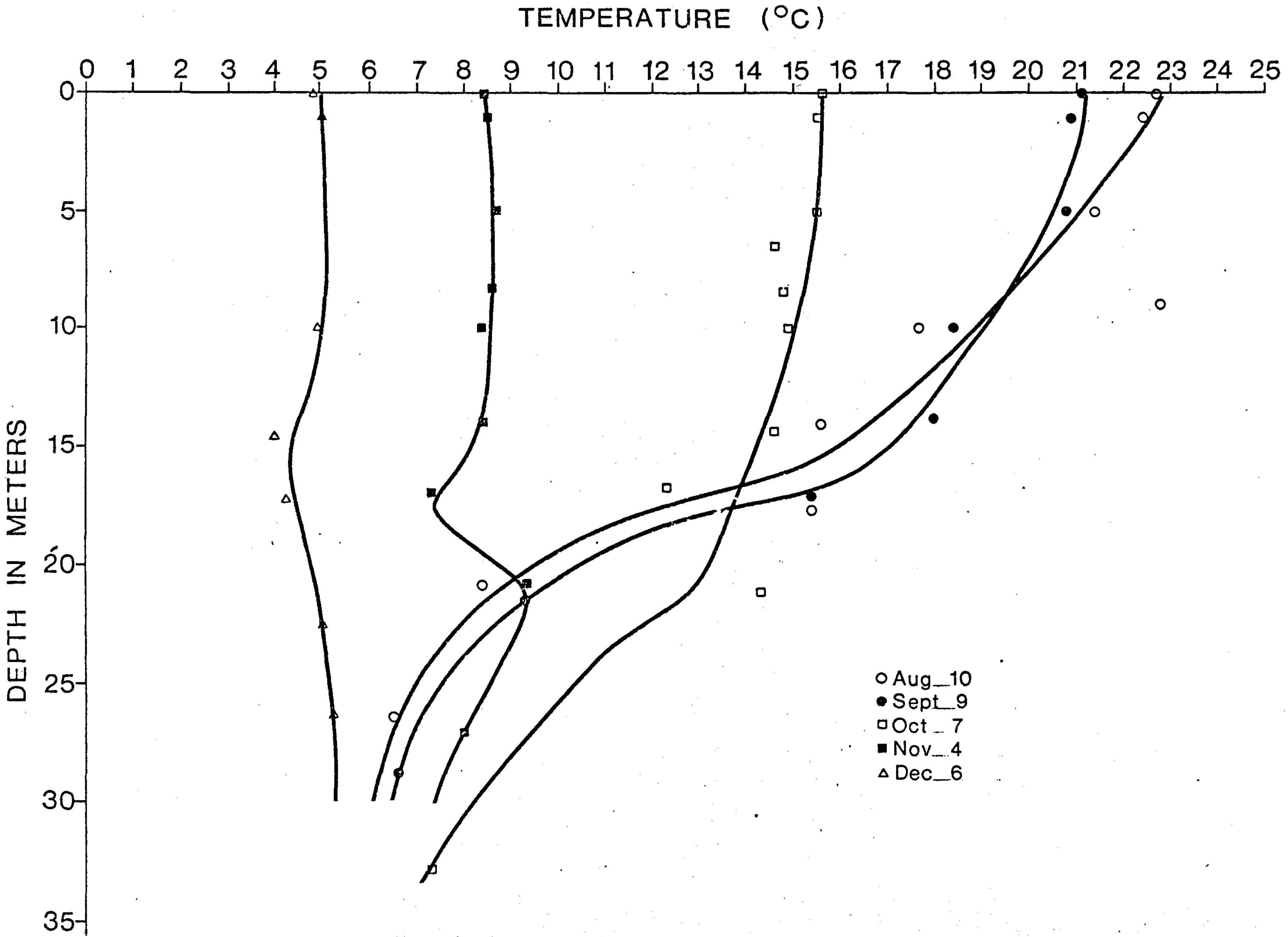


FIGURE A-5a

TEMPERATURE PROFILES LAKE ONTARIO 1970

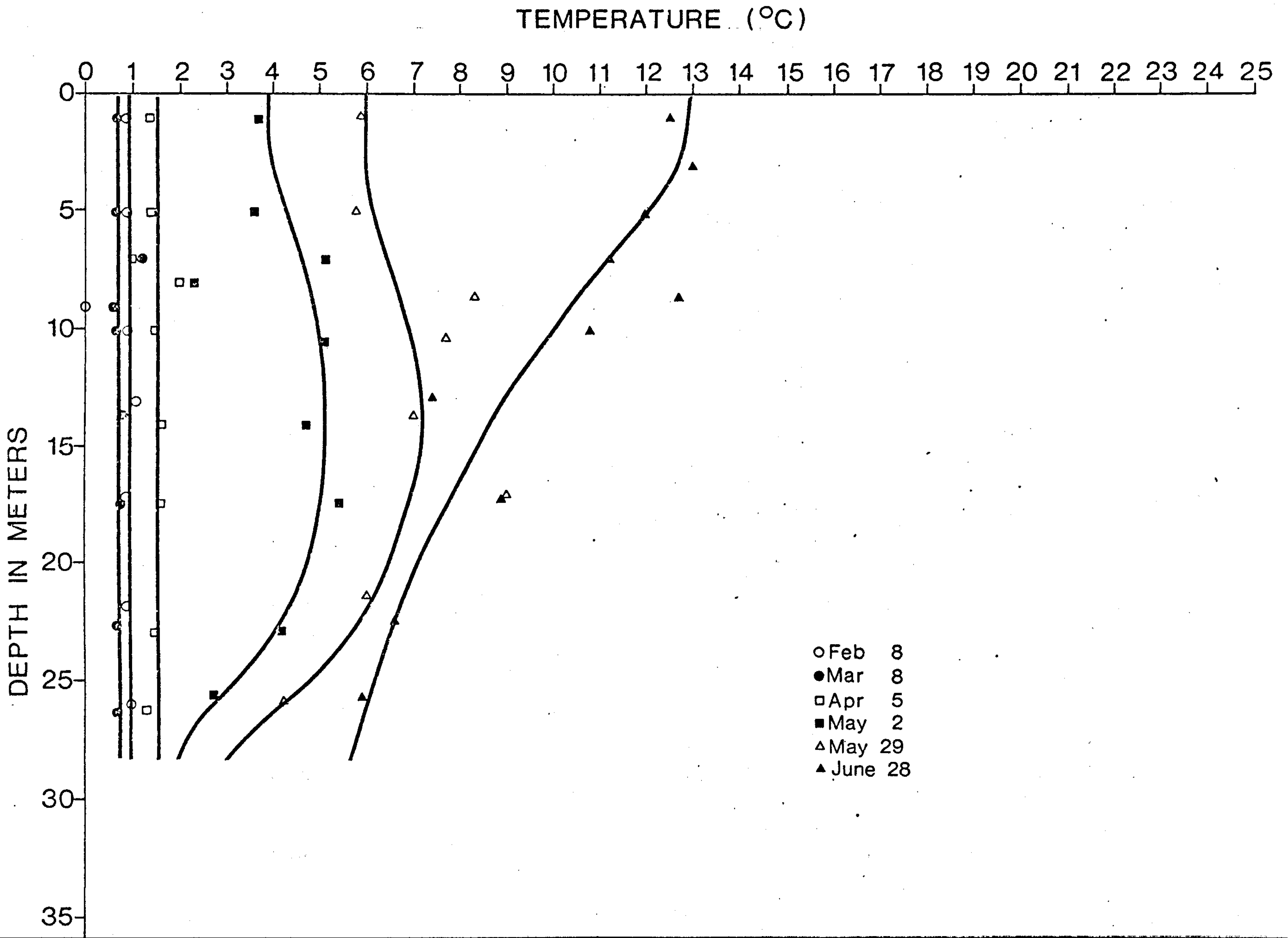


FIGURE A-5b

TEMPERATURE PROFILES LAKE ONTARIO 1970

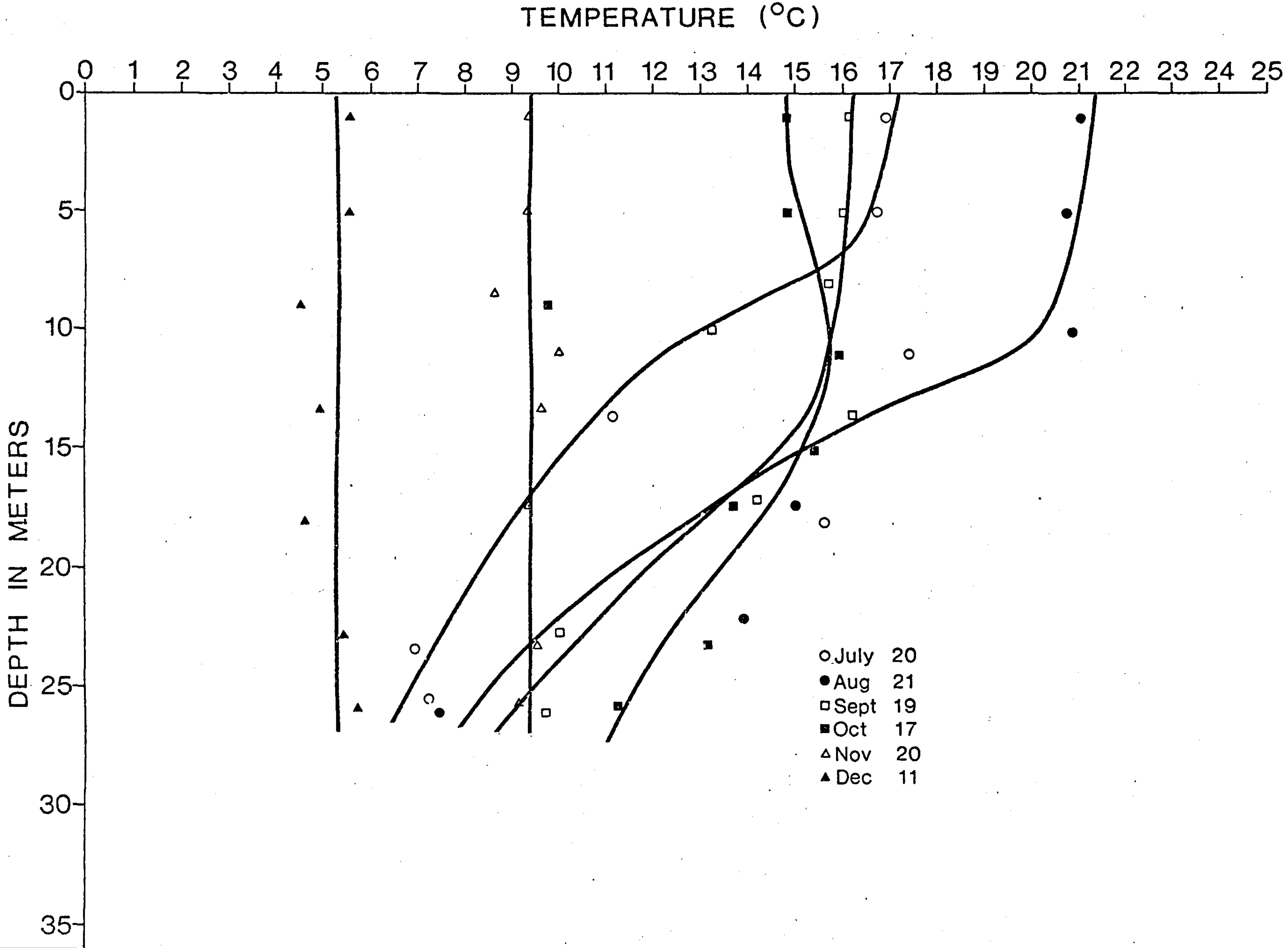


FIGURE A-6a

TEMPERATURE PROFILES LAKE ONTARIO 1972

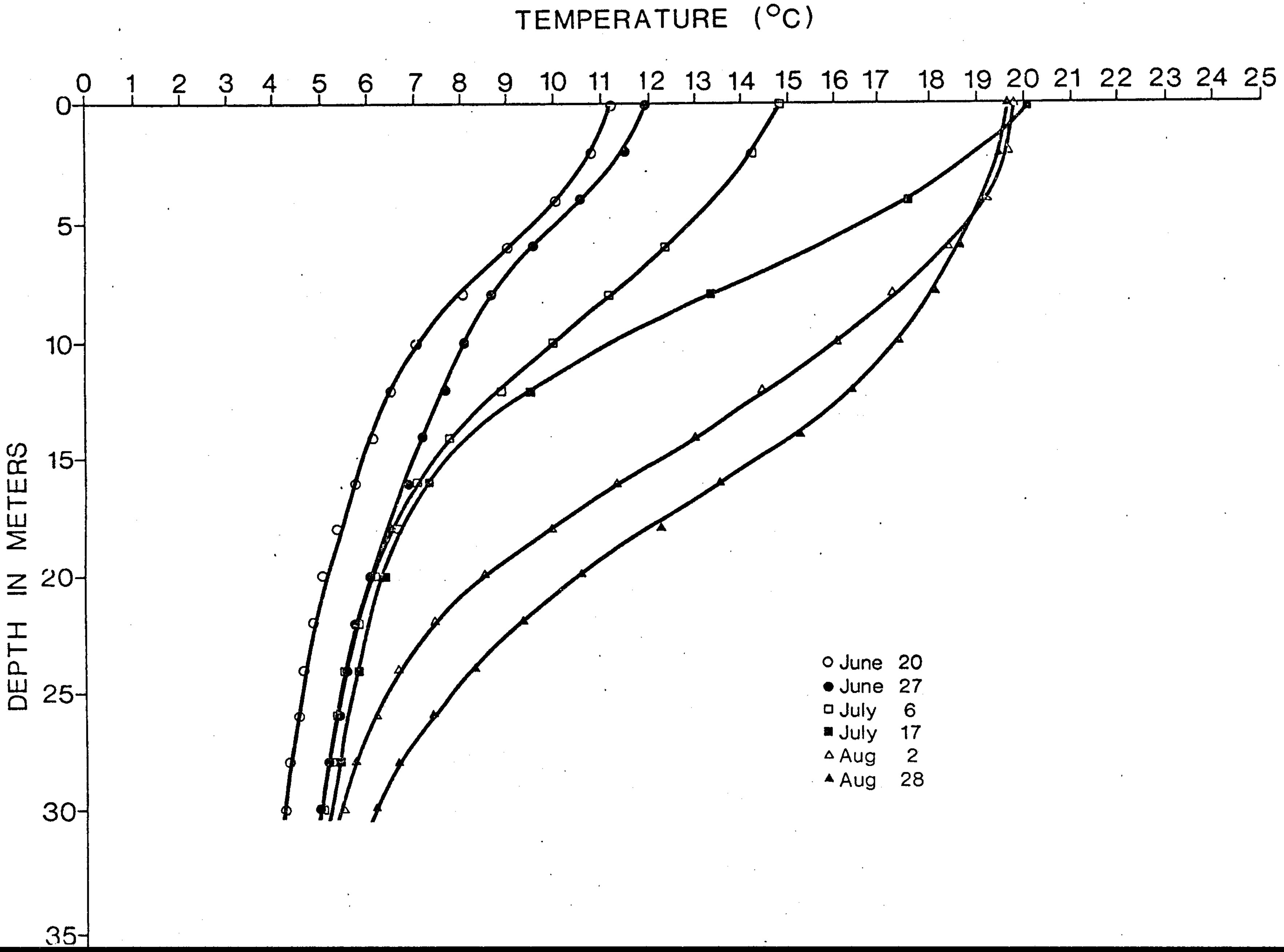
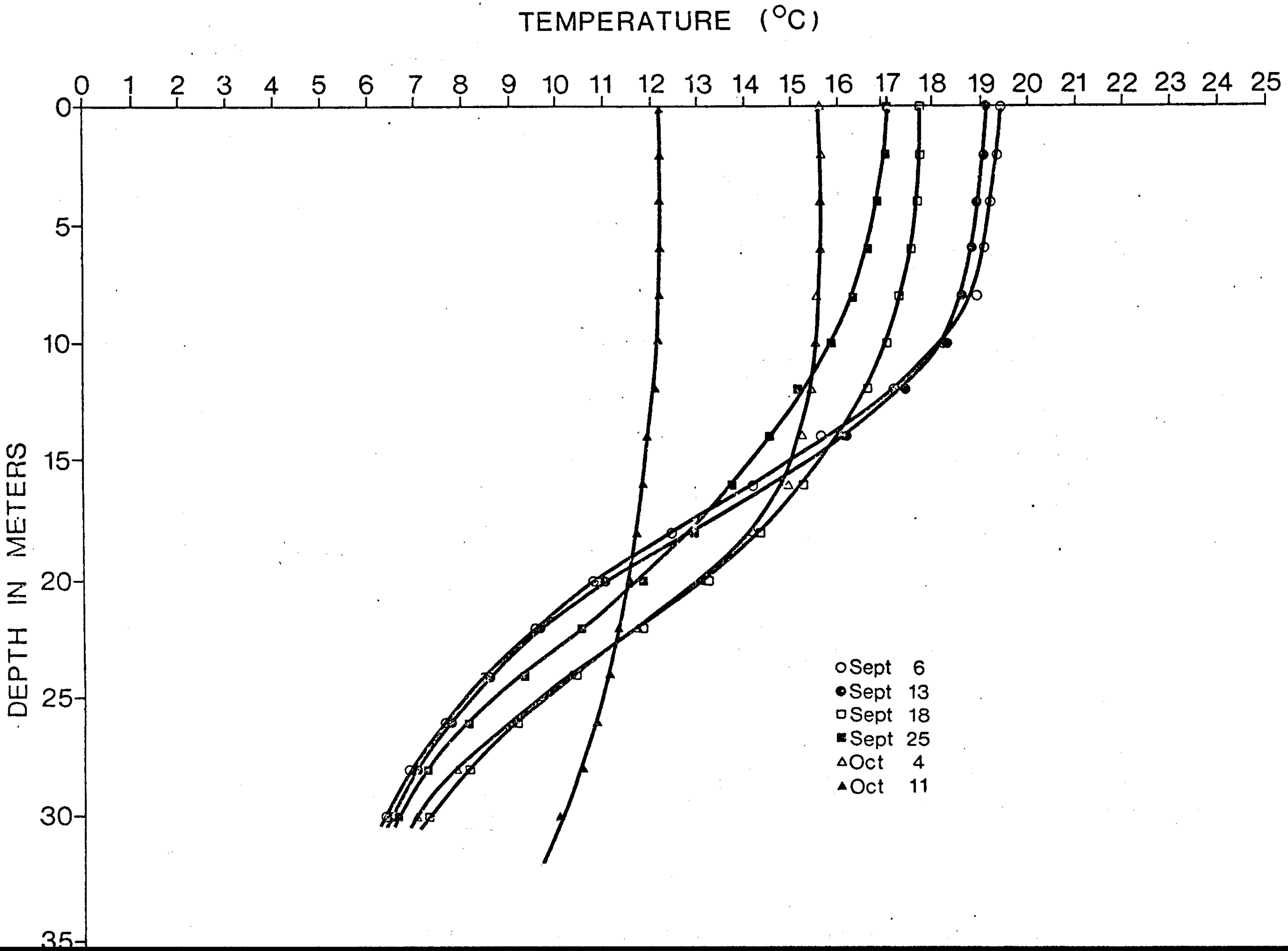


FIGURE A-6b

TEMPERATURE PROFILES LAKE ONTARIO 1972



APPENDIX B

CALCULATION OF $\hat{k}_{th}$ AT VARIOUS TIMES DURING STRATIFICATION IN 1966

TIME PERIOD 0-14 days

At 0 days $\bar{\theta}_e = 10.38\ {}^{o}C$

$\bar{\theta}_h = 4.48\ {}^{o}C$

At 14 days $\bar{\theta}_e = 16.05\ {}^{o}C$

$\bar{\theta}_h = 4.91\ {}^{o}C$

$\bar{V}_h = 15.957 \times 10^{11}\ m^3$

$\bar{A}_{th} = 0.1831 \times 10^{11}\ m^2$

$\hat{k}_{th} = 0.3141$ m/day

TIME PERIOD 14-28 days

At 14 days $\bar{\theta}_e = 12.70\ {}^{o}C$

$\bar{\theta}_h = 4.63\ {}^{o}C$

At 28 days $\bar{\theta}_e = 21.87\ {}^{o}C$

$\bar{\theta}_h = 5.13\ {}^{o}C$

$\bar{V}_h = 15.030 \times 10^{11}\ m^3$

$\bar{A}_{th} = 0.1800 \times 10^{11}\ m^2$

$\hat{k}_{th} = 0.2404$ m/day

TIME PERIOD 28-42 days

At 28 days $\bar{\theta}_e = 18.74\ {}^{o}C$

$\bar{\theta}_h = 4.55\ {}^{o}C$

At 42 days $\bar{\theta}_e = 19.52\ {}^{o}C$

$\bar{\theta}_h = 4.32\ {}^{o}C$

$\bar{V}_h = 13.765 \times 10^{11}\ m^3$

$\bar{A}_{th} = 0.1749 \times 10^{11}\ m^2$

$\hat{k}_{th} = 0.1051$ m/day

TIME PERIOD 42-70 days

At 42 days $\bar{\theta}_e = 17.93\ {}^{o}C$

$\bar{\theta}_h = 19.55\ {}^{o}C$

At 70 days $\bar{\theta}_e = 19.55\ {}^{o}C$

$\bar{\theta}_h = 4.81\ {}^{o}C$

$\bar{V}_h = 12.606 \times 10^{11}\ m^3$

$\bar{A}_{th} = 0.1683 \times 10^{11}\ m^3$

$\hat{k}_{th} = 0.626$ m/day

TIME PERIOD 70-98 days

At 70 days $\bar{\theta}_e = 18.38\ {}^{o}C$

$\bar{\theta}_h = 4.39\ {}^{o}C$

At 98 days $\bar{\theta}_e = 18.41\ {}^{o}C$

