OPTIMAL SCHEDULING OF CONVERTER AISLE OPERATION IN A NICKEL SMELTING PLANT

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OF CONVERTER AISLE OPERATION IN A NICKEL SMELTING PLANT

by

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ABSTRACT

The scheduling of the converter aisle of a nickel smelting plant is a non-trivial task with significant consequences to plant profitability and production. An optimization-based scheduling formulation is developed using a continuous-time paradigm to accurately represent event timings. The formulation accounts for environmental restrictions on sulfur dioxide emissions using event timing constraints. The formulation includes novel semi-continuous modeling to represent flash furnaces which operate with a continuous inlet flow and intermittent discrete material removal, as well as, a novel sequencing and symmetry-breaking scheme to account for identical units operating in parallel. A rolling horizon feature is included in the formulation to accommodate multi-period optimization. Tightening constraints are developed and used to improve the computational performance of the optimization and demonstrate the capacity of the proposed methodology to function as a real-time decision-support tool. A solution procedure is presented where an aggregate model is used to bound the objective function of the master problem in a two layer optimization scheme. Finally, a novel multi-tiered procedure is presented to enhance the optimization solution by re-optimizing for objectives of decreasing priority in order to minimize task start times and penalize deviations in the furnace flow rate.

To address the closed-loop properties of scheduling, a reactive scheduling mechanism is included to allow for rescheduling to account the impact of process disturbances on the operating schedule. A methodology for reducing radical scheduling changes due to the optimization during reactive scheduling is presented. The reactive scheduling algorithm utilizes a tiered optimization approach that progressively increases the degrees of freedom available, as required, in order to achieve a feasible production schedule. The use of the reactive scheduling algorithm demonstrates the ability to reject disturbances and transition plant operation in an agile manner.

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Chapter 1

Introduction

Enterprise-wide optimization (EWO) has become an increasingly relevant topic in the engineering and manufacturing world. EWO integrates chemical engineering expertise with operations research knowledge to develop sophisticated mathematical tools to improve decision making regarding the operation of a variety of manufacturing systems. EWO has become increasingly relevant as an operating strategy as the global market becomes increasingly competitive. EWO is meant to aid decision making with the objective of driving cost savings, profitability, and promote lean operation. A major component of EWO is the use of optimization-based scheduling to maximize operational efficiency. EWO spans many levels of the process control hierarchy including planning, scheduling, real-time optimization and control [Grossmann, 2012]. Viewed from the plant control hierarchy in Figure 1.1, scheduling falls between real-time optimization and production planning. The focus of this work is on a collaborative effort with Vale Base Metals Technology to design a mathematically based decision support tool to promote the efficient scheduling of a converter aisle in a nickel smelting plant.

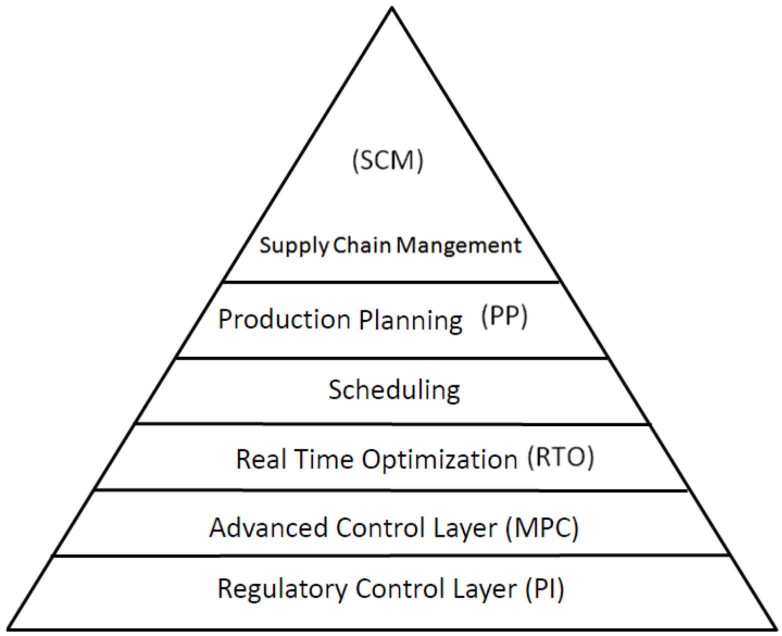

Figure 1.1: Process control hierarchy.

1.1 Motivation and Goals

Peirce-Smith converting is the predominant method by which nickel smelting is achieved. Converters are process units which are fed molten ore that is reacted with high purity oxygen blows to purify the material until a specific product quality is achieved. Converters must be co-ordinated with flash furnaces which melt and temporarily store solid ore. Efficient co-ordination of planning, scheduling, and resource allocation is crucial toward optimizing the process economics. The converter aisle of a nickel smelting plant is comprised of both discrete and continuous decisions with a large number of operational constraints that significantly complicate the generation of schedules by hand.

The objective of this work is to design a centralized optimization-based scheduling algorithm that will maximize production by coordinating multiple furnaces and several converters operating in parallel by making flash furnace matte allocation decisions to satisfy matte demand based on converter cycle progression. Matte allocation decisions are subject to restrictions such as shared crane usage, furnace capacity constraints, and a range of operating furnace flow rates. The scheduling optimization must account for the semi-continuous furnace operation due to the continuous inlet flow and discrete removal. Converters are characterized by both batch and cyclical operation where casting times must be offset so as to account for the limited casting availability. Furthermore, converter blowing must be sequenced and timed to achieve compliance with SO_2 emission tolerances which limit the number of converters operating in parallel. This work contributes toward the development of a real-time decision support tool to optimize production and systematically handle constraints.

The second objective of this work is to establish a mechanism for schedule updating that can make closed-loop decisions based on real-time information. A reactive scheduling approach will be demonstrated which reduces schedule changes while adapting to plant disturbances.

1.2 Process Description

The nickel smelting process is comprised of Flash Furnaces (FFs) and Peirce-Smith Converters (PSCs) [Warner *et al.*, 2005]. FFs receive a continuous stream of Dry Solid Charge (DSC) which is melted inside to produce Flash Furnace Matte (FFM). FFM is transferred to PSCs by a discrete drawing procedure referred to as tapping. Furnace taps are used to fill ladles which transport the FFM by means of cranes to the PSCs. Thus, FFs are operated semi-continuously due to a continuous inlet flow and a discrete tapping task. Figure 1.2 illustrates the process layout.

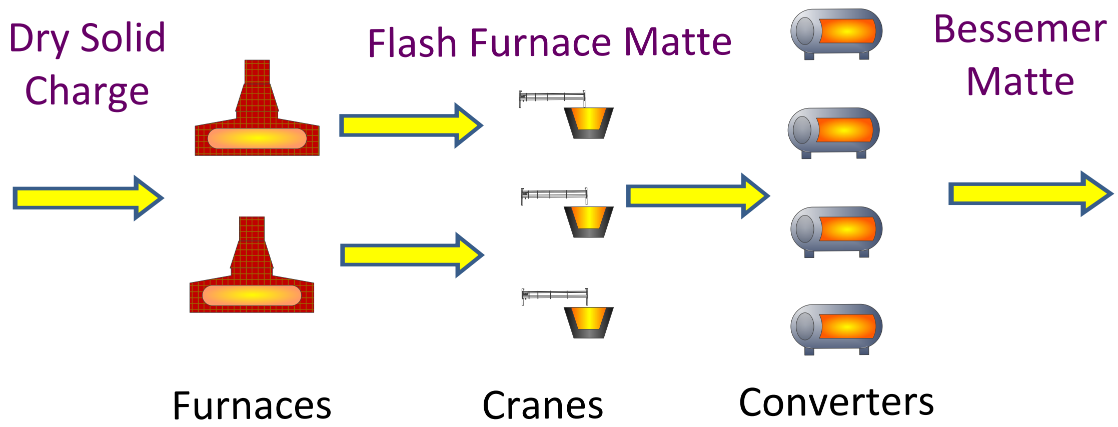

Figure 1.2: Nickel Smelting Process Flow Diagram.

PSCs follow a standard batch recipe where FFM tapped from FFs are poured into the PSC in an action referred to as charging. PSC charges are typically followed by high purity oxygen blows referred to as converter blows through nozzle-like emplacements referred to as tuyeres and a skimming action [Navarra, 2013]. Oxygen blows are employed in order remove impurities, the most common being sulfur and iron. After blowing, SO_2 is dissipated as a gas and sent to a super stack as an emission. In alternative processing environments it is possible to send SO_2 gas to an acid plant to recover acid as a secondary product. Mineral impurities in FFM separate from the FFM in a layer referred to as Converter Slag (CS) which rises to and is removed from the surface of the PSC by skimming which is then primarily disposed of [Hernandez, 1996]. In practice, it is possible to have a partial recycle of CS to the FFs to recover valuable minerals such as nickel; however, in this study CS recycle can be considered negligible in the FF mass balance as the mass contribution of the CS to the FF is not significant. Charges, blows, and skims are performed iteratively until the FFM inside the PSCs have been sufficiently purified at which point special finishing blows, a miss blow, and a nitrogen bath is used to cool the FFM. A miss blow is another emission producing blow that has a shortened duration in comparison to a regular blow. FFM is referred to as Bessemer Matte (BM) once it has been sufficiently purified, at which point the BM is sent to the caster. In practice it is possible to modify the standard PSC batch recipe by adding or removing a limited number of FFM ladles in the sequence in order to adhere to a schedule or achieve a certain product quality. However, modifying the number of PSC charges results in different casting sizes which is discouraged in an industrial environment. Furthermore, it is typically possible to vary the number of blows required in a standard batch as well as the rate of oxygen blowing. Varying the blowing rate affects the number of PSCs that are able to operate in parallel, pending environmental restrictions, as well as, the total cycle time for completion. In the following model we consider a constant blowing rate that will apply to all PSCs for all blows, and a standard batch recipe with a prescribed number of charges and blows. A cycle is comprised of five charges. Two charges are completed before the first blow, such that charges two through five are each accompanied by a blow. The first charge is not accompanied by a blow. Following an entire PSC cycle, a certain amount of time must be allocated before starting a new cycle on the same PSC. This period of converter inactivity is use to perform preventative maintenance tasks and cleaning, which is referred to as a standby phase. Additionally, the standard batch recipe can be modified wherein a special operation termed the Copper Washout (CuWa) can be used to shorten the total cycle time of a PSC [Welgama *et al.*, 1996]. Ladles of FFM are removed from the FFs and undergo special processing outside of the converter aisle. The removal of FFM for CuWa processing is omitted in the mass balance due to the infrequent operation of this task and the relatively negligible amount of material that is removed from the furnaces over the course of the relatively long time horizon. Ladles of CuWa can be used to substitute up to two ladles of FFM in a PSC charge and a maximum of one ladle of CuWa can be used in an entire PSC cycle. The benefit of the CuWa operation is that it reduces the duration of a single nominal blowing operation by over 80%. CuWa is allocated to the third charge in the cycle to shorten the duration of the second blow. Figure 1.3 explains the interaction of the furnace with the converter batch recipe.

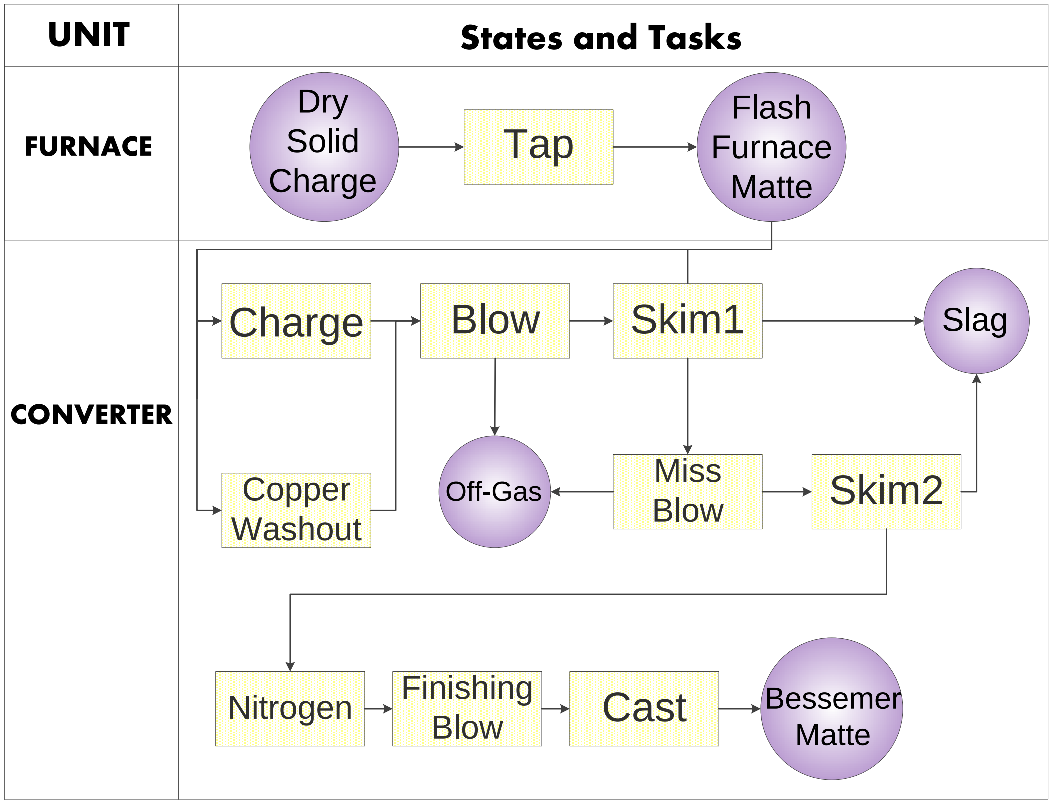

Figure 1.3: Process states and tasks.

A formulation that is able to capture the hybrid nature of the process and certain unique operating policies implemented in the industrial setting is required. Modeling challenges include: (i) accurately representing the FF semi-continuous operation and maintaining a FF inventory that lies within the upper and lower capacity limits, (ii) accounting for the PSC multistage batch recipe with a CuWa option, (iii) co-ordinating multiple converters operating in parallel with furnace taps, (iv) and including a standby phase in PSC operation. The optimization must also allow for both variable and constant FF flow rates. The optimization must be flexible enough to account for PSCs in mid-operation and allow PSC cycles to be partially completed, while driving production towards a maximum. Another significant modeling challenge is consideration of the environmental regulation on SO_2 emission limits which is another rate constraint that restricts the number of PSCs blowing and operating in parallel. Furthermore, it is necessary to capture crane utilization as a limited resource.

This limitation is treated as an upper bound on the rate of ladles leaving the furnaces per hour. Casting is also a limited resource which must be accounted for in the formulation such that the caster will not be overwhelmed with PSC cycles completing simultaneously. As PSCs require large amounts of energy to maintain operation, maximum cycle times must be constrained. Finally, this application is intended to be used ultimately as a real-time decision-support tool, and thus must be able to represent an industrial-scale process in real-time and generate a schedule that operators can readily implement. This suggests that the formulation must have expeditious solve times, must be of a reasonable problem size with appropriate tightening, and possess a solution process that achieves optimality while producing a sensible schedule.

1.3 Main Contributions

1. **Continuous-time, Slot-based Scheduling Optimization Formulation.** A novel continuous-time formulation that integrates the purely batch converter operation and semi-continuous furnace operation in a single framework is presented. The scheduling formulation allows for a range of flow rates or a constant flow rate to be specified a priori as a manipulated variable. The scheduling formulation also accounts for an emission restriction on sulfur dioxide blowing activities that represents a major process disturbance with significant consequences on production. The formulation incorporates a rolling horizon function that is used to allow for optimization is subsequent production periods by indicating which portion of the converter production cycle have yet to be completed and by re-initializing furnace inventories. The optimization formulation predicts furnace inventories, sulfur dioxide emissions, furnace flow rates, and converter production.

2. **Symmetry-breaking Scheme and Tightening Constraints.** As the plant configuration consists of a number of identical units acting in parallel, there exists a significant amount of symmetry in the optimization. A symmetry-breaking scheme is presented to improve the computational properties of the optimization and promote

efficient solve times. Application-specific tightening constraints are developed for the optimization to further improve the computational properties of the optimization.

3. **Schedule Enhancing Tiered Optimization.** A tiered optimization is employed to promote the integration of a secondary and tertiary objective. The additional objectives include the minimization of start times and move suppression, respectively. The additional objectives are solved without compromising production levels, or adding a significant computational load. The tiered optimization solves a mixed integer program in the first layer, then transforms into a linear program in the second layer by fixing the integer variables in the second layer, and transforms the optimization into a quadratic program in the third layer.

4. **Closed-loop Reactive Scheduling.** A novel reactive scheduling algorithm is employed to improve disturbance rejection and account for uncertainty. The algorithm exploits a combination of the dissociation of the event timings from binary variables and the rolling horizon ability to re-schedule production plans with a minimum of disruption.

1.4 Thesis Overview

Chapter 2 - Literature Review

Literature regarding smelting specific and converter-aisle scheduling applications is discussed in terms of the approaches used and the efficacy of the method used.

The literature reviews the state-of-the-art scheduling formulations and techniques. The scheduling literature reviews the different formulations based on the time representation, the major review papers, industrial applications of optimization in scheduling, and various rolling horizon strategies. The literature also includes papers on state-of-the-art tightening constraints to improve computational properties.

Finally, reactive scheduling approaches are also discussed in the literature review.

Chapter 3 - Optimization Formulation

An in-depth explanation of the mathematics governing the optimization is included along with a physical interpretation of each constraint and diagrams to explain.

Chapter 4 - Case Studies

Case studies demonstrating the key features of the optimization for a configuration of two furnaces and four converters are included for 24 hour time periods. The 24 hour time periods are more effective in demonstrating effects on production than 12 hour time horizons are. However, some of the cases in the 24 hour studies lead to a problem size that may prohibit the use of a 24 hour long time horizon in a practical implementation. For real-time implementation it is suggested that 12 hour time horizons be used along with the rolling horizon feature, as the computational load is reasonable enough to constitute a practical real-time strategy.

Chapter 5 - Reactive Scheduling

Rescheduling strategies using a reduced number of degrees of freedom are exploited to recover from process disruptions and uncertainty. Case studies focus on a 12 hour time horizon to demonstrate the efficacy of this method for real-time implementation. Cases study how the reactive scheduling algorithm copes with a mismatch in: predicted and actual event timings, furnace inventory, unit break-downs, and changes in the sulfur dioxide emission limits.

Chapter 6 - Conclusions and Recommendations

Conclusions on the ability of the formulation to represent the converter aisle and constitute a real-time strategy are made. The results of the reactive scheduling and rolling horizon feature are summarized in terms of their ability to regulate production. Potential research extensions are proposed.

Appendix A - Precursor Formulation

A formulation for a simplified process model and configuration is included. This precursor formulation was used to demonstrate the feasibility of using a mixed integer programming framework to schedule operation in the converting aisle. The mathematics presented here formed the initial basis for the formulation presented in Chapter 3.

Chapter 2

Literature Review

2.1 Optimization Applications in the Converter Aisle

Efforts to perform industrial optimization of smelting processes stretch back to as early as the 1970s as demonstrated by Templeton and Hankley [1970] where a systematic technique was demonstrated for optimally controlling the converter aisle of a copper smelter to predict system states. The problem was decomposed into smaller sub-problems to achieve tractability. The objectives of the work included scheduling blows to maximize copper production, and to control the converter furnaces in relation to blows based on a crane schedule. Early attempts at achieving efficient production planning include Rana and McCain [1983] who call for a methodology to diagnose and correct multivariable systems in order to rapidly and efficiently handle process disturbances using a predictive model and Neimanis [1979] who develop a mathematical model to schedule the converter aisle of a copper smelting plant to maximize the production rate. These early works were limited predominantly by both the hardware and software capabilities available at the time. However, with advances in computing and in mathematical programming, larger and more complex programs are feasible. More recent works toward scheduling in smelting environments include Ewaschuk *et al.* [2013] who develop a scheduling optimization using a continuous-time formulation for a simplified process model as a stepping stone to an optimal scheduling decision support system and

Harjunkoski *et al.* [2006] who use a standard continuous-time job-shop scheduling formulation to model a copper plant. Harjunkoski *et al.* [2006] discuss key features that the model must capture including: exact timing of processing tasks, tracking material transfer, reflecting the cyclic production, the availability of limited resources, and allowing enough slack time to integrate crane scheduling. Other efforts toward scheduling a smelting plant include Navarra [2013] who represent Peirce-Smith converting within a mixed integer linear program (MILP), where they model cranes and converters explicitly while accounting for the heat and material content of feed streams. Navarra [2013] employs a heuristic method to schedule converter blows under emission restrictions. While use of the heuristical method does not guarantee optimality, the solution can be used as a lower bound in a deterministic branch and bound search. Also, Prasad *et al.* [2006] schedule a multistage aluminum casting process containing parallel furnaces and casters. The authors account for the furnace mass balance by partitioning furnace operation into three phases consisting of a fill, hold, and draw. An algorithm that involved iteration between an aggregate model and the full detailed model was employed to achieve reduced solution times. This process bears a close resemblance to the nickel smelting process. Ng *et al.* [2005] develop a cost model for copper converter operation to evaluate the economic performance of different operating policies.

2.2 Scheduling Literature

In a broader sense, scheduling formulations can be classified based on the time representation, consisting of discrete-time and continuous-time formulations. Discrete-time formulations are characterized by a fixed grid where the time horizon is split into intervals of typically equal duration. Continuous-time formulations possess a flexible grid where event timings are represented by continuous variables that become an optimization decision.

Kondili *et al.* [1992] introduced a general framework for scheduling multipurpose and multiproduct plants using a discrete-time representation. The formulation uses states to represent materials, which are consumed or produced by tasks that are performed by units. This framework was termed the State-Task Network (STN), where the optimization was formulated as a mixed integer linear program (MILP). In a sequel, Shah *et al.* [1992] develop techniques to exploit problem characteristics with partial reformulation of constraints. This formulation was modified in Davies [2008] who extended the Kondili formulation to represent continuous processing characteristics within a discrete-time framework. The model included additional maintenance events and was applied to an industrial process in order to reduce material losses due to changeovers and plant failures.

Ierapetritou and Floudas [1998a] introduced the concept of unit-specific event-based continuous time formulations for the short-term scheduling of batch plants which resulted in models with a reduced number of binary variables, smaller integrality gaps, and fewer constraints. In Ierapetritou and Floudas [1998b], the authors extend the original unit-specific event-based formulation to model continuous processes. Finally, Ierapetritou *et al.* [1999] again extend the unit-specific event-based formulation to account for intermediate due dates and batch or semi-continuous operation. Maravelias and Grossmann [2003] contributed a global event-point continuous time model that was distinguished by having a time grid common to all units and a new class of valid inequalities to improve LP relaxations.

Slot-based formulations use a set of time intervals that are specified a priori to allow for unit allocation decisions. Unit allocation decisions can constitute matching batches to units, for example. Slot-based formulations otherwise function similarly to unit-specific event-based formulations, [Mendez *et al.*, 2006]. Erdirik-Dogan and Grossmann [2008] formulate a continuous-time asynchronous slot-based formulation with novel symmetry-breaking cuts to perform short-term scheduling of a multiproduct batch plant with parallel identical units.The model accounts for sequence-dependent changeovers and is solved by decomposing the problem into a higher level sequencing and lower level scheduling and sequencing problem. Sundaramoorthy and Karimi [2005] formulate a synchronous slot-based continuous-time model.

Maravelias [2005] propose a mixed time representation using a fixed time grid with variable processing times that are allowed to span a variable number of time periods. This paradigm contributes toward bridging the gap between discrete and continuous time models.

Sundaramoorthy and Maravelias [2010] address a major formulation challenge by bridging the gap between network and sequential processes using a general framework. Velez and Maravelias [2014] derive theoretical results for multiple, non-uniform, discrete-time grids that demonstrate that multi-grid models with sufficiently fine discretizations are guaranteed to achieve the same objective function value as single grid formulations.

Comprehensive reviews on optimization-based scheduling in process systems engineering include Floudas and Lin [2004] where discrete-time formulations are compared and contrasted against their continuous counterparts based on their strengths and limitations. The authors further discuss advances made in literature and identify areas for additional study and improvement. Shaik *et al.* [2006] compare the performance of several continuous-time models proposed in the literature, specifically slot-based, global event-based, and unit-specific event-based formulations. Mendez *et al.* [2006] provide a review of existing scheduling formulations and corresponding solution techniques, generate a classification system for various scheduling paradigms, and benchmark two literature examples. Harjunkoski *et al.* [2014] discuss the main characteristics of existing scheduling methodologies and their relation to industrial application. Furthermore, the authors identify highly relevant challenges in terms of translating academic knowledge into industrial value.

