Stochastic Inventory Models for Dual Channel Retailers

STOCHASTIC INVENTORY MODELS FOR DUAL CHANNEL RETAILERS

By

Mohamed Ibrahim Mahaboob, B.Tech.(Hons.), P.G.D.M.

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AUTHOR:	Mohamed Ibrahim Mahaboob B.Tech.(Hons.)-Elec.Engg. (Indian Institute of Technology, Kharagpur, India) P.D.G.M. (Indian Institute of Management, Calcutta, India)
SUPERVISOR:	Dr. Elkafi Hassini
COMMITTEE:	Dr. Elkafi Hassini Dr. Mahmut Parlar Dr. Sourav Ray
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To my son, Shafiq Syed Ibrahim and my dad, Abdul Syed Mahaboob

Abstract

We look at some inventory models for dual channel retailers who fulfill demands from both channels from the same pool of stock. Since the arrival of the internet, companies, particularly retailers, have been trying to figure out how best to take advantage of this new channel. This became acute for retailers when Amazon.com became the pre-dominant bookseller overcoming rivals like Barnes and Noble and Borders who had been operating for several decades. Firms in other categories were worried that a web only retailer like Amazon.com could end up dominating their sectors as well. To respond to such threats, existing retailers like Tesco and Lands End began offering their products through a web-site which became their web channel. To improve efficiency and to take advantage of specific characteristics of this channel, firms had to modify the operations of their existing channels. In this thesis, we look at retailers who took advantage of the web channel to better their operations for the entire firm.

We introduce the topic of the thesis in Chapter 1. In Chapter 2, we provide a comprehensive literature survey of the topics that are covered in the thesis. In Chapters 3 and 4, we study the impact of product substitution differences exhibited by customers in the two channels on the inventory decisions of the dual channel retailer. In Chapter 3, we study the inventory allocation decisions of a dual channel

retailer where the web demand is deterministic and the store demand is stochastic. The optimal allocation of inventory is impacted by the substitution decision of the retailer in the web channel and that of the consumer in the store channel. In Chapter 4, we extend this study to joint allocation and ordering decisions in the single period setting. We find that the pooling effect due to substitution between the products and the one-way substitution between channels can lead to profit gains for the retailer. Further, we introduce into the inventory substitution literature penalty costs based on customer segmentation. We divide customers into two classes, store loyal and brand loyal. Store loyal customers would substitute a product if their favourite product was missing rather than walk out of the store. Brand loyal customers, on the other hand, would look for their favourite product in another store in the event of a stockout.

In Chapter 5, we consider a problem faced by a dual channel catalog retailer in clearing stock using the two channels, the original catalog channel and the newer web channel. We develop a two-stage stochastic program with the first stage decisions being the prices and stock allocation between the two channels and the second stage decision being the clearance price for the web channel.

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I also thank my friends for their help, support, and presence in my life during hard times.

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Chapter 1

Introduction

1.1 Motivation

In 2012, the retail sector contributed 5.5% to the GDP in Canada, with a total sales of C\$328 billion and growth rate of 3%. The sector employed 2.2 million people in 2012, representing 12.6% of total employment. In the US total sales was \$2.744 trillion and the sector employed 14.8 million people, representing 10.3% of total employment. Among one of the important changes in this sector in the last decade is the adoption of e-commerce. The growth of e-commerce has been impressive. According to a report by Internet Retailer, a industry magazine, e-commerce sales in the USA have exploded from 58 billion in 2003 to 226 billion in 2012, a compounded growth rate (CAGR) of 16.3%. This compares to a CAGR of 3.5% for the entire retail sector during the same period of time. In 2003, e-commerce accounted for 2.6% of total retail sales of 2.3 trillion and this grew to 7.3% of total retail sales of 3.1 trillion in 2012. In 2012, the web is no longer the nascent channel that it was in 2003. It is now the primary growth channel for all retail firms, growing at the rate of 15.9%

and accounting for 25.4% of their growth in 2012. Thus a study of this channel is extremely important for the future growth and success of the firms in the retail sector.

Internet Retailer first produced a report on the top firms in the industry in 2004, titled the Top 300 Guide and in that they categorized the firms into four types based on the nature of their main channel for sales. These four types are Web only retailers, Retail Chain retailers - retail chains like Walmart, Best Buy Tesco, Catalog retailers such as L.L.Bean and Lands End, and Consumer Brand Manufacturers, like Dell and Apple. In 2012 the industry had grown so big, they increased the number of firms surveyed to 500 and called their report the Top 500 Guide. As shown in Figure 1.1, the market shares for these four types of firms in 2012 was 42.4%, 34.9%, 11.3% and 11.4%, respectively.

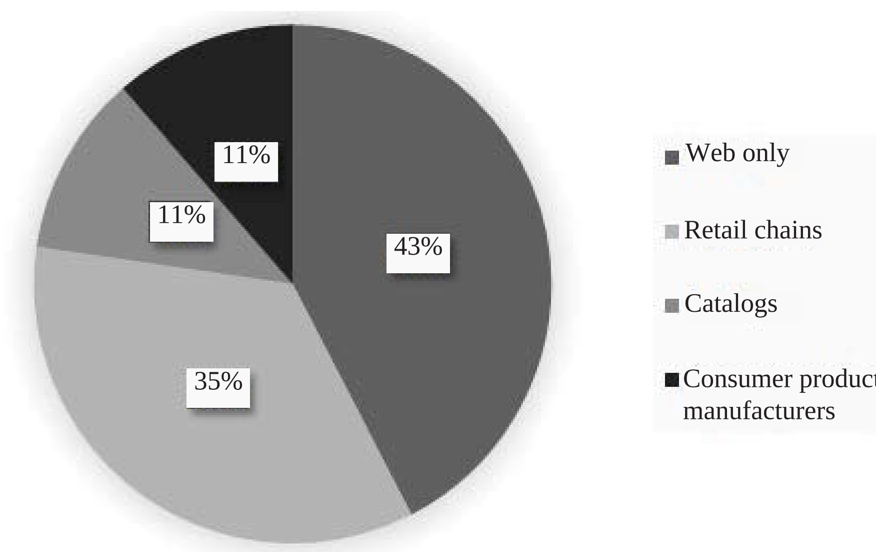


Figure 1.1: Market Share of Firm Categories

Due to the brutal competition that exists in the e-commerce sector, it is not surprising that only 146 firms from the initial Top 300 that Internet Retailer surveyed in 2003 survived. Considering only these survivors, the CAGR for the four types of firms for the period 2003 – 2012 is: Web only retailers: 27.1%, Retail chains: 16.7%, Catalogs: 17% and Consumer Brand Manufacturers: 14.7%. Excluding Amazon, the CAGR for this period for the Web only firms drops to 15.6%. Figure 1.2 provides a picture of the growth rates for the Retail Chains and Web only retailers, where the growth rate for the web only firms exceeded that of the retail chains.

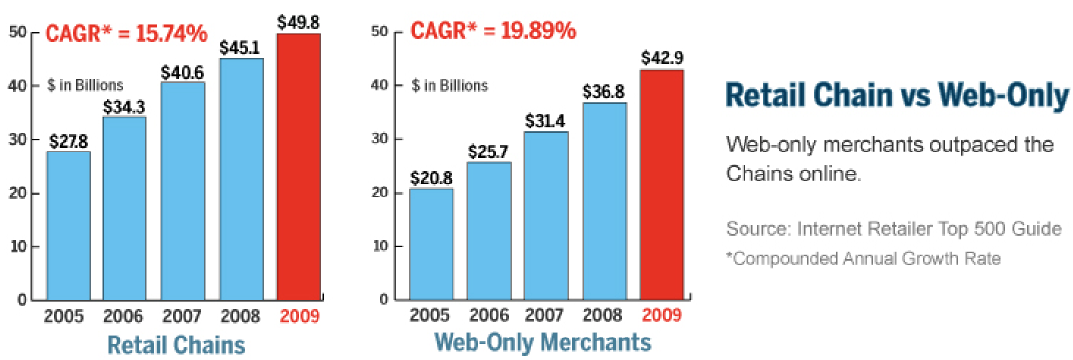


Figure 1.2: Retail Chains vs Web Only Retailers

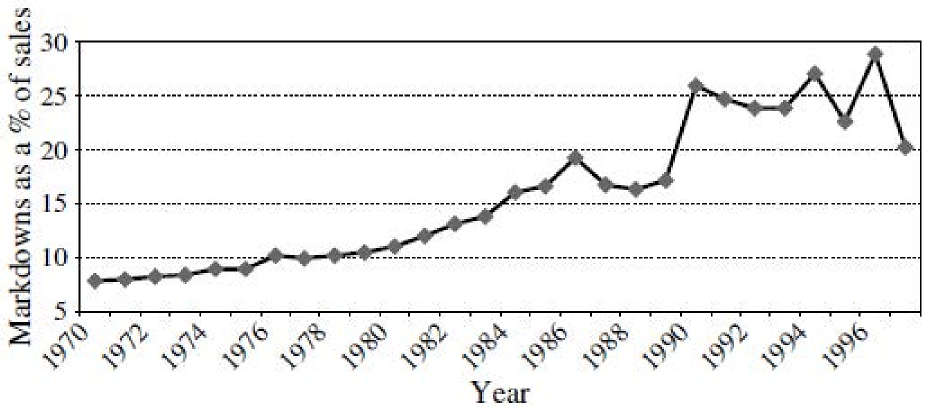
The growth rates for the web channel has been phenomenal as compared to their erstwhile channels, when retail is considered as a whole. From this it would appear that retailers should focus all their attention on the web channel alone but this would be misleading. Depending on the nature of goods sold, the penetration of the web channel has differed, ranging from less than 1% percent among Food/Drug and Hardware/Home Improvement retailers to close to 40% for Books/Music/Video retailers and 50% for Office Supplies. In fact the challenge for many firms in the industry is how best to manage their traditional channel and the newer web channel in a multichannel environment, often referred to as *o*mni-channel retailing. Hence, the situation is still fluid as to the channel strategy that will be the most profitable for the retailers in the years to come. In this thesis we look at retailers who operate dual channels, retail chains with stores as their traditional channel with a newer web channel and catalog merchants who traditionally sought sales by mailing catalogs to their customers and who are increasingly adding a web channel.

So all the doomsday scenario of the 1990s when every retailer was scared of being outsold by a web only retailer like Amazon turned out to be baseless. A further illustration of the competitiveness of the retail chains is borne out by the fact that eight of the top ten fastest growing e-retailers in the period 2003 to 2012 were retail chains. They did this by building web sites that would complement store shopping. Here is what Nicki Baird, managing partner of research and advisory firm Retail Systems Research had to say [116]:

> While there are exceptions, most of the retailers that have succeeded in the last 10 years have been aggressive about cross-channel initiatives. These guys are making investments that enable them to connect on-line demand to inventory in the store.

Given the importance of managing multiple channels for the retailer, in this thesis, we look at the operations of two of the four categories of retailers mentioned above, those that use more than one channel to fulfill demand. These would be Retail Chains and Catalog merchants.

There has been a growth in the use of optimization models by retailers to improve performance. Friend and Walker [49] cite markdown pricing as one significant use of optimization models by retailers. This has been necessitated by the increasing use of markdowns to clear inventory as is shown in Figure 1.3 taken from Fisher [48].



Source: National Retail Federation, Financial & Operating Results. After 1997 the NRF stopped reporting department store markdowns.

Figure 1.3: Department store markdowns as a percentage of sales.

Not only has the percentage amount of markdown on an average item been growing, the number of items that have to be marked down has also been increasing due to the increasing variety of goods sold. Figure 1.4 shows examples of markdowns for the different types of retailers taken from Hausman and Thorbeck [60].

Fast FASHION (ZARA)	Traditional European apparel retailer	Traditional US apparel retailer	US department store
15% of items 15% markdown	30–40% of items 30% markdown	50–60% of items 40% markdown	60–70% of items 40% markdown

Figure 1.4: Typical markdowns across retailer categories.

Earlier, the catalog merchants used printed catalogs mailed to consumers to generate sale orders by phone or mail. More of the catalog merchants are now using the printed catalogs as a brand ambassador while using the web as the primary means of transacting sales. According to the Direct Marketing Association's analysis of U.S. Postal Service reports, the number of catalogs mailed has dropped from 17.2 billion in 2003 to 12.5 billion in 2012, a drop of 27.3%. They have been more effective in converting web visitors into paying customers, a conversion rate of 5.1%, which is 50% better than the web only firms who come in next with a conversion rate of 3.4%. Their experience as catalog merchants has served them well. The challenge of clearing excess merchandise remains to be an important issue for these retailers. We study the use of the web channel by the catalog merchants to get rid of their excess merchandise.

1.2 Research problems and contributions

The first problem that we look at concerns the inventory ordering and allocation decisions of a retail chain with a web channel. Chain retailers, like Walmart, BestBuy and Longos use two sources for servicing web demand, the store and the warehouse. In our study of chain retailers, we concentrate on the strategy of retailers who service the web demand from the store. Depending on the product category, retailers have either asked the customer to pick up from the store or have them delivered to their homes. The first strategy is being used by mass market firms like Walmart and electronics chain firms like BestBuy. The second strategy is practiced by firms in the grocery business, like Safeway, Peapod and Tesco. This is when on-line customers send in their grocery list and the local store then has the items picked up and sent to the customer's home.

In Chapter 3 and 4, we study the impact of product substitution differences exhibited by customers in the two channels on the inventory decisions of the dual channel retailer. In Chapter 3, we study the inventory allocation decisions of a dual channel retailer where the web demand is deterministic and the store demand is stochastic. The optimal allocation of inventory is impacted by the substitution decision of the retailer in the web channel and the consumer in the store channel. In Chapter 4, we extend this study to joint allocation and ordering decisions in the single period setting. We find that the pooling effect due to substitution between the products and the one-way substitution between channels can lead to profit gains for the retailer. Further, we introduce into the inventory substitution literature, penalty costs based on customer segmentation. We divide customers into two classes, store loyal and brand loyal. Store loyal customers would substitute a product if their favourite product was missing rather than walk out of the store. Brand loyal customers, on the other hand, would look for their favourite product in another store in the event of a stockout.

In Chapter 5, we consider a problem faced by a catalog retailer in clearing stock from the two channels, the original catalog channel and a newer web channel. In a single period the retailer gets one price point opportunity to clear stock as against two price points on the web. For example, Landsend mails out overstock catalog to customers for a season. Since the price is printed in the catalog the retailer does not get to change the price based on sales feedback during the early part of the season. On the web, however, the retailer gets to reduce the price further based on sales feedback. For example, L.L.Bean another catalog retailer has two menu choices - New to Sale and Further Markdowns. Thus on the web the retailer can set prices in two stages. Considering the entire problem, we develop a two-stage stochastic program with the first stage representing the price for the initial price on the web and the price for the catalog and allocation ratios for the catalog and web channels while in the second stage the decision is on additional discount to be set to clear the stock in the web channel.

1.3 Overview of chapters

We present a review of the relevant literature in Chapter 2. In Chapter 3, we consider a single-period model where a retailer sells two substitutable products through web and store channels and determine the optimal stock allocations between the two channels. In Chapter 4, we extend the model in the previous chapter to include stock ordering and show the pooling benefit that accrues to the retailer due to product substitution and channel substitution of stock. In Chapter 5, we develop a two-stage stochastic model where a retailer sells a product through two channels, catalog and web, and prices to clear the inventory at two stages on the web. Finally in Chapter 6, we draw our conclusions and describe some future research directions.

Chapter 2

Literature review

As mentioned in the previous chapter, we are primarily interested in decisions made by dual channel retailers regarding their inventory and pricing. Within the broad field of inventory literature, we are more interested in the (1) product substitution subfield for we study the impact of product substitution between the two channels and (2) markdown and clearance pricing subfield as our second problem relates to clearance pricing for a catalog merchant. The study of inventory pooling is closely associated with the nature of substitution as practiced by web consumers. We keep in mind that both product substitution and dual channel demand fulfillment are in a way a means by which retailers practice inventory pooling.

Given that this field of inventory and price management in dual channel retailers is new, we look in depth at all the papers that have appeared in peer-reviewed journals that deal with inventory issues in such a setting.

2.1 Product substitution

In operations management, the literature on product substitution inventory would be included within the broad area of inventory pooling as the stocks of the substituted products are used as a common pool to satisfy customer demand. The concept of inventory pooling is simple. If there are several sources of demand and the correlation between these demands is not perfect, then a retailer would benefit by serving these demands from one pool of stock. This is shown by Eppen [44] who considers a multi-location newsvendor problem with the demand from each location being separate. He shows that centralization would lead to inventory savings for the retailer. Chen and Lin [30] generalize his results to any non-negative demand distributions with concave cost and penalty functions. Cherikh [31] considers the same system as above but with the excess demand in one location being reallocated to other locations with excess inventory and shows pooling reduces the risk from the uncertain demands leading to lower costs and higher profits. Gerchak and He [56] show that for most cases, increased variability between the various demands leads to greater savings from inventory pooling. There may however be situations where increased variability leads to reduced benefits from inventory pooling. Yang and Schrage [114] show that increased benefits from risk pooling need not necessarily mean reduced inventory stock. Munson et al. [84] provide some simple examples that can be used in classrooms to show that pooling of stocks is beneficial independent of the inventory model used, whether the model is the newsvendor model used by the above authors or a simple EOQ model or a continuous review model with normally distributed demands.

There are two types of product substitution. One is called stock-out-based substitution as exemplified in Parlar and Goyal [91] while the other is called assortment-based substitution as shown in Smith and Agrawal [103]. Stock-out-based substitution occurs when the customer wishing to purchase a product notices that the product has been sold out when visiting the store. The consumer then selects some other similar product whereas in assortment based substitution, the consumer has in mind some particular product but when he reaches the store finds that the store does not carry that product. The consumer then chooses some other similar product. In our study we are mainly concerned with stock-out-based product substitution.

Product substitution can also be classified in two other ways: customer substitution versus retailer / manufacturer substitution. The customer substitution is sometimes referred to in the literature as two-way substitution while the retailer / manufacturer substitution is referred as one-way substitution. In customer substitution, the customer makes the decision on which product to substitute with, given that a product is stocked out. A customer visiting Longo's grocery market in Canada to purchase 2% milk, may find it sold out and substitute with 1% milk. In another instance, the 1% milk may be sold out and the customer substitutes with the 2% milk in stock. Since we are dealing with customer choice, this kind of substitution is two-way. In retailer / manufacturer-based substitution, the retailer makes the choice for the customer and often it is one-way, i.e., the retailer substitutes with a more expensive product when a cheaper product is stocked out. A friend of mine went to purchase a Laptop with 80G of harddisk space and since the Laptop with that configuration was sold out, the retailer, FutureShop, an electronic retailer in Canada, provided my friend with a Laptop with 120G of harddisk space, with everything else being the same. Here the retailer provided the customer with a more expensive product so as to not lose the sale.

