OPTIMAL FISCAL STRATEGIES FOR ECONOMIC STABILIZATION

OPTIMAL FISCAL STRATEGIES FOR ECONOMIC STABILIZATION: AN ECONOMETRIC STUDY WITH ILLUSTRATIVE APPLICATION TO CANADA

By

OH YUL KWON, M.A.

A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree of Doctor of Philosophy

McMaster University

(November) 1972

DOCTOR OF PHILOSOPHY (1972)
(Economics)

McMASTER UNIVERSITY
Hamilton, Ontario.

TITLE: Optimal Fiscal Strategy for Economic Stabilization: An Econometric Study with Illustrative Application to Canada

AUTHOR: Oh Yul Kwon, B.Com. (Seoul National University)

M.A. (McMaster University)

SUPERVISOR: Professor F. T. Denton

NUMBER OF PAGES: ix, 133

ABSTRACT

The primary objective of this thesis is to develop a methodology for applying econometric models to problems of economic policy, and to illustrate it in terms of fiscal policy by applying it to problems of economic stabilization in Canada during the period 1967-69. It was assumed that the fiscal policy-maker in Canada has a preference (loss) function which is a weighted sum of squared deviations between the actual and the desired values of two target variables (changes in the GNE price deflator and the number of persons unemployed) and three instruments (changes in government expenditure on goods and services, and the personal and corporation income tax rates). Then, an intermediate sized annual econometric model of the Canadian economy was developed, subject to which the preference function was minimized. Since the parameters of the preference function were not given, thirty-six experiments of optimization were made under a range of plausible values of the relevant parameters. It was found that the numerical values of the target variables indicated by the optimal strategy were closer to the desired values and more stable in their movements than actual values. Also the results of various experiments suggested that the optimal strategy was relatively insensitive to changes in parameter values over a rather wide range.

ACKNOWLEDGMENTS

I am greatly indebted to the members of my doctoral committee for their valuable suggestions and criticisms. I am particularly indebted to Professor F. T. Denton, my thesis supervisor, for his excellent advice and patient guidance as well as his generous offering of time. I am also grateful to Professor D. M. Winch for not only his advice as a member of my thesis committee but for his classroom lectures which inspired this topic. To Professors B. G. Spencer and J. A. Johnson, I wish to express my gratitude for the valuable guidance they provided at many stages of the research and for the generous manner in which they gave of their time.

I wish to record my appreciation to Dr. J. R. Allan for first stimulating my interest in the problems of public finance and for the number of ways in which he helped me during the preparation of this work. I wish to thank Mrs. H. Hardy, one of my fellow students, for her assistance in improving the exposition of my arguments. Finally, my thanks go to my wife, Joanne, for undertaking the typing, arranging the bibliography, and patiently enduring my long and erratic study schedule for so many evenings.

For financial assistance during the doctoral portion of my studies, I am grateful to the Canada Council for its Doctoral Fellowship.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The Economic Council of Canada suggested, in its First Annual Review (1964), five "goals" of economic policy which are to be achieved "simultaneously and consistently". These goals are: full employment, a high rate of economic growth, reasonable stability of prices, a viable balance of payments, and an equitable distribution of rising income.[1] However, it appears that during the period from 1967 to 1969 these goals have not been achieved. As shown in Table 1.1, the annual unemployment rate averaged 4.5 percent and the consumer price index rose an average of 4.1 percent per annum during this period, leaving aside the other three goals.

Table 1.1

Unemployment Rates and Percentage Changes in the Consumer Price Index: 1967-1969

	1967	1968	1969	Average
Unemployment rate (%)	4.1	4.8	4.7	4.5
Percentage change in the consumer price index (%)	3.6	4.1	4.5	4.1

Source: Statistics Canada, Canadian Statistical Review.

[1]The Economic Council of Canada, First Annual Review, (Ottawa: Queen's Printer, 1964), p. 1.

"The dual objective of price-level stability and full employment is referred to usually as stabilization."[2] Fiscal policy for economic stabilization is primarily a matter of regulating aggregate demand so as to be compatible with the dual objectives. An expansionary policy is followed in order to reduce the unemployment rate, while a policy of contraction is adopted to moderate price inflation; the result may be the so-called "stop-and-go" policies of aggregate demand management. However, when the performance of both of the objective variables is unsatisfactory, as has occurred in Canada since 1967, conventional economic theory gives little guidance as to what policy actions would be appropriate. A question then arises as to what would have been the optimal policy for economic stabilization in Canada during this period. This dissertation, therefore, attempts (1) to review and describe the optimal strategy for achieving the policy objectives; (2) to illustrate the optimal strategy in terms of fiscal policy by applying it to economic stabilization in Canada during the period 1967-1969, thereby being able to compare the actual fiscal performance with the possible results from the optimal fiscal strategy during the same period; and (3) to make some suggestions regarding the use of fiscal policy for economic stabilization in Canada. However, in view of the fact that our model is highly simplified and aggregative, the results are intended to be illustrative of what might be done on a larger and more realistic scale for real policy purposes. Hence, this study should be viewed as experimental and indicative of a methodological approach rather than as being realistic

---

[2] R. A. Musgrave, The Theory of Public Finance, (Toronto: McGraw-Hill Book Co., 1959), p. 406.

and immediately applicable to real policy decisions.

We have an economic policy because we regard one state of the economy as being more "desirable" than another with regard to a given criterion. Thus, we must have at our disposal a "preference function" which we can regard as ranking the desirability of policy alternatives. The theory of economic policy should then be concerned with the problem of making a choice from among a set of alternative feasible government actions - a choice which presumably would seek to maximize the preference function, the feasibility requirement implying constraints. In this sense, the theory of economic policy parallels the theory of a consumer's or firm's behavior which deals with optimization under constraints.

Although the theory of economic policy has a long history, Tinbergen made the pioneering contribution to the integrated theory of quantitative economic policy.[3] Tinbergen distinguishes between quantitative and qualitative economic policy. The former involves manipulating certain instruments within the framework of a given economic structure; the latter refers to the changing of certain qualitative aspects of economic structure such as an institutional change.

Tinbergen has suggested that the problem of quantitative economic policy may be tackled in two different approaches:[4] fixed target

---

[3] J. Tinbergen, On the Theory of Economic Policy, (Amsterdam: North-Holland Publishing Co., 1952); Centralization and Decentralization in Economic Policy, (Amsterdam: North-Holland Publishing Co., 1954); and Economic Policy: Principles and Designs, (Amsterdam: North-Holland Publishing Co., 1967).

[4] The approach adopted in our study is the quantitative economic policy, and Tinbergen's theory of economic policy noted hereafter refers to his quantitative economic policy only.

approach and flexible target approach. In the fixed target approach, the targets, which are target variables given certain numerical values, are chosen directly, passing over the problems of determining a preference function and of maximizing it.[5] Therefore, the crucial problem of finding what values of the target variables maximize the preference function is assumed away in this approach; in other words, the maximization problem as such is eliminated. The policy problem thus involves solving for the values of instruments in the model subject to the boundary conditions. This requires the number of instruments to be not smaller than the number of targets, and thus the relative number of targets and instruments is broadly significant. Since the preference function is assumed away under this approach, it is implied that any instrument is a "free good" until a boundary condition imposed upon it is violated. This is not in general a realistic decision-making procedure, and it is desirable to indicate the cost of administering the instruments even within their boundaries. This requires a preference function into which the instruments enter as arguments. As a result, the fixed-target approach is far from being a complete theory of how economic policy is actually determined.

In the flexible target approach, instead of aiming at given values of target variables, the policy problem is to maximize a specified preference function in terms of both instruments and target variables, subject to the model and boundary conditions. In this context, Tinbergen's framework of quantitative economic policy consists of four

---

[5] J. Tinbergen, On the Theory of Economic Policy, p. 3

elements: (1) specification of the policy-maker's preference function; (2) specification of a quantitative economic model; (3) classification of the variables in the economic model; and (4) specification of the boundary conditions. Tinbergen's model specifies the set of quantitative structural relations among economic variables. These variables are classified into four different types: (i) "instruments" (x), which are the means available to the policy-maker and belong to the class of exogenous variables of the model; (ii) "target variables" (y), which are endogenous variables, being only indirectly affected by the policy-maker through the variations of instruments; (iii) "data" ($\hat{q}$), which are non-instrument exogenous variables of the model; and (iv) "irrelevant variables", which are non-target endogenous variables.

Tinbergen argues that economic policy is concerned with social welfare ("general interest") which must in principle depend not only on the individual utilities, but also on the way in which they are combined. The determination of the social welfare function, however, would involve insurmountable difficulties in estimating and combining the individual utilities. Because of these difficulties, he assumes the policy-maker's preference function, which is a function of both target variables and instruments. Finally, Tinbergen imposes boundary conditions on instruments beyond which the instruments cannot be varied. By sorting out the relationship among target variables, instruments, and data from a given economic model, Tinbergen's basic framework of economic policy can then be described as follows:

$$\text{maximize} \quad W = W(x, y) \qquad \text{(preference function)}$$

$$\text{subject to} \quad y = Rx + H_1\hat{q} \qquad \text{(economic model)}$$

$$x_{min} \leq x \leq x_{max} \qquad \text{(boundary condition)}$$

where R and H_1 are matrices of coefficients of appropriate order, and x, y, and $\hat{q}$ are appropriate column vectors of instruments, target variables, and "data", respectively.

There are various limitations to Tinbergen's formulation of economic policy from the point of view of its application to realistic policy determination. Realistically, the economic policy problem is that of optimal choice among alternative feasible policies on the basis of their effects on the characteristic dynamic performance of the economy. However, Tinbergen's framework is static in the sense that time patterns of the behavior of each variable do not enter into the model or investigation, and that there is no flexibility in policy-making procedures under changing conditions and as new information becomes available over time. Another limitation is that Tinbergen's formulation is based on a deterministic model, and thereby assumes away the stochastic elements in the economy. In reality, however, the determination of economic policy is carried out in situations characterized by uncertainty.

H. Theil has generalized the theory of economic policy.[6] He has incorporated into Tinbergen's theory of economic policy not only time patterns of the behavior of economic variables, but also the

---

[6] H. Theil, Economic Forecasts and Policy, (Amsterdam: North-Holland Publishing Co., 1961), and Optimal Decision Rules for Government and Industry, (Amsterdam: North-Holland Publishing Co., 1964).

necessary adjustments in response to changing conditions and new information over time. In this context, he has introduced a strategy approach to economic policy. He has also introduced uncertainty. At the same time, he has shown that under certain assumptions the policy-maker's optimal decisions are not affected if random variables in the system are replaced by their expected values. This is the so-called certainty equivalence theorem.

Theil has, however, arbitrarily chosen a single quadratic form for the policy-maker's preference function, including its parameters, from which he has derived the corresponding optimal policy decisions. Although, as will be shown in Chapter II, the quadratic form is an approximation of the general functional form, given the arguments, and has some practical convenience, the numerical values of the necessary parameters will not in general be given to us. Therefore, it would be desirable to show a set of the optimal policies under a range of values of the relevant parameters - rather than one optimal policy based on a single set of values of relevant parameters. It will then be expected that the policy-maker selects the optimal policy which is most compatible with his preference function. Using this approach, we can also show to which parameters the final results of optimization are most sensitive. This approach seems to be relevant, especially when it is impossible to achieve all policy targets simultaneously, and thus the policy-maker has to evaluate the relative cost of not achieving certain targets. Therefore, compared to standard policy simulation approaches which provide the results of a particular policy mix, this approach intends to provide answers to a question such as: "What particular policy mix is required in order to produce the optimal result?" Finally, Theil's method for computing solutions for optimization is burdensome. This can be improved by linking Theil's approach to the optimal policy to the concept of the generalized inverse of a matrix.

So far we have critically appraised Tinbergen's pioneering formulation of economic policy and referred briefly to Theil's approach. This has been done for the purpose of providing some historical background for our subsequent attempt to deal with an actual problem of economic policy. From this, we can conclude that to deal with the quantitative economic policy problem requires: (1) specification of target variables and instruments; (2) specification of a preference function as the criterion of optimality; (3) specification of a quantitative model of the economy; and (4) specification of constraints or boundary conditions.

We shall, therefore, discuss the specification of the policy problem in Chapter II, and in Chapter III, we shall derive an optimal strategy for economic policy based on the results from Chapter II. In Chapter IV, an annual econometric model of the Canadian economy will be developed, from which the linear constraints will be derived in Chapter V. Finally, in Chapter VI, the numerical values of the optimal strategy will be derived under various values of the parameters of the preference function, for the purpose of illustrating how such an approach might be used to determine the appropriate fiscal policy for economic stabilization. This will enable us to compare, in terms of stabilization, actual fiscal policy during the period from 1967 to 1969 with the "optimal policy" indicated by the analysis (bearing in mind, however, that the main purpose is to illustrate the application of the approach in question rather than to provide a fully realistic policy prescription for that period). A summary and conclusions are presented in Chapter VII.

CHAPTER II

SPECIFICATION OF THE POLICY PROBLEM

1. Target Variables and Instruments

Although the distinction between "target variables" and "instruments" is neither entirely unambiguous nor obvious, economic policy implies the existence of mutually independent target variables and of mutually independent instruments. As mentioned in Chapter I, stabilization policy involves two targets: full-employment and price-level stability. The method developed in this study requires the economic variables to be in the form of changes (first differences) in their levels. Hence, the changes in the number of persons unemployed and in the level of the GNE price deflator were chosen as the two target variables. These target variables are not directly controllable by the policy-maker, and they belong to a subset of the endogenous variables. The GNE price deflator is a more comprehensive measure of inflation than the consumer price index, because the former measures the rate of inflation for the entire economy. Thus, the GNE price deflator, rather than the consumer price index, was chosen as a target variable.

In an open economy such as Canada's, fluctuations in the balance of payments may reinforce domestic economic instability, while fiscal policy designed to achieve domestic stabilization may be inconsistent with policies designed to maintain the balance-of-payments equilibrium.

In Canada, however, maintenance of the balance-of-payments equilibrium does not seem to conflict with domestic stabilization objectives. Rather, stabilization policy is, in most cases, likely to contribute to the maintenance of a viable balance of payments.[1] Since, however, a sustained balance-of-payments disequilibrium might result in undesirable consequences with regard to national sovereignty as well as to the economy, the balance-of-payments equilibrium should be considered as a long-term constraint on the realization of domestic stabilization objectives rather than as a short-term objective in its own right. As a result, the idea of dealing with the balance of payments as a target has been considered and rejected in order to concentrate on the main interest of this study, i.e., full employment and a stable price level.[2]

In quantitative economic policy, instruments are regarded as economic variables which can be directly controlled by the policy-maker, and they are assumed to be effective in the sense that they are

---

[1]See, for example, T. R. Robinson, Foreign Trade and Economic Stability: Studies of the Royal Commission on Taxation, No. 5, (Ottawa: Queen's Printer, 1965), p. 170. Further, as shown by the import equation in our model, Canadian imports are highly sensitive to changes in GNP. Thus, the foreign trade sector of the economy seems to exert a stabilizing influence. This is also the opinion of the Royal Commission on Taxation. See the Royal Commission on Taxation, The Use of the Tax System to Achieve Economic and Social Objectives: Report of the Royal Commission on Taxation, Vol. 2, (Ottawa: Queen's Printer, 1966), Ch. V.

[2]Based on this account, the model developed does not explicitly contain equations for capital flows and the consequential flows of interest and dividends. Furthermore, monetary, rather than fiscal, policy seems to be an appropriate tool in Canada for the maintenance of the balance-of-payments equilibrium. See the Royal Commission on Taxation, The Use of the Tax System, Ch. V.

capable of influencing the target variables to the necessary degree. In the public finance literature, overall fiscal policy is sometimes divided into "automatic fiscal policy" and "discretionary fiscal policy". The former is regarded as a means of enhancing the responsiveness of expenditures and revenues to changes in GNP or in other economic variables. The latter indicates the policy-maker's discretionary manipulation of revenues and expenditures in order to achieve the desired values of some target variables. As the term "strategy" implies continuous action according to new information, our "optimal fiscal strategy for stabilization" corresponds to a discretionary fiscal policy.

There are various fiscal instruments which the fiscal policy-maker can manipulate or control for the purpose of discretionary fiscal stabilization policy. The instruments chosen for this study are changes in the personal income tax rate, in the corporation income tax rate, and in government expenditure on goods and services, including government fixed capital formation.[3] These three instruments appear to be those most commonly used and most effective fiscal tools in Canada for the purpose of economic stabilization.

2. Specification of the Model and the Constraints

The present study is concerned with problems of quantitative economic policy, which presumes the existence of a given economic structure. The economic structure, it is assumed, can be represented

---

[3]These three instruments were also recommended by the Royal Commission on Taxation as discretionary fiscal weapons. See the Royal Commission on Taxation, The Use of the Tax System, Ch. III.

by a certain number of linear equations in certain real-valued variables, including instruments and target variables. Linearity may be justified either because it is a special case, or because it is an adequate approximation when small variations in instruments are involved.

The theory of economic policy is, in essence, concerned with the analysis of policy-maker's decisions in the context of their effects on the characteristic dynamic performance of the economy. Therefore, the theory of economic policy must be in the context of dynamics in two senses: (i) the policy model contains information about the time pattern of behavior of economic variables; and (ii) the theory shows decision-making procedures for a number of successive periods under changing conditions associated with the sequence of new information.[4]

In the dynamic context, we can derive the relations among instruments and target variables over a horizon of T consecutive periods from a given (estimated) structural model. This can be written as[5]

$$y = Rx + Hs + Ju \tag{2.1}$$

where y is a column vector of nT target variables (n target variables over T periods), x is a column vector of mT instruments (m instruments over T periods), s is a column vector of hT non-instrument predetermined

---

[4]The underlying essence in our study is decision-making under uncertainty - decision theory. The application of decision theory to problems of macroeconomic policy has been developed by H. Theil, Economic Forecasts and Policy, (Amsterdam: North-Holland, 1961).

[5]This system is derived from the reduced form of an estimated structural model, and thus we assume that the necessary condition for deriving the reduced form is satisfied. An actual derivation of this system will be done in Chapter V.

variables, including additive constant terms, and u is a column vector of kT disturbance terms in structural equations, H is an nT x hT matrix of coefficients, relating non-instrument predetermined variables to target variables, J is an nT x kT matrix of coefficients which relates structural disturbance terms to target variables, R is an nT x mT matrix of coefficients, each of which indicates the effect of an instrument in a given period on a target variable in a given period, and all the variables are in the form of first differences.[6]

We can then partition vectors and matrices in (2.1) by periods, so that all variables and coefficients in (2.1) corresponding to the same period are grouped together:

$$(2.2)\qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_T \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_T \end{bmatrix} \qquad s = \begin{bmatrix} s_1 \\ s_2 \\ \cdot \\ \cdot \\ \cdot \\ s_T \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_T \end{bmatrix}$$

$$(2.3)\qquad R = \begin{bmatrix} R_{11} & R_{12} \cdots & R_{1T} \\ R_{21} & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ R_{T1} & \cdots\cdots & R_{TT} \end{bmatrix} \qquad H = \begin{bmatrix} H_{11} & H_{12} \cdots & H_{1T} \\ H_{21} & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ H_{T1} & \cdots\cdots & H_{TT} \end{bmatrix} \qquad J = \begin{bmatrix} J_{11} & J_{12} \cdots & J_{1T} \\ J_{21} & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ J_{T1} & \cdots\cdots & J_{TT} \end{bmatrix}$$

---

[6]If any structural equation is estimated in terms of levels, the corresponding u is then the first difference of the structural disturbance term. Since we have specified two target variables and three instruments in conjunction with a three-year strategy period, the vector of y and x are of orders 6 x 1 and 9 x 1 respectively, and the matrix R is of order 6 x 9. However, for a convenient exposition of optimization theory in Chapter III, we shall use a general notation.

The subvectors x_t, y_t, s_t, and u_t are now of order m x 1, n x 1, h x 1, and k x 1, respectively. The submatrices $R_{tt'}$, $H_{tt'}$, and $J_{tt'}$ are of order n x m, n x h, and n x k respectively; and $R_{tt'}$, for instance, indicates the effects of instruments in the t'^{th} period on target variables in the t^{th} period.[7] Hence, the submatrices on the diagonal of the R matrix in (2.3) indicate the effects of instruments on the target variables in the same period, the submatrices below the diagonal, the effects of instruments on target variables in later periods, and the submatrices above the diagonal, the effects of instruments on target variables in earlier periods. This also holds for the submatrices of matrices H and J with respect to non-instrument predetermined variables and structural disturbance terms, respectively. However, target variables of the present period are not expected to be affected by instruments, non-instrument predetermined variables, and disturbance terms of later periods, because there will be no "retroactive" effect. Thus, the coefficient submatrices $R_{tt'}$, $H_{tt'}$, and $J_{tt'}$ are zero matrices, as long as $t < t'$ for all t and t'. Therefore, the submatrices above the diagonal of the R, H and J matrices in (2.3) are zero matrices of

---

[7] For instance, $R_{tt'}$ can be written as

$$R_{tt'} = \begin{bmatrix} R^{11}_{tt'}, & R^{12}_{tt'} \cdots & R^{1m}_{tt'} \\ R^{21}_{tt'}, & \cdots\cdots & \cdots \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ R^{n1}_{tt'} & \cdots\cdots & R^{nm}_{tt'} \end{bmatrix}$$

where the first superscripts refer to target variables, and the second superscripts, to instruments.

order n x m, n x h, and n x k, respectively.

The coefficients of the structural equations are assumed to be time-invariant, and this implies that the reduced-form coefficients are constant over time. Hence, the coefficients on the diagonal of the R, H, and J matrices are all equal. Similarly, R_{21}, $R_{32}, \ldots, R_{TT-1}$ (or H_{21}, $H_{32}, \ldots, H_{TT-1}$; or J_{21}, $J_{32}, \ldots, J_{TT-1}$) are all identical, and so on. As a result, matrices R, H, and J can be rewritten as

$$(2.4) \qquad R = \begin{bmatrix} R_1 & 0 & \ldots & 0 \\ R_2 & R_1 & \ldots & 0 \\ \vdots & & & \vdots \\ R_T & \ldots & R_2 & R_1 \end{bmatrix} \qquad H = \begin{bmatrix} H_1 & 0 & \ldots & 0 \\ H_2 & H_1 & \ldots & \vdots \\ \vdots & & & \vdots \\ H_T & \ldots & H_2 & H_1 \end{bmatrix} \qquad J = \begin{bmatrix} J_1 & 0 & \ldots & 0 \\ J_2 & J_1 & \ldots & \vdots \\ \vdots & & & \vdots \\ J_T & \ldots & J_2 & J_1 \end{bmatrix}$$

where R_t indicates the effects of instruments on target variables t-1 years later; similarly, H_t and J_t represent the effects of non-instrument predetermined variables and the disturbance terms, respectively, on target variables t-1 years later.

A note about the disturbance term (u) is now in order. Taking (2.3) and the submatrices above the diagonal being zero matrices into account, we can rewrite (2.1) as

$$(2.5) \qquad y_t = \sum_{t'=1}^{t} R_{tt'} x_{t'} + \sum_{t'=1}^{t} H_{tt'} s_{t'} + \sum_{t'=1}^{t} J_{tt'} u_{t'} \quad \text{for } t = 1, \ldots, T$$

This implies that u_t can be determined as residuals from y_t, $x_1, \ldots, x_t$, and $s_1, \ldots, s_t$, the relevant coefficient matrices being assumed to be known, i.e., the past subvector of $u(u_1, \ldots, u_{t-1})$ is known as soon as the corresponding values of target variables, instruments and non-instrument predetermined variables are known. This implies, in turn, that the policy-maker can in principle take into account the past sub-vector of u when he makes the current period's decision, so that there is a dependence between the u-subvector and a later x-subvector.[8] However, the u-subvector is assumed to be independent of the present and past x-subvectors. In other words, it is assumed that the kT elements of u are subject to a probability distribution with three properties: (i) the distribution of the subvector u_t is independent of $x_{t'}$ for all t and t' such that $t \geq t'$; (ii) the expected values of the current and future subvectors are all zero;[9] and (iii) the variances and covariances of the kT elements are finite.

In sum, the vector relation among the x's and y's implied by equation (2.1) describes each target variable as an explicit linear function of instruments, non-instrument predetermined variables, and stochastic disturbance terms over T consecutive periods. As a result, the target variables in this model are also subject to a probability distribution.

3. Specification of a Preference Function

The problem of an optimal strategy requires a preference or criterion function. In this context, we would think that, although

---

[8]This possibility must be accepted as long as we are dealing with strategy, because the past-subvector provides an information for the current period's decision. This will be discussed further in Chapter III.

[9]This implies the assumption that the disturbances are uncorrelated over time.

the policy-maker is interested in controlling instruments so as to achieve certain desired values of target variables, he is probably unwilling (or unable) to vary his instruments without limit because of accompanying political and/or administrative cost. Hence, we assume that the policy-maker has a loss-function type of preference function in which both target variables and instruments appear as arguments in deviation form from their desired values. It is further assumed that this preference function is well behaved and cardinal in nature.

