AN INVESTIGATION INTO THE RELATIVE PRICE OF INVESTMENT

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By

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A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfillment of the Requirements

for the Degree

Doctor of Philosophy

McMaster University

DOCTOR OF PHILOSOPHY (2014)
(Economics)

McMaster University
Hamilton, Ontario

Title:	An Investigation into the Relative Price of Investment
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Number of Pages:	xvi, 215

Abstract

In this thesis, I examine the conventional assumption of identifying investment-specific technology by the inverse of the relative price of investment. Linking prices to technology in this fashion implies that the relative price is orthogonal to any other form of economic disturbance. However, recent research has found that both neutral technology and the relative price of investment are cointegrated in the postwar US. In the chapters that follow, I explore the impact of this identification by either linking the relative price of investment to total factor productivity, or by allowing this relative price to vary depending on investment demand. In all three chapters, I find that loosening this restriction has a sizable effect on the outcome of my research as it compares to the current literature.

In the second chapter, I investigate the effects of incorporating financial frictions into a two-country, two-good international business cycle model. The model is set up such that any changes in the relative price of investment arise endogenously. We find that the relative price of investment is positively correlated across countries in our model, much as it is in detrended US-Europe data. We also find that financial frictions tend to increase the volatility of the terms of trade and raise the international correlations of consumption, hours worked, output and investment.

Chapter 3 presents a news shock model adapted to reproduce the cointegrating relationship between total factor productivity and the relative price of investment. With cointegrated neutral and investment-specific technology, anticipated shocks to the common stochastic trend explain a substantial portion of the volatility in consumption, output, hours worked, and investment growth in the US. To the best of my knowledge, no other paper has look at the effects of news shocks in a set-up where neutral and investment-specific technology processes are cointegrated.

Chapter 4 takes a full-information model-based approach rather than a vector autoregressive empirical analysis to evaluate the link between investment-specific technology and the inverse of the relative price of investment. The two-sector model presented includes monopolistic competition, where firms can vary the markup charged on their product depending on the number of firms competing. In addition, we allow for flexibility in the return-to-scale parameter in the investment sector, which allows for curvature in investment production. With approximately half of the volatility in the relative price of investment determined by non-investment-specific technology, this chapter adds to the growing list of research that questions the legitimacy of the quality-adjusted relative price of investment as an indicator of investment-specific technology.

Acknowledgments

Above all attributes, perseverance is by far the most valued for a PhD student. From perseverance comes the strength to understand new material and to gain new skills through learning new software, and creates the drive required to attain goals that are years in the making. For my PhD, my determination to persevere came from those who supported and encouraged my ambition as I attempted to excel in my field and contribute to the scholarly work of my peers.

Of the people I wish to thank in reaching this milestone, I first and foremost want to thank my supervisor, Marc-Andre Letendre. Without his advice and the efforts he undertook to improve upon my work, I would not be here. He has been an excellent model of professionalism in both research and teaching, and it is for these reasons I chose him to be my example. In addition, I would like to thank the other members of my research committee, Alok Johri and William Scarth. The wealth of expertise they brought to my research has been a tremendous asset to my thesis as it progressed.

In addition to the aforementioned, I want to thank my wife, Ashleigh Wagner, whose encouragements continue to be instrumental. I am thankful for the sacrifices she has made so that I could reach my goals, and I look forward to raising our expanding family together. I want to thank my family, and particularly my parents, who together encouraged me to reach my goals. I am grateful for the loving support of my mother, who spent countless hours improving my math and reading skills during my early years, and of my father, who just a few years ago completed his PhD in economics as well. I will continue to seek his advice in the challenges ahead. I want to thank my in-laws, who generously provided a home away from home for me during my studies at McMaster. I also want to thank my nephews and nieces, who despite my efforts continue to call me Doctor Uncle Joel. I am thankful for my Mennonite/Baptist community, who encouraged my growth, both mentally and spiritually. Lastly, I want to thank my friends, who continue to mention my education path as an example to students with learning disabilities who doubt their abilities. I will not fail them.

Contents

List of Figures

List of Tables

Declaration of Academic Achievement

Chapter 2 was co-authored with my supervisor, Marc-Andre Letendre. Together we planned collaboratively the structure of the paper. I performed model simulations, which he would then critique and revise. A similar approach was taken in the write-up of this chapter.

I am the sole authors of all chapters other than chapter 2.

Chapter 1

Introduction

What drives business cycles? Which forces perpetuate the boom-bust cycle observed in a majority of macroeconomic aggregates? Understanding the determinants of these economic fluctuations has been the subject of debate, with the current business cycle literature still wavering on their origins. Coming out of the 2007 US recession, which depressed economies worldwide, the question "what drives the business cycle" is as relevant now as it has ever been. Knowing how to read the current economic environment and predict movements in macroeconomic aggregates allows households and firms to be better prepared for economic downturns, while simultaneously informing policy makers on the best decisions to mitigate recessions.

The business cycle literature has debated over time with respect to which external factors and which model setup best describes the data. Kydland and Prescott (1982) first identified changes in productivity as a potential source of business cycle volatility. Their neutral technology shocks defined by the Solow Residual are technology shocks which affect productivity economy-wide. The conclusion that economic decline results from technological regression proved to be too unsavoury for most macro-economists, leading others to suggest alternative explanations. Greenwood, Hercowitz, and Huffman (1988) introduced the idea that economic fluctuations are the result of productivity gains specific to the investment sector, rather than neutral technology shocks that affect productivity economy-wide, as suggested by Kydland and Prescott (1982). With even a temporary investment-specific technology (IST) shock, there is a long-lasting effect on macroeconomic aggregates, causing Greenwood, Hercowitz, and Krusell (1997) to attribute over half of the growth of output in postwar US to growth in investment technology.

As research in this field evolved, so did the view of IST as a potential source of business cycle volatility. Early research by Fisher (2006), who adapted the Greenwood, Hercowitz, and Krusell (1997) model to allow for a stochastic growth in IST, found that these shocks explain a sizable fraction of hours worked and output in the short-run. Justiniano and Primiceri (2008) argue that IST could be used to explain the Great Moderation observed in the US during the mid-1980's. Justiniano, Primiceri and Tambalotti (2010) argue that investment-specific shocks impact high frequency variability in output and hours worked through the inclusion of imperfect competition in their model. Later work, however, such as that by Justiniano, Primiceri, and Tambalotti (2011), Beaudry and Lucke (2009) as well as Schmitt-Grohe and Uribe (2012), come to a different conclusion. Each concludes that only a trivial percentage of the volatility in economic aggregates over the business cycle can be explained by shifts in IST. Since their work, business cycle literature has converged on the idea that IST shocks are incapable of explaining either low, or high-frequency volatility in any economic aggregate.

Despite IST's fall from glory, Greenwood, Hercowitz, and Huffman's (1988) IST shocks have become a staple feature in the business cycle literature. Likewise, identification of IST by the inverse of the relative price of investment (RPI) has remained more or less unchallenged. The assumption is simple. With a linear technology in the conversion of consumption goods into investment goods, any inflection in the RPI must be due to changes in technology specific to that sector. Thus, IST can be identified by the inverse of the RPI. The key consequence of this assumption is the orthogonality of the RPI with all other forms of economic disturbances, both technological and otherwise. Fisher (2009) highlights the orthogonality condition as a potentially damaging assumption whenever there exists any asymmetries between consumption and investment production. Kim (2009) applies a structural vector autoregression (SVAR) to assess the relative importance of IST shocks in explaining movement in the RPI. He concludes that the orthogonality condition does not hold with approximately 63 percent of the volatility in the RPI from 1955:I-2000:IV due to shocks typically assumed to be orthogonal to the RPI. Furthering this discussion, Basu et al. (2013) tackle the identification of IST by utilizing micro-level data rather than the RPI to identify productivity improvements in this sector. Their conclusion is that the RPI is slow to respond to changes in IST, with a typical lag time of up to three quarters. Lastly, Schmitt-Grohe and Uribe (2011) show that the orthogonality assumption regarding the RPI can be safely abandoned after finding that Total Factor Productivity (TFP) and the RPI are cointegrated in the postwar US. Following this train of thought, the three chapters included in this thesis tackle the prevailing assumption proposed by Greenwood, Hercowitz, and Huffman (1988) and a majority of literature that follows.

Chapter 2 tackles the challenge of correctly identifying the co-movement of key macroeconomic aggregates within a standard two-country, two-good business cycle framework. Chapter 2 adds to this model framework, first developed by Backus, Kehoe, and Kydland (BKK) (1993), by adapting it to include financial frictions in the production of investment goods à la Carlstrom and Fuerst (1997). These financial frictions are included by assuming that (1) entrepreneurs require external financing for the production of investment goods, and (2) lenders can observe the idiosyncratic productivity of the entrepreneur at a cost. This costly state verification framework implies that investment goods sell at a premium whenever there is a chance of default by entrepreneurs. By modifying the BKK model in this way, cross-country correlations in key macroeconomic aggregates increase and bring these values more in line with the data. Furthermore, there is also a sizable increase in the volatility of the terms of trade, bringing it closer to the data. Additionally, by incorporating financial frictions à la Carlstrom and Fuerst (1997), investment prices move together across countries. All of this happens without requiring technological spillovers across countries.

In the standard two-country, two-good real business cycle model, a Hicksian-neutral productivity shock for the home country boosts that country's intermediate goods production, and causes households within that country to increase both consumption and investment. With agency costs incorporated, entrepreneurs who lack the necessary funds required to self-finance, respond to the increased demand for investment goods by increasing their reliance on external funds. This leads to an increase in the interest rate charged on borrowed funds and, consequentially, the relative price of investment goods rises. With a relatively high RPI, households reduce investment until entrepreneurs raise the required internal funds to self-finance, resulting in a drop in the RPI. This temporary lag in investment spending causes a surge in the quantity of the home country's intermediate good available in international markets. Consequentially, there is a drop in export prices. For the foreign country, the drop in import prices encourages an increase in final good production, leading to an increase in consumption and investment in the foreign country, thus boosting cross-country correlations for these variables. Likewise, increased investment in the foreign country increases agency costs in that sector, and as a result, investment prices are also positively correlated across countries, matching closely the correlation observed between the US and the Euro area.

Chapter 2 focuses on the impact of unanticipated shocks on economic activity. However, it has been a long-standing belief among macroeconomists that economic activity responds to both expected and unexpected economic disturbances. Beaudry and Portier (2006) validated this idea by identifying anticipated technology shocks in the US economy. Through their cointegrated SVAR-based approach, they find that anticipated shocks to TFP explain over half of the volatility in output. With this research, Beaudry and Portier (2006) co-created a new focus on expectation-driven business cycles, referred to here and elsewhere as the news (or anticipated) shocks literature. However, recent research by both Schmitt-Grohe and Uribe (2012) as well as Khan and Tsoukalas (2012) comes to a very different conclusion. By applying a full-information model-based approach rather than a SVAR-based empirical analysis to the data, they find that anticipated technology shocks are not important in generating business cycle volatility. Rather, they find that non-technological disturbances play a significant role in generating business cycle volatility.

As mentioned earlier, the business cycle literature has traditionally assumed that any inflection in the RPI must be due to a shift in technology specific to the investment sector. This assumption is vital, as it allows the identification of IST by the inverse of the RPI. Recent research by Schmitt-Grohe and Uribe (2011) has shown that contrary to the previous assumption, log RPI and log TFP are in fact cointegrated. Chapter 3 adapts a standard news shock model to replicate the cointegrating relationship between the two variables. After a careful Bayesian estimation followed by a variance decomposition, my results indicated that anticipated technology shocks are an important source of business cycle volatility with approximately 30 percent of the volatility of output, consumption, and investment growth due to shifts in anticipated technology. These results contrast with recent findings by Schmitt-Grohe and Uribe (2012) and Khan and Tsoukalas (2012), mentioned above, that anticipated technology shocks are not important in generating any business cycle volatility.

The common stochastic trend shared between TFP and the RPI implies that any deviation from the long-run relationship shared between these two variables must generate a counteracting response in the short-run so as to maintain this relationship. This conclusion is central to the results listed above since any anticipated shift in neutral technology must be followed shortly after by a similar increase in IST. Therefore, agents anticipating an increase in productivity will also anticipate a decline in the RPI shortly thereafter. With these two technologies moving in tandem, anticipated TFP shocks regain their relevance as they now explain approximately 30 percent of the volatility in output consumption and investment growth over the business cycle. With these findings, this chapter argues that future news cycle research should account for the cyclical pattern in the RPI when analyzing and comparing the relative importance of one economic disturbance in generating business cycle volatility.

Chapters 2 and 3 both challenge the current assumption that IST, and therefore the RPI are orthogonal to any and all forms of economic disturbances. Chapter 2 challenges this assumption by allowing the RPI to vary depending on the entrepreneur's reliance on external financing. Thus, the RPI moves endogenously with the business cycle. Chapter 3 challenges the orthogonal assumption by showing that neutral and investment-specific technologies follow a common stochastic trend. Adapting the standard news shock model to allow for cointegration between these technologies implies that variability in the RPI over the business cycle is determined exogenously, as it is pinned down to another exogenous shock, TFP. This also implies that the RPI does not respond to shifts in investment demand. This approach is referred to as the exogenous approach to replicating movement in the RPI in this thesis. Does the endogenous approach (see chapter 2 for example) outperform the exogenous approach? Is it adequate to replicate movement in the RPI over the business cycle exogenously by linking IST and TFP together? These are the questions asked and answered in chapter 4.

Chapter 4 puts forward a two-sector model extended to allow for endogenous movement in the RPI. The first of these extensions is the inclusion of endogenous price markups. This is done by assuming that each sector is populated by a finite number of monopolistically competitive firms operating within an infinite number of industries. With a finite number of firms operating within both the consumption and investment sectors, firms choose the price they charge based on number of firms competing, rather than opting for a fixed markup based on the substitutability of their product over another. With firm entry (and exit) determined by the demand for their product, any disturbance, technological or otherwise, that increases (decreases) demand for their product leads to a decline (rise) in the markup charged over marginal costs. Whenever the relative demand for investment goods out-paces demand for consumption, there is therefore a decline in the RPI. With endogenous price markups, this implies that the RPI moves counter-cyclically as observed in the US data.

The second extension is to allow flexibility in the linearity assumption adopted by Greenwood, Hercowitz, and Krusell (1997) by allowing curvature in investment production. When this assumption is relaxed, the RPI moves in response to changes in TFP as the productivity gain leads to a shift in labour and capital services to the more productive sector. With a decline in the RPI in the postwar US, one would be tempted to confer increasing returns-to-scale in the production of investment goods. When the two-sector model mentioned above is adapted to allow for flexibility in the returns-to-scale in investment production, we find that there is evidence that the decline of the RPI in the US could be explained in part by increasing returns-to-scale in that industry.

When both of these extensions are made, we find that over half of the volatility in the RPI can be explained by shocks other than IST. When compared to a two-sector model, where the RPI is determined exogenously via cointegrated technologies and technological spillovers, this value declines to approximately 40 percent. Furthermore, with approximately 15 percent of the volatility in the RPI due to non-technological disturbances such as wage markup shocks, preference shocks, and shocks to the marginal efficiency of investment, these results indicate that non-technological disturbances are a crucial determinant of the RPI through their effect on relative demand. Thus, merely adapting the exogenous processes for TFP and IST to replicate movement in the RPI is inadequate. Furthermore, this research puts us closer to the estimate found by Kim (2009), who through his SVAR-based empirical approach found that as little as 27 percent of the RPI in the US can be attributed to IST.

The commonality shared by each of these three chapters is the challenge to the orthogonality condition first assumed by Greenwood, Hercowitz, and Krusell (1997). In chapter 2, this assumption is challenged by allowing the RPI to vary depending on the entrepreneurs' reliance on external financing. Chapter 3 challenges this assumption by allowing TFP and IST to follow a common stochastic trend within a standard news cycle model. Chapter 4 then challenges the current literature by comparing the two possible approaches (exogenous or endogenous) for replicating low and high-frequency volatilities observed in the RPI. In chapter 2, we find that inclusion of agency costs allows positive international co-movement without relying on technological spillovers, along with increased volatility in the terms of trade. Chapter 3 finds that the relative importance of anticipated technology shocks are impacted by whether the cointegrating nature of TFP and IST is properly addressed in modeling news shocks. Chapter 4 then finds that the RPI responds in part to changes in non-technological disturbances, thus suggesting that the endogenous-based approach outperforms any model where the RPI is tied to TFP. Within all of these chapters, the overarching challenge is the orthogonality assumption, and it is confirmed that this assumption is inadequate at best.

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Chapter 2

Agency Costs and International Cycles

Marc-André Letendre and Joel Wagner McMaster University

2.1 Introduction

In the simplest stochastic dynamic general equilibrium models (*e.g* the standard RBC model and many of its extensions) a representative agent can convert one unit of consumption good into one unit of capital ready for production. This very simple capital accumulation process can be made more realistic by breaking it into two stages. In the first, a technology is employed to convert consumption goods into investment goods. Shocks hitting that technology are known as investment-specific technology (IST) shocks. For example, shocks that lower costs in the investment sector, such as advancements in computer processing or improvements in production methods are of the IST type. In the second stage, investment goods are transformed into productive capital before entering the production process. Shocks that hit this transformation process are known as shocks to the marginal efficiency of investment (or MEI shocks). For example, shocks which affect the dissemination of investment goods, such as new forms of transportation or troubles in the financial sector are of the MEI type.

One of the most obvious impediment in acquiring capital is the credit needed to purchase investment goods. Disturbances to the credit market, including the recent financial crisis, affect the ability of firms to acquire capital, with consequences for the course of the economy. In their empirical work using US data, Justiniano, Primiceri, and Tambalotti (2011) identify MEI shocks as the primary source of business cycle volatility, accounting for 60% of the variation in US GDP, while IST shocks account for only a small fraction of that variance. Moreover, Schmitt-Grohé and Uribe (2012) estimate a closed-economy model including an extensive list of anticipated and unanticipated shocks. They conclude that MEI shocks are "estimated to explain a significant fraction of variation in output (28%) and investment (63%)" in the US.[1]

[1]Schmitt-Grohé and Uribe (2012) p. 2759

Given those empirical results it is of interest to see how financial frictions change the dynamics of an international business cycle model. This paper contributes to the international business cycles literature by presenting a detailed analysis of a two-good two-country international business cycle model à la Backus, Kehoe and Kydland (BKK) (1994) augmented with financial frictions à la Carlstrom and Fuerst (CF) (1997).[2] To the best of our knowledge, this has not been done before. Our model diverges from models commonly adopted in the international business cycle literature as we include entrepreneurs producing investment goods. They do not have enough wealth to entirely finance their investment projects so they must borrow to be able to produce. Households lend to entrepreneurs via financial intermediaries. We follow the agency model in CF,[3] which can be summarized as follows: if (i) entrepreneurs are reliant on external financing to fund their investment projects, (ii) there is asymmetric information between borrowers (entrepreneurs) and lenders, and (iii) it is costly for lenders to verify each entrepreneur's behavior, then there is a risk premium charged to entrepreneurs relying on borrowed funds. When a shock leads to an increase in the demand for investment goods the agency problem just described leads to a spike in the risk premium, which makes entrepreneurs reluctant to dramatically increase the size of their investment projects. Hence the supply of investment goods responds less than in the BKK model. The resulting excess demand for capital goods yields an increase in the relative price of investment (RPI).

[2] We elected to use CF (1997) model over other costly state verification models since it allows for endogenous movement in the relative price of investment.

[3] An alternative is to follow Bernanke and Gertler (1989).

As opposed to what is typically seen in the international business cycles literature, our model does not need to rely on capital adjustment costs or investment shocks to produce changes in the RPI. As the previous paragraph makes clear changes in the latter are purely endogenous and are the product of financial frictions. This is in sharp contrast with the canonical model where consumption goods are transformed into new capital one for one, implying that the capital supply curve is perfectly elastic at unity. It is also in contrast with papers which have shocks specific to the production of investment or new capital determine exogenously the rate at which consumption goods are converted into new capital.[4] It is worth noting that there is evidence that total factor productivity and RPI are interrelated in the US, Europe and Canada (*e.g.* Schmidt-Grohé and Uribe (2011) and Wagner (2013)). These two variables are intimately related in our theory as TFP shocks eventually leads to changes in RPI.

As highlighted above, our setup allows us to study how financial frictions in the form of agency costs produce endogenous changes in the price of investment. It also allows us to study how agency costs are transmitted across countries and how they influence the international co-movements of both investment and its price. Our main finding relates to the correlation of the RPI across countries. In our US-Europe data that correlation is small and positive. Remarkably, our model is capable of reproducing such a correlation despite the fact that innovations to total factor productivity are

[4]See Greenwood, Hercowitz and Huffman (1988) for an early contribution to the one-country business cycle literature as well as Letendre and Luo (2007) and Raffo (2010) for early contributions to the international business cycles literature.

not correlated across countries and that productivity shocks do not spill over across countries.

To understand why RPI is positively correlated across countries suppose that an unexpected positive productivity shock raises total factor productivity in country 1 in period 1. Since this shock is persistent households desire to increase their capital stock, hence they demand more investment goods. In light of this, entrepreneurs increase the size of their projects. Given that their net worth is mostly pre-determined they must borrow more. This greater reliance on external funds raises agency costs and the price of investment goods in terms of the final good shoots up. This tempers country 1's demand for investment goods leading to more units of that country's intermediate good finding their way to the world market. This results in a more important depreciation of the terms of trade than in the BKK model. The significant drop in the price of country 1's good prompts country 2 to buy more of it to raise consumption and investment (consumption smoothing). This increase in the demand for investment goods in country 2 pushes agency costs up, which raises the RPI in that country. These initial positive responses of the price of investment in both countries translate into a positive correlation in our simulations.

Furthermore, our analysis contributes to the literature by showing that adding financial frictions à la Carlstrom and Fuerst (1997) to a BKK-type model raises the international correlations of output, consumption, investment and labour[5] in addi-

[5] Yao (2012) documents the effects a leverage constraint has on international correlations in an

tion to enhancing the volatility of the terms of trade. In our simulations output, consumption, labour and investment are positively correlated across countries, terms of trade are highly volatile and net exports are countercyclical. Therefore, by adding a financial intermediation sector and investment goods producers we make the structure of the model more realistic with the added benefit of addressing some of the key weaknesses of the seminal BKK model.

The remainder of the paper is organized as follows. Section 2.2 lays out our model with agency costs. We explain how we select parameter values in Section 2.3. Section 2.4 discusses the model's implications and sensitivity analysis. Section 2.5 concludes.

2.2 Model

Our model is based on the two-good international real business cycle model of Backus, Kehoe and Kydland (1994). It consists of two countries that trade intermediate goods and financial assets. Each country has a large number of identical households, a large number of perfectly competitive and identical intermediate good producers and a large number of perfectly competitive and identical final good producers. Each country produces a single intermediate good. Intermediate goods from both countries are needed to produce final goods. Final goods are not traded internationally and can be either consumed or used as input in the production of new physical capital.

extended one-good BKK model.

We extend the BKK model by adding to each country (i) a financial sector which contains a large number of perfectly competitive and identical capital mutual funds (CMFs) and (ii) a set of entrepreneurs who are the producers of physical capital in the economy. As in CF the idiosyncratic productivity of entrepreneurs is private information which gives rise to agency costs. These agency costs lead to endogenous changes in the RPI.

Households in both countries have access to a complete set of state-contingent securities. The next few subsections provide a detailed description of the model. Flow charts depicting the model's structure and flow of goods can be found in appendix 2.A.2.

2.2.1 Intermediate Goods Sector

Let l index countries, where $l \in \{1, 2\}$. Within each country there is a large number of identical and perfectly competitive intermediate good producers. Hence we restrict our attention to a representative producer. Each country specializes in the production of an intermediate good. Country 1 (or the home country) produces intermediate good a (one can think of this as aluminum) and country 2 (or the foreign country) produces good b (one can think of this as bricks). These goods are traded across countries. We let q_{lt} (q^*_{lt}) denote the price of the intermediate good produced in country l in terms of the final good in country 1 (country 2).

Production of intermediate goods requires capital and labour, which are assumed to be immobile across countries. The representative producer in country l rents the economy's entire capital stock[6] available at the beginning of period t (K_{lt}) and hires the labour services supplied by households (H_{lt}) to produce intermediate goods according to the production function

$$Y_{lt} = Z_{lt} K_{lt}^{\alpha_K} (H_{lt})^{\alpha_H}, \qquad l = \{1, 2\} \tag{2.1}$$

where $0 < \alpha_k, \alpha_H < 1$, $\alpha_k + \alpha_H = 1$ and Z_{lt} denote an unanticipated stationary productivity shock. The representative intermediate good producer in country 1 maximizes

$$\Pi_{1t} = q_{1t} Y_{1t} - w_{1t} H_{1t} - r_{1t} K_{1t} \tag{2.2}$$

subject to (2.1) for $l = 1$. The producer rents capital from both households and entrepreneurs at a rental rate r_1 and hires household labour at the wage rates w_1. Due to the constant returns-to-scale production function and perfectly competitive markets, the wage and rental rates of labour and capital are equal to their corresponding marginal product.

[6]Upper case variables denote aggregate variables, whereas lowercase variables are specific to an individual agent.

2.2.2 Households

Each country is populated by a large number of identical, infinitely lived households. We assume that the household population in each country has measure one. The representative household in country $l = \{1, 2\}$ has the following expected lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_{lt}^H - \psi {H_{lt}}^{\nu} X_{lt}]^{1-\gamma}}{1-\gamma}, \qquad 0 < \beta < 1,\ \nu > 1,\ \psi > 0 \tag{2.3}$$

where

$$X_{lt} = {C_{lt}^{H}}^{\eta} X_{lt-1}^{1-\eta}, \qquad 0 \leq \eta \leq 1. \tag{2.4}$$

We denote consumption by C_{lt}^H and labour supply by H_{lt}. Whether or not consumption and hours worked by the household are time separable is determined by η, through the inclusion of X_{lt} in the households utility function. These preferences were first proposed by Jaimovich and Rebelo (2009). They allow one to dial up or down the strength of the wealth effect on the household's labour supply decision. When $\eta = 1$ (full wealth effect) these preferences become equivalent to the widely used preferences of King, Plosser, Robelo (KPR) (1988). When $\eta = 0$ (no wealth effect) they become equivalent to those in Greenwood, Hercowitz, and Huffman (GHH) (1988).

The representative household in country 1 must satisfy the budget constraint

$$C_{1t}^H + P_{1t}I_{1t}^H + q_{1t}\Sigma_{s_{t+1}} M(s^t, s_{t+1})D_1(s^t, s_{t+1}) = w_{1t}H_{1t} + r_{1t}K_{1t}^H + q_{1t}D_1(s^{t-1}, s_t) + PR_t. \tag{2.5}$$

The left side of the equation reflects that the household uses its income for three purposes; to consume, invest, and insure. Households can increase their capital stock by purchasing I_{1t}^H units of new investment good at a unit cost P_{1t}. Here P_{1t} represents the price of new investment goods in country 1 relative to the price of that country's final good. The price of new investment goods is taken as given by households (just like all other prices). Lastly, households trade state-contingent (Arrow-Debreu) securities. We can imagine that households are the ones deciding how many units of intermediate goods to import or export. Accordingly, they will trade contingent claims with households in the other country to cover any trade balance deficit. Let $D_1(s^t, s_{t+1})$ be the quantity of contingent claims purchased by households in country 1 after history s^t and that pays one unit of good a in period $t+1$ when the state of the economy is s_{t+1}. We denote the price of these contingent claims (in units of good a) by $M(s^t, s_{t+1})$

The right side of (2.5) shows that the representative household income in period t consists of its labour income $w_{lt}H_{lt}$, rental income $r_{lt}K_{lt}^H$, payoffs $D_1(s^{t-1}, s_t)$ from the relevant state-contingent security purchased in the previous period and collective profits of all domestic firms. The budget constraint of the representative household in country 2 is analogous.

The representative household capital accumulation equation is

$$K^H_{lt+1} = (1-\delta)K^H_{lt} + I^H_{lt} \tag{2.6}$$

where δ is the depreciation rate. The representative household chooses the sequences of consumption, labour, contingent claims and investment that maximize their lifetime utility (3.5) subject to (2.4)-(2.6), taking all prices as given.

2.2.3 Final Good Sector

Each country has a large number (measure one in each country) of identical and perfectly competitive final good[7] producers. Hence, we restrict our attention to a representative final good producer. Each country specializes in the production of a non-traded final good, which is used for household and entrepreneurial consumption and as an input in the production of investment goods. Production of the final good requires both domestic and imported intermediate goods. Country 1's production of final goods is given by the following Armington (1969) aggregator

$$G(a_{1t}, b_{1t}) = [\kappa a_{1t}^{\frac{\sigma-1}{\sigma}} + (1-\kappa) b_{1t}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}, \qquad 0 < \kappa < 1, \quad \sigma > 0. \tag{2.7}$$

[7]We use "final goods" and "consumption goods" interchangeably.

When $\kappa > 0.5$ each country has a bias towards the domestically produced intermediate good. Here σ represents the elasticity of substitution between domestic and foreign goods.

The representative final good producer in country 1 chooses quantities a_1 and b_1 to maximize its profits given by $G(a_{1t}, b_{1t}) - q_{1t}a_{1t} - q_{2t}b_{1t}$. Similarly, the foreign final good producer chooses a_2 and b_2 to maximize $G(b_{2t}, a_{2t}) - q^*_{1t}a_{2t} - q^*_{2t}b_{2t}$ where

$$G(b_{2t}, a_{2t}) = [\kappa b_{2t}^{\frac{\sigma-1}{\sigma}} + (1-\kappa)a_{2t}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}. \tag{2.8}$$

2.2.4 The Entrepreneurs

Each country is populated by an infinite number of entrepreneurs (measure one in each country). Entrepreneurs are not identical. The efficiency of the process employed by an entrepreneur to transform consumption goods (the final good in this model) into investment goods is specific to each entrepreneur and is determined by the random variable ω. The latter is assumed to be independently and identically distributed across entrepreneurs and across time periods with cumulative distribution function $\Phi(\omega)$ and density function $\phi(\omega)$. More specifically, an entrepreneur with ability or productivity ω transforms (within one period) i units of the consumption good into ωi units of investment goods. The resources needed to fund the investment projects come from both entrepreneurial wealth n (internal funds), as well as funds borrowed (via intra-period loans) from financial intermediaries (CMFs). Intermediaries obtain funds from households.

To introduce agency costs into our model, we assume that each entrepreneur's random productive potential, ω, is private information. If a CMF wishes to observe an entrepreneur's level of productivity it must incur a monitoring cost μi, which is measured in units of investment goods. Due to this asymmetric information, there exists a moral hazard problem between borrowers/entrepreneur and their potential lenders. The model is set up such that entrepreneurs will always truthfully report their ω.

An individual entrepreneur living in country l maximizes his expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} (\beta\Gamma)^t c_{lt}^E \qquad 0 < \Gamma < 1 \tag{2.9}$$

where c_{lt}^E is the entrepreneur's level of consumption. The assumption $0 < \Gamma < 1$ implies that entrepreneurs are relatively more impatient than households. This assumption, which is common in this literature, is made to avoid scenarios where entrepreneurs fully finance their investment projects through dramatically reducing their level of consumption.

Entrepreneurs face a budget constraint. An important component of that constraint is net worth. At the beginning of period t, an entrepreneur rents out his current capital stock (denoted k_{lt}^E) to local intermediate good producers which generates rental income $r_{lt}k_{lt}^E$. Then, the entrepreneur sells off all of his undepreciated capital stock $(1-\delta)k_{lt}^E$ to the local CMF which pays $P_{lt}(1-\delta)k_{lt}^E$ units of consumption goods to the entrepreneur. After all these transactions an entrepreneur in country $l = \{1, 2\}$ has total net worth (measured in units of domestic consumption goods)

$$n_{lt} = r_{lt}k_{lt}^E + P_{lt}(1-\delta)k_{lt}^E. \tag{2.10}$$

In the next section we outline the financial contract between entrepreneurs and the CMF. But for now it is enough to state that an entrepreneur's net worth is inversely related to the interest rate charged on his loan. Hence a rational risk neutral entrepreneur always chooses to sells off all of his undepreciated capital supply to bolster net worth. The income earned from the production of investment goods, discussed in the next subsection, constitutes the last element required to determine their budget.

Financial Contract and Investment Decision

Since country l entrepreneurs are dealing with country l CMF only (and *vice versa*) we momentarily omit the country subscript on variables to ease notation. Similarly, since loan contracts are entirely "resolved" within a period we also omit time subscripts. Entrepreneurs are indexed by j. For example, entrepreneur j's productivity is denoted ω_j.

Asymmetric information in our model becomes relevant only when an entrepreneur's net worth is small enough that it must partially rely on external financing from the CMF. In that context an entrepreneur with net worth n_j who invests i_j must borrow $(i_j - n_j)$ units of consumption goods from the CMF in order to start production. The sequence of events within a period is borrowed from CF:

1. Entrepreneur j and a CMF enter into a contractual agreement whereby investment i_j is determined. Given entrepreneurial net worth n_j, the amount borrowed $(i_j - n_j)$ is also determined.

2. Productivity ω_j is realized. Based on this realization the entrepreneur decides whether to honour his contract. The latter decision rests on a threshold productivity level $\bar{\omega}_j$ defined below. More specifically, if $\omega_j \geq \bar{\omega}_j$ the entrepreneur repays the CMF. Otherwise the entrepreneur defaults on his loan which triggers monitoring by the CMF and confiscation of all of the investment goods just produced by the entrepreneur.

3. Entrepreneurs who did not default make their consumption decision.

Despite the heterogeneity of the entrepreneurial population asymmetric information implies that the CMF charges a common interest rate r^k on all funds borrowed by entrepreneurs. Thus, an entrepreneur who borrows $(i_j - n_j)$ units of consumption goods from the CMF in any given period (and who does not default) has to pay back

$(i_j - n_j)(1 + r^k)$ units of investment goods to the CMF in that same period.

As mentioned in point 2 above, there exists a productivity threshold $\bar{\omega}_j$ at which entrepreneur j is indifferent between defaulting or paying back his loan to the CMF. This indifference point is reached when the quantity of investment goods produced by the entrepreneur is exactly equal to the amount he has to pay back to the CMF. Hence, $\bar{\omega}_j$ is the value of ω_j for which

$$i_j \bar{\omega}_j = (i_j - n_j)(1 + r^k) \quad \Rightarrow \quad \bar{\omega}_j = (1 + r^k)(1 - n_j/i_j). \tag{2.11}$$

Any productivity realization ω_j less than $\bar{\omega}_j$ leads entrepreneur j to default.

Given the definition of $\bar{\omega}$ provided above, we can now calculate entrepreneur j's expected net income (before idiosyncratic risk is resolved, *i.e.* before ω_j is observed) associated with the production of investment goods

$$F_j^E \equiv P \left(\int_{\bar{\omega}_j}^{\infty} \omega \ i_j \ \phi(\omega) \mathrm{d}\omega - (1 - \Phi(\bar{\omega}_j))(1 + r^k)(i_j - n_j) \right). \tag{2.12}$$

The first term in parentheses calculates the expected output of investment goods when entrepreneur j does not default. From this quantity we subtract the expected amount that must be paid back to the CMF. Recall that an entrepreneur who defaults on his loan sees his entire output confiscated by the CMF, so his net income is zero.

Expected income can be written as follows using (2.11)

$$F_j^E = P\ i_j \underbrace{\left(\int_{\bar{\omega}_j}^{\infty} \omega\phi(\omega)\mathrm{d}\omega - [1-\Phi(\bar{\omega}_j)]\bar{\omega}_j\right)}_{\equiv f(\bar{\omega}_j)} = P\ i_j f(\bar{\omega}_j). \tag{2.13}$$

Similarly the expected income of the CMF in its dealing with entrepreneur j is given by

$$F_j^{CMF} \equiv P\left(\int_0^{\bar{\omega}_j} \omega i_j\phi(\omega)\mathrm{d}\omega - \Phi(\bar{\omega}_j)\mu i_j + (1-\Phi(\bar{\omega}_j))(1+r^k)(i_j - n_j)\right) \tag{2.14}$$

where income is calculated in units of consumption goods. The first two terms in parentheses calculate the expected amount the CMF confiscates when the entrepreneur defaults, net of the expected monitoring costs. The last term accounts for the expected payment made by entrepreneur j when he does not default.

Expected income can be written as follows using (2.11)

$$F_j^{CMF} = P\ i_j \underbrace{\left(\int_0^{\bar{\omega}_j} \omega\phi(\omega)\mathrm{d}\omega - \Phi(\bar{\omega}_j)\mu\ + [1-\Phi(\bar{\omega}_j)]\bar{\omega}_j\right)}_{\equiv g(\bar{\omega}_j)} = P\ i_j g(\bar{\omega}_j). \tag{2.15}$$

Quantities $f(\bar{\omega}_j)$ and $g(\bar{\omega}_j)$ refer to the fraction of expected net capital output going to the entrepreneur and the CMF respectively. Assuming $E[\omega] = 1$ (a normalization) one can easily show that

$$f(\bar{\omega}_j) + g(\bar{\omega}_j) = 1 - \Phi(\bar{\omega}_j)\mu. \tag{2.16}$$

So far we have setup the general form of the contract, fully identified by the level of consumption goods used in investment production (i_j), and the threshold $\bar{\omega}_j$ with a gross interest rate charged on borrowed funds of

$$1 + r^k = \frac{\bar{\omega}_j}{1 - n_j/i_j} \tag{2.17}$$

which is implied by (2.11).

