EFFECT OF FLOOR SLABS AND FLOOR BEAMS ON STATIC AND DYNAMIC BEHAVIOUR OF SHEAR WALL STRUCTURES

by

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SCOPE AND CONTENTS:

This thesis studies the effect of floor slabs on the static and dynamic behaviour of the shear wall structure. A single component has been analysed using the 'Matrix Transfer' technique along with Vlasov's thin walled elastic beam theory. Experimental verification was done on a small scale plexiglas eight storey model in the form of a channel section for both static and dynamic loading.

The thesis also deals with the analysis of the non-planar shear walls coupled through floor beams subjected to static loading. The continuum approach along with Vlasov's theory has been used in the analysis. Experimental verification was done on a small scale plexiglas model in the form of two equal angles connected by eight floor beams at equal spacing.
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NOTATIONS

The following symbols are used throughout this thesis without further definition. Other symbols are defined when used.

\( E \) Modulus of Elasticity
\( G \) Modulus of rigidity
\( v \) Poisson's ratio
\( X,Y \) Orthogonal axis
\( Z \) Vertical axis
\( \omega \) Principal Sectorial co-ordinate
\( A \) cross sectional area
\( I_x,I_y \) Moment of Inertia about \( X \) and \( Y \) axes
\( J \) Torsional rigidity
\( I_p \) Polar moment of inertia about shear center
\( I_\omega \) Sectorial moment of inertia
\( \theta \) Rotation about \( Z \) axis
\( \theta' \) Rate of change of rotation about \( Z \) axis
\( u,v \) Displacement of shear center in \( x \) and \( y \) directions
\( u',v' \) Slope in \( zx \) and \( zy \) planes
\( V \) Shear Force
\( M \) Moment
\( H \) Torque
\( B \) Bimoment
\( N \) Axial Force
\( a_x,a_y \) Co-ordinate of shear center
\( \rho \) Mass density of material
\( w \) Natural frequency
\( (') \) Differentiation with respect to time
\( (') \) Differentiation with respect to space.
CHAPTER I
INTRODUCTION

1.1 Description of Shear Wall

As buildings increase in height, it becomes necessary to ensure adequate lateral stiffness. This stiffness may be achieved in various ways of which the use of shear wall is very common and popular.

Although shear walls can be arranged in a building in innumerable ways, they can be broadly classified into two basic types. In an apartment building, shear walls are used alone and located on both sides of the corridor as shown in Fig. 1.1.1. In an office building, shear walls are located in the center to form a service core for staircases, elevators etc. This core is surrounded by a structural framing which are interconnected as shown in Fig. 1.1.2. In both the above types, shear walls serve the multipurpose function of supporting vertical and lateral loads, acting as partition walls and serving other useful functions.

Shear walls are normally interconnected by floor slabs at each floor level. These floor slabs act as highly rigid diaphragm in their own plane and bend and twist out of plane. Therefore, the slabs transmit and distribute lateral loads among the walls and also provide some resistance.
Typical Apartment Building With Shear Wall

FIG. 1.1.1

Typical Office Building With Shear Walls in the central Core Surrounded by Structural Framing

FIG. 1.1.2
to the deformation of the walls. The effect of overall interaction between the walls and the floors is to increase the lateral stiffness of the building and to reduce stress level in the walls.

Very often shear walls are pierced to provide openings for doors, windows or corridors. The arrangement may be thought of as two or more sets of walls connected by beams. These beams resist the deformation of the wall and increase the stiffness of the assembly.

Frequently the section of shear walls are in the form of open thin walled sections. Such beams are distinguished from solid beams by experiencing longitudinal stress as a result of torsion due to warping. Appropriate theory should be considered for dealing with such sections.

For these reasons, a complete analysis of a building as shown in Figs. 1.1.1 and 1.1.2 is a most complex problem encountered in structural engineering practice. The complexity is due to various interacting elements. The dynamic analysis is even more complex. Approximate design methods, neglecting complex interaction can be used for proportioning elements which often under estimates the stiffness of building. Therefore, more sophisticated techniques of analysis are required.

The general purpose of research on shear wall structure is firstly to understand fully the behaviour of different elements and secondly, to develop more realistic methods of
1.2 Shear Wall Project

The Canada Emergency Measures Organization is sponsoring an extensive program into behaviour of shear wall building. This project is conducted in the Department of Civil Engineering and Engineering Mechanics at McMaster University. The experimental part of the project consists of building small scale shear wall structures and studying their response due to static and dynamic lateral loadings. The theoretical part of the project consists of developing theories to explain the behaviour of shear wall structures, comparing theoretical results with experiments and developing simplified design method.

Tests have been carried on an eight feet model having E shaped section and made of non-reinforced micro-concrete. Afsar(1), Quareshi(2), Speirs(3), Raina(4) and Swift(5) studied different aspects of behaviour of shear wall structures.

1.3 Review of Past Works

Coull and Smith (6) compiled a comprehensive summary of the published literature concerning shear wall buildings.

Winokur and Gluck (7) developed a method to form lateral stiffness matrix of asymmetric building by combining lateral stiffness matrix of each element. Transverse stiffness of slab and warping torsional stiffness of individual
elements has been neglected.

The phenomenon of warping has been known to the aeronotical engineers for a long time but its application to shear wall structure is rather recent. Vlasov (8) developed the theory of thin walled open beams. Zbirohowski-Koscia (9) presented Vlasov's theory in simpler way and with an aim to make it usable by practicing engineers.

Afsar (1) has outlined various analytical and experimental approaches used in the shear wall study.

Quareshi (2) analysed the shear wall with rows of opening by frame analogy method and also conducted experiments on small scale models.

Speirs (3) studied the behaviour of floor slabs introduced in shear wall structure. His theoretical analysis is mainly based on the initial parameter approach of Vlasov.

Raina (4) studied response of shear wall structure under dynamic loading.

Swift (5) developed computer program based on matrix method, to solve asymmetric coupled shear wall. He also developed a program to analyse shear wall with floors.

Qadeer (10) and Qadeer and Smith (11) discussed the interaction between walls and slabs in a cross wall structure. Curves are given for equivalent width of slab. Experimental
work on a model was done for verification of the theory.

Taranath (12) studied open section with and without floors. Finite element treatment for floor slab is used. Multiple open section core structure coupled through floor slab is also examined for the case of static loading.

Beck (13), Rosman (14) analysed plane coupled shear wall by continuum method. The connecting beams are replaced by independently acting laminae. Coull and Choudhury (15), (16) developed design curves for different types of loading based on the continuous method of analysis.

Choudhury (17) discussed the solution single shear wall with openings by continuous method, equivalent frame method and finite element method. The behaviour of walls interconnected through floor slab is also examined. A method of complete analysis of shear wall/frame buildings taking into account their three-dimensional behaviour is presented.

Michael (18) made torsion analysis of a core wall consisting of two equal channels tied by beams at equal spacing by the continuum approach.

Jenkins and Harrison (19) analysed tall building with shear walls under bending and torsion. His bending analysis is based on stiffness matrix approach and torsion analysis is based on the theorem of minimum potential energy. The
warping stiffness of the open sections are neglected. Experiments are carried out on small scale plexiglas model in different stages.

Holmes and Astill (20) conducted experiments on a small scale shear wall structure under simulated wind load. Comparison of experimental values are made with theoretical consideration of simplified structure using Rosman's (14) theory.

Rosman (21) presented analysis of pierced torsion boxes subjected to torsion loading, arbitrarily distributed along the height. Treatment for two channel box and four angle box is done. Determination of approximate fundamental period of torsional vibration is also included.

Gluck (22) presented a lateral load analysis by three dimensional continuous method for structures consisting of simple or coupled, prismatic or non-prismatic, shear walls and frames arranged asymmetrically in floor plan. Connecting beam on the shear wall is replaced by an 'elastic media' of known stiffness properties. Treatment for thin walled open section is included. Differential equations are obtained for three-generalised displacements. In his derivation of stiffness matrix for 'elastic media', slight inconsistency of the use of 'thin walled beam theory' was noticed. Modification of few elements of matrix has been suggested by Biswas and Tso (24) in a discussion of Gluck's (22) paper.
Macleod (23) commented on the limitation of the use of continuum method when the bending stiffness of the wall approach that of connecting beams. A criterion is developed for assessing when this effect may be important. Comparison is made with more accurate frame analysis.

Coull and Irwin (28) presented a method for the analysis of the distribution of load amongst the shear walls of a three dimensional multistorey building subjected to bending and torsion. The method is based on the continuum approach.

1.4 Present Investigation

In the second and third chapters of this thesis, particular interest is given on shear wall with floors. A shear wall structure consisting of channel section with floor slabs is analysed for static and dynamic loading using the 'Matrix Transfer' method. To the best of the author's knowledge, this method has not been used to solve similar problems before. Experimental study was conducted on a small scale plexiglas model. This model was subjected to lateral loading at different floor levels when the recorded deflections and strains were studied. It was then subjected to lateral vibration to determine the resonant frequencies. The relative strain distribution at resonance is also studied.

In the fourth chapter, particular interest is on the nonplanar shear walls coupled by floor beams where warping
due to torsion of piers is taken into account. Differential equations are obtained using the continuum approach. The present formulation is applicable to two shear walls connected by one row of beams and subjected to forces and torques distributed along its height. The experimental study was performed on a small scale plexiglas model consisting of two equal angle sections connected by beams at equal spacing. It was subjected to a force and a torque at top. The resulting strains and deflections are then analysed. A comparison of the experimental results with the theory is made in all three chapters.
CHAPTER II
STATIC STUDY OF SHEAR WALL WITH FLOORS

2.1 Summary

In this chapter, a shear wall structure consisting of a channel section with floor slab is analysed for static lateral load. The 'Matrix Transfer' technique is used. An experiment performed on a small scale plexiglas model Fig. (2.1.1) is described and the experimental results are compared with theoretical predictions.

2.2 Matrix Transfer Method

This method was originally developed by Holtzer(25) for treating torsional vibrations of shafts with lumped system. Myklestad(26) used a similar method for study of beam vibration problems. It was modified by Thomson(27) to extend its applicability to more general problems. Application of such method to static problem is less common.

Consider a system with n points and n elements along its length as shown in Fig.(2.2.1). For any point i there are two sub-point (i)_ and (i)_+ denoting position before and after the ith point. Generalised force and displacement quantities of a sub-point is assembled in a column matrix called state vector \( \{Z\} \). The part of the structure between (i)_ and (i-1)_+ is defined as ith field and that between (i)_ and (i)_+ is defined as ith point. Field transfer matrix \( [F_i] \) relates the state vectors of two subpoints in ith field and is obtained from

10
TEST STRUCTURE

FIG. 2.1.1

shear center

center of gravity

floor height 6"
floor slab 0.5" th.
\{Z_i^+\} State Vector at Point i & Sub point +

\[F_i\] Field Transfer Matrix at i th Field

\[P_i\] Point Transfer Matrix at i th Point

\[L_i\] Load Vector at i th Point

FIG. 2.2.1
solution of differential equation of the element. Point transfer Matrix \([P_i]\) relates the state vectors of two subpoints in \(i\)th point. Load vector at \(i\)th point is defined as \({L_i}\). The relations can be expressed as

\[
\{Z^+_i\} = [F_i]\{Z^-_{i-1}\} \quad (2.2.1)
\]

\[
\{Z^+_i\} = [P_i]\{Z^-_i\} + \{L_i\} \quad (2.2.2)
\]

where \(i = 1, 2, \ldots n\)

From these two sets of eqs. (2.2.1) and (2.2.2) it is possible to eliminate the state vectors of the inner points and get a relation between the state vectors of the extreme points \((0)_+\) and \((n)_+\)

\[
\{Z^+_n\} = [A]\{Z^+_0\} + \{N\} \quad (2.2.3)
\]

Where \([A]\) is combined transfer matrix and \(\{N\}\) is combined load vector obtained from multiplication of appropriate matrices.

Mathematically, they are given by

\[
[A] = [P_n][F_n] \ldots [P_2][F_2][P_1][F_1] \quad (2.2.3a)
\]

\[
\{N\} = [P_n][F_n] \ldots [P_2][F_2]\{L_1\} \\
+ [P_n][F_n] \ldots [P][F_3]\{L_2\} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\]

\[
+ [P_n][F_n]\{L_{n-1}\} [F_{n-1}]\{L_{n-2}\} \quad (2.2.3b)
\]

\[
+ [P_n][F_n]\{L_{n-1}\} + \{L_n\}
\]

The next step is to substitute the boundary conditions in the boundary state vectors, namely \(\{Z^+_n\}\) and \(\{Z^+_0\}\) in eq. 2.2.3.
Simplification of this matrix equation will yield a set of linear simultaneous equation which can be solved. The solution will give the values of boundary state vectors \{Z_n^+\} and \{Z_o^+\}. The state vector at other points follow from the eq. 2.2.1 and 2.2.2 as,

\[
\begin{align*}
\{Z_1^-\} &= [F_1][Z_o^+] \\
\{Z_1^+\} &= [P_1][F_1][Z_o^+] + \{L_1\} \\
\{Z_2^-\} &= [F_2][P_1][F_1][Z_o^+] + [F_2]\{L_1\} \\
\{Z_2^+\} &= [P_2][F_2][P_1][F_1][Z_o^+] + [P_2][F_2]\{L_1\} + \{L_2\} \\
\{Z_n^-\} &= [F_n][P_{n-1}][F_{n-1}]\ldots[P_1][F_1][Z_o^+] \\
&\quad + [F_n][P_{n-1}][F_{n-1}]\ldots[P_2][F_2]\{L_1\} \\
&\quad + [F_n][P_{n-1}][F_{n-1}]\ldots[P_3][F_3]\{L_2\} \\
&\quad \ldots \ldots \ldots + [F_n]\{L_{n-1}\}
\end{align*}
\]

(2.2.4)

2.3 Application of Matrix Transfer Method

The structure considered consists of a prismatic monosymmetric section with equally spaced slabs. The floor slab represents 'point' and the part of the beam in between floor slabs represents 'field' as defined earlier. The simplified model to be used in Matrix Transfer method is shown in Fig. 2.3.1. In this case n equals eight and all field transfer matrices and points transfer matrices are identical.
SIMPLIFIED MODEL

FIG. 2.3.1

FORCE & DISPLACEMENT QUANTITIES

FIG. 2.3.2
2.3.1 State Vector

The state vector is an eighth order column matrix consisting of the following terms

\[
\begin{pmatrix}
V \\
V' \\
M \\
V \\
\theta \\
\theta' \\
B \\
H
\end{pmatrix}
\]

The notations are explained and illustrated in fig. 2.3.2.

2.3.2 Field Transfer Matrix

The field is a prismatic thin walled beam of length \( \ell \) and its transfer matrix is obtained from the solution of the differential equations. When referred to principal axes, the uncoupled differential equations for bending in \( y \)-direction and rotation are (Eq. A.4, Appendix-A):

\begin{align*}
\frac{EI_x}{V} V'' &= 0 \quad (2.3.1) \\
\frac{EI_\omega}{V} \theta'' - \frac{GJ\theta'''}{V} &= 0 \quad (2.3.2)
\end{align*}

The solution of the first equation yields the following expressions for displacement, slope, moment and shear.

\[
\begin{align*}
v(Z) &= D_1 Z^3/6 + D_2 Z^2/2 + D_3 Z + D_4 \\
v'(Z) &= D_1 Z^2/2 + D_2 Z + D_3 \\
M(Z)/EI_x &= v'' = D_1 Z + D_2 \\
V(Z)/EI_x &= -v''' = -D_1
\end{align*}
\]
Where \( D_1, D_2, D_3 \) and \( D_4 \) are constants of integration determined from the boundary condition at \( Z=0 \) namely

\[
\begin{align*}
\nu &= \nu(0), \nu' = \nu'(0), M = M(0) \text{ and } \psi = \psi(0) \\
The state vector at \( Z = l \) can be expressed in terms of state vector at \( Z = 0 \) by the relation,
\end{align*}
\]

\[
\begin{align*}
\nu(l) &= (0) + \frac{\ell^2}{EI} M(0) - \frac{\ell^3}{6EI} \psi(0) \\
\nu'(l) &= \nu'(0) + \frac{\ell}{EI} M(0) - \frac{\ell^2}{2EI} \psi(0) \\
M(l) &= M(0) + \ell \psi(0) \\
\psi(l) &= \psi(0)
\end{align*}
\] (2.3.4)

The solution of the second equation yields the following expressions for rotation, warping, bimoment and torque.

\[
\begin{align*}
\theta(Z) &= c_1 + c_2 Z + c_3 \sinh KZ + c_4 \cosh KZ \\
\theta'(Z) &= c_2 + c_3 K \cosh KZ + c_4 K \sinh KZ \\
B(Z)/EI &= -\theta'' = -c_3 K^2 \sinh KZ - c_4 K^2 \cosh KZ \\
H(Z) &= -EI \Omega^{111} + CJ\Omega^1 = C_2GJ
\end{align*}
\] (2.3.5)

Where \( K = \sqrt{GJ/\Omega} \)

The constants \( c_1, c_2, c_3 \) and \( c_4 \) are constants of integration determined from the boundary condition at \( Z = 0 \) namely

\[
\begin{align*}
\theta &= \theta(0), \theta' = \theta'(0), B = B(0) \text{ and } H = H(0) \\
The boundary condition at \( Z = l \) are
\end{align*}
\]

\[
\begin{align*}
\theta &= \theta(l), \theta' = \theta'(l), B = B(l) \text{ and } H = H(l)
\end{align*}
\]
They can be expressed in terms of \( \theta(0) \), \( \theta'(0) \), \( B(0) \) and \( H(0) \) by the following expressions:

\[
\begin{align*}
\theta(\ell) &= \theta(0) - \frac{1}{K} \sinh K\ell \theta'(0) - \frac{1}{GJ}(1 - \cosh K\ell) B(0) \\
&\quad + \frac{1}{GJ} (\ell - \frac{1}{K} \sinh K\ell) H(0) \\
\theta'(\ell) &= \cosh K\ell \theta'(0) - \frac{K}{GJ} \sinh K\ell B(0) \\
&\quad + \frac{1}{GJ} (1 - \cosh K\ell) H(0) \\
B(\ell) &= -\frac{GJ}{K} \sin K\ell \theta'(0) + \cosh K\ell B(0) \\
&\quad + \frac{1}{K} \sinh K\ell H(0) \\
H(\ell) &= H(0)
\end{align*}
\] (2.3.6)

