Haze Removal Based on Chrominance and Sparsity Priors of Natural Images

HAZE REMOVAL BASED ON CHROMINANCE AND SPARSITY PRIORS OF NATURAL IMAGES

BY

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A THESIS

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To my family and friends

Abstract

This thesis addresses the problem of haze removal from a single image. Like all image restoration tasks, haze removal is an underdetermined inverse problem whose solution hinges on valid image priors or models. In this work, two new physically based models are proposed for image dehazing: 1. Gaussian mixture model of chrominance distribution of outdoor scenes; 2. Piecewise linear model of transmittance map. The first model is learnt using a large training set; the second model is derived from observations that object surfaces outdoors are either planar or of small curvatures. These two models can be combined themselves or used in conjunction with other dehazing techniques, such as the dark channel method, to recover the clear image via a sparsity-based image restoration process.

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Notation and abbreviations

GMM: Gaussian Mixture ModelDCP: Dark Channel PriorICA: Independent Component AnalysisHSI: Hue-Saturation-Intensity

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Chapter 1

Introduction

1.1 Problem Background

Photographs of outdoor scenes are often veiled because of undesired atmospheric phenomena like haze, fog, fume, etc. When travelling through atmospheric medium, lights are absorbed and scattered by water droplets, dust, and other particles. The interactions of lights with aerosols degrade image quality, in form of reduced contrast, diluted color saturation, and loss of details. These degradation effects are not only visually unpleasant but also detrimental in many computer vision applications like surveillance, object recognition and classification etc. Those automatic systems usually assume the clear visibility of input images. In case of bad atmospheric conditions where this assumption does not hold, their performances may severely decline. For decades, defogging/dehazing has been studied as a critical problem for removing the weather effects from degraded images.

In addition to removal of atmospheric veils, image dehazing techniques have potential applications in many other domains, for instance, distance measurement. Traditional distance measurement techniques, such as laser ranger and infrared ranger, work poorly in bad weathers due to the absorption and scattering by atmospheric medium. However, most dehazing methods must first reliably estimate the haze transmittance, which correlates with the distance to an object, before removing haze. The same distance estimation techniques can also be employed as a supplement of traditional distance measurement techniques in non-ideal weather conditions.

Moreover, the frameworks for dehazing methods can be generalized for other image restoration problems, such as color demosaicking, white balancing, color image interpolation etc. The demand for superior visual quality along with other versatile applications makes image dehazing an important topic.

1.2 Previous Works

To remove the undesired veil from images, many dehazing techniques have been proposed [1, 2, 3, 4, 5, 6, 7, 8, 9].

The first dehazing method used to remove effects of bad weather is proposed in [1]. In this work, Treibitz and Schechner use two or more images of the same scene with different polarization degrees to recover the contrast and hue of input images. Different polarization degrees are acquired by rotating a polarizing filter attached to the camera.

Shortly afterwards, more dehazing methods based on multiple images are proposed. For example, in [3], two images of the same scene captured under different weather conditions are jointly exploited to estimate the depth map and remove haze.

However, in spite of improved dehazing performance, those methods cannot be applied to many realistic cases such as dynamic scenes. Most of the time, it is difficult to acquire multiple images of the same scene, with different parameters and in different environmental conditions. Recently, there has been a growing interest in single image haze removal.

A well cited technique of haze removal was developed by Fattal [7]. This is an ICA estimation technique to infer the light transmittance in haze, under the assumption that the shading and light transmission functions are statistically independent. Fattal's method works well when the haze density is relatively low, but performs unsatisfactorily if the light attenuation is too much.

The most influential among the techniques for single image haze removal is perhaps the patch-wise method based on a dark-channel prior (DCP) by He et al. [5]. Many variants of the DPC method have been subsequently published [10, 11, 12, 13, 14, 15]. This family of dehazing methods rely on two assumptions: 1. the piecewise smoothness of depth images; 2. the existence of so-called dark points in local patches. However, the second assumption may be significantly off from the physical reality as most colors in outdoor natural scenes are unsaturated (e.g., the sky); chances are that none of the R, G, B values in a patch of the latent image is close to zero. This is why the restored images by the DCP methods are prone to saturation and hue distortions.

Some authors already noticed the color distortions and tried to eliminate them. For example, in [14], Chen et al. employ DCP as a foundation of their dehazing method and use an iterative algorithm to rectify the over-saturated colors. Eventually a relatively ideal compromise between natural color and image definition can be achieved. However this method does not recover the actual color of outdoor scenes. Its result is no more than a "compromise" to please human eyes, which is not the real haze-free images.

Interestingly, in a very recent work, Gibson and Nguyen interpret many previous dehazing methods including Fattal's as embodiments of a common approach to using dark channel prior [13].

1.3 Contribution of the thesis

Previous authors rely on dark points to estimate light transmittance coefficients, which are vital information for haze removal. This thesis introduces a new alternative technique of estimating light transmittance that can complement the DPC methods in regions where dark points are absent. The new technique is conceived from a chrominance aspect of natural image statistics.

High-frequency features in images predominantly result from interactions between lights and surface geometries (discontinuities in particular). But chrominance signals are primarily determined by spectral qualities of object surfaces. As most object surfaces consist of a uniform material or a uniform composite of materials, the two chrominance channels of a color image ought to be piecewise constant. This point is illustrated by an example image of grasses in Fig.1.1: the luminance channel contains busy edges and textures resulted from random orientation and placement of grass blades, whereas the two chrominance channels are almost flat.

Because natural image statistics exhibit much less variability in a 2D chrominance space than in a combined luminance-chrominance space, we explore the chrominance space in search for useful image priors.



Figure 1.1: Example of natural scene in YCbCr space; (a) original RGB image; (b) Y luminance channel; (c) Cb chromiance channel.

For most outdoor scenes, the joint distribution of chrominance values in a patch can be well fit by a Gaussian mixture model (GMM). This is a result of our physical world: the number of different surface materials in outdoor scenes (for examples, urban and rural landscapes) is relatively small, and they have distinct spectral signatures. We build this Gaussian mixture model of chrominance distribution using a training set of fog-free color images. Considering that different types of scenes have different chrominance characteristics, the training images can be classified into different types, say, urban and rural. Multiple GMM models can be built, one for each type. The GMM model is used in the dehazing process to produce maximum likelihood estimates of transmittance coefficients on a patch by patch basis.

In estimating the transmittance map, we apply the DCP and GMM methods judiciously, choosing the one that is more reliable. But there are patches for which none of the two methods can work; This happens if there are no dark points in the patch so that the DCP method does not work; at the same time the color of the input patch is missing in the training set or the input patch is corrupted by noises. In the latter case the GMM model cannot make estimates of sufficiently high likelihood. The question is how to restore a complete transmittance map **T** from available estimates that sparsely populate the image space. Our solution to the underlying restoration problem is also novel; it is derived from another physically based prior: surfaces of outdoor objects are either planar or of small curvatures. This geometrical prior allows us to prove that the transmittance map **T** is a 2D piecewise linear function, or the sparsity of the Laplacian signal $\nabla^2 \mathbf{T}$. Gaining the above insight leads to a new sparsity-based restoration algorithm to refine thinly sampled, noisy raw transmittance map **T**. The restoration algorithm is a mixed ℓ_1 - ℓ_2 minimization process guided by the input hazy RGB image.

The above sketch of the new haze removal technique will be refined in detail in



Figure 1.2: Rectification of hue distortion; (a) a hazy image; (b) dehazed image by this work; (c) dehazed image by the DCP method.

the rest of the thesis.