Following CF we assume that the lender is unaware of the entrepreneur's previous history, eliminating any repeated game scenario. Recall our assumption that ω is i.i.d. over time. Therefore, each contract is relevant for the current period only. From one of the many potential contracts, there exists an optimal contract which maximizes the expected income to the entrepreneur while leaving the lender indifferent between lending or retaining the necessary funds. Therefore, the optimal contract is the pair $(i_j, \bar{\omega}_j)$ maximizing F_j^E subject to

$$F_j^{CMF} \geq (i_j - n_j). \tag{2.18}$$

This optimization problem implies

$$i_j = \frac{n_j}{(1 - Pg(\bar{\omega}_j))} \tag{2.19}$$

and

$$P\left\{1 - \Phi(\bar{\omega}_j)\mu + \frac{\phi(\bar{\omega}_j)\mu f(\bar{\omega}_j)}{f'(\bar{\omega}_j)}\right\} = 1 \tag{2.20}$$

Since each entrepreneur takes the RPI as given equation (2.20) implies that $\bar{\omega}_j$ is an implicit function of the relative price P. Since the relative price is the same for all entrepreneurs, we have that $\bar{\omega}_j = \bar{\omega}$ for all j. Furthermore, given $\bar{\omega}(P)$ equation (2.19) determines the optimal amount of investment by the entrepreneur. This can be written as a function of the entrepreneur's wealth, and the relative price of investment

$$i_j = \frac{n_j}{1 - Pg(\bar{\omega}(P))} \equiv i(P, n_j). \tag{2.21}$$

Before turning to the consumption decision of an entrepreneur, we derive the aggregate amount of investment goods produced and supplied within a country. Entrepreneur j's choice $i(P, n_j)$ indicates the quantity of *consumption goods* the entrepreneur decides to use as input in the production of investment goods. The expected quantity of *investment goods* produced and supplied by entrepreneur j, net of monitoring costs when $\omega_j < \bar{\omega}$, is given by

$$i^s(P, n_j) \equiv i(P, n_j)\left\{1 - \mu\Phi(\bar{\omega}(P))\right\} = \frac{1 - \mu\Phi(\bar{\omega}(P))}{1 - Pg(\bar{\omega}(P))} n_j \tag{2.22}$$

Since the price of capital P is taken parametrically, an entrepreneur's choice of $i^s(P, n_j)$ is a linear function of his net worth n_j, which is convenient from the point of view of aggregation. Given our assumption of an infinite number of entrepreneurs (with measure unity), the law of large numbers implies that the above formula can be interpreted as the total amount of investment goods supplied by the population of entrepreneurs when total entrepreneurial net worth turns out to be n_j. Accordingly, denoting total entrepreneurial net worth by N, the supply of investment good in the entire country is given by

$$I^S(P, N) = \frac{1 - \Phi(\bar{\omega}(P))\mu}{1 - Pg(\bar{\omega}(P))} \cdot N \tag{2.23}$$

Bringing back country and time subscripts we have that the supply of investment goods in country l in period t is given by

$$I^S_{l,t} = \frac{1 - \Phi(\bar{\omega}_{lt})\mu}{1 - P_{lt}g(\bar{\omega}_{lt})} N_{lt} \tag{2.24}$$

where $\bar{\omega}_{lt}$ is shorthand notation for $\bar{\omega}(P_{lt})$.

The model is set up such that an entrepreneur always wishes to fully disinvest his entire net worth to fund investment projects. Accordingly, one can calculate an entrepreneur's gross expected return on internal funds (ERIF) as the ratio of $P_{lt}f(\bar{\omega}_{lt})i_{lt}$ (the expected income generated by capital production by an entrepreneur) to net worth n_{lt} (the amount of his own funds the entrepreneur places in capital production). Using equation (2.21) we write

$$ERIF_{lt} = \frac{P_{lt} f(\bar{\omega}_{lt}) i_{lt}}{n_{lt}} = \frac{P_{lt} f(\bar{\omega}_{lt})}{1 - P_{lt} g(\bar{\omega}_{lt})} \tag{2.25}$$

which is always greater than the return to external funds due to agency costs.

Budget Constraint and Consumption Decision

Here we must take into account of the heterogeneity across entrepreneurs. Let's first consider the case of an entrepreneur who draws a period-t level of productivity $\omega_{jt} < \bar{\omega}_t$. He defaults on his loan contract and sees the CMF take away his entire output of investment goods $\omega_{jt} i_{jt}$. He has no income left to finance current consumption and has no capital goods to carry into the next period.

Consider now the case of an entrepreneur who does not default ($\omega_{jt} > \bar{\omega}_t$). He produces $\omega_{jt} i_{jt}$ units of investment goods. Some of that, $(1 + r_t^k)(i_{jt} - n_{jt})$, goes back to the CMF to honour the debt agreement. What is left is allocated between (i) current-period consumption (c_{jt}^E) and capital carried into the future period (k_{jt+1}^E). Evidently, the choice of k_{jt+1}^E influences next-period net worth n_{jt+1}, which influences the expected income next period F_{jt+1}^E.[8] Therefore, when making his decision

[8]The expected income in period $t + 1$ denominated in units of $t + 1$ investment goods is $F_{jt+1}^E / P_{t+1} = i_{jt+1} f(\bar{\omega}_{jt+1})$. This takes into account of (i) repayment of the loan to the CMF when the realized productivity turns out to be greater than $\bar{\omega}_{jt+1}$ and (ii) the possibility that the

about k^E_{jt+1} in period t the entrepreneur takes into account the following $t+1$ budget constraint

$$\frac{c^E_{jt+1}}{P_{t+1}} + k^E_{jt+2} \leq i_{jt+1} f(\bar{\omega}_{jt+1}). \tag{2.26}$$

Updating the formulas for investment (2.21) and net worth (2.10) by one period in the future we can re-write the above budget constraint as

$$\frac{c^E_{jt+1}}{P_{t+1}} + k^E_{jt+2} \leq (r_{t+1} k^E_{jt+1} + P_{t+1}(1-\delta) k^E_{jt+1}) \frac{f(\bar{\omega}_{t+1})}{1 - P_{t+1} g(\bar{\omega}_{t+1})}. \tag{2.27}$$

The solvent entrepreneur's problem is to maximize expected lifetime utility (2.9) subject to his sequence of budget constraints. Assuming an interior solution, this problem yields the following Euler equation (now including country subscript l)

$$P_{lt} = \beta \Gamma E_t \left\{ [r_{lt+1} + P_{lt+1}(1-\delta)] \underbrace{\frac{P_{lt+1} f(\bar{\omega}_{lt+1})}{1 - P_{lt+1} g(\bar{\omega}_{lt+1})}}_{\mathrm{ERIF}_{lt+1}} \right\} \tag{2.28}$$

This Euler equation has a natural interpretation. The left side reflects the cost incurred by the entrepreneur to increase k^E_{jt+1} by one unit. The right side shows the expected discounted benefit of increasing k^E_{jt+1}. Such an increase raises the entrepreneur's net worth next period by an amount given by the term in square brackets. This additional net worth will be entirely used to finance investment projects in $t+1$, which is why it is multiplied by the gross expected return on internal funds (displayed

entrepreneur have a level of productivity lower than $\bar{\omega}_{jt+1}$, in which case he does not pay back his loan to the CMF and sees his output of investment goods confiscated by it.

in (2.25)).

To complete this section, we derive the aggregate budget constraint of the entire group of entrepreneurs. Recall that entrepreneur j has expected net income (in period t) given by $i_{jt}f(\bar{\omega}_{jt})$. This takes into account (i) repayment of the loan to the CMF when the realized productivity turns out to be greater than $\bar{\omega}_{lt}$ and (ii) the possibility that the entrepreneur has a level of productivity lower than $\bar{\omega}_{lt}$, in which case he does not pay back his loan to the CMF and sees his output of investment goods confiscated by it. Now let I_t denote aggregate "per entrepreneur" investment in the production of investment goods. With an infinite number of entrepreneurs and given the linearity of the expected income formula above, the law of large numbers implies that we can interpret $I_tf(\bar{\omega}_t)$ as aggregate "per entrepreneur" net income in period t. At the end of the period the group of entrepreneurs sell C^E_t/P_t units of capital back to the CMF to fund their period-t consumption, where C^E_{lt} denotes "per entrepreneur" consumption. Remaining units of investment goods are carried into the following period and constitute K^E_{lt+1}, the "per entrepreneur" stock of capital at the beginning of period $t+1$. Then, aggregating over all entrepreneurs budget constraints yields

$$\frac{C^E_t}{P_t} + K^E_{t+1} \leq I_tf(\bar{\omega}_t). \tag{2.29}$$

Using the aggregate equivalent of (2.21) and aggregate "per entrepreneur" net worth we can re-write the above budget constraint as (now adding back the country subscript

$l)$

$$\frac{C_{lt}^{E}}{P_{lt}} + K_{lt+1}^{E} \leq (r_{lt}K_{lt}^{E} + P_{lt}(1-\delta)K_{lt}^{E})\frac{f(\bar{\omega}_{lt})}{1 - P_{lt}g(\bar{\omega}_{lt})} \tag{2.30}$$

2.2.5 The Representative Capital Mutual Fund

The final component of our model is the intermediation sector, where there is a large number of identical and perfectly competitive capital mutual funds. All loanable funds in our model ultimately come from households who forward some of their savings to the representative CMF in return for investment goods. The CMF then lends to entrepreneurs these units of consumption goods received from households. We assume that all CMFs are risk neutral. Is this assumption appropriate given that CMFs allocate households savings and that household are risk averse? It turns out that it is. First, note that there is no uncertainty over the length of the contracts between CMFs and entrepreneurs. Second, there is an infinitely large number of entrepreneurs and there is no aggregate uncertainty about the distribution of ω. This implies that CMFs face no uncertainty about the total quantity of investment goods they are going to get from the population of entrepreneurs. Hence, CMFs can guarantee households that they will receive I^h units of investment goods for sure in exchange for PI^h units of consumption goods. Thus, we can think of households as being risk neutral in their willingness to loan funds.

We now describe all flows of consumption and investment goods going in and coming out of the financial intermediation sector. Recall that a detailed flow chart is provided in appendix 2.A.2.[9] We begin by accounting for the flow of consumption goods in country l. The items in the list below are arranged chronologically.

- Households pay $P_{lt}I_{lt}^{H}$ to CMFs to purchase investment goods which will be handed out to the household later in period t.
- CMFs pay $P_{lt}(1-\delta)K_{lt}^{E}$ to entrepreneurs when purchasing their undepreciated capital stock.
- CMFs lend $I_{lt}-N_{lt}$ to entrepreneurs as per their contracts. Aggregating up (2.21) we can write $I_{lt}-N_{lt}=P_{lt}g(\bar{\omega}_{lt})I_{lt}$.
- CMFs sell C_{lt}^{E} to entrepreneurs for their period t consumption.

These flows of consumption goods in country l implies that

$$\underbrace{P_{lt}I_{lt}^{H}}_{inflow}=\underbrace{P_{lt}(1-\delta)K_{lt}^{E}+P_{lt}g(\bar{\omega}_{lt})I_{lt}+C_{lt}^{E}.}_{outflow} \tag{2.31}$$

Let's now look at flows of investment goods in country l, again arranged chronologically.

- CMFs buy $(1-\delta)K_{lt}^{E}$ from entrepreneurs.

[9]Recall that CMFs produce neither consumption nor investment goods.

- A solvent entrepreneur j pays back $(1 + r_t^k)(i_{jt} - n_{jt})$ to CMFs to honour his debt contract. Alternatively, CMFs get $(\omega_j - \mu)i_j$ from entrepreneur j when he defaults. In the aggregate, the law of large numbers imply that CMFs receive from the entire group of entrepreneurs $g(\bar{\omega}_{lt})I_{lt}$ units of investment goods.

- Households receive the I_{lt}^H units of investment goods they paid for earlier in the period.

- Entrepreneurs pay C_{lt}^E/P_{lt} for the purchase of C_{lt}^E units of consumption goods.

Therefore, it must be the case that

$$\underbrace{(1-\delta)K_{lt}^E + g(\bar{\omega}_{lt})I_{lt} + \frac{C_{lt}^E}{P_{lt}}}_{inflow} = \underbrace{I_{lt}^H}_{outflow}. \tag{2.32}$$

Therefore, total profits (denominated in consumption goods) earned by the financial intermediaries in country l (inflows less outflows) are $\Pi_{lt}^{CMF} =$

$$\begin{aligned} &\left\{ P_{lt} I_{lt}^H + P_{lt}\left[(1-\delta)K_{lt}^E + g(\bar{\omega}_{lt})I_{lt} + \tfrac{C_{lt}^E}{P_{lt}}\right]\right\} \\ &- \left\{ P_{lt}(1-\delta)K_{lt}^E + P_{lt}g(\bar{\omega}_{lt})I_{lt} + C_{lt}^E + P_{lt}[I_{lt}^H]\right\} \end{aligned} \tag{2.33}$$

which are equal to zero as implied by (2.31) and (2.32).

2.2.6 Shocks

The productivity shocks driving total factor productivity (TFP) of intermediate goods producers are assumed to follow a stationary first-order vector autoregressive process given by

$$\begin{bmatrix} \ln Z_{1t} \\ \\ \ln Z_{2t} \end{bmatrix} = \begin{bmatrix} \rho_z & \rho_s \\ \\ \rho_s & \rho_z \end{bmatrix} \begin{bmatrix} \ln Z_{1t-1} \\ \\ \ln Z_{2t-1} \end{bmatrix} + \epsilon_t \tag{2.34}$$

$$E(\epsilon_t \epsilon_t') = \Sigma \tag{2.35}$$

where the vector of innovations $\epsilon_t = [\varepsilon_{1t} \;\; \varepsilon_{2t}]'$ is realized at the beginning of period t (before any decisions are made). Innovations are normally distributed and are independent over time. Parameters ρ_z and ρ_s govern the degrees of persistence and international spillovers of the shocks, respectively. Σ denotes the variance-covariance matrix of ϵ.

2.2.7 Market Clearing Conditions and Other Variables

The total amount of intermediate goods produced in period t in country 1 is allocated as follows

$$Y_{1t} = a_{1t} + a_{2t} \tag{2.36}$$

where a_{1t} is the number of units of good a used in the production of country 1's final good, and a_{2t} is the number of units exported to country 2, where it is used in the production of that country's final good. In the same way, country 2's output of good b is allocated as

$$Y_{2t} = b_{1t} + b_{2t} \tag{2.37}$$

where b_{2t} is the number of units of good b used in the production of country 2's final good, and b_{1t} is the number of units exported to country 1, where it is used in the production of that country's final good.

Variable H_{lt} appearing in the representative household's utility function and the representative intermediate good producer's production function represents both total household labour supplied and total demand for household-type labour by intermediate firms in country l, thus clearing the labour market.

For capital markets to clear, capital supply $K_{lt}^E + K_{lt}^H$ must equal the total capital demanded by the intermediate firm K_{lt}

$$K_{lt} = K_{lt}^E + K_{lt}^H. \tag{2.38}$$

The representative household's capital accumulation equation (2.6) and the overall entrepreneurial budget constraint (2.29) imply the accumulation equation

$$K_{lt+1}^H + K_{lt+1}^E = (1-\delta)K_{lt}^H + I_{lt}^H + f(\bar{\omega}_{lt})I_{lt} - \frac{C_{lt}^E}{P_{lt}}. \tag{2.39}$$

Substituting out I_{lt}^H using equation (2.32) yields

$$K_{lt+1}^H + K_{lt+1}^E = (1-\delta)K_{lt}^H + (1-\delta)K_{lt}^E + [g(\bar{\omega}_{lt}) + f(\bar{\omega}_{lt})]I_{lt} + \frac{C_{lt}^E}{P_{lt}} - \frac{C_{lt}^E}{P_{lt}}. \quad (2.40)$$

Using (2.38) and (2.16) gives us the aggregate capital accumulation equation

$$K_{lt+1} = (1-\delta)K_{lt} + [1 - \Phi(\bar{\omega}_{lt})\mu]I_{lt} \quad (2.41)$$

As for the market clearing condition for final goods we have

$$C_{1t}^H + C_{1t}^E + I_{1t} = G(a_{1t}, b_{1t}), \qquad C_{2t}^H + C_{2t}^E + I_{2t} = G(b_{2t}, a_{2t}) \quad (2.42)$$

We assume complete markets in state-contingent claims available to the households to diversify country-specific risks. These assets are traded exclusively between domestic and foreign households. Market clearing requires that the following condition holds

$$D_1(s^t, s_{t+1}) + D_2(s^t, s_{t+1}) = 0 \quad \text{for all } s_{t+1}. \quad (2.43)$$

When these assets are available, households perfectly diversify country-specific risk and the equilibrium allocations are such that in all states of the world the utility gained from an additional unit of good a in country 1 is exactly the same as the utility gained in country 2. The same holds for good b.

There are some key variables that we have not yet defined. Unless otherwise indicated all of our consumption statistics are calculated using aggregate consumption measured as $C_{lt} = C_{lt}^{H} + C_{lt}^{E}$.

A country's terms of trade are defined as the price of that country's imports relative to that of its exports. When we refer to the terms of trade (TOT) we are implicitly referring to country 1's terms of trade, which is given by q_{2t}/q_{1t}. Using the representative final good producer's first-order conditions we find

$$TOT_t = \frac{q_{2t}}{q_{1t}} = \frac{\partial G_1(a_{1t}, b_{1t})/\partial b_{1t}}{\partial G_1(a_{1t}, b_{1t})/\partial a_{1t}} = \frac{1-\kappa}{\kappa}\left(\frac{a_{1t}}{b_{1t}}\right)^{\frac{1}{\sigma}} \tag{2.44}$$

Gross domestic product in country 1 in units of final good is $GDP_{1t} = q_{1t}Y_{1t}$ while for country 2 we have $GDP_{2t} = q_{2t}^{*}Y_{2t}$

Country 1's ratio of net exports to GDP is calculated as

$$NX_{1t} = \frac{q_{1t}a_{2t} - q_{2t}b_{1t}}{GDP_{1t}} = \frac{a_{2t} - TOT_t b_{1t}}{Y_{1t}} \tag{2.45}$$

Following Raffo (2008) we also look at the trade balance at constant price (prices are set to their steady-state values). For the home country we have

$$NXQTY_{1t} = \frac{a_{2t} - T\bar{O}T b_{1t}}{Y_{1t}} \tag{2.46}$$

where $\bar{TOT}$ denotes the steady-state value of TOT.

2.3 Model Solution and Parameter Values

We linearize the model around a symmetric deterministic steady state, where both countries net exports equal zero. An approximate linear solution to our model is obtained using the method outlined in King, Plosser and Rebelo (2002). The KPR solution method requires us to assign values to the model's parameters. The parameter values we select are commonly used in both agency costs and international real business cycles literature. Just like BKK and their many offsprings, we have in mind a model where one country represents the U.S., the other represents Europe and a period is one quarter of a year. Table 2.1 summarizes the parameter values we use in our benchmark case. We now explain how we assign values to the parameters in our international business cycle model.

2.3.1 Household Preferences

The value of the representative household's discount factor is $\beta = 0.985$. The household's coefficient of risk aversion is set equal to $\gamma = 2$. The values for β and γ are well within the acceptable range found in the literature. The parameter ν governs the curvature of the household's preferences. When preferences are of the GHH type

($\eta = 0$) the labour supply elasticity is given by $1/(\nu - 1)$. Jaimovich and Rebelo (2009) set $\nu = 1.4$, Letendre and Luo (2007) use 1.7 while Johri, Letendre and Luo (2011) use 3. Raffo (2010) points out that when preferences are of the GHH type $\nu = 1.63$ implies the same Frisch elasticity of labour supply (1.5) as is considered by BKK (1994). We set $\nu = 1.63$ but also consider a higher value in our sensitivity analysis. Parameter η influences the intensity of the wealth effect on household labour supply. GHH preferences are seen more and more in the international macro literature. An early example is Correia, Neves and Rebelo (1995) and a recent example is Mandelman *et al* (2011). In the spirit of Jaimovich and Rebelo (2009) we select a small value for η so that we do not entirely shut down the income effect on labour supply like GHH preferences do. We use $\eta = 0.10$ but also consider $\eta = 1$. Parameter ψ is set to insure that the representative household spends thirty percent of its time working in the steady state.

2.3.2 External Shock Process

Parameter ρ_z represents the degree of persistence of productivity shocks. For quarterly models, estimated values for this parameter range from 0.906 (Backus, Kehoe and Kydland (1992)) all the way up to unity (*e.g.* Baxter and Crucini (1995)). We pick the mid-point of this range and use $\rho_z = 0.95$ in our benchmark case. Given that we want to study the model's ability to produce positive international comovements in the RPI we do not allow for any connections between the two countries shocks in our benchmark case. Accordingly, we set $\rho_s = 0$ and set the correlation of ε_1 and ε_2 to zero. We consider other values for ρ_z, ρ_s and $correl(\varepsilon_1, \varepsilon_2)$ in our sensitivity analysis. The standard deviation of ε_1 and ε_2 is set to match output volatility in the US.

2.3.3 Intermediate Goods

We set the capital share to $\alpha_K = 1/3$. We preserve a constant returns-to-scale set up by having $\alpha_H = 1 - \alpha_K$.

2.3.4 Final good Production

Production of the final good requires both domestically produced and imported intermediate goods. Recall that σ denotes the elasticity of substitution between domestic and imported intermediate goods and that κ determines home bias. Values of σ employed in the literature vary greatly. For example, BKK (1994), Chari, Kehoe and McGrattan (2002) and Raffo (2008) use 1.5. Heathcote and Perri (2002) use time-series data and got an estimated elasticity of 0.9. Mandelman, Rabanal, Rubio-Ramirez and Vilán (2011) use 0.62. Others, such as Hooper, Johnson and Marquez (2000) estimate a short-run trade elasticity in the U.S. of around 0.6 and less for other G7 countries. Even short-run trade elasticities as low as 0.22, as used in Taylor (1993), have even been suggested, inferring that traded goods are highly complementary. Corsetti, Dedola and Leduc (2007), Benigno and Thoenisssen (2008) and Raffo (2010) all allow for an elasticity of substitution at or below 0.5.[10] In line with current research we use an elasticity of substitution $\sigma = 0.5$. We also use Heathcote and Perri's (2002) estimated elasticity in our sensitivity analysis. As for home bias we set κ such that the share of imports to total intermediate good output (denoted im) is always equal to 0.15 as in BKK (1994) among others. With import share im, and the value assigned to σ this implies a home bias of

$$\kappa = \left(1 + \left(\frac{1 - im}{im}\right)^{(1/\sigma)}\right)^{-1}, \tag{2.47}$$

which is the same for both countries.

2.3.5 Stochastic Entrepreneurial Productivity

Recall that an entrepreneur is given a productivity level ω drawn from a distribution $\phi(\omega)$. These idiosyncratic productivities must be non-negative, and could range in value from 0 to ∞. The baseline model assumes the distribution of entrepreneurial

[10] This lack of consensus on the appropriate value of σ led Bodenstien (2011) to publish an article listing the consequences of varying this elasticity.

productivity is a lognormal distribution with a mean of 1. Accordingly we use

$$\phi(\omega) = \frac{1}{\omega\sqrt{2\pi\sigma_\omega^2}} e^{-\frac{(ln(\omega)-\mu_\omega)}{2\sigma_\omega^2}}, \qquad \Phi(\omega) = \frac{1}{2} + \frac{1}{2} ERF\left[\frac{ln(\omega) - \mu_\omega}{\sqrt{2\sigma_\omega^2}}\right] \tag{2.48}$$

where μ_ω and σ_ω refer respectively to the mean and standard deviation of the variable's natural logarithm.

When an entrepreneur's realized productivity is below the threshold $\bar{\omega}$ he defaults on his contract, which prompts monitoring by the CMF. In the aggregate the monitoring costs incurred by the CMF equal $\mu Pi\Phi(\bar{\omega})$. These losses can be anything from legal fees to lost sales. Given the broad interpretation of what could be included in these losses, the possible range for the monitoring cost μ is quite broad. Carlstrom and Fuerst (1997) give a possible range for the value of μ from as low as 0.2 to as high as 0.36. For ease of comparison we adopt the same monitoring cost as them, setting μ equal to 0.25.

We choose standard deviation σ_ω to match the 0.974 quarterly bankruptcy rate as reported by Fisher (1999). We follow CF and set $1/\Gamma$ equal to the return on internal investment. Notice that when there are agency costs the internal rate of return is always greater than the rate of return on external funds due to the risk premium charged on borrowed funds. If the discount factor Γ is too high, the entrepreneur continues to accumulate capital until the point he is self financed, and there would be no agency costs in the steady state. If the discount rate is too low, the entrepreneur heavily discounts future consumption opting for current consumption, in which case the entrepreneur chooses to hold no capital in the steady state. Therefore, in order for there to be a stable equilibrium, the internal rate of return must equal $1/\Gamma$ or

$$P_l \frac{f(\bar{\omega})}{1 - P_l g(\bar{\omega})} = \frac{1}{\Gamma}. \tag{2.49}$$

Accordingly, we set $\Gamma = 0.947$. This value implies an entrepreneurial discount factor of $\beta\Gamma = 0.937$. The steady state relative price of capital $\bar{P}$ is equal to 1.024, with entrepreneur's share of investment production $f(\bar{\omega}) = 0.39$.

2.4 Results

2.4.1 Business Cycles Statistics

We simulate the model 1000 times, with 200 periods in each simulation. Within each simulation we trim the first 100 observations to circumvent any potential biases caused by our choice of initial conditions. For each simulation, artificial data are passed through the Hodrick-Prescott filter (with a smoothing parameter of 1600) before calculating statistical moments. The moments presented in Table 2.2 are averages over 1000 simulations.

The "Data" column reproduces the statistics reported in BKK (1993). We calculated the statistics related to the RPI (indicated with a superscript LW) using the data described in appendix 2.A.1. That column also displays the statistics pertaining to the terms of trade and net exports reported in Raffo (2008) (indicated with a superscript R).

The column "Benchmark" reports the results obtained when using the parameter values described in Section 2.3. Our model matches exactly the standard deviation of GDP since the calibration of the variance of technology shocks targeted that moment. The first autocorrelation of GDP is large and positive (0.72 vs 0.86 in the data). The "within country" moments are broadly consistent with the data. Consumption and hours are less volatile than GDP while investment is significantly more variable than GDP in the US. Those three variables are very highly correlated with GDP.

The model is also doing a good job at producing realistic international business cycles. Terms of trade are highly volatile. Their relative standard deviation (1.70) falls in the range of estimates reported in the literature (1.12 in Raffo and 1.92 in BKK). The same thing holds for net exports. The correlation of net exports with GDP (-0.49) is in the range reported in the literature (-0.51 in Raffo and -0.37 in BKK) while net exports at constant prices is not as countercyclical as in the data (Raffo reports a correlation with GDP of -0.41 and in our model it is -0.18). All cross-country correlations are positive but do not numerically match the corresponding numbers in the data. It is true that the cross-country correlations of investment and consumption are rather low. But one has to remember that as opposed to what is usually done in the literature, productivity shocks are not correlated across countries. The third column of numbers displays the statistics implied by the model when the correlation between ε_1 and ε_2 is 0.26. Not surprisingly, international correlations increase and are now all significantly positive.

As is well known, the BKK model struggles to replicate some key empirical regularities. Most notably, the positive correlation of output, consumption, labour and investment across countries as well as the volatility of the terms of trade. A number of extensions have been proposed to address these shortcomings. A recent example is Raffo (2008) who shows that using GHH preferences enables the model to produce a positive cross-country correlation of labour. A detailed sensitivity analysis focussing on international correlations and volatility of terms of trade can be found in Heathcote and Perri (2002). They show that lowering the elasticity of substitution between intermediate goods (σ) and the degree of spillovers in productivity shocks raise the international correlations while also increasing moderately the variance of the terms of trade. Our paper contributes to the international business cycles literature by showing that a financial friction à la Carlstrom and Fuerst helps address (to some limited extent) some of the glaring shortcomings of the canonical BKK model. Compare the numbers in Table 2.2 under Benchmark and NFF (No Financial Frictions).[11] Notice

[11] The only difference between the benchmark model and the NFF model is that the latter has no entrepreneurs nor CMFs. Therefore, future capital can be increased by one unit simply by reducing consumption by one unit. We simulate the NFF model using the very same parameter values as in our benchmark.

how the cross-country correlations all increase a little and how the standard deviation of the terms of trade also increases when financial frictions are added to the model.[12] While these changes are not spectacularly large, notice that they occur in a set up where the standard deviation of the RPI relative to that of output in the model is only two thirds of its counterpart in our U.S. data (0.26 vs 0.39). Hence, the model has potential to do somewhat better.

A central contribution of our paper is to propose a model where the cross-country correlation of the RPI is endogenously determined by the model. As a result of international trade in intermediate goods the model delivers a small positive cross-country correlation for the RPI, which is close to what is seen in US-Europe data. In Section 2.4.2 we use impulse response functions to explain how the model generates this positive correlation.

Sensitivity Analysis

Table 2.3 shows that our main result is robust to changes in parameters. We first change parameter η, which controls the strength of the income effect on labour supply (see equation (2.4)). By setting $\eta = 1$ we effectively revert to a KPR utility function. We see that the cross-country correlation of RPI remains unchanged. Parameter ν determines the labour supply elasticity. We raise ν from 1.64 to 3 with little effect on

[12] Most other moments remain basically unchanged except the correlation of GDP with net export at constant prices which increases unfortunately.

the cross-country correlation of RPI. Given that cross-country correlations of output, consumption, labour and investment have been found to be sensitive to the elasticity of substitution in intermediate goods we re-simulate the model this time raising the elasticity to 0.9 (value estimated by Heathcote and Perri, 2002).[13] Again, the cross-country correlation of RPI changes very little. In the next case, we increase the persistence of productivity shocks to 0.99. Such a change strengthens the international comovement of RPI. Finally, we adopt the stochastic process estimated by Heathcote and Perri (2002). Shocks persistence is set to 0.97, the degree of spillovers is set to 0.025, and the correlation between ε_1 and ε_2 is set to 0.29. Once more, the cross-country correlation of RPI remains positive.

We now look at the case where the density function of productivity ω is the uniform distribution instead of the lognormal distribution. Details about how we proceed for that robustness test can be found in appendix 2.A.3. As the last column of Table 2.3 shows, the model is doing just as well with the uniform distribution. Consequently, we conclude that the positive correlation of RPI across countries is a robust implication of our model.

BKK (1993) found that there is a structural trade off between the volatility of the terms of trade and the volatility of what they call the import ratio (b_1/a_1). As they demonstrate, this relationship depends on the elasticity of substitution between

[13] To avoid having complex roots in our linearized system we have to increase β a little, using 0.99 instead of 0.985.

domestic and foreign intermediate goods. The trade off implies that although a higher elasticity of substitution σ implies more trade as goods become substitutable, it also implies that prices are less volatile as any price change would causes producers to shift to the cheaper good. This trade off is also in operation in our model too but it operates on a scale where volatilities are closer to the data. More specifically, when BKK (1993) set the elasticity of substitution at $\sigma = 0.5$ their model produces a standard deviation of the terms of trade around 0.75 and a standard deviation of the import ratio less than 0.5.[14] In our benchmark case the standard deviation of the import ratio is 1.63 and that of the terms of trade is 3.26. When we increase the elasticity from 0.5 to 0.9 the standard deviations are both around 2. So while our model does not reproduce the high volatility of both TOT and the import ratio, it still does much better than BKK's model.

Investment-Specific Shocks

As we have shown, our benchmark model generates a positive cross-country correlation in the relative price of investment similar to that observed in the detrended US-Europe data. We now seek to determine whether our results are the consequence of our inclusion of financial frictions and not simply due to cross-country co-movements in the relative price of investment that it generates. To this end, we

[14]The empirical evidence they report is that the import ratio is a little more volatile than the terms of tarde.

add investment-specific technology shocks (denoted V_{1t} and V_{2t}) to our model without financial frictions. That involves replacing the resource constraints shown in equation (2.42) with $C^H_{1t} + C^E_{1t} + \frac{I_{1t}}{V_{1t}} = G(a_{1t}, b_{1t})$ and $C^H_{2t} + C^E_{2t} + \frac{I_{2t}}{V_{2t}} = G(b_{2t}, a_{2t})$ and the vector autoregressive process assumed for shocks in (2.50) with

$$\begin{bmatrix} \ln Z_{1t} \\ \ln Z_{2t} \\ \ln V_{1t} \\ \ln V_{2t} \end{bmatrix} = \begin{bmatrix} \rho_z & 0 & 0 & 0 \\ 0 & \rho_z & 0 & 0 \\ 0 & 0 & \rho_v & 0 \\ 0 & 0 & 0 & \rho_v \end{bmatrix} \begin{bmatrix} \ln Z_{1t-1} \\ \ln Z_{2t-1} \\ \ln V_{1t-1} \\ \ln V_{2t-1} \end{bmatrix} + \epsilon_t \tag{2.50}$$

$$E(\epsilon_t \epsilon_t') = \Sigma \tag{2.51}$$

where the vector of innovations is now $\epsilon_t = [\epsilon_{z1t}, \epsilon_{z2t}, \epsilon_{v1t}, \epsilon_{v2t}]'$. The persistence parameter ρ_v is set to 0.99. The standard deviations of ϵ_{v1t} and ϵ_{v2t} and their correlation are calibrated to insure that the model with IST shocks reproduces the relative standard deviation of RPI and the cross-country correlation of RPI reported in Table 2.2. The volatility of GDP increases when we include IST shocks. Accordingly, we lower the standard deviation of TFP shocks by twenty percent to insure that the model reproduces the standard deviation of GDP measured in US data (1.92). All other parameters retain the values reported in Table 2.1.

We find that the cross-country correlations of output, hours and investment fall significantly with the inclusion of IST shocks in the model without financial frictions.

The cross-country correlation of GDP falls from 0.42 (last column of Table 2.2) to 0.09. The cross-country correlation of hours worked falls from 0.36 to -0.01. The cross-country correlation of investment falls from -0.03 to -0.40. Hence, IST shocks move the cross-country correlations in the wrong direction. Thus the inclusion of financial frictions in our benchmark model are vital in producing our benchmark results. The next section uses impulse responses to explain how financial frictions influence our benchmark model's behaviour.

2.4.2 Impulse Response Functions

We use impulse response functions to explain how our model operates. We pay special attention to the behavior of investment and of its relative price. The parameter values used are exactly the same as those in our benchmark case. The economy is in the steady state in period 0. The only shock hitting the economy is a 1% unexpected increase in the home country's total factor productivity (Z_1) in period 1. Recall that our stochastic process is such that the first-order autocorrelation of Z_1 is 0.95. Therefore, total factor productivity in country 1 remains above its steady-state value for several periods (half life of 14 periods). In all figures the variables are represented in percent deviations from the steady state and have not been filtered in any ways.

Figures 2.1-2.3 show that output, hours, investment and consumption in both countries increase in response to a shock in country 1. Figures 2.4-2.6 show the same responses but for the NFF model (no financial frictions). The responses of output, consumption and hours in Figures 2.1 and 2.2 are fairly similar to the corresponding ones in Figures 2.4 and 2.5. The initial responses of home variables are weaker in our model than in the NFF model and they have a hump shape.[15] The shape of these responses are not surprising in light of the work of Carlstrom and Fuerst (1997).

Figures 2.3 and 2.6 show that the response of investment in country 1 is slightly weaker when financial frictions are present. Interestingly, the sign of the initial response of investment in country 2 differs in the two models. We witness a small positive response in our model and a slightly negative response in the NFF model. Figure 2.7 shows the responses of the terms of trade in our model as well as the NFF and BKK models. As the discussion below makes clear, financial frictions, the high volatility of the terms of trade and the response of investment are intimately related. We now provide some economic intuition connecting the model to the impulse responses.

Recall that productivity shock Z_1 appears in the production function of intermediate good producers in country 1 and nowhere else. These producers use capital and labour to produce good a. A positive shock to Z_1 increases the marginal product of capital and labour, which drives up the demand for those factors of production. Hence, wages and rents on labour and capital services increase. Since entrepreneurs

[15] The observation that GDP responds in a hump shape manner to a temporary productivity shock goes at least as far back as Cogley and Nason (1995).

supply capital to the intermediate goods producer, the increased return on capital generates an immediate increase in the entrepreneurs net worth (see equation (2.10)). Since the bulk of an entrepreneur's net worth is determined by the value of their capital stock (a state variable) the overall increase in net worth is relatively small in the period of the shock. Figure 2.8 shows that net worth increases by about 0.5% in period 1, which is much smaller than the peak increase in net worth (3.75% in period 3).

