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PUBLIC AND PRIVATE HEALTH CARE FINANCING WITH ALTERNATE PUBLIC RATIONING RULES

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Abstract. We develop a model to analyze health care financing arrangements and under alternative public sector rationing rules. Health care is demanded by individuals varying in income and severity of illness. There is a limited supply of health care resources used to treat individuals, causing some individuals to go untreated. We examine outcomes under full public finance, full private finance, and mixed, parallel public and private finance under two rationing rules for the public sector: needs-based rationing and random rationing. Insurers (both public and private) must bid to obtain the necessary health care resources to treat their beneficiaries. While the public insurer's ability-to-pay is limited by its (fixed) budget, the private insurer's willingness-to-pay reflects the individuals' willingness-to-pay for care. When permitted, the private sector supplies supplementary health care to those willing and able to pay. The introduction of private insurance diverts treatment from relatively poor to relatively rich individuals. Moreover, if the public insurer allocates care according to need, the average severity of the untreated is higher in a mixed system than in a pure public system. While we can unambiguously sign most comparative static effects for a general set of distribution functions for income and severity, a complete analysis of the relationship between public sector rationing and the scope for a private health insurance market requires distributional assumptions. For a bivariate uniform distribution function we find that the private health insurance market is smaller when the public sector rations according to need as compared to random allocation of health care.

Keywords: health care financing, rationing rules

JEL Classifications: I11, I18

1. Introduction

Debate about the relative sizes and roles of the public and private sectors in financing health care persists in most countries. Although public financing dominates in most industrialized countries, pressure on public health care budgets has prompted increasing calls for an expanded role for private health care finance (see, e.g., Mossialos et al. 2002; OECD 2004). This expanded private sector role in health care finance takes three basic forms: increased user charges within public insurance systems; a more limited basket of services covered by public insurance (either through de-listing or greater scrutiny of potential new services); and greater reliance on parallel private finance whereby individuals can purchase private insurance to cover the costs of obtaining (publicly insured) care outside the public plan. Each of these raises distinct analytic issues for the overall efficiency and equity of health care systems.

The effects of parallel finance on equity and efficiency are particularly debated. Advocates of parallel private finance, for instance, argue that it can reduce wait times within the public system, reduce financial pressure within the public system, increase access to needed care, and increase quality of care (Globerman and Vining 1998; Crowley 2003; Montreal Economic Institute 2005; Esmail 2006). Opponents, of course, argue the opposite: by drawing both resources and support away from the publicly financed system, parallel private finance can increase wait times in the public system, reduce access for many in society, and reduce quality in the public system (Yalnizyan 2006; Canadian Health Coalition 2006).

This debate has taken on increased salience in a number of countries. In Canada, for instance, a recent Supreme Court decision struck down a law prohibiting parallel private insurance based in part on the argument that such insurance would increase access to care without harming the public insurance system (Chaoulli vs Government of Quebec 2005; Flood et al. 2005).[1] Australia and Portugal subsidize the purchase of parallel private insurance in the belief that such a policy will reduce wait times in its public system, increase

[1]Canada is an outlier internationally in the extent to which it limits parallel private finance for publicly insured physician and hospital services.

access and reduce public expenditures (Healy et al. 2006; Mossialos and Thomson 2004). The more dominant trend in recent years has been for countries to reduce or eliminate tax subsidies to such private insurance in the belief that such subsidies are not effective, e.g., Austria, Greece, Ireland, Italy, Spain, and the United Kingdom (Mossialos and Thomson 2004).

Increasing our understanding of the effects of parallel finance is therefore of considerable importance. The empirical literature investigating the effects of parallel finance provides limited guidance because it often lacks rigorous design and questionable generalizability across systems that differ in institutional design.[2] A small but growing analytic literature investigates a number of aspects of parallel finance.[3] Iversen (1997) and Olivella (2003), for example, analyze how public sector waiting time changes when a parallel system of finance is instituted and find that waiting time may go up. Hoel and Sæther (2003) argue that public-sector waiting lists may be an effective sorting device facilitating income redistribution. Using an optimal income taxation framework Marchand and Schroyen (2005) show that if income inequality is sufficiently large a mixed health care system (with a large public system) where the 'rich' opt for private care in order to prevent waiting can be socially desirable.

A switch from purely public to parallel finance may trigger behavioral responses by health care providers, especially if the policy change allows physician dual practice. Gonzalez (2004) argues that under parallel finance a physician has an incentive to over-provide public services in order to increase his/her private sector prestige and with it private earnings. Biglaiser and Ma (2006) analyze the quality incentives of physicians and distinguish between dedicated doctors and moonlighters. Even if not in the interest of patients, moonlighters may divert patients to their private practice should they find that beneficial. Brekke and Sørgard (2007) focus on providers' labor supply decisions, highlighting the fact that the public and private sectors compete for the same set of health care resources. Finally, providers may engage in

[2] Tuohy et al. (2004) survey the empirical evidence.

[3] Eggleston and Bir (2006) provide a partial survey of this analytic literature.

cream skimming, where private doctors only select the mild cases as in Barros and Olivella (2005) and Gonzalez (2005).

In this paper we contribute to this analytic literature by developing a simple model to investigate the effects of alternative public and private financing arrangements under different public section rationing rules. We consider that individuals differ in both health status and income and thereby capture the two most important dimensions of concern for equity in health care. With the exception of Iversen (1997) and Olivella (2003) the existing literature has considered only heterogeneity in one or the other. Society does not have enough real health care resources to treat all those who are ill, so each period some individuals must go without treatment. Under these assumptions we examine outcomes under three financing arrangements – public finance only, private finance only, and mixed, parallel public and private finance – and two alternative allocation rules within the public sector – needs-based allocation and random allocation – resulting in five distinct regimes. The analysis focuses on the following outcomes: the average severity of the treated and untreated; the average income of treated and untreated; the price insurers pay for the health care resource; and in the mixed system, the size of the private sector. In addition, in each case we consider the impact on key outcomes of both changes in the amount of available health care resources and the size of the public health care budget.

A joint analysis of alternative financing arrangements and public sector rationing rules has not previously been conducted. Although our basic setup shares some important characteristics with Iversen's (1997) framework, we address different questions and obtain a number of new results. While Iversen (1997) concentrates on wait times under needs-based rationing, we analyze who receives treatment under different institutional designs and demonstrate that outcomes – including the scope for a private health care system – crucially depend on the allocation rule adopted in the public system. Although our focus is not on wait times, our results are directly relevant for this strand of the literature: since the public sector rationing rule is an important determinant of the willingness to pay for private health insurance it will also affect public sector wait times (see the discussion section below for more details).

Additional aspects that differentiate our paper from Iversen (1997) are that (i) we incorporate the competition between public and private insurers for health care resources and (ii) individual severity is not observed ex ante, thereby creating a rationale for private health insurance.

We find that compared to a pure public system the average income of the treated is higher under both a pure private insurance system and a mixed public and private health system. The rich purchase private insurance in order to guarantee treatment. This result holds independently of the rationing rule applied in the public system. In contrast, the average severity of the treated depends on both the public sector's allocation method and the financing of health care. When the public sector allocates according to need, the introduction of private finance decreases the average severity of the treated and increases the average severity of the untreated. To be able to sign all comparative static effects and, more importantly, to facilitate comparison of the key variables under different allocation methods, we examine the special case of uniform distributions for income and for severity. We find that the private market in a mixed system will be smaller when public resources are allocated according to need rather than at random.

The paper is organized as follows. In Section 2 we present our basic framework and discuss the different public rationing rules in Section 3. Section 4 considers health care allocation in a pure public system. We contrast this outcome with a pure private system in Section 5 in order to develop a benchmark for a mixed system analyzed in Section 6. In Section 7 we consider the special case of uniform distributions for income and severity. In Section 8 we offer a comprehensive discussion of our results. All proofs are relegated to the Appendix.

2. The Model

Consider a continuum of individuals who differ in both their income and severity levels. For convenience we normalize the total population to unity. Incomes, Y, are distributed in the population on the interval $[\underline{Y}, \overline{Y}]$ according to the cumulative distribution function $G(Y)$ with $G(\underline{Y}) = 0$, $G(\overline{Y}) = 1$, and $G'(Y) = g(Y) > 0$ for all Y. Severity levels, s, are distributed in the population on the interval $[0,1]$ according to the cumulative distribution function $F(s)$ with $F(0)=0$, $F(1)=1$, and $F'(s)=f(s)>0$ for all s. We can think about the total population as follows:

$$\int_{\underline{Y}}^{\overline{Y}} \int_0^1 dF dG = 1.$$

We assume that income and severity are independently distributed.[4] Individuals know their income but are unaware of their severity level. An individual's severity level can be interpreted as the fraction of the individual's income that will be lost if the individual does not receive treatment, i.e., the loss is given by sY.[5] This implies that for a given severity level the income loss from not being treated is greater for high-income individuals.

