AUTOMATED RESOURCE ALLOCATION

A STUDY AND DESIGN
OF AN AUTOMATED RESOURCE ALLOCATION SYSTEM

By

DAVID JAMES BONHAM, B.Sc.

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AUTHOR: David James Bonham, B.Sc. (Queen's University)

SUPERVISOR: Professor W. R. Newcombe

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SCOPE AND CONTENTS

It has been the purpose of this work to design a system, suitable for the digital computer, to allocate individual activities (uncured tires) to a fixed and limited resource (tire-curing presses).

A comprehensive study of the requirements of the system has been conducted in the field. The results of this study and a review of existing methods of allocation are presented.

A thorough literature search in the area of Operations Research and Systems Engineering has been completed with primary attention given to computer adaptable mathematical programming techniques for the optimal solution of both linear and nonlinear assignment problems.

The tire-curing resource allocation problem has been formulated as a classical quadratic assignment problem. The logic and theory behind this formulation are covered.

Two distinct suboptimal algorithms have been programmed. Included is a discussion of the logic of these programs with the theory employed by them. Also, a full listing in FORTRAN IV for a computer program embodying the logic of the solution determined in this design project is presented.

Realistic problems have been tested; the results of which,

complete with analysis, are presented.

Conclusions drawn to the extension of this system to encompass the entire production facility are discussed as well as conclusions concerning the feasibility of using these processes for production control.

ACKNOWLEDGEMENTS

The author wishes to thank the GOODYEAR TIRE AND RUBBER COMPANY OF CANADA LIMITED, whose co-operation made this work possible.

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I ABSTRACT

The purpose of this work has been to generate a method by which an automobile tire manufacturer can optimally allocate its weekly production ticket to its automatic tire-curing presses.

The problem is of interest for the reason that the value of the objective function is markedly affected by the relative locations of tires amongst themselves. This consideration has negated the possibility of a solution being effected by the application of an algorithm for the classical linear assignment problem. In this work the problem has been formulated and solved as a quadratic assignment problem.

The logic of this method of solution has been programmed and subsequently used to solve example problems, the results of which are extremely encouraging.

II INTRODUCTION

II.1 General

In the past decade there has been a significant increase in the implementation of computer-controlled industrial systems. The areas that have enjoyed, perhaps, the most notable assist by the employment of automatic systems have been Process Control, Industrial Engineering and Management Science.

The computer has given rise to an area of Industrial or Systems Engineering that concerns itself with the practical application of the theories being generated by operations research scientists and applied mathematicians. Prior to the advent of high-speed computing devices with large memory capacity, some of the more promising concepts of operations research for industrial applications were impracticable from computation aspects and for that reason were studied out of theoretical interest only. As recently as 1958, T. C. Koopmans and M. Beckmann (1) in their paper "Assignment Problems and the Location of Economic Activities" stated in their conclusions on the formulation of what has become known as the Quadratic Assignment Problem that "the computational difficulties of finding a solution" to the above problem have "so far been insurmountable".[1]

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[1]T. C. Koopmans and N. Beckmann, "Assignment Problems and the Location of Economic Activities", Econometrica, 25 (1957), 52-76.

II.2 Outline of the Problem

The underlying theory of, and a suboptimal algorithm for, the solution to the Quadratic Assignment Problem form the basis on which this design project has created an automated system for the solution of a real resource-allocation problem that occurs in the tire manufacturing industry.

This work was undertaken with the co-operation of the GOODYEAR TIRE AND RUBBER CO., who supplied the facilities for investigation into the required design criteria. Since this system deals with the final construction stage of an automobile tire and since it directly affects the production capabilities of the plant, it will be used as the basis upon which other systems will be designed to deal with the manufacturing stages upstream from this process. It is feasible that this system could grow until it virtually encompassed the whole of the tire manufacturing process. Thus, the output from this system will serve as the input to the system for the next upstream manufacturing process.

GOODYEAR TIRE and McMaster University have different computing devices, and it was not feasible to utilize GOODYEAR TIRE's computerized information files. Thus the primary purpose of this work has been to demonstrate by means of realistic examples, considering all the known variables, that a solution to this resource-allocation problem can be provided by the methods of operations research. Thus, the logic involved rather than the detailed program is the essence of this thesis.

The primary requirement of this resource-allocation system is to translate management's sale orders, or listing of quantities and models of tires formulated from predicted market conditions, into an optimal production schedule by assigning these tires to the manufacturing equipment so as to minimize production costs.

II.3 Design Considerations

Since rapidly changing service requirements of tires have meant a considerable change in manufacturing methods, it has therefore been a requirement of this design to allow the system to be adjusted with minimal disruption to the program. It should also be noted that since the production facility under consideration has been Goodyear Tire's Toronto plant, the design has naturally been orientated towards its requirements. An attempt has been made to formulate as general a design as possible, and where this has not been possible it has been indicated. Also there may be installation dependant parameters not anticipated in this design.

The program was written for execution on a Control Data Corporation model 6400 computer.

III THE TIRE MANUFACTURING PROCESS

The initial phase of tire production consists of the assemblage of the ingredients in correct proportions to give the desired properties to the finished product. The prime component, of course, is rubber--both natural and synthetic. A change in the proportion of these ingredients can markedly affect the cured characteristics of the product. Each subsection of the tire has different property requirements. For example, the tread stock compound requires high abrasion resistance while the bead compound must possess good adhesion properties. To accommodate these diverse requirements, the tire companies employ a batch-mixing system which enables different compounds to be used in the component manufacturing processes (see Fig. 1). This mixing operation is effected in large counter-rotating worm screw mixers called Banburies.

The second production stage is a multi-stream component manufacturing process. One of these components is the ply stock which is produced from woven tire cord materials such as rayon, nylon and more recently polyester and Fibreglas. These materials are coated with rubber on both sides by a vertical three-roll scraper mill known as a calender. From the calenders the ply stock is skived to the proper length at an angle known as the bias angle. An example of one extreme is the radial-ply tire which has a bias angle of ninety degrees.

Another major component is the sidewall-tread combination which is extruded through two opposed screw extruders working

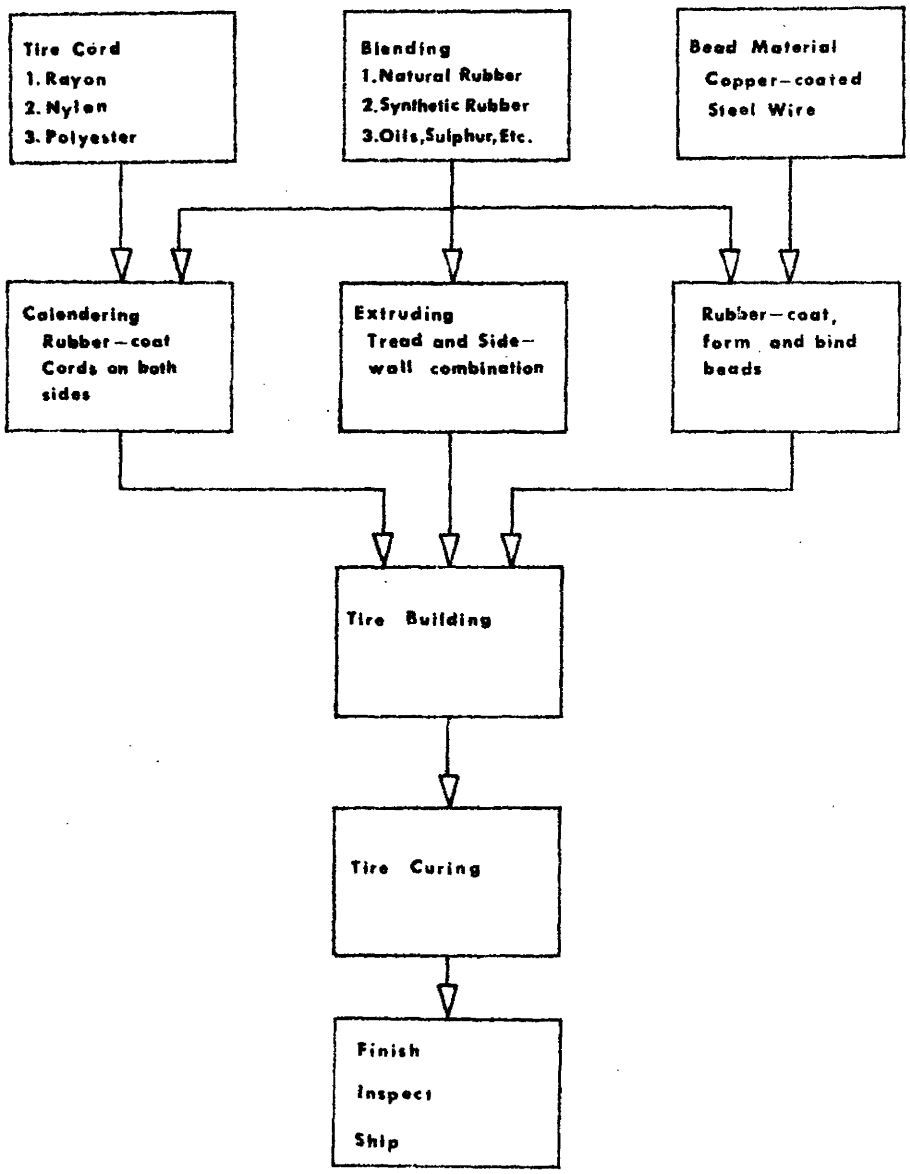

Figure 1 The Tire Manufacturing Process

through a common die. Using the two extruders it is possible to utilize different compounds for the two portions--the tread and the sidewall. This continuous extrusion is then skived to the proper component length for a particular tire line.

The third stage of production is the union of the ply stock, beads, tread and sidewall components on a tire-building machine (see Fig. II). These machines consist primarily of a collapsible drum upon which the components are "layed up" by a semi-automatic interaction of man and machine. The final product of the tire-building machine is a cylindrical structure known in the industry as a "green tire" (see Fig. III).

The fourth and final construction process for the tire is the molding and vulcanization during which the green tires are shaped into their final form while under elevated temperature and pressure conditions. It is at this time that the tire acquires its tread pattern as well as the aesthetic impressions and protusions on the sidewalls. The production machines employed to effect these changes are dual position tire-curing presses. These machines are supplied to virtually all North American tire manufactures by two major rubber equipment companies.

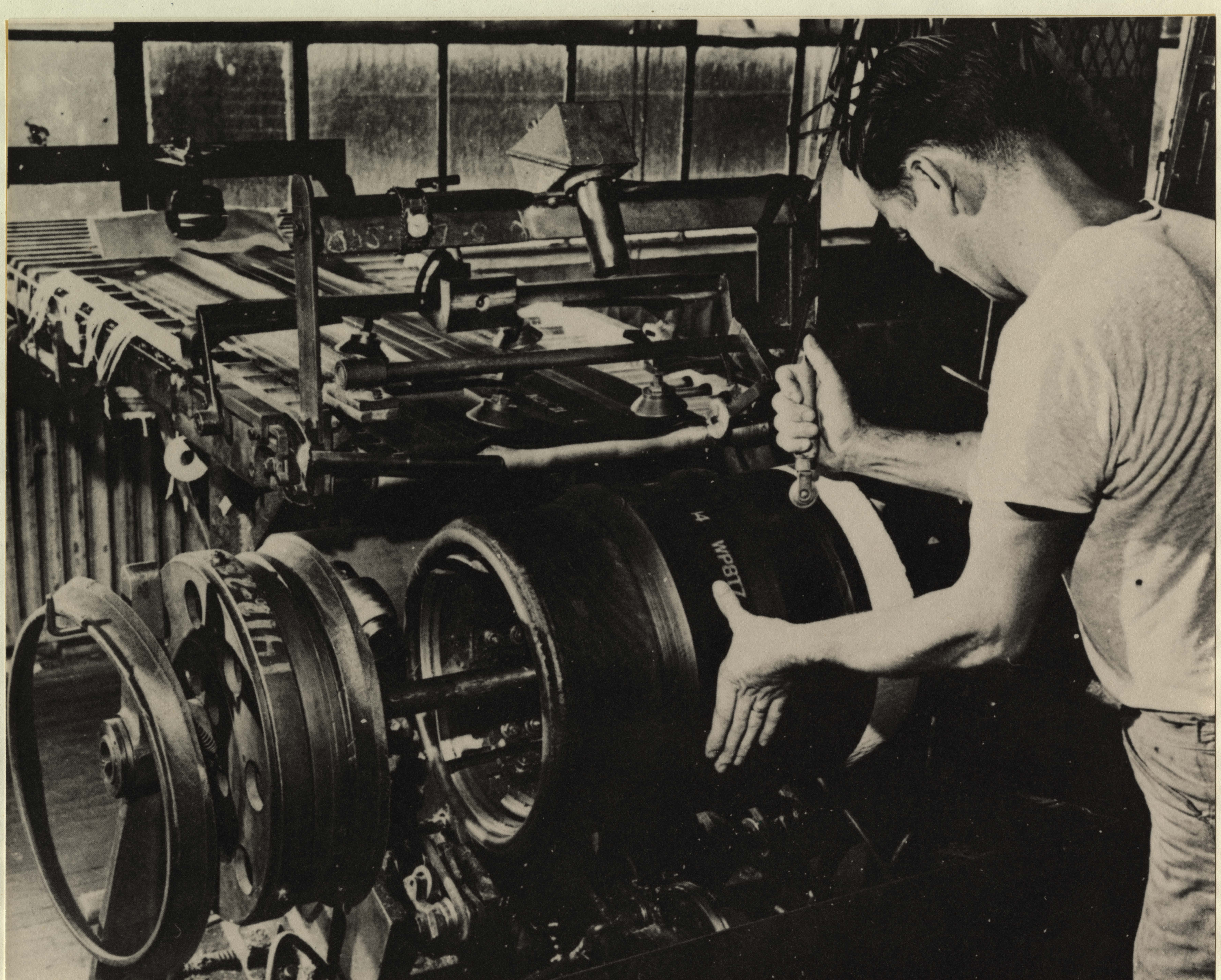

FIGURE II TIRE BUILDING MACHINE

FIGURE III A Green Tire.

IV THE PROBLEM IN DETAIL

IV.1 The Equipment

During the past two decades there has been a very significant change in the vulcanization procedures employed by the tire industry. The basic technique of forming a tire in a mold under elevated temperature and pressure has remained, but the equipment employed to accomplish this has been changed dramatically. The original process consisted of two separate operations, first, shaping, and then, curing. The green tire was formed into a shape approximating that of a finished tire by the insertion of a heavy rubber curing tube. This assembly of tire and curing tube was inserted into a tire mold. After several molds had been filled, they were lowered into a large diameter vertical pot heater where the tires were molded by the introduction of high pressure steam into the enclosed curing bag. A hydraulic ram counter-balanced the internal steam pressure of the tires being vulcanized and kept the molds closed. Upon completion of the specified curing cycle the molds were removed and the tires were stripped. During both the insertion and the removal of the curing tube, the tires were subjected to undesirable distortion.

Except for the very large tire sizes, the pot heater curing technique has been replaced by the automatic tire curing press. The necessity for the implementation of this change in process equipment has been based on both quality and production considerations. These machines consist of a stationary base portion that contains the lower half sections of two tire-curing molds, a rubber bladder, instrumentation, and a movable dome portion that contains the upper half sections of the two tire-curing molds (see Fig. IV). The changing design of the automotive tire plus the ever-present need for increased productivity have contributed to a significant evolution of these machines during the past decade. An example of a tire parameter change that has directly affected the press requirements has been the industry's tendency to lower profile designs. The standard tire of two years ago had a height-to-width ratio of 0.82. This ratio on contemporary tires has been reduced to 0.78 with speculation of further reductions. This, coupled with the introduction of Wide Tread high performance tires with height-to-width ratios of 0.70 and even 0.60, has meant an increase in mold thickness over conventional tires with similar overall diameters. Thus, the new presses must have the capability of accommodating these tires. The net result of this evolution has been that the tire manufacturers are faced with a conglomeration of different models of presses with overlapping but different capabilities.

The capability differences, as well as differences due to technical innovation in these presses, have considerable influence on how the product is allocated to them. The previously mentioned dimensional considerations of the machine usually result in "GO" or "NO GO" decision criteria. An example of a "NO GO" situation would occur if the tire being evaluated for a particular curing location has a mold height that was in excess of the maximum mold clearance in the press. The effects of other changes are less direct, but are equally important. The later model presses have been equipped

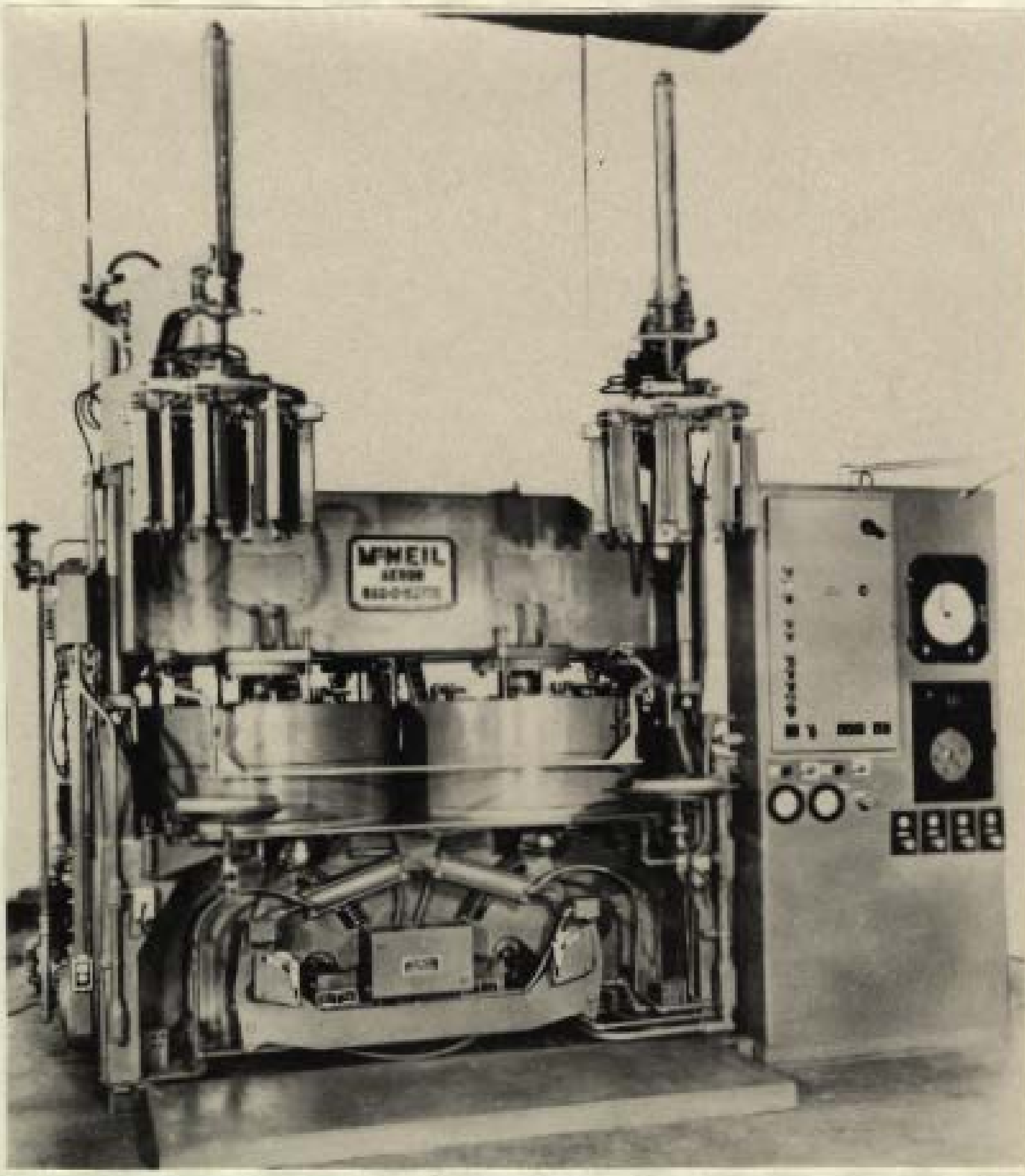

FIGURE IV(a) AUTOMATIC TIRE CURING PRESS

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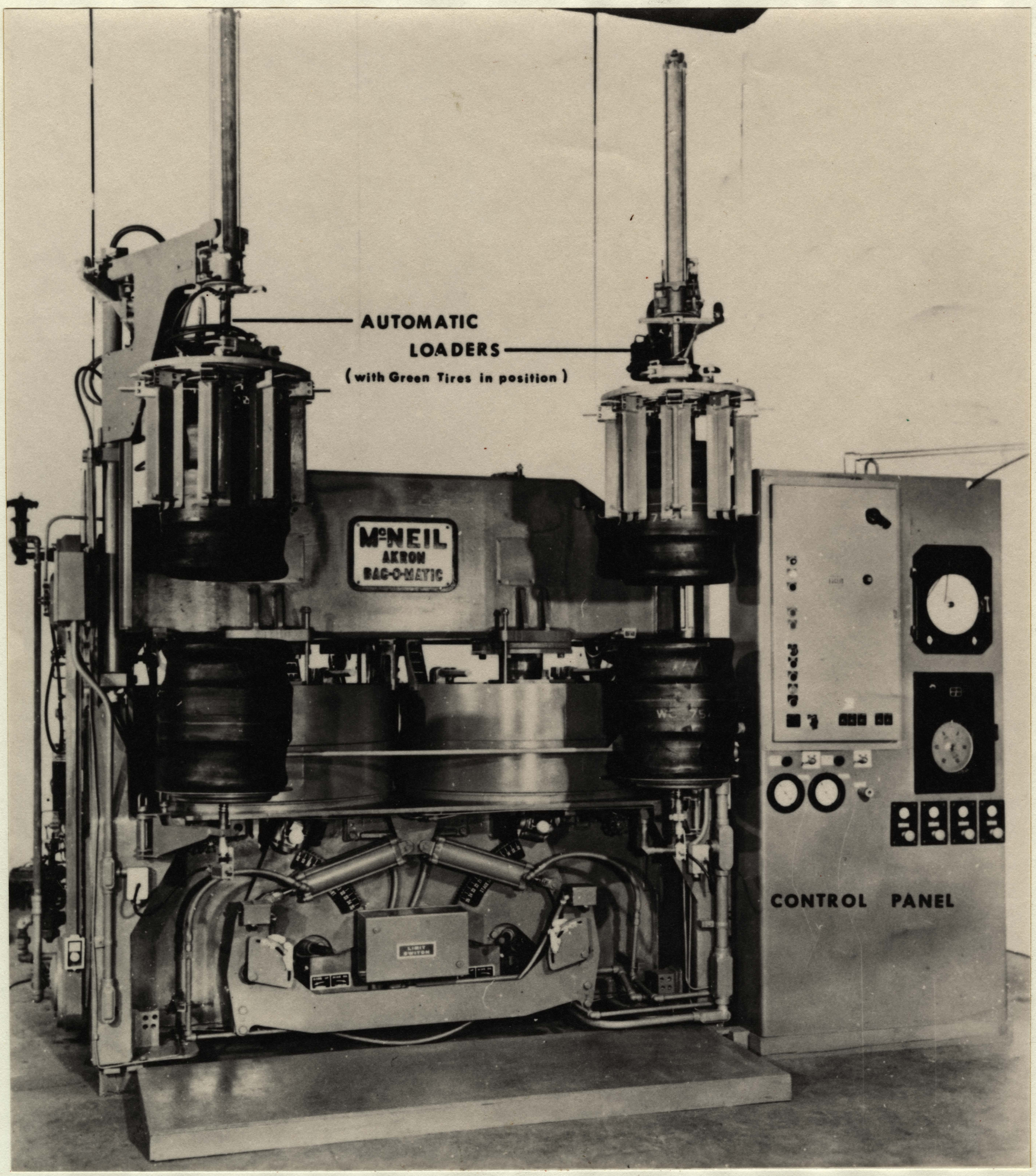

FIGURE IV(b) AUTOMATIC TIRE CURING PRESS

(Green tires in position to be loaded)

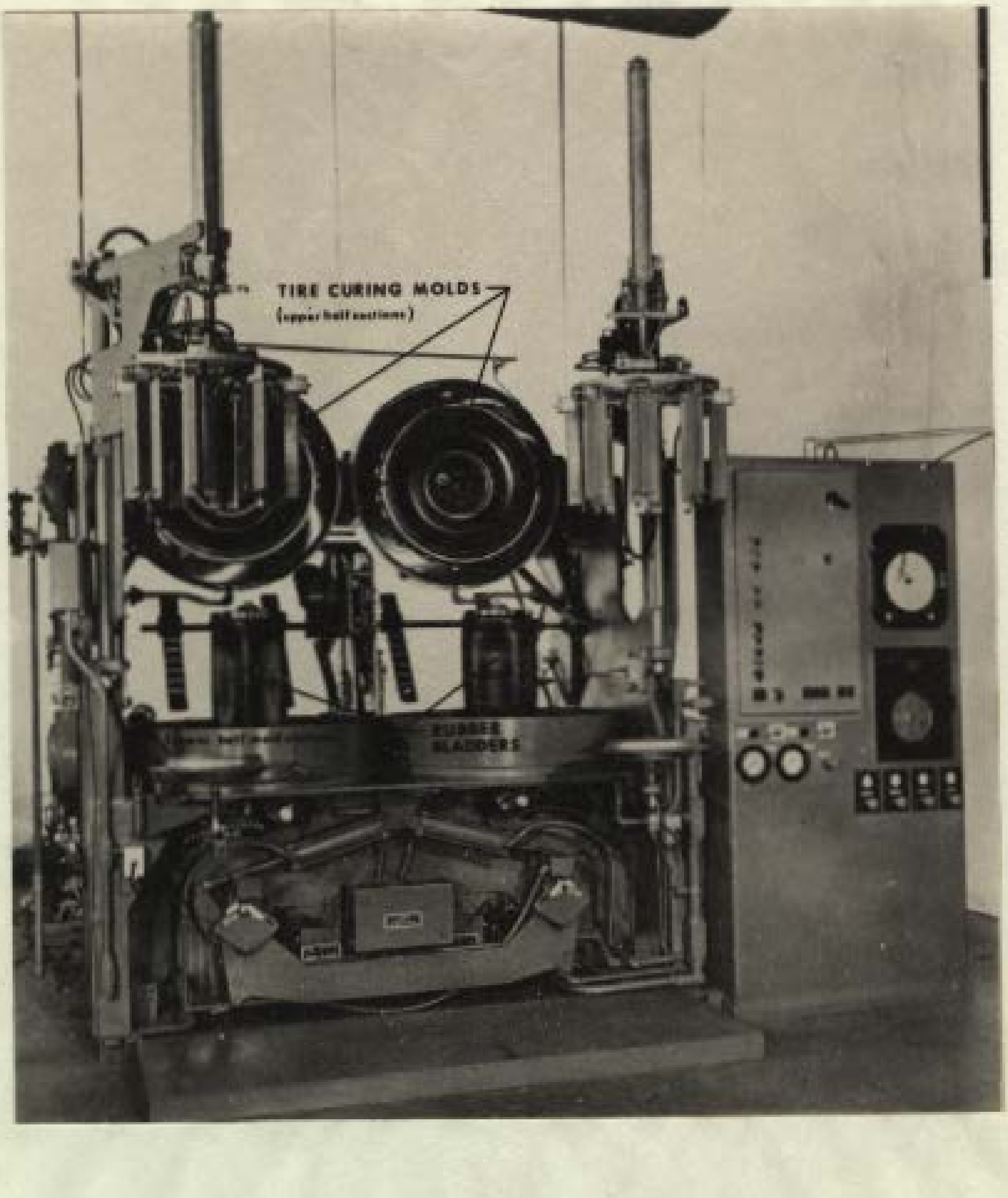

FIGURE IV(c) AUTOMATIC TIRE CURING PRESS
(open showing molds and bladders)

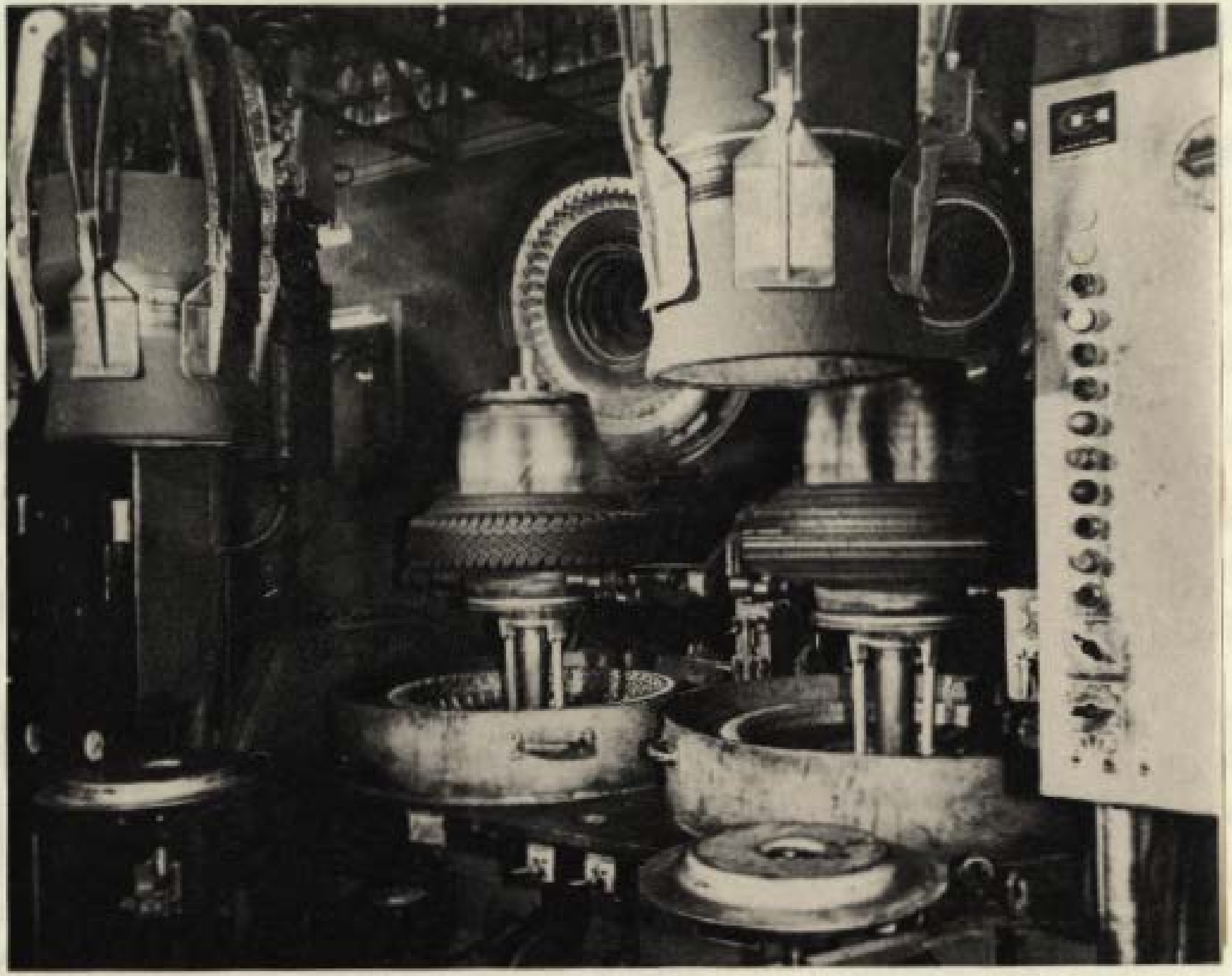

FIGURE IV(d) CLOSE-UP OF MOLD AREA AT CURE CYCLE COMPLETION with automatic loaders which enable the lines of such presses to be scheduled in a manner that allows completely random operation. This is possible since these machines do not require a machine operator to be present at the completion of the cure cycle. The lines of presses not equipped with automatic loaders can not be scheduled as freely. Here, an attempt must be made to keep tires with similar overall cure cycle times in those press lines. This consideration permits the line to be operated sequentially which, in turn, allows the operator to tend to each machine in an orderly manner (i.e., as he completes the loading of a press, the next press in the line has its cycle ending, allowing the operator to complete its unloading and subsequent loading and so on, until he has completely progressed down the press line). Another allocation criteria that is equipment-dependent concerns the available post cure inflation equipment. The newer presses are equipped with post cure inflators which, upon completion of the cure cycle, automatically mount and inflate the hot tires on rims. On the older presses the tires are manually removed from the presses, mounted on rims (resembling road rims), and inflated. Certain tire types do not lend themselves to easy manual mounting and are thus preferably located on presses equipped with the automatic inflation equipment (see Fig. V).

