Optimization-based Operability Analysis of Process Supply Chains
Optimization-based Operability Analysis of Process Supply Chains

by

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ABSTRACT

The North American forest products industry is primarily commodity-based and faces challenges. This has led to the proposal of a shift toward revenue diversification through the production of high-value specialty products along with the conventional commodity products. A key consideration for this new business strategy to remain competitive and sustainable is that the forest products supply chain designs must perform satisfactorily under the dynamic market conditions. The notion of supply chain operability attempts to characterize the ability of a supply chain to perform satisfactorily in the face of uncertainty. However, limited quantitative analysis is available in the current body of literature.

In this work, the concepts originated within the context of process systems engineering are adapted to develop optimization-based frameworks in order to characterize supply chain operability measures, in particular, supply chain flexibility and dynamic responsiveness. Although motivated by the forest products industry, the practical mathematical formulations presented are widely applicable to general process supply chains in other industries.

This thesis aims to extend the supply chain flexibility analysis formulation established by Mastragostino [2012] to include additional quantitative flexibility measures. The resulting framework provides a quantitative mapping to various types of flexibility frequently discussed in the operations research literature. Two case studies are included to illustrate the application of this framework for analyzing the flexibility of existing supply chain processes, as well as utilizing it in supply chain design. The work also builds on the analysis framework established by Mastragostino and Swartz [2014] to assess supply chain responsiveness, and to configure the framework in preparation for tackling design problems under uncertainty. Then a composite operability analysis framework is proposed to address both flexibility and responsiveness metrics simultaneously in forest products supply chain design and operation. A comprehensive case study based on a forest product company is performed and the trade-offs among flexibility, responsiveness and economics are examined.
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Chapter 1

Introduction

1.1 Motivation and Problem Statement

The North American forest products industry (FPI) is primarily commodity-based, and is facing enormous challenges from the global low-cost competitors, leading to a sustained decline in the demands of pulp and paper products and over-capacity of the existing production facilities. The global competitiveness is a result of the fundamental disadvantage due to the high labour and wood cost in North America compared to the major competitors from tropical countries, who also have built mills with greater capacities and thus enjoy the benefits from economy of scale [Van Heiningen 2006]. In order to combat the situation, capital and operational costs, as well as research and development activities have been reduced, resulting in obsolete pulp and paper mills [Wising and Stuart 2006]. This has led to the proposal for a shift toward revenue diversification, and the manufacture of bio-fuels as well as other higher-value specialty products [Wising and Stuart 2006]. New process technologies need to be incorporated into the existing pulp and paper mills, and essentially transform them into integrated forest biorefineries (IFBRs). By retrofitting them to the existing mills, as illustrated in Fig. 1.1, the obsolete mills can be transformed into more sustainable IFBRs to promote product variety [Chambost et al. 2007]. The additional capital cost is justified by improved robustness against market volatility.
The implicated business models must consider the increased mass customization for both existing as well as potentially new products. A key consideration in the product, process and supply chain designs should have adequate flexibility to changes in market conditions. This is required both at the strategic design level, where market changes sustained over long periods are considered, as well as at the tactical and/or operation level, where responsiveness to short-term variations is desired.

The implementation of this strategy may be phased into three stages (Fig. 1.2) as suggested by Mastragostino [2012] based on the concepts advocated in Mansoornejad et al. [2010]. The first phase involves product portfolio development and process technology selection in accordance to the market conditions. The second phase is then to design supply chain network that can carry these processes and deliver the selected products. The third phase aims to assess the supply chain performance under criteria from both operability and customer satisfaction aspects. Such assessment also provides strategic vision on the two previous phases. There is fruitful research related to the first and second phases, but very limited for phase three on the quantitative methodologies of supply chain operability analysis.

Being able to respond to changes in market conditions and their consequences, such as price
volatility, is critical for sustainable business concepts. This study will focus on evaluating operability measures in new and existing supply chains so that such responsiveness can be assured. The overall objective is to develop a framework for inclusion of operability considerations in the process and supply chain design for product diversification in the FPI.

1.2 Research Objectives and Main Contributions

The operability considerations on the improvement of forest products supply chains have motivated the main goal of this research, which is to further develop operability analysis suitable for general process supply chains. To achieve research goal, two sub-objectives are further defined. One is to extend the existing flexibility analysis formulations established by Mastragostino [2012], to include additional flexibility metrics, as well as to demonstrate its applicability to design and operation of general process supply chain in various industrial sectors through more comprehensive case studies. The second objective aims to integrate the flexibility analysis with the dynamic responsiveness analysis formulations proposed by Mastragostino and Swartz [2014]. The combined framework is able to simultaneously address both long-term sustained changes and short-term fluctuations in supply chain operations.
This thesis has unique contributions to supply chain optimization and operability analysis. The first contribution is to build on the supply chain flexibility analysis established by Mastragostino [2012], by extending the formulations to include additional flexibility metrics discussed in operations research literature. A second key contribution is the development of a practical mathematical programming model that is capable of addressing both flexibility and responsiveness considerations in forest products supply chains. Although motivated by the FPI, the outlined formulations and methodology are widely applicable to general process supply chains in other industries.

1.3 Thesis Organization

This thesis is organized into the following chapters:

Chapter 2 Literature Review

A highlight of relevant literature is presented.

Chapter 3 Supply Chain Flexibility Analysis Framework

An optimization-based flexibility analysis framework for supply chain networks is developed for the inclusion of flexibility considerations in supply chain design problems. Illustrative case studies are presented to demonstrate the applicability of the proposed methodology.

Chapter 4 Supply Chain Responsiveness Analysis Framework

An optimization-based formulation for supply chain responsiveness analysis is presented along with the illustrative case studies in the context of forest products industry to demon-
strate the applicability of the proposed methodology.

Chapter 5 Combined Supply Chain Operability Analysis Framework

Both flexibility and dynamic responsiveness analysis are included in a single framework to accommodate long-term sustained changes as well as short-term fluctuations in the market conditions. A comprehensive case study with both commodity and value-added specialty forest products is examined to illustrate the applicability of the combined framework.

Chapter 6 Conclusions and Recommendations

The main results are summarized and the key contributions are highlighted with concluding remarks on proposed frameworks. Future potential research directions are identified.
Chapter 2

Literature Review

A brief review of some of the key concepts relevant to this project is presented in this chapter. Concepts covered in this chapter include: (1) supply chain optimization; (2) forest products supply chains; (3) operability analysis. Readers can refer to the cited studies for more information.

2.1 Supply Chain Optimization

A supply is a network within an organization or between multiple organizations that involves the flows of products, finances and information, and typically includes a series of activities such as the procurement of raw materials, conversion from raw materials to final products, and distribution of final products to markets [Simchi-Levi et al. 2003; Papageorgiou 2009].

A comprehensive review article by Grossmann [2005] proposed the concept of enterprise-wide optimization (EWO) that lies at the interface of chemical engineering and operations research. Although the terms supply chain management (SCM) and EWO are sometimes used interchangeably in the literature, key differences between SCM and EWO have been identified. SCM focuses on logistics and distribution, usually characterized by linear models, and is traditionally the domain of operations research and management science. The em-
phasis of EWO is on the manufacturing facilities with a major consideration on planning, scheduling and control which often requires nonlinear process models and hence typical knowledge of chemical engineering.

The decision-making hierarchy across a supply chain is commonly divided into three levels given the complex and interactive nature of supply chains: strategic, tactical, and operational decisions, with respective long term (several years), intermediate term (weeks to months) and short term (days to weeks) planning horizons. The strategic level features decisions on opportunities related to capital investment and general network design, such as facility location, product selection, process technology, and configurations of supply chain structure. The tactical level includes high-level planning and distribution decisions among echelons such as raw material procurement, inventory policies, and in-bound and out-bound logistics. The operational level focuses on detailed operational decisions, such as weekly or monthly production planning, and daily scheduling and sequencing of production tasks [Grossmann, 2005].

Analogous to chemical processes, supply chain processes operate under dynamically changing market environments facing various types of uncertainty, which can be categorized as demand, process, supply, and control uncertainty [Geary et al., 2002]. Uncertainty is a significant issue in supply chain optimization, and can lead to sub-optimal decision making if not appropriately addressed. Supply chains are required to satisfactorily meet customer demand to exploit external opportunities in a cost-effective way. A key feature for businesses nowadays is that it is the entire supply chain that competes, rather than the individual components in it, and the success of the entire supply chains is determined by the end consumers [Christopher, 2005].

Introductory tutorials by Sen and Higle [1999] and Higle [2005] provide understanding of stochastic programming and modeling of future recourse to deal with uncertainty. In the framework of two-stage stochastic programming, decision variables are characterized into two categories, depending on whether they are implemented before or after the uncertainty is resolved. First-stage variables, are decided prior the realization of uncertain parame-
ters (“here-and-now”) and are typically related to decisions with design and allocation of resources. The implementations of second-stage variables, are delayed until further information on the uncertainty is available (“wait-and-see”) and are typically associated with operating or control decisions [Birge and Louveaux, 1997]. The objective function attempts to minimize the deterministic cost associated with the first-stage design variables and expected cost resulting from second-stage operating decisions while complying with constraints required for feasible operations.

The evaluation of the expected second-stage cost is a major challenge and two approaches are often employed in two-stage stochastic programming. The scenario-based approach uses scenario analysis to approximate the expectation; while the probabilistic approach directly applies the probability distribution of the uncertain parameter in the formulation to evaluate the expectation with an analytical integration method [Gupta and Maranas, 2000]. Liu and Sahinidis [1996] propose a two-stage stochastic programming approach for process planning under uncertainty, where first-stage variables include capital investment decisions, and second-stage variables include operating decisions. In a similar study, Tsiakis et al [2001] consider the design of a multi-product, multi-echelon supply chain through a mixed-integer linear programming (MILP) formulation, where two-stage stochastic programming is applied for the treatment of demand uncertainty. Scenarios for capturing demand uncertainty are generated to represent both optimistic and pessimistic situations within a risk analysis strategy. You et al [2009] present a stochastic mid-term tactical planning model which captures uncertainty in demand and freight rates using a two-stage stochastic programming approach, with inclusion of risk-management measures. Through simulations they quantify an average cost savings to be expected when using the stochastic approach as compared to a deterministic approach.

### 2.2 Forest Biomass Supply Chains

Biomass-based products, fuels and energy have been proposed as a sustainable solution to reduce the dependence on petroleum-based natural resource and to limit the effects of green
house gas emissions on climate change. Marquardt et al. [2010] predict that the transition from the petroleum-based to a bio-economy is expected to evolve continuously across all process industries in the next few decades. The authors also point out that the sustainable solutions require a much wider perspective and the model-based synthesis of the entire value chain is considered to be the key methodology. Hence, the leadership from process system engineers and operations research practitioners is desirable to address this multi-disciplinary decision making problem in collaboration with scientists and policy makers.

A recent comprehensive review by Yue et al. [2014] outlines the key challenges and opportunities in modeling and optimization of biomass-to-bioenergy supply chains. It provides an overview of major energy pathways from various types of biomass along with the related technologies and implementation approaches. It also reviews a large body of recent literature regarding the existing contributions on bioenergy supply chain optimization. The authors emphasize the importance of multi-scale modeling and optimization, which allows the integration of decision making involved in different levels.

Bioenergy and biomass-based products also offer promising opportunities for enterprise transformation of the forest product industry. The forest biorefineries, which can be integrated into the existing equipment and operations of the pulp and paper industry, are considered by many forestry companies as a strategy to diversifying their traditional business model [Stuart, 2006]. However, the successful implementation of forest biorefineries is not trivial and requires systematic analysis on product portfolio and technology selection as well as synthesis of the entire value chain. As part of the considerations for the first stage of the phased implementation, Chambost et al. [2007] introduce a systematic approach for product and process design to help pulp and paper industry explore the promising opportunities associated with forest biorefineries and to identify the necessary steps of implementing biorefineries in existing pulp and paper production facilities. Mansoornejad et al. [2010] propose a design decision making framework to link the product and process design with the supply chain design to bridge the considerations of the first and second stages.

An article by Shabani et al. [2013] reviews the mathematical models to optimize forest
biomass supply chains, including both deterministic and stochastic approaches. The optimization models reviewed are to provide optimal decisions related to technology and process choice, network design, the size and location of plant and storage, logistic options and material flows. The objectives involved are mostly economic criteria. In a more recent review article by the same research group, Cambero and Sowlati [2014] also include a review of studies that address other aspects of the sustainability for forest biomass supply chains, such as environmental and social considerations of biorefinery design and operation. The authors discover the trend in recent literature of considering multiple sustainability dimensions and integrating economic, environmental and social aspects in the analysis and optimization of forest biomass supply chains [Kanzian et al., 2013; Sacchelli et al., 2014].

2.3 Operability Analysis Overview

2.3.1 Process Operability

Process operability reflects the ability of a process system to perform satisfactorily under conditions different from the nominal operating and/or design conditions [Grossmann and Morari, 1984]. These operating characteristics of process plants have been studied in the literature over the past several decades. Efficiency, flexibility, controllability, resiliency, and reliability are several of the key aspects of operability commonly discussed in literature. Two of these aspects, flexibility and dynamic responsiveness, will be addressed in this study.

Flexibility is the steady-state notion of the ability to remain feasible under different operating conditions; while responsiveness is the dynamic notion of the ability for satisfactory transition between operating points. The design of a process can have a significant effect on its operability. The traditional approach for design under uncertainty problem is to first solve the design problem deterministically for given nominal values of the uncertainty parameters, and then empirical overdesign factors are applied to account for the uncertainty in parameters [Nishida et al., 1974]. This method guarantees neither optimality nor feasibility of the design solution and could as well lead to adverse effects or cost inefficiencies.
Therefore, the motivation was established to develop rigorous and systematic approaches for addressing operability considerations during process design.

**Process Flexibility**

The concept of process flexibility was proposed to mitigate uncertainty in process design. Grossmann and coworkers [Halemane and Grossmann 1983; Swaney and Grossmann 1985] developed a framework for flexibility analysis within the context of chemical process operation and design. Flexibility is defined as the ability to maintain feasible steady-state operation for all parameter values within a specified range. Two types of flexibility analysis problems are developed for process systems to evaluate whether a design is feasible for a known range of uncertainty and to compute a quantitative metric that denotes how flexible a design is. A recent comprehensive overview of the process flexibility analysis framework, extensions and application is given in [Grossmann et al. 2014].

To further introduce this framework mathematically, consider a process model and constraints represented by

\[
\begin{align*}
    h(d, x, z, \theta) &= 0 \\
    g(d, x, z, \theta) &\leq 0
\end{align*}
\]  

(2.1)  

(2.2)

where \( h \) represents steady-state model equations such as material and energy balances, \( g \) is a set of operating and/or design constraints, \( d \) is a vector of design variables, \( x \) is a vector of model states of the same dimension as \( h \), and \( z \) is a vector of control variables that can be adjusted in response to a realization of a set of uncertain parameters, \( \theta \). Using Eq. (2.1) to express the state variables as a function of \( d, z \) and \( \theta \), and substituting into Eq. (2.2), the feasible region can be described in the reduced variable space by

\[
f(d, z, \theta) \leq 0.
\]

The mathematical formulation of the feasibility test problem discussed in [Halemane and
Grossmann [1983] takes the following general form,

\[ X(d) = \max_{\theta \in T} \min_z \max_{j \in J} f_j(d, z, \theta) \leq 0 \quad (2.3) \]

If the objective function, \( X(d) \), is less than or equal to zero, the system is feasible for the entire range of parameter uncertainty denoted by the set \( T \) under the design specified by \( d \).