Eq. 2.3.4 and eq. 2.3.6 can be combined in a single matrix eq. as:

\[
\{Z(\ell)\} = [F]\{Z(0)\}
\] (2.3.7)

where

\[
\begin{align*}
\{v(\ell)\} &= \{v(0)\} \\
\{v'(\ell)\} &= \{v'(0)\} \\
\{M(\ell)\} &= \{M(0)\} \\
\{V(\ell)\} &= \{V(0)\} \\
\{Z(\ell)\} &= \{Z(0)\}\text{ and } Z(0) = \begin{bmatrix} \theta(0) \\ \theta'(0) \\ B(0) \\ H(0) \end{bmatrix}
\end{align*}
\]
Field transfer matrix is an eigth order square matrix

\[
[F] = \begin{pmatrix}
  f_{11} & f_{12} & f_{13} & f_{14} & 0 & 0 & 0 & 0 \\
  0 & f_{22} & f_{23} & f_{24} & 0 & 0 & 0 & 0 \\
  0 & 0 & f_{33} & f_{34} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & f_{44} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & f_{55} & f_{56} & f_{57} & f_{58} \\
  0 & 0 & 0 & 0 & 0 & f_{66} & f_{67} & f_{68} \\
  0 & 0 & 0 & 0 & 0 & f_{76} & f_{77} & f_{78} \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & f_{88}
\end{pmatrix}
\]

(2.3.8)

Where the non zero elements are

\[
\begin{align*}
f_{11} &= 1, & f_{12} &= \zeta, & f_{13} &= \zeta^2/2EI_x, & f_{14} &= -\zeta^3/6EI_x, \\
\end{align*}
\]

\[
\begin{align*}
f_{22} &= 1, & f_{23} &= \zeta/EI_x, & f_{24} &= -\zeta^2/2EI_x, \\
\end{align*}
\]

\[
\begin{align*}
f_{33} &= 1, & f_{34} &= \zeta, & f_{44} &= 1, \\
\end{align*}
\]

\[
\begin{align*}
f_{55} &= 1, & f_{56} &= \sinh \kappa \zeta/K, \\
\end{align*}
\]

\[
\begin{align*}
f_{57} &= (1 - \cosh \kappa \zeta)/GJ, & f_{58} &= (\zeta - \frac{1}{K} \sinh \kappa \zeta)/GJ, \\
\end{align*}
\]

\[
\begin{align*}
f_{66} &= \cosh \kappa \zeta, & f_{67} &= -K \sinh \kappa \zeta/GJ, \\
\end{align*}
\]

\[
\begin{align*}
f_{68} &= (1 - \cosh \kappa \zeta)/GJ, & f_{76} &= -GJ \sin \kappa \zeta/K, \\
\end{align*}
\]

\[
\begin{align*}
f_{77} &= \cosh \kappa \zeta, & f_{78} &= \sinh \kappa \zeta/K, & f_{88} &= 1.
\end{align*}
\]
2.3.3 Action of Floor Slab

Vlasov (1) considered the effect of a diaphragm on the behaviour of a thin walled beam. In a shear wall structure, floor slab is equivalent to diaphragm. Vlasov assumed that the diaphragm acts as a plate in torsion and derived the following relationship for the bimoment applied to the shear wall by the action of the slab (Fig. 2.3.4(a)).

\[ B_t = \frac{Et^3bd}{6(1+\nu)} \theta' \]  
\[ (2.3.9) \]

Where:
- \( b \) = width of slab
- \( d \) = length of slab
- \( t \) = thickness of slab
- \( \nu \) = Poisson's ratio
- \( E \) = modulus of elasticity of slab
- \( \theta' \) = warping of the shear wall at the level of slab.

In this derivation Vlasov neglected the effect of bending of slab (Fig. 2.3.4(b)) due to fixity of walls. This can be considered by treating the slab as a series of beams running between the flanges. The center line of the beams is the locus of point of contraflexure for each beam. If a cut is made along that line, there will be relative displacement to the left and right of the cut. (Fig. 2.3.4(c)). Shear force \( q \) will develop along the line to maintain continuity (Fig. 2.3.4 (d)). For an element of beam at a distance \( \xi \) from the wall, the sectorial areas at the center
(a) Torsion of slab  
(b) Bending of slab  
(c) Slab cut at center  
(d) Shear force  
(e) Sectorial area  
(f) Bimoment from slab
(Fig. 2.3.4(e)) considering rigid arms attached to thin walled beam are:

\[ \omega_r = \xi d, \omega_l = -\xi d. \]

Discrepancy of displacement is

\[ \delta = (\omega_r - \omega_l) \theta' = 2\xi d \theta' \]

Shear force develops to maintain continuity considering bending deformation only

\[ q = \frac{2Et^3\xi}{(1-v^2)d^2} \theta' \]

The bimoment due to the shear force is

\[ dB = q(\omega_r - \omega_l) d\xi = 2q \xi d d\xi \]

The total bimoment due to bending is obtained on integration

\[ B_b = \int_0^b dB = \frac{4Et^3b^3}{3d(1-v^2)} \theta' \]  \hspace{1cm} (2.3.10)

The combined bimoment due to torsion and bending is

\[ B_s = B_t + B_b = D \theta' \]  \hspace{1cm} (2.3.11)

Where

\[ D = \left( \frac{Et^3bd}{6(1+v)} + \frac{4Et^3b^3}{3d(1-v^2)} \right) \]  \hspace{1cm} (2.3.12)

From Fig. 2.3.4 (a), the bimoment contribution from slab is related to the bimoments in the walls immediately above and below the floor slab.

\[ B_+ = B_- - B_s = B_- - D \theta' \]  \hspace{1cm} (2.3.13)

2.3.4. Point Transfer Matrix and Load Vector

The Point transfer matrix [P] is a square matrix of order eighth. It is obtained from the consideration of
equilibrium and compatibility:

\[
[P] = \\
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -D & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

The load vector is a column matrix of order eight.

\[
{L_i} = \begin{pmatrix}
0 \\
0 \\
0 \\
-P_i \\
0 \\
0 \\
-Q_i
\end{pmatrix}
\]

Where \( i = 1, 2, \ldots, 8 \)

and \( P_i \) is applied load

and \( Q_i \) is the applied
torque at ith level.

2.3.5 Boundary Conditions

The shear wall is fixed at the base and free at top.

Therefore, the state vectors at the base and the top
can be written as
\[
\{Z^+_o\} = \begin{pmatrix}
0 \\
0 \\
M^+_o \\
V^+_o \\
0 \\
0 \\
B^+_o \\
H^+_o \\
\end{pmatrix}
\quad \text{and} \quad
\{Z^+_8\} = \begin{pmatrix}
v^+_8 \\
v^+_8' \\
0 \\
0 \\
0 \\
\theta^+_8 \\
\theta^+_8' \\
0 \\
\end{pmatrix}
\]

Substituting the above conditions in eq. 2.2.3 and making some rearrangement of terms the following eq. is obtained.

\[
[B] \{Y\} = \{N\} \quad (2.3.14)
\]

Where

\[
\{Y\} = \begin{pmatrix}
v^+_8 \\
v^+_8' \\
M^+_o \\
V^+_o \\
\theta^+_8 \\
\theta^+_8' \\
B^+_o \\
H^+_o \\
\end{pmatrix}
\quad \text{and} \quad \{N\} \text{ is defined in eq. 2.2.3b}
\]

and

\[
[B] = \begin{pmatrix}
1 & 0 & -a_{13} & -a_{14} & 0 & 0 & 0 & 0 \\
0 & 1 & -a_{23} & -a_{24} & 0 & 0 & 0 & 0 \\
0 & 0 & -a_{33} & -a_{34} & 0 & 0 & 0 & 0 \\
0 & 0 & -a_{43} & -a_{44} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -a_{57} & -a_{58} \\
0 & 0 & 0 & 0 & 0 & 1 & -a_{67} & -a_{68} \\
0 & 0 & 0 & 0 & 0 & 0 & -a_{77} & -a_{78} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_{87} & -a_{88} \\
\end{pmatrix}
\]
Here 'a' denotes the elements of matrix [A] as defined in eq. 2.2.3 (a). The solution is obtained by inversion

\[ \{Y\} = [B]^{-1}\{N\} \]  

(2.3.15)

Knowing \{Y\}, \{Z^+_O\} and \{Z^+_B\} can be formed. State vector at other points are obtained from expressions in eq. 2.2.4.

2.4 Computer Program

A computer program based on the above analysis has been written. The input data are the geometric and elastic properties and loading of the structure. The output quantities are the state vectors at all floor levels. The present program is for identical floor slabs, equal storey heights and prismatic section. Extension for stepped cases or different storey heights and floor slabs can be made with little modification.

The flow chart is given in Fig. 2.4.1 and the computer program is included in Appendix-B.

2.5 Experiment

An experiment was done on a small scale plexiglas model (Fig. 2.1.1). It was made by assembling different components representing walls and floors. The base of the model was connected to a thick base plate which in turn was fixed to two heavy I sections to achieve fixity (Fig. 2.5.1). It was loaded at the 8th, 6th and 4th floor respectively, one floor at a time, by hanging weights over
START

READ & PRINT INPUT DATA

FORM TRANSFER MATRICES P & F

FORM MATRICES A & N

FORM MATRIX B

INVERT MATRIX B

CALCULATE STATE VECTORS

PRINT STATE VECTORS

STOP

FLOW CHART

FIG. 2.4.1
EXPERIMENTAL SET UP

FIG. 2.5.1
FIG. 2.5.2

(a) Strain Gauge Positions

(b) Dial Gauge Positions
Fig. 2.5.3

EXPERIMENTAL SET UP FOR STATIC TEST
a pulley. Strain gauges were attached at the middle of 1st, 2nd and 5th storey. Leads from the strain gauges were hooked up to a strain indicator through switch boxes; and strain readings at every increment of loading were taken. Deflections are measured from readings of dial gauges mounted at different points of the structure (Fig. 2.5.2). The strain gauge and dial gauge readings are tabulated in Appendix C. Fig. 2.5.3 shows the experimental set up for the case with loading at top.

The following is a list of equipment and materials used in this experimental work.

A. Model Material: Plexiglas
   Elastic properties: $E = 0.40 \times 10^6$ psi, $\nu = 0.35$

B. Electric Resistance Strain Gauges
   Make: Micro Measurement
   Type: EA-41-25086-120
   suitable for plastic
   Resistance: $120 \ \Omega \pm 0.15\%$
   Gauge Factor: $2.01 \pm 0.5\%$

C. Dial Gauges:
   Make: Baty
   Reading: .001 in and .0001 in

D. Strain Indicator:
   Make: Budd Corporation
   Reading: Directly calibrated to strain in $\mu$in/in
   Range: $\pm 40,000 \ \mu$in/in
2.6 Results and Discussion

The linearity of the test structure is checked in fig. 2.6.3. The comparison of theoretical and experimental data as plotted in fig. 2.6.3 to 2.6.11 shows that the structure is not so stiff as predicted by considering both torsional and flexural stiffness of floor slabs. If only the torsional stiffness of floor slab is taken, the theoretical analysis gives a mathematical model which is more flexible than the actual structure. The difference between theory and experiment attributed to the local bending of the wall section at the joint of the floor slab as shown in fig. 2.6.1.

As a result of this bending, the joint is not rigid which in turn reduces the shear force $q$ at the centerline of slab (Fig. 2.3.4d). To allow for this effect, the bimoment contribution from flexure of the floor slab is modified by a factor $K$. The effect of the floor is then expressed as:

$$D = \left[ \frac{Et^3bd}{6(1+v)} + K \frac{4Et^3b^3}{3d(1-v^2)} \right]$$

An approximate method to assess the value of $K$ is given below. Consider a one bay multistory frame as in fig. 2.6.1. Assuming points of contraflexure are at the center of storey height. Let $M$ be the moment induced at the end of the slab strip if the joints do not rotate locally. Due to the local bending of the joints the final moment is $KM$ where $K$ is obtained as
Deflected Shape in Theory

Actual Deflected Shape

Thin Strip

FIG. 2.6.1

Distributed Factor

MOMENT DISTRIBUTION

FIG. 2.6.2
Experimental Point o
Theoretical for Slab
Torsion Only
Torsion + Eff. Bending
Torsion + Bending
Neglecting Slab

Load at 8th Floor

12.5 lbs

FIG. 2.6.3
FIG. 2.6.4

Experimental Point ○
Theoretical for Slab
Torsion Only ———
Torsion + Eff. Bending ———
Torsion + Bending ———
Neglecting Slab ———

Load at 8th Floor

HEIGHT (inch)

DEFLECTION OF q (inch)
Figure 2.6.5

- Experimental Point
- Theoretical for Slab
- Torsion Only
- Torsion + Eff. Bending
- Torsion + Bending
- Neglecting Slab
Experimental Point

Theoretical for Slab

Torsion Only

Torsion + Eff. Bending

Neglecting Slab

Load at 5th Floor

12.5 lbs
Load at 6th Floor

Experimental Point ○
Theoretical for Slab
Torsion Only ———- ———-
Torsion + Eff. Bending ———- ———-
Torsion + Bending ———- ———-
Neglecting Slab ———- ———-

FIG. 2.6.7
Experimental Point O
Theoretical for Slab
Torsion Only -----
Torsion + Eff. Bending------
Torsion + Bending -------
Neglecting Slab -------

FIG. 2.6.8
Load at 4th Floor

Experimental Point ○
Theoretical for Slab
Torsion Only ———
Torsion + Eff. Bending —— —
Torsion + Bending ————
Neglecting Slab ————

FIG. 2.6.9
Experimental Point ○

Theoretical for Slab

Torsion Only ————

Torsion + Eff. Bending ————

Torsion + Bending ————

Neglecting Slab ————

Load at 4th Floor

FIG. 2.6.10
Theoretical for Slab
Torsion Only
Torsion + Eff. Bending
Torsion + Bending
Neglecting Slab

Load at 4th Floor

FIG. 2.6.11
Experimental Point  ○
Theoretical for Slab
Torsion Only ————
Torsion + Eff. Bending ————
Torsion + Bending ————
Neglecting Slab ————

Load at 8th Floor

STRAIN DISTRIBUTION AT LEVEL A

FIG. 2.6.12
Experimental Points
Theoretical for Slab
Torsion Only
Torsion + Eff. Bending
Torsion + Bending
Neglecting Slab

Load at 8th Floor

STRAIN DISTRIBUTION AT LEVEL B

FIG. 2.6.13
Theoretical for Slab
Torsion Only
Torsion + Eff. Bending
Torsion + Bending
Neglecting Slab

Experimental Point

Load at 6th Floor

STRAIN DISTRIBUTION AT LEVEL A

FIG. 2.6.14
Experimental Point
Theoretical for Slab
Torsion Only
Torsion + Eff. Bending
Torsion + Bending
Neglecting Slab

Load at 6th Floor

STRAIN DISTRIBUTION AT LEVEL B

FIG. 2.6.15
Experimental Point ○
Theoretical for Slab
Torsion Only
Torsion + Eff. Bending
Torsion + Bending
Neglecting Slab

Load at 4th Floor

Fig. 2.6.16
Experimental Point ○
Theoretical for Slab
Torsion Only
Torsion + Eff. Bending
Torsion + Bending
Neglecting Slab

Load at 4th Floor

STRAIN DISTRIBUTION AT LEVEL B

FIG. 2.6.17
LOAD vs. STRAIN DIAGRAM FOR MODEL WITH FLOORS

FIG. 2.6.18
\[ K = \frac{1}{1 + \left( \frac{t_s}{t_b} \right)^3 \frac{h}{2d}} \]  

(2.6.2)

Where \( t_s \) and \( t_b \) are thickness of slab and beam flange respectively. The procedure of obtaining \( K \) is by a moment distribution scheme as shown in fig. 2.6.2.

If the ratio \( t_s/t_b \) is very small, the value of \( K \) is unity and the total flexural stiffness of the slab is effective. On the other hand if \( t_s/t_b \) is large, there will be large local bending of the flange which will violate the hypothesis of non deformable section of Vlasov. Therefore in order to use Vlasov's theory, the expression of \( K \) is valid only for \( t_s/t_b < 3 \).

For the model structure the value of \( K \) is 0.2545 and the displacement/rotation plots are found to have reasonable agreement with the experiment.

The strain distribution are plotted in fig. 2.6.12 to 2.6.17 and found to have reasonable agreement with the experiment.
CHAPTER III

DYNAMIC STUDY OF SHEAR WALL WITH FLOORS

3.1 Summary

In this chapter, the same shear wall structure treated in Chapter 2 is analysed, for dynamic loading. The 'Matrix Transfer' method is used in the analysis. A dynamic test was carried out to determine natural frequencies. The strain distribution at resonance was also determined. The experimental values are compared with the theoretical predictions.

3.2 Matrix Transfer Method

As in static case, field transfer matrix \([F_i]\) relates the state vector \(\{z\}\) of two subpoints in ith field. The point transfer matrix \([P_i]\) relates the state vector \(\{z\}\) of two subpoints in ith point. The application of the method is the same in the dynamic case except the external loading is replaced by inertial forces which appear in the point transfer matrix \([P_i]\). Consideration of equation for vibration is necessary for obtaining field transfer matrix \([F_i]\). The relation between state vectors can be expressed as

\[
\{z_i^-\} = [F_i]\{z_{i-1}^+\} \tag{3.2.1}
\]

\[
\{z_i^+\} = [P_i]\{z_i^-\} \tag{3.2.2}
\]

Where \(i = 1,2;...n\)

From these relations, the state vectors at inner points can be eliminated and the relation between state vectors of extreme
points \((0)_{+}\) and \((n)_{+}\) can be expressed as:

\[
\{z^+\}_n = [A]\{z^+\}_o
\] (3.2.3)

Where \([A]\) is the combined transfer matrix and is defined as

\[
[A] = [P_n][F_n]...[P_2][F_2][P_1][F_1]
\] (3.2.3a)

Substituting boundary conditions in the boundary state vectors namely \(\{z^+_n\}\) and \(\{z^+_o\}\) in eq. 3.2.3, a set of homogeneous linear simultaneous equations are obtained. They can be expressed in matrix form as

\[
[R]\{X\} = 0
\] (3.2.4)

Where \(\{X\}\) is a vector formed by collecting non zero terms of state vector \(\{z^+_o\}\). The matrix \([R]\) is obtained from matrix \([A]\) depending on boundary condition.

For non trivial solution, the determinant of \(R\) must vanish. Thus

\[
|R| = 0
\] (3.2.5)

The eq. 3.2.5 is the condition to determine natural frequency of vibration of the structure.