Responding to an increase in the return to physical capital households desire to acquire more capital goods. The slow response of net worth implies that entrepreneurs who want to increase the size of their projects need more external financing. As a result, agency costs and hence the price of investment increase (see Figure 2.8 again). Since the return to internal funds (2.25) is increasing in P, entrepreneurs respond to the increased demand for capital goods by significantly reducing their consumption (it falls by 25%) in response to the TFP shock in order to bolster internal funds. Consequently, entrepreneurial net worth increases importantly in period 2. As entrepreneurial wealth increases, entrepreneurs can respond to the increased demand for capital while relying less on external financing. This drives down both agency costs and the RPI in period 2, making for a short-lived increase in the relative price of investment.

One may wonder why I_2 increases in the model with financial frictions given that the price of investment P_2 increases in response to the shock, whereas that very same price is constant in the model without financial frictions. The answer lies in the response of another relative price: the terms of trade. Given the increase in P_1, the home country faces higher costs of producing investment goods in our model than in the NFF model hence it desires to invest less. Therefore, country 1 needs fewer units of final goods, which means it requires fewer units of their own intermediate good. Hence, a larger quantity of good a ends up being supplied to the world markets. This leads to a sharper fall in the price of that good and hence a more important increase in country 1's terms of trade (as Figure 2.7 makes clear). Figures 2.7 and 2.8 show a spike in the terms of trade at the very same time that P_1 spikes.

When the price of good a plummets, country 2 sees an opportunity to purchase a greater quantity of it which can then be bundled with its own intermediate good to raise the quantity of final goods produced there. Being consumption smoothers, households in country 2 consume some of that additional quantity of final goods and invest some of it in physical capital to raise future consumption. Raising the amount of final goods in country 2 requires that intermediate producers increase their output of good b since goods a and it are not perfect substitutes. Given that Z_2 does not change, more output of good b requires more labour and capital. The stronger demand for factors of production raises wage and rental rates. Consequently, the wealth of country 2 entrepreneurs increases but not enough to match the increase in the size of their projects. Thus, they must rely more heavily on external funding which raises agency costs in country 2 as well as P_2. To highlight this finding, Figure 2.9 plots the impulse response function of the home and foreign country's RPI in response to an unexpected 1 % increase in the home country's TFP. Overall, our theoretical setup suggests that agency costs can be transmitted from one country to another, leading to a positive co-movement of the RPI across countries.

2.5 Conclusion

The effects agency costs have on economic aggregates in a closed economy have been well established (early examples are Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997)). However, little research has been done on the effects of agency costs in the context of international business cycle models. We contribute to the international business cycle literature by adding a financial sector in each country of a two-good two-country BKK style model. Our add-on is inspired by the work of Carlstrom and Fuerst (1997). Our main finding is that our model produces a small positive cross-country correlation for the relative price of investment (in line with our Europe-US data). It is important to note that our model produces fluctuations and positive international co-movements in the relative price of investment in an entirely endogenous fashion. First, our model does not have any capital adjustment costs nor does it have any investment-specific technology shocks. Second, productivity shocks in our model are not correlated across countries and we do not allow for any type of international spillovers in total factor productivity. We show that this positive international correlation of the relative price of investment is remarkably robust to changes in parameter values.

We also contribute to the international business cycles literature by showing that financial frictions have the ability to increase the variance of the terms of trade as well as the international correlations of output, consumption, labour, and investment. Extending the model in ways that enhance the variability of the relative price of investment is left for future work.

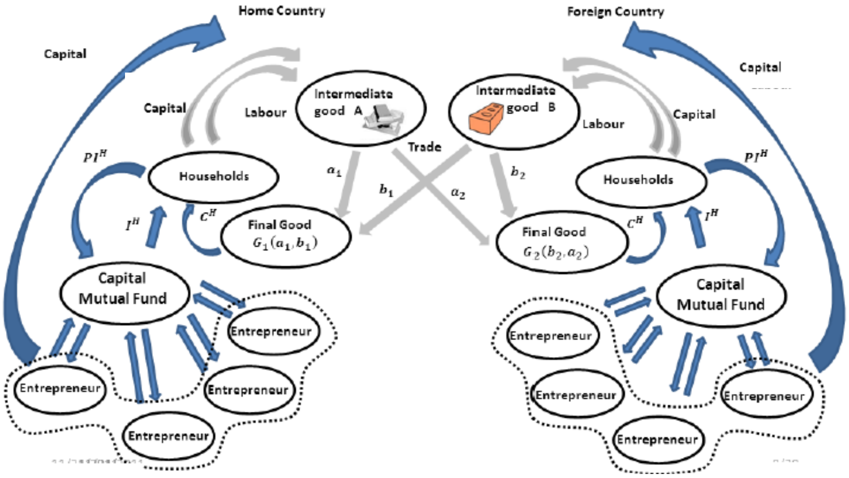
Appendix

2.A.1 Data

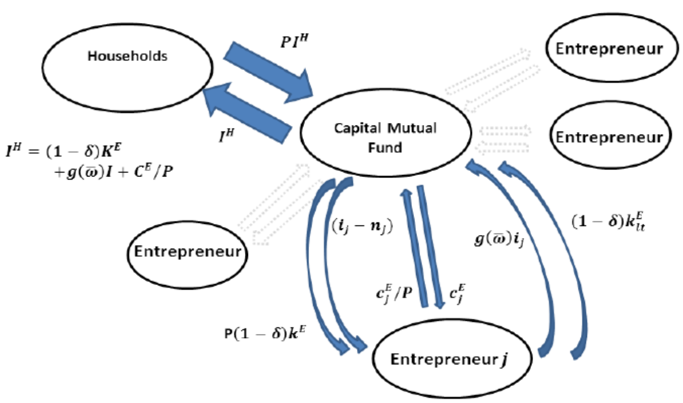
The relative price of investment is measured as the price deflator for investment goods divided by the price deflator for consumption goods. The variables considered in our consumption deflator includes consumption of non-durables, as well consumer services as listed in the National Income and Product Accounts provided by the Bureau of Economic Analysis. The investment deflator considered in this paper is the quality adjusted price deflator for producer durable equipment calculated by Gordon (1990). Unlike equipment price deflators from the NIPA tables, Gordon's (1990) series is adjusted for changes in equipment quality, such as faster computer processing speeds, or more energy efficient vehicles, both of which expand the production possibility of equipment, and hence the real value of these investment goods. Gordon's (1990) quality adjusted investment price deflator is reported annually from 1949:1 until 1983, and is extended to span from 1949:1-2006:4 using Fisher's (2006) technique which applies the work of Gordon's time series as well as Cummins and Violante (GCV) (2002) to expand the range of the dataset. Annual data is disaggregated into quarterly data via a splice interpolation.

2.A.2 Flow Charts

LW Model



Capital Mutual Fund



2.A.3 Uniform Distribution

As a robustness check, we also simulate the model using a uniform distribution with the same variance as the lognormal distribution. With positive probabilities assigned to values of ω between lower bound a and and upper bound b the uniform distribution is

$$\phi(\omega) = \frac{1}{b-a}. \tag{2.52}$$

with a cumulative distribution of

$$\Phi(\omega_t) = \frac{\omega_t - a}{b-a}. \tag{2.53}$$

This implies $f(\bar{\omega}_t)$ and $g(\bar{\omega}_t)$ equal

$$f(\bar{\omega}) = 1 - \frac{\bar{\omega}_t^2 - a^2}{2(b-a)} - \left(1 - \frac{\bar{\omega}_t - a}{b-a}\right)\bar{\omega}_t \tag{2.54}$$

$$g(\bar{\omega}) = \frac{\bar{\omega}_t^2 - a^2}{2(b-a)} - \frac{\bar{\omega}_t - a}{b-a}\mu + \left(1 - \frac{\bar{\omega}_t - a}{b-a}\right)\bar{\omega}_t. \tag{2.55}$$

It is necessary to change some parameter values in order to evaluate the effect of switching between lognormal and the uniform distributions. These changes often are required in order for their to be no growth in the steady state. In particular, when comparing these two distributions the value of the entrepreneur's discount rate Γ is adjusted in order to encourage the entrepreneur in this model to fully divest their net worth into their loan contract. For a uniform distribution with a standard deviation σ_ω of 0.207, and a mean of 1 the lower bound a is 0.642 and the upper bound b is 1.359, with a value for $\bar{\omega} = 0.6485$ implying that in steady state $\phi_{unif}(\bar{\omega}) = 0.0098$, which is equal to the same value with a lognormal distribution. With these values, the entrepreneurial discount rate is $\Gamma_{unif} = 0.6484$, with a steady state relative price of investment $P = 1.144$ and a steady state entrepreneurial share of investment $f(\bar{\omega}) = 0.352$.

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Table 2.1

Parameter Values

Benchmark Calibration	
β	0.985
γ	2
ν	1.64
ψ	2.5465237
η	0.1
ρ_z	0.95
ρ_s	0
$corr(\varepsilon_1, \varepsilon_2)$	0
σ_ε	$\sqrt{0.000145}$
δ	0.025
α_K	1/3
α_H	$1 - \alpha_K$
σ	0.5
$b_1/y_1 = a_2/y_2$	0.15
κ	0.9698
$\phi(\omega)$	lognormal($1,\sigma_w^2$)
σ_w	0.207
Γ	0.947
μ	0.25

Table 2.2

Business Cycle Statistics

	Data	**Benchmark**	**Correlated Shocks**	**No Financial Frictions**
SD(GDP)	1.92	1.92	2.04	1.89
Correl(GDP_t, GDP_{t-1})	0.86	0.72	0.72	0.68
RelSD(C)	0.75	0.56	0.54	0.55
RelSD(H)	0.61	0.54	0.53	0.57
RelSD(I)	3.27	3.59	3.40	3.56
RelSD(TOT)	1.12^{R},1.92	1.70	1.38	1.46
RelSD(NX)	0.27,0.30^{R}	0.26	0.21	0.27
RelSD(RPI)	0.39^{LW}	0.26	0.25	-
Correl(GDP, C)	0.82	0.96	0.97	0.98
Correl(GDP, H)	0.88	0.99	0.99	0.98
Correl(GDP, I)	0.94	0.96	0.97	0.97
Correl(GDP, NX)	-0.51^{R},-0.37	-0.49	-0.40	-0.53
Correl($GDP, NXQTY$)	-0.41^{R}	-0.18	-0.15	-0.40
Correl(GDP, TOT)	-0.20,0.12^{R}	0.45	0.36	0.49
Correl(GDP_1, GDP_2)	0.66	0.49	0.67	0.42
Correl(C_1, C_2)	0.51	0.21	0.44	0.17
Correl(I_1, I_2)	0.53	0.06	0.32	-0.03
Correl(H_1, H_2)	0.33	0.45	0.64	0.36
Correl(RPI_1, RPI_2)	0.12^{LW}	0.22	0.45	-

Notes:SD(x)=standard deviation of x. RelSD(x)=SD(x)/SD(GDP). Correl(x, y)=correlation between x and y. Numbers in the "Data" columns are from BKK (1993) except for numbers with a superscript. Moments for the RPI (bearing superscript LW) are based on our own calculations using data described in appendix 2.A.1. Numbers with superscript R are from Raffo (2008).

Table 2.3

Sensitivity Analysis

	$\eta = 1$	$\nu = 3$	$\sigma = 0.9$ $\beta = 0.99$	$\rho_z = 0.99$	$\rho_z = 0.97$ $\rho_s = 0.025$ $cor(\varepsilon_1, \varepsilon_2) = .29$	Uniform Distrib.
SD(GDP)	1.44	1.56	2.04	1.80	2.05	1.90
Correl(GDP_t, GDP_{t-1})	0.72	0.71	0.73	0.71	0.71	0.73
RelSD(C)	0.54	0.54	0.48	0.72	0.68	0.65
RelSD(H)	0.19	0.30	0.53	0.54	0.54	0.53
RelSD(I)	3.68	3.70	3.21	2.75	2.92	3.49
RelSD(TOT)	1.98	1.95	1.04	2.37	1.31	1.74
RelSD(NX)	0.27	0.28	0.15	0.24	0.21	0.26
RelSD(RPI)	0.26	0.27	0.23	0.21	0.22	0.31
Correl(GDP, C)	0.97	0.96	0.96	0.97	0.99	0.98
Correl(GDP, H)	0.94	0.99	0.99	0.97	0.98	0.98
Correl(GDP, I)	0.97	0.97	0.98	0.97	0.93	0.97
Correl(GDP, NX)	-0.49	-0.50	-0.55	-0.39	-0.40	-0.49
Correl($GDP, NXQTY$)	0.03	-0.01	-0.03	0.31	-0.19	-0.17
Correl(GDP, TOT)	0.46	0.47	0.52	0.38	0.36	0.44
Correl(GDP_1, GDP_2)	0.50	0.47	0.31	0.68	0.67	0.49
Correl(C_1, C_2)	0.16	0.13	0.28	0.32	0.66	0.20
Correl(I_1, I_2)	0.09	0.06	0	0.35	0.07	0.06
Correl(H_1, H_2)	0.72	0.45	0.31	0.61	0.65	0.45
Correl(RPI_1, RPI_2)	**0.22**	**0.18**	**0.20**	**0.49**	**0.24**	**0.27**

Figure 2.1

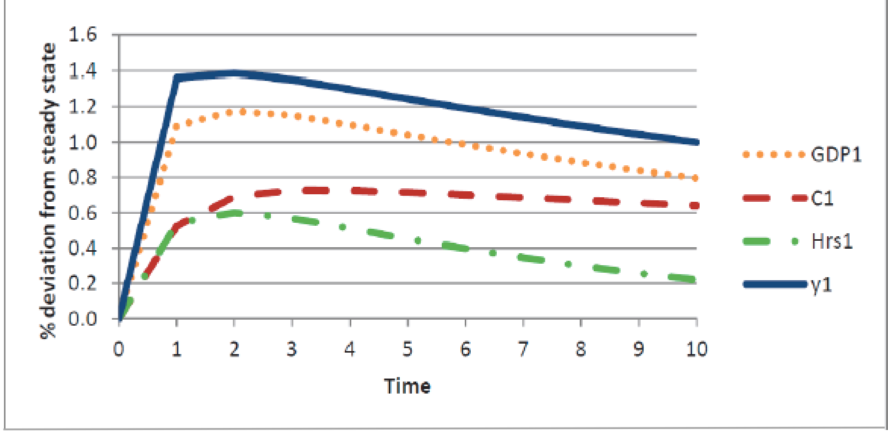


Figure 2.2

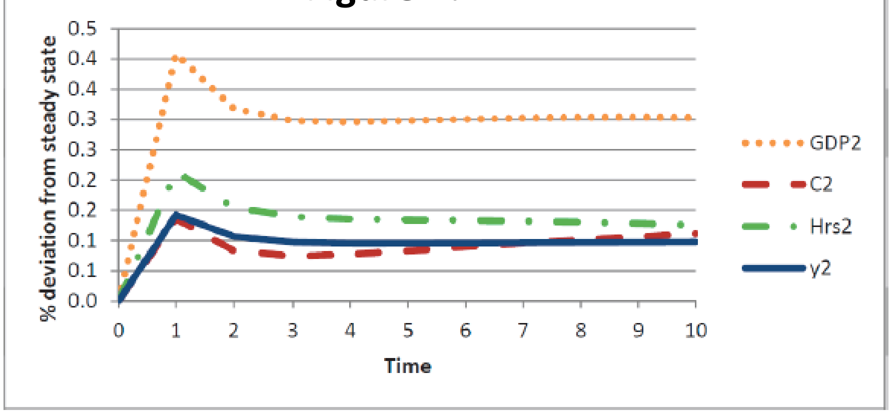


Figure 2.3

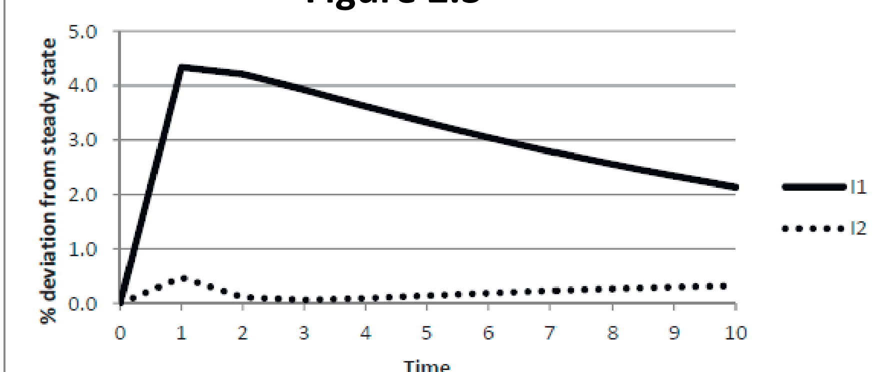
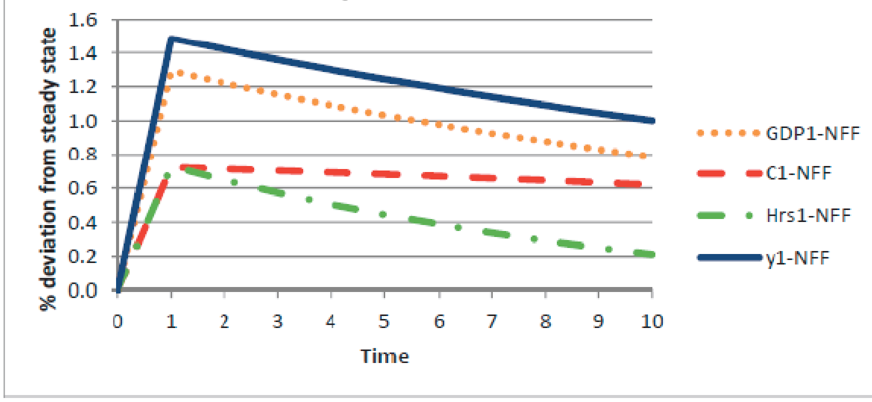


Figure 2.4



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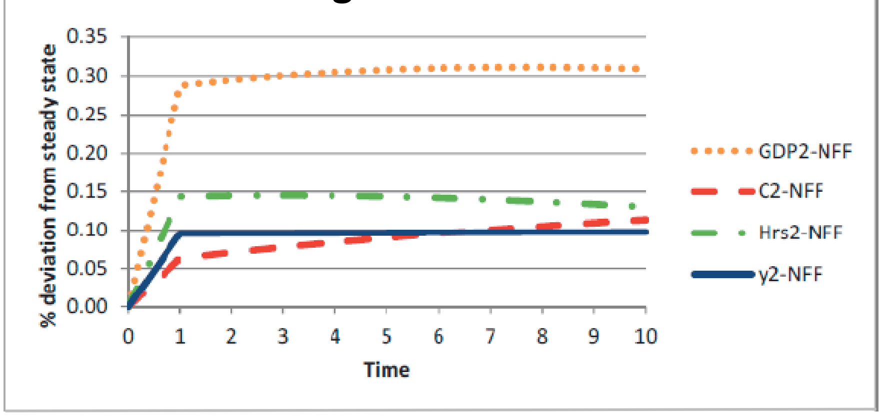
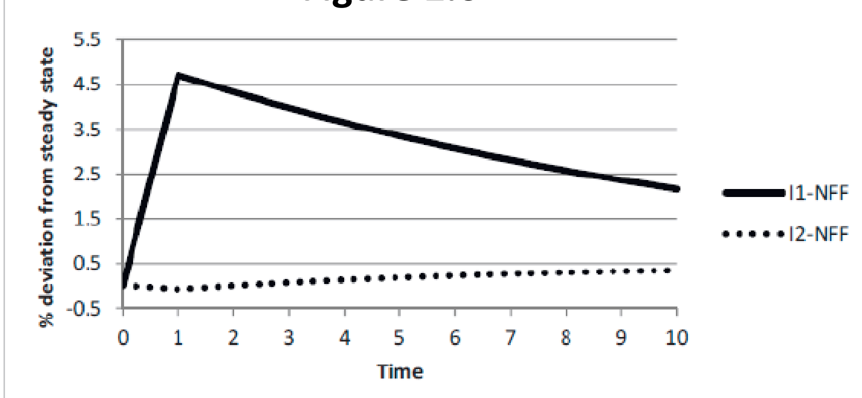


Figure 2.6



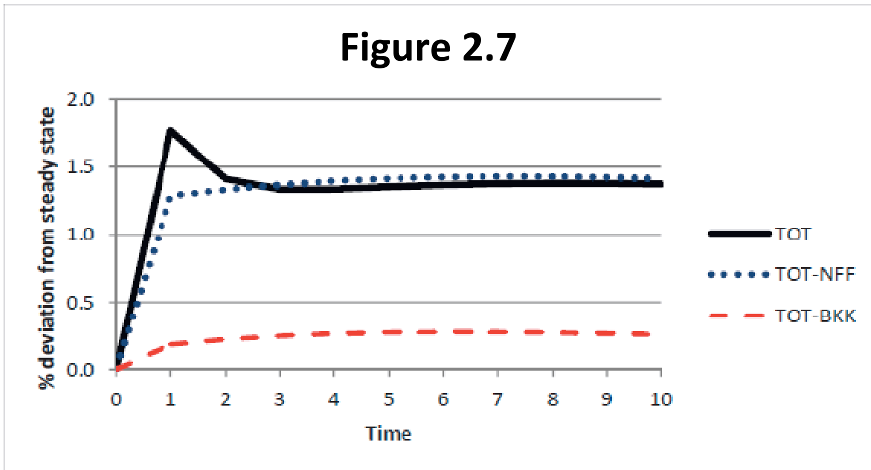


Figure 2.7

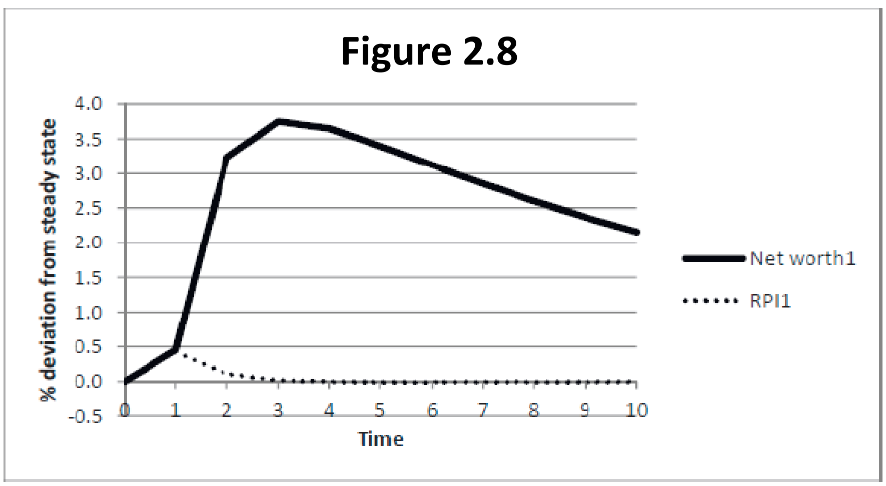


Figure 2.8

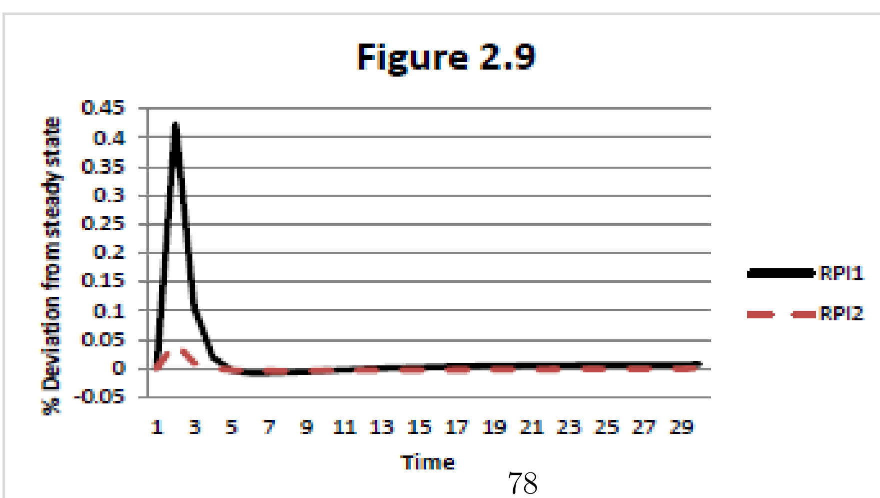


Figure 2.9

Chapter 3

Recycling Yesterday's News

3.1 Introduction

The vast majority of business cycle research, has considered neutral and investment-specific technology as two completely independent processes. However Schmitt-Grohe and Uribe (2011) show that total factor productivity (TFP) and the relative price of investment goods (RPI) have followed a common stochastic trend in post war US. When a standard RBC model is adapted to replicate the cointegrating relationship between neutral technology and the RPI, Schmitt-Grohe and Uribe (2011) find that surprise shocks to the common stochastic trend shared between these two exogenous processes contribute significantly to the volatility of output, consumption, hours worked and investment growth. We contribute to this line of research in two ways. First, we explore whether the cointegrating relationship found by Schmitt-Grohe and

Uribe (2011) between total factor productivity and the RPI is replicated in Canada and the Euro area. What we find is that this relationship is reproduced in both our Canadian data, as well as in our Euro Area data, thus suggesting that the cointegrating relationship found between TFP and the RPI in the US is not an isolated incident. Second, we incorporate a common stochastic trend shared between TFP and IST into a standard news shock model to replicate the cointegrating relationship between TFP and the RPI found in post war US data. Our business cycle model includes Jaimovich and Rebelo (2009) preferences with habit persistence in consumption, along with variable capital utilization rates and investment adjustment costs. The model is estimated using Bayesian methods and a variance decomposition is performed. Interestingly, when a vector error correction model (VECM) is incorporated into a model with these components, news shocks to the common stochastic trend contribute considerably to business cycle volatility. With cointegration, anticipated shocks to the common trend explain 26%, 27%, 16% and 26% of the growth rates of output, investment, hours and consumption respectively in our variance decomposition. These results suggest that anticipated technological shocks play a significantly larger role in business cycles when TFP and RPI are cointegrated than when they are not (as is Schmitt-Grohe and Uribe (2012) and Khan and Tsoukalas (2012)).

We find that when TFP and IST share a common stochastic trend, a positive anticipated shock to non-stationary TFP is accompanied by an anticipated boost in investment-specific technology and as a result, there is an anticipated drop in the

RPI. With the knowledge that the relative price drops when both shocks materialize, capital utilization rates jump and remain high even after the shocks are realized. During the interim this implies a high capital utilization rate, which generates an increase in output prior to the shock being realized. Interestingly, we find that when TFP and IST are cointegrated, anticipated technology shocks regain their relevance in generating business cycle volatility. In fact, when we allow for cointegration between these two technologies, the share of output, investment, consumption and hours variance explained by shocks to the common stochastic trend is greater than the sum of variance explained by TFP and IST without cointegration. This leads us to believe that the relative importance of these two shocks can only be fully appreciated when they are allowed to move together over the business cycle.

Thus our research begins in section 3.2 by first exploring the relationship between total factor productivity and investment-specific technology in the US, Canada and Euro area. Section 3.3 then lays out the dynamic stochastic general equilibrium model. Section 3.4 presents the Bayesian estimation method used to estimate our model. Section 3.5 discusses the model's results and how they compare to Schmitt-Grohe and Uribe (2012) and Khan and Tsoukalas (2012). Section 3.6 concludes.

3.2 Empirical Work

The theoretical foundation of this research rests on the idea that technological growth in the consumption and investment good sector move together over time. There are many manufacturing improvements and transportation improvements for example that improve productivity across multiple sectors. Guerrieri et al. (2010) argue the existence of precursor shock such as a multi-factor productivity shock which improves productivity in both sectors. Another possibility is that there are underlying mechanisms that cause both the TFP and the RPI to move together. For example, as Basu and Thoenissen (2009) show, when production of consumption and investment goods vary in their reliance on imported intermediates, then changes in the terms of trade, induced by a change in TFP would lead to movement in the RPI. Alternatively, as suggested by Floetotto, Jamiovich, and Pruitt (forthcoming) the RPI could move endogenously over the business cycle in a two-sector model with monopolistic competition. In their paper, movements by firms in and out of each sector would cause markup variation between sectors over the business cycle. Given that the standard approach is to measure investment-specific technology as the inverse of the RPI, both of these scenarios could lead the RPI and TFP to move together over the business cycle. Furthermore, prior to Bayesian estimation, it was common to include spillovers between IST and TFP shocks when more than one shock is included in a first order autoregressive (AR(1)) setup. The rational was simply to replicate the countercyclical nature of the RPI. Given the list of theoretical reasons why TFP and the RPI should move together over time, this section empirically investigates the relationship between TFP and the RPI, and show that these two time series do share a common trend that results in both TFP and IST moving together over time. The empirical methodology Schmitt-Grohe and Uribe (2011) apply in their research is applied here.

We estimate TFP using US non-farm business cycle data from 1949:1 to 2006:4 with the capital stock adjusted for capital utilization from the Bureau of Economic Analysis' National income and Product Accounts. The methods used to estimate TFP in the US are those used by Beaudry and Portier (2006) and is outlined in the data section of the appendix. As for the RPI, we adopt the same methodologies applied by Fisher (2006) to create a quality adjusted RPI (which was also used by SGU (2011)). Fisher's (2006) technique uses the Gordon, Cummins and Violante (GCV) (2002) equipment price deflator along with NIPA's estimates for the price of consumption goods to derive quarterly time series data on the RPI. We also include data on the TFP and the RPI in Canada and Europe for comparison, where the RPI is not adjusted for quality due to insufficient data. With this data, we take our first step in determining whether cointegration exists between the RPI and TFP in each country/area, by first applying an Augmented Dickey Fuller test to determine if these time series are non stationary.

The Augmented Dickey Fuller test is applied to assess whether a given time series follows a unit root process. Letting X_t be the time series in question, we apply the Augmented Dickey Fuller (ADF) test to the following equation

$$\Delta X_t = \alpha + \beta t + \gamma X_t + \delta_1 \Delta X_{t-1} + \delta_2 \Delta X_{t-2} + \ \dots \ + \delta_p \Delta X_{t-p} \tag{3.1}$$

where p is the number of lagged differences used in the test. The null hypothesis is $\gamma = 0$. The alternative is $\gamma < 0$, in which case X_t is a stationary time series. Failure to reject the null hypothesis, implies that time series X_t is non-stationary and follows a random walk. When this test is applied to our TFP and the RPI series, we fail to reject the null hypothesis at the 5 % significance level, thus implying that both technologies are non-stationary, as shown in table 3.1[1].

Given that both these time series are non-stationary in all countries included in our analysis, the next step is to test for cointegration. Johansen's test for cointegration can be thought of as a generalization of the Dicky fuller test. With TFP^i and RPI^i for country $i \in \{US, Canada, EuroArea\}$ then

$$\begin{bmatrix} TFP_t^i \\ RPI_t^i \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} TFP_{t-1}^i \\ RPI_{t-1}^i \end{bmatrix} + \begin{bmatrix} \epsilon_t^{TFP^i} \\ \epsilon_t^{RPI^i} \end{bmatrix} \tag{3.2}$$

[1]All Tables and Figures can be found at the end of the paper.

Alternatively we can rewrite this equation as

$$\begin{bmatrix} \Delta TFP_t^i \\ \\ \Delta RPI_t^i \end{bmatrix} = \Pi \begin{bmatrix} TFP_{t-1}^i \\ \\ RPI_{t-1}^i \end{bmatrix} + \begin{bmatrix} \epsilon_t^{TFP^i} \\ \\ \epsilon_t^{RPI^i} \end{bmatrix} \tag{3.3}$$

where

$$\Pi = \begin{bmatrix} a_{11}-1 & a_{12} \\ \\ a_{21} & a_{22}-1 \end{bmatrix} \tag{3.4}$$

Johansen's test determines the level of cointegration between TFP^i and the RPI^i by examing the rank of matrix Π (denoted by r) by running two tests. The first test assesses whether there is any cointegration at all by testing the null hypothesis that $r = 0$ against the general alternative. If the null hypothesis is rejected, a second test is conducted, this time testing the null hypothesis that $r \leq 1$. If the null hypothesis is accepted, than these two time series are cointegrated. Through Johansen's test for cointegration, we find that in each country/area, TFP and the RPI are cointegrated. Results of this test are shown in Table 3.2. By definition if two or more time series are cointegrated, then these series follow a common stochastic trend. This exercise demonstrates that the cointegrating relationship between TFP and the RPI in the US uncovered both here and by SGU (2011) is not unique to US data, but is found in both Canada and the Euro Area.

3.3 Model

As shown in the previous section, TFP and the RPI are cointegrated and follow a common stochastic trend in two of the three countries/areas considered in this paper. We now set up our benchmark DSGE model incorporating both cointegration between TFP and IST as well as a mechanism to allow households to anticipate and respond to future changes in their fundamentals. This model has all the usual components found in a real business cycle model, including households, a consumption goods producing firm and a capital goods producing firm. In addition to this general setup, there is also a labour union whose function is discussed later. Households in this model purchase consumption and investment goods from their respective producers, and are the sole providers of labour and capital services to the consumption good producing firm. This firm then uses these inputs to produce a consumption good that can either be consumed as is, or used to purchase investment goods from an investment producer. The investment good producer converts investment goods into capital goods via a linear production function. These investment goods, fresh off the production line then are matched with producers through a marginal efficiency of investment (MEI) process. Last of all there is a labour union which levies union dues on the households wage income, which are then returned back to the household as a lump sum. These four components make up the benchmark DSGE model used to determine the relevance of news shocks when both IST and TFP shocks are cointegrated.

In addition to the model setup described above there is also a variety of both anticipated and unanticipated shocks. The menu of shocks included in this paper consist of wage markup shocks, preference shocks, MEI shocks as well as stationary TFP and IST shocks. Each of these shocks listed are subject to both anticipated and unanticipated disturbances. However, unlike many news shock papers, this paper incorporates both anticipated and unanticipated shocks to cointegrated non-stationary TFP and IST. Our benchmark model is based on the results found by SGU *What's New's In Business Cycles* and hence shares many similarities with their work. Each of the components listed above are now discussed in turn.

3.3.1 Households

The economy is populated with a large number of identical infinitely lived households which each period consume C_t consumption goods and provide H_t units of labour. Their lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t b_t \frac{[C_t - \chi C_{t-1} - \phi {H_t}^{\theta} X_t]^{1-\sigma}}{1-\sigma} \tag{3.5}$$

$$X_t = (C_t - \chi C_{t-1})^{\eta} X_{t-1}^{1-\eta} \quad \text{with} \quad 0 \leq \eta \leq 1, \tag{3.6}$$

with $0 < \beta < 1$, $\phi > 0$, $\theta > 1$ and $\sigma > 0$. Here β is the households subjective discount factor, σ determines the curvature of the household utility, θ determines the level of labour supply elasticity and χ is the habit persistence parameter. We include a preference shock b_t which captures changes in household preferences over time. These preferences were first developed by Jaimovich and Rebelo (2009) and has since been coined Jaimovich and Rebelo (JR) preferences. The remaining parameter η, and the latent variable X_t are the distinctive elements that make up JR preferences. Parameter η (bound between 0 and 1) governs the sensitivity of the household's labour supply decision to changes in wealth. When the value of η approaches zero, we have preferences similar to those used by Greenwood Hercowitz and Huffman (1988) where the wealth effect on labour supply has been removed. These preferences are similar to the case with both home and market production, where changes in market wages cause households to shift labour between labour markets. When η is close to 1 we have King, Plosser and Rebelo (KPR) (1988) preferences, which exhibit a strong wealth effect on labour supply.

These preferences have become common in news shock research. This is due to their ability to generate an increase in labour supply in response to positive news of future productivity. As shown by Beaudry and Portier (2006) the standard DSGE model with KPR preferences is unable to generate a boom in economic activity in response to news of future productivity gains. One of the problems lies in the fact that both consumption and leisure are normal goods. With KPR preferences, this would cause the household to increase both their consumption and their leisure with an increase in lifetime income. Therefore, when wealth effects are at their fullest

(η close to 1), output decreases on impact due to the fact that households increase both consumption and leisure. Since consumption increases while output is falling, investment necessarily falls as well during the interim period. Absent any wealth effects, households increases their labour supply rather than decrease, in response to news of future labour productivity. Alternatively, one could add labour adjustment costs to encourage households to increase their labour supply in the interim.