Although, in principle, this preference function can take on any one of an infinite variety of forms, we shall assume, following Frisch and Theil, a quadratic form.[10] The assumption of a quadratic preference function is, of course, restrictive, but it is an approximation of the general form.[11] It also has the appealing property of giving a heavier weight, the greater the deviation of an outcome from the desired level.

Since dynamic relations link the actions taken in one period with a chain of consequences and thus actions in succeeding periods, we need in making the decision for one period to consider a whole consequence of future decisions. Consequently, the preference function should reflect future as well as present preference. Hence, the general

---

[10] R. Frisch, Numerical Determination of a Quadratic Preference Function for Use in Macroeconomic Programming, Memorandum, Institute of Economics, Oslo University, (Oslo University, 1957); and H. Theil, Optimal Decision Rules. In addition to Frisch and Theil, the quadratic preference function has been used in the economic literature by numerous authors.

[11] This will be shown in Appendix A. A further justification of a quadratic form on grounds of convenience will be discussed in Chapter III. Since it leads to a reasonable conclusion in our study, it is felt that its use can be justified — at least suggestive as an approximation.

quadratic form of the preference function in the dynamic context can be written in the matrix form as [12]

$$(2.6) \qquad W(X, Y) = a'X + b'Y + \tfrac{1}{2}(X'AX + Y'BY + X'CY + Y'C'X)$$

where $X(= x - x^*$, x^* being the desired value of x) is a column vector of mT instruments in deviation form from their desired values; $Y(= y-y^*$, y^* being the desired value of y) is a column vector of nT target variables in deviation form from their desired values; a and b are vectors of fixed elements of order mT x 1 and nT x 1 respectively; A, B, and C are matrices of fixed elements of order mT x mT, nT x nT and mT x nT respectively; and the primes are the transpose operators.

As shown above, our specified model is a stochastic model so that the target variables are stochastic functions of instruments. Consequently, the preference function contains stochastic elements. Stochastic preference functions have been investigated by a number of authors.[13] The approach used in this study is that of von Neumann and Morgenstern.[14] Using some axioms, they deduced a relationship such that the preference level W is given by

$$(2.7) \qquad W = EW(X, Y)$$

---

[12] For the derivation of this general quadratic form, see Appendix A. The "preference function" in (2.6) is rather a "disutility function", hence it is expected to be minimized. We can have maximization simply by adding the negative sign. However, no gain will be obtained by changing the name or adding the negative sign.

[13] See K. J. Arrow, "Alternative Approaches in the Theory of Choice in Risk-Taking Situations", Econometrica, XIX (1951), 404-437.

[14] J. von Neumann, and O. Morgenstern, Theory of Games and Economic Behavior, (Princeton: Princeton University Press, 1944).

where E is the expectation operator. From this, they derived the conclusion that the optimal behavior is to maximize (or minimize for the case of preference function of loss-function type) the expectation of the preference function.[15] We shall assume for our purposes that the policy-maker's preference function obeys the von Neumann and Morgenstern axioms.

Finally, since the preference function contains instruments and target variables in deviation form, it is necessary to transform the instruments and target variables in the constraints into deviation form. From (2.1) we get

$$(2.8) \qquad y - y^* = R(x - x^*) - y^* + Rx^* + Hs + Ju$$

where y^* and x^* are column vectors of, respectively, nT desired values of target variables and mT desired values of instruments. For the sake of convenience, we add the last four terms on the right-hand side of (2.8), and then we can write

$$(2.9) \qquad Y = RX + Z$$

---

[15]An easy explanation of von Neumann and Morgenstern's axioms and derivation of the hypothesis is in H. Chernoff and L. E. Moses, Elementary Decision Theory, (New York: John Wiley and Sons, Inc., 1959), especially Ch. 4 and Appendix F_2. The axioms outlined by Chernoff and Moses are as follows: An individual faced with two prospects (P_1 and P_2) can order them according to increasing preference (comparative), and such ordering is transitive for, say, three prospects (P_1, P_2, and P_3). If P_1 is preferred to P_2 which is preferred to P_3, then there is a mixture of P_1 and P_3 which is preferred to P_2, and there is a mixture of P_1 and P_3 over which P_2 is preferred. Finally, if the individual prefers P_1 to P_2, and P_3 is another prospect, then he will prefer a mixture of P_1 and P_3 to the same mixture of P_2 and P_3.

where Z is a column vector of nT additive terms, including the stochastic disturbance terms. Z is then subject to a probability distribution with the same properties as u has, except that the mean values of Z are not zero.

We have assumed the preference function of the policy-maker in which the arguments are deviations of actual values from desired values of both instruments and target variables. The desired values, either of instruments or target variables, may in reality correspond to certain boundaries (ceilings or floors). For example, the desired value of the employment level may be full employment. The policy problem is to minimize the deviations of the actual values from the desired values. Furthermore, since the deviations enter the preference function as quadratic terms, the seriousness of a deviation increases more than proportionally with the deviation itself. Under this setting of the problem, the possible inequality constraints, which would be imposed on instruments, may not be, in fact, essential for the optimal policy problem. Therefore, the only constraints imposed are those implied by the linear constraints as represented by equation (2.9).

In summary, we have now defined the economic policy problem with which we shall be concerned. For this purpose, we have specified stochastic constraints which represent the structural relationships among the instruments and target variables. We have also specified a quadratic preference function, the arguments of which are deviations of the actual from the desired values of both the instruments and target variables. Because of this specification of the preference function, we may not need inequality constraints, and the only constraints are those represented by the linear equality constraints. Thus, the policy problem is reduced to one of minimizing the expectation of the quadratic preference function subject to the constraints, i.e.,

$$\text{to minimize} \qquad W = EW(X, Y)$$

$$\text{subject to} \qquad Y = RX + Z$$

where the notation is as indicated earlier. Based on this basic characterization of the economic policy problem, we now turn to derive the optimal strategy for the achievement of economic objectives.

CHAPTER III

DERIVATION OF THE OPTIMAL STRATEGY

1. A Note on Strategy

In a dynamic context, the decision-making procedure requires the use of a strategy. A strategy contains a complete specification of a chain of decisions or "moves" to be taken for all successive strategy periods of time as a function of the state of information possessed by the "player" at the time when the action is to be decided upon. Later decisions are made contingent upon the information which the "player" will have at the time. Thus, when using a strategy, the policy-maker's present actions must recognize that additional information will be available in the future, and that he must be prepared to react to this information by revising his earlier plans. Clearly, the adoption of a strategy does not imply any restriction on the policy-maker's future actions. On the contrary, the strategy specifies only conditional plans for future periods under all conceivable conditions.[1]

The information can be distinguished into two categories: past

[1] It will be noticed that a strategy is different from planning, since planning is based only on the information which is available at the beginning of the first period of the planning. A strategy is also different from decision for each period based on information of that period, without taking account of the known fact that certain information will be available in the future and that future final decisions will be made dependent on this information. For an example of the latter difference, see H. Theil, J. C. G. Boot, and T. Kloek, Operations Research and Quantitative Economics, (New York: McGraw-Hill Book Co., 1965), pp. 133-135.

information, and future information. In this study, the former contains the numerical values of the subvectors $X_1, \ldots, X_{t-1}$, $Y_1, \ldots, Y_{t-1}$, and $Z_1, \ldots, Z_{t-1}$. The latter contains the numerical values of the subvectors $Z_t, Z_{t+1}, \ldots, Z_T$.[2] Bccause of the random elements, Z_t, $Z_{t+1}, \ldots,$ Z_T, the future information is subject to a conditional probability distribution, given the past information. As a result, in a dynamic context, the strategy itself is random.

It will be noted that at the beginning of period t+1, new information becomes available, i.e., the numerical values of Y_t, X_t, and Z_t. The policy-maker is then expected to take these values into account when he makes (or revises) his decision for period t+1, as long as he applies a strategy.[3]

A number of strategies can be characterized in the above manner. Our objective is to find a strategy which minimizes the expectation of the preference function (2.6), subject to the equality constraints (2.9). This strategy is called the "optimal strategy" in our study. We, therefore, wish to find the conditions which a numerically specified strategy (numerical values of instruments for consecutive strategy periods) should meet in order to minimize the preference function subject to the constraints.

---

[2]The coefficients of the preference function and the model will also be a part of information. Yet, they are assumed to be known constants throughout T consecutive periods.

[3]Z_t is in principle no longer stochastic at period t+1; rather, it is a vector of known fixed elements. This was indicated in Chapter II.

2. A Linear Decision Rule

The optimal strategy is derived by minimizing (2.6) subject to (2.9). This can be done in two alternative ways: (i) by treating X's and Y's symmetrically and applying the Lagrangean technique; and (ii) by substituting (2.9) into (2.6) to eliminate Y in the preference function, and minimize the preference function unconditionally with respect to the X's. Although these two ways yield exactly the same results, we will follow the latter method.

To illustrate, if we substitute (2.9) into (2.6), we obtain[4]

$$W = W(X,\ RX + Z) = K_o + K_1'X + \tfrac{1}{2}X'K_2X \tag{3.1}$$

where

$$\begin{aligned} K_o &= b'Z + \tfrac{1}{2}Z'BZ \\ K_1' &= a' + b'R + Z'(BR + C')' \\ K_2 &= A + R'BR + CR + R'C' \end{aligned} \tag{3.2}$$

It will be noted that K_o is a scalar, K_1' is a vector of order 1 x mT, and K_2 is a non-stochastic and symmetric matrix of order mT x mT.[5] We have thus eliminated the Y's from the preference function and have obtained an unconstrained quadratic preference function in terms of X's and Z's. Since the preference function contains the random vector (Z), the expected

---

[4]The exposition of this part is based on H. Theil, <u>Optimal Decision Rules</u>, Ch. IV.

[5]That K_2 is symmetric can be shown as follows:

$K_2 = A + R'BR + CR + R'C' = \begin{bmatrix} I & R' \end{bmatrix} \begin{bmatrix} A & C \\ C' & B \end{bmatrix} \begin{bmatrix} I \\ R \end{bmatrix}$ where I is an identity matrix of order mT x mT.

preference level will be

$$(3.3) \qquad EW(X,\ RX + Z) = E(K_0) + E(K_1'X) + \tfrac{1}{2}E(X'K_2X)$$

Suppose that an optimal strategy, minimizing (3.3), exists. Let us denote it by $\hat{X}$. Then, any other strategy (X) can be described as[6]

$$(3.4) \qquad X = \hat{X} + eV$$

where e is a scalar and V is a vector of mT elements which are in turn functions of information.

The preference function has a (local) minimum at $X = \hat{X}$, if $W(X,\ RX + Z) > W(\hat{X},\ R\hat{X} + Z)$ holds for all feasible vectors (X) (in the neighborhood of $\hat{X}$). Then, from (3.3) and (3.4), we get

$$(3.5) \quad EW(X,\ RX + Z) - EW(\hat{X},\ R\hat{X} + Z) = E\left[(K_1 + K_2\hat{X})'(X-\hat{X})\right] + \tfrac{1}{2}E\left[(X-\hat{X})'K_2(X-\hat{X})\right]$$

$$= eE\left[(K_1 + K_2\hat{X})'V\right] + \tfrac{1}{2}e^2E(V'K_2V)$$

It is clear from (3.5) that if $\hat{X}$ minimizes (3.3), the first term in the second line of (3.5) must vanish and the second term must be positive for any values of e and V. For suppose that $eE\left[(K_1 + K_2\hat{X})'V\right] \neq 0$, 0 being a zero vector, for any one of the mT elements of V. Then, as we reduce e to the direction of zero while maintaining V unchanged, the second term becomes negligible in absolute value as compared with the first term, so

[6]R. Courant, Differential and Integral Calculus, Vol. II, trans., by E. J. McShane, (New York: John Wiley and Sons, 1936), pp. 495-497; and H. Theil, Optimal Decision Rules, pp. 132-135.

that we can make the right-hand side negative. Hence, $\hat{X}$ cannot be a minimizing value. Thus, the necessary condition is

$$(3.6) \qquad eE\left[(K_1 + K_2\hat{X})'V\right] = 0$$

Since $eE\left[(K_1 + K_2\hat{X})'V\right] = 0$ by (3.6), the second term should always be positive, and this requires that K_2 be positive definite.[7]

Now suppose that all the vectors V are non-stochastic. This implies that the strategy (X) differs from the optimal strategy ($\hat{X}$) by an arbitrary vector of fixed numbers. Then, (3.6) takes the form $e\left[E(K_1 + K_2\hat{X})\right]'V = 0$, for which a necessary and sufficient condition is that $E(K_1 + K_2\hat{X}) = 0$. Making use of the fact that K_2 is non-stochastic and assuming that K_2 is non-singular, we obtain

$$(3.7) \qquad E(\hat{X}) = -K_2^{-1}E(K_1)$$

This is the expected value of the optimal strategy. The optimal strategy $\hat{X}$ contains random elements as explained above, and hence the optimal strategy with which we are concerned is the expected optimal strategy.

By expanding K_1 in (3.2), we get

$$(3.8) \qquad E(\hat{X}) = -K_2^{-1}\left[a + R'b + (R'B + C)E(Z)\right]$$

It can then be seen that $E(\hat{X})$ in (3.8) is linear in the expected values

---

[7]Whenever $\hat{X}$ is such that there is no feasible vector in the immediate neighbourhood which gives a lower expected value to the preference function, $\hat{X}$ is called a local minimum. If K_2 is positive definite, then $\hat{X}$ is in fact the global minimum and unique. For proof, see K. Lancaster, Mathematical Economics, (Toronto: McMillan, 1968), Ch. II; and J. C. G. Boot, Quadratic Programming, (Amsterdam: North-Holland Publishing Co., 1964), Ch. 2.

of the random term (Z) of the constraints. Hence, this is called a "linear decision rule".[8]

Out of the optimal strategy for the entire strategy period, the policy-maker's immediate concern is the first-period decision of the strategy. This is called the "first-period optimal strategy decision". The first-period optimal strategy decision is the subvector consisting of the first m elements of $E(\hat{X})$ in (3.8), and it is also a linear function of the expectation of Z.

3. The Certainty Equivalence Theorem

Now, suppose that the policy-maker decides upon his optimal strategy under the assumption that all the random elements coincide with their expectations, i.e., $Z = E(Z)$. This is the certainty case, because the expected values of Z are assumed to be known constants. Substituting E(Z) for Z, the preference function (3.1) can be written as

$$W[X, RX + E(Z)] = b'E(Z) + \tfrac{1}{2}E(Z')BE(Z) + E(K_1')X + \tfrac{1}{2}X'K_2X \tag{3.9}$$

The first two terms in the right-hand side of (3.9) are constant from the policy-maker's point of view. Hence, by differentiating (3.9) with respect to X, we get the necessary condition for minimization:

$$\bar{X} = -K_2^{-1}E(K_1) \tag{3.10}$$

where $\bar{X}$ is the optimal strategy obtained under the condition of $Z = E(Z)$.

---

[8] C. C. Holt, "Linear Decision Rules for Economic Stabilization and Growth", Quarterly Journal of Economics, LXXVI (1962), 20-45; and H. Theil, Optimal Decision Rules, p. 136.

We can see that (3.10) is identical with (3.7). This demonstrates that if the policy-maker behaves as though the uncertain (random) Z equals its expectation, the optimal decision chosen under certainty is identical with the optimal decision derived from minimizing the expected value of the preference function which contains uncertainty. This is called the "certainty equivalence".[9]

Since we are basically concerned with the optimal values of the instruments and accompanying target variables, rather than with the level of the preference function, (3.10) is the form which satisfies our purpose. Since the expected values of the random terms - but not the random terms themselves - are assumed to be known fixed numbers, we make use of the certainty equivalence approach for the actual computation of the optimal strategy.

The first m elements of $\bar{X}$ in (3.10) are the results of the first-period certainty equivalence decision. The optimal strategy for the second and later periods need not be computed immediately because it does not have to be made until the beginning of the second and later periods, respectively. However, the first-period certainty procedures can also be used for the second and later periods by shifting the period

---

[9]The certainty equivalence theorem was discovered by H. A. Simon, and generalized by H. Theil. See H. A. Simon, "Dynamic Programming under Uncertainty with a Quadratic Criterion Function", Econometrica, XXIV (1956), 74-81; H. Theil, "A Note on Certainty Equivalence in Dynamic Planning", Econometrica, XXV (1957), 346-349. The basic assumptions for the certainty equivalence are: (i) linear constraints; (ii) a quadratic preference function; and (iii) the distribution of Z_t independent of X_t with a finite covariance matrix. If any one of these assumptions is relaxed, the certainty equivalence will not hold. This can be easily proved, and is not pursued here. See H. Theil, Optimal Decision Rules, pp. 52-59.

of the problem. For instance, the second-period, at its beginning, is regarded as the first-period with a (T-1) period horizon, and the same procedures as for the first period are then applied to the second period. It should be noted that for the optimal decision for the second and later periods, the first-period decisions are regarded as implemented, and its additive term (Z_1) becomes in principle no longer random, but is now a known constant.

4. An Application of the Generalized Inverse of a Matrix to the Optimal Strategy

We have now derived the optimal strategy and the certainty equivalence using a conventional optimization technique. If, however, the preference function is a weighted sum of squared deviations between the actual and the desired values of target variables and instruments, we can apply the concept of the generalized inverse of a matrix in order to derive the optimal strategy.[10] This form of preference function requires that in (2.6) a and b both be zero vectors, C be a zero matrix, and A and B be diagonal matrices. Then, the preference function, corresponding to (2.6), is

$$W = \tfrac{1}{2}(x - x^*)'A(x - x^*) + \tfrac{1}{2}(y - y^*)'B(y - y^*) \tag{3.11}$$

[10]This is the form of preference function which will actually be used in the numerical calculation of the optimal strategy in Chapter VI. This is also a form of preference function used quite often in empirical work. See, for example, P. J. M. van den Bogaard and H. Theil, "Macrodynamic Policy-making: An Application of Strategy and Certainty Equivalence Concepts to the Economy of the United States, 1933-1936", in A. Zellner, ed., Readings in Economic Statistics and Econometrics, (Boston: Little, Brown and Company, 1968), pp. 660-680.

where A and B are diagonal matrices of order mT x mT and nT x nT respectively, and x and y are column vectors for mT instruments and nT target variables, respectively, measured from their original origins - not in deviation form, the asterisks indicating desired values of corresponding variables.

Assuming away a perverse case, the diagonal elements of A and B are all positive. Therefore, (3.11) can be rewritten as

$$W = \tfrac{1}{2}(\alpha x - \alpha x^*)'(\alpha x - \alpha x^*) + \tfrac{1}{2}(\beta y - \beta y^*)'(\beta y - \beta y^*) \tag{3.12}$$

where α and β are mT x mT and nT x nT diagonal matrices, respectively, with diagonal elements equal to the square roots of the corresponding diagonal elements of A and B. Recall that the constraints measured from the original origins of variables are as specified in (2.1), i.e., $y = Rx + Hs + Ju$.

The desired level of preference (or disutility) is assumed to be achieved when the actual values of target variables and instruments coincide with their desired values.[11] Substituting the constraints (2.1) into (3.12) yields the system of linear equations which is required to achieve the desired level of preference:[12]

---

[11] As seen by (3.12), this desired preference level corresponds to the absolute minimum (zero).

[12] Since we are concerned with the certainty equivalence, the random term u is replaced by its expected value. This system is in general inconsistent in the sense that if x takes on x^*, the upper part of this system does not in general hold. This says that if the policy-maker desires x^* for instruments and y^* for target variables, but if he decides on x^*, he does not get y^*, rather he obtains $Rx^* + Hs + JE(u)$, which is in general different from y^*. The difference between y^* and $Rx^* + Hs + JE(u)$ is called "inconsistency" of the policy-maker's desires. H. Theil, "Linear Decision Rules for Macrodynamic Policy Problems", in B. G. Hickman, ed., <u>Quantitative Planning for Economic Policy</u>, (New York: The Brookings Institution, 1965), pp. 18-42.

$$(3.13)\qquad \begin{aligned} \beta Rx &= \beta y^* - \beta Hs - \beta JE(u) \\ \alpha x &= \alpha x^* \end{aligned}$$

where only the x's are unknown. this system (3.13) can be rewritten in a simplified form as

$$(3.14)\qquad Dx = Q \quad \text{where} \quad D = \begin{bmatrix} \beta R \\ \alpha \end{bmatrix} \qquad Q = \begin{bmatrix} \beta y^* - \beta Hs - \beta JE(u) \\ \alpha x^* \end{bmatrix}$$

Hence, D is an (nT + mT) x mT matrix, and Q is an (nT + mT) x 1 vector. Note that D is a rectangular matrix and its rank is not specified yet.

The solution of a system of linear equations is well known if the multiplicative coefficient matrix is a square non-singular matrix. However, there are cases where the coefficient matrix, like D in (3.14), is neither square nor non-singular. In these cases, there may still be a solution to the system. This is the case where the generalized inverse of a matrix is used.[13]

Let us consider the matrix D in (3.14). We know that the constraints (2.1) consist of a consistent system of equations and that the x's are mutually independent instruments. Thus, mT columns of R are linearly independent. Since β is a diagonal matrix of order nT x nT, mT columns of βR are also linearly independent. Furthermore, since α is

---

[13] A systematic and compact presentation of the generalized inverse of a matrix will be found in F. A. Graybill, Introduction to Matrices with Application in Statistics, (Belmont: Wadsworth Publishing Co., 1969), Ch. 6 and 7; and J. C. G. Boot, Quadratic Programming, pp. 40-48.

an mT x mT diagonal matrix, we can see that the matrix D is of full-column rank - the rank of D being equal to mT.

It can be proved that if D is of full-column rank, its generalized inverse is [14]

$$(3.15) \qquad D^{-g} = (D'D)^{-1}D'$$

where D^{-g} denotes the generalized inverse of matrix D. As a result, the solution of the system (3.14) is

$$(3.16) \qquad \hat{x} = D^{-g}Q = (D'D)^{-1}D'Q$$

where $\hat{x}$ is an mT x 1 vector of optimal strategy decisions of instruments. Substituting (3.14) into (3.16), and making use of the relation $\alpha'\alpha = A$ and $\beta'\beta = B$, we get

$$(3.17) \qquad \hat{x} = \left[\begin{pmatrix}\beta R\\ \alpha\end{pmatrix}'\begin{pmatrix}\beta R\\ \alpha\end{pmatrix}\right]^{-1}\begin{pmatrix}\beta R\\ \alpha\end{pmatrix}'\begin{bmatrix}\beta y^* - \beta Hs - \beta JE(u)\\ \alpha x^*\end{bmatrix}$$

$$= (R'BR + A)^{-1}(R'By^* - R'BHs - R'BJE(u) + Ax^*)$$

The first-period optimal strategy decision is the subvector consisting of the first m elements of (3.17). This is in fact the first-period certainty equivalence because the random terms of the constraints are replaced by their expected values. It can easily be shown that (3.10) becomes identical with (3.17) if the preference function is a weighted sum of squared deviations between the actual and the desired values of

---

[14] See F. A. Graybill, Introduction to Matrices, pp. 99-100.

target variables and instruments, although it is not pursued here.

In this chapter we have derived the general rule for the optimal strategy, the first-period optimal strategy decision, and the first-period certainty equivalence. We have also shown that if the preference function is a weighted sum of squared deviations of target variables and instruments from their desired values, we can apply the concept of the generalized inverse of a matrix to the optimization problem. We shall apply these results to the analysis of fiscal policy for economic stabilization. For this purpose, we need first to construct a model of the Canadian economy. This will be done in the next chapter.

CHAPTER IV

SPECIFICATION OF THE MODEL

1. Introduction

The theory of economic policy is essentially concerned with the theory of optimal choice, as is economics itself. Hence, one ultimate purpose of economic research is to generate information that can be used to improve economic decision-making or strategy formation. Economic theory has been developed in terms of sets of relationships among economic variables which jointly determine their values. Quantitative economic policy which is our concern, requires quantitative relationships connecting economic variables. This can be sought by formulating an econometric model.

Econometric models for the purpose of economic policy should explicitly contain target variables and instruments, and spell out the relations between them by appropriately specifying the structural relationships among measurable economic variables. As indicated in Chapter II, the target variables chosen here are changes in the number of persons unemployed and in the GNE price deflator, and the instruments are changes in government expenditure on goods and services, in the personal income tax rate, and in the corporation income tax rate. Although a number of econometric models of the Canadian economy containing these target variables and instruments have been constructed by various persons and institutions at various times, they do not appear to be appropriate for our purposes. As a result, a relatively small-sized aggregate annual model of the Canadian economy was developed. Stabilization policy implies changes in the tax-expenditure system, either automatically or at the discretion of the government, to offset excesses or deficiencies in private demand. Thus, the model developed is a demand-oriented model which concentrates on the expenditure side of the national accounts. Since the ultimate objective in the use of this model is to measure the effects of discretionary fiscal policy on the economic system, especially on the dual target variables of stabilization policy, at the initial impact stage and over time, the three instruments and the two target variables mentioned above are explicitly incorporated into the model.