Examples of scheduling in industrial environments include Castro *et al.* [2013] who demonstrate a generic scheduling framework that explicitly accounts for energy constraints in the melt shop of a steel plant. They use three alternative models to demonstrate the tradeoff between computational performance and model fidelity using a discrete-time resource-task network. Harjunkoski and Grossmann [2001] present a decomposition algorithm for the short-term scheduling a steel plant. The master problem is disaggregated into MILP sub-problems based on the product families into groups, independent scheduling of the groups, followed by scheduling of all the groups, followed by a solution improvement stage using a linear program (LP). One of the techniques proposed in the paper includes manually fixing the use of electric arc furnaces (EAF)s in an alternating sequence. The solution methodology does not allow for direct comparison of the objective function achieved with the decomposition method to that of the full problem; however optimality gaps are estimated to lie between 1-3%. Additional examples of successful continuous-time formulations include Hazaras *et al.* [2012] who present a global-time formulation to address the combined production and maintenance scheduling of an industrial food manufacturer and then exploit the problem structure by using a product family grouping aggregation and partial reformulation to improve computation efficiency in [Hazaras *et al.*, 2014]. Moon and Hrymak [1999a] developed a slot-based model with a pre-ordering procedure to reduce problem symmetry which is applied to a batch annealing process to maximize material throughput in [Moon and Hrymak, 1999b].

Rolling horizon formulations have been successful in the scheduling of dynamic environments. Elia *et al.* [2014] formulate a MILP model to represent a network of natural gas to liquids system. To cope with the nearly intractable large-scale model the authors resort to using a rolling horizon strategy to demonstrate the economic performance over a long time horizon. Janak *et al.* [2006a] employ a decomposition consisting of dividing a long time horizon into a series of smaller time horizons which are solved sequentially with a novel continuous-time formulation. The formulation accounts for a large number of processing recipes, intermediate due dates, and multiple storage policies. Kostin *et al.* [2011] adopt a similar methodology in order to perform supply chain optimization in the sugar cane optimization. Lima *et al.* [2011] present three rolling horizon strategies with different models and time aggregation in the case of extended time horizons for producing glass.

2.3 Reactive Scheduling Literature

Cott and Macchietto [1989] propose the development of a real-time online schedule modification algorithm to reduce the effects of process variability on batches in a simple flowshop example. Two of the rescheduling algorithms proposed are a simple shift modification algorithm, and a projected operation modification algorithm. The projected operation algorithm was shown to improve performance by maintaining implemented batch times close to the nominal values.

Janak *et al.* [2006b] develop a reactive scheduling framework which determines which production tasks are unaffected by disturbances and which tasks require rescheduling. The authors consider reactive scheduling to correct for unit shutdowns and modified orders by rescheduling a subset of the unfinished tasks.

Bose and Pekny [2000] present a model predictive approach to planning and scheduling which forecasts target inventories in future periods to ensure desired customer service while reducing unnecessary inventory. A scheduling model attempts to schedule production tasks to achieve the inventory targets in future periods. solving this problem.

van den Heever and Grossmann [2003] address the integration of production planning and reactive scheduling to optimize a hydrogen supply network. The optimization is formulated as a multiperiod mixed integer nonlinear programming to consider a monthly horizon divided into 12 h time periods. The planning layer is computed every 12 h, whereas the scheduling layer is re-computed hourly using the previous solution to warm start the optimization.

Munawar and Gudi [2005] consider an integrated multilevel, control-theoretic framework to integrate planning, scheduling, and rescheduling. Examples focus around the cyclic scheduling of a simple refinery flow sheet where the authors first attempt to reject disturbances on the lower scheduling layer before they propagate to the planning layer.

Henning and Cerda [2000] present a knowledge-based framework for creating schedules for real-world applications. The authors discuss several requirements that contribute to an effective scheduling system including: enhancing human decision making without replacing domain experts, capturing intricate process dynamics, capturing the essential details of the process configuration, and the flexible adaptation of schedules using algorithm and expert knowledge.

Subramanian *et al.* [2012] demonstrate how a mixed integer program meant to represent a scheduling model can be re-formulated in a state-space format. Scheduling disturbances are included in the state-space representation to improve the optimizations' decision making abilities. The authors suggest that some concepts from Model Predictive Control can be adapted to improve scheduling and even study the stability of a system for a well-cited scheduling problem from literature.

Kopanos and Pistikopoulos [2014] develop a state-space representation for the scheduling problem that uses multiparametric programming to facilitate a rolling horizon. The use of multiparametric programming avoids the need for online optimization.

Li and Ierapetritou [2008a] review the main methods that exist for production scheduling under uncertainty and identify existing challenges. The authors identify reactive scheduling methodologies that are commonly used to treat faults in process unit and uncertainty in product demands and due dates.

Li and Ierapetritou [2008b] use a multiparametric programming approach to address reactive scheduling. The approach is meant to account for all possible future outcomes by solving a multiparametric problem. The method provides a direct approach to obtain reactive scheduling strategies from parametric solutions.

Bemporad and Morari [1999] present a method for integrating physical laws, logic rules, and operating constraints into a single unified framework for modeling and control. The authors denote the framework as mixed logical dynamical (MLD) systems and use it to develop a predictive control scheme to stabilize a system toward a reference trajectory to achieve set-point tracking while obeying constraints.

Nie *et al.* [2014] schedule a mixed batch/continuous process using a discrete time resource task network. The authors formulate the optimization in a state-space form for reactive scheduling purposes which potentially allows for the opportunity to investigate the stability properties of the closed-loop scheduling. The authors make use of variables to represent tasks that have been performed such that the formulation records history.

Pula [2009] uses a discrete-time formulation to address the reactive scheduling of furnaces in an ethylene plant. The scheduling formulation accounts for integrated process operation and maintenance, using a rolling horizon strategy. The author includes move suppression in the scheduler to reduce excessive fluctuation in the processing rates being fed to the furnaces.

Chapter 3

Optimization Formulation

A slot-based continuous-time rolling-horizon formulation is presented which accurately models the dynamics of the plant environment. The model is formulated as a MILP.

3.1 Formulation Overview

Maravelias [2012] defines the flexible flow-shop environment to encompass a predetermined sequence of processing steps which is common to a set of parallel units. The author further defines a process as sequential when it produces a single material where batch mixing and splitting are not permitted. Using this classification, the process configuration can be considered a flexible flow-shop, where processing occurs within a sequential, multistage framework. The process is operated cyclically, where batch cycles can be initialized and completed mid-way, with identical units operating in parallel.

While this is a short-term scheduling problem, the processing times of various tasks do not result in a greatest common factor that is readily amenable to a discrete-time formulation for the length of the time horizon being considered. Coupled with a requirement for precise event timing, a continuous-time formulation with rolling horizon capabilities would appear to more closely suit the unique needs and characteristics of this process.

From the process description and the required scheduling paradigm, it is clear that a mix of continuous and discrete decision variables will be required for modeling. Discrete variables will be required to model the allocation of furnace taps to converter charges, the number of charges performed on the converter, and the number of converter cycles performed. Discrete variables will also be required to map the allocation of CuWa to converters. Continuous decision variables will be required to model event timings on furnaces and converters, furnace inventory, furnace flow rates, and the amount of DSC added to the furnace.

3.2 Continuous Time Scheduling Formulation

Each converter cycle consists of five charges that remove a different number of ladles from the furnace. One furnace tap corresponds to one converter charge. Two charges occur before the first blow. Charges two through five are followed by an emission producing blow, as well as a slag removing skim. The fifth charge includes an extra emission producing blow referred to as the miss blow. The miss blow is necessary in order to achieve the desired removal of impurities. Following the miss blow is another skim and the final step which includes a non-emission producing finishing blow, a nitrogen bath, and casting.

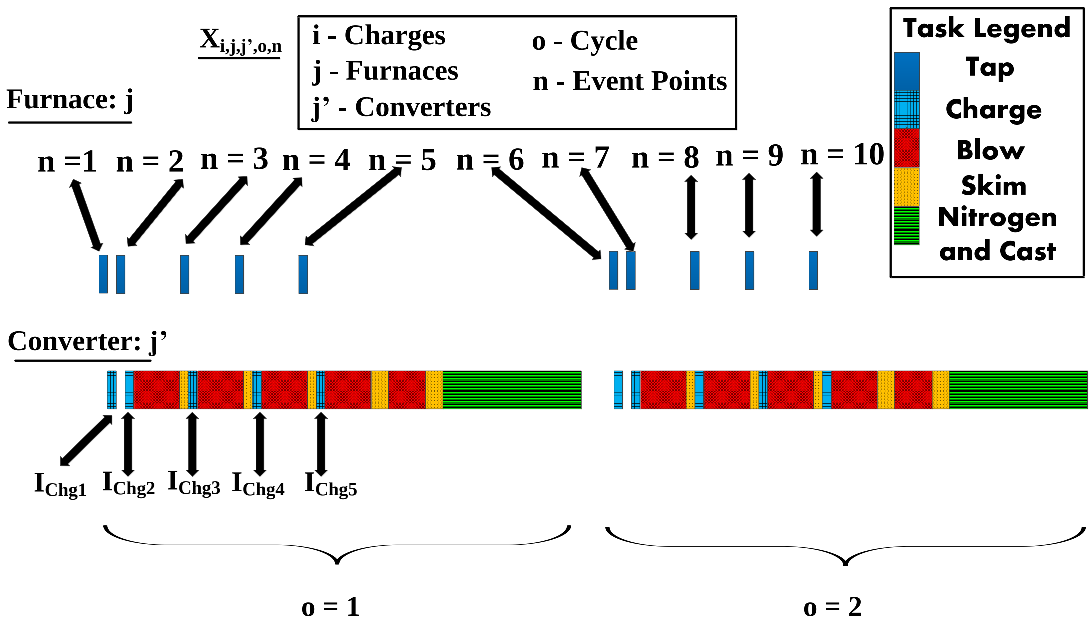

Figure 3.1: Binary charge allocation variable.

The formulation includes logic constraints to represent event sequencing, production levels, and matte allocation. Timing constraints are used to represent event timings. Blowing constraints are used to reflect restrictions on the production levels due to emission limits. Furnace capacities and mass balances are formulated to account for the semi-continuous filling of the flash furnaces. Tightening constraints are employed to improve the computational properties of the optimization. Finally, a tiered optimization procedure is used to improve solution quality for additional objectives.

Variable $x_{i,j,j',o,n}$ is used to allocate charges from furnaces to converters. Figure 3.1 illustrates the relationship between furnace tapping and converter charging. The binary variable $x_{i,j,j',o,n}$ takes on the value 1 if furnace j is tapped and converter j' receives FFM during the corresponding charge i in converter cycle o at event point n, and 0 otherwise. The binary $x_{i,j,j',o,n}$ is used to indicate the production level achieved by each converter.

3.2.1 Logic Constraints

Equation (3.1) states that at most, a single furnace j can be tapped and a corresponding converter j' during cycle o can receive charge i for each event point n. Each charge will be allocated once at most among all furnace taps.

$$\sum_{i \in I_{Chg}} \sum_{j \in J_F} \sum_{j' \in J_C} \sum_{o \in O} x_{i,j,j',o,n} \leq 1 \; \forall \; n \in N \tag{3.1}$$

Equation (3.2) states that for each furnace j, for each event point n, one corresponding converter j' during cycle o can receive charge i at most. Although, Equation (3.1) implies Equation (3.2), the addition of the constraint improved solve times.

$$\sum_{i \in I_{Chg}} \sum_{j' \in J_C} \sum_{o \in O} x_{i,j,j',o,n} \leq 1 \; \forall \; j \in J_F, \; n \in N \tag{3.2}$$

In Equation (3.3), each charge i on each converter j' can be performed once at most for every converter cycle o.

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \leq 1 \; \forall \; i \in I_{Chg}, \; j' \in J_C, \; o \in O \tag{3.3}$$

In Equation (3.4), $P_{i,j'}$ is a user specified parameter that assumes the value 1 if charge i has been performed on converter j' during an incomplete cycle in a previous schedule, indicating that the corresponding tap and charge need not be performed in the first converter cycle of the current schedule, and 0 otherwise.

$$P_{i,j'} + x_{i,j,j',o,n} \leq 1 \; \forall \; i \in I_{Chg}, \; j \in J_F, \; j' \in J_C, \; o = O_{first}, \; n \in N \tag{3.4}$$

The copper washout is modeled in such a way that ladles of FFM required to produce CuWa are assumed to already have been taken from the furnace and processed. Due to the relatively small amount of FFM removed for this purpose the CuWa operation is not accounted for in the mass balance. The operation is modeled to allow the user to specify the time at which ladles of CuWa are available. The CuWa can be employed any time after the user-specified time, where the CuWa replaces the third charge of the converter cycle which is represented in Equation (3.6). The binary variable $w_{j',o}$ takes on the value 1 if converter j' receives a ladle of copper washout in converter cycle o and 0 otherwise. Additionally, the parameter $P_{i,j'}$ is used to indicate that a copper washout or a charge has already been performed in Equation (3.5) and if activated will prevent allocation of the copper washout in the current scheduling horizon. Finally, ladles of CuWa must be utilized when made available, represented by Equation (3.7). The parameter L^{CW} specifies the number of ladles of copper washout available within the current scheduling horizon.

$$P_{i,j'} + w_{j',o} \leq 1 \; \forall \, i \in I_{Chg3}, \; j' \in J_C, \; o = O_{first} \tag{3.5}$$

$$w_{j',o} + \sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \leq 1 \; \forall \, i \in I_{Chg3}, \; j' \in J_C, \; o \in O \tag{3.6}$$

$$\sum_{j' \in J_C} \sum_{o \in O} w_{j',o} = L^{CW} \tag{3.7}$$

Equation (3.8) through Equation (3.14) specify the converter cycle sequence. The sequence consists of five charges, where the third charge can be substituted with a CuWa. Equation (3.8) and Equation (3.9) specify that the second and fifth charge are contingent upon the completion of the first and fourth charge, respectively. Due to the CuWa feature, modified constraints are required to model the precedence relationships. The progression parameter $P_{i,j'}$ is applied to the first cycle of each converter to indicate whether the corresponding charge has been previously completed. The progression parameter $P_{i,j'}$ allows for the initialization of the converters mid-cycle and facilitates a rolling horizon function.

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \leq P_{i',j'} + \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} \; \forall \; k \in \{2,5\}, \; i \in I_{Chg}(k), \; i' \in I_{Chg}(k-1), \; j' \in J_C, \; o = O_{first} \tag{3.8}$$

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \leq \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} \ \forall\, k \in \{2,5\},\ i \in I_{Chg}(k),\ i' \in I_{Chg}(k-1),\ j' \in J_C,$$
$$o \in O : o > O_{first} \tag{3.9}$$

Equation (3.10) and Equation (3.11) state that the completion of the either the third charge or corresponding CuWa is contingent on the completion of the second charge.

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} + w_{j',o} \leq P_{i',j'} + \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} \ \forall\, i \in I_{Chg3},\ i' \in I_{Chg2},$$
$$j' \in J_C,\ o = O_{first} \tag{3.10}$$

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} + w_{j',o} \leq \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} \ \forall\, i \in I_{Chg3},\ i' \in I_{Chg2},\ j' \in J_C,$$
$$o \in O : o > O_{first} \tag{3.11}$$

Equation (3.12) and Equation (3.13) state that the completion of the fourth charge is contingent upon the completion of either the third charge or the CuWa.

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \leq P_{i',j'} + \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} + w_{j',o} \ \forall\, i \in I_{Chg4},\ i' \in I_{Chg3},$$
$$j' \in J_C,\ o = O_{first} \tag{3.12}$$

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \leq \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} + w_{j',o} \ \forall\, i \in I_{Chg4},\ i' \in I_{Chg3},\ j' \in J_C,$$
$$o \in O : o > O_{first} \tag{3.13}$$

Equation (3.14) and Equation (3.15) state that a new cycle $o+1$ can only be started on converter j' once cycle o has been completed on the same converter.

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o+1,n} \leq \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} + P_{i',j'} \; \forall \; i \in I_{Chg1}, \; i' \in I_{Chg5}, \; j' \in J_C,$$
$$o = O_{first} : o < O_{last} \tag{3.14}$$

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o+1,n} \leq \sum_{j \in J_F} \sum_{n \in N} x_{i',j,j',o,n} \; \forall \; i \in I_{Chg1}, \; i' \in I_{Chg5}, \; j' \in J_C,$$
$$o \in O : O_{first} < o < O_{last} \tag{3.15}$$

The performance of the converter blows are contingent on the completion of the previous charges in the converter cycle sequence. Charges two through five are aggregated with emission producing blows, whereas the first charge is not. The number of converters allowed to blow and operate in parallel is indicated by the integer parameter l^{Max}. When l^{Max} is 0, the emission limits are so restrictive that they prevent any blows from being performed. This behaviour is modeled by Equation (3.16) which specifies that charges two through five and their associated emission producing blows cannot be performed.

$$x_{i,j,j',o,n} = 0 \; \forall \; i \in I_{Chg}, \; j \in J_F, \; j' \in J_C, \; o \in O, \; n \in N \; : l^{Max} = 0, \; i > I_{Chg1} \tag{3.16}$$

3.2.2 Time Constraints

Event timing variables for furnace taps are denoted by $T^{S,Furn}_{i,j,n}$ and $T^{F,Furn}_{i,j,n}$ for the respective start and end times. Equation (3.17) states that the end of tap i on furnace j is equal to the beginning of the furnace tap plus the task duration when converter j' for cycle o at event point n is charged. Equation (3.17) is used to model the timing and duration of furnace taps. The parameter $\alpha_{i,j}$ defines that duration of task i on unit j.

$$T^{F,Furn}_{i,j,n} = T^{S,Furn}_{i,j,n} + \alpha_{i,j} \sum_{i' \in I_{Chg}} \sum_{j' \in J_C} \sum_{o \in O} x_{i',j,j',o,n} \; \forall \; i \in I_{Tap}, \; j \in J_F, \; n \in N \tag{3.17}$$

Event timing variables for converter cycle tasks are denoted by $T^{S,Conv}_{i,j',n}$ and $T^{F,Conv}_{i,j',n}$ for the respective start and end times. Equation (3.18) states that the end of charge i on converter j' for cycle o is equal to the beginning of the converter charge plus the task duration when furnace j is tapped at event point n.

$$T^{F,Conv}_{i,j',o} = T^{S,Conv}_{i,j',o} + \alpha_{i,j'} \sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \; \forall \; i \in I_{Chg}, \; j' \in J_C, \; o \in O, \; i \neq I_{Chg3} \quad (3.18)$$

The same set of time variables in Equation (3.19) that are used to track the allocation of CuWa are also used to track the third charge as both tasks consume the same time duration.

$$T^{F,Conv}_{i,j',o} = T^{S,Conv}_{i,j',o} + \alpha_{i,j'} (\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} + w_{j',o}) \; \forall \; i \in I_{Chg3}, \; j' \in J_C, \; o \in O \quad (3.19)$$

The duration of skimming times are aggregated with their respective blows. The task I_F encompasses the miss blow, skim, nitrogen, finishing blow and casting operation. Aggregating several tasks and using fewer event timing variables was found to improve the computational properties of the optimization. Equation (3.20) states that the end of the blows or nitrogen, finishing blows, and casting i on converter j' for cycle o is equal to the beginning of the converter charge plus the task duration when furnace j is tapped at event point n. In Equation (3.21), the duration of the second blow is contingent on the performance of the previous CuWa or third charge. Blows, skims and nitrogen and casting, have been aggregated with the converter charge variable to reduce the number of integer variables used to represent the problem. While in reality these tasks result in material production or consumption, the problem requirements state that it is sufficient to model these tasks in terms of their time allocation in the schedule and does not require that these tasks also be modeled with a mass balance. The set $I_{ConverterTasks} \in \{I_{Chg1}, I_{Chg2}, I_{Blow1}, I_{Chg3}, I_{Blow2}, I_{Chg4}, I_{Blow3}, I_{Chg5}, I_F\}$ is a an ordered set containing all the tasks performed in a cycle, including the charges, emission producing blows, skims, a non-emission producing finishing blow, a nitrogen blow, and casting. Equation (3.20) specifies that each blow, including the finishing blow, is dependant on the completion of the previous charge in the cycle, according to the set $I_{ConverterTasks}$.

$$T^{F,Conv}_{i,j',o} = T^{S,Conv}_{i,j',o} + \alpha_{i,j'} \sum_{j \in J_F} \sum_{n \in N} x_{i-1,j,j',o,n} \; \forall \; i \in I_{Blow} \cup I_F, \; j' \in J_C, \; o \in O, \; i \neq I_{Blow2} \tag{3.20}$$

$$T^{F,Conv}_{i,j',o} = T^{S,Conv}_{i,j',o} + \alpha_{i,j'} \sum_{j \in J_F} \sum_{n \in N} x_{i-1,j,j',o,n} + \alpha_{CW} w_{j',o} \; \forall \; i \in I_{Blow2}, \; j' \in J_C, \; o \in O \tag{3.21}$$

Eqs. (3.22)-(3.25) state that all time variables must be constrained between the beginning and end of the time horizon.

$$H_0 \leq T^{S,Conv}_{i,j',o} \leq H_f \; \forall \; i \in I_{Chg} \cup I_{Blow} \cup I_F, \; j' \in J_C, \; o \in O \tag{3.22}$$

$$H_0 \leq T^{F,Conv}_{i,j',o} \leq H_f \; \forall \; i \in I_{Chg} \cup I_{Blow} \cup I_F, \; j' \in J_C, \; o \in O \tag{3.23}$$

$$H_0 \leq T^{S,Furn}_{i,j,n} \leq H_f \; \forall \; i \in I_{Tap}, \; j \in J_F, \; n \in N \tag{3.24}$$

$$H_0 \leq T^{F,Furn}_{i,j,n} \leq H_f \; \forall \; i \in I_{Tap}, \; j \in J_F, \; n \in N \tag{3.25}$$

Equation (3.26) states that each task i in converter j' for each cycle o must start after the end time of the previous task in the sequence.

$$T^{S,Conv}_{i,j',o} \geq T^{F,Conv}_{i-1,j',o} \; \forall \; i \in I_{ConverterTasks}, \; j' \in J_C, \; o \in O \; : i > I_{Chg1} \tag{3.26}$$

Equation (3.27) states that the beginning of a new converter cycle on the same converter can only start after the standby period, the duration of which is denoted by the parameter *Stndby* which is used to perform preventative maintenance and cleaning.

$$T^{S,Conv}_{i,j',o} \geq T^{F,Conv}_{i',j',o-1} + Stndby \sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \; \forall \; i \in I_{Chg1}, \; i' \in I_F, \; j' \in J_C, \; o \in O \; : \; o > O_{first} \tag{3.27}$$

Equation (3.28) states that subsequent taps on the same furnace are not allowed to overlap.

$$T^{S,Furn}_{i,j,n+1} \geq T^{F,Furn}_{i,j,n} \; \forall \; i \in I_{Tap}, \; j \in J_F, \; n \in N, \; n < N_{last} \tag{3.28}$$

It is essential to represent the behaviour of the cranes despite the fact that they are not modeled explicitly. This limitation is posed as a rate constraint on the number of ladles leaving the FFs per hour, however modeling a rate constraint is potentially nonlinear. To avoid nonlinearity in the formulation, the constraint is instead expressed as a limitation on how frequent taps across all furnaces are allowed to be performed, which captures the most essential dynamics of the cranes. Equation (3.29) expresses the crane limitation where the parameter DT is the minimum time that furnace taps must be separated by, so as to avoid overwhelming the cranes utilization and converter aisle. If both furnaces are filled to capacity, the forced separation of taps might contribute to infeasibility. However, if constraint Equation (3.29) is relaxed to exclude the first taps, a feasible solution may be found in the case where both furnaces are filled to capacity. Equation (3.29) implies Equation (3.28), making Equation (3.28) redundant. However, if Equation (3.29) is not used, Equation (3.28) becomes necessary to include in the formulation. The transfer of FFM ladles is modeled in Equation (3.30) and Equation (3.31) where the logic disjunctions are modeled with big-M constraints. When the binary variable $x_{i',j,j',o,n}$ assumes a value of one, the end of the furnace tap and the beginning of a converter charge must align within a time window of between the parameters $Trnsfr^{Min}$ and $Trnsfr^{Max}$. FFM must be transferred within $Trnsfr^{Max}$ to prevent solidification of the molten material in the ladle.

$$T^{S,Furn}_{i,j,n+1} \geq T^{F,Furn}_{i,j,n} + DT \; \forall \; i \in I_{Tap}, \; j \in J_F \; n \in N, \; n < N_{last} \tag{3.29}$$

$$T^{S,Conv}_{i',j',o} \; - \; T^{F,Furn}_{i,j,n} \geq Trnsfr^{Min}(x_{i',j,j',o,n}) \; - \; (H_f - H_0)(1 - x_{i',j,j',o,n}) \; \forall \; i \in I_{Tap}, \; i' \in I_{Chg}, \; j \in J_F, \; j' \in J_C, \; o \in O, \; n \in N \tag{3.30}$$

$$T^{S,Conv}_{i',j',o} \; - \; T^{F,Furn}_{i,j,n} \leq Trnsfr^{Max}(x_{i',j,j',o,n}) \; + \; (H_f - H_0)(1 - x_{i',j,j',o,n}) \; \forall \; i \in I_{Tap},$$
$$i' \in I_{Chg}, \; j \in J_F, \; j' \in J_C, \; o \in O, \; n \in N \qquad (3.31)$$

Equation (3.32) uses a big-M constraint to specify that any CuWa that is assigned to a converter must be allocated only after the CuWa is made available as specified by the parameter T^{CW}.

$$T^{S,Conv}_{i,j',o} \; - \; T^{CW} \geq - \; (H_f - H_0)(1 - w_{j',o}) \; \forall \; i \in I_{Chg3}, \; j' \in J_C, \; o \in O \qquad (3.32)$$

3.2.3 Symmetry-Breaking Scheme

Since the process operates four converters with identical processing characteristics which contribute equally in the objective function, there exists a significant amount of symmetry in the optimization which burdens the computation. In order to achieve a timely solution to allow for real-time implementation, it becomes necessary to develop a symmetry breaking scheme. It is possible to exploit symmetry as all the converters are virtually identical, save for their position in the plant and proximity to the crane track, which is not modeled in the optimization. The symmetry-breaking scheme proposed includes assigning a task and charging precedence that is assigned to converters based on how far along a converter has progressed in any individual cycle. Consider Table 3.1 where four converters are designated as converter A through D in the plant. At plant start up -Period 1- the converters do not contain any ladles of FFM and have not begun any step of the batch recipe. Typically, matte is allocated to the converter furthest in its cycle. Since the converters are not yet in operation it becomes possible to assign an arbitrary order in which converters should start and complete their respective cycles and receive FFM charges without compromising optimality. As such, it is possible for the operators or any intended user to assign converter A as the first in the queue, converter B as the second, and C and D as the third and fourth, respectively. At the end of the first period, converters A and B complete their cycles leaving

C and D unfinished due to resource limitations. In the time horizon following period 1 denoted Period 2, C and D are assigned to the start of the queue followed by A and B. At the end of the second period converter C completes processing, however D is unfinished and is the converter left furthest along in the cycle. As a results, D is assigned the highest priority and precedence in period 3. Also, because converter A and B were not started in Period 2, it becomes possible to interchange their order in the third period.

