A CANDU-PHW REACTOR WITH AN ANNULUS OF ENRICHED URANIUM
STUDIES OF A CANDU-PHW REACTOR CORE
CONTAINING AN ANNULUS OF ENRICHED URANIUM

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ABSTRACT

Computer studies are made of a CANDU-PHW reactor core containing an annulus of enriched uranium around a central zone of natural uranium. For hybrid cores of this type with a maximum radial form factor, the uranium requirements, fuel costs, stability, and power peaking upon refuelling are investigated. It is found that these hybrid cores offer potential savings of 10% to 20% in fuel costs and uranium utilization compared to the present CANDU-PHW core, and are only slightly less stable. However, power peaking upon refuelling is a problem with these cores.
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1. INTRODUCTION

This report examines the effect of modifying a CANDU-PHW reactor core by introducing an annulus of enriched uranium around the central region of natural uranium. The incentive for such a study is the possibility of improving the uranium resource utilization and reducing costs.

For various outer zone enrichments and inner zone radii, the variation of the power radial form factor (RFF, the ratio of the average to maximum channel powers) with inner and outer exposures was determined, for the purpose of maximizing the RFF. For hybrid cores having a near maximum RFF, the natural uranium requirements and fuel costs were determined. Power peaking after refuelling a channel in the enriched region was investigated. Finally, the subcriticality of the first radial and azimuthal flux modes was computed for a few of these hybrid cores to give a measure of their stability.

This study was made with the use of three computer codes: LATREP(1), FORGOD(2) and MICRETE(3). LATREP was used to determine the reactor physics homogenized cell parameters (such as cross sections, $k_\infty$, $L_s^2$, etc.) for fuel of a given exposure and enrichment. FORGOD is a fast,
simple, one-dimensional, two group, finite difference neutron diffusion code. It was used in the RFF study to determine flux and power shapes, the reactivity and the RFF. The regional powers from FORGOD were used in the uranium utilization and economic studies. MICRETE is a two-dimensional, two group, heterogeneous, source sink code. Power peaking resulting from refuelling a single channel was investigated with this code. MICRETE was also used to determine the eigenvalues of some of the higher harmonic flux modes, in the stability study.
2. DESCRIPTION OF THE REACTOR CORE

The reactor cores studied (see Figure 1) were right cylinders with the same overall dimensions as the Gentilly-2 (G-2) reactor core. The cores contained two regions—an inner zone of natural uranium with exposure $w_I$ (n/kb) and a radius of $R$ cm, and an outer zone of U-235 enriched uranium of enrichment $E\%$, exposure $w_0$, and a radius of 314 cm. The core was surrounded by a heavy water reflector with an outer radius of 380 cm. The axial extrapolation length was 606 cm.

Four inner zone radii ($R$) were studied: 196 cm, 226 cm, 255 cm and 285 cm. Enrichments of 0.71% (natural), 1.0%, 1.2%, 1.4%, 2.0% and 3.0% were considered.

A G-2, two region, natural uranium core was used as a reference. The inner region had a discharge exposure of 2.0 n/kb (8.35 MW.d/kgU) and extended to a radius of 196 cm. The outer region, which extended to a radius of 314 cm, contained natural uranium of exposure 1.644 n/kb (6.895 MW.d/kgU). A heavy water reflector surrounded the core to a radius of 380 cm.
3. COMPUTATIONAL METHODS USED

3.1 LATREP

LATREP is a cell homogenization code which computes equivalent two-group parameters. From the detailed fuel cell, containing the uranium fuel bundle (consisting of 37 zirconium clad, uranium oxide fuel elements), pressure tube, calandria tube, and heavy water moderator and coolant, LATREP determines two group parameters which are spatially constant over the cell, and which (ideally) yield the same average reaction rates as the actual position dependent parameters, which may vary greatly from point to point within the cell. Table 1 lists the physical details of the cell which are input as data into LATREP.

The cell parameters obtained from LATREP were burnup averaged, using Simpson's integration at 0.2 n/kb exposure intervals from 0.0 to 6.0 n/kb. The use of burnup averaged parameters allows a simple approximation to the actual situation, where fuel with a range of burnups are present. The burnup averaged two group parameters produced by LATREP were converted into the cross-sectional data required by FORGOD by use of the relationships shown in Appendix 2.
LATREP was run once for each enrichment used, and the fast and thermal group parameters were stored on permanent file. Subsequent FORGOD runs retrieved material properties for the reflector, and fuel of a certain enrichment and exposure, from this file.

3.2 FORGOD

FORGOD is a whole reactor computer code which solves the steady state, one dimensional diffusion equation for up to four energy groups. Two energy groups were used in this study, and 100 mesh points. FORGOD computes the core reactivity, fluxes and powers, and the RFF.

The input to FORGOD includes the LATREP homogenized cell parameters for the inner and outer core regions and the reflector, as well as the physical dimensions of the core.

Axial leakage was approximated by the inclusion in the diffusion equation of a buckling term, \(( \frac{\pi}{L} )^2\), where the axial extrapolation length of the reactor, \( L \), was 606 cm.

3.3 MICRETE

MICRETE is a two group, two dimensional heterogeneous code in which individual fuel channels are represented as sources and sinks of fast and thermal neutrons. It treats the reactor core as a lattice of rods.
immersed in a heavy water moderator. The core is surrounded by a heavy water reflector. The flux at any rod is the sum of the contributions to that flux by all the other rods, including itself. It is assumed that the core is axially uniform, and that the axial fluxes vary as \( \cos \left( \frac{\pi z}{L} \right) \).

The input to MICRETE includes the homogenized cell parameters generated by LATREP, the location coordinates \((x,y)\) and type of each fuel cell, as well as the core dimensions.