By incorporating robust, physical based priors into the haze removal process, the proposed algorithm can noticeably improve the visual quality of existing haze removal methods; in particular, it successfully rectifies the problem of hue distortions in areas with no dark points (refer to Fig.1.2 and note the reddish tone of the sky in the output image of the dark-channel method, which is removed by our new method).

1.4 Organization of the thesis

In the next chapter, image dehazing is formulated as a maximum likelihood inference problem based on chrominance statistics of outdoor scenes. Chapter III introduces and justifies the use of Gaussian mixture model for estimating the transmittance map **T**. It is also discussed how the model can be constructed via machine learning. An hybrid dehazing method mixing DCP with GMM are introduced in Chapter IV. In Chapter V the piecewise linearity of **T** is first proved assuming small surface curvatures. Then, the novel restoration algorithm is developed to refine the initially estimated **T** of Chapter III based on the sparsity of the Laplacian $\nabla^2 \mathbf{T}$ and the edge information of the input hazy image. Analysis of distortions in dehazed images is given in Chapter VI. Experimental results are reported and discussed in Chapter VII, and conclusions drawn in Chapter VIII.

Chapter 2

Problem Formulation

A widely used (albeit greatly simplified) model for photographs degraded by haze or other atmospheric medium is expressed as follow [16, 17, 18].

$$i_{\lambda} = tj_{\lambda} + (1-t)a_{\lambda} \tag{2.1}$$

where i_{λ} and j_{λ} are respectively the acquired pixel value through haze and the pertaining pixel value if without haze in wavelength λ ; t is the light transmittance coefficient, which is a function of the distance between the imaged object and the camera (in this work, t is assumed not depending on λ practicality sake); a_{λ} is the global atmospheric light strength in wavelength λ , assumed a constant in space. In Eq.(2.1), the air light strength a is relatively easy to estimate [5, 6, 7]. The more difficult and mission critical task is to estimate the space-varying light transmittance t. Our approach to estimating t is conceptually simple. For a hypothetic t for pixel i, taking an $m \times m$ patch **i** centered at i, restore the corresponding tentative dehazed patch \mathbf{j} by applying (1) in R, G and B channels, respectively

$$\mathbf{j}_{\lambda}(t) = (\mathbf{i}_{\lambda} - \mathbf{a}_{\lambda})/t + \mathbf{a}_{\lambda}, \quad \lambda \in \{R, G, B\}$$
(2.2)

Here t is assumed to be the same in patch **i**. Next, two chrominance channels $(\mathbf{c}_1(t), \mathbf{c}_2(t))$ are extracted from the restored color image patch $\mathbf{j}(t)$

$$(\mathbf{c}_{1}(t), \mathbf{c}_{2}(t)) = (\mathbf{j}_{R}(t), \mathbf{j}_{G}(t), \mathbf{j}_{B}(t)) \begin{pmatrix} \alpha_{R} & \beta_{R} \\ \alpha_{G} & \beta_{G} \\ \alpha_{B} & \beta_{B} \end{pmatrix}$$
(2.3)

where vectors $(\alpha_R, \alpha_G, \alpha_B)'$ and $(\alpha_R, \alpha_G, \alpha_B)'$ are the two chrominance bases of the chosen color space, say YUV or YC_rC_b .

Stacking the two resulting spatial-spectral vectors $\mathbf{c}_1(t)$ and $\mathbf{c}_2(t)$ into a $2m^2$ dimensional vector $\mathbf{c}(t) = (\mathbf{c}'_1(t), \mathbf{c}'_2(t))$ and letting $\Pr(\mathbf{c})$ be the joint probability of the pertaining random vector, the optimal estimate of the light transmittance t^* for patch **i** is then determined by

$$t^* = \underset{0 \le t \le 1}{\operatorname{arg\,max}} \Pr(\mathbf{c}(t)) \tag{2.4}$$

The proposed estimation scheme is patch based for the reason that a small locality tends to have a nearly same depth.

Chapter 3

Haze Removal using Chrominance Statistics

3.1 Gaussian mixture model of chrominance

Now let us investigate the joint distribution $Pr(\mathbf{c})$ of chrominance patches in outdoor scenes. As explained in the introduction, the chrominance channels of an image pertain to spectral qualities of object surfaces in the scene. In a natural environment, particularly outdoor scenes (arguably the only type of images for which haze/fog removal may be required), object surfaces consist of a relatively small number of natural and man-made materials, such as earth, rock, wood, vegetation, water, sky, glass, concrete, metal. Because these natural materials tend to have fairly distinct spectral signatures, the joint distribution $Pr(\mathbf{c})$ of chrominance patches can be satisfactorily fit by a Gaussian mixture model, with each of the component distributions in the Gaussian mixture being associated with the spectral signature of a physical substance.

The above-discussed chrominance statistics of natural scenes can be visualized in



Figure 3.1: Cluster property of chrominance distribution; (a) an outdoor image without haze; (b) its chrominance distribution.

Fig.3.1, where the intensity in (b) represents the probability density. The color pixels of the scenery image exhibit themselves as distinct Gaussian point clouds in the 2D chrominance space, corresponding to earth, sky, mountain and vegetation. Here, for the convenience of graphical representation, we plot the distribution for single-pixel patches. It is evident in Fig.3.1 (b) that the chrominance distribution of natural images $Pr(\mathbf{c})$ has well-defined population centers or clusters (member distributions in the Gaussian mixture model). As we will present in the next section, the Gaussian mixture model of chrominance $Pr(\mathbf{c})$ is used to test hypotheses of the light transmittance; the resulting maximum likelihood estimate t^* generates the dehazed patch $\mathbf{j}(t^*)$. Now we discuss how to build the required Gaussian mixture model using a set of haze-free outdoor images. A Gaussian mixture model consists of K component Gaussian probability density functions:

$$f_k(\mathbf{c}) = \frac{1}{\sqrt{(2\pi)^{2m^2} |\mathbf{\Sigma}_k|}} e^{-\frac{1}{2}(\mathbf{c}-\mu_k)' \mathbf{\Sigma}_k^{-1}(\mathbf{c}-\mu_k)}$$

= $N_k(\mu_k, \mathbf{\Sigma}_k)$ (3.1)

where **c** is, in our problem setting, the $2m^2$ -dimensional chrominance vector of an m×m patch. However, we only know the training chrominance vectors drawn from

the distribution of the Gaussian mixture

$$P_r(\mathbf{c}|\theta) = \sum_{k=1}^{K} \lambda_k N(\mu_k, \boldsymbol{\Sigma}_k)$$
(3.2)

where θ is the parameter set $\theta = \{\mu_k, \sum_k, \lambda_k\}_{k=1}^K$.

$$\theta^* = \arg \max_{\theta} \sum_{i=1}^{N} \log[P_r(\mathbf{c}_i | \theta)]$$

=
$$\arg \max_{\theta} \sum_{i=1}^{N} \log\left[\sum_{k=1}^{K} \lambda_k \mathcal{N}_{\mathbf{C}_i}(\mu_k, \sum_k)\right]$$
(3.3)

where $\mathbf{c_i}$ is the chrominance vector of a patch and N is the number of training patches. The maximum likelihood estimate t^* can be obtained by the expectation maximization (EM) algorithm [19]. Although the EM-based learning process with a large training set is computationally costly, it is done once and for all. Considering that $Pr(\mathbf{c})$ can vary significantly in natural (rural) and artificial (urban) outdoor scenes, two Gaussian mixture models are built in this work, using two different training sets.