Our approach in developing this model was to consider, for each hypothetical structural equation, a variety of explanatory variables as they are to be found in the usual sources - literature on economic theory or applications of statistical analysis and, in particular, earlier econometric models.[1] The choice of variables to be included in the structural equation must be acceptable from the point of view of economic

---

[1] Out of numerous models available, the following were found to be especially helpful: J. H. Helliwell, L. H. Officer, H. T. Shapiro, and I. A. Stewart, The Structure of RDX1, (Ottawa: The Bank of Canada, 1969); N. K. Choudhry, Y. Kotowitz, J. A. Sawyer, and J. W. L. Winder, The Annual Econometric Model of the Canadian Economy, (Toronto: University of Toronto, 1969); H. Tsurumi, A Four-Sector Econometric Model of the Canadian Economy, (Kingston: Queen's University, 1969); T. M. Brown "A Forecast Determination of National Product, Employment, and Price Level in Canada, from an Econometric Model", in the National Bureau of Economic Research, Models of Income Determination, (Princeton: Princeton University Press, 1964), pp. 59-96; and S. May "Dynamic Multipliers and Their Use for Fiscal Decision-Making", in the Economic Council of Canada, Conference on Stabilization Policies, (Ottawa: Queen's Printer, 1966), pp. 155-195.

theory. However, there are numerous theories for most of the equations, and thus, in most cases, economic theory alone does not provide a clear basis for choosing one set of variables over another in constructing econometric models.

Most of the work in developing this model was, therefore, devoted to a systematic refining and testing of hypotheses underlying the structural equations. For this purpose, various estimates of the parameters of the structural equations corresponding to various, mutually consistent, combinations of hypotheses and their relevant variables were made from annual data for the period 1950-69 except for the corporation income tax rate which starts from 1952.[2] The best one, two, or even a few combinations for each equation, judged on grounds of compatibility with economic theory, significance of parameter estimates, the coefficient of determination, the standard error of estimate, and the degree of autocorrelation of random terms, were then selected for further consideration. The ordinary least-squares method was used for this purpose, because in this preparatory stage the large number of possible combinations of hypotheses and relevant variables simply precluded the use of simultaneous methods.

Of those hypotheses for each equation selected by the ordinary least-squares method, one was chosen for each equation. Then, all of

---

[2]The period 1950-69 was chosen as the sample period because some of the revised national accounts data started from 1950. Also, starting in 1950 avoids possible difficulties arising from the fact that the years immediately following World War II constituted an unusual period. However, the equations with one-year lagged variables or in the form of first differences were estimated for the period 1951-69; the corporation income tax equation was estimated for the period 1952-69.

the parameters were re-estimated by the "structurally ordered instrumental variables" (SOIV) method proposed by F. M. Fisher and B. M. Mitchell, because of the well-known dangers of applying the ordinary least-squares method to the estimation of structural parameters of the simultaneous equation system.[3] The method of instrumental variables involves first-stage regressions of the right-hand side endogenous variables on the predetermined variables of the model. However, frequently in econometric models, as in this case, the number of predetermined variables exceeds the sample size, and thus a first-stage regression is not possible. Therefore, the first problem arising from the use of the instrumental variable method is the selection of instrumental variables for each right-hand side endogenous variable for the first-stage regression - the number of instrumental variables thus chosen being, of course, less than the sample size.,

In this context, the SOIV method suggests an elaborate technique, which is basically intended to combine the structural information of the model with information obtained from the data, for selecting a set of predetermined variables. Suppose that the complete model is normalized with different endogenous variables on the left-hand side of each equation. Then, behavioral equations are interpreted as causal relations,

---

[3]F. M. Fisher, "Dynamic Structure and Estimation in Economy-Wide Econometric Models", in J. Duesenberry, G. Fromm, L. R. Klein and E. Kuh, eds., The Brookings Quarterly Econometric Model of the United States, (Chicago: Rand-McNally, 1965), pp. 588-635; B. M. Mitchell and F. M. Fisher. "The Choice of Instrumental Variables in the Estimation of Economy-Wide Econometric Models: Some Further Thoughts", International Economic Review, XI (1970), 226-234; and B. M. Mitchell, "Estimation of Large Econometric Models by Principal Component and Instrumental Variable Method", Review of Economics and Statistics, LIII (1971), 140-146.

with causation running from right to left. From this idea, for each right-hand side endogenous variable in the model, a preference causal ordering of instrumental variables is constructed as follows. In each normalized equation explaining one of the right-hand side endogenous variables, there will typically be both predetermined variables and other endogenous variables. These predetermined variables are to be of first causal order. Then, additional predetermined variables are obtained by considering the equations explaining the right-hand side endogenous variables in this equation, and the predetermined variables from those equations are of second causal order. This procedure is continued until no new right-hand side endogenous variables are encountered, and thereby the predetermined variables are ranked according to their associated causal ordering numbers for each right-hand side endogenous variable. From these ordered predetermined variables, multicollinear variables, as judged by the information from their sample correlations, are deleted. The rest of the ordered predetermined variables are finally tested, beginning with those of lowest preference causal order, for contribution to the coefficient of determination ($\bar{R}^2$) corrected for the degree of freedom; and only those predetermined variables contributing "significantly", which are then the first-stage regressors, are retained.

Predetermined variables which are potential instrumental variables consist of both current and lagged exogenous variables and lagged endogenous variables. For our model at this stage, there are approximately twenty-two current exogenous variables, seven one-year lagged exogenous variables, and fifteen one-year lagged endogenous variables.[4] The seven exogenous variables which are used as lagged and as the current period's variables show quite a stable time trend except that for accrued net income of farm operators from farm production (YF). However, the estimated correlation coefficients between accrued net farm income and most of the endogenous variables are quite low. Hence, lagged exogenous variables were excluded from a set of eligible instrumental variables. Since the model is an annual model, lagged endogenous variables were included in the eligible list. Thus, out of the current exogenous variables and lagged endogenous variables, a set of instrumental variables for each right-hand side endogenous variable was chosen according to the SOIV method as explained in the preceding paragraph. Finally, each equation was estimated by replacing the right-hand side endogenous variables by their estimated values through the first-stage regressions on the instrumental variables thus chosen, and regressing the "dependent" variable of the equation on those estimated values and predetermined variables, if any. When any equation turned out to be poorly fitted when it was re-estimated by the SOIV method, a different form of the same structural equation already chosen by the OLS method was substituted, and all the structural equations were re-estimated again, simultaneously.

It will be noted that the structural equations were estimated in terms either of levels or of annual first differences. The use of first differences has certain advantages. First, positive autocorrelation

---

[4]The number of predetermined variables is not yet finally decided upon; it depends on the form of equation, for a certain structural equation, chosen from those estimated by the OLS method.

of random disturbances is apt to be present ~~in time~~ series regressions. First differences tend to remove such positive autocorrelation. Thus, if autocorrelation is present in a time series, the use of first differences will generally increase the likelihood that the standard significance tests are meaningful.[5] Secondly, some equations involve stock variables for which data are not currently available. The change in stock is composed of current additions less withdrawals. If the latter tends to be a smooth series, the first difference in stock may be well represented by gross additions. Thirdly, many of the economic investigations are concerned with changes in economic variables between two periods, so that the use of first differences serves to focus the analysis directly on these changes. Finally, the constant terms in first difference equations directly represent smooth changes over time without explicitly introducing factors for them.

However, some of the equations can be better estimated by using the level forms. The equations estimated in terms of levels can be converted easily into first difference form and, in some cases, they provide better forecasts of first differences than the estimates based directly on first-difference equations.[6] Furthermore, if positive autocorrelation of random disturbances is not serious when raw data is used in fitting an equation, the use of first differences may produce

---

[5] Since positive autocorrelation of residuals from time-series regressions causes a downward bias in calculated standard errors, giving an exaggerated appearance of significance of the estimated coefficients, seemingly good results might be misleading if autocorrelation is present. See D. B. Suits, "Forecasting and Analysis with an Econometric Model", American Economic Review, LII (1962), 104-132.

[6] A. S. Goldberger, Impact Multipliers and Dynamic Properties of the Klein-Goldberger Model, (Amsterdam: North-Holland Publishing Co., 1959), p. 52

rather serious negative autocorrelation. In this case, equations using levels are more appropriate than equations using first differences. As a consequence, the model developed was estimated using both levels and first differences, and for individual equations the final choice was made after comparing the results.

So far we have endeavoured to refine and test, in isolation, hypotheses about each structural equation. It is then necessary to see how the whole system will work. The dynamic performance of the complete system is particularly important for policy purposes. In order to evaluate the model's performance, a controlled simulation experiment was made for the period 1954-69 using the actual values of all exogenous variables and replacing the lagged endogenous variables by the computed values as the latter became available year by year.[7] The coefficients of the structural equations estimated by the SOIV method were used for the simulation.

As an evaluation criterion, Theil's inequality coefficient was used.[8] The numerical value of the inequality coefficient varies between one and zero, and the closer it is to zero, the better the predictions, on average, although there is no statistical significance test available for the coefficient. If the model's performance seemed to be unsatisfactory, as indicated by an inequality coefficient for an endogenous

---

[7]This simulation program used here is available at the Canadian Government's Department of Finance. This program, which is based on the Gauss-Seidel method of solving a system, is the simulation package of the Wharton School in the University of Pennsylvania modified by J. Kuiper and E. Piekaar. It can solve the model for the periods up to only 16 years.

[8]H. Theil, Economic Forecasts and Policy, pp. 31-33.

variable larger than .10 (except for non-farm business inventory changes), we returned to the results of the OLS estimation stage and chose a different equation form with which to repeat the experiments - re-estimation by the SOIV method and simulation. Using this iterative procedure yielded one final form for each equation; and 18 stochastic equations thus chosen and 8 identities constitute the final form of the model. The computed numerical values of Theil's inequality coefficients for twenty-six endogenous variables will be shown after a description of the final form of the model. Most of the values computed for the endogenous variables seem to be sufficiently close to the actual values, at least for present purposes. So far we have described our overall procedures in developing the model. In Section 2, we shall present a description and discussion of the structure of the model, including the final estimated form of each equation.

2. Discussion of the Model

A. Consumer Expenditure

The estimated consumption function (E.1) describes total consumer expenditure linearly in terms of real disposable income less net income received by farm operators from farm production, and lagged consumer expenditure.[9] Net farm income was subtracted from total disposable income because farm income is defined to include farm inventory change which

---

[9] "Net income received by farm operators from farm production" differs from "accrued net income of farm operators from farm production". The former excludes the adjustment which has been made to take account of the accrued net earnings arising out of the operations of the Canadian Wheat Board and the Canadian Co-operative Wheat Producers. See Statistics Canada, National Income and Expenditure Accounts: Third Quarter, 1971, p. 19.

may have little relevance to consumer expenditure.[10] As in most annual models, lagged consumption shows a statistically significant positive influence on current consumption expenditure.[11] The estimated consumption function can be explained by the hypothesis that consumption depends on expected real disposable income, which is in turn assumed to be a weighted average of the expectation held last period and the actual level observed currently. That is, suppose that consumption (CE) and expected disposable income (YD^e) are related by a simple linear equation

$$(4.1) \qquad CE_t = \gamma_o + \gamma_1 YD_t^e$$

The assumption made above for expected disposable income can be written as[12]

$$(4.2) \qquad YD_t^e = \lambda YD_{t-1}^e + (1 - \lambda)YD_t, \qquad 0 \leq \lambda < 1$$

where the weight (λ) is assumed to be non-negative and less than one. Assuming that the relation (4.2) holds true over time, we can easily derive a relation connecting the expectation hypothesis and a distributed-

---

[10] This is also S. May's approach. See S. May, "Dynamic Multipliers".

[11] It may be noted that in the presence of lagged dependent variables the Durbin-Watson statistic is biased toward the acceptance of the null hypothesis of non-autocorrelation of the disturbance term, and that in such a case the power of this statistic is low.

[12] This is the expectation hypothesis proposed initially by P. Cagan. See P. Cagan, "The Monetary Dynamics of Hyperinflation", in M. Friedman, ed., Studies in the Quantity Theory of Money, (Chicago: The University of Chicago Press, 1956), pp. 25-117. See also C. F. Christ, Econometric Models and Methods, (New York: John Wiley and Sons, 1966), pp. 204-208.

lag equation as follows:

$$(4.3) \qquad YD_t^e = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i YD_{t-i}$$

By substituting (4.3) into (4.1) and rearranging terms, we can get

$$(4.4) \qquad CE_t = \gamma_0(1 - \lambda) + \gamma_1(1 - \lambda)YD_t + \lambda CE_{t-1}$$

This is the form used for estimation of the consumption function.[13] As shown by the estimated consumption function, the results obtained by the OLS method and the SOIV method are quite alike as are those for the other equations.[14]

B. Business Fixed Capital Formation

The basic hypothesis adopted with regard to the investment function (E.2) is that business investment depends on the expected change in business output, which is in turn assumed, as was done for expected disposable income, to be a weighted average of the expectation held last period and the actual change observed currently. Hence, both changes in business output and lagged investment appear in the investment function. The sign of lagged investment is positive as expected. Real business output (ONG/Py) is calculated by subtracting from gross national product

---

[13] This form of consumption function can also be explained by the "habit persistence" hypothesis. See M. K. Evans, Macroeconomic Activity, (New York: Harper and Row Publishers, 1969), pp. 19-34.

[14] This phenomenon also occurred in the RDX1 econometric model. J. F. Helliwell, et.al, The structure of RDX1. It will be noted that the t-test is not strictly valid for testing coefficients estimated other than by the OLS method, but the "t-ratio" may be a useful indicator, though an approximation, of statistical significance. Note also that a precise interpretation of the D/W and $\bar{R}^2$ statistics is difficult in the context of the instrumental variable method of estimation. With regard to the foregoing, see C. F. Christ, Econometric Models and Methods, Ch. X.

the accrued net income of farm operators from farm production, and the government component of output (represented by the government wage bill and government capital consumption allowances). Net farm income is excluded for the reason given above. The estimated investment equation also includes non-government capital consumption allowances plus undistributed corporation profits, both being deflated by the deflator for business gross fixed capital formation and lagged one year. They are included on the ground that they correspond to a source of funds that can be used for investment.

It is certainly conceivable that the degree of capacity utilization will influence business investment. Since no data for it are available, the unemployment rate is included as a proxy variable for capacity utilization. The unemployment rate, lagged one year, has a negative effect on investment, as expected. Finally, the long-term rate of interest, lagged one year, is included with the aim of linking the goods market and the money market. Its coefficient bears the expected sign. Actual investments are likely to take some time after they have been projected. Hence, one-year lagged variables reflecting fund availability, capacity utilization, and investment costs seem to be more reasonable than those for the current period. Thus, business capital consumption allowances and undistributed corporation profits, the unemployment rate, and the long-term interest rate are all included as one-year lagged variables.[15]

---

[15]Since the output variable (ONG/Py) appears in the form of first differences, its effect is in fact delayed by six months.

C. Change in Non-farm Business Inventory

A stock adjustment model of inventory change is adopted here. It is postulated that the difference between desired and actual inventory stock will be adjusted only partially rather than totally in a given year.[16] Thus, the function takes the form

$$(4.5) \qquad (JNFB/Py)_t = \delta(JST^*_t - JST_{t-1})$$

where δ represents the fraction of the difference between the desired inventory stock at the beginning of the current period (JST^*_t) and the actual stock at the beginning of the previous period (JST_{t-1}), which is adjusted in the current period; $JNFB_t$ is the change in non-farm business inventories; and Py is the GNE deflator.[17] It is further assumed that the desired stock of inventory depends on last year's sales, expressed as

$$(4.6) \qquad JST^*_t = \gamma_o + \gamma_1\{(ONFB/Py)_{t-1} - (JNFB/Py)_{t-1}\}$$

where ONFB/Py is real non-farm business output computed by substracting from total business output (ONG) wage payments in the agricultural sector, and deflating the result by the GNE deflator. Sales is defined as non-farm business output less inventory change. It is assumed that

---

[16]This is called the flexible accelerator principle in inventory investment. See M. K. Evans, Macroeconomic Activity, pp. 204-206.

[17]The national accounts implicit price deflator for the change in inventories takes positive and negative values, small and large, or even indeterminate, because it is obtained as the ratio of the changes in current and constant-dollar stocks. Hence, it is not meaningful, so that it does not appear in the national accounts. Therefore, the GNE price deflator, which includes the implicit price deflator for inventories, is used in deflating the change in non-farm business inventories. See Dominion Bureau of Statistics, National Accounts Income and Expenditure: 1926-1956, (Ottawa: Queen's Printer, 1962), pp. 177-184.

the desired stock of inventory at the beginning of the period will be determined by past sales experience and, more specifically, that the desired stock depends on last year's sales - not on the current year's. By substituting (4.6) into (4.5), we get

$$(4.7)\qquad (JNFB/Py)_t = \gamma_o\delta + \gamma_1\delta\{(ONFB/Py)_{t-1} - (JNFB/Py)_{t-1}\} - \delta JST_{t-1}$$

This is the basic relationship underlying the estimated equation for change in real non-farm business inventory (E.3). The data for inventory stock were created from

$$(4.8)\qquad JST_t = \sum_{j=1}^{\infty} (JNFB/Py)_{t-j}$$

where JST_t has an arbitary origin of zero in the first year (1950). In addition to sales and lagged inventory stock, the change in imports and the change in the unemployment rate are also included in the equation. We assume in the case of imports that inventories of imported goods are likely to have a different pattern than inventories of domestically produced goods, because of different periods of time associated with orders and deliveries of imported goods. The change in the unemployment rate is included as a proxy for changes in business confidence; it bears the expected sign.

D. Imports

The explanatory variables in the import equation (E.4) are the level of domestic economic activity, the difference between domestic and foreign prices, the unemployment rate, and the lagged level of real imports. As a measure of the level of domestic economic activity, we chose deflated gross national product less accrued farm income (O/Py). The lagged import term may be relevant because a large part of purchases of imported capital and consumer goods would be similar to domestic investment and consumption;[18] the equation can then be explained by an expectation hypothesis, as was done with regard to the consumption and investment function, and thus the lagged level of imports appears with a positive sign, as expected. The unemployment rate is included as a proxy for the capacity utilization index and/or excess demand. Since a certain proportion of imported goods are capital goods, imports of those goods will decline as the unemployment rate increases, indicating a reduction in capacity utilization. On the other hand, when excess demand, indicated by a low unemployment rate, exists in the domestic economy, imports will increase. As a result, the unemployment rate is expected to reflect variables which have a negative effect on imports.

The implicit price index used for imports of goods and services is the import price deflator from the national accounts. Thus, it reflects not only the change in foreign prices but also the change in exchange rates. Although a flexible exchange rate system existed for a about half of our sample period, the present model is based on a fixed exchange rate system since that is what existed during the period 1967-69 to which the model is to be applied. Therefore, the import price index is treated as an exogenous variable throughout the whole sample period. The level of domestic prices is indicated by the GNE price deflator which, in turn, represents the prices of import-competing goods. Thus, the

[18] M. K. Evans, Macroeconomic Activity, p. 224.

difference between the two prices is expected to have a positive effect on imports.

E. Demand for Paid Workers in the Private Sector

Demand for and supply of labour are treated separately, with the number of unemployed being determined as a residual. Since the present model is concerned mainly with short-term stabilization policy, the total supply of labour is regarded as an exogenous variable. For the same reason, the number of employers and self-employed in the private sector is also treated as an exogenous variable. Since direct employment in the government could be a policy instrument, the number of government civilian employees is also regarded as exogenous, and is excluded from paid workers. Total paid workers in the private sector comprises, on the average, slightly more than 75 percent of total employment throughout the sample period.

The equation (E.5) representing the demand for paid workers attempts to explain it linearly in terms of real output in the private sector (ONG/Py), the annual average hourly real wage rate (WR/Py), and the previous period's level of paid workers. The quantity of labour demanded would depend on the output to be produced and the cost of labour. The latter is expected to be real labour cost, and hence the money wage rate is deflated by the GNE deflator. The sign of the coefficient is negative, as expected.

The one-period lagged level of paid workers also seems to be an important explanatory variable. When the economy slows down, private firms would not cut their labour force accordingly. Rather they would use their workers less intensively, reducing working hours and fostering some paid unemployment. This procedure followed by firms is reasonable because of the possible high cost to firms of firing, rehiring, and losing competent employees when the economy recovers. Hence, the lagged level of paid workers appears in the equation with a positive sign, as expected. The estimated equation can also be explained by an expectation hypothesis, as was done with regard to the consumption function. This expectation hypothsis may be helpful because hiring, layoff, or firing decisions depend in part on expected output.

F. Wage Rate

The wage rate function (E.6) developed here is basically a disequilibrium function that relates changes in the annual average hourly wage rate to excess demand in the labour market. Excess demand in the labour market would be represented by the unemployment rate. It is generally thought that the change in money wages is a non-linear function of the unemployment rate; when the unemployment rate is high, a small change in the unemployment rate might be expected to have little impact on the wage rate; on the other hand, when the unemployment rate is low, a small variation in this rate might be expected to have a large impact. This relation is often represented by making the wage change variable a linear function of the reciprocal of the unemployment rate.

Some refinement might be added to the linear relation between the change in the wage rate and the reciprocal of the unemployment rate. In most cases, wages are in effect determined, among other things, by the bargaining positions held in the labour market by trade unions and employers or their associations. The factors affecting the bargaining position of either party may be expressed by means of past profits and the demand prospects for labour (or the unemployment rate). If profits are high, employers will be less reluctant to comply with the demand for higher wages than in periods when profits are low. The level, rather than the change, of profits, lagged one year, is used because the existence of profits might act as a target for wage demands. Trade unions, on the other hand have to reckon not only with the level of profits, but also with the existing level of the unemployment rate. In many cases, wage bargaining is carried out at infrequent intervals, and wages for a year or more are determined by a single contract. As a result, the unemployment rate lagged one year seems to be more appropriate than the current rate. Apart from the unemployment rate and the level of profits, trade unions might take into account changes in the general price level for goods and services in their wage demands. Hence, the change in the price deflator for consumer expenditure is introduced into the wage equation.[19]

G. Prices

The implicit price deflator for gross national expenditure is used as the general indicator of price behaviour in the economy. The price (GNE deflator) equation used here (E.7) is based on an idea similar to the one underlying the wage equation; it relates changes in prices to excess demand in the economy's product market. Although excess demand in the product market is not exactly the same as excess demand in the

---

[19] It may be noted that the price deflator for consumer expenditure is in first difference form, and thus its effect is partially delayed.

labour market, for the economy as a whole there is generally a close relationship between the rate of unemployment and excess demand in the product market.[20] The relationship between excess demand in the product market and changes in the price level seems to be non-linear, indicating that, as the excess demand becomes negative, prices will not change by very much, while as excess demand increases, prices will rise rapidly. Because of this non-linear relationship, the unemployment rate appears in the equation as its inverse with a positive sign, as expected. In the present model, therefore, the inverse of the unemployment rate serves as a proxy for the influence of demand pressures both in the labour market and in the product market.

The change in the wage rate is also included in the GNE price deflator equation. The change in the wage rate adjusted for productivity changes appears to be more appropriate than the unadjusted change. The change in the wage rate can be adjusted, approximately, for the productivity change by subtracting the latter from the former, both being weighted somehow. Such an adjustment for the productivity change can then appear in this equation separately from changes in the wage rate itself. Furthermore, changes in productivity arise from technical progress which, for short-run purposes at any rate, may be regarded as an exogenous variable occuring more or less regularly over time for the economy as a whole. Therefore, the negative constant term in the estimated price equation in terms of first differences may reflect the annual change in

---

[20] R. G. Bodkin, E. P. Bond, G. L. Reuber, and T. R. Robinson, Price Stability and High Employment: The Options for Canadian Economic Policy, (Ottawa: Economic Council of Canada, 1966), p. 19.

productivity. A substantial part of Canada's imports is likely to be input for domestic production, so that changes in import prices would be reflected in domestic price changes. In addition, a large proportion of corporations in Canada are foreign owned, mainly being the U. S., subsidiaries, and their pricing policies may closely coordinate with those of their parent companies. As a result, the change in the import price index appears in the equation with a positive sign, as expected.

As indicated above, the GNE price deflator serves as the basis for determining prices in a particular sector of the model. Because elsewhere in the model we have used price indices for consumer expenditure and business investment in order to convert current-dollar values into constant-dollar values, we need to determine these two price indices. In the present model, it is assumed that the price index for consumer expenditure (E.8) follows movements in the general price level, as represented by the GNE price deflator; in a large model, one might wish to have a more detailed specification of the ways in which this price index is determined.

