A COMPARISON OF KINEMATIC FLOOD ROUTING METHODS

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KINEMATIC FLOOD ROUTING METHODS

by

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ABSTRACT:

To provide a logical framework for the comparison of various methods of kinematic flood routing a general method of kinematic flood routing is developed. After presenting the general framework, the properties of the numerical model are investigated by:

1. Algebraic examination of the finite difference scheme.
2. Numerical experiments using a high speed digital computer.
3. Comparison of the kinematic flood routing results with results of simulations using the complete one dimensional dynamic representation.

Particular facets of the numerical kinematic model that were studied included:

1. The stability of the numerical schematizations.
2. The degree of approximation with the finite difference system.

3. The applicability of kinematic methods to unsteady flow systems.

4. Methods of extending the kinematic solutions to predict attenuation as well as translation of the flood wave through the channel systems.

The results indicate that kinematic flood routing methods differ primarily in the point about which the finite difference equation is formulated, hereafter termed the nucleus, and that the general framework is capable of emulating such methods as the Muskinghum Method, other non-linear kinematic methods and reservoir routing. By varying the location of the nucleus the stability and degree of approximation is significantly altered. This results in the outflow hydrograph being sensitive to the location of the nucleus and the size of the finite difference steps.

To facilitate further research and application of the methods outlined in the thesis, a computer program was developed to enable kinematic flood routing to be performed in a natural channel with arbitrary geometry. Furthermore, the data is compatible with a program that is capable of performing a flood routing analysis using a numerical solution of the complete Saint-Venant equations. Documentation of the computer program for kinematic analysis is included with this thesis.

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It is difficult to repay my thesis supervisor, Dr. A. A. Smith for the many hours of reading drafts, and his appropriate advice. One particular piece of advice is worthy of being repeated. He warned that finishing a thesis and working full time is a very difficult task. He's right.

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CHAPTER 1

INTRODUCTION

Flood routing is the process of calculating the deformation and position of a flood wave as it passes through a body of water with a free surface. Fluctuations in the level of this surface provide temporary changes in storage which in turn give rise to some reduction in the flood peak. In the recent past there has been a considerable amount of effort directed towards the development of efficient methods of performing these types of calculations. The main reason for the interest in this particular type of unsteady flow phenomena may be the multitude of ways that this natural and sometimes man made occurrence affects the life patterns of humanity.

The primary usage of flood routing techniques is frequently associated with catastrophic floods, the objective being to estimate the magnitude and/or depth of the flows that may be expected at particular locations. The utilization of flood routing calculations for flood warning purposes is however just one of a number of functions that these mathematical tools can perform. For example, the analysis of unsteady flow in river systems may be a vital part of the day to day operation of a hydro-electric scheme on a multi-purpose waterway. The use of these algorithms as an operational tool may also aid in the efficient manipulation of flow control structures so that water is available for power generation, shipping and recreational purposes, without causing undue fluctuations in water levels or causing sudden surges in the channels.

The use of flood routing techniques as a planning and design tool must not be overlooked. Flood routing techniques are beneficial in analyzing a river system which is being developed or modified and thus ensure that adverse side effects and environmental impact are minimized. Alternatively, the analysis of an existing watershed may be useful in the development of land zoning bylaws to prevent the construction of expensive structures in areas subject to flooding.

Currently there is increasing interest in the establishment of such flood plain maps in semi-urban areas. In such locations the incidence of highway culverts and bridges result in the formation of a chain of "reservoirs" along short, relatively steep watercourses and it is essential to have an economic and reliable tool for the analysis of such systems.

Just as there are a large number and diverse types of problems that are tackled by flood routing algorithms, there are numerous approaches to performing the actual computation. Two general classifications may be used to identify flood routing techniques. These are:

1. Hydraulic techniques. The methods that fall into this classification are usually founded on the two laws that govern unsteady flow; conservation of mass and conservation of energy or momentum.
2. Hydrologic techniques. These algorithms frequently do not employ the rigorous equations describing unsteady flow, but instead attempt to model the system by the use of equations that yield results similar to the observed phenomena.

The correct classification of a particular approach to flood routing may be very dependent on the system being simulated. For example, if a flood routing method is formulated from the partial differential equations which describe unsteady flow, but with several of the time variant terms ignored, the approach could be classified as hydraulic if the exclusion of the terms is justifiable. However, if a system were encountered where it was not realistic to neglect some terms, the algorithm would fall into the hydrologic classification. Kinematic flood routing methods are a particular example of techniques which may be classified as either hydraulic or hydrologic.

Kinematic flood routing is a generic term which identifies a broad class of numerical methods used to route flood waves through a channel or waterway. Flood waves which behaved in a kinematic fashion were observed on the Mississippi River by Seddon (1900).

The term kinematic was applied to the particular phenomena by Lighthill and Whitham (1955) for the reason that the equations are described in terms of velocities rather than forces.

Prior to Lighthill and Whitham's presentation, storage-routing methods, which are special cases of kinematic routing, were employed by engineers and researchers. This was necessitated by the prohibitively large computational load imposed by a solution of the equations which describe the conservation of energy and mass. Even the simplified approaches were oftimes fraught with problems associated with excessive amounts of calculations. This led to the use of large time and distance steps in the numerical methods which in turn created stability problems in the numerical calculations.

With the advent of the high speed digital computer, it became feasible to tackle unsteady flow problems using a hydraulic approach. At the same time, attention was directed towards programming kinematic algorithms so that solutions could be obtained using the computer. This has resulted in a variety of techniques for approaching the problems. Because the hydraulic methods of analysis are usually founded on the momentum and continuity equations, these methods have a relatively unified basis on which they may be compared.

It seems that kinematic flood routing algorithms are not as clearly set in a logical framework for comparison, even though they are all founded on the continuity equation. Thus, the primary purpose of this thesis is to develop a general framework that may be used to compare the various kinematic algorithms.

The usefulness of a numerical technique depends not only on how well it models the physical phenomena; but also on the familiarity of the user with the characteristics of the algorithm. This aids the user by providing information that will allow the user to identify the limitations of the model and particular properties of the model that may be used to an advantage. In addition, a thorough understanding of the computational tool being employed is necessary to differentiate between physical phenomena that are predicted and numerical phenomena, the result of instability, numerical error or poor convergence characteristics, which do not truthfully represent the actual physical system. Thus a second objective of this thesis, after developing the general framework, is to investigate the numerical characteristics of the algorithm.

To provide a verification of the theoretical studies of the general kinematic flood routing method, a series of numerical experiments are presented for purposes of demonstration and comparison. These computer simulations are also utilized to demonstrate the accuracy of kinematic simulations when representing dynamic physical systems. These numerical experiments provide a comparison to determine the limits of kinematic algorithms and this investigation of the practical limitations of the kinematic methods is another of the thesis objectives. An explicit finite difference representation of the momentum and continuity equation is utilized to provide a data base for comparison purposes.

The application of a kinematic method to a particular problem may require the use of specific types of kinematic flood routing, for example, lag and route, Muskinghum flood routing, or reservoir routing. After studying the general method, several chapters are devoted to studying non-linear cases of these problems and a comparison of the results with dynamic simulations. The primary objective is to determine not only the useful ness of the various methods; but also to identify the effect of varying the step size in finite difference analyses and the effects of non-linearity.

The final objective of the study is to provide an efficient and versatile computer program that will enable a user to perform the various kinematic flood routing computations. This has been accomplished and a chapter is devoted to describing the development and use of the computer program. Designed to be used in a time-shared mode, the program is compatible with the Civil Engineering Program Library to allow easy access to the completed program.

CHAPTER 2

THE DYNAMIC SOLUTION

To provide a precise data base against which alternative flood routing methods may be compared it is necessary to have a means of generating the time history of flows resulting from inflow of a specified hydrograph to a known channel system. The use of natural channel systems to provide a data base can be immediately ruled out due to the complexity of both system and input, the inability of controlling flow parameters and the expense of monitoring flows at downstream sections A laboratory facility of sufficient scope and flexibility was not available and it was therefore decided to substitute a numerical model capable of generating the required data base with reasonable accuracy for systems of simple geometry.

The mathematical analysis of unsteady flow phenomena is founded upon the partial differential equations based on the laws of conservation of mass and linear momentum. Methods of solving these equations to yield values of flow depth and quantity at any desired point in space and time are numerical in nature (as opposed to analytical) and consist of the solution of finite difference formulations of the partial differential equations. This chapter is concerned with the discussion of the method adopted in this study to obtain such a solution.

A brief description of the literature and alternate schemes for analyzing unsteady flow is provided prior to presenting the method used in this study. Following the description of the finite difference formulation, the sensitivity tests used to verify the numerical model are outlined. The conclusions summarize the findings that the algorithm was capable of providing the data necessary for comparing alternative methods of flood routing.

2.1 THE EQUATIONS

The equations describing unsteady flow may be written as follows:

The Momentum Equation:

$$\frac{\delta h}{\delta x} + \frac{\delta}{\delta x}\left(\frac{Q^2}{2gA^2}\right) + \frac{1}{gA}\frac{\delta Q}{\delta t} + Sf = 0 \qquad (2.1)$$

The Continuity Equation:

$$\frac{\delta Q}{\delta x} + T_w \frac{\delta h}{\delta t} = \bar{q} \qquad (2.2)$$

Where: x = distance

t = time

h = water surface elevation

Q = flow rate

A = area of cross section

g = acceleration of gravity

T_w = surface width

$\bar{q}$ = rate of lateral inflow

Sf = slope of the friction line

The derivation of these equations, often called De Saint Venant's equations, may be found in numerous books. Several of these are Stoker (1957), Chow (1959), and Henderson (1965). The first two terms in equation 2.1 describe effects caused by nonuniform flow. Results of unsteady conditions are reflected in the third term which describes temporal acceleration. The effect of friction is modelled by the slope of the friction line. Terms in the continuity equation (equation 2.2) describe the change in flow rate along the channel, the rate of change of storage, and the amount of lateral inflow.

Because no analytic solution has been found for the partial differential equations which describe unsteady flow conditions, finite difference techniques must be used. Previous to the advent of the high speed digital computer it was not practically feasible to obtain solutions of these equations. In special cases, the method of characteristics was employed. However, the amount of calculations required for this relatively rapid method of solution limited its use. With the increased availability of high speed digital computers, it has become possible to solve the partial differential equations using numerical techniques. Still, the use of these methods may be rather difficult and expensive due to problems of stability with some formulations and the expense involved in programming and operating the computer.

Numerical solutions of De Saint Venant's equations do have several advantages. These equations provide an accurate description of the one dimensional flow system and enable the user to obtain very detailed information about the wave shape and its position during the time in which the wave is being propogated along the channel. It is beyond the scope of this chapter to provide an exhaustive comparison of all the various techniques used to solve the equations which describe unsteady flow. However, a short section giving a general description of several techniques follows.

2.2 DISCUSSION OF NUMERICAL METHODS

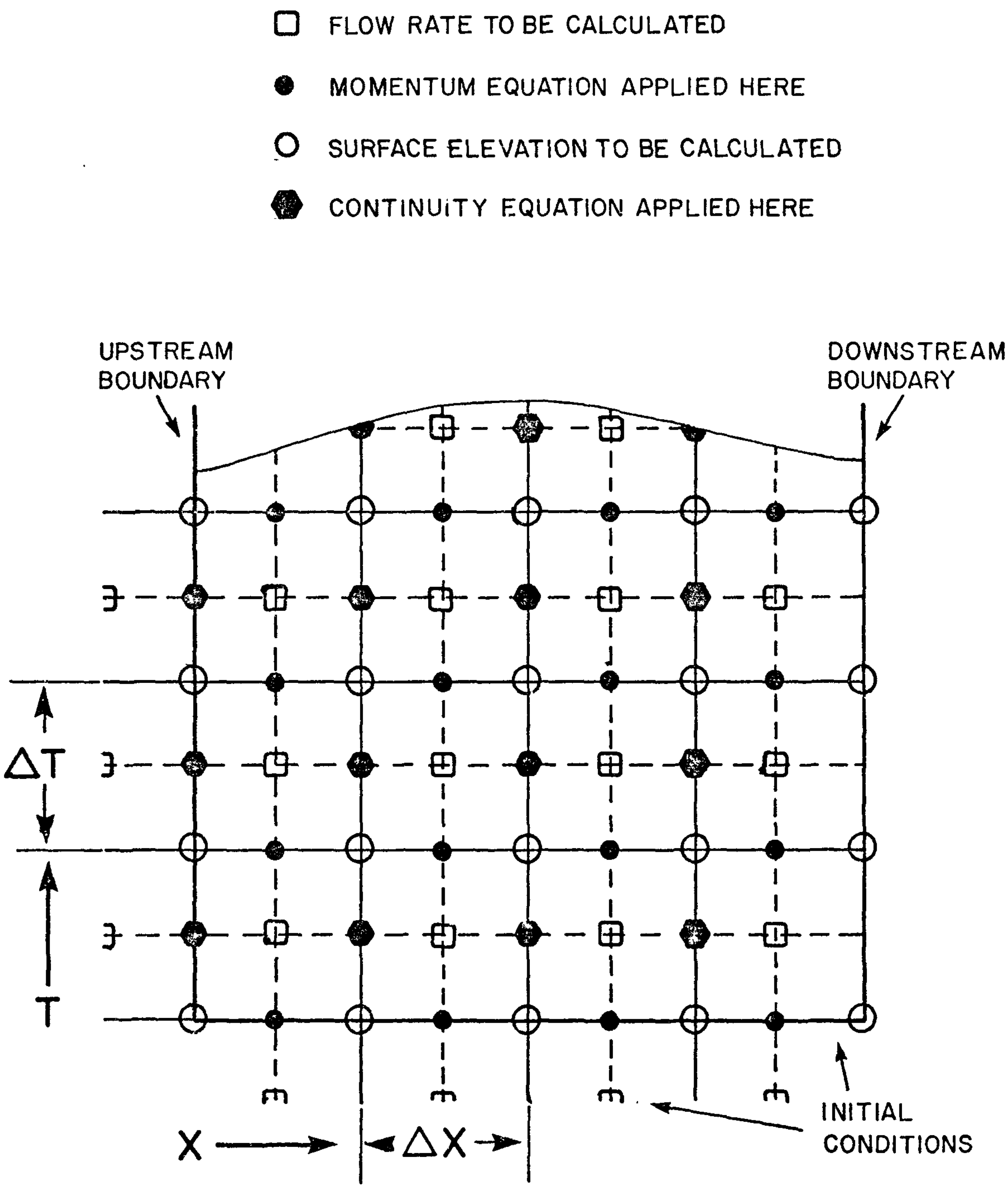
There exist two general methods of obtaining solutions to the partial differential equations. These are direct methods and characteristics methods. The partial differential equations describing unsteady flow may be rearranged in the form of four ordinary differential equations. Two of these equations define characteristic lines, paths of energy transfer, while the two other equations define energy change along the characteristic lines. Applying finite difference techniques to these equations is termed the characteristics method. Alternately, finite difference formulations of the partial differential equations may be used to provide solutions. This is known as the direct method.

The finite difference techniques used to solve either characteristics or direct methods may in turn be classified as explicit or implicit. An explicit method provides for a specific solution for an unknown quantity while an implicit technique requires the solution of several similtaneous equations to provide the values of a number of quantities. In both methods, the object is to obtain values of flow rate (or velocity) and water surface elevation (or water depth) at discrete points on a space-time diagram. Figure 2.1 shows a space-time diagram with a staggered rectangular grid. The solution progresses from the known initial conditions through successive increments in time. Thus, conditions along the channel at time "t" are used in conjunction with the boundary conditions to find the solution at time "$t + \Delta t$".

The type of boundary conditions encountered depends on the physical system being simulated. If flow is subcritical there is a

FIGURE 2.1

SPACE-TIME DIAGRAM



boundary condition at the upstream and downstream limits. Typically, the upstream boundary would be an inflow hydrograph while the downstream condition would be water surface elevation as a function of time and/or flow rate. When supercritical flow is encountered the boundary conditions are found only at the upstream limit.

The type of grid used on the space-time diagram is related to the type of solutions used. A characteristics solution, which results from applying finite difference methods to characteristic equations may be used with an irregular grid defined by intersections of characteristic lines or may be applied to a rectangular grid. The direct methods are usually used with a regular rectangular grid; the staggered rectangular grid being used primarily with explicit formulations of the direct method.

The attributes and drawbacks of various methods are briefly outlined in the following paragraphs.

2.2.1 Characteristics Method.

This method is believed to solve a given space-time diagram in the least time. More accuracy is claimed, especially where the flow varies quite rapidly as the solution progresses along the paths of energy transfer (characteristic lines). The answers are not provided at fixed points in time or space, which is a disadvantage when information at a particular time or location is required. This difficulty can be overcome by using a method of characteristics which solves for fixed points on a space-time diagram. Several other factors that favour the use of characteristics methods are:

1. The solution is more stable when flow conditions are supercritical.
2. The case of a flood wave propagating down an initially dry stream bed is more correctly modelled.
3. Characteristics methods are the most accurate methods of modelling rapidly varied flow as the characteristics lines are closer in regions of rapid variation.

References are: Amein (1966), Henderson (1965), Woolhiser and Liggett (1967), and Yevjevich and Barnes (1970).

2.2.2 Direct Method.

This method is widely used due to the relative ease of algebraically expressing the various equations and the subsequent reduction of programming difficulties. Answers are provided at fixed points in time and space which is convenient for interpretation of results. The disadvantages of direct finite difference techniques are that explicit versions are subject to stability problems especially when there are rapid variations of flow or if supercritical conditions are encountered.

Stability problems have been overcome by using implicit algorithms.

References are : Woolhiser and Liggett (1967), Smith (1968), Amein (1968), Amein and Fang (1970), Yevjevich and Barnes (1970), and Walden (1973).

2.3 NUMERICAL METHOD USED IN THIS STUDY

In choosing a numerical method to use as a base for comparison with approximate methods, preference was given to an algorithm which would provide the necessary accuracy with a minimal amount of computer programming. As initial tests were going to be made using rectangular channels, stability would not be as difficult a problem as would be encountered with a natural non-prismatic channel. Thus, an explicit method which used a staggered mesh on the time-space diagram was employed. This scheme had been successfully used in a similar situation where the channels were very nearly prismatic. James and Horne (1969), Smith (1968). In addition experience had been obtained with this method in conjunction with classroom studies. Thus, a small computer program was available which could be easily adapted to the present study.

The staggered mesh used by this method is shown in figure 2.1. An inflow hydrograph was used as the upstream boundary condition while the downstream condition for subcritical flow was assumed to be uniform flow. Initial conditions consist of a horizontal row of known flow rates at $t = -\Delta T/2$ and a horizontal row of water surface elevations specified at t = o. The row of flow rates is displaced $\Delta X/2$ upstream from the known water surface elevations.

For time $\Delta T/2$, the momentum equation is applied to a point under the first unknown flow rate downstream of the upstream limit. The unknown value is calculated using the initial conditions and the upstream boundary. This calculation is repeated as the process moves in the downstream direction. The previously unknown value is treated as the upstream boundary condition in calculating the next unknown flow rate. Figure 2. 2 shows the way in which the dynamic equation is applied. When the downstream limit is reached, time is incremented by $\Delta T/2$, the downstream boundary value is obtained and a series of calculations is begun in the upstream direction. This time, the continuity equation is applied to a point below the first unknown water surface elevation upstream of the downstream limit. After the unknown elevation is determined, the calculation is repeated at the next upstream location. Again the previously unknown value is used as the downstream boundary condition in predicting the next unknown. Figure 2. 3 portrays the application of the equation of continuity. When the upstream limit is reached, time is incremented by $\Delta T/2$

FIGURE 2.2

FINITE DIFFERENCE MOLECULE FOR THE MOMENTUM EQUATION

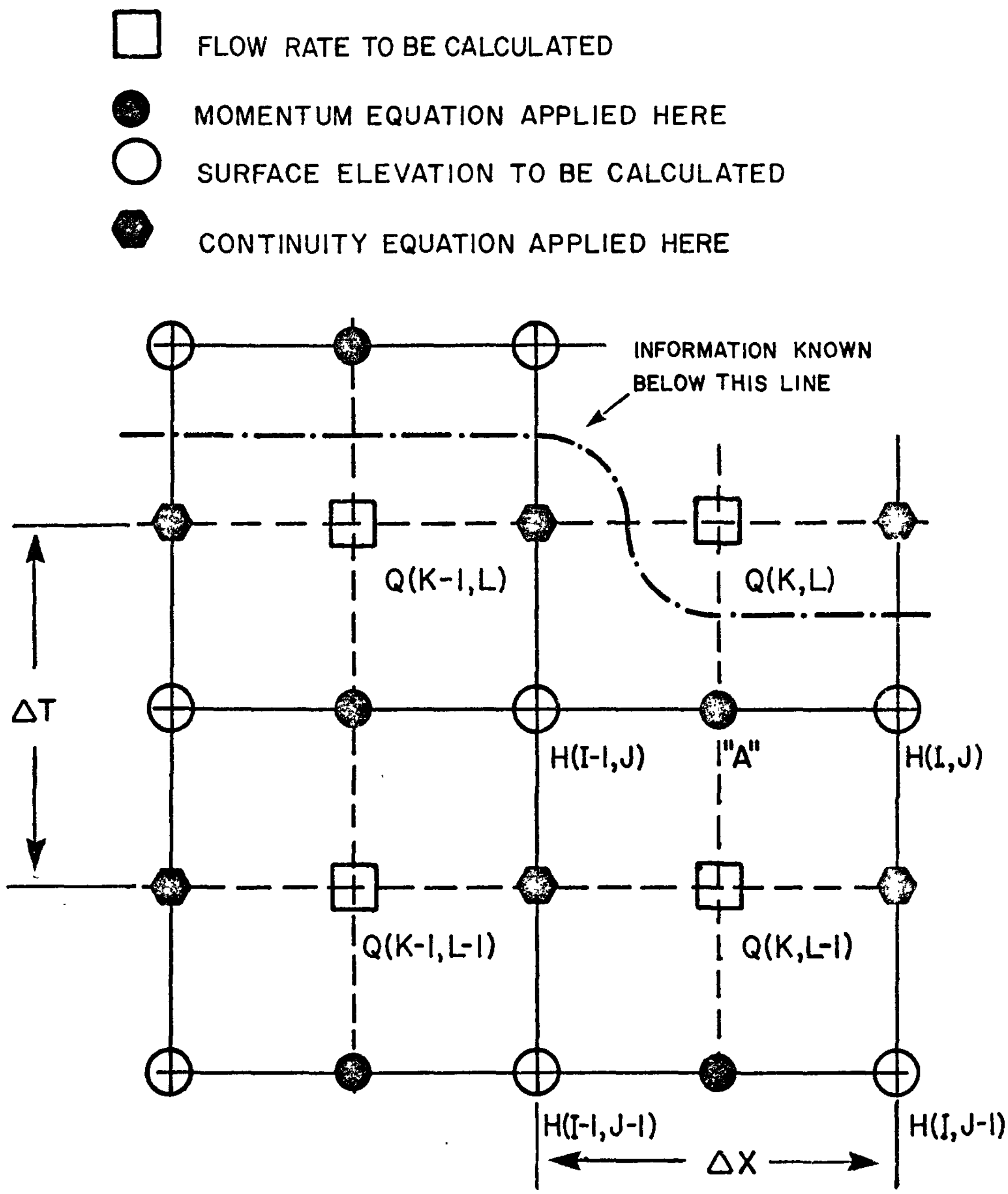
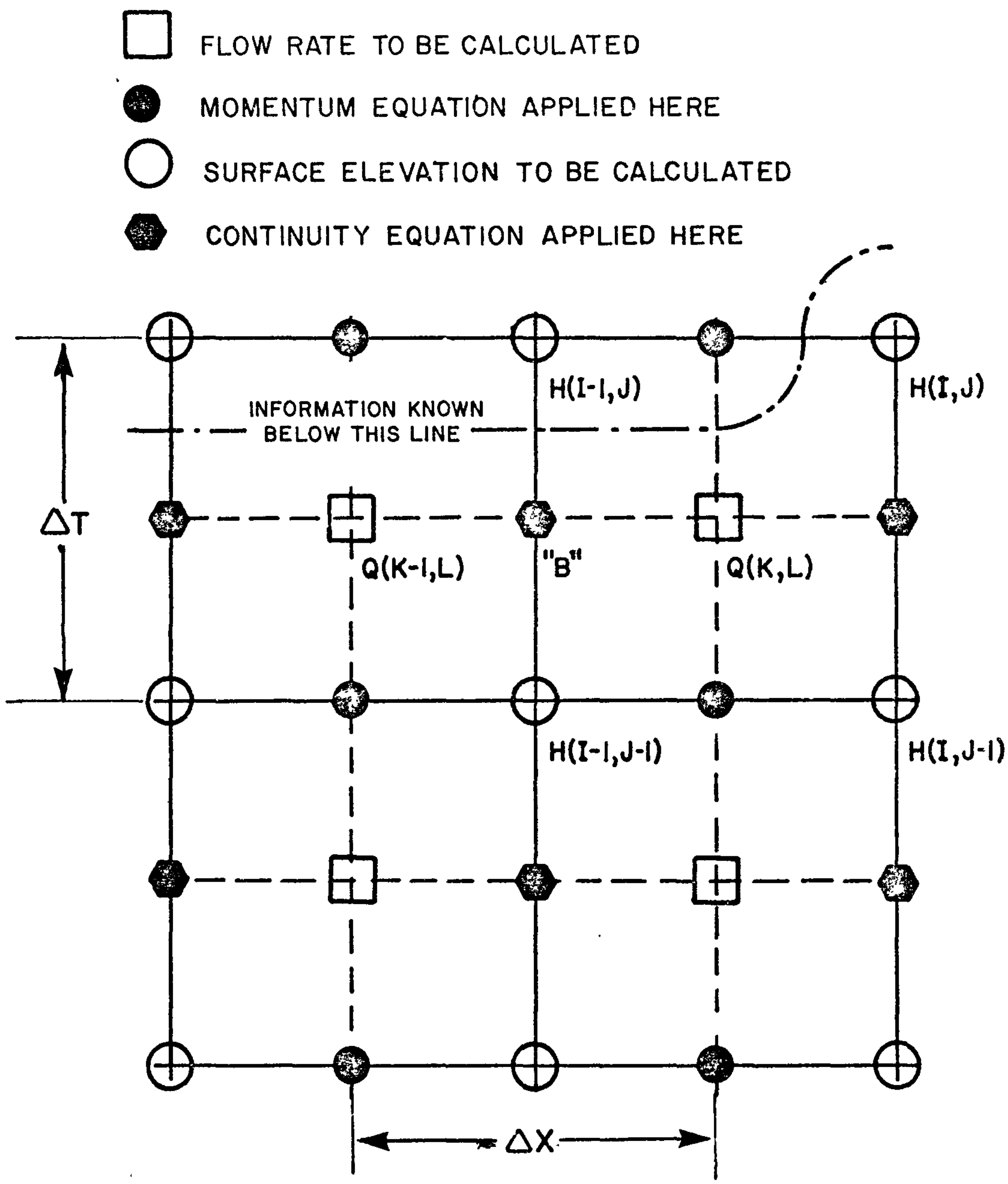


FIGURE 2.3

FINITE DIFFERENCE MOLECULE FOR THE CONTINUITY EQUATION



a new value for the upstream boundary is obtained, and the cycle begins again with the application of the momentum equation in successive steps moving downstream.

By repeating the previously described cycle, a time history of flow conditions along the channel may be obtained. The computation is stopped when a defined time is reached or when a nearly steady state is reached after a flood wave has passed through the channel. The cost of computing is related to the number of iterations required to fill the time space diagram and the amount of calculations in each cycle. When only the outflow hydrograph is desired, this method may seem to be quite wasteful due to the amount of unnecessary data which must be generated. However, it will be shown later that some methods of finite difference solution of unsteady flow can be relatively inexpensive.

2.4 FINITE DIFFERENCE FORMULATION OF THE UNSTEADY FLOW EQUATIONS

The first step in the solution of a problem requiring the solving of partial differential equations by a finite difference method is to express the partial differentials as finite differences. This was done in the following manner. The momentum equation is:

$$\frac{\delta h}{\delta x} + \frac{\delta}{\delta x}\left(\frac{Q^2}{2gA^2}\right) + \frac{1}{gA}\frac{\delta Q}{\delta t} + Sf = 0 \qquad (2.3)$$

Where:

$$\frac{\delta}{\delta x}\left(\frac{Q^2}{2gA^2}\right) = -\frac{Q^2}{gA^3}\frac{\delta A}{\delta x} + \frac{Q}{gA^2}\frac{\delta Q}{\delta x} \qquad (2.4)$$

and

$$\left.\frac{\delta A}{\delta x}\right|_{t=constant} = T_w\frac{\delta h}{\delta x} + \left.\frac{\delta A}{\delta x}\right|_{h=constant} \qquad (2.5)$$

Substituting equation 2. 5 into equation 2. 4

$$\frac{\delta}{\delta x}\left(\frac{Q^2}{2gA^2}\right) = \frac{Q^2 T_w \delta h}{g A^3 \delta x} - \left.\frac{Q^2}{gA^3}\frac{\delta A}{\delta x}\right|_{h=constant} + \frac{Q}{gA^2}\frac{\delta Q}{\delta x} \qquad (2.6)$$

From the continuity equation, assuming lateral inflow is equal to zero.

$$\frac{\delta Q}{\delta x} + T_w\frac{\delta h}{\delta t} = 0 \qquad (2.7)$$

$$\frac{\delta Q}{\delta x} = -T_w\frac{\delta h}{\delta x} \qquad (2.8)$$

Thus

$$\frac{Q}{gA^2}\frac{\delta Q}{\delta x} = -\frac{QT_w}{gA^2}\frac{\delta h}{\delta t} \qquad (2.9)$$

The dynamic equation may now be rewritten in the following manner

$$\frac{\delta h}{\delta x} - \frac{Q^2 T_w \delta h}{g A^3 \delta x} - \left.\frac{Q^2}{gA^3}\frac{\delta A}{\delta x}\right|_{h=constant} - \frac{QT_w}{gA^2}\frac{\delta h}{\delta t} + \frac{1}{gA}\frac{\delta Q}{\delta t} + Sf = 0 \qquad (2.10)$$

Figure 2. 2 shows an enlarged portion of the space-time diagram. The momentum equation is applied at point "A". Q(K, L) is unknown and all the data is known for points on rows below Q(K, L). In addition, information is available for points on the L row to the left of Q(K, L) thus the following approximations can be made

$$\frac{\delta h}{\delta x} = \frac{H(I,J) - H(I-1,J)}{\Delta X} \qquad (2.11)$$

$$\frac{\delta h}{\delta t} = \frac{H(I-1,J) - H(I-1,J-1) + H(I,J) - H(I,J-1)}{2\Delta T} \qquad (2.12)$$

$$\frac{\delta Q}{\delta t} = \frac{Q(K,L) - Q(K,L-1)}{\Delta T} \qquad (2.13)$$

As this portion of the study is limited to rectangular channels with uniform slopes, the following approximations are appropriate.

$$A = T_w * (H(I,J) - B(I) + H(I-1,J) - B(I-1))/2 \qquad (2.14)$$

$$\frac{\delta A}{\delta x} = T_w \frac{B(I) - B(I-1)}{\Delta X} \qquad (2.15)$$

Where:

B = invert elevation at the section.

Describing the slope of the friction line by Mannings equation results in the following expression.

$$Sf = \frac{Q(K,L) \times |Q(K,L-1)| \times n^2 \times P^{4/3}}{2.21 \times A^{10/3}} \qquad (2.16)$$

Where: n = Mannings roughness coefficient

$$p = T_w + (H(I,J) + H(I-1,J) - B(I) - B(I-1))$$

Using the absolute value of the known flow rate gives the energy slope term the same sign as the unknown flow rate. Thus, flow reversals which do not occur in a rapid fashion can be modelled with this scheme.

Rewriting the momentum equation in terms of finite differences yields the following:

$$\left(1 - \frac{Q(K,L)\,Q(K,L-1)\,T_w}{g A^3}\right) \times \frac{H(I,J) - H(I-1,J)}{\Delta X}$$

$$- \frac{Q(K,L)\,Q(K,L-1)}{g A^3} \times \frac{B(I) - B(I-1)}{\Delta X}$$

$$- \frac{Q(K,L) + Q(K,L-1)}{2} \times \frac{T_w}{g A^2} \times \frac{H(I,J) - H(I,J-1) + H(I-1,J) - H(I-1,J-1)}{2 \Delta T}$$

$$+ \frac{1}{g A} \times \frac{Q(K,L) - Q(K,L-1)}{\Delta T}$$

$$+ \frac{Q(K,L) \times |Q(K,L-1)| \times n^2 \times P^{4/3}}{2.21 \times A^{10/3}} = 0 \qquad (2.17)$$

Solving for the unknown flow rate produces the following expression.

$$Q(K,L) = \left(\frac{H(I,J)-H(I-1,J)}{\Delta X} - Q(K,L-1)\frac{1}{gA\Delta T} \right.$$

$$\left. + \frac{Q(K,L-1)T_w}{2gA^2} \times \frac{H(I-1,J)-H(I-1,J-1)+H(I,J)-H(I,J-1)}{2\Delta T} \right) \Big/$$

$$\left(\frac{Q(K,L-1)T_w}{gA^3} \times \frac{H(I,J)-H(I-1,J)}{\Delta X} + \frac{Q(K,L-1)}{gA^3} \times \frac{B(I)-B(I-1)}{\Delta X} \right.$$

$$+ \frac{T_w}{2gA^2} \times \frac{H(I-1,J)-H(I-1,J-1)+H(I,J)-H(I,J-1)}{2\Delta T}$$

$$\left. - \frac{1}{gA\Delta T} - \frac{|Q(K,L-1)| \times n^2 \times P^{4/3}}{2.21 \times A^{10/3}} \right) \qquad (2.18)$$

The continuity equation is applied in a similar fashion. Assuming no lateral inflow, the continuity equation is:

$$\frac{\delta Q}{\delta x} + T_w \frac{\delta h}{\delta t} = 0 \qquad (2.19)$$

Applying this equation at point "B" on the space-time diagram of figure 2.3 allows the following approximations to be made:

$$\frac{\delta Q}{\delta x} = \frac{Q(K,L) - Q(K-1,L)}{\Delta X} \qquad (2.20)$$

$$\frac{\delta h}{\delta t} = \frac{H(I-1,J) - H(I-1,J-1)}{\Delta T} \qquad (2.21)$$

Thus the continuity equation in finite difference form is:

$$\frac{Q(K,L) - Q(K-1,L)}{\Delta X} + T_w \frac{H(I-1,J) - H(I-1,J-1)}{\Delta T} = 0 \qquad (2.22)$$

The unknown water level is given by the expression:

$$H(I-1,J) = H(I-1,J-1) - \frac{\Delta T(Q(K,L) - Q(K-1,L))}{T_w \Delta X} \qquad (2.23)$$

2.5 DISCUSSION OF THE FINITE DIFFERENCE SOLUTION

After a finite difference scheme has been developed, it is necessary to determine the limitations of the method. Numerical stability, convergence properties, the degree of approximation and the discretization errors are factors which influence the way in which the algorithm may be used. Numerical stability is a property of the numerical method which keeps errors from concealing the true solution. Convergence is a measure of the accuracy with which a finite difference equation will represent the partial differential equation as ΔX and ΔT approach zero. Another measure of the accuracy of the numerical solution is the degree of approximation. The discretization errors represent errors caused by replacing a derivative (a tangent) with a finite difference (a chord).

The finite difference algorithm must first be numerically stable to be useful as a tool. An unstable formulation will allow small errors to grow unbounded which in turn will mask the true solution. Some algorithms are unstable, others are conditionally stable. Stability in explicit formulations of finite differences is largely dependent on the size of time step used in the calculation. This has been demonstrated by the Courant Condition which is:

$$(V + C)\frac{\Delta T}{\Delta X} \leq 1 \tag{2.24}$$

Garrison et. al. (1969) reported that another condition must be satisfied for a particular explicit scheme to be stable. The additional constraint is:

$$(V + C)\frac{\Delta T}{\Delta X} \leq 1 - \frac{gn^2 |V| \Delta T}{2.21 R^{2/3}} \tag{2.25}$$

For the present study, a pragmatic approach to determining the stability properties of the numerical scheme was used. A short description of the problem used in the tests is outlined in the following paragraphs.

To facilitate easy computation, a rectangular channel was used as a prototype. Figure 2.4 shows a picture of the channel similar to the one used in this study. Channel properties were as follows:

Length = 50,000 ft.

Width = 100 ft.

Depth = 20 ft.

Slope = varied for various executions

n = varied for various executions

The upstream boundary condition was a symmetrical triangular hydrograph. Figure 2.5 shows the characteristics of this hydrograph. Downstream control was assumed to be uniform flow depth for the flow rate of the previous time step.

Equal increments of ΔX were used in this analysis. By entering the number of subreaches into the program, ΔX was computed by dividing the total length by the number of subreaches. Further documentation of the computer program is provided in Appendix "D".

FIGURE 2.4

PICTORIAL VIEW OF SYSTEM ONE

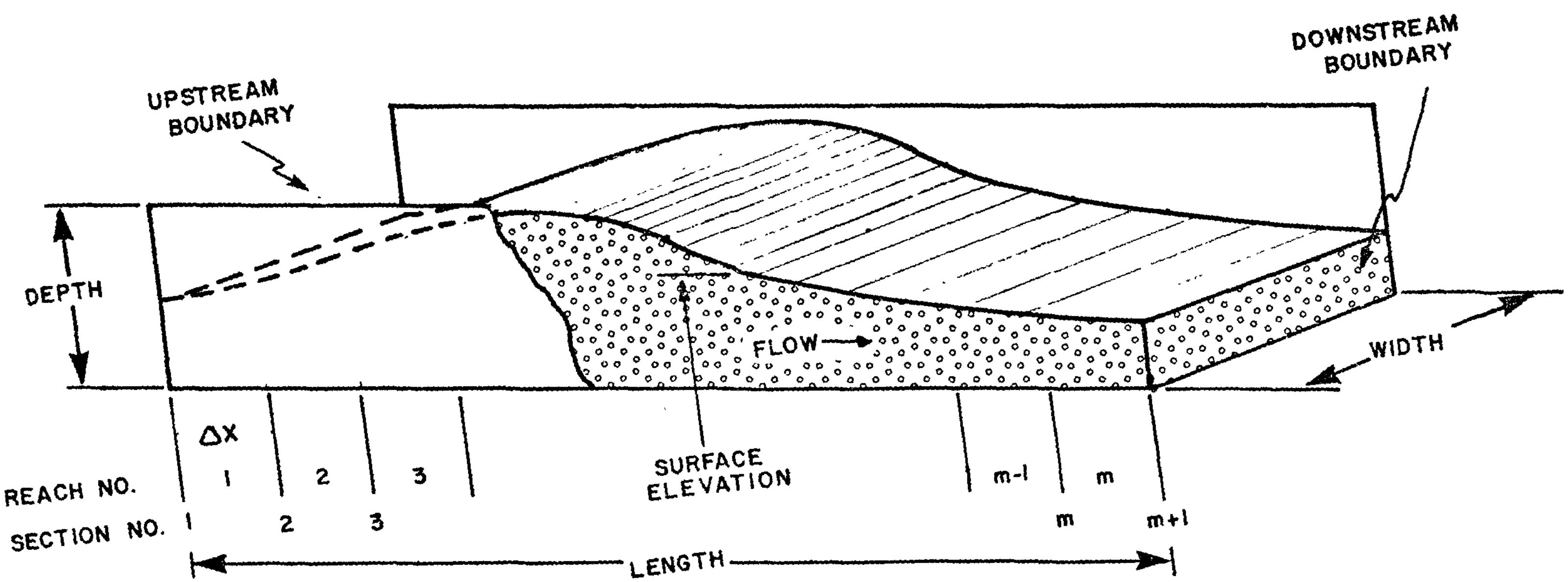
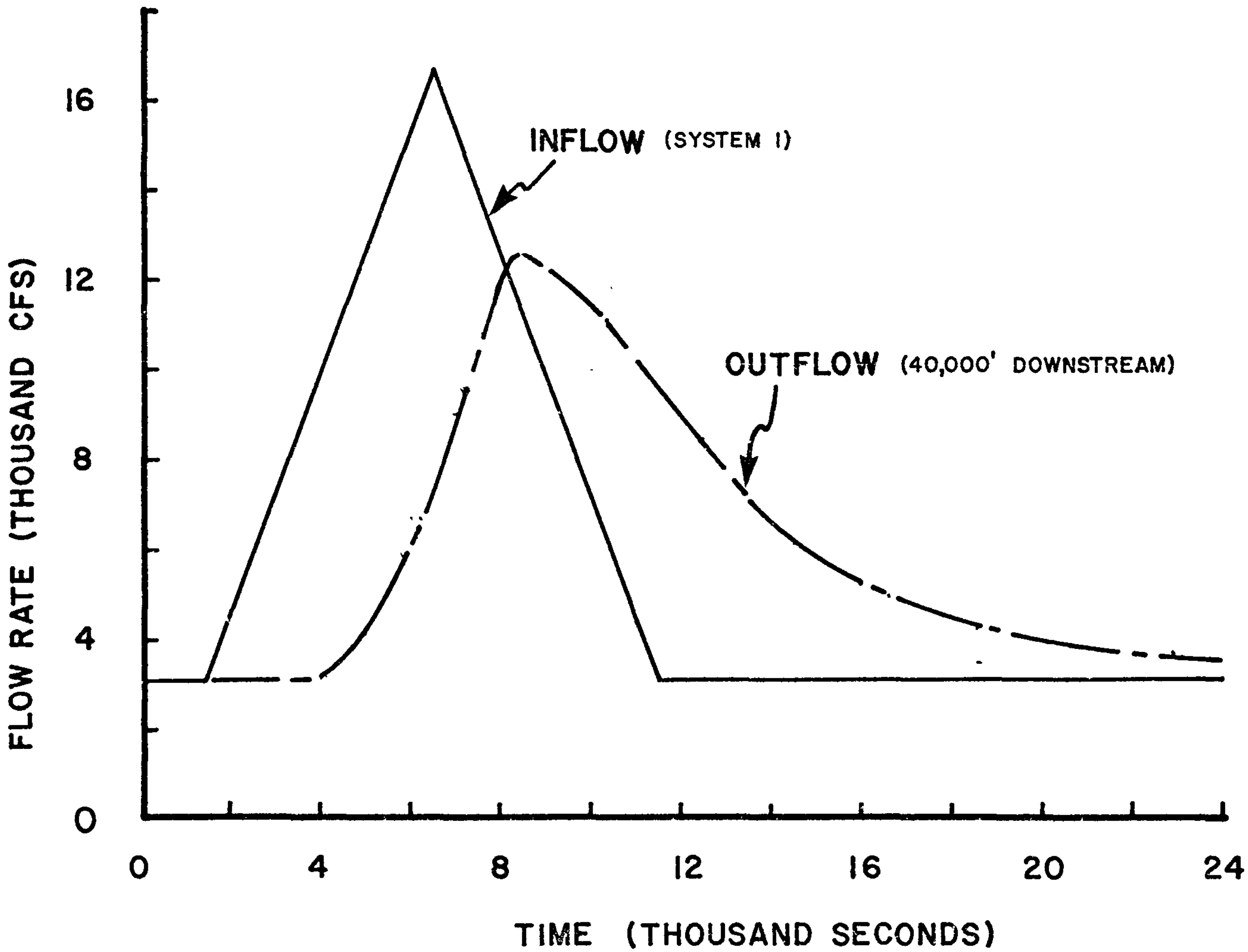


FIGURE 2.5

TYPICAL RESULTS OF FINITE DIFFERENCE ANALYSIS



During the execution of the program, ΔT was held fixed at a value determined by the following relationship:

$$(V+C)\frac{\Delta T}{\Delta X} \leq Z \qquad (2.26)$$

Where: v = full bank velocity

c = full bank celerity

Z = an arbitrarily chosen constant between 0 and 1

The constant, Z, is known as the Courant Number. It is defined as the time step used for the computation divided by the time step which satisfies the Courant Condition. The value of the Courant Number was reduced until no instabilities were detected. Results of several runs are shown in table 2.1. With Z = 0.67 the first test showed signs of instability on the falling limb of the hydrograph. Putting Z = 0.5 resulted in a hydrograph which showed no signs of instability. Increasing the slope to 0.001 and executing the program with Z=0.5 resulted in an unstable solution which terminated the job. The problem was successfully tackled with Z=0.25. This does not appear to agree with the stability condition as reported by Garrison. As the slope increases, the size of the Z value should decrease, this is in agreement with the condition of Garrison. However, their formula predicts that as the roughness coefficient n increases, the value of Z should also decrease. These tests indicate that as n increases

TABLE 2.1

STABILITY TESTS OF EXPLICIT METHOD

Slope	n	Z	Comments
0.0002	0.0149	0.67	slight signs of instability
0.0002	0.0149	0.50	stable
0.0010	0.0149	0.50	unstable
0.0010	0.0149	0.25	stable
0.0010	0.0322	0.50	stable

the value of Z should also increase. It appears that:

$$Z = f\left(\frac{\sqrt{S}}{n}\right) \qquad (2.27)$$

The sensitivity of the solution to changes in ΔX and ΔT is related to the way that the finite difference equation approximates the differential equation and to the discretization errors.

The degree of approximation is obtained by substituting a Taylor's Series expansion into the finite difference solution and observing how well it represents the partial differential equation. Appendix "B" contains the calculations which show that the momentum equation is represented by the finite difference equation in the following manner:

$$\frac{\delta h}{\delta x} + \frac{\delta}{\delta x}\left(\frac{Q^2}{2gA^2}\right) + \frac{1}{gA}\frac{\delta Q}{\delta t} + Sf + O(\Delta X^2, \Delta T^2) = 0 \qquad (2.28)$$

Similarly the representation of the continuity equation is:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + O(\Delta X^2, \Delta T^2) = \bar{q} \qquad (2.29)$$

Several executions of the computer program were performed to determine the effects of approximation errors and discretization errors as ΔX and ΔT vary.

Table 2.2 contains vital data from several executions which demonstrate the changes caused by varying ΔX and ΔT. Based on a comparison of peak values, it appears that the solution is sensitive only to the value of ΔX used. The lack of sensitivity to changes in ΔT can be attributed to the fact that stability criteria are a more stringent constraint than are convergence requirements. The stability criteria also makes ΔT a function of ΔX with the result that convergence appears dependent on ΔX.

Figure 2.5 shows the inflow hydrograph and a typical outflow hydrograph obtained using the finite difference analyses.

A check was provided by comparing the results with an implicit method developed by Walden (1973) with those provided by the explicit method described in the report. These results are presented in table 2.2.

Further checks were provided by using an example proposed by Thomas (1934). Amein (1967) shows the results of routing a flood through a very wide channel using characteristics, a direct explicit method and a direct implicit technique. The channel had a slope of one foot per mile and a Mannings n approximately equal to 0.03. The inflow hydrograph was sinusoidal with an initial flow of 50 cfs/ft width, a peak flow of 200 cfs/ft. width, and a time base of 96 hours.

TABLE 2.2

SENSITIVITY TESTS OF THE EXPLICIT METHOD
AND COMPARISON WITH IMPLICIT METHOD

ΔX (Ft)	Z	ΔT (Secs.)	Time of Maximum Flow (hrs.)	Maximum Flow (cfs)	Time of Maximum Depth (hrs.)	Maximum Depth (ft.)
1000'	0.50	14.8	2.274	12537	2.620	15.858
2000	0.50	29.7	2.290	12577	2.636	15.841
5000	0.50	74.2	2.348	12619	2.636	15.850
2000	0.25	14.8	2.295	12551	2.636	15.843
2000	0.50	29.7	2.290	12577	2.636	15.841
2000	0.67	39.7	2.296	12602	2.628	15.845
*2000	10.2	600	2.330	12565	2.50	15.14

Slope = 0.0002
n = 0.0149

All data from a station located 40,000' from the start.

*Implicit method

These results are presented in table 2.3 along with results obtained from the direct explicit method utilized in this report. Based on these results it can be seen that the method used in this thesis are less sensitive to change in ΔX and ΔT than the other methods reported by Amein (1968). This may be due to the size of binary word used by the computer. A large binary word will reduce errors due to truncation in the numerical calculation. Other differences may be explained as follows: Using a staggered mesh, which displaces the inflow hydrograph $\Delta X/2$ upstream, may explain why the peak is predicted at a slightly later time. Reducing the size of ΔX causes the peak to occur at an earlier time. Varying the time step has only a small effect on the time of the peak.

2.6 CONCLUSIONS

After studying the various numerical methods of solving the partial differential equations which describe unsteady flow, a direct explicit scheme was utilized to provide a precise data base for comparison of alternative flood routing methods. From tests to determine the stability and convergence properties of the finite difference formulation the following observations have been drawn:

1. To insure stability of the numerical solution, the size of the time increment must be reduced to a value which is smaller than

TABLE 2.3

COMPARISON OF RESULTS FOR FLOOD ROUTING METHODS

DEPTH AND TIME OF ARRIVAL OF PEAK FLOW
AT 200 MI STATION

Method	ΔX Mi	ΔT Hrs	Depth Ft	Time of Arrival Hrs
Explicit (As per Amein)	5	0.05	28.6	76.0
	5	0.10	28.6	76.0
	10	0.15	26.9	76.0
	10	0.20	26.0	76.0
Characteristics	--	0.2, 0.4 0.8, 1.2	29.0	76.0
	--	2.0	28.6	76.0
	--	2.5	26.9	76.0
		3.3	26.0	76.0
Implicit	5	0.5, 1.0, 1.5 2.0, 3.0	29.11	76.0
	5	5.0	29.13	76.0
	5	10.0	29.20	76.0
	10	0.5, 1.0 1.5, 2.0	29.11	76.0
Explicit (As used in this report)	5	0.10	29.18	76.70
	10	0.15	29.18	76.95
	10	0.20	29.18	77.00

that required to satisfy the Courant Condition. The Courant Number was used to define the size of the time step. This number, which is the size of the time step used divided by the time step implied by the Courant Condition appears to be inversely proportional to $\sqrt{S}/n$. As n increases, the Courant Number may also increase and as $\sqrt{S}$ increases, the Courant Number must decrease to insure stability.

While a smaller size of time step increased computation costs, it was still economically feasible to perform the required computations. Using a CDC 6400, only 36 seconds of central processor time were required to route a flood down the rectangular channel when it was divided into 25 sections each 2000 feet long and with Z = 0.5.

2. Sensitivity tests and theoretical analysis show that the solution of the equation is sensitive to the size of ΔX and ΔT. However, the variation of the solution was deemed sufficiently accurate for the purposes of this study. The variation in the peak value was less than 1%. Also, the variations between alternate flood routing techniques are on a larger order of magnitude.

3. Comparisons with other finite difference schemes, which have been successfully employed, show that the method used in this study compares very favourably in representing unsteady flow in a system with simple geometry.

Thus it was concluded that the numerical model will provide the data base which is required in the study of alternative flood routing methods.

CHAPTER 3

THE KINEMATIC SOLUTION

Kinematic flood routing is a generic term that identifies a class of methods for calculating the deformation of a flood wave as it passes through an open channel such as a river reach or man made conduit. These methods are based primarily upon the continuity equation and the assumption that there exists a single valued function relating flow rate to the physical properties of the channel. The popularity of these techniques can be attributed to the relative simplicity and low cost of obtaining solutions, especially before the widespread availability of the high speed digital computer lightened the computational load imposed by a complete solution of the momentum and continuity equations.

A literature review revealed numerous approaches to solving the continuity equation using either direct finite difference methods or characteristics techniques. This chapter attempts to provide a standard basis for comparison of the various algorithms by developing a general framework against which the different kinematic flood routing techniques may be compared. The first portion provides a review of the theoretical background of waves and shows the development of a general direct numerical method of kinematic flood routing. After discussing several special cases of kinematic flood routing and showing how they fit into the general method, a section is devoted to sensitivity tests of the numerical algorithm to determine limitations imposed by convergence properties, numerical stability, the degree of approximation and discretization errors.

Studies were carried out, by the writer, of the practical limitations of kinematic techniques in modelling physical systems and the validity of assuming single valued rating curves. These, however, are not discussed in this chapter but are reported later in the thesis. The chapter ends with several suggestions for further study and a summary of the results of various theoretical considerations of the general kinematic method.

3.1 THEORY

Seddon (1900) was one of the first to report on what are now termed kinematic waves. His observations of flood waves on the Mississippi River formed the basis of his classical report. The term "kinematic wave" was applied by Lighthill and Whitham (1955) in a paper which provided a thorough discussion of the theoretical background for the phenomena. This paper outlined the basic assumptions used in their study of kinematic waves.

Briefly these assumptions are as follows:

1. The channel flows may be assumed to be incompressible and one-dimensional, i.e., zero flow component normal to the flow direction.
2. For each point in the channel there exists a single valued relationship between the flow rate and the cross section area

The continuity equation which describes the phenomena is written in the following manner:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} = \bar{q} \tag{3.1}$$

Where: Q = flow rate

A = Area of cross section

$\bar{q}$ = rate of lateral inflow

x = distance

t = time

For the case of no lateral inflow, equation 3. 1 can be written as:

$$\frac{dQ}{dy}\frac{\delta y}{\delta x} + T_w \frac{\delta y}{\delta t} = 0 \tag{3.2}$$

Where: T_w = top width

y = depth of water

To an observer moving downstream with a speed equal to the velocity of propagation of the wave, both depth, y and discharge, Q will appear to remain constant. Thus the total derivative of depth dy/dt is zero. Therefore:

$$\frac{dy}{dt} = 0 = \frac{dx}{dt}\frac{\delta y}{\delta x} + \frac{\delta y}{\delta t} \tag{3.3}$$

Substituting for dy/dt in equation 3.2 yields:

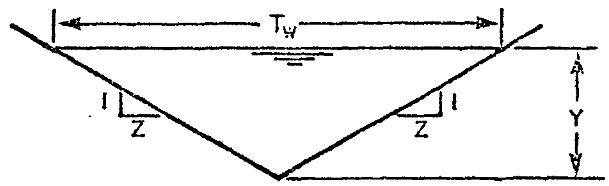
$$\frac{dx}{dt} = \frac{1}{T_w}\frac{dQ}{dy} = \frac{dQ}{dA} \tag{3.4}$$

That is, the wave velocity is equal to the slope of the curve relating flow rate and cross section area. (See Figure 3.1 for example.)

In Chapter 2, characteristic lines were defined as the paths in a space-time co-ordinate system along which changes in flow parameters may be described by ordinary differential equations. For flow situations described by the dynamic equation it can be shown that two sets of characteristic lines exist, being projected in the upstream and downstream direction respectively.

FIGURE 3.1

RELATION BETWEEN WAVE VELOCITY AND STAGE FOR A TRIANGULAR CROSS SECTION



AREA $A = ZY^2$

PERIMETER $P = 2\sqrt{Z^2+1}\;Y$

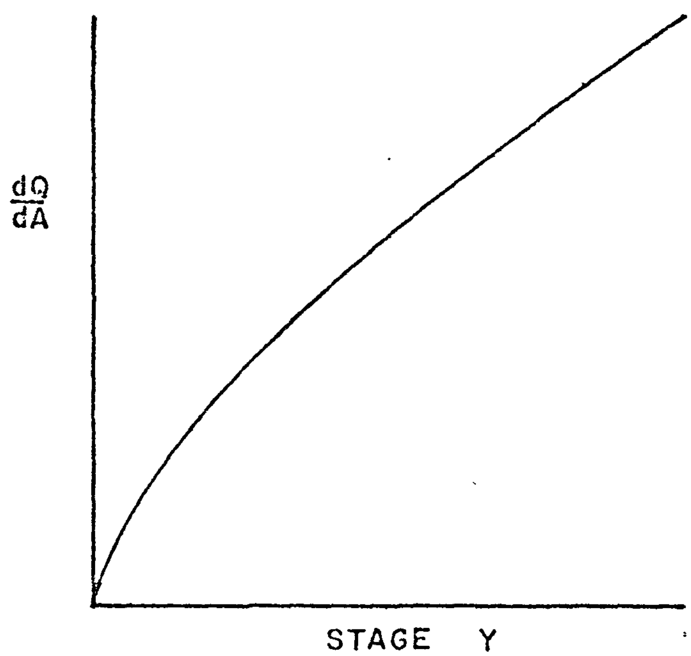
WIDTH $T_w = 2ZY$

BY MANNING:

$$Q = ZY^2 \frac{1.49}{n}\left(\frac{ZY^2}{2\sqrt{Z^2+1}\;Y}\right)^{\frac{2}{3}}\sqrt{S}$$

$$\frac{dQ}{dY} = \frac{8}{3}\,\frac{1.49}{n}\left(\frac{Z}{2\sqrt{Z^2+1}}\right)^{\frac{2}{3}}\sqrt{S}\;Y^{\frac{5}{3}}$$

$$\frac{dX}{dT} = \frac{dQ}{dA} = \frac{1}{T_w}\cdot\frac{dQ}{dY} = \frac{4}{3}\cdot\frac{1.49}{n}\left(\frac{Z}{2\sqrt{Z+1}}\right)^{\frac{2}{3}}\sqrt{S}\;Y^{\frac{2}{3}}$$



$$\left(\frac{dx}{dt}\right)^{+} = C^{+} = V + \sqrt{\frac{gA}{T_w}} \tag{3.5}$$

$$\left(\frac{dx}{dt}\right)^{-} = C^{-} = V - \sqrt{\frac{gA}{T_w}} \tag{3.6}$$

In the kinematic wave situation only one set of characteristic lines exists--those projected in the downstream direction along paths defined by

$$\left(\frac{dx}{dt}\right)^{+} = \frac{dQ}{dA} \tag{3.7}$$

From equation 3.4 it may be seen that wave velocity is a function of the depth of flow alone and therefore kinematic waves are non-dispersive and do not attenuate. However, they do change shape as a result of a variation of wave velocity with depth. The slope of the stage-discharge curves dQ/dA is usually steeper with higher flowrates. Thus higher flow rates (stages) move downstream faster than low flow rates. This variation in wave velocity can result in the intersection of characteristic lines from low flows and high flows. When this occurs a kinematic shock wave is formed. More discussion of kinematic shock waves is provided by Lighthill and Whitham (1955) and by Henderson (1966).

Lighthill and Whiteham also reported an additional criterion that must be satisfied in order for kinematic wave theory to apply. To prevent the formation of a hydraulic bore, the dynamic wave must attenuate. This attenuation will occur if the rate of change is given by:

$$\frac{dy}{dt} = \frac{gyoSo(2 - Fr)(1 + Fr)}{3Vo} \qquad (3.8)$$

Where: So = Bed slope

yo = Depth

Vo = Velocity

Fr = Froude number = $Vo/\sqrt{gyo}$

This imposes an upper limit on the rate of change of depth in the rising limb of the flood wave.

3.2 APPLICATION OF KINEMATIC WAVE THEORY

The application of kinematic wave theory can be carried out by using either (i) a characteristics solution or (ii) by the direct application of finite difference methods to the continuity equation. Characteristic solutions have been proposed by several authors. Lighthill and Whitham (1955); Henderson and Wooding (1964). These algorithms are very simple and efficient for systems with simple geometry and constant lateral inflow; the main advantage is that they may be solved using analytic techniques. The extension of characteristic methods to allow for variation of lateral inflow in time and space could be accomplished, but this would prohibit the use of analytic techniques for obtaining solutions.

Numerous direct finite difference methods have been proposed and implemented. Several of these are reported by Kibler and Woolhiser (1970) and Brakensiek (1967). Direct methods offer a very flexible algorithm which provides the best approach to the development of a general kinematic method suitable for sections of arbitrary geometry.

The continuity equation expressed earlier is:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} = \bar{q} \qquad (3.1)$$

and may be approximated by the finite difference expression.

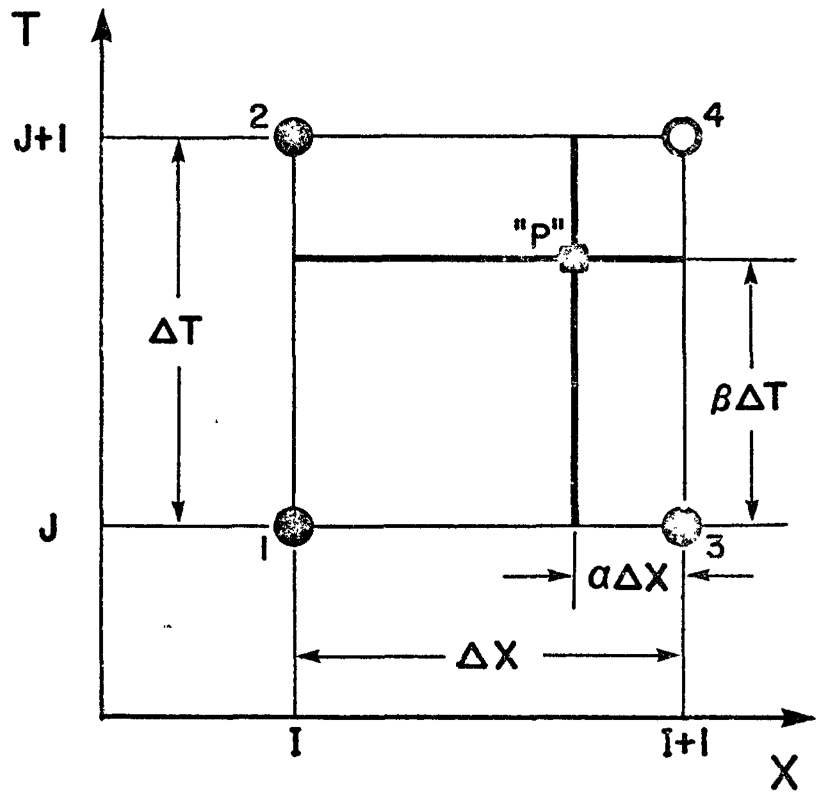
$$\frac{\Delta Q}{\Delta X} + \frac{\Delta A}{\Delta T} = \bar{q} \qquad (3.9)$$

Figure 3.2 shows a portion of a time-space diagram which is typical of those used in direct solutions of the continuity equation. In reviewing the approaches used by various workers, the basic difference appeared to be in the definition of the finite difference molecule. With the exception of the Lax-Wendroff method, all the techniques use

FIGURE 3.2

GENERAL FINITE DIFFERENCE SCHEME

GRAPHICAL REPRESENTATION



NUMERICAL APPROXIMATIONS

$$\frac{\delta A}{\delta T} = \frac{(1-\alpha)(A_4 - A_3) + \alpha(A_2 - A_1)}{\Delta T}$$

$$\frac{\delta Q}{\delta X} = \frac{\beta(Q_4 - Q_2) + (1-\beta)(Q_3 - Q_1)}{\Delta X}$$

MOLECULE THE SPACE-TIME ELEMENT BOUNDED BY POINTS 1243

NUCLEUS THE POINT "P" ABOUT WHICH THE FINITE DIFFERENCE EQUATION IS APPLIED a rectangular "molecule" defined by the ΔX and ΔT steps of the space-time grid. The equation is applied at a point somewhere within or on the boundary of the rectangle. A molecule was, therefore, chosen which allows the point of application of the continuity equation to be varied and defined by two parameters α and β as shown in figure 3.2. The algorithm based on this molecule provides a general framework which allows comparison with the other methods by the simple device of adjusting the values of the α and β parameters.

The general method used to describe points in the space-time co-ordinate system usually employs a double subscript notation as indicated in figure 3.2.

Thus the upstream points of the molecule are located at points I, J and I, J+1 while the downstream points are at I + 1, J and I + 1, J + 1. As a convenient short hand notation, the points are also numbered 1 through 4 as defined in figure 3.2 The unknown quantities are at location I + 1, J + 1 or in short hand form, point 4. Point "P" defines the location about which the continuity equation is applied, hereafter termed the nucleus.

Rewriting the continuity equation in terms of the approximations shown in figure 3.2 yields the following equation:

$$\frac{\beta(Q_4 - Q_2) + (1-\beta)(Q_3 - Q_1)}{\Delta X} + \frac{(1-\alpha)(A_4 - A_3) + \alpha(A_2 - A_1)}{\Delta T} = \bar{q} \quad (3.10)$$

Multiplying by ΔX and collecting unknowns leads to:

$$\beta Q_4 + (1-\alpha) A_4 \frac{\Delta X}{\Delta T} = (\beta - 1)(Q_3 - Q_1) + \alpha (A_1 - A_2) \frac{\Delta X}{\Delta T}$$

$$+ (1-\alpha) A_3 \frac{\Delta X}{\Delta T} + \beta Q_2$$

$$+ \bar{q} \Delta X \qquad (3.11)$$

Further rearranging gives:

$$\beta Q_4 + (1-\alpha) A_4 \frac{\Delta X}{\Delta T} = \beta Q_3 + (1-\alpha) A_3 \frac{\Delta X}{\Delta T} - Q_3$$

$$- \beta Q_1 + \alpha A_1 \frac{\Delta X}{\Delta T} + \beta Q_2 - \alpha A_2 \frac{\Delta X}{\Delta T}$$

$$+ Q_1 + \bar{q} \Delta X \qquad (3.12)$$

In this equation the two unknowns Q_4 and A_4 are defined implicitly. Another equation relating flow rate and cross section area at point 4 allows a solution to be obtained for Q_4 and A_4 by iteration or by means of a technique which employs functional relationships and interpolation.

These functional relationships are written as:

$$f(Q) = \beta Q - \alpha A \frac{\Delta X}{\Delta T} \qquad (3.13)$$

for the upstream points and

$$g(Q) = \beta Q + (1-\alpha)A\frac{\Delta X}{\Delta T} \qquad (3.14)$$

for the downstream locations.

Thus equation 3. 12 can be rewritten as:

$$g(Q_4) = g(Q_3) - Q_3 + f(Q_2)$$

$$- f(Q_1) + Q_1 + \bar{q}\Delta X \qquad (3.15)$$

The solution of the unknown is obtained by computing g(Q) from the known quantities on the right hand side of the equation 3. 15 and obtaining Q_4 by interpolation from a curve relating Q_4 and $g(Q_4)$. The solution may be continued either to the next molecule in the downstream direction or to the succeeding time increment. Thus the method may be used to define the conditions throughout the system at one time step or a complete time history can be determined for each elementary reach.

3. 3 A CONCEPTUAL MODEL

In comparing special cases of kinematic flood routing methods, a conceptual model is helpful in visualizing the relationship of one algorithm to another. The elementary reach which served as an example is shown in figure 3.3. Traditionally, storage in a channel is assumed to consist of two components, prism storage and wedge storage. Prism storage is considered to be a function of outflow whereas wedge storage is related to the amount by which inflow and outflow differ. However, in this study storage was thought of as two prisms. The storage in the downstream section is related to outflow, with storage in the upstream section a function of inflow. Storage in the upstream section is shown in figure 3.3 between sections "a" and "b" and is labelled ST_I. The volume of water stored in the reach between section "b" and "c" is labelled ST_O, and is a function of the outflow. Examining equation 3.12 reveals that it can be rewritten with the following substitutions.

$$\alpha A_1 \Delta X = ST_{I1} \quad (3.16)$$

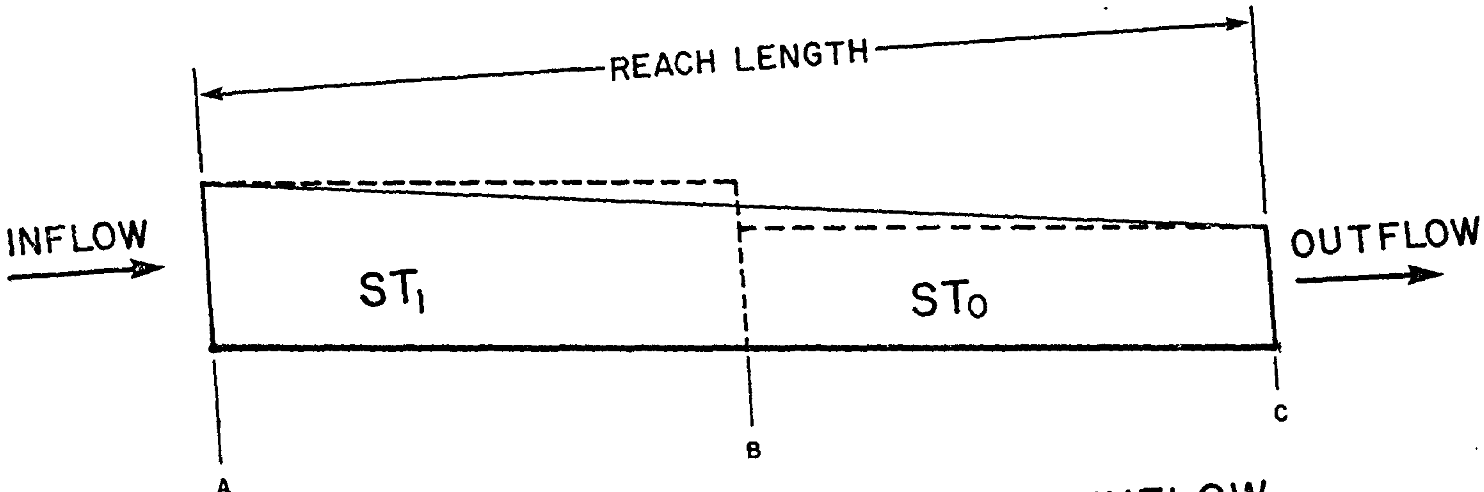
$$\alpha A_2 \Delta X = ST_{I2} \quad (3.17)$$

$$(1-\alpha) A_3 \Delta X = ST_{O3} \quad (3.18)$$

$$(1-\alpha) A_4 \Delta X = ST_{O4} \quad (3.19)$$

FIGURE 3.3

REPRESENTATION OF AN ELEMENTARY REACH



$ST_I = \emptyset_1(I) =$ STORAGE RELATED TO INFLOW

$ST_O = \emptyset_2(Q) =$ STORAGE RELATED TO OUTFLOW

Thus equation 3. 12 can be written as:

$$\beta Q_4 + \frac{ST_{04}}{\Delta T} = \beta Q_3 + \frac{ST_{03}}{\Delta T} - Q_3 - \beta Q_1 + \frac{ST_{I1}}{\Delta T} + Q_1$$

$$+ \beta Q_2 - \frac{ST_{I2}}{\Delta T} + \bar{q}\Delta X \qquad (3.20)$$

This shows the portions of the continuity equation which accounts for storage in the system and is helpful in considering two commonly used flood routing techniques which are special cases of the general kinematic method.

3. 3. 1 THE MUSKINGHUM METHOD AS A SPECIAL CASE OF KINEMATIC ROUTING.

The derivation of the Muskinghum method begins with the continuity equation expressed in the following manner:

$$I = O + \frac{\Delta ST}{\Delta T} \qquad (3.21)$$

employing the notation conventionally applied to the method,

I = Inflow

O = Outflow

ST = Storage

T = Time

Another equation relating the amount of water stored in an elementary reach to the inflow and outflow is defined in the following manner:

$$ST = K\bar{O} + Kx(I - O) \qquad (3.22)$$

The parameter K is a constant which relates storage to flow rate, while x is a factor which determines how much of the storage is related to outflow and how much is related to inflow. Prism storage is determined by the first term in the above equation and wedge storage is accounted for by the second term. This equation can be rewritten as:

$$ST = (1 - x)KO + xKI \qquad (3.23)$$

Setting the parameter x equal to zero makes storage a function of outflow alone. Storage is equally dependent on inflow and outflow when x = 0.5.

Expressing the change of storage ΔST as a function of the flows defined on the space-time diagram of figure 3.2 yields the following:

$$\Delta ST = (1-x)KQ_4 + xKQ_2 - (1-x)KQ_3 - xKQ_1 \qquad (3.24)$$

which may be expressed using expressions similar to those defined in equation 3.16 - 3.19.

$$\Delta ST = ST_{04} + ST_{I2} - ST_{03} - ST_{I1} \qquad (3.25)$$

Substituting equation 3.25 into equation 3.21 and defining inflow and outflow using the notation defined on figure 3.2 leads to:

$$\frac{Q_2 + Q_1}{2} + \bar{q}\Delta X = \frac{Q_4 + Q_3}{2} + \frac{ST_{04} + ST_{I2} - ST_{03} - ST_{I1}}{\Delta T} \qquad (3.26)$$

From which the following is obtained:

$$0.5Q_4 + \frac{ST_{04}}{\Delta T} = 0.5Q_3 + \frac{ST_{03}}{\Delta T} - Q_3 - 0.5Q_1 + \frac{ST_{I1}}{\Delta T}$$

$$+ Q_1 + 0.5Q_1 - \frac{ST_{I2}}{\Delta T} + \bar{q}\Delta X \qquad (3.27)$$

Comparing equation 3.27 with equation 3.20 reveals that they have a similar form. This shows that the Muskinghum method in a special case of the general method with the cross section area and flow rate related by a linear function which results in dQ/dA being constant for all values of Q.

The parameter x has the same meaning as the α parameter in the general method. Values of the parameters are:

$$\beta = 0.5 \qquad (3.28)$$

$$0.0 \leq \alpha \leq 0.5 \qquad (3.29)$$

3.3.2 RESERVOIR ROUTING AS A SPECIAL CASE OF KINEMATIC ROUTING.

The derivation of the numerical methods for reservoir routing begins with the same equation as is used for Muskinghum routing. That is:

$$I = O + \frac{\Delta ST}{\Delta T} \qquad (3.21)$$

The storage in a reservoir is usually a function of the outflow alone, but for completeness storage will be expressed as a function of inflow and outflow using the following equation:

$$ST = (1-\alpha)ST_O + \alpha ST_I \qquad (3.30)$$

In order for storage to be a function of only outflow, α must equal zero.

