BENDING MOMENTS AND DEFORMATIONS OF THIN ELASTIC CONICAL SHELLS ON EULER-WINKLER ELASTIC FOUNDATION

by

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SCOPE AND CONTENTS:

Various analytical methods for studying the behaviour of shallow conical shells on Euler-Winkler elastic foundation are presented.

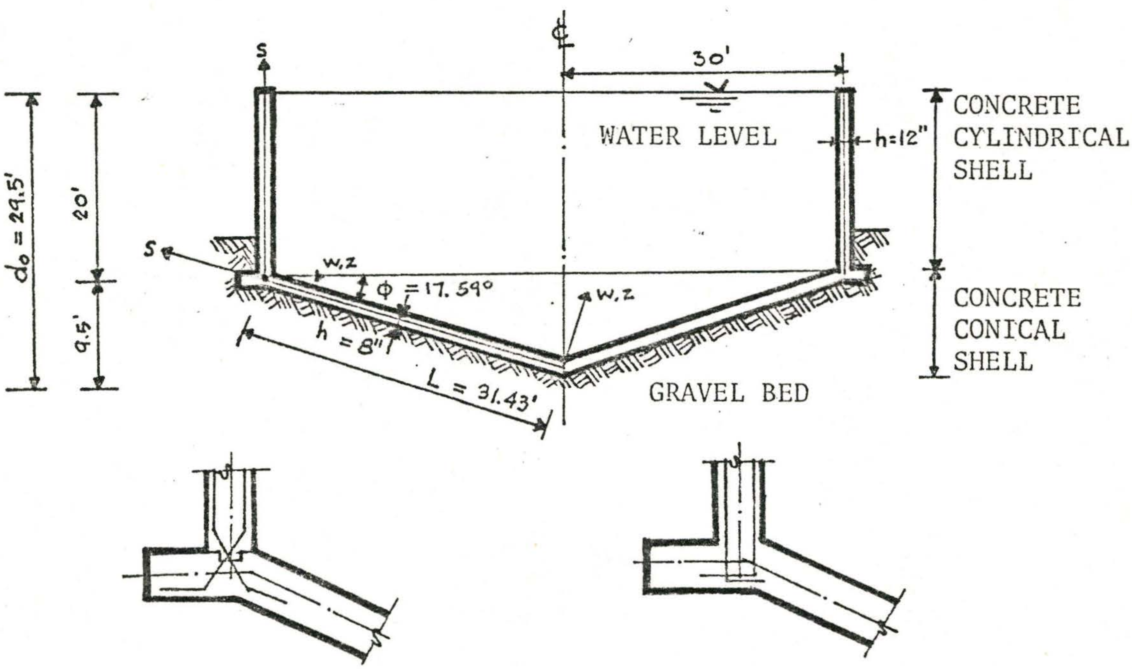
To account for the nature of concrete and the geometric properties of the shallow conical shell, Poisson's ratio and certain radial and circumferential deformations of the middle surface are neglected in deriving the basic differential equation. Analytical methods employed in the solution of this shell problem are the GECKELER and asymptotic types of approximations.

The presentations of various methods of analysis are made for a representative case of dimensions and loadings of the conical shell to make them as applicable as possible to the cases of thin conical shell commonly encountered in industry.

The shell structure studied is a tank in the form of a rotationally symmetrical cylindrical shell supported by a shallow conical shell foundation. The construction joint between the conical shell and the cylindrical shell is either monolithic or hinged.

The analytical results of this water tank supported on Euler-Winkler elastic foundation are compared with the corresponding findings of W. Flügge, who assumed a uniform soil bearing pressure acting on the conical shell structure.

The method of analysis which possesses obvious advantages over the other methods studied is selected to examine the effect of different elastic stiffness coefficients of the soil. The validity of simplifying the soil bearing pressure to a uniform distribution by most designers can consequently be studied by comparing it to the bearing pressures of an ideal elastic soil which is postulated to react to its deformation like a bed of independent elastic springs.



Meridional Cross-Section of Water Tank

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NOMENCLATURE

A	= Surface Area or Constant
A,a,B,b,C,c	= Constants
D	= Bending Stiffness Modulus of Shell
d	= Depth of Cylindrical Shell
E	= Modulus of Elasticity
F,H	= Force
h	= Thickness of the Shell
i	= Imaginary Number, $\sqrt{-1}$
k	= Foundation Stiffness Constant
L	= Meridional Length of Conical Shell
$M_{\theta\theta}$	= Circumferential Stress Couple
$M_{ss}=M_{\phi\phi}$	= Meridional Stress Couple
$N_{ss}=N_{\phi\phi}$	= Meridional Stress Resultant
n	= Variable
P,p	= Force
$Q_{sz}=Q_{\phi z}$	= Transverse Shear Stress Resultant
R,r	= Radii
S,s	= Co-ordinates
U	= $r_\theta Q_{\phi z}$, Dependent Variable
u,v,w	= Displacement Component
x,y,z	= Co-ordinates
α	= Constant
γ	= Density
Δ	= Horizontal Displacement

ε	= Strain or Variable
η	= Variable
θ	= Circumferential Co-ordinate
K	= Extensional Stiffness of Shell
λ	= Damping Coefficient of Conical Shell
λ'	= Damping Coefficient of Cylindrical Shell
σ,τ	= Stress
ν	= POISSON's Ratio
ϕ	= Angle
Φ	= Variable
$\chi_{\theta\theta}$	= Curvature Change of the Circumference
$\chi_{\phi\phi} = \frac{1}{r}\frac{d\chi}{d\phi}$	= Curvature Change of the Meridian
ψ	= Angle

CHAPTER I

INTRODUCTION

Even for the simplest types of shells, general solutions for the associated differential equations are fairly complicated. Assumptions are often made to simplify the differential equations and to facilitate the integrations by approximations. Simplifications are further justified on the ground that the minimum concrete thickness is often controlled by the construction practice, by the buckling properties of the shell, by the minimum reinforcement, and by the temperature and shrinkage requirements.

Thin shell analysis is commonly based on the following assumptions:

(1) The material is homogeneous, isotropic, and linearly elastic.

(2) The system behaves according to the small-deflection theory, which essentially requires that the deflection under the load be small enough so that changes in the shell geometry do not alter the static equilibrium. The usual measure of validity for this criterion is that the transverse displacements of the shell be small compared with the thickness of the shell.

(3) Only static loading and static structural responses are considered.

Many designers further simplify the situation by assuming that the soil bearing pressure is uniformly distributed. This assumption implies that the ground is similar to a bed of linearly elastic springs whereas the shell is infinitely rigid. The validity of such an assumption is very doubtful, especially in the vicinity of the boundaries where the effects of the edge loads and edge displacements are particularly pronounced. A study of this question might clarify some aspects of this problem.

Based on various considerations, a number of differential equations for shallow conical shells have been suggested.

The most simplified form of the solution is to consider the shallow conical shell similar to a circular plate in its structural behaviour. The longitudinal element of the conical shell can be regarded as a beam of linearly varying width supported on elastic foundation. The resulting fourth order differential equation has the form:

$$\frac{d^2}{dx^2} (EI \frac{d^2v}{dx^2}) + bkv = 0$$

where E is the modulus of elasticity, I is the moment of inertia, k is the foundation stiffness, and b is the variable width of the beam.

Though this differential equation is not difficult to solve, its assumption limits its application to very shallow conical shells.

The most general form of the differential equation for the shallow conical shell can be derived from the first and second fundamental linear shallow shell equations of KARL MARGUERRE:

$$D\nabla^4 w - \nabla_k^2 F = p_z$$

$$\nabla^4 F + Eh\nabla_k^4 = 0$$

where ∇ and ∇_k are the two-dimensional differential operators, D is the bending stiffness modulus, E is the modulus of elasticity, F is the stress function, h is the thickness of the shell, p_z is the normal load, and w is the transverse displacement of the shell.

These two equations ought to be reduced to a shallow conical shell differential equation of the eighth order, but it is apparent that the differential equation of such high order is extremely difficult to establish and to solve. Therefore it is more appropriate to establish the simplified differential equation for the shallow conical shells directly from the special equations for rotationally symmetrical shells.

CHAPTER II

STRESSES IN CONICAL SHELL SUPPORTED ON EULER-WINKLER ELASTIC FOUNDATION

2.1 EULER-WINKLER ELASTIC FOUNDATION

Euler-Winkler elastic foundation makes the assumption that each point of the soil exerts an independent reaction on the foundation. This means that the foundation is made out of discrete linear springs and does not represent a linearly elastic continuum. One spring and its deformation exerts no effect on the neighbouring springs. This assumption, while ignoring the shearing stiffness of the foundation, provides a great simplification to the extremely complicated soil compression problem.

The reactive pressure exerted by the load can be given by means of Euler-Winkler soil stiffness coefficient, k, and displacement, w: $p_f = -kw$. The hypothesis of the Euler-Winkler foundation is fairly acceptable for sandy soils, but is not well adaptable to clayey soils. To determine the elastic stiffness constant, small plates are loaded with a weight P in order to measure the displacement of the plate, as $k = P/(Aw)$ where A is the area of the plate. It is apparent that the perimetral shear effect is considerable because k changes when larger plates are used in this loading test. For clayey soils, the situation is further complicated by varying soil inhomogeneities and by plastic flow, slips in soil grains, and many other fortuitous properties.

As an approximate preliminary design criterion, the Euler-Winkler foundation can be used with profit. However, it should be noted that a soil with shearing stiffness would stretch out the boundary disturbance zone of the conical shell.

2.2 STRESSES AND STRAINS IN ELASTIC SHELL OF ROTATION*

A shell can be considered a naturally curved thin plate. This definition implies that the thickness of the thin shell is small compared with its other two dimensions, but it does not require that the shell be made of an elastic material. We generally assume that the material is elastic according to Hooke's law.

The stress acting in a shell can be expressed by a system of stress resultants with respect to the middle surface. If the shell is sufficiently thin and the equilibrium condition due to elasticity holds, the equilibrium problem of the shell can be made equipollent to the equilibrium problem of the middle surface.

The derivations of stresses and strains in the shell of rotation can be found in numerous standard works: (1), (2), (3). They are therefore not elaborated here. The following is a summary of all the forces and moments exerting on a shell element to serve as a background for developing the equilibrium equations.

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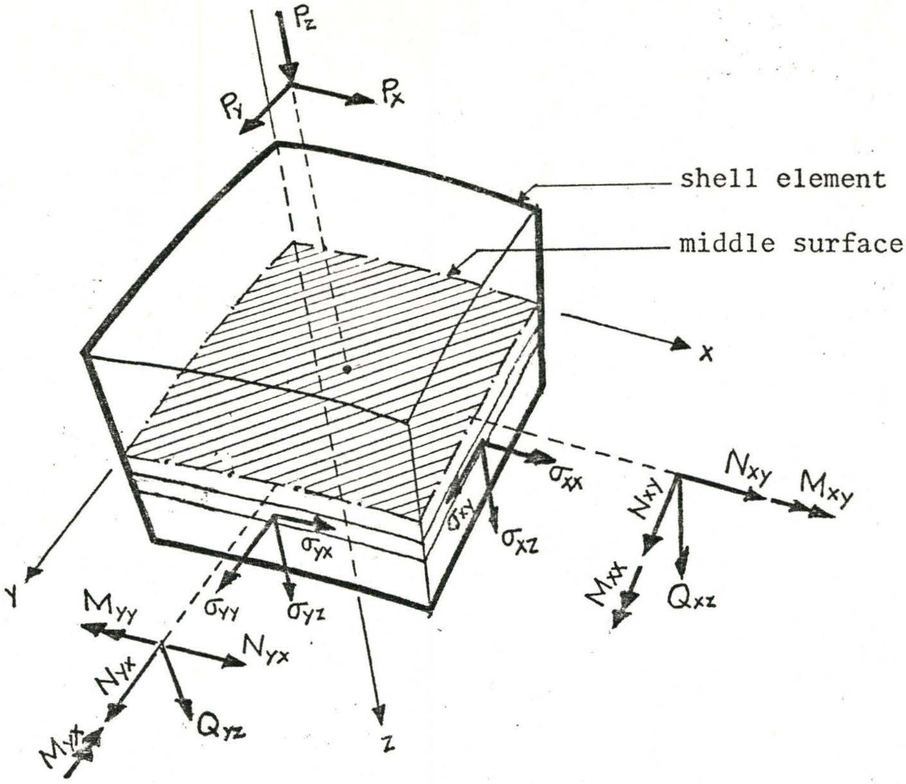
*For a critical historical review of the rotational shell theory, see APPENDIX I.



σ, stresses
p, external load
N,Q, stress resultants
M, stress couples

FIG. 2.1 Stress Resultants and Stress Couples

<u>Force Components</u>

Transverse Shear Stress Resultants:

$Q_{\theta z}$: x-component: $-d\phi d\theta Q_{\theta z} r_{\theta} \sin\phi$

z-component: $d\phi d\theta (\partial Q_{\theta z}/\partial\theta) r_{\phi}$

$Q_{\phi z}$: y-component: $-d\phi d\theta Q_{\phi z} r$

z-component: $d\phi d\theta (\partial Q_{\phi z}/\partial\phi)$

Circumferential Stress Resultants:

$N_{\theta\theta}$: radial component: $d\phi d\theta N_{\theta\theta} r_\phi$

x-component: $d\phi d\theta (\partial N_{\theta\theta}/\partial\theta) r_\phi$

y-component: $-d\phi d\theta N_{\theta\theta} r_\phi \cos\phi$

z-component: $d\phi d\theta N_{\theta\theta} r_\phi \sin\phi$

Meridional Stress Resultants:

$N_{\phi\phi}$: y-component: $d\phi d\theta (\partial(N_{\phi\phi} r)/\partial\phi)$

z-component: $d\phi d\theta N_{\phi\phi} r$

Shear Stress Resultants:

$N_{\theta\phi}$: x-component: $d\phi d\theta N_{\phi\theta} r_\phi \cos\phi$

y-component: $d\phi d\theta (\partial(N_{\theta\phi} r_\phi)/\partial\theta)$

$N_{\phi\theta}$: x-component: $d\phi d\theta (\partial(N_{\phi\theta} r)/\partial\phi)$

Moment Components:

Moment Resultants:

$M_{\theta\theta}$: x-component: $d\phi d\theta M_{\theta\theta} r_\phi \cos\phi$

y-component: $d\phi d\theta (\partial M_{\theta\theta}/\partial\theta) r_\phi$

$M_{\phi\phi}$: x-component: $-d\phi d\theta (\partial M_{\phi\phi}/\partial r_\phi)$

$M_{\theta\phi}$: x-component: $d\phi d\theta M_{\theta\phi} r_\phi \cos\phi$

y-component: $-d\phi d\theta (\partial(M_{\phi\theta} r)/\partial\phi)$

z-component: $-d\phi d\theta M_{\phi\theta} r$

<u>Moment Components of Forces:</u>

p_z, p_ϕ, p_θ:	x-component:	0
	y-component:	0
$Q_{\phi z}$:	x-component:	$d\phi d\theta r_\phi r Q_\phi$
	y-component:	0
$Q_{\theta z}$:	x-component:	0
	y-component:	$-d\phi d\theta r r_\phi Q_{\theta z}$
$N_{\theta\phi}$:	z-component:	$d\phi d\theta r r_\phi N_{\theta\phi}$
$N_{\phi\theta}$:	z-component:	$-d\phi d\theta r r_\phi N_{\phi\theta}$

In all cases infinitesimals of the third order are neglected as comparatively insignificant in equations of equilibrium.

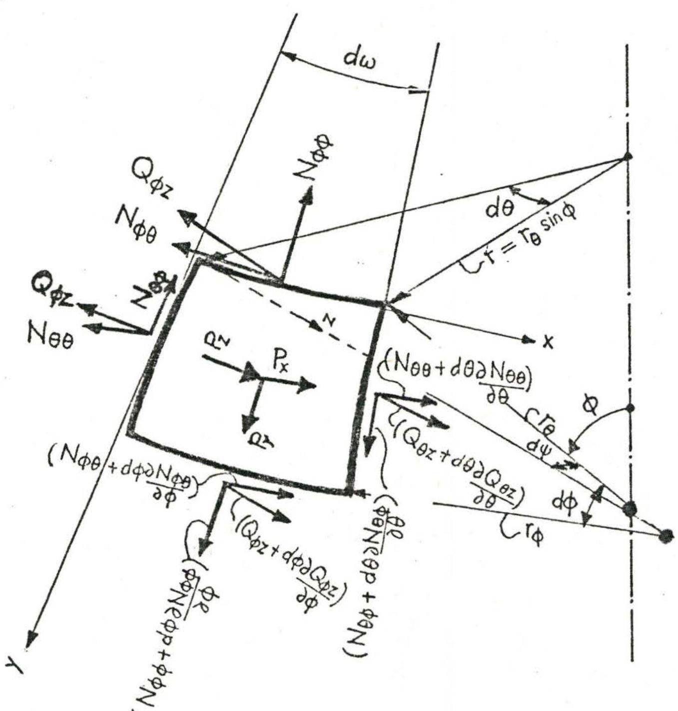


FIG. 2.2 Middle Surface Element Subject to Stress Resultants and Surface Tractions

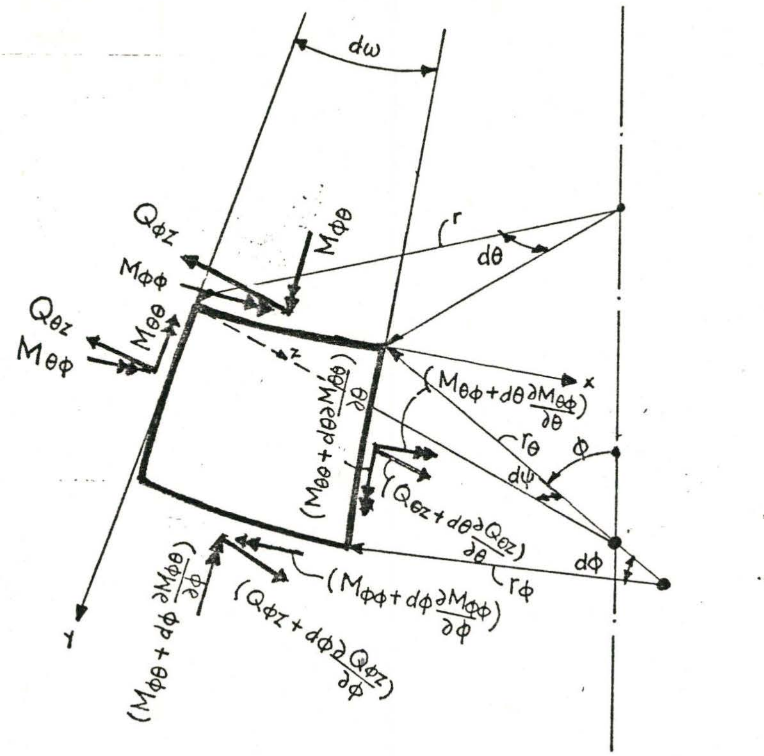


FIG. 2.3 Middle Surface Element Subject to the Stress Couples and Transverse Shear Stress Resultants

Local Force Equilibrium Condition

$$\Sigma F_x: \quad r_\phi\frac{\partial N_{\Theta\Theta}}{\partial\theta} + \frac{\partial(rN_{\phi\theta})}{\partial\phi} + r_\phi\cos\theta N_{\theta\phi} - r_\phi\sin\phi Q_{\theta z} + rr_\phi p_\theta = 0 \quad (2\text{-}1)$$

$$\Sigma F_y: \quad \frac{\partial(rN_{\phi\phi})}{\partial\phi} + r_\phi\frac{\partial N_{\theta\phi}}{\partial\theta} - r_\phi\cos\phi N_{\theta\theta} - rQ_{\phi z} + rr_\phi p_\phi = 0 \quad (2\text{-}2)$$

$$\Sigma F_z: \quad r_\phi\frac{\partial Q_{\theta z}}{\partial\theta} + \frac{\partial(rQ_{\phi z})}{\partial\phi} + r_\phi\sin\phi N_{\theta\theta} + rN_{\phi\phi} + rr_\phi p_z = 0 \quad (2\text{-}3)$$

Local Moment Equilibrium Condition

x-component (moment about x-axis):

$$r_\phi\frac{\partial M_{\theta\phi}}{\partial\Theta} - \frac{\partial(rM_{\phi\phi})}{\partial\phi} + r_\phi\cos\phi M_{\theta\theta} + rr_\phi Q_{\phi z} = 0 \quad (2\text{-}4)$$

y-component (moment about y-axis):

$$r_\phi \frac{\partial M_{\theta\theta}}{\partial\theta} - \frac{\partial (rM_{\phi\theta})}{\partial\phi} - r_\phi \cos\phi M_{\theta\phi} - rr_\phi Q_{\theta z} = 0 \qquad (2\text{-}5)$$

z-component (moment about z-axis):

$$rr_\phi N_{\theta\phi} - rr_\phi N_{\phi\theta} - r_\phi \cos\phi M_{\theta\phi} - rM_{\phi\theta} = 0 \qquad (2\text{-}6)$$

<u>Equilibrium Equations for Rotationally Symmetrical Conical Shell</u>

If LOVE's First Approximation for thin shells applies, $z/r_x \ll 1 \gg z/r_y$, then $N_{\theta\phi} \simeq N_{\phi\theta}$, $M_{\theta\phi} \simeq M_{\phi\theta}$. For rotationally symmetrical loading,

$$p_\theta = \partial N_{\theta\theta}/\partial\theta = \partial M_{\theta\theta}/\partial\theta = 0$$

$$N_{\phi\theta} = N_{\theta\phi} = M_{\phi\theta} = M_{\theta\phi} = 0$$

and

$$Q_{\theta z} = 0 \text{ from (2-1).}$$

The remaining equations from (2-1) to (2-6) are:

$$\frac{d(rN_{\phi\phi})}{d\phi} - r_\phi \cos\phi N_{\theta\theta} - rQ_{\phi z} + rr_\phi p_\phi = 0$$

$$\frac{d(rQ_{\phi z})}{d\phi} + r_\phi \sin\phi N_{\theta\theta} + rN_{\phi\phi} + rr_\phi p_z = 0$$

$$\frac{d(rM_{\phi\phi})}{d\phi} - r_\phi \cos\phi M_{\theta\theta} - rr_\phi Q_{\phi z} = 0$$

The soil reaction is $p_f = -kw$, and the tangential traction $p_\phi = 0$. As POISSON's ratio for concrete is negligibly small, $M_{\theta\theta} \simeq \nu M_{\phi\phi} = 0$. Consequently the circumferential bending stiffness of the thin shell is neglected. Introducing the meridional co-ordinate s for conical shells by letting $ds = r_\phi d\phi$ when $r_\phi \to \infty$ as $d\phi \to 0$, the equilibrium equations become:

$$\frac{d(N_{ss}s)}{ds} - N_{\theta\theta} = 0 \tag{2-7}$$

$$\frac{d(Q_{sz}s)}{ds} + N_{\theta\theta}\tan\phi - skw + sp_z = 0 \tag{2-8}$$

$$\frac{d(M_{ss}s)}{ds} - Q_{sz}s = 0 \tag{2-9}$$

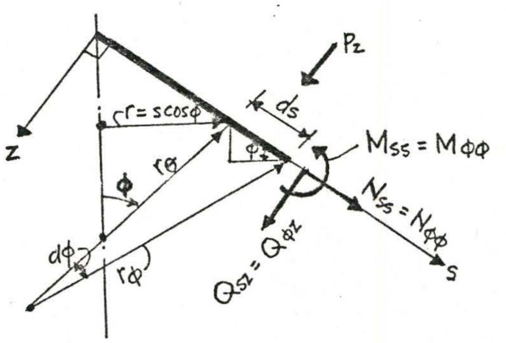


FIG. 2.4 Meridional Section of the Middle Surface of the Conical Shell with Co-ordinate s

Four unknowns in three equations make the system internally statically indeterminate to the first order. It is necessary to derive the additional equation from the deformation of the shell. The kinematics of deformation of the rotationally symmetrical shell are based on KIRCHOFF's hypothesis, which states that normals to the middle surface of the shell remain normals to the deformed middle surface and undergo no stretching.

The displacement-strain relations discussed in various standard works, (3, 4), are summarized as follows:

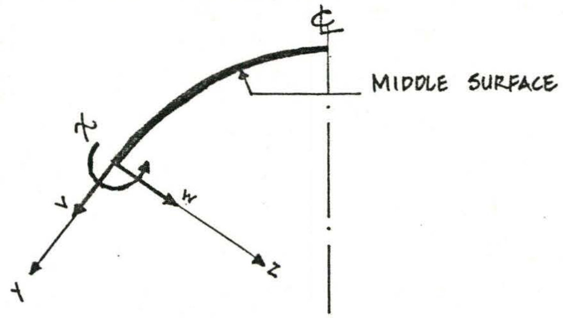


FIG. 2.5 Convention for Deformations: Meridional Section of the Middle Surface of the Shell

Curvature Change:

The change of slope of meridian at ϕ:

$$\chi = \frac{v}{r_\phi} + \frac{1}{r_\phi}\frac{dw}{d\phi}$$

The normal curvature change of the meridian due to deformation:

$$\chi_{\phi\phi} = \frac{d\chi}{ds_\phi} = \frac{d^2w}{ds^2}$$

The circumferential curvature change:

$$\chi_{\theta\theta} = \chi\cos\phi/r$$

The torsion of the middle surface:

$$\chi_{\phi\theta} = \chi_{\theta\phi} = 0$$

Strain:

The linear strain in the meridional direction (y-axis) at the point z:

$$\varepsilon_{\phi\phi}(z) \simeq \frac{1}{r_\phi}\frac{dv}{d\phi} - \frac{w}{r_\phi}$$

The linear strain in the circumferential direction (x-axis) at the point z:

$$\varepsilon_{\theta\theta}(z) = \frac{v\cos\phi}{r} - \frac{w}{r_\theta}$$

The plane strain in the parallel surface:

$$\varepsilon_{\phi\theta} = 0$$

The stress-strain relations for isotropic linearly elastic materials for rotationally symmetrical shells are:

$$\varepsilon_{\phi\phi} = (\sigma_{\phi\phi} - \nu\sigma_{\theta\theta})/E$$

$$\varepsilon_{\theta\theta} = (\sigma_{\theta\theta} - \nu\sigma_{\phi\phi})/E$$

$$\varepsilon_{\phi\theta} = \sigma_{\phi\theta}/2G = 0$$

Substituting these expressions for stresses, while the strains at z are given by:

$$\varepsilon_{\phi\phi}(z) = \varepsilon_{\phi\phi} - z\chi_{\phi\phi}$$

$$\varepsilon_{\theta\theta}(z) = \varepsilon_{\theta\theta} - z\chi_{\theta\theta}$$

into the cross-sectional resultants of stresses yields the constitutive equations:

$$N_{\phi\phi} = \frac{Eh}{1-\nu^2}(\varepsilon_{\phi\phi} - \nu\varepsilon_{\theta\theta}) \qquad M_{\phi\phi} = -D(\chi_{\phi\phi} + \nu\chi_{\theta\theta})$$

$$N_{\theta\theta} = \frac{Eh}{1-\nu^2}(\varepsilon_{\theta\theta} + \nu\varepsilon_{\phi\phi}) \qquad M_{\theta\theta} = -D(\chi_{\theta\theta} + \nu\chi_{\phi\phi})$$

$$N_{\phi\theta} = N_{\theta\phi} = Eh\varepsilon_{\phi\theta}/2(1+\nu) = 0 \qquad M_{\phi\theta} = M_{\theta\phi} = D(1-\nu)\chi_{\phi\theta} = 0$$

where the bending stiffness or the modulus of flexural rigidity D is $Eh^3/12(1-\nu^2)$, POISSON's ratio is ν, and the imposed LOVE's First Approximation for thin shells is $z/r_\phi << 1 >> z/r_\theta$.

2.3 DIFFERENTIAL EQUATION OF THE CONCRETE CONICAL SHELL ON EULER-WINKLER ELASTIC FOUNDATION

Substituting the equilibrium equation (2-9) into (2-8) gives:

$$\frac{d}{ds}\left[\frac{d}{ds}(M_{ss}s)\right] + N_{\theta\theta}\tan\phi - skw + sp_z = 0$$

From the constitutive equations and kinematic equations,

$$M_{ss} = M_{\phi\phi} = -D(\chi_{ss}) = -Dd^2w/ds^2$$

$$N_{\theta\theta} = Eh\varepsilon_{\theta\theta} = K\varepsilon_{\theta\theta} = -Kw\tan\phi/s$$

Since the POISSON's ratio $\nu \simeq 0$ is an admissible approximation for concrete, the extensional stiffness of the conical shell becomes

$$K = Eh/(1-\nu^2) \simeq Eh$$

the equation above becomes

$$-D\frac{d}{ds}\left[\frac{d}{ds}\left(\frac{d^2w}{ds^2}\right)\right] - K\frac{w}{s}\tan^2\phi - skw + sp_z = 0$$

where $\lambda^4 = K\tan^2\phi/D$ and $\kappa^4 = k/D$, or

$$\frac{d^2}{ds^2}\left(s\frac{d^2w}{ds^2}\right) + \left(\frac{1}{s}\lambda^4 + s\kappa^4\right)w = s\frac{p_z}{D}$$

The differential equation for the conical shell on Euler-Winkler elastic foundation is:

$$s^2\frac{d^4w}{ds^4} + 2s\frac{d^3w}{ds^3} + (\lambda^4 + s^2\kappa^4)w = (s^2p_z/D) \qquad (2\text{-}10)$$

where $\lambda^4 = 12\tan^2\phi/h^2$ and $\kappa^4 = 12k/(Eh^3)$, the flexural stiffness of the concrete shell with $\nu \simeq 0$ is $D \simeq Eh^3/12$, the modulus of elasticity of concrete is E, the thickness of the concrete shell is h, the stiffness coefficient of the EULER-WINKLER foundation is k, the normal loading is

p_z, the meridional coordinate is s, the transverse displacement is w, and the meridional angle is ϕ.

CHAPTER III

SOLUTIONS OF THE DIFFERENTIAL EQUATION OF THE CONICAL SHELL ON EULER-WINKLER ELASTIC FOUNDATION

3.1 APPROXIMATE COMPLEMENTARY SOLUTIONS

Differential equations derived in engineering mechanics may be of any order and type. The solution of many engineering problems, however, depends on the discovery of solutions of certain types of ordinary linear differential equation which are generally of the second order.

3.1.1 GECKELER TYPE OF APPROXIMATION

As in the region of the shell where $\phi > 25°$, $\cot\phi$ has a rather moderate value and $s\kappa^4 >> \lambda^4 = 12\tan^2\phi/h^2$, then the fourth order differential equation (2-10) with one dependent variable can be reduced to two conjugate second order differential equations. The solution of one such conjugate second order differential equation yields all the independent solutions of the fourth order differential equation.

Introducing the differential operator $L \equiv s\frac{d^2(\ )}{ds^2}$

then

$$LL(w) \equiv s\frac{d^2}{ds^2}\left(s\frac{d^2w}{ds^2}\right)$$

and the differential equation (2-10) can be expressed in the homogeneous form:

$$LL(w) + (s^2\kappa^4 + \lambda^4)w = 0$$

To reduce the fourth order differential equation to the second order, the assumption $s\kappa^4 >> \lambda^4$ can be made for the shallow, thin conical shell. Hence,

$$LL(w) + s^2\kappa^4 = 0 \qquad (3\text{-}1)$$

Setting $\lambda^4 \simeq 0$ means that the normal component of $N_{\theta\theta}$ for the shallow shell ($\tan\phi \simeq \sin\phi$ for small ϕ) is neglected, as from Chapter II, (2.3),

$$-sN_{\theta\theta}\tan\phi/D = Eh\tan^2\phi w/D = \lambda^4 w$$

If the reduced equation has a form

$$L(w) + (as + b)w = 0$$

then

$$L(w) = -(as + b)w \text{ and } LL(w) = (as + b)^2 w$$

because

$$L(as + b) = 0, \text{ consequently}$$

$$LL(w) + s^2\kappa^4 w = [(as + b)^2 + s^2\kappa^4]w = 0$$

The non-trivial solution, $w \neq 0$, is satisfied if and only if

$$a^2 + \kappa^4 = 0,\ 2ab = 0 \text{ and } b^2 = 0$$

Hence

$$a = \pm i\kappa^2,\ b = 0$$

The differential equation of the fourth order can be reduced to the solution of two conjugate second order differential equations:

$$L(w_j) \pm si\kappa^2 w_j = 0$$

where

$$\kappa^2 = \sqrt{12k/Eh^3} = 2K^2$$

Then,

$$L(w_j) \pm i2sK^2 w_j = 0$$

or

$$\frac{d^2 w_j}{ds^2} + i2K^2 w_j = 0$$

Applying EULER's particular solution $w_j = e^{ms}$ reduces the differential to

$$(m^2 \pm i2K^2)e^{ms} = 0$$

as

$$e^{ms} \mp 0$$

then,

$$m_1 = \sqrt{-i}\,\sqrt{2}\,K$$

and

$$m_2 = \sqrt{i}\,\sqrt{2}\,K$$

The solution is:

$$w = w_1(m_1) + w_2(m_2) = e^{-Ks}(Xe^{iKs} + Ze^{-iKs}) + e^{Ks}(Ye^{-iKs} + We^{iKs})$$

By the EULER formula and for complex X, Z, Y and W,

$$w = e^{Ks}(C_1 \cos Ks + C_2 \sin Ks) + e^{-Ks}(C_3 \cos Ks + C_4 \sin Ks)$$

As the e^{-Ks} terms are related to internal boundary near s = 0 and have minimal effects on the outside boundary zone, the solution for the outside boundary zone is:

$$w \simeq e^{Ks}(C_1 \cos Ks + C_2 \sin Ks) \qquad (3\text{-}2)$$

Substituting (3-2) into (2-9), we obtain the transverse shear stress resultant and the meridional stress couple as shown on the next page:

$$Q_{sz} = -\frac{Eh^3}{12}\left(\frac{d^3w}{ds^3} + \frac{1}{s}\frac{d^2w}{ds^2}\right)$$

$$= -\frac{Eh^3}{12}(2K^2e^{Ks})\{C_2[K(\cos Ks - \sin Ks) + (\cos Ks/s)]$$

$$- C_1[K(\cos Ks + \sin Ks) + (\sin Ks/s)]\} \qquad (3\text{-}3)$$

and

$$M_{ss} = -\frac{Eh^3}{12}\left(\frac{d^2w}{ds^2}\right)$$

$$= -\frac{Eh^3}{12}(2K^2e^{Ks})(C_2\cos Ks - C_1\sin Ks) \qquad (3\text{-}4)$$

3.1.2 BAUERSFELD-GECKELER TYPE OF APPROXIMATION

Substituting the dependent variable w in the differential equation (2-10) by $w = \phi(s)u(s)$, we obtain:

$$D(w) = s^2\frac{d^4w}{ds^4} + 2s\frac{d^3w}{ds^3} + (\lambda^4 + s^2\kappa^4)w$$

$$= s^2\phi\frac{d^4u}{ds^4} + \left(4s^2\frac{d\phi}{ds} + 2s\phi\right)\frac{d^3u}{ds^3} + 6s^2\left(\frac{d^2\phi}{ds^2} + \frac{d\phi}{ds}\right)\frac{d^2u}{ds^2}$$

$$+ \left(4s^2\frac{d^3\phi}{ds^3} + 6s\frac{d^2\phi}{ds^2}\right)\frac{du}{ds} + \left[s^2\frac{d^4\phi}{ds^4} + 2s\frac{d^3\phi}{ds^3} + (s^2\kappa^4 + \lambda^4)\phi\right]u=0$$

Setting $4s^2\frac{d\phi}{ds} + 2s\phi = 0$, and integrating $d\phi/\phi = -ds/(2s)$ yields the expression $\phi = 1/\sqrt{s}$, which when substituted in the differential equation above yields the normalized differential equation:

$$D(u) = \frac{d^4u}{ds^4} + \frac{3}{2}\frac{1}{s^2}\frac{d^2u}{ds^2} - \frac{3}{s^3}\frac{du}{ds} + \left(\frac{45}{6}\frac{1}{s^4} + \frac{\lambda^4}{s^2} + \kappa^4\right)u = 0$$

By the normalizing transformation, a larger magnitude difference has been created between the highest derivative d^4u/ds^4 and the next derivative d^2u/ds^2, and now $d^4u/ds^4 >> d^2u/ds^2$ for solutions in the form of heavily damped oscillations, yet an additional first order derivative $(3/s^3)du/ds$ and the function $(45/6)u/s^4$ have been added. Inasmuch as higher order derivatives have larger magnitudes, lesser magnitude quantities are neglected when $(3/2s^2)d^2u/ds^2 - (3/s^3)du/ds$ is neglected in $D(u) = 0$ than when $2s(d^3w/ds^3)$ is neglected in $D(w) = 0$. This simplification represents the so-called GECKELER's approximation.(*) The BAUERSFELD-GECKELER type of differential equation emerges by introducing the GECKELER approximation in the differential equation above, which gives:

$$\frac{d^4u}{ds^4} + \left(\frac{45}{6}\frac{1}{s^4} + \frac{\lambda^4}{s^2} + \kappa^4\right)u = 0$$

and by imposing the BAUERSFELD approximation for a <u>narrow</u> boundary zone,

$$\frac{45}{6}\frac{1}{s^4} + \frac{\lambda^4}{s^2} + \kappa^4 \simeq \frac{45}{6}\frac{1}{L^4} + \frac{\lambda^4}{L^2} + \kappa^4$$

The BAUERSFELD-GECKELER approximation differential equation is:

$$\frac{d^4u}{ds^4} + 4K^{**4} u = 0$$

where

$$K^{**4} = \frac{1}{4}\left(\frac{45}{6}\frac{1}{L^4} + \frac{\lambda^4}{L^2} + \kappa^4\right)$$

For cases when $45/6(1/L^4) << \lambda^4/L^2$, then $K^{**} \simeq 1/4(\lambda^4/L^2 + \kappa^4) = K^*$, and, thus

$$\frac{d^4u}{ds^4} + 4K^{*4} u = 0$$

(*) see APPENDIX I for historical notes.

The solution of this equation is similar to the solution of the equation (3-1):

$$u = e^{K^*s} (C_1 \cos K^*s + C_2 \sin K^*s)$$

As

$$w = \phi(s)u(s) = u(s)/\sqrt{s}$$

then

$$w = \frac{1}{\sqrt{s}}[e^{K^*s} (C_1 \cos K^*s + C_2 \sin K^* s)]$$

where

$$K^{*4} = \frac{1}{4} (\frac{\lambda^4}{L^2} + \kappa^4)$$

Considering the magnitude of the coefficient K^* for some problems, a further simplification becomes possible when $\lambda^4/L^2 << \kappa^4$:

$$K^{***} \simeq \kappa^4/4$$

The transverse shear stress resultant is:

$$Q_{sz} = -\frac{Eh^3}{12}\frac{e^{K^{**}s}}{\sqrt{s}} \Big[C_1 \{ [-2K^{**3} + \frac{5K^{**}}{4s^2} - \frac{9}{8s^3}] \cos K^{**}s$$

$$+ [-2K^{**2} + \frac{K^{**}}{s} - \frac{5}{4s^2}]K^{**} \sin K^{**}s\}$$

$$+ C_2 \{[-2K^{**3} + \frac{5K^{**}}{4s^2} - \frac{9}{8s^3}] \sin K^{**}s$$

$$- [-2K^{**2} - \frac{K^{**}}{s} - \frac{5}{4s^2}]K^{**} \cos K^{**}s \}\Big]$$

(3-6)

The meridional stress couple is:

$$M_{ss} = -\frac{Eh^3}{12}\frac{e^{K^{**}s}}{\sqrt{s}}\{C_1[(\frac{3}{4s} - K^{**})\frac{\cos K^{**}s}{s} - (-\frac{1}{s} + 2K^{**})K^{**}\sin K^{**}s]$$

$$+ C_2[(\frac{3}{4s} - K^{**})\frac{\sin K^{**}s}{s} + (-\frac{1}{s} + 2K^{**})K^{**}\cos K^{**}s]\}$$

(3-7)

3.1.3 ASYMPTOTIC SOLUTION FOR LARGE ARGUMENTS s(*)

From (2-10)

$$s^2\frac{d^4w}{ds^4} + 2s\frac{d^3w}{ds^3} + (\lambda^4 + s^2\kappa^4)w = 0$$

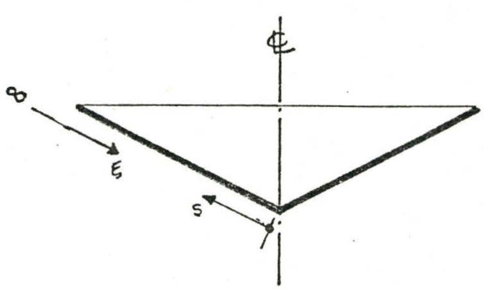


FIG. 3.1 Conical Shell: Meridional Section with Co-ordinates s and ξ.

Setting $s \simeq 1/\xi$ (for $s\to\infty$, $\xi\to 0$) to study the nature of the singularity at $s=\infty$ gives the transformed differential equation:

$$\frac{d^4w}{d\xi^4} + \frac{10}{\xi}\frac{d^3w}{d\xi^3} + \frac{24}{\xi^2}\frac{d^2w}{d\xi^2} + \frac{36}{\xi^3}\frac{dw}{d\xi} + (\frac{\lambda^4}{\xi^6} + \frac{\kappa^4}{\xi^8})w = 0$$

which has at $\xi=0$ (and, therefore, at $s\to\infty$) an irregular singular point.

(*) see APPENDIX II for notes on asymptotic solution.