3.3 Theoretical Analysis

The state vector is the same as used in Chapter 2. Fig. 2.3.1 and Fig. 2.3.2 are refered to for the simplified model and the illustration of notations.

3.3.1 Field Transfer Matrix

The field transfer matrix for a thin walled beam of length \(l\) is determined from the solution of differential equation of free vibration. For mono-symmetric section, the
equations are obtained by substituting \( a_y = 0 \) in the eq. A.5 (Appendix A). The first and the third equations are uncoupled and represent independent extensional vibration and flexural vibration in \( x \)-direction. The remaining equations representing coupled torsional and flexural vibration in \( y \)-direction are:

\[
E I_x v^{IV} + \rho A v^{\dddot{}} - \rho I_x v^{\dfrac{\dddot{}}{2}} - \rho A a_x \theta^{\dddot{}} = 0
\]  
(3.3.1a)

\[
E I_\omega \theta^{IV} - GJ\theta^{\dddot{}} + \rho I_\theta^{\dddot{}} - \rho I_\omega \theta^{\dfrac{\dddot{}}{2}} - \rho A a_x v^{\dfrac{\dddot{}}{2}} = 0
\]  
(3.3.1b)

Assuming periodic solution of the form

\[
\begin{bmatrix} v \\ \theta \end{bmatrix} = \begin{bmatrix} y \\ \phi \end{bmatrix} e^{i \omega t}
\]  
(3.3.2a)

and substituting eq. 3.3.2 in eq. 3.3.1, there is obtained

\[
E I_x y^{IV} - w^2 \rho A y + w^2 \rho I_x y^{\dfrac{\ddot{}}{2}} + w^2 \rho A a_x \phi = 0
\]  
(3.3.3a)

\[
E I_\omega \phi^{IV} - GJ\phi^{\dddot{}} - w^2 \rho I_\theta \phi + w^2 \rho I_\omega \phi^{\dfrac{\ddot{}}{2}} + w^2 \rho A a_x y = 0
\]  
(3.3.3b)

Expressing \( \phi \) in terms of \( y \) from eq. 3.3.3a there is obtained

\[
\phi = B_1 y^{IV} + B_2 y^{\dfrac{\ddot{}}{2}} + B_3 y
\]  
(3.3.4)

Where

\[
B_1 = - E I_x (w^2 \rho A a_x)
\]
\[
B_2 = - I_x (A a_x); \quad B_3 = 1/a_x
\]

Eliminating \( \phi \) from eq. 3.3.3b using eq. 3.3.4, there is obtained

\[
B_4 y^{VIII} + B_5 y^{VI} + B_6 y^{IV} + B_7 y^{\dfrac{\dddot{}}{2}} + B_8 y = 0
\]  
(3.3.5)
Where
\[ B_4 = -E^2 I_\omega I_x \]
\[ B_5 = E GJ I_x - 2w^2 \rho E I_\omega I_x \]
\[ B_6 = w^2 \rho \left( E I_\omega A + E I_p I_x + GJ I_x \right) - w^4 \rho^2 I_\omega I_x + (3.3.5a) \]
\[ B_7 = -w^2 \rho GJA + w^4 \rho^2 \left( I_p I_x + I_\omega A \right) \]
\[ B_8 = w^4 \rho^2 \left( -I_p A + A^2 a_x^2 \right) \]

Assuming solution of the form
\[ y = c e^{mz} \]

The characteristic equation is
\[ B_4 m^8 + B_5 m^6 + B_6 m^4 + B_7 m^2 + B_8 = 0 \quad (3.3.6) \]

Let the eight roots of this polynomial are
\[ m_1, m_2, m_3, \ldots, m_8. \]

The solution can then be expressed as
\[ v(z) = e^{iwt} \sum_{i=1}^{8} K_i e^{m_i z} \]
\[ v'(z) = e^{iwt} \sum_{i=1}^{8} K_i m_i e^{m_i z} \]

\[ M(z) = E I_x v'' = e^{iwt} \sum_{i=1}^{8} \left( \sum_{i=1}^{3} K_i m_i^2 m_i^i z \right) E I_x \]

\[ V(z) = -E I_x v''' = -e^{iwt} \sum_{i=1}^{8} \left( \sum_{i=1}^{3} K_i m_i^3 m_i^i z \right) E I_x \]

\[ \theta(z) = e^{iwt} \sum_{i=1}^{8} \left( B_1 m_i^4 + B_2 m_i^2 + B_3 \right) K_i e^{m_i z} \]

\[ \theta'(z) = e^{iwt} \sum_{i=1}^{8} \left( B_1 m_i^5 + B_2 m_i^3 + B_3 m_i \right) K_i e^{m_i z} \]

\[ B(z) = -E I_\omega \theta''' = -e^{iwt} \sum_{i=1}^{8} \left( \sum_{i=1}^{6} K_i m_i^6 + B_2 m_i^4 + B_3 m_i^2 \right) \]

\[ \times K_i e^{m_i z} \]
\[ H(z) = -E I \omega \theta'\prime\prime' + GJ \theta' \]

\[ = e^{i\omega t} \sum_{i=1}^{8} \left\{ -E I \omega B_i m_i^7 + (-E I \omega B_2 + GJB_1) m_i^5 ight\} e^{i\omega t} \]

\[ + (-E I \omega B_3 + GJB_2) m_i^3 + GJB_3 m_i \} e^{i\omega t} \]

Where \( K_1, K_2, \ldots, K_8 \) are the constants to be determined from boundary conditions.

The above equations can be expressed in matrix form as

\[
\{ Z(z) \} = [C][D(z)]\{K\} e^{i\omega t} \quad (3.3.8)
\]

Where \( Z(z) \) is the state vector at a distance \( z \) from the origin and defined as

\[
Z(z) = \begin{bmatrix}
V(z) \\
V'(z) \\
M(z) \\
V(z) \\
\theta(z) \\
\theta'(z) \\
B(z) \\
H(z)
\end{bmatrix} \quad (3.3.8a)
\]

\([C]\) is a 8 x 8 square matrix with elements as follows

\[
c(1,i) = 1 \\
c(2,i) = m_i \\
c(3,i) = E I x m_i^2 \\
c(4,i) = - E I x m_i^3 \\
c(5,i) = B_1 m_i^4 + B_2 m_i^2 + B_3 \\
c(6,i) = B_1 m_i^5 + B_2 m_i^3 + B_3 m_i \\
c(7,i) = - E I \omega (B_1 m_i^6 + B_2 m_i^4 + B_3 m_i^2) \
\]
\[ c(i,8) = -E I_{\omega} B_{1} m_{i}^{7} + (-E I_{\omega} B_{2} + GJB_{1}) m_{i}^{5} \]
\[ + (-E I_{\omega} B_{3} + GJB_{2}) m_{i}^{3} + GJB_{3} m_{i} \]

where \( i = 1, 2, \ldots, 8 \)

\([D(z)]\) is a 8 \( \times \) 8 diagonal matrix with diagonal elements as
\[ m_{i}^{z} \]
\[ d(i,i) = e^{m_{i}^{z}}, \text{where } i=1,2\ldots8 \]

\{K\} is a column matrix consisting of constants \( K_{i} \) \( i=1,2\ldots8 \)

The boundary conditions at the base \( z = 0 \) are
\[ v = v(0), v' = v'(0), M = M(0), V = V(0) \]
\[ \theta = \theta(0), \theta' = \theta'(0), B = B(0), H = H(0) \]

Substituting these conditions in eq. 3.3.8, there is obtained
\[ \{Z(0)\} = [C][I]{K} e^{iwt} \quad (3.3.9) \]

The constants can be determined by matrix inversion in eq. 3.3.9
\[ \{K\} = [C]^{-1}\{Z(0)\} \quad e^{-iwt} \quad (3.3.9a) \]

The boundary conditions at \( z = \ell \) are
\[ v = v(\ell), v' = v'(\ell), M = M(\ell), V = V(\ell) \]
\[ \theta = \theta(\ell), \theta' = \theta'(\ell), B = B(\ell), H = H(\ell) \]

Substituting these conditions in eq. 3.3.8 and using eq. 3.3.9 to eliminate \( \{K\} \)
\[ \{Z(\ell)\} = [C][D(\ell)][C]^{-1}\{Z(0)\} \quad (3.3.10) \]

The field transfer matrix is a 8 \( \times \) 8 square matrix obtained from matrix multiplication
\[ [F] = [C][D(\ell)][C]^{-1} \quad (3.3.11) \]
3.3.2 **Point Transfer Matrix**

The displacement of the center of gravity of the slab is shown in Fig. 3.3.1a. The inertia forces due to motion of the slab is shown in Figs. 3.3.1b to 3.3.1d. Stiffening action of the slab to contribute bimoment is shown in Fig. 3.3.1e.

Consideration of equilibrium of the slab element yields the following equations.

\[
V_+ = V_- + mv'' + am\theta'' \\
H_+ = H_- + J_m \theta'' + amv'' + a^2m\theta'' \\
M_+ = M_- + J_x \psi'' = M_- + J_x v'' + aJ_x \theta'' \\
B_+ = B_- - B_s = B_- - D\theta'
\]  

(3.3.12)

Notations used in the above eqs. are:

- \(a\) Distance between shear center of the section and the center of gravity of the slab
- \(m\) Mass of the slab
- \(J_m\) Polar mass moment of inertia of the slab about an axis through the center of gravity.
- \(J_x\) Mass moment of inertia of the slab about an axis parallel to \(x\) and passing through the center of gravity.
- \(D\) Bimoment contribution factor defined in eq. 2.3.12
shear center of section

centroid of slab

\[ \psi = v' + a \varepsilon' \]

(a) Displacement of Centroid of Slab

(b) Shear Force from Slab

(c) Torque from Slab

(d) Moment from Slab

(e) Bimoment from Slab

FIG. 3.3.1
For a periodic vibration of frequency \( w \)
\[
\begin{bmatrix}
  v(z,t) \\
  v'(z,t) \\
  \theta(z,t) \\
  \theta'(z,t)
\end{bmatrix}
= e^{iwt}
\begin{bmatrix}
  y(z) \\
  y'(z) \\
  \phi(z) \\
  \phi'(z)
\end{bmatrix}
\]
Differentiating twice with respect to time
\[
\begin{bmatrix}
  \dddot{v}(z,t) \\
  \dddot{v}'(z,t) \\
  \dddot{\theta}(z,t) \\
  \dddot{\theta}'(z,t)
\end{bmatrix}
= -w^2
\begin{bmatrix}
  v(z,t) \\
  v'(z,t) \\
  \theta(z,t) \\
  \theta'(z,t)
\end{bmatrix}
\]
Substituting in eq. 3.3.12, there is obtained
\[
\begin{align*}
V_+ &= V_- - w^2mv - w^2am\theta \\
H_+ &= H_- - w^2amv - w^2(J_m + a^2m)\theta \\
M_+ &= M_- - w^2J_xv' - w^2aJ_x\theta' \\
B_+ &= B_- - D\theta'
\end{align*}
\] (3.3.14)
Compatibility conditions give
\[
\begin{align*}
v_+ &= v_- \\
v'_+ &= v'_- \\
\theta_+ &= \theta_- \\
\theta'_+ &= \theta'_-
\end{align*}
\] (3.3.15)
Point transfer matrix \([P]\) can there be formed from eq. 3.3.14 and eq. 3.3.15 as
The elements of matrix are

\[ \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & P_{32} & 1 & 0 & 0 & P_{36} & 0 & 0 \\
P_{41} & 0 & 0 & 1 & P_{45} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & P_{76} & 1 & 0 \\
P_{81} & 0 & 0 & 0 & P_{85} & 0 & 0 & 1 \\
\end{bmatrix} \]

(3.3.16)

The elements of matrix are

\[ P_{32} = -w^2 J_x, \quad P_{36} = -w^2 a J_x \]
\[ P_{41} = -w^2 m, \quad P_{45} = -w^2 a m \]
\[ P_{76} = -D, \quad P_{81} = -w^2 a m \]
\[ P_{85} = -w^2 (J_m + a^2 m) \]

3.3.3 Boundary Conditions

The boundary conditions for the shear wall fixed at base and free at top can be written as

\[ \{ \mathbf{z}_o^+ \} = \begin{bmatrix}
0 \\
0 \\
M^+ \\
V_o^+ \\
0 \\
0 \\
B_o^+ \\
H_o^+ \\
o \\
\end{bmatrix} \quad \text{and} \quad \{ \mathbf{z}_8^+ \} = \begin{bmatrix}
v_8^+ \\
v_8' \\
0 \\
\theta_8^+ \\
\theta_8' \\
0 \\
0 \\
\end{bmatrix} \]
Substituting these conditions in eq. 3.2.3, there is obtained

\[
\begin{pmatrix}
    v_{8}^+ \\
v_{8}^+ \\
    0 \\
    0 \\
    \theta_{8}^+ \\
    \theta_{8}^+ \\
    0 \\
    0
\end{pmatrix}
= 
\begin{pmatrix}
    0 \\
    0 \\
    M_{0}^{+} \\
    V_{0}^{+} \\
    0 \\
    0 \\
    B_{0}^{+} \\
    H_{0}^{+}
\end{pmatrix}
\]

(3.3.17)

Re-arrangement of terms of the above eq. yields

\[ [R] \{X\} = 0 \]  

(3.3.18)

Where

\[ [R] = \begin{pmatrix}
    a_{33} & a_{34} & a_{37} & a_{38} \\
    a_{43} & a_{44} & a_{47} & a_{48} \\
    a_{73} & a_{74} & a_{77} & a_{78} \\
    a_{83} & a_{84} & a_{87} & a_{88}
\end{pmatrix} \]

Here 'a' denotes elements of matrix [A].

\[ \{X\} = \begin{pmatrix}
    M_{0}^{+} \\
    V_{0}^{+} \\
    B_{0}^{+} \\
    H_{0}^{+}
\end{pmatrix} \]

The condition to determine the natural frequencies is

\[ |R| = 0 \]  

(3.3.19)
After determining the natural frequencies, the relative values of the elements in vector \( \{X\} \) can be determined. The mode shape of the structure follows by back substitution of the vector \( \{X\} \).

### 3.4 Computer Program

A computer program based on the above analysis has been written. The input data are the geometric and elastic properties of the shear wall structure. The trial frequency \( w \) is increased from an initial value and \(|R|\) (eq. 3.3.19) is calculated. If \(|R| \neq 0\) another value of \( w \) is tried. The same procedure is repeated until \(|R|\) is reasonably small to be considered as zero. The next higher frequency is then determined following the same scheme. These natural frequencies obtained are then used as inputs in a second program to determine associated mode shapes.

The flow chart for the first program is shown in fig. 3.4.1 and the computer program is included in Appendix B.

### 3.5 Experiment

A dynamic experiment was carried out on the model (Fig. 2.1.1). The model was fixed on the shaking table and subjected to lateral vibration of known amplitude while the frequency is gradually swept from an initial value upwards. In all the dynamic tests, the shaking table was subjected
Start

Read and Print Input Data

Initialise Frequency $w$

Form Transfer Matrices $P$ & $F$

Calculate Matrix $A$

Form Matrix $R$ & Calculate $|R|$?

Increase $w$

$|R|$ Changes Sign

Yes

No

2

1
Iterate in a small range to find reasonable accuracy

Required No of Frequencies are obtained

Yes

Print Frequency

Stop

FLOW CHART

FIG. 3.4.1
to a constant displacement of 0.005 inch from 10 cps to 44 cps. After cross over frequency of 44 cps it was subjected to constant acceleration of 0.5 g. The response of the accelerometers fixed at different points of the model was studied.

The experimental set up is shown in fig. 3.5.1 to 3.5.3. The output from the accelerometers was viewed in an oscilloscope. The RMS response of the accelerometers were plotted in a XY recorder. D.C. voltage proportional to RMS acceleration was fed in X ordinate and Y ordinate was adjusted in a suitable time scale. The arrangement of instruments is shown in fig. 3.5.4. Arked increase in response is noticed in the frequency response plots (Fig. 3.6.1 to 3.6.4) at the resonant frequencies. For locating the resonant peak more accurately, the frequency was manually changed around each resonance zone and the response was monitored on a RMS voltmeter. The strains at resonance were determined from plotting outputs from the strain gauges in the Visicorder.

The arrangement of instrument used is shown in fig. 3.5.5. Relative strain distribution is drawn from these plots and shown in fig. 3.6.7 to fig. 3.6.10.

The following is a list of different instruments used in the experiment.

The dynamic testing set-up consists of
A. Sweep Oscillo for SD104A-5D

Make: Spectal Dynamic Corporation
Fig. 3.5.3
ACCELEROMETERS ATTACHED TO THE MODEL
ARRANGEMENT FOR RESONANCE SEARCH

FIG. 3.5.4

ARRANGEMENT FOR MEASURING STRAINS

FIG. 3.5.5
B. Amplitude Servo/Monitor SD105A
   Make: Spectal Dynamic Corporation.

D. Accelerometer source follower SFA-100
   Make: Ling Electronics

E. Accelerometer Normalizing Amplifier ANA-101
   Make: Ling Electronics

F. Power Amplifier CP-5/6
   Make: Ling Electronics

G. Shaker B 290
   Make: Ling Electronics

H. Vibraglide Splitable SINGCO 30-30
   Make: Marshall Research and Development Corporation

For monitoring and plotting the response the following instruments were employed.

I. Dual Beam Oscilloscope
   Make: Tectronix Inc.

J. Accelerometer

K. Laboratory amplifier 2616B
   Make: Endevco Corporation

L. D.C. Amplifier, High Gain Type 1-165
   Make: Endevco Corporation

M. Bridge Amplifier
   Make: Ellis Associates

N. RMS Voltmeter
   Make: Hewlett Packard

O. Visicorder (2 channels)
   Make: Honeywell Controls Ltd.
P. Digital Counter
Make: Hewlet Packard.

3.6 Results and Discussions

The frequency response plots as obtained from the dynamic test for the accelerometers located at different positions are shown in fig. 3.6.1 to 3.6.4. The figures near the peaks are the experimental resonant frequencies. Average of all the experimental frequencies together with the theoretical frequencies for different consideration of floor slabs are shown in table 3.6.1.

It can be seen that for the first and third modes the experimental freq. lies between the theoretical predicted value when the torsional restraining effect of the slab is considered and the theoretical predicted value when both the torsional and effective bending restraining effect of the slab is considered. The difference between the theoretical and experimental values is about 5%.