3.2 GMM-based estimation of transmittance

Having built the Gaussian mixture model $Pr(\mathbf{c})$ by off-line learning, we can then search through all component distributions $\mathcal{N}_k(\mu_k, \sum_k)$, $1 \le k \le K$, in the pre-built Gaussian mixture model, and through the t value range $0 < t \le 1$, to find

$$\{t^*, k^*\} = \underset{0 < t \le 1, 1 \le k \le K}{\arg \max} \Pr(\mathbf{c}(t)|k) \Pr(k)$$
(3.4)

which maximizes the probability of the chrominance vector $\mathbf{c}(t)$ of the dehazed patch $\mathbf{j}(t)$.

In the two dimensional search domain (t, k), the optimal value t^* is the extreme

point of $\mathcal{N}_k(\mu_k, \sum_k)$. Thus, the closed form solution of t^* given k can be found as follows

$$\frac{dN_{\mathbf{c}(t)}[\mu_{k}, \boldsymbol{\Sigma}_{k}]}{dt} = 0$$

$$\Rightarrow \frac{d(|\boldsymbol{\Sigma}_{k}|^{-0.5}e^{-0.5(\mathbf{c}(t)-\mu_{k})'\boldsymbol{\Sigma}_{k}^{-1}(\mathbf{c}(t)-\mu_{k}))}}{dt} = 0$$

$$\Rightarrow \frac{d((\mathbf{c}(i,t)-\mu_{k})'\boldsymbol{\Sigma}_{k}^{-1}(\mathbf{c}(i,t)-\mu_{k}))}{dt} = 0$$
(3.5)

Referring to notations in Eq.(2.2) and Eq.(2.3), we have

$$(\mathbf{c}(t) - \mu_k)' \boldsymbol{\Sigma}_k^{-1} (\mathbf{c}(t) - \mu_k)$$

= $(\mathbf{Q}(\mathbf{i} - \mathbf{a})/t + \mathbf{Q}\mathbf{a} - \mu_k)' \boldsymbol{\Sigma}_k^{-1}$ (3.6)
 $(\mathbf{Q}(\mathbf{i} - \mathbf{a})/t + \mathbf{Q}\mathbf{a} - \mu_k)$

where $\mathbf{j}, \mathbf{i}, \mathbf{a}$ are reshaped to $3m^2 \times 1$ vectors: $\mathbf{j} = [\mathbf{j}_R(t)', \mathbf{j}_G(t)', \mathbf{j}_B(t)']', \mathbf{i} = [\mathbf{i}_R(t)', \mathbf{i}_G(t)', \mathbf{i}_B(t)']', \mathbf{a} = [\mathbf{a}_R(\mathbf{t})', \mathbf{a}_G(\mathbf{t})', \mathbf{a}_B(\mathbf{t})']'$. **Q** is a $2m^2 \times 3m^2$ matrix that projects 3D color points in the RGB space to a chrominance plane:

$$Q = \begin{bmatrix} \alpha_R & \alpha_G & \alpha_B \\ \alpha_R & \alpha_G & \alpha_B \\ & \dots & & \\ & \alpha_R & \alpha_G & \alpha_B \\ & \beta_R & \beta_G & \beta_B \\ & \beta_R & \beta_G & \beta_B \\ & \dots & & \\ & & \beta_R & \beta_G & \beta_B \end{bmatrix}$$
(3.7)

The solution of Eq.(3.5) given k can be expressed as

$$t_k^* = \mathbf{M}' \mathbf{\Sigma}_k^{-1} \mathbf{M} / [\mathbf{M} \mathbf{\Sigma}_k^{-1} (\mu_k - \mathbf{B})]$$
(3.8)

where $\mathbf{M} = \mathbf{Q}(\mathbf{i} - \mathbf{a}), \ \mathbf{B} = \mathbf{Q}\mathbf{a}.$

For each k, a corresponding t_k^* is calculated; the K candidate values of t^* are compared to obtain the optimal pair $\{t^*, k^*\}$ that maximizes Eq.(3.4).

The proposed GMM estimation method is based on chrominance statistics of natural images and it can still work where dark points are absent; however, its performance drops if patch **i** consists of color patterns that are not well represented in the training set or if **i** has strong noises. In these patches the DCP methods may work better provided that there exist dark points. Therefore, the GMM and DCP methods can be used in conjunction to complement one the other. In the next chapter, a more sophisticated haze-removal method which combines GMM and DCP methods will be proposed.

Chapter 4

An Hybrid Dehazing Method Mixing DCP with GMM

Different from most of the existing dehazing methods, the newly proposed GMM method does not rely on the dark-channel prior and hence is applicable to a much larger class of scenes. However, compared with DCP based methods, the GMM method has weaknesses in some aspects. For example, the GMM method is prone to estimation error when patch **i** consists of color patterns that are not well represented in the training set or if **i** has strong noises. Table 4.1 shows a comparison between the strengths and weaknesses of GMM and DCP methods.

	the DCP method	the GMM method
strengths	In areas with dark- points, the DCP method is both fast and precise.	It's applicable to all kinds of areas even those without dark-points.
weaknesses	DCP method causes distortion in areas without dark-points.	The GMM method is prone to estimation errors.

Table 4.1: Comparison between DCP and GMM methods



Figure 4.1: Examples of GMM, DCP and hybrid methods; (a) original hazy image; (b) transmittance image of GMM; (c) transmittance image of DCP; (d) dehazed image of GMM; (e) dehazed image of DCP; (f) transmittance of hybrid method; (g) dehazed image of hybrid method; (h) selection map.

To visualize the characteristics of DCP and GMM methods, a set of sample outputs by the two methods is presented in Fig. 4.1. In this example, both methods share a same problem — their transmittance images are too coarse causing visible block and ring artifacts in the dehazed images such as the white edge around the boundaries between mountain and sky. This problem will be addressed using a sparsity-based transmittance image refinement method in the next chapter. In comparison, the transmittance image generated by the GMM method does not look as clean as which by the DCP method, because the GMM method is more prone to estimation error. However, it is possible to roughly calculate the accuracy of the GMM method's estimation in each region using the likelihood measurement introduced in Eq. (3.4). Therefore, intuitively, if the likelihood level is not convincing enough, the GMM method should not be used.

On the other hand, DCP method assumes that at least one pixel in each patch has a zero or near zero intensity level in one of the RGB color channels. Thus, in places where this assumption does not hold, the DCP method often incorrectly estimates haze transmittance, resulting in obvious color distortion (e.g. the clouds, the shed roof and white wood in Fig.4.1). The proposed GMM method, however, is free of this problem and performs much better in terms of visual quality than the DCP method in areas with no dark-point.

To take the advantages of both methods, we can stitch the two methods together, and selectively use the result by either DCP or GMM method based on whether one of the methods would likely work well for each region. This idea results a new hybrid dehazing approach as follows,

- For each patch, if the highest saturation value of all pixels is above a threshold, we adopt the DCP method's estimates.
- Otherwise, we estimate this patch using the GMM method and calculate its likelihood using Eq.(3.4).
- 3) If the likelihood is above a threshold, we adopt the GMM method's estimation.
- 4) Otherwise, we skip this block and leave its transmittance as "undecided".
- 5) Undecided transmittance patches will be estimated later using an aforementioned

sparsity-based refinement method.

Fig.4.1(f) (h) demonstrates the results by the above hybrid approach, where (f) is the transmittance map, (g) is the dehazed image, and (h) is the selection map indicating which method is used in the hybrid approach for each patch. In the selection map, the gray and white regions represent the uses of GMM and DCP methods, respectively; and the dark regions are undecided and should be estimated by the sparsity-based refinement method. As shown in the figure, regions without dark-points tend to employ GMM such as the sky and the sheds, while in other areas like the ground and the lawn, DCP method is more likely to be adopted.