Individuals can be treated and cured by the application of a health care service. As we will see below, it will be convenient to view this health care service as a specialist service rather than a service offered by a general practitioner. In fact, parallel private insurance is most commonly purchased to gain better access to specialist services (Mossialos and Thomson 2004; Foubister et al. 2006). One unit of health care is necessary and sufficient to cure any individual regardless of severity. Health care cures the patient immediately, so a treated patient suffers no income loss. One unit of a health care service requires one unit of a health care resource. One can think of this resource variously as being health care providers, capital or equipment required to deliver services such as an operating theater, or a health care good such as drugs. Regardless, there are not enough health care resources in the economy to treat everyone.[6]

[4]This assumption makes the model tractable. Income and severity, however, are likely to be negatively correlated. We discuss how such correlation would impact our results in Section 8.

[5]A more general monetary loss function from not being treated could be assumed, for example, $L(s,Y)$. Provided $L_{sY}>0$ all of our results hold. Note that this includes functions in which there is an income-independent severity loss, such as $L(s,Y)=g(s,Y)+k(s)$ with $k'(s)>0$.

[6]Given that the health care resources available are not sufficient to treat all patients, the public insurer has to ration public care. Effectively, there are two waiting times in public system; zero and infinity. Individuals are either treated right away or not treated at all. Standard rationing mechanisms are waiting times and criteria for public-sector treatment eligibility. We consider the latter and analyze the impact of different allocation rules and financing arrangements on the outcome of the health care system. In Section 8 we discuss how our results extend to waiting time frameworks.

There is a fixed supply of health care resources, denoted by H.[7] We assume that $H < 1$ so only a fraction of the total population can be treated with the existing health care resources.[8] These resources are offered on a competitive market. Health care is financed through one or both of a public insurance plan or private insurance. Both public and private insurers contract with suppliers of the health care resource to provide services to their respective beneficiaries. The public insurer bids for the health care resources according to its ability-to-pay, as determined by the public health care budget. Private insurers similarly bid for sufficient resources to cover the insurance contracts they have sold. The private insurers' willingness-to-pay is based on individuals' willingness-to-pay for private insurance that guarantees access to care regardless of severity level. Hence, when both public insurer and private insurers are present, the two sectors compete directly for the limited health care resource. This results in a market-clearing equilibrium price per provider, P, at which all H of the health care resource is allocated across the public and private sectors.[9] We analyze the composition of the untreated and treated populations and investigate how it depends on the health care financing arrangement and the rationing rule used to allocate public resources.[10] We first describe the possible rationing rules used by the public insurer and then consider the different health care financing arrangements.

3. Public Rationing Rules

Given the limited supply of health care resources, the public insurer must decide which persons to treat, that is, how to ration access to publicly insured services. Making this decision requires value judgements that, in general, will involve many dimensions of public

[7]Allowing for an elastic supply of health care resources would complicate the analysis without qualitatively changing our results.

[8]Given our population normalization, one can think of H as the number of individuals who can be treated with the available health care resources.

[9]Private insurers are assumed to be unable to price discriminate so there is a single price for health care resources.

[10]The market structure of private insurance is formally identical to one in which individuals bid directly for contracts with the suppliers of health care resources, i.e., individuals 'self-insure' by paying out of pocket for health care resources (ex ante). For the remainder of the paper we refer to the institutional arrangement as private insurance.

concern, e.g., ethics, equity, and efficiency. The optimal rationing rule will depend in part on the information available to the public insurer.

To fix ideas we consider two extreme rationing rules. First we assume that the public insurer rations health care according to need. That is, the public insurer treats the most severe cases first. Rationing by need requires that the public insurer observe individual severity levels prior to treatment. One could imagine a system with strong General Practitioner (GP) gatekeeping in which access to specialist care requires a referral from a GP: GPs diagnose severity and base referral to specialists on patient severity. Allocation according to need is the stated objective of many publicly financed health care systems (van Doorslaer et al. 1993), and is often motivated by one, or both, of efficiency goals (maximize the health gain with a given set of health care resources) and equity concerns (give priority to those most in need).[11] Need itself has been variously interpreted as an individual's health status (lower health status implies greater need), ability to benefit from health care (greater ability to benefit implies greater need) and the resources required to exhaust benefit from health care (more resources implies greater need) (Culyer and Wagstaff 1993). In our set-up, allocation by severity levels is consistent with allocation according to need under each of these conceptions of need (though by the third definition all individuals would be deemed as having the same need). Allocation according to severity in our context also advances both efficiency and equity; there is no conflict between the two.[12]

Second, we assume that the public insurer rations such that access to services is random. Under this rationing rule, the probability of treatment is independent of both income and severity. Rationing by random allocation would arise if the public insurer was unable to observe individual severities prior to allocating public health care services. One could imagine, for instance, a system in which there is no GP gatekeeping. Rather, individuals have direct access to specialists, who treat individuals on a first-come, first-serve basis. The specialist

[11] Health gain is measured solely by the severity of the individual's illness and not by the income the individual losses if left untreated.

[12] Allocation according to severity in our model is also consistent with allocating health services to reduce (avoidable) inequality in the distribution of health in the population, which has recently been adopted as the stated goal of the UK National Health Service (Hauck et al. 2001).

both diagnoses a patient's severity and treats the patient in a single visit. If people arrive at the specialists at random, allocation will be random.

In reality allocation within publicly financed health systems lies somewhere in between these two extremes. Some randomness of access can arise even in systems that strive to allocate health care according to need. GP gatekeeping is always imperfect even where present; further, in many systems specialists manage their own wait lists, which can vary considerably across specialists; and specialists may have differential access to required resources such as operating theaters. Consequently, even if referral to specialty care is based on severity, in the absence of system-wide coordination less severe patients of specialists with shorter lists can have better access than other, more severe cases of other specialists. Further, need is often ranked only within service areas (e.g., those with heart disease; those with breast cancer), with limited scope to rank need across conditions. So the allocation process ends up being a mixture of our two extreme rules – needs-based allocation and random allocation – where the weight given to each depends on the emphasis placed on need and the specific institutional arrangements of a health care system.

We now consider three alternative financing arrangements: public insurance only (denoted by the subscript "b" below); private insurance only (denoted by the subscript "r" below); and a mixed system of parallel public and private finance (denoted by the subscript "m" below).

4. Public Health Care Finance Only

Assume first that only public insurance is available. The public insurer has an exogenously determined budget, B, measured in dollars and would like to treat as many people as possible.[13] Denote M as the number (or, fraction of population) treated by the public sector. The public insurer's ability-to-pay (ATP) per treatment is given by

$$ATP_b = B/M. \tag{1}$$

[13]The size of the budget can be interpreted as the outcome of some political process that is independent of the problem at hand.

which is increasing in its budget and decreasing in the total number of treatments. Since the public insurer is the only demander of health care resources, the price per provider that clears the health care resource market is given by[14]

$$P_b = \frac{B}{H}. \tag{2}$$

The public insurer purchases all of the available health care resources, so H individuals are treated and $1 - H$ individuals remain untreated. Increasing the supply of the health care resource increases the number of people treated by the public insurer and reduces the equilibrium price the public insurer pays providers for the health care resource. Changes in the public budget, on the other hand, do not affect the number of individuals treated and only affect the rents received by the providers of the health care resource.

Who receives treatment from the public insurer will depend on how the public insurer allocates health care. As discussed above, we consider two allocation mechanisms or rationing rules: rationing by need and rationing by random allocation. Because it is the simpler case to solve, we begin the analysis by considering random allocation.

4.1. **Rationing by Random Allocation.** Let π_b denote the proportion of individuals treated in the public sector when public health care is allocated randomly. Then, we have

$$\pi_b = H. \tag{3}$$

In this case, the probability of treatment, π_b, is equal to H. The expected severity level of those treated (and of those untreated) is simply the unconditional expected severity level, $E(s)$, which by definition is

$$E(s) = \int_0^1 s dF. \tag{4}$$

[14]Throughout the paper we assume that the providers' reservation prices for the health care resource are below the equilibrium price.