In addition to the households ability to consume and supply labour is their ability to purchase and accumulate capital. This capital is used as an input in the production of our economy's consumption good. The household can increase their capital stock by purchasing investment goods I_t^g. Investment goods are produced using consumption goods as an input according to the following linear production function

$$I_t^g = A_t X_t^A I_t. \tag{3.7}$$

Here, I_t is the quantity of consumption goods used as an input in the production of investment goods in period t and $A_t X_t^A$ represents the level of investment-specific technology. Shocks to investment-specific technology can be divided into two components, a stationary component A_t and a non-stationary component X_t^A. Since investment production is linear, the relative price of the investment good in period t

P_t is equal to

$$P_t = \frac{1}{A_t X_t^A}. \tag{3.8}$$

With the wage and rental income earned by the household in compensation for the labour and capital services provided to consumption good producers, we can write the household's budget constraint as

$$C_t + I_t = \frac{W_t}{\mu_t^W} H_t + R_t U_t K_t + \Phi_t + \Pi_t. \tag{3.9}$$

Here, W_t and R_t are respectively the wage and rental rates paid by the firm for H_t hours worked and $U_t K_t$ capital services provided. As mentioned earlier in this section's preamble, there exists a labour union which collects a portion $1/\mu^w \leq 1$ of the households wage earning as labour dues. The revenue collected by the labour union Φ_t is then rebated back to the household. Since households own the consumption goods producing firm, the last remaining component of the households income is the profits earned by each producer Π_t owned by the household. With this income a representative household can either buy consumption goods C_t or purchase investment goods I_t, which are measured in units of consumption goods. The household accumulates this capital according to the following capital accumulation equation

$$K_{t+1} = (1 - \delta(U_t))K_t + v_t I_t^g (1 - S(\frac{I_t^g}{I_{t-1}^g})), \tag{3.10}$$

where K_t is the households fixed stock of capital in period t and as mentioned above, I_t^g represents the quantity of investment goods purchased in real terms in period t. Here, the utilization rate of capital U_t is assumed to be flexible within each period, where K_tU_t measures the amount of capital services provided by the household. Included in the capital accumulation equation is also an investment adjustment cost function $S()$

$$S(x) = \frac{\kappa}{2}(x - \mu^{i^g})^2, \tag{3.11}$$

where $\kappa \geq 0$, and $\bar{\mu}^{i^g}$ is the growth rate of real investment along a balanced growth path. Note that in the steady state $S = S' = 0$ and $S'' > 0$. We allow the rate of capital depreciation $\delta(U_t)$ to increase with the rate of capital utilization, assuming the following convex function.

$$\delta(U_t) = \delta_0 + \delta_1(U_t - 1) + \frac{\delta_2}{2}(U_t - 1)^2 \tag{3.12}$$

with δ_0, δ_1, and $\delta_2 > 0$. Last of all, υ_t indicates the marginal efficiency of investment (MEI) at period t. These MEI shocks, first suggested by Justiniano, Primiceri, and Tambalotti (2009), are introduced by conceptually dividing the process of creating capital into two stages of production. The first phase of production involves transforming consumption goods into investment goods, which is affected by investment-specific technology. When production technology is linear, investment-specific technology is exactly identified by the RPI. However, these investment goods, fresh off the production line remain idle until matched with a consumption goods producing firm. Shocks which affect this latter conversion are referred to as our MEI shocks. As an example, the firm's ability to access capital can affect the rate of conversion of investment goods to productive capital.

3.3.2 Firm

The consumption good in this economy, which will be considered as the numeraire good, are produced by an infinite number of identical and perfectly competitive firms according to the following production function

$$Y_t = Z_t (U_t K_t)^{\alpha} (X_t^Z H_t)^{1-\alpha}. \tag{3.13}$$

Here, α is between 0 and 1, implying constant returns-to-scale, H_t and $U_t K_t$ denote labour and capital services used by the firm. As was the case in the investment good sector, the total level of productivity in this sector consists of a stationary component Z_t with only a transitory effect on TFP, and a non-stationary trend component X_t^Z.

3.3.3 Exogenous shock process

Thus far six exogenous shocks appeared in the model presented above. There are four stationary shocks: Z_t, and A_t , a shock to household preferences b_t and a wage markup shock μ_t^w. Each of these exogenous processes is subject to both anticipated and unanticipated shocks. In general a stationary shock $x_t \in \{Z, A, b, \mu_w, v_t\}$, which evolves as

$$ln(\frac{x_t}{\mu^x}) = \rho_x ln(x_{t-1}) + \epsilon_t^{x0} + \epsilon_{t-4}^{x4} + \epsilon_{t-8}^{x8}, \quad (3.14)$$

where $0 \leq \rho < 1$ is the level of persistence and e_t^{x0} is an unanticipated shock to x_t . There are two news shocks denoted ϵ_{t-4}^{x4} and ϵ_{t-8}^{x8}, which are anticipated four and eight quarters in advance. The timing of our anticipated shocks follow the timing adopted by both SGU (2012) and Khan and Tsoukalas (2012). μ_x is the steady state value of variable x_t.

Common trend component

So far we have focused on those shocks which have only a temporary effect on the level of productivity in each sector. However as mentioned earlier, we cannot reject the presence of a unit root process in both TFP and the RPI in Canada, the US and the Euro Area. Therefore, as alluded to earlier, we incorporate two types of shocks into our model for both types of technology. These include a stationary component, which can only have a transitory effect on technology as well as a non stationary component which has a permanent effect on technology. Furthermore, we have shown through Johansen's test for cointegration, we cannot reject the hypothesis that TFP and the RPI are cointegrated in all three of the three countrys/areas included in our empirical analysis. As can be seen in equation (3.13), the level of productivity in the consumption good sector equals

$$TFP_t = Z_t(X_t^Z)^{1-\alpha}, \tag{3.15}$$

which has a stationary component Z_t and a non-stationary component X_t^Z. Likewise for the investment sector, investment-specific technology at time t equals

$$IST_t = A_t X_t^A, \tag{3.16}$$

where again A_t is the stationary component, and X_t^A is the non-stationary component of IST. As outlined in Section 3.2, there is strong empirical evidence that the logarithms of TFP and the RPI in the United States are I(1) cointegrated. Thus there exists a scalar Γ such that the combination of non-stationary TFP_t and the non-stationary RPI P_t

$$TFP_t^{\Gamma} P_t \tag{3.17}$$

is a stationary I(0) process. With the definition for the P_t in equation (3.8) and TFP_t in equation (3.15) we can rewrite (3.17) as

$$\frac{X_t^{Z(1-\alpha)}}{X_t^A}, \tag{3.18}$$

which is also a stationary I(0) process. With TFP and the RPI cointegrated, then it must also hold that X_t^Z and X_t^A must also be cointegrated.

Letting μ^Z equal to growth rate in X^Z and μ^A the growth rate of X^A, we have

$$\mu_t^Z = \frac{X_t^Z}{X_{t-1}^Z} \quad and \quad \mu_t^A = \frac{X_t^A}{X_{t-1}^A}. \tag{3.19}$$

The growth rates μ^Z and μ^A observe the following law of motion

$$\begin{bmatrix} ln(\mu_t^Z/\bar{\mu}^Z) \\ ln(\mu_t^A/\bar{\mu}^A) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} ln(\mu_{t-1}^Z/\bar{\mu}^Z) \\ ln(\mu_{t-1}^A/\bar{\mu}^A) \end{bmatrix} + \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} x_{t-1}^{co} + \begin{bmatrix} 1 & 0 \\ d_{21} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{\mu^Z}^0 \epsilon_{\mu^Z,t}^0 \\ \sigma_{\mu^A}^0 \epsilon_{\mu^A,t}^0 \end{bmatrix} + \Gamma_t^4 + \Gamma_t^8 \tag{3.20}$$

where

$$\Gamma_t^4 = \begin{bmatrix} \sigma_{\mu^Z}^4 \epsilon_{\mu^Z,t-4}^4 \\ \sigma_{\mu^A}^4 \epsilon_{\mu^A,t-4}^4 \end{bmatrix}, \Gamma_t^8 = \begin{bmatrix} \sigma_{\mu^Z}^8 \epsilon_{\mu^Z,t-8}^8 \\ \sigma_{\mu^A}^8 \epsilon_{\mu^A,t-8}^8 \end{bmatrix}. \tag{3.21}$$

and the error correction term x_t^{co} is calculated as

$$x_t^{co} = \psi ln(X_t^Z) - ln(X_t^A). \tag{3.22}$$

Here $\bar{\mu}^Z$ and $\bar{\mu}^A$ are the growth rates of TFP and IST along a balanced growth path. $\rho_{11} < 1$ and $\rho_{22} < 1$ determine the level of persistence for growth rates of TFP and IST respectively, while ρ_{12} and ρ_{21} determining the spillover between these two growth rates. In addition, we also include a coefficient matrix multiplying into the vector of unanticipated shocks. This allows for some correlation between innovations. This implies that innovations to growth rate of TFP could have an immediate effect on both IST and TFP. Coefficients κ_1 and κ_2 determine the impact changes in the common trend has on growth rates μ_z and μ_a respectively. Their value will be discussed in our estimation process. $\epsilon^0_{\mu^z t}$ and $\epsilon^0_{\mu^a t}$ are unanticipated shocks to μ^z_t and μ^a_t respectively, and $\epsilon^k_{\mu^i t}$ are anticipated shocks to μ^i_t for $i = \{A, Z\}$ observed k period(s) in advance. For each shock we assume a mean 0 and standard deviation of 1. The VECM setup in this paper builds on the one presented by SGU (2011) who do not include anticipated shocks.

3.3.4 The Detrended Model

With both TFP and IST following a common stochastic trend many of the economic aggregates are non-stationary. The trend in output, which also equals the trend in consumption, nominal investment and the wage and rental rates equals

$$X^Y_t = X^Z_t (X^A_t)^{\frac{\alpha}{1-\alpha}}. \tag{3.23}$$

Whereas the trend of the capital stock, and the level of real investmentl is

$$X^I_t = X^K_t = X^Y_t X^A_t. \tag{3.24}$$

There is no growth in hours and utilization. The later is normalized to 1 in the steady state. For the remainder of the paper we work with the detrended version of our model, where all variables are measured as deviations from the balanced growth path. Since we include multiple stochastic trends, the evolution of a variable is a combination of a variable's deviation from the its balanced growth path as well as the evolution of the stochastic trends X_t^Z and X_t^A. The detrended system of equations of our DSGE model consists of the following equations.

$$y_t = c_t + i_t \tag{3.25}$$

$$y_t = Z_t((u_t \frac{k_t}{\mu_t^k})^\alpha) h_t^{1-\alpha} \tag{3.26}$$

$$r_t = \alpha y_t \left(\frac{u_t k_t}{\mu_t^k} \right)^{-1} \tag{3.27}$$

$$w_t = Z_t(1-\alpha)((\frac{u_t k_t}{\mu_t^k})^\alpha) h_t^{-\alpha} \tag{3.28}$$

$$r_t = q_t(\delta_1 + \delta_2(u_t - 1)) \tag{3.29}$$

$$i_t^g = A_t i_t \tag{3.30}$$

$$b_t \theta \phi h_t^{\theta-1} x_t (c_t - \frac{\chi}{\mu_t^y} c_{t-1} - \phi h_t^\theta x_t)^{-\sigma} = \lambda_t \frac{w_t}{\mu_t^w} \tag{3.31}$$

$$\begin{aligned}\lambda_t &= b_t(c_t - \frac{\chi}{\mu_t^y}c_{t-1} - \phi h_t^\theta x_t)^{-\sigma} - E_0 {\mu_{t+1}^y}^{-\sigma}\beta\chi b_{t+1}(c_{t+1} - \frac{\chi}{\mu_{t+1}^y}c_t - \phi h_{t+1}^\theta x_{t+1})^{-\sigma}\dots \\ &-\lambda_{2t}\eta\mu_t^{y\eta-1}(c_t - \frac{\chi}{\mu_t^y}c_{t-1})^{\eta-1}x_{t-1}^{1-\eta}\dots \\ &+E_0{\mu_{t+1}^y}^{1-\sigma}\beta\lambda_{2t+1}\eta{\mu_{t+1}^y}^{\eta-1}\frac{\chi}{\mu_{t+1}^y}(c_{t+1} - \frac{\chi}{\mu_{t+1}^y}c_t)^{\eta-1}x_t^{1-\eta}\end{aligned} \tag{3.32}$$

$$\phi h_t^\theta b_t(c_t - \frac{\chi}{\mu_t^y}c_{t-1} - \phi h_t^\theta x_t)^{-\sigma} = \lambda_{2t} - \beta E_0{\mu_{t+1}^y}^{1-\sigma}\lambda_{2t+1}(1-\eta){\mu_{t+1}^y}^{\eta-1}(c_{t+1} - \frac{\chi}{\mu_{t+1}^y}c_t)^\eta x_t^{-\eta} \tag{3.33}$$

$$x_t = (c_t - \frac{\chi}{\mu_t^y}c_{t-1})^\eta(x_{t-1}^{1-\eta})(\mu_t^y)^{\eta-1} \tag{3.34}$$

$$k_{t+1} = (1 - \delta_0 - \delta_1(u-1) - \frac{1}{2}(\delta_2)(u-1)^2)\frac{k_t}{\mu_t^k} + v_t i_t^g(1 - (\frac{\kappa_k}{2})(\mu_t^k\frac{i_t^g}{i_{t-1}^g} - \bar{\mu}^k)^2) \tag{3.35}$$

$$\begin{aligned}\frac{\lambda_t}{A_t} &= \lambda_t q_t v_t(1 - \frac{\kappa_k}{2}(\mu_t^k\frac{i_t^g}{i_{t-1}^g} - \bar{\mu}^k)^2 - \mu_t^k\frac{i_t^g}{i_{t-1}^g}(\kappa_k)(\mu^k\frac{i_t^g}{i_{t-1}^g} - \bar{\mu}_t^k)) + \dots \\ &\dots + \beta E_t\frac{1}{\mu_{t+1}^A}((\mu_{t+1}^y)^{-\sigma})\lambda_{t+1}q_{t+1}v_{t+1}((\frac{i_{t+1}^g}{i_t^g})^2)(\mu_{t+1}^k)^2\kappa_k(\mu_{t+1}^k\frac{i_{t+1}^g}{i_t^g} - \bar{\mu}^k\end{aligned} \tag{3.36}$$

$$\begin{aligned}\lambda_t q_t &= \beta E_t\frac{\lambda_{t+1}}{\mu_{t+1}^A}{\mu_{t+1}^Y}^{-\sigma}(r_{t+1}u_{t+1} + q_{t+1}(1 - \delta_0 - \delta_1(u_{t+1}-1)\dots \\ &\dots - \frac{1}{2}(\delta_2)(u_{t+1}-1)^2))\end{aligned} \tag{3.37}$$

$$ln(Z_t) = \rho_Z ln(Z_{t-1}) + \sigma_Z^0\epsilon_{Z,t}^0 + \sigma_Z^4\epsilon_{Z,t-4}^4 + \sigma_Z^8\epsilon_{Z,t-8}^3 \tag{3.38}$$

$$ln(A_t) = \rho_A ln(A_{t-1}) + \sigma_A^0\epsilon_{A,t}^0 + \sigma_A^4\epsilon_{A,t-4}^4 + \sigma_A^8\epsilon_{A,t-8}^8 \tag{3.39}$$

$$ln(v_t) = \rho_v ln(v_{t-1}) + \sigma_v^0\epsilon_{v,t}^0 + \sigma_v^4\epsilon_{v,t-4}^4 + \sigma_v^8\epsilon_{v,t-8}^8 \tag{3.40}$$

$$ln(b_t) = \rho_b ln(b_{t-1}) + \sigma_b^0 \epsilon_{b,t}^0 + \sigma_b^4 \epsilon_{b,t-4}^4 + \sigma_b^8 \epsilon_{b,t-8}^8 \tag{3.41}$$

$$ln(\frac{\mu^w}{\bar{\mu}^w}) = \rho_{\mu^w} ln(\frac{\mu_{t-1}^w}{\bar{\mu}^w}) + \sigma_{\mu^w}^0 \epsilon_{\mu^w,t}^0 + \sigma_{\mu^w}^4 \epsilon_{\mu^w,t-4}^4 + \sigma_{\mu^w}^8 \epsilon_{\mu^w,t-8}^8 \tag{3.42}$$

$$\begin{bmatrix} ln(\mu_t^Z/\bar{\mu}^Z) \\ ln(\mu_t^A/\bar{\mu}^A) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} ln(\mu_{t-1}^Z/\bar{\mu}^Z) \\ ln(\mu_{t-1}^A/\bar{\mu}^A) \end{bmatrix} + \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} x_{t-1}^{co} + \begin{bmatrix} 1 & 0 \\ d_{21} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{\mu^Z}^0 \epsilon_{\mu^Z,t}^0 \\ \sigma_{\mu^A}^0 \epsilon_{\mu^A,t}^0 \end{bmatrix} + \Gamma_t^4 + \Gamma_t^8 \tag{3.43}$$

$$\Gamma_t^4 = \begin{bmatrix} \sigma_{\mu^Z}^4 \epsilon_{\mu^Z,t-4}^4 \\ \sigma_{\mu^A}^4 \epsilon_{\mu^A,t-4}^4 \end{bmatrix}, \Gamma_t^8 = \begin{bmatrix} \sigma_{\mu^Z}^8 \epsilon_{\mu^Z,t-8}^8 \\ \sigma_{\mu^A}^8 \epsilon_{\mu^A,t-8}^8 \end{bmatrix}. \tag{3.44}$$

$$x_t^{co} = \left(\psi ln(\mu_t^z) - ln(\mu_t^A)\right) + x_{t-1}^{co}. \tag{3.45}$$

Variables λ_t and $q_t\lambda_t$ and τ_t denote the lagrangian multiplier for the households budget constraint, capital accumulation the law of motion for the Jaimovich and Rebelo (2009) variable X_t respectively.

3.4 Model Estimation

3.4.1 Parameterization

As mentioned in the introduction, a majority of the parameter values used in the benchmark model are obtained by means of Bayesian estimation, while setting some of the better understood parameters ourselves. Simply put, Bayesian estimation allows the user to determine the likely value of a parameter set given our prior knowledge of the parameters in question, the model, and the data set collected.

Formally within the context of macroeconomics, Bayesian estimation requires the following three components. First and foremost, Bayesian estimation requires a dynamic stochastic general equilibrium (DSGE) model which we denote by M. Secondly, this process needs a list of the parameters Θ to be estimated. Third, prior distributions $P(\Theta|M)$ for those variables estimated. Not all parameters need to be estimated. Last of all Bayesian estimation needs sample time series data, which is reffered to as Y_T, which consists of time series data of aggregate variables included in the model. Ultimately, we are interested in the posterior density, which we denote by $P(\Theta_M|Y_T)$, which gives the probability that the set of parameters take on a given set of values within a pre-established parameter space. Before discussing the results of the Bayesian estimation, we give a brief introduction of the processes. Further details on Bayesian estimation can be found in An and Schorfheide's (2006) paper *Bayesian Analysis of DSGE Models*, or through the DYNARE website listed in the appendix.

Prior to Bayesian estimation, there were two commonly used methods in the field of macroeconomics used to establish model parameters. The first being direct calibration, and rationalizing their value given micro-level data and long-run times series evidence. When the exact value of a parameter is not known, it is traditional to discuss the robustness of the model results by calibrating the model for the given range of parameter values. The second method is maximum likelihood. Unlike direct calibration, maximum likelihood can be used to assess the likelihood that a parameter takes a specific value within a given parameter space. Arguments have been made that suggest that this method is more robust than direct calibration as it is more transparent and less vulnerable to researcher bias. Bayesian estimation can be thought of as a compromise between these two traditional methods.

The objective of Bayesian estimation is to find the posterior density $P(\Theta_M|Y_T, M)$ for a given parameter set, which can be interpreted as the probability parameter Θ_M takes on a certain vector value given model M and the sample data Y_T. Remember Y_T consists of a subset of observables linked to the variables included in the model. Given Y_T we can calculate the following likelihood:

$$L(\Theta_M|Y_T, M) \equiv P(Y_t|\Theta_M, M). \tag{3.46}$$

We can interpret $P(Y_t|\Theta_M, M)$ as the likelihood of observing our data sample given the vector of the estimated parameters and the model M where

$$L(\Theta_M|Y_T, M) = P(y_0|\Theta_M, M)\prod_{t=1}^{T} P(y_t|Y_{T-1}, \Theta_M, M) \tag{3.47}$$

Thus far we have a prior distribution $P(\Theta_M|M)$, which assigns a probability of parameter values within a given parameter space, and the likelihood of the sample $P(Y_T|\Theta_M, M)$; however as mentioned earlier we are ultimately interested in the posterior density $P(\Theta_M|Y_T, M)$. We can calculate the posterior distribution by taking advantage of Bayes theorem, through the following two equations:

$$P(\Theta_M|Y_T, M) = \frac{P(\Theta_M \cap Y_T|M)}{P(Y_T|M)}. \tag{3.48}$$

$$P(Y_T|\Theta_M, M) = \frac{P(Y_T \cap \Theta_M|M)}{P(\Theta_M|M)} \tag{3.49}$$

By replacing $P(\Theta_M \cap Y_T|M)$ in equation (3.48) by (3.49) we can calculate the posterior density $P(\Theta_M|Y_T, M)$

$$P(\Theta_M|Y_T, M) = \frac{P(Y_T|\Theta_M, M)P(\Theta_M|M)}{P(Y_T|M)}, \tag{3.50}$$

where $P(Y_T|M)$ is the probability of observing the data sample Y_T, conditional on the model selected, which equals

$$P(Y_T|M) = \int_{\Theta_M} P(\Theta_M \cap Y_T|M)d\Theta_M. \tag{3.51}$$

Last of all, with the likelihood function (3.47), our priors $P(\Theta_M|M)$, and given the fact that (3.51) is a constant for any given parameter Θ_M we can write the posterior density

$$P(\Theta_M|Y_T, M) \propto P(Y_T|\Theta_M, M)P(\Theta_M|M). \tag{3.52}$$

In other words, given the probabilities assigned to a given parameter space, and a set of observables, a maximum likelihood approach is used to determine the probability of observing the data, given a set of parameters. Given these two components, it calculates the probability a parameter takes on a specific value given the data observed along with the model structure. By matching the likelihood function in equation (3.47) with our priors $P(\Theta_M|M)$, the Bayesian estimation assigns a posterior probability to all the parameters included. It is worth noting that proper priors are essential in generating accurate model results. If, for example the prior chosen assigns a probability of zero for a given parameter space, then the Bayesian estimation process will also assign a posterior distribution with zero probability over the same parameter space, regardless of how likely it contains the true value.

This is what it meant when we mentioned earlier that Bayesian estimation is a combination of both maximum likelihood and direct calibration. For example, given a prior distribution $P(\cdot)$ of a parameter ϕ with mean μ_ϕ and a variance σ_ϕ, if we use a non-assuming uniform prior distribution with the variance $\sigma_\phi \to \infty$, the value estimated by our Bayesian estimation process will converge to the maximum likelihood estimate. Whereas, if we assume a tighter prior distribution with $\sigma_\phi \to 0$, we are closer to the direct calibration technique. With the prior distributions directly effecting the posterior distribution, a proper choice of priors is essential. Last of all, through use of a Kalman filter, we can generate estimates of the unknown likelihood function which can then be used to estimate posterior density through a Metropolis-Hastings Algorithm. The Metropolis-Hastings Algorithm generates random samples of these estimates through Monte Carlo Markov Chain. This of course is only a generalization of the inner workings of the Bayesian estimation process, and anyone interested in a more in-depth understanding of the methodology used to conduct a

Bayesian estimation can refer to An and Schorfheide (2006), and or the user guide available for DYNARE.

The list of the estimated parameters Θ include the preference parameters: θ, h^{ss}, η and χ. θ determines the elasticity of labour supply, h^{ss} the level of hours worked in the steady state, η determines the level of inter-temporal substitution in consumption and χ is the habit persistence parameter for consumption. Parameters governing the accumulation of capital, including δ_2, which determines how capital utilization impacts the depreciation of capital, and κ_k, which is the investment adjustment cost parameter is also estimated.

In addition to these parameters, the variance and persistence parameters governing the five stationary shocks are also estimated. These include parameters ρ_z, ρ_a,ρ_v, ρ_b, ρ_{μ^w}, which govern the persistence of the TFP shock, the IST shock, the MEI shock, the preference shock and the wage markup shock respectively. Furthermore, we estimate the relative size of both unanticipated and anticipated shocks to these five stationary series, which are listed in Table 3.4. For the non-stationary shock process we estimate the persistence parameters ρ_{11} and ρ_{22}, the spillover parameters ρ_{12}, ρ_{21} and d_{21}, the cointegration coefficients κ_1 and κ_2 as well as the variance of the innovations, both anticipated and unanticipated. Aside from the parameters mentioned above, there are some parameters which are calibrated directly. These include δ_0, δ_1, u^{ss},β,α, ϕ, σ, $\bar{\mu^w}$, $\bar{\mu_Y}$ and $\bar{\mu_A}$.

Akin to SGU (2011), we normalize the steady state utilization rate to 1 and set the parameter δ_0 such that the quarterly depreciation rate in the steady state is equal to 0.025. In addition, we set the household discount rate β equal to 0.975, and a value of 1 for the risk aversion parameter σ. α is set to 0.37 such that labour's share of output is equal to 0.63.

We use quarterly seasonally adjusted non-farm output from 1949:1 to 2006:4, available through the Bureau of Labour Statistics to estimate the growth rate of output. With this information we calculate the quarterly growth rate of output $\bar{\mu^y}$ equal to 1.0049. As in our empirical work in section 3.2, the same methodologies applied here are those applied by Fisher to create a quality adjusted RPI. Fisher's (2006) technique, which utilizes the Gordon-Cummins-Violante equipment price deflator along with Bureau of Labour Statistics National Income and Product Accounts Table estimates for the price of consumption goods to derive quarterly time series data on the RPI. With this information, the estimated growth in the RPI, is set equal to 0.9957 which implies a 0.0043 percent drop in the price of investment each quarter on average. With constant returns-to-scale in investment production[2], this implies estimated value $\bar{\mu}^A$ equal to 1.0043. With our estimates for $\bar{\mu}^A$ and $\bar{\mu}^Y$ we set the value of $\bar{\mu}^Z$ such that the growth of output matches the data along a balanced growth path. Given the model setup, and the values above for β, $\bar{\mu}^Y$, and $\bar{\mu}^A$, we set

$$\delta_1 = \frac{1}{\beta}(\bar{\mu}^Y)^\sigma \bar{\mu}^A + \delta_0 - 1 \tag{3.53}$$

in order for both the first order condition for capital and the first order condition for utilization to be satisfied.

Given our estimates for $\bar{\mu}_A$, and the implied value for $\bar{\mu}^z$, we set ψ, equal to $ln(\bar{\mu}^A)/ln(\bar{\mu}^Z)$

[2]By replacing equation (3.7) with $I_t^g = A_t X_t^A I_t^\zeta$, SGU (2011) estimate the curvature of investment production, where they conclude that ζ equals 1 and investment production is linear.

such that the common trend component of IST and TFP disappears in the steady state. The steady state wage markup rate $\bar{\mu}^W$ is set equal to 1.1, which is the value used by SGU (2011). Since the Bayesian estimation process is used here to determine the value for steady state hours in this model, ϕ is set within the Bayesian estimation process so that the labour first order condition is satisfied.

As mentioned earlier, in order to use Bayesian estimation, we require a time series dataset covering a subset of the variables included in our model setup. For our estimation, we include the log difference in gross domestic product, consumption and real investment, where each variable just mentioned is divided by the US population 16 and over. We also include the log difference of the RPI as well as the log difference in hours in our list of observables. Growth in output, investment, consumption and hours worked are included in the set of observables to capture the general movement of the economy. The growth rate for the RPI is included in the set of observables as to pin down possible movements in investment-specific technology. As demonstrated by Justiniano, Primiceri and Tambalotti (2011), the relative importance of investment-specific technology in generating business cycle volatility is heavily dependent on whether growth rates in the RPI are included in the set of observables. When the growth rate of the RPI is included in Y_T, the Bayesian estimation pins down movements of investment-specific technology to the inverse of the RPI. In fact, when their model is reestimated with the RPI included in the set of observables, investment-specific technology loses its ability to explain business cycle dynamics. As done by SGU (2012), we include the growth rate of TFP in the set of observables, which allows the estimation process to guide our estimates of the cointegration coefficients κ_1 and κ_2.

Each of these time series are observed quarterly. For comparability with other research, the following sections focus on the impact the VECM setup introduced in section 3.3 has on the US. Altogether we include six observables in our Bayesian estimation. Thus the dataset used in the Bayesian estimation Y_T is

$$Y_T = \begin{bmatrix} \Delta ln(Y_t) \\ \Delta ln(C_t) \\ \Delta ln(I_t) \\ \Delta ln(P_t) \\ \Delta ln(h_t) \\ \Delta ln(TFP_t) \end{bmatrix} \times 100 \tag{3.54}$$

Bayesian estimation requires accurate prior distributions for the estimated parameters. Many of the prior distributions used in this section are the same ones used by SGU (2012) which has a similar model structure to their other work SGU (2011). These priors are outlined in Table 3.4. As for the persistence parameters for the five stationary shock processes and the two non-stationary processes which include ρ_z, ρ_a, ρ_v,ρ_b, ρ_{μ^w}, ρ_{11}, ρ_{11}, we use beta distributions as our prior, with a mean 0.7, variance 0.2 and bound this distribution between 0 and 1. In addition to these parameters we estimate ρ_{12}, ρ_{21} and d_{21}. For ρ_{12}, and ρ_{21}, a uniform distribution is chosen between 0 and 1.5. The parameter d_{21} is also given a uniform distribution between -2 and 2. We set the standard deviation of the unanticipated and anticipated stationary TFP shocks such that in total, the sum of the standard deviations of these shocks add up to a value similar to that estimated by Kydland and Prescott

(1982). Given this goal, we choose an inverse gamma distribution with a mean of 0.5 and a variance of 2 for the standard deviation of an unanticipated shock to stationary TFP. For anticipated disturbances to stationary TFP we likewise assume a inverse gamma distribution with a mean of 0.1 and a variance of 2. By choosing these values, we assume that unanticipated shocks account for over 90% of the volatility of TFP. For transparency, we use the same distributions described above for both anticipated and unanticipated shocks for the standard deviation of all other shock processes used in our Bayesian estimation.

For the preferences parameter θ, we assume a gamma distribution with mean 3 and variance 0.75. For the habit persistence parameter χ we assume a beta distribution with mean of 0.5 and a variance of 0.1. As for the Jaimovich and Rebelo (2009) parameter η, we assume a rather non-presumptuous uniform distribution bounded by 0.001 and 0.999. As for the steady state hours we use a normal distribution with mean 0.3 and variance 0.1, bounded between 0 and 1. Last of all the investment adjustment parameters κ_k and the depreciation parameter δ_2 are given a gamma distribution, with a mean of 2.5 and variance of 8 and a uniform distribution between 0.01 and 10 respectively. Given the model introduced in section 3.3, as well as the dataset Y_T and listed set of priors for those parameters being estimated, we have all the elements required to estimate the posterior density of our parameter-set Θ.

3.4.2 Estimation Results and Posterior Distributions

The estimation process employed by DYNARE applies a random walk Metropolis-Hastings Markov Chain (MCMC) Algorithm to estimate the posterior distribution of our estimated variables. In the case of our analysis, we run five parallel chains of these MCMC and run 200 000 simulations, where the first half of the draws are discarded. The results of our Bayesian estimation of the model parameters listed above are available in Table 3.5 which recount the prior and posterior distribution along with a 90% confidence interval around the posterior mean. Of particular interest is the Jaimovich and Rebelo preference parameter η. With $\eta = 0.525$, the effect wealth has on household labour supply is substantial. With an estimated value of χ of 0.51, there is some habit persistence in the households consumption. For the remaining preference parameter θ and the steady state hours h^{ss} we estimate a value of 3.14 and 0.29 respectively, which put these estimates roughly in line with those in the literature. The estimated value for the investment adjustment cost parameter κ is 22.23. The high cost of adjusting capital is due to the increased volatility of investment in our model. The depreciation parameter δ_2 has an estimated value of 0.09 reflecting curvature in the depreciation function. The persistence parameter for the five stationary shocks (ρ_Z,ρ_A, ρ_v,ρ_{μ^w}, and ρ_b) range between 0.64 for stationary TFP persistence to as high as 0.97 for wage markup shocks.

For the parameters governing the movement of our two non-stationary components X_t^Z and X_t^A. Estimates for the persistence parameters ρ_{11} and ρ_{22} suggest that growth in TFP and IST have a moderate level of persistence with a value of 0.5 and 0.61 respectively. These values are roughly in line with those values found by SGU (2011) using maximum likelihood. Perhaps the most important are the values estimated for κ_1 and κ_2, which determine the impact our common trend component x_t has on both growth rates. For κ_1 and κ_2 we estimate values of 0.09 and 0.13 respectively. These estimates imply that both growth rates move together over time. Estimates for the yet to be discussed standard deviations for both anticipated and unanticipated shocks are available in Table 3.5.

3.5 Model Results

3.5.1 Variance Decomposition

With the parameter set estimated in the previous section, we can now begin our analysis regarding the impact of both anticipated and unanticipated shocks have in our model. The variance decomposition of the benchmark model is outlined in Table 3.6.[3] Anticipated and unanticipated shocks to both stationary TFP and IST explain very little of the variance of the observables included in our estimation. Anticipated and unanticipated preference shocks are also not important. Wage markup shocks are an important source of volatility in the growth rate for hours, with 7% of the volatility explained by unanticipated wage markup shocks and roughly 30% explained by anticipated wage markup shocks. Lastly, unanticipated MEI shocks explain 7% of the volatility of investment growth, which is much

[3]For each variance decomposition calculated in this section, the parameter values are set at the mean of each parameter's posterior distributions. Each Variance decomposition mentioned throughout this chapter and the next are contemporaneous in that they focus on the explanatory power of each shock in explaining volatility in observables and a not forecast error variance decompositions

lower than the results found by both SGU (2012) and KT (2012).

Table 3.6 shows that in the context of our model with cointegrated TFP and IST, shocks to the common trend in TFP and IST are important. Taken together, anticipated and unanticipated shocks to the common trend explain 86% -99% of the variance in investment growth, TFP growth, output growth, consumption growth and growth in the RPI. They explain about 50% of the variance in hours growth.

Our main finding is that anticipated shocks to the common trend in TFP and IST is an important source of business cycle volatility despite the inclusion of wage markup shocks in our model. Anticipated shocks to the common trend explain 27% of the variance of investment growth and of the RPI, 25% of TFP growth, and between 16% and 26% of the variance of output growth, consumption growth and hours growth. This finding contrasts with those of SGU (2012) and KT (2012) who find that anticipated technology shocks have a very small role to play in business cycles fluctuations. Therefore, our analysis suggests that allowing for cointegration is important to the study of technology news shocks.

No Cointegration

To test whether cointegration is important in generating the variance decomposition listed in Table 3.6, we set $\kappa_1 = \kappa_2 = \rho_{12} = \rho_{21} = d_{21} = 0$ in equation (3.20) and re-estimated our benchmark model with these new restrictions. As can be seen in Table 3.7, anticipated shocks to either TFP or IST, stationary or otherwise play a more limited role in generating business cycle volatility. Instead, anticipated MEI shocks, as found by SGU (2012) have regained their relevance in explaining business cycle volatilities with anticipated MEI shocks now account for 26% of the volatility of investment growth. The relative importance of anticipated wage markup shocks in explaing volatility in hours growth has dropped down to 10%. These results will differ from SGU (2012) due to two reasons; (i) our inclusion of stationary IST shocks, (ii) our exclusion of measurement error for all variables included in our set of observables while their research allows for some measurement error for output growth. We consider measurement errors later on in this section. But first, to help illustrate the impact cointegration between TFP and IST has on the time path of these two variables, we now look at the impulse response functions.

3.5.2 Impulse Responses

Figure 3.1 plots the impact of a one-standard-error innovation to $\epsilon^4_{\mu^Z,t}$ (left panel) and $\epsilon^4_{\mu^A,t}$ (right panel) on TFP growth μ^Z, IST growth μ^A and the common trend x^{co}. With cointegration between non-stationary TFP and IST, an anticipated change in the non-stationary TFP generates both an increase in productivity growth as well as an expectation that the RPI will fall.[4] As can be seen in Figure 3.1 both technologies increase in response one standard error shock to non-stationary TFP. The impulse response functions for consumption, hours, investment, output and capital utilization are shown in Figure 3.2, where variables are plotted in percent deviation from the balanced growth path. As pointed out by Beaudry and Portier (2006), news of future productivity is met with an increase in all variables observed. Like the vast majority of models driven by news shocks our model includes endogenous

[4] Recall that the RPI varies inversely with μ^A_t.

capital utilization, JR preferences and investment adjustment costs. These three features have been shown to promote a positive response of hours worked, investment and output in reaction to a positive anticipated technology shock (see JR 2009 for example). Other mechanisms exist that can generate an increase in output prior to the realized technology shock. For example, the inclusion of knowledge capital by Johri and Gunn (2011) generates an increase in output in response to news of future productivity gains. Given that a μ^z news shock announces a fall in the RPI, there is an additional incentive to increase capital utilization since it will soon be much cheaper to replace depreciated capital. This immediate positive response of utilization raises output right away. This increase in output is large enough to accommodate an increase in consumption and an increase in investment in the interim period.

As expected, the responses in Figure 3.2 show that prior to the shock being realized, output, consumption, capital utilization and investment all increase despite the slight drop in hours worked. To get a sense of the effects of co-integration Figure 3.3 shows the impulse responses implied by a four-period ahead anticipated shock to μ^z when there is no cointegration (dashed line). These simulated responses are calculated using the parameter values reported in Table 3.5 except for κ_1 and κ_2, which are restricted to be zero. The solid lines in Figure 3.3 simply reproduce the responses in Figure 3.2. Figure 3.3 makes clear that cointegration enhances the responses to a news shock about future TFP. Notice the much stronger response of capital utilization when TFP and IST are cointegrated. As explained above, when TFP and IST are cointegrated, an anticipated increase in TFP comes with an anticipated decrease in the RPI which makes utilization less costly.