Chapter 5

Sparsity-based Restoration of Transmittance Map

The 2D transmittance map \mathbf{T} estimated by the proposed GMM method, the aforementioned hybrid mehtod or by any other methods for this matter, is flawed, with noises, missing samples, and block artifacts. In this section we develop a new method to repair these defects and restore the initially estimated \mathbf{T} . The restoration of \mathbf{T} exploits a physically based prior: surfaces of outdoor objects are either planar or of small curvatures. With the above geometrical prior we can show that the 2D signal \mathbf{T} is piecewise linear, i.e., \mathbf{T} being sparse in the Laplacian space.

5.1 Piecewise linearity of transmittance map

Given the distance d between the camera and the imaged object, the light transmittance coefficient t is

$$t = e^{-\beta d} \tag{5.1}$$



Figure 5.1: The distance $f(\theta)$ from the camera to the imaged object varies from different view directions θ .

where β is a parameter related to size, material type and density of aerosols and to the wavelength of the light; but whatever complex relationship β has with the environment it does not affect the following analysis.

A simplified camera model is shown in Fig.5.1. Suppose the distance $f(\theta)$ from the camera to a point on the imaged object is a function of the relative angle θ between the ray to the point and the normal of the camera as illustrated in Fig. 5.1. If the neighbouring points, meaning the points with angle $\theta + \alpha$ for $\alpha \in [-\varepsilon, \varepsilon]$ where ε is a small positive constant, are all on the same object surface of small curvature, then

light transmittance function,

$$t(\theta + \alpha) = e^{-\beta f(\theta + \alpha)}, \qquad (5.2)$$

is nearly linear to $\theta + \alpha$ in the small region. This property can be shown using the Taylor series for $t(\theta + \alpha)$,

$$t(\theta + \alpha)$$

$$= t(\theta) + \frac{t'(\theta)}{1!}\alpha + \frac{t''(\theta)}{2!}\alpha^2 + O(\alpha^3)$$

$$= e^{-\beta f(\theta)} - e^{-\beta f(\theta)}\beta f'(\theta)\alpha$$

$$+ \frac{1}{2}e^{-\beta f(\theta)}\{[\beta f'(\theta)\alpha]^2 - \beta f''(\theta)\alpha^2\}$$

$$+ O(\alpha^3)$$

$$= t(\theta)\{1 - \beta f'(\theta)\alpha + \frac{1}{2}[\beta f'(\theta)\alpha]^2$$

$$- \frac{1}{2}\beta f''(\theta)\alpha^2\} + O(\alpha^3).$$
(5.3)

Considering that α is small, i.e., $|\alpha| \ll 1$, if we assume that the following equations are also true,

$$\begin{cases} |\beta f'(\theta)\alpha| \le c, \\ |\beta f''(\theta)\alpha| \le c, \end{cases}$$
(5.4)

for a small positive constant c (e.g., c = 0.1), then we have $|\beta f'(\theta)\alpha| \gg |\frac{1}{2}[\beta f'(\theta)\alpha]^2|$ and $|\frac{1}{2}\beta f''(\theta)\alpha^2| \ll 1$; therefore, it follows from the Taylor series as in Eq. 5.3 that

$$t(\theta + \alpha) \approx t(\theta)[1 - \beta f'(\theta)\alpha], \qquad (5.5)$$

which is a linear function of $\theta + \alpha$. The next step is to validate the assumptions in Eq. 5.4, i.e., both $|f'(\theta)|$ and $|f''(\theta)|$ being less than $c/|\beta\alpha|$. Attenuation coefficient β in the atmosphere is determined by many different factors, however, it is usually much

less than 0.01. On the other hand, angle of view α for each pixel neighbourhood in our application is also small. For example, if the image captured by a 35mm camera with a 28mm wide angle lens is divided into 100 small neighbourhoods horizontally, then the angle of view covered by each neighbourhood is,

$$2\varepsilon = \frac{1}{100} \cdot 2 \arctan \frac{36}{2 \cdot 28} \approx 0.0151$$
 (5.6)

Thus, $|\alpha|$ can only be as large as $\varepsilon \approx 0.00775$. Suppose c = 0.1, then

$$c/|\beta\alpha| > 0.1/0.01/0.00775 \approx 1325.$$
 (5.7)

If imaged object surface is sufficiently smooth, then $|f''(\theta)|$ is usually small and very unlikely exceeds $c/|\beta\alpha|$; and $|f'(\theta)|$ can only be very large when the surface is nearly parallel to the view direction, but in that case, the surface is barely visible hence irrelevant to the discussion. Therefore, the 2D transmittance map **T** is piecewise linear.

5.2 Sparsity space of transmittance image

The success of any sparsity-based image restoration technique primarily depends on the choice of sparsity space in which the restoration is performed. In search for a recovery space in which the transmittance image exhibits a high level of sparsity, we refer to the new prior proposed above.

Being piece-wise linear, the two-dimensional signal \mathbf{T} can be modeled as a collection of linear segments $T_k(x, y) = ax + by + c$, k being the segment index.

Immediately following the above proof is a much desired sparsity property for the underlying restoration task: a typical transmittance image \mathbf{T} is highly sparse

in the Laplacian space; in other words, the Laplacian (the second derivative) of the transmittance image is zero or near zero in all pixel locations except for the boundaries of different segments.

Denote by $\tilde{\mathbf{T}}$ the irregularly undersampled, noisy transmittance map produced by the combined GMM-DCP method,

$$\tilde{\mathbf{T}} = \mathbf{DT} + \mathbf{n} \tag{5.8}$$

where **D** is the downsampling operator that selects the spatial positions of initially available t estimates, and **n** is the estimation noise. The clean, fully sampled transmittance map **T** can be restored from \tilde{T} based on the sparsity of **T** in the Laplacian space, i.e., **T** is the solution of the following fixed ℓ_1 - ℓ_2 minimization problem:

$$\min \left\|\nabla^{2}\mathbf{T}\right\|_{\ell 1} \ s.t. \left\|\mathbf{D}\mathbf{T} - \tilde{\mathbf{T}}\right\|_{\ell 2} \le \sigma$$
(5.9)

with σ being the variance of the noise **n**.

5.3 Structured sparsity induced by observed image

The above is a baseline scheme of transmittance image restoration in the sparse space of Laplacian. The objective function Eq.(5.9) does not exploit the strong correlation between the transmittance map \mathbf{T} and the corresponding color image \mathbf{I} . This correlation introduces the following additional structures into the sparsity of the Laplacian space. We know not only the 2D Laplacian signal $\nabla^2 \mathbf{T}$ is mostly zero or near zero but also where it is likely to deviate greatly from zero. Indeed, $\nabla^2 \mathbf{T}$ tends to become non-zero either at the intersections of different surfaces or at where object obstructions happen. Both types of discontinuity correspond to edges in the captured image I. Therefore, the edge trajectories in I can predict the positions of non-zero elements of the 2D Laplacian signal $\nabla^2 \mathbf{T}$.