Thus:

$$ST = ST_O \qquad (3.31)$$

and

$$\Delta ST = ST_{O4} - ST_{O3} \qquad (3.32)$$

Substituting equation 3.32 into equation 3.21 and expressing the terms as defined on the space-time diagram of figure 3.2 yields:

$$\frac{Q_2+Q_1}{2} + \bar{q}\Delta X = \frac{Q_4+Q_3}{2} + \frac{ST_{O4}-ST_{O3}}{\Delta T} \qquad (3.33)$$

which can be expressed as:

$$0.5Q_4 + \frac{ST_{O4}}{\Delta T} = 0.5Q_3 + \frac{ST_{O3}}{\Delta T} - 0.5Q_1 + Q_1 + 0.5Q_2 + \bar{q}\Delta X - Q_3 \qquad (3.34)$$

The above equation is also similar in form to equation 3.20 with $\beta = 0.5$ and the storage independent of inflow. (i.e. $\alpha = 0.0$.)

Setting $\alpha = 0.0$, the functional relationships defined earlier (equations 3.13 and 3.14) become:

$$f(Q) = \beta Q \tag{3.35}$$

and

$$g(Q) = \beta Q + \frac{ST}{\Delta T} \tag{3.36}$$

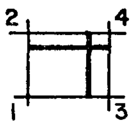
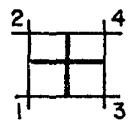
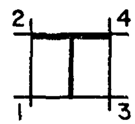
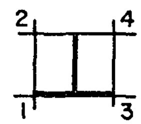
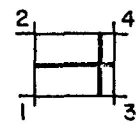
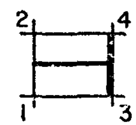
Thus the general kinematic method may be used to route a flood through a reservoir where the characteristics of the storage element can be defined as a function of outflow. Traditionally the value of β chosen for this type of analysis is 0.5.

3.3.3 FURTHER COMPARISONS.

Table 3.1 provides further comparisons of the general kinematic method with several other methods in addition to the two cases discussed earlier. The first of these methods (1, 2 and 3) were proposed by Brakensiek (1967) in a paper which reported the results of numerical experiments to determine the properties of the three formulations.

TABLE 3.1

COMPARISON OF FINITE DIFFERENCE SCHEMES

SCHEME	GENERAL	METHOD 1	METHOD 2	METHOD 3	MUSKINGHUM	RESERVOIR
MOLECULE						
α	$0.0 \le \alpha \le 1.0$	0.5	0.5	0.5	$0.0 \le \alpha \le 0.5$ (DEFINED AS X)	0.0
β	$0.0 \le \beta \le 1.0$	0.5	1.0	0.0	0.5	0.5
$\frac{\delta Q}{\delta x}$	$\frac{\beta(Q_4-Q_2)+(1-\beta)(Q_3-Q_1)}{\Delta x}$	$\frac{Q_4-Q_2+Q_3-Q_1}{2\Delta x}$	$\frac{Q_4-Q_2}{\Delta x}$	$\frac{Q_3-Q_1}{\Delta x}$	SAME AS METHOD 1	SAME AS METHOD 1
$\frac{\delta A}{\delta t}$	$\frac{(1-\alpha)(A_4-A_3)+\alpha(A_2-A_1)}{\Delta x}$	$\frac{A_4-A_3+A_2-A_1}{2\Delta x}$	SAME AS METHOD 1	SAME AS METHOD 1	SAME AS GENERAL METHOD	$\frac{A_4-A_3}{\Delta x}$

He found that the location of the nucleus has a marked affect on the behaviour of the algorithm and the solutions provided by the analysis. The three approaches can be modelled using the general kinematic method by defining α and β as follows:

Method 1

$\alpha = 0.5$

$\beta = 0.5$

Method 2

$\alpha = 0.5$

$\beta = 1.0$

Method 3

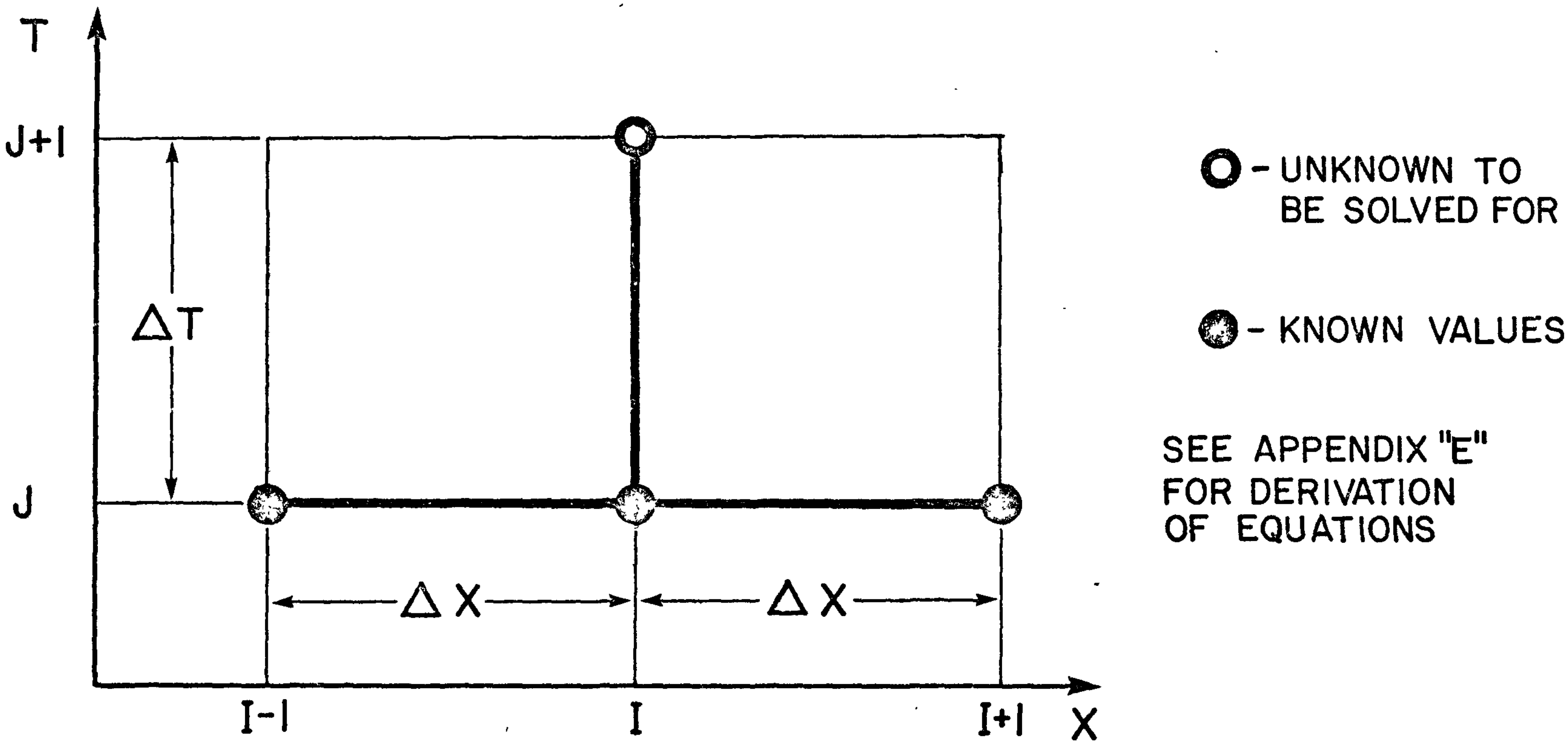
$\alpha = 0.5$

$\beta = 0.0$

Another approach to formulating a finite difference approximation to the continuity equation is known as the Lax-Wendroff method. This method employs four of the six nodal points defined by a double molecule and it is therefore not practical to provide a direct comparison with the general kinematic method. The arrangement of the molecule is shown in figure 3.4 and the finite difference equation is derived in Appendix E.

FIGURE 3.4

FINITE DIFFERENCE MOLECULE FOR LAX-WENDROFF METHOD



3.4 GENERAL SIGNIFICANCE OF THE α AND β PARAMETERS.

By detailed analytic comparison with reservoir and Muskinghum methods and by graphical representation and comparison with other methods as in table 3.1, it has been shown that most methods previously documented are special cases of the general method derived herein. By simple variation of the parameters α and β one is able to emulate the numerical behaviour of these other methods. It is, therefore, instructive to attempt to gain some insight as to the physical and numerical significance of varying α and β between their extreme values.

3.4.1 SPATIAL DERIVATIVES.

$\delta Q / \delta x$ is the gradient of Q with respect to distance and due to the wave passing along the channel, must clearly be different at time t = T and time t = T + ΔT.

The choice of β determines the manner in which the partial derivative $\delta Q / \delta x$ is approximated in terms of the nodal values. With β = 0.0, the gradient $\delta Q/\delta x$ is approximated using the conditions at time t = T. When β = 1.0 conditions at time t = T + ΔT are used to describe $\delta Q/\delta x$ and with β = 0.5 averages of the two previously mentioned values are used.

3.4.2 TIME DERIVATIVES.

The second term in the continuity equation $\delta A/\delta t$ describes the rate of change of cross section areas with respect to time. The way in which $\delta A/\delta t$ is specified by the physical conditions is determined by the value of the parameter α.

Using a value of $\alpha = 0.0$ the gradient $\delta A/\delta t$ is determined by conditions which exist at a position $x = X + \Delta X$ while setting $\alpha = 1.0$ will define $\delta A/\delta t$ by conditions at $x = X$. Using $\alpha = 0.5$ provides a value for $\delta A/\delta t$ which is a simple average of the conditions at the ends of the increment being considered.

The physical significance of describing $\delta A/\delta t$ at different locations within the molecule has been discussed previously in connection with the comparison of the Muskinghum method. Briefly, however, defining $\delta A/\delta t$ at the downstream location in the molecule ($\alpha = 0.0$ or $x = X + \Delta X$) describes the storage in the reach of length ΔX as a function of outflow alone. Similarly with $\alpha = 1.0$ storage is dependent only on the inflow. When $\alpha = 0.5$, $\delta A/\delta t$ is obtained as a simple average of the values at each end of the molecule and storage is equally dependent on inflow and outflow.

3.5 TESTING OF THE KINEMATIC FLOOD ROUTING TECHNIQUES

There are four properties which determine the suitability of a finite difference scheme in representing a differential equation.

These are:
1. Stability
2. Degree of approximation
3. Discretization errors
4. Convergence

Before implementing a finite difference algorithm, it is necessary to check the suitability of the method in each of these four respects. Each of these questions must be answered through engineering judgment based on mathematical analysis and experience. The next portion of this chapter provides some discussion of analytic investigations and numerical experiments which demonstrate the applicability of and differences between the alternative systems.

A portion of the following text is devoted to a discussion of each of the properties which describe the performance of a finite difference scheme.

3.5.1 STABILITY

A numerically stable procedure is a method in which small errors introduced into the calculations at a particular point in the x-t plane are not amplified as the solution is advanced through space and time. Errors caused by an unstable finite difference scheme will destroy the usefulness of the solution, thus the stability of the general kinematic method was the first item investigated. An analytic method of determining the stability of a finite difference scheme was successfully employed by several researchers, Walden (1973), Strelkoff (1970), Kibler et al (1970). This procedure, known as Van Neumann analysis, involves the investigation of a locally linearized version of the finite difference scheme on the assumption that the more complex non-linear system will behave in a similar manner to the linearized model.

Appendix A contains a detailed description of the various steps involved in determining a stability criterion. Briefly these steps are as follows. The finite difference scheme is first expressed in terms of the errors, which in turn are expressed as a Fourier Series. Because the system is linear the principle of superposition is applicable and only one component of the system need by examined at a time. If none of the harmonics of the Fourier Series are amplified in the succeeding computations, stability is attained.

The condition for stability may be expressed as follows:

$$\left|\frac{\bar{A}_4}{\bar{A}_3}\right| \leq 1.0 \qquad (3.37)$$

Where: $\bar{A}_4$ = Error of the unknown

$\bar{A}_3$ = Error of the value preceeding the unknown .

The results of this analysis are tabulated in table 3. 2 along with results by Kibler et al (1970) which provides the data for the Lax-Wendroff Scheme. These results indicate a trend toward decreasing stability as the nucleus moves upstream and toward earlier time levels. The worst condition was found to occur when the equation is applied so that the finite difference approximations are obtained by backward differences, that is when α = 1.0 and β = 0.0. The solution is unstable at this point regardless of the size of time and distance step used for computation.

Other points were located that provided solutions which were unconditionally stable as well as conditionally stable as shown in table 3. 2.

3. 5. 2 DEGREE OF APPROXIMATION

The degree of approximation provides a measure of how well the finite difference scheme represents the differential equation. If the differential equation is first order and finite difference schemes involve errors which are on the order of ΔX and ΔT, the finite differential equation may not converge to the differential equation as ΔX and ΔT approach zero.

TABLE 3.2

RESULTS OF STABILITY ANALYSIS

$$KN = C\frac{\Delta T}{\Delta X}$$

GENERAL METHOD:

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	$\infty \geq KN \geq 1$	$\infty \geq KN \geq \frac{3}{4}$	$\infty \geq KN \geq \frac{1}{2}$	$\infty \geq KN \geq \frac{1}{4}$	$\infty \geq KN \geq 0$
0.75	UNSTABLE	KN = 1	$2 \geq KN \geq \frac{2}{3}$	$3 \geq KN \geq \frac{1}{3}$	$4 \geq KN \geq 0$
0.50	UNSTABLE	UNSTABLE	KN = 1	$\frac{3}{2} \geq KN \geq \frac{1}{2}$	$2 \geq KN \geq 0$
0.25	UNSTABLE	UNSTABLE	UNSTABLE	KN = 1	$\frac{4}{3} \geq KN \geq 0$
0.00	UNSTABLE	UNSTABLE	UNSTABLE	UNSTABLE	$1 \geq KN \geq 0$

LAX-WENDROFF: $KN \leq 1$

Analyzing a finite difference scheme involves expanding the finite difference equation using a Taylor series and determining what terms are truncated. Appendix C shows the process by which the order of approximation of the general kinematic method was determined.

The investigation showed that the degree of approximation involves first and higher order terms in ΔX and ΔT as shown by the error terms in the modified continuity equation (3.37).

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + (2\alpha - 1)O(\Delta X) + (1 - 2\beta)O(\Delta T) + O(\Delta X^2, \Delta T^2) = \bar{q} \qquad (3.37)$$

The finite difference scheme provides the best approximation to the differential equation when $\alpha = 0.5$ and $\beta = 0.5$. When $\alpha = 0.5$ and $\beta = 0.5$, the equation for the finite difference representation of the differential equation becomes:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + O(\Delta X^2, \Delta T^2) = \bar{q} \qquad (3.38)$$

As the parameters α and β are varied and the nucleus moves off the centre point of the molecule, errors on the order of ΔX and ΔT are introduced. The absolute value of errors on the order of ΔX and ΔT introduced by varying these parameters increases linearly as the nucleus moves away from the centre of the molecule.

Kibler et al (1970) reported that the Lax-Wendroff method represented the continuity equation in the following manner:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + O(\Delta X^2, \Delta T^2) = \bar{q} \tag{3.39}$$

Thus the Lax-Wendroff method can be expected to behave in a manner similar to the general method with the equations applied to the centre of the molecule. However, the Lax-Wendroff molecule cannot be applied to obtain the downstream points on the time-space diagram because of the inverted "T" shape of the molecule. One of the other methods must be used for this portion of the grid.

3.5.3 DISCRETIZATION ERROR

Discretization errors are a result of replacing a tangent (differential) with a chord (finite difference). These discrepancies are often analyzed using several numerical experiments to determine the sensitivity of the solution to the size of increments used in the computation. As the size of the finite difference increases the accuracy of the aforementioned approximation decreases.

This type of study is closely related to the analysis of the degree of approximation. Thus, based on the results of the previous section it is reasonable to expect the computer solutions of the system to be sensitive to the position of the nucleus as well as to the size of time and distance steps used.

Chapter 2 contains a description of two physical systems which were used to test the finite difference scheme for solving the dynamic equations. The results reported in this section are based on the first system described in the previous chapter. The problem can be described as a wide, rectangular channel which is subject to an inflow defined by a trapezoidal hydrograph. The particular values used in the simulation were:

Width = 100'

Depth = 20'

Length = 50,000'

The inflow hydrograph is shown in figure 2.5.

A computer program was written which provided a solution using the general kinematic flood routing method. For each execution of this program, fixed values of ΔX and ΔT were specified, which the routine used to provide twenty-five solutions with various values of the parameters α and β. Several executions of this program provided the data necessary to carry out the sensitivity tests which demonstrated the effect of varying key parameters.

Tables 3.3, 3.4, and 3.5 summarize the results of three computer runs. The elements of the matrix define positions of the nucleus

TABLE 3. 3

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 1

40, 000 FT DOWNSTREAM

ΔT = 200 SECONDS

ΔX = 2500 FEET

β \ α	1. 0	0. 75	0. 50	0. 25	0. 00
1. 0	unstable	0. 927	0. 894	0. 867	0. 843
0. 75	unstable	unstable	0. 925	0. 892	0. 865
0. 50	unstable	unstable	0. 988	0. 924	0. 890
0. 25	unstable	unstable	unstable	0. 984	0. 922
0. 00	unstable	unstable	unstable	unstable	0. 981

TABLE 3.4

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 1

40,000 FT DOWNSTREAM

ΔT = 200 SECONDS

ΔX = 5000 FEET

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	unstable	0.969	0.894	0.844	0.804
0.75	unstable	unstable	0.924	0.865	0.821
0.50	unstable	unstable	0.965	0.890	0.840
0.25	unstable	unstable	unstable	0.920	0.861
0.00	unstable	unstable	unstable	0.961	0.886

TABLE 3.5

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 1

40,000 FT DOWNSTREAM

ΔT = 200 SECONDS

ΔX = 10,000 FEET

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	unstable	unstable	0.888	0.806	0.748
0.75	unstable	unstable	0.914	0.822	0.579
0.50	unstable	unstable	0.941	0.840	0.771
0.25	unstable	unstable	0.973	0.859	0.784
0.00	unstable	unstable	unstable	0.881	0.799

and each is characterized by a parameter equal to the peak flow divided by full bank flow. Only stable solutions are reported.

Examination of the results show that the peak value decreased as the value of α was reduced and/or as β increased. The rate at which the peak decreased or increased was related to size of the increments used in the computation.

This agrees with the results predicted by the analysis of the degree of approximation. As the size of the distance step increased, the peak value provided by the general method with the equation applied at a point where α was less than 0.5, decreased.

Similarly, for a fixed value of ΔX, the peak value provided by the solution decreased as α was varied from 0.5 towards 0.0. The decrease was not linear as predicted by the previous analysis. However, this prediction was based on a linear system. It is encouraging to note that the non-linear system does behave in a manner similar to the linear system.

Table 3.6 shows the ratios of peak outflow 40,000' downstream of the point of inflow obtained using the following fixed parameters:

$$\alpha = 0.5$$

$$\beta = 0.5$$

$$\Delta T = 200 \text{ seconds}$$

Values of ΔX used were 2,500, 5,000 and 10,000 feet.

TABLE 3.6

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 1

40,000 FEET DOWNSTREAM

ΔT = 200 SECONDS

α = 0.5

β = 0.5

ΔX (FT)	PEAK FLOW RATIO
2500	0.988
5000	0.965
10000	0.941

Comparing the peak flow rates at the outflow with the peak flow rate modified to account for the error in interpreting the hydrograph reveals that discretization errors and other types of inaccuracies have not caused any error of the peak flow ratio at the location where the outflow was recorded with $\Delta X = 2,500'$. However, when the size of ΔX was doubled to 5,000', a 2.4% reduction was made to the peak value. Furthermore, increasing the distance step to 10,000' introduced another 2.4% error to which is a total error of 4.8% from the value predicted by kinematic wave theory.

Thus, based on comparisons of the peak value of the outflow hydrographs, using a solution with the continuity equation centred in the middle of the molecule and time and distance steps of:

$$\Delta T = 200 \text{ seconds}$$

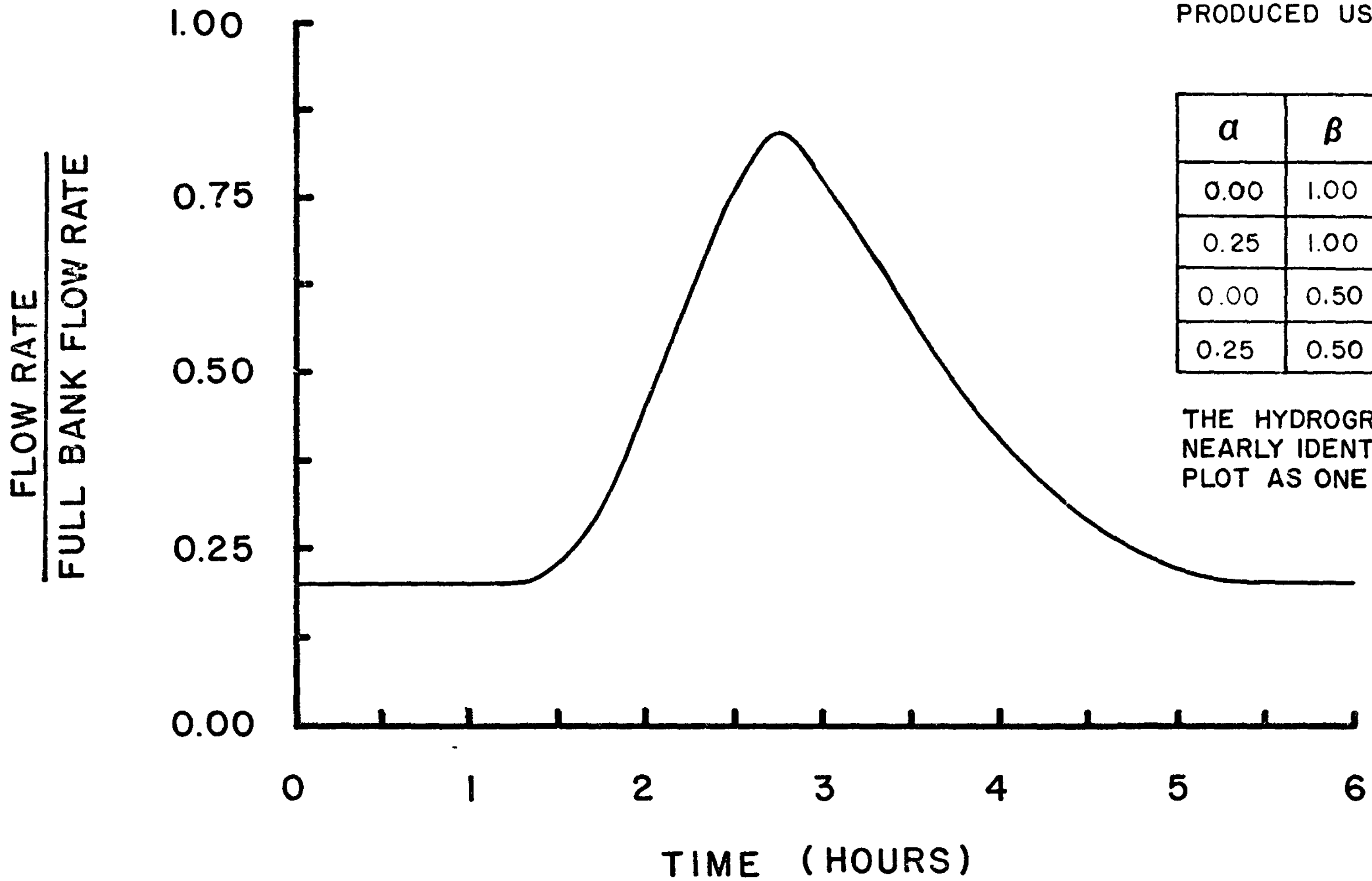
$$\Delta X = 2,500 \text{ feet.}$$

the solution appears to be free of discretization errors.

It may be argued that considering only the peak value of the outflow hydrograph does not give a complete picture of the behaviour of the solution in reproducing a particular shape of hydrograph. To compare the shape of the hydrographs, several plots were made of hydrographs which had peaks that were approximately equal.

These hydrographs are shown in figures 3.5 and 3.6. Only one hydrograph was plotted on each graph because the results from the

FIGURE 3.5
COMPARISON OF HYDROGRAPH SHAPES

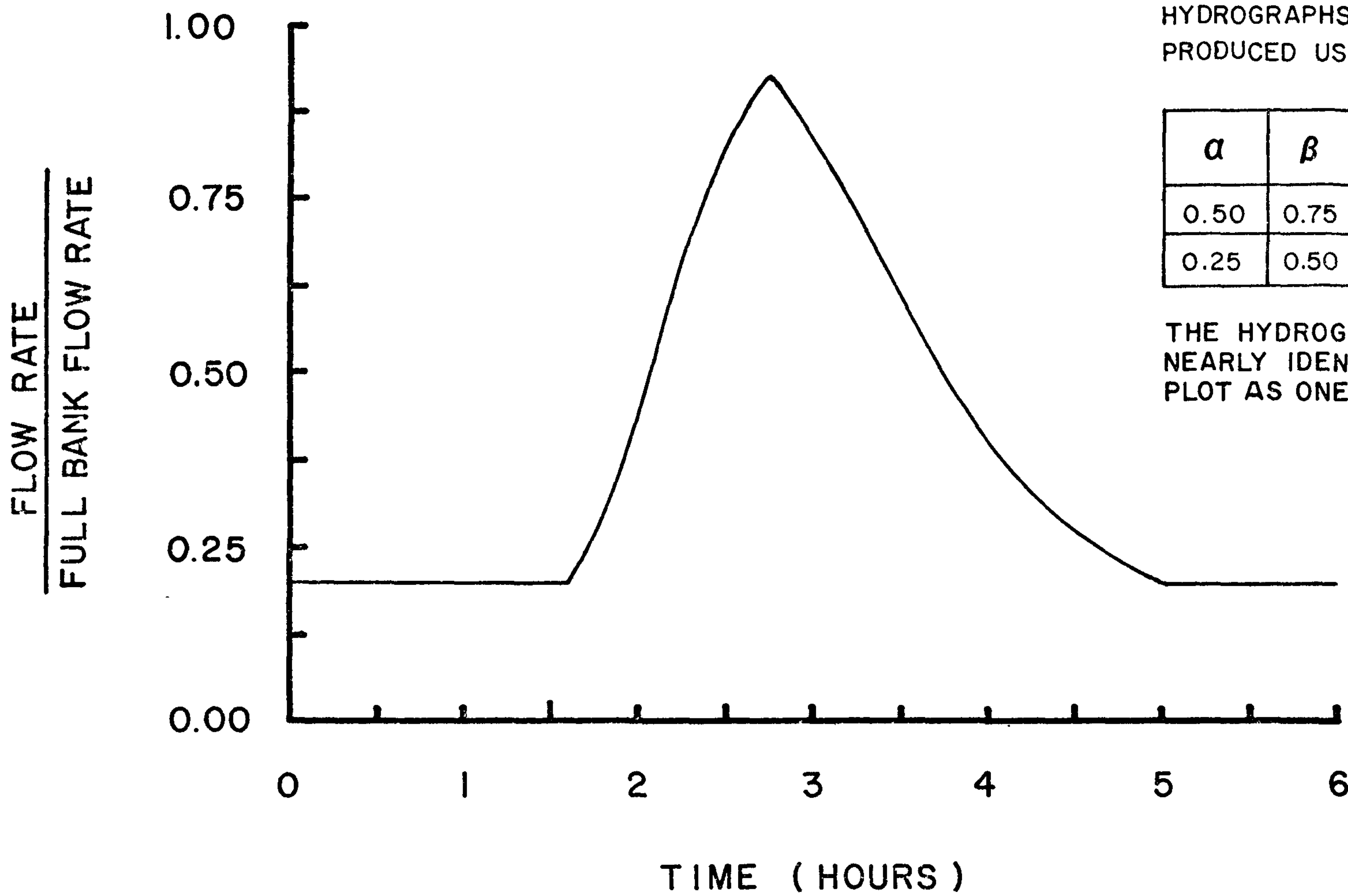


HYDROGRAPHS WERE PRODUCED USING:

α	β	ΔX
0.00	1.00	2500
0.25	1.00	5000
0.00	0.50	5000
0.25	0.50	10000

THE HYDROGRAPHS ARE NEARLY IDENTICAL AND PLOT AS ONE CURVE.

FIGURE 3.6
COMPARISON OF HYDROGRAPH SHAPES



HYDROGRAPHS WERE PRODUCED USING:

α	β	ΔX
0.50	0.75	2500
0.25	0.50	2500

THE HYDROGRAPHS ARE NEARLY IDENTICAL AND PLOT AS ONE CURVE.

different simulations compared so closely. The values of the parameters for these different simulations are shown on the plots.

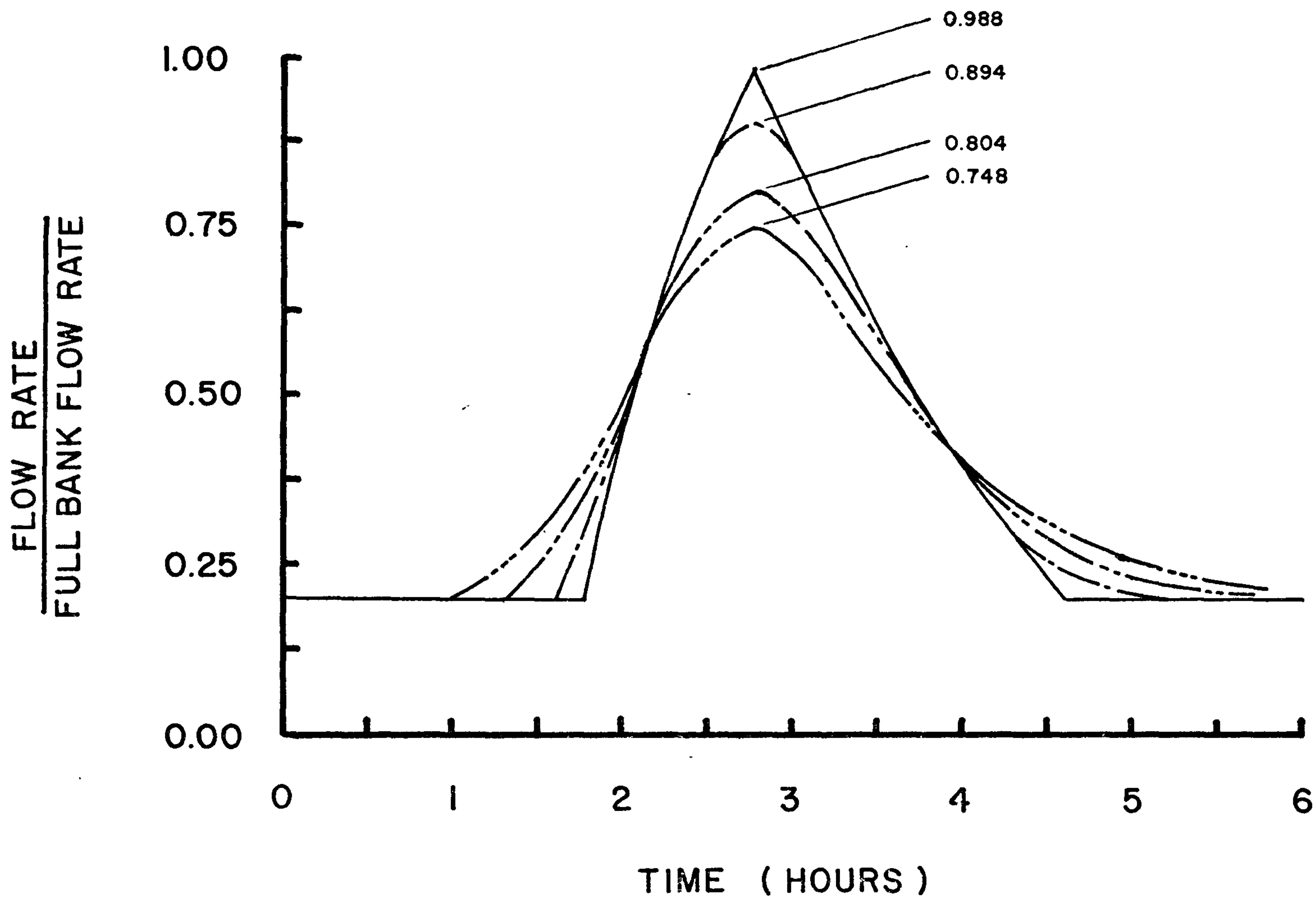
As a result of these comparisons, it was concluded that for the purposes of this study, comparing the peak value of the hydrograph was a reasonable method of quickly determining the sensitivity of the solution to changes in the size of increments used in the solution and to the location of the point where the continuity equation was applied to the finite difference molecule. To provide a visual method of correlating the peak value of the shape of the hydrograph, figure 3.7 is provided. This graph contains outflow hydrographs which have a range of peak values from 0.748 to 0.988, and shows the variety of shapes obtained for the outflow hydrograph. Generally, as the peak value reduced the rising limb of the hydrograph started earlier and the falling limb dropped less rapidly

3.5.4 CONVERGENCE

Convergence is a measure of how well a finite difference equation approximates the differential equation as the size of ΔX and ΔT approach zero. Finite difference schemes may be classified as convergent, conditionally convergent or non-convergent.

If the finite difference scheme converges to the differential equation as ΔX and ΔT approach zero, the scheme is termed

FIGURE 3.7
COMPARISON OF HYDROGRAPH SHAPES



convergent. A conditional convergent algorithm is one which only converges for particular values of ΔX and ΔT, while a non-convergent scheme will not accurately represent the differential equation regardless of the size of the finite difference steps.

Examination of the results of the degree of approximation analysis shows that the terms of the continuity equation are approximated by the finite difference scheme in the following manner:

$$\frac{\delta Q}{\delta x} = \frac{\delta Q}{\delta x} + \frac{1}{2}(2\alpha - 1)\Delta X \frac{\delta^2 Q}{\delta x^2} + \frac{1}{6}(1 - 3\alpha + 3\alpha^2)\Delta X^2 \frac{\delta^3 Q}{\delta x^3} + \frac{1}{2}(\beta - \beta^2)\Delta T^2 \frac{\delta^3 Q}{\delta x \delta t^2} \cdots \tag{3.40}$$

$$\frac{\delta A}{\delta t} = \frac{\delta A}{\delta t} + \frac{1}{2}(1 - 2\beta)\Delta T \frac{\delta^2 A}{\delta t^2} + \frac{1}{6}(1 - 3\beta + 3\beta^2)\Delta T^2 \frac{\delta^3 A}{\delta t^3} + \frac{1}{2}(\alpha - \alpha^2)\Delta X^2 \frac{\delta^3 A}{\delta x^2 \delta t} \cdots \tag{3.41}$$

These equations show that as ΔX and ΔT approach zero, the finite difference equation will converge to differential equation. Furthermore, convergence of general kinematic flood routing method is independent of the values of the parameters α and β and the relative sizes of ΔX and ΔT.

The fact that convergence is independent of α and β verifies a conclusion made by Dooge (1959) that a series of reservoirs can be used to model a kinematic channel; further discussion of this will be provided in a later chapter.

3.6 VERIFICATION OF THE GENERAL KINEMATIC FLOOD ROUTING METHOD

The previous sections have each dealt with a particular aspect of finite difference techniques which are used to gauge the performance of a numerical solution of a differential equation. This section provides a comparison of a solution obtained from a finite difference algorithm with the results of an analytic solution as verification of the numerical method.

The analytic solution was obtained in the following manner: Kinematic waves have one set of characteristics which travel downstream with a velocity.

$$\frac{dx}{dt} = \frac{dQ}{dA} \qquad (3.4)$$

Along a characteristic line, the flow rate Q is constant. Thus, associated with each flow rate there is a particular wave velocity. Knowing the wave velocity and the length between the point of inflow and the location where the outflow is being measured, the time between inflow and outflow for a particular value of Q can be obtained.

Using Mannings equation, the flow rate can be expressed in the following manner:

$$Q = \frac{1.49}{n} \frac{A^{5/3}}{P^{2/3}} \sqrt{S} \tag{3.42}$$

Where:
n = roughness coefficient

A = cross section area

P = wetted perimeter

S = slope

For the rectangular channel being studied:

$$A = T_w x\ Y \tag{3.43}$$

$$P = T_w + 2Y \tag{3.44}$$

Where:
T_w = top width

Y = depth of flow

From which:

$$\frac{dQ}{dA} = \frac{1.49\sqrt{S}}{n T_w} \frac{d}{dY}((T_w Y)^{5/3}(T_w + 2Y)^{-2/3}) \tag{3.45}$$

$$= \frac{1.49\sqrt{S}}{n T_w}\left(\frac{5}{3}\frac{T_w^{5/3} Y^{2/3}}{(T_w+2Y)^{2/3}} + (T_w Y)^{5/3}(-\frac{4}{3})(T_w+2Y)^{-5/3}\right) \tag{3.46}$$

$$= \frac{Q}{T_w}\left(\frac{5}{3Y} - \frac{4}{3P}\right) \tag{3.47}$$

A computer program was utilized to solve for flow rate and kinematic wave velocity as a function of depth. Using the results of this program, a curve of wave velocity vs. flow rate/full bank flow rate was plotted. This is shown in Appendix F along with a listing of the computer program and results.

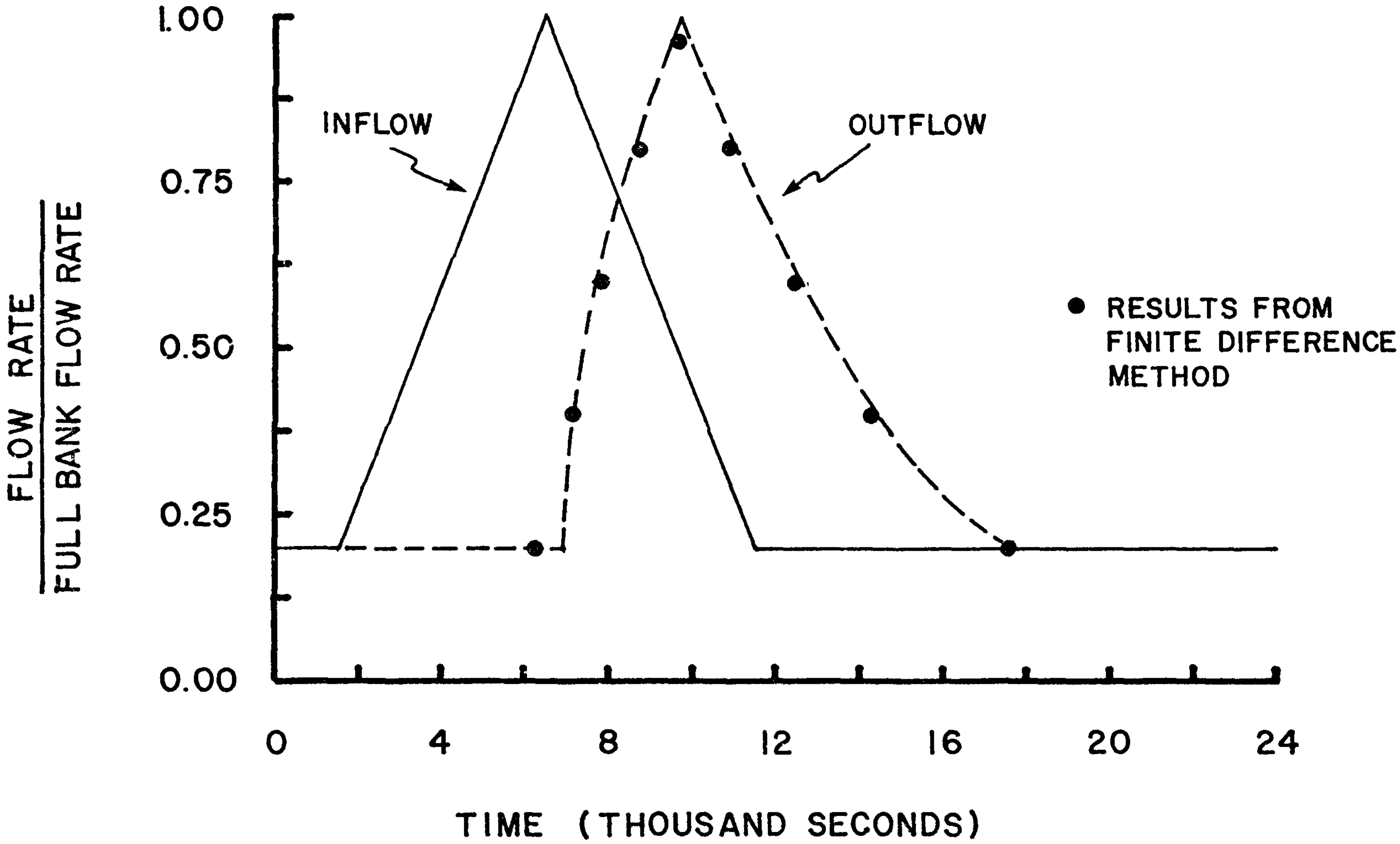
The inflow time of a particular flow rate was determined, and the wave velocity of the flow rate obtained from the graph. Multiplying the wave velocity by the length of the channel and adding the time of inflow, provided the time at which outflow of the particular flow rate would occur. By repeating this operation for a number of values in the range of flows specified by the inflow hydrograph, the outflow hydrograph was obtained.

Figure 3.8 shows a comparison of results obtained using both the analytic technique and the general kinematic flood routing method. These results compare very favourably and verify the useful ness of the general kinematic flood routing method in simulating the particular system under consideration when $\alpha = 0.5$ and $\beta = 0.5$.

3.7 CONCLUSIONS

This chapter has provided a discussion of kinematic wave theory and has shown the development of a general method of routing a flood using kinematic wave theory.

FIGURE 3.8
RESULTS USING CHARACTERISTICS METHOD SYSTEM I



With the framework provided by the general method of kinematic flood routing, it has been possible to compare several commonly used methods of flood routing. These techniques include reservoir routing, Muskinghum routing and several methods proposed by Brakensiek (1967).

The main feature of the general kinematic method, which allows flexibility in the choice of simulation method, is the ability to move the nucleus to any location on or within the boundaries of the finite difference molecule. This point is specified by two parameters α and β. The value of α specifies the position of the nucleus in the space domain while β locates the point in time. Most routing techniques utilize a finite difference molecule with $\beta = 0.5$. The parameter α is set equal to zero when a reservoir routing technique is used. When the widely known Muskinghum method is simulated, β is set equal to 0.5 and α is given the value of the Muskinghum weight factor X.

Analytic studies and numerical experiments have been employed to determine the numerical behaviour of the general kinematic method. The two parameters α and β have a very pronounced effect on the solutions provided with this method. Numerical errors and apparent attenuation may result depending on the values chosen for the α and β parameters. Amplification of the wave, as predicted by theoretical studies, was not reproduced by the numerical experiments due to a peculiarity of the computer program which constrained the wave to full bank flow conditions. Further study should be undertaken utilizing a routine free from this constraint so that the theoretical predictions of amplification may be verified.

Stability also has been found to be dependent on the choice of α and β . For example, with the molecule defined to route a flood using the reservoir technique, the scheme is conditionally stable. However, when the space and time derivatives are evaluated using a backwards differencing method ($\alpha = 1.0$, $\beta = 0.0$), the algorithm is unstable regardless of the size of ΔX or ΔT used in the simulation. A typical location of the nucleus, which defines a conditionally stable routine, is found at the centre of the molecule ($\alpha = 0.5$, $\beta = 0.5$).

With other properties being affected by the choice of α and β , it is reasonable to expect the degree of approximation of the finite difference scheme to be sensitive to these two parameters. This has been verified through analysis of the degree of approximation. Introductions of errors on the order of ΔX and ΔT appear to be the cause of the pseudo-attenuation and amplification mentioned earlier. The amount of attenuation is related not only to the choice of α and β , but also to the size of the finite space and time steps used in the simulation. It has been shown that the best representation of a kinematic wave in an open channel is obtained by locating the nucleus in the centre of the finite difference molecule. With α = 0.5 and β = 0.5, the degree of approximation contains only second order and higher terms, no first order errors being introduced into the computations. This is comparable with the accuracy obtained using the Lax-Wendroff method.

Having studied the numerical properties of the general kinematic flood routing method, the next direction for further regard may be a comparison of the kinematic solutions with the more rigorous dynamic method.

From these comparisons, it may be possible to determine guidelines for choosing appropriate values of α and β so that the properties of the general method may be more efficiently utilized in the simulation of dynamic flow systems. In addition, information may be obtained to determine the practical limitations of kinematic techniques.

CHAPTER 4

COMPARISON OF COMPLETE AND KINEMATIC SOLUTIONS

The two previous chapters have each dealt with a particular method of flood routing. The solution of the momentum and continuity equations describing unsteady flow was discussed in Chapter 2, and tests showed that it provided reasonable answers when modelling unsteady flow in open channels with simple geometry. Chapter 3 presented the theory and applications of kinematic methods used to simulate unsteady flow phenomena. The question now arises, " Which method is appropriate for the analysis of a specific physical system? "

In an attempt to answer this question, two different physical systems were modelled using both dynamic and kinematic analysis techniques. This chapter provides a comparison of these results and a discussion of the similarities and discrepancies of the results.

It has been pointed out in an earlier chapter that the dynamic method in general provides a more accurate answer than the kinematic solution. However, certain classes of problems may be identified in which the solutions obtained by both methods are not significantly different and it is not clear that any advantage results from the use of more rigorous dynamic methods. One of the objectives in this chapter will be to investigate criteria which may aid in the classification of such problems.

The approach taken to achieve these objectives may be outlined in the following manner. First, a review of the momentum equation and the various terms of that equation is made, and a method for performing an order of magnitude analysis of the terms is presented. Secondly, the two physical systems that were utilized in the study are introduced and the importance of the terms in the momentum equation are discussed. Thirdly, the results of the simulations are presented and discussed. Also, a critique of current discussion in the literature is provided along with the framework developed in this chapter.

In conclusion, a brief review of the chapter is provided and suggestions for further study are made.

4.1 ORDER OF MAGNITUDE ANALYSIS

Computation of flowrate in an open channel may be obtained typically by means of Manning's equation: i.e.

$$Q = A\,\frac{1.49}{n}\left(\frac{A}{P}\right)^{\frac{2}{3}}\sqrt{S_f} \qquad (4.1)$$

Where: Q = flowrate

A = Cross section area

P = Wetted perimeter

S_f = Slope of the friction line

n = roughness coefficient

If a steady flow regime in a prismatic channel is being considered and the flow is assumed to be uniform, then the slope of the friction line must be the same as the bed slope. However, when an unsteady flow system is being considered, the friction slope is dependent on spatial and temporal acceleration, the variation of flow depth along the channel as well as the bed slope.

This can be seen by examining the dynamic equation expressed earlier in this thesis. It is:

$$\frac{\delta h}{\delta x} + \frac{\delta}{\delta x}\left(\frac{Q^2}{2gA^2}\right) + \frac{1}{gA}\frac{\delta Q}{\delta A} + S_f = 0 \qquad (2.1)$$

Where: x = distance

t = time

h = water surface elevation

Q = flow rate

A = cross section area

g = acceleration of gravity

S_f = slope of the friction line

Expanding this equation leads to:

$$\frac{\delta y}{\delta x} - S_o + \frac{Q}{gA^2}\frac{\delta Q}{\delta x} - \frac{\delta y}{\delta x}\frac{Q^2 T_w}{g A^3} + \frac{1}{gA}\frac{\delta Q}{\delta t} + S_f = 0 \qquad (4.2)$$

Where: y = depth of flow

So = bed slope

T_w= surface width

The other variables have the same meaning as previously defined. Viewing each of the terms individually in the order they appear, they may be described as: (i) the rate of change of depth along the channel, (ii) the bed slope, (iii) the spatial acceleration, (iv) the rate of change of velocity head, (v) the temporal acceleration and (vi) the friction slope.

Solving for friction slope yields:

$$S_f = S_o - \frac{\delta y}{\delta x} + \frac{\delta y}{\delta x}\frac{Q^2 T_w}{g A^3} - \frac{Q}{gA^2}\frac{\delta Q}{\delta x} - \frac{1}{gA}\frac{\delta Q}{\delta t} \qquad (4.3)$$

This may also be expressed in the form

$$S_f = S_o - \frac{\delta y}{\delta x}(1 - F_R^2) - \frac{Q}{gA^2}\frac{\delta Q}{\delta x} - \frac{1}{gA}\frac{\delta Q}{\delta t} \qquad (4.4)$$

Where:

F_R = Froude number = $\dfrac{Q}{\sqrt{g\dfrac{A^3}{T_w}}}$

Substituting equation 4.4 into equation 4.1 gives

$$Q = A\frac{1.49}{n}\left(\frac{A}{P}\right)^{\frac{2}{3}}\sqrt{S_o - \frac{\delta y}{\delta x}(1-F_R^2) - \frac{Q}{gA^2}\frac{\delta Q}{\delta x} - \frac{1}{gA}\frac{\delta Q}{\delta t}} \qquad (4.5)$$

Equation 4.5 shows the terms mentioned earlier in this section which describe the slope of the friction line which in turn is used to compute the flowrate for an unsteady flow system.

When a kinematic flood routing technique is employed, flowrate is assumed to be a function only of y and x, that is no time variation of the stage discharge relation occurs. Expressing this in a slightly different manner; flow rate at any section of the reach is assumed to be dependent only on the stage. Thus, for a single value stage-discharge curve to adequately describe a physical system, so that a kinematic solution is appropriate, it appears that terms other than the bed slope must be relatively small.

Spatial acceleration $\dfrac{Q}{gA^2}\dfrac{\delta Q}{\delta x}$ and temporal acceleration $\dfrac{1}{gA}\dfrac{\delta Q}{\delta t}$ are functions of time because they vary as the flood wave passes through a section. However, variation of the spatial acceleration may also occur due to lateral inflow or outflow which may or may not be time dependent.

Similarly, the term describing the rate of change of specific energy due to a variation in depth with respect to distance will be a function of time as the flood passes, but it is also a description of the non-uniformity of flow under steady conditions. Thus, it is not necessary to assume that the single valued stage-discharge must be described by uniform flow conditions (i. e. S_f = So). (Henderson, 1966.)

For the present discussion, all the terms describing the slope of the friction line will be considered time dependent with the exception of the bed slope.

Henderson (1963) reported an analysis that was made to compare the relative order of magnitude of the four slope terms in an equation similar to equation 4. 5 using the kinematic wave as an approximation of the wave under consideration. The ensuing discussion follows the analysis presented by Henderson.

Defining kinematic wave velocity as c, equation 3. 3 may be written as:

$$\frac{dy}{dt} = \frac{\delta y}{\delta t} + C\frac{\delta y}{\delta x} = 0 \qquad (4.6)$$

This can be rewritten as:

$$-\frac{\delta y}{\delta x} = \frac{1}{C}\frac{\delta y}{\delta t} \tag{4.7}$$

For a system with a given inflow hydrograph Q = Q (t), equation 4. 7 can in turn be redefined in the following manner:

$$-\frac{\delta y}{\delta x} = \frac{1}{C^2}\frac{\delta y}{\delta t}\frac{\delta Q}{T_w \delta y} = \frac{1}{C^2}\frac{\delta Q}{T_w \delta t} \tag{4.8}$$

Henderson's analysis continues by considering a wide rectangular channel in which the flow resistance is defined by the Chezy equation. The following result was obtained:

$$\frac{\delta y/\delta x}{S_o} \;\alpha\; S_o^{-5/3} \times \frac{1}{q}\frac{\delta q}{\delta t} \tag{4.9}$$

Where: q = flow rate per unit width

When considering channels with arbitrary geometry and utilizing more complex resistance laws it is more difficult to obtain results in a form similar to equation 4. 9. However, the results shown by equation 4. 9 can also be deduced from the more general equation 4. 8. As the bed slope is increased. equation 4. 9 shows that the relative importance of $\delta y/\delta x$ will decrease. Similarly, as q increases or the rate of change of q with respect to time gets smaller, the importance of $\delta y/\delta x$ is reduced.

In general, c and T_w increase when flow rate increases; also, c increases when the slope is increased. Thus as slope or flow rate increase, the absolute value of the $\delta y/\delta x$ term will decrease. Decreasing the rate of change of flow rate with respect to time will also reduce the absolute value of the $\delta y/\delta x$ term.

Comparing the spatial acceleration $\frac{Q}{gA^2}\frac{\delta Q}{\delta x}$ with $\delta y/\delta x$ yields

$$\frac{\frac{Q}{gA^2}\frac{\delta Q}{\delta x}}{\frac{\delta y}{\delta x}} = \frac{Q}{gA^2}\frac{\delta Q}{\delta y} = O(F_R^2) \tag{4.10}$$

Where:

F_R = Froude number = $\dfrac{Q}{\sqrt{g\frac{A^3}{T_w}}}$

Also:

$$\frac{1}{gA}\frac{\delta Q}{\delta t} = O\left(\frac{C}{gA}\frac{\delta Q}{\delta x}\right) = O\left(\frac{Q}{gA^2}\frac{\delta Q}{\delta x}\right) \tag{4.11}$$

Thus the two acceleration terms $\frac{Q}{gA^2}\frac{\delta Q}{\delta x}$ and $\frac{1}{gA}\frac{\delta Q}{\delta t}$ are of the same order of magnitude and are of no higher order than $\delta y/\delta x$

unless $F_R >> 1$ which would occur only on extremely steep slopes such as mountain torrents.

In a system that has a channel with a very gentle bed slope, $F_R << 1$. Thus, while $\frac{\delta y}{\delta x} / S_o$ may be appreciable, the acceleration terms could be negligible.

This led Henderson to classify open channel, unsteady flow systems into three categories depending on the importance of the four slope terms.

The first of these categories was the steep sloped system. In this case, all the terms except the bed slope are negligible when describing friction slope. The second category consists of an intermediate sloped system in which all four of the terms defining friction slope are necessary to provide an accurate description of the flow rate. The third classification consists of a gentle sloped system where the acceleration terms are negligible and the friction slope is defined only by the bed slope So and $\frac{\delta y}{\delta x}$.

While these classifications are termed according to the bed slope, this is not the only factor which determines the relative importance of the various terms. These other factors include channel roughness, geometry of the channel, and the inflow hydrograph. It may be possible for a particular physical system to be classified as being steep if the inflow hydrograph rises and falls very slowly or it could be classified as intermediate if the hydrograph is very rapid and the flows are such that F_R^2 is large enough so that acceleration terms are important. If the flows are in a range where F_R^2 is low and the acceleration terms are negligible, but the changes in flow rate are rapid, the same system may be classified as gentle sloped. A later portion of this chapter describes an attempt to use these criteria to determine the importance of the various terms in the dynamic equation and to demonstrate the method of classifying open channel unsteady flow systems.

4.2 THE SYSTEMS STUDIED

The two examples used for the comparison of dynamic and kinematic flood routing techniques were very similar in some aspects, but quite diverse in other areas.

The first system has been discussed earlier in the sections dealing with sensitivity tests of the two methods of analysis, and comprised a wide rectangular channel having a downstream boundary condition of uniform flow and subject to a triangular inflow hydrograph as shown typically in figure 2.5 This system was used with several bed slopes. In each case a combination of bed slope and roughness was utilized which provided a consistent stage-discharge curve for all the results, i.e. $\sqrt{S_0}/n$ = constant. All comparisons were made at a section some distance upstream from the downstream boundary in an attempt to eliminate errors arising from the assumption of an unnatural downstream control.

The second system studied was taken from the problem first proposed by Thomas in 1934. Amein (1967) reports that the channel was assumed to be very wide with a sinusoidal inflow hydrograph starting from a base flow of 50 cfs/ft., and peaking at 200 cfs/ft. width. A time base of 96 hours was used with this hydrograph.

The main differences between the two systems were a difference of scale.

1. The physical length of the first problem is of the order of 1/40 that of Thomas' problem.
2. Although the flow rate per unit width of the inflow hydrographs were of the same order of magnitude, the first system employs a hydrograph with a time base of 2.77 hours compared to 96 hours for the second example. The effect of this is to greatly change the magnitude of the acceleration terms in the momentum equation.

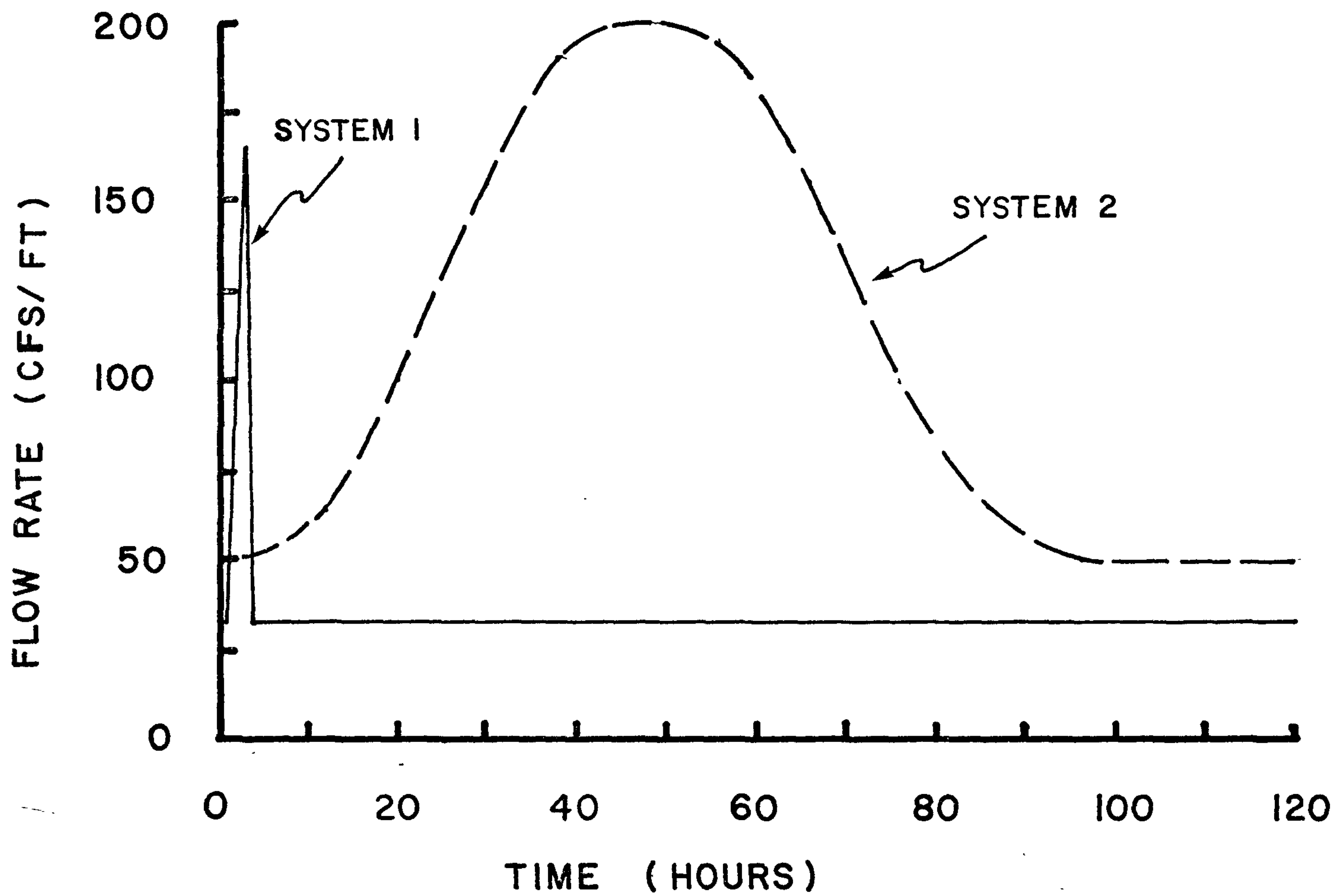
A concise description of the variables used for each problem is provided in table 4.1 A comparison of the inflow hydrographs is shown in figure 4.1

TABLE 4.1

COMPARISON OF SYSTEMS

VARIABLE	SYSTEM 1	SYSTEM 2
distance to outflow position	40,000 ft	300 miles
width	100 ft	very wide
depth	20 ft	30.1 ft
slope	0.0002 - 0.0100	1/5280
roughness	0.0149 - 0.1050	0.02985
INFLOW HYDROGRAPH		
shape	triangular	sinusoidal
time base	2.77 hrs	96 hrs
peak	166.7 cfs/ft	200 cfs/ft
baseflow	33.3 cfs/ft	50 cfs/ft

FIGURE 4.1
INFLOW HYDROGRAPHS



The result of an order of magnitude analysis for the two systems is shown in table 4.2 From this analysis, the systems may be classified in the following manner. When a bed slope of 0.0002 is used in System 1, all the terms of the momentum are of the same order of magnitude or greater than the bed slope, thus this system would be termed as an intermediate sloped system. With a bed slope of 0.001, the importance of the acceleration term is diminished and the system may be classified as either intermediate or gentle sloped, depending on what criterion is used to define terms as being significant. The simulation with the bed slope of 0.002 can be classified as gentle sloped because the acceleration terms are only approximately 5% of the bed slope and the $\frac{\delta y}{\delta x}$ term is still significant. This demonstrates the statement made earlier that bed slope is not the only criterion for determining how a system is to be classified. Because the roughness coefficient was increased as the slope increased, the gentle sloped system was encountered at a slope greater than the intermediate sloped system.

To demonstrate the results obtained for a steep sloped system, where the bed slope is much larger than the $\frac{\delta y}{\delta x}$ or acceleration terms, an execution of the program was made with So = 0.0100.

The order of magnitude analysis shows that System 2 would also be classified as a system with an intermediate slope, and from these

TABLE 4.2

ORDER OF MAGNITUDE ANALYSIS

So	$\left\lvert \frac{\delta y}{\delta x} \right\rvert$	$\left\lvert \frac{1}{gA} \frac{\delta Q}{\delta t} \right\rvert$	F_R^2	$\left\lvert \frac{\delta y}{\delta x} \right\rvert / So$	$\left\lvert \frac{1}{gA} \frac{\delta Q}{\delta t} \right\rvert / \left\lvert \frac{\delta y}{\delta x} \right\rvert$
SYSTEM 1					
0.0002	1.75×10^{-4} — 4.74×10^{-4}	0.41×10^{-4} — 1.16×10^{-4}	0.099 — 0.107	0.875 – 2.37	0.087 — 0.67
0.0010				0.175 – 0.474	
0.0020				0.0875 – 0.237	
0.0100				1.75×10^{-2} – 4.74×10^{-2}	
SYSTEM 2					
0.00019	2.4×10^{-3}	0.28×10^{-4}	0.035 – 0.045	1.28×10^{-1}	0.11×10^{-1}

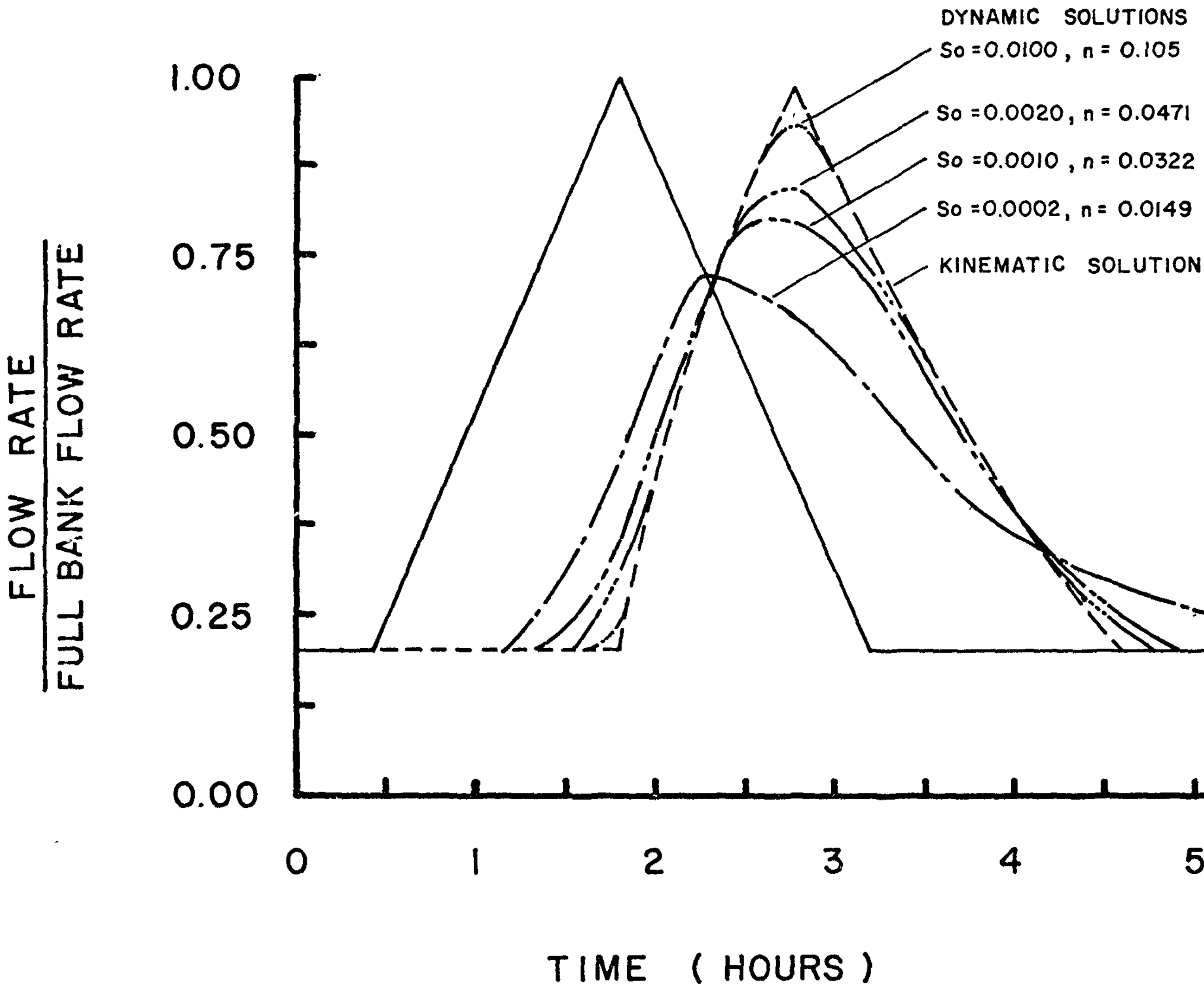
results, one would expect less attenuation of the flood wave in System 1 as the bed slope is increased. While a comparison between the two systems is difficult due to the differences of scale, one would expect a less marked attenuation of the flood wave in System 2 when compared with the attenuation obtained with System 1 using a bed slope of 0.0002, due to the relative sizes of $\frac{\delta y}{\delta x}$ and bed slope.

4.3 COMPARISON OF RESULTS FOR SYSTEM 1

The general kinematic flood routing technique with the nucleus located at the centre of the molecule $(\alpha = 0.5, \beta = 0.5)$ was used to obtain a kinematic solution for System 1. Because the ratio of bed slope and roughness was adjusted to maintain a constant ratio of $\sqrt{S_0}/n$, only one kinematic solution was required for the various bed slopes.

Figure 4.2 contains a comparison of the outflow hydrographs obtained from the four dynamic simulations of System 1, and from the kinematic solution. This clearly demonstrates the pronounced effect of the time variant terms on the resultant outflow hydrograph. The simulation of System 1 with a bed slope of 0.0002 shows the effect of terms which are of the same order of magnitude as the bed slope. In this case, the wave reached the outflow quicker than the other waves and subsided less rapidly. As slope increased, the attenuation of the

FIGURE 4.2
OUTFLOW HYDROGRAPHS 40,000' DOWNSTREAM



wave decreased and the wave form approached the shape predicted by kinematic wave theory.

Another demonstration of the effects of the terms in the dynamic equation can be seen by examining the stage-discharge curves in figure 4.3. With increased bed slope and roughness, the slope of the energy line was dominated by the bed slope term. Thus the hysteresis loop in the stage-discharge decreases as shown in the figure.

4.4 COMPARISON OF RESULTS FOR SYSTEM 2

An approach, similar to the one utilized for comparing the results of System 1, was used to compare the results of this system. However, hydrographs at two sections were recorded and peak values at a point half way between the two hydrographs were also printed to provide further comparisons. This allowed the results of this study to be viewed in conjunction with the results of two other studies, Amein (1967), and Garrison (1968).

A comparison of the hydrograph 100 miles downstream from the upper end of the channel as obtained by kinematic and dynamic solutions is shown in figure 4.4. The attenuation of the flood wave is apparent; but differences in the wave shape are relatively negligible.

However, by the time the flood wave has reached a point 300 miles downstream, there are considerable differences in the wave

FIGURE 4.3
STAGE-DISCHARGE CURVES
SYSTEM I 40,000' DOWNSTREAM

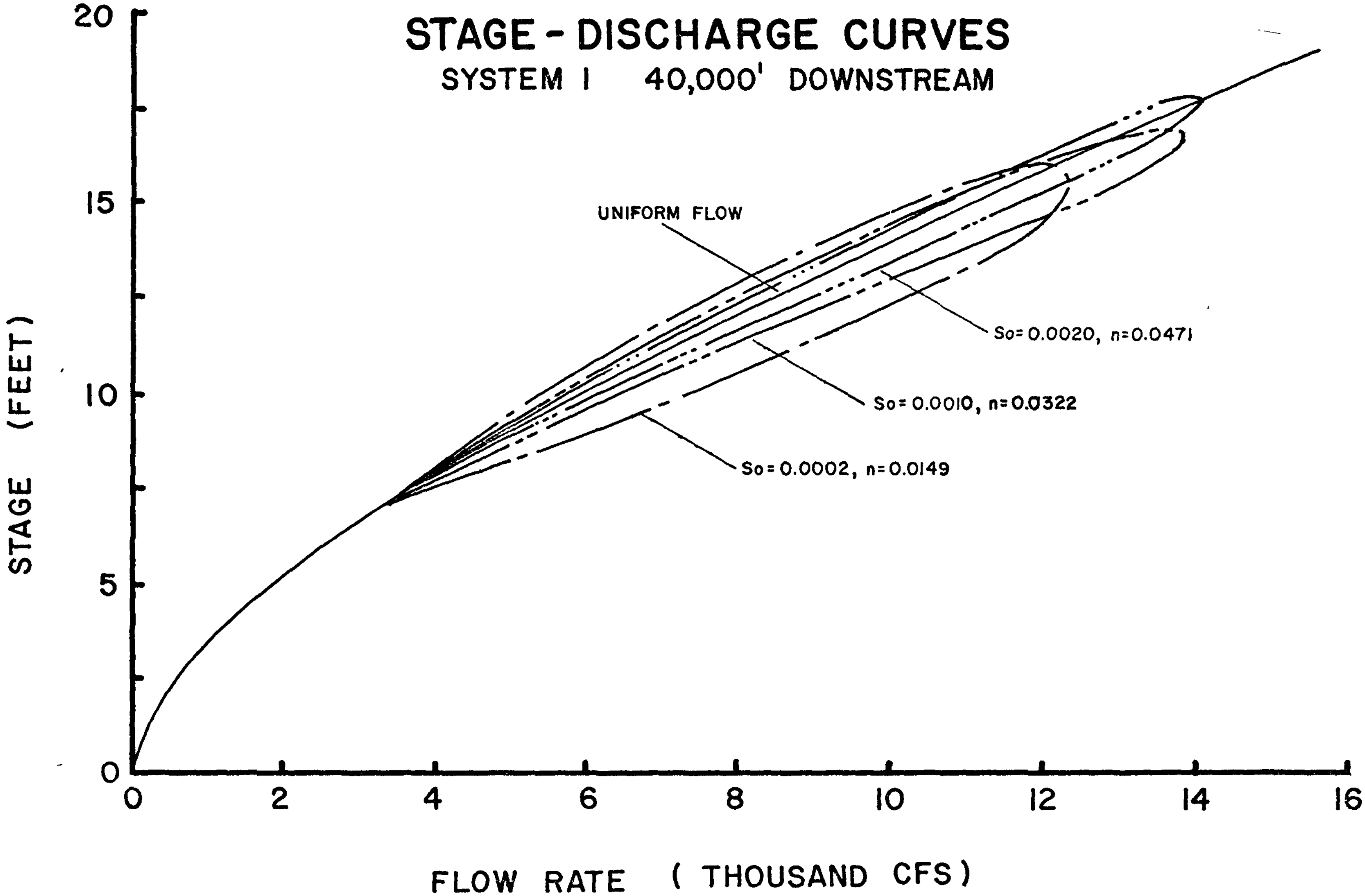
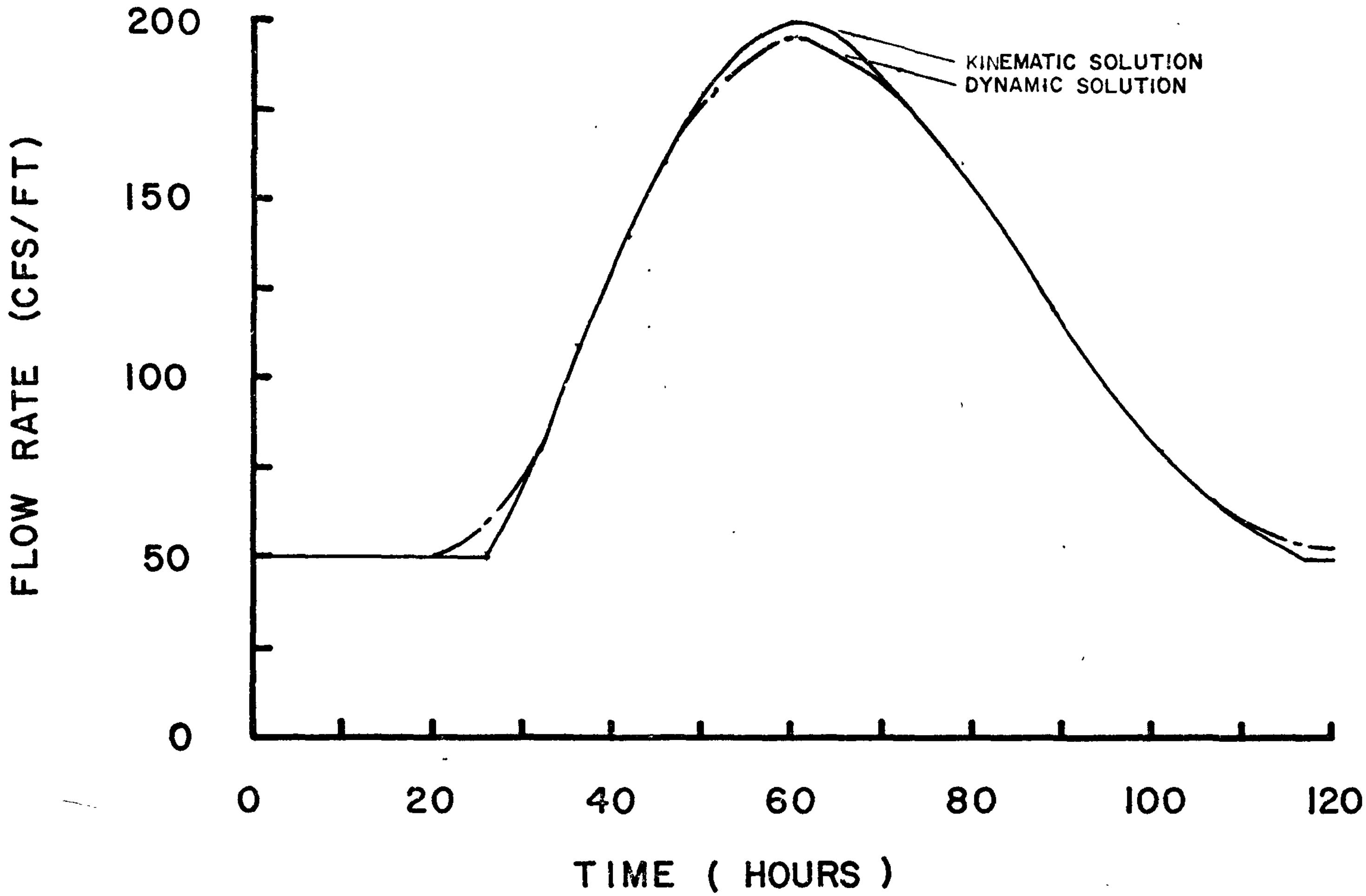


FIGURE 4.4
OUTFLOW HYDROGRAPHS
SYSTEM 2 100 MI. DOWNSTREAM



shapes resulting from the two methods of analysis as shown in figure 4.5. The wave front on the kinematic solution is much steeper and no attenuation has taken place. Conversely, the dynamic solution shows a wave which has steepened slightly, and has attenuated. The stage-discharge curve for this system 300 miles downstream is shown in figure 4.6.