4.2. **Rationing by Need.** When the public insurer rations care according to need, it specifies a severity threshold s_b such that all individuals with severity $s \geq s_b$ are treated and all those with $s < s_b$ are not treated. The severity threshold thus satisfies

$$1 - F(s_b) = H \tag{5}$$

which yields

$$s_b = F^{-1}(1 - H). \tag{6}$$

The expected severity conditional on being treated is equal to

$$E(s|s \geq s_b) = \frac{\int_{s_b}^{1} s dF}{1 - F(s_b)} \tag{7}$$

where $1 - F(s_b)$ is the probability of having a severity greater than the threshold. When there are sufficient health care resources available to treat all patients, the severity threshold is zero and the expected severity of the treated is equal to the unconditional expected severity $E(s)$. With $H \in (0, 1)$, however, the severity cut-off s_b will be strictly positive. Since public sector resources are targeted to high-severity patients the expected severity of the treated is greater than the unconditional mean, $E(s|s \geq s_b) > E(s)$.[15] This allows us to state the following:

Result 1. ***Public Health Care Finance Only***

a. *The average severity of those treated will be lower under random rationing than under rationing according to need. Under random allocation, the average severity of those treated is independent of the fixed supply of health care resources, H; under rationing by need the average severity level of those treated is decreasing in H.*

[15] This follows from differentiating (7) with respect to s_b to obtain $\left[f(s_b)/(1 - F(s_b))^2\right] \left[\int_{s_b}^{1} sdF - s_b(1 - F(s_b))\right] > 0$ for $s_b \in (0, 1)$.

b. *Under both random and needs-based rationing, an increase in the supply of health care reduces the equilibrium price of health care and increases the number of individuals that receive treatment.*

c. *Under both random and needs-based rationing there is no relationship between income and treatment.*

d. *Under both random and needs-based rationing an increase in the public budget increases the equilibrium price of health care and has no effect on either the number of individuals who receive treatment or the average severity of those treated.*

5. Private Health Care Finance Only

Assume now health care is financed wholly by private insurance. Prior to learning their severity, individuals can choose to purchase a private insurance policy. Private insurance guarantees treatment regardless of a person's severity level. Private insurers charge individuals actuarially fair premiums, so in this case all individuals face the same insurance premium (i.e., the cost of the health care resource).[16]

Consider individuals with income Y deciding whether to purchase private insurance. At the time of purchase individuals do not know their severity level and therefore can expect to lose $E(s)Y$ of their income if they do not purchase insurance. Suppose further that individuals are risk neutral and care only about their income. It follows that individuals with given income Y will have a maximum willingness-to-pay for treatment of

$$WTP = E(s)Y. \tag{8}$$

Individuals are willing to pay more to guarantee treatment the greater their expected loss without treatment, i.e., the higher their income and/or the higher the unconditional expected severity level.

[16] We are abstracting away from any possible strategic interactions between health care suppliers and private insurers. We discuss the implications of such strategic behaviour in section 8.

The private insurers bid for health care resources based on individuals' willingness-to-pay for private insurance. We can think about P, the price per health care provider, as the price for private health insurance. Using equation (8), all individuals with income higher than $P/E(s)$ would like to purchase insurance. Health care services will be allocated according to maximum willingness-to-pay. Given perfect competition among private insurers, however, individuals who purchase insurance pay the equilibrium price P rather than their maximum willingness-to-pay, and insurance companies make zero profits.[17] Thus, the number of individuals who purchase private insurance at a given price P is given by

$$\int_{P/E(s)}^{\overline{Y}} dG = 1 - G\left(P/E(s)\right). \tag{9}$$

Only H individuals can be treated. The equilibrium price per provider, P_r, that clears the health care resource market is defined implicitly by the following condition

$$1 - G\left(P_r/E(s)\right) = H. \tag{10}$$

Hence, the equilibrium price per provider (or, equivalently for private insurance) is

$$P_r = E(s)G^{-1}(1 - H). \tag{11}$$

Again, the proportion of individuals not being treated is $1-H$. In this case, all individuals with incomes $Y \geq Y_r$ are treated and all those individuals with incomes $Y < Y_r$ are not treated where

$$Y_r = \frac{P_r}{E(s)} = G^{-1}(1 - H). \tag{12}$$

We can now state the following result:

[17] Alternatively, the single price could result from insurance regulation that requires community-rated premiums. Community rating is found in a number of private (and public) insurance markets internationally.

Result 2. ***Private Insurance Only***

a. *The average or expected severity level of those treated is equal to the unconditional expected severity level, $E(s)$, and is independent of the supply of health care resource.*

b. *An increase in the supply of the health care resource reduces the equilibrium price of health care, increases the number of individuals who receive treatment, and reduces the average income of those treated.*

c. *There is a positive association between treatment and income.*

Note that private-only average severity among the treated equals the average severity of those treated in the public-only system with random rationing but is less than that for public-only with needs-based rationing. Further, the income-treatment relationship differs from the public-only case, for which treatment was independent of income.

6. Mixed, Parallel Public and Private Finance

We now consider the case of mixed, parallel finance with both a public insurer and a private insurance sector. Given that individuals know their own income we consider the following sequence of events:

1. Each individual draws a random severity level but does not learn it.
2. The public insurer and the private insurers submit bids for the health care resource (the bids of the private insurers reflect the willingness-to-pay of individuals for treatment)
3. The suppliers allocate health care resources according to the willingness-to-pay of the public and private insurers, and a price per provider is determined that clears the resource market.
4. Individuals who obtain a private insurance contract receive treatment from a supplier contracted with the private insurer. The health care resources secured by the public insurer are used to treat a fraction of those individuals who do not have private insurance.

To determine their willingness-to-pay for insurance individuals must form expectations about the probability of being treated in the public sector and about their severity level if they are not treated in the public system. We assume that all individuals form the same expectations. This expectation will depend on how the public insurer rations the resources it controls. As before, we consider two possibilities: random allocation (i.e., no gatekeeper) and allocation according to need (i.e., GP gatekeeper model). Consider first the case of random rationing.

6.1. **Rationing by Random Allocation.** Under this rationing rule, the expected severity conditional on not being treated is *independent* of the probability of being treated by the public sector. Hence, individuals form expectations about the probability of being treated in the public sector, denoted by π^e, while recognizing that under this type of allocation mechanism their expected severity if they are not treated in the public sector is simply the unconditional expected severity level, $E(s)$, as given by (4).

Consider now individuals with a given income Y deciding whether to purchase private insurance. If they do not purchase private insurance, then their expected income loss will be

$$(1-\pi^e)E(s)Y. \tag{13}$$

It follows that their maximum willingness-to-pay for treatment will be

$$WTP^R = (1-\pi^e)E(s)Y. \tag{14}$$

which is decreasing in their expected probability of being treated in the public sector and increasing in their income.[18]

Using equation (14), the number of individuals who have a maximum willingness-to-pay greater or equal to a given price of private insurance P is given by

$$\int_{P/[(1-\pi^e)E(s)]}^{\overline{Y}} dG = 1 - G\left(\frac{P}{(1-\pi^e)E(s)}\right). \tag{15}$$

[18]The superscript R is used to denote variables in the random allocation scenario.

The public insurer's ability-to-pay is still given by expression (1). The health care resource is allocated to insurers according to their willingness/ability-to-pay. The price paid by insurers (given expectations) that clears the health care resource market is implicitly defined by the following:

$$1 - G\left(\frac{P}{(1-\pi^e)E(s)}\right) + \frac{B}{P} = H \tag{16}$$

Only the relatively rich, $1 - G(\cdot)$, find it beneficial to guarantee themselves access to health care through a private health insurance contract. The relatively poor, $G(\cdot)$, go without private insurance and rely on the public insurer. Since there are not sufficient resources available, public-sector supply, B/P, is only sufficient to treat a fraction of those without private insurance; there remain $1 - H$ untreated individuals. Equation (16) yields a price that is increasing in B and decreasing in both H and π^e.