The price index for business fixed capital formation (E.9) is treated somewhat differently than is that for consumer expenditure. Since the demand and supply curves for fixed investment are presumably much more inelastic than for consumer goods, small shifts in the demand and supply curves for fixed investment are likely to have substantially greater effects on the price of investment goods than would be true in the consumer-goods sector.[21] This inelasticity is represented in this

---

[21] M. K. Evans, Macroeconomic Activity, p. 307

equation by the ratio of business fixed capital formation (ING) to non-farm output (O); this ratio represents the proportion of non-farm output being used in fixed capital formation. As a result, the equation for the change in the price index for business fixed capital formation includes not only the change in the GNE price deflator but also the change in the ratio (ING/O). Both of the explanatory variables have positive signs, as expected.

H. Corporation Profits before Taxes

The estimated equation for corporation profits before taxes (E.10) relates this variable linearly to non-farm business sales, lagged corporation profits before taxes, and the change in the unemployment rate. Since data for sales by incorporated businesses are not available, non-farm business sales, which is defined as non-farm business output (ONFB) less non-farm business inventory change (JNFB), was used. Since corporation profits before taxes are measured in current dollars, the sales variable used here is also in terms of current dollars. This variable thus differs from the (real) sales variable used in the equation for non-farm business inventory change. Corporation profits are highly sensitive to cyclical fluctuations in aggregate demand. These fluctuations can be reasonably approximated by changes in the unemployment rate. Lagged corporation profits with a positive effect distribute over time the effects of cyclical fluctuations.

I. Undistributed Corporation Profits[22]

[22] The exposition in this section follows closely C. F. Christ, Econometric Models and Methods, pp. 585-586.

Corporation dividend payments presumably depend in part on the expected level of future corportion profits after taxes. The expectation hypothesis adopted here, which is similar to those used in other equations in this model, postulates that the expected level of future corporation profits after taxes is a weighted average of the expectation held last period and the actual value observed currently. From this hypothesis, we derived an equation that relates dividends linearly to current corporation profits after taxes and lagged dividends. This can be expressed as

$$(4.9) \qquad DIV_t = \rho_o + \rho_1 CPA_t + \rho_2 DIV_{t-1}$$

where DIV_t denotes corporation dividend payments, CPA_t denotes corporation profits after taxes; and ρ_o could be either positive or negative, while ρ_1 and ρ_2 are both positive. Since the sum of dividends and undistributed corporation profits (CPU_t) equals corporation profits after taxes, i.e., $DIV_t + CPU_t = CPA_t$, both current and lagged dividend payments in the above equation can be replaced by current and lagged undistributed corporation profits. Thus, we get

$$(4.10) \qquad CPU_t = - \rho_o + (1 - \rho_1)CPA_t - \rho_2(CPA_{t-1} - CPU_{t-1})$$

Based on this hypothesis, the estimated equation (E.11) relates undistributed corporation profits to corporation profits after taxes, which are defined as corporation profits before taxes (CPB) less corporation income tax (TC), and the difference between lagged corporation profits after taxes and lagged undistributed corporation profits. All of the signs are as expected.[23]

J. The Government Sector

In the present model, government expenditures and revenues are both treated solely in current-dollar terms. All government expenditures of all levels of government, except unemployment insurance benefits, are treated as exogenous variables. The total tax receipts of all levels of government, excluding the withholding taxes imposed on non-residents, are allocated to personal income tax (TP), other personal direct taxes (TPO), corporation income tax (TC), and indirect taxes (TI). Revenue from other personal direct taxes (TPO) is insignificant - about 5 percent of total personal direct tax revenue - and thus it is treated as exogenous. The other three are treated as endogenous variables. Hence, the present model includes explicitly equations for the major categories of tax revenue. It will be noted that corporation income tax is recorded in the national accounts on an accrual basis, while the other three are on a collection basis.

A policy-oriented model should incorporate policy instruments as explicitly as possible so that the policy-maker can experiment easily with alternative values of the instruments. The policy instruments

---

[23] When the regression was run separately on CPA_{t-1} and CPU_{t-1} their coefficients were nearly identical:

$$CPU = -\underset{(2.36)}{61.298} + \underset{(27.87)}{.814}\,(CPB - TC) - \underset{(6.09)}{.480}\,(CPB - TC)_{-1} + \underset{(3.83)}{.433}\,CPU_{-1}$$

OLS: $\bar{R}^2 = .998$ D/W = 1.59

$$CPU = -\underset{(2.35)}{61.086} + \underset{(27.21)}{.811}\,(CPB - TC) - \underset{(6.01)}{.476}\,(CPB - TC)_{-1} + \underset{(3.82)}{.433}\,CPU_{-1}$$

SOIV: $\bar{R}^2 = .998$ D/W = 1.61

related to personal and corporation income taxes are the statutory tax rates. Therefore, we have incorporated the statutory rates of these two taxes into their equations. Unlike most of the other fitted equations, tax equations are, by their nature, fundamentally institutional relationships which are determined by the relevant regulations. Thus, the specification of tax equations is rather straightforward; it involves simply incorporating the complexities of the tax structure into manageable equation forms without losing its essential nature.

(i) Personal Income Tax[24] As mentioned above, personal income tax receipts are recorded in the national accounts on a collection basis. Relating taxes collected to personal income involves four conceptual steps. The first step is to relate personal income as recorded in the national accounts to income as defined for tax purposes because there are a number of differences between the two definitions of income. For example, most of the transfer payments to persons are not considered to be income for tax purposes but are included in personal income in the national accounts. The second step is to relate income assessed for tax purposes to taxable income by deducting all exemptions and deductions. The third step is to determine the accrual of tax liability on the basis of taxable income times applicable tax rates. The final step is to relate tax collections to tax accruals.

Let us consider the derivation of appropriate tax rates from the numerous statutory rates. To keep the model simple, we derived a weighted

---

[24] Our equations for personal and corporation income taxes are based on the approach developed by the Bank of Canada for the RDX1-Model; J. F. Helliwell, et.al, The Structure of RDX1, pp. 18-20.

overall average rate which applies to taxable income. Such a weighted rate can be derived in the following way. First, taxable income is disaggregated into a manageable number of classes. Secondly, mean taxable income in each class is found by dividing taxable income by the number of taxable returns in each class. The current statutory rate on the mean taxable income in each class is then regarded as the average class rate. Finally, the average class rate is weighted by the ratio of taxable income in each class to total taxable income, and summed over all classes to obtain a weighted overall average rate.[25] Thus,

$$(4.11) \qquad TPR_t = \sum_{i=1}^{n} CR_{it} \frac{YPT_{it}}{YPT_t}$$

where TPR_t = weighted average personal income tax rate in year t

CR_{it} = statutory rate applicable to mean taxable income in class i, year t

YPT_{it} = taxable income in class i, year t

YPT_t = total taxable income in year t

Therefore, a weighted average rate calculated by the above procedure takes explicitly into account the statutory rates. The weights used are regarded as exogenous because they would be determined entirely by income distribution which is beyond the scope of this study.

Based on basically the same procedure as above, the Bank of Canada has already calculated a weighted overall average rate which will

---

[25]Data required for the method described above are available in the Department of National Revenue, <u>Taxation Statistics</u>, (Ottawa: The Department of National Revenue).

be used for our personal income tax model.[26] The weighted average rate by the Bank of Canada does not include provincial levies in excess of the standard federal rates. The amount would not be great, however, because provincial rates over the standard federal rates are in most provinces not significantly high.[27] The weighted average rates estimated by the Bank of Canada are as shown in Table 4.1

Table 4.1

Weighted Average Personal Income Tax Rate

Year	Rate	Year	Rate	Year	Rate
1950	17.3	1957	17.4	1964	19.1
1951	19.0	1958	16.3	1965	18.7
1952	21.1	1959	16.9	1966	18.6
1953	20.3	1960	17.7	1967	20.1
1954	19.2	1961	17.8	1968	21.0
1955	18.3	1962	17.9	1969	23.4*
1956	17.4	1963	18.0		

*our estimates

We have described four steps involved in a personal income tax model. To keep the model simple, however, the first step - relating personal income in the national accounts to personal income assessed for tax purposes - will not be taken into account. Most of the elements, except unemployment insurance benefits, explaining the difference between

[26] J. H. Helliwell, R. G. Evans, F. W. Gorbet, R. F. S. Jarrett, and D. R. Stephenson, Government Sector Equations for Macroeconomic Models, (Ottawa: The Bank of Canada, 1969), p. 144.

[27] See Canadian Tax Foundation, The National Finances: 1970-71, (Toronto: Canadian Tax Foundation, 1970), pp. 39-43.

the two concepts of income would have a more or less steady time trend. Similarly, viewing total exemptions and deductions during our sample period indicates that they also have steady time trends.[28] Hence, we related taxable income (YPT) directly to personal income from the national accounts less unemployment insurance benefits (YP - UIB), and a time trend, as shown by the equation (E.12).[29]

We then estimated a tax accrual equation (E.13) relating tax accruals (TPAC) to the product of taxable income (YPT) and the weighted average personal income tax rate (TPR). This equation is estimated without a constant term, and the coefficient (.931) is close to one, as hoped for. Finally, tax collections (TP) are related to tax accruals, as shown by equation (E.14). As a whole, the personal income tax model developed here, consisting of three equations, seems to be satisfactory.

(ii) Corporation Income Tax Corporation income tax is recorded in the national accounts on an accrual basis. Hence, a derivation of corporation income tax requires two steps. The first step is to generate

---

[28]See the Department of National Revenue, Taxation Statistics: 1971, (Ottawa: The Department of National Revenue, 1971), p. 152.

[29]The results for the first and the second steps explained above are as follows:

$$YPAS = -\underset{(79.51)}{4238.304} + \underset{(43.22)}{.836}\ YP - \underset{(2.20)}{97.125}\ TIME \qquad \bar{R}^2 = .999 \quad D/W = 1.486$$

$$YPT = -\underset{(20.49)}{2974.600} + \underset{(60.94)}{.846}\ YPAS - \underset{(11.10)}{280.266}\ TIME \qquad \bar{R}^2 = .999 \quad D/W = .926$$

where YPAS is personal income assessed for personal income tax, and TIME is a time trend equal to 1 in 1950, 2 in 1951, etc.

taxable corporation profits from corporation profits before taxes in the national accounts. The second step is to relate tax accruals to the product of taxable corporation profits and an appropriate tax rate. The second step requires a weighted tax rate. A weighted rate is easier to calculate for corporation income tax than for personal income tax because there are only two marginal corporation income tax rates on the federal level and a single rate on the provincial level. For instance, the corporation income tax rates for 1969 were 21.54% on the first \$35,000 of taxable profits and 51.41% on that portion in excess of \$35,000, both rates including a 3% levy for the Old Age Security Tax and a 3% surtax rate. Difficulties in deriving a weighted average rate arise because some corporations pay the tax at the low rate on some portion of their profits and at the high rate on the rest. Thus, a weighted average high rate (WHR) applicable to these firms paying taxes at both rates is needed. This weighted average high rate (WHR) can be expressed as:[30]

$$(4.12) \qquad WHR = \frac{LR(LRM)(NHRF) + HR\{CPTH - (LRM)(NHRF)\}}{CPTH}$$

where LR = the low tax rate

HR = the high tax rate

LRM = the cut-off line for the two rates which is, in effect, allowable maximum profits per firm taxable at the low rate

NHRF = the number of firms that pay tax at both rates

---

[30] J. F. Helliwell, et.al, Government Sector Equations, pp. 148-153.

CPTH = taxable corporation profits of firms that pay at the high rate

The first term in the numerator indicates the tax payable at the lower rate by the firms whose taxable profits are more than the cut-off line and the second term indicates the tax payable on the portion of taxable profits applicable to the high rate. Given the weighted average high rate (WHR), the weighted average federal rate is:

$$(4.13) \qquad TCRF = \frac{WHR(CPTH) + LR(CPTL)}{CPT}$$

where TCRF = overall weighted average federal corporation income tax rate

CPTL = taxable corporation profits of firms that pay at the low rate

CPT = total taxable corporation profits; CPT = CPTL + CPTH

The first term in the numerator indicates the tax payable by firms that pay tax at both rates, and the second term indicates the tax payable by firms that pay only at the low rate. Substituting (4.12) into (4.13) yields

$$(4.14) \qquad TCRF = \frac{(HR)(CPTH) + LR(CPTL)}{CPT} - \frac{(HR - LR)(LRM)(NHRF)}{CPT}$$

In order to get a weighted average rate for both federal and provincial taxes we must add a weighted provincial rate for any provincial levies that exceed allowable provincial tax credits.[31] A weighted provincial excess can be found by weighing the provincial excess by the proportion of total taxable corporation profits in each of the respective provinces. This is added to (4.14) in order to arrive at a weighted average (federal and provincial) rate.

No data are available for the separation of taxable corporation profits between high-rate (CPTH) and low-rate (CPTL) firms. Therefore, we used the estimates of CPTH and CPTL made by the Bank of Canada for the period 1952-69. We also used the weighted provincial excess (WPE) estimated by the Bank. The rest of the data required for deriving (4.14) are available, from which, together with the estimates of CPTH, CPTL, and WPE, we calculated a weighted average (federal and provincial) corporation income tax rate (TCR) for the period 1952-69, as shown in Table 4.2.

Table 4.2

Weighted Average Corporation Income Tax Rate

Year	Rate	Year	Rate	Year	Rate
1952	50.7	1958	43.3	1964	46.2
1953	45.7	1959	46.4	1965	46.2
1954	45.2	1960	45.8	1966	45.8
1955	43.0	1961	45.7	1967	45.3
1956	42.6	1962	45.9	1968	46.8
1957	43.1	1963	46.2	1969	46.7

Once the weighted average corporation income tax rate was calculated, the rest of the procedure for the corporation income tax model

[31]For provincial levies, see relevant issues, Canadian Tax Foundation, The National Finances, (Toronto: Canadian Tax Foundation).

was rather straightforward, as in the case of the personal income tax model. First, we found the relationship between taxable corporation profits and corporation profits before taxes (E.15), and then corporation income tax accruals (TC) were regressed against the product of taxable corporation profits and the weighted average tax rate derived above. The estimated equations are quite satisfactory. The estimated coefficient of the equation for corporation income tax accruals is not significantly different from one, as hoped for.

(iii) Indirect Taxes Indirect taxes include many different types, making it virtually impossible to derive a composite statutory rate. A large part of indirect tax revenue, other than custom import duties, is related to sales. Thus, sales and imports were chosen as tax bases. Because of high serial correlation, the indirect tax equation (E.17) was estimated in terms of annual first differences.

(iv) Unemployment Insurance Benefits . Out of the numerous transfer payment programs, only unemployment insurance benefits are treated as endogenous. With regard to the other transfer payments, some of them are of little importance in scope, while some depend mainly on variables which are exogenous in the present model, such as demographic variables. Hence, these transfer payments can be treated as exogenous, without doing much harm.

Unemployment insurance benefit payments depend on the number of benefit recipients and the benefit rate at which each recipient is paid. The benefit rate is determined by the recipient's income, up to a maximum level, and by whether or not he has dependents. Because a consistently high proportion of recipients receive benefit payments at the maximum rate, this maximum rate was chosen to represent the whole structure. Furthermore, it has been found that the ratio of recipients with dependents to single recipients is quite stable at 53:47.[32] Using this ratio as the weight, the annual composite maximum rate of benefit payments was derived, as shown in Table 4.3.

Table 4.3

The Composite Maximum Rate of Unemployment Insurance Benefit Payments

Year	Rate	Year	Rate	Year	Rate
1950	17.61	1957	26.71	1964	31.77
1951	18.74	1958	26.71	1965	31.77
1952	19.75	1959	27.98	1966	31.77
1953	20.76	1960	31.77	1967	31.77
1954	20.76	1961	31.77	1968	39.80
1955	22.25	1962	31.77	1969	47.83
1956	26.71	1963	31.77		

On the other hand, the number of benefit recipients depends more or less on the number of persons unemployed. Thus, the number of unemployed and the annual composite maximum rate of benefit payments were chosen as the two explanatory variables in the estimated unemployment insurance benefit equation (E.18).

In addition to the 18 fitted equations which have been discussed thus far, there are 8 identities (E.19 - E.26). They are required either for closing the model or for defining variables. As a result, the model consists of 18 stochastic equations and 8 identities; and it contains 26 current endogenous variables, 23 current exogenous variables, (including

[32] J. F. Helliwell, et.al, Government Sector Equations, p. 93

the constant term), 15 one-year lagged endogenous variables, and 7 one-year lagged exogenous variables, the lags being at most one year.

Since the model contains ratios of endogenous variables, the entire system was solved by a modified Gauss-Siedel method for the period 1954-69. As a set of starting values of endogenous variables for the first iteration, the actual values of 1954 were used. As an evaluation criterion of the model's performance, Theil's inequality coefficients for the 26 endogenous variables were computed, as shown in Table 4.4.

All of the computed values of the coefficients are less than .08, except for non-farm business inventory changes. This would indicate that the model's operation is satisfactory at least for our purposes. Having now specified our model, together with a discussion of its special features, we will derive the constraints in Chapter V.

Table 4.4

Theil's Inequality Coefficients for Sample Period Controlled Solution

Variables	Coefficients	Variables	Coefficients
CE	.011	TP	.019
ING	.033	CPT	.021
JNFB	.298	TC	.026
M	.023	TI	.012
NEPW	.005	UIB	.076
WR	.009	YGNE	.010
Py	.009	YP	.009
Pc	.008	YD	.009
Pi	.014	O	.010
CPB	.033	ONG	.010
CPU	.063	ONFB	.011
YPT	.018	UL	.075
TPAC	.023	JST	.034

3. Structural Equations[33]

Consumer Expenditure

(E.1)

$$CE/Pc = \underset{(5.90)}{19.1228} + \underset{(7.08)}{.7286}\,(YD - YPF)/Pc + \underset{(1.57)}{.1907}\,(CE/Pc)_{-1} + u(ce)$$

$$ \underset{(5.71)}{18.6530} + \underset{(6.82)}{.7104} \qquad\qquad + \underset{(1.73)}{.2122}$$

O.L.S. $\bar{R}^2 = .9991$, D/W = 1.399 S.O.I.V. $\bar{R}^2 = .9991$, D/W = 1.432

Business Fixed Capital Formation

(E.2)

$$ING/Pi = \underset{(2.68)}{10.1915} + \underset{(3.71)}{.3510}\,\Delta(ONG/Py) + \underset{(2.88)}{.5044}\,\{(CCANG + CPU)/Pi\}_{-1}$$

$$ \underset{(2.76)}{10.5314} + \underset{(3.50)}{.3342} \qquad\qquad + \underset{(2.98)}{.5242}$$

$$- \underset{(2.14)}{119.1785}\,(UL/N)_{-1} - \underset{(2.07)}{3.2767}\,RL_{-1} + \underset{(6.29)}{.6947}\,(ING/Pi)_{-1} + u(i)$$

$$- \underset{(2.19)}{112.1418} \qquad - \underset{(2.11)}{3.3592} \qquad + \underset{(6.19)}{.6853}$$

O.L.S. $\bar{R}^2 = .9852$, D/W = 1.809 S.O.I.V. $\bar{R}^2 = .9852$, D/W = 1.802

---

[33]Following the conventional notations, $\bar{R}^2$ and D/W denote, respectively, the coefficient of determination corrected for degrees of freedom and the Durbin-Watson statistic. The figure in parentheses below each estimated coefficient indicates the value of the t-statistic. The u symbols accompanied with letters in parentheses indicate the disturbance terms. The coefficients estimated by the O.L.S. method appear above, and those by the S.O.I.V. method appear below.

Change in Non-Farm Business Inventory

$$
\begin{array}{llllll}
\text{(E.3)} & JNFB/Py = & -\ 21.1020 & +\ .1208\ (ONFB/Py - JNFB/Py)_{-1} & -\ .5967\ JST_{-1} \\
 & & (3.83) & (4.37) & (4.44) \\
 & & -\ 21.3013 & +\ .1217 & -\ .6034 \\
 & & (3.86) & (4.40) & (4.48)
\end{array}
$$

$$
\begin{array}{lll}
+\ .3893\ \Delta(M/Pm) & -\ 78.6216\ \Delta(UL/N) & +\ u(j) \\
(3.70) & (1.44) & \\
+\ .4034 & -\ 69.2906 & \\
(3.89) & (1.26) &
\end{array}
$$

O.L.S. $\bar{R}^2 = .7982$ S.O.I.V. $\bar{R}^2 = .7978$

$D/W = 2.463$ $D/W = 2.449$

Imports

$$
\begin{array}{llllll}
\text{(E.4)} & M/Pm = & 6.5609 & +\ .1118\ (O/Py) & +\ .4290\ (Py - Pm) & -\ 276.5404\ (UL/N) \\
 & & (1.16) & (3.37) & (1.24) & (4.46) \\
 & & 6.5348 & +\ .1115 & +\ .4279 & -\ 275.8841 \\
 & & (1.15) & (3.35) & (1.24) & (4.43)
\end{array}
$$

$$
\begin{array}{ll}
+\ .6395\ (M/Pm)_{-1} & +\ u(m) \\
(4.04) & \\
+\ .6409 & \\
(4.05) &
\end{array}
$$

O.L.S. $\bar{R}^2 = .9869$ S.O.I.V. $\bar{R}^2 = .9869$

$D/W = 1.892$ $D/W = 1.896$

Demand for Paid Workers in the Private Sector

$$
\begin{array}{llllll}
\text{(E.5)} & NEPW = & 1741.0149 & +\ 6.7730\ (ONG/Py) & -\ 64423.472\ (WR/Py) & +\ .3498\ NEPW_{-1} \\
 & & (8.84) & (9.63) & (3.41) & (3.55) \\
 & & 1758.0920 & +\ 6.8480 & -\ 64733.987 & +\ .3411 \\
 & & (8.79) & (9.48) & (3.43) & (3.40)
\end{array}
$$

$$+\ u(nepw)$$

O.L.S. $\bar{R}^2 = .9988$ S.O.I.V. $\bar{R}^2 = .9988$

$D/W = 1.554$ $D/W = 1.526$

Wage Rate (Hourly)

$$
\begin{array}{llllll}
\text{(E.6)} \quad \Delta WR = & -\ .0531 & +\ .0123\ \Delta Pc & +\ .0022\ (N/UL)_{-1} & +\ .00002\ (CPB - TC)_{-1} & +\ u(wr) \\
 & (2.39) & (4.60) & (3.20) & (5.20) & \\
 & & & & & \\
 & -\ .0531 & +\ .0123 & +\ .0022 & +\ .00002 & \\
 & (2.39) & (4.59) & (3.20) & (5.21) &
\end{array}
$$

O.L.S. $\bar{R}^2 = .8321$ S.O.I.V. $\bar{R}^2 = .8321$
D/W = 1.358 D/W = 1.356

The GNE Price Deflator

$$
\begin{array}{llllll}
\text{(E.7)} \quad \Delta Py = & -\ 2.3680 & +\ 27.1290\ \Delta WR & +\ .2217\ \Delta Pm & +\ .0991\ (N/UL) & +\ u(py) \\
 & (3.01) & (5.37) & (2.72) & (2.96) & \\
 & & & & & \\
 & -\ 2.4583 & +\ 26.5724 & +\ .2222 & +\ .1052 & \\
 & (3.12) & (5.22) & (2.73) & (3.13) &
\end{array}
$$

O.L.S. $\bar{R}^2 = .7880$ S.O.I.V. $\bar{R}^2 = .7875$
D/W = 1.826 D/W = 1.847

Price Deflator for Consumer Expenditure

$$
\begin{array}{ll}
\text{(E.8)} \quad \Delta Pc = & .8320\ \Delta Py + u(pc) \\
 & (9.60) \\
 & \\
 & .8319 \\
 & (15.87)
\end{array}
$$

O.L.S. $\bar{R}^2 = .8429$ S.O.I.V. $\bar{R}^2 = .8428$
D/W = 1.863 D/W = 1.863

The Price Deflator for Business Fixed Capital Formation

$$
\begin{array}{llll}
\text{(E.9)} \quad \Delta Pi = & .9681\ \Delta Py & +\ 72.5288\ \Delta(ING/O) & +\ u(pi) \\
 & (16.41) & (4.13) & \\
 & & & \\
 & .9688 & +\ 73.7909 & \\
 & (16.42) & (4.19) &
\end{array}
$$

O.L.S. $\bar{R}^2 = .8692$ S.O.I.V. $\bar{R}^2 = .8691$
D/W = 1.615 D/W = 1.605

Corporation Profits before Taxes

(E.10)
$$CPB = \underset{(1.71)}{278.8701} + \underset{(3.00)}{.0559}\,(ONFB - JNFB) + \underset{(2.92)}{.5183}\,CPB_{-1} - \underset{(4.60)}{24659.344}\,\Delta(UL/N)$$
$$\underset{(1.71)}{278.7359} + \underset{(2.99)}{.0558} \qquad + \underset{(2.92)}{.5190} \qquad - \underset{(4.59)}{24697.112}$$
$$+\ u(cpb)$$