One of the more recent innovations in the vulcanization process has been the introduction of an incremental shaping process during which the green tire is deformed into the finished tire shape in phases rather than in one continuous procedure. It has been

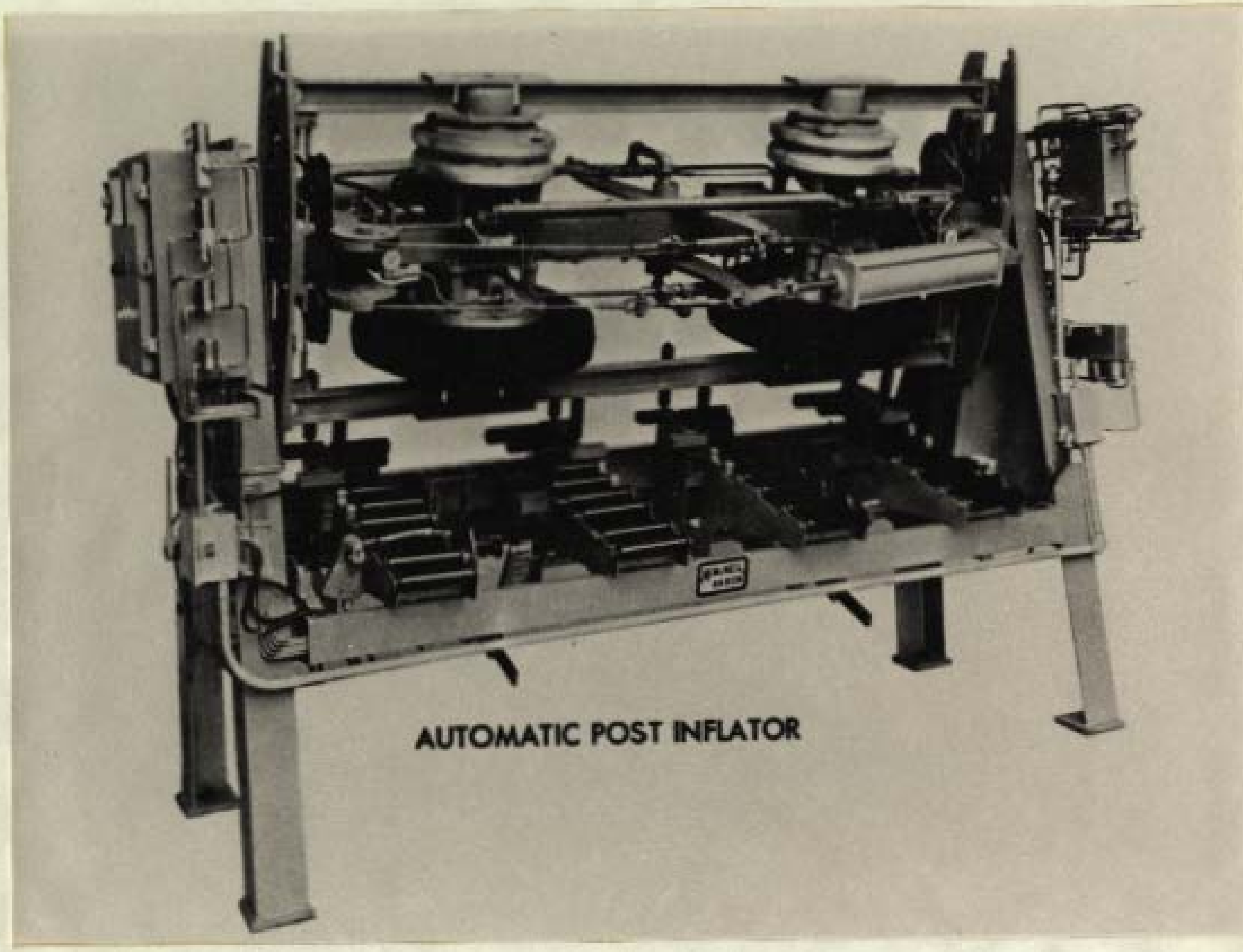

FIGURE V(a)

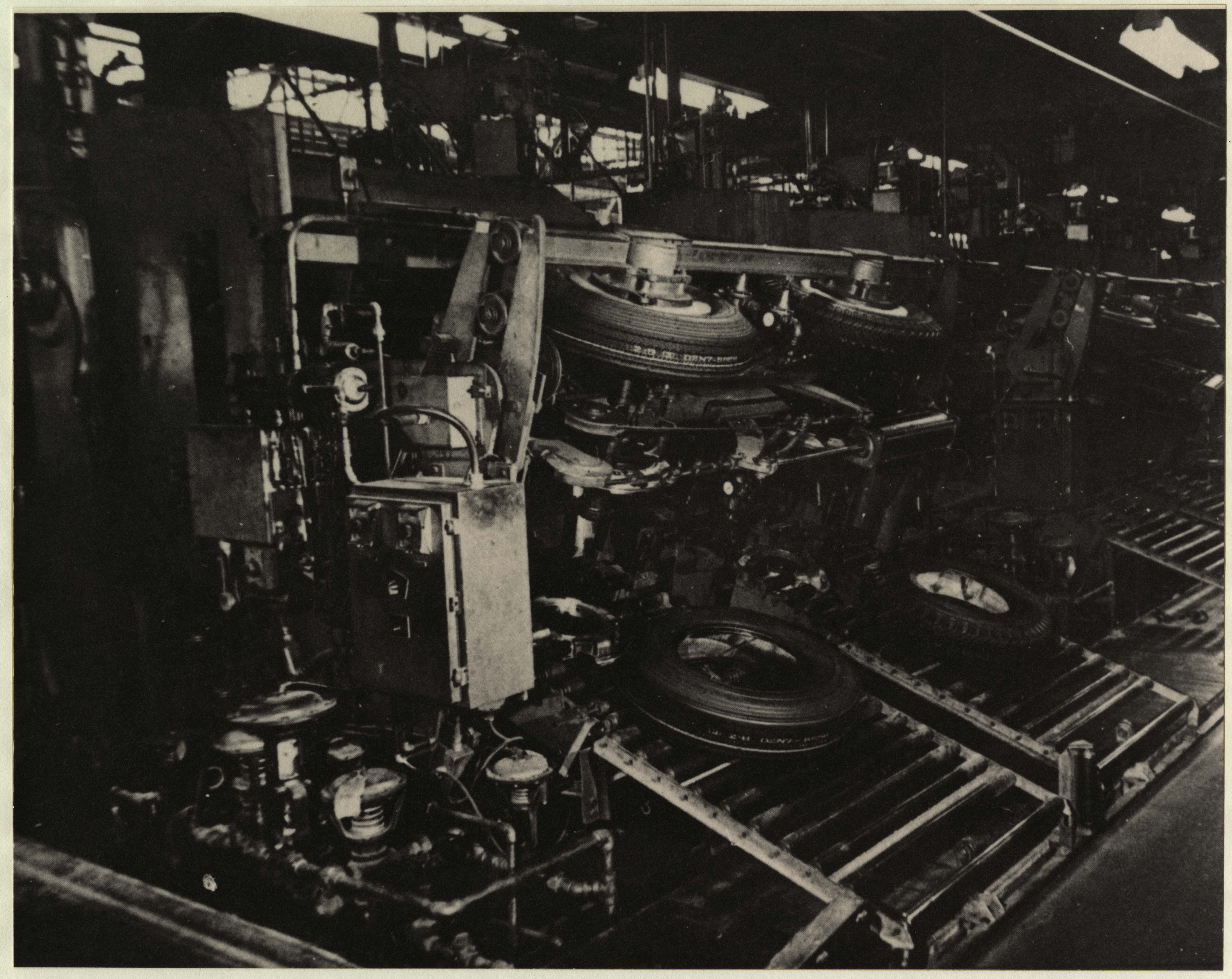

FIGURE V(b) Automatic Post Cure Inflator (Installed on A Press) established that by allowing the tire to stabilize in several stages of its deformation results in less variation in the radial force, a force evaluated about the tire's circumference, that is required to produce a standard deformation in the tire. The newer presses have this staged shaping incorporated into their cycle control, making them capable of single, double, or triple phase shaping. The older machines lack this facility. The earliest models of curing presses were equipped only with a single cycle controller to regulate the temperature and pressure cycles for both tire-curing positions. This means both tires must be identical in every aspect of their curing cycle. On the newer models, the manufacturers have included a separate controller for each mold position on the press. The result of the above considerations is an inconsistency in that the large volume models that require more than one curing location best fulfill the identical cycle requirements of the older machines. Unfortunately these are also the tires that should, for quality reasons, be allocated to the newer presses.

Different models of presses, due to inherent design features, produce varying degrees of quality or consistency in the cured product. For this reason the premium lines of tires should be allocated to the machines that consistently produce the best quality cure. Reliability is another factor that has to be considered when tires are being scheduled. The most important and high volume tires should, all other factors being equal, be scheduled on those presses that maintain the best service records.

IV.2 The Product

The large tire manufacturers market different lines of tires each consisting of a variety of sizes which in combination can result in approximately two thousand items in the product line-up for the company. This myriad of products can be broadly subdivided into three major classifications based on the tire construction: radial ply, Fibreglas belted, and conventional. Different cure processes are required for the different tire constructions. Thus, an obvious consideration for an allocation system is that the curing location being evaluated must be capable of fulfilling the specified curing requirements of the tire. (See Figure VI.)

Since the tire receives its external distinguishing features during the curing operation, there are cases where the different finished products are produced from the same green tire. The importance of this fact becomes significant in the allocation decision process in consideration of preprocess storage locations for the green tires (see Fig. VII). It is desirable to locate all common green tires in the general vicinity of each other.

For purposes of allocation of two tires to any one facility, the scheduler or scheduling system must also consider the relative heights of the green tires. If one of the tires is significantly longer than the other it would be impossible to arrive at a compatible shaping cycle for the two tires. This consideration arises from the fact that the taller tire has both beads in contact with the mold for a significantly longer period of time than has the shorter tire. The

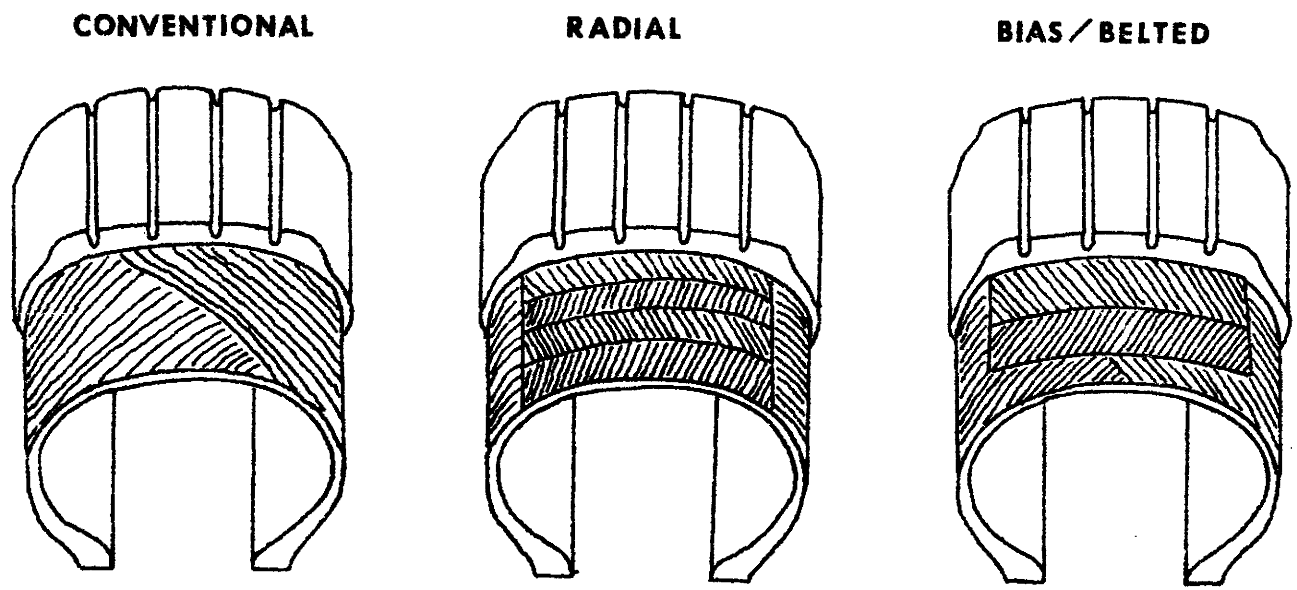

FIGURE VI Tire Constructions

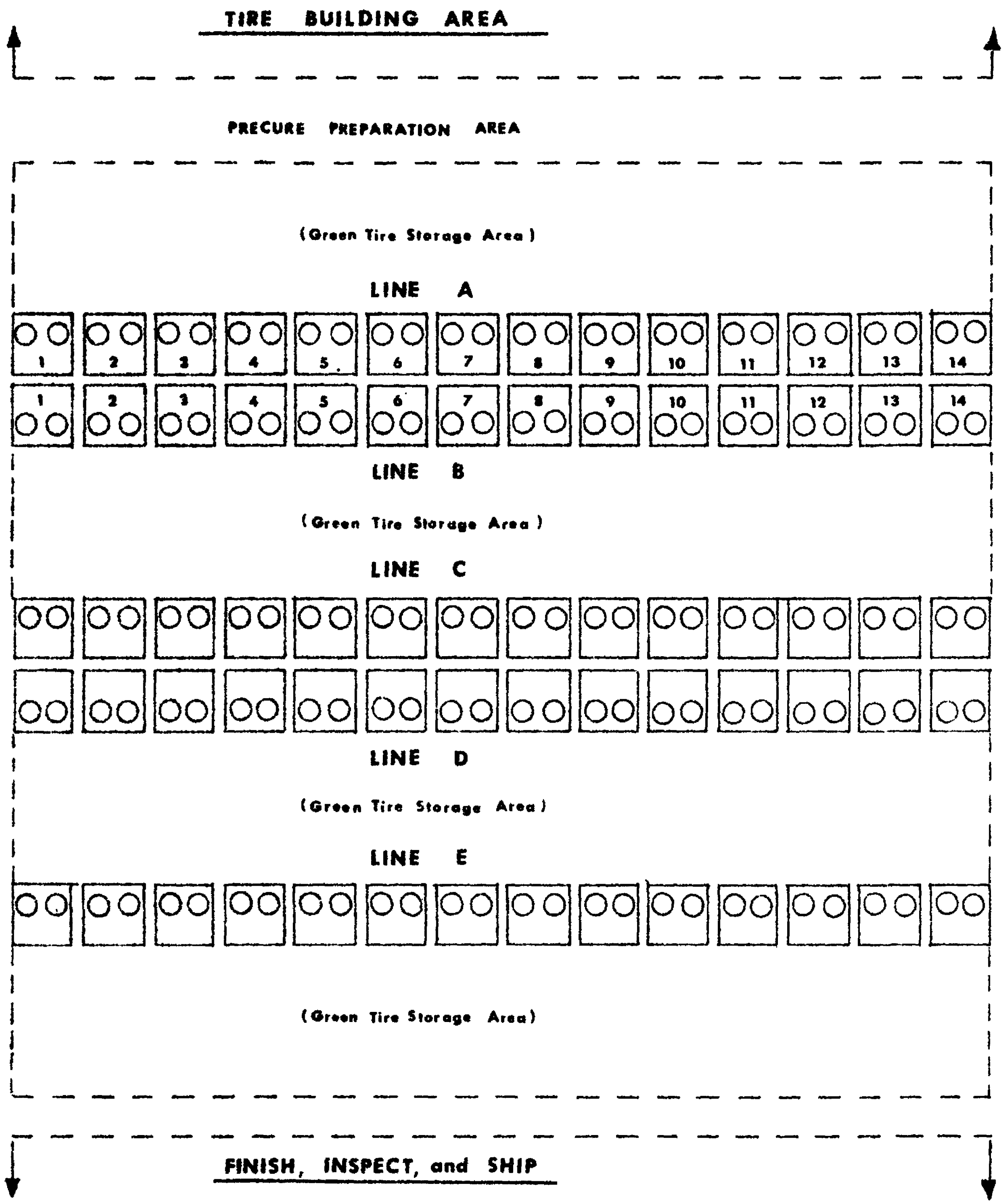

Figure VII Typical Cure Department Layout effect of failing to comply with this consideration is that by the completion of the first stage of deformation of the longer tire, the shorter tire has not been deformed enough to warrant a shaping pause. Similarly, if tires with bead diameter differences exceeding a prescribed limit are cured together, the result would be that the smaller diameter tire would have been deformed by the expanding bladder significantly in advance of the tire with the greater bead diameter.

The most important curing parameter to be considered by this system is the cycle time. If it is possible to satisfy the above requirements, it is most important that tires be matched by utilizing the base cure times. It is possible to give a tire a satisfactory cure at different overall cycle times by manipulating the other cure variables. An example would be to decrease the curing temperature and correspondingly increase the cycle time. There is a base cure cycle for each tire in the line. It is the cycle that has the shortest overall time while still maintaining a satisfactory cure. It is, therefore, imperative to attempt to have every tire curing on its base cycle, and consequently where this condition cannot be attained there will be wasted production machine time.

IV.3 The Procedure

The merchandise distribution department prepares a list of tires it requires to meet its distribution commitments and these tires then constitute the "on" tire list. Accordingly, since production positions are fixed in number, there must be an equal number of tires removed from production and these tires constitute the "off" tire list. After compilation these lists pass to the production control department whose responsibility it is to assign the "on" tires to the production equipment. In the present manual system, the cure scheduler obtains all the pertinent curing information for each "on" tire from catalogues. At this point, all "off" tires are removed from the master press schedule, and the scheduler attempts to fill these vacant cure positions with the "on" tires in accordance with the criteria discussed above. This procedure necessarily results in considerable relocation of tires already in the cure and a continual re-shuffling of the "on" tires. The scheduler, at the same time, must also attempt to minimize mold relocations since those changes are carried out at premium time on the weekend shifts. After a schedule has been compiled, it is returned to the merchandise distribution department with the suggested changes and deletions that the scheduler considers are required to complete an acceptable assignment. The schedule is then usually subjected to further modifications by the merchandise distribution people. Once a finalized schedule has been settled upon, it is returned to the production control department which uses this schedule of sizes, makes and quantities to schedule the tire-building machines. This process continues until all the upstream activities have been scheduled.

IV.4 Summary of the Problem Requirements

Upon receipt of a proposed production list of tires, it is necessary to make an optimal assignment of these tires to the manufacturing equipment. It is therefore necessary to schedule the individual tire to the press that has the most suitable attributes to cure the tire. The considerations to be made in this evaluation are the dimensional capabilities of the machine, the type of post-cure inflation equipment, the make and model of the press, the press operating condition, the service record, the cycle capabilities of the machine and the type of controls on the press, single or dual.

A further complication to this assignment is the duality feature of the press. It is the scheduler's responsibility also to consider that the tires being scheduled for any one press are themselves compatible in terms of green tire height, bead diameter, construction, and cure cycle.

It is also necessary to consider the relative locations of common green tires within the cure. In addition other considerations such as quality and machine reliability must be adhered to.

IV.5 Objectives of the Automated System

Since the cure is the final production stage of the tire, it is the first stage to be scheduled. This, by definition, means that the degree of closeness of this schedule to the optimum assignment has a very direct influence on the production of the entire plant. The aim of this project then is to utilize an optimization routine to determine the best possible assignment of tires to curing positions.

It should be noted that the techniques demonstrated by this system are also applicable to many of the upstream systems, most notable of which is the tire-to-tire machine allocation system. It is conceivable that this system could form the basis of a master scheduling system to control all the scheduling operations for the entire manufacturing process.

The necessity of finalizing the cure schedule before the rest of the operation can be scheduled, means a lengthy lead time from demand to implementation. This procedure invites what are termed "emergency changes". These are legitimate changes based on an unforeseen alteration in the market position of the company. It is hoped that the implementation of an efficient automated optimal system could substantially reduce this lead time. Such a reduction would benefit the company by allowing it to operate closer to the market with reduced inventories.

V LITERATURE SURVEY

V.1 Classical Methods - Development

Many of the problems of distribution and allocation of products have been formulated and solved by the application of linear programming techniques. An early realization that linear optimization methods could be gainfully employed in the allocation or "optimal utilization of machinery" was shown by the Russian Professor Kantorovich (1) in 1939. In his paper the author formulates three problems, termed A, B, and C, of which the first two, as shown by T. C. Koopmans (2), make use of a coefficient matrix which exhibits a form similar to the "effectiveness" matrix used in the transportation method of linear programming. Kantorovich further envisaged the application of the three problems to situations that include the assignment of items or tasks to machines in metalworking, in the plywood industry, in earth-moving, in trimming problems of sheet metal, in lumber, in paper, in oil refinery operations, in allocation of fuels to different uses, in allocation of lands to crops and on transportation equipment to freight flows.

Dantzig in a classical work in 1947 developed a numerical iterative technique known as the simplex method to solve linear optimization problems. As Thompson (16) notes the simplex technique is a general method by which any linear programming problem can be solved. The resource allocation problem lends itself more readily to one of several special procedures that simplifies the problem-solving process. One of these procedures is known as the transportation method of linear programming. The greatest advantage of this method is its computational simplicity. Bowman (12) cites an example of a problem-solving session at the Massachusetts Institute of Technology that demonstrated an increase of up to six hundred percent in the time to solve a problem that fit within the scope of the transportation method by using the simplex procedure. Several variations of the transportation procedure have been developed. The methods most frequently employed have been the "stepping-stone" suggested by Charnes and Cooper (17) and an algorithm suggested by Munkres (10).

The transportation method may be stated generally as follows:

if $N = \{1,2,\ldots\ldots,n\}$ and $M = \{1,2,\ldots\ldots,m\}$

$$\text{minimize } Z = \Sigma_i \Sigma_j a_{ij} x_{ij} \qquad (i \in M,\ j \in N) \qquad 5.1$$

subject to

$$\Sigma_j x_{ij} = p_i \qquad (i \in M) \qquad 5.2$$

$$\Sigma_i x_{ij} = b_j \qquad (j \in N) \qquad 5.3$$

$$x_{ij} > 0 \qquad (i \in M,\ j \in N) \qquad 5.4$$

For the case of balanced supply and demand, commonly referred to as the Hitchcock Distribution Problem, the following constraint is also applicable.

$$\Sigma_i p_i = \Sigma_j b_j \qquad 5.5$$

It should be noted that this constraint is required for the application of the above solution techniques. Thus, on occasion a slack destination or source must be added.

The transportation problem, as its name suggests, was first formulated as a special technique for determining a minimal cost program for transporting a product from several factories or manufacturing points to several distribution points or warehouses.

V.2 The Linear Assignment Problem

The assignment problem constitutes an allocation problem in which N activities are to be allocated to N facilities, and each facility can accommodate only one activity. If we consider each facility to be designated by $i = 1,2,\ldots\ldots,n$ and each activity designated by $j = 1,2,\ldots\ldots,n$, it is possible to construct an $N \times N$ matrix A where a_{ij} represents the productivity or effectiveness of activity i on facility j. The problem thus becomes one of assigning all activities to different facilities to optimize the overall effectiveness. An example application of this formulation that occurs is the assignment of personnel where N persons are to fill N jobs. In this case the effectiveness matrix A would be a measure of the individual's abilities at the task; such measures could be the number of man-hours required to perform the tasks, or, perhaps, the scores attained by the candidates on a set of aptitude tests. If the assumption is made that the activity is free to utilize any resource for any part of its total assignment, let x_{ij} be the fraction of time that activity i should utilize resource j, or in terms of the example cited, the fraction of time person i should perform task j.

The formulation of this problem thus becomes:

$$\text{minimize } Z = \Sigma_i \Sigma_j a_{ij} x_{ij} \qquad i \in N,\ j \in N \tag{5.7}$$

subject to

$$\Sigma_j x_{ij} = 1 \qquad j \in N \tag{5.8}$$

$$\Sigma_i x_{ij} = 1 \tag{5.9}$$

$$i \in N$$

$$x_{ij} > 0 \tag{5.10}$$

It can be seen from the above expressions that the assignment problem is a special case of the transportation problem with balanced demand and supply in which $M = N$, $a_i = 1$ $(i \varepsilon N)$ and $b_j = 1$ $(j \varepsilon N)$. Although the above equations represent a system of 2N constraint equations in N^2 variables, one of the constraint equations is not independent because of the known condition of balanced supply and demand. Thus, the classical assignment problem may be regarded as a linear programming problem having $(2N - 1)$ constraint equations in N^2 variables. There will be only $(2N - 1)$ basic variables in the optimal solution. The remaining nonbasic variables must be zero. Since the number of nonbasic variables is the difference between the total number of variables (N^2) and the number of basic variables $(2N - 1)$ then

$$N^2 - (2N - 1) = (N - 1)^2 \qquad 5.11$$

The above assignment problem is of a form that can be readily solved by the simplex method of linear programming although there is a possibility of considerable increase in computation if this is attempted. The most popular algorithm formulated for the solution of this linear assignment problem has been given by Kuhn (19). This algorithm is known as the "Hungarian Method of Assignment". Variants of this algorithm have been given by Munkres (10) and Flood (18).

Simple combinatorial algebra gives, for an N-dimensional assignment, N! distinct solutions. Munkres (10) shows that by assuming the worst possible conditions at each stage of his algorithm, the maximum number of operations needed is

$$(11N^3 + 12N^2 + 31N)/6 \qquad 5.12$$

This maximum is of theoretical interest since it is so much smaller than the number of operations necessary to formulate the N! possible solutions.

If the allocation of activities is limited to facilities such that each facility is utilized singularly by each activity, we have replaced the inequality constraint of $x_{ij} > 0$ by the integer constraint $x_{ij} = 0$, or 1. In a suitable and, perhaps more significant notation, the problem can be restated as:

$$\text{minimize } Z = \Sigma_i a_i \rho(i) \qquad 5.13$$

where ρ is a square permutation matrix of dimension N.

Balas (11) presents an interesting algorithm for the solution of a general linear program with zero-one variables. Essentially, this algorithm employs a tree-search technique that uses information generated in the search to exclude portions of the tree from consideration. This algorithm, from the examples included by Balas, seems to be a very efficient method for the solution of this type of problem. Balas cites a particularly ill-behaved example with 12 variables and 6 constraints that necessitated the investigation of 39 of a possible 4096 solutions.