To have an equilibrium, expectations must be confirmed, that is,

$$\pi^e = \pi, \tag{17}$$

where π is the proportion of individuals in the public system who are actually treated. This proportion is given by the health care resources available to the public system, B/P, divided by the number of individuals who rely solely on the public insurer, $G(\cdot)$:[19]

$$\pi = \frac{B/P}{G\left(P/[(1-\pi^e)E(s)]\right)}. \tag{18}$$

The equilibrium is characterized by equations (16), (17), and (18) and yields equilibrium values for P, π^e, and π. From (17) and (18), there is a unique π such that expectations are

[19] We assume that privately insured individuals obtain services only through the private insurance contract. They do not use the public system. They have not formally opted out because they are assumed to still contribute to public financing, i.e., their purchase of private insurance does not affect the public insurer's budget B.

confirmed.[20] Therefore, the following two conditions can be solved for the equilibrium values $P_m^R(B,H)$ and $\pi_m^R(B,H)$:

$$(19) \qquad 1-H = G\left(\frac{P}{(1-\pi)E(s)}\right) - \frac{B}{P},$$

$$(20) \qquad \pi = \frac{B/P}{G\left(P/[(1-\pi)E(s)]\right)}.$$

A few things are worth noting about the possible equilibria. First, both public and private insurers will be active only if $B/H < (1-H)E(s)\overline{Y}$. If this assumption does not hold, the public insurer would be able to contract with all available suppliers of the health care resource at a price greater than the highest-income individual's maximum willingness-to-pay, thereby out bidding private insurers for health care resources. Another extreme can also never occur, namely, $\pi = 0$. While the private maximum willingness-to-pay is always bounded from above the public ability-to-pay is not. From equation (1) we have that $ATP \to \infty$ as $M \to 0$, that is, at $\pi = 0$ the public insurer would be willing to spend an infinite amount for the first marginal unit, contradicting $\pi = 0$ as an equilibrium.[21] To have an equilibrium, the above curves must cross in (π, P)-space. Both of these curves will be strictly downward-sloping on $\pi \in (0, H)$ so it is possible that they do not cross. It can be shown, however, that if the curves do cross they will cross only once and there will be a unique equilibrium as shown in the Appendix.

To determine how changes in the fixed supply of health care resources or in the size of the public budget affect the equilibrium price and equilibrium probability of treatment in

[20]To see this, note that (17) is simply the 45 degree line in (π^e, π)-space and (18) is a strictly downward sloping curve in (π^e, π)-space. Evaluating (18) at $\pi^e = 0$ and $\pi^e = H$ yields $\pi > 0$ and $\pi < H$, respectively. Therefore, (18) lies above the 45 degree line when $\pi^e = 0$ and lies below the line when $\pi^e = H$. By continuity, (18) will cross the 45 degree line only once in (π^e, π)-space.

[21]This is an artifact of continuity. In a discrete model, the public insurer would be willing to spend its entire budget for the first unit of the health care resource. If the budget is sufficiently large, but not too large, then there will always be an interior solution with both public and private insurers.

the public sector we can totally differentiate (19) and (20) to obtain the following partial derivatives (see Appendix for details):

$$\frac{\partial P_m^R(B,H)}{\partial H} < 0, \frac{\partial P_m^R(B,H)}{\partial B} \gtreqqless 0, \frac{\partial \pi_m^R(B,H)}{\partial H} > 0, \frac{\partial \pi_m^R(B,H)}{\partial B} > 0. \tag{21}$$

As one would expect, an increase in the fixed supply of health care resources reduces the equilibrium price, and increases the equilibrium probability of treatment by the public insurer. When the public insurer's budget increases its ability to pay for health care resources improves and therefore the equilibrium probability of treatment by the public insurer goes up. The effect on the equilibrium price, however, is ambiguous: the increase in the public insurer's ability to pay has a direct positive effect on the price but at the same time there is an indirect effect that works in the opposite direction. The increase in the probability of treatment in the public sector reduces the expected loss when relying on the public system and with it the willingness to pay for private insurance. Since the bids of private insurers reflect the individual willingness to pay the higher public treatment probability softens competition for health care resources. Which of the two effects dominates depends on the assumed income distribution as shown in the Appendix.

How does the population with private insurance compare to those who rely solely on the public insurer? To answer this, we note that individuals with a willingness-to-pay equal or greater than the equilibrium price purchase private insurance (recall that the insurance premium equals the resource price given that each individual requires one unit of health care resource). Denote by Y_m^R the income level of the person with the maximum willingness-to-pay exactly equal to the equilibrium price. Then, those individuals with income $Y \geq Y_m^R$ purchase private insurance and those with $Y < Y_m^R$ do not. Using (14) and (17), Y_m^R is given by

$$Y_m^R = \frac{P_m^R}{(1-\pi_m^R)E(s)}. \tag{22}$$

An increase in either the equilibrium price of the health care resource or the probability of public treatment will increase Y_m^R and reduce the number of people purchasing private insurance. Neither the fixed supply of health care resources nor the public budget has a direct effect on the size of the privately insured population. These variables only indirectly affect the size of the privately insured population through their effects on the equilibrium values of both price and probability of treatment in the public system. Using the expressions from (21), we obtain the following result:[22]

Result 3. ***Parallel Financing, Public Random Rationing***

a. *The average severity of those treated is equal to $E(s)$ and is independent of both the fixed supply of health care resources and the public insurer's budget.*
b. *Under a mixed system of finance with parallel public and private insurance and public random allocation, an increase in the fixed supply of health care resource will reduce the equilibrium price of health care, increase the probability of being served in the public system, and increase (decrease) the number of individuals purchasing private insurance when $\pi_m^R > (<) 1/2$.*
c. *There will be a positive association between income and treatment arising from those who purchase private insurance.*
d. *An increase in the public insurer's budget will have an ambiguous effect on the price of health care, increase the probability of treatment in the public system and reduce the number of individuals purchasing private insurance.*

Compared to the public-only case, allowing for private insurance bids resources away from the public sector and gives higher-income individuals the opportunity to guarantee themselves treatment at the expense of reduced access for lower-income individuals.

[22]See Appendix for details.

6.2. **Rationing By Need.** Suppose now that the public insurer rations public health care resources according to need. We again begin by deriving individuals' maximum willingness-to-pay for private insurance. Under this rationing rule, the expected severity conditional on not being treated is no longer independent of the probability of being treated in the public sector. Individuals form expectations about the severity threshold for treatment in the public system, denoted by s_m^e, which determines their expectations about both the probability of not being treated by the public insurer, $F(s_m^e)$, and their expected severity if they are not treated in the public system, $E(s|s < s_m^e)$, where by definition

$$E(s|s < s_m^e) = \frac{\int_0^{s_m^e} sdF}{F(s_m^e)}. \tag{23}$$

Again, we assume all individuals form the same expectations regarding the public sector.

Consider now individuals with given income Y. If they do not purchase private insurance, then their expected income loss will be $F(s_m^e)E(s|s < s_m^e)Y$ which can rewritten using equation (23) as

$$\left(\int_0^{s_m^e} sdF\right) Y. \tag{24}$$

Therefore, the individuals' maximum willingness to pay is

$$WTP^N = \left(\int_0^{s_m^e} sdF\right) Y \tag{25}$$

which is increasing in both the (expected) severity threshold for treatment in the public system, s_m^e, and their income.[23]

Using equation (25), the number of individuals who have a maximum willingness-to-pay greater or equal to a given price of private insurance P is given by

$$\int_{P/\int_0^{s_m^e} sdF}^{\overline{Y}} dG = 1 - G\left(\frac{P}{\int_0^{s_m^e} sdF}\right). \tag{26}$$

[23]The superscript N is used to denote variables in the need-based allocation scenario.

The public insurer's maximum ability-to-pay is still given by expression (1). Health care resources are allocated according to insurers' willingness-to-pay and the equilibrium price (given expectations) that clears the health care resource market is implicitly defined by

$$1 - G\left(\frac{P}{\int_0^{s_m^e} sdF}\right) + \frac{B}{P} = H. \tag{27}$$

Again, to have an equilibrium, expectations must be confirmed, that is,

$$s_m^e = s_m \tag{28}$$

where s_m is implicitly defined by the following expression

$$\frac{B}{P} = (1 - F(s_m))G\left(\frac{P}{\int_0^{s_m^e} sdF}\right). \tag{29}$$

As there are not enough resources to treat everyone in the population, the number of individuals relying in the public sector, $G(\cdot)$, will exceed the publicly available resources, B/P. The public insurer then uses the severity threshold, s_m, to adjust the public sector treatment eligibility criterion and with it the probability of public treatment, $1 - F(s_m)$, such that it uses up all of its available resources and treats those with the highest severity levels. To have an equilibrium with both the public and private insurers it must be that this probability of treatment by the public insurer is positive but less than H.[24]

Given that there is a unique s_m such that expectations are confirmed,[25] the following two conditions can be solved for the equilibrium $P_m^N(B, H)$ and $s_m^N(B, H)$:

$$1 - H = G\left(\frac{P}{\int_0^{s_m} sdF}\right) - \frac{B}{P}, \tag{30}$$

[24]In other words, $1 - F(s_m) \in (0, H)$ since if $F(s_m) = 1$, then no one is treated in the public system and if $F(s_m) = 1 - H$ then the no one receives private insurance. Assumptions on the public insurer's budget are made to ensure an interior equilibrium is feasible, that is, $(B/H) < \left(\int_0^{F^{-1}(1-H)} sdF\right)\overline{Y}$.