The problem of consumer substitution for products with stochastic demand was introduced by McGillivray and Silver [81], who consider a two-product case with identical costs in a periodic order-up-to-S inventory setup. For fixed substitution probabilities, they develop heuristics for determining the optimal order-up-to levels. Using the case with substitution probability of 1 and comparing this to the no substitution case, they determine the limits on the potential benefits achievable from substitution. Parlar and Goyal [91], look at the same problem in a single period setting. They model the problem as an extension of the newsvendor problem and look at profit maximization. The profit function is shown to be concave for a large range of values for the price and substitution probabilities. Parlar is the first author to introduce penalty costs in substitution inventory literature in his study of competing retailers in [90]. Ernst and Kouvelis [45] look at a problem with three products, one of which is a bundle of the other two, with partial substitution and distribute the penalty costs in such a way that the shortage is proportional between the demand from the original purchasers of a product and the demand coming from the substitute purchasers of the product. Rajaram and Tang [95] use an approximation to develop a heuristic to analyze the effects of the degree of substitutability and the levels of demand variation and correlation on the optimal order quantity of each product. Netessine and Rudi [88] prove that the profit function with substitution between more than two products is not concave. Nagarajan and Rajagopalan [86] develop optimal inventory policies for two products that are negatively correlated and show that the optimal inventory of one is not dependent on the inventory level of the other provided the substitution rate is below some threshold. Stavrulaki [106] considers the effect of inventory dependent demand in conjunction with product substitution between two products on a retailer's inventory decisions.

The retailer / manufacturer substitution problem is handled by Bassok et al. [10] who discuss the infinite horizon multi-product problem with identical substitution costs. They show that a myopic base stock policy is optimal. Hsu and Bassok [63] provide the same analysis for a production system in which a single item can be converted into several products that satisfy several demand classes, with a nested one-way substitution structure. Axsäter [7] determines the optimal substitution rates for a retailer using continuous review order-up-to-S inventory policy and a retailer substitution policy. Rao et al. [96] develop heuristics based on the Wagner-Whitin algorithm to deal with the same problem dealt by Bassok et al. [10] but with non-identical substitution costs. They develop a two-stage model where the first stage decisions deal with which products to produce and the second stage decisions deal with which products to be substituted. Eynan and Foque [46] develop a model for a retailer who can influence customers to switch from one product to another even when the product desired by the customer is in stock. Liu et al. [74] looks at three different substitution policies that retailers may use in making one-way substitution and evaluate each on several performance measures like average inventory level, average backlogged demand and fill rate. Shah and Avittathur [100] consider demand cannibalization in addition to retailer product substitution when developing their model. In an interesting twist, Roychowdhury [98] considers the case where the product being substituted is the superior one, i.e. the less expensive product is used to substitute demand for the more expensive product. When the consumer demand cannot be identified with any specific probability distribution but is imprecise, then product substitution in such a scenario is handled by Dutta and Chakraborty [39]. Lu et al. [75] extend the analysis of one-way substitution with two products with one supplier to two suppliers, an unreliable supplier and an expensive reliable supplier.

Prior to the above models being used by retailers, one has to have a good idea of the demand function for the products sold by the retailer. There have been several studies in recent years that have looked at the problem of developing effective demand models from consumer observations through sales transactions. Anupindi et al. [5] provide a method for estimating the demand rates for products when consumers substitute a product with another when faced with a stock-out. Kök and Fisher [69] generalize the above model to take care of dynamic consumer choice under a shelf-space availability constraint and report on the implementation details of their procedure in a large grocery chain. Gilland and Heese [57] in addition to considering a shelf-space constraint also take into account the sequence of customer arrivals in deriving the optimal shelf-space to be allocated to the two substitutable products. Vulcano et al. [112] use the aggregate market share information coupled with observed availability of products along with actual sales to formulate a demand model with estimated arrival rates and product substitution rates. Tan and Karabati [109] use point-of-sale data to determine the order up-to-levels for inventory when considering multiple substitutable products in a multi-period framework under service constraints.

Mishra and Raghunathan [83] study the issue of vendor-managed inventory and show that information sharing between a retailer and a manufacturer which normally leads to better supply chain coordination, is diluted by product substitution. Ganesh et al. [52] show that this reduction in the value of information sharing increases with the degree of substitution. In a follow-up paper, [53], they extend this result to supply chains with more than two levels. Li et al. [73] quantify the impact of the bull-whip effect on the upstream firm due to product substitution.

The product substitution literature is seeing an increasing interest where both pricing and inventory are decision variables. A representative set of papers would include Karakul and Chan [66], Dong et al. [38], Akcay et al. [3] and Ceryan et al. [25]. For a recent review of articles on multi-product pricing, not necessarily product substitutes, see Soon [105].

A stream of research which is quite close to substitution is the assortment problem where the retailer has to decide on the optimal assortment of products along with the optimal inventory levels for each product that forms part of the assortment. Papers considering assortment and inventory issues are Pentico [94], van Ryzin and Mahajan [111], Agrawal and Smith [2] , Cachon et al. [22] and Gaur and Honhon [55].

2.2 Markdown and clearance pricing

The literature on markdown pricing has picked up in the last decade since the publication of the survey article by Elmaghraby and Keskinocak [41]. In their survey, Elmaghraby and Keskinocak, divide the literature on the basis of replenishment of inventory. In this thesis, we are concerned with dynamic pricing in the absence of inventory replenishment. Retailers offer two types of markdown pricing, a temporary markdown especially during specific holidays like Halloween and Thanksgiving and a permanent one near the end of a season when they need to get rid of existing stock to make way for goods for the next season. The economics literature attempts to determine the causes of markdown and clearance pricing by retailers while the operations management literature looks at the mechanics of markdown and clearance pricing in the presence of inventory considerations by the retailer.

Lazear [71] develops a theory of clearance pricing based on demand uncertainty of products. He postulates that with the increasing product variety, the retailer's knowledge of customer demand for the products has decreased. With this increase in demand uncertainty, the retailer is forced to mark-up all the varieties by more than necessary and then mark-down those that did not sell. In a pair of papers, Pashigian ([92], [93]) uses empirical data from the 1930s to the 1980s to find support for Lazear's theory of fashion and demand uncertainty. Nocke and Peitz [89] show that clearance sales is the optimal pricing policy for a retailer in contrast to earlier literature that showed that a constant price plan is optimal when considering inter-temporal pricing. Herbon et al. [61] counter that a fixed price policy would be preferable in an environment where there is a large variance in the valuation of customers and they carry out a simulation experiment to show this.

Feng and Gallego [47] determine the optimal time to markdown when the retailer knows the two prices, before and after markdown price. Bitran and Mondschein [17] study a markdown pricing problem where the retailer starts with a fixed inventory that has to be liquidated within a specific time period. They model the stochastic arrival of customers with heterogeneous valuations for the product and determine the optimal price path that the retailer should take in clearing the inventory. They find that a discrete set of markdown prices is almost as good as a continuous policy of price and inventory. They extend the method to coordinating the prices across multiple stores in Bitran et al. [16]. In a pair of papers ([104], [102]), Smith develops an optimization model for clearance sales for a retailer where the demand rate is a function of price as well as the remaining inventory. He and Achabal [102] develop the framework for clearance markdowns based on the following four ways in which clearance prices are different from other retail pricing decisions: (1) Clearance markdowns are permanent, i.e., that prices are not permitted to increase, (2) as the assortments are broken near the end of the season (not all colors or sizes may be available), demand is lower, (3) clearance prices are not advertised or if advertised, then the advertisement is carried for a few items only and (4) the clearance period is so short that the retailer has little time to correct for his error. Mantrala and Rao [80] develop a computer based decision support system to help retailers with developing various clearance pricing scenarios. Chan et al. [27] provide a comprehensive technical review of the important papers published on markdown and clearance pricing in the operations management literature prior to 2003.

Chou and Parlar [33] study the issue of optimal pricing when the inventory is fixed at the start of a finite horizon and the decision of the retailer is to determine the optimal price in each period. They assume a non-negative demand that is a function of price only. Gupta et al. [59] consider the same problem with demand that is stochastic and arbitrarily correlated across the planning periods. Unable to develop optimal pricing policies, they derive bounds on the optimal expected revenue and optimal prices. Kogan and Spiegel [68] consider demand to be a function of time and price, where demand decreases with time. They provide an example of a bookstore retailer that implemented their model. Cachon and Kök [20] study clearance pricing using the newsvendor model with the salvage value being the clearance price. They develop several methods of estimating this salvage value and then determine the estimate that leads to a near optimal ordering decision for a newsvendor. As compared to the previous work, where the demand function for clearing the inventory is deterministic even when the regular demand is stochastic, Karakul [65] considers a stochastic formulation for the demand for the clearance market and the regular season. Aydin and Ziya [9] develop optimal pricing policies for non-replenishable products when the demand is a function of time, inventory level and personal information about a consumer. Caro and Gallien [23] develop a clearance model for a fast-fashion retailer and share the details of their implementation methodology in Zara, the firm that introduced fast-fashion in the retail market.

There are some specialized models for clearance pricing with demand learning - Chung et al. [34], Gaul and Azizi [54] and Kwon et al. [70].

Aydin and Ziya [8], Gandhi et al. [51] and Coşgun et al. ([36], [37]) extend the analysis of markdown pricing from single products to multiple products.

Lee [72] develops a model to study coordination when the product is sold in two periods by two different firms, during the normal period by the manufacturer directly and during the markdown period through a discount retailer. Chen [29] deals with coordinating decisions between a retailer and a manufacturer under pre-determined markdowns and promotional effort by the retailer. Nair and Closs [87] use simulation to look at various aspects of supply chain coordination when a retailer deals with markdowns. Chung et al. [35] study the stocking and pricing decisions in a three-tier supply chain when the markdown is initiated by the firm at the top-most tier.

There has been a flurry of research considering strategic consumer behavior in the context of dynamic pricing and quite a few papers deal specifically with markdown and clearance pricing. For an overview of this literature, see Shen and Su [101], who cover papers before 2006 and Gönsch et al. [58] who in addition cover the papers between the period 2006 to 2012. In our review, we consider some of the more relevant papers. Strategic consumer behavior is taken to mean that the consumer takes into account the possibility of future price breaks in considering when to purchase a product. Su [107] considers the heterogeneity of customers along two dimensions - along product valuation and waiting time costs in his model of strategic consumer behavior when a retailer has to dispose a fixed inventory in a finite time horizon. In a follow-up paper, Su and Zhang [108] look at the possible recourse actions available to a retailer facing strategic consumers to improve profits. Elmaghraby et al. [40] consider strategic behavior of consumers who want to procure multiple units rather than a single unit. Yin et al. [115] consider the role of stock information on the strategic behavior of consumers.

The above papers dealt with strategic behavior by consumers who have no inkling of the exact future price that the retailer would seek. But there are some retailers who post future prices. Elmaghraby et al. [42] show that such a policy may not necessarily be advantageous to the consumer. Aviv and Pazgal [6] look at fixed-discount strategies employed by the retailer against contingent pricing and determine the conditions when one is preferable to the other. Gallego et al. [50] show that a two-stage markdown is optimal when not all consumers are strategic. Zhang and Cooper [117] assume that the retailer may choose not to make available the product in the second period but find that such a policy offers substantial benefits only in a few cases. Cachon and Swinney [21] consider a markdown problem with three classes of customers - myopic, bargain-hunting and strategic. The myopic consumer purchases at full price, the bargain-hunting consumer buys if the discounted price is sufficiently low and the strategic consumer who chooses between a purchase at full price or waits to purchase at an uncertain markdown price in the future. At full price, the consumer is assured of getting the product which is not the case if she chooses to wait to purchase at the markdown price. Mersereau and Zhang [82] consider the situation when the retailer knows that some of his customers are strategic but does not know the proportion of customers who act strategically.

2.3 Inventory and pricing models for dual channels

The literature on inventory models for dual channels is fairly recent, having developed only in the last four years. In this section, we take a detailed look at each of the papers that are in this field. Agatz, Fleischmann and van Nunen [1] provide a recent survey on the literature in this area.

Bendoly [13] is possibly the first paper to look at inventory issues related to satisfying dual-channel demand from the same inventory stock. He develops a simulation model to understand two pooling effects - pooling of inventory in a single store to satisfy the two sources of demand and pooling of inventory across several stores by means of transshipment to satisfy web demand. Bendoly et al. [14] look at the same two strategies that dual channel retailers use in servicing web demand - a decentralized strategy of servicing from a firm's stores and a centralized strategy of satisfying the web demand from a central warehouse. Using numerical experiments for a two-echelon order-up-to inventory system, they find that when the web demand is a small portion of total demand, the firm should use a decentralized strategy to service web demand. When the proportion exceeds a certain threshold then the firm should use a centralized strategy. The threshold is a function of total system demand and desired service levels. Bretthauer et al. [19] develop a branch-and-bound methodology to solve the above problem. Mahar et al. [76], consider the aspect of dynamically assigning fulfillment locations for the on-line order. Whereas previous research had looked at determining the optimal static assignment policy of on-line sales to store locations, Mahar et al. show that firms can improve their competitive position by investing in technology that provides continuous monitoring of on-line demands and inventory positions. As in previous research, they find that the percentage of sales occurring on-line plays a critical role in determining the size of the economic benefit. They extend their model to a multi-period setting in Mahar et al. [77]. In a related paper, Mahar and Wright [79] look at the possibility of delaying the assignment decision till sales have accumulated at a location rather than assigning the fulfillment location as soon as a sale is made. As expected this leads to reduced inventory costs at the fulfillment sites and the magnitude of the benefit is again dependent on the proportion of sales occurring on-line. In a subsequent paper, Mahar et al. [78] show that a dual-channel is better off allowing its web customers to pick-up their orders from a selected set of stores rather than from each and every store that it controls. Seifert et al. [99] take a comprehensive look at the various issues considered by the authors cited in this section.

The above papers deal with inventory issues related to dual-channel retailers. But, how important is availability of stock to the consumer? And can it become a competitive weapon in the hands of a chain retailer against a web-only retailer? These issues are discussed next. Bendoly et al. [15] determine that dual-channel firms should encourage the transparency of their efforts to integrate their channels in ways that assuage the consumer of greater product availability in his firm as compared to a competitor. Cattani et al. [24] look at the choices that a dual-channel retailer can make in their choice of types of products to offer, perishable or non-perishable, to the customers and determine which of two strategies for fulfillment, using current stores or develop a separate fulfillment warehouse, is suited to each offering. Hu and Li [64] consider the effect of service efforts that cannot be measured in the on-line channel as compared to the physical channel, such as the ability for the consumer to touch and feel the product, and develop a model to study these issues for the retailer. Basu [11] looks at the problem of coordinating prices in the on-line and physical channel for a dual-channel retailer.

In contrast to the study of dual channels operating on a single level in the above models, Alptekinoglu and Tang [4] develop a general dual-channel two-echelon model based on the newsvendor model of Eppen and Schrage [43]. They do this by decomposing the multiple warehouses and multiple stores problem into several single warehouse and multiple store problems and then determining the optimal allocation among the several warehouses. Inventory is held only in the stores and the warehouse is just a cross-docking facility. With this model in place, the authors study the trade-offs involved when satisfying the web channel from either the existing warehouses or the existing stores. Using numerical experiments, they show that depending on the correlation of demand between the web and store channel, there is a threshold of correlation below which it is more cost-effective to satisfy the web demand from the stores. Chiang and Monahan [67] study the centralized strategy of servicing web demand from the upper echelon warehouse while servicing the store demand from the lower level echelon using a Markov model with a one-for-one inventory policy. They allow for customers to shift to the other channel with fixed probabilities when their normal channel is stocked out. They are able to find numerically the base stock levels for both echelons. Using numerical experiments, they find that increasing the willingness of customers to switch channels when a stock out occurs can possibly increase total inventory costs. Chiang [32] uses the above model to study how the nature of product availability is affected by product substitution when the two levels are with different firms rather than a single centralized firm. Nagao et al. [85] assess the cost of tracking information to the above model to develop the inventory control policy of the retailer. Chen et al. [28] extend the analysis to the situation where there are three firms, two retailers competing to supply the product to the customer of the third firm, who carries no inventory but utilizes the services of the two firms to have the product drop-shipped. Hsiao et al. [62] consider the strategic reasons a retailer could have in expanding from a chain retailer to a dual-channel retailer and look at the specific interactions between a chain retailer expanding into the web with a consumer manufacturer who is considering selling direct.

Chapter 3

Stock allocation for two substitutable products

3.1 Introduction

With Tesco's success in the on-line grocery market (sales of 1 Billion in 2004 [97]), interest has again risen in this sector of the industry. With traditional grocers struggling from lackluster growth in their primary market, the lure of a fast growing segment has induced many traditional grocery retailers (Tesco, Sainsbury's in the UK, Albertsons, Safeway and Peapod in USA) to setup an Internet channel. Peopod (USA) a subsidiary of Royal Ahold (Netherlands) states on its website that it grew sales by 25% in the past few years. The retailers use the same store to satisfy both channels - the web consumers and the traditional store customers who visit the store physically. The advantage that accrue to the retailer from using the same stock flow from two differences that web customers exhibit - the order quantities are known ahead of time and the retailer is allowed to make substitutions to their product choice. Thus giving the retailer the opportunity to save on inventory costs. Our objective in this study is to develop models that would allow the retailer to optimally benefit from this opportunity.

Our work is motivated by the following real life scenario. A grocery store serves customers from two channels — store and web. The web customer orders from the retail website and the store fulfills the order by having it delivered to the customer's home. The customer has to finalize his order by midnight of the day the order has to be delivered. Customers who physically go to the store to make purchases do so during the hours the store is open. The store opens in the morning and stays open till late night. Store customers who do not find the product they are looking for due to stock out may choose to buy a substitute brand. Not all customers who come to purchase a brand will buy a substitute but for purposes of this paper, we assume a fixed percentage does. This assumption is common in the literature (e.g. Parlar and Goyal (1984) [91] and Netessine and Rudi (2003) [88]). This percentage will vary with the different brands. For the web consumer the situation is different. The retailer makes the substitution for the customer. Because the customer is more likely to accept the product if the substitute is of better quality, the retailer offers the more expensive product at the same price of the ordered product to increase the substitution acceptance rate. (For example, part of Peapod's substitution policy states unequivocally "Whenever we make a substitution, you will never be charged more than the price of the original item ordered.")