Table 3.1: Converter Precedence Assignment

Optimization Assignment j′	**Period**		
	1	**2**	**3**
1	A	C	D
2	B	D	B
3	C	A	A
4	D	B	C

Without compromising optimality, it is possible to assign a precedence to each converter and their respective cycles which reflects an operating policy where it is preferable to perform one cycle on each converter before starting a second cycle on one which has already been completed. This precedence allocation scheme is used in several constraints, including Eqs. (3.33)-(3.36) and Eqs. (3.49)-(3.53) and is summarized in Table 3.2. Converter precedence and queuing are reflected by the set Z. The number of elements in Z reflect the number of operating converters multiplied by the number of cycles possible based on maximum cycle time within the given time horizon. In a pre-processing step, the time horizon is divided by the minimum cycle time to estimate the number cycles that can be completed. Using the estimated number of cycles as an upper bound on production, the set Z is formed. Consider, Table 3.2 where it has been estimated that two cycles of production can be completed per converter on a time basis. Table 3.2 consider that the plant has four converters for a maximum of eight cycles of production. Based on the precedence scheme, the first cycle of converter one will be initiated first, then the first cycle of converter two and so on. Equation (3.33) sequences the converter tasks event timing according to the precedence scheme discussed in Table 3.2. Where the sets J_z and O_z represent the converter and cycle corresponding to the position in the ordered set Z.

Table 3.2: Converter Set Assignment

Converter Set Assignment		
Converter j'	Cycle o	Precedence Z
1	*1*	1
2	*1*	2
3	*1*	3
4	*1*	4
1	*2*	5
2	*2*	6
3	*2*	7
4	*2*	8

$$T^{S,Conv}_{i,j'',o'} \geq T^{S,Conv}_{i,j',o} \ \forall \ i \in I_{Chg} \cup I_{Blow}, \ z \in Z, \ j' \in J_z, \ o \in O_z \ j'' \in J_{z+1}, \ o' \in O_{z+1} \ : \ z < Z_{last} \tag{3.33}$$

As the batch sizes of BM produced by the converters remain relatively constant, a mass balance relating the transfer of BM to the caster is not explicitly required. Rather, it is necessary to represent the times at which the caster is used such that casting cycles do not overlap. The duration of the casting time is denoted by the parameter $Cast$ which is represented by Equation (3.34).

$$T^{S,Conv}_{i,j'',o'} \geq T^{S,Conv}_{i,j',o} + Cast \sum_{j \in J_F} \sum_{n \in N} x_{i-1,j,j'',o',n} \ \forall \ i \in I_F, \ z \in Z, \ j' \in J_z, \ o \in O_z \ j'' \in J_{z+1}, \ o' \in O_{z+1} \ : \ z < Z_{last} \tag{3.34}$$

3.2.4 Blowing Constraints

The environmental restriction on blowing emissions are a significant bottleneck that limit production in the plant. Due to governmental regulations, compliance with this limitation is necessary for continued plant operation. The specified acceptable emission rate is subject to change and can be enforced on the plant with short notice. When this happens, schedules that were feasible under less restrictive blowing conditions can be made infeasible and schedules produced in restrictive conditions may be sub-optimal when emission tolerances are raised. Emission regulations are a major process disturbance that impact plant operation and necessitate agile production plan transitions to promote an efficient and profitable enterprise. Constraints Equation (3.35) and Equation (3.36) are both used to model the blowing constraints.

$$T^{F,Conv}_{i,j',o} - \alpha_{skim2} - \alpha_{NFC} \leq T^{S,Conv}_{i',j'',o'} \ \forall \ i \in I_F, \ i' \in I_{Blow1}, \ z \in Z, \ j' \in J_z, \ o \in O_z, \\ j'' \in J_{z+l^{Max}} \ , o' \in O_{z+l^{Max}} : z \leq z^{Max} - l^{Max} \tag{3.35}$$

$$T^{F,Conv}_{i,j',o} - \alpha_{skim} \sum_{j \in J_F} \sum_{n \in N} x_{i'-1,j,j'',o',n} \leq T^{S,Conv}_{i',j'',o'} \ \forall \ i \in I_{Blow3}, \ i' \in I_{Blow1}, \ z \in Z, \\ j' \in J_z, \ o \in O_z, j'' \in J_{z+l^{Max}} \ , o' \in O_{z+l^{Max}} : z \leq z^{Max} - l^{Max} \tag{3.36}$$

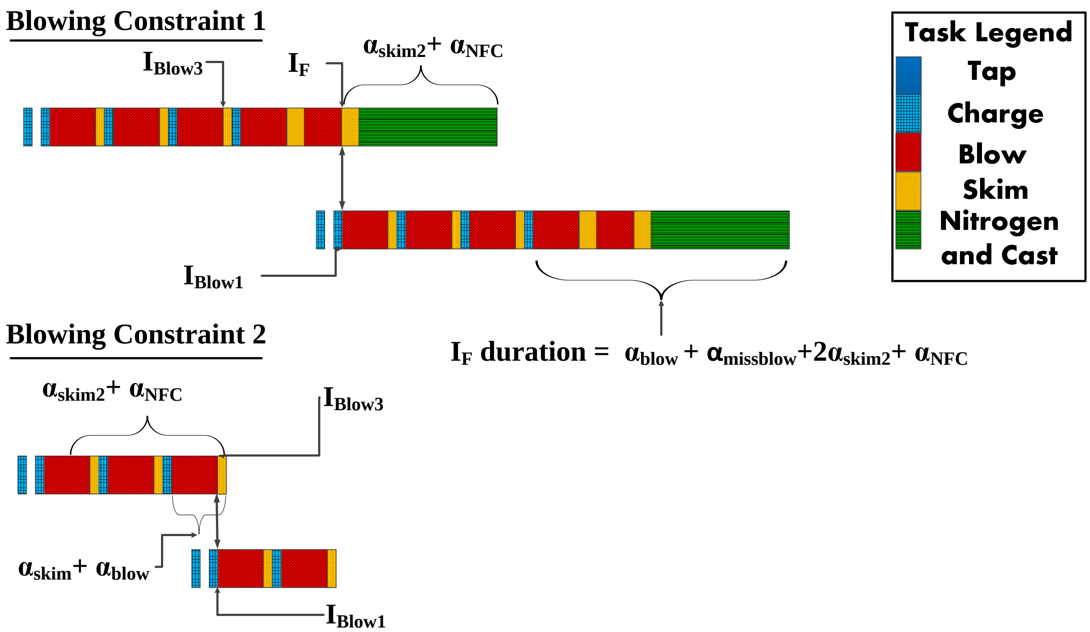

Figure 3.2: Illustration of timing constraints to represent blowing restriction.

Equation (3.35) specifies that the start of the first blow on converter j'' must commence after the end of the miss blow on converter j'. The end of the miss blow signifies the beginning of the final skim with a duration α_{skim2}, and is followed by the finishing blow, nitrogen bath, and casting steps which have a combined duration of α_{NFC}. The parameter l^{Max} is used to specify how many converters are allowed to blow simultaneously. The l^{Max} parameter is used to re-state which converters the constraints Equation (3.35) and Equation (3.36) apply to and then sequences the event timings such that the converters adhere to the blowing restrictions. The parameter q_{StrC} denotes the number of converters operating in the aisle and is used to scale sets and constraints as in Equation (3.35) and Equation (3.36). While Equation (3.35) is necessary to offset the blowing event times, it is not alone sufficient and also requires constraint Equation (3.36). Constraint Equation (3.36) accounts for the case where the last charge, is not performed. Due to the fact that Equation (3.35) uses the task duration of I_F which is greater than duration of regular blows, Equation (3.36) is required. Due to the fact that the first three blows share the same duration, Equation (3.36) is able to represent the blowing restriction in any of the cases where the last charge is not performed; otherwise the converter blows would overlap due to the greater task duration in I_F. The physical interpretation of the mathematical constraints is illustrated in Figure 3.2. The use of both constraints ensures the proper converter blow timings. Conceptually, constraints Equation (3.35) and Equation (3.36) time the beginning and end of the emission producing blows on each converter is such a way as to avoid blowing too many converters simultaneously. For example, consider the case where four converters are in operation and the l^{Max} parameter specifies that only one converter can blow at a time. In this simplified example, consider a single blow during each cycle. The blowing constraints would use the precedence scheme in Table 3.2 to compare the blow timings of converter one to converter two in cycle one. Likewise, the completion time of the blows on converter two would be compared to the beginning of the blow time on converter three. The effect of l^{Max} and the constraints for the case where one converter and then two converters are allowed to blow is illustrated in Figure 3.3.

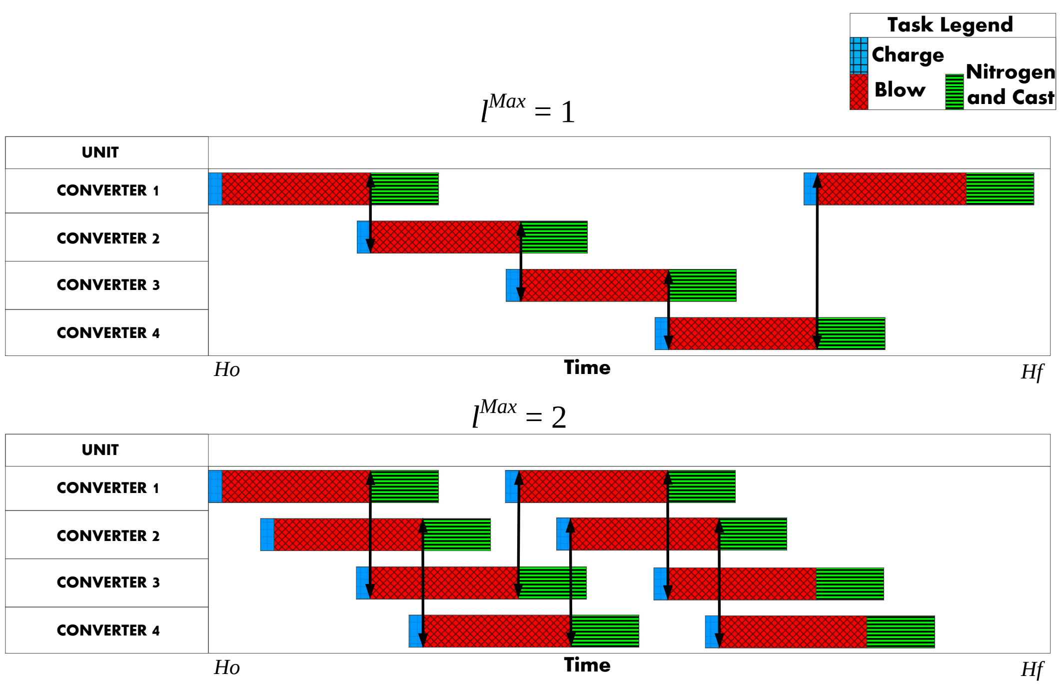

Figure 3.3: Illustration of blowing emission constraints mechanism.

3.2.5 Furnace Side Modeling

A mass balance is explicitly required on the furnaces due to the semi-continuous furnace operation, the capacity limitations, and the range of flow rates that determine the furnace inventory. Furnace taps must be co-ordinated with converter charges in order to achieve effective production planning. However, converters do not require a mass balance as the converter capacity is typically more than adequate in accommodating the standard batch recipe. Batches are commonly used to represent production without the necessity for a mass balance [Floudas and Lin, 2004], as is the case here.

Constraints Eqs. (3.37)-(3.39) state the the furnace inventory at the start and of a tap and end at the end of the horizon, denoted $M^S_{j,n}$, $M^F_{j,n}$, M^H_j, respectively must lie between the upper and lower capacity limits for each furnace j.

$$M_j^{Min} \leq M^S_{j,n} \leq M_j^{Max} \; \forall \; j \in J_F, \; n \in N \tag{3.37}$$

$$M_j^{Min} \leq M^F_{j,n} \leq M_j^{Max} \; \forall \; j \in J_F, \; n \in N \tag{3.38}$$

$$M_j^{Min} \leq M^H_j \leq M_j^{Max} \; \forall \; j \in J_F \tag{3.39}$$

Equation (3.40) states that the furnace inventory at the beginning of the tap at the first event point $M^S_{j,n}$ is equal to the initial furnace inventory $M_j(0)$ plus to the DSC added in between the beginning of the time horizon to the time of the first tap $B^S_{j,n}$. Equation (3.41) states that the furnace inventory at the end of each tap $M^F_{j,n}$ is equal to the furnace inventory at the beginning of the tap plus the DSC added between the beginning and end of the tap $B^F_{j,n}$ less the amount of material removed by the tap into the ladles. Equation (3.42) states that the furnace inventory at the beginning of each tap excluding the first is equal to the furnace inventory at the end of the previous tap in addition to the DSC added in between the end of the previous tap and the beginning of the current tap. Equation (3.43) states that the furnace inventory at the end of the time horizon is equivalent to the furnace inventory at the end of the last tap plus the DSC added between the end of the last tap and the end of the time horizon. Figure 3.4 illustrates the relation of the furnace taps to the material inventory variables and the material quantities of DSC added over each time interval. It is also necessary to account for the crane material capacity in the mass balances. The crane capacity is accounted for by the $\rho_{i,j}$ parameter which has a maximum value of the combined crane capacities and interacts with the furnaces to decide how much FFM is tapped from the furnaces.

$$M^S_{j,n} = M_j(0) + B^S_{j,n} \;\forall\; j \in J_F,\; n = N_{first} \tag{3.40}$$

$$M^F_{j,n} = M^S_{j,n} + B^F_{j,n} - \sum_{i \in I_{Chg}} (\rho_{i,j} \sum_{j' \in J_C} \sum_{o \in O} x_{i,j,j',o,n}) \;\forall\; j \in J_F,\; n \in N \tag{3.41}$$

$$M^S_{j,n} = M^F_{j,n-1} + B^S_{j,n} \;\forall\; j \in J_F,\; n \in N \;: n > N_{first} \tag{3.42}$$

$$M^H_j = M^F_{j,n} + B^H_j \;\forall\; j \in J_F,\; n = N_{last} \tag{3.43}$$

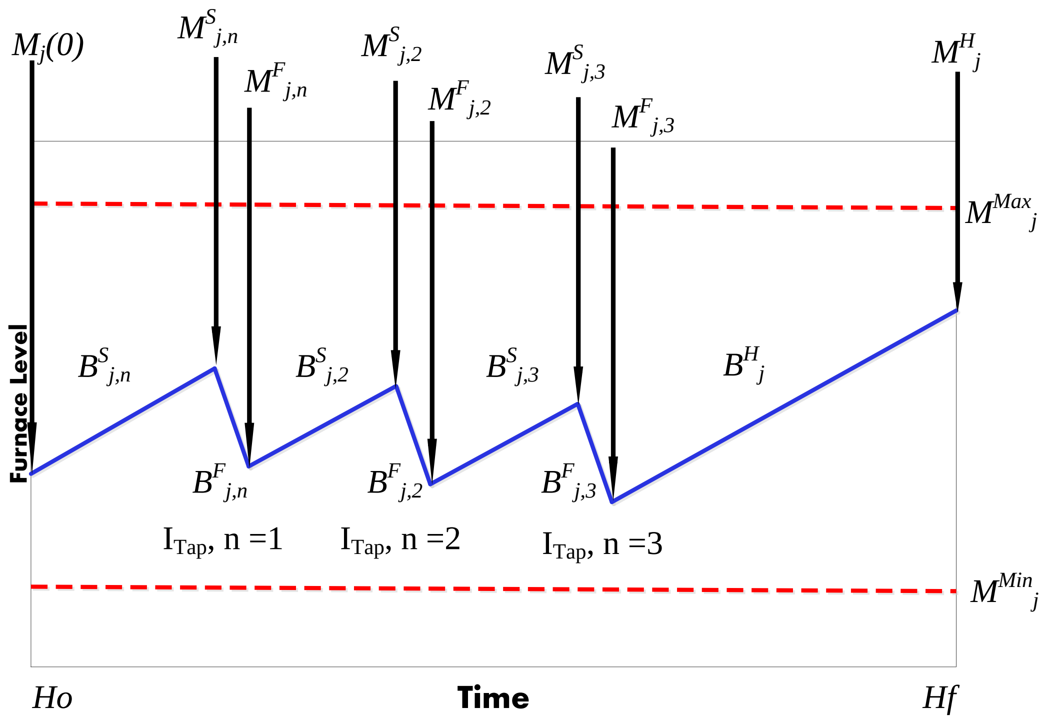

Figure 3.4: Furnace mass balance constraints.

Constraints Eqs. (3.44)-(3.47) use inequalities to relate the amount of DSC added over each time interval. Ierapetritou and Floudas [1998b] and Ierapetritou *et al.* [1999] model continuous flow using inequalities similar to those presented here. However, the constraints proposed by Ierapetritou *et al.* [1999] only apply to tasks performed during event points, whereas the constraints presented here extend the constraint to model flow during event points as well as in-between event points to represent continuous flow throughout the time horizon. Using inequalities avoids expressing the flow rate nonlinearly and allows the optimization to implicitly choose the effective flow rate over each time interval. When the maximum flow rate parameters $FR^{F,Max}_{j,n}$, $FR^{S,Max}_{j,n}$, $FR^{H,Max}_{j}$ are greater than the minimum flow rate parameter $FR^{F,Min}_{j,n}$, $FR^{S,Min}_{j,n}$, $FR^{H,Min}_{j}$ the flow rate implicitly becomes an optimization decision. When the two flow rate parameters are equal, the flow rate becomes a constant value during the entire time horizon. The formulation allows the assignment of different flow rates to different furnaces and different time intervals. With the DSC flow rates variable within each interval, the problem is a MILP. This is also true with fixed, specified DSC rates (setting the upper and lower bounds equal). However, for a uniform DSC into that is not specified a priori, the problem becomes nonlinear. Equation (3.44) models the flow rate between the initial time horizon H_0 and the first tap. Equation (3.45) models the flow rate between the beginning and end of each tap. Equation (3.46) models the flow rate between the beginning of each tap and the end of the previous tap. Equation (3.47) models the flow rate between the end of the last tap and the end of the time horizon H_f.

$$(T_{i,j,n}^{S,Furn} - H_0)FR_{j,n}^{S,Min} \leq B_{j,n}^{S} \leq FR_{j,n}^{S,Max}(T_{i,j,n}^{S,Furn} - H_0)\ \forall\, i \in I_{Tap},\ j \in J_F,\ n = N_{first} \tag{3.44}$$

$$(T_{i,j,n}^{F,Furn} - T_{i,j,n}^{S,Furn})FR_{j,n}^{F,Min} \leq B_{j,n}^{F} \leq FR_{j,n}^{F,Max}(T_{i,j,n}^{F,Furn} - T_{i,j,n}^{S,Furn})\ \forall\, i \in I_{Tap},\ j \in J_F,\ n \in N \tag{3.45}$$

$$(T_{i,j,n}^{S,Furn} - T_{i,j,n-1}^{F,Furn})FR_{j,n}^{S,Min} \leq B_{j,n}^{S} \leq FR_{j,n}^{S,Max}(T_{i,j,n}^{S,Furn} - T_{i,j,n-1}^{F,Furn})\ \forall\, i \in I_{Tap},\ j \in J_F,\ n \in N:\ n > N_{first} \tag{3.46}$$

$$(H_f - T_{i,j,n}^{F,Furn})FR_{j}^{H,Min} \leq B_{j}^{H} \leq FR_{j}^{H,Max}(H_f - T_{i,j,n}^{F,Furn})\ \forall\, i \in I_{Tap},\ j \in J_F,\ n = N_{last} \tag{3.47}$$

3.2.6 Binary Tightening Constraints

A series of tightening constraints are developed in order to reduce the computational time to solve all case studies to optimality. Equation (3.48) specifies that each event point n must be used sequentially. This serves to reduce the various combinations of solutions that might evaluate to the same objective function. Equation (3.48) serves to reduce the branch and bound nodes required to solve to optimality, as well as, guide the tree search.

$$\sum_{i \in I_{Chg}} \sum_{j \in J_F} \sum_{j' \in J_C} \sum_{o \in O} x_{i,j,j',o,n} \geq \sum_{i \in I_{Chg}} \sum_{j \in J_F} \sum_{j' \in J_C} \sum_{o \in O} x_{i,j,j',o,n+1} \; \forall \, n \in N : n \; < \; N_{last} \quad (3.48)$$

Janak and Floudas [2008] proposed a methodology to reduce the integrality gap for large scale continuous time problems using preprocessing steps to improve the computational performance of unit-specific event-based formulations. In their paper, they develop novel tightening constraints and extend unit event based formulations to account for task splitting in order to model finite intermediate storage. One of the tightening constraints that they propose makes use of preprocessing and prior knowledge to cancel certain tasks at certain event points where a task will have no option but to be inactive. The concept of cancelling binaries using prior knowledge is adapted and employed here in Equation (3.49) which is used to cancel converter cycles over a specific subset of event points. Using Equation (3.49) reduces the problem size and the required search effort where the set N_z specifies which event points are cancelled. This constraint applies to the case where all the converters are starting their cycles fresh, rather completing partial cycles. For example, based on the precedence scheme the first tap on converter one should always be allocated to the first event point. In recognizing this fact, it can be concluded that the binary variables for all the other charges on the rest of the converters will take a value of zero for the first event point.

$$x_{i,j,j',o,n} = 0 \; \forall \; i \in I_{Chg}, \; z \in Z, \; j \in J_F, \; j' \in J_z, \; o \in O_z, \; n \in N_z \qquad (3.49)$$

Constraint Eqs. (3.50)-(3.53) specify a precedence in which converter charges must be allocated. These constraints reduce the branch and bound search by specifying the order of matte allocation. Figure 3.5 summarizes the effect of the constraints. Equation (3.50) and Equation (3.51) are used to specify converter precedence for all charges except the third, which requires a different set to account for the CuWa. The constraints specify that charges must be done in the proper sequence and will prevent the following charge from being triggered if the current charge has not been completed. Equation (3.50) account for the first set of converter cycles which use the $P_{i,j'}$ parameter to specify progression, while Equation (3.51) accounts for the remainder of the cycles.

$$\sum_{j\in J_F}\sum_{n\in N} x_{i,j,j',o,n} + P_{i,j'} \geq \sum_{j\in J_F}\sum_{n'\in N} x_{i,j,j'',o',n'} \;\forall\; i \in I_{Chg},\; z \in Z,\; j' \in J_z,\; o \in O_z,$$
$$j'' \in J_{z+1},\; o' \in O_{z+1} : \; i \neq I_{Chg3},\; z \leq q_{StrC},\; z < Z_{last} \tag{3.50}$$

$$\sum_{j\in J_F}\sum_{n\in N} x_{i,j,j',o,n} \geq \sum_{j\in J_F}\sum_{n'\in N} x_{i,j,j'',o',n'} \;\forall\; i \in I_{Chg},\; z \in Z,\; j' \in J_z,\; o \in O_z,\; j'' \in J_{z+1},$$
$$o' \in O_{z+1} \; : \; i \neq I_{Chg3},\; q_{StrC} < z < Z_{last} \tag{3.51}$$

Equation (3.52) and Equation (3.53) are used to specify converter precedence for the third charge and CuWa. Equation (3.52) accounts for the first set of converter cycles which use the $P_{i,j'}$ parameter to specify progression, while Equation (3.53) accounts for the remainder of the cycles.

$$\sum_{j\in J_F}\sum_{n\in N} x_{i,j,j',o,n} + P_{i,j'} + w_{j',o} \geq \sum_{j\in J_F}\sum_{n'\in N} x_{i,j,j'',o',n'} + w_{j'',o} \;\forall\; i \in I_{Chg3},\; z \in Z,\; j' \in J_z,$$
$$o \in O_z, j'' \in J_{z+1},\; o' \in O_{z+1} \; : \; z \leq q_{StrC},\; z < Z_{last} \tag{3.52}$$

$$\sum_{j\in J_F}\sum_{n\in N} x_{i,j,j',o,n} + w_{j',o} \geq \sum_{j\in J_F}\sum_{n'\in N} x_{i,j,j'',o',n'} + w_{j'',o} \;\forall\; i \in I_{Chg3},\; z \in Z,\; j' \in J_z,\; o \in O_z,$$
$$j'' \in J_{z+1},\; o' \in O_{z+1} \; : \; q_{StrC} < z < Z_{last} \tag{3.53}$$

Cycle	o = 1					o = 2				
Converter j' = 1	$I_{Chg1} + P_1$	$I_{Chg2} + P_2$	$I_{Chg3} + P_3$	$I_{Chg4} + P_4$	$I_{Chg5} + P_5$	I_{Chg1}	I_{Chg2}	I_{Chg3}	I_{Chg4}	I_{Chg5}
	$\geq$	$\geq$	$\geq$	$\geq$	$\geq$	$\geq$	$\geq$	$\geq$	$\geq$	$\geq$
Converter j' = 2	I_{Chg1}	I_{Chg2}	I_{Chg3}	I_{Chg4}	I_{Chg5}	I_{Chg1}	I_{Chg2}	I_{Chg3}	I_{Chg4}	I_{Chg5}

Figure 3.5: Converter precedence.