$\bar{\theta}_h = 4.66\ {}^{o}C$

$\bar{V}_h = 11.585 \times 10^{11}\ m^3$

$\bar{A}_{th} = 0.162 \times 10^{11}\ m^2$

$\hat{k}_{th} = 0.0495$ m/day

APPENDIX C

CALCULATION OF THE AVERAGE VERTICAL EXCHANGE COEFFICIENT, $\hat{k}_{th}$, OVER THE STRATIFICATION PERIOD 1966-1970,1972

CALCULATION OF OVERALL $\hat{k}_{th}$ FOR SUMMER OF 1966

t(o): 8 June, 1966
t(t): 14 Sept., 1966

Δt = 98 days

Avg. Depth of Thermocline: 10.35 m

For t=0 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP.
0 - 0.3	.003	12.5	.0375
0.3 - 0.7	.005	11.0	.0550
0.7 - 1.85	.012	9.6	.1152
1.85 - 3.40	.018	8.6	.1548
3.40 - 5.90	.027	7.5	.2025
5.9 - 10.25	.048	6.5	.3120
	.113		.8770
10.25-17.1	.069	5.5	.3795
17.1 -32.1	.138	4.5	.6210
32.1-bottom	.688	4.0	2.7200
	.887		3.7205

$\bar{\theta}_e = 7.76$

$\bar{\theta}_h = 4.19$

For t=98 days

0 - 1.3	.015	20.1	0.3015
1.3 - 6.8	.060	19.5	1.1700
6.8 - 10.3	.039	18.65	0.7274
	.114		2.1989
10.34-12.4	.022	18.15	0.3993
12.4 -17.25	.053	16.6	0.8798
17.75-18.7	.009	14.6	0.1314
18.7 -19.7	.011	13.0	0.1430
19.7 -20.25	.006	11.0	0.0660
20.25-21.9	.015	9.0	0.1350
21.9 -23.1	.010	2.5	0.0750
23.1 -28.5	.048	6.0	0.2880
28.5 -50.0	.186	4.5	0.8370
50.0-bottom	.526	4	2.1040
	.886		5.0585

$\bar{\theta}_e = 19.29$

$\bar{\theta}_e = 5.71$

$$\hat{k}_{th} = \frac{13.662}{.1736}\left(\frac{5.71 - 4.19}{98(13.52 - 4.95)}\right) = 0.1424$$

CALCULATION OF OVERALL $\hat{k}_{th}$ FOR SUMMER OF 1967

t(o): 14 June, 1967
t(t): 3 Oct., 1967

Δt = 111 days

Avg. Depth of Thermocline: 11.0 m

For t=0 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP.
0 - 0.6	.007	16.5	.1155
0.6 - 2.2	.017	15.0	.2550
2.2 - 3.8	.018	13.0	.2340
3.8 - 5.4	.018	11.0	.1980
5.4 - 6.3	.011	9.5	.1045
6.3 - 7.4	.011	8.5	.0935
7.4 - 8.6	.014	7.5	.1050
8.6 - 11.0	.025	6.4	.1600
	.121		1.2655
11.0 - 15.0	.041	5.4	.2214
15.0 - 29.6	.135	4.5	.6075
29.6 - bottom	.703	4	2.8120
	.879		3.6409