Full use is made of core symmetry. Open centered, or channel centered lattices can be set up.
4. MAXIMIZING THE RADIAL FORM FACTOR

4.1 Burnup Flattening

4.1.1 Introduction

In reactor design and fuel management, maximizing the radial form factor (subject to other constraints) is of primary importance. The maximum channel power is limited to prevent dryout. Thus, for a given maximum channel power, a higher RFF would result in a greater average channel power and total reactor power, and hence, a lower unit energy cost. Alternatively, for a given total reactor power, a higher RFF would result in a greater safety margin between the maximum channel power and the maximum allowable channel power (ie. a larger critical power ratio).

4.1.2 Method Used

FORGOD was run for the G-2 reference core (with no reactivity load), and the resultant reactivity, about 12.8 mk, was used as the reactivity load for the hybrid cores. It was assumed that the 12.8 mk reactivity load was distributed uniformly over the hybrid cores, and so would not affect the flux or power profiles. For each hybrid core (characterized by a certain enrichment and inner region radius), FORGOD was used to determine the reactivity and
RFF for inner exposures between 1.6 n/kb and 6.0 n/kb, at increments of 0.4 n/kb. The result was a two-dimensional map of RFF and reactivity as a function of \( w_1 \) and \( w_0 \), for each core studied. From these maps, inner and outer exposures giving a reactivity of 12.8 mk were determined using linear interpolation, and the 12.8 mk reactivity contour was drawn. FORGOD was then run for several points on the 12.8 mk reactivity contour for each core, and the inner and outer exposures yielding the best RFF were determined.

4.1.3 Results and Discussion

4.1.3.1 Natural Uranium Reference Case

The reactivity of the G-2 reference case determined using LATREP and FORGOD—about 12.8 mk—is lower than the reactivity load of about 21 mk specified in the Station Manual,\(^4\) which is comprised of 15 mk worth in the adjuster rods for xenon override, 3 mk worth in the inner region for the zone controllers (which have a total worth of 7.5 mk but which normally operate only 2/5 full), and 3 mk worth of parasitic absorption uniformly distributed throughout the core.

In G-2, the reactivity load is used for flux shaping. The RFF with the 21 mk reactivity load is 0.823, as stated in the Station Manual. A 12.8 mk reactivity load
was simulated in FORGOD by increasing the thermal absorption cross section of the inner zone until the resultant reactivity was approximately zero. The resultant RFF was 0.817.

The flux and power maps before and after the 12.8 mk reactivity load has been added to the central region are shown in Figures 2 and 3 for the natural uranium G-2 reference core. The maximum power occurs in the center of the core. There is a very small (1%) discontinuity in the power at the interface of the inner and outer regions, due to the different power to flux ratios in the two regions.

4.1.3.2 Variation of the RFF with Inner and Outer Exposures

Figure 4 shows a few iso-reactivity and RFF contours for a hybrid core with 1.4% enrichment, and an inner zone radius of 196 cm. As one moves along the 12.8 mk reactivity contour in either direction, the RFF increases, reaches a maximum, then decreases. The flux and power profiles for inner and outer exposures of 2.20 n/kb (9.15 MW.d/kgU) and 4.24 n/kb (23.74 MW.d/kgU) respectively, for which the RFF is near maximum, are shown in Figure 5. Increasing $w_I$ (decreasing $w_O$), from the value giving a near maximum RFF results in the flux peaking in the outer region, as shown in Figure 6 for $w_I = 3.60$ n/kb. Similarly, decreasing $w_I$ (increasing $w_O$) results in the flux peaking in the inner region, as shown in Figure 7 for $w_I = 1.63$ n/kb. Of particular
importance is the discontinuity in the power at the inter-
face, to be discussed in greater detail in another section.

The variation of the RFF with inner and outer
exposures is shown in Figures 8 and 9 respectively, for an
enrichment of 1.4% and the four radii studied, with a
reactivity of 12.8 mk.

4.1.3.3 Variation of the Maximum RFF with Enrichment

To accurately determine the maximum RFF achievable
for each of the hybrid cores, the RFF/reactivity maps were
scanned for the exposures $w_1$, $w_0$ giving the largest RFF.
FORGOD was then run for this value of $w_0$, and for several
values of $w_1$, at 0.02 n/kb increments, until the RFF passed
through a maximum. The maximum RFF determined using this
fine mesh was on the average 0.7% (and never more than 2%)
greater than that determined using the coarser grid to
obtain the near maximum RFF for a reactivity of 12.8 mk.

The resultant maximum RFF is plotted as a function
of enrichment for each of the four radii studied, in
Figure 10. The RFF is maximized for an inner zone radius
of 255 cm. It decreases with increasing enrichment.

For the hybrid cores, the maximum RFF can be
achieved at any reactivity—ie. the maximum RFF contour
passes through all of the iso-reactivity contours, as can be
seen in Figure 4. However, for the natural uranium core,
with $R = 226$ cm, a maximum RFF of 0.909 can only be achieved for reactivities less than about 11 mk; for 12.8 mk the maximum RFF that can be achieved is 0.841. For $R = 255$ cm, the maximum RFF achievable for a reactivity of 12.8 mk is only 0.695 at $(w_I, w_0) = (2.02, 0.4)$ n/kb, compared to an RFF of 0.938 achievable for cores with very low (negative) reactivities.

This anomalous behaviour for the natural uranium cores is simply due to the peaking of the reactivity at a non-zero burnup for natural uranium fuel, due to the plutonium buildup, as shown in Figure 11. As a result, the iso-reactivity lines "turn over" at low outer exposures, as shown in Figure 13. For a constant $w_I$, decreasing $w_0$ at first results in an increase in reactivity, until a maximum is reached at about 0.4 n/kb (about 2.0 MW.d/kgU), at which point, decreasing $w_0$ further results in a decrease in reactivity.

For the natural uranium core with inner zone radius 196 cm, the RFF is maximized at $(w_I, w_0) = (2.32, 1.14)$ n/kb, and yet the reactor is fuelled with inner and outer exposures of 2.0 n/kb and 1.644 n/kb respectively. As will be apparent later, one reason for this is in the fuel economics.