O.L.S. $\bar{R}^2 = .9830$, D/W = 2.008 S.O.I.V. $\bar{R}^2 = .9830$, D/W = 2.011

Undistributed Corporation Profits

(E.11)
$$CPU = -\underset{(2.22)}{49.3787} + \underset{(32.80)}{.7995}\,(CPB - TC) - \underset{(7.58)}{.5154}\,(CPB - TC - CPU)_{-1} + u(cpu)$$
$$-\underset{(1.76)}{42.6631} + \underset{(25.96)}{.8125} \qquad - \underset{(6.27)}{.5534}$$

O.L.S. $\bar{R}^2 = .9982$, D/W = 1.701 S.O.I.V. $\bar{R}^2 = .9981$, D/W = 1.609

Taxable Personal Income

(E.12)
$$YPT = -\underset{(34.14)}{6417.7611} + \underset{(41.76)}{.7021}\,(YP - UIB) - \underset{(8.84)}{337.7091}\,TIME + u(ypt)$$
$$-\underset{(28.65)}{6335.8702} + \underset{(31.16)}{.6917} \qquad - \underset{(6.36)}{315.2139}$$

O.L.S. $\bar{R}^2 = .9986$, D/W = 1.744 S.O.I.V. $\bar{R}^2 = .9985$, D/W = 1.674

Personal Income Tax Accrued

(E.13)
$$\Delta TPAC = \underset{(43.74)}{.9313}\,\Delta(.01\ YPT.TPR) + u(tpac)$$
$$\underset{(43.59)}{.9292}$$

O.L.S. $\bar{R}^2 = .9839$, D/W = 1.874 S.O.I.V. $\bar{R}^2 = .9839$, D/W = 1.879

Personal Income Tax Collected

(E.14) $TP = -45.6773 + 1.1615\ TPAC + u(tp)$
$(1.75) \quad (118.60)$

$-45.8724 + 1.1616$
$(1.75) \quad (118.24)$

O.L.S. $\bar{R}^2 = .9987$ $D/W = 1.508$ S.O.I.V. $\bar{R}^2 = .9987$ $D/W = 1.509$

Taxable Corporation Profits

(E.15) $CPT = 887.1464 + .6198\ CPB + u(cpt)$
$(8.48) \quad (28.72)$

$862.3180 + .6253$
$(7.95) \quad (27.81)$

O.L.S. $\bar{R}^2 = .9775$ $D/W = .866$ S.O.I.V. $\bar{R}^2 = .9774$ $D/W = .860$

Corporation Income Tax Accrued

(E.16) $TC = 1.0441\ (.01\ CPT.TCR) + u(tc)$
(148.54)

1.0441
(148.54)

O.L.S. $\bar{R}^2 = .9908$ $D/W = 1.760$ S.O.I.V. $\bar{R}^2 = .9908$ $D/W = 1.760$

Indirect Tax

(E.17) $\Delta TI = 67.7995 + .0770\ \Delta(O - JNFB) + .2195\ \Delta M + u(ti)$
$(1.33) \quad (3.47) \quad (4.34)$

$67.9879 + .0764 + .2214$
$(1.33) \quad (3.44) \quad (4.36)$

O.L.S. $\bar{R}^2 = .8917$ $D/W = 1.645$ S.O.I.V. $\bar{R}^2 = .8917$ $D/W = 1.641$

Unemployment Insurance Benefits

(E.18) $\Delta UIB = 3.9593 + 1.0494\ \Delta UL + 3.9409\ \Delta UIBR + u(uib)$

$\qquad\qquad\quad (.77) \qquad (14.83) \qquad\qquad (2.33)$

$\qquad\qquad\quad 3.9750 + 1.0474 \qquad\qquad + 3.9438$

$\qquad\qquad\quad (.78) \qquad (14.74) \qquad\qquad (2.33)$

O.L.S. $\bar{R}^2 = .9269$ $\quad$ D/W = 1.763

S.O.I.V. $\bar{R}^2 = .9269$ $\quad$ D/W = 1.766

Identities

(E.19) YGNE = CE + ING + JNFB + JFG + GE + XPT - M + YRE

(E.20) YP = YGNE - CCA - TI - TC - CPU + TR + UIB + YPRE

(E.21) YD = YP - TP - TPO

(E.22) O = YGNE - YF

(E.23) ONG = O - GWCCA

(E.24) ONFB = ONG - WF

(E.25) UL = N - NEPW - NEG - NES

(E.26) $JST = JST_{-1} + JNFB/Py$

The Glossary of Symbols*

Symbol	Description
CCA	Capital consumption allowances and miscellaneous valuation adjustments, millions of current dollars.
CCANG	Non-government capital consumption allowances and miscellaneous valuation adjustments, millions of current dollars.
*CE	Personal expenditure on consumer goods and services, millions of current dollars.
*CPB	Corporation profits before taxes, millions of current dollars.
*CPT	Taxable corporation profits, millions of current dollars.
*CPU	Undistributed corporation profits, millions of current dollars.
GE	Government expenditure on goods and services including government gross fixed capital formation, millions of current dollars.
GWCCA	Government wage payments and capital consumption allowances, millions of current dollars.
*ING	Business gross fixed capital formation, millions of current dollars.
JFG	Change in inventories in agriculture and government, millions of current dollars.
*JNFB	Change in non-farm business inventories, millions of current dollars.
*JST	Stock of non-farm business inventories at beginning of year, millions of 1961 dollars; 1961 = 100.0. For the way of constructing JST, see the text on page 48.
*M	Imports of goods and services, millions of current dollars.
N	Civilian labor force, thousands of persons.

*A variable with an asterisk is endogenous variable to the system.

NEG	Government civilian employees, thousands of persons.
*NEPW	Paid workers in the private sector, thousands of persons.
NES	Employers and self-employed in the private sector, thousands of persons.
*O	Gross National Product (GNP) less accrued net income of farm operators from farm production (YF), millions of current dollars.
*ONFB	Non-farm business output (see equation 24), millions of current dollars.
*ONG	Total business output (see equation 23), millions of current dollars.
*Pc	Price deflator for consumption expenditure; 1961 = 100.0.
*Pi	Price deflator for business gross fixed capital formation; 1961 = 100.0.
Pm	Price deflator for imports; 1961 = 100.0.
*Py	Price deflator for Gross National Expenditure; 1961 = 100.0.
RL	Long-term average yield of Government of Canada bonds; 10 years and over, percentage.
*TC	Corporation income tax accrued, millions of current dollars.
TCR	Weighted corporation income tax rate, percentage.
*TI	Indirect taxes collected, millions of current dollars.
TIME	Time trend; equals 1 in 1950, 2 in 1951, etc.
*TP	Personal income tax collected, millions of current dollars.
*TPAC	Personal Income tax accrued, millions of current dollars.
TPO	Other personal direct taxes collected, millions of current dollars.
TR	Transfer payments to persons less unemployment insurance benefits, millions of current dollars.
u(ce)	Disturbance term in consumption function; similarly for other stochastic equations with relevant letters in parentheses.
*UIB	Unemployment insurance benefits, millions of current dollars.

UIBR	Average maximum rate of unemployment insurance benefits, current dollars.
*UL	Number of persons unemployed, thousands of persons.
WF	Wage payments to paid workers in the agricultural sector, millions of current dollars.
*WR	Average annual hourly wage rate in manufacturing, current dollars.
XPT	Exports of goods and services, millions of current dollars.
*YD	Personal disposable income, millions of current dollars.
YF	Accrued net income of farm operators from farm production, millions of current dollars.
*YGNE	Gross national expenditure, millions of current dollars.
*YP	Personal income, millions of current dollars.
YPF	Net income received by farm operators from farm production, millions of current dollars.
YPRE	Miscellaneous elements of personal income, millions of current dollars (see equation E.20).
*YPT	Taxable personal income, millions of current dollars.
YRE	Residual error of estimate of gross national expenditure, millions of current dollars.

CHAPTER V

DERIVATION OF THE CONSTRAINTS

1. Linearization of a Non-Linear System

Recall that the constraints (2.1) are $y = Rx + Hs + Ju$. This can be written for the three-year period as

$$(5.1)\qquad \begin{bmatrix} y_t \\ y_{t+1} \\ y_{t+2} \end{bmatrix} = \begin{bmatrix} R_1 & 0 & 0 \\ R_2 & R_1 & 0 \\ R_3 & R_2 & R_1 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t+1} \\ x_{t+2} \end{bmatrix} + \begin{bmatrix} H_1 & 0 & 0 \\ H_2 & H_1 & 0 \\ H_3 & H_2 & H_1 \end{bmatrix} \begin{bmatrix} s_t \\ s_{t+1} \\ s_{t+2} \end{bmatrix} + \begin{bmatrix} J_1 & 0 & 0 \\ J_2 & J_1 & 0 \\ J_3 & J_2 & J_1 \end{bmatrix} \begin{bmatrix} u_t \\ u_{t+1} \\ u_{t+2} \end{bmatrix}$$

where the symbols are as explained in Chapter II. In the present study, each subvector (y_t) of y consists of two target variables: the changes in the GNE price deflator and in the number of persons unemployed, and each subvector (x_t) of x consists of three instruments: the changes in government expenditure on goods and services, in the personal income tax rate, and in the corporation income tax rate. Thus

$$(5.2)\qquad y_t = \begin{bmatrix} \Delta Py_t \\ \Delta UL_t \end{bmatrix} \qquad x_t = \begin{bmatrix} \Delta GE_t \\ \Delta TPR_t \\ \Delta TCR_t \end{bmatrix} \qquad t = 1, 2, 3.$$

where the symbols are as explained in Chapter IV.

We then note that the equation system which was specified in

Chapter IV is not in the required form, as shown by the constraints (2.1), i.e., it does not have all of the target variables on the left-hand side and the instruments, other predetermined variables, and disturbance terms on the right-hand side. Moreover, some of the structural equations contain ratios of variables, whereas the constraints are required to be linear in the original variables. Hence, we need two remedies: linearization and derivation of reduced forms. This can be introduced in the following way.

Suppose a non-linear structural system is written as

$$f(z, g) = v \tag{5.3}$$

where f is a matrix of functional operators, and z, g, and v are vector of endogenous variables, predetermined variables, and disturbance terms respectively. By totally differentiating (5.3) and rearranging terms, we get

$$dz = -\left(\frac{\partial f}{\partial z}\right)^{-1}\left(\frac{\partial f}{\partial g}\right)dg + \left(\frac{\partial f}{\partial z}\right)^{-1}dv \tag{5.4}$$

In the non-linear case, the partial derivatives in (5.4) will not be constants; however, when they are evaluated at some point (z_o, g_o), they become a set of constants, so that (5.4) is a linearized version of the reduced-form of the system (5.3). Since we are concerned with year-to-year changes in the variables, the relation (5.4), after the partial derivatives have been evaluated, can be written approximately in the form of first differences as

$$\Delta z = \pi\Delta g + \theta\Delta v \tag{5.5}$$

where π and θ are matrices of relevant orders, whose elements are all constant, and the sign (Δ) indicates first differences.

In this linearized version of the reduced-form derived from a non-linear system, the response of an endogenous variable to changes in the predetermined variables depends on the point at which the partial derivatives are evaluated. Hence, the reduced-form coefficients thus derived are, strictly speaking, valid only for small changes about the evaluation point. This implies that when we apply the reduced-form coefficients thus computed to the strategy period, the closer the point of evaluation is to that of the strategy period, the more valid the coefficient will be. Therefore, we chose the evaluation point corresponding to the values of the variables in 1966.[1]

2. The Final Form of the Constraints: 1967-1969

Since our model contains one-year lagged endogenous and exogenous variables, the linear reduced-form equations derived as above can typically be written as

$$(5.6) \qquad p_t = Lp_{t-1} + Fq_t + Gq_{t-1} + Su_t$$

where L, F, G, and S are matrices of the reduced form coefficients of relevant order, p_t and p_{t-1} are vectors of current and one-year lagged endogenous variables respectively, q_t and q_{t-1} are vectors of current and one-year lagged exogenous variables respectively, u_t is a vector of

---

[1]This idea of deriving the reduced form coefficients from a non-linear system has been used by A. S. Goldberger, Impact Multipliers and Dynamic Properties of the Klein-Goldberger Model, (Amsterdam: North-Holland Publishing Co., 1959), pp. 14-20.

structural disturbance terms, and all the variables are in the form of first differences. It will be noted that the linear reduced-form of our model was derived with a condensed version of the model, in which the number of equations was reduced from twenty-six to twenty-one. This was done by combining six identities (E.19 - E. 24). As a result, p_t, p_{t-1}, q_t, q_{t-1}, and u_t are, respectively, vectors of order 21 x 1, 15 x 1, 23 x 1, 7 x 1, and 21 x 1 (three of the twenty-one elements of u_t being zero); and the matrices L, F, G, and S are, respectively, of order 21 x 15, 21 x 23, 21 x 7, and 21 x 21; and their numerical values are shown in Table 5.2, 5.3, 5.4, and 5.5 respectively, at the end of this chapter.

In the case of a three-year strategy, we must know the effects of the instruments on the target variables not only in the same year but also in the next year and two years later. The coefficient matrices L and F in (5.6) represent the effect on the current year's endogenous variables of the previous year's endogenous variables and the current year's exogenous variables, respectively. It is not true, however, that G measures the effects of the previous year's exogenous variables on the current year's endogenous variables. This is because q_{t-1} affects p_t not only directly, as indicated by G, but also indirectly through p_{t-1}. In other words, although the matrix of coefficients F represents the effect of the exogenous variables on the endogenous variables in the same year, it does not specify explicitly the effect one year and two years later. This problem can be solved in the following way.

The estimated coefficients of the model are assumed to be time-invariant. This implies that the relation (5.6) holds over time.

Hence, we can substitute a one-year lagged version of (5.6) for p_{t-1} in (5.6), and by repeating this process r times, we get[2]

$$(5.7)\quad p_t = L^{r+1}p_{t-r-1} + Fq_t + \sum_{i=1}^{r} L^{i-1}(LF + G)q_{t-i} + L^{r}Gq_{t-r-1} + \sum_{i=0}^{r} L^{i}Su_{t-i}$$

From (5.7), we can easily derive the equation system for a three-year period as

$$(5.8)\quad \begin{bmatrix} p_t \\ p_{t+1} \\ p_{t+2} \end{bmatrix} = \begin{bmatrix} F & 0 & 0 \\ F_2 & F & 0 \\ F_3 & F_2 & F \end{bmatrix} \begin{bmatrix} q_t \\ q_{t+1} \\ q_{t+2} \end{bmatrix} + \begin{bmatrix} G \\ LG \\ L^2G \end{bmatrix} q_{t-1} + \begin{bmatrix} L \\ L^2 \\ L^3 \end{bmatrix} p_{t-1} + \begin{bmatrix} S & 0 & 0 \\ LS & S & 0 \\ L^2S & LS & S \end{bmatrix} \begin{bmatrix} u_t \\ u_{t+1} \\ u_{t+2} \end{bmatrix}$$

where F, $F_2 = (LF + G)$, and $F_3 = L(LF + G)$ specify the effects on the changes in the endogenous variables of the changes in the exogenous variables, respectively, of the same year, one year, and two years later; the matrices S, LS and L^2S will be similarly interpreted with respect to the disturbance terms; finally, the coefficient matrices for q_{t-1} (G, LG, and L^2G) and for p_{t-1} (L, L^2 and L^3) represent the effects of the changes in the one-year lagged exogenous and endogenous variables, respectively, in the current year, one year, and two years later.

We are concerned with the reduced-form equations for the two target variables. Hence, the constraints consist of only six rows of

---

[2] If the matrix L is such that the limit of L^r for $r \to \infty$ is a zero matrix, the first and fourth terms on the right-hand side of (5.7) are eliminated, and the remaining form is called the "final form" of the equation system. H. Theil and J. C. G. Boot, "The Final Form of Econometric Equation System", in A. Zellner, ed., Readings in Economic Statistics and Econometrics, pp. 611-630.

(5.8), two for each year, corresponding to the two target variables. The current exogenous variables contains three instruments. Hence, the matrix R_1 in (5.1) is a 2 x 3 submatrix of the matrix F corresponding to the three instruments in the relevant two rows of p_t. Similarly, R_2 and R_3 in (5.1) are 2 x 3 submatrices of the F_2 and F_3 matrices respectively. Finally, the final form of the constraints for the three-year strategy (1967-1969) may be written as[3]

$$(5.9) \qquad y = Rx + H_1\hat{q} + H_2q_6 + H_3p_6 + Ju$$

where

$$y = \begin{bmatrix} y_7 \\ y_8 \\ y_9 \end{bmatrix} \quad x = \begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} \quad \hat{q} = \begin{bmatrix} \hat{q}_7 \\ \hat{q}_8 \\ \hat{q}_9 \end{bmatrix} \quad u = \begin{bmatrix} u_7 \\ u_8 \\ u_9 \end{bmatrix} \quad R = \begin{bmatrix} R_1 & 0 & 0 \\ R_2 & R_1 & 0 \\ R_3 & R_2 & R_1 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} \hat{F} & 0 & 0 \\ \hat{F}_2 & \hat{F} & 0 \\ \hat{F}_3 & \hat{F}_2 & \hat{F} \end{bmatrix} \quad H_2 = \begin{bmatrix} \bar{G} \\ \bar{L}\bar{G} \\ \bar{L}^2\bar{G} \end{bmatrix} \quad H_3 = \begin{bmatrix} \bar{L} \\ \bar{L}^2 \\ \bar{L}^3 \end{bmatrix} \quad J = \begin{bmatrix} J_1 & 0 & 0 \\ J_2 & J_1 & 0 \\ J_3 & J_2 & J_1 \end{bmatrix}$$

$\hat{q}$ represents twenty non-instrument exogenous variables for three years (1967-69); the elements $\hat{F}$, $\hat{F}_2$, and $\hat{F}_3$ of matrix H_1 represent the two relevant rows, respectively, of F, F_2, and F_3 in (5.8) for non-instrument exogenous variables; the elements of matrices H_2 and H_3 with bars above them represent the two relevant rows of corresponding matrices in (5.8);

---

[3]We can see that Hs in (2.1) corresponds to the sum of $H_1\hat{q}$, H_2q_6 and H_3p_6 in (5.9).

and the subscripts 6, 7, 8, and 9 represent, respectively, the years 1966, 1967, 1968 and 1969.[4]

3. Numerical Specification of the Constraints for the First-Year Optimal Strategy

For the three-year strategy period (1967-69), the coefficients of R_1, R_2, and R_3 were computed, as shown in Table 5.1, and these, in turn, were graphed in Figure 5.1. The effect of an increase in

Table 5.1

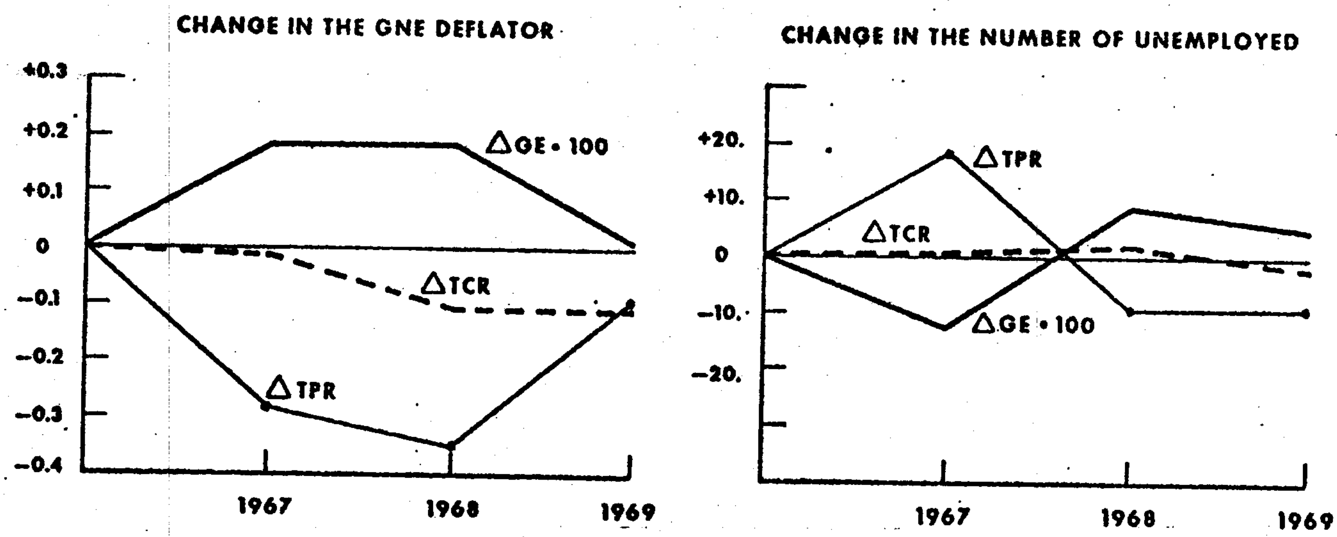
The Estimated Values of the Submatrices R_1, R_2, and R_3

Target variables	Instruments		
	ΔGE	ΔTPR	ΔTCR
		R_1	
ΔPy	.0019	-.2799	-.0110
ΔUL	-.1251	18.6165	.7337
		R_2	
ΔPy	.0019	-.3412	-.1040
ΔUL	.0913	-9.6342	2.5351
		R_3	
ΔPy	.0001	-.0882	-.1060
ΔUL	.0490	-9.3458	-1.7861

[4] The orders of vectors and matrices in (5.9) are:
$y = 6 \times 1$; $x = 9 \times 1$; $\hat{q} = 60 \times 1$; $q_6 = 7 \times 1$; $p_6 = 15 \times 1$; $u = 63 \times 1$; $R = 6 \times 9$; $H_1 = 6 \times 60$; $H_2 = 6 \times 7$; $^6H_3 = 6 \times 15$; $^6J = 6 \times 63$.

Figure 5.1

Time Paths of Target Variables in Response to One-Shot Increase in Instruments: (Increases in the Two Tax Rates are Unit Increases; and the Increase in Government Expenditure is 100 Units).



government expenditure on price changes is positive throughout the three-year period; but the effect is high in the first two years and reduces almost to zero at the third year. The effects of increases in the two tax rates on price changes are more or less opposite; their negative effects are high in the first two years, and then decline (or remain approximately the same as in the second year in the case of the corporation income tax rate).

An increase in government expenditure has a negative effect on the number of persons unemployed in the first year and a minor positive effect in the last two years; thus, the cumulative effect during the three years is almost negligible. This may be explained in the following way. In our model, government expenditure increases real business output and the number of paid workers, and thus it decreases the unemployment rate. A lower unemployment rate brings forth a higher wage rate which, in turn, causes a decrease in the number of paid workers and an increase in the unemployment rate. This oscillating process will continue until the wage rate and prices have stabilized at higher levels and real output is no longer higher.[5]

An increase in the personal income tax rate has a positive effect on the number of persons unemployed in the first year, then minor negative effects in the following two years, thereby making its cumulative effects for the three years more or less negligible. Similarly, an increase in the corporation income tax rate has positive effects during the first two years and a negative effect in the third year, with a minor cumulative positive effect.

An increase in the personal income tax rate increases personal income tax collections. This decreases disposable income, consumer expenditure, real business output, and the number of paid workers, thereby resulting in a high unemployment rate. The unemployment rate will decline again as a result of a lower wage rate. Similarly, an increase in the corporation income tax rate increases corporation income tax. This will decrease consumer expenditure due to lower personal income, and private business investment as a result of lower undistributed corporation profits. These will decrease real business output and the number

---

[5]While this is not a thorough explanation of the effects of government expenditure on the number of unemployed, tracing out all of the channels is not required for the present purpose.

of paid workers, thereby resulting in a higher unemployment rate such as occurs with an increase in the personal income tax rate.

As opposed to the process in the context of government expenditure, the process following increases in the two tax rates will continue until the wage rate and prices have stabilized at lower levels, and real income is no longer lower. All of the three instruments affect the price level through changes in aggregate demand and then in the unemployment rate. Hence, the speed with which a price change occurs appears to be slower than that for a change in the number of persons unemployed. As a result, although our system is likely to be stable, the effects of instruments on the target variables will oscillate, and they appear to converge - not explode - towards zero.[6]

It will be noted that the values of all elements of q_6 and p_6 in (5.9) are, in principle, known at the beginning of the strategy period 1967-69; they contain values taken by exogenous and endogenous variables in the year (1966) preceding the strategy period. The values of non-instrument exogenous variables (elements of $\hat{q}$ in (5.9)) must be predicted at the beginning of the strategy period, although we will use the actual values for the present purposes.[7] Thus, the numerical values of the three

---

[6] It has not been investigated as to how long the oscillating processes will take until they arrive at their ultimate destination. Multiplier paths oscillating toward zero seem to be common features of various models. See C. F. Christ, "Econometric Models of the Financial Sector", Journal of Money, Credit and Banking, III (1971), Part II, 419-449, and E. M. Gramlich, "Comments on Discussion of the Carl Christ Paper", Journal of Money, Credit and Banking, III (1971), Part II, 464-468.