The feasibility test problem is only able to determine whether or not a design is feasible in the face of parameter uncertainty, and therefore it is desirable to develop a measure reflecting the degree of flexibility of a process design. A quantitative analysis of process flexibility was developed by Swaney and Grossmann [1985], who presented an index of flexibility to quantify the flexibility through the size of the region of feasible steady-state operation in the uncertain parameter space. The basis is to define a range of uncertain parameters, \( T \), as given in Eq. (2.6), where \( \theta^N \) is the nominal value of the uncertain parameters, and \( \Delta \theta^- \) and \( \Delta \theta^+ \) are specified upper and lower bounds for the uncertain parameters. The flexibility index, \( F \), corresponds to the largest value of \( \delta \), such that the design is feasible over the range of uncertain parameters, \( \theta \), with the general mathematical formulation given as follows,

\[
F = \max \delta \\
\text{s.t. : } \quad X(d) = \max_{\theta \in T} \min_z \max_{j \in J} f_j(d, z, \theta) \leq 0 \\
\quad T(\delta) = \{ \theta | \theta^N - \delta \Delta \theta^- \leq \theta \leq \theta^N + \delta \Delta \theta^+ \} \quad (2.6)
\]

This index corresponds to the maximum scaled deviation of uncertain parameters from their nominal values for which feasible operation can be guaranteed by proper manipulation of the control variables. The geometric interpretation of the index is the size of the largest possible hyperrectangle centered at the nominal operating conditions that can be inscribed within the feasible operation region. In addition, computation of the flexibility index can reveal the critical parameter combinations which limit the feasibility of a given design. The magnitude of \( F \) compared with one reveals quantitative information of flexibility of a process.

Both feasibility test and flexibility index problems involve the max-min-max formulation which is very difficult to solve due to its discontinuous nature. These problems can be
greatly simplified for the cases when the critical points that limit the flexibility correspond
to extreme values of the uncertain parameter set $T$ [Halemane and Grossmann, 1983].
Swaney and Grossmann [1985] prove that in order for this condition to be true, the feasible
region of a process needs to be one-dimensional quasi-convex (1-DQC) in the uncertainty
space, i.e. the feasible region has to be at least convex in the directions parallel to coordinate
axes. Based on these assumptions, the flexibility index problem can be simplified to the
following.

$$
\delta^k = \max \delta
$$

subject to:

$$
f_j(d, z, \theta) \leq 0, \; \forall j \in J
$$

$$
\theta = \theta^N + \delta \Delta \theta^k
$$

where $\Delta \theta^k, k \in V$ denotes the vertex directions from nominal point to the vertices since
the critical points will lie on vertices. The maximum deviation, $\delta^k$, is determined as by the
above optimization problem. The flexibility index is then determined as the smallest $\delta^k$
value among all vertices,

$$
F = \min_{k \in V} \{ \delta^k \}.
$$

This approach is referred as the vertex enumeration strategy (VES) because the algorithm
needs to visit every vertex defined by the hyperrectangle. For a system with $n$ uncertain
parameters, the number of vertices where the optimization problem needs to be solved is
$2^n$. Grossmann and Floudas [1987] reformulated the max-min-max formulation as mixed-
integer optimization problem based on identifying the set of active constraints that limit
flexibility. The active set method (ASM) is more general and has the advantage of not
relying on the assumption that the critical parameter values lie at the vertices.

**Dynamic Operability**

Flexibility serves as a prerequisite for dynamic operability as the feasibility must be guaran-
teed for an operating condition before considering other criteria related to the transitional
behavior towards this operating condition. However, flexibility itself is not sufficient to depict the dynamic behavior of a process moving between operating conditions, such as slow dynamics, violation of constraints, or actions to reject disturbances.

Nishida and Ichikawa [1975] recognize the needs to take into account of the effects of dynamic behavior at the stage of process design rather than designing the system for steady-state operations and then compensating the dynamic effects by process controls. The authors model the behavior of a processing unit by a set of ODEs and solve it using the gradient method for optimality. Similar to dynamic operability, the term “dynamic resilience” is used in Morari [1983] to describe the ability of a plant to move fast and smoothly between operating conditions and to effectively deal with disturbances. The author also shows that dynamic resilience does not depend on the controller structure or type implemented, but is determined by inherent features of the system, such as limitations of controller actions, stability, realizability, and robustness of the system under uncertainty and modeling errors.

Optimization-based methods are effective both for quantitative assessment of process dynamic operability, and for optimal process design with operability and economical considerations. Swartz [1996] proposes a computational framework for plant operability assessment, which uses Q-parametrization to represent feedback controllers within an optimization formulation to provide a measure of operability. The author then includes model uncertainty within the proposed framework in his later work [Ross and Swartz, 1997] to highlight the needs for considerations of all performance limiting factors mentioned in Morari [1983] simultaneously. Uztürk and Georgakis [2002] propose a measure of dynamic operability, evaluated using the solution of an open-loop optimal control scheme, and thus represents the upper bound on the achievable control performance. A dynamic operability index (dDOI) is also defined based on this measure to quantify the dynamic performance over the entire operating ranges. Other than optimization-based formulations, a new development for plant-wide operability analysis by Bao and coworkers is based on passivity and dissipativity. A detailed introduction of this methodology can be found in Bao and Lee [2007].

In a recent review article by Vega et al. [2014], the approaches to integrate process design
and control are surveyed and categorized into two types: the projecting methods, where
dynamic operability measures are monitored during the process design to predict the trade-
offs between design and control; and the integrated-optimization methods, which solve the
process design and control simultaneously within a single optimization framework. Dynamic
optimization formulations have been presented in wide range of literature and considered ef-
effective to address dynamic performance considerations during process design and operation.

Sanchez-Sanchez and Ricardoz-Sandoval 2013.

2.3.2 Supply Chain Operability

Backx et al. 1998 suggest that the cooperative interactions between partners in the supply
chain are beneficial for plant optimization and the system should lead from plant-focused
to supply chain-conscious to improve plant and supply chain performance. The authors
also identify the general goal of operable supply chains as being able to satisfy the needs of
the major stakeholders (market, individual customers, corporate, and society) and further
translate this goal into the needs for different (agile, lean, responding, smooth, and predic-
tive) supply chains. Significant attention has been drawn in recent decades to the idea of
“lean manufacturing” and the even wider concept of “lean enterprise”. The key philosophy
of the lean approach is to effectively reduce or eliminate waste during the activities within
supply chains – an approach that works well with relatively stable demands and predictable
market conditions Christopher 2000. However, the lean approach is not as desirable un-
der more volatile demand with high product variety, where a much greater level of agility
is required to sustain the robustness of the supply chain operations. Agility refers to the
capability of an organization to adapt to changes related to organizational structures, in-
formation systems, logistics processes and the ability to exploit profitability under volatile
market conditions Christopher 2005 Naylor et al. 1999. Agility, flexibility, and respon-
siveness and have been widely discussed within the operations research and management
science literature. The terms are sometimes used interchangeably (e.g. Tang and Tomlin
2008: Towill and Christopher 2002), and sometimes both flexibility and responsiveness
are considered as means and reflections of agility [Gunasekaran and Ngai, 2005].

Holweg [2005] proposes a balanced conceptual model of responsiveness beyond the qualitative description, based on the contributing factors of product, process and volume. Reichhart and Holweg [2007] extend the prior study and define different types of responsiveness, both in terms of the unit of change and in terms of the time horizon affected. Based on this, a conceptual framework is proposed to differentiate the factors that require a supply chain to be responsive and those that enable it to be responsive. In addition, authors recognize that a supply chain can exhibit different levels of responsiveness, depending on the location of responsiveness measurement.

A comprehensive review by Stevenson and Spring [2007] identifies the gaps in the literature as to view flexibility assessment as a unit analysis and fail to explore the full impacts of supply chain flexibility between multiple organizations. Sethi and Sethi [1990], in a comprehensive review on manufacturing flexibility, describe flexibility as a complex, multidimensional concept that is hard to capture, and refer to the fact that at least 50 different terms for various types of flexibility have appeared in the manufacturing literature. However, most of these discussions are conceptual and qualitative, with precise definitions largely elusive.

There are a number of contributions featuring quantitative metrics. Beamon [1999] identified flexibility as one of three key types of supply chain performance measures, and proposed metrics for the evaluation of four types of flexibility (volume, delivery, mix, and new product flexibility). Product demand flexibility of a manufacturing system is quantified in Son and Park [1987] as the ratio of total output to the inventory cost of finished products. Tang and Tomlin [2008] discuss flexibility strategies for reducing the impact of several supply chain risks they identify. They also provide quantitative analysis under simplifying assumptions in order to develop relationships between the level of flexibility and its mitigating effects. The strategies addressed include multiple suppliers, flexible supply contracts, flexible manufacturing processes, flexibility via postponement, and flexibility via responsive pricing. Mansoornejad et al. [2011] identify four major types of flexibility studied within the chemical engineering literature as recipe, product, volume and process flexibility. In later work by
the same authors, they propose metrics to quantify volume flexibility and economic robustness, evaluated through deviations from nominal conditions in a scenario-based approach [Mansoornejad et al., 2013].

A few studies have addressed supply chain responsiveness captured by lead time within mathematical programming frameworks. The lead time is defined as time from the moment the customer places an order to the moment it is received by the customer, and a longer lead time implies a less responsive supply chain system. The lead time is closely related to all delays incurred during supply chain activities, as well as the inventory levels of materials available. The worst case lead time, used by You and Grossmann [2007] as a quantitative responsiveness measure, is the longest response time taken on a linear supply chain path from a supplier to a customer. The worst case lead time also corresponds to the response time when there are zero inventories in the system, which is also known as the “pull” system or make-to-order supply chain. The authors then propose a bi-criterion MINLP optimization framework to include the considerations of economics and responsiveness on the design and operation of a multi-echelon process supply chain. Building on their prior work, You and Grossmann [2008] employ a probabilistic model for stock-out and the expected lead time is proposed as the quantitative reflection of supply chain responsiveness. An MINLP model with both responsiveness and economics criteria is proposed to determine the optimal network structure, transportation links, and inventory levels of a multi-echelon supply chain under demand uncertainty.

Mastragostino and Swartz [2014] recently establish a mathematical framework for dynamic operability analysis of supply chain using a dynamic optimization formulation. A multi-objective optimization formulation is posed, where the uncertain demand scenarios are considered using a two-stage stochastic programming formulation to explore the trade-offs between economic and responsiveness criteria.

To summarize, a brief overview of the relevant literature in supply chain optimization under uncertainty and the relevant mathematical programming techniques is presented. Then the focus is shifted to forest biomass supply chains to familiarize the readers with the ap-
plication context of this work. The development of operability analysis in process plants is introduced, and both conceptual and analytical approaches on supply chain operability are reviewed. Our work builds on and extends the aforementioned studies by Swartz and coworkers \cite{Mastragostino2012, Mastragostino2014} and synthesizes a mathematical framework to include both aspects of the operability considerations to support the forest products industry transformation.
Chapter 3

Supply Chain Flexibility Analysis Framework

3.1 Supply Chain Flexibility Analysis

This work considers a supply chain network containing multiple echelons (suppliers, manufacturing sites, distribution centers and markets), multiple production schemes and multiple products, as shown in Fig. 3.1. Raw materials can either be purchased directly from suppliers and then shipped to manufacturing sites, or produced within the sites as intermediate products of other processes. A set of multi-scheme processes convert raw materials into products within manufacturing plants. Products are then either shipped to distribution centers or consumed in other processes as inputs. A tactical-level steady-state supply chain model is presented, with modeling details adapted from Bok et al. [2000] and Mastragostino [2012]. The steady-state supply chain model assumes no accumulation of materials (raw materials, intermediate products and final products) at any facilities of the supply chain network during operation. Following the steady-state model, the equations related to the supply chain profit and costs are presented, as a part of the integration of the proposed framework with economics criteria.
3.1.1 System Description

An order of chemical \( j \in J \), \( O_{j,h,m} \), is made to suppliers \( h \in H_j \) from manufacturing plants \( m \in M_j \), where \( H_j \) represents the set of suppliers supplying chemical \( j \) and \( M_j \) is the set of plants involving chemical \( j \). In manufacturing sites, the chemical \( j \) is then converted into either intermediate products for further processing, or final products shipped to distribution centers to be distributed to satisfy demands from the market. The chemical flows of \( j \) involved in scheme \( k \) of the manufacturing process \( i \) within the plant \( m \) are \( W_{j,k,i,m} \). \( F_{j,m,v} \) represents the direct withdrawal of final product \( j \) shipped from the plant \( m \) to distribution center \( v \in V_j \), to fulfill a demand of \( D_{j,v} \).

3.1.2 Steady-state Supply Chain Model

The steady-state material balance of chemical \( j \) at manufacturing plant \( m \) is given in Eq. (3.1). The generation of \( j \) on the left hand side, including purchase from all suppliers \( H_j \) \( (\sum_{h \in H_j} O_{j,h,m}) \) and production during manufacturing \( (\sum_{i \in I_m} \sum_{k \in K_j} W_{j,k,i,m}) \), equals to the removal of \( j \), including the consumption in other manufacturing processes.
(\sum_{i \in I_m} \sum_{k \in K_{ji}^P} W_{j,k,i,m}) \text{ along with the shipment to distribution centers } (\sum_{v \in V_j} F_{j,m,v}).

The sets \( K_{ji}^P \) and \( K_{ji}^C \) represent the production schemes of process \( i \) that produce and consume chemical \( j \), respectively, and \( I_m \) is the set of processes available in site \( m \). Note that \( K_{ji}^P \) and \( K_{ji}^C \) are subsets of all production schemes available in process \( i \), denoted as \( K_i \).

\[
\sum_{h \in H_j} O_{j,h,m} + \sum_{i \in I_m} \sum_{k \in K_{ji}^P} W_{j,k,i,m} = \sum_{i \in I_m} \sum_{k \in K_{ji}^C} W_{j,k,i,m} + \sum_{v \in V_j} F_{j,m,v} \quad \forall j \in J, m \in M_j
\] (3.1)

Eq. (3.2) depicts the conversion of chemicals in relation to the main product for each of the schemes (defined as the set \( J_{ki}^M \)). The material balance coefficient, \( \eta_{j,k,i,m} \), is the flow ratio of of chemical \( j \) to the main product involved in scheme \( k \) of process \( i \). \( J_{ki} \) is the set of all chemicals involved in scheme \( k \) of process \( i \). By this definition, the material balance coefficients of main chemicals in each scheme is one (when \( j = j' \)).

\[
W_{j,k,i,m} = \eta_{j,k,i,m} W_{j',k,i,m} \quad \forall j \in J_{ki}, j' \in J_{ki}^M, k \in K_i, i \in I_m, m \in M_j
\] (3.2)

The material balance of final products is given in Eq. (3.3), where all products arriving at distribution center are distributed and \( Q_{j,v} \) is quantity of final product \( j \) distributed from distribution center \( v \). The demand satisfaction of product is fulfilled by this distribution as in Eq. (3.4).

\[
\sum_{m \in M_j} F_{j,m,v} = Q_{j,v} \quad \forall j \in J, v \in V_j
\] (3.3)

\[
D_{j,v} = Q_{j,v} \quad \forall j \in J, v \in V_j
\] (3.4)

The availability of chemical \( j \) from supplier \( h \) is subjected to an upper bound, \( O_{j,h}^{max} \), as shown in Eq. (3.5).

\[
\sum_{m \in M_j} O_{j,h,m} \leq O_{j,h}^{max} \quad \forall j \in J, h \in H_j
\] (3.5)

Regardless of the production schemes, the production quantity of main products from each scheme must be bounded by the production capacity of the corresponding process. \( X_{k,i,m} \) in Eq. (3.6) is a binary variable, and Eq. (3.7) restricts only one scheme to be operated for each of the processes at a given time, if needed at all. If \( X_{k,i,m} = 1 \), scheme \( k \) of process \( i \) is in operation and the main product flow cannot exceed the capacity, and otherwise the right side of the equation will force the flow to be zero.

\[
W_{j,k,i,m} \leq C_{i,m}^P X_{k,i,m} \quad \forall j \in J_{ki}^M, k \in K_i, i \in I_m, m \in M_j
\] (3.6)
\[
\sum_{k \in K_i} X_{k,i,m} \leq 1 \quad \forall i \in I_m, m \in M
\] (3.7)

### 3.1.3 Supply Chain Costs

The operating profit of the supply chain of interest is the difference between the revenue generated by selling products to fulfill market demands and the operating costs incurred during the production of final products.

\[
\text{Profit} = \text{Revenue} - \text{Cost}
\] (3.8)

The operating revenue in Eq. (3.9) is computed as the sum of all products sold at price \( P_{\text{Sale}}^{j,v} \) to satisfy the demand of product \( j \).

\[
\text{Revenue} = \sum_{j \in J} \sum_{v \in V_j} P_{\text{Sale}}^{j,v} Q_{j,v}
\] (3.9)

The operating cost includes raw material purchase cost, fixed operating cost of production schemes, and variable production cost proportional to the main product flows as shown in Eq. (3.10), where \( c_{j,h}^{PRC} \) is the cost per unit of raw material \( j \) purchased from supplier \( h \), and \( c_{k,i,m}^{\text{FIX}} \) and \( c_{k,i,m}^{PRD} \) are fixed and variable production cost for scheme \( k \) of process \( i \) in site \( m \).

\[
\text{Cost}^{\text{OPR}} = \sum_{j \in J} \sum_{h \in H_j} \sum_{m \in M_j} c_{j,h}^{PRC} O_{j,h,m} + \sum_{m \in M} \sum_{i \in I_m} \sum_{k \in K_i} c_{k,i,m}^{\text{FIX}} X_{k,i,m} + \sum_{j \in J} \sum_{m \in M_j} \sum_{i \in I_m} \sum_{k \in K_i} c_{k,i,m}^{PRD} W_{j,k,i,m}
\] (3.10)

The total supply chain cost considers both the capital and operating costs discussed above, where the capital cost is determined by the sizing of the production capacity in Eq. (3.11). Parameter \( c_{i,m}^{CP} \) is cost to build capacity for each unit of main chemical produced from process \( i \) of site \( m \).

\[
\text{Cost}^{\text{CAP}} = \sum_{i \in I_m} \sum_{m \in M} c_{i,m}^{CP} C_{i,m}^{CP}
\] (3.11)
3.1.4 Flexibility Analysis Formulation

In this section, the methodology to evaluate flexibility of process supply chains is introduced. The development of the analysis is motivated by the flexibility index problem in Swaney and Grossmann [1985]. The flexibility index of supply chain (denoted as $F$) is proposed to quantify the level of flexibility of a supply chain network in terms of the range of uncertainty. It can be interpreted as the maximum deviation from the nominal design and/or operation parameters that can be tolerated for steady state operation.