V.3 The Quadratic Assignment Problem

One of the basic assumptions of the linear assignment problem is that the assignment of any one activity to utilize a facility in no way affects the economic return or effectiveness of any other activity on any other resource; or simply, there is no interaction between activities. Within the frame of reference previously cited, it is assumed that the personnel do not interact with each other in a manner so as to affect the overall efficiency of the total assignment.

The first published statement of the quadratic assignment problem was presented by Koopmans and Beckman (3), in the context of an analysis of economic acitivity. In this paper the quadratic assignment problem has been formulated for application to the assignment of manufacturing plants to geographical locations. Koopmans and Beckman state:

> The assumption that the benefit from an economic activity at some location does not depend on the uses of other locations is quite inadequate to the complexities of Locational decisions.[2]

The criteria considered in this analysis of the problem has been the cost of interplant material flows. In this formulation they consider the allocation of N plants to N locations. In the A matrix, referred to previously in the discussion of the linear problem, the element a_{ij} represented a net revenue. In this example the element a_{ij} of A represents a "semi-net" revenue from the operation of plant i at location j; that is, gross revenue less cost of primary inputs, but before subtracting the cost of transportation of intermediate products between plants. Thus, this semi-net revenue is still independent of the assignment of other plants to other locations. To express this interplant transportation cost they used two symmetric matrices, C and D, where the element c_{ij} of matrix C represents the commodity flows (in weight units) from plant i to plant j and element d_{ij} of matrix D

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[2]T. C. Koopmans and N. Beckmann, "Assignment Problems and the Location of Economic Activities", Econometrica, 25 (1957), 52-76.

represents the cost of transportation for the unit flow from location i to location j. The flow coefficients c_{ij} are assumed to be independent of the plant assignment, and are applicable to all amounts and compositions of flows. Koopmans and Beckmann made the further stipulation that the cost coefficients d_{ij} satisfy the triangular inequality

$$d_{ij} \leq d_{ik} + d_{kj} \qquad (i,j,k \in N) \tag{5.14}$$

which simply states that transportation from location i to location j via a third location k is not cheaper than direct transportation. The notation used here, for the purpose of continuity is the same as Gilmour, not that used by Koopmans and Beckmann. To find the total interplant transportation cost, it is necessary to begin with a known permutation ρ to evaluate the expression.

$$\Sigma_i \Sigma_j c_{ij} d\rho(i)\rho(j) \tag{5.15}$$

It follows that the total net revenue for an assignment thus becomes

$$Z = \Sigma a_i \rho(i) - \Sigma c_{ij} d\rho(i)\rho(j) \qquad (i,j \in N) \tag{5.16}$$

The quadratic assignment problem formulated is thus the maximization of the above expression. Koopmans and Beckmann point out that the designation of the quadratic assignment problem is rather arbitrary. The justification for this designation stems from the fact that the maximand contains a term in the second degree in the unknown permutation. This is more obvious in the notation of Koopmans and Beckmann where the equivalent expression to the above can be written

$$Z = \Sigma_k \Sigma_i a_{ki} p_{ki} - \Sigma_{kL} \Sigma_{ij} c_{kL} p_{ki} d_{ij} p_{ij} \qquad 5.17$$

where p is the desired permutation matrix.

Conway and Maxwell (8) suggest an approach to the problem of facility assignment that assumed no economic advantage of any location over any other location for an activity. In other words, they were primarily concerned with the interaction costs or interplant commodity transportation costs, between the facilities. Again, this paper formulates the problem by measuring the value of an assignment by summing, as the interaction costs, the permuted products of traffic and distance matrices. The approach utilized minimizes the value of this summation. In the case of the symmetric "distance" and "traffic" matrices there are $n = (N - 1)N$ different paths between pairs of locations. Let $D_1, D_2, \ldots\ldots, D_n$ represent the lengths of these paths ranked so that $D_1 > D_2 > D_3 \ldots\ldots > D_n$. Consider the $n = (N - 1)N$ pair of facilities; let $C_1, C_2, \ldots\ldots, C_n$ represent the traffic between these pairs ranked so that $C_1 < C_2 < \ldots\ldots < C_n$. Using these values construct an $n \times n$ matrix X such that

$$X_{ij} = D_i \cdot C_j \qquad 5.18$$

Thus, there will be an assignment that consists of selecting n elements of the X matrix, one element per row and one element per column. The sum of these n elements will then be the value of that assignment. The converse of the above statement is not true. Any n elements with one in each row and one in each column do not necessarily correspond to an assignment due to the possibility of an incompatible arrangement of facilities. Thus, the problem can be stated as: find the n elements of X, taking one from each row and one from each column which correspond to an assignment of facilities to locations, whose sum is minimal. Thus, a solution can be effected if the partial sets of n elements from X can be determined. The successive sets could be examined starting with the set with the smallest sum until a set that constitutes an assignment is discovered. Conway and Maxwell point out that the minimum sum set of n elements from X (one from each row and column) are the elements of the principal diagonal. They further point out that if by luck this set of elements corresponds to an assignment, the problem is solved. Since this will not always occur they have utilized an additional matrix Y whose elements are defined as:

y_{ij} = the minimum sum of n elements of the X matrix, one from each row and one from each column which includes x_{ij}.

This Y matrix demonstrates the following characteristics for any element y_{ab} on or above the principal diagonal, $y_{ab} < y_{ij}$ for all $i < a$, $j > b$. For any element $y_{mn} < y_{ij}$ for all $i > m$, $j < n$. Practically this Y matrix would not be computed but rather the properties discussed would be used to suggest that the search for a small sum set, which corresponds to an assignment, commence in the neighborhood of the principal diagonal and work out systematically from there. This solution technique still requires considerable luck and is computationally prohibitive.

Land (7) considered the problem of inter-related costs under the simplifying assumption of no economic advantage to any facility in any location, and formulated a numerical routine that utilized a systematic exploration of a limited part of the solution space. To illustrate his development, Land considered the problem of locating manufacturing departments in evaluating a plant layout. This problem is easily seen to be analogous to the previous problem of plant location. The method postulated by Land consists of constructing a cost matrix A which shows for each pair of departments, i and j, the cost which would be incurred if they were allocated to locations p and q. Thus, the problem has been reduced to the form of a linear assignment problem, and can be stated as:

$$Z = \Sigma a_i \rho(i) \qquad 5.19$$

where ρ is again a permutation matrix with one non-zero element in each row and column. Since there are n(n - 1)/2 possible pairs of departments, n(n - 1)/2 possible combinations of locations, the cost matrix A is square and of dimension n(n - 1)/2. Unfortunately, a solution of the above problem, for the same reason as Maxwell's formulation, is not necessarily a solution to the real problem. An example of this would be an assignment that gave an optimum cost that included department pair A and B in location pair 1 and 2, and department pair A and C in location pair 3 and 4. This, of course, is incompatible. Land's numerical routine is based on the following logic:

> Consider any one variable X_{wt} which assigns departments k and L to locations u and v, i.e., w = (kL) and t = (uv), then

$$\text{if } X_{(kL)(uv)} = 1 \qquad 5.20$$

$$X_{(kj)(pq)} = 0 \quad \text{or} \quad X_{(jk)(pq)} = 0^* \qquad 5.21$$

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*Either the left column equals zero or the right column equals zero.

$$X_{(iL)(pq)} = 0 \quad \text{or} \quad X_{(Li)(pq)} = 0 \qquad 5.22$$

$$X_{(ij)(uq)} = 0 \quad \text{or} \quad X_{(ij)(qu)} = 0 \qquad 5.23$$

$$X_{(ij)(pv)} = 0 \quad \text{or} \quad X_{(ij)(pv)} = 0 \qquad 5.24$$

$$k \neq i \neq L \qquad 5.25$$

$$k \neq j \neq L \qquad 5.26$$

$$u \neq p \neq v \qquad 5.27$$

$$u \neq q \neq v \qquad 5.28$$

There will be a minimum cost of the simple assignment problem under each of the mutually exclusive assumptions $X_{(wt)} = 0$ and $X_{(wt)} = 1$. The linear assignment costs on the basis of these two assumptions will represent lower bounds. Thus a decision tree can be created by eliminating areas of search. Pairs of departments can be assigned and a lower bound calculated. Thus, each branch of this "branch and bound" technique need only to be pursued until the lower bound exceeds the value of a known assignment. It is impossible to predict the length of the computation; in the extreme case where all costs are equal, the routine will generate all n! possible solutions.

The problem of scheduling classes in a university was the motivation for the investigation by Carlson and Nemhauser (4) of an algorithm to solve a quadratic assignment problem. The formulation presented had the following two major conditions. First, each activity must be scheduled on exactly one facility, and second, any number of activities can be scheduled on any single facility. Every combination of two activities scheduled on the same facility gives rise to an interaction cost. This technique has a particular application for assignments where the real assignment is not subjected to the constraint of one and only one activity or even a single pair of activities being allocated to one facility.

The fact that the quadratic assignment problem can be formulated as an equivalent integer linear program was demonstrated by Lawler (6). His approach defined a linear problem of n^4 variables y_{ijpq} from the quadratic problem of n^2 variables x_{ij} where, effectively

$$y_{ijpq} = x_{ij} x_{pq} \tag{5.29}$$

The objective function then becomes:

$$Z = \Sigma_{ijpq} c_{ijpq} y_{ijpq} \qquad \begin{array}{l} N = \{1,2,\ldots\ldots,n\} \\ (i,j,p,q \ \varepsilon\ N) \end{array} \tag{5.30}$$

subject to the constraints

$$\Sigma_j x_{ij} = 1 \qquad (j \ \varepsilon\ N) \tag{5.31}$$

$$\Sigma_i x_{ij} = 1 \qquad (i \ \varepsilon\ N) \tag{5.32}$$

$$\Sigma_{ijpq} y_{ijpq} = n^2 \tag{5.33}$$

$$x_{ij} + x_{pq} - 2y_{ijpq} > 0 \qquad (i,j,p,q \ \varepsilon\ N) \tag{5.34}$$

$$x_{ij} = 0, \text{ or } 1 \qquad (i,j \ \varepsilon\ N) \tag{5.35}$$

$$y_{ijpq} = 0, \text{ or } 1 \qquad (i,j,p,q \ \varepsilon\ N) \tag{5.36}$$

Reference (6) shows a proof that a feasible solution of the above linear problem corresponds to a feasible solution of the equivalent quadratic problem. Although the above formulation of the problem is theoretically interesting, it is not computationally useful considering the present state of integer linear programming.

Lawler realized the disadvantages of this technique of solution, and as an alternative suggested a technique of calculating a lower bound for the assignment as a basis for an algorithm to solve the quadratic problem. He further discussed the extension of this formulation to cover cubic.....n-adic problems. A discussion of the theory employed in the development of this method is omitted at this point, since, independently, Gilmore (9) and Lawler arrived at algorithms that are essentially identical when applied to the Koopman-Beckmann problem. Thus, discussion of this algorithm is included with the discussion of Gilmore's work.

Lawler did discuss several extensions of the application of the quadratic assignment problem. These were the "candidates problem" (see ref. (6)), the minimization of latency in magnetic drum computers, placement of electronic assemblies so as to minimize wire lengths, and various problems in the synthesis of sequential switching circuits.

Further and independent work on the placing of electronic modules on a computer backboard, in a manner so as to minimize the total wire lengths, has been performed by Steinberg (5). He formulated a solution based on a quadratic assignment problem; again, considering that one location could not attain an economic advantage by having any particular module scheduled to it. Steinberg used two matrices, C and D, where element c_{ij} represents the number of wires connecting module i to module j, and the element $d_{\alpha\beta}$ represents the distance from location P_α to P_β. From this it can be seen that matrices C and D must be symmetric with their main diagonals all zero. Steinberg's algorithm proved very successful in this particular application.

As an extension to the work of Steinberg, and independently parallel to the work of Lawler, Gilmore (9) formulated three distinct algorithms for the solution of the quadratic assignment problem. One of these algorithms is an optimal solution technique while the other two are suboptimal. Gilmore based his algorithms on a minor revision to the original formulation by Koopmans and Beckmann. The revision consists of minimizing not just an interaction sum but the interaction sum and the sum for an ordinary assignment problem. This can be stated as:

$$\text{minimize } Z = \Sigma_i a_i \rho(i) + \Sigma_{ij} c_{ij} d\rho(i)\rho(j) \qquad 5.37$$

where C and D are two matrices such as those employed by Steinberg in the wiring problem.

Gilmore's algorithm, similar to Lawler's, utilizes a branch and bound technique to limit the area of search for an optimal solution. Let α be a partial permutation for which ρ is a completion. By a partial permutation of $1,2,\ldots\ldots,n$ means a 1-1 map of some subset of $\{1,2,\ldots\ldots,n\}$ into $\{1,2,\ldots\ldots,n\}$. An extension of a partial permutation is a permutation or partial permutation ρ such that $\alpha(i) = \rho(i)$ for all i for which α is defined. If Z_o is the smallest possible $Z(\rho)$ then is a satisfactory solution if and only if $Z_o = Z(\rho)$.

If we assume we are given any permutation ρ, not knowing Z_o to determine whether ρ is satisfactory, or not, it is necessary to determine whether there exists a permutation π such that $Z(\pi) < Z(\rho)$. If α is any partial permutation for which $Z(\alpha) > Z(\rho)$, then no completion π of α can satisfy $Z(\pi) < Z(\rho)$ since necessarily $Z(\pi) > Z(\alpha)$. Gilmore's algorithm exploits this simple fact by generating a succession of permutations $\pi_1, \pi_2, \ldots\ldots, \pi_k$ such that $Z(\rho) > Z(\pi_1)$, $Z(\pi_1) > Z(\pi_2)$, $\ldots\ldots, Z(\pi_{k-1}) > Z(\pi_k)$, and such that π_k is satisfactory.

The efficiency of this algorithm is dependent upon the closeness of the initial permutation ρ to the actual minimum. The two suboptimal algorithms given by Gilmore could be used as a means to determine the initial permutation. The suboptimal algorithms are based on the premise that the problem of determining a permutation ρ to minimize $Z(\rho)$ could be regarded as an $(N - 1)$ stage decision process where at each stage two numbers, i and j, must be chosen by some guiding decision criteria until a complete permutation ρ has been constructed. These suboptimal algorithms when applied to Steinberg's wiring problem compete very well with Steinberg's algorithm. Details of these algorithms are included in Appendix A of this thesis.

VI MATHEMATICAL FORMULATION

For this particular problem where tires are to be allocated to tire-curing presses, a typical problem of scheduling N tires on N/2 presses can be considered. The first and most obvious decision criteria in evaluating an allocation arrangement is the suitability of the tire being scheduled to the tire-curing press. As a brief review the parameters to be considered in this evaluation are as follows:

(1) overall mold height;

(2) overall mold diameter;

(3) shaping phases required;

(4) post-cure inflation equipment required; and

(5) press rating.

If the facility is considered as consisting of N curing locations rather than N/2 presses, it is possible to formulate a simple linear assignment problem where the objective function to be minimized would be the unused machine attributes. This is a valid aspiration on the basis that it is consistent with the secondary consideration of forcing the higher precision tires towards the newer and more versatile machines. Thus, the formulation consists of measuring the "cost" resulting from scheduling any tire on any facility, and of arranging these "costs" into an assignment matrix A. The term "cost" as used in this context is not a cost in monetary terms; rather, a measure of the suitability of a tire to a press. Thus element a_{ij} is the cost of scheduling tire i into curing location j.

As mentioned the cost is a measure of the machine attributes used and the more closely a tire comes to making full use of the machine, the lower the cost.

This can be stated as

$$\text{minimize } Z = \Sigma_i a_i \rho(i) \qquad (i \in N) \tag{6.1}$$

Again ρ is an $N \times N$ permutation matrix with one non-zero element per row and per column.

An element a_{ij} of matrix A is evaluated from the following relations:

$$d_1 = P_{MHT_j} - T_{MLDH_i}{}^{*} \tag{6.2}$$

$$d_2 = P_{MW_j} - T_{MLDW_i} \tag{6.3}$$

$$d_3 = P_{SHP_j} - SHAPE_i \tag{6.4}$$

$d_4 = 0$ (if press rating and tire rating are compatible) 6.5

$d_4 = 100$ (if above is not true) 6.6

$d_5 = 0$ (if press post cure inflation equipment and the tire requirements are compatible) 6.7

$d_5 = 100$ (if above is not true) 6.8

$$a_{ij} = d_1 x w_1 + d_2 x w_2 + d_3 x w_3 + d_4 x w_4 + d_5 x w_5 \tag{6.9}$$

The variables w_ρ ($\rho = 1,2,\ldots,5$) are weighting factors that consist of two components: a magnitude adjustment factor and an influence

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*See Appendix C for variable definitions.

factor. The use of these variables will be explained more fully later by means of an example. (See also Section C.3.)

If any of relations 6.2, 6.3, and 6.4 are negative, a "NO GO" condition exists and the value of a_{ij} is arbitrarily set at 10000. Thus a "NO GO" situation is formally treated as a high cost situation.

The linear assignment problem, stated above, makes the assumption that the economic benefit of a tire in a curing location does not depend on the uses of the other locations. This is obviously inadequate for the real problem. The overall cure efficiency is greatly affected by the interaction of the tires scheduled into adjacent curing locations. The extreme case of this occurs when a curing location becomes invalid, even with a low attributes cost, to a tire on the basis of violating one of the compatibility considerations listed below and described previously. There is also an interaction that occurs from the relative positions of similar (in the extreme "common green") tires in the adjacent presses and press lines. Thus an interaction "cost" can be considered to exist between tires based on their similarities. Again, the term "interaction cost" is a relative one that is a measure of the similarities of two tires. Therefore, all the possible comparisons of N tires can be expressed in an N × N matrix C where c_{ij} is the interaction cost of tire i and tire j. The value of C is small for tires that are similar and high for dissimilar tires. In the limit $c_{ij} = 0$, $i \neq j$, and i and j are common green tires.

A brief review of the parameters evaluated for compatibility considerations follows:

(1) bead diameter;

(2) construction;

(3) post-cure inflation pressure;

(4) number of shaping phases;

(5) cure times (basic); and

(6) green tire heights.

The element c_{ij} of the C matrix is evaluated by means of the following relationships:

$$h_1 = ABS(BW_i - BW_j) \tag{6.10}$$

$$h_2 = 0 \text{ (if the construction of tires i and j are the same)} \tag{6.11}$$

$$h_2 = 10 \text{ (if the above condition is not true)} \tag{6.12}$$

$$h_3 = ABS(PCIP_i - PCIP_j) \tag{6.13}$$

$$h_4 = ABS(SHAPE_i - SHAPE_j) \tag{6.14}$$

$$h_5 = ABS(CT_i - CT_j) \tag{6.15}$$

$$h_6 = ABS(GTHT_i - GTHT_j) \tag{6.16}$$

$$\text{If } h_6 > 1.5, \text{ then } h_6 = 10 \times h_6 \tag{6.17}$$

$$c_{ij} = h_1 x w_6 + h_2 x w_7 + h_3 x w_8 + h_4 x w_9 + h_5 x w_{10} + h_6 x w_{11} \tag{6.18}$$

The C matrix, in this case representing the interaction of activities, is comparable to Steinberg's (5) matrix C that represented the number of wires connecting each module to every other module. In this problem, consistent with the "backboard wiring problem", the C matrix has all zeros on the principal diagonal, and it is symmetrical.

To complete the synthesis of the tire scheduling problem as a quadratic assignment problem, it is necessary to consider the interrelationships of the tire-curing locations. The considerations suggested in the previous discussion of the problem are mainly locational in nature.

A brief review of these locational considerations follows:

Are locations i and j

(1) on the same press?

(2) in the same press line?

(3) in adjacent press lines?

(4) on the same make of press? or

(5) on the same model of press?

Thus, the relationships between each cure location can be established and described by an N × N matrix D where element d_{ij} is a numerical representation of the relation of location i to location j.

Element d_{ij} is calculated as follows:

f_1 = 100 (if location i and j are on the same press) 6.19

f_1 = 0 (if above relation is not true) 6.20

f_2 = 20 (if i and j are in the same press line) 6.21

f_2 = 15 (if i and j are in adjacent press lines) 6.22

f_2 = 5 (if i and j are separated by at least one press line) 6.23

f_3 = 10 (if i and j are on same make of press) 6.24

f_3 = 5 (if i and j are on different makes of presses) 6.25

f_4 = 10 (if i and j are on same model of press) 6.26

f_4 = 5 (if i and j are on different models of presses) 6.27

$$d_{ij} = f_1 x w_{12} + f_2 x w_{13} + f_3 x w_{14} + f_4 x w_{15} \qquad 6.28$$

From the above relationships it can be seen that the highest value of element d_{ij} occurs when location i and j are a pair on the same press. The lowest value occurs if i and j are on dissimilar presses in different and not adjacent press lines. The D matrix used in this problem corresponds in nature and usage to the distance matrix D of Steinberg's wiring problem.

It is now possible to write the objective function as:

$$\text{minimize } Z = \Sigma_i a_{i\rho(i)} + \Sigma_i \Sigma_j c_{ij} d_{\rho(i)\rho(j)} \qquad 6.29$$

which is similar to the form presented by Gilmore (9).

In order to minimize Z it is necessary to minimize both terms contained in it. The first term, as mentioned previously, is a minimum when the fewest machine attributes remain unused. It meets the initial requirement that each of the elements a_{ij} of the assignment matrix is the benefit obtained by locating tire i in facility j and is independent of the assignment of other tires to other facilities. Therefore, the minimization of this quantity drives the solution to allocate the tires to the most suited facility.

The second term, the quadratic term, consists of the sum of the products of the interaction costs of tires i and j in the element c_{ij} and the interaction costs d_{ij} of the facilities on which tires i and j are scheduled. Simple combinatorial analysis shows that a minimum occurs for the sum of a permuted product when the largest of one element is combined with the smallest of the other. In other words, the minimum occurs when the tires with the lowest interaction cost c_{ij} are scheduled to the pairs of facilities with the highest interaction cost d_{ij}, and the pair of tires with the highest interaction cost c_{ij} are scheduled to the facilities with the lowest interaction cost d_{ij}. Therefore, the minimization of this second term will drive the tires that have the greatest number of similarities towards a single press since the highest value of d_{ij} occurs when i and j are on the same facility. The minimization will also drive similar tires to facilities in one area of the department since the next highest value of d_{ij} occurs when i and j are in the same line of presses. The next increase in the value of d_{ij} occurs when i and j are on the same make and model of press. The result of this is a tendency of the solution to keep the similar tires grouped in the same area within a line of machines.

To summarize, the minimization of the optimization function results in:

(1) matching tires to presses on the basis of minimizing unused press attributes;

(2) matching tire to tire on basis of similarities in construction, model, cure parameters, and styles;

(3) grouping similar tires in an area in the cure; and

(4) grouping the most alike tires on a single press.

The action of this solution technique on this problem is analogous to the backboard wiring problem with the added consideration of an advantage gained by the location of a module to a particular position on the backboard. As stated previously, c_{ij} represents the number of wires from module i to module j and d_{ij} the distance between location i and j. To minimize wire the modules with the highest number of wires connecting them must be located close to each other.

At the same time the advantage gained by the location of these modules in these positions must be considered. Therefore, as with the tire problem, the minimum occurs when the highest c_{ij} is combined with the smallest d_{ij} and vice versa.

VII THE SYSTEM

The primary purpose of this thesis has been to demonstrate that the principles of formulating the tire allocation problem as a quadratic assignment problem could be employed as the basis of an automatic system to solve a real problem for The Goodyear Tire and Rubber Co. of Canada Limited.

It has been the intent of this project to generate as general a solution as possible, while still satisfying the requirements of the particular system under study. It must be emphasized that the discrete values used to demonstrate various points are ficticious and in no manner represent the operating position of the Goodyear Tire and Rubber Co.

The overall design considerations of the system are these:

(1) To arrive at an optimum allocation of tires to tire-curing presses, the optimum being a function of both quality and quantity.

(2) To minimize expensive internal mold relocation.

(3) To design the input and output operations such that the system can be utilized as a "black box" routine which does not require the user to have a complete understanding of its mode of operation.

Consideration of the second requirement, the minimization of internal mold relocations, suggests that as an initial trial the oncoming N tires should be tried in the N vacant cavities. Thus, if an optimal arrangement occurred it would represent the most ideal condition--optimum production with no rearrangement of existing curing positions. However, this is an unlikely occurrence, and the method of allocation on the surface appears to be inconsistent with the optimization of the entire operating position of the plant. This inconsistency can be dealt with by several different techniques. One of these would be the addition of any tires in the present cure to both the "on" and "off" lists. This procedure would effectively bring these tires into consideration by the system. The limit of these additions would be the inclusion of the balance of the cure to these lists, and the subsequent re-allocation of all the tires in an optimal manner would occur. The difficulty, obviously, would be the unprofitable number of mold changes that would occur. The cost of mold changes would likely exceed the increase in profit derived by the total re-allocation.

Another approach would be to apply a suboptimal system that evaluates every tire on the production list and which arbitrarily decreases the linear independent assignment cost a_{ij} if tire i in the previous assignment occupied facility j. The arbitrary constant could be adjusted by the user until, by experience, the number of mold changes occurring would be economical. Thus, each tire would have a preference for its present position. It would only be re-located if in the newly assigned position a more beneficial return was produced than in its compensated present position. The advantage of this system is the inclusion of every tire in the cure for the evaluation of the interaction cost matrix C.

The approach utilized in this system introduces the assumption that there will be a value that assesses the performance of each press in curing the tires assigned to it. If the least desirable assessment is represented by a high numerical value, then an additional assumption is that there is a value of this parameter that can be set by each plant below which a press can be considered to be operating satisfactorily. This is effectively establishing an allowable tolerance to the optimum operating position within which the system is considered to be functioning acceptably. With this consideration, the first allocation attempt made by this system is to fit N tires specified on the list of additions to the locations vacated by the N tires being removed from production. This does not exclude the possibility of supplementing additional tires from the present cure to these lists for consideration by the system. However, no attempt is made to deter the relocation of these additional tires. A first fit is attempted and tested according to the above criteria; if acceptable the system terminates. If the results are not satisfactory, the effectiveness of every press is evaluated, and those that exceed the user set minimum are then re-evaluated amongst themselves. This procedure is then repeated until either the condition for every press having an acceptable effectiveness coefficient or the number of mold relocations has exceeded a user set value, or there has been no further improvement in the overall cure.

The attempt to fit the new tires to the vacant locations is not an unreasonable starting condition because the partial problem does not require the relative line positions of various tires to be considered. For this installation the exclusion of this consideration is not serious since like machines are themselves grouped. Therefore, the machine suitability considerations tend to drive the solution to schedule similar tires in the general proximity of each other--although this is not guaranteed. In the general case, the consideration of relative positions may have more importance. However, it is unlikely that these considerations would take preference over the suitability or combinatorial considerations, neither of which is affected by dealing with partial solutions. Thus, if an optimal routine was employed on their initial allocation and the previous schedule had been satisfactory, this approach will at least not drive the solution into an infeasible area on the basis of previously allocated tires. The advantages of employing this approach are as follows:

(1) It enables the user to control directly such things as the number of allowable mold changes.

(2) It enables him to observe the fit that occurs at each stage.

(3) The size of the problem is substantially reduced at any one stage.

As example of this last point, an installation consisting of P locations (P/2 presses), the previous approaches would necessitate the evaluation of three $P \times P$ matrices. For a typical Canadian plant this could easily exceed the capacity of most industrial computing devices. The above approach only requires the evaluation of three $N \times N$ matrices, where N is usually in the order of P/10 - P/4 for an average plant.

The algorithms chosen to solve the quadratic assignment problem formulated by this system were the two suboptimal algorithms described by Gilmore (9). These algorithms were chosen in preference to Gilmore's or Lawler's optimal algorithm, based on the computation of lower bounds since Gilmore states that this algorithm becomes unrealistic for fifteen or more activities. The problem here is consistently larger in magnitude than this limit allows.

The Carlson-Nemhauser (4) algorithm was considered because it is based on the concept of assigning activities considering there to be an interaction cost arising from every combination of two activities allocated to the same facility. This appears to be exactly the case with the tire assignment problem. However, the algorithm made the additional assumption that any number of activities could be scheduled on any single facility. This, of course, is a violation of a real constraint of this problem.

A further, rather arbitrary, decision made in the design of this system was the selection of an algorithm for the solution of the linear assignment problem, formulated by and necessary for the solution of Gilmore's algorithm for the quadratic problem. The algorithm selected was Munkre's (10). The reason is no more sophisticated than that it was the one suggested by Gilmore. It is believed that any one of several excellent algorithms for this linear problem could have been utilized with equal success.

The system, presented here relies on the user to supply, by means of punched cards, all of the pertinent cure facts for every tire to be scheduled. In a real installation this information could easily be stored in a random access file and the information could be referenced by part number.

Because Goodyear Tire and McMaster University have different computer systems it became prohibitive to attempt to duplicate Goodyear's files at McMaster's machine. Therefore, to demonstrate the workability of the solution, ficticious inputs have been used for the sample problem solved in Appendix D.

A system flow-chart follows (see Fig. VIII).

VII.1 Sample Calculation

The purpose of this section is to clarify the description of the system given in the preceeding section by means of a numerical example. Where possible the variables here have the same meaning as the variables used in the program in Appendix C.

VII.1(a) The Problem

Consider the problem of scheduling ten new tires in an installation similar in arrangement to the one shown in Figure VII.

(a) The Equipment: The pertinent information about each press is given in Table I.

(b) The ON and OFF Tire Lists: These lists, including the relevant cure cycle information for each ON tire, are given in Table II.

STEP 1.