3.5.3 Robustness Check

Robustness to Measurement Error

As can be seen in equation (3.54), some deviation from the set of observables could be allowed through the inclusion of measurement errors. Measurement errors could be justified in our Bayesian estimation due to our simple model setup. With no price rigidities, no government, no trade, etc, there could be economic disturbances which affect our set of observables which are not included in our parsimonious model. Since these components have a non-trivial impact on an economy, it could lead to patterns in the data which our model would not be able to explain. Furthermore, since the data used is not 100% accurate, including measurement errors can account for any mis-measurement in the data. Including measurement errors therefore allows for some slackness in the Bayesian estimation process. We treat all observables equally and include measurement errors for each observable (as in Ireland (2004) for example) according to the following function

$$Y_T = \begin{bmatrix} \Delta ln(Y_t) \\ \Delta ln(C_t) \\ \Delta ln(I_t) \\ \Delta ln(P_t) \\ \Delta ln(h_t) \\ \Delta ln(TFP_t) \end{bmatrix} \times 100 + \begin{bmatrix} \epsilon^{ME}_{Y,t} \\ \epsilon^{ME}_{C,t} \\ \epsilon^{ME}_{I,t} \\ \epsilon^{ME}_{P,t} \\ \epsilon^{ME}_{h,t} \\ \epsilon^{ME}_{TFP,t} \end{bmatrix} \qquad (3.55)$$

where ϵ_{it}^{ME} is the measurement error of observable i and are bound between zero and one quarter of the variance of the respective observable with a uniform prior. To test whether our model results are sensitive to exclusion of measurement errors, the baseline model is run again with measurement errors for all observables. The results of our Bayesian estimation when measurement errors are included as in equation (3.55) are available in Table 3.8. Table 3.9 displays the variance decomposition. As can be seen in Table 3.9, including these measurement errors increases the relative importance on both anticipated MEI shocks as well as anticipated wage markup shocks, bringing their relative importance in our variance decomposition closer to those found by SGU (2012). Despite the increased importance of these two shocks when measurement errors are included, anticipated shocks to the common stochastic trend still contribute significantly to the volatilities of all observables included except hours growth. The ability for anticipated shocks to the common stochastic trend to explain movement in this observable drops from 16% without measurement errors to roughly 4% with measurement errors. Hence our main results are robust to the inclusion of measurement errors.

Priors

One notable difference separating our work and that done by KT (2012) are the priors used for the five stationary shocks and two non stationary shock processes. In our Bayesian estimation process outlined in section 3.4 we assumed that the persistence parameters, as well as the volatilities of the anticipated and unanticipated shocks were the same across all shocks included in our model. These priors were chosen such that no shock was given an inherent advantage over another in the estimation process. However others, such as KT (2012) have chosen priors for the persistence and volatility of shocks that vary across the various shocks included in their model. To assess whether our results are sensitive to the priors chosen in Section 3.4, we reestimate the model with their priors for the five shocks processes that our model and theirs have in common. The variance decomposition outlined in Table 3.11 indicates that even when we adopt those priors used by KT (2012), we still find that anticipated shocks to the common stochastic trend are a relevant source of volatility for the observables included in our Bayesian estimation. Thus our results are robust to variation in the priors used in our Bayesian estimation.

3.6 Comparison to Current Research

At the forefront of news shock research is the work done by Schmitt-Grohe and Uribe's (2012) in their paper *What's News in Business Cycles.* Their model is similar to the benchmark model established in section 3.3 with the following exceptions. First, their research only has disturbances to the stochastic growth rate of investment technology, while our research includes both stationary and non-stationary IST shocks. Second, their model has decreasing returns-to-scale in consumption production, unlike the constant returns-to-scale in equation (3.26). Third, their Bayesian estimation allows for measurement error in growth in output while our benchmark model does not include any measurement error for any observable included in equation (3.54). Last of all our benchmark includes cointegration between TFP and IST while their work has these two series evolving independently.

To a lesser extent we also compare our benchmark model results to Khan and Tsoukalas' (2012) who draw similar conclusions to SGU (2012) but unlike our benchmark model, they incorporate a series of nominal frictions into their news shock model. These two papers challenge the empirical findings of Beaudry and Portier (2006) who through a VAR empirical exercise find that approximately half of the volatility of output can be explained by anticipated changes in either type of technology. Schmitt-Grohe and Uribe (2012) as well as Khan and Tsoukalas (2012) find that anticipated changes in productivity regardless of the sector, were unable to generate substantial volatility in hours, consumption, investment and output. Both papers find that only unanticipated technology shocks, not anticipated technology shocks are relevant to understanding business cycle dynamics. To help understand the relevance of including cointegration into a standard news shock model, this section compares the model results outlined in the previous section to those found by these studies. We begin our discussion by first comparing our results with those found by Schmitt-Grohe and Uribe (2012).

As can be seen in Table 3.6, like SGU (2012), we find that shocks to stationary investment-specific technology are not important, while both anticipated and unanticipated wage markups shocks are important in explaining volatility in the growth rate of hours. However, unlike Schmitt-Grohe and Uribe (2012) the variance decomposition finds anticipated technology shocks are a relevant source of business cycle volatility, with anticipated shocks to non-stationary TFP assigned a non-trivial weight in the variance decomposition. By endorsing the VECM relationship between non-stationary TFP and non-stationary IST, news of future TFP growth is always accompanied by a drop in the RPI. The drop in the price of investment goods and the subsequent increase in investment when the shock is realized imply that the response of capital utilization, consumption and output to a non-stationary TFP shock is substantially larger than the response to non-stationary TFP shock when these shocks are not cointegrated.

As mentioned above, SGU (2012) find that anticipated shocks to both stationary and non-stationary TFP and IST are able to explain very little of the variation in the observables included in their estimation. These results are demonstrated Table 3.6 in their paper with 1%-2% of the volatility of each of their observables explained by anticipated shocks to either TFP or IST. Of notable exception is the high weight assigned to anticipated shocks to MEI, with 19% of the volatility of investment growth due to movements in this variable. SGU (2012) do find however that anticipated shocks to non-technology elements and particularly anticipated preference and wage markup shocks as a source of volatility in their DSGE model. Of particular interest is the ability of anticipated wage markup shocks to explain growth rates in output, consumption, investment and hours worked, with 17%, 18%, 12% and 67% of the volatility of these observables explained by anticipated shocks to wage markups in their model. SGU (2012) argue that the high weight assigned to anticipated wage markup shocks in their variance decomposition may be due to the history of prolonged negotiations between workers and their employers. An anticipated rise in the wage markup (a drop in the real wage income earned by the household) implies that output, investment and consumption all fall in the interim. Since their Bayesian estimation assigns a value for the JR parameter η close to zero, the wealth effect of a future drop in wages does not impact the households labour supply decision prior to the shock being is realized.

As mentioned earlier in section 3.5, SGU's (2012) results are close to those found when the benchmark model has all components linking non-stationary TFP and IST removed from the VECM model outlined in equation (3.20), with 10% of the volatility of hours growth due to anticipated wage markup shocks, and roughly 26% of the volatility of investment growth attributable to anticipated MEI shocks. As demonstrated in Table 3.9 however, inclusion of measurement errors for all observables causes the relative importance of anticipated wage markup shocks to increase substantially while unanticipated shocks to the common stochastic trend drop in their ability to explain volatility in the observables. With measurement errors included for all observables anticipated wage markup shocks now explain roughly 10% of the volatility of both output and consumption growth, and 75% of the volatility of hours growth. Anticipated MEI shocks have also increased in significance, explaining 7%, 9%, 12% and 58% of the volatility of the growth rates of hours, output, consumption and investment respectively. These values are roughly in line with those found by SGU (2012). However, as noted to earlier, even when measurement errors are included in our Bayesian estimation, anticipated shocks to the common stochastic trend are still a relevant source of business cycle volatility explaining 20% to 24 % of the volatility of output, consumption and investment.

In contrast to SGU (2012), Khan and Tsoukalas (2012) present an alternative DSGE model with nominal frictions in both prices and wages. They conclude, like Schmitt-Grohe and Uribe (2012) that the majority of movement in output growth can be attributed to changes in the marginal efficiency of investment, explaining an estimated 47% of the unconditional variance of output growth in their model. However, unlike SGU (2012) KT

(2012) conclude that anticipated shocks to MEI lack the ability to cause volatility in any of the observables included in their paper. In addition, anticipated changes in wage markups are again like SGU (2012) found to be a source of business cycle volatility and explain approximately 8%,14%, 3% and 60% of the variance of output growth, consumption growth, investment growth and hours respectively. These estimates can be found in the variance decomposition outlined in Table 3 of their paper.

The low value assigned to anticipated technology shocks by KT (2012) along with SGU (2012) in their variance decomposition relies heavily on the menu of shocks included in their DSGE model. In Table 3.11 we show that the relevance of news shocks depends in particular on whether preference and wage markup shocks are included in the DSGE setup. Table 3.11 outlines the variance decomposition for an alternative parsimonious model without cointegration or spillovers between technology shocks (by setting $\kappa_1 = \kappa_2 = \rho_{12} = \rho_{21} = d_{21} = 0$ in equation (3.20)) and without wage markup and preference shocks. When we remove both wage markup and preference shocks, anticipated shocks to non-stationary technology shocks reappear as potential source of business cycle volatilities. Comparing the benchmark model without cointegration (Table 3.7) to the model with neither cointegration nor wage markup or preference shocks (Table 3.11), one can see that the relative importance of anticipated technology shocks relies heavily on whether wage markup and preference shocks are included in the set an exogenous disturbance. As an example, for the benchmark model without cointegration, the relative importance of anticipated non-stationary IST shocks drop from 7% to less than 1% when these shocks included. This result also holds true for growth rates for hours, investment and consumption, where in each case the relative importance of anticipated technology shocks drop with the inclusion of these shocks. However, as Table 3.6 demonstrates, when we allow for cointegration between non-stationary TFP and non-stationary IST, the relative important of anticipated shocks to non-stationary TFP remains even when we include anticipated and unanticipated wage markup and preference shocks.

This chapter assumes that there exists a single common stochastic trend between TFP and IST, as is done by Schmitt-Grohe and Uribe (2011). Fisher (2009), in his comment of the work done by Beaudry and Lucke (2009) calls to our attention that the chosen number of cointegrating relationships can have important implications for the relative importance of one shock over another when analyzing the variance decomposition. While making the assumption of a single cointegrating relationship, this chapter further assumes that the remaining shocks do not share a common stochastic trend(s), nor have a cointegrated relationship with either TFP or the RPI. For example, their could exist a cointegrating relationship between neutral technology and wage markup shocks, or between MEI and the RPI. Extending this research to include multiple cointegrating relationships will have important implications not only for this research, but also for the work done by Schmitt-Grohe and Uribe (2012) as well as Khan and Tsoukalas (2012). This is left for future research.

3.7 Conclusion

This research began by asking whether the cointegrating relationship shared by TFP and RPI challenged our current understanding of how anticipated shocks generate volatility in US data. In answering this question, we first addressed whether this cointegrating relationship first found by SGU (2011) in post war US is evident in other countries/areas outside the US. Tests for cointegration in the Canadian, and Euro Area data found that there is clear evidence that the cointegrating relationship is not limited to US data. Second, we adapted a canonical RBC model to include both anticipated shocks as well as replicate the cointegrating relationship observed between TFP and the RPI in the US. We estimate this new model using Bayesian methods and find overwhelmingly that anticipated shocks to the common stochastic trend account for a sizable portion of business cycle volatility. With cointegration, anticipated technology shocks matter with roughly 16% to 27% of the volatility of output, consumption and investment and hours growth explained by anticipated shocks to the common stochastic trend. Without cointegration, these values drop to roughly 1% to 4% roughly matching those results found by SGU (2012) and KT (2012) who both found that anticipated technology shocks (of any kind) do not generate business cycle volatility. Thus the answer to the question we set out to answer is unequivocally yes. Correctly reproducing the cointegrating relationship between TFP and the RPI challenges our current understanding of the relative importance of news shocks with anticipated technology shocks regaining their relevance explaining business cycle volatility in the US.

3.8 Appendix

United States Data

Time series data pertaining to growth in output, investment, hours worked, consumption, TFP and the RPI are gathered from the dataset provided by Schmitt-Grohe and Uribe (2011) paper *Busiess Cycles With A Common Trend in Neutral and Investment-Specific Productivity.* Access to this website is made available through the Authors website.

Euro Area

Time series data for output, capital, capital utilization, hours, the consumer price index, the investment price index and the population for the Euro Area as defined by Ireland (2013) were gathered from the dataset provided by Ireland (2004) paper *A Method for Taking Models to the Data.* Access to this website is made available through the Authors website.

2.02

Canadian Data

CANSIM-Canadian socioeconomic database from Statistics Canada

Variable	Data Source
Output	Cansim II: v1992067 Table 380-0002: Gross domestic product, expenditure-based; Canada; Chained (2002) dollars; Seasonally adjusted at annual rates; Gross domestic product (GDP) at market prices (x 1,000,000)
Capital	Cansim II: v1070258 Table 031-0002 Flows and stocks of fixed non-residential capital, by North American Industry Classification System (NAICS) and asset, Canada, provinces and territories; Canada; Current prices; Total all industries; Straight-line end-year net stock; Total assets (x 1,000,000) (annual, 1955 to 2011)
Capacity Utilization	Cansim II: Table 028-0001 Industrial capacity utilization rates, by Standard Industrial Classification, 1980 (SIC), quarterly (percent) and Table 028-0002 Industrial capacity utilization rates, by North American Industry Classification System (NAICS), quarterly (percent)
Hours	Cansim II: v1409155 Table 383-0008: Indexes of labour productivity, unit labour costs and related variables;Canada; Business sector; Hours worked or Table 282-0015 Labour force survey estimates (LFS), by usual hours worked, main or all jobs, sex and age group, unadjusted for seasonality, monthly
Consumer Price Index	Cansim II: Table 326-0021 Consumer Price Index (CPI), 2009 basket, annual (2002=100)
Investment Price Index	Cansim II: Table 329-0045 Industry price indexes for machinery and equipment, motor vehicles and other transport equipment, quarterly (index, 1997=100)(1)
Population	Cansim II: Table 051-0001 Estimates of population, by age group and sex for July 1, Canada, provinces and territories, annual (persons)

3.8.1 DYNARE

Information on DYNARE software is available at www.dynare.org. This website provides information on the mathematical processes applied throughout this paper.

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Figure 3.1
Benchmark Model

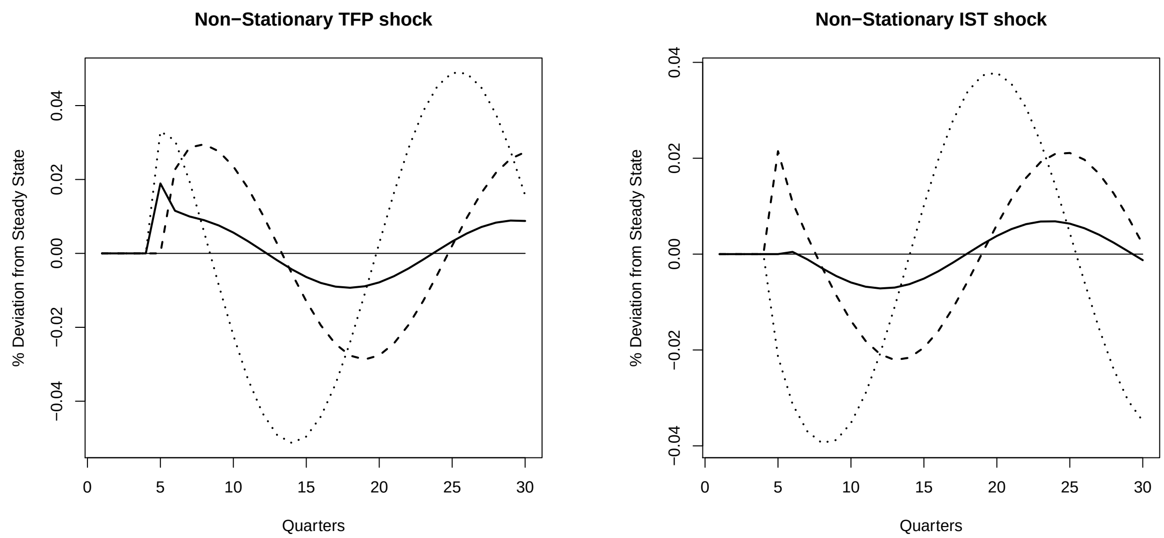

Figure 3.1: Impulse response function of μ^Z (solid line) and μ^A (dashed line) and the cointegrating term x^{co} (dotted line) to a one standard error innovation to $\epsilon^4_{\mu^Z,t}$ (left) and a one standard error innovation to $\epsilon^4_{\mu^A,t}$ (right).

Figure 3.2

Impulse Responses to 4 Quarter anticipated News shocks to μ^Z and μ^A
Benchmark model

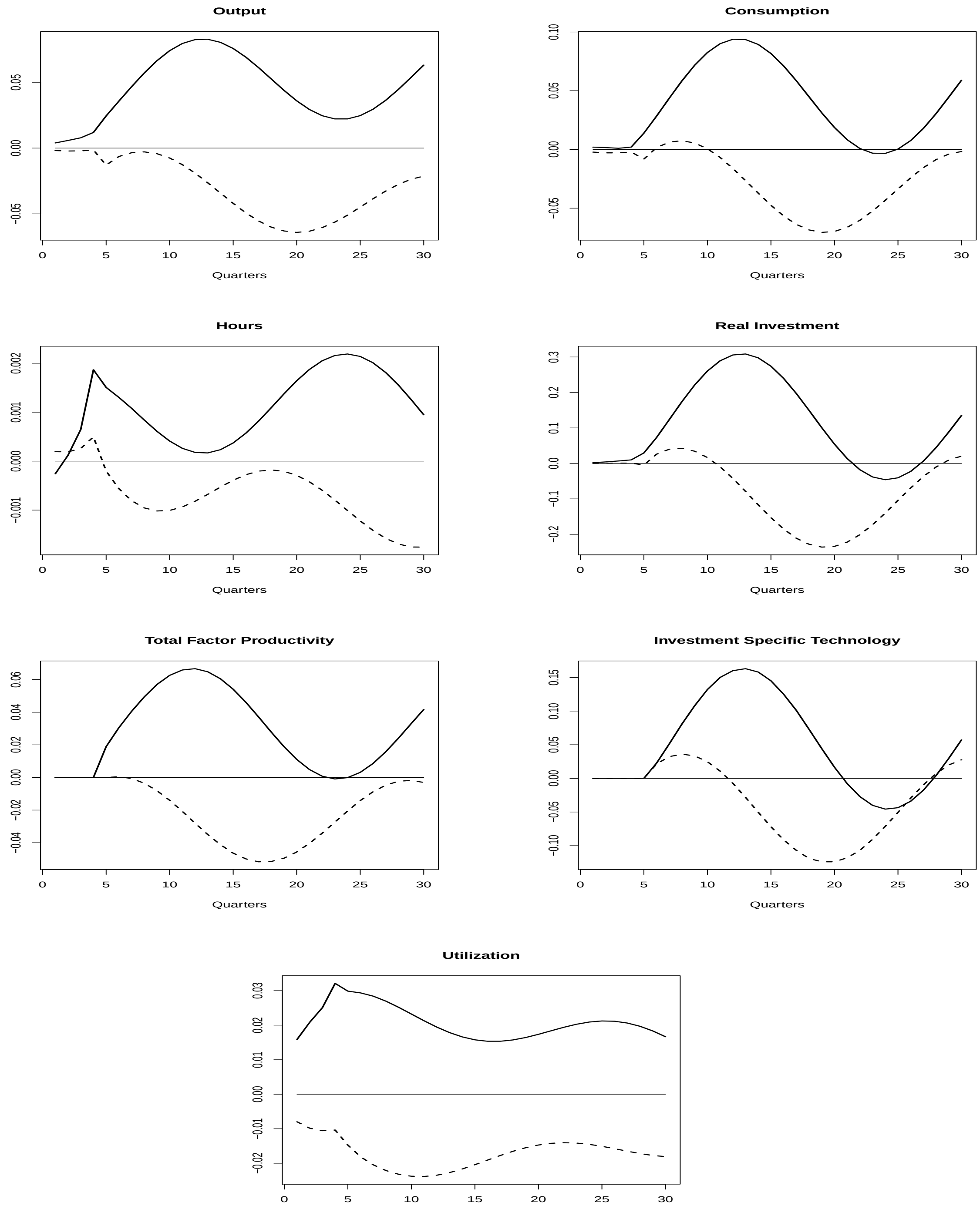

Figure 3.2: Impulse responses function of a one standard error innovation to $\epsilon^4_{\mu^Z,t}$ (solid) and a one standard error innovation to $\epsilon^4_{\mu^A,t}$ (dashed), measured as a percent deviation from the respective balanced growth path.

Figure 3.3

Impulse Responses to 4 Quarter anticipated News shocks to μ^Z
With and Without Cointegration

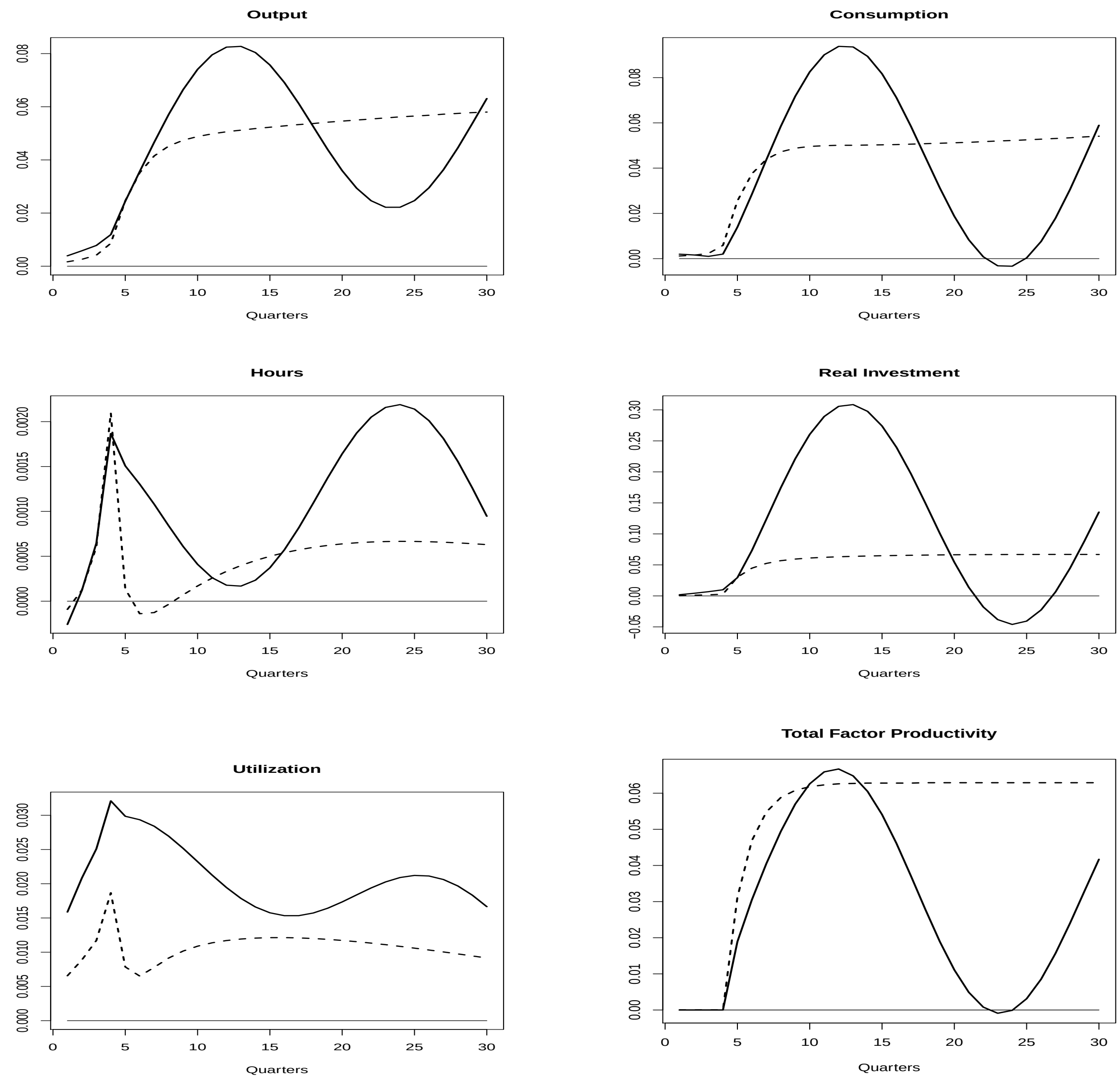

Figure 3.3: Impulse responses function of a one standard error innovation to $\epsilon^4_{\mu^Z,t}$ (solid) with cointegration between TFP and IST and a one standard error innovation to $\epsilon^4_{\mu^Z,t}$ (dashed) with no cointegration between these two technologies. Each impulse response is measured as a percent deviation from the respective balanced growth path.

Table 3.1
Test for non-stationarity of TFP and the RPI

Test	Variable	Test Statistic	Critical Value (5 percent)	Reject Null Hypothesis
United States				
ADF	$ln(TFP_{US})$	-2.3947	-3.43	No
ADF	$ln(RPI_{US})$	-0.3085	-3.43	No
Euro Area				
ADF	$ln(TFP_{EA})$	-2.0926	-3.43	No
ADF	$ln(RPI_{EA})$	-2.4784	-3.43	No
Canada				
ADF	$ln(TFP_{CDN})$	-1.7659	-3.43	No
ADF	$ln(RPI_{CDN})$	-0.2496	-3.43	No

Table 3.1: The Null hypothesis of the Agmented Dickey Fuller Test (ADF) is that time series is non-stationary.

Table 3.2
Test for Cointegration between TFP and the RPI

Null Hypothesis	Alternative Hypothesis	Test Statistic	Critical Value (5 percent)	Reject Null Hypothesis
United States				
$r = 0$	$r > 0$	37.83	19.96	Yes
$r \leq 1$	$r > 1$	7.83	9.24	No
Euro Area				
$r = 0$	$r > 0$	39.67	19.96	Yes
$r \leq 1$	$r > 1$	8.73	9.24	No
Canada				
$r = 0$	$r > 0$	36.96	19.96	Yes
$r \leq 1$	$r > 1$	7.52	9.24	No

Table 3.2: A Johansen test for cointegration tests whether TFP and RPI are cointegrated. Here r identifies the number of cointegration relations. If the Null hypothesis $r \leq 1$ is accepted, then there is at least one cointegrating relationship.

Table 3.3: Calibrated Parameters

Parameter	Value	Description
σ	1	Risk aversion
$\bar{\mu}^W$	1.10	steady state wage markup
$\bar{\mu}^Y$	1.0049	Per capita output growth along a balanced growth path
δ_0	0.025	Depreciation rate in steady state
$\bar{\mu}^A$	1.0043	Per capita IST growth along a balanced growth path
$\bar{\mu}^Z$	1.0023	Per capita TFP growth along a balanced growth path
β	0.975	Subjective discount factor
u^{ss}	1.00	Steady state capital utilization rate
α	0.37	Capital Share of Output
$\psi = ln(\bar{\mu}^A)/ln(\bar{\mu}^Z)$	1.8758	Cointegration Coefficient

Table 3.4: Priors

Parameter	Discription	Prior Distribution	Lower Bound	Upper Bound	Mean	Variance
θ		Gamma			3	0.75
χ		Beta			0.5	0.1
δ_2		Uniform	0.05	0.15		
η		Uniform	0.01	0.99		
h^{ss}		Normal			0.3	0.03
κ		Gamma			2.45	8
ρ_Z		Beta			0.70	0.20
ρ_A		Beta			0.70	0.20
ρ_v		Beta			0.70	0.20
ρ_b		Uniform	0	0.99		
ρ_{μ^W}		Beta			0.70	0.20
ρ_{11}		Beta			0.70	0.20
ρ_{22}		Beta			0.70	0.20
ρ_{12}		Uniform			0	1.5
ρ_{21}		Uniform			0	1.5
d_{21}		Uniform			-2	2
κ_1		Uniform	-0.5	0.5		
κ_2		Uniform	-0.95	0.95		
σ_i^0		Inv Gamma			0.5	2
σ_i^k		Inv Gamma			0.1	2

Table 3.4: σ_i^0 refers to the variance of an unanticipated shock to disturbance $i = \{Z, A, b, v, \mu^W, \mu^Z, \mu^A\}$, σ_i^k refers to the variance of an anticipated shock to disturbance i known $k = \{4, 8\}$ periods in advance.

Table 3.5
Bayesian Estimation

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
θ	Normal	3	0.75	3.1449	3.0667	3.2231
χ	Gamma	0.5	0.1	0.5093	0.5076	0.511
δ_2	Uniform	0.08	0.0404	0.0887	0.086	0.0913
η	Uniform	0.504	0.2855	0.5251	0.5208	0.5295
h^{ss}	Normal	0.3	0.03	0.2817	0.2808	0.2827
κ	Gamma	2.46	8	22.235	22.1983	22.2717
ρ_Z	Beta	0.7	0.2	0.6318	0.6269	0.6368
ρ_A	Beta	0.7	0.2	0.7336	0.7097	0.7576
ρ_v	Beta	0.7	0.2	0.9416	0.9278	0.9554
ρ_b	Uniform	0.7	0.2	0.7644	0.7548	0.7741
ρ_{μ^W}	Beta	0.7	0.2	0.9709	0.9644	0.9775
ρ_{11}	Beta	0.7	0.2	0.504	0.4842	0.5239
ρ_{22}	Beta	0.7	0.2	0.6105	0.6087	0.6124
ρ_{21}	Uniform	0.75	0.4327	0.9684	0.934	1.0029
ρ_{12}	Uniform	0.75	0.4327	0.0775	0.0761	0.0788
κ_1	Uniform	0	0.2887	0.0913	0.0901	0.0924
κ_2	Uniform	0	0.5485	0.1325	0.1299	0.1351
d_{21}	Uniform	0	1.1547	0.8789	0.8489	0.9088
σ_z^0	Inverse Gamma	0.5	2	0.0658	0.0613	0.0703
σ_a^0	Inverse Gamma	0.5	2	0.0678	0.0621	0.0735
σ_v^0	Inverse Gamma	0.5	2	0.1261	0.0869	0.1653
σ_b^0	Inverse Gamma	0.5	2	0.1328	0.0653	0.2002
$\sigma_{\mu^w}^0$	Inverse Gamma	0.5	2	0.2017	0.1305	0.2729
$\sigma_{\mu^Z}^0$	Inverse Gamma	0.5	2	0.082	0.0728	0.0912
$\sigma_{\mu^A}^0$	Inverse Gamma	0.5	2	0.0774	0.0671	0.0877
σ_Z^1	Inverse Gamma	0.1	2	0.0155	0.0144	0.0166
σ_Z^2	Inverse Gamma	0.1	2	0.0298	0.0207	0.0389
σ_A^1	Inverse Gamma	0.1	2	0.0582	0.0319	0.0845
σ_A^2	Inverse Gamma	0.1	2	0.0265	0.0205	0.0326

Table 3.5 Continued
Bayesian Estimation

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
σ_v^1	Inverse Gamma	0.1	2	0.0733	0.0479	0.0988
σ_v^2	Inverse Gamma	0.1	2	0.1687	0.1243	0.2131
σ_b^1	Inverse Gamma	0.1	2	0.0822	0.0211	0.1433
σ_b^2	Inverse Gamma	0.1	2	0.0762	0.0163	0.1361
$\sigma_{\mu^W}^1$	Inverse Gamma	0.1	2	0.3116	0.0822	0.5409
$\sigma_{\mu^W}^2$	Inverse Gamma	0.1	2	0.3031	0.2733	0.3329
$\sigma_{\mu^A}^1$	Inverse Gamma	0.1	2	0.0610	0.0233	0.0987
$\sigma_{\mu^A}^2$	Inverse Gamma	0.1	2	0.0343	0.0239	0.0446
$\sigma_{\mu^Z}^1$	Inverse Gamma	0.1	2	0.0311	0.0233	0.0389
$\sigma_{\mu^Z}^2$	Inverse Gamma	0.1	2	0.0649	0.033	0.0968

Table 3.6
Variance Decomposition: Benchmark model

	g^y	g^c	g^i	g^h	g^{rpi}	g^{tfp}
Stationary TFP						
ϵ_Z^0	3.09	1.81	0.09	7.5	0	6.03
$\sum_{i=4,8} \epsilon_Z^i$	0.4	0.22	0.02	4.76	0	1.22
Stationary IST						
ϵ_A^0	0.15	0.03	2.93	0.74	0.28	0
$\sum_{i=4,8} \epsilon_A^i$	0.06	0	0.85	0.38	0.08	0
Preferences						
ϵ_b^0	0.15	0.13	0.08	0.04	0	0
$\sum_{i=4,8} \epsilon_b^i$	0.12	0.08	0.02	0.4	0	0
Wage Markup						
$\epsilon_{\mu^W}^0$	0.22	0.11	0.04	6.95	0	0
$\sum_{i=4,8} \epsilon_{\mu^W}^i$	0.71	0.38	0.17	28.76	0	0
MEI						
ϵ_v^0	0.82	0.17	6.73	0.64	0	0
$\sum_{i=4,8} \epsilon_v^i$	0.25	0.13	2.61	0.32	0	0
Common Trend						
$\epsilon_{\mu^Z}^0 + \epsilon_{\mu^A}^0$	68.31	70.88	59.08	33.77	72.7	67.36
$\sum_{i=4,8}\{\epsilon_{\mu^Z}^i + \epsilon_{\mu^A}^i\}$	25.72	26.05	27.39	15.73	26.94	25.38

Here, g^i is the growth rate of variable $i = \{y, c, i, h, rpi, tfp\}$.

Table 3.7

No Cointegration

$\kappa_1 = \kappa_2 = 0 = \rho_{21} = \rho_{12} = d_{21} = 0$

	g^y	g^c	g^i	g^h	g^{rpi}	g^{tfp}
Stationary TFP						
ϵ_Z^0	79.49	81.83	29.55	76.48	0	92.7
$\sum_{i=4,8} \epsilon_Z^i$	0.46	0.43	0.36	2.3	0	0.9
Stationary IST						
ϵ_A^0	0.27	0.04	9.18	0.31	41.34	0
$\sum_{i=4,8} \epsilon_A^i$	0.1	0	2.43	0.12	9.9	0
Preferences						
ϵ_b^0	0.39	0.6	0.16	0.02	0	0
$\sum_{i=4,8} \epsilon_b^i$	0.22	0.3	0.04	0.12	0	0
Wage Markup						
$\epsilon_{\mu^W}^0$	0.45	0.42	0.37	3.47	0	0
$\sum_{i=4,8} \epsilon_{\mu^W}^i$	0.88	0.79	1.04	10.04	0	0
MEI						
ϵ_v^0	0.14	0.02	3.29	0.04	0	0
$\sum_{i=4,8} \epsilon_v^i$	0.67	0.46	25.77	0.45	0	0
Non Stationary TFP						
$\epsilon_{\mu^Z}^0$	15.17	13.78	14.27	3.15	0	5.41
$\sum_{i=4,8} \epsilon_{\mu^Z}^i$	1.38	1.06	2.22	2.5	0	0.99
Non Stationary IST						
$\epsilon_{\mu^A}^0$	0.26	0.21	8.96	0.81	39.19	0
$\sum_{i=4,8} \epsilon_{\mu^A}^i$	0.13	0.03	2.37	0.19	9.56	0

Here, g^i is the growth rate of variable $i = \{y, c, i, h, rpi, tfp\}$.