From the order of magnitude analysis and a comparison with simulations of the first system, more attenuation may have been expected because of the high value associated with the $\frac{\delta y}{\delta x}$ term. However, this may be explained by considering the manner in which the order of magnitude of the $\frac{\delta y}{\delta x}$ term was obtained. In System 1, the trapezoidal inflow hydrograph provided a rate of change of flow rate $\frac{\delta Q}{\delta t}$ that was constant when the hydrograph was rising or falling. With the sinusoidal inflow hydrograph, $\frac{\delta Q}{\delta t}$ varied with time and the maximum rate of change of flow rate was used to determine the order of magnitude of the $\frac{\delta y}{\delta x}$ term. During the major portion of the simulation, $\frac{\delta y}{\delta x}$ was actually much less than the value specified in table 4.2. This may account for the relatively small amount of attenuation demonstrated in the results of this simulation.

4.5 FURTHER COMPARISONS

Kinematic solutions of unsteady open channel flow problems were first utilized by engineers and researchers before the advent of high

FIGURE 4.5
OUTFLOW HYDROGRAPHS
SYSTEM 2 300 MI. DOWNSTREAM

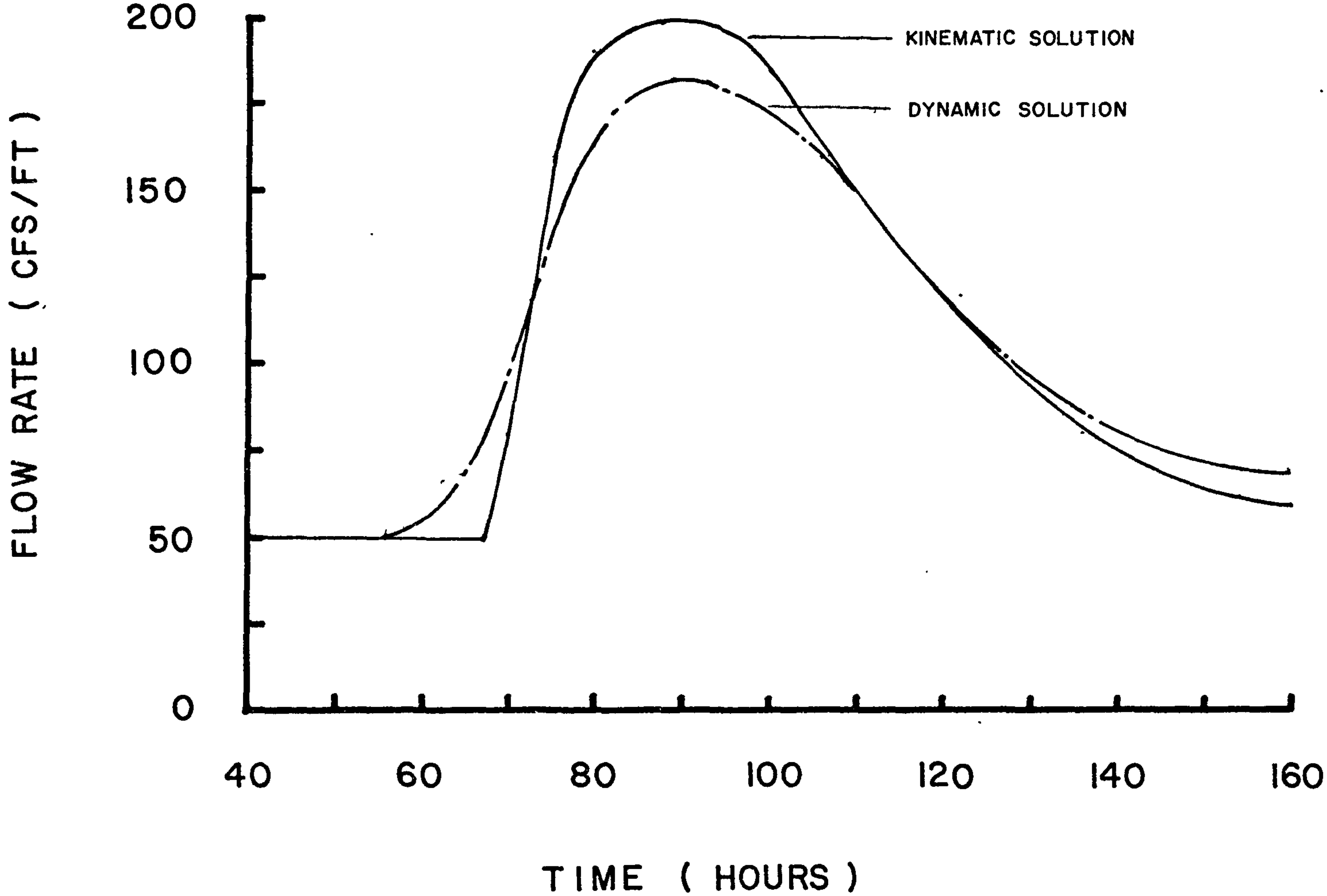
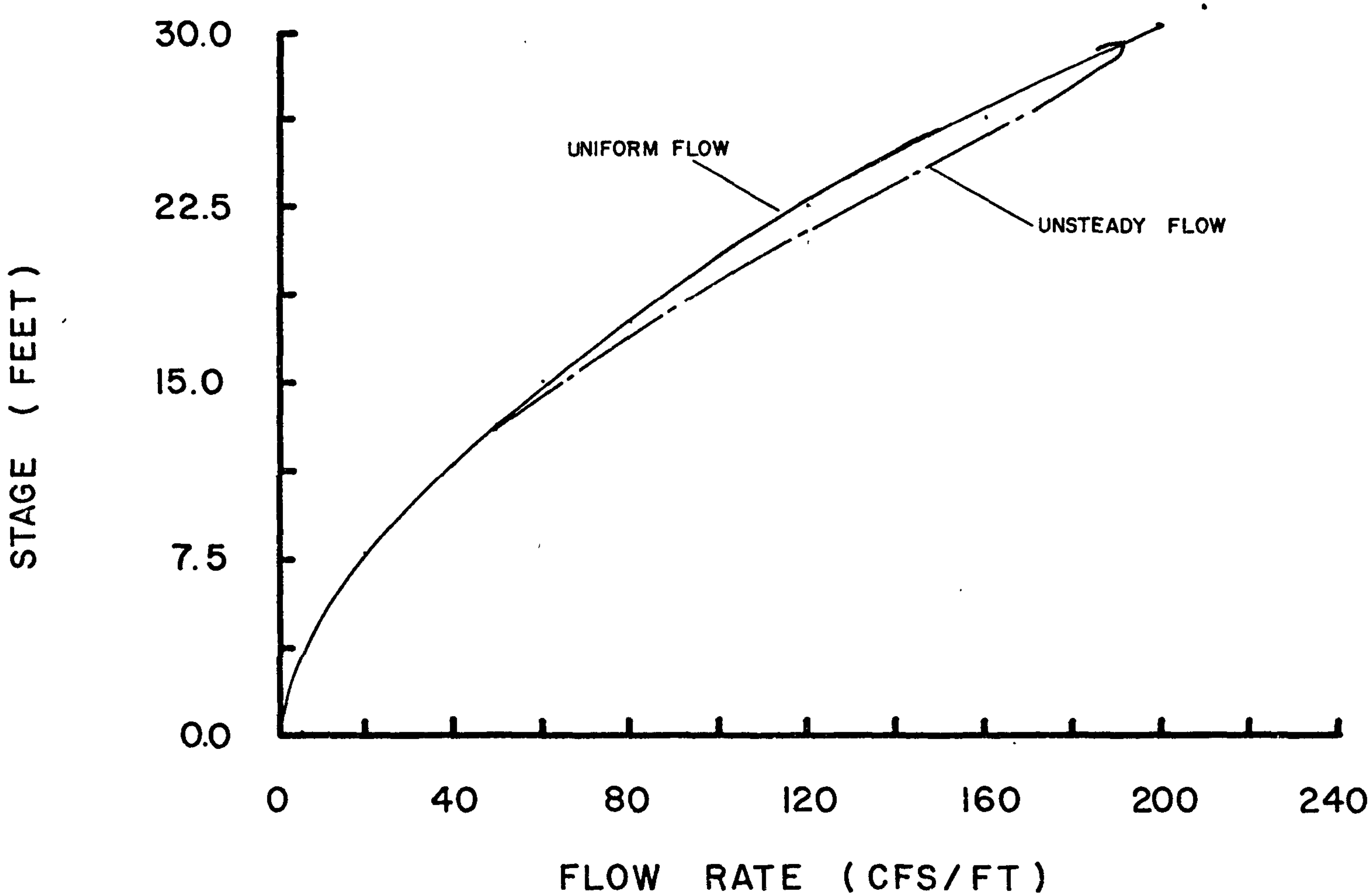


FIGURE 4.6

STAGE-DISCHARGE CURVE

SYSTEM 2 300 MI. DOWNSTREAM



speed digital computers due to the enormous amount of calculations required to solve the dynamic equations. While these shorthand methods reduced the computational load, it was still necessary to have large increments in time and space to reduce the number of calculations required for the numerical solution by a kinematic technique. The use of step sizes which were too large may have provided solutions which were not realistic or which gave results that were extremely sensitive to the step sizes in time or distance.

Recording the peak values of depth and flowrate and the times at which these maximums occurred at a section 200 miles downstream in System 2, allowed a comparison to be made with results of the studies by Amein (1968) and Graves (1967). Table 4.3 contains a table which shows a comparison of peak values for flowrate and depths obtained using the various techniques.

When a kinematic solution was used with the nucleus in the centre of the molecule, no attenuation resulted. Moving the nucleus to the downstream boundary so that the channel was simulated by a series of reservoirs, caused some attenuation of the flood wave. The size of ΔX dictated the amount of attenuation that was manifested in the kinematic solution with the nucleus located off of the centre of the molecule. There appears to be considerable difference in the timing of the peak values as the sizes of ΔX and ΔT were varied. This,

TABLE 4.3

COMPARISONS TWO HUNDRED MILES DOWNSTREAM SYSTEM 2

METHOD	ΔX (MILES)	ΔT (HOURS)	FLOW RATE PEAK (CFS/FT)	FLOW RATE TIME (HOURS)	DEPTH PEAK (FEET)	DEPTH TIME (HOURS)
KINEMATIC $\alpha = 0.50$ $\beta = 0.50$	10	2	199.9	75.1	30.06	75.1
		3	200.0	74.6	30.07	74.6
	20	2	200.2	74.8	30.07	74.8
		3	200.0	74 8	30.06	74.8
KINEMATIC $\alpha = 0.00$ $\beta = 0.50$	10	2	194.2	75.1	29.54	75.1
		3	194.4	75.2	29.55	75.2
	20	2	188.0	75.8	28.97	75.8
		3	188.1	75.9	28.98	75.9
EXPLICIT DYNAMIC SOLUTION	10	0.15	191.0	75.1	29.18	76.9
STORAGE ROUTING AMEIN (1968) METHOD PROPOSED BY THOMAS(1934)	25	12	—	—	29.2	78.0
		24	—	—	27.8	90.0
	50	12	—	—	27.7	78.0
		24	—	—	26.0	84.0
GRAVES (1968)	50	1	193.9	76.0	29.5	77.6

however, is due to the relatively flat top of the wave which resulted in the peak being ill defined. The depth did not vary by more than a few hundredths of a foot several time steps before or after the peak. The average value of the time of peak is 75.25 hours and the values obtained for individual runs do not vary from this value by more than one time step. The results obtained with the kinematic solution also compare favourably with the answers provided by the finite difference solution of the dynamic equations. While the attenuation was not accurately modelled, the times to the peak values compare reasonably well.

Amein (1968) reports the results of Thomas' storage-routing technique in a paper describing an implicit finite difference solution of the dynamic equation. He pointed out that these results seem to indicate that the storage routing technique is unacceptable due to the variation of the values recorded for peak stage and the time of peak. These problems may be symptoms of using large steps resulting in the solution becoming unstable. In all of the cases cited, the time of peak did not vary from the peak predicted using the dynamic solution by more than one time step. Thus the kinematic and the dynamic solutions are similar in that there are maximum sizes of ΔX and ΔT that can be used to achieve satisfactory results.

Graves (1967) used a flood routing technique based on the continuity equation and a stage discharge curve that was a function of the steady flow conditions and the rate of change of flow with respect to time. This type of solution is neither kinematic or dynamic in a strict theoretical sense. However, it is noteworthy in that it is an example of methods which attempt to bridge the gap between kinematic solutions and the complete solution of the dynamic equation. The results of his study show a peak flow rate of 193.9 cubic feet per second per foot channel width occurring at a point 200 miles downstream 76 hours after the start of the simulation. The maximum stage was 29.46 feet 78 hours after the start of the simulation. These results compare quite favourably with the dynamic solution.

In making these comparisons, two problems have been manifested which are worthy of further discussion. First, when comparisons of results are obtained using numerical flood routing models, it is sometimes advantageous to compare stages while other situations warrant a comparison of flow rates. In reporting results of models which do not assume a single valued stage-discharge relation, it may be expedient to report both stage and flow rates. The other problem encountered dealt with the size of the time step used in the computation. Difficulties associated with using large step sizes in a finite difference scheme are well documented and are usually guarded against. However, the type of flood wave that was encountered in System 2 had a relatively flat top which made locating the true time of peak a difficult task.

Using smaller time steps would reduce this uncertainty; but would increase the computation costs. Another approach to overcoming this problem would be to fit a curve to the data points in the region of the peak using either graphical methods or a numerical technique. The latter angle of attack was used with data where the peak did not appear to fall close to one of the time increments. A second order curve was utilized in this attempt. The equation has the form:

$$Q = a + bt + ct^2 \qquad (4.12)$$

Where: a, b, c, = coefficients determined from the data

Q = flowrate

t = time

The results shown in table 4.3 have been refined by this interpolation technique.

4.6 CONCLUSIONS

In this chapter, a review of the terms in the momentum equation has been made and a method for determining the order of magnitude of these terms has been investigated. From this study, several broad qualitative classifications of unsteady flow phenomena have been demonstrated. These form a preliminary basis for determining the suitability of applying a kinematic solution to a particular unsteady flow situation.

The classifications, first proposed by Henderson, are:

(1) STEEP SLOPE - Only the bed slope is significant in this type of system. The absolute value of $\frac{\delta y}{\delta x}$ is estimated using the equation:

$$\left|\frac{\delta y}{\delta x}\right| = \left|\frac{1}{C^2}\frac{\delta Q}{T_w \delta t}\right| \qquad (4.8)$$

If the quantity

$$\left|\frac{\delta y}{\delta x}\right| \Big/ S_o << 1 \qquad (4.13)$$

the system may be classified as steep.

(2) GENTLE SLOPE - With this classification, only the bed slope and $\frac{\delta y}{\delta x}$ terms of the dynamic equation are significant. It has been shown that if the system does not fit into the steep slope classification and

$$F_R^2 << 1 \qquad (4.14)$$

the system may be classified as gentle sloped.

(3) INTERMEDIATE SLOPE - This system is encountered when all the terms in the momentum equation are important. A

system which cannot be classified with the two previous categories will fit under this title.

Results of several simulations were presented which showed the comparisons of dynamic and kinematic solutions for various classifications of systems. As the bedslope became more significant in relation to the other terms, the solutions tended toward a kinematic solution. However, attenuation, which resulted from the dynamic effects in the system, was detected even when the bedslope was two orders of magnitude greater than the $\frac{\delta y}{\delta x}$ term. It is felt that this observation is not a general observation that can be applied to other cases of unsteady flow phenomena. Other systems, which have a longer wave length hydrograph, may have much less attenuation.

It should also be noted that the classifications steep, intermediate or gentle slope are not the result of a particular bedslope in a channel. These classifications also reflect the flow resistance, channel geometry, ranges of flows encountered in the hydrograph and the rate of change of the flow rates. This was demonstrated by studying a system which, for a particular inflow hydrograph, changed from intermediate slope to gentle slope and finally to a steep slope system, as the bedslope and roughness were increased.

Comparisons of results from this study were made with results reported by Amein (1968) and Graves (1967) to demonstrate problems associated with using excessively large time steps. The difficulties were manifested in the form of instabilities and in data interpretation. This demonstrates that both the kinematic methods and the dynamic methods have limitations to the sizes of ΔX and ΔT that can be successfully employed to provide a solution. An interpolation technique was employed to overcome problems associated with data interpretation.

The method proposed by Graves is interesting in that it is an example of methods which are employed to model the attenuation and movement of a flood wave with a quasi-kinematic approach. Another example of a technique which employs an approximation in conjunction with kinematic flood routing to model movement and attenuation is the kinematic solution with the nucleus moved from the centre of the molecule. These types of models have not been discussed at this point in the thesis. However, the next chapters present a discussion of several methods that may be utilized to extend the usefulness of kinematic models.

CHAPTER 5

ATTENUATION AND KINEMATIC METHODS

This chapter provides a discussion of a method of modelling attenuation in conjunction with the general kinematic flood routing algorithm. The previous chapter, which contains a comparison of kinematic and dynamic solutions of two physical systems, demonstrates the ability of the two methods to model attenuation. The dynamic solution follows from a rational numerical representation of the actual physical phenomena. Kinematic methods, per se, do not predict attenuation but depend on a manipulation of the computational scheme to approximate the reduction of the flood wave, which may not, in fact, truly represent the process that is occurring in the prototype.

Thus, while dynamic techniques provide a more realistic method of predicting attenuation, it may be possible and computationally advantageous to utilize a modified kinematic routine, if the solutions can be shown, by a calibration process, to be valid.

In presenting this discussion three components of modelling attenuation will be explored. These are:

1. "Molecule effect:" the attenuation introduced by moving the nucleus from the centre of the molecule.
2. "Cascade effect:" the way in which attenuation is affected by varying the number of storage units in series.
3. "Storage effect:" the dependance of the attenuation on the relative magnitude of live storage and volume in the flood hydrograph.

To show the usefulness of this approach in the simulation of unsteady open channel flow, a number of numerical experiments are presented and the results are compared with the corresponding dynamic solutions. A description of the way in which an engineer might calibrate a model of this type is included along with the concluding remarks.

5.1 THE "MOLECULE EFFECT" - MODELLING ATTENUATION BY MOVING THE NUCLEUS

In designing the numerical experiments used with this study, two distinct physical systems were employed. These systems, described in detail earlier in this thesis were:

(i) a rectangular channel 20' deep and 100 ' wide subjected to a triangular shaped inflow hydrograph

(ii) a very wide channel subjected to a sinusoidal inflow hydrograph.

The primary difference between the two systems was the scale. The time base of the inflow hydrograph for the first system was 2.77 hours while the second hydrograph had a time base of 96 hours. The length of the channel for the first system was 40,000 feet whereas a channel 300 miles long was used in the second system.

To examine the sensitivity of the flood routing solution to the position of the nucleus within the molecule, a program was employed which for fixed values of ΔX and ΔT, carried out a routing analysis for a range of values of α and β. Both parameters were varied between 0.0 and 1.0 with increments of 0.25, so that a total of 25 possible solutions were obtained. It was recognized at the outset, that a significant number of these nucleus positions would not result in an acceptable solution, because of numerical instability at certain values of α and β.

The results for the first system are shown in Tables 3.3 and 3.4 and 3.5 of Chapter 3. The elements of the matrix define the positions of the nucleus and the values contained in each element are the ratios of the peak outflow divided by the full bank flow rate. A similar set of results was compiled for the second system in tables 5.1, 5.2, 5.3 and 5.4.

TABLE 5.1

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 2

300 MILES DOWNSTREAM

ΔT = 2 HOURS

ΔX = 10 MILES

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	0.976	0.954	0.932	0.910	0.888
0.75	unstable	0.986	0.964	0.942	0.919
0.50	unstable	unstable	0.996	0.975	0.952
0.25	unstable	unstable	unstable	unstable	0.986
0.00	unstable	unstable	unstable	unstable	unstable

TABLE 5.2

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 2

300 MILES DOWNSTREAM

ΔT = 2 HOURS

ΔX = 20 MILES

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	unstable	0.975	0.931	0.887	0.849
0.75	unstable	unstable	0.963	0.918	0.875
0.50	unstable	unstable	0.997	0.951	0.904
0.25	unstable	unstable	unstable	0.985	0.938
0.00	unstable	unstable	unstable	1.000	0.972

TABLE 5.3

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 2

300 MILES DOWNSTREAM

ΔT = 3 HOURS

ΔX = 10 MILES

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	0.946	0.924	0.902	0.880	0.860
0.75	unstable	0.970	0.949	0.927	0.903
0.50	unstable	unstable	1.000	0.975	0.954
0.25	unstable	unstable	unstable	unstable	1.000
0.00	unstable	unstable	unstable	unstable	unstable

TABLE 5.4

PEAK VALUES OF THE OUTFLOW HYDROGRAPH

SYSTEM 2

300 MILES DOWNSTREAM

ΔT = 3 HOURS

ΔX = 20 MILES

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	unstable	0.944	0.901	0.860	0.825
0.75	unstable	0.990	0.948	0.903	0.860
0.50	unstable	unstable	0.995	0.952	0.904
0.25	unstable	unstable	unstable	0.999	0.956
0.00	unstable	unstable	unstable	unstable	1.00

Before proceeding with the theoretical aspects of the "molecule effect" it is vital that the reader obtain a clear picture of the phenomena that is manifested by these results. If a three dimensional plot were made, showing the peak outflow versus α and β, a smooth surface would be depicted. This surface may be visualized by imagining contour lines across the tables which contain these values. For example, the results shown in table 3.3 for System 1, would form a surface with the line of steepest descent going from the central position toward the upper right hand corner of the table ($\alpha = 0.0$, $\beta = 1.0$) with the gradient of this line becoming shallower in the proximity of the corner. Similarly, table 5.1, which shows some results from System 2, presents data which would form a surface with the steepest descent upwards and to the right. If the strike of the surface were plotted to determine the direction of the dip, the strike being a horizontal line perpendicular to the dip or line of steepest descent, the direction of the dip would be found to be three increments upwards and two increments to the right. Examination of the other tables (i.e. 5.2 to 5.4) reveals a similar result, although it can be seen that as the size of ΔX and ΔT varies, the direction of the dip changes. When ΔX is made larger, the dip swings towards the right hand side; but if ΔT is increased, the dip points toward the upper portion of the matrix.

If the necessary key factors could be identified enabling an engineer or researcher to describe the characteristics of these surfaces without having to generate numerous computer simulations, it would then be possible to preselect the necessary values of α, β etc. in order to produce attenuation of a required amount.

The following theoretical investigation is directed towards identification of the parameters which characterize the P(α , β) surfaces described above and towards rationalizing the introduction of numerical error, or "molecule effect" that is brought about by moving the nucleus away from the centre of the molecule.

Examination of the finite difference approximation of the continuity equation in differential form has revealed that the representation is given by equation 5. 1.

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + \frac{1}{2}(2\alpha-1)\Delta X\frac{\delta^2 Q}{\delta x^2} + \frac{1}{2}(1-2\beta)\Delta T\frac{\delta^2 A}{\delta t^2} + O(\Delta X^2, \Delta T^2) = \bar{q} \qquad (5.1)$$

The mathematical proof of this statement is presented in Appendix C.

Ignoring error terms higher than first order allows the equation to be written as:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + \epsilon = \bar{q} \qquad (5.2)$$

where:

$$\epsilon = \frac{1}{2}(2\alpha-1)\Delta X\frac{\delta^2 Q}{\delta x^2} + \frac{1}{2}(1-2\beta)\Delta T\frac{\delta^2 A}{\delta t^2} \tag{5.3}$$

To calculate the sensitivity of the size of the error terms to variations of α, β, ΔX and ΔT, it is necessary to analyze the nature of the mathematical model. This information can be obtained by determining the relative size of the terms $\frac{\delta^2 Q}{\delta x^2}$ and $\frac{\delta^2 A}{\delta x^2}$.

Describing the kinematic wave by the partial differential equation:

$$\frac{\delta y}{\delta t} + C\frac{\delta y}{\delta x} = 0 \tag{5.4}$$

implicitly defines the wave celerity as

$$C = \frac{-\delta y/\delta t}{\delta y/\delta x} = -\frac{\delta x}{\delta t} \tag{5.5}$$

Rearranging the differential form of the continuity equation yields:

$$\frac{\delta Q}{\delta x} = \bar{q} - \frac{\delta A}{\delta t} \tag{5.6}$$

or:

$$\frac{\delta A}{\delta t} = \bar{q} - \frac{\delta Q}{\delta x} \tag{5.7}$$

Further differentiation of equations 5.6 and 5.7 leads to the second order terms of 5.8 and 5.9.

$$\frac{\delta^2 Q}{\delta x^2} = -\frac{\delta^2 A}{\delta x \delta t} \qquad (5.8)$$

$$\frac{\delta^2 A}{\delta t^2} = -\frac{\delta^2 Q}{\delta t \, \delta x} \qquad (5.9)$$

Multiplying equation 5.8 by the kinematic wave velocity gives:

$$C\frac{\delta^2 Q}{\delta x^2} = -C\frac{\delta^2 A}{\delta x \delta t} \qquad (5.10)$$

from which:

$$C\frac{\delta^2 Q}{\delta x^2} = -\left(-\frac{\delta x}{\delta t}\right)\frac{\delta A}{\delta x \delta t} \qquad (5.11)$$

$$C\frac{\delta^2 Q}{\delta x^2} = \frac{\delta^2 A}{\delta t^2} \qquad (5.12)$$

This leads to several important points. First, by substituting equation 5. 12 into the error expression 5. 3 the following equation is obtained:

$$\epsilon = (2\alpha - 1)\frac{\Delta X}{2}\frac{\delta^2 Q}{\delta x^2} + (1-2\beta)\frac{C\Delta T}{2}\frac{\delta^2 Q}{\delta x^2} \tag{5.13}$$

$$= \frac{\delta^2 Q}{\delta x^2}\left((\alpha - 0.5)\Delta X + (0.5 - \beta)C\Delta T\right) \tag{5.14}$$

This demonstrates the qualitative dependance of the error term on linear terms in α, β.

The second item that can be developed from the previous algebraic manipulations is a finite difference representation of the error terms.

By referring to figure 3.2 for a definition of nodal values in the finite difference molecule, it can be seen that:

$$\frac{\delta^2 A}{\delta t^2} = -\frac{\delta^2 Q}{\delta x \delta t} = -\left(\frac{\frac{(Q_4 - Q_3)}{\Delta T} - \frac{(Q_2 - Q_1)}{\Delta T}}{\Delta X}\right) \tag{5.15}$$

$$= -\left(\frac{Q_4 - Q_3 + Q_1 - Q_2}{\Delta X \Delta T}\right) \tag{5.16}$$

Similarly:

$$\frac{\delta^2 Q}{\delta x^2} = -\frac{\delta^2 A}{\delta x\,\delta t} = -\left(\frac{\frac{(A_4-A_3)}{\Delta T} - \frac{(A_2-A_1)}{\Delta T}}{\Delta X}\right) \tag{5.17}$$

$$= -\left(\frac{A_4 - A_3 + A_1 - A_2}{\Delta X\,\Delta T}\right) \tag{5.18}$$

Using the following finite difference approximations:

$$\frac{\delta Q}{\delta x} = \frac{Q_4 + Q_3 - Q_2 - Q_1}{2\Delta X} \tag{5.19}$$

and:

$$\frac{\delta A}{\delta t} = \frac{A_4 + A_2 - A_3 - A_1}{2\Delta T} \tag{5.20}$$

equation 5. 1 may be rewritten in finite difference form. The equation is:

$$\frac{Q_4 + Q_3 - Q_2 - Q_1}{2\Delta X} + \frac{A_4 + A_2 - A_3 - A_1}{2\Delta T}$$

$$+\frac{1}{2}(2\alpha - 1)\Delta X\left(\frac{-(A_4 - A_3 + A_1 - A_2)}{\Delta X\,\Delta T}\right)$$

$$+\frac{1}{2}(1 - 2\beta)\Delta T\left(\frac{-(Q_4 - Q_3 + Q_1 - Q_2)}{\Delta X\,\Delta T}\right) = \bar{q} \tag{5.21}$$

Collecting terms yields:

$$\frac{Q_4 + Q_3 - Q_2 - Q_1}{2\Delta X} + \frac{1-2\beta}{2} \frac{Q_3 + Q_2 - Q_4 - Q_1}{\Delta X}$$

$$\frac{A_4 + A_2 - A_3 - A_1}{2\Delta T} + \frac{2\alpha - 1}{2} \frac{A_3 + A_2 - A_4 - A_1}{\Delta T} = \bar{q} \qquad (5.22)$$

Further simplification gives:

$$\frac{\beta(Q_4 - Q_2) + (1-\beta)(Q_3 - Q_1)}{\Delta X}$$

$$+ \frac{(1-\alpha)(A_4 - A_3) + \alpha(A_2 - A_1)}{\Delta T} = \bar{q} \qquad (5.23)$$

It can be seen that equation 5.23 is precisely the same as equation 3.10. This leads to the conclusion that the error term related to the selection of α and ΔX has physical significance in that it indicates the amount of reservoir type storage, storage which is a function of outflow alone, that is assigned to the elementary reach.

Furthermore, because the error terms are linear additions to the continuity equation, the introduction of error related to the choice of β and ΔT causes the model to behave in a manner similar to the introduction of reservoir storage in the channel.

Thus, by the use of equation 5.14, the first order errors associated with ΔT may be converted to equivalent reservoir storage.

This property has already been demonstrated in pictorial fashion in a previous chapter, where the shape of the outflow hydrograph for the first system was shown to be related to the peak of the hydrograph. (Figure 3.6) If the peak attenuated to a particular value, the flood wave appeared to have the same shape regardless of whether the attenuation resulted from the introduction of error in the order of ΔX or error in the order of ΔT.

A qualitative description of the equivalent reservoir storage in the channel may be obtained in the following manner. Dividing equation 5.12 by $\frac{\Delta X}{2}\frac{\delta^2 Q}{\delta x^2}$ yields:

$$\frac{2\epsilon}{\Delta X \frac{\delta^2 Q}{\delta x^2}} = S = (2\alpha - 1) + (1 - 2\beta)\frac{C\Delta T}{\Delta X} \qquad (5.24)$$

where: S = stability number defined in Appendix A.

Using the dimensionless number, S, it is possible to describe the amount of equivalent reservoir storage in an elementary reach. If the nucleus is in the centre of the molecule, S = 0.0 and there is no reservoir storage in the channel. However, when the nucleus is located at the downstream boundary (α = 0.0, β = 0.5) the value of S is -1.0 and the elementary unit is modelled as a reservoir.

The amount of apparent reservoir storage in the elementary reach can be increased even further by selecting a value of β greater than 0.5. The exact amount by which the apparent reservoir storage will increase depends upon the values of c, ΔX and ΔT. If $C \frac{\Delta T}{\Delta X}$ is precisely equal to unity, the value of S will vary in direct proportion to both $(2\alpha - 1)$ and $(1-2\beta)$.

For a linear channel the kinematic wave velocity is constant with respect to y and there is no difficulty in determining the value of c to use in calculating S. In non-linear channel elements in which c is a function of the stage y, it is necessary to choose some value of stage or discharge for which c may be computed. Ideally this stage should be able to be selected from a knowledge of the inflow hydrograph. However, until some suitable guidelines are available to aid in this selection it is preferable to determine the appropriate value of c by simulation. Because the attenuation is a function of S it is possible to determine $C\frac{\Delta T}{\Delta X}$ from the data presented in the tables:

For System 1, $C\frac{\Delta T}{\Delta X}$ may be calculated in the following manner:

(a) The time steps chosen for the simulation caused the peak of the inflow hydrograph to be truncated to 0.984 times the maximum full bank flow. Thus, the values chosen to determine c should be divided by 0.984.

(b) Using data from table 3.3, the sensitivity to changes of α at the centre of the molecule are:

$$\frac{\Delta P}{\Delta(2\alpha-1)} = \frac{1.00 - \frac{0.924}{0.984}}{-0.50} = -0.122$$

The sensitivity to changes in β are:

$$\frac{\Delta P}{\Delta(1-2\beta)} = \frac{1.00 - \frac{0.925}{0.984}}{-0.50} = -0.120$$

Thus, S is slightly more sensitive to the value of α and

$$C\frac{\Delta T}{\Delta X} = \frac{-0.120}{-0.122}$$

or

$$C = \frac{-0.120}{-0.122} \times \frac{2500}{200} = 12.3 \text{ fps}$$

(c) This type of calculation can be repeated for all of the data available and an average value determined. However, when applying this method it should be noted that attenuation may not, and indeed, usually will not, be a linear function of S. Thus, the sensitivity tests described above should be made using values of the attenuated peak outflow that are approximately equal.

If we replace the parameters α and β by equivalent variables with the origin at the centre of the molecule we obtain

$$\theta = 2\alpha - 1 \ ; \quad \phi = 1 - 2\beta$$

Then the desired value of c may be expressed as

$$C = \frac{\Delta X}{\Delta T} \frac{\frac{\delta P}{\delta \phi}}{\frac{\delta P}{\delta \theta}} \tag{5.25}$$

If P is assumed to be 1.0 at the point $\alpha = 0.5$, $\beta = 0.5$ ($\theta = 0$, $\phi = 0$) then two simulation runs will provide the forward difference values for $\frac{\delta P}{\delta \theta}$ and $\frac{\delta P}{\delta \phi}$ from which c may be evaluated.

5.2 "CASCADE EFFECT"

The "cascade effect" is defined as the extent to which overall attenuation is increased as the number of subreaches of constant ΔX

are increased. Dooge (1959) demonstrated that the peak outflow issuing from a series of linear reservoirs may be related to the Poisson (N-1, N-1). Evaluation of the Poisson function reveals that the term P(N-1, N-1) is proportional to $\sqrt{N}$ where N is the number of elements chained together. A similar situation may be shown to exist for chains of non-linear reservoirs.

Several numerical experiments were performed to determine the relationship between the attenuation, (1-P), and the number of elements n when the constraint of linearity is removed. For these experiments, the cross section area from System 1 was utilized with reach lengths of 1, 250 and 5, 000 feet. The inflow hydrograph was defined as the triangular hydrograph used for System 1. The hydrograph was routed through various cascades with 2, 4, 8, 16, 32 and 64 units in each cascade.

In order to introduce a significant degree of attenuation and vary the molecule effect the computations were made with the following positions of the nucleus.

POSITION / PARAMETER	1	2	3
α	0.0	0.0	0.5
β	0.5	1.0	1.0

The results of these simulations are presented in figure 5. 1. which shows a plot of attenuation (1-P), versus the number of cascaded reaches, N, in the system. Each of these numerical experiments resulted in a set of points that could be approximated by a straight line on a log-log plot. After fitting the straight lines to the data it could be seen that a family of nearly parallel curves resulted.

The slope of the various lines was determined to provide an estimate of the exponent, e, in the general equation

$$(1-P) = k N^{e} \qquad (5.26)$$

where:

P = peak outflow ratio

k = a coefficient

N = number of reaches in the simulation

e = an exponent

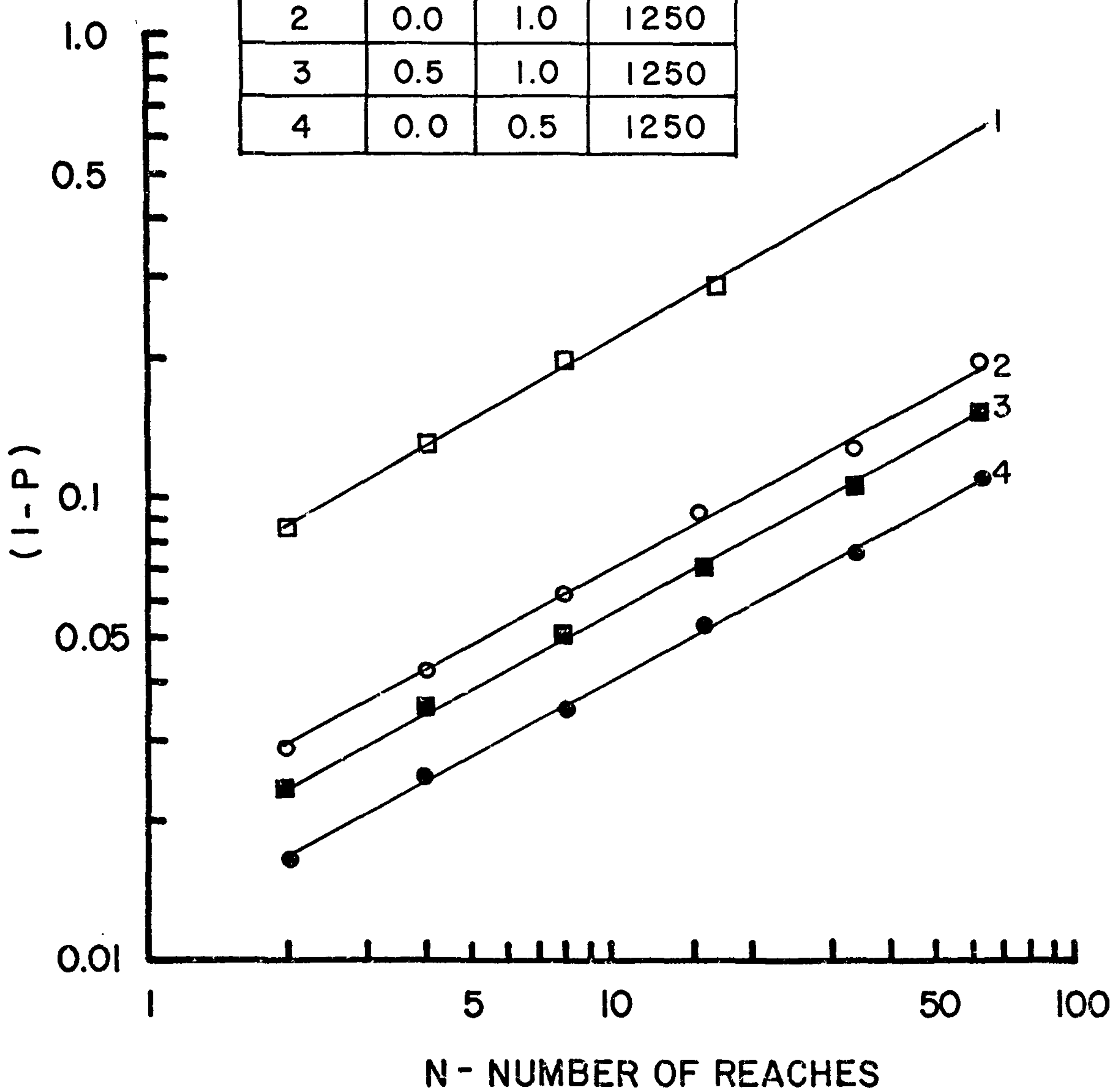
The results indicated a range of results from 0. 542 to 0. 574 for e as shown on figure 5. 1.

These results are presented to demonstrate an interesting feature of the system and simulation. It is also interesting to note that if a linear system was utilized, the exponent would be of the order of 0. 5. More tests are required to delineate the relationship of the exponent e to factors such as the input, and system

FIGURE 5.1

ATTENUATION Vs NUMBER OF REACHES

CURVE N°.	α	β	ΔX
1	0.0	1.0	5000
2	0.0	1.0	1250
3	0.5	1.0	1250
4	0.0	0.5	1250



characteristics. Further investigations into these items and the variance of the exponent, e, could provide information which may quantify the effect of system non-linearity on the response characteristics.

5.3 "STORAGE EFFECT"

"Storage effect" is defined as the attenuation that results from a finite quantity of live storage expressed as a proportion of the volume in the flood hydrograph. It is intuitive that more attenuation will occur as reservoir volume increases.

Since a constant inflow hydrograph was employed in all tests, this effect was studied by running further numerical experiments with System 1 in which the attenuation, 1-P, was dependant on the reach length ΔX .

Following empirical evaluation of the molecule and cascade effect it was then intended to relate the attenuation as a function of the live storage. Although it may be anticipated that this relationship is highly non-linear over a wide range of attenuation values, it is reasonable to assume that within the relatively small range of ΔX values examined the function would not depart significantly from a straight line plot.

5.4 ANALYSIS OF NUMERICAL EXPERIMENTS

For the purpose of evaluating the several functional relations described above, attention was directed to the result of numerical experiments on System 1 as recorded in tables 3.3 - 3.5. The various effects were isolated in a slightly different order to that in which they have been considered up to this point, i.e. cascade effect and molecule effect and storage effect.

5.4.1 CASCADE EFFECT

In order to study the effect of increasing the number of serial elements in a system, the program was modified slightly so that ΔX, α and β could be held constant, while n was increased geometrically. The results for various values of ΔX, α and β are tabulated in table 5.5.

Following on the argument of section 5.2, a log-log plot of the attenuation (1-P) versus n (number of elements) revealed that the value of e in equation 5.26 was essentially constant with a mean value of 0.55. The slope of the various plots is indicated if figure 5.1 and ranges from 0.542 to 0.576.

The effect of this correlation is to allow the attenuation of a single reach to be determined as a function of

$$\frac{(1-P)}{N^{0.55}}$$

TABLE 5.5

TYPICAL RESULTS OF CASCADE EFFECT EXPERIMENTS

SYSTEM 1

ΔT = 200 SECONDS

ΔX = 1250 FEET

α	β	N	(1-P)
		2	0.0164
0.00	0.50	4	0.0247
		8	0.0381
		16	0.0525
		32	0.0755
		64	0.1089

where P is the resultant relative peak flow issuing from a cascade of N elements. Armed with this information it was now possible to extend the analysis to the molecule and storage effects.

5.4.2 MOLECULE EFFECT

As discussed in section 5.1, the attenuation (1-P) may be related to the stability parameter S which is shown to be a function of α, β, ΔX, ΔT and c by equation 5.24.

Figure 5.2 shows a log-log plot of $(1-P)/N^{.55}$ as a function of S. Each line represents values obtained for constant values of ΔX. Once again the relations appear to be simple exponentials with the range of experiments covered. However, the exponent of the different experiments varies between 0.5 and 0.6. The average of these values is 0.55. The general result is that for constant values of N and ΔX, the attenuation (1-P) is proportional to approximately $S^{0.55}$. Thus for constant N and ΔX

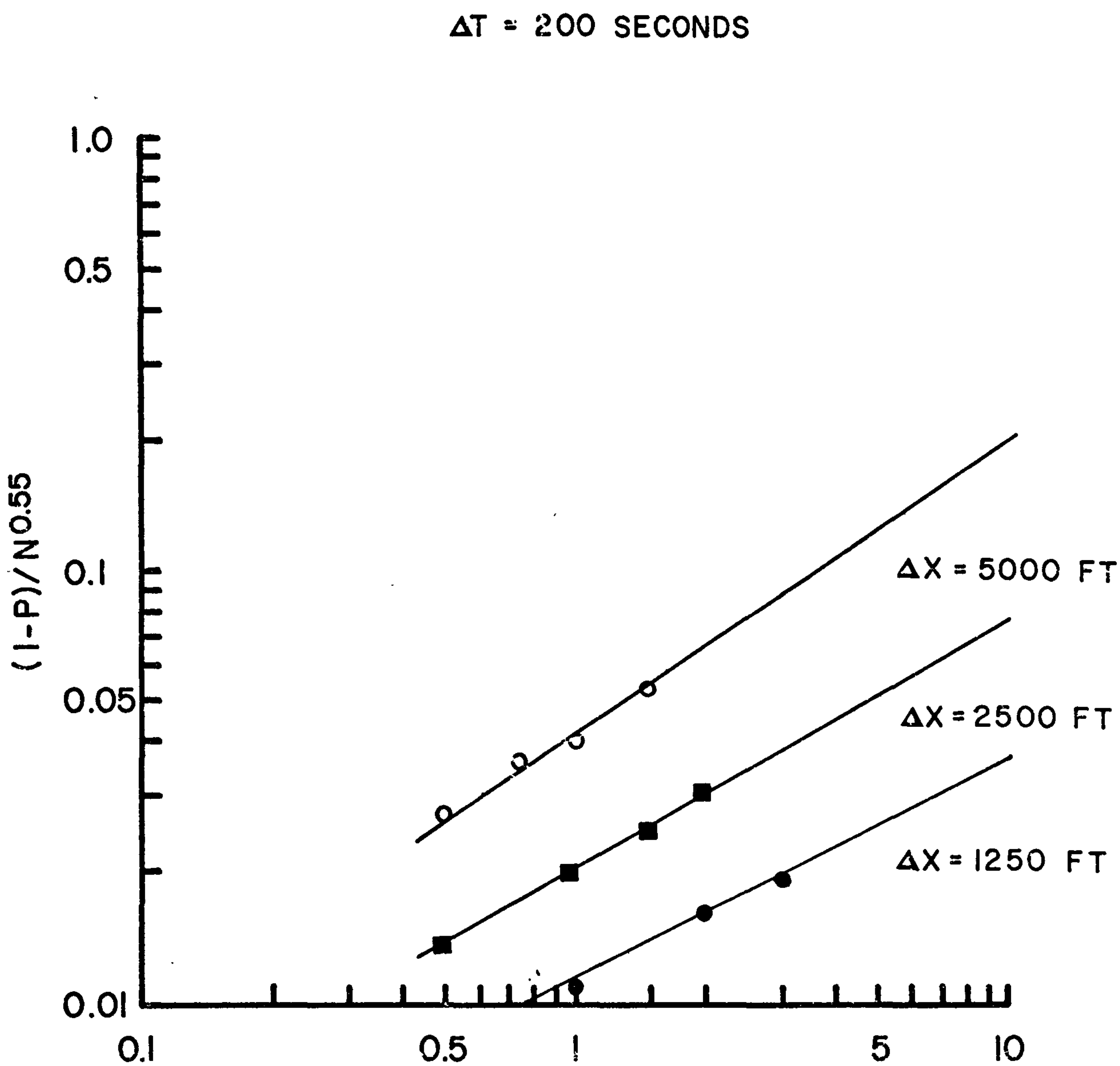
$$\frac{1-P}{-S^{0.55}} = \text{CONSTANT} \qquad (5.27)$$

5.4.3 STORAGE EFFECT

Using the correlations determined in the foregoing two sections it is now possible to isolate the dependence of attenuation on

FIGURE 5.2

ATTENUATION PER ELEMENTARY REACH Vs STABILITY NUMBER



$$-S = -\left[(2\alpha-1)+(1-2\beta)\frac{C\Delta T}{\Delta X}\right]$$

the live storage. This was achieved by reducing the results of the numerical tests to obtain values of

$$\frac{(1-P)}{-S^{0.55}N^{0.55}} = f(\Delta X) \qquad (5.28)$$

To express the relative live storage as a volumetric ratio, it is convenient to use as abscissa the non-dimensional term

$$\frac{\Delta X \cdot \Delta A}{Vol}$$

where: $\Delta A = A(Qmax) - A(Qbase)$

Vol = volume of inflow hydrograph above the base flow

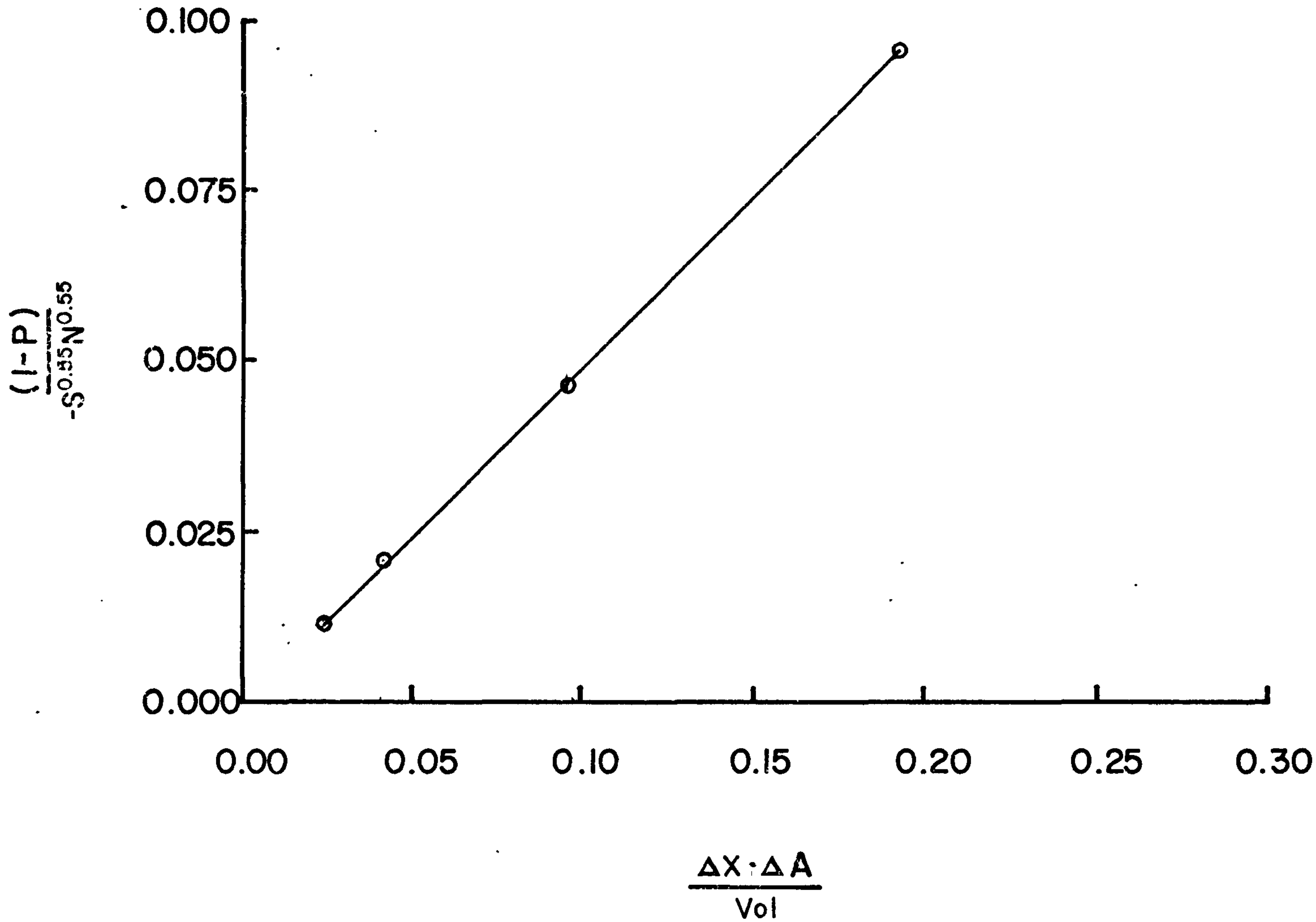
This choice of parameter is somewhat arbitrary and valid here only because of the constancy of the inflow hydrograph employed and the prismatic nature of the channel. The results of this analysis are shown in figure 5.3 in which it may be seen that the "reduced" attenuation is approximately a linear function of the relative live storage parameter.

$$\frac{(1-P)}{-S^{0.55}N^{0.55}} = k\frac{\Delta X \cdot \Delta A}{Vol} \qquad (5.29)$$

where: k = 0.485 for the results examined

FIGURE 5.3

ATTENUATION PER ELEMENTARY REACH DUE TO STORAGE EFFECT



5.4.4 EXTENSION TO OTHER SYSTEMS

The results of the foregoing analysis are of interest but are rather dependent on the assumptions and constraints of the particular system from which the numerical data were obtained. It must be recognized that a more generally applicable relation may require inclusion of the effect of the hydrograph shape (defined perhaps by higher statistical moments) and the degree to which the expression $\Delta X \cdot \Delta A$ truly represents the live storage in the system.

It must be recognized that the functional relations developed above are valid only for the system from which numerical data were obtained and that application of these correlations to other systems is unwarranted until such time as further numerical tests can be carried out. Until such time, however, it is desirable to develop some simple guidelines for the development, use and calibration of kinematic routing models. The following section attempts to develop such a methodology on the basis of the analysis presented earlier and the application in various circumstances is illustrated.

5.5 GUIDELINES FOR CALIBRATION OF KINEMATIC ROUTING MODELS

Since it is recognized that no attempt has been made to quantify the dependance of attenuation on hydrograph shape and storage effect, it follows that the engineer engaged in constructing a kinematic routing model must make use of trial numerical experiments as an aid to calibration. This being so, it is desirable to identify a compound parameter which combines the several separate effects described above and which will enable computational effort to be minimized in the process of model development and adjustment.

In the process of calculating the relation of attenuation to S, ΔX and N, it was shown that a correlation existed which promised to become a useful tool to efficiently calibrate a numerical model.

Rearranging equation 5.14 indicates that

$$2\epsilon = \frac{\delta^2 Q}{\delta x^2} \cdot S \cdot \Delta X \qquad (5.30)$$

The term $\frac{\delta^2 Q}{\delta x^2}$ is a function of the system and the inflow hydrograph and remains fixed for any independant values of α, β, ΔX and ΔT. Assuming that the attenuation is some function of the error term, a correlation between P and-$S\Delta X$ should exist. Figure 5.4 contains a plot of P versus-$S\Delta X$ for System 1; a similar plot for System 2 is shown in figure 5.5. The results plotted on these curves are shown in tables 5.6 and 5.7.

FIGURE 5.4

ATTENUATION DUE TO MOLECULE EFFECT

SYSTEM I

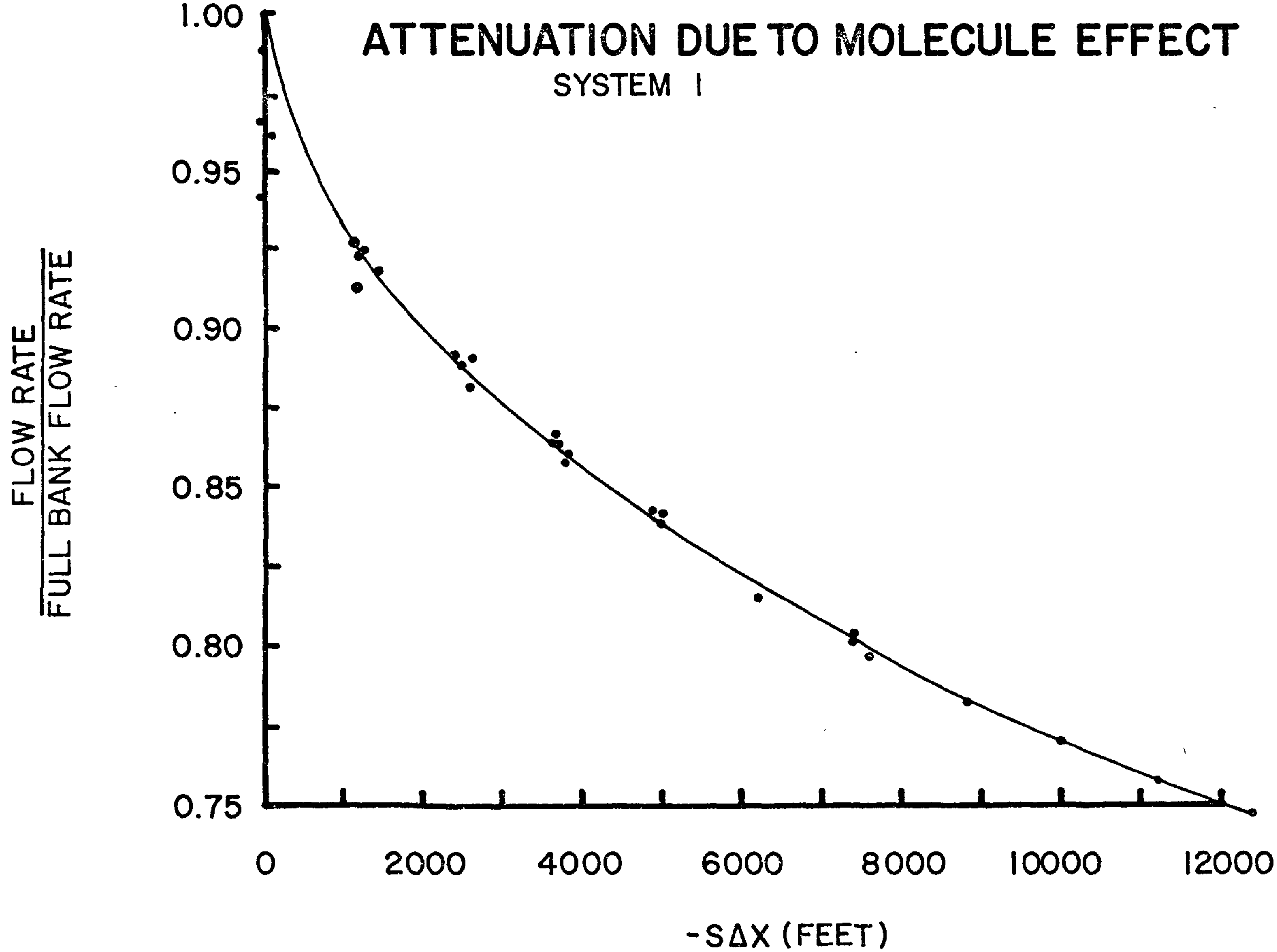


FIGURE 5.5

ATTENUATION DUE TO MOLECULE EFFECT

SYSTEM 2

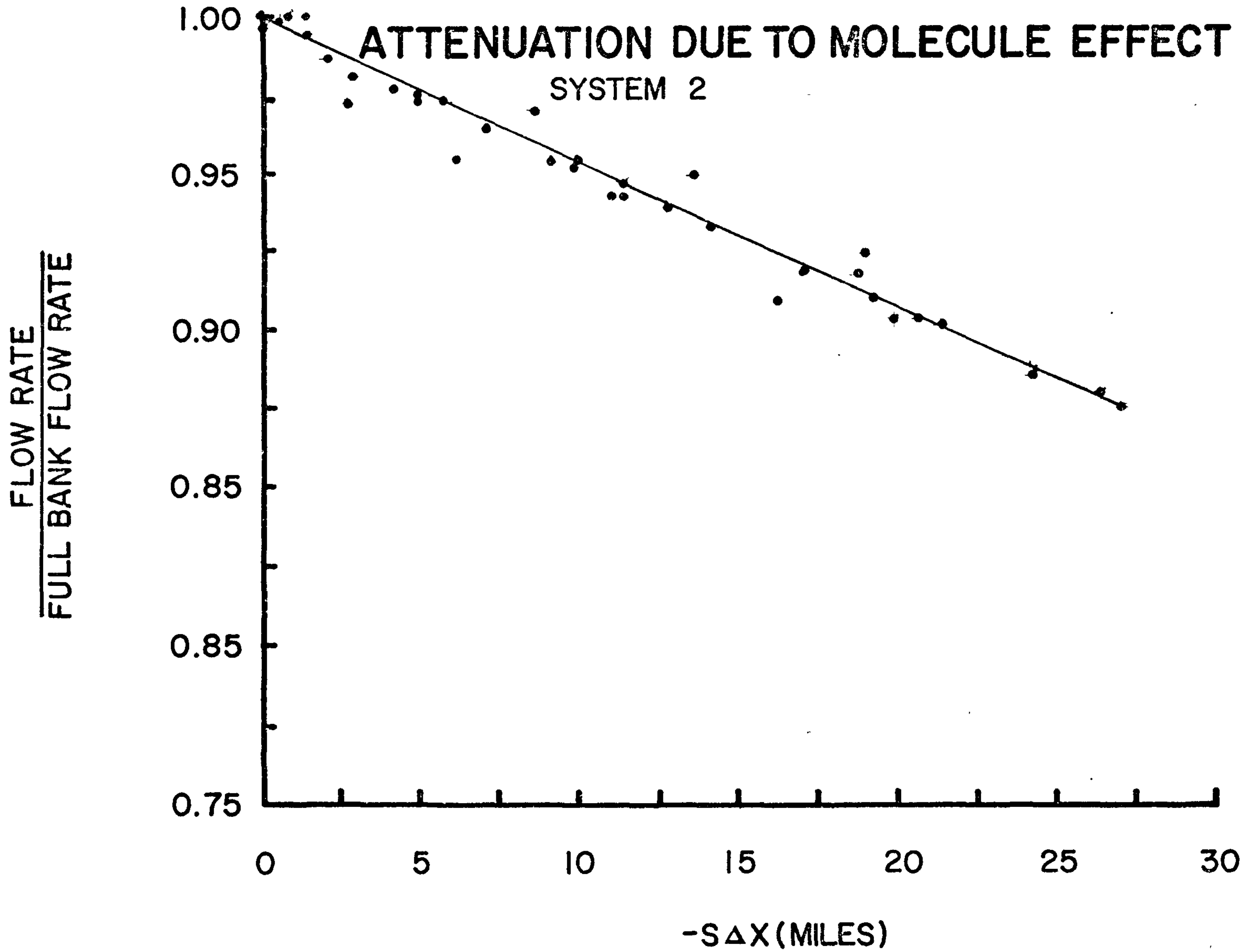


TABLE 5.6

TYPICAL VALUES USED TO DEFINE

THE P VS SΔX CURVE

SYSTEM 1

40,000 FT. DOWNSTREAM

ΔT (SEC.)	ΔX (FT.)	α	β	S	SΔX (FT.)	PEAK
		0.50		0.0	0	0.988
	2,500	0.25		-0.5	-1250	0.924
		0.00		-1.0	-2500	0.890
		0.50		0.0	0	0.965
200	5,000	0.25	0.5	-0.5	-2500	0.890
		0.00		-1.0	-5000	0.840
		0.50		0.0	0	0.941
	10,000	0.25		-0.5	-5000	0.840
		0.00		-1.0	-10000	0.771

TABLE 5.7

TYPICAL VALUES USED TO DEFINE

THE P VS $S\Delta X$ CURVE

SYSTEM 2

300 MILES DOWNSTREAM

ΔT (HRS.)	ΔX (MI.)	α	β	S	$S\Delta X$ (MI.)	PEAK
2	10	0.00	0.25	-0.285	- 2.85	0.986
			0.50	-1.000	-10.00	0.952
			0.75	-1.715	-17.15	0.919
			1.00	-2.430	-24.30	0.888
		0.25	0.50	-0.500	- 5.00	0.975
			0.75	-1.215	-12.15	0.942
			1.00	-1.930	-19.30	0.910
		0.50	0.50	0.000	0.00	0.996
			0.75	-0.715	- 7.15	0.964
			1.00	-1.430	-14.30	0.932
		0.75	0.75	-0.215	- 2.15	0.986
			1.00	-0.930	- 9.30	0.954
		1.00	1.00	-0.430	- 4.30	0.976
3	20	0.00	0.25	-0.463	- 5.26	0.956
			0.50	-1.000	-20.00	0.904
			0.75	-1.537	-30.74	0.860
			1.00	-2.075	-41.50	0.825
		0.25	0.25	+0.037	+ 0.74	0.999
			0.50	-0.500	-10.00	0.952
			0.75	-1.037	-20.74	0.903
			1.00	-1.537	-30.74	0.860
		0.50	0.50	0.000	00.00	0.995
			0.75	-0.537	-10.74	0.948
			1.00	-1.075	-21.50	0.901
		0.75	0.75	-0.037	- 0.74	0.990
			1.00	-0.575	-11.5	0.944

It seems reasonable to assume that similar curves may be obtained for any system. A procedure may therefore be developed whereby these curves may be obtained from two or more simulation runs with carefully selected parameters. This curve may then be used as a guideline for the selection of the required value of $S\Delta X$ corresponding to the observed attenuation. A suggested procedure for the selection of parametric values is given below in the form of a check test.

1) Select an appropriate value of ΔT to model the inflow hydrograph. If the channel is arbitrarily shaped it may be necessary to specify the ΔX increments due to the constraints of geometric data.

2) Obtain an estimate for the kinematic wave velocity, c, based on the system and the inflow hydrograph.

3) Assuming that the ΔX steps can be varied, select a value of $\Delta X \doteq C\Delta T$ (say) with the constraint that $\frac{L}{\Delta X}$ is an integer. In the first simulation selecting $\alpha = 0.0$, $\beta = 1.0$ will provide the maximum amount of attenuation that is possible with the specified ΔX and ΔT In addition, the numerical calculations will be unconditionally stable.

4) The above steps would define $S\Delta X$ and performing the necessary mathematical calculations would provide a result with an attenuated outflow hydrograph.

5) If the predicted attenuation is greater than the observed peak then adopt a smaller value of $S\Delta X$, say one half of the previous value. There are a number of ways in which $S\Delta X$ may be varied by adjusting one of four variables α, β, ΔX or ΔT. Unless the value of c is well established, it is suggested that α and β be adjusted and ΔX and ΔT held fixed. This may provide more information which may be helpful in determining c.

6) Repeat the calculations with the reduced value of c to obtain a new estimate of the peak outflow and hence define another point on the relationship

$$P = f(S\Delta X) \qquad (5.31)$$

from which it may be possible to approximate the $S\Delta X$ required to provide the needed amount of attenuation.

7) If the first value of the attenuated hydrograph is less than the desired amount, it is necessary to increase $S\Delta X$. This can be done by increasing either ΔX or ΔT. If the channel has been defined with arbitrary cross sections it would be necessary to adjust ΔT. Otherwise an

adjustment of ΔX may provide the most efficient solution.

8) Successive adjustments of $S\Delta X$ would produce the relationship of P to $S\Delta X$ and allow the user to calibrate the model.

There may be other variations on the previously described calibration process that may be employed to achieve the desired results. For example, if the value of c is unknown, the preliminary calculations may be performed with $\beta = 0.5$ and thus eliminate the need to have a value for the kinematic wave velocity. Another numerical experiment with $\alpha = 0.5$ and $\beta > 0.5$ would yield the data required to determine c. In selecting values for the parameters α and β and step sizes ΔX and ΔT the user should remember to follow stability criteria guidelines (see Chapter 3) and must also utilize sound engineering judgment. This will ensure that unreasonably large numerical errors and approximations are not introduced into the calculations.

5.6 DISCUSSION OF HYDROGRAPHS RESULTING FROM ATTENUATED KINEMATIC SOLUTIONS

The results obtained from the kinematic simulations are shown in figures 5.6 — 5 8 for System 1 and in figure 5.9 for System 2.