[25]To see this, note that (28) is just the 45 degree line in (s_m^e, s_m)-space and (29) is a strictly decreasing curve in (s_m^e, s_m)-space. To show that there is a unique s_m such that expectations are confirmed, we need to show that at some s_m^e (29) is above the 45 degree line and at some other s_m^e (29) is below the 45 degree line. Suppose $s_m^e = 1$, then from (29) $1 - F(s_m) = P/E(s) > 0$ and $s_m < 1$. As s_m^e gets very close to zero, the term $P/(\int_0^{s_m^e} sdF)$ gets very large and $G(\cdot)$ approaches one. From (29), $1 - F(s_m)$ approaches B/P and since under a mixed, parallel finance system, $B/P \leq H < 1$, s_m will be strictly greater than zero.

$$\text{(31)} \qquad \frac{B}{P} = (1 - F(s_m))G\left(\frac{P}{\int_0^{s_m} sdF}\right).$$

Both (30) and (31) exhibit a strictly positive relationship between P and s_m but the shape of these curves will depend on the assumed distributions of both severity levels and income, so the curves could conceivably cross zero, one or multiple times in (s_m, P)-space. As such, for any given severity and income distributions we cannot say definitively how changes in the fixed supply of health care resources or the size of the public insurer's budget affect the equilibrium price and equilibrium probability of treatment in the public sector unless we make some assumptions about the relative slopes of the two curves if, and when, they cross.[26]

We can, however, say the following: the public threshold will be higher under the mixed, parallel finance with both a public insurer and a private insurance sector than when there is only a public insurer. With a public insurer only and rationing by need, $1-F(s_b) = H$. Under a mixed system with rationing by need, it follows from (30) and (31) that $1-F(s_m^N)G(Y_m^N) = H$. Together, these two expressions imply that $s_m^N > s_b$. Consequently, expected severity of those untreated in the public sector, $E(s|s \leq s_m^N)$, will be higher than under a public-only system of finance.[27] Because higher-income individuals purchase private insurance; under the mixed system, those who rely on the public insurer will on average have lower-incomes than when there is no private insurance.

As before, we can determine the income cut-off for purchasing private insurance denoted in this case by Y_m^N. Under rationing by need, individuals with $Y \geq Y_m^N$ will purchase private insurance and those with $Y < Y_m^N$ will not purchase private insurance where using (25) and (28)

$$\text{(32)} \qquad Y_m^N = \frac{P_m^N(B,H)}{\int_0^{s_m^N(B,H)} sdF}.$$

[26]See Appendix for details.

[27]Differentiating the expression for $E(s|s \leq s_m)$ given by (23) with respect to s_m yields $[f(s_m)/F(s_m)^2][s_m F(s_m) - \int_0^{s_m} sdF] > 0$.

Not surprisingly, an increase in the equilibrium price of the health care resource or a reduction in expected loss of not being treated in the public system will reduce private insurance purchases. We are unable to state definitively how the parameters H and B affect private insurance purchases. While we cannot rule out multiple equilibria for any distribution functions, we find a unique solution for uniform distributions and obtain unambiguous comparative statics in Section 7 below.

Result 4. ***Parallel Financing, Public Needs-Based Rationing***

a. *The average severity of those treated is higher than under random rationing with mixed finance but less than needs-based rationing under public-only insurance.*
b. *The impact of a change in the fixed supply of the health care resource on the equilibrium price of health care, the probability of treatment in the public sector, and the number of individuals purchasing private insurance is determined only by making specific distributional assumptions.*
c. *There is a positive association between income and treatment arising from those who purchase private insurance.*
d. *The impact of a change in the public sector budget on the equilibrium price of health care, the probability of treatment in the public sector, and the number of individuals purchasing private insurance is determined only by making specific distributional assumptions.*

Under a mixed system of finance, the average incomes of those individuals treated will be higher than under a public-only system of finance regardless of how the public insurer rations health care. The average severity of those not treated in the public sector, however, will be higher (unchanged) when the public insurer rations health care by need (randomly).

7. Example

To ease comparison of the two rationing scenarios, we now turn to the special case of independent uniform distributions for both income and severity levels.[28] This allows us to obtain explicit expressions for equilibrium prices and quantities. Assume that $\underline{Y} = 0$. We have that $F(s) = s$, $f(s) = 1$, $G(Y) = Y/\overline{Y}$, and $g(Y) = 1/\overline{Y}$. Further,

$$E(s) = \int_0^1 sdF = s^2/2|_0^1 = 1/2$$

and

$$\int_0^{\bar{s}} sdF = \bar{s}^2/2.$$

Consider each of the three financing arrangements in turn.

7.1. **Public Health Care Finance Only.**

As in the general model, the equilibrium price per provider is given by $P_b = B/H$ and the probability of treatment under random rationing is $\pi_b = H$. With a uniform severity distribution and rationing by need, access to public health care is restricted to those individuals with severity levels at least as large as $s_b = 1-H$. Under both rationing rules, the probability of receiving treatment is equal to H. The average or expected severity of those treated under random rationing is $1/2$ and the expected severity of those treated under rationing by need is $[\int_{1-H}^1 sdF]/H = (1-(1-H)^2)/(2H) = (2-H)/2 > 1/2$. Result 1 holds.

7.2. **Private Health Care Finance Only.**

With a uniform severity distribution, individuals' maximum willingness-to-pay for private health insurance will be equal to one half of their income. The price that clears the health care resources market is equal to $P_r = (1-H)\overline{Y}/2$. All individuals with incomes at least as large as $Y_r = (1-H)\overline{Y}$ purchase private insurance and receive health care services. All those individuals with income less than Y_r do not receive

[28]Solving the model with uniform distributions for both income and severity is relatively straightforward. Alternate income distributions, such as a Pareto distribution, would complicate the analysis without providing additional insight.

any health care. The expected severity of those treated is equal to 1/2. Likewise, Result 2 holds.

7.3. **Mixed, Parallel Public and Private Health Care Finance.** We consider each of the public rationing rules in turn.

7.3.1. *Rationing by Random Allocation.* The equilibrium conditions determining P and π are given by (19) and (20). Solving, and using $E(s) = 1/2$, we obtain

$$P_m^R = (1 - H)\overline{Y}/2, \tag{33}$$

$$\pi_m^R = \frac{2B}{2B + (1 - H)^2\overline{Y}}. \tag{34}$$

Under this type of public rationing rule, the expected severity of those treated both in the public and the private sectors is the same and equal to 1/2. The equilibrium price for a unit of health care resource is also the same as under private-only health care finance. The main difference between this mixed system and the public-only system is that individuals with incomes greater than

$$Y_m^R = \frac{2B}{(1 - H)} + (1 - H)\overline{Y} \tag{35}$$

purchase private insurance. Under a public-only system, the average or expected income of those treated is equal to $\overline{Y}/2$ whereas under this mixed health care system, the average income of those treated privately is equal to $(\overline{Y} + Y_m^R)/2$ and the expected income of someone who remains in the public sector is $Y_m^R/2$. Since those in the public sector are equally likely to be treated, the average or expected income of those treated under the mixed system is higher than in a purely public system. Introducing a private health care insurance alongside a public insurer does not change the number of individuals who receive treatment due to the fixed supply of health care resources; nor does it change the expected severity of those treated. It does, however, reduce the likelihood that a lower-income person receives health care.

The comparative statics in this case are straightforward. Differentiating (33) and (34), we obtain

$$\frac{dP_m^R}{dH} < 0, \frac{dP_m^R}{dB} = 0, \frac{d\pi_m^R}{dH} > 0, \frac{d\pi_m^R}{dB} > 0. \tag{36}$$

An increase in the fixed supply of health care resources reduces the price of the health care resource and increases the probability of treatment in the public sector. These two effects counteract one another such that the effect of an increase in H on equation (35) or the demand for private insurance is ambiguous and will depend on the assumed parameter values. An increase in the size of the public budget has no effect on the equilibrium price but increases the probability of public treatment. Therefore, an increase in B reduces the number of individuals purchasing private health care, that is, Y_m^R is increasing in B.