Two categories of flexibility indices are defined. Individual flexibility indices intend to map out the different types of flexibility discussed qualitatively in the operations research literature, which aims to evaluate the flexibility of supply chain under a specific type of uncertainty. In addition, an overall flexibility index is defined to reflect the flexibility of the entire supply chain network where all types of uncertainty are considered.

**Individual Flexibility Metric**

Individual flexibility indices $F_k$ are defined in Eqs. (3.12) to (3.15) as the maximum deviation that can be sustained for feasible steady-state supply chain operation from the nominal conditions of uncertainty $\theta_k$ in both positive and negative directions, $\Delta \theta_k^+$ and $\Delta \theta_k^-$. $h_i$ and $g_j$ are the collection of equality and inequality constraints representing the steady state supply chain model discussed previously in Eqs. (3.1) to (3.7).

\[
F_k = \max \delta_k
\]  \hspace{1cm} (3.12)

subject to:

\[
h_i(d, x, z, \theta_k) = 0 \quad \forall i \in I \tag{3.13}
\]

\[
g_j(d, x, z, \theta_k) \leq 0 \quad \forall j \in J \tag{3.14}
\]

\[
\theta_k^N - \delta_k \Delta \theta_k^- \leq \theta_k \leq \theta_k^N + \delta_k \Delta \theta_k^+ \tag{3.15}
\]

$d$ is a representation of the supply chain design variables. $x$ and $z$ are the collection of supply chain model variables, with no explicit distinctions between state variables and...
control variables.

Table 3.1 lists a few examples of individual flexibility indices within the operations research literature defined with the proposed formulation. These indices are defined by the corresponding uncertainty they assess.

<table>
<thead>
<tr>
<th>Flexibility ($F_k$)</th>
<th>Uncertainty ($\theta_k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand flexibility, $F_d = \max \delta_d$</td>
<td>Product demand</td>
</tr>
<tr>
<td>Supply flexibility, $F_s = \max \delta_s$</td>
<td>Raw material supply</td>
</tr>
<tr>
<td>Yield flexibility, $F_y = \max \delta_y$</td>
<td>Production yield</td>
</tr>
<tr>
<td>Market flexibility, $F_m = \max \delta_m$</td>
<td>Price of final product or costs</td>
</tr>
</tbody>
</table>

When uncertainty $\theta_k$ lies in product demands, demand flexibility index, $F_d$, can be interpreted as the maximum deviation in demand that can be satisfied by a supply chain while still maintaining feasible operation by adjusting the volume of product output. Demand flexibility is also studied quantitatively in operations research and management science literature. Son and Park [1987] quantify product demand flexibility as the ratio of total output of a manufacturing system to the inventory cost of finished products. This approach considers the inventory cost as an opportunity cost and treats demand flexibility as a responsiveness measure of a manufacturing system to match output levels with demands in the effort to reduce cost. Another flexibility measure that is often discussed in relation to demand flexibility is volume flexibility, defined as the ability of a supply chain to operate profitably at different levels of output. Beamon [1999], Mansoornejad et al. [2013] quantify volume flexibility as the deviation from nominal production rate in a dimensionless form. Gerwin [1993] views volume flexibility as an adaptive response to uncertainty in demand as it permits changes in the aggregated production level. The possession of sufficient volume flexibility can be considered as a mitigation in face of demand uncertainty, which is equivalent to the control variables $z$ in the proposed formulation as shown in Eqs. (3.13) and (3.14). Our proposed methodology originates from the feasibility test problem .
Grossmann, 1985 and therefore recognizes flexibility as a steady-state concept to capture the range of feasibility.

Similar to demand, another external source of uncertainty is supply uncertainty. The supply flexibility index, $F_s$, can be evaluated via the proposed formulation to capture the degree of variation in the quantity supplied that can be sustained by the feasible downstream supply chain operations. Recipe flexibility, defined by Mansoornejad et al. 2013 as the ability to utilize a set of adaptable material recipes to achieve different products and/or outputs, is therefore a moderation of the impacts caused by supply-related uncertainties. Variation in raw material quality, reduction of production cost by selecting cheaper recipes, and substitution due to material unavailability are the three main reasons that recipe flexibility is desired during manufacturing processes. In addition, Ferrer-Nadal et al. 2008 also demonstrate that the adoption of flexible recipes has high benefit in risk management associated with uncertain demands. Therefore, the recipe flexibility also shares some connection with the demand uncertainty.

The production yield is the ability of a process to produce, hence, with the same inputs, a process with a higher yield can generate more products. In a supply chain network, yield is recognized as an internal/endogenous source of uncertainty and its corresponding flexibility index, $F_y$, intends to measure the possible range of process yield variations without causing infeasibility of other components in the supply chain. The yield flexibility is usually related to the discussion of production or process flexibility as surveyed in Sethi and Sethi, 1990.

The final entry in Table 3.1, market flexibility, $F_m$, is defined using the proposed formulation with an additional constraint to ensure that the profit of the entire supply chain, as defined in Eq. (3.8), does not go below zero, because a losing supply chain is not sustainable in the long run. A more conceptual definition in Sethi and Sethi, 1990 refers market flexibility as the adaptability to a changing market environment. This flexibility index can be applied to all market related parameters defined in Eqs. (3.9) and (3.10). For instance, the price flexibility index computed as a result of the proposed formulation can be used to further calculate the lowest sale price of final product to remain profitable; the index computed
from production cost uncertainty can be used to derive the highest production cost that a supply chain can remain profitable. Mansoornejad et al. [2013] also present a metric of robustness which is essentially an assessment of a supply chain’s ability to remain profitable. It is computed as the percentage of aggregate deviation from the base case profit for all scenarios with lower than nominal profit.

**Overall Flexibility Metric**

The individual flexibility metrics introduced previously provide insights on how flexible a process is from a certain aspect uncertainty, while the overall flexibility of a supply chain reflects the potential of the entire network when facing multiple types of uncertainty simultaneously. A overall flexibility index, $F$, is defined with the similar formulation as in Eqs. (3.16) to (3.19), but $\theta$ here represents the collection of all uncertain parameters (i.e. $\theta = [\theta_1, \theta_2, \ldots, \theta_k]$).

$$ F = \max \delta $$

subject to:

$$ h_i(d, x, z, \theta) = 0 \quad \forall i \in I \quad (3.17) $$

$$ g_j(d, x, z, \theta) \leq 0 \quad \forall j \in J \quad (3.18) $$

$$ \theta^N - \delta \Delta \theta^- \leq \theta \leq \theta^N + \delta \Delta \theta^+ \quad (3.19) $$

It is noted that both individual and overall flexibility metrics are not only suitable for flexibility evaluation, but also can be embedded within a design under uncertainty formulation. Both applications are considered and illustrated in the case studies.

**3.2 Case Studies**

Two case studies are presented to illustrate the proposed flexibility analysis framework. All simulations were performed on a 3.4 GHz Intel® Core™ i7-2600 CPU DELL Optiplex
990 machine with 8 GB of RAM, running Windows 7 Professional 64-bit OS. All graphical results were generated in MATLAB R2013a with outputs from AMPL simulations.

### 3.2.1 Case Study 1

The first set of studied considered under this subject is adapted from Mastragostino [2012] with the supply chain network shown in Fig. 3.2. A single supplier delivers one type of raw material to both manufacturing sites, plant 1 and plant 2, where three final products, A, B and C are produced in corresponding continuous dedicated processes. All final products are first shipped to four distribution centers and then distributed to customers to fulfill various demands. The processes producing product B are available at both plants, but the processes for A and C are exclusive to plant 1 and plant 2, respectively. The key model parameters are listed in Tables 3.2 and 3.3.
Table 3.2: Production capacity ($C_{i,m}$) in units produced in process $i$ at site $m$

<table>
<thead>
<tr>
<th>site ($m$)</th>
<th>process ($i$)</th>
<th>IA</th>
<th>IB</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>IA</td>
<td>140</td>
<td>30</td>
<td>–</td>
</tr>
<tr>
<td>M2</td>
<td>IA</td>
<td>–</td>
<td>100</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 3.3: Material balance coefficients ($\eta_{j,k,i,m}$) of chemical $j$ in process $i$ at site $m$

<table>
<thead>
<tr>
<th>chemical ($j$)</th>
<th>process ($i$)</th>
<th>IA</th>
<th>IB</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>JRM</td>
<td>IA</td>
<td>6.6</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>JA</td>
<td>IA</td>
<td>1</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>JB</td>
<td>IA</td>
<td>–</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>JC</td>
<td>IA</td>
<td>–</td>
<td>–</td>
<td>1</td>
</tr>
</tbody>
</table>

Evaluation of Flexibility

First, the flexibility of the supply chain in Fig. 3.2 is analyzed under uncertainties in product demand and raw material availability. To simplify the problem, only critical points corresponding to the upper extreme value of the uncertain demand and the lower limits of the uncertainty in the raw material availability are addressed in the flexibility index problem. The inclusion of only the upper bound in demand uncertainty is based on the assumption that demand satisfaction can be guaranteed once highest possible demand is fulfilled. Similarly, if the supply chain can remain feasible with the minimal amount of raw materials available from the supplier, such a supply chain should be able to meet demands when more raw materials are available for purchase. These assumptions are reasonable for a supply chain system with demand and/or supply uncertainty without considering inventory levels. The flexibility indices are evaluated with the proposed formulation, modeled as an LP and solved using CPLEX 12.6 in AMPL. The numerical values of individual flexibility as well as overall flexibility indices are determined as follows.
Table 3.4: Demand parameters in units of final product $j$ at distribution center $v$

<table>
<thead>
<tr>
<th>Nominal demand ($D_{j,v}$)</th>
<th>product ($j$)</th>
<th>distribution center ($v$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VA</td>
<td>40</td>
<td>JA</td>
</tr>
<tr>
<td>VB</td>
<td>35</td>
<td>JB</td>
</tr>
<tr>
<td>VC</td>
<td>–</td>
<td>JC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand deviation ($\Delta D_{j,v}$)</th>
<th>product ($j$)</th>
<th>distribution center ($v$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VA</td>
<td>14</td>
<td>JA</td>
</tr>
<tr>
<td>VB</td>
<td>20</td>
<td>JB</td>
</tr>
<tr>
<td>VC</td>
<td>–</td>
<td>JC</td>
</tr>
<tr>
<td>VD</td>
<td>10</td>
<td>–</td>
</tr>
</tbody>
</table>

- Demand flexibility index, $F_d = 0.91$, considering only uncertainty in demand for scenarios tabulated in Table 3.4. The demand flexibility index is less than one, indicating that this supply chain configuration does not possess sufficient flexibility to accommodate all possible realizations of demand uncertainty. Production capacity of product A in plant 1 is then identified as the network bottleneck restricting the flexibility, and thus an improved flexibility index is expected when a greater capacity for this process is incorporated in a design retrofit.

- Supply flexibility index, $F_s = 1.8$, based on uncertain availability of raw material with nominal value $O_{j,h}^{max} = 1500$ and deviation $\Delta O_{j,h}^{max} = 50$. The supply flexibility index is greater than one so it implies that this supply chain not only can meet the upper extreme of the availability uncertainty, but also is capable of accommodating the level of availability as low as $O_{j,h}^{max} - F_s \Delta O_{j,h}^{max} = 1410$ units of raw material.

- Market flexibility index, $F_m = 3.87$, based on the selling price uncertainty scenarios listed in Table 3.5 and cost parameters in Table 3.6 under the constraint of non-
Table 3.5: Price parameters in $/unit of final product \( j \) at distribution center \( v \)

<table>
<thead>
<tr>
<th>Nominal price ( P^{Sale}_{j,v} ) product ( j )</th>
<th>JA</th>
<th>JB</th>
<th>JC</th>
</tr>
</thead>
<tbody>
<tr>
<td>distribution center ( v )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VA</td>
<td>10</td>
<td>15</td>
<td>–</td>
</tr>
<tr>
<td>VB</td>
<td>15</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>VC</td>
<td></td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>VD</td>
<td>25</td>
<td>–</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price deviation ( \Delta P^{Sale}_{j,v} ) product ( j )</th>
<th>JA</th>
<th>JB</th>
<th>JC</th>
</tr>
</thead>
<tbody>
<tr>
<td>distribution center ( v )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VA</td>
<td>-2</td>
<td>-3</td>
<td>–</td>
</tr>
<tr>
<td>VB</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
</tr>
<tr>
<td>VC</td>
<td></td>
<td>-1</td>
<td>-5</td>
</tr>
<tr>
<td>VD</td>
<td>-5</td>
<td>–</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 3.6: Production cost \( C_{i,m}^{prd} \) in $/unit of final product produced in process \( i \) at site \( m \)

<table>
<thead>
<tr>
<th>process ( i )</th>
<th>IA</th>
<th>IB</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>site ( m )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>0.5</td>
<td>0.7</td>
<td>–</td>
</tr>
<tr>
<td>M2</td>
<td></td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

- The combined index for demand and market flexibility for multiple products, \( F_{md}^i = 0.34 \), under demand uncertainty for all products at distribution centers \( A \) and \( C \) and price uncertainty for all products at \( B \) and \( D \).

- The combined index for demand and market flexibility for a single product, \( F_{md}^s = 0.97 \), under both demand and price uncertainty considered simultaneously only for product \( A \) distributed at distribution center \( A \). Both demand and price for product \( A \) are assumed to vary independently from each other. This results in an NLP formula-
tion (solved with IPOPT 3.8) due to the bilinear term depicting revenue as a product of price and demand as shown previously in Eq. (3.9).

- Overall flexibility index, \( F = 0.28 \), given the presence of 3 types of uncertainty simultaneously: demand (for all products distributed at distribution centers \( A \) and \( C \)), price (for all products distributed at distribution centers \( B \) and \( D \)) and supply uncertainty simultaneously. The overall flexibility of entire supply chain is significantly worse than any of the individual indices under the uncertainty due to the combination of the limiting conditions from individual uncertainty.