Constraint Equs. 3.54-3.57 are valid inequalities that serve to further guide the branch and bound search and were found to improve computational time. The constraints are summarized graphically in Figure 3.6. Equation (3.54) and Equation (3.55) are used to specify that each charge for the previous cycle for each converter must be completed before starting a charge on the next cycle for all charges except the third, which requires a different set to account for the CuWa. Equation (3.54) account for the first set of converter cycles which use the $P_{i,j'}$ parameter to specify progression, while Equation (3.55) accounts for the remainder of the cycles.

$$\sum_{j\in J_F}\sum_{n\in N} x_{i,j,j',o,n} + P_{i,j'} \geq \sum_{j\in J_F}\sum_{n'\in N} x_{i,j,j',o+1,n'} \;\forall\, i \in I_{Chg},\; j' \in J_C,\; o = O_{first} \;:\; i \neq I_{Chg3},\; o < O_{last} \tag{3.54}$$

$$\sum_{j\in J_F}\sum_{n\in N} x_{i,j,j',o,n} \geq \sum_{j^*\in J_F}\sum_{n'\in N} x_{i,j,j',o+1,n'} \;\forall\, i \in I_{Chg},\; j' \in J_C,\; o \in O \;:\; i \neq I_{Chg3},\; O_{first} < o < O_{last} \tag{3.55}$$

Equation (3.56) and Equation (3.57) are used to specify that the third charge and CuWa must be performed on the previous cycle before starting the new cycle. Equation (3.56) accounts for the first set of converter cycles which use the $P_{i,j'}$ parameter to specify progression, while Equation (3.57) accounts for the remainder of the cycles.

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} + P_{i,j'} + w_{j',o} \geq \sum_{j \in J_F} \sum_{n' \in N} x_{i,j,j',o+1,n'} + w_{j',o+1} \ \forall \ i \in I_{Chg3}, \ j' \in J_C, \ o = O_{first}, o < O_{last} \tag{3.56}$$

$$\sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} + w_{j',o} \geq \sum_{j \in J_F} \sum_{n' \in N} x_{i,j,j',o+1,n'} + w_{j',o+1} \ \forall \ i \in I_{Chg3}, \ j' \in J_C, \ o \in O \ : \ O_{first} < o < O_{last} \tag{3.57}$$

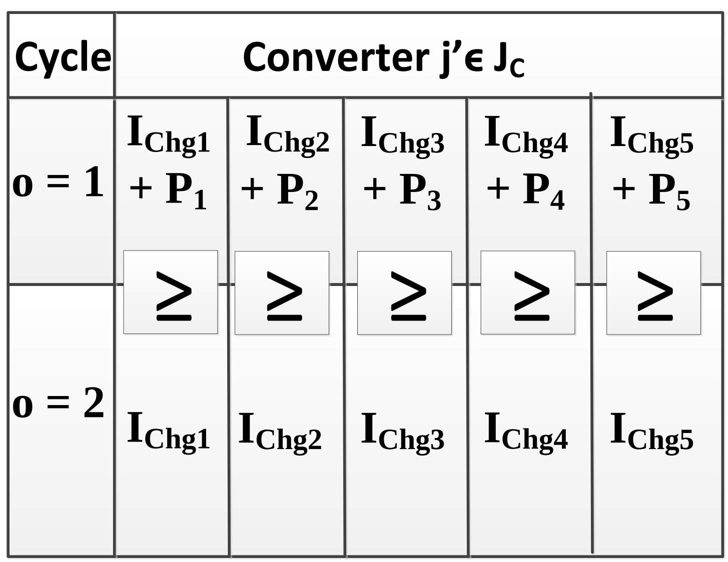

Figure 3.6: Cycle precedence.

Merchan and Maravelias [2014] and Velez and Maravelias [2013b] propose the use of integer variables to represent the number of batches and tasks completed which demonstrate improvement in computational times. The proposed constraints are adapted here in Equation (3.58) and Equation (3.59) which track the number of tasks performed and improved some solution times by as much as two orders of magnitude which is consistent with the results demonstrated in Merchan and Maravelias [2014] and Velez and Maravelias [2013b].

$$y_{i,j',o} = \sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} \; \forall \; i \in I_{Chg}, \; j' \in J_C, \; o \in O, \; i \neq I_{Chg3} \tag{3.58}$$

$$y_{i,j',o} = \sum_{j \in J_F} \sum_{n \in N} x_{i,j,j',o,n} + w_{j',o} \; \forall \; i \in I_{Chg3}, \; j' \in J_C, \; o \in O \tag{3.59}$$

3.2.7 Flow Rate, Mass Balance Tightening Constraints

Constraint Eqs. (3.60)-(3.62) introduce valid inequalities that were found to reduce both the branch and bound nodes searched and the number of simplex iterations required for solution to optimality. The constraints are inspired by a physical observation regarding furnace operation. The observation is that the time between tapping events is limited by the minimum flow rate and the furnace capacity. Using this information we formulate inequalities to tighten the formulation. Equation (3.60) limits the difference between start time of the first tap and the start of the time horizon times the minimum flow rate to be less than the maximum furnace capacity less the starting inventory. The interpretation is that tapping must start before the furnace begins to overfill. Equation (3.61) states that tapping must begin after the previous tap, before the maximum furnace capacity is reached. Equation (3.62) states that the end of the last tap must be performed such that the furnace is not overwhelmed by the end of the time horizon.

$$(T^{S,Furn}_{i,j,n} - H_0) FR^{S,Min}_{j,n} \leq \; M^{Max}_j - M_j(0) \; \forall \; i \in I_{Tap}, \; j \in J_F, \; n = N_{first} \tag{3.60}$$

$$(T^{S,Furn}_{i,j,n} - T^{F,Furn}_{i,j,n-1}) FR^{S,Min}_{j,n} \leq \; M^{Max}_j - M^{Min}_j \; \forall \; i \in I_{Tap}, \; j \in J_F, \;\; n \in N : \; n > N_{first} \tag{3.61}$$

$$(H_f - T^{F,Furn}_{i,j,n}) FR^{H,Min}_{j,n} \leq \; M^{Max}_j - M^{Min}_j \; \forall \; i \in I_{Tap}, \; j \in J_F, \; n = N_{last} \tag{3.62}$$

Constraint Eqs. (3.63)-(3.66) similarly limit the amount of FFM added to the furnace to be less than the total furnace capacity.

$$0 \leq B^{S}_{j,n} \leq M^{Max}_{j} - M_j(0)\ \forall\ j \in J_F,\ n = N_{first} \tag{3.63}$$

$$0 \leq B^{S}_{j,n} \leq M^{Max}_{j} - M^{Min}_{j}\ \forall\ j \in J_F,\ n \in N\ : n > N_{first} \tag{3.64}$$

$$0 \leq B^{H}_{j} \leq M^{Max}_{j} - M^{Min}_{j}\ \forall\ j \in J_F \tag{3.65}$$

$$0 \leq B^{F}_{j,n} \leq M^{Max}_{j} - M^{Min}_{j} + \max_{i \in I_{Chg}} (\rho_{i,j})\ \forall\ j \in J_F,\ n \in N \tag{3.66}$$

Equation (3.67) states that the initial furnace inventory in addition to the minimum FFM added throughout the entire time horizon minus the total number of furnace taps performed must be less than the entire furnace capacity. In effect, the constraint specifies a minimum production level that will avoid overfilling each furnace.

$$M(0)_j - M^{Min}_{j} + FR^{H,Min}_{j}(H_f - H_o) - \sum_{i \in I_{Chg}} (\rho_{i,j} \sum_{j' \in J_C} \sum_{o \in O} \sum_{n \in N} x_{i,j,j',o,n}) \leq M^{Max}_{j} - M^{Min}_{j}$$
$$\forall\ j \in J_F \tag{3.67}$$

Equation (3.68) specifies an upper bound on the number of possible taps based on the maximum amount of FFM that is added throughout the time horizon.

$$\sum_{i \in I_{Chg}} (\rho_{i,j} \sum_{j' \in J_C} \sum_{o \in O} \sum_{n \in N} x_{i,j,j',o,n}) \leq M(0)_j - M^{Min}_{j} + FR^{H,Max}_{j}(H_f - H_o)\ \forall\ j \in J_F \tag{3.68}$$

Together, Equation (3.67) and Equation (3.68) place an upper and lower bound on production. Velez and Maravelias [2013c] and Velez and Maravelias [2013a] similarly propose tightening constraints based in the minimum production levels required.

3.2.8 Time Tightening Constraints

Constraint Equation (3.69) specifies that when a charge has been performed, as indicated by $P_{i'',j'}$ the next unfinished task takes precedence over each charge in the remaining converters. This serves to further specify the converter sequencing and eliminate symmetric solutions. In Equation (3.69), if charge i'' is unfinished the $P_{i'',j'}$ parameter will be equal to one and will force the start time of the first charge on the next converter to be greater than the start time of the charge i''.

$$T^{S,Conv}_{i,j',o} \leq T^{S,Conv}_{i',j'+1,o} + (H_f - H_0)(1 - P_{i'',j'}) \; \forall \; k = 2..5 \; , i \in I_{Chg}(k), \; i'' \in I_{Chg}(k-1)$$
$$i' \in I_{Chg1}, \; j' \in J_C, \; o \in O_{first} : j' < J_{C\,last} \tag{3.69}$$

In practice, is it desirable to specify a maximum cycle time for converters to complete. If cycles do not meet this criteria, it is industrially preferable to delay the entire cycle or cancel the cycle entirely. This behavior is captured by the parameter $MaxTime$ in constraint Equation (3.70). If the cycle time is defined too tightly, feasible solutions can be eliminated. However, if the cycle tolerance is defined too broadly the constraint will have little impact on the solutions found and the computational performance of the optimization.

$$T^{F,Conv}_{i,j',o} - T^{S,Conv}_{i',j',o} \leq MaxTime \; \forall \; i \in I_F, \; i' \in I_{Chg1}, \; j' \in J_C, \; o \in O \tag{3.70}$$

3.2.9 Objective Function

Equation (3.71) states the objective function as a maximization of the weighted sum of the binaries so as to achieve an optimal production level for a given time horizon. The weighting factor $\gamma_{i,o}$ is assigned increasing parameter values with increasing i and o. The purpose of assigning this weighting is two-fold. The first purpose is to achieve a symmetry-breaking scheme such that converter charges will avoid being viewed as having an identical value in the objective function. This should contribute to improved computational behaviour during the solution procedure. The second purpose is an industrial insight that was shared during the course of this work, where charges are generally allocated to the converter which has progressed furthest into a cycle. That is to say that it is more preferable to complete converter cycles in progress, rather than initialize a new cycle, in general. Since the CuWa substitutes the third charge, it is allocated the same weighting factor in the objective function. The objective function is formulated to maximize production within a given time horizon, and to drive converter cycle progression to a maximum when completion of a cycle is limited by the available resources. The maximization of the copper washout motivates the allocation of the CuWa, when resources allow for its use.

$$\text{Maximize Production} = \max\Big(\sum_{o \in O}\Big(\sum_{i \in I_{Chg}} \gamma_{i,o} \sum_{j \in J_F} \sum_{j' \in J_C} \sum_{n \in N} x_{i,j,j',o,n} + \sum_{i' \in I_{Chg3}} \gamma_{i',o} \sum_{j' \in J_C} w_{j',o}\Big)\Big) \tag{3.71}$$

3.3 Solution Procedure

A slot-based continuous time formulation which is formulated as a MILP for the purposes of converter aisle scheduling was presented in the previous section. The formulation models furnaces which are operated semi-continuously and are used to melt and store FFM which is fed batch-wise to converters in the plant. The formulation allows for a rolling horizon capability in order to optimize consecutive time periods by parameterizing the subset of converter charges that have been completed in previous time periods. Furthermore, the formulation accounts for the blowing emission disturbance by timing converter blows to avoid violating the emission restriction.

Optimizations were performed on a 2.33 GHz IntelrCoreTM 2 Quad processor with 6 GB of RAM, running Windows 7 Professional 64-bit. The formulation was modeled using AMPL and solve with CPLEX 12.5.

3.3.1 Tiered Optimization

The formulation to this point has focused on a production maximization. While this is the objective with the highest priority, other objectives of lower priority may be desirable to facilitate ease of industrial implementation. Swartz [1995] and Chong and Swartz [2013] propose a methodology using multi-tiered optimization to prioritize among competing objectives where feasibility in the first tier guarantees feasibility in all the remaining subproblems. A solution enhancing procedure is presented here where a multi-tiered approach is exploited. The first layer focuses on the production maximization, while the second layer minimizes the start time of each task, and the third layer minimizes unnecessary flow rate changes. The solution procedure is illustrated in Figure 3.7.

Tier	Problem Type	DOF	Locked
1.) **Production Maximization** Lock in Production Level	MILP	X, Y, W, T, B, M	
2.) **Minimize Start Times** Lock in Schedule Event Timing	LP	T, B, M	X, Y, W
3.) **Minimize Flow Rate Changes** Schedule Enhanced	QP	B, M, FR	X, Y, W, T

Figure 3.7: Tiered optimization procedure.

Once the production maximization has been solved, it is possible to fix the binaries in order to lock in the production level and treat the binaries as parameters. Once the binaries are locked in, the remaining degrees of freedom include: the event timing variables, material inventory variables, and the amounts of FFM added. As event timings were not included in the objective function of the first tier, the first tier optimization may be time insensitive to a degree. In an industrial setting it is preferable to start each task as early as possible, rather than delay tasks. As such, the second tiered optimization minimizes the start time of each event. While it is possible to use a multi-objective function rather than a tiered approach, a tiered approach was found to be more efficient computationally. Also, rather than fixing the binaries it is also possible to simply lock in the objective function value of the first tier and solve for a schedule with a global minimum in terms of start times. The reason that the binaries are fixed rather than the objective function is to transform the MILP into an LP which significantly reduces the computational effort. Fixing the objective function value would translate into solving two MILPs sequentially which could potentially increase computation time past the tolerance for a real-time application. Furthermore, achieving a global minimum in terms of start time is merely a secondary objective that does not justify the additional computation of solving a second MILP.

$$\text{Minimize Start Times} = \min\Big(\sum_{i\in I_{Tap}}\sum_{j\in J_F}\sum_{n\in N} T^{S,Furn}_{i,j,n} + \sum_{i'\in I_{Chg}\cup I_{Blow}\cup I_F}\sum_{j'\in J_C}\sum_{o\in O} T^{S,Conv}_{i',j',o}\Big) \tag{3.72}$$

Once the second tier has been solved, the event timing variables are fixed in addition to the binaries. When the flow rates are allowed to vary within a given time horizon, the optimization will take advantage of this feature to find an optimal production level. However, there exist several non-unique furnace flow rate profiles that might satisfy an optimal production schedule, some of which might require more adjustments to the flow rate than others. Penalizing the changes in flow rate will result in a schedule that is easier for operators to follow and will require a minimum of adjustment. However, penalizing the manipulated input in a way that is analogous to Model Predictive Control (MPC) in the first or second tier might result in potentially introducing non-linearity into the formulation. A computationally efficient method to penalize the change in input moves is required.

Constraints Equation (3.73) through to Equation (3.79) are used to explicitly model the flow rate over each now fixed time intervals using the variables $FR^{S}_{j,n}$, $FR^{F}_{j,n}$, $FR^{H}_{j,n}$ both during, between, and after each tap. Equation (3.73), Equation (3.74), Equation (3.75) constrain the flow rate between the minimum and maximum values.

$$FR^{S,Min}_{j,n} \leq FR^{S}_{j,n} \leq FR^{S,Max}_{j,n} \; \forall \; j \in J_F, \; n \in N \tag{3.73}$$

$$FR^{F,Min}_{j,n} \leq FR^{F}_{j,n} \leq FR^{F,Max}_{j,n} \; \forall \; j \in J_F, \; n \in N \tag{3.74}$$

$$FR^{H,Min}_{j} \leq FR^{H}_{j} \leq FR^{H,Max}_{j} \; \forall \; j \in J_F \tag{3.75}$$

Constraint Equs. 3.76-3.79 are used to assign the continuous flow rate variables as an equality to the amount of FFM added to each furnace over each non-zero time interval. The flow rate variables are in a sense dummy variables which are used to assign equality constraints in a way that avoids a division by zero. If a time interval has a zero value, which would result in a division by zero, the equality is not assigned. Because the event timings were fixed in the second tier, the time variables are now treated as parameters and the equality assignments are linear rather than nonlinear. Equation (3.76) assigns the flow rate variable to the first event point subject to the $\phi^{S}_{j,n}$ parameter being non-zero, indicating a non-zero denominator. Equation (3.77) assigns the flow rate equality of each tap excluding the first, subject to the $\phi^{S}_{j,n}$ parameter being non-zero. Equation (3.78) assigns the flow rate equality between each tap, subject to the $\phi^{F}_{j,n}$ parameter being non-zero. Equation (3.79) assigns the flow rate equality between the end of the last tap and the end of the horizon, subject to the ϕ^{H}_{j} parameter being non-zero.

$$FR^{S}_{j,n} = \frac{B^{S}_{j,n}}{T^{S,Furn}_{i,j,n} - H_0} \; \forall \; i \in I_{Tap}, \; j \in J_F, \; n = N_{first} \; : \; \phi^{S}_{j,n} \neq 0 \tag{3.76}$$

$$FR^S_{j,n} = \frac{B^S_{j,n}}{T^{S,Furn}_{i,j,n} - T^{F,Furn}_{i,j,n-1}} \;\forall\, i \in I_{Tap},\; j \in J_F,\; n \in N \;:\; n \neq N_{first},\; \phi^S_{j,n} \neq 0 \tag{3.77}$$

$$FR^F_{j,n} = \frac{B^F_{j,n}}{T^{F,Furn}_{i,j,n} - T^{S,Furn}_{i,j,n}} \;\forall\, i \in I_{Tap},\; j \in J_F,\; n \in N \;:\; \phi^F_{j,n} \neq 0 \tag{3.78}$$

$$FR^H_j = \frac{B^H_j}{H_f - T^{F,Furn}_{i,j,n}} \;\forall\, i \in I_{Tap},\; j \in J_F,\; n = N_{last} \;:\; \phi^H_j \neq 0 \tag{3.79}$$

The new objective function in Equation (3.80) is formulated as the quadratic minimization of the change in flow rate over each event point interval. In this manner we have presented a methodology to effectively minimize the change in flow rate to the furnace by transforming a MILP to a linear program (LP) into a quadratic program (QP). The QP has the added benefit of efficient solve times and so does not increase computation in any significant way. This method penalizes changes in the input profile in a manner analogous to MPC.

$$\text{Minimize Flow Rate Deviation} = \min\Big(\sum_{j\in J_F}\sum_{n\in N}(FR^F_{j,n} - FR^S_{j,n})^2 + \sum_{j\in J_F}\sum_{n\in N:n\neq N_{last}}(FR^S_{j,n+1} - FR^F_{j,n})^2 + \sum_{j\in J_F}\sum_{n=N_{last}}(FR^H_j - FR^F_{j,n})^2\Big) \tag{3.80}$$

3.3.2 Objective Bounding Procedure

In practice there are two furnaces that feed the four converters in the plant. The addition of each furnace results in a significant increase in the number of integer variables in the optimization. The increased number of integer variables result in a significant increase in the solution time. Rather than solve the two furnace and four converter problem at each instance, a bounding procedure is used to reduce the time required to solve the two furnace problem. The combined capacity limits and flow rates of the two identical furnaces are approximated by one aggregated furnace with an equivalent capacity and flow rate range.

The solution of the aggregated furnace problem in terms of converter production is used to provide a tight upper bound on the production level in the converters for the two furnace problem. Based on this methodology, the optimal solution of the aggregated furnace problem is used to provide a constraint on the upper production level of the two furnace problem. This procedure was found to improve the computational properties of the optimization is many cases.

Chapter 4

Case Studies

The following case studies will investigate a configuration consisting of two furnaces with identical unit capacities which service four converters operating batch wise. A base case will be established to identify what an optimal production schedule should achieve in ideal conditions. Scenarios will identify what happens to production when a constant flow rate is permitted to the furnaces, how the schedule changes when a copper washout is provided, how restrictive emission limits impact production, and how the rolling horizon function allows for continued planning in subsequent periods, in comparison to the base case.

There are three major aspects that must be presented to operators for the optimization to be valuable. The first item is the generation of a Gantt chart to relay event timings, unit sequencing, CuWa utilization, and blow timings that satisfy environmental restrictions. The second item required is a control chart for the material inventory in the furnaces. The last item required is a prediction of the flow rates required to satisfy the production schedule based on the current flow rate limits. The following case studies demonstrate the various features of the scheduling optimization and the efficacy of the methodology presented.

4.1 Results

Furnace taps that must be spaced out according to the rate limitation previously discussed in Equation (3.29) and due to the furnace inventory and filling constraints. Converter charges must be performed to within a tolerance of $Trnsfr^{Min}$ and $Trnsfr^{Max}$ in relation to furnace taps by Equation (3.30) and Equation (3.31). The boxes around each converter cycle represent that the entire cycle has been completed within the current schedule. Converters are assigned an event timing and charging precedence according to the sequencing scheme described in Table 3.2. Casting times are partially offset from each other in order to represent the casting unit availability as modeled by Equation (3.34). The last 60 minutes of the I_F task represents the casting operation. It is the casting times that are aggregated with the nitrogen, miss blow, skim tasks that must be offset. The space in between cycles on the same converter is mandated by Equation (3.27) for the use of preventative maintenance and cleaning to a minimum duration of *Stndby*. Converter cycles are only able to take a maximum duration of $MaxTime$.

4.1.1 Case 1 - Base Case

Figure 4.1 contains three plots that include the furnace inventory, predicted flow rate, and a Gantt chart. The Gantt chart on the top of Figure 4.1 relates the event timing of each converter and displays the completion of seven converter cycles and the partial completion of an additional cycle with four charges performed. In this case converters are assumed to be starting without any charges having been performed. The objective function has been purposely weighted to drive converters to perform as many charges as possible in the case that they are unable to complete an entire cycle. The optimization is unable to complete the final cycle due to the time limitations imposed by the tapping rate, the standby constraint, and the casting offset. Were the entire horizon to be extended, the optimization would be able to accommodate the last charge on the last cycle which includes the casting and finishing phase of the cycle. The furnace inventory chart in the middle of Figure 4.1 displays that the furnace inventory begins from the maximum and minimum furnace capacity limits and maintains furnace operation within the operating limits. The furnaces are meant to act as buffers that service converter demand based on when converters are ready to receive new charges. The flow rate in this case is allowed to vary within the maximum and minimum flow rate limits. Due to the lack of a penalty in changes to the flow rate in the objective function or hard constraints to regulate the flow rate, the flow rate cycles between the limits frequently and likely unnecessarily. However, specifying a penalty or explicit flow rate constraints in the the production maximization optimization is complicated by the potential to introduce nonlinearity in the optimization and division by zero. This is due to the bilinear manner by which the flow rate interacts with time and mass.