FIGURE 5.6
OUTFLOW HYDROGRAPHS
SYSTEM I

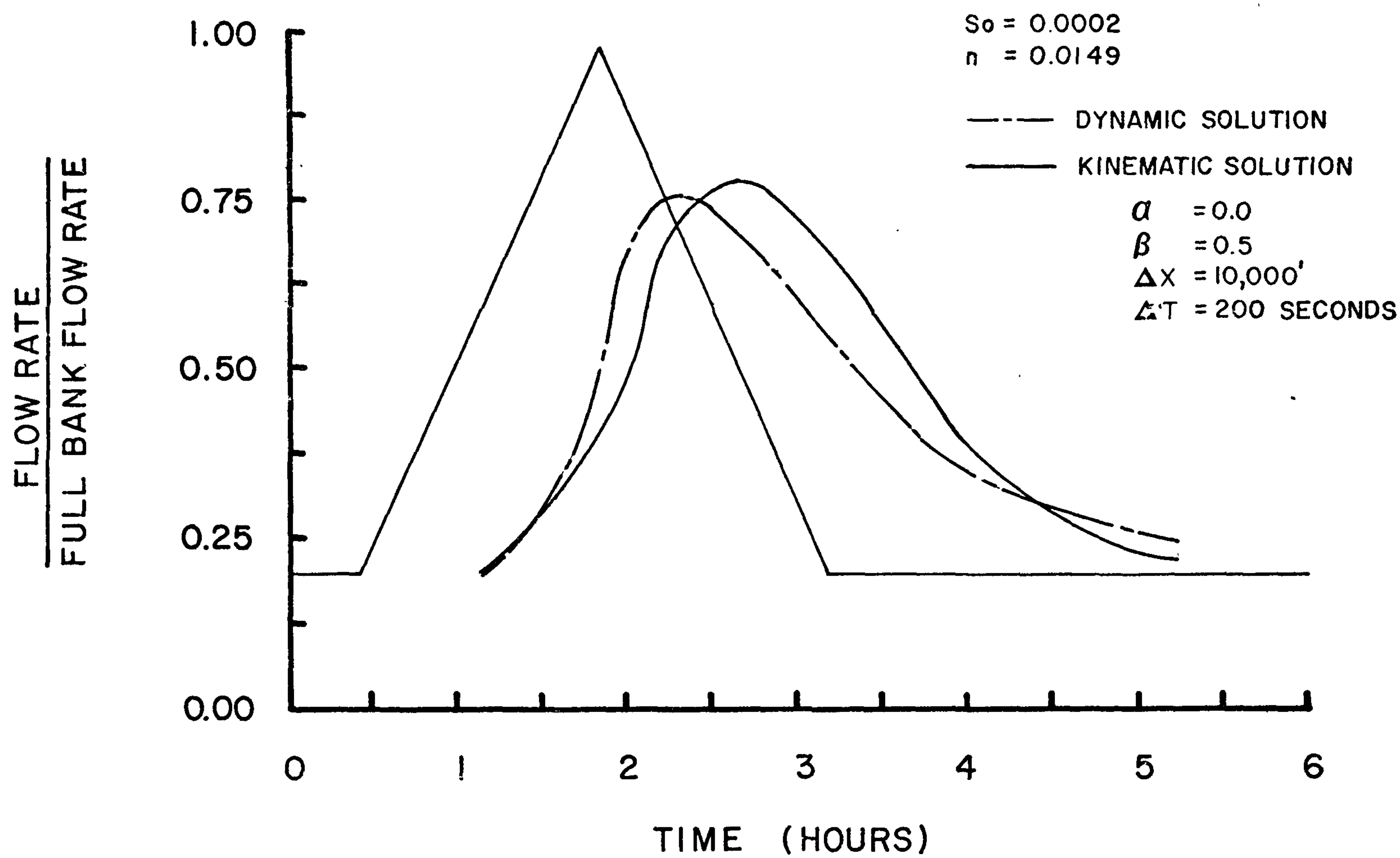


FIGURE 5.7
OUTFLOW HYDROGRAPHS
SYSTEM I

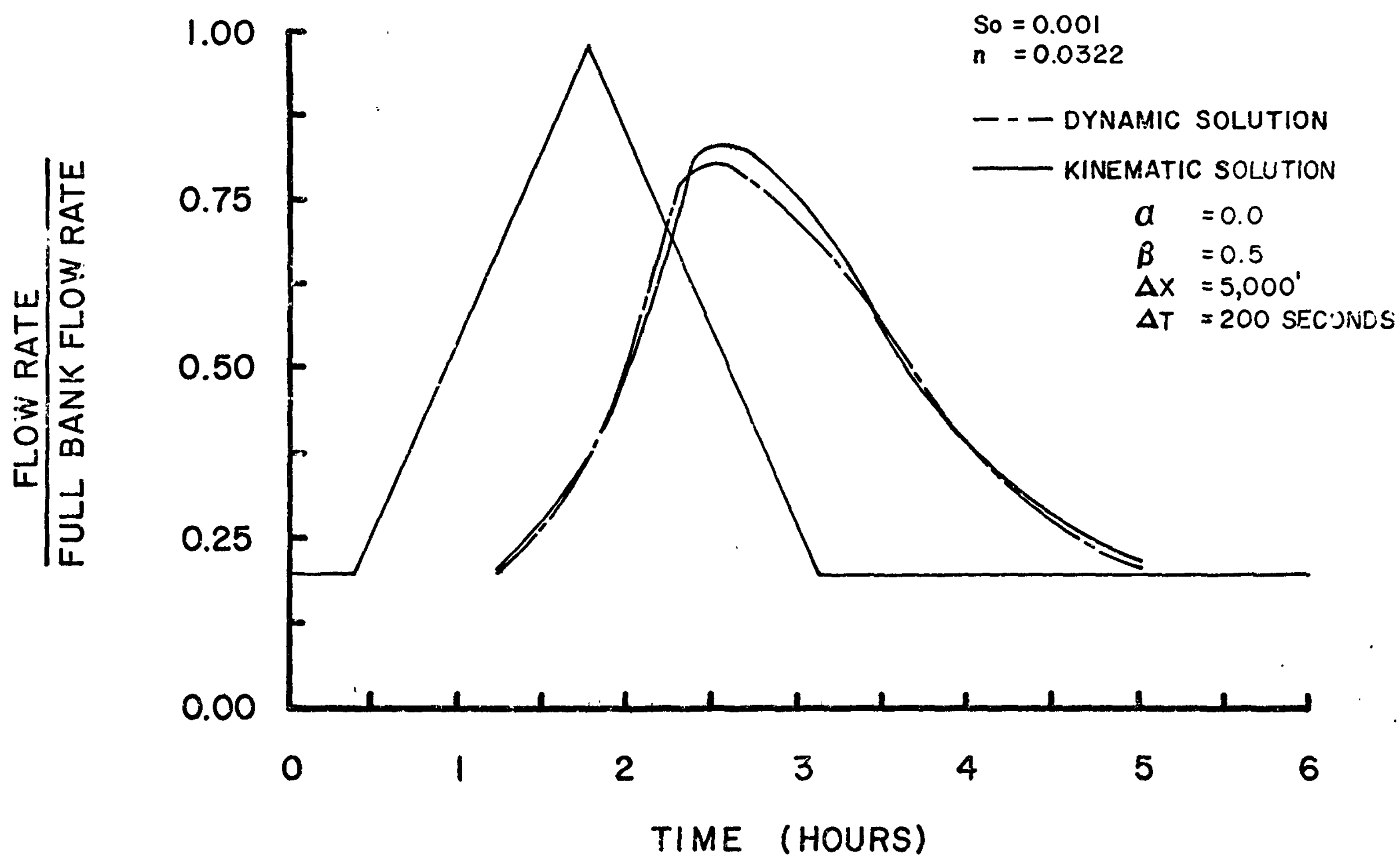


FIGURE 5.8

OUTFLOW HYDROGRAPHS

SYSTEM I

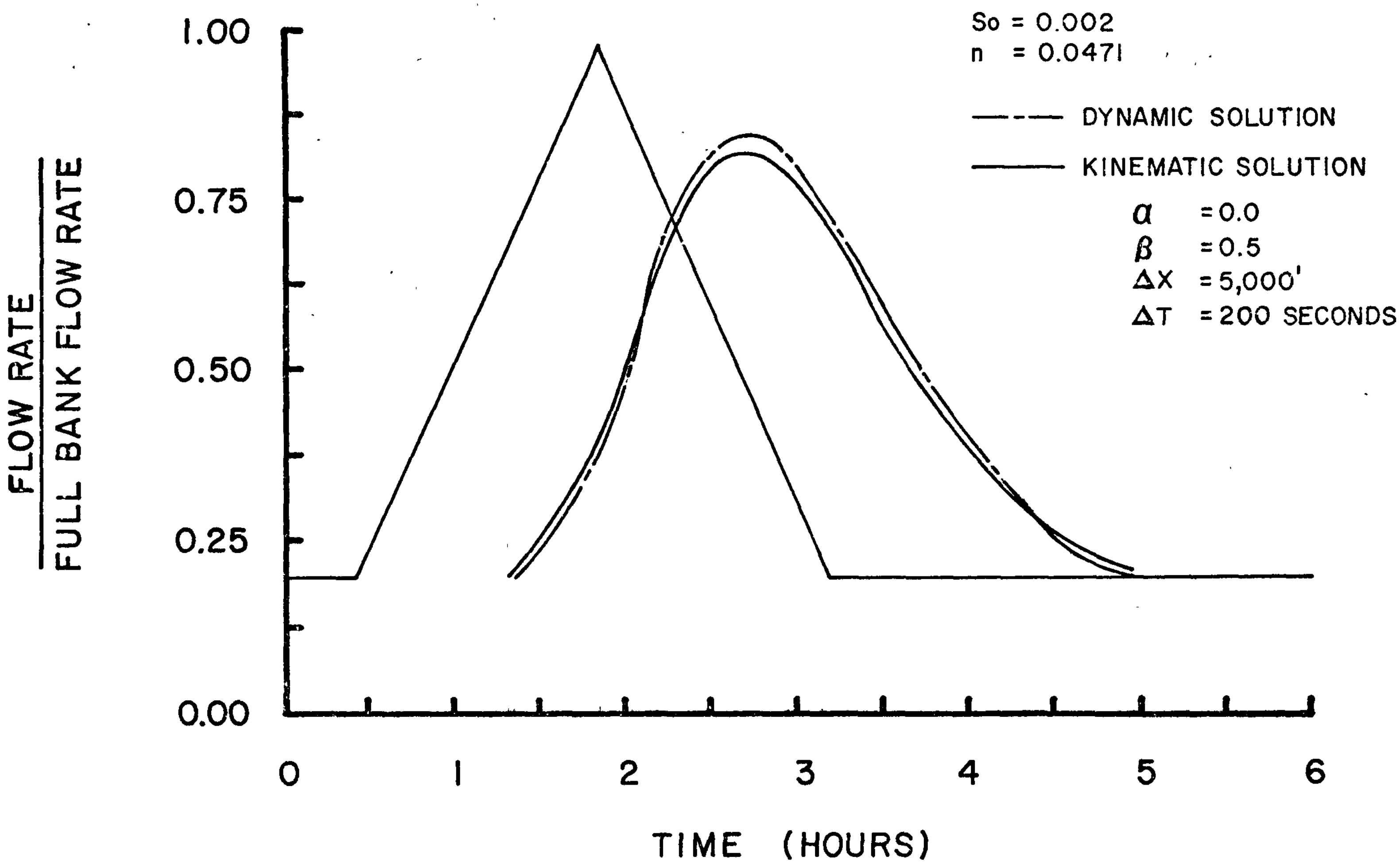
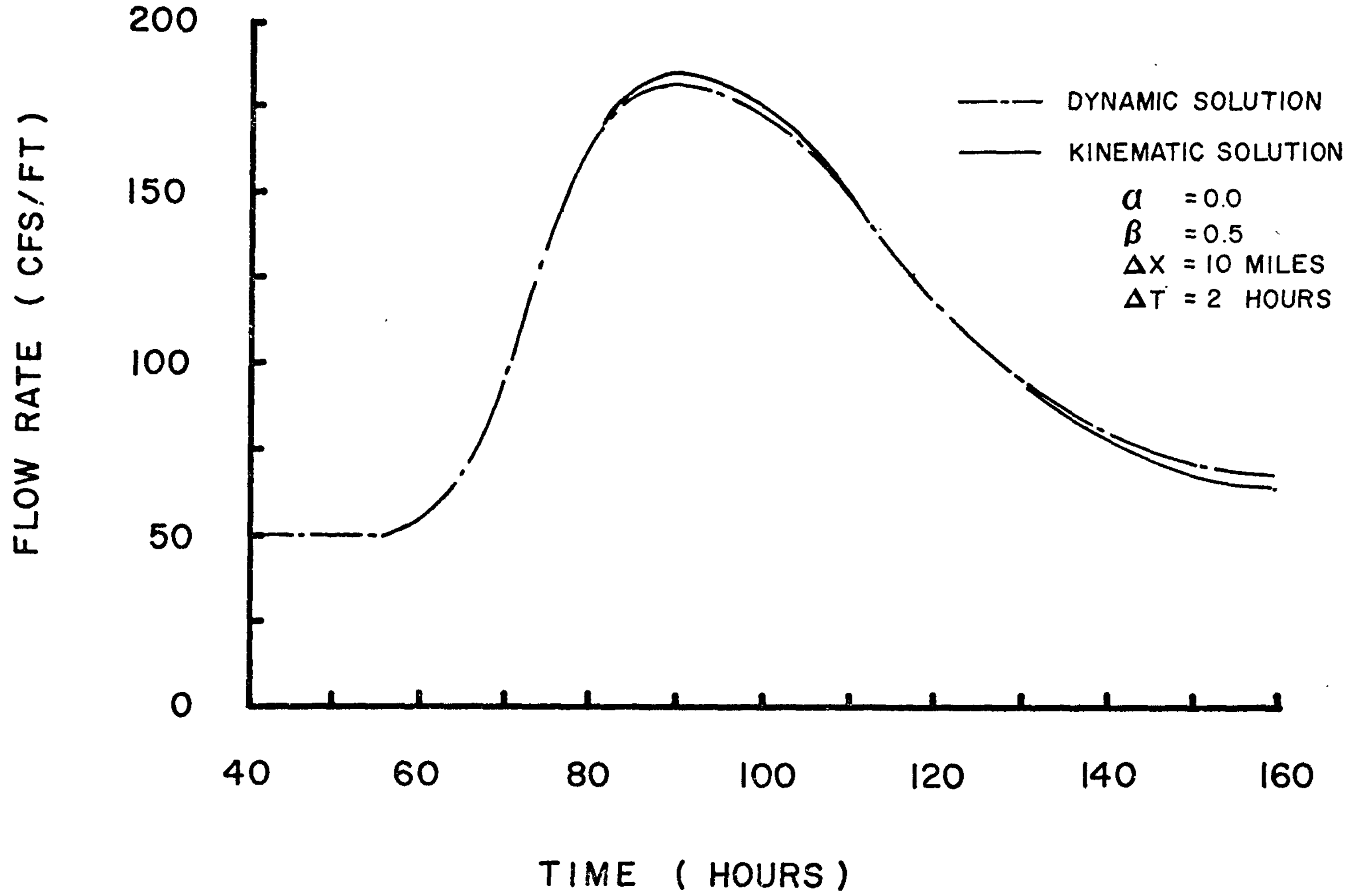


FIGURE 5.9

OUTFLOW HYDROGRAPHS

SYSTEM 2 300 MI. DOWNSTREAM



When the bed slope was very shallow, dynamic effects were significant and the kinematic solution was lagged relative to the dynamic solution as shown in figure 5.6. However, as the bed slope was increased and dynamic effects became less significant the dynamic solution and the attenuated kinematic solution were very close as demonstrated on figures 5.7 and 5.8. Figure 5.9 illustrates that with System 2 there was a very close correspondence of the results from the two methods of analysis. Further discussion of criteria relating to the applicability of kinematic routing is found in Chapter 4.

5.7 CONCLUSIONS

Numerical experiments have been performed to study the way in which attenuation is modelled by the:

1) Molecule effect
2) Cascade effect
3) Storage effect

Results of using the general kinematic flood routing method to simulate the above effects reveal that in certain cases it is possible to accurately model unsteady open channel flow. In the past kinematic techniques have been subject to criticism due to the sensitivity of the solution to the size of ΔX and ΔT. However, investigating this sensitivity and determining how it affects the solution, provides valuable insight into a potentially useful numerical tool. Varying the size of ΔX changes the molecule effect, depending on the selection of α and β, it changes the storage effect and for a fixed total reach length it affects the cascade effect.

Similarly the selection of ΔT is a significant parameter in determining the molecule effect if the nucleus is not located on the centre of the molecule.

In the course of the numerical experiments, it was noted that the peak outflow, if plotted over the finite difference molecule, forms a relatively smooth surface. A limited relationship has been developed which can be used to predict the value of the peak at any point over the molecule after calibration. With this relationship, which combines molecule, cascade and storage effects, it is possible to establish a kinematic model based on results of a dynamic model and/or actual recorded input and output of the prototype. Using S and ΔX it is possible to select values of α , β , ΔX and ΔT to produce a particular outflow hydrograph, provided the dynamic effects are not too dominant.

Further research is required to determine the influence of other factors such as the inflow hydrograph characteristics and physical properties of the system on the outflow hydrograph. This may enable a user to reduce the number of numerical experiments required to calibrate a model and may make it possible to predict the outflow hydrographs after the system has been altered without subsequent recalibration.

CHAPTER 6

MODELLING ATTENUATION WITH AN IMAGINARY RESERVOIR

The previous chapter was devoted to a discussion of apparent attenuation which resulted from the inclusion of storage in an elementary reach. This storage was introduced by modifying the computational approach so that combination channel-reservoir units were modelled. This chapter is devoted to studying a system where the channel and reservoirs were modelled as two distinct phenomena.

The prime objectives of this chapter are; (i) to provide a picture of the basic idea of a channel and reservoir in series, (ii) to study the sensitivity of the solution to the location of the reservoir and (iii) to investigate differences resulting from simulations where the reservoirs have different degrees of nonlinearity.

The results of the study are summarized in the concluding remarks. As well, several suggestions are included which point out areas where future research of this topic may be beneficial and where the concept of a channel and imaginary reservoir in series may be useful.

6.1 THE PHYSICAL SYSTEM:

Using a channel and an imaginary reservoir in series to simulate a natural channel system is an often used hydrologic tool. This technique, commonly known as the lag and route method, utilizes two components in performing the numerical simulation. These are (i) a linear channel and (ii) a linear reservoir. A linear channel is defined as a channel in which the flow rate is a linear function of the cross-sectional area. Thus, the kinematic wave velocity $\frac{dQ}{dA}$ is a constant for any specific cross-section independent of flow rate (or stage). Translation of the wave is modelled by this portion of the model. The linear reservoir, which produces the attenuation of the flood wave, is defined as a reservoir in which the storage is a linear function of the outflow.

There are several characteristics of linear systems that allows the simulation process to be simplified to a degree. Firstly, when linear components are utilized in the study the response of the system and therefore the solution are not sensitive to the input and it is possible to utilize the principle of superposition. Thus, it is not necessary to consider the order in which the components occur.

The present study enlarges upon the basic lag and route approach by allowing for the inclusion of non-linear channel and non-linear reservoir elements in the system. In particular, the following variables were considered:

1. The location of the reservoir.
2. The degree of non-linearity of the reservoir.
3. The magnitude of live storage associated with the reservoir.

In studying the sensitivity of the solution to the location of the reservoir, the limiting cases were investigated. The traditional lag and route method, with the reservoir located at the downstream end of the channel was used along with what is hereafter defined as the route and lag method, a system with the imaginary reservoir at the upstream limit of the channel.

System 1 was utilized for this portion of the study. The channel was modelled as eight elementary reaches, each 5000 feet long. The single imaginary reservoir was described using the following equation:

$$ST = KQ^{w} \qquad (6.1)$$

where:

ST = storage

Q = flow rate

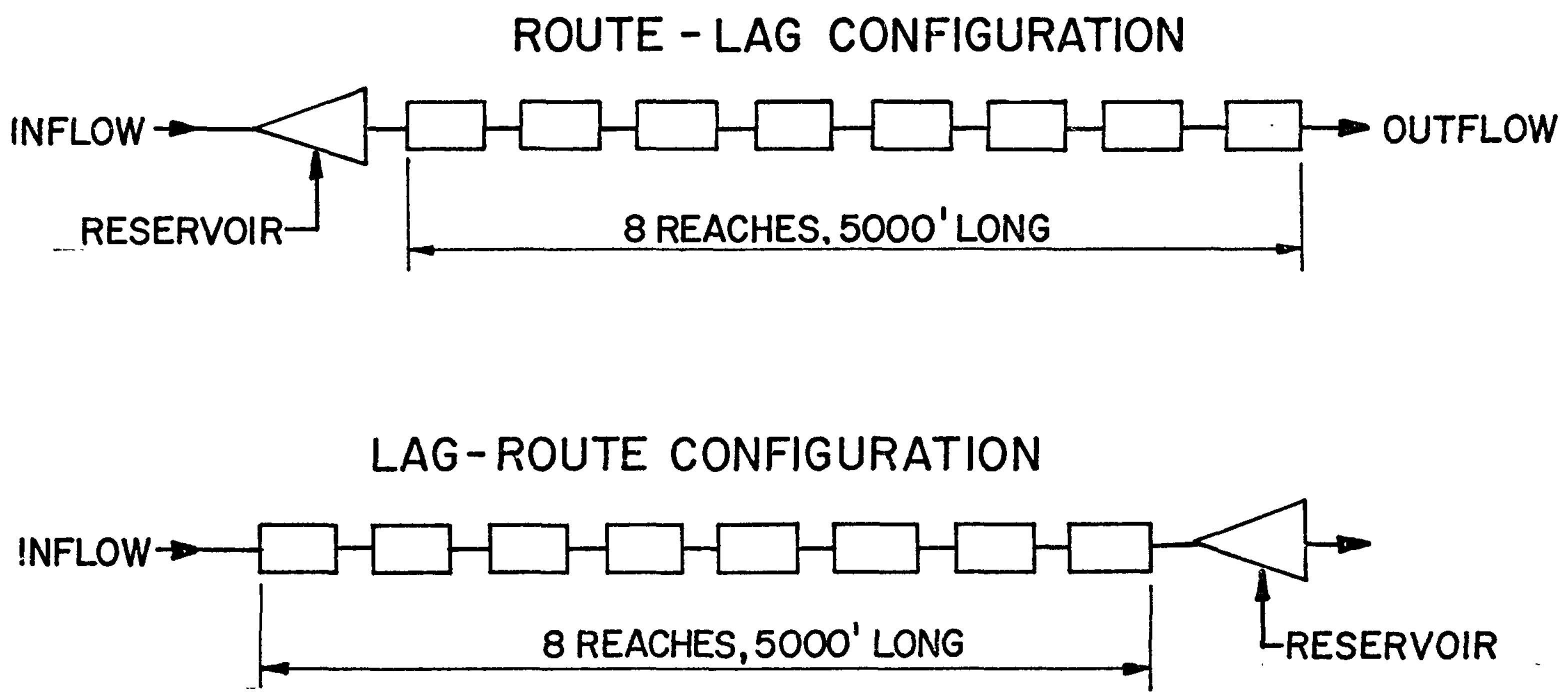
K = a parameter

w = a parameter

A schematic representation of the system is shown in figure 6. 1.

When studying System 1 using the finite difference solutions of the momentum and continuity equations, attenuated peaks of the order of 0. 7 and 0. 8 were obtained. By trial and error, an approximate res-

FIGURE 6.1
SCHEMATIC OF CHANNEL - RESERVOIR SYSTEM



ervoir size was determined which would result in attenuation of approximately the same magnitude. This was used to study the sensitivity of the solution to the location of the reservoir.

6.2 THE POSITION OF THE RESERVOIR

Table 6.1 contains a set of results obtained from a series of simulations using the route and lag analysis (reservoir upstream) while table 6.2 contains a set of results for similar simulations using the lag and route (reservoir downstream) method of analysis.

For each system studied, nine reservoir types were examined using value of K and w as follows:

$$K = 12000,\ 14000,\ 16000$$

$$w = 0.8,\ 0.9\ ,\ 1.0$$

For each system and reservoir type, the attenuated peak, P, and two measures of lag were recorded. The lag parameters used were defined as follows:

Tp = time to peak (hr)

Tc = time to centroid (hrs) of hydrograph above baseflow

The values of P, Tp, Tc are given for each system and reservoir type in tables 6.1 and 6.2. As might be expected, the attenuation P varies inversely with K and w since each of these terms tend to increase the sensitivity of live storage to the discharge Q (see equation 6.1). The lag terms similarly increase with increase of storage

TABLE 6. 1

DATA FROM RESERVOIR -- CHANNEL SIMULATION

SYSTEM 1

40, 000 FT DOWNSTREAM

*Truncated

K \ w	0. 8	0. 9	1. 0
12000	P = 0. 833 Tp = 1. 24 Tc = 1. 47	P = 0. 632 Tp = 1. 69 Tc = 1. 91	P = 0. 437 Tp = 2. 13 Tc = 1. 89*
14000	P = 0. 808 Tp = 1. 30 Tc = 1. 54	P = 0. 599 Tp = 1. 74 Tc = 1. 93	P = 0. 412 Tp = 2. 19 Tc = 1. 87*
16000	P = 0. 780 Tp = 1. 36 Tc = 1. 60	P = 0. 570 Tp = 1. 80 Tc = 1. 96	P = 0. 391 Tp = 2. 19 Tc = 1. 85*

P = Peak Flow/Full Bank Flow

Tp = Time to Peak (hrs)

Tc = Time to Centroid (hrs)

See figure 6. 3 for definition of above terms.

TABLE 6.2

DATA FROM CHANNEL--RESERVOIR SIMULATION

SYSTEM 1

40,000 FT DOWNSTREAM

*Truncated

K \ w	0.8	0.9	1.0
12000	P = 0.818 Tp = 1.13 Tc = 1.42	P = 0.616 Tp = 1.58 Tc = 1.80	P = 0.429 Tp = 2.02 Tc = 1.80*
14000	P = 0.791 Tp = 1.19 Tc = 1.49	P = 0.584 Tp = 1.63 Tc = 1.78*	P = 0.405 Tp = 2.08 Tc = 0.80*
16000	P = 0.766 Tp = 1.25 Tc = 1.56	P = 0.556 Tp = 1.69 Tc = 1.74*	P = 0.386 Tp = 2.08 Tc = 0.10*

P = Peak Flow/Full Bank Flow

Tp = Time to Peak (hrs)

Tc = Time to Centroid (hrs)

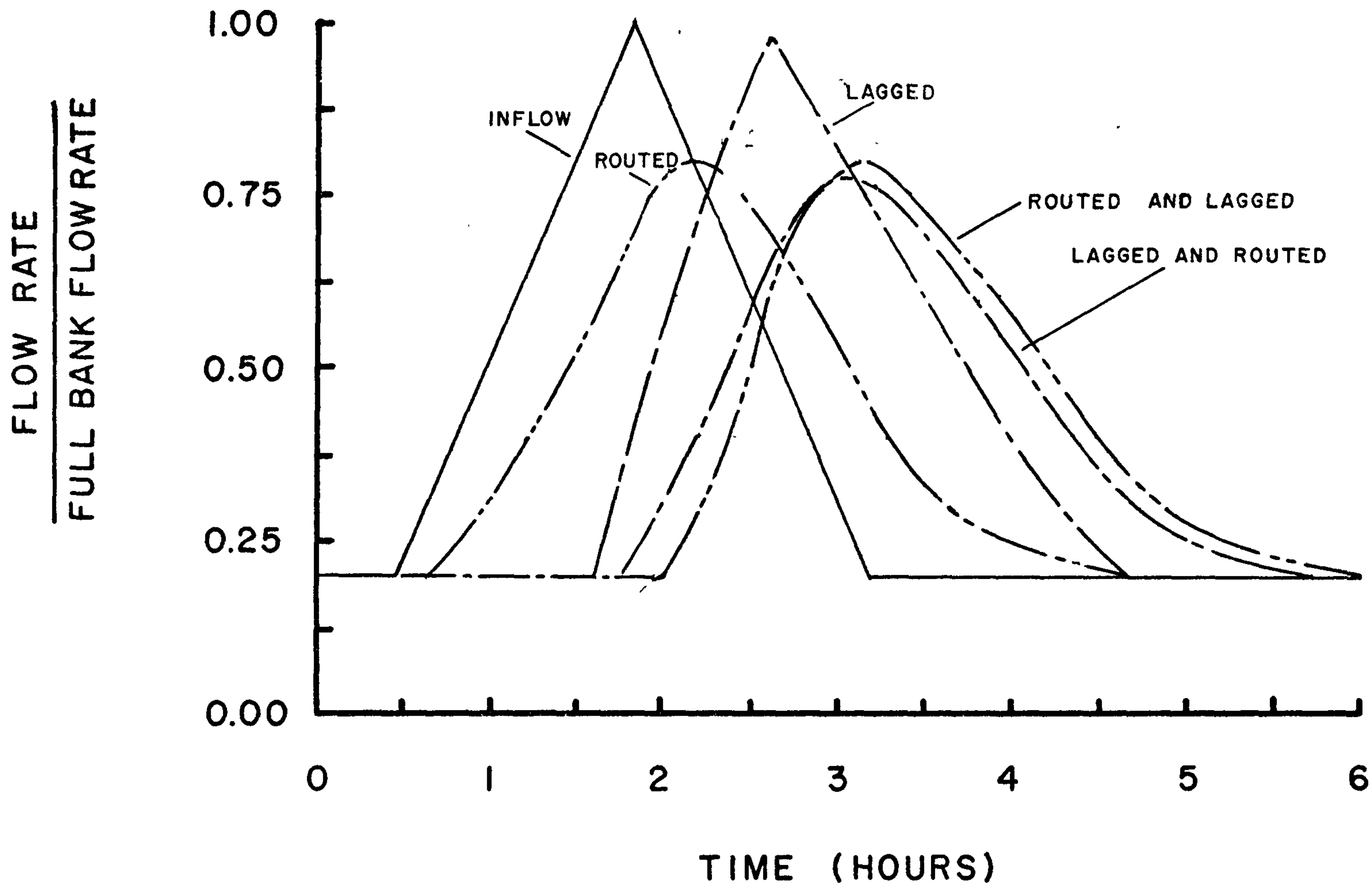
See figure 6.3 for definition of above terms.

(K and w increasing) although with large amounts of lag and attenuation, the outflow hydrograph is significantly truncated resulting in serious error in evaluation of Tc. These cases are marked in tables 6.1 and 6.2.

The initial observation is that whereas the value of P is sensitive to values of K and w, the sensitivity to the position of the reservoir is low. In general, values of P in table 6.2 are 2% lower than in table 6.1. The effect of moving the reservoir downstream is small and even this small difference is due not to any fundamental change in the performance of the model, but rather to numerical "softening" of the peak in the channel routing stage which is more pronounced when the unattenuated hydrograph is routed down the channel. Thus, for this particular system, the attenuation is quite insensitive to the location of the reservoir. This can be attributed to the fact that the modulating effect of the reservoir is dependent upon wave shape, among other things, and there is no significant change in the wave shape as it passes down the channel.

While there was no significant differences in the amount of attenuation when the reservoir location was varied from the upstream end of the channel to the downstream limit, there was an appreciable change in the shape of the outflow hydrograph as shown in figure 6.2. With a route and lag system, the peak outflow occurred later and the

FIGURE 6.2
TYPICAL HYDROGRAPHS, CHANNEL-RESERVOIR
K=14000 W=0.8



rising limb of the hydrograph was steeper than the simulation of the lag and route configuration. This phenomena results from the variation of kinematic wave velocity with stage.

For the system studied, higher stages have a higher kinematic wave velocity than lower stages or flow rates, thus, the peak of the unattenuated wave would travel down the channel faster and arrive at the downstream end passing through the reservoir sooner than the wave which was attenuated by a reservoir and then flowed down the channel. This variation in kinematic wave velocity also causes waves to steepen; but with the reservoir at the downstream end, the steepening caused in the channel was smoothed out. However, the waves from the reservoir located at the upstream end of the channel were steepened by the passage down the channel.

A comparison of the kinematic solutions with the dynamic solutions indicates that there is a considerably longer lag in the outflow from the kinematic simulations. This can be attributed to the introduction of the imaginary reservoir and the fact that the kinematic wave velocity of the channel is less than the dynamic wave velocity.

Although these preliminary tests confirmed the general qualitative dependence on K and w, further tests were used to extend this knowledge. In these tests the route and lag configuration was used to minimize the errors that were introduced into the kinematic channel routing calculations.

6.3 MEASUREMENT AND DEFINITION OF THE RESPONSE CHARACTERISTICS

In order to provide a quantitative base for the comparison of the results of the numerous simulations that were made during the process of studying the effects of varying K and w, the following items were quantified:

1. Peak outflow.
2. Centroidal lag of the outflow hydrograph.
3. Skewness of the outflow hydrograph.

The first two items were computed in the simulation program and obtained from the computer printouts. A triangular inflow hydrograph, defined earlier in the description of System 1 was used in this study. As a result of the discretization of the hydrograph to conform with the time steps used in the numerical calculations, the peak of the hydrograph was truncated. Therefore, the inflow hydrograph was trapezoidal in shape with a peak value of 0.984 times the peak of the triangular hydrograph. However, this truncation was ignored and no corrections were made to the data produced by the computer. The centroidal lag value calculated by the computer included all of the base flow. Corrections were made to the data generated by the computer to obtain the centroid of the outflow hydrograph alone. A graphical representation of these and other values used to describe the hydrograph is contained in figure 6. 3.

The third parameter used to describe the hydrograph was an attempt to quantify the general shape of the hydrograph in relation to the centroid and the peak discharge. This empirical skew factor was defined as:

$$SF = \frac{Tc - Tp}{T_b(P - 0.2)} \qquad (6.2)$$

where: SF = skew factor

P = peak flow ratio

Tc = time of centroid

Tp = time of peak discharge

Tb = time base of inflow hydrograph.

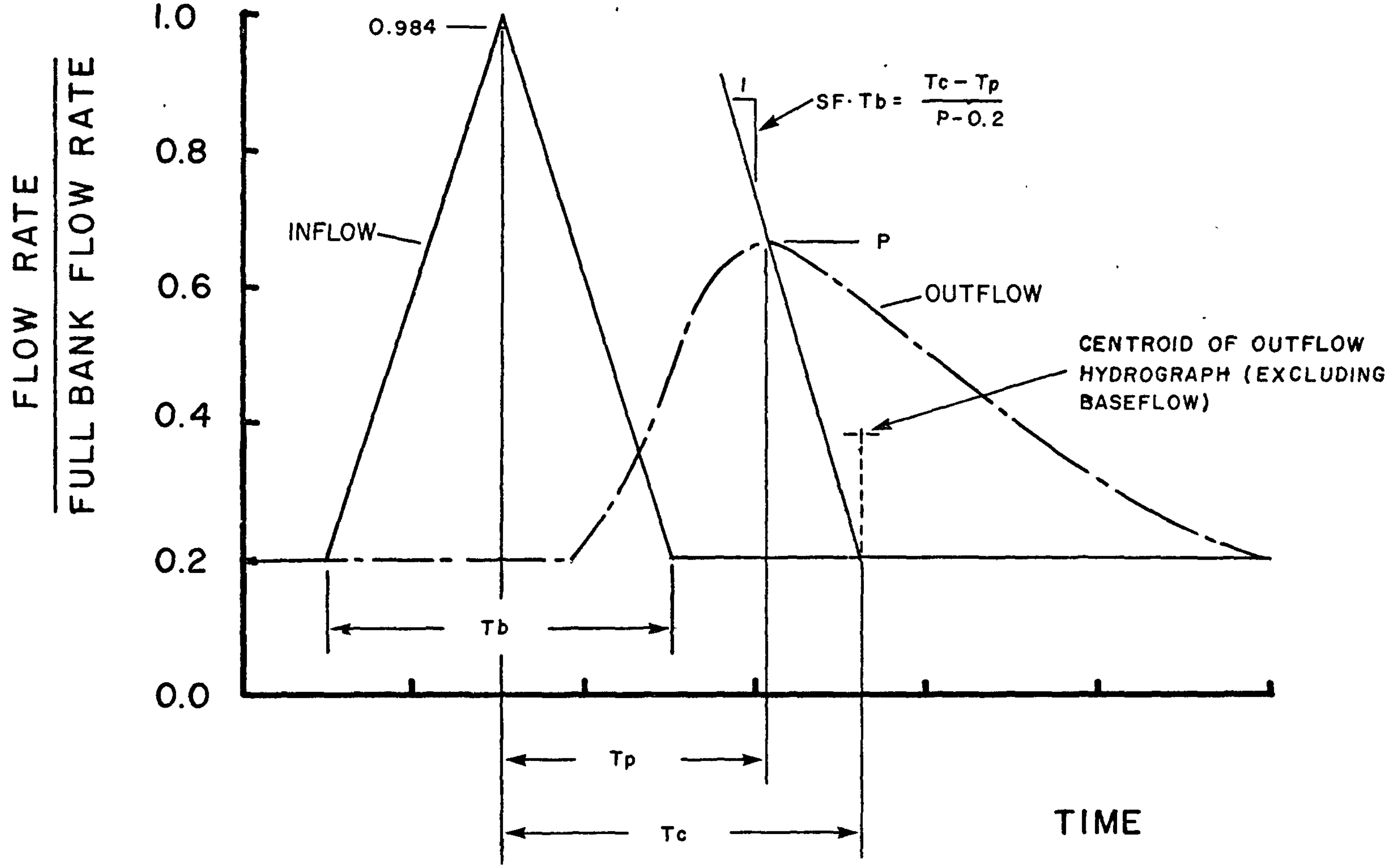
0. 2= base flow rate ratio

The skew factor describes the slope of the line from the base flow rate and the centroid of the hydrograph to the peak outflow and time of peak outflow as shown in figure 6. 3.

6. 4 SYSTEM PARAMETERS.

A measure of the system parameters (independent variables) was required to complement the response variables (dependent

FIGURE 6.3
DEFINITION OF RESPONSE PARAMETERS



variables) so that the effects of non-linearity and the size of the reservoir could be studied.

The channel properties and the configuration of the system were held constant. The system was characterized by the ST versus Q curve and the two parameters that were utilized to describe this curve were the slope of the chord and the "bow" or departure from linearity in the curve over the range of flow rate that was experienced by the system. These parameters are respectively:

$$\text{CHORD SLOPE} = \frac{\Delta ST}{\Delta Q} \qquad (6.3)$$

where: ST = storage

Q = flow rate

and

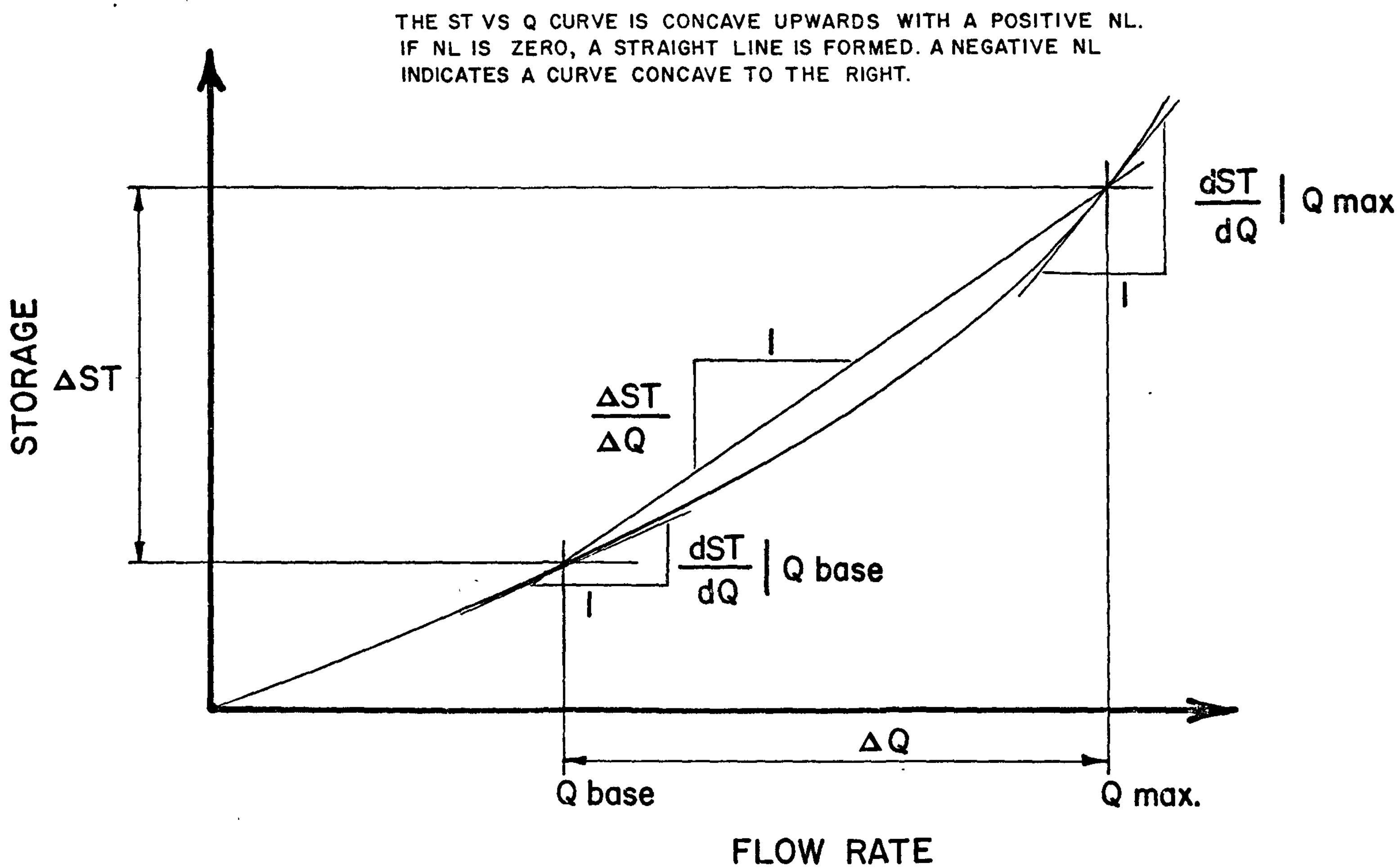
$$NL = \frac{\left.\frac{dST}{dQ}\right|_{QMAX} - \left.\frac{dST}{dQ}\right|_{QBASE}}{\frac{\Delta ST}{\Delta Q}} \qquad (6.4)$$

where:

$$\frac{dST}{dQ} = wKQ^{w-1}$$

A definition sketch of these parameters is included in figure 6.4.

FIGURE 6.4
DEFINITION OF SYSTEM PARAMETERS



Because the range of flows that were encountered was constant for all of the tests NL reduced to a function of w, however, if the range of flows varied, NL would reflect the change in the curvature of the ST versus Q curve for different ranges of flows.

6.5 DISCUSSION OF THE RESULTS

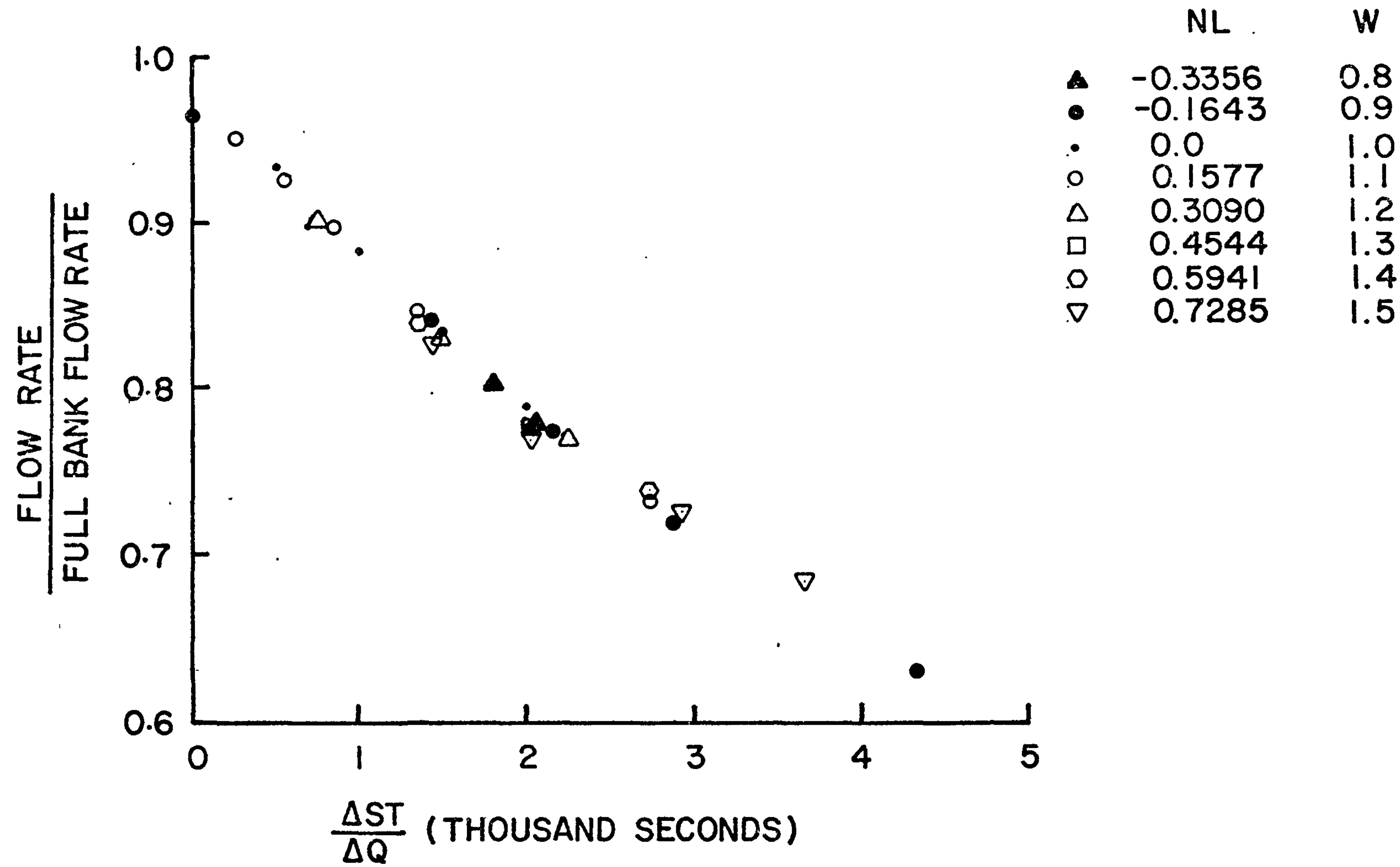
To determine the relationship of the response characteristics to the system parameters, a number of plots were made to show the relationship of P, Tc and SF to $\frac{\Delta ST}{\Delta Q}$ and NL. In each of the cases, $\frac{\Delta ST}{\Delta Q}$ was found to be the dominant physical parameter. The solutions were sensitive to the non-linearity of the reservoir as will be shown in the discussion of the various graphs.

6.5.1 RELATION OF PEAK FLOW RATIO, P, TO SYSTEM PARAMETERS.

Figure 6.5 contains a plot of P versus $\frac{\Delta ST}{\Delta Q}$. Each of the co-ordinate pairs plotted was identified to show the NL number for the particular simulation from which the value was obtained. This figure indicates that as $\frac{\Delta ST}{\Delta Q}$ increases, P decreases. In other words, as the surface area of the reservoir increases, there is a larger amount of attenuation.

There was a small amount of scatter in the points that were plotted. By identifying each of the points according to the non-

FIGURE 6.5
PEAK OUTFLOW Vs CHORD SLOPE



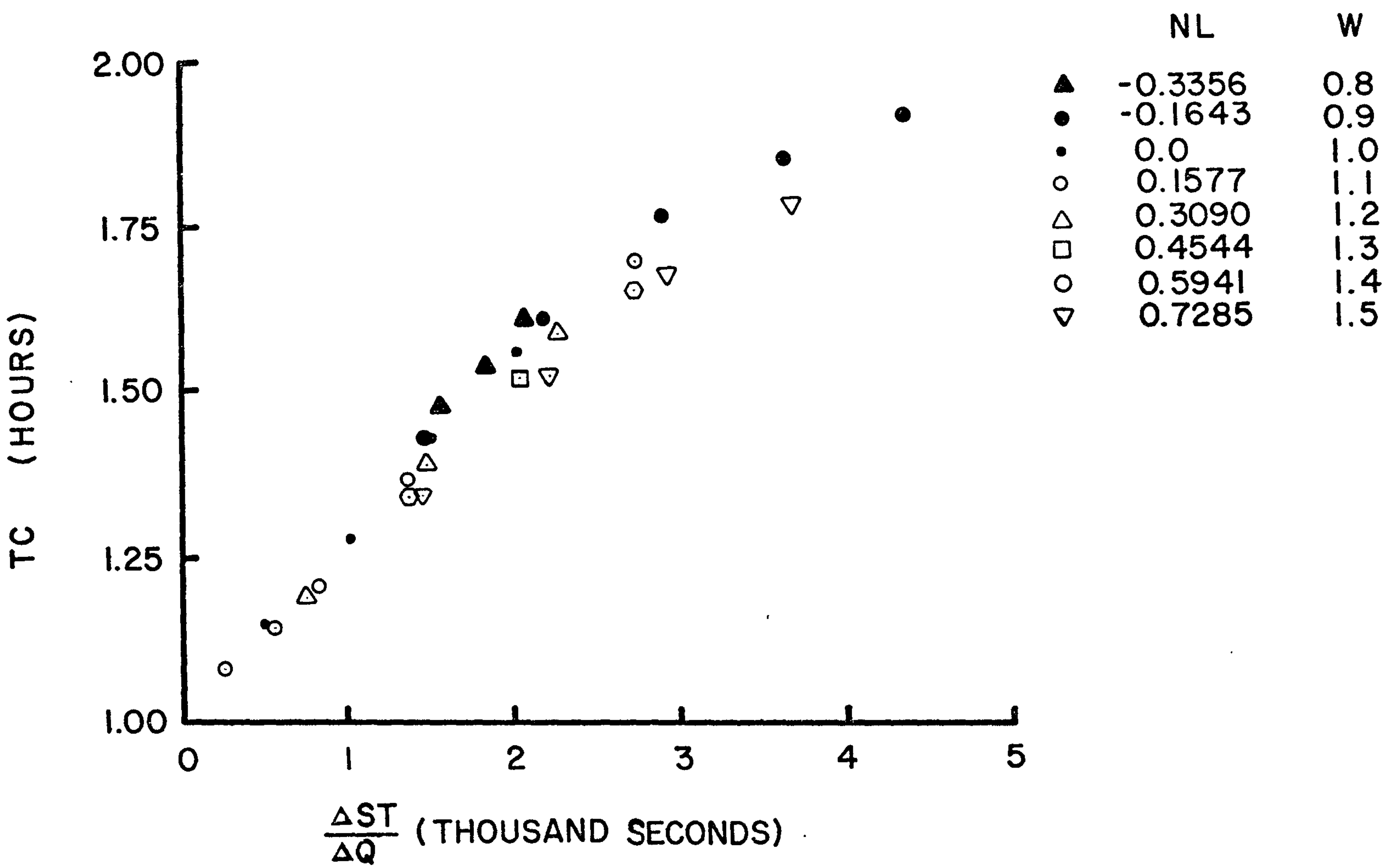
linearity of the reservoir, it was obvious that this scatter could be correlated to NL.

As the non-linearity number, NL, increased algebraically from negative values to positive values, a larger $\frac{\Delta ST}{\Delta Q}$ was required to yield a given amount of attenuation. It should be noted that $\frac{\Delta ST}{\Delta Q}$ was calculated on the basis of base flow and the maximum flow rate of the input, which in this study was full bank flow. If $\frac{\Delta ST}{\Delta Q}$ had been calculated using base flow and the peak of the attenuated outflow hydrograph, the variation of the values of $\frac{\Delta ST}{\Delta Q}$ with respect to NL may have been reduced.

6.5.2 RELATION OF TIME OF CENTROID, Tc, TO SYSTEM PARAMETERS

The plot of Tc versus $\frac{\Delta ST}{\Delta Q}$ shown in figure 6.6 indicates that as $\frac{\Delta ST}{\Delta Q}$ increases there is a corresponding increase in the centroidal lag. This is a result which is intuitively correct for as the size of a reservoir increases, it is reasonable to expect an increase in the delay caused by storage. The plot showed a small amount of scatter that again could be attributed to the non-linearity of the reservoir. With an algebraic increase of NL, there was a decrease in the cetroidal lag associated with a particular value of $\frac{\Delta ST}{\Delta Q}$. This can be attributed to the properties of a storage unit which has a proportionally larger storage with high flow rates when compared

FIGURE 6.6
CENTROIDAL LAG Vs CHORD SLOPE



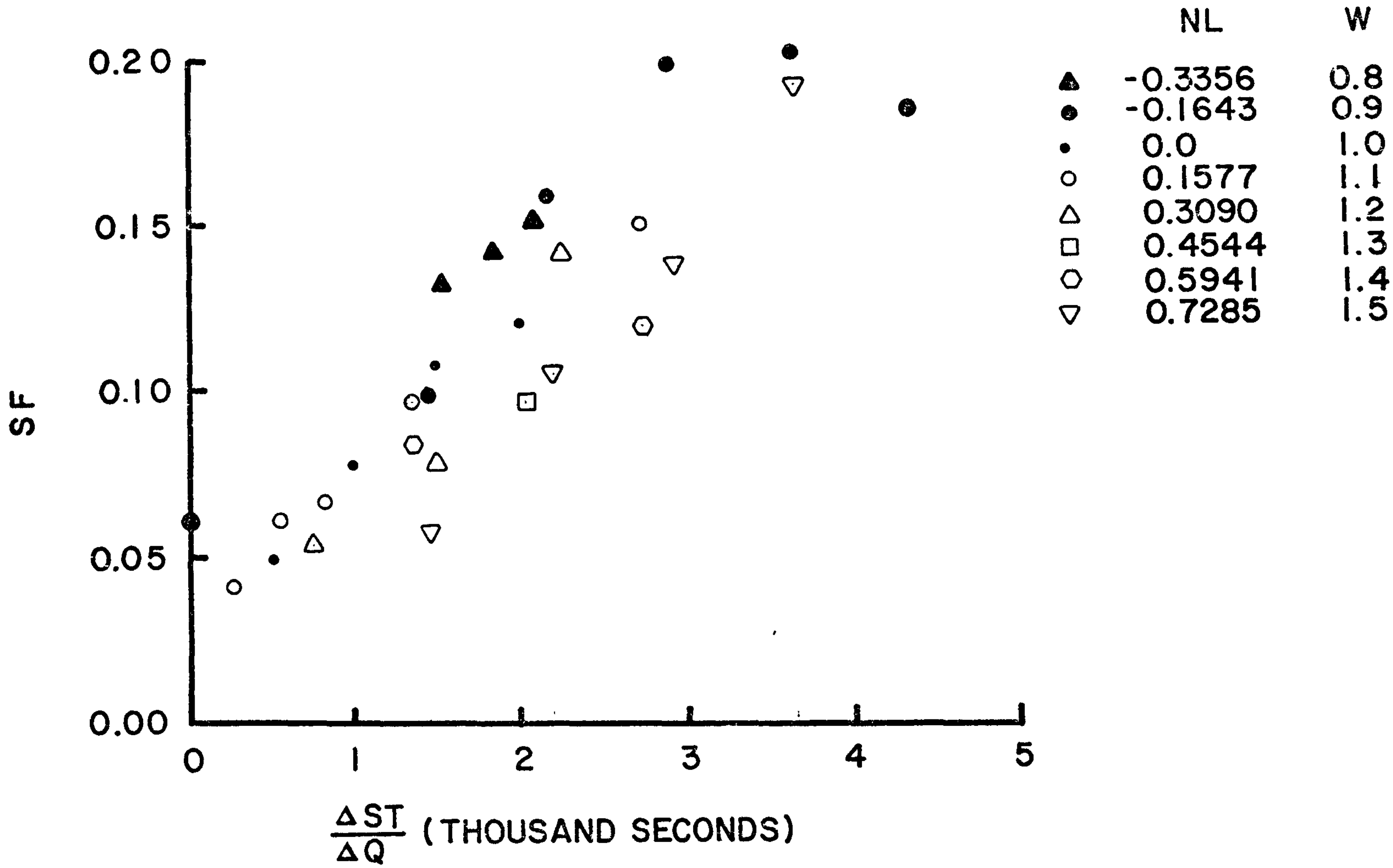
with lower discharges. During the early portions of the inflow hydrograph, when the flows are low, there is no delay in the flow through the storage element. This contributes to the earlier centroid of the outflow hydrograph.

6.5.3 RELATION OF SKEW FACTOR, SF, TO SYSTEM PARAMETERS

The plot of SF versus $\frac{\Delta ST}{\Delta Q}$ is shown in figure 6.7. The two previous graphs have shown relationships that were very nearly single valued. However, there is a large amount of scatter that can be attributed to the non-linearity of the reservoir and to what appears to be numerical errors in the computation of the skewness factor. This numerical error is a result of truncation in the computer output. From the data available, it appears that as the non-linearity increases, the skewness decreases. An increase of $\frac{\Delta ST}{\Delta Q}$ causes a larger skewness. It will be shown later that the time of the peak outflow is a unique function of P. Thus, for a given P value, the time of the peak will be fixed, but the centroid of the hydrograph will vary in relation to NL. As NL increases and Tc decreases, SF will decrease.

The curves that are presented in the previous discussion are only valid over a finite range. This is due to the fact that as $\frac{\Delta ST}{\Delta Q}$

FIGURE 6.7
SKEW FACTOR Vs CHORD SLOPE



increased, there was an increase in the length of the falling limb of the outflow hydrograph, which in turn resulted in a truncation of a portion of the hydrograph due to the termination of the simulation. The curve showing P versus $\frac{\Delta ST}{\Delta Q}$ is not subject to errors of this nature, but the plots of the centroidal lag and the skewness factor are influenced by this error when $\frac{\Delta ST}{\Delta Q}$ is larger than approximately 4, 000 seconds.

Reductions in the amount of scatter in the figures which show peak outflow, centroidal lag and skewness factor as a function of $\frac{\Delta ST}{\Delta Q}$ may be obtained if the chord slope is computed using the base flow and the peak outflow rather than base flow and peak inflow. However, the use of the latter technique may remain the most useful due to the fact that peak outflow is an unknown until the simulation is completed.

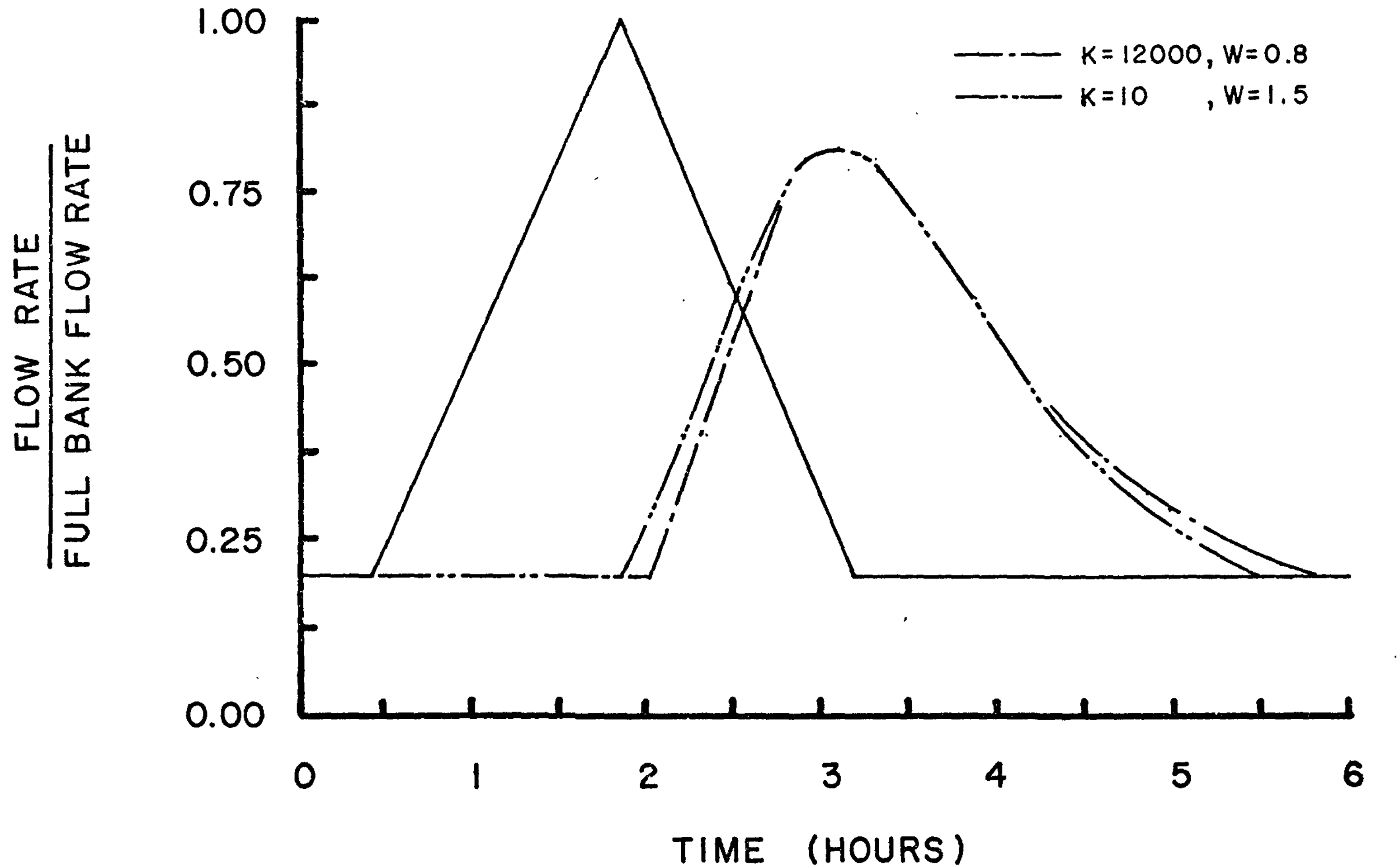
6.6 FURTHER COMPARISONS

To provide a comparison of the shapes of the variance of the hydrograph shape as the parameters K and w were varied, two of the many hydrographs obtained from these simulations were plotted on figure 6.8. Each of these hydrographs had a peak ratio of approximately 0. 83. With K = 10 and w = 1. 5, the hydrograph rose slightly quicker and dropped slightly sooner than the simulated hydrograph that was obtained using K = 12, 000 and

FIGURE 6.8

OUTFLOW HYDROGRAPHS

RESERVOIR-CHANNEL SYSTEM



w = 0.8. In the region of the peak there was no significant difference between the hydrographs and the two plots coincide.

The reason for the very close correspondence of the two floodwaves in the region of the peak can be attributed to the characteristics of the system being studied. For a given inflow hydrograph, kinematic channel system and imaginary reservoir the time at which a particular peak outflow occurs is constant regardless of the value K and w chosen to simulate the reservoir. For example, with the reservoir located at the upstream end of the channel, the hydrograph will be attenuated and the peak will occur at the intersection of the outflow hydrograph and the falling limb of the inflow hydrograph, thus uniquely determining the time of peak outflow from the reservoir independent of the reservoir characteristics, though the reservoir properties determine the peak. Similarly, the travel time for a specific flow rate is constant for a particular kinematic channel. A plot of the peak outflow ratio was plotted along with the times that these peaks occur. This data is contained in figure 6.9. As the peak outflow increased the speed at which the wave propagated increased and thus the time lag decreased.

Scatter resulted from the discrete representation of the hydrograph which caused the occurences to be represented at intervals one time step apart.

FIGURE 6.9

PEAK OUTFLOW VS Tp

RESERVOIR - CHANNEL, SYSTEM I

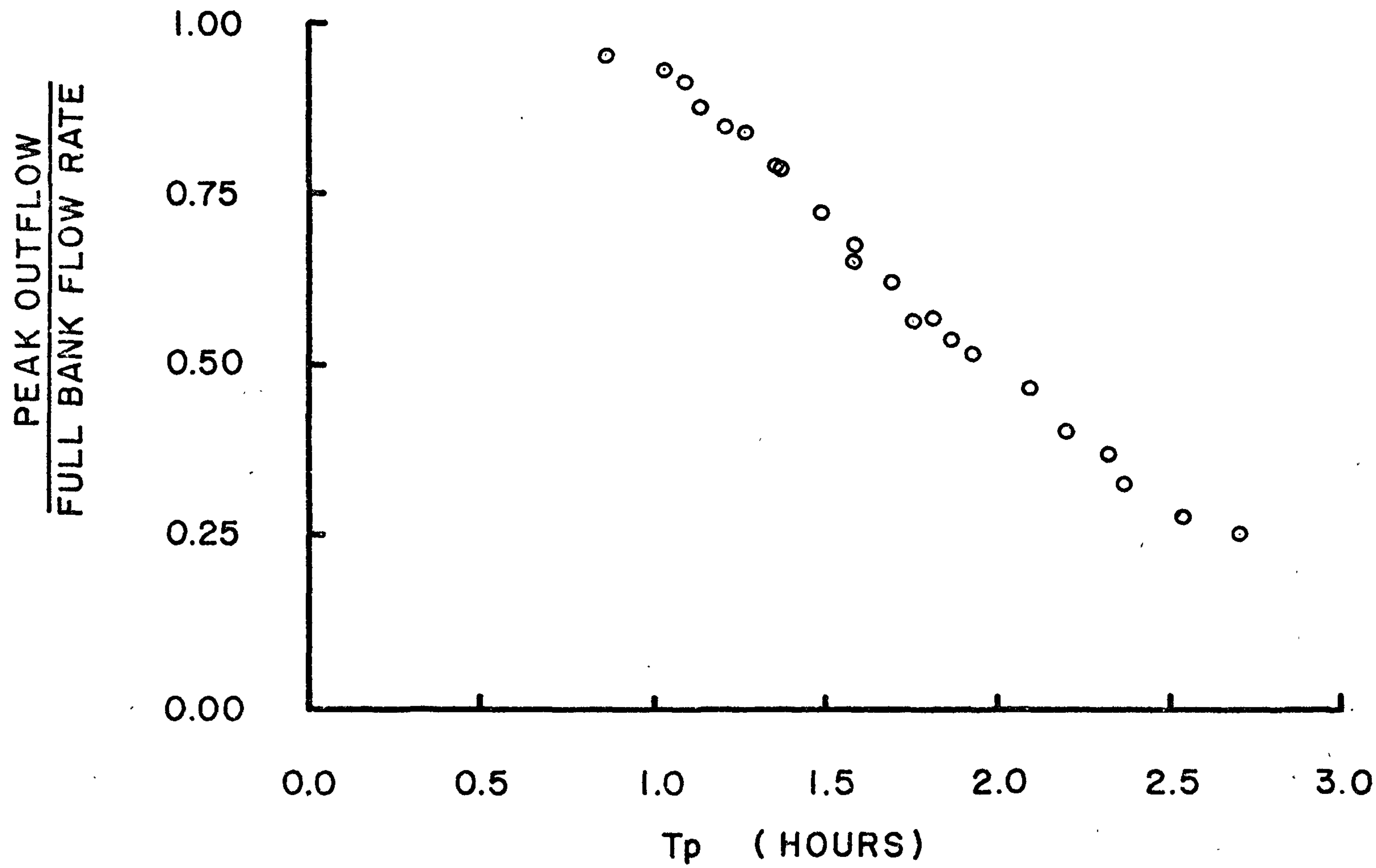
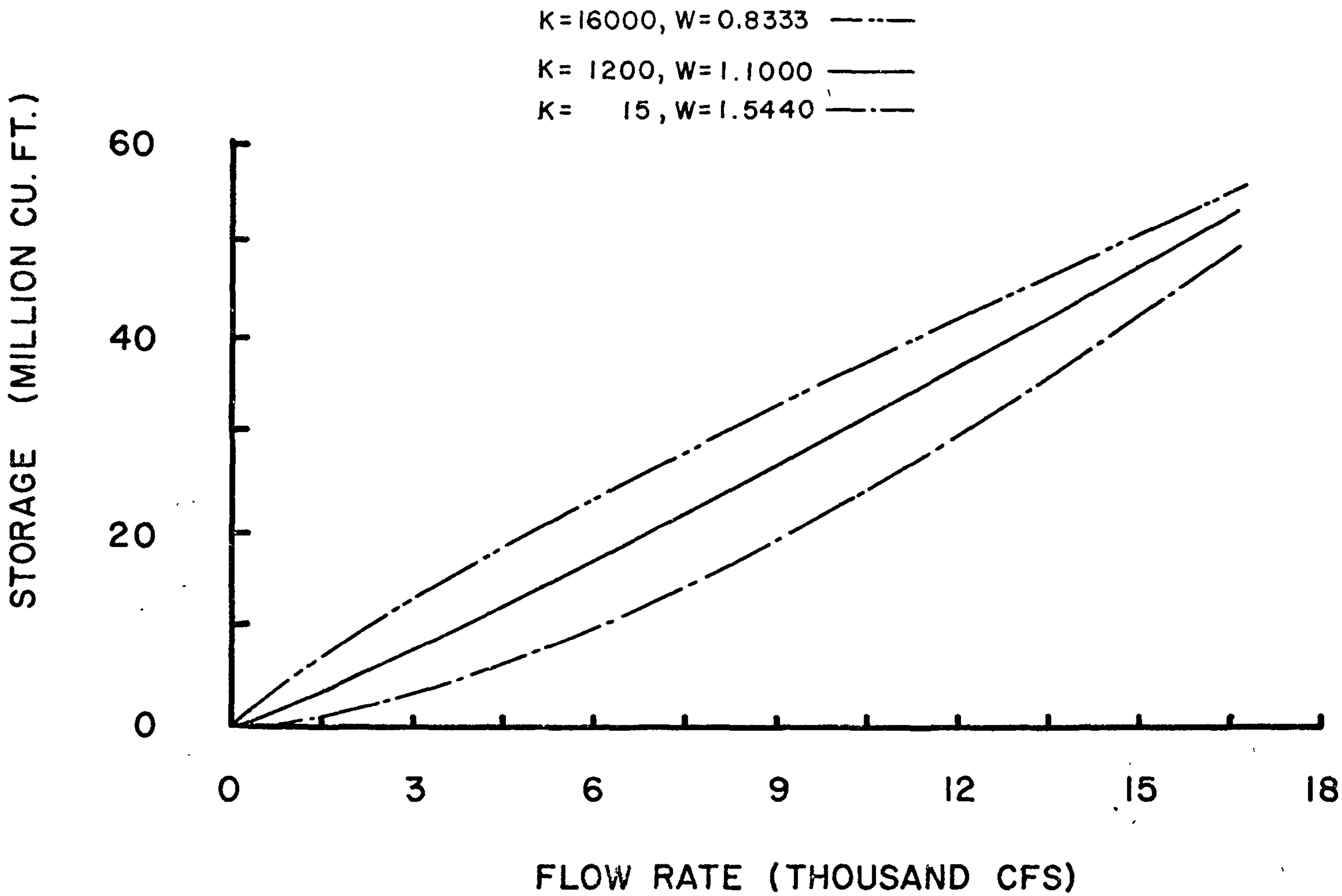


Figure 6.10 shows three storage versus flow rate curves for reservoirs which attenuated the flood wave to a peak ratio of 0.7 or 11,672 cfs. It can be seen that the maximum amount of storage utilized varied significantly as the non-linearity of the reservoir characteristics varied. With K = 15 and w = 1.544, the maximum storage required was twenty seven million cubic feet. Forty two million cubic feet of storage was required with K = 16,000 and w = 0.833. This fact may not be particularly important where the reservoir is imaginary, however, if the reservoir was being constructed to provide maximum attenuation, it would be prudent to design and operate the control structures so that the storage-outflow relationship is defined by an equation with a high index, w. In essence, this means that the reservoir should not be filled during the lower flows of the rising limb of the hydrograph, and the available storage should be reserved for larger flows immediately before and after the peak inflow.

6.7 CONCLUSIONS

This chapter has provided a brief review of lag-route and route-lag methods of simulated unsteady flow in open channels. It may be of interest to note that the use of a reservoir in series with a kinematic channel is analogous to the continuously stirred tank

FIGURE 6.10
TYPICAL RESERVOIR RATINGS



reactor and the plug flow tank reactor utilized in chemical engineering simulations. With reservoir and channel systems the objective is to predict flow rates while the concentration of a tracer is the important variable with the tank reactor systems.

The basic properties of the reservoir channel system that were studied included:

1. The location of the reservoir.
2. The degree of non-linearity of the reservoir.
3. The magnitude of live storage associated with the reservoir.

Reservoir characteristics were described using the following equation:

$$ST = KQ^{w} \tag{6.1}$$

where:
- ST = storage
- Q = flow rate
- K = a parameter
- w = a parameter

The general kinematic flood routing method was utilized to perform the channel routing computations.

The results of the study may be summarized as follows:

1. The general kinematic flood routing technique seems to perform more satisfactorily when the flood wave is less peaked.

2. For the particular system studied, the position of the imaginary reservoir affects the shape and timing of the hydrograph. Moving the reservoir toward the downstream boundary results in an outflow hydrograph that occurs sooner and which rises more slowly. The amount of attenuation predicted by the simulations indicates that the reduction of the flood peak is relatively insensitive to the location of the reservoir.

3. The non-linearity of the reservoir affects the peak of the outflow hydrograph, the centroidal lag of the hydrograph and the skewness factor. However, the chord slope of the relationship between storage and discharge, $\frac{\Delta ST}{\Delta Q}$, appears to be the dominant factor in determining values of these response characteristics. A comparison of several hydrographs, which had approximately the same peak outflow ratio indicated a relatively small change in the outflow hydrograph as a result of varying the parameters K and w. This lends substance to the statement of Dooge and Harley (1967) that the assumption of a linear reservoir for simulation purposes does not appear to be particularly restrictive.

4. The simulation of System 1 by means of the dynamic or "complete" solution with So = 0.0002 and n = 0.0149 yields an outflow hydrograph which is attenuated to a peak value approximately equal to 0.75 times full bank flow. Comparisons with the results of the route and lag method indicate that some thirty to forty million cubic feet of storage would be required to produce an attenuation equal to that provided by dynamic dispersion. If the construction of a reservoir on a channel similar to System 1 was being contemplated, the attenuation of the flood wave as it passes down the river must be considered. Ignoring this attenuation would result in the dam being larger than required to produce a given reduction of the flood wave. However, if the channel attenuation is accounted for, it is possible that the size of the dam could be reduced knowing that the flood wave would be attenuated to the required amount before flowing out of the system.

The comparison of the route-lag and lag-route results with the data obtained from the dynamic simulation indicates that the lag predicted by the complete solution is significantly shorter than the lag predicted by either of the other two methods. This can be attributed to the lag that was introduced by the inclusion of the imaginary reservoir and the fact that the kinematic wave velocity is slower than the dynamic wave velocity. The route-lag model has the longest lag of the systems studied.

More study is required to provide guidelines for selecting the appropriate parameters to use in describing the imaginary reservoir for a particular physical system. However, it appears that utilizing a linear reservoir and calibrating the simulation tool with recorded or simulated data is a viable and extremely useful tool for hydrologic simulation.

CHAPTER 7

A KINEMATIC FLOOD ROUTING MODEL

Comparisons of kinematic flood routing simulations with rigorous solutions of the momentum and continuity equations describing unsteady flow phenomena have revealed that unsteady flow problems may be classed in three different categories. These are:

1. Situations where kinematic theory provides as accurate an answer as could be obtained by the use of the more expensive rigorous solution.
2. Cases where a good approximation is obtained by using the kinematic theory in conjunction with another approximation such as an imaginary reservoir in series with the channel to account for attenuation.
3. Problems where the changes in flow are so rapid or the slope so shallow that the kinematic wave theory provides answers which are markedly different from those observed or predicted by a dynamic solution.

At present, criteria for differentiating between the various classes of problems are more qualitative than quantitative and these criteria require further study. Development of an efficient and versatile computer program to perform kinematic flood routing calculations can be justified on the grounds that it can be used as a research tool to help establish quantitative guidelines for determining into which of the above mentioned categories a given physical problem falls. Further comparisons between kinematic wave solutions and results obtained by solving the momentum and continuity equations could provide the key to development of the rules for successfully applying kinematic wave methods. The use of the program as an engineering tool must not be overlooked and, to this end the program should be capable of handling natural channels defined by arbitrary geometry.

This chapter deals with the objectives and development of an interactive computer program capable of performing kinematic flood routing calculations for systems of arbitrary geometry. After outlining the objectives of the program and elementary operations used to route a flood, several applications of the program are shown. Concluding remarks are provided, which outline specific aspects of the method and the computer program that could be developed further.

7.1 OBJECTIVES

The primary objective of this aspect of the study was to develop a computer program capable of being used as an engineering or research tool. To accomplish this goal, the following specific program capabilities were identified.

