THE ANALYSIS OF INTERREGIONAL FISCAL POLICY:

A SIMULATION APPROACH

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ABSTRACT

Conventional wisdom is that fiscal policy at the regional level is ineffective. Recent concern about stability of bond-financed fiscal policy imposes an additional constraint on the effectiveness of interregional fiscal policy. In the conventional macroeconomic model, regional public sectors are ignored, or are at most a subset of the national model. Fiscal and financial interrelationships among different levels of government have not been investigated thoroughly in the literature.

The purpose of this dissertation is to provide a theoretical framework for the analysis of interregional fiscal policy. We argue that all government budget constraints must be explicitly included in the model, and regions become the major building blocks of the system. Stability of the system then depends on the fiscal and financial interrelationships among different levels of government.

We examine a once-and-for-all fiscal policy change in the interregional model with and without the federal sector. The simulated results based on an acceptable range of parameter values show that the system cannot generate a stable long-run equilibrium. At best, a quasi-equilibrium is attainable in that only the overall government budget constraint is satisfied.

A once-and-for-all policy change is not only irrelevant in reality since public sectors react to actual economic situations, but also becomes a source of instability in an interregional model.

The final version of the interregional model incorporates an endogenous fiscal policy. Government expenditure becomes an endogenous variable and fiscal policies are target-oriented. The income level and balanced-budget are the main targets. A system of government expenditure reaction functions is built into the model with each government adopting an active fiscal policy in order to achieve income and balanced-budget targets. The public sector adjusts its fiscal policy according to the last period's economic situation. The extent of these government expenditure changes is governed by the target-adjustment parameters. Each government has its own priority or objective in determining the target-adjustment parameter values.

The simulated results show that the interregional model can generate a stable long-run equilibrium, regardless of the mode of federal financing policy. The effectiveness of an active fiscal policy and the critical limits of these target-adjustment parameters are investigated. Of prime importance is the finding that an independent regional fiscal policy cannot generate a stable long-run equilibrium. Only when all governments cooperate actively in fiscal management can the system achieve the targets. Thus, the final version of the interregional model not only rejects independent regional fiscal policies, but requires coordination and cooperation among all governments in devising a viable fiscal policy. Our simulation findings therefore strengthen the case for fiscal federalism.

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Introduction

1.1 Nature of the Problem

Musgrave (1959) proposes that the public sector has three primary economic objectives in achieving a welfare optimum: First, the establishment of an efficient pattern of resource use (the allocation objective); Second, the attainment of the most equitable distribution of income (the distribution objective); and Third, the maintainence of high employment with stable prices (the stabilization objective).[1]

The Musgravian analytical framework has become the standard terms of reference for theoretical studies of public finance. However, this pure analytical approach is generally carried out on the premise of a single, presumably the national, public sector, and thus ignores the institutional complications a federal state may face in fiscal management.

In a federal state, the lower levels of government may not be in a position to pursue all three desirable economic objectives. Musgrave argues that in a federal state, the distribution and stabilization functions should be federal government's responsibility, while the allocation function can be decentralized to the lower levels of government.[2]

In the context of a simple local income determination model, Oates (1968) offers his justification for the stabilization function being centralized. He argues that local government can employ only fiscal stabilization tools, and problems of openness and external indebtedness seriously impair the freedom and effectiveness with which these tools can be used. Hence he concludes that:

> "The case for having the central government assume primary responsibility for the stabilization function appears, therefore, to rest on a firm economic foundation. Our local income and payments model suggests that local government cannot use conventional tools to much effect and must instead rely mainly on beggar-thy-neighbor policies, which from a national standpoint are likely to produce far from the desired results. The central government, on the other hand, is free to adopt monetary policies and fiscal programs involving deficit finance; consequently, the stabilization problem must be resolved primarily at the central government level."[3]

Engerman (1965) reaches a somewhat similar conclusion:

> "Thus, as long as stabilization measures are left to particular states, there can be no expectation of an optimum national policy, for there may be either smaller or larger changes in demand than would be considered desirable. In the contemporary situation, given both financial constraints and interstate strategy, the presumption that stabilization measures will be insufficient if they are left to lower-level governments appears most reasonable."[4]

The conventional wisdom has thus established that lower level governments in a federal setting should not pursue active stabilization fiscal policy. In the literature of fiscal federalism, many writers use the term regional, local, state or provincial to denote the lower-level government. For consistent and simple reasons, in the following discussion of this thesis, we use the term regional to represent lower level governments.

Barber (1966) does not feel the barriers to regional fiscal policy are insurmountable. He argues that:

> "While in certain respects a province is not in as strong a position to pursue an <u>independent</u> fiscal policy as the federal government, the differences are mainly of degree rather than kind and even where the province lacks certain powers that reside in the federal sphere this would not prevent the use of fiscal policy by a province."[5]

However, Barber offers no firm theoretical justification, comparable to the Oates model, to substantiate his argument. The Musgravian classification or functional assignment has therefore dominated the theory of fiscal federalism for nearly two decades. A balanced budget becomes the prudent fiscal policy for any non-federal government.

With increasing regional fiscal automony, large regions, especially, have implemented their own fiscal policy. To date there is no firm theoretical justification to refute Oates's conclusion. At most, the Oates model is being criticized as being too crude.[6]

Instead of construction of a theoretical framework for analysing interregional fiscal policy, the emphasis has shifted from discussion of its feasibility to the actual measurement of its impact on the national fiscal policy.[7] Auld (1975), using the "Weighted Budget Result" method, investigates the the question of whether the effects of Ontario fiscal policy between the 1960-1967 period counteract or reinforce the federal fiscal objective. What makes his finding interesting is his argument that a regional fiscal policy may be perverse with respect to a federal fiscal objective, but stabilizing in terms of a regional fiscal objective. This gives the regional fiscal objective an independent identity, and is the natural outcome of increasing regional fiscal autonomy.

Auld views the stabilization problem from a broader perspective which includes both federal and regional governments. However, the "Weighted Budget Result" method considers only the first round effect of the expenditure and tax items and may give misleading results because the long-run effects are omitted.

Wilson (1977), in the discussion of regional fiscal policy, emphasizes the long-run financial implications of regional fiscal policy on the federal and other regional governments. He points out that under a fixed exchange rate system, a regional government issuing new bonds in the foreign capital market to finance its expansionary fiscal policy, may require the central bank to purchase the foreign exchange in order to maintain the exchange rate.[8] Moreover, induced exchange rate changes due to expansionary fiscal policy of one region will spread to other regions as well. Hence "external" positive or negative effects may spill over to other regions. The interregional financial influence is another form of leakage which must be added explicitly to the model. Wilson concludes that discretionary fiscal policy by a large region such as Ontario has generally played a constructive role, and recommends that discretionary fiscal policy by Ontario should be planned in consultation with the federal government and other major regions.

An analysis of financial implications of regional fiscal policy is important because it provides a basis to evaluate and revise Oates's conclusion. As Auld (1979) puts it:

> "What is important is that once the financial implications of fiscal policy are recognized, the question of federal versus provincial fiscal policy takes on another dimension. The federal government can co-ordinate its fiscal policy with monetary policy (subject to Balance of Payments constraints) while the Province cannot; although this fact does not preclude non-federal fiscal policy, it certainly adds a constraint."[9]

The recent trend of decentralization and increasing regional fiscal automony provides economic as well as political reasons for regions to pursue their fiscal policies. A viable approach or guideline for regional fiscal policy must be developed to aid both federal and regional governments in their economic planning.

1.2 Purpose and Methodology of the Study

The primary purpose of this dissertation is to develop a theoretical framework for the study of interregional fiscal policy in a federal setting. It is hoped that the interregional model used in this study will not only serve as an instrument in the analysis of fiscal federalism, but will also suggest some new insights for future research. To achieve this goal, we emphasize the following points in the construction and analysis of our interregional model:

First, we emphasize an interregional rather than an independent regional fiscal policy. The basic nature of the Oates model is an income determination model of a small local sector. No federal and other local sectors appear in the Oates model. The national variables are assumed exogenous. Without considering intergovernmental or intersectoral influences, we thus label it as a "small region's" fiscal policy. Barber also emphasizes the feasibility of independent fiscal policy for the

Province of Ontario. However, there is no such thing as a completely independent regional fiscal policy. The financial implications to other regions and their response, as discussed by Wilson (1977), play an important role in determining the effectiveness of any fiscal policy pursued by a region. Any discussion of regional fiscal policy must be within an interregional setting. Every government, regional and federal, must participate and cooperate in fiscal management.

Second, we emphasize the importance of government budget constraints. One basic limitation in Oates' analysis is the lack of any dynamic system explaining how the long-run equilibrium condition is derived. Oates imposes the long-run equilibrium condition in the model without questioning the stability of the system. It is meaningless to discuss the long-run multiplier effect if the system is unstable. The dynamics of the interregional model can be generated by the government budget constraints. Being a stock-flow relationship, a government budget constraint turns the model into a dynamic one even though the basic behavioral relationships are static.[10]

Third, we emphasize the fiscal and financial relationships between levels of government. One special feature of our interregional model is the financial leakages and repercussions between regions. In our model, we assume each region is large enough to affect the financial and monetary variables. Moreover, the dynamic system of the interregional model now consists of more than one government budget constraint, i.e., one budget constraint for each government. The stability conditions then become more complicated and stringent than in a simple public-sector model. When one government implements its fiscal policy, the tax base and tax revenue of other governments are affected, which necessitates other governments adopting an accommodating fiscal policy to balance their budgets. These fiscal and financial considerations between two levels of government are then the main determinant of the success of interregional fiscal policy.

Fourth , we emphasize target-oriented fiscal policy in all public sectors. The conventional approach treats government expenditure as an exogenous variable. Fiscal policy is expressed as a once-and-for-all change of government expenditures and the public sector does not respond to the evolving economic situation. In the interregional model, this turns out to be a source of instability. We thus assume a target-oriented fiscal policy in all public sectors. Our target-oriented government expenditure reaction function involves both exogenous and endogenous components.

Analysis of the fiscal and financial interrelationships among governments in a federal setting has not been carried out extensively in the literature. Most studies are limited to one region, or at most two regions, and the federal sector and national variables are treated as exogenous. We thus develop an interregional model which includes at least two regional sectors and a federal sector. Budget constraints of all governments must be included explicitly. Our model, incorporating the government budget constraints, is represented by a system of non-linear simultaneous equations. Stock-flow relationships determine the dynamics of the model.

Within the interregional framework, we discuss fiscal policy by applying the conventional approach of treating the government expenditures as exogenous variables. We use the interregional model with and without a federal sector to demonstrate the basic nature of the stability problem. We argue that the use of a target-oriented government expenditure reaction function is a viable approach in the analysis of interregional fiscal policy. This approach provides a theoretical framework for the study of interregional fiscal policy and, it is hoped, offers a better alternative to the Oates model.

The long-run character of the model depends on the stability of the system. Stability analysis, therefore, occupies a major role in our study. Despite the model's conceptual simplicity, it involves many variables and equations. The overall size of the model necessitates the use of computer simulation techniques to trace out the time path of the system after an exogenous shock.

Since our model is hypothetical, in that it does not relate to any particular country, parameter values and initial conditions can be assigned arbitrarily. However, to make them as representative of a real economy as possible, we use actual data whenever possible. Canadian data have been used in setting initial conditions for the variables of the model and as a basis for choosing values for a number of the parameters. Although a standard set of parameter values has been calculated, sensitivity analysis is carried out to assess the effects of alternative choices of parameter values.

The simulation experiments lead to a number of conclusions of general interest. Of prime importance is the finding that an independent regional fiscal policy cannot generate a stable long-run equilibrium. Only when all governments are actively involved in fiscal management can the system achieve the targets. Thus our findings are in sharp contrast to Oates' conclusion that regions should not pursue any stabilization policy. The simulated results show that uncoordinated fiscal policies among governments would produce undesirable effects. Thus the IRFG model does not only reject independent regional fiscal policy, but also requires coordination and cooperation among governments in devising a viable fiscal policy. The simulation findings based on the IRFG model therefore strengthen the case for fiscal federalism.

1.3 Organization of the Dissertation

The dissertation is an attempt to apply simulation techniques to analyse interregional fiscal policy in a federal setting. In chapter 2, the literature is reviewed with the focus on an examination of theories which explore the viability of an active interregional fiscal policy. The chapter opens with a critical review of the Oates model. This is followed by a discussion of financing interregional fiscal policy, with emphasis on the economics of bond-financing. The last section summarizes the conclusions we derived from the literature.

Chapter 3 discusses the fiscal policy within a setting in which only one public sector, the national government, exists. The purpose of such an aggregate national model is to provide a framework with which an interregional model can be developed. An aggregate national model of a Keynesian aggregate-demand type is constructed. Fiscal policy based on pure bond-financing and pure money-financing, with and without the Christ adjustment method,[11] is discussed. Various aspects of the simulation approach, as well as the need for introducing regional government sectors, are discussed.

An interregional model without a federal sector (IR) is presented in chapter 4. We start with the description of the model. This is followed by an analysis of fiscal policy diagrammatically and mathematically. A simulation approach is discussed, along with the theoretical basis for deriving regional functional forms and initial conditions and parameter values, from the national model. Simulation results are presented and discussed. The last section suggests the importance of introducing the federal sector.

Chapter 5 presents and describes an interregional model with a federal sector (IRF). The discussion is concerned with both interregional and federal fiscal policies, and these are examined with regard to two different modes of federal financing policy. The later part of the chapter considers and discusses some other fiscal schemes that can produce stable results.

After examining the difficulties encountered, especially the difficulty of meeting the stability condition in the previous models, the final version of the interregional model (IRFG) is presented and described in chapter 6. The meaning of the government expenditure reaction function is discussed. Special attention is given to the different interpretations of fiscal policy in the model. Base-run simulation results are presented and discussed.

Chapter 7 consists of a detailed analysis of the simulation results obtained with the full model under various assumptions about the values of target-adjustment parameters and the size of regions. The objectives of an active fiscal policy are discussed in the second section. We then present the simulation results for the following two cases: (1) identical regions case; and (2) unequal regions case. We consider the effects of an increase of 5 percent in income target in both regions, following the initial equilibrium. The effectiveness of various active fiscal policies is examined, and tests are conducted to determine the critical limits of the target-adjustment parameters. The last section of the chapter generalizes the findings and discusses their implications in the formation of a viable fiscal policy.

Chapter 8 provides a summary and a statement of conclusions. Policy implications are discussed and the scope for further research is outlined.

Footnotes to Chapter 1

(1). Musgrave (1959), pp.3-27.

(2). Ibid., pp.179-182.

(3). Oates (1968). p.44.

(4). Engerman (1965) p.56.

(5). Barber (1966) p.29.

(6). Auld (1976). He finds that in the balanced budget case, expansion will occur provided that public sector spends less, at the margin, on foreign goods than does the private sector.

(7). For the measurement of the impact of regional fiscal policy on the national fiscal objective, see also Rafuse Jr. (1966), Robinson and Courchene (1969), and Jones, Bardecki and Hull (1977). The purpose of these studies is to examine whether regional fiscal behavior is perverse to the national fiscal objective, which is the so-called "Fiscal Perversity Hypothesis" proposed by Hansen and Perloff (1944).

(8). In an open-economy model under a fixed exchange rate system, the purchases of foreign exchange becomes one of the uses of government funds. Under a fixed exchange rate system, the sales of bonds by a regional government in the foreign capital market increase the capital inflow. The federal government must purchase the foreign exchange in order to maintain the exchange rate. Thus the federal monetary policy is indirectly affected by the regional government's financing policy. See Scarth (1975), pp.13-14.

(9). Auld (1978) p.275.

(10).See Turnovsky (1978), Chapter 4. He explains how the government budget constraint can turn a static model into a dynamic one.

(11).See Christ (1979) and Chapter 2 of this thesis. Christ treats total government expenditures as constant. Any changes in the interest payments will be offset by the government expenditures on goods and services. Christ argues that this fiscal scheme will generate a stable long-run equilibrium even under bond-financed fiscal policy. Such a scheme is termed here the "Christ adjustment method".

Chapter 2

Review of Literature

2.1 Introduction

Fiscal policy has long been regarded as the sole responsibility of the federal government and conventional wisdom is that there is little if any role for interregional fiscal policy. It has been proposed by Musgrave and substantiated by Oates that fiscal policy at the regional level is ineffective. More recent concern about the stability problem associated with bond-financing casts doubt on the feasibility of interregional fiscal policy in which bond-financing is the only means available for a regional government to finance its deficit.

This chapter reviews and assesses these arguments regarding the efficacy of interregional fiscal policy. Section 2.2 is devoted to a critical review of the Oates model. In section 2.3 we examine the financing aspect of interregional fiscal policy, with particular attention given to the debt-limit theory. Section 2.4 reviews the economics of bond-financing, emphasizes its unstable nature and discusses the Christ adjustment method. Section 2.5 gives some conclusions which follow from the review of the literature.

2.2 Critical Review of the Oates Model

Oates[1] (1968) uses a simple income determination model of a local public sector to demonstrate the ineffectiveness of local fiscal policy. The Oates model has commodities and financial asset markets and a trade balance equilibrium equation. The level of output is assumed perfectly elastic at the given price level. The rate of interest and the amount of exports are exogenously given. Most important is the assumption that government expenditures are divided between imports and domestically produced goods in the same proportion as private expenditures.

Description of the Oates Model

(2.1)	$C(YD,R,A)+G+X-I(YD,R,A,G)-Y = 0$	Commodities Market
(2.2)	$L(YD,R,A) - A = 0$	Financial Asset Market
(2.3a)	$X - I(YD,R,A,G) = 0$	Trade Balance Equation (balanced-budget)
(2.3b)	$X - I(YD,R,A,G) + D = 0$	Trade Balance Equation (deficit-financing)
(2.3c)	$D = G - T$	Budget Deficit

Notation :

Y — real income

YD — disposable income

C — total private expenditures

R — rate of interest(exogenously determined)

X — flow of exports

I — flow of imports

G — local government expenditures (exogenously determined)

A — real value of the net financial asset holdings of the private sector

T — taxes(exogenously determined)

D — government budget deficit

Under the above assumptions, the debt-financing deficit multiplier is given by:[2]

(2.4) $dY/dG = (1 - a)/a$

where "a" is the percentage of consumption on imports. The multiplier thus depends on the relative openness of the local economy. The more open the local economy, i.e., the larger the "a", the less expansionary the impact of deficit spending on the equilibrium level of income. The multiplier becomes unity when "a" equals 0.5. Oates argues that because of the high degree of openness of the local economy and the resultant high import leakages, the multiplier is not likely to be large.

However, if we relax Oates' crucial assumption about the same proportions of private and public spending on foreign and domestic goods, we derive the following multiplier:[3]

(2.5) $dY/dG = (1 - a)/b$

where "a" and "b" are the marginal propensity to import of the public and private sectors, respectively. Given "b", the multiplier will be larger the smaller is "a". One way the local government can minimize the import leakage is to spend its money on projects which create jobs directly for local unemployed workers, such as public works. Another method is to adopt a "buy at home" policy.[4] In the Oates model, local fiscal policy would be successful only if the import leakage were minimized. The size of the multiplier would then depend on the structure and development of the local economy. In this sense, the efficacy of local fiscal policy becomes an empirical question. In the limit, the multiplier approaches 1/b when the government buys no imports.

The Oates model has been regarded as the main theoretical argument against interregional fiscal policy. Yet the appropriateness of the Oates methodology for analysing the long-run properties of the model is under question. There is also criticism of the Oates "proportionality" assumption. His conclusion is basically derived in the context of long-run steady-state equilibrium, and no explicit consideration has been given to the intrinsic dynamics of the system and the stability of such an adjustment process. Oates adds the trade balance equilibrium condition to the model without any derivation of it.

The validity of Oates' conclusions depend crucially on the trade balance equilibrium equation, which we argue is misspecified. We focus on the deficit-financing case (2.3b) only. Since Oates states the trade balance equilibrium equation without any derivation, it is necessary to discuss the exact meaning of this equation.

By substituting (2.3c) and (2.1) into (2.3b), we have:

$$
\begin{aligned}
&(2.3b) \quad && X - IM + D = 0 \\
& && X - IM + (G - T) = 0 \\
& && (G + X - IM) - T = 0 \\
& && (Y - C) - T = 0 \\
&(2.6) && YD - C = 0
\end{aligned}
$$

In footnote 3 of the 1968 paper, Oates argues that it is suffice for purposes of his study to assume the level of taxes as predetermined. and $YD=Y-T$. Thus the trade balance equation is equivalent to saying that the difference between disposable income and total private expenditures must be zero. However, (YD-C) equals savings and can be considered as the change of private wealth $(A-A_{-1})$, which is also the definition of the private-sector budget constraint.[5] Therefore (2.6) implies that in long-run equilibrium the private-sector budget constraint must be satisfied.

One basic characteristic of the one-sector macro-model is that the private (public)-sector budget constraint does not need to appear explicitly in the model since it is implied by the public (private)-sector budget constraint and other relationships of the model.[6] In the Oates model, if (2.3b) is considered as the government budget constraint in long-run equilibrium, then (2.3b), together with (2.1), can imply the private-sector budget constraint (2.6). However, the government budget constraint (2.3b) includes only the trade account and ignores the interest payments and the capital account of the balance of payments. Consequently, the private-sector budget constraint (2.6) also excludes interest income that should be considered as an income component. As we will show in Section 2.4, the stability and the long-run properties of the system under a bond-financing policy, depend crucially on the behavior of this "interest payments" term.

The overall trade and capital accounts relating to the balance of payments will affect the purchase of the foreign exchange so as to maintain the exchange rate. If one argues that maintaining the exchange rate is the sole responsibility of the federal government, then the trade account deficit should not be included in (2.3b), and the federal sector should be introduced in the Oates model. Even in this case, the "interest payments" of the local government sector must be added to the local government budget constraint since it is the local government's obligation to service the debt, and the federal government budget constraint must include the "changes in foreign exchange". Thus we conclude that (2.3b) is not the correct long-run equilibrium condition, even if stability of the system is assumed.

Another inconsistency in the Oates model is in the analysis of the financial asset market. Oates argues that since local governments have no monetary powers, and since the price of bonds in terms of money is fixed by the externally determined rate of interest, it is convenient to aggregate the money and bonds markets into a single financial asset market.[7] The net financial asset holdings (A) equal the sum of money and bonds (M + B).

In (2.2), L(Y,A,R) denotes the total demand function for financial assets (M + B). However, such an aggregate demand function is not warranted because the coefficients representing the responses to the exogenous variables are not the same for money and bond demand functions. Bonds bear interest while money does not. This different characteristic necessitates a separate demand function for each financial asset. In the Oates small local macro-model with perfect capital mobility, only domestic and foreign bonds are perfect substitutes. One can no longer specify separate demand functions for these two types of bonds; they are indistinguishable. Oates also argues that according to Walras' Law, there are only two independent equations in the model, and the financal asset market (M + B) must be redundant. However, our argument suggests that we need to specify only one asset demand function explicitly and that the other will be determined from the stock constraint (A = M + B). Thus only one, not both, of the financial asset demand functions is redundant.

Moreover, Oates argues that the private sector still attempts to establish a portfolio balance between money and bonds, but since this can be done by trading with <u>nonresidents</u> at a fixed price, the model need not explicitly take account of this phenomenon.[8] However, Oates does not define the exact meaning of "nonresidents". They may be either foreigners or residents of other regions of the same country. If residents adjust their portfolios in the foreign capital market, effects on the capital account of the balance of payments, ignored by the Oates model, must be taken into account. If residents adjust their portfolios with other regions within the federation, other regions, which are ignored in the Oates model, must then be considered. Thus Oates suggests the existence of the interregional financial relationships but fails to recognize their importance in the study of interregional fiscal policy.

The Oates model deals with only one local public sector. There are no other regional or federal sectors involved. Judging from the fact that import leakages play an important role in the Oates model, there is no reason why other interregional financial influences or relationships should be excluded. Federal-regional fiscal arrangements also have a great impact in determining the feasibility of interregional fiscal policy. Thus, an interregional model must include these fiscal and financial relationships.

2.3 Financing Interregional Fiscal Policy

The basic argument against interregional fiscal policy is that a region has limited power to finance an expansionary fiscal policy. Since the monetary power is exercised by the federal government, the only alternative for the regional government is to finance a deficit is to issue bonds.

There are several theories explaining that there exists a certain debt-limit which a regional government can reach without incurring an excess burden. One such theory adopted by the Ontario Government says that the ratio of regional debt to regional income should not exceed the combined growth rate of regional income and labor force.[9] The rationale underlying this theory is that a growing population and rising income are the main demand determinants for more regional public goods and social capital facilities. Any evaluation of the burden imposed by debt-financing must therefore be related to the growth of these two factors. The major weakness of this theory is the overemphasis of the demand side and neglect of the supply constraint. The debt-limit would be meaningless if no more credit were available or could be obtained only under unacceptable terms.

Another view of the debt-limit has been proposed by Barth (1977) using the concept of "potential-oriented debt".[10] He argues that the success (or limit) of potential-oriented debt-financing depends on the interest rate being lower than the growth rate of potential output when there is a cyclically neutral rise in the price level. If the interest rate is higher, interest expenditure for the growing public debt will come to exceed the spending made possible by a given amount of credit. In the long run it will not be possible for the government to carry out its fiscal policy.

Using a dynamic aggregate demand and supply model of firms, households and a regional government sector, Swan (1977) employs a portfolio approach to analyse the debt-limit, which is the condition for interregional fiscal policy. He also concludes that fiscal policy in a region would be successful provided that the real rate of interest on outside borrowing is <u>less</u> than the aggregate real growth rate of aggregate output and labor supply in the region.[11]

Swan's model assumes away the federal government sector. Only the regional government sector is presented. Since the regional government has no authority to print new money, Swan assumes households, firms and the regional government issue bonds to the foreign capital market in order to acquire money balances.[12] Trade and capital movements are carried out only between the region and the rest of the world. Swan correctly includes the interest payments in the regional government budget constraint, but the "changes of foreign exchange" term is excluded. Such exclusion is not justified unless the trade surplus offsets the capital inflow in each period, which is not guaranteed in Swan's model.

One advantage of Swan's model is the introduction of the supply side to the system. However, the Swan model, besides the above criticism about the validity of the budget constraints, also commits the common error of assuming that long-run equilibrium exists without addressing the stability question.

One interesting policy implication of the Swan model deserves more discussion. In the Swan model, the market rate of interest is an increasing function of the proportion of interest payments to the outside world relative to regional income. The private sector as well as the public sector can influence the market rate of interest through sales of bonds in the foreign capital market. This implies that the public sector should take action to offset any private sector action unfavorable to regional stabilization objectives.

To improve the realism of the model in a federal state setting, it is necessary to add one more region to the Swan model. By virtue of the perfect capital mobility assumption, there is only one market rate of interest prevailing in both regions. It follows immediately from the Swan model that a bond-financing deficit in any region will have monetary effects on the others, notably by inducing a higher rate of interest. Whether these effects are desirable or detrimental to the other regions, either in the short run or the long run, will depend on the structure of the interregional economy.

These spillover or external effects are not confined only to other regions. The federal government would also be affected. Wilson (1977) argues that in a fixed exchange rate system, the sale of regional government bonds to finance the deficit in the foreign capital market would require the central bank to purchase foreign exchange in order to maintain the exchange rate. Therefore, a fixed exchange rate regime does put the central bank in the position of having to respond to regional action.[13]

In practice, regional governments do not follow the above theories in framing their fiscal and debt management policies. For example, Ontario Government pegs a constant ratio of net debt to gross regional product as the debt-limit.[14] This ratio or ceiling then becomes a generally accepted expression of the region's indebtedness and a measure of the tone of fiscal management. However, maintaining the debt ceiling would imply that in a recession, as the economy slows down, the regional government must reduce the bond-financing deficit. This may be a sound policy in terms of financial management, but it is not countercyclical from a stabilization perspective.

Recently, various proposals have been made for increasing Canadian regional fiscal autonomy. One interesting proposal suggests that regional governments have limited access to the central bank for the purpose of financing their deficits.[15] To share the monetary power between the federal and the regional governments within confederation is an innovative idea in the theory of fiscal federalism. Devising such a scheme acceptable to all governments and conducive to national and regional objectives would involve a greater degree of fiscal coordination. Nonetheless, such a scheme would give regional governments more scope and flexibility in pursuing their fiscal policies.

Despite the possibility of limited access to the money supply, issuing bonds is still the main source of financing regional deficits. Recognizing the intrinsic instability of bond financing, and the fiscal and financial interrelationships among various levels of government, interregional fiscal policy takes on a new dimension.

2.4 Economics of Bond-Financing

The concept of a "government budget constraint (GBC)" was first introduced by Ott and Ott (1965) and Christ (1967). The GBC is the requirement that the uses of government funds must equal the sources of government funds. In other words, total government expenditures must equal total government tax and other revenues, including newly printed money and newly issued bonds.

The GBC can also be considered as a relationship between the stocks of financial assets and flows of claims on output. Being a stock-flow relationship, it imposes a dynamic structure on the system even if the underlying behavioral relationships are static. The analysis of the stability condition then becomes a vital element in assessing the long-run or steady-state effects of any policy changes. If the system is such that, once disturbed from its initial equilibrium, it cannot converge to a new steady state, i.e., the system is "unstable", then it is inappropriate to discuss the comparative statics or long-run effects of any policy changes.

Consider the following simple GBC:

(2.7) $\dot{M} + \dot{B}(1/R) = G + B - u(Y + B)$

Assuming all bonds are consols paying one dollar per period, then (1/R) is the price of a bond where R is the market rate of interest, and the revenue to the government from a new bond issue is $\dot{B}(1/R)$. The dot over a variable indicates a change of stock between two consecutive periods. Also B represents both the number of bonds and the interest on the bonds. The left-hand side of (2.7) shows the sources of government funds. The government can either print new money $\dot{M}$ or issue new bonds $\dot{B}$. The right-hand side of (2.7) shows the uses of government funds which is expressed in terms of a budget deficit. The total expenditures consist of expenditures on goods and services (G) and debt interest payments (B). The tax revenue is obtained by applying the proportional tax rate (u) to total factor and interest income.

One implication of (2.7) is that, depending on the mode of financing, either $\dot{M}$ or $\dot{B}$, or any combination of them, must become an endogenous variable. The government, in setting and financing its fiscal policy, cannot determine the time path of either $\dot{M}$ or $\dot{B}$ independently; it must be determined in the context of the whole system (not specified here). Whenever the system evolves such that the budget is in deficit or surplus, then either $\dot{M}$ or $\dot{B}$ must change to satisfy the GBC.

Let us assume that the system, which is not specified here, is so simple that the only dynamic equation is the GBC. Further, let us define the system to be in long-run equilibrium when all the changes of endogenous variables are zero and all exogenous variables are constant. Then this requires that either $\dot{M}$ or $\dot{B}$ be zero, depending on the mode of financing. It means that in the long run government must balance its budget. Note that in some dynamic models the steady-state does not require a balanced budget.[16]

In our simple system, consider a once-and-for-all increase in exogenous government expenditures. If the condition that either $\dot{M}$ or $\dot{B}$ is zero and the other stated conditions are satisfied, then the system is said to be in "stable long-run equilibrium". This can be illustrated more clearly by the following conditions:

For pure money-financing (i.e., $\dot{B} = 0$)

(2.8) $d\dot{M}/dM < 0$ Stable

(2.9) $d\dot{M}/dM > 0$ Unstable

For pure bond-financing (i.e., $\dot{M} = 0$)

(2.10) $d\dot{B}/dB < 0$ Stable

(2.11) $d\dot{B}/dB > 0$ Unstable

In words, the condition says that the stability of the system depends on whether the financing of the deficit would reduce the further need to finance the deficit. Since bonds are the main source of deficit financing in regional governments, we therefore concentrate on the discussion of the stability of bond-financing.

Assuming pure bond-financing and differentiating (2.7) with respect to B, we have:

(2.12) $d\dot{B}/dB = R(1-u((dY/dB)+1)) + (G+B-u(Y+B))(dR/dB)$

$= R(1-u((dY/dB)+1))$ in the neighborhood of equilibrium .

Thus the necessary and sufficient condition for local stability is:

(2.13) $1 - u((dY/dB)+1) < 0$

and this implies

(2.14) $dY/dB > (1 - u)/u$

If the proportional tax rate is 0.25, for example, then dY/dB must be greater than 3 in order for the system to be stable. Whether condition (2.14) is satisfied would certainly depend on the structure of the model. Thus Blinder and Solow and others argue that stability of bond-financing is an empirical question.[17]

One particular characteristic of bond-financing deserves more discussion. The government has an obligation to service the ongoing interest payments on the debt. This is the term B in the GBC. Due to this basic nature of bond-financing, stability condition (2.14) may be difficult to satisfy. An intuitive explanation can demonstrate the intrinsic instability of bond-financing. Assuming a once-and-for-all increase in exogenous expenditures, there will be a rise in national income through a multiplier effect. However, tax revenues increase by less than government expenditures as long as the marginal propensity to spend and marginal tax rate are less than one. Therefore, bonds must be issued to cover the budget deficit. This causes a further increase in national income and in the budget deficit. National income increases because households spend a fraction of their interest income. The budget deficit increases since the interest obligations go up by the increase in B while tax revenue goes up only by a fraction of the increase in B. Therefore more bonds must be issued indefinitely. There may be some set of parameter values such that the system can generate a large u((dY/dB)+1)dB effect to offset the increase in interest payments dB. Otherwise the system is unstable. This is the main reason for arguing that stability of bond-financing is an empirical question.

Specification of the structural model would certainly alter the requirements for stability under bond-financing. For example, Blinder and Solow (1973) use a simple model in which the price level and capacity output are asssumed constant. They claim that empirical evidence for the United States suggests that the model under bond-financed fiscal policy is stable.[18] However, Scarth (1976) points out an error in the stability condition reported by Blinder and Solow (1973), and demonstrates that the crowding-out effect cannot be comfortably dismissed as Blinder and Solow have suggested.[19] Tobin and Buiter (1976) also show that stability under bond-financing in the Blinder and Solow model (1973) is possible only under rather extreme assumptions.[20]

In their extended model, with full employment, flexible price and allowance for capacity growth as a result of capital accumulation, Tobin and Buiter (1976) and Buiter (1977) analyse three simple mechanisms for generating price expectation. They argue that bond-financed fiscal policy is unstable in cases of static expectations. Further, it may be potentially but not necessarily stable in cases of myopic perfect foresight. In the intermediate case of adaptive expectations, stability requires, but is not guaranteed by, a finite minimum speed of adaptation.[21]

Scarth (1975,1977) assumes productive capacity is fixed and examines fiscal policy under alternative exchange rate systems. He finds that the system is likely non-convergent, no matter which exchange rate policy is undertaken.[22] When endogenous fiscal policy is introduced in the simple model, Scarth (1979) argues that bond-financed fiscal policy is likely to be unstable.[23]

Christ (1978) and Niehans (1978) analyse a steady-inflation equilibrium. They find that the system is unlikely to be stable when bonds are the endogenous variable.[24]

Pyle and Turnovsky (1977) present an intermediate-run model in which the capacity effect of investment is included. They argue that the best way to incorporate this effect is to respecify the system in a per-unit-of-capital form. They find that pure bond-financing is much more likely to be unstable than is a pure money-financed budget. They also argue that the stability problem under bond-financing is an open empirical question.[25]

One extensive search for the parameter set that can satisfy the stability condition under bond-financing has been carried out by Nguyen and Turnovsky (1979). Their model incorporates capacity output and endogenizes the price variable. They first set up the possible range of parameter values and variables for a hypothetical economy. To test the global stability of the system under various policies, they choose about 120,000 samples involving different combinations of parameter values. They compare not only the effects of alternative policy specifications, but also the roles of several key parameters. Only about 10 percent of all the cases considered ensure stability under bond-financing. They also find some interesting conditions which will increase the possibility of stability under bond-financing:[26]

(1) the wealth effect in the real expenditure function is positive;

(2) the wealth effect in the demand-for-money function is zero or very small and the income effect in the same function is large;

(3) the fixed real stock of money is small.

However, Nguyen and Turnovsky conclude that increases in the wealth effect in the demand for money function make the system very unstable under bond-financing. Their finding supports the proposition discussed by Turnovsky (1979) that a positive wealth effect in the demand for money function exerts a destabilizing effect under bond-financing, while a positive wealth effect in the real expenditure function is necessary for stability under the same policy.[27]

The main source of instability, as pointed out before, is the ongoing payment of interest. One possible way to get rid of this source of instability is to adjust the ongoing interest payments in such a way that the total government expenditures, including the interest payments, are kept constant. As the deficit decreases, the system moves towards equilibrium. Buiter (1977) uses the following government expenditure variable (G_1), which consists of the government expenditures on goods and services and the government interest payments net of tax ($(1-u)B$).

(2.15) $G_1 = G + (1-u)B$

Rewriting GBC in terms of G_1, we have:

(2.16) $\dot{B}(1/R) = G + B - u(Y+B)$

$= G_1 - uY$

However, Christ (1979) argues that this assumption will not bring stability to the system.[28] But Christ is incorrect in arguing that the deficit (G_1-uY) does not change as more bonds are issued. If G_1 is held constant and more bonds are issued to finance the deficit, then the national income (Y) may increase due to an increase in the stock of financial wealth and the associated wealth effect. On the other hand, the negative effect of larger bond issues in the national income identity will decrease the national income. Thus Y may change, and consequently the magnitude of the deficit may change.

Christ (1979) suggests another form of government expenditure variable which can increase the speed of reduction of the deficit, and consequently increase the possibility of stability under bond-financing. He defines his government expenditure variable (G_2) as the sum of expenditures on goods and services and the debt interest payments gross of tax.[29]

(2.17) $G_2 = G + B$

Rewriting the GBC in terms of G_2, we have:

(2.18) $\dot{B}(1/R) = G + B - u(Y+B)$

$= G_2 - u(Y+B)$

In this case, when bonds are issued to finance a deficit with G_2 constant, the term $-uB$ decreases algebraically, which decreases the deficit and moves the system towards equilibrium.

Applying the modified G_2 method in the Blinder and Solow model, Christ concludes that bond-financing is definitely stable. While he cannot establish stability conclusively in the Tobin-Buiter static expectations model, he weakens the Tobin-Buiter conclusion that the stability determinants are negative.

Using Christ's G_2 method (called the Christ adjustment method, hereafter) gives some hope for stability under bond-financing. Of course, the cost is that the effectiveness of fiscal policy must be reduced due to the adjustment of government expenditures on goods and services required to counteract any increase in interest payments. Rewriting (2.17), we have:

(2.19) $\quad G = G_2 - \dot{B}$

Since G_2 is held constant, any increase in B will be offset by an equal decrease in G. Consequently aggregate demand will be decreased and the national income will be affected adversely.

This has an important implication for the discussion of interregional fiscal policy. The conventional wisdom regarding interregional fiscal policy is rather negative in that, while the policy is feasible, it is not advisable for a regional government to pursue any sort of bond-financed fiscal policy, simply because the system is unstable. It is therefore not surprising that regional governments are included in the private sector in the literature of macro-model and government budget constraints, which completely eliminates the possibility and responsibility of regional government in regard to the pursuit of fiscal policy.[30] To exaggergate the point, it implies that there is no need for a regional government to be held accountable for any cyclical change in the regional economy; it is the federal government that must be held responsible.

To include regional governments in the private sector for the reasons of instability of bond-financing and lack of recourse to the central bank is totally unjustified. In view of the fact that in Canada, for example, regional government budgets are far greater than the budget of the federal government, any discussion of fiscal policy which ignores the regional governments is unrealistic. When a regional government sector is included in the model, one can foresee that the requirement to satisfy more than one government budget constraint will change the course of analysis of fiscal policy.

2.5 Conclusion

In this chapter we have examined and criticized the validity of the Oates model in terms of methodology and specification. Using the concept of the private and public sector budget constraints, we argue that the "trade balance equation" in the Oates model is misspecified. We emphasize the importance of considering the dynamics of the system in determining its stability and long-run properties. This approach is completely ignored in the Oates model, which emphasizes only the long-run equilibrium properties without asking whether the system is stable. The same methodological error is observed in the Swan model.

The financing aspect of an interregional fiscal policy and the related topic of the economics of bond-financing have been reviewed. The importance of the Christ adjustment method in solving the instability problem of bond-financing has also been discussed.

Footnotes to Chapter 2

(1). The Oates 1968 paper is developed from a paper written jointly by McKinnon and Oates (1966). The 1966 paper deals with a small national economy. In the 1968 paper, Oates uses the same structural model and notation, except that the national public sector is changed to a local public sector.

(2). Oates (1968), p.43.

(3). In the case of bond-financing deficit, the Oates model becomes:

(2.20) $Y = C(YD, A, R) + G + X - I(YD, A, R, G)$

(2.21) $X - I(YD, A, R, G) + G - T = 0$

where the variables are defined as follows:

Y—Regional Income
YD—Disposable Income
C—Private Expenditure
G—Public Expenditure
A—Financial Asset
T—Taxes
R—Rate of Interest
X—Exports
I—Imports

Assume: $dI/dA = b(dC/dA)$
$dI/dYd = b(dC/dYd)$
$dI/dG = a$

Differentiate the system and solve to obtain:

$$dY/dG = ((dI/dG)(dC/dA) - dC/dA)/(-dI/dA)$$
$$= (a(dC/dA) - (dC/dA))/(-b(dC/dA))$$
$$= (1 - a)/b$$

If government buys no imports (i.e., $a = 0$), then $dY/dG = 1/b$.

(4). For example, Ontario and Quebec have a purchasing policy favoring local firms. In Quebec, the policy provides that unless the lowest bid by a Quebec firm is substantially higher than the lowest bid by non-Quebec firms, the Province will buy tangible goods from the Quebec firm. In Ontario, a ten percent preference is given to Canadian firms. See Trebilcock (1977), pp.105-106.

(5). Since the rate of interest is assumed exogenous in the Oates model, there would be no capital gain or loss in the private sector budget constraint.

(6). See Scarth (1975), pp.13-14.

(7). Oates (1968), p.39.

(8). Ibid., p.39.

(9). Barber (1966), p.60 and p.71. See also <u>The Ontario Economy 1977-1987</u>, p.40.

(10). Barth (1977), pp.12-22. This is an associated concept derived from the Cyclically Neutral Budget(CNB). According to the concept of CNB, government borrowing is cyclically neutral in so far as it is necessary to finance that part of the CNB which is not covered by taxes and other revenues.

(11).Swan (1977), p.18.

(12).Ibid., p.5.

(13).See footnote 8 of Chapter 1.

(14).See Ontario Budget 1978 , pp.26-27.

(15).Auld (1979), p.281.

(16).For example, a dynamic steady-state growth equilibruim path for aggregate real variables and a constant price level would require steady-state growth of nominal stocks and flows at a rate approximately the sum of the growth rates of real variables and prices. See Turnovsky (1978).

(17).Blinder and Solow (1973), p.316.

(18).Ibid., p.316.

(19).Scarth (1976), p.387.

(20).Tobin and Buiter (1976), p.290.

(21).Buiter (1977), p.354 and Tobin and Buiter (1976), p.290. They consider the following three simple mechanisms for generating price expectation: (a). Static expectation $x(t) = 0$; (b). Myopic perfect foresight $x(t) = \dot{p}(t)/p(t)$; (c). Adaptative expectation $\dot{x}(t) = b(\dot{p}/p-x(t))$, $b > 0$; where x denotes the expected instantaneous proportional rate of change of the price level p.

(22).Scarth (1977), p.150.

(23).Scarth (1979), p.108.

(24).Christ (1978), p.69.

(25).Pyle and Turnovsky (1977), p.427.

(26).Nguyen and Turnovsky (1979), p.270.

(27).Turnovsky (1979), p.41.

(28).Christ (1979), p.534.

(29).Ibid., p.534.

(30).Christ (1979), p.526.

Chapter 3

Analysis Based on a National Model

3.1 Introduction

We have reviewed, in the last chapter, some of the concepts, theories and issues of interregional fiscal policy, and discussed the financial implications of bond-financed fiscal policy. In particular, we emphasized the requirement for satisfying more than one government budget constraint when two or more levels of government are involved.

In this chapter we construct a simple aggregate National Model. This model is used as a reference model from which an interregional model can be developed. The simulation approach is also introduced. The numerical findings based on the analysis of bond-financed fiscal policy with one public sector provide some information beyond what we have reviewed in the existing literature. However, readers who are familiar with the literature can skip this chapter without loss of continuity.

In the next section of this chapter we present an overview of the National Model, and indicate different versions associated with bond or money-financing policy. The subsequent section provides a detailed analysis of fiscal policy. Problems of formal stability analysis and the need for a simulation approach are also discussed. In section 3.4 we describe the various steps in a simulation analysis and discuss some of the results obtained. Section 3.5 introduces the Christ adjustment method, and analyses the results of applying this method to the bond-financing case. Section 3.6 presents an argument for introducing the regional government sector into the model. Section 3.7 gives a summary statement of the conclusions of the chapter.

3.2 The National Model

The National Model is a simple dynamic, closed-economy macro-model. The notation is defined immediately following the equations, and the main features of the structural model are then explained.

(3.1) $$Y = E + G$$

(3.2) $$E = E(YD, W, R)$$

(3.3) $$YD = (1 - u)(Y + TR + RB)$$

(3.4) $$M = L(Y, W, R)$$

(3.5) $$W = M + B$$

(3.6) $$\dot{M} + \dot{B} = G + TR + RB - u(Y + TR + RB)$$

(3.7a) $\dot{M} = 0$ federal pure bond-financing policy
(3.7b) $\dot{B} = 0$ federal pure money-financing policy

Notation :

B — Value of outstanding government bonds[1]
E — Private expenditures
G — Government expenditures on goods and services
L — Demand for money
M — Money supply
R — Rate of interest
TR— Transfer payments
u -- Proportional tax rate
W — Private financial wealth
Y — Gross national product
YD— Private disposable income
• -- Dot above variable denotes derivative with respect to time

Endogenous variables: Y, E, YD, R, W, B, M.

Predetermined variables: G, TR, B, M.

Predetermined parameter: u

The assumptions about the partial derivatives are as follows:

$$0 \leqslant E_{YD} \leqslant 1 \qquad E_R \leqslant 1 \qquad E_W \geqslant 0$$

$$L_Y \geqslant 0 \qquad L_R \leqslant 0 \qquad 0 \leqslant L_W \leqslant 1$$

Equation (3.1) describes the equilibrium condition in the product market, or the national income identity. We follow Oates' assumption that the level of output Y is perfectly elastic at the given price level (for simplicity, assume unitary price level) and that it adjusts, in a Keynesian fashion, to the level of aggregate demand. Equation (3.2) is the private expenditure function; private expenditure increases with private disposable income and wealth, and decreases with the rate of interest. Private disposable income is defined in equation (3.3) as income received from production (i.e., factor income), plus transfer payments and interest income on government bonds, minus proportional taxes.[2] We assume that only the government issues bonds and only the private sector holds bonds.

Equation (3.4) is the equilibrium condition in the money market. It states that the demand for money balances depends upon the level of income, the rate of interest and the stock of wealth. The demand determinants reflect the transaction, speculative and portfolio allocation effects. The amount held for transaction purposes depends mainly on the aggregate income level Y, but will depend also upon the cost of holding money, i.e., the rate of interest. Likewise, the amount held for speculative purposes depends mainly on the rate of interest but varies with income. The wealth variable reflects the portfolio framework in which the demand for money is considered along with other financial assets.

Financial wealth is defined in equation (3.5) as consisting of the stock of money and outstanding government bonds. Because of this stock constraint, only one financial market need be considered, and as usual we focus our attention on the money market. The wealth constraint equation (3.5), together with the assumption of equilibrium in the money market (3.4), implies that residents own precisely their desired holdings of bonds.

Equation (3.6) is the government budget constraint (GBC) which determines the dynamics of the model. The GBC stipulates that the sources and uses of government funds must be in balance. The uses are expenditures on goods and services, transfer payments and interest payments on outstanding bonds. The sources are current tax revenues and positive growth in the money supply and in the stock of bonds.

To complete the model we must specify the government financing policies. We have chosen to consider the two policies. Equation (3.7a) defines a pure bond-financing policy; we set $\dot{M} = 0$ and let $\dot{B}$ be determined endogenously. Equation (3.7b) describes a pure money-financing policy; we set $\dot{B} = 0$ and let $\dot{M}$ be determined endogenously.

The GBC has three important effects on the model:

First, the financial flows ($\dot{M}$ and $\dot{B}$) not only imply claims on current output but also affect the financial stocks (M and B) in later periods. Being a relationship between the stock of financial assets and the flow of claims on output, the GBC imposes a dynamic structure on the system even though the underlying behavioral relationships are static. The discussion of stability conditions, long-run multipliers, and the nature of equilibrium becomes indispensible in any analysis of policy changes. The effectiveness of any policy change depends on the mode of financing which directly affects the stability condition of the system.

Second, with the incorporation of the GBC equation, one of the policy variables must be determined by the joint action of the GBC and rest of the structural model. Consider the case of a once-and-for-all increase in G financed by a new money supply. Such a change specifies that two policy variables, G and $\dot{M}$, must be changed initially, and also that $\dot{M}$ is endogenous in the system. Whenever the system changes in such a way that the government budget is not balanced, $\dot{M}$ must also be changed in order to satisfy the GBC.

Third, the presence of the GBC raises the question of whether a finance constraint should be imposed on the private sector in order to have a consistent specification of the behavior of both sectors in the economy. The finance constraint on the private sector does exist implicitly in the model. This can be demonstrated as follows: Using equation (3.1) and substituting G into (3.6), we have:

$$
\begin{aligned}
(3.8) \qquad \dot{W} &= \dot{M} + \dot{B} \\
&= G + TR + RB - u(Y + TR + RB) \\
&= (Y - E) + TR + RB - u(Y + TR + RB) \\
&= (Y + TR + RB) - E - u(Y + TR + RB) \\
&= (1 - u)(Y + TR + RB) - E \\
&= \text{private saving}
\end{aligned}
$$

Equation (3.8) shows that a change of private wealth is equal to the difference between disposable income and expenditures, which is equivalent to saying that a change of private wealth is equal to private saving. This is the financial constraint on the private sector, and is already implied by the model. Thus there is no need to add another constraint. By virtue of (3.5), the GBC also determines the rate of wealth accumulation by the private sector.

Following Oates (1968), Christ (1978,1979) and Turnovsky (1978,1979), we make the simplifying assumption of abstracting from the dynamics of physical capital accumulation in our model. Pyle and Turnovsky (1977) have argued that the procedure of including the capital stock by simply adding the identity relating the rate of capital accumulation to investment is somewhat unsatisfactory. To incorporate capital adequately involves complicating the model significantly by explicitly introducing a labor market, production function, capital-labor substitution, firms' financial constraint, and so forth. They argue that the results are quite similar to those obtained from a more complex model, and therefore little generality would be lost in abstracting from such consideration.[3] In view of the possible complexities we would face when restructuring the National Model into an interregional model, we adopt this simplifying assumption of abstracting from the dynamics of capital accumulation in order to keep our analysis as tractable as possible.