Each morning, before serving the store customers, the store manager has to make decisions on how to fulfill the web orders. At the time of opening, she knows the available stock for each product and the size of the web orders. The manager does not know how many people who visit the store that day will purchase a given product but she knows the demand distribution for each product. Given the uncertainty in the demand from customers visiting the store and the quantity available, the store manager may consider it a prudent business decision to not satisfy the entire web demand but keep the stock aside for store customers. If she decides to keep more stock of a product for the store customers, she will need to decide whether to forgo the web sale or to satisfy the demand with a more expensive product as mentioned above. If she decided to substitute with a more expensive brand, she will lose on the margin. If she chooses not to make the sale, she will lose the customers' goodwill.

To model the above scenario we consider a stylized model of a retailer selling two substitutable products 1 and 2. The retailer has to decide on how to allocate the inventory so as to satisfy the web orders for both the products and the quantity to set aside for the store customers. Web orders for product 1 can be satisfied by giving the web customer either product 1 or product 2, both at the lower price of product 1. Web orders for product 2 can be satisfied only by giving the web customer product 2. Note that we are assuming that all web customers are willing to accept substitution. The case where only a fraction will do can be easily handled by our model.

The remainder of the chapter is organized as follows: In §3.2 we develop our model and in §3.3 we present some structural properties of the solution and develop some insights from numerical experiments. Finally we conclude in §3.4 with some ideas for future research.

3.2 Model development

We consider a single-period model where a retailer sells two products through two channels - web and store. The same inventory stock is used to satisfy demand in both channels. The two products, 1 and 2, differ in quality and this is reflected in the selling price, with the higher quality product having the higher selling price. The two products are assumed to be substitutes: when one product is stocked out, a fraction of the consumers would purchase the other one instead.

The exact substitution pattern depends on the channel. In the web channel the retailer decides on the substitution while in the store channel the consumer decides on the substitution. Store consumers substitute when their favorite product is stocked out. Not all store consumers who came to purchase a product will necessarily substitute but we assume there is a fixed proportion who do. Since the substitution decision lies with the consumer, the price paid by the consumer is the selling price of the product.

The retailer substitution on the web is a one-way substitution, with the retailer substituting the less expensive product with the more expensive one. Since the customer does not choose the substitute product, the retailer offers the more expensive product at the lower selling price. Also, the retailer need not be stocked out to make the substitution. She may choose to strategically substitute for the sole purpose of keeping more inventory of product 1 for the store customers.

The two channels also differ in the nature of demand. The web demand for both products is known at the beginning of the period. The store demand for both products is uncertain with known probability distributions. As in previous studies, Parlar and Goyal (1984) [91] and Netessine and Rudi (2003) [88], we assume that the demands for the products are independent.

3.2.1 Customer segments based penalty costs

In our model we have chosen to take into account the differences in utility exhibited by the two segments of customers - store loyal and brand loyal. For any customer when their favourite product is stocked out, we assume they have two choices - switch products or switch stores. In both choice there is disutility for the customer from carrying out that task. For store loyal customers, the disutility from switching products is less as compared to the disutility from switching stores and vice versa for brand loyal customers. Hence we assign different penalty costs for the store loyal and brand loyal customers. We take the service level to be the probability of no stock out. Thus for brand loyal customers this is the probability that their brand is not stocked out. For store loyal customers this will be the probability that both brands are not stocked out. Hence for any given service level for a brand loyal customer, the corresponding service level for the store loyal customer will be higher. In reverse, the penalty costs for the store loyal customer will be lower than the penalty costs for the brand loyal customer for the same service level.

3.2.2 Notation and assumptions

We use the following notation:

S_i	random store demand for product i with density $f_i(.)$, $i = 1, 2$
w_i	deterministic web demand for product i, $i = 1, 2$
Q_i	stock in hand for product i, $i = 1, 2$
r_i	product i's selling price, $i = 1, 2, r_2 > r_1$
v_i	product i's salvage value, $i = 1, 2$
p_w	penalty cost incurred for web customers
p_s	penalty cost incurred for store loyal customers
p_b	penalty cost incurred for brand loyal customers, $p_b > p_s$
δ_i	proportion of product i's store customers who are store loyal, $i = 1, 2$
$(1 - \delta_i)$	proportion of product i's store customers who are brand loyal, $i = 1, 2$.

Decision variables

x_{ij} proportion of web demand for product i satisfied from product j stock $i \leq j; i, j = 1, 2$.

In the current literature on substitution the penalty cost has been assigned on a per product basis. Since we are interested in the inventory allocation between channels, we differentiate between customers seeking each channel and hence determine penalty costs based on customer segmentation. In addition, to differentiate between customers who show differences in their substitution behavior in the store, we further segment this class of customers into store loyal and brand loyal customers. Store loyal customers would rather substitute with the other product than leave the store. Brand loyal customers on the other hand will walk out of the store if their favorite product is missing. Thus we end up with three classes of customers: the web customer, the store loyal customer and the brand loyal customer. For the web channel, the retailer is penalized, only, when the web consumer is not serviced with a product. This means that for a web customer of product 1 serviced with product 2, there is no additional penalty beyond the loss in margin, $r_2 - r_1$, suffered by the retailer. For a store loyal customer the penalty cost is assessed only if both products are stocked out and for the brand loyal customer, the penalty cost is assessed when the product sought is stocked out. As in previous studies (e.g. [91], [88]) we assume that the demands for the products are independent.

The primary decision of the retailer is to decide on the allocation of the on-hand inventory for both products to the two channels. She has to make three decisions: (1) the proportion of product 1 web demand that should be satisfied from product 1 stock, x_{11}, (2) the proportion of product 1 web demand satisfied from product 2 stock, x_{12}, and, (3) the proportion of product 2 web demand to be satisfied from product 2 stock, x_{22}. Any remaining portions of web demand for either product that is not satisfied is considered a lost sale. The decisions are to be made at the beginning of the period before the store demands are realized but after the web demands are known.

To illustrate the trade-offs involved in these decisions, we provide a simple example that ignores penalty costs (without loss of generality, we assume the purchase costs for both products are zero and that there are no other costs).

Example 1. *Assume there was 1 unit of product 2 and 1 unit of product 1 in stock and 1 unit of web demand for product 1 at the beginning of the period. By satisfying the web demand for product 1 with product 2, the retailer makes the unit of product 1 available to the store. Now assume that the store demand for the period turns out to be* 1 *for product 1 and* 0 *for product 2. In this case the retailer would get a total* *revenue of* $2r_1$*. However, if the store demand had turned out to be* 0 *for product 1 and* 1 *for product 2, then the retailer revenue would be* $(1+\delta_2)r_1$*. Suppose now that the retailer had instead used product 1 to satisfy the web demand for product 1. The retailer would then have realized a total of* $r_1+\delta_1 r_2$ *if the store demand had turned out to be* 1 *for product 1 and* 0 *for product 2 and* r_1+r_2 *if the store demand had turned out to be* 0 *for product 1 and* 1 *for product 2. Thus we see that even for such a simple problem, the retailer could end up with four different revenue levels given the allocation by the retailer and the subsequent store demand realizations. These different scenarios are illustrated in Table 1 using our notation.*

Allocation (x_{11}, x_{12}, x_{22})	*Store Demand Realization* (s_1, s_2)	*Revenue*
$(0,1,-)$	$(1,0)$	$2r_1$
	$(0,1)$	$(1+\delta_2)r_1$
$(1,0,-)$	$(1,0)$	$r_1+\delta_1 r_2$
	$(0,1)$	r_1+r_2

Table 3.1: Different revenue scenarios for Example 1, $Q_1=1, Q_2=1, w_1=1, w_2=0$.

With penalty costs, the scenarios are more complex and thus the need for a model which will provide this analysis for the retailer.

3.2.3 Expected revenue function

The retailer derives revenue from the web and store customers. The expected revenue derived from web customers is

$$R_w = r_1 x_{11} w_1 + r_1 x_{12} w_1 + r_2 x_{22} w_2 - p_w[(1 - x_{11} - x_{12})\, w_1 + (1 - x_{22})\, w_2].$$

The first term on the right is the revenue from servicing web demand for product

1 with product 1 stock, the second term is the revenue from servicing web demand for product 1 with product 2 stock, the third term is the revenue generated from servicing web demand for product 2 and the fourth and fifth terms are the penalty costs for any shortage in product 1 and product 2 respectively.

To determine the revenue from store customers, we consider four cases. Depending on the realized values of the store demand for product 1 and product 2, s_1 and s_2 respectively, the revenue function takes one of these expressions. We let Q_i^s be the stock of product i that is left in store after satisfying web demand, i.e., $Q_1^s = Q_1 - x_{11}w_1, Q_2^s = Q_2 - x_{12}w_1 - x_{22}w_2$.

Case 1: $s_1 \leq Q_1^s, \quad s_2 \leq Q_2^s$

$$R_s^1 = r_1 s_1 + r_2 s_2 + v_1(Q_1^s - s_1) + v_2(Q_2^s - s_2)$$

There is excess stock of both products and hence there will be no substitution.

Case 2: $s_1 > Q_1^s, \quad s_2 > Q_2^s$

$$\begin{aligned} R_s^2 &= r_1 Q_1^s + r_2 Q_2^s - p_s[\delta_1(s_1 - Q_1^s) + \delta_2(s_2 - Q_2^s)] \\ &-p_b[(1-\delta_1)(s_1 - Q_1^s) + (1-\delta_2)(s_2 - Q_2^s)] \end{aligned}$$

Both products have excess demand. We assumed the shortage to be allocated proportionately between store loyal and brand loyal customers.

Case 3: $s_1 > Q_1^s, \quad s_2 \leq Q_2^s$

There is a shortage in product 1 and an excess of stock in product 2. We distinguish between the case where all store loyal customers are satisfied (Case 3a) and where they are not (Case 3b).

Case 3a) $\delta_1(s_1 - Q_1^s) \leq (Q_2^s - s_2)$

$$R_s^{3a} = r_1 Q_1^s + r_2 s_2 + r_2 \delta_1 (s_1 - Q_1^s) + v_2[Q_2^s - s_2 - \delta_1(s_1 - Q_1^s)] - p_b(1-\delta_1)(s_1 - Q_1^s)$$

Case 3b) $\delta_1(s_1 - Q_1^s) > (Q_2^s - s_2)$

$$R_s^{3b} = r_1 Q_1^s + r_2 s_2 + r_2 (Q_2^s - s_2) - p_s[\delta_1(s_1 - Q_1^s) - (Q_2^s - s_2)] - p_b(1-\delta_1)(s_1 - Q_1^s)$$

Case 4: $s_1 \leq Q_1^s, \quad s_2 > Q_2^s$

There is a shortage in product 2 and an excess of stock in product 1. We distinguish between the case where all store loyal customers are satisfied (Case 4a) and where they are not (Case 4b).

Case 4a) $\delta_2(s_2 - Q_2^s) \leq (Q_1^s - s_1)$

$$R_s^{4a} = r_1 s_1 + r_2 Q_2^s + r_1 \delta_2 (s_2 - Q_2^s) + v_1[Q_1^s - s_1 - \delta_2(s_2 - Q_2^s)] - p_b(1-\delta_2)(s_2 - Q_2^s)$$

Case 4b) $\delta_2(s_2 - Q_2^s) > (Q_1^s - s_1)$

$$R_s^{4b} = r_1 s_1 + r_2 Q_2^s + r_1 (Q_1^s - s_1) - p_s[\delta_2(s_2 - Q_2^s) - (Q_1^s - s_1)] - p_b(1-\delta_2)(s_2 - Q_2^s)$$

After integrating over the four regions and performing some algebra, the expected store revenue is

$$ER_s = \int_0^{Q_1^s} r_1 s_1 f_1(s_1) ds_1 + \int_0^{Q_2^s} r_2 s_2 f_2(s_2) ds_2$$

$$
\begin{aligned}
&+\int_{Q_1^s}^{\infty}[r_1Q_1^s - p_b(1-\delta_1)(s_1-Q_1^s)]f_1(s_1)ds_1 + \int_{Q_2^s}^{\infty}[r_2Q_2^s - p_b(1-\delta_2)(s_2-Q_2^s)]f_2(s_2)ds_2 \\
&+\int_0^{Q_1^s}\int_{Q_2^s}^{\phi_2(s_1)} \delta_2(r_1-v_1)(s_2-Q_2^s)f_1(s_1)f_2(s_2)ds_2ds_1 \\
&+\int_0^{Q_1^s}\int_0^{\phi_2(s_1)} v_1(Q_1^s-s_1)f_1(s_1)f_2(s_2)ds_2ds_1 \\
&+\int_0^{Q_1^s}\int_{\phi_2(s_1)}^{\infty} r_1(Q_1^s-s_1)-p_s[\delta_2(s_2-Q_2^s)-(Q_1^s-s_1)]f_1(s_1)f_2(s_2)ds_2ds_1 \\
&+\int_0^{Q_2^s}\int_{Q_1^s}^{\phi_1(s_2)} \delta_1(r_2-v_2)(s_1-Q_1^s)f_1(s_1)f_2(s_2)ds_1ds_2 \\
&+\int_0^{Q_2^s}\int_0^{\phi_1(s_2)} v_2(Q_2^s-s_2)f_1(s_1)f_2(s_2)ds_1ds_2 \\
&+\int_0^{Q_2^s}\int_{\phi_1(s_2)}^{\infty} r_2(Q_2^s-s_2)-p_s[\delta_1(s_1-Q_1^s)-(Q_2^s-s_2)]f_1(s_1)f_2(s_2)ds_1ds_2 \\
&-\int_{Q_1^s}^{\infty}\int_{Q_2^s}^{\infty}[p_s\delta_1(s_1-Q_1^s)+p_s\delta_2(s_2-Q_2^s)]f_1(s_1)f_2(s_2)ds_2ds_1
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_1(s_2) &= Q_1^s + (Q_2^s - s_2)/\delta_1 \\
\phi_2(s_1) &= Q_2^s + (Q_1^s - s_1)/\delta_2.
\end{aligned}
$$

The first expression, $\phi_1(s_2)$, can be thought of as the number of product 1 store consumers needed to sell all product 1 stock and the leftover product 2 stock. Similarly, the second expression, $\phi_2(s_1)$, is the number of product 2 store consumers needed to sell all product 2 stock and the leftover product 1 stock.

The retailer's expected revenue is determined by adding the expected revenue from the store channel to the revenue from the web channel.

$$
ER = R_w + ER_s
$$

In Theorem 2, we show that the expected revenue function is concave under a specific condition.

Theorem 2. *The retailer's expected revenue function is concave when* $(r_1 + p_s) \geq \delta_1(r_2 + p_s)$.

Proof. See Appendix A. □

The concavity conditions involve the product prices, the cheaper product substitution rate and the store penalty costs. The condition $(r_1 + p_s) \geq \delta_1(r_2 + p_s)$ is satisfied when $\delta_1 r_2 \leq r_1$, i.e., the price of the cheap product is at least δ_1% of the price of the expensive product. Given that various estimates of the substitution rates in cases of stock-outs have been found to be more that 40% [110], this implies that r_1 should be at least $0.4r_2$. We have studied the ratio of the lowest to highest prices of different product families in an on-line grocery store and found that the lowest ratio is 0.43 with an average of 0.79. Our findings are summarized in Table 3.2.

Product	**Lowest Price** (r_1)	**Highest Price** (r_2)	**Price Ratio** (r_1/r_2)
Cola (Regular or Diet)			
Cans (12x355mL)	4.69	5.29	0.89
Pasta Noodles			
Spaghetti (900g)	1.69	2.09	0.81
Lasagne (500g)	2.19	2.89	0.76
Baby Needs			
Canned beginner food (128ml)	0.59	0.69	0.86
Formula Concentrate (12x385mL)	32.99	38.99	0.85
Rice Cereal (227g)	3.19	4.29	0.74
Size 2 diapers (unit)	0.29	0.35	0.83
Dish Detergent (100ml)	0.23	0.53	0.43
Cat food (10g)	0.16	0.28	0.57
Ice Cream 2L	5.49	7.49	0.73
Strawberry Jam 500 ml	4.49	4.69	0.96

Table 3.2: Ratio of highest and lowest product prices in an on-line grocery store.

It is also worth mentioning that the condition in Theorem 2 is a sufficient but not necessary condition. Thus the range of values for which the revenue function is concave is larger. When these conditions are not met, we would use global optimization routines to solve the problem.

3.2.4 Model

We maximize the retailer's expected revenue given the proportion and stock constraints. The problem can be formulated as follows:

$$\begin{aligned} &\max \quad ER \\ &\text{s. t.} \\ &\qquad x_{11} + x_{12} \leq 1 \\ &\qquad x_{22} \leq 1 \\ &\qquad x_{11}w_1 \leq Q_1 \\ &\qquad x_{12}w_1 + x_{22}w_2 \leq Q_2 \\ &\qquad x_{11}, x_{12}, x_{22} \geq 0. \end{aligned} \tag{P}$$

The first two constraints together with the non-negativity constraint make sure that the decision variables, x_{11}, x_{12} and x_{22} lie between 0 and 1. The third and fourth constraints make sure that the web demand that is satisfied does not exceed the available stock.

3.3 Analysis

3.3.1 Structural Results

In this section we assume that the concavity conditions in Theorem 2 holds. We define the following

$$F_{11} = \frac{\partial ER}{\partial x_{11}}, F_{12} = \frac{\partial ER}{\partial x_{12}}, F_{22} = \frac{\partial ER}{\partial x_{22}}.$$

These first derivatives will enable us to see the conditions under which the retailer would benefit from using the stock of each product to satisfy web demand as opposed to keeping it aside for the store demand.