**Flexibility under sustained unit shutdown**

The second part of flexibility analysis is to examine the how the demand flexibility is impacted by a sustained unit shutdown. Extended from the previous case, the same supply chain network now experiences a shutdown of production B in plant 1, i.e. the capacity of that process is reduced to zero. At the same uncertainty level, the demand flexibility decreases significantly from 0.91 to 0.31. A decline is expected as now the supply chain has to operate under more stringent conditions and needs to counteract the same demand volatility with some production capacity unavailable due to a unit shutdown. This part of the case study also demonstrates an important application of the flexibility index analysis. The index is scalar valued and its magnitude is more meaningful when used to compare different design options.

### 3.2.2 Case Study 2

The second case study will demonstrate the application of the flexibility analysis framework to solve a design problem. In principle, design decisions cannot be easily modified during operations once the supply chain configuration is determined, and thus it is important to design the supply chain in a manner that feasible operation can be maintained under different uncertainty scenarios by adjusting operation variables.
A process supply chain network partially adapted from Bok et al. [2000] is considered. This network has one supplier providing raw materials to one manufacturing plant, and final products are distributed directly to market without the use of any distribution facilities. The plant consists of a set of manufacturing processes that are interconnected in a finite number of ways as shown in Fig. 3.3. These processes are either dedicated or flexible involving a set of chemicals (raw materials, intermediates, and products). The chemicals J1, J2, J4 and J6 can either be purchased from suppliers or produced as intermediate products from other processes. The chemicals J3 and J5 can be sold as final products to market, or J3 can be consumed in other processes as an intermediate product. The set of manufacturing processes involved are described individually below, and the material balance coefficients for all the schemes for each of the processes are listed in Table 3.7.

- **Process 1** is a dedicated continuous process producing chemical J3 from chemicals J1 and J6.
- **Process 2** is a flexible continuous process with two production schemes. Scheme 1 produces chemical J3 from chemical J1, while scheme 2 produces chemical J4 from chemicals J1 and J6.
- **Process 3** is a flexible continuous process with four production schemes. Both schemes 1 and 2 use chemical J1 to produce chemical J3 and J4, respectively. Schemes 3 and
Process 4 is a flexible continuous process with two production schemes. Scheme 1 produces chemical J5 and J6 from chemical J3, while scheme 2 produces the same products from chemical J4.

Table 3.7: Material balance coefficients ($\eta_{j,k,i,m}$) for chemical j in scheme k of process i in the plant

<table>
<thead>
<tr>
<th>process (i)</th>
<th>scheme (k)</th>
<th>chemical (j)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>J1</td>
</tr>
<tr>
<td>I1</td>
<td>–</td>
<td>1.05</td>
</tr>
<tr>
<td>I2</td>
<td>K1</td>
<td>1.02</td>
</tr>
<tr>
<td>I2</td>
<td>K2</td>
<td>1.10</td>
</tr>
<tr>
<td>I3</td>
<td>K1</td>
<td>1.10</td>
</tr>
<tr>
<td>I3</td>
<td>K2</td>
<td>1.20</td>
</tr>
<tr>
<td>I3</td>
<td>K3</td>
<td>–</td>
</tr>
<tr>
<td>I3</td>
<td>K4</td>
<td>–</td>
</tr>
<tr>
<td>I4</td>
<td>K1</td>
<td>–</td>
</tr>
<tr>
<td>I4</td>
<td>K2</td>
<td>–</td>
</tr>
</tbody>
</table>

There are two sources of uncertainty for this system: the maximum availability of raw materials (supply uncertainty) and the demand of products (demand uncertainty). The capacity of each production process is to be determined so that this network can perform satisfactorily under both demand and supply uncertainties in the economically optimal manner.

The steady-state supply chain model described in Eqs. (3.1) to (3.7) is used with capacity $C_{i,m}^P$ of process i at site m as the design variable. Operating variables include the purchase quantity $O_{j,h,m}$ of raw material, chemical flow $W_{j,k,i,m}$, shipment $F_{j,m,v}$ from site m to distribution center v, distribution quantity $Q_{j,v}$ from distribution center v to customers, and binary variables $X_{k,i,m}$ to select production schemes during operations. Due to the
existence of the binary variable and bilinear term in Eq. (3.6), this supply chain model takes the form of a mixed-integer nonlinear program (MINLP), which is less desirable to solve as an optimization problem. Thus this equation is reformulated to avoid nonlinearity using Eqs. (3.20) and (3.21) below.

\[
W_{j,k,i,m} \leq C_{i,m}^{\text{max}} X_{k,i,m} \quad \forall j \in J_{k,i}^M, k \in K_i, i \in I_m, m \in M 
\]  
(3.20)

\[
W_{j,k,i,m} \leq C_{i,m}^{P} \quad \forall j \in J_{k,i}^M, k \in K_i, i \in I_m, m \in M
\]  
(3.21)

Eq. (3.20) becomes active when the binary variable is zero, and the production capacity is bounded by Eq. (3.21) when the binary variable is one. Parameter \(C_{i,m}^{\text{max}}\) denotes the maximum capacity that can be allocated for process \(i\) in site \(m\).

The objective is to minimize the expected total cost, \(E(\text{Cost})\), as the uniformly weighted average of optimal solutions from all scenarios. The overall optimization formulation is comprised of equations below, where \(N\) represents the number of uncertain scenarios (indexed by \(s \in S\)). Three uncertain parameters are considered, demands of chemical \(J_3\) and \(J_5\), as well as the maximum availability of chemical \(J_6\), corresponding nominal and extreme values are listed in Table 3.8. The cost-related parameters can be found in Tables 3.9 to 3.12. In the proposed formulation, equations set representing the supply chain model need to be reiterated for every scenario, but the subscript \(s\) is omitted for the purpose of clarity.

\[
\min_{z,X_{k,i,m}} E(\text{Cost}) = \frac{1}{N} \sum_{s \in S} \text{Cost}^{\text{OPR}}_s + \text{Cost}^{\text{CAP}}_s 
\]  
(3.22)

subject to:

supply chain model Eqs. (3.1)-(3.5), and (3.20)-(3.21)

supply chain cost Eqs. (3.11) and (3.10)

\[
O_{j,h}^{\text{max}} = O_{j,h}^{\text{max}}^N \pm F \Delta O_{j,h}^{\text{max}} \quad \forall j \in J, h \in H_j 
\]  
(3.23)

\[
D_{j,v} = D_{j,v}^N \pm F \Delta D_{j,v} \quad \forall j \in J, v \in V_j 
\]  
(3.24)

\[
X_{k,i,m} \in \{0, 1\} \quad \forall k \in K_i, i \in I_m, m \in M 
\]  
(3.25)
\[ z \geq 0, \text{ where } z = \begin{cases} O_{j,h,m}, & \forall \ j \in J, h \in H_j, m \in M_j \\ W_{j,k,i,m}, & \forall \ j \in J, k \in K_i, i \in I_m, m \in M \\ F_{j,m,v}, & \forall \ j \in J, m \in M, v \in V_j \\ Q_{j,v}, & \forall \ j \in J, v \in V_j \\ C_{i,m}, & \forall \ i \in I_m, m \in M \\ O_{j,h}^\text{max}, & \forall \ j \in J, h \in H_j \\ D_{j,v}, & \forall \ j \in J, v \in V_j \end{cases} \] 

(3.26)

Table 3.8: Maximum availability \( O_{j,h}^\text{max} \) in units of raw material and demand \( D_j \) in units of final product with nominal values and extreme bounds if uncertain

<table>
<thead>
<tr>
<th>chemical ( j )</th>
<th>Nominal ( O_{j,h}^\text{max} )</th>
<th>Upper bound ( \Delta O_{j,h}^\text{max}^+ )</th>
<th>Lower bound ( \Delta O_{j,h}^\text{max}^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>80</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>J2</td>
<td>100</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>J4</td>
<td>100</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>J6</td>
<td>10</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>( D_j^\text{N} )</td>
<td>( \Delta D_j^+ )</td>
<td>( \Delta D_j^- )</td>
</tr>
<tr>
<td>J3</td>
<td>50</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>J5</td>
<td>30</td>
<td>18</td>
<td>6</td>
</tr>
</tbody>
</table>

Fig. 3.4 displays both the expected total cost and the optimal capacity sizing for this supply

Table 3.9: Raw material cost \( c_{j,h}^\text{PRC} \) in \$/unit of raw material \( j \) ordered from supplier \( h \)

<table>
<thead>
<tr>
<th>raw material ( j )</th>
<th>supplier ( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>H1</td>
</tr>
<tr>
<td>J2</td>
<td>0.75</td>
</tr>
<tr>
<td>J4</td>
<td>0.5</td>
</tr>
<tr>
<td>J6</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Table 3.10: Capacity building cost ($c_{CP}^{i,m}$) in $/unit of capacity for process $i$ in the plant

<table>
<thead>
<tr>
<th>process ($i$)</th>
<th>$C_{i}^{Cap}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>1</td>
</tr>
<tr>
<td>I2</td>
<td>1.2</td>
</tr>
<tr>
<td>I3</td>
<td>1.5</td>
</tr>
<tr>
<td>I4</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 3.11: Variable operation cost ($c_{PRD}^{k,i,m}$) in $/unit produced in scheme $k$ of process $i$ in the plant

<table>
<thead>
<tr>
<th>process ($i$)</th>
<th>scheme ($k$)</th>
<th>K1</th>
<th>K2</th>
<th>K3</th>
<th>K4</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>I2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
<tr>
<td>I2</td>
<td>I2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
<tr>
<td>I3</td>
<td>I3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
<tr>
<td>I4</td>
<td>I4</td>
<td>0.2</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 3.12: Fixed operation cost ($c_{FIX}^{k,i,m}$) in $ if production scheme $k$ of process $i$ in operation

<table>
<thead>
<tr>
<th>process ($i$)</th>
<th>scheme ($k$)</th>
<th>K1</th>
<th>K2</th>
<th>K3</th>
<th>K4</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>I1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
<tr>
<td>I2</td>
<td>I2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
<tr>
<td>I3</td>
<td>I3</td>
<td>0.1</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
<tr>
<td>I4</td>
<td>I4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.12</td>
<td>0.1</td>
</tr>
</tbody>
</table>
chain with flexibility index ranging from 0.1 to 2. At each of the flexibility level, an MILP problem with 81 binary and 347 linear variables is solved using Gurobi 5.5. The plot exhibits a positive correlation between expected total cost and flexibility requirement of the supply chain. A higher flexibility level requirement results in greater capacities for all processes within the network to accommodate the more volatile demands and more limited supply. As a direct result of greater capacity, there is a positive correlation between the flexibility requirement and the overall capital cost.

![Figure 3.4: Capacity (bar) and expected total cost (*) as a function of flexibility requirement](image)

In terms of individual capacity, a positive correlation is observed between process 4 capacity and desirable flexibility level. The trend for each of the first 3 processes are not strictly monotonic, but the sum of these 3 capacity increases with the flexibility requirement. Process 4 is the only process that can produce chemical J5, a final product requested by market, so its capacity must increase accordingly to accommodate the greater production level. Some of the inputs to process 4 are contributed by the first 3 processes. Therefore, a clearly greater combined capacity is observed to support the downstream production.

The capacity of process 2 stops decreasing at flexibility index equal to 1.9 and in the meantime the capacity of process 3 stops increasing with flexibility requirement, which indicates a capacity relocation strategy switching. As explained in the process description,
both processes 2 and 3 produce the same product following similar recipes. However, the capital cost for introducing additional capacity for process 3 is more expensive. Taking into consideration the other operating costs as well, it is more economically favorable to substitute the production capability of process 3 with that of process 2 at flexibility index of 1.9 onwards. This results in a lower total cost of entire supply chain operation.

At a flexibility index of one, the capacity sizing of each process is given in Table 3.13 and the operations of this design under two selected uncertain scenarios are detailed in Figs. 3.5 and 3.6. The chemical flows and schemes that are in use remain highlighted while the rest are in grey. The numbers labeled next to each colored arrow represent the corresponding chemical flows.

Table 3.13: Optimal production capacity \( (C_{i,m}^P) \) in units produced in process \( i \) in the plant at flexibility index \( F = 1 \)

<table>
<thead>
<tr>
<th>process ( i )</th>
<th>( C_{i,m}^P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>9</td>
</tr>
<tr>
<td>I2</td>
<td>34</td>
</tr>
<tr>
<td>I3</td>
<td>12</td>
</tr>
<tr>
<td>I4</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 3.5: Operation of scenario 1: J3 and J5 demands high, J6 supply low
Fig. 3.5 depicts the operating decisions when both demands of J3 and J5 are at the upper extreme value, and the maximum availability of J6 is at its lower extreme value. This is also the so-called “worst-case” scenario since the supply chain needs to satisfy the highest demands with the least amount of resource. Processes 1, 2, and 3 all exert their full capacity to produce J3 in order to fulfill the high demand. J5 is produced through scheme 2 of process 4 by using raw material J4 completely from a supplier as no upstream processes can spare their capacity to produce any J4. As a byproduct from scheme 2 of process 4, a small amount of J6 is produced and used to supply process 1 so that the necessary purchase of J6 is minimized.

Fig. 3.6: Operation of scenario 2: J3 and J5 demands low, J6 supply low

Fig. 3.6 illustrates the production paths when both demands of J3 and J5 are low, and the availability of J6 is also at the lower extreme level. The lower demand of J3 is fulfilled by processes 1 and 2. Process 3 is now switched to actively supply J4 for the downstream process 4 to produce required product J5 from it for market. As a result, the purchase of J4 from a supplier is significantly less compared to that in the worst-case scenario as J4 can now be replenished from the internal manufacturing processes. Another consequence of the lower throughput of process 4 is that the production of J6 is also reduced. Therefore, more J6 needs to be purchased from a supplier to ensure sufficient supply to process 1 so it can run at its full capacity to fulfill the J3 demand.
To remark on one of the key assumptions made for this case study, we assume the feasible region of this problem is continuous and one dimensional quasi-convex (1-DQC). This problem takes the form of an MILP, which does not imply convexity. We have looked into it further to generate the quilt plot of the feasible region as previously done in Sirdeshpande et al. [2005]. Under a fixed capacity sizing design determined by the flexibility level, a feasibility test is performed by sampling a large number of different combinations of uncertain parameters. The feasible region is therefore graphically reconstructed showing no discontinuity within the feasible region and the region is 1-DQC. The vertex enumeration strategy (VES) approach we adopted in this case study only guarantees the feasibility of the extreme points of the uncertain parameter sets. At this point, feasibility of the entire range cannot be proved analytically, the optimal solutions obtained serve as the necessary condition for the feasibility of a specific capacity sizing determined by flexibility.

3.3 Summary

In this chapter, the steady-state attribute of operability, flexibility, is quantified for process supply chains. The development of flexibility analysis in chemical plants is reviewed and the application of the flexibility analysis has been extended in supply chains under uncertainty. Two types of flexibility measures, individual and overall flexibility indices, are quantified using this framework under various supply chain uncertainties. Two illustrative case studies are presented to demonstrate the application of this framework on such aspects, including evaluation of individual and overall flexibility indices for an existing supply chain, and the examination of the trade-offs between economic criteria and flexibility in a design problem involving production capacity sizing and binary operating variables. Overall, this chapter provides a detailed understanding of the flexibility framework and quantitative application to supply chain networks, and offers a decision-making framework for optimal supply chain design using a quantitative measure of flexibility.
Chapter 4

Supply Chain Responsiveness Analysis Framework

4.1 Supply Chain Responsiveness Analysis

A tactical-level supply chain model is developed to represent a discrete-time multi-period formulation, where the horizon is divided into equal length intervals, $\Delta t$, indexed by $t \in T = \{1, \ldots, H\}$, where $H$ is the horizon length. The chemicals $j \in J$ involved in this model can be categorized into three types: raw materials ($J_{RM}$) which cannot be produced within the supply chain and are only available through purchasing from suppliers, final products ($J_{FP}$) which are produced to be sold to customers, and finally the intermediate products ($J_{IP}$) which are produced and then either consumed in other processes or sold to customers as final products. Both raw materials and intermediate products can be stored on site within the manufacturing site, and the final products are stored in distribution centers.