For the hybrid cores studied, the maximum RFF obtained was 0.918, for an enrichment of 1.0% and $R = 255$ cm. This is 12% greater than the RFF of 0.817 for the natural
uranium reference case. Thus, if the hybrid core with 1.0% enrichment and \( R = 255 \text{ cm} \) could be operated at the same maximum power as the natural uranium reference core, a 12% increase in the output power could be achieved.

4.1.3.4 Variation with Enrichment of the Inner and Outer Exposures Giving a Maximum RFF and 12.8 mk Reactivity

Included in Table 3 are the inner and outer exposures which give a near maximum RFF for each of the hybrid cores studied. The inner exposure giving a maximum RFF decreases a very small amount (about 6%) from about 2.27 n/kb at 1.0% enrichment to 2.14 n/kb at 3.0% enrichment. One cannot draw any conclusions about the variation of \( w_1 \) with inner zone radius.

The outer exposure giving a maximum RFF shows a strong dependence on enrichment and radius, as shown in Figure 14. The outer exposure increases with increasing enrichment, and decreases with increasing radius, as would be expected intuitively. The variation of the outer exposure with enrichment is important from an economic viewpoint, since it is these two parameters, along with the power in the outer region, which determine the unit energy cost of the fuel in this region.
4.1.3.5 Equilibrium Power Peaking

As has been mentioned with respect to Figures 5, 6, and 7, there is a discontinuity in the power at the interface between the inner and outer regions for the hybrid cores. Since the flux is everywhere continuous in the core, this discontinuity is due solely to the different fission power to flux ratios of the fuel in the two regions (see Figure 12). The percent discontinuity in power at the boundary is plotted against enrichment for the four radii studied, in Figure 15. These cores have inner and outer enrichments which give a near maximum RFF. The power peaking increases with increasing enrichment and increasing inner zone radius.

For the enrichments less than 2% that were studied, the maximum channel power occurred at the boundary between the inner and outer zones, the power at the boundary being no more than 5% greater than the power at the center of the core. Thus, the peak power in the enriched zone may limit the total reactor power.

The actual discontinuity in power at the interface will be less than that indicated by FORGOD, since the fuel is localized in discrete channels rather than being continuously distributed throughout each of the two regions of the core. Results from MICRETE (in the power peaking after refuelling study) showed the same trend of increasing power discontinuity with increasing enrichment and inner
zone radius, but the magnitude of the discontinuity was a factor of two lower than that indicated by FORGOD.

4.2 Absorber Flattening

4.2.1 Introduction

For a hybrid core with a given enrichment and inner region radius, there are an infinite number of inner and outer exposure combinations which will give a certain specified reactivity for the reactivity load. If one uses burnup flattening to shape the flux, the inner and outer exposures could be chosen to give a maximum RFF. However, it may be desirable to choose inner and outer exposures which maximize another parameter, in which case it may be necessary to use absorber flattening to shape the flux. The feasibility of using absorber flattening for flux shaping has been investigated for one hybrid core.

4.2.2 Method Used

For a hybrid core with 1.2% enrichment, and an inner region radius of 196 cm, a few inner and outer exposures were chosen which gave 12.8 mk reactivity. To simulate 3.5 mk worth of parasitic thermal absorption distributed uniformly throughout the core, the thermal absorption cross section in each region was increased by 1.0035. About 8 mk worth of thermal absorption was added to the cores for flux
shaping, using two methods. In the first method, the thermal absorption cross section of either the inner or outer region was increased until the core reactivity was about 1 mk. In the second method, each region of the core was divided roughly in half, giving four core regions of outer radii 96 cm, 196 cm, 255 cm and 314 cm. The thermal absorption cross section of each region was increased according to the average power in the region—the thermal absorption cross section of the region having the largest average power was increased the most, while that of the region having the smallest average power was not changed at all.

4.2.3 Results and Discussion

The results of this study are shown in Table 2. The shape of the flux before and after the absorption was added to the core can be inferred from the ratio of the flux at the center of the core ($\phi_0$) to the flux at a radius of 198 cm ($\phi_{198}$), and by the location of the maximum flux, $R_{\text{max}}$. It is evident that for this core, absorber flux flattening is most successful for inner exposures between 2.0 n/kb and 2.1 n/kb—the flattest flux and highest RFF being obtained for $w_1 = 2.1$ n/kb. For smaller exposures, the flux is initially strongly peaked in the center of the core, so that 8 mk of additional thermal absorption in the
central region does not significantly improve the flux or RFF. For slightly larger exposures, the addition of 8 mk thermal absorption in the central region results in a flux depression there. For inner exposures greater than about 2.4 n/kb, the flux is initially depressed in the inner zone, so that any additional absorption must be added to the outer region. However, adding the additional core absorption uniformly to the outer region is not efficient, since the flux peaks near the interface of the inner and outer regions for cores with these higher inner exposures, and much of the absorber worth is wasted near the outer region of the core.

As would be expected, the linear power weighting scheme in the four region core proved to be a better method of distributing the extra core absorption, resulting in a better RFF in all cases except one, where the RFF decreased slightly.

The best RFF obtained using burnup flattening alone, for a core with enrichment of 1.2% and an inner region radius of 195 cm was 0.833 (Figure 10), with inner and outer exposures of 2.24 n/kb and 3.72 n/kb respectively (Table 3). Using absorber flattening and a linear power weighting scheme, a RFF of 0.847 was obtained with an inner exposure of 2.4 n/kb.
Thus, absorber flattening can be used for a wide range of exposures to shape the flux and improve the RFF. The degree of success depends to some extent on how well the absorber distribution is matched with the power distribution.
5. NATURAL URANIUM REQUIREMENTS

For each of the cores studied, the natural uranium requirements were computed. The formula used is listed in Appendix 3: the inverse burnup in each region is weighted by the fraction of the total power produced in that region. The results are shown in Table 3, and are plotted in Figure 16. The natural uranium requirements decrease with decreasing inner region radius, and are minimized over a fairly large range of enrichments. The uranium requirements increase rapidly for high enrichments.