The normalized differential equation from Section 3.1.2 is:

$$D(u) = \frac{d^4u}{ds^4} + \frac{3}{2}\frac{1}{s^2}\frac{d^2u}{ds^2} - \frac{3}{s^3}\frac{du}{ds} + \left[\frac{45}{6}\frac{1}{s^4} + \frac{\lambda^4}{s^2} + \kappa^4\right] u = 0$$

is derived from (2-10), it can be satisfied by THOMÉ's normal solution series as an asymptotic expansion about the irregular singular point at $s=\infty$,

$$u = Ae^{Q(s)}s^c\Phi(s) = e^{P(s)}s^c\Phi(s)$$

where $Q(s)$ is a polynomial in s and $\Phi(s) = a_o + \frac{a_1}{s} + \frac{a_2}{s^2} + \cdots$

If the polynomial is linear and limited to a single term

$P(s) = ms$, (m = constant)

then the normal solution series is

$$u = e^{ms}\sum_{n=0}^{\infty} a_n s^{-n+c} = e^{ms}\sum_{n=0}^{\infty} a_n s^{-(n-c)}$$

where

$s>0$, $a_o \neq o$

Substituting u into the normalized differential equation and shifting the index,

$$D(u) = \sum_{n=0}^{\infty} (m^4 + \kappa^4) a_n s^{-n} - \sum_{n=1}^{\infty} 4m^3(n-1-c)a_{n-1}s^{-n}$$

$$+ \sum_{n=2}^{\infty} \{m^2[6(n-2-c)(n-1-c) + \frac{3}{2}] + \lambda^4\}a_{n-2}s^{-n}$$

$$- \sum_{n=3}^{\infty} m\{(n-3-c)[4(n-2-c)(n-1-c) + 3] + 3\}a_{n-3}s^{-n}$$

$$+ \sum_{n=4}^{\infty} \{(n-4-c)[(n-3-c)(n-2-c)(n-1-c) + \frac{3}{2}(n-3-c) + 3] + \frac{45}{6}\}a_{n-4}s^{-n} = 0$$

An inverse power series in terms of (1/s) vanishes, if and only if all coefficients of $(1/s)^n$ vanish:

$$n = 0: \quad (m^4 + \kappa^4)a_o = 0$$

As $a_o \neq 0$, the characteristic equation

$$m_4 + \kappa^4 = 0$$

results, where the roots are

$$m_{1,2,3,4} = \sqrt[4]{-1}\kappa = \pm\sqrt{\pm i}\kappa$$

All the roots of this indicial equation are distinct, therefore, the normal series solution can be applied to yield all pertinent asymptotic solutions for large arguments of s:

$$n = 1: \qquad (m^4 + \kappa^4)a_1 + 4m^3ca_o = 0$$

As $a\neq 0$, $m\neq 0$, $(m^4 + \kappa^4) \neq 0$, then $c = 0$.

$$n = 2: \quad (m^4 + \kappa^4)a_2 - 4m^3(1-c)a_1 + \{m^2\ [-c(1-c) + \tfrac{3}{2}] + \lambda^4\}a_o = 0$$

then

$$a_1 = [\frac{3}{8m} + \frac{m}{4}(\frac{\lambda^4}{m^4})\ ]a_o = [\frac{3}{8m}\ \frac{\tilde{m}}{m} - \frac{m}{4}\ \alpha^4]a_o$$

where $\tilde{m}$ is the conjugate complex of m.

$$n = 3: \quad (m^4 + \kappa^4)a_3 - 4m^3(2)a_2 + \{m^2[6(1)(2) + \tfrac{3}{2}] + \lambda^4\}a_1$$

$$-m(0)(3)a_o = 0$$

then

$$a_2 = (\frac{81}{128}\frac{1}{m^2} - \frac{15}{34}\alpha^4 + \frac{m^2\alpha^8}{32})a_o$$

$$n = 4: \quad (m^4 + \kappa^4)a_4 - 4m^3(3)a_3 + \{m^2[6(2)(3) + \frac{3}{2}] + \lambda^4\}a_2$$

$$-m\{[4(2)(3) + 3] + 3\}a_1 + (\frac{45}{6})a_o = 0$$

then

$$a_3 = (\frac{75}{24}\frac{1}{m} - \frac{m\alpha^4}{12})(\frac{81}{128}\frac{1}{m^2} - \frac{15\alpha^4}{32} + \frac{m^2\alpha^8}{32})a_o$$

$$-(\frac{5}{2}\frac{1}{m^2})(\frac{3}{8}\frac{1}{m} - \frac{m\alpha^4}{4} - \frac{5}{8}\frac{1}{m^3})a_o$$

etc., where

$$\alpha^4 = (\frac{\lambda}{\kappa})^4 = (\frac{\lambda}{-m})^4$$

This asymptotic solution is valid only for large values of s. Consequently the boundary conditions at small s are meaningless and are to be ignored. Therefore, only that part of the solution associated with external boundary where s is large shall be retained. As the external boundary conditions are associated with functions which increases with increasing s, only the roots m_1 and m_4 that yield exponential functions e^{ms} which increase with s are consistent with this asymptotic solution, and therefore are pertinent to the bending problem of the closed conical shell. As

$$m^4 = \kappa^4, \quad (\kappa>0)$$

then,

$$m_{1,2,3,4} = \frac{1}{\sqrt{2}}(\pm 1 \pm i)\kappa$$

Substituting m_1 and m_2 into u gives

$$u = e^{(\kappa/\sqrt{2})s} e^{(i\kappa/\sqrt{2})s} \{1 + [\frac{3}{8} \frac{1}{\kappa(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})} - \frac{\kappa}{4} (\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}) \alpha^4] \frac{1}{s}$$

$$+ [\frac{81}{128} \frac{1}{\kappa^2 i} - \frac{15\alpha^4}{32} + \frac{i\kappa^2\alpha^8}{32}] \frac{1}{s^2} + \cdots \} a_o$$

$$+ e^{(\kappa/\sqrt{2})s} e^{(-i\kappa/\sqrt{2})s} \{1 + [\frac{3}{8} \frac{1}{\kappa(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})} - \frac{\kappa}{4} (\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}) \alpha^4] \frac{1}{s}$$

$$+ [- \frac{81}{128} \frac{1}{\kappa^2 i} - \frac{15\alpha^4}{32} + \frac{i\kappa^2\alpha^8}{32}] \frac{1}{s^2} + \cdots \} b_o$$

In EULER form,

$$u = e^{(\kappa/\sqrt{2})s} [\cos(\frac{\kappa s}{\sqrt{2}}) + i\sin(\frac{\kappa s}{\sqrt{2}})] \{1 + [\frac{3(1-i)}{8\sqrt{2}\kappa} - \frac{\kappa(1-i)\alpha^4}{4\sqrt{2}}] \frac{1}{s}$$

$$+ [\frac{-81}{128} \frac{1}{\kappa^4} - \frac{15\alpha^4}{32} + \frac{i\kappa^2\alpha^8}{32}] \frac{1}{s^2}$$

$$+ \cdots \} a_o$$

$$+ e^{(\kappa/\sqrt{2})s} [\cos(\frac{\kappa s}{\sqrt{2}}) - i\sin(\frac{\kappa s}{\sqrt{2}})] \{1 + [\frac{3(1+i)}{8\sqrt{2}\kappa} - \frac{\kappa(1-i)\alpha^4}{4\sqrt{2}}] \frac{1}{s}$$

$$+ [\frac{81}{128} \frac{i}{\kappa^4} - \frac{15\alpha^4}{32} - \frac{i\kappa^2\alpha^8}{32}] \frac{1}{s^2}$$

$$+ \cdots \} b_o$$

In algebraic form,

$$u = e^{(\kappa/\sqrt{2})s}\{[(a_o + b_o)\cos(\frac{\kappa s}{\sqrt{2}}) + i(a_o - b_o)\sin(\frac{\kappa s}{\sqrt{2}})]$$

$$+\Big[[(A - iB)a_o + (A + iB)b_o]\cos(\frac{\kappa s}{\sqrt{2}})$$

$$+i[(A - iB)a_o + (A + iB)b_o]\sin(\frac{\kappa s}{\sqrt{2}})\Big]\frac{1}{s}$$

$$+\Big[[(C - iD)a_o + (C + iD)b_o]\cos(\frac{\kappa s}{\sqrt{2}})$$

$$+i[(C - iD)a_o - (C + iD)b_o]\sin(\frac{\kappa s}{\sqrt{2}})\Big]\frac{1}{s^2}$$

$$+ \ldots\}$$

Hence,

$$A = \frac{1}{\sqrt{2}}(\frac{3}{8\kappa} - \frac{\kappa\alpha^4}{4}), \quad B = \frac{1}{\sqrt{2}}(\frac{3}{8\kappa} + \frac{\kappa\alpha^4}{4}),$$

$$C = -\frac{15\alpha^4}{32}, \quad D = \frac{81}{128\kappa^2} - \frac{\kappa^2\alpha^8}{32},$$

If

$$a_o + b_o = C_1 \text{ and } i(a_o + b_o) = C_2$$

then,

$$a_o = (C_1 - iC_2)/2 \text{ and } b_o = (C_1 + C_2)/2$$

Substituting A,B,C,D,a_o and b_o into u and collecting cosine and sine terms,

$$u=e^{(\kappa/\sqrt{2})s}\{C_1[(1 + \frac{A}{s} + \frac{C}{s^2} + \cdots)\cos(\frac{\kappa s}{\sqrt{2}}) + (\frac{B}{s} + \frac{D}{s^2} + \cdots)\sin(\frac{\kappa s}{\sqrt{2}})]$$

$$+C_2[-(\frac{B}{s} + \frac{D}{s^2} + \cdots)\cos(\frac{\kappa s}{\sqrt{2}}) + (1 + \frac{A}{s} + \frac{C}{s^2} + \cdots)\sin(\frac{\kappa s}{\sqrt{2}})]\}$$

As $w = u/\sqrt{s}$, $\kappa = \sqrt{2}K$ and $\alpha = \lambda/\kappa$, then,

$$w = \frac{e^{Ks}}{\sqrt{s}} \Bigg[C_1 \{ [1 - \frac{K\alpha^4}{4}(1 - \frac{3}{4}\frac{1}{K^2\alpha^4})\frac{1}{s} - \frac{15}{32}\frac{\alpha^4}{s^2} + \dots] \cos Ks$$

$$+ [\frac{K\alpha^4}{4}(1 + \frac{3}{4}\frac{1}{K^2\alpha^4})\frac{1}{s} - \frac{K^2\alpha^8}{16}(1 - \frac{81}{16}\frac{1}{K^4\alpha^8})\frac{1}{s^2} + \dots] \sin Ks \}$$

$$C_2 \{ -[\frac{K\alpha^4}{4}(1 + \frac{3}{4}\frac{1}{K^2\alpha^2})\frac{1}{s} - \frac{K^2\alpha^8}{16}(1 - \frac{81}{16}\frac{1}{K^4\alpha^8})\frac{1}{s^2} + \dots] \cos Ks$$

$$+ [1 - \frac{K\alpha^4}{4}(1 - \frac{3}{4}\frac{1}{K^2\alpha^4})\frac{1}{s} - \frac{15\alpha^4}{32\, s^2} + \dots] \sin Ks \} \Bigg] \qquad (3\text{-}8)$$

Then the stress couple,

$$M_{ss} = - \frac{Eh^3}{12}\frac{d^2w}{ds^2}$$

$$= - \frac{Eh^3}{12} \Bigg[C_1 \{ [\frac{K}{2}(-\frac{5}{4} + K^2\alpha^4)\frac{1}{s} + \frac{1}{16}(\frac{33}{8} - 2K^4\alpha^8)\frac{1}{s^2} + \dots] \cos Ks$$

$$- [2K^2 - \frac{K}{2}(\frac{5}{4} + K^2\alpha^4)\frac{1}{s} + \frac{9}{16}\frac{K^2\alpha^4}{s^2} + \dots] \sin Ks \}$$

$$+ C_2 \{ [2K^2 - \frac{K}{2}(\frac{5}{4} + K^2\alpha^4)\frac{1}{s} + \frac{9}{16}\frac{K^2\alpha^4}{16\, s^2} + \dots] \cos Ks$$

$$+ [\frac{K}{2}(-\frac{5}{4} + K^2\alpha^4)\frac{1}{s} + \frac{1}{16}(\frac{33}{8} - 2K^4\alpha^8)\frac{1}{s^2} + \dots] \sin Ks \} \Bigg]$$

$$(3\text{-}9)$$

and the transverse shear stress resultant,

$$Q_{sz} = -\frac{Eh^3}{12}\left(\frac{d^3w}{ds^3} + \frac{1}{s}\frac{d^2w}{ds^2}\right)$$

$$= -\frac{Eh^3}{12}\frac{e^{Ks}}{\sqrt{s}} K\Big[C_1\{ [-2K^2 + \frac{K^3\alpha^4}{s} + (\frac{73}{128} - \frac{13K^2\alpha^4}{16} - \frac{K^4\alpha^8}{8})\frac{1}{s^2} + \cdots]\cos Ks$$

$$-[2K^2 - \frac{K}{4s} + (\frac{73}{128} + \frac{13K^2\alpha^4}{16} - \frac{K^4\alpha^8}{8})\frac{1}{s^2} + \cdots]\sin Ks \}$$

$$+C_2\{[2K^2 - \frac{K}{4s} + (\frac{73}{128} + \frac{13K^2\alpha^4}{16} - \frac{K^4\alpha^8}{8})\frac{1}{s^2} + \cdots]\cos Ks$$

$$+ [-2K^2 + \frac{K^3\alpha^4}{s} + (\frac{73}{128} - \frac{13K^2\alpha^4}{16} - \frac{K^4\alpha^8}{8})\frac{1}{s^2} + \cdots]\sin Ks\}\Big]$$

(3-10)

(a) Second Order Asymptotic Solution

If the coefficient series in (3-8), (3-9) and (3-10) are truncated to exclude terms with $(1/s)^n$ for n=3,4, ..., then it represents the second order asymptotic solution of the differential equation.

(b) First Order Asymptotic Solution

Truncating the asymptotic solution to retain terms up to (1/s) order gives the asymptotic solution of the first order.

3.2 PARTICULAR SOLUTION

John-Erik EKSTRÖM in 1932 demonstrated that the complementary solution of GECKELER was quite accurate for thin shells, but that the membrane solution as a representative of the particular solution in some cases led to errors as large as 30 per cent.

The membrane solution can be considered as a simplified particular solution when in the differential equation the bending stiffness is reduced to zero, and the resulting simplified differential equation is solved for w^p:

$$s^2 \frac{d^4w^p}{ds^4} + 2s\frac{d^3w^p}{ds^3} + (\lambda^4 + s^2\kappa^4)w^p = s^2 \frac{p_z}{D}$$

This means that

$$N_{\theta\theta}\tan\phi + \frac{d}{ds}\left[\frac{d}{ds}(M_{ss})\right] - skw + sp_z = 0$$

if $M_{ss} \simeq 0$, the transverse bending is assumed vanishingly small, and $w^p \rightarrow w^m$ (w^m is the lateral displacement).

$$N_{\theta\theta}\tan\phi - skw^m + sp_z = - Ehtan^2\phi\frac{w^m}{s} - skw^m + sp_z = 0$$

As

$$N_{\theta\theta} = Ehtan\phi w/s$$

then

$$[(Ehtan^2\phi/s) + sk]w^m + sp_z = 0$$

and

$$w^m = \frac{p_z}{\frac{Ehtan^2\phi}{s^2} + k}$$

As was pointed out by EKSTRÖM, the use of $w = w^c + w^m$ may occasionally lead to large errors; it is therefore advisable to calculate the particular solution from the original differential equation:

$$D(w^p) = \frac{d^4w^p}{ds^4} + \frac{2}{s}\frac{d^3w^p}{ds^3} + \left(\frac{\lambda^4}{s^2} + \kappa^4\right) w^p = \frac{p_z}{D}$$

Introducing the LAURENT series as an asymptotic particular solution about the irregular singular point at $s = \infty$,

$$w^p = \sum_{k=-\infty}^{\infty} a_k s^k = \cdots + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_o (a_{-1}/s) + (a_{-2}/s^2)$$

$$+ (a_{-3}/s^3) + (a_{-4}/s^4) + (a_{-5}/s^5) + (a_{-6}/s^6) + \cdots$$

Substituting w^p into the differential equation,

$$D(w^p) = \cdots + (\kappa^4 a_4 + \cdots) s^4 + (\kappa^4 a_3 + \cdots) s^3 + (\kappa^4 a_2 + \lambda^4 a_4) s^2$$

$$+ (\kappa^4 a_1 + \lambda^4 a_3) s + (\kappa^4 a_o + \lambda^4 a_2 + 72 a_4) + [(\kappa^4 a_{-1} + \lambda^4 a_1 + 12 a_3)/s]$$

$$+ [(\kappa^4 a_{-2} + \lambda^4 a_o)/s^2] + [(\kappa^4 a_{-3} + \lambda^4 a_{-1})/s^3] + [(\kappa^4 a_{-2} + \lambda^4 a_{-4})/s^4]$$

$$+ [(\kappa^4 a_{-3} + 12 a_{-1} + \cdots)/s^5] + [(72 a_{-2} + \lambda^4 a_{-4} + \cdots)/s^6] + \cdots$$

We want a particular solution for the domain $s_1 < s \leqslant L$, where s_1 is sufficiently large, so that all values of $s > s_1$ are large. This series is semi-convergent and it is used in its truncated form inclusive of terms $k = -2$ only as a rough approximation near the boundary. As the hydrostatic load is almost uniform, the influence of w^P is of subordinate significance. Then the particular solution as a truncated LAURENT series has the form:

$$w^p \simeq \frac{1}{4K^4} \left[Cs^2 + Bs + (A - \alpha^4 C) - \frac{\alpha^4 B}{s} - \frac{\alpha^4 (A - \alpha^4 C)}{s^2} \right]$$

in which

$$\alpha = \frac{\lambda}{\kappa},$$

and

$$\kappa^4 = 4K^4 \qquad (3\text{-}11)$$

As for hydrostatic loading,

$$-\frac{12}{Eh^3}\gamma_w(d_o - s\sin\phi) = \frac{p_z}{D} = A + Bs + Cs^2$$

hence,

$$A = -\frac{12}{Eh^3}\gamma_w d_o, \quad B = \frac{12}{Eh^3}\gamma_w \sin\phi, \quad C = 0$$

where γ_w is the specific weight of the liquid and d_o is the height of liquid column.

CHAPTER IV

SOLUTIONS OF CONICAL SHELL UNDER UNIFORM SOIL BEARING PRESSURE AND OF ROTATIONALLY SYMMETRICAL CYLINDRICAL SHELL UNDER HYDROSTATIC PRESSURE

4.1 TRANSFORMED EQUILIBRIUM EQUATIONS

From Chapter II (2.2), the three remaining equilibrium equations are:

$$\frac{d(rN_{\phi\phi})}{d\phi} - r_{\phi}\cos\phi N_{\theta\theta} - rQ_{\phi z} + rr_{\phi}p_{\phi} = 0$$

$$\frac{d(rQ_{\phi z})}{d\phi} + r_{\phi}\sin\phi N_{\theta\theta} + rN_{\theta\theta} + rr_{\phi}p_{z} = 0$$

$$\frac{d(rM_{\phi\phi})}{d\phi} - r_{\phi}\cos\phi M_{\theta\theta} - rr_{\phi}Q_{\phi z} = 0$$

Multiplying the first equation by $\sin\phi$ and the second equation by $\cos\phi$, and then adding and integrating with respect to ϕ gives,

$$N_{\phi\phi} = -\cot\phi Q_{\phi z} + \frac{1}{r_{\theta}}\frac{F(\phi)}{\sin^2\phi} \qquad (4\text{-}1)$$

where

$$F(\phi) = C - \int r_{\phi}r_{\theta}\sin\phi(p_{\phi}\sin\phi + p_{z}\cos\phi)d\phi$$

This equation represents the axial equilibrium of forces of a finite free-body of the rotational shell isolated by a horizontal section at any ϕ. Multiplying the second equation by $\sin\phi$, and setting $r = r_{\theta}\sin\phi$, and substituting from (4-1) yields,

$$N_{\theta\theta} = -\frac{1}{r_{\phi}}\frac{d(r_{\theta}Q_{\phi z})}{d\phi} + \frac{H(\phi)}{r_{\phi}} \qquad (4\text{-}2)$$

where

$$H(\phi) = \frac{F(\phi)}{\sin^2\phi} + r_\phi r_\theta P_z$$

(4-1), (4-2) and the third equilibrium equation form the transformed equilibrium equations which are more appropriate for applications.

4.2 COMPATIBILITY CONDITIONS FOR SYMMETRICAL DEFORMATION OF THE ROTATIONAL SHELL

From Chapter II, the kinematic equations of deformation are:

$$\varepsilon_{\phi\phi} = \frac{1}{r_\phi}\frac{dv}{d\phi} - \frac{w}{r_\phi}$$

$$\varepsilon_{\theta\theta} = \frac{\cos\phi\, v}{r} - \frac{w}{r_\theta}$$

$$\chi = \frac{v}{r_\phi} + \frac{1}{r_\phi}\frac{dw}{d\phi}$$

Multiplying $\varepsilon_{\phi\phi}$ by r_ϕ and $\varepsilon_{\theta\theta}$ by r_θ, subtracting one equation from another, multiplying the result by the integrating factor $(1/\sin\phi)$, and then integrating and solving for v yields:

$$v = \sin\phi\left(\int\frac{r_\phi\varepsilon_{\phi\phi} - r_\theta\varepsilon_{\theta\theta}}{\sin\phi}d\phi + C\right)$$

Substituting v into $\varepsilon_{\theta\theta}$ and solving for w, gives

$$w = v\cos\phi - r_\theta\varepsilon_{\theta\theta}$$

The compatibility condition of the kinematical quantities $\varepsilon_{\phi\phi}$, $\varepsilon_{\theta\theta}$, and χ is insured if a differential equation can be derived by combining the three displacement definitions into one relation. The compatibility condition for consistent deformation can be obtained by substituting $(dw/d\phi)$ and $(dv/d\phi)$ into χ,

$$\chi = \frac{1}{Ehr_\phi}\{[N_{\phi\phi}(r_\phi + \nu r_\theta) - N_{\theta\theta}(r_\theta + \nu r_\phi)]\cot\phi + \frac{r_\theta}{h}\frac{dh}{d\phi}(N_{\theta\theta} - \nu N_{\phi\phi})$$

$$- \frac{d}{d\phi}[r_\phi(N_{\theta\theta} - \nu N_{\phi\phi})]\} \qquad (4\text{-}3)$$

as

$$r_\phi \varepsilon_{\phi\phi} = \frac{r_\phi}{Eh}(N_{\phi\phi} - \nu N_{\theta\theta}) \text{ and } r_\theta \varepsilon_{\theta\theta} = \frac{r_\theta}{Eh}(N_{\theta\theta} - \nu N_{\phi\phi})$$

4.3 MEMBRANE THEORY OF ROTATIONALLY SYMMETRICAL SHELL

If the rotationally symmetrical shell is so thin that its bending stiffness may be neglected, then,

$$M_{\phi\phi} = M_{\theta\theta} = M_{\theta\phi} = M_{\phi\theta} = 0$$

and from (2-4), $Q_{\phi z} = 0$. Consequently (4-1) becomes

$$N^m_{\phi\phi} = \frac{F(\phi)}{r_\theta \sin^2\phi} \qquad (a)$$

(4-2) becomes

$$N^m_{\theta\theta} = -\frac{H(\phi)}{r_\phi} \qquad (b)$$

and (4-3) becomes

$$\frac{N^m_{\phi\phi}}{r_\phi} + \frac{N^m_{\theta\theta}}{r_\theta} + p_z = 0 \qquad (c)$$

where

$$r = r_\theta \sin\phi$$

Conical shells represent limiting cases of the general rotational shells when $r_\phi d_\phi \to ds_\phi = ds$, $r_\phi \to \infty$ and $\phi \to$ constant.

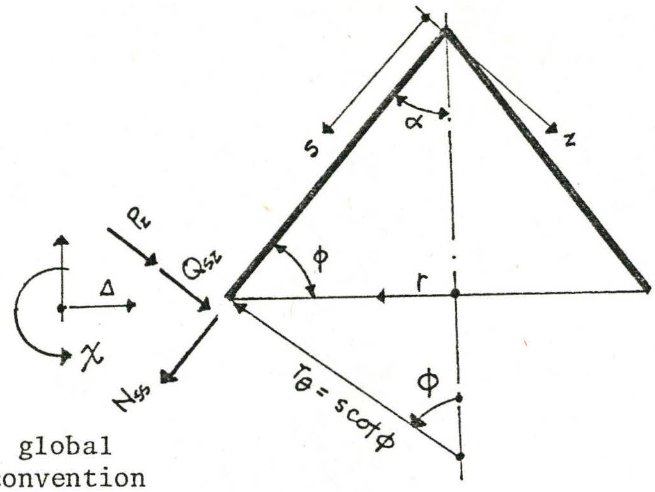


FIG. 4.1 Meridional Section of a Conical Shell.

From (4-1) and (a)

$$N_{ss} = -\cot\phi\frac{U}{r_\theta} + N^m_{ss} = (-U/s) + N^m_{ss} \tag{4-4}$$

from (4-2) and (b)

$$N_{\theta\theta} = -\frac{1}{r\phi}\frac{dU}{d\phi} + N^m_{\theta\theta} = (-dU/ds) + N^m_{\theta\theta} \tag{4-5}$$

where

$$N_{\phi\phi} = N_{ss},\ N^m_{\phi\phi} = \frac{F(\phi)}{r_\theta \sin^2\phi} = N^m_{ss},\ U = r_\theta Q_{sz} = s\cot\phi Q_{sz},\ N^m_{\theta\theta} = H(\phi)/r_\phi$$

4.4 SOLUTIONS OF THE CONICAL SHELL UNDER UNIFORM SOIL BEARING PRESSURE

Substituting (4-4) and (4-5) into (4-3) and introducing the MEISSNER differential operator,

$$L(\ ) \equiv -\frac{r_\theta d^2(\ )}{r_\phi d\phi^2} + \left[\frac{r_\theta \cot\phi}{r_\phi} + \frac{d}{d\phi}\left(\frac{r_\theta}{r_\phi}\right)\right]\frac{d(\ )}{d\phi} - \frac{r_\phi \cot^2\phi(\ )}{r_\theta}$$

into this equation gives

$$L(U) + \nu U + \frac{1dh}{hd\phi}(\nu\cot\phi U - \frac{r_\theta}{r_\phi}\frac{dU}{d\phi}) = Ehr_\phi(\chi - \chi^m) \qquad (A)$$

The second equation is obtained from the equilibrium equation in Chapter II, (2.2),

$$\frac{d(rM_{\phi\phi})}{d\phi} - r_\phi \cos\phi M_{\theta\theta} - rr_\phi Q_{\phi z} = 0$$

where

$$r = r_\theta \sin\phi$$

Substituting from the constitutive equations

$$M_{\phi\phi} = - D(\chi_{\phi\phi} + \nu\chi_{\theta\theta}) \quad \text{and} \quad M_{\theta\theta} = -D(\chi_{\theta\theta} + \nu\chi_{\phi\phi})$$

and

$$U = r_\theta Q_\phi$$

then, introducing the MEISSNER differential operator, L, gives,

$$L(\chi) - \nu\chi + \frac{3}{h}\frac{dh}{d\phi}(\nu\cot\phi\chi + \frac{r_\theta}{r_\phi}\frac{d\chi}{d\phi}) = -\frac{r_\phi U}{D} \qquad (B)$$

For conical shells the MEISSNER differential operator becomes,

$$L_s = \lim_{r_\phi \to \infty} [L(\ )/r_\phi] = \cot\phi\ [s\frac{d^2(\ )}{ds^2} + \frac{d(\ )}{ds} - \frac{1}{s}(\ )]$$

In order that the limits of the MEISSNER equations (A) and (B) be finite, it was necessary to multiply both equations by the reciprocal of r_ϕ before proceeding to the limit $r_\phi \to \infty$. For the conical shell with constant wall thickness h, dh/ds = 0, the homogeneous equation for the boundary perturbation is obtained for $\chi^m=0$,

$$L_s(U) = Eh\tan\phi \qquad (A)$$

$$L_s(\chi) = -\tan\phi U/D \qquad (B)$$

which can be uncoupled into two higher order differential equations,

$$L_sL_s(U) + \mu^4 U = 0 \qquad (A)$$

$$L_sL_s(V) + \mu^4 V = 0 \qquad (B)$$

where

$$\mu^4 = Eh\tan^2\phi/D = 12(1-\nu^2)\tan\phi/h^2$$

As

$$L_sL_s(\Phi) + \mu^4\Phi = L_s[L_s(\Phi)\pm i\mu^2\Phi]\pm i\mu^2[L_s(\Phi) \pm i\mu^2\Phi] = 0$$

where Φ represents either U or χ, then (A) and (B) can be reduced to second order equations

$$L_s(U)\pm i\mu^2 U = 0 \qquad (A)$$

$$L_s(\chi)\pm i\mu^2\chi = 0 \qquad (B)$$

Introducing a new independent variable $\eta = 2\mu\sqrt{s}$, these uncoupled equations become:

$$\eta^2 \frac{d^2U}{d\eta^2} + \eta\frac{dU}{d\eta} + (-2^2 \pm i\eta^2)U = 0$$

$$\eta^2 \frac{d^2\chi}{d\eta^2} + \eta\frac{d\chi}{d\eta} + (-2^2 \pm i\eta^2)\chi = 0$$

The differential equation

$$\eta^2 \frac{d^2U}{d\eta^2} + \eta\frac{dU}{d\eta} - (2^2 + i\eta^2)U = 0$$

is satisfied by the KELVIN functions $(ber_2\eta + ibei_2\eta)$ and $(ker_2\eta + ikei_2\eta)$.

Applying the recurrence relations of KELVIN functions to a shell closed at the apex gives,

$$U = \cot\phi\, sQ_{sz} = C_1(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta) + C_2(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta)$$

From (4-5),

$$N_{\theta\theta} = -\frac{dU}{ds} = -\mu_s^2[\,C_1(\frac{4}{\eta^2}\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{ber}'\eta - \frac{8}{\eta^3}\mathrm{bei}'\eta) + C_2(\frac{4}{\eta^2}\mathrm{bei}\eta - \frac{2}{\eta}\mathrm{bei}'\eta + \frac{8}{\eta^3}\mathrm{ber}'\eta)\,] \quad (4\text{-}6)$$

and from (4-4),

$$N_{ss} = -\frac{U}{s} = \mu_s^2\,[C_1(-\frac{4}{\eta^2}\mathrm{ber}\eta + \frac{8}{\eta^3}\mathrm{bei}'\eta) + C_2(\frac{4}{\eta^2}\mathrm{bei}\eta - \frac{8}{\eta^3}\mathrm{ber}'\eta)] \quad (4\text{-}7)$$

From (A),

$$\chi = \frac{1}{Eh\tan\phi}\,(s\frac{d^2U}{ds^2} + \frac{dU}{ds} - \frac{U}{s})$$

as

$$\eta = 2\mu_s\sqrt{s}$$

and

$$\mu_s = \sqrt{(12(1-\nu^2)\tan\phi)/h}$$

The meridional rotation:

$$E\chi = -\frac{\sqrt{12(1-\nu^2)}}{h^2}\,[C_1(\mathrm{bei}\eta) + \frac{2}{\eta}\mathrm{ber}'\eta) - C_2\,(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta)] \quad (4\text{-}8)$$

The transverse shear stress resultant:

$$Q_{sz} = \frac{\tan\phi}{s}\,[C_1\,(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta) + C_2(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta)] \quad (4\text{-}9)$$

The meridional stress-couple:

$$M_{ss} = -D\left(\frac{d\chi}{ds} + \nu\frac{\chi}{s}\right)$$

$$= 2\frac{\tan\phi}{\eta^2}\left\{C_1\left[\eta \mathrm{bei}'\eta - 2(1-\nu)\left(\mathrm{bei}\,\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)\right]\right.$$

$$\left. -C_2\left[\eta \mathrm{ber}'\eta - 2(1-\nu)\left(\mathrm{ber}\,\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right)\right]\right\} \quad (4\text{-}10)$$

The circumferential stress-couple:

$$M_{\theta\theta} = -D\left(\frac{\chi}{s} + \nu\frac{d\chi}{ds}\right)$$

$$= 2\frac{\tan\phi}{\eta^2}\left\{C_1\left[\nu\eta \mathrm{bei}'\eta + 2(1-\nu)\left(\mathrm{bei}\,\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)\right]\right.$$

$$\left. -C_2\left[\nu\eta \mathrm{ber}'\eta + 2(1-\nu)\left(\mathrm{ber}\,\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right)\right]\right\} \quad (4\text{-}11)$$

The radial displacement:

$$\Delta = -\frac{s\cos\phi}{Eh}N_{\theta\theta}$$

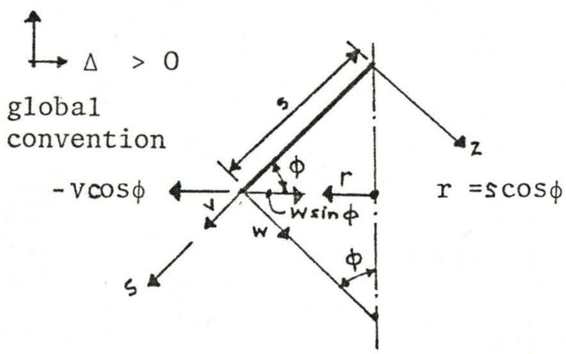
$$= -\mu^2\frac{s\cos\phi}{Eh\eta}\left\{C_1\left[-2\mathrm{ber}'\eta + \frac{4}{\eta}\mathrm{ber}\,\eta - \frac{8}{\eta^2}\mathrm{bei}'\eta\right]\right.$$

$$\left. +C_2\left[-2\mathrm{bei}'\eta + \frac{4}{\eta}\mathrm{bei}\,\eta + \frac{8}{\eta^2}\mathrm{ber}'\eta\right]\right\} \quad (4\text{-}12)$$

where $\nu \simeq 0$ and

$$\mu^2\frac{s\cos\phi}{Eh\eta} = \eta\frac{\cos\phi}{4h}$$

The meridional displacement:



$$\Delta = -v\cos\phi + w\sin\phi$$

$$\Delta = -r\varepsilon_{\theta\theta} = -r\left(\frac{N_{\theta\theta} - \nu N_{ss}}{Eh}\right) \simeq -s\cos\phi\frac{N_{\theta\theta}}{Eh}$$

as

$$\nu \simeq 0, \text{ and } \varepsilon_{\theta\theta} > 0$$

then

$$-v\cos\phi + w\sin\phi = -s\cos\phi\,\frac{N_{\theta\theta}}{Eh}$$

or

$$N_{\theta\theta} = (v - w\tan\phi)\,\frac{Eh}{s}$$

From equilibrium equation (2-8) in z-direction:

$$N_{\theta\theta}\tan\phi + \frac{d}{ds}\,(Q_{sz}s) + (p_z(s) - kw)s = 0$$

Thus, the meridional displacement can be calculated by successive approximations from the formula

$$v(s) = \frac{h^2}{12\tan\phi}\left(s^2\,\frac{d^4w}{ds^4} + 2s\,\frac{d^3w}{ds^3}\right) + \left(\frac{k}{Eh\tan\phi} + \tan\phi\right)w - \frac{s^2}{Eh\tan\phi}\,p_z(s)$$

(4-13)

For membrane state neglect bending stiffness

$$\frac{h^2}{12} \to 0: \quad v^m(s) = \left(\frac{K}{Eh\tan\phi} + \tan\phi\right)w^m - \frac{s^2}{Eh\tan\phi} p_z(s) \qquad (4\text{-}14)$$

4.5 SOLUTIONS OF ROTATIONALLY SYMMETRICAL CYLINDRICAL SHELL SUBJECT TO HYDROSTATIC PRESSURE

If a rotational shell has a cylindrical surface, then $r_\phi \to \infty$, $r_\theta \to R$, $\phi \to \pi/2$, and $r_\phi d\phi \to ds_\phi = ds$

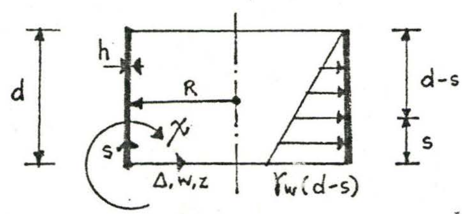


FIG. 4.2 The Longitudinal Section of a Cylindrical Shell.

The MEISSNER differential operator becomes,

$$\lim_{r_\phi \to \infty} \frac{L(\ )}{r_\phi} = \lim_{r_\phi \to \infty} \left\{ r_\theta \frac{d^2(\ )}{ds^2} + \left[\frac{r_\theta}{r_\phi}\cos\phi + \frac{d}{ds}\left(\frac{r_\theta}{r_\phi}\right)\right]\frac{d(\ )}{ds} - \frac{1}{r_\theta}\cot^2\phi(\ ) \right\} = R\frac{d^2(\ )}{ds^2}$$

for h = constant, the MEISSNER equations are:

$$R\frac{d^2U}{ds^2} = Eh\,(\chi - \chi^m) \qquad (A)$$

$$R\frac{d^2\chi}{ds^2} = -\frac{U}{D} \qquad (B)$$

where $U = RQ_{sz}$

The uncoupled equations are

$$\frac{d^4U}{ds^4} + 4\lambda'^4 U = -\frac{Eh}{r^2}\frac{d^2\chi^m}{ds^2}$$

$$\frac{d^4\chi}{ds^4} + 4\lambda'^4 \chi = \frac{\chi^m}{R^2D}$$

where

$$\lambda'^4 = \frac{Eh}{4R^2D} = \frac{3(1-\nu^2)}{R^2h^2}$$

as

$$\chi = \frac{dw}{ds}$$

the homogeneous equation when $\chi^m = 0$ is:

$$\frac{d^4w}{ds^4} + 4\lambda'^4 w = 0$$

which has the solution

$$w^c = e^{\lambda' s}(C_1\cos\lambda' s + C_2\sin\lambda' s) + e^{-\lambda' s}(C_3\cos\lambda' s + C_4\sin\lambda' s)$$

Long enough cylindrical shells with $\lambda' d>6$ can be considered as semi-infinite. In such a thin cylindrical shell one boundary effect does not appreciably affect the other boundary. For this reason the boundary effects of this cylindrical shell can be separated, which results in considerable simplification of the boundary conditions of the cylindrical shell. The lower edge effect can therefore be studied separately from that of the upper edge. The part of the homogeneous solution which is pertinent for the lower edge is,

$$w^c = e^{-\lambda' s}(C_3\cos\lambda' s + C_4\sin\lambda' s)$$

For most practical loads

$$p_s(s) = p_o + p_1 s + p_2 s^2 + p_3 s^3$$

where p_i are constants, the particular solution in the form of

$$w^p = a_o + a_1 s + a_2 s^2 + a_3 s^3$$

is adequate, as $d^4w^p/ds^4 = 0$,

$$w^p = R^3 p_s(s)/Eh$$

As the differential equation is exact for rotationally symmetrical cylindrical shells, there are no approximations involved in this solution.

The complete solution is:

$$w(s) = w^c(s) + w^p(s)$$

$$= e^{-\lambda' s}(C_3 \cos\lambda' s + C_4 \sin\lambda' s) - \frac{R^2}{Eh}\gamma_w (d-s) \qquad (4\text{-}15)$$

where γ_w denotes the specific weight of the liquid.

The longitudinal stress-couple:

$$M_{ss} = - D\left(\frac{d^2 w}{ds^2} + \nu \frac{d^2 w}{ds_\theta^{\,2}}\right)$$

$$= - \frac{Eh^3}{12}\left[2\lambda'^2 e^{-\lambda' s}(C_3 \sin\lambda' s - C_4 \cos\lambda' s)\right] \qquad (4\text{-}16)$$

where

$$\nu \simeq 0.$$

The transverse shear stress resultant:

$$Q_{sz} = - D(\frac{d^3w}{ds^3} + \nu\frac{d^2w}{ds^2}) = -D\frac{d^3w}{ds^3}$$

$$= - \frac{Eh^3}{12} \{2\lambda'^3 e^{-\lambda's} [C_3(\cos\lambda's-\sin\lambda's) + C_4(\sin\lambda's+\cos\lambda's]\} \qquad (4\text{-}17)$$

as

$$\nu \frac{d^2w}{ds^2} = 0, \text{ as } \nu \simeq 0.$$

The meridional rotation:

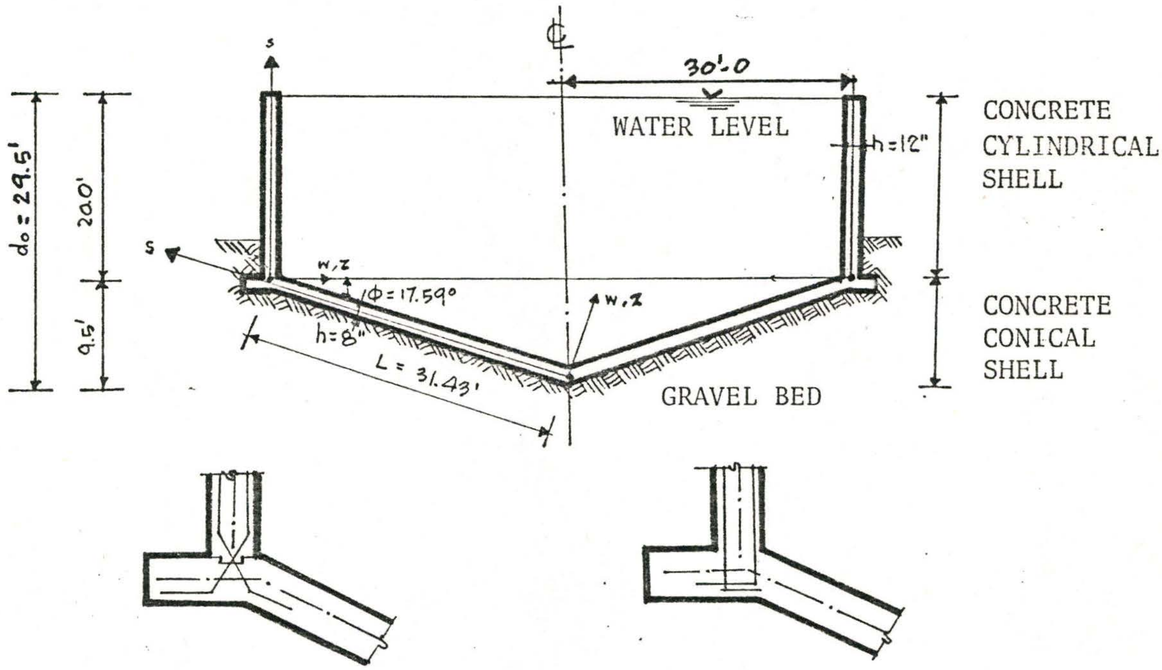
$$\chi = \frac{dw}{ds} = \lambda' e^{-\lambda's}[-C_3(\cos\lambda's + \sin\lambda's) + C_4(\cos\lambda's - \sin\lambda's)] \qquad (4\text{-}18)$$

CHAPTER V

CASE STUDY: WATER TANK

5.1 GENERAL DESCRIPTION OF THE PROBLEM

A cylindrical concrete tank with internal radius of 30 feet and wall thickness of 12 inches is supported by a conical concrete shell with wall thickness 8 inches and meridional slope of 17.59 degrees as shown in Fig. 5.1. The tank is filled with water and rests on a gravel bed foundation. The specific weight of concrete $\gamma = 150$ pcf and the modulus of elasticity of concrete $E = 30 \times 10^6$ psi. The specific weight of water $\gamma_w = 62.5$ pcf. The well-graded gravel bed foundation is assumed to respond to loading in two different ways: firstly, it is assumed to behave like an Euler-Winkler elastic foundation with the foundation stiffness constant $k = 500$ pci, and secondly, it is assumed to exert uniform soil pressure as if the conical shell were infinitely rigid. The analytical solutions used for the conical shell on the Euler-Winkler foundation are the asymptotic and the Geckeler type of approximation. Different analytical results on the transverse bending of the tank are compared. The effect of different foundation stiffness constants on the bending of the tank is examined.



NON-MONOLITHIC JOINT DETAIL

MONOLITHIC JOINT DETAIL

For Concrete	γ,	Specific Weight = 150pcf
	E,	Modulus of Elasticity = 3 X 10^6psi
Gravel	k,	Foundation Stiffness Constant
		Well-Graded Gravel k = 500pci
Water	γ_w,	Specific Weight = 3.617 X 10^{-2}pci
		= 62.5pcf

FIG. 5.1 Meridional Cross-Section of Water Tank

5.2 WATER TANK WITH MONOLITHIC JOINT

5.2.1 CONICAL SHELL SUPPORTED BY EULER-WINKLER ELASTIC FOUNDATION

In the case of a water tank, the conical shell constituting the structural base or foundation of the tank may rest on an approximately elastic foundation. The simplest assumption for the response of the supporting soil is that the stiffness of the subgrade is proportional to the deflection w of the conical shell. This assumption is analogous to presuming the shell to rest on a bed of distributed discrete linear elastic springs. All such springs are regarded as compressive because the soil's capacity to resist tensile stresses is negligible. The soil reaction intensity is then given by $p_f = -kw$. The constant k, the elastic foundation stiffness constant, is expressed in pounds per square inch per inch of deflection. The numerical value of the stiffness constant depends largely on the properties of the subgrade. These values may be roughly estimated by means of the table of the elastic stiffness constants of the soil foundation.

In this analysis the tank is filled with water. The water pressure produces a circumferential stress resultant in the cylindrical shell, which increases from zero at the water level to the maximum at the bottom of the shell structure. The corresponding deformation of the lower edge of the rotationally symmetrical cylindrical shell is described by its radial displacement, Δ_{cyl} and by its meridional rotation, χ_{cyl}.

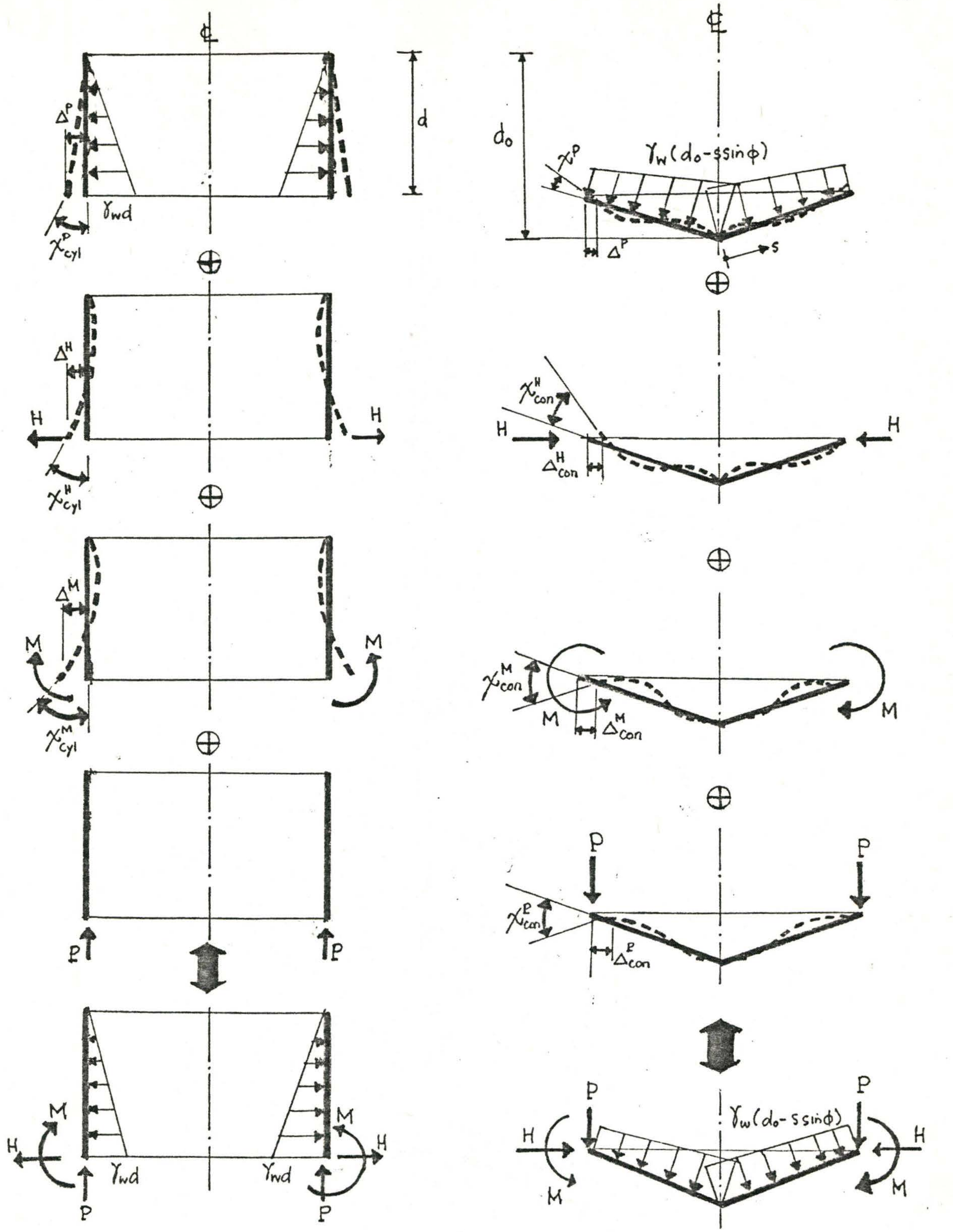


FIG. 5.2 Water Tank on Elastic Foundation

The conical base as the structural foundation of the tank carries two loads: the water pressure and the weight of the wall of the cylindrical shell. It may be assumed that the water pressure on the tank bottom is transmitted directly across the shell wall by transverse compression of the wall to the ground, and hence does not create membrane forces in the conical shell. The weight of the cylindrical shell is applied to the conical shell as a vertical distributed edge load P. The conical shell is supported by the elastic reaction of the ground.

Owing to the continuity of the deformation, the resultant horizontal displacements and the rotational deformations of the cylindrical shell and of the conical shell are assumed to be equal at the intersection of the two shells which represent the compatibility conditions of the deformation of the structure and determine the redundant quantities: the radial reaction H and the transverse stress-couple M of the statically indeterminate shell structure.

(A) COMPLEMENTARY AND PARTICULAR SOLUTIONS FOR ROTATIONALLY SYMMETRICAL CYLINDRICAL SHELL

Transverse Displacement:

$$w(s) = e^{-\lambda' s}(C_3\cos\lambda' s + C_4\sin\lambda' s) - \frac{R^2}{Eh}\gamma_w(d-s) \qquad (4\text{-}15)$$

where

$$\lambda' = \sqrt[4]{3/(Rh^2)}$$

Boundary Conditions for Complementary Solutions

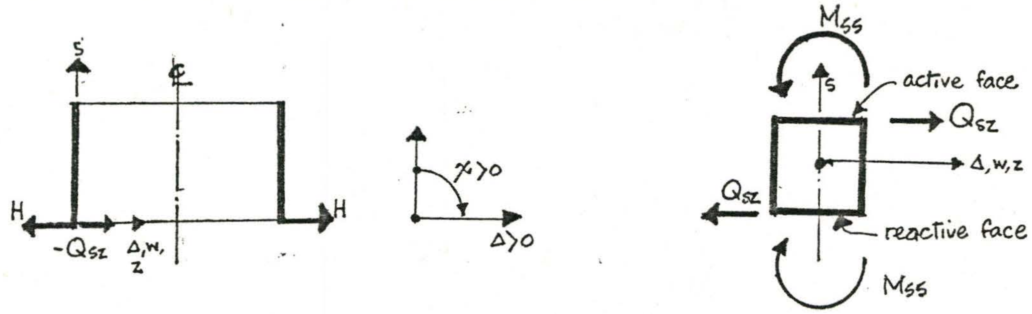


FIG. 5.3 Boundary Forces: Cylindrical Shell Section

Boundary Force H at s = 0:

(a) $-Q_{sz} = -H$

$$D\frac{d^3w^c}{ds^3} = D[2\lambda'^3(C_3 + C_4)] = -H$$

(b) $-M_{ss} = 0$

$$D\frac{d^2w^c}{ds^2} = D[2\lambda'^3(-C_4)] = 0$$

From (a) and (b), $C_4 = 0$, $C_3 = -H/(2\lambda'^3 D)$.