There are three types of delays considered in this model. The procurement delay accounts for the time between order placement and arrival of the raw material at the manufacturing site available for processing. The production delay for each of the process is for capturing the time between the consumption of inputs and the generation of outputs of a process,
which can also be interpreted as the residence time of a processing unit. The transportation delay is mainly affected by the distance between the origin and destination, for instance, manufacturing site and distribution center. All delays are assumed to take a value of multiple of $\Delta t$ to avoid a fractional time index in all related variables as described later.

4.1.1 Multi-period Supply Chain Model

The multi-period supply chain model presented here is an extension of the steady-state model developed in Section 3.1.2. The major difference is the inclusion of the change in inventory levels in the material balances, as well as the time delays involved in the process. Since the application focus will be on integrated forest biorefineries, we refer to manufacturing sites as biorefineries.

The mass balance of raw materials and intermediate products at a biorefinery is given by Eq. (4.1). $\delta_{j,h,m}^{PRC}$ represents the procurement delay between when an order is made to a supplier $h$ and when the material arrives on site and can be processed by a processing unit at biorefinery $m$. The current inventory level of chemical $j$ at biorefinery $m$, $I_{j,m,t}^M$, is the summation of last period’s ending inventory and current period inflows less the outflows within the time period. Inflows of chemical $j$ include the generation from production ($\sum_{i \in I_m} \sum_{k \in K^P_{ji}} W_{j,k,i,m,t}$) and the purchase placed $\delta_{j,h,m}^{PRC}$ periods ago but arrived in the current period ($\sum_{h \in H_j} O_{j,h,m,(t-\delta_{j,h,m}^{PRC})/\Delta t}$). The outflows of the chemical consist of consumption ($\sum_{i \in I_m} \sum_{k \in K^C_{ji}} W_{j,k,i,m,t}$) and shipments to distribution centers ($\sum_{v \in V_j} F_{j,m,v,t}$).

$$\sum_{h \in H_j} O_{j,h,m,(t-\delta_{j,h,m}^{PRC})/\Delta t} + \sum_{i \in I_m} \sum_{k \in K^P_{ji}} W_{j,k,i,m,t} - \sum_{i \in I_m} \sum_{k \in K^C_{ji}} W_{j,k,i,m,t} - \sum_{v \in V_j} F_{j,m,v,t} + I^M_{j,m,t-1} = I^M_{j,m,t} \quad \forall j \in J_{RM} \cup J_{IP}, m \in M_j, t \in T$$ (4.1)

The raw material for forest products is often biomass, which is considered perishable. Some studies consider the deterioration of biomass with time by including a deterioration rate in the inventory balances [Eksioglu et al., 2009; Mastragostino and Swartz, 2014], but such phenomenon is not considered in the current model of our study.

All final products produced are not stored on site at the biorefineries as shown in Eq. (4.2)
and are grouped by destination and shipped to various distribution centers. At distribution centers, incoming products received from biorefineries are stored and distributed to customers \( c \in C_j \), who request product \( j \). It is also noted that Eq. (4.2) is consistent with Eq. (4.1) written for final products, \( j \in J_{FP} \), where both of the purchase and consumption terms of \( j \) are zero.

\[
\sum_{v \in V_j} F_{j,m,v,t} = \sum_{i \in I_m} \sum_{k \in K_{ji}} W_{j,k,i,m,t} \quad \forall j \in J_{FP}, m \in M_j, t \in T \quad (4.2)
\]

The inventory balance of final products at distribution centers is shown in Eq. (4.3), where \( F_{j,m,v,t} \) represents the quantity of final product \( j \) shipped from biorefinery \( m \) to distribution center \( v \), and \( Q_{j,c,v,t} \) stands for the quantity of final products distributed to customer \( c \) from distribution center \( v \) during each time period. \( \delta_{m,v}^{TRP} \) represents the transportation delay of final products shipped from biorefinery \( m \) to distribution center \( v \), and it is assumed that no delays are considered for products to reach customers from distribution centers.

\[
\sum_{m \in M_j} F_{j,m,v,(t-\delta_{m,v}^{TRP}/\Delta t)} - \sum_{c \in C_j} Q_{j,c,v,t} + I_{j,v,t}^V = I_{j,v,t}^{V-1} \quad \forall j \in J_{FP}, v \in V_j, t \in T \quad (4.3)
\]

Eqs. (4.4) and (4.5) relate the chemicals involved to the main product defined in each of the schemes.

\[
W_{j,k,i,m,t,(t-\delta_{k,i}^{PRD}/\Delta t)} = \eta_{j,k,i,m} W_{j',k,i,m,t} \quad \forall j \in J_{ki}^C, j' \in J_{ki}^M, k \in K_i, i \in I_m, m \in M_j \quad (4.4)
\]

\[
W_{j,k,i,m,t} = \eta_{j,k,i,m} W_{j',k,i,m,t} \quad \forall j \in J_{ki}^P, j' \in J_{ki}^M, k \in K_i, i \in I_m, m \in M_j \quad (4.5)
\]

Sets \( J_{ki}^C \) and \( J_{ki}^P \) represent the chemicals that are consumed and produced respectively in scheme \( k \) of process \( i \). \( J_{ki}^M \), a subset of \( J_{ki}^P \), is the set of main chemical produced in scheme \( k \) of process \( i \). Eq. (4.4) captures the relationship between the inputs and the main chemical, where \( \delta_{k,i}^{PRD} \) is the production delay for scheme \( k \) of process \( i \). Eq. (4.5) relates the output chemicals to the main chemical in each scheme, and thus the output flows are not affected by the production delays.

Unlike the steady-state model used for the flexibility analysis, the demands may be only partially fulfilled for some time periods. The unmet demands of product \( j \) from customer \( c \), \( B_{j,c,t} \), can be either considered as lost sale or logged as back orders to be fulfilled later.
Eq. (4.6) captures the demand fulfillment which in particular depicts the scenario of back order accumulation. This equation would be suitable for the scenario counting unmet demands as lost sale, if the term $B_{j,c,t-1}$ is omitted from the left side of the equation, because no previous unmet demands are accumulated throughout the time horizon.

$$D_{j,c,t} - \sum_{v \in V_j} Q_{j,c,v,t} + B_{j,c,t-1} = B_{j,c,t} \quad \forall j \in J_{FP}, c \in C_j, t \in T$$

(4.6)

Similar to the steady-state supply chain model, two capacity constraints are also included. All raw materials ordered for all the biorefineries from any supplier during a time period are bounded by a maximum availability as given in Eq. (4.7). The total output volume of the main product in process $i$ at biorefinery $m$ is constrained by the production capacity $C_{i,m}$ as in Eq. (4.8).

$$\sum_{m \in M_j} O_{j,h,m,t} \leq O_{j,h}^{max} \quad \forall j \in J_{RM}, h \in H_j, t \in T$$

(4.7)

$$\sum_{k \in K_i} W_{j,k.i,m,t} \leq C_{i,m}^{P} \quad \forall j \in J_{ki}^M, i \in I_i, m \in M_j, t \in T$$

(4.8)

In addition to those present in the steady-state model, the capacity constraints related inventory levels are also included in the following equations. For both raw materials and intermediate products stored at a biorefinery, Eq. (4.9) depicts the inventory capacity constraint. For final products stored at distribution centers, Eq. (4.10) captures such constraints.

$$I_{j,m,t}^{M} \leq C_{j,m}^{M} \quad \forall j \in J_{RM} \cup J_{IP}, m \in M_j, t \in T$$

(4.9)

$$I_{j,v,t}^{V} \leq C_{j,v}^{V} \quad \forall j \in J_{FP}, v \in V_j, t \in T$$

(4.10)

4.1.2 Dynamic Responsiveness Formulation

Building on the formulation presented in Mastragostino and Swartz [2014], the basis of quantitative characterization of dynamic responsiveness is to evaluate the lead time of a supply chain process in response to a step change in demand. The lead time is computed by tracking the time between when a change in demand occurs and when the new demand level is fulfilled and sustained for the rest of the optimization horizon.
Figure 4.1: Illustration of lead time evaluation: (a) unmet demand logged as back order; (b) unmet demand considered as lost sale

Fig. 4.1 conceptually illustrates the approach, where in both sub-figures (a) and (b) the lead time is delay between the moment the step change takes place, and the time when demand can be satisfied and sustained. Sub-figure (a) considers the scenario when the unmet demand is logged as back orders, which need to be delivered in later times, while sub-figure (b) depicts the scenario when the unmet demand is regarded as lost sale, such as products sold to a spot market, which are not needed in later time periods if not fulfilled in the current time period. The demand response has an overshoot in sub-figure (a), and the additional delivery is to fulfill the back orders accumulated during the lead time. This is also consistent with the response of inventory levels, indicating a faster return to setpoint in sub-figure (b) as the inventory starts to replenish right after the new demand is met and sustained.

It is necessary to include a binary variable, $Y_{j,c,t}^{\text{met}}$, in the formulation to track the demand fulfillment. $Y_{j,c,t}^{\text{met}} = 1$ if the demand of product $j$ requested by customer $c$ can be served and sustained at time $t$ and $Y_{j,c,t}^{\text{met}} = 0$ otherwise.

The inventory setpoint of chemicals, $\bar{I}$, and the inventory offset as the deviation below the setpoint, $E$, are introduced as a part of the formulation to minimize the inventory offset. The variables are non-negative so the inequalities ensure that the offsets do not count when
inventory is above the setpoint as shown in the following equations.

\[ E_{j,m,t}^M \geq \bar{I}_M - I_{j,m,t} \quad \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j, t \in T \]  
(4.11)

\[ E_{j,v,t}^V \geq \bar{I}_V - I_{j,v,t} \quad \forall \ j \in J_{FP}, v \in V_j, t \in T \]  
(4.12)

The following disjunction formulation allows demand to be partially satisfied at time period \( t \) through the variable \( K_{j,c,t}^1 \), and restricts the variable \( K_{j,c,t}^2 \) to be zero when \( Y_{j,c,t}^{\text{met}} = 0 \). When \( Y_{j,c,t}^{\text{met}} = 1 \), \( K_{j,c,t}^1 \) is forced to be 0, and \( K_{j,c,t}^2 \) serves the demand at time period \( t \) in full.

\[ \sum_{v \in V_j} Q_{j,c,v,t} = K_{j,c,t}^1 + K_{j,c,t}^2 \quad \forall \ j \in J_{FP}, c \in C_j, t \in T \]  
(4.13)

\[ K_{j,c,t}^1 \leq D_{j,c,t}(1 - Y_{j,c,t}^{\text{met}}) \quad \forall \ j \in J_{FP}, c \in C_j, t \in T \]  
(4.14)

\[ K_{j,c,t}^2 = D_{j,c,t}Y_{j,c,t}^{\text{met}} \quad \forall \ j \in J_{FP}, c \in C_j, t \in T \]  
(4.15)

As shown in Eqs. (4.14) and (4.15), the system exerts its effort to only fulfill the demand at the current time period, without accounting the accumulated back orders till the current time period. This is consistent with treating the unmet demand as lost sale (depicted in Fig. 4.1 (b)) and is appropriate to model specialty products demand with volatile patterns or spot markets in this manner. On the other hand, the above formulation can be easily adapted to the situations when the unmet demand is treated as back orders and needs to be supplied to customers by the end of the optimization horizon (depicted in Fig. 4.1 (a)). Eqs. (4.13) and (4.14) remain unchanged but the following constraint in Eq. (4.16) is included to replace Eq. (4.15).

\[ D_{j,c,t}Y_{j,c,t}^{\text{met}} \leq K_{j,c,t}^2 \leq \alpha Y_{j,c,t}^{\text{met}} \quad \forall j \in J_{FP}, c \in C_j, t \in T \]  
(4.16)

This constraint allows distribution centers to distribute more products to customers than what request in a time period when \( Y_{j,c,t}^{\text{met}} = 1 \), and \( \alpha \) is chosen as a large number relative to \( D_{j,c,t} \). The additional amount distributed is to fulfill the back orders accumulated so all back orders can be cleared by the end of the optimization horizon by including an addition constraint in the model, \( B_{j,c,t} = 0 \). When \( Y_{j,c,t}^{\text{met}} = 0 \), the demand is not fully served so \( K_{j,c,t}^2 = 0 \).
Eqs. (4.17) and (4.18) restrict $Y_{j,c,t}^{\text{met}}$ to only change from 0 to 1, since demand fulfillment is required to be sustained over the optimization horizon. The nonnegative continuous variable $A_{j,c,t}$ takes a value of 1 to indicate when $Y_{j,c,t}^{\text{met}}$ has switched from 0 to 1.

$$Y_{j,c,t}^{\text{met}} - Y_{j,c,t-1}^{\text{met}} = A_{j,c,t} \quad \forall j \in J_{FP}, c \in C_j, t = 2 \ldots H$$  \hspace{1cm} (4.17)

$$Y_{j,c,t}^{\text{met}} = A_{j,c,t} \quad \forall j \in J_{FP}, c \in C_j, t = 1$$  \hspace{1cm} (4.18)

If the worst case lead time of the system is expected to be smaller than the optimization horizon length, a tightening constraint shown in Eq. (4.19) can be applied. The parameter $\tau$ indicates the time period by which the demand must be met, and $\tau \leq H$. $\tau$ can be determined appropriately to enhance computational performance by exploiting system knowledge. $Y_{j,c,t}^{\text{met}}$ is also fixed at 1 from the tightening constraint as shown in Eq. (4.20) to indicate that the satisfaction of demand is sustained.

$$\sum_{t=1}^{t=\tau} A_{j,c,t} = 1 \quad \forall j \in J_{FP}, c \in C_j$$  \hspace{1cm} (4.19)

$$Y_{j,c,t}^{\text{met}} = 1 \quad \forall j \in J_{FP}, c \in C_j, t = \tau \ldots H$$  \hspace{1cm} (4.20)

The following equations describe how the system behaves at the beginning and the end of the time horizon. The inventory setpoints are time independent, equated to the inventory position at $t = 0$ (Eqs. (4.21) and (4.22)). In addition, Eqs. (4.23) and (4.24) enforce the inventory levels to return to the corresponding setpoints after the response to demand step change, at the end of the optimization horizon.

$$\bar{I}_{j,m}^M = I_{j,m,t=0}^M \quad \forall j \in J_{RM} \cup J_{IP}, m \in M_j,$$  \hspace{1cm} (4.21)

$$\bar{I}_{j,v}^V = I_{j,v,t=0}^V \quad \forall j \in J_{FP}, v \in V_j,$$  \hspace{1cm} (4.22)

$$\bar{I}_{j,m}^M \leq I_{j,m,t=H}^M \quad \forall j \in J_{RM} \cup J_{IP}, m \in M_j,$$  \hspace{1cm} (4.23)

$$\bar{I}_{j,v}^V \leq I_{j,v,t=H}^V \quad \forall j \in J_{FP}, v \in V_j,$$  \hspace{1cm} (4.24)

The lead time to completely satisfy the demand of chemical $j$ requested by customer $c$ is defined in Eq. (4.25) as the time elapsed before the demand is fulfilled.

$$L_{j,c} = \sum_t (t-1)A_{j,c,t}\Delta t \quad \forall j \in J_{FP}, c \in C_j$$  \hspace{1cm} (4.25)
The system lead time (SLT) given in Eq. (4.26) is depicted as a weighed-average of lead times for all products reaching all customers via different paths. \( \omega_{jc} \) is the weight of lead time assigned for final product \( j \) requested by customer \( c \), and \( \sum_{j \in J_{FP}} \sum_{c \in C_j} \omega_{jc} = 1 \).

\[
SLT = \sum_{j \in J_{FP}} \sum_{c \in C_j} \omega_{jc} L_{j,c} \tag{4.26}
\]

### 4.1.3 Multi-period Supply Chain Economics Formulation

The operational profit of successfully converting raw materials into final products in order to meet demands is described in Eq. (4.27) as the difference between revenue from product sales (as computed in Eq. (4.28)) and operating costs.

\[
\text{Profit} = \text{Revenue} - \text{Total Cost} \tag{4.27}
\]

\[
\text{Revenue} = \sum_{t \in T} \sum_{j \in J_{FP}} \sum_{c \in C_j} \sum_{v \in V_j} p_{Sale}^{j,c} Q_{j,c,v,t} \tag{4.28}
\]