3.3 Study of Fiscal Policy

The usual approach in macroeconomic analysis of fiscal policy is to consider the static (short-run) and the steady-state (long-run) equilibria, and to use conventional comparative static techniques to analyse the short-run and long-run effects of various policy changes. Stability conditions must be satisfied in order to be in long-run equilibrium. Assuming a once-and-for-all increase in government expenditures, the analysis of fiscal policy is presented as follows:

3.3.1 Static System

In the short run, all the stock variables (M, W, B) are constant and the dynamic equation does not come into play in the model. Further simplifying the system through substitutions, the short-run system, (3.1) to (3.5), then reduces to two equations:

$$Y = E((1-u)(Y + TR + RB), W, R) + G \tag{3.9}$$

$$M = L(Y, W, R) \tag{3.10}$$

Equation (3.9) corresponds to the IS curve and equation (3.10) to the LM curve of the system. This short-run system determines endogenous variables Y and R in terms of predetermined variables and parameter G, TR, u, B, M and W. ((3.11) and (3.12), respectively):

$$Y = Y(G, TR, u, B, M, W) \tag{3.11}$$

$$R = R(G, TR, u, B, M, W) \tag{3.12}$$

By differentiating the static system and solving for multiplier effects, we have (see Appendix 3.1):

$$dY/dG = \frac{1}{1-E_{YD}(1-u)(1-B(L_Y/L_R))+E_R L_Y/L_R} \tag{3.13}$$

$$dR/dG = \frac{-L_Y/L_R}{1-E_{YD}(1-u)(1-B(L_Y/L_R))+E_R L_Y/L_R} \tag{3.14}$$

$$dY/dM = \frac{E_W+(1-L_W)((E_{YD}(1-u)B+E_R)/L_R)}{1-E_{YD}(1-u)(1-B(L_Y/L_R))+E_R L_Y/L_R} \tag{3.15}$$

$$dR/dM = \frac{1-L_W-L_Y(dY/dM)}{L_R} \tag{3.16}$$

$$dY/dB = \frac{E_W-(E_R L_W/L_R)+E_{YD}(1-u)(R-(BL_W/L_R))}{1-E_{YD}(1-u)(1-BL_Y/L_R)+E_R L_Y/L_R} \tag{3.17}$$

$$(3.18)\quad dR/dB = -\frac{L_W(1-E_RL_Y/L_R)/L_R + L_Y(E_W+E_{YD}(1-u)(R-BL_W/L_R))/L_R}{1-E_{YD}(1-u)(1-BL_Y/L_R)+E_RL_Y/L_R}$$

An examination of the partial derivatives leads us to expect that the signs of the multiplier effects will be as follows:

Table 3.1 The National Model
Signs of Short-Run Multiplier Effects

Derivative of	With respect to G	M	B
Y	+	+	?
R	+	-	+

3.3.2 Long-Run System

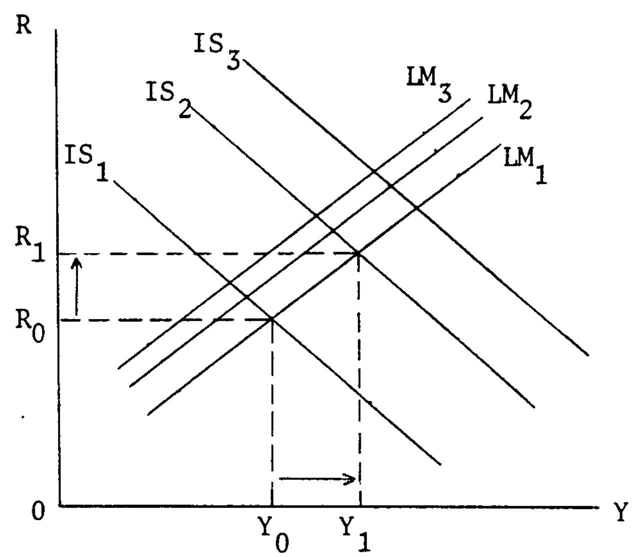
In the long run, we must consider the full model, including the GBC. Changes of M or B would affect the financial stock at later periods; interest payments affect not only private disposable income, but also the GBC itself. The long-run equilibrium is then defined as the condition that exists when all changes of endogenous variables are zero, and all predetermined variables remain constant. It thus requires the government budget to be balanced.

The concept of "stability" and the dynamic adjustment process can best be illustrated by using the IS-LM framework. Consider the case of a once-and-for-all increase in government expenditures. In Fig.3.1, the deficit is financed by issuing new bonds. In the first period, the IS curve shifts from IS_1 to IS_2 due to expansionary fiscal policy.
In the second period, the IS curve shifts further rightward, to IS_3, because government expenditure stays at a higher level and disposable

income is increased because of higher interest income obtained from the bond-financed fiscal policy. The magnitude of the rightward shift of IS mainly depends on the magnitude of the deficit. In each subsequent period, if the budget is not balanced, the IS curve will shift further to the right. Stability requires that each subsequent shift of the IS curve be smaller than the previous one.

Figure.3.1

Bond-Financed Fiscal Policy in the National Model

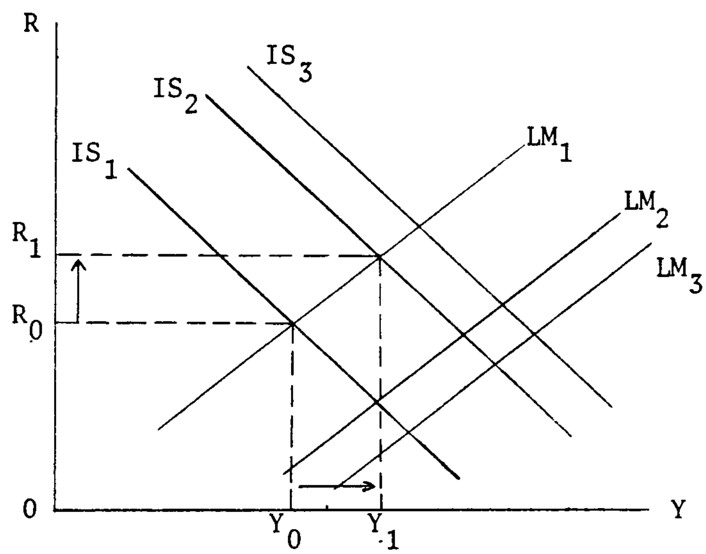


On the other hand, greater private wealth resulting from new bond issuing will increase the demand for money, at any level of income and interest rate, in the second and subsequent periods. Therefore the LM curves will shift leftward. Stability requires that each shift of the IS curve dominates each shift of the LM curve in terms of movement to the right, and that each subsequent shift of the LM and IS curves be smaller than the previous one. The budget deficit must decline over time and the system will approach a long-run equilibrium in which the budget is balanced.

When the deficit is financed by printing new money, the adjustment process is represented by Fig.3.2. In the first period, the IS curve shifts from IS_1 to IS_2. Thus the short-run effects are the same regardless of the mode of financing. However, in the second period, the money supply must be increased to finance the deficit, and hence the LM curve shifts rightward to LM_2. In each subsequent period, if the budget is not balanced, the IS and LM curves will both shift rightward.

Figure.3.2

Money-Financed Fiscal policy in the National Model



Stability requires that each subsequent shift of the IS-LM curves be smaller than the pervious one. The budget deficit must decline over time and the system will again approach a long-run equilibrium in which the budget is balanced.

Applying the stability critera defined by equations (2.8) to (2.11), we find that, under pure money-financing,

(3.19) $\dot{M} = G + TR + RB - u(Y + TR + RB)$

By defferentiating (3.19) with respect to M, we have

(3.20) $\partial\dot{M}/\partial M = B(\partial R/\partial M) - u((\partial Y/\partial M) + B(\partial R/\partial M))$

$$= B(1 - u)(\partial R/\partial M) - u(\partial Y/\partial M) < 0$$

From the comparative static results, we can be sure that $(\partial\dot{M}/\partial M)$ is negative. Thus, the system is stable under pure money-financing. Under pure bond-financing,

(3.21) $\dot{B} = G + TR + RB - u(Y + TR + RB)$

By differentiating (3.21) with respect to B, we have:

(3.22) $\partial\dot{B}/\partial B = B(\partial R/\partial B) + 1 - u((\partial Y/\partial B) + B(\partial R/\partial B) + R)$

$$= 1 + (1 - u)B(\partial R/\partial B) - u((\partial Y/\partial B) + R) \lesseqqgtr 0$$

From the comparative static result, we still cannot sign $(\partial\dot{B}/\partial B)$. Stability requires that

(3.23) $1 + (1-u)B(\partial R/\partial B) - u((\partial Y/\partial B) + R) < 0$

(3.23) can be expressed as follows:

(3.24) $\partial Y/\partial B > ((1/u) - R + (1/u)(1-u)B(\partial R/\partial B))$

Thus, (3.24) is the necessary and sufficient condition for local stability under bond-financing policy. However, the actual $(\partial Y/\partial B)$ depends on the structure of the whole system. It is for this reason that Blinder and Solow argue that stability under bond-financed fiscal policy is an empirical question.[4]

Provided that the system is stable, the long-run system is our full model together with the stability condition that $\dot{M}$ (or $\dot{B}$) equals zero. Under a pure money-financing policy, the long-run system becomes

(3.1) $Y = E + G$

(3.2) $E = E(YD, W, R)$

(3.3) $YD = (1 - u)(Y + TR + RB)$

(3.4) $M = L(Y, W, R)$

(3.5) $W = M + B$

(3.25) $0 = G + TR + RB - u(Y + TR + RB)$

Endogenous variables: Y, E, YD, R, M, W.

Predetrmined variables: G, TR, B.

Predetermined parameter: u

Under a pure bond-financing policy, the long-run system is the same as under a pure money-financing policy, except that now B becomes an endogenous variable and M changes to a predetermined variable.

Endogenous variables: Y, E, YD, R, B, W.

Predetermined variables: G, TR, M.

Predetermined parameter: u

Our theoretical analysis does not differ from the Blinder and Solow model (1973) except that we assume one period bonds instead of consols.

Without resorting to the simulation, two qualitative conclusions can be drawn:

(1). Money-financed fiscal policy is stable but stability under bond-financed fiscal policy is an empirical question.

(2). When the system is stable, the long-run multiplier of a bond-financed fiscal policy is greater than the long-run multiplier of a money-financed fiscal policy.

The exact time profile of the adjustment process and the effects of different structural coefficients on the stability can only be obtained through simulation analysis (e.g., Table 3.2).

3.3.3 Need for a Simulation Approach to the Analysis of Fiscal Policy in the National Model

The formal approach of calculating the short-run and long-run effects of policy change has the following limitations:

First , it is difficult to sign the comparative static results when the model becomes more complicated.

Second , except for the simplest model, involving only a first or second-order dynamic system, stability analysis is a complicated exercise. Even when one can write down the formal stability conditions, one may not be able to give a definite answer to the question of the actual stability of the system..The complexities of formal stability analysis do not allow us to give the formal stability conditions a simple intuitive interpretation.

Third , stability analysis of the dynamic system gives us little information about the time profile of the adjustment path. The speed of adjusting from one steady state to another is of prime interest to the policy decision-maker, since it indicates the effectiveness of the policy. Though the steady-state equilibrium may take longer to be achieved, we still want to know the effects at different points in time after the initial policy change, and to compare these with the long-run effects. Yet the conventional method provides little information about the speed of the adjustment process.

One possible method of tracing the time profile of the adjustment process is by the application of simulation techniques which can be used to supplement the formal comparative static method. The following sections describe a simulation approach to the investigation of the time profile in a dynamic macro-model.

3.4 A Simulation Approach to the Analysis of Fiscal Policy in the National Model

In this section we describe the detailed steps involved in applying simulation techniques to analyse the effects of fiscal policy. The procedure involves the specification of the model in discrete time. Functional forms must be specified, initial conditions prescribed, and values assigned to the parameters and exogenous variables of the model.

3.4.1 The National Model in a Discrete Time Framework

To facilitate computer simulation, our continuous National Model must be rewritten for discrete time periods:

(3.26) $Y_t = E_t + G_t$

(3.27) $E_t = E(YD_t, W_t, R_t)$

(3.28) $YD_t = (1-u_t)(Y_t + TR_t + R_t B_t)$

(3.29) $M_t = L(Y_t, W_t, R_t)$

(3.30) $W_t = M_t + B_t$

(3.31) $DM_t + DB_t = G_t + TR_t + R_t B_t - u_t(Y_t + TR_t + R_t B_t)$

(3.32) $DM_t = M_{t+1} - M_t$

(3.33) $DB_t = B_{t+1} - B_t$

Federal financing policy options:

(3.34a) $DM_t = 0$ federal pure bond-financing policy

(3.34b) $DB_t = 0$ federal pure money-financing policy

The subscript t denotes time period in the case of flow variables or rates and beginning of time period in the case of stocks. We assume that all decisions are made at the beginning of a period.

The full National Model in discrete time consists of equations (3.26) to (3.34), in which the endogenous and predetermined variables are as follows:

Endogenous variables: $Y_t, E_t, YD_t, R_t, W_t, DM_t, DB_t, M_{t+1}, B_{t+1}$

Predetermined variables: G_t, TR_t, u_t, M_t, B_t.

The general character of the National Model is the same in discrete time as in continue time. The GBC (3.31) is precisely analogous to equation (3.6) in the continuous-time model. The government will print new money DM_t or issue new bonds DB_t in the next period after the policy change. This time lag reflects the fact that in the initial period, the endogenous variables are the same regardless of the financing policy. The analogy also carries over to the private-sector budget constraint, which as we have demonstrated, is implicit in the model.

3.4.2 Specification of Functional Forms

For simulation, we have to specify the functional forms for the private expenditure and demand-for-money functions. Of course, each of these relationships could assume many specific forms but for our purposes a simple form is sufficient. Thus we assume that both the private expenditures, imports, and demand-for-money functions are linear. The analysis of micro-foundation of these macro-functions is presented in Appendix 3.2.

3.4.3 Specification of Initial Values

For the initial conditions, we can assign any arbitrary values to the variables of the hypothetical economy, as long as the values are internally consistent. However, we consider it is interesting, for illustrative purposes, to employ Canadian data. The following initial values are based on averages of Canadian data for the five-year period

1970-1974. (The purpose of taking a five-year average is to level out cyclical fluctuations.) Since our National Model is a simple closed-economy type, it is therefore necessary to eliminate the net exports, though the actual 1970-1974 average of net exports is relatively small. Our approach is to allocate net exports to private and public expenditures pro rata.[5] The GNP is normalized to 100 and the other variables are adjusted in the same proportion. The resulting initial values are as follows:[6]

$Y_t = 100.0$	$E_t = 80.0$	$G_t = 20.0$
$YD_t = 80.0$	$TR_t = 15.6$	$R_t = 0.05$
$M_t = 31.5$	$B_t = 75.5$	$W_t = 107.0$
$u_t = 0.33$	$DM_t = 0.0$	$DB_t = 0.0$

We assume that the budget is balanced in the initial period, and the proportional tax rate is chosen so as to satisfy this requirement. For the financial variables, we assume the rate of interest is 5 percent. We assume that only the government issues bonds, and only the private sector holds bonds. It may be pointed out that in the closed-economy model with a balanced budget in the initial equilibrium, private disposable income should equal to private expenditure.[7]

3.4.4 Specification of Parameter Values

We have to assign parameter values to the private expenditure and demand-for-money functions. A range of possible parameter values will be considered. Our approach will be to consider first the most appropriate values and then to specify additional values which are above and below the original ones.

Existing literature provides us with valuable information about these parameter values. Two early papers by Christ (1967,1968) and one recently by Nguyen and Turnovsky (1979) use a simulation approach to analyse fiscal policy and model stability. They assign various sets of parameter values to private expenditure and demand for money functions. Nguyen and Turnovsky postulate a very wide range of values to test for stability. We use a similar approach.

The following parameter values are derived with reference to the above cited papers, and to other papers which have analysed the Canadian economy. We shall try to justify these figures as far as possible. For present purposes it seemed preferable to work with elasticities first and then derive the corresponding slope coefficients for the given set of initial values.

We start with the private expenditure function. It is widely agreed that the elasticity of private expenditure with respect to disposable income is in the range of 0.75 to 0.9. We thus pick N(E,YD)=0.8. (We use N(•) to denote the elasticity.) The slope coefficient a_1 then become

$$a_1 = N(E,YD)(E/YD) = 0.8(80/80) = 0.8$$

Determination of a value for the elasticity of private expenditure with respect to wealth, N(E,W), is less straightforward. Nguyen and Turnovsky (1979) assign a wide range, from 0.14 to 0.54, while Christ provides no figure. More values of the elasticity of consumption with respect to wealth, N(C,W), have been postulated by Dolde (1975) and Smith (1978). The values derived from their model are 0.208 and 0.25, respectively,[8] and we employ these values in our analysis. To derive the N(E,W), an adjustment must be made since consumption is only part of total private expenditure. Our model abstracts from the complexities of investment determination and the dynamics of physical capital accumulation by allowing only for total private expenditure. Care must be taken in deriving N(E,W) since wealth is not a determinant of investment(I). Expressing N(E,W) in terms of its basic components, we have

$$(3.35)\quad N(E,W) = (\partial E/\partial W)(W/E)$$
$$= (\partial C/\partial W)(W/C)(C/E) + (\partial I/\partial W)(W/I)(I/E)$$
$$= N(C,W)(C/E) + N(I,W)(I/E)$$
$$= N(C,W)(C/E) \quad \text{since } N(I,W) = 0$$

If we use Dolde's figure and the initial values obtained earlier, we then have

$$N(E,W) = 0.2(0.71478) = 0.1429$$

which is equivalent to the lower bound of the Nguyen and Turnovsky (1979) range of values. The slope coefficient a_2 is then derived as

$$a_2 = N(E,W)(E/W) = 0.1429(80/107) = 0.107$$

In the context of the Canadian economy, Scarth (1973) estimated a significant wealth effect (0.1405) on Canadian consumption expenditures.[9] Based on this value, the N(C,W) is then calculated by applying the mean values of C and W:

$$N(C,W) = 0.1405(18562.55/6895.99) = 0.378$$

Then the N(E,W) becomes:

$$N(E,W) = 0.378(0.71478) = 0.27$$

and the slope coefficient becomes:

$$a_2 = N(E,W)(E/W) = 0.27(80/107) = 0.2$$

We will consider this as the upper bound.

With regard to the N(E,R) value, we can express N(E,R) in terms of its consumption and investment elements:

$$(3.36) \quad N(E,R) = (\partial E/\partial R)(R/E)$$
$$= (\partial C/\partial R)(R/C)(C/E) + (\partial I/\partial R)(R/I)(I/E)$$
$$= N(C,R)(C/E) + N(I,R)(I/E)$$

We assume -0.08 and -0.14 for N(C,R) and N(I,R), respectively, as in Dolde's model. Fitting in our initial values, we then have

$$N(E,R) = -0.08(0.71478)-0.14(0.285722) = -0.097$$

In the context of the Canadian economy, we calculate the following elasticities from the Scarth (1973) model:

N(I,R) = -0.1448

N(RC,R)= -0.0725 where RC is investment in residential construction

Then N(E,R) = -0.0725(0.71478)-0.1448(0.285722) = -0.093

These calculated figures are similar to Christ's (1968) assigned value of -0.08, and is in the Nguyen and Turnovsky range of -0.14 to 0.[10] We thus conclude that N(E,R) = -0.09 is within the acceptable range. The a_3 slope coefficient is then derived as

$$a_3 = N(E,R)(E/R) = -0.09(80/0.05) = -144.0$$

The intercept term is calculated as a residual. The private expenditure function becomes

$$E_t = 11.751+0.8YD_t+0.107W_t-144.0R_t$$

As for the money demand function, there are various hypotheses about the value of demand elasticities and various empirical estimates. Here we use Tobin's hypothesis (1969). He postulates that the fraction of wealth held in money depends on the rate of return (R) and the income wealth ratio (Y/W):

(3.39) $M/W = L(R, Y/W)$

This function implies that the sum of demand elasticities with respect to W and Y is equal to unity.[11] The next task is to determine the individual elasticities. Smith (1978) specifies N(M,Y)=0.64, N(M,W)=0.36, N(M,R)=-0.5, while Dolde (1975) specifies N(M,Y)=0.65, N(M,W)=0.56 and N(M,R)=-0.47. Other estimates are provided by Chow (1966), from his regression results he calculates N(M,Y)=0.66, N(M,W)=0.36 and N(M,R)=-0.627.[12] From these similar estimates and from the elasticity property of (3.37), we assign the following elasticities to be used in our simulations:

$$N(M,Y) = 0.6 \quad N(M,W) = 0.4 \quad N(M,R) = -0.6$$

where $N(M,Y) + N(M,W) = 1$

The slope coefficients are derived as follows:

$$a_5 = N(M,Y)(M/Y) = 0.6(31.5/100) = 0.189$$

$$a_6 = N(M,W)(M/W) = 0.4(31.5/107) = 0.11775$$

$$a_7 = N(M,R)(M/R) = -0.6(31.5/0.05) = -378.0$$

$$M_t = 18.9+0.189Y_t+0.11775W_t-378.0R_t$$

Other possible parameter sets

By changing some underlying assumptions about the slopes of the IS-LM curves, we can generate other possible parameter sets that can give rise to different responses or different time profiles in the adjustment process. The following sets are considered.

Parameter Set (2)

Consider the case of a flatter LM curve. The economists who believe fiscal policy matters argue that the flatter the LM curve, the more effective the fiscal policy. Thus we divide a_3 of parameter set (1) by 3 and multiply a_7 by 3. The constant terms adjust accordingly.

$E_t = 6.951 + 0.8YD_t + 0.107W_t - 48.0R_t$

$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$

Parameter Set (3)

Consider the case of a steeper IS curve. Increase a_1 of parameter set (2) by 0.1:

$E_t = -1.049 + 0.9YD_t + 0.107W_t - 48.0R_t$

$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$

Parameter Set (4)

Consider the case of a larger wealth effect. Increase a_2 of parameter set (3) by 0.093 to 0.2:

$E_t = -11.0 + 0.9YD_t + 0.2W_t - 48.0R_t$

$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$

Some other combinations are as follows:

Parameter Set (5)

$E_t = 18.951 + 0.65YD_t + 0.107W_t - 48.0R_t$

$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$

Parameter Set (6)

$E_t = 11.751 + 0.8YD_t + 0.107W_t - 144.0R_t$

$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$

Parameter Set (7)

$E_t = 26.15 + 0.8YD_t + 0.107W_t - 432.0R_t$

$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$

Parameter Set (8)

$E_t = 18.151 + 0.9YD_t + 0.107W_t - 432.0R_t$

$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$

Parameter Set (9)

$$E_t = 8.2 + 0.9YD_t + 0.2W_t - 432.0R_t$$

$$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$$

Parameter Set (10)

$$E_t = 38.18 + 0.65YD_t + 0.107W_t - 432.0R_t$$

$$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$$

Parameter Set (11)

$$E_t = 3.751 + 0.9YD_t + 0.107W_t - 144.0R_t$$

$$M_t = 18.9 + 0.189Y_t + 0.11775W_t - 378.0R_t$$

Parameter Set (12)

$$E_t = 23.75 + 0.65YD_t + 0.107W_t - 144.0R_t$$

$$M_t = 18.9 + 0.189Y_t + 0.11775W_t - 378.0R_t$$

Parameter Set (13)

$$E_t = 23.75 + 0.65YD_t + 0.107W_t - 144.0R_t$$

$$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$$

Parameter Set (14)

$$E_t = 3.75 + 0.9YD_t + 0.107W_t - 144.0R_t$$

$$M_t = 56.7 + 0.189Y_t + 0.11775W_t - 1134.0R_t$$

3.4.5 Simulation Results

Table 3.2 presents the effects of a once-and-for-all increase in government expenditures on goods and services by 10 percent above its initial equilibrium level. Several findings are of interest:

First, there is no crowding-out effect observed in the impact period. The short-run income multiplier effects range from 1.5 to 2.5.

Second, under a money-financing policy, the long-run equilibrium is stable regardless of which parameter set is used, but the long run stability under a bond-financing policy depends on the parameter values, i.e., on the structure of the system. Under a bond-financing policy, a flatter LM curve increases the likelihood of achieving stability, but stability also depends on the shape of the IS curve. A flatter IS curve intersecting with a flatter LM curve would not necessarily import stability to the system. This is evidenced by the results for parameter sets (7), (10), (12) and (13). A steeper IS curve along with a flatter LM curve would ensure stability to the system under a bond-financing policy, as indicated by the results for parameter sets (2), (3), (4), (5), (9) and (14).

Third, the last column of Table 3.2 indicates the time periods required for the endogenous variables attaining a new constant (steady-state) level to the fourth decimal point. Depending on the accuracy (i.e., to the n'th decimal point) required, the rate of approach to the steady state will be different. Therefore the time period required to approach stability is an approximation. This interpretation applies to all tables reported in this thesis.

Table 3.2: The National Model
Simulated Effects of a Once-and-for-all Increase in Gl_t by 10%

Parameter Set	Financing Policy	Period	ΔE	ΔR	ΔY	ΔRB/T	Stability at (Time Period)
(1)		Impact	1.8787	0.0019	3.8783	0.0958	
	Bond	Long Run	--	--	--	--	Unstable
	Money	Long Run	3.5873	-0.0031	5.5873	0.0861	20
(2)		Impact	2.2999	0.0007	4.2999	0.0958	
	Bond	Long Run	5.7063	0.0025	7.7063	0.1086	220
	Money	Long Run	3.5776	-0.0032	5.5776	0.0859	80
(3)		Impact	3.0344	0.0008	5.0344	0.0958	
	Bond	Long Run	4.8839	0.0017	6.8839	0.1000	140
	Money	Long Run	3.8557	-0.0014	5.8557	0.0889	80
(4)		Impact	3.0344	0.0008	5.0344	0.0958	
	Bond	Long Run	4.5433	0.0014	6.5433	0.0964	70
	Money	Long Run	3.9919	-0.0005	5.9919	0.0904	50
(5)		Impact	1.5279	0.0006	3.5279	0.0958	
	Bond	Long Run	7.8008	0.0045	9.8008	0.1299	310
	Money	Long Run	3.2284	-0.0054	5.2284	0.0821	110
(6)		Impact	2.1569	0.0007	4.1569	0.0958	
	Bond	Long Run	6.2034	0.0030	8.2034	0.1138	290
	Money	Long Run	3.7287	-0.0022	5.7287	0.0876	60
(7)		Impact	1.7798	0.0006	3.7798	0.0958	
	Bond	Long Run	11.2592	0.0076	13.2592	0.1629	700
	Money	Long Run	3.8903	-0.0011	5.8903	0.0893	40
(8)		Impact	2.3359	0.0007	4.3359	0.0958	
	Bond	Long Run	7.0133	0.0038	9.0133	0.1220	400
	Money	Long Run	3.9930	-0.0005	5.9930	0.0905	40
(9)		Impact	2.3359	0.0007	4.3359	0.0958	
	Bond	Long Run	5.0325	0.0018	7.0325	0.1016	160
	Money	Long Run	4.0313	-0.0002	6.0313	0.9009	30
(10)		Impact	1.1700	0.0005	3.1700	0.0958	
	Bond	Long Run	--	--	--	--	Unstable
	Money	Long Run	3.7459	-0.0021	5.7459	0.0878	50
(11)		Impact	2.4834	0.0022	4.4834	0.0958	
	Bond	Long Run	11.2054	0.0170	13.2054	0.1624	590
	Money	Long Run	3.8601	-0.0013	5.8601	0.0890	30
(12)		Impact	1.2261	0.0016	3.2261	0.0958	
	Bond	Long Run	--	--	--	--	Unstable
	Money	Long Run	3.2434	-0.0053	5.2434	0.0823	40
(13)		Impact	1.4310	0.0006	3.4310	0.0958	
	Bond	Long Run	9.3257	0.0059	11.3257	0.1448	460
	Money	Long Run	3.4701	-0.0039	5.4701	0.0848	80
(14)		Impact	2.8395	0.0008	4.8395	0.0958	
	Bond	Long Run	5.1533	0.0020	7.1533	0.1028	180
	Money	Long Run	3.9237	-0.0009	5.9237	0.0897	60

Note: The last column of the table indicates the time periods required to approach a steady state to the accuracy of the fourth decimal point.

Fourth , the system reaches a long run equilibrium faster under a money-financing than under a bond-financing policy, regardless of which parameter set is used. The rate of approach to stability depends also on the structure of the system. It exerts opposite effects on bond and money-financing policies (e.g., compare the results for parameter sets (3), (8), and (14)). A higher interest rate coefficient in the IS curve results in a higher long-run rate of interest which will eventually increase the tax base through higher interest income. This increases the speed of reduction of the deficit and the rate of approach to a stable equilibrium. The results show that the speed of approach to stability is greater for parameter set (8), followed by parameter sets (14) and (3). On the other hand, a higher rate of interest under bond-financing means a higher interest obligation and a higher deficit. More time is then required to reduce the higher deficit. This is shown in Table 3.2 which indicates that the speed of approach to equilibrium is greater in parameter set (3), followed by parameter sets (14) and (8). This is a complete reversal of the results obtained under a money-financing policy.

Table 3.3 The National Model
Multiplier Effects of a Once-and-for-all
Increase in G_t by 10 Percent

Period	Money-Financing	Bond-Financing
Impact	1.5 -- 2.5	1.5 -- 2.5
Long Run	2.6 -- 3.0	3.5 -- 6.5

Fifth , the impact income multiplier, which is the same under either financing policy, ranges from 1.5 to 2.5. The long-run income multipliers under a money-financing policy range from 2.61 to 3.0. Because of increased interest payments, the long-run income multipliers under a bond-financing policy range from 3.5 to 6.5, higher than those obtained under a money-financing policy. Of course, consideration must be given to the rate of approach to stability when interpreting and comparing long-run multiplier effects.

Fig.3.3 and Fig.3.4 depict the time paths of several important endogenous variables under money and bond-financing policies. The results for only five parameter sets (1-5) are presented in Fig.3.3, the money-financing case; and only the three parameter sets that can produce a stable equilibrium are considered in the bond-financing case. Results for the latter parameter sets (2-4) are depicted in Fig.3.4. One significant difference between money and bond-financing policies is that the speed of approach to a stable equilibrium, i.e., the speed of approach to a balanced budget, is faster in the money than in the bond-financing case. This is evident from a comparison of the final diagrams of Fig.3.3 and Fig.3.4.

The simulation results verify the proposition that stability under bond-financed fiscal policy is an empirical question. It is worthwhile to examine the mechanism of the Christ adjustment method in eliminating the intrinsically unstable chacter of a bond-financed fiscal policy.

Figure 3.3: The National Model
Simulated Effects of a Once-and-for-all Increase in G_t by 10%
Money-Financing Policy

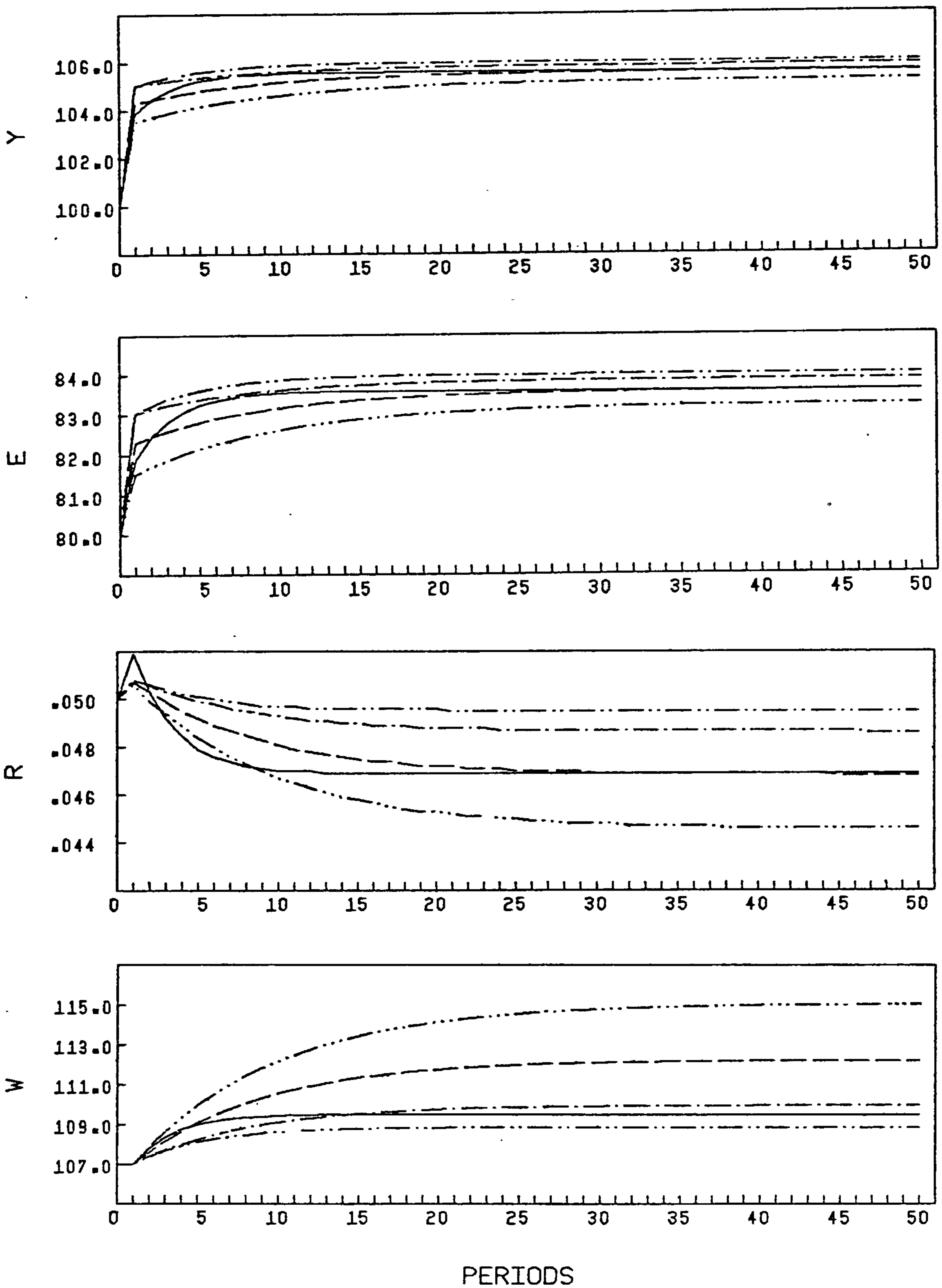
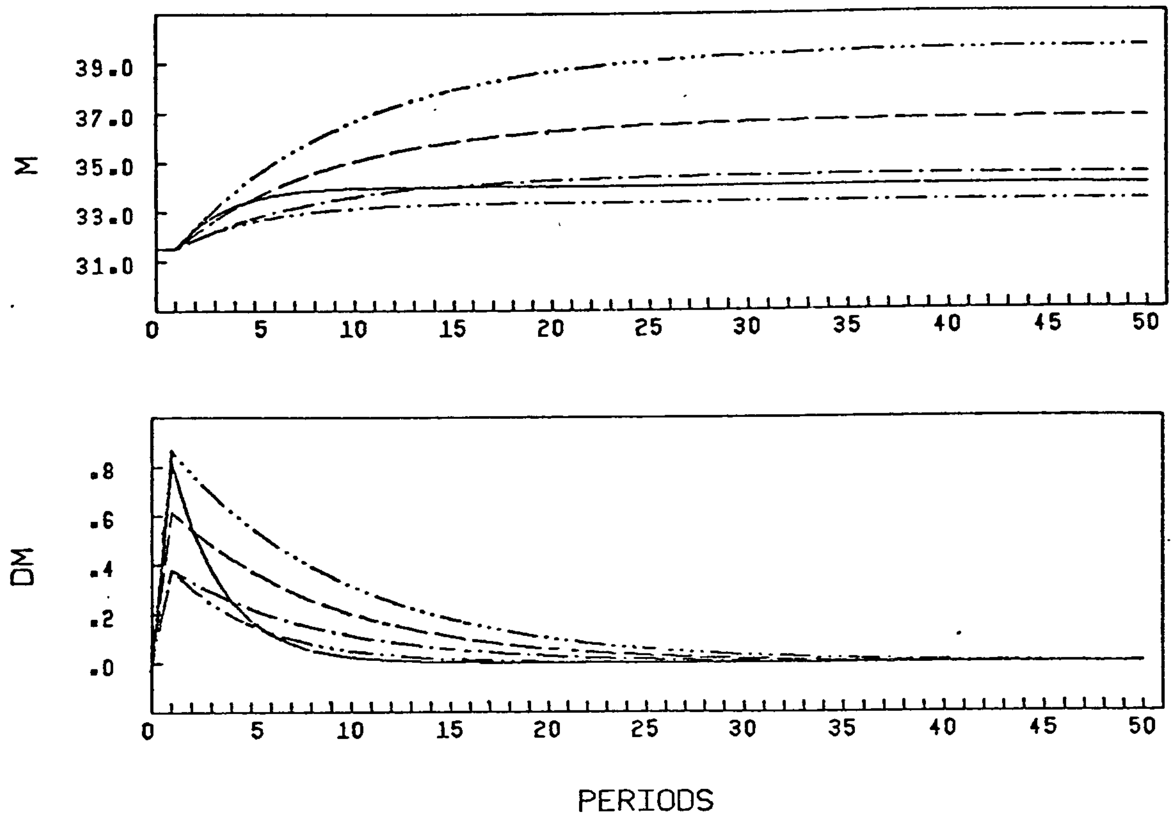


Figure 3.3(Contd.)



Parameter Set(1)

Parameter Set(2)

Parameter Set(3)

Parameter Set(4)

Parameter Set(5)

Figure 3.4: The National Model
Simulated Effects of a Once-and-for-all Increase in G_t by 10%
Bond-Financing Policy

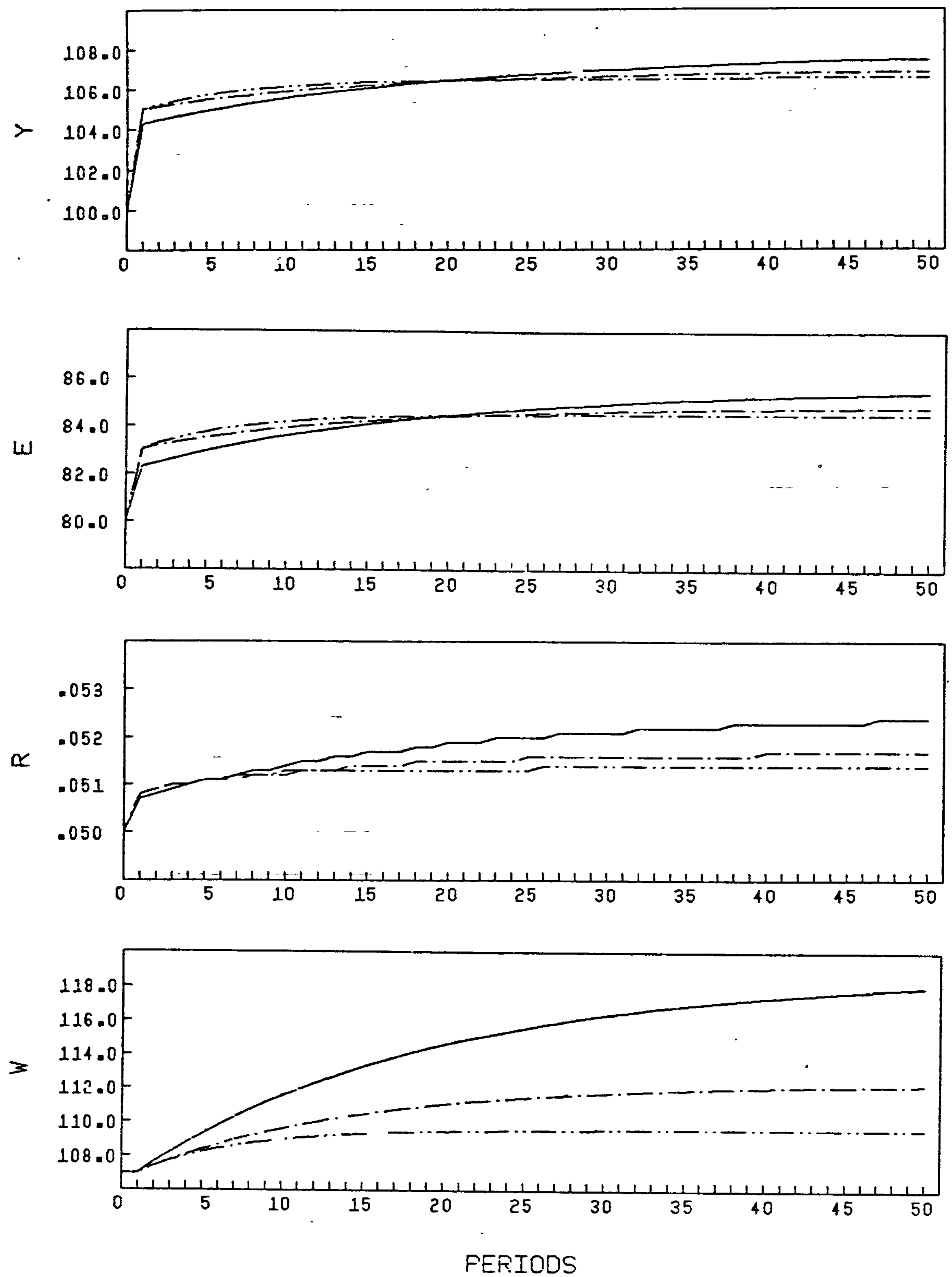
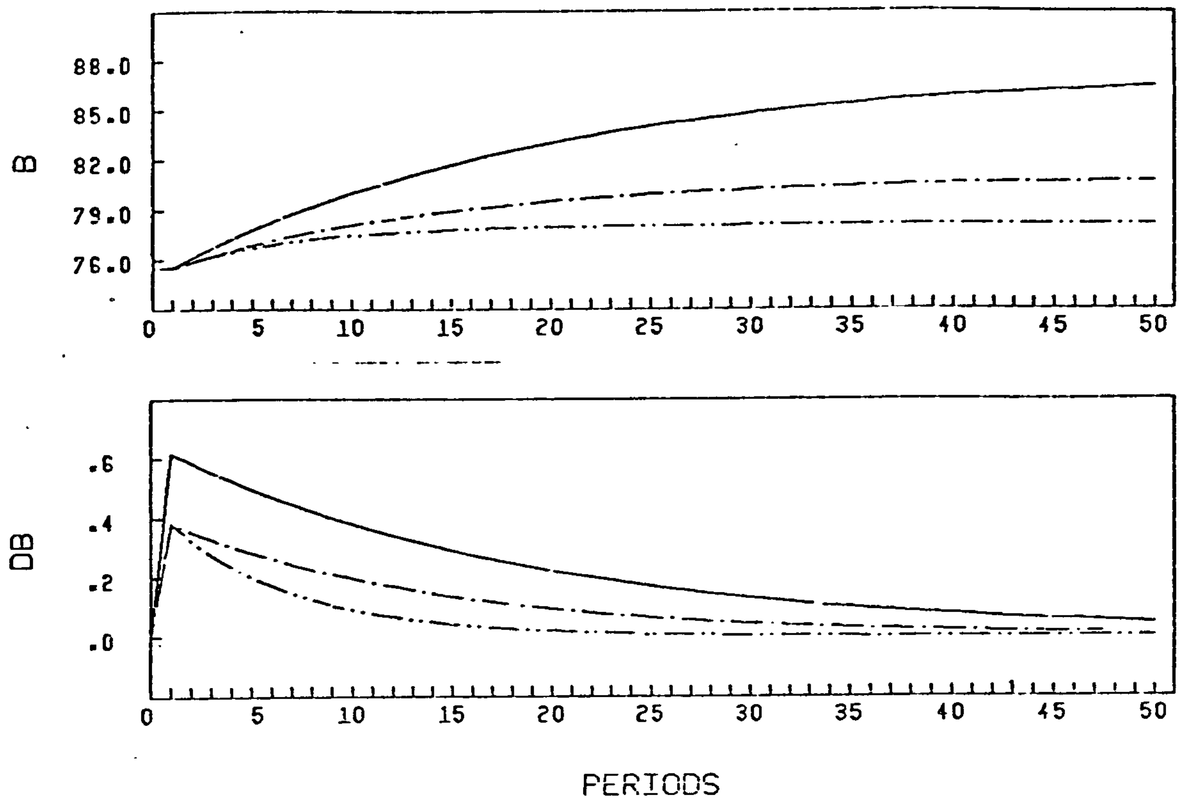


Figure 3.4(Contd.)



——————— Parameter Set(2)
— — —— — Parameter Set(3)
——— – – ——– – Parameter Set(4)

3.5 Applying the Christ Adjustment Method to the National Model

The basic intrinsic instability associated with bond-financing arises from the interest payments term in the GBC. Christ (1979) suggests that this intrinsic instability can be eliminated simply by treating the total government expenditures in the GBC as fixed. Any increase in interest payments due to a bond-financed deficit will be offset exactly by government expenditures on goods and services in the national income identity. To demonstrate, let GT represent fixed total government expenditures:

(3.37) $GT = G + TR + RB$

(3.38) $G = GT - TR - RB$

The national income identity and the GBC have to be respecified as:

(3.39) $Y = E + (GT - TR - RB)$

(3.40) $\dot{B} = GT - u(Y + TR + RB)$

By differentiating (3.40) with respect to B, we have:

(3.41) $\partial\dot{B}/\partial B = -u((\partial Y/\partial B)+B(\partial R/\partial B)+R)$

< 0 since $((\partial Y/\partial B)+B(\partial R/\partial B)+R)$ becomes positive

Thus the system becomes stable. But $(\partial Y/\partial B)$ is less than before because any increase in B would exert a deflationary effect on aggregate demand.

Two characteristics of the Christ adjustment method should be noted:[13]

First, all parameter sets produce the same change in private expenditures. This can be attributed to the basic nature of the Christ adjustment method and the definition of long-run equilibrium. There will be no new bond issuing in long-run equilibrium, and hence the government budget constraint can be represented as

(3.42) $GT - u(Y + TR + RB) = 0$

or as

(3.43) $GT = u(Y + TR + RB)$

In long-run equilibrium, tax revenue must be equal to total government expenditures, which is an assumed constant in the Christ adjustment method. Given a constant proportional tax rate, the tax base must be at the level which can generate the exact amount of tax revenue required to balance the budget. Equation (3.43) implies that private disposable income must also be a constant. Recalling from footnote 7 the proposition that private expenditure must equal disposable income under the assumption of a balanced budget in initial equilibrium, we derive the proposition that in long-run equilibrium, under the Christ adjustment method, private expenditure must be a constant. Its magnitude can be represented by the following equation:

(3.44) $E = YD = (1-u)(Y+TR+RB) = \text{constant}$

In the ordinary case without adjustment, the government expenditures on goods and services are fixed at a higher level after a once-and-for-all increase, leaving the remaining changes in aggregate demand accounted for by the new private expenditure level. When the Christ adjustment method is applied, the private expenditures become constant at a level determined by (3.44), leaving the remaining changes of aggregate demand accounted for by the new level of government expenditures on goods and services. One main feature of the Christ adjustment method is that government expenditures on goods and services adjust downward continuously, offsetting any increase in interest obligation. The larger the unstable force, the larger the downward adjustment that would be required.

Second, due to the elimination of the unstable force, the length of time required to achieve stability is reduced significantly. The impact and long-run multipliers are less than before because of the dampening effect due to the downward adjustment of government expenditures on goods and services. Moreover, the difference between the impact and the long-run multiplier effects are reduced significantly.

3.6 Need for an Interregional Model

With the prevailing wisdom that fiscal policy is mainly a federal economic responsibility, theoretical models which can be used to analyse interregional fiscal policy have not been well developed. At most, the regional government sector becomes a component of a national model, without any feedback effects upon the national variables. This asymmetric approach has been based mainly on the assumption either that the impact of interregional fiscal policy on the national economy is negligible, or that there is no scope whatsoever for regional governments to have any active fiscal policy. Perhaps the strongest argument against an active interregional fiscal policy is that bond-financing, the only means available to a regional government for financing deficit may be unstable. However, it is now generally accepted that the stability of bond-financing policy is an empirical question. Even if bond-financing policy may be unstable, the Christ adjustment method would at least partly solve the problem.

One common approach in a conventional macro-model is to include regional governments in the private sector and restrict the public sector to the federal government and the central bank.[14] We have demonstrated that the private-sector budget constraint (3.8) does not appear explicitly in the model because it has already been implied by other relationships of the model. However, if we rewrite (3.8) in discrete time, we have

(3.45) $DW_t = Y_t - E_t + TR_t + R_tB_t - u_t(Y_t + TR_t + R_tB_t)$

By definition, the left-hand side of (3.45) can be expressed as

(3.46) $DW_t = W_{t+1} - W_t$

Substituting (3.46) to (3.45), we have

(3.47) $W_{t+1} - W_t = Y_t - E_t + TR_t + R_tB_t - u_t(Y_t + TR_t + R_tB_t)$

or as

(3.48) $W_{t+1} + E_t + u_t(Y_t + TR_t + R_tB_t) = Y_t + TR_t + R_tB_t + W_t$

Equation (3.48) says that income flows and wealth in period t (the right-hand side), i.e., the sources of funds, determine the private expenditures and tax payments in period t and wealth at the beginning of period t+1 (the left-hand side), i.e., the uses of funds. If a regional government is included in the private sector, this behavioral relationship (3.48) will not necessarily be true, since interregional fiscal policy in period t can determine income in the same period and wealth at the start of the next period.

Grouping the general public and the regional governments into a single sector simply implies that regional governments should adopt the same spending policy as the general public. There is a fundamental difference in the philosophy of expenditure and financial considerations between the private and public sectors, regardless of whether the latter is a regional or federal government. The conventional approach is that the government sector appears in the national income identity including the expenditures of goods and services by all levels of government, while the GBC considers only the federal government sector. The regional governments are included in the private sector and their budget constraints do not appear explicitly in the model. Grouping the regional government with the private sector therefore bypasses the important question of interregional fiscal and financial relationships, which is crucial to the stability of the system, we want to address.

The public sector <u>per se</u> should include both the federal and regional governments. Considering the difference of financial means available to these governments and the increasing fiscal autonomy that regional government have, for example, in Canada, we should have separate government budget constraints for different levels of government. It may be justifiable to group all regional governments together as a total non-federal government sector on the grounds that they all face the same financing policy constraint, namely either a balanced-budget or a bond-financing constraint. Winer and Sheikh (1977) use this approach to examine the differential effects of federal and regional policies. However, such an approach cannot give the regional government any guideline with respect to macro-policy. Moreover, such an approach gives no information about the impact of one region's fiscal policy on the other regions, since allowance is made only for federal and total non-federal government sectors and the endogenous income variable is total GNP.

Thus individual regions become the major building blocks of our expanded model. We assume the economy to be composed of two regions. Of course, there may be more than two regions in a federal state. But in an economic sense, if we want to evaluate the fiscal policy of one region, then we can treat all the others as if they were one, and no generality will be lost. National effects are derived by aggregating regional effects. Furthermore, the federal variables are decomposed by region. Thus separate allowance is made for federal expenditure and revenue in each region. Government budget constraints are introduced explicitly for federal and regional governments. It is only in this context that we can evaluate realistically the fiscal and financial relationships among governments and the actual effects of fiscal policy.

3.7 Conclusion

The purpose of this chapter has been to present a simple national model from which an interregional model can be developed. The national model has also been used to illustrate the analysis of fiscal policy under money and bond-financing policies. A simulation approach has been introduced, and the Christ adjustment method has been discussed. Finally, we have argued for introducing regional governments explicitly into the model. The next chapter is devoted to the construction of an interregional model which involves separate regions but, for initial simplicity, involves no federal sector.

Footnotes to Chapter 3

(1). We assume here B is a one-period bond. It thus differs from the consols assumed in the Blinder and Solow model (1973).

(2). Interest income on outstanding government bonds must be added to factor income in calculating the tax base. This is not double-counting. Since interest payments on outstanding government bonds are transfer payments, they do not relate to any transactions involving goods and services, and therefore are not included as part of government expenditures on goods and services (G).

(3). Pyle and Turnovsky (1976) argue that the most satisfactory way to incorporate capital accumulation into an essentially Keynesian model is to define the dynamics relative to the stock of capital. In this case, the GBC is augmented to include a "growth tax" term. They argue that the results are similar to those obtained by abstracting from the dynamics of physical capital accumulation, and little is lost through such abstraction.

(4). See Blinder and Solow (1973), p.316.

(5). Statistics Canada. System of National Accounts, National Income and Expenditure Accounts, Catalogue 13-201 Annual.

(6). The components of some of the aggregate variables after normalization are as follows: GF=6.0, G1+G2=14.0, TRF=9.6, TR1+TR2=6.0, BF=41.3, and B1+B2=34.2, where F denotes the federal sector and 1,2 indicate regions 1 and 2, respectively.

(7). This is demonstrated by substituting national income identity (3.26) and balanced budget (3.49), into disposable income equation (3.28).

$$Y_t = E_t + G_t \tag{3.26}$$

$$G_t+TR_t+R_tB_t = u_t(Y_t+TR_t+R_tB_t) \tag{3.49}$$

$$\begin{aligned} YD_t &= (1-u_t)(Y_t+TR_t+R_tB_t) \\ &= Y_t+TR_t+R_tB_t-u_t(Y_t+TR_t+R_tB_t) \\ &= E_t+(G_t+TR_t+R_tB_t-u_t(Y_t+TR_t+R_tB_t)) \\ &= E_t + 0 \\ &= E_t \end{aligned} \tag{3.28}$$

(8). Dolde (1975), pp.7-8 and 21-22, and Smith (1978), pp.5-7.

(9). Scarth (1973), p.304.

(10). Nguyen and Turnovsky (1979), pp.266-267.

(11).Given $M/W = L(R, Y/W)$, then $\partial M/\partial R = WL_R$, $\partial M/\partial Y = L_Y$

and $N(M,R) = WL_R(R/M)$, $N(M,Y) = L_Y(Y/M)$

thus $\partial M/\partial W = L-(WL_Y Y)/W^2 = (M-L_Y Y)/W$

and $N(M,W) = (M-L_Y Y)/W(W/M) = 1 - N(M,Y)$

or $N(M,W) + N(M,Y) = 1$

(12).Chow (1966), pp.111-124.

(13).When applying the Christ adjustment method to all previous unstable parameter sets (1), (10), and (12), they all generate stable results(not reported in the chapter).

(14).See Christ (1967,1979), Blinder and Solow (1973).

Chapter 4

Analysis Based on an Interregional Model Without a Federal Sector

4.1 Introduction

The preceding chapter focused on the analysis of federal fiscal policy in a simple national model in which regional government sectors were ignored. We have pointed out that such an asymmetric approach does not only undermine the importance of interregional fiscal policy, but more important, assumes away the fiscal and financial implications and interactions of the government budget constraints between two levels of government. We thus argue that regional government sectors should be included, and the analysis of such financial implications and fiscal policies should be carried out in an interregional framework, including a federal government sector.