(a) Search the existing cure for the tires listed on the OFF tire list. These locations are then numbered 1 to N (N = 10 for this problem). See Table II for a list of available locations.

STEP 2.

From Table II it can be seen that the first six tires being removed empty 3 complete presses. Similarly, tires 9 and 10 empty a complete press. However, tire 7 empties only half of press B14

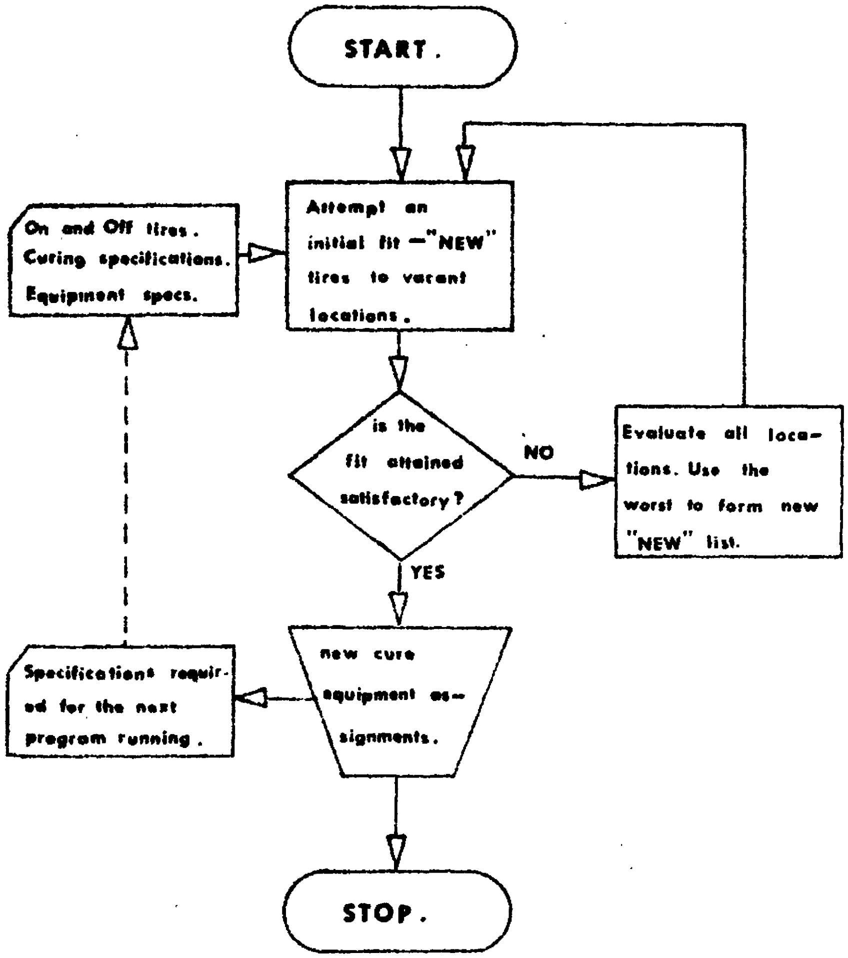

Figure VIII The System

and tire 8 empties only half of press C14. In order to be able to compute the compatibility relations for the establishment of matrix C, it is necessary to add the tires left on these presses to the ON tire list and increase N by 2. (The cure parameters for these tires are given in Table 2.)

STEP 3.

The problem is now to be formulated as a quadratic assignment problem. The first step in this formulation is the evaluation of the suitability relations 6.2 to 6.9 in order to complete matrix A.

Sample calculations for element a_{25} follows:

$$d_1 = PMHT_5 - TMLDH_2$$
$$= 11.5 - 10$$
$$= 1.5$$

$$d_2 = PMW_5 - TMLDW_2$$
$$= 38 - 35$$
$$= 3$$

$$d_3 = PSHP_5 - SHAPE_2$$
$$= 3 - 3$$
$$= 0$$

Since tire 2 (TIRE N202) is a high quality tire requiring precision curing and since press 5 (B13) has only got a medium rating, then $d_4 = 100$.

Since tire 2 requires automatic post-cure inflation and since press B13 is equipped with manual inflation equipment, then $d_5 = 100$.

Therefore

$$a_{25} = d_1 x w_1 + d_2 x w_2 + d_3 x w_3 + d_4 x w_4 + d_5 x w_5$$
$$= (1.5 \times 100) + (3 \times 100) + (100 \times 1) + (100 \times 1) = 650$$

The purpose of the weighting factors is to allow the user to adjust the affect of each of the above factors on the assignment. As an example of this a user may deem a difference in the required number of shaping phases and the available number of phases of one to be twice as important as a three-inch difference in available and required mold heights. Therefore, using the given value of w_1, w_3 should equal 6.

$$w_3 = 2 \times (w_1 \times d_1)/100$$
$$= 2 \times (100 \times 3)/100$$
$$= 6$$

A table of the values of the weighting factors used in this example is given in table III.

The remaining elements of matrix A are shown in Table IV.

The next step required to formulate this problem as a quadratic assignment problem is the evaluation of the compatibility relations 6.10 to 6.18.

Sample calculations for element c_{18} (tire 1 compared to tire 8) follows:

$$h_1 = |Bw_1 - Bw_8|$$
$$= 11 - 10$$
$$= 1$$

Since the construction type of tire 1 (TIRE N201) is the same as the construction type of tire 8 (TIRE N208), then $h_2 = 0$.

$$h_3 = |(PCIP_1 - PCIP_8)|$$
$$= |40 - 35|$$
$$= 5$$

TABLE I - EQUIPMENT

PRESS	MAKE*	MODEL (MOD)	RATING (IPRAT)	MAX. MOLD HEIGHT (PMHT)	MAX. MOLD DIAMETER (PMW)	NUMBER OF SHAPING PHASES (PSHP)	TYPE OF CONTROL (ICONTR) **	TYPE OF PCI UNIT (IPPCI) **
A1-A2	BOM	M1	1	10.5	43	1	S	M
A3-A4	BOM	M1	1	10.5	38	2	S	M
A5-A6	BOM	M1	1	11.5	38	2	S	M
A7-A13	BOM	M1	1	10.5	43	1	S	M
A14	BOM	M1	2	10.5	38	3	D	M
B1	BOM	M1	2	12.5	38	3	D	M
B2-B14	BOM	M1	2	11.5	38	3	D	M
C1-C2	NRM	46	3	14.5	46	3	D	A
C3	NRM	40	3	14.5	40.5	3	D	A
C4	NRM	46	3	14.5	46	3	D	A
C5-C6	NRM	40	3	14.5	40.5	3	D	A
C7-C11	BOM	M1	2	11.5	38	3	D	A
C12-C13	NRM	40	3	14.5	40.5	3	D	A
C14	BOM	M12	1	13	40	2	D	M
D1-D5	BOM	M12	1	13	40	2	D	M
D6-D14	BOM	M10	3	13	40	3	D	A
E1-E4	BOM	M10	3	13	40	3	D	A
E5-E6	BOM	M12	1	13	40	2	D	A
E7-E14	BOM	M10	3	13	40	3	D	A

*BOM: McNIEL CORPORATION - BAG-O-MATIC PRESS.
NRM: NATIONAL RUBBER MACH. CORPORATION - AUTOFORM VULCANIZER.

**S - SINGLE CONTROL
D - DUAL CONTROL

***M - MANUAL POST INFLATOR
A - AUTOMATIC POST INFLATOR

TABLE II - ON TIRE LIST

PART NUMBER	FACTORY CODE NO.	GREEN TIRE HEIGHT (GTHT)	CURE TIME (CT)	NO. OF SHAPING PHASES (SHAPE)	MOLD HEIGHT (TMLDH)	MOLD DIAMETER (TMLDW)	BEAD DIAMETER (BW)	PCI PRESSURE (PCIP)	RATING (IRAT)	TYPE OF PCI IPCI*	CONSTRUCTION (CONST)	CURE (CURE)	REF. NO.
TIREN201	4201	20	14.1	3	12	40	11	40	2	1	STD	BOM	1
TIREN202	4202	22	13	3	10	35	10	35	2	1	STD	BOM	2
TIREN203	4203	18	12.2	3	11	33	13	35	2	1	STD	BOM	3
TIREN204	4204	20	14.1	3	12	40	11	40	2	1	STD	BOM	4
TIREN205	4205	23	15	3	11	33	13	35	2	1	STD	BOM	5
TIREN206	4206	18	12.2	3	11	33	13	35	2	1	STD	BOM	6
TIREN207	4207	21	13.8	2	10.5	31	12	35	1	1	STD	BOM	7
TIREN208	4208	22	13	3	10	35	10	35	2	1	STD	BOM	8
TIREN209	4209	23	15	3	11	33	13	35	2	1	STD	BOM	9
TIREN210	4210	21	13.8	2	10.5	31	12	35	1	1	STD	BOM	10

ADDITIONAL TIRES FROM PRESSES B14 AND C14

TIREN091	4091	20	15	3	11.5	38	14	35	2	0	STD	BOM	11
TIREN093	4093	20	15	2	13.0	40	14	35	2	0	STD	BOM	12

THE OFF TIRE LIST

PART NUMBER	FACTORY CODE	IN PRESENT LOCATION (AVAILABLE)	REFERENCE NUMBER
TIREN061	4061	D1	1
TIREN061	4061	D1	2
TIREN117	4117	D14	3
TIREN117	4117	D14	4
TIREN033	4033	B13	5
TIREN033	4033	B13	6
TIREN091	4091	B14	7
TIREN093	4093	C14	8
TIREN076	4076	E6	9
TIREN076	4076	E6	10

TABLE III WEIGHTING FACTORS

W_1 = (mold height differences) = 100

W_2 = (mold diameter differences) = 100

W_3 = (shaping phase differences) = 100

W_4 = (rating differences) = 1

W_5 = (P.C. I. equipment differences) = 1

W_6 = (bead diameter differences) = 20

W_7 = (construction differences) = 1

W_8 = (P.C. I. pressure differences) = 1

W_9 = (shaping phase differences) = 40

W_{10} = (cure cycle time differences) = 100

W_{11} = (green tire height differences) = 20

W_{12} = (same press for two locations) = 1

W_{13} = (relative press lines) = 1

W_{14} = (makes of presses) = 1

W_{15} = (models of presses) = 1

TABLE IV   THE A MATRIX

100	100	200	200	10000	10000	10000	100	100	100	10000	100
800	800	900	900	650	650	650	800	800	800	650	800
900	900	1000	1000	750	750	750	900	900	900	750	900
100	100	200	200	10000	10000	10000	100	100	100	10000	100
900	900	1000	1000	750	750	750	900	900	900	750	900
900	900	1000	1000	750	750	750	900	900	900	750	900
1250	1250	1250	1250	1100	1100	1100	1250	1250	1250	1100	1250
800	800	900	900	650	650	650	800	800	800	650	800
900	900	1000	1000	750	750	750	900	900	900	750	900
1250	1250	1250	1250	1100	1100	1100	1250	1250	1250	1100	1250
450	450	550	550	200	200	200	450	450	450	200	450
200	200	300	300	10000	10000	10000	200	200	200	10000	200

TABLE V. THE C MATRIX

0	535	995	0	1095	995	115	535	1095	115	2295	2335
535	0	1480	535	820	1480	540	0	820	540	2600	2640
995	1480	0	995	1280	0	820	1480	1280	820	3280	3320
0	535	995	0	1095	995	115	535	1095	115	2295	2335
1095	820	1280	1095	0	1280	580	820	0	580	3200	3240
995	1480	0	995	1280	0	820	1480	1280	820	3280	3320
115	540	820	115	580	820	0	540	580	0	2580	2540
535	0	1480	535	820	1480	540	0	820	540	2600	2640
1095	820	1280	1095	0	1280	580	820	0	580	3200	3240
115	540	820	115	580	820	0	540	580	0	2580	2540
2295	2600	3280	2295	3200	3280	2580	2600	3200	2580	0	40
2335	2640	3320	2335	3240	3320	2540	2640	3240	2540	40	0

$$h_4 = |SHAPE_1 - SHAPE_8|$$
$$= |3 - 3|$$
$$= 0$$

$$h_5 = |CT_1 - CT_8|$$
$$= |14.1 - 13|$$
$$= 1.1$$

$$h_6 = |GTHT_1 - GTHT_8|$$
$$= |20 - 22|$$
$$= 2$$

Since h_6 is greater than a limit of 1.5, it is penalized by a factor of 10. Therefore:

$$h_6 = 10 \times h_6 = 20$$

Therefore:

$$C_{18} = h_1 x w_6 + h_2 x w_7 + h_3 x w_8 + h_4 x w_9 + h_5 x w_{10} + h_6 x w_{11}$$
$$= (1 \times 20) + (0 \times 1) + (5 \times 1) + (0 \times 40) + (1.1 \times 100) + (20 \times 20)$$
$$= 535$$

The complete C matrix is given in Table V.

To complete the formulation of this problem as a quadratic assignment problem it is necessary to determine the value of the locational relations 6.18 to 6.27 to complete matrix D.

Sample calculations for element d_{14}, Location 1 (PRESS D1) compared to Location 4 (PRESS D14) follow:

$f_1 = 0$, since locations 1 and 4 are not on the same press.

$f_2 = 20$, since locations 1 and 4 are on the same press line.

$f_3 = 10$, since locations 1 and 4 are on the same make of press.

$f_4 = 5$, since locations 1 and 4 are not on the same model of press.

Therefore:

$$d_{14} = f_1 x w_{12} + f_2 x w_{13} + f_3 x w_{14} + f_4 x w_{15}$$
$$= (0 \times 1) + (20 \times 1) + (10 \times 1) + (5 \times 1)$$
$$= 35$$

The complete D matrix is given in Table VI.

STEP 4.

The problem is now in a form suitable for one of the system algorithms for the quadratic assignment problem. The details of this solution are omitted at this point and are assumed complete. The resulting permutation matrix α is given in Table VII. The permutation matrix α, consisting of zero or one values, indicates the assignment determined by this system. It is interpreted as follows:

The first non-zerp element of α is in position 1,1. This signifies that tire 1 is scheduled to position 1, or, in the terms of this specific problem, TIREN201 is scheduled on PRESS D1. The next non-zero element occurs in position 2,3 indicating that tire 2 (TIREN202) is scheduled into location 3 (PRESS D14). The balance of this initial assignment is given in Table VIII.

STEP 5.

At this point in the solution, it becomes necessary to evaluate the effectiveness of all the curing locations. The vector Z is used for this purpose. Z_1 for example is the effectiveness of press 1 (PRESS A1 in this example) to cure the two tires assigned to it.

$$Z_i = a_{xi} + a_{yi} + c_{xy}$$

where: a_{xi} = suitability coefficient for tire x on facility i evaluated by the relations 6.1 to 6.9;

a_{yi} = similar to above for tire y on facility i; and

c_{xy} = compatibility of tires x and y to be cured on the same facility, evaluated by the relations 6.10 to 6.18.

It is assumed for purposes of this example that experience has shown that a press can function acceptably with a value of Z that is less than 1100.0 (ZMAX). The cure is now researched. Those presses with an effectiveness coefficient in excess of Z_{max} are re-allocated, by repeating the procedure from Step 3.

For purposes of this example, a previous cure was contrived in which press A3 was operating ineffectively. The remaining presses were all operating acceptably with varying degrees of effectiveness. The review of the cure found that there were 3 presses operating unacceptably. These were A3, C14 and D14. The tires on C14 and D14 are part of the new assignment.

Repeating the above procedure it indicates that it would be advantageous to relocate the tires now on press A3 to presses C14 and D14. The new tires originally assigned to C14 and D14 are now assigned to press E6. A further re-evaluation indicates that the cure is now operating entirely satisfactorily. The format of the program results are shown in Table IX.

This example was solved on McMaster's computer where it took 17.3 seconds of central processor time to complete this solution.

TABLE VI THE D MATRIX

0	240	35	35	20	20	20	35	35	35	20	35
240	0	35	35	20	20	20	35	35	35	20	35
35	35	0	240	20	20	20	30	30	30	20	30
35	35	240	0	20	20	20	30	30	30	20	30
20	20	20	20	0	240	40	30	20	20	40	30
20	20	20	20	240	0	40	30	20	20	40	30
20	20	20	20	40	40	0	30	20	20	240	30
35	35	30	30	30	30	30	0	25	25	30	240
35	35	30	30	20	20	20	25	0	240	20	25
35	35	30	30	20	20	20	25	240	0	20	25
20	20	20	20	40	40	240	30	20	20	0	30
35	35	30	30	30	30	30	240	25	25	30	0

TABLE VII THE PERMUTATION MATRIX

1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0

TABLE VIII

INITIAL PLACING OF "ON" TIRES

TIRE TIREN201 TO BE SCHEDULED ON PRESS D1

TIRE TIREN202 TO BE SCHEDULED ON PRESS D14

TIRE TIREN203 TO BE SCHEDULED ON PRESS B14

TIRE TIREN204 TO BE SCHEDULED ON PRESS D1

TIRE TIREN205 TO BE SCHEDULED ON PRESS B13

TIRE TIREN206 TO BE SCHEDULED ON PRESS B14

TIRE TIREN207 TO BE SCHEDULED ON PRESS C14

TIRE TIREN208 TO BE SCHEDULED ON PRESS D14

TIRE TIREN209 TO BE SCHEDULED ON PRESS B13

TIRE TIREN210 TO BE SCHEDULED ON PRESS C14

TABLE IX

<u>THE FOLLOWING TIRES ARE RELOCATED</u>

TIRE TIREN003 RELOCATED FROM PRESS A3 TO PRESS D14

TIRE TIREN091 RELOCATED FROM PRESS B14 TO PRESS E6

TIRE TIREN093 RELOCATED FROM PRESS C14 TO PRESS E6

<u>THE "ON" TIRE LOCATIONS</u>

TIRE TIREN201 TO BE SCHEDULED ON PRESS D1

TIRE TIREN202 TO BE SCHEDULED ON PRESS C14

TIRE TIREN203 TO BE SCHEDULED ON PRESS B14

TIRE TIREN204 TO BE SCHEDULED ON PRESS D1

TIRE TIREN205 TO BE SCHEDULED ON PRESS B13

TIRE TIREN206 TO BE SCHEDULED ON PRESS B14

TIRE TIREN207 TO BE SCHEDULED ON PRESS D14

TIRE TIREN208 TO BE SCHEDULED ON PRESS C14

TIRE TIREN209 TO BE SCHEDULED ON PRESS B13

TIRE TIREN210 TO BE SCHEDULED ON PRESS A3

VIII CONCLUSIONS

The value of an automated optimizing system to allocate tires to tire-curing presses becomes meaningful when considering the number of operations required to evaluate all the possible unique solutions. For example, in the relatively minor problem of assigning ten new tires to ten vacant curing locations on five curing presses, there are factorial ten possible assignments of tires to locations. Each tire to location evaluation consists of the comparison of eight tire parameters to the corresponding machine parameters. Similarly, in assessing the value of each combination of tires it is necessary to compare six additional parameters. Considering each parameter comparison to be an operation, the result of this is 3,628,800 unique assignments requiring 50,803,000 operations to complete the evaluation. Even if 99% of these assignments were obviously invalid for one reason or another, there are still 36,288 possible arrangements requiring 50,803 operations. These figures, based on the simplifying assumption that the ten vacant locations are contained on five presses, preclude the consideration of the additional parameter comparisons required to assess a new tire relative to one left on a press. Remembering this, and extending the problem to cover as many as twenty-five new tires, it can safely be assumed that a manual and explicit approach is not adequate.

As a comparison, in a sample problem of assigning ten new tires to ten locations contained on fifteen presses, including a second run consisting of sixteen tires from eight presses, this system reached the known optimum assignment in 18.49 seconds of central processor time.

The implementation of an automated system, such as this one, could conceivably contribute to the productive capability of a manufacturer in the following manner:

(1) by virtue of a significant decrease in the time required to effect a schedule, it would be possible for the company to decrease the lead time between market requirements and implementation of production. This, in turn, would allow the company to operate closer to the market with reduced inventories.

(2) The likelihood of reducing the number of position relocations presently required to accomplish an acceptable assignment is greatly enhanced. This possibility arises since the optimal system can examine, for the initial fit, a far greater number of possible combinations out of which the most advantageous are selected.

The logic of the algorithms used for the solution of both the quadratic and linear assignment problems has been incorporated into individual, and very general subroutines. Two reasons governed this choice, the first of which has been to allow the user to modify the main or executive portion of the program without being concerned with the destruction of the logic of the assignment portion. The second reason, and, perhaps, the more significant reason, is to allow these subroutines to be employed by a more general system concerned with the assignment of components or tires to every stage in the tire-manufacturing process. One of the more obvious extensions of the logic of this system is to incorporate the tire to the tire-building machine system. This system appears linear since there exists a one-to-one assignment condition (i.e., one tire per machine). However, an interaction amongst the relative locations assigned to tires does occur if one considers the grouping of tires using common or similar components.

The desirability of applying an optimal or suboptimal technique for the types of system discussed so far is dependent upon the nature of the facility and the product line. A system such as the tire-to-tire press system is highly desirable in an older installation with vastly different curing facilities and which manufactures a diversified product line. This desirability decreases as the homogeneity of a manufacturing plant's equipment increases, and similarly as the number of product lines decreases.

It is not within the scope of this thesis to calculate the expected savings that will be realized by the implementation of this automated system. The major return will be an improved operating efficiency for the entire plant stemming from a more efficient use of existing equipment. One measure that can be estimated is the difference in the cost to formulate an assignment. The present manual system requires 1.5 man-days directly with approximately an additional 0.5 to 1.0 man-day of interaction with advisory personnel. Assuming a cost of $6.00 per hour for the manual system, this amounts to an annual cost of $6250.00 to schedule the cure. On the other hand, the system described by this project required 76.4 seconds of central processor time to complete an assignment of twenty ON tires. At the rate of $9.16 per minute, the annual cost to schedule automatically would be about $650.00. Thus an estimated annual saving of about \$5600.00 would be realized. However, as mentioned this would likely be trivial in comparison to the improved operating conditions.

IX RECOMMENDED CHANGES AND ADDITIONS

One of the benefits anticipated from the use of an automatic optimal assignment system is the possibility of decreasing the scheduling period (i.e., change from weekly scheduling to daily scheduling). One result of this would be the reduction of the size of the problem for any one assignment. This, in turn, may reduce the problem to the point (less than 15 tires) where it would be feasible to employ an optimal algorithm for the assignment. Either Lawler's or Gilmore's algorithm, under these conditions, would be a valuable addition to the program. The option to utilize this routine could easily be made automatically by the system when the size of the assignment warranted it.

This system has been formulated on the premise that tire-curing times are continuously variable between an upper and lower limit. While this is essentially true, there are cases where tires have approved cures only at discrete times within the above range. This introduces a complication in the sense that two tires being evaluated for a single press may have over-lapping time ranges and yet still not have a common approved cycle time. This complication can best be handled during the evaluation of the C, or compatibility, matrix by the addition of a simple routine that compares all approved cycle times for each tire, until it finds the lowest common time. A penalty should then be added to the value of c_{ij} the corresponding element of matrix C. This penalty should increase in value as the difference between the common time and the shorter of the two base times increases. It will also be necessary to add an additional matrix T, where element t_{ij} would be the common curing time of tires i and j.

The system in its present form requires a great deal of core for a reasonable size assignment (approximately $14{,}500_{10}$ words for a twenty-tire problem). One of the factors contributing to this, however, has been the need to carry in the program the files of curing parameters for every tire in the previous assignment. If keyed random access disk files were utilized the core requirements would be substantially reduced.

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APPENDIX A

The Quadratic Assignment Problem

Gilmore's Algorithm

A.1 General

A mathematical statement of the quadratic assignment problem follows:

$$\text{minimize } Z = \Sigma_i a_i \rho(i) + \sum_{i \neq j} c_{ij} d_{\rho(i)\rho(j)}$$

Although the tire assignment problem consisted of symmetric matrices C and D, these algorithms can be applied to the more general problem using non-symmetric matrices.

The presentation here is essentially the same as Reference 9.

A.2 A Lower Bound

Some definitions are in order before proceeding with the determination of a lower bound on Z for completions ρ of a partial permutation α.

(1) A partial permutation of $1,\ldots\ldots,n$ is a 1-1 map of some subset of $\{1,\ldots\ldots,n\}$ into $\{1,\ldots\ldots,n\}$.

(2) An extension of a partial permutation α is a partial permutation β such that $\alpha(i) = \beta(i)$, for all i for which α is defined. By a completion of a partial permutation α is meant a permutation which is an extension of α.

(3) Given two vectors, $V = (v_1, v_2, \ldots\ldots, v_m)$ and $W = (w_1, \ldots\ldots, w_m)$ of non-negative elements, the problem of determining a permutation ρ of $1,\ldots\ldots,m$ for which the permuted dot product

$$\Sigma_i v_i w_{\rho(i)}$$

is a minimum is solved by matching the smallest v_i with the largest w_j, the second smallest v_i with the second largest w_j and so forth. For any two vectors V and W of the same dimension. $P(v,w)$ will denote the minimum permuted dot product.

If lower bounds are calculated separately for the two terms

$$\Sigma_i a_{i\rho(i)}$$

and

$$\Sigma_{i\neq j} c_{ij} d_{\rho(i)\rho(j)}$$

appearing in Z. Then a lower bound for Z will be the sum of the two bounds. A lower bound for the first term is easily calculated, since the problem of finding such a permutation is to solve an ordinary assignment problem. A lower bound for the second term can also be found by solving an ordinary assignment problem. Let c_i be the ith row of C and c_i' the i'th column each with c_{ii} deleted and let d_i and d_i' be similarly defined, then if τ is such that $0 \leqq \tau \leqq 1$ and if E' is the $n \times n$ matrix $\tau||P(c_i,d_j)|| + (1 - \tau)||P(c_i',d_j')||$ a lower bound for the second term results if the matrix E' is solved as an ordinary assignment problem.

A lower bound for Z can also be determined for completions ω of a partial permutation α. This can be done directly by letting $c(i,\alpha)$ and $c(\alpha,i)$ for $i \notin \mathrm{dom}(\alpha)$, and let $d(j,\alpha)$ and $d(\alpha,j)$ for $j \notin \mathrm{ran}(\alpha)$ be vectors of all elements respectively d_{jk} and d_{kj}, $j \neq k$ and $k \notin \mathrm{ran}(\alpha)$. Further for $i \notin \mathrm{dom}(\alpha)$ and $j \notin \mathrm{ran}(\alpha)$ let

$$e_{ij} = a_{ij} + \tau P[c(i,\alpha),d(j,\alpha)] + (1 - \tau)P[c(\alpha,i),d(\alpha,j)] + \Sigma_{m \in \mathrm{dom}(\alpha)}(c_{im}d_{j\alpha(m)} + c_{mi}d_{\alpha(m)j})$$

If $\mathrm{dom}(\alpha)$ has m members then there are $n - m$ values of i and j for which e_{ij} is defined so that they can be regarded as the elements of an $(n - m) \times (n - m)$ matrix E. If σ ranges over all completions of α and

$$b(\alpha) = \Sigma_{i \in dom(\alpha)} a_{i\alpha(i)} + \Sigma_{i \neq j; i,j \in dom(\alpha)} c_{ij} d_{\alpha(i)\alpha(j)}$$
$$+ \min_\sigma \Sigma_{i \notin dom(\alpha)} e_{i\sigma(i)}$$

then $Z(\rho) \geqq b(\alpha)$. Therefore $b(\alpha)$ is a lower bound for $Z(\rho)$ for completions α.

A.3 The Algorithm

The problem of determining a permutation ρ to minimize Z can be regarded as an $(n - 1)$ stage decision process where at each stage two numbers i and j must be chosen, $1 \leqq i,j \leqq n$, the number chosen for i not having been chosen at an earlier stage for i and the number for j not having been chosen at an earlier stage for j. Thus a sequence $\alpha_0, \alpha_1, \alpha_2, \ldots, {}_{n-1}$ of partial permutations is determined by the $n - 1$ decisions where for each k, $0 \leqq k \leqq n-1$, α_k is of rank k, α_{k+1} is an extension of α_k, the sequence determining the single completion α_n of the partial permutation α_{n-1}.

For any k, $0 \leqq k \leqq n-2$, let E_k be the $(n - k) \times (n - k)$ matrix E defined in the computation of the lower bound $b(\alpha)$. The choice of an i and j at the $(k+1)^{st}$ decision stage is then the choice of an element from E_k.

There are many ways in which E_k can be used to choose i and j. The two recommended by Gilmore, and used in this system, are as follows:

(A) A max-min choice: determine for E_k the minimum of each row and column and take the maximum of the minimums.

(B) A maximum of an assignment minimum choice: solve the ordinary assignment problem for E_k and choose the largest of the $n - k$ elements of E_k appearing in the assignment problem solution.

Decision method (A) requires of the order of $(n - k)^2$ elementary operations. (Where an elementary operation is taken as any one of multiplication, addition or comparison of two numbers.) An algorithm bases on this method then would require of the order of $\Sigma_{k=0}^{n-1} (n-k)^3$ elementary operations or of the order of n^4.

Using Munkres' algorithm (see Appendix B) for the solution of linear assignment problem, method (B) requires of the order of $(n-k)^4$ elementary operations. Therefore an algorithm based on this method requires of the order of $\Sigma_{k=1}^{n-1} (n-k)^4$ elementary operations or of the order of n^5.

A flow diagram for this algorithm is given in Figure IX.