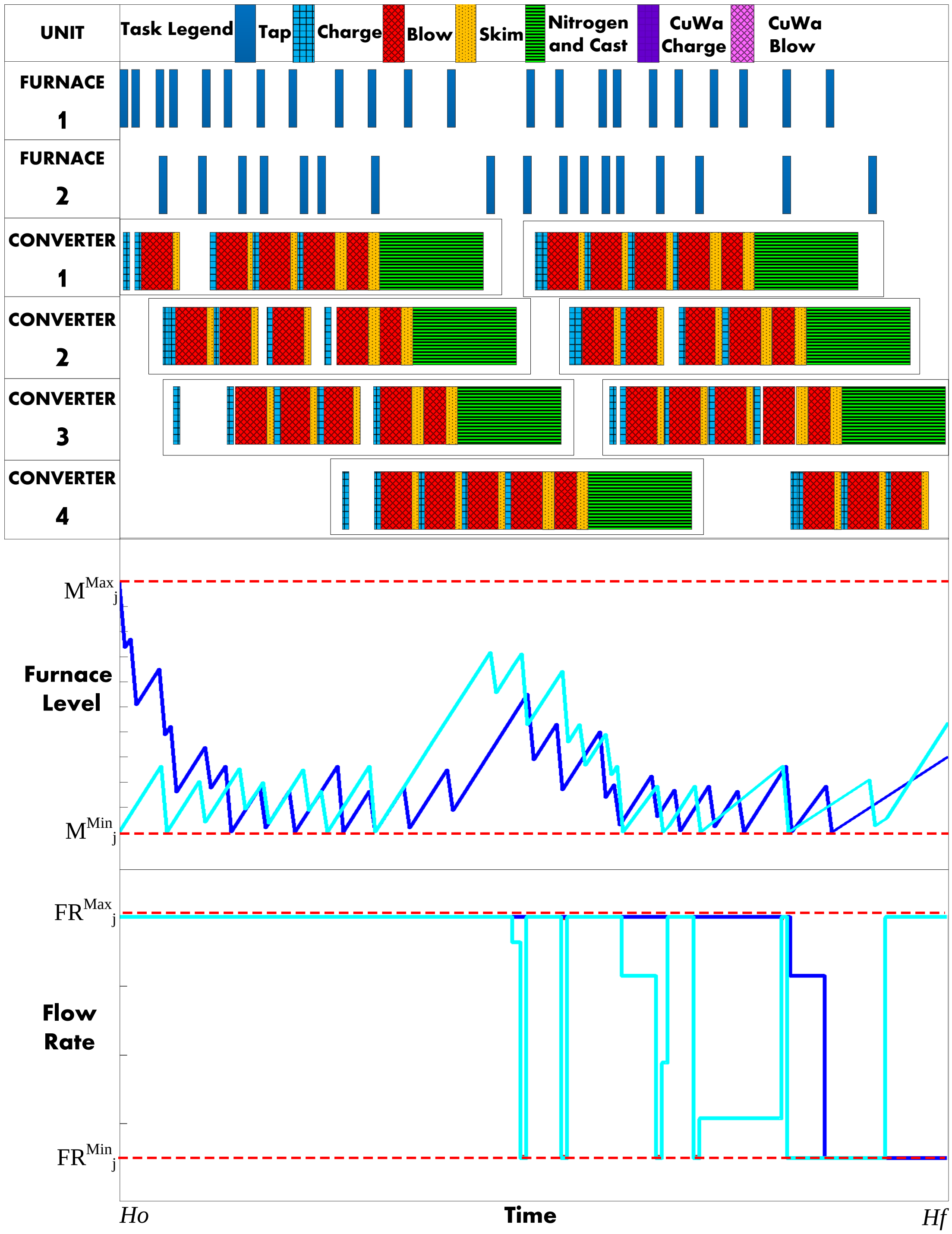

Figure 4.1: Case 1. Time horizon 24 h; Variable flow rate available; Copper washout unavailable; No blowing restriction; Tiered optimization not applied.

4.1.2 Case 2 - Availability and Utilization of Copper Washout

Figure 4.2 illustrates the use of the copper washout operation, where the allocation of CuWa is encircled. Three ladles of copper washout are made available within the time horizon, where the operating conditions of the optimization are otherwise identical to that of Figure 4.1. In this case the CuWa ladles are made available immediately. Despite the reduced cycle times, the optimization is unable to achieve a higher level of production in this case, as compared to the base case. Conceivably, there might be cases where the use of the CuWa operation allows for greater production levels due to the reduction in the total converter cycle time. This might be especially true in cases where the initial furnace inventories are both very near to their upper capacity limit. While the production schedule was optimal in Figure 4.1, there are secondary and tertiary objectives to fulfill that would enhance operation and the practicality of the schedule. Figure 4.2 demonstrates the use of a solution enhancing procedure where a multi-tiered optimization is employed to improve the schedule. The multi-tiered procedure is summarized in Equation (3.80). The first tier maximizes production and then locks the integer variables. This is demonstrated by the same number of converters being completed and number of partial cycles initialized. The second tier minimizes the start times of each task. The last layer penalizes changes in the flow rate. The resulting flow rate control chart shows a significant reduction of the cycling behavior experienced in Figure 4.1. As a result of the combined time minimization and penalization of changes to the flow rate, the flow rate over most of the horizon is higher. The higher flow rate results in an increased ending furnace inventory. A higher ending inventory is a desirable result as it allows for greater production in the next time period.

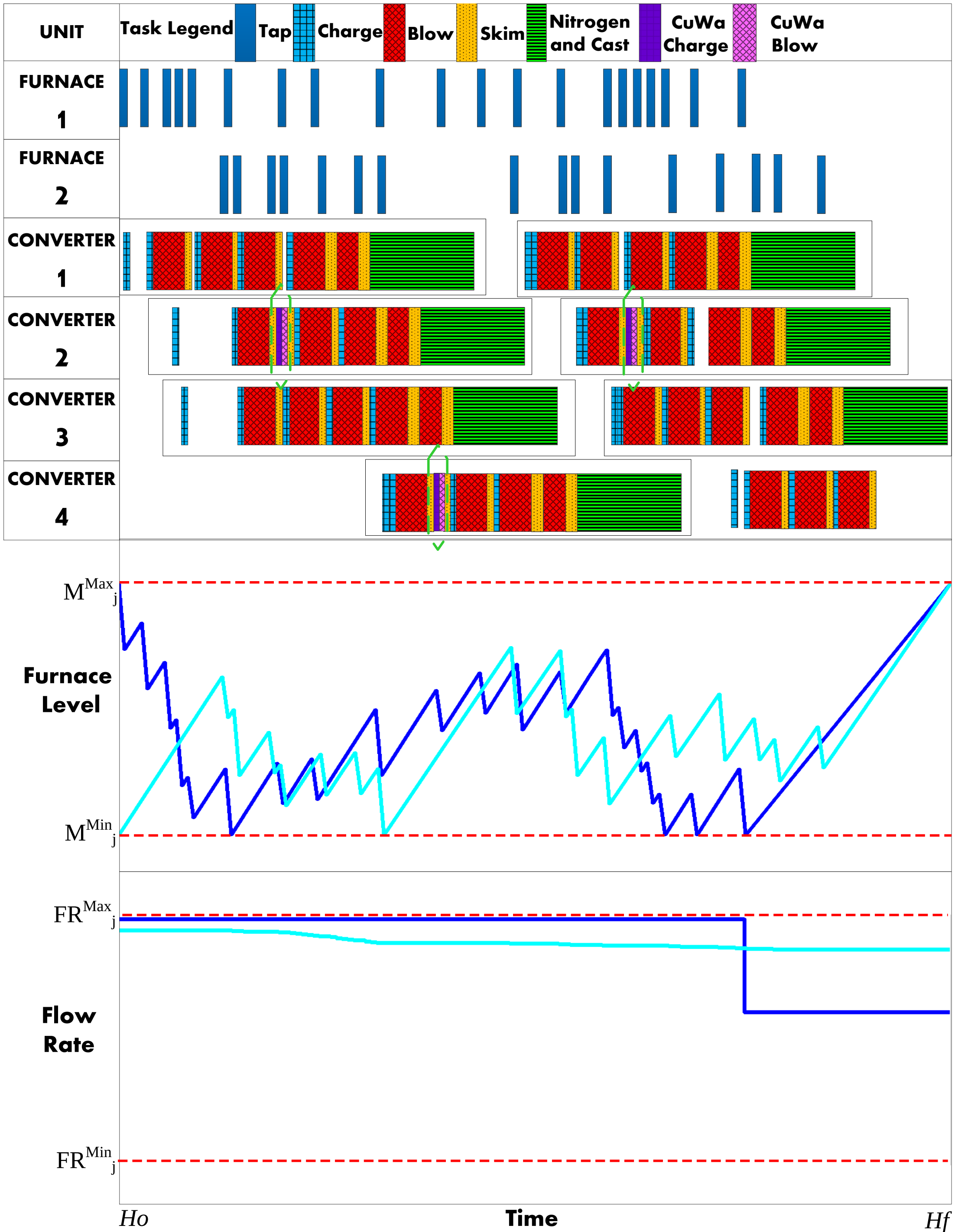

Figure 4.2: Case 2. Time horizon 24 h; Variable flow rate available; Three ladles of copper washout available at time H_0; No blowing restriction; Tiered optimization is applied.

4.1.3 Case 3 - Constant Flow Rate to both Furnaces

Figure 4.3 illustrates the case where there is a constant flow rate specified in the flow rate parameters. The result in comparison to Figure 4.1, is a significant decrease in production where only two cycles are completed and three partial cycles are performed. The FFM supply is decreased due to a lower constant flow rate. This results in a decrease in the total number of charges that can be performed and thus decreases the number of complete cycles. Due to the fact that the converter timings are not placed in the objective function in the production maximization layer, event timings do not contribute to the objective function. As a result, event timings are not initiated earlier in the time horizon, though in practice it might be possible to. As a result, the optimization delays the production cycles, as starting them earlier does not improve the objective function.

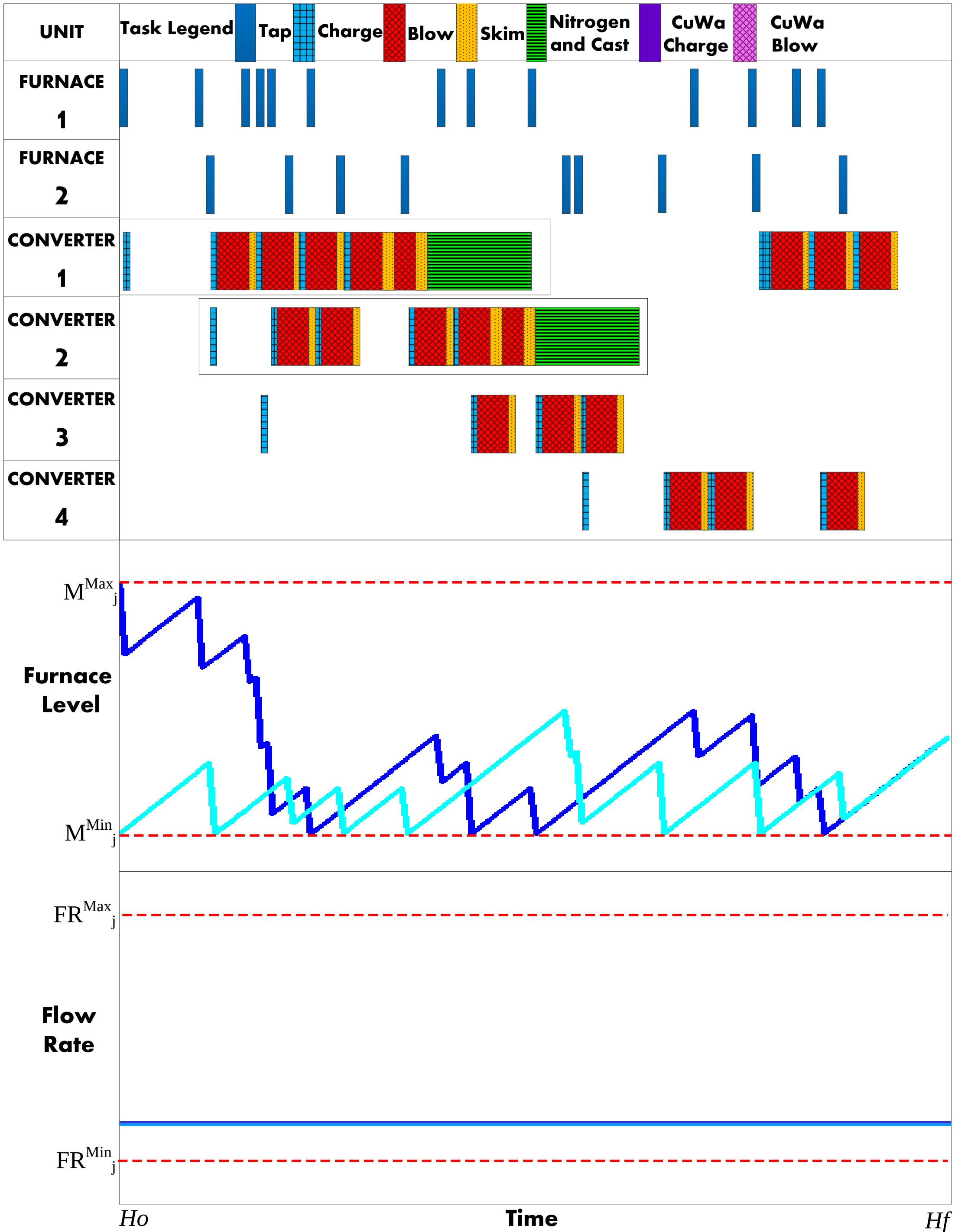

Figure 4.3: Case 3. Time horizon 24 h; Constant flow rate specified; Copper washout unavailable; No blowing restriction; Tiered optimization is applied.

4.1.4 Case 4 - Emission Limits Restrict Production

The distinguishing feature of Figure 4.4 is the limitation on blowing activities, where the full flow rate range is available for use. The emission restriction effectively limits production to allow for only two converters to blow simultaneously. The black arrows are visual aids meant to illustrate the beginning and end of some of the converter blows. The periods within the black arrows depict that a maximum of two converters are blowing at any given time, whereas the previous case studies had no such restriction. Accounting for the blowing restriction is achieved with appropriately timing and sequencing the converters using Equation (3.35) and Equation (3.36). Despite the blowing restriction, the optimization is able to complete six entire cycles and two partial cycles. The optimization is unable to achieve the same production level of Figure 4.1 due to the blowing restriction.

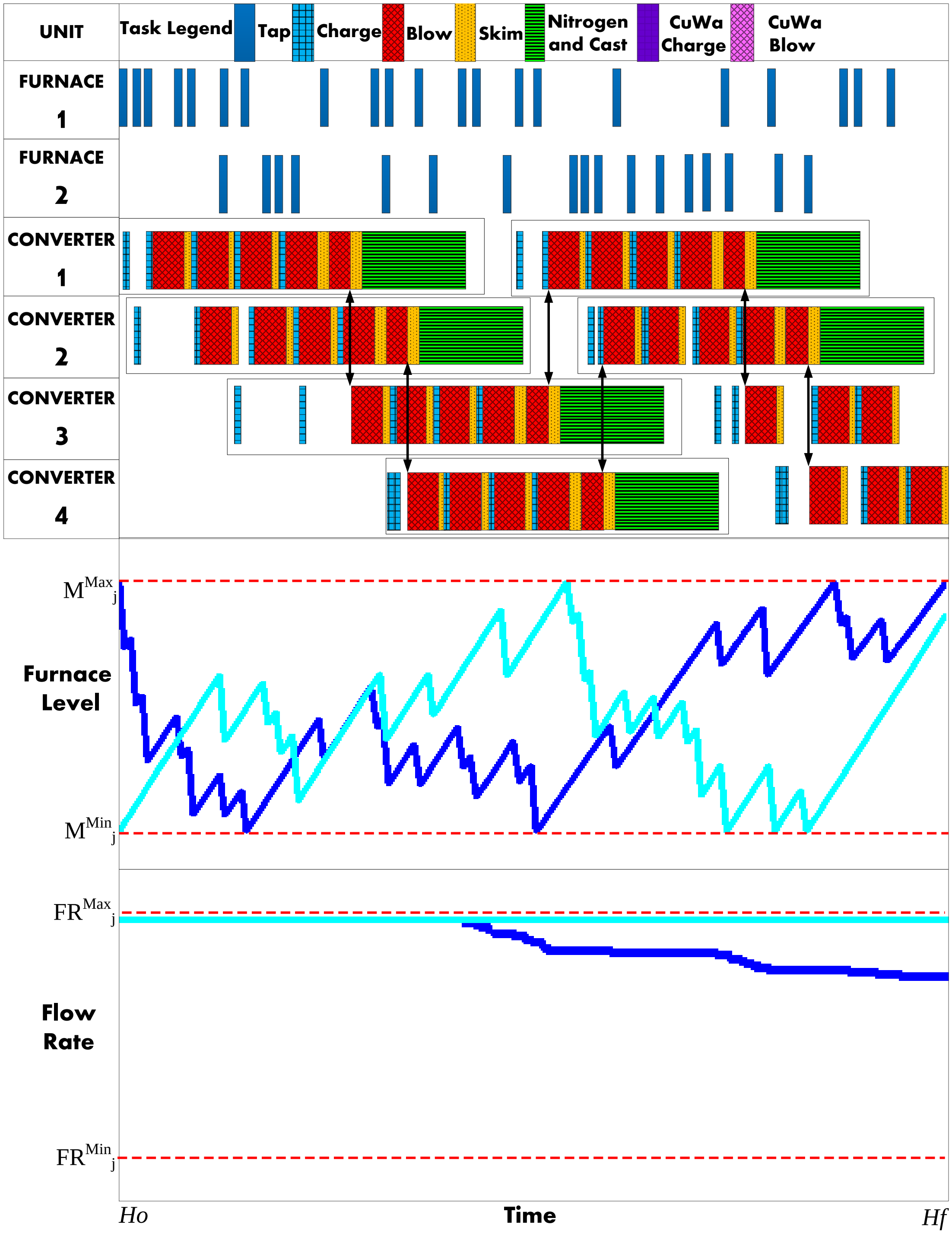

Figure 4.4: Case 4. Time horizon 24 h; Variable flow rate available; Copper washout unavailable; Blowing limit restricts production to only two converters blow at most at a time; Tiered optimization is applied.

4.1.5 Case 5 - Illustrating the Rolling Horizon function

Figure 4.5 illustrates the rolling horizon capabilities that are incorporated in the formulation. The case continues the next 24 hours following Figure 4.3. Three converters have had the first four charges completed and so Figure 4.5 begins by completing the the final charge and casting step for the first three converters. In completing a charge it is also assumed the any corresponding blows and skims have also been completed. According to the ordering scheme presented in Table 3.1 we assign the converter with unfinished cycles as converters one through three. Based on the specified ordering, the optimization automatically assigns the converters to the set notation discussed in Table 3.2. Completion of the relevant charges is indicated using the $P_{i,j'}$ parameter, which is used in Equation (3.69) to specify that the next unfinished converter charge should begin before all other charges on the next converter in the queue. The same flow rate in Figure 4.3 was used in this case. Furthermore, the ending inventories in Figure 4.3 were used as the initial conditions in the case. The resulting optimization was able to complete the unfinished cycles, complete three entire cycles, and initialize two partial cycles. According to the rolling horizon strategy, the furnace inventory from end of the horizon would be used as the initial inventory for the next period; the converters would be re-sequenced based on the ordering algorithm and the appropriate $P_{i,j'}$ parameters would be set to either zero or one, based on the converter progression. For example, converter three would be re-designated as converter one with four charges completed and converter four would re-designated as converter two with one charge completed. Converter one and two would become converter three and four, respectively. Any changes in the availability of CuWa, emission restrictions, or the flow rate limits would likewise be updated and the optimization would be re-computed.

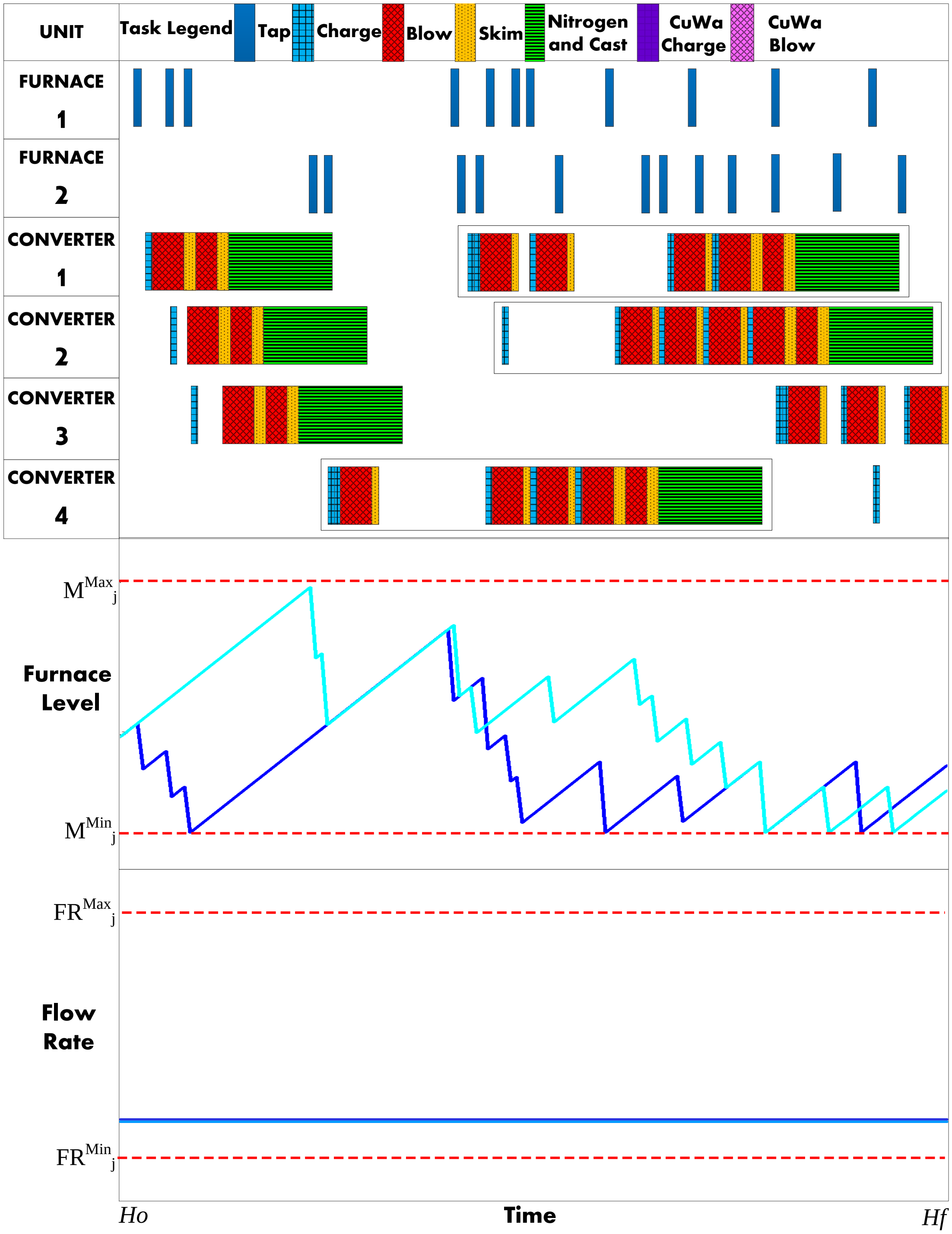

Figure 4.5: Case 5. Time horizon 24 h; Constant flow rate specified; Copper washout unavailable; No blowing restriction; Tiered optimization is applied.

Table 4.1 and Table 4.2 summarize the computation required to solve each problem to optimality and the problem size of each case study, respectively. Table 4.3 summarizes the furnace capacities and the number of ladles each charge removes from the furnaces.

Table 4.1: Computational performance

Case Study	CPU Time (s)	BB Nodes	Simplex Iterations
1	3,556.59	48,104	4,044,552
2	14,516.6	232,277	15,865,124
3	250.522	11,617	217,572
4	1,639.54	26,418	1,777,187
5	503.524	8,438	280,853

Table 4.2: Case study optimization characteristics

Case Study	Binary Variables	Continuous Variables	Constraints
1	1,310	613	3,790
2	1,268	577	3,626
3	1,320	625	3,833
4	1,320	625	3,845
5	1,596	484	4,116

Table 4.3: Parameter values

I_{Chg}	$\rho_{i,j}$
1,2	3
3,4,5	2
M^{Min}	M^{Max}
2	12

Chapter 5

Reactive Scheduling

The scheduling previously presented has been performed in an open loop manner. In this chapter a reactive scheduling approach is presented in which scheduling is performed in a closed loop setting by using real time event triggers and plant states to correct and adapt schedules when disturbances impact the plant. The proposed methodology for reducing radical scheduling changes uses a tiered optimization approach that progressively increases the degrees of freedom available as required, in order to achieve a feasible production schedule.

The reactive scheduling procedure that will be outlined is meant to be event-driven rather than being employed at every sampling instance. In solving an MILP scheduling problem with symmetry, it is possible to obtain radical changes in sequencing and event timing without a change in the objective function. This artifact of optimization leads to difficulty in plant implementation that could lead to plant operators being required to unnecessarily follow production plans that would otherwise require far less modification and operator intervention. For this reason, the system is meant to be event-driven so as to avoid modifying the schedule when it is unnecessary. That is to say, that as long as the plant-model mismatch is tolerable, the schedule should not be re-computed. Thus, it is desirable for the reactive scheduling to modify schedules as little as possible in an attempt to sustain feasible operation. Furthermore, depending on the sampling rate, the computational load required to accommodate a re-scheduling approach at each sampling time may be too great to allow for a feasible implementation.

5.1 Closed loop Scheduling Paradigm

The reactive scheduling is meant to be used much like model predictive control. The formulation presented allows for a predicted schedule and predicted production level to be obtained where event timing and unit sequencing, as well as, flow rates substitute a sequence of input moves. Periodically, the true plant schedule and expected plant schedule should be compared to determine whether they meet a tolerance that has been defined a priori. One metric that might be compared is whether the predicted event times are within tolerance of their actual execution. Another metric might include whether the furnace inventories match the scheduling prediction to within a tolerance. Other conditions that might necessitate a complete re-optimization might include: whether the expected unit sequencing has changed, a change in the emission limit, or a process unit failure. The realization of these events can then be accounted for through re-optimization in order to transition production schedules in an efficient and agile manner. Using real-time measurements of plant states and production conditions for use in re-optimization constitutes the closed-loop reactive scheduling paradigm.