Algorithmically, we can integrate the above structured sparsity of $\nabla^2 \mathbf{T}$ into the transmittance image restoration process in Eq.(5.9) by applying different weights $\mathbf{w} = (w_1, w_2, ..., w_n)$ to individual elements of the Laplacian signal $\nabla^2 \mathbf{T}$ prior to the ℓ_1 minimization; weight value w_n is determined by the likelihood that the Laplacian signal $\nabla^2 \mathbf{T}$ has a large amplitude at pixel position n. Weight w_n is lowered if $\nabla^2 \mathbf{T}$ has higher likelihood to be large in amplitude. The resulting structured sparsity-based objective function for transmittance image restoration becomes

$$\min \left\| \mathbf{W} \cdot \nabla^2 \mathbf{T} \right\|_{\ell 1} s.t. \left\| \mathbf{DT} - \tilde{\mathbf{T}} \right\|_{\ell 2} \le \sigma$$
(5.10)

In Eq.(5.10), adaptively varying weights in the Laplacian space provides a mechanism of increasing or decreasing the penalty for non-zero elements to occur at given locations. As discussed earlier, the two-dimensional spare signal $\nabla^2 \mathbf{T}$ is far more likely to switch from zero to nonzero on or near the edges of the input image I than in other locations. One way of using this statistical prior to aid the restoration of \mathbf{T} is as follows. First, an edge detection algorithm (e.g., Canny's edge detector) is applied to I and extracts an edge map E. Then, a spatially varying weighting map W is generated from the edge map E by dilating the edge skeletons of E and increasing the weights as moving away from the edge locations.

Two examples of the proposed transmittance image restoration process are given in Fig.5.2 and Fig.5.3. As clearly demonstrated, the initially estimated transmittance images $\tilde{\mathbf{T}}$ with noises, "holes" and block artifacts are well restored.



Figure 5.2: The effects of the proposed transmittance image restoration technique; (a) hazy input image; (b) restored transmittance image; (c) initially estimated transmittance image, in which the red points mark missing estimates; (d) restored image.



Figure 5.3: The effects of the proposed transmittance image restoration technique; (a) hazy input image; (b) restored transmittance image; (c) initially estimated transmittance image, in which the red points mark missing estimates; (d) restored image.

5.4 Introduction to previous refinement methods

In addition to the sparsity-based refinement method introduced above, previous papers also exploit various methods to refine the initial transmittance image. In [5] a soft-matting optimization approach is employed. In this approach, a mating Laplacian matrix is designed to enforce the transmittance value as a local linear transform of the input image colors.

Since this approach is too time-consuming to be used in real applications, He et al. replace it with the "guided filter" in [8]. The guided filter is a translation-variant filter derived from a local linear model. It generates the filtering output based on the content of another image, which is called "guidance image". In the scenario of haze removal, the hazy input image is often chosen as the guidance. Guided filter generates almost the same results as the soft-matting approach. But its time complexity is O(n), which makes it more suitable to refine the initial transmittance image.

The key assumption of the guided filter is a local linear model between the guidance \mathbf{G} and the output transmittance image \mathbf{T} , which is expressed in Eq.(5.11)

$$\mathbf{T}_{\mathbf{i}} = a_k \mathbf{G}_i + b_k, \forall i \in \omega_k \tag{5.11}$$

where ω_k is a patch centered at the pixel k.

To determine the linear coefficients a_k and b_k , He et al seek a solution to Eq.(5.11) that minimizes the difference between output **T** and input $\tilde{\mathbf{T}}$. Specifically they minimize the following cost function in a patch,

$$E(a_k, b_k) = \sum_{i \in \omega_k} \left((a_k \mathbf{G}_i + b_k - \tilde{\mathbf{T}}_i)^2 + \varepsilon a_k^2 \right)$$
(5.12)

where ε is a regularization parameter for avoiding too large values of a_k .

This local linear model is applied to all patches in the entire image. And the final value of a pixel is calculated as the average of its different estimates in overlapping windows, which is expressed as

$$\mathbf{T}_{\mathbf{i}} = \frac{1}{|\omega|} \sum_{k:i \in \omega_k} (a_k \mathbf{G}_i + b_k), \forall i \in \omega_k \tilde{T}$$

= $\bar{a}_i \mathbf{G}_i + \bar{b}_i$ (5.13)

where $\bar{a}_i = \frac{1}{|\omega|} \sum_{k \in \omega_k} a_k$ and $\bar{b}_i = \frac{1}{|\omega|} \sum_{k \in \omega_k} b_k$.

It is proved that the guided filter has two critical properties [8]:

- 1) $\nabla \mathbf{T} \approx \bar{a} \nabla \mathbf{G}$, meaning that abrupt intensity changes in the guidance image \mathbf{G} can be mostly maintained in the output transmittance image.
- 2) Patches in the guidance image G with large variance, introduce nearly no change to the transmittance values in the output T. Whereas in flat patches where G has low variances, the transmittance values in T correspond to the average of nearby values.

5.5 Comparison between the guided filter and the sparsity-based refinement method

The aforementioned properties make guided filter an excellent filter for edgepreserving image smoothing. However in our application of transmittance refinement, it leads to wrong transmittance values since it imposes "edges" on the refined result. Examples are shown in Fig.5.4 and Fig.5.5. Obvious intensity changes around the texts on the boards and graffiti drawings in Fig.5.4(c) and Fig.5.5(c) respectively can be seen. This distortion is absurd since the backgrounds and the contents on them should be equally distant to the observer. In contrast, the newly proposed sparsity-based refinement method generates much more reasonable transmittance images in Fig.5.4(d) and Fig.5.5(d). The theoretical basis of the sparsity-based method is "piece-wise linearity of transmittance". This basis does not impose any "edge-preserving" requirement on the output. Even though a weighting vector \mathbf{W} extracted from the input image is applied to assist the transmittance image restoration, it is merely a "proposal on potential edge positions". In areas where \mathbf{W} has small values, intensity constraint on the output would be relaxed, but no abrupt intensity changes are enforced.



Figure 5.4: Comparison between guided filter and sparsity-based refinement method. (a) the input foggy image; (b) the initial transmittance map; (c) refined result using guided filter; (d) refined result using sparsity-based method.



Figure 5.5: Comparison between guided filter and sparsity-based refinement method. (a) the input foggy image; (b) the initial transmittance map; (c) refined result using guided filter; (d) refined result using sparsity-based method.

Chapter 6

Analysis of Hue and Saturation Distortions in Dehazed Images

As pointed out in previous chapters, the DCP method is prone to make erroneous estimates of transmittance. In this chapter we study whether and how the estimation errors in transmittance distort the saturation and hue of the dehazzed images. Quite surprisingly, no such a study has been reported in the large existing body of literature on image dehazing. This was probably because previous works were primarily motivated by consumer applications; image quality assessment was biased toward the improvement of contrast brought by dehazing at the expense of color fidelity. However, in many scientific and professional applications, such as remote sensing, accurate recovery of spectral properties of object surfaces in haze removal becomes very important, making it necessary to investigate the capability of haze removal techniques to restore the original spectral information.

In this paper, we set out to answer the questions whether and how the use of the DCP prior in haze removal results in spectral distortions. Our analysis shows that dehazing methods based on the DCP prior can generate substantial distortions in

both hue and saturation in their results if the so-called dark points do not exist.

6.1 Problem Statement and Dark Point

As is mentioned in Chapter 1, a widely used, simplified model for photographs degraded by haze or other atmospheric medium is expressed as follow [16, 17, 18].

$$i_{\lambda} = tj_{\lambda} + (1-t)a_{\lambda} \tag{6.1}$$

where i_{λ} and j_{λ} are respectively the acquired pixel value through haze and the pertaining pixel value if without haze in wavelength λ ; t is the light transmittance coefficient, which is a function of the distance between the imaged object and the camera (in this work, t is assumed not depending on λ for practicality sake); a_{λ} is the global atmospheric light strength in wavelength λ , assumed a constant in space.

In Eq.(6.1), the airlight color a_{λ} is relatively easy to estimate [5, 6, 7]. The more difficult and mission critical task is to estimate the space-varying light transmittance t.