For the natural uranium reference case, the uranium requirements are 161 g/kW.a. For a hybrid core with 1.2% enrichment, and an inner region radius of 196 cm, the uranium requirements are 129 g/kW.a -- 21% lower.
6. FUEL COST

6.1 Cores with Maximized RFF

The unit energy fuel cost was determined for each of the cores studied, with maximized RFF, using the formula listed in Appendix 4. The results are tabulated in Table 6, and plotted in Figure 17. As with the uranium utilization, the unit energy fuel cost decreases with decreasing inner region radius, and has a minimum with respect to enrichment which depends on the inner region radius. The unit energy fuel cost for a core with enrichment between 1.0% and 1.2%, and an inner region radius of 196 cm is 2.26 mills/kW.h(e), 15% lower than the fuel cost for the natural uranium reference case—2.62 mills/kW.h(e). A 13% reduction in fuel costs could be achieved with a core with 1.4% enrichment, and an inner region radius of 255 cm. It should be emphasized that the hybrid cores studied have a near maximum RFF, so that potential economic savings in fuel costs would be in addition to any capital cost savings resulting from an improved RFF.

6.2 Cores with Constant Reactivity

Another important economic consideration is the following: for a core with a given outer zone enrichment, and inner zone radius, how does the unit energy fuel cost
vary with inner and outer exposures, for a given reactivity. In the last section, only the fuel cost for cores having a near maximum RFF was determined. However, it may be necessary or advantageous, to choose inner and outer exposures for which the RFF is not maximized, and improve the RFF using absorber flattening.

To look at this question, the unit energy fuel cost was determined for several pairs of inner and outer exposures giving a reactivity of 12.8 mk, for the natural uranium reference core, and for hybrid cores having enrichments of 1.0% and 2.0% and an inner region radius of 196 cm. The unit energy fuel cost is plotted as a function of inner exposure in Figure 18. The curve for the natural uranium core is markedly different from the curves for the hybrid cores. For the natural uranium core, the unit energy fuel cost is minimized for an inner exposure of about 1.7 n/kb, and increases rapidly for larger or smaller exposures. The curves for the hybrid cores, on the other hand, have a very broad minimum; the unit energy fuel cost varies little over a large range of inner exposures.

Thus, it is evident that for the natural uranium core, exposures must be chosen to give a near minimum unit energy fuel cost, with the RFF being improved by absorber flattening. The unit energy fuel cost for the G-2 natural uranium core with an inner exposure of 2.0 n/kb is
2.64 mills/kW.h(e) while the unit energy cost for an inner exposure of 2.32 n/kb, where the RFF is maximized, is 3.26 mills/kW.h(e)—24% higher.

One has much more flexibility in an economic sense, in choosing the exposures for the hybrid cores. One can maximize the RFF using only burnup flattening, through the proper choice of inner and outer exposures, and obtain near minimum fuel cost for the same exposures.
7. **POWER PEAKING AFTER REFUELLING**

7.1 **Introduction**

When a reactor fuel channel containing enriched uranium is refuelled, power peaking can be expected to occur for two reasons. First, the fresh fuel has a higher fission power per unit flux ratio than the equilibrium burnup fuel, due to the greater amount of fissionable material present in the fresh fuel; hence, the power would rise even if there was no local increase in the flux. There is also a local increase in flux, caused by the additional reactivity added to the core by the fresh fuel. This flux-reactivity relationship is non-linear.

Power peaking upon refuelling can also be expected for natural uranium fuel. However, because of the importance of the plutonium buildup in natural uranium fuel, the reactivity and power to flux ratio do not vary as greatly between equilibrium and zero burnup fuel, as for enriched fuel. Thus, power peaking will not be as severe.

These comments are illustrated in Figures 11 and 12, where the burnup averaged reactivity, and burnup averaged power per unit flux ratio are plotted against burnup for natural and 2.0% enriched uranium. The reactivity curve
for natural uranium peaks, due to plutonium buildup. The fission power per unit flux for natural uranium decreases slowly with increasing burnup.

The maximum channel power is limited to prevent dryout and fuel failure. In G-2, the nominal maximum channel power is 6.5 MW(th). However, when determining the probability of fuel failure, it is not only the absolute value of the channel power during a power change which is important, but also the change in power, the fuel burnup, and the dwell time at the higher power. During a power ramp, the probability of fuel failure for a given burnup increases with increasing final power and power change.\(^5\)

The computer code MICRETE was used to determine the power peaking that would result from refuelling a channel of enriched uranium for various reactor cores having a maximum RFF.

7.2 Method Used

MICRETE was used to investigate the power peaking resulting from refuelling a channel of enriched uranium near the interface between the inner and outer core regions. The refuelled channel was chosen at the boundary of the inner and outer regions because the flux (and power) in the outer region is highest there. A range of enrichments was considered, from 0.7\% (natural) to 3.0\%, and two inner region radii—196 cm and 255 cm.
To decrease computer time, a core with 8-fold
reflectional symmetry was used. This meant that four channels
were actually refuelled—one along the diagonal in each of
the four quadrants in the core. This will result in an
increase in core reactivity upon refuelling which will be
four times larger than that which would result from
refuelling a single channel. However, the effect on the
power of the refuelled channel should be small, since the
other four channels are far away and will contribute little
to the flux at the refuelled channel. Another minor
consequence of using 8-fold symmetry is that only 376 of the
380 channels could be represented. This will result in a
slightly lower core reactivity.

The method of calculating the power using MICRETTE is
discussed in Appendix 5.

A grid of mesh points was used in which the odd numbered
x,y coordinates represented fuel channels. In the cores
having an inner region radius of 196 cm, channel (11,11)
was refuelled, while in the cores having an inner region
radius of 255 cm, channel (13,13) was refuelled. Material
properties of the fuel—equilibrium burnup fuel, and fresh
fuel with and without xenon poisons—were obtained from
LATREP.