More detailed analysis can be done by rearranging the various terms that make up the first derivative expressions. We start with

$$\begin{aligned} F_{11} = w_1 \Bigg[& (r_1 - v_1 + p_w) - (r_1 - v_1 + \delta_1 p_s + (1-\delta_1)p_b) \int_{Q_1^s}^{\infty} f_1(s_1)ds_1 \\ & + \delta_1 (r_2 - v_2 + p_s) \int_0^{Q_2^s} \int_{Q_1^s}^{\phi_1(s_2)} f_1(s_1) f_2(s_2) ds_1 ds_2 \\ & - (r_1 - v_1 + p_s) \int_0^{Q_1^s} \int_{\phi_2(s_1)}^{\infty} f_1(s_1) f_2(s_2) ds_2 ds_1 \Bigg]. \end{aligned} \tag{3.1}$$

The first term is the marginal revenue that the retailer would get from satisfying web demand for product 1. It consists of the actual revenue r_1 minus the salvage value v_1 plus the penalty cost p_w that is avoided. The second term is the marginal revenue lost from not serving the store demand. There are four components to this term - the actual revenue lost, r_1, the salvage value obtained, v_1, the partial penalty cost due to not serving the store loyal customer and the partial penalty cost from not serving the brand loyal customer. The third term is the additional marginal revenue from servicing the store loyal customer who substitutes with product 2 when product 1 has been used to service the web demand. This term arises due to the fact that because one unit of product 1 was made available to the web customer, the retailer has now an opportunity to sell a unit of product 2 to the store loyal customer. The final term is the loss in marginal revenue from being unable to service the substitute demand from product 2 store customers. The third and fourth terms show how substitution results in additional revenues gained and lost.

For F_{22}, the full expression is

$$\begin{aligned} F_{22} = w_2 \Bigg[&(r_2 - v_2 + p_w) - (r_2 - v_2 + \delta_2 p_s + (1-\delta_2)p_b) \int_{Q_2^s}^{\infty} f_2(s_2) ds_2 \\ &+\delta_2(r_1 - v_1 + p_s) \int_0^{Q_1^s} \int_{Q_2^s}^{\phi_2(s_1)} f_1(s_1) f_2(s_2) ds_1 ds_2 \\ &-(r_2 - v_2 + p_s) \int_0^{Q_2^s} \int_{\phi_1(s_2)}^{\infty} f_1(s_1) f_2(s_2) ds_2 ds_1 \Bigg]. \quad (3.2) \end{aligned}$$

The analysis is similar to that of F_{11} except that now the focus is on the use of product 2's stock to service product 2 web demand as opposed to keeping product 2 stock for store customers.

Finally, we examine F_{12}.

$$F_{12} = w_1 \Bigg[(r_1 - v_2 + p_w) - (r_2 - v_2 + \delta_2 p_s + (1-\delta_2) p_b) \int_{Q_2^s}^{\infty} f_2(s_2) ds_2 + \delta_2 (r_1 - v_1 + p_s) \int_0^{Q_1^s} \int_{Q_2^s}^{\phi_2(s_1)} f_1(s_1) f_2(s_2) ds_1 ds_2 - (r_2 - v_2 + p_s) \int_0^{Q_2^s} \int_{\phi_1(s_2)}^{\infty} f_1(s_1) f_2(s_2) ds_2 ds_1 \Bigg]. \tag{3.3}$$

The value of F_{12} determines the difference in value from using product 2 to satisfy web demand for product 1 over keeping it in stock for servicing store customers. Here the first term is the marginal revenue from the web customer for product 1, the second term is the marginal revenue lost from the store customer with intent to purchase product 2, the third term is the marginal revenue from store customer who substituted with product 1 when product 2 is out of stock and the final term is the marginal revenue lost from store customers who substitute with product 2 when product 1 is sold out. The reason for the third term to be positive is because when product 2 is used to service web demand an opportunity arises to sell product 1 to a product 2 customer. The second and fourth terms are negative since they represent the loss from being unable to sell product 2 in the store to either the store customer who came looking for product 2 or was willing to substitute for it.

In Proposition 3 we show that product 2 can be used to satisfy web demand for product 1 only after serving the web demand for product 2.

Proposition 3. *a) When $w_2 \geq Q_2$, then $x_{12} = 0$.*

b) When $w_2 < Q_2$ and $x_{22} \neq 1$, then $x_{12} = 0$.

Proof. All KKT conditions are satisfied when $x_{12} = 0$ for the instance mentioned above. □

Proposition 3 is useful as it facilitates in solving Problem P. Depending on whether $x_{22} = 1$ or not problem P reduces to a couple of problems in two decision variables.

For purposes of the proofs in the next three propositions, we associate the following Lagrange multipliers with their respective constraints: λ_1 with $x_{11} + x_{12} \leq 1$, λ_2 with $x_{11}w_1 \leq Q_1$, λ_3 with $x_{12}w_1 + x_{22}w_2 \leq Q_2$ and λ_4 with $x_{22} \leq 1$.

When the web demand for product 1 exceeds its stock and the web penalty cost is higher than a given threshold, then we show in following proposition that a simple closed form allocation exists.

Proposition 4. *When penalty costs satisfy the following two conditions:*

1. $p_w > \delta_1 p_s + (1 - \delta_1)p_b$ *and*

2. $p_w > \delta_2 p_s + (1 - \delta_2)p_b + (r_2 - r_1)$,

the inventory allocation is calculated easily for the following cases:

			x_{11}	x_{12}	x_{22}
$w_1 > Q_1$	$w_2 < Q_2$	$w_1 + w_2 = Q_1 + Q_2$	$\frac{Q_1}{w_1}$	$\frac{(Q_2 - w_2)}{w_1}$	1
$w_1 \geq Q_1$	$w_2 \geq Q_2$	$w_1 + w_2 \geq Q_1 + Q_2$	$\frac{Q_1}{w_1}$	0	$\frac{Q_2}{w_2}$

Proof. For the four cases listed here, (i) $\lambda_1 \neq 0, \lambda_2 \neq 0, \lambda_3 \neq 0, \lambda_4 \neq 0$, (ii) $\lambda_1 \neq 0, \lambda_2 \neq 0, \lambda_3 \neq 0, \lambda_4 = 0$, (iii) $\lambda_1 = 0, \lambda_2 \neq 0, \lambda_3 \neq 0, \lambda_4 \neq 0$ and (iv) $\lambda_1 = 0, \lambda_2 \neq 0, \lambda_3 \neq 0, \lambda_4 = 0$, all KKT conditions are satisfied when $p_w > \delta_1 p_s + (1 - \delta_1)p_b$ and $p_w > \delta_2 p_s + (1 - \delta_2)p_b + (r_2 - r_1)$. Under these same conditions the values for x_{11}, x_{12} and x_{22} are as stated above. □

With the next proposition, we look at the feasibility of satisfying web demand for product 1 with product 2 stock prior to using product 1 stock. If the amount of product 2 stock is greater than the web demand for both products, as is often the case, then the retailer gets to benefit by keeping aside more product 1 stock for the store customer and using up product 2 stock for servicing both products' web demands. This is profitable for the retailer only under the conditions specified below.

Proposition 5. *When either of the following two sets of conditions hold, (i) $w_2 < Q_2$, $w_1 + w_2 \leq Q_1 + Q_2$ and $F_{12} > F_{11} > 0$ or (ii) $w_1 + w_2 \leq Q_2$, $F_{12} > F_{11}$ and $F_{12} > 0$, then the retailer should allocate as follows:*

$$
\begin{aligned}
x_{22} &= 1 \\
x_{12} &= \min\left(\frac{Q_2 - w_2}{w_1}, 1\right) \\
x_{11} &= 1 - x_{12}
\end{aligned}
$$

Proof. For the case where $\lambda_1 \neq 0, \lambda_2 = 0, \lambda_3 \neq 0, \lambda_4 \neq 0$, we find that when $w_2 < Q_2$, $w_1 + w_2 \leq Q_1 + Q_2$ and $F_{12} > F_{11} > 0$, all KKT conditions are satisfied for the values of $x_{11} = (w_1 + w_2 - Q_2)/w_1 = 1 - x_{12}$, $x_{12} = (Q_2 - w_2)/w_1$ and $x_{22} = 1$. Similarly for the case where $\lambda_1 \neq 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 \neq 0$, we find that when $w_1 + w_2 \leq Q_2$, $F_{12} > F_{11}$ and $F_{12} > 0$, all KKT conditions are satisfied for the values of $x_{11} = 0$, $x_{12} = 1$ and $x_{22} = 1$. Putting these two conditions together, we get the result in the proposition. □

In words, when the conditions for proposition 5 are satisfied, the retailer should allocate as follows:

- Satisfy all web demand for product 2 from product 2 stock.

- With the remaining product 2 stock, satisfy the web demand for product 1 till either all demand is satisfied or the product 2 stock runs out.
- Satisfy the remaining web demand for product 1 from product 1's stock.
- Leave the remaining stock for both products for store customers.

In the following proposition, we find conditions where the retailer is better off not servicing a product's web demand. We consider three scenarios, scenario a) where the retailer should not service only the web demand for product 1, scenario b) under which the retailer should not service the web demand for product 2 and scenario c) where it makes sense for the retailer to ignore the web demands for both products. In a) the entire stock of product 1 is kept aside for store customers but the web demand for product 2 is satisfied from product 2 stock. In b) the entire stock of product 2 stock is kept for store customers but the web demand for product 1 is satisfied from product 1 stock. In c) the entire stock of product 1 and product 2 stock are kept for store customers.

Proposition 6. *We have three cases when at least web demand for one product is not served:*

a) $x_{11} = x_{12} = 0$*, web demand for product 1 should not be satisfied, if (i)* $w_2 \geq Q_2$*,* $F_{11} \leq 0$ $F_{22} > 0$*, where* $x_{22} = Q_2/w_2$*, or (ii)* $w_2 < Q_2$*,* $F_{11} \leq 0$ *and* $(r_2 - r_1) \geq F_{22} > 0$*, where* $x_{22} = 1$*.*

b) $x_{22} = 0$*, web demand for product 2 should not be satisfied, if* $F_{11} > 0$ *and* $F_{22} < 0$*, where* $x_{12} = 0$*. In this case* x_{11} *can either be* 1 *or* Q_1/w_1*.*

c) $x_{11} = x_{12} = x_{22} = 0$*, web demand for product 1 and product 2 should not be satisfied, if* $F_{11} \leq 0$ *and* $F_{22} \leq 0$*.*

Proof. a) For case (i) either $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 \neq 0, \lambda_4 \neq 0$ or $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 \neq 0, \lambda_4 = 0$, we find that the KKT conditions are satisfied when $w_2 \geq Q_2$, $F_{11} \leq 0$ and $F_{22} > 0$, and the values for x_{11}, x_{12} and x_{22} are $0, 0, Q_2/w_2$ respectively. Similarly, for case (ii) $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 \neq 0$, when $w_2 < Q_2$, $F_{11} \leq 0$ and $(r_2 - r_1) \geq F_{22} > 0$, x_{11}, x_{12} and x_{22} take on the values $0, 0, 1$.

b) Under similar reasoning, all KKT conditions are satisfied when $F_{11} > 0$ and $F_{22} < 0$ for the following three cases (i) $\lambda_1 \neq 0, \lambda_2 \neq 0, \lambda_3 = 0, \lambda_4 = 0$, (ii) $\lambda_1 \neq 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0$ and (iii) $\lambda_1 = 0, \lambda_2 \neq 0, \lambda_3 = 0, \lambda_4 = 0$. For the three cases, the values for x_{11}, x_{12} and x_{22} are $(1, 0, 0)$, $(1, 0, 0)$ and $(Q_1/w_1, 0, 0)$ respectively. When $x_{22} = 0$, we know that the web demand for Product 2 is not satisfied.

c) For the case of $\lambda_1 \neq 0, \lambda_2 \neq 0, \lambda_3 \neq 0, \lambda_4 \neq 0$, when $F_{11} \leq 0$ and $F_{22} \leq 0$, the values of x_{11}, x_{12} and x_{22} are all 0. Hence, no web demand is satisfied and all the stock is kept for the store customers. □

3.3.2 Numerical Experiments

Our objective for this section is two-fold:

1. To study how the allocation proportions, x_{11}, x_{12} and x_{22}, and revenue change with substitution rates.

2. To study the difference in revenue when considered against a model where the retailer does not substitute.

We take the following parameters for the numerical experiments. Note that these settings lead to $x_{22} = 1$. We focus on this case to show the effect of retailer substitution. We chose the Erlang distribution as it represents the real demand situation better (e.g when compared to Uniform or Exponential) and is numerically more tractable than the Normal distribution. In addition, with the Erlang distribution we do not have to worry about negative values. Previous studies that have used the Erlang distribution, e.g. [26].

$r_1 = 9, r_2 = 10, p_w = 5, p_s = 3, p_b = 4, Q_1 = 80, Q_2 = 100, w_1 = 30, w_2 = 20$. For the demand, parameters for the Erlang distribution are as follows, $S_1 = Erlang(3, 0.06), S_2 = Erlang(3, 0.06)$, where the first parameter denotes the shape and the second parameter denotes the rate of the probability distribution.

Substitution rates

Figure 3.5 shows the change in decision variables with the change in substitution rates and Figure 3.6 shows the change in revenue with the change in substitution rates.

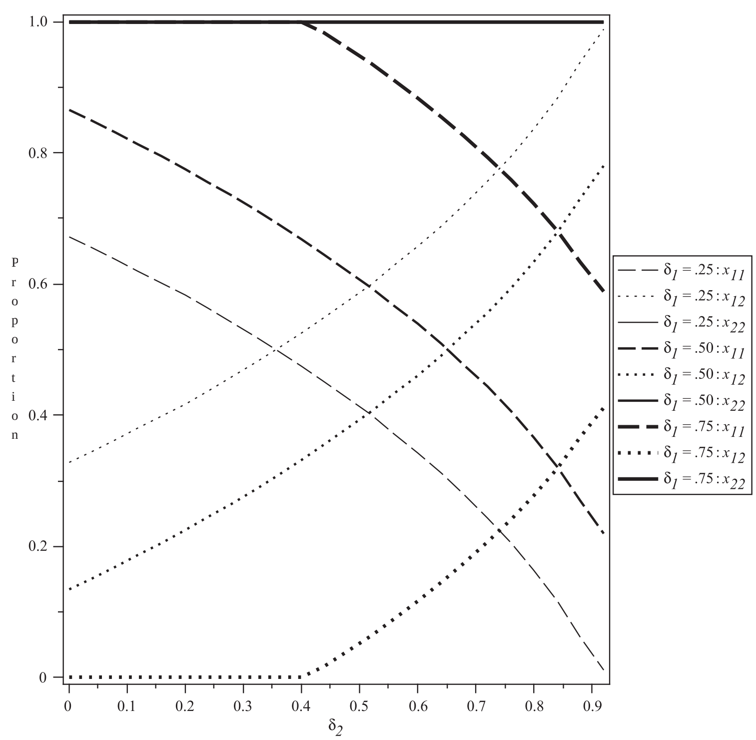


Figure 3.5: Decision variables vs substitution rates

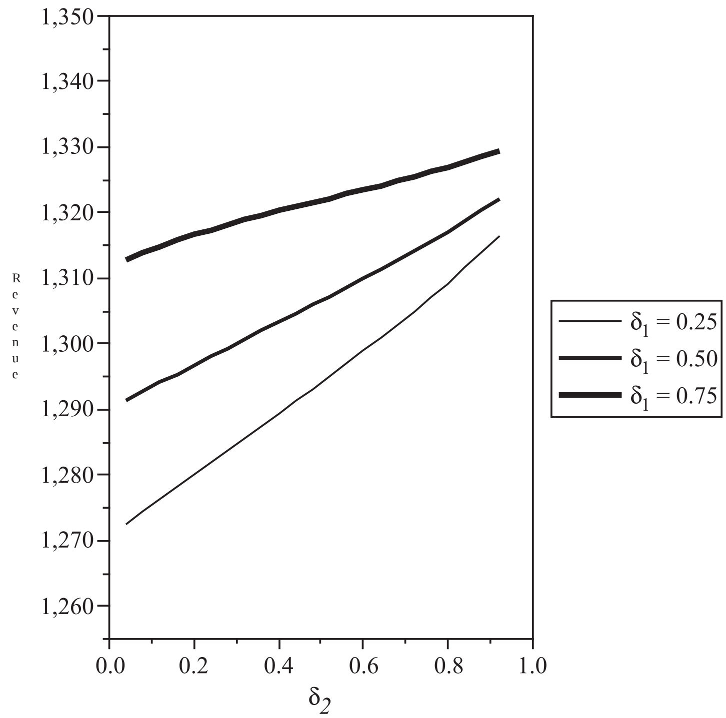


Figure 3.6: Revenue against substitution rates

We observe the following:

- As the proportion of store loyal customers for product 1 increases, the proportion of product 1's web demand satisfied by product 2 decreases and that satisfied by product 1 increases.

- As the proportion of store loyal customers for product 2 increases, the proportion of product 1's web demand satisfied by product 2 increases and the proportion of product 1's web demand satisfied by product 1 decreases.

- As the proportion of store loyal customers for either product increases, the retailer's revenue increases.

We first explain the substitution pattern. As the proportion of store loyal customers increases for product 1, the retailer needs to make less of product 1 available for the store. Being store loyal, they will substitute with product 2 if product 1 is stocked out. Hence the retailer has more of product 1 to sell to the web customer. As the proportion of store loyal customers of product 2 increases, the retailer needs less of product 2 to make available for the store. Being store loyal, they will substitute with product 1 if product 2 is stocked out. This allows the retailer to make more of product 2 available to satisfy the web demand - first for product 2 and then as retailer-substituted demand for product 1.

With the increase in the proportion of store loyal customers, there are less customers walking out of the store and hence the revenue increase.

Comparison with no web substitution

Figure 3.7 shows the revenue difference between the web substitution model (WS) and the no web substitution model (NWS).

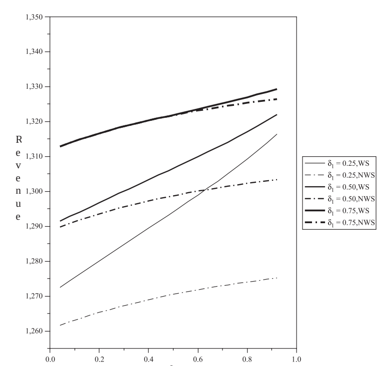


Figure 3.7: Revenue Comparison

We observe that in all cases the revenue in the web substitution model (WS) is at least as good as the revenue in the no web substitution model (NWS). We also observe that the revenue difference is maximum when δ_2 is high and δ_1 is low and lowest when δ_2 is low and δ_1 is high. In our numerical study we find that the revenue saving could be upto 3.5% (significant knowing that profit margins in grocery sector are 1-2% of sales [18]).

A possible explanation follows. The fact that the web demand happens prior to the store demand allows the retailer to realign the stock in hand with the expected distribution of store demand. This gain more than makes up for the loss suffered in margin when the retailer substitutes with the more expensive product a portion of the web demand for the cheaper product.

3.4 Conclusion

In this paper we have considered an inventory allocation model in a dual channel setting: a traditional store and a web channel. The store customer can substitute for stocked out items while the retailer makes the substitution for the web customer. In addition, we introduce penalty costs based on customer segmentation into the inventory literature as opposed to product-based penalty costs. We show that the retailer can benefit by servicing the web customer by the more expensive product at the price of the cheaper one. Numerically, we show that these savings are substantial in the grocery market.