The total operational costs can be further divided into the following terms:

- Procurement costs for raw materials from suppliers (Eq. (4.29))
- Inventory costs for raw materials, intermediate products, and final products (Eq. (4.30))
- Variable production costs proportional to the main product of the production processes (Eq. (4.31))
- Transportation cost of final products from biorefinery to distribution centers (Eq. (4.32))

\[
C_{PRC} = \sum_{t \in T} \sum_{j \in J_{RM}} \sum_{h \in H_j} \sum_{m \in M_j} c_{j,h}^{PRC} Q_{j,h,m,t} \tag{4.29}
\]

\[
C_{INV} = \sum_{t \in T} \sum_{j \in J_{RM} \cup J_{IP}} \sum_{m \in M_j} c_{j,m}^{IP} M_{j,m,t} + \sum_{t \in T} \sum_{j \in J_{FP}} \sum_{v \in V_j} c_{j,v}^{IV} V_{j,v,t} \tag{4.30}
\]

\[
C_{PRD} = \sum_{t \in T} \sum_{j \in J_{RM}} \sum_{m \in M_j} \sum_{i \in I_m} \sum_{k \in K_i} c_{i}^{PRD} W_{j,k,i,m,t} \tag{4.31}
\]

\[
C_{TRP} = \sum_{t \in T} \sum_{j \in J_{FP}} \sum_{m \in M_j} \sum_{v \in V_j} c_{j,m,v}^{TRP} F_{j,m,v,t} \tag{4.32}
\]
To ensure that the inventory level returns to its setpoint when the new demand is met, a penalty term is also included in the economics formulation to penalize the negative deviation away from the setpoint, as shown in Eq. (4.33). The positive deviation is minimized via the inventory cost term in Eq. (4.30), so the system will avoid storing more materials than necessary for the cost benefit.

\[ P_{INV} = \sum_{t \in T} \sum_{j \in J_{RM} \cup J_{IP}} \sum_{m \in M_j} c_{EM,j,m} E_{M,j,m,t}^M + \sum_{t \in T} \sum_{j \in J_{FP}} \sum_{v \in V_j} c_{EV,j,v} E_{V,j,v,t}^V \]  

The capital cost may be computed based on the capacity of the processing units and storage space of inventory using the following equation.

\[ C_{CAP} = \sum_{m \in M} \sum_{i \in I_m} c_{CP,i,m} C_{P,i,m}^P + \sum_{j \in J_{RM} \cup J_{IP}} \sum_{m \in M_j} c_{CIM,j,m} C_{M,j,m}^M + \sum_{j \in J_{FP}} \sum_{v \in V_j} c_{CIV,j,v} C_{V,j,v}^V \]  

### 4.1.4 Optimization Formulation for Responsiveness Analysis

To evaluate the expected lead time under an uncertain demand pattern, a two-stage stochastic programming approach is adopted. The uncertain demand pattern is treated as multiple step changes (denoted by \( s \)) with uncertain magnitudes. The system lead time is computed for each of the scenarios considered, and the expected value is taken as the measure of responsiveness.

The responsiveness evaluation for an existing supply chain system given the fixed supply chain design (topology, delays and capacities) and initial states, is equivalent to solving the second-stage problem of the two-stage stochastic formulation as all the first-stage decisions are already determined. The first-stage decisions get included as model variables when considering the economic criteria in the model, such as inventory setpoints, which are common for all uncertain scenarios. In the next chapter, some of the design decisions such as capacities, will be relaxed, and this framework is going to be adapted so the design decisions can act as first-stage variables.

In order to merely assess the responsiveness of a supply chain system using the system lead time as described in Eq. (4.26), it does not require consideration of the supply chain...
economics. However, the solutions resulting from such an approach may not be cost-optimal, because there are no cost incentives to prevent over-designing or to select cheaper options among all the feasible solutions. Therefore, an economic objective needs to be included in this responsiveness analysis framework.

Maximizing responsiveness and minimizing costs are two conflicting objectives. Many of the attempts on improving the responsiveness, such as holding more inventory, shortening process and transportational delays, would very likely be costly. One of the remedies is to include the responsiveness measure as a constraint of the cost minimization problem. This can be implemented using the \( \epsilon \)-constraint method by specifying that the system lead time must be less than a parameter \( \epsilon \), and then solving the problem with a series of \( \epsilon \) values to perform Pareto analysis to investigate the trade-offs.

The following formulation evaluates the supply chain responsiveness, and the subscript \( s \) denoting scenarios is omitted for clarity purpose.

\[
\min_{z, Y_{j,c,t}} \mathbb{E}(SLT) \tag{4.35}
\]

subject to:

supply chain model: Eqs. (4.1) to (4.10); and

responsiveness criteria: Eqs. (4.11) to (4.26)
\[ z \geq 0, \text{where } z = \begin{cases} A_{j,c,t} & \forall \ j \in J_{FP}, c \in C_j, t \in T \\ B_{j,c,t} & \forall \ j \in J_{FP}, c \in C_j, t \in T \\ C_{i,m}^p & \forall \ m \in M, i \in I_m \\ C_{j,m}^M & \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j \\ C_{j,v}^V & \forall \ j \in J_{FP}, v \in V_j \\ E_{j,m,t}^M & \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j, t \in T \\ E_{j,v,t}^V & \forall \ j \in J_{FP}, v \in V_j, t \in T \\ F_{j,m,v,t} & \forall \ j \in J_{FP}, m \in M_j, v \in V_j, t \in T \\ I_{j,m,t}^M & \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j, t \in T \\ I_{j,v,t}^V & \forall \ j \in J_{FP}, v \in V_j, t \in T \\ K_{j,c,t}^1, K_{j,c,t}^2 & \forall \ j \in J_{FP}, c \in C_j, t \in T \\ L_{j,c} & \forall \ j \in J_{FP}, c \in C_j \\ O_{j,h,m,t} & \forall \ j \in J_{RM}, h \in H_j, m \in M_j, t \in T \\ W_{j,k,i,m,t} & \forall \ j \in J_{ki}, m \in M_j, i \in I_m, k \in K_i, t \in T \end{cases} \] (4.36)

To include the considerations of economic criteria, the inventory setpoints of all materials \((\bar{I}_{j,m} \forall j \in J_{RM} \cup J_{IP}, m \in M_j \text{ and } \bar{I}_{j,v}^V \forall j \in J_{FP}, v \in V_j)\) are now used as first-stage variables rather than model parameters. The objective function below is used instead to account for expected total cost and penalty cost. Eqs. (4.29) to (4.32) are also included, as well as a constraint, \(E(\text{SLT}) \leq \epsilon\), so the value of \(\epsilon\) can be set that the optimal solutions are achieved for each of the responsiveness levels specified.

\[
\min_{z, Y_{j,c,t}} \mathbb{E}(\text{Total Cost} + P_{INV}) \quad (4.37)
\]
4.2 Case studies

A simplified representation of a forest products supply chain is considered for this case study as shown in Fig. 4.2, where the blocks represent processes and triangles represent inventory. An integrated biorefinery receives biomass from suppliers as raw materials, which is then converted into a series of products. The woody biomass is a mixture of 3 major components: cellulose, lignin and hemicellulose. The lignin is separated from hemicellulose and can be used further for value-added products, but is not considered in this part of the case study. The hemicellulose is stored as an intermediate product on site in the biorefinery, and can be either processed further in the fermentation process or sold as a final product distributed to customers from distribution centers. The cellulose component is also stored on site as an intermediate product and can be consumed in the fermentation process to produce bioethanol.

4.2.1 Problem Description

An integrated biorefinery receiving biomass from two suppliers and serving two customers is considered. The system is assumed to operate at steady state at the beginning of the time horizon, which means all processing units operate at a constant production volumes to satisfy customer demands, and there is no accumulation of materials at any of the inventory facilities. There are three types of delays involved in the supply chain operation.

Figure 4.2: Biorefinery supply chain configuration for case study
Table 4.1: Procurement delay ($\delta_{j,h,m}^{PRC}$) in days

<table>
<thead>
<tr>
<th>raw material ($j$)</th>
<th>biorefinery ($m$)</th>
<th>supplier ($h$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biomass</td>
<td>M1</td>
<td>S1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.2: Production delay ($\delta_{i}^{PRD}$) in days

<table>
<thead>
<tr>
<th>process ($i$)</th>
<th>production delay ($\delta_{i}^{PRD}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation</td>
<td>1</td>
</tr>
<tr>
<td>Preparation</td>
<td>0</td>
</tr>
<tr>
<td>Fermentation</td>
<td>3</td>
</tr>
</tbody>
</table>

For this case, as listed in Tables 4.1 to 4.3, the procurement delay, production delay and the transportation delay, but only the transportation delay between the biorefinery and distribution centers is considered, and no delays are assumed for customers to access the final products in the distribution centers.

Table 4.4 summarizes the material balance coefficients for all processes involved in the biorefinery. The coefficients for lignin separation and cellulose preparation processes are based on the information from [Sannigrahi and Ragauskas 2013] and the average biomass composition reported by [McKendry 2002]. The coefficients of fermentation process are in the reference of the theoretical yield by [Humbird et al. 2011] and the average yields determined by [Lin and Tanaka 2006].

At $t = 0$, the supply chain system is assumed to operate optimally at steady state to satisfy

Table 4.3: Transportation delay ($\delta_{m,v}^{TRP}$) in days

<table>
<thead>
<tr>
<th>biorefinery ($m$)</th>
<th>distribution center ($v$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>V1</td>
</tr>
<tr>
<td></td>
<td>V2</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
Table 4.4: Material balance coefficients ($\eta_{j,k,i,m}$) of chemical $j$ in process $i$ of the biorefinery

<table>
<thead>
<tr>
<th>process ($i$)</th>
<th>chemical ($j$)</th>
<th>Biomass</th>
<th>Cellulose</th>
<th>Hemicellulose</th>
<th>Bioethanol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation</td>
<td>2.8 1 -</td>
<td>1 -</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preparation</td>
<td>2.2 1 -</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fermentation</td>
<td>- 2.5 1 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Nominal demands fulfilled by steady-state operation

<table>
<thead>
<tr>
<th>customer ($c$)</th>
<th>chemical ($j$)</th>
<th>Hemicellulose</th>
<th>Bioethanol</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>120</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>150</td>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>

the nominal demand levels as listed in Table 4.5 and the initial inventory levels determined by inventory setpoints.

Two sub-cases are considered based on this network configuration. Firstly, the expected lead time, as a measure of supply chain responsiveness is evaluated by considering different step sizes in demand change using a multi-scenario approach. Then the trade-offs between two conflicting objectives, minimizing expected operating costs and minimizing expected lead time, are investigated.

### 4.2.2 Case Study 1: Evaluation of Expected Lead Time

For this part of the case study, the system is assumed to run at nominal production rates while meeting all demand levels with zero level of inventory to start for all materials. There are 101 scenarios of uncertain step changes in hemicellulose demand considered, including 50 positive step sizes and 50 negative step sizes from the nominal levels, as well as the step size of zero to represent the nominal scenario. The step sizes range from -100 to 100 with
Section 4.2

an interval of 2, resulting in the final demands ranging from 20 to 220 units for customer 1. The step sizes for customer 2 range from -50 to 50 with an interval of 1, resulting in the final demands ranging from 100 to 200 units of hemicellulose.

The expected lead time for these 101 scenarios is 0.35 days as an average of satisfying both hemicellulose customers. All the non-zero lead times are contributed by the scenarios with a positive step change in the demand, as the excess final products due to declined demand can be stored as inventory and used to fulfill demands in later time periods without causing further delays in delivering the products.

If only the scenarios with positive step change in demand are counted, the expected lead time is 0.7 days, which is still significantly less than the sum of minimal delays (0 days in procurement from supplier $S^2 + 1$ day in separation process + 2 days in transportation = 3 days) incurred to produce and deliver hemicellulose. The sum of minimal delays would have been observed if the system were to start from idle without any nominal production levels or inventory levels, because the system would become a complete “pull” system (made-to-order) and experience every necessary delay during the supply chain activities. The system starting with some levels of production has the option to store the products as inventory for several time periods, so the amount of products available to be drawn from inventory in the next period can suffice the new demand. This “self-conservation” feature is unique to the supply chains starting with material flows in the system and is advantageous in terms of responding to demand changes compared to a complete “pull” system. The similar analogy can be found in chemical plant operations such that it usually takes a shorter time to transit to another operating point from the current operating point than from start-up.

4.2.3 Case Study 2: Economic Trade-offs of Responsiveness

This case study aims to explore the trade-offs between responsiveness and the total operational cost of the supply chain over 20 operating periods. To simplify, 10 uncertain demand scenarios are considered as listed: step sizes ranged from -100 to +80 with an interval of 20 units for customer 1, and steps ranged from -35 to +28 with an interval of 7 units for
Table 4.6: Raw material purchase cost ($c_{PRC}^{j,h}$) in $/unit of raw material $j$ purchased from supplier $h$

<table>
<thead>
<tr>
<th>raw material ($j$)</th>
<th>supplier ($h$)</th>
<th>S1</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biomass</td>
<td>S1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7: Production cost ($c_{PRD}^{i}$) in $/day/unit produced in process $i$

<table>
<thead>
<tr>
<th>process ($i$)</th>
<th>production cost ($c_{PRD}^{i}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation</td>
<td>10</td>
</tr>
<tr>
<td>Preparation</td>
<td>20</td>
</tr>
<tr>
<td>Fermentation</td>
<td>50</td>
</tr>
</tbody>
</table>

customer 2. The parameter $\tau$ is set to 4 so the expected lead time for hemicellulose to reach the customers cannot exceed 3 days.

The cost parameters used are listed in Tables 4.6 to 4.8 for procurement, production and transportation costs, respectively. The inventory costs are presented in Tables 4.9 and 4.10.

Fig. 4.3 displays the expected total operating cost (line plot) and the inventory setpoint of hemicellulose stored in the distribution center (bar plot) for each of the expected lead times specified ranging from 0 to 3 days at an interval of 0.1 days. The inventory setpoint displayed serves as a first-stage design decision and thus is common to all demand scenarios.

Table 4.8: Transportation cost ($c_{TRP}^{j,m,v}$) in $/unit of final product $j$ shipped from biorefinery $m$ to distribution center $v$

<table>
<thead>
<tr>
<th>biorefinery ($m$)</th>
<th>distribution center ($v$)</th>
<th>chemical ($j$)</th>
<th>Hemicellulose</th>
<th>Bioethanol</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>V1</td>
<td>12</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>V2</td>
<td>-</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.9: Inventory cost \( (c_{IM}^{j,m}) \) in $/day/unit of raw material or intermediate product \( j \) stored in biorefinery \( m \)

<table>
<thead>
<tr>
<th>chemical ( j )</th>
<th>biorefinery ( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biomass</td>
<td>1</td>
</tr>
<tr>
<td>Hemicellulose</td>
<td>4</td>
</tr>
<tr>
<td>Cellulose</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4.10: Inventory cost \( (c_{IV}^{j,v}) \) in $/day/unit of final product \( j \) stored in distribution center \( v \)

<table>
<thead>
<tr>
<th>distribution center ( v )</th>
<th>chemical ( j )</th>
<th>Hemicellulose</th>
<th>Bioethanol</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>5</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>V2</td>
<td>–</td>
<td>8</td>
<td>–</td>
</tr>
</tbody>
</table>
Figure 4.3: Optimal operating costs with specified lead time requirement
to guarantee feasibility. It is noted that even though the lead times for individual customers
must be integer by definition, the expected lead time, as an equally weighted average of all
scenarios can be fractional.

The expected total operating cost exhibits a decreasing trend in response to the greater
expected lead time allowed. The decline becomes less dramatic for expected lead time
greater than 0.3 days. Such a trend observed in operating cost is consistent with the
requirement of initial inventory of final products as shown by the bar plot. The level of
initial inventory, captured as the inventory setpoint, increases to accommodate the more
stringent requirement of the responsiveness. For instance in the worst case scenario, when
demands for both customers step up to the maximum levels, $80 + 28 = 108$ extra units of
hemicellulose need to be supplied in addition to the nominal demands in each time period.
The transportation delay is 2 days, thus the initial inventory of 216 units is exactly sufficient
to supply the additional demand during the 2 days of delay, and meanwhile the upstream
processes can ramp up production to catch up with the new demand.