In this chapter, we construct an interregional model, but without the federal sector (IR). This model can be considered as the analogue of the simple national model without regional government sectors. The purpose is to focus attention on the interregional relationships themselves before incorporating the additional complexities associated with the federal sector.

The stability conditions under the IR model are more stringent than those of the National Model. Results of simulation tell us about the stability of the system.

The IR model is presented and discussed in section 4.2. Section 4.3 is devoted to the analysis of fiscal policy, based on this model. Simulation procedures and results are discussed in section 4.4 and section 4.5 focuses on the application of the Christ adjustment method to the model. Section 4.6 discusses the need for introducing the federal government sector and the final section of the chapter presents some concluding remarks.

4.2 An Interregional Model Without a Federal Sector

The IR model is presented first; notation and explanations then follow:

Equations of the IR Model :

(4.1) $Y1_t = E1_t + G1_t + IM2_t - IM1_t$

(4.2) $Y2_t = E2_t + G2_t + IM1_t - IM2_t$

(4.3) $E1_t = E1(YD1_t, W1_t, R_t)$

(4.4) $E2_t = E2(YD2_t, M_t+B1_t+B2_t-W1_t, R_t)$

(4.5) $IM1_t = IM1(YD1_t, W1_t, R_t)$

(4.6) $IM2_t = IM2(YD2_t, M_t+B1_t+B2_t-W1_t, R_t)$

(4.7) $YD1_t = (1 - u1_t)(Y1_t + TR1_t + R_tH1_t)$

(4.8) $YD2_t = (1 - u2_t)(Y2_t + TR2_t + R_t(B1_t+B2_t-H1_t))$

(4.9) $W1_t - H1_t = L1(Y1_t, W1_t, R_t)$

(4.10) $M_t - W1_t + H1_t = L2(Y2_t, M_t+B1_t+B2_t-W1_t, R_t)$

(4.11) $DW1_t = G1_t+TR1_t+R_tH1_t+IM2_t-IM1_t - u1_t(Y1_t+TR1_t+R_tH1_t)$

(4.12) $DB1_t = G1_t + TR1_t + R_t B1_t - u1_t(Y1_t + TR1_t + R_t H1_t)$

(4.13) $DB2_t = G2_t + TR2_t + R_t B2_t - u2_t(Y2_t + TR2_t + R_t(B1_t + B2_t - H1_t))$

(4.14) $DW1_t = W1_{t+1} - W1_t$

(4.15) $DB1_t = B1_{t+1} - B1_t$

(4.16) $DB2_t = B2_{t+1} - B2_t$

Endogenous variables: $Y1_t, Y2_t, E1_t, E2_t, IM1_t, IM2_t, H1_t, R_t, YD1_t,$
$YD2_t, DW1_t, DB1_t, DB2_t, W1_{t+1}, B1_{t+1}, B2_{t+1}$.

Predetermined variables: $G1_t, G2_t, TR1_t, TR2_t, u1_t, u2_t, W1_t, M_t, B1_t, B2_t$.

Assumptions about partial derivatives are as follows:

$0 \leqslant E1_{YD1} \leqslant 1$	$E1_R \leqslant 1$	$E1_{W1} \geqslant 0$
$0 \leqslant E2_{YD2} \leqslant 1$	$E2_R \leqslant 1$	$E2_{W2} \geqslant 0$
$0 \leqslant IM1_{YD1} \leqslant 1$	$IM1_R \geqslant 1$	$IM1_{W1} \geqslant 0$
$0 \leqslant IM2_{YD2} \leqslant 1$	$IM2_R \geqslant 1$	$IM2_{W2} \geqslant 0$
$L1_{Y1} \geqslant 0$	$L1_R \leqslant 0$	$0 \leqslant L1_{W1} \leqslant 1$
$L2_{Y2} \geqslant 0$	$L2_R \leqslant 0$	$0 \leqslant L2_{W2} \leqslant 1$

Notation :

1 — Region 1

2 — Region 2

B — Outstanding regional government bonds

DB — Changes of outstanding regional government bonds

DW — Changes of domestic financial wealth

E — Regional private expenditures

G — Regional government expenditures on goods and services

H — Domestic holding of outstanding bonds

IM — Regional imports

L — Money demand function

M — Fixed money supply

R — Rate of interest

TR — Transfer payments

W — Domestic financial wealth

Y — Gross regional product

YD — Regional private disposable income

The IR model is a two-region, four-sector aggregate demand model. The four sectors are the private and government sectors in Region 1 and Region 2. The behavior of the regional government sectors is assumed to be exogenous, while the behavior of the private sector is fully explained within the model.

The IR model makes the same assumption of a closed-economy with respect to the rest of the world as did the National Model. Within one confederation, there are no restrictions on trade and capital movements between regions. Thus the IR model is assumed closed with respect to the rest of the world, but open with respect to the rest of the national economy. Each region can affect the other through expenditure policy. There is one monetary system controlled by the federal government, which in this IR model is assumed to exist implicitly in that it supplies a fixed level of money.[1] This assumption, along with the assumption of perfect capital mobility, implies that there is a common market rate of interest prevailing in both regions. However, each region is assumed to be large enough to influence the market rate of interest. We also assume the same fixed price level (for simplicity, assume unitary price level) and perfectly elastic supply in both regions.

To façilitate the use of computer technique in determining the stability of the system, we assume discrete time in the IR model.

Because the financial implications play a critical role in the discussion of the stability of the system and the effectiveness of interregional fiscal policy, it is worthwhile at the outset to define clearly the financial variables of the system. Denoting B as the number and value of bonds, and RB as the interest on B, and also assuming again that only governments issue bonds and only the private sector holds bonds, we have:

$B1_t$-- Bonds issued by the first regional government

$B2_t$-- Bonds issued by the second regional government

$H1_t$-- Bonds held by residents of the first region

$H2_t$-- Bonds held by residents of the second region

(4.17) $H1_t + H2_t = B1_t + B2_t$

(4.18) $H2_t = B1_t + B2_t - H1_t$

We also have:

$M1_t$-- Money stock (issued by the federal government) held by residents of the first region

$M2_t$-- Money stock (issued by the federal government) held by residents of the second region

(4.19) $M_t = M1_t + M2_t$

From the definition of financial wealth, we have:

(4.20) $W1_t = M1_t + H1_t$

(4.21) $W2_t = M2_t + H2_t$

To simplify the system through substitution, we use the left-hand side expressions of equations (4.22) to (4.24) to represent the right-hand side variables in the IR model.[2]

(4.22) $W1_t - H1_t = M1_t$

(4.23) $M_t - W1_t + H1_t = M2_t$

(4.24) $M_t + B1_t + B2_t - W1_t = W2_t$

The model starts with the definition of gross regional product (GRP) in each region, as determined by (4.1) and (4.2), respectively. The government expenditures on goods and services are assumed exogenous. Note that in the IR model, one region's exports are equivalent to the other region's imports. It is inappropriate to treat exports as exogenous, as in some small-country open-economy models, since now $Y1_t$ and $Y2_t$ must be determined simultaneously within the whole system.

Equations (4.3) and (4.4) incorporate the private expenditure functions of the two regions. It is assumed that private expenditure is a function of disposable income, wealth and the common rate of interest. The wealth variable is the sum of the total private holdings of money and bonds in each region. Equations (4.5) and (4.6) incorporate the import functions of the two regions. Note that imports depend upon the same determinants as private expenditures. This reflects the assumption that imports are determined jointly with private expenditures, and therefore depend upon the same variables. Equations (4.7) and (4.8) are definitional equations of disposable income in the two regions.

Equations (4.9) and (4.10) are regional equilibrium conditions in the money market. One special characteristic of the IR model is that while the total stock of money is fixed throughout the analysis, the distribution of the money stock between two regions is not fixed.[3] It is endogenous within the system, both in the short run and the long run. The link between (4.7) and (4.8) is the amount of $H1_t$, the stock of bonds held by residents of the first region. In the short run, when the stock of wealth is assumed constant, any changes of $H1_t$ (or $H2_t$)

must be offset by opposite changes of $M1_t$ (or $M2_t$). When there is fiscal expansion in the first region, $H1_t$ must decline in order to get more $M1_t$ to accommodate the increasing economic activity. This is accomplished by selling $H1_t$ to the second region. In the short run, the distribution of M_t is therefore implied by the amount of $H1_t$. In the long run, both $W1_t$ and $H1_t$, as well as the distribution of M_t between the two regions, are endogenous in the system.

The stock variables do not stay constant through time. They are driven by the dynamic role of the budget constraint. Equations (4.12) and (4.13) are the budget constraints of the first and second regional governments. These equations stipulate that the uses and sources of government funds must be in balance. The uses are expenditures on goods and services, transfer payments and interest on outstanding debt. The sources are current tax revenue and the positive growth in the stock of bonds.

Equation (4.11) is the private sector budget constraint in the first region. As we have demonstrated in (3.8) of Chapter 3 a change of private wealth is equivalent to the private saving. Thus (4.11) is derived as follows:

$$
\begin{aligned}
(4.25)\quad DW1_t &= YD1_t - E1_t \\
&= (1 - u1_t)(Y1_t + TR1_t + R_tH1_t) - E1_t \\
&= (Y1_t - E1_t) + TR1_t + R_tH1_t - u1_t(Y1_t + TR1_t + R_tH1_t) \\
&= G1_t + IM2_t - IM1_t + TR1_t + R_tH1_t - u1_t(Y1_t + TR1_t + R_tH1_t)
\end{aligned}
$$

One significant difference between the IR model and the National Model is that in the latter, the private-sector budget constraint does not appear explicitly, but is implied by the GBC and other relationships.

(See section 3.2 and (3.8).) This feature does not carry over to the IR model. This point can be demonstrated by working with equation (4.11). Through substitution, we have

$$(4.26)\quad DW1_t = G1_t+IM2_t-IM1_t+TR1_t+R_tH1_t - u1_t(Y1_t+TR1_t+R_tH1_t)$$
$$= (G1_t + TR1_t + R_tB1_t - u1_t(Y1_t+TR1_t+R_tH1_t))$$
$$+ R_t(H1_t-B1_t) + IM2_t - IM1_t$$
$$= DB1_t + (R_t(H1_t-B1_t) + IM2_t - IM1_t)$$

Equation (4.26) shows that the first regional GBC ($DB1_t$), together with the second term on the right-hand side, which is the current-account balance of payments between the regions, implies the private sector budget constraint in the first region. The public sector budget constraint alone does not rule out the need for an explicit private sector budget constraint in the IR model. The above conclusion can be attributed to the special nature of this model, which is closed with respect to the rest of the world, but open within the national economy. Under the same monetary system and within the confederation, there is no reason for the federal authority to implement any policy to change the parity rate between regions in order to solve a balance-of-payments problem. Any surplus or deficit in the balance of payments represents a flow of wealth between regions, and must be represented by the private-sector budget constraint.

By the same token, the private-sector budget constraint for the second region can be expressed as

$$(4.27)\quad DW2_t = G2_t+TR2_t+R_t(B1_t+B2_t-H1_t)+IM1_t-IM2_t$$
$$- u2_t(Y2_t+TR2_t+R_t(B1_t+B2_t-H1_t))$$

The dynamic system of the IR model consists of two public and two private budget constraints. The long-run equilibrium of the system requires that $DW1_t$, $DW2_t$, $DB1_t$ and $DB2_t$ be equal to zero. However, according to Walras' Law, only three of these values are independent. If any three are zero, then the fourth one must be zero also. This can be demonstrated by the following substitution:

From (4.11) and assuming $DW1_t = 0$, we have:

(4.28) $IM1_t - IM2_t = G1_t + TR1_t + R_t H1_t - u1_t (Y1_t + TR1_t + R_t H1_t)$

Substituting (4.28) into (4.27), we have:

$$
\begin{aligned}
(4.29)\quad DW2_t &= G2_t + TR2_t + R_t (B1_t + B2_t - H1_t) + (G1_t + TR1_t + R_t H1_t \\
&\quad - u1_t (Y1_t + TR1_t + R_t H1_t)) - u2_t (Y2_t + TR2_t + R_t (B1_t + B2_t - H1_t)) \\
&= DB1_t + DB2_t \\
&= 0 \text{ if } DB1_t = DB2_t = 0
\end{aligned}
$$

Thus we need to examine only three dynamic equations in the system. In the above equations of the IR model, $DW2_t$ does not appear explicitly.

4.3 Study of Interregional Fiscal Policy

In this section we discuss the short-run and long-run effects of interregional fiscal policy. Formal stability analysis will be undertaken in the last part of the section.

4.3.1 Static System

In the short run, all the stock variables are constant, and all the dynamic equations need not be considered. The static system then consists of the first ten equations, (4.1) to (4.10). The endogenous and predetermined variables are listed below:

Endogenous variables: $Y1_t, Y2_t, E1_t, E2_t, IM1_t, IM2_t, YD1_t, YD2_t, H1_t, R_t$.

Predetermined variables: $G1_t, G2_t, TR1_t, TR2_t, u1_t, u2_t, uF_t, W1_t, M_t, B1_t, B2_t$.

Equations (4.1) to (4.10) can be simplified, through substitution, to four equations determining $Y1_t$, $Y2_t$, $H1_t$ and R_t:

(4.30) $Y1_t$ = Y1(predetermined variables)

(4.31) $Y2_t$ = Y2(predetermined variables)

(4.32) $H1_t$ = H1(predetermined variables)

(4.33) R_t = R(predetermined variables)

We then differentiate the above system and solve for the following multipliers. (See Appendix 4.1.):

(4.34) $dY1_t/dG1_t = e_3/(e_1e_3-e_2e_4)$

(4.35) $dY1_t/dG2_t = e_2/(e_1e_3-e_2e_4)$

(4.36) $dY2_t/dG1_t = e_4/(e_1e_3-e_2e_4)$

(4.37) $dY2_t/dG2_t = e_1/(e_1e_3-e_2e_4)$

(4.38) $dR_t/dG1_t = -(L_{11}e_3+L_{21}e_4)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(4.39) $dR_t/dG2_t = -(L_{11}e_2+L_{21}e_1)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(4.40) $dH1_t/dG1_t = -(L_{11}L_{23}e_3-L_{21}e_4)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(4.41) $dH1_t/dG2_t = -(L_{11}L_{23}e_2-L_{21}e_1)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

where the summary coefficients (e's) are defined in Appendix 4.1. Since the signs of all the e's are indeterminate, we cannot sign $(e_1e_3-e_2e_4)$ a priori. However, from assumptions about magnitudes of partial derivatives in all parameter sets considered, we would expect that the following results (Table 4.1) will hold. (See Appendix 4.1.):

Table 4.1: The IR Model
Signs of Short-Run Multiplier Effects

Derivative of	With Respect to $G1_t$	$G2_t$
$Y1_t$	+	+
$Y2_t$	+	+
R_t	+	+
$H1_t$	-	+

The short-run effect of a region's fiscal policy on its own income is definitely positive, but its effect on the income of the other region depends on the slope of the IS-LM curves and the initial holding of financial stock in each region. Based on the four parameter sets considered in the chapter, we obtain positive income effects for both regions.[4] The short-run effect on the rate of interest is positive for an expansionary fiscal policy initiated by any region. Of particular interest is the movement of $H1_t$. Any expansionary fiscal policy in the first region would reduce the holdings of $H1_t$, while any expansionary fiscal policy in the second region would increase the holdings of $H1_t$, i.e., reduce the holdings of $H2_t$. Both effects reflect the same proposition discussed in connection with the money market equilibrium equation. Any increase in the demand for money in the short run would be offset by a corresponding decrease in the holding of bonds.

The following diagrams illustrate the direction of movement suggested by Table 4.1

Fig.4.1 and Fig.4.2: The IR Model

Short-Run Multiplier Effects

Fig.4.1

Region 1

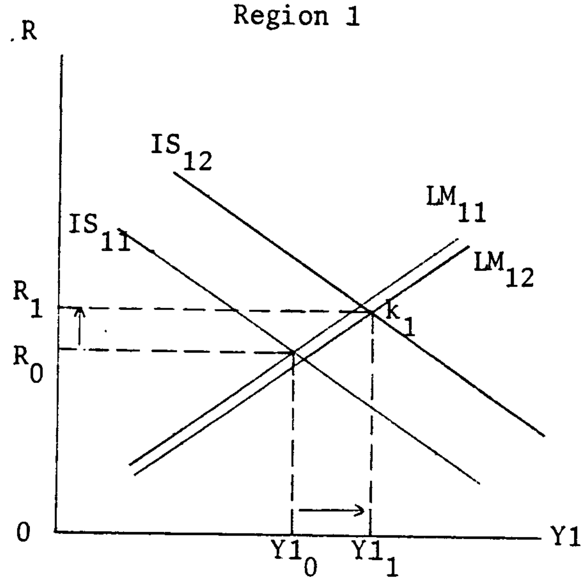
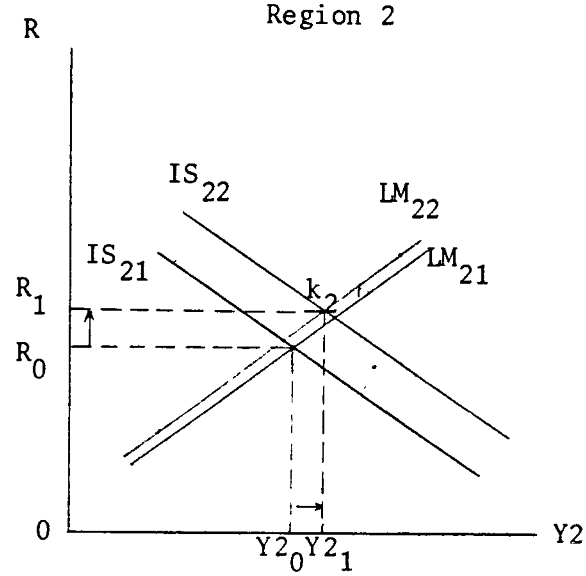


Fig.4.2

Region 2



Consider a once-and-for-all increase in Gl_t in the first region. We assume both regions are identical, having the same shape of IS-LM curves. In the interregional model, the IS-LM curves in both regions are interrelated. Now the first regional government raises its expenditure and the IS curve shifts to IS_{12}. The IS curve in the second region also shifts rightward to IS_{22}, partly because of the trade surplus that ensures, partly because of induced private expenditures. The extent of the shifting of the IS curve in the region of fiscal expansion is greater than the shifting of IS curve in the response region. In the first region, Hl_t is reduced in order to acquire higher money balances. Thus LM_{11} shifts rightward to LM_{12}, while

LM_{21} shifts to LM_{22} by exactly the smae amount. To accommodate these increased economic activities, the holding of bonds must be reduced to obtain a higher money balance. Therefore, with a fixed money stock and increasing money demand from both regions, the rate of interest must be higher.

The new equilibria in the short-run are k_1 and k_2. The same rate of interest prevails in the two regions, a result of the basic assumption of perfect capital mobility. For the system to attain a new equilibrium, LM_{11} must shift rightward and LM_{21} must shift leftward. The system will not produce any short-run equilibrium if LM_{11} shifts leftward and LM_{21} shifts rightward because the rightward shift of IS_{11} is greater than the shift of IS_{21}.

4.3.2 Long-Run System

In the long-run, we must consider the full model, including both private and public-sector budget constraints. Changes of $DW1_t$, $DB1_t$ and $DB2_t$ would affect the financial stocks and interest payments in later periods. The long-run equilibrium is then defined as the situation that exists when all changes of endogenous variables are zero and all exogenous variables remain constant. Long-run equilibrium thus requires that the government budget be balanced and that changes of private wealth be zero.

Fig.4.1 and Fig.4.2 can also be used to analyse the dynamics or stability of the system. Assume a once-and-for-all increase in government expenditures in the first region. The impact effects are represented by equilibrium points k_1 and k_2 for regions 1 and 2, respectively.

In the first region, the IS curve will shift to the right, since government expenditures stay at the higher level. Higher disposable income and wealth due to new bond issues reinforce the rightward movement of the IS_1 curve. However, higher taxes are an offsetting force which tends to reduce the deficit, and indirectly, to reduce the magnitude of each subsequent shift of the IS_1 curve. If the tax revenues in the first region can reduce a budget deficit over time, then the first region may approach a long-run equilibrium.

An important feature of the IR model is the role of interregional influences in determining the final outcome of any policy change. Though the budget deficit in the first region may diminish over time, the possibility of attaining a long-run equilibrium still depends on the movement of the LM curves in both regions. If the LM curve in the first region does not remain stable in each subsequent period, then it is impossible for the first region to reach a long-run equilibrium.

The movement of LM curves is determined by the interaction between regional bond markets and the balance of payments. The rightward shift in LM in the first region means it sells bonds to the second region in order to acquire more money balances so that it can pay for its balance-of-payments deficit. For a given M_t throughout the analysis, any rightward shift of the LM curve in one region must be accompanied by a corresponding leftward shift of the LM curve in the other region.

To determine the direction of movement of the LM curves, we must analyse the induced effects on the second region. In the impact period, the second region also realizes a higher income due to expansionary fiscal policy implemented in the first region. In the second and subsequent periods, the budget of the second region is in surplus since the increasing tax revenues are higher than the induced interest payments changes. To balance its budget, the second regional government must retire some of its outstanding bonds. The induced shift of IS_2 will continue as long as IS_1 is moving rightward. With even higher tax revenues in each subsequent period, the second regional government must retire outstanding bonds at a faster rate. After retiring all of its outstanding bonds, the second regional government will use the increased tax revenues to buy bonds issued by the first regional government. This process will continue as long as the budgets are not balanced. When the bonds needed to be retired by the second region are not equal the bonds needed to be issued by the first region, the LM curves in the two regions will adjust continuously, and the system is unstable. In Table 4.2 of the next section, the parameter set (1) reflects this instability.

Depending on the slope of the IS-LM curves, there is also the possibility that when the value of bonds issued by the first region equals exactly the value of bonds retired by the second region, then the LM curves will have no tendency to shift. Provided also that the IS curves become stable in each subsequent period, the system will stay at a new equilibrium. This is reflected by the parameter sets (2) to (4) of Table 4.2.

However, we are inclined not to label such a situation as "steady-state equilibrium". We call it instead "quasi-equilibrium". Though every endogenous variable except $DB1_t$ and $DB2_t$ remains unchanged at a higher level, the definition of steady-state equilibrium is not fulfilled because both regional government budgets are not in balance. It may be possible for the second regional government to buy more and more bonds from the first region with ever-increasing tax revenues, but it may be politically and economically infeasible for the first regional government to issue bonds indefinitely. Realistically, there is a debt-limit which would not be exceeded by any responsible government. Moreover, we assume that only the private sector holds financial wealth. Any increase in financial wealth held by the second regional government must then be distributed to its residents. An increase in $W2_t$ would generate another cyclical fluctuation that would certainly make the system more likely to be unstable.

4.3.3 Formal Stability Analysis

This section describes the steps involved in conducting formal stability analysis. The dynamic system of the IR model consists of a set of difference equations. According to the Schur Theorem,[5] the problem of stability for difference equations becomes one of finding conditions for which the absolute values of the characteristic roots are less than unity.

Consider the dynamic system (4.11) to (4.16) of the IR model. Through substitution, we express the dynamic system in terms of the following system of linear difference equations:

$$(4.42) \quad \begin{bmatrix} W1_{t+1} \\ B1_{t+1} \\ B2_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} W1_t \\ B1_t \\ B2_t \end{bmatrix}$$

The characteristic equation of (4.42) is expanded and we have

$$(4.43) \quad c_0\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0$$

The Schur conditions can be shown as:

$$(4.44)\quad \Delta_1 = \begin{vmatrix} c_0 & c_3 \\ c_3 & c_0 \end{vmatrix} > 0$$

$$(4.45)\quad \Delta_2 = \begin{vmatrix} c_0 & 0 & c_3 & c_2 \\ c_1 & c_0 & 0 & c_3 \\ c_3 & 0 & c_0 & c_1 \\ c_2 & c_3 & 0 & c_0 \end{vmatrix} > 0$$

$$(4.46)\quad \Delta_3 = \begin{vmatrix} c_0 & 0 & 0 & c_3 & c_2 & c_1 \\ c_1 & c_0 & 0 & 0 & c_3 & c_2 \\ c_2 & c_1 & c_0 & 0 & 0 & c_3 \\ c_3 & 0 & 0 & c_0 & c_1 & c_2 \\ c_2 & c_3 & 0 & 0 & c_0 & c_1 \\ c_1 & c_2 & c_3 & 0 & 0 & c_0 \end{vmatrix} > 0$$

For the system to be stable, it requires that $|\lambda_i| < 1$. And $|\lambda_i| < 1$ when $\Delta_1 > 0$, $\Delta_2 > 0$, and $\Delta_3 > 0$.

Because of the complexities of the expressions in the a and c terms, it is virtually impossible to determine the stability property simply by looking at the signs of the partial derivatives. Again we must use a computer simulation to trace the time path of the system.

4.4 A Simulation Approach to the Analysis of Fiscal Policy in the IR Model

4.4.1 Specification of Functional Form

We assume the same functional form as in the National Model. The analysis of micro-foundation of these macro-functions is presented in Appendix 4.2.

4.4.2 Specification of Initial Values

The initial values of the IR model are presented below and discussion follows:

$$Y1_t = Y2_t = 35.0 \qquad E1_t = E2_t = 28.0$$

$$G1_t = G2_t = 7.0 \qquad IM1_t = IM2_t = 8.4$$

$$YD1_t = YD2_t = 28.0 \qquad TR1_t = TR2_t = 3.0$$

$$B1_t = B2_t = 17.1 \qquad u1_t = u2_t = 0.279$$

$$W1_t = W2_t = 28.125 \qquad H1_t = H2_t = 17.1$$

$$DW1_t = DB1_t = DB2_t = 0 \qquad R_t = 0.05 \qquad M_t = 22.05$$

The initial values of the IR model are derived from the National Model. In the simplest case, we assume identical regions. Since the IR model omits the federal sector, the federal component of the national aggregate variables, except the money stock, is first deleted. However, we assume each region maintains the same ratio of government expenditures on goods and services to aggregate demand as in the National Model. Private expenditure is an endogenous variable to be determined within the system. It would be inappropriate to delete the federal expenditure item from each regional income identity and leave regional private expenditures intact.[6]

Interregional trade is denoted by $IM1_t$ and $IM2_t$. The trade balance is assumed initially to be zero. We assume that imports in each region account for 30 percent of private expenditure.[7]

The financial wealth consists of the regional government bonds and the fixed money stock. Though the IR model assumes away the federal sector, the money stock controlled by the federal authorities must remain in the model to facilitate economic transactions. The money stock is the only federal variable allowed for in the IR model, and is assumed fixed. The new money stock is derived by multiplying the new income figures by the income velocity obtained from the National Model.

The proportional tax rate is derived under the assumption of a balanced budget in the initial equilibrium state. Note that in the initial equilibrium, with balanced budget and balanced interregional current account, private disposable income should equal private expenditure.[8]

4.4.3 Specification of Parameter Values

Using the same elasticities as those derived for the National Model, and adopting the same assumptions about alternative parameter values, we construct the following parameter sets for the IR model:

Parameter Set (1)

$E1_t = 6.19 + 0.8YD1_t + 0.107W1_t - 72.0R_t$

$IM1_t = 1.86 + 0.24YD1_t + 0.0321W1_t - 21.6R_t$

$W1_t - H1_t = 10.55 + 0.189Y1_t + 0.12W1_t - 189.0R_t$

Same parameter values for region 2.

Parameter Set (2)

$El_t = 3.79 + 0.8YDl_t + 0.107Wl_t - 24.0R_t$

$IMl_t = 1.13 + 0.24YDl_t + 0.0321Wl_t - 7.2R_t$

$Wl_t - Hl_t = 29.5 + 0.189Yl_t + 0.12Wl_t - 567.0R_t$

Same parameter values for region 2.

Parameter Set (3)

$El_t = 0.99 + 0.9YDl_t + 0.107Wl_t - 24.0R_t$

$IMl_t = 0.29 + 0.27YDl_t + 0.0321Wl_t - 7.2R_t$

$Wl_t - Hl_t = 29.5 + 0.189Yl_t + 0.12Wl_t - 567.0R_t$

Same parameter values for region 2.

Parameter Set (4)

$El_t = -1.625 + 0.9YDl_t + 0.2Wl_t - 24.0R_t$

$IMl_t = -0.48 + 0.27YDl_t + 0.06Wl_t - 7.2R_t$

$Wl_t - Hl_t = 29.5 + 0.189Yl_t + 0.12Wl_t - 567.0R_t$

Same parameter values for region 2.

4.4.4 Simulation Results

Consider a once-and-for-all increase in Gl_t of 10 percent in the first region, after the initial equilibrium. The simulation generates the resulting time paths of the endogenous variables. The results are presented in Table 4.2. Several conclusions can be drawn from these results:

First , the long-run system is unstable for the first parameter set.

Second , for parameter sets (2) to (4), with flatter LM and steeper IS curves, a quasi-equilibrium is produced, in the following sense:

(i) $DWl_t = 0$

(ii) $DBl_t + DB2_t = 0$

(iii) $DB1_t > 0$

(iv) $DB2_t < 0$

(v) $DB1_t = -DB2_t$

(vi) $DB1_t$ increases and $DB2_t$ decreases indefinitely

(vii) $B1_t$ and R_tB1_t increase while $B2_t$ and R_tB2_t decrease indefinitely

(viii) Other endogenous variables stay at a new level.

Third, the long-run income multiplier effects are greater than corresponding impact effects. The long-run multiplier effect in the first region is greater than in the second region.

Table 4.3: The IR Model
Multiplier Effects of a Once-and-for-all
Increase in Gl_t by 10 Percent

Period	Region 1	Region 2
Impact	1.7 — 2.1	0.4 — 0.7
Long Run (quasi-equilibrium)	2.6 — 3.1	1.2 — 1.6

Fourth, the steeper the IS curve, the faster the system approaches its quasi-equilibrium. The longer the time period required to reach a quasi-equilibrium, the more bonds that must be issued to cover the deficit, resulting in a higher rate of interest, private financial wealth, and income.

The simulation results show that in the long run only a quasi-equilibrium is achieved. The nature of such a quasi-equilibrium has been discussed in the last section. In the next section we will apply the Christ adjustment method in an attempt to produce a stable equilibrium.

Table 4.2 The IR Model

Simulated Effects of a Once-and-for-all Increase in Gl_t by 10%

Parameter Set	Financing Policy	Period	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability at (Time Period)
(1)		Impact	0.6297	0.1117	0.7414	0.0007	1.1743	0.2671	1.4414	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
(2)		Impact	0.7264	0.2083	0.9347	0.0003	1.2710	0.3637	1.6347	
	Bond	Long Run	1.6887	0.9246	2.6133	0.0011	2.1594	1.1538	3.3132	200 (Q.E.)
(3)		Impact	0.9365	0.3313	1.2678	0.0003	1.4549	0.5128	1.9677	
	Bond	Long Run	1.4543	0.7243	2.1786	0.0007	1.9353	0.9433	2.8786	140 (Q.E.)
(4)		Impact	0.9365	0.3313	1.2978	0.0003	1.4549	0.5128	1.9677	
	Bond	Long Run	1.3599	0.6445	2.0044	0.0006	1.8452	0.8591	2.7043	70 (Q.E.)

Note: Q.E. denotes a quais-equilibrium.

Table 4.4 The IR Model
Simulated Effects of a Once-and-for-all Increase in Gl_t by 10%
Applying the Christ Adjustment Method

Parameter Set	Financing Policy	Period	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability at (Time Period)
(1)		Impact	0.6171	0.0991	0.7162	0.0007	1.1498	0.2426	1.3924	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
(2)		Impact	0.7202	0.2022	0.9224	0.0003	1.2602	0.3530	1.6132	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
(3)		Impact	0.9265	0.3213	1.2478	0.0003	1.4394	0.4973	1.9367	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
(4)		Impact	0.9265	0.3213	1.2478	0.0003	1.4394	0.4973	1.9367	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable

4.5 Applying the Christ Adjustment Method to the IR Model

Applying the Christ adjustment method, we set total government expenditures (GT) at a constant level, we then have

(4.47) $GT1_t = G1_t + TR1_t + R_tB1_t$

(4.48) $GT2_t = G2_t + TR2_t + R_tB2_t$

(4.49) $G1_t = GT1_t - TR1_t - R_tB1_t$

(4.50) $G2_t = GT2_t - TR2_t - R_tB2_t$

The regional income identity and the dynamic equations are adjusted in the following fashion, other equations of the IR model remaining intact:

(4.51) $Y1_t = E1_t+(GT1_t-TR1_t-R_tB1_t)+IM2_t-IM1_t$

(4.52) $Y2_t = E2_t+(GT2_t-TR2_t-R_tB2_t)+IM1_t-IM2_t$

(4.53) $DW1_t = (GT1_t-TR1_t-R_tDB1_t)+TR1_t+IM2_t-IM1_t-u1_t(Y1_t+TR1_t+R_tH1_t)$

(4.54) $DB1_t = GT1_t-u1_t(Y1_t+TR1_t+R_tH1_t)$

(4.55) $DB2_t = GT2_t-u2_t(Y2_t+TR2_t+R_t(B1_t+B2_t-H1_t))$

The results of Table 4.4 show that the long-run system becomes unstable, regardless of which parameter set is used. Even a quasi-equilibrium is not attainable.

The basic idea underlying the Christ adjustment method is that by treating total government expenditures constant in the GBC, any changes of interest obligation would be adjusted in the regional income identity, leaving the GBC free from any source of instability. The Christ adjustment method works in the National Model but fails in the IR model. It is worthwhile to examine the basic reason underlying this failure.

Consider again a once-and-for-all increase in $G1_t$ of 10 percent, i.e., $GT1_t$ is increased by the same amount as $G1_t$. Concentrating on the fiscal relationships between two regional governments, we examine the two government budget constraints. For the first region, a once-and-for-all increase in $GT1_t$ necessitates the issuing of new bonds in the second and subsequent periods. Since now $GT1_t$ becomes constant at a higher level, the budget will be in balance provided that the tax revenues are large enough to cover the higher level of expenditures. Therefore the Christ adjustment method would seem workable for the first region.

For the second region, $GT2_t$ remains at its initial level, and any changes of tax revenues will be offset by corresponding opposite changes in the outstanding stocks of bonds. Higher regional income will generate a higher tax revenue in the second region. With $GT2_t$ constant throughout the analysis, bonds ($B2_t$) must be retired in later periods in the second region. $GT2_t$ remains constant throughout the analysis, therefore What makes (4.55) different from (4.13) is that interest payments do not appear in (4.55). Thus (4.55) partially eliminates an aggravating force which results in an ever-increasing retirement of $B2_t$, a major source of instability in the second region and in the system as a whole.

Unfortunately, this is not the end of the story. The influence on the regional income identity must also be considered. The regional aggregate demand is affected inversely by any changes in the region's interest obligation. Higher interest payments due to more bond-issuing by the first region exert a dampening effect on the first region's aggregate demand, thus reducing the tendency to generate enough tax revenues to cover the higher level of total government expenditures. On the other hand, lower interest obligations due to retirement of outstanding bonds in the second region exert an expansionary effect on the second region's aggregate demand, thereby increasing its tax revenues. Higher tax revenues necessitate an even larger amount of retirement of the second region's outstanding bonds. The system then becomes unstable.

The Christ adjustment method works well for the National Model as discussed in Section 3.5 of Chapter 3, but fails in the IR model simply because of the conflicting requirements for stability in the two regions.

4.6 Need for Introducing the Federal Sector

After applying the Christ adjustment method, fiscal policy, including pure bond-financing policy, works for the National Model. However, even if the Christ adjustment method is applied, fiscal policy fails to generate a stable equilibrium in the IR model. Both models represent extreme special cases of the complete interregional model with a federal sector. The above conclusions, although they may be of interest in their own right, should not be given too much weight in appraising the actual effects of fiscal policy.

The fact is that the National Model considers only one federal government budget constraint while the IR model examines only the regional government budget constraints, and both are therefore unable to tell the whole story of fiscal and financial interrelationships between two levels of government. The success of fiscal policy in the aggregate National Model may be attributed to the suppression of certain intrinsic unstable forces, or the failure of fiscal policy in the IR model may be due to the omission of some variables or relationships which may bring stability to the system.

The Oates model considers only a small local economy and assumes the existence of a long-run equilibrium, or the stability of the system, without proving it. Even if we apply the Christ adjustment method to Oates' one-region government sector model to ensure stability, the Oates model at best approximates our National Model, which we have argued is an extreme case of the more general interregional model.

Only when we put the government budget constraints of all governments explicitly into an interregional framework, can we investigate realistically the actual fiscal effects and stability of such a system. Two possibilities may arise:

(1) The system is stable because the strength of stable federal fiscal policy outweighs the unstable interregional fiscal policy.

(2) The system is unstable because the strength of unstable interregional fiscal policy outweighs the stable federal fiscal policy.

These possibilities are investigated in the next chapter.

4.7 Conclusion

In this chapter we have presented our interregional model without the federal sector, and discussed interregional fiscal policy in that context. We find that the system is in a quasi-equilibrium, in that the aggregate regional budget is balanced, but the individual regional budgets are not. The acceptability of such a quasi-equilibrium has to be rejected. The system is unstable if the Christ adjustment method is applied. It reflects the conflicting requirements for stability in the two regions. Thus we argue that for a realistic and complete model, the federal sector should be added to the interregional model.

Footnotes to Chapter 4

(1). This model is an interregional model without a federal sector. We assume away any federal influence except that a fixed level of money stock is provided by the central bank The reason for this federal influence is that regional governments cannot control the money supply. In this sense, the federal presence is slipped in through the back door. However, no federal influence is exerted in the model.

(2). Through substitution, we have

$$\begin{aligned} M2_t &= M_t - M1_t \\ &= M_t - (W1_t - H1_t) \\ &= M_t - W1_t + H1_t \end{aligned}$$

$$\begin{aligned} W2_t &= M2_t + H2_t \\ &= M_t - W1_t + H1_t + (B1_t + B2_t - H1_t) \\ &= M_t + B1_t + B2_t - W1_t \end{aligned}$$

(3). The endogeneity property of money in the Oates model is different from the situation in the IR model. In the Oates model, the endogenous variable A (i.e., net financial asset) consists of money and bonds. Oates argues that residents can adjust their portfolios through transactions with nonresidents. Since the rate of interest is assumed constant or exogenously determined, Oates argues that the portfolio adjustment need not be dealt with explicitly. Thus in the Oates model, only the endogenous variable A is determined, but the exact distribution of M and B is indeterminate. In the IR model, the endogeneity of M1 and H1 acts as a market force to achieve a unique rate of interest in both regions. Therefore we can determine the exact distribution of the fixed sum of money across regions.

(4). See Appendix 4.1 for the discussion of the determinants of the signs of e_2 and e_4.

(5). See Yamane (1962), pp.365-378.

(6). In this case it would mean $E1_t = E2_t = 40$ and the total private expenditures account for 85.1% of aggregate demand, a ratio higher than in the National Model.

(7). We assume that only the private sector is involved in interregional trade. The public sector also consumes imports but does not import directly from the other region.

(8). This is demonstrated by substituting the gross regional income identity and the balanced budget into the disposable income equation:

$$\begin{aligned} (4.7).\ YD1_t &= (1-u1_t)(Y1_t+TR1_t+R_tH1_t) \\ &= Y1_t+TR1_t+R_tH1_t-u1_t(Y1_t+TR1_t+R_tH1_t) \\ &= (E1_t+G1_t+IM2_t-IM1_t)+TR1_t+R_tH1_t-u1_t(Y1_t+TR1_t+R_tH1_t) \end{aligned}$$

$$= E1_t + (IM2_t - IM1_t + R_t(H1_t - B1_t))$$

$$+ (G1_t + TR1_t + R_tB1_t - u1_t(Y1_t + TR1_t + R_tH1_t))$$

$$= E1_t + 0 + 0$$

$$= E1_t$$

Similarly, it can be shown that $YD2_t = E2_t$.

Chapter 5

Analysis Based on an Interregional Model With a Federal Sector

5.1 Introduction

The preceding chapter provided an analysis of interregional fiscal policy based on a model with no federal sector. The main purpose was to examine the complexities involved when interregional connections are brought into consideration. Stability conditions become more stringent when two more budget constraints must be added to the dynamic system. As pointed out in the last chapter, the system has intrinsic instability which works against the attainment of a long-run steady-state equilibrium, and only a quasi-equilibrium can be achieved.

One positive finding from the last chapter is the important role that the federal sector may play in the success of interregional fiscal policy. This does not mean a retreat to the conventional wisdom that only the federal government can implement stabilization policy. Rather, in the new setting, where regional and federal governments interact within the whole system, there is no such thing as "independent" fiscal policy, either federal or regional.[1] The importance of a government budget constraint, therefore, constitutes not just part of the dynamics of the macroeconomic model, but more important, it also introduces a basic conceptual change in the philosophy of fiscal policy.

In the interregional model, regional income is the main welfare indicator for each region. The federal sector acts as a coordinator, helping a regional government to achieve its fiscal objectives. Whether the introduction of a federal sector into the interregional model can produce a stable long-run equilibrium is a novel idea to pursue.

In the next section of this chapter we present and discuss the interregional model with a federal sector (IRF). Section 5.3 is devoted to the analysis of the interplay between regional and federal fiscal policies. Section 5.4 describes the simulation approach to be followed, and simulation results are discussed. In section 5.5 we apply the Christ adjustment method to the expanded system. In section 5.6 we present two ad hoc fiscal schemes which could bring stability to the system. Their implications for the construction of an interregional model and the analysis of interregional fiscal policy are discussed. Section 5.7 summarizes the conclusions which follow from the earlier sections.

5.2 An Interregional Model With a Federal Sector

The interregional model with a federal sector (IRF) is an extended version of the IR model developed in Chapter 4, obtained by adding a federal sector to each region. The IRF model is a two-region, six-sector aggregate demand model. The six sectors of the model are the private sector, regional government sector, and the federal sector in each of the two regions. The status of the federal sector in each region is defined by the federal budgetary position in the region. The behavior of the government sectors is assumed to be exogenous, while the behavior of the other sectors is fully explained within the model. All assumptions in the IR model will apply to the IRF model as well. The IRF model represents the extension of the National Model into an interregional framework.

We present the equations of the IRF model, along with notation and explanations.

Equations of the IRF Model

(5.1) $Y1_t = E1_t + G1_t + GF1_t + IM2_t - IM1_t$

(5.2) $Y2_t = E2_t + G2_t + GF2_t + IM1_t - IM2_t$

(5.3) $E1_t = E1(YD1_t, W1_t, R_t)$

(5.4) $E2_t = E2(YD2_t, M_t + B1_t + B2_t + BF_t - W1_t, R_t)$

(5.5) $IM1_t = IM1(YD1_t, W1_t, R_t)$

(5.6) $IM2_t = IM2(YD2_t, M_t + B1_t + B2_t + BF_t - W1_t, R_t)$

(5.7) $YD1_t = (1 - u1_t - uF_t)(Y1_t + TR1_t + TRF1_t + R_t H1_t)$

(5.8) $YD2_t = (1 - u2_t - uF_t)(Y2_t + TR2_t + TRF2_t + R_t(B1_t + B2_t + BF_t - H1_t))$

(5.9) $W1_t - H1_t = L1(Y1_t, W1_t, R_t)$

(5.10) $M_t - W1_t + H1_t = L2(Y2_t, M_t + B1_t + B2_t + BF_t - W1_t, R_t)$

(5.11) $DW1_t = G1_t + GF1_t + TR1_t + TRF1_t + R_t H1_t + IM2_t - IM1_t$
$- (u1_t + uF_t)(Y1_t + TR1_t + TRF1_t + R_t H1_t)$

(5.12) $DB1_t = G1_t + TR1_t + R_t B1_t - u1_t(Y1_t + TR1_t + TRF1_t + R_t H1_t)$

(5.13) $DB2_t = G2_t + TR2_t + R_t B2_t - u2_t(Y2_t + TR2_t + TRF2_t + R_t(B1_t + B2_t + BF_t - H1_t))$

(5.14) $DM_t + DBF_t = GF1_t + GF2_t + TRF1_t + TRF2_t + R_t BF_t - uF_t(Y1_t + Y2_t$
$+ TR1_t + TR2_t + TRF1_t + TRF2_t + R_t(B1_t + B2_t + BF_t))$

(5.15) $DW1_t = W1_{t+1} - W1_t$

(5.16) $DB1_t = B1_{t+1} - B1_t$

(5.17) $DB2_t = B2_{t+1} - B2_t$

(5.18) $DM_t = M_{t+1} - M_t$

(5.19) $DBF_t = BF_{t+1} - BF_t$

Federal financing policy option:

(5.20a) $DM_t = 0$ federal pure bond-financing policy

(5.20b) $DBF_t = 0$ federal pure money-financing policy

Endogenous variables: $Y1_t, Y2_t, E1_t, E2_t, IM1_t, IM2_t, YD1_t, YD2_t, R_t, H1_t, DW1_t,$
$DB1_t, DB2_t, DBF_t, DM_t, W1_{t+1}, B1_{t+1}, B2_{t+1}, BF_{t+1}, M_{t+1}.$

Predetermined variables: $G1_t, G2_t, GF1_t, GF2_t, TR1_t, TR2_t, TRF1_t, TRF2_t,$
$u1_t, u2_t, uF_t, W1_t, B1_t, B2_t, BF_t, M_t.$

Assumptions about partial derivatives are as follows:

$0 \leqslant E1_{YD1} \leqslant 1$	$E1_R \leqslant 1$	$E1_{W1} \geqslant 0$
$0 \leqslant E2_{YD2} \leqslant 1$	$E2_R \leqslant 1$	$E2_{W2} \geqslant 0$
$0 \leqslant IM1_{YD1} \leqslant 1$	$IM1_R \geqslant 1$	$IM1_{W1} \geqslant 0$
$0 \leqslant IM2_{YD2} \leqslant 1$	$IM2_R \geqslant 1$	$IM2_{W2} \geqslant 0$
$L1_{Y1} \geqslant 0$	$L1_R \leqslant 0$	$0 \leqslant L1_{W1} \leqslant 1$
$L2_{Y2} \geqslant 0$	$L2_R \leqslant 0$	$0 \leqslant L2_{W2} \leqslant 1$

Notation :

1 — Region 1

2 — Region 2

B — Stock of outstanding government bonds

DB — Changes of stock of outstanding government bonds

DM — Changes of money stock

DW — Changes of domestic financial wealth

E — Private expenditures

F — The federal sector

G — Government expenditures on goods and services

H — Domestic holding of government bonds

M — Outstanding money stock

R — Market rate of interest

t -- Time period

TR — Transfer payments

u -- Proportional tax rate

W — Domestic financial wealth

Y — Gross regional product

YD — Private disposable income

We include the federal expenditures on goods and services in the regional income identity (5.1) and (5.2). Note that only those federal expenditures on goods and services in the regions (i.e., $GF1_t$, $GF2_t$) are included in the calculation of regional aggregate demand. Equations (5.3) to (5.10) are self-explanatory, having the same interpretation as in the IR model. Note also that the same federal proportional tax rate is applied in both regions. The $H1_t$ and $W1_t$ variables are augmented by the bonds issued by the federal government. Additional notation is as follows:

$B1_t$ -- Bonds issued by the first regional government

$B2_t$ — Bonds issued by the second regional government

BF_t -- Bonds issued by the federal government

$H1_t$ -- Bonds held by residents of the first region

$H2_t$ -- Bonds held by residents of the second region

M_t -- Money stock issued by the federal government

$M1_t$ -- Money stock held by residents of the first region

$M2_t$ — Money stock held by residents of the second region

We then have

(5.21) $H1_t + H2_t = B1_t + B2_t + BF_t$

(5.22) $M_t = M1_t + M2_t$

(5.23) $W1_t = M1_t + H1_t$

(5.24) $W2_t = M2_t + H2_t$

To simplify the system through substitution, we use the left-hand side expressions of equations (5.25) to (5.28) to represent the right-hand side variables in the IRF model.[2]

(5.25) $W1_t - H1_t$ for $M1_t$

(5.26) $M_t - W1_t + H1_t$ for $M2_t$

(5.27) $B1_t + B2_t + BF_t - H1_t$ for $H2_t$

(5.28) $M_t + B1_t + B2_t + BF_t - W1_t$ for $W2_t$

The private-sector budget constraint $DW1_t$ is augmented by federal budgetary position in the region, while the regional government budget constraint remains intact. The whole dynamic system is augmented by the addition of the federal government budget constraint (5.14) and its associated financing policy equations (5.20a) and (5.20b). The federal government can print new money or issue new bonds to finance its budget deficit. Note that there is only one budget constraint for the federal government. Though we have a federal sector in both regions, it does not necessarily follow that we should have a budget constraint for the federal sector in each region because, from the federal government perspective, it is only the overall federal accounts that are subject to a binding constraint. Imposing regional federal sector budget constraints would reduce the federal government's degrees of freedom in pursuing its fiscal policy. To complete the model, (5.20a) specifies the pure federal bond-financing policy while (5.20b) specifies the pure federal money-financing policy.

In the IR model, we have demonstrated that only one of the private-sector budget constraints need to appear explicitly in the model. In the IRF model, we have a federal sector in both regions, but have only one aggregate federal government budget constraint. We may suspect whether the arguments presented in the IR model would apply to the IRF model as well. To demonstrate, following the same argument as presented in the last chapter, we have

$$\begin{aligned}(5.29)\quad DW1_t &= YD1_t - E1_t \\ &= (1-u1_t-uF_t)(Y1_t+TR1_t+TRF1_t+R_tH1_t)-E1_t \\ &= (Y1_t-E1_t)+TR1_t+TRF1_t+R_tH1_t-(u1_t+uF_t)(Y1_t+TR1_t+TRF1_t+R_tH1_t) \\ &= G1_t+GF1_t+IM2_t-IM1_t+TR1_t+TRF1_t+R_tH1_t-(u1_t+uF_t)(Y1_t+TR1_t \\ &\quad +TRF1_t+R_tH1_t)\end{aligned}$$

By the same token, we have

$$\begin{aligned}(5.30)\quad DW2_t &= G2_t+GF2_t+IM1_t-IM2_t+TR2_t+TRF2_t+R_t(B1_t+B2_t+BF_t-H1_t) \\ &\quad -(u2_t+uF_t)(Y2_t+TR2_t+TRF2_t+R_t(B1_t+B2_t+BF_t-H1_t))\end{aligned}$$

To examine whether the public sector budget constraints and the rest of the system have already implied the private sector budget constraint, we investigate (5.11) again:

$$\begin{aligned}(5.31)\quad DW1_t &= G1_t+GF1_t+TR1_t+TRF1_t+IM2_t-IM1_t+R_tH1_t \\ &\quad -(u1_t+uF_t)(Y1_t+TR1_t+TRF1_t+R_tH1_t) \\ &= (G1_t+TR1_t+R_tB1_t-u1_t(Y1_t+TR1_t+TRF1_t+R_tH1_t)) \\ &\quad +(GF1_t+TRF1_t+R_tBF1_t-uF_t(Y1_t+TR1_t+TRF1_t+R_tH1_t)) \\ &\quad +(R_t(H1_t-B1_t-BF1_t)+IM2_t-IM1_t) \\ &= DB1_t+DBF1_t+(R_t(H1_t-B1_t)+IM2_t-IM1_t)\end{aligned}$$

Equation (5.31) is derived under the federal pure bond-financing policy.

If the federal government adopts a pure money-financing policy, we then have

(5.32) $DW1_t = DB1_t + DM1_t + (R_t(H1_t - B1_t) + IM2_t - IM1_t)$

Though we have argued that there is no compelling reason that we should have a separate budget constraint for the federal sector in each region, the use of $BF1_t$, $DBF1_t$, and $DM1_t$ is for expository purposes. Equation (5.31) indicates that the budget constraints of the regional government and the federal government for a region along with the current balance of payments between regions, imply the private sector budget constraint. The public sector budget constraints therefore do not rule out the need for an explicit private sector budget constraint in the IRF model.

To examine whether we must consider both of the private sector budget constraints in the dynamic system, we set $DW1_t$ to zero and set an expression for the trade balance between regions (i.e., $IM1_t - IM2_t$). We then substitute this into (5.30):

(5.33) $$\begin{aligned} DW2_t &= G2_t + GF2_t + (G1_t + GF1_t + TR1_t + TRF1_t + R_tH1_t - (u1_t + uF_t)(Y1_t + TR1_t \\ &\quad + TRF1_t + R_tH1_t)) + R_t(B1_t + B2_t + BF_t - H1_t) - (u2_t + uF_t)(Y2_t + TR2_t \\ &\quad + TRF2_t + R_t(B1_t + B2_t + BF_t - H1_t)) \\ &= DB1_t + DB2_t + DM_t + DBF_t + DW1_t \end{aligned}$$

If any four of the dynamic equations satisfy the equilibrium condition, the fifth one must also satisfy the equilibrium condition. Thus the dynamic system consists of only four independent equations. In the above equations of the IRF model, $DW2_t$ does not appear explicitly.