1. Kinematic routing methods must be able to be applied to natural channels of arbitrary geometry. The general kinematic method outlined in Chapter 3 was used as the theoretical basis and the numerical calculations were performed using a direct finite difference technique.
2. It has been shown in Chapters 3 and 5 that the values selected for the two parameters α and β have a very noticeable effect on the results of the computer simulation. To enable the use of this program as a research aid, it is vital that there be flexibility in the selection of α and β. Also, changing the two parameters may be beneficial in engineering applications.
3. To predict the attenuation of the flood wave, it may be necessary to insert a reservoir in series with the channel. This storage unit may be included either by variation of the parameters in the kinematic routing algorithm, as outlined above, or by placing an imaginary reservoir in series with the channel.
4. A command oriented procedure is desirable to enable the user to have maximum flexibility in the manner in which the particular system is analyzed.

5. The program should be designed to take advantage of the facilities of time sharing since this mode of operation allows the maximum amount of flexibility and interaction with the numerical representation of the system as the flood is being routed along the channel.

7.2 DEVELOPMENT OF PROGRAM PROCEDURE

The development of a program that fulfils the above objectives was carried out within the context of the Civil Engineering Program Library (CEPL), Smith (1970), Smith (1974), Walden (1973). This allowed the existing resources of the program library to be utilized and, hopefully, will ensure that the finished program is accessible for use by others.

Currently, the CEPL is comprised of FORTRAN routines capable of analyzing a number of elementary engineering programs, and designed to perform a number of basic operations. These programs may be classified into three general categories.

1. Basic routines provided to perform, one elementary function or calculation. A subroutine, which determines the cross-section properties, falls into this category.
2. Slightly more comprehensive routines which require calls of the lower level subroutines to perform an elementary operation fall into the second group.

3. Managerial routines which utilize the other routines in the process of solving a particular problem.

A "building block" approach may be taken to develop a new program utilizing small modules. In this way, documentation is simplified and users can easily apply the routines.

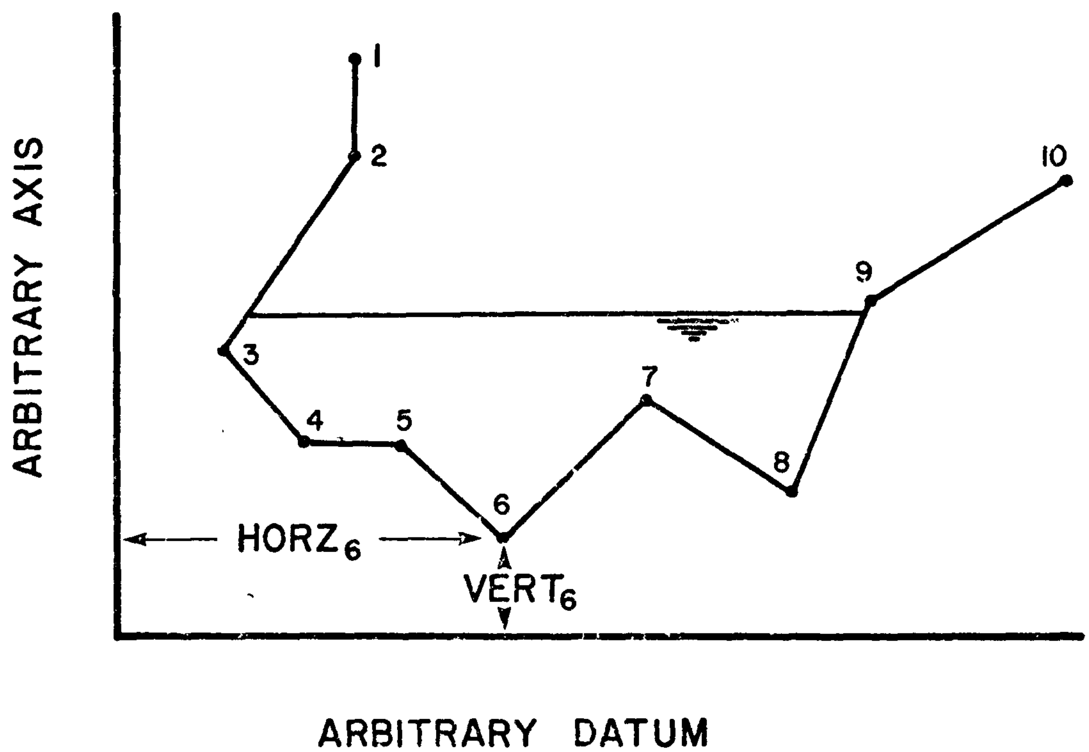
One of the features of the CEPL, which is particularly useful in achieving the objectives outlined earlier, is the method of defining the arbitrary geometry of a natural channel. These properties are specified by section number, chainage from the upstream end and a roughness measure. The cross-section data is entered by specifying a series of coordinates which define the outline of the channel. Figure 7.1 contains a pictorial representation of the manner in which the cross-section is defined.

The computation of steady state profiles is a vital step in the process of routing a flood using the kinematic method outlined in Chapter 3.

A number of routines are available in the CEPL to calculate these flow profiles. For example, subroutine EZRA performs the backwater calculation for each reach between two cross-sections. However, subroutines CRITIC and CONTRO are available to calculate the flow depth that may occur at a channel transition such as a bridge or weir.

FIGURE 7.1

DEFINITION OF THE CHANNEL CROSS-SECTION



New subroutines, known as KINRUT and RESVOR, designed to be consistent with CEPL, were written to perform the kinematic flood routing and the reservoir routing. Documentation of these subroutines is provided in Appendix D. KINRUT routes the flood through one elementary reach using the functional relationships which are defined prior to calling the subroutine. This allows flexibility in the selection of values for the parameters α and β. The other method of including a pseudo-reservoir into the simulation is facilitated by RESVOR. Storage in the imaginary reservoir is defined using the following equation:

$$ST = KQ^{W} \tag{7.1}$$

where: ST = Storage

Q = Flowrate

K = A parameter

w = A parameter

7.3 THE PROGRAM

The program developed to analyze the passage of a flood wave in a channel using kinematic wave theory is predominently a subroutine of the "managerial" type. A small driver program serves merely to dimension the arrays required for the calling statement. The chief function of the managerial subroutine is to provide a forum for its interactive operation of the various elementary computational subroutines through the use of operational data entered from a time-shared console.

The flow diagram in figure 7.2 shows the general operation of this routine. After the execution of the program begins, the required storage space is dimensioned in the main program. Subsequently, subroutine RIVER3 is called and the geometric data is read in from a previously defined tape. Further operation of the program requires explicit direction by commands entered in the operational data.

The operational data includes commands which direct the program to allow entry of data describing the downstream water levels, flow rates along the channel and the resistance law to use for computation of the steady flow state profiles, among other things. Data describing the system geometry may be printed and this information may be adjusted, if desired. The extent of the options available is best described by the list of commands available in RIVER3 shown in figure 7.3.

7.4 USE OF THE PROGRAM

Before presenting an example of the use of the program, a brief discussion of the steps required to perform the kinematic

FIGURE 7.2

PROGRAM FLOW CHART

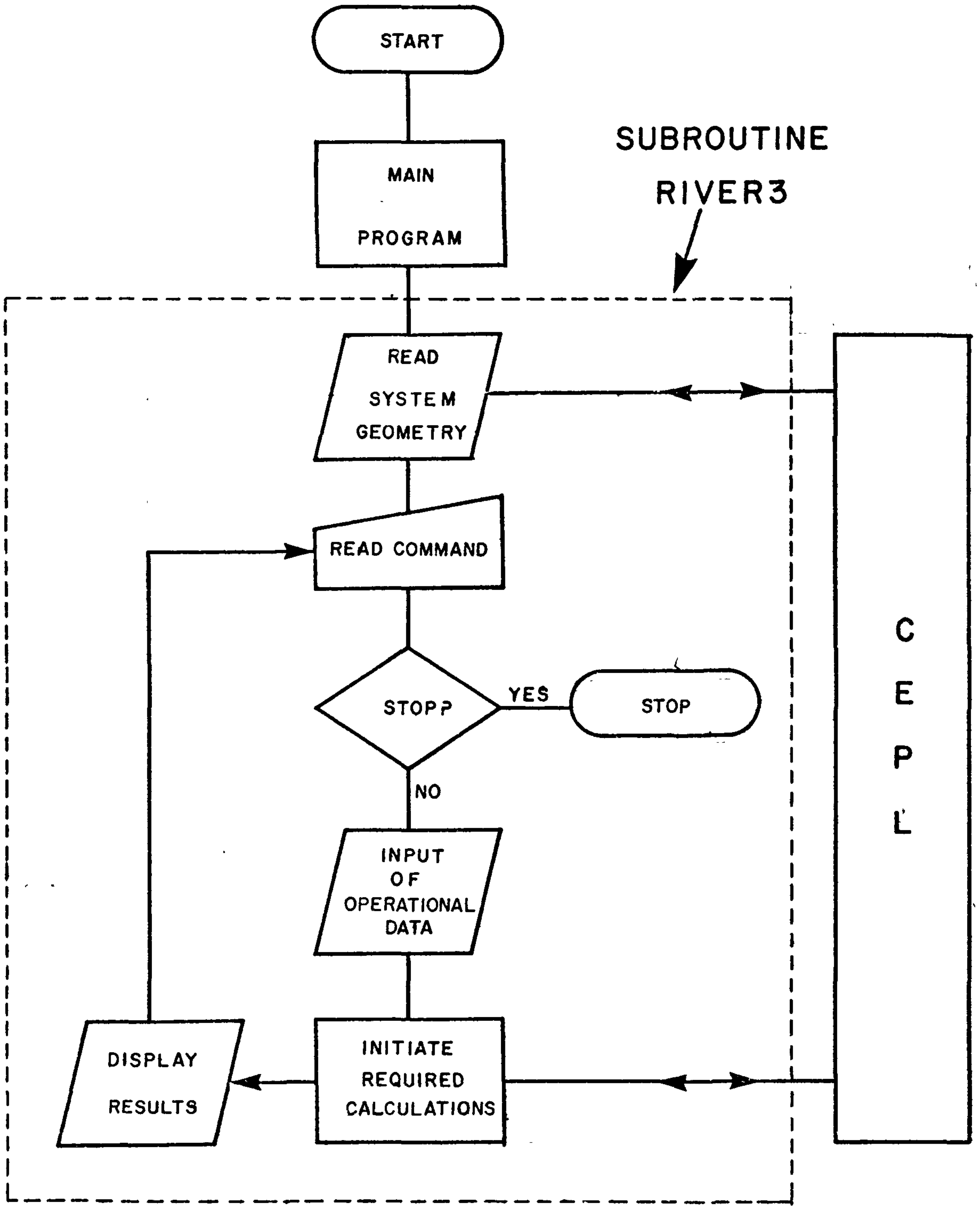


FIGURE 7.3

LIST OF COMMANDS

```
AFTER INVITATION TO TYPE ":-" GIVE ONE OF
THE FOLLOWING COMMANDS....
DISCHARGE....TO SPECIFY FLOW
D/S WL.......TO DEFINE DOWNSTREAM CONTROL LEVEL
INFLOW.......TO DEFINE INFLOW HYDROGRAPH
RESISTANCE...TO SET FLOW RESISTANCE LAW
OLD SECTION..TO PRINT COORDS OF A SECTION
NEW SECTION..TO REDEFINE COORDS OF A SECTION
OLD COEFF....TO PRINT ROUGHNESS MEASURE
NEW COEFF....TO REDEFINE ROUGHNESS MEASURE
CRITIC.......TO COMPUTE CRITICAL DEPTH AT A SECTION
CHANGES......TO PRINT CHANGES OF COORDS OR ROUGHNESS
TABLE........TO PRINT TABLE OF ALL SECTIONS DATA
PROFILES.....TO PRINT OUT SURFACE PROFILES
COMPUTE......TO COMPUTE PROFILES
ROUTE........TO ROUTE THE FLOOD
RESERVOIR....TO ROUTE THROUGH A RESERVOIR
RESTART......TO BEGIN AGAIN
HELP.........FOR COMMAND OPTIONS
STOP.........TO TERMINATE
```

routing computations will be discussed. To calculate the functional relationships required for the flood routing, it is necessary to perform a steady state analysis of the watercourse and to calibrate the model.

The data required, and the order of operations is generally as follows:

1. Define the downstream control by specifying the stage-discharge curve using a series of coordinates.
2. Define the lowest and highest flow rates to be used for computing the flow profiles and the number of profiles to be computed. These flow rates may be varied along the channel to account for lateral inflow or outflow.
3. Define the resistance law to be used in the computations.
4. After entry of the above data, the profiles may be computed.
5. If a printout of the surface profiles is desired, it may be obtained using the PROFILES command.

At this point, the user is provided with data describing the steady state performance of the system and field observations, if available, may be compared with the profiles. If the two sets of information show significant discrepancies, physical data, such as the roughness coefficients, may be varied and the profiles recalculated. This process could then be repeated if necessary until the computed results agree with the observed profiles. Alternately, the user may wish to use only the steady state capabilities of the program and could study the effects of channel modifications by varying the cross-section data or could discontinue the execution of the program.

If the steady-state calculations are found to be satisfactory after completing an examination and/or calibration process, the unsteady analysis may be started. This involves the separate operations of (i) defining the inflow hydrograph and (ii) performing the required flood routing.

Currently the program is capable of handling only one inflow hydrograph; but this hydrograph may be defined at any section specified by the user in the operational data.

7.4.1 Definition of Inflow Hydrograph

The inflow hydrograph is described by time and flow rate coordinates. Straight lines are assumed between these points. The flow rate for time less than the first coordinate is assumed to be the flow rate of the first coordinate and the flow rate for time greater than the time of the last coordinate is assumed to be the flow rate of the last coordinate.

7.4.2 Performing the Routing

Kinematic or reservoir routing may be performed in any order. The only limitation, at present, is that the routing must start at the location where the inflow hydrograph is defined or where the last routing ended. This allows a user to route a flood for several reaches, insert a reservoir, if desired, and/or continue with the kinematic routing. Alternately, the process may be restarted at the location where the inflow hydrograph was defined.

If the kinematic routing through the channel is used, an option allows the user to redefine the parameters α and β for a number of elementary reaches. The default value for these two parameters is 0.5. Other information defining the time step, sections where the routing is to begin and end, and the start and finish of the time period under consideration must also be entered. The reservoir routing option requires the same type of data regarding time step, as well as start and finish time, but the size of the reservoir, as defined by the parameters K and w, and the location of the reservoir must be specified.

Using these features, a calibration of the response hydrograph may be performed or sensitivity tests may be performed. In general, the other command options may be used at any time to provide information or to modify data. Checks are built into the options to warn the user if other data must be entered before the specified operation may be carried out or if changes have been made, say to cross-section data, and vital information such as surface profiles, has not been recalculated. The exception to this rule is the STOP command. This command may be used only at the end of the computations. No other commands may be entered after STOP.

7.5 EXAMPLE APPLICATIONS

This section of the chapter is devoted to demonstrating the use of RIVER3 in providing an effective means of studying kinematic flood waves in natural channels for research or engineering purposes. Two systems and the results from each are presented. Each description outlines the physical system and results from the various numerical experiments.

7.5.1 Application One

The first system studied consisted of approximately 10 miles of channel defined by 58 cross-sections. The profile of the invert level along the watercourse is shown in figure 7.4 along with some of the typical cross-sections. Appendix G contains a listing of the geometric data used to describe the channel.

An inflow hydrograph with a trapezoidal shape having a base flow of 350 cfs and a peak of 4,000 cfs was utilized. Figure 7.5 shows this hydrograph and the outflow hydrograph obtained using an implicit dynamic analysis. Walden (1973).

FIGURE 7.4
INVERT PROFILE AND CROSS-SECTIONS
APPLICATION ONE

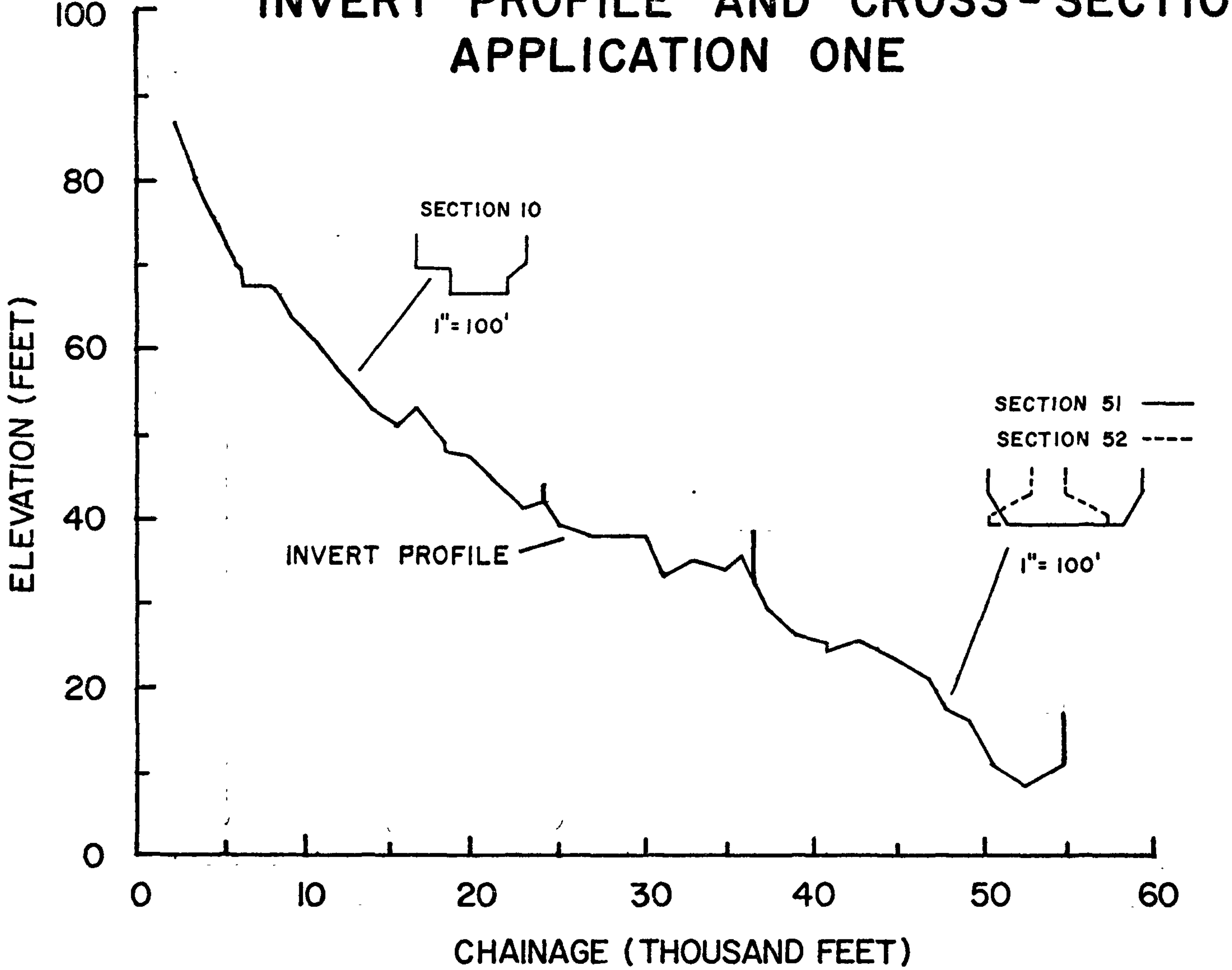
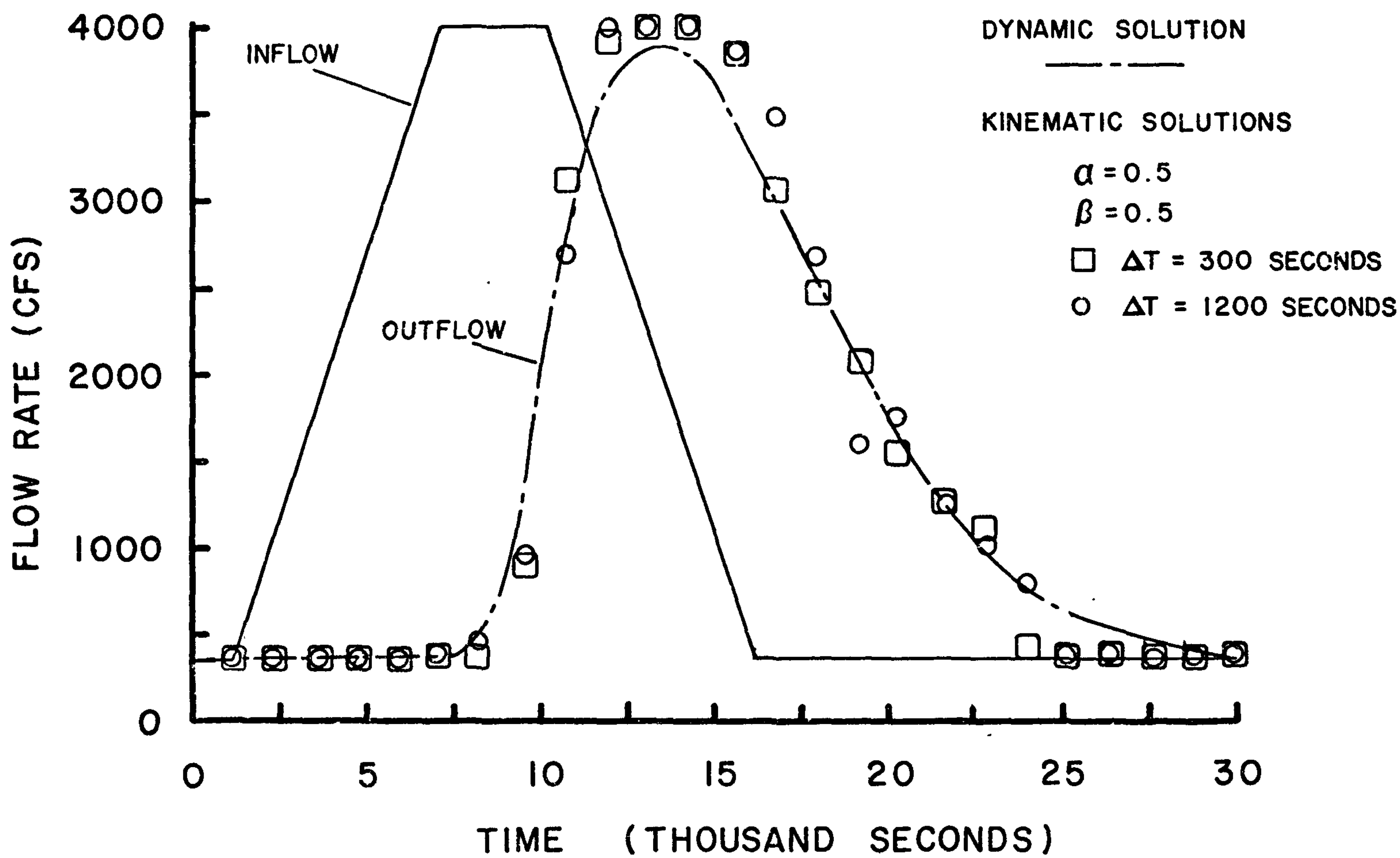


FIGURE 7.5
HYDROGRAPHS FOR APPLICATION ONE



This particular physical system showed very little attenuation, which indicates that a kinematic solution may prove to be a very good approximation to the more complete dynamic solution. The first tests performed were used to study the effects of changing the size of the time step. The values chosen for the two tests were 1, 200 seconds and 300 seconds. The outflow hydrographs predicted from these simulations are shown on figure 7.5 along with the solution predicted using the dynamic analysis. Both of the kinematic solutions predicted the general shape of the outflow hydrograph. However, there were signs of slight instability on the falling limb of the hydrograph with $\Delta T = 1,200$ seconds. Neither kinematic solution correctly predicted the peak outflow nor did they simulate the dynamic effects demonstrated by the earlier rise of the outflow hydrograph.

To simulate the attenuation of the floodwave, several tests were performed with the reaches between section numbers 41 and 45 simulated using:

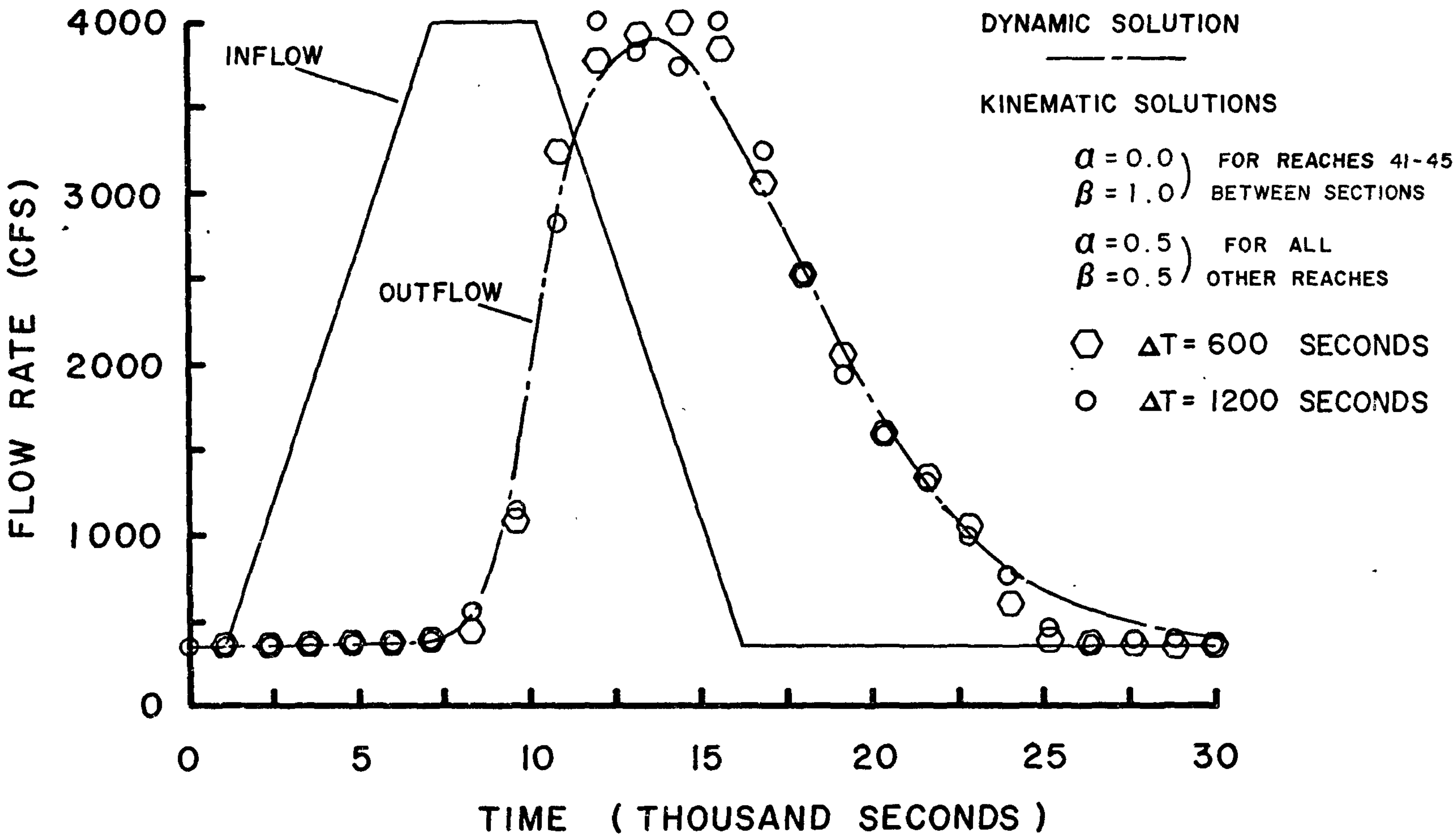
$$\alpha = 0.0$$

$$\beta = 0.5$$

The other reaches were simulated using the standard default values of 0.5 for these parameters. The results of these tests, which did not successfully predict the attenuation of the flood wave as it passed through the channel are shown in figure 7.6. This may have been due, in part, to slight instability of the computation in the region of the peak. Nevertheless, the general shape of the outflow hydrograph was modelled.

A number of other tests were performed using various values for the parameters and time steps, etc. In general, these tests did not successfully model large amounts of attenuation. This was not a severe restriction in application one due to the fact that the physical prototype did not appear to manifest large amounts of attenuation. The introduction of a significant amount of attenuation, if that was deemed necessary, could be made by the inclusion of an imaginary reservoir in series with the channel. The simulation shown in application one was relatively successful in simulating the shape of the outflow hydrograph. This was due largely to the fact that the system was relatively steep and a kinematic analysis was a good approximation. The useful ness of the kinematic analysis as an engineering tool relies upon the verification of the results. In the present study, this verification was provided by a more rigorous dynamic analysis. Other means of substantiating the results could be (i) verification by comparison of the computed results with hydrologic data recorded in the system or (ii) development of guide-

FIGURE 7.6
HYDROGRAPHS FOR APPLICATION ONE



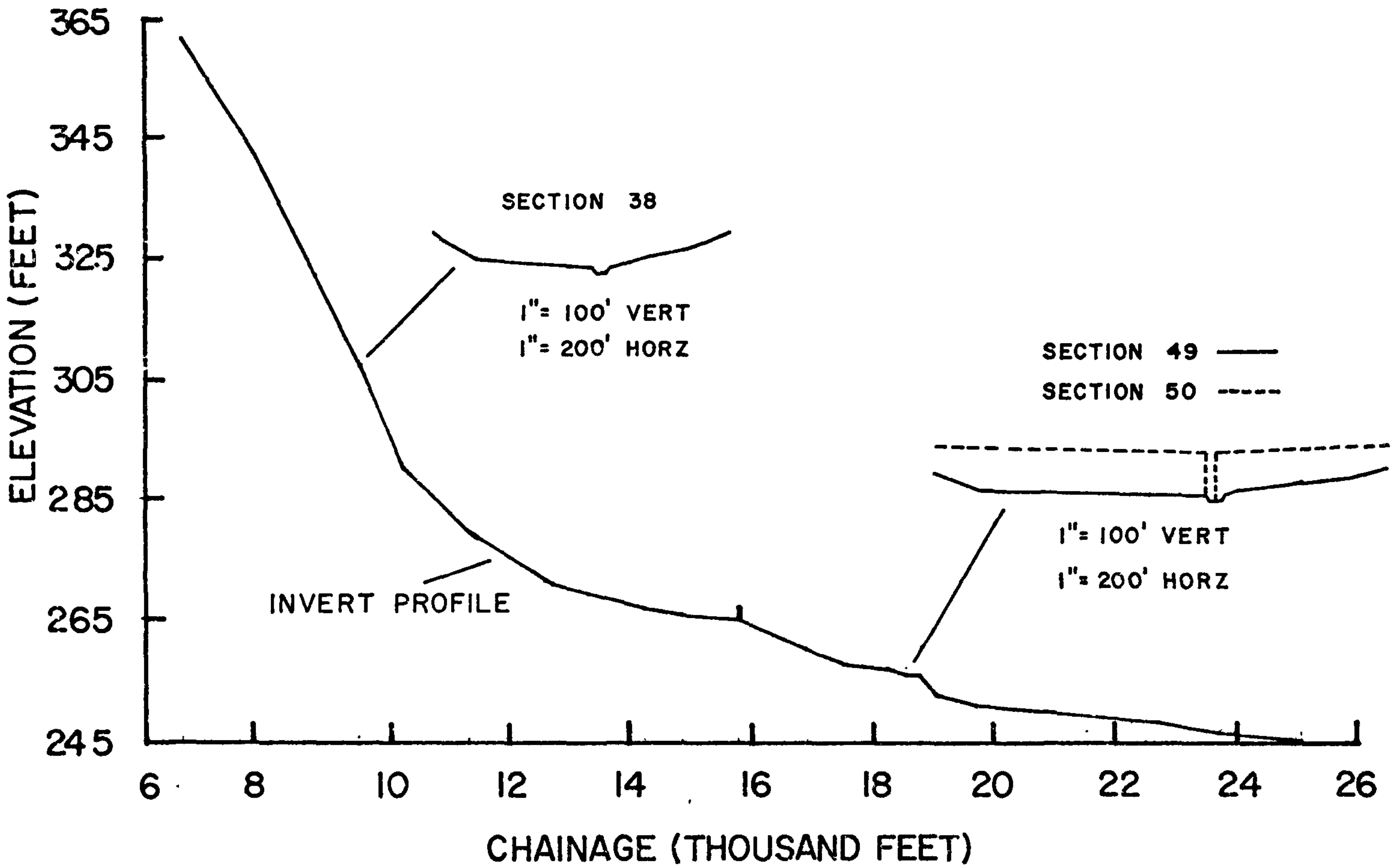
lines that would enable a user to determine the applicability of kinematic analysis to the particular system without the use of the dynamic analysis or extensive hydrologic data. A preliminary basis for these guidelines is presented in Chapter 4.

7.5.2 Application Two

The second prototype that was modelled using RIVER3 was a natural channel that had two major constrictions in the lower reaches. Figure 7.7 contains a profile of the invert elevations of application two and some typical cross sections of this waterway, the geometric data is listed in Appendix G. These constrictions were large embankments with relatively small culverts extending through the lower portions. During a major flood, substantial ponding of water would occur behind these embankments, particularly the upstream embankment. This ponding would result in the attenuation of a flood wave if the embankment was stable under the severe load imposed by the innundation of the upstream side. The system described above exists, in the Lower Ancaster Creek near Hamilton, Ontario. The upstream embankment carries a railroad track and a highway traverses the downstream embankment.

In this study, several modifications were made to the data to make the problem more tractable. Firstly, during a major flood, the culverts would be flowing full, the assumption of open channel flow

FIGURE 7.7
INVERT PROFILE AND CROSS-SECTIONS
APPLICATION TWO



is not valid. Thus, instead of culverts through embankments, the structures were modelled as embankments with deep, narrow slots, through which the water flowed. Secondly, Sulphur Creek, a major tributary, joins the main stream above the first large constriction.

The program was designed to handle a single water course as opposed to a network. The complication arising from the existence of Sulphur Creek tributary was avoided by ignoring the tributary inflow and modifying the storm hydrograph accordingly.

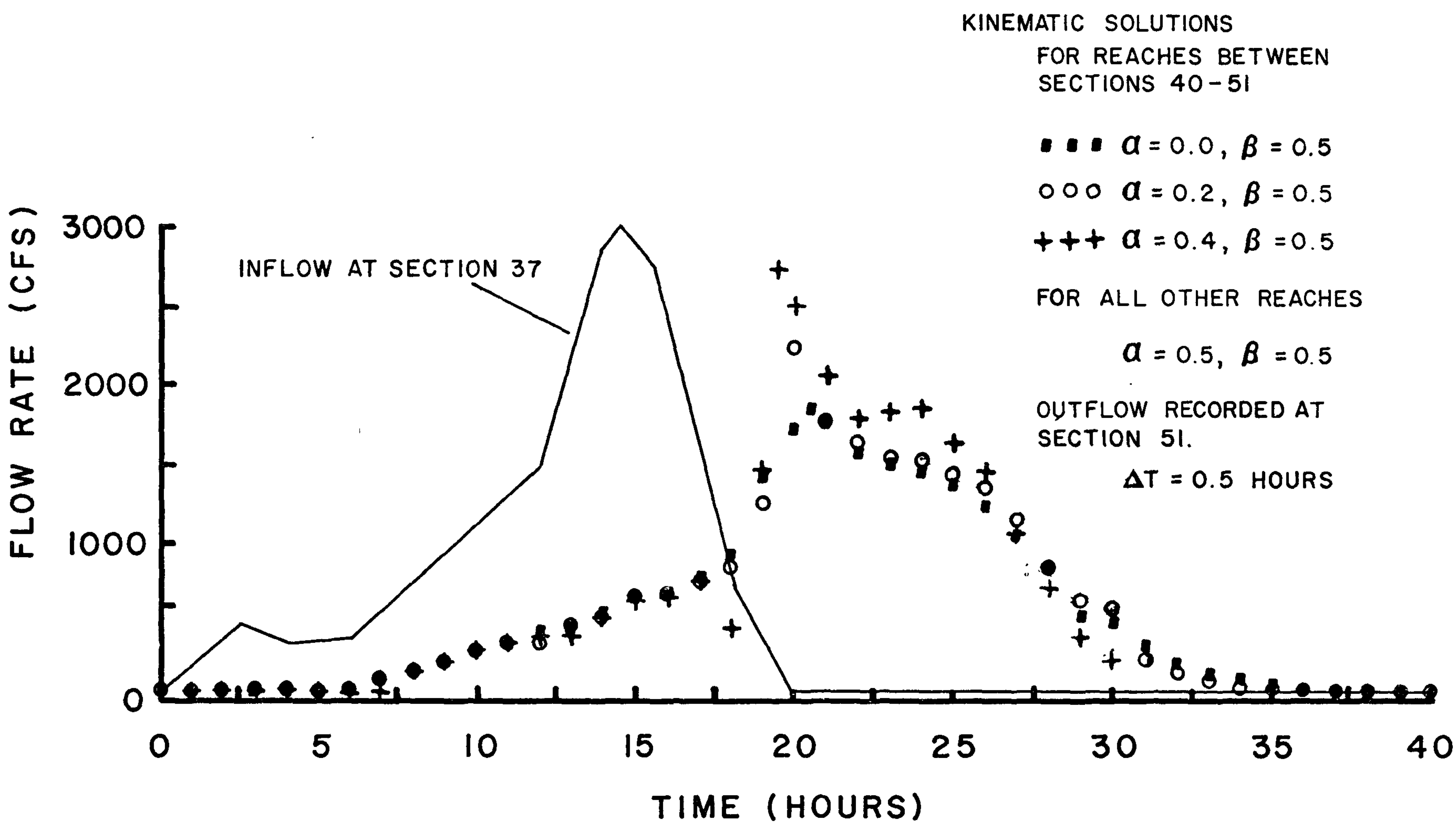
It should be noted that these difficulties were a result of restrictions in the CEPL routines designed to compute the steady state profiles rather than limitations of the flood routing algorithm. Further development of the computer program could lead to a solution of these problems without the above mentioned simplifications.

The primary purpose of application two was to demonstrate the use of RIVER3 in a situation where a significant amount of attenuation would be manifested.

The input hydrograph, shown in figure 7.8 was routed from section number 37 to the first embankment at section 51 and thence to section 64, the downstream limit of the stream.

The first routing, using the nucleus in the centre of the finite difference molecule proved to be unstable. To provide a solution which was stable, the nucleus of the molecule was moved downstream

FIGURE 7.8
HYDROGRAPHS FOR APPLICATION TWO



(i. e. maintaining $\beta = 0.5$). The results of these simulations are shown in figure 7. 8.

With α set to 0. 0, the outflow hydrograph rose slowly and peaked at a value of approximately 1, 850 cfs. There were no signs of numerical instability. Using a value of $\alpha = 0.2$, a similar soluation was obtained, except there was a sharp rise to a peak of about 2, 200 cfs., and then a drop to approximately the same hydrograph predicted by the solution with $\alpha = 0.0$. The last solution shown on figure 7. 8 was obtained using $\alpha = 0.4$. In the early portions of the outflow hydrograph, there is no significant differences between it and the other two solutions. However, in the region of the peak there are indications of numerical instability. The tests results shownon figure 7. 8 were compiled using a time step of one half an hour. To determine the sensitivity of the numerical analysis to changes in the size of the time step, two tests were performed with $\alpha = 0.5$ and $\beta = 1.0$ These results are shown in figure 7. 9. It can be seen that with both of the simulations, a relatively long, well rounded hydrograph resulted and the peak outflow was attenuated to approximately one-half of the peak inflow. Using $\Delta T = 1$ hour resulted in the flood wave being attenuated to a peak of 1, 420 cfs. while the peak obtained using $\Delta T = \frac{1}{2}$ hr. was 1, 530 cfs.

FIGURE 7.9
HYDROGRAPHS FOR APPLICATION TWO

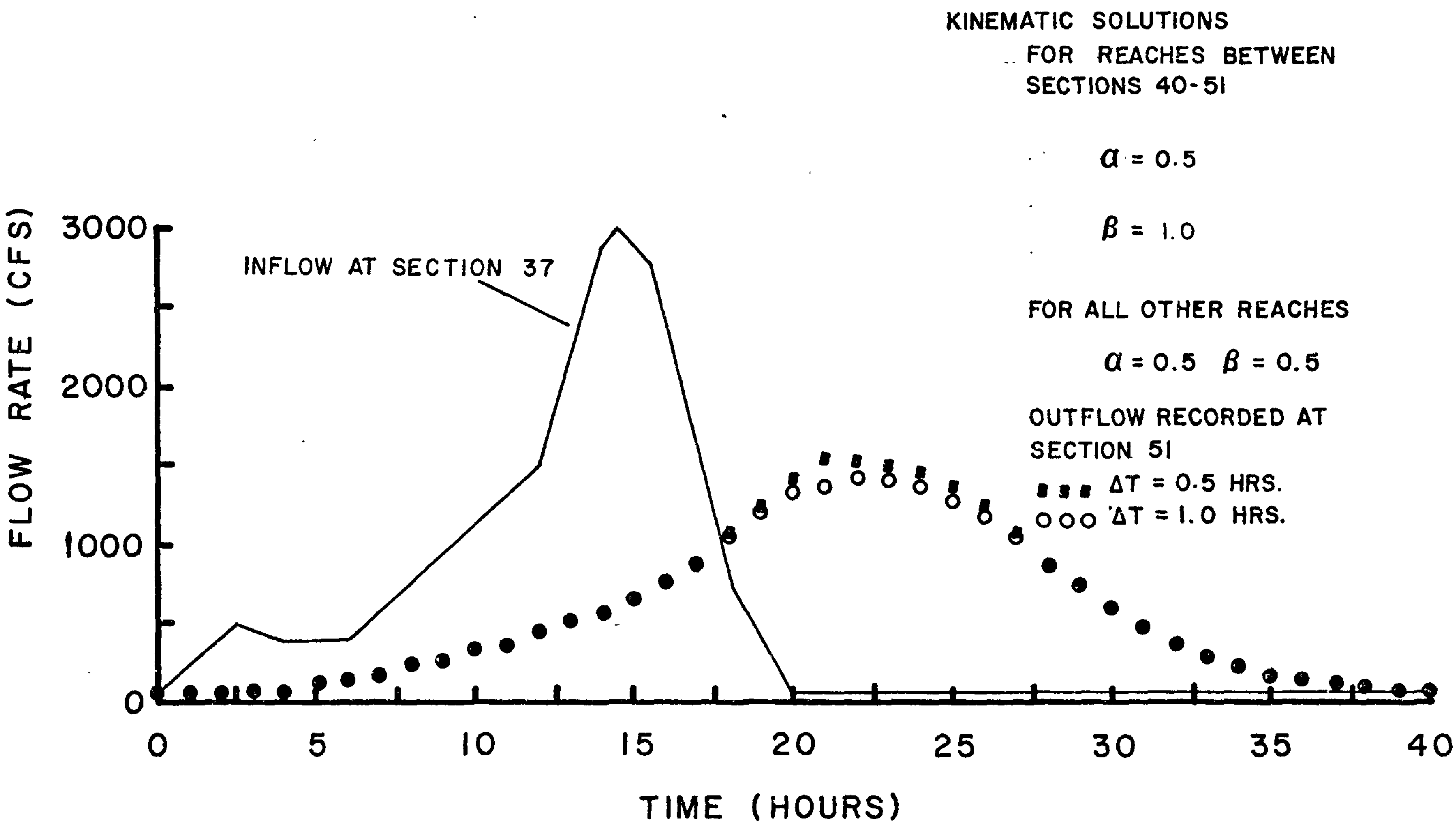
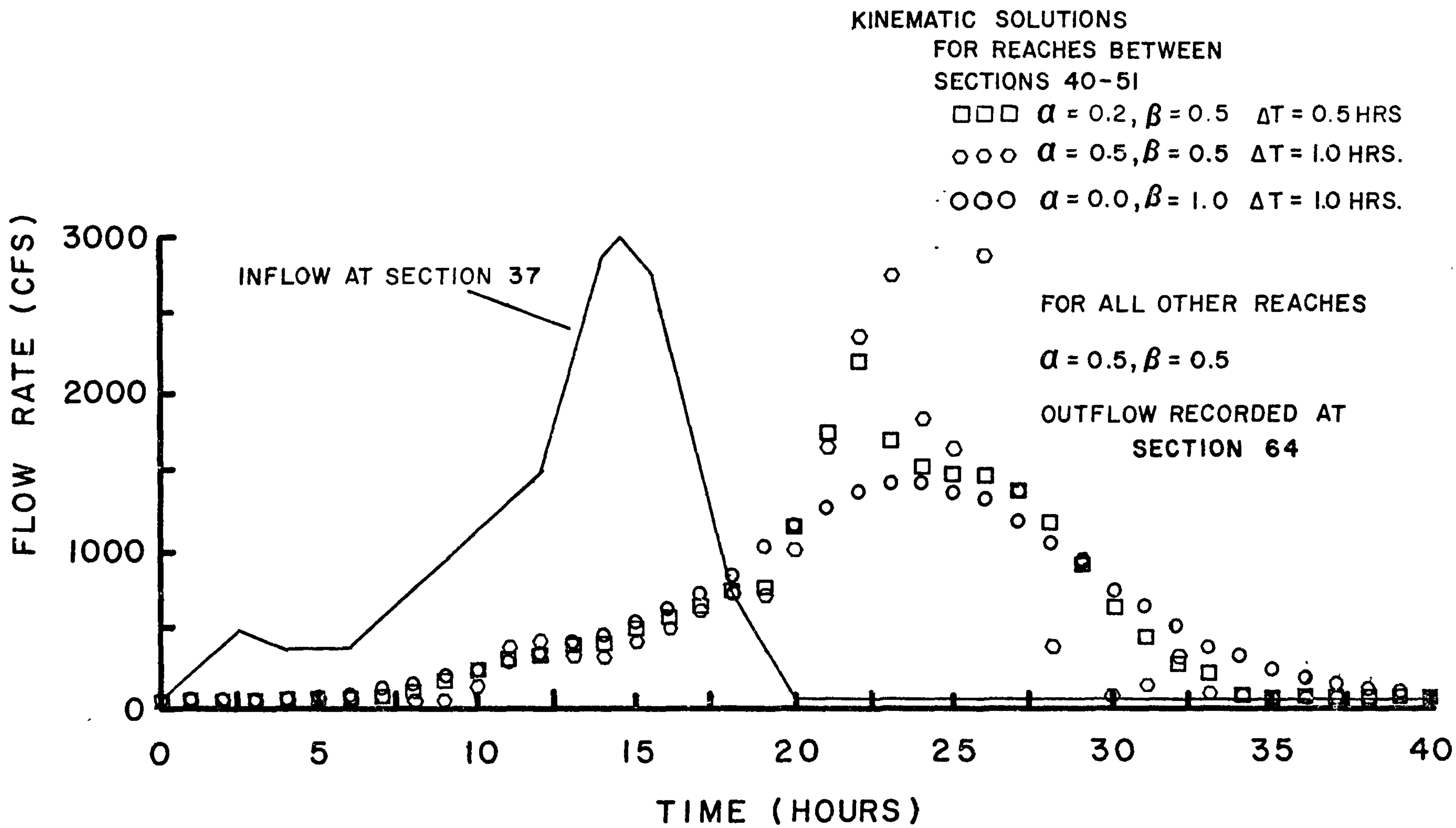


Figure 7.10 contains a comparison of several outflow hydrographs at the downstream limit of the stream. When the nucleus was located in the centre of the molecule for all of the reaches in the simulation, some numerical instability appeared after the peak. The same simulation was performed using $\Delta T = \frac{1}{2}$ hour, and the solution showed even greater numerical errors, therefore, the results were not plotted. In general, numerical stability problems were reduced when the simulation was performed with the nucleus shifted from the centre of the molecule for at least a few of the reaches upstream of the first large embankment.

The results of these simulations indicate that it is possible to model a system where the flood wave is attenuated as it passes through the system. However, it is necessary that the results be verified by a more rigorous analysis.

An attempt to provide the data to verify the kinematic model using an implicit dynamic technique was unsuccessful. Amein (1969), Walden (1973). The computer program utilized a four point implicit method to approximate the differential equations governing unsteady flow in open channels. The system of non-linear equations that resulted from the finite difference equations was solved using a Newton-Raphson iteration technique. However, for this application, the interative procedure failed to converge to a solution and execution

FIGURE 7.10
HYDROGRAPHS FOR APPLICATION TWO



was abnormally halted. It appears that alternate methods of solving the system of equations must be employed in order to obtain a solution.

Application two may be handled by an alternate approach which involves assuming a reservoir in the reaches above the first embankment. The properties of the reservoir would be determined by the volume of water stored in the valley as a function of outflow. This type of approach could possibly yield a more valid solution than the previous method which considered the large volume in the valley as a series of channel reservoirs. If the valley section was simulated as one unit, more attenuation may result and the time of travel through the valley would probably be reduced. This remains an area that requires further study.

7.6 CONCLUSIONS AND DISCUSSION OF RESULTS

The computer program has been developed to allow a user to route a flood through an open channel system defined by arbitrary geometry. The routine is designed to be utilized in time sharing mode with operational data entered on a teletype console. The results are printed on the same unit. Execution of the program is directed by commands entered as operational data. The general capabilities of the program include the ability to perform steady state analysis of a system and store the data for use during the analysis of unsteady flow conditions. Additional commands are included to provide efficient data handling capabilities and to allow modification of the geometric data during the execution of the program.

Several applications of the program have been presented and have yielded results which indicate that the kinematic analysis is useful in analyzing physical systems provided the solutions can be verified by either (i) a more rigorous analysis or by (ii) recorded hydrologic data. Attenuation may be simulated by varying the values of α and β or by the introduction of an imaginary reservoir in series with the channel. Commands incorporated in RIVER3 facilitate both of these devices.

Stability was one of the prime factors which governed the success of the various numerical experiments that were performed to demonstrate the two applications. Numerical instability may be reduced by positioning the nucleus away from the centre of the molecule in a more stable region. However, another approach may be utilized to improve stability. Because the program analyzes a complete time history of one reach before computing the conditions in the next portion of the watercourse, it is possible to use a time step that varies as a function of the reach length and the kinematic wave velocity through that section of the stream. The output hydrograph would be reduced to a common time step via an interpolation routine. This requires further study and would involve some rearrangement of the routines currently being employed in RIVER3. Some experimentation with the selection of the value of ΔT may result in a solution which is stable and yet avoids the need to have a variable time step. Alternately, the flood may be routed through one series of reaches using one time step and then routed through another series using a different increment of time. One unique feature of kinematic routing is the fact that, in some cases, a large time step may produce more stable results.

Compatibility with the CEPL has been one of the major guidelines utilized in the development of RIVER3, and it is hoped that an improved version will become an addition to this library of routines. There are several specific areas that should be investigated and possibly improved. These are outlined in the following paragraphs.

The improvements that are envisaged for RIVER3 are primarily rearrangements of the input and output modes to allow greater flexibility and easier operation. For example, the channel analysis routines are capable of handling cross sections defined by arbitrary geometry, however, the reservoir characteristics must be defined by the two parameters K and w of the exponential relation (7.1). By providing the necessary input options, it would be possible to define arbitrary reservoir characteristics by using co-ordinates to define storage as a function of flow rate. RESRUT, a subroutine currently available in the CEPL, is capable of employing this type of data, or a simple rearrangement of the data, to perform a reservoir routing.

The basic command options that are available in RIVER3 have, by and large, been adopted from other similar programs currently found in the CEPL. However, several of the commands have been enlarged and other options have been added. As a result, the amount of operational data has increased. To alleviate the amount of input required from the teletype terminal, some of the data may need to be incorporated in files similar to the data file used for geometric data. Particular examples of this include the values of α and β which may be included directly in the cross-section data. Reservoir data and inflow hydrograph information may be stored on files and read in at the user's command.

There are several suggestions that may also be investigated as major improvements to extend the power of RIVER3 and the CEPL. The item which could receive first priority is the provision of graph plotting capabilities for the program. This could be done either as a graph on the remote terminal or via storage of the necessary data to provide plots on the off line plotter associated with a computer facility. Other improvements could allow watershed (overland flow) simulation by the inclusion of routines to handle the data that describes the watershed as a large number of surfaces specified by length, width, slope and roughness. In addition, the routine would then require additional input capabilities to allow definition of the rainfall hyetograph(s). The third improvement would enable a user to verify the results obtained using a kinematic analysis by utilizing the dynamic flood routing capabilities of the CEPL. This might be done by programming a command option into RIVER3 which would allow the user to generate the input cards necessary to perform the more rigorous analysis in a batch mode, for example.

There are many other items that could be included in a routine such as RIVER3, however, the selection of the items and the approach used to carry out these functions would depend upon individual tastes, and the degree of sophistication desired in the program. The application and further development of RIVER3 remains an almost limitless area for future work.

CHAPTER 8

CONCLUSIONS

The preceeding chapters include detailed conclusions relating to the specific topics. This final chapter contains only a summary of the major points.

The objectives of this thesis were as follows:

(1) Provide a precise data base which can be utilized to compare various flood routing algorithms.

(2) Investigate kinematic flood routing methods to develop a general framework for comparison of the variety of techniques found in the literature.

(3) Determine the numerical characteristics of the general kinematic flood routing method. This will help determine the limitations of the finite difference schemes.

(4) Compare the results of the kinematic simulations with the data base in an attempt to determine practical limitations of the kinematic algorithms.

(5) Investigate methods of modelling attenuation with kinematic flood routing methods.

(6) Provide a versatile computer program that will enable a user to apply kinematic flood routing techniques and the methods of modelling attenuation in channel systems of arbitrary geometry.

These objectives have been achieved in the following manner.

Chapter 2 outlines the steps that were taken to provide the data base. An explicit staggered mesh finite difference scheme was used to provide a numerical solution to the differential equations describing the conservation of linear momentum and mass in prismatic channels of simple geometry. Tests were performed on the stability and convergence of the calculations to ensure as far as possible that the results of the numerical model were indeed reliable. Comparisons with other finite difference schemes which had been employed successfully show that the explicit method compared very well in representing unsteady flow in a system with simple geometry.

The general framework to compare the various kinematic algorithms was developed in Chapter 3. This method employs a rectangular finite difference molecule with the system variables defined at the corners of the molecule. (See figure 3.2) The difference between this method and the various other approaches to kinematic flood routing lies in the selection of the point about which the continuity equation is expanded; a further distinction lies in the relationship adopted between the flow rate and the cross section area. Applying the continuity equation about the centre of the finite difference molecule models the elementary channel unit as an ideal channel section where storage is related equally to inflow and outflow. A reservoir is simulated when the continuity equation is expanded about a point on the downstream boundary of the molecule. The stability of the numerical schemes is affected significantly by the location of the nucleus, the point about which the continuity equation is expanded. Three areas of stability were identified, a zone of unconditional instability, a zone of conditional stability, and a point of unconditional stability. (See table A-1 in Appendix A.)

When the continuity equation is applied to a point on the downstream edge of the molecule and on the highest time level of the molecule, the scheme is unconditionally stable. The definition of the stable and unstable zones of the molecule was shown to depend on the size of the space and time incremental values of α and β.

Moving the nucleus from the centre of the molecule results in an increase in the amount of error that is introduced in the numerical calculations. An analytic investigation of the finite difference equation of continuity indicates that first order errors are not introduced into the calculations when the nucleus is located at the centre of the molecule. However, as the nucleus is moved in a downstream direction or to an increasing level of time, there is an increase in the amount of first order error introduced into the computations. These errors are always negative in sign and introduce a pseudo-attenuation into the kinematic representation of a flood wave passing through a channel system.

Chapter 4 contains the results of a number of kinematic simulations compared with results produced by the solution of the momentum and continuity equations. An investigation was made of the order of magnitude of the various terms of the momentum equation to help evaluate the differences between the various simulations. The results show that as the order of magnitude of the terms reduce in comparison to the bedslope, the kinematic results agree closely with the more precise results produced by the numerical solution of the momentum and continuity equation. No guidelines were developed to indicate the difference in the relative size of the various terms in order to have close agreement between the results of the two methods of simulations. Preliminary results indicate that as the time base of the inflow hydrograph increases, the difference in the relative size of the terms can decrease in comparison with the relative size necessary to produce good results with a peaky, fast rising hydrograph.

Without intuitive or pre-calibrated use of the pseudo-attenuation phenomena referred to above, kinematic flood routing methods do not correctly predict attenuation of the flood wave as it passes through the channel. Chapters 5 and 6 outline modifications that can be made to the numerical model to introduce attenuation into the simulated flood wave. The method outlined in Chapter 5 is the introduction of a calibrated numerical error into the computational scheme. Alternately the procedure used, moving the nucleus away from the centre of the finite difference molecule, may be viewed as modelling the elementary reaches as combination channel-reservoir units. The results of the numerical experiments indicate that in certain cases it is possible to simulate unsteady open channel systems accurately. This is true when dynamic effects demonstrated by an early rise of the outflow hydrograph, are not the dominant process in the physical system.

The series of numerical experiments demonstrated that the values of attenuated peak outflow obtained as function of the coefficients α and β, defining displacement of the nucleus from the centre of the molecule, constituted a set of points comprising a well conditioned and slightly concave surface denoted by $P(\alpha, \beta)$. The properties of this surface were examined and it was demonstrated to be a useful tool in the calibration of kinematic flood routing models based on results of either a dynamic (or 'complete') solution and/or actual prototype observations. The calibrated model may then in turn be used for further simulations. A detailed example of the use of this tool is included in Chapter 5.

Experiments, of a preliminary nature, were performed with System 1 to determine the effect of arranging identical elementary reaches in a cascade. The results, for the system modelled, indicate that as the number of elementary reaches increased the total attenuation increased. The amount of attenuation was found to vary with the number of reaches to the power of 0.55. (See equation 5.26)

Chapter 6 was devoted to studying the inclusion of a reservoir in series with a kinematic channel as a tool to model attenuation. The device of combining a channel and an imaginary reservoir in series is commonly used to model a hydrologic system. The method is known as the lag and route technique. The study reported in Chapter 6 utilized non-linear components in an attempt to identify the sensitivity of the solution to the reservoir location and different degrees of non-linearity in the reservoir. The numerical experiments indicated that the position of the imaginary reservoir affects the timing and shape of the hydrograph. Moving the reservoir toward the downstream boundary results in an outflow hydrograph that occurs sooner and rises more slowly. The simulations indicate that the amount of attenuation is relatively insensitive to the location of the reservoir. Furthermore, the non-linearity of the reservoir affects the peak of the outflow hydrograph, the centroidal lag of the hydrograph and the skewness factor. However, the chord slope of the relationship between storage and discharge (as illustrated in figure 6.4) appears to be the dominant factor in determining the response characteristics.

Finally, Chapter 7 describes the development of an interactive computer model that is capable of performing kinematic flood routing in natural channels using the general method outlined in Chapter 3 as well as allowing for the inclusion of a reservoir at any section along the channel. Execution of the program is directed by commands entered as operational data. The general capabilities of the computer program include the ability to perform steady state analysis as well as allowing for the modification of the geometric data, which describes the channel, during the execution of the program.

Two examples of the program are included in the thesis. Both of these experiments demonstrate the effect of moving the nucleus from the centre of the molecule. Not only was there an increase of attenuation as the nucleus was moved from the centre; there was also an increase in the stability of the numerical analysis.

This computer program, known as RIVER3, has been designed to be compatible with the Civil Engineering Program Library. This has been done to facilitate easy access by potential users of this program. There are an almost limitless number of modifications that can be made to RIVER3 to improve and expand its capabilities. Hopefully, this thesis has provided some insight into these various possibilities and enhances the understanding of kinematic flood routing techniques as very useful hydraulic and hydrologic simulation tools.

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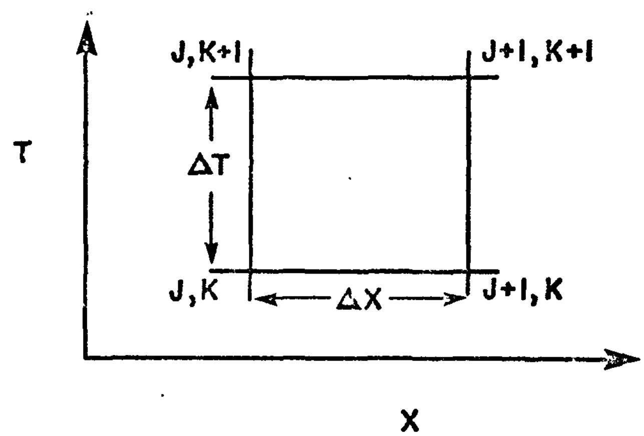
APPENDIX A

STABILITY ANALYSIS OF THE GENERAL KINEMATIC METHOD

This appendix is devoted to the stability analysis of the general kinematic method. Stability is a primary consideration used to evaluate the performance of a numerical solution. An unstable scheme will cause small errors to amplify and dominate, thus, masking the solution.

This analysis is based on a linearized version of the continuity equation. For nonlinear equations, the method is not exact, but if the increment being considered is small and the coefficients of the derivatives are smooth functions, the approximation of constant coefficients is reasonable. Hopefully, this analysis will identify schemes which are obviously unstable.

The finite difference grid used to solve the continuity equation is:



The continuity equation is:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} = \bar{q} \qquad (A.1)$$

Where: Q = flow rate

A = cross section area

x = distance

t = time

$\bar{q}$ = rate of lateral inflow

Because:

$$c = \frac{dQ}{dA} \qquad (A.2)$$

Where: c = kinematic wave velocity

The continuity equation may be rewritten as:

$$c\frac{\delta A}{\delta x} + \frac{\delta A}{\delta t} = \bar{q} \qquad (A.3)$$

At any point J, K the numerical solution A_J^K is equal to the true solution $A(J\Delta X, K\Delta T)$ plus an error term $\bar{A}_J^K$ or:

$$A_J^K = A(J\Delta X, K\Delta T) + \bar{A}_J^K \qquad (A.4)$$

Because the system is linear, the error term can be written as one term of a Fourier Series:

$$\bar{A}_J^K = A_o \, EXP(i(m\sigma\Delta X + n\gamma\Delta T)) \qquad (A.5)$$

Where: A_o = constant

σ, γ = wave numbers in space and time

$i = \sqrt{-1}$

By writing the linearized finite difference equation of continuity in terms of the numerical solution and subtracting the true solution, the finite difference equation in terms of the errors is obtained. For the general kinematic method this equation is:

$$C\left(\frac{\beta(\bar{A}_{J+1}^{K+1} - \bar{A}_J^{K+1}) + (1-\beta)(\bar{A}_{J+1}^{K} - \bar{A}_J^{K})}{\Delta X} \right)$$

$$+ \frac{(1-\alpha)(\bar{A}_{J+1}^{K+1} - \bar{A}_{J+1}^{K}) + \alpha(\bar{A}_J^{K+1} - \bar{A}_J^{K})}{\Delta T} = 0 \qquad (A.6)$$

For the point J, K, m and n may be assumed equal to zero, with no loss of generality. Thus:

$$\bar{A}_J^K = Ao \tag{A.7}$$

$$\bar{A}_J^{K+1} = Ao\,EXP(i\gamma\Delta T) \tag{A.8}$$

$$\bar{A}_{J+1}^K = Ao\,EXP(i\sigma\Delta X) \tag{A.9}$$

$$\bar{A}_{J+1}^{K+1} = Ao\,EXP(i(\sigma\Delta X + \gamma\Delta T)) \tag{A.10}$$

Solving equation A.6 for the unknown $\bar{A}_{J+1}^{K+1}$ and multiplying by ΔT yields:

$$C\frac{\Delta T}{\Delta X}\beta\bar{A}_{J+1}^{K+1} + (1-\alpha)\bar{A}_{J+1}^{K+1} = \alpha\bar{A}_J^K + (1-\beta)C\frac{\Delta T}{\Delta X}\bar{A}_J^K$$

$$- C\frac{\Delta T}{\Delta X}(1-\beta)\bar{A}_{J+1}^K + (1-\alpha)\bar{A}_{J+1}^K$$

$$+ C\frac{\Delta T}{\Delta X}\beta\bar{A}_J^{K+1} - \alpha\bar{A}_J^{K+1} \tag{A.11}$$

Collecting terms:

$$\left((1-\alpha) + C\beta\frac{\Delta T}{\Delta X}\right)\bar{A}_{J+1}^{K+1} = \left(\alpha + (1-\beta)C\frac{\Delta T}{\Delta X}\right)\bar{A}_{J}^{K}$$

$$+ \left((1-\alpha) - (1-\beta)C\frac{\Delta T}{\Delta X}\right)\bar{A}_{J+1}^{K}$$

$$+ \left(-\alpha + \beta C\frac{\Delta T}{\Delta X}\right)\bar{A}_{J}^{K+1} \qquad (A.12)$$

To facilitate easy algebraic manipulation, the following substitutions are employed, the D terms corresponding to the numbered nodes of Figure 3.2.

$$D1 = \alpha + (1-\beta)C\frac{\Delta T}{\Delta X} \qquad (A.13)$$

$$D2 = -\alpha + \beta C\frac{\Delta T}{\Delta X} \qquad (A.14)$$

$$D3 = (1-\alpha) - (1-\beta)C\frac{\Delta T}{\Delta X} \qquad (A.15)$$

$$D4 = (1-\alpha) + \beta C\frac{\Delta T}{\Delta X} \qquad (A.16)$$

Thus, equation A.12 becomes:

$$D4\bar{A}_{J+1}^{K+1} = D1\bar{A}_{J}^{K} + D2\bar{A}_{J}^{K+1} + D3\bar{A}_{J+1}^{K} \qquad \text{(A.17)}$$

To ensure stability, errors must not amplify, thus:

$$\left|\frac{\bar{A}_{J+1}^{K+1}}{\bar{A}_{J+1}^{K}}\right| = 1 \qquad \text{(A.18)}$$

The error term must lie within the unity circle. That is:

$$|EXP(i(\sigma\Delta X + \gamma\Delta T))| = \left|\frac{D3\bar{A}_{J+1}^{K}}{D4}\right|\left|\frac{D1\bar{A}_{J}^{K} + D2\bar{A}_{J}^{K+1} + D3\bar{A}_{J+1}^{K}}{D3\bar{A}_{J+1}^{K}}\right| \leq 1 \qquad \text{(A.19)}$$

Inserting the error terms written as components of a Fourier Series gives:

$$|EXP(i(\sigma\Delta X + \gamma\Delta T))| = \left|\frac{D1 + D2EXP(i\gamma\Delta T) + D3EXP(i\sigma\Delta X)}{D4}\right| \leq 1 \qquad \text{(A.20)}$$

The following trignometric identities may be utilized in equation A. 20:

$$EXP(i\sigma\Delta X) = COS\,\sigma\Delta X + iSIN\,\sigma\Delta X \quad (A.21)$$

$$EXP(i\gamma\Delta T) = COS\,\gamma\Delta T + iSIN\,\gamma\Delta T \quad (A.22)$$

Substituting equations A. 21 and A. 22 into equation A. 20 yields:

$$\left|EXP(i(\sigma\Delta X + \gamma\Delta T))\right| = \frac{1}{|D4|}\left|D1 + D2(COS\,\gamma\Delta T + iSIN\,\gamma\Delta T)\right.$$

$$\left. + D3(COS\,\sigma\Delta X + iSIN\,\sigma\Delta X)\right| \leq 1 \quad (A.23)$$

The worst case will occur when:

$$COS\,\sigma\Delta X + iSIN\,\sigma\Delta X = \pm 1 \quad (A.24)$$

and

$$COS\,\gamma\Delta T + iSIN\,\gamma\Delta T = \pm 1 \quad (A.25)$$

Thus, to ensure stability, the following equation should be satisfied:

$$\frac{|D1| + |D2| + |D3|}{|D4|} \leq 1 \qquad \text{(A.26)}$$

Rearranging equation A. 26 gives:

$$|D1| + |D2| + |D3| - |D4| \leq 0 \qquad \text{(A.27)}$$

Expanding equation A. 27 leads to:

$$\left| \alpha + (1-\beta) C \frac{\Delta T}{\Delta X} \right| + \left| -\alpha + \beta C \frac{\Delta T}{\Delta X} \right|$$

$$+ \left| (1-\alpha) - (1-\beta) C \frac{\Delta T}{\Delta X} \right| - \left| (1-\alpha) + \beta C \frac{\Delta T}{\Delta X} \right| \leq 1 \qquad \text{(A.28)}$$

For: $0 \leq \alpha \leq 1$

$0 \leq \beta \leq 1$

D_1 and D_4 will always be positive thus:

$$\left| -\alpha + \beta C \frac{\Delta T}{\Delta X} \right| + \left| (1-\alpha) - (1-\beta) C \frac{\Delta T}{\Delta X} \right|$$

$$+ \alpha + (1-\beta) C \frac{\Delta T}{\Delta X} - 1 + \alpha - \beta C \frac{\Delta T}{\Delta X} \leq 0 \qquad \text{(A.29)}$$

Or:

$$\left| -\alpha + \beta C \frac{\Delta T}{\Delta X} \right| + \left| (1-\alpha) - (1-\beta) C \frac{\Delta T}{\Delta X} \right|$$

$$+ (2\alpha - 1) + (1 - 2\beta) C \frac{\Delta T}{\Delta X} \leq 0 \qquad \text{(A.30)}$$

If the first two terms of equation A. 30 are of the same sign, then:

$$\left| (1-2\alpha) + (2\beta - 1) C \frac{\Delta T}{\Delta X} \right|$$

$$+ (2\alpha - 1) + (1 - 2\beta) C \frac{\Delta T}{\Delta X} = 0 \qquad \text{(A.31)}$$

Defining:

$$S = (2\alpha - 1) + (1 - 2\beta) C \frac{\Delta T}{\Delta X} \qquad \text{(A.32)}$$

Where: S = Stability number

Equation A.31 may be rewritten as:

$$\left| -S \right| + S \leq 0 \tag{A.33}$$

Thus stability could be achieved if:

$$S \leq 0 \tag{A.34}$$

and if the following two terms have the same sign.

$$C1 = -\alpha + \beta C \frac{\Delta T}{\Delta X} \tag{A.35}$$

$$C2 = (1-\alpha) - (1-\beta) C \frac{\Delta T}{\Delta X} \tag{A.36}$$

If the two conditions do not have the same sign then the following equation must be satisfied in order to ensure stability.

$$\left| C1 \right| + \left| C2 \right| + S = 0 \tag{A.37}$$

Several examples are presented to demonstrate the application of these rules.

Consider the location of the nucleus at a point specified by

$$\alpha = 0.0$$

$$\beta = 0.0$$

At that point:

$$C_1 = 0.0$$

$$C_2 = 1 - C \frac{\Delta T}{\Delta X}$$

Thus the two terms have the same sign and the only condition for stability is equation A. 34:

$$S = -1 + C\frac{\Delta T}{\Delta X} \leq 0$$

Or:

$$C\frac{\Delta T}{\Delta X} \leq 1$$

If the nucleus was located at a point specified by:

$$\alpha = 0.0$$

$$\beta = 0.5$$

then:

$$C_1 = 0.5\, C\frac{\Delta T}{\Delta X}$$

$$C_2 = 1 - 0.5\, C\frac{\Delta T}{\Delta X}$$

The two conditions will have the same sign if

$$C\frac{\Delta T}{\Delta X} \leq 2$$

and it will be necessary to satisfy only equation A.34.

$$S = -1 \leq 0$$

For $C\frac{\Delta T}{\Delta X} \geq 2$, equation A. 37 must be satisfied:

$$\left|0.5\, C\frac{\Delta T}{\Delta X}\right| + \left|1 - 0.5\, C\frac{\Delta T}{\Delta X}\right| - 1 \leq 0$$

Or:

$$C\frac{\Delta T}{\Delta X} - 2 \leq 0$$

This condition is satisfied only when $C\frac{\Delta T}{\Delta X} \leq 2.0$. Thus it was concluded that the solution is stable only under the following condition.

$$C\frac{\Delta T}{\Delta X} \leq 2$$

This type of analysis was continued for the various points specified in table A.1.

TABLE A.1

RESULTS OF STABILITY ANALYSIS

$$KN = C\frac{\Delta T}{\Delta X}$$

GENERAL METHOD:

β \ α	1.00	0.75	0.50	0.25	0.00
1.00	$\infty \geq KN \geq 1$	$\infty \geq KN \geq \frac{3}{4}$	$\infty \geq KN \geq \frac{1}{2}$	$\infty \geq KN \geq \frac{1}{4}$	$\infty \geq KN \geq 0$
0.75	UNSTABLE	KN = 1	$2 \geq KN \geq \frac{2}{3}$	$3 \geq KN \geq \frac{1}{3}$	$4 \geq KN \geq 0$
0.50	UNSTABLE	UNSTABLE	KN = 1	$\frac{3}{2} \geq KN \geq \frac{1}{2}$	$2 \geq KN \geq 0$
0.25	UNSTABLE	UNSTABLE	UNSTABLE	KN = 1	$\frac{4}{3} \geq KN \geq 0$
0.00	UNSTABLE	UNSTABLE	UNSTABLE	UNSTABLE	$1 \geq KN \geq 0$

LAX-WENDROFF: $KN \leq 1$

APPENDIX B

DEGREE OF APPROXIMATION FOR THE DYNAMIC ANALYSIS

By expanding the differential formulations of the momentum and continuity equations using a Taylor Series and observing the terms that are truncated when the finite difference approximations are made, it is possible to determine the degree of approximation.