Table 3.8
Bayesian Estimation With Measurement Error

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
θ	Normal	3	0.75	2.8538	2.8218	2.8896
χ	Gamma	0.5	0.1	0.4408	0.4362	0.4439
δ_2	Uniform	0.08	0.0404	0.0857	0.0849	0.0868
η	Uniform	0.504	0.2855	0.591	0.5809	0.601
h^{ss}	Normal	0.3	0.03	0.301	0.2989	0.303
κ	Gamma	2.46	8	11.0775	10.4688	11.8829
ρ_Z	Beta	0.7	0.2	0.68	0.6688	0.689
ρ_A	Beta	0.7	0.2	0.7255	0.7112	0.739
ρ_v	Beta	0.7	0.2	0.6451	0.6308	0.6511
ρ_b	Uniform	0.7	0.2	0.7401	0.7323	0.7485
ρ_{μ^W}	Beta	0.7	0.2	0.9813	0.9689	0.99
ρ_{11}	Beta	0.7	0.2	0.4723	0.457	0.4857
ρ_{22}	Beta	0.7	0.2	0.5844	0.5801	0.589
ρ_{21}	Uniform	0.75	0.4327	1.2623	1.2517	1.2733
ρ_{12}	Uniform	0.75	0.4327	0.2694	0.2552	0.2846
κ_1	Uniform	0	0.2887	0.1553	0.1495	0.1606
κ_2	Uniform	0	0.5485	0.5545	0.5286	0.5766
d_{21}	Uniform	0	1.1547	1.5759	1.5292	1.6126
σ_z^0	Inverse Gamma	0.5	2	0.0608	0.0603	0.0614
σ_a^0	Inverse Gamma	0.5	2	0.0623	0.0603	0.0647
σ_v^0	Inverse Gamma	0.5	2	0.1116	0.086	0.138
σ_b^0	Inverse Gamma	0.5	2	0.0806	0.0664	0.0944
σ_{muw}^0	Inverse Gamma	0.5	2	0.1418	0.0986	0.1768
$\sigma_{\mu^Z}^0$	Inverse Gamma	0.5	2	0.0616	0.0603	0.0632
$\sigma_{\mu^A}^0$	Inverse Gamma	0.5	2	0.0622	0.0603	0.0645
σ_Z^1	Inverse Gamma	0.1	2	0.0181	0.015	0.0214
σ_Z^2	Inverse Gamma	0.1	2	0.0185	0.0149	0.0218
σ_A^1	Inverse Gamma	0.1	2	0.0209	0.016	0.0257
σ_A^2	Inverse Gamma	0.1	2	0.0243	0.0185	0.0302

Table 3.8 Continued
Bayesian Estimation

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
σ_v^1	Inverse Gamma	0.1	2	1.2954	1.2168	1.3875
σ_v^2	Inverse Gamma	0.1	2	0.8815	0.7794	1.0334
σ_b^1	Inverse Gamma	0.1	2	0.0248	0.0182	0.0318
σ_b^2	Inverse Gamma	0.1	2	0.0272	0.0192	0.0347
$\sigma_{\mu^W}^1$	Inverse Gamma	0.1	2	0.0536	0.0289	0.078
$\sigma_{\mu^W}^2$	Inverse Gamma	0.1	2	0.0689	0.0273	0.107
$\sigma_{\mu^A}^1$	Inverse Gamma	0.1	2	0.0239	0.0178	0.0303
$\sigma_{\mu^A}^2$	Inverse Gamma	0.1	2	0.0314	0.0212	0.0417
$\sigma_{\mu^Z}^1$	Inverse Gamma	0.1	2	0.0161	0.0133	0.0189
$\sigma_{\mu^Z}^2$	Inverse Gamma	0.1	2	0.0255	0.0216	0.0293
σ_Y^{ME}	Uniform	$\frac{1}{8}\hat{\sigma_Y}$	0.072	0.2503	0.2501	0.2505
σ_I^{ME}	Uniform	$\frac{1}{8}\hat{\sigma_I}$	0.1819	0.3224	0.3189	0.3262
σ_C^{ME}	Uniform	$\frac{1}{8}\hat{\sigma_C}$	0.0392	0.1184	0.115	0.1213
σ_{RPI}^{ME}	Uniform	$\frac{1}{8}\sigma_{\hat{RPI}}$	0.0301	0.0702	0.0695	0.0709
σ_H^{ME}	Uniform	$\frac{1}{8}\hat{\sigma_h}$	0.286	0.4604	0.4414	0.4756
σ_{TFP}^{ME}	Uniform	$\frac{1}{8}\hat{\sigma_h}$	0.1819	0.3412	0.3345	0.3484

Table 3.9

Measurement Errors Included for all Observables

	g^y	g^c	g^i	g^h	g^{rpi}	g^{tfp}
Stationary TFP						
ϵ_Z^0	7.47	7.66	0.24	2.07	0	26.72
$\sum_{i=4,8} \epsilon_Z^i$	0.94	0.92	0.05	1.49	0	5.26
Stationary IST						
ϵ_A^0	0.29	0.09	1.3	0.25	2.14	0
$\sum_{i=4,8} \epsilon_A^i$	0.14	0.02	0.44	0.13	0.63	0
Preferences						
ϵ_b^0	0.47	0.9	0.14	0.04	0	0
$\sum_{i=4,8} \epsilon_b^i$	0.59	0.88	0.04	0.34	0	0
Wage Markup						
$\epsilon_{\mu^W}^0$	1.69	1.68	0.08	10.26	0	0
$\sum_{i=4,8} \epsilon_{\mu^W}^i$	10.86	10.93	0.78	72.53	0	0
MEI						
ϵ_v^0	0.05	0.03	0.25	0.02	0	0
$\sum_{i=4,8} \epsilon_v^i$	9.37	12.67	57.99	6.61	0	0
Common Trend						
$\epsilon_{\mu^Z}^0 + \epsilon_{\mu^A}^0$	44.16	42.46	18.39	2.49	57.71	40.69
$\sum_{i=4,8}\{\epsilon_{\mu^Z}^i + \epsilon_{\mu^A}^i\}$	23.93	21.76	20.3	3.57	39.51	27.26

Here, g^i is the growth rate of variable $i = \{y, c, i, h, rpi, tfp\}$.

Table 3.10
No Wage Markup Shocks, No Preference Shocks and No Cointegration

	g^y	g^c	g^i	g^h	g^{rpi}	g^{gtfp}
Stationary TFP						
ϵ_Z^0	81.22	86.56	22.18	69.34	0	97.02
$\sum_{i=4,8}\epsilon_Z^i$	0.04	0.04	0.02	0.16	0	0.07
Stationary IST						
ϵ_A^0	1.47	0.23	9.04	2.06	17.97	0
$\sum_{i=4,8}\epsilon_A^i$	1.24	0.17	6.23	1.85	9.78	0
Preferences						
ϵ_b^0	-	-	-	-	-	-
$\sum_{i=4,8}\epsilon_b^i$	-	-	-	-	-	-
Wage Markups						
$\epsilon_{\mu^w}^0$	-	-	-	-	-	-
$\sum_{i=4,8}\epsilon_{\mu^w}^i$	-	-	-	-	-	-
MEI						
ϵ_v^0	0.2	0.21	2.12	0.13	0	0
$\sum_{i=4,8}\epsilon_v^i$	0.32	0.55	3.57	0.38	0	0
Non Stationary TFP						
$\epsilon_{\mu^Z}^0$	1.73	1.73	0.61	0.27	0	0.7
$\sum_{i=4,8}\epsilon_{\mu^Z}^i$	2.87	2.28	1.99	4.68	0	2.21
Non Stationary IST						
$\epsilon_{\mu^A}^0$	3.8	2.86	20.85	9.33	33.06	0
$\sum_{i=4,8}\epsilon_{\mu^A}^i$	7.11	5.37	33.38	11.79	39.19	0

Here, g^i is the growth rate of variable $i = \{y, c, i, h, rpi, rw, tfp\}$.

Table 3.11
Khan and Tsoukalas (2012) Priors

	g^y	g^c	g^i	g^h	g^{rpi}	g^{tfp}
Stationary TFP						
ϵ_Z^0	8.62	12.61	0.06	3.91	0	37.67
$\sum_{i=4,8} \epsilon_Z^i$	5.43	7.69	0.08	6.66	0	36
Stationary IST						
ϵ_A^0	0.9	0.24	2.75	0.35	7.29	0
$\sum_{i=4,8} \epsilon_A^i$	2.77	0.29	6.46	1.07	16.55	0
Preferences						
ϵ_b^0	0.13	0.24	0	0	0	0
$\sum_{i=4,8} \epsilon_b^i$	0.39	0.69	0	0.04	0	0
Wage Markup						
$\epsilon_{\mu^W}^0$	0.51	0.71	0.01	1.68	0	0
$\sum_{i=4,8} \epsilon_{\mu^W}^i$	17.7	23.45	0.67	76.13	0	0
MEI						
ϵ_v^0	21.95	4.02	33.7	1.33	0	0
$\sum_{i=4,8} \epsilon_v^i$	7.38	8.82	25.71	1.78	0	0
Common Trend						
$\epsilon_{\mu^Z}^0 + \epsilon_{\mu^A}^0$	20.27	31.89	13.96	3.23	35.65	13.63
$\sum_{i=4,8}\{\epsilon_{\mu^Z}^i + \epsilon_{\mu^A}^i\}$	13.93	9.35	16.6	3.8	40.51	12.71

Here, g^i is the growth rate of variable $i = \{y, c, i, h, rpi, tfp\}$. We adopt the priors for the peristance parameters, as well as the volatilities for anticipated and unanticated innovation for the six types of shocks that are included in both the benchmark model and KT (2012). As for the persistance and the volatility for anticipated and unanticipated shocks to stationary IST, we adopt the same priors as used for the stationary TFP process by KT (2012).

Chapter 4

The Endogenous Relative Price of Investment

4.1 Introduction

Since Greenwood, Hercowitz, and Huffman (GHH) (1988) first identified investment-specific technology (IST) as a potential source of business cycle volatility, this type of shock has become a common feature in the business cycle literature. Likewise, identification of IST has remained roughly in line with the method used by GHH (1998). Since their seminal work, the business cycle literature has shifted over time in its assessment of the relative importance of IST. At first, research such as that by Fisher (2006) as well as Justiniano, Primiceri, and Tambalotti (2010), to name a few, found IST to be an important source of both low-frequency and high-frequency volatility. Each time, the relative importance of IST is assessed by either analyzing the variance decomposition, or by growth accounting as done by Fisher (2006). Recent research, such as that of Justiniano et al. (2011) and Schmitt-Grohe and Uribe (2011), has however, found that IST, when correctly adapted to reflect movement in the relative price of investment (RPI), lacks the ability to generate any business cycle volatility.

Beaudry and Lucke (2009) take an alternative approach. In their research, rather than analyzing a dynamic stochastic general equilibrium model's variance decomposition, they quantitatively assess the relative importance of IST against a menu of alternative shocks using an approach based on a cointegrated structural vector autoregression (SVAR). They conclude that expected changes in neutral technology, as well as preference and monetary shocks play a far more significant role in explaining high-frequency movements in the data in their forecast error variance decomposition than IST. All of the aforementioned research relies heavily on the assumption that IST can be uniquely identified by the inverse of the RPI. Using micro-level data, Basu et al. (2013) show that the RPI responds slowly to changes in IST often taking up to three quarters for the effect of an IST shock to impact the RPI. This could be due to either sticky investment prices, or, alternatively, investment prices that are driven by forces other than IST. Fisher (2009) highlights the identification of IST by the inverse of the RPI could be problematic whenever there exists any further asymmetries between consumption and investment producing sectors. Through a SVAR-based approach, Kim (2009) finds that IST shocks could at most explain 27 percent of the

RPI 1955:I-2000:IV. The assumption that IST is an independent stochastic process implies that the RPI is orthogonal to any other type of economic disturbance, such as neutral technology shocks, wage shocks, or preference shocks, which are commonly included in the literature. Therefore, the adequacy of the RPI to correctly indicate movements in IST, could, for example, be assessed by the independence of the RPI with any one of these disturbances. If the inverse of the RPI, as GHH (1988) suggested, is a good indication of IST, then these technology shocks should in theory be unrelated to neutral technology as measured by total factor productivity (TFP).

As can be seen in Figure 4.1, upturns in *tfp* are typically followed by a decrease in the *rpi*. The *tfp* plotted in Figure 4.1 is calculated as in Beaudry and Lucke (2009) (the log of non-farm output less the log of both non-farm hours and capital services, each scaled by its share of output).[1] As for the *rpi*, we use the quality-adjusted *rpi* time series as calculated by Fisher (2006). This data series adjusts the relative price of equipment estimated by using the Gordon-Cummins-Violante (GCV) equipment price deflator and divides it by the quarterly price deflator for consumption goods found in the NIPA tables. With these two time series, Fisher (2006) obtains a quarterly measure of the *rpi* adjusted for changes in quality. Information on the data used is available in the Data Used section of the appendix.

[1] Data on the Real Non-Farm Gross Value-Added Output is calculated by the Bureau of Economic Analysis 1947:1-2013:4. Non-farm hours worked calculated by the Bureau of Labour Statistics (BLS) 1947:1-2013:4. Capital services time series are calculated from the (BLS) private sector Non-Farm Business Sector (NAICS 113-81) 2009 index 1987-2012.

With a correlation between detrended *tfp* and *rpi* of approximately -0.216, it would appear that the *rpi* moves countercyclically to *tfp*. This fact has been addressed in countless papers, such as that of Letendre and Luo (2007), who adapt the standard AR(1) setup to allow for spillovers between *tfp* and *rpi* in order to replicate the countercyclical nature of the *rpi*. Thus, it appears that in the short-run, the theory suggested by GHH (1988) that relative prices can be used to determine changes in relative technologies across sectors is less than robust.

Schmitt-Grohe and Uribe (2011) have furthered the disconnect between IST and the *rpi* by also demonstrating that *tfp* and the *rpi* are cointegrated in the long run. With both *tfp* and the *rpi* integrated of order 1 stationary in the US, they apply a Johansen's test for cointegration in which they show that in addition to *tfp* and *rpi* being non-stationary, they are also cointegrated.[2] With both *tfp* and the *rpi* cointegrated, then there exists a cointegration coefficient β such that the difference in levels between each time series remains I(0) stationary in the long run. To highlight this fact, Figure 4.2 plots *tfp* along with the inverse of the *rpi* adjusted by the cointegration coefficient $\beta = 0.623$. Figure 4.2 demonstrates that these two times series follow a common stochastic trend. Given the assumption made by GHH (1988) that relative technologies across sectors is reflected in the relative price, it would be expected that these two time series would not follow a common stochastic trend, as both cointegra-

[2]Schmitt-Grohe and Uribe (2011) apply both Augmented Dickey Fuller Test (ADF) as well as a Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test to determine the stationarity of both *tfp and the rpi. As can be seen in Tables 1 and 2 of their paper, respectively, both of these tests conclude that these two series are non-stationary.*

tion tests, as well as Figure 4.2 appear to suggest. As is shown in section 4.4, when the standard business cycle model is adapted to replicate the co-movement of *tfp* and the *rpi*, 39 percent of *rpi* growth from one period to the next can be explained by shifts in neutral technology.

Given the aforementioned relationships between *tfp* and the *rpi*, both in the long run as well as over the business cycle, the assumption that the *rpi* is orthogonal to any form of economic disturbance can be safely rejected. Further tests could be done to assess the orthogonality of the *rpi* with any other form of economic disturbances, such as wage markup, preference shocks or shifts in the marginal efficiency of investment.

Is the cyclical movement in the *rpi* a technological phenomenon, or are movement in the *rpi* due to changes in the relative demand for investment goods over consumption goods? In response to this question, this paper proposes a two-sector model adapted to incorporate both endogenous markup variation as well as transforming investment production to allow for either increasing or decreasing returns-to-scale in the production of investment goods. Endogenous price markups are incorporated by assuming that each sector (consumption and investment) is populated by a finite number of firms each selling a differentiable good. Each of these firms is capable of not only influencing its own price, but also the price charged across all firms. Curvature in investment production is introduced by allowing the sum of factor shares to be greater or less than 1. An alternative model is presented in section 4.4, where movement in the *rpi* is generated entirely by technological spillovers. As section 4.4 demonstrates, when the assumption of orthogonality between technologies is relaxed, approximately 39 percent of the *rpi* can be attributed to shifts in *tfp*. In contrast, when the *rpi* moves in response to changes in demand as is the case in the benchmark model, the explanatory power of IST drops further to 52 percent. Stationary and non-stationary *tfp* shocks explain approximately 32 percent of the volatility of the *rpi*. Non-technological shocks contribute 16 percent. With the vast majority of business cycle research assuming that the *rpi* is determined exogenously, the results of this paper are particularly poignant.

These two approaches are compared to Kim's (2009) research to assess whether one approach outperforms the other. As outlined in section 4.4, our results indicate that an endogenous approach to modeling movement in the *rpi* outperforms the exogenous-based approach due to the impact non-technological shocks have on the relative demand for investment goods over consumption goods. Through the endogenous-based approach, 52 percent of the volatility of the *rpi* is explained by unanticipated changes in IST. These results brings into question the current convention in linking IST to the inverse of the *rpi*.

The remainder of the paper is organized as follows. Section 4.2 outlines the benchmark model, consisting of the varying elements that allow the *rpi* to move endogenously over time. Section 4.3 outlines the Bayesian estimation process, which is used to estimate the parameter values. Section 4.4 outlines the results of the benchmark model with variations of this model to assess the relative importance in each aspect in generating these results. Section 4.5 concludes.

4.2 Benchmark Model

The benchmark model for this paper involves a two-sector real business cycle model with monopolistic competition in both consumption and investment good-producing sectors as well as the possibility of increasing or decreasing returns to scale (IRS/DRS) in the production of investment goods. This model is set up in such a way that firms are able to vary the markup charged above production costs depending on the number of competing firms within that industry. We begin with an outline of the various stages of production in the consumption sector.

Production of each good can be divided into three stages of production. These stages include a finite number of monopolistically competitive firms that produce their product using both capital and labour inputs. These goods are then aggregated at an industry level by firms that assemble them into a composite good to be sold at the sector-level. Lastly, there is a perfectly competitive firm that purchases these industry-level goods and assembles them into a composite good ready to sell to consumers. For ease of illustration, we begin our dissection of the various stages of consumption production at the sector level.

4.2.1 Consumption Sector

Sector-Level

At the aggregate level, the consumption good produced in this economy C_t is a composite good consisting of a continuum of unit measure one industry-level goods produced using the following constant returns-to-scale production function.

$$C_t = \left[\int_0^1 Q_t^c(j)^{\omega} \mathrm{d}j \right]^{\frac{1}{\omega}}, \tag{4.1}$$

where $Q_t^c(j)$ refers to quantity of output produced in industry j, with the elasticity of substitution between industry-level goods equal to $\frac{1}{1-\omega}$. The total profit earned by assembling these industry-level goods at the sector-level Π_t^c is equal to

$$\Pi_t^c = \left\{ P_t^c C_t - \int_0^1 P_t^c(j) Q_t^c(j) \mathrm{d}j \right\}, \tag{4.2}$$

where P_t^c is the price of the sector-level consumption good and $P_t^c(j)$ is the price paid for industry j's composite good. Solving the production problem for the sector-level consumption goods producer implies a conditional input demand of

$$Q_t^c(j) = \left(\frac{P_t^c(j)}{P_t^c} \right)^{\frac{1}{\omega - 1}} C_t \tag{4.3}$$

of industry j's good by the sector-level producer, where the price index P_t^c is equal to

$$P_t^c = \left[\int_0^1 P_t^c(j)^{\frac{\omega}{\omega - 1}} \mathrm{d}j \right]^{\frac{\omega - 1}{\omega}}. \tag{4.4}$$

Industry-Level

The industry-level consumption good is produced using a constant returns-to-scale production function which aggregates output produced by a finite number of firms within industry j. Firm i within industry j produces a differentiable good $x_t^c(j, i)$. This good is used as an input at the industry-level through the following production function

$$Q_t^c(j) = \left[N_t^c(j) \right]^{1 - \frac{1}{\tau}} \left[\sum_{i=1}^{N_t^c} x_t^c(j, i)^\tau \right]^{\frac{1}{\tau}}. \tag{4.5}$$

$N_t^c(j)$ denotes the number of firms competing in industry j and $1/(1 - \tau)$ is the elasticity of substitution between industry-level goods. Given this production function, the profit function for the firm producing the industry j good $\Pi_t^c(j)$ is determined as

$$\Pi_t^c(j) = \left\{ P_t^c(j)Q_t^c(j) - \sum_{i=1}^{N_t^c(j)} x_t^c(j,i)p_t^c(j,i) \right\}, \tag{4.6}$$

where $P_t^c(j,i)$ denotes the price of firm i's output in industry j. This profit function implies a conditional demand

$$x_t^c(j,i) = \Big(\frac{P_t^c(j,i)}{P_t^c(j)}\Big)^{\frac{1}{\tau-1}} \frac{Q_t^c(j)}{N_t^c} \tag{4.7}$$

by industry j for firm i's product. Analogous to the sector level of production, the industry j consumption good price index is equal to

$$P_t^c(j) = N_t^c(j)^{\frac{1}{\tau}-1} \left[\sum_{i=1}^{N_t^c(j)} P_t^c(j,i)^{\frac{\tau}{\tau-1}} \right]^{\frac{\tau-1}{\tau}}. \tag{4.8}$$

Firm-Level

The last stage of production consists of a finite number of monopolistically competitive firms within each industry. These firms produce a good using both capital and labour as inputs. We assume that these firms can costlessly differentiate their product, thus, given a finite number of firms competing, have the ability to not only influence its own price $P_t^c(j,i)$, but also the industry-level price $P_t^c(j)$. While firm i in industry j has the ability to influence $P_t^c(j,i)$ as well as $P_t^c(j)$, it does not, however, have the ability to influence the sector-level price P_t^c. In industry j, firm i's good is produced using the following constant returns-to-scale production function

$$x_t^c(j,i) = Z_t\big(k_t^c(j,i)\big)^{\alpha}\big(X_t^z h_t^c(j,i)\big)^{1-\alpha} - \phi^c, \quad \textit{where} \quad \phi^c > 0, 0 < \alpha < 1 \tag{4.9}$$

where $k_t^c(j,i)$ and $h_t^c(j,i)$ denote the capital and labour used by firm i in industry j respectively, α is capital share of output, and ϕ^c denotes the fixed cost of production. We assume there are two types of technology shocks affecting production of consumption goods. These include a stationary technology shock, Z_t, and a non-stationary labour-augmenting technology, X_t^z, where the stochastic growth rate of X_t^Z is given by

$$\mu_t^z \equiv \frac{X_t^z}{X_{t-1}^z}. \tag{4.10}$$

TFP in the consumption sector is

$$TFP_t = Z_t\big(X_t^z\big)^{1-\alpha} \tag{4.11}$$

Given the conditional input demand for industry-level consumption goods by the sector-level firm (equation (4.3)) and industry j's conditional input demand by industry j for firm i's consumption good (equation (4.7)), we can write the conditional demand for firm i's good as

$$x_t^c(j,i) = \left[\frac{P_t^c(j,i)}{P_t^c(j)}\right]^{\frac{1}{\tau-1}} \left[\frac{P_t^c(j)}{P_t^c}\right]^{\frac{1}{\omega-1}} \frac{C_t}{N_t^c(j)}. \tag{4.12}$$

Thus, firm i maximizes profits

$$\Pi_t^c(j,i) = \{P_t^c(j,i)x_t^c(j,i) - w_t^c h_t^c(j,i) - r_t^c k_t^c(j,i)\} \tag{4.13}$$

by choosing its capital and labour demand as well as a price $P_t^c(j,i)$, subject to its production function (4.9).

Solving the firm-level problem, we get

$$P_t^c(j,i) = \mu_t^c(N_t^c(j))MC_t^c(j,i) = \frac{(1-\omega)N_t^c(j) - (\tau-\omega)}{\tau(1-\omega)N_t^c(j) - (\tau-\omega)}MC_t^c(j,i), \tag{4.14}$$

where $MC_t^c(j,i)$ is the marginal cost of production by firm i in sector j and $\mu_t^c(N_t^c(j))$ is the markup charged by this firm. The firm's optimal labour demand implies a wage rate in the consumption sector

$$w_t^c = \frac{P_t^c(j,i)}{\mu_t^c(j,i)} \alpha Z_t^c \left(\frac{k_t^c(j,i)}{h_t^c(j,i)} \right)^{\alpha} X_t^{z1-\alpha} \tag{4.15}$$

and a rental rate

$$r_t^c = \frac{P_t^c(j,i)}{\mu_t^c(j,i)} (1-\alpha) Z_t^c \left(\frac{k_t^c(j,i)}{h_t^c(j,i)} \right)^{\alpha-1} X_t^{z1-\alpha}. \tag{4.16}$$

The markup charged over production costs by this firm is determined by both the number of firms competing in their industry as well as the substitutability of its goods both within and across industries.

Without loss of generality, we assume that firm-level technology is identical both within and across industries in the consumption sector. This assumption implies that for every firm $i \in [0, N_t^c(j)]$ and for every industry $j \in [0,1]$, firms make identical decisions when choosing both labour and capital services ($h_t^c(j,i) = h_t^c, k_t^c(j,i) = k_t^c$). This implies the quantity of goods produced by each firm will also be the same across all firms ($x_t^c(j,i) = x_t^c$) . Furthermore, with this assumption we can generalize the price charged by firms along with the price index at both an industrial level (equation (4.8)), as well as at a sector level (equation (4.4)) implying $P_t^c(j,i) = P_t^c(j) = P_t^c$.

As mentioned earlier, the firm incurs a fixed cost of production ϕ_t^c, which we set according to the following zero-profit condition

$$\phi_t^c = x_t^c(\mu_t^c - 1) \tag{4.17}$$

along a balanced growth path (BGP). Given N_t^c firms in each industry, we can calculate the quantity of consumption goods produced C_t as

$$C_t = Q_t^c = N_t^c x_t^c = \frac{Z_t}{\mu_t^c}(k_t^c)^\alpha (X_t^z h_t^c)^{1-\alpha}. \tag{4.18}$$

With this equation along the zero profit condition outlined in equation (4.17) we can calculate the total number of firms operating within each industry as

$$N_t^c = \frac{\mu_t^c - 1}{\mu_t^c \phi^c} Z_t^c (k_t^c)^\alpha (X_t^Z h_t^c)^{1-\alpha}. \tag{4.19}$$

4.2.2 Investment Sector

Thus far we have outlined the various stages of production in the consumption good sector. The investment sector shares a similar structure to the consumption good sector, having a finite number of monopolistically competitive firms selling their differentiable products to a continuum of unit measure one industry-level firms, who in turn sell these goods to the sector-level producer. Similar to the consumption sector, we begin our description of the investment good sector by first starting at the sector-level.

Sector-Level

Sector level investment goods are produced by amalgamating a continuum of industry-level investment goods according to the following constant returns-to-scale production function

$$I_t = \left[\int_0^1 Q_t^I(j)^{\omega} \mathrm{d}j \right]^{\frac{1}{\omega}}. \tag{4.20}$$

As was the case in the consumption sector, the final good produced in the investment sector I_t is a composite good consisting of a continuum of industry-level investment goods $Q_t^I(j)$ of unit measure 1. The profit function for the investment good producer at the sector level is

$$\Pi_t^I = \left\{ P_t^I I_t - \int_0^1 P_t^I(j) Q_t^I(j) \mathrm{d}j \right\}, \tag{4.21}$$

where $P_t^I(j)$ is the price of industry j's investment good and $Q_t^I(j)$ denotes the quantity of investment goods produced in industry j. As was the case in the consumption sector, industry-level investment goods are not perfect substitutes but rather have an elasticity of substitution determined by $1/(1-\omega)$.

Industry-Level

At the industry level, the investment good sector is symmetric in construction to the consumption good sector at the same level of production. Production of the industry level composite good is given by the following constant returns-to-scale production function

$$Q_t^I(j) = \left(N_t^I(j)\right)^{1-\frac{1}{\tau}} \left[\sum_{i=1}^{N_t^I(j)} x_t^I(j,i)^{\tau} \right]^{\frac{1}{\tau}}. \tag{4.22}$$

The conditional input demand for the firm-level good $x_t^I(j,i)$ by industry j is then calculated as

$$x_t^I(j,i) = \left(\frac{P_t^I(j,i)}{P_t^I(j)} \right)^{\frac{1}{\tau-1}} \frac{Q_t^I(j)}{N_t^I} \tag{4.23}$$

with the price index $P_t^I(j)$ in industry j equal to

$$P_t^I(j) = N_t^I(j)^{\frac{1}{\tau}-1} \left[\sum_{i=1}^{N_t^I(j)} P_t^I(j,i)^{\frac{\tau}{\tau-1}} \right]^{\frac{\tau-1}{\tau}}. \tag{4.24}$$

4.2.3 Firm-Level

Production of the firm-level investment good, $x_t^I(j,i)$, follows the following production function

$$x_t^I(j,i) = Z_t A_t \left(k_t^I(j,i)^{\alpha} \left(X_t^Z X_t^A h_t^I(j,i)\right)^{1-\alpha}\right)^{\xi} - \phi^I \tag{4.25}$$

where $k_t^I(j,i)$ and $h_t^I(j,i)$ denote the capital and labour services used by firm i in industry j, ϕ^I is the fixed cost of production, and $\alpha\xi$ denotes capital share of output. As was the case in the consumption good sector, technology in the investment sector can be broken down into two separate components. There is a stationary IST shock A_t as well as the stationary *tfp* shock Z_t. There is also a non-stationary labour-augmenting technology $X_t^A(j)$ specific to the investment sector along with the neutral technology $X_t^Z(j)$. The non-stationary IST is assumed to follow a stochastic growth rate, defined as follows.

$$\mu_t^A \equiv \frac{X_t^A}{X_{t-1}^A}. \tag{4.26}$$

IST is measured as

$$IST_t = A_t \left(X_t^A\right)^{(1-\alpha)\xi} \tag{4.27}$$

With each firm i selling a differentiable good in industry j, firms compete on price, thus allowing investment firms to sell their product at a markup μ_t^I above their respective marginal cost $MC_t^I(j,i)$

$$P_t^I(j,i) = \mu_t^I(N_t^I(j))MC_t^I(j,i) = \frac{(1-\omega)N_t^I - (\tau-\omega)}{\tau(1-\omega)N_t^I - (\tau-\omega)}MC_t^I(j,i). \tag{4.28}$$

As was the case in the consumption sector, with symmetric technologies across industries, we can drop all indexes. The fixed cost of production is set equal to

$$\phi_t^I = x_t^I(\mu^I - 1). \tag{4.29}$$

This is used to remove firm profits along a BGP. With this information, we can now calculate the number of firms in the investment sector as

$$N_t^I = \left(\frac{(\mu_t^I - 1)}{\mu_t^I \phi^I} Z_t A_t\right)^{\frac{1}{\xi}} k_t^{I\alpha}\left(X_t^Z X_t^A h_t^I\right)^{1-\alpha}. \tag{4.30}$$

This implies a total output in the investment sector of

$$I_t = \frac{Z_t A_t}{\mu_t^I}\left(k_t^{I\alpha}\left(X_t^Z X_t^A h_t^I\right)^{1-\alpha}\right)^{\xi}. \tag{4.31}$$

The real wage and rental rates in the investment sector are

$$w_t^I = \frac{P^I}{\mu_t^I}\alpha Z_t A_t k_t^{I\,\alpha\xi} h_t^{I\,(1-\alpha)\xi-1} X_t^Z X_t^{A\,(1-\alpha)\xi} \tag{4.32}$$

$$r_t^I = \frac{P^I}{\mu_t^I}(\alpha\xi - 1) Z_t A_t k_t^{I\,\alpha\xi-1} (X_t^Z X_t^A h_t^I)^{(1-\alpha)\xi}. \tag{4.33}$$

With both labour and capital perfectly mobile between sectors, we have

$$w_t^I = w_t^C \quad and \quad r_t^I = r_t^C. \tag{4.34}$$

Dividing the wage rate in the investment sector by the wage rate in the consumption sector, we can estimate the rpi as

$$\frac{P_t^I}{P_t^c} = \frac{\mu_t^I}{\mu_t^C}\frac{1}{A_t}\left(\frac{k_t^C}{h_t^C}\right)^{\alpha}\left(\frac{k_t^I}{h_t^I}\right)^{-\alpha\xi} h_t^{I\,1-\xi} X_t^{Z\,(1-\alpha)(1-\xi)} X_t^{A\,(\alpha-1)\xi}. \tag{4.35}$$

4.2.4 Households

The economy consists of a large number of identical and infinitely lived households who, by choosing consumption C_t and hours worked H_t, maximize their expected lifetime utility subject to their budget constraint, with a lifetime utility of

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \tag{4.36}$$

where $0 < \beta < 1$, is the subjective discount factor. The households' periodic utility function is represented using Jamiovich and Rebelo preferences

$$U(C_t, H_t) = \frac{b_t(C_t - \chi C_{t-1} - \Gamma H_t^{\Theta} X_t)^{1-\sigma} - 1}{1-\sigma} \tag{4.37}$$

$$X_t = (C_t - \chi C_{t-1})^{\eta} X_{t-1}^{1-\eta}. \tag{4.38}$$

Here $\Gamma > 0$, $\Theta > 1$, $\sigma > 0, \chi > 0$, and $0 < \eta < 1$. Here Θ determines the level of labour supply elasticity and σ determines the curvature of household utility, χ is the habit persistence parameter, and η determines the effect wealth has on household labour supply decisions. The elements included in the periodic utility function that are distinctive to the style of preferences used by Jaimovich and Rebelo (2009) are the preference parameter η and the latent variable X_t. These preferences have become popular due to their ability to dial up or dial down the wealth effect on labour supply. When η is close to 1, we have King, Plosser, and Rebelo (1988) preferences (strong wealth effect). When η is closer to 0, we have GHH (1988) preferences, with a limited wealth effect on labour supply. Lastly, we allow for preference shocks b_t which alter the households' intertemporal consumption and labour supply decisions.

Households can accumulate capital according to the following capital accumulation equation

$$K_{t+1} = (1-\delta)K_t + v_t I_t \tag{4.39}$$

where K_t is the households' capital stock and I_t is the real quantity of investment goods purchased in period t. Lastly, we include a marginal efficiency of investment (MEI) v_t. These shocks have become popular in the literature since Justiniano et al. (2011) demonstrated that they are an important determinant of volatility in investment growth over the business cycle. The households' labour and capital services are used by both capital and consumption goods-producing firms. The household budget constraint is given by the following formula.

$$P_t^c C_t + P_t^I I_t = \frac{w_t H_t}{\mu^w} + r_t K_t^H + \Pi_t^C + \Pi_t^I + \Psi_t. \tag{4.40}$$

Given wages earned in each sector are equal (labour supply is perfectly mobile), the household earns a labour income of $w_t H_t/\mu^w$ for hours worked in each sector, where H_t denotes the number of hours supplied by households in period t, w_t denotes the wages paid, and w_t/μ^w denotes the wages earned by the household adjusted by a wage markup shock. Here I assume that the portion of wages taken from the household through the wage markup shock are rebated back to the household via a lump sum transfer Ψ_t. Households' also earns a rental income from capital services provided to both sectors $r_t K_t$. Lastly, since households own both consumption and investment-goods producing firms, any profits Π_t^C and Π_t^I are accrued to the household. Given the prices P_t^c and P_t^I for consumption and investment goods respectively, households purchase C_t consumption goods and I_t investment goods, all measured in real terms.

With households as the only source of labour in this model, the market-clearing conditions in the labour markets imply that labour supply H_t equals the sum of labour demand in both sectors. With N_t^C firms operating within the consumption sector and N_t^I firms within the investment sector, this equilibrium condition implies

$$H_t = N_t^C h_t^C + N^I h_t^I. \tag{4.41}$$

Normalizing the population of entrepreneurs to 1, the capital market clears when

$$K_t = N_t^C k_t^C + N_t^I k_t^I. \tag{4.42}$$

Last of all, with all firms within each sector identical, total amount of consumption and investment goods produced is calculated as follows

$$C_t = N_t^C x_t^C \tag{4.43}$$

and

$$I_t = N_t^I x_t^I. \tag{4.44}$$

4.2.5 Exogenous Shocks

Altogether we have seven types of shocks. There are technology shocks, which include both stationary and non-stationary *tfp* as well as stationary and non-stationary IST shocks. The non-technology shocks include wage markup, preference, and MEI, each of which is assumed to be stationary. For stationary shocks Z_t and A_t, we assume the following AR(1) processes

$$ln(Z_t) = \rho_Z ln(Z_{t-1}) + \epsilon_t^Z \tag{4.45}$$

$$ln(A_t) = \rho_A ln(A_{t-1}) + \epsilon_t^A \tag{4.46}$$

where $0 < \rho_Z < 1$, $0 < \rho_A < 1$ refers to the level of persistence for each shock, while e_t^Z and ϵ_t^A are unanticipated shock to $ln(Z_t)$, and $ln(A_t)$ respectively. The steady state values of Z_t and A_t are normalized to 1. Lastly, innovations ϵ_t^Z and ϵ_t^Z have an expected value of zero, with variance σ_t^Z and σ_t^A respectively. As for the non-stationary components for neutral and investment-specific technology, we assume each technology follows a stochastic growth rates according to the following laws of motion

$$ln(\mu_t^Z/\bar{\mu}^Z) = \rho_{\mu^Z} ln(\mu_{t-1}^Z/\bar{\mu}^Z) + \sigma^{\mu^Z} \epsilon_t^{\mu^Z} \tag{4.47}$$

$$ln(\mu_t^A/\bar{\mu}^Z) = \rho_{\mu^A} ln(\mu_{t-1}^A/\bar{\mu}^A) + \sigma^{\mu^A} \epsilon_t^{\mu^A}, \tag{4.48}$$

where the growth rates in TFP and IST are calculated as in equations (4.10) and (4.26) respectively. The persistence of each disturbance ρ_μ^z and ρ_μ^A is assumed to be between 0 and 1. The innovations in *tfp* growth $\epsilon_t^{\mu^z}$ and IST growth $\epsilon_t^{\mu^A}$ are unanticipated, with a standard deviation $\sigma_t^{\mu^Z}$ and $\sigma_t^{\mu^A}$ respectively. Lastly, $\bar{\mu^Z}$ and $\bar{\mu^A}$ denote the steady state values of μ_t^Z and μ_t^A, which are discussed in the next section.

There are three stationary non-technological shocks, including wage markup, preference, and MEI shocks, which move according to the following laws of motion, respectively.

$$ln(\frac{\mu^w}{\bar{\mu}^w}) = \rho_{\mu^w} ln(\frac{\mu^w_{t-1}}{\bar{\mu}^w}) + \sigma^{\mu^w} \epsilon_t^{\mu^w} \tag{4.49}$$

$$ln(b_t) = \rho_b ln(b_{t-1}) + \sigma_b \epsilon_t^b \tag{4.50}$$

$$ln(v_t) = \rho_v ln(v_{t-1}) + \sigma_v \epsilon_t^v. \tag{4.51}$$

Each of the persistence parameters ρ_{μ^w}, ρ_b, and ρ_v are between 0 and 1. Last of all, each innovation listed above is assumed to be i.i.d with mean 0 and variance of 1, where σ_{μ^w}, σ_b, and σ_v are the standard deviation.