5.3 Study of Fiscal Policy

In this section, we study an interregional as well as a federal fiscal policy.

5.3.1 Static System

The static system consists of the first ten equations, (5.1) to (5.10), with the following endogenous and predetermined variables:

Endogenous variables: $Y1_t, Y2_t, E1_t, E2_t, IM1_t, IM2_t, YD1_t, YD2_t, H1_t, R_t$.

Predetermined variables: $G1_t, G2_t, GF1_t, GF2_t, TR1_t, TR2_t, TRF1_t, TRF2_t,$
$u1_t, u2_t, uF_t, B1_t, B2_t, BF_t, M_t, W1_t$.

Through substitutions, equation (5.1) to (5.10) can be reduced to the following simple form::

(5.34) $Y1_t$ = Y1(predetermined variables)

(5.35) $Y2_t$ = Y2(predetermined variables)

(5.36) $H1_t$ = H1(predetermined variables)

(5.37) R_t = R(predetermined variables)

We next differentiate the above system and solve for multiplier effects:[3]

(5.38) $dY1_t/dG1_t=dY1_t/dGF1_t=e_3/(e_1e_3-e_2e_4)$

(5.39) $dY1_t/dG2_t=dY1_t/dGF2_t=e_2/(e_1e_3-e_2e_4)$

(5.40) $dY2_t/dG1_t=dY2_t/dGF1_t=e_4/(e_1e_3-e_2e_4)$

(5.41) $dY2_t/dG2_t=dY2_t/dGF2_t=e_1/(e_1e_3-e_2e_4)$

(5.42) $dH1_t/dG1_t=dH1_t/dGF1_t=(L_{13}L_{21}e_4-L_{11}L_{23}e_3)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(5.43) $dH1_t/dG2_t=dH1_t/dGF2_t=(L_{13}L_{21}e_1-L_{11}L_{23}e_2)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(5.44) $dR_t/dG1_t=dR_t/dGF1_t=-(L_{11}e_3+L_{21}e_4)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(5.45) $dR_t/dG2_t=dR_t/dGF2_t=-(L_{11}e_2+L_{21}e_1)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

The short-run effects of fiscal policy are summarized in Table 5.1:

Table 5.1: The IRF Model
Signs of Short-Run Multiplier Effects

Derivative of	With Respect to $G1_t=GF1_t$	$G2_t=GF2_t$
$Y1_t$	+	+
$Y2_t$	+	+
$H1_t$	-	+
R_t	+	+

In qualitative forms, the short-run effects are virtually the same as in the IR model, since all the dynamic equations are irrelevant for short-run analysis. The diagramatic approach of Fig.4.1 and Fig.4.2 can also be applied here. It is only the long-run analysis that differs between the IR and IRF models.

5.3.2 Long-Run System

In the long run we must consider the full model, including the private and public sector budget constraints. The long-run equilibrium is defined as the situation that exists when all exogenous variables remain constant and all changes of endogenous variables are zero. Having one more equation in the dynamic system would make the stability conditions more stringent, especially when the federal policy is pure bond-financing. Since the IRF model can be considered as a combination of the National and the IR models, it is interesting to consider whether the stable force of federal money-financing can outweigh the intrinsic instability of the IR model.

In the complete interregional model in which the budget constraints of all governments interact, we want to examine not only whether the introduction of a federal sector can outweigh the unstable forces, but also whether the intrinsically stable federal money-financed fiscal policy would be hampered by the intrinsically unstable nature of interregional fiscal relationships. These questions, particularly the last one, will have important implications for the federal-regional fiscal relationships.

5.3.2.1 Interregional Fiscal Policy

Suppose there is a once-and-for-all increase in government expenditures in the first region ($G1_t$). The long-run effect will be analysed under different assumptions about federal financing policy.

(a) Federal Pure Bond-Financing Policy

Stability of the system requires that all private and public sector budget constraints be balanced. In the IS-LM framework, this requirement means that in the long run, the IS-LM curves of both regions must be stable. Under the assumption of a federal pure bond-financing policy, the money stock remains unchanged throughout the analysis. The shift of the LM curve in one region will be completely offset by an opposite shift of the LM curve in the other region. The LM curves will remain stable if the market rate of interest is the same in both regions and there is no incentive for any changes in the stocks of bonds in the two regions.

For the first region, a once-and-for-all increase in government expenditure will increase regional income through multiplier effects. Of course, regional bonds must be issued in the second and subsequent periods to cover the higher level of government expenditures. One condition for stability is that the required new bond issues decrease in each subsequent period. Whether this condition is satisfied depends on whether the increased tax revenues would cover the higher level of regional government expenditures without new regional bond issues.

The second region also benefits from the expansionary fiscal policy in the first region through a higher export surplus. Higher tax revenues which result from higher incomes would create a budget surplus in the second region. To balance its budget, the second regional government must retire outstanding bonds so that total interest payments will be reduced.

The federal government also experiences a budget surplus because incomes are increasing in both regions. To balance its budget, the federal government must also retire its outstanding bonds so that its interest payments will be reduced.

Taking all three government budget constraints together, we have the first regional government issuing new bonds while the federal and the second regional government are retiring outstanding bonds. What is the total impact on the private-sector budget constraint, and in general, the stability of the system? These questions are answered by examining the LM curves. The LM curve in the first region will shift rightward through sales of Hl_t to the second region for the purpose of acquiring more money balances to accommodate increasing economic activity. The LM curve in the second region will shift leftward by an equal magnitude through the retirement of outstanding bonds and capital movements to the first region so as to keep the rate of interest at the same level in both regions. The retirement of federal bonds will reinforce the rightward shifting of the LM_1 curve if the federal government uses all the increased tax revenues to retire the outstanding federal bonds in the first region. On the other hand, if the federal government uses all the increased tax revenues to retire the outstanding bonds in the second region, the rightward shifting of LM_1 will be reduced. The composition of financial assets in each region will therefore be affected by the federal government's policy.

The above adjustment process has one intrinsic unstable force working against the attainment of a stable equilibrium. In the first region, unless the increase in tax revenues is higher than the increase in interest obligations on outstanding bonds, the first regional government must issue more new bonds in each subsequent period. In the case of the second regional government and federal government, unless the increase in tax revenues is equal to the decrease in interest obligations due to the retirement of outstanding bonds, these two governments have to retire more outstanding bonds in each subsequent period.

The private sector budget constraint will be satisfied, if either no new bonds are issued or the aggregate of new bonds issued is zero. The system then works in such a way that the private holdings of bonds (Hl_t) remain unchanged at their new level. We have argued in the last chapter that the second condition is not an acceptable definition of stability. In this case, the government budget constraints are not all satisfied, but a positive increase in the bonds of any one government due to exogenous increase in its expenditures will be offset exactly by the combined decrease in the bonds of the other two governments. As Hl_t remains constant in the long run, it means that the other two governments are continuously buying the additional bonds. However, this explanation contradicts our premise that only residents hold government bonds. Moreover, requiring the aggregate of new bonds issued being zero as a condition for stability implies that we consider only one aggregate government budget constraint. It would be incompatible with the theme of studying interregional fiscal policy.

With all these conflicting forces interacting within the system, we do not know whether the introduction of the federal sector will reinforce the tendency for instability or produce a stable equilibrium. In view of the fact that bond-financing under the National Model is very sensitive to the structure of the system, various sets of parameter values will be used in the simulations to investigate the stability problem. In Table 5.2A of the next section, parameter sets (2) to (4) produce quasi-equilibria while parameter set (1) generates unstable results.

(b) Federal Pure Money-Financing Policy

The shifting of the LM curves is affected significantly when the federal government adopts a pure money-financing policy. As in the last section, the first regional government issues new bonds while the second regional government retires outstanding bonds. The federal government, with increasing tax revenues and a budget surplus, must contract the outstanding money stock to balance its budget.

With a smaller money stock available, the market rate of interest would be higher than in the last case. The interest obligation for the first regional government would be greater, thus increasing the likelihood of instability. The second regional government would then have to retire its outstanding bonds at an even faster rate in order to shift a higher portion of money stock to the first region for its fiscal expansion.

This adjustment process is hindered by two factors:

First, due to the fiscal expansionary policy implemented in the first region, the increase in regional income is greater in the first than in the second region. Consequently the increasing money demand in the first region is greater than the amount which can be released by, and shifted from, the second region.

Second, the contraction of an outstanding money stock by the federal government to balance its budget leaves no room for the regional government to maneuver, except in bidding up the market rate of interest to whatever level it requires to balance the regional budget. We might arrive at a point at which the interest rate is so high that even when all the money stock shifts to the first region, the stability condition cannot be satisfied.

Contrary to our original thought that the introduction of a federal sector may produce a stable equilibrium, we find that under federal pure money-financing policy, the tendency towards instability is increased because of the contraction of the outstanding money stock. In Table 5.2A of the next section, all parameter sets produce unstable results under federal money-financing policy.

5.3.2.2 Federal Fiscal Policy

Suppose there is a once-and-for-all increase in federal spending in the first region $GF1_t$. The long-run effects will be analysed for different modes of federal financing policies.

(a) Federal Pure Bond-Financing Policy

For the first region, a once-and-for-all increase in federal government expenditures will increase income. Tax revenues will be increased. Realizing a budget surplus, the first regional government will have to retire its outstanding bonds in order to balance its budget.

For the second region, income will be increased through a higher export surplus. Thus, with a budget surplus, the second regional government must retire its outstanding bonds, though to a lesser extent than the government of the first region.

For the federal government, bonds must be issued in the second and subsequent periods in order to finance a once-and-for-all increase in $GF1_t$. The system will be stable provided that all budgets are balanced, and the federal budget constraint plays a dominant role. If the federal tax revenue is high enough to cover the higher level of federal spending, there is no need for further bond issuing. Whether the federal budget is balanced or not depends on the interaction of the whole system.

To satisfy the regional government budget constraints, changes of regional government revenues and expenditures must be zero. Though there are no exogenous changes in government expenditures on goods and services in both regions, the induced change in interest payments, through changes in the market rate of interest and retirement of outstanding bonds, may become a source of instability. The market rate of interest is determined by the interaction of IS-LM curves, but there are conflicting forces in the bond market associated with the increase in the number of federal bonds and the retirement of outstanding regional bonds. If in aggregate the new bonds issued are greater than the outstanding bond retirements, then the market rate of interest will rise. A higher rate of interest, along with the reduction of outstanding regional bonds, may cause interest payments to decrease, to increase, or to remain unchanged.

The decreasing interest obligations of both regions over time imply budget surpluses which would lead to further bond retirement, thus aggravating the disequilibrium condition. Therefore the system is unstable for the case of decreasing interest obligation in both regions. If the interest obligations remain constant, the system is still unstable because higher regional tax revenues necessitate further retirement of bonds. Only in the case of increasing interest obligations would there be the possibility of a stable equilibrium. This is evident from the government budget constraint: if the increasing interest payments exactly offset the increasing tax revenues, then there will be no need for further issuing of regional bonds.

On the other hand, if in aggregate the new bonds issued are fewer than the outstanding bonds retired, the equilibrium market rate of interest will depend on the magnitude of the shifting of the IS curves. Stability in this case is less likely.

Bond-financed federal fiscal policy in the National Model, depending on the structure of the system, can generate stable equilibrium. The analysis of bond-financed federal fiscal policy in an interregional setting with three more budget constraints does not rule out the possibility, however remote it might be, of reaching a stable equilibrium.

There is also the possibility that the system will achieve quasi-equilibrium, i.e., that the private sector budget constraint and the aggregate of all public sector budget constraints will be satisfied. This outcome is possible because of the perfect substitutability among the bonds of the three governments. As discussed before, the requirement for ever-increasing or decreasing government bonds is unacceptable for political reasons. In Table 5.2B of the next section, parameter sets (2) to (4) produce quasi-equilibria while parameter set (1) generates unstable results.

(b) Federal Pure Money-Financing

The regional governments, as in the last case, will retire their outstanding bonds in order to balance their respective budgets.

For the federal government, new money must be printed in the second and subsequent periods in order to finance a once-and-for-all increase in $GF1_t$. Pure money-financing fiscal policy in the National Model is stable. However, in the IRF model the stability of the whole system depends on whether the regional government budget constraints are satisfied. We have argued that the rate of interest must be high enough to offset the effect of the retirement of outstanding bonds in order to generate an increase in interest obligations, which is the only possible way that the regional government budget constraints can be satisfied.

This condition, however, cannot be achieved under federal pure money-financed policy since the market rate of interest decreases over time when new money is injected into the economy to finance higher federal spending. These combined effects reduce the interest obligation for both regional governments over time. Consequently internal pressure for further bond retirement is generated in both regions.

A pure money-financed federal fiscal policy in the National Model is stable. The pure money-financed federal fiscal policy in an interregional setting, surprisingly, is unstable . The intrinsically unstable nature of interregional bond-financing eventually overshadows the stablizing tendency of federal money-financing. Table 5.2B of the next section shows that all parameter sets fail to generate a stable equilibrium under a federal money-financing policy.

5.3.3 Formal Stability Analysis

The formal stability analysis of the IRF model follows the same path as in the IR model except that the system is augmented by BF_{t+1} (or M_{t+1}) under federal bond (or money)-financing policy. Because of the complexities of the expressions for the a and c terms, the formal tests for stability are not carried out.

5.4 A Simulation Approach to the Analysis of Fiscal Policy in the IRF Model

5.4.1 Specification of Functional Form

We assume the same functional forms as adopted in the IR model.

5.4.2 Specification of Initial Values

The following initial values are derived from the National Model are divided equally between the two regions, which are assumed to be identical. Some of the regional data are presented in footnote 6 of Chapter 3. It is assumed that the system is in an initial stationary equilibrium, with a balanced budget for each government, and a zero balance of payments between regions. The values of the proportional tax rate follow from the balanced-budget assumption.

$Y1_t = Y2_t = 50.0$ $\qquad E1_t = E2_t = 40.0$

$G1_t = G2_t = 7.0$ $\qquad GF1_t = GF2_t = 3.0$

$YD1_t = YD2_t = 40.0$ $\qquad IM1_t = IM2_t = 12.0$

$TR1_t = TR2_t = 3.0$ $\qquad TRF1_t = TRF2_t = 4.8$

$B1_t = B2_t = 17.1$ $\qquad u1_t = u2_t = 0.1818$

$R_t = 0.05$ $\qquad W1_t = 53.5$ $\qquad H1_t = 37.5$

$M_t = 31.5$ $\qquad BF_t = 41.3$ $\qquad uF_t = 0.1415$

$DW1_t = DB1_t = DB2_t = DBF_t = DM_t = 0$

5.4.3 Specification of Parameter Values

Following assumptions and procedures similar to those adopted in the last chapter, four parameter sets are constructed. Each region has the same parameter sets.

Parameter Set (1)

$$E1_t = 5.87+0.8YD1_t+0.107W1_t-72.0R_t$$

$$IM1_t = 1.77+0.24YD1_t+0.0321W1_t-21.6R_t$$

$$W1_t-H1_t = 9.45+0.189Y1_t+0.12W1_t-189.0R_t$$

The second region has the same parameter values.

Parameter Set (2)

$$E1_t = 3.47+0.8YD1_t+0.107W1_t-24.0R_t$$

$$IM1_t = 1.05+0.24YD1_t+0.0321W1_t-7.2R_t$$

$$W1_t-H1_t = 28.35+0.189Y1_t+0.12W1_t-567.0R_t$$

The second region has the same parameter values.

Parameter Set (3)

$$E1_t = -0.5+0.9YD1_t+0.107W1_t-24.0R_t$$

$$IM1_t = -0.15+0.27YD1_t+0.0321W1_t-7.2R_t$$

$$W1_t-H1_t = 28.35+0.189Y1_t+0.12W1_t-567.0R_t$$

The second region has the same parameter values.

Parameter Set (4)

$$E1_t = -5.5+0.9YD1_t+0.2W1_t-24.0R_t$$

$$IM1_t = -1.65+0.27YD1_t+0.06W1_t-7.2R_t$$

$$W1_t-H1_t = 28.35+0.189Y1_t+0.12W1_t-567.0R_t$$

The second region has the same parameter values.

5.4.4 Simulation Results

Table 5.2A shows the results of a once-and-for-all increase of 10 percent in the expenditures of first regional government, while Table 5.2B shows the results of a once-and-for-all increase in federal government expenditures in the first region of 10 percent. Each table reports the simulated effects of different sets of parameter values.

In the initial case, tax rates are exogenous and there is no adjustment through the government expenditure function or the dynamic equations. In Table 5.2A, we observe that for parameter set (1), the system is unstable regardless of which mode of federal financing is used. While parameter sets (2), (3) and (4) produce a long-run equilibrium only under a federal bond-financing policy, it is a quasi-equilibrium , in the sense in which we have defined that term. A similar outcome is also observed in Table 5.2B, pertaining to the analysis of federal fiscal policy.

The simulation results confirm our analysis of the long-run stability of the system. The introduction of a federal sector does not offset the destabilizing influence of interregional fiscal policy. Indeed, even in isolation, federal fiscal policy fails to produce stable results.

Table 5.2.A: The IRF Model
Simulated Effects of a Once-and-for-all Increase in Gl_t by 10%

Parameter Set	Federal Financing Policy	Period	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability
(1)		Impact	0.5648	0.0928	0.6576	0.0007	1.1232	0.2344	1.3576	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(2)		Impact	0.6385	0.1665	0.8050	0.0003	1.1969	0.3081	1.5050	
	Bond	Long Run	1.3318	0.6466	1.9784	0.0009	1.8264	0.8521	2.6785	Quasi-Equilibrium
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(3)		Impact	0.8058	0.2563	1.0621	0.0003	1.3409	0.4211	1.7620	
	Bond	Long Run	1.1811	0.5234	1.7045	0.0006	1.6839	0.7206	2.4045	Quasi-Equilibrium
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(4)		Impact	0.8058	0.2563	1.0621	0.0003	1.3409	0.4211	1.7620	
	Bond	Long Run	1.1171	0.4713	1.5884	0.0005	1.6233	0.6651	2.2884	Quasi-Equilibrium
	Money	Long Runm	--	--	--	--	--	--	--	Unstable

Table 5.2B: The IRF Model
Simulated Effects of a Once-and-for-all Increase in GFl_t by 10%

Parameter Set	Federal Financing Policy	Period	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability
(1)		Impact	0.2421	0.0398	0.2819	0.0003	0.4814	0.1005	0.5819	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(2)		Impact	0.2736	0.0714	0.3450	0.0001	0.5130	0.1320	0.6450	
	Bond	Long Run	0.5695	0.2760	0.8455	0.0004	0.7815	0.3640	1.1455	Quasi-Equilibrium
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(3)		Impact	0.3454	0.1098	0.4552	0.0001	0.5747	0.1805	0.7552	
	Bond	Long Run	0.5059	0.2240	0.7299	0.0003	0.7213	0.3085	1.0298	Quasi-Equilibrium
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(4)		Impact	0.3454	0.1098	0.4552	0.0001	0.5747	0.1805	0.7552	
	Bond	Long Run	0.4786	0.2019	0.6805	0.0002	0.6956	0.2849	0.9805	Quasi-Equilibrium
	Money	Long Run	--	--	--	--	--	--	--	Unstable

5.5 Applying the Christ Adjustment Method to the IRF Model

As before, one possibility is that the intrinsic instability problem can be avoided by employing the Christ adjustment method and treating total government expenditures as constant for each government. Any changes in interest payments would then be offset by corresponding changes in government expenditures on goods and services.

Letting GT stand for total government expenditures of a particular government, we have:

(5.46) $GT1_t = G1_t+TR1_t+R_tB1_t$

(5.47) $GT2_t = G2_t+TR2_t+R_tB2_t$

(5.48) $GTF1_t = GF1_t+TRF1_t+R_tBF1_t$

(5.49) $GTF2_t = GF2_t+TRF2_t+R_tBF2_t$

Treating the left-hand side of equations (5.46) to (5.49) as constant, we then express G in the following manner:

(5.50) $G1_t = GT1_t-TR1_t-R_tB1_t$

(5.51) $G2_t = GT2_t-TR2_t-R_tB2_t$

(5.52) $GF1_t = GTF1_t-TRF1_t-R_tBF1_t$

(5.53) $GF2_t = GTF2_t-TRF22_t-R_tBF2_t$

All the government expenditure items in the income identity and dynamic equations are replaced by equations (5.50) to (5.53). Rewriting the income identity and dynamic equations, we have

(5.54) $Y1_t = GT1_t-TR1_t-R_tB1_t+GTF1_t-TRF1_t-R_tBF1_t+E1_t+IM2_t-IM1_t$

(5.55) $Y2_t = GT22_t-TR2_t-R_tB2_t+GTF2_t-TRF2_t-R_tBF2_t+E2_t+IM1_t-IM2_t$

(5.56) $DW1_t = GT1_t+GTF1_t+R_t(H1_t-B1_t-BF1_t)+IM2_t-IM1_t-(u1_t+uF_t)(Y1_t$
$+TR1_t+TRF1_t+R_tH1_t)$

$$(5.57)\quad DB1_t = GT1_t - u1_t(Y1_t+TR1_t+TRF1_t+R_tH1_t)$$

$$(5.58)\quad DB2_t = GT2_t - u2_t(Y2_t+TR2_t+TRF2_t+R_t(B1_t+B2_t+BF_t-H1_t))$$

$$(5.59)\quad DM_t+DBF_t = GTF1_t+GTF2_t-uF_t(Y1_t+Y2_t+TR1_t+TR2_t+TRF1_t+TRF2_t +R_t(B1_t+B2_t+BF_t))$$

The primary purpose of the Christ adjustment method is to avoid the instability that arises from changes in debt interest payments. The method works in the National Model but fails in an interregional setting. In the interregional setting, the Christ method eliminates one source of instability, but at the same time introduces a new source of instability. This can be demonstrated by examining equations (5.54) to (5.59).

In the case of a federal fiscal policy, as discussed in the previous section, outstanding regional bonds are retired. Keeping GT constant at the initial value, the regional government budget constraint would be balanced if the tax revenue remained unchanged from its initial level. This requires that the tax bases for the two regional governments remain constant. With higher income in both regions, it then implies that interest income must decrease by the same amount in order to keep the tax base constant at its initial value. The interest income decreases if the market rate of interest declines and the outstanding stock of bonds is reduced. Thus, the possibility that this objective will be achieved is relatively greater under federal money-financing than under bond-financing. On the other hand, the retirement of outstanding regional bonds initiated in (5.57) and (5.58), reduces interest obligations in both regions. The lower $B1_t$ and $B2_t$ values when transmitted to (5.50) and (5.51), would generate higher $G1_t$ and $G2_t$ in both regions, and this will augment $Y1_t$ and $Y2_t$. The increase in aggregate demand will be even greater under federal pure bond-financing policy. Of course, this will also result in higher interest rate. Since regional bonds are retired, interest obligations may be reduced or may rise slightly due to the resulting higher interest rate. On the whole, the increase in $Y1_t$ and $Y2_t$ will be greater than the decrease in interest obligations. Further retirement of regional bonds will be needed and this will have a further destabilizing influence. In short, the system is unstable and tends to explode.

The analogy carries over to the case of interregional fiscal policy. Consider the case in which the first regional government initiates a once-and-for-all increase in government expenditures. Bonds must then be issued in the first region, but bonds must be retired by the second region and the federal government, if pure federal bond-financing is chosen. For budget constraints (5.58) and (5.59) to be satisfied, the tax bases must be unchanged. As $Y1_t$ and $Y2_t$ are increasing, it follows that interest income must be reduced. Any reduction in interest income would mean a higher level of government expenditures in (5.51), (5.52) and (5.53). Thus again the system is unstable.

The system is unstable even when the Christ adjustment method is employed. Only when the basic destabilizing forces are eliminated within the system can we expect the system to generate a stable equilibrium solution. Tables 5.3A and 5.3B depict the results of applying the Christ adjustment method to the cases dealt with in Tables 5.2A and 5.2B. The system is seen to be unstable regardless of the type of federal financing policy.

5.6 Two Fiscal Schemes for Solving the Stability Problem

Following the argument in the last section, the simplest way to satisfy the budget constraints of those governments which do not implement expansionary fiscal policy is by imposing a balanced-budget policy. There are two fiscal schemes for implementing such a policy. One involves endogenizing the tax rates; and the other involves only government expenditures. Both are considered below:

Tables 5.4A to 5.4C present simulation results for the endogenous tax rate scheme. Table 5.4A shows results for the case of a once-and-for-all increase in $G1_t$ of 10 percent, with the federal and second regional governments endogenizing the $u2_t$ and uF_t to balance their respective budgets in each period, while $u1_t$ remains fixed. The system becomes stable only for parameter sets (3) and (4). The rate of approach to equilibrium is rather slow. Table 5.4B shows the case of a once-and-for-all increase in $GF1_t$ by 10 percent, with the two regional governments adopting balanced-budget policies while uF_t remains fixed. Except for parameter set (1), the system is stable. Table 5.4C shows the case of a once-and-for-all increase in $GF1_t$ and $GF2_t$ by 10 percent with $u1_t$ and $u2_t$ endogenized.

The results of Tables 5.4A to 5.4C indicate that even under the extreme assumption of balanced-budgets in those governments which do not implement expansionary fiscal policy, the stability of the system still depends on the slope of the IS-LM curves, i.e., the structure of the economy.

The balanced-budget policy adopted by those governments which do not implement expansionary fiscal policy not only eliminates the conflicting requirements for stability, but also generates secondary effects which reinforce the movement of the system towards a stable equilibrium. This secondary effect is associated with the induced aggregate demand generated by higher disposable income, and is a direct result of the lower tax rate required to balance the budget. With regional income increased even further, the tax revenue is high enough to cover interest obligations even in the bond-financing case.

The major weakness of this scheme is the need for two governments to adjust the tax rate in each time period which seems unrealistic, since changes of tax rates require a certain amount of time for legislative or parliamentary approval.

It is easier to adjust government expenditures than the tax rate. We consider then, the scheme in which the government expenditure is used to pursue a balanced-budget policy. A new variable called "endogenous government expenditures" is introduced. Its main purpose is to balance the budget. Now government expenditure consists of two components: exogenous (G_t) and endogenous (GEN_t). Take the case in which the federal government increases its expenditures ($GF1_t$) by 10 percent. Now $GEN1_t$ and $GEN2_t$ are endogenized in the regional government budget constraints so as to bring $DB1_t$ and $DB2_t$ to zero in each period. The income identity and the dynamic equations are adjusted as follows:

(5.60) $Y1_t = G1_t + GF1_t + GEN1_t + GENF1_t + E1_t + IM2_t$

(5.61) $Y2_t = G2_t + GF2_t + GEN2_t + GENF2_t + E2_t + IM1_t$

(5.62) $DW1_t = G1_t+GF1_t+GEN1_t+GENF1_t+TR1_t+TRF1_t+R_tH1_t$

$-(u1_t+uF_t)(Y1_t+TR1_t+TRF1_t+R_tH1_t)$

(5.63) $DB1_t = G1_t+GEN1_t+TR1_t+R_tB1_t-u1_t(Y1_t+TR1_t+TRF1_t+R_tH1_t)$

(5.64) $DB2_t = G2_t+GEN2_t+TR2_t+R_tB2_t-u2_t(Y2_t+TR2_t+TRF2_t+R_t(B1_t+B2_t+BF_t-H1_t))$

(5.65) $DM_t+DBF_t = GF1_t+GF2_t+GFEN1_t+GFEN2_t+TRF1_t+TRF2_t+R_tBF_t$

$-uF_t(Y1_t+Y2_t+TR1_t+TR2_t+TRF1_t+TRF2_t+R_t(B1_t+B2_t+BF_t))$

And the new endogenous expenditure functions are

(5.66) $GEN1_t = u1_t(Y1_t+TR1_t+TRF1_t+R_tH1_t)-G1_t-TR1_t-R_tB1_t$

(5.67) $GEN2_t = u2_t(Y2_t+TR2_t+TRF2_t+R_t(B1_t+B2_t+BF_t-H1_t))-G2_t-TR2_t-R_tB2_t$

(5.62) $GENF1_t = -GEN1_t$

(5.63) $GENF2_t = -GEN2_t$

There are various ways of specifying such a fiscal scheme. Here we set the endogenous federal expenditure in each region equal to the negative value of regional endogenous expenditures. The purpose is to offset any increase in regional endogenous expenditures in the income identity. If $GENF_t$ is not used, then the GEN_t may overshoot the system and become unstable.

Table 5.4D shows that under the endogenous expenditure scheme, all parameter sets, regardless of the mode of financing, produce stable long-run results.

We can devise other fiscal schemes as well, but it is not worthwhile to pursue this analysis further because the following theoretical limitations restrict its acceptability:

First , the fiscal schemes require full cooperation from the governments which do not pursue an expansionary fiscal policy and are required to adopt a passive balanced-budget fiscal policy. The fiscal schemes prescribe a passive philosophy of fiscal management which is incompatible with the current Canadian theme of increasing decentralization and regional fiscal autonomy. Under these schemes, two governments must passively adjust their expenditures or tax rates in each period so as to balance their respective budgets, without any concern about evolving economic conditions at the regional or the national level.

Second , the fiscal schemes produce a balanced budget in each period for each of two government, thus bypassing the relationships of financial constraints among the three governments that we want to address.

Nevertheless, one main conclusion can be drawn from the analysis based on the IRF model, that is, government expenditures must be endogenized in the system in order to generate a stable long-run equilibrium. The introduction of an acceptable rule or reaction function for accomplishing this is the main objective of the next chapter.

Table 5.3A: The IRF Model
Simulated Effects of a Once-and-for-all Increase in Gl_t by 10%
Applying the Christ Adjustment Method

Parameter Set	Federal Financing Policy	Period	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability
(1)		Impact	0.5423	0.0704	0.6127	0.0006	1.0769	0.1881	1.2650	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(2)		Impact	0.6270	0.1490	0.7760	0.0002	1.1754	0.2482	1.4596	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(3)		Impact	0.7881	0.2322	1.0203	0.0003	1.3117	0.3893	1.7010	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(4)		Impact	0.7861	0.2258	1.0119	0.0003	1.3084	0.3843	1.6927	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable

Table 5.3B: The IRF Model
Simulated Effects of a Once-and-for-all Increase in GFl_t by 10%
Applying the Christ Adjustment Method

Parameter Set	Federal Financing Policy	Period	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability
(1)		Impact	0.2323	0.0302	0.2626	0.0003	0.4615	0.0806	0.5421	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(2)		Impact	0.2682	0.0598	0.3280	0.0001	0.5028	0.1195	0.6223	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(3)		Impact	0.3370	0.0951	0.4321	0.0001	0.5609	0.1641	0.7250	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable
(4)		Impact	0.3350	0.0887	0.4237	0.0001	0.5577	0.1591	0.7168	
	Bond	Long Run	--	--	--	--	--	--	--	Unstable
	Money	Long Run	--	--	--	--	--	--	--	Unstable

Table 5.4A: The IRF Model
Simulated Effects of a Once-and-for-all Increase in $G1_t$ by 10%
Endogenized $u2_t$ and uF_t Fiscal Scheme

Parameter Set	Period	ΔuF	Δu2	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability (Time Period)
(1)	Impact	-0.00205	-0.00139	0.7813	0.3927	1.1740	0.0009	1.3647	0.5093	1.8740	
	Long Run	--	--	--	--	--	--	--	--	--	Unstable
(2)	Impact	-0.00268	-0.00213	0.9665	0.6231	1.5896	0.0004	1.5635	0.7261	2.2896	
	Long Run	--	--	--	--	--	--	--	--	--	Unstable
(3)	Impact	-0.00392	-0.00372	1.4758	1.1913	2.6671	0.0006	2.0905	1.2766	3.3671	
	Long Run	-0.02060	-0.02612	8.4294	10.3822	18.8116	0.0053	9.7150	9.7966	19.5216	600
(4)	Impact	-0.00115	-0.00372	1.4758	1.1913	2.6671	0.0006	2.0905	1.2766	3.3671	
	Long Run	-0.01151	-0.01386	4.5046	5.1018	9.6064	0.0023	5.3837	4.9226	10.2063	180

Table 5.4B: The IRF Model
Simulated Effects of a Once-and-for-all Increase in $GF1_t$ by 10%
Endogenized $u1_t$ and $u2_t$ Fiscal Scheme

Parameter Set	Federal Financing Policy	Period	Δu1	Δu2	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability (Time Period)
(1)		Impact	-0.0017	-0.0005	0.3825	0.1078	0.4930	0.0004	0.6020	0.1910	0.7930	
	Bond	Long Run	--	--	--	--	--	--	--	--	--	Unstable
	Money	Long Run	-0.0036	-0.0021	0.9423	0.5156	1.4579	-0.0013	1.1143	0.6435	1.7578	85
(2)		Impact	-0.0019	-0.0007	0.4541	0.1766	0.6307	0.0002	0.6708	0.2598	0.9306	
	Bond	Long Run	-0.0097	-0.0083	3.0252	2.5954	5.6206	0.0022	3.1963	2.7243	5.9206	300
	Money	Long Run	-0.0036	-0.0021	0.9396	0.5130	1.4526	-0.0013	1.1117	0.6409	1.7526	107
(3)		Impact	-0.0012	-0.0012	0.6475	0.3202	0.9677	0.0002	0.8493	0.4184	1.2677	
	Bond	Long Run	-0.0053	-0.0039	1.5782	1.1742	2.7524	0.0008	1.7571	1.2954	3.0525	190
	Money	Long Run	-0.0036	-0.0022	1.0016	0.5980	1.5996	-0.0006	1.1805	0.7191	1.8996	99
(4)		Impact	-0.0025	-0.0012	0.6475	0.3202	0.9677	0.0002	0.8493	0.4184	1.2677	
	Bond	Long Run	-0.0044	-0.0030	1.2944	0.9010	2.1954	0.0005	1.4764	1.0190	2.4954	92
	Money	Long Run	-0.0037	-0.0023	1.0328	0.6394	1.6722	-0.0003	1.2173	0.7687	1.9860	54

Note: The last column of the table indicates the time periods required to approach a steady state to the accuracy of the fourth decimal point.

Table 5.4C: The IRF Model
Simulated Effects of a Once-and-for-all Increase in $GF1_t$ and $GF2_t$ by 10%
Endogenized $u1_t$ and $u2_t$ Fiscal Scheme

Parameter Set	Federal Financing Policy	Period	Δu1	Δu2	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability (Time Period)
(1)		Impact	-0.0023	-0.0023	0.4930	0.4930	0.9860	0.0008	0.7930	0.7930	1.5860	
	Bond	Long Run	--	--	--	--	--	--	--	--	--	Unstable
	Money	Long Run	-0.0056	-0.0056	1.4579	1.4579	2.9178	-0.0026	1.7579	1.7579	3.5158	60
(2)		Impact	-0.0027	-0.0027	0.6306	0.6306	1.2612	0.0003	0.9306	0.9306	1.8612	
	Bond	Long Run	-0.0190	-0.0190	6.2429	6.2429	12.4858	0.0048	6.5429	6.5429	13.0858	600
	Money	Long Run	-0.0056	-0.0056	1.4526	1.4526	2.9052	-0.0027	1.7526	1.7526	3.5052	140
(3)		Impact	-0.0037	-0.0037	0.9676	0.9676	1.9352	0.0004	1.2676	1.2676	2.5252	
	Bond	Long Run	-0.0091	-0.0091	2.7746	2.7746	5.5492	0.0016	3.0746	3.0746	6.1492	260
	Money	Long Run	-0.0058	-0.0058	1.5996	1.5996	3.1992	-0.0013	1.8996	1.8996	3.7992	100
(4)		Impact	-0.0037	-0.0037	0.9676	0.9676	1.9352	0.0004	1.2676	1.2676	2.5252	
	Bond	Long Run	-0.0073	-0.0073	2.2002	2.2002	4.4004	0.0011	2.5002	2.5002	5.0004	100
	Money	Long Run	-0.0059	-0.0059	1.5159	1.5159	3.0318	-0.0005	1.9721	1.9721	3.9442	60

Table 5.4D: The IRF Model
Simulated Effects of a Once-and-for-all Increase in $GF1_t$ by 10%
Endogenized $GEN1_t$ and $GEN2_t$ Fiscal Scheme

Parameter Set	Federal Financing Policy	Period	ΔGEN1	ΔGEN2	ΔE1	ΔE2	Δ(E1+E2)	ΔR	ΔY1	ΔY2	Δ(Y1+Y2)	Stability (Time Period)
(1)		Impact	0.0842	0.0156	0.2420	0.0398	0.2818	0.0003	0.4814	0.1005	0.5819	
	Bond	Long Run	0.3630	0.2837	1.7115	1.4157	3.1272	0.0061	1.9228	1.5044	3.4272	600
	Money	Long Run	0.1207	0.0412	0.4156	0.1225	0.5381	-0.0005	0.6277	0.2104	0.8381	58
(2)		Impact	0.0919	0.0232	0.2736	0.0714	0.3450	0.0001	0.5230	0.1320	0.6450	
	Bond	Long Run	0.1482	0.0685	0.5695	0.2760	0.8455	0.0003	0.7815	0.3640	1.1455	162
	Money	Long Run	0.1207	0.0412	0.4149	0.1218	0.5367	-0.0005	0.6270	0.2096	0.8366	68
(3)		Impact	0.1029	0.0319	0.3453	0.1098	0.4551	0.0001	0.5747	0.1815	0.7552	
	Bond	Long Run	0.1329	0.0564	0.5058	0.2240	0.7298	0.0003	0.7213	0.3085	0.7552	105
	Money	Long Run	0.1201	0.0437	0.4301	0.1483	0.5784	-0.0002	0.6456	0.2328	0.8784	72
(4)		Impact	0.1029	0.0319	0.3453	0.1098	0.4551	0.0001	0.5747	0.1805	0.7552	
	Bond	Long Run	0.1264	0.0513	0.4786	0.2019	0.6805	0.0002	0.6956	0.2849	0.9805	51
	Money	Long Run	0.1200	0.0449	0.4377	0.1610	0.5987	-0.0001	0.6547	0.2441	0.8988	32

Note: The last column of the table indicates the time periods required to approach a steady state to the accuracy of the fourth decimal point.

5.7 Conclusion

In this chapter we have analysed the effects of fiscal policy at the federal and interregional levels, using the IRF model. We find that the combination of federal and interregional fiscal policies cannot produce long-run stable results. At best, only a quasi-equilibrium is attainable. The Christ adjustment method, which works in the National Model, fails to generate stable equilibrium in the interregional model, with or without a federal sector: it introduces more instability than it eliminates. In the last section, we considered two schemes for balancing the budgets of those governments which do not implement an expansionary fiscal policy. The results, limitations and implications for the interregional model of these schemes were discussed.

Footnotes to Chapter 5

(1). Barber (1966) argues that economically important regions should have their own independent fiscal policies. However, once interregional influence is recognized, there is no such thing as independent fiscal policy. This is the perfect analogy of the argument put forward by Christ (1979) that there is no "pure" monetary or fiscal policy since one cannot be insulated from the other.

(2). Through substitution, we have:

$$
\begin{aligned}
M2_t &= M_t - M1_t \\
&= M_t - (W1_t - H1_t) \\
&= M_t - W1_t + H1_t
\end{aligned}
$$

$$
\begin{aligned}
W2_t &= M2_t + H2_t \\
&= (M_t - W1_t + H1_t) + (B1_t + B2_t + BF_t - H1_t) \\
&= M_t + B1_t + B2_t + BF_t - W1_t
\end{aligned}
$$

(3). See section 4.3.3 of Chapter 4. The derivation of the short-run multiplier effects is exactly the same as in Appendix 4.1 except that now we have to include uF_t in the "tax rate" term.

Chapter 6

Analysis Based on an Interregional Model With a Federal Sector and Government Expenditure Reaction Functions

6.1 Introduction

Our previous models (except in Section 6.5) have been based on the assumption that expenditures at both levels of government are exogenous. Governments do not react to evolving economic conditions. There is no mechanism in the system to ensure that complementary fiscal actions would be taken by either level of government, either to achieve the target objectives, or to satisfy the stability conditions required for long-run stable equilibrium. In our system, the private and public budget constraints simply trace out the time path of the endogenous variables. Such passive fiscal policy may be appropriate (though this is arguable) for a National Model, but definitely not for an interregional model. It is evident from Tables 5.2A and 5.2B that a very peculiar fiscal arrangement has emerged, with one government issuing bonds and the other two governments buying new bonds indefinitely. The fiscal relationships among these governments are unstable and therefore the system as a whole is unstable, even though the other endogenous variables exhibit stablility. Unless we impose balanced-budget fiscal schemes (Section 5.6) on the governments not pursuing expansionary fiscal policy, we will not be able to obtain a long-run stable equilibrium, regardless of any financing policy. Such fiscal schemes would certainly limit the scope of interregional fiscal policy, and it is unrealistic to assume that they would be adopted.

There are fiscal arrangements between regional and federal governments such as grants, equalization payments, and so on. It is reasonable to introduce government expenditure functions to take into account these arrangements and their induced changes. Our purpose is not only to construct a system which will satisfy the stability condition, but also to extend our investigation of the efficacy of interregional fiscal policy.

The interregional model with a federal sector and government expenditure reaction function (IRFG) will first be presented in the next section, along with the relevant notation and explanation. In section 6.3 fiscal policy will be discussed. Section 6.4 presents and discusses the simulation results. Section 6.5 summarizes the major findings of the chapter.

6.2 Equations of the IRFG Model

(6.1) $Y1_t = E1_t + G1_t + GF1_t + IM2_t - IM1_t$

(6.2) $Y2_t = E2_t + G2_t + GF2_t + IM1_t - IM2_t$

(6.3) $E1_t = E1(YD1_t, W1_t, R_t)$

(6.4) $E2_t = E2(YD2_t, M_t + B1_t + B2_t + BF_t - W1_t, R_t)$

(6.5) $IM1_t = IM1(YD1_t, W1_t, R_t)$

(6.6) $IM2_t = IM2(YD2_t, M_t + B1_t + B2_t + BF_t - W1_t, R_t)$

(6.7) $YD1_t = (1 - u1_t - uF_t)(Y1_t + TR1_t + TRF1_t + R_t H1_t)$

(6.8) $YD2_t = (1 - u2_t - uF_t)(Y2_t + TR2_t + TRF2_t + R_t(B1_t + B2_t + BF_t - H1_t))$

(6.9) $W1_t - H1_t = L1(Y1_t, W1_t, R_t)$

$$(6.10)\quad M_t - W1_t + H1_t = L2(Y2_t, M_t + B1_t + B2_t + BF_t - W1_t, R_t)$$

$$(6.11)\quad \frac{G1_t - G1_{t-1}}{G1_{t-1}} = A1\left(\frac{\overline{Y1} - Y1_{t-1}}{\overline{Y1}}\right) - A2\left(\frac{DB1_{t-1}}{\overline{Y1}}\right)$$

$$(6.12)\quad \frac{G2_t - G2_{t-1}}{G2_{t-1}} = A3\left(\frac{\overline{Y2} - Y2_{t-1}}{\overline{Y2}}\right) - A4\left(\frac{DB2_{t-1}}{\overline{Y2}}\right)$$

$$(6.13)\quad Y_t = Y1_t + Y2_t$$

$$(6.14)\quad \overline{Y} = \overline{Y1} + \overline{Y2}$$

$$(6.15)\quad \frac{GF_t - GF_{t-1}}{GF_{t-1}} = A5\left(\frac{\overline{Y} - Y_{t-1}}{\overline{Y}}\right) - A6\left(\frac{DM_{t-1} + DBF_{t-1}}{\overline{Y}}\right)$$

$$(6.16)\quad P1_t = P1_{t-1} + A7\left(\frac{\overline{Y1}}{\overline{Y}}\right)\left(\frac{\overline{Y1} - Y1_{t-1}}{\overline{Y1}}\right) - A8\left(\left(\frac{G1_{t-1} + GF1_{t-1}}{\overline{Y1}}\right) - \left(\frac{G2_{t-1} + GF2_{t-1}}{\overline{Y2}}\right)\right)$$

$$(6.17)\quad P2_t = P2_{t-1} + A9\left(\frac{\overline{Y2}}{\overline{Y}}\right)\left(\frac{\overline{Y2} - Y2_{t-1}}{\overline{Y2}}\right) + A10\left(\left(\frac{G1_{t-1} + GF1_{t-1}}{\overline{Y1}}\right) - \left(\frac{G2_{t-1} + GF2_{t-1}}{\overline{Y2}}\right)\right)$$

$$(6.18)\quad p1_t = P1_t / (P1_t + P2_t)$$

$$(6.19)\quad p2_t = P2_t / (P1_t + P2_t)$$

$$(6.20)\quad GF1_t = p1_t GF_t$$

$$(6.21)\quad GF2_t = p2_t GF_t$$

$$(6.22)\quad DW1_t = G1_t + GF1_t + TR1_t + TRF1_t + R_t H1_t + IM2_t - IM1_t - (u1_t + uF_t)(Y1_t + TR1_t + TRF1_t + R_t H1_t)$$

$$(6.23)\quad DB1_t = G1_t + TR1_t + R_t B1_t - u1_t(Y1_t + TR1_t + TRF1_t + R_t H1_t)$$

$$(6.24)\quad DB2_t = G2_t + TR2_t + R_t B2_t - u2_t(Y2_t + TR2_t + TRF2_t + R_t(B1_t + B2_t + BF_t - H1_t))$$

$$(6.25)\quad DM_t + DBF_t = GF1_t + GF2_t + TRF1_t + TRF2_t + R_t BF_t - uF_t(Y1_t + Y2_t + TR1_t + TR2_t + TRF1_t + TRF2_t + R_t(B1_t + B2_t + BF_t))$$

(6.26) $DW1_t = W1_{t+1} - W1_t$

(6.27) $DB1_t = B1_{t+1} - B1_t$

(6.28) $DB2_t = B2_{t+1} - B2_t$

(6.29) $DM_t = M_{t+1} - M_t$

(6.30) $DBF_t = BF_{t+1} - BF_t$

Federal financing policy options:

(6.31a) $DM_t = 0$ federal pure bond-financing policy

(6.31b) $DBF_t = 0$ federal pure money-financing policy

Endogenous variables: $Y1_t, Y2_t, E1_t, E2_t, IM1_t, IM2_t, YD1_t, YD2_t, R_t, H1_t, G1_t,$
$G2_t, Y_t, \overline{Y}, GF_t, P1_t, P2_t, p1_t, p2_t, GF1_t, GF2_t, DW1_t,$
$DB1_t, DB2_t, DBF_t, DM_t, W1_{t+1}, B1_{t+1}, B2_{t+1}, BF_{t+1}, M_{t+1}.$

Predetermined variables: $G1_{t-1}, G2_{t-1}, GF_{t-1}, Y1_{t-1}, Y2_{t-1}, \overline{Y1}, \overline{Y2}, DB1_{t-1}, DB2_{t-1},$
$DM_{t-1}, DBF_{t-1}, P1_{t-1}, P2_{t-1}, GF1_{t-1}, GF2_{t-1}, TR1_t, TR2_t,$
$TRF1_t, TRF2_t, u1_t, u2_t, uF_t, W1_t, B1_t, B2_t, BF_t, M_t,$
A1, A2, A3, A4, A5, A6, A7, A8, A9, A10.

Assumptions about partial derivatives are as follows:

$0 \leqslant E1_{YD1} \leqslant 1$	$E1_R \leqslant 1$	$E1_{W1} \geqslant 0$
$0 \leqslant E2_{YD2} \leqslant 1$	$E2_R \leqslant 1$	$E2_{W2} \geqslant 0$
$0 \leqslant IM1_{YD1} \leqslant 1$	$IM1_R \geqslant 1$	$IM1_{W1} \geqslant 0$
$0 \leqslant IM2_{YD2} \leqslant 1$	$IM2_R \geqslant 1$	$IM2_{W2} \geqslant 0$
$L1_{Y1} \geqslant 0$	$L1_R \leqslant 0$	$0 \leqslant L1_{W1} \leqslant 1$
$L2_{Y2} \geqslant 0$	$L2_R \leqslant 0$	$0 \leqslant L2_{W2} \leqslant 1$

Notation :

1 — Region 1
2 -- Region 2
A1 — Income target-adjustment parameter in Region 1.
A2 -- Balanced-budget target-adjustment parameter in Region 1.
A3 — Income target-adjustment parameter in Region 2.
A4 — Balanced-budget target-adjustment parameter in Region 2.
A5 — Income target-adjustment parameter of the federal sector.
A6 -- Balanced-budget target-adjustment parameter of the federal sector.
A7 -- Federal expenditure distribution parameter for Region 1 with respect to the income target.
A8 -- Federal expenditure distribution parameter for Region 1 with respect to the equal public expenditure ratio target.
A9 -- Federal expenditure distribution parameter for Region 2 with respect to the income target.
A10— Federal expenditure distribution parameter for Region 2 with respect to the equal public expenditure ratio target.
B — Value of the outstanding bonds.
DB — Changes of the outstanding stock of bonds.
DM — Changes of the money stock.
DW — Changes of stock of private financial wealth.
E — Private expenditures.
F — Federal sector.
G — Government expenditures on goods and services.
H — Private holdings of outstanding stock of bonds.
IM — Imports.
M — Money stock.
P — Unnormalized proportion of federal spending in region.
p -- Normalized proportion of federal spending in region.
R — Rate of interest.
t -- Time period.
TR — Transfer payments.[1]
u -- Proportional tax rate.
W — Private financial wealth.
Y — Gross national product.
$\bar{Y}$ — Income target(capacity output).
YD — Disposable income.

The explanations of equations (6.1) to (6.10) are the same as for the IRF model except now G_t and GF_t become endogenous variables. Equations (6.11) to (6.21) are the government expenditure reaction functions which are the new features of the IRFG model. The concept of government expenditure reaction function is developed from Scarth's (1979) paper on "Bond-financed Fiscal Policy and the Problem of Instrument Instability". He argues that a once-and-for-all policy is irrelevant since the actual government reacts to changes in national income. He therefore proposes the following endogenous fiscal policy rule:[2]

$$(6.32) \qquad \dot{G} = a(Y^f - Y) \qquad\qquad a > 0$$

The above rule specifies that government raises (lowers) its spending whenever actual output is below (above) its target value (Y^f). Full equilibrium in the model exists when all endogenous variables are constant. This implies that Y^f=Y and that the government budget is balanced. In a simple closed-economy model, ignoring interest rate and wealth effects, he concludes that while endogenous fiscal policy can keep output near its target value, it cannot control the size of the national debt.[3] The basic version of the system must be unstable with respect to bonds and government spending. When the wealth effect is added, stability is possible.[4]

To parallel the label of the Christ adjustment method, I call the endogenous fiscal policy rule (6.32) the Scarth adjustment method. There are two special features of the Scarth adjustment method: (1), fiscal policy is target-oriented, and (2), endogenous fiscal policy does not introduce an additional destabilizing forces into the system. It is in this sense that the Scarth adjustment method is to be preferred to the Christ adjustment method which treats total government expenditures as constant, and allows government expenditures on goods and services to passively adjust to the changes of interest payments. As we have demonstrated in the IR and IRF models, the Christ adjustment method introduced more destabilizing forces than it eliminated.

Applying and elaborating the Scarth adjustment method to our interregional model, we incorporate more targets into the government expenditure reaction functions because there are other objectives to be secured in a federal system. Most important is the inclusion of the balanced-budget target. The stability of the system depends crucially on interest payments under a bond-financed fiscal policy. We argue, however, that interest payments are also affected by the amount of government expenditures in each period. One possible way to eliminate the instability is by dealing with its basic source. Therefore a balanced-budget target is added to the income target in the government expenditure reaction function. Our system of government expenditure reaction functions will be discussed under the headings of "Government Targets" and "Federal Distribution Scheme".

6.2.1 Government Expenditure Reaction Function and Government Targets

Equations (6.11), (6.12) and (6.15) are government expenditure reaction functions for regional and federal governments. The aim of fiscal policy is the pursuit of a higher target level of income as well as a budget that is balanced in the long run. In accordance with these objectives, (6.11), (6.12) and (6.15) specify that the proportionate change of government expenditures be a linear function of two terms: one is the deviation of the last period's income from its target level; the other is the ratio of the last period's deficit to the target income level. Each government adjusts its expenditures in the current period to the situation that existed in the previous period. If last period's income was less than the target value, government expenditures will be increased in this period. By the same token, if there was a deficit (i.e., a positive DB or DM) last period, the government will reduce its expenditure in the current period.