[7] In principle, the value of $\hat{q}_7$ in (5.9) is a predicted value when it is used for the strategy in 1967, but its actual value must replace the predicted value in 1968 and 1969 because the former will then be available. Similarly, the predicted value of $\hat{q}_8$ must be replaced by its actual value for the strategy in 1969.

terms $H_1\hat{q}$, H_2q_6, and H_3p_6 of (5.9) were specified as

$$(5.10) \quad H_1\hat{q} + H_2q_6 + H_3p_6 = \begin{bmatrix} 3.6478 \\ 115.9464 \\ 2.7485 \\ 60.9812 \\ 2.1865 \\ 81.2486 \end{bmatrix}$$

The disturbance terms (u) in (5.9) were replaced by their expected values. The residuals in the estimated equations are more or less uncorrelated over time. Hence, we regarded the expected values of the elements of u as zero in accordance with our assumption made in Chapter II. As a result, we have now completed the numerical specification of the constraints for the first-period optimal strategy (certainty equivalence decision).

4. Numerical Specification of the Constraints for Later Years

It was pointed out in Chapter III that the first-period certainty equivalence procedure would be used for the second and later periods by shifting the period of the problem. That is, for the second year (1968), we proceeded as if the year 1968 was the first year of a two-year strategy period. At the beginning of the second strategy period, however, new information becomes available. That is, u_7 in (5.9) is a vector of residuals which are, in principle, known at the beginning of

1968, and similarly u_8 is a vector of known residuals at the beginning of 1969. Therefore, u_7 must be taken into account as known residuals for the second strategy period, and u_7 and u_8 are both known residuals for the strategy in 1969. We can see from (5.9) that for the second-period strategy (1968-1969), the constraints contain the products of $\bar{u}_7$ ($\bar{u}_7$ being the realized disturbances of u_7) and coefficient matrices J_2 and J_3 in the form as shown in (5.11) below. The numerical values of the terms were specified as

$$(5.11) \qquad \begin{bmatrix} J_2\bar{u}_7 \\ J_3\bar{u}_8 \end{bmatrix} = \begin{bmatrix} -1.1121 \\ -48.5806 \\ 1.8247 \\ 7.3865 \end{bmatrix}$$

Similarly, for the last-period strategy (1969), the constraints contain the sum of $J_3\bar{u}_7$ and $J_2\bar{u}_8$ ($\bar{u}_8$ being the realized disturbance of u_8). Its numerical values were specified as

$$(5.12) \qquad \begin{bmatrix} J_3\bar{u}_7 + J_2\bar{u}_8 \end{bmatrix} = \begin{bmatrix} -2.4636 \\ 52.9696 \end{bmatrix}$$

We can also see from (5.9) that for the second-period strategy, the constraints contain the products of instruments for 1967 (x_7) and coefficient matrices R_2 and R_3, i.e., R_2x_7 and R_3x_7. Also for the last-period strategy, the constraints contain R_3x_7 and R_2x_8. Since it was assumed that the optimal strategy had been implemented in 1967, the first year's values of the instruments (x_7), which will be used in the second-period strategy, must be implemented optimal values.

Similarly, for the last-period strategy (1969), the instruments (x_8) for 1968, in addition to x_7, must be implemented optimal values of them. Therefore, although we had already specified the constraints for the first-period strategy, we could not specify numerically the constraints for the second-period strategy until we had implemented the first-period optimization. Nor could we specify numerically the third-year constraints until we had implemented the second-period optimization.

We have now derived the constraints from the model and completed the numerical specification of the constraints for the first-period. The policy-maker is expected to minimize the preference function subject to the constraints year by year. This is the procedure for arriving at an optimal policy. In the following chapter, an optimal fiscal policy for economic stabilization for the three year period 1967-69 will be determined. We can then compare the actual fiscal policy during this period with the fiscal policy indicated as being optimal by our model.

Table 5.2

Numerical Values of the Reduced-Form Coefficients of the Model: For the One-Year Lagged Endogenous Variables (Ł-Matrix in (5.6))

Current Endogenous Variables	One-Year Lagged Endogenous Variables*														
	CF_{-1}	ING_{-1}	$JNFB_{-1}$	M_{-1}	$NEPW_{-1}$	WR_{-1}	Py_{-1}	Pc_{-1}	Pi_{-1}	CPB_{-1}	CPU_{-1}	TC_{-1}	$YGNE_{-1}$	UL_{-1}	JST_{-1}
CE	.487	1.024	-.179	-1.250	-1.664	2737.930	445.746	-285.827	-173.186	.399	.007	-1.033	-.331	-7.719	-101.370
ING	.189	1.497	-.117	-.793	-1.125	1865.196	333.955	-93.623	-253.294	.120	.844	-.366	-.628	-5.350	-66.163
JNFB	.105	.390	-.186	-.590	.249	-411.891	-.973	-26.939	-65.978	.016	.132	-.152	-.008	-.196	-105.687
M	.171	.638	-.105	-.076	.218	-361.439	87.314	-46.007	-107.861	.030	.215	-.252	-.212	-2.108	-59.842
NEPW	.027	.100	-.017	-.113	.130	-215.035	6.413	-2.894	-16.946	-.004	.034	-.031	-.033	-.231	-9.402
WR	.000	.000	-.000	-.000	.000	.967	.015	-.017	-.003	.000	.000	-.000	-.000	-.000	-.001
Py	.000	.002	-.000	-.002	.002	-3.233	1.470	-.493	-.255	.001	.001	-.001	-.001	-.012	-.141
Pc	.000	.001	-.000	-.001	.002	-2.690	1.223	-.410	-.212	.001	.000	-.001	-.000	-.010	-.118
Pi	.000	.003	-.000	-.002	.001	-2.142	1.639	-.488	-.451	.001	.001	-.001	-.001	-.015	-.157
CPB	.118	.439	-.066	-.485	.265	-438.782	60.022	-28.257	-74.282	.532	.148	-.167	-.153	1.886	-37.357
CPU	.066	.245	-.037	-.271	.148	-245.035	33.519	-15.780	-41.483	-.256	.636	.460	-.085	1.053	-20.862
YPT	.278	1.038	-.182	-1.279	-2.036	3374.921	387.458	-205.478	-175.623	.384	-.033	-1.059	-.334	-7.769	-103.413
YPAC	.048	.179	-.032	-.221	-.352	583.315	66.967	-35.514	-30.354	.066	-.006	-.183	-.058	-1.343	-17.874
TP	.056	.208	-.037	-.257	-.409	677.570	77.788	-41.253	-35.259	.077	-.007	-.213	-.067	-1.560	-20.762
CPT	.074	.275	-.041	-.303	.166	-274.358	37.530	-17.668	-46.447	.333	.093	-.104	-.095	1.179	-23.359
TC	.037	.137	-.021	-.152	.083	-137.207	18.769	-8.836	-23.228	.167	.046	-.052	-.048	.590	-11.682
TI	.076	.285	-.038	-.167	-.182	301.007	72.259	-35.675	-48.237	.044	.096	-.144	-.104	-1.305	-21.481
UIB	-.028	-.105	.017	.118	-.136	225.231	-6.717	3.031	17.750	.004	-.035	.032	.035	.263	9.848
YGNE	.609	2.273-	-.376	-2.556	-2.759	4572.674	691.414	-360.382	-384.596	.505	.767	-1.298	-.755	-11.157	-213.377
UL	-.027	-.100	.017	.113	-.130	215.035	-6.413	-2.894	16.946	.004	-.034	.031	.033	.251	9.402
JST	.001	.003	-.002	-.005	.002	-3.344	-.124	-.197	-.556	.000	.001	-.001	-.000	-.001	.088

* All the variables are in the forms of first differences

Table 5.3

Numerical Values of the Reduced-Form Coefficients of the Model: For Current Exogenous Variables (F-Matrix in (5.6))*

Current Endogenous Variables	GE	TPR	TCR	CCA	CCANG	GWCCA	JNFG	N	NEG	NES	Pm	RL	YPRE	TINT	TPO	TR	UIBR	WF	XPT	YF	YPF	YRE	CONST
	Current Exogenous Variables																						
CE	1.292	-341.210	-13.448	-1.402	.000	-.149	1.292	4.748	-4.878	-4.878	-100.032	.000	1.402	103.039	-1.628	1.402	5.530	.068	1.292	.110	-1.628	1.292	-89.752
INC	.890	-132.444	-5.220	-.544	.000	-.521	.890	3.198	-3.299	-3.299	-97.342	.000	.544	39.996	-.632	.544	2.146	.027	.890	-.345	-.632	.890	-34.838
JNFB	.492	-73.276	-2.888	-.301	.000	-.223	.492	-.690	.729	.729	-78.445	.000	.301	22.128	-.350	.301	1.188	.015	.492	-.191	-.350	.492	-19.274
M	.805	-119.792	-4.721	-.492	.000	-.331	.805	-.596	.639	.639	-13.669	.000	.492	36.175	-.572	.492	1.941	.024	.805	-.312	-.572	.805	-31.310
NEPW	.126	-18.821	-.742	-.077	.000	-.069	.126	.600	-.620	-.620	-16.895	.000	.077	5.684	-.090	.077	.305	.004	.126	-.049	-.090	.126	-4.931
WR	.000	-.003	-.000	-.000	.000	-.000	.000	-.000	.000	.000	.001	.000	.000	.001	-.000	.000	.000	.000	.000	-.000	-.000	.000	-.001
Py	.002	-.283	-.011	-.002	.000	-.001	.002	-.006	.006	.006	.051	.000	.001	.086	-.001	.001	.005	.000	.002	-.001	-.001	.002	-.074
Pc	.002	-.235	-.009	-.001	.000	-.001	.002	-.005	.005	.005	.043	.000	.001	.071	-.001	.001	.004	.000	.002	-.001	-.001	.002	-.062
Pi	.002	-.314	-.012	-.001	.000	-.002	.002	-.004	.004	.004	.005	.000	.001	.095	-.002	.001	.005	.000	.002	-.001	-.002	.002	-.083
CPB	.554	-82.499	-3.252	-.339	.000	-.304	.554	-.734	.776	.776	-66.580	.000	.339	24.913	-.394	.339	1.337	-.039	.554	-.215	-.394	.554	-21.701
CPU	.310	-46.071	-43.383	-.189	.000	-.170	.310	-.410	.433	.433	-37.182	.000	.189	13.913	-.220	.189	.747	-.022	.310	-.120	-.220	.310	-12.119
YPT	1.310	-195.049	-14.322	-1.493	.000	-.087	1.310	5.804	-5.969	-5.969	-117.164	.000	1.493	-256.313	-.931	1.493	5.889	.073	1.310	.183	-.931	1.310	-95.584
TPAC	.227	146.699	-2.475	-.258	.000	-.015	.227	1.003	-1.032	-1.032	-20.251	.000	.258	-44.301	-.161	.258	1.018	.013	.227	.032	-.161	.227	-16.521
TP	.263	170.403	-2.875	-.300	.000	-.017	.263	1.165	-1.198	-1.198	-23.523	.000	.300	-51.459	-.187	.300	1.182	.015	.263	.037	-.187	.263	-19.190
CPT	.347	-51.584	-2.033	-.212	.000	-.190	.347	-.459	.485	.485	-41.631	.000	.212	15.578	-.246	.212	.836	-.025	.347	-.135	-.246	.347	-13.569
TC	.173	-25.797	50.142	-.106	.000	-.095	.173	-.230	.243	.243	-20.820	.000	.106	7.790	-.123	.106	.418	-.012	.173	-.067	-.123	.173	-6.784
TI	.360	-53.572	-2.111	-.220	.000	-.099	.360	.521	-.532	-.532	-17.069	.000	.220	16.178	-.256	.220	.868	.011	.360	-.140	-.256	.360	53.896
UIB	-.132	19.713	.777	.081	.000	.072	-.132	.419	-.398	-.398	17.696	.000	-.081	-5.953	.094	-.081	3.624	-.004	-.132	.051	.094	-.132	9.160
YGNE	2.869	-427.137	-16.835	-1.755	.000	-.562	2.869	7.853	-8.088	-8.088	-262.151	.000	1.755	128.988	-2.038	1.755	6.922	.085	2.869	-.114	-2.038	2.869	-112.356
UL	-.126	18.821	.742	.077	.000	.069	-.126	.400	-.380	-.380	16.895	.000	-.077	-5.684	.090	-.077	-.305	-.004	-.126	.049	.090	-.126	4.931
JST	.004	-.618	-.024	-.003	.000	-.002	.004	-.006	.006	.006	-.689	.000	.003	.187	-.003	.003	.010	.000	.004	-.002	-.003	.004	-.163

* All variables are in the forms of first differences

Table 5.4

Numerical Values of the Reduced-Form Coefficients of the Model: For One-Year Lagged Exogenous Variables (G-Matrix in (5.6)) *

Current Endogenous Variables	One-Year Lagged Exogenous Variables						
	$CCANG_{-1}$	$GWCCA_{-1}$	N_{-1}	Pm_{-1}	RL_{-1}	WF_{-1}	YF_{-1}
CE	.783	.331	.282	100.042	-585.928	-.179	.296
ING	1.145	.628	.197	97.348	-856.954	-.117	.577
JNFB	.298	.008	.008	78.444	-223.218	-.186	-.006
M	.488	.212	.079	13.665	-364.920	-.105	.190
NEPW	.077	.033	.010	16.895	57.333	-.017	.030
WR	.000	.000	.000	-.001	-.009	-.000	.000
Py	.001	.001	.000	-.051	-.862	-.000	.000
Pc	.001	.000	.000	-.043	-.717	-.000	.000
Pi	.002	.001	.001	-.005	-1.526	-.000	.001
CPB	.336	.153	-.069	66.584	-251.315	-.066	.138
CPU	.188	.085	-.039	37.184	-140.346	-.037	.077
YPT	.794	.334	.284	117.174	-594.174	-.182	.299
TPAC	.137	.058	.049	20.252	-102.696	-.032	.052
TP	.159	.067	.057	23.525	-119.290	-.037	.060
CPT	.210	.095	-.043	41.633	-157.140	-.041	.086
TC	.105	.048	-.022	20.821	78.586	-.021	.043
TI	.218	.104	.048	17.070	-163.196	-.038	.094
UIB	-.080	-.035	-.010	-17.696	60.052	.017	-.031
YGNE	1.738	.755	.408	262.169	-1301.180	-.376	.677
UL	-.077	-.033	-.010	-16.895	57.333	.017	-.030
JST	.003	.000	.000	.689	-1.882	-.002	-.000

* All variables are in the forms of first differences

Table 5.5

Numerical Values of the Reduced-Form Coefficients of the Model: For Disturbance Terms (S-Matrix in (5.6))*

Current Endogenous Variables	u(ce)	u(ing)	u(jnfb)	u(m)	u(nepw)	u(wr)	u(py)	u(pc)	u(pi)	u(cpb)
CE	252.121	174.423	168.007	-107.521	-4.878	10272.988	282.814	319.587	160.586	-1.221
ING	97.864	255.104	109.656	-66.269	-3.299	2579.282	26.873	106.728	234.867	-.474
JNFB	54.144	66.449	175.163	9.530	.729	-593.899	-6.850	34.189	61.178	-.262
M	88.513	108.632	99.181	49.459	.639	-810.994	-16.918	57.860	100.014	-.429
NEPW	13.907	17.067	15.583	-9.417	.380	-479.779	-9.963	6.756	15.713	-.067
WR	.002	.003	.002	-.001	.000	1.300	.013	.018	.002	-.000
Py	.209	.257	.234	-.142	.006	29.283	1.224	.521	.236	-.001
Pc	.174	.214	.195	-.118	.005	24.362	1.018	1.433	.197	-.001
Pi	.232	.454	.260	-.157	.004	27.749	1.125	.519	1.418	-.001
CPB	40.959	74.813	61.915	-43.856	.776	-837.142	-14.992	36.420	68.878	.705
CPU	34.042	41.779	34.576	-24.491	.433	-467.497	-8.372	20.338	38.465	.394
YPT	144.123	176.878	171.394	-110.351	-5.969	9293.924	222.750	224.776	162.846	-1.301
TPAC	24.910	30.571	29.623	-19.073	-1.032	1606.343	38.500	38.850	28.146	-.225
TP	28.935	35.511	34.410	-22.155	-1.198	1865.906	44.721	45.128	32.694	-.261
CPT	38.116	56.779	38.714	-27.422	.485	-523.441	-9.374	22.772	43.068	.441
TC	19.062	23.394	19.361	-13.714	.243	-261.773	-4.688	11.388	21.538	.220
TI	39.585	48.581	35.603	-6.115	-.532	864.891	21.221	40.976	44.727	-.192
UTB	-14.566	-17.877	-16.321	9.864	-.398	502.528	10.436	-4.982	-16.459	.071
YGNE	313.614	387.344	353.646	-213.718	-8.088	13069.365	319.756	402.645	356.616	-1.329
UL	-13.907	-17.067	-15.583	9.417	-.380	479.779	9.963	-4.756	-15.713	.067
JST	.457	.560	1.512	.094	.006	-7.479	-.156	.238	.516	-.002

Current Endogenous Variables	u(cpu)	u(ypt)	u(tpac)	u(tp)	u(cpt)	u(tc)	u(ci)	u(ifb)	u(ygne)	u(ul)	u(jst)
CE	-1.402	-.327	-1.891	-1.628	-.132	-.263	-1.402	1.402	1.292	4.878	.000
ING	-.544	-.127	-.734	-.632	-.051	.102	.544	.544	.890	3.299	.000
JNFB	-.301	-.070	-.406	-.350	-.028	-.057	-.301	.301	.492	-.729	.000
M	-.492	-.115	-.664	-.572	-.046	-.092	-.492	.492	.805	-.639	.000
NEPW	-.077	-.018	-.104	-.090	-.307	-.015	-.077	.077	.126	.620	.000
WR	-.000	-.000	-.000	-.000	-.000	-.000	-.000	.000	.000	-.000	.000
Py	-.001	-.000	-.002	-.001	-.000	-.000	-.001	.001	.002	-.096	.000
Pc	-.001	-.000	-.001	-.001	-.000	-.000	-.001	.001	.002	-.005	.000
Pi	-.001	-.000	-.002	-.002	-.000	-.000	-.001	.001	.002	-.004	.000
CPB	-.339	-.079	-.457	-.394	-.032	-.064	-.339	.339	.354	-.776	.000
CPU	.311	-.044	-.255	-.220	-.424	-.848	-.189	.189	-.310	-.433	.000
YPT	-1.493	.813	-1.081	-.931	-.140	-.280	-1.493	1.493	1.310	5.969	.000
TPAC	-.258	.141	.813	-.161	-.026	-.048	-.258	.258	227	1.032	.000
TP	-.300	.163	.945	.813	-.028	-.056	-.300	.300	.263	1.198	.000
CPT	-.212	-.049	-.286	-.246	.980	-.040	-.212	.212	.347	-.485	.000
TC	-.106	-.025	-.143	-.123	.450	.980	-.106	.106	.173	-.243	.000
TI	-.220	-.051	-.297	-.256	-.021	-.041	.780	.220	.360	.532	.000
UTB	.081	.019	.109	.094	.008	.015	.081	.919	-.132	.398	.000
YGNE	-1.755	-.409	-2.368	-2.038	-.165	-.329	-1.755	1.755	2.859	9.088	.000
UL	.077	.018	.104	.090	.007	.015	.077	-.077	-.126	.380	.000
JST	-.003	-.001	-.003	-.003	-.000	-.001	-.003	.003	.004	-.006	1.000

* All variables are in the forms of first differences

CHAPTER VI

OPTIMAL FISCAL STRATEGY FOR ECONOMIC STABILIZATION IN CANADA

1. Desired Values of Target Variables and Instruments

We will attempt in this chapter to calculate an optimal fiscal strategy for economic stabilization in Canada based on the theory of optimal strategy (outlined in Chapter III) in conjunction with the constraints (presented in Chapter V) using the period 1967-69 for illustration. We assume that the fiscal policy-maker in Canada, being interested in stabilizing the Canadian economy, formulates a loss-function type of quadratic preference function which he wishes to minimize subject to the constraints under which the Canadian economy operates. Since the two target variables and the three instruments specified earlier enter the preference function in the form of deviations of the actual from the desired values, we must specify desired values for both the target variables and instruments in each of the three years 1967-69. Since any attempt at derivation of the desired values directly from the policy-maker is far beyond the scope of this study, they will be selected in a subjective but, it is hoped, reasonable way, as follows.

Economic policy choices are not made in a vacuum but in a political, social and cultural context. This being the case, choices of the desired values of target variables are restricted. For instance, in a market economy only a particular rate of unemployment may be socially acceptable together with some particular rate of price increase. What is true for target variables is, to a large extent, also true for instruments. Extravagant government expenditure or extremely low tax rates may be self-defeating if they increase the rate of price increase. If tax rates are too high, taxpayer resistance to the payment of taxes may become a problem. Similarly, extremely low government expenditure may be politically or economically undesirable. Thus, because of institutional and structural factors, there may be a number of floor or ceiling restrictions to the target variables and instruments. The above restrictions circumscribe the range in which choices of the desired values can be made. As a consequence, we will first specify only a probable range of the desired values in the case of the two target variables which are of prime importance, and then we will choose a few specific values within this range with which to conduct our experiments. In the case of the three instruments, we will choose one specific value for each instrument, as will be shown shortly.

With regard to the unemployment rate, it is assumed that the desired value of the unemployment rate lies between one and three percent. We then chose three specific unemployment rates: one, two, and three percent. Similarly, with regard to the rate of inflation, it is assumed that the desired increase belongs in the range of a zero to three percent annual increase in the GNE price deflator, and we then chose four specific per annum percentage increases as desired values for further experiments: zero, one, two and three percent.[1]

Assuming that the policy-maker is willing to approach these goals gradually, these desired values are assumed to be the goals which the policy-maker wishes to achieve by the third year of the strategy period (1969), rather than in the first or second year. Hence, we must also specify the desired values for the intermediate two years - 1967 and 1968. This is done by a simple linear interpolation between the actual values in the base year (1966) and the corresponding desired values in 1969. Thus,

$$\text{(6.1)} \quad \begin{aligned} UR^*_{6+t} &= UR_6 + \frac{t}{3}(UR^*_9 - UR_6) \\ PAR^*_{6+t} &= PAR_6 + \frac{t}{3}(PAR^*_9 - PAR_6), \quad t = 1, 2, 3. \end{aligned}$$

where UR represents the unemployment rate and PAR represents the percentage rate of annual change in the GNE deflator, the terms with asterisks representing the desired values, and the subscripts 6 and 9 indicating the years 1966 and 1969, respectively.

Since our model regards the change in the number of persons unemployed, rather than the unemployment rate, as the relevant target

---

[1] In Canada there are a number of discussions and examples of desired values for the two target variables. For example, the Economic Council of Canada chose a 3% unemployment rate, 1.4% for the desired per annum price increase for consumer prices, and 2.0% for the overall (GNE) price level, the Economic Council of Canada, Second Annual Review, (Ottawa: Queen's Printer, 1965), p. 7. The Royal Commission on Taxation suggested a 3.5% unemployment rate, and a 1.5-2.0% annual increase in the consumer price index as two targets; the Royal Commission on Taxation, Report of the Royal Commission on the Taxation, (Ottawa: Queen's Printer, 1966), Vol. II, p. 53.

variable, it is necessary to translate one into the other. This can be done easily, if the total civilian labor force is given, by multiplying the number of persons in the civilian labor force by the unemployment rate and then obtaining first differences. Since, in principle, the actual number in the civilian labor force during the strategy period 1967-69, will be unknown at the beginning of the strategy period, the labor force, estimated by a semilogarithmic linear trend line fitted for the period 1961-69, was used for the present purpose.[2] The desired changes in the number of persons unemployed, based on the estimated labor force for the three-year period (1967-69) and corresponding to the three specific rates of unemployment, are shown in Table 6.1.

Table 6.1

Desired Changes in the Number of Persons Unemployed, Corresponding to the Three Desired Unemployment Rates, and Actual Changes: 1967-1969

Year	Unemployment Rates and Changes in the Number of Unemployed			
	Desired Unemployment Rates			
	1%	2%	3%	Actual
	Changes in the Level (Thousands of Persons)			
1967	-61	-30	-7	48
1968	-56	-40	-8	67
1969	-69	-35	-9	0

[2] The growth rate of the labor force estimated by the semilogarithmic trend line fitting for the period 1961-69 was 2.99%. As shown below, the labor force thus estimated is quite close to the actual labor force for the period 1967-69.

	1967	1968	1969
Estimated labor force(thousands of persons)	7656	7885	8121
Actual labor force (thousands of persons)	7694	7919	8162

Similarly, the desired percentage changes in the GNE price deflator specified above should also be translated into the desired changes in the level. This translation requires only the level of the GNE price deflator in the base year (1966) because the desired percentage change is in terms of annual change over the previous year. The desired annual changes in the GNE price deflator computed according to the four specific percentage changes are shown in Table 6.2. Since we have specified three desired values for the change in the number of unemployed and four desired values for the change in the GNE price deflator, we have twelve different combinations of the desired values of the target variables for our experiments.