In the second mode, the experimental value is 16% lower than the theoretical calculated value. Since the second mode is a bending predominant mode, in this case, the larger difference may be caused be neglecting shear deformation in the mathematical model. The importance of considering shear deformation for bending predominant mode was noted by Tso (29).
<table>
<thead>
<tr>
<th>NO. FREQ</th>
<th>THEORETICAL FREQUENCY (cps)</th>
<th>EXPERIMENTAL FREQUENCY (cps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Floor-less Structure</td>
<td>Torsion Only of Slab</td>
</tr>
<tr>
<td>1</td>
<td>16.5</td>
<td>14.5</td>
</tr>
<tr>
<td>2</td>
<td>65.6</td>
<td>56.7</td>
</tr>
<tr>
<td>3</td>
<td>93.9</td>
<td>71.8</td>
</tr>
</tbody>
</table>

COMPARISON OF FREQUENCIES

TABLE 3.6.1
FREQUENCY RESPONSE PLOT

FIG. 3.6.1
FREQUENCY RESPONSE PLOT

FIG. 3.6.2
FREQUENCY RESPONSE PLOT

ACCELERATION (g)

FREQUENCY (cps)

16.1
48.9
73.8

ACCELEROMETER POSITION

FIG 3.6.3
FREQUENCY RESPONSE PLOT

FIG. 3.6.4
**Fig. 3.6.5**

(a) **Mode Shape for First Freq.**

(b) **Mode Shape for Second Freq.**

(c) **Mode Shape for Third Freq.**
RELATIVE ROTATION & DISPLACEMENT AT 8TH. FLOOR
FIG. 3.6.6
RELATIVE STRAIN DISTRIBUTION AT LEVEL A
AT FIRST FREQUENCY

FIG. 3.6.7
RELATIVE STRAIN DISTRIBUTION AT LEVEL B
AT FIRST FREQUENCY

FIG. 3.6.8
RELATIVE STRAIN DISTRIBUTION AT LEVEL A
AT SECOND FREQUENCY

FIG. 3.6.9
RELATIVE STRAIN DISTRIBUTION AT LEVEL B
AT SECOND FREQUENCY

FIG. 3.6.10
The associated mode shapes of the structure for first three modes obtained theoretically are plotted in fig. 3.6.5. It shows from these plots that the first mode consists of torsion predominant displacement and the second mode consists of bending predominant displacements.

It is also noted that in first mode displacement and rotation are in phase. In second mode they are out of phase. In third mode they are in phase in lower part of the structure but out of phase in top part.

The displacements and rotations at the top of the structure for different modes are plotted in fig. 3.6.6. The values of \( \psi \) and \( \theta \) written on the figure are the values of mode shapes curve (Fig. 3.6.5) at 48 inch levels are relative values only. In other words the values for the first mode does not have any relation with that for the second or the third mode.

It is seen from the frequency response plots (fig. 3.6.1 to fig. 3.6.4) that at any resonant frequency, the response varies depending on the position of the accelerometer. For example in the third resonance the accelerometers mounted at points q and r show lower response than that mounted at points p and s. In the second resonance, the accelerometers mounted at points q and r shows higher response than that mounted at points p and s. In the first resonance, the response of accelerometers mounted at p and s shows higher response than that mounted at q and r. These
type of behaviour is due to the in phase or out of phase nature of displacements and rotations in different modes and can be explained from fig. 3.6.6. It is seen from the plot that points p and s undergo maximum translational displacement due to combined action of v and θ in the third and the first mode. Whereas points q and r undergo maximum translational displacement in second mode only.

The relative strain distribution at level A and level B (refer fig. 2.5.2 for level A and B) at the first and the second resonance are plotted (Fig. 3.6.7 to Fig. 3.6.10). Theoretical distribution are drawn from the calculated mode shapes. Since the mode shapes are defined by relative values only the theoretical strain diagram is made to pass through one experimental point. A reasonable agreement between the theory and experiment is shown in these plots.
CHAPTER IV

STATIC FORMULATION OF NON PLANAR COUPLED SHEAR WALL

4.1 Summary

In this chapter, the nonplanar coupled shear wall is analysed using the continuum approach. Differential equations are developed for such wall subjected to lateral forces and torques distributed along the height. Special configurations of shear walls are shown and the modification on differential equations and solution is indicated. The experimental study was performed on a small scale plexiglas model consisting of two angles connected by beams at equal spacing (Fig. 4.4.1). The model was subjected to a force and a torque at the top storey and strains and deflections are measured. The experimental results are then compared with the theoretical predictions.

4.2 Theoretical Analysis

Consider two nonplanar piers which are connected by floor beams at equal spacing (Fig. 4.2.1). In the analysis the center of the connecting beam 0 is taken as the reference point. The differential equations are derived in terms of the displacement variables of point 0, namely u, v and θ. The external forces and torques are also referenced to the same point 0. The theory is based on two assumptions:

(i) The deformation of the connecting beams due to bending in horizontal plane is restricted.
GEOMETRY OF COUPLED SHEAR WALL

FIG. 4.2.1
(ii) Points of contraflexure for the connecting beams due to bending in vertical plane are taken to be at the center.

In addition to the above assumptions, Vlasov's theory is taken to be valid for individual section constituting the coupled wall.

Using the continuum approach, the connecting beams are replaced by independently acting laminae of appropriate stiffness (Fig. 4.2.2a).

4.2.1 Notations Used

The notations used in the present analysis are listed below and illustrated in fig. 4.2.1.

\( X_1, Y_1 \)  Orthogonal principal axes for pier 1.

\( u_1, v_1, \theta_1 \)  Generalised displacements of the shear center of pier 1.

\( X, Y, Z \)  Orthogonal global axes with origin at point 0. X is parallel to the length of the beam.

\( X_1, Y_1 \)  Orthogonal axes parallel to \( X \) and \( Y \) and passing through centroid of pier 1 (Fig. 4.2.4).

\( u, v, \theta \)  Generalised global displacements of point 0.

\( e_{x1}, e_{y1} \)  Co-ordinates of the centroid of pier 1 referred to global axes.

\( c_{x1}, c_{y1} \)  Co-ordinate of the shear center of pier 1 referred to global axes.

\( p_{x1}, p_{x2} \)  Distances of the centroid of pier 1 from point 0 measured parallel to \( X_1, Y_1 \) axes.

\( q_{x1}, q_{x2} \)  Distances of the shear center of pier 1 from point 0 measured parallel to \( X_1, Y_1 \) axes.

\( \phi, \omega_1 \)  Angle between \( X_1 \) and \( X \) axes.

\( \omega_1 \)  Sectoral ordinate of pier 1 at the point 0 referred to shear center of the same pier.
FIG. 4.2.2

(a) Replaced Structure

(b) Deformed Structure

(c) Shear Force & Axial Force
Moment of inertia of pier 1 about $x_1$, $y_1$ axes.

Moment of inertia of pier 1 about $x_1$ and $y_1$ axes (Fig. 4.2.4).

Product moment of inertia of pier 1 about axes parallel to $X,Y$ and passing through centroid i.e. $x_1,y_1$ axes respectively.

Principal sectorial moment of inertia.

External moment about $X$ axis

External moment about $Y$ axis

External torque about point $O$.

Moment of inertia of the connecting beam

Area of the connecting beam

Clear span of connecting beam.

Storey height

Note:--The subscript 1 in the above notations are replaced by 2 for pier 2.

4.2.2. Geometric Relations

The global axes of the structure are $X$, $Y$ and $Z$ are the reference axes about which the displacements and forces are referred. The $X$ axis is parallel to the longitudinal axis of the connecting beam. The principal axis of the piers are inclined at angles $\phi_1$ and $\phi_2$ to the $X$ axis (Fig. 4.2.1).

The $Z$ axis is the vertical axis through point $O$.

The assumption (i) along with Vlasov's hypothesis of non deformable cross section leads to a rigid section of the coupled wall, for which the following geometric relation are valied for the transformation of displacement of the cross section from one reference point to another.
The Transfer matrices $[R_j]$ and $[T_j]$ can be defined as:

$$[R_j] = \begin{pmatrix} \cos \phi_j & \sin \phi_j & 0 \\ -\sin \phi_j & \cos \phi_j & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ (4.2.1)

$$[T_j] = \begin{pmatrix} 1 & 0 & -c_{yi} \\ 0 & 1 & c_{xj} \\ 0 & 0 & 1 \end{pmatrix}$$ (4.2.2)

$(j = 1,2)$

The relationship between the global displacement variables $u$, $v$ and $\theta$ and the displacement variables of the piers are:

$$\begin{pmatrix} u_j \\ v_j \\ \theta_j \end{pmatrix} = [R_j][T_j] \begin{pmatrix} u \\ v \\ \theta \end{pmatrix}$$ (4.2.3)

$(j = 1,2)$

Other geometric relations which relate the distances measured along the principal axes of the piers to the global directions are:

$$\begin{pmatrix} p_{xj} \\ p_{yi} \end{pmatrix} = \begin{pmatrix} \cos \phi_j & \sin \phi_j \\ -\sin \phi_j & \cos \phi_j \end{pmatrix} \begin{pmatrix} e_{xj} \\ e_{yj} \end{pmatrix}$$ (4.2.4)

$$\begin{pmatrix} q_{xj} \\ q_{yj} \end{pmatrix} = \begin{pmatrix} \cos \phi_j & \sin \phi_j \\ -\sin \phi_j & \cos \phi_j \end{pmatrix} \begin{pmatrix} c_{xj} \\ c_{yj} \end{pmatrix}$$ (4.2.5)

in which $j = 1,2$
4.2.3 Displacement Consideration

An imaginary cut is made along the center line of the laminae. Due to deflection of the piers, there is a relative displacement $\delta_1$ to the left and the right of the cut as shown in fig. 4.2.3(a).

$$\delta_1 = u_2' p_{x2} - u_1' p_{x1} + v_2' p_{y2} - v_1' p_{y1}$$

$$-\theta_2' I_w - \theta_1' I_w$$

(4.2.6)

Using relations in eq. 4.2.3 and 4.2.4 this equation becomes:

$$\delta_1 = u'a + v'b + \theta'(w+d)$$

(4.2.7)

Where

$$a = e_{x2} - e_{x1}$$

$$b = e_{y2} - e_{y1}$$

$$\omega = \omega_1 - \omega_2$$

$$d = c_{x2} e_{y2} - c_{y2} e_{x2} + c_{y1} e_{x1} - c_{x1} e_{y1}$$

The shear force distribution $q$ induced at the center of laminae produces compressive force on one pier and tensile force in the other. This axial force $T$ in pier is related to $q$ (Fig. 4.2.2c) as:

$$q = -\frac{dT}{dz}$$

(4.2.8)

This axial force $T$ produces axial deformation of the pier which decreases the relative displacement at center of laminae by $\delta_2$.

$$\delta_2 = \int_0^Z \left( \frac{1}{E A_1} + \frac{1}{E A_2} \right) T(\xi) d\xi$$

(4.2.9)
(a) Gap due to Deflection of Piers
(b) Gap due to Axial Deformation of Piers
(c) Gap due to Deformation of Laminas

FIG. 4.2.3

TRANSFORMATION OF MOMENT OF INERTIA

FIG. 4.2.4
Finally the force \( q \) will produce in the laminae a deformation \( \delta_3 \) due to bending and shear.

\[
\delta_3 = \frac{qc^3}{12EJ_b} \tag{4.2.10}
\]

Where \( J_b \) is the equivalent moment of inertia of laminas taking into account shear deformation of beam.

\[
J_b = \frac{I_b h^2}{12EI_b} \tag{4.2.11}
\]

To satisfy the condition of compatibility it is required

\[
\delta_1 = \delta_2 + \delta_3
\]

Substituting above expressions for \( \delta_1, \delta_2, \) and \( \delta_3 \)

\[
u'a + v'b + \theta'(w+d)
\]

\[
= \int_0^L \left[ \frac{1}{E} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) T(\xi) d\xi + \frac{qc^3}{12EJ_b} \right] \tag{4.2.12(a)}
\]

Differenting once

\[
u''a + v''b + \theta''(w+d)
\]

\[
= \frac{1}{E} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) T + \frac{c^3}{12EJ_b} \frac{d}{dz} \tag{4.2.12}
\]

4.2.4 Force Equilibrium Conditions

The internal moments acting on the different components of coupled shear wall are shown in fig. 4.2.5. These internal moments along with couple produced by axial force \( T \) balances the external moment. For equilibrium of moments about \( Y \) and \( X \) axis, the following expressions can be derived.
INTERNAL MOMENTS

FIG. 4.2.5

INTERNAL TORQUES & SHEAR FORCES

FIG. 4.2.6
The relations between the moment of inertia with reference to different axes (Fig. 4.2.4) are:

\[ EI_{y_1}u'' \cos \phi_1 - EI_{x_1}v'' \sin \phi_1 + EI_{y_2}u'' \cos \phi_2 = -EI_{x_2}v'' \sin \phi_2 + Ta = M_y \]  
(4.2.13)

\[ EI_{y_1}u'' \sin \phi_1 + EI_{x_1}v'' \cos \phi_1 + EI_{y_2}u'' \sin \phi_2 + EI_{x_2}v'' \cos \phi_2 + Tb = M_x \]  
(4.2.14)

The relations between the moment of inertia with reference to different axes (Fig. 4.2.4) are:

\[ S_{xj} = I_{yj} \sin^2 \phi_j + I_{xj} \cos^2 \phi_j \]
\[ S_{yj} = I_{yj} \cos^2 \phi_j + I_{xj} \sin^2 \phi_j \]
\[ S_{xyj} = (I_{yj} - I_{xj}) \sin \phi_j \cos \phi_j \]
(4.2.15)

in which \( j = 1,2 \)

Using relations in eq. 4.2.3 and 4.2.15, the above equations are simplified as follows.

\[ E S_{y}u'' + E S_{xy}v'' - E S_{yc} \theta'' + Ta = M_y \]  
(4.2.16)

\[ E S_{xy}u'' + E S_{x}v'' + E S_{xc} \theta'' + Tb = M_x \]  
(4.2.17)

Where

\[ S_y = S_{y1} + S_{y2} \]
\[ S_x = S_{x1} + S_{x2} \]
\[ S_{xy} = S_{xy1} + S_{xy2} \]
\[ S_{yc} = c_{y1} S_{y1} + c_{y2} S_{y2} - c_{x1} S_{xy1} - c_{x2} S_{xy2} \]
\[ S_{xc} = c_{x1} S_{x1} + c_{x2} S_{x2} - c_{y1} S_{xy1} - c_{y2} S_{xy2} \]  
(4.2.18)
TORQUES & SHEAR FORCES DUE TO BEAM SHEAR $q$

FIG. 4.2.7'

FORCES ACTING ON AN ELEMENT

FIG. 4.2.8
The internal torques and shear forces acting on different components of coupled shear wall are shown in fig. 4.2.6. Let $Q_t$ be the resultant torque of all these forces about point $O$. Therefore:

$$Q_t = -EI_{x_1}v'''q_{x_1} + EI_{y_1}u'''q_{y_1} - EI_{x_2}v'''q_{x_2}$$
$$+ EI_{y_2}u'''q_{y_2} - E(I_{\omega_1} + I_{\omega_2})\theta''' + G(J_1 + J_2)\theta'$$

(4.2.19)

Using relations in eq. 4.2.3 and 4.2.15 the above equation is simplified as follows:

$$\bar{Q}_t = ES_{yc}u''' - E S_{xc}v''' - EI_{\omega}\theta''' + GJ\theta'$$

(4.2.20)

Where

$$I_{\omega} = I_{\omega_1} + I_{\omega_2} + c_{x_1}^2 I_{x_1} + c_{x_2}^2 I_{x_2} + c_{y_1}^2 I_{y_1}$$
$$+ c_{y_2}^2 I_{y_2} - 2c_{x_1}c_{y_1} I_{x} I_{y_1} - 2c_{x_2}c_{y_2} I_{x} I_{y_2}$$

(4.2.21)

$$J = J_1 + J_2$$

Additional shear forces $Q_{x_1}, Q_{y_1}, Q_{x_2}$ and $Q_{y_2}$ and torques $Q_{t_1}$ and $Q_{t_2}$ develop in the section due to the shear force $q$ in the laminae (Fig. 4.2.7). These forces can be expressed in terms of $q$ from consideration of equilibrium of an element as shown in fig. 4.2.8. Thus,

$$Q_{x_1} = -qP_{x_1}; \quad Q_{y_1} = -qP_{y_1};$$
$$Q_{x_2} = qP_{x_2}; \quad Q_{y_2} = qP_{y_2}$$
$$Q_{t_1} = q\omega_1; \quad Q_{t_2} = -q\omega_2$$

(4.2.22)
The resultant torque \( \overline{Q}_t \) about the point 0 then becomes

\[
\overline{Q}_t = Q_{t1} + Q_{t2} - Q_{x1} q_{y1} + Q_{y1} q_{x1} - Q_{x2} q_{y2} + Q_{y2} q_{x2} \quad (4.2.23)
\]

Using relations in eq. 4.2.4, eq. 4.2.5 and eq. 4.2.22, the above equation is simplified as follows:

\[
\overline{Q}_t = q(\omega+d) = -(\omega+d) \frac{dT}{dz} \quad (4.2.24)
\]

Equilibrium of torque about 0 gives the internal torque \( \overline{Q}_t \) together with torque due to shear force \( \overline{Q}_t \) must balance the external torque \( Q_t \). Thus

\[
\overline{Q}_t + \overline{Q}_t = Q_t
\]

or

\[
-E S_{x c} v'''' + E S_{y c} u''' - EI_\omega \theta''' + GJ\theta''
- (\omega+d) \frac{dT}{dz} = Q_t \quad (4.2.25)
\]

### 4.2.5 Differential Equations

The compatibility condition (eq. 4.2.12) and the three force equilibrium condition in eq. 4.2.16, eq. 4.2.17 and eq. 4.2.25 are four equations relating the unknown of the problem \( u, v, \theta \) and \( T \). In the following paragraphs, simplification is made to reduce the four coupled equations to a single equation in \( \theta \). From eq. 4.2.16 and 4.2.17 the following expressions are obtained by algebraic elimination.