Displacement due to Boundary Reaction H:

$$w^H(s) = -(\frac{2\lambda' R^2 e^{-\lambda' s}}{Eh} \cos\lambda' s)H$$

Stress Couple due to H:

$$M^H_{cyl}(s) = (\frac{1}{\lambda'} e^{-\lambda' s} \sin\lambda' s)H$$

Influence Coefficients for H=1 at s=0:

$$\Delta^H_{cyl} = w^H = -2\lambda' R^2 H/(Eh)$$

$$\chi^H_{cyl} = \frac{dw^H}{ds} = 2\lambda'^2 R^2 H/(Eh)$$

Boundary Moment M at s = 0;

(a) $-M_{ss} = -M$

$$D\frac{d^2w^c}{ds^2} = D(2\lambda'^2 C_4) = M$$

(b) $-Q_{sz} = 0$

$$D\frac{d^3w^c}{ds^3} = D[2\lambda'^3(C_3 + C_4)] = 0$$

From (a) and (b), $C_3 = -2\lambda'^2R^2M/(Eh)$, $C_4 = 2\lambda'^2R^2M/(Eh)$

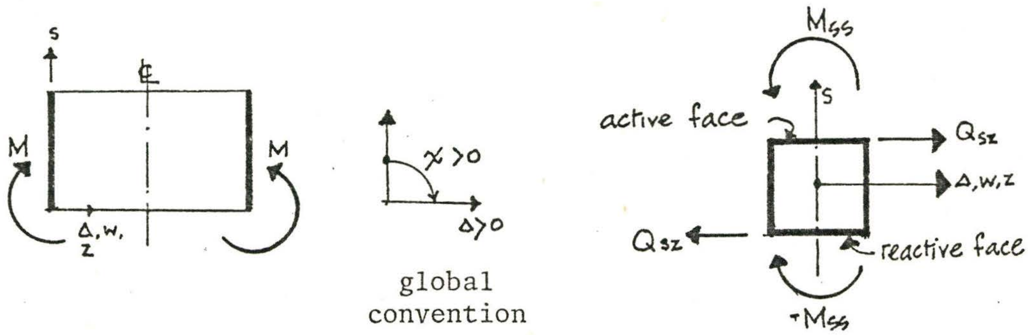


FIG. 5.4 Boundary Moment: Cylindrical Sectional Shell

Displacement due to M:

$$w^M(s) = -[\frac{2\lambda'^2R^2e^{-\lambda's}}{Eh}(\cos\lambda's - \sin\lambda's)]\ M$$

Stress Couple due to M:

$$M^M_{cyl}(s) = [e^{-\lambda's}(\sin\lambda's + \cos\lambda's)]\ M$$

Influence Coefficients for M=1 at s=0:

$$\Delta^M_{cyl} = w^M = -2\lambda'^2R^2M/(Eh)$$

$$\chi^M_{cyl} = \frac{dw^M}{ds} = 4\lambda'^3R^2M/(Eh)$$

The influence coefficients should satisfy the BETTI Reciprocal Work Principle, and the MAXWELL's Reciprocal Displacement Principle:

$$(-H)\Delta^M = (-H)\frac{(2\lambda'^2R^2M)}{Eh} = -\frac{(2\lambda'^2R^2)}{Eh}HM$$

$$(M)\chi^H = M\frac{(2\lambda'^2R^2H)}{Eh} = \frac{(2\lambda'^2R^2)}{Eh}MH$$

Let M = 1, and H = -1, then $\Delta^M = \chi^H$.

Boundary Conditions for Particular Solution

Boundary deformation at s = 0:

$$\Delta^p_{cyl} = w^p = -R^2\gamma_w(d-s)/(Eh)$$

$$\chi^p_{cyl} = \frac{dw^p}{ds} = -\frac{R^2\gamma_w}{Eh}\frac{d(d-s)}{ds} = \frac{R^2\gamma_w}{Eh}$$

Case Study Numerical Values

$$E\Delta^H_{cyl} = -2\lambda' R^2H/h = -432.0H \text{ lb/in}$$

$$E\Delta^M_{cyl} = -2\lambda'^2R^2M/h = -8.650M \text{ lb/in}$$

$$E\Delta^p_{cyl} = -R^2d\gamma_w/h = -9.375X10^4 \text{ lb/in}$$

$$E\chi^H_{cyl} = 2\lambda'^2R^2H/h = 8.650H \text{ lb/in}^2$$

$$E\chi^M_{cyl} = 4\lambda'^3R^2M/h = 0.346M \text{ lb/in}^2$$

$$E\chi^p_{cyl} = R^2\gamma_w/h = 390.6 \text{ lb/in}^2$$

where

$\lambda' = \sqrt[4]{3/(R^2h^2)} = 2.00X10^{-2}$ in^{-1}, R = 360 in, h = 12 in

$\gamma_w = 3.617X10^{-2}$lb/in^3, d = 240 in, H = lb/in, M = in-lb/in

(B) COMPLEMENTARY AND PARTICULAR SOLUTIONS FOR CONICAL SHELL

(a) GECKELER APPROXIMATE SOLUTION

Transverse Displacement:

From equations (3-2) and (3-11)

$$w = w^c + w^p$$

$$\simeq e^{Ks}(C_1 \cos Ks + C_2 \sin Ks) + \frac{1}{4K^4}(Bs + A - \frac{\alpha^4 B}{s} - \frac{\alpha^4 A}{s^2})$$

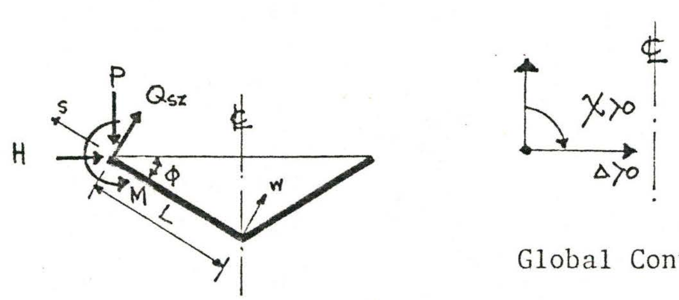
where,

$$A = -12\gamma_w d_o/(Eh^3)$$

$$B = 12\gamma_w \sin\phi/(Eh^3)$$

and

$$\alpha = \frac{\lambda}{K}$$



Global Convention

FIG. 5.5 Boundary Forces and Moments: Meridional Section of the Conical Shell

Boundary Conditions for Complementary Solutions

Boundary Forces H and P at s = L:

(a) $Q_{sz} = H\sin\phi - P\cos\phi$

(b) $M_{ss} = 0$

From (a) and (3-3)

$$-\frac{Eh^3}{12}(2K^2e^{KL})\left\{-C_1\left[K(\cos KL + \sin KL) + \frac{\sin KL}{L}\right] + C_2\left[K(\cos KL - \sin KL) + \frac{\cos KL}{L}\right]\right\} = H\sin\phi - P\cos\phi \quad (1)$$

From (b) and (3-4)

$$-\frac{Eh^3}{12}(2K^2e^{KL})\left[-C_1\sin KL - C_2\cos KL\right] = 0 \quad (2)$$

Substituting (2) into (1) yields,

$C_1 = a_1H + b_1P$ and $C_2 = a_2H + b_2P$

where

$a_1 = \sin\phi/B_1$

$b_1 = -\cos\phi/B_1$

with

$$B_1 = -\frac{Eh^3}{12}(2K^2e^{KL})\left[-K(\cos KL + \sin KL \quad \tan KL)\right]$$

and

$a_2 = \sin\phi/B_2$

$b_2 = \cos\phi/B_2$

with

$$B_2 = -\frac{Eh^3}{12}(2K^2e^{KL})\left[-K(\cot KL \quad \cos KL + \sin KL)\right]$$

Displacement due to H and P:

From (3-2)

$$w^{H,P}(s) = e^{Ks}\,[(a_1\cos Ks + a_2\sin Ks)H + (b_1\cos Ks + b_2\sin Ks)P]$$

Stress Couple due to H and P:

From (3-4),

$$M_{ss}^{H,P}(s) = -\frac{Eh^3}{12}(2K^2 e^{Ks})\,[(-a_1\sin Ks + a_2\cos Ks)H + (-b_1\sin Ks + b_2\cos Ks)P]$$

Influence Coefficients for H = 1 at s = L:

$$E\Delta_{con}^{H} = Ew^{H}\sin\phi \qquad E\chi_{con}^{H} = E\frac{dw^{H}}{ds}$$

Dead weight of wall section per linear inch is $P=\gamma h_c d$, where γ is the specific weight of concrete, d is the height of the cylindrical shell wall, and h_c is the depth of the cylindrical shell wall.

Influence Coefficients for Deformations due to P at s = L:

$$E\Delta_{con}^{Pp} = Ew^{Pp}\sin\phi \qquad E\chi_{con}^{Pp} = E\frac{dw^{Pp}}{ds}$$

Boundary Moment at s = L:

(a) $M_{ss} = M = -\dfrac{Eh^3}{12}\dfrac{d^2w}{ds^2}$

(b) $Q_{sz} = 0$

From (a) and (3-4),

$$-\frac{Eh^3}{12}(2K^2 e^{KL})(-C_1\sin KL + C_2\cos KL) = M \qquad (1)$$

From (b) and (3-3),

$$-\frac{Eh^3}{12}(2K^2 e^{KL})\,\{-C_1\,[K(\cos KL + \sin KL) + (\sin KL/KL)]$$

$$+C_2\,[K(\cos KL - \sin KL) + (\cos KL/KL)]\} = 0 \qquad (2)$$

Substituting (2) into (1) yields,

$$C_1 = c_1 M \quad \text{and} \quad C_2 = c_2 M$$

where

$$c_1 = c_2 c_3$$

$$c_2 = 1/ \left\{ -\frac{Eh^3}{12} (2K^2 e^{KL}) \left[\frac{K\sin^2 KL}{K(\cos KL + \sin KL) + \frac{\sin KL}{L}} \right] \right\}$$

$$c_3 = \frac{K(\cos KL - \sin KL) + \frac{\cos KL}{L}}{K(\cos KL + \sin KL) + \frac{\sin KL}{L}}$$

Displacement due to M:

From (3-2)

$$w^M(s) = e^{Ks}(c_1 \cos Ks + c_2 \sin Ks)M$$

Stress couple due to M:

From (3-4)

$$M_{ss}(s) = -\frac{Eh^3}{12} (2K^2 e^{KL}) (-c_1 \sin Ks + c_2 \cos Ks)M$$

Influence coefficients for M = 1 at s = L:

$$E\Delta^M_{con} = Ew^M \sin\phi \qquad\qquad E\chi^M_{con} = E\frac{dw^M}{ds}$$

According to MAXWELL's Reciprocal Displacement Principle, influence coefficients $E\Delta^M_{con}$ and $E\chi^H_{con}$ should be equal in absolute numerical values.

<u>Boundary Conditions for Particular Solution</u>

From (3-11) at s = L,

$$w^P(L) = \frac{1}{4K^4} \left(BL + A - \frac{\alpha^4 B}{L} - \frac{\alpha^4 A}{L^2} - \frac{\alpha^4 B}{L^3} - \frac{\alpha^8 A}{L^4} \cdots\right)$$

where

$$A = -12\gamma_w d_o/(Eh^3)$$

$$B = 12\gamma_w \sin\phi/(Eh^3)$$

$$\alpha = \lambda/\kappa$$

$$\lambda = \sqrt[4]{12\tan^2\phi/h^2}$$

$$\kappa = \sqrt[4]{12k/(Eh^3)}$$

Hence influence coefficients are:

$$E\Delta^p_{con} = Ew^p \sin\phi \qquad E\chi^p_{con} = E\frac{dw^p}{ds}$$

(C) COMPATIBILITY EQUATIONS FOR CYLINDRICAL AND CONICAL SHELLS

To satisfy the continuity criterion, the corresponding resultant deformations of the cylindrical shell and the conical shell, i.e. the horizontal displacements and the rotations, should be equal at the joint of the two shells.

Conical Shell: Cylindrical Shell:

$$E\Delta: \quad E\Delta^H_{con} + E\Delta^{Pp}_{con} + E\Delta^M_{con} + E\Delta^p_{con} = E\Delta^H_{cyl} + E\Delta^M_{cyl} + E\Delta^p_{cyl}$$

$$E\chi: \quad E\chi^H_{con} + E\chi^{Pp}_{con} + E\chi^M_{con} + E\chi^p_{con} = E\chi^H_{cyl} + E\chi^M_{cyl} + E\chi^p_{cyl}$$

From the above linear equations, H and M can be found. Substituting H and M into previous equations, it is possible to determine stress couples, transverse shear stress resultants, displacements, and rotations along the generators of the cylindrical and the conical shell.

Calculations for various approximate solutions have been worked out by a computer programme written in PLC language. Results for the case study and descriptions of the programme are summarized in APPENDIX III.

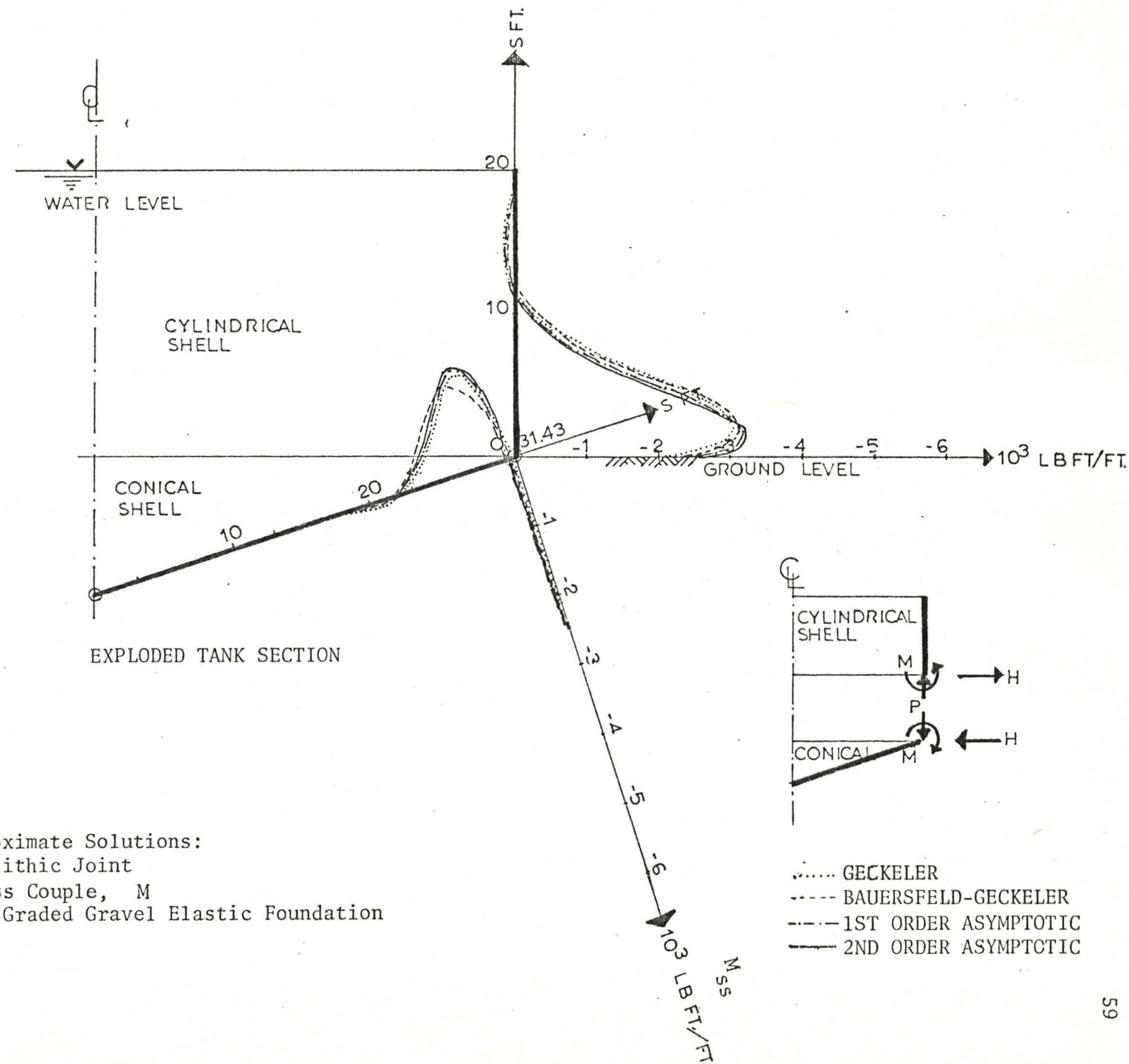


FIG. 5.6 Approximate Solutions:
Monolithic Joint
Stress Couple, M
Well-Graded Gravel Elastic Foundation

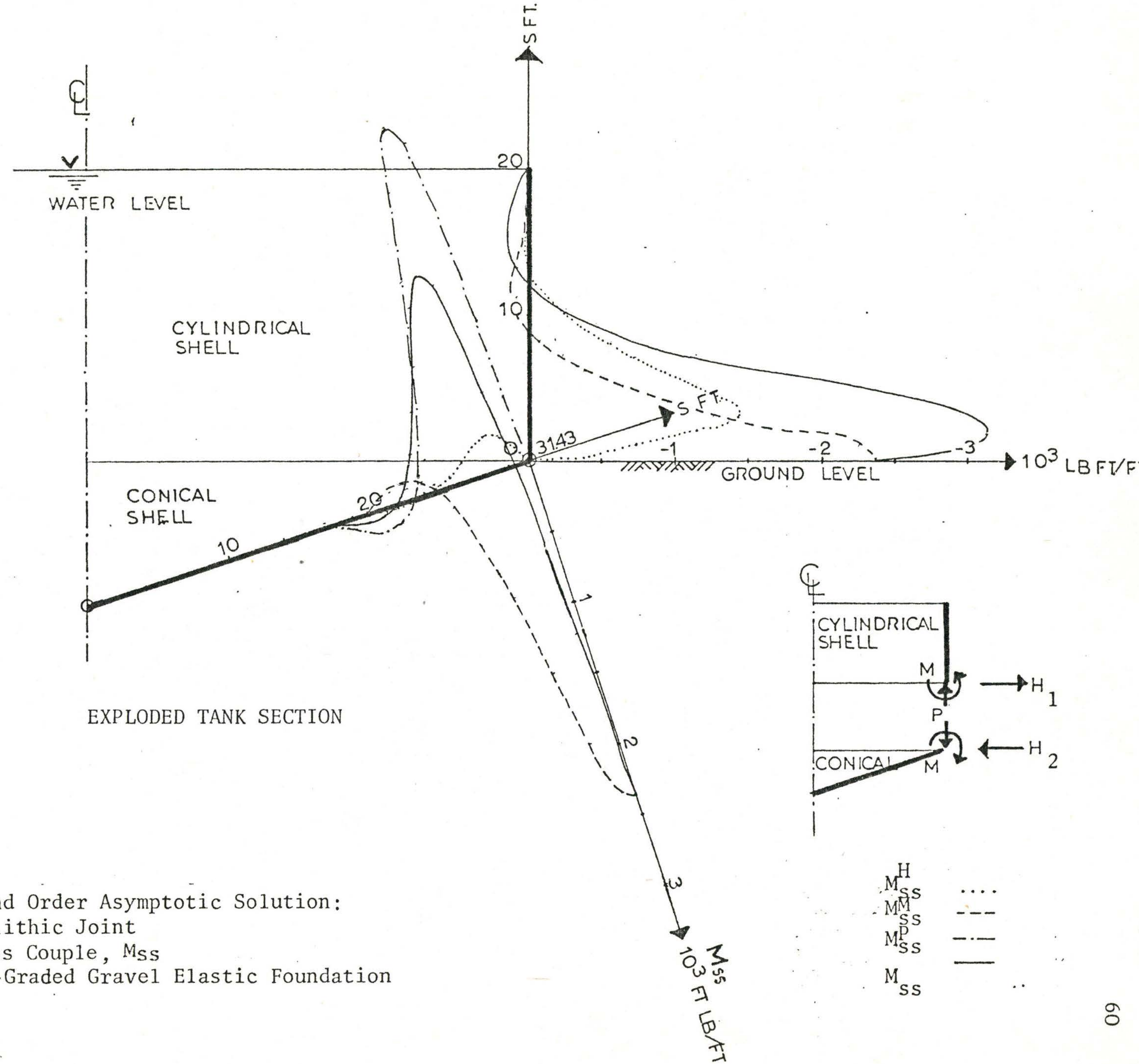


FIG. 5.7 Second Order Asymptotic Solution:
Monolithic Joint
Stress Couple, M_{ss}
Well-Graded Gravel Elastic Foundation

5.2.2 THE CONICAL SHELL SUPPORTED BY A UNIFORM SOIL BEARING PRESSURE FOUNDATION (W. FLÜGGE'S SOLUTION)

As the water tank now rests on a foundation postulated to exert a uniform soil bearing pressure, the conical shell is supported by a reaction of the ground which is a uniform load p_z normal to the shell. In order to provide compatibility of the cylindrical and conical shells at their juncture, interactive reaction forces and moments must be applied to the edges of both shells in the exploded structure. For the cylindrical shell, there is a radial load H_1 and a moment M. For the conical shell, we have the same moment acting in the opposite direction and a radial load H_2 of such magnitude that H_2, H_1 and external force $P\cot\phi$ satisfies the action-reaction principle. The external force $P\cot\phi$ is created by the weight of the cylindrical shell. As the conical shell cannot carry a vertical edge load P by membrane forces alone, a radial load $P\cot\phi$ is introduced so that the resultant force $(P/\sin\phi)$acts in the direction of the meridian. As this additional radial load does not actually exist, it is compensated by the appropriate difference of the radial reactive forces H_1 and H_2.

(A) COMPLEMENTARY AND PARTICULAR SOLUTION FOR ROTATIONALLY SYMMETRICAL CYLINDRICAL SHELL

The influence coefficients for the rotationally symmetrical cylindrical shell were calculated in Chapter V, (5.2.1).

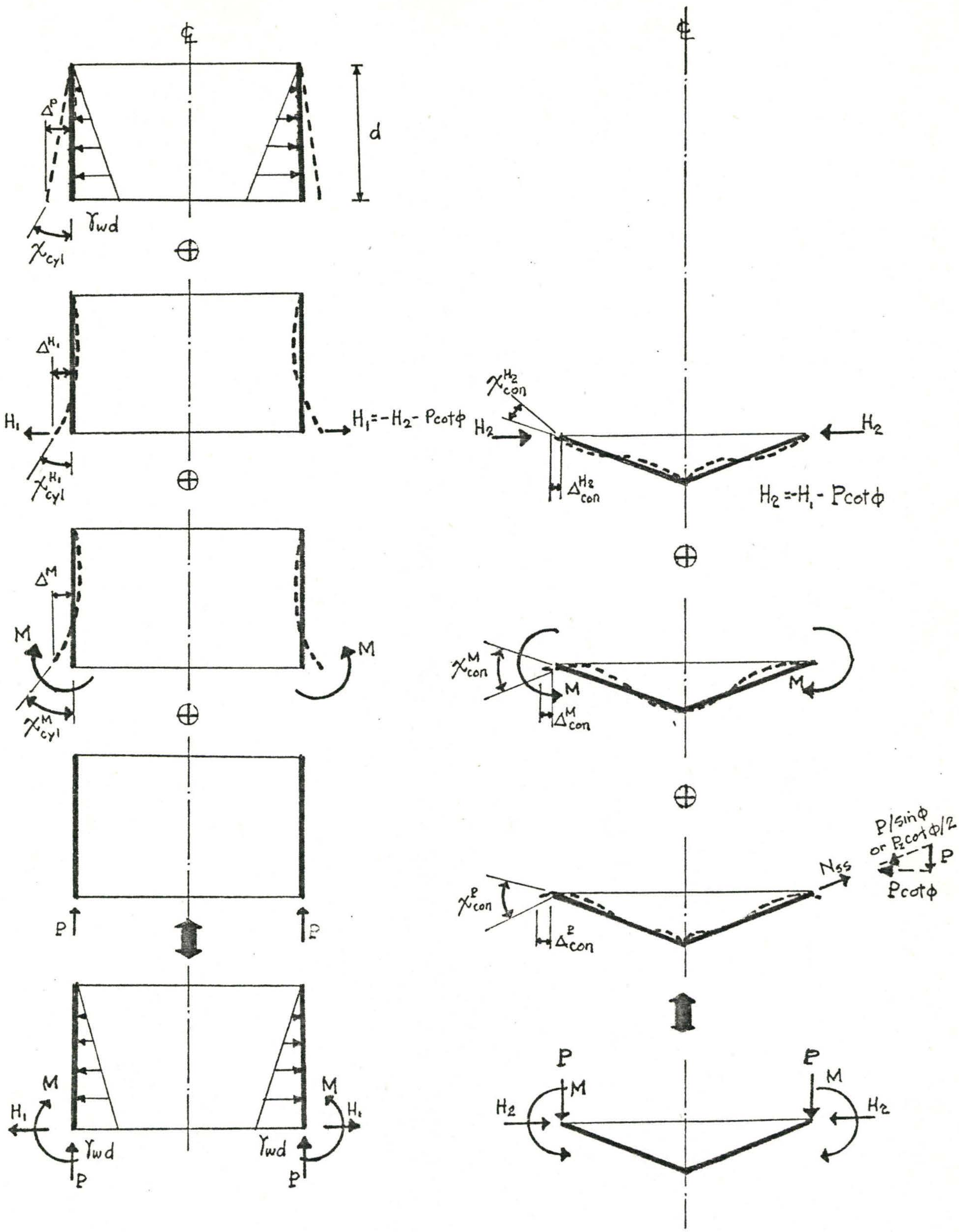


FIG. 5.8 Water Tank on Uniformly Varying Soil Bearing Pressure Foundation

(B) COMPLEMENTARY AND MEMBRANE SOLUTIONS FOR CONICAL SHELL

From Chapter IV, (4.4):

$$N_{\theta\theta} = -\mu_s^2 \left[C_1\left(\frac{4}{\eta^2}\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{ber}'\eta - \frac{8}{\eta^3}\mathrm{bei}'\eta\right) + C_2\left(\frac{4}{\eta^2}\mathrm{bei}'\eta - \frac{2}{\eta}\mathrm{bei}'\eta + \frac{8}{\eta^3}\mathrm{ber}'\eta\right)\right] \quad (4\text{-}6)$$

$$N_{ss} = -\mu_s^2 \left[C_1\left(\frac{4}{\eta^2}\mathrm{ber}\eta - \frac{8}{\eta^3}\mathrm{bei}'\eta\right) + C_2\left(\frac{4}{\eta^2}\mathrm{bei}\eta + \frac{8}{\eta^3}\mathrm{ber}'\eta\right)\right] \quad (4\text{-}7)$$

$$Q_{sz} = \frac{\tan\phi}{s}\left[C_1\left(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right) + C_2\left(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)\right] \quad (4\text{-}9)$$

$$M_{ss} = \frac{2\tan\phi}{\eta^2}\left\{C_1\left[\eta\mathrm{bei}'\eta - 2\left(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)\right] - C_2\left[\eta\mathrm{ber}'\eta - 2\left(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right)\right]\right\} \quad (4\text{-}10)$$

$$E\chi_{con} = -\frac{\sqrt{12}}{h^2}\left[C_1\left(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right) - C_2\left(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right)\right] \quad (4\text{-}8)$$

$$E\Delta_{con} = -\frac{\eta\cos\phi}{4h}\left\{C_1\left[-2\mathrm{ber}'\eta + \frac{4}{\eta}\mathrm{ber}\eta - \frac{8}{\eta^2}\mathrm{bei}'\eta\right] + C_2\left[-2\mathrm{bei}'\eta + \frac{4}{\eta}\mathrm{bei} + \frac{8}{\eta^2}\mathrm{ber}'\eta\right]\right\} \quad (4\text{-}12)$$

where

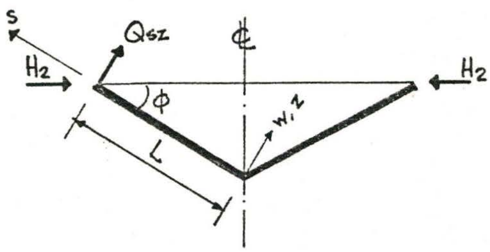
$$\eta = 2\mu\sqrt{s}; \quad \mu = \sqrt[4]{12\tan^2\phi/h^2}$$

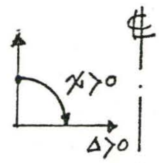
and

$$\nu \simeq 0 \text{ for concrete}$$

Tabular values of KELVIN functions for $\eta > 10$ are obtained from "TABLES OF THE BESSEL-KELVIN FUNCTIONS ber,bei,ker,kei, AND THEIR DERIVATIVES FOR THE ARGUMENT RANGE 0(0.01)107.50" by H. A. LOWELL, NASA TR R-32, 1959.

Boundary Conditions for Complementary Solutions





Global Convention

FIG. 5.9 Boundary Forces: Meridional Section of the Conical Shell

Boundary Force H_2 at s = L:

(a) $Q_{sz} = H_2 \sin\phi$

(b) $M_{ss} = 0$

From (b) and (4-10),

$$\frac{2\tan\phi}{\eta^2}\{C_1[\eta \mathrm{bei}'\eta - 2(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta)] - C_2[\eta \mathrm{ber}'\eta - 2(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta)]\} = 0 \qquad (1)$$

From (a) and (4-9),

$$\frac{\tan\phi}{s}[C_1(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta) + C_2(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta)] = H_2 \sin\phi \qquad (2)$$

Substituting (1) into (2) yields:

$C_1 = a_1 H_2$ and $C_2 = a_2 H_2$

where

$$a_1 = \frac{\sin\phi}{\{\frac{\tan\phi}{L}[(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta) + \frac{(\eta \mathrm{bei}'\eta - 2\mathrm{bei}\eta - \frac{4}{\eta}\mathrm{ber}'\eta)}{(\eta \mathrm{ber}'\eta - 2\mathrm{ber}\eta + \mathrm{bei}'\eta)}(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta)]\}}$$

$$a_2 = \left[\frac{(\eta \mathrm{bei} - 2\mathrm{bei}\eta - \frac{4}{\eta}\mathrm{ber}'\eta)}{(\eta \mathrm{ber}'\eta - 2\mathrm{ber}\eta + \frac{4}{\eta}\mathrm{bei}'\eta)}\right] a_1 .$$

Normal Displacement due to H_2:

From (4-12),

$$w^{H_2}(s) = \frac{\Delta_{con}}{\sin\phi}$$

$$= -\frac{1}{\sin\phi}\left(\frac{\eta\cos\phi}{4hE}\right)\left[a_1\left(-2ber'\eta + \frac{4}{\eta}ber\eta - \frac{8}{\eta^2}bei'\eta\right)\right.$$

$$\left. + a_2\left(-2bei'\eta + \frac{4}{\eta}bei\eta + \frac{8}{\eta^2}ber'\eta\right)\right]H_2$$

Stress Couple Due to H_2:

From (4-10),

$$M_{ss}(s) = \frac{2\tan\phi}{\eta^2}\left\{a_1\left[\eta bei'\eta - 2\left(bei\eta + \frac{2}{\eta}ber'\eta\right)\right]\right.$$

$$\left. -a_2\left[\eta ber'\eta - 2\left(ber\eta - \frac{2}{\eta}bei'\eta\right)\right]\right\}H_2$$

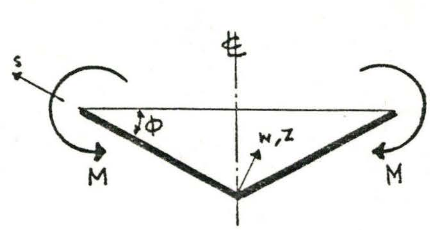
Influence coefficients for $H_2 = 1$ at $s = L$:

From (4-8) and (4-12),

$$E\Delta_{con}^{H_2} = -\frac{\eta\cos\phi}{4h}\left[a_1\left(-2ber'\eta + \frac{4}{\eta}ber\eta - \frac{8}{\eta^2}bei'\eta\right)\right.$$

$$\left. +a_2\left(-2bei'\eta + \frac{4}{\eta}bei + \frac{8}{\eta^2}ber'\eta\right)\right]$$

$$E\chi_{con}^{H_2} = -\frac{\sqrt{12}}{h^2}\left[a_1\left(bei\eta + \frac{2}{\eta}ber'\eta\right) - a_2\left(ber\eta - \frac{2}{\eta}bei'\eta\right)\right]$$



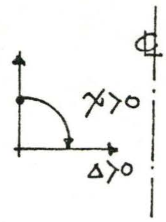

Global Convention

FIG. 5.10 Boundary Moments: Meridional Section of the Conical Shell

Boundary Moment at s = L:

(a) $M_{ss} = M$

(b) $Q_{sz} = 0$

From (b) and (4-9),

$$\frac{\tan\phi}{s}\left[C_1\left(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right) + C_2\left(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)\right] = 0 \tag{1}$$

From (a) and (4-10),

$$\frac{2\tan\phi}{\eta^2}\left\{C_1\left[\eta\mathrm{bei}'\eta - 2\left(\mathrm{bei}\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)\right] - C_2\left[\eta\mathrm{ber}'\eta - 2\left(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right)\right]\right\} = M \tag{2}$$

Substituting (1) into (2) yields

$$C_1 = c_1 M \qquad C_2 = c_2 M$$

where

$$c_1 = \frac{1}{\left[\frac{2\tan\phi}{\eta^2}\left(\eta\mathrm{bei}'\eta - 2\mathrm{bei}\eta - \frac{4}{\eta}\mathrm{ber}'\eta\right) - \frac{\left(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right)}{\left(\mathrm{ber}'\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)}\left(\eta\mathrm{ber}'\eta - 2\mathrm{ber}\eta + \frac{4}{\eta}\mathrm{bei}'\eta\right)\right]}$$

$$c_2 = \left[\frac{\left(\mathrm{ber}\eta - \frac{2}{\eta}\mathrm{bei}'\eta\right)}{\left(\mathrm{bei}'\eta + \frac{2}{\eta}\mathrm{ber}'\eta\right)}\right]c_1$$

Displacement due to M:

From (4-12),

$$w^M(s) = \frac{\Delta_{con}}{\sin\phi}$$

$$= -\frac{1}{\sin\phi}\left(\frac{\eta\cos\phi}{4hE}\right)\Big[C_1(-2ber'\eta + \frac{4}{\eta}ber\eta - \frac{8}{\eta^2}bei'\eta)$$

$$+C_2(-2bei'\eta + \frac{4}{\eta}bei\eta + \frac{8}{\eta^2}ber'\eta)\Big]M$$

Stress Couple due to M:

From (4-10),

$$M_{ss}^M(s) = \frac{2\tan\phi}{\eta^2}\Big\{C_1[\eta bei'\eta - 2(bei\eta + \frac{2}{\eta}ber'\eta)]$$

$$-C_2[\eta ber'\eta - 2(ber\eta - \frac{2}{\eta}bei'\eta)]\Big\}M$$

Influence Coefficients for M = 1 at s = L:

From (4-8) and (4-12),

$$E\Delta_{con}^M = -\frac{\eta\cos\phi}{4h}\Big[C_1(-ber'\eta + \frac{4}{\eta}ber\eta - \frac{8}{\eta^2}bei'\eta)$$

$$+C_2(-2bei'\eta + \frac{4}{\eta}bei + \frac{8}{\eta^2}ber'\eta)\Big]$$

$$E\chi_{con}^M = -\frac{\sqrt{12}}{h^2}\Big[C_1(bei\eta + \frac{2}{\eta}ber'\eta) - C_2(ber\eta - \frac{2}{\eta}bei'\eta)\Big]$$

Boundary Conditions for Membrane Solution

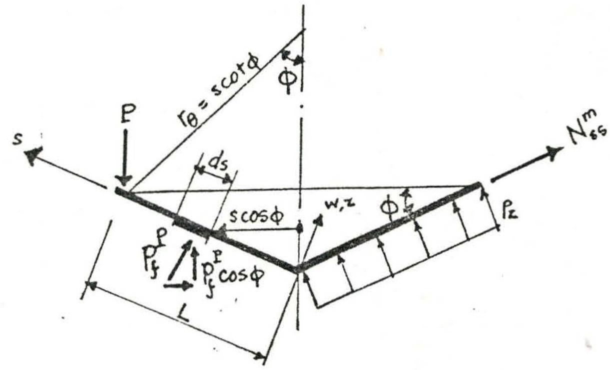


FIG. 5.11 Soil Pressure Distribution

Soil Reaction:

$$\int_0^L (p_f^P \cos\phi) 2\pi s \cos\phi ds = \pi p_f^P L^2 \cos^2\phi$$

Applied Load:

$$2\pi(L\cos\phi)P$$

Equilibrium Equation:

$$2\pi(L\cos\phi)P - \pi p_f^P L^2 \cos^2\phi = 0$$

Hence,

$$p_f^P = \frac{2P}{L\cos\phi} = p_z$$

Equilibrium Force Components:

$$\frac{N^m{}_{ss}}{r_s} + \frac{N^m{}_{\theta\theta}}{r_\theta} + p_z = 0 \qquad \text{(From Chapter IV,(4.3))}$$

As for conical shell,

$$r_\theta = s\cot\phi \text{ and } r_s = \infty$$

then

$$N^m_{\theta\theta} = -p_z r_\theta = -p_z s \cot\phi$$

and

$$N^m_{ss} = - \frac{\int 2\pi s \cos\phi (p_z \cos\phi) ds}{2\pi s \sin\phi \cos\phi} = - \frac{p_z s}{2} \cot\phi$$

or in terms of P,

$$N^m_{ss} = \frac{P}{\sin\phi}$$

The membrane meridional rotation from (4-3)

$$E\chi^m = \frac{1}{h}[(N^m_{ss} - \nu N^m_{\theta\theta}) - \frac{r_\theta}{r_s}(N^m_{\theta\theta} - N^m_{ss})] \cot\phi - \frac{1}{r_s}\frac{d}{d\phi}[r_\theta(N^m_{\theta\theta} - \nu N^m_{ss})]$$

$$= - \frac{\cot\phi}{h}[(N^m_{ss} - N^m_{\theta\theta}) + s\frac{dN^m_{\theta\theta}}{ds}]$$

As $s = s(\phi)$, $r_s = \frac{ds}{d\phi} \to \infty$ for conical shell, and $\nu \simeq 0$ for concrete,

$$r_\theta = s \cot\phi, \quad \frac{d}{d\phi} = r_s \frac{d}{ds}$$

Substituting for N^m_{ss} and $N^m_{\theta\theta}$, the meridional rotation is

$$E\chi^m_{con} = \frac{3}{2} p_z \frac{s\cot^2\phi}{h}$$

Membrane horizontal displacement:

From (4-12),

$$E\Delta^m_{con} \simeq - \frac{s\cos\phi}{h} N^m_{\theta\theta} = - \frac{p_z s^2}{h} \cos\phi\cot\phi$$

<u>(C) COMPATIBILITY CONDITIONS FOR CYLINDRICAL AND CONICAL SHELLS</u>

Cylindrical Shell: Conical Shell:

$$E\Delta : \quad E(\Delta^p_{cyl} + \Delta^{H_1}_{cyl} + \Delta^M_{cyl}) = E(\Delta^m_{con} + \Delta^{H_2}_{con} + \Delta^M_{con}) \qquad (1)$$

$$E\chi : \quad E(\chi^p_{cyl} + \chi^{H_1}_{cyl} + \chi^M_{cyl}) = E(\chi^m_{con} + \chi^{H_2}_{con} + \chi^M_{con}) \qquad (2)$$

$$H_2 = H_1 - P\cot\phi$$

The results for the case study are summarized in APPENDIX III.

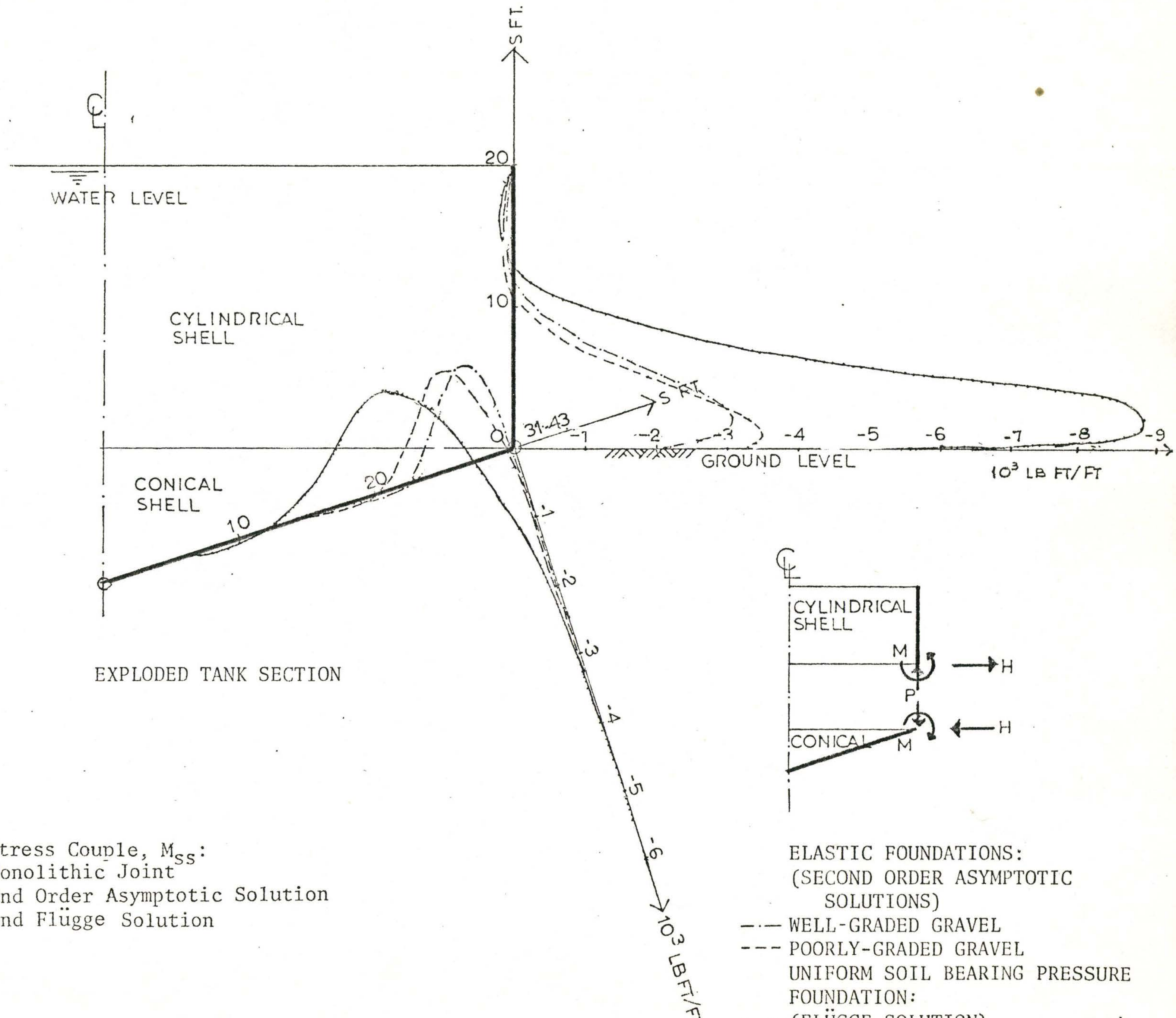


FIG. 5.12 Stress Couple, M_{ss}:
Monolithic Joint
2nd Order Asymptotic Solution
and Flügge Solution

ELASTIC FOUNDATIONS:
(SECOND ORDER ASYMPTOTIC SOLUTIONS)
–·– WELL-GRADED GRAVEL
- - - POORLY-GRADED GRAVEL
UNIFORM SOIL BEARING PRESSURE FOUNDATION:
—— (FLÜGGE SOLUTION)

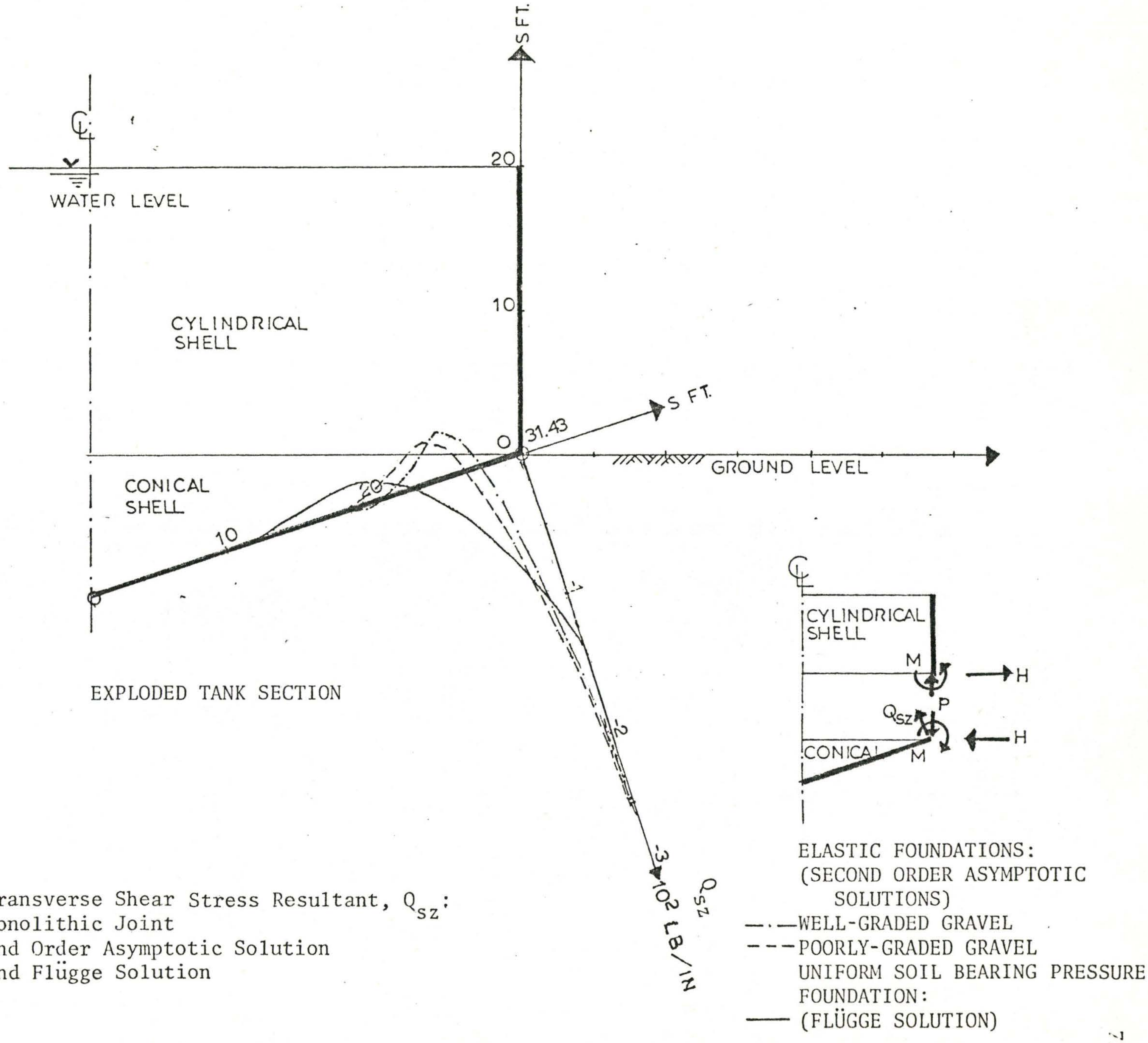
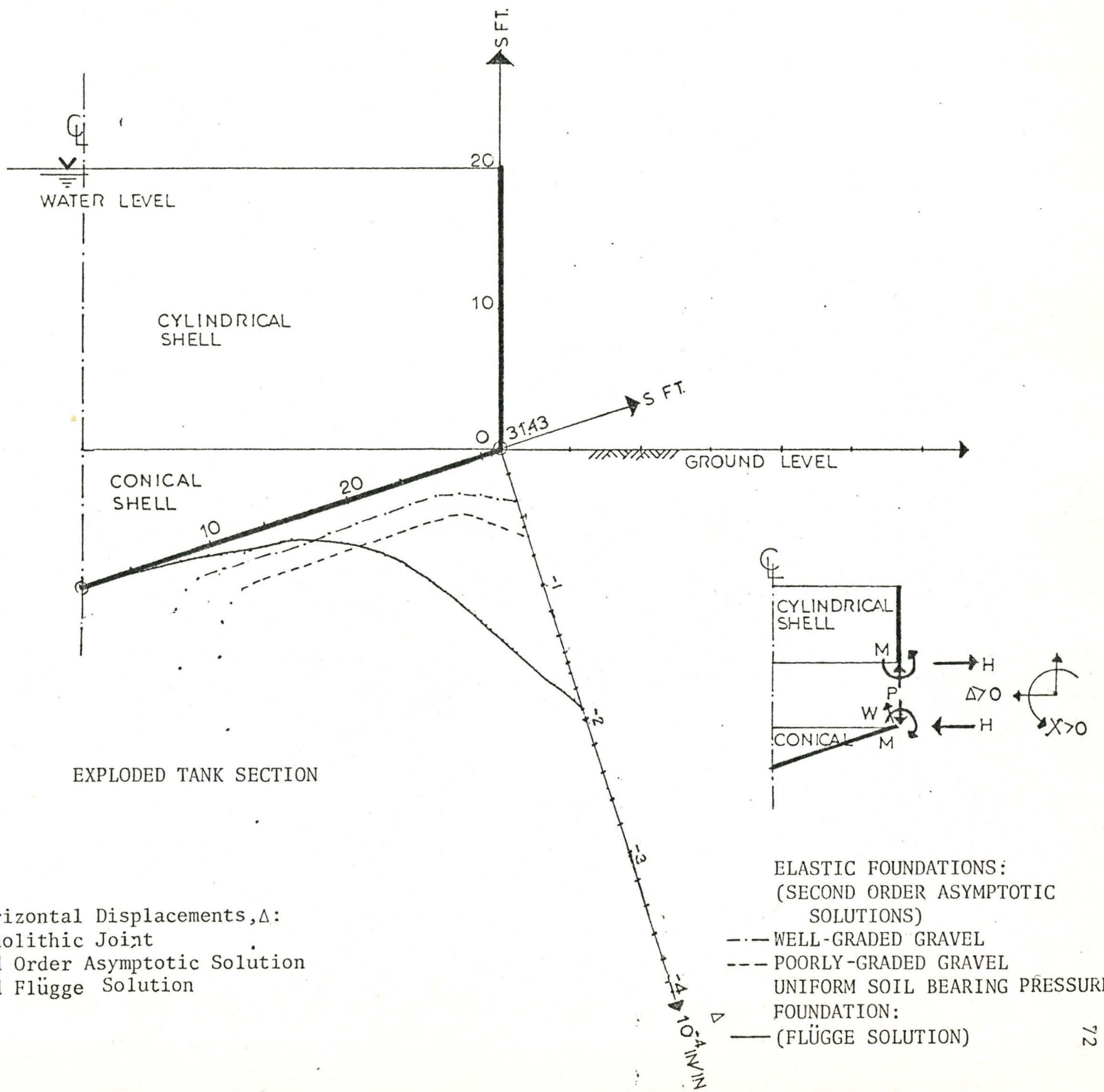