Table 6.2

Desired Changes in the GNE Price Deflator, Corresponding to the Four Desired Values of Annual Percentage Changes, and Actual Change: 1967-1969 (the deflator being 100.0 in 1961)

Year	Annual Percentage Changes and Changes in the Level				
	Desired Annual Percentage Changes				
	0%	1%	2%	3%	Actual
	Changes in the Level				
1967	3.5	3.9	4.2	4.7	3.9
1968	1.8	2.6	3.4	4.2	4.2
1969	0	1.2	2.4	3.7	5.8

The desired changes in government expenditure are based on the assumption that maintaining its rate of growth is desirable. As a consequence, the annual growth rate of government expenditure was estimated by fitting a semilogarithmic linear trend for the period

1961-69. The implied growth rate of 10.0% thus obtained was extrapolated from the actual government expenditure in 1966 for the three years (1967-69) in order to get the desired levels for that period. The annual changes of the desired levels are assumed to be the desired changes for government expenditure for our purposes. In view of the political difficulties which might accompany frequent changes in the personal and corporation income tax rates, we shall assume that maintaining the two tax rates unchanged is desirable. Hence, the desired values of instruments for the three-year strategy period are specified, as shown in Table 6.3.

Table 6.3

Desired and Actual Changes in Government Expenditure, in the Personal Income Tax Rate, and in the Corporation Income Tax Rate: 1967-1969
(millions of current dollars for government expenditure, and percentage rates for the two tax rates)

Year	Instruments*		
	ΔGE	ΔTPR	ΔTCR
1967	1267 (1238)	0 (1.5)	0 (-.5)
1968	1393 (1250)	0 (.9)	0 (1.5)
1969	1532 (1579)	0 (2.4)	0 (-.1)

*Figures in the parentheses are actual changes

2. Coefficients of the Preference Function

We have thus far completed the numerical specifications of the desired values of target variables and instruments. The specification of the preference function will be complete if the coefficients of the preference function are specified. We recall that the general quadratic form of the preference function is

$$W(X, Y) = a'X + b'Y + \tfrac{1}{2}(X'AX + Y'BY + X'CY + Y'C'X) \tag{6.2}$$

where no numerical values of the coefficients are given. In the interest of simplicity in conjunction with our purpose which is to illustrate the use of an optimization technique, all the elements of a, b, and C were defined to be zero, as were all of the off-diagonal elements of A and B. This indicates that the preference function is a type of weighted sum of squares of deviations between the actual and the desired values of the target variables and instruments for the three-year period.

Such a preference function can be written for a three-year period as

$$W = \tfrac{1}{2} \sum_{t=1}^{3} \{B_1(\Delta Py_t - \Delta Py_t^*)^2 + B_2(\Delta UL_t - \Delta UL_t^*)^2 + A_1(\Delta GE_t - \Delta GE_t^*)^2 + A_2(\Delta TPR_t - \Delta TPR_t^*)^2 + A_3(\Delta TCR_t - \Delta TCR_t^*)^2\} \tag{6.3}$$

where the A's and B's with subscripts represent the weights for the arguments. These weights were selected in the following subjective but, it is hoped, reasonable fashion.

By assuming that these weights are all positive (a non-perversity case), the preference function (6.3) can be rewritten as

$$(6.4)\quad W = \tfrac{1}{2}\sum_{t=1}^{3}\left\{\left(\frac{\Delta Py_t - \Delta Py_t^*}{\pi_1}\right)^2 + \left(\frac{\Delta UL_t - \Delta UL_t^*}{\pi_2}\right)^2 + \left(\frac{\Delta GE_t - \Delta GE_t^*}{\rho_1}\right)^2 + \left(\frac{\Delta TPR_t - \Delta TPR_t^*}{\rho_2}\right)^2 + \left(\frac{\Delta TCR_t - \Delta TCR_t^*}{\rho_3}\right)^2\right\}$$

where π_1, and π_2 are the reciprocals of the square roots of B_1 and B_2 respectively, and ρ_1, ρ_2, and ρ_3 are those for A_1, A_2, and A_3 respectively. The preference function (6.4) says that, for instance, the disutility caused by a deviation of ΔPy_t from its desired value (ΔPy_t^*) by π_1, is equal to the disutility caused by a deviation of π_2 between ΔUL_t and ΔUL_t^*. Hence, it is clear that only relative size of π_1 and π_2 (and ρ_1, ρ_2, and ρ_3) are important. Thus, we can choose an arbitrary value for one of the weights and, accordingly, relative values for the rest of the weights.

First, we assigned a one-unit value to the deviation of ΔPy_t from ΔPy_t^*, which amounts to one percent of the 1966 GNE price deflator, and then the same one-unit value was assigned to six different deviations of ΔUL_t from ΔUL_t^*, which amount to .25%, .5%, .75%, 1.0%, 1.5%, and 2.0% of the 1966 labor force. It is then clear that π_1 and π_2 correspond to the 1966 levels of the GNE price deflator and the labor force respectively. As a consequence, given a one-unit value to one percent of the 1966 GNE price deflator, six different levels of the 1966 labor force, corresponding to the six unemployment rates specified above, were divided by the one percent level of the 1966 GNE price deflator, and the reciprocals of the squared quotients are the weights for the term $(\Delta UL_t - \Delta UL_t^*)^2$. They are shown in Table 6.4.

Table 6.4

Weights on the Squared Deviations between the Desired and Actual Number of Unemployed, Corresponding to Six Different Unemployment Rates, Given a Unit Value for $(\Delta Py_t - \Delta Py_t^*)^2$

Terms	Weights (Unemployment Rates)					
	BI(.25%)	BII(.5%)	BIII(.75%)	BIV(1%)	BV(1.5%)	BVI(2%)
$(\Delta Py_t - \Delta Py_t^*)^2$	1	1	1	1	1	1
$(\Delta UL_t - \Delta UL_t^*)^2$	.00381	.00095	.00042	.00024	.00011	.00006

To obtain the weights on the deviations of instruments from their respective desired values, the following hypothetical question is posed: "ceteris paribus, what deviations of instruments from their desired values would increase total disutility by the same amount that a one percentage point decrease, with respect to the 1966 level, in the deviation of the GNE price deflator from its desired value would reduce it?" An answer for the "equivalent" deviations was as follows: (i) ±$100 million from the desired value of government expenditure (ii) ±.5 percentage points from the desired value of the personal income tax rate; and (iii) ±1.0 percentage points from the desired value of the corporation income tax rate. Similarly, in addition to this, three other sets of equivalent deviations were also suggested for our experiments, as shown in Table 6.5.

The weights on the squared deviations of the instruments were then calculated in the same way as in the context of the number of unemployed. The weights thus derived are shown in Table 6.5, together with the corresponding equivalent deviations of the three instruments from their respective desired values.

Table 6.5

Weights on the Squared Deviations of the Three Instruments from Their Respective Desired Values, Given a Unit Value for $(\Delta Py_t - \Delta Py_t^*)^2$

Term	Equivalent deviations	Weights	Equivalent deviations	Weights
		(AI)		(AII)
$(\Delta GE_t - \Delta GE_t^*)^2$	\$100	.00013	\$200	.00003
$(\Delta TPR_t - \Delta TPR_t^*)^2$	.5	5.24411	1.0	1.31103
$(\Delta TCR_t - \Delta TCR_t^*)^2$	1.0	1.31103	2.0	.32776
		(AIII)		(AIV)
$(\Delta GE_t - \Delta GE_t^*)^2$	\$300	.00002	\$400	.00001
$(\Delta TPR_t - \Delta TPR_t^*)^2$	1.5	.58268	2.0	.32776
$(\Delta TCR_t - \Delta TCR_t^*)^2$	3.0	.14567	4.0	.08194

We have now numerically specified six different sets of weights on the deviations of the target variables, and four different sets of weights on the deviations of instruments. As a result, twenty-four different combinations of weights will be used in the experiments. It will be noted that the weights thus derived are different for different variables, but equal for different years. Hence, B in (6.2) is a 6 x 6 diagonal matrix, with diagonal elements consisting of one of the weights for the target variables (e.g., BI) for three consecutive years. For instance, BI is

$$
(6.5) \qquad BI = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & .00381 & & & & \cdot \\ \cdot & & 1 & & & \cdot \\ \cdot & & & .00381 & & \cdot \\ \cdot & & & & 1 & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & .00381 \end{bmatrix}
$$

where the off-diagonal elements are all zero, and the three diagonal elements, representing one variable for three consecutive years, are the same. Similarly A in (6.2) is a 9 x 9 diagonal matrix, with the diagonal elements consisting of one of the weights for the instruments (e.g., AI) for three consecutive years.

3. Numerical Derivation of the Optimal Strategy

So far we have prepared all of the necessary steps for derivation of the first-period certainty equivalence decision; we have specified the preference function and the constraints. Since the preference function adopted is a type of weighted sum of squared deviations, we are able to make use of the concept of the generalized inverse of a matrix which is highly efficient from the computational point of view. Taking into account the constraints specified in (5.9), the optimal strategy can be written in the form of the generalized inverse of a matrix as[3]

---

[3] Fortunately, when a matrix is of full-column rank like matrix D in (6.6), a program of the generalized inverse of a matrix is available in A.P.L. (A Programming Language) by I. P. Sharp Associates, Ltd., Toronto.

(6.6) $\hat{x} = D^{-g}Q$

where $D = \begin{bmatrix} \beta R \\ \alpha \end{bmatrix}$ $Q = \begin{bmatrix} \beta(y^* - H_1\hat{q} - H_2q_6 - H_3p_6 - JE(u)) \\ \alpha x^* \end{bmatrix}$

It is clear from (3.12) and (6.3) that α and β in (6.6) are 9 x 9 and 6 x 6 diagonal matrices, respectively, with diagonal elements equal to the square roots of the corresponding diagonal elements of matrices A and B. We have also specified twelve different sets of values for y^* and a single set of values for x^*.

As the first experiment, we chose the weights AII and BIII (for α and β) and the desired values of the GNE price deflator and the number of unemployed corresponding, respectively, to the two percentage annual change in the deflator and the two percent rate of unemployment. For the certainty equivalence procedure, $E(u) = 0$, where 0 is a zero vector. As a result, all of the components in (6.6), except x of course, are now numerically specified. Then, the optimal strategic values of instruments were calculated by directly using the generalized inverse of the matrix D in (6.6):

(6.7) $$\hat{x}_1 = \begin{bmatrix} \Delta GE_7 \\ \Delta TPR_7 \\ \Delta TCR_7 \\ \Delta GE_8 \\ \Delta TPR_8 \\ \Delta TCR_8 \\ \Delta GE_9 \\ \Delta TPR_9 \\ \Delta TCR_9 \end{bmatrix} = \begin{bmatrix} 1005 \\ 1.25 \\ 1.86 \\ 1082 \\ 1.25 \\ .97 \\ 1428 \\ .39 \\ .06 \end{bmatrix}$$

where $\hat{x}_1$ represents a complete specification of a chain of optimal decisions to be taken for the three consecutive strategy periods (from 1967 to 1969), and the subscripts 7, 8, and 9 represent, respectively, the years 1967, 1968, and 1969.

Out of this chain of decisions ($\hat{x}_1$) for the entire period specified at the beginning of 1967, the first three elements of (6.7) are to be actually implemented in 1967. That is,

$$(6.8) \qquad \hat{x}_7 = \begin{bmatrix} \Delta GE_7 \\ \Delta TPR_7 \\ \Delta TCR_7 \end{bmatrix} = \begin{bmatrix} 1005 \\ 1.25 \\ 1.86 \end{bmatrix}$$

This is the first-period certainty equivalence decision. It should be noted that the first-period optimal strategic values of instruments above are decided with clear plans for the future values of instruments for the rest of the strategy period, given the information available at the beginning of the strategy period (1967). These future values are, however, subject to adjustment as new information becomes available in the future.

For the second year (1968), we proceeded as if 1968 was the first year of a two-year strategy period, and the realized values of the structural disturbances ($\bar{u}_7$) in 1967 and the 1967 optimal values of instruments ($\hat{x}_7$) were taken as given. Using (5.9) the constraints for 1968 took the form:

$$(6.9)\qquad \begin{bmatrix} y_8 \\ y_9 \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ R_2 & R_1 \end{bmatrix} \begin{bmatrix} x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} J_1 & 0 \\ J_2 & J_1 \end{bmatrix} \begin{bmatrix} u_8 \\ u_9 \end{bmatrix} + \begin{bmatrix} H_8 \\ H_9 \end{bmatrix}$$

where

$$\begin{bmatrix} H_8 \\ H_9 \end{bmatrix} = \begin{bmatrix} \hat{F}_2 & \hat{F} & 0 \\ \hat{F}_3 & \hat{F}_2 & \hat{F} \end{bmatrix} \begin{bmatrix} \hat{q}_7 \\ \hat{q}_8 \\ \hat{q}_9 \end{bmatrix} + \begin{bmatrix} \bar{E}\bar{G} \\ \bar{L}^2\bar{G} \end{bmatrix} q_6 + \begin{bmatrix} \bar{L}^2 \\ \bar{L}^3 \end{bmatrix} p_6 + \begin{bmatrix} R_2\hat{x}_7 \\ R_3\hat{x}_7 \end{bmatrix} + \begin{bmatrix} J_2\bar{u}_7 \\ J_3\bar{u}_7 \end{bmatrix}$$

The numerical values of this were calculated as

$$(6.10)\qquad \begin{bmatrix} H_8 \\ H_9 \end{bmatrix} = \begin{bmatrix} 2.32 \\ 93 \\ 3.45 \\ 93 \end{bmatrix}$$

For the second-period certainty equivalence, the expectation of u_8 and u_9 were assumed to be zero vectors. We then obtained the optimal strategy at the beginning of 1968 as

$$(6.11)\qquad \hat{x}_2 = \begin{bmatrix} \Delta GE_8 \\ \Delta TPR_8 \\ \Delta TCR_8 \\ \Delta GE_9 \\ \Delta TPR_9 \\ \Delta TCR_9 \end{bmatrix} = \begin{bmatrix} 1011 \\ 1.50 \\ 1.35 \\ 1366 \\ .61 \\ .10 \end{bmatrix}$$

The optimal values for the second-period are represented by the first three elements of (6.11). It will be noted that, as shown in (6.7), the optimal values of the three instruments for 1968 were planned at the beginning of 1967 with the information available at the time. These have been changed at the beginning of 1968, when the decisions are actually to be taken. Similarly, the plans for 1969 made in 1968 are also different from those made in 1967.

The derivation of the last-period (1969) optimal strategy is now relatively straightforward; we proceeded as if 1969 was the first-year of a one-year strategy period, and the realized values of the structural disturbances in 1967 and 1968, and the 1967 and 1968 optimal values of instruments were taken as given. By substituting these given values into the constraints (5.9), we obtained the constraints for the 1969 strategy, based on which we obtained the 1969 optimal strategy through the certainty equivalence procedure:

$$\hat{x}_9 = \begin{bmatrix} \Delta GE_9 \\ \Delta TPR_9 \\ \Delta TCR_9 \end{bmatrix} = \begin{bmatrix} 1584 \\ -.19 \\ -.03 \end{bmatrix} \tag{6.12}$$

We have now derived the optimal strategic values of the three instruments for the three-year period. These optimal values may be compared with their actual values. They are shown in Table 6.6.

A question then arises as to what would be the values of the two target variables which are of prime importance if the optimal strategic values of the instruments were actually implemented. These "optimal" values of the target variables are then to be compared with their values derived from the actual instruments. Before pursuing this, it is necessary to compute the "controlled" values of the two target variables

Table 6.6

Actual (Controlled), Desired, and Optimal Values of the Instruments
and the Target Variables, with the Coefficients
AII and BIII of the Preference Function

Year	Instruments: Actual	Instruments: Optimal	Instruments: Desired	Target Variables: Controlled	Target Variables: Optimal	Target Variables: Desired*
		Government Expenditure			GNE Price Deflator	
1967	1238	1005	1267	5.58	5.18	4.2
1968	1250	1011	1393	6.73	4.38	3.4
1969	1579	1584	1532	6.45	3.81	2.4
		Personal Income Tax Rate			Number of Unemployed	
1967	1.5	1.25	0	-13	14	-30
1968	.9	1.56	0	18	-1	-40
1969	2.4	-.19	0	85	46	-35
		Corporation Income Tax Rate				
1967	-.5	1.86	0			
1968	1.5	1.35	0			
1969	-.1	-.03	0			

*These desired values correspond to the 2% change in the GNE price deflator and the 2% unemployment rate.

through the numerically specified constraints (5.9) using the actual values of the instruments as shown in Table 6.6. The comparison of the time paths of the controlled values of the target variables with those of their optimal values then indicates the period-by-period effects of the optimal strategy.[4] These optimal values can be obtained by substituting into the constraints (5.9) the optimal values of instruments and the lagged realized values of the structural disturbances, if required. In other words, in order to obtain the 1968 optimal values of the target variables, the 1967 realized values of the disturbances are required as are the values of the disturbances realized in 1967 and 1968 for the optimal values of the target variables in 1969. The optimal values of the target variables thus calculated were shown in Table 6.6.

For the first period (1967), we can see from Table 6.6 that the optimal value of the change in government expenditure in 1967 is an increase of \$1,005 million, as compared to the actual increase of \$1,238 million. The optimal value of the change in the personal income tax rate is slightly lower than the actual value. The corporation income tax rate increases by 1.86 percentage points under the optimal policy, as compared to the actual decrease of .5 percentage points. Therefore, the 1967 optimal strategy is a contractive policy, as compared to the actual policy. This seems to be reasonable because the imminent stabilization problem in 1967 was inflation rather than

---

[4] See C. F. Christ, "Econometric Models of the Financial Sector", Journal of Money, Credit and Banking, III (1971), Part II, 420-421.

unemployment. With the optimal values of the instruments, the GNE price deflator increases by 5.18 points in the index, while the controlled value increases by 5.58 points. The number of unemployed increases by 14 thousand under the optimal policy, as compared to the controlled decrease of 13 thousand. It appears that the effect of the 1967 anti-inflationary optimal policy is not strong. This would occur because the effects of the instruments on the price level is delayed, as shown by Table 5.1 or Figure 5.1 in Chapter V.

For the second period (1968), the optimal policy is still geared to be anti-inflationary; the optimal increase in government expenditure is lower than the actual, while the personal income tax rate is higher than the actual, the corporation income tax rate being almost the same as the actual. Under this policy the optimal value of the change in the GNE price deflator is an increase of 4.38 points in the index, as compared to the controlled increase of 6.73 points, while the number of unemployed decreases by 1 thousand, as compared to the controlled increase of 18 thousand. A smaller increase in the optimal value of the GNE price deflator in the second year, as compared to that in the first year, would be partly attributed to the delayed effects of the first year's optimal policy. The anti-inflationary policy in the first year might result in a lower increase in the number of unemployed in the second year because of the oscillating effects.

For the third period (1969), the optimal policy is geared to be expansionary; the optimal value of government expenditure is slightly higher than the actual value, and the optimal and actual changes in the corporation income tax are almost the same, whereas the optimal change in the personal income tax rate is a decrease of .19 percentage points, as compared to the actual increase of 2.4 percentage points. Because of this expansionary policy, the number of unemployed increases by only 46 thousand over the previous year's level, while its controlled value is an increase of 85 thousand. The optimal value of the change in the GNE price deflator is also lower than the controlled value; the optimal value is an increase of 3.81 points in the index, as compared to the controlled value of 6.45 points. An anti-inflationary policy in 1968 might attribute to a smaller increase in the GNE price deflator in 1969. As a whole, the optimal values of the two target variables seem to be more "desirable" than the controlled values during the three-year strategy period.

4. Optimal Strategies under Various Parameter Values of the Preference Function

A. Under Different Desired Values of the Two Target Variables, Given Coefficients (AII and BIII) of the Preference Function

We have now specified the optimal strategy and corresponding values of the target variables for the three-year period, based on one specific set of subjectively chosen parameters (the desired values of a two percent annual change in the GNE price deflator and a two percent unemployment rate, desired values for the instruments specified in Table 6.3, and the coefficients AII and BIII of the preference function). We will now test the sensitivity of the optimal strategy to changes in either the coefficients or in the desired values.

Maintaining the coefficients AII and BIII of the preference function, together with the same set of the desired values of the instruments, we obtained, through the same procedure followed earlier, the optimal strategies and corresponding values of the two target variables with twelve different combinations of the desired values of the target variables. These are shown in Table 6.7.

Suppose we change the desired value of the number of unemployed in such a way that it amounts to a change from a three percent to a one percent unemployment rate (with respect to the 1966 labor force), together with a given desired value of the annual change in the GNE price deflator of, say, a two percent. The result may be seen by comparing column (9) with column (7) in Table 6.7. Note that the two target variables remain almost unchanged in the first two years, and in the third year the number of unemployed decreases by only 12 thousand (only about .2 percent of the 1966 labor force) and the GNE deflator increases by only .24 points in the index (about .2 percentage points of the 1966 level). The three instruments also remain almost unchanged in the first two years, and then in the third year government expenditure increases and the two tax rates decrease slightly. This phenomenon seems to hold for any desired values of the GNE deflator within the range from zero to three percent. Therefore, the results of the optimal strategy seem to be quite stable under various desired values of the number of unemployed.

Suppose now that we change the desired value of the GNE price deflator from a three percent to a zero percent annual change (with respect to the 1966 level), given a desired value of the number of unemployed, say, the one corresponding to the two percent unemployment

Table 6.7

Optimal Strategies under Various Desired Values of the Two Target Variables, Given the Coefficients (AII and BIII) of the Preference Function and the Desired Values for Instruments Shown in Table 6.3: 1967-1969

Percentage Change in Py / Unemployment Rate	0%			1%			2%			3%			Actual Values	Controlled Values
	1%	2%	3%	1%	2%	3%	1%	2%	3%	1%	2%	3%		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1967														
ΔGE	940	933	941	978	970	978	1012	1005	1013	1053	1045	1054	1238	
ΔTPR	1.61	1.63	1.62	1.41	1.44	1.42	1.23	1.25	1.24	1.02	1.04	1.03	1.5	
ΔTCR	2.61	2.60	2.62	2.24	2.23	2.25	1.87	1.86	1.88	1.49	1.48	1.50	-.5	
ΔPy	4.95	4.93	4.95	5.08	5.06	5.08	5.20	5.18	5.20	5.35	5.32	5.34	3.9	5.58
ΔUL	29	31	29	21	22	21	12	14	13	3	5	3	48	-13
1968														
ΔGE	890	899	887	947	955	943	1003	1011	1000	1062	1071	1059	1250	
ΔTPR	2.06	2.06	2.12	1.81	1.81	1.87	1.56	1.56	1.62	1.30	1.30	1.36	.5	
ΔTCR	1.80	1.88	1.93	1.54	1.61	1.67	1.28	1.35	1.41	1.00	1.07	1.12	1.5	
ΔPy	3.68	3.67	3.65	4.04	4.03	4.01	4.39	4.38	4.36	4.77	4.76	4.74	4.2	6.73
ΔUL	17	15	19	9	7	11	1	-1	3	-7	-9	-5	67	18
1969														
ΔGE	1525	1488	1460	1574	1536	1508	1622	1584	1556	1674	1636	1609	1579	
ΔTPR	.02	.16	.27	-.15	-.02	.09	-.33	-.19	-.09	-.52	-.39	-.28	2.4	
ΔTCR	.00	.03	.04	-.02	-.00	.01	-.05	-.03	-.01	-.03	-.06	-.04	-.1	
ΔPy	3.06	2.96	2.82	3.49	3.38	3.25	3.91	3.81	3.67	4.36	4.26	4.13	5.8	6.45
ΔUL	35	43	48	37	45	49	39	46	51	40	48	53	0	85

rate. Now compare columns (11) and (2) in Table 6.7. The number of unemployed increases slightly (on average, about .3 percent of the 1966 labor force) in the first two years, but remains almost unchanged in the third year. The GNE price deflator decreases by only 1.3 points in the index (or about one percentage point of the 1966 level) in the third year (about .8 percentage points of the 1966 level on the annual average for the three years). On the instrument side, government expenditure decreases on the annual average by about $144 million, while the personal and corporation income tax rates increase, respectively, by about .6 and .7 percentage points on the annual average. As a result, we may say that the results of the optimal strategy will not be affected to a large extent by different desired values of the GNE price deflator. Relatively speaking, however, the optimal strategy seems to be more sensitive to the different desired values of the GNE price deflator than to those of the number of unemployed. This analysis also shows a somewhat clear "trade-off" relation not only between the target variables but also between target variables and instruments.