The expected total operating cost plotted in Fig. 4.3 excludes the penalty cost associated
with the inventory setpoint deviations. For the expected lead times less than 0.3 days, the
major differences in operating cost are due to maintaining the inventory levels determined by the optimal inventory setpoints during the optimization horizon as the objective function minimizes any negative deviations from the inventory setpoints.

After 0.3 days of expected lead time, the decline in total operating cost is more linear. The reason for this steady decline is that fewer time periods are fulfilled out of the fixed time horizon, when the expected lead time is allowed to be longer. This explanation is confirmed with the constant normalized daily operating cost depicted in Fig. 4.4 where the cost is the expected daily operating cost for steady-state operation sustained at the new demand, and computed based on the average of a 5-day period of steady-state operation to fulfill the new demand. However, despite the fact that it seems to have a cost benefit to elongate the expected lead time during supply chain operation, the revenue for satisfying customer demands on time also reduces significantly. Moreover, the consequences for failing to meet the customer demands on time are not limited to the lost revenue in the nowadays competitive business environment. Potential lost orders, lost customers, damaged reputation and credibility, reduced market share, and other liabilities to customers and partners, are some
examples of the risks that can harm the sustainability and competitiveness of the business with lack of sufficient responsiveness.

4.3 Summary

In this chapter, an analytical framework is developed to quantify responsiveness, a dynamic aspect of supply chain operability. The expected lead time responding to demand step changes, computed using a multi-scenario approach, is used as a measure of supply chain responsiveness. The framework extends from the formulation established by Mastragostino and Swartz [2014] and adopts a supply chain model with multiple production schemes. In addition, the framework is more general in handling unmet demands with the distinction of contractual and spot market demands. A case study on a forest products supply chain is presented to demonstrate the applicability of the proposed framework in two aspects. The framework is first used to evaluate the responsiveness of the supply chain system with pre-existing configuration, initial states and inventory setpoints. Then the inventory setpoints are relaxed and used as first-stage design decisions in a cost minimization problem. Overall, this chapter offers a quantitative methodology to include the consideration of dynamic operability in supply chain design and operation when experiencing short-term fluctuations in product demands.
Chapter 5

Composite Supply Chain
Operability Analysis Framework

5.1 Composite Supply Chain Operability Analysis

A practical mathematical programming formulation is presented so the quantitative considerations of flexibility and responsiveness can be addressed simultaneously within a single optimization-based framework. The key principle underlying this composite framework is to assess the flexibility using the steady-state formulation and meanwhile to evaluate the responsiveness with the dynamic formulation on the same system.

5.1.1 System Description

A multi-echelon and multi-product biomass supply chain is considered. A discrete-time multi-period supply chain model formulation is presented with the time horizon divided into equal length intervals, $\Delta t$, indexed by $t \in T = \{1, \ldots, H\}$, where $H$ is the horizon length. The chemicals $j \in J$ involved in this model can be categorized into three types: raw materials ($J_{RM}$), intermediate products ($J_{IP}$), and final products ($J_{FP}$) as previously
defined in Chapter 4. Both raw materials and intermediate products can be stored on site within the biorefineries, and the final products are stored in distribution centers. The final products are categorized further into two families: the commodity products (denoted as set \( J_{FP}^C \)), whose market conditions are expected to be predictable in the short term but to experience sustained changes in the long run, and specialty products (denoted as set \( J_{FP}^S \)), characterized by high profit margins and volatile demand patterns.

5.1.2 Supply Chain Model

Due to the need to incorporate flexibility and responsiveness analysis simultaneously, both the steady-state version and discrete-time version of the supply chain models developed in the previous chapters are included. The steady-state model are captured in Eqs. (3.1) to (3.6) and its discrete-time counterpart in Eqs. (4.1) to (4.10).

5.1.3 Mathematical Programming Formulation

The composite framework described below, on one hand evaluates flexibility of commodity product demands using the steady-state part of the model with the assumption that the demands of specialty products remain constant at nominal levels. On the other hand, the supply chain responsiveness is assessed on the specialty products using the discrete-time model, assuming the demands of commodity products remain unchanged from their nominal levels.

The formulation takes the form of two-stage stochastic programming where the first stage design decisions are capacity sizings and the inventory setpoints, then the uncertain demand is revealed using the multi-scenario approach. The overall mathematical formulation of the composite framework that integrates both flexibility and responsiveness measures takes the following form.

\[
\min_{d,z,Y^{mext}} \mathbb{E}(\text{Total Cost} + P_{INV}) - C_{CAP} \tag{5.1}
\]

subject to:
steady supply chain model: Eqs. (3.1) to (3.6);

discrete-time supply chain model Eqs. (4.1) to (4.10);

responsiveness criteria: Eqs. (4.11) to (4.26);

economic criteria: Eqs. (4.29) to (4.34);

\[ SFI \geq \epsilon_f \quad (5.2) \]
\[ ELT = E(SLT) \leq \epsilon_r \quad (5.3) \]

\[ D_{j,c,vs} = \mu D_{j,c} + SFI \Delta D_{j,c,vs} \quad \forall j \in J_{FP}, c \in C_j, vs \in VS \quad (5.4) \]
\[ D_{j,c,vs} = \mu D_{j,c} \quad \forall j \in J_{FP}, c \in C_j, vs \in VS \quad (5.5) \]

\[ D_{j,c,t,s} = \mu D_{j,c} + \frac{s}{NS} SFI \Delta D_{j,c} \quad \forall j \in J_{FP}, c \in C_j, t \geq 1, s \in S \quad (5.6) \]
\[ D_{j,c,t,s} = \mu D_{j,c} \quad \forall j \in J_{FP}, c \in C_j, t = 0, s \in S \quad (5.7) \]
\[ D_{j,c,t,s} = \mu D_{j,c} \quad \forall j \in J_{FP}, c \in C_j, t \in T, s \in S \quad (5.8) \]

\[ d \geq 0, \text{ where } d = \left\{ \begin{array}{l}
C_{i,m}^P \quad \forall \ m \in M, i \in I_m \\
C_{a,m}^M \quad \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j \\
C_{j,v}^V \quad \forall \ j \in J_{FP}, v \in V_j \\
I_{j,m}^M \quad \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j \\
I_{j,v}^V \quad \forall \ j \in J_{FP}, v \in V_j 
\end{array} \right. \quad (5.9) \]

\[ z \geq 0, \text{ where } z = \left\{ \begin{array}{l}
A_{j,c,t} \quad \forall \ j \in J_{FP}, c \in C_j, t \in T \\
B_{j,c,t} \quad \forall \ j \in J_{FP}, c \in C_j, t \in T \\
E_{j,m,t}^M \quad \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j, t \in T \\
E_{j,v,t}^V \quad \forall \ j \in J_{FP}, v \in V_j, t \in T \\
F_{j,m,v,t} \quad \forall \ j \in J_{FP}, m \in M_j, v \in V_j, t \in T \\
I_{j,m,t}^M \quad \forall \ j \in J_{RM} \cup J_{IP}, m \in M_j, t \in T \\
I_{j,v,t}^V \quad \forall \ j \in J_{FP}, v \in V_j, t \in T \\
K_{j,c,t}^1, K_{j,c,t}^2 \quad \forall \ j \in J_{FP}, c \in C_j, t \in T \\
L_{j,c} \quad \forall \ j \in J_{FP}, c \in C_j \\
O_{j,h,m,t} \quad \forall \ j \in J_{RM}, h \in H_j, m \in M_j, t \in T \\
Q_{j,c,v,t} \quad \forall \ j \in J_{FP}, v \in V_j, c \in C_j, t \in T \\
W_{j,k,i,m,t} \quad \forall \ j \in J_{ki}, m \in M_j, i \in I_m, k \in K_i, t \in T
\end{array} \right. \quad (5.10) \]
The objective function in Eq. (5.1) minimizes the total capital cost related to building the necessary capacity for processes and inventory, as well as the expected value of the second stage objective, the sum of operating cost and penalty cost. The flexibility and responsiveness criteria are captured through Eqs. (5.2) and (5.3) as the $\epsilon$-constraints.

In addition, Eqs. (5.4) to (5.8) are included to represent the realizations of demand uncertainty using a multi-scenario approach for both commodity and specialty products. For a commodity product, $j \in J^C_{FP}$, only the flexibility measure is considered in a steady-state manner during the design stage. A combination of vertex scenarios for product demands, indexed by $vs \in VS$, are used in conjunction with the steady-state supply chain model. Eq. (5.4) ensures that the optimal design is feasible for all possible combinations of the demand levels determined by these vertex scenarios. In the meantime, the demands of specialty products are assumed to remain at their nominal levels as in Eq. (5.5).

For a specialty product $j \in J^S_{FP}$, the responsiveness measure is considered by evaluating the expected lead time for multiple scenarios of step changes in demands, indexed by $s \in S$. The feasibility to achieve the new demand levels serves as the prerequisite for responsiveness. Hence, the magnitudes of step changes in demands of specialty products are set to depend on the level of flexibility of the entire system. The same flexibility index is also shared with commodity products to guarantee that the resultant design is sufficient for the demand variations of all final products. All the scenarios are the representation of the uniformly increasing step sizes from nominal demand to maximum in terms of the flexibility level. As shown in Eq. (5.6), for the demand scenario of $s = NS$, it implies the lead time taken to meet the maximum step change in demand determined by the flexibility criteria.

5.2 Case Study

This case study aims to demonstrate the application of the composite analysis framework formulation and its adaptation within a design problem. The practical mathematical formulation outlined is tested on a comprehensive case study adapted from Dansereau et al.
which involves a North American forest product company exploring the options of implementing forest biorefinery process. A simplified representation of the supply chain configuration including a subset of the processes and products from the original study is presented in Fig. 5.1. The dark-colored blocks represent the paper production processes, and the light-colored ones represent the biorefinery processes.

The biorefinery is retrofitted from a paper mill with a thermomechanical pulping (TMP) process, capable of processing wood chips to make pulp, and a deinking pulping (DIP) process, consuming recycled newsprint and magazine paper by removing the ink from the fibre to produce pulp. The paper making process then converts pulp into newsprint products. The biorefinery line starts with a biomass organosolv fractionation process as a pretreatment to solubilize the lignin and hemicellulose components in the woody biomass so the remaining cellulose component can be isolated and become more accessible by the downstream processes \cite{Sannigrahi2013}. The lignin component of the biomass is separated from hemicellulose and can be used to produce lignin-derived products or consumed to produce energy but is not considered within the scope of this case study. Hemicellulose is then stored as an intermediate product and can be further processed into furfural products or bioethanol via the fermentation process. The cellulose component is also considered as an intermediate product and stored on site to be further converted into bioethanol via fermentation. The fermentation process can produce bioethanol from either of the feedstocks.
with different yields and microorganisms as catalyst, and it is assumed to be able to operate with any ratios of hemicellulose to cellulose as inputs.

The three final products, newsprint, bioethanol and furfural products are all shipped to corresponding distribution centers, where the demands from different customers for these products are fulfilled.

### 5.2.1 Problem Description

An integrated biorefinery plant receiving raw materials from two suppliers and serving two customers is considered. A time horizon of 20 days with time interval of 1 day is used in this case study. The system is assumed to operate at the nominal production rate at the beginning of the time horizon, which means all processing units operate at a constant production volumes to satisfy nominal demand levels. Tables 5.1 to 5.3 list all the parameters related to the delays considered during the supply chain operation, including procurement delay, production delay and transportation delay between the biorefineries and distribution centers, with no delays assumed for customers to access the final products in the distribution centers.

The commodity products in this case are newsprint, with an expectation of a declined demand, and bioethanol, with an expectation of a growing demand as a substitution of fossil fuels in the future. The furfural products are considered as the high-value specialty products with a volatile demand pattern. The outlined formulation is applied to determine the optimal plant sizing by minimizing the expected economic objective under these demand
Table 5.2: Production delay ($\delta_{i}^{PRD}$) in days

<table>
<thead>
<tr>
<th>process (i)</th>
<th>production delay ($\delta_{i}^{PRD}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deinking pulping</td>
<td>0</td>
</tr>
<tr>
<td>Thermomechanical pulping</td>
<td>0</td>
</tr>
<tr>
<td>Paper making</td>
<td>0</td>
</tr>
<tr>
<td>Fractionation</td>
<td>0</td>
</tr>
<tr>
<td>Separation</td>
<td>1</td>
</tr>
<tr>
<td>Preparation</td>
<td>0</td>
</tr>
<tr>
<td>Fermentation</td>
<td>3</td>
</tr>
<tr>
<td>Furfural production</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.3: Transportation delay ($\delta_{m,v}^{TRP}$) in days

<table>
<thead>
<tr>
<th>biorefinery (m)</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 5.4: Material balance coefficients \((\eta_{j,k,i,m})\) of chemical \(j\) in process \(i\)

<table>
<thead>
<tr>
<th>Process ((i))</th>
<th>RP</th>
<th>WC</th>
<th>PP</th>
<th>BM</th>
<th>HC</th>
<th>CL</th>
<th>NP</th>
<th>BE</th>
<th>FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deinking</td>
<td>1.2</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMP</td>
<td></td>
<td></td>
<td></td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paper making</td>
<td></td>
<td></td>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fractionation</td>
<td></td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separation</td>
<td></td>
<td></td>
<td></td>
<td>2.8</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preparation</td>
<td></td>
<td></td>
<td></td>
<td>2.2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fermentation</td>
<td></td>
<td></td>
<td></td>
<td>2.5</td>
<td></td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Furfural production</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Uncertainties. The plant sizing decisions include:

- the optimal processing capacity for all paper making and biorefinery processes
- the optimal storage capacity for all raw materials and intermediate products stored in biorefinery, and final products stored in distribution centers
- the inventory setpoints to indicate the initial inventory levels for all storage facilities

Table 5.4 summarizes the material balance coefficients for all processes involved in the biorefinery. The data for paper line processes are collected from Bajpai [2011]. The data for fractionation, separation and cellulose preparation processes are based on the information from Sannigrahi and Ragauskas [2013] and the average biomass composition reported by McKendry [2002]. The coefficients of fermentation process are in the reference of the theoretical yield by Humbird et al. [2011] and the average yields determined by Lin and Tanaka [2006]. The furfural production coefficients are estimated using the theoretical yield by Mandalika and Runge [2012].