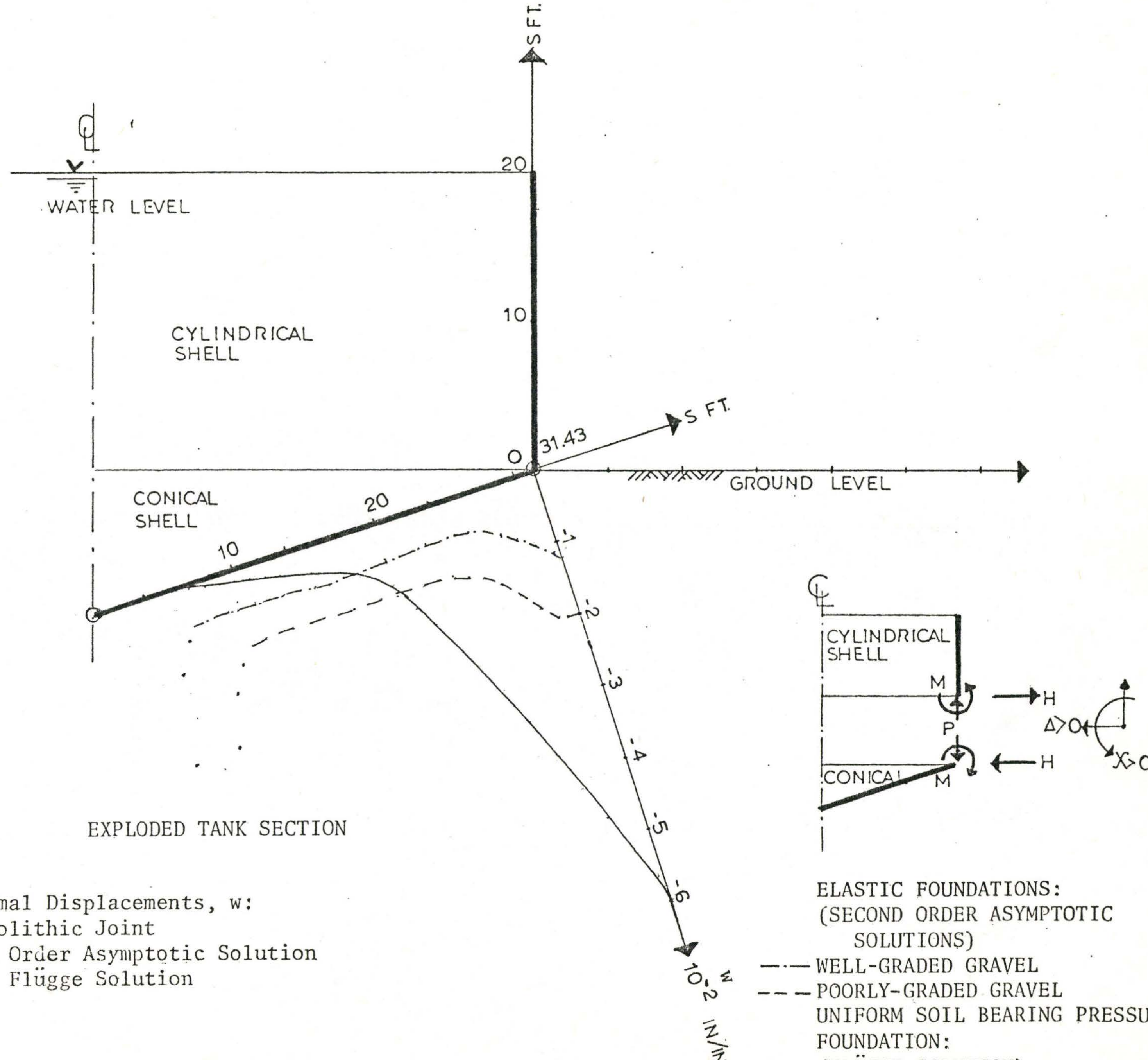
FIG. 5.13 Transverse Shear Stress Resultant, Q_{sz}: Monolithic Joint
2nd Order Asymptotic Solution
and Flügge Solution



EXPLODED TANK SECTION

FIG. 5.14 Horizontal Displacements, Δ:
Monolithic Joint
2nd Order Asymptotic Solution
and Flügge Solution

ELASTIC FOUNDATIONS:
(SECOND ORDER ASYMPTOTIC SOLUTIONS)
—·— WELL-GRADED GRAVEL
--- POORLY-GRADED GRAVEL
UNIFORM SOIL BEARING PRESSURE FOUNDATION:
—— (FLÜGGE SOLUTION)



EXPLODED TANK SECTION

FIG. 5.15 Normal Displacements, w:
Monolithic Joint
2nd Order Asymptotic Solution
and Flügge Solution

ELASTIC FOUNDATIONS:
(SECOND ORDER ASYMPTOTIC SOLUTIONS)
—·— WELL-GRADED GRAVEL
– – – POORLY-GRADED GRAVEL
UNIFORM SOIL BEARING PRESSURE FOUNDATION:
——— (FLÜGGE SOLUTION)

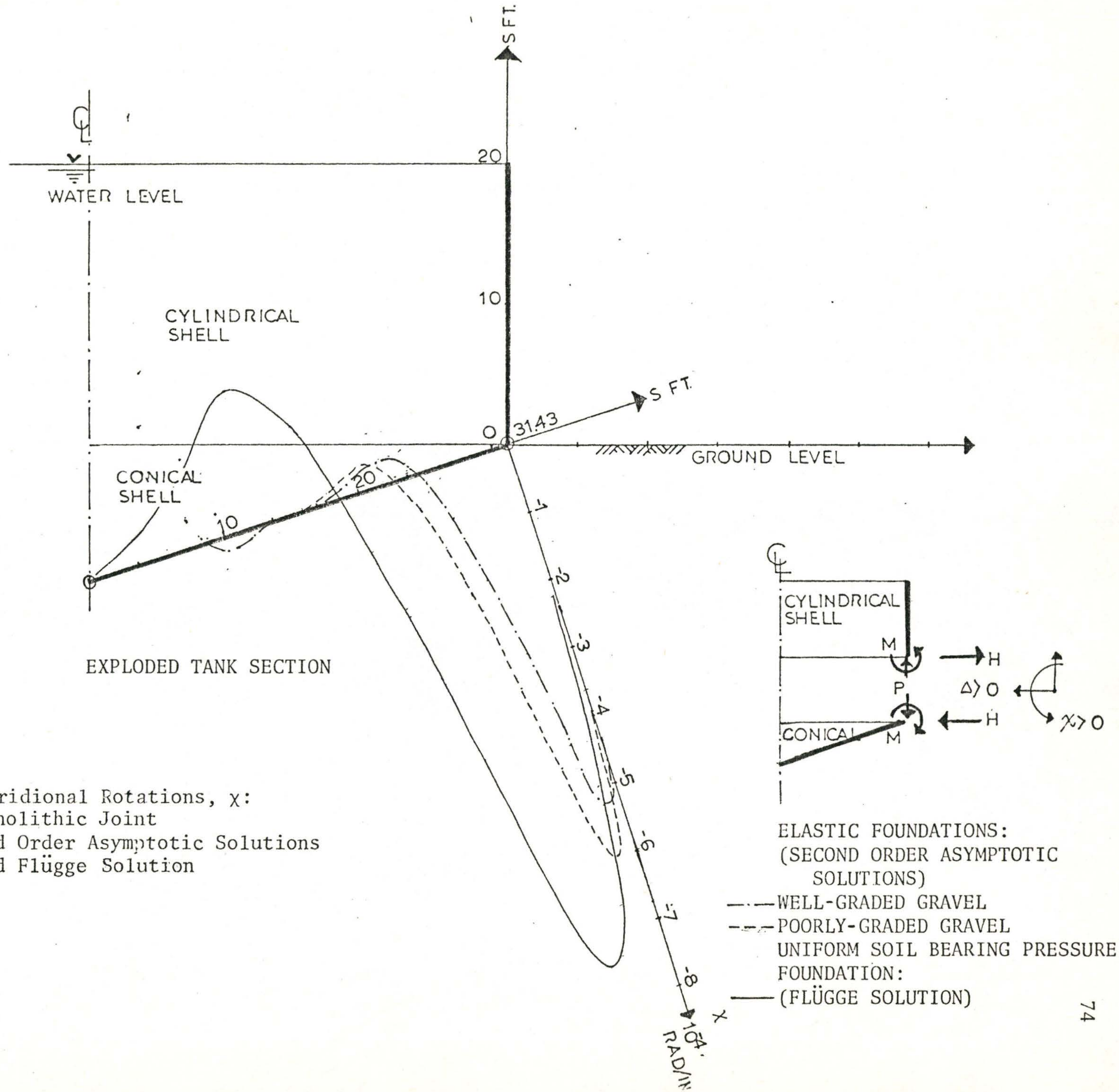


FIG. 5.16 Meridional Rotations, χ:
Monolithic Joint
2nd Order Asymptotic Solutions
and Flügge Solution

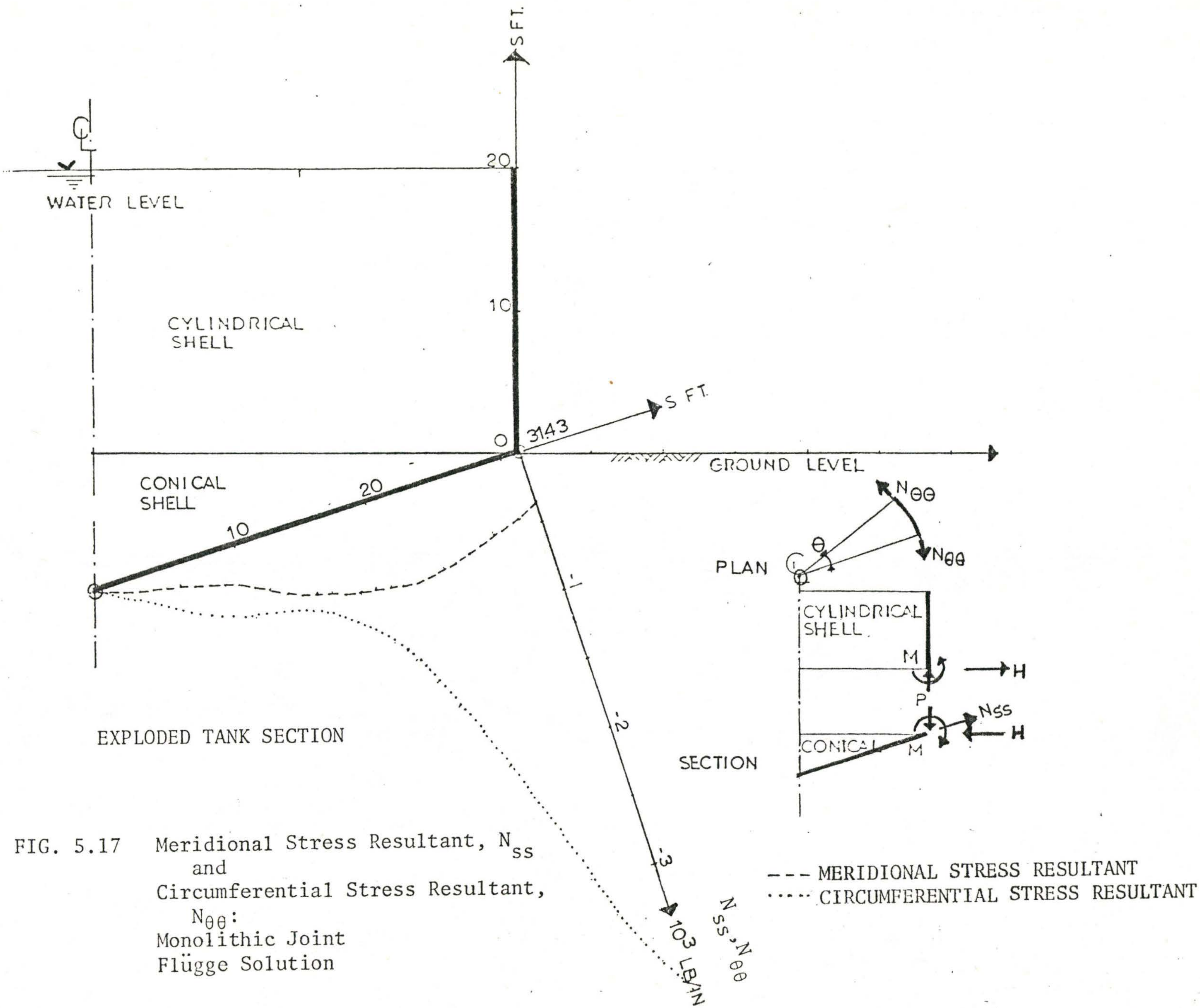


FIG. 5.17 Meridional Stress Resultant, N_{ss} and Circumferential Stress Resultant, $N_{\theta\theta}$: Monolithic Joint Flügge Solution

5.2.3 EFFECTS OF FOUNDATION STIFFNESS

Since various approximate solutions lead to similar results, we select the more accurate second order asymptotic solution to study the effects of foundation stiffness. The formerly assumed well-graded gravel elastic foundation with k=500pci is now changed to poorly-graded with k=250pci. Results for elastic foundations are then compared with those of the uniformly varying soil bearing pressure foundation. Numerical values are tabulated in APPENDIX III.

5.2.4 SOIL BEARING PRESSURE

In this section, we attempt to study the soil bearing pressure distribution over the conical shell. The results derived from second order asymptotic solution are compared with the uniformly varying soil bearing pressure foundation assumed by FLÜGGE. At this stage, we are in the position to check partially the validity of our differential equations since the total soil bearing pressure should agree closely with the total applied load.

For elastic foundation:

$$p_f = -kw$$

where $w = w^c + w^p$ is the normal displacement and k is the elastic foundation stiffness (k = 500pci for well-graded gravel and k = 250pci for poorly-graded gravel).

For uniformly varying soil bearing pressure foundations:

$$p_f = p_f^P + p_f^W$$

where

$$p_f{}^P = \frac{2P}{L\cos\phi}$$

is soil reaction due to edge load and

$$p_f{}^w = \gamma_w\, [d_o - (\sin\phi)s]$$

is reaction due to the weight of water transmitted directly to the foundation.

The results are summarized in APPENDIX III.

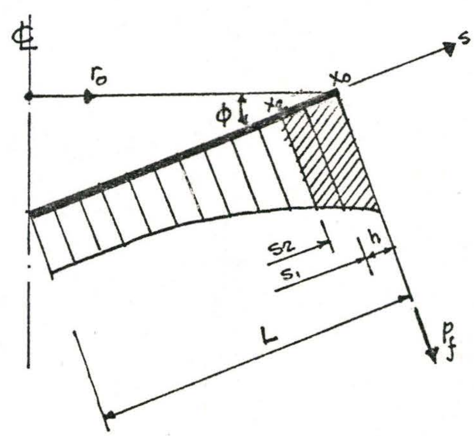


FIG. 5.18 Total Soil Bearing Pressure: Conical Shell, Meridional Section

Total soil bearing pressure is estimated by SIMPSON's rule:

For interval x_0 to $x_2 = x_0 + 2h$

$$\int_{x_2}^{x_0 + 2h} p_f dx = [\frac{h}{3}(p_{f_0} + 4p_{f_1} + p_{f_2})]\ 2\pi r_o \frac{s_1}{L}$$

For interval x_2 to $x_4 = x_2 + 2h$:

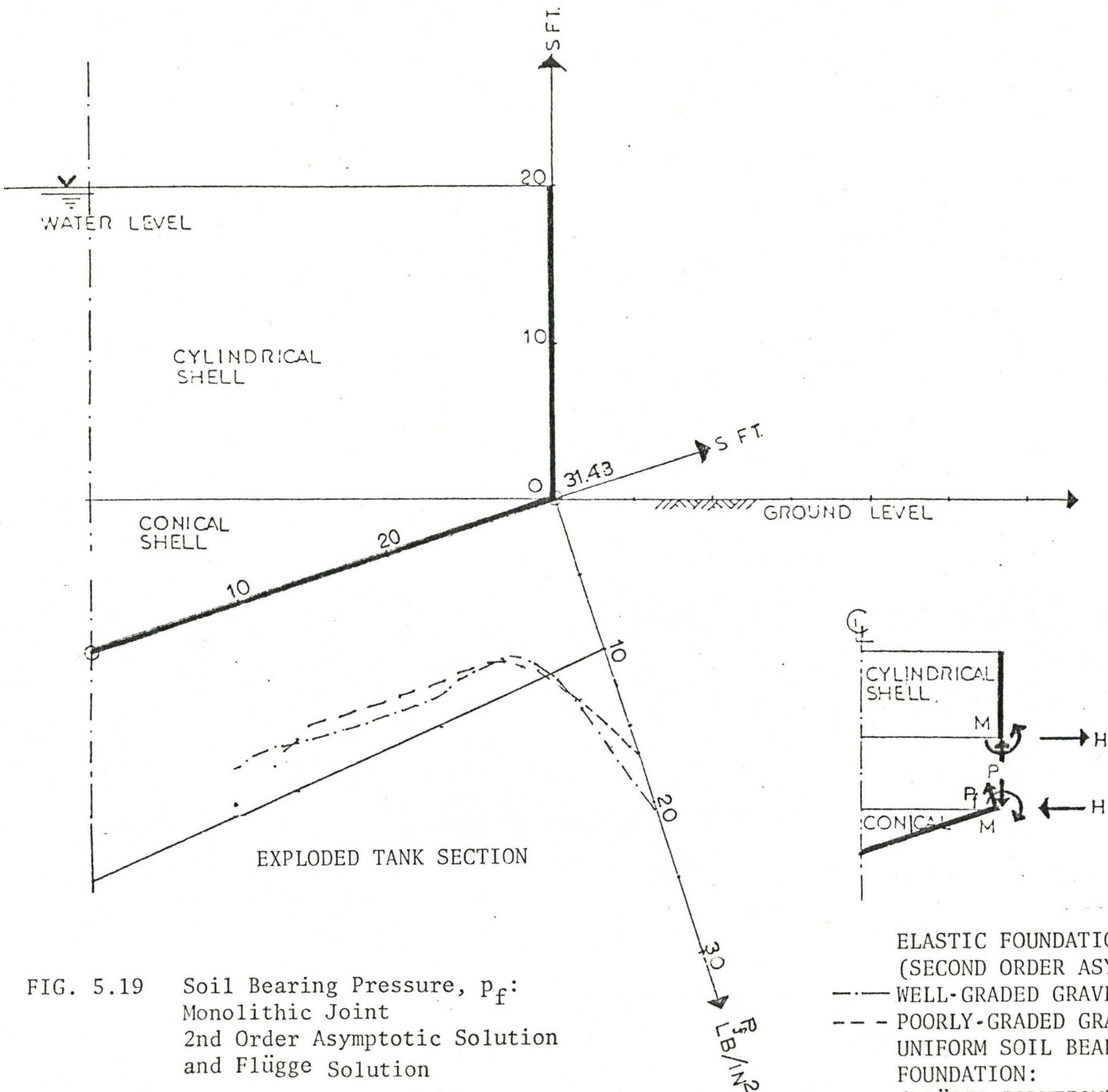
$$\int_{x_2}^{x_2+2h} p_f dx = [\frac{h}{3}\ (p_{f_2} + 4p_{f_3} + p_{f_4})]\ 2\pi r_o \frac{s_2}{L}$$

and so on. Adding all such expressions from x_0 to x_n, where n is even, we have,

$$\int_{x_0}^{x_0 + nh} p_f dx = 2\pi r_o \frac{h}{3L}[(p_{f_0} + 4p_{f_1} + p_{f_2})\, s_1 + (p_{f_2} + 4p_{f_3} + p_{f_4})\, s_3 + (p_{f_4} + 4p_{f_5} + p_{f_6}) s_5 \ldots]$$

5.3 WATER TANK WITH NON-MONOLITHIC JOINT

The analysis of water tank with non-monolithic joint is similar to that of monolithic joint described in Chapter V, Section 2.1. The same equations are applicable as in the monolithic case, except that M=0 and there is no $E\chi$ compatibility equation. Results are summarized in APPENDIX III.



ELASTIC FOUNDATIONS:
(SECOND ORDER ASYMPTOTIC SOLUTIONS)
—·— WELL-GRADED GRAVEL
– – – POORLY-GRADED GRAVEL
UNIFORM SOIL BEARING PRESSURE FOUNDATION:
——— (FLÜGGE SOLUTION)

FIG. 5.19 Soil Bearing Pressure, p_f: Monolithic Joint 2nd Order Asymptotic Solution and Flügge Solution

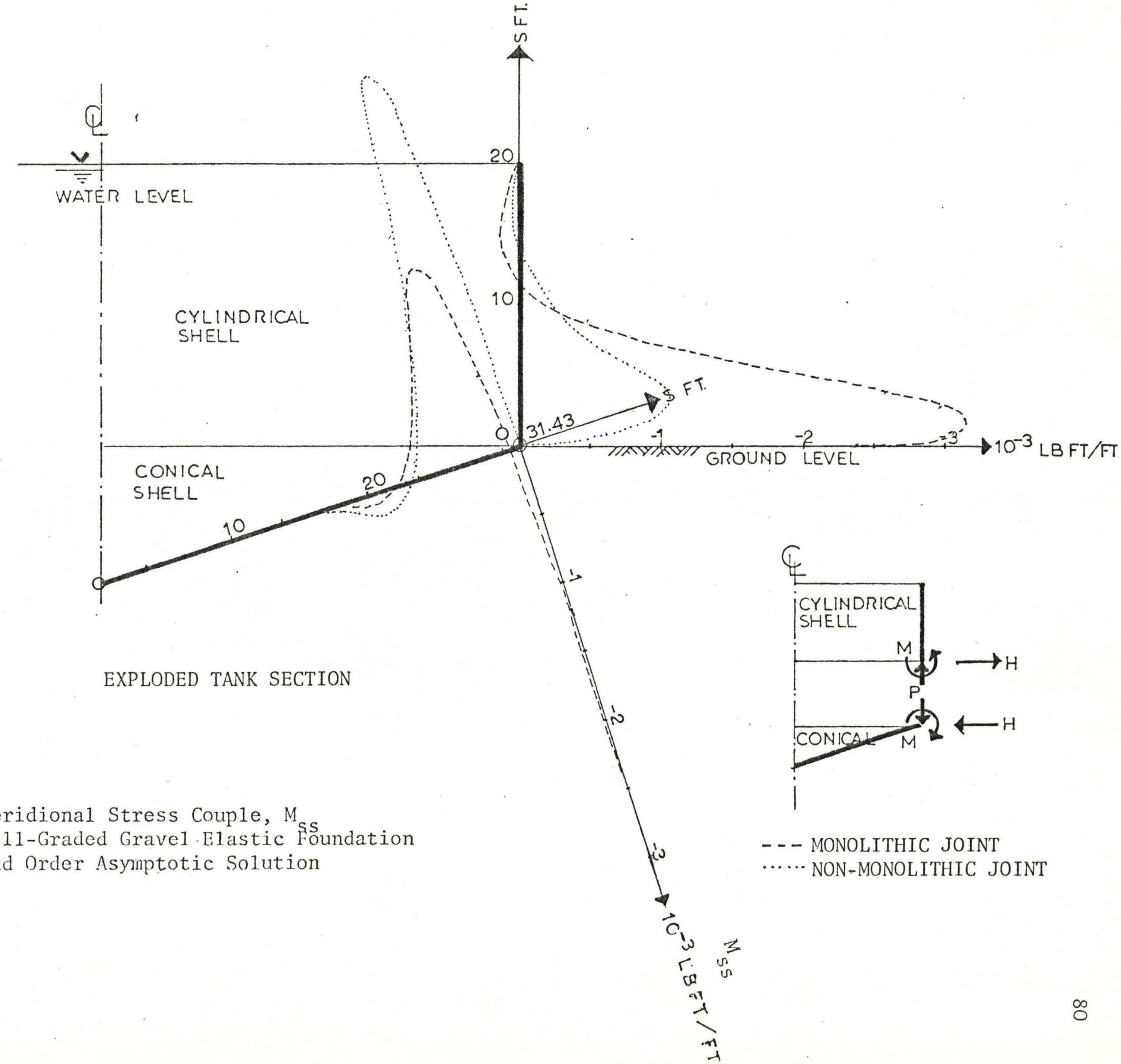


FIG. 5.20 Meridional Stress Couple, M_{ss}
Well-Graded Gravel Elastic Foundation
2nd Order Asymptotic Solution

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

INTRODUCTION

The underlying theme of this project is to study the validity of the uniformly varying soil bearing pressure assumption employed by engineering designers as a simplification to the complex elastic foundation on which the conical shell is supported.

This assumption is exemplified by Flügge's approximation, which, as noted previously, suggests that the conical shell is infinitely rigid and suffers no relative normal displacement while generating the soil bearing pressure. On the basis of this idea, the soil bearing pressure can be determined directly from the equilibrium equations without relating it to the soil stiffness. This idea however contradicts the general understanding of the behaviour of the conical shell which implies that the state of stress at the boundary region is dominantly transverse bending and therefore requires that normal displacements at the edge of the conical shell cannot be neglected. It is therefore worthwhile to examine the significance of the foundation stiffness in the design of the conical shell.

COMMENTS ON THE DIFFERENTIAL EQUATION (EQ. 2-10) FOR CONICAL SHELL ON THE EULER-WINKLER ELASTIC FOUNDATION

In the process of reducing the order of the differential equation to a manageable degree, we have made several assumptions following KANN (40) which should be taken into account in the study of calculated results:

1. Since the circumferential moment, $M_{\theta\theta}$, is assumed to be negligibly small, the differential equation of the conical shell represents the shell as one which is less stiff than is actually the case. Both the magnitude and the rate of change of the displacement along the meridian can be expected to be slightly exaggerated.

2. The Euler-Winkler springs of elastic foundation are considered to be discrete units. Hence, the presence of any shear resistance between the springs, i.e. between the contiguous soil, will tend to smooth out the rate of change in the reactive soil bearing pressures.

3. The differential equation does not take into account the deformation effects of meridional stress resultant at the boundary. We have assumed that u = o even though du/ds is not necessarily a small strain:

$$N_{ss} = \frac{Eh}{1-\nu^2}(\varepsilon_{ss} + \nu\varepsilon_{\theta\theta}) \simeq Eh\varepsilon_{ss} = Eh\frac{du}{ds}$$

At s = L

$$\frac{du}{ds} \simeq \frac{N_{ss}}{Eh} = -\frac{H\cot\phi + P\sin\phi}{Eh} \neq 0$$

CONCLUSIONS

1. A conical shell with $\phi < 10^\circ$ behaves in some respects somewhat similarly to a circular plate, whereas a conical shell with $\phi > 30^\circ$ behaves in some respects similarly to a cylindrical shell.

2. The differential equation (2-10) becomes most accurate for conical shells with $\phi > 30^\circ$ since the circumferential moment in the theory used was assumed to contribute little to the stiffness of the shell.

3. The various approximate solutions for the differential equation (2-10) yield quite similar results. Using Maxwell's Reciprocal Displacement Principle:

$$|E\Delta^M_{con}| = |E\chi^H_{con}|$$

as a theoretical criterion, we can establish that the asymptotic solution best satisfies this relation. For the case with well-graded gravel elastic foundation and monolithic joint:

Approximation	$\lvert E\Delta^M_{con}\rvert$	$\lvert E\chi^H_{con}\rvert$	$(\lvert E\Delta^M_{con}\rvert - \lvert E\chi^H_{con}\rvert)$
Geckeler	3.350	3.308	0.042
Bauersfeld-Geckeler	3.701	3.681	0.020
1st Order Asymptotic	3.778	3.740	0.038
2nd Order Asymptotic	3.660	3.676	0.016

4. Lowering the soil stiffness coefficient causes a stress redistribution to balance the applied loads. The result of this is a lateral shift of the positive maximum bending moment towards the apex. For extremely soft foundation, we can expect the momentless region to be very narrow.

5. By neglecting the dead weight of the conical shell, it is found that the total soil bearing pressure of the elastic foundation derived from the second order Asymptotic Solution agrees with the estimated total applied load. This can be considered as a partial corroboration on the validity of the differential equation (2.10)

Total applied loads:

water loads	4.54×10^6 lb	(vertical)
wall load (cylindrical shell)	5.80×10^5 lb	(vertical)
$\Sigma P_v =$	5.12×10^6 lb	(vertical)

Total vertical resultant soil bearing pressures for the elastic foundation (second order asymptotic solution) is 5.41×10^6 lb (normal) or 5.16×10^6 lb (vertical). Assuming the base is wet, hence frictional force, $p_s=0$; and the soil bearing pressure from s=0 to 3 ft. can be estimated by linear extrapolation.

6. Instability has not been considered in the study. It is only appropriate for conical shells which are very thin, flat or made of a low-modulus concrete. Concrete creep may also contribute to larger deformation, but this aspect of the problem has not been considered in this report.

RECOMMENDATIONS

1. The Asymptotic Solution can be approximated by the simpler Geckeler approximation when higher degree of accuracy is not required.

2. The accuracy of the Asymptotic Solution can be improved by retaining higher order terms.

3. The uniformly varying soil bearing pressure foundation corresponds to the case with low foundation stiffness coefficient. This assumption however does not necessarily lead to a conservative design. Reviewing the structure with monolithic joint in the case study, it is apparent that for some occasions both the stress and the deformation at any point can be underestimated.

	Well-Graded Gravel Elastic Foundation (A)	Uniform Soil Bearing Pressure Foundation (B)	Difference $\frac{(B-A)x100}{A}$
Bending Moment (in lb/in)			
Max. +	1427 (s=336")	1327 (s=276")	-7.0%
Max. -	-2387 (s=377.2)	-5485 (s=377.2")	129.8%
Transverse Shear Force (lb/in)			
Max. +	31.9 (s=312")	17.6 (s=228")	-44.8%
Max. -	-263.5 (s=377.2")	-147.9 (s=377.2")	-43.9%
Normal Displacement (in/lb)			
Max. -	$-4.09x10^{-2}$ (s=377.2")	$-1.94x10^{-1}$ (s=377.2")	-52.6%
Rotation*			
Max. +	$3.95x10^{-5}$ (s=264")	$3.614x10^{-5}$ (s=377.2")	-8.5%
Max. -	$-5.27x10^{-4}$ (s=360")	$-7.54x10^{-4}$ (s=324")	43.1%

*Neglecting results where s < 120 in.

4. The meridional stress resultant derived from the equilibrium equation is extremely difficult to integrate as the displacement w has factors such as e^{Ks}, $1/(s^K)$, cosKs, and sinKs. As from equilibrium equation

$$\frac{d}{ds}(N_{ss}s) = N_{\theta\theta} = -\frac{Eh\tan\phi w}{s}$$

then,

$$N_{ss} = -Eh\tan\phi \frac{1}{s}\int \frac{w}{s}\,ds + C$$

However, it is possible to obtain some good estimates for N_{ss} by successive approximation methods.

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APPENDIX I

HISTORICAL NOTES: THEORY OF SHELLS

Leonhard EULER [1776] postulated the shell to consist of individual rings of a double layer of meridional and tangential beams, but correct results were not reached by such theoretical models.

The first theoretical analysis of thin shells based upon theory of elasticity was given by the French engineer Émile CLAPEYRON and Gabriel LAMÉ in 1828 (4). They considered the rotationally symmetrical shell under axially symmetrical loading, and made the assumption that stresses are virtually uniform across the small thickness of the thin shell. In such a case, the constitutive equations are not necessary and the shell can be analyzed on the basis of equilibrium equations alone. This theory of momentless equilibrium of thin shells is now called the "membrane" theory of shells. However, the "membrane" state of stress creates a deformation in shells that ordinarily cannot be made to conform to the support conditions of the shell, and as a result transverse bending near the support of the thin shell is induced.

A consistent theory of the bending of plates was presented by German theoretical physicist Gustav KIRCHHOFF in 1850 (27) which was a great improvement over the theory of plates as given by the French engineers and mathematicians Henri L. NAVIER in 1821 (1), Augustin CAUCHY in 1828 (2), and Denis S. POISSON in 1829 (3) as it established the proper boundary conditions associated with the bending and stretching of thin elastic plates. KIRCHHOFF maintained that normals to the middle surface of the thin shell remain normal to the deformed middle surface and suffer no deformation.

In 1862, German mathematician Alfred CLEBSCH (7) adopted KIRCHHOFF's hypothesis and established equations of equilibrium for plates which fall into two separate sets of equations: one set describes the equilibrium of stress-resultants and applied loads; the other set stands for the moment equilibrium of stress couples and transverse shear stress resultants.

In 1874, the German electrical engineer and industrialist Heinrich ARON (8) gave the first general elastic theory of thin shells. He described the geometrical properties of the middle surface and the deformation of the middle surface of the shell in terms of two Gaussian parametric coordinates and analyzed it as a problem of differential geometry of surfaces. ARON incorporated KIRCHHOFF's hypothesis of plates in bending and adopted the method of CLEBSCH in thin plates theory to the analysis of thin elastic shells. ARON arrived at an expression for the potential energy of the strained thin shell which is analogous in form to that obtained by KIRCHHOFF for thin plates.

In 1882 Lord RAYLEIGH proposed an inextentional theory of vibration for thin shells (9), which was based upon the assumption that the middle surface of the deforming shell in vibrating motion remains essentially unstretched, an idea obtained from physical reasoning. In 1883, the French mathematician Êmile MATHIEU, in contrast to RAYLEIGH, adopted the method used by POISSON in the vibration of thin plates in 1829, and assumed that stretching of the middle surface is mainly important in the vibration of thin shells (11). MATHIEU's fundamental equations could be deduced from the ARON's potential energy quantity if only the middle surface stretching terms are retained.

A great progress within shell analysis was achieved by the English mathematician Augustus E. H. LOVE in 1888 (10). LOVE applied the KIRCHHOFF Hypothesis and derived the basic thin shell equations together with the associated boundary conditions (5). LOVE's potential energy expression for thin shells had a form similar to that of ARON. LOVE attempted to make the various approximations used in the construction of the constitutive equations and the kinematical descriptions of strains more systematic by introducing the so-called LOVE's First and Second Approximations, and he corrected some faults and inconsistencies in the shell theory of ARON. LOVE demonstrated that RAYLEIGH's inextensional theory does not permit proper satisfaction of the boundary conditions of the vibrating thin elastic shell.

In 1889 LOVE published his theory of thin elastic spherical shells (12).

Before the epochal memoir of LOVE in 1888, German engineers Emil WINKLER in 1860 (6) and Franz GRASHOF in 1878, had made attempts to solve thin shell problems of industrial importance without the aid of general theory of thin shells with some success.

In 1890, English mathematician Albert B. BASSET (13), and Australian mathematician Horace LAMB (14) both had demonstrated that A.E.H. LOVE's [1888] "extensional" strain in the thin shell in most cases is confined to a rather narrow region near the edges of the shell as it is implemental in satisfying the boundary conditions of the shell. The rest of the shell beyond this narrow boundary zone may be considered approximately to be "inextensional" as Lord RAYLEIGH had assumed in 1882. These facts led engineers to the concept of the "boundary disturbance" or "perturbation" zone in the deformation of thin shells. The boundary condition can be satisfied by superimposing the deformation efforts of the transverse bending and of the membrane state of stress at the boundary. This approach to shell analysis represents a method of successive corrections. It has been found that the thinner the shell, the narrower is the so-called boundary disturbance zone.

LOVE's general thin shell theories (9) led to such complicated differential equations and intricate solutions that only shells with the simplest middle surface geometries could be analyzed at all by its means.

After LOVE's general theory of thin shells had been published, many German engineers made elaborate attempts to apply the ideas of LOVE's general theory of thin shells to important industrial problems, such as pressure vessel and tank problems: W. SCHÜLE in 1900 and 1911 (16), Carl RUNGE in 1904 (19), L. MAURER in 1904 (18), C. PFLEIDERER in 1908 and 1910 (23), Heinrich MÜLLER-BRESLAU and Hans REISSNER in 1908 (26), R. LORENZ in 1908 (21), Phillip FORCHHEIMER in 1904 (20) and 1910 (9), and Karl FEDERHOFER in 1909 (24) by a graphical method.

These investigators were able to obtain some approximate solutions for the industrial shell problems, and exact solutions for the cylindrical shell problems.

An accelerated interest in thin rotationally symmetrical shell problems was given by Aurel STODOLA, professor at the Swiss Federal Institute of Technology in Zurich, Switzerland, with the publication of the fourth edition of his famous book Steam Turbines in 1910, as it contained a stress analysis with numerical results for the conical pressure vessel head by ZIEGLER, an assistant and student of Professor STODOLA. ZIEGLER derived two simultaneous differential equations of the second and third orders for the thin conical shell from LOVE's general shell equations, and he integrated these equations by elaborate method of series. STODOLA's doctoral students, Huldreich KELLER in 1912 (21) and Eduard FANKHAUSER in 1913 (28), developed similar solutions from LOVE's theory for spherical and conical pressure vessel heads and dished heads with variable wall thickness and meridional curvature, but their methods were intricate and extensive as they used methods of finite differences and step-by-step integration. KELLER published fully calculated results for a large number of different cases of thin shells.

At the time KELLER and FANKHAUSER were working on their thesis, a new era in thin shell theory began with the publication of the epoch-making paper (26) of Hans REISSNER on thin spherical shells in 1912. Hans REISSNER demonstrated how two symmetrical second order differential equations can be constructed for the rotationally symmetrical problem of the thin spherical shell on the basis of LOVE's theory of thin shells when two parametric variables, the transverse shear stress resultant and the meridional rotation, are introduced as dependent variables.

In the same year, Otto BLUMENTHAL (25) showed how asymptotic integration of the REISSNER differential equations, the reduced differential equation of which contains a large scalar parameter, can be carried out. Moreover, BLUMENTHAL also established the numerical bounds for the error committed by any truncated asymptotic solution of a differential equation containing a large scalar parameter when the solution represents the parametric asymptotic expansion.

Ernst MEISSNER, a co-supervisor of KELLER's doctoral thesis, suggested to the latter to try to take advantage of the symmetrical construction of the differential equations of thin shells in the manner of Hans REISSNER, and then to construct the reduced differential equation which could be integrated by conventional methods, but KELLER was not successful in this effort. Consequently MEISSNER took up the study of the rotationally symmetrical shell problem himself.

MEISSNER was able to establish the general symmetrical equations to rotationally symmetrical shells in 1913 (29). He demonstrated how these equations, which can be reduced to a fourth order differential equation in one dependent variable, can be reduced to two conjugate second order differential equations for shells of constant meridional curvature, such as spherical, conical, and toroidal shells. He showed that in this case the solution of one second order differential equation yields all the solutions of the problem. In 1915 (30) MEISSNER showed that the symmetrical form of the differential equations can be achieved for arbitrary rotationally symmetrical shells with an appropriate choice of the dependent variables. MEISSNER reduced the differential equation of the rotational shells with constant curvature to hypergeometric differential equations which possess notoriously slow-converging series solutions that have small radii of convergence.

MEISSNER directed a research of a number of his doctoral students, who produced a series of theses on the rotational shell problems: R. ZOELLY wrote a thesis on buckling of spherical shells in 1915 (31), L. BOLLE a thesis on the bending of spherical shells in 1916 (32), E. WISSLER a thesis on toroidal shells in 1916 (33), F. DUBOIS a thesis on conical shells in 1917 (34), and E. HONEGGER a thesis on conical shells with linearly varying wall thickness in 1919 (39).

In 1916, Karl FEDERHOFER wrote a paper on the stability of shallow spherical shells, published in 1916 and 1917 (37), in which he reduced the shallow spherical shell equations to a BESSEL differential equation, the solutions of which were given in terms of BESSEL functions. For large arguments, FEDERHOFER introduced the first term asymptotic approximation of the BESSEL functions.

In 1917, Hans REISSNER's doctoral student, Edmund SCHWERIN (38), wrote a thesis on the asymptotic solution of the spherical shell under symmetrical and antisymmetrical loadings, which was carried out to numerical results. He published this work in 1919 (38).

In 1917, W. EFFENBERGER (35) introduced the solution of cylindrical shells with slightly varying wall thickness in the form of the perturbation series in powers of a small parameter ε: $h(x) = h_o + \varepsilon x$ and $h(x) = h_o + \varepsilon x^2$. In the perturbation method, the solution is expanded into a power series of a small parameter ε:

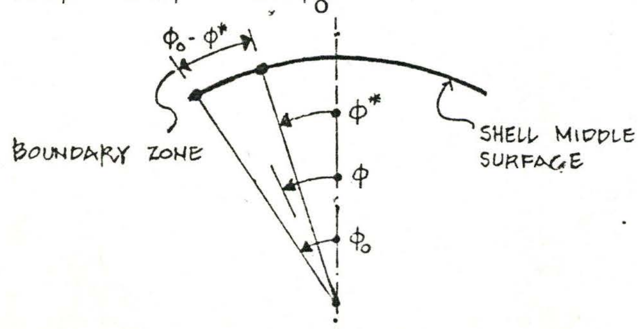
$$w(x) = w_o(x) + \varepsilon w_1(x) + \varepsilon^2 w_2 + \ldots\ldots.$$

EFFENBERG obtained a simple differential equation with explicit solutions $w_o(x)$, $w_1(x)$, $w_2(x)$...., to be determined from the solution of a set of differential equations. The solution converges rapidly for small ε, and already the first term approximation $w(x) = w_o + \varepsilon w_1$ was sufficient for applications.

There was a need to have simpler, yet for engineering applications sufficiently accurate solutions for shell problems. In 1921, F. KANN published an investigation (40) on the conical shell problems which admits BESSEL series solutions for a simplified differential equation, the dependent variable of which is the normal displacement w. KANN assumed the POISSON's ratio $\nu \simeq 0$, the circumferential displacement $u \simeq 0$, and the meridional displacement v to be negligibly small and, furthermore, he neglected also the circumferential bending stiffness of the conical shell. These simplifications enabled KANN to transform the conical shell equation into a BESSEL differential equation in which the transverse displacement w is the dependent variable. These simplifications are also used in the present work.

In 1925 (42), MEISSNER was able to demonstrate that for a very thin shell of rotation, the boundary bending of which diminishes rapidly with the distance from the shell's edge, the form of its meridian and its meridional curvature are of minor significance to its boundary deformation problem.

In 1924, German engineering physicist Walther BAUERSFELD of the firm ZEISS in Jena based his reasoning on the fact discovered by BARNARD and LAMB that for a sufficiently thin spherical shell the boundary effect is limited to a rather narrow boundary zone around the edge of the shell. He reasoned that the coefficient $\cot\phi$ of the pertinent differential equation does not change appreciably over the boundary zone defined by $\phi_o - \phi^*$, therefore, $\phi \simeq \phi^* \simeq \phi_o$ and $\cot\phi \simeq \cot\phi^* \simeq \cot\phi_o$.



MERIDIONAL SHELL SECTION

On this basis the differential equations of the spherical shell can be approximated by one with constant coefficients, and, therefore, they could be easily solved by standard methods of integration. This type of solution of thin shell by BAUERSFELD was patented and therefore was not published.

In 1926 (45), physicist Joseph W. GECKELER, who collaborated in the design of the ZEISS dome with BAUERSFELD and a well-known shell design pioneer Franz DISCHINGER, saw another possible simplification. GECKELER reasoned that if the boundary zone of transverse bending is small, then the solution must consist of strongly damped functions; hence the derivatives of higher order are numerically much larger than the derivatives of lower order as long as their coefficients are of comparable orders of magnitude in the differential equation. Therefore GECKELER was able to neglect all first order derivatives in his two second order differential equations for non-shallow thin rotational shells. This method of integration is called the GECKELER approximate solution.

Peter L. PASTERNAK (43, 44) gave several simplified methods for the analysis of rotational shells and tanks in 1925 and 1926.

In 1928 and 1929 (46), MEISSNER pointed out that a toroidal shell can be used as a replacement to the actual shell for the purpose of the boundary effect calculations. He demonstrated in his paper how HORN's asymptotic method could be used to integrate approximately such toroidal shell equations, and how their solution can be given in terms of BESSEL functions. MEISSNER also showed how the spherical shell solutions could be given in terms of LEGENDRE's polynomials and approximated by asymptotic series restricted to one term.

In 1930 (47), GECKELER published a solution for shallow rotationally symmetrical shells.

In 1932 (42), E. LICHTENSTERN published his investigations on the bending of conical shells with linearly varying wall thickness.

In 1933 (49), the Swedish engineer John-Erik EKSTRÖM studied the general problem of various shells of rotation, among others the conical shell. In 1935 (50), EKSTRÖM investigated conical shell frustrums and their use in finding foundation pressures under circular foundation plates.