\[
u'' = C_4 \theta'' + C_2 T + C_3 M_y + C_4 M_x \quad (4.2.26)
\]

\[
v'' = C_5 \theta'' + C_6 T + C_7 M_y + C_8 M_x \quad (4.2.27)
\]
Where

\[ SS_{xy} = S_x S_y - S_{xy}^2; \]
\[ C_1 = (S_{xy} S_{xc} + S_x S_{yc})/SS_{xy}; \]
\[ C_2 = -(a S_x - b S_{xy})/E SS_{xy}; \]
\[ C_3 = S_x/E SS_{xy}; C_4 = -S_{xy}/E SS_{xy}; \]
\[ C_5 = -(S_y S_{xc} + S_{xy} S_{yc})/SS_{xy}; \]
\[ C_6 = -(b S_y - a S_{xy})/E SS_{xy}; \]
\[ C_7 = -S_{xy}/E SS_{xy}; C_8 = S_y/E SS_{xy} \]

(4.2.28)

Differeniating eq. 4.2.26 and eq. 4.2.27 and substituting in eq. 4.2.25,

\[ -q = \frac{dT}{dZ} = -\frac{\bar{I}_w}{r_1} \theta'' + \frac{GJ}{r_1} \theta' + C_9 M_y' + C_{10} M_x' + C_{11} \Omega_t \]

(4.2.29)

Where

\[ \bar{I}_w = I_w - (S_y S_{xc}^2 + S_x S_{xc}^2)/SS_{xy} \]
\[ r_1 = \omega + d - S_{xc} (b S_y - a S_{xy})/SS_{xy} \]
\[ + S_{yc} (a S_x - b S_{xy})/SS_{xy} \]
\[ C_9 = (S_{xy} S_{xc} + S_x S_{yc})/SS_{xy} r_1 \]
\[ C_{10} = -(S_y S_{xc} + S_{xy} S_{yc})/SS_{xy} r_1 \]
\[ C_{11} = -1/r_1. \]

Substituting eq. 4.2.26 and eq. 4.2.27 in eq. 4.2.12,

\[ E r_2 \theta'' - \frac{1}{A} T + C_{12} M_y + C_{13} M_x + \gamma \frac{d^2T}{dZ^2} = 0 \]

(4.2.30)

Where

\[ r_2 = \omega + d + a(S_{xy} S_{xc} + S_x S_{yc})/SS_{xy} \]
\[ -b(S_y S_{xc} + S_{xy} S_{yc})/SS_{xy} \]
\[
1/A = 1/A_1 + 1/A_2 + a(a s_x - b s_{xy})/SS_{xy} + \\
+ b(b s_y - a s_{xy})/SS_{xy}
\]

\[
C_{12} = (a s_x - b s_{xy})/SS_{xy} \\
C_{13} = (b s_y - a s_{xy})/SS_{xy}
\]

\[
\gamma = c^3/12J_b
\]

Differentiating eq. 4.2.31 once

\[
E \frac{r^3}{2} \theta''' - \frac{1}{A} \frac{dT}{dZ} + C_{12} M'_y + C_{13} M'_x + \gamma \frac{d^3T}{dZ^3} = 0 \quad (4.2.32)
\]

Eliminating \(\frac{dT}{dZ}\) and \(\frac{d^3T}{dZ^3}\) using eq. 4.2.29 the following fifth order differential equation in \(\theta\) is obtained.

\[
\beta_1 \theta^\gamma - \beta_2 \theta''' + \beta_3 \theta'' = C_{14} M'_y + C_{15} M'_x + C_9 M'''
\]

\[
+ C_{10} M''_x + C_{16} Q_t - Q''_t \quad (4.2.33)
\]

Where

\[
\begin{align*}
\beta_1 &= \frac{E I \theta}{w} \\
\beta_2 &= \frac{E I \theta + GJ + \frac{E r \cdot r^2}{\gamma}}{A \gamma} \\
\beta_3 &= \frac{GJ/A \gamma; C_{16} = 1/A \gamma} \\
C_{14} &= \frac{C_{12} r^1_1}{\gamma} - \frac{C_9}{AY} \\
C_{15} &= \frac{C_{13} r^1_1}{\gamma} - \frac{C_{10}}{\gamma}
\end{align*}
\]

\[
(4.2.34)
\]

The Eq. 4.2.33 along with eq. 4.2.26, eq. 4.2.27 and eq. 4.2.29 are the final equations used in the analysis.
4.2.5 Boundary Conditions

For no rotation and displacement at base
\[ \theta(0) = 0 \quad (4.2.41(a)) \]
\[ u(0) = 0 \quad (4.2.41(b)) \]
\[ v(0) = 0 \quad (4.2.41(c)) \]

For no slope and warping at base
\[ \theta'(0) = 0 \quad (4.2.42(a)) \]
\[ u'(0) = 0 \quad (4.2.42(b)) \]
\[ v'(0) = 0 \quad (4.2.42(c)) \]

For no moment and bi-moment at top
\[ \theta''(H) = 0 \quad (4.2.43(a)) \]
\[ u''(H) = 0 \quad (4.2.43(b)) \]
\[ v''(H) = 0 \quad (4.2.43(c)) \]

Substituting eq. 4.2.42 in eq. 4.2.12a the following condition is obtained at \( z = 0 \).
\[ q(0) = -\frac{dT}{dz} \bigg|_{z=0} = 0 \]

Using eq. 4.2.29
\[
\frac{EI}{T} \left[ \frac{\theta'''}{r_1} - \frac{GJ}{r_1} \theta' - C_9 M'_y - C_{10} M'_x - C_{11} Q' \right] \bigg|_{z=0} = 0
\]

Axial force \( T \) can be expressed in terms of \( q \) by the following relation
\[ T = \int_{z}^{H} q(\xi)d\xi \quad (4.2.45) \]

From which at \( z = H, T = 0 \) (4.2.46)

Substituting eq. 4.2.43 and 4.2.46 in eq. 4.2.12
\[
\frac{dq}{dz} \bigg|_{z=H} = -\frac{dT}{dz} \bigg|_{z=H} = 0 \quad (4.2.47)
\]
Differentiating expression in eq. 4.2.29 and using in eq. 4.2.47

\[
\left. \frac{E I}{r_1} \theta'' - \frac{G J}{r_1} \theta'' - C_9 \frac{M''}{y} - C_{10} \frac{M'}{x} - C_{11} Q \right|_{x=H} = 0
\] (4.2.48)

The solution of the differential equation subjected to the above boundary conditions are the complete solution to the problem.

4.2.7 Solution

The solution of the differential equation (eq. 4.2.33) will consist the complimentary solution \( \theta_C \) and a particular integral \( \theta_P \). Thus

\[ \theta = \theta_C + \theta_P \]

The complimentary part \( \theta_C \) will satisfy the following equation

\[ \beta_1 \theta'' - \beta_2 \theta''' + \beta_3 \theta' = 0 \] (4.2.50)

Assuming solution of the form \( \theta_C = Ke^{mz} \) the following characteristic equation is obtained.

\[ \beta_1 m^5 - \beta_2 m^3 + \beta_3 m = 0 \] (4.2.51)

The roots are: \( m_1 = 0 \)

\[ m_2, m_3 = \pm \sqrt[3]{\frac{\beta_2^2 - 4 \beta_1 \beta_3}{2 \beta_1}} \]
In the above expressions \( \beta_1, \beta_2 \) and \( \beta_3 \) are positive. It can be shown that
\[
\beta_2^2 > 4\beta_1\beta_3
\]
Therefore all roots are real.

Defining
\[
\alpha_1 = |m_2| = |m_3|
\]
\[
\alpha_2 = |m_4| = |m_5|
\]

The solution is of the following form.
\[
\theta_c = \kappa_1 + \kappa_2 \cos \alpha_1 z + \kappa_3 \sin \alpha_1 z + \kappa_4 \cos \alpha_2 z \\
+ \kappa_5 \sin \alpha_2 z
\]  
\hspace{10cm} (4.2.52)

The particular integral \( \theta_p \) depends on loading.

Case-I, Concentrated load \( W_x, W_y \) and torque \( W_t \) at top. The loads are acting in the +ve directions and refered to the center line of beams. For such loading
\[
M_y = W_x (H-Z)
\]
\[
M_x = W_y (H-Z)
\]
\[
Q_t = W_t
\]

Particular integral for this case is:
\[
\theta_p = K_p z
\]
Where

\[
K_p = - (C_{14} w_x + C_{15} w_y - C_{16} w_t)
\]  \hspace{1cm} (4.2.53)

Case-II, Uniformly distributed load \( w_x, w_y \) and torque \( w_t \) acting throughout the height. The loads are acting in the positive directions and referred to the center line of beams. For such loading:

\[
M_y = w_x (H-Z)^2 / 2 \\
M_x = w_y (H-Z)^2 / 2 \\
Q_t = w_t Z
\]

Particular integral for this case is:

\[
\theta_p = K_{p1} Z^2 + K_{p2} Z
\]

Where

\[
K_{p1} = (C_{14} w_x + C_{15} w_y - C_{16} w_t) / \beta_3 \\
K_{p2} = - H(C_{14} w_x + C_{15} w_y - C_{16} w_t) / \beta_3 \]  \hspace{1cm} (4.2.54)

The constants \( \kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5 \) \hspace{1cm} (eq. 4.2.52) are determined from the boundary condition in eq. 4.2.41(a), eq. 4.2.42(a), eq. 4.2.43(a), eq. 4.2.44 and eq. 4.2.48.

After obtaining complete solution for \( \theta \), the expression for shear force \( q \) is determined from eq. 4.2.29. The expression for axial force \( T \) is determined from direct integration of the expression for \( q \) subjected to boundary condition as in eq. 4.2.46.
The expression of displacements $u$ and $v$ are determined from direct integration of the expressions in eq. 4.2.26 and eq. 4.2.27 subjected to boundary condition as in eq. 4.2.41(b), eq. 4.2.41(c), eq. 4.2.42(b) and eq. 4.2.42(c).

The displacement of the individual piers $u_j$, $v_j$ and $\theta_j$ (where $j = 1, 2$) are determined from the relation in eq. 4.2.3. The average moment and bimoment of the individual piers are determined from the following relations.

$$M_{yj} = EI_{yj} u_j''', \quad M_{xj} = EI_{xj} v'''$$
$$B_j = - EI_{w, j} \theta_j''' \quad \text{(4.2.55)}$$

where $j = 1, 2$

4.2.8 Special Configurations

For a mono symmetric configuration of piers as shown in Case-A (fig. 4.2.11) $b = 0; S_{xy} = 0; S_{xc} = 0$. Substituting these conditions in the governing equations (eq. 4.2.12, eq. 4.2.16, eq. 4.2.17 and eq. 4.2.25) they are reduced to

$$u''a + \theta''(\omega + d) = \frac{1}{E} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) T + \frac{c^3}{12 J_0 E} \frac{d\theta}{dz} \quad \text{(4.2.56)}$$
$$E S_{y} u''' + E S_{yc} \theta''' + Ta = M_y \quad \text{(4.2.57)}$$
$$E S_{x} v''' = M_x \quad \text{(4.2.58)}$$
$$E S_{yc} u'''' - EI_{w} \theta'''' + G J \theta' - (\omega + d) \frac{dT}{dz} \frac{d\theta}{dz} = Q_t \quad \text{(4.2.59)}$$
CASE - A

CASE - B

CASE - C

FIG. 4.2.11
The eq. 4.2.58 is uncoupled and represents independent bending of the structure about the X axis. The other equations can be combined in a single differential equation in $\theta$ and solved following the same scheme as before.

For a mono symmetric configuration of piers as in Case-B (fig. 4.2.11) $b = 0$, $S_{xy} = 0$, $S_{xc} = 0$; $(\omega+d) = 0$. Substituting these conditions the governing equations (eq. 4.2.12, eq. 4.2.16, eq. 4.2.17 and eq. 4.2.25) they are reduced to.

\[
\begin{align*}
  u''a &= \frac{1}{EA_1} + \frac{1}{EA_2}d + \frac{c}{I_2E_J_b}d^2 \\
  E S_y u'' + T a &= M_y \\
  E S_x v'' + E S_{xc} \theta'' &= M_x \\
  -E S_{xc} v''' - EI_\omega \theta''' + GJ \theta' &= Q_t
\end{align*}
\] (4.2.60)

\[\text{(4.2.61)}\]
\[\text{(4.2.62)}\]
\[\text{(4.2.63)}\]

In the above equations $u$ and $T$ are coupled in the first two equations. In the last two equations $v$ and $\theta$ are coupled but independent of $u$ and $T$. The first two equations (eq. 4.2.60 and eq. 4.2.61) represent plane coupled case for bending about y axis but the bending about x axis and rotation are coupled in the other two equations.

For a symmetric configuration of piers as shown in Case-C (fig. 4.2.11) $b = 0$, $I_{xy} = 0$, $I_{xc} = 0$; $I_{yc} = 0$; $(\omega+d) = 0$. Substituting these conditions in the governing equations (eq. 4.2.12, eq. 4.2.16, eq. 4.2.17 and eq. 4.2.25),
they are reduced to:

\[ u'' \ a = \frac{1}{E} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) T + \frac{c^3}{12 E J_{\nu}} \ \frac{dq}{dz} \]  
\[ (4.2.64) \]

\[ E S_{yc} \ u'' + T \ a = M_y \]  
\[ (4.2.65) \]

\[ E S_{x} \ v'' = M_x \]  
\[ (4.2.66) \]

\[ -E I_{\omega} \theta'''' + G J \theta' = Q_t \]  
\[ (4.2.67) \]

The first two equations (eq. 4.2.64 and eq. 4.2.65) represent the plane coupled case for bending about y axis. The other two equations (eq. 4.2.66 and eq. 4.2.67) are uncoupled representing independent rotation and bending about x axis. It should be noted that though the individual piers do not have any sectorial moment of inertia, the group has an equivalent \( I_{\omega} \) as defined in eq. 4.2.21. In this case

\[ I_{\omega} = C_{x1}^2 I_{x1} + C_{x2}^2 I_{x2} \]  
\[ (4.2.68) \]

Therefore, the torsional resistance of the combined structure is substantially larger than the sum of individual resistance.

4.2.9 Effect of Neglecting Axial Deformation of Piers

Gluck (22) assumed the axial deformation of piers to be very small and neglected it in his analysis. It is of interest to indicate the present analysis is reducible to the equation given by Gluck.
If axial deformation of piers is neglected, \( \delta_2 = 0 \) (eq. 4.2.9). For which the compatibility equation (eq. 4.2.12a) is reduced to,

\[
\frac{c^3}{12EJ_b}q = \frac{c^3}{12EJ_b}q (4.2.69)
\]

Differentiating eq. 4.2.16 and eq. 4.2.17 and using eq. 4.2.8

\[
E S_y u''' + S_{xy} v''' - E S_{yc} \theta''' - aq = M_y' \tag{4.2.70}
\]

\[
E S_{xy} u''' + E S_x v''' + E S_{xc} \theta''' - bq = M_x' \tag{4.2.71}
\]

Using eq. 4.2.8, eq. 4.2.25 becomes

\[
-E S_{xc} v''' + E S_{yc} u''' - EI \omega''' + GJ\theta' = (\omega+d)q = Q_t \tag{4.2.72}
\]

Eliminating eq. q from eq. 4.2.70, eq. 4.2.71 and eq. 4.2.72 by the help of eq. 4.2.69 and expressing in matrix form:

\[
\begin{bmatrix}
E S_y & E S_{xy} & -E S_{yc} \\
E S_{xy} & E S_x & E S_{xc} \\
-E S_{yc} & E S_{xc} & EI \omega
\end{bmatrix}
\begin{bmatrix}
u''' \\
v'''
\theta'''
\end{bmatrix} =
\begin{bmatrix}
\begin{bmatrix}
u' \\
v' \\
0
\end{bmatrix} \\
0 \\
GJ \begin{bmatrix}
\theta'
\end{bmatrix}
\end{bmatrix}
\[
+ \frac{12EJb}{c^3} \begin{bmatrix}
a^2 & ab & a(\omega d)
\ab & b^2 & b(\omega d)
\a(\omega d) & \b(\omega d) & (\omega d)^2
\end{bmatrix}
\begin{bmatrix}
u'
\v'
\theta'
\end{bmatrix} = \begin{bmatrix}
-M'_y
-M'_x
Q_t
\end{bmatrix} \tag{4.2.73}
\]

This equation is same as Gluck (22) except the factor \( d \) which is omitted. The error has been noted and suggested by Biswas and Tso (24).

4.3 Computer Program

A computer program based on the analysis has been written. This program covers the general configuration and the special configuration in Case-A (Fig. 4.2.11). subject to concentrated force and/or torque at top. The input data are the geometric and elastic properties of the shear wall. The program determines the value of constants \( \kappa_1, \kappa_2, \ldots, \kappa_5 \) (eq. 4.2.52) by solving a set of linear simultaneous equation obtained from the boundary conditions. The output consists of the generalised displacement of reference point 0, shear force in the connecting beams and bending moment of the individual piers at chosen levels. The computer program is included in Appendix B.

4.4 Experiment

An experiment was done on a small scale model (fig. 4.4.1). It consisted of two equal angles connected by floor beams at equal spacing. The model was made from plexiglas sheet. It was loaded by a concentrated force at the top by hanging weights over a pulley system (Fig. 4.4.2). A
TEST MODEL

FIG. 4.4.1
Fig. 4.4.2

EXPERIMENTAL SET UP
FIG. 4.4.4

(a) Strain Gauge Positions

(b) Dial Gauge Positions
second loading configuration consists of a torque applied at top of the same structure. This was done by applying two equal but opposite forces as shown in fig. 4.4.3. Strain gauges and dial gauges were fixed at different points of the model (Fig. 4.4.4) and readings were taken at every increment of loading. Strain gauge and dial gauge readings are tabulated in Appendix-C. The same set of instruments used for static test of model with floor (Chapter 2) was used.

4.5 Results and Discussion

The linearity of the test structure is checked in fig. 4.8.10. The rotation and displacement of the model subjected to concentrated load and torque at top are plotted in fig. 4.5.1 to fig. 4.5.4. The moment in pier 1 in the principal directions and distributed shear force $q$ in the connecting beam are plotted in fig. 4.5.5. From the moment diagram, the theoretical strains at different points in level AA and BB (Fig. 4.4.4) are determined. The strain distribution thus obtained together with the experimental strains are plotted in Fig. 4.5.6 to 4.5.9.