$\bar{\theta}_e$ = 10.46

θ_h 4.14

For t=111 days

0 - 2.9	.031	16.25	.5038
2.9 - 8.2	.060	15.5	.9300
8.2 - 11.0	.030	14.7	.4410
	.121		1.8748
11.0 - 12.3	.015	14.2	.2130
12.3 - 17.8	.054	13.0	.7020
17.8 - 19.4	.016	11.5	.1840
19.4 - 20.8	.014	10.5	.1478
20.8 - 22.0	.011	9.5	.1045
22.0 - 24.1	.017	8.5	.1445
24.1 - 30.8	.062	7.0	.4340
30.8 - 46.5	.135	5.0	.6750
64.5 - bottom	.555	4.0	2.2200
	.879		4.8240

$\bar{\theta}_e$ = 15.49

$\bar{\theta}_h$ = 5.49

$$\hat{k}_{th} = \frac{13.3770}{.1715}\left(\frac{5.49 - 4.14}{111(12.975 - 4.815)}\right) = 0.1163$$

CALCULATION OF OVERALL $\hat{k}_{th}$ FOR SUMMER OF 1968

t(o): 6 July, 1968
t(t): 22 Aug., 1968

Δt = 47 days

Avg. Depth of Thermocline: 15.75 m

For t=0 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP.
0 - 10.0	.111	11.75	1.3042
10.0 - 12.0	.022	11.25	.2475
12.0 - 15.0	.029	10.5	.3045
15.0 - 15.75	.007	9.85	.0690
	.169		1.9252
15.75- 17.5	.018	9.35	.1683
17.5 - 20.0	.025	8.5	.2125
20.0 - 28.2	.073	7.0	.5110
28.2 - 37.0	.080	5.0	.4000
37.0 - bottom	.635	4.0	2.5400
	.831		3.8318

$\bar{\theta}_e = 11.39$

$\bar{\theta}_h = 4.61$

For t=47 days

0 - 8.8	.098	18.7	1.8326
8.8 - 13.0	.044	17.5	.7700
13.0 - 15.75	.027	16.45	.4442
	.169		3.0468
15.75- 17.2	.015	15.55	.2332
17.2 - 17.8	.006	14.5	.870
17.8 - 19.0	.012	13.0	.1560
19.0 - 21.5	.025	11.0	.2750
21.5 - 25.6	.035	9.0	.3150
25.6 - 31.0	.050	7.0	.3500
31.0 - 36.7	.050	5.0	.2500
36.7 - bottom	.638	4.0	2.5520
	0.831		4.2182

$\bar{\theta}_e = 18.03$

$\bar{\theta}_h = 5.08$

$$\hat{k}_{th} = \frac{12.1229}{.1655}\left(\frac{5.08 - 4.61}{47(14.71 - 4.845)}\right) = 0.07425$$

CALCULATION OF OVERALL $\hat{k}_{th}$ FOR SUMMER OF 1969

t(o): 13 June, 1969
t(t): 7 Oct., 1969

Δt = 116 days

Avg. Depth of Thermocline: 16.5 m

For t=0 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP
0 - 7	.077	10.3	.7931
7 - 10.2	.036	9.5	.3420
10.2 - 10.8	.006	8.5	.0510
10.8 - 11.4	.007	7.5	.0525
11.4 - 12.8	.014	6.5	.0910
12.8 - 16.5	.037	5.65	.2090
	.177		1.5386
16.5 - 21.0	.045	5.15	.2318
21.0 - 37.6	.148	4.5	.6660
37.6 - bottom	.630	4.0	2.5200
	.823		3.4178

$\bar{\theta}_e = 8.69$

$\bar{\theta}_h$ 4.15

For t=116 days

0 - 10.8	.119	15.3	1.8207
10.8 - 16.5	.058	14.4	.8353
	.177		2.6559
16.5 - 21.4	.049	13.4	.6566
21.4 - 23.8	.019	12.5	.2375
23.8 - 25.0	.012	11.5	.1380
25.0 - 26.6	.013	10.5	.1365
26.6 - 30.6	.037	9.0	.3330
30.6 - 38.0	.066	7.0	.4620
30.0 - 46.0	.067	5.0	.3350
46.0 - bottom	.560	4.0	2.2400
	.823		4.5386

$\bar{\theta}_e = 15.00$

$\bar{\theta}_h = 5.51$

$$\hat{k}_{th} = \frac{11.9262}{.1645}\left(\frac{5.51 - 4.15}{116(11.845 - 4.83)}\right) = 0.1212$$

CALCULATION OF OVERALL $\hat{k}_{th}$ FOR SUMMER OF 1970

t(o): 28 June, 1970
t(t): 19 Sept., 1970

Δt = 83 days

Depth of Avg. Thermocline: 9 m

For t=0 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP.
0 - 5.0	.055	12.5	.6875
5.0 - 7.4	.027	11.5	.3105
7.4 - 9.0	.018	10.7	.1926
	.100		1.1906
9.0 - 10.0	.011	10.2	.1122
10.0 - 12.8	.029	9.5	.2755
12.8 - 16.4	.036	8.5	.3060
16.4 - 20.4	.039	7.5	.2925
20.4 - 26.0	.050	6.5	.3250
26.0 - 38.3	.110	5.0	.5500
38.3 - bottom	.625	4.0	2.5000
	.900		4.3612

$\bar{\theta}_e = 11.91$

$\bar{\theta}_h$ 4.84

For t=83 days

0 - 9.0	.100	16.05	1.6050
	.1		1.6050
9.0 - 14.4	.055	15.45	.8498
14.4 - 16.2	.019	14.5	.2755
16.2 - 19.7	.035	13.0	.4550
19.7 - 23.8	.036	11.0	.3960
23.8 - 28.6	.043	9.0	.3870
28.6 - 33.3	.043	7.0	.3010
33.3 - 38.2	.043	5.0	.2150
38.2 - bottom	.626	4.0	2.5040
	.900		5.3833

$\bar{\theta}_e = 16.05$

$\bar{\theta}_h = 5.98$

$$\hat{k}_{th} = \frac{14.096}{.1762}\left(\frac{5.98 - 4.84}{83(13.98 - 5.41)}\right) = 0.1282$$

CALCULATION OF OVERALL $\hat{k}_{th}$ FOR SUMMER OF 1972

t(o) : June 20, 1972
t(t) : Sept. 20, 1972

Δt = 92 days

Avg. Depth of Thermocline: 13.25 m

For t=0 days

DEPTH IN METERS	VOL. FRACN.	AVG. TEMP.	VOL. x TEMP.
0 - 1.3	.015	11.1	.1665
1.3 - 4.1	.030	10.5	.3150
4.1 - 6.0	.022	9.5	.2090
6.0 - 7.8	.020	8.5	.1700
7.8 - 10.2	.026	7.5	.1950
10.2 - 13.25	.031	6.6	.2046
	.144		1.2601
13.25 - 14.1	.008	6.1	.0488
14.1 - 20.0	.060	5.5	.3300
20.0 - 42.0	.194	4.5	.8730
42.0 - bottom	.594	4.0	2.6278
	.856		3.6278

$\bar{\theta}_e = 8.75$

$\bar{\theta}_h = 4.24$

For t=92 days

0 - 9.5	.106	17.35	1.8391
9.5 - 13.25	.038	16.6	.6308
	.144		2.4699
13.25 -14.0	.007	16.1	.1127
14.0 - 16.6	.026	15.5	.4030
16.6 - 18.6	.020	14.5	.2900
18.6 - 20.2	.016	13.5	.2160
20.2 - 21.7	.015	12.5	.1875
21.7 - 23.0	.010	11.5	.1150
23.0 - 24.6	.015	10.5	.1575
24.6 - 28.2	.032	9.0	.2880
28.2 - 34.4	.057	7.0	.3990
34.4 - 42.0	.064	5.5	.3520
42.0 - bottom	.594	4.0	2.3760
	.856		4.8967

$\bar{\theta}_e = 17.15$

$\theta_h = 5.72$

$$\hat{k}_{th} = \frac{12.7942}{.1689}\left(\frac{5.72 - 4.24}{92(12.95 - 4.98)}\right) = 0.1529$$

APPENDIX D

LAKEWIDE AVERAGES OF DO AND P ACTUAL DATA

RESULTS - 1966

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

	STRATIFICATION PERIOD		PHOSPHORUS AS P		
DATE OF CRUISE	TEMP. (^{o}C)	DO (mg/l)	TFP (μg/l)	SRP (μg/l)	TP (μg/l)
June 25 = 5.83	E 17.49	12.42			
	H 4.83	12.71			
June 30 = 6.00	E 21.48	11.23			
	H 5.62	12.39			
July 10 = 6.33	E 18.04	10.87			
	H 4.82	12.45			
July 15 =	E				
	H	12.01			
July 24 = 6.80	E 18.80	9.65			
	H 5.17	12.40			
Aug. 7 = 7.23	E 17.52	10.02			
	H 4.73	12.41			
Aug. 19 = 7.63	E 18.20	9.41			
	H 5.17	11.85			
Sept. 2 = 8.07	E 17.01	9.71			
	H 4.69	12.17			
Sept.11 = 8.37	E				
	H	10.38			
Sept.16 = 8.53	E 16.83	9.45			
	H 4.51	12.05			
Sept.24 = 8.80	E 11.49	10.13			
	H 4.36	11.93			
Sept.29 = 8.97	E 12.91	10.09			
	H 5.01	11.87			
Oct. 3 = 9.10	E 11.99				
	H 4.88	12.23			

E - epilimnion
H - hypolimnion
TFP - total filtered phosphorus
SRP - soluble reactive phosphorus
TP - total phosphorus

RESULTS - 1967

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

	STRATIFICATION PERIOD		PHOSPHORUS AS P		
DATE OF CRUISE	TEMP. (°C)	DO (mg/l)	TFP (μg/l)	SRP (μg/l)	TP (μg/l)
June 17 = 5.57	E 14.17	13.15			
	H 4.26	13.09			
June 29 = 5.97	E 14.92	11.90			
	H 5.06	12.49			
July 13 = 6.43	E 19.65	10.08			
	H 4.95	11.66			
Aug. 25 = 7.83	E 20.30	8.77			
	H 5.44	11.32			
Sept. 9 = 8.30	E 18.15	9.53			
	H 6.33	11.04			
Sept.21 = 8.70	E 18.48	9.64			15.1
	H 5.53	10.95			24.7
Oct. 6 = 9.20	E 12.26	10.15			17.9
	H 5.61	11.35			23.1
	NON-STRATIFICATION PERIOD				
Oct. 21 = 9.70	6.46	11.23			22.3
Nov. 2 = 10.07	6.29	11.26			16.6

RESULTS - 1968

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

	STRATIFICATION PERIOD		PHOSPHORUS AS P		
DATE OF CRUISE	TEMP. (^{o}C)	DO (mg/l)	TFP (µg/l)	SRP (µg/l)	TP (µg/l)
July 6 = 6.20	E 13.00	12.59		6.9	21.6
	H 4.75	12.90		10.7	
July 27= 6.90	E 16.52	10.96			
	H 4.80	12.56			
Aug. 22 = 7.73	E 17.14	9.64			
	H 5.05	11.95			
Sept.12 = 8.40	E 17.5	9.27			
	H 4.95	11.59			
Oct. 8 = 9.27	E 14.41	10.13		7.1	17.0
	H 6.85	11.77		15.3	21.4
	NON-STRATIFICATION PERIOD				
May 5 = 4.17	3.31	13.72			
May 30 = 5.00	4.14	13.61			
Oct.31 =10.00	6.84	11.56		15.9	18.4
Nov.22 =10.73	5.69	11.45		2.7	5.6

RESULTS - 1969

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

	STRATIFICATION PERIOD		PHOSPHORUS AS P		
DATE OF CRUISE	TEMP. (^{o}C)	DO (mg/l)	TFP (μg/l)	SRP (μg/l)	TP (μg/l)
June 21= 5.70	E 10.16 H 4.18	13.48 12.76			
July 13= 6.43	E 14.72 H 4.66	11.82 12.54		3.0 9.9	21.7 19.7
Aug. 10= 7.33	E 19.28 H 4.97	9.07 11.81		4.9 13.2	20.8 24.8
Aug. 22= 7.73	E 11.79 H 4.13	11.58 11.59			
Sept.9 = 8.30	E 18.72 H 4.78	9.23 11.35		3.1 10.0	22.6 21.7
Oct. 7 = 9.23	E 14.46 H 5.15	9.9 11.04		2.1 10.1	18.4 27.8
	NON-STRATIFICATION PERIOD				
April 16 = 3.53	2.43	13.69		13.8	
May 17 = 4.57	3.93	13.67		8.9	23.8
June 4 = 5.13	4.80	13.28			
June 13 = 5.43	5.02	13.24		6.3	23.8
Nov. 4 =10.13	6.1	11.79		12.1	26.3
Dec. 6 =11.20	4.79	12.16		12.2	23.7