Since 1944, a number of papers (51, 52, 53) and many others have been published on rotational shells and conical shells, but none of these works dealt with conical shells on elastic foundation.

APPENDIX II

HISTORICAL NOTES: ASYMPTOTIC SOLUTION

Asymptotic integration methods emerged from the efforts of the mathematicians and scientists to find approximate solutions to ordinary differential equations with error estimates for large values of the independent variable or for a large parameter present in the differential equation.

Solutions About the Singularity Points

Lazarus FUCHS in 1866 launched the basic construction of solutions for general ordinary differential equations.

$$P_n(x)\frac{d^n u}{dx^n} + P_{n-1}(x)\frac{d^{n-1} u}{dx^{n-1}} + P_{n-2}(x)\frac{d^{n-2} u}{dx^{n-2}} + \cdots + P_o(x)u = 0$$

where $P_r(x)$ are polynomials

$$P_r(x) = a_m x^m + a_{m-1} x^{m-1} + \cdots + a_o \qquad (r = 0, 1, \ldots, n)$$

According to FUCHS' theory, the differential equations can have singularities at the roots of $P_n(x) = 0$ and at $x = 0$. FUCHS demonstrated that the solution of the differential equation around a regular point $x = 0$ possesses a formal power series solution at and around this point. This series is absolutely and uniformly convergent and its circle of convergence extends to the nearest singular point of $P_n(x)$.

This equation can be put into the form:

$$\frac{d^n u}{dx^n} + \frac{P_{n-1}(x)}{P_n(x)} \frac{d^{n-1} u}{dx^{n-1}} + \cdots + \frac{P_o(x)u}{P_n(x)} = 0$$

A regular point is a point at which all the coefficients are analytic (finite, single-valued and possess a unique derivative at x, and in its neighbourhood).

A shift of variable $x = \xi - \xi_o$ gives a differential equation:

$$\frac{d^n u}{d\xi^n} + p_{n-1}(\xi) \frac{d^{n-1} u}{d\xi^{n-1}} + \cdots + p_o(\xi)u = 0$$

in which the coefficients can be expressed in the form,

$$p_r(\xi) = \frac{P_r(\xi)}{(\xi-\xi_o)^{n-r}} \qquad r = n-1,\ n-2, \ldots, 0$$

The singular points at $\xi = \xi_o$ are points for which any

$$\lim_{\xi \to \xi_o} p_r(\xi) = \lim_{\xi \to \xi_o} P_r(\xi)/(\xi-\xi_o)^{n-r} \to \infty$$

Regular singular points are points where $n > n-r$ $(r = n-1, n-2, \ldots, 0)$, e.g., the coefficient $p_3(\xi)$ has the first order of singularity at $\xi = \xi_o$: if $r = 3$, $n = 4$, $n-r = 4-3 = 1$. Irregular singular points are points where $n-r > n$.

FUCHS considered the properties which the differential equation must have in order that its solution at the singular point $\xi = \xi_o$ has the converging series solution of the form:

$$u(\xi) = (\xi-\xi_o)^{C_i} \sum_{k=-\infty}^{+\infty} b_k (\xi-\xi_o)^k$$

where $\sum_{k=-\infty}^{+\infty} b_k (\xi-\xi_o)^k$ is a convergent LAURENT series.

By substituting the derivatives of the given $u(\xi)$ into the differential equation and collecting terms with like powers, the indicial equation is obtained for $n = 0$.

$$[C]_n + [C]_{n-1}p_{n-1}(\xi_o) + \cdots p_o(\xi_o) = 0$$

where

$$[C]_n = C(C - 1) \cdots (C - k + 1)$$

If the roots C_i of the indicial equation are distinct and no two roots C_i and $C_j (j \neq i)$ differ by an integer, then the fundamental solution set of differential equation contains the above solution. In all other cases the solution may have the form:

$$u(\xi) = (\xi-\xi_o)^C \{\sum_{k=-\infty}^{+\infty} C_{Ok}(\xi-\xi_o)^k + \sum_{k=-\infty}^{+\infty} C_{1k} (\xi-\xi_o)^k \ln(\xi-\xi_o)$$

$$+ \cdots + \sum_{k=-\infty}^{+\infty} C_{(p-1)k}(\xi-\xi_o)^k [\ln(\xi-\xi_o)]^{p-1}\}$$

where $p \leqslant l, l$ denotes the multiplicity of roots or the number of roots which differ by an integer. At a regular singular point the differential equation of the n-th order has a set of n linearly independent solutions of either of the above solutions. At a regular singular point $\xi = \xi_o$, the LAURENT series in these solutions have a finite number of negative power terms.

At regular singular points with finite number of negative powers of $(\xi-\xi_o)$, all the coefficients of the LAURENT series can be determined by recursion equations obtained from the differential equation after formal substitution of the series.

However, at irregular singular points, all solutions contain an infinite number of negative powers of $(\xi-\xi_o)$ in the LAURENT series in the solution, and therefore the aforementioned method is not valid, as the coefficients of the LAURENT series cannot be determined by substituting the series into the differential equation and equating equal powers of the variables. A method of determining these coefficients at an irregular singular point was first given by the German mathematician Ludwig Wilhelm THOMÉ in 1877. A differential equation of the n-th order has the regular singularity at $\xi = \infty$ if it has a form:

$$\frac{d^n u}{d\xi^n} + \frac{1}{\xi} f_{n-1}\left(\frac{1}{\xi}\right) \frac{d^{n-1} u}{d\xi^{n-1}} + \cdots + \frac{1}{\xi^n} f_o\left(\frac{1}{\xi}\right) u = 0$$

when its coefficients $f_i(1/\xi)$ are analytic functions at $\xi = \infty$.

THOMÉ showed that in addition to the possible convergent series solutions, a differential equation with an irregular singular point at $\xi = \infty$,

$$D(u) = \frac{d^n u}{d\xi^n} + p_{n-1}(\xi) \frac{d^{n-1} u}{d\xi^{n-1}} + \cdots + p_1(\xi) u = 0$$

in which the coefficients are developable into a descending power series, has normal series solutions about $\xi = \infty$ of the form:

$$u_i(\xi) = e^{Q_k(\xi)}\, v(\xi) = e^{Q_k(\xi)} \xi^{C_i} (a_o + a_1 \xi^{-1} + a_2 \xi^{-2} + \cdots)$$

$$= e^{Q_k(\xi)} \xi^{C_i} \sum_{m=0}^{\infty} a_m \xi^{-m}$$

where $Q_k(\xi) = C \frac{i\xi^k}{k} + C_{i1}\xi^{k-1} + \cdots + C_{ik}\xi$ is a polynomial of degree k in ξ and the leading coefficient C_i is a root of the characteristic equation obtained by substituting $u(\xi)$ into the differential equation, collecting like terms of $a_m\xi^{-m}$ and setting $m = 0$. k is the order of the normal solution and is the integer equal to, or immediately larger than, the largest difference:

$$N_r = (M_r - M_n)/n-r = \text{maximum} \qquad (r = 0, 1, 2, \ldots, n)$$

where M_r denotes the degree of the polynomial $p_r(\xi)$.

The normal series solutions usually diverge at an irregular singular point of the differential equation. If the roots of the characteristic equation differ by an integer then the logarithmic normal series solutions apply:

$$u(\xi) = e^{Q_k(\xi)} \xi^{C_i} [\psi_o(\tfrac{1}{\xi}) + \ln\xi\psi_1(\tfrac{1}{\xi}) + \cdots + \ln^r\xi\psi_r(\tfrac{1}{\xi})]$$

where the integer r is to be determined.

If the indicial equation has l multiple roots among all its n roots $C_1, \ldots, C_i, \ldots, C_n$, then the differential equation has the anormal series solution of order k,

$$u_i(\xi) = e^{Q_k(1/m\sqrt{\xi})} \xi^{C_i} \psi^{(1/m\sqrt{\xi})}$$

where $Q_k(1/m\sqrt{\xi})$ is an increasing power series of $1/m\sqrt{\xi}$ and the integer $m \leqslant l$. If the degree of $Q_k(1/m\sqrt{\xi})$ is greater than (k-1)m and less than km, then this series is an anormal series solution of order k. The anormal series solution was first given by Charles-Eugene FABRY in his doctoral thesis, in 1885.

Each normal series solution of the differential equation

$$\frac{d^n u}{d\xi^n} + p_{n-1}(\xi) \frac{d^{n-1} u}{d\xi^{n-1}} + \cdots + p_o u = 0$$

though it diverges, constitutes asymptotically one integral of that differential equation for large values of x.

Any irregular singular point can be transferred by a linear transformation, the so-called translation, to $x = \infty$ without altering the differential equation.

In general, the normal, anormal and logarithmic normal series solutions diverge, and therefore no important advancement in the solution of differential equations with irregular singular points was made until Henri POINCARÉ through his introduction of the theory of asymptotic series succeeded in isolating the essential property of these semi-divergent series that furnishes useful approximations to the solutions of differential equations for large values of the variable.

Asymptotic Expansions

Any functions, such as the solution of the differential equation, $u(\xi)$, can be represented by asymptotic power series for large values of ξ,

$$u(\xi) \sim \sum_{n=0}^{m} a_n/\xi^n$$

if

$$\lim_{\xi\to\infty} \xi^{m+1}[u(\xi) - \sum_{n=0}^{m} a_n/\xi^n] \to a_{m+1}$$

This series, in general, is divergent, but may, in special cases, be convergent. The order of the magnitude of the error at any truncation of the series is less than the magnitude of the first term omitted:

$$|u(\xi) - \sum_{n=0}^{m} a_n/\xi^n| < |a_{m+1}/\xi^{m+1}|$$

German mathematician Adolph KNESER showed from 1896 to 1899 how the normal solutions of certain differential equations can be used to construct the actual solutions. His results were of fundamental importance, but limited in scope, as it is necessary to prove that the normal series solutions are asymptotic to the actual solutions without actually integrating the differential equations exactly, a task that may be very difficult or even impossible.

The most important results in the asymptotic solutions of differential equations for large values of the independent variable x, and for a large parameter, were established by a German mathematician Jakob HORN (1867-1946) in a sequence of papers from 1897 to 1910. In these papers HORN made use of a number of methods in various combinations whereby he was able to bring POINCARÉ's ideas on asymptotic integration to their full power.

In 1897, HORN employed KNESER's method to find asymptotic solutions for differential equations of the second order and higher rank irregular singularity. HORN used an idea of Heinrich WEBER in 1890 by assuming that the actual solutions of this differential equation can be expressed in terms of finite sums of the normal series solutions and some other functions $v_{in}(x)$:

$$u_i(\xi) = C^{Q_{k+1}(\xi)} \xi^{P_i}[C_1 + \cdots + C_{in}\xi^{-n}] + v_{in}\xi^{-n} \quad (i = 1,2)$$

where

$$Q(\xi) = \alpha_i \xi^{k+1}(k+1)^{-1} + \alpha_{i1}\xi^k + \cdots + \alpha_{ik}\xi$$

Then, $v_{in}(x)$ must be the solution of a certain differential equation. By the study of this differential equation he was able to show that the normal series solutions are the asymptotic series solutions of this differential equation.

For differential equations with equal characteristic roots $\alpha_1 = \alpha_2$, HORN used KNESER's transformation for variable

$$v(\xi) = \xi[u(\xi) - \alpha_1]$$

in the differential equation to which the previous results apply, except that now ξ must be replaced by $\sqrt{\xi}$.

In 1900 and 1910, HORN was able to prove that his results applied to the n-th order differential equations, and that its normal series solutions are in fact the asymptotic series solutions for large real ξ.

HORN was also the first mathematician to apply asymptotic series methods to the solution of differential equations containing a large parameter in 1899. He demonstrated that the concepts of the large independent variable problem can also be applied to the asymptotic methods of solution of differential equations containing a large parameter. HORN made use of successive approximation methods in the construction of actual solutions for his normalized differential equation; that was a novel approach, and proved that the normal series solutions in terms of inverse powers of the parameter represent the asymptotic solutions of the differential equation with a large parameter. In the asymptotic integration of differential equations containing a large parameter, HORN was a pioneer.

POINCARÉ proved that the sum, the difference, the product, and the quotient of two functions are represented asymptotically by the sum, the difference, the product and the quotient of their individual asymptotic series, provided that the constant term in the divisor series expansions can be integrated and differentiated term by term.

For large positive ξ, the normal series solution of the differential equation of the n-th order represents its asymptotic solution in co-ordinate expansion:

$$u(\xi) \sim e^{Q(\xi)} A\xi^{C} \sum_{n=0}^{\infty} a_n/\xi^n$$

where

$$\lim_{\xi\to\infty} \xi^{m+1}[A\xi^{-C} e^{-Q(\xi)} u(\xi) - \Sigma a_n/\xi^n] \to a_{m+1}$$

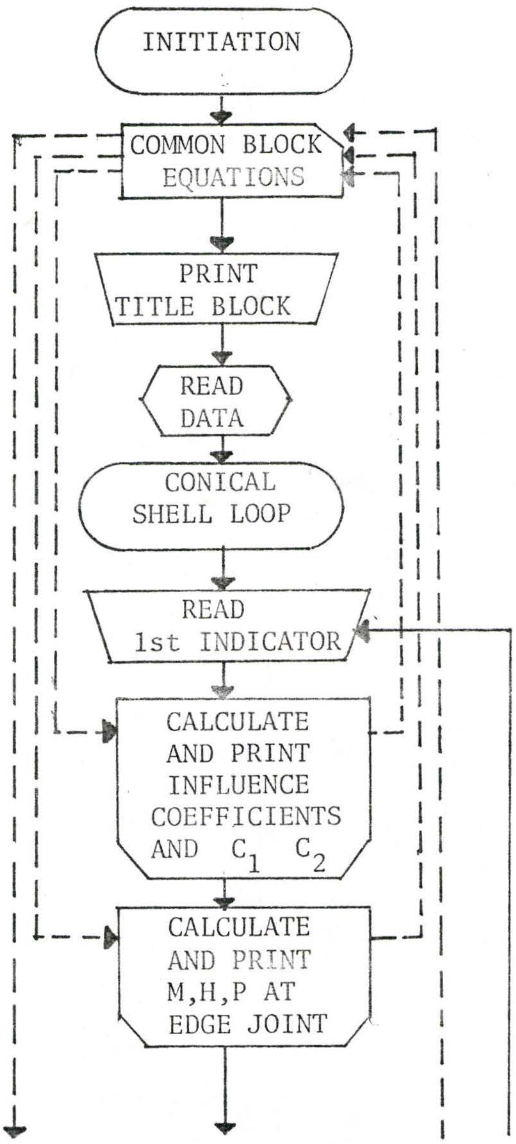
POINCARÉ noted that if any asymptotic series represents the same function in a complete neighbourhood of $\xi = \infty$, then it is convergent.

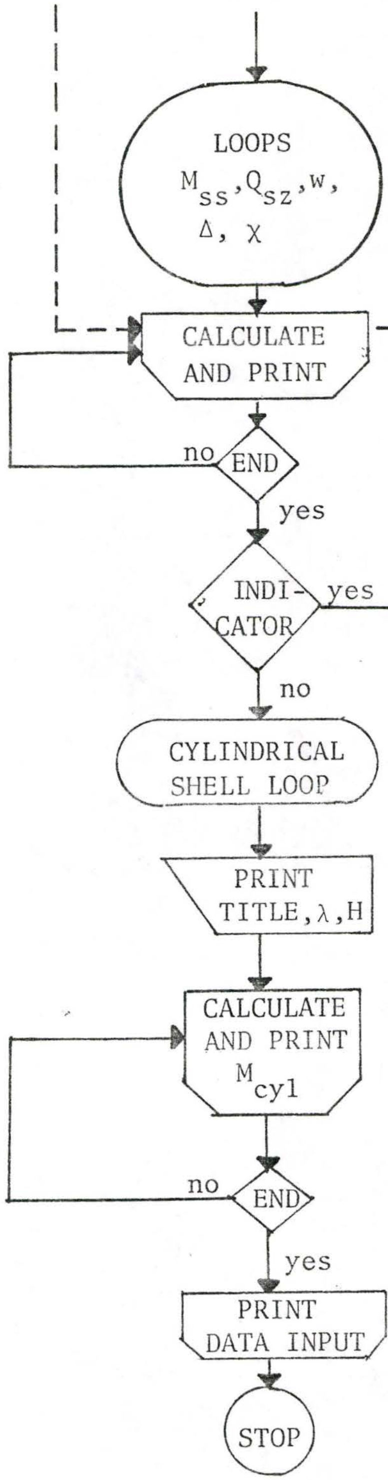
In the case of differential equations which contain a large parameter, the asymptotic solution requires an expansion in terms of the inverse of this parameter and represents its asymptotic parametric expansion. As the shell problem studied here is not related to a large parameter, the historical review of this type of solution is not included.

APPENDIX III

COMPUTER PROGRAMME AND CALCULATED RESULTS

FLOW CHART




LOOPS
M_{ss}, Q_{sz}, w,
Δ, χ
CALCULATE AND PRINT
no
END
yes
INDI-CATOR
yes
no
CYLINDRICAL SHELL LOOP
PRINT TITLE, λ, H
CALCULATE AND PRINT M_{cyl}
no
END
yes
PRINT DATA INPUT
STOP

PROGRAMME AND CALCULATED RESULTS

The programme is written in PLC computer language.

Inputs:
1. Approximate Method Indicator
2. Kelvin Functions: berη, beiη, ber'η, bei'η

   for η = 14.38 to 0.0
3. λ, K, E, h, s at edge joint, ϕ, P, α^4,
4. Influence Coefficients:

   $E\Delta^{p}_{con}$, $E\chi^{p}_{con}$, $E\Delta^{p}_{cyl}$, $E\chi^{p}_{cyl}$, $E\Delta^{H}_{cyl}$,

   $E\chi^{H}_{cyl}$, $E\Delta^{M}_{cyl}$, $E\chi^{M}_{cyl}$.

Outputs: Influence Coefficients for Conical and Cylindrical Shells,

(A) Conical Shell:

1. C_1, C_2
2. M, H, P at edge joint
3. Bending Moments

   M^{H}_{ss}con, M^{P}_{ss}con, M^{M}_{ss}con, M_{ss}con
4. Transverse Shear Force

   Q^{H}_{sz}con, Q^{P}_{sz}con, Q^{M}_{sz}con, Q_{sz}con
5. Normal and Horizontal Displacements and Soil Bearing Pressures

   w^{H}_{con}, w^{P}_{con}, w^{M}_{con}, w^{pP}_{con}, w_{con}, Δ_{con}, $p_f$con
6. Rotations

   χ^{H}_{con}, χ^{P}_{con}, χ^{M}_{con}, χ^{pP}_{con}, χ_{con}

7. Additional results for Flügge's Approximation:

   Circumferential Shear Force

   $N^{H}_{\theta\theta}con$, $N^{P}_{\theta\theta}con$, $N^{M}_{\theta\theta}con$, $N^{pP}_{\theta\theta}con$, $N_{\theta\theta}con$

   Meridional Shear Force

   $N^{H}_{\phi\phi}con$, $N^{P}_{\phi\phi}con$, $N^{M}_{\phi\phi}con$, $N^{pP}_{\phi\phi}con$, $N_{\phi\phi}con$

(B) Cylindrical Shell:

1. λ
2. H at edge joint
3. Bending Moments

   M^{M}_{cyl}, M^{H}_{cyl}, M_{cyl}

(C) Data Input

k, E, h, s, ϕ, P, α^2, K, K**

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                    **********************************************************************************
                    * PLC       RLSE 7.5.                                    MONITOR,VERSION 3C     *
                    *                              K. CHUNG                                         *
                    * TUESDAY OCTOBER 30, 1979.                                         6:37 PM     *
                    **********************************************************************************

D='K. CHUNG'

ROCEDURE OPTIONS(MAIN);                                           PL/C-R7.5-326  10/11/79 23:10 PAG

LEVEL NEST BLOCK MLVL  SOURCE TEXT

                         XNAME:PROCEDURE OPTIONS(MAIN);
   1          1             DECLARE(NAM,NAMES(9))CHARACTER(40)VARYING;
   1          1             DECLARE(III,E#,F#)FIXED;
   1          1             DECLARE(QHCON,QPCON,QMCON,QPPCON,QCON,HK,
                                   WHCON,WPCON,WMCON,WPPCON,WCON,
                                   XHCON,XPCON,XMCON,XPPCON,XCON,
                                   CHCON,CPCON,CMCON,CPPCON,CCON,
                                   NHCON,NPCON,NMCON,NPPCON,NCON,
                                   DCON,PF,HHH)FLOAT;
   1          1             DECLARE(A,B,D,E,F,G,H,K,M,P,S,T,Z)FLOAT;
   1          1             DECLARE(AA,BB,CC,DD,EE,FF,GG,HH,II,JJ,KK,LL,MM,NN,RR,UU,VV,WW,XX,YY,ZZ)FLOAT;
   1          1             DECLARE(E0,ES,KS,KKS,A4,K0,H0,KKK)FLOAT;
   1          1             DECLARE(EDHCON,EXHCON,EDPCON,EXPCON,EDMCON,EXMCON,
                                   EDHCYL,EXHCYL,EDPCYL,EXPCYL,EDMCYL,EXMCYL)FLOAT;
   1          1             DECLARE(LS,EKS,EKKS)FLOAT;
   1          1             DECLARE(LAMDA)FLOAT;
   1          1             DECLARE(MMCYL,MHCYL,MCYL)FLOAT;
   1          1             DECLARE(SPRIME)FLOAT;
   1          1             DECLARE(LOOP)BIT(1);
   1          1             DECLARE (A1,A2,A3) ENTRY(FIXED)RETURNS(FLOAT);
   1          1             DECLARE(BER(32),BEI(32),BERI(32),BEII(32))FLOAT;
   1          1             ON ENDFILE(SYSIN) LOOP='0'B;

   2          2             A1:PROCEDURE(F#)RETURNS(FLOAT);
   2          3                DECLARE(A)FLOAT;
   2          3                DECLARE(F#)FIXED;

   2          3                BEGIN;
   3          4                DECLARE CASE(20) LABEL;
   3          4                DECLARE(E,H)FLOAT;
   3          4                E=E0;  H=H0;

   3          4                GOTO CASE((E# - 1) * 4 + F#);

   3          4                CASE( 1):
                                  A=EKS;
   3          4                   GOTO END_CASE;

   3          4                CASE( 2):
                                  A=-E * H**3E0 * K*K * EKS / 6E0;
   3          4                   GOTO END_CASE;

   3          4                CASE( 3):
                                  A=-E * H**3E0 * K*K * EKS / 6E0;
   3          4                   GOTO END_CASE;

   3          4                CASE( 4):
                                  A=K * EKS;
   3          4                   GOTO END_CASE;
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 3        4                CASE( 5):
 3        4                   IF S=0E0 THEN A=0E0;
```



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EVEL NEST BLOCK MLVL  SOURCE TEXT

 3        4                   ELSE
                              A=EKKS / SQRT(S);
 3        4                   GOTO END_CASE;

 3        4                CASE( 6):
 3        4                   IF S=0E0 THEN A=0E0;
 3        4                   ELSE
                              A=-E * H**3E0 * EKKS / (12E0 * SQRT(S));
 3        4                   GOTO END_CASE;

 3        4                CASE( 7):
                              A=-E * H**3E0 * EKKS / (12E0 * SQRT(S));
 3        4                   GOTO END_CASE;

 3        4                CASE( 8):
 3        4                   IF S=0E0 THEN A=0E0;
 3        4                   ELSE
                              A=EKKS / SQRT(S);
 3        4                   GOTO END_CASE;

 3        4                CASE( 9):
                              A=EKS / SQRT(S);
 3        4                   GOTO END_CASE;

 3        4                CASE(10):
                              A=-E * H**3E0 * EKS * K / (12E0 * SQRT(S));
 3        4                   GOTO END_CASE;

 3        4                CASE(11):
                              A=-E * H**3E0 * EKS     / (12E0 * SQRT(S));
 3        4                   GOTO END_CASE;

 3        4                CASE(12):
                              A=EKS / SQRT(S);
 3        4                   GOTO END_CASE;

 3        4                CASE(13):
                              A=EKS / SQRT(S);
 3        4                   GOTO END_CASE;

 3        4                CASE(14):
                              A=-E * H**3E0 * EKS * K / (12E0 * SQRT(S));
 3        4                   GOTO END_CASE;

 3        4                CASE(15):
                              A=-E * H**3E0 * EKS     / (12E0 * SQRT(S));
 3        4                   GOTO END_CASE;

 3        4                CASE(16):
                              A=EKS / SQRT(S);
 3        4                   GOTO END_CASE;
 3        4                CASE(17):
                              A = -2E0 * U * SQRT(S) * COS(T) / (4E0 * H0 * E0 * SIN(T));
 3        4                   GOTO END_CASE;
```

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LEVEL NEST BLOCK MLVL   SOURCE TEXT

  3          4              CASE(18):
  3          4                 IF S=0E0 THEN A=0E0;
  3          4                 ELSE
                                  A = TAN(T) / S;
  3          4                 GOTO END_CASE;

  3          4              CASE(19):
  3          4                 IF S=0E0 THEN A=0E0;
  3          4                 ELSE
                                  A = 2E0 * TAN(T) /(2E0 * U * SQRT(S))**2E0;
  3          4                 GOTO END_CASE;

  3          4              CASE(20):
                                  A = -SQRT(12E0) / (H0 * H0 * E0);
  3          4                 GOTO END_CASE;

  3          4              END_CASE:;

  3          4              END;
  2          3              RETURN(A);
  2          3              END;

  1          1           A2:PROCEDURE(F#)RETURNS(FLOAT);
  2          5              DECLARE(A)FLOAT;
  2          5              DECLARE(F#)FIXED;
  2          5              DECLARE CASE(20) LABEL;

  2          5              GOTO CASE((E# - 1) * 4 + F#);

  2          5              CASE( 1):
                               A=COS(KS);
  2          5                 GOTO END_CASE;

  2          5              CASE( 2):
  2          5                 IF S=0E0 THEN A=0E0;
  2          5                 ELSE
                               A=-K * (COS(KS) + SIN(KS)) + SIN(KS) / S;
  2          5                 GOTO END_CASE;

  2          5              CASE( 3):
                               A=-SIN(KS);
  2          5                 GOTO END_CASE;

  2          5              CASE( 4):
                               A=COS(KS) - SIN(KS);
  2          5                 GOTO END_CASE;

  2          5              CASE( 5):
                               A=COS(KKS);
  2          5                 GOTO END_CASE;

  2          5              CASE( 6):
  2          5                 IF S=0E0 THEN A=0E0;
  2          5                 ELSE
                         A=(-2E0*KK**3E0+5E0*KK/(4E0*S*S)-9E0/(8E0*S*S*S))*COS(KKS)+
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LEVEL NEST BLOCK MLVL   SOURCE TEXT

                               (-2E0*KK**3E0+KK**2E0/S-5E0*KK/(4E0*S*S))*SIN(KKS);
  2          5                    GOTO END_CASE;

  2          5                 CASE( 7):
                                  A=(.75/S - KK) * COS(KKS)/S -(-1E0/S + 2E0*KK) * KK * SIN(KKS);
  2          5                    GOTO END_CASE;

  2          5                 CASE( 8):
  2          5                    IF S=0E0 THEN A=0E0;
  2          5                    ELSE
                                  A=(KK - 1E0/(2E0*S)) * COS(KKS) - KK * SIN(KKS);
  2          5                    GOTO END_CASE;

  2          5                 CASE( 9):
                                  A=(1E0 - K*A4/4E0*(1E0 - .75/(K*K*A4))/S) * COS(KS) +
                                            (K*A4/4E0*(1E0 + .75/(K*K*A4))/S) * SIN(KS);
  2          5                    GOTO END_CASE;

  2          5                 CASE(10):
                                  A=(-2E0 * K*K + K**3E0 * A4/S) * COS(KS) -
                                    ( 2E0 * K*K - K/(4E0*S)) * SIN(KS);
  2          5                    GOTO END_CASE;

  2          5                 CASE(11):
                                  A=(          K/2E0 * (-1.25 + K*K*A4)/S) * COS(KS) -
                                    (2E0*K*K-K/2E0 * ( 1.25 + K*K*A4)/S) * SIN(KS);
  2          5                    GOTO END_CASE;

  2          5                 CASE(12):
                                  A=(K - .125/S) * COS(KS) - (K - K*K*A4/(2E0*S)) * SIN(KS);
  2          5                    GOTO END_CASE;

  2          5                 CASE(13):
                                  A=(1E0 - K*A4/4E0 * (1E0 - .75/(K*K*A4))/S - 15E0/32E0 *
                                     A4/(S*S)) * COS(KS) +
                                           (K*A4/4E0 * (1E0 + .75/(K*K * A4))/S - K*K*A4*A4/16E0 *
                                     (1E0 - 81E0/(16E0 * K**4E0 *A4*A4))/(S*S)) * SIN(KS);
  2          5                    GOTO END_CASE;

  2          5                 CASE(14):
                                  A=(    -2E0 * K*K + K**3E0 * A4/S +
                                    (73E0/128E0 - 13E0 * K*K * A4/16E0 - K**4E0 * A4*A4/8E0)/(S*S))
                                     * COS(KS) -
                                    (2E0 * K*K - K/(4E0*S) +
                                    (73E0/128E0 + 13E0 * K*K * A4/16E0 - K**4E0 * A4*A4/8E0)/(S*S))
                                     * SIN(KS);
  2          5                    GOTO END_CASE;

  2          5                 CASE(15):
                                    A=(K/2E0 * (-1.25 + K*K * A4)/S + 1E0/16E0 *
                                    (33E0/8E0 -2E0 * K**4E0 * A4*A4)/(S*S)) * COS(KS) -
                                    (2E0 * K*K - K/2E0 * (1.25 + K*K * A4)/S + 9E0/16E0 *
                                     K*K * A4/(S*S)) * SIN(KS);
  2          5                    GOTO END_CASE;
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LEVEL NEST BLOCK MLVL   SOURCE TEXT

  2          5                     CASE(16):
                                      A=(K - .125/S - K*A4/16E0 * (1.5 - 9E0/(16E0 * K*K * A4) + K*K * A4) /
                                        (S*S)) * COS(KS) -
                                        (K - K*K * A4/(2E0*S) - K*A4/16E0 * (1.5 + 9E0/(16E0 * K*K * A4) -
                                         K*K * A4)/(S*S)) * SIN(KS);
  2          5                        GOTO END_CASE;

  2          5                     CASE(17):
  2          5                        IF S=0E0 THEN A=0E0;
  2          5                        ELSE
                                         A = -2E0 * BERI(L) + 4E0 * BER(L) /(2E0 * U * SQRT(S)) -
                                             8E0 *BEII(L) / (2E0 * U * SQRT(S))**2E0;
  2          5                        GOTO END_CASE;

  2          5                     CASE(18):
  2          5                        IF S=0E0 THEN A=0E0;
  2          5                        ELSE
                                         A =  BER(L) - 2E0 * BEII(L) /(2E0 *U*SQRT(S));
  2          5                        GOTO END_CASE;

  2          5                     CASE(19):
  2          5                        IF S=0E0 THEN A=0E0;
  2          5                        ELSE
                                         A = 2E0 * U * SQRT(S) * BEII(L) - 2E0 * (BEI(L) + 2E0 * BERI(L) /
                                             (2E0 * U * SQRT(S)));
  2          5                        GOTO END_CASE;

  2          5                     CASE(20):
  2          5                        IF S=0E0 THEN A=0E0;
  2          5                        ELSE
                                         A = BEI(L) + 2E0 * BERI(L) /(2E0 * U * SQRT(S));
  2          5                        GOTO END_CASE;

  2          5                     END_CASE:;
  2          5                     RETURN(A);
  2          5                     END;

  1          1                  A3:PROCEDURE(F#)RETURNS(FLOAT);
  2          6                     DECLARE(A)FLOAT;
  2          6                     DECLARE(F#)FIXED;
  2          6                     DECLARE CASE(20) LABEL;

  2          6                     GOTO CASE((E# - 1) * 4 + F#);

  2          6                     CASE( 1):
                                      A=SIN(KS);
  2          6                        GOTO END_CASE;

  2          6                     CASE( 2):
  2          6                        IF S=0E0 THEN A=0E0;
  2          6                        ELSE
                                      A=K * (COS(KS) - SIN(KS)) + COS(KS) / S;
  2          6                        GOTO END_CASE;

  2          6                     CASE( 3):
```

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LEVEL NEST BLOCK MLVL  SOURCE TEXT

                             A=COS(KS);
  2          6               GOTO END_CASE;

  2          6            CASE( 4):
                             A=COS(KS) + SIN(KS);
  2          6               GOTO END_CASE;

  2          6            CASE( 5):
                             A=SIN(KKS);
  2          6               GOTO END_CASE;

  2          6            CASE( 6):
  2          6               IF S=0E0 THEN A=0E0;
  2          6               ELSE
                             A=(-2E0*KK**3E0+5E0*KK/(4E0*S*S)-9E0/(8E0*S*S*S))*SIN(KKS)+
                             -(-2E0*KK**3E0+KK**2E0/S-5E0*KK/(4E0*S*S))*COS(KKS);
  2          6               GOTO END_CASE;

  2          6            CASE( 7):
                             A=(.75/S - KK) * SIN(KKS)/S +(-1E0/S + 2E0*KK) * KK * COS(KKS);
  2          6               GOTO END_CASE;

  2          6            CASE( 8):
  2          6               IF S=0E0 THEN A=0E0;
  2          6               ELSE
                             A=(KK - 1E0/(2E0*S)) * SIN(KKS) + KK * COS(KKS);
  2          6               GOTO END_CASE;

  2          6            CASE( 9):
                             A=      -(K*A4/4E0*(1E0 + .75/(K*K*A4))/S) * COS(KS) +
                               (1E0 - K*A4/4E0*(1E0 - .75/(K*K*A4))/S) * SIN(KS);
  2          6               GOTO END_CASE;

  2          6            CASE(10):
                             A=( 2E0 * K*K - K/(4E0*S)) * COS(KS) +
                               (-2E0 * K*K +K**3E0*A4/S)*SIN(KS);
  2          6               GOTO END_CASE;

  2          6            CASE(11):
                             A=(2E0*K*K-K/2E0 * ( 1.25 + K*K*A4)/S) * COS(KS) +
                               (          K/2E0 * (-1.25 + K*K*A4)/S) * SIN(KS);
  2          6               GOTO END_CASE;

  2          6            CASE(12):
                             A=(K - .125/S) * SIN(KS) + (K - K*K*A4/(2E0*S)) * COS(KS);
  2          6               GOTO END_CASE;

  2          6            CASE(13):
                             A=(1E0 - K*A4/4E0 * (1E0 - .75/(K*K*A4))/S - 15E0/32E0 *
                                A4/(S*S)) * SIN(KS) -
                                    (K*A4/4E0 * (1E0 + .75/(K*K * A4))/S - K*K*A4*A4/16E0 *
                                (1E0 - 81E0/(16E0 * K**4E0 *A4*A4))/(S*S)) * COS(KS);
  2          6               GOTO END_CASE;

  2          6            CASE(14):
```

```
                              A=(-    2E0 * K*K + K**3E0 * A4/S +
                                (73E0/128E0 - 13E0 * K*K * A4/16E0 - K**4E0 * A4*A4/8E0)/(S*S))
                                 * SIN(KS) +
                                (2E0 * K*K - K/(4E0*S) +
                                (73E0/128E0 + 13E0 * K*K * A4/16E0 - K**4E0 * A4*A4/8E0)/(S*S))
                                 * COS(KS);
  2          6                GOTO END_CASE;

  2          6             CASE(15):
                                A=(K/2E0 * (-1.25 + K*K * A4)/S + 1E0/16E0 *
                                (33E0/8E0 -2E0 * K**4E0 * A4*A4)/(S*S)) * SIN(KS) +
                                (2E0 * K*K - K/2E0 * (1.25 + K*K * A4)/S + 9E0/16E0 *
                                 K*K * A4/(S*S)) * COS(KS);
  2          6                GOTO END_CASE;

  2          6             CASE(16):
                              A=(K - .125/S - K*A4/16E0 * (1.5 - 9E0/(16E0 * K*K * A4) + K*K * A4) /
                                (S*S)) * SIN(KS) +
                                (K - K*K * A4/(2E0*S) - K*A4/16E0 * (1.5 + 9E0/(16E0 * K*K * A4) -
                                 K*K * A4)/(S*S)) * COS(KS);
  2          6                GOTO END_CASE;


  2          6             CASE(17):
  2          6                IF S=0E0 THEN A=0E0;
  2          6                ELSE
                                 A = -2E0 * BEII(L) + 4E0 * BEI(L) /(2E0 * U * SQRT(S)) +
                                     8E0 *BERI(L) / (2E0 * U * SQRT(S))**2E0;
  2          6                GOTO END_CASE;

  2          6             CASE(18):
  2          6                IF S=0E0 THEN A=0E0;
  2          6                ELSE
                                 A =  BEI(L) + 2E0 * BERI(L) /(2E0 *U*SQRT(S));
  2          6                GOTO END_CASE;

  2          6             CASE(19):
  2          6                IF S=0E0 THEN A=0E0;
  2          6                ELSE
                                 A =-2E0 * U * SQRT(S) * BERI(L) + 2E0 * (BER(L) - 2E0 * BEII(L) /
                                     (2E0 * U * SQRT(S)));
  2          6                GOTO END_CASE;

  2          6             CASE(20):
  2          6                IF S=0E0 THEN A=0E0;
  2          6                ELSE
                                 A =-BER(L) + 2E0 * BEII(L) /(2E0 * U * SQRT(S));
  2          6                GOTO END_CASE;
  2          6             END_CASE:;
  2          6             RETURN(A);

  2          6             END;

  1          1             Z1:PROCEDURE;
  2          7             PUT SKIP(2) LIST('   S IN INCHES');
```

```
LEVEL NEST BLOCK MLVL  SOURCE TEXT

  2          7        PUT SKIP LIST('CIRCUMFERENTIAL FORCE IN POUND /INCH');
  2          7        PUT SKIP(2) EDIT('S','CHCON','CPCON','CMCON','CPPCON','CCON')
                                (X(9),A,X(17),A,X(15),A,X(15),A,X(15),A,X(15),A);
  2          7        PUT SKIP LIST(' ');
  2          7        IF E# = 1 THEN
  2          7        I=-12;
  2          7        ELSE
                      I=0;
  2          7        S=SPRIME;
  2          7        L=1;
  2          7        DO III=372 TO I BY -12;
  2     1    7           N=2E0*U*SQRT(S);
  2     1    7           X=4E0*BER(L)/(N*N)-2E0*BERI(L)/N-8E0*BEII(L)/(N*N*N);
  2     1    7           Y=4E0*BEI(L)/(N*N)-2E0*BEII(L)/N+8E0*BERI(L)/(N*N*N);
  2     1    7           Z=-U*U;
  2     1    7           CPCON=0E0;
  2     1    7           CHCON=Z*(A*X+D*Y)*H;
  2     1    7           CMCON=Z*(F*X+G*Y)*M;
  2     1    7           CPPCON=-PF*S/TAN(T);
  2     1    7           CCON=CHCON + CPCON + CMCON+CPPCON;
  2     1    7           PUT SKIP EDIT(S,CHCON,CPCON,CMCON,CPPCON,CCON)
                         (X(5),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),
                         X(10),E(10,3) );
  2     1    7           S=III;
  2     1    7           L=L+1;
  2     1    7           END;
  2          7        PUT SKIP(2) LIST('    S IN INCHES');
  2          7        PUT SKIP LIST('MERIDIONAL FORCE IN POUND/INCH');
  2          7        PUT SKIP(2) EDIT('S','NHCON','NPCON','NMCON','NPPCON','NCON')
                                (X(9),A,X(17),A,X(15),A,X(15),A,X(15),A,X(15),A);
  2          7        PUT SKIP LIST(' ');
  2          7        IF E# = 1 THEN
  2          7        I=-12;
  2          7        ELSE
                      I=0;
  2          7        S=SPRIME;
  2          7        L=1;
  2          7        DO III=372 TO I BY -12;
  2     1    7           N=2E0*U*SQRT(S);
  2     1    7           X=4E0*BER(L)/(N*N)-8E0*BEII(L)/(N*N*N);
  2     1    7           Y=4E0*BEI(L)/(N*N)+8E0*BERI(L)/(N*N*N);
  2     1    7           Z=-U*U;
  2     1    7           NPCON=0E0;
  2     1    7           NHCON=Z*(A*X+D*Y)*H;
  2     1    7           NMCON=Z*(F*X+G*Y)*M;
  2     1    7           NPPCON=-PF*S/(TAN(T)*2E0);
  2     1    7           NCON=NHCON + NPCON + NMCON+NPPCON;
  2     1    7           PUT SKIP EDIT(S,NHCON,NPCON,NMCON,NPPCON,NCON)
                         (X(5),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),
                         X(10),E(10,3) );
  2     1    7           S=III;
  2     1    7           L=L+1;
  2     1    7           END;
  2          7        END Z1;
```

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EVEL NEST BLOCK MLVL  SOURCE TEXT

  1          1          NAM='ORDER ASYMPTOTIC SOLUTION';
  1          1          NAMES(1)='GECKELER APPROXIMATION';
  1          1          NAMES(2)='BAUERSFELD-'|| NAMES(1);
  1          1          NAMES(3)='1ST' || NAM;
  1          1          NAMES(4)='2ND' || NAM;
  1          1          NAMES(5)='FLUGGE APPROXIMATION';
  1          1          NAMES(8)='CONICAL SHELL:';
  1          1          NAMES(9)='CYLINDRICAL SHELL';
  1          1          DO L=1 TO 32;
  1     1    1             GET LIST(BER(L),BEI(L),BERI(L),BEII(L));
  1     1    1             END;
  1          1             GET LIST(LAMDA);
  1          1          LOOP='1'B;
  1          1             GET LIST(K0,E0,H0,S,T,P,A4);
  1          1             GET LIST(NN,HH,II,JJ,RR,KKK,LL,MM);
  1          1             GET LIST(E#,F#);
  1          1                IF E#=5 THEN
  1          1                GET LIST(A,F,D,G,M,  HHH,PF,WW,XX,YY,UU,HK);
  1          1                IF MM=0E0 THEN
  1          1                NAMES(6)='NON-MONOLITHIC JOINT';
  1          1                ELSE
                                NAMES(6)='MONOLITHIC JOINT';
  1          1                IF E#=5 THEN
  1          1                NAMES(7)='UNIFORM SOIL BEARING PRESSURE FOUNDATION';
  1          1                ELSE
                                IF K0=500E0 THEN
  1          1                NAMES(7)='WELL-GRADED GRAVEL ELASTIC FOUNDATION,';
  1          1                ELSE
                                NAMES(7)='POORLY-GRADED GRAVEL ELASTIC FOUNDATION';
  1          1          SPRIME=S;
  1          1          DO WHILE(LOOP);
  1     1    1          S=SPRIME;
  1     1    1             E0=E0;
  1     1    1             K0=K0;
  1     1    1             H0=H0;
  1     1    1             U = (12E0 * TAN(T)**2E0 / H0**2E0)**0.25;
  1     1    1             PUT PAGE LIST(NAMES(6));
  1     1    1             PUT LIST (NAMES(7));
  1     1    1             PUT SKIP(2) LIST(NAMES(8));
  1     1    1             PUT LIST(NAMES(E#));
  1     1    1             IF E#=5 THEN DO;
  1     2    1             EDHCON=WW;
  1     2    1             EXHCON=XX;
  1     2    1             EDMCON=YY;
  1     2    1             EXMCON=UU;
  1     2    1             EDPPCON=0E0;
  1     2    1             EXPPCON=0E0;
  1     2    1             P=0E0;
  1     2    1             B=0E0;
  1     2    1             E=0E0;
  1     2    1             END;
  1     1    1             ELSE DO;
  1     2    1             K=(3E0 * K0/(E0 * H0**3E0))**0.25;
  1     2    1             KK=(0.25 * (7.5/S**4E0 + 12E0 * TAN(T)**2E0/(H0*H0 * S*S) +
                                12E0 * K0/(E0 * H0**3E0)))**0.25;
```

```
LEVEL NEST BLOCK MLVL  SOURCE TEXT

  1     2     1              KS=K * S;
  1     2     1              KKS=KK * S;
  1     2     1              EKS=EXP(KS);
  1     2     1              EKKS=EXP(KKS);
  1     2     1              EE=-A3(3)*A2(2)+A3(2)*A2(3);
  1     2     1              DD=A1(2)*EE;
  1     2     1              FF=A1(3)*(-EE);
  1     2     1              A=-A3(3)*SIN(T)/DD;
  1     2     1              B= A3(3)*COS(T)/DD;
  1     2     1              D= A2(3)*SIN(T)/DD;
  1     2     1              E=-A2(3)*COS(T)/DD;
  1     2     1              IF MM=0E0 THEN
  1     2     1              F=0E0;
  1     2     1              ELSE
                             F=-A3(2)/FF;
  1     2     1              IF MM=0E0 THEN
  1     2     1              G=0E0;
  1     2     1              ELSE
                             G=A2(2)/FF;
  1     2     1              EDHCON=E0*SIN(T)*A1(1)*(A*A2(1)+D*A3(1));
  1     2     1              EXHCON=E0*A1(4)*(A*A2(4)+D*A3(4));
  1     2     1              EXPPCON=E0*A1(4)*(B*A2(4)+E*A3(4))*P;
  1     2     1              EDPPCON=E0*SIN(T)*A1(1)*(B*A2(1)+E*A3(1))*P;
  1     2     1              EDMCON=E0*SIN(T)*A1(1)*(F*A2(1)+G*A3(1));
  1     2     1              EXMCON=E0*A1(4)*(F*A2(4)+G*A3(4));
  1     2     1              END;
  1     1     1              EDPCON=NN;
  1     1     1              EXPCON=RR;
  1     1     1              EDPCYL=HH;
  1     1     1              EXPCYL=KKK;
  1     1     1              EDHCYL=II;
  1     1     1              EXHCYL=LL;
  1     1     1              EDMCYL=JJ;
  1     1     1              EXMCYL=MM;
  1     1     1              PUT SKIP(2) EDIT ('C1=',A,'H + ',B,'P + ',F,'M')
                                         (X(5),A,E(10,3),A,E(10,3),A,E(10,3),A);
  1     1     1              PUT EDIT('C2=',D,'H + ',E,'P + ',G,'M')
                                         (X(5),A,E(10,3),A,E(10,3),A,E(10,3),A);
  1     1     1              PUT SKIP(2) EDIT('EDHCON=',EDHCON,'EDHCYL=',EDHCYL)
                                        (X(5),A,E(10,3),X(10),A,E(10,3));
  1     1     1              PUT SKIP(2) EDIT('EDPCON=',EDPCON,'EDPCYL=',EDPCYL)
                                        (X(5),A,E(10,3),X(10),A,E(10,3));
  1     1     1              PUT SKIP(2) EDIT('EDMCON=',EDMCON,'EDMCYL=',EDMCYL)
                                        (X(5),A,E(10,3),X(10),A,E(10,3));
  1     1     1              PUT SKIP(2) EDIT('EDPPCON=',EDPPCON)(X(5),A,E(10,3));
  1     1     1              PUT SKIP(2) EDIT('EXHCON=',EXHCON,'EXHCYL=',EXHCYL)
                                        (X(5),A,E(10,3),X(10),A,E(10,3));
  1     1     1              PUT SKIP(2) EDIT('EXPCON=',EXPCON,'EXPCYL=',EXPCYL)
                                        (X(5),A,E(10,3),X(10),A,E(10,3));
  1     1     1              PUT SKIP(2) EDIT('EXMCON=',EXMCON,'EXMCYL=',EXMCYL)
                                        (X(5),A,E(10,3),X(10),A,E(10,3));
  1     1     1              PUT SKIP(2) EDIT('EXPPCON=',EXPPCON)(X(5),A,E(10,3));
  1     1     1              IF E#=5 THEN DO;
  1     2     1              IF MM=0E0 THEN
  1     2     1              M=0E0;
```