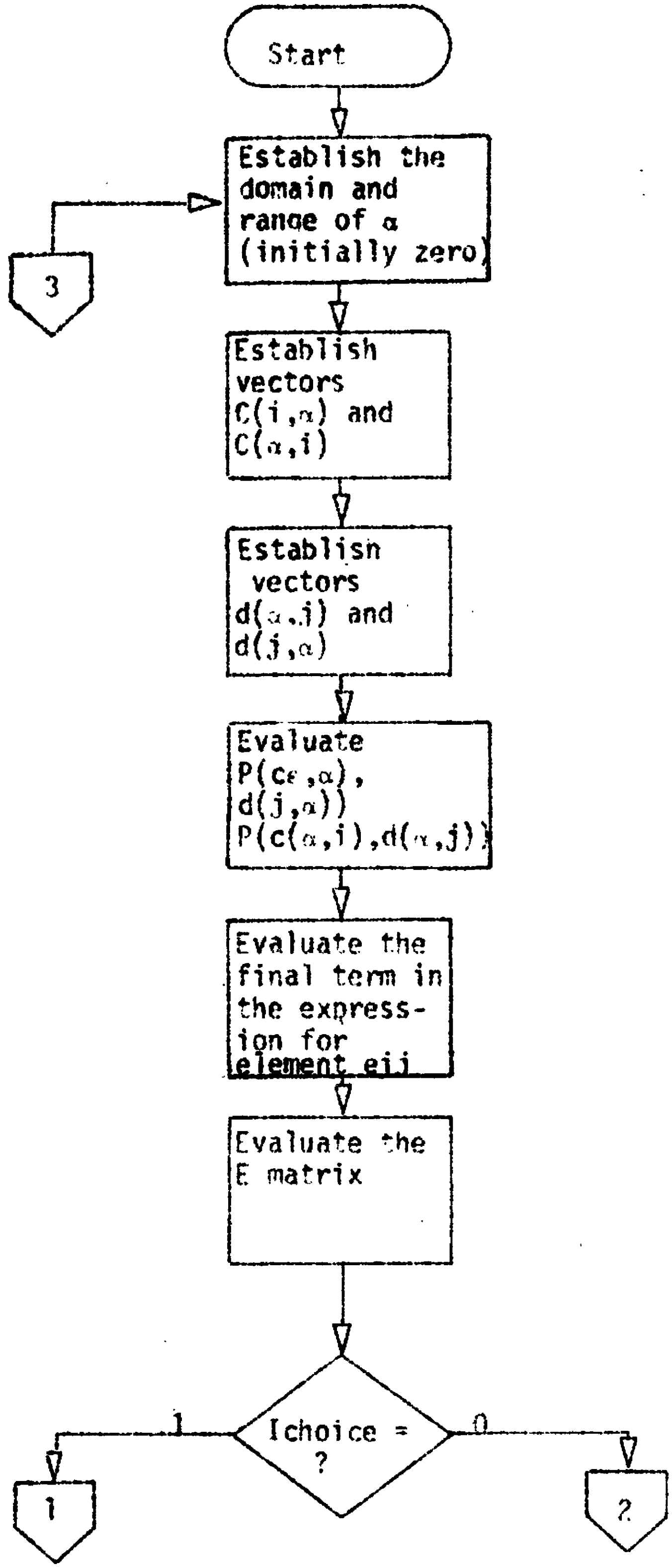

FIGURE IX GILMORE'S ALGORITHM

Selection Method A

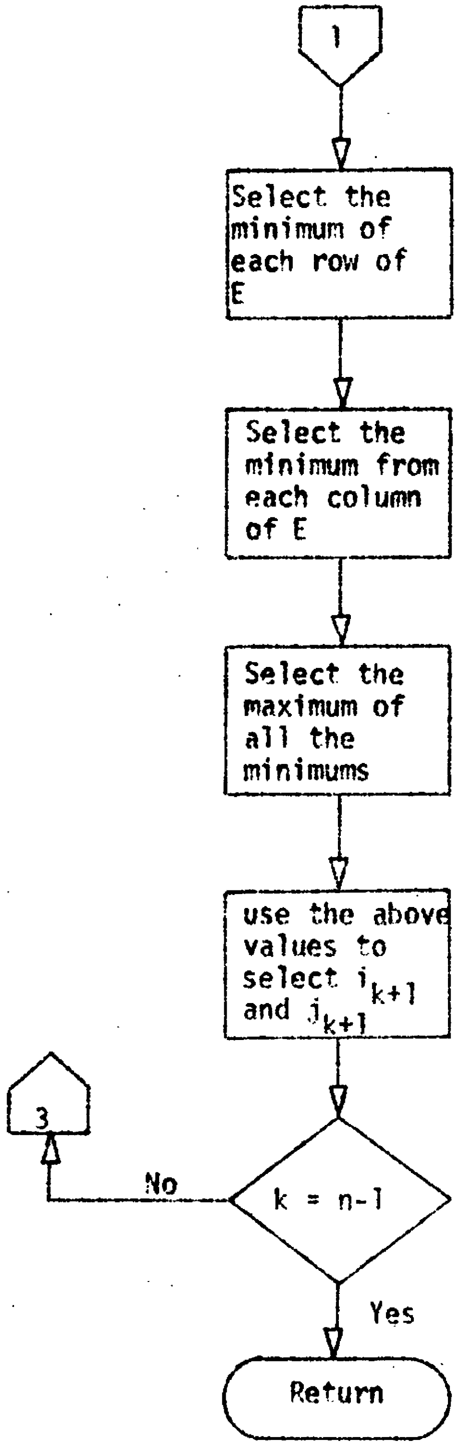

FIGURE IX continued

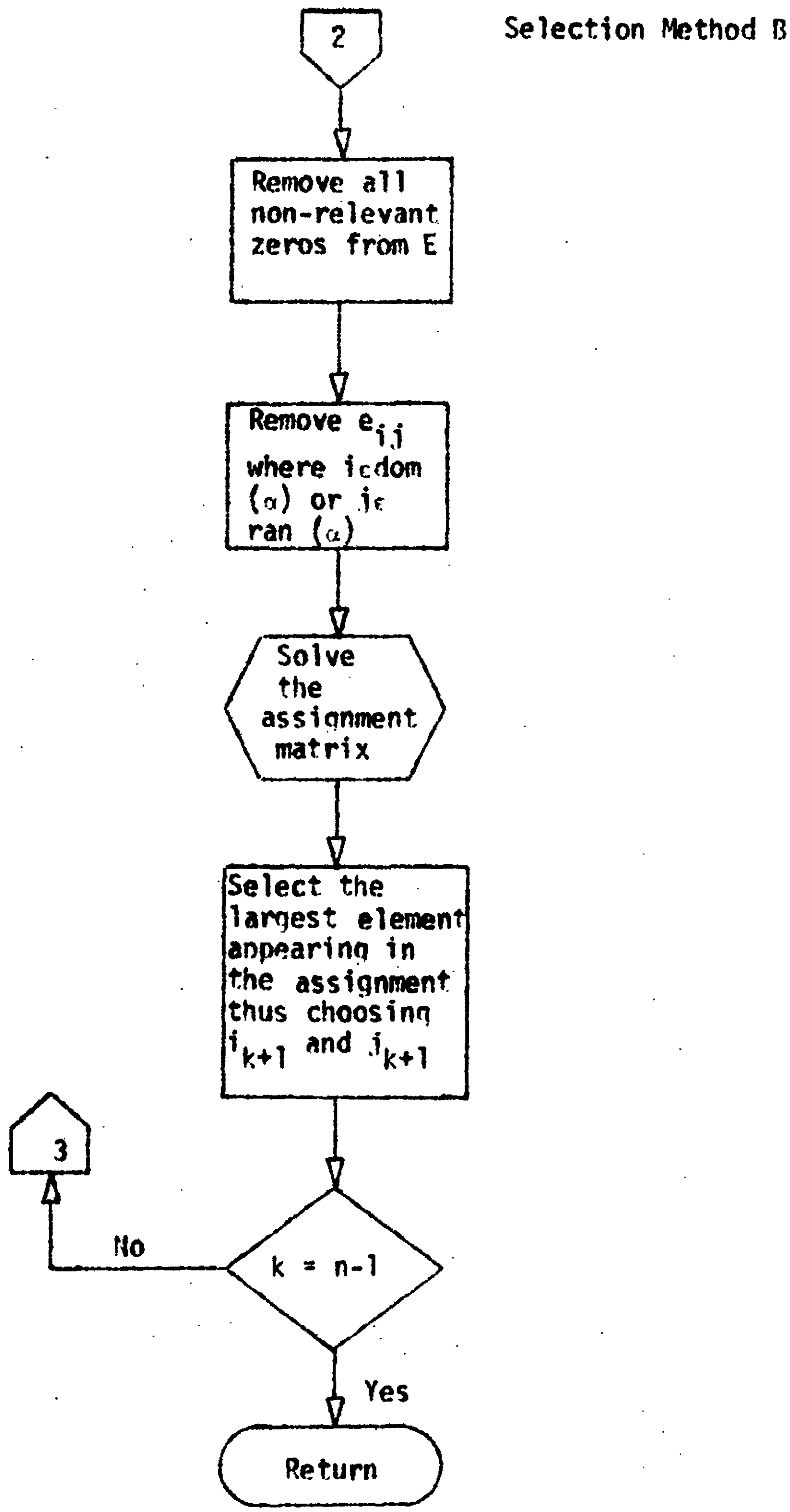

FIGURE IX continued

APPENDIX B

The Linear Assignment Problem

Munkres' Algorithm

B.1 General

This appendix presents Munkres' algorithm for the solution of a linear assignment problem. In this system this routine is employed to determine a permutation matrix under Method B of Gilmore's algorithm for the quadratic assignment problem. The presentation here is essentially the same as in Reference 10.

B.2 The Algorithm

A mathematical statement of the problem follows:

$$\text{minimize } Z = \Sigma_i a_{i\rho(i)}$$

Two remarks are in order: (1) There is a theorem of König which states: If A is a matrix, and m is the maximum number of independent zero elements of A, then there are m lines which contain all the zero elements of A. (A set of elements of a matrix are said to be independent if no two of them lie in the same line where the word line applies both to the rows and columns of a matrix.) (2) It is readily seen that the solution of this problem is not changed if an arbitrary constant is subtracted from every element of the matrix.

In the course of the problem, certain lines will be distinguished: they will be referred to as covered lines. An element of a matrix is said to be non-covered, once-covered, or twice-covered, accordingly as it lies in precisely none, one, or two covered lines. Some zeros are distinguished by means of asterisks and some by primes (there are respectively "starred zeros" and "primed zeros").

No lines are covered; no zeros are starred or primed. For each row of matrix A, subtract the smallest element in that row. Do the same for each column of the matrix.

Consider a zero Z of the matrix. If there is no starred zero in its row and none in its column, star Z. Repeat, considering each zero in the matrix in turn. Cover every column containing a starred zero. (These starred zeros are independent.)

STEP 1.

Choose a non-covered zero and prime it. Consider the row containing it. If there is no starred zero in this row, go at once to STEP 2. If there is a starred zero Z in this row, cover this row and uncover the column of Z. Repeat until all zeros covered. Go to STEP 3.

STEP 2.

There is a sequence of alternating starred and primed zeros, constructed as follows: Let Z_0 denote the uncovered zero prime. (There is only one.) Let Z_1 denote the zero starred in Z_0's column (if any). Let Z_2 denote the zero primed in Z_1's column (if any). Similarly continue until the sequence stops at a zero primed, Z_{2k}, which has no zero starred in its column. (Note that no column contains more than one zero starred and no row more than one zero primed so that the sequence is uniquely specified.)

Unstar each starred zero of the sequence and star each primed zero of the sequence. (The resulting set of starred zeros is independent. It is larger than the previous set of independent zeros by one.) Erase all primes, uncover every row, and cover every column containing a zero starred. If all columns are covered, the starred zeros form the desired independent set. Otherwise, return to STEP 1.

STEP 3.

[At this point, all the zeros of the matrix are covered. Each zero starred is covered by precisely one line, so there are exactly as many covered lines as there are starred zeros.] Let h denote the smallest non-covered element of the matrix; it will be positive. Add h to each covered row; then subtract h from each uncovered column. Return to STEP 1, without altering any asterisks, primes, or covered lines.

A flow diagram for this algorithm is given in Figure X .

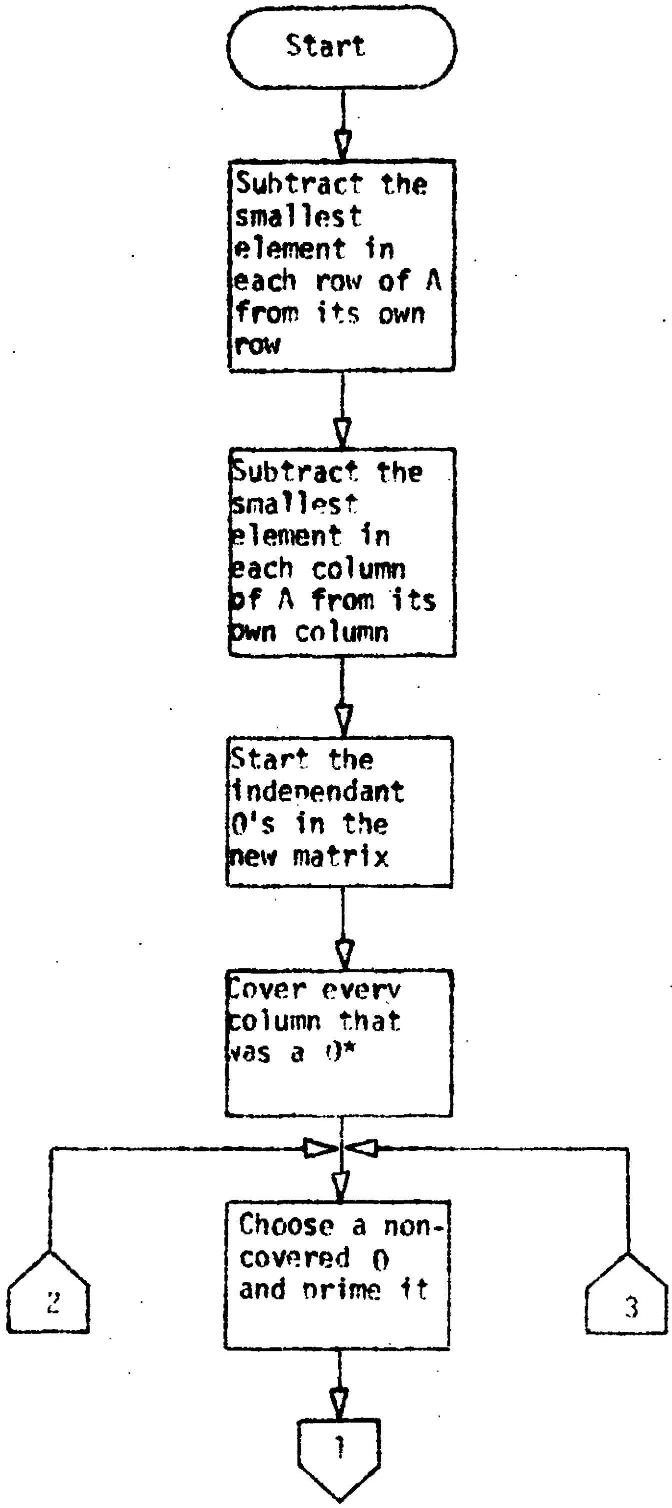

FIGURE X MUNKRES' ALGORITHM

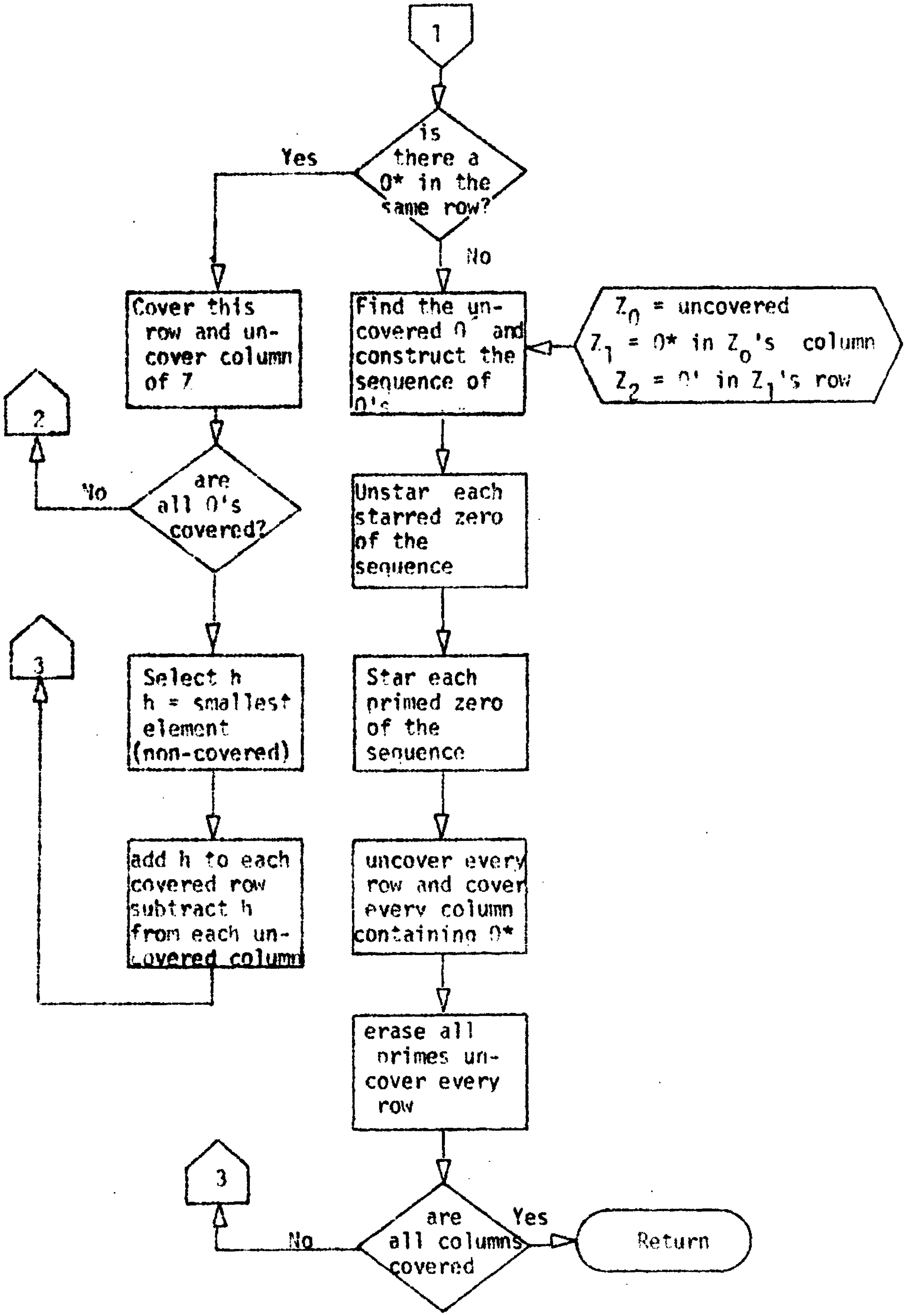

FIGURE X continued

APPENDIX C

The Program

C.1 General

This program consists of a main executive program and four subroutines. The logic of the formulation of the quadratic assignment problem as well as the execution of the input and output operations are contained in the main program. The subroutines are used to execute the iterative solution technique and to perform service functions.

C.2 Variable Thesaurus

A(I,J) - Matrix of effectiveness coefficients, evaluated by relations 6.1 to 6.9.

ALPHA(I,J) - Permutation matrix evaluated in SUBROUTINE OPTI.

BW(I) - Bead diameter of tire I, Example 14", 15", etc.

C(I,J) - Matrix of compatibility coefficients.

CONST(I) - Construction type of tire I, Example Radial Ply, Belted, or Conventional.

CT(I) - Base cure cycle time for tire I.

CTME - Cure cycle time for tires on the Semi-Automatic Presses.

CURE(I) - Used to designate type of cure required by tire I. Example $Cure_2$ = NRM - Tire 2 is restricted to an NRM press.

D(I,J) - Matrix of locational coefficients.

E(I,J) - Working array established by SUBROUTINE OPTI. Used as input assignment matrix for SUBROUTINE LINAS.

EBW(I) - Same as BW(I) except that the "E" prefix means that tire I has an assigned position in the cure. Similarly for ECT(I), EGTHT(I), ECONST(I), ECURE(I), EPCIP(I), ESHAPE(I), ETMLDH(I), ETMLDW(I), IEPCI(I), and IERAT(I).

GTHT(I) - Total overall length of a green tire.

IBOM(I) - Working vector used in SUBROUTINE LINAS.

IC(I)	- Working vector used in SUBROUTINE OPTI.
ICNT	- Counter used in the main program.
ICHOICE	- Used to select algorithm for the problem solution. = 1 - Method A selected. = 0 - Method B selected.
ICONTR(I)	- Used to designate type of control on press I. = 2 - Position I has an independent temperature and pressure control from its adjacent position. = 1 - not independent.
IDAN(I)	- Working vector used in SUBROUTINE LINAS.
IDATA	- Used to output data for observation purposes if desired. = 1 - Data is printed out. = 0 - Data is not printed out.
IOFC(I)	- Four digit green tire designation number for tire I on the OFF tire list.
IDIAG	- Used to output intermediate calculations for observation - if desired. = 1 - Calculations are printed. = 0 - Calculations are not printed.
IDOM(I)	- Working vector used in SUBROUTINE OPTI.
IFC(I)	- Four digit green tire designation number for tire I on the ON tire list.
IK(I)	- Working vector used in the main program.
IPCI(I)	- Used to denote whether tire I requires automatic post-cure inflation. = 1 - Tire requires automatic post-cure inflation. = 0 - Tire does not require automatic post-cure inflation.
IPPCI(I)	- Indicates type of post-cure inflation equipment is available at location I. = 1 - Press is equipped with manual P.C.I. equipment. = 2 - Press is equipped with automatic P.C.I. equipment.
IPRAT(I)	- User set rating value for performance of position I. Example: IPRAT(2) = 1 - Position 2 gives poor overall performance. IPRAT(6) = 3 - Position 6 gives excellent performance.

IPUNCH - Used to output punched cards containing the relevant cure parameters for tires in the new assignment.
= 1 - punched output.
= 0 - no punched output.

IRAN(I) - Working vector used in SUBROUTINE OPTI.

IRAT(I) - User set measure of the quality of cure required by tire I.
= 1 - Tire requires precision curing.
= 2 - Tire requires standard curing.
= 3 - Tire does not require precision curing.

ITOPC(I) - Four digit green tire designation for tire assigned to location I.

IZ(I) - Working vector used in SUBROUTINE LINAS.

JJK(I) - Working vector used in the main program.

LCTNS - Total number of curing locations = 2 x number of presses.

MAKE(I) - Manufacturer of curing location I.

MOD(I) - Manufacturer's designation for curing location I.

N - Number of tires on ON tire list.

NN - Allotted array size.

NIEL(I) - Number of curing locations in press line I.

NLINES - Number of lines of presses.

NMAX - Maximum number of mold changes to be allowed.

NRUN - Maximum number of cycles through the program to be allowed.

NSEMI - Number of semi-automatic presses to be allowed.

OFF(I) - Eight digit alpha-numeric designation for tire I on the OFF tire list.

ON(I) - As above, except for ON tire list.

ONOR(I) - Working vector used to store original ON tire list.

PCIP(I) - Post cure inflation pressure required by tire I.

PMHT(I) - The largest mold height that position I can accommodate.

PMW(I) - The largest mold diameter that position I can accommodate.

PRESS(I) - Departmental nomenclature used to designate position I. Example Al, C14, etc.

PSHP(I) - Number of shaping phases location I can accommodate.

SHAPE(I) - Number of shaping phases required by tire I.

TMLDH(I) - Mold height for tire I.

TMLDW(I) - Mold diameter for tire I.

TOP(I) - Eight digit alpha-numeric designation for the tire assigned to location I.

VCAI(I,J) - Working array used in SUBROUTINE OPTI.

VCIA(I,J) - As above.

VDAJ(I,J) - As above.

VDJA(I,J) - As above.

W(I) - Weighting factors used to accent various assignment considerations. (These will be explained in detail later.)

WORKI(I) - Working array used in SUBROUTINE OPTI.

WORK2(I) - As above.

WORK3(I,J) - As above.

Z(I) - Effectiveness coefficient used as a measure of how effectively Press I is curing the two tires assigned to it.

C.3 Weighting Factors

In selecting a value for each of the 14 weighting factors used in this program, the user must consider their importance relative to each other. There may be factors that are not very important in an absolute sense; however, these should not be assigned a zero value. To do so would negate the factor for which the zero weighting factor has been assessed from consideration.

These weighting factors are as follows:

W(1) = Relative importance of difference in mold heights between press maximum and the tire under consideration. Note if this value is negative then this position is not considered valid.

W(2) = As above but considers mold widths.

W(3) = Relative importance between number of phases of shaping available on the press and the number of phases of shaping required by the tire under consideration. Note if this value is negative the press location is not considered valid for that tire.

W(4) = Importance of difference in cure time between two tires. Note in the program only the primary times are considered.

W(5) = Relative importance of height differential between two tires. Note if difference is 1.5 inches or greater this is considered an invalid condition.

W(6) = Relative importance of difference of number of phases of shaping required between two tires.

W(7) = Relative importance of the difference in bead widths of tires.

W(8) = Weighting factor used to accentuate the increase in the value of D(I,J) when curing locations I and J are on the same press. D(I,J) is increased by 200*W(8) if above condition is true.

W(9) = Weighting used to accentuate the increase in the value of D(I,J) when curing locations I and J are in the same line of presses. D(I,J) is increased by 20*W(9) if the above condition is true.

W(10) = Weighting factor used to accentuate the increase in the value of D(I,J) when curing locations I and J are on the same make of press. D(I,J) is increased by 10*W(10) if the above condition is true.

W(11) = Weighting factor used to accentuate the increase in the value of D(I,J) when curing locations I and J are on the same model of press. D(I,J) is increased by 10*W(11) if the above condition is true.

W(12) = Weighting factor used to accentuate the effect on A(I,J) if tire I requires a high quality cure and location J is a high quality curing location. A(I,J) is reduced by 100*W(12) if the condition is true.

W(13) = Weighting factor used to accentuate the effect on A(I,J) if tire I requires automatic post cure inflation and location J is equipped with the same. A(I,J) is reduced by 100*W(13) if the above condition is true.

W(14) = Weighting factor used to accentuate the difference in post cure inflation pressures required for tire I and tire J.

C.4 DIMENSION Statement

Throughout the program all subscripted variables have variable dimensioning in order to simplify the program usage. All arrays and vectors have been carried back from each subroutine to the main program thus requiring the user to enter the required dimensions in only one place. The required DIMENSION statement is shown in Figure XI.

Example. If in the main program arrays A and X appear in the dimension statement, and if they also are used in a subroutine, then the following form has been utilized.

```
 DIMENSION PMHT( XX),PMW( XX),PRESS( XX),TOP( XX),ITOPC( XX),IPPCI(
1 XX),ICONTR( XX),IPRAT( XX),MOD( XX),MAKE( XX),ON( X),IFC( X),BW(
2X),PCIP( X),IRAT( X),CONST( X),CURE( X),IPCI( X),PSHP( XX),CT( X),
3GTHT( X),SHAPE( X),TMLDH( X),TMLDW( X),A( X, X),C( X, X),D( X, X),
4E( X, X),IDOM( X),IRAN( X),VCIA( X, X),VCAI( X, X),VDAJ( X, X),VDJ
5A( X, X),WORK1( X),WORK2( X),IC( X),JR( X),WORK3( X, X),IZ(2X),JZ(
62X),IBOM( X),IDAN( X),OFF( X),IOFC( X),JK( X),NIEL(K),IK( X),EGTHT
7( XX),ECT( XX),ESHAPE( XX),ETMLDH( XX),EBW( XX),EPCIP( XX),IERAT(
8XX),IEPCI( XX),ECONST( XX),ECURE( XX),UNOR( X),JJK( X),ETMLDW( XX)
9,Z(KK),W(14)
 INTEGER ALPHA( X, X),ZSTAR( X, X),ZPRIM( X, X)
```

WHERE

X=THE NUMBER OF TIRES ON THE NEW SCHEDULE.

XX=THE TOTAL NUMBER OF CURING LOCATIONS.

K=THE NUMBER OF LINES OF PRESSES.

KK=THE TOTAL NUMBER OF PRESSES.

FIGURE XI THE DIMENSION STATEMENT

```
MAIN PROGRAM
DIMENSION A(15.15), X(10)
M = 15
CALL SUBROUTINE B(A,X,M)
END
SUBROUTINE B(A,X,M)
DIMENSION A(M,1), X(1)
ETC.
```

C.5 Main Program

The purposes of the main program are to formulate the quadratic assignment problem and to execute all input and output operations. Special attention should be given to the following options which are available when using this program. Flow diagram is given in Figure XII.

IDATA = 1 - All the input data will be printed for observation purposes.
= 0 - Data is omitted.

ICHOICE = 1 - Algorithm A is selected as the solution technique for the quadratic assignment problem.
= 0 - Algorithm B is selected.

IDIAG = 1 - Internal calculations will be output for purposes of observation.
= 0 - No internal calculations are output.

IPUNCH = 1 - Data for next run is punched out.
= 0 - No data is punched out.

The following limit parameters must be set by the user.

NRUN = Number of allowable program executions.

NMAX = Maximum allowable number of mold changes.

ZMAX = Maximum allowable effectiveness coefficient.