Chapter 4

Use of the web channel to re-balance stocks

4.1 Introduction

Retailers are forever looking to increase market share in the face of ever increasing competition. New technologies enable a new entrant to dramatically gain market share and reshape an entire industry. The arrival of the Internet in the mid-nineties was one such technology. It allowed Amazon.com, a new player in the book industry to become the market leader in a matter of years. Retailers in other industries observed this event with alarm. They did not want to be 'Amazoned' in their own markets by an upstart. So without understanding the technology and its effect on customers, firms blindly created separate businesses to cater to this consumer segment. Often entirely separate business structures were put in place to satisfy this market. Most were money losing propositions for existing market leaders, since there was no synergy between their current businesses and the newer web business. Gradually some of these firms got to understand how the Internet channel customers were different from their other customers and they began to exploit this knowledge.

One of the firms that understood the Internet and knew how to synergise its current business with this new business and at the same time exploit the differences exhibited by the web customers is Tesco. Tesco, the largest grocer in the UK, when they got into the on-line grocery business decided to do things differently from web pioneers like Webvan and other grocers in the US market. Instead of using a centralized warehouse, they used their existing stores that serviced their regular clientle, to service the web customers. Orders of web customers were serviced from the store closest to the customer. They have been extremely successful and are the largest e-retailer in the UK. Their success drove other stores like the Safeway and Albertsons in the US to copy their model. The following two characteristics of the on-line grocery business affected the nature of demand and by extension the inventory held in the store. First, customers had to place their order ahead of time. Deliveries for orders on the next day had to be submitted by midnight of the previous day. Second, the web customer gave the power of selecting the substitute product to the retailer when faced with a stock-out. Given the large number of SKU's carried by a grocery store, there is no easy way for the customer to tell the retailer what to substitute when their ordered item is not available. The retailer seeks the approval of the customer to make the substitution on the customer's behalf and the default is for the retailer to make the substitution.

4.2 The problem

We use a similar setup to Chapter 3 here. The retailer derives revenue from the web and store channels. At the time of ordering both web and store demand is stochastic. We assume that delivery of the stock is immediate. By design, the web channel provides the retailer with a window of decision opportunities: The time from when customers place their orders to the time the orders are fulfilled. During this time the retailer allocates the two products stocks between the two channels. She has to make three allocation decisions: (1) the proportion of product 1 web demand to be satisfied from product 1 stock, x_{11}, (2) the proportion of product 1 web demand to be satisfied from product 2 stock, x_{12}, and, (3) the proportion of product 2 web demand to be satisfied from product 2 stock, x_{22}. Any remaining portion of web demand for either product that is not satisfied is considered a lost sale.

We maximize the retailer's expected profit given the sequence of events outlined in Figure 4.8 by the following two-stage stochastic program formulation.

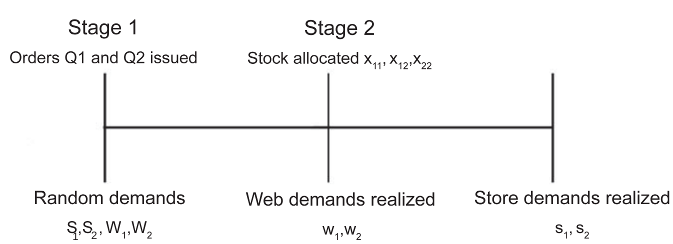


Figure 4.8: A time-line of the decisions in joint ordering and allocation

Decisions are made in two stages. In the first stage or at the beginning of the period, the retailer knows only the distributions of the demand in the two channels for each product. She decides on the ordering quantities, Q_1 and Q_2, knowing that an opportunity to re-allocate the stocks between the two channels will arise before the demands are satisfied. This second stage happens when the web orders are received and the distributions for the store demands are updated. At this stage (second stage), the retailer looks at the stock in hand for the two products, Q_1 and Q_2, the realized web demands, w_1 and w_2, and then makes the allocation decisions mentioned earlier.

4.2.1 Notation and assumptions

We use the following notation:

S_i	random store demand for product i with density $f_i(.)$, $i = 1, 2$
W_i	random web demand for product i with density $g_i(.)$ and mean μ_{wi}, $i = 1, 2$
D_i	total retailer demand for product i, $i = 1, 2$
$\theta(s_i, w_i)$	joint density of S_i and W_i
$h_i(s_i\|w_i)$	updated store density when the observed value of web demand is w_i, $i = 1, 2$
	$= \frac{\theta(s_i,w_i)}{g_i(.)}$
x_{ij}	proportion of web demand for product i
	satisfied from product j stock, $i \leq j; i, j = 1, 2$
Q_i	ordering quantity for product i, $i = 1, 2$
Q_i^s	stock of product i that is left in store after satisfying web demand
r_i	product i's selling price, $i = 1, 2, r_2 > r_1$
v_i	product i's salvage value, $i = 1, 2, v_i \leq r_i$
p_w	penalty cost for web customers, $p_w \geq 0$
p_s	penalty cost for store loyal customers, $p_s \geq 0$
p_b	penalty cost for brand loyal customers, $p_b \geq 0$

c_i	purchase cost for product i, $c_i < r_i$, $i = 1, 2$
δ_i	substitution rate for product i, $i = 1, 2$
E_W	expectation over the web demand distributions
ES	expected revenue in the second stage

4.2.2 Value function

We first develop the formulation for the entire problem and then show in some detail the formulation for the second stage.

$$\Pi(Q_1^*, Q_2^*) = \max_{\substack{Q_i \geq 0 \\ i=1,2}} \quad \{\Pi(Q_1, Q_2) = -\sum_{i=1}^{2} c_i Q_i + E_W[\Omega(Q_1, Q_2, w_1, w_2)]\} \tag{P}$$

where $\Omega(Q_1, Q_2, w_1, w_2)$ is the optimal revenue of the second stage problem as formulated below.

$$\max \quad ES \tag{4.4}$$

s. t.

$$x_{11} + x_{12} \leq 1 \tag{4.5}$$

$$x_{22} \leq 1 \tag{4.6}$$

$$x_{11} w_1 \leq Q_1 \tag{4.7}$$

$$x_{12} w_1 + x_{22} w_2 \leq Q_2 \tag{4.8}$$

$$x_{11}, x_{12}, x_{22} \geq 0. \tag{4.9}$$

Second stage problem formulation

The second stage problem is a problem of optimal allocations between the two products in the two channels. We develop the expression for the expected retailer revenue by determining the revenue from the web channel first. The stock left from servicing the web channel is used to satisfy the store channel.

Depending on the realized values of the web demands for product 1 and product 2, w_1 and w_2 respectively, the revenue from the web takes the following expression:

$$R_w = r_1 x_{11} w_1 + r_1 x_{12} w_1 + r_2 x_{22} w_2 - p_w(1 - x_{11} - x_{12})w_1 - p_w(1 - x_{22})w_2 \quad (4.10)$$

The quantities left for the store customers can be calculated as follows:

$Q_1^s = Q_1 - x_{11}w_1, \quad Q_2^s = Q_2 - x_{12}w_1 - x_{22}w_2$

To determine the revenue from store customers, R_s, we look at four cases of realized store demand values:

Case 1: $s_1 \leq Q_1^s, \quad s_2 \leq Q_2^s$

$$R_s^1 = r_1 s_1 + r_2 s_2 + v_1(Q_1^s - s_1) + v_2(Q_2^s - s_2)$$

There is excess stock of both products and hence there will be no substitution.

Case 2: $s_1 > Q_1^s, \quad s_2 > Q_2^s$

$$\begin{aligned} R_s^2 &= r_1 Q_1^s + r_2 Q_2^s - p_s[\delta_1(s_1 - Q_1^s) + \delta_2(s_2 - Q_2^s)] \\ &- p_b[(1-\delta_1)(s_1 - Q_1^s) + (1-\delta_2)(s_2 - Q_2^s)] \end{aligned}$$

Both products have excess demand. The shortage is allocated proportionately between store loyal and brand loyal customers.

Case 3: $s_1 > Q_1^s, \quad s_2 \leq Q_2^s$

There is a shortage in product 1 and an excess of stock in product 2. We distinguish between the case where all store loyal customers are satisfied (Case 3a) and where they are not (Case 3b).

Case 3a) $\delta_1(s_1 - Q_1^s) \leq (Q_2^s - s_2)$

$$R_s^{3a} = r_1 Q_1^s + r_2 s_2 + r_2 \delta_1(s_1 - Q_1^s) + v_2[Q_2^s - s_2 - \delta_1(s_1 - Q_1^s)] - p_b(1 - \delta_1)(s_1 - Q_1^s)$$

Case 3b) $\delta_1(s_1 - Q_1^s) > (Q_2^s - s_2)$

$$R_s^{3b} = r_1 Q_1^s + r_2 s_2 + r_2(Q_2^s - s_2) - p_s[\delta_1(s_1 - Q_1^s) - (Q_2^s - s_2)] - p_b(1 - \delta_1)(s_1 - Q_1^s)$$

Case 4: $s_2 > Q_2^s, \quad s_1 \leq Q_1^s$

There is a shortage in product 2 and an excess of stock in product 1. We distinguish between the case where all store loyal customers are satisfied (Case 4a) and where they are not (Case 4b).

Case 4a) $\delta_2(s_2 - Q_2^s) \leq (Q_1^s - s_1)$

$$R_s^{4a} = r_2 Q_2^s + r_1 s_1 + r_1 \delta_2(s_2 - Q_2^s) + v_1[Q_1^s - s_1 - \delta_2(s_2 - Q_2^s)] - p_b(1 - \delta_2)(s_2 - Q_2^s)$$

Case 4b) $\delta_2(s_2 - Q_2^s) > (Q_1^s - s_1)$

$$R_s^{4b} = r_2 Q_2^s + r_1 s_1 + r_1(Q_1^s - s_1) - p_s[\delta_2(s_2 - Q_2^s) - (Q_1^s - s_1)] - p_b(1 - \delta_2)(s_2 - Q_2^s)$$

After integrating over the six regions, the expected store revenue is

$$
\begin{aligned}
ER_s = & \int_0^{Q_1^s} \int_0^{Q_2^s} R_s^1 h_2(s_2|w_2) ds_2 h_1(s_1|w_1) ds_1 \\
& + \int_{Q_1^s}^{\infty} \int_{Q_2^s}^{\infty} R_s^2 h_2(s_2|w_2) ds_2 h_1(s_1|w_1) ds_1 \\
& + \int_{Q_1^s}^{\phi_1} \int_0^{Q_2^s} R_s^{3a} h_2(s_2|w_2) ds_2 h_1(s_1|w_1) ds_1 \\
& + \int_{\phi_1}^{\infty} \int_0^{Q_2^s} R_s^{3b} h_2(s_2|w_2) ds_2 h_1(s_1|w_1) ds_1 \\
& + \int_0^{Q_1^s} \int_{Q_2^s}^{\phi_2} R_s^{4a} h_2(s_2|w_2) ds_2 h_1(s_1|w_1) ds_1 \\
& + \int_0^{Q_1^s} \int_{\phi_2}^{\infty} R_s^{4b} h_2(s_2|w_2) ds_2 h_1(s_1|w_1) ds_1
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_1 &= Q_1^s + \frac{Q_2^s - s_2}{\delta_1} \\
\phi_2 &= Q_2^s + \frac{Q_1^s - s_1}{\delta_2}
\end{aligned}
$$

The revenue for the dual channel retailer in the second stage is obtained by summing the revenue from the web and store channels.

$$
ES = R_w + ER_s \tag{4.11}
$$

From this point onwards, we use independent demand distributions, in our analysis.

4.3 Solution method

We use the following method to solve the above problem.

1. Exhaustively search over the positive quadrant of Q_1 and Q_2 values for the pair that obtains the highest profit. For practical purposes, limit the values of Q_1 and Q_2 from 0 to Q_1^{ul} and 0 to Q_2^{ul} respectively. Q_1^{ul} and Q_2^{ul} are set to the maximum amounts that can be ordered from the supplier, limits set by the supplier.

2. For each pair of Q_1 and Q_2 values, determine the profit, from evaluating the following equation.

$$\Pi(Q_1^{ii}, Q_2^{jj}) = -c_1 Q_1^{ii} - c_2 Q_2^{jj} + E_W[\Omega(Q_1^{ii}, Q_2^{jj}, w_1, w_2)]$$
$$\forall \quad Q_1^{ii} = 0, \dots, Q_1^{ul}, \quad Q_2^{jj} = 0, \dots, Q_2^{ul} \tag{PIII}$$

3. We use the Monte Carlo method to evaluate the integral, $E_W[\Omega(Q_1^{ii}, Q_2^{jj}, w_1, w_2)]$. To do so, we select a random sample of N pairs of values from the web demand distributions and obtain an estimate for $E_W[\Omega(Q_1^{ii}, Q_2^{jj}, w_1, w_2)]$, thereby approximating the above problem to:

$$\hat{\Pi}(Q_1^{ii}, Q_2^{jj}) = -c_1 Q_1^{ii} - c_2 Q_2^{jj} + N^{-1} \sum_{k=1}^{N} \Omega(Q_1^{ii}, Q_2^{jj}, w_1^k, w_2^k)$$
$$\forall \quad Q_1^{ii} = 0, \dots, Q_1^{ul}, \quad Q_2^{jj} = 0, \dots, Q_2^{ul} \tag{PIIIa}$$

By the Strong Law of Large Numbers, $E_W[\Omega(Q_1^{ii}, Q_2^{jj}, w_1, w_2)]$ converges to $N^{-1} \sum_{k=1}^{N} \Omega(Q_1^{ii}, Q_2^{jj}, w_1^k, w_2^k)$ with probability one, as $N \to \infty$ at the rate of

$O_p(N^{-1/2})$ when choosing an iid sample. We can determine the confidence interval for the difference between $\hat{\Pi}(Q_1^{ii}, Q_2^{jj})$ and $\Pi(Q_1^{ii}, Q_2^{jj})$ (for details, see Bayraksan and Morton [12]).

4. We established the concavity conditions for $\Omega(Q_1^{ii}, Q_2^{jj}, w_1^k, w_2^k)$ in Chapter 3. We use the method described in Chapter 3 to solve for $\Omega(Q_1^{ii}, Q_2^{jj}, w_1^k, w_2^k)$.

4.4 Gain from re-balancing of stocks

To determine the gain from the re-balancing of stocks, we follow the steps enumerated below.

1. Solve the expected value problem, formulated below, in Section 4.4.1 for $Q_1 = Q_1^*, Q_2 = Q_2^*$. Let, the expected profit of the expected value problem be denoted by $\Pi(Q_1^{**}, Q_2^{**})$.

2. Gain from re-balancing is given by $[\Pi(Q_1^*, Q_2^*) - \Pi(Q_1^{**}, Q_2^{**})]$.

4.4.1 Expected value problem

The expected value problem is obtained by replacing the web demands by the mean of their distributions.

Model formulation

The retailer maximizes profit, subtracting the costs from the revenue obtained in the two channels.

$$\max_{\substack{Q_i, x_{ij} \\ i \leq j \\ i,j=1,2}} E\Pi' = -\sum_{i=1}^{2} c_i Q_i^* + \Omega(Q_1 = Q_1^*, Q_2 = Q_2^*, w_1 = \mu_{w1}, w_2 = \mu_{w2}) \tag{EVP}$$

Under the following assumptions, we determine the concavity of the above objective function.

1. $r_1 + p_w - c_1 > 0$
2. $r_2 + p_w - c_2 > 0$
3. $r_1 + p_w - c_2 > 0$

The first two are obvious. The retailer would only sell a product if she makes money on the sale. Hence, the cost of the product has to be less than the loss to the retailer in not making the sale. The third assumption makes sure that the retailer only substitutes with the more expensive product if marginal revenue from selling the expensive product at the cheaper price is more than the cost of the expensive product.

In Theorem 7, we show that the expected profit function is concave under two conditions.

Theorem 7. *The retailer's expected profit function in EVP is concave when $r_2 \leq r_1/\delta_1$ and $p_b \geq p_s$.*

Proof. See Appendix B. □

4.5 Numerical results

Our objective for this section is two-fold:

1. To study how the optimal ordering quantities, Q_1^* and Q_2^*, and optimal profit, $\Pi(Q_1^*, Q_2^*)$, change with substitution rates.
2. To study the effect of variance on the re-balancing gain.

We take the parameters in Table 4.3 for the numerical experiments.

r_2	r_1	c_2	c_1	p_w	p_b	p_s	v_2	v_1
10	8	5	4	5	4	3	0	0

Table 4.3: Parameter values

4.5.1 Substitution rates

In this section, we look at the effect of substitution rates on the optimal ordering quantities and profit. We assume exponential independent demands with values as in Table 4.4.

Demand	S1	S2	W1	W2
Mean	30	30	20	10
Variance	900	900	400	100

Table 4.4: Independent Exponential Demands

We look at the change in optimal values, Q_1^*, Q_2^* and $\Pi(Q_1^*, Q_2^*)$ across δ_1 and across δ_2 in Figures 4.9, 4.10 and 4.11 respectively. We also look at the percentage gain due to re-balancing in Figure 4.12.

We observe the following:

- The profit increases with *i*) increase in the substitution rate for product 1, δ_1 and *ii*) increase in the substitution rate for product 2, δ_2.
- With the increase in substitution rate for product 1, δ_1, the ordering quantity for product 1, Q_1, decreases and the ordering quantity for product 2, Q_2, increases.
- With the increase in substitution rate for product 2, δ_2, the ordering quantity for product 1, Q_1, increases and the ordering quantity for product 2, Q_2, decreases.
- With the increase in δ_1 and/or δ_2, the % gain decreases.

The increase in profits with the increase in the substitution rate can be explained by the increase in the effective demand for the product, as fewer customers leave the store due to stock-out. This leads to increased profits.

The behavior of the ordering quantities is interesting and at first glance puzzling. One possible explanation could be that when δ_1 increases, the number of customers of product 1 willing to substitute increases and the retailer exploits this fact and orders more Q_2 which has a better margin than Q_1. But there does not seem to be a reasonable explanation for the decrease in the ordering quantity for Q_2 upon increasing δ_2.