```
LEVEL NEST BLOCK MLVL  SOURCE TEXT

  1     2     1          H=HHH;
  1     2     1          END;
  1     1     1          ELSE DO;
  1     2     1          IF MM=0E0 THEN
  1     2     1          M=0E0;
  1     2     1          ELSE
                         M=((EXPCON + EXPPCON - EXPCYL) + (EXHCON - EXHCYL) *
                               (EDPCON + EDPPCON - EDPCYL) / (EDHCYL - EDHCON)) /
                            ((EXMCYL - EXMCON) - (EDMCON - EDMCYL) / (EDHCYL - EDHCON));
  1     2     1          H=((EDPCON + EDPPCON - EDPCYL) + ( EDMCON - EDMCYL) * M)
                            /(EDHCYL - EDHCON);
  1     2     1          END;
  1     1     1          PUT SKIP(2) EDIT('M=',M,'H=',H,'P=',P)
                                   (X(5),A,E(10,3),X(10),A,E(10,3),X(10),A,E(10,3));
  1     1     1          PUT SKIP(2) LIST('    S IN INCHES');
  1     1     1          PUT SKIP LIST('    BENDING MOMENTS IN INCH-POUND/INCH');
  1     1     1          PUT SKIP(2) EDIT('S','MHCON','MPCON','MMCON','MCON')
                                   (X(9),A,X(17),A,X(15),A,X(15),A,X(15),A);
  1     1     1          PUT SKIP LIST(' ');
  1     1     1          IF E# = 1 THEN
  1     1     1          I=-12;
  1     1     1          ELSE
                         I=0;
  1     1     1          L=1;
  1     1     1          IF E#=5 THEN DO;
  1     2     1          K=0E0;
  1     2     1          KK=0E0;
  1     2     1          END;
  1     1     1          DO III=372 TO I BY -12;
  1     2     1             Z=A1(3);
  1     2     1             MHCON=Z*(A*A2(3)+D*A3(3))*H;
  1     2     1             MPCON=Z*(B*A2(3)+E*A3(3))*P;
  1     2     1             MMCON=Z*(F*A2(3)+G*A3(3))*M;
  1     2     1             MCON=MHCON + MPCON + MMCON;
  1     2     1             PUT SKIP EDIT(S,MHCON,MPCON,MMCON,MCON)
                          (X(5),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3));
  1     2     1             S=III;
  1     2     1             L=L+1;
  1     2     1             KS=K * S;
  1     2     1             KKS=KK * S;
  1     2     1             EKS = EXP(KS);
  1     2     1             EKKS = EXP(KKS);
  1     2     1             END;
  1     1     1          AA=-0.3021/E0;
  1     1     1          BB=0.0002562/E0;
  1     1     1          PUT SKIP(2) LIST('    S IN INCHES');
  1     1     1          PUT SKIP    LIST('TRANSVERSE SHEAR FORCE IN POUND/INCH');
  1     1     1          PUT SKIP(2) EDIT('S','QHCON','QPCON','QMCON','QCON')
                                   (X(9),A,X(17),A,X(15),A,X(15),A,X(15),A);
  1     1     1          PUT SKIP LIST(' ');
  1     1     1          IF E# = 1 THEN
  1     1     1          I=-12;
  1     1     1          ELSE
                         I=0;
  1     1     1          S=SPRIME;
```

```
LEVEL NEST BLOCK MLVL  SOURCE TEXT

  1    1    1             KS=K * S;
  1    1    1             KKS=KK*S;
  1    1    1             EKS = EXP(KS);
  1    1    1             EKKS = EXP(KKS);  .
  1    1    1          L=1;
  1    1    1          DO III=372 TO I BY -12;
  1    2    1             Z=A1(2);
  1    2    1             QHCON=Z*(A*A2(2)+D*A3(2))*H;
  1    2    1             QPCON=Z*(B*A2(2)+E*A3(2))*P;
  1    2    1             QMCON=Z*(F*A2(2)+G*A3(2))*M;
  1    2    1             QCON=QHCON + QPCON + QMCON;
  1    2    1             PUT SKIP EDIT(S,QHCON,QPCON,QMCON,QCON)
                          (X(5),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3));
  1    2    1             S=III;
  1    2    1             L=L+1;
  1    2    1             KS=K * S;
  1    2    1             KKS=KK * S;
  1    2    1             EKS = EXP(KS);
  1    2    1             EKKS = EXP(KKS);
  1    2    1             END;
  1    1    1          PUT SKIP(2) LIST('    S IN INCHES');
  1    1    1          PUT SKIP LIST('W PREFIX FOR NORMAL      DISPLACEMENTS IN INCH/INCH');
  1    1    1          PUT SKIP LIST('D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH');
  1    1    1          PUT SKIP LIST('PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE');
  1    1    1          PUT SKIP(2) EDIT('S','WHCON','WPCON','WMCON','WPPCON','WCON','PF','DCON')
                                 (X(9),A,X(12),A,X(10),A,X(10),A,X(10),A,X(10),A,X(10),A,
                                 X(10),A);
  1    1    1          PUT SKIP LIST(' ');
  1    1    1          IF E# = 1 THEN
  1    1    1          I=-12;
  1    1    1          ELSE
                       I=0;
  1    1    1          S=SPRIME;
  1    1    1             KS=K * S;
  1    1    1             KKS=KK*S;
  1    1    1             EKS = EXP(KS);
  1    1    1             EKKS = EXP(KKS);
  1    1    1          L=1;
  1    1    1          DO III=372 TO I BY -12;
  1    2    1             PF=1.391;
  1    2    1             Z=A1(1);
  1    2    1             WHCON=Z*(A*A2(1)+D*A3(1))*H;
  1    2    1             WPCON=Z*(B*A2(1)+E*A3(1))*P;
  1    2    1             WMCON=Z*(F*A2(1)+G*A3(1))*M;
  1    2    1             IF E#=5 THEN
  1    2    1             WPPCON=-PF*COS(T)*S**2E0/(HO*SIN(T)*EO*TAN(T));
  1    2    1             ELSE DO;
  1    3    1             IF E#=2 THEN K=KK;
  1    3    1             ELSE K=K;
  1    3    1             IF S=0E0 THEN WPPCON=0E0;
  1    3    1             ELSE
                          WPPCON=1E0/(4E0*K**4E0)*(BB*S+AA-A4*BB/S-A4*AA/S**2E0+A4*BB/S**3E0+
                          A4*A4*AA/S**4E0);
  1    3    1             END;
  1    2    1             WCON=WHCON + WPCON + WMCON+WPPCON;
```

```
LEVEL NEST BLOCK MLVL   SOURCE TEXT

  1    2    1              DCON=WCON*SIN(T);
  1    2    1              PF=1.391;
  1    2    1              IF E#=5 THEN PF=PF+0.03617*(354E0-SIN(T)*S);
  1    2    1              ELSE
                           PF=-WCON*K0;
  1    2    1              PUT SKIP EDIT(S,WHCON,WPCON,WMCON,WPPCON,WCON,PF,DCON)
                           (X(5),E(10,3),X(5 ),E(10,3),X(5 ),E(10,3),X(5 ),E(10,3),X(5 ),E(10,3),
                           X(5 ),E(10,3),X(5 ),E(10,3),X(5),E(10,3));
  1    2    1              S=III;
  1    2    1              L=L+1;
  1    2    1              KS=K * S;
  1    2    1              KKS=KK * S;
  1    2    1              EKS = EXP(KS);
  1    2    1              EKKS = EXP(KKS);
  1    2    1              END;
  1    1    1          PUT SKIP(2) LIST('   S IN INCHES');
  1    1    1          PUT SKIP LIST('ROTATIONS IN RADIAN/INCH');
  1    1    1          PUT SKIP(2) EDIT('S','XHCON','XPCON','XMCON','XPPCON','XCON')
                                    (X(9),A,X(17),A,X(15),A,X(15),A,X(15),A,X(15),A);
  1    1    1          PUT SKIP LIST(' ');
  1    1    1          IF E# = 1 THEN
  1    1    1          I=-12;
  1    1    1          ELSE
                       I=0;
  1    1    1          S=SPRIME;
  1    1    1              KS=K * S;
  1    1    1              KKS=KK*S;
  1    1    1              EKS = EXP(KS);
  1    1    1              EKKS = EXP(KKS);
  1    1    1              L=1;
  1    1    1          DO III=372 TO I BY -12;
  1    2    1              Z=A1(4);
  1    2    1              XHCON=Z*(A*A2(4)+D*A3(4))*H;
  1    2    1              XPCON=Z*(B*A2(4)+E*A3(4))*P;
  1    2    1              XMCON=Z*(F*A2(4)+G*A3(4))*M;
  1    2    1              PF=1.391;
  1    2    1              IF E#=5 THEN
  1    2    1              XPPCON=1.5*PF*S/(E0*TAN(T)**2E0*H0);
  1    2    1              ELSE DO;
  1    3    1              IF E#=2 THEN K=KK;
  1    3    1              ELSE K=K;
  1    3    1              IF S=0E0 THEN XPPCON=0E0;
  1    3    1              ELSE
                           XPPCON=1E0/(4E0*K**4E0)*(BB+A4*BB/S**2E0+2E0*A4*AA/S**3E0+
                           3E0*A4*BB/S**4E0-4E0*A4*A4*AA /S**5E0);
  1    3    1              END;
  1    2    1              XCON=XHCON + XPCON + XMCON+XPPCON;
  1    2    1              PUT SKIP EDIT(S,XHCON,XPCON,XMCON,XPPCON,XCON)
                           (X(5),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3),
                           X(10),E(10,3) );
  1    2    1              S=III;
  1    2    1              L=L+1;
  1    2    1              KS=K * S;
  1    2    1              KKS=KK * S;
  1    2    1              EKS = EXP(KS);
```

```
LEVEL NEST BLOCK MLVL   SOURCE TEXT

  1     2     1              EKKS = EXP(KKS);
  1     2     1              END;
  1     1     1           IF E#=5 THEN CALL Z1;

  1     1     1           PUT SKIP(10)LIST('        '||NAMES(9));
  1     1     1           PUT SKIP(2) EDIT('LAMDA=',LAMDA)(X(5),A,E(10,3));
  1     1     1               IF E#=5 THEN H=HK;
  1     1     1           PUT EDIT('H=',H )(X(5),A,E(10,3));
  1     1     1           PUT SKIP(2) LIST('     S IN INCHES');
  1     1     1           PUT SKIP LIST('    BENDING MOMENTS IN INCH-POUND/INCH');
  1     1     1           PUT SKIP(2) EDIT('S','MMCYL','MHCYL','MCYL')
                                      (X(9),A,X(17),A,X(15),A,X(15),A);
  1     1     1           PUT SKIP LIST(' ');
  1     1     1           DO III=0 TO 240 BY 12;
  1     2     1              S=III;
  1     2     1              LS=LAMDA*S;
  1     2     1              MMCYL=EXP(-LS)*(SIN(LS)+COS(LS))*M;
  1     2     1              MHCYL=EXP(-LS)*SIN(LS)*H/LAMDA;
  1     2     1              MCYL=MMCYL+MHCYL;
  1     2     1              PUT SKIP EDIT(S,MMCYL,MHCYL,MCYL)
                             (X(5),E(10,3),X(10),E(10,3),X(10),E(10,3),X(10),E(10,3));
  1     2     1              END;
  1     1     1           PUT SKIP(2) EDIT('K0=',K0,'E0',E0,'H0=',H0,'S=',S,'T=',T,'P=',P,
                                          'ALPHA4=',A4,'K=',K,'KK=',KK)
                                       (X(5),A,E(10,3),X(10),A,E(10,3),
                                       X(10),A,E(10,3),X(10),A,E(10,3),X(10),A,E(10,3),
                                       X(10),A,E(10,3),X(10),A,E(10,3),
                                       X(10),A,E(10,3),X(10),A,E(10,3));
  1     1     1           GET LIST(E#,F#);
  1     1     1           END;
  1           1           END XNAME;

WARNINGS DETECTED DURING CODE GENERATION:

ARNING: NO FILE SPECIFIED. SYSIN/SYSPRINT ASSUMED. (CG0C)
```

MONOLITHIC JOINT WELL-GRADED GRAVEL ELASTIC FOUNDATION,

CONICAL SHELL: GECKELER APPROXIMATION

C1= 1.387E-10H + -5.952E-10P + -3.815E-11M C2=-1.622E-10H + 5.116E-10P + -4.179E-12M

EDHCON= 3.180E+01 EDHCYL=-4.320E+02

EDPCON=-1.538E+04 EDPCYL=-9.375E+04

EDMCON=-3.350E+00 EDMCYL=-8.650E+00

EDPPCON=-2.508E+04

EXHCON= 3.308E+00 EXHCYL= 8.650E+00

EXPCON= 5.485E+01 EXPCYL= 3.906E+02

EXMCON=-7.212E-01 EXMCYL= 3.460E-01

EXPPCON=-2.609E+03

M=-2.160E+03 H=-9.022E+01 P= 2.500E+02

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MHCON	MPCON	MMCON	MCON
3.772E+02	0.000E+00	0.000E+00	-2.160E+03	-2.160E+03
3.720E+02	1.090E+02	9.600E+02	-2.090E+03	-1.021E+03
3.600E+02	2.390E+02	2.095E+03	-1.685E+03	6.490E+02
3.480E+02	2.540E+02	2.220E+03	-1.166E+03	1.308E+03
3.360E+02	2.110E+02	1.845E+03	-6.940E+02	1.362E+03
3.240E+02	1.490E+02	1.309E+03	-3.370E+02	1.121E+03
3.120E+02	9.100E+01	8.010E+02	-1.030E+02	7.890E+02
3.000E+02	4.600E+01	4.060E+02	2.600E+01	4.780E+02
2.880E+02	1.600E+01	1.410E+02	8.200E+01	2.390E+02
2.760E+02	-1.000E+00	-1.100E+01	9.300E+01	8.100E+01
2.640E+02	-9.000E+00	-8.000E+01	7.900E+01	-1.000E+01
2.520E+02	-1.100E+01	-9.700E+01	5.700E+01	-5.100E+01
2.400E+02	-9.000E+00	-8.600E+01	3.600E+01	-5.900E+01
2.280E+02	-7.000E+00	-6.400E+01	1.900E+01	-5.200E+01
2.160E+02	-4.000E+00	-4.100E+01	7.000E+00	-3.800E+01
2.040E+02	-2.000E+00	-2.200E+01	0.000E+00	-2.400E+01
1.920E+02	-1.000E+00	-9.000E+00	-3.000E+00	-1.300E+01
1.800E+02	0.000E+00	-1.000E+00	-4.000E+00	-5.000E+00
1.680E+02	0.000E+00	2.000E+00	-3.000E+00	-1.000E+00
1.560E+02	0.000E+00	4.000E+00	-2.000E+00	2.000E+00
1.440E+02	0.000E+00	3.000E+00	-1.000E+00	2.000E+00
1.320E+02	0.000E+00	3.000E+00	-1.000E+00	2.000E+00
1.200E+02	0.000E+00	2.000E+00	0.000E+00	2.000E+00
1.080E+02	0.000E+00	1.000E+00	0.000E+00	1.000E+00
9.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6.000E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4.800E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

1.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

S IN INCHES
TRANSVERSE SHEAR FORCE IN POUND/INCH

S	QHCON	QPCON	QMCON	QCON
3.772E+02	-2.726E+01	-2.383E+02	0.000E+00	-2.656E+02
3.720E+02	-1.949E+01	-1.704E+02	-1.709E+01	-2.069E+02
3.600E+02	-6.253E+00	-5.465E+01	-3.722E+01	-9.812E+01
3.480E+02	1.200E+00	1.049E+01	-3.938E+01	-2.769E+01
3.360E+02	4.480E+00	3.916E+01	-3.270E+01	1.094E+01
3.240E+02	5.159E+00	4.509E+01	-2.319E+01	2.706E+01
3.120E+02	4.480E+00	3.916E+01	-1.421E+01	2.943E+01
3.000E+02	3.292E+00	2.878E+01	-7.231E+00	2.484E+01
2.880E+02	2.095E+00	1.831E+01	-2.556E+00	1.735E+01
2.760E+02	1.125E+00	9.836E+00	1.284E-01	1.109E+01
2.640E+02	4.502E-01	3.935E+00	1.357E+00	5.742E+00
2.520E+02	4.451E-02	3.891E-01	1.665E+00	2.099E+00
2.400E+02	-1.557E-01	-1.361E+00	1.484E+00	-3.282E-02
2.280E+02	-2.203E-01	-1.926E+00	1.109E+00	-1.037E+00
2.160E+02	-2.086E-01	-1.823E+00	7.177E-01	-1.314E+00
2.040E+02	-1.630E-01	-1.425E+00	3.954E-01	-1.192E+00
1.920E+02	-1.102E-01	-9.633E-01	1.684E-01	-9.051E-01
1.800E+02	-6.415E-02	-5.607E-01	3.026E-02	-5.946E-01
1.680E+02	-3.009E-02	-2.630E-01	-3.954E-02	-3.326E-01
1.560E+02	-8.210E-03	-7.177E-02	-6.396E-02	-1.439E-01
1.440E+02	3.744E-03	3.273E-02	-6.262E-02	-2.615E-02
1.320E+02	8.736E-03	7.636E-02	-4.969E-02	3.540E-02
1.200E+02	9.490E-03	8.295E-02	-3.402E-02	5.842E-02
1.080E+02	8.099E-03	7.080E-02	-2.023E-02	5.866E-02
9.600E+01	5.955E-03	5.206E-02	-1.007E-02	4.794E-02
8.400E+01	3.855E-03	3.370E-02	-3.618E-03	3.393E-02
7.200E+01	2.163E-03	1.891E-02	-1.501E-04	2.092E-02
6.000E+01	9.718E-04	8.494E-03	1.265E-03	1.073E-02
4.800E+01	2.272E-04	1.986E-03	1.472E-03	3.684E-03
3.600E+01	-1.850E-04	-1.617E-03	1.102E-03	-7.008E-04
2.400E+01	-3.954E-04	-3.456E-03	5.508E-04	-3.301E-03
1.200E+01	-5.769E-04	-5.043E-03	-9.403E-06	-5.629E-03
0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

S IN INCHES
W PREFIX FOR NORMAL DISPLACEMENTS IN INCH/INCH
D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH
PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE

S	WHCON	WPCON	WMCON	WPPCON	WCON	PF	DCON
3.772E+02	-3.164E-03	-2.766E-02	7.983E-03	-1.697E-02	-3.981E-02	1.991E+01	-1.203E-02
3.720E+02	-2.655E-03	-2.321E-02	5.525E-03	-1.706E-02	-3.740E-02	1.870E+01	-1.130E-02
3.600E+02	-1.583E-03	-1.384E-02	1.436E-03	-1.728E-02	-3.126E-02	1.563E+01	-9.448E-03
3.480E+02	-7.699E-04	-6.729E-03	-7.677E-04	-1.749E-02	-2.576E-02	1.288E+01	-7.783E-03
3.360E+02	-2.372E-04	-2.073E-03	-1.854E-03	-1.769E-02	-2.166E-02	1.083E+01	-6.545E-03
3.240E+02	5.960E-05	5.210E-04	-1.748E-03	-1.789E-02	-1.906E-02	9.529E+00	-5.760E-03
3.120E+02	1.875E-04	1.639E-03	-1.451E-03	-1.808E-02	-1.771E-02	8.853E+00	-5.351E-03
3.000E+02	2.111E-04	1.845E-03	-1.026E-03	-1.826E-02	-1.724E-02	8.618E+00	-5.209E-03
2.880E+02	1.810E-04	1.582E-03	-6.286E-04	-1.844E-02	-1.730E-02	8.651E+00	-5.229E-03
2.760E+02	1.315E-04	1.149E-03	-3.180E-04	-1.860E-02	-1.764E-02	8.818E+00	-5.330E-03
2.640E+02	8.255E-05	7.218E-04	-1.099E-04	-1.875E-02	-1.805E-02	9.026E+00	-5.455E-03

2.520E+02	4.343E-05	3.796E-04	9.426E-06	-1.888E-02	-1.845E-02	9.223E+00	-5.575E-03
2.400E+02	1.655E-05	1.447E-04	6.370E-05	-1.899E-02	-1.877E-02	9.384E+00	-5.672E-03
2.280E+02	6.815E-07	5.937E-06	7.679E-05	-1.909E-02	-1.900E-02	9.502E+00	-5.743E-03
2.160E+02	-6.914E-06	-6.044E-05	6.796E-05	-1.916E-02	-1.916E-02	9.578E+00	-5.789E-03
2.040E+02	-9.137E-06	-7.966E-05	5.054E-05	-1.919E-02	-1.923E-02	9.617E+00	-5.812E-03
1.920E+02	-8.398E-06	-7.341E-05	3.249E-05	-1.920E-02	-1.925E-02	9.624E+00	-5.817E-03
1.800E+02	-6.412E-06	-5.605E-05	1.765E-05	-1.916E-02	-1.921E-02	9.604E+00	-5.805E-03
1.680E+02	-4.226E-06	-3.694E-05	7.234E-06	-1.908E-02	-1.912E-02	9.558E+00	-5.777E-03
1.560E+02	-2.374E-06	-2.075E-05	9.214E-07	-1.895E-02	-1.897E-02	9.485E+00	-5.733E-03
1.440E+02	-1.039E-06	-9.083E-06	-2.227E-06	-1.876E-02	-1.877E-02	9.387E+00	-5.674E-03
1.320E+02	-2.087E-07	-1.824E-06	-3.273E-06	-1.853E-02	-1.854E-02	9.268E+00	-5.602E-03
1.200E+02	2.226E-07	1.945E-06	-3.126E-06	-1.829E-02	-1.829E-02	9.146E+00	-5.528E-03
1.080E+02	3.819E-07	3.333E-06	-2.447E-06	-1.814E-02	-1.814E-02	9.071E+00	-5.482E-03
9.600E+01	3.823E-07	3.342E-06	-1.650E-06	-1.835E-02	-1.835E-02	9.173E+00	-5.544E-03
8.400E+01	3.079E-07	2.691E-06	-9.530E-07	-1.962E-02	-1.962E-02	9.810E+00	-5.929E-03
7.200E+01	2.128E-07	1.860E-06	-4.393E-07	-2.400E-02	-2.400E-02	1.200E+01	-7.252E-03
6.000E+01	1.265E-07	1.106E-06	-1.120E-07	-3.796E-02	-3.796E-02	1.898E+01	-1.147E-02
4.800E+01	6.128E-08	5.356E-07	6.379E-08	-8.593E-02	-8.592E-02	4.296E+01	-2.597E-02
3.600E+01	1.863E-08	1.628E-07	1.340E-07	-2.890E-01	-2.890E-01	1.445E+02	-8.734E-02
2.400E+01	-5.065E-09	-4.427E-08	1.409E-07	-1.621E+00	-1.621E+00	8.106E+02	-4.899E-01
1.200E+01	-1.522E-08	-1.330E-07	1.166E-07	-2.809E+01	-2.809E+01	1.404E+04	-8.489E+00
0.000E+00	-1.702E-08	-1.433E-07	8.243E-08	0.000E+00	-8.338E-08	4.169E-05	-2.520E-08

S IN INCHES
ROTATIONS IN RADIAN/INCH

S	XHCON	XPCON	XMCON	XPPCON	XCON
3.772E+02	-9.948E-05	-8.695E-04	5.194E-04	1.828E-05	-4.313E-04
3.720E+02	-9.713E-05	-8.490E-04	4.334E-04	1.813E-05	-4.946E-04
3.600E+02	-7.956E-05	-6.954E-04	2.546E-04	1.774E-05	-5.026E-04
3.480E+02	-5.577E-05	-4.874E-04	1.208E-04	1.730E-05	-4.051E-04
3.360E+02	-3.369E-05	-2.945E-04	3.431E-05	1.680E-05	-2.771E-04
3.240E+02	-1.674E-05	-1.463E-04	-1.306E-05	1.623E-05	-1.599E-04
3.120E+02	-5.496E-06	-4.804E-05	-3.279E-05	1.557E-05	-7.076E-05
3.000E+02	6.626E-07	7.539E-06	-3.567E-05	1.481E-05	-1.245E-05
2.880E+02	3.635E-06	3.221E-05	-3.006E-05	1.393E-05	1.977E-05
2.760E+02	4.293E-06	3.752E-05	-2.155E-05	1.292E-05	3.318E-05
2.640E+02	3.741E-06	3.270E-05	-1.335E-05	1.173E-05	3.482E-05
2.520E+02	2.751E-06	2.405E-05	-6.884E-06	1.034E-05	3.026E-05
2.400E+02	1.749E-06	1.528E-05	-2.530E-06	8.717E-06	2.325E-05
2.280E+02	9.360E-07	8.131E-06	4.926E-08	6.804E-06	1.597E-05
2.160E+02	3.711E-07	3.244E-06	1.239E-06	4.549E-06	9.403E-06
2.040E+02	3.305E-08	2.889E-07	1.556E-06	1.895E-06	3.773E-06
1.920E+02	-1.324E-07	-1.157E-06	1.402E-06	-1.217E-06	-1.105E-06
1.800E+02	-1.843E-07	-1.611E-06	1.056E-06	-4.827E-06	-5.567E-06
1.680E+02	-1.723E-07	-1.510E-06	6.867E-07	-8.921E-06	-9.918E-06
1.560E+02	-1.336E-07	-1.168E-06	3.792E-07	-1.334E-05	-1.426E-05
1.440E+02	-8.914E-08	-7.792E-07	1.606E-07	-1.759E-05	-1.830E-05
1.320E+02	-5.082E-08	-4.442E-07	2.646E-08	-2.035E-05	-2.082E-05
1.200E+02	-2.288E-08	-2.000E-07	-4.179E-08	-1.832E-05	-1.859E-05
1.080E+02	-5.272E-09	-4.608E-08	-6.573E-08	-3.248E-06	-3.365E-06
9.600E+01	4.036E-09	3.528E-08	-6.416E-08	4.642E-05	4.640E-05
8.400E+01	7.625E-09	6.665E-08	-5.090E-08	1.908E-04	1.908E-04
7.200E+01	7.625E-09	6.840E-08	-3.473E-08	6.156E-04	6.157E-04
6.000E+01	6.391E-09	5.587E-08	-2.034E-08	1.985E-03	1.985E-03
4.800E+01	4.470E-09	3.907E-08	-9.610E-09	7.233E-03	7.233E-03
3.600E+01	2.694E-09	2.355E-08	-2.686E-09	3.444E-02	3.444E-02
2.400E+01	1.333E-09	1.165E-08	1.094E-09	2.835E-01	2.835E-01
1.200E+01	4.326E-10	3.782E-09	2.658E-09	9.496E+00	9.496E+00

0.000E+00	-7.514E-11	-6.568E-10	2.875E-09	0.000E+00	2.143E-09

CYLINDRICAL SHELL

LAMDA= 2.002E-02 H=-9.022E+01

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	-2.160E+03	0.000E+00	-2.160E+03
1.200E+01	-2.055E+03	-8.432E+02	-2.898E+03
2.400E+01	-1.803E+03	-1.288E+03	-3.091E+03
3.600E+01	-1.483E+03	-1.446E+03	-2.930E+03
4.800E+01	-1.151E+03	-1.413E+03	-2.564E+03
6.000E+01	-8.408E+02	-1.264E+03	-2.105E+03
7.200E+01	-5.728E+02	-1.037E+03	-1.630E+03
8.400E+01	-3.550E+02	-8.333E+02	-1.188E+03
9.600E+01	-1.881E+02	-6.192E+02	-8.073E+02
1.080E+02	-6.780E+01	-4.305E+02	-4.983E+02
1.200E+02	1.277E+01	-2.747E+02	-2.620E+02
1.320E+02	6.144E+01	-1.535E+02	-9.203E+01
1.440E+02	8.597E+01	-6.453E+01	2.144E+01
1.560E+02	9.333E+01	-3.664E+00	8.967E+01
1.680E+02	8.941E+01	3.431E+01	1.237E+02
1.800E+02	7.887E+01	5.469E+01	1.336E+02
1.920E+02	6.519E+01	6.232E+01	1.275E+02
2.040E+02	5.081E+01	6.139E+01	1.122E+02
2.160E+02	3.731E+01	5.524E+01	9.255E+01
2.280E+02	2.557E+01	4.642E+01	7.199E+01
2.400E+02	1.599E+01	3.675E+01	5.274E+01

KO= 5.000E+02 EO 3.000E+06 HO= 8.000E+00 S= 2.400E+02 T= 3.070E-01 P= 2.500E+02
ALPHA4= 4.323E+03 K= 3.144E-02 KK= 3.170E-02

MONOLITHIC JOINT   WELL-GRADED GRAVEL ELASTIC FOUNDATION.

CONICAL SHELL: BAUERSFELD-GECKELER APPROXIMATION

C1= 4.072E-09H + -1.284E-08P + -7.191E-10M   C2=-2.608E-09H + 8.225E-09P + -1.398E-10M

EDHCON= 3.512E+01   EDHCYL=-4.320E+02

EDPCON=-1.533E+04   EDPCYL=-9.375E+04

EDMCON=-3.701E+00   EDMCYL=-8.650E+00

EDPPCON=-2.770E+04

EXHCON= 3.081E+00   EXHCYL= 8.650E+00

EXPCON= 5.485E+01   EXPCYL= 3.906E+02

EXMCON=-7.737E-01   EXMCYL= 3.460E-01

EXPPCON=-2.903E+03

M=-2.388E+03   H=-8.318E+01   P= 2.500E+02

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MHCON	MPCON	MMCON	MCON
3.772E+02	0.000E+00	0.000E+00	-2.388E+03	-2.388E+03
3.720E+02	1.100E+02	1.046E+03	-2.360E+03	-1.204E+03
3.600E+02	2.430E+02	2.307E+03	-1.979E+03	5.710E+02
3.480E+02	2.600E+02	2.472E+03	-1.415E+03	1.317E+03
3.360E+02	2.190E+02	2.077E+03	-8.720E+02	1.424E+03
3.240E+02	1.570E+02	1.489E+03	-4.430E+02	1.203E+03
3.120E+02	9.700E+01	9.210E+02	-1.510E+02	8.670E+02
3.000E+02	4.900E+01	4.700E+02	1.800E+01	5.370E+02
2.880E+02	1.700E+01	1.620E+02	9.700E+01	2.760E+02
2.760E+02	-1.000E+00	-1.700E+01	1.170E+02	9.900E+01
2.640E+02	-1.000E+01	-1.010E+02	1.040E+02	-7.000E+00
2.520E+02	-1.300E+01	-1.230E+02	7.800E+01	-5.800E+01
2.400E+02	-1.100E+01	-1.100E+02	5.000E+01	-7.100E+01
2.280E+02	-8.000E+00	-8.300E+01	2.700E+01	-6.400E+01
2.160E+02	-5.000E+00	-5.400E+01	1.100E+01	-4.800E+01
2.040E+02	-3.000E+00	-3.000E+01	1.000E+00	-3.200E+01
1.920E+02	-1.000E+00	-1.200E+01	-4.000E+00	-1.700E+01
1.800E+02	0.000E+00	-1.000E+00	-5.000E+00	-6.000E+00
1.680E+02	0.000E+00	4.000E+00	-5.000E+00	-1.000E+00
1.560E+02	0.000E+00	6.000E+00	-4.000E+00	2.000E+00
1.440E+02	0.000E+00	6.000E+00	-3.000E+00	3.000E+00
1.320E+02	0.000E+00	4.000E+00	-1.000E+00	3.000E+00
1.200E+02	0.000E+00	3.000E+00	0.000E+00	3.000E+00
1.080E+02	0.000E+00	2.000E+00	0.000E+00	2.000E+00
9.600E+01	0.000E+00	1.000E+00	0.000E+00	1.000E+00
8.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6.000E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4.800E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

1.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

S IN INCHES
TRANSVERSE SHEAR FORCE IN POUND/INCH

S	QHCON	QPCON	QMCON	QCON
3.772E+02	-2.514E+01	-2.383E+02	0.000E+00	-2.634E+02
3.720E+02	-1.753E+01	-1.662E+02	-2.203E+01	-2.058E+02
3.600E+02	-4.681E+00	-4.438E+01	-4.862E+01	-9.768E+01
3.480E+02	2.418E+00	2.293E+01	-5.212E+01	-2.677E+01
3.360E+02	5.383E+00	5.103E+01	-4.380E+01	1.261E+01
3.240E+02	5.735E+00	5.434E+01	-3.139E+01	2.923E+01
3.120E+02	4.872E+00	4.619E+01	-1.938E+01	3.169E+01
3.000E+02	3.499E+00	3.317E+01	-9.851E+00	2.682E+01
2.880E+02	2.164E+00	2.052E+01	-3.347E+00	1.934E+01
2.760E+02	1.103E+00	1.046E+01	4.522E-01	1.202E+01
2.640E+02	3.769E-01	3.574E+00	2.216E+00	6.166E+00
2.520E+02	-4.895E-02	-4.641E-01	2.655E+00	2.142E+00
2.400E+02	-2.478E-01	-2.349E+00	2.367E+00	-2.303E-01
2.280E+02	-2.983E-01	-2.828E+00	1.778E+00	-1.348E+00
2.160E+02	-2.668E-01	-2.530E+00	1.154E+00	-1.643E+00
2.040E+02	-2.011E-01	-1.907E+00	6.293E-01	-1.479E+00
1.920E+02	-1.310E-01	-1.242E+00	2.526E-01	-1.120E+00
1.800E+02	-7.178E-02	-6.806E-01	1.883E-02	-7.335E-01
1.680E+02	-2.899E-02	-2.749E-01	-1.008E-01	-4.047E-01
1.560E+02	-2.268E-03	-2.150E-02	-1.418E-01	-1.656E-01
1.440E+02	1.155E-02	1.095E-01	-1.362E-01	-1.517E-02
1.320E+02	1.640E-02	1.555E-01	-1.084E-01	6.346E-02
1.200E+02	1.587E-02	1.505E-01	-7.454E-02	9.182E-02
1.080E+02	1.274E-02	1.207E-01	-4.373E-02	8.975E-02
9.600E+01	8.836E-03	8.377E-02	-1.997E-02	7.263E-02
8.400E+01	5.230E-03	4.958E-02	-3.071E-03	5.084E-02
7.200E+01	2.392E-03	2.268E-02	5.298E-03	3.037E-02
6.000E+01	4.182E-04	3.965E-03	9.562E-03	1.395E-02
4.800E+01	-8.115E-04	-7.694E-03	1.062E-02	2.119E-03
3.600E+01	-1.551E-03	-1.470E-02	1.020E-02	-6.053E-03
2.400E+01	-2.388E-03	-2.264E-02	1.106E-02	-1.396E-02
1.200E+01	-1.007E-02	-9.547E-02	4.535E-02	-6.019E-02

S IN INCHES
W PREFIX FOR NORMAL DISPLACEMENTS IN INCH/INCH
D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH
PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE

S	WHCON	WPCON	WMCON	WPPCON	WCON	PF	DCON
3.772E+02	-3.222E-03	-3.055E-02	9.749E-03	-1.641E-02	-4.043E-02	2.022E+01	-1.222E-02
3.720E+02	-2.700E-03	-2.560E-02	6.820E-03	-1.650E-02	-3.798E-02	1.899E+01	-1.148E-02
3.600E+02	-1.598E-03	-1.515E-02	1.854E-03	-1.671E-02	-3.160E-02	1.580E+01	-9.549E-03
3.480E+02	-7.583E-04	-7.189E-03	-9.005E-04	-1.691E-02	-2.576E-02	1.288E+01	-7.785E-03
3.360E+02	-2.069E-04	-1.962E-03	-2.060E-03	-1.711E-02	-2.134E-02	1.067E+01	-6.449E-03
3.240E+02	9.976E-05	9.438E-04	-2.227E-03	-1.730E-02	-1.848E-02	9.242E+00	-5.586E-03
3.120E+02	2.294E-04	2.175E-03	-1.882E-03	-1.749E-02	-1.697E-02	8.483E+00	-5.127E-03
3.000E+02	2.485E-04	2.356E-03	-1.355E-03	-1.766E-02	-1.641E-02	8.207E+00	-4.960E-03
2.880E+02	2.104E-04	1.905E-03	-8.402E-04	-1.783E-02	-1.647E-02	8.233E+00	-4.976E-03
2.760E+02	1.518E-04	1.439E-03	-4.294E-04	-1.799E-02	-1.683E-02	8.413E+00	-5.085E-03
2.640E+02	9.431E-05	8.942E-04	-1.472E-04	-1.813E-02	-1.729E-02	8.644E+00	-5.225E-03
2.520E+02	4.831E-05	4.581E-04	1.870E-05	-1.826E-02	-1.773E-02	8.867E+00	-5.359E-03
2.400E+02	1.662E-05	1.576E-04	9.647E-05	-1.837E-02	-1.810E-02	9.049E+00	-5.469E-03

2.280E+02	-2.033E-06	-1.975E-05	1.165E-04	-1.846E-02	-1.836E-02	9.182E+00	-5.550E-03
2.160E+02	-1.091E-05	-1.034E-04	1.044E-04	-1.853E-02	-1.854E-02	9.268E+00	-5.602E-03
2.040E+02	-1.322E-05	-1.233E-04	7.881E-05	-1.856E-02	-1.862E-02	9.312E+00	-5.628E-03
1.920E+02	-1.189E-05	-1.127E-04	5.139E-05	-1.857E-02	-1.864E-02	9.321E+00	-5.633E-03
1.800E+02	-9.012E-06	-8.544E-05	2.817E-05	-1.853E-02	-1.860E-02	9.300E+00	-5.621E-03
1.680E+02	-5.903E-06	-5.597E-05	1.134E-05	-1.845E-02	-1.850E-02	9.252E+00	-5.592E-03
1.560E+02	-3.253E-06	-3.034E-05	8.028E-07	-1.832E-02	-1.836E-02	9.179E+00	-5.548E-03
1.440E+02	-1.318E-06	-1.250E-05	-4.664E-06	-1.814E-02	-1.816E-02	9.082E+00	-5.489E-03
1.320E+02	-3.553E-08	-9.037E-07	-6.592E-06	-1.792E-02	-1.793E-02	8.965E+00	-5.418E-03
1.200E+02	5.470E-07	5.192E-06	-6.389E-06	-1.769E-02	-1.769E-02	8.845E+00	-5.346E-03
1.080E+02	7.822E-07	7.416E-06	-5.144E-06	-1.755E-02	-1.754E-02	8.771E+00	-5.301E-03
9.600E+01	7.670E-07	7.272E-06	-3.568E-06	-1.774E-02	-1.774E-02	8.870E+00	-5.361E-03
8.400E+01	6.255E-07	5.940E-06	-2.144E-06	-1.898E-02	-1.897E-02	9.486E+00	-5.733E-03
7.200E+01	4.452E-07	4.220E-06	-1.003E-06	-2.321E-02	-2.321E-02	1.160E+01	-7.013E-03
6.000E+01	2.726E-07	2.585E-06	-2.100E-07	-3.671E-02	-3.671E-02	1.835E+01	-1.109E-02
4.800E+01	1.322E-07	1.253E-06	2.762E-07	-8.310E-02	-8.310E-02	4.155E+01	-2.511E-02
3.600E+01	2.928E-08	2.776E-07	5.315E-07	-2.795E-01	-2.795E-01	1.397E+02	-8.447E-02
2.400E+01	-4.183E-08	-3.966E-07	6.439E-07	-1.568E+00	-1.568E+00	7.839E+02	-4.738E-01
1.200E+01	-9.879E-08	-9.366E-07	7.257E-07	-2.717E+01	-2.717E+01	1.358E+04	-8.210E+00

S IN INCHES
ROTATIONS IN RADIAN/INCH

S	XHCON	XPCON	XMCON	XPPCON	XCON
3.772E+02	-1.021E-04	-9.675E-04	6.159E-04	1.768E-05	-4.360E-04
3.720E+02	-9.970E-05	-9.453E-04	5.198E-04	1.753E-05	-5.076E-04
3.600E+02	-8.196E-05	-7.770E-04	3.138E-04	1.716E-05	-5.280E-04
3.480E+02	-5.767E-05	-5.467E-04	1.542E-04	1.673E-05	-4.335E-04
3.360E+02	-3.489E-05	-3.308E-04	4.760E-05	1.625E-05	-3.018E-04
3.240E+02	-1.720E-05	-1.630E-04	-1.305E-05	1.569E-05	-1.776E-04
3.120E+02	-5.347E-06	-5.070E-05	-3.991E-05	1.506E-05	-8.090E-05
3.000E+02	1.416E-06	1.342E-05	-4.531E-05	1.432E-05	-1.614E-05
2.880E+02	4.427E-06	4.198E-05	-3.929E-05	1.348E-05	2.059E-05
2.760E+02	5.049E-06	4.786E-05	-2.889E-05	1.249E-05	3.652E-05
2.640E+02	4.392E-06	4.164E-05	-1.834E-05	1.135E-05	3.904E-05
2.520E+02	3.240E-06	3.071E-05	-9.721E-06	1.000E-05	3.424E-05
2.400E+02	2.064E-06	1.957E-05	-3.673E-06	8.431E-06	2.639E-05
2.280E+02	1.099E-06	1.042E-05	-1.942E-08	6.580E-06	1.808E-05
2.160E+02	4.192E-07	3.974E-06	1.776E-06	4.399E-06	1.057E-05
2.040E+02	6.222E-09	5.899E-08	2.326E-06	1.833E-06	4.224E-06
1.920E+02	-1.985E-07	-1.882E-06	2.162E-06	-1.177E-06	-1.095E-06
1.800E+02	-2.628E-07	-2.492E-06	1.679E-06	-4.668E-06	-5.744E-06
1.680E+02	-2.460E-07	-2.333E-06	1.128E-06	-8.628E-06	-1.008E-05
1.560E+02	-1.924E-07	-1.324E-06	6.462E-07	-1.290E-05	-1.427E-05
1.440E+02	-1.303E-07	-1.235E-06	2.869E-07	-1.701E-05	-1.809E-05
1.320E+02	-7.546E-08	-7.154E-07	5.426E-08	-1.968E-05	-2.042E-05
1.200E+02	-3.418E-08	-3.241E-07	-7.293E-08	-1.772E-05	-1.815E-05
1.080E+02	-7.141E-09	-6.770E-08	-1.245E-07	-3.141E-06	-3.340E-06
9.600E+01	7.932E-09	7.521E-08	-1.290E-07	4.489E-05	4.485E-05
8.400E+01	1.432E-08	1.358E-07	-1.091E-07	1.845E-04	1.846E-04
7.200E+01	1.522E-08	1.443E-07	-8.044E-08	5.954E-04	5.955E-04
6.000E+01	1.322E-08	1.253E-07	-5.241E-08	1.919E-03	1.919E-03
4.800E+01	1.012E-08	9.599E-08	-2.972E-08	6.995E-03	6.995E-03
3.600E+01	7.108E-09	6.739E-08	-1.406E-08	3.331E-02	3.331E-02
2.400E+01	4.954E-09	4.697E-08	-6.052E-09	2.741E-01	2.741E-01
1.200E+01	5.364E-09	5.085E-08	-1.162E-08	9.184E+00	9.184E+00

CYLINDRICAL SHELL

LAMDA= 2.002E-02   H=-8.318E+01

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	-2.388E+03	0.000E+00	-2.388E+03
1.200E+01	-2.271E+03	-7.774E+02	-3.049E+03
2.400E+01	-1.993E+03	-1.188E+03	-3.180E+03
3.600E+01	-1.639E+03	-1.334E+03	-2.973E+03
4.800E+01	-1.272E+03	-1.303E+03	-2.575E+03
6.000E+01	-9.295E+02	-1.165E+03	-2.095E+03
7.200E+01	-6.332E+02	-9.747E+02	-1.608E+03
8.400E+01	-3.925E+02	-7.683E+02	-1.161E+03
9.600E+01	-2.079E+02	-5.703E+02	-7.788E+02
1.080E+02	-7.494E+01	-3.969E+02	-4.719E+02
1.200E+02	1.412E+01	-2.533E+02	-2.392E+02
1.320E+02	6.792E+01	-1.415E+02	-7.358E+01
1.440E+02	9.503E+01	-5.949E+01	3.533E+01
1.560E+02	1.032E+02	-3.378E+00	9.979E+01
1.680E+02	9.884E+01	3.164E+01	1.305E+02
1.800E+02	8.719E+01	5.042E+01	1.376E+02
1.920E+02	7.207E+01	5.746E+01	1.295E+02
2.040E+02	5.617E+01	5.660E+01	1.128E+02
2.160E+02	4.124E+01	5.093E+01	9.217E+01
2.280E+02	2.826E+01	4.280E+01	7.106E+01
2.400E+02	1.767E+01	3.388E+01	5.155E+01

KO= 5.000E+02   EO 3.000E+06   HO= 8.000E+00   S= 2.400E+02   T= 3.070E-01   P= 2.500E+02
ALPHA4= 4.823E+03   K= 3.170E-02   KK= 3.170E-02

IN STMT 521 PROGRAM RETURNS FROM MAIN PROCEDURE.

MONOLITHIC JOINT WELL-GRADED GRAVEL ELASTIC FOUNDATION,

CONICAL SHELL: 1STORDER ASYMPTOTIC SOLUTION

C1= 4.168E-09H + -1.315E-08P + -3.978E-10M C2=-4.246E-09H + 1.339E-08P + 2.759E-11M

EDHCON= 3.610E+01	EDHCYL=-4.320E+02
EDPCON=-1.538E+04	EDPCYL=-9.375E+04
EDMCON=-3.778E+00	EDMCYL=-8.650E+00
EDPPCON=-2.847E+04	
EXHCON= 3.740E+00	EXHCYL= 8.650E+00
EXPCON= 5.485E+01	EXPCYL= 3.906E+02
EXMCON=-7.820E-01	EXMCYL= 3.460E-01
EXPPCON=-2.950E+03	

M=-2.426E+03 H=-8.136E+01 P= 2.500E+02

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MHCON	MPCON	MMCON	MCON
3.772E+02	0.000E+00	0.000E+00	-2.425E+03	-2.425E+03
3.720E+02	1.080E+02	1.050E+03	-2.397E+03	-1.239E+03
3.600E+02	2.390E+02	2.317E+03	-2.007E+03	5.490E+02
3.480E+02	2.550E+02	2.480E+03	-1.432E+03	1.303E+03
3.360E+02	2.140E+02	2.081E+03	-8.790E+02	1.416E+03
3.240E+02	1.530E+02	1.437E+03	-4.420E+02	1.198E+03
3.120E+02	9.400E+01	9.130E+02	-1.460E+02	8.610E+02
3.000E+02	4.700E+01	4.590E+02	2.400E+01	5.300E+02
2.880E+02	1.500E+01	1.510E+02	1.020E+02	2.680E+02
2.760E+02	-2.000E+00	-2.600E+01	1.200E+02	9.200E+01
2.640E+02	-1.100E+01	-1.070E+02	1.050E+02	-1.300E+01
2.520E+02	-1.200E+01	-1.250E+02	7.800E+01	-5.900E+01
2.400E+02	-1.100E+01	-1.100E+02	4.900E+01	-7.200E+01
2.280E+02	-8.000E+00	-8.100E+01	2.600E+01	-6.300E+01
2.160E+02	-5.000E+00	-5.100E+01	9.000E+00	-4.700E+01
2.040E+02	-2.000E+00	-2.600E+01	0.000E+00	-2.800E+01
1.920E+02	0.000E+00	-9.000E+00	-5.000E+00	-1.400E+01
1.800E+02	0.000E+00	0.000E+00	-6.000E+00	-6.000E+00
1.680E+02	0.000E+00	5.000E+00	-5.000E+00	0.000E+00
1.560E+02	0.000E+00	6.000E+00	-4.000E+00	2.000E+00
1.440E+02	0.000E+00	6.000E+00	-2.000E+00	4.000E+00
1.320E+02	0.000E+00	4.000E+00	-1.000E+00	3.000E+00
1.200E+02	0.000E+00	2.000E+00	0.000E+00	2.000E+00
1.080E+02	0.000E+00	1.000E+00	0.000E+00	1.000E+00
9.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

6.000E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4.800E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
1.200E+01	0.000E+00	1.000E+00	0.000E+00	1.000E+00

S IN INCHES
TRANSVERSE SHEAR FORCE IN POUND/INCH

S	QHCON	QPCON	QMCON	QCON
3.772E+02	-2.459E+01	-2.383E+02	-8.944E-16	-2.629E+02
3.720E+02	-1.716E+01	-1.663E+02	-2.231E+01	-2.058E+02
3.600E+02	-4.584E+00	-4.443E+01	-4.926E+01	-9.827E+01
3.480E+02	2.374E+00	2.301E+01	-5.278E+01	-2.740E+01
3.360E+02	5.279E+00	5.117E+01	-4.431E+01	1.214E+01
3.240E+02	5.667E+00	5.493E+01	-3.167E+01	2.892E+01
3.120E+02	4.762E+00	4.615E+01	-1.944E+01	3.147E+01
3.000E+02	3.404E+00	3.299E+01	-9.766E+00	2.663E+01
2.880E+02	2.088E+00	2.024E+01	-3.187E+00	1.914E+01
2.760E+02	1.046E+00	1.014E+01	6.228E-01	1.181E+01
2.640E+02	3.373E-01	3.289E+00	2.334E+00	5.960E+00
2.520E+02	-7.296E-02	-7.074E-01	2.741E+00	1.960E+00
2.400E+02	-2.587E-01	-2.507E+00	2.397E+00	-3.687E-01
2.280E+02	-2.991E-01	-2.899E+00	1.764E+00	-1.433E+00
2.160E+02	-2.605E-01	-2.525E+00	1.111E+00	-1.674E+00
2.040E+02	-1.908E-01	-1.849E+00	5.747E-01	-1.465E+00
1.920E+02	-1.192E-01	-1.155E+00	1.989E-01	-1.076E+00
1.800E+02	-6.054E-02	-5.868E-01	-2.505E-02	-6.724E-01
1.680E+02	-1.963E-02	-1.903E-01	-1.305E-01	-3.404E-01
1.560E+02	4.528E-03	4.388E-02	-1.566E-01	-1.082E-01
1.440E+02	1.562E-02	1.514E-01	-1.380E-01	2.896E-02
1.320E+02	1.738E-02	1.742E-01	-1.006E-01	9.161E-02
1.200E+02	1.543E-02	1.496E-01	-6.084E-02	1.042E-01
1.080E+02	1.036E-02	1.053E-01	-2.765E-02	8.649E-02
9.600E+01	6.112E-03	5.924E-02	-4.354E-03	6.099E-02
8.400E+01	2.174E-03	2.108E-02	9.140E-03	3.239E-02
7.200E+01	-5.735E-04	-5.559E-03	1.460E-02	8.463E-03
6.000E+01	-2.134E-03	-2.068E-02	1.428E-02	-8.539E-03
4.800E+01	-2.687E-03	-2.604E-02	1.016E-02	-1.856E-02
3.600E+01	-2.425E-03	-2.350E-02	3.379E-03	-2.255E-02
2.400E+01	-1.316E-03	-1.275E-02	-7.047E-03	-2.112E-02
1.200E+01	2.348E-03	2.276E-02	-3.367E-02	-8.568E-03

S IN INCHES
W PREFIX FOR NORMAL DISPLACEMENTS IN INCH/INCH
D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH
PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE

S	WHCON	WPCON	WMCON	WPPCON	WCON	PF	DCON
3.772E+02	-3.239E-03	-3.140E-02	1.011E-02	-1.697E-02	-4.150E-02	2.075E+01	-1.254E-02
3.720E+02	-2.714E-03	-2.631E-02	7.058E-03	-1.706E-02	-3.903E-02	1.951E+01	-1.179E-02
3.600E+02	-1.605E-03	-1.535E-02	1.687E-03	-1.728E-02	-3.255E-02	1.627E+01	-9.836E-03
3.480E+02	-7.583E-04	-7.330E-03	-9.803E-04	-1.749E-02	-2.658E-02	1.329E+01	-8.032E-03
3.360E+02	-2.022E-04	-1.959E-03	-2.181E-03	-1.769E-02	-2.204E-02	1.102E+01	-6.659E-03
3.240E+02	1.066E-04	1.033E-03	-2.343E-03	-1.789E-02	-1.909E-02	9.547E+00	-5.770E-03
3.120E+02	2.359E-04	2.286E-03	-1.970E-03	-1.808E-02	-1.753E-02	8.765E+00	-5.298E-03
3.000E+02	2.533E-04	2.455E-03	-1.409E-03	-1.826E-02	-1.696E-02	8.482E+00	-5.127E-03
2.880E+02	2.129E-04	2.064E-03	-8.641E-04	-1.844E-02	-1.702E-02	8.512E+00	-5.145E-03
2.760E+02	1.521E-04	1.475E-03	-4.318E-04	-1.860E-02	-1.740E-02	8.702E+00	-5.259E-03
2.640E+02	9.310E-05	9.023E-04	-1.374E-04	-1.875E-02	-1.789E-02	8.944E+00	-5.406E-03
2.520E+02	4.624E-05	4.482E-04	3.304E-05	-1.888E-02	-1.835E-02	9.176E+00	-5.546E-03
2.400E+02	1.434E-05	1.390E-04	1.101E-04	-1.899E-02	-1.873E-02	9.365E+00	-5.660E-03

2.280E+02	-4.093E-06	-3.967E-05	1.266E-04	-1.909E-02	-1.900E-02	9.502E+00	-5.743E-03
2.160E+02	-1.237E-05	-1.199E-04	1.102E-04	-1.916E-02	-1.918E-02	9.589E+00	-5.796E-03
2.040E+02	-1.404E-05	-1.361E-04	8.064E-05	-1.919E-02	-1.926E-02	9.632E+00	-5.822E-03
1.920E+02	-1.213E-05	-1.176E-04	5.030E-05	-1.920E-02	-1.928E-02	9.639E+00	-5.826E-03
1.800E+02	-8.813E-06	-8.542E-05	2.539E-05	-1.916E-02	-1.923E-02	9.616E+00	-5.812E-03
1.680E+02	-5.425E-06	-5.258E-05	7.980E-06	-1.908E-02	-1.913E-02	9.566E+00	-5.782E-03
1.560E+02	-2.655E-06	-2.573E-05	-2.327E-06	-1.895E-02	-1.898E-02	9.489E+00	-5.735E-03
1.440E+02	-7.237E-07	-7.063E-06	-7.069E-06	-1.876E-02	-1.878E-02	9.388E+00	-5.674E-03
1.320E+02	3.987E-07	3.864E-06	-8.069E-06	-1.853E-02	-1.854E-02	9.268E+00	-5.601E-03
1.200E+02	8.992E-07	8.715E-06	-6.951E-06	-1.829E-02	-1.829E-02	9.145E+00	-5.527E-03
1.080E+02	9.770E-07	9.470E-06	-4.939E-06	-1.814E-02	-1.814E-02	9.068E+00	-5.481E-03
9.600E+01	8.150E-07	7.399E-06	-2.828E-06	-1.835E-02	-1.834E-02	9.171E+00	-5.543E-03
8.400E+01	5.513E-07	5.344E-06	-1.053E-06	-1.962E-02	-1.962E-02	9.808E+00	-5.928E-03
7.200E+01	2.763E-07	2.678E-06	2.185E-07	-2.400E-02	-2.400E-02	1.200E+01	-7.252E-03
6.000E+01	3.868E-08	3.749E-07	9.816E-07	-3.796E-02	-3.796E-02	1.898E+01	-1.147E-02
4.800E+01	-1.439E-07	-1.395E-06	1.314E-06	-8.593E-02	-8.593E-02	4.296E+01	-2.597E-02
3.600E+01	-2.760E-07	-2.675E-06	1.310E-06	-2.890E-01	-2.890E-01	1.445E+02	-8.734E-02
2.400E+01	-3.825E-07	-3.707E-06	1.001E-06	-1.621E+00	-1.621E+00	8.106E+02	-4.899E-01
1.200E+01	-5.550E-07	-5.379E-06	-6.372E-08	-2.809E+01	-2.809E+01	1.404E+04	-8.489E+00

S IN INCHES
ROTATIONS IN RADIAN/INCH

S	XHCON	XPCON	XMCON	XPPCON	XCON
3.772E+02	-1.014E-04	-9.832E-04	6.324E-04	1.828E-05	-4.339E-04
3.720E+02	-9.911E-05	-9.606E-04	5.334E-04	1.813E-05	-5.082E-04
3.600E+02	-8.143E-05	-7.892E-04	3.211E-04	1.774E-05	-5.318E-04
3.480E+02	-5.720E-05	-5.544E-04	1.567E-04	1.730E-05	-4.376E-04
3.360E+02	-3.446E-05	-3.340E-04	4.702E-05	1.680E-05	-3.046E-04
3.240E+02	-1.682E-05	-1.630E-04	-1.509E-05	1.623E-05	-1.787E-04
3.120E+02	-5.033E-06	-4.879E-05	-4.226E-05	1.557E-05	-8.051E-05
3.000E+02	1.648E-06	1.598E-05	-4.726E-05	1.481E-05	-1.483E-05
2.880E+02	4.571E-06	4.430E-05	-4.057E-05	1.393E-05	2.224E-05
2.760E+02	5.107E-06	4.949E-05	-2.948E-05	1.292E-05	3.804E-05
2.640E+02	4.379E-06	4.244E-05	-1.839E-05	1.173E-05	4.017E-05
2.520E+02	3.177E-06	3.080E-05	-9.429E-06	1.034E-05	3.489E-05
2.400E+02	1.976E-06	1.915E-05	-3.237E-06	8.717E-06	2.661E-05
2.280E+02	1.006E-06	9.743E-06	4.130E-07	6.804E-06	1.797E-05
2.160E+02	3.357E-07	3.254E-06	2.116E-06	4.549E-06	1.025E-05
2.040E+02	-5.773E-08	-5.595E-07	2.537E-06	1.895E-06	3.814E-06
1.920E+02	-2.394E-07	-2.320E-06	2.246E-06	-1.217E-06	-1.531E-06
1.800E+02	-2.816E-07	-2.729E-06	1.661E-06	-4.827E-06	-6.177E-06
1.680E+02	-2.466E-07	-2.390E-06	1.043E-06	-8.921E-06	-1.051E-05
1.560E+02	-1.381E-07	-1.746E-06	5.297E-07	-1.334E-05	-1.474E-05
1.440E+02	-1.103E-07	-1.074E-06	1.678E-07	-1.759E-05	-1.861E-05
1.320E+02	-5.351E-08	-5.186E-07	-4.735E-08	-2.035E-05	-2.097E-05
1.200E+02	-1.355E-08	-1.313E-07	-1.459E-07	-1.832E-05	-1.861E-05
1.080E+02	9.712E-09	9.413E-08	-1.653E-07	-3.248E-06	-3.309E-06
9.600E+01	1.969E-08	1.908E-07	-1.392E-07	4.642E-05	4.649E-05
8.400E+01	2.057E-08	1.994E-07	-9.363E-08	1.908E-04	1.909E-04
7.200E+01	1.616E-08	1.566E-07	-4.546E-08	6.156E-04	6.158E-04
6.000E+01	9.317E-09	9.030E-08	-4.017E-09	1.985E-03	1.985E-03
4.800E+01	1.817E-09	1.762E-08	2.717E-08	7.233E-03	7.233E-03
3.600E+01	-5.716E-09	-5.540E-08	4.843E-08	3.444E-02	3.444E-02
2.400E+01	-1.437E-08	-1.393E-07	6.292E-08	2.835E-01	2.835E-01
1.200E+01	-3.305E-08	-3.203E-07	7.853E-08	9.496E+00	9.496E+00

CYLINDRICAL SHELL

LAMDA= 2.002E-02 H=-8.136E+01

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	-2.426E+03	0.000E+00	-2.426E+03
1.200E+01	-2.307E+03	-7.605E+02	-3.068E+03
2.400E+01	-2.024E+03	-1.162E+03	-3.186E+03
3.600E+01	-1.665E+03	-1.304E+03	-2.970E+03
4.800E+01	-1.292E+03	-1.274E+03	-2.567E+03
6.000E+01	-9.442E+02	-1.140E+03	-2.084E+03
7.200E+01	-6.432E+02	-9.535E+02	-1.597E+03
8.400E+01	-3.987E+02	-7.515E+02	-1.150E+03
9.600E+01	-2.112E+02	-5.584E+02	-7.696E+02
1.080E+02	-7.613E+01	-3.882E+02	-4.644E+02
1.200E+02	1.434E+01	-2.478E+02	-2.334E+02
1.320E+02	6.899E+01	-1.384E+02	-6.941E+01
1.440E+02	9.653E+01	-5.820E+01	3.833E+01
1.560E+02	1.048E+02	-3.304E+00	1.015E+02
1.680E+02	1.004E+02	3.095E+01	1.313E+02
1.800E+02	8.857E+01	4.932E+01	1.379E+02
1.920E+02	7.321E+01	5.621E+01	1.294E+02
2.040E+02	5.706E+01	5.536E+01	1.124E+02
2.160E+02	4.189E+01	4.982E+01	9.171E+01
2.280E+02	2.871E+01	4.186E+01	7.057E+01
2.400E+02	1.795E+01	3.314E+01	5.109E+01

KO= 5.000E+02 EO 3.000E+06 HO= 8.000E+00 S= 2.400E+02 T= 3.070E-01 P= 2.500E+02
ALPHA4= 4.823E+03 K= 3.144E-02 KK= 3.170E-02

MONOLITHIC JOINT WELL-GRADED GRAVEL ELASTIC FOUNDATION,

CONICAL SHELL: 2NDORDER ASYMPTOTIC SOLUTION

C1= 4.159E-09H + -1.312E-08P + -8.899E-10M C2=-4.143E-09H + 1.307E-08P + 1.809E-11M

EDHCON= 3.496E+01 EDHCYL=-4.320E+02

EDPCON=-1.338E+04 EDPCYL=-9.375E+04

EDMCON=-3.660E+00 EDMCYL=-8.650E+00

EDPPCON=-2.757E+04

EXHCON= 3.676E+00 EXHCYL= 8.650E+00

EXPCON= 5.485E+01 EXPCYL= 3.906E+02

EXMCON=-7.714E-01 EXMCYL= 3.460E-01

EXPPCON=-2.899E+03

M=-2.388E+03 H=-8.327E+01 P= 2.500E+02

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MHCON	MPCON	MMCON	MCON
3.772E+02	0.000E+00	0.000E+00	-2.387E+03	-2.387E+03
3.720E+02	1.100E+02	1.045E+03	-2.360E+03	-1.205E+03
3.600E+02	2.430E+02	2.307E+03	-1.978E+03	5.720E+02
3.480E+02	2.610E+02	2.472E+03	-1.414E+03	1.319E+03
3.360E+02	2.190E+02	2.078E+03	-8.700E+02	1.427E+03
3.240E+02	1.570E+02	1.488E+03	-4.400E+02	1.205E+03
3.120E+02	9.600E+01	9.170E+02	-1.480E+02	8.650E+02
3.000E+02	4.900E+01	4.650E+02	2.100E+01	5.350E+02
2.880E+02	1.600E+01	1.570E+02	9.900E+01	2.720E+02
2.760E+02	-2.000E+00	-2.200E+01	1.180E+02	9.400E+01
2.640E+02	-1.100E+01	-1.050E+02	1.040E+02	-1.200E+01
2.520E+02	-1.300E+01	-1.250E+02	7.800E+01	-6.000E+01
2.400E+02	-1.100E+01	-1.100E+02	5.000E+01	-7.100E+01
2.280E+02	-8.000E+00	-8.200E+01	2.600E+01	-6.400E+01
2.160E+02	-5.000E+00	-5.200E+01	1.000E+01	-4.700E+01
2.040E+02	-2.000E+00	-2.800E+01	0.000E+00	-3.000E+01
1.920E+02	-1.000E+00	-1.000E+01	-4.000E+00	-1.500E+01
1.800E+02	0.000E+00	0.000E+00	-6.000E+00	-6.000E+00
1.680E+02	0.000E+00	5.000E+00	-5.000E+00	0.000E+00
1.560E+02	0.000E+00	6.000E+00	-4.000E+00	2.000E+00
1.440E+02	0.000E+00	6.000E+00	-2.000E+00	4.000E+00
1.320E+02	0.000E+00	4.000E+00	-1.000E+00	3.000E+00
1.200E+02	0.000E+00	3.000E+00	0.000E+00	3.000E+00
1.080E+02	0.000E+00	1.000E+00	0.000E+00	1.000E+00
9.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6.000E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4.800E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

1.200E+01	0.000E+00	-4.000E+00	1.000E+00	-3.000E+00

S IN INCHES
TRANSVERSE SHEAR FORCE IN POUND/INCH

S	QHCON	QPCON	QMCON	QCON
3.772E+02	-2.516E+01	-2.383E+02	5.502E-17	-2.635E+02
3.720E+02	-1.757E+01	-1.684E+02	-2.204E+01	-2.060E+02
3.600E+02	-4.708E+00	-4.459E+01	-4.871E+01	-9.800E+01
3.480E+02	2.420E+00	2.292E+01	-5.227E+01	-2.693E+01
3.360E+02	5.408E+00	5.122E+01	-4.397E+01	1.266E+01
3.240E+02	5.820E+00	5.511E+01	-3.151E+01	2.943E+01
3.120E+02	4.904E+00	4.644E+01	-1.942E+01	3.192E+01
3.000E+02	3.519E+00	3.332E+01	-9.826E+00	2.702E+01
2.880E+02	2.171E+00	2.056E+01	-3.275E+00	1.945E+01
2.760E+02	1.098E+00	1.040E+01	5.437E-01	1.204E+01
2.640E+02	3.646E-01	3.453E+00	2.303E+00	6.121E+00
2.520E+02	-8.366E-02	-8.029E-01	2.722E+00	2.055E+00
2.400E+02	-2.612E-01	-2.473E+00	2.406E+00	-3.285E-01
2.280E+02	-3.081E-01	-2.918E+00	1.791E+00	-1.435E+00
2.160E+02	-2.723E-01	-2.579E+00	1.146E+00	-1.705E+00
2.040E+02	-2.027E-01	-1.919E+00	6.094E-01	-1.512E+00
1.920E+02	-1.295E-01	-1.227E+00	2.276E-01	-1.129E+00
1.800E+02	-6.853E-02	-6.490E-01	-5.323E-03	-7.228E-01
1.680E+02	-2.508E-02	-2.376E-01	-1.204E-01	-3.830E-01
1.560E+02	1.439E-03	1.363E-02	-1.551E-01	-1.401E-01
1.440E+02	1.453E-02	1.376E-01	-1.433E-01	8.863E-03
1.320E+02	1.844E-02	1.747E-01	-1.103E-01	8.281E-02
1.200E+02	1.700E-02	1.610E-01	-7.277E-02	1.052E-01
1.080E+02	1.312E-02	1.243E-01	-3.990E-02	9.748E-02
9.600E+01	8.714E-03	8.253E-02	-1.540E-02	7.534E-02
8.400E+01	4.843E-03	4.588E-02	4.306E-04	5.114E-02
7.200E+01	1.938E-03	1.836E-02	9.109E-03	2.940E-02
6.000E+01	2.392E-05	2.266E-04	1.290E-02	1.315E-02
4.800E+01	-1.128E-03	-1.068E-02	1.434E-02	2.530E-03
3.600E+01	-1.990E-03	-1.884E-02	1.668E-02	-4.149E-03
2.400E+01	-3.962E-03	-3.752E-02	2.848E-02	-1.300E-02
1.200E+01	-2.007E-02	-1.901E-01	1.240E-01	-8.608E-02

S IN INCHES
W PREFIX FOR NORMAL DISPLACEMENTS IN INCH/INCH
D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH
PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE

S	WHCON	WPCON	WMCON	WPPCON	WCON	PF	DCON
3.772E+02	-3.211E-03	-3.041E-02	9.640E-03	-1.697E-02	-4.095E-02	2.048E+01	-1.238E-02
3.720E+02	-2.637E-03	-2.545E-02	6.718E-03	-1.706E-02	-3.348E-02	1.924E+01	-1.163E-02
3.600E+02	-1.538E-03	-1.530E-02	1.775E-03	-1.728E-02	-3.208E-02	1.604E+01	-9.695E-03
3.480E+02	-7.450E-04	-7.056E-03	-9.531E-04	-1.749E-02	-2.624E-02	1.312E+01	-7.930E-03
3.360E+02	-1.963E-04	-1.859E-03	-2.087E-03	-1.769E-02	-2.183E-02	1.092E+01	-6.599E-03
3.240E+02	1.066E-04	1.011E-03	-2.231E-03	-1.789E-02	-1.900E-02	9.502E+00	-5.743E-03
3.120E+02	2.327E-04	2.204E-03	-1.870E-03	-1.808E-02	-1.752E-02	8.758E+00	-5.293E-03
3.000E+02	2.468E-04	2.334E-03	-1.333E-03	-1.826E-02	-1.699E-02	8.497E+00	-5.136E-03
2.880E+02	2.081E-04	1.971E-03	-8.154E-04	-1.844E-02	-1.707E-02	8.537E+00	-5.160E-03
2.760E+02	1.432E-04	1.403E-03	-4.064E-04	-1.860E-02	-1.745E-02	8.727E+00	-5.274E-03
2.640E+02	9.036E-05	8.558E-04	-1.292E-04	-1.875E-02	-1.793E-02	8.965E+00	-5.418E-03
2.520E+02	4.473E-05	4.236E-04	3.031E-05	-1.888E-02	-1.838E-02	9.190E+00	-5.555E-03
2.400E+02	1.367E-05	1.313E-04	1.018E-04	-1.899E-02	-1.875E-02	9.373E+00	-5.665E-03

2.280E+02	-3.826E-06	-3.623E-05	1.166E-04	-1.909E-02	-1.901E-02	9.505E+00	-5.745E-03
2.160E+02	-1.157E-05	-1.103E-04	1.009E-04	-1.916E-02	-1.918E-02	9.588E+00	-5.795E-03
2.040E+02	-1.317E-05	-1.247E-04	7.334E-05	-1.919E-02	-1.926E-02	9.630E+00	-5.820E-03
1.920E+02	-1.129E-05	-1.069E-04	4.539E-05	-1.920E-02	-1.927E-02	9.636E+00	-5.824E-03
1.800E+02	-8.111E-06	-7.681E-05	2.273E-05	-1.916E-02	-1.923E-02	9.613E+00	-5.810E-03
1.680E+02	-4.927E-06	-4.666E-05	7.135E-06	-1.908E-02	-1.913E-02	9.563E+00	-5.780E-03
1.560E+02	-2.371E-06	-2.246E-05	-1.889E-06	-1.895E-02	-1.897E-02	9.487E+00	-5.734E-03
1.440E+02	-6.379E-07	-6.041E-06	-5.854E-06	-1.876E-02	-1.877E-02	9.387E+00	-5.674E-03
1.320E+02	3.352E-07	3.175E-06	-6.499E-06	-1.853E-02	-1.853E-02	9.267E+00	-5.601E-03
1.200E+02	7.266E-07	6.881E-06	-5.343E-06	-1.829E-02	-1.829E-02	9.145E+00	-5.527E-03
1.080E+02	7.392E-07	7.000E-06	-3.509E-06	-1.814E-02	-1.814E-02	9.069E+00	-5.481E-03
9.600E+01	5.501E-07	5.210E-06	-1.708E-06	-1.835E-02	-1.834E-02	9.172E+00	-5.544E-03
8.400E+01	2.909E-07	2.755E-06	-3.159E-07	-1.962E-02	-1.962E-02	9.809E+00	-5.929E-03
7.200E+01	4.570E-08	4.328E-07	5.282E-07	-2.400E-02	-2.400E-02	1.200E+01	-7.252E-03
6.000E+01	-1.391E-07	-1.317E-06	8.147E-07	-3.796E-02	-3.796E-02	1.898E+01	-1.147E-02
4.800E+01	-2.391E-07	-2.265E-06	5.578E-07	-8.593E-02	-8.593E-02	4.296E+01	-2.597E-02
3.600E+01	-2.241E-07	-2.122E-06	-3.705E-07	-2.890E-01	-2.890E-01	1.445E+02	-8.734E-02
2.400E+01	5.310E-08	5.029E-07	-2.877E-06	-1.621E+00	-1.621E+00	8.106E+02	-4.899E-01
1.200E+01	2.198E-06	2.081E-05	-1.574E-05	-2.809E+01	-2.809E+01	1.404E+04	-8.489E+00

S IN INCHES
ROTATIONS IN RADIAN/INCH

S	XHCON	XPCON	XMCON	XPPCON	XCON
3.772E+02	-1.020E-04	-9.663E-04	6.140E-04	1.828E-05	-4.361E-04
3.720E+02	-9.964E-05	-9.436E-04	5.178E-04	1.813E-05	-5.073E-04
3.600E+02	-8.180E-05	-7.747E-04	3.118E-04	1.774E-05	-5.270E-04
3.480E+02	-5.745E-05	-5.441E-04	1.524E-04	1.730E-05	-4.318E-04
3.360E+02	-3.464E-05	-3.280E-04	4.610E-05	1.680E-05	-2.998E-04
3.240E+02	-1.694E-05	-1.605E-04	-1.413E-05	1.623E-05	-1.753E-04
3.120E+02	-5.129E-06	-4.858E-05	-4.052E-05	1.557E-05	-7.866E-05
3.000E+02	1.574E-06	1.491E-05	-4.548E-05	1.481E-05	-1.419E-05
2.880E+02	4.516E-06	4.276E-05	-3.912E-05	1.393E-05	2.210E-05
2.760E+02	5.069E-06	4.801E-05	-2.849E-05	1.292E-05	3.750E-05
2.640E+02	4.359E-06	4.129E-05	-1.784E-05	1.173E-05	3.953E-05
2.520E+02	3.173E-06	3.005E-05	-9.229E-06	1.034E-05	3.434E-05
2.400E+02	1.983E-06	1.878E-05	-3.264E-06	8.717E-06	2.622E-05
2.280E+02	1.021E-06	9.673E-06	2.646E-07	6.804E-06	1.776E-05
2.160E+02	3.557E-07	3.369E-06	1.923E-06	4.549E-06	1.020E-05
2.040E+02	-3.717E-08	-3.520E-07	2.362E-06	1.895E-06	3.367E-06
1.920E+02	-2.210E-07	-2.093E-06	2.111E-06	-1.217E-06	-1.419E-06
1.800E+02	-2.670E-07	-2.528E-06	1.575E-06	-4.827E-06	-6.047E-06
1.680E+02	-2.367E-07	-2.241E-06	1.004E-06	-8.921E-06	-1.040E-05
1.560E+02	-1.748E-07	-1.656E-06	5.262E-07	-1.334E-05	-1.465E-05
1.440E+02	-1.097E-07	-1.039E-06	1.889E-07	-1.759E-05	-1.855E-05
1.320E+02	-5.576E-08	-5.281E-07	-1.277E-08	-2.035E-05	-2.094E-05
1.200E+02	-1.819E-08	-1.722E-07	-1.070E-07	-1.832E-05	-1.862E-05
1.080E+02	3.642E-09	3.449E-08	-1.288E-07	-3.248E-06	-3.333E-06
9.600E+01	1.304E-08	1.235E-07	-1.096E-07	4.642E-05	4.645E-05
8.400E+01	1.406E-08	1.331E-07	-7.350E-08	1.908E-04	1.909E-04
7.200E+01	1.040E-08	9.845E-08	-3.654E-08	6.156E-04	6.157E-04
6.000E+01	4.882E-09	4.624E-08	-7.856E-09	1.985E-03	1.985E-03
4.800E+01	-5.136E-10	-4.864E-09	7.500E-09	7.233E-03	7.233E-03
3.600E+01	-4.242E-09	-4.017E-08	4.195E-09	3.444E-02	3.444E-02
2.400E+01	-2.886E-09	-2.733E-08	-3.902E-08	2.835E-01	2.835E-01
1.200E+01	3.902E-08	3.695E-07	-3.303E-07	9.496E+00	9.496E+00

CYLINDRICAL SHELL

LAMDA= 2.002E-02 H=-8.327E+01

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	-2.388E+03	0.000E+00	-2.388E+03
1.200E+01	-2.271E+03	-7.783E+02	-3.049E+03
2.400E+01	-1.992E+03	-1.189E+03	-3.181E+03
3.600E+01	-1.639E+03	-1.335E+03	-2.974E+03
4.800E+01	-1.272E+03	-1.304E+03	-2.576E+03
6.000E+01	-9.293E+02	-1.167E+03	-2.096E+03
7.200E+01	-6.331E+02	-9.758E+02	-1.609E+03
8.400E+01	-3.924E+02	-7.691E+02	-1.162E+03
9.600E+01	-2.079E+02	-5.715E+02	-7.794E+02
1.080E+02	-7.493E+01	-3.974E+02	-4.723E+02
1.200E+02	1.412E+01	-2.536E+02	-2.395E+02
1.320E+02	6.791E+01	-1.417E+02	-7.374E+01
1.440E+02	9.501E+01	-5.956E+01	3.545E+01