The government expenditure reaction functions can be rewritten in the following way:

$$(6.33)\quad G1_t = G1_{t-1}\left(1+A1\left(\frac{\overline{Y1}-Y1_{t-1}}{\overline{Y1}}\right)-A2\left(\frac{DB1_{t-1}}{\overline{Y1}}\right)\right)$$

$$(6.34)\quad G2_t = G2_{t-1}\left(1+A3\left(\frac{\overline{Y2}-Y2_{t-1}}{\overline{Y2}}\right)-A4\left(\frac{DB2_{t-1}}{\overline{Y2}}\right)\right)$$

$$(6.35)\quad GF_t = GF_{t-1}\left(1+A5\left(\frac{\overline{Y}-Y_{t-1}}{\overline{Y}}\right)-A6\left(\frac{DM_{t-1}+DBF_{t-1}}{\overline{Y}}\right)\right)$$

Equations (6.33) to (6.35) specify that the current level of government expenditures is based on the last period's level, adjusted for differences between actual and target values. This new scheme is much more realistic than the one adopted in the last chapter which requires an adjustment in each period to balance the budget.

In our scheme, each regional government adjusts its expenditures according to its income and balanced-budget targets, while the federal government adjusts its expenditures according to the overall income and its own balanced-budget target. Since the overall income target is only the sum of regional income targets, the federal government is helping each region to reach the income target at a faster speed. However, the regional balanced-budget target is basically the regional government's responsibility. The federal government is helping each regional government to balance its budget indirectly, though, through aiding in reaching the regional income target faster, and thus reducing the amount of deficit financing.

The driving force of the government expenditure reaction function is the income target, which is assumed exogenous to the system. The income target can be considered as the capacity output level. Government expenditures will be increased as long as a new target has been set, or the targets have not been achieved. That is why we regard government expenditure as an endogenous variable. Actually the government expenditure reaction functions (6.11), (6.12) and (6.15), consist of both endogenous and exogenous components. The former have been defined as deviations of last period's value from target values. The exogenous component reflects the target-adjustment parameters, which are the policy variables determined by each government. A higher target-adjustment parameter means a more active fiscal policy. Therefore the public sectors neither react entirely passively to the system, nor do they engage in unconstrained fiscal policy. Rather, under the expenditure scheme (6.11), (6.12) and (6.15), they are active and responsive in fiscal management.

Equation (6.14) specifies that the aggregate income target ($\overline{Y}$) is the sums of the regional income targets ($\overline{Y1}+\overline{Y2}$). Thus the aggregate income target must be consistent with the regional income targets. For example, if both $\overline{Y1}$ and $\overline{Y2}$ are increased by 5 percent, then $\overline{Y}$ would also increase by 5 percent. If regions have income targets different from the federal government's targets for them, then (6.14) will be violated. To illustrate, we denote

$\overline{YF1}$ — the federal government's income target for Region 1

$\overline{YF2}$ — the federal government's income target for Region 2

$\overline{YR1}$ — the first regional government's income target

$\overline{YR2}$ — the second regional government's income target

We then rewrite (6.14) as

(6.14a) $\overline{Y} = \overline{YF1} + \overline{YF2}$

and substitute $\overline{Y1}$ and $\overline{Y2}$ by $\overline{YR1}$ and $\overline{YR2}$ in other equations of the model. In this revised model, when we have different income targets ($\overline{YF1} \neq \overline{YR1}$, $\overline{YF2} \neq \overline{YR2}$), the system will not produce a stable long-run equilibrium. The differences in income targets will force government expenditures $G1_t$, $G2_t$, and GF_t to adjust indefinitely.[5] Thus the IRFG model requires full coordination of the regional and federal governments in calculating the consistent estimates of the income targets. In the IRFG model, we use only $\overline{Y1}$ and $\overline{Y2}$ as the consistent estimates of income targets.

6.2.2 Government Expenditure Reaction Function and the Federal Distribution Scheme

Equations (6.16) to (6.21) describe the scheme for distributing federal government expenditures to the two regions. $P1_{t-1}$ and $P2_{t-1}$ can be considered as the proportional shares of federal spending in two regions at an initial equilibrium. Once a new target income level is set, the current proportional shares also have to adjust over time, according to the extent that the targets are being achieved. The last two terms on the right hand side of (6.16) and (6.17) are the factors of adjustment to the proportional shares of the last period.

These adjustment factors can also be considered as the targets in a federal setting. The first objective is to help a regional government reach its income target. The federal government distributes its expenditures to regions not only according to the relative deviations from income targets, i.e., $((\overline{Y1}-Y1_{t-1})/\overline{Y1})$ and $((\overline{Y2}-Y2_{t-1})/\overline{Y2})$, but also according to the relative sizes of the regions, i.e., $(\overline{Y1}/\overline{Y})$ and

$(\overline{Y2}/\overline{Y})$. If the sizes of the two regions are different, more government expenditures are required to achieve a one percent reduction of unemployment or a one percent increase of income in the larger region than in the smaller region. The combined effects then reduce to $((\overline{Y1}-Y1_{t-1})/\overline{Y})$ and $((\overline{Y2}-Y2_{t-1})/\overline{Y})$ as illustrated in equations (6.36) and (6.37).

$$(6.36)\qquad \frac{\overline{Y1}-Y1_{t-1}}{\overline{Y}} = \left(\frac{\overline{Y1}}{\overline{Y}}\right)\left(\frac{\overline{Y1}-Y1_{t-1}}{\overline{Y1}}\right)$$

$$(6.37)\qquad \frac{\overline{Y2}-Y2_{t-1}}{\overline{Y}} = \left(\frac{\overline{Y2}}{\overline{Y}}\right)\left(\frac{\overline{Y2}-Y2_{t-1}}{\overline{Y2}}\right)$$

The second target (or the second adjustment factor) originates from the need to reduce or eliminate disparities in regional fiscal capacity. The fiscal capacity (or residuum) is defined as the difference between per person benefits and costs of local public goods.[6] If we include federal expenditures in the regions, then the regional tax revenues become inappropriate as a basis for comparison. A different criterion is then used. The federal government tries to equalize the ratio of total government expenditures to income target across regions. If one region has a higher ratio, the federal government will reduce its expenditures in that region. When this objective is achieved, the supply of public services per unit of capacity output will be the same in both regions.

Other targets can be brought into the distribution scheme as well. However, some trade-off must be made between the desirability of these targets and the possible effects on stability of the system. Since (6.15) considers only the balanced-budget target of the federal government, there is the possibility that the distribution of federal expenditures to two regions may overshoot or undershoot the targets.

The current unnormalized $P1_t$ and $P2_t$ may not add up to unity. An increase in $P1_t$ does not necessarily equal the decrease in $P2_t$. A simple procedure for normalizing the ratios is therefore introduced, in equations (6.18) and (6.19).

The other elements of the system are the same as in the IRF model. Note that the assumptions underlying previous models apply in this model. The IRFG model can also be viewed as composed of three recursive systems or blocks of equations. The government expenditure reaction functions are determined by the last period's results. Thus equations (6.11) to (6.21) constitute the first recursive block. Once the values of government expenditure variables are determined, equations (6.1) to (6.10) can then be used to solve for ten endogenous variables. Equations (6.1) to (6.10) are the non-linear simultaneous equation system of the model, and thus constitute the second recursive block. When the results of the first two recursive systems are obtained, equations (6.22) to (6.31) determine the changes of financial stocks in the next period, and these then exert an influence through the government expenditure reaction functions. Thus equations (6.22) to (6.31) constitute the third recursive block of the system.

6.3 Study of Fiscal Policy

A significant feature of the IRFG model is the presence of the government expenditure reaction functions. The driving force of the model is the attempt to achieve the exogenous income target (capacity output) levels. Government expenditure is increased whenever a new income target has been set. Each government adopts an active fiscal policy in pursuing its target. The analysis of fiscal policy is therefore quite different from the conventional method of shocking the system with a once-and-for-all increase in government expenditures, and letting the dynamic system trace out the time path of the system. As we have demonstrated in the previous chapter, when expansionary fiscal policy is implemented by one government, the other two government budget constraints come into play, and governments react passively to balance their budgets, without any objective or target to achieve, other than balancing the budget. A balanced-budget target is one, but not necessary the only target. On the other hand, the government which has implemented an expansionary fiscal policy also reacts passively to balance its budget, though the main thrust of an active fiscal policy is to achieve a higher income level. The previous systems are so rigid that all government expenditures on goods and services are assumed exogenous, leaving the asset market and interest payments as the only channels available to achieve the balanced-budget target. The momentum of the passive-reactive fiscal policies at all levels of government is so strong, and the stability requirements derived from these passive-reactive policies so conflicting and diverse, that an internal inconsistency is built into the system.

With government expenditures endogenized, the IRFG model avoids this internal inconsistency by requiring that every government adopt an active fiscal policy to achieve the income target as well as the balanced-budget target. Because of the special nature of endogenized government expenditures, interpretation of the short-run and long-run multipliers will be different from the intrepretation in the models constructed in previous chapters, and in the models presented in the existing literature.

6.3.1. Static System

In the short run, all dynamic equations are ignored. The static system thus consists of (6.1) to (6.21), with the following endogenous and predetermined variables:

Endogenous variables: $Y1_t, Y2_t, E1_t, E2_t, IM1_t, IM2_t, YD1_t, YD2_t, R_t, H1_t,$
$G1_t, G2_t, GF_t, GF1_t, GF2_t, Y_t, \overline{Y}, P1_t, P2_t, p1_t, p2_t.$

Predetermined variables: $G1_{t-1}, G2_{t-1}, GF_{t-1}, Y1_{t-1}, Y2_{t-1}, \overline{Y1}, \overline{Y2}, DB1_{t-1}, DB2_{t-1}$
$P1_{t-1}, P2_{t-1}, GF1_{t-1}, GF2_{t-1}, TR1_t, TR2_t, TRF1_t, TRF2_t,$
$DM_{t-1}, DBF_{t-1}, u1_t, u2_t, uF_t, W1_t, B1_t, B2_t, BF_t, M_t,$
$A1, A2, A3, A4, A5, A6, A7, A8, A9, A10.$

Through substitutions, equations (6.1) to (6.21) can be simplified to obtain functions involving only predetermined variables:

(6.38) $Y1_t = Y1(\text{predetermined variables})$

(6.39) $Y2_t = Y2(\text{predetermined variables})$

(6.40) $H1_t = H1(\text{predetermined variables})$

(6.41) $R_t = R(\text{predetermined variables})$

(6.42) $G1_t = G1(\text{predetermined variables})$

(6.43) $G2_t = G2(\text{predetermined variables})$

(6.44) $GF_t = GF(\text{predetermined variables})$

(6.45) $GF1_t = GF1(\text{predetermined variables})$

(6.46) $GF2_t = GF2(\text{predetermined variables})$

We differentiate the above system and solve for the following short-run multiplier effects (See Appendix 6.1.):

$$(6.47)\quad dY1_t/d\overline{Y1} = (\partial Y1_t/\partial G1_t)(\partial G1_t/\partial \overline{Y1}) = \frac{f_3+f_1f_7}{1-f_1f_5}$$

$$(6.48)\quad dY1_t/d\overline{Y2} = (\partial Y1_t/\partial G2_t)(\partial G2_t/\partial \overline{Y2}) = \frac{f_4+f_1f_8}{1-f_1f_5}$$

$$(6.49)\quad dY1_t/d\overline{Y} = (\partial Y1_t/\partial GF1_t)(\partial GF1_t/\partial GF_t)(\partial GF_t/\partial \overline{Y}) + (\partial Y1_t/\partial GF2_t)(\partial GF2_t/\partial GF_t)(\partial GF_t/\partial \overline{Y}) = \frac{f_2+f_1f_6}{1-f_1f_5}$$

$$(6.50)\quad dY2_t/d\overline{Y1} = (\partial Y2_t/\partial G1_t)(\partial G1_t/\partial \overline{Y1}) = \frac{f_7+f_3f_5}{1-f_1f_5}$$

$$(6.51)\quad dY2_t/d\overline{Y2} = (\partial Y2_t/\partial G2_t)(\partial G2_t/\partial \overline{Y2}) = \frac{f_8+f_4f_5}{1-f_1f_5}$$

$$(6.52)\quad dY2_t/d\overline{Y} = (\partial Y2_t/\partial GF1_t)(\partial GF1_t/\partial GF_t)(\partial GF_t/\partial \overline{Y}) + (\partial Y2_t/\partial GF2_t)(\partial GF2_t/\partial GF_t)(\partial GF_t/\partial \overline{Y}) = \frac{f_6+f_2f_5}{1-f_1f_5}$$

$$(6.53)\quad dR_t/d\overline{Y1} = (\partial R_t/\partial G1_t)(\partial G1_t/\partial \overline{Y1}) = \frac{a_{18}(f_3+f_1f_7)+a_{22}(f_7+f_3f_5)}{(a_{20}+a_{24})(1-f_1f_5)}$$

$$(6.54)\quad dR_t/d\overline{Y2} = (\partial R_t/\partial G2_t)(\partial G2_t/\partial \overline{Y2}) = \frac{a_{18}(f_4+f_1f_8)+a_{22}(f_8+f_4f_5)}{(a_{20}+a_{24})(1-f_1f_5)}$$

$$(6.55)\quad dR_t/d\overline{Y} = (\partial R_t/\partial GF_t)(\partial GF_t/\partial \overline{Y}) = \frac{a_{18}(f_2+f_1f_6)+a_{22}(f_6+f_2f_5)}{(a_{20}+a_{24})(1-f_1f_5)}$$

$$(6.56)\quad dH1_t/d\overline{Y1} = (\partial H1_t/\partial G1_t)(\partial G1_t/\partial \overline{Y1}) = \frac{a_{20}a_{22}(f_7+f_3f_5)-a_{18}a_{24}(f_3+f_1f_7)}{(a_{20}+a_{24})(1-f_1f_5)}$$

$$(6.57)\quad dH1_t/d\overline{Y2} = (\partial H1_t/\partial G2_t)(\partial G2_t/\partial \overline{Y2}) = \frac{a_{20}a_{22}(f_8+f_4f_5)-a_{18}a_{24}(f_4+f_1f_8)}{(a_{20}+a_{24})(1-f_1f_5)}$$

$$(6.58)\quad dH1_t/d\overline{Y} = (\partial H1_t/\partial GF_t)(\partial GF_t/\partial \overline{Y}) = \frac{a_{20}a_{22}(f_6+f_2f_5)-a_{18}a_{24}(f_2+f_1f_6)}{(a_{20}+a_{24})(1-f_1f_5)}$$

In Table 6.1, the exogenous variables are $\overline{Y1}$, $\overline{Y2}$, and $\overline{Y}$. They affect the system through their influences on government expenditures.

Equations (6.44) to (6.55) represent partial short-run multiplier effects. The last column of Table 6.1 presents the aggregate (or combined) short run multiplier effects.

Table 6.1: The IRFG Model

Signs of Short-Run Multiplier Effects

Derivative of	With $\overline{Y1}$	$\overline{Y2}$	Respect $\overline{Y}$	to $(\overline{Y1}+\overline{Y2}+\overline{Y})$
$Y1_t$	+	+	+	+
$Y2_t$	+	+	+	+
R_t	+	+	+	+
$H1_t$	-	+	?	?

A significant difference between Table 5.1 and Table 6.1 is that in Table 5.1 (IRF model), only one of the government expenditures is increased in a once-and-for-all fashion, but in Table 6.1 (IRFG model), all government expenditures are changing. The endogenous variables are affected not only by one, but by all of the exogenous income target variables ($\overline{Y1}$, $\overline{Y2}$ and $\overline{Y}$). This is the reason for a combined multiplier effect in the last column. The aggregate short-run multiplier effects are the sums of all the partial ones:

$$(6.59)\quad dY1_t/d\overline{Y} + dY1_t/d\overline{Y1} + dY1_t/d\overline{Y2} = \frac{f_2+f_3+f_4+f_1(f_6+f_7+f_8)}{(1-f_1f_5)}$$

$$(6.60)\quad dY2_t/d\overline{Y} + dY2_t/d\overline{Y1} + dY2_t/d\overline{Y2} = \frac{f_6+f_7+f_8+f_5(f_2+f_3+f_4)}{(1-f_1f_5)}$$

$$(6.61)\quad dR_t/d\overline{Y} + dR_t/d\overline{Y1} + dR_t/d\overline{Y2}$$

$$= \frac{a_{18}(f_2+f_3+f_4+f_1(f_6+f_7+f_8))+a_{22}(f_6+f_7+f_8+f_5(f_2+f_3+f_4))}{(a_{20}+a_{24})(1-f_1f_5)}$$

(6.62) $dH1_t/d\overline{Y} + dH1_t/d\overline{Y1} + dH1_t/d\overline{Y2}$

$$= \frac{a_{20}a_{22}(f_6+f_7+f_8+f_5(f_2+f_3+f_4))-a_{18}a_{24}(f_2+f_3+f_4+f_1(f_6+f_7+f_8))}{(a_{20}+a_{24})(1-f_1f_5)}$$

Fig.6.1 and Fig.6.2: The IRFG Model
Short-Run Multiplier Effects
Case I: $\overline{Y1}$, $\overline{Y2}$ Increased by the Same Percentage
IS_1, IS_2 Shifted by the Same Magnitude

Fig. 6.1

Region 1

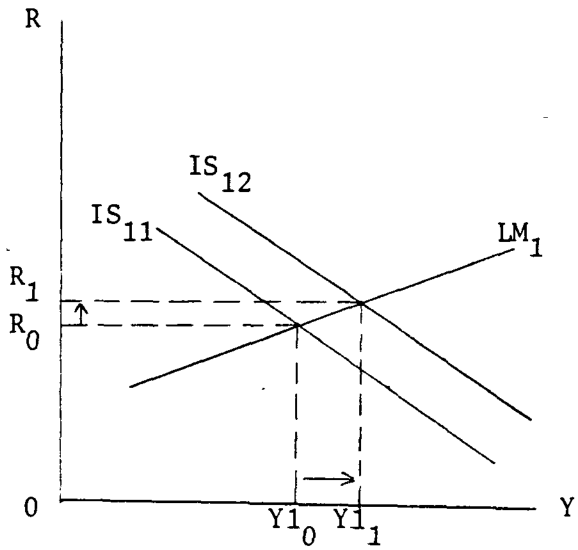


Fig.6.2

Region 2

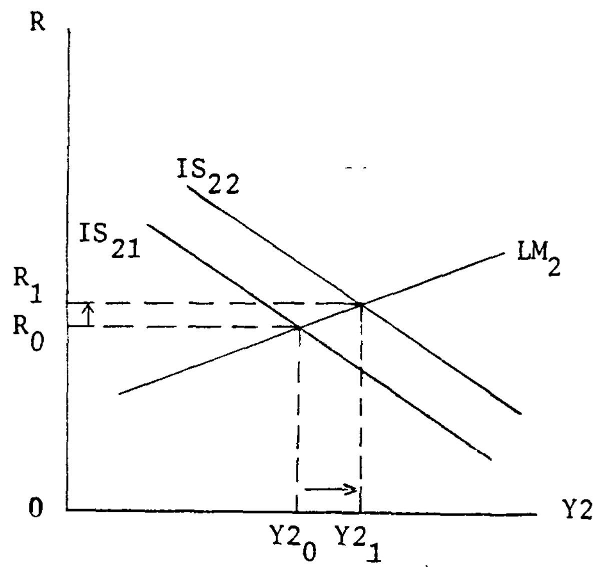


Fig.6.1 and Fig.6.2 show the short-run multiplier effects in the case where both regional income targets are increased by the same percentage. We assume, as before, two identical regions. The shifts in the IS curves show the net total increase in aggregate demand. They are determined by the total government expenditures in each region. The movement of the LM curves depends on the relative shifting of the IS curves. If both IS curves shift rightward by the same magnitude, the LM curves are not at all affected. This is completely different from the situation depicted in Fig.4.1 and Fig.4.2 where LM_2 must shift

leftward when the first region acquires more money balances to accommodate increasing economic activity by selling bonds to the second region. In Fig.6.1 and Fig.6.2, both regions experience increasing aggregate demands of the same magnitude. This magnitude is determined not only by the percentage increase in income targets, but also by the distribution of federal expenditures between the two regions. A higher proportion of federal spending in one region would increase the aggregate demand in that region. With both regions experiencing the same increase in aggregate demand and competing for more money balances from the fixed money stock, the final outcome is a higher interest rate but no movement of the LM curves. This implies that in the short run the domestically held stock of bonds in both regions remains intact.

There are two other possibilities indicated in Table 6.1. We will consider the case in which the regional income targets increase by different percentages in the two regions, and the IS curves shift rightward by different magnitudes, or any combination of distribution schemes that would generate different increases in aggregate demand in the two regions.

Fig.6.3 and Fig.6.4: The IRFG Model
Short-Run Multiplier Effects
Case II: $\overline{Y1}$, $\overline{Y2}$ Increased by Different Percentage
IS_1, IS_2 Shifted by Different Magnitude

Fig.6.3

Region 1

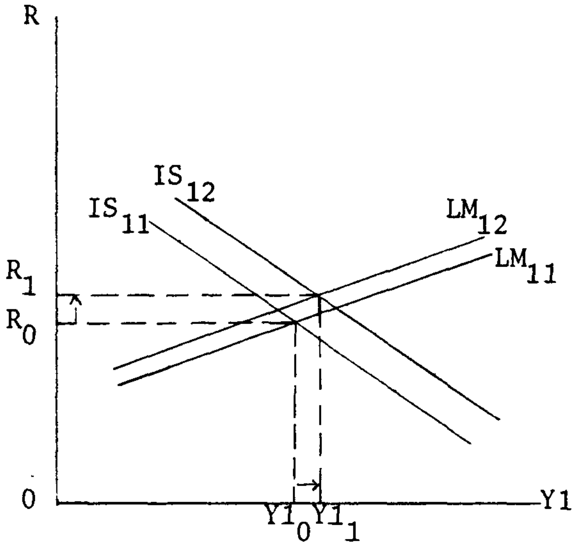
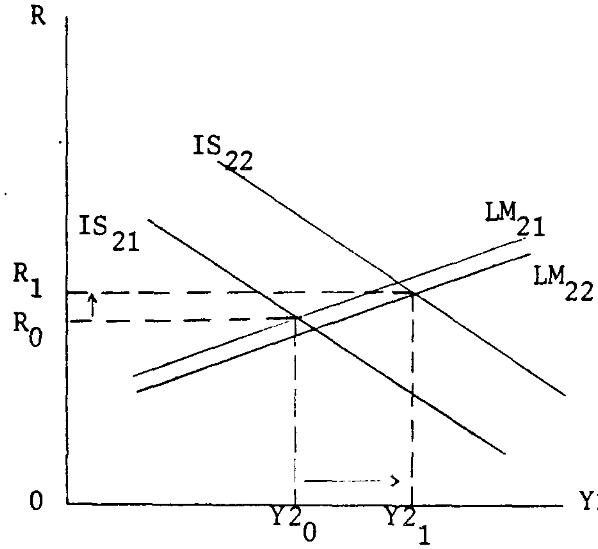


Fig.6.4

Region 2



In Fig.6.3 and Fig.6.4, the rightward shifting of IS_2 is greater than the rightward shifting of IS_1. Thus, the demand for more money balances to accommodate increasing economic activity is higher in the second region than in the first one. In the short run, the total money stock remains unchanged. A region can get more money stock by selling bonds to the other region. This is indicated by the movement of the LM curves. Bond holdings in the second region fall by the shifting of LM_2 rightward while bond holdings in the first region increase by the shifting of LM_1 leftward.

6.3.2. Long-Run System

In analysing the long-run effect we must consider the full model, including the government budget constraints. Long-run equilibrium, in the IRFG model, is defined as the situation that exists when the targets are reached, all changes of endogenous variables are zero, and all predetermined variables remain constant. The system will not be in equilibrium at incomes which are either higher or lower than the targets. As long as the targets are not reached, the system will generate dynamic forces which push it towards the targets. It is therefore not possible that one region would satisfy its target while the other region was forced to adjust its expenditures indefinitely.

There is no distinction in terms of stability between interregional fiscal policy and federal fiscal policy. Each becomes part of the fiscal process in moving towards the targets, and interregional and federal fiscal policies reinforce rather than counteract each other.[7] The likelihood of moving toward stability is therefore much higher than in the previous model.

A balanced budget is one of the main targets of fiscal policy. The IRFG model is constructed under the assumption that a balanced budget must be maintained even at the expense of prolonging the time required to reach the income target. In this sense the likelihood of moving toward stability is much higher than in the previous model. However, this does not mean stability would automatically be achieved.

Though the driving force of the IRFG model is the thrust towards achievement of the income targets, the time required to reach these targets is governed by the target-adjustment parameters (A's). The larger the A values, the faster the approach to the targets. These parameters can be viewed as policy parameters. Each level of government has its own objective in determining the possible range of A's. It is possible that the A values, if outside some specific range, may result in overshooting or undershooting of the targets, and the system may be unstable. Special attention must be paid to the distribution parameters A7, A8, A9 and A10 since they affect the shares of federal government expenditures going to the two regions. Too high or too low a level of federal expenditures in a region will affect aggregate demand and income in both regions, and thus indirectly influence the regional governments' expenditures in the next period. Given the same ratio of total government expenditures to regional income targets, the regional income target can be achieved with smaller regional government expenditures and higher federal government expenditures in a given region, or vice versa. This will affect the interest burden ratios in both regions.

The formal stability analysis of the IRFG model follows the same procedure as with the IRFG model, except that now all G variables are endogenous. Adding several endogenous variables and equations would further complicate the expression for each term of the Schur conditions discussed in Section 4.3.3. Exploration of the stability of the system is confined again to analysis by simulation.

The discussion of long-run multiplier effects is meaningful if and only if the system is stable. Provided that the system is stable, the long-run system becomes

(6.63) Y1 = E1+G1+GF1+IM2-IM1

(6.64) $Y2 = E2+G2+GF2+IM1-IM2$

(6.65) $E1 = E1(YD1,W1,R)$

(6.66) $E2 = E2(YD2,M+B1+B2+BF-W1,R)$

(6.67) $IM1 = IM1(YD1,W1,R)$

(6.68) $IM2 = IM2(YD2,M+B1+B2+BF-W1,R)$

(6.69) $YD1 = (1-u1-uF)(Y1+TR1+TRF1+RH1)$

(6.70) $YD2 = (1-u2-uF)(Y2+TR2+TRF2+R(B1+B2+BF-H1))$

(6.71) $W1-H1 = L1(Y1,W1,R)$

(6.72) $M-W1+H1 = L2(Y2,M+B1+B2+BF-W1,R)$

(6.73) $0 = G1+GF1+TR1+TRF1+RH1+IM2-IM1-(u1+uF)(Y1+TR1+TRF1+RH1)$

(6.74) $0 = G1+TR1+RB1-u1(Y1+TR1+TRF1+RH1)$

(6.75) $0 = G2+TR2+RB2-u2(Y2+TR2+TRF2+R(B1+B2+BF-H1))$

(6.76) $0 = GF1+GF2+TRF1+TRF2+RBF-uF(Y1+Y2+TR1+TR2+TRF1+TRF2+R(B1+B2+BF))$

In long-run equilibrium, all changes in the stocks of financial assets become zero when the budgets are balanced. Thus "zero" appears on the left-hand sides of the four budget constraints (6.73) to (7.76). Note also that time subscripts do not appear in any of the equations because, by definition, the system is in long-run equilibrium only when changes of time do not matter. The endogenous and predetermined variables are then as follows:

Endogenous variables: Y1,Y2,E1,E2,IM1,IM2,YD1,YD2,H1,R,W1,B1,B2, BF(or M, depending on the federal financing policy)

Predetermined variables: G1,G2,GF,GF1,GF2,u1,u2,uF,TR1,TR2,TRF1,TRF2, M(or BF, depending on the federal financing policy)

In the IRFG model, the interpretation of the long-run multiplier effects is different from the conventional wisdom based on analysis of once-and-for-all changes of expenditures of a specific government. In the IRFG model, expenditures on goods and services of all governments are changing in different magnitudes in each period, according to (6.11) to (6.21). Government expenditures will be constant if and only if all the targets are achieved. If all the income targets are achieved, and the budgets are balanced, then from equations (6.33) to (6.35), we have

(6.77) $G1_t = G1_{t-1}$

(6.78) $G2_t = G2_{t-1}$

(6.79) $GF_t = GF_{t-1}$

In equations (6.77) to (6.79), the right-hand side values are last period's government expenditures, and these will remain unchanged when the targets are reached. If in a long-run stable equilibrium, with targets achieved (e.g., $dY1_t = d\overline{Y1}$), we write the long-run partial multiplier in the same manner as the short-run partial multiplier (6.47), it becomes

(6.80) $dY1_t/d\overline{Y1} = (\partial Y1_t/\partial G1_t)(\partial G1_t/\partial \overline{Y1}) = 1$

Equation (6.80) implies a long-run partial multiplier of unity for Region 1 with respect to its own regional goverment expenditure, and therefore the increase in $G1_t$ must be the same as in $\overline{Y1}$. Equation (6.80) also implies either that $\overline{Y2}$ and $\overline{Y}$ have no effect on $Y1_t$ or that the combined effect of $\overline{Y2}$ and $\overline{Y}$ on $Y1_t$ is zero.

However, the long-run partial multiplier effect expressed in (6.80) is not warranted for the following reasons:

First , the long-run partial multiplier effect should be determined by the extent of government expenditure changes initiated by the higher income target, rather than by direct comparison of changes of actual and target income levels. (In this case the $dY1_t/d\overline{Y1}=1$, as stipulated in (6.80).) It is only through changes of government expenditures that actual income will be brought to its new target level.

Second, the long-run partial multiplier effect expressed as the middle term of (6.80) is inappropriate for the IRFG model. The confusion arises from the conventional wisdom of what is meant by a long-run multiplier and the basic nature of government expenditures in the IRFG model. In conventional analysis, a change of government expenditures is fixed by a once-and-for-all increase, while a change of income is variable, depending on the multiplier effect. In the IRFG model, if the target were reached, then actual income would equal the new fixed level of target income. The government expenditures on goods and services adjust over time and become constant only when the target income level is achieved. Moreover, this new constant level of government expenditures on goods and services is determined by the target adjustment parameters. Different combinations of target-adjustment parameters may result in different levels of government expenditures. Using the multiplier formula expressed in the middle term of (6.80) would mean different long-run partial multiplier effects for the same income target level.[8]

The unreasonable conclusion derived from (6.80) is the result of applying the conventional analysis to the IRFG model in which the government expenditures adjust over time. To capture the basic nature of the endogenous government expenditures, the long-run partial multiplier effects should be calculated as follows:

(6.81) $dY1_t/d\overline{Y1} = \sum_{t=1}^{\infty} (\partial Y1_t/\partial G1_t)(\partial G1_t/\partial \overline{Y1})$

(6.82) $dY1_t/d\overline{Y2} = \sum_{t=1}^{N} (\partial Y1_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})$

(6.83) $dY1_t/d\overline{Y} = \sum_{t=1}^{N} (\partial Y1_t/\partial GF1_t)(\partial GF1_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$
$+ \sum_{t=1}^{N} (\partial Y1_t/\partial GF2_t)(\partial GF2_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

(6.84) $dY2_t/d\overline{Y1} = \sum_{t=1}^{N} (\partial Y2_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$

(6.85) $dY2_t/d\overline{Y2} = \sum_{t=1}^{N} (\partial Y2_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})$

(6.86) $dY2_t/d\overline{Y} = \sum_{t=1}^{N} (\partial Y2_t/\partial GF1_t)(\partial GF1_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$
$+ \sum_{t=1}^{N} (\partial Y2_t/\partial GF2_t)(\partial GF2_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

(6.87) $dR_t/d\overline{Y1} = \sum_{t=1}^{N} (\partial R_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$

(6.88) $dR_t/d\overline{Y2} = \sum_{t=1}^{N} (\partial R_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})$

(6.89) $dR_t/d\overline{Y} = \sum_{t=1}^{N} (\partial R_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

(6.90) $dH1_t/d\overline{Y1} = \sum_{t=1}^{N} (\partial H1_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$

(6.91) $dH1_t/d\overline{Y2} = \sum_{t=1}^{N} (\partial H1_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})$

(6.92) $dH1_t/d\overline{Y} = \sum_{t=1}^{N} (\partial H1_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

where "N" denotes the number of time period required to approach the targets to the fourth decimal point. Strictly speaking, the targets are reached only approximately. The higher degree of accuracy (i.e., to the n'th decimal point) required, the larger the N would result. Our criterion of "to the fourth decimal point" is a reasonable approximation. The long-run aggregate multiplier effects are then as follows:

(6.93) $(dY1_t/d\overline{Y1})+(dY1_t/d\overline{Y2})+(dY1_t/d\overline{Y}) = \sum_{t=1}^{N} (\partial Y1_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$
$+ \sum_{t=1}^{N} (\partial Y1_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2}) + \sum_{t=1}^{N} (\partial Y1_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

(6.94) $(dY2_t/d\overline{Y1})+(dY2_t/d\overline{Y2})+(dY2_t/d\overline{Y}) = \sum_{t=1}^{N} (\partial Y2_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$
$+ \sum_{t=1}^{N} (\partial Y2_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2}) + \sum_{t=1}^{N} (\partial Y2_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

(6.95) $(dR_t/d\overline{Y1})+(dR_t/d\overline{Y2})+(dR_t/d\overline{Y}) = \sum_{t=1}^{N} (\partial R_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$
$+ \sum_{t=1}^{N} (\partial R_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2}) + \sum_{t=1}^{N} (\partial R_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

(6.96) $(dH1_t/d\overline{Y1})+(dH1_t/d\overline{Y2})+(dH1_t/d\overline{Y}) = \sum_{t=1}^{N} (\partial H1_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$
$+ \sum_{t=1}^{N} (\partial H1_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2}) + \sum_{t=1}^{N} (\partial H1_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

Because of the large number of equations, the substitutions required to derive expressions in terms only of exogenous variables are numerous, and the final expressions are very complicated. Only some general conclusions can be drawn. For example, the own-expenditure income multipliers are greater than the cross-expenditure multipliers.

Another difference between the conventional and the IRFG model is in the relative strengths of the two federal modes of financing policies. Christ (1979) argues that money-financing policy is unambiguously more powerful than bond-financing policy.[9] However, it is not necessarily true in the IRFG model. The long-run federal bond-financing multiplier effects may be greater than their pure money-financing policy counterparts, or vice versa . This can be attributed to the values of the target-adjustment parameters which determine the amounts of government expenditures and the long-run multiplier effects. Which target-adjustment parameter is predominant in determining the strength of the long-run multiplier effects is an empirical question. As we emphasized at the outset, the prime objective of an active fiscal policy is the attainment of targets and the long-run multiplier effects are less important. The main concern of the public sector shifts to the interest burden ratio, which is defined as the ratio of interest payments on outstanding bonds to tax revenues, associated with deficit financing. This will be further discussed in the next chapter.

6.4 A Simulation Approach to the Analysis of Fiscal Policy in the IRFG Model

The forms of the functions for private expenditures, imports and money demand remain the same as in the IRF model. Now, however, all government expenditures on goods and services become endogenous. We assume that the system is in a stationary equilibrium initially and consider two cases, one with identical regions and the other with regions with unequal size.

The case of equi-sized regions was considered in the last chapter. For the case in which the regions are of unequal size, we assume that the first region is larger than the second in the ratio of 0.7 to 0.3. The functional forms remain unchanged. The micro-foundation of macro-functions follows the same approach as discussed in Appendix 4.2. initial values and parameter sets are presented as follows:

$Y1_t = 70.0$	$Y2_t = 30.0$	$E1_t = 56.0$	$E2_t = 24.0$
$G1_t = 9.8$	$G2_t = 4.2$	$GF1_t = 4.2$	$GF2_t = 1.8$
$YD1_t = 56.0$	$YD2_t = 24.0$	$IM1_t = 16.8$	$IM2_t = 16.8$
$TR1_t = 4.2$	$TR2_t = 1.8$	$TRF1_t = 6.72$	$TRF2_t = 2.88$
$B1_t = 23.94$	$B2_t = 10.26$	$BF_t = 41.3$	$M_t = 31.5$
$W1_t = 74.9$	$H1_t = 52.85$	$u1_t = 0.18$	$u2_t = 0.18$
$uF_t = 0.15$	$R_t = 0.05$	$DW1_t=DB1_t=DB2_t=DBF_t=DM_t= 0$	

Parameter Set (1)

$$E1_t = 8.23+0.8YD1_t+0.107W1_t-100.8R_t$$

$$IM1_t = 2.47+0.24YD1_t+0.032W1_t-30.24R_t$$

$$W1_t-H1_t= 13.2+0.189Y1_t+0.12W1_t-264.6R_t$$

$$E2_t = 3.52+0.8YD2_t+0.107(M_t+B1_t+B2_t+BF_t-W1_t)-43.2R_t$$

$$IM2_t = 2.47+0.56YD2_t+0.075(M_t+B1_t+B2_t+BF_t-W1_t)-30.24R_t$$

$$M_t - W1_t + H1_t = 5.67 + 0.189Y2_t + 0.12(M_t + B1_t + B2_t + BF_t - W1_t) - 113.4R_t$$

Parameter Set (2)

$$E1_t = 4.86 + 0.8YD1_t + 0.107W1_t - 33.6R_t$$

$$IM1_t = 1.46 + 0.24YD1_t + 0.032W1_t - 10.1R_t$$

$$W1_t - H1_t = 39.7 + 0.189Y1_t + 0.12W1_t - 793.8R_t$$

$$E2_t = 2.08 + 0.8YD2_t + 0.107(M_t + B1_t + B2_t + BF_t - W1_t) - 14.1R_t$$

$$IM2_t = 1.46 + 0.56YD2_t + 0.075(M_t + B1_t + B2_t + BF_t - W1_t) - 10.1R_t$$

$$M_t - W1_t + H1_t = 17.0 + 0.189Y2_t + 0.12(M_t + B1_t + B2_t + BF_t - W1_t) - 340.2R_t$$

Parameter Set (3)

$$E1_t = -0.73 + 0.9YD1_t + 0.107W1_t - 33.6R_t$$

$$IM1_t = -0.22 + 0.27YD1_t + 0.032W1_t - 10.08R_t$$

$$W1_t - H1_t = 39.7 + 0.189Y1_t + 0.12W1_t - 793.8R_t$$

$$E2_t = -0.31 + 0.9YD2_t + 0.107(M_t + B1_t + B2_t + BF_t - W1_t) - 10.1R_t$$

$$IM2_t = -0.22 + 0.63YD2_t + 0.075(M_t + B1_t + B2_t + BF_t - W1_t) - 10.08R_t$$

$$M_t - W1_t + H1_t = 17.0 + 0.189Y2_t + 0.12(M_t + B1_t + B2_t + BF_t - W1_t) - 340.2R_t$$

Parameter Set (4)

$$E1_t = -7.7 + 0.9YD1_t + 0.2W1_t - 33.6R_t$$

$$IM1_t = -2.3 + 0.27YD1_t + 0.06W1_t - 10.1R_t$$

$$W1_t - H1_t = 39.7 + 0.189Y1_t + 0.12W1_t - 793.8R_t$$

$$E2_t = -3.3 + 0.9YD2_t + 0.2(M_t + B1_t + B2_t + BF_t - W1_t) - 14.1R_t$$

$$IM2_t = -2.3 + 0.63YD2_t + 0.14(M_t + B1_t + B2_t + BF_t - W1_t) - 10.1R_t$$

$$M_t - W1_t + H1_t = 17.0 + 0.189Y2_t + 0.12(M_t + B1_t + B2_t + BF_t - W1_t) - 340.2R_t$$

After the initial stationary equilibrium is obtained, governments then calculate the new capacity output levels which would be feasible as income targets. The system, shocked by the changes in income target levels, will generate higher levels of government expenditures in order to reach the new target.

Table 6.2 and Table 6.3 indicate the effects when income targets are increased by 5 percent in both regions (i.e., Y1 and Y2 are each increased by 2.5, and Y is increased by 5.0, in Case I; Y1 is increased by 3.5, Y2 by 1.5, and Y by 5.0, in Case II). As before, we consider four parameter sets, under each of federal bond-financing and money-financing policies. In each case, stable long-run equilibrium is achieved.

In the base-run simulations, we assume the values of all of the target-adjustment parameters equal to one. A unitary target-adjustment means that government expenditure changes in exactly the same percentage as the actual value deviated from its target level. Thus the base-run simulations can be used as a reference for comparison and discussion of the effectiveness of an active fiscal policy.

Because of the assumption of identical regions, along with the same percentage income target inceases and the same target-adjustment parameters, the changes of endogenous variables will be the same in both regions in Case I. From Tables 6.2 and 6.3, the following conclusions can be drawn:

Table 6.2: The IRFG Model, Identical-Regions
Simulated Effects when Income Targets Increased by 5% in Both Regions
Unitary Target-Adjustment Parameter Values

Parameter Set	Federal Financing Policy	Period	ΔE1 (=ΔE2)	ΔR	ΔY1 (=ΔY2)	ΔGF1 (=ΔGF2)	ΔG1 (=ΔG2)	ΔENDG1 (=ΔENDG2)	ΔRBT1 (=ΔRBT2)	ΔRBTF	Stability (Time Period)
(1)		Impact	0.4473	0.0009	0.9235	0.1429	0.3333	0.4762			
	Bond	Long Run	2.3094	0.0089	2.5000	-0.0765	0.2671	0.1906	0.0270	0.0564	400
	Money	Long Run	1.9329	0.0030	2.5000	0.3642	0.2029	0.5671	0.0246	0.0014	80
(2)		Impact	0.5476	0.0003	1.0238	0.1429	0.3333	0.4762			
	Bond	Long Run	1.8487	0.0016	2.5000	0.3953	0.2560	0.6513	0.0181	-0.0038	150
	Money	Long Run	1.8653	0.0019	2.5000	0.3724	0.2623	0.6347	0.0179	-0.0009	80
(3)		Impact	0.7225	0.0004	1.1987	0.1429	0.3333	0.4762			
	Bond	Long Run	1.7730	0.0012	2.5000	0.4557	0.2713	0.7270	0.0151	-0.0119	170
	Money	Long Run	1.8442	0.0025	2.5000	0.3551	0.3007	0.6558	0.0141	0.0005	70
(4)		Impact	0.7225	0.0004	1.1987	0.1429	0.3333	0.4762			
	Bond	Long Run	1.7356	0.0010	2.5000	0.4850	0.2794	0.7644	0.0136	-0.0159	180
	Money	Long Run	1.8302	0.0030	2.5000	0.3441	0.3257	0.6698	0.0116	0.0014	30

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

Table 6.3: The IRFG Model, Unequal-Regions
Simulated Effects when Income Target Increased by 5% in Both Regions
Unitary Target-Adjustment Parameter Values

Parameter Set	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1
(1)		Impact	0.6263	0.2684	0.0009	1.2929	0.5541	0.2000
	Bond	Long Run	3.2332	1.3856	0.0089	3.5000	1.5000	-0.1072
	Money	Long Run	2.7061	1.1598	0.0030	3.5000	1.5000	0.5099
(2)		Impact	0.7666	0.3286	0.0003	1.4333	0.6143	0.2000
	Bond	Long Run	2.5882	1.1092	0.0016	3.5000	1.5000	0.5533
	Money	Long Run	2.6114	1.1192	0.0019	3.5000	1.5000	0.5213
(3)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6379
	Money	Long Run	2.5819	1.1065	0.0025	3.5000	1.5000	0.4971
(4)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000
	Bond	Long Run	2.4299	1.0414	0.0010	3.5000	1.5000	0.6788
	Money	Long Run.	2.5623	1.0981	0.0029	3.5000	1.5000	0.4817

ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
0.4667	0.6667	0.0857	0.2000	0.2857				
0.3741	0.2669	-0.0460	0.1603	0.1143	0.0270	0.0270	0.0564	400
0.2840	0.7939	0.2185	0.1217	0.3402	0.0246	0.0246	0.0014	80
0.4667	0.6667	0.0857	0.2000	0.2857				
0.3585	0.9118	0.2371	0.1536	0.3908	0.0181	0.0181	-0.0038	150
0.3673	0.8886	0.2234	0.1574	0.3808	0.0179	0.0179	-0.0009	80
0.4667	0.6667	0.0857	0.2000	0.2857				
0.3799	1.0177	0.2734	0.1628	0.4362	0.0151	0.0151	-0.0119	180
0.4210	0.9181	0.2130	0.1804	0.3935	0.0141	0.0141	0.0005	80
0.4667	0.6667	0.0857	0.2000	0.2857				
0.3913	1.0701	0.2909	0.1677	0.4586	0.0136	0.0136	-0.0159	190
0.4560	0.9377	0.2065	0.1954	0.4019	0.0116	0.0116	0.0014	30

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

First, the changes of government expenditures in the impact period in each public sector are the same for all four sets of parameter values, because the target-adjustment parameters remain unchanged. However, the changes of other endogenous variables in the impact period depend on the structure of the model. The steeper the IS curve and the flatter the LM curves, the greater the short-run multiplier effects. The flatter the IS curves and the steeper the LM curves, the greater the change in the rate of interest.

Second, as far as stability is concerned, the system is stable for all parameter sets, all financing policies, and both regional-size cases. The rate of approach to the targets is always slower under a federal bond-financing than under a money-financing policy. This finding confirms the proposition that interest payments on outstanding bonds constitute an extra source of instability that could be eliminated only in a longer time period, given the same target adjustment parameters in the IRFG model. The differences in the rate of approach to equilibrium vary according to the structure of the economy. For parameter sets (2) and (3), the rates are slower with federal bond-financing than with money-financing. While in extreme cases, like parameter set (1), in which a steeper LM curve intersects a flatter IS curve, and parameter set (4), in which a steeper IS curve intersects a flatter LM curve, the rates are much slower with federal bond-financing than with federal money-financing (roughly six times). On the other hand, the differences in the rates of approach to equilibrium are very small for parameter sets under a federal money-financing policy. Thus our base-run simulation results verify the important contribution of the balanced-budget target-adjustment parameters in bringing the system to a stable long-run equilibrium.

<u>Third</u> , the changes of government expenditures in the long-run are different from their short-run counterparts. Though the target-adjustment parameters remain unchanged, the levels of aggregate demand, which consist of private and public expenditures, must be determined by the structure of the economy. Once again, they depend also on how fast the system approaches its new target levels. For example, the slower the rate of approach to equilibrium generated by the first parameter set, bonds must be issued by each of the three governments, thereby bidding up the rate of interest by almost 1 percent, which is largest increase in all of the cases considered. More outstanding bonds, together with a higher rate of interest, produces a higher interest burden ratio in all governments, indicating that interest payments grow more as compared to tax revenues. This is evidenced by a larger increase in the interest burden ratio, (2.7 percent and 3.6 percent for regional and federal governments, respectively) as compared to other cases. Higher interest burden ratios in the public sector mean more interest income for the private sector, and consequently result in a greater increase in private expenditures. The extent of the increase in public expenditures must then be reduced in order to produce the new fixed level of aggregate demand.

<u>Fourth</u> , the results for parameter sets (2) to (4) show that the increase in the rate of interest is higher with a federal money-financing policy than with a federal bond-financing policy. This result is in complete contrast to the conventional analysis in which the opposite is the case.[10] In the IRFG model, federal tax revenues increase far more than expenditures when income targets in both regions are reached, thereby necessitating either the retirement of outstanding bonds, under a bond-financing policy, or the contraction of money supply, under a

money-financing policy. In the former case, the upward pressure on the interest rate is reduced, since now the federal government retires its outstanding bonds, leaving only the two regional governments to compete for sales of new issues in the bond market. However, in the latter case, when the federal government contracts the money supply, more upward pressure is placed on the interest rate. For this reason there is a greater increase in the rate of interest in the case of a federal money-financing policy.

Fifth , the larger tax base of the federal government also affects its interest burden ratio. Except in the case of bond-financing with the first parameter set, all other cases show that the increase in the interest burden ratio is substantially smaller for the federal government (RBTF) than for the regional governments (RBT1, RBT2). This puts the federal government in a stronger position for implementing an active fiscal policy. We will consider this matter more thoroughly in the next chapter.

Sixth , another finding is that the income target is approached rather rapidly. We observe in every case that over 90 percent of the increased income target level (i.e., the gap between the new and previous income targets) is reached after five periods. This is because expenditures of all governments are adjusting so as to approach the income targets faster in the IRFG model. However, the need to balance the budgets then reduces the rate of approach in later periods.

Fig.6.5 and Fig.6.6 depict the time paths of endogenous variables for both the identical and unequal-region cases, assuming that the income targets are increased by 5 percent in both regions. Only results based on parameter set (3) are presented. In each figure, we present results for both the money-financing (solid line) and bond-financing (broken line) cases. In the unequal-regions case, endogenous variables of both regions are presented. The aggregate federal variables are the same for both the identical and unequal-regions cases; thus we observe the same DM, DBF and RBTF curves in both Fig.6.5 and Fig.6.6. Total government expenditures in a region (ENDG) is the sum of the federal and regional government expenditures in the region.

In this model, with government expenditure reaction functions, the different effects of money-financing and bond-financing policies on regional endogenous variables are very small, compared with the results displayed in Fig.3.3 and Fig.3.4, based on the National Model. For example, the regional income lines are so similar that it becomes difficult to distinguish them. For the federal government, the big difference between money-financing and bond-financing policies is not only in the rates of approach to targets, but also in the effects on the federal interest burden ratio (RBTF). While the regional interest burden ratios (RBT1, RBT2) remain roughly the same under both federal financing policies, the federal interest burden ratio shows a 1 percent difference. The more aggressive or active the federal fiscal policy, the wider this difference.

Figure 6.5: The IRFG Model
Identical-Regions, Base-Run Simulated Effects
of an Increase in Income Target by 5% in Both Regions

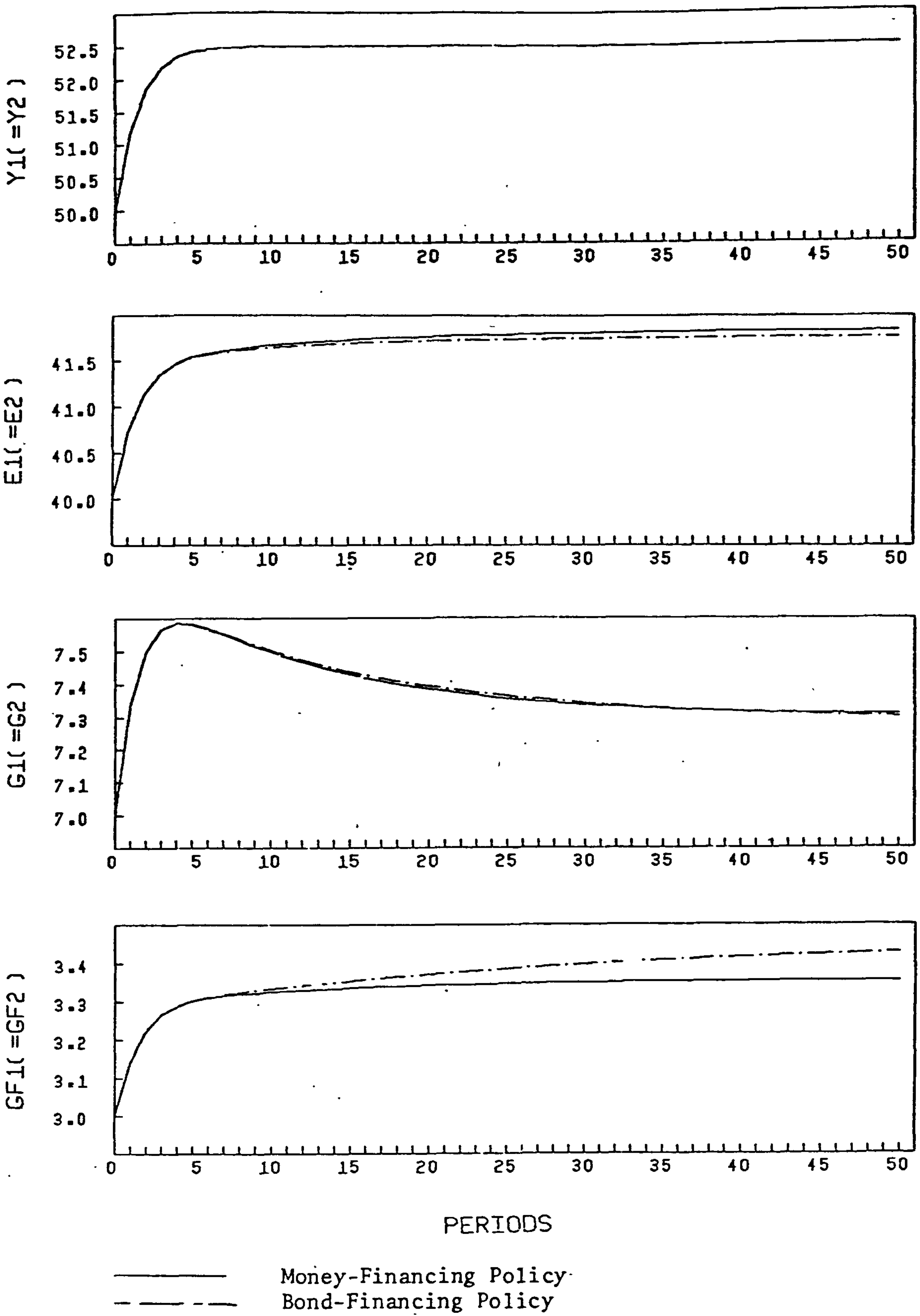


Figure 6.5(Contd.)