In order to estimate the transmittance, the first step in most DCP-based methods is search for the so-called "dark-channel" in a local patch Ω . The dark-channel in patch Ω is defined to be the color band τ such that

$$i_{\tau} = \arg\min_{\lambda \in (R,G,B)} \left(\min_{i \in \Omega} \left(\frac{i_{\lambda}}{a_{\lambda}} \right) \right)$$
(6.2)

accordingly, the pixel with intensity i_{τ} in band τ is assumed a dark point.

In Eq.(6.1), i_{λ} consists of two parts: the airlight a_{λ} and the radiance j_{λ} from the object; radiance $j_{\lambda} = 0$ holds only on true dark point. In the absence of any true dark point in a local patch, the DCP method forces $j_{\lambda} = 0$ on the pixel of intensity

 i_{τ} ; namely, it attributes intensity i_{τ} completely to the airlight, i.e., $i_{\tau} = (1 - \tilde{t})a_{\tau}$, and estimates the transmittance of the patch to be

$$\tilde{t} = 1 - \frac{i_\tau}{a_\tau} \tag{6.3}$$

To prevent \tilde{t} from being zero, some previous works [5, 14] introduce a close-toone multiplier ω to Eq.(6.3) and express \tilde{t} as $1 - \omega \cdot \frac{i_{\tau}}{a_{\tau}}$. However, this is merely an expediency without physical justification. In this work, we assume $\omega = 1$.

Let j_{τ} be the non-zero ground truth radiance of the assumed dark point i_{τ} such that $i_{\tau} = tj_{\tau} + (1-t)a_{\tau}$. Then, the estimation error in transmittance is

$$\Delta t = t - \tilde{t} = \frac{tj_{\tau}}{a_{\tau}} > 0. \tag{6.4}$$

In other words, the DCP-type of dehazing methods always underestimate the transmittance, or overestimates the thickness of air particles.

Next we analyze how the estimation errors in transmittance distort the saturation and hue of the dehazed images. In the analysis we need to select a suitable color space. Many color spaces, such as RGB, YUV, HIS, etc. have been proposed for various applications; each has its own advantages and disadvantages. We choose, among all color spaces, the Hue-Saturation-Intensity (HSI) color space for its intuitive interpretation and explicit representations of color attributes hue and saturation. In the three-dimensional HSI color space, hue and saturation are two different dimensions that are orthogonal to the intensity dimension. The transform from the common RGB color space to the HIS space is given in Eq.(6.5).

$$I = \frac{R+G+B}{3}$$

$$S = 1 - 3 \cdot \frac{\min(R,G,B)}{R+G+B}$$

$$H = \begin{cases} \frac{180^{o}}{\pi} \cos^{-1} \frac{0.5[(R-G)+(R-B)]}{[(R-G)^{2}+(R-B)(G-B)]^{0.5}}, B \le G \\ 360^{o} - \frac{180^{o}}{\pi} \cos^{-1} \frac{0.5[(R-G)+(R-B)]}{[(R-G)^{2}+(R-B)(G-B)]^{0.5}}, B > G \end{cases}$$
(6.5)

6.2 Saturation Distortion

Given an estimated transmittance \tilde{t} , according to Eq.(6.1) and Eq.(6.5), the saturation value $S(\tilde{t})$ of a dehazed pixel \tilde{j} can be expressed as

$$S(\tilde{t}) = 1 - 3 \cdot \frac{\tilde{j}_0}{\sum_{\lambda} j_{\lambda}}$$

= $1 - 3 \cdot \frac{\frac{i_0 - a_0}{\tilde{t}} + a_0}{\sum_{\lambda} \left(\frac{i_{\lambda} - a_{\lambda}}{\tilde{t}} + a_{\lambda}\right)}$
= $1 - 3 \cdot \frac{i_0 - (1 - \tilde{t})a_0}{\sum_{\lambda} i_{\lambda} - (1 - \tilde{t}) \cdot \sum_{\lambda} a_{\lambda}}$ (6.6)

where $\tilde{j}_o = min(\tilde{j}_R, \tilde{j}_G, \tilde{j}_B)$.

In presence of transmittance error Δt , the saturation error ΔS can be expressed as follows,

$$\Delta S = S(\tilde{t}) - S(t)$$

$$= S(t - \Delta t) - S(t)$$

$$= 3 \cdot \frac{a_0 \sum_{\lambda} i_{\lambda} - i_0 \sum_{\lambda} a_{\lambda}}{\left[\sum_{\lambda} i_{\lambda} - (1-t) \cdot \sum_{\lambda} a_{\lambda}\right] \cdot \left[\frac{\sum_{\lambda} i_{\lambda} - (1-t) \cdot \sum_{\lambda} a_{\lambda}}{\Delta t} - \sum_{\lambda} a_{\lambda}\right]}.$$
(6.7)

In most cases, the airlight is close to white, meaning $a_R \approx a_G \approx a_B = a$. The

numerator of ΔS in Eq.(6.7) can be simplified to

$$a_{0} \sum_{\lambda} i_{\lambda} - i_{0} \sum_{\lambda} a_{\lambda}$$

$$\approx a \sum_{\lambda} i_{\lambda} - 3ai_{0}$$

$$> 0$$
(6.8)

which is always positive.

The denominator in Eq.(6.7) can be factorized into three terms:

$$\begin{bmatrix} \sum_{\lambda} i_{\lambda} - (1-t) \cdot \sum_{\lambda} a_{\lambda} \end{bmatrix} \begin{bmatrix} \frac{\sum_{\lambda} i_{\lambda} - (1-t) \cdot \sum_{\lambda} a_{\lambda}}{\Delta t} - \sum_{\lambda} a_{\lambda} \end{bmatrix}$$

= $\frac{1}{\Delta t} \cdot \sum_{\lambda} [i_{\lambda} - (1-t) \cdot a_{\lambda}] \cdot \sum_{\lambda} [i_{\lambda} - (1-t+\Delta t) \cdot a_{\lambda}]$ (6.9)

By Eq.(6.1), the second term in Eq.(6.9) is equal to $t \cdot \sum_{\lambda} j_{\lambda}$, which is always positive. Now let us examine the third term. Base on the definition of the dark point and how the DCP method estimates the transmittance, for any channel $\lambda \in \{R, G, B\}$ and pixel $i \in \Omega$, we have the following inequalities:

$$i_{\lambda} - (1 - t + \Delta t) \cdot a_{\lambda}$$

$$= i_{\lambda} - (1 - \tilde{t}) \cdot a_{\lambda}$$

$$= a_{\lambda} \cdot \left(\frac{i_{\lambda}}{a_{\lambda}} - (1 - \tilde{t})\right)$$

$$\geq a_{\lambda} \cdot \left(\frac{i_{\tau}}{a_{\tau}} - (1 - \tilde{t})\right)$$

$$= 0$$
(6.10)

Therefore, the third term in Eq.(6.9) is nonnegative; it equals zero only if i is the dark point and $i_R = i_G = i_B$, $a_R = a_G = a_B$. (In case it equals zero, a small positive ε can be added to avoid "divide-zero" error.) By now we have shown that both the numerator and denominator of Eq.(6.7) are positive, and hence ΔS is positive. In

other words, the saturation of a dehazed pixel by the DCP method is always greater than the ground truth saturation, when the airlight is close to white. This explains why many users like the results of the DCP-based haze removal, although inaccurate, because more vivid colors are much preferred to faint ones.