The experimental results of rotation and displacement compared reasonably well with the theory except for the case of displacement measured due to an applied torque as shown in fig. 4.5.3. The probable reason for difference in fig. 4.5.3 may be due to the imperfection of the torque applying device. It is conceivable that some lateral load may develop in addition to the applied torque during the test.
Experimental Point ○
Theoretical Considering Axial Deformation ———
Theoretical Neglecting Axial Deformation ———

FIG. 4.5.1
Experimental Point ○
Theoretical Considering Axial Defomation ——
Theoretical Neglecting Axial Deformation ——

FIG. 4.5.2

25 lbs
Experimental Point ○
Theoretical Considering Axial Deformation ———
Theoretical Neglecting Axial Deformation ———

FIG. 4.5.3
Experimental Point
Theoretical Considering Axial Deformation
TheoreticalNeglecting Axial Deformation

125 lb-in

FIG. 4.5.4
FIG. 4.5.5

DUE TO CONCENTRATED FORCE 25 lbs AT TOP

DUE TO CONCENTRATED TORQUE 125 lb-in AT TOP
STRAIN DISTRIBUTION AT LEVEL AA

FIG. 4.5.6
STRAIN DISTRIBUTION AT LEVEL BB

FIG. 4.5.7
STRAIN DISTRIBUTION AT LEVEL AA

FIG. 4.5.8
STRAIN DISTRIBUTION AT LEVEL BB

FIG. 4.8.9
LOAD vs. STRAIN DIAGRAM FOR MODEL WITH BEAMS

FIG. 4.8.10
The effect of axial deformation of piers is found to have no effect on results for loading case with applied torque. However, the effect is considerable for in the case of lateral loading. The displacement at top decreases by 37% and rotation at top decreases by 7% as a result of neglecting axial deformation. The error introduced by neglecting axial deformation becomes significant if the axial force in the piers is large as in the case of the lateral loading. A comparison from the shear (q) plot in fig. 4.5.5 shows that the shear is about five times in the case of applied loading as compared to the case of applied torque. Since the axial force in the piers is the sum of the distributed shear $q$, neglecting the deformation due to axial force in the case of applied loading affect the results considerably.

In the strain diagrams (Fig. 4.5.6 to 4.5.9) the comparison between the experiment and the theory is less favourable. Since strain is a local measure, the experimental results are affected by the local imperfection of the test model. The continuum approach used in the analysis is expected to give results to the overall behaviour of the structure but with less accurate results to the local behaviour. Nevertheless, the trend of the strain distribution as predicted by the theory is verified by the experimental points.

In general it is expected that the continuum approach of analysis is best suited for structures with a large number of stories. In the present study, the model used in the experiment consists of eight floor beams only. Yet, the results obtained from the analysis compare well with the
experimental points.
CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The results from the static analysis and the testing of shear wall model with floors show that floor slab can provide considerable torsional stiffness by providing restraint against warping. Neglect of the floor slab stiffness will underestimate the stiffness of the structure. Moreover the actual structure is not as stiff as predicted by considering both torsional and flexural stiffness of the slabs. The bending stiffness of the slab is less effective due to the local rotation of the joints and a modified bending stiffness is to be considered. This is taken into account by a factor $K$ in eq. 2.6.2. Theoretical values thus obtained compare reasonably well with experiments for displacement and strain measurements.

In the dynamic study of shear wall with floor slabs, the mass and mass moment of inertia of the slabs have to be considered in addition to the stiffness provided. The vibration is in general coupled. The first mode is a torsion predominant one and the second mode is bending predominant. The theoretical frequency compared reasonably with the experimental values except for the second frequency which is 16% higher. This is due to the neglect of shear deformation in the theory, which have considerable effect on the frequency for bending predominant modes.
In the static and dynamic analysis of shear walls with floors, the 'Matrix Transfer' method has been used. The method is ideal for continuous systems with discrete points. The main advantage is that the size of the matrix handled in the analysis is independent of the number of floors. An increase of the number of floors will only increase the number of matrix multiplication which can be done easily in a digital computer.

The non-planar coupled shear wall was analysed using the continuum method. It is shown that the effect of neglecting axial deformation of piers will lead to gross overestimation of stiffness of the structure for certain cases. Simplicity can be achieved by this assumption but should always be done with caution. The theoretical analysis shows reasonable agreement with the experimental results for displacements but less favourable for strains. So it can be concluded that continuum method gives good results for overall behaviour but is less accurate for local behaviour of a structure.

5.2 Recommendations

The present study of shear wall structures with special interest on floor slabs and floor beams are carried out on simple structures. The arrangement of the shear walls in an actual multi-storey building is very complex. The analysis is also complex due to various interacting elements. Simplicity can be achieved by assumption but a complete understanding of the behaviour of different interacting elements
is necessary for making reasonable assumptions.

More experimental and theoretical study on the behaviour of floor slab with different geometrical arrangement is necessary. In such cases, it may not be justified to treat the floor slab as a series of beams but as an elastically supported plate.

Extension of the formulation on non-planar coupled shear wall is necessary for more generalised cases. The present analysis is applicable for two shear walls connected by a single row of beams. Extension for cases with more piers and more rows of connecting beam is required. A study on the dynamic analysis for non-planar coupled shear walls is also recomended.

In general, the analytical tools presently available to design engineers are very limited and most of the time very restricted in its applications. Therefore, more theoretical work supported by experimental data is necessary in the shear wall field to assist the practicing engineers to analyse and design structures which will be safe at the same time economic.
APPENDIX A

VLASOV'S THEORY OF THIN WALLED BEAM

The method of analysis used in the present work is based on the theory presented by Vlasov (8). Vlasov's theory is based on two geometric hypothesis:

(a) a thin walled beam of open section can be considered as a shell of rigid (undeformable) cross section.

(b) the shear deformation of the middle surface (characterising change in the angle between the co-ordinate line) can be neglected.

In shear wall structure, the concrete clear wall can be treated as thin walled beams connected by floor slabs which are normally located at regular intervals. The action of the floor slab is to prevent any deformation of the section which supports the hypothesis (a). Hypothesis (b) requires shear deformation to be negligible compared with the torsional and flexural deformations. Vlasov states that this is satisfied if for the structure shown in Fig. A.1

\[ \frac{t}{d} \leq 0.1 \text{ and } \frac{d}{\lambda} \leq 0.1 \]

For components of tall building this conditions are satisfied.

The expression of longitudinal stress in Vlasov's theory is

\[ \sigma = \frac{N}{A} - \frac{M_y \cdot x}{I_y} - \frac{M_x \cdot y}{I_x} + \frac{B \cdot \omega}{I_\omega} \]  \hspace{1cm} (A.1)

The first three terms coincide with the equation known from elementary theory. The last term of the expression is
Dimensional Limitations

FIG. A 1

FIG. A 2

shear center

Centroid

x & y Ordinates

Sectorial Coordinate $\omega$
Co-ordinate System

FIG. A 3

Generalised Displacements

FIG. A 4

Generalised Forces

FIG. A 5
the longitudinal stress due to warping. The notations of the last terms are explained as:

Bimoment $B$ is a generalised balanced system of forces statically equivalent to zero. Units are force x (length)$^2$ e.g. lbs. in$^2$.

$\omega$ is the sectorial area of the point on the section where the stress is being measured. Units are (length)$^2$ e.g. in$^2$.

$I_\omega$ is sectorial moment of Inertia of the section and is defined as $\int\omega^2 dA$ units are (length)$^6$ e.g. in$^6$.

The distribution of sectorial co-ordinate for some open section is shown in Fig. A.2. Right hand co-ordinate system used in the present work (Fig. A.3). The generalised displacement variables are shear center displacements $u, v$ and $\theta$ in $xy$ plane and centroidal displacement $s$ in $z$ direction (Fig. A.4). Sign convention for generalised forces are shown in Fig. A.5.

The relation between the generalised forces and displacement variables are:

$$\begin{align*}
N &= E\alpha' s', & M_x &= EI_x v'', & M_y &= EI_y u'', \\
B &= -EI_\omega \theta''', & H &= -EI_\omega \theta'''' + GJ\theta', \\
V_y &= -EI_x v''', & V_x &= -EI_y u'''.
\end{align*}$$

(A.2)

Of these quantities, axial force and bending moments are referred to the centroid and shear forces and torque are referred to the shear center of the section.

The longitudinal stress at any point can be expressed
in terms of displacement variables as:
\[
\sigma = E[s' - u''x - v''y - \theta''\omega]
\]  \hspace{1cm} (A.3)
Displacement of any point in z direction is obtained as:
\[
\delta = s - u'x - v'y - \theta\omega
\]  \hspace{1cm} (A.3a)
Tensile stress and displacements in the direction of positive \(x, y\) & \(z\) are positive in eq. A.3 and eq. A.3a. When referred to principal generalised co-ordinates of an open section, the differential equation of a thin walled beam statically loaded at its ends are:
\[
\begin{align*}
EAs'' &= 0 \\
EI_x u'' &= 0 \\
EI_y u'' &= 0 \\
EI_{\omega \theta}'' - GJ\theta'' &= 0
\end{align*}
\]  \hspace{1cm} (A.4)
Differential equations for free vibration of a thin walled beam are:
\[
\begin{align*}
EAs'' - \rho A s'' &= 0 \\
EI_x u'' + \rho A u'' - \rho I_x u'' - \rho A_\theta u'' &= 0 \\
EI_y u'' + \rho A u'' - \rho I_y u'' + \rho A_\theta u'' &= 0 \\
EI_{\omega \theta}'' - GJ\theta'' + \rho I \theta'' - \rho I_{\omega} \theta'' + \rho A_\theta u'' - \rho A_x u'' &= 0
\end{align*}
\]  \hspace{1cm} (A.5)
Eq. A.4 and eq. A.5 are referred in Chapter 2 and Chapter 3 and solved accordingly as they appeared in the analysis.
APPENDIX - B

COMPUTER PROGRAMS
PROGRAM TST (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)
STATIC ANALYSIS OF SHEAR WALL WITH FLOORS
M. ENG. THESIS BISWA

COMPUTER NOTATIONS USED
XI MOMENT OF INERTIA OF SECTION ABOUT X AXIS
YI MOMENT OF INERTIA OF SECTION ABOUT Y AXIS
WI SECTORIAL MOMENT OF INERTIA OF SECTION
DI POLAR MOMENT OF INERTIA ABOUT SHEAR CENTER
E MODULUS OF ELASTICITY
G MODULUS OF RIGIDITY
H FLOOR HEIGHT
NF NUMBER OF STOREY
DM BIMOMENT CONTRIBUTION FACTOR
PLY(I) LATERAL LOAD AT I TH FLOOR IN Y DIRECTION
PLR(I) APPLIED TORQUE AT I TH FLOOR

DIMENSION P(8,8), F(8,8), PFA(8,H), PFH(8,H), Al(8,8), T(8,8),
1At(8,1), bT(8,1), Nl(8), WORK(8), PLY(50), PLR(50), S(8,8), PS(8,1)

INPUT DATA

000003 XI = 42.7089
000005 YI = 11.0322
000006 WI = 108.6931
000010 DI = .097670
000011 E = .4406
000013 PSN = .35
000014 G = E/(2.*(1+PSN))
000020 NF = 8
000041 H = 6
000022 DM = 20.109E+04
000024 DM = 124.909E+04
000025 DM = 0
000026 DO 75 I=1, NF
000027 DO 75 I=1, NF
000030 DO 75 I=1, NF
000033 PLR(I) = 0.
000035 PLR(I) = 0.
000036 PLY(I) = 0.
000038 PLY(I) = 12.5
000040 PLR(I) = 6.25*12.5

PRINTING OUT INPUT DATA

WHITE (6,1)
000044 WRITE (6,1) * INPUT DATA *
000042 WRITE (6,2) * NF,H,XI,YI, WI,DI,PSN,E,G,D,DM
000074 WRITE (6,3) *
000074 WRITE (6,4) *
000074 WRITE (6,5) *
000074 WRITE (6,6) *
000074 WRITE (6,7) *
0000100 DO 70 I=1, NF

FIELD TRANSFER MATRIX F

WHITE (6,7) * F(I,J)= PLY(I)*PLR(J)

0000102 GU= 0
0000108 AK = H*SQRT((6D/(E*WI)))
0000128 AL= AK/H
0000127 DU 4 I= 1, 8
0000130 DU 4 J= 1, 8
0000131 IF (I.NE.J) F(I,J) = 0.
0000134 F(J,J)= 1.
F(1, 2) = H
F(1, 3) = -(H^4)/(2 - 6*E*XI)
F(1, 4) = -(H^4)/(2 - 6*E*XI)
F(2, 3) = -(H^4)/(2 - 6*E*XI)
F(2, 4) = -(H^4)/(2 - 6*E*XI)
F(3, 4) = H
F(5, 6) = SINH(AK)/AL
F(5, 7) = (1 - COSH(AK))/GD
F(5, 8) = (H * SINH(AK))/AL
F(6, 8) = COSH(AK)
F(6, 9) = -(AL * SINH(AK))/GD
F(6, 10) = (1 - COSH(AK))/GD
F(7, 9) = (1U * SINH(AK))/AL
F(7, 10) = COSH(AK)
F(7, 11) = SINH(AK)/AL

POINT TRANSFER MATRIX

DU 6 I = 1, 8
DU 6 J = 1, 8
IF (I = NE = 3, P(I, J) = 0.
6 P(J, J) = 1.
P(7, b) = -DM
32 CALL FLOOR(P, F, NF, A, PFA, PFB, PF)
DU 10 I = 1, 8
10 AT(I, 1) = 0
AT(4, 1) = -PLY(NF)
AT(8, 1) = -PLR(NF)
WRITE (6, 9)
9 FORMAT (1X, * FIELD TRANSFER MATRIX *)
WRITE (6, 5) ((F(I, J), J = 1, 8), I = 1, 8)
WRITE (6, 13)
13 FORMAT (1X, * POINT TRANSFER MATRIX *)
WRITE (6, 5) ((P(I, J), J = 1, 8), I = 1, 8)
5 FORMAT (1X, 8E15.6)

NC = 1
11 CALL FLOOR(P, F, NC, T, PFA, PFB, PF)
NCC = NF = NC
DU 12 I = 1, 8
12 AT(I, 1) = AT(I, 1) + T(I, 4) * PLY(NCC) = T(I, 8) * PLR(NCC)
NC = NC + 1
IF (NC.EQ.1) GO TO 14
14 DU 16 I = 1, 8
DU 16 J = 1, 8
16 A(I, J) = A(I, J)
DU 18 I = 1, 4
DU 18 J = 1, 2
K = I + 4
L = J + 4
A(I, J) = 0.
A(I, L) = 0.
A(I + 1, I) = 1.
A(2, 2) = 1.
A(3, 5) = 1.
A(6, 6) = 1.
CALL MINVSE(A, 8, B, 1, E = 0, IERR, N, WORK)
IF (IERR.EQ.0) GO TO 30
WHITE (6, 5)
57 FORMAT (5A, * UNSUCCESSFUL INVERSION *)
WHITE (6, 5)
GO TO 100
30 CALL GMPSU(A, AT, BT, 8, 8, 1)
00 432 BT(I+1) = 0
00 433 BT(2*1) = 0
00 434 BT(5*1) = 0
00 435 BT(6*1) = 0

C
C AS THE STATE VECTOR AT BASE
00 436 WRITE (6, 31)
00 441 31 FROLL (1X)
00 445 WRITE (6, 37)
*6X, *TORQUE*)
00 449 WRITE (6, 33) ((UT(I,J), J = 1, 8), I = 1, 8)
00 453 33 FORMAT (4X, *O0I4X, 8E14_4)
00 456 NA = 1
00 459 17 DO 27 I = 1, 8
00 462 27 AI(I+1) = 0
00 465 AT(4*I) = ATL(NA)
00 472 AT(8*1) = ATL(NA)
00 475 CALL GMPSU(F, BT, S, 8, 8, 1)
00 501 WRITE (6, 41) NA, ((S(I,J), J = 1, 1), I = 1, 8)
00 524 20 FORMAT (4X, *3, *BELOW*, 8E14_4)
00 527 CALL GMPSU(F, S, P$8, 8, 1)
00 529 DO 22 I = 1, 8
00 532 22 PS(I+1) = PS(I, 1) + AT(1, 1)
00 534 WRITE (6, 21) NA, ((PS(I,J), J = 1, 1), I = 1, 8)
00 554 21 FORMAT (4X, *3, *AFTER*, 8E14_4)
00 556 IF (NA < EQ. NF) GO TO 100
00 556 24 UU = 24 I = 1, 8
00 560 BT(I+1) = PS(I, 1)
00 565 NA = NA + 1
00 566 GO TO 17
00 567 100 STOP
00 571 END

UNUSED COMPILER SPACE
00 4000
SUBROUTINE FLOOR(P, F, N, T, PFA, PFH, PF)
MULTIPLICATION OF MATRICES ACCORDING TO NO OF FLOOR
P, F ARE THE INPUT MATRICES
N IS THE NO OF FLOORS
T IS THE OUTPUT MATRIX
PFA, PFH, PF ARE THE WORKING MATRICES OF SIZE 8 * 8
DIMENSION P(8,8), F(8,8), T(8,8), PFA(8,8), PFH(8,8), PF(8,8)

KOUNT = 1
IF (N .GT. 1) GO TO 4

DU 10 I = 1, 8
DU 10 J = 1, 8
10 T(I, J) = PF(I, J)

GO TO 30

DU 5 I = 1, 8
DU 5 J = 1, 8
5 PFA(I, J) = PE(I, J)

CALL MULT(PFA, PF, PFB, 8)

KOUNT = KOUNT + 1

DU 7 I = 1, 8
DU 7 J = 1, 8
7 PFA(I, J) = PFB(I, J)

IF (KOUNT .EQ. N) GO TO 9

GO TO 6

9 DU 12 I = 1, 8
DU 12 J = 1, 8
12 T(I, J) = PFB(I, J)

30 RETURN

END
SUBROUTINE MULT(A,B,C,N)

MULTIPLICATION OF SQUARE MATRICES

DIMENSION A(N,N),B(N,N),C(N,N)

DO 1 I=1,N

DO 1 J=1,N

C(I,J)=0.