With both non-stationary neutral and investment-specific technology, each variable discussed thus far must be detrended wherever a trend is present. With the trend in neutral technology denoted by X_t^Z, the trend in output X_t^Y has the following form

$$X^Y = \left(X_t^Z\right)^{\frac{1-\alpha}{1-\alpha\xi}} \left(X_t^A\right)^{\frac{(1-\alpha)\xi\alpha}{1-\alpha\xi}} \tag{4.52}$$

where consumption, nominal investment, output, and the fixed cost of production in the consumption sector all share this same trend. As for the trend of the capital stock X_t^k, the trend in the fixed cost of investment production, and the trend of real investment X_t^I, we have

$$X^I = \left(X_t^Z X_t^A\right)^{\frac{(1-\alpha)\xi}{1-\alpha\xi}}. \tag{4.53}$$

We normalize the price of consumption goods P_t^c to 1. The trend in the *rpi* is equal to

$$X_t^{PI} = \left(X_t^A\right)^{\frac{-(1-\alpha)^2\xi}{1-\alpha\xi}} \left(X_t^Z\right)^{\frac{(1-\alpha)(1-\xi)}{1-\alpha\xi}}. \tag{4.54}$$

There is no growth in hours, price markups, or the number of firms within an industry. Letting $\mu^Y \equiv X_t^Y/X_{t-1}^Y$, and $\mu^K \equiv X_t^K/X_{t-1}^K$, the system of equations for the detrended model includes

$$\tilde{Y}_t = \tilde{C}_t + \tilde{P}_t^I \tilde{I}_t \tag{4.55}$$

$$\tilde{C}_t = \frac{Z_t}{\mu^c}\left(\frac{\tilde{K}_t^c}{\mu^K}\right)^{\alpha} H_t^{C\,1-\alpha} \tag{4.56}$$

$$\tilde{I}_t = \frac{Z_t A_t}{\mu^I}\left(\frac{\tilde{K}_t^I}{N_t^I \mu^K}\right)^{\alpha}\left(\frac{H^I}{N_t^I}\right)^{1-\alpha} \tag{4.57}$$

$$\tilde{K}_{t+1} = (1-\delta)\frac{\tilde{K}_t}{\mu^K} + v_t \tilde{I}_t \tag{4.58}$$

$$N_t^C = \left(\frac{\mu_t^C - 1}{\mu_t^C \tilde{\phi}^C}\right) Z_t \left(\frac{\tilde{K_t^C}}{\mu_t^Y}\right)^{\alpha} H_t^{C\,1-\alpha} \tag{4.59}$$

$$N_t^I = \left(\frac{\mu_t^I - 1}{\mu_t^I \tilde{\phi}^I} Z_t A_t\right)^{\frac{1}{\xi}} \left(\frac{\tilde{K^I}_t}{\mu_t^K}\right)^{\alpha} H_t^{I\,1-\alpha} \tag{4.60}$$

$$\tilde{\lambda}_t \tilde{P}_t^I = E_t\left\{\frac{\tilde{\lambda}_{t+1} \beta \mu_{t+1}^{Y\;1-\sigma}}{\mu_{t+1}^K}\left(\tilde{r}_{t+1} + \tilde{P_{t+1}^I}(1-\delta)\right)\right\} \tag{4.61}$$

$$\tilde{w}_t = \frac{\tilde{P_t^I}}{\mu_t^I}(1-\alpha)\xi Z_t A_t \left(\frac{\tilde{K_t^I}}{N_t^I \mu_t^K}\right)^{\alpha\xi} \left(\frac{H_t^I}{N_t^I}\right)^{(1-\alpha)\xi-1} \tag{4.62}$$

$$\tilde{w}_t = \frac{1}{\mu_t^C}(1-\alpha) Z_t \left(\frac{\tilde{K_t^C}}{\mu_t^K H_t^C}\right)^{\alpha} \tag{4.63}$$

$$\tilde{r}_t = \frac{\tilde{P_t^I}}{\mu_t^I} \alpha Z_t A_t \left(\frac{\tilde{K^I}_t}{N_t^I \mu_t^K}\right)^{\alpha\xi-1} \left(\frac{H_t^I}{N_t^I}\right)^{(1-\alpha)\xi} \tag{4.64}$$

$$\tilde{r}_t = \frac{1}{\mu_t^C} \alpha Z_t \left(\frac{\tilde{K_t^C}}{\mu_t^K H_t^C}\right)^{\alpha-1} \tag{4.65}$$

$$b_t \Theta \Gamma H_t^{\Theta-1} \tilde{X}_t \left(C_t - \chi \frac{\tilde{C_{t-1}}}{\mu^Y} - \Gamma H_t^{\Theta} \tilde{X}_t\right)^{-\sigma} = \frac{\tilde{\lambda}_t \tilde{w}_t}{\mu^w} \tag{4.66}$$

$$\begin{aligned}
\tilde{\lambda_t} \;=\; & b_t(\tilde{C}_t - \frac{\chi}{\mu_t^y}\tilde{C_{t-1}} - \Gamma H_t^{\theta} \tilde{X}_t)^{-\sigma} - E_0 b_{t+1} \mu_{t+1}^{y\;-\sigma} \beta \chi (\tilde{C_{t+1}} - \frac{\chi}{\mu_{t+1}^y}\tilde{C}_t - \Gamma H_{t+1}^{\theta} \tilde{X_{t+1}})^{-\sigma} \ldots \\
& - \tilde{\lambda_{2t}} \eta \mu_t^{y\eta-1} (\tilde{C}_t - \frac{\chi}{\mu_t^y}\tilde{C_{t-1}})^{\eta-1} \tilde{X_{t-1}}^{1-\eta} \ldots \\
& + E_0 \mu_{t+1}^{y\;1-\sigma} \beta \tilde{\lambda_{2t+1}} \eta \mu_{t+1}^{y\;\eta-1} \frac{\chi}{\mu_{t+1}^y} (\tilde{C_{t+1}} - \frac{\chi}{\mu_{t+1}^y}\tilde{C}_t)^{\eta-1} \tilde{X}_t^{1-\eta}
\end{aligned} \tag{4.67}$$

$$b_t \Gamma H_t^{\theta} (\tilde{C}_t - \frac{\chi}{\mu_t^y}\tilde{C_{t-1}} - \Gamma H_t^{\theta} \tilde{X}_t)^{-\sigma} = \tilde{\lambda_{2t}} - \beta E_0 \mu_{t+1}^{y\;1-\sigma} \tilde{\lambda_{2t+1}} (1-\eta) \mu_{t+1}^{y\;\eta-1} (\tilde{C_{t+1}} - \frac{\chi}{\mu_{t+1}^y}\tilde{C}_t)^{\eta} \tilde{X}_t^{-\eta} \tag{4.68}$$

$$\tilde{X}_t = (\tilde{C}_t - \frac{\chi}{\mu_t^y}\tilde{C_{t-1}})^{\eta}(\tilde{X_{t-1}}^{1-\eta})(\mu_t^y)^{\eta-1} \tag{4.69}$$

$$\mu_t^I = \frac{(1-\omega^I)N_t^I - (\tau^I - \omega^I)}{\tau^I(1-\omega^I)N_t^I - (\tau^I - \omega^I)} \tag{4.70}$$

$$\mu_t^C = \frac{(1-\omega^C)N_t^C - (\tau^C - \omega^C)}{\tau^c(1-\omega^C)N_t^C - (\tau^C - \omega^C)} \tag{4.71}$$

$$H = H_t^C + H_t^I \tag{4.72}$$

$$ln(Z_t) = \rho_Z ln(Z_{t-1}) + \sigma^Z \epsilon_t^Z \tag{4.73}$$

$$ln(A_t) = \rho_A ln(A_{t-1}) + \sigma^A \epsilon_t^A \tag{4.74}$$

$$ln(v_t) = \rho_v ln(v_{t-1}) + \sigma_v \epsilon_t^v \tag{4.75}$$

$$ln(b_t) = \rho_b ln(b_{t-1}) + \sigma_b \epsilon_t^b \tag{4.76}$$

$$ln(\frac{\mu^w}{\bar{\mu}^w}) = \rho_{\mu^w} ln(\frac{\mu_{t-1}^w}{\bar{\mu}^w}) + \sigma^{\mu^w} \epsilon_t^{\mu^w} \tag{4.77}$$

$$ln(\mu_t^Z/\bar{\mu}^Z) = \rho_{\mu^Z} ln(\mu_{t-1}^Z/\bar{\mu}^Z) + \sigma^{\mu^Z} \epsilon_t^{\mu^Z} \tag{4.78}$$

$$ln(\mu_t^A/\bar{\mu}^A) = \rho_{\mu^A} ln(\mu_{t-1}^A/\bar{\mu}^A) + \sigma^{\mu^A} \epsilon_t^{\mu^A} \tag{4.79}$$

where λ and λ_2 are Lagrangian multipliers.

4.3 Model Estimation

We use a Bayesian estimation process to determine the value of the majority of the parameters included in benchmark model, while calibrating some of the more well-known parameters directly. This method is now widely used in the business cycle literature due to its ability to take the best aspects of maximum likelihood estimation and direct calibration. The Bayesian estimation process involves three components, which include a list of observables, the model, and a set of priors. The priors are chosen based on either micro-level data and/or economic theory which assigns a higher weight to a given area of the parameter subspace. It is with these priors that the Bayesian estimation can be understood as bridging both maximum likelihood and direct calibration. As the proportion of the parameter subspace included within the prior distribution decreases, the Bayesian estimation becomes akin to direct calibration. Conversely, as the given area of the parameter subspace increases to infinity, the Bayesian estimation will be where the log-likelihood function peaks, thus maximum likelihood. For the more frequently estimated parameters, we choose priors that match those used in the literature. To facilitate our Bayesian estimation, we will be using DYNARE. For readers who are interested in a more in-depth discussion into the mechanisms involved in the Bayesian estimation process, we recommend An and Schorfheide (2007).

The list of observables included in our Bayesian estimation process includes log differences in output, investment, consumption, hours worked, and the *rpi*, all measured in percentage terms. Letting Υ_t denote the vector of observables, we have

$$\Upsilon_t = \begin{bmatrix} \Delta ln(Y_t) \\ \Delta ln(C_t) \\ \Delta ln(I_t) \\ \Delta ln(H_t) \\ \Delta ln(RPI_t) \end{bmatrix} \times 100 + \begin{bmatrix} \epsilon^{ME}_{Y,t} \\ \epsilon^{ME}_{C,t} \\ \epsilon^{ME}_{I,t} \\ \epsilon^{ME}_{H,t} \\ \epsilon^{ME}_{RPI,t} \end{bmatrix} \tag{4.80}$$

where measurement errors are included for all observables, following Ireland (2004).

Thus far, for notational simplicity we have assumed that the elasticities of substitution between firm-level and industry-level goods were identical across sectors. However, this assumption could be potentially restrictive, hence from this point on we assume that each sector differs in its elasticity of substitution, both between industry, and firm-level goods. Thus, τ_c and τ_i govern the elasticity of substitution between firm-level goods in the consumption and investment sectors, respectively. Likewise, ω_c and ω_i govern the elasticity of substitution between industry-level consumption goods and industry-level investment goods. As done by Floetotto et al. (2009), we assume that the elasticity of substitution in both sectors must be greater at a firm-level than at the industry level ($\frac{1}{1-\omega^C} < \frac{1}{1-\tau^C}$ and $\frac{1}{1-\omega^I} < \frac{1}{1-\tau^I}$). As pointed out by Floetotto et al. (2009), there is no clear estimate for these elasticities in the literature. The value assigned to these elasticities depends heavily on the markup charged above marginal costs within each industry along with the number of firms which either enter or exit each industry. Combining equations (4.43) and (4.44), the zero-profit conditions for each sector described in equations (4.17) and (4.29) and equations (4.14) and (4.28), we can calculate the percentage change in markup charged in both sectors, denoted by $\hat{\mu_t^C}$ and $\hat{\mu_t^I}$ respectively, as follows

$$\hat{\mu_t^C} = \frac{(1 - \tau^C \bar{\mu_t^C})}{\tau^C \bar{\mu_t^C}} \hat{C}_t \tag{4.81}$$

$$\hat{\mu_t^I} = \frac{(1 - \tau^I \bar{\mu_t^I})}{\tau^I \bar{\mu_t^I}} \hat{I}_t. \tag{4.82}$$

Log linearizing equations (4.14) and (4.28), we can calculate the percentage change in markup charged by firms as a function of the number of firms competing within each industry

$$\hat{\mu_t^C} = \frac{\tau^C(\bar{\mu^C} - 1)(\bar{\mu^C}\tau^C - 1)}{\bar{\mu^C}\tau^C(\tau^C - 1)} \hat{N_t^C} \tag{4.83}$$

$$\hat{\mu_t^I} = \frac{\tau^I(\bar{\mu^I} - 1)(\bar{\mu^I}\tau^{-}1)}{\bar{\mu^I}\tau^C(\tau^I - 1)} \hat{N_t^I}. \tag{4.84}$$

Combining equations (4.81) with (4.83) and (4.82) with (4.84), we can then estimate the values τ^c and τ^i with data on the number of firms within each sector N_t^I and N_t^C as well as data on both consumption and investment. To calculate the number of firms operating within each sector, we (1) estimate the number of firms operating within each of the non-agriculture SIC supersectors, (2) scale each sector by its average contribution to total payroll and then (3) subdividing each sector by its contributions to either consumption or investment production by using data from the input-output use tables available through the Bureau of Economic Analysis.[3] A detailed list of the data used and the steps involved in estimating the number of firms competing within each sector appears in the appendix. With data on the number of firms competing within each sector $\hat{N^I}$ and $\hat{N^C}$ from 1997 to 2012 in the US accompanied with data on aggregate consumption and investment, we can estimate the value of τ^C and τ^I.

Floetotto and Jamiovich (2008) estimate the firm-level markup μ in their one-sector model as low as 1.05 using value-added data and as high as 1.4 using data they collected on gross output. Given this range, we set the steady state markups $\bar{\mu^C}$ and $\bar{\mu^I}$ equal to 1.3 as done by Floetotto et al. (2009). With these values for $\bar{\mu^C}$ and $\bar{\mu^I}$, we regress $\hat{N_t^C}$ with $\hat{C_t}$ and $\hat{N_t^I}$ with $\hat{I_t}$ as suggested above and use the coefficient estimates to estimate the value for τ^c and τ^i as listed in Table 4.1. Given this information, a normal prior distribution is chosen for τ^C and τ^I with a mean and standard deviation equal to their estimated value and standard error estimated

[3]The method we use to estimate the number of firms operating within each sector is the same approach used by Floetotto et al. (2009).

in Table 4.1. Governed by the assumption that the elasticity of substitution between firm-level goods is greater then the elasticity of substitution across industries, ω^I and ω^C are set equal to 0.6. The value of these parameters do not impact our results.

Moving on to the preference parameters, we assume a Gamma distribution with mean 3 and variance 0.75 for θ, which determines the elasticity of labour supply. The habit persistence parameter χ is assigned a Beta distribution with mean of 0.5 and variance equal to 0.1. As for η, which determines the wealth effect on labour supply, we assign a uniform distribution between 0 and 1. Lastly, since the steady state hours are left to be estimated in our Bayesian estimation, they are assigned a normal distribution around a mean of 0.3 with a standard deviation of 0.03.

Through the parameter ξ, the returns-to-scale in the investment sector could differ from a constant returns-to-scale assumed in the consumption sector. A prior is chosen with a mean of 1, which reflects the standard convention of constant returns-to-scale but allows for the data to choose a value for ξ other than 1 by assuming a normal distribution with a standard deviation equal to 0.1. For observable $i \in \{Y, I, C, H, RPI\}$, the measurement error ϵ_{it}^{ME} has a mean of zero and standard deviation σ_i^{ME} governed by a uniform prior distribution bound between 0 and one quarter of the standard deviation of the observable. The remaining parameters to be estimated include the persistence and variance for the seven shocks discussed in the previous section. The priors chosen for these parameters along with all other priors used in the Bayesian estimation are available in Table 4.2.

As outlined in section 4.2, the growth rate of the *rpi* is equal to

$$\mu_t^{RPI} = \left(\mu_t^A\right)^{\frac{-(1-\alpha)^2\xi}{1-\alpha\xi}} \left(\mu_t^Z\right)^{\frac{(1-\alpha)(1-\xi)}{1-\alpha\xi}}, \tag{4.85}$$

while the growth rate of output is equal to

$$\mu^Y = \left(\mu_t^Z\right)^{\frac{1-\alpha}{1-\alpha\xi}} \left(\mu_t^A\right)^{\frac{(1-\alpha)\xi\alpha}{1-\alpha\xi}}. \tag{4.86}$$

The steady state growth rate of the *rpi* is calculated using the time series for the quarterly *rpi* adjusted for changes in quality from 1948:1 to 2006:3 mentioned in the empirical section of this paper. Using this time series the estimated growth rate of the *rpi* equals 0.9957. As for the gross growth rate of output, we calculate the steady state quarterly growth rate of output μ^Y using seasonally adjusted non-farm output from 1949:1 to 2006:3 available through the Bureau of Labour Statistics. With this data, we estimate an average quarterly growth rate of output equal to 1.0049. With these two growth rates at hand, we choose a growth rate for non-stationary neutral and investment-specific technology that matches the growth rates of both output and the *rpi*.

The parameters that have yet to be discussed are those directly calibrated. The preference parameter σ, governing the households risk aversion is set equal to 1 which implies logarithmic preferences. The households' quarterly discount parameter β is set equal to 0.99. The Cobb-Douglas parameter α, is set equal to 0.31. The quarterly depreciation rate δ is set equal to 0.02. All calibrated parameters are shown in Table 4.3.

Given the benchmark model M outlined in section 4.2, the set of observables Υ_t, and a vector of parameters, Θ_M, we can begin our Bayesian estimation process. Using these components, along with the likelihood function $L(\Theta_M, \Upsilon_t, M)$ calculated as

$$L(\Theta_M|\Upsilon_T, M) = p(\upsilon_0|\Theta_M, M) \prod_{t=1}^{T} p(\upsilon_t|\Upsilon_{T-1}, \Theta_M, M) \tag{4.87}$$

and with a Kalman filter to calculate the unknown likelihood function along with Metropolis-Hastings algorithm, which generates a random sample of these estimates through a Monte Carlo Markov Chain, we calculate the posterior density $P(\Theta_M|\Upsilon_t, M)$. The results of our Bayesian estimation are available in Table 4.4.

4.4 Model Results

As the benchmark model of this paper establishes, the cyclical nature of the *rpi* can be reproduced by allowing it to respond to changes in the relative demand of investment goods to consumption goods, in addition to changes in technology. This method is referred to as the *endogenous approach* since the *rpi* is treated as an endogenous variable. A second approach could alternatively have movements in the *rpi* modeled exogenously by assuming technologies across sectors move together over time, rather than endogenously. This method will be referred to as the *exogenous approach* as the *rpi* is determined completely by changes in either neutral and investment-specific technology. Both approaches are valid choices and are evaluated in this paper to assess whether one approach outperforms the other. Contrasting these two methods will determine whether future research should model the countercyclical pattern observed in the *rpi* endogenously, or exogenously. The following section presents a model where cyclical movements in the *rpi* are entirely exogenous.

4.4.1 Two-Sector Model with Cointegrated TFP and IST

As outlined in section 4.2, the benchmark model assumes that the *rpi* moves in response to changes in *tfp* and the non-technological disturbances (wage markup, preference, and MEI) through the inclusion of endogenous price markups and IRS/DRS in the production of investment goods. Rather than have the *rpi* move endogenously, one might be interested in modeling the relationship between *tfp* and the *rpi* exogenously. As demonstrated by Schmitt-Grohe and Uribe (2011), *tfp* and *rpi* in postwar US are best characterized by a cointegrating relationship in postwar US. With both *tfp* and *rpi* cointegrated, then any deviation from the equilibrium long-run relationship between *tfp* and the *rpi* by one of either technologies will generate a counteracting response in the other technology so as to maintain the long-run relationship between these two time series. Furthermore, Schmitt-Grohe and Uribe (2011), Wagner (2013) have shown that cointegration impacts the relative importance of technology shocks when analyzing the variance decomposition. Given our attempt to replicate the true data-generating process governing the co-movement of *tfp* and the *rpi*, along with the research listed above, then it seems natural to allow *tfp* and the *rpi* to follow a common stochastic trend in our assessment. To clarify, this model does not include IRS/DRS in the investment sector by fixing $\xi = 1$ nor does it allow for movements of firms in and out of each sector, thus cutting off any endogenous movement in the price markups in each sector. Including cointegration between these sectors can be achieved by updating equations (4.47) and (4.48) to the following

$$\begin{bmatrix} ln(\mu_t^Z/\bar{\mu}^Z) \\ ln(\mu_t^A/\bar{\mu}^A) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} ln(\mu_{t-1}^Z/\bar{\mu}^Z) \\ ln(\mu_{t-1}^A/\bar{\mu}^A) \end{bmatrix} + \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} x_{t-1}^{co} + \begin{bmatrix} \epsilon_t^{\mu^Z} \\ \epsilon_t^{\mu^A} \end{bmatrix}, \quad (4.88)$$

where, as before, μ_t^Z and μ_t^A are the growth rates of the non-stationary neutral technology in the consumption and investment sectors respectively and x_t^{co} is the cointegrating term which equals

$$x_t^{co} = \nu ln(X_t^Z) - ln(X_t^A), \quad (4.89)$$

where ν is calibrated such that x_t^{co} equals zero in steady state. As before, ρ_{μ^Z} and ρ_{μ^A} determine the level of persistence while κ_1 and κ_2 determine the impact changes in the common trend have on μ^Z and μ^A respectively; these are the cointegration coefficients. As before, $\epsilon_t^{mu^Z}$ and $\epsilon_t^{\mu^A}$ are unanticipated shocks to μ_t^Z and μ_t^A respectively. In addition, equations (4.45) and (4.46) are replaced by

$$\begin{bmatrix} ln(Z_t) \\ ln(A_t) \end{bmatrix} = \begin{bmatrix} \rho_Z \\ \rho_A \end{bmatrix} \begin{bmatrix} ln(Z_{t-1}) \\ ln(A_{t-1}) \end{bmatrix} + \begin{bmatrix} s_Z & s_{Z,A} \\ s_{A,Z} & s_A \end{bmatrix} \begin{bmatrix} \epsilon_t^Z \\ \epsilon_t^A \end{bmatrix}, \tag{4.90}$$

where, as before, ρ_Z and ρ_A determine the level of persistence while $s_{Z,A}$ and $s_{A,Z}$ determine the potential spillover between innovations for these two technologies.

This model develops from the one-sector dynamic stochastic general equilibrium model studied by Schmitt-Grohe and Uribe (2011). Estimating this model involves a different set of parameters than that included in the benchmark model. For those parameters that are shared between this model and the benchmark, we assume the same prior distribution. For the cointegration coefficients κ_1 and κ_2, we assume a prior with a mean of zero, with lower and upper bounds of -0.4 and 0.4 for both κ_1 and κ_2. The correlation between innovations in neutral and investment-specific technology is given a normal prior distribution with mean of -0.13 and a variance of 0.1, allowing some flexibility in the estimate. The results of the Bayesian estimation are available in Table 4.8.

This model when estimated captures the cyclical nature of the *rpi* along with the long-run trend in the *rpi* exogenously. It therefore provides some measure of the effectiveness of the endogenous approach to modeling movement in the *rpi* as is done the benchmark. The results of the variance decomposition in a two-sector model with co-movement is available in Table 4.7. Of notable interest is the high weight assigned to *tfp* in generating volatility in the *rpi*. What can we learn from this experiment? First of all, the assumption that IST can be identified by the inverse of the *rpi* is unreliable at best. With 39 percent of the *rpi* explained by neutral technology shocks, the classical assumption that neutral technology shocks which impact productivity across all sectors do not affect relative prices is invalid.[4]

When looking closer at the variance decomposition of the benchmark model (shown in Table 4.6) another interesting conclusion can be drawn. When compared to the variance decomposition for the model which takes the exogenous based approach (Table 4.7), wage markup shocks, rather than stationary and non-stationary *tfp* shocks become the dominant source of volatility in output, investment and hours growth. This results suggests that how we generate co-movement between *tfp* and *rpi* has important implications for how we interpret the relative importance of one shock over another. Furthermore, nearly 48 percent of the *rpi* is determined by non-IST shocks in the endogenous based approach (See Table 4.5). Approximately 32 percent of that value can be attributed to changes in *tfp*, with the remaining explained by movements

[4]This assumption was first proposed by GHH (1988) and later adopted by Fisher (2006), Beaudry and Lucke (2009), as well as Justiniano et al. (2009), to name a few.

in wage markup shocks, preference shocks, and shocks to the MEI. These results suggest that movement in the *rpi* is not merely a technological phenomenon, but rather is in part determined by changes in aggregate consumption and investment. With 48 percent of the *rpi* determined by shocks that are not investment-specific, these findings approach those found by Kim (2009), who finds in his SVAR approach that only 27 percent of the *rpi* can be explained by IST.

4.4.2 The Relative Price of Investment

Along with the exogenous approach mentioned above, we simulate three other alternate versions of our benchmark model with various elements removed to highlight the relative importance of each component in explaining volatility in the *rpi*. As was demonstrated in section 4.2, the *rpi* is calculated as

$$\frac{P_t^I}{P_t^c} = \frac{\mu_t^I}{\mu_t^C}\frac{1}{A_t\xi}\left(\frac{k_t^C}{h_t^C}\right)^{\alpha}\left(\frac{k_t^I}{h_t^I}\right)^{-\alpha\xi}\left(\frac{h_t^I}{N_t^I}\right)^{1-\xi} X_t^{Z(1-\alpha)(1-\xi)} X_t^{A(\alpha-1)\xi}. \tag{4.91}$$

Linearizing equation (4.91) we get

$$\hat{P}_t^I = \left(\hat{\mu_t^I} - A\hat{\mu_t^C}\right) - \hat{A}_t + (1-\alpha)\xi\hat{X_t^A} + \alpha(1-\xi)\left(\frac{\hat{K_t}}{H_t}\right) + (1-\xi)\hat{H}_t^I - (1-\xi)\hat{N}_t^I + (1-\alpha)(1-\xi)\hat{X_t^Z}. \tag{4.92}$$

Note that the *rpi* can be broken down into several separate components, including stationary IST, the difference in price markups between sectors, the trend in the IST, and lastly the capital labour ratio reflecting the increasing returns-to-scale in the investment sector. In total, approximately 52 percent of the *rpi* variance is explained by movements in IST. This measures against the 100 percent used in most of the literature, with the remaining proportion split between endogenous movements in price markups, the number of firms operating within the investment sector, the capital labour ratio, and lastly *tfp*.

To understand the contribution of both endogenous price markups and IRS/DRS in generating movement in the *rpi*, we isolate each component and assess its impact on *rpi* volatility. To isolate the effect of markups on *rpi* volatility, we set $\xi = 1$ so as to imply constant returns-to-scale in the investment sector. As can be seen in Table 4.5, when the benchmark model is simulated with IRS/DRS removed, we find that roughly 46 percent of the *rpi* can be explained by non-IST shocks. Thus, it appears that endogenous price markups are important in facilitating endogenous movement in the *rpi*.

To identify the proportion of *rpi* variability explained by the inclusion of ξ in our benchmark model, we remove endogenous movement in price markups by restricting movements of firms in and out of each sector, thus pinning down the price markup charged in both sectors to $1/\tau^C$ in the consumption sector and $1/\tau^I$ in the investment sector.[5] When the benchmark model is simulated with endogenous price markups removed from the model, we find that the proportion of *rpi* volatility explained by non-IST drops from 48 percent to roughly 4 percent, suggesting that the difference can be attributed to the presence of endogenous price markups.

What do we learn from this experiment? Given the results of our various variance decomposition, endogenous price markups are vital in explaining endogenous movement in the *rpi*. Of particular interest is the ability of endogenous price markups to translate the non-technological shocks into movement in the *rpi* in the benchmark model. In this model, preference shocks, wage markup shocks, and MEI shocks combined explain 16 percent of *rpi* from one period to the next. This suggests the overall impact of demand shocks occur through changes in the price markups over the business cycle.

Figure 4.3 plots the impulse responses of output, investment, consumption, hours worked, and the *rpi* to both a one-standard-error innovation to ϵ_t^z and a one-standard-

[5]To accomplish this, we remove both the number of firms in both sectors (N_t^C and N_t^I) as well as markups μ^C and μ^I as endogenous variables while removing equations governing the number of firms (4.59) and (4.60) as well as the markup equations (4.70) and (4.71) from the system of equations listed at the end of section 4.2. Thus, each markup is set equal to its steady state value and will not fluctuate over the business cycle.

error innovation to $\epsilon_t^{\mu^Z}$. As Figure 4.3 shows, a positive shock to stationary *tfp* generates an expansion in output, hours, and investment along with a decline in the *rpi*. The immediate increase in production along with the decline in the *rpi* generates a higher-than-normal response in investment, leading to a reduction in household consumption as they accommodate the increase in investment. A non-stationary *tfp* shock generates a similar response in output, hours, and investment, with a much more muted response in the *rpi*. The *rpi* declines by less as households respond to a permanent increase in productivity by increasing consumption immediately due to the permanent income hypothesis.

The decline in the *rpi* occurs for the following reasons. First, as seen in equations (4.30) and (4.19), there is an increase in the number of firms operating within each industry as the direct effect of increased productivity would cause firms to enter the market. With $\xi \neq 1$, the number of firms entering into the investment sector N_t^I outpaced the number of firms entering into the consumption sector, which, given (4.28) and (4.14) causes the markup charged in the investment sector to decrease by more than in the consumption sector. Indirectly, with an increase in *tfp*, the demand for investment increases, implying both an increase in the number of firms within the investment sector N_t^I as well as a decline in the price of investment goods P_t^I due to the drop in the price markup μ_t^I that results from increased competition in this sector. Declining demand for consumption goods leads to a decline in the number of firms competing within this sector, counteracting the initial increase in the number of firms operating due to increased productivity. With the decline in price of investment goods, the net response to a stationary *tfp* shock is a decline in the *rpi*. Thus far, we have discussed the implications of a temporary shock to *tfp*. In response to a permanent increase in *tfp*, there is an increase in both consumption (permanent income hypothesis) and investment, and consequentially shocks to the growth rate attribute less to the overall variance decomposition of the growth rate of the *rpi*. However, with $\xi > 1$, we can contribute a fraction of the decline in the *rpi* over the past 60 years in the US to the increase in overall neutral technology over the same period. Since we include multiple stochastic trends, the deviation of the variable from its BGP includes both the variation of the variable from its respective BGP, along with the variation of the stochastic trend itself.

Figure 4.4 plots the impulse responses of output, investment, consumption, hours worked, and the *rpi* to both a one standard-error-innovation to ϵ_t^A and a one-standard-error innovation to $\epsilon_t^{\mu^A}$. As can be seen in Figure 4.4, a positive shock to stationary IST generates an expansion in output, hours, and investment along with a decline in the *rpi*. The decline in the *rpi* in response to shock to stationary IST occurs through the following channels. With an increase in IST, the profitability of production in the investment sector causes firms to enter into the investment sector and therefore drives down the markup charged on investment goods. This is the direct effect on the number of firms operating in the investment sector. The second effect comes from households switching from consumption goods to the now relatively cheap investment good, driving up the number of firms entering into the market and drives down the markup charged in this sector. With a decline in consumption, firms exit the consumption sector and we observe an increase in the markup charged by the remaining firms. The overall effect is for the *rpi* to decrease by more than if markups were constant. The same logic holds true for a permanent shift in IST, with an increase rather than a decrease in consumption as households' lifetime permanent disposable income increases. The effect of increased consumption with a permanent shift in IST is a gradual decline in the *rpi* rather than an immediate decline, as is observed in response to a temporary IST shock.

There are three non-technological disturbances included in the benchmark model. These are preference shocks, wage markup shocks, and MEI shocks. Figure 4.5 plots the impulse responses of output, investment, consumption, hours worked, and the *rpi* to the standard error innovation to each of the three non-technological disturbances.

The increase in the *rpi* that occurs in response to shocks to either preferences or wage markups occurs through the following channels. With a positive preference shock, households increase their demand for consumption goods over investment goods, thus driving down the markup charged on consumption goods and driving up the markup on investment goods and hence an increase in *rpi*. With wage markup shocks, both consumption and investment fall in response to a drop in household income. However, due to consumption smoothing, the response in consumption demand and consequently the markup charged are an order of magnitude smaller than the responses in investment. Through the steps listed above, this results in a rise in the *rpi*. In response to an MEI shock, the decline in the *rpi* is due to an increase in investment demand and a decrease in demand for consumption goods. Investment demand increases as households realize that a given amount of savings can be converted into a greater amount of investment goods. This leads households to reduce consumption to free up resources for further investment. Thus, MEI shocks have the exact opposite impact on the *rpi* when compared to preference and wage markup shocks.

Figure 4.6 simulates the impulse response functions of the *rpi* in the benchmark model in response to each type of shock and compares these to simulations with various components removed, according to the processes listed earlier. In each alternative case, the parameter estimates used are those of the benchmark model outlined in Table 4.4[6]

The impact MEI shocks have on the *rpi* via shifts in the relative demand for investment goods over consumption goods is of particular interest. As mentioned in the introduction, the traditional assumption in the business cycle literature has been to assume a one-for-one transformation in the conversion of consumption goods

[6]For no IRS/DRS in the investment sector, ξ is set equal to 1. For the model with no movement in markups, equations (4.59), (4.60), (4.70), and (4.71) are removed from the system of equations listed at the end of section 4.2. Last of all, for the model with either component removed, the IRF of the *rpi* equals the inverse of IST.

into investment goods. However, as Justiniano et al. (2011) argued, a more realistic version of this transformation would involve two steps, the first being a transformation of consumption goods into investment goods, which is altered by shifts in IST. The second transformation involves taking capital goods fresh off the production line and converting these goods into active capital. Shocks to this mechanism are referred to as changes in the MEI. Both of these steps are included in the benchmark model. Justiniano et al. (2011), however, assume that the IST can be identified by the inverse of the *rpi* while changes in the MEI are driven by changes in the firms' ability to access credit. They make this assertion by linking movements in the MEI in their model to the spread between high yield and AAA corporate bonds (a measure of risk premium). Incidentally, they assume that changes in the firms' ability to access credit does not impact the *rpi*. However, as can be seen in Figure 4.6, shocks to the MEI do impact the *rpi* as expected. Following Justiniano et al.'s (2011) interpretation of MEI shocks, a positive shock to MEI increases the accessibility of credit, leading to an increase in investment sales and a decline in the *rpi*. While this challenges the framework of Justiniano et al. (2011), only 6% of the growth rate of the *rpi* can be attributed to MEI shocks.

Thus far no sensitivity analysis has been done on the given parameter choices listed in Table 4.3. For example, the parameter value for the steady state markups for consumption and investment ($\bar{\mu}^I$ and $\bar{\mu}^C$ respectively) are set to 1.3, matching those used by Floetotto and Jaimovich (2008). In their research, which parallels the work done here, they explore the ability of endogenous price markups to explain movement in neutral technology. Their sensitivity analysis shows that when these values are increased, the proportion of neutral technology explained endogenous increase as well. Given the similarity in market structure, it is likely that there exists a postiive relationship between steady state markups, and the proportion of the *rpi* explained endogneously. Sensitivity analysis of these results to changes in the steady state markups, or any of remaining calibrated parameters is left to future research.

4.5 Conclusion

Since the seminal work of GHH (1988), IST has become a common feature in most of the business cycle literature. Likewise, the convenient assumption assumed by GHH (1988) that IST can be identified by the inverse of the *rpi* has also remained the same. Assuming that the *rpi* is orthogonal to the business cycle eliminates any possibility that the *rpi* moves in response to changes in the relative demand for investment goods over consumption goods. With 48 percent of the *rpi* determined by non-IST shocks via the endogenous price mechanisms listed above, our model approaches those results found by Kim (2009), who finds that IST in the US explains at most 27 percent of the volatility of the *rpi* in the SVAR estimation, with non-technological disturbances having significant explanatory power. As the benchmark model demonstrates, when the *rpi* moves in part due to changes in aggregate demand via endogenous price markups, IST accounts for less than half of the volatility in the *rpi*. Furthermore, non-technological shocks, such as preference, wage markup, and MEI shocks have an important source of business cycle volatility through their effect on aggregate demand. With an estimate for the IRS/DRS parameter $\xi = 1.042$, our benchmark results can in part explain the downward trend in the *rpi* observed from 1949-2006. However, inclusion of this parameter did not significantly contribute to endogenous movement in the *rpi*. Lastly, the sizable fraction of the *rpi* explained by non-IST warrants serious skepticism regarding the interpretation of business cycle research where the *rpi* is modeled exogenously. Such results may be misleading since the *rpi* may not move in tandem with the business cycle. Given these results, future business cycle research tackling questions regarding the relative importance of IST requires the incorporation of a mechanism to generate endogenous movements in the *rpi* to changes in the relative demand for consumption goods to investment goods.