RESULTS - 1970

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

DATE OF CRUISE		STRATIFICATION PERIOD		PHOSPHORUS AS P	
	TEMP. (^oC)	DO (mg/l)	TFP (µg/l)	SRP (µg/l)	TP (µg/l)
June 28 = 5.93	E 12.54	12.40			
	H 5.30	12.70			
July 20 = 6.67	E 16.82	10.61			
	H 5.36	12.10			
Aug. 21 = 7.70	E 20.77	8.76			
	H 5.66	11.10			
Sept.19 = 8.63	E 16.05	9.85			
	H 5.74	11.35			
	NON-STRATIFICATION PERIOD				
Feb. 8 = 1.27	1.92	13.14		14.6	24.7
Mar. 8 = 2.27	1.57	13.39		13.3	23.5
Apr. 5 = 3.17	1.81	13.52			
May 2 = 4.07	3.00	13.64			
May 29 = 4.97	4.26	13.35			
Oct. 17 = 9.57		10.83			
Nov. 20 =10.67	6.84	11.14			
Dec. 11 =11.37	5.11	11.84			

RESULTS - 1971

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

DATE OF CRUISE	STRATIFICATION PERIOD TEMP. (^{o}C)	 DO (mg/l)	PHOSPHORUS AS P TFP (μg/l)	 SRP (μg/l)	 TP (μg/l)
Aug. 13 = 7.43	E 11.50	10.40			23.3
	H 5.12	12.17			24.3
	NON-STRATIFICATION PERIOD				
Apr. 3 = 3.10	1.36	13.65		13.5	27.4
May 6 = 3.47					
Nov. 19 =10.63	5.93		13.9	12.7	16.9

RESULTS - 1972

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

	STRATIFICATION PERIOD		PHOSPHORUS AS P		
DATE OF CRUISE	TEMP. (^{O}C)	DO (mg/l)	TFP (μg/l)	SRP (μg/l)	TP (μg/l)
July 1 = 6.03	E 9.89	13.91	11.1	4.4	21.0
	H 4.68	13.56	14.5	9.6	20.0
July 29= 6.97	E 17.25	11.72	8.6	2.0	18.4
	H 3.94	13.03	14.7	10.9	19.0
Sept.16= 8.53	E 19.05	9.80	7.1	1.2	14.7
	H 4.85	12.21	16.5	11.0	20.2
Sept.23=8.77	E 16.72	9.66			
	H 5.46	11.28			
	NON-STRATIFICATION PERIOD				
Apr. 22 = 3.73	1.98	13.63	16.6	13.2	23.1
June 3 = 5.10		13.81			
Oct. 28 = 9.93	6.0	11.76	15.5	9.9	19.8
Dec. 2 =11.07	5.70	12.14	17.3	10.5	21.3

RESULTS - 1973

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

	NON-STRATIFICATION PERIOD		PHOSPHORUS AS P		
DATE OF CRUISE	TEMP. (°C)	DO (mg/l)	TFP (µg/l)	SRP (µg/l)	TP (µg/l)
Jan. 19 = 0.63	3.15	12.80	19.8	13.7	23.6
Mar. 17 = 2.57	2.19	13.47	20.3	15.0	23.2
Mar. 24 = 2.80	2.20	13.52	19.2	16.0	24.7
Apr. 28 = 3.93	3.12	13.56	20.0	13.9	31.5
Dec. 6 =11.20	5.60	11.72	15.3	12.1	18.8
Nov. 3 =10.10		11.30			

RESULTS - 1974

LAKE-WIDE AVERAGES OF DO AND PHOSPHORUS

	STRATIFICATION PERIOD		PHOSPHORUS AS P		
DATE OF CRUISE	TEMP. (°C)	DO (mg/l)	TFP (μg/l)	SRP (μg/l)	TP (μg/l)
June 20= 5/67	E 11.88				
	H 3.96	13.38			
July 5 = 6.17	E 15.04				
	H 4.06	12.49			24.1
July 28= 6.93	E 19.17				
	H 4.00	12.06			
Aug.10 = 7.33	E 20.08				
	H 3.92	11.93			
Aug.16 = 7.53	E 20.93				
	H 4.00	11.42			
Aug.22 = 7.73	E 23.02				
	H 4.35	11.84			
Sept.7 = 8.23	E 19.23				
	H 4.32	11.20			28.4
Sept.20= 8.67	E 15.83				
	H	11.57			
Oct. 5 = 9.17	E 11.21				
	H 3.88	11.49			
	NON-STRATIFICATION PERIOD				
Apr. 4 = 3.13	1.96	13.48	19.3	14.7	24.8
Apr. 19 = 3.63	2.56				
May 2 = 4.07	3.22	13.92			
May 16 = 4.53	3.70	13.7			
June 7 = 5.23		13.38			
Oct. 18 =		11.19			
Nov. 29 =10.97	5.29	12.66			23.8
Dec. 18 =11.60	4.92				