B. Under Different Sets of Coefficients of the Preference Function, Given a Set of Desired Values of Target Variables and Instruments

Maintaining the desired values of the two target variables, corresponding to the two percent annual change in the GNE price deflator and the two percent unemployment rate, and of the instruments specified in Table 6.3, we obtained the optimal strategies and corresponding values of the two target variables with twenty-four different combinations of the coefficients of the preference function. These are shown in

Table 6.8.

First, suppose we change the coefficients for the target variables from BI to BII, with the coefficients AII for the instruments. This says that the relative weight of one to four for a one percent deviation of the change in the GNE price deflator and in the number of unemployed from their respective desired values, with respect to the 1966 level of GNE deflator and labor force, changes to one to two. This is to compare column (7) to column (8) in Table 6.8. Then, the number of unemployed increases by 30 thousand in the third year (or an annual average of about 15 thousand for the three years), while the price decreases by .85 points in the index in the third year (or about .41 points on the annual average). In the first two years, the three instruments do not change significantly; and in the third year, government expenditure decreases $199 million, the personal income tax rate increases .73 percentage points, and the corporation income tax rate increases insignificantly.

As we further increase the weight for the GNE price deflator relative to that for the number of unemployed, the target variables and the instruments change further in the same direction as described above but, as it is hoped for, the extent of the changes is gradually declining, as shown by Table 6.8. Therefore, the changes in the coefficients influence the results of the optimal strategy in a stable fashion, and the changes over a certain range appear to have only minor influence on the results of the optimal strategy.

Now suppose we reduce the weights for the instruments relative to the target variables in the interest of achieving certain "goals" of

Table 6.8

Optimal Strategies Under Various Sets of Coefficient Values of the Preference Function
Given a Set of Desired Values for Target Variables and Instruments, 1967-1969
(the 2% Price Change and the 2% Unemployment Rate, and Desired Values of Instruments Shown in Table 6.3)

Instruments Target Variables	AI						AII					
	BI (1)	BII (2)	BIII (3)	BIV (4)	BV (5)	BVI (6)	BI (7)	BII (8)	BIII (9)	BIV (10)	BV (11)	BVI (12)
1967												
GE	1088	1139	1162	1172	1180	1183	1002	985	1005	1020	1036	1043
TPR	.75	.58	.50	.47	.44	.43	1.12	1.30	1.25	1.21	1.16	1.14
TCR	.65	.70	.70	.70	.70	.70	1.65	1.82	1.86	1.88	1.89	1.89
Py	5.50	5.64	5.71	5.74	5.76	5.77	5.22	5.13	5.18	5.22	5.27	5.29
UL	-7	-17	-21	-23	-24	-25	12	17	14	11	8	7
1968												
GE	1166	1216	1238	1248	1256	1259	1047	1004	1011	1021	1031	1036
TPR	.76	.67	.62	.59	.58	.57	1.22	1.52	1.56	1.56	1.54	1.54
TCR	.11	.37	.45	.49	.51	.52	.81	1.19	1.35	1.43	1.50	1.53
Py	5.38	5.64	5.77	5.83	5.87	5.89	4.61	4.33	4.38	4.44	4.51	4.54
UL	-27	-29	-30	-30	-30	-30	-12	-3	-1	-0	0	0
1969												
GE	1768	1605	1552	1530	1513	1507	1867	1668	1584	1541	1504	1488
TPR	-.87	-.27	-.07	.01	.07	.09	-1.23	-.50	-.19	-.03	.10	.16
TCR	-.14	-.04	-.01	.00	.01	.01	-.19	-.08	-.03	-.01	.02	.03
Py	5.33	4.87	4.78	4.74	4.71	4.70	4.92	4.07	3.81	3.69	3.60	3.57
UL	40	82	97	104	108	110	-2	28	46	57	67	71

Instruments Target Variables	AIII						AIV					
	BI (13)	BII (14)	BIII (15)	BIV (16)	BV (17)	BVI (18)	BI (19)	BII (20)	BIII (21)	BIV (22)	BV (23)	BVI (24)
1967												
GE	980	902	903	915	933	943	983	869	854	861	879	890
TPR	1.23	1.69	1.74	1.72	1.68	1.65	1.28	1.91	2.04	2.06	2.01	1.98
TCR	2.97	2.84	2.81	2.80	2.79	2.79	4.68	3.88	3.61	3.50	3.43	3.40
Py	5.13	4.85	4.84	4.87	4.92	4.94	5.10	4.71	4.65	4.66	4.71	4.74
UL	17	36	37	35	32	30	19	45	49	48	45	42
1968												
GE	1024	896	874	875	883	889	1046	862	818	811	819	828
TPR	1.32	1.99	2.16	2.20	2.22	2.22	1.31	2.24	2.51	2.60	2.63	2.63
TCR	1.81	2.11	2.25	2.34	2.43	2.47	3.08	3.07	3.12	3.17	3.24	3.28
Py	4.31	3.58	3.48	3.49	3.55	3.59	4.15	3.18	2.98	2.96	3.02	3.07
UL	-6	12	17	19	21	21	-4	19	26	29	30	31
1969												
GE	1904	1691	1612	1570	1528	1509	1934	1707	1637	1603	1567	1548
TPR	-1.37	-.39	-.40	-.14	.01	.08	-1.54	-.67	-.40	-.27	-.13	-.06
TCR	-.22	-.09	-.05	-.02	.00	.01	-.24	-.11	-.06	-.04	-.02	-.01
Py	4.69	3.52	3.17	3.02	2.91	2.86	4.52	3.18	2.80	2.66	2.57	2.53
UL	-15	2	14	23	33	38	-20	-10	-4	2	11	16

the target variables during the strategy period. If we change the weights for the instruments from AI to AIV, maintaining the weights BIII for the target variables, we can see that government expenditure changes gradually away from its desired value, while the two tax rates change quite significantly from their desired values. On the other hand, the GNE price deflator and the number of unemployed decrease toward their desired values. However, as it is hoped for, the extent of the decreases declines quite rapidly. For instance, when we compare columns (3) and (9) for the third year, the GNE price deflator decreases by .97 points in the index, and the number of unemployed decreases by 51 thousand. However, if we compare columns (9) and (15), the GNE deflator and the number of unemployed decreases by .64 points and 32 thousand, respectively, and finally, only .37 points and 18 thousand, respectively, as the weights change from AIII to AIV (columns (15) and (21)).

As a result, increasing the weights for the two target variables relative to the three instruments would help to change the target variables toward their desired values, but with accompanying costs of changing the instruments away from their desired values. However, a further increase in the weights for the target variables relative to those for the instruments over a certain range would not change, to a significant extent, the target variables toward their desired values.

In sum, we have now seen that the changes in the parameters of the preference function affect the results of the optimal strategy in a stable fashion, and insignificantly for some cases. Furthermore, as it is hoped for, the effective changes are limited within a certain range in the sense that further changes over the range do not affect significantly the results of the optimal strategy. This would imply that a slight mis-specification of the preference function may not be all important and a less than perfect function could still be useful for the analysis.[6]

[6] This was pointed out by Roskamp from a theoretical point of view. Our results would support his argument from the empirical side. See K. W. Roskamp, "Multiple Fiscal Policy Objectives and Optimal Budget: A Programming Approach", Public Finance, XXVI (1971), 361-374.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Fiscal policy for economic stabilization is primarily a matter of regulating aggregate demand so as to be compatible with the dual targets of full employment and price stability; an expansionary policy is applied in order to increase employment, while a contractive policy is adopted to moderate price inflation. However, it has been recognized that a gain on the employment front usually implies a loss on the anti-inflationary front. If it is impossible to achieve both targets of stabilization simultaneously, what would be the "best" policy to be adopted? For this question to be answered, the policy-maker must evaluate the relative "cost" of unemployment and inflation, and make the "best" combination of them over time subject to the constraints imposed by the given economic structure. This is the basic subject of the present study.

In Chapter I, we proposed that economic policy is in essence an optimization - maximization (or minimization) of a criterion function subject to the constraints, and then we surveyed the basic economic policy problem from the historical perspective. In Chapter II, we specified the two target variables which are in general referred to as the dual objectives of stabilization. They are, in our study, the changes in the GNE price deflator and in the number of persons unemployed.

We also specified three effective instruments: changes in government expenditure on goods and services including government fixed capital formation, in the personal income tax rate, and in the corporation income tax rate. We specified linear constraints which could be derived from a given (estimated) structural model. In the constraints, the target variables are explained by the instruments, non-instrument predetermined variables, and random disturbance terms, the target variables thereby being subject to a probability distribution. We then assumed a loss-function type of preference function in a quadratic form in which both the target variables and the instruments enter as arguments in the deviation forms from their desired values. It is also assumed that the preference function is well behaved and cardinal in nature. Since the preference function contains the target variables which are, in turn, subject to a probability distribution, the preference function itself is subject to uncertainty, and hence we proposed, as the optimal policy, to minimize the expectation of the preference function subject to the constraint.

Chapter III was devoted to the explanation of the optimal strategy in a general form originated by H. Theil, and discussions of the certainty equivalence theorem. Finally, we linked Theil's approach to the optimal strategy to the concept of the generalized inverse of a matrix. This linkage is of interest in itself; and, in addition it provides a method for computing solutions for optimization which seems to be more efficient than Theil's.

In Chapter IV, we developed an intermediate size econometric model of the Canadian economy (eighteen stochastic equations and eight identities). All the stochastic equations were estimated by the least-squares method and the structurally ordered instrumental variable method proposed by F. M. Fisher and B. M. Mitchell. The model as presented above is complete with respect to structure. The solution to the model does exist, and the whole system works sufficiently well, at least for our purposes, as shown by a simulation based on a Gauss-Seidel technique. Construction of a model not only provides the constraints required for the optimal strategy, but it is also interesting and important in itself, for the model portrays the structure of the Canadian economy as specified. However, the model developed is not in the required form for the linear constraints; it contains ratios of variables, and it does not have target variables on the left-hand side and the instruments, other predetermined variables, and disturbance terms on the right-hand side. Hence, we derived a linearized reduced form of the model, from which we obtained the "final form" of the linear constraints for the period 1967-69. This was done in Chapter V.

In Chapter VI, we combined all of the preparations made prior to this chapter, and derived numerically the optimal strategy for the period 1967-69 under certainty equivalence decisions using a (programmed) technique of the generalized inverse of a matrix. For this we had to numerically specify the parameters of the preference function: desired values of the target variables and the instruments, and the coefficients. We first specified subjectively a range in which the desired values would belong. In the case of the change in the GNE price deflator, we chose a range from a zero to three percent annual change, and then chose four specific values corresponding to a zero, one, two, and three percent annual change. Similarly, in the case of the number of unemployed, we chose a range from a one to three percent rate of unemployment, and then chose three specific values corresponding to a one, two, and three percent rate of unemployment. In the case of the three instruments, we chose one specific desired value for each instrument. By combining them, we experimented with twelve different sets of desired values.

In the interest of simplicity, we assumed that the preference function is a weighted sum of squared deviations of the target variables and the instruments from their respective desired values for the three-year period. Then, since we are concerned only with relative weights, we assigned, first, a unit value to a one percent deviation of the GNE price deflator from its desired value, with respect to the 1966 level. Then six different weights were tried for the deviations of the number of unemployed from its desired values: .25%, .5%, .75%, 1.0%, 1.5%, and 2.0%, with respect to the 1966 labor force. Similarly, given a unit value to the deviation of the GNE price deflator, four different weights were tried for the instruments, maintaining the relative weights among the three instruments approximately unchanged.

For the first experiment, we arbitrarily chose a specific set of parameters for the preference function but, it was hoped, it would consist of a typical value for each parameter. The parameters were: (i) the desired values of the two target variables, corresponding to a two percent annual change in the GNE price deflator and a two percent rate of unemployment, and the one set of desired values of instruments; and (ii) the weights for the arguments of the preference function such that, given a unit value on a one percent deviation of the GNE price deflator with respect to the 1966 level, the deviations of the variables from their respective desired values which amounted to a .75 percent rate of unemployment with respect to the 1966 labor force, $200 million of government expenditure, one percentage point of the personal income tax rate, and two percentage points of the corporation income tax rate were all assigned a unit value.

Based on these parameters of the preference function and the constraints, the numerical solution of the optimal strategy was found. The numerical values of the two target variables derived from the optimal strategy appeared to be "better" than their "controlled" values in the sense that the optimal values were closer to the desired values as a whole and more stable in their movements than the controlled values. Also, the numerical values of the instruments move under the optimal strategy in a more compatible way with improving economic stabilization than the actual policy; if the future as well as present stabilization problem is inflation, they become restrictive, while they become expansionary if high unemployment is the future as well as the present main stabilization problem. The future stabilization problem is also involved because the current optimal strategy is determined with full anticipation of the future economic development.

Our results seem to demonstrate that not only the model developed but also the present approach of the optimal strategy for economic stabilization are operational, and this approach to fiscal policy decisions seems to be more efficient in the sense that the standard policy simulation approach could, in principle, provide any feasible path and, in particular, the optimal paths of the target variables, but they involve a large number of simulation runs in the case of multiple instruments, while the present approach provides the optimal paths with considerably less computational effort.

Secondly, we then carried out experiments of optimization under different parameters of the preference function. We obtained optimal strategies under three different desired values for the number of unemployed and four different desired values for the GNE deflator, maintaining the rest of the parameters as used in the first experiment. It was found that the results of the optimal strategy were quite stable under the three desired values for the number of unemployed. The results of optimization were also stable under the four desired values for the GNE deflator, though to a lesser degree than under the different desired values for the number of unemployed.

Thirdly, we experimented with optimization under various coefficients of the two target variables, maintaining the rest of the parameters as used in the first experiment. It was then found that the changes in the coefficients of the two target variables influenced the results of the optimal strategy in a quite stable fashion, and the changes over a certain range had little impact on the optimal decisions.

Finally, we increased the coefficients for the two target variables relative to those for the three instruments. It was then found that the target variables moved, not however to a large extent, toward their desired values, but this raised the cost of changing the instruments away from their desired values. Moreover, further increases in the relative weights for the target variables did not change significantly the target variables toward their desired values.

The results of our various experiments appear to lead to the conclusion that the effective changes in the parameters are, more or less, limited within a certain range in the sense that further changes over the range do not significantly affect the results of the optimal strategy. This would imply that a slight mis-specification of the preference function - the weakest element in the optimal strategy approach - may not be all that important and a less than perfect function could still be useful for the analysis.

APPENDIX A

DERIVATION OF A LOSS-FUNCTION TYPE PREFERENCE FUNCTION

Suppose that the policy-maker has a preference function as

(A.1) $U = U(x, y)$

where, it should be noticed, x and y are redefined as an instrument and a target variable, respectively. We shall assume that the preference function is comparative and transitive, so that the set of all fully specified arguments has a complete preference ordering.

It is assumed that the policy-maker has certain desired values for the instrument and the target variable and the corresponding desired value of his preference function. However, such desired values of the instrument and the target variable may not in general be realized simultaneously because they are constrained by the economic model. In other words, the desired instrument value may not be compatible with the desired value of the target variable within a given economic structure. This incompatibility may be in general unavoidable as soon as we take into account uncertainty, which is represented by the stochastic terms in the economic model. This occurs because the deterministic desired value of the target variable is unlikely to be identical with the stochastic value that follows from the economic model, given the desired instrument value. Under these circumstances, rational behavior which has been introduced into the economic literature is to minimize the deviations between the desired values and actual values.[1] Thus, the new preference function, which is the difference between the desired and actual values of the original preference function, becomes a function of deviations between the desired and actual values of the instrument and the target variable. This can be proved in the following way. Suppose the desired value of the preference function is given by

$$(A.2) \qquad U^* = U(x^*, y^*)$$

where x^* and y^* are desired values of the instrument and the target variable respectively, and U^* is the corresponding value of the policy-maker's preference function. Subtracting (A.2) from (A.1), we get

$$(A.3) \qquad W = U - U^* = U(x, y) - U(x^*, y^*)$$

If we expand $U = U(x, y)$ by means of the Taylor expansion, we obtain

$$\begin{aligned}(A.4) \qquad U(x, y) &= U(x^*,y^*) + U_x(x^*,y^*)(x - x^*) + U_y(x^*,y^*)(y - y^*) \\ &+ \tfrac{1}{2}\{U_{xx}(x^*,y^*)(x - x^*)^2 + U_{yy}(x^*,y^*)(y - y^*)^2 \\ &+ U_{xy}(x^*,y^*)(x - x^*)(y - y^*) + U_{yx}(x^*,y^*)(y - y^*)(x - x^*)\} \\ &+ 1/3!\{U_{xxx}(x^*,y^*)(x - x^*)^3,..\}..+ 1/n!\{U_{x...x}(x^*,y^*)(x - x^*)^n..\}\end{aligned}$$

where $U_x = \frac{\partial U}{\partial x}$, $U_{xx} = \frac{\partial^2 U}{\partial x^2}$, $U_{xy} = \frac{\partial^2 U}{\partial x \partial y}$, etc. From this we can see easily that $W = U(x, y) - U(x^*,y^*)$ is a function of deviations of instruments and target variables from their desired values. The two arguments x and

---

[1] H. A. Simon, "A Behavioral Model of Rational Choice," in Models of Man, (New York: John Wiley and Sons, 1957), Ch. 14.

y can be extended to m instruments and n target variables. The quadratic preference function involves the first three terms of (A.4), excluding the term $U(x^*,y^*)$, and hence using the expansion only as far as the quadratic term.

BIBLIOGRAPHY

Arrow, Kenneth J. "Alternative Approaches in the Theory of Choice in Risk-Taking Situation". Econometrica, IX (1951), 404-437.

Bodkin, Ronald G., E. P. Bond, G. L. Reuber, and T. R. Bobinson. Price Stability and High Employment: The Options for Canadian Economic Policy. Ottawa: The Economic Council of Canada, 1966.

Bogaard, P. J. M. van den, H. Theil, "Macrodynamic Policy-Making: An Application of Strategy and Certainty Equivalence Concepts to the Economy of the United States, 1933-1936", in A. Zellner, ed., Readings in Economic Statistics and Econometrics. Boston: Little, Brown and Co., 1968, pp. 660-680.

Boot, John C. G. Quadratic Programming. Amsterdam: North-Holland, 1964.

Brown, T. M. "A Forecast Determination of National Product, Employment, and Price Level in Canada, from an Econometric Model", in Models of Income Determination, edited by the National Bureau of Economic Research, Princeton: Princeton University Press, 1964, pp. 59-96.

Cagan, P. "The Monetary Dynamics of Hyperinflation", in M. Friedman, ed., Studies in the Quantity Theory of Money. Chicago: University of Chicago Press, 1956, pp. 25-117.

Canada, The Department of National Revenue. Taxation Statistics. Ottawa: The Department of National Revenue, 1950-1971.

Canadian Tax Foundation. The National Finances. Toronto: Canadian Tax Foundation, 1952-1971.

--------. The National Finances: 1970-71. Toronto: Canadian Tax Foundation, 1970.

Chernoff, H. and L. E. Moses. Elementary Decision Theory. New York: John Wiley and Sons, 1959.

Choudhry, Nanda K., Y. Kotowitz, J. A. Sawyer, and J. W. L. Winder. An Annual Econometric Model of the Canadian Economy. Toronto: The University of Toronto, 1969.

Christ, Carl F. Econometric Models and Methods. New York: John Wiley and Sons, 1966.

Christ, Carl F. "Econometric Models of the Financial Sector", Journal of Money, Credit and Banking, III (1971), Part II, 419-449.

Courant, R. Differential and Integral Calculus. Vol. II. trans., by E. J. McShane, New York: John Wiley and Sons, 1936.

Culbertson, John M. Macroeconomic Theory and Stabilization Policy. New York: McGraw-Hill, 1968.

Duesenberry, James S., G. Fromm, L. R. Klein, and E. Kuh, ed. The Brookings Quarterly Econometric Model of the United States. Chicago: Rand-McNally, 1965.

The Economic Council of Canada. First Annual Review. Ottawa: Queen's Printer, 1964.

--------. Second Annual Review. Ottawa: Queen's Printer, 1965.

Evans, Michael K. Macroeconomic Activity. New York: Harper and Row Publishers, 1969.

Fisher, Franklin M. "Dynamic Structure and Estimation in Economy-Wide Econometric Models", in J. Duesenberry et al ed., The Brookings Quarterly Econometric Model of the United States, Chicago: Rand-McNally, 1965, pp. 588-635.

Fox, Karl A., J. K. Sengupta, and E. Thorbecke. The Theory of Quantitative Economic Policy with Applications to Economic Growth and Stabilization. Amsterdam: North-Holland Publishing Co., 1966.

Frisch, T. Numerical Determination of a Quadratic Preference Function for Use in Macroeconomic Programming. Oslo: Oslo University, 1957.

Goldberger, Arthur S. Impact Multipliers and Dynamic Properties of the Klein-Goldberger Model. Amsterdam: North-Holland, 1959.

Gramlich, Edward M. "Comments on Discussion of the Carl Christ Paper", Journal of Money, Credit and Banking, III (1971), Part II, 464-468.

Graybill, Franklin A. Introduction to Matrices with Application in Statistics. Belmont: Wadsworth Publishing Co., 1969.

Helliwell, John F., L. H. Officer, H. T. Shapiro, and I. A. Stewart. The Structure of RDX1. Ottawa: The Bank of Canada, 1969.

--------., R. G. Evans, F. W. Gorbet, R. S. Jarrett, and D. R. Stepehnson. Government Sector Equations for Macroeconomic Models. Ottawa: The Bank of Canada, 1969.

Holt, Charles C. " Linear Decision Rules for Economic Stabilization and Growth", Quarterly Journal of Economics, LXXIV (1962), 20-45.

Lancaster, Kelvin. Mathematical Economics. New York: The MacMillan Co., 1968.

May, Sidney. "Dynamic Multipliers and Their Use for Fiscal Decision-Making", in Conference on Stabilization Policies, edited by the Economic Council of Canada. Ottawa: Queen's Printer, 1966, pp. 155-195.

Mitchell, Bridger M. "Estimation of Large Econometric Models by Principal Component and Instrumental Variable Method", Review of Economics and Statistics, LIII (1971), 140-146.

--------., and F. M. Fisher. "The Choice of Instrumental Variables in the Estimation of Economy-Wide Econometric Models: Some Further Thoughts", International Economic Review, XI (1970), 226-233.

Musgrave, Richard A. The Theory of Public Finance. New York: McGraw-Hill, 1959.

Neumann, J. von, and O. Margenstern. Theory of Games and Economic Behavior. Princeton: Princeton University Press, 1944.

Robinson, T. Russel. Foreign Trade and Economic Stability. Studies of the Royal Commission on Taxation. No. 5. Ottawa: Queen's Printer, 1965.

Roskamp, Carl W. "Multiple Fiscal Policy Objectives and Optimal Budget: A Programming Approach", Public Finance, XXVI (1971), 361-374.

The Royal Commission on Banking and Finance. Report of the Royal Commission on Banking and Finance: 1964. Ottawa: Queen's Printer, 1964.

The Royal Commission on Taxation. The Use of the Tax System to Achieve Economic and Social Objectives: Report of the Royal Commission on Taxation, Vol. II. Ottawa: Queen's Printer, 1966.

Simon, Herbert A. "Dynamic Programming under Uncertainty with a Quadratic Criterion Function", Econometrica, XXIV (1956), 74-81.

--------., Models of Man. New York: John Wiley and Sons, 1957.

Statistics Canada (Dominion Bureau of Statistics). National Accounts Income and Expenditure: 1926-1956. Ottawa: Queen's Printer, 1962.

Suits, Daniel B. "Forecasting and Analysis with an Econometric Model", American Economic Review, LII (1962), 104-132.

Theil, Henri. Optimal Decision Rules for Government and Industry. Amsterdam: North-Holland, 1968.

--------. Economic Forecasts and Policy. Amsterdam: North-Holland, 1961.

Theil, Henri. "Economic Model and Welfare Maximization", Weltwirtschaftliches Archiv, LXXII (1954), 60-83.

--------., J. C. G. Boot and T. Kloek. Operations Research and Quantitative Economics. New York: McGraw-Hill, 1965.

--------. "A Note on Certainty Equivalence in Dynamic Planning", Econometrica, XXV (1957), 346-349.

--------., and J. C. G. Boot. "The Final Form of Econometric Equation System", in A Zellner, ed., Readings in Economic Statistics and Econometrics. Boston: Little, Brown and Co., 1968, pp. 611-630.

--------., "Linear Decision Rules for Macrodynamic Policy Problems", in B. G. Hickman, ed., Quantitative Planning for Economic Policy. New York: The Brookings Institution, 1965, pp. 18-42.

Tinbergen, J. On the Theory of Economic Policy. Amsterdam: North-Holland, 1952.

--------. Centralization and Decentralization in Economic Policy. Amsterdam: North-Holland, 1954.

--------. Economic Policy: Principles and Designs. Amsterdam: North-Holland, 1967.

Tsurumi, Hiroki. A Four-Sector Econometric Model of the Canadian Economy. Kingston: Queen's University, 1969.