Power peaking for the hybrid cores was compared to
peaking in the natural uranium G-2 reference core. The
refuelling of two different channels in the natural uranium
core was investigated—a channel in the center of the core, where the flux (and power) is highest, and channel (11,11). A different core layout was used to refuel the central channel, as refuelling channel (1,1) would result in four channels at the center of the core being refuelled. In the new core, the fuel channels were located at even (x,y) coordinate number mesh points, and channel (0,0) was refuelled.

A diagram of the core layouts is shown in Figure 19.

7.3 Results and Discussion

A measure of the power peaking upon refuelling which is independent of the normalization, is the change in the relative difference between the power of the refuelled channel and the power of channel (1,1) upon refuelling. This quantity is plotted in Figure 20 as a function of enrichment, for the two radii studied. This measure of power peaking increases almost linearly with increasing enrichment, reaching a maximum of 275% at 3.0% enrichment. Refuelling channel (0,0) in the G-2 reference resulted in a 9% increase in the relative difference between the power at channel (0,0) and (10,10) when fresh fuel containing no xenon was used, and a 2% increase when fresh fuel with equilibrium xenon was used.

For all the hybrid cores studied, the final powers after refuelling exceed 6.5 MW(th). The powers range from 6.9 MW after refuelling channel (13,13) in a core with
with 1.0% enrichment and an inner region radius of 255 cm with fuel containing equilibrium xenon poisons, to a maximum of 19 MW after refuelling channel (11,11) with completely fresh fuel containing no xenon in a core with 3.0% enrichment and an inner region radius of 196 cm.

The equilibrium power distribution before refuelling corresponds to a time-averaged situation, which would be quite different from the instantaneous power distribution. In reality there would be variations in channel power about the equilibrium power, and one would observe even greater power peaking upon occasion.

Thus, power peaking is a problem with these hybrid cores.

7.4 Reducing Channel Power Peaking

There are several strategies that could be used to reduce the channel power peaking following refuelling. For small enrichments, the percentage change in the ratio of the power of the refuelled channel to the power at channel (1,1) upon refuelling is smaller for larger inner radii. As well, the absolute value of the final power is lower for the larger inner radii cores, because of the smaller flux. Thus, power peaking can be reduced by going to cores with a large inner region radius.

In practice, one would not refuel an entire channel of enriched uranium at once. Refuelling only a fraction of
a channel at a time would significantly reduce the channel power peaking. For instance, right at the interface one could refuel only a single bundle at a time, replacing a larger fraction of the channel elsewhere in the core.

Negative reactivity devices, such as zone controllers, could be used to lower the flux in the vicinity of a channel which was about to be refuelled.

Changes in the fuel design could also lead to a reduction in power peaking. For example, a burnable poison could be added to the fuel.

If none of the above means were sufficient to provide a large enough critical ratio, the entire reactor power could be derated to allow for safe refuelling.

Future reactors could be designed to allow for power peaking during refuelling, by increasing the total number of channels, thereby reducing the average channel power. The economic drawback of increased capital costs due to the larger size may be offset by reduced fuelling costs, due to increased burnup and lower leakage.
8. REACTOR STABILITY

8.1 Introduction

The separation of the eigenvalues of the higher harmonics from the fundamental is one measure of reactor stability. A subcriticality of less than about 30 mk for any higher order flux mode indicates an inherent instability against xenon-induced oscillations (6).

The computer code MICRETE was used for a number of hybrid cores to determine the eigenvalues corresponding to the first radial and azimuthal modes (see Figure 21) and hence, the subcriticality of these modes.

8.2 Results and Discussion

The subcriticality of the first radial and azimuthal flux modes is listed in Table 7 for the cores studied. For cores with a given enrichment, the subcriticality of these flux modes decreases with increasing inner region radius. This is to be expected, since increasing the inner region radius results in a flatter flux (and higher RFF) and a greater flux gradient in the outer region, which decreases the reactor stability. One cannot expect an exact scaling of the subcriticality with RFF, since the RFF depends on the power shape, while the subcriticality depends on the
flux shape.

Note that the subcriticality of the first radial mode decreases by only 0.9 mk in going from the unflattened to flattened G-2 natural uranium core, but decreases by about 6.0 mk in going from an inner region radius of 196 cm to 255 cm for the hybrid cores.

The subcriticality of the first radial and azimuthal flux modes is smaller for the hybrid cores than for the natural uranium core, indicating that these cores are less stable. However, the subcriticality of the first radial mode is still large for the hybrid cores, and this mode is still stable against xenon oscillations. The subcriticality of the first azimuthal mode decreases by at most 0.5 mk for the hybrid cores and this is a small absolute change.

Thus, the hybrid cores studied are only slightly less stable than the G-2 natural uranium core. The introduction of an annulus of enriched uranium in the CANDU-PHW reactor core is therefore a means of significantly improving the power RFF (by producing a discontinuity in the power at the interface between the inner and outer regions) without significantly changing the flux shape and decreasing the reactor stability.
9. **CONCLUSIONS**

This report has explored several aspects of a hybrid CANDU-PHW reactor core containing an annulus of enriched uranium around a central zone of natural uranium. It has been determined that the radial form factor could be increased by up to 12% with such a core, with only a slight decrease in stability. Improvements in the uranium utilization and fuel economics of between 10% and 20% could be attained. Power peaking upon refuelling a channel of enriched uranium in the hybrid core can be severe.

The choice of enrichment, inner region radius, and burnups for the hybrid core would depend upon the importance assigned to the parameters discussed, as one cannot simultaneously achieve a maximum radial form factor, minimum uranium requirements and costs, a minimum power peaking, and maximum stability. However, uranium enrichment between 1.0% and 1.4% and an inner region radius no greater than 255 cm would probably yield satisfactory values for all of the above parameters.

Future studies of this hybrid core should concentrate on the problem of power peaking upon refuelling. A three dimensional code (or a two dimensional code in which one dimension is the axial dimension) should be used to determine
whether the power peaking problem could be overcome through the use of a proper fuel management scheme, and whether such a scheme would be economically feasible.
APPENDICIES
APPENDIX 1

List of Symbols

The subscripts I and 0 refer to the inner and outer regions of the core respectively. The subscript T stands for total.