APPENDIX E

PART 1: DISSOLVED OXYGEN vs TIME FOR DIFFERENT YEARS

PART 2: CALCULATION OF OBSERVED HYPOLIMNETIC DO FLUXES

FIGURE E-1
DISSOLVED OXYGEN VS TIME
OBSERVED DATA 1966 STRATIFICATION

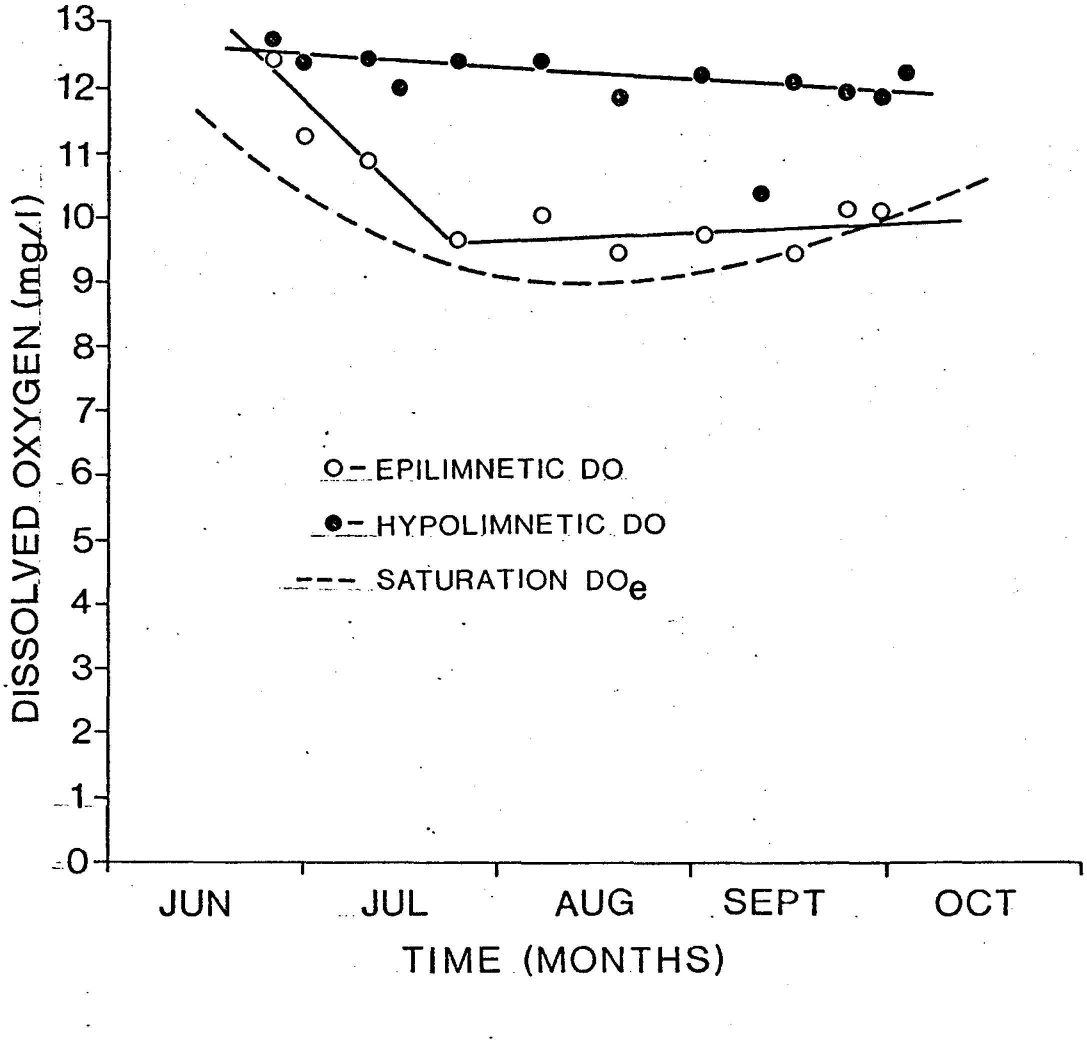


FIGURE E-2
DISSOLVED OXYGEN VS TIME
OBSERVED DATA 1967 STRATIFICATION

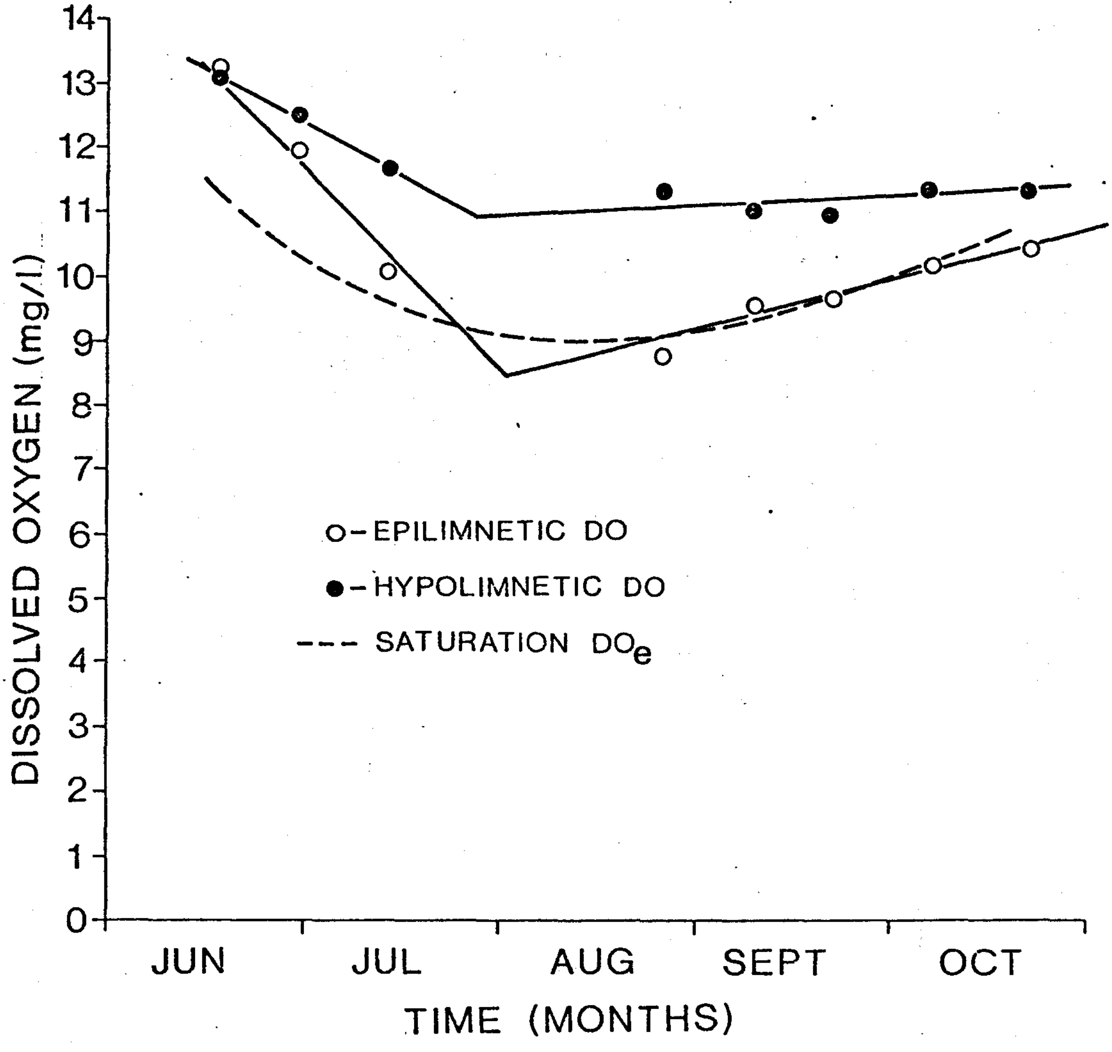


FIGURE E-3
DISSOLVED OXYGEN VS TIME
OBSERVED DATA 1968 STRATIFICATION

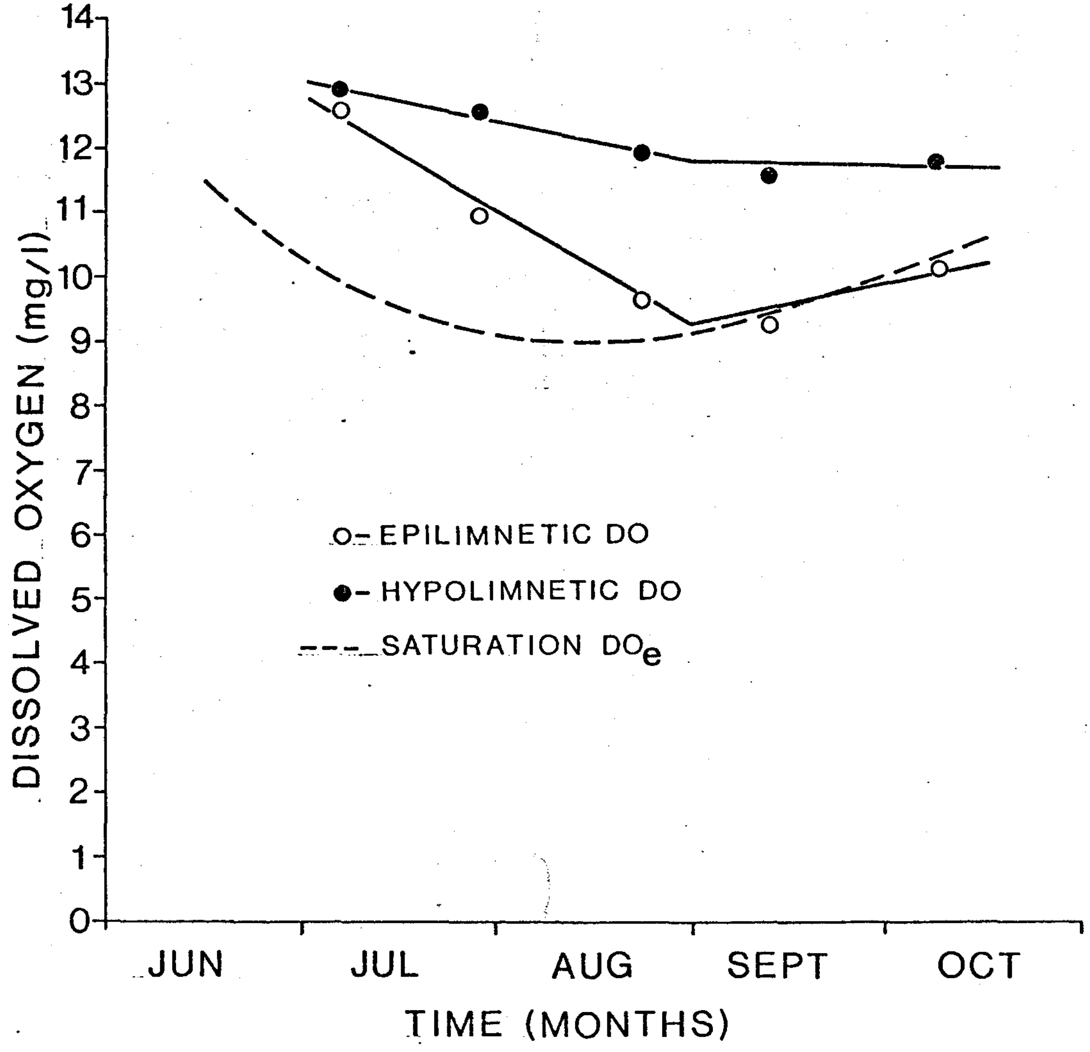


FIGURE E-4
DISSOLVED OXYGEN VS TIME
OBSERVED DATA 1969 STRATIFICATION

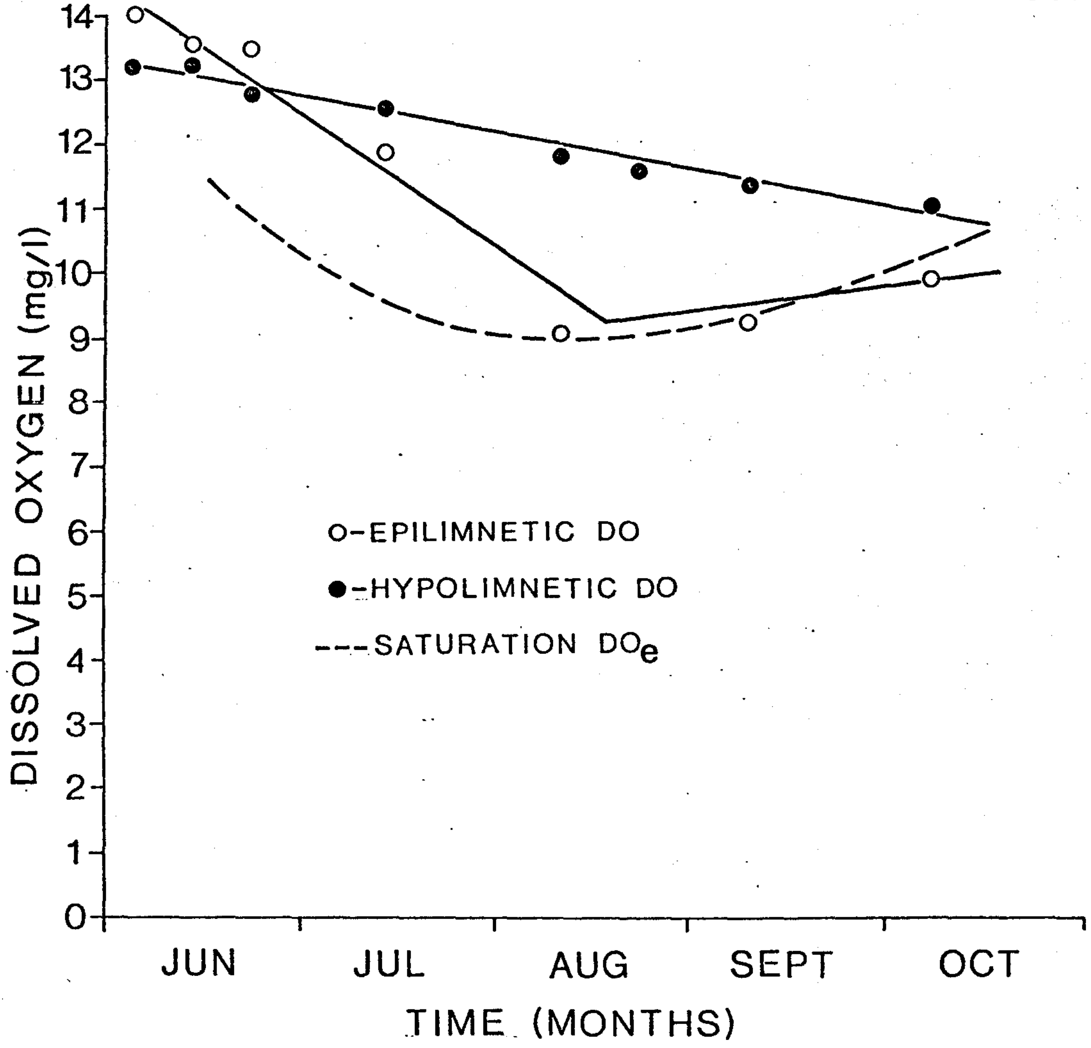


FIGURE E-5
DISSOLVED OXYGEN VS TIME
OBSERVED DATA 1970 STRATIFICATION

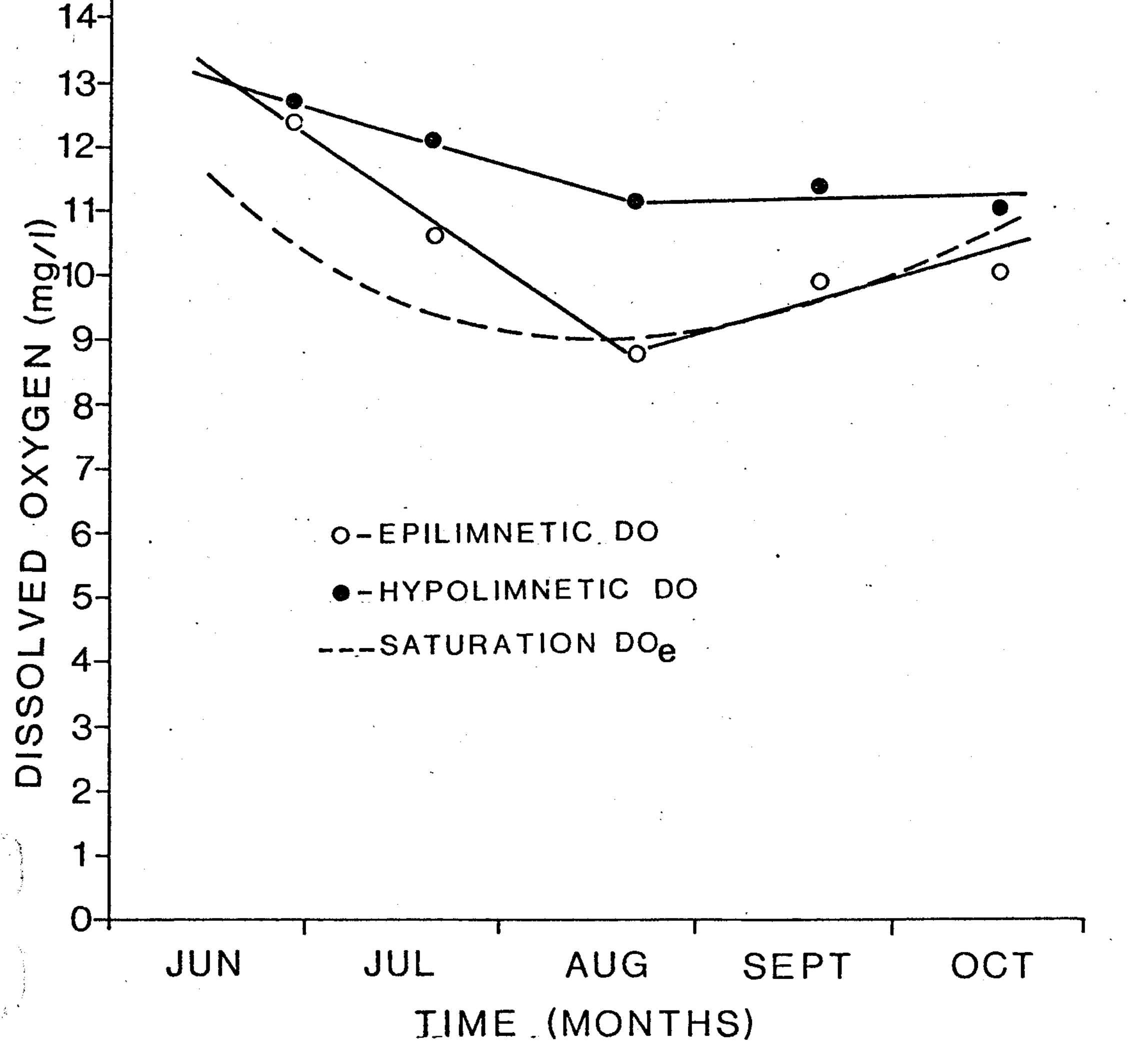
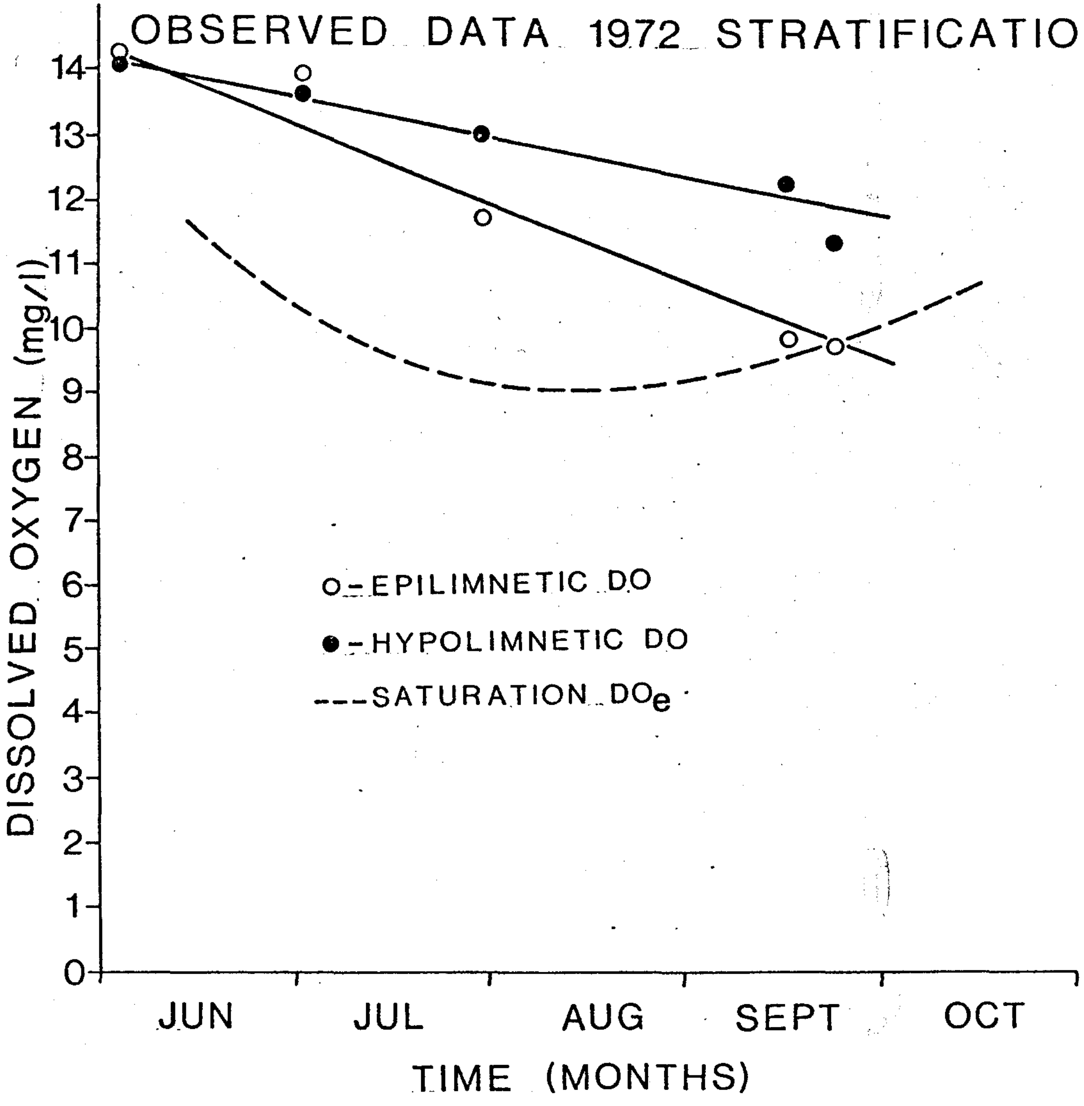