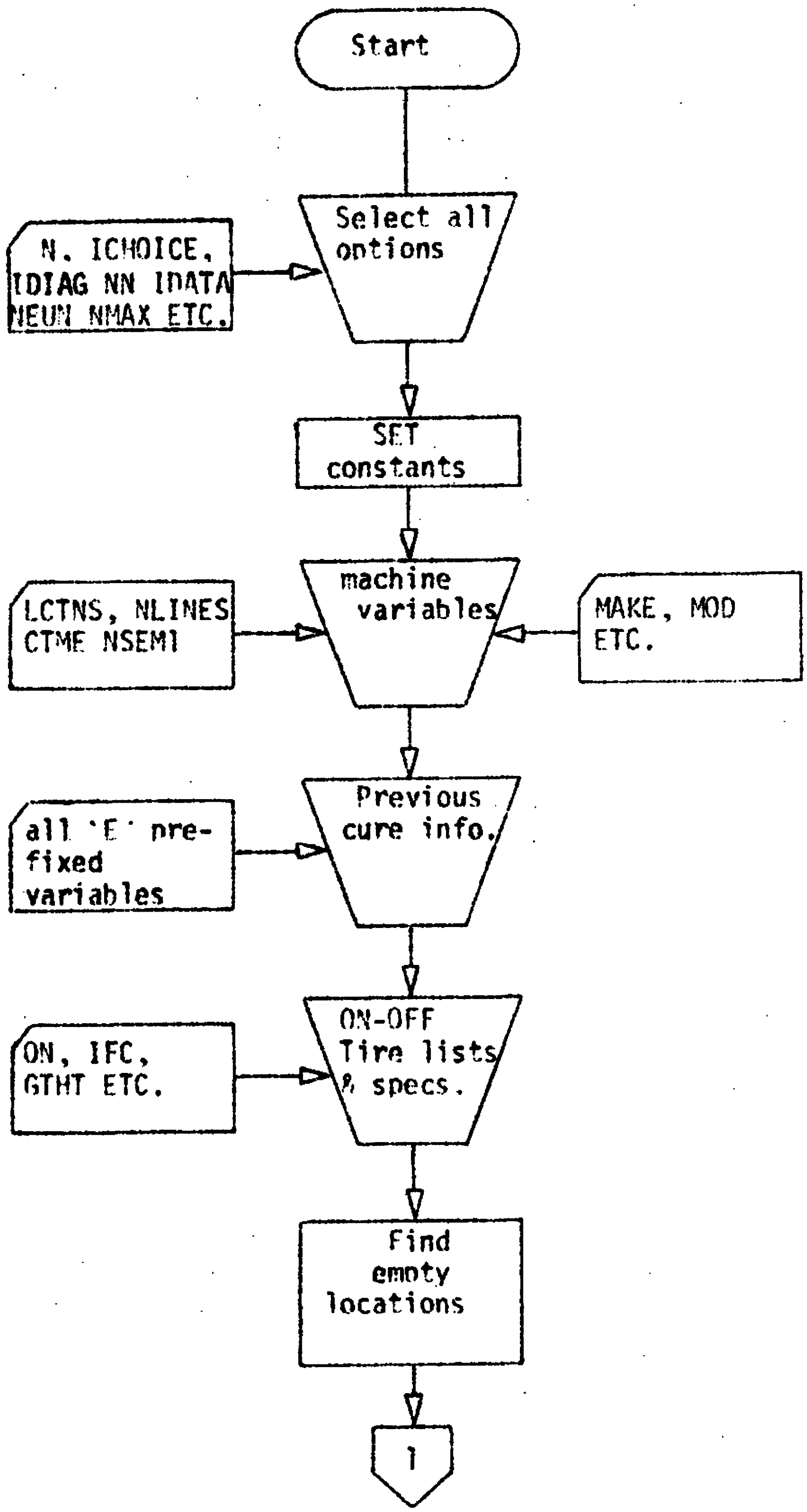

FIGURE XII THE MAIN PROGRAM

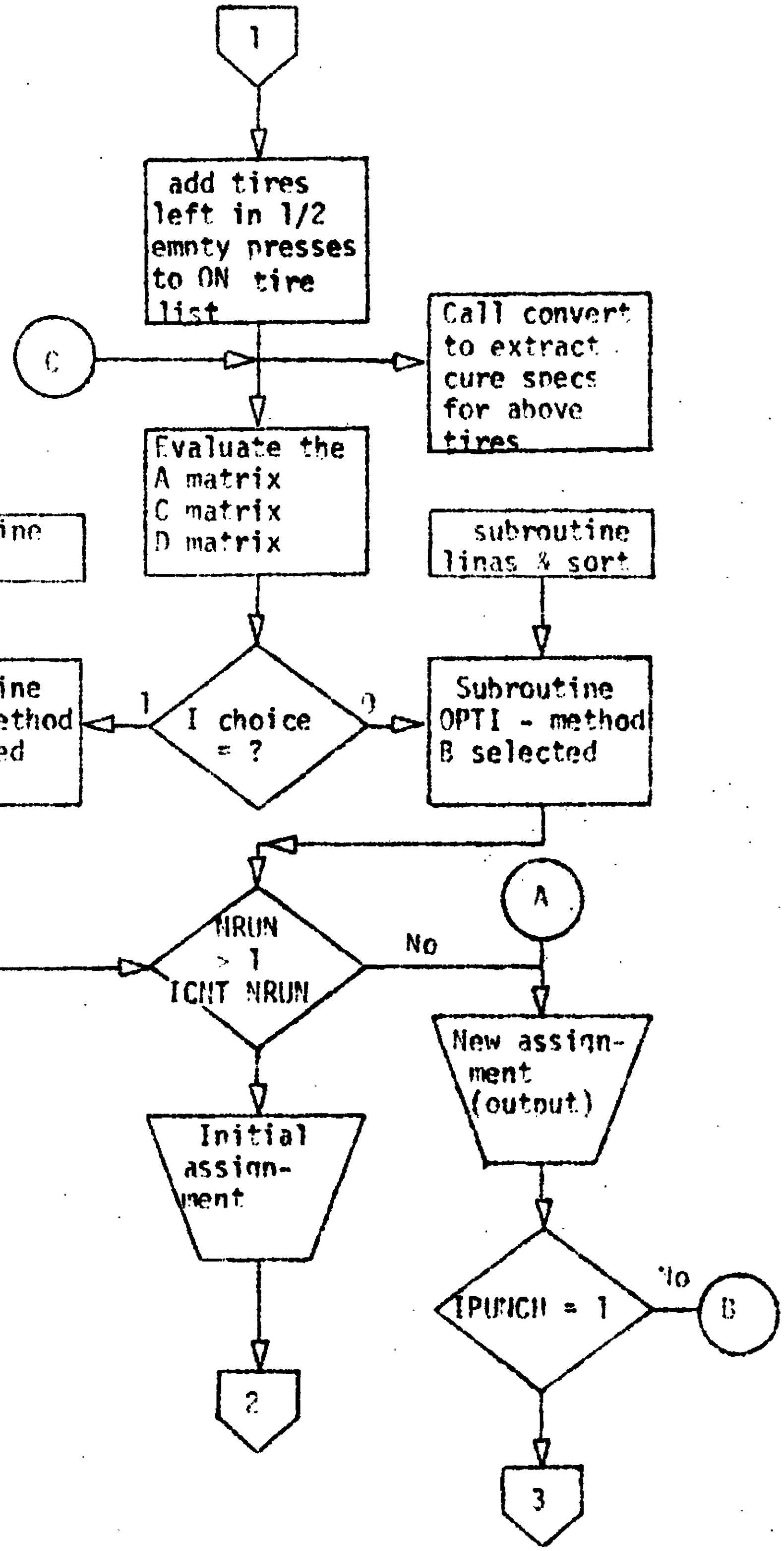

FIGURE XII continued

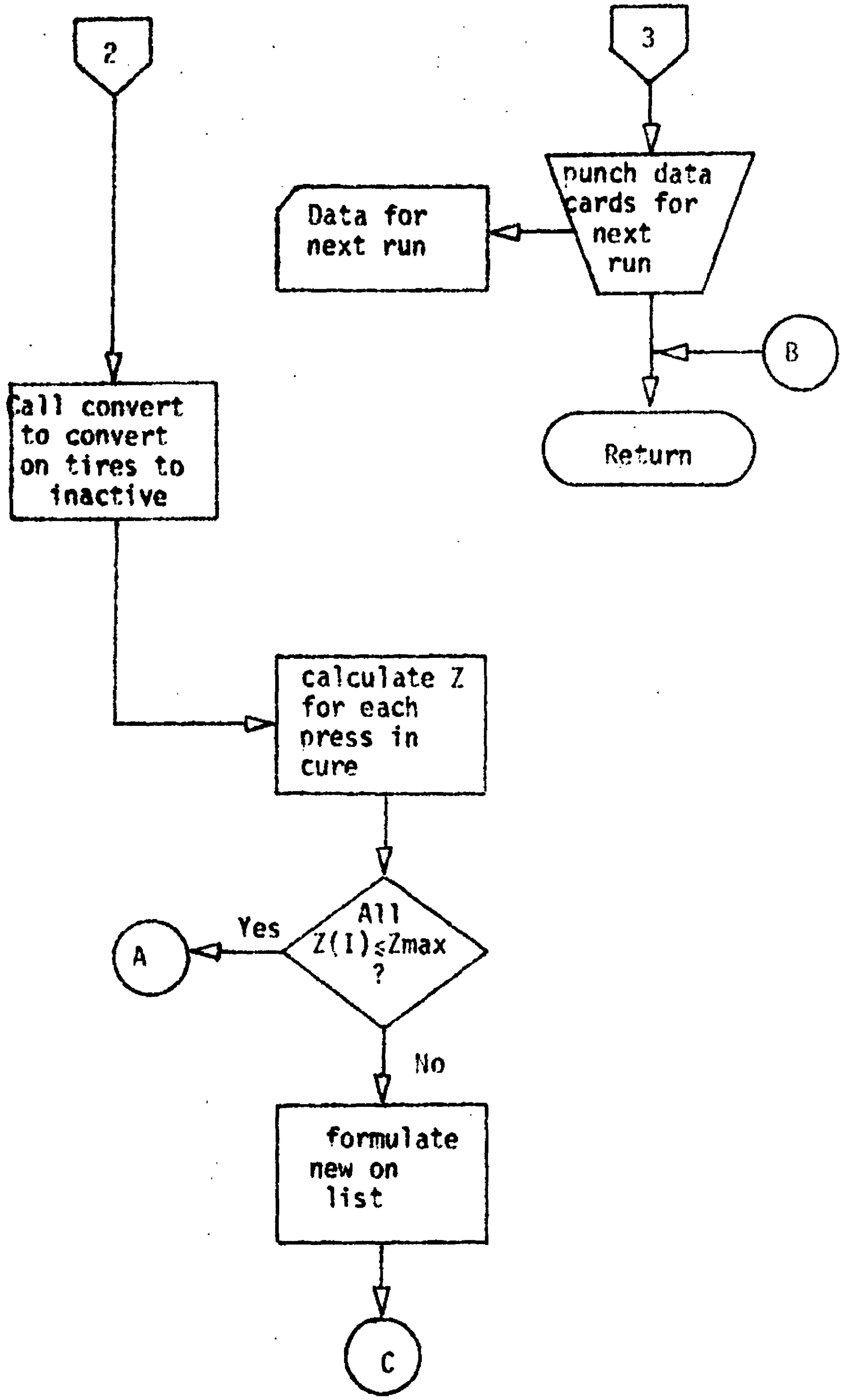

FIGURE XII continued

C.6 How to set up a Data Deck (see Figure XIII)

VARIABLE	NO. OF CARDS	FORMAT
N, ICHOICE, IDIAG, NN, IDATA, NRUN, NMAX, ZMAX, IPUNCH	1	7I8, F10.0, I8
LCTNS, NLINES, CTME, NSEMI	1	2I5, F10.2, I5
NIEL(I)	(NLINES)	I10
W(I)	14	F15.3
MAKE(I), MOD(I), IPRAT(I), PMHT(I), PMW(I), PSHP(I), ICONTR(I), IPPCI(I)	(LCTNS)	A3, 3X, A3, 3X, I3, F10.3, 2I3
PRESS(I), TOP(I), ITOPC(I)	(LCTNS)	A4, 6X, A8, 5X, I5
EGTHT(I), ECT(I), ESHAPE(I), ETMLDH(I), ETMLDW(I), EBW(I), EPCIP(I), IERAT(I), IEPCI(I), ECONST(I), ECURE(I)	(LCTNS)	7F9.2, 2I3, 2A5
ON(I), IFC(I)	(N)	A8, 6X, I4
GTHT(I), CT(I), SHAPE(I), TMLDH(I), TMLDW(I), BW(I), PCIP(I), IRAT(I), IPCI(I), CONST(I), CURE(I)	(N)	7F12.3, 2I10, 2A10
OFF(I), IOFC(I)	(N)	A8, 6X, I4

C.7 Service Subroutines

Two of the four SUBROUTINES used in this program serve essentially a service role. These are:

(1) SUBROUTINE CONVERT: The purpose of this SUBROUTINE is to convert the cure parameters from active form to inactive, inactive to active, and from one inactive location to another. ("E" prefix denotes inactive form.)

(2) SUBROUTINE SORT: This SUBROUTINE is used to sort singly dimensional arrays into either ascending or descending order of magnitude. SORT is

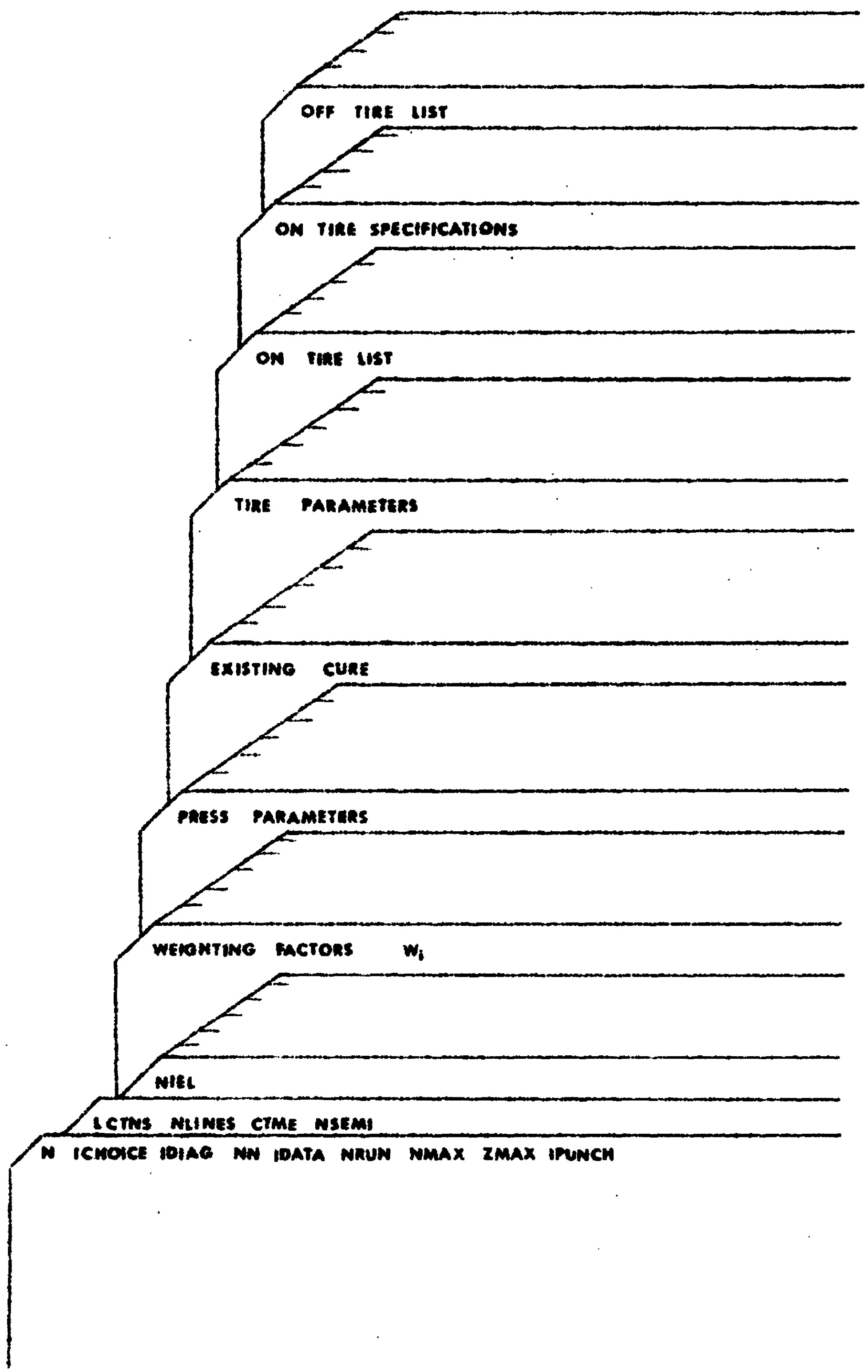

FIGURE XIII SET UP OF A DATA DECK

called by SUBROUTINE OPTI. This SUBROUTINE is unchanged from the SUBROUTINE of the same name in the library of McMaster's C.D.C. 6400 computer.

Flow diagrams for these service SUBROUTINES are given in Figure XIV and XV.

C.8 SUBROUTINE OPTI

The SUBROUTINE OPTI contains the logic of the two suboptimal algorithms for the solution of the quadratic assignment problem. Here, as in Appendix A, the two methods are referred to as Method A and Method B. The choice of which algorithm is utilized is governed by the value of ICHOICE in the main program.

Although the quadratic assignment problem formulated by this system has two symmetric matrices, C and D, the SUBROUTINE OPTI has been written to handle the more general non-symmetric case.

A complete discussion of these algorithms including a flowchart is given in Appendix A.

C.9 SUBROUTINE LINAS

The SUBROUTINE LINAS contains the logic of an algorithm for the solution to a linear assignment problem. The particular algorithm used here was derived by Munkres. A complete discussion of this algorithm is given in Appendix B.