DO 1 K=1,N

C(I,J)= C(I,J)+ A(I,K)* B(K,J)

CONTINUE

RETURN

END

UNUSED COMPILER SPACE

009500
PROGRAM TST (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)
DYNAMIC ANALYSIS OF SHEAR WALL WITH FLOORS
M. ENG. T. HENRIS BISWAS

COMPUTER NOTATIONS USED
E MODULUS OF ELASTICITY
G MODULUS OF RIGIDITY
PNU POISONS RATIO
RO MASS DENSITY OF MATERIAL
XI MOMENT OF INERTIA OF SECTION ABOUT X AXIS
D1 TORSION FACTOR OF THE SECTION
PI POLAR MOMENT OF INERTIA OF SECTION ABOUT SHEAR CENTER
AX COORDINATE OF SHEAR CENTER
XJ MASS MOMENT OF INERTIA OF PLATE ABOUT X AXIS
ZJ POLAR MASS MOMENT OF INERTIA OF PLATE ABOUT SHEAR CENTER
AP DISTANCE BETWEEN SHEAR CENTER AND CENTROID OF PLATE
H HEIGHT OF EACH STOREY
NFLOOR NUMBER OF FLOORS
W STARTING FREQUENCY
WINC INCREMENT OF FREQUENCY

DIMENSION TS(4,4), TF(4,4)
DIMENSION C(8,8), D(8,8), TR(8,8)
DIMENSION RA(9), RI(8), RL(8)
DIMENSION P(8,8), CP(8,8), NI(8), TEMP(8), W(8,8)
DIMENSION F(8,8), PF(8,8), PPA(8,8), PDF(8,8), T(8,8), TT(4,4)

COMPLEX C, I, CT, IC, TEMP, RC

INPUT DATA
000003 000004 000005 000006 000007 000008 000009 000010 000011 000012 000013 000014 000015 000016 000017 000018 000019 000020 000021 000022 000023 000024 000025 000026 000027 000028 000029 000030 000031 000032 000033 000034 000035 000036 000037 000038
NFLOOR= 8
RU= 1.11E-04
DN= 4.5767E-04
AM= 1.84089E-04
XJ= 8.7445E-04
ZJ= 5.24879E-04
AP= 5.4412
AX= -2.3192
H= 6
E= 5.0E-06
PNU= .35
G= E/(2.*PNU))
A= 4.2706
X1= 4.27089
W= 108.6931
DI= .97576
PI= 100.7907
NFQ= 3
W= 50.
WINC= 100.

WRITE (6,3) H, E, G, RO
WRITE (6,4) 3 FLOOR HEIGHT = FLOOR1 = F1 = F15 = F9 = F7 = F5 = F3 = F1
WRITE (6,5) 10F ELASTICITY = E15 = E13 = E9 = E7 = E5 = E3 = E1
WRITE (6,6) 2 DENSITY OF MATERIAL = D15 = D13 = D9 = D7 = D5 = D3 = D1
WRITE (6,7) NFLOOR
WRITE (6,8) 7 NUMBER OF FLOORS = 4
WRITE (6,9) 6, 1 A, X1, W, X, PI, AX, AM, XJ, ZJ, UM, AP

139
000116 2 FORMAT (1X, SECTION PROPERTIES, 9x, AREA =, F9.4/9x, MOMENT OF
1 INERTIA ABOUT X =, F9.4/9x, SECTORIAL MOMENT OF INERTIA =, F9.4/
29x, TORSION FACTOR =, F9.4/9x, POLAR MOMENT OF INERTIA =, F9.4/
39x, ORDI NATE OF SHEAR CENTRE =, F9.4/1a, DIAPHRAGM PROPERTIES =, F9
4x, MASS OF PLATE =, E15.6/9x, MASS MOMENT OF INERTIA ABOUT X =, E15
5E15.6/9x, POLAR MASS MOMENT OF INERTIA =, E15.6/9x, BIMOMENT FAC
6TOR =, E15.6/9x, SHEAR CENTRE TO C.G. OF THE PLATE =, F9.4)
000116 1 FORMAT (6F9.4/7, F9.4/JE15.6)
000116 WHITE (6, 42)
000122 42 FORMAT (1X, * RESULTS FOR FIXED FREE CASE *)
000123 KKKK = 333
000124 NCOUNT = 6666
000125 DET0 = 0.
000125 N FRE = 0.
000126 C POINT TRANSFER MATRIX P
000126 40 DU 4 I = 1, 8
000128 DU 4 J = 1, 8
000130 IF (J.I.E. J) P(I, J) = 0.
000134 4 P(J, J) = 1.
000143 P(1, 2) = -XJ#W#W
000146 P(3, 6) = -W#AM#W#W
000150 P(4, 1) = -AP#AM#W
000152 P(4, 5) = -AP#AM#W#2
000154 P(7, 6) = -DM
000155 P(8, 1) = -W#AP#W#2
000157 P(8, 9) = -VJ#W#W#2
000161 C COEFFICIENTS OF CHARACTERISTICS EQUATION
000162 6 AA(I) = 0.
000165 AA(1) = -W*IXI*E#W
000170 AA(5) = W*IXI*E#G*2.*W*IXI*RO*E#W*G2.*W*IXI*(W#W)*(RO#W)
000175 AA(7) = -DI*AP#G#(W#W) + (PI*AI*W#A)*(W#W)*(RO#W)
000183 AA(9) = (PI#A + I#AX)*W#(W#G) + (W#W)*(RO#W)
000187 C SOLUTION BY SUBROUTINE BAIRST
000187 CALL BAIRST (AA#RR, RI, 8)
000192 DU 5 I = 1, 8
000205 DU 5 J = 1, 8
000208 C FIELD TRANSFER MATRIX F
000208 5 DU 11 J = 1, 8
000212 C(1, J) = CMPLX(1, 0)
000216 C(2, J) = RC(J)
000220 C(3, J) = E*IXI*(RC(J)*#2)
000224 C(4, J) = E*IXI*(RC(J)*#3)
000228 C(5, J) = (E*IXI)/(W#W)*RC(A#AX)*(RC(J)*#4)-(XI/(A#AX))*(RC(J)*#2
000232 1 + 1/A#X)
000235 C(6, J) = (E*IXI)/(W#W)*RC(A#AX)*(RC(J)*#5)-(XI/(A#AX))*(RC(J)*#3
000239 1 + 1/A#X)*RC(J)
000243 C(7, J) = (E*IXI)/(W#W)*RC(A#AX)*(RC(J)*#6)+(E*IXI)/
000247 1(A#AX)*RC(J)*#4 + (E*IXI)/(A#AX)*RC(J)*#2
000251 C(8, J) = (E*IXI)/(W#W)*RC(A#AX)*(RC(J)*#7)+(E*IXI)/
000255 1(A#AX)*(E*IXI)/(W#W)*RO#A#AX)*(RC(J)*#5 + (E*IXI)/(A#AX)*
000260 2XI)/(A#AX)*RC(J)*#3 + (G#D1)/AX)*RC(J)
000264 C CONTINUE
000268 11 DU 13 J = 1, 8
000272 DU 13 J = 1, 8
000276 IF (J.I.E. J) D(I, J) = 0.
000280 000282 D(I, J) = EXP*(H#RC(J))
13  B(I,J)=C(I,J)
0008553  CALL INVCPX(B B, B, 1, E=B, IERR, NI, TEMP )
000652  CALL MATMULT(C, U, CD, B)
000655  CALL MATMULT(C, B, Q, B)
000670  DO 24 J=1,B
000672  DO 24 J=1,B
000673  24  F(I,J)=REA1(Q(I,J))
000705  CALL FLOOR(P; F; N; FLOOR, T; PFA+PH, PF )
000713  CALL UCXFR(I, T)
000715  CALL VETVAI(T, I, DET, 4)
000720  46  PROD= DETO*DET
000722  IF (NCOUNT.EQ, 5555) GO TO 47
000724  IF (PHOO*GE.0) GO TO 8
000726  DETN=DET
000730  45  WAP= w+(w-WO) DETO/(DETO-DETN)
000737  PC= (WAP-WO)/WAP
000741  IF (PC.LT.0.0) GO TO 55
000743  W= WAP
000745  NCOUNT= 5555
000746  GO TO 40
000747  47  IF (PHOO.LT.0.0) GO TO 44
000751  DETO= DET
000752  KKK= 222
000753  WO= w
000755  W= W
000756  GO TO 45
000756  55  NFREQ= NFREQ+1
000760  WW= WAP/6.28
000762  WRITE (6, 56) NFREQ, WW
000771  60  FORMAT (7X. *FREQ *.13, 5X, F10.4 )
000771  8  DETO= DET
000773  IF (KKK.EQ.222 ) DETO= DETN
000776  KKK= 111
000777  WO= w
000777  NCOUNT= 3333
000777  W= W+ WINC
000783  IF (NFREQ.EQ.NFQ) GO TO 50
000785  GO TO 40
000786  50  STOP
000787  END

UNUSED  COMPIER SPACE
003100
SUBROUTINE INVCX(A,N,NN,ZERO,IERR,NI,TEMP)

REM THE ROUTINE INVERSES THE MATRIX BY A PIvoTAL METHOD
REM USING THE LARGEST ELEMENT IN THE NEXT ROW AS A PIvoT. THE
REM FIRST PART DOES THE INVERSION AND THE SECOND PART
REM HARRANGES THE ROWS AND COLUMNS TO TAKE ACCOUNT OF THE
REM FACT THAT THE PIvoTS WERE NOT ON THE DIAGONAL.
REM THIS SUBROUTINE IS CALLED BY CALL INVMA PL(A*NN,NN,ZERO,IERR,NI,TEMP) WHERE A IS AN NN BY NN MATRIX IN AN N BY N ARRAY.
REM ZERO IS A TEST VALUE BELOW WHICH AN ELEMENT IS CONSIDERED TO BE ZERO, IER IS AN INDICATOR WHICH IS 0 IF THE INVERSE
REM IS FOUND AND NON-ZERO OTHERWISE, NI IS A WORKING ARRAY.
REM OF NN ELEMENTS AND TEMP IS ANOTHER.

DIMENSION A(N,N),NI(N,NN),TEMP(NN)
COMPLEX AA,NI,TEMP

DO 10 I=1,NN
  DO 30 J=1,NN
    NI(J)=0
  DO 20 J=1,NN
    AAA=V
    IF (NI(J)*NE.0) GO TO 30
    C=CIABS(A(K,J))
    IF (C.GT.*AAA) GO TO 30
    AAA=C
    JJ=J
  CONTINUE
  IF (AAA.GT.*ZERO) GO TO 70
  IERR=K
  RETURN

  NI(JJ)=K
  AA=A(K,J)/C
  A(K,J)=1.
  DO 50 J=1,NN
    AA=A(K,J)*AA
    DO 60 I=1,NN
      IF (I.EQ.K) GO TO 60
      AAA=A(I,J)
      IF (AAA.GT.*ZERO) GO TO 50
    CONTINUE
  CONTINUE
  IERR=0

  DO 90 I=1,NN
    IF (NI(I)*NE.1) GO TO 95
    RETURN
  CONTINUE

  DO 95 I=1,NN
    NI(J)=K
    DO 80 J=1,NN
      TEMP(K)=A(I,J)
      DO 80 J=1,NN
        A(I,J)=TEMP(J)
      CONTINUE
    DO 80 J=1,NN
      A(I,J)=TEMP(J)
    CONTINUE
  CONTINUE

10 CONTINUE
20 CONTINUE
30 CONTINUE
50 CONTINUE
60 CONTINUE
70 CONTINUE
80 CONTINUE
90 CONTINUE
SUBROUTINE FLOOR(P,F,N,T,PFA,PFB,PF)
MULTIPLICATION OF MATRICES ACCORDING TO NO OF FLOOR
P, F ARE THE INPUT MATRICES
T IS THE OUTPUT MATRIX
PFA, PFB, PF ARE THE WORKING MATRICES OF SIZE 8*8
DIMENSION P(8,8), F(8,8), T(8,8), PFA(8,8), PFB(8,8), PF(8,8)
CALL MULT(P,F,PF,8)
KOUNT = 1
IF (N>1) GO TO 4
DO 10 I=1,8
DO 10 J=1,8
T(I,J) = PF(I,J)
10 GO TO 30
DO 4 I=1,8
DO 4 J=1,8
PFA(I,J) = PF(I,J)
5 CALL MULT(PFA,PF,PFB,8)
6 KOUNT = KOUNT+1
DO 7 J=1,8
7 PFA(I,J) = -PFB(I,J)
IF (KOUNT EQ N) GO TO 9
DO 9 J=1,8
9 GO TO 6
DO 12 J=1,8
12 T(I,J) = PFB(I,J)
DO 12 I=1,8
12 RETURN
END

UNUSED COMPILER SPACE
SUBROUTINE MATMULT (A,B,C,N)
MULTIPLICATION OF COMPLEX MATRICES
DIMENSION A(N,N),B(N,N),C(N,N)
COMPLEX A,B,C
DO 1 I=1,N
DO 1 J=1,N
C(I,J)=0.
DO 1 K=1,N
C(I,J)=C(I,J)+A(I,K)*B(K,J)
1 CONTINUE
RETURN
ENDD

SUBROUTINE BCXFR(T,TI)
BOUNDARY CONDITION ONE END FIXED OTHER END FREE
DIMENSION T(8,8),TI(4,4)
DO 30 I=1,4
DO 30 J=1,4
30 CONTINUE
RETURN
ENDD

UNUSED COMPILER SPACE
009400
SUBROUTINE DETVAL(A,DET,M)  
EVALUATION OF DETERMINANT  
DIMENSION A(N,M)  
NE=0  
MM=M-1  
DO 8 J=1,MM  
JJ=J+1  
DO 30 JB=JJ,M  
IF (ABS(A(J,J)) .GE. ABS(A(JB,J))) GO TO 30  
NE=NE+1  
IF (NE.EQ.2) NE=0  
DO 28 KK=1,M  
HOLD=A(J,KK)  
A(J,KK)=A(JB,KK)  
28 A(JB,KK)=HOLD  
DO 30 CONTINUE  
30 CONTINUE  
N=NN=M  
C=A(NN,J)/A(J,J)  
DO 4 N=1,M  
A(NN,N)=A(NN,N)-A(J,N)*C  
4 CONTINUE  
DETr=DET+T  
6 CONTINUE  
RETURN  
END  

SUBROUTINE MULT(A,B,C,N)  
multiplication of square matrices  
DIMENSION A(N,N),B(N,N),C(N,N)  
DO 1 I=1,N  
DO 1 J=1,N  
C(I,J)=0.  
1 CONTINUE  
RETURN  
END  

UNUSED COMPILER SPACE
PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
NON PLANAR COUPLED SHEAR WALL
M. ENG. THESIS BISWAS
COMPUTER NOTATIONS USED
SX1, SX2, SY1, SY2, SXYY1, SXYY2, SW1, SW2
SY1, SY2, SY2, SW1, SW2
SECTORIAL MOMENT OF INERTIA OF PIERS W.R.T SHEAR CENTRE
SJ1, SJ2
TORSION FACTORS OF PIERS
EX1, EY1, EX2, EY2
COORDINATES OF C.G. OF PIERS W.R.T GLOBAL AXES
CX1, CY1, CX2, CY2
COORDINATES OF S.C. OF PIERS W.R.T GLOBAL AXES
A1, A2
AREA OF THE PIERS
W1, W2
SECTORIAL COORDINATES
E, G, PSN
MODULUS OF ELASTICITY, MODULUS OF RIGIDITY, POISONS RATIO
L
CLEAR SPAN OF THE CONNECTING BEAM
SB
MOMENT OF INERTIA OF CONNECTING BEAM
AST
EFFECTIVE SHEAR AREA OF CONNECTING BEAM
M
SPACING OF CONNECTING BEAM
NF
NUMBER OF FLOORS
PX
CONCENTRATED LOAD AT TOP IN X DIRECTION
PY
CONCENTRATED LOAD AT TOP IN Y DIRECTION
QT
CONCENTRATED TORQUE AT TOP
HT
TOTAL HEIGHT OF STRUCTURE
DIMENSION AAA(5,5), BBB(5)

000003

INPUT DATA
SX1 = 9.377
SX2 = 9.377
SY1 = 2.289
SY2 = 2.289
SXYY1 = -2.594
SXYY2 = 2.594
SW1 = 0.
SW2 = 0.
SJ1 = 1.159
SJ2 = 1.159
EX1 = -4.313
EX2 = 4.313
EY1 = 1.687
EY2 = 1.687
CX1 = -4.804
CX2 = 4.804
CY1 = 0.
CY2 = 0.
A1 = 3.128
A2 = 3.128
W1 = 0.
W2 = 0.
E = 4E+06
PSN = 0.35
G = E/(2.*(1.+PSN))
L = 4.
SB = 1.0997
AST = 0.5865
H = 6.
NF = 8
C

000056    PX= 25.
000060    PY= 0.
000061    QT= 0.
C

000062    COMPUTATION STARTS
000064    MT= FLOAT(NF)*H
000064    D= CX2*EY2- CY2*EX2+ CY1*EX1- CX1*EY1
000074    A= EX2- EX1
000075    B= EY2- EY1
000076    W= N2- W1
000100    RFACT= 1.+ (12.*E*SBJ)/(L*L*G*AST)
000110    SJB= SB/(H*RFACT)
000112    SJ= SJ1+ SJ2
000114    SX= SX1+ SX2
000116    SY= SY1+ SY2
000120    SX*Y= SX1+ SX2+ SY1+ SY2
000122    SYC= CY1*SY1+ CY2*SY2- CX1*SXY1- CX2*SXY2
000132    SXC= CX1*SX1+ CX2*SX2- CY1*SXY1- CY2*SXY2
000141    SW= CX1*SX1+ CX2*SX2+ SY1*SY1+ CY1*SY1+ CY2*SY2+ SW1+ SW2
000163    A= SX*Y+ SX*Y
000167    BB= SX*Y+ SX*Y
000172    CC= A*SXC+ B*SYC
000174    SXP= SX1+ SX2
000176    SYP= SY1+ SY2
000200    SWP= SW- (SXC*SXC)/SX- (SYC*SYC)/SY
000206    IF (AA.EQ.0.) GO TO 40
000207    SX= SX- SX*Y*(BB/AA)
000212    SY= SY- SY*Y*(AA/BB)
000214    AP= A- SXY*(CC/BB)
000217    BP= B- SY*Y*(CC/AA)
000222    R= (SXP*SYC*SYC)/(SY*BB)+ (SXY*SXC*SXC)/(SX*AA)
000233    Q= (SYC*SXC*SYC)/(SY*AA)+ (SXY*SYC*SYC)/(SY*BB)
000244    R= (SXP*SYC*SYC)/(SY*BB)- (SXY*SXC*SYC)/(SX*AA)
000256    S= (SXP*SXC*SYC)/(SX*AA)- (SXY*SYC*SYA)/(SY*BB)
000270    GO TO 42

000271    SX= SXP
000273    SY= SYP
000274    AP= A
000276    BP= B
000277    P= SYC/SY
000301    D= 0.
000302    R= A/SY
000303    S= 0.