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4.6 Appendix

4.6.1 Data Used

1. Output Y_t Non-Farm Gross Value-Added NIPA Table 1.3.5 Row 3 1947:Q1 - 2013:Q4 Chained 2009 Dollars Seasonally Adjusted at Annual Rates

2. Consumption C_t Real Personal Consumption Expenditure BEA 1947:Q1 -2013:Q4 Chained 2009 Dollars PCECC96 Seasonally Adjusted at Annual rates

3. Investment I_t Real Investment Expenditure BEA 1947:Q1 -2013:Q4 Chained 2009 Dollars GPDIC96 Seasonally Adjusted at Annual rates

4. Private Non-Farm Hours Worked Major Sector Multisector Productivity Index Base Year 2009 1947:Q1 -2013:Q4 BLS PRS85006033 Seasonally Adjusted at Annual Rates

5. Private Non-Farm Business Sector Capital Services Index 100 2009 BLS MPU4910042 1987-2012 BLS PRS85006033 Annual data on capital services are converted into quarterly data assuming a constant growth rate between quarters.

6. Annual Payroll Information Small Business Administration Data for 20 industry groups. Used to weigh the relevance of each sector.

7. Input-Output Use Table BEA Before Redefinitions 1997-2012. Used to determine the proportion of goods from each industry going to investment projects

and consumption goods production.

8. Number of firms within each SIC Supersector via the BLS data by adding Expansions (firms that hired), Contractions (firms that laid off employees) plus Openings (new startups) less Closures (firms that closed).

9. Real Consumption Expenditure Non-Durables and Services NIPA Table 1.1.6 Real Gross Domestic Product Chained 2009 Dollars Seasonally Adjusted at Annual Rates

10. Real Investment Goods Expenditure on Equipment and Consumer Durables NIPA Table 1.1.6 Real Gross Domestic Product Chained 2009 Dollars Seasonally Adjusted at Annual Rates

11. Calculate the number of firms within each industry N_t^I and N_t^C by multiplying the number of firms within each industry (8) by their contribution to total payroll (6) then subdividing each sector by its contributions to either consumption or investment production.

12. Calculate the elasticities τ^I and τ^C through the following manipulations:

    (a) Calculate $\hat{I}_t$, $\hat{C}_t$, $\hat{N}_t^C$ and $\hat{N}_t^I$ as their log deviation from the HP trend.

    (b) Regress $\hat{N}_t^C$ on $\hat{C}_t$ and $\hat{N}_t^I$ on $\hat{I}_t$

    (c) Using the conditions

$$\hat{C}_t = \frac{(\tau^C(\mu^C - 1))}{(1 - \tau^C)} \hat{N}_t^C \tag{4.93}$$

$$\hat{I}_t = \frac{(\tau^I(\mu^I - 1))}{(1 - \tau^I)}\hat{N}_t^I \tag{4.94}$$

and set $\mu^I = \mu^C = 1.3$ and calculate values τ^C and τ^I

13. Calculate $\hat{\mu_t^C}$ and $\hat{\mu_t^I}$ by equations

$$\hat{\mu_t^C} = \frac{(1 - \tau^C\mu^C)}{(\tau^C\mu^C)}\hat{C}_t \tag{4.95}$$

$$\hat{\mu_t^I} = \frac{(1 - \tau^I\mu^I)}{(\tau^I\mu^I)}\hat{I}_t \tag{4.96}$$

Figure 4.1

Detrended Log TFP and the Inverse of Detrended Log RPI

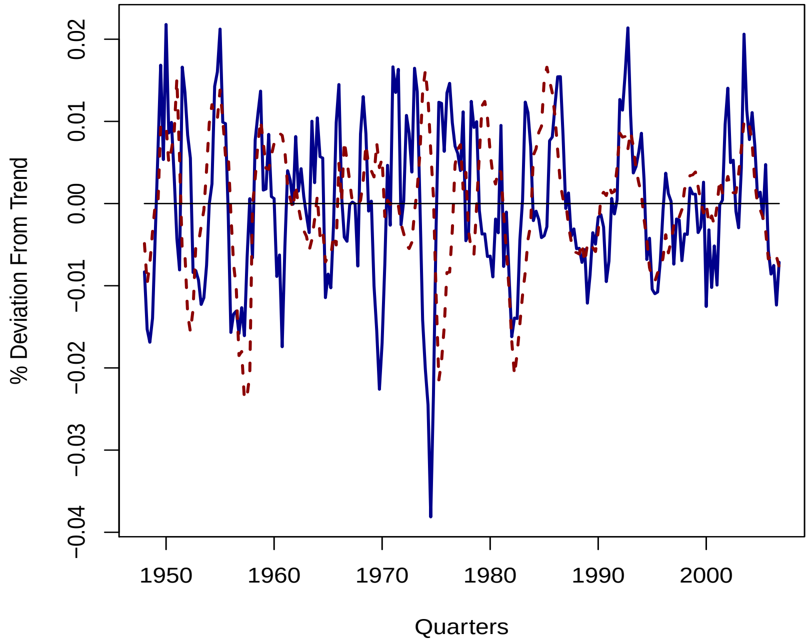

Figure 4.1: Plots of log TFP (*tfp*) along with the log RPI (*rpi*) for the US from 1948:1 to 2006:3, where each is detrended using a HP filter. The solid line indicates the log detrended *tfp* while the dashed line indicates the inverse of the detrended quality-adjusted *rpi* over the same period, where the inverse of the log detrended RPI is plotted so as to illustrate the link between these two technologies.

Figure 4.2
Demeaned Log TFP and the Inverse of Demeaned Log RPI Adjusted by the Cointegration Coefficient

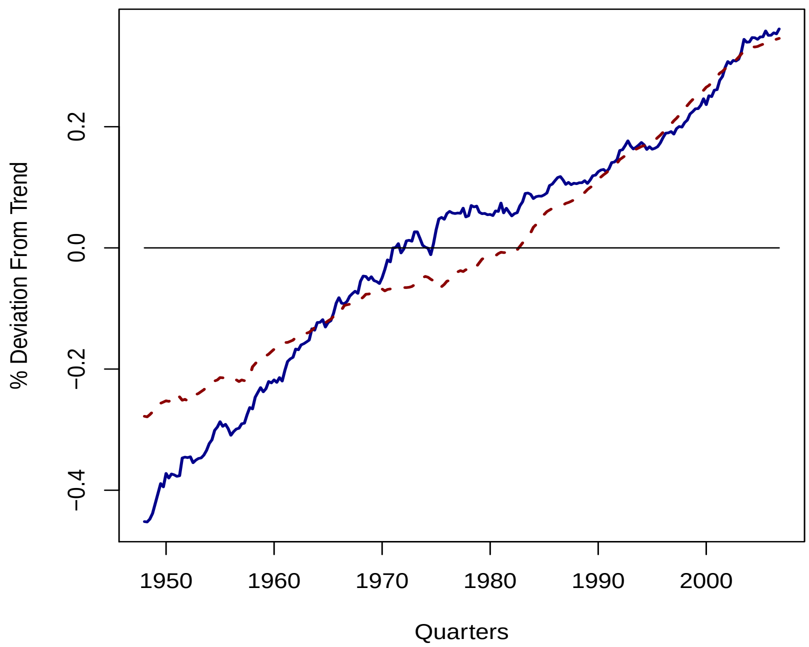

Figure 4.2: Plots of *tfp* along with the *rpi* for the US from 1948:1 to 2006:3. The solid line indicates the *tfp* while the dashed line indicates the inverse of the quality-adjusted emphrpi over the same period, where the inverse of the *rpi* is plotted so as to illustrate the link between these two time series. Each time series has been demeaned

Figure 4.3

Impulse Responses to Neutral Technology Shock
Benchmark Model

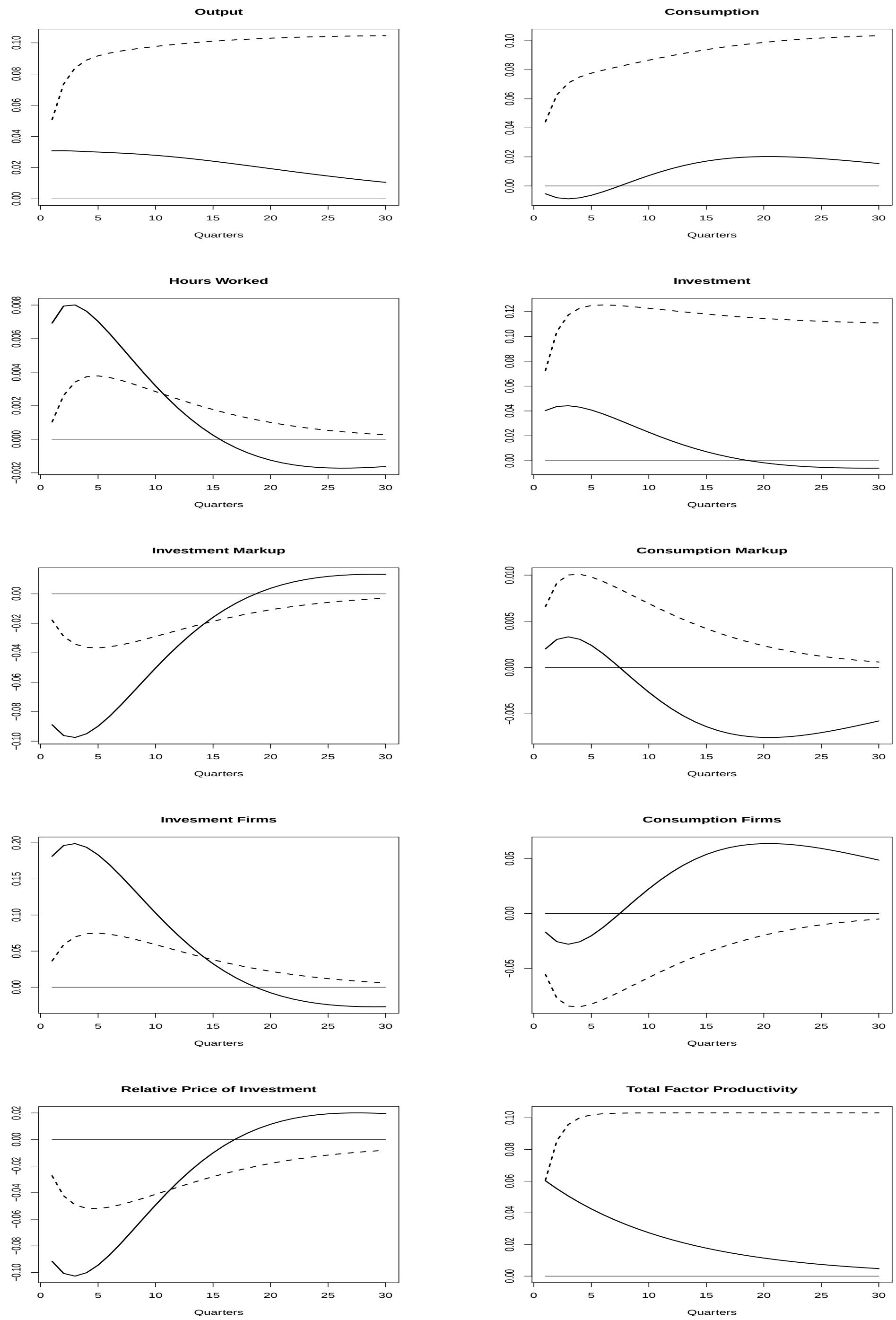

Figure 4.3: Impulse responses to a one-standard-error innovation to ϵ_t^Z (solid) and a one-standard-error innovation to $\epsilon_t^{\mu^Z}$ (dashed), measured as a percent deviation from the respective balanced growth path.

Figure 4.4

Impulse Responses to IST Shock
Benchmark Model

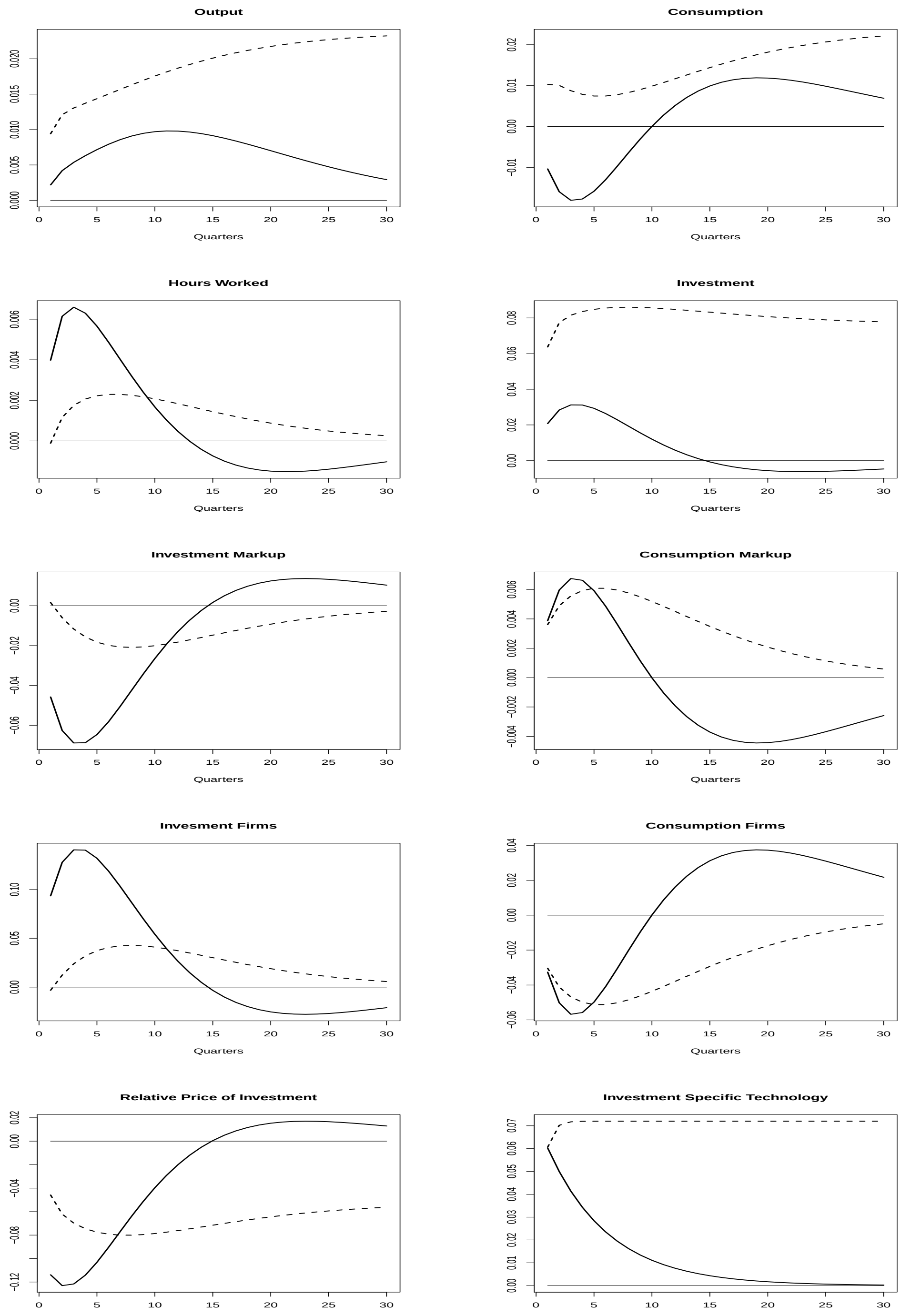

Figure 4.4: Impulse responses to a one-standard-error innovation to ϵ_t^Z (solid) and a one-standard-error innovation to $\epsilon_t^{\mu^Z}$ (dashed), measured as a percent deviation from the respective BGP.

Figure 4.5

Impulse Responses to Non-Technology Shocks
Wage Markup, Preference, and MEI
Benchmark Model

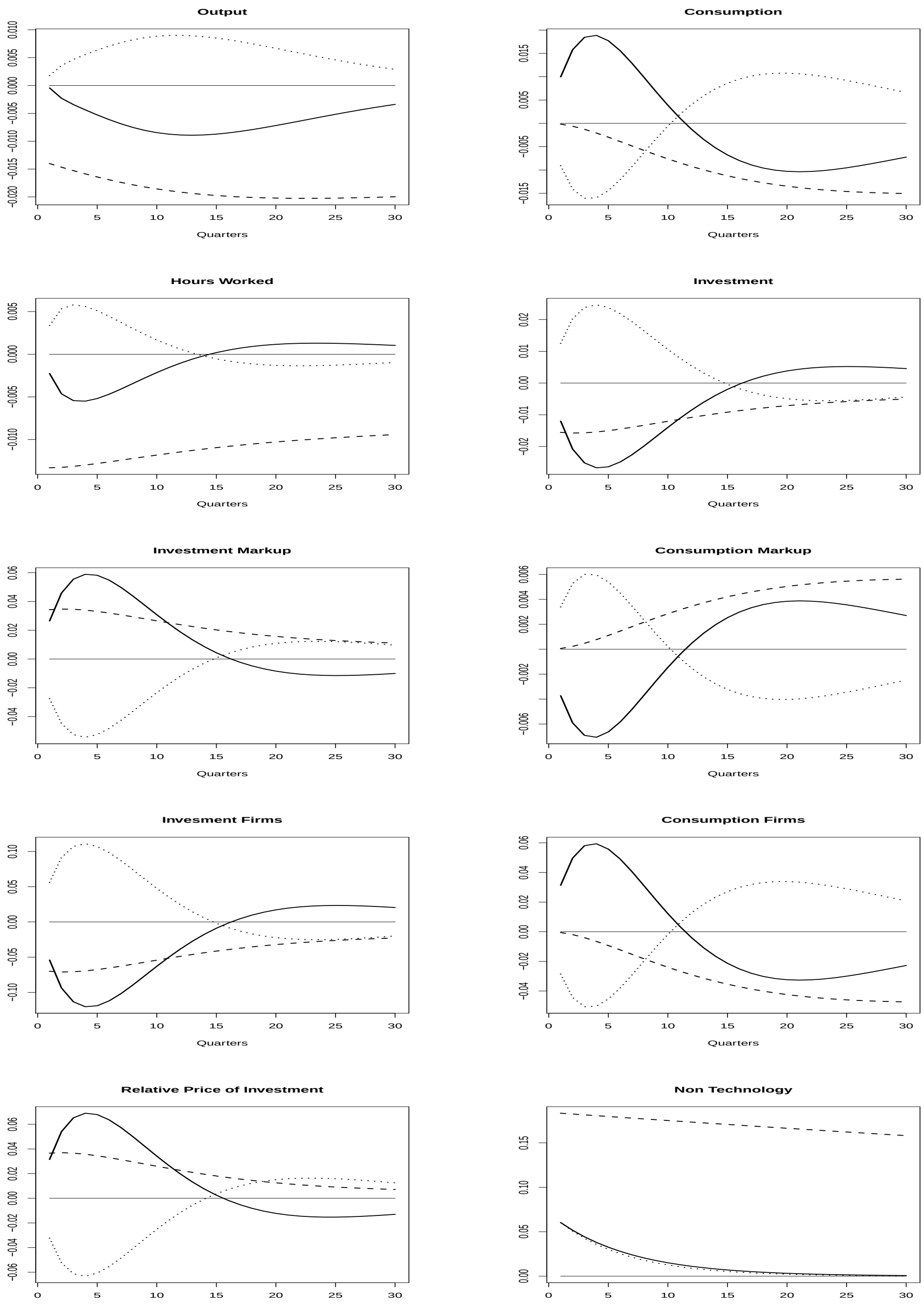

Figure 4.5: Impulse responses to a one-standard-error innovation to ϵ_t^b (solid), a one-standard-error innovation to $\epsilon_t^{\mu^W}$ (dashed), and a one-standard-error innovation to ϵ_t^V (dotted), measured as a percent deviation from its respective steady state.

Figure 4.6

Impulse Responses of the *rpi*

With Various Components of the Benchmark Model Removed

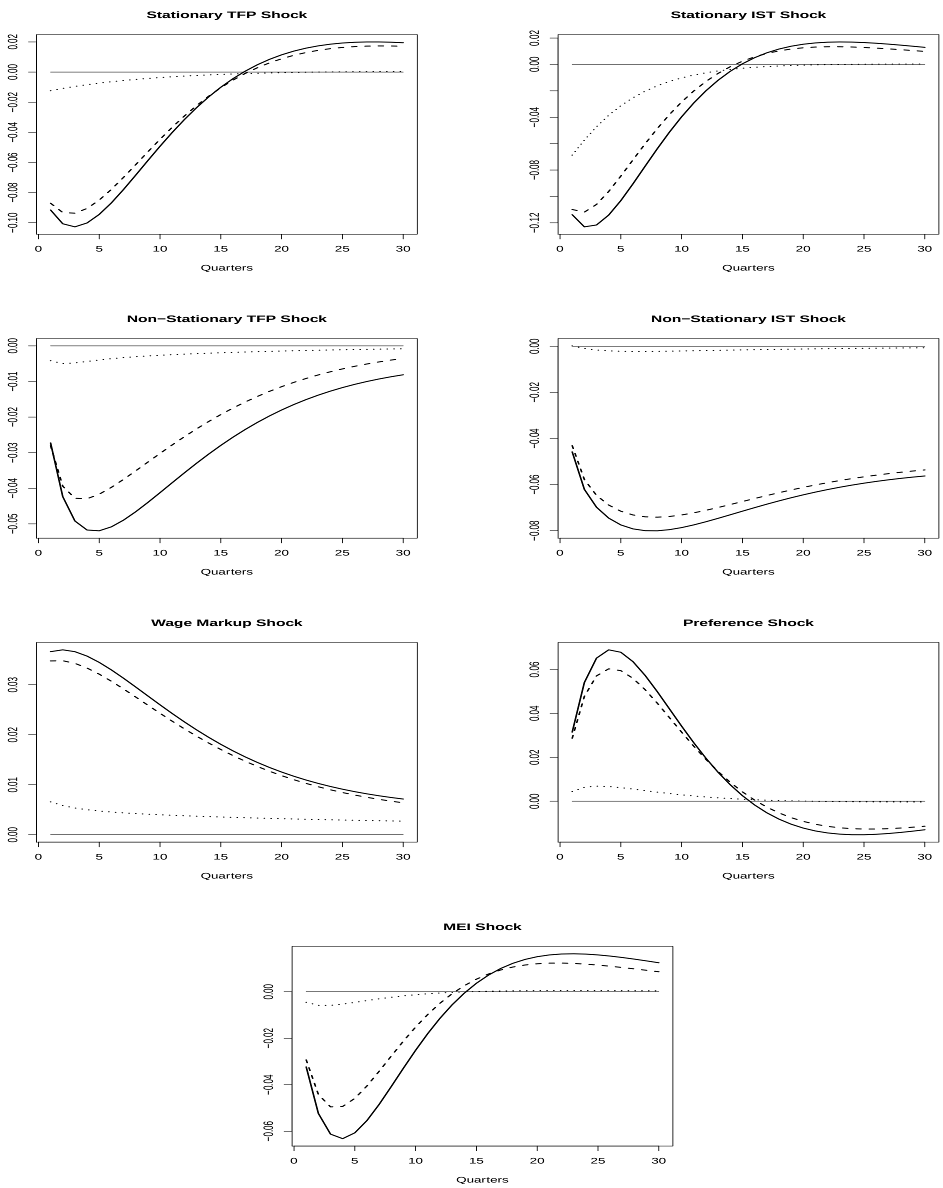

Figure 4.6: Impulse responses to a one-standard-error innovation to ϵ_t^k, where $k \in \{z, a, \mu^Z, \mu^A, b, \mu^W, V\}$ for the following models: benchmark (solid), a model with only endogenous price markups (dashed), and a model with only IRS/DRS in the investment sector (dotted). Each impulse response function is measured as a percent deviation from the respective balanced growth path.

Table 4.1

Precursor OLS Estimates for τ^C τ^I

Sector	Consumption	Investment
Theory	$\hat{N_t^C} = \left(\frac{\tau^C(\mu^C-1)}{1-\tau^C}\right)\hat{C}_t$	$\hat{N_t^I} = \left(\frac{\tau^I(\mu^I-1)}{1-\tau^I}\right)\hat{I}_t$
Data	$\hat{N_t^C} = \underset{(0.001)}{-6.745e^{-19}} + \underset{(0.11)}{1.123^{***}}\ \hat{C}_t$	$\hat{N_t^I} = \underset{(0.001)}{-3.66e^{-19}} + \underset{(0.027)}{0.394^{***}}\ \hat{I}_t$
R^2	0.5648	0.7252

Table 4.1: Data on both the dependent and independent variables are outlined in the Data Used section of the appendix. Each variable accompanied by a hat is the log deviation from its Hodrick-âĂŞPrescott filter trend, where λ = 1600 with no drift. Data ranges from 1992:Q3 to 2013:Q4. Significance codes: '***' denotes 0.001, '**' 0.01, and '*' 0.05.

Table 4.2

Priors

Parameter	Prior Distribution	Lower Bound	Upper Bound	Mean	Variance
ξ	normal			1	0.1
τ^I	normal			0.92	0.08
τ^C	normal			0.87	0.09
ω^I	normal			0.78	0.05
ω^C	normal			0.70	0.05
θ	gamma			3	0.75
χ	beta			0.5	0.1
η	Uniform	0.01	0.99		
h^{ss}	normal			0.3	0.03
ρ_Z	Beta			0.85	0.20
ρ_A	Beta			0.85	0.20
ρ_v	Beta			0.70	0.20
ρ_b	Beta			0.70	0.20
ρ_{μ^w}	Beta			0.70	0.20
ρ_{μ^Z}	Beta			0.40	0.20
ρ_{μ^A}	Beta			0.20	0.10
σ^i	Inv Gamma			0.5	2
σ^k	Uniform	0	$\frac{1}{4}\sigma^{obs}$		

Table 4.2: σ^i refers to the variance of an unanticipated shock to $i = \{Z, A, b, V, \mu^W, \mu^Z, \mu^A\}$ σ^k variance of the Measurement Error for the observable $k = \{Y, I, C, H, RPI\}$.

Table 4.3

Calibrated Parameters

Parameter	Value	Description
σ	1	Risk aversion
$\bar{\mu}^Y$	1.0049	Per capita output growth along a BGP
$\bar{\mu}^{RPI}$	0.9957	Per capita *rpi* growth along a BGP
δ	0.02	Depreciation rate in steady state
$\bar{\mu}^w$	1.10	Steady state wage markup
β	0.99	Subjective discount factor
α	0.31	Capital Share of Output
$\bar{\mu}^I, \bar{\mu}^C$	1.3	Steady state markup

Table 4.3: Parameter values governing the per capita output growth, IST growth, steady state markup, and households' risk aversion are those used by Schmitt-Grohe and Uribe (2011).

Table 4.4

Bayesian Estimation

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
τ^I	normal	0.9	0.04	0.9154	0.9151	0.9157
τ^c	normal	0.78	0.4	0.8445	0.8432	0.8458
ξ	Normal	1	0.1	1.0424	1.0417	1.0431
θ	Normal	3	0.75	3.9606	3.9473	3.9739
χ	Gamma	0.5	0.1	0.6654	0.6633	0.6675
η	Uniform	0.504	0.2855	0.6257	0.6213	0.6301
h^{ss}	Normal	0.3	0.03	0.2425	0.2418	0.2432
ρ_Z	Beta	0.9	0.05	0.9024	0.9019	0.9029
ρ_A	Beta	0.9	0.05	0.8094	0.8078	0.811
ρ_V	Beta	0.8	0.1	0.7758	0.7727	0.7789
ρ_b	beta	0.8	0.1	0.823	0.8207	0.8253
ρ_{μ^W}	Beta	0.8	0.1	0.9948	0.9895	1.0001
ρ_{11}	Beta	0.3	0.2	0.2658	0.2625	0.2691
ρ_{22}	Beta	0.3	0.1	0.1725	0.1715	0.1735
σ_z^0	Inverse Gamma	0.5	2	0.0608	0.0591	0.0625
σ_a^0	Inverse Gamma	0.5	2	0.0607	0.059	0.0624
σ_V^0	Inverse Gamma	0.5	2	0.0625	0.0565	0.0685
σ_b^0	Inverse Gamma	0.5	2	0.0612	0.0592	0.0632
$\sigma_{\mu^w}^0$	Inverse Gamma	0.5	2	0.1928	0.1691	0.2165
$\sigma_{\mu^Z}^0$	Inverse Gamma	0.5	2	0.0639	0.0597	0.0681
$\sigma_{\mu^A}^0$	Inverse Gamma	0.5	2	0.0625	0.0588	0.0662

Table 4.4: σ^i refers to the variance of an unanticipated shock to $i = \{Z, A, b, V, \mu^W, \mu^Z, \mu^A\}$.

Table 4.5
Variance Decomposition: RPI Growth

	Benchmark	Endogenous Markups	IRS/DRS	Two-Sector
Stationary TFP (Z_t)	28	28	2	0
Stationary IST (A_t)	43	44	65	70
Non-Stationary TFP (μ_t^Z)	4	4	1	0
Non-Stationary IST (μ_t^A)	10	10	31	30
Preference (b_t)	6	5	0	0
Wage Markup (μ_t^W)	4	4	1	0
MEI (v_t)	6	5	0	0

Table 4.5: Each cell refers to the percentage of the volatility of the *rpi* explained by the shock listed on the left, by the model listed above. There are three model variants compared to the benchmark model. These include; a variant of the benchmark model without curvature in investment production, a variant with curvature in investment production without endogenous price markups, and lastly a two-sector model with neither curvature in investment nor endogenous price markups.

Table 4.6
Variance Decomposition: Benchmark Model

	g^y	g^c	g^i	g^h	g^{rpi}
Stationary TFP (Z_t)	61	14	50	18	28
Stationary IST (A_t)	1	28	17	9	43
Non-Stationary TFP (μ_t^Z)	24	6	8	2	4
Non-Stationary IST (μ_t^A)	1	3	2	1	10
Preference (b_t)	1	26	9	5	6
Wage Markup (μ_t^W)	13	1	7	59	4
MEI (V_t)	1	22	8	6	6

Table 4.6: The column headers are defined as follows: g^y growth rate of output, g^c growth rate of consumption, g^i growth rate of investment, g^h growth rate of hours, g^{rpi} growth rate of the *rpi*.

Table 4.7
Variance Decomposition: Model With Cointegration

	g^y	g^c	g^i	g^h	g^{rpi}
Stationary TFP (Z_t)	33	9	31	2	0
Stationary IST (A_t)	0	12	6	0	39
Non-Stationary TFP (μ_t^Z)	12	22	12	0	39
Non-Stationary IST (μ_t^A)	0	3	2	0	22
Preference (b_t)	0	16	2	0	0
Wage Markup (μ_t^W)	55	31	45	97	0
MEI (V_t)	0	7	2	0	0

Table 4.7: The column headers are defined as follows: g^y growth rate of output, g^c growth rate of consumption, g^i growth rate of investment, g^h growth rate of hours, g^{rpi} growth rate of the *rpi*.

Table 4.8

Bayesian Estimation

With Technological Spillovers and Cointegration

Parameter	Distribution	Prior mean	Standard Deviation	Posterior mean	5%	95%
θ	Normal	3	0.75	3.8728	3.8663	3.8793
χ	Gamma	0.5	0.1	0.6064	0.6055	0.6073
η	Uniform	0.504	0.2855	0.1898	0.1864	0.1932
h^{ss}	Normal	0.3	0.03	0.2546	0.2542	0.255
ρ_Z	Beta	0.9	0.05	0.9153	0.915	0.9156
ρ_A	Beta	0.9	0.05	0.864	0.8635	0.8645
ρ_V	Beta	0.8	0.1	0.6763	0.6755	0.6771
ρ_b	beta	0.8	0.1	0.823	0.8949	0.8989
ρ_{μ^W}	Beta	0.8	0.1	0.8539	0.8532	0.8546
ρ_{11}	Beta	0.3	0.2	0.3455	0.3434	0.3476
ρ_{22}	Beta	0.3	0.1	0.3948	0.3925	0.3971
ρ_{12}	Normal	0	0.1	-0.0259	-0.0282	-0.0236
ρ_{21}	Normal	0	0.1	-0.0051	-0.0074	-0.0028
κ_1	Uniform	0	0.23	-0.0716	-0.0737	-0.0695
κ_2	Uniform	0	0.23	0.3954	0.3919	0.3989
$corr(Z, A)$	Normal	-0.13	0.1	-0.0052	-0.0053	-0.0051
σ_z^0	Inverse Gamma	0.5	2	0.0613	0.0595	0.0631
σ_a^0	Inverse Gamma	0.5	2	0.061	0.0592	0.0628
σ_V^0	Inverse Gamma	0.5	2	0.0623	0.0552	0.0694
σ_b^0	Inverse Gamma	0.5	2	0.0612	0.0591	0.0635
$\sigma_{\mu^w}^0$	Inverse Gamma	0.5	2	0.1928	0.4426	0.4764
$\sigma_{\mu^Z}^0$	Inverse Gamma	0.5	2	0.0639	0.0601	0.0617
$\sigma_{\mu^A}^0$	Inverse Gamma	0.5	2	0.0626	0.0592	0.066

Table 4.8: All forms of endogenous movement in the *rpi* have been removed. Prior distributions for parameters shared with the benchmark model remain as described in section 4.3. σ^i refers to the variance of an unanticipated shock to $i = \{Z, A, b, V, \mu^W, \mu^Z, \mu^A\}$ σ^k variance of the measurement error for the observable $k = \{Y, I, C, H, RPI\}$.

Chapter 5

Conclusion

Since the seminal work of Greenwood, Hercowitz, and Huffman (1988) first introduced us to IST, these shocks have become a staple of business cycle literature. Likewise, identification of IST has remained in line with Greenwood, Hercowitz, and Huffman's (1988) first proposal. The logic is simple, since neutral technology shocks improve productivity economy-wide, there should not be any impact on relative prices. Therefore, any movement in the RPI should be due to innovations that are specific to the investment sector. However, identifying IST in this manner implies that the RPI is orthogonal to any and all other forms of economic disturbance. In this thesis, I tackle the implications of relaxing this assumption in both open and closed economies.

In my second chapter I set up a two-sector real business cycle model adapted to allow for financial frictions in the production of investment goods à la Carlstrom and Fuerst (1997). In this chapter, entrepreneurs and their privately observed idiosyncratic productivity are charged with the task of producing investment goods within their own borders. These entrepreneurs lack the necessary funds to self-finance and thus need to borrow from a financial intermediary. Adapting the standard two-country, two-good model developed by BKK (1993) in this manner provided some interesting results. First and foremost, with financial frictions in investment production, the RPI moves endogenously to shifts in investment demand, rather than to shifts in IST. Furthermore, this model demonstrates that financial frictions can correct some of the shortfalls observed in the standard BKK model. With financial frictions, we find that country-specific technology shocks increase output, consumption, and investment across all countries rather than just where productivity is the highest. This leads to positive cross-country correlations in output, consumption, and investment despite the exclusion of technological spillovers across countries. Lastly, it has been shown that financial frictions lead to an increase in both exports and terms of trade volatility over the business cycle. Overall, the inclusion of financial frictions, and the implications this has on RPI, significantly improves upon the ability of the BKK model to match the data.

In the third chapter of this thesis, I adapt the standard news cycle model to replicate the cointegrating relationship between TFP and the RPI observed in the US data. As mentioned throughout this thesis, there are many reasons to be suspicious of the orthogonality restriction on the RPI proposed by Greenwood, Hercowitz, and Krussel (1997). The recent work by Schmitt-Grohe and Uribe (2011) is a case in point, where the authors show that TFP and the RPI follow a common stochastic trend in the postwar US. After performing a Bayesian estimation and then a variance decomposition, my results conclude that the relative importance of anticipated technology shocks depends on whether one has properly accounted for the cointegrated relationship between TFP and the RPI. When the standard news shock model is adapted in this manner, approximately 30 percent of the volatility of output, consumption and investment can be attributed to changes in anticipated technology. This is a significant finding since the current news cycle literature, such as Schmitt-Grohe and Uribe (2012) and Khan and Tsoukalas (2012), has converged on the idea that anticipated technology shocks are not an important determinant of business cycle volatility in any of the key economic aggregates.

In chapter four, I adapt a two-sector model to include both endogenous price markups and curvature in investment production. This model is then compared to an alternative version, where movements in the RPI are replicated by allowing both TFP and IST to move together over time. The endogenous-based approach outperforms the exogenous-based approach, with over 15 percent of the RPI explained by non-technological disturbances. These results indicate that the standard method of identifying IST with the inverse of the RPI is flawed as it does not reflect how relative prices vary depending on relative demand for investment goods over consumption goods. Thus far, this research has focused on a closed, rather than an open economy framework. Recent research by Mandelman et al. (2011) and Ireland (2013) demonstrates that TFP and IST are cointegrated across countries in their two-country stochastic growth models. With future research I hope to apply the lessons learned in chapter 4 in an international environment to explain this phenomenon. Overall, this thesis as a whole finds that future research should acknowledge the fact the RPI is a poor indicator of IST.

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