The Taylor Series for a function of two variables is:

$$f(x+g, t+h) = f(x,t) + g\frac{\delta f}{\delta x} + h\frac{\delta f}{\delta t} + \frac{g^2}{2}\frac{\delta^2 f}{\delta x^2} + \frac{h^2}{2}\frac{\delta^2 f}{\delta t^2} + gh\frac{\delta^2 f}{\delta x\,\delta t} + \frac{g^3}{6}\frac{\delta^3 f}{\delta x^3} + \frac{g^2 h}{2}\frac{\delta^3 f}{\delta x^2\,\delta t} + \frac{gh^2}{2}\frac{\delta^3 f}{\delta x\,\delta t^2} + \frac{h}{6}\frac{\delta^3 f}{\delta x^3} + \cdots \quad (B.1)$$

For the continuity equation:

$$g = \Delta X/2$$

$$h = \Delta T/2$$

$$\frac{\delta Q}{\delta x} \cong \frac{Q(x+\Delta X/2,\, t) - Q(x-\Delta X/2,\, t)}{\Delta X}$$

$$\cong \frac{1}{\Delta X}\left(\Delta X\frac{\delta Q}{\delta x} + \frac{\Delta X^3}{24}\frac{\delta^3 Q}{\delta x^3} \cdots\right) \quad (B.2)$$

$$\frac{\delta A}{\delta t} \cong \frac{A(x, t+\Delta T/2) - A(x, t-\Delta T/2)}{\Delta T}$$

$$\cong \frac{1}{\Delta T}\left(\Delta T \frac{\delta A}{\delta t} + \frac{\Delta T^3}{24}\frac{\delta^3 A}{\delta t^3} \cdots \right) \qquad (B.3)$$

Substituting into the continuity equations yields:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + O(\Delta X^2, \Delta T^2) = \bar{q} \qquad (B.4)$$

For the momentum equation, the following approximations will be found:

$$\frac{\delta Q}{\delta t} \cong \frac{1}{\Delta T}\left(\Delta T \frac{\delta Q}{\delta t} + \frac{\Delta T^3}{24}\frac{\delta^3 Q}{\delta t^3} \cdots \right) \qquad (B.5)$$

$$\frac{\delta h}{\delta x} \cong \frac{1}{\Delta X}\left(\Delta X \frac{\delta h}{\delta x} + \frac{\Delta X^3}{24}\frac{\delta^3 h}{\delta x^3} \cdots \right) \qquad (B.6)$$

and

$$\frac{\delta h}{\delta t} \cong \frac{1}{\Delta T}\left(\Delta T \frac{\delta h}{\delta t} + \frac{\Delta T^3}{24}\frac{\delta^3 h}{\delta t^3} + \frac{\Delta T \Delta X^2}{8}\frac{\delta^3 h}{\delta x^2 \delta t} \cdots \right) \qquad (B.7)$$

Therefore the approximation is in the order of ΔT^2 and ΔX^2. However, another approximation should be analyzed. That is the manner in which Q^2 terms are represented.

$$Q(x,t)^2 \cong Q(x, t + \Delta T/2)\, Q(x, t - \Delta T/2) \qquad \text{(B.8)}$$

$$Q(x,t)^2 \cong Q(x,t) - \frac{\Delta T^2}{4}\left(\frac{\delta Q}{\delta t}\right) + \cdots \qquad \text{(B.9)}$$

Thus, the approximation of the momentum equation is of the order of ΔX^2 and ΔT^2.

APPENDIX C

DEGREE OF APPROXIMATION FOR THE GENERAL KINEMATIC ROUTING METHOD

Analyzing the degree of approximation involves expanding the differential equation into a Taylor Series and determining what terms are truncated when the finite difference approximations are made.

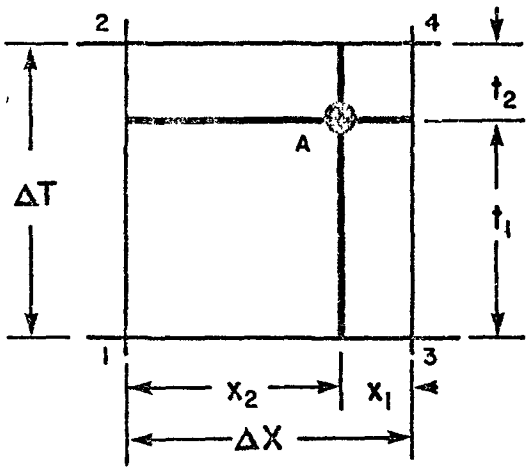
For a function, f(x, t), the value of the function at f(x+g, t+h) is given by the Taylor Series:

$$f(x+g,t+h) = f(x,t) + g\frac{\delta f}{\delta x} + h\frac{\delta f}{\delta t} + \frac{g^2}{2}\frac{\delta^2 f}{\delta x^2}$$

$$+ gh\frac{\delta^2 f}{\delta x\,\delta t} + \frac{h^2}{2}\frac{\delta^2 f}{\delta t^2} + \frac{g^3}{6}\frac{\delta^3 f}{\delta x^3}$$

$$+ \frac{g^2 h}{2}\frac{\delta^3 f}{\delta x^2 \delta t} + \frac{g h^2}{2}\frac{\delta^3 f}{\delta x\,\delta t^2} + \frac{h^3}{6}\frac{\delta^3 f}{\delta t^3} + \cdots \qquad (C.1)$$

To obtain the general kinematic flood routing method, the continuity equation is applied using finite difference methods, to a space time diagram as shown below.



The finite difference approximations are:

$$\frac{\delta Q}{\delta x} = \frac{\beta(Q_4 - Q_2) + (1-\beta)(Q_3 - Q_1)}{\Delta X} \qquad (C.2)$$

$$\frac{\delta A}{\delta t} = \frac{(1-\alpha)(A_4 - A_3) + \alpha(A_2 - A_1)}{\Delta T} \qquad (C.3)$$

Expanding about point A:

$$Q_4 - Q_2 = (x_1 + x_2)\frac{\delta Q}{\delta x} + \frac{1}{2}(x_1^2 - x_2^2)\frac{\delta^2 Q}{\delta x^2}$$

$$+ (x_1 t_2 + x_2 t_2)\frac{\delta^2 Q}{\delta x\,\delta t} + \frac{1}{6}(x_1^3 + x_2^3)\frac{\delta^3 Q}{\delta x^3}$$

$$+ \frac{1}{2}(x_1^2 t_2 - x_2^2 t_2)\frac{\delta^3 Q}{\delta x^2\,\delta t} + \frac{1}{2}(x_1 t_2^2 + x_2 t_2^2)\frac{\delta^3 Q}{\delta x\,\delta t^2} + \cdots \quad (C.4)$$

$$Q_3 - Q_1 = (x_1 + x_2)\frac{\delta Q}{\delta x} + \frac{1}{2}(x_1^2 - x_2^2)\frac{\delta^2 Q}{\delta x^2}$$

$$- (x_1 t_1 + x_2 t_1)\frac{\delta^2 Q}{\delta x\,\delta t} + \frac{1}{6}(x_1^3 + x_2^3)\frac{\delta^3 Q}{\delta x^3}$$

$$- \frac{1}{2}(x_1^2 t_1 - x_2^2 t_1)\frac{\delta^2 Q}{\delta x^2\,\delta t} + \frac{1}{2}(x_1 t_1^2 + x_2 t_1^2)\frac{\delta^3 Q}{\delta x\,\delta t^2} + \cdots \quad (C.5)$$

Substituting expressions for $Q_4 - Q_2$ and $Q_3 - Q_1$ into equation C-2, the following expression is obtained:

$$\frac{\delta Q}{\delta x} = \frac{1}{\Delta X}\Bigg(\Delta X \frac{\delta Q}{\delta x} + \frac{1}{2}(x_1^2 - x_2^2)\frac{\delta^2 Q}{\delta x^2} + \frac{1}{6}(x_1^3 + x_2^3)\frac{\delta^3 Q}{\delta x^3}$$

$$+ \beta(x_1 t_2 + x_2 t_2)\frac{\delta^2 Q}{\delta x\,\delta t} - (x_1 t_1 + x_2 t_1)\frac{\delta^2 Q}{\delta x\,\delta t}$$

$$+ \beta(x_1 t_1 + x_2 t_1)\frac{\delta^2 Q}{\delta x\,\delta t} + \frac{\beta}{2}(x_1^2 t_2 - x_2^2 t_2)\frac{\delta^3 Q}{\delta x^2\,\delta t}$$

$$- \frac{1}{2}(x_1^2 t_1 - x_2^2 t_1)\frac{\delta^3 Q}{\delta x^2\,\delta t} + \frac{\beta}{2}(x_1^2 t_1 - x_2^2 t_1)\frac{\delta^3 Q}{\delta x^2\,\delta t}$$

$$+ \frac{\beta}{2}(x_1 t_2^2 + x_2 t_2^2)\frac{\delta^3 Q}{\delta x\,\delta t^2} + \frac{1}{2}(x_1 t_1^2 + x_2 t_1^2)\frac{\delta^3 Q}{\delta x\,\delta t^2}$$

$$+ \frac{\beta}{2}(x_1 t_1^2 + x_2 t_1^2)\frac{\delta^3 Q}{\delta x\,\delta t^2} + \cdots \Bigg) \qquad (C.6)$$

The values of x_1, x_2, t_1, t_2, etc. can be expressed in the following manner:

$x_1 = a\Delta X$ $\qquad$ $x_1^2 = a^2 \Delta X^2$

$x_2 = (1 - a)\Delta X$ $\qquad$ $x_2^2 = (1 - 2a + a^2)\Delta X^2$

$t_1 = \beta \Delta T$ $\qquad$ $t_1^2 = \beta^2 \Delta T^2$

$t_1^2 = (1 - \beta)\Delta T$ $\qquad$ $t_2^2 = (1 - 2\beta + \beta^2)\Delta T^2$

This yields:

$$\frac{\delta Q}{\delta x} = \frac{1}{\Delta X}\left(\Delta X \frac{\delta Q}{\delta x} + \frac{1}{2}(2\alpha - 1)\Delta X^2 \frac{\delta^2 Q}{\delta x^2}\right.$$

$$\left. + \frac{1}{6}(1 - 3\alpha + 3\alpha^2)\Delta X^3 \frac{\delta^3 Q}{\delta x^3} + \frac{1}{2}(\beta - \beta^2)\Delta X \Delta T^2 \frac{\delta^3 Q}{\delta x \delta t^2} + \cdots\right) \quad (C.7)$$

Thus:

$$\frac{\delta Q}{\delta x} \cong \frac{\delta Q}{\delta x} + O(\Delta X^2, \Delta T^2) \quad (C.8)$$

when

$\alpha = 0.5$

If $\alpha \neq 0.5$

$$\frac{\delta Q}{\delta x} \cong \frac{\delta Q}{\delta x} + O(\Delta X, \Delta T^2) \quad (C.9)$$

The magnitude of the $O(\Delta X)$ error is related to $(2\alpha - 1)$.

The general description of truncation error is:

$$\frac{\delta Q}{\delta x} \cong \frac{\delta Q}{\delta x} + (2\alpha - 1)\,O(\Delta X) + O(\Delta X^2, \Delta T^2) \quad (C.10)$$

Similarly expanding the $\frac{\delta A}{\delta t}$ term about point A:

$$\frac{\delta A}{\delta t} = \frac{1}{\Delta T}\left(\Delta T \frac{\delta A}{\delta t} + \frac{1}{2}(1-2\beta)\Delta T^2 \frac{\delta^2 A}{\delta t^2}\right.$$

$$\left. + \frac{1}{6}(1-3\beta+3\beta^2)\Delta T^3 \frac{\delta^3 A}{\delta t^3} + \frac{1}{2}(\alpha-\alpha^2)\Delta X^2 \Delta T \frac{\delta^3 A}{\delta x^2 \delta t} \cdots\right) \quad \text{(C.11)}$$

When $\beta = 0.5$ the approximation will be

$$\frac{\delta A}{\delta t} = \frac{\delta A}{\delta t} + O(\Delta X^2, \Delta T^2) \quad \text{(C.12)}$$

If $\beta \neq 0.5$

$$\frac{\delta A}{\delta t} = \frac{\delta A}{\delta t} + O(\Delta X^2, \Delta T) \quad \text{(C.13)}$$

Generally, the approximation may be written as

$$\frac{\delta A}{\delta t} = \frac{\delta A}{\delta t} + (1-2\beta)\,O(\Delta T) + O(\Delta X^2, \Delta T^2) \quad \text{(C.14)}$$

The expression for the general kinematic flood routing method can be written as follows:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} + (2\alpha - 1)O(\Delta X) + (1 - 2\beta)O(\Delta T)$$

$$+ O(\Delta X^2, \Delta T^2) = \bar{q} \qquad \text{(C.15)}$$

As the location about which the equation of continuity is moved further away from the centre of the molecule, the amount of error introduced by approximations on the order of ΔX and ΔT increases linearly.

As α is varied, terms on the order of ΔX are modified while varying β changes the way the errors on the order of ΔT are introduced.

A plot of $(2\alpha - 1)$ over the molecule will reveal a plane sloping toward the downstream side of the molecule (denoted by points 3 and 4). This is shown in figure C-1. Similarly $(1-2\beta)$ will be a plane tipped in the direction of increasing time (denoted by points 2 and 4).

Because the errors $O(\Delta X)$ and $O(\Delta T)$ introduced by varying α or β respectively, are independent of each other, it would be expected that a plot of errors would be a plane which has a dip and strike determined by the relative size of the errors as well as the sign of the errors.

FIGURE C.1

PLOT OF $(2\alpha-1)$

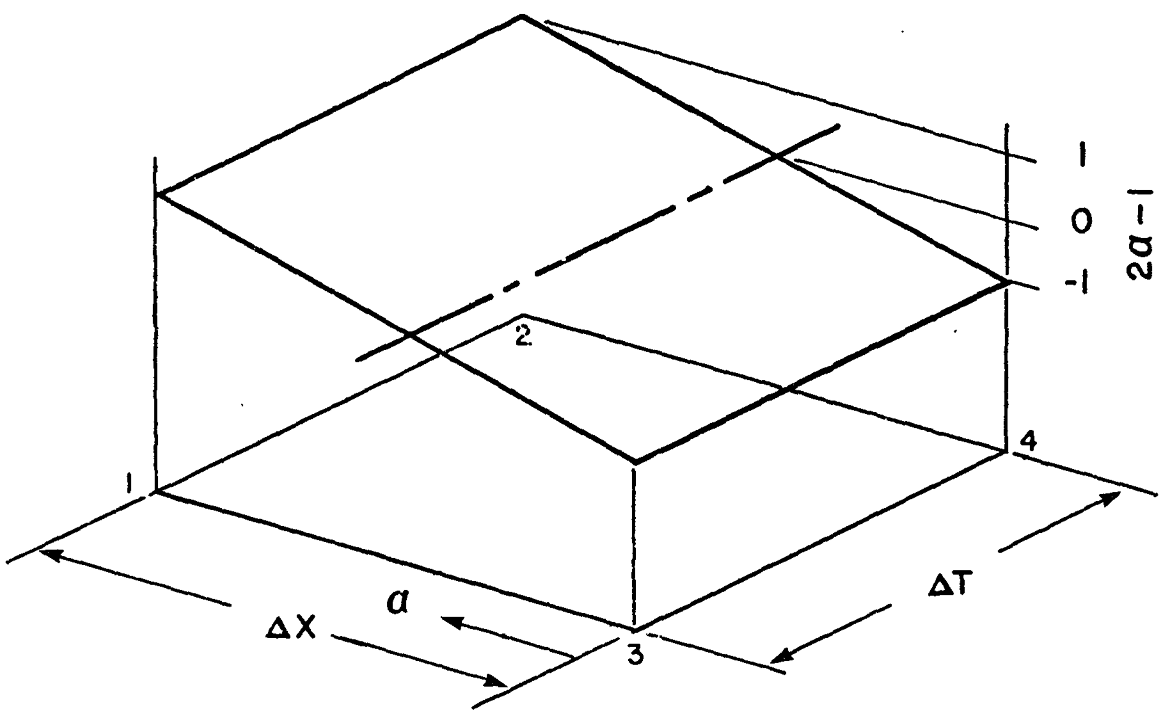
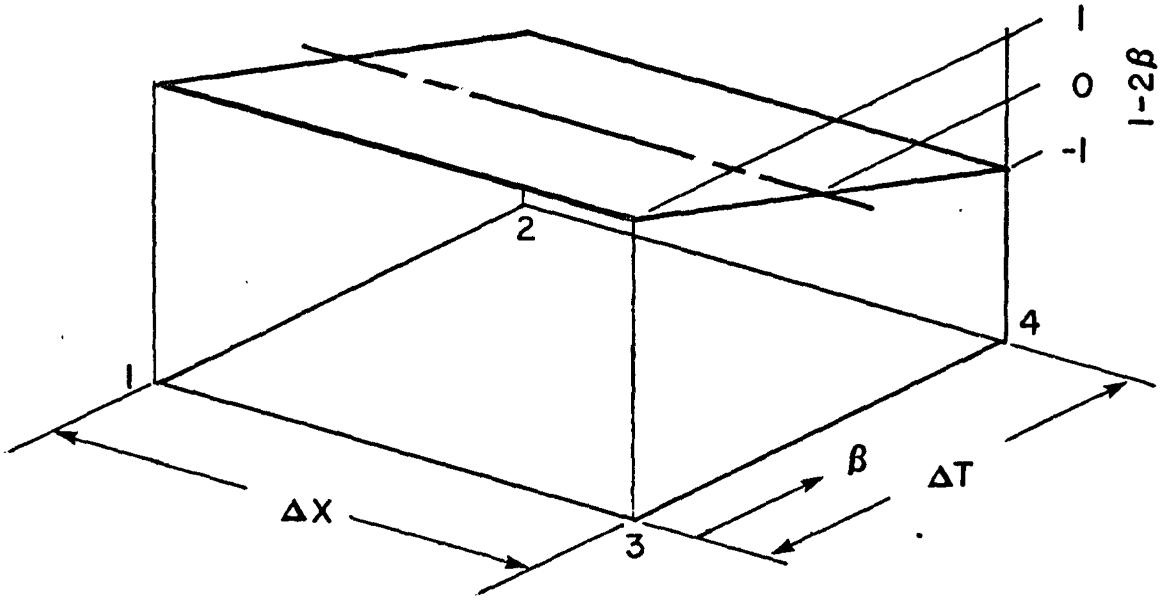


FIGURE C.2

PLOT OF (1-2β)



APPENDIX D

DOCUMENTATION OF

COMPUTER ROUTINES

***FINDIF

***KINDIF

HPLOT

KINFUN

KINRUT

RESVOR

RIVER3

UNSTEADY FLOW ANALYSIS

(1) PURPOSE: This program provides a finite difference solution of the partial differential equations describing one-dimensional unsteady flow in a rectangular channel.

(2) METHOD: The input data is supplied to the program using data cards. Problem variables are initialized and the input is printed to provide a check before the solution of the equations is begun. An explicit finite difference scheme based on a staggered mesh space-time diagram is utilized to provide the solution. Output is in the form of printer plots which show hydrographs at any two sections defined by the user. Depth versus time curves are also plotted for any two sections which are specified at the time of execution. Similarly, a stage discharge curve is plotted for a user defined section. Options are available to have the hydrographs, etc., punched on cards for utilization with other programs and for storage purposes.

(3) PROGRAM:

(a) DECK NAME: FINDIF

(b) CALLING SEQUENCE: None. This is a driving program.

(c) INPUT:

Data Card Type	Data	Format
1	Length of channel, width,	F10.1, F10.2
	Mannings n, slope, depth	2F10.8, F10.2
2	Number of reaches, Courant Number for choice of time increment, time at which the analysis is to be terminated (seconds).	I3, 2F10.2
3	Test number (for identification)	I3
4	Punch code to produce hydrographs and depth versus time plots. Punch code for stage-discharge curve. (Enter 0 for no cards, 1 for punched output.)	2I3
5	Section for which stage-discharge curve should be plotted.	I3
6	Section numbers for which hydrographs are to be plotted.	2I3

7	Section numbers for which stage versus time curves are to be plotted.	2I3
8	Number of points defining inflow hydrograph.	I3
9	The ratio of flow to full bank flow and time (seconds) for one point on the inflow hydrograph.	F10.2, F10.0

Note: Repeat card 9 for each of the points which define the inflow hydrograph.

(d) OUTPUT: The output consists of a listing of various variables for the problem being tackled as well as two hydrographs, two stage versus time curves, and a stage discharge curve. Options allow the stage discharge curve, hydrographs, and depth versus time curves to be punched on cards for future use. (Also see discussion.)

(e) RESTRICTIONS: This program is limited to the solution of problems involving uniform rectangular channels where flow resistance is defined by Mannings equation. The time step is determined from full bank flow conditions and the Courant Number. Stability problems may be encountered if the Courant Number is too large. (The program is unable to simulate supercritical flow.) A uniform flow depth is assumed as the downstream control. (See discussion.)

(f) OTHER DECKS: INTERI, MANNGQ, PLOTPT, OUTPLT

(4) EXAMPLE:

(a) THE PROBLEM: A rectangular channel 50,000' long, 100' wide and 20' deep is subject to a triangular inflow hydrograph. Twenty-five reaches are utilized in the analysis and bedslope = 0.0002 is specified with a Mannings n = 0.0149. The Courant Number was set equal to 0.5 and the analysis was carried out over a period of 24,000 seconds.

A copy of the data cards and sample output is shown in Appendix G of the source.

(5) DISCUSSION: New users should refer to documentation of all related program and routines. Further details of the method of analysis is available from the source.

A variation of this program was developed to perform the same type of calculations with a wide channel. This program was further modified to provide output which could be used to generate an animated movie of the flood wave moving down the channel.

(6) SOURCE: "A Comparison of Kinematic Flood Routing Methods". by Fred Biesenthal
(A Master of Engineering Thesis)
McMaster University
Hamilton, Ontario

UNSTEADY FLOW ANALYSIS

(1) PURPOSE: This program models the movement of a flood wave down a uniform rectangular channel using a kinematic flood routing technique. For each problem tackled, twenty-five solutions are provided. This enables a user to study the effects of varying the position of the "nucleus" within the finite difference "molecule".

(2) METHOD: After obtaining the input data, the program variables are initialized, and the flood routing portion is carried out. The actual flood routing calculations are performed by subroutine "KINRUT" and a further description of the algorithm may be obtained from the documentation of that subroutine as well as from the source. The flood routing computations are incorporated within nested "DO" loops so that the parameters ALPHA and BETA are systematically varied to provide solutions with twenty-five positions of the nucleus. With each solution a title page and two graphs are provided to document the results of the simulations.

(3) PROGRAM:

(a) DECK NAME: KINDIF

(b) CALLING SEQUENCE: None, this is a driving program.

(c) INPUT:

Data Card Type	Data	Format
1	Length of channel, width, Manning n, slope, depth	F10.1, F10.2, 2F10.8, F10.2
2	Number of reaches, time step (seconds). Time at which analysis is to be terminated (seconds).	I3, 2F10.2
3	Test number (for identification)	I3
4	Punch code to produce hydrographs on cards.	I3
5	Section numbers for which hydrographs are to be plotted	2I3
6	Section numbers for which stage versus time curves are to be plotted.	2I3
7	Number of points defining inflow hydrograph	I3
8	The ratio of flow to fullbank flow and time (seconds) for one point on the inflow hydrograph.	F10.2, F10.0

Note: Repeat card 8 for each of the points which define the inflow hydrograph.

(d) OUTPUT: The output consists of a listing of various parameters for the problem being tackled in the form of a title block as well as two hydrographs and two stage - time curves for each of the solutions attempted. An option is incorporated in the program to allow a user to record the hydrographs on punched cards.

(e) RESTRICTIONS: This program is limited to the solution of problems involving uniform rectangular channels where flow resistance is defined by Mannings equation. (See discussion.)

(f) OTHER DECKS: HPLOT, INTER1, MANNGQ, KINRUT, KINFUN

(4) EXAMPLE:

(a) THE PROBLEM: A rectangular channel 50,000' long 100' wide and 20' deep is subject to a triangular inflow hydrograph. Twenty reaches are used to analyze the channel which has a bedslope of 0.0002 and a Mannings "n" of 0.0149. The time step was set equal to 200 seconds and the analysis was made for a period of 24,000 seconds. A copy of the data input cards and a sample output is shown in Appendix G of the source.

(5) DISCUSSION: New users should refer to documentation of all related programs and routines. Further details of the method of analysis is available from the source.

A variation of this program was developed to perform the same calculations with a very wide channel.

Another variation was developed which allowed the user to install an imaginary reservoir in series with the channel. This reservoir could be located at a section in the channel and the size of the reservoir was defined in the input data. The "nucleus" was located in the centre of the finite difference molecule when the channel was analyzed. Nested "DO" loops were incorporated to allow several simultation to be performed with different sizes of reservoirs.

(6) SOURCE: "A Comparison of Kinematic Flood Routing Methods", by Fred Biesenthal
(A Master of Engineering Thesis).
McMaster University
Hamilton, Ontario

HYDROGRAPH PLOTS

(1) PURPOSE: This subroutine plots two hydrographs on a set of axis.

(2) METHOD: A grid which is 50 printer lines high and 118 spaces wide, is utilized. The hydrograph is specified by a series of zeros or plus signs positioned in the grid. Automatic scaling is provided within the subroutine and the calibration marks are placed on the scales. Time units are plotted (and labeled) as hours) across the bottom scale. An interpolation routine allows hydrographs with two different time steps to be plotted on the same scale. (The largest time step is used for the time scale.)

For further details of the method refer to the subroutine listing.

(3) PROGRAM:

(a) DECK NAME: HPLOT

(b) CALLING SEQUENCE: HPLOT (QOUT2D. DTIMAR, NQOUT, PEAKAR, TSTART)

QOUT2D = A two dimensional floating

point array containing the flowrates of the two hydrographs to be plotted. The flow rates are specified at equal time intervals though the time step for the two hydrographs need not be the same value.

DTIMAR = A floating point array containing two values. The first value is the time step for the first hydrograph and the second value contains the time step for the second hydrograph.

NQOUT = A floating point array defining the number of points in each hydrograph.

PEAKAR = A floating point array which defines the peak flow rate of each hydrograph.

TSTART = A floating point value which defines the start time of the hydrograph.

(c) OUTPUT: A call of this subroutine results in a plot of two hydrographs on one page of computer paper. A line printer which is at least 130 spaces wide is required.

(d) RESTRICTIONS: Negative values will not be plotted correctly.

(e) OTHER DECKS REQUIRED: None

(4) EXAMPLE: The sample printouts available from the source provide examples of this subroutine.

(5) DISCUSSION: The use of the subroutine is restricted to batch mode operations. It is possible, with further modification to utilize this approach with interactive terminals.

(6) SOURCE: Modified from Williams, Jimmy R. and Roy W. Hann, "HYMO, A Problem Oriented Language for Building Models", Water Resources Research, Vol. 8, No. 1, 79 - 86, February, 1972.

FLOOD ROUTING USING THE CONTINUITY EQUATIONS

(1) PURPOSE: This routine calculates the functional relationships of an elementary river reach prior to the routing of a floodwave using subroutine "KINRUT".

(2) METHOD: The functional relationships are defined as follows:

FUS = BETA * QIN - ALPHA * STOR/DTIM

for the upstream section and

GDS = BETA * QOUT + (1.-ALPHA) * STOR/DTIM

for the downstream section.

It should be noted that STOR is the storage in the total length of the elementary reach with a steady state condition. In the case of FUS(QIN), the flow rate along the elementary reach is assumed to be that of the inflow while a steady flow rate of QOUT along the elementary reach is assumed when calculating GDS(QOUT).

(3) PROGRAM:

(a) DECK NAME: KINFUN

(b) CALLING SEQUENCE: CALL KINFUN (ALPHA, BETA, DTIM, STORUS, QUSAR, NFUS, STORDS, QDSAR, NFDS, FUSAR, GDSAR)

WHERE

ALPHA = Floating point variable containing ALPHA.

BETA = Floating point variable containing BETA.

DTIM = Floating point variable containing the value of the time increment.

STORUS = Floating point array of size NFUS which contains values for steady state storage to define coordinates of storage - inflow relationship.

QUSAR = Floating point array of size NFUS which contains values of inflow to define coordinates of storage - inflow relationship.

NFUS = Integer describing the number of points which define the relationship between steady state storage and inflow.

STORDS = Floating point array of size NFDS which contains values for steady state storage to define coordinates of storage - outflow relationship.

QDSAR = Floating point array of size NFDS which contains values of outflow to define coordinates of storage - outflow relationship.

NFDS = Integer describing the number points which define the relationship between steady state storage and outflow.

FUSAR = Floating point array of size NFUS containing values of the function relationship associated with inflow.

GDSAR = Floating point array of size NFDS containing values of the function relationship associated with outflow.

(c) OUTPUT FORMAT: The function relations are assigned to array FUSAR for the upstream section and GDSAR for the downstream section.

(d) OTHER DECKS REQUIRED: None

(e) RESTRICTIONS: The relationship between storage and flow rate must be a single valued function.

(4) EXAMPLE: Refer to the source for an example.

(5) DISCUSSION: None

(6) SOURCE: "A Comparison of Kinematic Flood Routing Methods", by Fred Biesenthal
(A Master of Engineering Thesis),
McMaster University
Hamilton, Ontario

FLOOD ROUTING USING THE CONTINUITY EQUATIONS

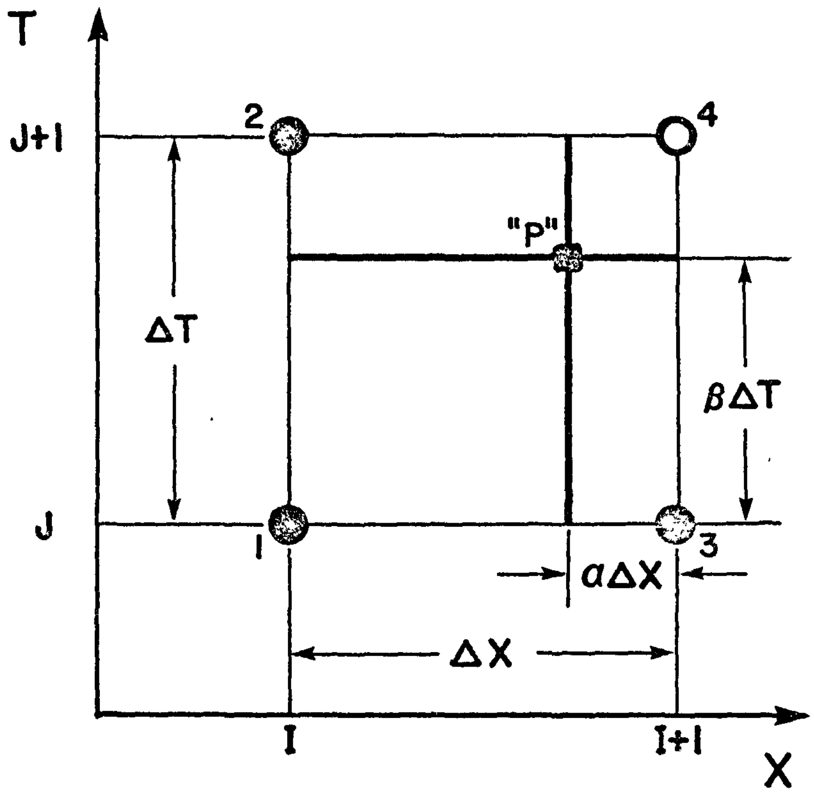
(1) PURPOSE: This routine is used to determine the outflow from a system where the unsteady flow regime can be reasonably modelled using kinematic wave theory.

(2) METHOD: The routine uses a finite difference technique to solve the continuity equation. Two parameters ALPHA and BETA are used to determine the way in which the finite differences are calculated. Figure KINRUT1 shows the finite difference approximations and a portion of the space time grid. The calculations are made using functional relationships of inflow and outflow. The process of routing a flood through an elementary reach is very similar to the process of routing a flood through a reservoir using the storage - indication method. A user should refer to the source for a more complete description of the numerical system used and the properties of the finite difference scheme. Refer to the documentation of KINFUN for the definition of the functional relationships.

FIGURE KINRUT I

GENERAL FINITE DIFFERENCE SCHEME

GRAPHICAL REPRESENTATION



NUMERICAL APPROXIMATIONS

$$\frac{\delta A}{\delta T} = \frac{(1-\alpha)(A_4 - A_3) + \alpha(A_2 - A_1)}{\Delta T}$$

$$\frac{\delta Q}{\delta X} = \frac{\beta(Q_4 - Q_2) + (1-\beta)(Q_3 - Q_1)}{\Delta X}$$

MOLECULE THE SPACE-TIME ELEMENT BOUNDED BY POINTS 1243

NUCLEUS THE POINT "P" ABOUT WHICH THE FINITE DIFFERENCE EQUATION IS APPLIED

(3) PROGRAM:

(a) DECK NAME: KINRUT

(b) CALLING SEQUENCE: CALL KINRUT (GDSAR, QDSAR, NFDS, FUSAR, QUSAR, NFUS, QLINAR, TIMARL, NPTSL, DX, DTIM, NPTS, QINAR, QOUTAR)

WHERE

GDSAR = Floating point array of size NFDS containing values for the functional relation of outflow.

QDSAR = Float point array of size NFDS containing values of outflow related to the array GDSAR.

NFDS = Integer defining number of points which describe the function relation of outflow.

FUSAR = Floating point array of size NFUS containing values for the functional relation of inflow.

QUSAR = Floating point array of size NFUS containing values for inflow related to the array FUSAR.

NFUS = Integer defining the number of points which describe the function relation of inflow.

QLINAR = Floating point array of size NPTSL containing the values of flow per unit length which describe the lateral inflow hydrograph.

TIMARL = Floating point array of size NPTSL containing the coordinates of time which describe the lateral inflow hydrograph.

NPTSL = Integer defining the number of points which describe the lateral inflow hydrograph.

DX = Floating point variable describing the length of the channel or reservoir which is subject to lateral inflow.

DTIM = Floating point variable containing the time increment.

NPTS = Integer defining the number of points which describe the inflow and outflow hydrographs.

QINAR = Floating point array of size NPTS which contains the points describing the inflow hydrograph.

QOUTAR = Floating point array of size NPTS which contains the computed outflow hydrograph.

(c) OUTPUT FORMAT: The outflow hydrograph is assigned to array QOUTAR. Points of the outflow hydrograph are specified at equal time increments of value DTIM.

(d) OTHER DECKS REQUIRED: INTER 1

(e) RESTRICTIONS: The user must specify the first value of the outflow hydrograph before calling KINRUT, and the points in the inflow hydrograph must be defined at equal time increments. Coordinates of the outflow hydrograph will correspond to the points on the inflow hydrograph.

If lateral inflow is not a factor in the particular problem, it will be set to zero by defining the first value of TIMARL as a real negative value.

The functional relationships between flow and storage must be defined before calling KINRUT.

(4) EXAMPLE; Refer to the Source.

(5) DISCUSSION: None

(6) SOURCE: "A Comparison of Kinematic Flood Routing Methods", by Fred Biesenthal
(A Master of Engineering Thesis,)
McMaster University
Hamilton, Ontario

RESERVOIR ROUTING

(1) PURPOSE: This subroutine generates the functional rating curves used by "KINRUT" to route a flood through a reservoir. A call of "KINRUT" performs the actual routing.

(2) METHOD: For a detailed description of the method used to route the flood refer to documentation of "KINRUT" and "KINFUN" as well as the source. The rating of the reservoir is given by the equation

STOR = FPVK*QOUT**FPVW

WHERE:

STOR = Storage

FPVK = A constant

QOUT = Flow rate

FPVW = A constant

(3) PROGRAM:

(a) DECK NAME; RESVOR

(b) CALLING SEQUENCE: CALL RESVOR, (FPVK, FPVW, FLOWAR, NFLOW, DTIM, NQIN, QINAR, QOUTAR)

WHERE: FPVK = A floating point variable containing the value of the constant K.

FPVW	=	A floating point variable containing the value of the constant W.
FLOWAR	=	A floating point array containing the flow rates to be used when the functional relationships are to be calculated.
NFLOW	=	An integer value which specifies the number of values in the vector FLOWAR.
DTIM	=	A floating point variable which specifies the time step used in the calculations. (Same units as time unit of flow rates.)
NQIN	=	An integer value which specifies the number of points in the flow hydrographs.
QINAR	=	A floating point array which contains the flow rates of the inflow hydrograph specified at equal time intervals defined by DTIM.

QOUTAR = A floating point array which contains the flow rates of the outflow hydrograph specified at equal time intervals as defined by DTIM.

(c) OUTPUT: The routed flow is returned to the calling sequence via QOUTAR. A plot of the inflow and outflow hydrographs is provided by a call of subroutine "HPLOT".

(d) RESTRICTIONS: To provide a dimensionless representation of the hydrograph, a floating point value is transfered to the subroutine via a COMMON statement labelled "FLOW". The flows were divided by a flowrate so that the result was a value of Q where:

$$1 \geq Q \geq 0$$

(e) OTHER DECKS REQUIRED: KINRUT, HPLOT

(4) EXAMPLE: An example of the use of this program is available from the source.

(5) DISCUSSION: The main purpose of this subroutine is to route a flood through an imaginary reservoir. If it is more advantageous to utilize reservoir rating curves which are storage as a function

of elevation and outflow as a function of elevation see RESRUT.

(6) SOURCE: "A Comparison of Kinematic Flood Routing Methods", by Fred Biesenthal
(A Master of Engineering Thesis)
McMaster University
Hamilton, Ontario

STEADY AND UNSTEADY RIVER FLOW SIMULATION

(i) PURPOSE: This program is designed specifically for use in a time sharing mode and is intended to set up, calibrate and subsequently modify a numerical model of a natural river channel under steady and unsteady flow conditions.

A stretch of river is described by a number of cross-sections each of which is defined by a series of points. the initial co-ordinate values of which are referred to arbitrary datums for level and horizontal distance.

Each cross-section is identified by a fixed chainage (and section number), and is also assigned an initial value of roughness coefficient. The resistance to flow may be defined by any one of a number of laws; the choice being made during the run.

The program operates on the system thus defined and for specific values of discharge and downstream rating curve computes the water surface elevation(s) and energy level(s) at the cross sections specified during execution.

In the course of execution the user has the option to vary the discharge along the channel, the number of profiles calculated. the location and rating curve of the down-stream control. the portion of the river over which the profile(s) is to be computed, the print out of the profile data, the rough-ness coefficient and the geometry of any selected section or any combinations of the options mentioned above. In addition, there are several other features of the program devoted to steady state analysis which enables the user to calculate the critical flow depth at a selected section, list any changes that have been made to the data or to print the existing cross section data.

An initially defined system may therefore be adjusted to correctly reproduce an observed flow profile and then be used to examine the effect of a chosen design flow and to experiment with changes to the cross section geometry. The profiles that are calculated may be utilized in the unsteady analysis described below.

The analysis of water profiles incorporates a method of handling transition sections, such as weirs or bridges, which may occur along the channel.

The unsteady analysis incorporated into this subroutine allows a hydrograph of varying flow to be routed down portions of the channel using kinematic wave theory. An option is available to allow a user to route the flood through a reservoir which may be incorporated within the channel. Data which describes the hydrograph and other variables such as the time step for computation purposes are all entered during execution of the program.

(2) METHOD: The calculation consists essentially of repeated applications of the subroutines EZRA and CONTRO, starting from the farthest downstream reach and proceeding upstream for each of the profiles.

Flood routing is performed using kinematic wave theory as formulated in the subroutine KINRUT .

(3) PROGRAM:

(a) DECK NAME: RIVER3

(b) CALLING SEQUENCE: RIVER3 (HORZ2D, VERT2D, NPTSAR, CHAINR, RCAR, NXSEC, MAXPTS, G, NR1, NR2, NW, NF, AREAR, ELEVAR, FLOW2D, WLAR, QDAR, FUSAR, GDSAR, QUSAR, QDSAR, QRAR)

WHERE:

HORZ2D = Two-dimensional array containing the horizontal coordinates for the points describing the series of cross-sections. The first subscript of the array represents the section number and the second subscript represents one of the horizontal coordinates.

VERT2D = Two-dimensional array containing the vertical coordinates for the points describing the series of cross-sections. The first subscript of the array represents the section number and the second subscript represents one of the vertical coordinates.

NPTSAR	=	One-dimensional array containing the number of coordinate points for each cross-section in the series of sections.
CHAINR	=	One-dimensional array containing the chainage values for the series of cross-sections.
RCAR	=	One-dimensional array containing the roughness coefficients for each cross-section in a series of sections.
NXSEC	=	The number of cross-sections in the river channel.
MAXPTS	=	The maximum number of coordinate points required to describe any cross-section for the series of sections.
G	=	Gravitational acceleration constant.

NR1	=	Periph. Device No. for Geometry file.
NR2	=	Periph. Device No. for keyboard input.
NW	=	Periph. Device No. for terminal output.
NF	=	The number of points in the working arrays and tables define cross-section properties versus flow rate.
AREAR	=	Two-dimensional array containing cross-section area for each section and flow used to describe the steady state performance. Dimensioned NF*NXSEC.
ELEVAR	=	Two-dimensional array containing water surface elevation for each section and flow used to describe the steady state performance. Dimensioned NF*NXSEC.

FLOW2D	=	Two-dimensional array containing the flowrates at each section used to calculate the steady state performance.
WLAR	=	Vector array which contains the water level coordinates for the rating curve of the downstream section.
QDAR	=	Vector array which contains the discharge coordinates for the rating curve of the downstream section.
FUSAR	=	Vector array which contains the function coordinates of the rating curve for the upstream section used to perform the kinematic flood routing.
GDSAR	=	Vector array which contains the function coordinates of the rating curve for the downstream section used to perform the kinematic flood routing.

QUSAR	=	Vector array which contains the flow coordinates of the rating curve for the upstream section used to perform the kinematic flood routing.
QDSAR	=	Vector array which contains the flow coordinates of the rating curve for the downstream section used to perform the kinematic flood routing.
QRAR	=	Vector array which contains the flow coordinates of the rating curve used for reservoir routing.

(c) OUTPUT: The output of the subroutine consists of invitations to enter data, such as questions requesting input, as well as the results of the various calculations. Generally, information provided consists of section number, chainage, water surface elevation and energy level for steady state analysis and section number and time and flow rates for a hydrograph calculated during unsteady flow analysis.

(d) RESTRICTIONS: The flow resistance law must be chosen from among the following: Chezy, Manning, Strickler, Colebrook-White, Nikuradse's logarithmic Smooth Turbulent or Nikuradse's logarithmic Rough Turbulent. It is the user's responsibility to ensure that the selected law is appropriate both to the river system and to the roughness coefficient contained in the geometric data.

If the smooth turbulent law is used, the roughness coefficient is ignored but arbitrary data must still be provided.

The number of cross-sections cannot be varied from that defined in the data file. Neither can the initially defined maximum number of points per section be exceeded. The discharge is assumed to be uniform along each sub-reach. However, a number of sub-reaches may be defined within the river system. When performing the calculations a sub-critical flow regime is assumed.

(e) OTHER DECKS REQUIRED: READXS, SELSEC, PROPS, CRITIC, BOTTOM, SFROMQ, CHEZYQ, MANNGQ, STRICQ, COLEQ, SMOTHQ, ROUGHQ, EZRA, CONTRO, KINRUT

(4) EXAMPLE:

(a) INPUTS: A rectangular channel 50,000' long, 100' wide and 20' deep is subject to a triangular inflow hydrograph. Eleven sections with with four points each were utilized to describe the channel which had a bed slope = 0.0002. The Mannings roughness coefficient was n = 0.0149.

The geometrical data was stored on a file and was read in as TAPE1. For each section the data comprises:

(i) Section number (sections must be numbered sequentially from No. 1 at the upstream end, but need not be read in that order.)

(ii) distance

(iii) roughness measure

(iv) Number of points used to describe the section.

(v) A sequence of pairs of coordinates defining the cross-section geometry.

A copy of TAPE1 is shown in Appendix G of the source. The format of this input file is dictated by the format used for the read statements in subroutine READXS. Other information relating to discharge, control level, inflow hydrographs and subsequent system changes etc., is input from the console during the run. Each input is detailed in the sample output which is largely self-explanatory.

(b) OUTPUT: A sample output is shown in Appendix G of the source.

(5) DISCUSSION: New users should refer to documentation of all related programs and routines. Further details of the method used for analysis of unsteady flow are available from the source.

(6) SOURCE: Modified from RIVER2 (Dr. A. A. Smith. McMaster University, Hamilton, Ontario.) "A Comparison of Kinematic Flood Routing Methods", by Fred Biesenthal (A Master of Engineering Thesis) McMaster University Hamilton, Ontario

APPENDIX E

DERIVATION OF THE LAX-WENDROFF METHOD

The continuity equation is:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} = \bar{q} \qquad (E.1)$$

This equation can be written in the conservation form:

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} - \bar{q} = 0 \qquad (E.2)$$

Expanding $A(X, T + \Delta T)$ using a Taylor Series, the following is obtained:

$$A(X, T+\Delta T) = A(X,T) + \Delta T \frac{\delta A}{\delta t} + \frac{\Delta T^2}{2} \frac{\delta^2 A}{\delta t^2} + O(\Delta T^3) \qquad (E.3)$$

From equation E. 2

$$\frac{\delta A}{\delta t} = -\left(\frac{\delta Q}{\delta x} - \bar{q} \right) \qquad (E.4)$$

and

$$\frac{\delta^2 A}{\delta t^2} = -\frac{\delta}{\delta x}\left(\frac{\delta Q}{\delta t}\right) + \frac{\delta \bar{q}}{\delta t} \tag{E.5}$$

Assuming a singled valued relationship between Q and A:

$$\frac{\delta Q}{\delta t} = f(A)\frac{\delta A}{\delta t} \tag{E.6}$$

Substituting equations E. 6 and E. 4 into equation E. 5 yields:

$$\frac{\delta^2 A}{\delta t^2} = \frac{\delta}{\delta x}\left(f(A)\frac{\delta Q}{\delta x} - f(A)\bar{q}\right) + \frac{\delta \bar{q}}{\delta t} \tag{E.7}$$

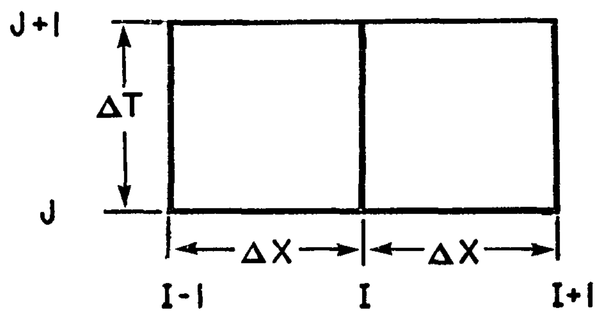
Therefore:

$$A(X,T+\Delta T) = A(X,T) - \Delta T\left(\frac{\delta Q}{\delta x} - \bar{q}\right)$$

$$+\frac{\Delta T^2}{2}\left(\frac{\delta}{\delta x}\left(f(A)\frac{\delta Q}{\delta x} - f(A)\bar{q}\right) + \frac{\delta \bar{q}}{\delta t}\right) \tag{E.8}$$

Using the space-time grid shown in figure 3.2 equation E.8 can be expressed in terms of finite differences.

Figure 3.2 is reproduced below for reference purposes.



The equation in finite difference form is

$$A(I,J+1) = A(I,J) - \Delta T\left(\frac{Q(I+1,J) - Q(I-1,J)}{2\Delta X} - \frac{1}{2}(\bar{q}(I+1,J) + \bar{q}(I-1,J))\right) + \frac{\Delta T^2}{2\Delta X}\left(f(A)\frac{Q(I+1,J) - Q(I,J)}{\Delta X} - \frac{f(A)}{2}(\bar{q}(I+1,J) + \bar{q}(I,J)) - f(A)\frac{Q(I,J) - Q(I-1,J)}{\Delta X} + \frac{f(A)}{2}(\bar{q}(I,J) + \bar{q}(I-1,J))\right) + \frac{\Delta T}{2}\left(\bar{q}(I,J+1) - \bar{q}(I,J)\right) \qquad (E.9)$$

It should be noted that the value f(A) is a unique value for each position on the space time diagram. That is,

$$f(A(X,T)) = \frac{Q(X,T)}{A(X,T)} \tag{E.10}$$

Substituting equation E. 10 into equation E. 9 gives:

$$\begin{aligned} A(I,J+1) = A(I,J) - \Delta T\Bigg(& \frac{Q(I+1,J)-Q(I-1,J)}{2\Delta X} \\ & -\frac{1}{2}\left(\bar{q}(I+1,J)+\bar{q}(I-1,J)\right)\Bigg) + \frac{\Delta T^2}{2\Delta X}\Bigg(\frac{Q(I+1,J)^2}{A(I+1,J)\Delta X} \\ & -2\frac{Q(I,J)^2}{A(I,J)\Delta X} + \frac{Q(I-1,J)^2}{A(I-1,J)\Delta X} - \frac{Q(I+1,J)\,\bar{q}(I+1,J)}{2A(I+1,J)} \\ & + \frac{Q(I-1,J)\,\bar{q}(I-1,J)}{2A(I-1,J)}\Bigg) + \frac{\Delta T}{2}\left(\bar{q}(I,J+1)-\bar{q}(I,J)\right) \end{aligned} \tag{E.11}$$

After deriving A(I, J+1), Q(I, J+1) is obtained from the relationship between flow rate and area. This function is single valued for kinematic waves.

Using the above formulation, the solution advances downstream on a particular time level. Suceeding time levels are considered in following passes down the channel.

APPENDIX F

KINEMATIC FLOOD ROUTING--METHOD OF CHARACTERISTICS

Kinematic waves have one set of characteristics which travel downstream with a velocity, C , where

$$C = \frac{dQ}{dA} \qquad (F.1)$$

Q = flow rate

A = area

To route a flood wave through a channel, using a kinematic method, it is necessary to calculate only the wave velocity for various flow rates, determine how much time is required for the particular flow rate to move through the length of channel and thus plot points which determine the time history of the outflow hydrograph.

Determination of

Using Mannings Formula:

$$Q = \frac{1.49}{n} A R^{\frac{2}{3}} \sqrt{S} \qquad (F.2)$$

$$= \frac{1.49}{n} \sqrt{S} \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \qquad (F.3)$$

$$K = \frac{1.49}{n} \sqrt{S} \qquad \text{(F.4)}$$

FOR A RECTANGULAR CHANNEL:

$$A^{5/3} P^{-2/3} = (T_w X Y)^{5/3} (T_w + 2Y)^{-2/3} \qquad \text{(F.5)}$$

T = TOPWIDTH

Y = DEPTH

$$\frac{dQ}{dA} = \frac{1}{T_w} \frac{dQ}{dY} \qquad \text{(F.6)}$$

$$\frac{dQ}{dY} = \frac{d}{dY}\left(K (T_w Y)^{5/3} (T_w + 2Y)^{-2/3} \right) \qquad \text{(F.7)}$$

$$\frac{dQ}{dY} = K\left(\frac{5}{3} \frac{T_w^{\frac{5}{3}} Y^{\frac{2}{3}}}{(T_w + 2Y)^{2/3}} - \frac{4}{3} \frac{(T_w Y)^{5/3}}{(T_w + 2Y)^{2/3}} \right) \qquad \text{(F.8)}$$

$$\frac{dQ}{T_w dY} = \frac{K (T_w Y)^{5/3}}{T_w (T_w + 2Y)^{2/3}} \left(\frac{5}{3Y} - \frac{4}{3(T_w + 2Y)} \right) \qquad \text{(F.9)}$$

$$\frac{dQ}{dA} = \frac{Q}{T_w} \left(\frac{5}{3Y} - \frac{4}{3(T_w + 2Y)} \right) \qquad \text{(F.10)}$$

The following program was written to calculate flow rate, ration of flow rate to full depth flow rate and kinematic wave velocity all as a function depth.

```
      PROGRAM KIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      S=0.0002
      RC=0.0149
      G=32,2
      W=100.0
      DMAX=20.0
      DH=DMAX/40.
      P=DMAX*2.0+100.0
      A=W*DMAX
      CALL MANNGQ(A,P,S,RC,G,QMAX)
      H=0.0
      DO 10 I=1,40
      H=H+DH
      A=H*W
      P=W+2.0*H
      CALL MANNGQ(A,P,S,RC,G,Q)
      VK=Q/W*(5.0/(3.0*H)-4.0/(3.0*W+6.0*H))
      QR=Q/QMAX
   10 WRITE(6,100) H,Q,QR,VK
  100 FORMAT(1X,4F12.4)
      STOP
      END
```

The results are shown in table F. 1

TABLE F-1

FLOW RATE, FLOW RATIO AND KINEMATIC WAVE VELOCITY

AS A FUNCTION OF DEPTH

SYSTEM I

DEPTH (ft)	FLOW RATE (cfs)	Q / Q full bank	C (fps)
.5000	44.3121	.0027	1.4712
1.0000	139.7612	.0084	2.3111
1.5000	272.9276	.0164	2.9972
2.0000	438.0072	.0263	3.5939
2.5000	631.2877	.0379	4.1284
3.0000	850.0642	.0510	4.6157
3.5000	1092.2224	.0655	5.0650
4.0000	1356.0353	.0813	5.4827
4.5000	1640.0486	.0983	5.8736
5.0000	1943.0102	.1165	6.2412
5.5000	2263.8242	.1358	6.5881
6.0000	2601.5186	.1560	6.9167
6.5000	2955.2223	.1772	7.2288
7.0000	3324.1481	.1993	7.5258
7.5000	3707.5789	.2223	7.8092
8.0000	4104.8584	.2462	8.0800
8.5000	4515.3820	.2708	8.3391
9.0000	4938.5908	.2962	8.5875
9.5000	5373.9658	.3223	8.8259
10.0000	5821.0237	.3491	9.0549
10.5000	6279.3127	.3766	9.2752
11.0000	6748.4098	.4047	9.4873
11.5000	7227.9175	.4334	9.6917
12.0000	7717.4617	.4628	9.8889
12.5000	8216.6895	.4927	10.0791
13.0000	8725.2675	.5232	10.2629
13.5000	9242.8803	.5543	10.4406
14.0000	9769.2287	.5858	10.6124
14.5000	10304.0287	.6179	10.7787
15.0000	10847.0104	.6505	10.9397
15.5000	11397.9171	.6835	11.0957
16.0000	11956.5040	.7170	11.2470
16.5000	12522.5375	.7509	11.3936
17.0000	13095.7949	.7853	11.5360
17.5000	13676.0631	.8201	11.6741
18.0000	14263.1384	.8553	11.8083
18.5000	14856.8258	.8909	11.9386
19.0000	15456.9383	.9269	12.0653
19.5000	16063.2967	.9633	12.1885
20.0000	16675.7291	1.0000	12.3083

This data was used to plot a curve of flow rate versus wave velocity. (Figure F.1) By dividing wave velocity for a particular flow rate into the length of the channel in feet, the lag between inflow and outflow of that flow rate can be determined. Points on an inflow hydrograph can be transposed downstream and the outflow hydrograph can be determined.

System 1 was analyzed using this method. The results of the analysis are given in table F.2 and the hydrograph is plotted in figure 3.8.

TABLE F-2

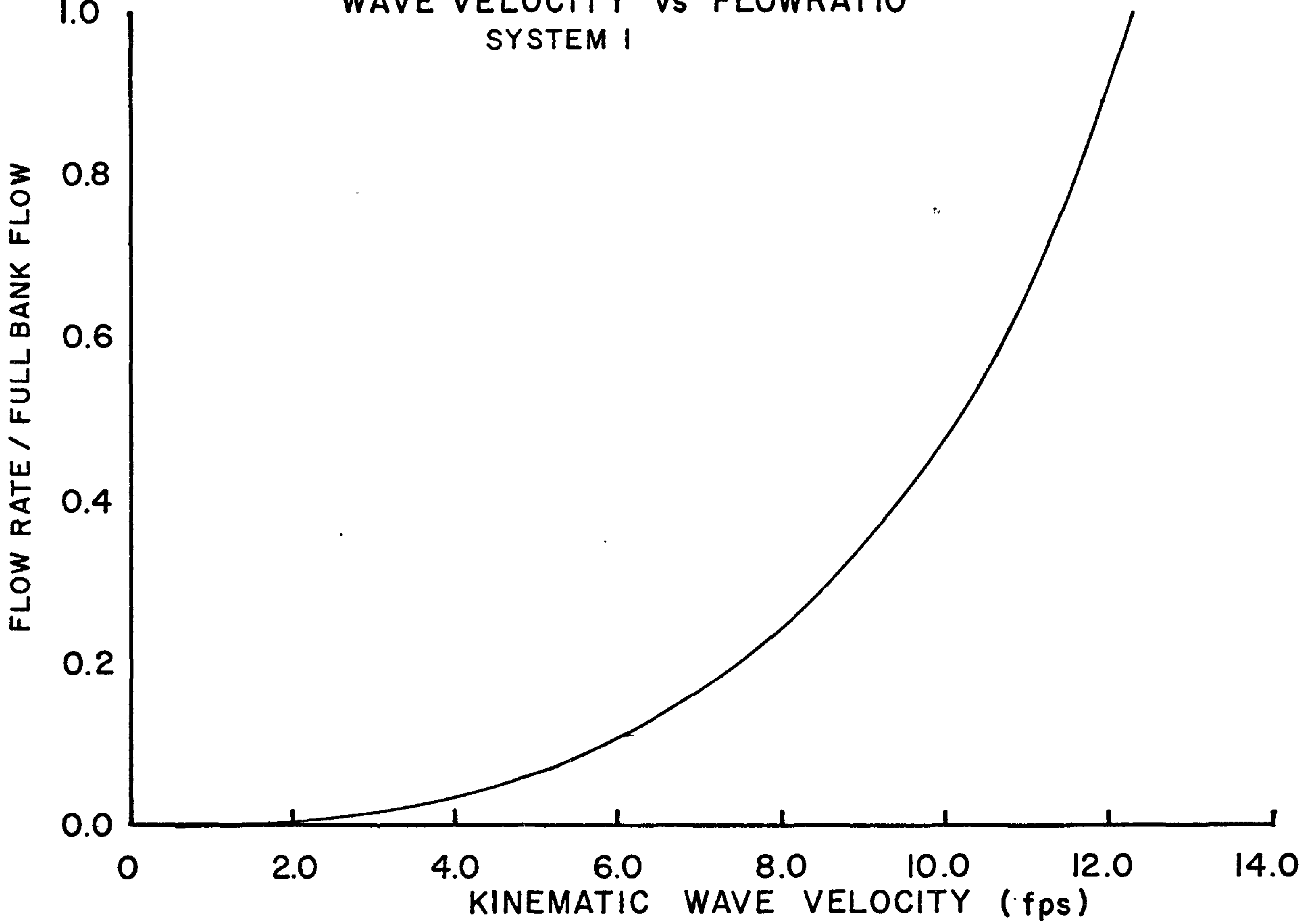
RESULTS OF KINEMATIC FLOOD ROUTING SYSTEM I

Inflow Ratio	Velocity (ft/sec)	Travel Time (sec)	Inflow Time (sec)	Outflow Time (sec)
0.2	7.52	5319	1500	6820
0.4	9.46	4228	2750	6980
0.6	10.73	3728	4000	7730
0.8	11.60	3448	5250	8700
1.0	12.31	3249	6500	9750
0.8	11.60	3448	7750	11200
0.6	10.73	3728	9000	12730
0.4	9.46	4228	10250	14480
0.2	7.52	5319	11500	16820

FIGURE F.1

WAVE VELOCITY vs FLOWRATIO

SYSTEM I



APPENDIX G

LISTINGS OF COMPUTER INPUT FILES, ROUTINES AND OUTPUT

INPUT FILES: FINDIF
- KINDIF
- TAPE1
- ANCDAT
- WHTDAT

ROUTINES: FINDIF
- KINDIF
- KINFUN
- KINRUT
- HPLOT
- RESVOR
- RIVER3

OUTPUT: FINDIF
- KINDIF
- RIVER3

TYPICAL INPUT FILE: FINDIF

```
100=HRAG,T400.
BIESENTHAL
110=ATTACH,RIVERF,ID=HRAG,MR=1.
120=ATTACH,CIVLIB,ID=HRAG,MR=1.
130=FTN(I=RIVERF)
140=LDSET(LIB=CIVLIB)
150=LGO.
151=*EOR
170=50000.     100.      0.0149     0.0002     20.
180= 250.5        24000.
190=190
200=  1  1
210= 21
220=  1 21
230=  1 21
240=  3
250=0.2        1500.
260=1.0        6500.
270=0.2        11500.
271=*EOR
272=*EOF
```

TYPICAL INPUT FILE: KINDIF

```
100=HRAG,T400.
BIESENTHAL
110=ATTACH,KINDIF,ID=HRAG,MR=1.
120=ATTACH,CIVLIB,ID=HRAG,MR=1.
130=FTN(I=KINDIF)
140=LDSET(LIB=CIVLIB)
150=LGO.
151=*EOR
170=50000.     100.       0.0149     0.0002     20.
180= 20200.       24000.
190=190
200=  1   1
220=  1  17
230=  1  17
240=  3
250=0.2        1500.
260=1.0        6500.
270=0.2        11500.
271=*EOR
272=*EOF
```

GEOMETRIC DATA FILE : TAPE1

```
  1      0.0 .0149  4
    0.00130.00   0.00110.00 100.00110.00 100.00130.00
  2   5000.0 .0149  4
    0.00129.00   0.00109.00 100.00109.00 100.00129.00
  3  10000.0 .0149  4
    0.00128.00   0.00108.00 100.00108.00 100.00128.00
  4  15000.0 .0149  4
    0.00127.00   0.00107.00 100.00107.00 100.00127.00
  5  20000.0 .0149  4
    0.00126.00   0.00106.00 100.00106.00 100.00126.00
  6  25000.0 .0149  4
    0.00125.00   0.00105.00 100.00105.00 100.00125.00
  7  30000.0 .0149  4
    0.00124.00   0.00104.00 100.00104.00 100.00124.00
  8  35000.0 .0149  4
    0.00123.00   0.00103.00 100.00103.00 100.00123.00
  9  40000.0 .0149  4
    0.00122.00   0.00102.00 100.00102.00 100.00122.00
 10  45000.0 .0149  4
    0.00121.00   0.00101.00 100.00101.00 100.00121.00
 11  50000.0 .0149  4
    0.00120.00   0.00100.00 100.00100.00 100.00120.00
```

INPUT FILE: ANCDAT (GEOMETRIC DATA FOR APPLICATION TWO)

```
1          0.0  .0500 10                                                                             000100
    0.0 720.0    32.0 715.0    55.0 710.0    91.0 707.0    95.0 703.0   105.0 703.0000110
  109.0 707.0   168.0 710.0   192.0 715.0   312.0 720.0                               000120
2        243.0  .0500 10                                                                             000130
    0.0 705.0    10.0 700.0    20.0 695.0    36.0 692.0    40.0 688.0    50.0 688.0000140
   54.0 692.0    60.0 695.0    83.0 700.0   120.0 705.0                               000150
3        243.0  .0150 13                                                                             000160
    5.5 710.0     5.5 700.0     2.0 698.0     1.1 696.0      .2 691.0     0.0 689.5000170
    5.5 688.0    11.0 689.5    10.8 691.0     9.9 696.0     9.0 698.0     5.6 700.0000180
    5.6 710.0                                                                                        000190
4        320.0  .0150 13                                                                             000200
    5.5 710.0     5.5 699.0     2.0 697.0     1.1 695.0      .2 690.0     0.0 688.5000210
    5.5 687.0    11.0 688.5    10.8 690.0     9.9 695.0     9.0 697.0     5.6 699.0000220
    5.6 710.0                                                                                        000230
5        320.0  .0500 10                                                                             000240
    0.0 705.0    36.0 700.0    45.0 695.0    57.0 690.0    69.0 687.0    80.0 687.0000250
   80.0 690.0    80.0 695.0    80.0 700.0    80.0 705.0                               000260
6        382.0  .0500 10                                                                             000270
    0.0 705.0    23.0 700.0    35.0 695.0    45.0 690.0    89.0 685.0   101.0 685.0000280
  105.0 690.0   115.0 695.0   115.0 700.0   115.0 705.0                               000290
7        382.0  .0150 16                                                                             000300
    0.0 705.0    23.0 700.0    35.0 695.0    38.0 693.5    95.0 693.4    95.0 692.5000310
   89.3 692.4    89.3 685.0   100.8 685.0   100.8 692.4    95.1 692.5    95.1 693.4000320
  112.0 693.5   115.0 695.0   115.0 700.0   115.0 705.0                               000330
8        400.0  .0500 10                                                                             000340
    0.0 705.0    23.0 700.0    35.0 695.0    45.0 690.0    89.0 685.0   101.0 685.0000350
  105.0 690.0   115.0 695.0   115.0 700.0   115.0 705.0                               000360
9        600.0  .0400 10                                                                             000370
    0.0 685.0    23.0 680.0    35.0 675.0    36.0 673.0    45.0 668.0    54.0 668.0000380
   65.0 673.0    80.0 675.0   110.0 680.0   140.0 685.0                               000390
10       600.0  .0150 16                                                                             000400
    0.0 685.0    23.0 680.0    35.0 675.0    36.0 674.0    49.5 673.9    49.5 673.1000410
   45.0 673.0    45.0 668.0    54.0 668.0    54.0 673.0    49.6 673.1    49.6 673.9000420
   72.0 674.0    80.0 675.0   110.0 680.0   140.0 685.0                               000430
11       670.0  .0400 10                                                                             000440
    0.0 685.0    23.0 680.0    35.0 675.0    36.0 673.0    45.0 668.0    54.0 668.0000450
   65.0 673.0    80.0 675.0   110.0 680.0   140.0 685.0                               000460
12      1020.0  .0400 10                                                                             000470
    0.0 672.0    20.0 670.0    40.0 665.0    42.5 660.0    45.0 655.0    55.0 655.0000480
   72.5 660.0   100.0 665.0   142.0 670.0   142.0 672.0                               000490
13      1020.0  .0200 22                                                                             000500
    0.0 672.0    20.0 670.0    40.0 665.0    50.0 664.9    50.0 659.5    53.2 659.4000510
   53.2 657.0    57.2 657.0    57.2 659.4    60.4 659.5    60.4 664.9    70.4 665.0000520
   70.4 664.5    70.9 664.4    70.9 664.0    79.9 664.0    79.9 664.4    80.5 664.5000530
   80.5 665.0   100.0 665.1   142.0 670.0   142.0 672.0                               000540
14      1050.0  .0200  8                                                                             000550
    0.0 670.0    25.0 665.0    36.0 663.0    36.0 657.0    44.0 657.0    44.0 663.0000560
   90.0 665.0   140.0 670.0                                                           000570
15      1200.0  .0200  6                                                                             000580
    0.0 643.0    60.0 640.0    60.0 637.0    68.0 637.0    68.0 640.0    80.0 643.0000590
16      1260.0  .0800  6                                                                             000600
    0.0 625.0    65.0 620.0    70.0 616.0    80.0 616.0    85.0 620.0    97.0 625.0000610
17      1455.0  .0800  6                                                                             000620
    0.0 615.0     3.5 610.0     6.0 609.0    16.0 609.0    18.5 610.0    22.0 615.0000630
18      1455.0  .0240 15                                                                             000640
    0.0 615.5    45.0 615.0    45.0 614.6    42.4 613.6    40.3 612.6    39.8 610.9000650
   40.4 610.0    43.7 609.0    47.1 610.0    47.7 610.9    47.2 612.6    43.7 613.6000660
   45.1 614.6    45.1 615.0    90.0 615.5                                             000670
19      1500.0  .0500  6                                                                             000680
    0.0 615.0    35.0 610.0    42.5 607.0    47.5 607.0    55.0 610.0    90.0 615.0000690
20      1860.0  .0500  6                                                                             000700
    0.0 610.0   501.0 605.0   503.0 602.0   510.0 602.0   512.0 605.0   931.0 610.0000710
21      2370.0  .0500  8                                                                             000720
    0.0 605.0   240.0 600.0   314.0 597.0   315.0 595.0   325.0 595.0   326.0 597.0000730
  410.0 600.0   500.0 605.0                                                           000740
22      2370.0  .0400 12                                                                             000750
    0.0 605.0   240.0 600.0   314.0 595.0   314.0 593.5   317.0 593.4   317.0 593.0000760
  323.5 593.0   323.5 593.4   327.0 593.5   327.0 595.0   410.0 600.0   500.0 605.0000770
```

INPUT - ANCDAT/2

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23      2400.0  .0500   8                                                             000780
   0.0 605.0  240.0 600.0   314.0 595.0   315.0 590.0   325.0 590.0   326.0 595.0000790
 410.0 600.0  500.0 605.0                                                           000800
24      2920.0  .0600  12                                                            000810
   0.0 590.0   15.0 585.0    27.5 580.0    37.5 575.0    47.5 570.0    55.0 566.0000820
  65.0 566.0   72.5 570.0    82.5 575.0    92.5 580.0   105.0 565.0   120.0 590.0000830
25      3000.0  .1500   6                                                             000840
   0.0 570.0   16.0 560.0    29.5 550.0    49.5 550.0    63.0 560.0    79.0 570.0000850
26      3420.0  .0700   7                                                             000860
   0.0 501.0   28.0 495.0    57.0 494.0    67.0 489.0    82.0 489.0    97.0 495.0000870
 180.0 501.0                                                                         000880
27      3420.0  .0150  12                                                            000890
   0.0 502.0    5.0 501.1    80.0 501.0    80.0 496.1    72.5 496.0    72.5 489.0000900
  87.5 489.0   87.5 496.0    80.1 496.1    80.1 501.0   185.0 501.1   200.0 502.0000910
28      3460.0  .0700   6                                                             000920
   0.0 500.0    0.0 487.0    15.0 487.0    21.0 490.0    31.0 495.0    41.0 500.0000930
29      3960.0  .0900   8                                                             000940
   0.0 460.0   40.0 455.0    69.0 453.0    75.0 450.0    85.0 450.0    91.0 453.0000950
 115.0 455.0  175.0 460.0                                                           000960
30      4420.0  .0800  12                                                            000970
   0.0 450.0   55.0 445.0    83.0 440.0   105.0 435.0   139.0 429.0   145.0 426.0000980
 155.0 426.0  161.0 429.0   290.0 435.0   387.0 440.0   417.0 445.0   470.0 450.0000990
31      5075.0  .0800  12                                                            001000
   0.0 420.0   63.0 415.0   170.0 410.0   193.0 405.0   217.0 404.0   223.0 401.0001010
 233.0 401.0  239.0 404.0   279.0 405.0   327.0 410.0   337.0 415.0   364.0 420.0001020
32      5485.0  .1000  11                                                            001030
   0.0 410.0    9.0 405.0    17.0 400.0    42.0 395.0    78.0 394.0    85.0 391.0001040
  95.0 391.0  102.0 395.0   142.0 400.0   210.0 405.0   262.0 410.0                001050
33      5465.0  .0150  16                                                            001060
   0.0 411.0    7.0 406.0    17.0 401.0    20.0 398.0    26.0 397.0    26.0 395.6001070
  20.0 395.5   20.0 391.0    32.0 391.0    32.0 395.5    26.1 395.6    26.1 397.0001080
 137.0 398.0  197.0 401.0   302.0 406.0   330.0 411.0                               001090
34      5550.0  .0150  15                                                            001100
   0.0 410.0    7.0 405.0    17.0 400.0    20.0 397.0    26.0 396.0    26.0 394.6001110
  20.0 394.5   20.0 390.0    32.0 390.0    32.0 394.5    26.1 394.6    26.1 396.0001120
 137.0 397.0  197.0 400.0   302.0 405.0   330.0 410.0                               001130
35      5550.0  .1500  12                                                            001140
   0.0 410.0    7.0 405.0    17.0 400.0    22.0 395.0    28.0 393.0    35.0 390.0001150
  45.0 390.0   52.0 393.0   127.0 395.0   197.0 400.0   302.0 405.0   330.0 410.0001160
36      6580.0  .1200  12                                                            001170
   0.0 385.0   10.0 380.0    20.0 375.0    30.0 370.0    92.0 365.0    95.0 362.0001180
 105.0 362.0  108.0 365.0   175.0 370.0   194.0 375.0   256.0 380.0   309.0 385.0001190
37      7780.0  .1200  12                                                            001200
   0.0 365.0   14.0 360.0    40.0 355.0    72.0 350.0   148.0 346.0   152.0 343.0001210
 172.0 343.0  176.0 346.0   512.0 350.0   540.0 355.0   572.0 360.0   782.0 365.0001220
38      9460.0  .1200  12                                                            001230
   0.0 325.0   15.0 320.0    32.0 315.0    50.0 310.0   182.0 305.0   185.0 302.0001240
 195.0 302.0  198.0 305.0   240.0 310.0   282.0 315.0   308.0 320.0   330.0 325.0001250
39     10240.0  .1000  12                                                            001260
   0.0 310.0    8.0 305.0    16.0 300.0    50.0 295.0   149.0 294.0   153.0 290.0001270
 163.0 290.0  167.0 294.0   195.0 295.0   225.0 300.0   236.0 305.0   261.0 310.0001280
40     11300.0  .0900  12                                                            001290
   0.0 300.0    9.0 295.0    21.0 290.0    26.0 285.0    78.0 284.0    83.0 279.0001300
  93.0 279.0   98.0 284.0   116.0 285.0   163.0 290.0   187.0 295.0   200.0 300.0001310
41     12758.0  .0900  12                                                            001320
   0.0 290.0   10.0 285.0    19.0 280.0    31.0 275.0    48.0 274.0    54.0 271.0001330
  64.0 271.0   71.0 274.0   161.0 275.0   277.0 280.0   299.0 285.0   309.0 290.0001340
42     14250.0  .0800  11                                                            001350
   0.0 285.0   20.0 280.0    65.0 275.0   175.0 271.0   223.0 270.0   235.0 267.0001360
 245.0 267.0  265.0 271.0   494.0 275.0   510.0 280.0   524.0 285.0                001370
43     14900.0  .0800  11                                                            001380
   0.0 285.0   12.0 280.0    25.0 275.0   175.0 269.0   179.0 266.0   199.0 266.0001390
 203.0 269.0  389.0 270.0   401.0 275.0   417.0 280.0   447.0 285.0                001400
44     15800.0  .0700  10                                                            001410
   0.0 285.0    0.0 274.0    84.0 270.0    88.0 267.0    98.0 267.0   100.0 270.0001420
 120.0 271.0  130.0 275.0   132.0 280.0   138.0 265.0                               001430
```

INPUT - ANCDAT/3