FIGURE E-6
DISSOLVED OXYGEN VS TIME
OBSERVED DATA 1972 STRATIFICATION



APPENDIX E - PART 2

CALCULATION OF OBSERVED HYPOLIMNETIC DO FLUXES

a) CALCULATION OF TOTAL FLUX OF OXYGEN FROM THE HYPOLIMNION

Method:

$$\text{Total Flux} = \frac{\text{Total Depletion over } \Delta t}{\Delta t} \times \text{Average depth of hypolimnion (at } \bar{t}\text{)}$$

$$\Delta t = t_2 - t_1$$

$\bar{t}$ = time at middle of time period

$$\text{Total Depletion} = DO_h(t_2) - DO_h(t_2)$$

$$\text{Average Depth of Hypolimnion} = \frac{V_h}{A_{th}} \text{ at time } \bar{t}$$

1966 - Middle of time period (102 days) - 51 days or Aug. 10
7.33 months

$Z_e = 16.315$ m
$V_e = 2.82 \times 10^{11} m^3$
$V_h = 13.476 \times 10^{11} m^3$ → $Z_h = 82.91$ m
$A_{th} = 1.6254 \times 10^{10} m^2$

$$TF = (12.6 - 11.95) \times \frac{82.91}{102} = 0.53$$

1967 - Middle of time period (112 days) - 56 days or Aug. 15
7.50 months

$Z_e = 9.48$ m
$V_e = 1.679 \times 10^{11} m^3$
$V_h = 14.617 \times 10^{11} m^3$ → $Z_h = 85.4$ m
$A_{th} = 1.7116 \times 10^{10} m^2$

$$TF = (12.9 - 11.1) \times \frac{85.4}{112} = 1.37$$

1968 - Middle of time period (102 days) - 51 days or Aug. 20
7.67 months

$Z_e = 16.387$
$V_e = 2.831 \times 10^{11} m^3$
$V_h = 13.4647 \times 10^{11} m^3$ $\rightarrow Z_h = 82.88$ m
$A_{th} = 1.6245 \times 10^{10} m^2$

$$TF = (13.05 - 11.7) \times \frac{82.88}{102}$$

$$= 1.10$$

1969 - Middle of time period (122 days) - 61 days or Aug. 10
7.33 months

$Z_e = 15.677$ m
$V_e = 2.716 \times 10^{11} m^3$
$V_h = 13.580 \times 10^{11} m^3$ $\rightarrow Z_h = 83.14$ m
$A_{th} = 1.6335 \times 10^{10} m^2$

$$TF = (13.1 - 10.9) \times \frac{83.14}{122}$$

$$= 1.50$$

1970 - Middle of time period (102 days) - 51 days or Aug. 10
7.33 months

$Z_e = 9.688$ m
$V_e = 1.715 \times 10^{11} m^3$
$V_h = 14.581 \times 10^{11} m^3$ $\rightarrow Z_h = 85.32$ m
$A_{th} = 1.7089 \times 10^{10} m^2$

$$TF = (13.0 - 11.2) \frac{85.32}{102}$$
$$= 1.51$$

1972 - Middle of time period (92 days) - 46 days or Aug. 5
7.17 months

$Z_e = 13.059$ m
$V_e = 2.284 \times 10^{11} m^3$
$V_h = 14.012 \times 10^{11} m^3$ $\rightarrow Z_h = 84.08$ m
$A_{th} = 1.6665 \times 10^{10} m^2$

$$TF = (13.75 - 11.) \frac{84.08}{92}$$

$$= 1.69$$

b) CALCULATION OF VERTICAL TRANSPORT FLUX OF OXYGEN FROM THE HYPOLIMNION

Method:

Vertical Transport Flux $= VT_f = (\overline{DO}_h - \overline{DO}_e)\ \hat{K}_{th}$

where $\overline{DO}_h$ and $\overline{DO}_e$ are averaged over the time period.

Note: When two time periods are required over the entire stratification, the calculation is performed for each time period. A weighted average (over time) of the two results is used as an estimate for the total time of stratification.

1966: June 20 - July 24, $\Delta t_1 = 34$ days; $\overline{DO}_h = 12.5\ \frac{g}{m^3}$; $\overline{DO}_e = 11.15\ \frac{g}{m^3}$

July 24 - Sept. 30, $\Delta t_2 = 68$ days; $\overline{DO}_h = 12.15\ \frac{g}{m^3}$; $\overline{DO}_e = 9.8\ \frac{g}{m^3}$

$$VT_f^1 = (12.5 - 11.15)\ \frac{g}{m^3} \times 0.1424\ \text{m/day}$$
$$= 0.19\ \frac{g}{m^2 day}$$

$$VT_f^2 = (12.15 - 9.8)\ \frac{g}{m^3} \times 0.1424\ \text{m/day}$$
$$= 0.33$$

Weighted Avg: $$\overline{VT}_f = 0.19 \times \frac{34}{102} + 0.33 \times \frac{68}{102}$$
$$= 0.29\ \frac{g}{m^2 day}$$

1967: June 20 - Aug. 1, $\Delta t_1 = 42$ days; $\overline{DO}_h = 11.8$; $\overline{DO}_e = 10.6$

Aug. 1 - Oct. 10, $\Delta t_2 = 70$ days; $\overline{DO}_h = 11.1$; $\overline{DO}_e = 9.3$

$$\overline{VT}_f = [(11.8 - 10.6) + 0.1163]\ \frac{42}{112} + [11.1 - 9.3)$$
$$\times\ 0.1163]\ \frac{70}{112}$$
$$= 0.18\ \frac{g}{m^2\ day}$$

1968: June 30 - Aug. 31 $\overline{DO}_h = 12.4$ $\overline{DO}_e = 11.05$
$\Delta t_1 = 62$ days

Aug. 31 - Oct. 10 $\overline{DO}_h = 11.7$ $\overline{DO}_e = 9.7$

$$\overline{VT}_f = [(12.4 - 11.05) \times 0.0742]\ \frac{62}{102} + [(11.7 - 9.7)$$
$$\times\ 0.0742]\ \frac{40}{102}$$
$$= 0.12\ \frac{g}{m^2\ day}$$

1969: June 10 - Aug. 18 $\overline{DO}_h = 12.5$ $\overline{DO}_e = 11.5$
$\Delta t_1 = 69$ days

Aug. 18 - Oct. 10 $\overline{DO}_h = 11.4$ $\overline{DO}_e = 9.6$
$\Delta t_2 = 53$ days

$$\overline{VT}_f = [(12.5 - 11.5) \times 0.1212]\ \frac{69}{102} + [(11.4 - 9.6)$$
$$\times\ 0.1212]\ \frac{53}{122}$$
$$= 0.16\ \frac{g}{m^2 day}$$

1970: June 20 - Aug. 20 $\overline{DO}_h = 12.1$ $\overline{DO}_e = 10.8$
$\Delta t_1 = 61$ days

Aug. 20 - Sept. 30 $\overline{DO}_h = 11.15$ $\overline{DO}_e = 9.4$
$\Delta t_2 = 41$ days

$$\overline{VT}_f = [(12.1 - 10.8) \times 0.1282]\ \frac{61}{102} + [(11.15 - 9.4)$$
$$\times\ 0.1282]\ \frac{41}{102}$$
$$= 0.19\ \frac{g}{m^2\ day}$$

<u>1972:</u> June 20 - Sept. 20 $\overline{DO}_h = 12.85$ $\overline{DO}_e = 11.7$
$\Delta t = 92$ days