Under the assumption of white airlight, the expression of ΔS in Eq.(6.7) can be further simplified to

$$\Delta S = S(\tilde{t}) - S(t)$$

$$= 3 \cdot \frac{a_0 \sum_{\lambda} i_{\lambda} - i_0 \sum_{\lambda} a_{\lambda}}{\left[\sum_{\lambda} i_{\lambda} - (1-t) \cdot \sum_{\lambda} a_{\lambda}\right] \cdot \left[\frac{\sum_{\lambda} i_{\lambda} - (1-t) \cdot \sum_{\lambda} a_{\lambda}}{\Delta t} - \sum_{\lambda} a_{\lambda}\right]}$$

$$\approx \frac{3a \cdot \left(\sum_{\lambda} i_{\lambda} - 3i_0\right)}{\left[\sum_{\lambda} i_{\lambda} - 3(1-t)a\right] \left[\frac{\sum_{\lambda} i_{\lambda} - 3(1-t)a}{\Delta t} - 3a\right]}$$

$$> 0$$
(6.11)

To investigate the growth trend of ΔS , the first derivative of ΔS with respect to Δt is expressed as follows,

$$\frac{d\Delta S}{d\Delta t} = \frac{3a \cdot \left(\sum_{\lambda} i_{\lambda} - 3i_{0}\right)}{\sum_{\lambda} i_{\lambda} - 3(1 - t)a} \cdot \left(\frac{\Delta t}{\sum_{\lambda} i_{\lambda} - 3(1 - t)a - 3a\Delta t}\right)' = \frac{3a \cdot \left(\sum_{\lambda} i_{\lambda} - 3i_{0}\right)}{\left(\sum_{\lambda} i_{\lambda} - 3(1 - t + \Delta t)a\right)^{2}}$$
(6.12)

The denominator $\sum_{\lambda} i_{\lambda} - 3(1 - t + \Delta t)a$, as we mentioned in Eq.(6.10), is nonnegative and approaching to zero when Δt increases. Therefore, the growth of ΔS accelerates as Δt increases.

To be more explicit, the Taylor series expansion of ΔS with respect to Δt is given

as follow,

$$\Delta S = \frac{3a \cdot \left(\sum_{\lambda} i_{\lambda} - 3i_{0}\right)}{\left[\sum_{\lambda} i_{\lambda} - 3(1-t)a\right] \left[\frac{\sum_{\lambda} i_{\lambda} - 3(1-t)a}{\Delta t} - 3a\right]}$$

$$= 3a \cdot \left(\sum_{\lambda} i_{\lambda} - 3i_{0}\right) \cdot \left\{\frac{1}{B^{2}}\Delta t + \frac{3a}{B^{3}}\Delta t^{2} + \frac{9a^{2}}{B^{4}}\Delta t^{3} + \dots\right\}$$

$$(6.13)$$

where $B = \sum_{\lambda} i_{\lambda} - 3(1-t)a$.

It can be seen that if Δt is close to zero, then the first-order term in Eq.(6.13) dominates and ΔS can be approximated by a liner function. However, when Δt increases, higher order terms in Eq.(6.13) need to be kept and the growth trend of ΔS in Δt is much faster than linear growth. This error behavior can be seen in Fig.6.1, which plots ΔS versus Δt for an example case where t = 0.85, a = 255 and $\mathbf{i} = [170; 155; 140]$.



Figure 6.1: Error behavior of ΔS versus Δt .

6.3 Hue distortion

To analyze the hue distortion, let us refer to an illustration of the HSI color space in Fig.6.2.



Figure 6.2: (a) HSI color model; (b) Hue-saturation plane.

Similar to notations used in the previous section, in Fig.6.2(b), **a** is the airlight color; $\mathbf{j} = \mathbf{j}(\mathbf{t})$ and $\mathbf{\tilde{j}} = \mathbf{j}(\mathbf{\tilde{t}})$ represent the dehazed pixel colors with accurate and estimated transmittance. $\boldsymbol{a}, \boldsymbol{j}$ and $\mathbf{\tilde{j}}$ are their projection points on the hue-saturation plane; **o** is the intersection of the hue plane and the intensity axis. Given an estimated \tilde{t} , the hue of $\mathbf{\tilde{j}}$ is equal to the angle $\angle \mathbf{\tilde{j}ob}$ and the resulting hue distortion ΔH is expressed as follow.

$$\Delta H = |\angle \tilde{\boldsymbol{\jmath}} \mathbf{o} \mathbf{b} - \angle \boldsymbol{\jmath} \mathbf{o} \mathbf{b}|$$

= $\angle \tilde{\boldsymbol{\jmath}} \mathbf{o} \boldsymbol{\jmath}$ (6.14)

By trigonometry,

$$\cos(\Delta H) = \frac{|\tilde{\boldsymbol{j}}\mathbf{o}|^2 + |\boldsymbol{j}\mathbf{o}|^2 - |\tilde{\boldsymbol{j}}\boldsymbol{j}|^2}{2 \cdot |\tilde{\boldsymbol{j}}\mathbf{o}| \cdot |\boldsymbol{j}\mathbf{o}|}$$
(6.15)

where $|\mathbf{p_1p_2}|$ represents the distance between points $\mathbf{p_1}$ and $\mathbf{p_2}$. In space geometry



Figure 6.3: Affine relations between various color points in the HSI space.

those segment lengthes can be expressed as

$$|\mathbf{j}\mathbf{o}| = \left| \left(\frac{\mathbf{i}-\mathbf{a}}{t} + \mathbf{a} \right) \times \mathbf{N} \right|$$
$$|\mathbf{\tilde{j}}\mathbf{o}| = \left| \left(\frac{\mathbf{i}-\mathbf{a}}{t-\Delta t} + \mathbf{a} \right) \times \mathbf{N} \right|$$
$$|\mathbf{\tilde{j}}\mathbf{j}| = \left| \left(\frac{\mathbf{i}-\mathbf{a}}{t-\Delta t} - \frac{\mathbf{i}-\mathbf{a}}{t} \right) \times \mathbf{N} \right|$$
(6.16)

where $\mathbf{N} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ is the normal of the saturation-hue plane. Thus, we have

$$\Delta H = \cos^{-1} \left(\frac{|\mathbf{\tilde{jo}}|^2 + |\mathbf{jo}|^2 - |\mathbf{\tilde{jj}}|^2}{2|\mathbf{\tilde{jo}}| \cdot |\mathbf{jo}|} \right) = \cos^{-1} \left(\frac{\left| \left(\frac{\mathbf{i} - \mathbf{a}}{t - \Delta t} + \mathbf{a} \right) \times \mathbf{N} \right|^2 + \left| \left(\frac{\mathbf{i} - \mathbf{a}}{t} + \mathbf{a} \right) \times \mathbf{N} \right|^2 - \left| \frac{\Delta t \cdot (\mathbf{i} - \mathbf{a})}{t \cdot (t - \Delta t)} \times \mathbf{N} \right|^2} \right)$$
(6.17)

For white airlight, \boldsymbol{a} coincides with \mathbf{o} ; points $\boldsymbol{\jmath}$, $\boldsymbol{\tilde{\jmath}}$ and \mathbf{o} are collinear, $|\boldsymbol{\tilde{\jmath}}\mathbf{o}| = |\boldsymbol{\tilde{\jmath}}\boldsymbol{\jmath}| + |\boldsymbol{\jmath}\mathbf{o}|$. It follows from Eq.(6.17)

$$\Delta H = \cos^{-1} \left(\frac{|\tilde{\mathbf{jo}}|^2 + |\mathbf{jo}|^2 - |\tilde{\mathbf{jj}}|^2}{2|\tilde{\mathbf{jo}}| \cdot |\mathbf{jo}|} \right)$$

= $\cos^{-1} \left(\frac{|\tilde{\mathbf{jo}}|^2 + |\mathbf{jo}|^2 - (|\tilde{\mathbf{jo}}| - |\mathbf{jo}|)^2}{2|\tilde{\mathbf{jo}}| \cdot |\mathbf{jo}|} \right)$
= 0 (6.18)

If the airlight is white, regardless of the estimated transmittance \tilde{t} , the hue distortion $\Delta H = H(\tilde{t}) - H(t)$ is zero.