7.3.2. *Rationing By Need.* The equilibrium conditions determining P and s_m are given by (30) and (31). Solving, and using $E(s) = 1/2$, we obtain

$$P_m^N = (1 - H)\overline{Y}/2 - \frac{B}{1 - H}, \tag{37}$$

$$s_m^N = 1 - \frac{2B}{(1 - H)^2 \overline{Y}}. \tag{38}$$

When the public insurer allocates by need, the expected severity of those treated through private insurance is $1/2$ and the expected severity of those treated in the public sector is $(1+s_m^N)/2 > 1/2$. The availability of private insurance gives higher-income individuals access to health care regardless of severity. Since higher income individuals face *ex ante* a greater expected loss they are willing to pay more for private insurance. As such, those unable to afford private insurance rely on the public insurer. The income cut-off for private insurance is given by

$$Y_m^N = \frac{(1 - H)\overline{Y}}{1 - \frac{2B}{(1-H)^2\overline{Y}}}. \tag{39}$$

Consequently, the average income of those treated will be higher than in the public-only system. Further, under rationing by need the average severity of those treated will be higher. Introducing private finance alongside a public insurer in this model does not change the number of individuals who receive treatment (by assumption) but it does change the expected severity of those treated, i.e., the average severity of the treated drops and the average severity of the untreated rises.

The comparative statics in this case are also straightforward. Differentiating (37) and (38), we have

$$\frac{dP_m^N}{dH} < 0, \frac{dP_m^N}{dB} < 0, \frac{ds_m^N}{dH} < 0, \frac{ds_m^N}{dB} < 0. \tag{40}$$

The equilibrium price and the public severity threshold are all decreasing in both the fixed supply of health care resources as well as the size of the public budget.

7.3.3. *Comparison of Outcomes with Different Rationing Rules.* We can also compare the outcomes across the two different types of public sector rationing rules. Using the above expressions we obtain

$$P_m^R > P_m^N,$$

$$\pi_m^R < 1 - F(s_m^N),$$

$$Y_m^R < Y_m^N.$$

More people buy private insurance when the public insurer rations randomly than when it rations by need. Intuitively, the expected loss (given income) is lower under rationing by need so individuals are willingly to pay less for insurance. Consequently, more people rely on the public insurer under rationing by need. In addition the probability of being treated in the public system will be higher.

Another key difference between the outcomes with the different rationing rules is the effect of a change in the size of the public insurer's budget. Under random rationing, changing the public budget has no effect on the equilibrium price of the health care resource; in contrast, under rationing by need the equilibrium price is decreasing in the size of the public budget. Owners of health care resources (e.g., providers) would strictly prefer the mixed health care system under random rationing as it earns them the same rents as private-only financing system.

Recall that in a public-only system changing the rationing rule does not change either the price of the health care resource or the probability of treatment. Introducing private insurance alongside the public sector affects the outcomes of interest differently depending on the assumed rationing rule. In particular, we would expect to see fewer people purchasing private insurance, suppliers of the fixed health care resource earning smaller rents, and those who rely on public insurance facing a greater probability of being treated when the public insurer allocates according to need rather than allocating health care randomly.

8. Discussion

Overview of Findings. Table 1 summarizes key results from the model. The first row indicates how the average severity of those who receive treatment compares across models. Because treatment restores everyone to full health, the average severity can serve as an index of the total amount of health produced with the available resources. The average severity of treated, and by implication the total amount of health produced, is highest under public-only financing with needs-based rationing. Next is mixed public and private finance with needs-based rationing in the public sector. This generates less health than public only with needs-based rationing because some resources go to those with private insurance, for which access is based on willingness-to-pay rather than severity. Finally, private-only, public-only with random rationing and mixed public and private with random public-sector rationing all generate equal amounts of health. The health production under these arrangements are all equal because of the assumed independence of income and severity, which means that any arrangement in which access is determined by something other than severity will result in the same expected health impact.

Average severity of treated in population, $\overline{s}$	$\overline{s}_b^N > \overline{s}_m^N > \overline{s}_r = \overline{s}_b^R = \overline{s}_m^R$
Average income of treated in population, $\overline{y}$	$\overline{y}_r > \overline{y}_m^R > \overline{y}_m^N > \overline{y}_b^R = \overline{y}_b^N$
Impact of increase in supply of health care resource	• increases the number of people treated in all regimes • $P_b^R \downarrow, P_b^N \downarrow, P_r \downarrow, P_m^R \downarrow, P_m^N \downarrow\uparrow$ • impact on $\overline{s}$ depends on financing and allocation method • impact on $\overline{y}$ depends on financing and allocation method
Impact of increase in public sector budget	$P_b^R \uparrow, P_b^N \uparrow, P_m^R \downarrow\uparrow, P_m^N \downarrow\uparrow$

Notes: $\overline{s}$ and $\overline{y}$ denote the average severity of the treated and the average income of the treated, respectively. The scripts and superscripts correspond to those used in the text.

Table 1. Comparison of the key outcomes across the five regimes.

Because severity maps directly into need, the ranking according to the average severity of the treated also provides the ranking for need-related equity. That is, public-only with needs-based rationing would be judged most equitable according to the criterion of allocation according to needs, while private only, mixed random and public-only random are ranked least equitable by this criterion.

The second row summarizes our findings with respect to income-related equity. In this case, the average income of the treated serves as an index of income-related equity. Equity is achieved if there is no relationship between income and use. Not surprisingly, the private-only arrangement, in which access is determined solely by willingness-to-pay, is least equitable. Mixed financing with random rationing is next, followed by mixed financing with needs-based rationing. Mixed financing with random rationing is less equitable than mixed financing with needs-based rationing because the private insurance sector is larger under the former. The most equitable regimes are public only with either random or needs-based rationing. From the perspective of income-related equity the rationing rule does not matter in a public only system because income does not play a role in either case.

The next two rows summarize the impacts of increasing the supply of health care resources and increasing the public health care budget. Increasing the supply of the health care resource increases the proportion of the society that receives treatment regardless of the financing arrangement. It also decreases the price of the health care resource in all cases except the mixed system under needs-based allocation, for which the impact is ambiguous.[29] The impact of an increase of the health care resource on the average severity of those treated and the average income of those treated depends on the specific financing/rationing combination under consideration. For public-only finance with needs-based rationing, for instance, an increase in the supply of the health care resource will cause the average severity of both the treated and untreated to fall and the average income of the treated and untreated to be unchanged (and simply equal to the mean income in society). Under public-only finance with random rationing, however, an increase in the supply of the health care resource has no impact on either the average severity of the treated or the average income of the treated (and untreated). Increasing the supply in the private only system has no impact on the average severity of those treated, but it does lower the average incomes of both the treated and untreated. Finally, under a mixed system, the impact depends on which insurer – the public or the private – gets the marginal unit of the resource.

Increasing the public budget under public-only finance has no real impact; it simply increases the price of the health care resource and therefore the rents that accrue to providers. Finally, the impact of a budget increase under mixed finance is ambiguous. The intuition is straightforward: An increase in the public insurers budget directly increases the the public ability to pay for health care resources. This direct effect on the equilibrium price is clearly positive. But there is an additional indirect effect that works in the opposite direction. When the public insurer attracts more resources more patients can be treated in the public system, i.e. the probability of treatment in the public sector goes up. This, in turn, reduces the expected loss when relying on the public system and with it the willingness to pay for

[29]Note that we have assumed zero price-elasticity of supply, so the change in price does not affect the quantity supplied, i.e. there is no market entry (or exit).

private insurance. Since the bids of private insurers reflect the individual willingness to pay the higher public treatment probability softens competition for health care resources. Which of the two effects dominates depends on the distributional assumptions. For our example we found that these two effects exactly offset one another under random rationing while the negative effect dominates under needs-based allocation.

In our model, therefore, the system of financing and the public-sector rationing rule interact to generate the ultimate system effects on health production in society and income-related inequity of access. The introduction of parallel private insurance alongside a public health care system will create an income gradient in access to health as those with higher incomes have both greater means and greater incentive to purchase private insurance that guarantees access to care. But the magnitude of this gradient depends in part on the public sector rationing rule: other things equal, for individuals private insurance is more desirable under random rationing than needs-based rationing. Consequently, the size of the private insurance sector is larger under random rationing and so will be the income gradient in access. The impact of parallel finance on overall system health production also depends in part on the rationing rule used by the public sector. The introduction of private finance diverts resources from the public sector to the private sector, reducing the public sector's capacity to provide services. The opportunity cost (measured in health) of those resources in the public sector is greater when the public sector rations according to need than when it rations randomly.