Figure 5.1 describes the reactive scheduling framework in the context of a block diagram where d denotes a measured disturbance, u denotes the optimally computed inputs, y denotes the plant output, and $y_{predicted}$ denotes the predicted outputs. The objective of the controller is to achieve a state of optimal production levels which is sent to the scheduling optimization. The scheduling optimization functions as a controller in terms of computing event timing and unit sequencing which are meant to be implemented in the plant. The scheduling optimization is able to use real-time plant states and information regarding disturbances to compute optimal production level. Upon implementation, event triggers are periodically monitored to determine whether the scheduling optimization is required to take corrective action.

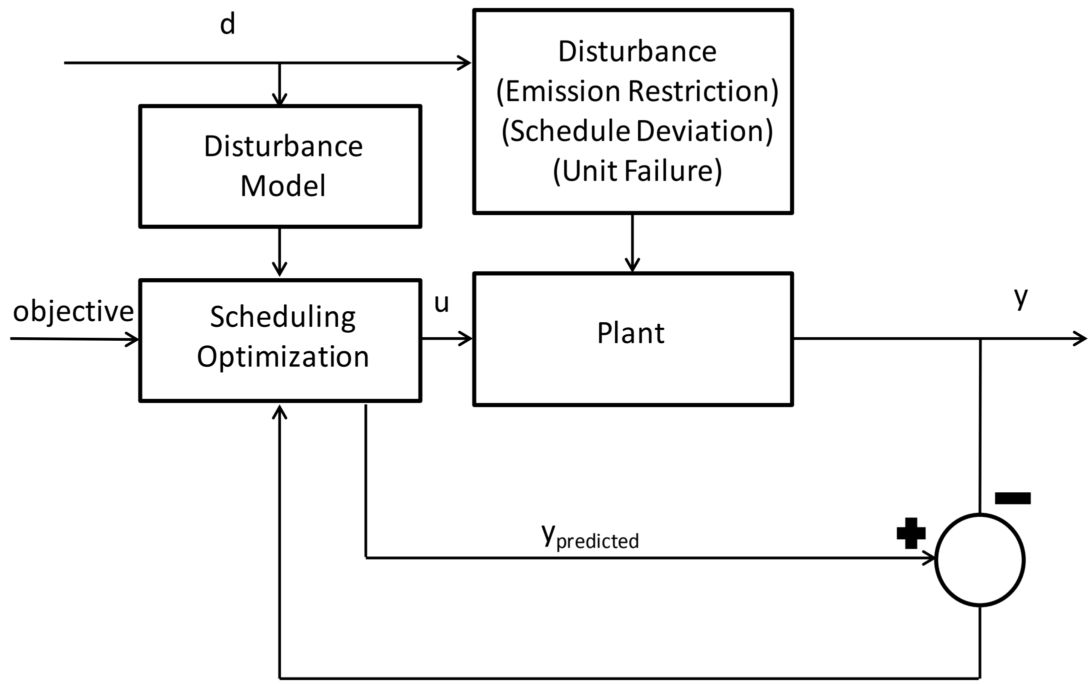

Figure 5.1: Scheduling optimization block diagram.

Using a closed loop reactive scheduling approach has many benefits. For example, the reactive scheduling has improved disturbance rejection properties for disturbances that are significant enough to measure, such as in the case of a change in the emission limits. Furthermore, reactive scheduling is able to compensate for when the plant-model mismatch is significant enough to warrant re-optimizing. The reactive scheduling is also meant to be used as a tool to account for uncertainty. Sources of uncertainty include the previously discussed plant-model mismatch, unit failure, and parameter uncertainty.

5.1.1 Reactive Scheduling Algorithm

A reactive scheduling algorithm is presented here which takes advantage of the the structure of the continuous-time formulation. The reactive scheduling is exploited by warm starting the optimization with the previous solution and initially re-optimizing a small subset of the degrees of freedom available. The degrees of freedom are subsequently increased in the optimization until a feasible schedule is obtained. Using this strategy should result in re-optimized schedules that deviate from a given nominal schedule in only as much is required to obtain feasibility.

A three step approach is employed. The reactive scheduling algorithm is illustrated in Figure 5.2. After obtaining a solution and detecting a fault, step two records the time at which a fault has taken place, as well as, an input from the user as to how much time is required before production can resume. The subset of the optimization that constituted the nominal schedule up to the current time is parameterized and effectively fixed. This, in effect, assumes that the schedule up until the time the fault was realized has been implemented and is now considered to be history. The subset of the optimization solution that follows the realization of the fault is now re-optimized where the flow rates and furnace inventories constitute degrees of freedom. Step two allows for the user to manually shift the schedule by a specified amount. As long as event timings do not exceed the length of the time horizon and the furnace capacity limits are not violated, the original production level should be maintained and a feasible schedule should be obtained. In step two Eqs. (3.70), (3.30), and (3.31) are relaxed to help find a feasible solution.

When re-optimization of the furnace flow rates and inventories is not sufficient to find a feasible solution, the event timings of each process unit are added as degrees of freedom in the optimization. Thus, step three includes more degrees of freedom than are available in step two. For example, should a user specify a start time that delays the production schedule in a way that exceeds the current time horizon in step two, an infeasible message should result. Upon the realization of an infeasible message, the optimization should progress to step three in the search to produce a feasible schedule. Step three should also maintain the nominal production level due to the fact that the assignment of binary variables and sequencing remains fixed during steps two and three.

When re-optimization of the event timings is insufficient in obtaining a feasible schedule, the rolling horizon function is employed to generate a new schedule that is complete with new unit sequencing and a new production level. In this case, the current furnace inventories, and completed converter charges are sent to the optimization to re-compute an entirely new schedule. Step one employs all of the degrees of freedom in the formulation to facilitate production plans throughout the duration of plant operation.

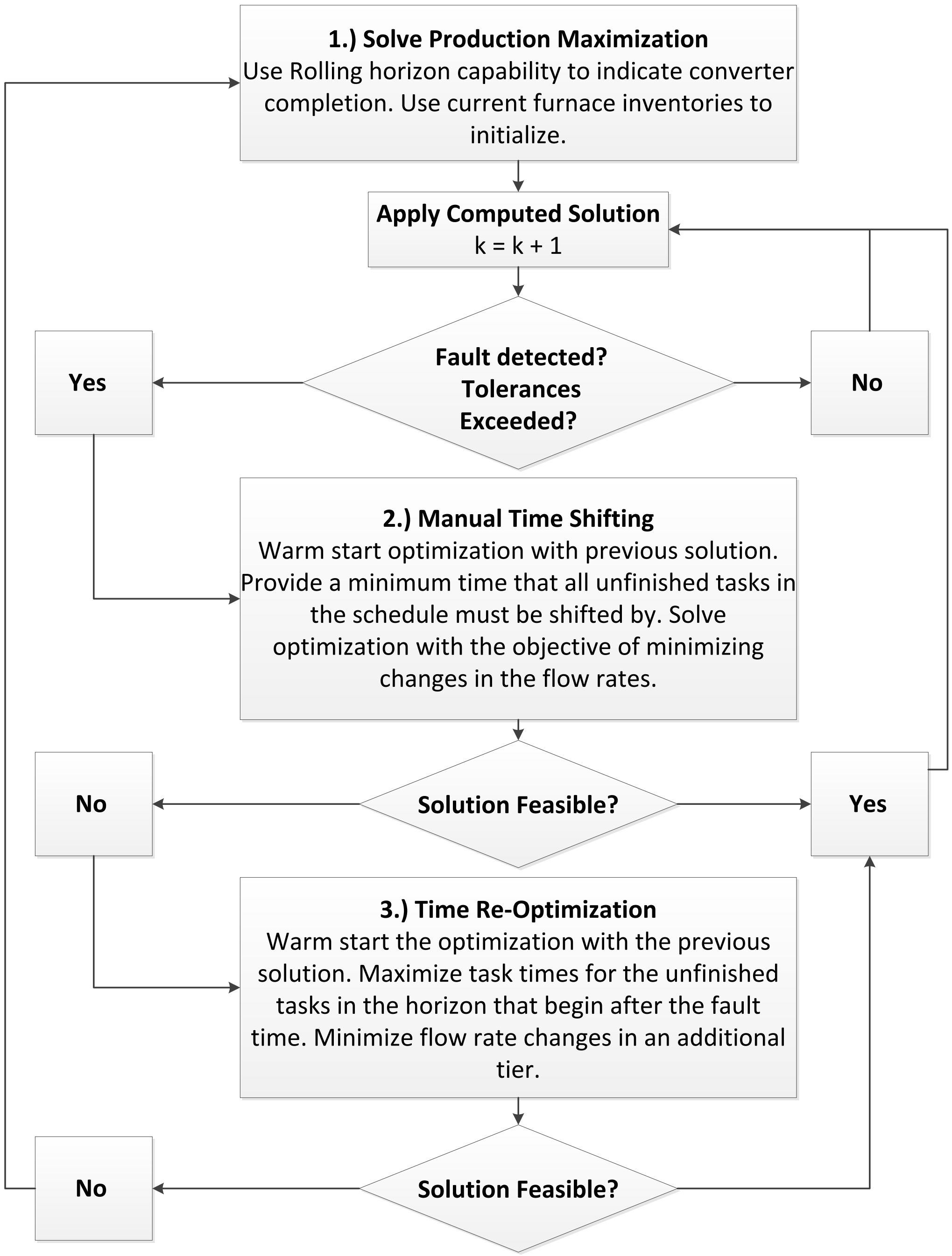

Figure 5.2: Reactive scheduling algorithm.

This approach is not possible using a discrete-time formulation as event timings and sequencing are intrinsically implied by the binary variables, whereas event timings and sequencing are somewhat more dissociated from the binary variables, by comparison, in continuous-time formulations. Suffice to say that, one set of variables are typically used to indicate event timings, unit sequencing, and production assignment in discrete-time formulations, whereas continuous-time formulations employ additional variables to represent the same aspects, thus allowing for the possibility of the re-optimization strategy presented.

5.2 Reactive Scheduling Case Studies

Considering twenty-four hour time horizons for real-time application is too computationally burdensome to implement in real-time for a configuration of two furnaces and four converters. Considering a horizon of twelve hours however, results in a reduced problem size that is consistently able to solve within very reasonable time frames and indicates the potential for plant applicability. For this reason, this chapter will focus on solving twelve hour time horizons.

The following cases study a nominal schedule which experiences a fault during the course of implementation in the time horizon. The cases that follow will illustrate the use of the reactive scheduling algorithm presented and the efficacy of the algorithm in recovering a schedule that has been disrupted by a fault. The cases will illustrate conditions where it sufficient to re-optimize only a subset of the degrees of freedom available in the optimization and conditions where it is necessary to increase the degrees of freedom until a feasible solution is obtained.

5.2.1 Nominal Schedule

The nominal schedule in Figure 5.3 illustrates the case where both furnaces are able to use the entire range of flow rates and a restriction on emission leads to a maximum of two converters blowing at a time. The tiering procedure presented in Figure 3.7 is used to minimize task start times and suppress moves in the furnace flow rates. At time 125 min we assume that an event occurs where the schedule implementation is detected to have deviated from the original schedule, and the reactive scheduling algorithm is called to adjust the event timings accordingly.

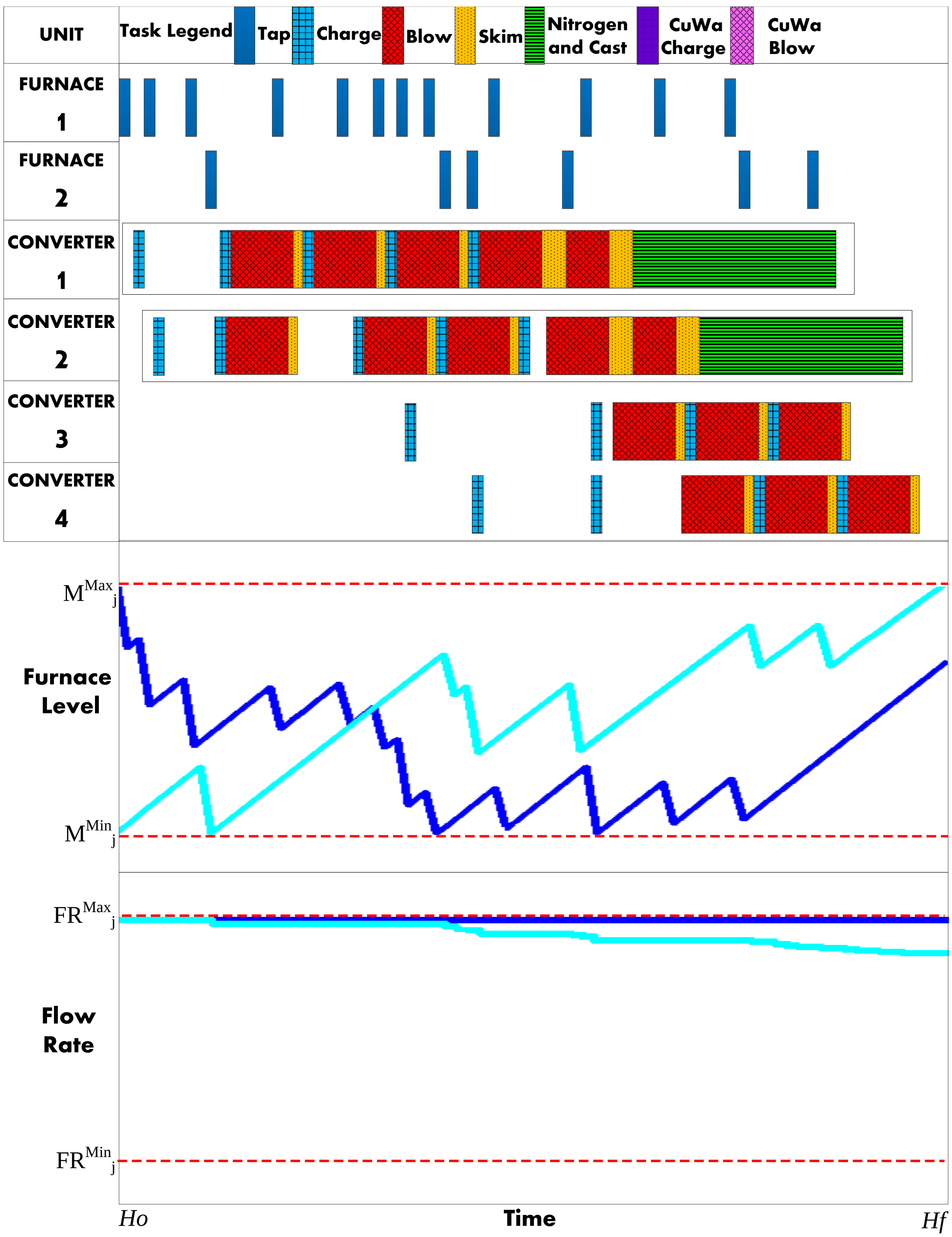

Figure 5.3: Nominal schedule, Time horizon 12 h; Variable flow rate available; Copper washout unavailable; Blowing limit restricts production to only two converters blow at most at a time; Tiered optimization is applied.

5.2.2 Time Shifting

In Figure 5.4, the user is able to specify that the event occurring after time 125 min must each be shifted by exactly 40 min. The previous solution is used to warm start the optimization and event timings occurring up to and including time 125 min are fixed. All of the binary variables are also held fixed, such that the production level is identical to that of the nominal schedule. Finding a feasible solution that allows for the event timings to be shifted by 40 min requires that the total horizon length is increased by 60 min. In this case the furnace inventories and flow rates are degrees of freedom where the objective function in this case is the minimization of flow rate changes. As a result, the furnace inventory and inlet flow rates differ than those in Figure 5.3. The extension of the total time horizon allows the optimization to find a feasible solution without sacrificing production. However, if the total horizon length was not increased, then shifting the tasks by 40 minutes would result in an infeasible result. At this point the reactive scheduling algorithm would move to the next stage and consider the event timings after 125 min to be additional degrees freedom.

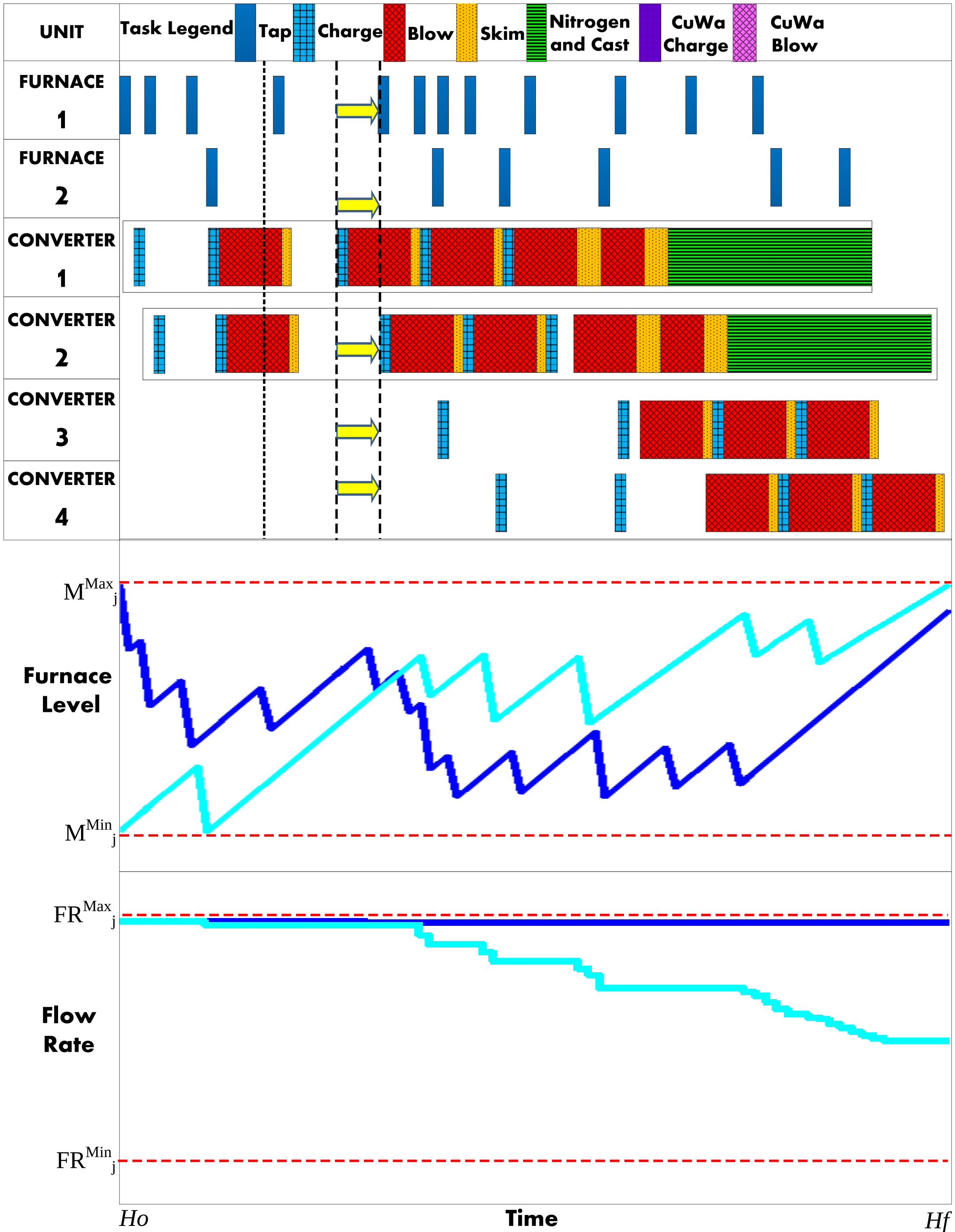

Figure 5.4: Time shifted schedule.

5.2.3 Time Re-optimization

In Figure 5.5, the event timings after time 125 min are treated as degrees of freedom. However, the integer variables remain fixed, as well as, the events preceding time 125 min. The objective in this optimization is to maximize start times, followed by an additional tier of penalizing flow rate changes. The start times are maximized in order to give the operators the maximum amount of time remaining in the horizon to recover and complete the schedule. The time re-optimization exploits the slack time in the horizon in order to complete the production schedule. The user specifies that the unstarted tasks following time 125 must begin no sooner that 28 min after time 125. The optimization was able to delay the schedule through re-optimization of the event timings, such that a feasible solution was obtained. If the user specified a time greater than 28 min, the optimization would be unable to recover a feasible solution. At this point the algorithm would resort to using the current furnace inventories and converter charge indicators to complete a total re-optimization. Cases that require, complete re-optimization include an increasingly restrictive change in the emission limits, significant deviations in the furnace inventory from the predicted inventory, unit breakdowns, and the on-time or early completion of the entire production schedule.

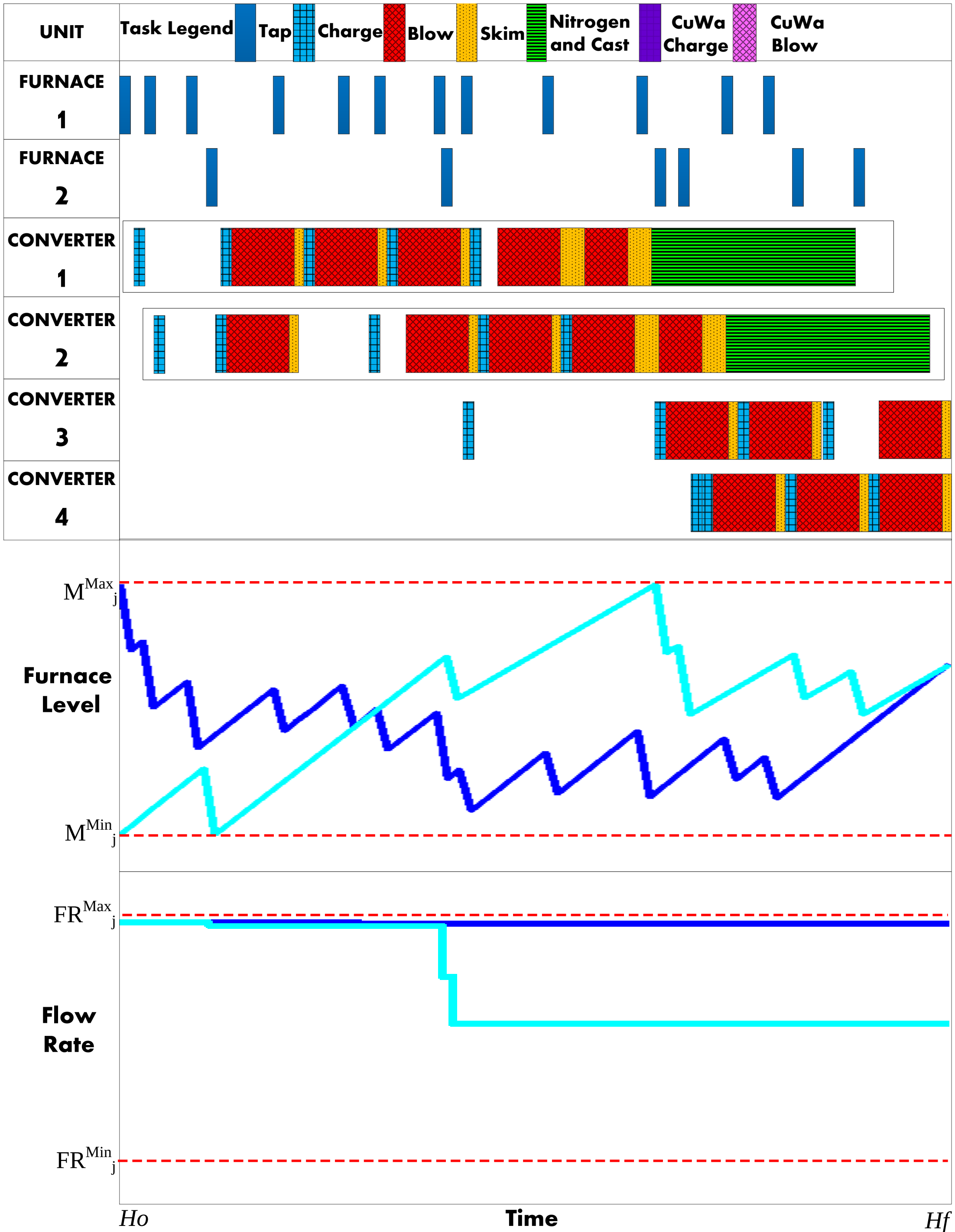

Figure 5.5: Time re-optimized schedule.

5.2.4 Full Re-optimization

In Figure 5.6, an increasingly restrictive emission limit is imposed on production which admits only one converter to blow at a maximum, at a time from time 125 onward. This disturbance requires complete re-optimization. The re-optimization uses the entire tiering procedure to improve the resulting schedule. The furnace inventories at time 125 are used to re-initialize the optimization. The indicator parameters are used to indicates that the first two charges on converter one and two have been completed.