The cost parameters used are listed in Tables 5.6 to 5.8 for procurement, production and transportation costs, respectively. The inventory costs are presented in Tables 5.9 and 5.10.
Table 5.5: Nominal demands in tonnes of final product $j$ fulfilled for customer $c$ by steady-state operation

<table>
<thead>
<tr>
<th>customer ($c$)</th>
<th>Newsprint</th>
<th>Bioethanol</th>
<th>Furfural products</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>400</td>
<td>500</td>
<td>200</td>
</tr>
<tr>
<td>C2</td>
<td>500</td>
<td>600</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 5.6: Raw material purchase cost ($c_{j,h}^{PRC}$) in $$/tonne of raw material $j$ purchased from supplier $h$

<table>
<thead>
<tr>
<th>raw material ($j$)</th>
<th>supplier ($h$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S1</td>
</tr>
<tr>
<td>Biomass</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 5.7: Production cost ($c_{i}^{PRD}$) in $$/day/tonne produced in process $i$

<table>
<thead>
<tr>
<th>process ($i$)</th>
<th>production cost ($c_{i}^{PRD}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deinking pulping</td>
<td>10</td>
</tr>
<tr>
<td>Thermomechanical pulping</td>
<td>10</td>
</tr>
<tr>
<td>Paper making</td>
<td>10</td>
</tr>
<tr>
<td>Fractionation</td>
<td>10</td>
</tr>
<tr>
<td>Separation</td>
<td>20</td>
</tr>
<tr>
<td>Preparation</td>
<td>20</td>
</tr>
<tr>
<td>Fermentation</td>
<td>30</td>
</tr>
<tr>
<td>Furfural production</td>
<td>50</td>
</tr>
</tbody>
</table>
Table 5.8: Transportation cost ($^{TRP}_{j,m,v}$) in $$/tonne of final produce \( j \) shipped from biorefinery \( m \) to distribution center \( v \)

<table>
<thead>
<tr>
<th>biorefinery ((m))</th>
<th>distribution center ((v))</th>
<th>Newsprint</th>
<th>Bioethanol</th>
<th>Furfural products</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>V1</td>
<td>8</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>M1</td>
<td>V2</td>
<td>–</td>
<td>35</td>
<td>–</td>
</tr>
<tr>
<td>M1</td>
<td>V3</td>
<td>–</td>
<td>–</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 5.9: Inventory cost ($^{JM}_{j,m}$) in $$/day/tonne of raw material or intermediate product \( j \) stored in biorefinery \( m \)

<table>
<thead>
<tr>
<th>chemical ((j))</th>
<th>biorefinery ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recycle paper</td>
<td>M1</td>
</tr>
<tr>
<td>Wood chip</td>
<td>3</td>
</tr>
<tr>
<td>Pulp</td>
<td>2</td>
</tr>
<tr>
<td>Biomass</td>
<td>5</td>
</tr>
<tr>
<td>Hemicellulose</td>
<td>8</td>
</tr>
<tr>
<td>Cellulose</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5.10: Inventory cost ($^{IV}_{j,v}$) in $$/day/tonne of final product \( j \) stored in distribution center \( v \)

<table>
<thead>
<tr>
<th>distribution center ((v))</th>
<th>Newsprint</th>
<th>Bioethanol</th>
<th>Furfural products</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>8</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>V2</td>
<td>–</td>
<td>10</td>
<td>–</td>
</tr>
<tr>
<td>V3</td>
<td>–</td>
<td>–</td>
<td>15</td>
</tr>
</tbody>
</table>
5.2.2 Results and Discussion

In order to examine the trade-offs in all dimensions of flexibility, responsiveness, and cost, a Pareto analysis is performed and a 3-D Pareto surface is generated using various levels of flexibility and expected lead time (ELT). The flexibility levels are specified from 0 to 2 with a uniform increment of 0.2, and the expected lead times used range from 0 to 5 days with a uniform increment of 0.5 days. Thus, there are in total 121 points evaluated to generate the Pareto surface displayed in Fig. 5.2. Each of the points represents the optimal expected operating cost determined over 10 scenarios of the step changes in furfural demand, and 4 vertex scenarios to cover all the possible combinations of newsprint and bioethanol demands.

The Pareto surface exhibits a general trend of the expected operating cost increasing with a greater level of flexibility and with a less expected lead time (a greater level of responsiveness). Hence the surface peaks at the point where the flexibility level is highest and the most stringent requirement of responsiveness (no lead time allowed) is imposed. The trade-offs between cost and flexibility have been discussed in Chapter 3, where the escalating operating cost is due to the fulfillment of greater demands. The increase in operating cost with greater responsiveness level is contributed by having a sufficient initial inventory level and maintaining such a level during the entire optimization horizon.

Fig. 5.3 plots the relationship of inventory setpoint with expected lead specified with a fixed level of flexibility. Conversely, Fig. 5.4 displays the relationship between inventory

---

Table 5.11: Demand deviation for commodity products \( j \) requested by customer \( c \) in tonnes/day

<table>
<thead>
<tr>
<th>Demand deviation(( \Delta D_{j,c} ))</th>
<th>chemical ( \langle j \rangle )</th>
<th>customer ( \langle c \rangle )</th>
<th>Newsprint</th>
<th>Bioethanol</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>-50</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>-30</td>
<td>80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.2: Optimal cost plotted as a Parento surface as a function of specified level of flexibility and responsiveness

setpoint and flexibility index with a fixed level of expected lead time. Both figures only show the inventory setpoints of hemicellulose, bioethanol and furfural products, and the rest of chemicals involved in this supply chain system have inventory setpoints of zero for all the levels of expected lead time investigated.

The significance of the inventory setpoint is to create an initial level of inventory as suggested in Eqs. (4.21) and (4.22) of the responsiveness formulation. The inventory setpoint is positively related to the responsiveness level, as reflected in Fig. 5.3 where more initial inventory is required when the expected lead time is short. When the expected lead time remains as zero days, all materials in Fig. 5.4 have inventory setpoint increasing with flexibility level. This observation is a result of the integration of flexibility with the step sizes by Eq. (5.6). At each of the flexibility levels, the setpoint of furfural products is the quantity of the step change sustained for 4 days, the sum of the production delay from input as
hemicellulose and the transportation delay to reach distribution centers.

It is counterintuitive to observe that both plots have nonzero levels of bioethanol inventory setpoint as the demand of bioethanol is constant for the time horizon. After a detailed investigation of other related variables, this amount of inventory is consumed because the production of bioethanol is not sufficient to fulfill the demand for some scenarios due to the lower cellulose production caused by the shortage of biomass into the cellulose preparation process. The biomass intended for bioethanol is used to generate hemicellulose instead so more furfural products are produced, in order to replenish the furfural inventory in time to avoid penalty. The trade-off existing between the inventory cost of storing bioethanol and inventory setpoint deviation penalty of furfural products determines that it is more cost-effective to keep inventory of bioethanol due to the high penalty cost.
5.3 Summary

A composite framework integrating flexibility and responsiveness is presented in this chapter to accommodate the considerations of long-term sustained changes and short-term fluctuations in market conditions simultaneously. Building on the individual analysis frameworks introduced in the previous chapters, the composite framework can also be reduced to either of the frameworks depending on the application context. A comprehensive case study of a forest products company implementing biorefineries is examined. The framework is utilized in designing process and storage capacities to satisfy the specified flexibility and responsiveness requirements in the most cost-effective manner.
Chapter 6

Conclusions and Recommendations

The intent of this chapter is to summarize the findings of this study and propose some potential topics for future research.

6.1 Conclusions

Flexibility, the steady-state notion of supply chain operability, reflects the ability of a supply chain to operate feasibly under conditions away from its design and/or operating conditions. The development of process plant flexibility analysis is reviewed and the application of the flexibility analysis has been extended in process supply chains under uncertainty. The quantitative methodology for supply chain flexibility analysis established by Mastragostino [2012] has been extended to include more flexibility metrics discussed in operations research literature. Two types of flexibility measures, individual and overall flexibility indices, are quantified using this framework under various supply chain uncertainties. A case study on a supply chain with pre-determined designs illustrates that the framework can be easily applied to assess flexibility under various sources of uncertainty individually, as well as simultaneously. The framework also demonstrates its applicability in another case study, where it is adopted in a flexible manufacturing process to determine the optimal production
capacity sizings in the most cost-effective manner. Due to the presence of binary variables, although the validity of convexity assumptions has not yet been analytically proven, the solutions posed fulfill necessary conditions for flexibility.

Responsiveness, the dynamic notion of supply chain operability, reflects the ability of a supply chain to transition satisfactorily between operating conditions. A computational methodology to evaluate responsiveness proposed by Mastragostino and Swartz [2014] has been adapted to a more general supply chain model, and to handle the unmet customer demands with differentiations between products sold to contractual customers and to spot markets. A biorefinery supply chain case study is presented to demonstrate the applicability of this framework in evaluating the expected lead time as a measure of supply chain responsiveness based on fixed topological designs and initial operating state. Then the inventory setpoint is relaxed as the first-stage design decision within the two-stage stochastic formulations to understand the trade-offs between operating costs and responsiveness.

To integrate both aforementioned operability considerations, a composite analysis framework is proposed to include both flexibility and responsiveness measures simultaneously in forest products supply chain design and operation. The steady-state formulation is responsible to assess flexibility under sustained changes, and meanwhile dynamic formulation is responsible to evaluate the responsiveness measures while constrained by the feasibility pre-requisite of the responsiveness. A comprehensive case study is included to determine the optimal capacity allocations for a forest products company when exploring options to invest and implement a biorefinery retrofitted to existing pulp and paper facilities. The trade-offs among flexibility, responsiveness and economics are examined. The composite framework can be reduced to either of the operability analysis formulation depending on the application context.

Overall, this study presents quantitative frameworks to evaluate operability considerations as a part of the decision making toolbox during the design stage of process supply chains. The practical formulations outlined are applicable to general supply chains and contribute to the competitiveness of industries operating under dynamic market conditions.
6.2 Recommendations of Further Work

As pointed out by Naylor et al. [1999], the use of either of the lean and agile paradigms in the manufacturing environment is too simplistic. Both paradigms have to be strategically combined within a supply chain and customized by the specific market knowledge. It is therefore crucial to determine the location at which the lean and agile manufacturing paradigms decouple. One of the direct extensions of this work is to utilize the composite analysis framework established to determine the position of the decoupling point by identifying the levels of inventory required along the supply chain. Moreover, the quantitative identification of the decoupling point also provides some insights on customization postponement and product differentiation during manufacturing processes. The customization and differentiation based on consumer needs require high responsiveness and agility to the market, while before that point, lean manufacturing is more desirable supporting the production of the common forms as intermediate products prior to final products.

In addition to operating cost, other aspects of the economic criteria, such as profitability, can also be included in future work when examining the trade-offs with operability. Net present value (NPV) and return on investment (ROI) are two of the typical profitability measures that can be considered. By including such criteria, the relationship between capital cost and operating profit will be revealed, and thus an optimum level of operability should be able to be determined through the proposed framework.
List of References


Nomenclature

Sets and Indices

Sets

\begin{itemize}
\item $j \in J$ \hspace{1cm} Chemicals
\item $k \in K$ \hspace{1cm} Production schemes
\item $i \in I$ \hspace{1cm} Processes
\item $m \in M$ \hspace{1cm} Plants
\item $h \in H$ \hspace{1cm} Suppliers
\item $c \in C$ \hspace{1cm} Customers
\item $v \in V$ \hspace{1cm} Distribution centers
\item $t \in T$ \hspace{1cm} Time periods
\item $s \in S$ \hspace{1cm} Scenarios
\item $vs \in VS$ \hspace{1cm} Vertex scenarios
\end{itemize}
Subsets

- $J_{RM}$: Raw materials purchased from suppliers
- $J_{IP}$: Intermediate products produced and consumed in manufacturing plants
- $J_{FP}$: Final products distributed to markets
- $J_{FP}^C$: Commodity products distributed to markets
- $J_{FP}^S$: Specialty products distributed to markets
- $J_{ki}^C$: Chemicals that are consumed in scheme $k$ of process $i$
- $J_{ki}^P$: Chemicals that are produced in scheme $k$ of process $i$
- $J_{ki}$: Chemicals that are involved in scheme $k$ of process $i$
- $J_{ki}^M$: Main chemicals that are produced in scheme $k$ of process $i$
- $H_j$: Suppliers that offer chemical $j$
- $C_j$: Customers that request chemical $j$
- $C_j^C$: Customers that request commodity chemical $j$
- $C_j^S$: Customers that request specialty chemical $j$
- $M_j$: Plants that process chemical $j$
- $K_i$: Production schemes in process $i$
- $I_m$: Processes that exist in biorefinery $m$
- $V_j$: Distribution centers that distribute chemical $j$
- $K_{ji}^C$: Production schemes in process $i$ that consume chemical $j$
- $K_{ji}^P$: Production schemes in process $i$ that produce chemical $j$
## Parameters

### Model Parameters

- **\( Q_{j,h}^{\text{max}} \)**: Maximum quantity of chemical \( j \) offered by supplier \( h \)
- **\( \eta_{j,k,i,m} \)**: Mass balance coefficient of chemical \( j \) in scheme \( k \) of process \( i \) at plant \( m \)
- **\( \delta_{j,h,m}^{\text{PRC}} \)**: Procurement delay of raw material \( j \) from supplier \( h \) to plant \( m \)
- **\( \delta_{k,i}^{\text{PRD}} \)**: Production delay of scheme \( k \) of process \( i \)
- **\( \delta_{j,m,v}^{\text{TRP}} \)**: Transportation delay of final product \( j \) from manufacturing site \( m \) to distribution center \( v \)
- **\( \omega_{j,c} \)**: System lead time weighting factor for product \( j \) reaching customer \( c \)
- **\( \tau \)**: Time period when product demand must be met
- **\( \Delta t \)**: Length of each time period
- **\( \mathcal{H} \)**: Optimization time horizon

### Cost Parameters

- **\( p_{j,c}^{\text{Sale}} \)**: Sale price of final product \( j \) to customer \( c \)
- **\( c_{i,m}^{\text{CP}} \)**: Capital cost per unit of capacity of process \( i \) built at manufacturing site \( m \)
- **\( c_{j,m}^{\text{CIM}} \)**: Capital cost per unit of inventory capacity of chemical \( j \) at manufacturing site \( m \)
- **\( c_{j,v}^{\text{CIV}} \)**: Capital cost per unit of inventory capacity of final product \( j \) in distribution center \( v \)
- **\( c_{j,h}^{\text{PRC}} \)**: Procurement cost raw material \( j \) to purchased from supplier \( h \)
- **\( c_{i,m}^{\text{PRD}} \)**: Variable cost per unit of product produced at manufacturing site \( m \)
- **\( c_{j,m,v}^{\text{TRP}} \)**: Transportation cost per unit of final product \( j \) shipped from manufacturing site \( m \) to distribution center \( v \)
- **\( c_{j,m}^{\text{IM}} \)**: Inventory cost per time period per unit of chemical \( j \) stored at manufacturing site \( m \)
- **\( c_{j,v}^{\text{IV}} \)**: Inventory cost per time period per unit of final product \( j \) stored at distribution center \( v \)
- **\( c_{j,m}^{\text{EM}} \)**: Inventory offset penalty cost per time period per unit of chemical \( j \) stored at manufacturing site \( m \)
- **\( c_{j,v}^{\text{EV}} \)**: Inventory offset penalty cost per time period per unit of final product \( j \) stored at distribution center \( v \)
Variables

\[ C_{i,m}^P \] Process capacity of process \( i \) at manufacturing site \( m \)
\[ C_{j,m}^M \] Inventory capacity of chemical \( j \) stored in manufacturing site \( m \)
\[ C_{j,v}^V \] Inventory capacity of final product \( j \) stored in distribution center \( v \)
\[ \bar{I}_{j,m}^M \] Inventory setpoint of chemical \( j \) at manufacturing site \( m \)
\[ \bar{I}_{j,v}^V \] Inventory setpoint of final product \( j \) at distribution center \( v \)
\[ I_{j,m}^M \] Inventory level of chemical \( j \) at manufacturing site \( m \) during time period \( t \)
\[ I_{j,v}^V \] Inventory level of final product \( j \) at distribution center \( v \) during time period \( t \)
\[ B_{j,c,t} \] Back order quantity of chemical \( j \) from customer \( c \) during time period \( t \)
\[ E_{j,m}^M \] Inventory offset of chemical \( j \) at manufacturing site \( m \) during time period \( t \)
\[ E_{j,v}^V \] Inventory offset of final product \( j \) at distribution center \( v \) during time period \( t \)
\[ K_{j,c,t}^1/K_{j,c,t}^2 \] Partial fulfillment of final product \( j \) requested by customer \( c \) during time period \( t \)
\[ L_{j,c,s} \] Lead time to satisfy demand of chemical \( j \) requested by customer \( c \)
\[ A_{j,c,t} \] Indication (= 1) of the time period \( t \) when demand of chemical \( j \) from customer \( c \)
\[ Y_{j,c,t}^{\text{met}} \] Binary indication for demand fulfillment of chemical \( j \) requested by customer \( c \)
\[ X_{k,i,m} \] Binary variable used to select scheme \( k \) operated in process \( i \) in manufacturing site \( m \)
\[ F_{j,m,v,t} \] Shipment of chemical \( j \) from manufacturing site \( m \) to distribution center \( v \) during time period \( t \)
\[ O_{j,h,m,t} \] Quantity of raw material \( j \) ordered by manufacturing site \( m \) from supplier \( h \) during time period \( t \) for scenario \( s \)
\[ Q_{j,c,v,t} \] Quantity of final product \( j \) distributed to customer \( c \) from distribution center \( v \) during time period \( t \)
\[ W_{j,k,i,m,t} \] Chemical flow of chemical \( j \) in scheme \( k \) of process \( i \) at manufacturing site \( m \) during time period \( t \)