\( w \) - exposure (n/kb)
\( B \) - burnup (kW.d/ gU)
\( R \) - radius of inner region (cm)
\( E \) - enrichment of outer region (%) 
\( P \) - power (MW); the total reactor power is 2180 MW (thermal) 
  \( P_I + P_0 = 2180 \) MW(th). The net electrical power is \( \eta \times P \).

\( F \) - feed factor (kg of natural uranium required to produce a kg of enriched uranium of enrichment \( E \). Assuming a tails enrichment of .25%, \( F = \frac{E - 0.25}{0.71 - 0.25} \)

\( \eta \) - station efficiency (30%)
\( X \) - fuel bundle cost ($/ gU)
\( C \) - unit energy fuel cost (mills/kW.h(e))

SWU - separative work unit
APPENDIX 2

Relationships for Converting LATREP Parameters to FORGOD Parameters

Included in the input data required by FORGOD are various cross sections for the fuel and reflector (moderator). However, LATREP generates four-factor parameters for the fuel and reflector. The two sets of parameters are related by the following relationships. (FORGOD parameters are on the left of the equality sign, LATREP parameters on the right.) For the fuel,

\[ D_1 = D_F \]
\[ D_2 = D_T \]
\[ \Sigma_{a1} = (1-p) \frac{D_F}{L_S} \]
\[ \Sigma_{a2} = \frac{D_T}{L^2} \]
\[ \Sigma_{R1} = p \frac{D_F}{L_S} \]
\[ \Sigma_{R2} = \nu \Sigma_{f1} = 0.0 \]
\[ \nu \Sigma_{f2} = (k_\infty/p) \times \left( \frac{D_T}{L^2} \right) \]

and similarly for the moderator, except that

\[ \Sigma_{a1} = \Sigma_{R1} = \nu \Sigma_{f1} = \nu \Sigma_{f2} = 0.0. \] The symbols have their usual meaning.
Calculating Natural Uranium Requirements

If the reactor core contains fuel of uniform enrichment and burnup, then the uranium requirements are simply \( \left( \frac{F}{B} \right) \) g/kW.d, for burnup B and feed F. However, for a two-zone reactor, the inverse burnup in each zone must be weighted by the fraction of the total power produced in that zone (7). FORGOD was run to determine the power in each region (see Appendix 5).

\[
\left[ \left( \frac{P_I}{P_T} \times \frac{1}{B_I} \right) + \left( \frac{P_0}{P_T} \times \frac{F}{B_0} \right) \right] \times \frac{365.25}{\eta} (gU/kW.a)
\]

\[
= \left[ \frac{P_I(x \times 10^3 \text{ MW})}{B_I(\text{kW.d/gU})} + \frac{P_0(x \times 10^3 \text{ MW}) \times F}{B_0(\text{kW.d/gU})} \right] \times \frac{365.25}{0.3 \times 2.18(x \times 10^3 \text{ MW})}
\]

\[
= 558.5 \left[ \frac{P_I(x \times 10^3 \text{ MW})}{B_I(\text{kW.d/gU})} + \frac{P_0(x \times 10^3 \text{ MW}) \times F}{B_0(\text{kW.d/gU})} \right] (gU/kW.a)
\]
APPENDIX 4

Calculating Fuel Costs

In a uniform reactor, the fuel cost would be given by \( \left( \frac{X}{B} \right) \) \$/kW.d. For a two region reactor, the fuel bundle cost and inverse burnup in each region must be weighted by the fraction of the total power contributed by that region, so that the total unit energy cost is

\[
C = \left( \frac{X_I \times P_I}{B_I} + \frac{X_0 \times P_0}{B_0} \right) \frac{1}{P_T \times n} \text{ mills/kW.d(e)}
\]

\[
= 0.0637 \left( \frac{X_I \times P_I}{B_I} + \frac{X_0 \times P_0}{B_0} \right) \text{ mills/kW.h(e)}
\]

with \( P \) in units of \((x10^3 \text{ MW})\), \( X \) in units of \((\text{mills/gU})\), and \( B \) in units of \((\text{kW.d/gU})\). 1 mill = 10^-3 dollars.

The fuel bundle costs were obtained using the ground rules listed in Table 4. The fuel bundle costs as a function of enrichment are shown in Table 5, and the inner and outer burnups and power were obtained from FORGOD and are tabulated in Table 3.
APPENDIX 5

Computing Power

In FORGOD, the cell power was determined by multiplying the average cell flux per unit volume computed by FORGOD, by the radius where the cell was located (to weight the volume) by the power per unit flux ratio. This last factor was obtained from LATREP as a product of terms:

\[
\text{cell power per unit flux} = \frac{\text{total power produced in the fuel}}{\text{average flux in the fuel}} \times \text{(ratio of average fuel to cell flux)}
\]

In MICRETE, the power was taken to be proportional to the fission rate per unit length of rod (8).
Table 1: TESHOM INPUT DATA FOR LATREP

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### TABLE 2

**Absorber Flattening**

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### List of Symbols

A - Absorption added to inner region (of 2 region core).
B - Absorption added to outer region (of 2 region core).

$w_I$ ($w_O$) - inner (outer) exposure.

$p$ - reactivity (mk)

Rmax - radius where maximum flux occurs.

$\phi_0$ ($\phi_{198}$) - flux at a radius of 0.0 cm (198 cm).

The subscript "i" refers to the core containing a uniformly distributed 3.5 mk reactivity load. The subscript "f" refers to the core after an additional 8 mk reactivity load has been added.