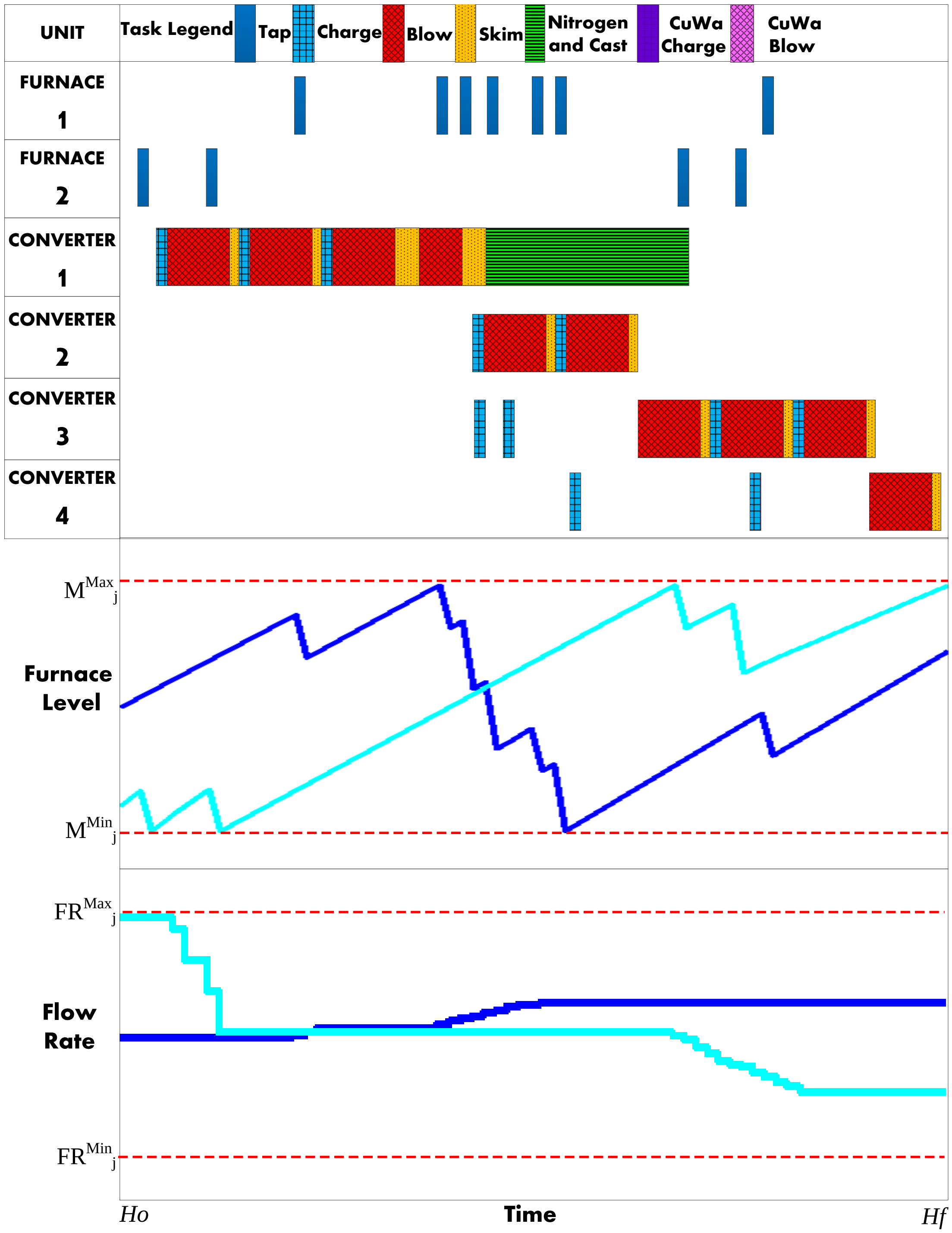

Figure 5.6: Fully re-optimized schedule.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

A novel continuous-time slot-based MIP representation for the scheduling of the converter aisle of a nickel smelting plant is presented. The formulation includes semi-continuous modeling of the furnaces as well as converter event timing and sequencing. The formulation accounts for the copper washout operation, blowing emission restriction, and includes a rolling horizon feature. Tightening constraints are developed and applied to improve the computational performance of the optimization to demonstrate the efficacy of the optimization as a real-time decision support tool. A methodology for applying a tight upper bound on production is achieved using an approximation of the two furnaces with an aggregated equivalent capacity is presented, with a solution enhancing procedure that uses a multi-tiered optimization.

A reactive scheduling algorithm was presented which is meant to adapt schedules when disturbances disrupt plant operation. The reactive scheduling algorithm uses a subset of the decision variables in an attempt to reduce significant changes to the schedule and progressively increases the degrees of freedom used until a suitable schedule can be found.

The combined rolling horizon formulation and reactive scheduling constitute a real-time decision support tool to maximize production efficiency in the nickel smelting plant.

6.2 Recommendations for Further Work

1. **Consideration of Uncertainty.** There exists, potentially, a significant degree of parameter uncertainty that can contribute to plant-model mismatch. Furthermore, due to the inherently stochastic nature of the plant, explicit consideration of uncertainty using robust optimization would undoubtedly improve decision-making in the plant. Two strategies to incorporate uncertainty in the formulation include the use of chance constraints or a two-stage stochastic optimization. Using stochastic optimization has the consequence of increasing the problem size and therefore the computational effort required to solve the optimization. Chance constraints might however introduce nonlinearity to the formulation, also causing challenges in achieving a solution efficiently and in drawing conclusions regarding global optimality.

2. **Application of Decomposition Algorithms.** Solving a configuration of two furnaces and four converters over a 24 hour time horizon can take significant effort in many cases. This might warrant investigation into decomposition algorithms to improve the computational time. One algorithm that might potentially be applicable is Lagrangean decomposition as it has been applied to unit-event based continuous-time formulations in [Wu and Ierapetritou, 2003]. Novel modification of this decomposition technique may yield computational improvements.

Nomenclature

Indicies/Sets

$i, i' \in I_{Tap}$	Task for the timing of FFM removal from a Furnace
$i, i' \in I_{Chg}$	Task for FFM removal from a Furnace during a Tap and addition of FFM in a Converter and the timing for the corresponding Charge
$i, i' \in I_{Blow}$	Task for the timing of Converter Blows
$i, i' \in I_F$	Task for the timing of the Finishing Blows, Nitrogen Blanket, and Casting
$i \in I_{ConverterTasks}$	An ordered set containing all the tasks performed in a cycle, including the charges, blows, skims, a finishing blow, a nitrogen blow, and casting
$j, j^* \in J_F$	Furnaces present in the Plant
$j' \in J_C$	Converters present in the Plant
$z \in Z$	Queuing Order Applied to Converters
$j' \in J_z$	Set of Converters indexed by z for use as a converter queuing system
$o \in O$	Number of Converter Cycles per Converter
$o \in O_z$	Set of Converter Cycles indexed by z for use as a converter queuing system
$n, n' \in N$	Event points
$n' \in N_z$	Event points where queue z is not permitted to exist

Binary Variables

$x_{i,j,j',o,n}$	1 if FFM is transferred during the task of Tapping and the corresponding Charge i from Furnace j to Converter j' during Converter sequence o at event point n and 0 otherwise
$w_{j',o}$	1 if a Ladle of Copper Washout is transferred to Converter j' during Converter cycle o at event point n and 0 otherwise
$y_{i,j',o}$	1 if Charge i on Converter j' during Converter sequence o is performed and 0 otherwise

Continuous Variables

$T^{S,Furn}_{i,j,n}$	Start Time of Tap i on Furnace j at event point n
$T^{F,Furn}_{i,j,n}$	End Time of Tap i on Furnace j at event point n
$T^{S,Conv}_{i,j',o}$	Start Time of task i on Converter j' during cycle o
$T^{F,Conv}_{i,j',o}$	End Time of task i on Converter j' during cycle o

$M^S_{j,n}$	Inventory of Furnace j at the start of event point n
$M^F_{j,n}$	Inventory of Furnace j at the end of event point n
M^H_j	Inventory of Furnace j at the end of the time horizon
$B^S_{j,n}$	Amount of FFM added to Furnace j between the end of event point $n-1$ and the start of event point n
$B^F_{j,n}$	Amount of FFM added to Furnace j between the start and end of event point n
B^H_j	Amount of FFM added to Furnace j between the end of the last event point and the end of the time horizon
$FR^S_{j,n}$	Flow rate into Furnace j between the end of event point $n-1$ and the start of event point n
$FR^F_{j,n}$	Flow rate into Furnace j between the start and end of event point n
FR^H_j	Flow rate into Furnace j between the end of the last event point and the end of the time horizon

Parameters

$\gamma_{i,o}$	Weighting of task i for cycle o in the objective function
$\alpha_{i,j}$	Duration of task i for unit j
α_{blow}	Duration of a regular blow
$\alpha_{missblow}$	Duration of a miss blow
α_{skim}	Duration of a type 1 skim
α_{skim2}	Duration of a type 2 skim
α_{NFC}	Duration of the finishing blows, nitrogen blow, and the casting$j$
α_{CW}	Duration of Blow with Copper Washout i for unit j
q_{StrC}	A positive integer parameter describing the number of available converters available for operation
H_0	Beginning of the Time Horizon
H_f	End of the Time Horizon
$M_j(0)$	Initial Inventory of Furnace j at time H_0
M^{Min}_j	Minimum Inventory Limit of Furnace j
M^{Max}_j	Maximum Capacity of Furnace j
$FR^{S,Min}_{j,n}$	Minimum Flow Rate into Furnace j between the end of event point $n-1$ and the start of event point n

$FR_{j,n}^{S,Max}$	Maximum Flow Rate into Furnace j between the end of event point $n-1$ and the start of event point n
$FR_{j,n}^{F,Min}$	Minimum Flow Rate into Furnace j between the start and end of event point n
$FR_{j,n}^{F,Max}$	Maximum Flow Rate into Furnace j between the start and end of event point n
$FR_{j}^{H,Min}$	Minimum Flow Rate into Furnace j between the end of the last event point and the end of the time horizon
$FR_{j}^{H,Max}$	Maximum Flow Rate into Furnace j between the end of the last event point and the end of the time horizon
$\phi_{j,n}^{S}$	A binary parameter that assumes the value of 1 if the duration of time on Furnace j between the end of event point $n-1$ and the start of event point n is greater than zero and 0 otherwise
$\phi_{j,n}^{F}$	A binary parameter that assumes the value of 1 if the duration of time on Furnace j between the start and end of event point n is greater than zero and 0 otherwise
ϕ_{j}^{H}	A binary parameter that assumes the value of 1 if the duration of time on Furnace j between the end of the last event point and the end of the time horizon is greater than zero and 0 otherwise
$\rho_{i,j}$	Number of Ladles that charge i removes from furnace j
DT	The time by which subsequent taps must be offset across all furnaces
$Stndby$	The time by which subsequent cycles must be offset on the same converter
$Cast$	The time by which subsequent converter castings must be offset across all converters
$Trnsfr^{Min}$	Minimum Time that a Converter Charge must begin after a Furnace Tap j
$Trnsfr^{Max}$	Maximum Time that a Converter Charge must begin after a Furnace Tap j
$MaxTime$	Maximum Time that a Converter cycle can occupy
$P_{i,j'}$	A binary parameter that takes on the value 1 if Charge i has been performed on Converter j' during converter cycle one in a previous time period and 0 otherwise
L^{CW}	A positive integer parameter that specifies the number of ladles of copper washout available and 0 if copper washout ladles are unavailable
T^{CW}	A parameter that specifies when ladles of copper washout will be made available for allocation to converters

l^{Max}	A positive integer parameter specifying the number of Converters allowed to blow simultaneously based on the emission limits

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Appendix A

Precursor Formulation

A.1 Introduction

The following formulation presents a simplified process model that was used as the precursor to the scheduling optimization presented. The precursor formulation differs from the main formulation in many ways. For example, the precursor formulation is based on a unit-specific event-based continuous-time formulation whereas, the main formulation is a slot-based continuous-time formulation. Furthermore, the precursor is configured specifically for one furnace and two converter whereas, the main formulation can accommodate a general number of converters and furnaces. Also, the blowing constraints are modeled using a combination of timing and binary variables whereas, the main formulation uses a combination of timing constraints and the symmetry-breaking precedence scheme to represent the blowing restrictions. Another significant difference is how the precursor formulation considers only a single charge at the beginning of the cycle whereas, the main formulation considers several charges during the course of the cycle. Several charges during the course of the cycle represents the true batch recipe more accurately. The following work is published in [Ewaschuk *et al.*, 2013].

A.1.1 Objectives

This section poses a continuous-time formulation based on the formulation by Ierapetritou and Floudas (1998a,b) that is applied to a simplified industrial process. The objective of this work is to develop a rigorous optimization formulation in order to maximize a given objective function, while satisfying furnace capacity constraints, and obeying environmental regulations on blowing activities through optimal scheduling.

A.2 Formulation

In the scheduling formulation i is the index for tasks within the set of available tasks i ∈ I, j is the index for available units in the plant j ∈ J, n is the index for event points in the given time horizon n ∈ N, and s is the index for the states that can be produced or consumed in terms of material in the plant s ∈ S. The set J_i denotes the set of units which are able to perform task i. The variable $w_{i,j,n}$ is binary and is used to describe whether task i is scheduled on unit j at event point n. If a task is scheduled, $w_{i,j,n}$ is equal to one and is zero otherwise.

A.2.1 Task Assignment

Equation (A.1) states that once a furnace is tapped the FFM must be transferred to charge a converter at every event point. The left hand side represents furnaces and the right hand side represents converters. I_j defines the set of tasks that can be performed on unit j. Set I_{tap} and I_{chg} are tapping and charging tasks. J_f and J_c The reader is referred to Bemporad and Morari (1999) and Mitra et al. (1994) for an extensive discussion on formulating logic constraints and embedding these constraints within optimization or control applications.

$$\sum_{j \in J_f} w_{i,j,n} = \sum_{j' \in J_c} w_{i',j',n} \; \forall \; i \in I_{tap}, \; i' \in I_{chg}, \; n \in N \tag{A.1}$$

$$w_{i,j,n} = w_{i',j,n} \; \forall \; j \in J_c, \; i \in I_{chg}, \; i' \in I_j \tag{A.2}$$

If a converter is selected to be charged in Equation (A.1), then all the remaining tasks that are performed during a blowing cycle must also be performed, according to Equation (A.2). If a converter is not selected for charging then the decision variable representing that task will be equal to zero and by Equation (A.2) all other tasks must also be equal to zero on that unit.

A.2.2 Production Constraints

The continuous variable, $B_{i,j,n}$ represents the batch size produced by task i, on unit j, at event point n. Batch size production is limited by the maximum and minimum capacity, denoted by $V_{s,i,j}^{max}$ and $V_{s,i,j}^{min}$, respectively. Furthermore, the batch size is limited by the decision to produce, such that when the optimization decides not to produce, represented by zero, the batch size will be driven to zero. These constraints are enforced in Equation (A.3). The set I_s contains the set of tasks that produce state s. S_{OfG} is the set of states for off-gas.

$$w_{i,j,n}V_{s,i,j}^{min} \leq B_{i,j,n} \leq w_{i,j,n}V_{s,i,j}^{max}\ \forall\ s \in S,\ i \in I_s,\ j \in J_i,\ n \in N \tag{A.3}$$

The batch size of off-gas produced must be treated using an alternate method that captures the continuous nature of this task. Off-gas is produced in a continuous manner, rather than discretely such that the constant blowing rate BR, multiplied by the duration of the blowing task yields the amount of off-gas produced in Equation (A.4). The continuous variables $T_{i,j,n}^{s}$ and $T_{i,j,n}^{f}$ represent the start and finish times of task i, on unit j, at event point n, and in this case represent the blowing task on the converters.

$$B_{i,j,n} = BR(T_{i,j,n}^{f} - T_{i,j,n}^{s})\ \forall\ s \in S_{OfG},\ i \in I_s,\ j \in J_i,\ n \in N \tag{A.4}$$

A.2.3 Mass Balance

The continuous variable $M_{s,n}$ is used to track the quantity of state s available at event point n. S_{nFFM} is the set of states that exclude FFM. Equation (A.5) outlines that quantity of material at each event point is equal to the quantity at the previous event point in addition to the quantity of material generated in each batch at the current event point. When written for the first event point, the quantity at the previous event point corresponds to the initial inventory.

$$M_{s,n} = M_{s,n-1} + \sum_{i \in I_s} \sum_{j \in J_i} B_{i,j,n} \; \forall \; s \in S_{nFFM}, \; n \in N \tag{A.5}$$

Equations (A.6) through (A.10) are required to model the semi-continuous nature of the furnace. Furnace operation consists of a continuous charge over the entire horizon and includes the drawing of FFM from the furnace as a discrete task. Equation (A.6) is used to model the continuous flow into the furnace from the start of the horizon to the end of the first tap, whereas Equation (A.7) is used to track the material inventory at the end of each tap for the interior event points. Equations (A.8) and (A.9) are used to track material inventory at the beginning of the taps using the variable $M^{st}_{s,n}$. Equation (A.10) is used to track the material accumulated at the end of the horizon. The parameters H_i and H_f define the time horizon over which the schedule is to be generated. S_{FFM} is the set of states that contains FFM. The parameter FR denotes the flow rate at which the furnace is filled. It is assumed in this study that the quantity of material transferred from the furnace is equivalent to the size of the batches in the converters.

$$M_{s,n} = M_{s,n-1} - B_{i,j,n} + (T^{f}_{i,j,n} - H_i)FR \; \forall \; s \in S_{FFM}, \; i \in I_s, \; j \in J_i, \; n = 1 \tag{A.6}$$

$$M_{s,n} = M_{s,n-1} - B_{i,j,n} + (T^{f}_{i,j,n} - T^{f}_{i,j,n-1})FR \; \forall \; s \in S_{FFM}, \; i \in I_s, \; j \in J_i, \; n \neq 1 \tag{A.7}$$

$$M^{st}_{s,n} = M_{s,n-1} + (T^{s}_{i,j,n} - H_i)FR \; \forall \; s \in S_{FFM}, \; i \in I_s, \; j \in J_i, \; n = 1 \tag{A.8}$$

$$M^{st}_{s,n} = M^{st}_{s,n-1} - B_{i,j,n-1} + (T^{s}_{i,j,n} - T^{s}_{i,j,n-1})FR \; \forall \; s \in S_{FFM}, \; i \in I_s, \; j \in J_i, \; n \in N \; : \; n \neq 1 \tag{A.9}$$

$$M^{end}_{s} = M^{st}_{s,n-1} + (H_f - T^{f}_{i,j,n-1})FR \; \forall \; s \in S_{FFM}, \; i \in I_s, \; j \in J_i, \; n = N \tag{A.10}$$

A.2.4 Storage Capacity Constraints

Equation (A.11) specifies that the amount of state s at the end of a tap must be between the minimum and maximum storage capacity and is used for all units.

$$M_s^{min} \leq M_{s,n} \leq M_s^{max} \; \forall \; s \in S, \; n \in N \tag{A.11}$$

Due to the semi-continuous behaviour of the furnace, Equation (A.11) is not sufficient to constrain furnace inventory within the furnace capacity limits. Equation (A.12) constrains the mass balance upon initiation of taps to maintain the material level within the capacity limits. Together, Equations (A.11), (A.12), and (A.13) ensure that taps are spread across the horizon to satisfy the furnace capacity limits at all times.

$$M_s^{min} \leq M_{s,n}^{st} \leq M_s^{max} \; \forall \; s \in S_{FFM}, \; n \in N \tag{A.12}$$

$$M_s^{min} \leq M_s^{end} \leq M_s^{max} \; \forall \; s \in S_{FFM} \tag{A.13}$$

A.2.5 Task Duration

The parameter $\alpha_{i,j}$ defines the duration that task i requires on unit j for completion in Equation (A.14). If task i is scheduled on unit j, at event point n, then the completion time becomes the summation of the start time and the task duration. Otherwise, the start and finishing times of a task are equivalent when the binary decision variable $w_{i,j,n}$ is zero. Though the start and finish times must be allocated by the optimization, if the task duration is zero then the task effectively does not occur, will not consume time within the horizon, nor will the task contribute to material production or consumption.

$$T_{i,j,n}^{f} = T_{i,j,n}^{S} + \alpha_{i,j} w_{i,j,n} \; \forall \; i \in I, \; j \in J_i, \; n \in N \tag{A.14}$$

A.2.6 Blowing Constraints

Equation (A.15) governs the environmental constraints that apply to blows and SO_2 emission. When the binary variable $w_{i,j,n}$ is one for blowing task i, the associated SO_2 emission must be less than an Atmospheric Pollution Index (API) value; otherwise the blow is not permitted. When blows are scheduled on contiguous event points, they will contribute to the blowing rate BR. When this summation exceeds the API limit the constraint has the option of activating the binary variable

$u_{s,i,i',j,j',n}$ such that the constraint in (16) will be satisfied. The variable $u_{s,i,i',j,j',n}$ tracks whether the instantaneous blowing rate exceeds the permissible environmental limits.

$$w_{i,j,n}BR \leq API \; \forall \; s \in S_{OfG}, \; i \in I_s, \; j \in J_i, \; n \in N \tag{A.15}$$

$$BR(w_{i,j,n} + w_{i',j',n+1} - u_{s,i,i',j,j',n}) \leq API \; \forall \; s \in S_{OfG}, \; i \in I_s, \; j \in J_i, \; i' \in I_s, \; j' \in J_{i'}, \; n \in N, \; j \neq j', \; n \neq N \tag{A.16}$$

When $u_{s,i,i',j,j',n}$ is equal to one, Equation (A.17) will force the blow at event point $n+1$ to be performed after the completion of the blow at event point n. When blows do not exceed the API limit, there is no incentive for the optimization to select $u_{s,i,i',j,j',n}$ to be one. When $u_{s,i,i',j,j',n}$ is equal to zero, it is permissible for blowing times on different units to overlap and Equation (A.17) will be slack.

$$T^s_{i',j',n+1} - T^s_{i,j,n} \geq \alpha_{i,j} u_{s,i,i',j,j',n} - (H_f - H_i)(1 - u_{s,i,i',j,j',n}) \; \forall \; s \in S_{OfG}, \; i \in I_s, \; j \in J_i, \; i' \in I_s, \; j' \in J_{i'}, \; n \in N, \; j \neq j', \; n \neq N \tag{A.17}$$

A.2.7 Sequence Constraints: Same Task in the Same Unit

Equation (A.18) is meant for use on the converters and specifies that the timings of the tasks that are incorporated in a converter cycle must occur in series and must begin immediately following the completion of the previous task in the cycle.

$$T^f_{i,j,n} = T^s_{i+1,j,n} \; \forall \; j \in J_c, \; i \in I_j, \; n \in N, \; i \neq Ic_{finish} \tag{A.18}$$

Ic_{finish} is the last task that occurs in a converter cycle. Equation (A.19) and (A.20) specify that start and finish times must be greater than or equal to the start time and finish time of the task at the previous event point, respectively.

$$T^s_{i,j,n} \geq T^s_{i,j,n-1} \; \forall \; i \in I, \; j \in J_i \; n \in N, \; n \neq 1 \tag{A.19}$$

$$T^{f}_{i,j,n} \geq T^{f}_{i,j,n-1} \; \forall \; i \in I, \; j \in J_i, \; n \in N, \; n \neq 1 \tag{A.20}$$

A.2.8 Equivalent Charging and Tapping times

When a furnace is tapped, a converter must be charged. When a converter is charged, $w_{i',j',n}$ (with i' corresponding to a charging task) will be equal to one such that the start time of the converter charging must be less than or equal to the start time of the tap on the furnace in Equation (A.21). Likewise, when $w_{i',j',n}$ is equal to one the finish time of the charge, $T^{f}_{i',j',n}$, must be greater than or equal to the finish time of the tap, $T^{f}_{i,j,n}$ in A.22. Effectively, A.21 and A.22 force the start and finish times of a converter selected for charging, to be equivalent to the start and finish times of the tap and are slack otherwise.

$$T^{s}_{i,j,n} - T^{s}_{i',j',n} \geq -(H_f - H_i)(1 - w_{i',j',n}) \; \forall \; i \in I_{tap}, \; j \in J_i, \; i' \in I_{chg} \; j' \in J_{i'}, \; n \in N \tag{A.21}$$

$$T^{f}_{i,j,n} - T^{f}_{i',j',n} \geq -(H_f - H_i)(1 - w_{i',j',n}) \; \forall \; i \in I_{tap}, \; j \in J_i, \; i' \in I_{chg} \; j' \in J_{i'}, \; n \in N \tag{A.22}$$

A.2.9 Sequence Constraints: Completion of Previous Tasks

Equation (A.23) is meant for use on the converters and specifies that a new converter cycle may only be started after the previous cycle has completed. Equation (A.24) specifies that the start time of a task must be greater than the finish of the same task at the previous event point for every task, on every unit.

$$T^{s}_{i,j,n} \geq T^{f}_{i',j,n-1} \; \forall \; i \in Ic_{start}, \; i' \in Ic_{finish}, \; j \in J_c, \; n \in N, \; n \neq 1 \tag{A.23}$$

$$T^{s}_{i,j,n} \geq T^{f}_{i,j,n-1} \; \forall \; i \in I, \; j \in J_i, \; n \in N, \; n \neq 1 \tag{A.24}$$

Ic_{start} is the first task that is performed in a converter cycle.