```
45   15800.0  .0700 17                                                                  001440
       0.0 285.0     5.0 280.0     12.0 275.0     25.0 271.0     95.0 270.0    100.0 265.0001450
     110.0 265.0   115.0 270.0    325.0 274.0    409.0 271.0    413.0 267.0    423.0 267.0001460
     425.0 270.0   445.0 271.0    455.0 275.0    457.0 280.0    463.0 285.0               001470
46   16900.0  .0700 12                                                                  001480
       0.0 260.0    18.0 275.0     25.0 270.0    360.0 266.0    410.0 265.0    420.0 260.0001490
     440.0 260.0   450.0 265.0    505.0 266.0    516.0 270.0    540.0 275.0    554.0 280.0001500
47   17560.0  .0700 12                                                                  001510
       0.0 280.0     4.0 275.0     22.0 270.0     32.0 265.0    388.0 263.0    398.0 258.0001520
     418.0 258.0   428.0 263.0    532.0 265.0    722.0 270.0    763.0 275.0    790.0 280.0001530
48   18205.0  .1500 10                                                                  001540
       0.0 275.0    10.0 270.0     40.0 265.0    408.0 260.0    413.0 257.0    428.0 257.0001550
     431.0 260.0   484.0 265.0    530.0 270.0    591.0 275.0                              001560
49   18575.0  .0900 10                                                                  001570
       0.0 275.0    20.0 270.0     55.0 265.0    298.0 260.0    303.0 256.0    318.0 256.0001580
     323.0 260.0   345.0 265.0    448.0 270.0    492.0 275.0                              001590
50   18575.0  .0300  6                                                                  001600
    -730.  331.0     0.0  285.      0.0  256.      4.5  256.      4.5  285.     250.  331.001610
51   18800.0  .0300  6                                                                  001620
    -730.  331.0     0.0  285.      0.0  256.      4.5  256.      4.5  285.     250.  331.001630
52   18800.0  .0800 10                                                                  001640
       0.0 275.0    20.0 270.0     47.0 265.0    335.0 260.0    332.0 254.0    355.0 254.0001650
     375.0 260.0   407.0 265.0    438.0 270.0    460.0 275.0                              001660
53   19070.0  .0700 12                                                                  001670
       0.0 275.0    16.0 270.0     26.0 265.0    344.0 260.0    389.0 257.0    397.0 253.0001680
     420.0 253.0   422.0 257.0    424.0 260.0    426.0 265.0    428.0 270.0    430.0 275.0001690
54   19070.0  .0310  6                                                                  001700
    -460.  300.0     0.0  280.      0.0  253.      6.0  253.      6.0  280.     130.  300.001710
55   19300.0  .0310  6                                                                  001720
    -460.  300.0     0.0  280.      0.0  253.      6.0  253.      6.0  280.     130.  300.001730
56   19300.0  .1000 10                                                                  001740
       0.0 270.0    25.0 265.0     55.0 260.0    403.0 258.0    408.0 252.0    424.0 252.0001750
     427.0 258.0   430.0 260.0    440.0 265.0    455.0 270.0                              001760
57   19700.0  .1500 12                                                                  001770
       0.0 270.0     3.0 265.0      5.0 260.0      7.0 259.0     10.0 255.0     12.0 251.0001780
      28.0 251.0    30.0 255.0     31.0 259.0    290.0 260.0    361.0 265.0    375.0 270.0001790
58   20900.0  .0700 11                                                                  001800
       0.0 270.0    19.0 265.0     29.0 260.0     57.0 258.0     61.0 250.1     69.0 250.0001810
      77.0 250.1    81.0 258.0    434.0 260.0    439.0 265.0    480.0 270.0               001820
59   22075.0  .0650 15                                                                  001830
       0.0 270.0    15.0 265.0     31.0 260.0     53.0 255.0    115.0 253.0    156.0 252.5001840
     159.0 248.5   165.0 248.4    171.0 248.5    174.0 252.5    215.0 253.0    618.0 255.0001850
     644.0 260.0   669.0 265.0    695.0 270.0                                             001860
60   23070.0  .0600 15                                                                  001870
       0.0 265.0    10.0 260.0     29.0 257.0     49.0 256.0     49.0 250.1     65.0 250.0001880
      65.0 247.2    75.0 247.1     85.0 247.2     85.0 250.0    114.0 250.1    114.0 256.0001890
     119.0 257.0   390.0 260.0    420.0 265.0                                             001900
61   23600.0  .0600 13                                                                  001910
       0.0 265.0    12.5 260.0     25.0 255.0     37.5 250.0                              001920
      67.0 246.2    73.5 246.3     76.5 249.0     96.5 250.0    109.0 255.0    121.5 260.0001930
     134.0 265.0                                                                        001940
62   23700.0  .0600 13                                                                  001950
       0.0 265.0    12.5 260.0     25.0 255.0     37.5 250.0                              001960
      67.0 246.2    73.5 246.3     76.5 249.0     96.5 250.0    109.0 255.0    121.5 260.0001970
     134.0 265.0                                                                        001980
63   23700.0  .0600 13                                                                  001990
       0.0 265.0    12.5 260.0     25.0 255.0     37.5 250.0     57.5 249.0     60.5 246.3002000
      67.0 246.2    73.5 246.3     76.5 249.0     96.5 250.0    109.0 255.0    121.5 260.0002010
     134.0 265.0                                                                        002020
64   24580.0  .0600 11                                                                  002030
       0.0 262.0    22.0 251.0     28.0 248.0     47.0 247.0     49.0 245.0     60.0 245.0002040
      62.0 247.0    81.0 247.0     87.0 251.0    177.0 252.0    317.0 262.0               002050
```

INPUT FILE:WHTDAT (GEOMETRIC DATA FOR APPLICATION ONE)

```
  1 5 1.00       2220.0                                                              000190
  0.0113.0   0.0 86.5 56.0 86.5 65.0 90.0 90.0115.0                                  000200
  2 8 1.00       3240.0                                                              000210
-11.0110.0-10.0 89.6   0.0 89.6  0.0 80.0 49.0 80.0 64.0 88.2 97.0 94.0 97.0110.0000220
  3 7 1.00       5040.0                                                              000230
-10.0 97.0  20.0 72.6 40.0 72.0 50.0 73.0 60.0 73.8 84.0 87.0 84.0 98.0              000240
  4 8 1.50       5960.0                                                              000250
  0.0100.0   0.0 80.0 12.0 79.7 24.0 72.5 48.0 70.0 70.0 70.4 80.0 87.5 80.0100.0000260
  5 6  .50       7260.0                                                              000270
  0.0 95.0   0.0 67.0 46.0 68.0 71.0 74.0 81.0 86.0 81.0 95.0                        000280
  6 7  .50       7260.0                                                              000290
 60.0 95.0  60.0 78.8 -1.0 78.7 -1.0 67.0 46.0 68.0 71.0 73.5 71.0 96.0              000300
  7 6  .20       7260.0                                                              000310
  0.0 95.0   0.0 67.0 46.0 68.0 71.0 74.0 81.0 86.0 81.0 95.0                        000320
  8 6  .20       7900.0                                                              000330
  0.0 95.0   0.0 70.0 30.0 67.0 59.0 69.0100.0 76.0100.0 95.0                        000340
  9 5  .40       9000.0                                                              000350
-10.0 94.0  20.0 65.0 60.0 64.0 80.0 67.0 80.0 94.0                                  000360
 10 7  .20      10530.0                                                              000370
  0.0 95.0   0.0 75.0 19.0 60.2 55.0 60.5 55.0 69.0 66.0 77.0 66.0 95.0              000380
 11 7  .50      11800.0                                                              000390
  0.0 97.0   0.0 77.0 28.0 64.0 28.0 58.0 78.0 58.0 88.0 71.0 88.0 97.0              000400
 12 8  .50      11800.0                                                              000410
 20.0 90.0  20.0 72.0  0.0 64.0  0.0 58.0 50.0 58.0 50.0 64.0 30.0 72.0 30.0 90.0000420
 13 7  .50      11800.0                                                              000430
  0.0 90.0   0.0 77.0 28.0 64.0 28.0 58.0 78.0 58.0 88.0 71.0 88.0 90.0              000440
 14 7  .50      12500.0                                                              000450
  0.0 90.0   0.0 68.0 19.0 57.6 29.0 56.5 69.0 58.0 89.0 65.0 94.0 90.0              000460
 15 5 0.90      13950.0                                                              000470
 15.0 90.0  15.0 73.0 75.0 53.0 98.0 54.0 98.0 90.0                                  000480
 16 6  .10      15340.0                                                              000490
  0.0 90.0   0.0 70.0 50.0 57.0 65.0 51.0 85.0 57.0 85.0 90.0                        000500
 17 6  .10      15340.0                                                              000510
 10.0 90.0  10.0 68.0 50.0 57.0 65.0 51.0 75.0 54.0 75.0 90.0                        000520
 18 6  .10      15340.0                                                              000530
  0.0 85.0   0.0 70.0 50.0 56.5 65.0 51.0 85.0 56.5 85.0 85.0                        000540
 19 6  .10      16500.0                                                              000550
  0.0 85.0   0.0 70.0 10.0 53.5 75.0 53.5 80.0 70.0 80.0 85.0                        000560
 20 8  .10      18200.0                                                              000570
 -8.0 74.0  12.0 50.0 12.0 49.0 54.0 49.0 70.0 52.0 80.0 57.0100.0 62.0100.0 74.0000580
 21 4  .10      18200.0                                                              000590
  0.0 80.0   0.0 49.2 62.0 49.2 62.0 80.0                                            000600
 22 8  .10      18200.0                                                              000610
-15.0 75.0  12.0 49.5 12.0 48.4 54.0 49.0 70.0 52.0 80.0 57.0100.0 62.0100.0 75.0000620
 23 6  .50      19800.0                                                              000630
  0.0 75.0   4.0 50.0 10.0 47.5 60.0 47.4 90.0 56.5100.0 75.0                        000640
 24 8  .50      21000.0                                                              000650
  0.0 75.0   0.0 59.5 15.0 49.0 24.0 45.5 84.0 44.5103.0 55.0116.0 56.0140.0 75.0000660
 25 7  .10      22850.0                                                              000670
  0.0 72.0   0.0 60.0 11.0 52.0 11.0 41.5 95.0 41.5 95.0 60.0 95.0 72.0              000680
 26 6  .10      24000.0                                                              000690
  0.0 70.0   0.0 49.0 30.0 42.0 84.0 42.0 96.0 47.0 97.0 70.0                        000700
 27 8  .01      24000.0                                                              000710
  0.0 70.0   0.0 49.0 14.0 45.0 30.0 44.0 84.0 44.0 84.0 47.0 96.0 47.0 97.0 70.0000720
 28 7  .10      24000.0                                                              000730
  0.0 70.0   0.0 49.0 30.0 40.5 84.0 40.3 84.0 47.0 96.0 47.0 97.0 70.0              000740
 29 8  .10      24900.0                                                              000750
  0.0 70.0   0.0 53.0 28.0 47.5 36.0 41.0 46.0 39.2 86.0 40.0 96.0 41.5131.0 70.0000760
 30 6  .10      26900.0                                                              000770
  0.0 70.0   0.0 49.4  7.0 47.0 19.0 38.0 74.0 38.0 75.0 70.0                        000780
```

INPUT - WHTDAT/2

```
  31 6  .10    29000.0                                                                   000790
-10.0 70.0 10.0 50.0 20.0 38.0 90.0 38.0100.0 50.0106.0 70.0                             000800
  32 4  .10    29000.0                                                                   000810
  0.0 70.0  0.0 38.2 70.0 38.2 70.0 70.0                                                 000820
  33 6  .10    29000.0                                                                   000830
-10.0 70.0 10.0 50.0 20.0 38.2 90.0 38.2100.0 50.0106.0 70.0                             000840
  34 7  .05    30070.0                                                                   000850
-18.0 65.0 20.0 41.6 27.0 38.0 72.0 38.0 86.0 42.0110.0 55.0110.0 65.0                   000860
  35 7  .10    31000.0                                                                   000870
  0.0 65.0  0.0 51.0 27.0 43.5 43.0 42.2 53.0 35.0100.0 33.0110.0 65.0                   000880
  36 7  .10    32910.0                                                                   000890
 -2.0 65.0 -2.0 50.0 24.0 37.5 35.0 35.6 81.0 35.6104.0 51.0104.0 65.0                   000900
  37 8  .10    34770.0                                                                   000910
  4.0 65.0  4.0 50.0 22.0 34.0 70.0 34.0100.0 43.0120.0 45.0135.0 50.0135.0 65.0000920
  38 8  .10    35800.0                                                                   000930
  0.0 65.0  0.0 50.0  5.0 43.5 30.0 40.5 34.0 35.5 92.0 35.5130.0 44.0130.0 65.0000940
  39 8  .10    36450.0                                                                   000950
  0.0 65.0  0.0 50.0 20.0 39.0 20.0 32.0100.0 32.0100.0 39.0110.0 50.0110.0 65.0000960
  40 6  .01    36450.0                                                                   000970
 -3.0 65.0 -3.0 50.0 20.0 38.6100.0 38.6110.0 43.5110.0 65.0                             000980
  41 8  .20    36450.0                                                                   000990
  0.0 65.0  0.0 50.0 23.0 38.6 23.0 30.6103.0 30.6103.0 38.6113.0 43.5113.0 65.0001000
  42 7  .20    37150.0                                                                   001010
  0.0 60.0  0.0 40.5 25.0 32.3 30.0 29.0 68.0 29.0 80.0 42.0 80.0 60.0                   001020
  43 7  .10    38100.0                                                                   001030
  0.0 60.0  0.0 44.0 17.0 29.0 58.0 29.0 70.0 37.8 85.0 41.5 85.0 60.0                   001040
  44 7  .10    38980.0                                                                   001050
  0.0 60.0  0.0 40.5 15.0 36.0 27.0 26.0 70.0 27.5 85.0 38.3 85.0 60.0                   001060
  45 6  .10    40800.0                                                                   001070
  0.0 58.0  0.0 42.0 10.0 25.5120.0 25.5130.0 42.0130.0 58.0                             001080
  46 4  .04    40800.0                                                                   001090
  0.0 58.0  0.0 24.5 73.0 24.5 73.0 58.0                                                 001100
  47 6  .05    40800.0                                                                   001110
  0.0 58.0  0.0 42.0 10.0 24.5120.0 24.5130.0 42.0130.0 58.0                             001120
  48 6  .05    42800.0                                                                   001130
  0.0 58.0  0.0 35.0 30.0 25.6 85.0 25.5100.0 34.0100.0 58.0                             001140
  49 8  .05    44800.0                                                                   001150
  0.0 56.0  0.0 43.0 11.0 32.0 25.0 32.0 30.0 23.6105.0 25.0145.0 32.0145.0 55.0001160
  50 6  .20    46800.0                                                                   001170
  0.0 50.0  0.0 36.5 10.0 20.4 65.0 20.6105.0 34.0105.0 50.0                             001180
  51 6  .10    47920.0                                                                   001190
  0.0 50.0  0.0 35.0 10.0 17.5 80.0 17.5 90.0 35.0 90.0 50.0                             001200
  52 8  .20    47920.0                                                                   001210
 25.0 50.0 25.0 34.0  0.0 21.5  0.0 16.5 70.0 16.5 70.0 21.5 45.0 34.0 45.0 50.0001220
  53 6  .20    47920.0                                                                   001230
  0.0 50.0  0.0 35.0 10.0 17.5 80.0 17.5 90.0 35.0 90.0 50.0                             001240
  54 8  .25    49000.0                                                                   001250
-10.0 48.0 10.0 28.0 38.0 23.0 47.0 16.0102.0 16.0110.0 21.5118.0 21.6155.0 48.0001260
  55 8  .40    50230.0                                                                   001270
  0.0 48.0  0.0 30.0 10.0 18.7 38.0 10.8 89.0 11.8 94.0 20.5120.0 30.0120.0 48.0001280
  56 7  .40    52465.0                                                                   001290
  0.0 48.0  0.0 30.0 20.0  8.2100.0  8.5105.0 23.5116.0 23.5116.0 48.0                   001300
  57 4  .20    54700.0                                                                   001310
  0.0 48.0  0.0 11.0170.0 11.0170.0 48.0                                                 001320
  58 5  .01    54700.0                                                                   001330
  0.0 48.0  0.0 17.0 10.0 17.3175.0 17.3175.0 48.0                                       001340
```

LISTING OF PROGRAM FINDIF

```
      PROGRAM FINDIF      73/73    OPT=0    TRACE                    FTN 4.0+P355        01/28/74  11.35.12.        PAGE    1

            PROGRAM FINDIF(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7=P000100
           1UNCH)                                                             000110
            DIMENSION H(200,2),Q(200,2),WL(20),TWL(20),X(200),BOT(200)       000120
            DIMENSIONHYD(200,2),DTHYD(2),PEAK(2),IEND(2)                      000130
            DIMENSION HDP(200,2),PEHYD(2),IEDP(2)                            000140
            DIMENSION QI(20),TI(20)                                          000150
      C     ---------------------------------------------------------------- 000160
      C     THIS PROGRAM ROUTES A FLOOD USING AN EXPLICIT FINITE DIFFERENCE   000170
      C     TECHNIQUE    A DICTIONARY OF THE VARIABLES FOLLOWS.               000180
      C     ---------------------------------------------------------------- 000190
      C     BOT(I) ELEVATIONS OF THE CHANNEL BOTTOMS AT THE SECTIONS          000200
      C     CEL  THE CELERITY AT BANK FULL NORMAL FLOW                       000210
      C     DT TIME INCREMENT OF EACH ITERATION                              000220
      C     DTMAX  THE MAXIMUM TIME STEP AT BANK FULL CONDITIONS             000230
      C     DX  THE LENGTH OF A REACH                                        000240
      C     H(I,J) WATER LEVELS AT THE SECTIONS                              000250
      C      NREACH  THE NUMBER OF REACHES IN THE CHANNEL                    000260
      C     NSECS NUMBER OF SECTIONS IN THE CHANNEL                          000270
      C     Q(I,J) FLOW RATES AT THE SECTIONS                                000280
      C     QIN  THE VOLUME OF THE WATER WHICH FLOWED INTO THE CHANNEL       000290
      C     QOUT  THE VOLUME OF WATER WHICH FLOWED OUT OF THE CHANNEL        000300
      C     RC  ROUGHNESS COEFFICIENT  MANNINGS N                            000310
      C     TIMEI  START TIME                                                000320
      C     TIMEF  THE FINISH TIME                                           000330
      C     TWL(I)  THE FLOWRATES OF THE POINTS IN A STAGE DISCHARGE CURVE    000340
      C     WL(I)  THE WATER LEVELS OF THE  POINTS IN STAGE DISCHARGE CURVE   000350
      C     X(I) DISTANCE FROM THE OUTFALL IN FEET                           000360
      C      XTOTAL  THE TOTAL LENGTH OF THE CHANNEL                         000370
      C                                                                      000380
      C     READ PROBLEM VARIABLES                                           000390
            READ(5,501) XTOTAL,T,RC,SLOPE,DEPTH                              000400
            READ(5,502) NREACH,DT,TIMEF                                      000410
            READ(5,503) ITEST                                                000420
            READ(5,506) NPU,NPU2                                             000430
            READ(5,503) NSTAGE                                               000440
            READ(5,506)NSEQ1,NSEQ2                                           000450
            READ(5,506)NSEU1,NSED2                                           000460
            READ(5,504) NQPTS                                                000470
            READ(5,505) (QI(I),TI(I),I=1,NQPTS)                              000480
      C                                                                      000490
      C     SET PROBLEM VARIABLES                                            000500
             PEAK(1)=PEAK(2)=PEHYD(1)=PEHYD(2)=1.                            000510
            FMOMI=FMOMO=0.0                                                  000520
            G=32.2                                                           000530
            A=T*DEPTH                                                        000540
            P=T+2.*DEPTH                                                     000550
            CALL MANNGO(A,P,SLOPE,RC,G,QMAX)                                 000560
      C     SET UP THE INFLOW HYDROGRAPH                                     000570
            DO 180 I=1,NQPTS                                                 000580
        180 QI(I)=QI(I)*QMAX                                                 000590
            DX=XTOTAL/FLOAT(NREACH)                                          000600
            NSECS=NREACH+1                                                   000610
            QIN=QOUT=0.                                                      000620
            NPWL=20                                                          000630
            IEND(1)=IEND(2)=IEDP(1)=IEDP(2)=0                                000640
            TIMEI=0.                                                         000650
            TIME=0.                                                          000660
```

LISTING - FINDIF/2


```
      CEL=SQRT(G*DEPTH)+QMAX/(DEPTH*T)
      DTMAX=DX/CEL
      DT=DT*DTMAX
C
C     DERIVE STAGE DISCHARGE CURVE
      DH=DEPTH/FLOAT(NPWL-1)
      WL(1)=0.
      TWL(1)=0.0
      DO 7 I=2,NPWL
      WL(I)=WL(I-1)+DH
      A=T*FLOAT(I-1)*DH
      P=T+2.*FLOAT(I-1)*DH
      CALL MANNGA(A,P,SLOPE,RC,G,QX)
      CALL PLOTPT(QX,WL(I),4)
    7 TWL(I)=QX
C
C     SET INITIAL CONDITIONS AT STEADY STATE
      CALL INTER1(QT,TI,NQPTS,TIME,Q(1,2))
      BOT(1)=XTOTAL*SLOPE+100.
      X(1)=0.0
      Q(1,1)=Q(1,2)
      CALL INTER1(WL,TWL,NPWL,Q(1,2),FLOWD)
      H(1,1)=H(1,2)=BOT(1)+FLOWD
      DROP=DX*SLOPE
      DO 6 I=2,NSECS
      Q(I,1)=Q(I,2)=Q(1,2)
      BOT(I)=BOT(I-1)-DROP
      X(I)=X(I-1)+DX
    6 H(I,1)=H(I,2)=BOT(I)+FLOWD
C     CALCULATE INTIAL STORAGE IN THE CHANNEL
      STORE1=XTOTAL*T*FLOWD
C     CALCULATE NUMBER OF STEPS BETWEEN PRINT OUT
      KSTEP=TIMEF/(118.*DT) +0.5
      IF(KSTEP.EQ.0) KSTEP=1
      ICOUNT=KSTEP
      DTHYD(1)=DTHYD(2)=FLOAT(KSTEP)*DT/3600.
C     WRITE THE CONDITIONS FOR THIS RUN
      WRITE(6,2001) XTOTAL,DEPTH,SLOPE,RC,QMAX,CEL
      WRITE(6,2008) T
      WRITE(6,2002) NREACH,DT,DTMAX
      WRITE(6,2006) ITEST
      WRITE(6,2004) Q(1,2)
      DO 100 I=1,NSECS
  100 WRITE(6,2005)X(I),BOT(I),H(I,2)
      IF(NPU2.EQ.1) WRITE(7,3007) ITEST
      IF(NPU.LE.0) WRITE(6,2012)
      IF(NPU2.LE.0) WRITE(6,2013)
C     INCREMENT TIME
      TIME=TIME+DT/2.
C
C     START CALCULATIONS OF Q
   60 CALL INTER1(QT,TI,NQPTS,TIME,QUS)
      Q(1,2)=QUS
      DO 20 I=2,NSECS
      HA=(H(I,2)+H(I-1,2)-BOT(I)-BOT(I-1))/2.
      DHDX=(H(I,2)-H(I-1,2))/(X(I)-X(I-1))
      FAC1=Q(I,1)/(G*T*T*HA**3.)*DHDX
```

LISTING - FINDIF/3


```
      FAC2=(H(I-1,2)-H(I-1,1)+H(I,2)-H(I,1))/(G*T*HA*HA*4.*DT)
      FAC3=Q(I,1)*(BOT(I-1)-BOT(I))/(G*T*T*HA**3.)/(X(I)-X(I-1))
      FAC4=1./(G*T*HA*DT)
      FAC5=ABS(Q(I,1))*RC*RC*(T+2.*HA)**(4./3.)/(1.49*1.49*(T*HA)**(10.
     */3.))
   20 Q(I,2)=(DHDX-Q(I,1)*(FAC2+FAC4))/(FAC1+FAC2+FAC3-FAC4-FAC5)
      QIN=QIN+(Q(1,1)+Q(1,2))/2.0
      QOUT=QOUT+(Q(NSECS,1)+Q(NSECS,2))/2.
      FMOMI=FMOMI+Q(NSEQ1,2)*TIME/3600.
      FMOMO=FMOMO+Q(NSEQ2,2)*TIME/3600.
C     TRANSFER INFORMATION IN THE H MATRIX
      DO 30 I=1,NSECS
   30 H(I,1)=H(I,2)
C     INCREMENT TIME
      TIME=TIME+DT/2.
C     DO THE CALCULATIONS FOR H
      CALL INTFH1(WL,TWL,NPWL,Q(NSECS,2),WLDS)
      H(NSECS,2)=WLDS+100.0
      DO 40 I=2,NSECS
      K=NSECS+2-I
      J=K-1
      IF(I.EQ.NSECS) DX=X(2)-X(1)
      IF(I.LT.NSECS) DX=(X(K)-X(NSECS-I))/2.
   40 H(J,2)=H(J,1)-DT/DX*(Q(K,2)-Q(J,2))/T
C
C     PRINT THE VALUES IF APPROPRIATE
      IF(TIME.LT.0.) GO TO 80
      IF(ICOUNT.NE.KSTEP) GO TO 170
      INTER=IEND(1)+1
      ICOUNT=0
   70 HYD(INTER,1)=(Q(NSEQ1,1)+Q(NSEQ1,2))/(QMAX*2.)
      HYD(INTER,2)=(Q(NSEQ2,1)+Q(NSEQ2,2))/(QMAX*2.)
      HDP(INTER,1)=(H(NSED1,1)-BOT(NSED1))/DEPTH
      HDP(INTER,2)=(H(NSED2,1)-BOT(NSED2))/DEPTH
      IEND(1)=IEND(2)=IEDP(1)=IEDP(2)=IEDP(2)+1
      HX=H(NSTAGE,1)-BOT(NSTAGE)
      QX=(Q(NSTAGE,1)+Q(NSTAGE,2)+Q(NSTAGE+1,1)+Q(NSTAGE+1,2))/4.
      SF=QX*QX*RC*RC*(T+2.*HX)**(4./3.)/(1.49*1.49*(T*HX)**(10./3.))
      DQ=(Q(NSTAGE,2)-Q(NSTAGE,1)+Q(NSTAGE+1,2)-Q(NSTAGE+1,1))/2.
      DV=DQ/(G*HX*T*DT)
      CALL PLOTPT(QX,HX,9)
      IF(NPU2) 170,170,150
  150 WRITE(7,3006) HX,QX,SF,DV
  170 ICOUNT=ICOUNT+1
C     TRANSFER INFORMATION IN THE Q MATRIX
   80 DO 50 I=1,NSECS
   50 Q(I,1)=Q(I,2)
C     INCREMENT TIME
      TIME=TIME+DT/2.
      IF(TIME.LE.TIMEF) GO TO 60
      TIMEI=TIMEI/3600.
      CALL HPLOT(HYD,DTHYD,IEND,PEAK,TIMEI)
      WRITE(6,2010) NSEQ1,NSEQ2
      CALL HPLOT(HDP,DTHYD,IEDP,PEHYD,TIMEI)
      WRITE(6,2011) NSED1,NSED2
C     COMPUTE STORAGE IN THE CHANNEL AT FINISH OF ROUTING
      STORE2=0.
```

LISTING - FINDIF / 4


```
      DO 90 I=2,NSECS
   90 STORE2=STORE2+H(I,2)-BOT(I)
      STORE2=STORE2+(H(1,2)-BOT(1)-H(NSECS,2)+BOT(NSECS))/2.0
      STORE2=STORE2*DX*T
      QIN=QIN*DT
      QOUT=QOUT*DT
C     CALCULATE CENTROIDS OF THE HYDROGRAPHS
      FMOMI=FMOMI*DT/QIN
      FMOMO=FMOMO*DT/QOUT
      ERROR=(STORE2-STORE1+QOUT-QIN)/QIN*100.
      WRITE(6,2003) ERROR
      WRITE(6,2007) NSEQ1,FMOMI,NSEQ2,FMOMO
      IF(NPU) 120,120,110
  110 WRITE(7,3001)DEPTH,T,SLOPE,RC
      IEN=IEND(1)
      DO 160 I=1,IEN
      HYD(I,1)=HYD(I,1)*QMAX
      HYD(I,2)=HYD(I,2)*QMAX
      HDP(I,1)=HDP(I,1)*DEPTH
  160 HDP(I,2)=HDP(I,2)*DEPTH
      QX=QIN
      NSEQ=NSEQ1
      NSED=NSED1
      DO 130 J=1,2
      WRITE(7,3002) J,BOT(J)
      DO 140 I=1,NPWL
      A=T*FLOAT(I-1)*DH
      E=BOT(J)+WL(I)
       WRITE(7,3003)E,A,TWL(I)
  140 CONTINUE
      WRITE(7,3004) J,NSEQ,DTHYD(J),QMAX,QX,IEND(J)
      WRITE(7,3005) (HYD(I,J),I=1,IEN)
      WRITE(7,3008) NSED,DTHYD(J),IEN
      WRITE(7,3009) (HDP(I,J),I=1,IEN)
      QX=QOUT
      NSEQ=NSEQ2
      NSED=NSED2
  130 CONTINUE
  120 CALL PLOTPT(0.,0.,9)
      CALL OUTPLT
      WRITE(6,2009) NSTAGE
      STOP
C
C     FORMAT STATEMENTS
C
C     FORMATS 500+  INPUT FORMATS
C     FORMATS 200 + OUTPUT (PRINTER) FORMATS
C     FORMATS 3000+ OUTPUT (PUNCH) FORMATS
C
  501 FORMAT(F10.1,F10.2,2F10.8,F10.2)
  502 FORMAT(I3,2F10.2)
  503 FORMAT(I3)
  504 FORMAT(I3)
  505 FORMAT(F10.2,F10.0)
  506 FORMAT(2I3)
 2001 FORMAT(1H1,///,20X,*FLOOD ROUTING USING THE EXPLICIT FINITE *,
     1  *DIFFERENCE METHOD*,///,25X,*CHANNEL PROPERTIES AND FLOW  *,
```

LISTING - FINDIF/5


```
                 2CONDITIONS*,//,35X,*TOTAL LENGTH        =*,F12.1,
                 3  * FT*,/,35X,*MAXIMUM DEPTH         =*,F12.1,* FT*,/,35X,*SLOPE*,
                 4   14X,*=*,F12.5,* FT/FT*,/,35X,*MANNINGS N*,9X,*=*,F12.5,/,35X,
                 5 *MAXIMUM FLOW*,7X,*=*,F12.1,* CUSECS*,/,35X,*CELERITY AT MAX
                 6*F12.2,*FT/SEC*)
            2002 FORMAT(1X,//,25X,*PROBLEM VARIABLES*,/,
                 1 35X,*NUMBER OF REACHES   =*9X,I3,/,35X,*TIME INCREMENT*,5X,*=*,
                 2 F12.1,* SECONDS*,/,35X,*MAX TIME INCREMENT =*,F12.1,* SECONDS*)
            2003 FORMAT(//,35X,*THE ERROR IN THE VOLUME WAS=*,F10.3,* PER CENT*)
            2004 FORMAT(//,25X,*INVERT ELEVATIONS AND INITIAL WATER LEVELS AT EACH
                 1SECTION*,/,37X,*INITIAL FLOW RATE=*,F12.1,* CUSECS*,/,37X,
                 2*CHAINAGE*,9X,*INVERT ELEVATION*,13X,*WATER LEVEL*)
            2005 FORMAT(35X,F12.1,2(12X,F12.2))
            2006 FORMAT(35X,*TEST NUMBER             =*,9X,I3)
            2007 FORMAT(35X,*CENTROID OF HYDROGRPH NO*,I3,*=*,F8.2,* HRS*,/,
                 1            35X,*CENTROID OF HYDROGRPH NO*,I3,*=*,F8.2,* HRS*)
            2008 FORMAT(35X,*WIDTH OF CHANNEL     =*,F12.1,* FT*)
            2009 FORMAT(/,35X,*THIS THE STAGE-DISCHARGE CURVE AT SECTION *,I3,
                 1/,40X,1H*,* UNIFORM FLOW*,/,40X,*. UNSTEADY FLOW*)
            2010 FORMAT(/,35X,*THIS IS A PLOT OF THE RATIO OF THE FLOW TO THE MAX*,
                 1* FLOW VS TIME*,/,35X,*000 AT SECTION NO=*,I4,6X,*... AT SECTION*,
                 2* NO=*,I4)
            2011 FORMAT(/,35X,*THIS IS A PLOT OF THE RATIO OF DEPTH TO MAX DEPTH*,
                 1* VS TIME*,/,35X,*000 AT SECTION NO=*,I4,6X,*... AT SECTION*,
                 2* NO=*,I4)
            2012 FORMAT(/,35X,*NO HYDROGRAPHS WERE PUNCHED*)
            2013 FORMAT(/,35X,*THE STAGE DISCHARGE CURVE WAS NOT PUNCHED*)
            3001 FORMAT(2X,*DEPTH=*,F7.2,* WIDTH=*,F7.2,* SLOPE=*,F10.8,*  RC=*,F10
                 1.8)
            3002 FORMAT(*STORE RATING CURVE  ID=*,I1,* VS=1  MIN E=*,F8.2)
            3003 FORMAT (20X,3(F14.3,1X))
            3004 FORMAT(*RECALL HYD*,11X,*ID=*,I1,*   HYD NO=*,I3,*   DT=*,F9.6,
                 1* HRS   QA=1.0*,/,21X,*PEAK=*,F7.1,*   RO=*,F12.0,*   NO PTS=*
                 I,I3)
            3005 FORMAT(25(/,25X,6(1X,F8.0),1X))
            3006 FORMAT(20X,2F15.3,2F15.9)
            3007 FORMAT(* TEST NUMBER =*,I3)
            3008 FORMAT(* DEPTH VS TIME SEC NO=*,I3,* DT=*,F9.6,* HRS*,
                 1 * NO PTS=*,I3)
            3009 FORMAT(10(1X,F7.2))
                 END
```

LISTING OF PROGRAM KINDIF

```
      PROGRAM KINDIF(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE7=PUNCH)
      DIMENSION H(200),Q(200),WL(20),TWL(20),X(200),BOT(200)
      DIMENSION AL(20),FUN(20),TWL2(20),FUN2(20),QINIT(200)
      DIMENSIONHYD(200,2),DTHYD(2),PEAK(2),IEND(2)
      DIMENSION HOP(200,2),PEHYD(2),IEDP(2)
      DIMENSION QI(20),TI(20)
      DIMENSION QLT(1),TLT(1),GUN(20)
C     ----------------------------------------------------------------
C     THIS PROGRAM ROUTES A FLOOD USING A KINEMATIC WAVE METHOD
C     A DICTIONARY OF THE VARIABLES FOLLOWS
C     ----------------------------------------------------------------
C     BOT(I) ELEVATIONS OF THE CHANNEL BOTTOMS AT THE SECTIONS
C     CEL  THE CELERITY AT BANK FULL NORMAL FLOW
C     DT TIME INCREMENT OF EACH ITERATION
C     DTMAX  THE MAXIMUM TIME STEP AT BANK FULL CONDITIONS
C     DX  THE LENGTH OF A REACH
C     H(I,J) WATER LEVELS AT THE SECTIONS
C      NREACH  THE NUMBER OF REACHES IN THE CHANNEL
C     NSECS NUMBER OF SECTIONS IN THE CHANNEL
C     Q(I,J) FLOW RATES AT THE SECTIONS
C     QIN  THE VOLUME OF THE WATER WHICH FLOWED INTO THE CHANNEL
C     QOUT  THE VOLUME OF WATER WHICH FLOWED OUT OF THE CHANNEL
C     RC  ROUGHNESS COEFFICIENT  MANNINGS N
C     TIMEI  START TIME
C     TIMEF  THE FINISH TIME
C     TWL(I)  THE FLOWRATES OF THE POINTS IN A STAGE DISCHARGE CURVE
C     WL(I)  THE WATER LEVELS OF THE  POINTS IN STAGE DISCHARGE CURVE
C     X(I) DISTANCE FROM THE OUTFALL IN FEET
C      XTOTAL  THE TOTAL LENGTH OF THE CHANNEL
C
C     READ PROBLEM VARIABLES
      READ(5,501) XTOTAL,T,RC,SLOPE,DEPTH
      READ(5,502) NREACH,DT,TIMEF
      READ(5,503) ITEST
      READ(5,506) NPU,NPU2
      READ(5,506)NSE01,NSE02
      READ(5,506)NSE01,NSE02
      READ(5,504) NQPTS
      READ(5,505) (QI(I),TI(I),I=1,NQPTS)
C
C     SET PROBLEM VARIABLES
      TLT(1)=-1.0
      G=32.2
      A=T*DEPTH
      P=T+2.*DEPTH
      CALL MANNGQ(A,P,SLOPE,RC,G,QMAX)
C     SET UP THE INFLOW HYDROGRAPH
      NPTS=TIMEF/DT
      DO 180 I=1,NQPTS
  180 QI(I)=QI(I)*QMAX
      DX=XTOTAL/FLOAT(NREACH)
      NSECS=NREACH+1
      NPWL=20
      IEND(1)=IEND(2)=IEDP(1)=IEDP(2)=NPTS
      TIME=0.
      TIMEI=0.0
      CEL=SQRT(G*DEPTH)+QMAX/(DEPTH*T)
      DTMAX=DX/CEL
      DO 190 I=1,NPTS
      TIMEQ=FLOAT(I-1)*DT
```

LISTING - KINDIF/2

```
      CALL INTERP1(QI,TI,NQPTS,TIMEQ,QINIT(I))
  190 CONTINUE
C
C     DERIVE STAGE DISCHARGE CURVE
      DH=DEPTH/FLOAT(NPWL-1)
      AL(1)=0.0
      WL(1)=0.
      TWL(1)=0.0
      DO 7 I=2,NPWL
      WL(I)=WL(I-1)+DH
      A=T*FLOAT(I-1)*DH
      AL(I)=A*DX
      P=T+2.*FLOAT(I-1)*DH
      CALL MANNGQ(A,P,SLOPE,RC,G,QX)
    7 TWL(I)=QX
C
C     SET INITIAL CONDITIONS AT STEADY STATE
      CALL INTEP1 (QI,TI,NQPTS,TIME,Q(1))
      BOT(1)=XTOTAL*SLOPE+100.
      X(1)=0.0
      CALL INTEP1 (WL,TWL,NPWL,Q(1),FLOWD)
      DROP=DX*SLOPE
      H(1)=BOT(1)+FLOWD
      DO 6 I=2,NSECS
      BOT(I)=BOT(I-1)-DROP
      X(I)=X(I-1)+DX
    6 H(I)=BOT(I)+FLOWD
C     CALCULATE INTIAL STORAGE IN THE CHANNEL
      STORE1=XTOTAL*T*FLOWD
      QTHYD(1)=QTHYD(2)=DT/3600.
      DO 200 IBETA=1,5
      BETA=1.0-FLOAT(IBETA-1)/4.0
      DO 210 IALFA=1,5
      ALF=FLOAT(IALFA-1)/4.0
C     WRITE THE CONDITIONS FOR THIS RUN
      WRITE(6,2001) XTOTAL,DEPTH,SLOPE,RC,QMAX,CEL
      WRITE(6,2008) T
      WRITE(6,2012) NREACH,DT,DTMAX
      QIN=QOUT=STORE2=TIME=TIMEI=FMOMI=FMOMO=0.0
      PEAK(1)=PEAK(2)=PEHYD(1)=PEHYD(2)=1.0
      PEAK1=PEAK2=0.0
      WRITE(6,2006) ITEST
      DO 220 INFLOW=1,NPTS
  220 Q(INFLOW)=QINIT(INFLOW)
      WRITE(6,2014) ALF,BETA
      WRITE(6,2014) Q(1)
      DO 100 I=1,NSECS
  100 WRITE (6,2005) X(I),BOT(I),H(I)
      IF(NPU2.EQ.1) WRITE(7,3007) ITEST
      IF(NPU.LE.0) WRITE(6,2012)
      IF(NPU2.LE.0) WRITE(6,2013)
      DO 30 I=1,NPWL
      FUN2(I)=AL(I)
   30 TWL2(I)=TWL(I)
      NPWL2=NPWL
      CALL KINFUN(ALF,BETA,DT,AL,TWL,NPWL,FUN2,TWL2,NPWL2,GUN,FUN)
      CALL INTER1(AL,TWL,NPWL,Q(NPTS),FAC4)
      STORE2=STORE2+FAC4*ALF
      DO 60 I=1,NREACH
      H(1)=Q(1)
      CALL KINRUT (FUN,TWL,NPWL,GUN,TWL2,NPWL2,QLT,TLT,1,DX,DT,NPTS,Q,H)
```

LISTING - KINDIF/3

```
      IF(I.NE.NSEQ1) GO TO 22
      DO 23 II=1,NPTS
      CALL INTER1 (WL,TWL,NPWL,Q(II),HD)
   23 HDP(II)=HD/DEPTH
   22 IF ((I+1).NE.NSEQ2) GO TO 24
      DO 25 II=1,NPTS
      CALL INTER1 (WL,TWL,NPWL,H(II),HD)
   25 HDP(II,2)=HD/DEPTH
   24 CONTINUE
      IF(I.NE.NSEQ1) GO TO 20
      DO 21 II=1,NPTS
      QIN=QIN+Q(II)
      FMOMO=FMOMO +Q(II)*FLOAT(II-1)*DT/3600.
      QQQ=Q(II)/QMAX
      IF(PEAK1.GT.QQQ) GO TO 21
      TIME1=FLOAT(II-1)*DT/3600.0
      PEAK1=QQQ
   21 HYD(II,1)=QQQ
   20 IF((I+1).NE.NSEQ2) GO TO 40
      DO 41 II=1,NPTS
      QOUT=QOUT+H(II)
      FMOMI=FMOMI+Q(II)*DT*FLOAT(II-1)/3600.
      QQQ=H(II)/QMAX
      IF(PEAK2.GT.QQQ) GO TO 41
      TIME2=FLOAT(II-1)*DT/3600.0
      PEAK2=QQQ
   41 HYD(II,2)=QQQ
   40 CONTINUE
      CALL INTER1 (FUN2,TWL2,NPWL,H(NPTS),FAC4)
      STORE2=STORE2+FAC4
      DO 50 II=1,NPTS
   50 Q(II)=H(II)
   60 CONTINUE
      CALL INTER1(FUN2,TWL2,NPWL,H(NPTS),FAC4)
      STORE2=STORE2-FAC4*ALF
      TIMEI=TIMEI/3600.
      CALL HPLOT(HYD,DTHYD,IEND,PEAK,TIMEI)
      WRITE(6,2010) NSEQ1,NSEQ2
      CALL HPLOT(HDP,DTHYD,IEDP,PEHYD,TIMEI)
      WRITE(6,2011) NSEQ1,NSEQ2
      QIN=QIN*DT
      QOUT=QOUT*DT
C     CALCULATE CENTROIDS OF THE HYDROGRAPHS
      FMOMI=FMOMI*DT/QIN
      FMOMO=FMOMO*DT/QOUT
      ERROR=(STORE2-STORE1+QOUT-QIN)/QIN*100.
      WRITE(6,2003) ERROR
      WRITE(6,2007) NSEQ1,FMOMO,NSEQ2,FMOMI
      WRITE(6,2015) NSEQ1,PEAK1,TIME1
      WRITE(6,2015) NSEQ2,PEAK2,TIME2
      IF(NPU) 120,120,110
  110 WRITE(7,3001)DEPTH,T,SLOPE,RC
      IEN=IEND(1)
      DO 160 I=1,IEN
      HYD(I,2)=HYD(I,2)*QMAX
  160 CONTINUE
      QX=QIN
      NSEQ=NSEQ2
      NSEQ=NSEQ2
      J=2
  140 CONTINUE
```

LISTING - KINDIF/4

```
      WRITE(7,3004) J,NSEQ,DTHYD(J),QMAX,QX,IEND(J)
      WRITE(7,3011)ALF,BETA
      WRITE(7,3005) (HYD(I,J),I=1,IEN)
      IFINIS=-1
      WRITE(7,3010) IFINIS
      QX=QOUT
      NSEQ=NSEQ2
      NSEQ=NSEQ2
  130 CONTINUE
  120 CONTINUE
  210 CONTINUE
  200 CONTINUE
      STOP
C
C     FORMAT STATEMENTS
C
C     FORMATS 500+  INPUT FORMATS
C     FORMATS 2000+ OUTPUT (PRINTER) FORMATS
C     FORMATS 3000+ OUTPUT (PUNCH) FORMATS
C
  501 FORMAT(F10.1,F10.2,2F10.8,F10.2)
  502 FORMAT(I3,2F10.2)
  503 FORMAT(I3)
  504 FORMAT(I3)
  505 FORMAT(F10.2,F10.0)
  506 FORMAT(2I3)
 2001 FORMAT(1H1,///,20X,*FLOOD ROUTING USING THE KINEMATIC WAVE*,
     1* METHOD*,///,25X,*CHANNEL PROPERTIES AND FLOW *,           *
     2CONDITIONS*,//,35X,*TOTAL LENGTH          =*,F12.1,
     3  * FT*,/,35X,*MAXIMUM DEPTH         =*,F12.1,* FT*,/,35X,*SLOPE*,
     4  14X,*=*,F12.5,* FT/FT*,/,35X,*MANNINGS N*,9X,*=*,F12.5,/,35X,
     5 *MAXIMUM FLOW*,7X,*=*,F12.1,* CUSECS*,/,35X,*CELERITY AT MAX    =
     6*F12.2,*FT/SEC*)
 2002 FORMAT(1X,//,25X,*PROBLEM VARIABLES*,/,
     1 35X,*NUMBER OF REACHES  =*9X,I3,/,35X,*TIME INCREMENT*,5X,*=*,
     2 F12.1,* SECONDS*,/,35X,*MAX TIME INCREMENT =*,F12.1,* SECONDS*)
 2003 FORMAT(//,35X,*THE ERROR IN THE VOLUME WAS=*,F11.3,* PER CENT*)
 2004 FORMAT(//,25X,*INVERT ELEVATIONS AND INITIAL WATER LEVELS AT EACH
     1SECTION*,/,37X,*INITIAL FLOW RATE=*,F12.1,* CUSECS*,/,37X,
     2*CHAINAGE*,9X,*INVERT ELEVATION*,13X,*WATER LEVEL*)
 2005 FORMAT(35X,F12.1,2(12X,F12.2))
 2006 FORMAT(35X,*TEST NUMBER          =*,9X,I3)
 2007 FORMAT(35X,*CENTROID OF HYDROGRPH NO*,I3,*=*,F8.2,* HRS*,/,
     1          35X,*CENTROID OF HYDROGRPH NO*,I3,*=*,F8.2,* HRS*)
 2008 FORMAT(35X,*WIDTH OF CHANNEL    =*,F12.1,* FT*)
 2010 FORMAT(/,35X,*THIS IS A PLOT OF THE RATIO OF THE FLOW TO THE MAX*,
     1* FLOW VS TIME*,/,35X,*OOO AT SECTION NO=*,I4,6X,*+++ AT SECTION*,
     2* NO=*,I4)
 2011 FORMAT(/,35X,*THIS IS A PLOT OF THE RATIO OF DEPTH TO MAX DEPTH*,
     1* VS TIME*,/,35X,*OOO AT SECTION NO=*,I4,6X,*+++ AT SECTION*,
     2* NO=*,I4)
 2012 FORMAT(/,35X,*NO HYDROGRAPHS WERE PUNCHED*)
 2013 FORMAT(/,35X,*THE STAGE DISCHARGE CURVE WAS NOT PUNCHED*)
 2014 FORMAT(35X,*ALFA*,15X,*=*,F12.3,/,35X,*BETA*,15X,*=*,
     1F12.3)
 2015 FORMAT(35X,*HYDROGRAPH NO*,I3,* PEAK=*,F6.4,* TIME=*,F6.3,* HRS*)
 3001 FORMAT(2X,*DEPTH=*,F7.2,* WIDTH=*,F7.2,* SLOPE=*,F10.8,*  RC=*,F10
     1.8)
 3002 FORMAT(*STORE RATING CURVE  ID=*,I1,* VS=1  MIN E=*,F8.2)
 3003 FORMAT (20X,3(F14.3,1X))
 3004 FORMAT(*RECALL HYD*,11X,*ID=*,I1,*   HYD NO=*,I3,*   DT=*,F9.6,
```

LISTING - KINDIF/5

```
     1* HRS    DA=1.0*,/,21X,*PEAK=*,F7.1,*   RO=*,F12.0,*   NO PTS=*
     1,I3)
3005 FORMAT(25(/,25X,6(1X,F8.0),1X))
3007 FORMAT(* TEST NUMBER =*,I3)
3008 FORMAT(* DEPTH VS TIME SEC NO=*,I3,* DT=*,F9.6,* HRS*,
    1 * NO PTS=*,I3)
3010 FORMAT(I3)
3011 FORMAT(F12.3,F12.3)
3009 FORMAT(10(1X,F7.2))
     END
```

LISTING OF SUBROUTINE KINFUN

```
      SUBROUTINE KINFUN      73/73    OPT=0    TRACE                FTN 4.0+P355       09/15/73  14.55.40.        PAGE     1

                  SUBROUTINE KINFUN(ALF,BETA,DT,SUS,QUS,NUS,SDS,QDS,NDS,GUS,FDS)     004180
                  DIMENSION SUS(NUS),QUS(NUS),SDS(NDS),QDS(NDS),GUS(NUS),FDS(NDS)    004190
                  DO 1 I=1,NUS                                                        004200
                1 GUS(I)=BETA*QUS(I)-ALF*SUS(I)/DT                                    004210
                  DO 2 I=1,NDS                                                        004220
                2 FDS(I)=BETA*QDS(I)+(1.0-ALF)*SDS(I)/DT                              004240
                  RETURN                                                              004250
                  END                                                                 004260
```

LISTING OF SUBROUTINE KINRUT

```
      SUBROUTINE KINRUT      73/73    OPT=0    TRACE                 FTN 4.0+P355        08/15/73  14.55.38.        PAGE     1

                       SUBROUTINE KINRUT (FDS,QDS,NDS,GUS,QUS,NUS,QLT,TLT,NLT,DX,DT,NIN,  003980
                      1QIN,QOUT)                                                          003990
                       DIMENSION FDS(NDS),GUS(NUS),QDS(NDS),QLT(NLT),QUS(NUS),QIN(NIN),   004000
                      1QOUT(NIN),TLT(NLT)                                                 004010
                C      QOUT(1) MUST BE DEFINED PRIOR TO CALLING THE SUBROUTINE.           004020
                       CALL INTER1 (FDS,QDS,NDS,QOUT(1),FAC3)                             004030
                       CALL INTER1 (GUS,QUS,NUS,QIN(1),GAC1)                              004040
                       DO 1 I=2,NIN                                                       004050
                       CALL INTER1 (GUS,QUS,NUS,QIN(I),GAC2)                              004060
                       QSIDE=0.0                                                          004070
                       IF (TLT(1).LT.0.0) GO TO 2                                         004080
                       TIME=FLOAT(I-1)*DT                                                 004090
                       CALL INTER1 (QLT,TLT,NLT,TIME,QSIDE)                               004100
                       QSIDE=QSIDE*DX                                                     004110
                     2 FAC4=FAC3-GAC1+GAC2-QOUT(I-1)+QIN(I-1)+QSIDE                       004120
                       CALL INTER1 (QDS,FDS,NDS,FAC4,QOUT(I))                             004130
                       FAC3=FAC4                                                          004140
                     1 GAC1=GAC2                                                          004150
                       RETURN                                                             004160
                       END                                                                004170
```

LISTING OF SUBROUTINE HPLOT

```
SUBROUTINE HPLOT        73/73    OPT=0    TRACE                FTN 4.0+P355        01/28/74  11.35.18.        PAGE    1

                SUBROUTINE HPLOT(OCFS,DT,IEND,PEAK,TIME)
        C       THIS SUBROUTINE PLOTS EITHER 1 OR 2 HYDROGRAPHS ON A SET OF AXIS
                DIMENSION OCFS(200,2),DT(2),IEND(2),PEAK(2),CFS(120),DATA(2)
                WRITE(6,100)
            100 FORMAT(1H1)
                ID1=1
                ID2=2
                DATA ZERO,PLUS,BLANK,DASH,DOT/1H0,1H+,1H ,1H-,1H./
                MRTO = 1
                XMRTO = 1.
                MAX = 118
                J = 1
        C       ARE THERE 1 OR 2 HYDROGRAPHS
                IF(ID2) 27, 27, 28
        C       DETERMINE HIGHEST PEAK IF 2 HYDROGRAPHS
             27 QMAX = PEAK(ID1)
                GO TO 37
             28 IF(PEAK(ID1) - PEAK(ID2)) 29, 29, 30
             29 QMAX = PEAK(ID2)
                GO TO 31
             30 QMAX = PEAK(ID1)
        C       IF 2 HYDROGRAPHS DETERMINE LARGEST DT AND INTERPOLATE OTHER
        C       HYDROGRAPH IF NECESSARY
             31 IF(DT(ID1) - DT(ID2)) 32, 33, 34
             32 L = ID1
                K = ID2
                GO TO 35
             34 L = ID2
                K = ID1
             35 M = IEND(L)
                TID = DT(K)
                TIDH = 0.
                DO 75 I = 2, M
                TIDH = TIDH + DT(L)
                IF(TID - TIDH) 76, 77, 75
             77 J = J + 1
                CFS(J) = OCFS(I,L)
                TID = TID + DT(K)
                GO TO 75
             76 J = J + 1
                CFS(J) = OCFS(I-1,L) +((TID - TIDH + DT(L)) / DT(L)) * (OCFS(I,L)
               1- OCFS(I-1,L))
                TID = TID + DT(K)
             75 CONTINUE
                IEND(L) = J
                DT(L) = DT(K)
                DO 78 I = 2, J
             78 OCFS(I,L) = CFS(I)
             33 IF(IEND(ID1) - IEND(ID2)) 37, 37, 38
             37 M = IEND(ID1)
                GO TO 39
             38 M = IEND(ID2)
             39 IF(M - MAX) 45, 45, 64
        C       DETERMINE TIME SCALE
             45 MRTO = MAX / M
                XMRTO = MRTO
                GO TO 4
```

LISTING - HPLOT/2

```
SUBROUTINE HPLOT       73/73    OPT=0    TRACE                    FTN 4.0+P355        01/28/74  11.35.18.       PAGE    2

         64 M = MAX
          4 YSCL = QMAX / 50.
C           PLOT HYDROGRAPHS
            DO 65 I = 1, MAX
         65 CFS(I) = DASH
            WRITE(6,57) QMAX, (CFS(I), I=1,MAX), DOT
         57 FORMAT(1X,F6.2,*        *,119A1)
            Q1 = QMAX
            J1 = 10
            DO 50 J = 1, 50
            IF(J - J1) 67, 68, 67
         68 DO 69 I = 1, MAX
         69 CFS(I) = DASH
            GO TO 71
         67 DO 70 I = 1, MAX
         70 CFS(I) = BLANK
         71 Q2 = Q1 - YSCL
            K = 1
            DO 51 I = 2, M
            K = K + MRTO
            IF(QCFS(I,ID1) - Q1  ) 46, 47, 51
         46 IF(QCFS(I,ID1) - Q2) 51, 51, 47
         47 CFS(K) = ZERO
         51 CONTINUE
            WRITE(6,60) Q1, (CFS(I), I=1,MAX), DOT
            IF(ID2) 52, 52, 53
         53 K = 1
            DO 54 I = 2, M
            K = K + MRTO
            IF(QCFS(I,ID2) - Q1  ) 66, 56, 5
         66 IF(QCFS(I,ID2) - Q2) 5 , 5 , 56
         56 CFS(K) = PLUS
            GO TO 54
          5 CFS(K) = BLANK
         54 CONTINUE
            WRITE(6,3)        (CFS(I),I=1,MAX)
          3 FORMAT(1H+,11X,118A1)
         52 IF(J - J1) 58, 59, 58
         59 J1 = J1 + 10
            WRITE(6,2) Q2
          2 FORMAT(1H+,F6.2)
         58 Q1 = Q2
         60 FORMAT(11X,120A1)
         50 CONTINUE
            CFS(1) = TIME
            DTT = DT(ID1) * 10. / XMRTO
C           PUT TIME ARRAY IN CFS AND WRITE TIME SCALE
            DO 61 I = 2, 12
         61 CFS(I) = CFS(I-1) + DTT
            WRITE(6,62) (CFS(I), I = 1, 12)
         62 FORMAT(6X,12F10.3)
            WRITE(6,63)
         63 FORMAT(49X,*TIME HOURS*///)
          6 RETURN
            END
```

LISTING OF SUBROUTINE RESVOR

```
SUBROUTINE RESVOR      73/73    OPT=0    TRACE              FTN 4.0+P355       01/09/74  14.23.49.       PAGE     1

          SUBROUTINE RESVOR(RK,XRES,TWL,NPWL,DT,NPTS,Q,H)                 004270
          COMMON /FLOW/ QMAX                                              004280
          DIMENSION GUS(20),FDS(20),TWL(20),WL1(20),Q(NPTS),H(NPTS)       004290
          DIMENSION QLT(1),TLT(1),PP(2),HH(200,2),II(2),DTH(2)            004300
          H(1)=Q(1)                                                       004310
          DX=1.0                                                          004320
          TLT(1)=-1.0                                                     004330
          DO 1 I=1,20                                                     004340
          GUS(I)=TWL(I)/2.0                                               004350
          WL1(I)=TWL(I)                                                   004360
        1 FDS(I)=WL1(I)/2.0+RK*WL1(I)**XRES/DT                            004370
          CALL KINOUT(FDS,WL1,20,GUS,TWL,20,QLT,TLT,1,DX,DT,NPTS,Q,H)     004380
          II(1)=II(2)=NPTS                                                004390
          PP(1)=PP(2)=1.0                                                 004400
          DTH(1)=DTH(2)=DT/3600.0                                         004410
          DO 2 I=1,NPTS                                                   004420
          HH(I,1)=Q(I)/QMAX                                               004430
        2 HH(I,2)=H(I)/QMAX                                               004440
          TIME=0.0                                                        004450
          CALL HPLOT(HH,DTH,II,PP,TIME)                                   004460
          WRITE(6,100)                                                    004470
      100 FORMAT(35X,*THIS IS THE HYDROGRAPHS AT THE IMAGINARY RESERVOIR*,    004480
         */,35X,*  INFLOW=000     OUTFLOW=*** *)                          004490
          DO 3 I=1,NPTS                                                   004500
        3 Q(I)=H(I)                                                       004510
          RETURN                                                          004520
          END                                                             004530
```

LISTING OF SUBROUTINE RIVER3 (AND THE DRIVER)

```
      PROGRAM PROFIL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
      DIMENSION B(64,22),H(64,22),X(64),RC(64),NPTS(64)
      DIMENSION AT(64,10),ELEV(64,10),QT(64,10)
      DIMENSION WLD(20),QD(20),WK1(20),WK2(20),WK6(20),WK7(20),WK9(20)
      CALL CONNEC(5LINPUT)
      CALL CONNEC(6LOUTPUT)
      WRITE(6,1)
1     FORMAT(* SUPPLY NO OF SECTIONS...I3*)
      READ(5,2)NSECS
2     FORMAT(I3)
      WRITE(6,3)
3     FORMAT(* SUPPLY MAX NO OF PTS...I3*)
      READ(5,4)MAXPTS
4     FORMAT(I3)
      NF1=10
      CALL RIVER3(B,H,NPTS,X,RC,NSECS,MAXPTS,32.2,1,5,6,NF1,AT,
     +ELEV,QT,WLD,QD,WK1,WK2,WK6,WK7,WK9)
      STOP
      END
      SUBROUTINE READXS(B,H,NPTS,X,RC,NSECS,MAXPTS,NREAD)
      DIMENSION B(NSECS,MAXPTS),H(NSECS,MAXPTS),X(NSECS),
     +     RC(NSECS),NPTS(NSECS)
      READ(NREAD,10)I,X(I),RC(I),NPTS(I)
      N=NPTS(I)
      READ(NREAD,20)(B(I,J),H(I,J),J=1,N)
   10 FORMAT(I3,F10.1,F6.4,I3)
   20 FORMAT(3X,2F6.2,1X,2F6.2,1X,2F6.2,1X,2F6.2,
     +     1X,2F6.2,1X,2F6.2)
      RETURN
      END
      SUBROUTINE RIVER3(B,H,NPTS,X,RC,NSECS,MAXPTS,G,NR1,NR2,NW,NF1,AT,
     +ELEV,QT,WLD,QD,WK1,WK2,WK6,WK7,WK9)
      DIMENSION B(NSECS,MAXPTS),H(NSECS,MAXPTS),NPTS(NSECS)
     +   ,X(NSECS),RC(NSECS)
      DIMENSION AT(NSECS,NF1),ELEV(NSECS,NF1),QT(NSECS,NF1)
      DIMENSION IIST(2,100),BI(30),HI(30),QS(20,2),IOFQ(20)
      DIMENSION WLD(NF1),QD(NF1),TI(20),QI(20)
      DIMENSION WK1(NF1),WK2(NF1),WK6(NF1),WK7(NF1),WK9(NF1)
      DIMENSIONWK3(200),WK4(200),WK5(200),WK8(2)
      DIMENSION NSA(20),ALF(20),BET(20)
      EXTERNAL CHEZYQ,MANNGQ,STRICQ,ROUGHQ,SMOTHQ,COLEQ
      DO 5 I=1,NSECS
5     IIST(1,I)=IIST(2,I)=0
      REWIND NR1
      DO 10 I=1,NSECS
10    CALL READXS(B,H,NPTS,X,RC,NSECS,MAXPTS,NR1)
      WRITE(NW,11)
11    FORMAT(* DO YOU WANT LIST OF COMMANDS?...YES/NO  *)
      READ(NR2,12)SKIP
12    FORMAT(A3)
      IF(SKIP.EQ.3HNO ) GOTO 21
13    WRITE(NW,20)
20    FORMAT(* AFTER INVITATION TO TYPE "X=" GIVE ONE OF*,/,
     +  * THE FOLLOWING COMMANDS.... * ,/,
     +  * DISCHARGE....TO SPECIFY FLOW * ,/,
     +  * D/S WL.......TO DEFINE DOWNSTREAM CONTROL LEVEL * ,/,
     +  * INFLOW.......TO DEFINE INFLOW HYDROGRAPH *,/,
     +  * RESISTANCE...TO SET FLOW RESISTANCE LAW * ,/,
     +  * OLD SECTION..TO PRINT COORDS OF A SECTION * ,/,
     +  * NEW SECTION..TO REDEFINE COORDS OF A SECTION * ,/,
     +  * OLD COEFF....TO PRINT ROUGHNESS MEASURE * ,/,
     +  * NEW COEFF....TO REDEFINE ROUGHNESS MEASURE *,/,
     +  * CRITIC.......TO COMPUTE CRITICAL DEPTH AT A SECTION*,/,
     +  * CHANGES......TO PRINT CHANGES OF COORDS OR ROUGHNESS*,/,
     +  * TABLE........TO PRINT TABLE OF ALL SECTIONS DATA*,/,
```

— DRIVER

— START OF RIVER3

LISTING - RIVER3/2

```
     +   * PROFILES.....TO PRINT OUT SURFACE PROFILES*,/,
     +   * COMPUTE......TO COMPUTE PROFILES*,/,
     +   * ROUTE........TO ROUTE THE FLOOD*,/,
     +   * RESERVOIR....TO ROUTE THROUGH A RESERVOIR*,/,
     +   * RESTART......TO BEGIN AGAIN*,/,
     +   * HELP.........FOR COMMAND OPTIONS*,/,
     +   * STOP.........TO TERMINATE*)
21    IC=IF1=IF2=IF3=IF5=IF6=IF7=IF8=0
25    WRITE(NW,30)
30    FORMAT(* %-*)
35    READ(NR2,40)COMND
40    FORMAT(A6)
      IC=0
      IF(COMND.EQ.6HDISCHA) IC=1
      IF(COMND.EQ.6HD/S WL) IC=2
      IF(COMND.EQ.6HRESIST) IC=3
      IF(COMND.EQ.6HOLD SE) IC=4
      IF(COMND.EQ.6HNEW SE) IC=5
      IF(COMND.EQ.6HOLD CO) IC=6
      IF(COMND.EQ.6HNEW CO) IC=7
      IF(COMND.EQ.6HCOMPUT) IC=8
      IF(COMND.EQ.6HSTOP  ) IC=9
      IF(COMND.EQ.6HCRITIC) IC=10
      IF(COMND.EQ.6HCHANGE) IC=11
      IF(COMND.EQ.6HTABLE ) IC=12
      IF(COMND.EQ.6HRESTAR) IC=13
      IF(COMND.EQ.6HHELP  ) IC=14
      IF(COMND.EQ.6HINFLOW) IC=15
      IF(COMND.EQ.6HPROFIL) IC=16
      IF(COMND.EQ.6HROUTE ) IC=17
      IF(COMND.EQ.6HRESERV) IC=18
      IF(IC.GT.0) GOTO 60
      WRITE(NW,70)
70    FORMAT(* BEG PARDON...PLEASE RETYPE*)
      GOTO 25
60    GOTO(210,220,230,240,250,260,270,280,290,
     +      310,320,320,4,13,400,425,475,502) IC
210   WRITE(NW,211)
      IF6=0
211   FORMAT(* ENTER NO.OF SUB-REACHES (I3)*,/,
     +   * WITH DIFFERENT DISCHARGES*)
      READ(NR2,212) NQS
212   FORMAT(I3)
      IF(NQS.LE.20) GOTO 213
      NQS=20
      WRITE(NW,214)
214   FORMAT(* NO.OF SUB-REACHES TRUNCATED TO 20*)
213   WRITE(NW,215)
215   FORMAT(* SUPPLY UPSTREAM SEC.NO. (I3) AND*,/,
     +   * DISCHARGES (2F9.3) FOR EACH SUB-REACH*,/,
     + * SUPPLY THE LOW VALUE FIRST*)
      DO 216 I=1,NQS
      WRITE(NW,217) I
217   FORMAT(* REACH NO.*,I3)
      READ(NR2,218) IOFQ(I),QS(I,1),QS(I,2)
 218  FORMAT(I3,2F9.3)
      IF(QS(I,1).GT.0.) GO TO 488
      QS(I,1)=1.0
      WRITE(NW,489)
 489  FORMAT(* NEATIVE OR ZERO FLOW SET EQUAL TO 1.0*)
 488  IF(QS(I,2).GT.0.) GO TO 216
      QS(I,2)=1.0
      WRITE(NW,489)
216   CONTINUE
      WRITE(NW,414)
 414  FORMAT(* ENTER NO. OF PTS IN FUNTION CURVES (I3)*)
      READ(NR2,212) NRPTS
      IF(NRPTS.LE.NF1) GO TO 415
      NRPTS=NF1
```

LISTING - RIVER3/3

```
      WRITE(NW,416) NF1
  416 FORMAT(* NO. OF PTS. TRUNCATED TO *,I3)
  415 IF1=1
      GOTO 25
 220  WRITE(NW,221)
 221  FORMAT(* DEFINE SEC. NO. WHERE D/S WATER*,/,
     +  * LEVEL TO BE SPECIFIED, I3  *)
      READ(NR2,222)NDS
 222  FORMAT(I3)
      IF(NDS.LE.NSECS.AND.NDS.GT.0) GOTO 223
      WRITE(NW,246) NDS,NSECS
      GOTO 220
  223 NPSP=NDS
      IF6=0
      WRITE(NW,412)
  412 FORMAT(* DEFINE NUMBER OF POINTS IN D/S WL RATING CURVE  (I3)*)
      READ(NR2,212) NPWL
      IF(NPWL.LT.NF1) GO TO 441
      NPWL=NF1
      WRITE(NW,442) NF1
  442 FORMAT(* NO. OF PTS TRUNCATED TO *,I3)
  441 WRITE(NW,225)
  225 FORMAT(* DEFINE D/S WL AND FLOWRATE 2F9.3*,/,
     +  * START AT THE LOWEST WATER LEVEL*)
      DO 413 I=1,NPWL
      WRITE(NW,407) I
  413 READ(NR2,226) WLO(I),QO(I)
 226  FORMAT(2F9.3)
      IF2=1
      GOTO 25
 230  IF(IF3.EQ.1) GOTO 236
      WRITE(NW,231)
 231  FORMAT(* SPECIFY RESISTANCE LAW BY TYPING*,/,
     +  * CHEZY, MANNING, STRICKLER, COLEBROOK*,/,
     +  * SMOOTH OR ROUGH.....P.S.   IS YOUR*,/,
     +  * ROUGHNESS MEASURE COMPATIBLE'*,/,* X=*)
      GOTO 232
 236  WRITE(NW,237)
 237  FORMAT(* RESPECIFY RESISTANCE LAW*,/,
     +  * SEE PREVIOUS NOTE *,/,* X=*)
 232  READ(NR2,233)UNAME
 233  FORMAT(A6)
      IU=0
      IF(UNAME.EQ.6HCHEZY ) IU=1
      IF(UNAME.EQ.6HMANNIN) IU=2
      IF(UNAME.EQ.6HSTRICK) IU=3
      IF(UNAME.EQ.6HCOLEBR) IU=4
      IF(UNAME.EQ.6HSMOOTH) IU=5
      IF(UNAME.EQ.6HROUGH ) IU=6
      IF(IU.GT.0) GOTO 235
      WRITE(NW,234)
 234  FORMAT(* NOT RECOGNIZED...PLEASE RETYPE*,/,* X=*)
      GOTO 232
 235  IF3=1
      IF6=0
      GOTO 25
 240  WRITE(NW,241)
 241  FORMAT(* DEFINE SECTION NO., I3   *)
      READ(NR2,242)NS
 242  FORMAT(I3)
      IF(NS.LE.NSECS.AND.NS.GT.0) GOTO 245
      WRITE(NW,246) NS,NSECS
 246  FORMAT(* SEC.NO.*,I5,* OUT OF RANGE 1-*,I3)
      GOTO 240
 245  WRITE(NW,243) NS,NPTS(NS)
 243  FORMAT(* SECTION NO.*,I3,* HAS*,I3,* PTS....COORDS ARE*)
      NP=NPTS(NS)
      WRITE(NW,244)(B(NS,I),H(NS,I),I=1,NP)
```

LISTING - RIVER3/4

```
244   FORMAT(5(F5.1,*,*,F5.1,2X))
      GOTO 25
250   WRITE(NW,251)
251   FORMAT(* DEFINE SECTION NO,  ; NO, OF POINTS IN SECTION, 2I3*)
      READ(NR2,252)NS,NP
      IF(NS.LE.NSECS.AND.NS.GT.0) GOTO 257
      WRITE(NW,246) NS,NSECS
      GOTO 250
257   IF(NP.LE.MAXPTS.AND.NP.GT.0) GOTO 259
      WRITE(NW,258) NP,MAXPTS
258   FORMAT(* NO.OF PTS,*,I5,* OUT OF RANGE 1-*,I3)
      GOTO 250
252   FORMAT(2I3)
259   ITST(2,NS)=1
      NPTS(NS)=NP
      IF6=0
      WRITE(NW,253)
253   FORMAT(* GIVE COORDS IN PAIRS,  2F6.1*)
      DO 254 I=1,NP
      WRITE(NW,255) I
255   FORMAT(* PT,*,I3,*!   *)
      READ(NR2,256)B(NS,I),H(NS,I)
256   FORMAT(2F6.1)
254   CONTINUE
      GOTO 25
260   WRITE(NW,261)
261   FORMAT(* GIVE SECTION NO,,   I3   *)
      READ(NR2,262) NS
262   FORMAT(I3)
      IF(NS.LE.NSECS.AND.NS.GT.0) GOTO 264
      WRITE(NW,246) NS,NSECS
      GOTO 260
264   WRITE(NW,263) NS,RC(NS)
263   FORMAT(* ROUGHNESS AT SECN,*,I3,* IS*,F6.3)
      GOTO 25
270   WRITE(NW,271)
271   FORMAT(* DEFINE SECTION NO, & COEFF, I3 & F5.3  *)
      READ(NR2,272) NS,RC(NS)
272   FORMAT(I3,F5.3)
      IF(NS.LE.NSECS.AND.NS.GT.0) GOTO 273
      WRITE(NW,246) NS,NSECS
      GOTO 270
273   ITST(1,NS)=1
      IF6=0
      GO TO 25