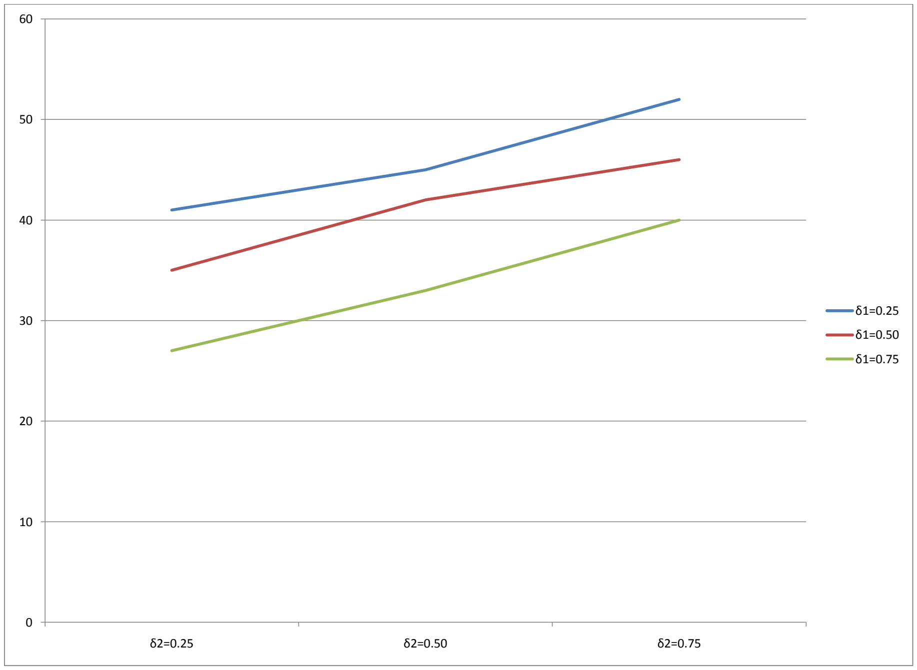


Figure 4.9: Optimal $Q1$

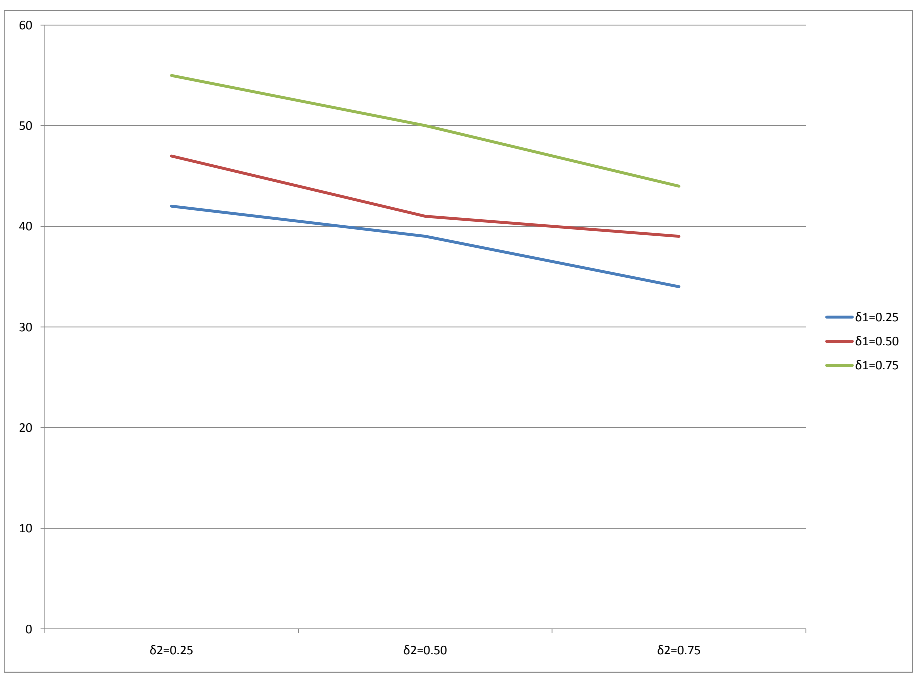


Figure 4.10: Optimal $Q2$

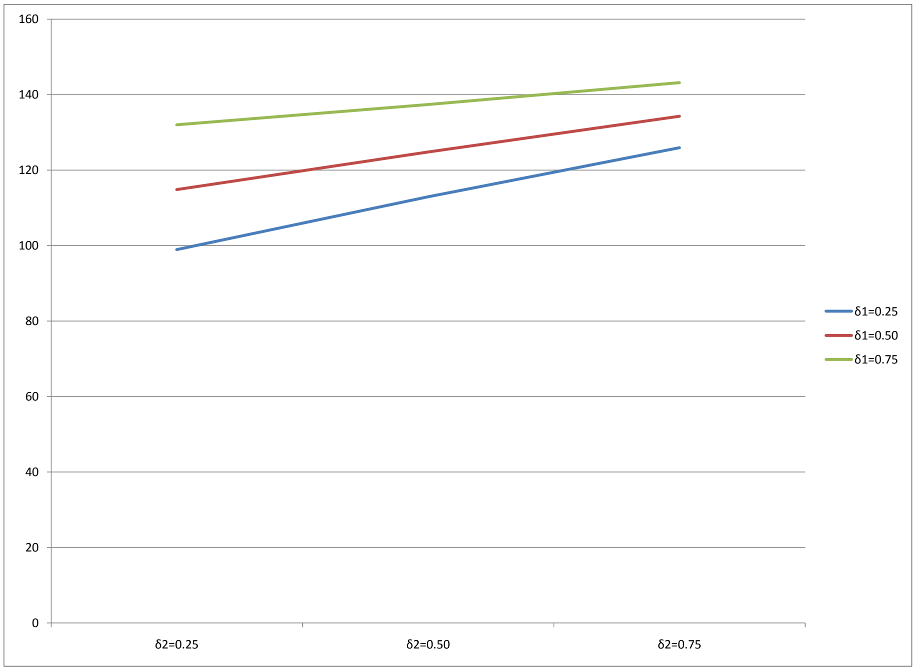


Figure 4.11: Optimal Profit

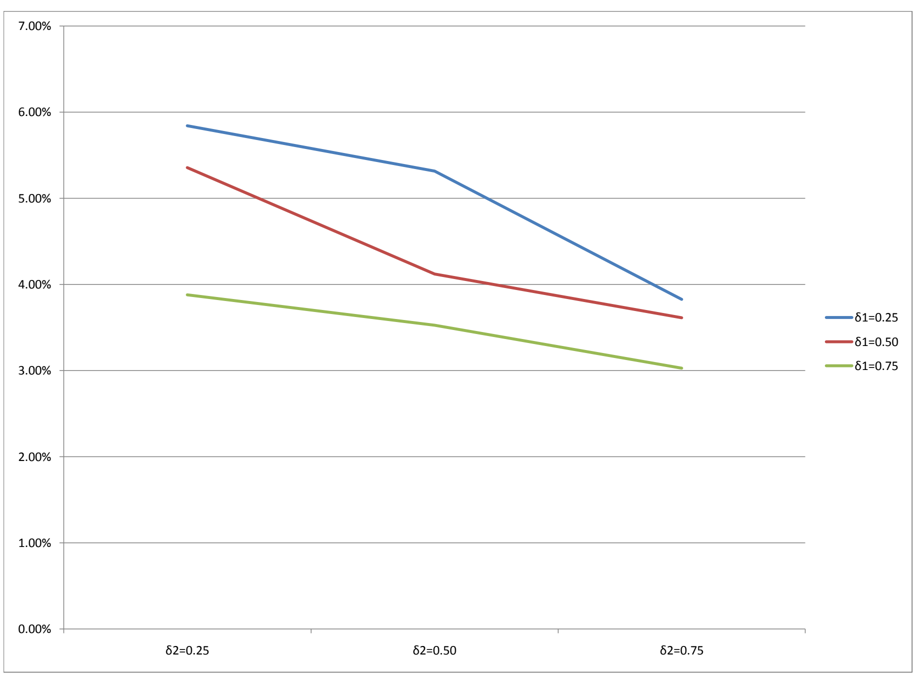


Figure 4.12: Effects of Substitution Rates on % Gain - Exponential Demands

4.5.2 Variance

Figure 4.13 shows the effect of variance on the re-balancing gains. We consider two levels of variance, low and high, using Erlang distributions (Table 4.5).

Demand	S1	S2	W1	W2
Mean	30	30	20	10
Low Variance	300	300	192	108
High Variance	900	900	576	324

Table 4.5: Independent Erlang Demands

We observe that the gain is higher when the variance among demands is high. This is to be expected as the re-balancing effort will decrease with lower variance among demands.

4.6 Conclusion

In this chapter we have considered an inventory ordering and allocation model in a dual channel setting: a traditional store and a web channel. The decisions are made in two stages - ordering in the first stage and allocation in the second stage. We considered two substitutable products with customer substitution in the store and retailer substitution on the web. We show that this substitution pattern for a dual channel retailer allows for a re-balancing of stocks enabling the retailer to make gains in profits.

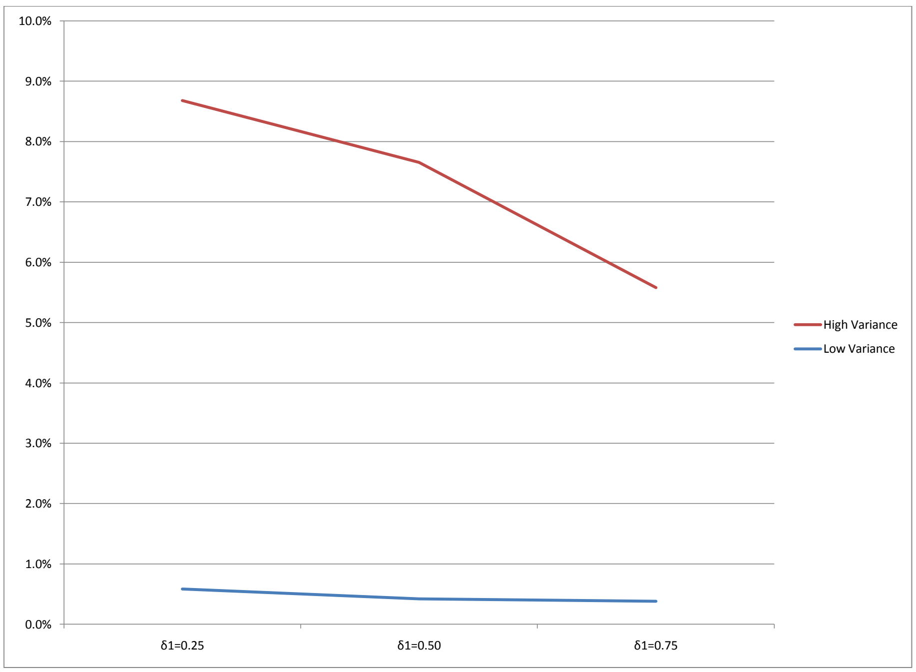


Figure 4.13: Effect of Variance on % Gain - Erlang Demands

Chapter 5

Clearance pricing

5.1 The Problem

We consider a problem faced by a catalog retailer in clearing stock from the two channels, the original catalog channel and a newer web channel. In a single period the retailer gets one price point opportunity to clear stock using the catalog as against two price points on the web. For example, Land's End mails out overstock catalog to customers for a season. Since the price is printed in the catalog the retailer does not get to change the price based on sales feedback during the early part of the season. On the web, however, the retailer gets to change the price based on sales feedback. For example, L.L.Bean another catalog retailer has two menu choices - New to Sale and Further Markdowns. Thus on the web the retailer can set prices in two stages. Considering the entire problem, we develop a two-stage stochastic program with the first stage representing the price for the first stage of the web and the price for the catalog and allocation ratios for the catalog and web channels while in the second stage the decision is on additional discount to be set to clear the stock in the web channel. The decisions in the two stages are determining the optimal allocation ratios between the two channels and percentage markdown of the product in the catalog channel and the web channel (first stage) and the optimal percentage markdown in the web channel (second stage). We assume that the two classes of customers are separate and have no knowledge of the prices in the other channel. Web consumers do not have access to the catalog as the catalog is mailed to existing customers only. Catalog customers may have access to the web but prefer the catalog for shopping purposes as a force of habit.

5.2 Model development

We consider a single period model where the catalog retailer uses two channels, web and catalog, to clear stock left over after regular sales for the season. For the catalog channel, the retailer publishes a separate catalog for clearance sales with the clearance price printed for each item. On the web, the retailer prices in two stages. In the first stage, the retailer posts a discounted price. Upon observing demand in the first stage, the retailer updates the web demand for the second stage and sets the discount to be charged in the second stage of web sales. We model a multiplicative form of demand. We model uncertainty only as regards the market potential and not with respect to price sensitivity.

5.2.1 Notation and assumptions

We use the following notation:

r	regular selling price of product for both catalog and web customers
η_g	markdown offered to catalog customers
η_{w_1}	markdown offered to web customers in the first stage
r_g	product's discounted selling price for catalog customers
	$= \eta_g r$
r_w	product's discounted selling price when first offered on the web
	$= \eta_{w_1} r$
η_{w_2}	markdown offered to web customers in the 2^{nd} stage
	(i.e. $\eta_{w_2} r_w$ is the clearance price)
p_g	penalty cost for catalog customers
p_{w_1}	penalty cost for first stage web customers
p_{w_2}	penalty cost for second stage web customers
Q	available stock of product
y_g	proportion of stock allocated to catalog consumers
y_w	proportion of stock allocated to web consumers
D_g	random catalog demand for product for the entire period
	$= \alpha_g r_g^{-\beta_g} G$ $(\alpha_g > 0, \beta_g > 1)$
	G is assumed normally distributed with mean μ_g and std. dev. σ_g
D_{w_1}	random web demand for product in first sub-period
	$= \alpha_w r_w^{-\beta_w} W_1$ $(\alpha_w > 0, \beta_w > 1)$
D_{w_2}	random web demand for product in second sub-period
	$= \alpha_w (\eta_{w_2} r_w)^- \beta_w W_2$
	W_1 and W_2 assumed normal with $(\mu_{w_1}, \sigma_{w_1})$ and $(\mu_{w_2}, \sigma_{w_2})$ respectively.
$D_{w_2\|w_1}$	updated web demand for 2^{nd} stage.

5.2.2 Derivation of expected revenue

Let $E_Z(.)$ be the expectation taken over random variable Z, $z^+ = \max(0, z)$ and $(z_1 \wedge z_2) = \min(z_1, z_2)$.

We use the following additional notation,

$Q_g = y_g Q$	the stock allocated to the catalog channel
$Q_{w_1} = y_w Q$	the stock allocated to the web channel
$Q_{w_2} = Q - Q_{w_1} - Q_g$	unallocated stock used to satisfy web demand in second stage
$I_{w_2} = (Q_{w_1} - D_{w_1})^+$	stock left over after satisfying web demand in first stage

We develop the expected revenue function for the dual channel catalog retailer. The retailer maximizes revenue

$$
\begin{aligned}
&ER \\
&\quad = E_G[r_g(D_g \wedge Q_g) - p_g(D_g - Q_g)^+] \\
&\quad + E_{W_1}[r_w(D_{w_1} \wedge Q_{w_1}) - p_{w_1}(D_{w_1} - Q_{w_1})^+ \\
&\quad + \max_{0 \le \eta_{w_2} \le 1} \quad E_{W_2|w_1}[\eta_{w_2} r_w\{D_{w_2|w_1} \wedge (I_{w_2} + Q_{w_2})\} - p_{w_2}\{D_{w_2|w_1} - (I_{w_2} + Q_{w_2})\}^+]]
\end{aligned}
$$

s.t.

$$y_g + y_w \le 1 \tag{5.12}$$

$$\eta_g, \eta_{w_1} \le 1 \tag{5.13}$$

$$y_g, y_w, \eta_g, \eta_{w_1} \ge 0 \tag{5.14}$$

We assume independent demands.

Theorem 8. *The second stage objective function, $E_{W_2|w_1}$, is concave, in η_{w_2}.*

Proof. The second order derivative is strictly negative. □

5.3 Solution method

Our solution method is based on searching the bounded real space defined by the constraints, (5.12)-(5.14), and determining the point that has the highest value of the objective function, ER. We need to overcome two hurdles in doing this. First, there are an infinite number of points in the real space. Second, the objective function cannot be evaluated exactly, since even though the second stage function, $E_{W_{2_j}|w_1}$, is concave in η_{w_2}, there is no closed form solution for η_{w_2}. Our procedure to arrive at the optimal values for the first stage variables, $(y_g^*, y_w^*, \eta_g^*, \eta_{w_1}^*)$ and the associated optimal value for the revenue at that point is described below:

1. First we ignore the constraint (5.12). The search space is now a unit hypercube in four dimensions, $(y_g^*, y_w^*, \eta_g^*, \eta_{w_1}^*)$.

2. In this unit hypercube, we take points along each dimension such that the unit distance is divided into κ equal parts. So, we end up with, $M = (\kappa-1)^4$ points. The coordinates for these M points are given by:

$$\left\{(x_1 = y_g, x_2 = y_w, x_3 = \eta_g, x_4 = \eta_{w1})\middle| x_j \in \left\{\frac{1}{2(\kappa-1)}, \frac{3}{2(\kappa-1)}, \cdots, \frac{(2\kappa-1)}{2(\kappa-1)}\right\}, j = 1\cdots 4\right\}$$

3. At each of these points, instead of the function ER, we evaluate the approximate

function given below:

$$\begin{aligned}
&\hat{ER} \\
&\quad = E_G[r_g(D_g \wedge Q_g) - p_g(D_g - Q_g^+)] \\
&\quad + E_{W1}[r_w(D_{w1} \wedge Q_{w1}) - p_{w1}(D_{w1} - Q_{w1})^+] \\
&\quad + N^{-1} \sum_{k=1}^{N} \max_{0 \le \eta_{w2} \le 1} E_{W2|w_1^k}[\eta_{w2} r_w \{D_{w2|w_1^k} \wedge (I_{w2} + Q_{w2})\} \\
&\quad - p_{w2}\{D_{w2|w_1^k} - (I_{w2} + Q_{w2})\}^+]
\end{aligned}$$

4. The second stage is evaluated by convex programming methods.

5. We use the method developed by Yakowitz et al. [113] to determine N. This method allows us to claim convergence of $\hat{ER}$ to ER in probability.

6. Once N is determined, we can reduce the number of points where $\hat{ER}$ is calculated to those points that satisfy all the constraints, (5.12) included.

7. The point with the highest estimated value of $\hat{ER}$ provides the optimal value of ER and the coordinates where this occurs provides the optimal values for the first stage variables, $(y_g^*, y_w^*, \eta_g^*, \eta_{w_1}^*)$.