$$\overline{VT}_f = (12.85 - 11.7)\ 0.1529$$

$$= 0.18\ \frac{g}{m^2\ \text{day}}$$

APPENDIX F

COMPUTER PROGRAM

```
      PROGRAM OXYGEN (INPUT,OUTPUT,PUNCH,TAPE1=INPUT,TAPE3=OUTPUT,TAPE7)
      DIMENSION X(6,200),Y(3,250),A(6,5),B(3,3),W(12)
      DIMENSION ARTIME(200),YXPPE(200),YXPPH(200),YXOPE(200),YXOPH(200),
     1YXDOE(200),YXDOH(200)
      DIMENSION TE(50),TH(50),ADOOE(50),ADOOH(50)
      DIMENSION TP(200),PPEM(200),PPHM(200),OPEM(200),OPHM(200),TPEM(200
     1),TPHM(200)
      REAL NRFLOW
      INTEGER YEAR
C  READ IN THE LAKE PARAMETERS THAT ARE CONSIDERED CONSTANT FOR THE
C  TIME PERIOD OVER WHICH THE MODEL IS USED.
    1 READ(1,5)Q,VL,VEU,ASURF,ASED,Z,ZEUPH
    5 FORMAT(E14.6)
C  READ IN THE LAKE PHOSPHORUS LOADING,NIAGARA RIVER INFLOW,AND THE
C  TRIBUTARY INFLOWS.
      READ(1,6)TRFLOW,NRFLOW,PHOSLD
    6 FORMAT(E14.6)
C  READ IN THE BIOLOGICAL REAERATION VERTICAL TRANSPORT,SEDIMENTATION
C  AND PARTICULATE PHOSPHORUS TO DISSOLVED OXYGEN EQUIVALENT
C  COEFFICIENTS. THESE MAY BE SUBSEQUENTLY CHANGED DURING SENSITIVITY
C  ANALYSIS.
      READ(1,7)CVTTH,CNPE,CDH,FAC,CSOD,CNPEU,CDECOMP,CSEP,CSAF,CFLOC,
     1CAER
    7 FORMAT(E14.6)
      READ(1,8)YEAR,NDOEDP,NDOHDP
    8 FORMAT(3I4)
C  READ IN INITIAL VALUES OF DISSOLVED OXYGEN AND PHOSPHORUS.
C  PRINT OUT THE LAKE PARAMETERS,LOADINGS,INPUT FLOWS,AND THE MODEL
C  COEFFICIENTS.
      WRITE(3,11)
   11 FORMAT(1H0,≠THE LAKE PARAMETERS INFLOWS AND LOADINGS ARE AS FOLLOW
     1S≠)
      WRITE(3,12)
   12 FORMAT(1H0,≠   LAKE OUTFLOW       NIAGARA R FLOW      TRIBUTARY F
     1LOW        LAKE VOLUME    VOL EUPHOTIC ZONE ≠)
      WRITE(3,13)Q,NRFLOW,TRFLOW,VL,VEU
   13 FORMAT(1H0,5F20.4)
      WRITE(3,14)
   14 FORMAT(1H0,≠   SURFACE AREA      AREA OF SEDIMENT      DEPTH OF L
     1AKE      DEPTH EUPH ZONE  ≠)
      WRITE(3,15)ASURF,ASED,Z,ZEUPH
   15 FORMAT(1H0,4F20.4)
      WRITE(3,16)
   16 FORMAT(1H0,≠       YEAR                 PHOS LOAD      ≠)
      WRITE(3,17)YEAR,PHOSLD
   17 FORMAT(1H0,8X,I4,8X,F20.4)
      WRITE(3,18)
   18 FORMAT(1H0,≠THE MODEL COEFFICIENTS(BIOLOGICAL,PHYSICAL RATE AND TR
     1ANSPORT) ARE AS FOLLOWS≠)
      WRITE(3,21)
   21 FORMAT(1H0,≠  COEFF OF V T TH   COEFF NET PROD EPI   COEFF DECOMP
     1 HYPO    DO-PHOS FACTOR     SED OXYGEN DEMAND ≠)
      WRITE(3,22)CVTTH,CNPE,CDH,FAC,CSOD
```

```
   22 FORMAT(1H0,6F20.4)
      WRITE(3,23)
   23 FORMAT(1H0,≠ COEFF NET PROD EUPH    COEFF DECOMP          COEFF SED EP
     1ILIM   COEFF SED ABS FLOC        COEFF FLOC          SURF AER COEFF
     2≠)
      WRITE(3,24)CNPEU,CDECOMP,CSEP,CSAF,CFLOC,CAER
   24 FORMAT(1H0,6F20.4)
      DO 90 I=1,6
      DO 90 J=1,6
      A(I,J)=0.
   90 CONTINUE
      DO 91 I=1,3
      DO 91 J=1,3
      B(I,J)=0.
   91 CONTINUE
      DO 92 I=1,12
      W(I)=0.
   92 CONTINUE
      IF(YEAR.EQ.1966)GO TO 59
C     WINTER MODEL PREDICTIONS
C     WINTER MODEL PREDICTIONS
C     WINTER MODEL PREDICTIONS
      TIME=0.0
      H=0.25
      J=1
      CAER=7.50
C     CALCULATE THE TERMS OF THE COEFFICIENT MATRIX--WINTER MODEL
      B(1,1)=-Q/VL-CDECOMP-(CSAF*(1.+CFLOC*(Z-ZEUPH)))*ASED/VL
      B(1,2)=CNPEU*VEU/VL
      B(2,1)=CDECOMP
      B(2,2)=-Q/VL-CNPEU*VEU/VL
      B(3,1)=-CDECOMP*FAC
      B(3,2)=(CNPEU*VEU/VL)*FAC
      B(3,3)=-CAER*ASURF/VL-Q/VL
      W(10)=PHOSLD/VL
      WRITE(3,70)
   70 FORMAT(1H0,18X≠PREDICTED WINTER RESULTS    ≠)
      WRITE(3,71)
   71 FORMAT(1H0,≠TIME(MONTHS)       PP             OP            DO        ≠)
      WRITE(3,72)TIME,(Y(JJ,J),JJ=1,3)
   72 FORMAT(1H0,4X,F5.2,3X,3(2X,F10.2))
      TP(1)=TIME
      PPEM(1)=Y(1,1)
      PPHM(1)=Y(2,1)
      OPEM(1)=Y(3,1)
      OPHM(1)=Y(4,1)
      ARTIME(1)=TIME
      YXPPE(1)=Y(1,1)
      YXPPH(1)=Y(1,1)
      YXOPE(1)=Y(2,1)
      YXOPH(1)=Y(2,1)
      YXDOE(1)=Y(3,1)
      YXDOH(1)=Y(3,1)
```

```
      DO 250 LLL=1,33
      IF(TIME.LE.5.31)GO TO 201
      IF(TIME.GT.5.31.AND.TIME.LE.6.61)GO TO 202
      IF(TIME.GT.6.61)GO TO 203
  201 D=TIME
      TSURF=((.1584*D-.5972)*D-.0716)*D+2.6058
      GO TO 205
  202 D=TIME-5.31
      TSURF=((-1.4288*D+1.9278)*D+6.9978)*D+9.1264
      GO TO 205
  203 D=TIME-6.61
      TSURF=((.4278*D-3.6162)*D+4.814)*D+18.3106
      GO TO 205
  205 DOSATN=14.48-.36*TSURF+.0043*TSURF**2.
      W(11)=(CAER*ASURF*DOSATN/VL)+(DOSATN*(TRFLOW+NFFLOW)/VL)-(CSOD*ASF
     1D/VL)
      DO 235 LL=1,5
      YP1=Y(1,J)
      YP2=Y(2,J)
      YP3=Y(3,J)
      DO 225 L=1,4
      Y1=YP1+(B(1,1)*YP1  + B(1,2)*YP2  + B(1,3)*YP3  + W(9))*(H/2.)
      Y2=YP2+(B(2,1)*YP1  + B(2,2)*YP2  + B(2,3)*YP3  + W(10))*(H/2.)
      Y3=YP3+(B(3,1)*YP1  + B(3,2)*YP2  + B(3,3)*YP3  + W(11))*(H/2.)
      YY1= Y(1,J)  +  (B(1,1)*Y1  + B(1,2)*Y2  + B(1,3)*Y3  + W(9))*H
      YY2= Y(2,J)  +  (B(2,1)*Y1  + B(2,2)*Y2  + B(2,3)*Y3  + W(10))*H
      YY3= Y(3,J)  +  (B(3,1)*Y1  + B(3,2)*Y2  + B(3,3)*Y3  + W(11))*H
      YP1=YY1
      YP2=YY2
      YP3=YY3
  225 CONTINUE
      Y(1,J+1)=YY1
      Y(2,J+1)=YY2
      Y(3,J+1)=YY3
      J=J+1
  235 CONTINUE
      TIME=0.0+LLL*5./30.
      CALL PLOTPT(TIME,Y(3,J),11)
      CALL PLOTPT(TIME,Y(3,J),12)
      GO TO 241
  241 WRITE(3,255)TIME,(Y(II,J),II=1,3)
  255 FORMAT(1H0,4X,F5.2,3X,3(2X,F10.2))
       JCOUNT=LLL+1
      ARTIME(JCOUNT)=TIME
      YXPPE(JCOUNT)=Y(1,J)
      YXPPH(JCOUNT)=Y(1,J)
      YXOPE(JCOUNT)=Y(2,J)
      YXOPH(JCOUNT)=Y(2,J)
      YXDOE(JCOUNT)=Y(3,J)
      YXDOH(JCOUNT)=Y(3,J)
      TP(JCOUNT)=TIME
      PPEM(JCOUNT)=Y(1,J)
      PPHM(JCOUNT)=Y(1,J)
```

```
          OPEM(JCOUNT)=Y(2, J)
          OPHM(JCOUNT)=Y(2, J)
      250 CONTINUE
          TIME=5.5
          H=0.25
          I=1
          CAEP=2.0
          X(1,I)= Y(1,J)
          X(2,I)=Y(1,J)
          X(3,I)=Y(2,J)
          X(4,I)= Y(2,J)
          X(5,I)= Y(3,J)
          X(6,I) = Y(3,J)
C     SUMMER MODEL PREDICTIONS
C     SUMMER MODEL PREDICTIONS
C     SUMMER MODEL PREDICTIONS
          ICOUNT=JCOUNT+1
          ARTIME(ICOUNT)=TIME
          YXPPE(ICOUNT)=X(1,1)
          YXPPH(ICOUNT)=X(2,1)
          YXOPE(ICOUNT)=X(3,1)
          YXOPH(ICOUNT)=X(4,1)
          YXDOE(ICOUNT)=X(5,1)
          YXDOH(ICOUNT)=X(6,1)
          TP(ICOUNT)=TIME
          PPEM(ICOUNT)=X(1,1)
          PPHM(ICOUNT)=X(2,1)
          OPEM(ICOUNT)=X(3,1)
          OPHM(ICOUNT)=X(4,1)
          GO TO 58
       59 TIME=5.50
          I=1
          H=0.25
          CAEP=2.0
          X(1,1)=4.00
          X(2,1)=4.00
          X(3,1)=20.00
          X(4,1)=20.00
          X(5,1)=13.10
          X(6,1)=13.10
          ARTIME(1)=TIME
          YXPPE(1)=X(1,1)
          YXPPH(1)=X(2,1)
          YXOPE(1)=X(3,1)
          YXOPH(1)=X(4,1)
          YXDOE(1)=X(5,1)
          YXDOH(1)=X(6,1)
          TP(1)=TIME
          PPEM(1)=X(1,1)
          PPHM(1)=X(2,1)
          OPEM(1)=X(3,1)
          OPHM(1)=X(4,1)
       58 WRITE(3,60)
```

```
   60 FORMAT(1H1,16X,≠PREDICTED SUMMER STRATIFICATION RESULTS≠)
      WRITE(3,61)
   61 FORMAT(1H0,≠TIME(MONTHS                 PP EP        PPHYP          OP EP
     1OP HYP         DO EP          DO HYP      ≠)
      WRITE(3,62)TIME,(X(II,1),II=1,6)
   62 FORMAT(1H0,4X,F5.2,3X,6(2X,F10.2))
C     CALCULATE TERMS OF COEFFICIENTS MATRIX--SUMMER MODEL
C     NOTE THAT ZEP,ZHYP,VEP,VHYPAND DO SATURATION ARE TIME DEPENDENT
      DO 200 KKK=1,60
      IF(YEAR.EQ.1966)ZE=5.5*TIME-24.
      IF(YEAR.EQ.1967)ZE=0.840*TIME+3.18
      IF(YEAR.EQ.1968)ZE=1.100*TIME+7.95
      IF(YEAR.EQ.1969)ZE=2.840*TIME-5.14
      IF(YEAR.EQ.1970)ZE=3.660*TIME-17.14
      IF(YEAR.EQ.1971)ZE=3.620*TIME-13.08
      IF(YEAR.EQ.1972)ZE=5.760*TIME-28.24
      IF(YEAR.EQ.1973)ZE=3.620*TIME-13.08
      IF(YEAR.EQ.1974)ZE=3.620*TIME-13.08
      VEP=(-63.*(ZE)**2.+18310.*ZE)*1000000.
      ATH=(-126.*ZE+18310.)*1000000.
      VHYP=VL-VEP
      ZH=VHYP/ATH
      IF(TIME.LE.5.31)GO TO 101
      IF(TIME.GT.5.31.AND.TIME.LE.6.61)GO TO 102
      IF(TIME.GT.6.61)GO TO 103
  101 D=TIME
      TSURF=((.1584*D-.5972)*D-.0716)*D+2.6058
      GO TO 105
  102 D=TIME-5.31
      TSURF=((-1.4288*D+1.9278)*D+6.9978)*D+9.1264
      GO TO 105
  103 D=TIME-6.61
      TSURF=((.4278*D-3.6162)*D+4.814)*D+18.3106