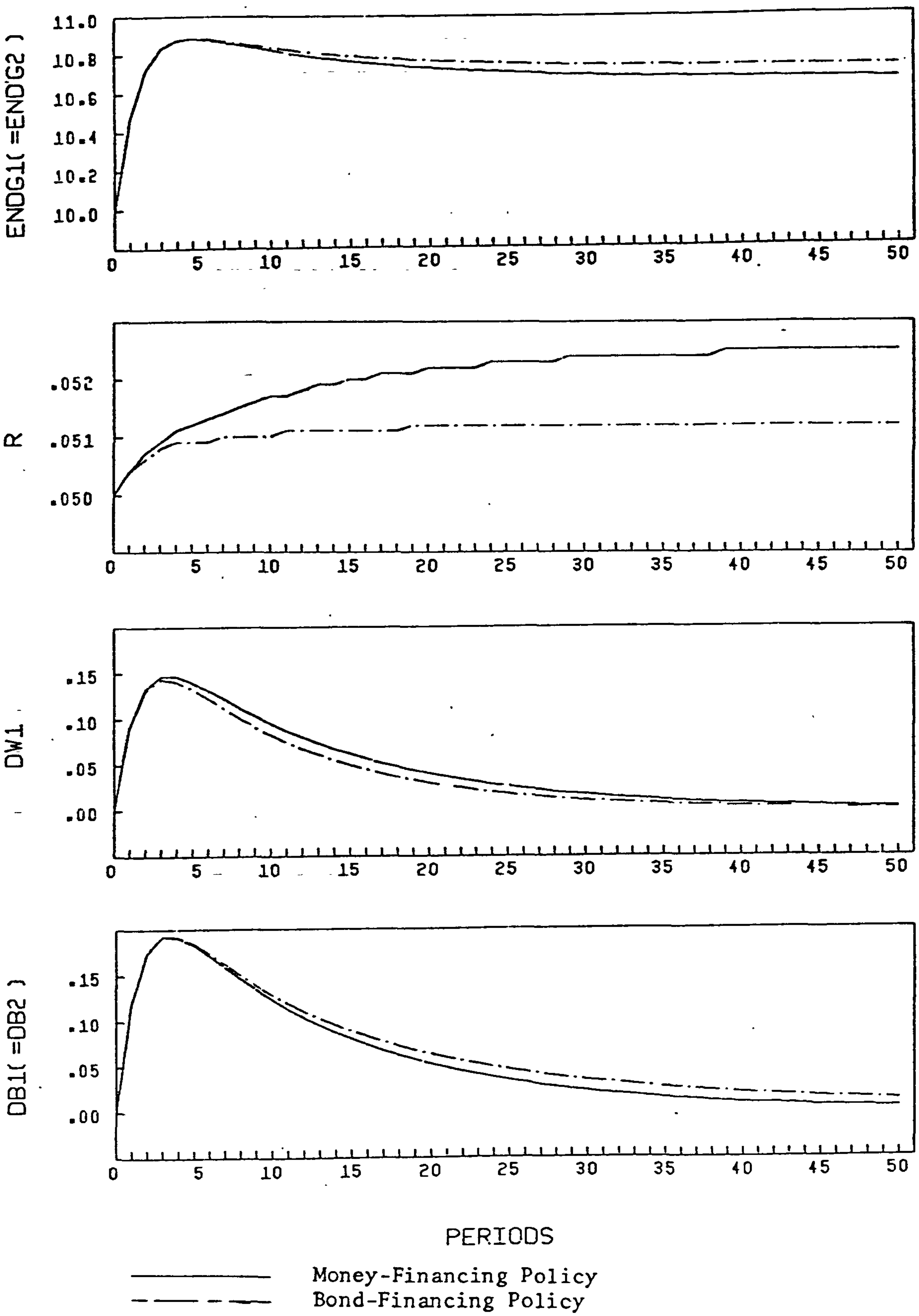
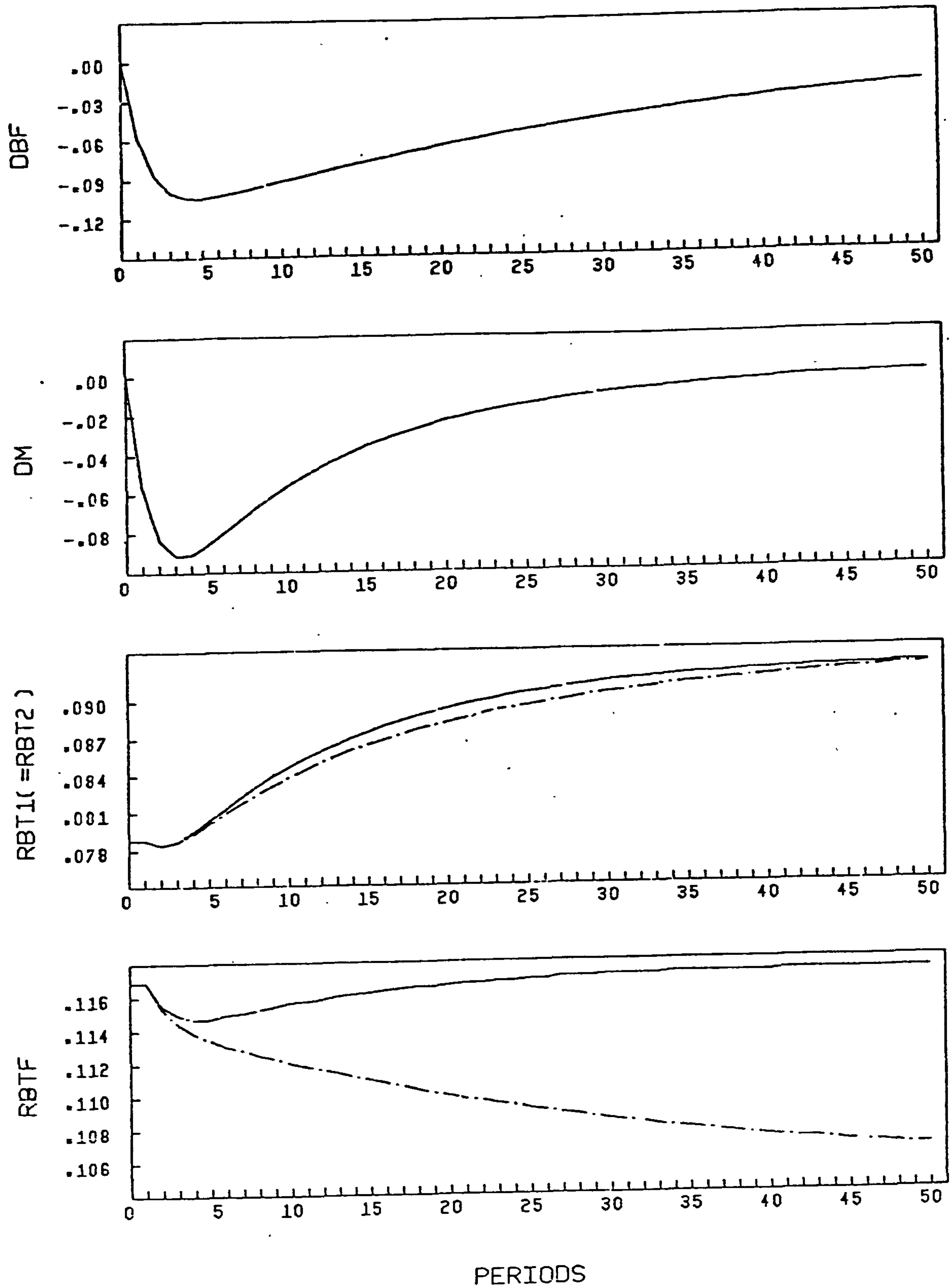


Figure 6.5(Contd.)



——— Money-Financing Policy

– — – — Bond-Financing Policy

Figure 6.6: The IRFG Model
Unequal-Regions, Base-Run Simulated Effects
of an Increase in Income Target by 5% in Both Regions

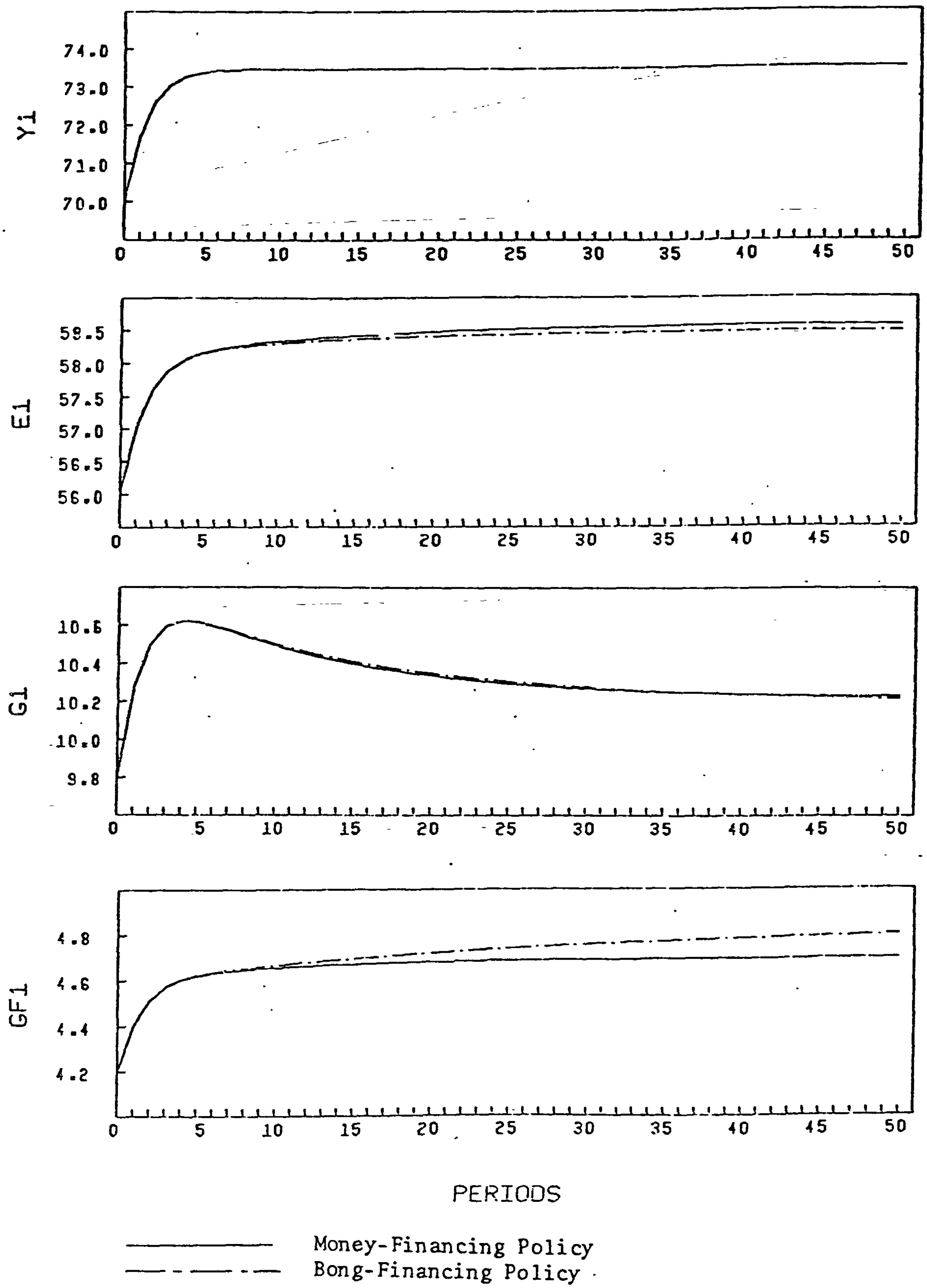


Figure 6.6(Contd.)

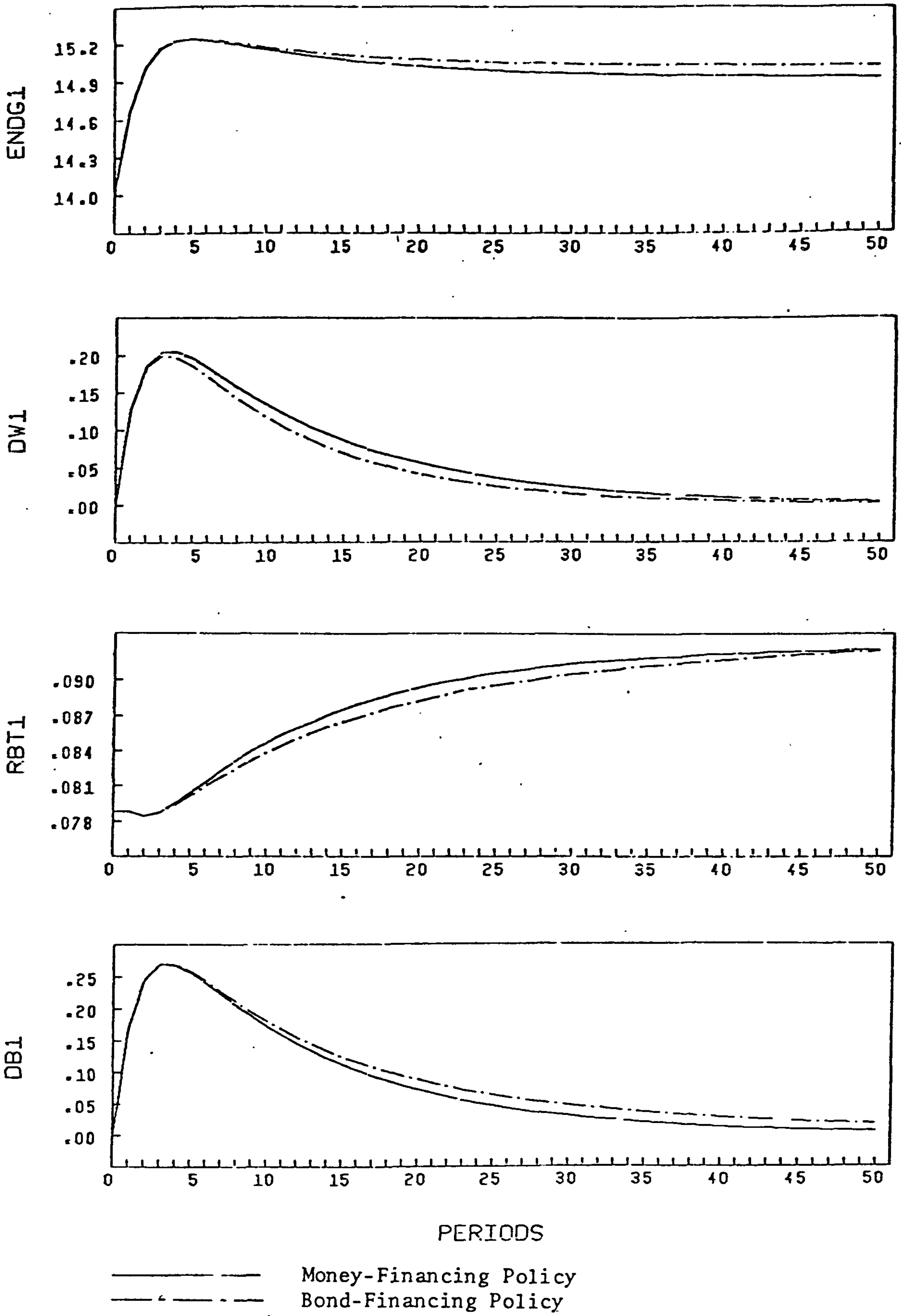


Figure 6.6(Contd.)

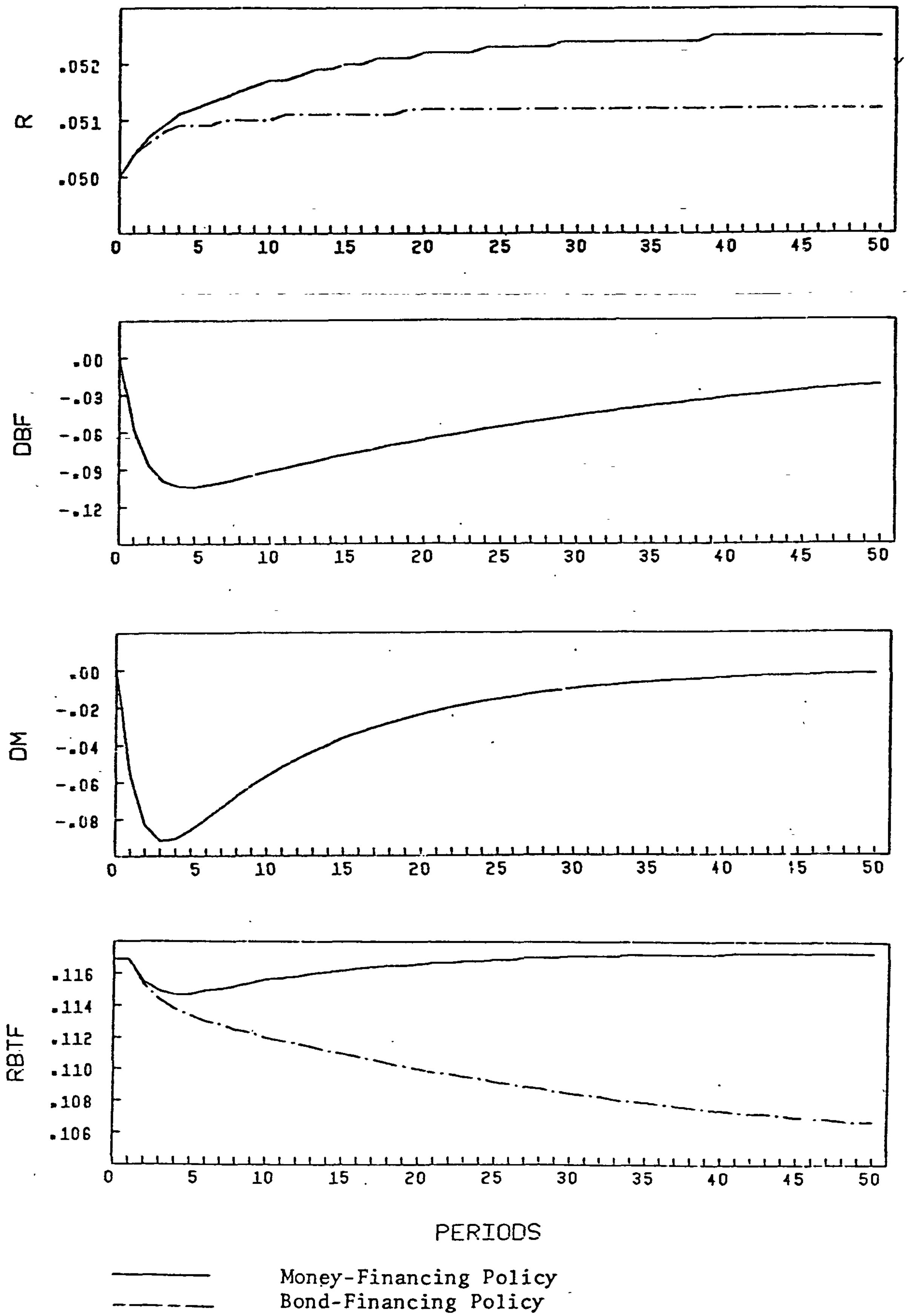
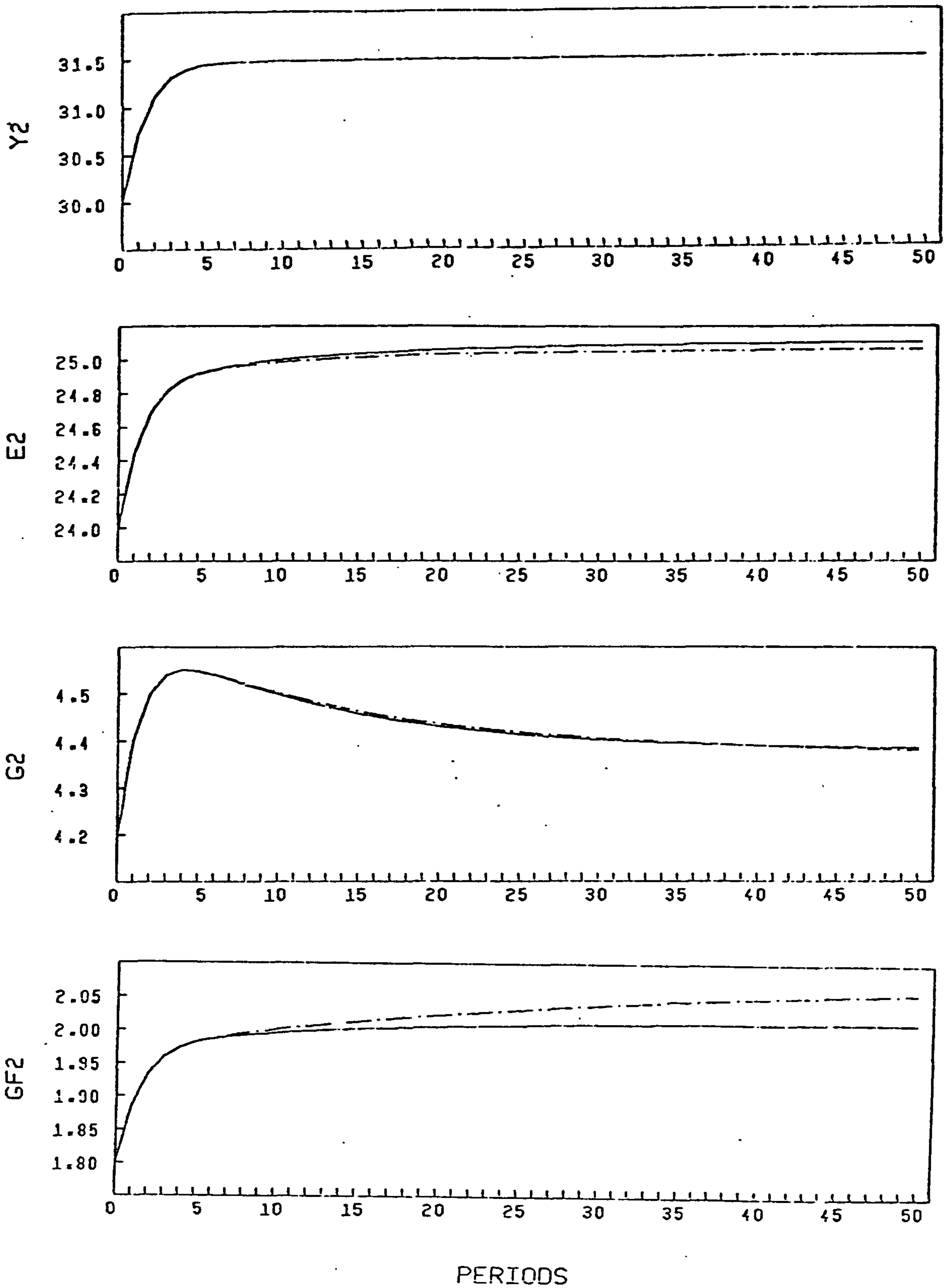


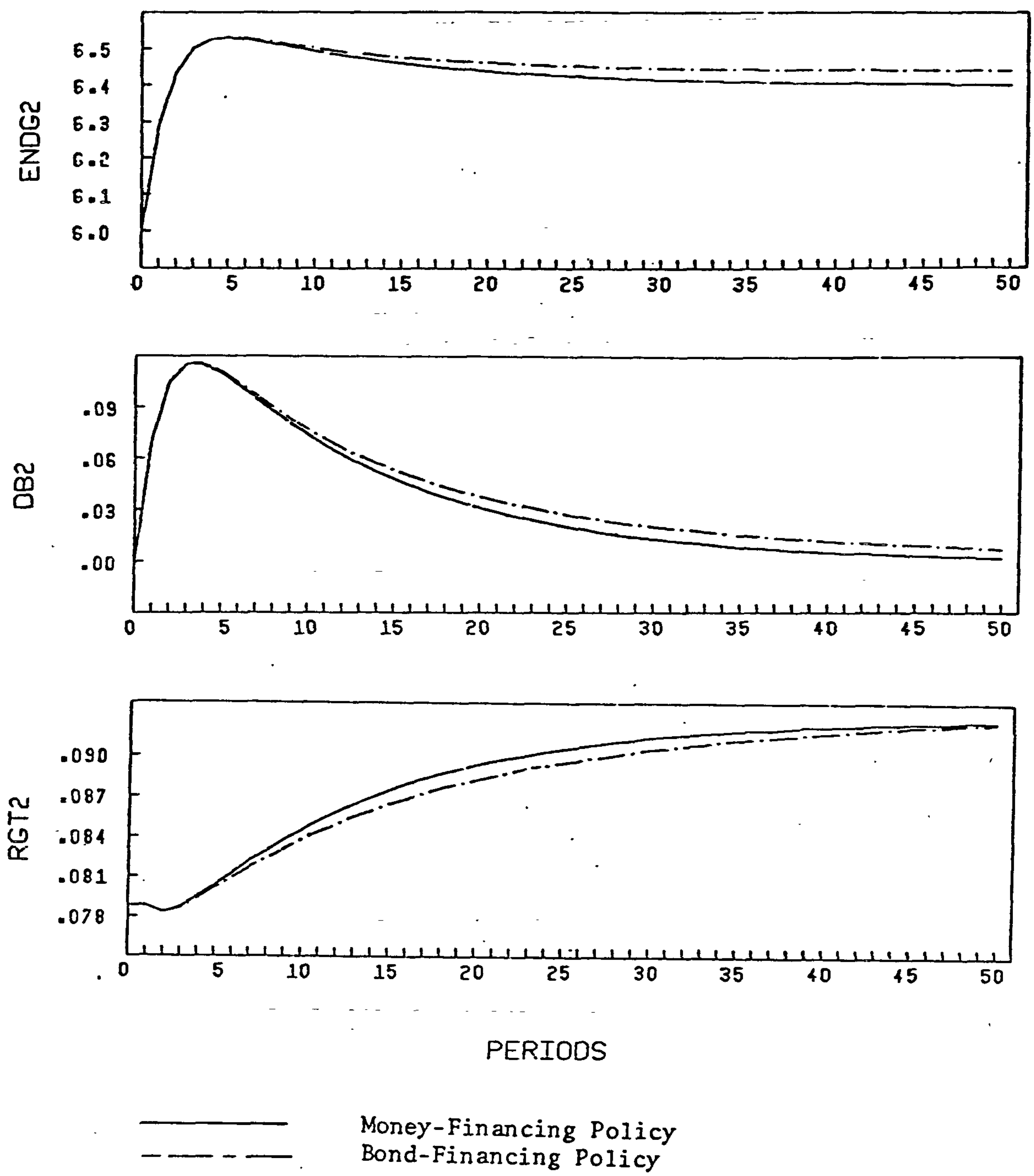
Figure 6.6(Contd.)



Money-Financing Policy

Bond-Financing Policy

Figure 6.6(Contd.)



6.5 Conclusion

In this chapter, we have presented and described the IRFG model. This model will be used in the simulation analysis of an active fiscal policy in the next chapter. As has been noted, the model endogenizes government expenditures, by making them responsive to differences from target level. In addition, the model also incorporates target-adjustment parameters in the government expenditure reaction functions, and this allows us to obtain further insight into the effectiveness of active fiscal policy.

Results for cases in which income targets are increased by 5 percent in both regions have been presented. The system was found to be stable for all four parameter sets, regardless of the federal financing policy.

Footnotes to Chapter 6

(1). Transfer payments are not included in the variables representing endogenous government expenditures because transfer payments are not a component of aggregate demand and therefore do not enter into the income identity equation. Assuming no population growth in our model, we thus keep transfer payments constant at the initial equilibrium level.

(2). The term "instrument instability" has been suggested by Holbrook (1972). He argues that attempts to offset completely the cumulative impact of past changes in the policy instrument may require ever greater changes in the future value of the instrument.

(3). See Scarth (1979), p.112.

(4). Ibid, p.113.

(5). Using the revised model described in Section 6.2.1 and assuming different YF and YR values, we experiment with different combinations of target-adjustment parameters. Results (not reported in the chapter) show that the system generates time paths that fluctuate around their target values indefinitely. Thus the system cannot produce a stable equilibrium.

(6). Buchanan argues that if we consider two regions with an identical output of public goods and services, the wealthier of the two regions will, other things equal, be able to meet its revenue requirements with lower tax rates. Raising a given amount of revenue per resident requires lower tax rates the higher the level of per capita income. The fiscal capacity in the two regions are not equal. See Buchanan (1950).

(7). We assume the income targets estimates are consistent for the regional and federal governments. See the argument presented in Section 6.2.1.

(8). This will be demonstrated in the analysis of an active fiscal policy in the next chapter.

(9). Christ (1979), p.537.

(10). For example, in Table 3.1, the long-run rate of interest declines under a federal pure money-financing policy but increases under a federal pure bond-financing policy.

Chapter 7

Simulation Analysis of an Active Fiscal Policy With the IRFG Model

7.1 Introduction

This chapter offers a detailed presentation and discussion of the results of simulation experiments in which the IRFG model is used to study the effects of interregional fiscal policies on endogenous variables. Each government may pursue an active fiscal policy through variations of the values of the target-adjustment parameters. We want to investigate whether an active fiscal policy implemented by one region benefits the whole economy, or only the implementing region. Especially, we want to address the question of whether the size of the region matters in carrying out an active fiscal policy. Two different cases; one with identical regions and a second with larger and smaller regions, are investigated in this study.

In section 7.2 we discuss the objectives of an active fiscal policy. In section 7.3, we present the simulation results of the identical-regions case in which the implications of different combinations of target-adjustment parameters are investigated. We focus on the effectiveness of an active fiscal policy and the testing for the critical limits of the target-adjustment parameters. Section 7.4 is devoted to the discussion of the unequal-regions case and section 7.5

offers some concluding remarks on the findings of this chapter. The simulated results show that an independent regional fiscal policy cannot generate a stable long-run equilibrium. Only when all governments participate in fiscal management can the targets be achieved. This is in sharp contrast to Oates' conclusion that regions should not pursue any fiscal policy. We also find that there are certain limits of decentralization. The IRFG model thus emphasizes coordination and cooperation among all governments to formulate a viable fiscal policy.

7.2 Objectives of an Active Fiscal Policy

As we have shown in the last chapter, the system is stable when unitary target-adjustment parameters (i.e., base-run simulation) are used. However, a more active fiscal policy implies larger target-adjustment parameter values. The government expenditure reaction function does not rule out an active fiscal policy. In fact, the government expenditure reaction function consists of both endogenous and exogenous components. The exogenous components, namely the target-adjustment parameters, are the policy parameters which each government can freely choose.

The objectives of an active fiscal policy, from the regional and federal government perspectives, will be examined by conducting several experiments, as follows:

First, we increase the speed in reaching the target values. As evidenced in Tables 6.2 and 6.3, the rates of approach to equilibrium are relatively slow, mainly because of the need for satisfying the balanced-budget target. By increasing the value of the balanced-budget target-adjustment parameter, it is possible that we can increase these rates. The budget surplus or deficit is basically determined by the income target-adjustment parameter values. A larger parameter value not only causes the income target to be reached faster, but also increases the deficit. Thus different values of the income target-adjustment parameters also affect the speed at which equilibrium is approached. A relatively slow speed would certainly limit the effectiveness of fiscal policy. It is presumably the objective of both the regional and federal governments to improve the effectiveness of fiscal policy.

Second , we reduce the interest burden ratio. The interest burden ratio is the prime concern of a regional government, which has no access to the central bank. To finance a deficit, the regional government must resort to the bond market. A higher interest burden ratio not only causes a stability problem, but also limits the feasibility of a regional government pursuing further expansionary fiscal policy. There is always political and economic concern that interest payments should not exceed a certain percentage of tax revenues. A political party may have difficulty in staying in power if such a limit is exceeded. The federal government, by virtue of its control over the money supply, is not in the same financial situation as the regional governments. However, helping the regional government to obtain a better financial position may also be an objective of federal fiscal policy, in addition to its prime concern of reaching the income targets. This consideration will have a bearing on which financing policy the federal government decides to use.

Third , we reduce the economic imbalance between regions. This is mainly a federal objective. The fiscal capacity of a poor region is obviously smaller than that of a rich region. In the IRFG model, we hypothesize that one federal objective is to equalize the ratios of the total federal and regional government expenditures to target income across regions.[1] This is represented by the equal expenditure ratio target-adjustment parameters in the federal expenditure distribution functions (i.e., A8 and A10). The federal government may want to help the poor region reach its income target faster by increasing the value of A10 in the federal expenditure distribution function.

These objectives are not without conflict, as far as stability is concerned. The following section investigates the effectiveness of an active fiscal policy and tests for the critical limits of target-adjustment parameter values.

7.3 Analysis of an Active Fiscal Policy: the IRFG Model

We assume, in all of the following simulation experiments, that the income target is increased by 5 percent in both regions after the initial equilibrium. Since the base-run simulation results show that all parameter sets can generate stable results, for convenience, we use only the parameter set (3) as a representative set. The experiments and discussion will be concentrated mainly on the examination of the effectiveness of an active fiscal policy and testing for the critical limits of target-adjustment parameters. The experiments are carried out by changing some of the base-run target-adjustment parameter values.

7.3.1 Identical-Regions Case: The Effectiveness of an Active Fiscal Policy

In this section, we present the following simulation experiments, numbered 1 to 10 (Table 7.1). We then summarize the results of the experiments. Detailed quantitative results are presented in Table 7.2.

Table 7.1: The IRFG Model
Identical-Regions Case
The Effectiveness of an Active Fiscal Policy
Listing of the Simulation Experiments

	Specification				of		Parameter		Values	
Experiments	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
(1)	2	2	1	1	1	1	1	1	1	1
(2)	3	3	1	1	1	1	1	1	1	1
(3)	3	1	1	1	1	1	1	1	1	1
(4)	3	1	1	1	1	1	2	2	1	1
(5)	2	1	2	1	2	1	1	1	1	1
(6)	3	1	3	1	3	1	1	1	1	1
(7)	1	2	1	2	1	2	1	1	1	1
(8)	2	2	2	2	2	2	1	1	1	1
(9)	1	1	1	1	1	1	2	2	1	1
(10)	1	1	1	1	2	2	1	1	1	1

In Experiment (1), we double the value of income and balanced-budget target-adjustment parameters in the first region,(i.e., A1=A2=2) while other "A's" retain their base-run values. In the impact period, the federal government expenditures in both regions, as well as the expenditures of the government of Region 2, remain unchanged from their respective base-run values, but the expenditures of the government of Region 1 double (A1 is increased by 100 percent). Because of the multiplier effect, the increase in income in the first region is more than the additional $G1_t$. Moreover, the second region also benefits from the expansionary fiscal policy implemented by the first region. We observe an almost two-fold increase in income in the second region.

Table 7.2: The IRFG Model, Identical-Regions
Simulated Effects when Income Targets Increased by 5% in Both Regions
The Effectiveness of An Active Fiscal Policy

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
Base Run		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.4762	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4557	0.2713	0.7270	0.4557	0.2713	0.7270	0.0151	0.0151	-0.0119	170
	Money	Long Run	1.8442	1.8442	0.0025	2.5000	2.5000	0.3551	0.3007	0.6558	0.3551	0.3007	0.6558	0.0141	0.0141	0.0005	70
(1)		Impact	1.1062	0.8445	0.0005	1.8372	1.3992	0.1429	0.6667	0.8096	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7730	1.7730	0.0021	2.5000	2.5000	0.5215	0.2055	0.7270	0.5215	0.2055	0.7270	0.0209	0.0131	-0.0142	180
	Money	Long Run	1.8638	1.8638	0.0029	2.5000	2.5000	0.4068	0.2294	0.6362	0.4068	0.2294	0.6362	0.0208	0.0112	0.0012	120
(2)		Impact	1.4898	0.9666	0.0007	2.4757	1.5997	0.1429	1.0000	1.1429	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.5338	0.1931	0.7270	0.4279	0.2990	0.7270	0.0220	0.0127	-0.0147	180
	Money	Long Run	1.8677	1.8677	0.0029	2.5000	2.5000	0.4170	0.2153	0.6323	0.2862	0.3461	0.6323	0.0221	0.0106	0.0014	130
(3)		Impact	1.4898	0.9666	0.0007	2.4757	1.5997	0.1429	1.0000	1.1429	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7731	1.7731	0.0012	2.5000	2.5000	1.6475	-0.9205	0.7270	0.0549	0.6721	0.7270	0.1203	-0.0198	-0.0548	450
	Money	Long Run	2.1775	2.1775	0.0082	2.5000	2.5000	1.3351	-1.0128	0.3224	-0.7113	1.0339	0.3224	0.1361	-0.0428	0.0121	650
(4)		Impact	1.5447	0.9117	0.0007	2.5675	1.5080	0.2127	1.0000	1.2127	0.0730	0.3333	0.4063				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	1.2713	-0.5444	0.7270	0.2489	0.4781	0.7270	0.0871	-0.0031	-0.0449	320
	Money	Long Run	2.0544	2.0544	0.0062	2.5000	2.5000	0.8218	-0.3762	0.4456	-0.1688	0.6144	0.4456	0.0779	-0.0089	0.0080	350
(5)		Impact	1.4449	1.4449	0.0008	2.3973	2.3973	0.2857	0.6667	0.9524	0.2857	0.6667	0.9524				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4551	0.2718	0.7270	0.4551	0.2718	0.7270	0.0151	0.0151	-0.0119	150
	Money	Long Run	1.8438	1.8438	0.0025	2.5000	2.5000	0.3551	0.3011	0.6562	0.3551	0.3011	0.6562	0.0140	0.0140	0.0005	60
(6)		Impact	2.1674	2.1674	0.0012	3.5960	3.5960	0.4286	1.0000	1.4286	0.4286	1.0000	1.4286				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4534	0.2736	0.7270	0.4534	0.2736	0.7270	0.0149	0.0149	-0.0117	140
	Money	Long Run	1.8424	1.8424	0.0025	2.5000	2.5000	0.3553	0.3023	0.6576	0.3553	0.3023	0.6576	0.0139	0.0139	0.0004	60
(7)		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.4762	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3579	0.3691	0.7270	0.3579	0.3691	0.7270	0.0065	0.0065	-0.0013	80
	Money	Long Run	1.7633	1.7633	0.0010	2.5000	2.5000	0.3676	0.3692	0.7367	0.3676	0.3692	0.7367	0.0063	0.0063	-0.0027	70
(8)		Impact	1.4449	1.4449	0.0008	2.3973	2.3973	0.2857	0.6667	0.9524	0.2857	0.6667	0.9524				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3576	0.3693	0.7270	0.3576	0.3693	0.7270	0.0065	0.0065	-0.0013	60
	Money	Long Run	1.7629	1.7629	0.0010	2.5000	2.5000	0.3676	0.3694	0.7371	0.3676	0.3694	0.7371	0.0063	0.0063	-0.0026	60
(9)		Impact	0.7773	0.6676	0.0004	1.2904	1.1069	0.2127	0.3333	0.5460	0.0730	0.3333	0.4063				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4035	0.3235	0.7270	0.5079	0.2190	0.7270	0.0105	0.0197	-0.0119	170
	Money	Long Run	1.8443	1.8443	0.0025	2.5000	2.5000	0.3045	0.3512	0.6557	0.4056	0.2501	0.6557	0.0096	0.0185	0.0005	120
(10)		Impact	0.9392	0.9392	0.0005	1.5583	1.5583	0.2857	0.3333	0.6190	0.2857	0.3333	0.6190				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3041	0.4229	0.7270	0.3041	0.4229	0.7270	0.0018	0.0018	0.0045	60
	Money	Long Run	1.7000	1.7000	-0.0001	2.5000	2.5000	0.3780	0.4220	0.8000	0.3780	0.4220	0.8000	0.0002	0.0002	-0.0051	50

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

In the long run, the results are different depending on the federal financing policy. Under a federal bond-financing policy when income targets are achieved in both regions, the total of new bond issues is the same as in the base-run, regardless of the different target-adjustment parameter values. This results because of the perfect substitutability of bonds issued by different governments. However, the distribution of this fixed total depends on the value of target-adjustment parameters. When we double A1, we observe that the interest burden ratio is increased by half of one percent in the first region while the second regional and federal governments reduce their respective interest burden ratios.

Because of the same increase in total outstanding bonds under federal bond-financing policy, as compared with the base-run level, the rate of interest remains at its base-run level, regardless of the combinations of target-adjustment parameters. Consequently, disposable income and private expenditures are also at their base-run values. By virtue of the equal expenditure ratio targets, the total public expenditures in both regions are the same. However, the distribution of the total between regional and federal components depends on all target-adjustment parameters. As has been noted in the last chapter, due to their endogenous nature, these variables do not have the same importance as in the conventional long-run analysis. Emphasis has been shifted to the interest burden ratio.

Under the federal money-financing policy, the active fiscal policy implemented by the first region affects the budget situation of each government. With current income approaching its target level faster than in the base-run and with a broader tax base, the federal government enjoys a budget surplus, thereby necessitating the contraction of the money supply at a greater rate than in the base run. Thus we observe a higher increase in the rate of interest and private expenditures than in the base run. As far as the interest burden ratio is concerned, there is not much difference in the effects in the first region from the two federal financing policies. However, the increase in the interest burden ratio under a federal money-financing policy is less for the second region, and higher for the federal government, than under a federal bond-financing policy.

Through its effect in raising the rate of interest and private expenditures, aggregate demand in the first region overshoots the income target level under a federal money-financing policy. However, the discrepancy is rather small, due to the higher balanced-budget target adjustment parameter A2.

The basic spirit of Experiment (2) is identical to that of Experiment (1), except that a more aggressive fiscal policy is adopted (A1=A2=3). We observe roughly the same results as obtained in Experiment (1). Even the rates of approach to equilibrium are the same as in Experiment (1).

However, compared with the base-run results, the rates of approach to equilibrium under a federal bond-financing policy in Experiments (1) and (2) have not decreased in any way, while the rates of approach to equilibrium actually become slower under a federal money-financing policy. This is because the overshooting of the system under a federal money-financing policy requires a longer time period approach to the balanced-budget target. Considering the adverse effect on the higher interest burden ratio associated with a higher deficit, it is not worthwhile for one region to pursue a too active fiscal policy, while other governments maintain their base-run target-adjustment parameter values.

Results of Experiments (1) and (2) show that while the first region bears the cost of an active fiscal policy in terms of a higher interest burden ratio, the second region and the federal government actually benefit from the first region's active fiscal policy. In Experiment (3), A1 increases to 3, while the other "A's" retain their base-run values. We want to investigate whether the first region benefits from its active fiscal policy, while the burden of balancing the budget is equally distributed to all governments (i.e., A2=A4=A6=1).

Experiment (3) produces an interesting result. It shows that an active fiscal policy implemented by one region without a correspondingly higher balanced-budget target-adjustment parameter would be worse than the base-run result. This is evidenced by the higher interest burden ratio in the first region (over 20 percent)[2] and the overall longer time period required to approach stability.

More surprisingly, it is the second region which overshoots the system while the first region approaches the income target level gradually. Under a federal money-financing policy, with A1=3 and A2=1, the deficit in the first region increases in the early stage, because A2 is not large enough to outweigh the strong A1 effect. The balanced-budget target is approached very slowly and hence there is pressure to decrease

$G1_t$. On the other hand, with A3=A4=1, the increase in income in the impact period is not as high for the second region as for the first. In each subsequent period $G2_t$ increases gradually. Moreover, since A7=A8=A9=A10=1, the federal government expenditures made in the second region are larger than the amount distributed to the first region because of a higher government expenditure level by the first region in the impact period. The combined effect is that total public expenditure is higher in the second than in the first region, thus overshooting the income target in the second region, while the first region approaches the income target slowly. A higher budget surplus in the second region would generate more $G2_t$, in accordance with the government expenditure reaction function (6.12). The overall income targets, however, are approached very slowly. Hence, the federal government expenditures actually increase. To achieve the equal expenditure ratio target, federal government expenditures in the second region must decrease while expenditures increase in the first region. Furthermore, the decrease in $GF2_t$ is greater than the increase in $G2_t$, and the increase in $GF1_t$ is greater than the decrease in $G1_t$. The system thus approaches its target level with the income in the second region falling and that in the first region increasing towards their respective income target values.

The same behavior pattern is observed under a federal bond-financing policy, except that the increase in the rate of interest is much smaller, thereby resulting in a more rapid approach to equilibrium. Consequently the interest burden ratio is also smaller in the first region as compared to that obtained under a federal money-financing policy.

Thus, we conclude that both income and balanced-budget targets must be considered in any active fiscal policy pursued by any government.

In Experiment (4) we double the values of A7 and A8 to 2 with A1=3. With heavy weights in the federal government distribution function for the first region, this means that the federal government partially takes up the responsibility for balancing the budget in the first region. We therefore observe much improved results over Experiment (3). However, the speed of approach to equilibrium is 2 times slower under a federal bond-financing policy and 5 times slower under a federal money-financing policy, as compared to the base run. The interest burden ratio for the first region is over 15 percent.

From Experiments (1) to (4), we find that the region which pursues a more active fiscal policy, while other governments maintain their base-run target-adjustment parameter values, bears the bulk of the cost. In Experiment (5), we extend the active fiscal policy to all governments by setting A1=A3=A5=2, with all other "A's" remaining at their base-run values. Interestingly enough, the results are almost the same as in the base run. Of special note is the fact that the interest burden ratios are unchanged. One basic improvement is that the rate of approach to equilibrium is slightly increased.

We increase the "aggressiveness" of fiscal policy in Experiment (6) by setting A1=A3=A5=3, while other "A's" remain at their base-run values. The results improve slightly over Experiment (5) with regard to both the interest burden ratios and the rate of movement towards equilibrium.

The major reason for the unimpressive performance of an active fiscal policy by all governments is that only income target-adjustment parameters are considered, leaving the balanced-budget target-adjustment parameters intact. The interest burden ratios and the speed of adjustment towards equilibrium are, to a large extent, determined by the balanced-budget targets. Thus in Experiment (7) we double the balanced-budget target-adjustment parameters, i.e. A2=A4=A6=2, with all other "A's" remaining at their base-run values. The results are very encouraging in that not only is the increase in interest burden ratios reduced to half of the increase in the base run, but also that the rate of movement towards equilibrium increases by 100 percent over the base run under a federal bond-financing policy.

Combining both Experiments (5) and (7), we have Experiment (8) in which A1=A2=A3=A4=A5=A6=2 and all other "A's" remain at their base-run values. The advantages of Experiments (5) and (7) are now compounded in Experiment (8). The interest burden ratios are exactly the same as in Experiment (7) while the rates of movement towards equilibrium are the same under either mode of federal financing policy. We therefore conclude that active fiscal policy works.

The importance of distribution target-adjustment parameters A7 to A10 has been observed in connection with Experiment (4). However, Experiment (4) is an asymmetric case in which only the income target-adjustment parameter in the first region increases to a higher value. In Experiment (9), we present a case in which only the distribution target-adjustment parameters in the first region are increased to A7=A8=2 with all other "A's" remaining at their base-run values. With income and balanced-budget target-adjustment parameters remaining unchanged, the total increase in new bond issues or new money supply, is the same as in the base run. However, because of different distribution target-adjustment parameters across regions, the distribution of federal government expenditures in each region is different, thereby affecting the expenditure level of the regional governments. The interest burden ratio remains unchanged for the federal government. On the other hand, while the total increase in regional interest burden ratios remains unchanged from the base-run level, the distribution of this increasing ratio lays a greater burden on the second region, which has been assigned smaller values for A9 and A10. Moreover, the rate of movement towards equilibrium becomes slower under a federal money-financing policy.

In this identical-regions case, there is no reason for the federal government to favor one region over another. It is worthwhile to investigate the active fiscal policy implemented by the federal government, not by mainipulating the distribution of expenditure, but by increasing the income and balanced-budget target-adjustment parameters. Thus in Experiment (10) we set A5=A6=2, with other "A's" remaining at their base-run values. The results are much better than those obtained in Experiment (8). The increase in the interest burden ratios in the two regions has been reduced significantly, by 80 percent under a federal bond-financing policy, and by nearly 100 percent under a federal money-financing policy. These reductions would undoubtedly increase the fiscal capacity in each region. As far as the federal government is concerned, the interest burden ratio increases only in the bond-financing case, while it decreases in the money-financing case. Furthermore, the speed of approach towards equilibrium increases in the money-financing policy case. The federal government, by virtue of its broader tax base and money-financing option, should pursue a more active fiscal policy.

7.3.2 Identical-Regions Case: Testing for the Critical Limits of Target-Adjustment Parameters

There are certain limits which the target-adjustment parameters cannot exceed, if the system is to be capable of attaining a stable equilibrium. Target-adjustment parameters which are too high or too low cause instability within the system, so that either the system fluctuates around its target values, or it becomes explosive. These limits also specify how actively the regional and federal governments can pursue their fiscal policies.

In the following experiments, we will investigate the question of critical limits for the following subsets of parameters:

(a). Income target-adjustment parameters A1, A3 and A5.

(b). Balanced-budget target-adjustment parameters A2, A4 and A6.

(c). Income and balanced-budget target-adjustment parameters A1, A2, A3, A4, A5 and A6.

(d). Federal income and balanced-budget target-adjustment parameters A5 and A6.

Table 7.3: The IRFG Model
Identical-Regions Case
Testing for the Critical Limits of Target-Adjustment Parameters
Listing of the Simulation Experiments

	Specification			of		Parameter		Values	
Experiments	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Testing for the critical limits of income target-adjustment parameters										
(11)	3.7	1	3.7	1	3.7	1	1	1	1	1
(12)	3.8	1	3.8	1	3.8	1	1	1	1	1
(13)	3.9	1	3.9	1	3.9	1	1	1	1	1
(14)	4.0	1	4.0	1	4.0	1	1	1	1	1
(15)	0.5	1	0.5	1	0.5	1	1	1	1	1
(16)	0.1	1	0.1	1	0.1	1	1	1	1	1
Testing for the critical limits of balanced-budget target-adjustment parameters										
(17)	1	0.6	1	0.6	1	0.6	1	1	1	1
(18)	1	0.5	1	0.5	1	0.5	1	1	1	1
(19)	1	0.4	1	0.4	1	0.4	1	1	1	1
(20)	1	0	1	0	1	0	1	1	1	1
(21)	1	5	1	5	1	5	1	1	1	1
(22)	1	10	1	10	1	10	1	1	1	1
Testing for the critical limits of income and balanced-budget target-adjustment parameters										
(23)	3	3	3	3	3	3	1	1	1	1
(24)	3.5	3.5	3.5	3.5	3.5	3.5	1	1	1	1
(25)	3.6	3.6	3.6	3.6	3.6	3.6	1	1	1	1
(26)	3.7	3.7	3.7	3.7	3.7	3.7	1	1	1	1
(27)	3.8	3.8	3.8	3.8	3.8	3.8	1	1	1	1
(28)	4	4	4	4	4	4	1	1	1	1
(29)	0.3	0.3	0.3	0.3	0.3	0.3	1	1	1	1
(30)	0.4	0.4	0.4	0.4	0.4	0.4	1	1	1	1
(31)	0.5	0.5	0.5	0.5	0.5	0.5	1	1	1	1
(32)	0.6	0.6	0.6	0.6	0.6	0.6	1	1	1	1
Testing for the critical limits of income and balanced-budget target-adjustment parameters for the federal sector										
(33)	1	1	1	1	3	3	1	1	1	1
(34)	1	1	1	1	5	5	1	1	1	1
(35)	1	1	1	1	8	8	1	1	1	1
(36)	1	1	1	1	8.8	8.8	1	1	1	1
(37)	1	1	1	1	8.9	8.9	1	1	1	1
(38)	1	1	1	1	9.3	9.3	1	1	1	1
(39)	1	1	1	1	9.4	9.4	1	1	1	1
(40)	1	1	1	1	9.5	9.5	1	1	1	1
(41)	1	1	1	1	0.1	0.1	1	1	1	1
(42)	1	1	1	1	0.2	0.2	1	1	1	1
(43)	1	1	1	1	0.3	0.3	1	1	1	1
(44)	1	1	1	1	0.4	0.4	1	1	1	1

Table 7.4: The IRFG Model, Identical-Regions
Simulated Effects when Income Targets Increased by 5% in Both Regions
Testing for the Critical Limits of Target-Adjustment Parameters

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
(11)		Impact	2.6731	2.6731	0.0015	4.4350	4.4350	0.5286	1.2333	1.7619	0.5286	1.2333	1.7619				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4360	0.2909	0.7269	0.4360	0.2909	0.7269	0.0134	0.0134	-0.0098	130
	Money	Long Run	1.8287	1.8287	0.0022	2.5000	2.5000	0.3574	0.3140	0.6713	0.3574	0.3140	0.6713	0.0126	0.0126	-0.0001	110
(12)		Impact	2.7454	2.7454	0.0015	4.5549	4.5549	0.5429	1.2667	1.8095	0.5429	1.2667	1.8095				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4105	0.3165	0.7270	0.4105	0.3165	0.7270	0.0111	0.0111	-0.0070	210
	Money	Long Run	1.8105	1.8105	0.0020	2.5000	2.5000	0.3602	0.3294	0.6895	0.3602	0.3294	0.6895	0.0108	0.0108	-0.0008	200
(13)		Impact	2.8176	2.8176	0.0015	4.6748	4.6748	0.5571	1.3000	1.8571	0.5571	1.3000	1.8571				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(14)		Impact	2.8899	2.8899	0.0016	4.7946	4.7946	0.5714	1.3333	1.9048	0.5714	1.3333	1.9048				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(15)		Impact	0.3612	0.3612	0.0002	0.5993	0.5993	0.0714	0.1667	0.2381	0.0714	0.1667	0.2381				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4557	0.2713	0.7269	0.4557	0.2713	0.7269	0.0151	0.0151	-0.0119	200
	Money	Long Run	1.8443	1.8443	0.0025	2.5000	2.5000	0.3550	0.3007	0.6557	0.3550	0.3007	0.6557	0.0141	0.0141	0.0005	90
(16)		Impact	0.0723	0.0723	0.00004	0.1199	0.1199	0.0143	0.0333	0.0476	0.0143	0.0333	0.0476				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.4556	0.2714	0.7270	0.4556	0.2714	0.7270	0.0151	0.0151	-0.0119	310
	Money	Long Run	1.8441	1.8441	0.0025	2.5000	2.5000	0.3550	0.3008	0.6559	0.3551	0.3008	0.6559	0.0141	0.0141	0.0005	220
(17)		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.4762	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	1.3987	-0.6718	0.7269	1.3987	-0.6718	0.7269	0.0985	0.0985	-0.1144	650
	Money	Long Run	1.9898	1.9898	0.0050	2.5000	2.5000	0.3347	0.1755	0.5102	0.3347	0.1755	0.5102	0.0282	0.0282	0.0057	160
(18)		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.4762	0.1429	0.3333	0.4762				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.0934	2.0934	0.0068	2.5000	2.5000	0.3217	0.0848	0.4066	0.3217	0.0848	0.4066	0.0384	0.0384	0.0093	240
(19)		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.4762	0.1429	0.3333	0.4762				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(20)		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.6762	0.1429	0.3333	0.6762				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(21)		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.4762	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3298	0.3971	0.7270	0.3298	0.3971	0.7270	0.0040	0.0040	0.0017	110
	Money	Long Run	1.7216	1.7216	0.0003	2.5000	2.5000	0.3744	0.4040	0.7784	0.3744	0.4040	0.7784	0.0023	0.0023	-0.0042	90

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

Table 7.4 Continued.