1.560E+02	1.031E+02	-3.382E+00	9.977E+01
1.680E+02	9.882E+01	3.167E+01	1.305E+02
1.800E+02	8.717E+01	5.048E+01	1.376E+02
1.920E+02	7.205E+01	5.752E+01	1.296E+02
2.040E+02	5.616E+01	5.666E+01	1.128E+02
2.160E+02	4.123E+01	5.098E+01	9.222E+01
2.280E+02	2.826E+01	4.284E+01	7.110E+01
2.400E+02	1.767E+01	3.392E+01	5.159E+01

KO= 5.000E+02 EO 3.000E+06 HO= 8.000E+00 S= 2.400E+02 T= 3.070E-01 P= 2.500E+02
ALPHA4= 4.823E+03 K= 3.144E-02 KK= 3.170E-02

IN STMT 521 PROGRAM RETURNS FROM MAIN PROCEDURE.

MONOLITHIC JOINT POORLY-GRADED GRAVEL ELASTIC FOUNDATION

CONICAL SHELL: 2NDORDER ASYMPTOTIC SOLUTION

C1=-6.330E-08H + 1.997E-07P + 3.474E-09M C2=-2.759E-08H + 8.703E-08P + 8.222E-09M

EDHCON= 5.734E+01 EDHCYL=-4.320E+02

EDPCON=-1.338E+04 EDPCYL=-9.375E+04

EDMCON=-5.056E+00 EDMCYL=-8.650E+00

EDPPCON=-4.522E+04

EXHCON= 5.140E+00 EXHCYL= 8.650E+00

EXPCON= 5.425E+01 EXPCYL= 3.906E+02

EXMCON=-9.136E-01 EXMCYL= 3.460E-01

EXPPCON=-4.053E+03

M=-3.277E+03 H=-4.368E+01 P= 2.500E+02

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MHCON	MPCON	MMCON	MCON
3.772E+02	0.000E+00	0.000E+00	-3.276E+03	-3.276E+03
3.720E+02	5.900E+01	1.068E+03	-3.260E+03	-2.133E+03
3.600E+02	1.400E+02	2.532E+03	-2.879E+03	-2.070E+02
3.480E+02	1.630E+02	2.958E+03	-2.255E+03	8.660E+02
3.360E+02	1.520E+02	2.759E+03	-1.590E+03	1.321E+03
3.240E+02	1.240E+02	2.251E+03	-9.980E+02	1.377E+03
3.120E+02	9.100E+01	1.649E+03	-5.320E+02	1.208E+03
3.000E+02	5.900E+01	1.082E+03	-2.020E+02	9.390E+02
2.880E+02	3.400E+01	6.170E+02	6.000E+00	6.570E+02
2.760E+02	1.500E+01	2.740E+02	1.180E+02	4.070E+02
2.640E+02	2.000E+00	4.800E+01	1.620E+02	2.120E+02
2.520E+02	-4.000E+00	-8.100E+01	1.620E+02	7.700E+01
2.400E+02	-7.000E+00	-1.390E+02	1.380E+02	-8.000E+00
2.280E+02	-8.000E+00	-1.510E+02	1.050E+02	-5.400E+01
2.160E+02	-7.000E+00	-1.350E+02	7.200E+01	-7.000E+01
2.040E+02	-5.000E+00	-1.060E+02	4.300E+01	-6.800E+01
1.920E+02	-4.000E+00	-7.500E+01	2.100E+01	-5.800E+01
1.800E+02	-2.000E+00	-4.700E+01	6.000E+00	-4.300E+01
1.680E+02	-1.000E+00	-2.500E+01	-2.000E+00	-2.800E+01
1.560E+02	0.000E+00	-9.000E+00	-7.000E+00	-1.600E+01
1.440E+02	0.000E+00	0.000E+00	-8.000E+00	-8.000E+00
1.320E+02	0.000E+00	5.000E+00	-8.000E+00	-3.000E+00
1.200E+02	0.000E+00	7.000E+00	-6.000E+00	1.000E+00
1.080E+02	0.000E+00	7.000E+00	-4.000E+00	3.000E+00
9.600E+01	0.000E+00	6.000E+00	-3.000E+00	3.000E+00
8.400E+01	0.000E+00	4.000E+00	-2.000E+00	2.000E+00
7.200E+01	0.000E+00	3.000E+00	-1.000E+00	2.000E+00
6.000E+01	0.000E+00	3.000E+00	-1.000E+00	2.000E+00
4.800E+01	0.000E+00	4.000E+00	-1.000E+00	3.000E+00
3.600E+01	0.000E+00	7.000E+00	-2.000E+00	5.000E+00
2.400E+01	1.000E+00	1.800E+01	-5.000E+00	1.400E+01

1.200E+01	4.000E+00	8.200E+01	-9.000E+00	7.700E+01

S IN INCHES
TRANSVERSE SHEAR FORCE IN POUND/INCH

S	QHCON	QPCON	QMCON	QCON
3.772E+02	-1.320E+01	-2.383E+02	2.464E-15	-2.515E+02
3.720E+02	-9.785E+00	-1.767E+02	-2.249E+01	-2.089E+02
3.600E+02	-3.633E+00	-6.596E+01	-5.342E+01	-1.230E+02
3.480E+02	2.125E-01	3.833E+00	-6.233E+01	-5.849E+01
3.360E+02	2.283E+00	4.130E+01	-5.846E+01	-1.487E+01
3.240E+02	3.089E+00	5.577E+01	-4.780E+01	1.106E+01
3.120E+02	3.080E+00	5.561E+01	-3.507E+01	2.362E+01
3.000E+02	2.630E+00	4.743E+01	-2.304E+01	2.707E+01
2.880E+02	2.004E+00	3.618E+01	-1.314E+01	2.504E+01
2.760E+02	1.371E+00	2.476E+01	-5.814E+00	2.032E+01
2.640E+02	8.204E-01	1.492E+01	-9.573E-01	1.479E+01
2.520E+02	4.077E-01	7.360E+00	1.349E+00	9.617E+00
2.400E+02	1.134E-01	2.138E+00	3.126E+00	5.383E+00
2.280E+02	-3.812E-02	-1.049E+00	3.379E+00	2.272E+00
2.160E+02	-1.473E-01	-2.659E+00	3.032E+00	2.257E-01
2.040E+02	-1.755E-01	-3.168E+00	2.403E+00	-9.400E-01
1.920E+02	-1.659E-01	-2.995E+00	1.709E+00	-1.452E+00
1.800E+02	-1.368E-01	-2.470E+00	1.078E+00	-1.530E+00
1.680E+02	-1.011E-01	-1.825E+00	5.710E-01	-1.355E+00
1.560E+02	-6.670E-02	-1.204E+00	2.059E-01	-1.065E+00
1.440E+02	-3.799E-02	-6.859E-01	-2.870E-02	-7.526E-01
1.320E+02	-1.650E-02	-2.980E-01	-1.575E-01	-4.720E-01
1.200E+02	-2.074E-03	-3.745E-02	-2.097E-01	-2.492E-01
1.080E+02	6.424E-03	1.160E-01	-2.126E-01	-9.014E-02
9.600E+01	1.052E-02	1.898E-01	-1.890E-01	1.131E-02
8.400E+01	1.177E-02	2.126E-01	-1.567E-01	6.769E-02
7.200E+01	1.168E-02	2.108E-01	-1.281E-01	9.439E-02
6.000E+01	1.164E-02	2.102E-01	-1.131E-01	1.037E-01
4.800E+01	1.344E-02	2.426E-01	-1.224E-01	1.337E-01
3.600E+01	2.084E-02	3.763E-01	-1.787E-01	2.184E-01
2.400E+01	4.889E-02	8.827E-01	-3.706E-01	5.610E-01
1.200E+01	2.409E-01	4.349E+00	-1.450E+00	3.140E+00

S IN INCHES
W PREFIX FOR NORMAL DISPLACEMENTS IN INCH/INCH
D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH
PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE

S	WHCON	WPCON	WMCON	WPPCON	WCON	PF	DCON
3.772E+02	-2.763E-03	-4.938E-02	1.827E-02	-3.293E-02	-6.729E-02	1.682E+01	-2.034E-02
3.720E+02	-2.375E-03	-4.233E-02	1.346E-02	-3.308E-02	-6.489E-02	1.622E+01	-1.981E-02
3.600E+02	-1.346E-03	-2.731E-02	4.857E-03	-3.344E-02	-5.793E-02	1.448E+01	-1.751E-02
3.480E+02	-3.628E-04	-1.533E-02	-5.061E-04	-3.377E-02	-5.072E-02	1.268E+01	-1.533E-02
3.360E+02	-3.694E-04	-6.669E-03	-3.331E-03	-3.409E-02	-4.446E-02	1.112E+01	-1.344E-02
3.240E+02	-4.809E-05	-8.682E-04	-4.366E-03	-3.439E-02	-3.967E-02	9.918E+00	-1.199E-02
3.120E+02	1.327E-04	2.393E-03	-4.279E-03	-3.466E-02	-3.641E-02	9.104E+00	-1.100E-02
3.000E+02	2.106E-04	3.303E-03	-3.596E-03	-3.491E-02	-3.449E-02	8.623E+00	-1.042E-02
2.880E+02	2.215E-04	3.996E-03	-2.691E-03	-3.513E-02	-3.360E-02	8.401E+00	-1.013E-02
2.760E+02	1.939E-04	3.501E-03	-1.798E-03	-3.531E-02	-3.342E-02	8.354E+00	-1.010E-02
2.640E+02	1.499E-04	2.708E-03	-1.046E-03	-3.546E-02	-3.365E-02	8.413E+00	-1.017E-02
2.520E+02	1.034E-04	1.866E-03	-4.812E-04	-3.557E-02	-3.408E-02	8.520E+00	-1.030E-02
2.400E+02	6.249E-05	1.123E-03	-1.036E-04	-3.563E-02	-3.454E-02	8.636E+00	-1.044E-02

2.280E+02	3.034E-05	5.508E-04	1.154E-04	-3.564E-02	-3.494E-02	8.735E+00	-1.056E-02
2.160E+02	9.022E-06	1.629E-04	2.147E-04	-3.561E-02	-3.522E-02	8.805E+00	-1.064E-02
2.040E+02	-4.037E-06	-7.378E-05	2.335E-04	-3.552E-02	-3.537E-02	8.842E+00	-1.069E-02
1.920E+02	-1.040E-05	-1.879E-04	2.054E-04	-3.540E-02	-3.539E-02	8.847E+00	-1.069E-02
1.800E+02	-1.231E-05	-2.138E-04	1.559E-04	-3.525E-02	-3.532E-02	8.830E+00	-1.067E-02
1.680E+02	-1.076E-05	-1.942E-04	1.028E-04	-3.512E-02	-3.522E-02	8.806E+00	-1.065E-02
1.560E+02	-8.123E-06	-1.467E-04	5.574E-05	-3.510E-02	-3.520E-02	8.801E+00	-1.064E-02
1.440E+02	-5.151E-06	-9.300E-05	2.033E-05	-3.536E-02	-3.544E-02	8.861E+00	-1.071E-02
1.320E+02	-2.489E-06	-4.493E-05	-2.527E-06	-3.624E-02	-3.629E-02	9.074E+00	-1.097E-02
1.200E+02	-4.729E-07	-8.538E-06	-1.399E-05	-3.843E-02	-3.846E-02	9.614E+00	-1.162E-02
1.080E+02	7.849E-07	1.417E-05	-1.634E-05	-4.338E-02	-4.339E-02	1.085E+01	-1.311E-02
9.600E+01	1.312E-06	2.308E-05	-1.217E-05	-5.431E-02	-5.430E-02	1.357E+01	-1.641E-02
8.400E+01	1.202E-06	2.171E-05	-3.813E-06	-7.886E-02	-7.884E-02	1.971E+01	-2.383E-02
7.200E+01	5.534E-07	1.003E-05	6.915E-06	-1.370E-01	-1.370E-01	3.425E+01	-4.140E-02
6.000E+01	-3.706E-07	-1.030E-05	1.896E-05	-2.884E-01	-2.883E-01	7.209E+01	-8.714E-02
4.800E+01	-2.281E-06	-4.118E-05	3.218E-05	-7.471E-01	-7.471E-01	1.866E+02	-2.258E-01
3.600E+01	-5.088E-06	-9.137E-05	4.784E-05	-2.537E+00	-2.537E+00	6.344E+02	-7.668E-01
2.400E+01	-1.129E-05	-2.038E-04	7.005E-05	-1.368E+01	-1.368E+01	3.420E+03	-4.135E+00
1.200E+01	-3.968E-05	-7.164E-04	9.991E-05	-2.283E+02	-2.283E+02	5.707E+04	-6.898E+01

S IN INCHES
ROTATIONS IN RADIAN/INCH

S	XHCON	XPCON	XMCON	XPPCON	XCON
3.772E+02	-7.483E-05	-1.331E-03	9.978E-04	3.066E-05	-3.975E-04
3.720E+02	-7.348E-05	-1.327E-03	8.653E-04	3.014E-05	-5.047E-04
3.600E+02	-6.340E-05	-1.145E-03	5.744E-04	2.883E-05	-6.049E-04
3.480E+02	-4.870E-05	-8.792E-04	3.332E-04	2.736E-05	-5.674E-04
3.360E+02	-3.362E-05	-6.070E-04	1.539E-04	2.571E-05	-4.610E-04
3.240E+02	-2.034E-05	-3.709E-04	3.441E-05	2.386E-05	-3.332E-04
3.120E+02	-1.045E-05	-1.887E-04	-3.532E-05	2.179E-05	-2.127E-04
3.000E+02	-3.437E-06	-6.242E-05	-6.787E-05	1.946E-05	-1.143E-04
2.880E+02	8.381E-07	1.513E-05	-7.549E-05	1.685E-05	-4.267E-05
2.760E+02	3.038E-06	5.486E-05	-6.846E-05	1.392E-05	3.354E-06
2.640E+02	3.778E-06	6.821E-05	-5.452E-05	1.067E-05	2.814E-05
2.520E+02	3.618E-06	6.528E-05	-3.886E-05	7.083E-06	3.711E-05
2.400E+02	2.987E-06	5.393E-05	-2.460E-05	3.184E-06	3.550E-05
2.280E+02	2.198E-06	3.969E-05	-1.319E-05	-9.466E-07	2.774E-05
2.160E+02	1.440E-06	2.599E-05	-5.052E-06	-5.127E-06	1.725E-05
2.040E+02	8.116E-07	1.465E-05	9.840E-08	-8.938E-06	6.575E-06
1.920E+02	3.491E-07	6.302E-06	2.843E-06	-1.181E-05	-2.315E-06
1.800E+02	4.728E-08	8.536E-07	3.858E-06	-1.223E-05	-7.469E-06
1.680E+02	-1.204E-07	-2.173E-06	3.775E-06	-7.659E-06	-6.177E-06
1.560E+02	-1.885E-07	-3.403E-06	3.105E-06	6.861E-06	6.374E-06
1.440E+02	-1.913E-07	-3.484E-06	2.221E-06	4.106E-05	3.954E-05
1.320E+02	-1.577E-07	-2.847E-06	1.365E-06	1.146E-04	1.130E-04
1.200E+02	-1.098E-07	-1.979E-06	6.688E-07	2.692E-04	2.678E-04
1.080E+02	-6.189E-08	-1.117E-06	1.879E-07	5.974E-04	5.964E-04
9.600E+01	-2.341E-08	-4.226E-07	-7.553E-08	1.321E-03	1.320E-03
8.400E+01	1.372E-09	2.477E-08	-1.474E-07	3.018E-03	3.018E-03
7.200E+01	1.068E-08	1.929E-07	-6.298E-08	7.388E-03	7.388E-03
6.000E+01	3.472E-09	6.283E-08	1.478E-07	2.026E-02	2.026E-02
4.800E+01	-2.358E-08	-4.255E-07	4.760E-07	6.663E-02	6.663E-02
3.600E+01	-8.395E-08	-1.516E-06	9.646E-07	2.968E-01	2.968E-01
2.400E+01	-2.392E-07	-4.318E-06	1.815E-06	2.342E+00	2.342E+00
1.200E+01	-1.021E-06	-1.844E-05	3.993E-06	7.664E+01	7.664E+01

CYLINDRICAL SHELL

LAMDA= 2.002E-02   H=-4.368E+01

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	-3.277E+03	0.000E+00	-3.277E+03
1.200E+01	-3.116E+03	-4.082E+02	-3.524E+03
2.400E+01	-2.734E+03	-6.237E+02	-3.357E+03
3.600E+01	-2.249E+03	-7.003E+02	-2.950E+03
4.800E+01	-1.745E+03	-6.841E+02	-2.429E+03
6.000E+01	-1.275E+03	-6.120E+02	-1.887E+03
7.200E+01	-6.637E+02	-5.118E+02	-1.381E+03
8.400E+01	-5.385E+02	-4.034E+02	-9.419E+02
9.600E+01	-2.853E+02	-2.998E+02	-5.851E+02
1.080E+02	-1.028E+02	-2.084E+02	-3.112E+02
1.200E+02	1.937E+01	-1.330E+02	-1.136E+02
1.320E+02	9.319E+01	-7.430E+01	1.888E+01
1.440E+02	1.304E+02	-3.124E+01	9.914E+01
1.560E+02	1.415E+02	-1.774E+00	1.398E+02
1.680E+02	1.356E+02	1.661E+01	1.522E+02
1.800E+02	1.196E+02	2.648E+01	1.461E+02
1.920E+02	9.887E+01	3.017E+01	1.290E+02
2.040E+02	7.707E+01	2.972E+01	1.068E+02
2.160E+02	5.658E+01	2.674E+01	8.333E+01
2.280E+02	3.878E+01	2.247E+01	6.125E+01
2.400E+02	2.425E+01	1.779E+01	4.204E+01

KU= 2.500E+02   EU 3.000E+06   HO= 8.000E+00   S= 2.400E+02   T= 3.070E-01   P= 2.500E+02
ALPHA4= 9.652E+03   K= 2.643E-02   KK= 2.687E-02

IN STMT 521 PROGRAM RETURNS FROM MAIN PROCEDURE.

MONOLITHIC JOINT   UNIFORM SOIL BEARING PRESSURE FOUNDATION

CONICAL SHELL: FLUGGE APPROXIMATION

C1=-6.545E-02H + 0.000E+00P + -5.783E-03M   C2=-1.959E-01H + 0.000E+00P + 1.205E-02M

EDHCON= 4.314E+02   EDHCYL=-4.320E+02

EDPCON=-1.538E+04   EDPCYL=-9.375E+04

EDMCON=-2.004E+01   EDMCYL=-8.650E+00

EDPPCON= 0.000E+00

EXHCON= 2.007E+01   EXHCYL= 8.650E+00

EXPCON= 5.485E+01   EXPCYL= 3.906E+02

EXMCON=-1.810E+00   EXMCYL= 3.460E-01

EXPPCON= 0.000E+00

M=-5.488E+03   H=-4.894E+02   P= 0.000E+00

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MHCON	MPCON	MMCON	MCON
3.772E+02	0.000E+00	0.000E+00	-5.485E+03	-5.485E+03
3.720E+02	1.966E+03	0.000E+00	-5.361E+03	-3.395E+03
3.600E+02	2.913E+03	0.000E+00	-5.067E+03	-2.154E+03
3.480E+02	3.496E+03	0.000E+00	-4.633E+03	-1.137E+03
3.360E+02	3.783E+03	0.000E+00	-4.113E+03	-3.300E+02
3.240E+02	3.835E+03	0.000E+00	-3.526E+03	3.090E+02
3.120E+02	3.689E+03	0.000E+00	-2.907E+03	7.820E+02
3.000E+02	3.404E+03	0.000E+00	-2.313E+03	1.091E+03
2.880E+02	3.030E+03	0.000E+00	-1.768E+03	1.262E+03
2.760E+02	2.592E+03	0.000E+00	-1.265E+03	1.327E+03
2.640E+02	2.124E+03	0.000E+00	-8.200E+02	1.304E+03
2.520E+02	1.672E+03	0.000E+00	-4.570E+02	1.215E+03
2.400E+02	1.241E+03	0.000E+00	-1.620E+02	1.079E+03
2.280E+02	8.510E+02	0.000E+00	6.100E+01	9.120E+02
2.160E+02	5.150E+02	0.000E+00	2.200E+02	7.350E+02
2.040E+02	2.470E+02	0.000E+00	3.150E+02	5.620E+02
1.920E+02	3.400E+01	0.000E+00	3.630E+02	3.970E+02
1.800E+02	-1.170E+02	0.000E+00	3.680E+02	2.510E+02
1.680E+02	-2.100E+02	0.000E+00	3.440E+02	1.340E+02
1.560E+02	-2.600E+02	0.000E+00	2.980E+02	3.800E+01
1.440E+02	-2.700E+02	0.000E+00	2.390E+02	-3.100E+01
1.320E+02	-2.500E+02	0.000E+00	1.760E+02	-7.400E+01
1.200E+02	-1.880E+02	0.000E+00	1.500E+02	-3.800E+01
1.080E+02	-1.620E+02	0.000E+00	6.500E+01	-9.700E+01
9.600E+01	-1.100E+02	0.000E+00	2.300E+01	-8.700E+01
8.400E+01	-6.200E+01	0.000E+00	-6.000E+00	-6.800E+01
7.200E+01	-2.200E+01	0.000E+00	-2.400E+01	-4.600E+01
6.000E+01	5.000E+00	0.000E+00	-3.000E+01	-2.500E+01
4.800E+01	2.000E+01	0.000E+00	-2.700E+01	-7.000E+00
3.600E+01	2.300E+01	0.000E+00	-1.800E+01	5.000E+00
2.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

1.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

S IN INCHES
TRANSVERSE SHEAR FORCE IN POUND/INCH

S	QHCON	QPCON	QMCON	QCON
3.772E+02	-1.479E+02	0.000E+00	-3.761E-04	-1.479E+02
3.720E+02	-3.989E+01	0.000E+00	-2.321E+01	-1.181E+02
3.600E+02	-5.601E+01	0.000E+00	-4.246E+01	-9.847E+01
3.480E+02	-2.771E+01	0.000E+00	-5.183E+01	-7.954E+01
3.360E+02	-4.758E+00	0.000E+00	-5.710E+01	-6.186E+01
3.240E+02	1.386E+01	0.000E+00	-5.897E+01	-4.511E+01
3.120E+02	2.801E+01	0.000E+00	-5.783E+01	-2.982E+01
3.000E+02	3.752E+01	0.000E+00	-5.442E+01	-1.690E+01
2.880E+02	4.315E+01	0.000E+00	-4.947E+01	-6.319E+00
2.760E+02	4.557E+01	0.000E+00	-4.325E+01	2.320E+00
2.640E+02	4.519E+01	0.000E+00	-3.624E+01	8.945E+00
2.520E+02	4.269E+01	0.000E+00	-2.921E+01	1.348E+01
2.400E+02	3.852E+01	0.000E+00	-2.223E+01	1.629E+01
2.280E+02	3.319E+01	0.000E+00	-1.564E+01	1.755E+01
2.160E+02	2.720E+01	0.000E+00	-9.714E+00	1.749E+01
2.040E+02	2.123E+01	0.000E+00	-4.793E+00	1.644E+01
1.920E+02	1.522E+01	0.000E+00	-6.516E-01	1.457E+01
1.800E+02	9.716E+00	0.000E+00	2.476E+00	1.219E+01
1.680E+02	5.099E+00	0.000E+00	4.579E+00	9.678E+00
1.560E+02	1.102E+00	0.000E+00	5.867E+00	6.969E+00
1.440E+02	-1.884E+00	0.000E+00	6.338E+00	4.454E+00
1.320E+02	-3.929E+00	0.000E+00	6.143E+00	2.214E+00
1.200E+02	-5.291E+00	0.000E+00	5.232E+00	-5.844E-02
1.080E+02	-5.393E+00	0.000E+00	4.433E+00	-9.596E-01
9.600E+01	-5.062E+00	0.000E+00	3.201E+00	-1.861E+00
8.400E+01	-4.237E+00	0.000E+00	1.945E+00	-2.292E+00
7.200E+01	-3.065E+00	0.000E+00	7.651E-01	-2.300E+00
6.000E+01	-1.774E+00	0.000E+00	-1.906E-01	-1.964E+00
4.800E+01	-5.016E-01	0.000E+00	-8.579E-01	-1.359E+00
3.600E+01	5.406E-01	0.000E+00	-1.142E+00	-6.013E-01
2.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
1.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

S IN INCHES
W PREFIX FOR NORMAL DISPLACEMENTS IN INCH/INCH
D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH
PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE

S	WHCON	WPCON	WMCON	WPPCON	WCON	PF	DCON
3.772E+02	-2.331E-01	0.000E+00	1.214E-01	-8.203E-02	-1.937E-01	1.007E+01	-5.853E-02
3.720E+02	-1.818E-01	0.000E+00	7.368E-02	-7.980E-02	-1.879E-01	1.013E+01	-5.678E-02
3.600E+02	-1.440E-01	0.000E+00	4.486E-02	-7.474E-02	-1.738E-01	1.026E+01	-5.254E-02
3.480E+02	-1.102E-01	0.000E+00	2.246E-02	-6.984E-02	-1.576E-01	1.039E+01	-4.763E-02
3.360E+02	-8.110E-02	0.000E+00	5.699E-03	-6.510E-02	-1.405E-01	1.052E+01	-4.240E-02
3.240E+02	-5.578E-02	0.000E+00	-6.678E-03	-6.054E-02	-1.230E-01	1.065E+01	-3.716E-02
3.120E+02	-3.461E-02	0.000E+00	-1.506E-02	-5.613E-02	-1.058E-01	1.078E+01	-3.197E-02
3.000E+02	-1.826E-02	0.000E+00	-1.984E-02	-5.190E-02	-9.000E-02	1.092E+01	-2.720E-02
2.880E+02	-6.140E-03	0.000E+00	-2.189E-02	-4.783E-02	-7.586E-02	1.105E+01	-2.293E-02
2.760E+02	2.660E-03	0.000E+00	-2.196E-02	-4.393E-02	-6.322E-02	1.118E+01	-1.911E-02
2.640E+02	8.503E-03	0.000E+00	-2.055E-02	-4.019E-02	-5.224E-02	1.131E+01	-1.579E-02
2.520E+02	1.174E-02	0.000E+00	-1.823E-02	-3.662E-02	-4.311E-02	1.144E+01	-1.303E-02
2.400E+02	1.309E-02	0.000E+00	-1.539E-02	-3.322E-02	-3.551E-02	1.157E+01	-1.073E-02

2.280E+02	1.299E-02	0.000E+00	-1.235E-02	-2.998E-02	-2.934E-02	1.170E+01	-8.866E-03
2.160E+02	1.139E-02	0.000E+00	-9.391E-03	-2.690E-02	-2.441E-02	1.183E+01	-7.377E-03
2.040E+02	1.022E-02	0.000E+00	-6.779E-03	-2.400E-02	-2.056E-02	1.197E+01	-6.212E-03
1.920E+02	8.241E-03	0.000E+00	-4.483E-03	-2.126E-02	-1.750E-02	1.210E+01	-5.289E-03
1.800E+02	6.241E-03	0.000E+00	-2.646E-03	-1.868E-02	-1.509E-02	1.223E+01	-4.560E-03
1.680E+02	4.450E-03	0.000E+00	-1.301E-03	-1.628E-02	-1.313E-02	1.236E+01	-3.967E-03
1.560E+02	2.854E-03	0.000E+00	-3.190E-04	-1.403E-02	-1.150E-02	1.249E+01	-3.475E-03
1.440E+02	1.637E-03	0.000E+00	2.811E-04	-1.196E-02	-1.007E-02	1.262E+01	-3.043E-03
1.320E+02	6.919E-04	0.000E+00	5.695E-04	-1.005E-02	-8.766E-03	1.275E+01	-2.649E-03
1.200E+02	7.025E-04	0.000E+00	2.847E-04	-8.304E-03	-7.317E-03	1.288E+01	-2.211E-03
1.080E+02	-2.343E-04	0.000E+00	6.273E-04	-6.726E-03	-6.333E-03	1.301E+01	-1.914E-03
9.600E+01	-3.706E-04	0.000E+00	4.979E-04	-5.315E-03	-5.187E-03	1.315E+01	-1.568E-03
8.400E+01	-3.691E-04	0.000E+00	3.433E-04	-4.069E-03	-4.095E-03	1.328E+01	-1.237E-03
7.200E+01	-2.937E-04	0.000E+00	1.984E-04	-2.989E-03	-3.062E-03	1.341E+01	-9.313E-04
6.000E+01	-1.864E-04	0.000E+00	8.834E-05	-2.076E-03	-2.174E-03	1.354E+01	-6.570E-04
4.800E+01	-9.039E-05	0.000E+00	1.894E-05	-1.329E-03	-1.400E-03	1.367E+01	-4.231E-04
3.600E+01	-2.499E-05	0.000E+00	-1.135E-05	-7.474E-04	-7.837E-04	1.380E+01	-2.368E-04
2.400E+01	0.000E+00	0.000E+00	0.000E+00	-3.322E-04	-3.322E-04	1.393E+01	-1.004E-04
1.200E+01	0.000E+00	0.000E+00	0.000E+00	-8.304E-05	-8.304E-05	1.406E+01	-2.509E-05

S IN INCHES
ROTATIONS IN RADIAN/INCH

S	XHCON	XPCON	XMCON	XPPCON	XCON
3.772E+02	-3.274E-03	0.000E+00	3.309E-03	3.262E-04	3.614E-04
3.720E+02	-3.145E-03	0.000E+00	2.600E-03	3.218E-04	-2.234E-04
3.600E+02	-2.906E-03	0.000E+00	2.094E-03	3.114E-04	-5.008E-04
3.480E+02	-2.598E-03	0.000E+00	1.631E-03	3.010E-04	-6.662E-04
3.360E+02	-2.255E-03	0.000E+00	1.221E-03	2.906E-04	-7.437E-04
3.240E+02	-1.889E-03	0.000E+00	8.551E-04	2.803E-04	-7.536E-04
3.120E+02	-1.520E-03	0.000E+00	5.406E-04	2.699E-04	-7.092E-04
3.000E+02	-1.179E-03	0.000E+00	2.905E-04	2.595E-04	-6.286E-04
2.880E+02	-8.756E-04	0.000E+00	9.901E-05	2.491E-04	-5.275E-04
2.760E+02	-6.065E-04	0.000E+00	-4.551E-05	2.387E-04	-4.133E-04
2.640E+02	-3.777E-04	0.000E+00	-1.461E-04	2.284E-04	-2.955E-04
2.520E+02	-1.979E-04	0.000E+00	-2.062E-04	2.180E-04	-1.860E-04
2.400E+02	-5.844E-05	0.000E+00	-2.353E-04	2.076E-04	-8.617E-05
2.280E+02	4.242E-05	0.000E+00	-2.397E-04	1.972E-04	-1.825E-08
2.160E+02	1.083E-04	0.000E+00	-2.254E-04	1.868E-04	6.991E-05
2.040E+02	1.441E-04	0.000E+00	-1.996E-04	1.765E-04	1.210E-04
1.920E+02	1.574E-04	0.000E+00	-1.660E-04	1.661E-04	1.575E-04
1.800E+02	1.529E-04	0.000E+00	-1.299E-04	1.557E-04	1.786E-04
1.680E+02	1.370E-04	0.000E+00	-9.599E-05	1.453E-04	1.863E-04
1.560E+02	1.132E-04	0.000E+00	-6.393E-05	1.349E-04	1.842E-04
1.440E+02	8.690E-05	0.000E+00	-3.746E-05	1.246E-04	1.740E-04
1.320E+02	6.092E-05	0.000E+00	-1.683E-05	1.142E-04	1.583E-04
1.200E+02	3.347E-05	0.000E+00	8.118E-07	1.038E-04	1.381E-04
1.080E+02	1.973E-05	0.000E+00	6.451E-06	9.342E-05	1.196E-04
9.600E+01	6.098E-06	0.000E+00	1.079E-05	8.304E-05	9.993E-05
8.400E+01	-2.436E-06	0.000E+00	1.153E-05	7.266E-05	8.170E-05
7.200E+01	-6.751E-06	0.000E+00	9.863E-06	6.228E-05	6.539E-05
6.000E+01	-7.536E-06	0.000E+00	6.986E-05	5.190E-05	5.135E-05
4.800E+01	-6.053E-06	0.000E+00	3.846E-06	4.152E-05	3.931E-05
3.600E+01	-3.489E-06	0.000E+00	1.258E-06	3.114E-05	2.891E-05
2.400E+01	0.000E+00	0.000E+00	0.000E+00	2.076E-05	2.076E-05
1.200E+01	0.000E+00	0.000E+00	0.000E+00	1.038E-05	1.038E-05

S IN INCHES
CIRCUMFERENTIAL FORCE IN POUND /INCH

S	CHCON	CPCON	CMCON	CPPCON	CCON
3.772E+02	-4.820E+03	0.000E+00	2.517E+03	-1.655E+03	-3.958E+03
3.720E+02	-3.789E+03	0.000E+00	1.539E+03	-1.632E+03	-3.882E+03
3.600E+02	-3.035E+03	0.000E+00	9.525E+02	-1.580E+03	-3.682E+03
3.480E+02	-2.534E+03	0.000E+00	5.298E+02	-1.527E+03	-3.551E+03
3.360E+02	-1.918E+03	0.000E+00	1.412E+02	-1.474E+03	-3.249E+03
3.240E+02	-1.343E+03	0.000E+00	-1.570E+02	-1.422E+03	-2.922E+03
3.120E+02	-8.500E+02	0.000E+00	-3.689E+02	-1.369E+03	-2.588E+03
3.000E+02	-5.009E+02	0.000E+00	-5.275E+02	-1.316E+03	-2.345E+03
2.880E+02	-1.749E+02	0.000E+00	-5.992E+02	-1.264E+03	-2.038E+03
2.760E+02	7.230E+01	0.000E+00	-6.177E+02	-1.211E+03	-1.756E+03
2.640E+02	2.455E+02	0.000E+00	-5.939E+02	-1.158E+03	-1.507E+03
2.520E+02	3.709E+02	0.000E+00	-5.822E+02	-1.106E+03	-1.317E+03
2.400E+02	4.282E+02	0.000E+00	-5.059E+02	-1.053E+03	-1.131E+03
2.280E+02	4.392E+02	0.000E+00	-4.183E+02	-1.000E+03	-9.794E+02
2.160E+02	4.483E+02	0.000E+00	-3.567E+02	-9.477E+02	-8.551E+02
2.040E+02	3.995E+02	0.000E+00	-2.662E+02	-8.951E+02	-7.618E+02
1.920E+02	3.340E+02	0.000E+00	-1.821E+02	-8.424E+02	-6.906E+02
1.800E+02	2.874E+02	0.000E+00	-1.234E+02	-7.898E+02	-6.258E+02
1.680E+02	2.135E+02	0.000E+00	-6.330E+01	-7.371E+02	-5.869E+02
1.560E+02	1.428E+02	0.000E+00	-1.633E+01	-6.845E+02	-5.580E+02
1.440E+02	9.406E+01	0.000E+00	1.486E+01	-6.318E+02	-5.229E+02
1.320E+02	4.275E+01	0.000E+00	3.518E+01	-5.792E+02	-5.012E+02
1.200E+02	4.536E+01	0.000E+00	1.813E+01	-5.265E+02	-4.630E+02
1.080E+02	-1.694E+01	0.000E+00	4.729E+01	-4.739E+02	-4.435E+02
9.600E+01	-3.000E+01	0.000E+00	4.055E+01	-4.212E+02	-4.107E+02
8.400E+01	-3.822E+01	0.000E+00	3.413E+01	-3.686E+02	-3.706E+02
7.200E+01	-3.173E+01	0.000E+00	2.176E+01	-3.159E+02	-3.259E+02
6.000E+01	-2.396E+01	0.000E+00	1.250E+01	-2.633E+02	-2.767E+02
4.800E+01	-1.460E+01	0.000E+00	3.088E+00	-2.106E+02	-2.221E+02
3.600E+01	-5.647E+00	0.000E+00	-2.496E+00	-1.580E+02	-1.661E+02
2.400E+01	0.000E+00	0.000E+00	0.000E+00	-1.053E+02	-1.053E+02
1.200E+01	0.000E+00	0.000E+00	0.000E+00	-5.265E+01	-5.265E+01

S IN INCHES
MERIDIONAL FORCE IN POUND/INCH

S	NHCON	NPCON	NMCON	NPPCON	NCON
3.772E+02	4.932E+02	0.000E+00	-1.630E+00	-8.274E+02	-3.359E+02
3.720E+02	2.961E+02	0.000E+00	9.157E+01	-8.161E+02	-4.285E+02
3.600E+02	1.783E+02	0.000E+00	1.349E+02	-7.898E+02	-4.766E+02
3.480E+02	1.017E+02	0.000E+00	1.815E+02	-7.634E+02	-4.803E+02
3.360E+02	1.852E+01	0.000E+00	1.945E+02	-7.371E+02	-5.241E+02
3.240E+02	-4.481E+01	0.000E+00	1.948E+02	-7.108E+02	-5.608E+02
3.120E+02	-8.926E+01	0.000E+00	1.847E+02	-6.845E+02	-5.890E+02
3.000E+02	-1.321E+02	0.000E+00	1.943E+02	-6.581E+02	-5.959E+02
2.880E+02	-1.478E+02	0.000E+00	1.703E+02	-6.318E+02	-6.090E+02
2.760E+02	-1.504E+02	0.000E+00	1.432E+02	-6.055E+02	-6.127E+02
2.640E+02	-1.434E+02	0.000E+00	1.151E+02	-5.792E+02	-6.075E+02
2.520E+02	-1.522E+02	0.000E+00	1.050E+02	-5.528E+02	-6.001E+02
2.400E+02	-1.318E+02	0.000E+00	7.636E+01	-5.265E+02	-5.817E+02
2.280E+02	-1.081E+02	0.000E+00	5.110E+01	-5.002E+02	-5.572E+02
2.160E+02	-1.011E+02	0.000E+00	3.689E+01	-4.739E+02	-5.380E+02
2.040E+02	-7.487E+01	0.000E+00	1.738E+01	-4.475E+02	-5.050E+02
1.920E+02	-5.604E+01	0.000E+00	2.387E+00	-4.212E+02	-4.695E+02
1.800E+02	-3.790E+01	0.000E+00	-8.499E+00	-3.949E+02	-4.413E+02
1.680E+02	-1.877E+01	0.000E+00	-1.585E+01	-3.686E+02	-4.032E+02
1.560E+02	-3.891E+00	0.000E+00	-1.936E+01	-3.422E+02	-3.655E+02

1.440E+02	6.293E+00	0.000E+00	-2.387E+01	-3.159E+02	-3.335E+02
1.320E+02	1.350E+01	0.000E+00	-2.161E+01	-2.896E+02	-2.977E+02
1.200E+02	1.709E+01	0.000E+00	-1.693E+01	-2.633E+02	-2.631E+02
1.080E+02	1.993E+01	0.000E+00	-1.872E+01	-2.369E+02	-2.337E+02
9.600E+01	1.702E+01	0.000E+00	-1.085E+01	-2.106E+02	-2.044E+02
8.400E+01	1.675E+01	0.000E+00	-8.012E+00	-1.843E+02	-1.755E+02
7.200E+01	1.059E+01	0.000E+00	-2.746E+00	-1.580E+02	-1.501E+02
6.000E+01	7.590E+00	0.000E+00	3.298E-01	-1.316E+02	-1.237E+02
4.800E+01	1.736E+00	0.000E+00	2.783E+00	-1.053E+02	-1.008E+02
3.600E+01	-1.673E+00	0.000E+00	4.174E+00	-7.898E+01	-7.648E+01
2.400E+01	0.000E+00	0.000E+00	0.000E+00	-5.265E+01	-5.265E+01
1.200E+01	0.000E+00	0.000E+00	0.000E+00	-2.633E+01	-2.633E+01

CYLINDRICAL SHELL

LAMDA= 2.002E-02 H= 2.992E+02 (This input is incorrect see P. 153 for H= -2.992E+02)

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	-5.488E+03	0.000E+00	-5.488E+03
1.200E+01	-5.219E+03	2.796E+03	-2.423E+03
2.400E+01	-4.579E+03	4.272E+03	-3.069E+02
3.600E+01	-3.767E+03	4.797E+03	1.030E+03
4.800E+01	-2.923E+03	4.686E+03	1.763E+03
6.000E+01	-2.136E+03	4.192E+03	2.056E+03
7.200E+01	-1.455E+03	3.506E+03	2.051E+03
8.400E+01	-9.019E+02	2.763E+03	1.862E+03
9.600E+01	-4.778E+02	2.053E+03	1.575E+03
1.080E+02	-1.722E+02	1.428E+03	1.255E+03
1.200E+02	3.244E+01	9.111E+02	9.436E+02
1.320E+02	1.561E+02	5.089E+02	6.650E+02
1.440E+02	2.184E+02	2.140E+02	4.324E+02
1.560E+02	2.371E+02	1.215E+01	2.492E+02
1.680E+02	2.271E+02	-1.138E+02	1.133E+02
1.800E+02	2.004E+02	-1.814E+02	1.900E+01
1.920E+02	1.656E+02	-2.067E+02	-4.107E+01
2.040E+02	1.291E+02	-2.036E+02	-7.450E+01
2.160E+02	9.477E+01	-1.832E+02	-8.841E+01
2.280E+02	6.495E+01	-1.539E+02	-8.899E+01
2.400E+02	4.061E+01	-1.219E+02	-8.126E+01

KO= 5.000E+02 EO 3.000E+06 HO= 0.000E+00 S= 2.400E+02 T= 3.070E-01 P= 0.000E+00
ALPHA4= 4.823E+03 K= 0.000E+00 KK= 0.000E+00

IN STMT 521 PROGRAM RETURNS FROM MAIN PROCEDURE.

CYLINDRICAL SHELL

LAMDA = 2.002E-02 H = -2.992E+02

S IN INCHES

BENDING MOMEMTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	-5.488E+03	-0.000E+00	-5.488E+03
1.200E+01	-5.219E+03	-2.796E+03	-8.015E+03
2.400E+01	-4.579E+03	-4.272E+03	-8.851E+03
3.600E+01	-3.767E+03	-4.797E+03	-8.564E+03
4.800E+01	-2.928E+03	-4.686E+03	-7.809E+03
6.000E+01	-2.136E+03	-4.192E+03	-6.328E+03
7.200E+01	-1.455E+03	-3.508E+03	-4.961E+03
8.400E+01	-9.019E+02	-2.763E+03	-3.665E+03
9.600E+01	-4.778E+02	-2.053E+03	-2.530E+03
1.080E+02	-1.722E+02	-1.428E+03	-1.600E+03
1.200E+02	3.244E+01	-9.111E+02	-8.787E+03
1.320E+02	1.561E+02	-5.089E+02	-3.508E+03
1.440E+02	2.184E+02	-2.140E+02	4.400E+00
1.560E+02	2.371E+02	-1.215E+01	2.250E+02
1.680E+02	2.271E+02	1.138E+02	3.409E+02
1.800E+02	2.004E+02	1.814E+02	3.818E+02
1.920E+02	1.656E+02	2.067E+02	3.723E+02
2.040E+02	1.291E+02	2.036E+02	3.327E+02
2.160E+02	9.477E+01	1.832E+02	2.780E+02
2.280E+02	6.495E+01	1.539E+02	2.188E+02
2.400E+02	4.061E+01	1.219E+02	1.625E+02

NON-MONOLITHIC JOINT   WELL-GRADED GRAVEL ELASTIC FOUNDATION,

CONICAL SHELL: 2NDORDER ASYMPTOTIC SOLUTION

C1= 4.159E-09H + -1.312E-08P + 0.000E+00M   C2=-4.143E-09H + 1.307E-08P + 0.000E+00M

EDHCON= 3.496E+01   EDHCYL=-4.320E+02

EDPCON=-1.532E+04   EDPCYL=-9.375E+04

EDMCON= 0.000E+00   EDMCYL= 0.000E+00

EDPPCON=-2.757E+04

EXHCON= 3.376E+00   EXHCYL= 8.650E+00

EXPCON= 5.435E+01   EXPCYL= 3.906E+02

EXMCON= 0.000E+00   EXMCYL= 0.000E+00

EXPPCON=-2.899E+03

M= 0.000E+00   H=-1.088E+02   P= 2.500E+02

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MHCON	MPCON	MMCON	MCON
3.772E+02	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3.720E+02	1.440E+02	1.045E+03	0.000E+00	1.189E+03
3.600E+02	3.180E+02	2.307E+03	0.000E+00	2.625E+03
3.480E+02	3.410E+02	2.472E+03	0.000E+00	2.813E+03
3.360E+02	2.860E+02	2.078E+03	0.000E+00	2.364E+03
3.240E+02	2.050E+02	1.488E+03	0.000E+00	1.693E+03
3.120E+02	1.260E+02	9.170E+02	0.000E+00	1.043E+03
3.000E+02	6.400E+01	4.650E+02	0.000E+00	5.290E+02
2.880E+02	2.100E+01	1.570E+02	0.000E+00	1.780E+02
2.760E+02	-3.000E+00	-2.200E+01	0.000E+00	-2.500E+01
2.640E+02	-1.400E+01	-1.050E+02	0.000E+00	-1.190E+02
2.520E+02	-1.700E+01	-1.250E+02	0.000E+00	-1.420E+02
2.400E+02	-1.300E+01	-1.100E+02	0.000E+00	-1.230E+02
2.280E+02	-1.100E+01	-8.200E+01	0.000E+00	-9.300E+01
2.160E+02	-7.000E+00	-5.200E+01	0.000E+00	-5.900E+01
2.040E+02	-3.000E+00	-2.800E+01	0.000E+00	-3.100E+01
1.920E+02	-1.000E+00	-1.000E+01	0.000E+00	-1.100E+01
1.800E+02	0.000E+00	0.000E+00	0.000E+00	0.000E+00
1.680E+02	0.000E+00	5.000E+00	0.000E+00	5.000E+00
1.560E+02	0.000E+00	6.000E+00	0.000E+00	6.000E+00
1.440E+02	0.000E+00	6.000E+00	0.000E+00	6.000E+00
1.320E+02	0.000E+00	4.000E+00	0.000E+00	4.000E+00
1.200E+02	0.000E+00	3.000E+00	0.000E+00	3.000E+00
1.080E+02	0.000E+00	1.000E+00	0.000E+00	1.000E+00
9.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7.200E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6.000E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4.800E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3.600E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2.400E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

1.200E+01	0.000E+00	-4.000E+00	0.000E+00	-4.000E+00

S IN INCHES
TRANSVERSE SHEAR FORCE IN POUND/INCH

S	QHCON	QPCON	QMCON	QCON
3.772E+02	-3.287E+01	-2.383E+02	0.000E+00	-2.712E+02
3.720E+02	-2.296E+01	-1.664E+02	0.000E+00	-1.894E+02
3.600E+02	-6.151E+00	-4.459E+01	0.000E+00	-5.074E+01
3.480E+02	3.161E+00	2.292E+01	0.000E+00	2.608E+01
3.360E+02	7.065E+00	5.122E+01	0.000E+00	5.828E+01
3.240E+02	7.603E+00	5.511E+01	0.000E+00	6.272E+01
3.120E+02	6.406E+00	4.644E+01	0.000E+00	5.285E+01
3.000E+02	4.597E+00	3.332E+01	0.000E+00	3.792E+01
2.880E+02	2.836E+00	2.056E+01	0.000E+00	2.339E+01
2.760E+02	1.435E+00	1.040E+01	0.000E+00	1.183E+01
2.640E+02	4.734E-01	3.453E+00	0.000E+00	3.930E+00
2.520E+02	-8.317E-02	-6.029E-01	0.000E+00	-6.861E-01
2.400E+02	-3.412E-01	-2.473E+00	0.000E+00	-2.815E+00
2.280E+02	-4.026E-01	-2.918E+00	0.000E+00	-3.321E+00
2.160E+02	-3.558E-01	-2.579E+00	0.000E+00	-2.935E+00
2.040E+02	-2.647E-01	-1.919E+00	0.000E+00	-2.184E+00
1.920E+02	-1.692E-01	-1.227E+00	0.000E+00	-1.396E+00
1.800E+02	-8.952E-02	-6.490E-01	0.000E+00	-7.385E-01
1.680E+02	-3.277E-02	-2.376E-01	0.000E+00	-2.703E-01
1.560E+02	1.880E-03	1.363E-02	0.000E+00	1.551E-02
1.440E+02	1.899E-02	1.376E-01	0.000E+00	1.566E-01
1.320E+02	2.409E-02	1.747E-01	0.000E+00	1.988E-01
1.200E+02	2.221E-02	1.610E-01	0.000E+00	1.832E-01
1.080E+02	1.714E-02	1.243E-01	0.000E+00	1.414E-01
9.600E+01	1.133E-02	8.258E-02	0.000E+00	9.391E-02
8.400E+01	6.327E-03	4.586E-02	0.000E+00	5.219E-02
7.200E+01	2.532E-03	1.836E-02	0.000E+00	2.089E-02
6.000E+01	3.125E-05	2.266E-04	0.000E+00	2.578E-04
4.800E+01	-1.474E-03	-1.068E-02	0.000E+00	-1.216E-02
3.600E+01	-2.599E-03	-1.884E-02	0.000E+00	-2.144E-02
2.400E+01	-5.176E-03	-3.752E-02	0.000E+00	-4.270E-02
1.200E+01	-2.622E-02	-1.901E-01	0.000E+00	-2.163E-01

S IN INCHES
W PREFIX FOR NORMAL DISPLACEMENTS IN INCH/INCH
D PREFIX FOR HORIZONTAL DISPLACEMENTS IN INCH/INCH
PF FOR NORMAL SOIL REACTIONS IN POUND/INCH SQUARE

S	WHCON	WPCON	WMCON	WPPCON	WCON	PF	DCON
3.772E+02	-4.196E-03	-3.041E-02	0.000E+00	-1.697E-02	-5.158E-02	2.579E+01	-1.559E-02
3.720E+02	-3.511E-03	-2.545E-02	0.000E+00	-1.706E-02	-4.602E-02	2.301E+01	-1.391E-02
3.600E+02	-2.069E-03	-1.500E-02	0.000E+00	-1.728E-02	-3.434E-02	1.717E+01	-1.038E-02
3.480E+02	-9.733E-04	-7.056E-03	0.000E+00	-1.749E-02	-2.552E-02	1.276E+01	-7.711E-03
3.360E+02	-2.864E-04	-1.839E-03	0.000E+00	-1.769E-02	-1.981E-02	9.904E+00	-5.986E-03
3.240E+02	1.395E-04	1.011E-03	0.000E+00	-1.789E-02	-1.674E-02	8.370E+00	-5.059E-03
3.120E+02	3.040E-04	2.204E-03	0.000E+00	-1.808E-02	-1.557E-02	7.787E+00	-4.707E-03
3.000E+02	3.248E-04	2.334E-03	0.000E+00	-1.826E-02	-1.559E-02	7.793E+00	-4.710E-03
2.880E+02	2.719E-04	1.971E-03	0.000E+00	-1.844E-02	-1.619E-02	8.097E+00	-4.894E-03
2.760E+02	1.936E-04	1.403E-03	0.000E+00	-1.860E-02	-1.700E-02	8.501E+00	-5.138E-03
2.640E+02	1.181E-04	8.553E-04	0.000E+00	-1.875E-02	-1.777E-02	8.886E+00	-5.371E-03
2.520E+02	5.844E-05	4.236E-04	0.000E+00	-1.888E-02	-1.840E-02	9.199E+00	-5.560E-03
2.400E+02	1.812E-05	1.313E-04	0.000E+00	-1.899E-02	-1.884E-02	9.422E+00	-5.695E-03

2.280E+02	-4.998E-06	-3.623E-05	0.000E+00	-1.909E-02	-1.913E-02	9.564E+00	-5.781E-03
2.160E+02	-1.524E-05	-1.105E-04	0.000E+00	-1.916E-02	-1.928E-02	9.641E+00	-5.827E-03
2.040E+02	-1.721E-05	-1.247E-04	0.000E+00	-1.919E-02	-1.934E-02	9.668E+00	-5.844E-03
1.920E+02	-1.475E-05	-1.069E-04	0.000E+00	-1.920E-02	-1.932E-02	9.660E+00	-5.839E-03
1.800E+02	-1.000E-05	-7.681E-05	0.000E+00	-1.916E-02	-1.925E-02	9.625E+00	-5.818E-03
1.680E+02	-6.436E-06	-4.666E-05	0.000E+00	-1.908E-02	-1.913E-02	9.567E+00	-5.783E-03
1.560E+02	-3.098E-06	-2.246E-05	0.000E+00	-1.895E-02	-1.897E-02	9.487E+00	-5.734E-03
1.440E+02	-8.334E-07	-6.041E-06	0.000E+00	-1.876E-02	-1.877E-02	9.384E+00	-5.672E-03
1.320E+02	4.379E-07	3.175E-06	0.000E+00	-1.853E-02	-1.853E-02	9.264E+00	-5.599E-03
1.200E+02	9.492E-07	6.881E-06	0.000E+00	-1.829E-02	-1.828E-02	9.142E+00	-5.526E-03
1.080E+02	9.656E-07	7.000E-06	0.000E+00	-1.814E-02	-1.813E-02	9.067E+00	-5.480E-03
9.600E+01	7.187E-07	5.210E-06	0.000E+00	-1.835E-02	-1.834E-02	9.171E+00	-5.543E-03
8.400E+01	3.800E-07	2.755E-06	0.000E+00	-1.962E-02	-1.962E-02	9.809E+00	-5.929E-03
7.200E+01	5.970E-08	4.323E-07	0.000E+00	-2.400E-02	-2.400E-02	1.200E+01	-7.252E-03
6.000E+01	-1.817E-07	-1.317E-06	0.000E+00	-3.796E-02	-3.796E-02	1.898E+01	-1.147E-02
4.800E+01	-3.124E-07	-2.265E-06	0.000E+00	-8.593E-02	-8.593E-02	4.296E+01	-2.597E-02
3.600E+01	-2.923E-07	-2.122E-06	0.000E+00	-2.890E-01	-2.890E-01	1.445E+02	-8.734E-02
2.400E+01	6.937E-08	5.029E-07	0.000E+00	-1.621E+00	-1.621E+00	8.106E+02	-4.899E-01
1.200E+01	2.871E-06	2.081E-05	0.000E+00	-2.809E+01	-2.809E+01	1.404E+04	-8.489E+00

S IN INCHES
ROTATIONS IN RADIAN/INCH

S	XHCON	XPCON	XMCON	XPPCON	XCON
3.772E+02	-1.333E-04	-9.663E-04	0.000E+00	1.828E-05	-1.081E-03
3.720E+02	-1.302E-04	-9.436E-04	0.000E+00	1.813E-05	-1.056E-03
3.600E+02	-1.069E-04	-7.747E-04	0.000E+00	1.774E-05	-8.638E-04
3.480E+02	-7.506E-05	-5.441E-04	0.000E+00	1.730E-05	-6.018E-04
3.360E+02	-4.525E-05	-3.280E-04	0.000E+00	1.680E-05	-3.565E-04
3.240E+02	-2.214E-05	-1.605E-04	0.000E+00	1.623E-05	-1.664E-04
3.120E+02	-6.701E-06	-4.838E-05	0.000E+00	1.557E-05	-3.971E-05
3.000E+02	2.057E-06	1.491E-05	0.000E+00	1.481E-05	3.178E-05
2.880E+02	5.899E-06	4.276E-05	0.000E+00	1.393E-05	6.260E-05
2.760E+02	6.623E-06	4.801E-05	0.000E+00	1.292E-05	6.755E-05
2.640E+02	5.693E-06	4.129E-05	0.000E+00	1.173E-05	5.871E-05
2.520E+02	4.145E-06	3.005E-05	0.000E+00	1.034E-05	4.454E-05
2.400E+02	2.591E-06	1.878E-05	0.000E+00	8.717E-06	3.009E-05
2.280E+02	1.334E-06	9.673E-06	0.000E+00	6.804E-06	1.781E-05
2.160E+02	4.647E-07	3.369E-06	0.000E+00	4.549E-06	8.382E-06
2.040E+02	-4.856E-08	-3.520E-07	0.000E+00	1.895E-06	1.494E-06
1.920E+02	-2.887E-07	-2.093E-06	0.000E+00	-1.217E-06	-3.598E-06
1.800E+02	-3.488E-07	-2.528E-06	0.000E+00	-4.827E-06	-7.704E-06
1.680E+02	-3.092E-07	-2.241E-06	0.000E+00	-8.921E-06	-1.147E-05
1.560E+02	-2.234E-07	-1.656E-06	0.000E+00	-1.334E-05	-1.523E-05
1.440E+02	-1.433E-07	-1.039E-06	0.000E+00	-1.759E-05	-1.877E-05
1.320E+02	-7.235E-08	-5.281E-07	0.000E+00	-2.035E-05	-2.095E-05
1.200E+02	-2.376E-08	-1.722E-07	0.000E+00	-1.832E-05	-1.852E-05
1.080E+02	4.755E-09	3.449E-08	0.000E+00	-3.248E-06	-3.208E-06
9.600E+01	1.703E-08	1.235E-07	0.000E+00	4.642E-05	4.656E-05
8.400E+01	1.837E-08	1.331E-07	0.000E+00	1.908E-04	1.910E-04
7.200E+01	1.358E-08	9.845E-08	0.000E+00	6.156E-04	6.158E-04
6.000E+01	6.378E-09	4.624E-08	0.000E+00	1.985E-03	1.985E-03
4.800E+01	-6.709E-10	-4.864E-09	0.000E+00	7.233E-03	7.233E-03
3.600E+01	-5.541E-09	-4.017E-08	0.000E+00	3.444E-02	3.444E-02
2.400E+01	-3.770E-09	-2.733E-08	0.000E+00	2.835E-01	2.835E-01
1.200E+01	5.037E-08	3.695E-07	0.000E+00	9.496E+00	9.496E+00

CYLINDRICAL SHELL

LAMDA= 2.002E-02   H=-1.088E+02

S IN INCHES
BENDING MOMENTS IN INCH-POUND/INCH

S	MMCYL	MHCYL	MCYL
0.000E+00	0.000E+00	0.000E+00	0.000E+00
1.200E+01	0.000E+00	-1.017E+03	-1.017E+03
2.400E+01	0.000E+00	-1.553E+03	-1.553E+03
3.600E+01	0.000E+00	-1.744E+03	-1.744E+03
4.800E+01	0.000E+00	-1.704E+03	-1.704E+03
6.000E+01	0.000E+00	-1.524E+03	-1.524E+03
7.200E+01	0.000E+00	-1.275E+03	-1.275E+03
8.400E+01	0.000E+00	-1.005E+03	-1.005E+03
9.600E+01	0.000E+00	-7.466E+02	-7.466E+02
1.080E+02	0.000E+00	-5.191E+02	-5.191E+02
1.200E+02	0.000E+00	-3.313E+02	-3.313E+02
1.320E+02	0.000E+00	-1.851E+02	-1.851E+02
1.440E+02	0.000E+00	-7.781E+01	-7.781E+01
1.560E+02	0.000E+00	-4.418E+00	-4.418E+00
1.680E+02	0.000E+00	4.138E+01	4.138E+01
1.800E+02	0.000E+00	6.594E+01	6.594E+01
1.920E+02	0.000E+00	7.515E+01	7.515E+01
2.040E+02	0.000E+00	7.402E+01	7.402E+01
2.160E+02	0.000E+00	6.681E+01	6.681E+01
2.280E+02	0.000E+00	5.597E+01	5.597E+01
2.400E+02	0.000E+00	4.431E+01	4.431E+01

K0= 5.000E+02   E0 3.000E+06   H0= 8.000E+00   S= 2.400E+02   T= 3.070E-01   P= 2.500E+02
ALPHA4= 4.823E+03   K= 3.144E-02   KK= 3.170E-02

IN STMT 521 PROGRAM RETURNS FROM MAIN PROCEDURE.

TOTAL SOIL BEARING PRESSURE CALCULATIONS

(1) Uniform Soil Bearing Pressure Foundation (Flügge's Solution)

From Chapter V.2.4,

$$p_f = p_f^P + p_f^W$$

where,

$$p_f^P = 2P\ (L\cos\phi)$$

$$p_f^W - \gamma_W(d_o - s\sin\phi)$$

Using SIMPSON's rule:

$$p_f = 2\pi r_o \frac{h}{3L}[(p_{f_o} + 4p_{f_1} + p_{f_2})s_1 + (p_{f_2} + 4p_{f_3} + p_{f_4})s_3 + \cdots]$$

$$= 23.99 \times 210{,}137.76 \text{ lb}$$

$$= 5{,}041{,}204.86 \text{ lb or } 5.04 \times 10^6 \text{ lb (vertical)}$$

Total Applied Loads:

Water Loads + Concrete Loads

$= 4.54 \times 10^6 + 5.80 \times 10^5$ lb

$= 5.12 \times 10^6$ lb (vertical)

Note: (1) 1.5% error is related to the method of approximation used.

(2) The base is wet, hence no frictional force, $p_s = 0$.

Simpson's Approximation

Monolithic Joint, Uniform Soil Bearing Pressure Foundation, Flügge's Solution

n	s (in.)	p (lb/in^2)	4p (lb/in^2)	$(p_{f_n} + 4p_{f_{n+1}} + p_{f_{n+2}})s_{n+1}$ (lb/in)
0		10.07		
1	372	10.13	40.52	22,636.20
2		10.26		
3	348	10.39	41.56	21,694.32
4		10.52		
5	324	10.65	42.60	20,703.60
6		10.78		
7	300	10.92	43.68	19,653.00
8		11.05		
9	276	11.18	44.72	18,514.08
10		11.31		
11	252	11.44	45.76	17,297.28
12		11.57		
13	228	11.70	46.80	16,005.60
14		11.83		
15	204	11.97	47.88	14,649.24
16		12.10		
17	180	12.23	48.92	13,208.40
18		12.36		
19	156	12.49	49.96	11,690.64
20		12.62		
21	132	12.75	51.00	10,098.00
22		12.88		
23	108	13.01	52.01	8,428.32
24		13.15		
25	84	13.28	53.12	6,693.12
26		13.41		
27	60	13.54	54.16	4,874.40
28		13.67		
29	36	13.80	55.20	2,980.80
30		13.93		
31	12	14.06	56.24	1,010.76
32		(14.06)		
				+) 210,137.76

Note: ( ) are estimated values by linear extrapolation.

(2) Elastic Foundation (Second Order Asymptotic Solution)

From Chapter V.2.4

p_f (normal) = -kw

where,

$w = w^c + w^p$

k = 500 pci

Using SIMPSON's rule:

$$p_f \text{ (normal)} = 2\pi r_o \frac{h}{3L} [(p_{f_o} + 4p_{f_1} + p_{f_2})s_1 + (p_{f_2} + 4p_{f_3} + p_{f_4})s_3 + \ldots]$$

= 23.99 x 225,477.72 lb

= 5,409,210.45 lb or 5.41 x 10^6 lb (normal)

p_f (vertical) = p_f (normal) x cosϕ = 5.16 x 10^6 lb (vertical)

Total Applied Load = 5.12 x 10^6 lb (vertical)

Note: (1) 0.8% error is related to the method of approximation used and the linear extrapolation required to estimate the soil bearing pressures around the apex zone (s = 0 to 3 ft.). This shall not be used to conclude that the Second Order Asymptotic Solution is more accurate than Flügge's Solution.

(2) The base is wet, hence no frictional force, p_s = 0.

Simpson's Approximation

Monolithic Joint, Well-graded Gravel Elastic Foundation
Second Order Asymptotic Solution

n	s (in.)	p (lb/in.^{2})	4p (lb/in.^{2})	$(p_{f_n} + 4p_{f_{n+1}} + p_{f_{n+2}}) s_{n+1}$ (lb/in.)
0		20.48		
1	372	19.24	76.96	42,214.56
2		16.04		
3	348	13.12	52.48	27,645.12
4		10.92		
5	324	9.50	38.01	18,691.56
6		8.76		
7	300	8.50	33.99	15,385.50
8		8.54		
9	276	8.73	34.91	14,465.71
10		8.97		
11	252	9.19	36.76	13,884.69
12		9.37		
13	228	9.51	38.02	12,991.67
14		9.59		
15	204	9.63	38.52	11,779.78
16		9.64		
17	180	9.62	38.45	10,376.82
18		9.56		
19	156	9.49	37.95	8,876.40
20		9.39		
21	132	9.27	37.07	7,339.46
22		9.15		
23	108	9.07	36.28	5,896.48
24		9.17		
25	84	9.81	39.24	5,074.61
26		12.00		
27	60	18.98	75.92	7,852.80
28		42.96		
29	36	(68.00)	272.00	14,614.56
30		(91.00)		
31	12	(115.00)	468.00	8,388.00
32		(140.00)		
				+) 225,477.72

Note: ( ) are estimated values by linear extrapolation.