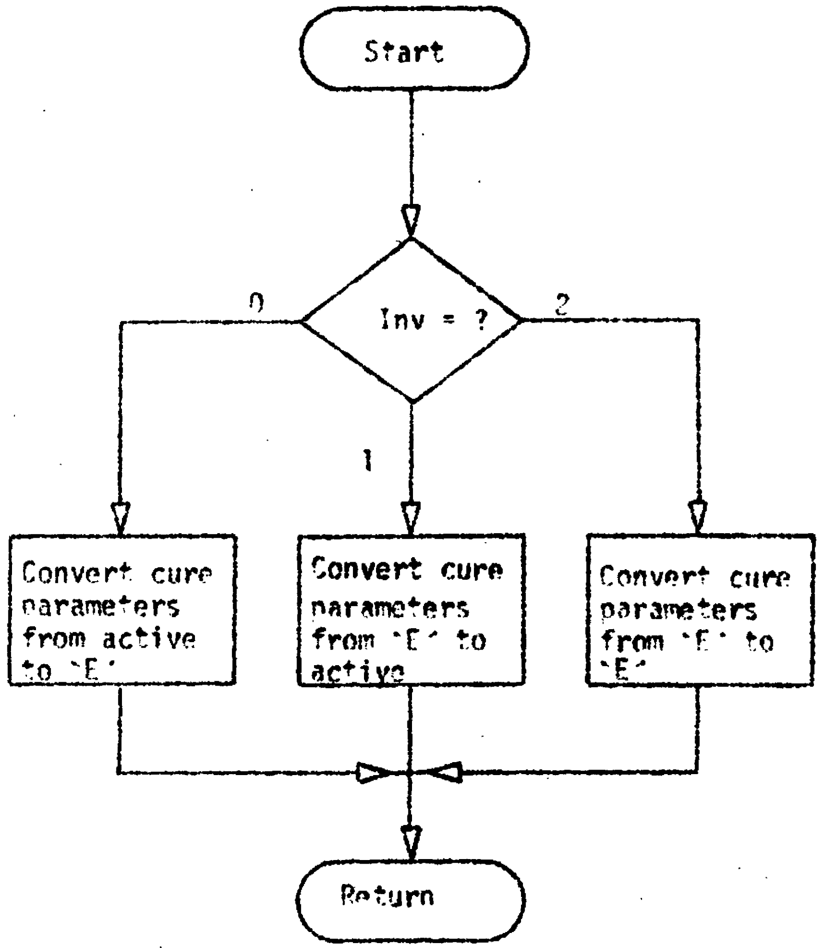

FIGURE XIV SUBROUTINE CONVERT

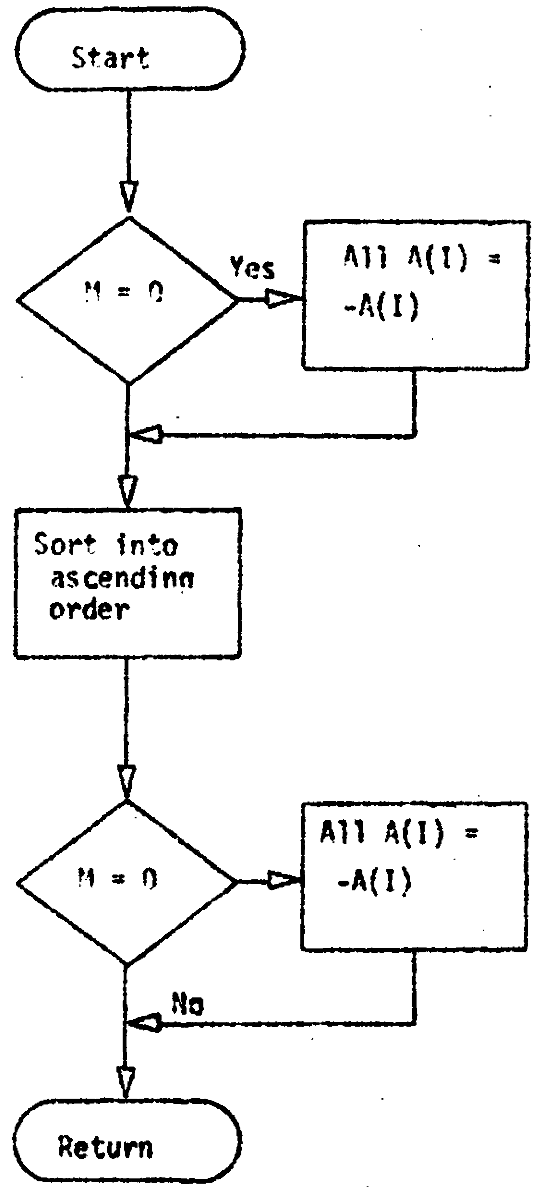

FIGURE XV SUBROUTINE SORT

C.10 FORTRAN IV LISTING

```
C
C     PROGRAM TO AUTOMATE THE SCHEDULING OF TIRE CURING PRESSES.
C
      DIMENSION PMHT( XX),PMW( XX),PRESS( XX),TOP( XX),ITOPC( XX),IPPCI(
     1 XX),ICONTR( XX),IPRAT( XX),MOD( XX),MAKE( XX),ON( X),IFC( X),BW(
     2X),PCIP( X),IRAT( X),CONST( X),CURE( X),IPCI( X),PSHP( XX),CT( X),
     3GTHT( X),SHAPE( X),TMLDH( X),TMLDW( X),A( X, X),C( X, X),D( X, X),
     4E( X, X),IDOM( X),IRAN( X),VCIA( X, X),VCAI( X, X),VDAJ( X, X),VDJ
     5A( X, X),WORK1( X),WORK2( X),IC( X),JR( X),WORK3( X, X),IZ(2X),JZ(
     62X),IBOM( X),IDAN( X),OFF( X),IOFC( X),JK( X),NIEL(K),IK( X),EGTHT
     7( XX),ECT( XX),ESHAPE( XX),ETMLDH( XX),EBW( XX),EPCIP( XX),IERAT(
     8XX),IEPCI( XX),ECONST( XX),ECURE( XX),ONOR( X),JJK( X),ETMLDW( XX)
     9,Z(KK),W(14)
      INTEGER ALPHA( X, X),ZSTAR( X, X),ZPRIM( X, X)


      WHERE
        X=THE NUMBER OF TIRES ON THE NEW SCHEDULE.
        XX=THE TOTAL NUMBER OF CURING LOCATIONS.
        K=THE NUMBER OF LINES OF PRESSES.
        KK=THE TOTAL NUMBER OF PRESSES.
C
C     READ IN THE NUMBER OF CHANGES,CHOICE OF METHOD ,CHOICE OF DIGNOS-
C     TICS,ARRAY SIZE, DATA OUTPUT.
C
      READ(5,100) N,ICHOICE,IDIAG,NN,IDATA,NRUN,NMAX,ZMAX,IPUNCH
C
C     SET CONSTANTS
C
      NPART=0
      K2=0
      ICNT=0
      IF(IDATA.EQ.1)WRITE(6,117)N,ICHOICE,IDIAG,NN,IDATA,NRUN,NMAX,ZMAX
C
C     READ IN NUMBER OF CURING LOCATIONS. NUMBER OF LINES OF PRESSES,CURE
C     TIME FOR THE SEMI AUTOMATIC PRESSES,NUMBER OF SEMI AUTOMATICS.
C
      READ(5,114)LCTNS,NLINES,CTME,NSEMI
      IF(IDATA.EQ.1) WRITE(6,113)LCTNS,NLINES
      DO 30 I=1,NLINES
      READ(5,121)NIEL(I)
   30 CONTINUE
C
C     READ IN THE WEIGHTING FACTORS W(1)----W(14).
C
      DO 66 I=1,14
      READ(5,140) W(I)
   66 CONTINUE
C
C     CLEAR ARRAYS
```

```
C
      DO 1 I=1,LCTNS
      PMHT(I)=0.
      PMW(I)=0.
      PRESS(I)=0.
      TOP(I)=0.
      ITOPC(I)=0
      IPPCI(I)=0
      ICONTR(I)=0
      IPRAT(I)=0
      MOD(I)=0
      MAKE(I)=0
      PSHP(I)=0.
      EGTHT(I)=0.
      ECT(I)=0.
      ESHAPE(I)=0.
      ETMLDH(I)=0.
      ETMLDW(I)=0.
      EBW(I)=0.
      EPCIP(I)=0.
      IERAT(I)=0
      IEPCI(I)=0
      ECONST(I)=0.
      ECURE(I)=0.
    1 CONTINUE
C
C     READ IN FILE OF PRESS PARAMETERS.
C
      IF(IDATA.EQ.1) WRITE(6,122)
      DO 2 I=1,LCTNS
      READ(5,101)MAKE(I),MOD(I),IPRAT(I),PMHT(I),PMW(I),PSHP(I),ICONTR(I
     1),IPPCI(I)
      IF(IDATA.EQ.0) GO TO 2
      WRITE(6,115)MAKE(I),MOD(I),IPRAT(I),PMHT(I),PMW(I),PSHP(I),ICONTR(
     1I),IPPCI(I)
    2 CONTINUE
C
C     READ IN THE EXISTING CURE - NOTE THIS INFORMATION IS DERIVED FROM
C     THE PREVIOUS RUNNING OF THIS PROGRAM.
C
      IF(IDATA.EQ.1)WRITE(6,123)
      DO 3 I=1,LCTNS
      READ(5,102)PRESS(I),TOP(I),ITOPC(I)
      IF(IDATA.EQ.0) GO TO 3
      WRITE(6,116) PRESS(I),TOP(I),ITOPC(I)
    3 CONTINUE
C
C     READ IN THE SPECIFIED CURE PARAMETERS FOR EACH TIRE IN THE
C     EXISTING SCHEDULE.
C
```

```
      DO 49 I=1,LCTNS
      READ(5,104) EGTHT(I),ECT(I),ESHAPE(I),ETMLDH(I),ETMLDW(I),EBW(I),E
     1PCIP(I),IERAT(I),IEPCI(I),ECONST(I),ECURE(I)
   49 CONTINUE
C
C     CLEAR ARRAYS
C
      DO 4 I=1,NN
      OFF(I)=0.
      ON(I)=0.
      IFC(I)=0
      IOFC(I)=0
      BW(I)=0.
      PCIP(I)=0.
      IRAT(I)=0
      CONST(I)=0.
      CURE(I)=0.
      IPCI(I)=0
      PSHP(I)=0.
      CT(I)=0.
      GTHT(I)=0.
      SHAPE(I)=0.
      TMLDH(I)=0.
      TMLDW(I)=0.
      IDOM(I)=0
      IRAN(I)=0
      WORK1(I)=0.
      WORK2(I)=0.
      IC(I)=0
      JR(I)=0
      IBOM(I)=0
      IDAN(I)=0
      DO 4 J=1,NN
      A(I,J)=0.
      C(I,J)=0.
      D(I,J)=0.
      E(I,J)=0.
      VCIA(I,J)=0.
      VCAI(I,J)=0.
      VDAJ(I,J)=0.
      VDJA(I,J)=0.
      WORK3(I,J)=0.
      ALPHA(I,J)=0
      ZSTAR(I,J)=0
      ZPRIM(I,J)=0
      ONOR(I)=0.
      IK(I)=0
      JJK(I)=0
    4 CONTINUE
C
```

```
C
C     READ THE -ON- TIRE LIST.
C     READ IN THE SPECIFIED CURE PARAMETERS FOR EVERY TIRE ON THE -ON-
C     TIRE LIST.
C
      IF(IDATA.EQ.1)WRITE(6,124)
      DO 5 I=1,N
      READ(5,103) ON(I),IFC(I)
      IF(IDATA.EQ.0) GO TO 5
      WRITE(6,118)ON(I),IFC(I)
    5 CONTINUE
      IF(IDATA.EQ.1)WRITE(6,125)
      DO 6 I=1,N
      READ(5,104) GTHT(I),CT(I),SHAPE(I),TMLDH(I),TMLDW(I),BW(I),PCIP(I)
     1,IRAT(I),IPCI(I),CONST(I),CURE(I)
      IF(IDATA.EQ.0) GO TO 6
      WRITE(6,119) GTHT(I),CT(I),SHAPE(I),TMLDH(I),TMLDW(I),BW(I),PCIP(I
     1),IRAT,IPCI(I),CONST(I),CURE(I)
    6 CONTINUE
C
C     READ OFF TIRE LIST
C
      IF(IDATA.EQ.1) WRITE(6,126)
      DO 7 I=1,N
      READ(5,103) OFF(I),IOFC(I)
      IF(IDATA.EQ.0) GO TO 7
      WRITE(6,118) OFF(I),IOFC(I)
    7 CONTINUE
      L=0
C
C     SEARCH ALL LOCATIONS TO DETERMINE THOSE BEING VACATED.
C
      DO 8 I=1,N
      DO 88 J=1,LCTNS
      IF(OFF(I).NE.TOP(J)) GO TO 88
      L=L+1
      JK(L)=J
      ITOPC(J)=0
      TOP(J)=8H   EMPTY
      GO TO 8
   88 CONTINUE
    8 CONTINUE
      IF(IDIAG.EQ.0) GO TO 10
      WRITE(6,120)(JK(I),I=1,N)
C
C     WRITE OUT THE EMPTY CAVITIES.
C
      WRITE(6,105)
      DO 9 I=1,N
      J=JK(I)
      WRITE(6,106) PRESS(J)
```

```
    9 CONTINUE
   10 CONTINUE
C
C     ADD TIRES IN THE HALF EMPTY PRESSES TO THE -ON- TIRE LIST.INCREASE
C     N BY THIS NUMBER.
C
      NORIG=N
      DO 35 II=1,NORIG
      I=JK(II)
      KK=(I/2)*2
      LE=I+1
      IF(KK.EQ.I) LE=I-1
      DO 34 JJ=1,NORIG
      IF(JK(JJ).EQ.LE) GO TO 35
   34 CONTINUE
      N=N+1
      JK(N)=LE
C
C     CONVERT THE CURE SPECS FOR THE ABOVE TIRES FROM THE -E- FORMAT TO
C     ACTIVE FORM.
C
      CALL CONVERT(N,LE,EGTHT,TOP,GTHT,ON,CONST,ECONST,PCIP,EPCIP,SHAPE,
     1ESHAPE,CT,ECT,TMLDH,ETMLDH,TMLDW,ETMLDW,IPCI,IEPCI,CURE,ECURE,IFC,
     2ITOPC,0)
   35 CONTINUE
      NEW=NORIG+1
      NORG2=N
      IF(IDIAG.EQ.0) GO TO 37
      WRITE(6,130)
      DO 36 I=NEW,N
      JJ=JK(I)
      WRITE(6,129) PRESS(JJ)
   36 CONTINUE
   37 CONTINUE
C
C     SET UP THE A MATRIX
C
   58 DO 12 I=1,N
      DO 12 J=1,N
      D6=0.
      JTEMP=JK(J)
      IF(K2.GT.0) JTEMP=JJK(J)
      IF(JTEMP.LE.NSEMI.AND.CT(I).NE.CTME) D6=1000.
      D1=PMHT(JTEMP)-TMLDH(I)
      IF(D1.LT.0.) GO TO 11
      D2=PMW(JTEMP)-TMLDW(I)
      IF(D2.LT.0.) GO TO 11
      D3=PSHP(JTEMP)-SHAPE(I)
      D4=100.
      D5=100.
```

```
      IF(IRAT(I).NE.1) GO TO 28
      IF(IPRAT(JTEMP).EQ.3) D4=0.
   28 IF(IPCI(I).NE.1) GO TO 29
      IF(IPPCI(JTEMP).EQ.2) D5=0.
   29 A(I,J)=D1*W(1)+D2*W(2)+D3*W(3)+D4*W(12)+D5*W(13)+D6
      GO TO 12
   11 A(I,J)=10000.
   12 CONTINUE
C
C     EVALUATE THE C MATRIX
C
      DO 14 I=1,N
      DO 14 J=1,N
      IF (I.EQ.J) GO TO 13
      H1=ABS(BW(I)-BW(J))
      IF(H1.GE.2.)H1=10.*H1
      H2=0.
      IF(CONST(I).NE.CONST(J)) H2=10.
      H3=ABS(PCIP(I)-PCIP(J))
      H4=ABS(SHAPE(J)-SHAPE(I))
      H5=ABS(CT(I)-CT(J))
      H6=ABS(GTHT(I)-GTHT(J))
      IF(H6.GT.1.5) H6=10.*H6
      C(I,J)=H5*W(4)+H6*W(5)+H4*W(6)+H1*W(7)+H3*W(14)
      GO TO 14
   13 C(I,J)=0.0
   14 CONTINUE
C
C     GENERATE THE D MATRIX
C
      DO 15 II=1,N
      DO 15 JJ=1,N
      IF(II.EQ.JJ) GO TO 27
C
C     ARE LOCATIONS I AND J ON THE SAME PRESS.
C
      F1=0.
      I=JK(II)
      J=JK(JJ)
      IF(K2.GT.0) I=JJK(II)
      IF(K2.GT.0) J=JJK(JJ)
      KK=(I/2)*2
      IF(KK.EQ.I) GO TO 21
      LL=I+1
      IF(J.EQ.LL) F1=200.
      GO TO 22
   21 LL=I-1
      IF(J.EQ.LL) F1=200.
C
C     ARE LOCATIONS I AND J IN THE SAME LINE OF PRESSES.
```

```
C
   22 ICK=0
      JCK=0
      DO 23 IT=1,NLINES
      ICK=ICK+NIEL(IT)
      IF(I.GT.ICK) GO TO 23
      LNI=IT
      GO TO 24
   23 CONTINUE
   24 DO 25 JT=1,NLINES
      JCK=JCK+NIEL(JT)
      IF(J.GT.JCK) GO TO 25
      LNJ=JT
      GO TO 26
   25 CONTINUE
   26 LDIF=IABS(LNI-LNJ)
      F2=5.
      IF(LDIF.EQ.0) F2=20.
      IF(LDIF.EQ.1) F2=15.
C
C     ARE LOCATIONS I AND J ON THE SAME MAKE OR MODEL OF PRESS.
C
      F3=5.
      F4=5.
      IF(MAKE(I).EQ.MAKE(J)) F3=10.
      IF(MOD(I).EQ.MOD(J)) F4=10.
      D(II,JJ)=F1*W(8)+F2*W(9)+F3*W(10)+F4*W(11)
      GO TO 15
   27 D(II,JJ)=0.
   15 CONTINUE
      IF(IDIAG.EQ.0) GO TO 19
      WRITE(6,108)
      DO 16 I=1,N
      WRITE(6,109)(A(I,J),J=1,N)
   16 CONTINUE
      WRITE(6,110)
      DO 17 I=1,N
      WRITE(6,109)(C(I,J),J=1,N)
   17 CONTINUE
      WRITE(6,111)
      DO 18 I=1,N
      WRITE(6,109)(D(I,J),J=1,N)
   18 CONTINUE
C
C     CALL SUBROUTINE OPTI TO PERFORM AN ITERATIVE ALGORITHM TO SOLVE
C     THE QUADRATIC ASSIGNMENT PROBLEM FORMULATED
C
   19 CONTINUE
      CALL OPTI(A,C,D,N,ICHOICE,ALPHA,IDIAG,E,IDOM,IRAN,VCIA,VCAI,VDAJ,V
     1DJA,WORK1,WORK2,IC,JR,WORK3,IZ,JZ,IBOM,IDAN,ZSTAR,ZPRIM,NN)
```

```
      ICNT=ICNT+1
      IF(IDIAG.EQ.0) GO TO 31
      WRITE(6,135)
      DO 20 I=1,N
      WRITE(6,112) (ALPHA(I,J),J=1,N)
   20 CONTINUE
   31 CONTINUE
      IF(ICNT.EQ.2) GO TO 60
      IF(NRUN.GT.1) GO TO 43
      WRITE(6,128)
   44 DO 33 I=1,NORIG
      DO 32 J=1,N
      IF(ALPHA(I,J).EQ.0) GO TO 32
      JJ=JK(J)
      IK(I)=JK(J)
      WRITE(6,127) ON(I),PRESS(JJ)
      GO TO 33
   32 CONTINUE
   33 CONTINUE
      IF(NPART.EQ.1) GO TO 45
      IF(N.NE.NORIG) GO TO 38
      WRITE(6,131)
      GO TO 42
   38 WRITE(6,132)
      LP=0
      DO 39 I=NEW,N
      DO 40 J=1,N
      IF(ALPHA(I,J).EQ.0) GO TO 40
      JJ=JK(J)
      IK(I)=JK(J)
      LP=1
      LJ=JK(I)
      OLD=PRESS(LJ)
      WRITE(6,133) ON(I),OLD,PRESS(JJ)
   40 CONTINUE
      IF(LP.EQ.0) WRITE(6,134)
   39 CONTINUE
      GO TO 42
C
C     CONVERT PARAMETERS OF -ON- TIRES TO THOSE OF TIRES IN THE CURE.
C
   43 WRITE(6,136)
      NPART=1
      GO TO 44
   45 DO 68 I=NEW,N
      DO 69 J=1,N
      IF(ALPHA(I,J).EQ.0) GO TO 69
      IK(I)=JK(J)
   69 CONTINUE
   68 CONTINUE
```

```
      DO 47 I=1,N
      DO 46 J=1,N
      IF(ALPHA(I,J).EQ.0) GO TO 46
      JJ=JK(J)
      CALL CONVERT(JJ,I,EGTHT,TOP,GTHT,ON,CONST,ECONST,PCIP,EPCIP,SHAPE,
     1ESHAPE,CT,ECT,TMLDH,ETMLDH,TMLDW,ETMLDW,IPCI,IEPCI,CURE,ECURE,IFC,
     2ITOPC,1)
   46 CONTINUE
   47 CONTINUE
C
C     CALCULATE A VALUE OF Z FOR EACH TIRE IN THE CURE.
C
      K=0
      DO 48 I=2,LCTNS,2
      K=K+1
      L=I-1
      DA11=ABS(PMHT(L)-ETMLDH(L))
      DA21=ABS(PMHT(I)-ETMLDH(I))
      DA12=ABS(PMW(L)-ETMLDW(L))
      DA22=ABS(PMW(I)-ETMLDW(I))
      DA13=ABS(PSHP(L)-ESHAPE(L))
      DA23=ABS(PSHP(I)-ESHAPE(I))
      A1=DA11*W(1)+DA12*W(2)+DA13*W(3)
      A2=DA21*W(1)+DA22*W(2)+DA23*W(3)
      H121=ABS(EBW(L)-EBW(I))
      IF(H121.GE.2.) H121=H121*10.
      H122=0.
      IF(ECONST(I).NE.ECONST(L)) H122=10.
      H123=ABS(EPCIP(L)-EPCIP(I))
      H124=ABS(ESHAPF(L)-ESHAPE(I))
      H125=ABS(ECT(L)-ECT(I))
      H126=ABS(EGTHT(L)-EGTHT(I))
      IF(H126.GT.1.5) H126=H126*10.
      C12=H125*W(4)+H126*W(5)+H124*W(6)+H121*W(7)+H123*W(14)
      Z(K)=A1+A2+C12
   48 CONTINUE
C
C
C     SELECT THE LARGEST VALUES OF Z FOR RE-OPTIMIZATION.
      LCTNS2=LCTNS/2
      DO 50 I=1,N
      ONOR(I)=ON(I)
   50 CONTINUE
      IF(IDIAG.EQ.0) GO TO 67
      WRITE(6,137)
      DO 51 I=2,LCTNS2,2
      K=I-1
      WRITE(6,138) Z(K),Z(I)
   51 CONTINUE
   67 K=0
```

```
      N=0
      DO 54 I=2,LCTNS,2
      K=K+1
      IF(Z(K).LT.ZMAX) GO TO 54
      LO=1
      N=N+2
      L=N
      K2=2*K
      GO TO 53
   52 LO=0
   53 CALL CONVERT(L,K2,EGTHT,TOP,GTHT,ON,CONST,ECONST,PCIP,EPCIP,SHAPE,
     1ESHAPE,CT,ECT,TMLDH,ETMLDH,TMLDW,ETMLDW,IPCI,IEPCI,CURE,ECURE,IFC,
     2ITOPC,0)
      JJK(L)=K2
      IF(N.GE.NN) GO TO 55
      L=N-1
      K2=K2-1
      IF(LO.EQ.1) GO TO 52
      IF(N.GE.NMAX) GO TO 56
   54 CONTINUE
      WRITE(6,141) N
      IF(N.GT.0) GO TO 56
      NPART=0
      N=NORG2
      GO TO 44
   55 WRITE(6,139)
      STOP
   56 DO 57 I=1,NN
      DO 57 J=1,NN
      E(I,J)=0.
      A(I,J)=0.
      C(I,J)=0.
      D(I,J)=0.
      ALPHA(I,J)=0
   57 CONTINUE
      GO TO 58
   60 WRITE(6,132)
      DO 59 I=1,N
      DO 61 J=1,N
      IF(ALPHA(I,J).EQ.0) GO TO 61
      DO 62 II=1,NORIG
      IF(ON(I).NE.ONOR(II)) GO TO 62
      IK(II)=JJK(J)
      GO TO 59
   62 CONTINUE
      DO 63 II=NEW,NORG2
      IF(ON(I).NE.ONOR(II)) GO TO 63
      IK(II)=JJK(J)
      GO TO 59
   63 CONTINUE
```

```
      LL=JJK(I)
      L=JJK(J)
      IF(TOP(LL).EQ.TOP(L)) GO TO 61
      WRITE(6,133) TOP(LL),PRESS(LL),PRESS(L)
      CALL CONVERT(L,LL,EGTHT,TOP,GTHT,ON,CONST,ECONST,PCIP,EPCIP,SHAPE,
     1ESHAPE,CT,ECT,TMLDH,ETMLDH,TMLDW,ETMLDW,IPCI,IEPCI,CURE,ECURE,IFC,
     2ITOPC,2)
   61 CONTINUE
   59 CONTINUE
      DO 64 I=NEW,NORG2
      LL=IK(I)
      LLL=JK(I)
      WRITE(6,133) ONOR(I),PRESS(LLL),PRESS(LL)
      CALL CONVERT(LL,I,EGTHT,TOP,GTHT,ONOR,CONST,ECONST,PCIP,EPCIP,SHAP
     1E,ESHAPE,CT,ECT,TMLDH,ETMLDH,TMLDW,ETMLDW,IPCI,IEPCI,CURE,ECURE,IF
     2C,ITOPC,1)
   64 CONTINUE
      WRITE(6,128)
      DO 65 I=1,NORIG
      JJ=IK(I)
      WRITE(6,127) ONOR(I),PRESS(JJ)
      CALL CONVERT(JJ,I,EGTHT,TOP,GTHT,ONOR,CONST,ECONST,PCIP,EPCIP,SHAP
     1E,ESHAPE,CT,ECT,TMLDH,ETMLDH,TMLDW,ETMLDW,IPCI,IEPCI,CURE,ECURE,IF
     2C,ITOPC,1)
   65 CONTINUE
C
C     WRITE OUT THE NEW CURE ASSIGNMENT.
C
      WRITE(6,144)
      KI=1
      DO 72 I=1,NLINES
      KO=KI+NIEL(I)-2
      WRITE(6,142) (PRESS(II),II=KI,KO,2)
      WRITE(6,143) (ITOPC(II),II=KI,KO,2)
      KL=KO+1
      KN=KI+1
      WRITE(6,143) (ITOPC(II),II=KN,KL,2)
      KI=KI+NIEL(I)
   72 CONTINUE
  144 FORMAT(1H0,23HTHE NEW CURE ASSIGNMENT/1H ,23H---------------------
     1--)
  142 FORMAT(1H0,15A8)
  143 FORMAT(1H0,I9,14I8////)
C
C     PUNCH THE CURE PARAMETERS FOR THE NEXT RUN.
C
      IF(IPUNCH.EQ.0) GO TO 42
      DO 70 I=1,LCTNS
      WRITE(4,102) PRESS(I),TOP(I),ITOPC(I)
   70 CONTINUE
```

```
      DO 71 I=1,LCTNS
      WRITE(4,104)EGTHT(I),ECT(I),ESHAPE(I),ETMLDH(I),ETMLDW(I),EBW(I),E
     1PCIP(I),IERAT(I),IEPCI(I),ECONST(I),ECURE(I)
   71 CONTINUE
  100 FORMAT(7I8,F10.0,I8)
  101 FORMAT(A3,3X,A3,3X,I3,3F10.3,2I3)
  102 FORMAT(A4,6X,A8,5X,I5)
  103 FORMAT(A8,6X,I4)
  104 FORMAT(7F9.2,2I3,2A5)
  105 FORMAT(1H0,21HEMPTY PRESS LOCATIONS)
  106 FORMAT(1H0,A4)
  107 FORMAT(10F8.0)
  108 FORMAT(1H1,12HTHE A MATRIX/1H ,12H------------)
  109 FORMAT(1H0,12F7.0)
  110 FORMAT(1H1,12HTHE C MATRIX/1H ,12H------------)
  111 FORMAT(1H1,12HTHE D MATRIX/1H ,12H------------)
  112 FORMAT(1H0,12+10)
  113 FORMAT(1H0,10HLOCATIONS=I5,10X,17HLINES OF PRESSES=,I5)
  114 FORMAT(2I5,F10.2,I5)
  115 FORMAT(1H ,A4,5X,A4,5X,I4,3F12.4,2I6)
  116 FORMAT(1H ,A4,7X,A8,6X,I4)
  117 FORMAT(1H0,49HVALUES OF N,ICHOICE,IDIAG,NN,IDATA,NRUN,NMAX,ZMAX/1H
     1 ,7I10,F20.3)
  118 FORMAT(1H ,A8,6X,I4)
  119 FORMAT(1H ,7F12.3,2I10,2A10)
  120 FORMAT(1H0,12HVALUES OF JK/1H0,10I12)
  121 FORMAT(I10)
  122 FORMAT(1H0,4HMAKE,4X,5HMODEL,5X,6HRATING,3X,7HMOLD HT,4X,8HMOLD DI
     1A,5X,7HSHAPING,2X,5HCONTR,2X,3HPCI)
  123 FORMAT(1H0,5HPRESS,8X,4HTIRE,8X,4HCODE)
  124 FORMAT(1H0,54HON TIRE LIST 8 DIGIT PART NO. AND 4 DIGIT FACTORY CO
     1DE)
  125 FORMAT(1H0,9X,4HGTHT,8X,9HCURE TIME,9X,5HSHAPE,9X,7HMOLD HT,7X,8HM
     1OLD DIA,7X,8HBEAD DIA)
  126 FORMAT(1H0,55HOFF TIRE LIST 8 DIGIT PART NO. AND 4 DIGIT FACTORY C
     1ODE)
  127 FORMAT(1H0,5HTIRE ,A8,26H TO BE SCHEDULED ON PRESS ,A4)
  128 FORMAT(1H0,23HTHE -ON- TIRE LOCATIONS/1H ,23H---------------------
     1--)
  129 FORMAT(1H0,5X,A4)
  130 FORMAT(1H0,37HADDITIONAL PRESS LOCATIONS CONSIDERED)
  131 FORMAT(1H0,22HNO TIRES ARE RELOCATED)
  132 FORMAT(1H0/1H0,33HTHE FOLLOWING TIRES ARE RELOCATED/1H ,33H-------
     1--------------------------)
  133 FORMAT(1H0,4HTIRE,1X,A8,1X,20HRELOCATED FROM PRESS,A4,9H TO PRESS,
     1A4)
  134 FORMAT(1H0,10X,4HNONE)
  135 FORMAT(1H1,30HTHE ALPHA (PERMUTATION) MATRIX)
  136 FORMAT(1H0,29HINITIAL PLACING OF -ON- TIRES/1H ,29H---------------
     1--------------)
```

```
137 FORMAT(1H0,11HVALUES OF Z)
138 FORMAT(1H ,2F20.4)
139 FORMAT(1H0,39HINSUFFICIENT ARRAY SIZING - JOB ABORTED)
140 FORMAT(F15.3)
141 FORMAT(1H0,2HN=,I5)
 42 STOP
    END
```

```
      SUBROUTINE OPTI(A,C,D,N,ICHOICE,ALPHA,IDIAG,E,IDOM,IRAN,VCIA,VCAI,
     1VDAJ,VDJA,WORK1,WORK2,IC,JR,WORK3,IZ,JZ,IBOM,IDAN,ZSTAR,ZPRIM,NN)
C
C     PURPOSE OF THIS SUBROUTINE IS TO EMPLOY A  SUBOPTIMAL ALGORITHM TO
C     DETERMINE AN ASSIGNMENT OF N ACTIVITIES ON N FACILITIES. IT IS
C     NECESSARY TO MINIMIZE THE SUM OF AN ORDINARY ASSIGNMENT PROBLEM
C     AND A QUADRATIC INTERACTION COST BETWEEN ACTIVITIES.
C
      DIMENSION A(NN,1),C(NN,1),D(NN,1),E(NN,1),IDOM(1),IRAN(1),VCIA(NN,
     11),VCAI(NN,1),VDAJ(NN,1),VDJA(NN,1),WORK1(1),WORK2(1),IC(1),JR(1),
     2WORK3(NN,1),IZ(1),JZ(1),IBOM(1),IDAN(1)
      INTEGER ALPHA(NN,1),ZSTAR(NN,1),ZPRIM(NN,1)
      KK=0
      T=0.5
C     CLEAR ALL WORKING ARRAYS
C
      DO 1 I=1,NN
      IDOM(I)=0
      IRAN(I)=0
      WORK1(I)=0.
      WORK2(I)=0.
      DO 1 J=1,N
      WORK3(I,J)=0.
      ALPHA(I,J)=0
      VCIA(I,J)=0.
      VCAI(I,J)=0.0
      VDAJ(I,J)=0.
      VDJA(I,J)=0.
      E(I,J)=0.
    1 CONTINUE
C
C     ESTABLISH DOM(ALPHA) AND RAN(ALPHA)
C
   20 LL=0
      DO 3 I=1,N
      DO 2 J=1,N
      IF(ALPHA(I,J).EQ.0) GO TO 2
      IDOM(I)=1
      IRAN(J)=1
      LL=LL+1
      GO TO 3
    2 CONTINUE
    3 CONTINUE
      IF(IDIAG.EQ.0) GO TO 24
      WRITE(6,200)(IDOM(I),I=1,N)
      WRITE(6,201)(IRAN(J),J=1,N)
C
C     ESTABLISH C(I,ALPHA) AND C(ALPHA,I) VECTORS AND THEN SORT INTO
C     ORDER OF ASCENDING MAGNITUDE
C
```

```
   24 DO 5 I=1,N
      IF(IDOM(I).EQ.1) GO TO 5
      M=0
      DO 4 K=1,N
      IF(IDOM(K).EQ.1) GO TO 4
      IF (K.EQ.I) GO TO 4
      M=M+1
      WORK1(M)=C(I,K)
      WORK2(M)=C(K,I)
    4 CONTINUE
      CALL SORT(WORK1,M,-1)
      CALL SORT(WORK2,M,-1)
      IF(IDIAG.EQ.0) GO TO 25
      WRITE(6,202)
   25 DO 5 K=1,M
      VCIA(I,K)=WORK1(K)
      VCAI(I,K)=WORK2(K)
      IF(IDIAG.EQ.0) GO TO 5
      WRITE(6,203) VCIA(I,K),VCAI(I,K)
    5 CONTINUE
  203 FORMAT(1H ,2F30.8)
C
C     CLEAR THE WORKING ARRAYS
C
      DO 6 I=1,NN
      WORK1(I)=0.
      WORK2(I)=0.
    6 CONTINUE
C
C     ESTABLISH D(J,ALPHA) AND D(ALPHA,J) VECTORS AND SORT INTO
C     DESCENDING ORDER OF MAGNITUDE
C
      DO 8 J=1,N
      IF(IRAN(J).EQ.1) GO TO 8
      MM=0
      DO 7 K=1,N
      IF(IRAN(K).EQ.1) GO TO 7
      IF(J.EQ.K) GO TO 7
      MM=MM+1
      WORK1(MM)=D(J,K)
      WORK2(MM)=D(K,J)
    7 CONTINUE
      CALL SORT(WORK1,MM,1)
      CALL SORT(WORK2,MM,1)
      IF(IDIAG.EQ.0) GO TO 26
      WRITE(6,204)
   26 DO 8 K=1,MM
      VDAJ(J,K)=WORK1(K)
      VDJA(J,K)=WORK2(K)
      IF(IDIAG.EQ.0) GO TO 8
```

```
      WRITE(6,203) VDAJ(J,K),VDJA(J,K)
    8 CONTINUE
      IF(MM.EQ.M) GO TO 9
      WRITE(6,100)
C
C     CALCULATE THE ELEMENTS OF THE E MATRIX
C
    9 DO 12 I=1,N
      IF(IDOM(I).EQ.1) GO TO 131
      DO 12 J=1,N
      IF(IRAN(J).EQ.1) GO TO 121
C
C     CALCULATE THE PERMUTED DOT PRODUCT OF P(C(I,ALPHA),D(J,ALPHA))
C     AND P(C(ALPHA,I),D(ALPHA,J))
C
      P1=0.0
      P2=0.0
      DO 10 K=1,M
      P1=P1+VCIA(I,K)*VDJA(J,K)
      P2=P2+VCAI(I,K)*VDAJ(J,K)
   10 CONTINUE
      FTERM=0.
C
C     EVALUATE THE FINAL TERM IN THE EXPRESSION FOR THE ELEMENT OF THE
C     MATRIX E
C
      DO 11 ME=1,N
      DO 11 MN=1,N
      IF(ALPHA(ME,MN).EQ.0) GO TO 11
      FTERM=FTERM+C(I,ME)*D(J,MN)+C(ME,I)*D(MN,J)
   11 CONTINUE
      E(I,J)=A(I,J)+T*P1+(1.-T)*P2+FTERM
      GO TO 12
  121 E(I,J)=0.0
      GO TO 12
  131 DO 12 JJ=1,N
      E(I,JJ)=0.0
   12 CONTINUE
      IF(IDIAG.EQ.0) GO TO 27
      WRITE(6,205)
      DO 351 I=1,N
      WRITE(6,206)(E(I,J),J=1,N)
  351 CONTINUE
C
C     SELECT THE CRITERIA FOR SELECTION OF I AND J TO FORM NEW PERM-
C     UTATION MATRIX ALPHA OF RANK K+1
C
   27 IF(ICHOICE.EQ.1) GO TO 13
C
C     SELECTION METHOD B -- IE I AND J SELECTED ON BASIS OF THE SOLUTION
C     OF A LINEAR ASSIGNMENT PROBLEM
```

```
C
      LN=0
      LC=0
C
C     RE-ARRANGE THE ELEMENTS OF THE E MATRIX TO FORM A MATRIX (WORK3)
C     THAT CONTAINS NO NON-RELEVANT ZEROS OR DOES NOT CONTAIN E(I,J)
C     WHERE I IS IN THE DOMAIN OF ALPHA OR J IS IN THE RANGE OF ALPHA
C
      DO 21 I=1,N
      IF(IDOM(I).EQ.1) GO TO 21
      LM=0
      LN=LN+1
      DO 21 J=1,N
      IF(IRAN(J).EQ.1) GO TO 21
      LC=LC+1
      LM=LM+1
      WORK3(LN,LM)=E(I,J)
   21 CONTINUE
C
C     CALL SUBROUTINE LINAS TO SOLVE THE LINEAR ASSIGNMENT PROBLEM
C     USING MATRIX E AS THE ASSIGNMENT MATRIX
C
      CALL LINAS(WORK3,LN,ZSTAR,IDIAG,N,IZ,JZ,IBOM,IDAN,ZPRIM,NN)
      LN=0
      LM=0
      BIG=0.0
C
C     SELECT THE LARGEST ELEMENT FROM THE MATRIX E THAT APPEARS IN THE
C     ASSIGNMENT SOLUTION. THIS CHOOSES THE VALUES OF I(K+1) AND J(K+1)
C     AND THUS DETERMINES THE NEXT PERMUTATION MATRIX ALPHA(K+1)
C
      DO 23 I=1,N
      IF(IDOM(I).EQ.1) GO TO 23
      LM=0
      LN=LN+1
      DO 22 J=1,N
      IF(IRAN(J).EQ.1) GO TO 22
      LM=LM+1
      IF(ZSTAR(LN,LM).EQ.0)  GO TO 22
      IF(E(I,J).LT.BIG) GO TO 23
      BIG=E(I,J)
      JK=J
      IK=I
      GO TO 23
   22 CONTINUE
   23 CONTINUE
      IF(IDIAG.EQ.0) GO TO 19
      WRITE(6,2077)IK,JK
      GO TO 19
   13 CONTINUE
```

```
C
C     SELECTION METHOD A -- IE I AND J SELECTED ON THE BASIS OF MAXIMUM
C     OF THE MIMIMUMS OF EACH ROW AND EACH COLUMN
C
      DO 14 I=1,N
      FMINR=10.E 6
      DO 14 J=1,N
      IF(E(I,J).EQ.0.0) GO TO 14
      IF(E(I,J).GT.FMINR) GO TO 14
      FMINR=E(I,J)
      JR(I)=J
   14 CONTINUE
      DO 15 J=1,N
      FMINC=10.E 6
      DO 15 I=1,N
      IF(E(I,J).EQ.0.0) GO TO 15
      IF(E(I,J).GT.FMINC) GO TO 15
      IC(J)=I
      FMINC=E(I,J)
   15 CONTINUE
C
C     SELECT THE MAX OF THE MIN OF THE ROWS
C
      FMAXR=0.0
      DO 16 I=1,N
      JJ=JR(I)
      IF(E(I,JJ).LT.FMAXR) GO TO 16
      FMAXR=E(I,JJ)
      IT=I
   16 CONTINUE
C
C     SELECT THE MAX OF THE MIN OF THE COLUMNS
C
      FMAXC=0.0
      DO 17 J=1,N
      II=IC(J)
      IF(E(II,J).LT.FMAXC) GO TO 16
      FMAXC=E(II,J)
      JT=J
   17 CONTINUE
      IF(FMAXC.GE.FMAXR) GO TO 18
      IK=IC(JT)
      JK=JT
      GO TO 19
   18 IK=IT
      JK=JR(IT)
   19 ALPHA(IK,JK)=1
      KK=KK+1
      IF(KK.EQ.N) RETURN
      GO TO 20
```

```
 100 FORMAT(1H0,36HSTOPPED DUE TO UNEQUAL SIZED VECTORS)
 200 FORMAT(1H0,4HIDOM/1H0,12I10)
 201 FORMAT(1H0,4HIRAN/1H0,12I10)
 202 FORMAT(1H0,21HVECTORS VCIA AND VCAI)
 204 FORMAT(1H0,13HVDAJ AND VDJA)
 205 FORMAT(1H0,12HTHE F MATRIX)
 206 FORMAT(1H ,12F10.1)
2077 FORMAT(1H0,9HIK AND JK/1H0,2I20)
     END
```

```
      SUBROUTINE LINAS(WORK3,LC,ZSTAR,IDIAG,N,IZ,JZ,IDOM,IRAN,ZPRIM,NN)
C
C     THE PURPOSE OF THIS SUBROUTINE IS TO FIND A MINIMAL ASSIGNMENT
C     OF LC ACTIVITIES ON LC FACILITIES. IN CASE WHERE LINAS IS CALLED
C     BY SUBROUTINE OPTI-- THE INPUT MATRIX WORK3 IS TREATED AS AN
C     ASSIGNMENT MATRIX.
      DIMENSION WORK3(NN,1),IZ(1),JZ(1),IDOM(1),IRAN(1)
      INTEGER ZSTAR(NN,1),ZPRIM(NN,1)
C
C     CLEAR THE WORKING ARRAYS
C
      DO 1 I=1,NN
      IDOM(I)=0
      IRAN(I)=0
      DO 1 J=1,NN
      ZSTAR(I,J)=0
      ZPRIM(I,J)=0
    1 CONTINUE
      LC2=LC*2
      DO 111 I=1,LC2
      IZ(I)=0
      JZ(I)=0
  111 CONTINUE
      IF(IDIAG.EQ.0) GO TO 32
      WRITE(6,453)
      DO 451 I=1,LC
      WRITE(6,452)(WORK3(I,J),J=1,LC)
  451 CONTINUE
C
C     SUBTRACT THE SMALLEST ELEMENT IN EACH ROW OF WORK3 FROM ITS OWN
C     ROW
C
   32 DO 3 I=1,LC
      SMALL=10.E 6
      DO 2 J=1,LC
      IF(WORK3(I,J).GT.SMALL) GO TO 2
      SMALL=WORK3(I,J)
      JT=J
    2 CONTINUE
      DO 3 L=1,LC
      WORK3(I,L)=WORK3(I,L)-SMALL
    3 CONTINUE
C
C     SUBTRACT THE SMALLEST ELEMENT OF EACH COLUMN OF WORK3 FROM ITS
C     OWN COLUMN
C
      DO 5 J=1,LC
      SMALL=10.E 6
      DO 4 I=1,LC
      IF(WORK3(I,J).GT.SMALL) GO TO 4
```

```
      SMALL=WORK3(I,J)
      IT=I
    4 CONTINUE
      DO 5 L=1,LC
      WORK3(L,J)=WORK3(L,J)-SMALL
    5 CONTINUE
C
C     STAR THE INDEPENDANT ZEROS IN THE NEW MATRIX WORK3
C
      DO 8 I=1,LC
      DO 88 J=1,LC
      IF (WORK3(I,J).GT.0.) GO TO 88
C
C     CONSIDER THE ROW IN WHICH THE ABOVE ZERO OCCURS
C
      DO 6 L=1,LC
      IF(ZSTAR(I,L).EQ.1) GO TO 8
    6 CONTINUE
C
C     CONSIDER THE COLUMN IN WHICH THE ABOVE ZERO OCCURS
C
      DO 7 L=1,LC
      IF(ZSTAR(L,J).EQ.1) GO TO 88
    7 CONTINUE
      ZSTAR(I,J)=1
   88 CONTINUE
    8 CONTINUE
      IF(IDIAG.EQ.0) GO TO 33
      WRITE(6,107)
      DO 108 I=1,LC
      WRITE(6,109)(ZSTAR(I,J),J=1,LC)
  108 CONTINUE
C
C     COVER THE COLUMNS THAT CONTAIN A STARRED ZERO. IF IRAN(J)=1 COLUMN
C     J IS UNCOVERED. IF IRAN(J)=1 COLUMN J IS COVERED
C
   33 DO 10 J=1,LC
      DO 9 I=1,LC
      IF(ZSTAR(I,J).EQ.0) GO TO 9
      IRAN(J)=1
      GO TO 10
    9 CONTINUE
   10 CONTINUE
C
C     STEP 1 --- CHOOSE A NON-COVERED ZERO AND PRIME IT.
C
   30 ISTEP=1
      IF(IDIAG.EQ.0) GO TO 34
      WRITE(6,104)
      DO 31 I=1,LC
```

```
      WRITE(6,103)(WORK3(I,J),J=1,LC)
   31 CONTINUE
   34 DO 13 I=1,LC
      DO 13 J=1,LC
      IF(WORK3(I,J).GT.0.0) GO TO 13
      IF(IRAN(J).EQ.1) GO TO 13
      IF(IDOM(I).EQ.1) GO TO 13
      ZPRIM(I,J)=1
C
C
C     CONSIDER ROW IN WHICH ABOVE ZERO OCCURS-ASCERTAIN WHETHER IT
C     CONTAINS A STARRED ZERO. IF YES - UNCOVER COLUMN AND COVER
C     ROW IN WHICH THE STARRED ZERO OCCURS
      DO 11 L=1,LC
      IF(ZSTAR(I,L).EQ.0) GO TO 11
      IDOM(I)=1
      IRAN(L)=0
      GO TO 13
   11 CONTINUE
      GO TO 14
   13 CONTINUE
      IF(IDIAG.EQ.0) GO TO 23
      WRITE(6,102)
      GO TO 23
   14 CONTINUE
C
C     STEP 2 -- FIND THE UNCOVERED PRIMED ZERO AND CONSTRUCT THE
C     SEQUENCE OF ZEROS
C
      ISTEP=2
      K=0
      DO 15 I=1,LC
      DO 15 J=1,LC
      IF(ZPRIM(I,J).EQ.0) GO TO 15
      IF(IDOM(I).EQ.1) GO TO 15
      IF(IRAN(J).EQ.1) GO TO 15
      K=K+1
      IZ(K)=I
      JZ(K)=J
      LEF=JZ(K)
      IF(IDIAG.EQ.0) GO TO 16
      WRITE(6,110)K,IZ(K),JZ(K)
      GO TO 16
   15 CONTINUE
      WRITE(6,100)
      RETURN
   16 DO 17 I=1,LC
      IF(ZSTAR(I,LEF).EQ.0) GO TO 17
      K=K+1
      IZ(K)=I
```

```
      JZ(K)=JZ(K-1)
      LEM=IZ(K)
      LEP=JZ(K)
      ZSTAR(I,LEP)=0
      IF(IDIAG.EQ.0) GO TO 18
      WRITE(6,110)K,IZ(K),JZ(K)
      GO TO 18
   17 CONTINUE
C
C     SEQUENCE OF ZEROS TERMINATED NORMALLY
C
      GO TO 20
   18 DO 19 L=1,LC
      IF(ZPRIM(LEM,L).EQ.0) GO TO 19
      K=K+1
      IZ(K)=IZ(K-1)
      LET=IZ(K)
      JZ(K)=L
      LEF=JZ(K)
      IF(IDIAG.EQ.0) GO TO 16
      WRITE(6,110)K,IZ(K),JZ(K)
      GO TO 16
   19 CONTINUE
C
C     THE SEQUENCE OF ZEROS TERMINATED ON A STARRED ZERO
C
      WRITE(6,101)
      RETURN
   20 CONTINUE
C
C     UNSTAR EACH STARRED ZERO OF THE SEQUENCE AND STAR EACH PRIMED
C     ZERO OF THE SEQUENCE
C
      DO 201 I=1,K,2
      L=IZ(I)
      LL=JZ(I)
      ZSTAR(L,LL)=1
  201 CONTINUE
      DO 21 I=1,LC
      DO 21 J=1,LC
      ZPRIM(I,J)=0
   21 CONTINUE
C
C     UNCOVER EVERY ROW AND COVER EVERY COLUMN CONTAINING A STARRED ZERO
C
      DO 22 I=1,LC
      DO 22 J=1,LC
      IF (ZSTAR(I,J).EQ.0) GO TO 22
      IDOM(I)=0
      IRAN(J)=1
```

```
   22 CONTINUE
      IF(IDIAG.EQ.0) GO TO 23
      WRITE(6,191)
      WRITE(6,112)(IDOM(I),I=1,LC)
      WRITE(6,112)(IRAN(I),I=1,LC)
      WRITE(6,113)
      DO 114 I=1,LC
      WRITE(6,109)(OSTAR(I,J),J=1,LC)
  114 CONTINUE
C
C     CHECK ON POSSIBLE CONDITION THAT ALL COLUMNS ARE COVERED
C
   23 DO 24 I=1,LC
      IF(IRAN(I).EQ.1) GO TO 24
      IF(ISTEP.EQ.1) GO TO 25
      GO TO 30
   24 CONTINUE
      IF(IDIAG.EQ.0) GO TO 35
      WRITE(6,106)
      DO 29 I=1,LC
      WRITE(6,105)(ZSTAR(I,J),J=1,LC)
   29 CONTINUE
   35 RETURN
C
C     STEP 3 --
C
   25 H=10.E 6
      DO 26 I=1,LC
      IF(IDOM(I).EQ.1) GO TO 26
      DO 26 J=1,LC
      IF(IRAN(J).EQ.1) GO TO 26
      IF(WORK3(I,J).GE.H) GO TO 26
      H=WORK3(I,J)
   26 CONTINUE
      DO 27 I=1,LC
      IF(IDOM(I).EQ.0) GO TO 27
      DO 27 J=1,LC
      WORK3(I,J)=WORK3(I,J)+H
   27 CONTINUE
      DO 28 J=1,LC
      IF(IRAN(J).EQ.1) GO TO 28
      DO 28 I=1,LC
      WORK3(I,J)=WORK3(I,J)-H
   28 CONTINUE
      GO TO 30
  100 FORMAT(1H0,46HNO UNCOVERED ZERO FOUND BY LINAS DURING STEP 2)
  101 FORMAT(1H0,42HSERIES OF ZEROS TERMINATED ON STARRED ZERO)
  102 FORMAT(1H0,29HCOULD NOT FIND UNCOVERED ZERO)
  103 FORMAT(1H ,12F10.2)
  104 FORMAT(1H0,16HTHE MATRIX WORK3)
```

```
105 FORMAT(1H ,12I6)
106 FORMAT(1H0,25HTEMPORARY OUTPUT OF ZSTAR)
107 FORMAT(1H0,21HINDEPENDANT 0 STARRED)
109 FORMAT(1H ,12I6)
110 FORMAT(1H0,2HK=,I5,5X,6HIZ(K)=,I5,5X,6HJZ(K)=,I5)
112 FORMAT(1H ,12I10)
113 FORMAT(1H ,33HZSTAR PRECEEDING RETURN TO STEP 1)
191 FORMAT(1H0,24HCOVERED ROWS AND COLUMNS)
452 FORMAT(1H ,12F10.2)
453 FORMAT(1H0,15HOUTPUT NUMBER 2)
    END
```

```
      SUBROUTINE CONVERT(I,J,EGTHT,TOP,GTHT,ON,CONST,ECONST,PCIP,EPCIP,S
     1HAPE,ESHAPE,CT,ECT,TMLDH,ETMLDH,TMLDW,ETMLDW,IPCI,IEPCI,CURE,ECURE
     2,IFC,ITOPC,INV)
      DIMENSION EGTHT(1),TOP(1),GTHT(1),ON(1),CONST(1),ECONST(1),PCIP(1)
     1,SHAPE(1),ESHAPE(1),CT(1),ECT(1),TMLDH(1),ETMLDH(1),TMLDW(1),ETMLD
     2W(1),IPCI(1),IEPCI(1),CURE(1),ECURE(1),IFC(1),ITOPC(1),EPCIP(1)
C
C     IF INV=0 CONVERSION IS ACTIVE TO -E-.
C     IF INV=1 CONVERSION IS -E- TO ACTIVE.
C     IF INV=2 CONVERSION IS -E- TO -E-
C
      IF(INV.NE.0) GO TO 10
      ON(I)=TOP(J)
      IFC(I)=ITOPC(J)
      GTHT(I)=EGTHT(J)
      CONST(I)=ECONST(J)
      PCIP(I)=EPCIP(J)
      SHAPE(I)=ESHAPE(J)
      CT(I)=ECT(J)
      TMLDH(I)=ETMLDH(J)
      TMLDW(I)=ETMLDW(J)
      IPCI(I)=IEPCI(J)
      CURE(I)=ECURE(J)
      GO TO 20
   10 IF(INV.EQ.2) GO TO 15
      TOP(I)=ON(J)
      ITOPC(I)=IFC(J)
      EGTHT(I)=GTHT(J)
      ECONST(I)=CONST(J)
      EPCIP(I)=PCIP(J)
      ESHAPE(I)=SHAPE(J)
      ECT(I)=CT(J)
      ETMLDH(I)=TMLDH(J)
      ETMLDW(I)=TMLDW(J)
      IEPCI(I)=IPCI(J)
      ECURE(I)=CURE(J)
      GO TO 20
   15 TOP(I)=TOP(J)
      ITOPC(I)=ITOPC(J)
      EGTHT(I)=EGTHT(J)
      ECONST(I)=ECONST(J)
      EPCIP(I)=EPCIP(J)
      ESHAPE(I)=ESHAPE(J)
      ECT(I)=ECT(J)
      ETMLDH(I)=ETMLDH(J)
      ETMLDW(I)=ETMLDW(J)
      IEPCI(I)=IEPC+(J)
      ECURE(I)=ECURE(J)
   20 RETURN
      END
```

```
      SUBROUTINE  SORT(A,N,M)
      DIMENSION A(1)
      IF(M.GT.0) GO TO 20
      DO 10 I=1,N
      A(I)=-A(I)
   10 CONTINUE
   20 ILO=2
      IHI=N
   30 II=0
      DO 40 I=ILO,IHI
      IF(A(I-1).GT.A(I)) GO TO 40
      II=I
      T=A(I)
      A(I)=A(I-1)
      A(I-1)=T
   40 CONTINUE
      IF(II.EQ.0) GO TO 60
      IHI=II
      II=0
      IADD=IHI+ILO
      DO 50 J=ILO,IHI
      I=IADD-J
      IF(A(I-1).GT.A(I)) GO TO 50
      II=I
      T=A(I)
      A(I)=A(I-1)
      A(I-1)=T
   50 CONTINUE
      ILO=II
      IF(ILO.NE.0) GO TO 30
   60 IF(M.GT.0) RETURN
      DO 70 I=1,N
   70 A(I)=-A(I)
      RETURN
      END
```