There are a couple of drawbacks of the above algorithm:

1. As the discretization level increases (i.e. κ), the value of N required for convergence increases exponentially. This means the simulation effort required to produce points closer to the optimal values in the real space will also grow exponentially.

2. Since we are dealing with quasi-random points, some points in the real space will never be sampled. Thus if the optimal points are never sampled, this method will give sub-optimal values.

5.3.1 An example

We use the following parameters for the problem, $r = 5, p_g = p_{w_1} = p_{w_2} = 0, Q = 1200$, in our example. The demand functions for the catalog, first period web and second period web are $200(5\eta_g)^{-1.6}G, 400(5\eta_{w_1})^{-1.6}W_1$ and $400(5\eta_{w_1}\eta_{w_2})^{-1.6}W_2$, respectively. G, W_1 and W_2 are normally distributed-with mean equal to 1 and standard deviation $0.1, 0.2$ and 0.3, respectively. For the algorithm, the parameters are $\kappa = 5, M = 256$ and $N = 30$. To obtain convergence with these values of M and N, we need to determine a constant C from the following equation (16 in [113]):

$$M = C * \left(\frac{MN}{\ln(M) + \ln(N)}\right)^{\frac{1}{2}}$$

The value of C that satisfies the above equation is 8.7373. The revenue found by the algorithm is \$932.36. The values for the first stage variables are $y_g^* = 0.375, y_w^* = 0.125, \eta_g^* = 0.125, \eta_{w_1}^* = 0.375$.

5.4 Conclusion

In this chapter, we looked at an operational problem for a dual channel retailer using a web and a catalog to clear stock. We formulated the problem as a two-stage stochastic program where the first stage decision variables are the allocation ratios for the two channels and the initial price discounts for the two channels. We updated the web demand in the second stage based on the first stage web demand and then determined the optimal price discount for the second stage. We developed a unique solution method based on discretization of the real space. We approximated the first stage expectation in real space to discrete values and used convex programming to solve the second stage problem.

Chapter 6

Summary and Future Research

In this dissertation, we have looked at inventory and pricing issues related to dual channel retailers. We have formulated two problems related to product substitution and one problem related to clearance pricing. In all our work, we use single-period stochastic inventory models.

We develop an understanding of the effect of product substitution on the profits of a dual channel retailer having two channels, store and web, in Chapter 3 and Chapter 4.

In Chapter 3, we began by analyzing the allocation decisions to be made by a dual channel retailer with stock in hand when the web orders are deterministic and store demands are stochastic. We model a situation where there are two substitutable products and the substitution pattern differs with the channel. For web customers, the retailer may substitute the cheaper product with the costlier product while for in the store, customers can substitute either product with some probability. We develop conditions to show when such retailer substitutions can profit the retailer.

In Chapter 4, we extend the model in Chapter 3 by determining the ordering quantities for the two substitutable products in addition to the stock allocations. We use a two-stage stochastic programming formulation where the ordering decisions are made in the first stage and allocation decisions are made in the second stage. We show that the re-balancing of stock results in an increase of profits for the retailer.

The work in Chapters 3 and 4 can be extended in several directions. Here are some of them:

1. Retailers have expanded their channels from two to three. The channels being store, catalog and web. An example of such a retailer is Sears. The coordination effort among the three channels is greater. There is considerable overlap of customers between the web and the catalog channel and this needs to be taken into account.

2. We have considered two products in Chapters 3 and 4. The analysis can be extended to n products. The analysis gets more complicated as the retailer has to now consider the optimal assortment to carry in addition to stock allocation, stock ordering and pricing. We will need to revisit our assumption, $p_b > p_s$, when considering multiple products more so when considered as assortments of substitutable products.

3. In all our studies we have considered independent demands. Correlated demands provide a means of conveying demand information from one stage to the other and can give more insights.

We develop a model for determining clearance prices on the web and catalog for a catalog dual channel retailer.

There are several extensions to Chapter 5 that are possible. Here are a few of them:

1. We need to consider uncertainty in price sensitivity in addition to uncertainty in market potential.

2. We need to consider that web customers could behave strategically, in that once aware prices could fall further, they may choose to wait rather than purchase immediately.

3. The two channels differ in the cost of ascertaining price in the other channel. This would need to be incorporated to get additional insights into the problem.

4. There are other solution methods available. We need to investigate and compare against the solution method suggested in this chapter.

In this thesis we focused on single period two stage stochastic inventory models. In situations that deal with products with periodic ordering, it makes sense to develop multi-period inventory ordering models with multi-stage allocations between the two channels considering product substitution. Retailers dealing with the problem of substitution for web customers are trying several different options that justifies their role in what is a customer decision. When retailers do make the substitution decision, they should now how much they stand to gain from these substitution decisions and hence can compensate the customer in other ways. For determining how much they stand to gain, we would need a more realistic and hence a multi-period model.

Appendix A

Appendix for Chapter 3

A.1 Proof of Theorem 2

Let $E_S(R|w_1, w_2) := g(x_{11}, x_{12}, x_{22})$ and H be the Hessian of g. Let the elements of the Hessian be H_{ij} where $i, j = 1, 2, 3$. Note that $H_{ij} = H_{ji}$, since g is continuous and differentiable in x_{ij}.

The first principal minors which are the elements in the leading diagonal of the Hessian are

$$
\begin{aligned}
H_{11} &= \frac{\partial^2 E(R)}{\partial {x_{11}}^2} = {w_1}^2(y_{11} + y_{12}) \\
H_{22} &= \frac{\partial^2 E(R)}{\partial {x_{12}}^2} = {w_1}^2(y_{21} + y_{22}) \\
H_{33} &= \frac{\partial^2 E(R)}{\partial {x_{22}}^2} = {w_2}^2(y_{21} + y_{22})
\end{aligned}
$$

where

$$y_{11} = -(1-\delta_1)(p_b - p_s)f_1(Q_1^s) - \{(r_1 + p_s) - \delta_1(r_2 + p_s)\}f_1(Q_1^s)F_2(Q_2^s)$$

$$y_{12} = -\delta_1(r_2 - v_2 + p_s)\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2 - \frac{r_1 - v_1 + p_s}{\delta_2}\int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1$$

$$y_{21} = -(1-\delta_2)(p_b - p_s)f_2(Q_2^s) - \{(r_2 + p_s) - \delta_1(r_1 + p_s)\}f_2(Q_2^s)F_1({Q_1}^s)$$

and

$$y_{22} = -\delta_2(r_1 - v_1 + p_s)\int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1 - \frac{r_2 - v_2 + p_s}{\delta_1}\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2.$$

Note that $y_{12} < 0$ and $y_{22} < 0$. Given that $r_2 > r_1$, one can easily show that $y_{11} < 0$ and $y_{21} < 0$ when $p_b \geq p_s$ and $(r_1 + p_s) \geq \delta_1(r_2 + p_s)$. Therefore, the first three principal minors are negative.

Now,

$$H_{12} = \frac{\partial^2 E(R)}{\partial x_{11}\partial x_{12}} = {w_1}^2 y_3$$

where

$$y_3 = [-(r_2 - v_2 + p_s)\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2 - (r_1 - v_1 + p_s)\int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1].$$

With the above, the Hessian can be written as

$$H = \begin{pmatrix} w_1^2(y_{11} + y_{12}) & w_1^2 y_3 & w_1 w_2 y_3 \\ w_1^2 y_3 & w_1^2(y_{21} + y_{22}) & w_1 w_2(y_{21} + y_{22}) \\ w_1 w_2 y_3 & w_1 w_2(y_{21} + y_{22}) & w_2^2(y_{21} + y_{22}) \end{pmatrix}$$

Note that the second principal minor obtained by removing the first row and column is zero and the other two principal minors are $w_1^2 w_2^2[(y_{11}+y_{12})(y_{21}+y_{22})-y_3^2]$ and $w_1^4[(y_{21}+y_{22})(y_{11}+y_{12})-y_3^2]$. With some algebra, we can show that $y_{12}y_{22}-y_3^2 = \frac{1}{\delta_1\delta_2}(1-\delta_1\delta_2)^2(r_2-v_2+p_s)(r_1-v_1+p_s)\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2 \int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1 \geq 0$. When $p_b \geq p_s$ and $(r_1+p_s) \geq \delta_1(r_2+p_s)$, $y_{11} < 0$ and $y_{21} < 0$. Therefore, the second principal minors are non-negative.

The third principal minor which is the determinant of the Hessian is 0.

Appendix B

Appendix for Chapter 4

B.1 Proof of Theorem 7

.

Let $E(\Pi') := \Xi(x_{11}, x_{12}, x_{22}, Q_1, Q_2)$ and H be the Hessian of Ξ. Let the elements of the Hessian be H_{ij} where $i, j = 1, 2, 3, 4, 5$. Note that $H_{ij} = H_{ji}$, since Ξ is continuous and differentiable in $x_{11}, x_{12}, x_{22}, Q_1, Q_2$. When the demands are independent, we replace the density functions $h_i(.|\mu_{w_i})$ by $f_i(.)$. The first principal minors which are the elements in the leading diagonal of the Hessian are

$$H_{11} = \frac{\partial^2 E(\Pi')}{\partial x_{11}^2} = w_1^2(y_{11} + y_{12})$$
$$H_{22} = \frac{\partial^2 E(\Pi')}{\partial x_{12}^2} = w_1^2(y_{21} + y_{22})$$
$$H_{33} = \frac{\partial^2 E(\Pi')}{\partial x_{22}^2} = w_2^2(y_{21} + y_{22})$$
$$H_{44} = \frac{\partial^2 E(\Pi')}{\partial Q_1^2} = (y_{11} + y_{12})$$

$$H_{55} = \frac{\partial^2 E(\Pi')}{\partial Q_2^2} = (y_{21} + y_{22})$$

where

$$y_{11} = -(\delta_1)(p_b - p_s)f_1(Q_1^s) - (r_1 + p_s)f_1(Q_1^s)F_2Q_2^s$$

$$y_{12} = -\delta_1(r_2 - v_2 + p_s)\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2 - \frac{r_1 - v_1 + p_s}{\delta_2}\int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1$$

$$y_{21} = -(1 - \delta_2)(p_b - p_s)f_2(Q_2^s) - (r_2 + p_s) - \delta_1(r_1 + p_s)f_2(Q_2^s)F_1(Q_1^s)$$

and

$y_{22} = -\delta_2(r_1 - v_1 + p_s)\int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1 \frac{r_2 - v_2 + p_s}{\delta_1}\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2.$

Note that $y_{12} < 0$ and $y_{22} < 0$. Given that $r_2 > r_1$, one can easily show that $y_{11} < 0$ and $y_{21} < 0$ when $p_b \geq p_s$ and $r_1 \geq \delta_1 r_2$. Therefore, the five principal minors are negative.

Now,

$$H_{12} = \frac{\partial^2 E(\Pi')}{\partial x_{11}\partial x_{12}} = w_1^2 y_3$$

where

$$y_3 = [-(r_2 - v_2 + p_s)\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2 - (r_1 - v_1 + p_s)\int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1]$$

With the above, the Hessian can be written as

$$H = \begin{pmatrix} w_1^2(y_{11}+y_{12}) & w_1^2 y_3 & w_1 w_2 y_3 & -w_1(y_{11}+y_{12}) & -w_1 y_3 \\ w_1^2 y_3 & w_1^2(y_{21}+y_{22}) & w_1 w_2(y_{21}+y_{22}) & -w_1 y_3 & -w_1(y_{21}+y_{22}) \\ w_1 w_2 y_3 & w_1 w_2(y_{21}+y_{22}) & w_2^2(y_{21}+y_{22}) & -w_2 y_3 & -w_2(y_{21}+y_{22}) \\ -w_1(y_{11}+y_{12}) & -w_1 y_3 & -w_2 y_3 & y_{11}+y_{12} & y_3 \\ -w_1 y_3 & -w_1(y_{21}+y_{22}) & -w_2(y_{21}+y_{22}) & y_3 & y_{21}+y_{22} \end{pmatrix}$$

There are ten second principal minors and they are of two forms - one of which leads to zero and the other can be expressed as: $\kappa[(y_{11}+y_{12})(y_{21}+y_{22}) - y_3^2]$ where κ is some positive constant. With some algebra, we can show that $y_{12}y_{22} - y_3^2 = \frac{1}{\delta_1\delta_2}(1-\delta_1\delta_2)^2(r_2 - v_2 + p_s)(r_1 - v_1 + p_s)\int_0^{Q_2^s} f_1(\phi_1)f_2(s_2)ds_2 \int_0^{Q_1^s} f_2(\phi_2)f_1(s_1)ds_1 \geq 0$. When $p_b \geq p_s$ and $(r_1 + p_s) \geq \delta_1(r_2 + p_s)$, $y_{11} < 0$ and $y_{21} < 0$. Therefore, the second principal minors are non-negative. All the third principal minors, of which there are ten, equal 0. Similarly, the five fourth principal minors, all equal 0. The fifth principal minor which is the determinant of the Hessian is 0.

B.2 Karush-Kuhn-Tucker Conditions for P'

The optimization problem (**??**) with constraints (**??**) – (**??**) can be transformed to the following unconstrained problem:

$$\begin{aligned} L(x_{11}, x_{12}, x_{22}, Q_1, Q_2) = E(\Pi') &- \lambda_1(x_{11} + x_{12} - 1) - \lambda_2(x_{11}\mu_{w1} - Q_1) \\ &- \lambda_3(x_{12}\mu_{w1} + x_{22}\mu_{w2} - Q_2) - \lambda_4(x_{22} - 1) \quad \text{(B.15)} \end{aligned}$$

With $x_{11}, x_{12}, x_{22}, Q_1, Q_2 \geq 0$, the Karush-Kuhn-Tucker conditions can be stated as follows:

$$\frac{\partial E(\Pi')}{\partial x_{11}} - \lambda_1 - \lambda_2\mu_{w1} \leq 0 \tag{B.16}$$

$$\frac{\partial E(\Pi')}{\partial x_{12}} - \lambda_1 - \lambda_3\mu_{w1} \leq 0 \tag{B.17}$$

$$\frac{\partial E(\Pi')}{\partial x_{22}} - \lambda_4 - \lambda_3\mu_{w2} \leq 0 \tag{B.18}$$

$$\frac{\partial E(\Pi')}{\partial Q_1} + \lambda_2 \leq 0 \tag{B.19}$$

$$\frac{\partial E(\Pi')}{\partial Q_2} + \lambda_3 \leq 0 \tag{B.20}$$

$$\lambda_1 \cdot (x_{11} + x_{12} - 1) = 0 \tag{B.21}$$

$$\lambda_2 \cdot (x_{11}\mu_{w1} - Q_1) = 0 \tag{B.22}$$

$$\lambda_3 \cdot (x_{12}\mu_{w1} + x_{22}\mu_{w2} - Q_2) = 0 \tag{B.23}$$

$$\lambda_4 \cdot (x_{22} - 1) = 0 \tag{B.24}$$

$$\left(\frac{\partial E(\Pi')}{\partial x_{11}} - \lambda_1 - \lambda_2\mu_{w1}\right) \cdot x_{11} = 0 \tag{B.25}$$

$$\left(\frac{\partial E(\Pi')}{\partial x_{12}} - \lambda_1 - \lambda_3\mu_{w1}\right) \cdot x_{12} = 0 \tag{B.26}$$

$$\left(\frac{\partial E(\Pi')}{\partial x_{22}} - \lambda_4 - \lambda_3\mu_{w2}\right) \cdot x_{22} = 0 \tag{B.27}$$

$$\left(\frac{\partial E(\Pi')}{\partial Q_1} + \lambda_2\right) \cdot Q_1 = 0 \tag{B.28}$$

$$\left(\frac{\partial E(\Pi')}{\partial Q_2} + \lambda_3\right) \cdot Q_2 = 0 \tag{B.29}$$

The values of all the Lagrange multipliers λ_1, λ_2, λ_3 and λ_4 can be either zero or greater than zero. Therefore, 16 possible combinations exist for the Lagrange multipliers λ_1, λ_2, λ_3 and λ_4. (see Table B.6)

Case	λ_1	λ_2	λ_3	λ_4
1	> 0	> 0	> 0	> 0
2	> 0	> 0	> 0	$= 0$
3	> 0	> 0	$= 0$	> 0
4	> 0	$= 0$	> 0	> 0
5	$= 0$	> 0	> 0	> 0
6	> 0	> 0	$= 0$	$= 0$
7	> 0	$= 0$	$= 0$	> 0
8	> 0	$= 0$	> 0	$= 0$
9	$= 0$	> 0	> 0	$= 0$
10	$= 0$	> 0	$= 0$	> 0
11	$= 0$	$= 0$	> 0	> 0
12	> 0	$= 0$	$= 0$	$= 0$
13	$= 0$	> 0	$= 0$	$= 0$
14	$= 0$	$= 0$	> 0	$= 0$
15	$= 0$	$= 0$	$= 0$	> 0
16	$= 0$	$= 0$	$= 0$	$= 0$

Table B.6: Lagrange Multipliers for P'

Bibliography

[1] N. A. Agatz, M. Fleischmann, and J. A. Van Nunen. E-fulfillment and multi-channel distribution–a review. *European Journal of Operational Research*, 187(2):339–356, 2008.

[2] N. Agrawal and S. A. Smith. Optimal retail assortments for substitutable items purchased in sets. *Naval Research Logistics (NRL)*, 50(7):793–822, 2003.

[3] Y. Akcay, H. P. Natarajan, and S. H. Xu. Joint dynamic pricing of multiple perishable products under consumer choice. *Management Science*, 56(8):1345–1361, 2010.

[4] A. Alptekinoğlu and C. S. Tang. A model for analyzing multi-channel distribution systems. *European Journal of Operational Research*, 163(3):802–824, 2005.

[5] R. Anupindi, M. Dada, and S. Gupta. Estimation of consumer demand with stock-out based substitution: An application to vending machine products. *Marketing Science*, 17(4):406–423, 1998.

[6] Y. Aviv and A. Pazgal. Optimal pricing of seasonal products in the presence of

forward-looking consumers. *Manufacturing & Service Operations Management*, 10(3):339–359, 2008.

[7] S. Axsäter. Evaluation of unidirectional lateral transshipments and substitutions in inventory systems. *European Journal of Operational Research*, 149(2):438–447, 2003.

[8] G. Aydin and S. Ziya. Pricing promotional products under upselling. *Manufacturing & Service Operations Management*, 10(3):360–376, 2008.

[9] G. Aydin and S. Ziya. Technical note-personalized dynamic pricing of limited inventories. *Operations Research*, 57(6):1523–1531, 2009.