A.2.10 Time Horizon Constraints

Equations (A.25) and (A.26) specify that all start and finish times must be scheduled within the available horizon.

$$H_i \leq T^s_{i,j,n} \leq H_f \; \forall \, i \in I, \; j \in J_i, \; n \in N \tag{A.25}$$

$$H_i \leq T^f_{i,j,n} \leq H_f \; \forall \, i \in I, \; j \in J_i, \; n \in N \tag{A.26}$$

A.2.11 Objective Function

Equation (A.27) maximizes the amount of BM cast within a given horizon. I_{NC} represents the final task in the converter cycle which is the nitrogen cooling and casting.

$$max \sum_{j \in J_i} \sum_{n \in N} B_{i,j,n} \; i \in I_{NC} \tag{A.27}$$

A.3 Case Study

The following case study considers an illustrative example based on a much simplified industrial process, which only consists of one furnace that feeds two converters with no recycle. The furnace is continuously fed with only FFM tapping performed. The converter cycle is simplified to include only initial charge up, first blow, skimming (without additional furnace matte charging), second blow, and nitrogen cooling and casting. Note that matte/slag mass changes due to adding flux and reaction of iron sulphide to iron oxide are not considered in this case study. A process diagram of the case study is shown in Figure A.1. The scheduling problem is modeled with AMPL and solved using CPLEX 12.5 with a solution time on the order of a tenth of a second. The computation was performed using a 2.33 GHz IntelrCoreTM 2 Quad processor with 6 GB of RAM, running Windows 7 Professional 64-bit.

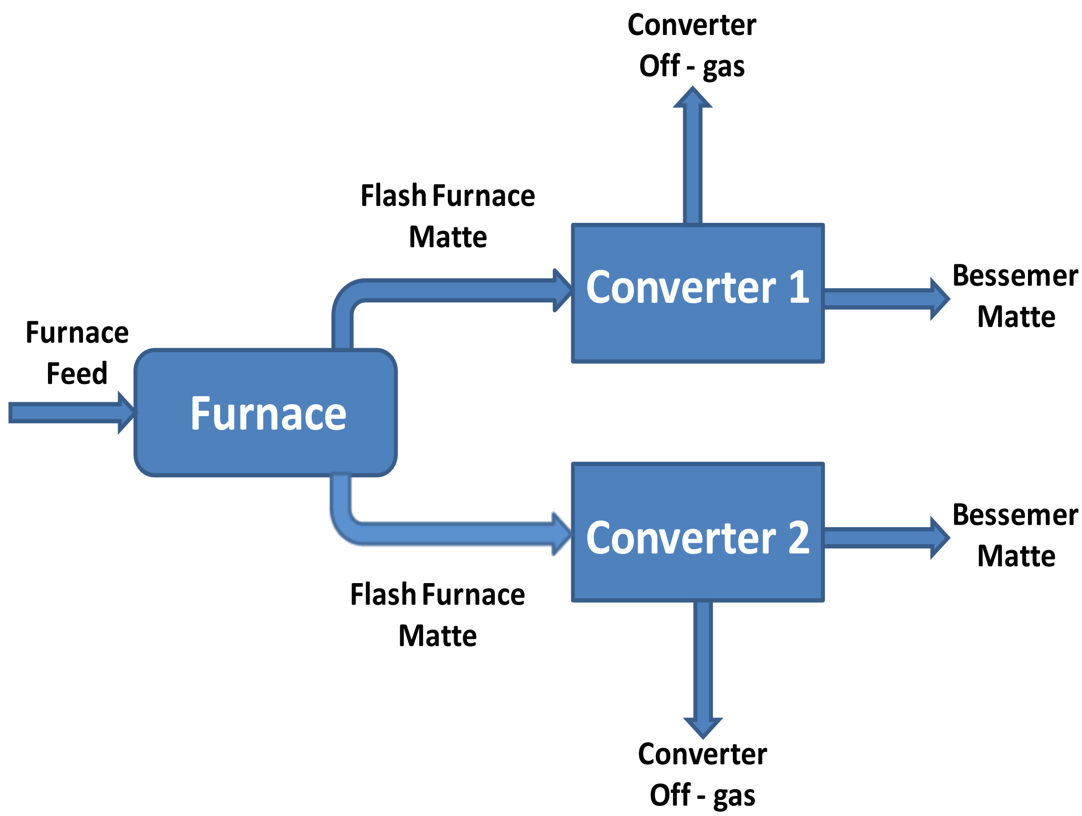

Figure A.1: Process Diagram of Nickel Smelting Case Study.

A.3.1 Case A: Restrictive API limits

In Figure A.2, the Gantt chart demonstrates that converter cycles are initiated by a tap on the furnace and a simultaneous charging on one of the two converters which is enforced by Equation (A.1). The timings of the tap and charge are governed by Equation (A.21) and Equation (A.22). Once a converter cycle begins, each task must be performed following the completion of the previous task in the cycle which is specified by Equation (A.2) and Equation (A.18). In case A the environmental blowing constraints are specified such that performing blows simultaneously on the two converters is not permissible. Nor, is it possible for the timing of the blows to overlap. While performing two blows simultaneously is not permissible, this does not necessarily imply that a blow must be cancelled. Rather, the optimization forces blows to be spread apart across the horizon so as to avoid overlapping. Since the objective is to maximize the quantity of Bessemer matte cast, converter cycles alternate on each unit for each tap to fulfil the objective, while respecting the blowing and casting constraints.

The schedule is generated such that blows are performed on one converter as non-blowing activities are performed on the other. This allows for the maximum production of BM cast over the given time horizon. Converter cycles are not permitted to begin simultaneously on both converters due to the fact that when the furnace is tapped only one converter can be charged during the duration of this task. Therefore, when scheduling the production, the converter cycles must be at least offset by the duration of a tap and a charge, and may be further offset, so as to avoid violating the blowing constraints. Were there an additional furnace available for charging and the API constraints were not restrictive, an additional converter cycle would be possible and the timing of the converter cycle on both converters would be identical. The units would operate in parallel and in-sync.

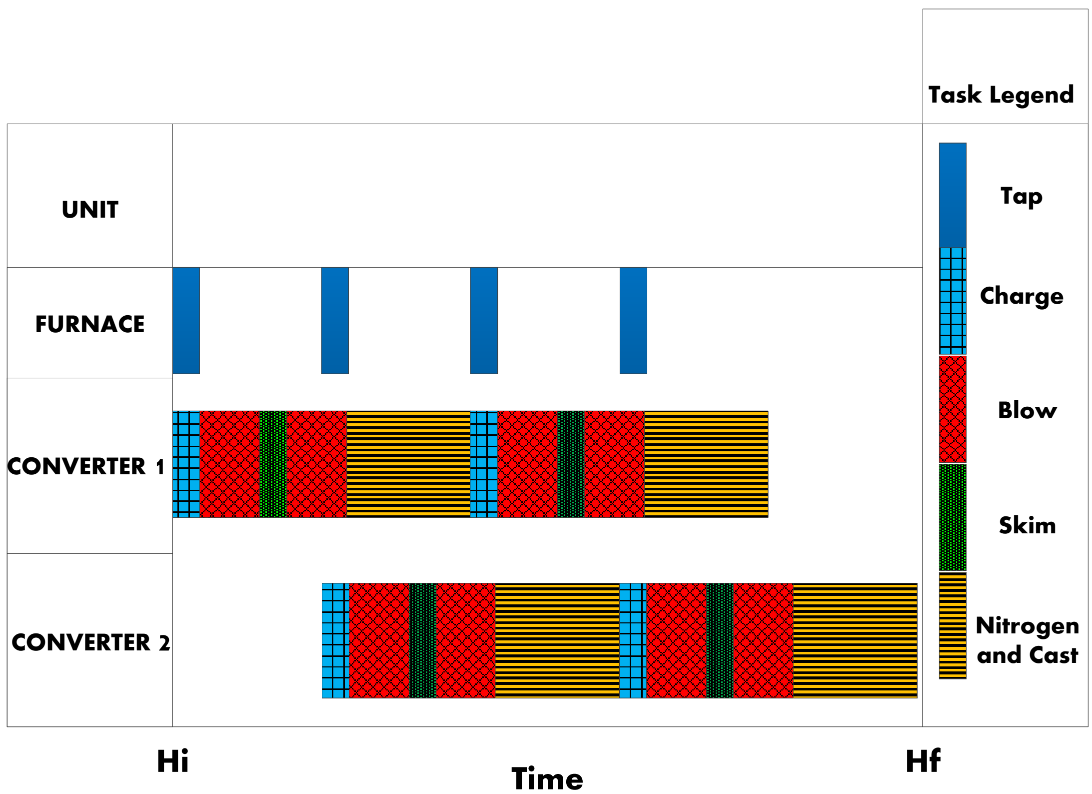

Figure A.2: Gantt Chart for Restrictive API constraint.

Figure A.3 contains the instantaneous blowing profiles over the time horizon. The dashed line represents the current upper bound on allowable blows. The plot illustrates that only a single blow may be performed at any given time.

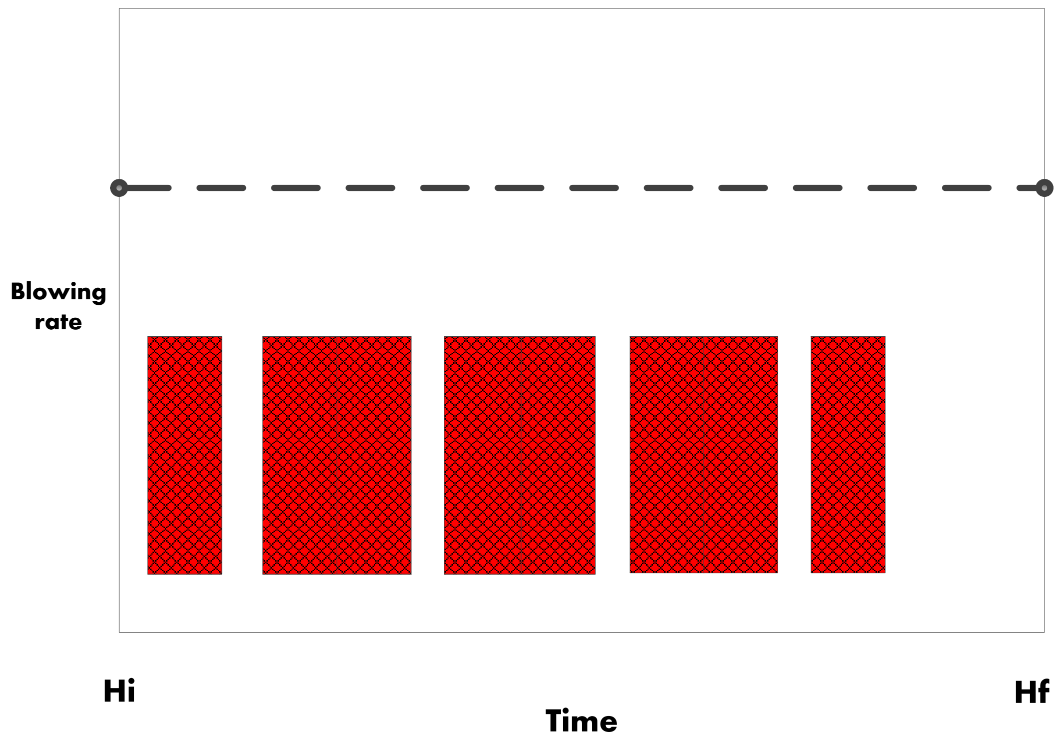

Figure A.3: Instantaneous blowing rate with restrictive API limits: Dashed line represents the API limit.

Figure A.4 illustrates the progression of furnace inventory over the time horizon. The quantity of material in the furnace increases until a tap is performed at which point material is drawn from the furnace. FFM in the furnace is able to stay well within the lower and upper capacity limits in Figure A.4.

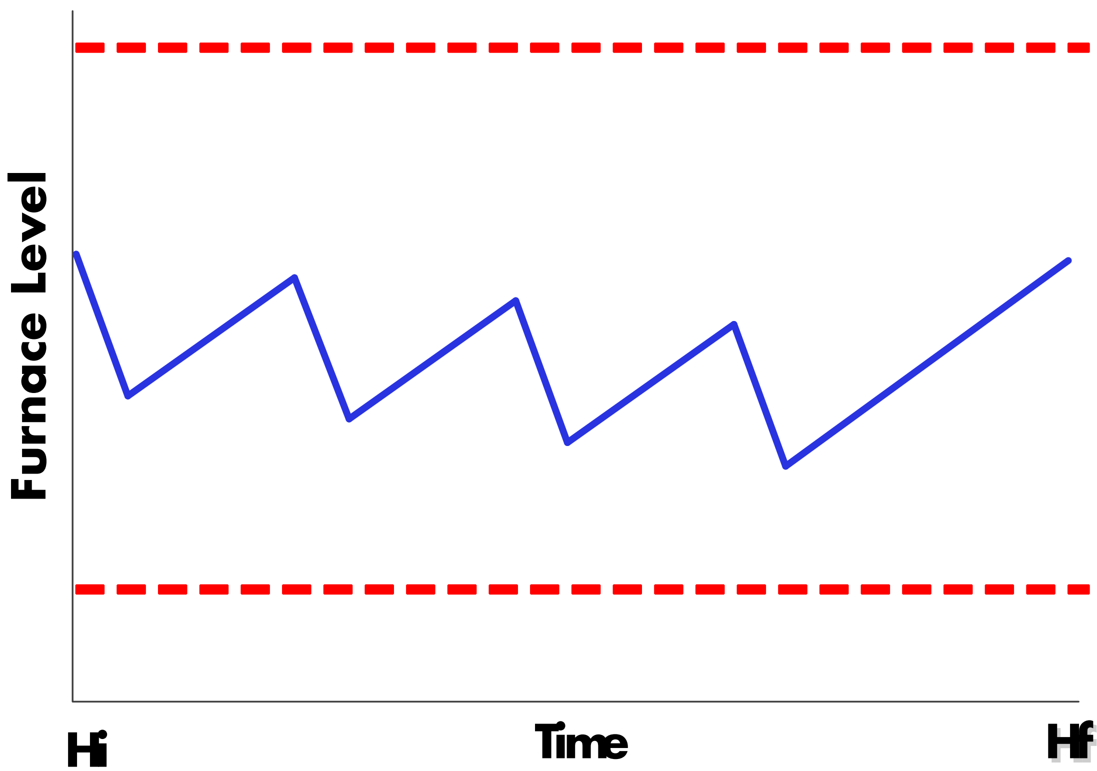

Figure A.4: Furnace level Progression under restrictive API limits: dashed lines denote furnace capacity limits.

A.3.2 Case B: Non - Restrictive API limits

In case B the optimization is performed while relaxing the environmental blowing constraints and allowing for the simultaneous blowing of both converters. All other parameters are kept the same as in the previous case. Figure A.5 contains the Gantt chart for case B. The optimization selects the furnace tapping to be performed consecutively now that the blowing constraints are no longer limiting. The new schedule is not able to cast more material in the same time horizon due to the offset required by the tapping. The offset reduces the remaining available time, such that completing an entire converter cycle in the remaining unscheduled time is not possible. This suggests that the final converter cycle performed on converter 2 can be performed at any time after the last cast on converter 1. However, were the time horizon constraints to be relaxed, the schedule would allow for two additional casts and the partial completion of two additional converter cycles, whereas the schedule in Figure A.2 would allow for only one additional cast and a partial converter cycle at most.

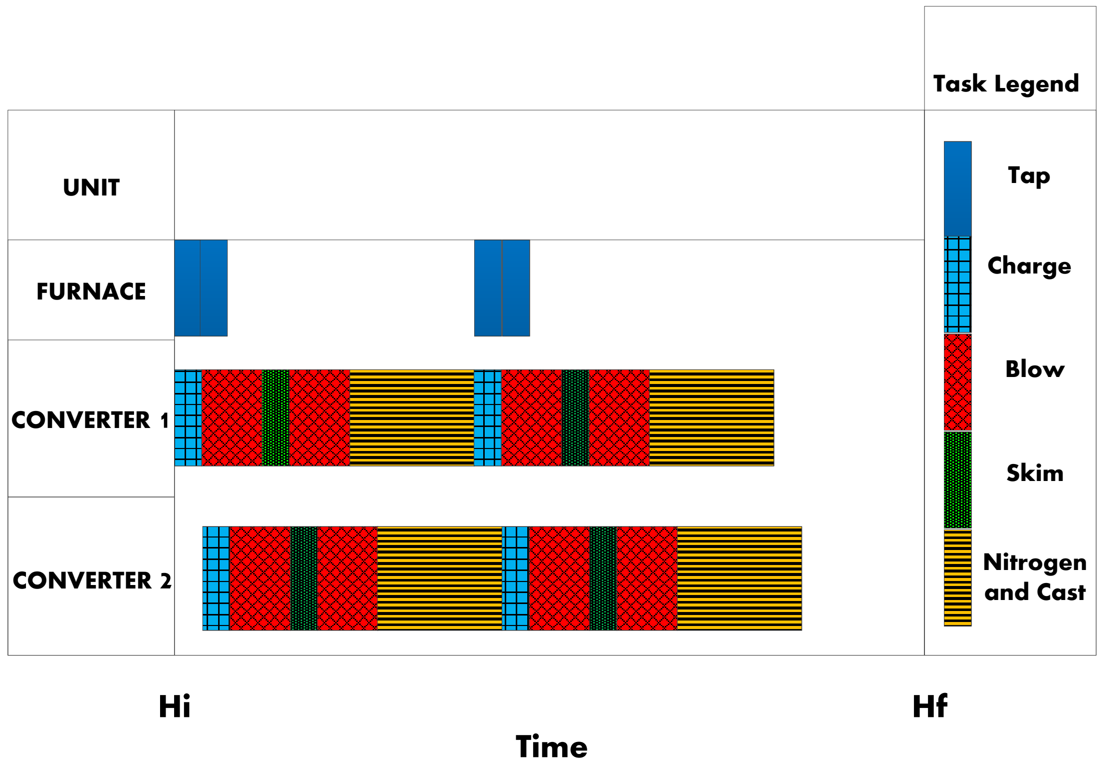

Figure A.5: Gantt Chart for Non-Restrictive API constraint.

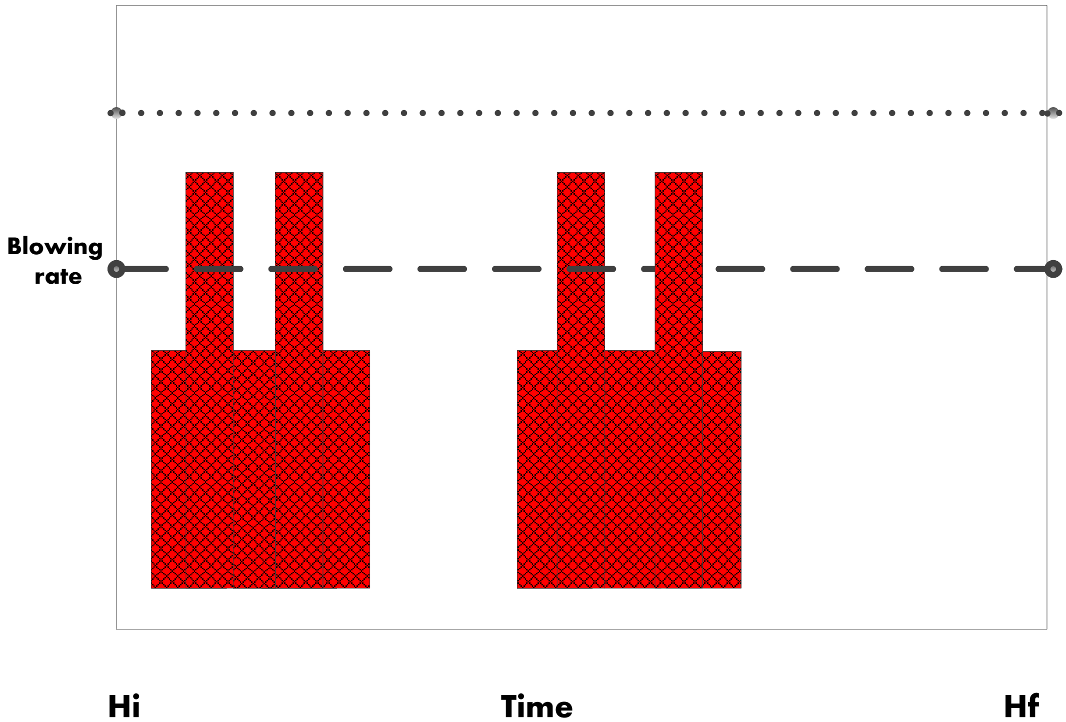

Figure A.6: Instantaneous blowing rate with non-restrictive API limits: Dashed line represents the previous API limit, the dotted line represents the current limit.

Since the API limit has been increased, simultaneous blows are now permissible. The overlapping blows present in Figure A.6 are below the new API limit, represented by the dotted line, and are above the API limit used in the previous case study, represented by the dashed line.

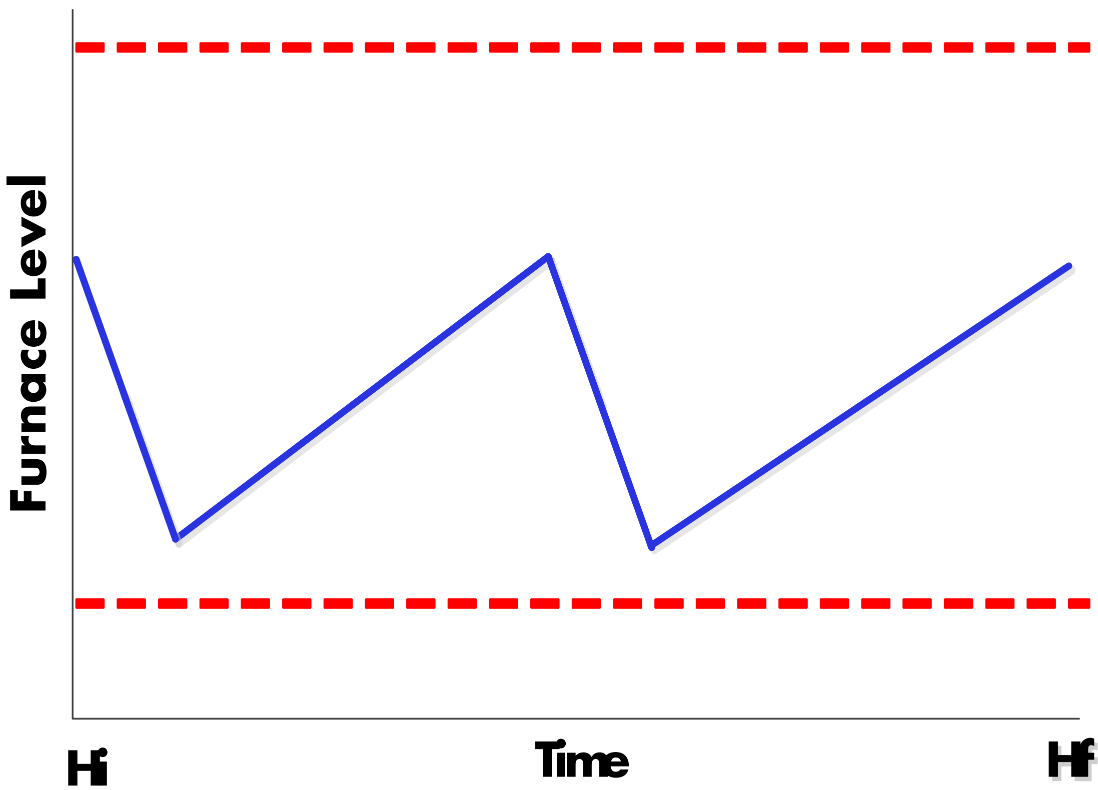

Figure A.7: Furnace level Progression under Non-restrictive API limits: dashed lines denote furnace capacity limits.

Figure A.7 presents a furnace accumulation profile different to the one in the previous case study. In comparison, Figure A.5 spreads the taps apart as opposed to performing them contiguously as in Figure A.2. Despite the difference in the timing of the taps, the two furnaces start with the same amount of inventory, have the same amount of material added and removed over the entire horizon, and therefore finish with the same amount of material. Likewise, the furnace inventory remains well within the lower and upper bound.

A.4 Conclusion

This section adapts the continuous-time formulations proposed by Ierapetritou and Floudas (1998a,b) and applies it to model a simplified converter aisle operation in a nickel smelting plant. The key features of the formulation in this section are consideration of the semi-continuous nature of furnace operation to retain operation within the available furnace capacity, and sequencing and timing of blows on converter in order to obey the environmental, regulatory constraints in order to fulfil a quantifiable objective. A study that demonstrates the effect of the API limits on the optimal schedule is included. This work is meant to have potential value to plant-floor operation as a decision support tool to aid operators in navigating the multivariate considerations that are required to generate an optimal schedule. Future work will focus on extending the current formulation to a large industrial scope including increased number of units and additional operating tasks. In addition, inclusion of recycle streams, the effect of synergistic operating policies on the nominal schedule, dynamic environmental regulations, and dynamic furnace flow rates will be considered. Finally, the deterministic case is treated in this section. Further work will include mechanisms for dealing with uncertainty.