APPENDIX D

Test Problems

D.1 General

The program of Appendix C has been used to determine solutions for several different problems, each of which has been designed to test different aspects of the system's capabilities.

D.2 Test Problem No. 1

The first problem attempted was designed to test the system's capacity to find a contrived optimum assignment.

The ON tire list used those tires shown in Table 2 of Section VII.1 in the main text of the thesis. Careful examination of these ON tires will reveal that the best assignment possible occurs when the following combinations occur:

TIREN201 is combined with TIREN204
TIREN202 is combined with TIREN208
TIREN203 is combined with TIREN206
TIREN205 is combined with TIREN209
TIREN207 is combined with TIREN210

In addition the ten OFF tires were contained on six different presses. The two remaining tires on the half-emptied presses are such that they also would make an ideal combination. The previous cure contrived for the problem was operating satisfactorily with the exception of press A3.

In order to be able to ascertain whether the suitability relations were being considered, the OFF tires were removed from presses with widely varying capabilities. Since the tires themselves are quite varied in press requirements, an inconsistency in assignment would easily be detected.

This system ascertained the optimum in 17.3 seconds of central processor time. The system completed the following assignments shown in Table X . The unexpected result of relocating tire (TIREN003) from press A3 to Press D15 to be combined with tire (TIRE210) and tire (TIREN003) from A3 to C15 to be combined with (TIREN207) results in a better overall cure assignment than that previously contrived. It should be noted that on the initial fit (ON tires to OFF tire locations) the pairwise assignment concurred exactly to the contrived pairwise arrangement.

D.3 <u>Test Problem No. 2</u>

The second test problem was contrived to test the capability of this system to handle realistic size problems. The particular problem examined was, in effect, twice test problem number 1. (Twenty ON tires to be assigned, the original ten tires plus an additional ten tires each matching one of the original ten in every aspect except designation number.) For this problem the original OFF list of ten tires was supplemented with ten additional tires taken from five identical presses. The purpose of using five identical presses was to determine how many tires are shifted about arbitrarily from the initial assignment on subsequent re-examination. This would indicate the locational influence exerted by additional tires being evaluated during this subsequent reassignment. These relocations, of course, do not require any physical relocation in the cure, but rather they are simply a change in assignment.

This system completed an assignment of these twenty tires, with

TABLE X

The New Assignments - Test Problem No. 1

THE FOLLOWING TIRES ARE RELOCATED

TIRE TIREN003 RELOCATED FROM PRESS A3 TO PRESS D15

TIRE TIREN003 RELOCATED FROM PRESS A3 TO PRESS C15

TIRE TIREN091 RELOCATED FROM PRESS B15 TO PRESS E6

TIRE TIREN093 RELOCATED FROM PRESS C15 TO PRESS E6

THE "ON" TIRE LOCATIONS

TIRE TIREN201 TO BE SCHEDULED ON PRESS D1

TIRE TIREN202 TO BE SCHEDULED ON PRESS B13

TIRE TIREN203 TO BE SCHEDULED ON PRESS A3

TIRE TIREN204 TO BE SCHEDULED ON PRESS D1

TIRE TIREN205 TO BE SCHEDULED ON PRESS B15

TIRE TIREN206 TO BE SCHEDULED ON PRESS A3

TIRE TIREN207 TO BE SCHEDULED ON PRESS C15

TIRE TIREN208 TO BE SCHEDULED ON PRESS B15

TIRE TIREN209 TO BE SCHEDULED ON PRESS B13

TIRE TIREN210 TO BE SCHEDULED ON PRESS D15 one review of the total cure, in 76.4 seconds of central processor time. The pairwise combinations were, in every case, identical tires. The machine suitability considerations seemed effective since no impossible assignments occurred. (It is very difficult to determine if these machines are optimally employed on a problem of this magnitude--the best that can be determined is that every position is acceptable.) There were four relocations of tires from the initial assignment. This was a wholesale interchange of tires on adjacent presses. It should be noted that this is the most likely occurrance and that in terms of the real problem is not serious.

APPENDIX E

Simplified Input Instructions

E.1 General

The purpose of this section is to present a series of questions which, when answered, constitute the required input information for the system. This procedure should simplify the operating procedure, and minimize the amount of detailed knowledge required for a user interested in testing the program before familiarizing himself with its details.

1. How many tires to be scheduled? N = ?

2. What is the allotted array size? NN = ?

   To ensure a safe value for NN, it is recommended that the user try an initial value of NN = 2 x N. (Note the program requires approximately the following storage space in words.)

   $$\text{MEMORY} = 22(\text{LCTNS}) + 11(\text{NN})^2 + 31(\text{NN}) + 8000$$

3. Which algorithm is to be used?

   ICHOICE = 1 - Method A is selected.

   ICHOICE = 0 - Method B is selected.

   It is recommended that the user try Method B for an initial attempt.

4. Do you want the internal calculations printed?

   IDIAG = 0 - No Calculations.

   IDIAG = 1 - Calculations printed out.

   WARNING: there is a great deal of output when IDIAG = 1.

5. Do you want the input data printed?

   IDATA = 0 - No data.

   IDATA = 1 - Data printed out.

6. What is the maximum number of iterations of that the entire system is to be allowed to complete? NRUN = ?

NOTE: NRUN = 1 (for first attempt)

7. How many mold changes will be accepted? NMAX = ?

8. What is allowable press operating coefficient? ZMAX = ?

   (This value should be set high for initial use.)

9. Do you require punched output of the new assignment?

   IPUNCH = 1 - Punched output.

   IPUNCH = 0 - No punched output.

10. How many curing positions? (2 x number of presses) LCTNS = ?

11. How many lines of presses? NLINES = ?

12. How many presses in each line?

    NIEL(1) = ?   NIEL(NLINES) = ?

13. What are the weighting factors? W(1).....W(14) (See Chapter VII.1.)

14. What are the press parameters for each curing position?

Example (given for position 1):

MAKE(1) = (make of press) e.g. BOM

MOD(1) = (model of press) e.g. M12

IPRAT(1) = (press rating) e.g. 3

PMHT(1) = (max. mold height) e.g. 12"

PMW(1) = (max. mold diameter) e.g. 44"

PSHP(1) = (shaping phases) e.g. 2

ICONTR(1) = (type of control) e.g. 2

IPPCI(1) = (type of PCI unit) e.g. 2

15. What was previous assignment? Part no. for each position? Code no. for each position? Example (location 1):

PRESS(1) = (department number) e.g. A3

TOP(1) = (part no.) e.g. TIRENO21

ITOPC(I) = (code no.) e.g. 4021

16. What are cure specifications for each of the above?

Example (location I):

EGTHT(I) = (green tire height) e.g. 22"

ECT(I) = (base cure time) e.g. 15 min.

ESHAPE(I) = (number of shaping phases) e.g. 3

ETMLDH(I) = (mold height) e.g. 11"

ETMLDW(I) = (mold diameter) e.g. 43"

EBW(I) = (bead diameter) e.g. 14"

EPCIP(I) = (PCI pressure) e.g. 30 psi

ECONST(I) = (construction) e.g. Radial ply

ECURE(I) = (special cure required) e.g. NRM

17. Which are new tires to be scheduled? Part no.? Code no.?

Example (Tire I):

ON(I) = (part no.) e.g. TIRENO33

IFC(I) = (code no.) e.g. 4033

18. What are cure specifications for above ON tires? Example (Tire I):

GTHT(I) = (green tire height) e.g. 22"

CT(I) = (base cure time) e.g. 14.1 min.

SHAPE(I) = (shaping phases) e.g. 3

TMLDH(I) = (mold height) e.g. 11"

TMLDW(I) = (mold diameter) e.g. 44"

BW(I) = (bead diameter) e.g. 13"

PCIP(I) = (post cure inflation pressure) e.g. 35 psi

IRAT(I) = (cure rating) e.g. 2

IPCI(I) = (PCI equipment) e.g. 1

CONST(I) = (construction) e.g. Radial ply

CURE(I) = (special cure) e.g. NRM

19. Which tires are being removed from the cure? Example (Tire I):

OFF(I) = (part no.) e.g. TIRENO44

IOFC(I) = (code no.) e.g. 4044

The figure showing the set up of the data deck is repeated here as Figure XVI. Similarly the required DIMENSION statement is repeated as Figure XVII.

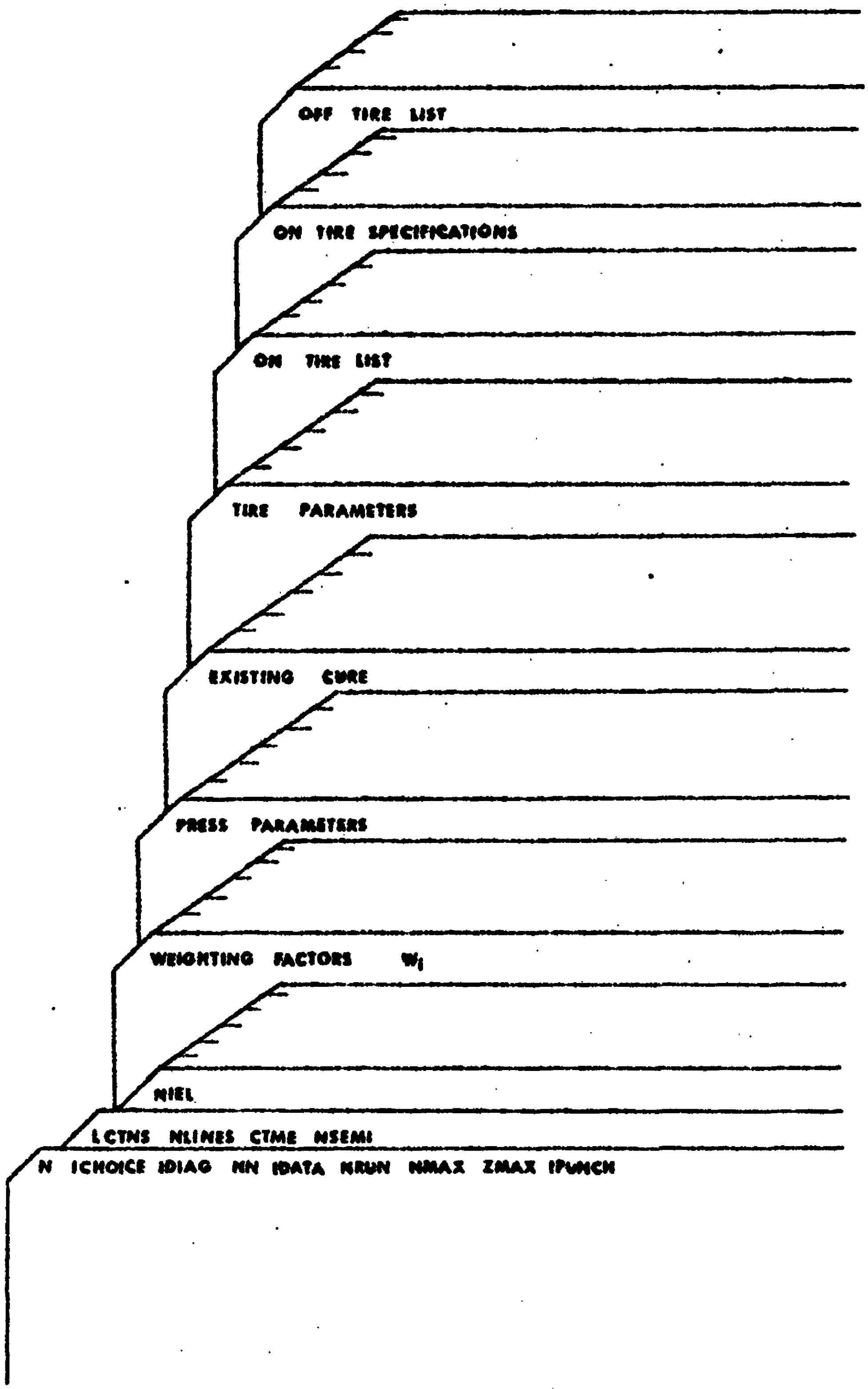

FIGURE XVI SET UP OF A DATA DECK

```
 DIMENSION PMHT( XX),PMW( XX),PRESS( XX),TOP( XX),ITOPC( XX),IPPCI(
1 XX),ICONTR( XX),IPRAT( XX),MOD( XX),MAKE( XX),ON( X),IFC( X),BW(
2X),PCIP( X),IRAT( X),CONST( X),CURE( X),IPCI( X),PSHP( XX),CT( X),
3GTHT( X),SHAPE( X),TMLDH( X),TMLDW( X),A( X, X),C( X, X),D( X, X),
4E( X, X),IDOM( X),IRAN( X),VCIA( X, X),VCAI( X, X),VDAJ( X, X),VDJ
5A( X, X),WORK1( X),WORK2( X),IC( X),JR( X),WORK3( X, X),IZ(2X),JZ(
62X),IBOM( X),IDAN( X),OFF( X),IOFC( X),JK( X),NIEL(K),IK( X),EGTHT
7( XX),ECT( XX),ESHAPE( XX),ETMLDH( XX),EBW( XX),EPCIP( XX),IERAT(
8XX),IEPCI( XX),ECONST( XX),ECURE( XX),ONOR( X),JJK( X),ETMLDW( XX)
9,Z(KK),W(14)
 INTEGER ALPHA( X, X),ZSTAR( X, X),ZPRIM( X, X)
```

WHERE

X=THE NUMBER OF TIRES ON THE NEW SCHEDULE.

XX=THE TOTAL NUMBER OF CURING LOCATIONS.

K=THE NUMBER OF LINES OF PRESSES.

KK=THE TOTAL NUMBER OF PRESSES.

FIGURE XVII THE DIMENSION STATEMENT