Our model assumes (largely for reasons of tractability) that income and severity are independently distributed. It is well-documented that, in reality, there is a large negative association between income and health status. Across many societies, the poor systematically have lower average health status (Evans et al. 1994). Such a negative association would tend to exacerbate the negative health and equity effects associated with parallel private finance when the public sector rations according to need but leave unchanged our conclusions when the public sector rations randomly.[30]

[30]The precise effect would depend on the strength of the negative association between health and income as well as the relative magnitudes of the income and need-related elasticities of demand for private insurance.

Our Analysis and Systems of Health Care Finance and Delivery. Evidence indicates that the demand for supplemental private insurance is strongly income-related, especially where the individual market dominates the insurance sector (Barret and Conlon 2001; Besley et al. 1999; Besley 2001; Propper 2000; Mossialos and Thomson 2004). Evidence regarding the relationship between wait times and demand for insurance is inconclusive, in part because of the difficulty of teasing apart the causal relationship. Our model suggests an additional factor that could affect the demand for parallel private insurance – the public sector rationing rule. Anecdotal evidence indicates that individuals are, within limits, willing to wait for a service if they believe that the public system is rationing according to need and their wait reflects the more serious needs of others. If, however, there is a perception that some are getting quicker access for reasons other than need, the tolerance for waiting falls and they are more willing to support private options. Hence, demand for privately financed care depends not just on the length of the wait, but on the process for allocating access.

The model predictions are broadly consistent with empirical analyses of income-related equity in the utilization of health care. Research based on the methods developed as part of the 'ECuity project' document that, for specialist services, countries with the largest parallel private sectors to the public or social insurance system demonstrate the highest degrees of pro-rich income-related inequity in use (van Doorslaer et al. 2004).

Our model has assumed two sets of passive stakeholders: suppliers of health care and private insurers. In reality their behaviour plays an important role in determining outcomes under alternative financing and allocation arrangements. In both cases, strategic response by these groups could exacerbate some of the effects of mixed systems of finance. If suppliers are able to establish dual practices in which they can selectively treat patients in the public system and in their private-pay practice, the strategic incentive to selectively recommend privately paid care for those who can afford it would strengthen the income gradient in access to care and weaken the relationship between need and use. Such strategic behaviour has been a persistent concern in parallel systems of finance (Mossialos and Thomson 2004; Tuohy et al. 2004). Private insurers that offer supplementary parallel insurance tend to focus on a small number relatively simple procedures, leaving the complicated, higher-level care for the public system (Mossialos and Thomson 2004). They can further attempt to cream-skim the relatively healthy within any risk category. The net effect of these and related responses is difficult determine, but many such strategic responses would likely exacerbate both the efficiency and equity concerns associated with private insurance. In this sense, our analysis can be seen as conservative since it ignores such responses.

The role of public sector waiting time. One of the main insights provided by our analysis is the close connection between health care financing and the public sector allocation rule. When waiting time is introduced to our framework there are essentially two closely related dimensions the public insurer has think about: waiting list admissions and individual waiting time. In a system without gatekeepers no severity information is available, one might expect waiting list admissions to be random and waiting time will be uniform. In contrast, (some) severity information is available in gatekeeping systems and needs-based allocation is feasible. This will typically lead to public waiting list admission of the most severe cases only. Additionally waiting times may not be uniform; when waiting costs are increasing with severity it may be desirable to have prioritization, i.e., lower than average waiting time for the most severe cases on the waiting list. This, in turn, would imply longer than average wait time for relatively mild cases. The willingness to pay for private insurance will depend on the allocation method adopted. How they relate to one another depends on several things: how waiting lists are managed; whether or not total waiting time in the public system is independent of the severity of publicly treated individuals; whether or not there are enough health care resources to treat all individuals (if yes, then all individuals would be admitted to the public waiting list); how severity and waiting time translate into actual (and expected) waiting cost.

To find out more about how the public allocation method affects the scope for a private health care market in parallel to the public system in the presence of (managed) waiting list thus requires careful modeling of the queuing procedure. This is beyond the scope of the current paper and is therefore left for future research.

Appendix

Mixed, Parallel Public and Private Finance with Random Rationing. Define the following two implicit functions using (19) and (20):

$$Z^1(P,\pi;B,H) = G\left(\frac{P}{(1-\pi)E(s)}\right) - \frac{B}{P} - (1-H)$$

$$Z^2(P,\pi;B,H) = \pi - \frac{B}{P}\frac{1}{G\left(P/[(1-\pi)E(s)]\right)}$$

The equilibrium is characterized by

$$Z^1(P,\pi;B,H) = 0$$

$$Z^2(P,\pi;B,H) = 0$$

Denote the explicit solution to these two conditions as $P_m^R = P^*(B,H)$ and $\pi_m^R = \pi^*(B,H)$. By the Implicit Function Theorem, we know that these explicit equations and subsequent partial derivatives are well-defined if the determinant of the Jacobian matrix is non-zero. The determinant of the Jacobian is

$$\det J = \begin{vmatrix} Z_P^1 & Z_\pi^1 \\ Z_P^2 & Z_\pi^2 \end{vmatrix}$$

where $Z_x = \partial Z/\partial x$ and from the above expressions

$$Z_P^1 = \frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{(1-\pi)E(s)} + \frac{B}{P^2} > 0$$

$$Z_\pi^1 = \frac{g\left(\frac{P}{(1-\pi)E(s)}\right)P}{(1-\pi)^2E(s)} > 0$$

$$Z_P^2 = \frac{B}{P^2}\frac{1}{G((P/[(1-\pi)E(s)])} + \frac{B}{P}\frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{[G((P/[(1-\pi)E(s)])]^2}\frac{1}{(1-\pi)E(s)} > 0$$

$$Z_\pi^2 = 1 + \frac{B}{P}\frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{[G((P/[(1-\pi)E(s)])]^2}\frac{P}{(1-\pi)^2E(s)} > 0$$

Substituting in the above expressions and using the equilibrium condition $Z^2(P,\pi;B,H)=0$, we have

$$\det J = Z_P^1 Z_\pi^2 - Z_\pi^1 Z_P^2 = \frac{B}{P^2} + \frac{g}{E(s)} > 0.$$

To solve for the partial derivatives, we differentiate the two equilibrium conditions and use Cramer's Rule to obtain

$$P_H^* = \frac{1}{\det J}\left[-Z_H^1 Z_\pi^2 + Z_H^2 Z_\pi^1\right] < 0, \tag{A.1}$$

$$P_B^* = \frac{1}{\det J}\left[-Z_B^1 Z_\pi^2 + Z_B^2 Z_\pi^1\right] \gtreqless 0, \tag{A.2}$$

$$\pi_H^* = \frac{1}{\det J}\left[-Z_H^2 Z_P^1 + Z_H^1 Z_P^2\right] > 0, \tag{A.3}$$

$$\pi_B^* = \frac{1}{\det J}\left[-Z_B^2 Z_P^1 + Z_B^1 Z_P^2\right] > 0. \tag{A.4}$$

where the remaining partial derivatives are

$$Z_H^1 = 1, \quad Z_H^2 = 0$$

$$Z_B^1 = -\frac{1}{P}, \quad Z_B^2 = -\frac{1}{PG\left(P/[(1-\pi)E(s)]\right)}.$$

To determine the sign of P_B^*, we can substitute in the relevant expressions to obtain the following:

$$\begin{aligned}
P_B^* &= \frac{1}{\det J}\left[-Z_B^1 Z_\pi^2 + Z_B^2 Z_\pi^1\right] \\
&= \frac{1}{\det J}\left[\frac{1}{P} Z_\pi^2 - \frac{1}{PG} Z_\pi^1\right] \\
&= \frac{1}{\det J}\left[\frac{1}{P}\left(1 + \frac{B}{P}\frac{g}{G^2}\frac{P}{(1-\pi)^2 E(s)}\right) - \frac{1}{PG}\frac{gP}{(1-\pi)^2 E(s)}\right] \\
&= \frac{1}{\det J}\left[\frac{1}{P} + \frac{B}{P}\frac{g}{G^2}\frac{1}{(1-\pi)^2 E(s)} - \frac{1}{G}\frac{g}{(1-\pi)^2 E(s)}\right] \\
&= \frac{1}{\det J}\frac{1}{P}\left[1 - \frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{G\left(\frac{P}{(1-\pi)E(s)}\right)}\frac{P}{(1-\pi)E(s)}\right] \gtreqless 0
\end{aligned}$$

The equilibrium price will be increasing (decreasing) in B if $[dG(x)/dx][x/G(x)]$ is less than (greater than) one. Under a uniform distribution, $[dG(x)/dx][x/G(x)] = 1$.