280   CONTINUE
      IF(IF1.EQ.0) GOTO 210
      IF(IF2.EQ.0) GOTO 220
      IF(IF3.EQ.0) GOTO 230
      WRITE(NW,278)
278   FORMAT(* SPECIFY UPSTREAM LIMIT OF PROFILE*,/,
     +  * BY SEC, NO, (I3)*)
      READ(NR2,279)NUS
279   FORMAT(I3)
      IF(NUS.LE.NDS.AND.NUS.GT.0) GOTO 293
      WRITE(NW,246)NUS,NDS
      GOTO 280
281   FORMAT(///,* SEC.   DISTANCE   DISCHARGE   WATER    ENERGY*,/,
     +  * NO.                             LEVEL      LEVEL*)
293   WRITE(NW,417)
417   FORMAT(* SPECIFY D/S STARTING POINT OF PROFILES*,/,
     +  * BY SEC, NO, (I3)*)
      READ(NR2,212)NSDS
      IF(NSDS.LE.NDS.AND.NSDS.GT.0) GO TO 418
      WRITE(NW,246) NSDS,NDS
      GO TO 293
```

LISTING - RIVER3/5

```
418  IF(NSDS.GE.NPSP) GO TO 419
     WRITE(NW,420) NSDS,NPSP,NDS
420  FORMAT(* STAGE DISCHARGE RELATION NOT DEFINED*,/,
    + * AT SECTION NO, *,I3,* RELATION DEFINED AT*,/,
    + * SECTIONS *,I3,* - *,I3)
     GO TO 293
419  NPSP=NUS
     IF(NSDS.NE.NDS) GO TO 421
     DO 422 I=1,NPWL
     WK1(I)=WLD(I)
422  WK2(I)=QD(I)
     NWK1=NPWL
     GO TO 433
421  DO 424 I=1,NRPTS
     WK1(I)=ELEV(NSDS,I)
424  WK2(I)=QT(NSDS,I)
     NWK1=NRPTS
433  NWK2=NRPTS-1
     IF7=1
     IF(NWK2.EQ.0) NWK2=1
     IF6=1
     DO 300 IJ=1,NRPTS
     DO 274 J=1,NQS
     JJ=NQS+1-J
     IF(NDS.GE.IOFQ(JJ)) GO TO 275
274  CONTINUE
275  Q=(QS(JJ,2)-QS(JJ,1))*FLOAT(IJ-1)/NWK2+QS(JJ,1)
     CALL INTER1(WK1,WK2,NWK1,Q,WLDS)
     DSWL=WLDS
     CALL SELSEC(B,H,NSECS,MAXPTS,NSDS,BI,HI)
     CALL PROPS (BI,HI,NPTS(NSDS),DSWL,A,T,P,AY)
     AT(NSDS,IJ)=A
     ELEV(NSDS,IJ)=WLDS
     QT(NSDS,IJ)=Q
     EN=WLDS+Q*Q/(2.0*G*A*A)
     NUS1=NUS+1
     DO 300 I2=NUS1,NSDS
     I=NSDS+NUS-I2
     DO 276 J=1,NQS
     JJ=NQS+1-J
     IF(I.GE.IOFQ(JJ)) GOTO 277
276  CONTINUE
277  Q=(QS(JJ,2)-QS(JJ,1))*FLOAT(IJ-1)/NWK2+QS(JJ,1)
     IF(X(I).EQ.X(I+1)) GO TO 326
     GO TO (284,285,286,287,288,289) IQ
284  CALL EZRA(B,H,NPTS,X,RC,NSECS,MAXPTS,I,WLDS,
    +  Q,G,CHEZYQ,WLUS,EN)
     GOTO 291
285  CALL EZRA(B,H,NPTS,X,RC,NSECS,MAXPTS,I,WLDS,
    +  Q,G,MANNGQ,WLUS,EN)
     GOTO 291
286  CALL EZRA(B,H,NPTS,X,RC,NSECS,MAXPTS,I,WLDS,
    +  Q,G,STRICQ,WLUS,EN)
     GOTO 291
287  CALL EZRA(B,H,NPTS,X,RC,NSECS,MAXPTS,I,WLDS,
    +  Q,G,COLEQ, WLUS,EN)
     GOTO 291
288  CALL EZRA(B,H,NPTS,X,RC,NSECS,MAXPTS,I,WLDS,
    +  Q,G,SMOTHQ,WLUS,EN)
     GOTO 291
289  CALL EZRA(B,H,NPTS,X,RC,NSECS,MAXPTS,I,WLDS,
    +  Q,G,ROUGHQ,WLUS,EN)
     GOTO 291
326  CALL SELSEC(B,H,NSECS,MAXPTS,I,BI,HI)
     GOTO 399
     WRITE(NW,327)
327  FORMAT(1X,* SPECIFY HEADLOSS COEFF. , F6.3 X=  *)
     READ(NR2,328) CL
328  FORMAT(F6.3)
```

LISTING - RIVER3/6

```
 399  CL=1.0
      CALL CONTRO(BI,HI,NPTS(I),WLDS,EN,G,Q,CL,WLUS,EUS)
      EN=EUS
      GO TO 291

 291  CALL SELSEC(B,H,NSECS,MAXPTS,I,BI,HI)
      CALL PROPS(BI,HI,NPTS(I),WLUS,A,T,P,AY)
      AT(I,IJ)=A
      ELEV(I,IJ)=WLUS
      QT(I,IJ)=Q
 300  WLDS=WLUS
292   FORMAT(////)
      GO TO 25
 310  CONTINUE
 317  WRITE(NW,311)
311   FORMAT(* SPECIFY SECTION NO,I3*)
      READ(NR2,312) ICRIT
312   FORMAT(I3)
      IF(ICRIT.LE.NSECS.AND.ICRIT.GT.0) GOTO 313
      WRITE(NW,246) ICRIT,NSECS
      GOTO 317
 313  WRITE(NW,316)
 316  FORMAT(* ENTER DISCHARGE  F9.3*)
      READ (NR2,318) Q
 318  FORMAT(F9.3)
      CALL SELSEC(B,H,NSECS,MAXPTS,ICRIT,BI,HI)
      CALL CRITIC(BI,HI,NPTS(ICRIT),Q,G,YCR,HCR)
      CALL BOTTOM (HI,NPTS(ICRIT),BOT,W(MAX)
      WRITE(NW,314) ICRIT,Q,YCR+BOT,HCR+BOT
314   FORMAT(* AT SEC*,I3,* WITH Q=*,F10.3,/,
     +       * CRITICAL WATER LEVEL=*,F8.3,* AND*,/,
     +        * CRITICAL ENERGY LEVEL=*,F8.3)

      GOTO 25
320   IF4=0
      IF (IC.EQ.11) WRITE(NW,329)
      IF (IC.EQ.12) WRITE(NW,330)

      DO 321 I=1,NSECS
      IF(IC.EQ.11.AND.ITST(1,I).EQ.0)GO TO 322
      WRITE(NW,323)I,X(I),RC(I),NPTS(I)
      IF4=1
322   IF(IC.EQ.11.AND.ITST(2,I).EQ.0) GO TO 321
      NP=NPTS(I)
      WRITE(NW,324)I,(B(I,J),H(I,J),J=1,NP)
      IF4=1
 323  FORMAT(I3,F10.1,F6.4,I3)
 324  FORMAT(I3,5(2F6.1,1X),2F6.1)
325   FORMAT(* NO CHANGES*)
 329  FORMAT(* SECTION CHANGES ARE...*)
 330  FORMAT(* CURRENT DATA IS...*)

321   CONTINUE
      IF(IF4.EQ.0) WRITE(NW,325)

       GO TO 25
 400  WRITE(NW,401)
 401   FORMAT(* HOW MANY POINTS DEFINE THE INFLOW HYD_..(I3)*)
      READ(NR2,212) NPHYD
      IF(NPHYD.LE.20) GO TO 486
      NPHYD=20
      WRITE(NW,487)
 487  FORMAT(* NO. OF PTS. TRUNCATED TO 20*)
 486  WRITE(NW,402)
 402  FORMAT(* ARE THE UNITS HOURS OR SECONDS_ *)
 404  READ(NR2,40) UNITS
      ITU=0
```

LISTING - RIVER3/7

```
      IF(UNITS.EQ.6HHOURS ) ITU=1
      IF(UNITS.EQ.6HSECOND) ITU=2
      IF(ITU.NE.0) GO TO 403
      WRITE(NW,70)
      GO TO 404
  403 WRITE(NW,405)
  405 FORMAT(* AFTER EACH "_" ENTER TIME AND FLOWRATE..(2F9.3)*)
      DO 406 I=1,NPHYD
      WRITE(NW,407) I
  407 FORMAT(* PT.*,I3,* _ *)
  406 READ (NR2,408) TI(I),QI(I)
  408 FORMAT(2F9.3)
  411 WRITE(NW,409)
  409 FORMAT(* SPECIFY THE SEC NO. AT WHICH INFLOW IS DEFINED...(I3)*)
      READ (NR2,212) NSINF
      IF(NSINF.GT.NSECS.OR.NSINF.LT.1) GO TO 410
      NFRP=NSINF
      IFR=0
      IF5=1
      GO TO 25
  410 WRITE(NW,246) NSINF,NSECS
      GO TO 411
  425 IF(IF7.EQ.1) GO TO 426
      WRITE(NW,427)
  427 FORMAT(* PROFILES ARE NOT COMPUTED YET*)
       GO TO 25
  426 IF(IF6.EQ.0) WRITE(NW,428)
  428 FORMAT(* WARNING, A CHANGE HAS BEEN MADE AND *,/,
     + * NOTHING HAS BEEN RECOMPUTED *)
  432 WRITE(NW,429)
  429 FORMAT(* SPECIFY U/S SEC. NO. OF PRINTOUT  (I3)*)
      READ(NR2,212) NUSPR
      IF(NUSPR.GE.NUS.AND.NUSPR.LE.NDS) GO TO 430
      WRITE(NW,431) NUS, NDS
  431 FORMAT(* SEC. NO. MUST BE IN THE RANGE *,I3,* - *,I3)
       GO TO 432
  430 WRITE(NW,482)
  482 FORMAT(* SPECIFY D/S SEC. NO. OF PRINTOUT (I3)*)
      READ (NR2,212) NDSPR
      IF(NDSPR.GE.NUS.AND.NDSPR.LE.NDS) GO TO 436
      WRITE( NW,431) NUS,NDS
       GO TO 430
  436 IF(NDSPR.GE.NUSPR) GO TO 434
      WRITE(NW,435)NDSPR,NUSPR
  435 FORMAT(* D/S SEC. *,I3,* IS SPECIFIED U/S OF U/S SEC. *,I3)
      GO TO 430
  434 WRITE(NW,281)
      DO 437 I=NUSPR,NDSPR
      EN=ELEV(I,1)+QT(I,1)**2.0/(2.0*G*AT(I,1)**2.0)
      WRITE(NW,439) I,X(I),QT(I,1),ELEV(I,1),EN
      DO 438 II=2,NRPTS
      EN=ELEV(I,II)+QT(I,II)**2.0/(2.0*G*AT(I,II)**2.0)
  438 WRITE(NW,440) QT(I,II),ELEV(I,II),EN
  437 CONTINUE
  439 FORMAT(/,I4,F10.1,3F10.3)
  440 FORMAT(14X,3F10.3)
      WRITE(NW,292)
      GO TO 25
  475 IF(IF7.NE.0) GO TO 483
      WRITE(NW,427)
      GO TO 25
  483 IF(IF5.NE.0) GO TO 480
      WRITE(NW,484)
  484 FORMAT(* THE INFLOW HYDROGRAPH IS NOT DEFINED*)
      GO TO 25
  480 IF(IF6.NE.0) GO TO 490
      WRITE(NW,428)
      WRITE(NW,443)
```

```
  443 FORMAT(* DO YOU WISH TO CONTINUE WITH THIS COMMAND_(YES/NO)*)
      READ(NR2,12) CONTU
      IF(CONTU.EQ.3HYES) GO TO 490
      GO TO 25
  490 IF(NSINF.GE.NPSP.OR.NFRP.GE.NPSP) GO TO 492
      WRITE(NW,491) NPSP
  491 FORMAT(* PROFILES ARE ONLY COMPUTED UP TO SEC. *,I3)
      GO TO 25
  492 IF(NSINF.LE.NDS.AND.NFRP.LE.NDS) GO TO 481
      WRITE(NW,493) NDS
  493 FORMAT(* PROFILES NOT COMPUTED D/S OF SEC. *,I3)
      GO TO 25
  481 ALFA=0.5
  444 WRITE(NW,446)
  446 FORMAT(* THE WEIGHT FACTORS ARE ASSUMED TO BE 0.5*,/,
     + * DO YOU WISH TO CHANGE ANY_ (YES/NO)*)
      NCHNGS=0
      READ(NR2,12) CONTU
      IF(CONTU.EQ.3HNO ) GO TO 447
      WRITE(NW,522)
  522 FORMAT(* HOW MANY SECTIONS ARE TO BE CHANGED_ (I3)*)
      READ(NR2,212) NCHNGS
      IF(NCHNGS.LE.20) GO TO 523
      WRITE(NW,494)
  494 FORMAT(* NUMBER OF CHANGES TRUNCATED TO 20*)
      NCHNGS=20
  523 WRITE(NW,495)
  495 FORMAT(* SPECIFY U/S SEC.NO. (I3),*,/,
     + * ALFA AND BETA (2F9.3) FOR EA. SECTION*)
      DO 496 I=1,NCHNGS
      WRITE(NW,497) I
  497 FORMAT(* CHANGE NO. *,I3)
  496 READ (NR2,498)NSA(I),ALF(I),BET(I)
  498 FORMAT(I3,2F9.3)
  447 WRITE(NW,449)
  449 FORMAT(* DEFINE U/S SEC. NO. WHERE ROUTING STARTS (I3)*)
      READ(NR2,212) NSR
      IF(NSR.GE.NPSP.AND.NSR.LT.NDS) GO TO 450
      WRITE(NW,431) NPSP,NDS
      GO TO 447
  450 IF(NSR.EQ.NFRP.OR.NSR.EQ.NSINF) GO TO 451
      WRITE(NW,452)NSINF,NFRP
  452 FORMAT(* THE ROUTING MUST START AT SEC. NO.*,/,
     +   1X,I3,* OR *,I3)
      GO TO 447
  451 WRITE(NW,453)
  453 FORMAT(* DEFINE D/S SEC. NO. WHERE ROUTING FINISHES (I3)*)
      READ(NR2,212) NFR
      IF(NFR.GE.NDR) GO TO 530
      WRITE(NW,435) NFR,NSR
      GO TO 451
  530 IF(NFR.GE.NSINF.AND.NFR.LE.NDS) GO TO 454
      WRITE(NW,455) NFR,NSINF,NDS
  455 FORMAT(* SEC. NO. *,I5,* OUT OF RANGE *,I3,* - *,I3)
      GO TO 451
  454 WRITE(NW,456)
  456 FORMAT(* DEFINE TIME STEP FOR COMPUTATION. USE *,/,
     + * SAME UNITS OF TIME AS USED FOR HYDROGRAPH (F9.4)*)
      READ(NR2,457) DELTA
  457 FORMAT(F9.4)
      WRITE(NW,458)
  458 FORMAT(* ENTER START TIME WITH CONSISTENT UNITS (F9.3)*)
      READ(NR2,459) TIMES
  459 FORMAT(F9.3)
  533 WRITE(NW,462)
  462 FORMAT(* ENTER FINISH TIME WITH CONSISTENT UNITS, (F9.3)*)
      READ(NR2,459) TIMEF
      IF(TIMEF.GE.TIMES) GO TO 531
      WRITE(NW,532) TIMEF,TIMES
  532 FORMAT(* FINISH TIME *,F9.3,* IS EARLIER THAN*,/,
     + * THE START TIME *,F9.3)
```

LISTING - RIVER3/9

```
      GO TO 533
  531 TT=TIMES
      WK8(1)=WK9(1)=-1.0
      IF(NSR.NE.NSINF) GO TO 466
      IF(IF8.EQ.0) GO TO 527
      WRITE(NW,528)
  528 FORMAT(* THE INFLOW HAS BEEN ROUTED THROUGH*,/,
     + * A RESERVOIR,DO YOU WISH TO ROUTE THE *,/,
     + * ORIGINAL INFLOW_ (YES/NO) *)
      READ(NR2,12) CONTU
      IF(CONTU.EQ.3HNO ) GO TO 466
      IF8=0
  527 NN=0
      DO 461 I=1,200
      CALL INTER1(QI,TI,NPHYD,TT,WK3(I))
      WK5(I)=TT
      IF(TT.GT.TIMEF) GO TO 463
      NN=NN+1
      TT=TT+DELTA
  461 CONTINUE
      GO TO 463
  466 NNN=0
      DO 465 I=1,200
      CALL INTER1(WK3,WK5,NN,TT,WK4(I))
      IF(TT.GT.TIMEF) GO TO 485
      TT=TT+DELTA
      NNN=NNN+1
  465 CONTINUE
  485 NN=NNN
      TT=TIMES
      DO 470 I=1,NN
      WK5(I)=TT
      TT=TT+DELTA
  470 WK3(I)=WK4(I)
  463 DT=DELTA
      IF(ITU.EQ.2) GO TO 467
      DT=DELTA*3600.
  467 NSR=NSR+1
      NFRP=NFR
      NWK2=NRPTS
      IF(NSR-1.EQ.NFR) GO TO 529
      IF8=0
      DO 468 I=NSR,NFR
      ALFA=0.5
      BETA=0.5
      IF(NCHNGS.EQ.0) GO TO 501
      DO 499 II=1,NCHNGS
      IF(NSA(II).EQ.I-1) GO TO 500
  499 CONTINUE
      GO TO 501
  500 ALFA=ALF(II)
      BETA=BET(II)
  501 DX=X(I)-X(I-1)
      IF(DX.EQ.0.) GO TO 468
      DO 469 II=1,NRPTS
      STOR=(AT(I,II)+AT(I-1,II))*DX/(2.0*DT)
      WK1(II)=BETA*QT(I-1,II)-ALFA*STOR
      WK2(II)=BETA*QT(I,II)+(1.0-ALFA)*STOR
      WK7(II)=QT(I-1,II)
  469 WK6(II)=QT(I,II)
      WK4(1)=WK3(1)
      CALL KINRUT(WK2,WK6,NRPTS,WK1,WK7,NWK2,WK8,WK9,1,DX,DT
     +,NN,WK3,WK4)
      DO 471 II=1,NN
  471 WK3(II)=WK4(II)
```

LISTING - RIVER3/10

```
468 CONTINUE
529 WRITE(NW,472) NFR
472 FORMAT(//,* HYDROGRAPH AT SEC. NO. *,I3,/,
   + *        TIME           FLOWRATE *)
    DO 473 I=1,NN
    TT=FLOAT(I-1)*DELTA+TIMES
473 WRITE(NW,474) TT,WK3(I)
474 FORMAT(1X,F11.3,1X,F11.3)
    GO TO 25
502 IF(IF5.NE.0) GO TO 503
    WRITE(NW,484)
    GO TO 25
503 IF(IF1.NE.0) GO TO 520
    WRITE(NW,521)
521 FORMAT(* SPECIFY THE DISCHARGES *)
    GO TO 25
520 WRITE(NW,504) NSINF,NFRP
504 FORMAT(* SPECIFY LOCATION OF RESERVOIR (I3)*,/,
   + * AT SEC. NO. * ,I3,* OR *,I3)
    READ(NR2,212) NRL
    IF(NRL.EQ.NSINF.OR.NRL.EQ.NFRP) GO TO 505
    WRITE(NW,506) NSINF,NFRP
506 FORMAT(* THE RESERVOIR MUST BE AT *,/,
   +   * SEC.NO. *,I3,* OR *,I3)
    GO TO 503
505 WRITE(NW,507)
507 FORMAT(* ENTER K,X, DESCRIBING THE RESERVOIR (2F9.3)*)
    READ (NR2,508) RK,RX
508 FORMAT(2F9.3)
    WRITE(NW,456)
    READ(NR2,457) DELTA
    WRITE(NW,458)
    READ(NR2,459) TIMES
534 WRITE(NW,462)
    READ(NR2,459) TIMEF
    IF(TIMEF.GE.TIMES) GO TO 535
    WRITE(NW,532) TIMEF,TIMES
    GO TO 534
535 TT=TIMES
    WK8(1)=WK9(1)=-1.0
    IF(NRL.NE.NSINF) GO TO 509
    NN=0
    IF8=1
    DO 511 I=1,200
    CALL INTER1(QI,TI,NPHYD,TT,WK3(I))
    WK5(I)=TT
    IF(TT.GT.TIMEF) GO TO 512
    NN=NN+1
    TT=TT+DELTA
511 CONTINUE
    GO TO 512
509 NNN=0
    DO 513 I=1,200
    CALL INTER1(WK3,WK5,NN,TT,WK4(I))
    IF(TT.GT.TIMEF) GO TO 514
    TT=TT+DELTA
    NNN=NNN+1
513 CONTINUE
514 NN=NNN
    TT=TIMES
    DO 515 I=1,NN
    WK5(I)=TT
    TT=TT+DELTA
515 WK3(I)=WK4(I)
512 DT=DELTA
    IF(ITU.EQ.2) GO TO 516
    DT=DELTA*3600.
516 NWK2=NRPTS
```

```
      DO 525 I=1,NQS
      JJ=NQS+1-I
      IF(NRL.GE.IOFQ(JJ)) GO TO 526
  525 CONTINUE
  526 NWK3=NRPTS-1
      IF(NWK3.EQ.0) NWK3=1.
      DO 517 I=1,NRPTS
      Q=(QS(JJ,2)-QS(JJ,1))*FLOAT(I-1)/NWK3+QS(JJ,1)
      STOR=RK*Q**RX
      WK1(I)=0.5*Q
      WK2(I)=0.5*Q+STOR/DT
  517 WK6(I)=WK7(I)=Q
      WK4(1)=WK3(1)
      CALL KINRUT(WK2,WK6,NRPTS,WK1,WK7,NWK2,WK8,WK9,1,1.,DT,
     +NN,WK3,WK4)
      WRITE(NW,518) NRL
  518 FORMAT (//,* HYDROGRAPH AFTER RESERVOIR AT SEC. *,I3,/,
     + *    TIME           FLOWRATE *)
      DO 519 I=1,NN
      TT=FLOAT(I-1)*DELTA+TIMES
      WK3(I)=WK4(I)
  519 WRITE(NW,474) TT,WK3(I)
      GO TO 25
  290 STOP
      END
```

TYPICAL OUTPUT FROM FINDIF

```
FLOOD ROUTING USING THE EXPLICIT FINITE DIFFERENCE METHOD

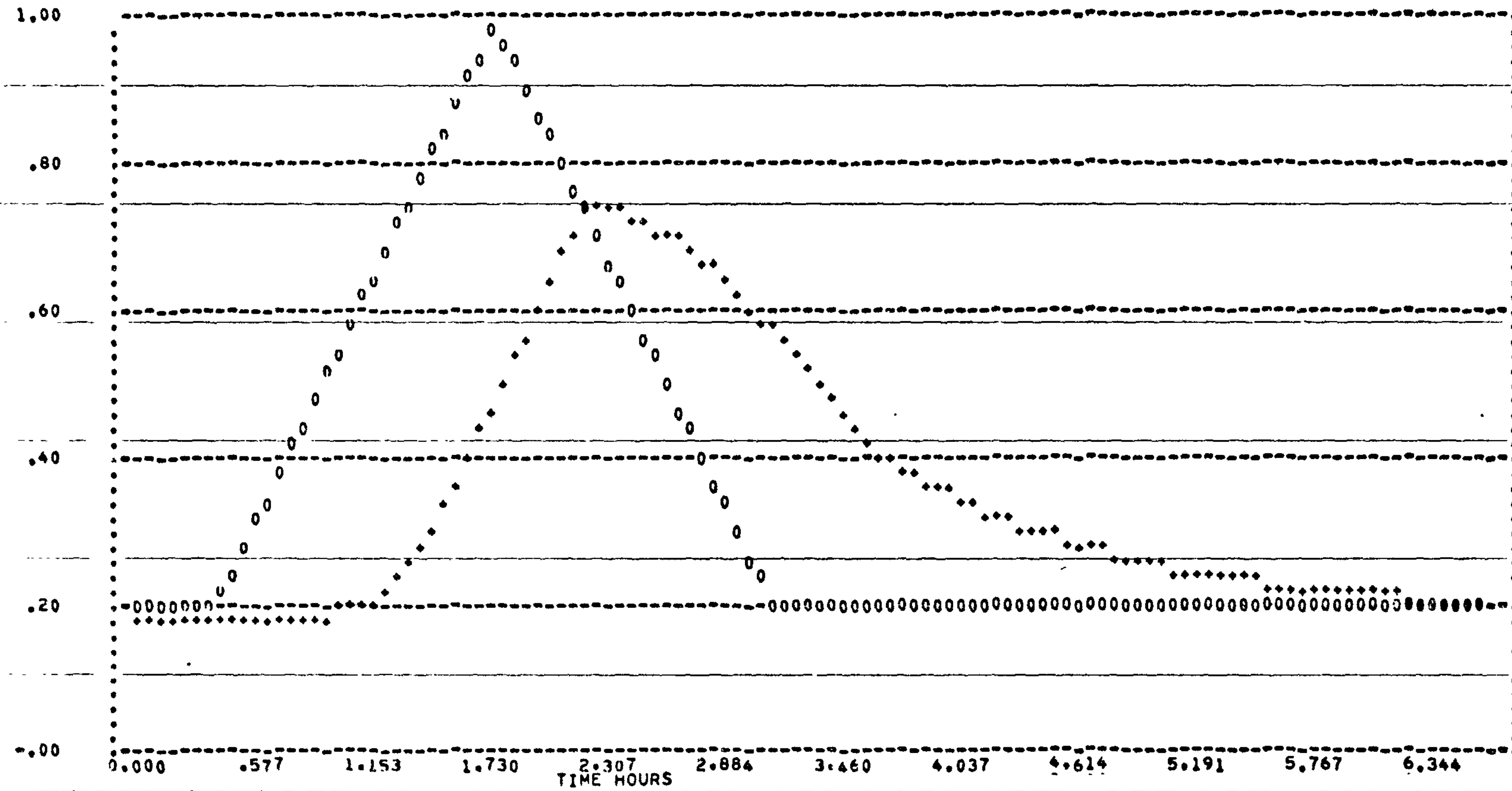
CHANNEL PROPERTIES AND FLOW CONDITIONS
          TOTAL LENGTH          =    50000.0 FT
          MAXIMUM DEPTH         =       20.0 FT
          SLOPE                 =     .00020 FT/FT
          MANNINGS N            =     .01490
          MAXIMUM FLOW          =    16675.7 CUSECS
          CELERITY AT MAX       =      33.72FT/SEC
          WIDTH OF CHANNEL      =      100.0 FT

PROBLEM VARIABLES
          NUMBER OF REACHES     =         25
          TIME INCREMENT        =       29.7 SECONDS
          MAX TIME INCREMENT    =       59.3 SECONDS
          TEST NUMBER           =        150

INVERT ELEVATIONS AND INITIAL WATER LEVELS AT EACH SECTION
            INITIAL FLOW RATE=      3335.1 CUSECS
```

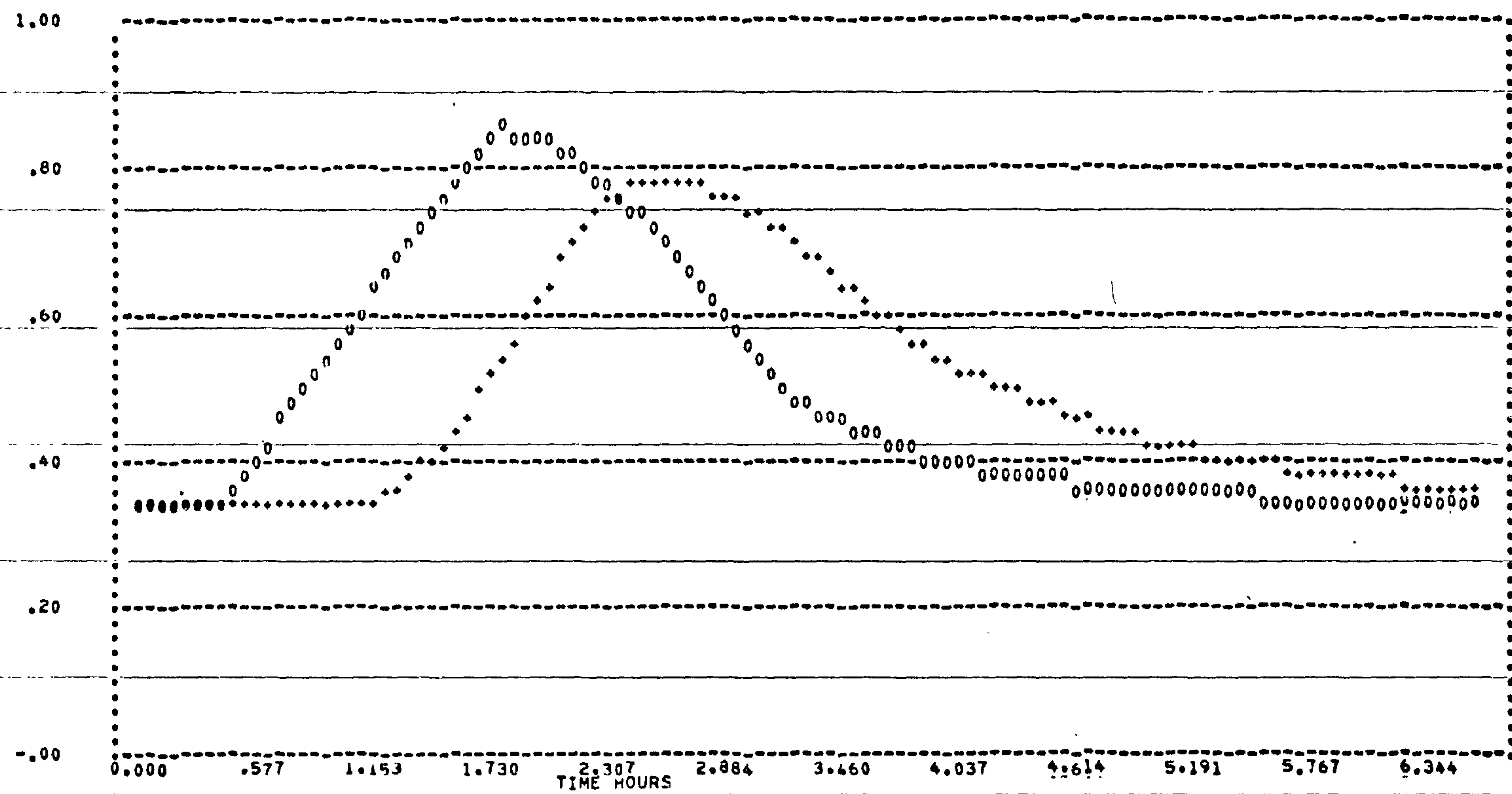
CHAINAGE	INVERT ELEVATION	WATER LEVEL
0.0	110.00	117.00
2000.0	109.60	116.60
4000.0	109.20	116.20
6000.0	108.80	115.80
8000.0	108.40	115.40
10000.0	108.00	115.00
12000.0	107.60	114.60
14000.0	107.20	114.20
16000.0	106.80	113.80
18000.0	106.40	113.40
20000.0	106.00	113.00
22000.0	105.60	112.60
24000.0	105.20	112.20
26000.0	104.80	111.80
28000.0	104.40	111.40
30000.0	104.00	111.00
32000.0	103.60	110.60
34000.0	103.20	110.20
36000.0	102.80	109.80
38000.0	102.40	109.40
40000.0	102.00	109.00
42000.0	101.60	108.60
44000.0	101.20	108.20
46000.0	100.80	107.80
48000.0	100.40	107.40
50000.0	100.00	107.00

OUTPUT-FINDIF/2



THIS IS A PLOT OF THE RATIO OF THE FLOW TO THE MAX FLOW VS TIME
000 AT SECTION NO= 1 +++ AT SECTION NO= 21

OUTPUT - FINDIF/3

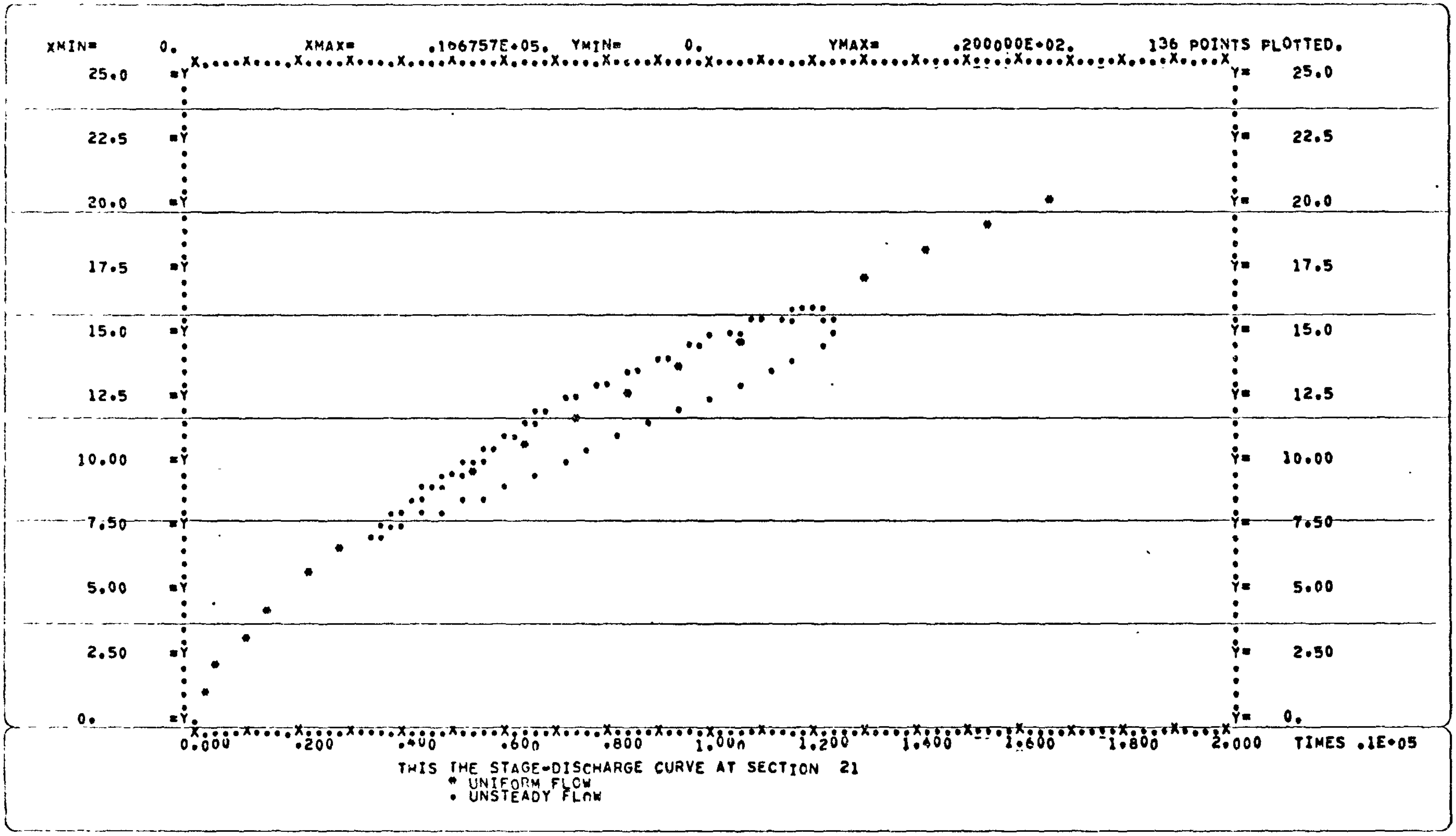


THIS IS A PLOT OF THE RATIO OF DEPTH TO MAX DEPTH VS TIME
000 AT SECTION NO= 1 +++ AT SECTION NO= 21

OUTPUT-FINDIF/4

```
THE ERROR IN THE VOLUME WAS=      .022 PER CENT
CENTROID OF HYDROGRPH NO  1=    2.64 HRS
CENTROID OF HYDROGRPH NO 21=    3.19 HRS
```

OUTPUT-FINDIF/5



TYPICAL OUTPUT FROM KINDIF

FLOOD ROUTING USING THE KINEMATIC WAVE METHOD

CHANNEL PROPERTIES AND FLOW CONDITIONS

TOTAL LENGTH	=	50000.0 FT
MAXIMUM DEPTH	=	20.0 FT
SLOPE	=	.00020 FT/FT
MANNINGS N	=	.01490
MAXIMUM FLOW	=	16675.7 CUSECS
CELERITY AT MAX	=	33.72FT/SEC
WIDTH OF CHANNEL	=	100.0 FT

PROBLEM VARIABLES

NUMBER OF REACHES	=	20
TIME INCREMENT	=	200.0 SECONDS
MAX TIME INCREMENT	=	74.2 SECONDS
TEST NUMBER	=	220
ALFA	=	0.000
BETA	=	1.000

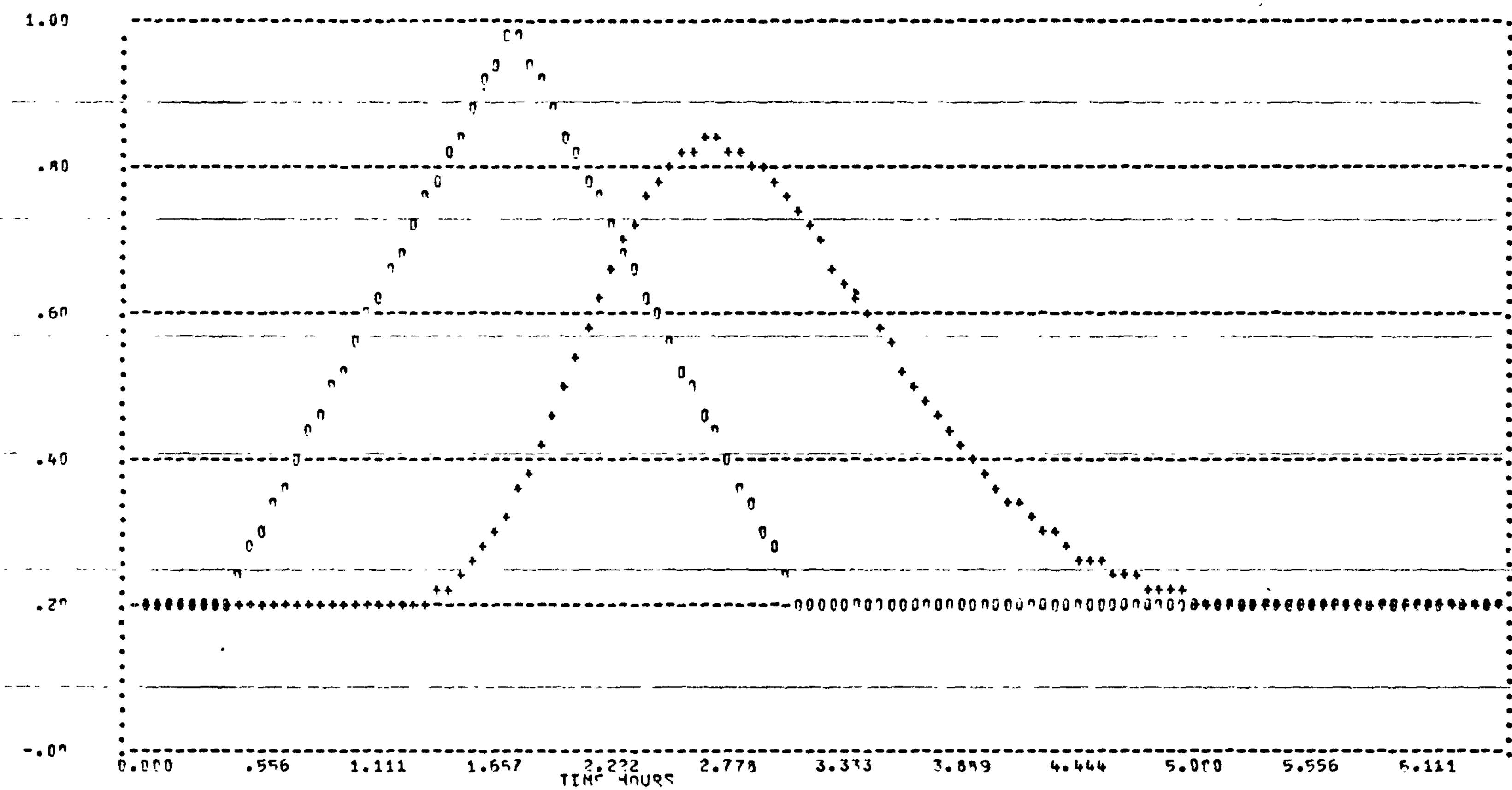
INVERT ELEVATIONS AND INITIAL WATER LEVELS AT EACH SECTION

INITIAL FLOW RATE= 3335.1 CUSECS

CHAINAGE	INVERT ELEVATION	WATER LEVEL
0.0	110.00	117.00
2500.0	109.50	115.50
5000.0	109.00	116.00
7500.0	108.50	115.50
10000.0	108.00	115.00
12500.0	107.50	114.50
15000.0	107.00	114.00
17500.0	106.50	113.50
20000.0	106.00	113.00
22500.0	105.50	112.50
25000.0	105.00	112.00
27500.0	104.50	111.50
30000.0	104.00	111.00
32500.0	103.50	110.50
35000.0	103.00	110.00
37500.0	102.50	109.50
40000.0	102.00	109.00
42500.0	101.50	108.50
45000.0	101.00	108.00
47500.0	100.50	107.50
50000.0	100.00	107.00

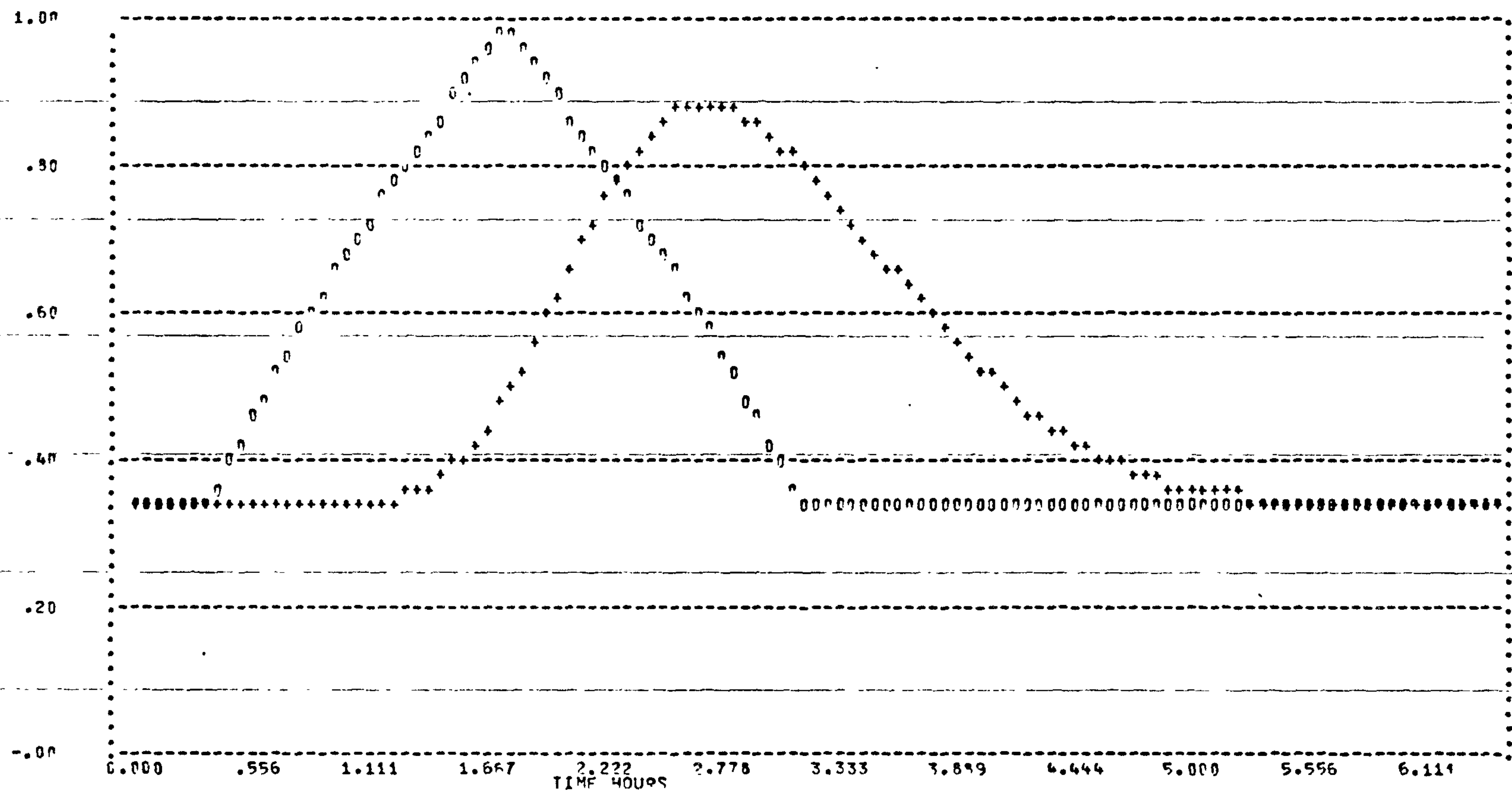
THE STAGE DISCHARGE CURVE WAS NOT PUNCHED

OUTPUT-KINDIF/2



THIS IS A PLOT OF THE RATIO OF THE FLOW TO THE MAX FLOW VS TIME
000 AT SECTION NO= 1 +++ AT SECTION NO= 17

OUTPUT-KINDIF/3



THIS IS A PLOT OF THE RATIO OF DEPTH TO MAX DEPTH VS TIME
000 AT SECTION NO= 1 +++ AT SECTION NO= 17

OUTPUT - KINDIF/4

```
THE ERROR IN THE VOLUME WAS=       .004 PER CENT
CENTROID OF HYDROGRAPH NO  1=    2.62 HRS
CENTROID OF HYDROGRAPH NO 17=    3.13 HRS
HYDROGRAPH NO  1 PEAK= .9847 TIME= 1.833 HRS
HYDROGRAPH NO 17 PEAK= .8434 TIME= 2.722 HRS
```

NOTE: THIS IS ONLY ONE OF TWENTY-FIVE SIMULATIONS PRODUCED BY A SINGLE EXECUTION OF "KINDIF."

TYPICAL OUTPUT FROM RIVER3

```
SUPPLY NO OF SECTIONS...I3  11
SUPPLY MAX NO OF PTS...I3    4
DO YOU WANT LIST OF COMMANDS\...YES/NO          YES
AFTER INVITATION TO TYPE ":-" GIVE ONE OF
THE FOLLOWING COMMANDS....
DISCHARGE....TO SPECIFY FLOW
D/S WL.......TO DEFINE DOWNSTREAM CONTROL LEVEL
INFLOW.......TO DEFINE INFLOW HYDROGRAPH
RESISTANCE...TO SET FLOW RESISTANCE LAW
OLD SECTION..TO PRINT COORDS OF A SECTION
NEW SECTION..TO REDEFINE COORDS OF A SECTION
OLD COEFF....TO PRINT ROUGHNESS MEASURE
NEW COEFF....TO REDEFINE ROUGHNESS MEASURE
CRITIC.......TO COMPUTE CRITICAL DEPTH AT A SECTION
CHANGES......TO PRINT CHANGES OF COORDS OR ROUGHNESS
TABLE........TO PRINT TABLE OF ALL SECTIONS DATA
PROFILES.....TO PRINT OUT SURFACE PROFILES
COMPUTE......TO COMPUTE PROFILES
ROUTE........TO ROUTE THE FLOOD
RESERVOIR....TO ROUTE THROUGH A RESERVOIR
RESTART......TO BEGIN AGAIN
HELP.........FOR COMMAND OPTIONS
STOP.........TO TERMINATE
:-      RESISTANCE
SPECIFY RESISTANCE LAW BY TYPING
CHEZY, MANNING, STRICKLER, COLEBROOK
SMOOTH OR ROUGH.....P.S.   IS YOUR
ROUGHNESS MEASURE COMPATIBLE\
:-      MANNING
:-      DISCHARGE
ENTER NO.OF SUB-REACHES (I3)
WITH DIFFERENT DISCHARGES     1
SUPPLY UPSTREAM SEC.NO. (I3) AND
DISCHARGES (2F9.3) FOR EACH SUB-REACH
SUPPLY THE LOW VALUE FIRST
REACH NO.  1        1 3000.    16675.
ENTER NO. OF PTS IN FUNTION CURVES (I3)         5
:-      COMPUTE
DEFINE SEC. NO. WHERE D/S WATER
LEVEL TO BE SPECIFIED, I3   11
DEFINE NUMBER OF POINTS IN D/S WL RATING CURVE  (I3)        3
DEFINE D/S WL AND FLOWRATE 2F9.3
START AT THE LOWEST WATER LEVEL
PT.  1 ?          107.     3000.
PT.  2 ?          113.     10000.
PT.  3 ?          120.     16675.
```

OUTPUT - RIVER3/2

```
:-      COMPUTE
SPECIFY UPSTREAM LIMIT OF PROFILE
BY SEC. NO. (I3)   3
SPECIFY D/S STARTING POINT OF PROFILES
BY SEC. NO. (I3)   9
STAGE DISCHARGE RELATION NOT DEFINED
AT SECTION NO.   9 RELATION DEFINED AT
SECTIONS  11 -  11
SPECIFY D/S STARTING POINT OF PROFILES
BY SEC. NO. (I3)  11
:-      COMPUTE
SPECIFY UPSTREAM LIMIT OF PROFILE
BY SEC. NO. (I3)   1
SPECIFY D/S STARTING POINT OF PROFILES
BY SEC. NO. (I3)   3
:-      PROFILES
SPECIFY U/S SEC. NO. OF PRINTOUT  (I3)          1
SPECIFY D/S SEC. NO. OF PRINTOUT (I3) 11
```

SEC. NO.	DISTANCE	DISCHARGE	WATER LEVEL	ENERGY LEVEL
1	0.0	3000.000	116.564	116.888
		6418.750	120.627	121.193
		9837.500	123.977	124.746
		13256.250	127.046	127.985
		16675.000	130.000	131.079
2	5000.0	3000.000	115.565	115.890
		6418.750	119.618	120.185
		9837.500	122.953	123.725
		13256.250	126.025	126.967
		16675.000	129.000	130.079
3	10000.0	3000.000	114.568	114.892
		6418.750	118.606	119.174
		9837.500	121.921	122.697
		13256.250	125.000	125.945
		16675.000	128.000	129.079
4	15000.0	3000.000	113.573	113.896
		6418.750	117.588	118.159
		9837.500	120.881	121.661
		13256.250	123.970	124.917
		16675.000	127.000	128.079

OUTPUT - RIVER3/3

5	20000.0	3000.000	112.581	112.904
		6418.750	116.563	117.137
		9837.500	119.828	120.614
		13256.250	122.932	123.884
		16675.000	126.000	127.079
6	25000.0	3000.000	111.595	111.916
		6418.750	115.529	116.106
		9837.500	118.759	119.553
		13256.250	121.885	122.842
		16675.000	125.000	126.079
7	30000.0	3000.000	110.618	110.937
		6418.750	114.480	115.062
		9837.500	117.669	118.474
		13256.250	120.828	121.791
		16675.000	124.000	125.079
8	35000.0	3000.000	109.658	109.973
		6418.750	113.410	114.000
		9837.500	116.550	117.368
		13256.250	119.756	120.728
		16675.000	123.000	124.079
9	40000.0	3000.000	108.725	109.034
		6418.750	112.308	112.910
		9837.500	115.389	116.228
		13256.250	118.667	119.649
		16675.000	122.000	123.079
10	45000.0	3000.000	107.832	108.132
		6418.750	111.159	111.779
		9837.500	114.170	115.036
		13256.250	117.556	118.551
		16675.000	121.000	122.079
11	50000.0	3000.000	107.000	107.285
		6418.750	109.930	110.579
		9837.500	112.861	113.769
		13256.250	116.415	117.428
		16675.000	120.000	121.079

OUTPUT - RIVER3/4

```
:-      ROUTE
THE INFLOW HYDROGRAPH IS NOT DEFINED
:-      INFLOW
HOW MANY POINTS DEFINE THE INFLOW HYD?..(I3)      3
ARE THE UNITS HOURS OR SECONDS?        SECONDS
AFTER EACH "?" ENTER TIME AND FLOWRATE..(2F9.3)
PT.  1 ?          1500.    3335.
PT.  2 ?          6500.    16675.
PT.  3 ?          11500.   3335.
SPECIFY THE SEC NO. AT WHICH INFLOW IS DEFINED...(I3)       1
:-      ROUTE
THE WEIGHT FACTORS ARE ASSUMED TO BE 0.5
DO YOU WISH TO CHANGE ANY? (YES/NO)  NO
DEFINE U/S SEC. NO. WHERE ROUTING STARTS (I3)     1
DEFINE D/S SEC. NO. WHERE ROUTING FINISHES (I3)  9
DEFINE TIME STEP FOR COMPUTATION.  USE
SAME UNITS OF TIME AS USED FOR HYDROGRAPH (F9.4)          200.
ENTER START TIME WITH CONSISTENT UNITS (F9.3)  0.0
ENTER FINISH TIME WITH CONSISTENT UNITS, (F9.3)15000.


HYDROGRAPH AT SEC. NO.    9
   TIME        FLOWRATE
      0.000    3335.000
    200.000    3335.000
    400.000    3335.000
    600.000    3335.000
    800.000    3335.000
   1000.000    3335.000
   1200.000    3335.000
   1400.000    3335.000
   1600.000    3335.796
   1800.000    3327.287
   2000.000    3359.838
   2200.000    3317.736
   2400.000    3282.170
   2600.000    3389.122
   2800.000    3417.854
   3000.000    3321.493
   3200.000    3209.806
   3400.000    3230.446
   3600.000    3339.303
   3800.000    3462.927
   4000.000    3520.652
   4200.000    3486.220
   4400.000    3374.778
   4600.000    3246.841
   4800.000    3147.967
   5000.000    3067.595
```

OUTPUT - RIVER3/5

5200.000	3030.879
5400.000	3000.000
5600.000	3000.000
5800.000	3000.000
6000.000	3226.843
6200.000	3848.215
6400.000	4647.208
6600.000	5550.326
6800.000	6472.276
7000.000	7405.371
7200.000	8292.711
7400.000	8941.942
7600.000	9495.465
7800.000	10307.126
8000.000	11005.271
8200.000	11419.812
8400.000	11749.979
8600.000	12251.178
8800.000	12985.908
9000.000	13858.984
9200.000	14707.368
9400.000	15397.740
9600.000	15851.727
9800.000	16065.916
10000.000	16046.515
10200.000	15856.087
10400.000	15556.072
10600.000	15179.402
10800.000	14753.309
11000.000	14297.967
11200.000	13816.157
11400.000	13270.195
11600.000	12733.183
11800.000	12221.965
12000.000	11759.379
12200.000	11241.673
12400.000	10788.580
12600.000	10411.226
12800.000	9854.774
13000.000	9194.578
13200.000	8653.719
13400.000	8267.276
13600.000	7981.947
13800.000	7720.165
14000.000	7437.375
14200.000	7127.168
14400.000	6768.503
14600.000	6352.529
14800.000	5926.439
15000.000	5468.961

OUTPUT-RIVER3/6

```
:-      RESERVOIR
SPECIFY LOCATION OF RESERVOIR (I3)
AT SEC. NO.   1 OR   9        9
ENTER K,X, DESCRIBING THE RESERVOIR (2F9.3)    10000.    1.1
DEFINE TIME STEP FOR COMPUTATION.  USE
SAME UNITS OF TIME AS USED FOR HYDROGRAPH (F9.4)          200.
ENTER START TIME WITH CONSISTENT UNITS (F9.3)  1800.
ENTER FINISH TIME WITH CONSISTENT UNITS, (F9.3)15000.


HYDROGRAPH AFTER RESERVOIR AT SEC.    9
  TIME          FLOWRATE
   1800.000      3327.287
   2000.000      3327.414
   2200.000      3327.503
   2400.000      3327.288
   2600.000      3327.353
   2800.000      3327.946
   3000.000      3328.271
   3200.000      3327.784
   3400.000      3326.945
   3600.000      3326.617
   3800.000      3327.198
   4000.000      3328.480
   4200.000      3329.843
   4400.000      3330.627
   4600.000      3330.472
   4800.000      3329.436
   5000.000      3327.709
   5200.000      3325.540
   5400.000      3323.124
   5600.000      3320.607
   5800.000      3318.109
   6000.000      3316.515
   6200.000      3318.237
   6400.000      3325.477
   6600.000      3339.291
   6800.000      3360.106
   7000.000      3387.985
   7200.000      3422.736
   7400.000      3463.202
   7600.000      3508.038
   7800.000      3557.841
   8000.000      3613.138
```

OUTPUT - RIVER3/7

```
   8200.000    3672.337
   8400.000    3733.976
   8600.000    3798.373
   8800.000    3867.082
   9000.000    3941.519
   9200.000    4022.080
   9400.000    4108.008
   9600.000    4197.724
   9800.000    4289.343
  10000.000    4381.007
  10200.000    4471.140
  10400.000    4558.660
  10600.000    4642.863
  10800.000    4723.283
  11000.000    4799.644
  11200.000    4871.759
  11400.000    4939.310
  11600.000    5002.116
  11800.000    5060.350
  12000.000    5114.337
  12200.000    5164.085
  12400.000    5209.665
  12600.000    5251.655
  12800.000    5289.681
  13000.000    5322.672
  13200.000    5350.727
  13400.000    5374.952
  13600.000    5396.372
  13800.000    5415.494
  14000.000    5432.346
  14200.000    5446.757
  14400.000    5458.451
  14600.000    5467.036
  14800.000    5472.274
  15000.000    5474.030
:-     ROUTE
THE WEIGHT FACTORS ARE ASSUMED TO BE 0.5
DO YOU WISH TO CHANGE ANY? (YES/NO)  NO
DEFINE U/S SEC. NO. WHERE ROUTING STARTS (I3)     9
DEFINE D/S SEC. NO. WHERE ROUTING FINISHES (I3) 11
DEFINE TIME STEP FOR COMPUTATION.  USE
SAME UNITS OF TIME AS USED FOR HYDROGRAPH (F9.4)          200.
ENTER START TIME WITH CONSISTENT UNITS (F9.3)  6400.
ENTER FINISH TIME WITH CONSISTENT UNITS, (F9.3)15000.
```

OUTPUT - RIVER3/8

HYDROGRAPH AT SEC. NO. 11

TIME	FLOWRATE
6400.000	3325.477
6600.000	3327.819
6800.000	3321.889
7000.000	3317.980
7200.000	3319.483
7400.000	3327.469
7600.000	3342.786
7800.000	3366.003
8000.000	3395.695
8200.000	3430.487
8400.000	3471.075
8600.000	3517.936
8800.000	3569.884
9000.000	3625.135
9200.000	3682.877
9400.000	3743.698
9600.000	3808.980
9800.000	3879.982
10000.000	3957.171
10200.000	4040.098
10400.000	4127.572
10600.000	4217.884
10800.000	4309.244
11000.000	4400.103
11200.000	4489.215
11400.000	4575.638
11600.000	4658.855
11800.000	4738.455
12000.000	4813.923
12200.000	4884.808
12400.000	4951.156
12600.000	5013.130
12800.000	5070.385
13000.000	5123.317
13200.000	5172.834
13400.000	5218.654
13600.000	5259.752
13800.000	5295.676
14000.000	5326.839
14200.000	5354.114
14400.000	5378.271
14600.000	5399.791
14800.000	5418.942
15000.000	5435.648

OUTPUT - RIVER3/9

```
?-      ROUTE
THE WEIGHT FACTORS ARE ASSUMED TO BE 0.5
DO YOU WISH TO CHANGE ANY? (YES/NO)  NO
DEFINE U/S SEC. NO. WHERE ROUTING STARTS (I3)     1
DEFINE D/S SEC. NO. WHERE ROUTING FINISHES (I3) 11
DEFINE TIME STEP FOR COMPUTATION.  USE
SAME UNITS OF TIME AS USED FOR HYDROGRAPH (F9.4)          200.
ENTER START TIME WITH CONSISTENT UNITS (F9.3)  1400.0
ENTER FINISH TIME WITH CONSISTENT UNITS, (F9.3)15000.
```

HYDROGRAPH AT SEC. NO. 11

TIME	FLOWRATE
1400.000	3335.000
1600.000	3335.135
1800.000	3333.147
2000.000	3344.816
2200.000	3312.288
2400.000	3344.725
2600.000	3382.342
2800.000	3288.979
3000.000	3272.541
3200.000	3364.068
3400.000	3441.552
3600.000	3379.674
3800.000	3267.358
4000.000	3201.949
4200.000	3237.249
4400.000	3344.666
4600.000	3463.018
4800.000	3519.677
5000.000	3486.291
5200.000	3410.329
5400.000	3298.021
5600.000	3207.041
5800.000	3120.187
6000.000	3067.452
6200.000	3026.419
6400.000	3010.347
6600.000	3000.000
6800.000	3000.000
7000.000	3179.674
7200.000	4026.367
7400.000	4919.684
7600.000	5996.061
7800.000	7085.805
8000.000	7776.079
8200.000	8429.149

OUTPUT - RIVER3/10

```
    8400.000     9241.573
    8600.000    10001.907
    8800.000    10486.739
    9000.000    10824.222
    9200.000    11162.359
    9400.000    11659.443
    9600.000    12359.981
    9800.000    13206.846
   10000.000    14065.501
   10200.000    14847.776
   10400.000    15453.160
   10600.000    15829.071
   10800.000    15987.072
   11000.000    15960.140
   11200.000    15784.936
   11400.000    15497.727
   11600.000    15174.797
   11800.000    14765.795
   12000.000    14289.758
   12200.000    13744.334
   12400.000    13259.507
   12600.000    12741.053
   12800.000    12156.478
   13000.000    11746.403
   13200.000    11369.349
   13400.000    10814.897
   13600.000    10143.846
   13800.000     9459.336
   14000.000     8892.297
   14200.000     8483.380
   14400.000     8155.858
   14600.000     7865.744
   14800.000     7581.413
   15000.000     7280.501
 :-      CRITIC
SPECIFY SECTION NO,I3         11
ENTER DISCHARGE  F9.3      5000.
AT SEC  1 WITH Q=  5000.000
CRITICAL WATER LEVEL= 114.266 AND
CRITICAL ENERGY LEVEL= 116.399
 :-      OLD SECTION
DEFINE SECTION NO., I3        3
SECTION NO.  3 HAS  4 PTS....COORDS ARE
  0.0,128.0     0.0,108.0  100.0,108.0  100.0,128.0
 :-      STOP
      STOP
       6.791 CP SECONDS EXECUTION TIME
```

APPENDIX H

SUMMARY OF NOTATION

This appendix provides a dictionary of the notation used in this thesis. It should be noted that several variables serve more than one purpose. The appropriate definition can be inferred from the context and from the description provided with the variables. Where there is more than one definition of a variable, the location of the less frequent definition(s) is identified.

A	=	area of the cross section
$\bar{A}$	=	an error, expressed as a term of a Fourier Series
A_o	=	a coefficient on a term of a Fourier Series
a	=	a coefficient of regression (equation 4.12)
B	=	invert elevation at a section
b	=	a coefficient of regression (equation 4. 12)
C	=	celerity of a wave relative to water (Chapter 2)
	=	kinematic wave velocity
C1	=	a stability condition variable
C2	=	a stability condition variable
c	=	a coefficient of regression (equation 4. 12)
D	=	a coefficient of an error term
e	=	an exponent (equation 5. 26)
Fr	=	Froude Number

g = acceleration of gravity

= a displacement on the distance axis from the point about which a Taylor Series is written (Appendix B, C)

H = water surface elevation, (usually associated with a position in the space-time diagram)

HORZ = a variable discribing the horizontal distance from an arbitrary axis to a coordinate describing the channel cross section

h = water surface elevation

= a displacement on the time axis from the point about which a Taylor Series is written (Appendix B, C)

I = a subscript to denote a position on a space-time diagram

= inflow (equations 3.21 - 3.24)

i = $\sqrt{-1}$

J = a subscript to denote a position on a space-time diagram

K = a subscript to denote a position on a space-time diagram (Chapter 2, Appendix A)

= a parameter in the equation describing storage in an imaginary reservoir or an elementary reach

= a coefficient (Appendix F)

KN = Kinematic Courant Number

k	=	a coefficient in the equation relating attenuation and the number of reaches in a simulation (equation 5.26)
	=	a coefficient in the equation relating "reduced" attenuation to relative live storage (equation 5.29)
L	=	a subscript to denote a position on a space-time diagram
	=	total length of the channel (Chapter 5)
m	=	a coefficient (Appendix A)
N	=	number of reaches in the total length of the channel
NL	=	a parameter which describes the "bow" in the relationship between storage and flow rate (defined in figure 6.4)
n	=	Mannings roughness coefficient
	=	a coefficient (Appendix A)
O	=	outflow
P	=	wetted perimeter of a cross section (Chapter 2, figure 3.1, equations 3.42 - 3.47, equations 4.1 - 4.5 Appendix F)
	=	peak outflow divided by peak inflow or full bank flow rate
Q	=	flow rate
q	=	flow rate per unit width
$\bar{q}$	=	rate of lateral inflow
R	=	hydraulic radius
S	=	slope (Chapter 2, figure 3.1, equations 3.42 - 3.46, Appendix F)

SF =	skew factor, a parameter which describes the slope of the line from the base flow rate and the time of the centroid of the hydrograph to the peak outflow and the time of peak outflow (Defined in figure 6.3)
Sf =	slope of the friction line
So =	bed slope
ST =	Storage in an elementary reach or reservoir
T =	time (usually associated with a finite difference step)
Tb =	time base of the inflow hydrograph
Tc =	time of the centroid of the outflow hydrograph measured from the time of the centroid of the inflow hydrograph
Tp =	time of peak outflow measured from the time of peak inflow
Tw =	surface width
VERT=	a variable describing the horizontal distance from an arbitrary axis to a coordinate describing the channel cross section
Vo =	flow velocity under steady state conditions
w =	an exponent in the equation describing storage in an imaginary reservoir
X =	distance (usually associated with a finite difference step)
x =	distance
=	a parameter used in Muskinghum Flood Routing (equations 3.22 - 3.24)

Y	=	vertical depth of water
y	=	vertical depth of water
yo	=	vertical depth of water under steady state conditions
Z	=	Courant Number (Chapter 2)
	=	side slope of a triangular cross section (Chapter 3)
α	=	a parameter which identifies the location of the nucleus of a molecule in the x axis. Referenced to the upstream, lowest time level of the molecule
β	=	a parameter which identifies the location of the nucleus of a molecule in the t axis. Referenced to the upstream, lowest time level of the molecule
ϵ	=	error term due to finite difference approximation.
γ	=	a wave number (Appendix A)
θ	=	a parameter which identifies the location of the nucleus of a molecule on the x axis. Referenced to the centre of the molecule
σ	=	a wave number (Appendix A)
ϕ	=	a parameter which identifies the location of the nucleus of a molecule on the t axis. Referenced to the centre of the molecule

APPENDIX I

To provide further clarification of the procedure that could be followed in calibrating a kinematic flood routing model, the following example is provided. The numerical values used in this example are taken from tables 3. 3, 3. 4 and 3. 5. However, it will be assumed that these results are available only after a simulation of the particular physical system. The channel employed has been described previously in Chapter 2 as System 1. The waterway is 50, 000 feet long, 100 feet wide, with a depth of 20 feet. A symetrical triangular inflow hydrograph was employed as a description of the time variant inflow. A peak outflow of 0. 840 times the full bank flow was assumed. Figure 4. 2 shows both the inflow hydrograph and various outflow hydrographs predicted using a numerical solution of the momentum and continuity equations. It is intended that this example will show the calibration of a kinematic model to emulate the peak outflow. A discussion of the wave shapes will be made in the conclusions of this appendix.

The procedure follows the description presented in Chapter 5 on pages 153 to 155.

1. To describe the hydrograph adequately, a time step ΔT , of 200 seconds was assumed. It should be

noted that this results in the truncation of the peak inflow to 0.984 times that specified in the inflow hydrograph.

2. From the results presented in Appendix F, a value of C, the kinematic wave velocity, of 12.3 feet per second is assumed. This is equivalent to the full bank flow kinematic wave velocity. (With the hindsight provided by results presented in Chapter 5, this is an accurate estimate of the effective kinematic wave velocity.)

3. An estimate of the size of the distance step, ΔX, can be obtained from setting

$$\Delta X \doteq C \Delta T$$

The first estimate of ΔX subject to the constraint that $\frac{L}{\Delta X}$ is an integer is:

$$\Delta X = 2500 \text{ FEET}$$

Performing a simulation with $\alpha = 0.0$ and $\beta = 1.0$ yields a peak outflow of 0.843 times full bank flow. Performing the calculations with the nucleus located in the upper right hand corner of the molecule defined in Figure 3.2, as specified above, results in the maximum amount of attenuation for the given ΔX and ΔT with a nucleus inside the molecule.

4. Because the peak outflow does not correspond with the objective, it is necessary to modify the value of $S\Delta X$. To provide an estimate of the next value of $S\Delta X$ to use in an attempt to achieve the desired attenuation, the following steps may be taken. First, calculate the value of $S\Delta X$ for the completed simulation.

$$S = (2\alpha - 1) + (1 - 2\beta) C \frac{\Delta T}{\Delta X}$$

$$= (-1) + (-1) 12.3 \frac{200}{2500}$$

$$= -1.984$$

Thus:

$$S\Delta X = -1.984 \times 2500 = -4960 \text{ FEET}$$

Assuming that no attenuation occurs with $S\Delta X = 0.0$, a straight line may be plotted through the points

$$P = 1.0 \, , \; -S\Delta X = 0.0$$

and

$$P = 0.843 \, , \; -S\Delta X = 4960$$

This straight line may be extended to obtain the next estimate of $S\Delta X$, approximately 5, 000 feet.

It is not possible to increase the value of $S\Delta X$ by varying α and β subject to the constraint that the n ucleus must remain inside the molecule with the previously defined parameters. Thus, it is necessary to vary the values of ΔX and/or ΔT . For a system of arbitrary geometry with sections defined at regular or irregular intervals, modification of ΔX may be impractical due to the fact that the system definition will be significantly altered. Thus, the variation of ΔT may prove to be the most fruitful alternative in altering S. The limiting constraints on the selection of ΔT (and ΔX) are the discretization error and the stability requirements.

5. For the second interation, the simulation was repeated with $\alpha = 0.0$, $\beta = 1.0$, $\Delta X = 5,000$ feet and $\Delta T = 200$ seconds. This results in a peak outflow of 0.804 times the full bank flow. Clearly this is an over estimate of $S\Delta X$. For this simulation:

$$S\Delta X = ((2\alpha - 1) + (1 - 2\beta) C \frac{\Delta T}{\Delta X}) \Delta X$$

$$= ((-1) + (-1) 12.3 \frac{200}{5000}) 5000$$

$$= -7460 \text{ FEET}$$

6. The above simulation defines another point on the which could be used as a guide for further refinements. The third simulation is attempted with $S\Delta X = -5,000$ feet. Again it should be noted that there are four variables that may be used to modify $S\Delta X$:

α, β, ΔX, ΔT

For example a few of the points are:

α	β	ΔX	ΔT
0.00	0.5	5,000	variable*
0.246	1.0	5,000	200
0.25	0.5	10,000	variable*
0.374	1.0	10,000	200

Two of the above points, each identified with an asterisk, are shown in tables 3.4 and 3.5. Interpolation to obtain the 0.840 contours on these tables will give an indication of the interelation of the various parameters which define $S\Delta X$. The choice of the appropriate values of α β, ΔX and ΔT remains at the discretion of the user who must observe the constraint of stability and discretization error.

Figure 5.8 shows a comparison of the kinematic simulation performed with

$$\alpha = 0.0$$

$$\beta = 0.5$$

$$\Delta X = 5,000 \text{ feet}$$

$$\Delta T = 200 \text{ seconds}$$

and a dynamic solution. The two hydrographs have the same general shape. The peak of the dynamic simulation is slightly higher than the peak of the kinematic solution. Plotting the first simulation presented in this appendix on the graph may yield a result which agrees more closely with the dynamic solution.

In certain instances, there may be significant differences in the shapes of the two hydrographs. For such cases further research to determine methods of modifying the shape of the hydrograph predicted by the kinematic solution would be required. Preliminary results presented in Chapter 3 on figures 3.5 and 3.6 indicate that the hydrograph is insensitive to the variables used to define $S\Delta X$.