42    WX= W+ (SY*BB)/(SX*SY)- D
000315    WB=-W+ (BP*SXC)/SX- (AP*SYC)*SY
000326    FF= (A*AP)/SY+ (B*BP)/SX+ 1./A1+ 1./A2
000336    F= 1./FF
000337    ALPH= (1.*L)/(12.*SJB)
000345    CLTH5= ALPH*E*SWP
000347    CLTH3= -(E*SWP)/F+ ALPH*G*SJB- E*WP*WB
000356    CLTH1= (G*SJB)/F
000360    CRMYP1= - (R*WB+P/F)
000366    CRMXP3= ALPH*P
000370    CRMXP3= ALPH*Q
000371    CRMXP2= ALPH
000372    CRMXP1= 1./F

147
WRITE (6,2) SX1,SX2,SY1,SY2,SYX1,SYX2,SW1,SW2,SJ1,SJ2,EX1,EX2,EY1,
1EY2,CX1,CX2,CY1,CY2,A1,A2,W1,W2,E,G,PSN,L,SB,AST,H
000471 WRITE (6,3)
000475 3 FORMAT (10X)
000475 WRITE (6, 2) SX,C,SY,C,SW,A,BB,CC,CX,SY,SX,SX,SY,SYP,AP,BP,SWP,WP,P,Q,
1WP,F,R,S,ALPH,CLTH5,CLTH3,CLTH1,CRMY1,CRMX1,CRMY3,CRMX3,
2CRQT,CRQTP2
000575 2 FORMAT (1X,6E15.6)
000575 RAS= (-CLTH3- SORT(CLTH3*CLTH3- 4.*CLTH5*CLTH1))/(2.*CLTH5)
000607 RBS= (-CLTH3- SORT(CLTH3*CLTH3- 4.*CLTH5*CLTH1))/(2.*CLTH5)
000621 RA= SORT(RAS)
000623 RB= SORT(RBS)
000625 CP= -(CPMY1*PX+ CRMXP1*PY)/CLTH1 + (CRQT*QT)/CLTH1
000635 ALP= ALPH*E*SWP
000637 BET= -(ALPH*G*SJ- E*WP*WB)
000645 RA5= RA*RA*RA*RA*RA
000647 RA4= RA*RA*RA*RA
000650 RA3= RA*RA*RA
000651 RA2= RA*RA
000652 RB5= RB*RB*RB*RB*RB
000654 RB4= RB*RB*RB*RB
000655 RB3= RB*RB*RB
000656 RB2= RB*RB
000657 PAT= RA*HT
000660 RBT= RB*HT
000661 WRITE (6,5)
000664 WRITE (6, 2) RAS,RBS,RA,RB,CP,BET,ALP,RAT,RBT
000712 DO 4 I= 1,5
000714 DO 4 J= 1,5
000715 4 AAA(I,J)= 0.
000724 AAA(1,1)= 1.
000725 AAA(1,2)= 1.
000726 AAA(1,3)= 1.
000727 AAA(2,3)= RA
000730 AAA(2,5)= RB
000732 AAA(3,3)= RA*RA*RA*E*SWP- G*SJ*RA
000732 AAA(3,5)= RB*RB*RB*E*SWP- G*SJ*RB
000734 AAA(4,2)= (ALP*RA4+ BET*RA2)*COSH(RAT)
000734 AAA(4,3)= (ALP*RA4+ BET*RA2)*COSH(RAT)
000736 AAA(4,4)= (ALP*RB4+ BET*RB2)*COSH(RBT)
000736 AAA(4,5)= (ALP*RB4+ BET*RB2)*COSH(RBT)
000771 AAA(5,2)= RA2*COSH(RAT)
000771 AAA(5,3)= RA2*SINH(RAT)
000772 AAA(5,4)= RB2*COSH(RBT)
000772 AAA(5,5)= RB2*SINH(RBT)
001000 BBB(1)= 0.
001016 BBB(2)= -CP
001017 BBB(3)= G*SJ*CP-P*PX+ Q*PY - QT
001027 BBB(4)= 0.
001030 BBB(5)= 0.
001031 WRITE (6,7) AAA
001036 7 FORMAT (3X,5E18.6)
001036 WRITE (6,7) BBB
001044 CALL SIMQAAA,BBB,5,KS
001047 WRITE (6,9) KS
001055 9 FORMAT (20X,3)
001055 WRITE (6,7) BBB
001063 C1 = B8B(1)
001065 C2 = B8B(2)
001066 C3 = B8B(3)
001070 C4 = B8B(4)
001071 C5 = B8B(5)
001073 Z = 0
001074 WRITE (6,11)
001077 11 FORMAT (10X)
001080 10 RA2 = RA*Z
001082 RBZ = RB*Z
001090 THE1 = C1 + C2*COSH(RAZ) + C3*SINH(RAZ) + C4*COSH(RBZ) + C5*SINH(RBZ) +
001098 CP*Z
001101 THETP1 = C2*RA2*SINH(RAZ) + C3*RA2*COSH(RAZ) + C4*RB2*SINH(RBZ) + C5*
001103 1RB*COSH(RBZ) + CP
001106 THETP2 = C2*RA2*COSH(RAZ) + C3*RA2*SINH(RAZ) + C4*RB2*COSH(RBZ) +
001108 1C5*RB2*SINH(RBZ)
001110 THETP3 = C2*RA2*SINH(RAZ) + C3*RA2*COSH(RAZ) + C4*RB3*SINH(RBZ) +
001113 1C5*RB3*COSH(RBZ)
001116 WRITE (6,12) THE1,THETP1,THETP2,THETP3
001120 12 FORMAT (1X,6E20.6)
001123 IF (Z.EQ.HT) GO TO 13
001126 Z = Z+H
001128 GO TO 10
001130 13 B1 = (C2*(-E*SWP*RA3 + G*SJ*RA))/WB
001132 B2 = (C3*(-E*SWP*RA3 + G*SJ*RA))/WB
001135 B3 = (C4*(-E*SWP*RB3 + G*SJ*RB))/WB
001138 B4 = (C5*(-E*SWP*RB3 + G*SJ*RB))/WB
001140 B5 = (G*SJ*GP-P*PX + Q*PY -QT)/WB
001143 D1 = -B1/RA
001146 D2 = -B2/RA
001149 D3 = -B3/RB
001152 D4 = -B4/RB
001155 D5 = -B5
001158 D6 = (B1*COSH(RAT) + B2*SINH(RAT))/RA + (B3*COSH(RBT) + B4*SINH(RBT))
001161 1/RB + B5*HT
001164 WRITE (6,19)
001167 19 FORMAT (10X)
001170 15 RAZ = RA*Z
001172 RBZ = RB*Z
001174 FQ = B1*SINH(RAZ) + B2*COSH(RAZ) + E3*SINH(RBZ) + B4*COSH(RBZ) + B5
001177 FT = D1*COSH(RAZ) + D2*SINH(RAZ) + D3*COSH(RBZ) + D4*SINH(RBZ) +
001180 1 D5*Z + D6
001183 THE2 = C1 + C2*COSH(RAZ) + C3*SINH(RAZ) + C4*COSH(RBZ) + C5*SINH(RBZ) +
001188 CP*Z
001191 THETP1 = C2*RA2*SINH(RAZ) + C3*RA2*COSH(RAZ) + C4*RB2*SINH(RBZ) + C5*
001194 1RB*COSH(RBZ) + CP
001197 THETP2 = C2*RA2*COSH(RAZ) + C3*RA2*SINH(RAZ) + C4*RB2*COSH(RBZ) +
001200 1C5*RB2*SINH(RBZ)
001203 THETP3 = C2*RA2*SINH(RAZ) + C3*RA2*COSH(RAZ) + C4*RB3*SINH(RBZ) +
001206 1C5*RB3*COSH(RBZ)
001209 BIMNT1 = -E*SW1*THETP2
001212 BIMNT2 = -E*SW2*THETP2
001215 TRQ1 = -E*SW1*THETP3 + G*SJ1*THETP4
001218 TRQ2 = -E*SW2*THETP3 + G*SJ2*THETP1
001221 WRITE (6,17) F0,FT,BIMNT1,BIMNT2,TRQ1,TRQ2
001224 17 FORMAT (1X,6E16.6)
001227 IF (Z.EQ.HT) GO TO 29
001565    Z = Z + H
001567    GO TO 15
001567    29 Z1 = (SYNC*C2*RA2)/SY = (AP*D1)/(E*SY)
001560    Z2 = (SYNC*C3*RA2)/SY = (AP*D2)/(E*SY)
001611    Z3 = (SYNC*C4*RB2)/SY = (AP*D3)/(E*SY)
001622    Z4 = (SYNC*C5*RB2)/SY = (AP*D4)/(E*SY)
001633    Z5 = -(AP*D5)/(E*SY) - (SPX*SYNC*PX)/(E*SY*B) + (SY*SYC*PY)/(E*SY*B)
001652    Z6 = -(AP*D6)/(E*SY) + (SPX*SYNC*HT*PX)/(E*SY*B) - (SY*SYC*HT*PY)
001672    Z7 = -Z2/RA = Z4/RA2
001672    Z8 = -Z1/RA2 = Z5/RA
001702    E1 = (SYNC*C2*RA2)/SX = (BP*D1)/(E*SY)
001713    E2 = (SYNC*C3*RA2)/SX = (BP*D2)/(E*SY)
001724    E3 = (SYNC*C4*RB2)/SX = (BP*D3)/(E*SY)
001735    E4 = (SYNC*C5*RB2)/SX = (BP*D4)/(E*SY)
001746    IF (AA.EQ.0.) GO TO 50
001747    E5 = -(BP*D5)/(E*SY) + (SY*SYC*PX)/(E*SY*AA) - (SY*SYC*PY)/
001765    1 (E*SY*AA)
002005    E6 = -(BP*D6)/(E*SY) - (SY*SYC*HT*PX)/(E*SY*AA) + (SY*SYC*HT*PY)
002005    1/(E*SY*AA)
002005    GO TO 52
002005    50 E5 = PY/(E*SY)
002006    E6 = (PY*HT)/(E*SY)
002014    E7 = -E2/RA - E4/RA
002020    E8 = -E1/RA2 - E3/RA
002024    Z = 0.
002025    WRITE(6,22)
002031    22 FORMAT(10X)
002031    20 RAZ = RA*Z
002033    RBZ = RB*Z
002035    ZETP2 = Z1*COSH(RAZ) + Z2*SINH(RAZ) + Z3*COSH(RBZ) + Z4*SINH(RBZ) +
002057    1 Z5*Z+ Z6
002057    ZETP1 = (Z1/RA)*SINH(RAZ) + (Z2/RA)*COSH(RAZ) + (Z3/RA)*SINH(RBZ) +
002113    1 (Z4/RA)*COSH(RBZ) + (Z5/2.)*Z*Z+ Z6*Z+ Z7
002113    ZET = (Z1/RA2)*COSH(RAZ) + (Z2/RA2)*SINH(RAZ) + (Z3/RA2)*COSH(RBZ) +
0022153   1 (Z4/RA2)*SINH(RBZ) + (Z5/6.)*Z*Z+ Z6/2.)*Z*Z+ Z7*Z+ Z8
0022153   ETAP2 = E1*COSH(RAZ) + E2*SINH(RAZ) + E3*COSH(RBZ) + E4*SINH(RBZ) +
0022153   1 E5*Z+ E6
0022153   ETAP1 = (E1/RA)*SINH(RAZ) + (E2/RA)*COSH(RAZ) + (E3/RA)*SINH(RBZ) +
0022232   1 + (E4/RA)*COSH(RBZ) + (E5/2.)*Z*Z+ E6*Z+ E7
0022232   ETA = (E1/RA2)*COSH(RAZ) + (E2/RA2)*SINH(RAZ) + (E3/RA2)*COSH(RBZ) +
002272   1 (E4/RA2)*SINH(RBZ) + (E5/6.)*Z*Z+ (E6/2.)*Z*Z+E7*Z+ E8
002272   WRITE (6,37) ZET, ZETP1, ZETP2, ETA, ETAP1, ETAP2
002312    37 FORMAT (1X,6E13.4)
002312    IF (Z.EQ.0.) GO TO 26
002314    Z = Z + H
002316    GO TO 20
002316    STOP
002320    26 RETURN
002320
UNUSED COMPILER SPACE
010700
APPENDIX-C

EXPERIMENTAL DATA
## TABLE C1

### STRAIN DATA FOR MODEL WITH FLOORS

strain in µ in/in for load at 8th floor

<table>
<thead>
<tr>
<th>Strain Gauge No.</th>
<th>Loading</th>
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<td>2.5 lbs.</td>
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<tr>
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<tr>
<td>4</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>+15</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>9</td>
<td>*</td>
</tr>
<tr>
<td>10</td>
<td>-22</td>
</tr>
<tr>
<td>11</td>
<td>+5</td>
</tr>
<tr>
<td>12</td>
<td>-12</td>
</tr>
<tr>
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<td>-14</td>
</tr>
<tr>
<td>16</td>
<td>-3</td>
</tr>
<tr>
<td>17</td>
<td>-14</td>
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</tr>
<tr>
<td>19</td>
<td>-8</td>
</tr>
<tr>
<td>20</td>
<td>+25</td>
</tr>
<tr>
<td>22</td>
<td>+16</td>
</tr>
<tr>
<td>23</td>
<td>+6</td>
</tr>
<tr>
<td>24</td>
<td>+14</td>
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* Strain Gauge out of order
<table>
<thead>
<tr>
<th>Strain Gauge No.</th>
<th>Loading</th>
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</tr>
<tr>
<td>7</td>
<td>+62</td>
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<tr>
<td>8</td>
<td>-38</td>
</tr>
<tr>
<td>10</td>
<td>-104</td>
</tr>
<tr>
<td>11</td>
<td>+8</td>
</tr>
<tr>
<td>12</td>
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<td>16</td>
<td>-15</td>
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<td>17</td>
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</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>23</td>
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</tr>
<tr>
<td>24</td>
<td>+80</td>
</tr>
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</table>

* Strain Gauge out of order
### TABLE C2

DEFLECTION DATA FOR MODEL WITH FLOOR

Deflection in inch for load at 8th floor

<table>
<thead>
<tr>
<th>Dial Gauge No.</th>
<th>2.5 lbs.</th>
<th>5 lbs.</th>
<th>7.5 lbs.</th>
<th>10 lbs.</th>
<th>12.5 lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.011</td>
<td>0.032</td>
<td>0.055</td>
<td>0.079</td>
<td>0.097</td>
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<tr>
<td>2</td>
<td>0.0285</td>
<td>0.0725</td>
<td>0.118</td>
<td>0.1695</td>
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<tr>
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<td>0.026</td>
<td>0.043</td>
<td>0.062</td>
<td>0.084</td>
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<tr>
<td>4</td>
<td>0.025</td>
<td>0.066</td>
<td>0.108</td>
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<td>0.187</td>
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<tr>
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<td>0.007</td>
<td>0.022</td>
<td>0.041</td>
<td>0.061</td>
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<td>0.122</td>
<td>0.152</td>
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<td>0.010</td>
<td>0.022</td>
<td>0.024</td>
<td>0.030</td>
</tr>
<tr>
<td>8</td>
<td>0.010</td>
<td>0.030</td>
<td>0.048</td>
<td>0.069</td>
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<tr>
<td>9</td>
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<td>0.004</td>
<td>0.006</td>
<td>0.009</td>
<td>0.011</td>
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<tr>
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<td>0.011</td>
<td>0.0175</td>
<td>0.025</td>
<td>0.031</td>
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</table>

<table>
<thead>
<tr>
<th>Dial Gauge No.</th>
<th>10 lbs.</th>
<th>7.5 lbs.</th>
<th>5 lbs.</th>
<th>2.5 lbs.</th>
<th>0</th>
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</thead>
<tbody>
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<td>0.0885</td>
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<tr>
<td>3</td>
<td>0.066</td>
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<tr>
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**TABLE C3**

**STRAIN DATA FOR MODEL WITH FLOORS**

Strain in $\mu$ in/in for load at 6th floor

<table>
<thead>
<tr>
<th>Strain Gauge No.</th>
<th>2.5 lbs.</th>
<th>5 lbs.</th>
<th>7.5 lbs.</th>
<th>10 lbs.</th>
<th>12.5 lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>+24</td>
<td>+44</td>
<td>+66</td>
<td>+86</td>
<td>+106</td>
</tr>
<tr>
<td>3</td>
<td>-24</td>
<td>-46</td>
<td>-70</td>
<td>-92</td>
<td>-116</td>
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<tr>
<td>4</td>
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<td>+18</td>
<td>+36</td>
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* Strain Gauge out of order
### TABLE C3 (continued)

**STRAIN DATA FOR MODEL WITH FLOORS**

Strain in \( \mu \text{in/in} \) for load at 6th floor

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* Strain Gauge out of order
## TABLE C4

**DEFLECTION DATA FOR MODEL WITH FLOORS**

Deflection in inch for load at 6th floor

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## TABLE C5

**STRAIN DATA FOR MODEL WITH FLOORS**

Strain in $\mu$ in/in for load at 4th floor

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* Strain Gauge out of order
### TABLE C5 (continued)

**STRAIN DATA FOR MODEL WITH FLOORS**

Strain in $\mu$ in/in for load at 4th floor

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TABLE C6

DEFLECTION DATA FOR MODEL WITH FLOOR

Deflection in inch for load at 4th floor

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<td>0.007</td>
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### TABLE C7

**STRAIN DATA FOR MODEL WITH BEAMS**

Strain in $\mu$ in/in for lateral load at top

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<th>Strain Gauge No.</th>
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<th>15 lbs.</th>
<th>20 lbs.</th>
<th>25 lbs.</th>
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<td>-76</td>
<td>-112</td>
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<td></td>
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</tr>
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</table>
### TABLE C7 (continued)

**STRAIN DATA FOR MODEL WITH BEAMS**

strain in \( \mu \) in/in for lateral load at top

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<td>+4</td>
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TABLE C9

STRAIN DATA FOR MODEL WITH BEAMS

Strain in $\mu$ in/in for torque at top

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TABLE C9 (continued)

STRAIN DATA FOR MODEL WITH BEAMS

Strain in µin/in for torque at top

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<tr>
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### TABLE Cl.0

DEFLECTION DATA FOR MODEL WITH FLOORS

Deflection in inch for torque at top

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<td>.0005</td>
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