[10] Y. Bassok, R. Anupindi, and R. Akella. Single-period multiproduct inventory models with substitution. *Operations Research*, 47(4):632–642, 1999.

[11] P. Basu. Coordinating multi-channel pricing of seasonal goods. *International Journal of E-Business Research (IJEBR)*, 8(4):34–56, 2012.

[12] G. Bayraksan and D. P. Morton. Assessing solution quality in stochastic programs. *Mathematical Programming*, 108(2-3):495–514, 2006.

[13] E. Bendoly. Integrated inventory pooling for firms servicing both on-line and store demand. *Computers & Operations Research*, 31(9):1465–1480, 2004.

[14] E. Bendoly, D. Blocher, K. M. Bretthauer, and M. A. Venkataramanan. Service and cost benefits through clicks-and-mortar integration: Implications for the centralization/decentralization debate. *European Journal of Operational Research*, 180(1):426–442, July 1, 2007 2007.

[15] E. Bendoly, J. Blocher, K. Bretthauer, S. Krishnan, and M. Venkataramanan. Online/in-store integration and customer retention. *Journal of Service Research*, 7(4):313–327, 2005.

[16] G. Bitran, R. Caldentey, and S. Mondschein. Coordinating clearance markdown sales of seasonal products in retail chains. *Operations research*, 46(5):609–624, 1998.

[17] G. R. Bitran and S. V. Mondschein. Periodic pricing of seasonal products in retailing. *Management Science*, 43(1):64–79, 1997.

[18] K. K. Boyer and G. T. M. Hult. Extending the supply chain: Integrating operations and marketing in the online grocery industry. *Journal of Operations Management*, 23(6):642, Sep 2005.

[19] K. Bretthauer, S. Mahar, and M. Venakataramanan. Inventory and distribution strategies for retail/e-tail organizations. *Computers & Industrial Engineering*, 58(1):119–132, 2010.

[20] G. P. Cachon and A. G. Kök. Implementation of the newsvendor model with clearance pricing: How to (and how not to) estimate a salvage value. *Manufacturing & Service Operations Management*, 9(3):276–290, 2007.

[21] G. P. Cachon and R. Swinney. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science*, 55(3):497–511, 2009.

[22] G. P. Cachon, C. Terwiesch, and Y. Xu. Retail assortment planning in the presence of consumer search. *Manufacturing and Service Operations Management*, 7(4):330–346, Fall 2005.

[23] F. Caro and J. Gallien. Clearance pricing optimization for a fast-fashion retailer. *Operations Research*, 60(6):1404–1422, 2012.

[24] K. Cattani, O. Perdikaki, and A. Marucheck. The perishability of online grocers. *Decision Sciences*, 38(2):329–355, 2007.

[25] O. Ceryan, O. Sahin, and I. Duenyas. Dynamic pricing of substitutable products in the presence of capacity flexibility. *Manufacturing & Service Operations Management*, 15(1):86–101, 2013.

[26] S. Cetinkaya and M. Parlar. Optimal myopic policy for a stochastic inventory problem with fixed and proportional backorder costs. *European Journal of Operational Research*, 110(1):20–41, 1998.

[27] L. M. Chan, Z. M. Shen, D. Simchi-Levi, and J. L. Swann. Coordination of pricing and inventory decisions: a survey and classification. *Handbook of Quantitative Supply Chain Analysis*, pages 335–392, 2004.

[28] J. Chen, Y. Chen, M. Parlar, and Y. Xiao. Optimal inventory and admission policies for drop-shipping retailers serving in-store and online customers. *IIE Transactions*, 43(5):332–347, 2011.

[29] L.-M. Chen. Joint order and promotion effort decisions for a fashion apparel commodity under pre-determined markdown. *Journal of Promotion Management*, 18(3):361–391, 2012.

[30] M. S. Chen and C. T. Lin. An example of disbenefits of centralized stocking. *Journal of the Operational Research Society*, 41(3):259–262, Mar 1990.

[31] M. Cherikh. On the effect of centralisation on expected profits in a multi-location newsboy problem. *Journal of the Operational Research Society*, 51(6):755–761, Jun 2000.

[32] W. Chiang. Product availability in competitive and cooperative dual-channel distribution with stock-out based substitution. *European Journal of Operational Research*, 200(1):111–126, 2010.

[33] F.-S. Chou and M. Parlar. Optimal control of a revenue management system with dynamic pricing facing linear demand. *Optimal Control Applications and Methods*, 27(6):323–347, 2006.

[34] C.-S. Chung, J. Flynn, and J. Zhu. The newsvendor problem with an in-season price adjustment. *European Journal of Operational Research*, 198(1):148–156, 2009.

[35] W. Chung, S. Talluri, and R. Narasimhan. Price markdown scheme in a multi-echelon supply chain in a high-tech industry. *European Journal of Operational Research*, 215(3):581–589, 2011.

[36] Ö. Coşgun, U. Kula, and C. Kahraman. Analysis of cross-price effects on markdown policies by using function approximation techniques. *Knowledge-Based Systems*, 53:173–184, 2013.

[37] Ö. Coşgun, U. Kula, and C. Kahraman. Markdown optimization via approximate dynamic programming. *International Journal of Computational Intelligence Systems*, 6(1):64–78, 2013.

[38] L. Dong, P. Kouvelis, and Z. Tian. Dynamic pricing and inventory control of substitute products. *Manufacturing & Service Operations Management*, 11(2):317–339, 2009.

[39] P. Dutta and D. Chakraborty. Incorporating one-way substitution policy into the newsboy problem with imprecise customer demand. *European Journal of Operational Research*, 200(1):99–110, 2010.

[40] W. Elmaghraby, A. Gülcü, and P. Keskinocak. Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands. *Manufacturing & Service Operations Management*, 10(1):126–148, 2008.

[41] W. Elmaghraby and P. Keskinocak. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science*, 49(10):1287–1309, 2003.

[42] W. Elmaghraby, S. A. Lippman, C. S. Tang, and R. Yin. Will more purchasing options benefit customers? *Production and Operations Management*, 18(4):381–401, 2009.

[43] G. Eppen and L. Schrage. *Centralized ordering policies in a multi-warehouse system with lead times and random demand.*, pages 51–67. TIMS Studies in Management Science. North-Holland, Amsterdam, 1981.

[44] G. D. Eppen. Effects of centralization on expected costs in a multi-location newsboy problem. *Management Science*, 25(5):498–501, 1979.

[45] R. Ernst and P. Kouvelis. The effects of selling packaged goods on inventory decisions. *Management Science*, 45(8):1142–1155, 1999.

[46] A. Eynan and T. Fouque. Capturing the risk-pooling effect through demand reshape. *Management Science*, 49(6):704–717, 2003.

[47] Y. Feng and G. Gallego. Optimal starting times for end-of-season sales and optimal stopping times for promotional fares. *Management Science*, 41(8):1371–1391, 1995.

[48] M. Fisher. OR FORUM-Rocket Science Retailing: The 2006 Philip McCord Morse lecture. *Operations Research*, 57(3):527–540, 2009.

[49] S. C. Friend and P. H. Walker. Welcome to the new world of merchandising. *Harvard Business Review*, 79(10):133–141, 2001.

[50] G. Gallego, R. Phillips, and Ö. Şahin. Strategic management of distressed inventory. *Production and Operations Management*, 17(4):402–415, 2008.

[51] A. K. Gandhi, S. K. Kumar, M. K. Pandey, and M. Tiwari. Empso-based optimization for inter-temporal multi-product revenue management under salvage consideration. *Applied Soft Computing*, 11(1):468–476, 2011.

[52] M. Ganesh, S. Raghunathan, and C. Rajendran. The value of information sharing in a multi-product supply chain with product substitution. *IIE Transactions*, 40(12):1124–1140, 2008.

[53] M. Ganesh, S. Raghunathan, and C. Rajendran. The value of information sharing in a multi-product, multi-level supply chain: Impact of product substitution, demand correlation, and partial information sharing. *Decision Support Systems*, 2013.

[54] W. Gaul and A. D. Azizi. A demand learning data based approach to optimize revenues of a retail chain. In *Classification as a Tool for Research*, pages 683–691. Springer, 2010.

[55] V. Gaur and D. Honhon. Assortment planning and inventory decisions under a locational choice model. *Management Science*, 52(10):1528–1543, Oct 2006.

[56] Y. Gerchak and Q. M. He. On the relation between the benefits of risk pooling and the variability of demand. *IIE Transactions*, 35(11):1027–1031, Nov 2003.

[57] W. G. Gilland and H. S. Heese. Sequence matters: Shelf-space allocation under dynamic customer-driven substitution. *Production and Operations Management*, 22(4):875–887, 2013.

[58] J. Gönsch, R. Klein, M. Neugebauer, and C. Steinhardt. Dynamic pricing with strategic customers. *Journal of Business Economics*, 83(5):505–549, 2013.

[59] D. Gupta, A. V. Hill, and T. Bouzdine-Chameeva. A pricing model for clearing end-of-season retail inventory. *European Journal of Operational Research*, 170(2):518–540, 2006.

[60] W. H. Hausman and J. S. Thorbeck. Fast fashion: Quantifying the benefits. In *Innovative Quick Response Programs in Logistics and Supply Chain Management*, pages 315–329. Springer, 2010.

[61] A. Herbon, U. Spiegel, and J. Templeman. Simulation study of the price differentiation effect in a stochastic deteriorating inventory with heterogeneous consumers–freshness sensitivity. *Applied Economics*, 44(24):3101–3119, 2012.

[62] L. Hsiao and Y.-J. Chen. Strategic motive for introducing internet channels in a supply chain. *Production and Operations Management*, 23(1):36–47, 2014.

[63] A. Hsu and Y. Bassok. Random yield and random demand in a production system with downward substitution. *Operations Research*, 47(2):277–290, 1999.

[64] W. Hu and Y. Li. Retail service for mixed retail and e-tail channels. *Annals of Operations Research*, 192(1):151–171, 2012.

[65] M. Karakul. Joint pricing and procurement of fashion products in the existence of clearance markets. *International Journal of Production Economics*, 114(2):487–506, 2008.

[66] M. Karakul and L. M. A. Chan. Analytical and managerial implications of integrating product substitutability in the joint pricing and procurement problem. *European Journal of Operational Research*, 190(1):179–204, 2008.

[67] W. Kevin Chiang and G. Monahan. Managing inventories in a two-echelon dual-channel supply chain. *European Journal of Operational Research*, 162(2):325–341, 2005.

[68] K. Kogan and U. Spiegel. Optimal policies for inventory usage, production and pricing of fashion goods over a selling season. *Journal of the Operational Research Society*, 57(3):304–315, 2006.

[69] A. G. Kök and M. L. Fisher. Demand estimation and assortment optimization under substitution: Methodology and application. *Operations Research*, 55(6):1001–1021, 2007.

[70] H. D. Kwon, S. A. Lippman, and C. S. Tang. Optimal markdown pricing strategy with demand learning. *Probability in the Engineering and Informational Sciences*, 26(01):77–104, 2012.

[71] E. P. Lazear. Retail pricing and clearance sales. *The American Economic Review*, 1986.

[72] C. H. Lee. Coordination on stocking and progressive pricing policies for a supply chain. *International Journal of Production Economics*, 106(1):307–319, 2007.

[73] X. Li, L. Song, and Z. Zhao. Quantifying the impact of demand substitution on the bullwhip effect in a supply chain. *Logistics Research*, 3(4):221–232, 2011.

[74] J. Liu and C.-G. Lee. Evaluation of inventory policies with unidirectional substitutions. *European Journal of Operational Research*, 182(1):145–163, 2007.

[75] M. Lu, S. Huang, and Z.-J. M. Shen. Product substitution and dual sourcing under random supply failures. *Transportation Research Part B: Methodological*, 45(8):1251–1265, 2011.

[76] S. Mahar, K. Bretthauer, and M. Venkataraman. The value of virtual pooling in dual sales channel supply chains. *European Journal of Operational Research*, 192:561–575, 2009.

[77] S. Mahar, K. M. Bretthauer, and M. Venkataramanan. An algorithm for solving the multi-period online fulfillment assignment problem. *Mathematical and Computer Modelling*, 50(9):1294–1304, 2009.

[78] S. Mahar, P. A. Salzarulo, and P. Daniel Wright. Using online pickup site

inclusion policies to manage demand in retail/e-tail organizations. *Computers & Operations Research*, 39(5):991–999, 2012.

[79] S. Mahar and P. D. Wright. The value of postponing online fulfillment decisions in multi-channel retail/e-tail organizations. *Computers & Operations Research*, 36(11):3061–3072, 2009.

[80] M. K. Mantrala and S. Rao. A decision-support system that helps retailers decide order quantities and markdowns for fashion goods. *Interfaces*, 31(3_supplement):S146–S165, 2001.

[81] A. R. McGillivray and E. Silver. Some concepts for inventory control under substitutable demand. *Infor*, 16(1):47–63, 1978.

[82] A. J. Mersereau and D. Zhang. Markdown pricing with unknown fraction of strategic customers. *Manufacturing & Service Operations Management*, 14(3):355–370, 2012.

[83] B. K. Mishra and S. Raghunathan. Retailer-vs. vendor-managed inventory and brand competition. *Management Science*, 50(4):445–457, 2004.

[84] C. L. Munson, J. L. Hu, and M. J. Rosenblatt. Teaching the costs of uncoordinated supply chains. *Interfaces*, 33(3):24–39, May-Jun 2003.

[85] H. Nagao, K. Takahashi, K. Morikawa, and D. Hirotani. An inventory control policy with tracking information for dual-channel supply chains. *Brazilian Journal of Operations & Production Management*, 8:121–146, 2011.

[86] M. Nagarajan and S. Rajagopalan. Inventory models for substitutable products: optimal policies and heuristics. *Management Science*, 54(8):1453–1466, 2008.

[87] A. Nair and D. J. Closs. An examination of the impact of coordinating supply chain policies and price markdowns on short lifecycle product retail performance. *International Journal of Production Economics*, 102(2):379–392, 2006.

[88] S. Netessine and N. Rudi. Centralized and competitive inventory models with demand substitution. *Operations Research*, 51(2):329–335, 2003.

[89] V. Nocke and M. Peitz. A theory of clearance sales*. *The Economic Journal*, 117(522):964–990, 2007.

[90] M. Parlar. Game theoretic analysis of the substitutable product inventory problem with random demands. *Naval Research Logistics*, 35(3):397–409, Jun 1988.

[91] M. Parlar and S. Goyal. Optimal ordering decisions for two substitutable products with stochastic demands. *Opsearch*, 21(1):1–15, 1984.

[92] B. P. Pashigian. Demand uncertainty and sales: A study of fashion and markdown pricing.

[93] B. P. Pashigian and B. Bowen. Why are products sold on sale?: Explanations of pricing regularities. *The Quarterly Journal of Economics*, 106(4):1015–1038, 1991.

[94] D. W. Pentico. Assortment problem with nonlinear cost functions. *Operations Research*, 24(6):1129–1142, 1976.

[95] K. Rajaram and C. S. Tang. The impact of product substitution on retail merchandising. *European Journal of Operational Research*, 135(3):582–601, 2001.

[96] U. S. Rao, J. M. Swaminathan, and J. Zhang. Multi-product inventory planning with downward substitution, stochastic demand and setup costs. *IIE Transactions*, 36(1):59–71, 2004.

[97] T. Richardson. Tesco.com bags 500m in online sales. *The Register*, February 2004.

[98] S. Roychowdhury. A multi-period profit maximization policy for a stochastic demand inventory system with upward substitution. *International Journal of Applied Mathematics & Computer Sciences*, 5(4), 2009.

[99] R. Seifert, U. Thonemann, and M. Sieke. Relaxing channel separation: Integrating a virtual store into the supply chain via transshipments. *IIE Transactions*, 38(11):917–931, 2006.

[100] J. Shah and B. Avittathur. The retailer multi-item inventory problem with demand cannibalization and substitution. *International Journal of Production Economics*, 106(1):104–114, 2007.

[101] Z.-J. M. Shen and X. Su. Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production and operations management*, 16(6):713–728, 2007.

[102] S. A. Smith and D. D. Achabal. Clearance pricing and inventory policies for retail chains. *Management Science*, 44(3):285–300, 1998.

[103] S. A. Smith and N. Agrawal. Management of multi-item retail inventory systems with demand substitution. *Operations Research*, 48(1):50–64, 2000.

[104] S. A. Smith, N. Agrawal, and S. H. McIntyre. A discrete optimization model for seasonal merchandise planning. *Journal of Retailing*, 74(2):193–221, 1998.

[105] W. Soon. A review of multi-product pricing models. *Applied Mathematics and Computation*, 217(21):8149–8165, 2011.

[106] E. Stavrulaki. Inventory decisions for substitutable products with stock-dependent demand. *International Journal of Production Economics*, 129(1):65–78, 2011.

[107] X. Su. Intertemporal pricing with strategic customer behavior. *Management Science*, 53(5):726–741, 2007.

[108] X. Su and F. Zhang. Strategic customer behavior, commitment, and supply chain performance. *Management Science*, 54(10):1759–1773, 2008.

[109] B. Tan and S. Karabati. Retail inventory management with stock-out based dynamic demand substitution. *International Journal of Production Economics*, 145(1):78–87, 2013.

[110] S. G. Timme and C. Williams-Timme. The real cost of holding inventory. *Supply Chain Management Review*, 7(4):30, 2003.

[111] G. van Ryzin and S. Mahajan. On the relationship between inventory costs and variety benefits in retail assortments. *Management Science*, 45(11):1496–1509, Nov 1999.

[112] G. Vulcano, G. Van Ryzin, and R. Ratliff. Estimating primary demand for substitutable products from sales transaction data. *Operations Research*, 60(2):313–334, 2012.

[113] S. Yakowitz, P. L'ecuyer, and F. Vázquez-Abad. Global stochastic optimization with low-dispersion point sets. *Operations Research*, 48(6):939–950, 2000.

[114] H. Yang and L. Schrage. Conditions that cause risk pooling to increase inventory. *European Journal of Operational Research*, 192(3):837–851, 2009.

[115] R. Yin, Y. Aviv, A. Pazgal, and C. S. Tang. Optimal markdown pricing: Implications of inventory display formats in the presence of strategic customers. *Management Science*, 55(8):1391–1408, 2009.

[116] S. Zaroban. Stores that take advantage of the web more than hold their own. *Internet Retailer*, (May 3), 2013.

[117] D. Zhang and W. L. Cooper. Managing clearance sales in the presence of strategic customers. *Production and Operations Management*, 17(4):416–431, 2008.