      GO TO 105
  105 DOSATN=14.48-.36*TSURF+.0043*TSURF**2.
      VTCE=(CVTTH*ATH)/VEP
      VTCH=(CVTTH*ATH)/VHYP
      A(1,1)=-Q/VEP-VTCE-CSEP*ATH/VEP
      A(1,2)=VTCE
      A(1,3)=CNPE
      A(2,1)=VTCH+CSEP*ATH/VHYP
      A(2,2)=-VTCH-(CSAF*(1.+CFLOC*ZH)*ASED/VHYP)-CDH
      A(3,3)=-Q/VEP-CNPE-VTCE
      A(3,4)=VTCE
      A(4,2)=CDH
      A(4,3)=VTCH
      A(4,4)=-VTCH
      A(5,3)=CNPE*FAC
      A(5,5)=-CAER*ASURF/VEP-VTCE-Q/VEP
      A(5,6)=VTCE
      A(6,2)=-CDH*FAC
      A(6,5)=VTCH
```

```
      W(3)=PHOSLD/VEP
      W(5)=(CAER*ASURF*DOSATN/VEP)+DOSATN*(TRFLOW+NPFLOW)/VEP
      W(6)=-CSOD*ASED/VHYP
      DO 150 KK=1,2
      XP1=X(1,I)
      XP2=X(2,I)
      XP3=X(3,I)
      XP4=X(4,I)
      XP5=X(5,I)
      XP6=X(6,I)
      DO 100 K=1,4
      X1=XP1+(A(1,1)*XP1 + A(1,2)*XP2 + A(1,3)*XP3 + A(1,4)*XP4 +
     1 A(1,5)*XP5 + A(1,6)*XP6 + W(1))*(H/2.)
      X2=XP2+(A(2,1)*XP1 + A(2,2)*XP2 + A(2,3)*XP3 + A(2,4)*XP4 +
     2 A(2,5)*XP5 + A(2,6)*XP6 + W(2))*(H/2.)
      X3=XP3+(A(3,1)*XP1 + A(3,2)*XP2 + A(3,3)*XP3 + A(3,4)*XP4 +
     3 A(3,5)*XP5 + A(3,6)*XP6 + W(3))*(H/2.)
      X4=XP4+(A(4,1)*XP1 + A(4,2)*XP2 + A(4,3)*XP3 + A(4,4)*XP4 +
     4 A(4,5)*XP5 + A(4,6)*XP6 + W(4))*(H/2.)
      X5=XP5+(A(5,1)*XP1 + A(5,2)*XP2 + A(5,3)*XP3 + A(5,4)*XP4 +
     5 A(5,5)*XP5 + A(5,6)*XP6 + W(5))*(H/2.)
      X6=XP6+(A(6,1)*XP1 + A(6,2)*XP2 + A(6,3)*XP3 + A(6,4)*XP4 +
     6 A(6,5)*XP5 + A(6,6)*XP6 + W(6))*(H/2.)
       XX1= XP1+ (A(1,1)*X1 + A(1,2)*X2 + A(1,3)*X3 + A(1,4)*X4 +
     1 A(1,5)*X5 + A(1,6)*X6 + W(1))*H
       XX2= XP2+ (A(2,1)*X1 + A(2,2)*X2 + A(2,3)*X3 + A(2,4)*X4 +
     2 A(2,5)*X5 + A(2,6)*X6 + W(2))*H
       XX3= XP3+ (A(3,1)*X1 + A(3,2)*X2 + A(3,3)*X3 + A(3,4)*X4 +
     1A(3,5)*X5 + A(3,6)*X6 + W(3))*H
       XX4=XP4 + (A(4,1)*X1 + A(4,2)*X2 + A(4,3)*X3 + A(4,4)*X4 +
     1 A(4,5)*X5 + A(4,6)*X6 + W(4))*H
       XX5=XP5 + (A(5,1)*X1 + A(5,2)*X2 + A(5,3)*X3 + A(5,4)*X4 +
     1 A(5,5)*X5 + A(5,6)*X6 + W(5))*H
       XX6=XP6 + (A(6,1)*X1 + A(6,2)*X2 + A(6,3)*X3 + A(6,4)*X4 +
     1 A(6,5)*X5 + A(6,6)*X6 + W(6))*H
       XP1=XX1
       XP2=XX2
       XP3=XX3
       XP4=XX4
       XP5=XX5
       XP6=XX6
  100  CONTINUE
       X(1,I+1)= XX1
       X(2,I+1)= XX2
       X(3,I+1)= XX3
       X(4,I+1)= XX4
       X(5,I+1)= XX5
       X(6,I+1)= XX6
       I= I+1
  150  CONTINUE
       TIME=5.50+KKK*2./30.
       CALL PLOTPT(TIME,X(5,I),11)
       CALL PLOTPT(TIME,X(5,I),12)
```

```
      WRITE(3,175)TIME,(X(II,I),II=1,6)
  175 FORMAT(1H0,4X,F5.2,3X,6(2X,F10.2))
      VEFLUX=CVTTH*(X(6,I)-X(5,I))
      DHFLUX=CDH*FAC*ZH*X(2,I)
      IF(KKK.EQ.1)WRITE(3,174)VEFLUX,DHFLUX
      IF(KKK.EQ.15)WRITE(3,174)VEFLUX,DHFLUX
      IF(KKK.EQ.30)WRITE(3,174)VEFLUX,DHFLUX
      IF(KKK.EQ.45)WRITE(3,174)VEFLUX,DHFLUX
      IF(KKK.EQ.60)WRITE(3,174)VEFLUX,DHFLUX
  174 FORMAT(1H0,4X,2F10.3)
      IF(YEAR.NE.1966)GO TO 199
      ICOUNT=KKK+1
      GO TO 198
  199 ICOUNT=KKK+35
  198 ARTIME(ICOUNT)=TIME
      YXPPE(ICOUNT)=X(1,I)
      YXPPH(ICOUNT)=X(2,I)
      YXOPE(ICOUNT)=X(3,I)
      YXOPH(ICOUNT)=X(4,I)
      YXDOE(ICOUNT)=X(5,I)
      YXDOH(ICOUNT)=X(6,I)
      TP(ICOUNT)=TIME
      PPEM(ICOUNT)=X(1,I)
      PPHM(ICOUNT)=X(2,I)
      OPEM(ICOUNT)=X(3,I)
      OPHM(ICOUNT)=X(4,I)
  200 CONTINUE
      IF(YEAR.NE.1966)GO TO 213
      J=1
      GO TO 214
  213 J=167
  214 Y(1,J)=  (X(1,I)*VEP+X(2,I)*VHYP)/(VEP+VHYP)
      Y(2,J)=  (X(3,I)*VEP+X(4,I)*VHYP)/(VEP+VHYP)
      Y(3,J)=  (X(5,I)*VEP+X(6,I)*VHYP)/(VEP+VHYP)
      CAER=7.50
C  WINTER MODEL PREDICTIONS
C  WINTER MODEL PREDICTIONS
C  WINTER MODEL PREDICTIONS
C  CALCULATE THE TERMS OF THE COEFFICIENT MATRIX--WINTER MODEL
      B(1,1)=-Q/VL-CDECOMP-(CSAF*(1.+CFLOC*(Z-ZEUPH)))*ASED/VL
      B(1,2)=CNPEU*VEU/VL
      B(2,1)=CDECOMP
      B(2,2)=-Q/VL-CNPEU*VEU/VL
      B(3,1)=-CDECOMP*FAC
      B(3,2)=(CNPEU*VEU/VL)*FAC
      B(3,3)=-CAER*ASURF/VL-Q/VL
      W(1,1)=PHOSLD/VL
      WRITE(3,80)
   80 FORMAT(1H0,18X≠PREDICTED WINTER RESULTS    ≠)
      WRITE(3,81)
   81 FORMAT(1H0,≠TIME(MONTHS)      PP             OP             CO      ≠)
      WRITE(3,82)TIME,(Y(JJ,J),JJ=1,3)
   82 FORMAT(1H0,4X,F5.2,3X,3(2X,F10.2))
```

```
      JCOUNT=ICOUNT+1
      ARTIME(JCOUNT)=TIME
      YXPPE(JCOUNT)=Y(1,J)
      YXPPH(JCOUNT)=Y(1,J)
      YXOPE(JCOUNT)=Y(2,J)
      YXOPH(JCOUNT)=Y(2,J)
      YXDOE(JCOUNT)=Y(3,J)
      YXDOH(JCOUNT)=Y(3,J)
      TP(JCOUNT)=TIME
      PPEM(JCOUNT)=Y(1,J)
      PPHM(JCOUNT)=Y(1,J)
      OPEM(JCOUNT)=Y(2,J)
      OPHM(JCOUNT)=Y(2,J)
      DO 251 LLL=1,15
      W(11)=(CAER*ASURF*DOSATN/VL)+(DOSATN*(TRFLOW+NFFLOW)/VL)-(CSOD*ASE
     1D/VL)
      DO 236 LL=1,5
      YP1=Y(1,J)
      YP2=Y(2,J)
      YP3=Y(3,J)
      DO 226 L=1,4
      Y1=YP1+(B(1,1)*YP1 + B(1,2)*YP2 + B(1,3)*YP3 + W(9))*(H/2.)
      Y2=YP2+(B(2,1)*YP1 + B(2,2)*YP2 + B(2,3)*YP3 + W(10))*(H/2.)
      Y3=YP3+(B(3,1)*YP1 + B(3,2)*YP2 + B(3,3)*YP3 + W(11))*(H/2.)
      YY1= Y(1,J) + (B(1,1)*Y1 + B(1,2)*Y2 + B(1,3)*Y3 + W(9))*H
      YY2= Y(2,J) + (B(2,1)*Y1 + B(2,2)*Y2 + B(2,3)*Y3 + W(10))*H
      YY3= Y(3,J) + (B(3,1)*Y1 + B(3,2)*Y2 + B(3,3)*Y3 + W(11))*H
      YP1=YY1
      YP2=YY2
      YP3=YY3
  226 CONTINUE
      Y(1,J+1)=YY1
      Y(2,J+1)=YY2
      Y(3,J+1)=YY3
      J=J+1
  236 CONTINUE
  240 TIME=9.50+LLL*5./30.
      CALL PLOTPT(TIME,Y(3,J),11)
      CALL PLOTPT(TIME,Y(3,J),12)
  341 WRITE(3,355)TIME,(Y(II,J),II=1,3)
  355 FORMAT(1H0,4X,F5.2,3X,3(2X,F10.2))
      IF(YEAR.NE.1966)GO TO 256
      JCOUNT=LLL+62
      GO TO 257
  256 JCOUNT=LLL+96
  257 ARTIME(JCOUNT)=TIME
      YXPPE(JCOUNT)=Y(1,J)
      YXPPH(JCOUNT)=Y(1,J)
      YXOPE(JCOUNT)=Y(2,J)
      YXOPH(JCOUNT)=Y(2,J)
      YXDOE(JCOUNT)=Y(3,J)
      YXDOH(JCOUNT)=Y(3,J)
      TP(JCOUNT)=TIME
```

```
          PPEM(JCOUNT)=Y(1,J)
          PPHM(JCOUNT)=Y(1,J)
          OPEM(JCOUNT)=Y(2,J)
          OPHM(JCOUNT)=Y(2,J)
          IF(TIME.LE.5.31)GO TO 301
          IF(TIME.GT.5.31.AND.TIME.LE.6.61)GO TO 302
          IF(TIME.GT.6.61)GO TO 303
      301 D=TIME
          TSURF=((.1584*D-.5972)*D-.0716)*D+2.6058
          GO TO 305
      302 D=TIME-5.31
          TSURF=((-1.4288*D+1.9278)*D+6.9978)*D+9.1264
          GO TO 305
      303 D=TIME-6.61
          TSURF=((.4278*D-3.6162)*D+4.814)*D+18.3106
          GO TO 305
      305 DOSATN=14.48-.36*TSURF+.0043*TSURF**2.
      251 CONTINUE
          Y(1,1)=Y(1,J)
          Y(2,1)=Y(2,J)
          Y(3,1)=Y(3,J)
          M=JCOUNT
          DO 1000 IC=1,JCOUNT,4
          ID=IC-1
          WRITE(7,500)(ARTIME(ID+J),YXPPE(ID+J),J=1,4)
     1000 CONTINUE
          DO 1001 IC=1,JCOUNT,4
          ID=IC-1
          WRITE(7,500)(ARTIME(ID+J),YXPPH(ID+J),J=1,4)
     1001 CONTINUE
          DO 1002 IC=1,JCOUNT,4
          ID=IC-1
          WRITE(7,500)(ARTIME(ID+J),YXOPE(ID+J),J=1,4)
     1002 CONTINUE
          DO 1003 IC=1,JCOUNT,4
          ID=IC-1
          WRITE(7,500)(ARTIME(ID+J),YXOPH(ID+J),J=1,4)
     1003 CONTINUE
          DO 1004 IC=1,JCOUNT,4
          ID=IC-1
          WRITE(7,500)(ARTIME(ID+J),YXDOE(ID+J),J=1,4)
     1004 CONTINUE
          DO 1005 IC=1,JCOUNT,4
          ID=IC-1
          WRITE(7,500)(ARTIME(ID+J),YXDOH(ID+J),J=1,4)
     1005 CONTINUE
      500 FORMAT(8F10.3)
          DO 600 IADE=1,NDOEDP
          READ(1,601)TE(IADE),ADODE(IADE)
      601 FORMAT(2F6.2)
          CALL PLOTPT(TE(IADE),ADODE(IADE),13)
      600 CONTINUE
```

```
          READ(1,602)TH(IADH),ADODH(IADH)
      602 FORMAT(2F6.2)
          CALL PLOTPT(TH(IADH),ADODH(IADH),14)
      650 CONTINUE
          CALL OUTPLT
          DO 2000 MM=1,M,2
          TPEM(MM)=PPEM(MM)+OPEM(MM)
          CALL PLOTPT(TP(MM),PPEM(MM),11)
          CALL PLOTPT(TP(MM),OPEM(MM),12)
          CALL PLOTPT(TP(MM),TPEM(MM),13)
     2000 CONTINUE
          CALL OUTPLT
          DO 2001 MN=1,M,2
          TPHM(MN)=PPHM(MN)+OPHM(MN)
          CALL PLOTPT(TP(MN),PPHM(MN),11)
          CALL PLOTPT(TP(MN),OPHM(MN),12)
          CALL PLOTPT(TP(MN),TPHM(MN),13)
     2001 CONTINUE
          CALL OUTPLT
          IF(YEAR.LE.1973)GO TO 1
          STOP
          END
```