Determination of the Relative Slopes. Using $Z^1(P,\pi) = 0$ and $Z^2(P,\pi) = 0$ to solve for π as a function of P yields $\pi^i(P)$ for $i = 1, 2$ where B and H have been suppressed. Differentiating each expressions and using the above expressions yields

$$\frac{d\pi^1}{dP} = -\frac{Z_P^1}{Z_\pi^1} < 0,$$

$$\frac{d\pi^2}{dP} = -\frac{Z_P^2}{Z_\pi^2} < 0,$$

and

$$\frac{d\pi^1}{dP} < \frac{d\pi^2}{dP}.$$

An equilibrium is given by $\pi^1(P) = \pi^2(P)$. Therefore, if the two curves cross in (π, P)-space, $\pi^1(P)$ will be steeper than $\pi^2(P)$ and they can only cross once.

Comparative Statics of the Private Insurance Income Cut-off.

$$Y_m^R(B,H) = \frac{P^*(B,H)}{(1-\pi^*(B,H))E(s)}$$

$$\begin{aligned}
\frac{\partial Y_m^R}{\partial H} &= P_H^* \frac{1}{(1-\pi)E(s)} + \frac{P}{(1-\pi)^2 E(s)}\pi_H^* \\
&= \frac{1}{\det J}\frac{1}{(1-\pi)E(s)}\left[-Z_\pi^2 + \frac{P}{(1-\pi)}Z_P^2\right] \\
&= \frac{1}{\det J}\frac{1}{(1-\pi)E(s)}\left[-1 + \frac{B}{P}\frac{1}{G((P/[(1-\pi)E(s)])\,(1-\pi)}\right] \\
&= \frac{1}{\det J}\frac{1}{(1-\pi)^2 E(s)}\,(2\pi-1) \gtreqless 0, \\
\frac{\partial Y_m^R}{\partial B} &= P_B^* \frac{1}{(1-\pi)E(s)} + \frac{P}{(1-\pi)^2 E(s)}\pi_B^* \\
&= \frac{1}{\det J}\frac{1}{(1-\pi)E(s)}\left[-Z_B^1 Z_\pi^2 + Z_B^2 Z_\pi^1 + \frac{P}{(1-\pi)}\left[-Z_B^2 Z_P^1 + Z_B^1 Z_P^2\right]\right] \\
&= \frac{1}{\det J}\frac{1}{(1-\pi)E(s)}\left[Z_B^1\left(\frac{P}{(1-\pi)}Z_P^2 - Z_\pi^2\right) + Z_B^2\left(Z_\pi^1 - \frac{P}{(1-\pi)}Z_P^1\right)\right] \\
&= \frac{1}{\det J}\frac{1}{(1-\pi)E(s)}\left[\frac{1}{P}\left(1 - \frac{B}{P(1-\pi)G}\right) + \frac{B}{P^2 G(1-\pi)}\right] \\
&= \frac{1}{\det J}\frac{1}{(1-\pi)E(s)}\frac{1}{P} > 0,
\end{aligned}$$

From the above expressions, we have that Y_m^R will be increasing (decreasing) in H if $\pi > (<)1/2$ and increasing in B.

Mixed, Parallel Public and Private Finance with Need-Based Rationing. Define the following two implicit functions using (30) and (31):

$$M^1(P, s_m; B, H) = G\left(\frac{P}{\int_0^{s_m} s dF}\right) - \frac{B}{P} - (1-H)$$

$$M^2(P, s_m; B, H) = (1 - F(s_m))G\left(\frac{P}{\int_0^{s_m} s dF}\right) - \frac{B}{P}$$

The equilibrium is characterized by

$$M^1(P, s_m; B, H) = 0$$

$$M^2(P, s_m; B, H) = 0$$

Denote the explicit solution to these two conditions as $P_m^N = P^*(B, H)$ and $s_m^N = s^*(B, H)$. By the Implicit Function Theorem, we know that these explicit equations and subsequent partial derivatives are well-defined if the determinant of the Jacobian matrix is non-zero. The determinant of the Jacobian in this case is

$$\det J = \begin{vmatrix} M_P^1 & M_{s_m}^1 \\ M_P^2 & M_{s_m}^2 \end{vmatrix}$$

where $M_x = \partial M/\partial x$ and from the above expressions

$$M_P^1 = \frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right)}{\int_0^{s_m} sdF} + \frac{B}{P^2} > 0$$

$$M_{s_m}^1 = -\frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right) P s_m f(s_m)}{\left(\int_0^{s_m} sdF\right)^2} < 0$$

$$M_P^2 = (1 - F(s_m))\left[\frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right)}{\int_0^{s_m} sdF}\right] + \frac{B}{P^2} > 0$$

$$M_{s_m}^2 = -f(s_m)G\left(\frac{P}{\int_0^{s_m^e} sdF}\right) - (1 - F(s_m))\frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right) P s_m f(s_m)}{\left(\int_0^{s_m} sdF\right)^2} < 0$$

Substituting in the above expressions and using the equilibrium condition $M^2(P, \pi; B, H) = 0$, we have

$$\det J = M_P^1 M_{s_m}^2 - M_{s_m}^1 M_P^2 \gtreqless 0.$$

$$\text{sign}[\det J] = -\text{sign}\left[\frac{P}{1 - F(s_m)}\frac{dx}{dP} + \frac{F(s_m)}{f(s_m)}\frac{dx}{ds_m} + \frac{G(x)}{g(x)}\right]$$

where $x = P/[\int_0^{s_m} sdF]$.

The sign of the determinant will depend on both the assumed income and severity distributions. The first and the third terms in the expression on the right-hand side were also obtained in the expression for the determinant under rationing by random allocation, noting that $\pi = 1 - F$ and x is the income cut-off for those purchasing private insurance. The second term is new, however, and reflects the fact that a change in the severity threshold affects the individuals expected severity if they remain publicly insured. This term has the opposite sign of the first and third term (which are both positive). That is, holding price fixed, an increase in threshold increases the demand for private insurance (reduces x). Under uniform distributions, this effect dominates and the determinant will be negative.

Assuming that the determinant is negative, $\det J < 0$, we can solve for the partial derivatives by differentiating the two equilibrium conditions and using Cramer's Rule to obtain

$$P_H^* = \frac{1}{\det J}\left[-M_H^1 M_{s_m}^2 + M_H^2 M_{s_m}^1\right] < 0, \tag{A.5}$$

$$P_B^* = \frac{1}{\det J}\left[-M_B^1 M_{s_m}^2 + M_B^2 M_{s_m}^1\right] \gtreqless 0, \tag{A.6}$$

$$s_H^* = \frac{1}{\det J}\left[-M_H^2 M_P^1 + M_H^1 M_P^2\right] < 0, \tag{A.7}$$

$$s_B^* = \frac{1}{\det J}\left[-M_B^2 M_P^1 + M_B^1 M_P^2\right] < 0. \tag{A.8}$$

The above partial would take the opposite sign if instead we assumed that $\det J > 0$.

Determination of the Relative Slopes. Using $M^1(P, s_m) = 0$ and $M^2(P, s_m) = 0$ to solve for s_m as a function of P yields $s_m^i(P)$ for $i = 1, 2$ where B and H have been suppressed. Differentiating each expressions and using the above expressions yields

$$\frac{ds_m^1}{dP} = -\frac{M_P^1}{M_{s_m}^1} > 0,$$

$$\frac{ds_m^2}{dP} = -\frac{M_P^2}{M_{s_m}^2} > 0,$$

and

$$\frac{ds_m^1}{dP} \gtreqless \frac{ds_m^2}{dP}.$$

An equilibrium is given by $s_m^1(P) = s_m^2(P)$. Even if the curves cross only once, it is possible that $s_m^1(P)$ is steeper or flatter than $s_m^2(P)$.

Comparative Statics of the Private Insurance Income Cut-off.

$$Y_m^N(P, s_m) = \frac{P}{\int_0^{s_m} sdF}$$

First assume that $\det J < 0$.

$$\frac{\partial Y_m^N}{\partial H} = P_H^* \frac{1}{\int_0^{s_m} sdF} + \frac{Ps_m f(s_m)}{\left(\int_0^{s_m} sdF\right)^2} s_H^* \gtreqless 0,$$
$$\frac{\partial Y_m^N}{\partial B} = P_B^* \frac{1}{\int_0^{s_m} sdF} + \frac{Ps_m f(s_m)}{\left(\int_0^{s_m} sdF\right)^2} s_B^* \gtreqless 0,$$

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