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
(22)		Impact	0.7225	0.7225	0.0004	1.1987	1.1987	0.1429	0.3333	0.4762	0.1429	0.3333	0.4762				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3227	0.4042	0.7270	0.3227	0.4042	0.7270	0.0034	0.0034	0.0025	140
	Money	Long Run	1.7086	1.7086	0.0003	2.5000	2.5000	0.3765	0.4149	0.7914	0.3765	0.4149	0.7914	0.0010	0.0010	-0.0047	110
(23)		Impact	2.1674	2.1674	0.0012	3.5960	3.5960	0.4286	1.0000	1.4286	0.4286	1.0000	1.4286				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3394	0.3876	0.7270	0.3394	0.3876	0.7270	0.0049	0.0049	0.0007	70
	Money	Long Run	1.7378	1.7378	0.0006	2.5000	2.5000	0.3717	0.3905	0.7622	0.3717	0.3905	0.7622	0.0038	0.0038	-0.0036	60
(24)		Impact	2.5286	2.5286	0.0014	4.1953	4.1953	0.5000	1.1667	1.6667	0.5000	1.1667	1.6667				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3291	0.3978	0.7269	0.3291	0.3978	0.7269	0.0040	0.0040	0.0018	100
	Money	Long Run	1.7231	1.7231	0.0003	2.5000	2.5000	0.3741	0.4028	0.7769	0.3741	0.4028	0.7769	0.0024	0.0024	-0.0041	110
(25)		Impact	2.6009	2.6009	0.0014	4.3152	4.3152	0.5143	1.2000	1.7143	0.5143	1.2000	1.7143				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3199	0.4070	0.7269	0.3199	0.4070	0.7269	0.0032	0.0032	0.0028	210
	Money	Long Run	1.7104	1.7104	0.0001	2.5000	2.5000	0.372	0.4133	0.7895	0.3762	0.4133	0.7895	0.0012	0.0012	-0.0046	230
(26)		Impact	2.6731	2.6731	0.0015	4.4350	4.4350	0.5286	1.2333	1.7619	0.5286	1.2333	1.7619				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(27)		Impact	2.7454	2.7454	0.0015	4.5549	4.5549	0.5429	1.2667	1.8096	0.5429	1.2667	1.8096				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(28)		Impact	2.8899	2.8899	0.0016	4.7946	4.7946	0.5714	1.3333	1.9048	0.5714	1.3333	1.9048				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(29)		Impact	0.2167	0.2167	0.0001	0.3596	0.3596	0.0429	0.1000	0.1429	0.0429	0.1000	0.1429				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(30)		Impact	0.2890	0.2890	0.0001	0.4795	0.4795	0.0571	0.1333	0.1905	0.0571	0.1333	0.1905				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.3573	2.3573	0.0110	2.5000	2.5000	0.2932	-0.1504	0.1428	0.2932	-0.1504	0.1428	0.0644	0.0644	0.0178	450
(31)		Impact	0.3612	0.3612	0.0002	0.5993	0.5993	0.0714	0.1667	0.2381	0.0714	0.1667	0.2381				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.0936	2.0936	0.0068	2.5000	2.5000	0.3217	0.0847	0.4064	0.3217	0.0847	0.4064	0.0384	0.0384	0.0090	260
(32)		Impact	0.4335	0.4335	0.0002	0.7192	0.7192	0.0857	0.2000	0.2857	0.0857	0.2000	0.2857				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	1.4112	-0.6843	0.7269	1.4112	-0.6843	0.7269	0.0995	0.0995	-0.1156	800
	Money	Long Run	1.9899	1.9899	0.0051	2.5000	2.5000	0.3347	0.1754	0.5101	0.3347	0.1754	0.5101	0.0282	0.0282	0.0057	180

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

Table 7.4 Continued.

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
(33)		Impact	1.1560	1.1560	0.0006	1.9179	1.9179	0.4286	0.3333	0.7619	0.4286	0.3333	0.7619				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.2742	0.4528	0.7270	0.2742	0.4528	0.7270	-0.0009	-0.0009	0.0078	60
	Money	Long Run	1.6521	1.6521	-0.0011	2.5000	2.5000	0.3683	0.4616	0.8479	0.3863	0.4616	0.8479	-0.0043	-0.0043	-0.0070	50
(34)		Impact	1.5894	1.5894	0.0009	2.6371	2.6371	0.7143	0.3333	1.0476	0.7143	0.3333	1.0476				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.2565	0.4705	0.7270	0.2565	0.4705	0.7270	-0.0024	-0.0024	0.0097	60
	Money	Long Run	1.6164	1.6164	-0.0018	2.5000	2.5000	0.3928	0.4909	0.8836	0.3928	0.4909	0.8836	-0.0077	-0.0077	-0.0084	50
(35)		Impact	2.2397	2.2397	0.0012	3.7158	3.7158	1.1429	0.3333	1.4762	1.1429	0.3333	1.4762				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.2847	0.4422	0.7270	0.2847	0.4422	0.7270	0.0001	0.0001	0.0066	60
	Money	Long Run	1.6582	1.6582	-0.0009	2.5000	2.5000	0.3852	0.4565	0.8418	0.3852	0.4565	0.8418	-0.0037	-0.0037	-0.0067	70
(36)		Impact	2.4130	2.4130	0.0013	4.0035	4.0035	1.2571	0.3333	1.5905	1.2571	0.3333	1.5905				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	0.3794	0.3475	0.7269	0.3794	0.3475	0.7269	0.0084	0.0084	-0.0037	660
	Money	Long Run	1.8971	1.8971	0.0035	2.5000	2.5000	0.3473	0.2555	0.6028	0.3473	0.2555	0.6028	0.0192	0.0192	0.0024	800
(37)		Impact	2.4347	2.4347	0.0013	4.0395	4.0395	1.2714	0.3333	1.6047	1.2714	0.3333	1.6047				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.0801	2.0801	0.0066	2.5000	2.5000	0.3230	0.0969	0.4199	0.3230	0.0969	0.4199	0.0370	0.0370	0.0088	1050
(38)		Impact	2.5214	2.5214	0.0014	4.1833	4.1833	1.4386	0.3333	1.6619	1.4386	0.3333	1.6619				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	3.5213	3.5213	0.0267	2.5000	2.5000	0.2199	-1.2412	-1.0213	0.2199	-1.2412	-1.0213	0.1713	0.1713	0.0480	1200
(39)		Impact	2.5431	2.5431	0.0014	4.2193	4.2193	1.3429	0.3333	1.6762	1.3429	0.3333	1.6762				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(40)		Impact	2.5648	2.5648	0.0014	4.2552	4.2552	1.3571	0.3333	1.6905	1.3571	0.3333	1.6905				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(41)		Impact	0.5274	0.5274	0.0003	0.8570	0.8570	0.0143	0.3333	0.3476	0.0143	0.3333	0.3476				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(42)		Impact	0.5491	0.5491	0.0003	0.9110	0.9110	0.0286	0.3333	0.3619	0.0286	0.3333	0.3619				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.9461	2.9461	0.0194	2.5000	2.5000	0.2502	-0.6964	-0.4462	0.2502	-0.6964	-0.4462	0.1229	0.1229	0.0342	320
(43)		Impact	0.5708	0.5708	0.0003	0.9469	0.9469	0.0429	0.3333	0.3762	0.0429	0.3333	0.3762				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.4865	2.4865	0.0129	2.5000	2.5000	0.2819	-0.2684	0.0135	0.2819	-0.2684	0.0135	0.0772	0.0772	0.0216	220
(44)		Impact	0.5924	0.5924	0.0003	0.9829	0.9829	0.0571	0.3333	0.3905	0.0571	0.3333	0.3905				
	Bond	Long Run	1.7730	1.7730	0.0012	2.5000	2.5000	4.0912	-3.3642	0.7269	4.0912	-3.3642	0.7269	0.3359	0.3359	-0.4061	800
	Money	Long Run	2.2610	2.2610	0.0095	2.5000	2.5000	0.3031	-0.0641	0.2390	0.3031	-0.0641	0.2390	0.0548	0.0548	0.0148	170

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

All other target-adjustment parameters not investigated remain at their base-run values. In the following experiments testing for the critical limits of target-adjustment parameters, we consider the accuracy of the limit up to one decimal point. Therefore the critical limits are only an approximation. Next, we summarize the results of the simulation experiments. Detailed quantitative results are presented in Table 7.4

(a). Upper and Lower Limits of A1, A3, and A5

Experiments (11) and (14) test for the upper limit of the overall income target-adjustment parameters A1, A3 and A5. We assume that they are of the same value, i.e., that the same "degree" of fiscal policy is adopted by each government. The system is stable when the income target-adjustment parameters increase up to 3.8, and unstable over and above this critical limit. When we increase the values of A1, A3 and A5 from 2 to 3, we get better results. However, when the values change from 3 to 3.8, we get much worse results than those of the base run. Given the unitary value of other target-adjustment parameters, 3.8 determines the upper limit of the income target-adjustment parameters.

The lower limit of A1, A3 and A5 is zero. In this case there would be no changes of government expenditures and consequently the system would stay at its initial equilibrium level even though the income targets have been raised. Thus we conclude that the exogenous component of the government expenditure reaction function exerts more influence than the endogenous component. Experiments (15) and (16) show the effects of reducing A1, A3 and A5 to 0.5 and 0.1. The lower the values, the slower the rate of movement towards equilibrium.

(b). <u>Upper and Lower Limits of A2, A4 and A6</u>

Experiments (17) to (19) test for the lower limits of A2, A4 and A6. When they are reduced from 1 to 0.4, the system becomes unstable under both federal financing policies. At the value of 0.5, the system is stable only under a federal money-financing policy, which has a very slow rate of movement towards equilibrium. The system is stable under both federal financing policies when the values of A2, A4 and A6 are equal to 0.6. Thus the critical lower limit of the balanced-budget target-adjustment parameters is reached when their values are reduced to approximately half of the base-run values.

For the balanced-budget target-adjustment parameter, the lower limit is of greater importance than for the upper limit. Regarding the income target-adjustment parameters, too low a value for A2, A4 and A6 implies that a dampening effect exerted by these balanced-budget target-adjustment parameters is not large enough to outweigh the expansionary effect initiated by the income target-adjustment parameters, and therefore the deficit grows in each subsequent period and the system is unstable. Experiment (20) shows that when we do not consider the balanced-budget target (i.e., A2=A4=A6=0), the system is unstable. This result demonstrates the importance of the balanced-budget target in approaching a stable equilibrium.

Experiments (21) and (22) test for the upper limit of A2, A4 and A6. There is no definite answer as to whether the results are better than in the base run or in Experiment (7). Improvement in one aspect is always accompanied by deterioration in others. The upper limit of A2, A4 and A6 is not as important as the lower limit. As long as the balanced-budget target-adjustment parameters are not less than the income target-adjustment parameters, the balanced-budget target-adjustment parameters will not contribute to instability in the system.

(c). <u>Upper and Lower Limits of A1, A2, A3, A4, A5 and A6</u>

Experiments (23) and (28) test for the upper limit of the income and balanced-budget target-adjustment parameters. When these values increase to 3, results are slightly better under a federal money-financing policy than in Experiment (8). However, it is hard to determine their superiority under a federal bond-financing policy. The system becomes unstable when the parameter values increase to 3.7. Therefore 3.6 becomes the upper limit for the income and balanced-budget target-adjustment parameters. As we emphasized in the discussion of an active fiscal policy, we are not interested in the lower limit of the A1 to A6 values, for this has little significance. Lowering these values below the base-run level would only reduce the speed of approach to the equilibrium. Experiments (29) to (32) show that under both financing policies, the system still can generate a stable equilibrium when the values of A1 to A6 are reduced to 0.6. Thus 0.6 becomes the lower limit of the income and balanced-budget target-adjustment parameters.

(d). <u>Upper and Lower Limits of A5 and A6</u>

The results of the testing for the critical limits of A5 and A6 are important. Experiments (36) and (40) show that the system becomes unstable under a federal bond-financing policy when A5 and A6 are increased to 8.9, and unstable under both federal financing policies when A5 and A6 are increased to 9.4. Thus 8.8 and 9.3 determine the upper limit for A5 and A6 values under federal bond-financing and money-financing policies, respectively. Experiments (26) to (28) show the effects of increasing the federal government expenditures. The results improve when A5 and A6 are increased from 3 to 5, but worsen when A5 and A6 are increased from 5 to 8. Thus there are certain limits beyond which a "too aggressive" federal fiscal policy would cause the system to overshoot the target values. On the other hand, there is also a lower limit (A5=A6=0.1) below which the system becomes unstable. Experiments (41) to (44) indicate the effects of reducing A5 and A6 values gradually. The lower limit is insignificant, as the main thrust of an active fiscal policy is to reach the target at a faster rate.

Another significant finding can be observed by comparing results of Experiments (23) and (33). In Experiment (23) all governments act together in pursuing the same active fiscal policy, while in Experiment (33) only the federal government implements a more active fiscal policy. Experiment (33) shows much better results than Experiment (23) as far as the interest burden ratios of the regional governments are concerned. In Experiment (23), these ratios increase by nearly half of one percent, while they drop by almost half of one percent in Experiment (33). The objective of improving regional governments' fiscal capacity is better achieved in Experiment (33) than in Experiment (23). Moreover, the rate of movement towards equilibrium is much faster in Experiment (33) than in Experiment (23).

7.3.3 Unequal-Regions Case: The Effectiveness of an Active Fiscal Policy

In this section, we summarize the results of Experiments (45) to (57). Detailed quantitative results are presented in Table 7.6.

Table 7.5: The IRFG Model
Unequal-Regions Case
The Effectiveness of An Active Fiscal Policy
Listing of the Simulation Experiments

	Specification			of		Parameter			Values	
Experiments	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
(45)	2	2	1	1	1	1	1	1	1	1
(46)	1	1	2	2	1	1	1	1	1	1
(47)	3	1	1	1	1	1	1	1	1	1
(48)	1	1	3	1	1	1	1	1	1	1
(49)	3	1	1	1	1	1	2	2	1	1
(50)	1	1	3	1	1	1	1	1	2	2
(51)	2	1	2	1	2	1	1	1	1	1
(52)	3	1	3	1	3	1	1	1	1	1
(53)	1	2	1	2	1	2	1	1	1	1
(54)	2	2	2	2	2	2	1	1	1	1
(55)	1	1	1	1	1	1	1	1	2	2
(56)	1	1	1	1	1	1	2	2	1	1
(57)	1	1	1	1	2	2	1	1	1	1

In Experiment (45), we examine the case of a more active fiscal policy implemented in the first region, indicated by a higher target-adjustment parameter value A1=A2=2. In Experiment (46), we study the effect of a higher A3=A4=2 value. The results of these more active fiscal policies do not show any improvement over the base run. In Experiments (47) and (48), we examine an active fiscal policy in terms of higher income target-adjustment parameters in the two regions. Once again, these asymmetric fiscal policies overshoot the system, thereby reducing the rate of movement towards equilibrium. Thus, the results are much worse than in the base run.

Table 7.6: The IRFG Model, Unequal-Regions
Simulated Effects when Income Targets Increased by 5% in Both Regions
The Effectiveness of an Active Fiscal Policy

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
Base Run		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6379	0.3799	1.0177	0.2734	0.1628	0.4362	0.0151	0.0151	-0.0119	180
	Money	Long Run	2.5819	1.1065	0.0025	3.5000	1.5000	0.4971	0.4210	0.9181	0.2130	0.1805	0.3935	0.0141	0.0141	0.0005	80
(45)		Impact	1.5907	0.5623	0.0006	2.6404	0.9316	0.2000	0.9333	1.1333	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.7153	0.3024	1.0177	0.2587	0.1175	0.4362	0.0200	0.0130	-0.0153	160
	Money	Long Run	2.6234	1.1243	0.0030	3.5000	1.5000	0.5324	0.3442	0.8766	0.1690	0.2067	0.3757	0.0196	0.0196	0.0016	110
(46)		Impact	1.1402	0.6082	0.0005	1.8905	1.0102	0.2000	0.4667	0.6667	0.0857	0.4000	0.4857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6238	0.3939	1.0177	0.3129	0.1232	0.4362	0.0142	0.0209	-0.0133	180
	Money	Long Run	2.5982	1.1135	0.0027	3.5000	1.5000	0.4567	0.4450	0.9018	0.2499	0.1366	0.3865	0.0128	0.0208	0.0009	110
(47)		Impact	2.1700	0.6910	0.0008	3.6027	1.1440	0.2000	1.4000	1.6000	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	1.5326	-0.5149	1.0177	0.1050	0.3312	0.4362	0.0715	-0.0096	-0.0513	270
	Money	Long Run	2.9065	1.2457	0.0065	3.5000	1.5000	0.7763	-0.2029	0.5934	-0.1488	0.4032	0.2544	0.0581	-0.0135	0.0087	210
(48)		Impact	1.2690	0.7828	0.0006	2.1029	1.3013	0.2000	0.4667	0.6667	0.0857	0.6000	0.6857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.3454	0.6724	1.0178	1.1057	-0.6695	0.4361	-0.0033	0.1375	-0.0413	390
	Money	Long Run	2.8751	1.2322	0.0061	3.5000	1.5000	-0.2620	0.8870	0.6250	0.9527	-0.6473	0.2678	-0.0106	0.1395	0.0080	530
(49)		Impact	2.2047	0.6563	0.0008	3.6608	1.0859	0.2581	1.4000	1.6581	0.0276	0.2000	0.2276				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	1.4426	-0.4249	1.0177	0.1523	0.2839	0.4362	0.0658	-0.0027	-0.0490	260
	Money	Long Run	2.8853	1.2365	0.0063	3.5000	1.5000	0.7376	-0.1229	0.6147	-0.0863	0.3497	0.2635	0.0528	-0.0060	0.0082	180
(50)		Impact	1.2337	0.8182	0.0006	2.0438	1.3605	0.1408	0.4667	0.6075	0.1449	0.6000	0.7449				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4887	0.5291	1.0177	0.8216	-0.3855	0.4362	0.0057	0.0957	-0.0335	260
	Money	Long Run	2.7760	1.1897	0.0050	3.5000	1.5000	0.6758	0.6483	0.7240	0.5955	-0.2852	0.3103	0.2282	0.0852	0.0055	240
(51)		Impact	2.0229	0.8670	0.0008	3.3562	1.4384	0.4000	0.9333	1.3333	0.1714	0.4000	0.5714				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6372	0.3805	1.0177	0.2731	0.1361	0.4362	0.0151	0.0151	-0.0019	160
	Money	Long Run	2.5814	1.1063	0.0025	3.5000	1.5000	0.6972	0.4215	0.9186	0.2131	0.1806	0.3937	0.0140	0.0140	0.0005	70
(52)		Impact	3.0344	1.3004	0.0012	5.0344	2.1576	0.6000	1.4000	2.0000	0.2571	0.6000	0.8571				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6347	0.3830	1.0177	0.2720	0.1642	0.4362	0.0149	0.0149	-0.0177	140
	Money	Long Run	2.5793	1.1054	0.0025	3.5000	1.5000	0.6975	0.4232	0.9207	0.2132	0.1814	0.3946	0.0139	0.0139	0.0004	60
(53)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.5010	0.5167	1.0177	0.2147	0.2214	0.4362	0.0065	0.0065	-0.0013	70
	Money	Long Run	2.4686	1.0580	0.0010	3.5000	1.5000	0.5146	0.5168	1.0314	0.2205	0.2215	0.4420	0.0063	0.0063	-0.0026	70
(54)		Impact	2.0229	0.8670	0.0008	3.3562	1.4384	0.4000	0.9333	1.3333	0.1714	0.4000	0.5714				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.5007	0.5171	1.0177	0.2146	0.2216	0.4362	0.0065	0.0065	-0.0013	60
	Money	Long Run	2.4681	1.0578	0.0010	3.5000	1.5000	0.5147	0.5172	1.0319	0.2206	0.2217	0.4422	0.0063	0.0063	-0.0026	60
(55)		Impact	0.9761	0.4689	0.0004	1.6189	0.7784	0.1408	0.4667	0.6075	0.1449	0.2000	0.3449				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6612	0.3565	1.0177	0.2502	0.1859	0.4362	0.0166	0.0177	-0.0119	180
	Money	Long Run	2.5819	1.1065	0.0025	3.5000	1.5000	0.5193	0.3988	0.9181	0.1908	0.2027	0.3952	0.0155	0.0108	0.0005	90
(56)		Impact	1.0462	0.3988	0.0004	1.7363	0.6611	0.2581	0.4667	0.7248	0.0276	0.2000	0.2276				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6057	0.4120	1.0177	0.3058	0.1304	0.4362	0.0131	0.0189	-0.0119	180
	Money	Long Run	2.5819	1.1065	0.0025	3.5000	1.5000	0.4660	0.4521	0.9181	0.2441	0.1493	0.3935	0.0121	0.0187	0.0005	110
(57)		Impact	1.3149	0.5623	0.0005	2.1816	0.9350	0.4000	0.4667	0.8667	0.1714	0.2000	0.3714				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4257	0.5920	1.0177	0.1825	0.2537	0.4362	0.0018	0.0018	0.0045	50
	Money	Long Run	2.3800	1.0200	-0.0001	3.5000	1.5000	0.5292	0.5908	1.1200	0.2268	0.2532	0.4800	0.0010	0.0010	-0.0051	50

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

Adopting the same approach as in the identical-regions case, we add a higher federal distribution parameter to the values in Experiments (47) and (48), which then become Experiments (49) and (50). There are some improvements over Experiments (47) and (48), but the effects are small and the analysis does not yield any significant policy implications.

In Experiments (51), (52) and (53), we consider the case of active fiscal policy pursued by all governments. Experiments (51) and (52) investigate changes in the income target-adjustment parameters while Experiment (53) studies the effect of changes in the balanced-budget target-adjustment parameters. Results show that the balanced-budget target-adjustment parameters play a crucial role in determining the rate of movement towards equilibrium. The higher the A2, A4 and A6 values, the faster the system approach towards equilibrium. Therefore the improvement observed in Experiment (53) is far better than in Experiments (51) and (52). This confirms the previous results of the identical-regions case.

Combining Experiments (51) and (53) in Experiment (54) shows that an active fiscal policy adopted by all governments is an improvement over the base run, especially with federal bond-financing.

Experiments (55) and (56) present the results of changing the distribution of federal expenditures between the two regions. The higher the federal expenditures in the region, the lower the increase in regional expenditures required to reach the target. Therefore the increase in the regional interest burden ratio would be lowered. However, since income and balanced-budget target-adjustment parameters remain at their base-run values, the speed of approach to the targets is not increased under either federal financing policy.

In Experiment (57) we double the A5 and A6 values to 2, while the other "A's" retain their base-run values. The results are better than in Experiment (54), where a more active fiscal policy is adopted by every government. As far as the speed of approach to the targets is concerned, it is even faster than in Experiment (10) for the identical-regions case.

7.3.4 Unequal-Regions Case: Testing for the Critical Limits of Target-Adjustment Parameters

In this section, we perform the same experiments (Table 7.7) as in the identical-regions case. We then summarize the results of the simulation experiments. Detailed quantitative results are presented in Table 7.8. Again, we consider the accuracy of the critical limits up to one decimal point.

Table 7.7: The IRFG Model
Unequal-Regions Case
Testing for the Critical Limits of Target-Adjustment Parameters
Listing of the Simulation Experiments

	Specification			of		Parameter		Values	
Experiments A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Testing for the critical limits of income target-adjustment parameters									
(58) 3.7	1	3.7	1	3.7	1	1	1	1	1
(59) 3.8	1	3.8	1	3.8	1	1	1	1	1
(60) 3.9	1	3.9	1	3.9	1	1	1	1	1
(61) 4	1	4	1	4	1	1	1	1	1
(62) 0.5	1	0.5	1	0.5	1	1	1	1	1
(63) 0.1	1	0.1	1	0.1	1	1	1	1	1
Testing for the critical limits of balanced-budget target-adjustment parameters									
(64) 1	0.3	1	0.3	1	0.3	1	1	1	1
(65) 1	0.4	1	0.4	1	0.4	1	1	1	1
(66) 1	0.5	1	0.5	1	0.5	1	1	1	1
(67) 1	0.6	1	0.6	1	0.6	1	1	1	1
(68) 1	5	1	5	1	5	1	1	1	1
(69) 1	10	1	10	1	10	1	1	1	1
Testing for the critical limits of income and balanced-budget target-adjustment parameters									
(70) 3	3	3	3	3	3	1	1	1	1
(71) 3.5	3.5	3.5	3.5	3.5	3.5	1	1	1	1
(72) 3.6	3.6	3.6	3.6	3.6	3.6	1	1	1	1
(73) 3.7	3.7	3.7	3.7	3.7	3.7	1	1	1	1
(74) 3.8	3.8	3.8	3.8	3.8	3.8	1	1	1	1
(75) 4.0	4.0	4.0	4.0	4.0	4.0	1	1	1	1
(76) 0.3	0.3	0.3	0.3	0.3	0.3	1	1	1	1
(77) 0.4	0.4	0.4	0.4	0.4	0.4	1	1	1	1
(78) 0.5	0.5	0.5	0.5	0.5	0.5	1	1	1	1
(79) 0.6	0.6	0.6	0.6	0.6	0.6	1	1	1	1
Testing for the critical limits of income and balanced-budget target-adjustment parameters in the federal sector									
(80) 1	1	1	1	3	3	1	1	1	1
(81) 1	1	1	1	5	5	1	1	1	1
(82) 1	1	1	1	8	8	1	1	1	1
(83) 1	1	1	1	8.8	8.8	1	1	1	1
(84) 1	1	1	1	8.9	8.9	1	1	1	1
(85) 1	1	1	1	9.3	9.3	1	1	1	1
(86) 1	1	1	1	9.4	9.4	1	1	1	1
(87) 1	1	1	1	9.5	9.5	1	1	1	1
(88) 1	1	1	1	0.1	0.1	1	1	1	1
(89) 1	1	1	1	0.2	0.2	1	1	1	1
(90) 1	1	1	1	0.3	0.3	1	1	1	1
(91) 1	1	1	1	0.4	0.4	1	1	1	1

Table 7.8: The IRFG Model, Unequal-Regions
Simulated Effects when Income Targets Increased by 5% in Both Regions
Testing for the Critical Limits of Target-Adjustment Parameters Case

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
(58)		Impact	3.7424	1.6039	0.0015	6.2090	2.6610	0.7400	1.7267	2.4667	0.3171	0.7400	1.0571				
	Bond	Long Run	2.4832	1.0638	0.0012	3.5000	1.5000	0.6105	0.4073	1.0178	0.2616	0.1745	0.4361	0.0134	0.0134	-0.0098	140
	Money	Long Run	2.5601	1.0972	0.0022	3.5000	1.5000	0.5003	0.4395	0.9399	0.2144	0.1884	0.4028	0.0126	0.0126	-0.0001	110
(59)		Impact	3.8435	1.6472	0.0015	6.3769	2.7329	0.7600	1.7733	2.5333	0.3257	0.7600	1.0857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.5744	0.2429	1.0177	0.2462	0.1898	0.4362	0.0111	0.0111	-0.0070	250
	Money	Long Run	2.5346	1.0863	0.0019	3.5000	1.5000	0.5042	0.4611	0.9654	0.2161	0.1976	0.4137	0.0108	0.0108	-0.0008	200
(60)		Impact	3.9447	1.6906	0.0015	6.5447	2.8049	0.7800	1.8200	2.6000	0.3343	0.7800	1.1143				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(61)		Impact	3.9542	1.7339	0.0016	6.7125	2.8768	o.8000	1.8667	2.6667	0.3429	0.8000	1.1428				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(62)		Impact	0.5058	0.2168	0.0002	0.8391	0.3596	0.1000	0.2333	0.3333	0.0429	0.1000	0.1429				
	Bond	Long Run	2.4822	1.0638	0.0012	3.5000	1.5000	0.6380	0.3798	1.0177	0.2734	0.1628	0.4362	0.0151	0.0151	-0.0119	.200
	Money	Long Run	2.5820	1.1066	0.0025	3.5000	1.5000	0.4971	0.4209	0.9180	0.2130	0.1804	0.3925	0.0141	0.0141	0.0005	90
(63)		Impact	0.1012	0.0434	0.00004	0.1678	0.0719	0.0200	0.0467	0.0667	0.0086	0.0200	0.0286				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6378	0.3799	1.0177	0.2734	0.1628	0.4362	0.0151	0.0151	-0.0119	330
	Money	Long Run	2.5818	1.1065	0.0025	3.5000	1.5000	0.4971	0.4211	0.9180	0.2130	0.1805	0.3935	0.0141	0.0141	0.0005	230
(64)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(65)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	3.3041	1.4160	0.0110	3.5000	1.5000	0.4105	-0.2146	0.1959	0.1759	-0.0920	0.0840	0.0646	0.0646	0.0178	600
(66)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.9308	1.2561	0.0068	3.5000	1.5000	0.4504	0.1188	0.5692	0.1930	0.0509	0.2439	0.0384	0.0384	0.0093	250
(67)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	1.9582	-0.9405	1.0177	0.8392	-0.4031	0.4362	0.0985	0.0985	-0.1144	650
	Money	Long Run	2.7857	1.1939	0.0050	3.5000	1.5000	0.2686	0.2457	0.7143	0.2008	0.1053	0.3061	0.0282	0.0282	0.0057	160
(68)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4617	0.5560	1.0177	0.1979	0.2383	0.4362	0.0040	0.0040	0.0017	110
	Money	Long Run	2.4103	1.0330	0.0003	3.5000	1.5000	0.5241	0.5656	1.0897	0.2246	0.2424	0.4670	0.0023	0.0023	-0.0042	90

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

Table 7.8 Continued.

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
(69)		Impact	1.0115	0.4335	0.0004	1.6781	0.7192	0.2000	0.4667	0.6667	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4518	0.5659	1.0177	0.1936	0.2425	0.4362	0.0034	0.0034	0.0025	140
	Money	Long Run	2.3920	1.0252	0.00003	3.5000	1.5000	0.5272	0.5808	1.1080	0.2259	0.2489	0.4748	0.0010	0.0010	-0.0047	120
(70)		Impact	3.0344	1.3004	0.0012	5.0344	2.1576	0.6000	1.4000	2.0000	0.2571	0.6000	0.8571				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4752	0.5426	1.0177	0.2036	0.4325	0.4362	0.0049	0.0049	0.0005	70
	Money	Long Run	2.4329	1.0427	0.0006	3.5000	1.5000	0.7204	0.5468	1.0671	0.2230	0.2343	0.4573	0.0038	0.0038	-0.0036	60
(71)		Impact	3.5401	1.5172	0.0014	5.8734	2.5172	0.7000	1.6333	2.3333	0.3000	0.7000	1.0000				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4608	0.5569	1.0177	0.1975	0.2387	0.4362	0.0040	0.0040	0.0018	110
	Money	Long Run	2.4124	1.0339	0.0003	3.5000	1.5000	0.5238	0.5639	1.0876	0.2245	0.2417	0.4662	0.0014	0.0014	-0.0041	110
(72)		Impact	3.6412	1.5605	0.0014	6.0412	2.5891	0.7200	1.6800	2.4000	0.3086	0.7200	1.0286				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4479	0.5609	1.0188	0.1920	0.2442	0.4362	0.0032	0.0032	0.0028	210
	Money	Long Run	2.3946	1.0263	0.0001	3.5000	1.5000	0.5267	0.5786	1.1053	0.2257	0.2480	0.4737	0.0012	0.0012	-0.0046	240
(73)		Impact	3.7424	1.0639	0.0015	6.2090	2.6610	0.7400	1.7267	2.4667	0.3171	0.7400	1.0571				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(74)		Impact	3.8435	1.6472	0.0015	6.3768	2.7329	0.7600	1.7733	1.5333	0.3257	0.7600	1.0857				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(75)		Impact	4.0458	1.7339	0.0016	6.7125	2.8768	0.8000	1.8667	2.6667	0.3429	0.8000	1.1429				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(76)		Impact	0.3035	0.1301	0.0001	0.5035	0.2158	0.0600	0.1400	0.2000	0.0257	0.0600	0.0857				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(77)		Impact	0.4046	0.1734	0.0001	0.6713	0.2877	0.0800	0.1867	0.2667	0.0343	0.0800	0.1143				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	3.3001	1.4144	0.0109	3.5000	1.5000	0.4105	-0.2107	0.1998	0.1759	-0.0903	0.0856	0.0644	0.0644	0.0178	410
(78)		Impact	1.5058	0.2168	0.0002	0.8391	0.3596	0.1000	0.2333	0.3333	0.0429	0.1000	0.1429				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.9282	1.2250	0.0068	3.5000	1.5000	0.4501	0.1217	0.5718	0.1029	0.0521	0.2450	0.0382	0.0382	0.0093	250
(79)		Impact	0.6069	0.2601	0.0002	1.0069	0.4315	0.1200	0.2800	0.4000	0.0514	0.1200	0.1714				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	1.9757	-0.9580	1.0177	0.8467	-0.4106	0.4362	0.0995	0.0995	-0.1156	800
	Money	Long Run	2.7859	1.1939	0.0051	3.5000	1.5000	0.4686	0.2456	0.7141	0.2008	0.1052	0.3061	0.0282	0.0282	0.0043	170

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

Table 7.8 Continued.

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
(80)		Impact	1.6183	0.6936	0.0006	2.6850	1.1507	0.6000	0.4667	1.0667	0.2571	0.2000	0.4571				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.3838	0.6339	1.0177	0.1645	0.2717	0.4362	-0.0009	-0.0009	0.0078	70
	Money	Long Run	2.3129	0.9912	-0.0011	3.5000	1.5000	0.5408	0.6462	1.1871	0.2318	0.2770	0.5088	-0.0043	-0.0043	-0.0070	50
(81)		Impact	2.2252	0.9537	0.0009	3.6919	1.5822	1.0000	0.4667	1.4667	0.4286	0.2000	0.6286				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.3591	0.6587	1.0177	0.1539	0.2823	0.4362	-0.0024	-0.0024	0.0097	60
	Money	Long Run	2.2629	0.9698	-0.0018	3.5000	1.5000	0.5499	0.6872	1.2371	0.2357	0.2945	0.5302	-0.0077	-0.0077	-0.0084	50
(82)		Impact	3.1355	1.3438	0.0012	5.2022	2.2295	1.6000	0.4667	2.0667	0.6857	0.2000	0.8857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.3986	0.6191	1.0177	0.1708	0.2653	0.4362	0.0001	0.0001	0.0066	60
	Money	Long Run	2.3215	0.9949	-0.0009	3.5000	1.5000	0.5393	0.6391	1.1785	0.2311	0.2739	0.5051	-0.0037	-0.0037	-0.0067	70
(83)		Impact	3.3783	1.4478	0.0014	5.6049	2.2421	1.7600	0.4667	2.2267	0.7543	0.2000	0.9543				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.5312	0.4866	1.0177	0.2276	0.2086	0.4362	0.0084	0.0084	-0.0037	680
	Money	Long Run	2.6561	1.1383	0.0035	3.5000	1.5000	0.4863	0.3576	0.8439	0.2084	0.1533	0.3617	0.0192	0.0192	0.0024	910
(84)		Impact	3.4086	1.4608	0.0014	5.6553	2.4237	1.7800	0.4667	2.2467	0.7629	0.2000	0.9269				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	2.9121	1.2486	0.0065	3.5000	1.5000	0.4522	0.1357	0.5879	0.1935	0.0579	0.2514	0.0370	0.0370	0.0088	1050
(85)		Impact	3.5300	1.5129	0.0014	5.8566	2.5100	1.8600	0.4667	2.3267	0.7971	0.2000	0.9971				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	4.9559	2.1182	0.0267	3.5000	1.5000	0.3091	-1.7650	-1.4559	0.1382	-0.7564	-0.6182	0.1713	0.1713	0.0480	1200
(86)		Impact	3.5603	1.5259	0.0014	5.9070	2.5316	2.0800	0.4667	2.3467	0.8057	0.2000	1.0057				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(87)		Impact	3.5907	1.5389	0.0014	5.9573	2.5531	1.9000	0.4667	2.3667	0.8143	0.2000	1.0143				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(88)		Impact	0.7384	0.3165	0.0003	1.2251	0.5250	0.0200	0.4667	0.4867	0.0085	0.2000	0.2085				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(89)		Impact	0.7587	0.3295	0.0003	1.2754	0.5466	0.0400	0.4667	0.5067	0.0171	0.2000	0.2171				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	4.1246	1.7677	0.0194	3.5000	1.5000	0.3503	-0.9749	-0.6246	0.1501	-0.4178	-0.2677	0.1229	0.1229	0.0342	330
(90)		Impact	0.7991	0.3425	0.0004	1.3257	0.5682	0.0600	0.4667	0.5267	0.0257	0.2000	0.2257				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	3.4792	1.4912	0.0129	3.5000	1.5000	0.3944	-0.3737	0.0207	0.1690	-0.1602	0.0089	0.0770	0.0770	0.0216	210
(91)		Impact	0.8294	0.3555	0.0003	1.3761	0.5897	0.0800	0.4667	0.5467	0.0342	0.2000	0.2342				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	5.7276	-4.7099	1.0177	2.4527	-2.0185	0.4362	0.3359	0.3359	-0.4061	800
	Money	Long Run	3.1652	1.3565	0.0095	3.5000	1.5000	0.4243	-0.0895	0.3348	0.1818	-0.0384	0.1435	0.0548	0.0548	0.0148	180

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

(a). <u>Upper and Lower Limits of A1, A3 and A5</u>

Experiments (58) and (59) show that when income target-adjustment parameter values are increased to 3.9, the system becomes unstable. Thus 3.8 becomes the upper limit of A1, A3 and A5. Experiments (62) and (63) show the effect of gradually reducing the values of A1, A3 and A5. The lower limit is zero, at which point all government expenditures remain unchanged at their initial equilibrium values.

(b). <u>Upper and Lower Limits of A2, A4 and A6</u>

Experiments (64) to (67) test for the lower limit of A2, A4 and A6. The system becomes unstable when these parameter values are reduced to 0.3 under both federal financing policies, but the lower limits for the two federal financing policies are not equal. The effect of lower A2, A4 and A6 values is to reduce the speed of approach towards equilibrium and to worsen the interest burden ratios. In Experiments (68) and (69), we present the effects of increasing the A2, A4 and A6 values to 5 and 10. Basically, the deficit is determined by the income target-adjustment parameters. Unless the balanced-budget target-adjustment parameters are extremely high, the stability of the system is not greatly affected.

(c). <u>Upper and Lower Limits of A1, A2, A3, A4, A5 and A6</u>

Experiments (70) to (75) test for the upper limit of the income and balanced-budget target-adjustment parameters. The system becomes unstable when the parameters are increased to 3.7. Experiments (76) to (79) test for the lower limit of the income and balanced-budget target-adjustment parameters. All these results show that target-adjustment parameter values below some level are incompatible with the objective of an active fiscal policy.

(d). Upper and Lower Limits of A5 and A6

Experiments (80) to (87) test for the upper limit of A5 and A6. The system becomes unstable under a federal bond-financing policy when A5 and A6 are increased to 8.9, and under a federal money-financing policy when A5 and A6 are increased to 9.4. Experiments (88) to (91) suggest the lower limit of A5 and A6. The results confirm the limit obtained in the identical-regions case.

7.3.5 Effectiveness of an Independent Regional Fiscal Policy

Oates[3] has demonstrated that a small region is not very effective in pursuing an active fiscal policy, while Barber[4] argues that a large region should engage more actively in fiscal management. They follow the same approach in investigating the effectiveness of an independent fiscal policy adopted by a single region. However, we have argued that when fiscal and financial relationships among governments are considered, there is no such thing as a completely independent regional fiscal policy. The IRFG model, with government expenditure reaction functions, asserts that every government has its own responsibility in pursuing an active fiscal policy so as to achieve the prescribed targets. To substantiate our argument, the following experiments (92) to (97) as defined in Table 7.9, investigate the effectiveness of an independent regional fiscal policy, based on the IRFG model. Detailed quantitative results are presented in Table 7.10.

Table 7.9: The IRFG Model
Effectiveness of an Independent Regional Fiscal Policy
Listing of the Simulation Experiments

Experiments	Specification A1	A2	A3	of A4	A5	Parameter A6	A7	A8	Values A9	A10
					Identical-Regions Case					
(92)	1	1	0	0	0	0	1	1	1	1
(93)	1	1	0	0	0	0	0	0	0	0
					Unequal-Regions Case					
(94)	1	1	0	0	0	0	1	1	1	1
(95)	1	1	0	0	0	0	0	0	0	0
(96)	0	0	1	1	0	0	1	1	1	1
(97)	0	0	1	1	0	0	0	0	0	0

In the IRFG model, an independent regional fiscal policy is defined as positive target-adjustment parameters for the implementing region and zero target-adjustment parameters for the other region and the federal government. Experiments (92) and (93) examine the identical-regions case, while Experiments (94) to (97) investigate the unequal-regions case. We also experiment with both unitary and zero federal distribution parameters (A7) to (A10) in both cases.

The simulated results of Experiments (92) to (97) show that the system cannot generate a stable long-run equilibrium. The underlying reason is that the budgets of those governments not pursuing an active fiscal policy are affected by induced changes in interest payments and tax revenues, though there are no changes in their expenditures on goods and services. With zero income and balanced-budget target-adjustment parameters for these governments, their budgets are not balanced at all. These destabilizing forces outweigh the stabilizing force exerted by a unitary balanced-budget target-adjustment parameter adopted in the policy-implementing region. Therefore the system cannot achieve the

Table 7.10: The IRFG Model
Simulated Effects when Income Targets Increased by 5% in Both Regions
Effectiveness of Independent Regional Fiscal Policy

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
							Identical-Regions Case										
(92)		Impact	0.3837	0.1220	0.0001	0.6385	0.2005	0.0000	0.3333	0.3333	0.0000	0.0000	0.0000				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(93)		Impact	0.3837	0.1220	0.0001	0.6385	0.2005	0.0000	0.3333	0.3333	0.0000	0.0000	0.0000				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
							Unequal-Regions Case										
(94)		Impact	0.5793	0.1288	0.0002	0.9623	0.2124	0.0000	0.4667	0.4667	0.0000	0.0000	0.0000				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(95)		Impact	0.5793	0.1288	0.0002	0.9623	0.2124	0.0000	0.4667	0.4667	0.0000	0.0000	0.0000				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(96)		Impact	0.1288	0.1747	0.0001	0.2124	0.2910	0.0000	0.0000	0.0000	0.0000	0.2000	0.2000				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(97)		Impact	0.1288	0.1747	0.0001	0.2124	0.2910	0.0000	0.0000	0.0000	0.0000	0.2000	0.2000				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

income and balanced-budget targets.

The simulated results of Experiments (92) to (97) substantiate the basic postulate of the IRFG model, i.e., every government must participate actively in fiscal management so as to achieve the targets. Independent regional fiscal policy in the sense of only one region implementing an active fiscal policy, while other governments adopt zero target-adjustment parameters, does not produce a stable long-run equilibrium.

7.3.6 Effects of Unequal-Regions on an Active Fiscal Policy

We have demonstrated in the last section that every region should involve actively in fiscal management. However, the question of whether the unequal sizes would affect the degree of activeness of fiscal policy in each region deserves more attention. If a larger region becomes more aggressive in fiscal management, then we have A1 and A2 values that are higher than the A3 and A4 values of the smaller region. In this section, we perform Experiments (98) to (104) as defined in Table 7.11. Detailed quantitative results are presented in Table 7.12.

Table 7.11: The IRFG Model
Effects of Unequal-Regions on an Active Fiscal Policy
Listing of the Simulation Experiments

	Specification			of		Parameter		Values		
Experiments	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
(98)	1	1	0.5	0.5	1	1	1	1	1	1
(99)	3	3	0.5	0.5	3	3	1	1	1	1
(100)	3	3	1	1	3	3	1	1	1	1
(101)	4	4	3	3	3	3	1	1	1	1
(102)	4	4	1	1	1	1	1	1	1	1
(103)	5	5	1	1	1	1	1	1	1	1
(104)	6	6	1	1	1	1	1	1	1	1

Table 7.12: The IRFG Model
Simulated Effects when Income Targets Increased by 5% in Both Regions
Effects of Unequal-Regions on an Active Fiscal Policy

Experiment	Federal Financing Policy	Period	ΔE1	ΔE2	ΔR	ΔY1	ΔY2	ΔGF1	ΔG1	ΔENDG1	ΔGF2	ΔG2	ΔENDG2	ΔRBT1	ΔRBT2	ΔRBTF	Stability (Time Period)
(98)		Impact	0.9471	0.3462	0.0003	1.5719	0.5737	0.2000	0.4667	0.6667	0.0857	0.1000	0.1857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.7432	0.2745	1.0177	-0.0253	0.4615	0.4362	0.0218	-0.0289	-0.0014	450
	Money	Long Run	2.4752	1.0608	0.0011	3.5000	1.5000	0.7516	0.2731	1.0248	-0.0180	0.4572	0.4392	0.0218	-0.0283	-0.0024	410
(99)		Impact	2.7124	0.8638	0.0009	4.5033	1.4300	0.6000	1.4000	2.0000	0.2571	0.1000	0.3571				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.6951	0.3226	1.0177	-0.2718	0.7080	0.4362	0.0207	-0.0650	0.0145	400
	Money	Long Run	2.2308	0.9561	-0.0022	3.5000	1.5000	0.8754	0.3938	1.2691	-0.0814	0.6253	0.5439	0.0103	-0.0571	-0.0094	310
(100)		Impact	2.7668	0.9511	0.0010	4.6090	1.5755	0.6000	1.4000	2.0000	0.2571	0.2000	0.4571				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.5136	0.5041	1.0177	0.1199	0.3163	0.4362	0.0073	-0.0074	0.0031	110
	Money	Long Run	2.3914	1.0249	0.0003	3.5000	1.5000	0.5951	0.5135	1.1086	0.1581	0.3170	0.4751	0.0053	-0.0090	-0.0047	110
(101)		Impact	3.6136	1.4292	0.0014	5.9966	2.3700	0.6000	1.8667	2.4667	0.2571	0.6000	0.8571				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.4201	0.5976	1.0177	0.2212	0.2150	0.4362	0.0014	0.0014	0.0027	170
	Money	Long Run	2.4002	1.0286	0.0001	3.5000	1.5000	0.4993	0.6005	1.0998	0.2519	0.2195	0.4714	-0.0001	0.0055	-0.0045	180
(102)		Impact	2.7492	0.8198	0.0010	4.5650	1.3564	0.2000	1.8667	2.0667	0.0857	0.2000	0.2857				
	Bond	Long Run	2.4823	1.0638	0.0012	3.5000	1.5000	0.7251	0.2926	1.0177	0.2575	0.1787	0.4362	0.0206	0.0128	-0.0158	150
	Money	Long Run	2.6306	1.1274	0.0031	3.5000	1.5000	0.5379	0.3315	0.8694	0.1620	0.2106	0.3726	0.0205	0.0104	0.0018	110
(103)		Impact	3.3284	0.9486	0.0012	5.5272	1.5688	0.2000	2.3333	2.5333	0.0857	0.2000	0.2857				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
(104)		Impact	3.9076	1.0774	0.0014	2.4895	1.7812	0.2000	2.8000	3.0000	0.0857	0.2000	0.2857				
	Bond	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable
	Money	Long Run	—	—	—	—	—	—	—	—	—	—	—	—	—	—	Unstable

Note: The last column of the table indicates the time periods required to approach the targets to the fourth decimal point.

In Experiments (98) and (99), we assign lower income and balanced-budget target-adjustment parameters to the second (i.e., small) region. With A3=A4=0.5, the rate of approach to equilibrium reduces considerably in comparing with the base run. In Experiment (99), when a more active fiscal policy is implemented by both the federal and the first regional governments, while the second regional government adopts a less active fiscal policy, the expansionary effect cannot outweigh the dampening effect created by a less than unitary A3 and A4 values.

When the values of A3 and A4 are increased to the base-run level, and more active fiscal policy is implemented by the federal and the first regional governments, the results, as shown in Experiment (100), are much better than those of Experiment (99). However, both Experiments (99) and (100) are inferior to Experiment (23) in which all governments adopt the same active fiscal policy. In Experiment (101), the first (i.e., large) region adopts a more active fiscal policy (A1=A2=4), while the second region and the federal government maintain their target-adjustment parameters at 3. Comparing the results in Experiment with those in Experiment (70) in which A1 to A6 are equal to 3, the speed of approach to equilibrium is much slower in the former experiment.

Moreover, when the first region adopts a more active fiscal policy, i.e., higher A1 and A2 values, while other governments retain their base-run target-adjustment parameters, the system generates more destabilizing forces. This is illustrated in Experiments (103) and (104) in which the system becomes unstable when the A1 and A2 values are increased to and above 5.

Perhaps one major advantage, from the small region's point of view, is the resulting smaller interest burden ratio which occurs in Experiments (92) to (94). However, this advantage must be weighed against the cost of the much longer period of time required for each government to adjust its expenditure level in order to approach both the income and balanced-budget targets.

Based on the results of Experiments (99) to (100) and Experiment (23) and (70), we conclude that a more active fiscal policy implemented by a larger region, while other target-adjustment parameters retain their base-run values, may generate undesirable effects, notably a slower speed of approach towards the targets. Moreover, there is also an upper limit beyond which a too active fiscal policy implemented by a larger region, while other target-adjustment parameters retain their base-run values, will not produce a stable long-run equilibrium.

In Fig.7.1 and Fig.7.2, we present a comparison of effects between the base run and Experiment (33) in which the target-adjustment parameters of the federal sector are three times of their base-run values (i.e., A5=A6=3). Fig.7.1 shows the case of a federal money-financing policy while Fig.7.2 depicts the case of a federal bond-financing policy. The solid lines represent the base-run values while the broken lines denote the Experiment (33) values.

In Fig.7.1 and Fig.7.2, we present only the deficit and the interest burden ratio values for the regional and federal governments, since these variables are the ones most affected by the more active federal fiscal policy. We observe a complete reversal of the base-run results in both cases. The regional governments now realize a budget surplus in the first five periods, and then a very small deficit in the course of reaching the balanced-budget target. This contributes to the reduction of the interest burden ratio (RBT1), by almost 2 percent, compared with the base-run value. Improvement in the regional fiscal and financial situations is basically attributable to the aggressive fiscal policy adopted by the federal government.