+ Linear power weighting in a 4 region core used.
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<tr>
<td>3.0</td>
<td>226</td>
<td>2.12</td>
<td>5.80</td>
<td>8.83</td>
<td>45.99</td>
<td>1.212</td>
<td>0.968</td>
<td>10.629</td>
<td>202</td>
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<td>3.0</td>
<td>255</td>
<td>2.15</td>
<td>5.30</td>
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<td>43.85</td>
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<td>0.721</td>
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<td>189</td>
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<td>3.0</td>
<td>285</td>
<td>2.14</td>
<td>4.20</td>
<td>8.91</td>
<td>38.68</td>
<td>1.77</td>
<td>0.410</td>
<td>10.629</td>
<td>174</td>
</tr>
</tbody>
</table>
### TABLE 4

**Ground Rules for Fuel Bundle Costs**

All costs are given in 1977 dollars.

<table>
<thead>
<tr>
<th>Natural Uranium Fuel</th>
<th>$/kg U</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellowcake</td>
<td>104</td>
</tr>
<tr>
<td>Fabrication</td>
<td>44</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>144 $/kg U</td>
</tr>
</tbody>
</table>

**Enriched U-235 Fuel**

<table>
<thead>
<tr>
<th></th>
<th>$/kg U</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellowcake</td>
<td>104 x F</td>
</tr>
<tr>
<td>Conversion (U_3O_6 \rightarrow UF_6)</td>
<td>4.4 x F</td>
</tr>
<tr>
<td>Enrichment (USAEC toll enrichment)</td>
<td>100 x SWU</td>
</tr>
<tr>
<td>Conversion (UF_6 \rightarrow UO_2)</td>
<td>10</td>
</tr>
<tr>
<td>Fabrication</td>
<td>40</td>
</tr>
</tbody>
</table>

**Total**

\[
(104xF) + (4.4xF) + (100xSWU) + 50 \quad ($/kgU)
\]

* Assuming tails enrichment of .25%.
**TABLE 5**

Fuel Bundle Costs

<table>
<thead>
<tr>
<th>Enrichment (% U-235)</th>
<th>Feed Factor</th>
<th>SWU(10)</th>
<th>Cost (1977 $/kgU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>natural</td>
<td>1.000</td>
<td>0</td>
<td>144.0</td>
</tr>
<tr>
<td>1.0</td>
<td>1.627</td>
<td>0.3177</td>
<td>258.1</td>
</tr>
<tr>
<td>1.2</td>
<td>2.061</td>
<td>0.5924</td>
<td>332.7</td>
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<tr>
<td>1.4</td>
<td>2.495</td>
<td>0.8959</td>
<td>410.0</td>
</tr>
<tr>
<td>2.0</td>
<td>3.796</td>
<td>1.915</td>
<td>653.0</td>
</tr>
<tr>
<td>3.0</td>
<td>5.965</td>
<td>3.811</td>
<td>1077.7</td>
</tr>
</tbody>
</table>
### Table 6

<table>
<thead>
<tr>
<th>E (%)</th>
<th>R (cm)</th>
<th>C_I</th>
<th>C_0</th>
<th>C_T (mills/kW.h(e))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>196</td>
<td>1.23</td>
<td>1.41</td>
<td>2.64</td>
</tr>
<tr>
<td>1.0</td>
<td>196</td>
<td>0.91</td>
<td>1.36</td>
<td>2.26</td>
</tr>
<tr>
<td>1.0</td>
<td>226</td>
<td>1.15</td>
<td>1.19</td>
<td>2.34</td>
</tr>
<tr>
<td>1.0</td>
<td>255</td>
<td>1.25</td>
<td>1.32</td>
<td>2.57</td>
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<td>0.94</td>
<td>1.32</td>
<td>2.26</td>
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<td>226</td>
<td>1.19</td>
<td>1.11</td>
<td>2.30</td>
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<td>1.2</td>
<td>255</td>
<td>1.46</td>
<td>0.92</td>
<td>2.39</td>
</tr>
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<td>1.77</td>
<td>0.88</td>
<td>2.65</td>
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<tr>
<td>1.4</td>
<td>196</td>
<td>0.97</td>
<td>1.34</td>
<td>2.30</td>
</tr>
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<td>1.4</td>
<td>226</td>
<td>1.18</td>
<td>1.14</td>
<td>2.32</td>
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<td>0.91</td>
<td>2.37</td>
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<td>1.76</td>
<td>0.74</td>
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<td>0.97</td>
<td>1.51</td>
<td>2.49</td>
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<tr>
<td>2.0</td>
<td>226</td>
<td>1.20</td>
<td>1.26</td>
<td>2.46</td>
</tr>
<tr>
<td>2.0</td>
<td>255</td>
<td>1.45</td>
<td>1.00</td>
<td>2.45</td>
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<td>2.0</td>
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<td>0.67</td>
<td>2.48</td>
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<td>0.96</td>
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<td>2.78</td>
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<td>3.0</td>
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<td>1.45</td>
<td>2.70</td>
</tr>
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<td>255</td>
<td>1.50</td>
<td>1.13</td>
<td>2.63</td>
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<tr>
<td>3.0</td>
<td>285</td>
<td>1.82</td>
<td>0.73</td>
<td>2.55</td>
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</table>
### TABLE 7
Subcriticality of First Azimuthal and First Radial Flux Modes

<table>
<thead>
<tr>
<th>Core</th>
<th>RFF</th>
<th>1st Radial</th>
<th>1st Azimuthal</th>
</tr>
</thead>
<tbody>
<tr>
<td>unflattened G-2</td>
<td>0.65</td>
<td>69.0</td>
<td>22.2</td>
</tr>
<tr>
<td>flattened G-2</td>
<td>0.82</td>
<td>68.1</td>
<td>17.7</td>
</tr>
<tr>
<td>E = 1%; R = 196 cm</td>
<td>0.84</td>
<td>66.2</td>
<td>16.8</td>
</tr>
<tr>
<td>E = 1%; R = 255 cm</td>
<td>0.92</td>
<td>59.6</td>
<td>16.3</td>
</tr>
<tr>
<td>E = 2%; R = 196 cm</td>
<td>0.80</td>
<td>64.0</td>
<td>17.6</td>
</tr>
<tr>
<td>E = 2%; R = 255 cm</td>
<td>0.87</td>
<td>58.5</td>
<td>16.4</td>
</tr>
</tbody>
</table>
FIGURES
Figure 1: HYBRID CORE MODEL

\[ R = 196 \text{ cm} \quad E = 0.7\% \\
226 \text{ cm} \quad 1.0\% \\
255 \text{ cm} \quad 1.2\% \\
285 \text{ cm} \quad 1.4\% \\
\]
Figure 2: RADIAL FLUX AND POWER FOR UNFLATTENED G-2
REFERENCE CORE