Given the object surface radiance \mathbf{j} and the airlight \mathbf{a} , the hue distortion ΔH depends on the estimation error Δt in transmittance. The relation between ΔH and Δt can be explained geometrically in Fig.6.3(a). Due to the affinity of the hazy image formation model of Eq.(6.19), points \mathbf{a} , \mathbf{i} , \mathbf{j} and $\mathbf{\tilde{j}}$ are colinear in the HSI space; and so are points \mathbf{a} , \mathbf{i} , \mathbf{j} and $\mathbf{\tilde{j}}$. As point $\mathbf{\tilde{j}}$ lies on the extension line of $\mathbf{a}\mathbf{i}$, we have

$$\mathbf{i} = t\mathbf{j} + (1 - t)\mathbf{a}$$

$$\tilde{\mathbf{j}} = \frac{\mathbf{i} - \mathbf{a}}{t - \Delta t} + \mathbf{a}$$

$$= \frac{t}{t - \Delta t} \cdot (\mathbf{j} - \mathbf{a}) + \mathbf{a}$$
(6.19)

This means that increasing the transmittance estimation error Δt moves color point $\tilde{\boldsymbol{j}}$ farther away from color point \boldsymbol{a} ; consequently, the hue distortion ΔH , which is the angle $\angle \boldsymbol{j} \boldsymbol{o} \tilde{\boldsymbol{j}}$, becomes larger.

In general, the more the airlight **a** deviates from white in color, the greater the hue distortion ΔH becomes, as shown in Fig.6.3(b). As the projection point of the airlight color changes from a_0 to a_1 , moving away from the white point **o** (i.e., a_0 is less saturated or closer to white than a_1), the projection point of the dehazed color moves accordingly from \tilde{j}_0 to \tilde{j}_1 according to Eq.(??)ptj). As a result, the hue distortion increases from $\angle j o \tilde{j}_0$ to $\angle j o \tilde{j}_1$ as marked in Fig.6.3(b).

6.4 Remarks and future work

The DCP family of single image dehazing methods have rapidly gained popularity and become main stream; in many cases they deliver good results. But the dark channel assumption made by the DCP methods may not always hold in real world photographs. We have analyzed the errors and their properties in the results of the DCP methods if there are no real dark points in the scene imaged. Our analysis reveals that the DCP methods can generate large errors in saturation and hue if the dark channel assumption is violated. One needs to exercise caution when interpreting the results of the DCP methods in professional and scientific applications where the fidelity criteria are more than meet the eyes, being aware of the causes and behaviors of spectral distortions of these methods.

In Fig.6.4, Fig.6.5 and Fig.6.6, some sample results of the DCP method [5] are presented to show the visual effects of the spectral distortions. In the dehazed forest image, the color of grass appears over saturated and the sky has a strange purple tone. Similar color distortions can be found in the dehazed flower field image, in which the flowers have unnaturally vivid color and the sky turns greenish. Those observations are in agreement with the above analysis.



original image

dehazed image

Figure 6.4: Examples of color distortions.



original image

dehazed image

Figure 6.5: Examples of color distortions.



original image

dehazed image

Figure 6.6: Examples of color distortions.

Chapter 7

Experimental Results and Remarks

The new haze removal technique is implemented and tested on a variety of hazy images, ranging from city skylines to rural landscapes; its performance is evaluated in comparison with the DCP method and Fattal's method. In the construction of the proposed GMM model of chrominance distributions, training images are classified into two types: city and rural scenes. A separate Gaussian mixture model is built for each scene type. Moreover, each Gaussian mixture model is split into 32 Gaussian components using the EM algorithm. The sparsity-based optimization algorithm for transmittance estimation is implemented using CVX toolbox.

Fig.7.1 through Fig.7.4 are samples of our experimental results. Because the proposed technique incorporates into the restoration process of dehazing new physically valid priors of natural images, it should be expected to outperform existing methods. Indeed, comparing the dehazed images generated by the different methods in Fig.7.1 through Fig.7.4, one can see that the proposed method reproduces more realistic, sharper images that are largely free of hue distortions.

The differences between the new dehazing technique and the DCP method are clearly visible in areas where the dark channel assumption is invalid. For instances, in Fig.7.1, both the DCP method and Fattal's method overestimate the transmittance and consequently make the trees in the background too dark; in Figs.7.1,7.2 and 7.4, the sky reproduced by the DCP method has incorrect hues.



Figure 7.1: Visual comparison of different methods. (a) input hazy image; (b) the DCP method; (c) Fattal's method; (d) the proposed method; (e) restored transmittance image by the proposed technique; (f) decision map in transmittance estimation, in which white for DCP, gray for GMM, and black for unestimated locations.



Figure 7.2: Visual comparison of different methods. (a) input hazy image; (b) the DCP method; (c) Fattal's method; (d) the proposed method; (e) restored transmittance image by the proposed technique; (f) decision map in transmittance estimation, in which white for DCP, gray for GMM, and black for unestimated locations.



Figure 7.3: Visual comparison of different methods. (a) input hazy image; (b) the DCP method; (c) Fattal's method; (d) the proposed method; (e) restored transmittance image by the proposed technique; (f) decision map in transmittance estimation, in which white for DCP, gray for GMM, and black for unestimated locations.



Figure 7.4: Visual comparison of different methods. (a) input hazy image; (b) the DCP method; (c) Fattal's method; (d) the proposed method; (e) restored transmittance image by the proposed technique; (f) decision map in transmittance estimation, in which white for DCP, gray for GMM, and black for unestimated locations.

Chapter 8

Conclusion and Future Work

The surface properties of our natural world afford us some strong, robust priors that can be exploited to solve the problem of haze removal. Two physically backed, previously unexplored priors, the clustering in chrominance distributions and the Laplacian sparsity of transmittance images are used to derive more robust estimates of light transmittance through air. Experimental results validate the competitive advantages of the new technique, particularly in preventing color distortions in the dehazed images.

When estimating the transmittance image, the proposed haze removal technique combines the dark-channel prior with the aforementioned prior on chrominance distributions. It does not necessarily rely on the existence of dark points and hence can be applied in a larger class of scenes.

Apparently the proposed haze removal technique has as good visual quality as the DCP method when the dark-channel prior holds. Moreover, the former is free of hue distortions suffered by the latter when the dark-channel prior is invalid. It is easy to see that the proposed Gaussian mixture model of chrominance statistics for outdoor scenes can be made more discriminative if it is further parameterized by scene classes,

such as natural scenery, city landscape, etc.

Another main contribution of this thesis is the proposal of a new sparsity-based optimization approach for transmittance image refinement. This new refinement method relies on an observation that most outdoor object surfaces are piece-wise linear. Compared to previous refinement methods like guided filter, soft-matting etc., the proposed refinement method is more physically justified and able to generate better refined outputs.

Last but not least, unlike the dark channel prior which is tailored to dehazing, the proposed chrominance priors have more general utilities, and can be applied to solve other image restoration problems, such as color demosaicking, white balancing, color image interpolation, distance measurement in bad weathers etc.

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