R=156.0CM; E=0.7%; W1=1.9544/N/KG; W0=1.6444/N/KG; RFF=.654; LCM=1.0125:2

- POWER

- FLUX
Figure 3: RADIAL FLUX AND POWER FOR FLATTENED G-2 REFERENCE CORE

\[ R = 136.0 \text{ cm}, \quad E = 0.7\%, \quad \text{W}_1 = 1.99 \text{N/kg}, \quad \text{W}_0 = 1.64 \text{N/kg}, \quad \text{RFF} = 0.817, \quad \text{LM} = 1.001365 \]

- POWER
- FLUX
Figure 4: ISO-RFF AND REACTIVITY CONTOURS: \( E=1.4\%, \ R=196 \text{ CM} \)

Radial Form Factor $\rho = 0.0 \text{ mk}$
$\rho = 12.8 \text{ mk}$
$\rho = 30.0 \text{ mk}$

Outer Exposure (N/KB)
Inner Exposure (W/KB)
Figure 5: RADIAL FLUX AND POWER (MAXIMIZED RFF):

\[ E = 1.4\%, \quad R = 196 \, \text{CM} \]

\[ R = 196.0 \, \text{CM}; \quad E = 1.4\%; \quad W1 = 2.20 \, \text{NAD}; \quad W0 = 4.24 \, \text{NAD}; \quad \text{RFF} = 0.823; \quad \text{LM} = 1.012373 \]

POWER

FLUX

FLUX AND POWER (ARBITRARY UNITS)

RADIUS (CM)
Figure 6: RADIAL FLUX AND POWER FOR LOW OUTER BURNUP:

E=1.4%, R=196 CM

R=196.0CM, E=1.4%, W1=3.60N KG, W0=3.71N KG, RFF=.746, LAM=1.012629

- POWER
- FLUX

FLUX AND POWER (ARBITRARY UNITS)

RADIUS (CM)
Figure 7: RADIAL FLUX AND POWER WITH LOW INNER BURNUP:

$E = 1.4\%$, $R = 196$ CM

$R = 196.0$ CM; $E = 1.4\%$; $W1 = 1.63$ KN/KB; $W0 = 5.20$ KN/KB; $RFF = 0.171$; $LM = 1.012$ SEC

---

FLUX AND POWER (ARBITRARY UNITS)

RADIUS (CM)
Figure 8: VARIATION OF RFF WITH INNER EXPOSURE: E=1.4%
Figure 9: VARIATION OF RFF WITH OUTER EXPOSURE: E=1.4%
Figure 10: MAXIMUM RFF ACHIEVABLE VS ENRICHMENT

The maximum RFF could be achieved for any reactivity of interest for the hybrid cores, but not for the natural U cores.

+ R=195 CM
+ R=226 CM
+ R=255 CM
+ R=285 CM

* G-2 Reference Case
Figure 11: VARIATION OF REACTIVITY WITH BURNUP

+ E=2%
X NAT U

Burnup-Averaged Reactivity

Burnup (MWD/kg)
Figure 12: VARIATION OF POWER PER UNIT FLUX RATIO (PF) WITH BURNUP

\[ \text{PF} \times 10^{14} \text{ KW/CM/UNIT FLUX} \]

\[ \text{BURNUP (MWD/ KG)} \]
Figure 13: ISO-RFF AND REACTIVITY CONTOURS:
NATURAL U, R=226 CM
Figure 14: VARIATION OF OUTER EXPOSURE WITH ENRICHMENT (MAXIMIZED RFF)

* G-2 Reference Case

OUTER EXPOSURE (N/KB)

ENRICHMENT (%)
Figure 15: POWER DISCONTINUITY BETWEEN THE INNER AND OUTER
REGIONS IN THE EQUILIBRIUM STATE

* G-2 Reference Case
Figure 16: NATURAL URANIUM REQUIREMENTS VS ENRICHMENT
(MAXIMIZED RFF)

+ R=196 CM
× R=225 CM
⊙ R=255 CM
★ R=269 CM

* G-2 Reference Case
Figure 17: UNIT ENERGY FUEL COSTS VS ENRICHMENT
(MAXIMIZED RFF)

G-2 Reference Case

\[ \begin{align*}
R &= 196 \text{ CM} \\
R &= 226 \text{ CM} \\
R &= 255 \text{ CM} \\
R &= 265 \text{ CM}
\end{align*} \]
Figure 18: UNIT ENERGY FUEL COSTS VS INNER EXPOSURE
(12.8 MK REACTIVITY)

* G-2 Reference Case

+ E=.71
× E=1.0; R=196
× E=2.0; R=196
Figure 19: **CORE LAYOUT FOR MICRETE**

(1/8th Core shown)

---

**R = 196 cm**

Channel (11,11) refuelled.

---

**R = 255 cm**

Channel (13,13) refuelled.

---

**R = 196 cm**

Channel (0,0) refuelled.
Figure 20: % Change in relative difference between the power of the refuelled channel and the power of channel (1,1) upon refuelling.

- + R=195; with XE
- x R=195; no XE
- - - x R=255; with XE
- --- x R=255; no XE
Figure 21: FLUX MODES

Fundamental Mode

First Azimuthal Mode

First Radial Mode
REFERENCES


4. Private communication from F.T. Clayton.


6. Private communication from F.N. McDonnell.

7. Private communication from B. Townes.


9. Private communication from B. Townes.

10. Private communication from J. Veeder.