The effects, of course, depend on the financing policy of the federal government. The regional governments benefit more from a federal money-financing policy than a bond-financing policy. Because of its aggressive fiscal policy, the budget deficit of the federal government increases sharply in the first three periods, and then approaches a balanced-budget level gradually. If this is the cost the federal government must assume, then we must examine the influence of the interest burden ratio (RBTF). We observe that the difference of RBTF between the new and the base-run levels is increased by almost 2 percent in the federal bond-financing case, while it is reduced by nearly 1 percent in the federal money-financing case. Even in the federal bond-financing case, the overall interest burden ratio is still an improvement over its base-run value. The superiority of the federal money-financing policy over the bond-financing policy is fully demonstrated in Fig.7.1 and Fig.7.2.

Figure 7.1: The IRFG Model
Identical-Regions, Comparison of Base-Run and
Experiment (33) Effects of an Increase in Income
Target by 5% in Both Regions, Federal Money-Financing Policy

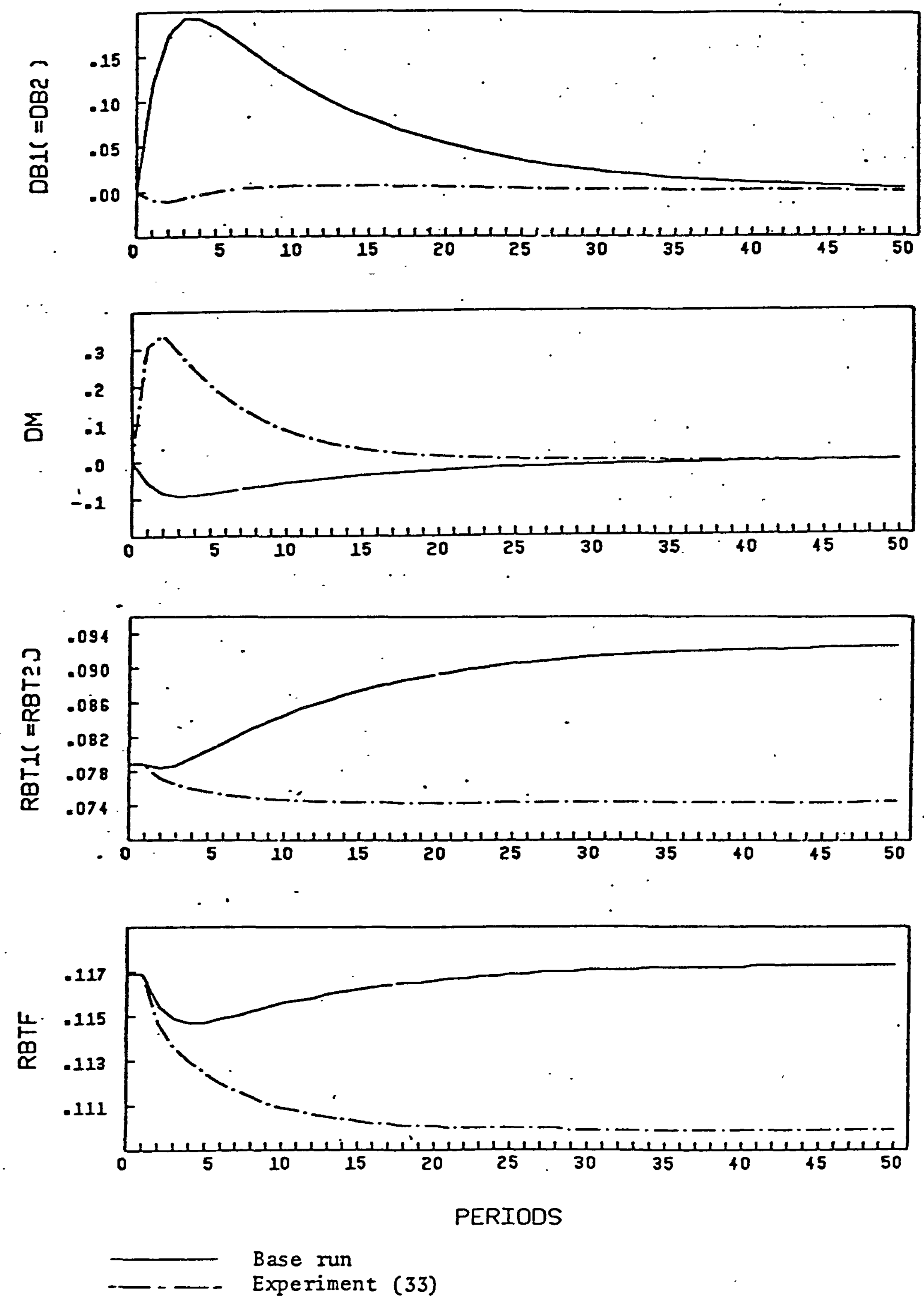
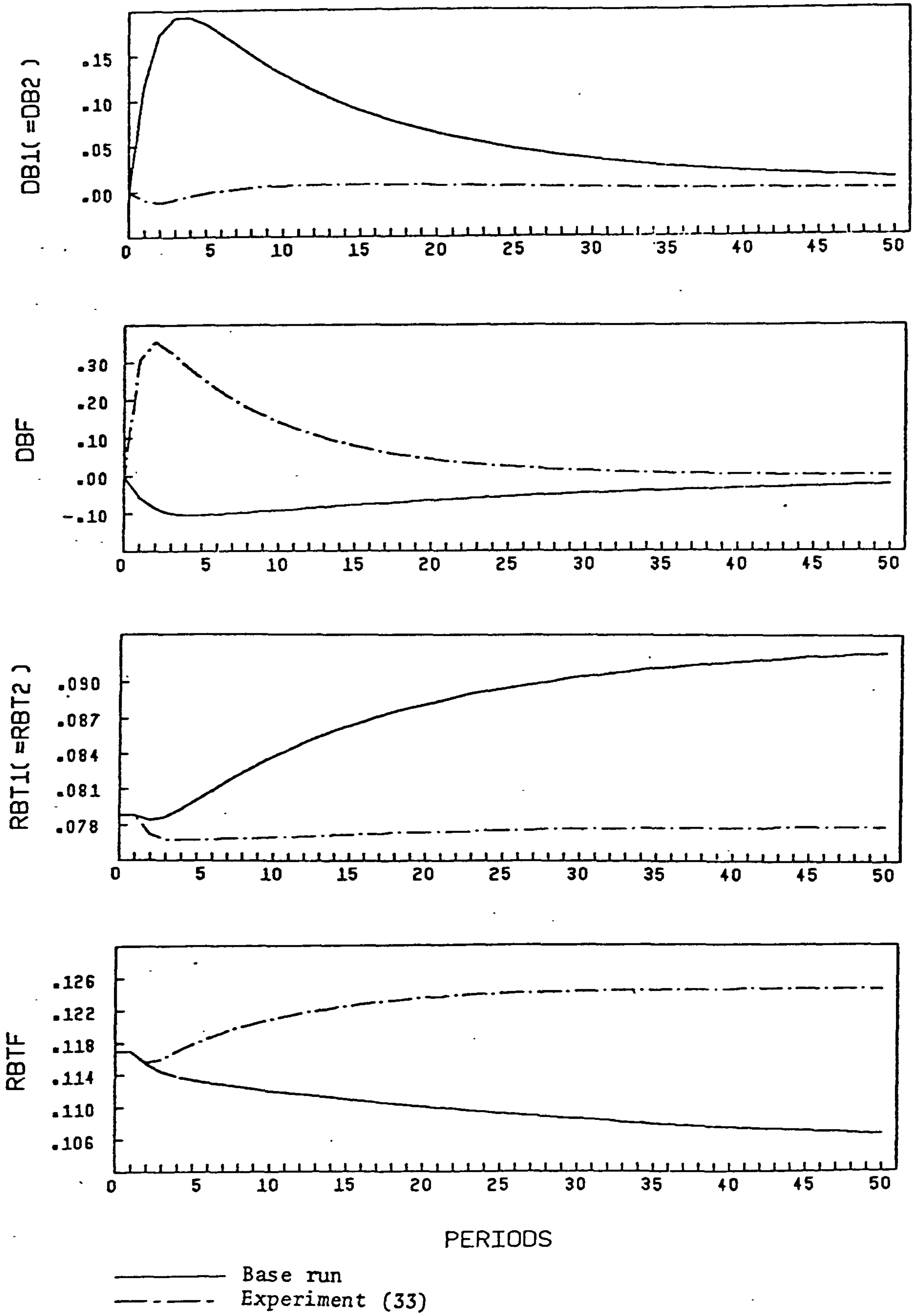


Figure 7.2: The IRFG Model
Unequal-Regions, Comparison of Base-Run and
Experiment (33) Effects of an Increase in Income
Target by 5% in Both Regions, Federal Bond-Financing Policy



7.4 General Discussion and Concluding Remarks

From the above simulation experiments, there follow several general conclusions:

First, simulated results show that in the IRFG model, an independent regional fiscal policy cannot generate a stable long-run equilibrium. This finding substantiates the basic postulate of the IRFG model that every government must participate actively in fiscal management so as to achieve the targets. However, this finding differs from Oates' conclusion that regions should not a pursue fiscal policy. Two important differences should be noted:

(i) Oates assumes a long-run equilibrium exists. Our experiments show that an independent regional fiscal policy cannot generate a stable long-run equilibrium. Our findings also disagree with Barber's conclusion that a large region should pursue an independent fiscal policy.

(ii) Oates argues that since a long-run multiplier effect is quite small for regional fiscal actions, regions should not engage in fiscal policy. Our experiments show that all governments must participate actively in fiscal management so as to achieve the targets.

Second, an active fiscal policy pursued by all governments can be implemented through higher income target-adjustment parameters, or through higher balanced-budget target-adjustment parameters. Combined higher income and balanced-budget target-adjustment parameters increase the speed of approach towards equilibrium.

Third , there are certain upper and lower limits to active fiscal policy, beyond which the system cannot generate a stable equilibrium.

Fourth , A too active fiscal policy implemented by a large region, while other target-adjustment parameters retain their base-run values, may reduce the speed of approach towards equilibrium, and may even generate instability to the system. Thus there are certain limits to decentralization beyond which the system cannot generate a stable long-run equilibrium.

Fifth , our simulated results show that different combinations of target-adjustment parameters produce different fiscal performances. Some combinations may increase the speed of approach to equilibrium, while others may reduce the speed of approach to equilibrium and increase the interest burden ratios, or may even not be able to generate a stable equilibrium. Since the choice of target-adjustment parameters is at the discretionary control of each government, uncoordinated fiscal policies among governments would produce undersirable effects in the achievement of the targets. Thus the IRFG model not only rejects independent regional fiscal policies, but also requires coordinated fiscal policies among governments.

A viable fiscal policy must consider the level of income target changes, the effects on the speed of approach to equilibrium, the interest burden ratios in the public sectors, and the distribution of aggregate demand between private and public sectors. Our simulated results demonstrate that there are always some trade-offs in these considerations when the degree of activeness of fiscal policy is increased. Thus the IRFG model requires cooperation and coordination among governments in devising a viable fiscal policy. Our findings therefore strengthen the case for fiscal federalism.

Footnotes to Chapter 7

(1). Not all economists agree with the idea of using federal fiscal policy in reducing the economic imbalance between regions. For example, in Canada, Courchene (1979) argues that such policies may prevent natural adjustment processes with the result that poor regions become more dependent on federal government. Thus instead of bringing about improvement, the federal fiscal policy maintains the status quo in economically depressed regions.

(2). The initial interest burden ratios are as follows: RBT1=RBT2=0.0788 and RBTF=0.1169.

(3). Oates (1968), p.44.

(4). Barber (1967), p.29.

Chapter 8

Summary and Outlook for Further Research

8.1 Introduction

This final chapter provides a brief summary and discussion of fiscal policy, as analysed using conventional models and using our IRFG model. It also presents an overview of the possible policy implications of the IRFG model and of the simulation experiments in which it has been used.

One major concern in a federal system is how far decentralization should go. There has always been a struggle over fiscal autonomy among regional governments as well as between regional and federal governments. The simulation experiments in this thesis were undertaken with the intention of evaluating the effectiveness of an active fiscal policy, in hopes that we could aid in solving the decentralization question stated above.

In the following section of this concluding chapter we present a brief summary and discussion of fiscal policy based on conventional models and on the final version of our model. In section 8.3 we discuss the possible implications of the model in a policy setting. Section 8.4 concludes the chapter with an overview of the outlook for further research.

8.2 Summary of Fiscal Policy Based on the Conventional and the IRFG Models

The purpose of this thesis has been to provide a theoretical framework for analysing interregional fiscal policy in a federal setting, with special emphasis on the government budget constraints and their fiscal and financial relationships among various levels of government.

In order to achieve this goal, a simple conventional closed-economy national model with one aggregate public sector was first developed, in Chapter 3. The simple national model was then further developed into an interregional model in Chapter 4, with the aggregate national variables distributed equally to the two regions, and incorporating regional government budget constraints. The interregional model is an open-economy type within the confederation but closed with respect to the rest of the world.

In these conventional national and interregional models, government expenditures on goods and services are treated as exogenous. An active fiscal policy is defined as involving changes in the exogenous government tax rates and expenditures. To analyse the long-run consequences of an active fiscal policy, we thus considered a once-and-for-all increase of government expenditures. Complexities of the model made the formal stability analysis untractable, and a simulation approach was used to trace out the time paths of endogenous variables after an exogenous fiscal shock to the system.

The simulation results of the National Model confirm the conventional wisdom that the system is stable under a money-financing policy, while stability under a bond-financing policy is an empirical question. When we have a flat IS and steep LM curve, the system is unstable under a bond-financing policy. However, applying the Christ adjustment method produces stable results, regardless of the slopes of the IS-LM curves.

If the system is stable, the long-run multiplier effects are greater under a bond-financing policy than under a money-financing policy. However, the speed of approach towards equilibrium is faster under money-financing than bond-financing policy. A slower rate of approach towards equilibrium implies that more bonds must be issued to finance the deficit, and consequently there is a larger stock of private wealth and a larger amount of interest income which contribute to larger multiplier effects under a bond-financed fiscal policy.

The interregional model without a federal sector focuses on the fiscal and financial relationships between two regions. With a fixed amount of money stock, each region can acquire more money balances only through sales of bonds to the other region. Consider an active fiscal policy implemented by the first region with a once-and-for-all increase in government expenditures on goods and services. Not only is the government budget constraint in the first region affected by its active fiscal policy, but also the second region is affected. The simulated results show that at best the system attains a quasi-equilibrium in which the increase in the new bonds issued in the first region equals the increase in the retirement of outstanding bonds in the second region, while the other endogenous variables remain at their new levels. The acceptability of such a quasi-equilibrium has been rejected because the government budget constraints are never balanced. Applying Christ's adjustment method to the system introduces more instability than it eliminates. The simulated results show that even a quasi-equilibrium is not then attainable.

The aggregate National Model and the interregional model without a federal sector (IR) are extreme cases of the full interregional model with a federal sector (IRF) which incorporates the budget constraints of all governments. The fiscal policy of the IRF model was discussed in Chapter 5. The hope that a stable federal money-financed fiscal policy could make interregional fiscal policy feasible, did not materialize. Even more surprising was the finding that federal fiscal policy, which works in the National Model, fails to generate a stable long-run equilibrium in the IRF system.

One basic reason underlying these unstable results is the different requirements for stability in the fiscal expansionary public sector and in the remaining public sectors, which are governed by their respective government budget constraints. At best, the system attains a quasi-equilibrium under a federal bond-financed fiscal policy. Applying the Christ adjustment method can only aggravate the instability.

Two other fiscal schemes which generate balanced budgets were examined as well. One scheme involves forcing the public sectors other than the fiscal expansionary public sector, to endogenize their tax rates so as to balance their budgets in each period. The other scheme involves the creation of an endogenous expenditure item in the government budget constraint in those public sectors other than the fiscal expansionary public sector, so that again they will automatically balance their budgets in each period. One major limitation of such schemes is the total elimination of the problem of interregional fiscal and financial relationships that we want to address.

These two compensating fiscal schemes reveal the basic weakness of the National and interregional models, in which there is no mechanism for the public sectors to respond to evolving economic situations. These models consider a once-and-for-all increase in government expenditures, and only allow for the creation, contraction, or retirement of money and bonds to secure a stable long-run equilibrium. This conventional approach follows the methodology employed by Christ (1968), Blinder and Solow (1973), Christ (1979) and other economists during more than a decade of establishing the theoretical significance of the government budget constraint when considering a once-and-for-all increase in government expenditures. Even the Christ adjustment method treats the total government expenditures as fixed and lets government expenditures on goods and services adjust for any changes in interest payments.

This conventional approach may work for a one-public-sector model, as illustrated in our National Model in Chapter 3, but it fails to generate a stable equilibrium in our interregional model with or without a federal sector, as demonstrated in Chapters 4 and 5. When more than one government budget constraint is involved, the Christ adjustment method introduces more instability than it eliminates.

The basic claim of Chapters 4 and 5 was that this once-and-for-all change policy is irrelevant in the studies of interregional fiscal policy since, in reality, governments react to evolving economic situations. In an effort to provide a more realistic theoretical framework for studying interregional fiscal policy, we elaborated the Scarth adjustment method of making fiscal policy endogenous in constructing an interregional model with a federal sector and government expenditure reaction functions, in Chapter 6. In our expanded (IRFG) model, an active fiscal policy must be target-oriented. In other words, fiscal policy is used only to achieve a prescribed target. The philosophy of fiscal policy underlying this model is that government should be responsible for achieving income and other targets. Thus expenditures of all governments are endogenized in the system, in the sense that the public sector must adopt an active fiscal policy when the income and balanced-budget targets are not achieved.

The government expenditure reaction function has both exogenous and endogenous components. The endogenous component is governed by the deviation of the last period's value from the target value, while the exogenous part includes the target-adjustment parameters which defines the policies adopted by each level of government. A more active fiscal policy is represented by higher target-adjustment parameter values.

Since in our IRFG system, all governments participate in active fiscal policy, the analysis of the short-run and long-run effects are quite different from conventional analysis in which only a single region raises its expenditures once and for all. Long-run equilibrium is defined as the situation that exists when the targets are achieved and the exogenous variables remain unchanged.

In the base-run simulations in which all target-adjustment parameters were assumed to have unitary value, the system attained its long-run stable equilibrium under either type of federal financing policy, for both identical or unequal-regions cases, and for all parameter sets. This stable result was attributed basically to the balanced-budget target imposed on the government expenditure reaction functions. Our general findings of Chapter 6 support the spirit of the Scarth adjustment method. We observe that fiscal policy is more effective under federal money-financing than under a bond-financing policy.

A more active fiscal policy is represented by higher target-adjustment parameter values. In Chapter 7 we experimented with different combinations of active fiscal policies, and tested for the critical limits of these target-adjustment parameters, in both the identical and unequal-regions cases.

Several findings are of general interest. Of prime importance is the finding that an independent regional fiscal policy cannot generate a stable long-run equilibrium but when all governments participate actively in fiscal management, the prescribed targets can be achieved. Thus our findings are in sharp contrast to Oates' conclusion that regions should not pursue any fiscal policy. Another interesting finding is that a too active fiscal policy implemented by a large region, while other target-adjustment parameters retain their base-run values, may reduce the speed of approach to equilibrium, and may even generate instability to the system. Thus there are certain limits to decentralization. Furhtermore, our simulated results show that different combinations of target-adjustment parameters will generate different fiscal performances. Since the choice of the target-adjustment parameters is at the discretionary control of each government, uncoordinated fiscal policies would produce undesirable effects to the achievement of the targets. Thus the IRFG model not only rejects independent regional fiscal policies, but also requires coordinated fiscal policies among governments. The simulation findings based on the IRFG model therefore strengthen the case for fiscal federalism.

8.3 Policy Implications of the IRFG Model

In this section we discuss some implications of applying the simulation results in policy-making. The IRFG model constructed in this thesis is theoretical and based on a closed economy with respect to the rest of the world. Therefore specific policy recommendations may not apply to the Canadian economy. However, the results of the simulation experiments are of general interest in that they shed some light on the basic fiscal and financial influences between regional and federal governments in a federal setting.

The model has established the fact that within a confederation, no one public sector can be insulated from the fiscal actions taken by the other public sectors, regardless of the size of the public sector. This puts pressure on all regions to pursue an active fiscal policy even though the conventional wisdom is that fiscal policy actions taken by regional governments are ineffective. Recognition of the fact that every regional government is also being responsible for achieving the targets in that region is an important development in the philosophy of fiscal management. Of course, the federal government assumes the responsibility for achieving the targets for the whole economy. Placing this responsibility partially on regional government also avoids the problem raised by Courchene (1979) that some poor regions become dependencies of the federal government.

The IRFG model emphasizes the need for cooperation and participation of all governments in active fiscal management. This is in sharp contrast to the conventional approach of analysing the efficacy of the one regional-public-sector fiscal policy.

The results of simulation experiments have also demonstrated the superiority of the federal government in pursuing an active fiscal policy by virtue of its broader tax base and its money financing option. The federal government, acting as a coordinator, implements fiscal policy for the attainment of targets for the whole economy. Therefore the federal government should engage more actively in fiscal management. Of course, the IRFG model requires every government implementing an active fiscal policy. Moreover, the federal government can incorporate other criteria in its distribution function so as to achieve more economic balance between regions. The model thus strengthens the case for fiscal federalism. This is further evidenced by our finding that too aggressive a fiscal policy pursued by a large region, while other governments retain their base-run target-adjustment parameter values, may not generate a stable long-run equilibrium. Thus we conclude that there are certain limits to decentralization beyond which the adverse effects may outweigh the perceived benefits.

8.4 Outlook for Further Research

The previous simulations experiments based on the IRFG model have focused mainly on changes of expenditures of all governments. Effects of changes in tax rates have not been analysed. Changes in tax rates would not only affect aggregate demand, but would also affect the factor markets and shift the aggregate supply curve as well. This would have a bearing on the choice of target-adjustment parameters, and consequently determines the efficacy of fiscal policy. Thus it is desirable to include the aggregate supply side in the IRFG model, and extend the simulation experiments to analyse effects of tax rate changes.

Another possible direction of research is to analyse the impacts of the private sector's action on the formation and efficacy of fiscal policy. Changes of private expenditures, imports, and demand for money functions could affect aggregate demand. These private sector's actions may contribute to the achievement of the target at a faster rate, or they may require complementary fiscal actions to counteract undersirable effects. Simulations of changes of the intercepts and/or slopes of these functions may give us valuable information in the formation of fiscal policy.

The interregional model emphasising fiscal and financial interrelationships has not been extensively investigated in the literature. The above brief discussion suggests a wide range of possibilities for extension of this thesis and, it may be hoped, provides some stimulation for further research and policy analysis in the area of fiscal federalism.

Appendix 3.1: The National Model

Derivation of Short-Run Multipliers

From (3.9) and (3.10):

(3.9) $Y = E((1-u)(Y+TR+RB),W,R)+G$

(3.10) $M = L(Y,W,R)$

Consider the fiscal policy. Taking total differentials of (3.9) and (3.10), we have:

(3.1.1) $dY = E_{YD}(1-u)(dY+BdR)+E_R dR+dG$

(3.1.2) $0 = L_Y dY+L_R dR$

From (3.1.2), we have:

(3.1.3) $dR = -(L_Y/L_R)dY$

Substituting (3.1.3) to (3.1.1), we have:

(3.1.1) $dY = E_{YD}(1-u)(dY+B(-L_Y/L_R)dY)+E_R(-L_Y/L_R)dY+dG$

(3.1.4) $dY(1-E_{YD}(1-u)(1-B(L_Y/L_R))+E_R L_Y/L_R) = dG$

(3.1.5) $$dY/dG = \frac{1}{1-E_{YD}(1-u)(1-B(L_Y/L_R))+E_R L_Y/L_R} > 0$$

Substituting (3.1.5) to (3.1.3), we have:

(3.1.6) $$dR/dG = \frac{-L_Y/L_R}{1-E_{YD}(1-u)(1-B(L_Y/L_R))+E_R L_Y/L_R} > 0$$

Consider the monetary policy. Taking total differentials of (3.9) and (3.10), we have:

(3.1.7) $dY = E_{YD}(1-u)(dY+BdR)+E_R dR+dG+E_W dM$

(3.1.8) $dM = L_Y dY+L_R dR+L_W dM$

From (3.1.8), we have:

(3.1.9) $dR = ((1-L_W)dM-L_Y dY)/L_R$

Substituting (3.1.9) to (3.1.7), we have:

$$dY = E_{YD}(1-u)(dY+B((1-L_W)dM-L_YdY)/L_R))+E_R((1-L_W)dM-L_YdY)/L_R)+dG+E_WdM$$

$$(3.1.10)\quad dY/dM = \frac{E_W+(1-L_W)((E_{YD}(1-u)B+E_R)/L_R)}{1-E_{YD}(1-u)(1-BL_Y/L_R)+E_RL_Y/L_R} > 0$$

Substituting (3.1.10) to (3.1.9), we have:

$$(3.1.11)\quad dR/dM = (1-L_W-L_{YD}dY/dM)/L_R < 0$$

Consider the open market operation:

$$(3.1.12)\quad dY = E_{YD}(1-u)(dY+BdR+RdB)+E_RdR+dG+E_WdB$$

$$(3.1.13)\quad 0 = L_YdY+L_RdR+L_WdB$$

From (3.1.13), we have:

$$(3.1.14)\quad dR = -(L_YdY+L_WdB)/L_R$$

Substituting (3.1.14) to (3.1.12), we have:

$$dY(1-E_{YD}(1-u)(1-BL_Y/L_R)+E_RL_Y/L_R) = dB(E_{YD}(1-u)(-BL_W/L_R+R)-E_RL_W/L_R+E_W)$$

$$(3.1.15)\quad dY/dB = \frac{E_W-E_RL_W/L_R+E_{YD}(1-u)(R-BL_W/L_R)}{1-E_{YD}(1-u)(1-BL_Y/L_R)+E_RL_Y/L_R} \gtrless 0$$

Substituting (3.1.15) to (3.1.14), we have:

$$(3.1.16)\quad dR/dB = -(L_W+L_YdY/dB)/L_R$$

or as

$$(3.1.17)\quad dR/dB = -\frac{(L_W(1-E_RL_Y/L_R)+L_Y(E_W+E_{YD}(1-u)(R-BL_W/L_R))/L_R}{1-E_{YD}(1-u)(1-BL_Y/L_R)+E_RL_Y/L_R} > 0$$

Appendix 3.2: The National Model

Micro-Foundation of Macro-Functions

Consider the private expenditure function, for example, at the micro level for a representative household:

(3.2.1) $$E_t/NH_t = \theta + \alpha YD_t/NH_t + \beta W_t/NH_t + \gamma R_t$$

where NH denotes total number of households. We assume that all households have the same private expenditure function. Note also that all variables in (3.2.1) are divided by the total numbers of households except for the intercept and the financial variable R. At the macro level, when we aggregate all the private expenditure functions, we have

(3.2.2) $$E_t = \theta NH_t + \alpha YD_t + \beta W_t + \gamma NH_t R_t$$

Comparing equations (3.2.1) and (3.2.2), coefficients α and β for disposable income and wealth variables are the same for both the macro and micro-functions, but the intercept and the interest rate coefficient are multiplied by the number of households. Another interesting feature is that the corresponding elasticities are the same for both the macro and micro-functions. Using N(·) to denote elasticity, we have

(3.2.3) $$N(E_t/NH_t, YD_t/NH_t) = ((YD_t/NH_t)/(E_t/NH_t)) * \partial((E_t/NH_t)/(YD_t/NH_t))$$
$$= \alpha (YD_t/E_t) = N(E_t, YD_t)$$

(3.2.4) $$N(E_t/NH_t, W_t/NH_t) = ((W_t/NH_t)/(E_t/NH_t)) * \partial((E_t/NH_t)/(W_t/NH_t))$$
$$= \beta (W_t/E_t) = N(E_t, W_t)$$

(3.2.5) $$N(E_t/NH_t, R_t) = ((R_t/(E_t/NH_t)) * \partial((E_t/NH_t)/R_t))$$
$$= \gamma NH_t (R_t/E_t) = N(E_t, R_t)$$

Thus the macro private expenditure function is totally compatible with the micro foundation. The analogy applies to the other functions as well. For convenience, the macro-functions are expressed as follows:

(3.2.6) $E_t = a_1 + a_2YD_t + a_3W_t + a_4R_t$

(3.2.7) $M_t = a_5 + a_6Y_t + a_7W_t + a_8R_t$

Appendix 4.1: The IR Model

Derivation of Short-Run Multipliers

For simplicity, the following derivation ignores time-period subscripts.

(4.1.1) $Y1 = E1((1-u1)(Y1+TR1+RH1), W1, R)+G1-IM1((1-u1)(Y1+TR1+RH1), W1, R)+IM2((1-u2)(Y2+TR2+R(B1+B2-H1), M+B1+B2-W1, R)$

(4.1.2) $Y2 = E2((1-u2)(Y2+TR2+R(B1+B2-H1), M+B1+B2-W1, R) +G2+IM1((1-u1)(Y1+TR1+RH1), W1, R) -IM2((1-u2)(Y2+TR2+R(B1+B2-H1), M+B1+B2-W1, R)$

(4.1.3) $W1-H1 = L1(Y1, W1, R)$

(4.1.4) $M-W1+H1 = L2(Y2, M+B1+B2-W1, R)$

Take total differentials of the above system. (Note: we use the second subscript to represent the partial derivative.)

From (4.1.1), we have:

(4.1.5) $dY1 = E_{11}((1-u1)(dY1+H1dR+RdH1)+E_{13}dR+dG1 -IM_{11}(1-u1)(dY1+RdH1+H1dR)-IM_{13}dR +IM_{21}(1-u2)(dY2+(B1+B2-H1)dR-RdH1)+IM_{23}dR$

(4.1.6) $(1-(E_{11}-IM_{11})(1-u1))dY1 = IM_{21}(1-u2)dY2+(E_{11}(1-u1)H1 +E_{13}-IM_{11}(1-u1)H1-IM_{13}+IM_{21}(1-u2)(B1+B2-H1) +IM_{23})dR+((E_{11}-IM_{11})(1-u1)R-IM_{21}(1-u2)R)dH1+dG1$

Using the "d'i" to represent the above bracketed terms, we have:

(4.1.7) $d_1 = 1-(E_{11}-IM_{11})(1-u1)$

(4.1.8) $d_2 = IM_{21}(1-u2)$

(4.1.9) $d_3 = E_{11}(1-u1)H1+E_{13}-IM_{11}(1-u1)H1-IM_{13}+IM_{21}(1-u2)(B1+B2-H1)+IM_{23}$

(4.1.10) $d_4 = (E_{11}-IM_{11})(1-u1)R-IM_{21}(1-u2)R$

From the plausible magnitudes of the partial derivatives described in the chapter, the signs of d_1 to d_4 are positive. Substituting all the d_i terms into (4.1.6), we have

(4.1.11) $d_1dY1 = d_2dY2+d_3dR+d_4dH1+dG1$

From (4.1.2), we have

(4.1.12) $dY2 = E_{21}(1-u2)(dY2+(B1+B2-H1)dR-RdH1)+E_{23}dR$
$+dG2+IM_{11}(1-u1)(dY1+RdH1+H1dR)+IM_{13}dR$
$-IM_{21}(1-u2)(dY2-RdH1+(B1+B2-H1)dR)-IM_{23}dR$

(4.1.13) $(1-(E_{21}-IM_{21})(1-u2))dY2 = IM_{11}(1-u1)dY1+(IM_{11}(1-u1)H1$
$+(E_{21}-IM_{21})(1-u2)(B1+B2-H1)+E_{23}+IM_{13}-IM_{23})dR$
$+(IM_{11}(1-u1)R+(IM_{21}-E_{21})(1-u2)R)dH1+dG2$

Using the "d'i" to represent the above bracketed terms, we have:

(4.1.14) $d_5 = 1-(E_{21}-IM_{21})(1-u2)$

(4.1.15) $d_6 = IM_{11}(1-u1)$

(4.1.16) $d_7 = IM_{11}(1-u1)H1+(E_{21}-IM_{21})(1-u2)(B1+B2-H1)+E_{23}+IM_{13}-IM_{23}$

(4.1.17) $d_8 = IM_{11}(1-u1)R+(IM_{21}-E_{21})(1-u2)R$

From the plausible magnitudes of the partial derivatives described in the chapter, we obtain positive signs for d_5 to d_7 and negative sign for d_8. Substituting the d_i terms into (4.1.13), we have

(4.1.18) $d_5dY2 = d_6dY1+d_7dR+d_8dH1+dG2$

From (4.1.3) and (4.1.4), we have:

(4.1.19) $-dH1 = L_{11}dY1+L_{13}dR$

(4.1.20) $dH1 = L_{21}dY2+L_{23}dR$

Adding (4.1.19) and (4.1.20), we have:

(4.1.21) $0 = L_{11}dY1+L_{21}dY2+(L_{13}+L_{23})dR$

(4.1.22) $dR = -(L_{11}dY1+L_{21}dY2)/(L_{13}+L_{23})$

Substituting (4.1.22) into (4.1.20), we have:

(4.1.23) $dH1 = L_{21}dY2-(L_{23}(L_{11}dY1+L_{21}dY2))/(L_{13}+L_{23})$

Substituting (4.1.22) and (4.1.23) into (4.1.11) and (4.1.18), we have:

(4.1.24) $d_1dY1 = d_2dY2-d_3(L_{11}dY1+L_{21}dY2)/(L_{13}+L_{23})$
$+d_4(L_{21}dY_2-(L_{23}(L_{11}dY1+L_{21}dY2)/(L_{13}+L_{23})))+dG1$

(4.1.25) $(d_1+(d_3L_{11}+d_4L_{11}L_{23})/(L_{13}+L_{23}))dY1 = (d_2-(d_3L_{21}$
$+d_4L_{21}L_{23})/(L_{13}+L_{23})+d_4L_{21})dY2+dG1$

Using the "e'i" to represent the above bracketed terms, we have:

(4.1.26) $e_1 = d_1+(d_3L_{11}+d_4L_{11}L_{23})/(L_{13}+L_{23})$

(4.1.27) $e_2 = d_2 - (d_3L_{21}+d_4L_{11}L_{23})/(L_{13}+L_{23}) +d_4L_{21}$

Substituting (4.1.26) and (4.1.27) to (4.1.25), we have

(4.1.28) $e_1dY1 = e_2dY2+dG1$

Note that the signs of all e's are indeterminate depending on the right-hand side terms. For example, the sign of e_2 depends on whether the negative middle term of e_2 is less or greater than the sum of other two positive terms. The magnitude of the negative middle term depends on the relative magnitude of d_3L_{21} and $d_4L_{21}L_{23}$. Moreover, the magnitude of d_3 depends mainly on the magnitudes of H1 and E_{13}. A small H1 and a high E_{13} (absolute) values imply a higher d_3, which may result in a negative e_2.

(4.1.29) $d_5dY2 = d_6dY1-(d_7(L_{11}dY1+L_{21}dY2)/(L_{13}+L_{23}))$
$+d_8(L_{21}dY2-L_{23}(L_{11}dY1+L_{21}dY2)/(L_{13}+L_{23}))+dG2$

Using the "e'i" to represent the above bracketed terms, we have

(4.1.30) $e_3 = d_5 + (d_7L_{21}+d_8L_{21}L_{23})/(L_{13}+L_{23}) + d_8L_{21}$

(4.1.31) $e_4 = d_6 - (d_7L_{11}+d_8L_{11}L_{23})/(L_{13}+L_{23})$

Substituting (4.1.30) and (4.1.31) to (4.1.29), we have

(4.1.32) $e_3dY2 = e_4dY1+dG2$

The explanation of the sign of e_2 also applies to e_4.

From (4.1.28) and (4.1.32), we have

(4.1.33) $(e_1e_3-e_2e_4)dY1 = e_3dG1+e_2dG2$

(4.1.34) $dY1/dG1 = e_3/(e_1e_3-e_2e_4)$

(4.1.35) $dY1/dG2 = e_2/(e_1e_3-e_2e_4)$

(4.1.36) $(e_1e_3-e_2e_4)dY2 = e_4dG1+e_1dG2$

(4.1.37) $dY2/dG1 = e_4/(e_1e_3-e_2e_4)$

(4.1.38) $dY2/dG2 = e_1/(e_1e_3-e_2e_4)$

Through substitution, we have:

(4.1.39) $dR/dG1 = -(L_{11}dY1/dG1 + L_{21}dY2/dG1)/(L_{13}+L_{23})$

$= -(L_{11}e_3+L_{21}e_4)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(4.1.40) $dR/dG2 = -(L_{11}dY1/dG2 + L_{21}dY2/dG2)/(L_{13}+L_{23})$

$= -(L_{11}e_2+L_{21}e_1)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(4.1.41) $dH1/dG1 = -(dY1/dG1)(L_{23}L_{11})/(L_{13}+L_{23})$

$+(L_{21}-L_{23}L_{21}/(L_{13}+L_{23}))(dY2/dG1)$

$= (L_{13}L_{21}e_4-L_{11}L_{23}e_3)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

(4.1.42) $dH1/dG2 = (dY1/dG2)(L_{11}L_{23})/(L_{13}+L_{23})$

$+(L_{21}-L_{21}L_{23}/(L_{13}+L_{23}))(dY2/dG2)$

$= (L_{13}L_{21}e_1-L_{11}L_{23}e_2)/(L_{13}+L_{23})(e_1e_3-e_2e_4)$

Since the signs of all the e's are indeterminate, we cannot sign $(e_1e_3-e_2e_4)$. However, based on the the coefficients of all parameter sets (1) to (4) described in the text of the chapter, we obtain the signs of short-run multiplier effects as shown in Table 4.1. (See p.84.)

Appendix 4.2: The IR Model

Micro-Foundation of Macro-Functions

We assume the same functional forms as in the National Model but the problem of decomposing from the national into interregional functions deserves more discussion. Consider, for example, the private expenditure functions. Equation (4.2.1) is the national E_t function, while equations (4.2.2) and (4.2.3) are the regional functions, respectively.

(4.2.1) $E_t = E(YD_t, W_t, R_t)$

(4.2.2) $E1_t = E1(YD1_t, W1_t, R_t)$

(4.2.3) $E2_t = E2(YD2_t, W2_t, R_t)$

The criteria for determining the division of E_t into $E1_t$ and $E2_t$

are: (i) income distributor (a_{11}, a_{12})

(ii) household distributor (a_{21}, a_{22}) whereas

(4.2.4) $a_{11}+a_{12} = 1 \qquad 0 \leqslant a_{11} \leqslant 1 \qquad 0 \leqslant a_{12} \leqslant 1$

(4.2.5) $a_{21}+a_{22} = 1 \qquad 0 \leqslant a_{21} \leqslant 1 \qquad 0 \leqslant a_{22} \leqslant 1$

Using the above distributors, the regional components can be expressed as

(4.2.6)
$$
\begin{aligned}
E1_t &= a_{11}E_t & E2_t &= a_{12}E_t \\
YD1_t &= a_{11}YD_t & YD2_t &= a_{12}YD_t \\
W1_t &= a_{11}W_t & W2_t &= a_{12}W_t \\
NH1_t &= a_{21}NH_t & NH2_t &= a_{22}NH_t
\end{aligned}
$$

The E_t function at the micro-level discussed in Appendix 3.2 is restated here as:

(4.2.7) $E_t/NH_t = \delta + \alpha(YD_t/NH_t) + \beta(W_t/NH_t) + \gamma R_t$

Applying the same procedure to the IR model, the $E1_t$ function at the micro-level then becomes:

$$(4.2.8)\quad E1_t/NH1_t = \theta 1 + \alpha 1(YD1_t/NH1_t) + \beta 1(W1_t/NH1_t) + \gamma 1/R_t$$

Substituting (4.2.6) into (4.2.8), we have

$$(4.2.9)\quad a_{11}E_t/a_{21}NH_t = \theta 1 + \alpha 1(a_{11}YD_t/a_{21}NH_t) + \beta 1(a_{11}W_t/a_{21}NH_t + \gamma 1/R_t$$

Multiplying (4.2.9) through by (a_{21}/a_{11}), we have

$$(4.2.10)\quad E_t/NH_t = \theta 1(a_{21}/a_{11}) + \alpha 1(YD_t/NH_t) + \beta 1(W_t/NH_t) + \gamma 1(a_{21}/a_{11})R_t$$

Comparing (4.2.7) with (4.2.10), we have

$$(4.2.11)\quad \begin{aligned} \theta &= (a_{21}/a_{11}) && \text{or } \theta 1 = (a_{11}/a_{21})\theta \\ \alpha &= \alpha 1 && \\ \beta &= \beta 1 && \\ \gamma &= (a_{21}/a_{11})\gamma 1 && \text{or } \gamma 1 = (a_{11}/a_{21})\gamma \end{aligned}$$

On the other hand, the macro-function of $E1_t$ can be derived from (4.2.8):

$$(4.2.12)\quad E1_t = \theta 1\, NH1_t + \alpha 1\, YD1_t + \beta 1\, W1_t + (\gamma 1\, NH1_t)R_t$$

Substituting (4.2.11) into (4.2.12), we have

$$(4.2.13)\quad E1_t = \theta(a_{11}/a_{21})NH1_t + \alpha\, YD1_t + \beta\, W1_t + \gamma(a_{11}/a_{21})R_t$$

The intercept and the interest rate coefficient of (4.2.13) are affected by $(a_{11}/a_{21})NH1_t$. For example, if a_{11}, a_{21}, and $NH1_t$ are half of the national values, then the intercept and interest rate coefficient of $E1_t$ are equal to half the magnitudes of their national function. By the same token, for the second region, we have

$$(4.2.14)\quad E2_t = \theta(a_{12}/a_{22})NH2_t + \alpha\, YD2_t + \beta\, W2_t + \gamma(a_{12}/a_{22})NH2_t R_t$$

The same interpretation applied to other functions as well. Note also that the corresponding elasticities are the same for both micro and macro-functions in both regions. To demonstrate, we have

$$(4.2.15)\quad \begin{aligned} N(E1,YD1) &= \partial(E1/YD1)*(YD1/E1) = \alpha 1(YD1/E1) \\ &= \partial((E1/NH1)/(YD1/NH1))*((YD1/NH1)/(E1/NH1)) \end{aligned}$$

$= N(E1/NH1, YD1/NH1)$

(4.2.16) $N(E1,W1) = \partial(E1/W1)*(W1/E1) = \beta/(W1/E1)$
$= \partial((E1/NH1)/(W1/NH1))*((W1/NH1)/(E1/NH1))$
$= N(E1/NH1, W1/NH1)$

(4.2.17) $N(E1,R) = \partial(E1/R)*(R/E1) = (\gamma/NH1)*(R/E1)$
$= \gamma/(R/(E1/NH1)) = \partial((E1/NH1)/R))*(R/(E1/NH1))$
$= N(E1/NH1, R)$

Note also that the corresponding elasticities are the same for the national and interregional models.

(4.2.18) $N(E1,YD1) = \alpha/(YD1/E1) = \alpha(a_{11}YD/a_{11}E) = \alpha(YD/E) = N(E,YD)$

(4.2.19) $N(E1,W1) = \beta/(W1/E1) = \beta(a_{11}W/a_{11}E) = \beta(W/E) = N(E,W)$

(4.2.20) $N(E1,R) = \gamma/NH1(R/E1) = \gamma(a_{11}/a_{21})(a_{21}NH)(R/a_{11}E)$
$= \gamma NH(R/E) = N(E,R)$

Including the second region, we have

(4.2.21) $N(E,YD) = N(E1,YD1) = N(E2,YD2)$

(4.2.22) $N(E,W) = N(E1,W1) = N(E2,W2)$

(4.2.23) $N(E,R) = N(E1,R) = N(E2,R)$

The above discussion of micro-foundation of macro-functions applies to the IRF and IRFG models as well.

Appendix 6.1: The IRFG Model

Derivation of Short-Run Multiplier

To simplify the notation of partial derivatives, we adopt the following expressions:

(6.1.1) $E1_t = a_1+a_2YD1_t+a_3W1_t-a_4R_t$

(6.1.2) $E2_t = a_5+a_6YD2_t+a_7(M_t+B1_t+B2_t+BF_t-W1_t)-a_8R_t$

(6.1.3) $IM_t = a_9+a_{10}YD1_t+a_{11}W1_t-a_{12}R_t$

(6.1.4) $IM_t = a_{13}+a_{14}YD2_t+a_{15}(M_t+B1_t+B2_t+BF_t-W1_t)-a_{16}R_t$

(6.1.5) $W1_t-H1_t = a_{17}+a_{18}Y1_t+a_{19}W1_t-a_{20}R_t$

(6.1.6) $M_t-W1_t+H1_t = a_{21}+a_{22}Y2_t+a_{23}(M_t+B1_t+B2_t-W1_t)-a_{24}R_t$

Rewriting the government expenditure reaction functions, we have:

(6.1.7) $G1_t = G1_{t-1}(1+A1((\overline{Y1}-Y1_{t-1})/\overline{Y1})-A2(DB1_{t-1}/\overline{Y1}))$

(6.1.8) $G2_t = G2_{t-1}(1+A3((\overline{Y2}-Y2_{t-1})/\overline{Y2})-A4(DB2_{t-1}/\overline{Y2}))$

(6.1.9) $GF_t = GF_{t-1}(1+A5((\overline{Y}-Y_{t-1})/\overline{Y})-A6(DM_{t-1}+DBF_{t-1})/\overline{Y}))$

Assuming A8=A10, we have

(6.1.10) $P1_t+P2_t = P1_{t-1}+P2_{t-1}+A7((\overline{Y1}-Y1_{t-1})/\overline{Y1})+A9((\overline{Y2}-Y2_{t-1})/\overline{Y2})$

(6.1.11) $GF1_t = (P1_t/(P1_t+P2_t))GF_t$

(6.1.12) $GF2_t = (P2_t/(P1_t+P2_t))GF_t$

Substituting the above equations into the regional income identity, we have

(6.1.13) $$Y1_t = a_1+a_2YD1_t+a_3W1_t-a_4R_t+G1_{t-1}(1+A1(\overline{Y1}-Y1_{t-1})/\overline{Y1})$$
$$-A2(DB1_{t-1}/\overline{Y1}))+(P1_t/(P1_t+P2_t)GF_t-a_9-a_{10}YD1_t-a_{11}W1_t$$
$$+a_{12}R_t+a_{13}+a_{14}YD2_t+a_{15}(M_t+B1_t+B2_t+BF_t-W1_t)-a_{16}R_t$$

(6.1.14) $$dY1_t = a_2(1-u1_t-uF_t)(dY1_t+R_tdH1_t+H1_tdR_t)-a_4dR_t$$
$$+G1_{t-1}(A1(Y1_{t-1}/\overline{Y1}^2)d\overline{Y1}+A2(DB1_{t-1}/\overline{Y1}^2)d\overline{Y1})$$
$$+(P1_t/(P1_t+P2_t))GF_{t-1}(A5(Y_{t-1}/\overline{Y}^2)dY+A6(DM_{t-1}+DBF_{t-1})/\overline{Y}^2)d\overline{Y})$$
$$+GF_{t-1}(1+A5(\overline{Y}-Y_{t-1})/\overline{Y}+A6(DM_{t-1}+DBF_{t-1})/\overline{Y})(P1_t+P2_t)*$$

$(A7(\bar{Y}d\bar{Y1}-(\bar{Y1}-Y1_{t-1})d\bar{Y}/\bar{Y}^2)-A8(-(G1_{t-1}+GF1_{t-1})d\bar{Y1}/\bar{Y1}^2$

$+(G2_{t-1}+GF2_{t-1})d\bar{Y2}/\bar{Y2}^2)) + P1_t(A7((\bar{Y}d\bar{Y1}+(\bar{Y1}-Y1_{t-1})d\bar{Y})/\bar{Y}^2$

$+A9(\bar{Y}d\bar{Y2}-(\bar{Y2}-Y2_{t-1})d\bar{Y})/\bar{Y}^2))/(P1_t+P2_t)^2 -a_{10}(1-u1_t-uF_t)*$

$(dY1_t+R_tdH1_t+H1_tdR_t)+a_{14}(1-u2_t-uF_t)(dY2_t-R_tdH1_t$

$-H1_tdR_t)-a_{16}dR_t$

(6.1.15) $dY1_t(1-(a_2-a_{10})(1-u1_t-uF_t)) = a_{14}(1-u2_t-uF_t)dY2_t$

$+((a_2-a_{10})(1-u1_t-uF_t)H1_t-a_4-a_{16}+a_{12}-a_{14}(1-u2_t$

$-uF_t)H1_t)dR_t + ((a_2-a_{10})(1-u1_t-uF_t)R_t-a_{14}(1-u2_t$

$-uF_t)R_t)dH1_t + ((P1_t/(P1_t+P2_t))GF_{t-1}((A5Y_{t-1}$

$+A6(DM_{t-1}+DBF_{t-1})\bar{Y}^2)-GF_t((P1_tA7(\bar{Y1}-Y1_{t-1})+P1_tA9$

$*(\bar{Y2}-Y2_{t-1})/\bar{Y}^2(P1_t+P2_t)^2))d\bar{Y} + (G1_{t-1}((A1Y1_{t-1}+A2$

$*DB1_{t-1})/Y1^2) + GF_t((P1_tA7\bar{Y}+(P1_t+P2_t)A8(G1_{t-1}+GF1_{t-1})$

$/\bar{Y1}^2(P1_t+P2_t)^2))d\bar{Y1} + (GF_t((P1_tA9\bar{Y}/\bar{Y}^2) + ((P1_t+P2_t)$

$*A8(G2_{t-1}+GF2_{t-1})/\bar{Y2}^2)/(P1_t+P2_t)^2)d\bar{Y2}$

Using d_1, d_2, and d_3 to represent the coefficients of $d\bar{Y}$, $d\bar{Y1}$, and $d\bar{Y2}$ in (6.1.15), we then have

(6.1.16) $dY1_t = (a_{14}(1-u2_t-uF_t)dY2_t+((a_2-a_{10})(1-u1_t-uF_t)H1_t$

$-a_4-a_{16}-a_{12}-a_{14}(1-u2_t-uF_t)H1_t)dR_t+((a_2-a_{10})(1-u1_t$

$-uF_t)R_t-a_{14}(1-u2_t-uF_t)R_tdH1_t+d_1d\bar{Y}+d_2d\bar{Y1}+d_3d\bar{Y2})$

$/(1-(a_2-a_{10})(1-u1_t-uF_t))$

Applying the same method to the second region, we have

(6.1.17) $dY2_t(1-(a_6-a_{14})(1-u2_t-uF_t)) = a_{10}(1-u1_t-uF_t)dY1_t$

$-((a_6-a_{14})(1-u2_t-uF_t)H1_t+a_8+a_{12}+a_{16}-a_{10}(1-u1_t$

$-uF_t)H1_t)dR_t + (a_{10}(1-u1_t-uF_t)R_t-(a_6-a_{14})(1-u2_t$

$-uF_t)R_t)dH1_t + ((P2_t/(P1_t+P2_t))GF_{t-1}((A5Y_{t-1}$

$+A6(DM_{t-1}+DBF_{t-1})/\bar{Y}^2)-GF_t((P2_tA7(\bar{Y1}-Y1_{t-1})+P2_tA9$

$*(\bar{Y2}-Y2_{t-1})/\bar{Y}^2(P1_t+P2_t)^2)d\bar{Y} + (GF_t((P2_tA7\bar{Y}/\bar{Y}^2)$

$-((P1_t+P2_t)A10(G1_{t-1}+GF1_{t-1})/\overline{Y1}^2)/(P1_t+P2_t)^2\ )d\overline{Y1}$

$+\ GF_{t-1}((A3Y2_{t-1}+A4DB2_{t-1})/\overline{Y2}^2)\ +\ GF_t((P1_t+P2_t)$

$*(G2_{t-1}+GF2_{t-1})+P2_tA9\overline{Y})/\overline{Y}^2(P1_t+P2_t)^2)\ )d\overline{Y2}$

Using d_4, d_5, and d_6 to represent the coefficients of $d\overline{Y}$, $d\overline{Y1}$, and $d\overline{Y2}$ in (6.1.17), we then have

(6.1.18) $dY2_t = (\ a_{10}(1-u1_t-uF_t)dY1_t-((a_6-a_{14})(1-u2_t-uF_t)H1_t$

$+a_8+a_{12}+a_{16}-a_{10}(1-u1_t-uF_t)H1_t)dR_t+(a_{10}(1-u1_t-uF_t)R_t$

$-(a_6-a_{14})(1-u2_t-uF_t)R_t)dH1_t+d_4d\overline{Y}+d_5d\overline{Y1}+d_6d\overline{Y2}\ )$

$/(1-(a_6-a_{14})(1-u2_t-uF_t))$

From (6.1.5) and (6.1.6), we have

(6.1.19) $-dH1_t = a_{18}dY1_t-a_{20}dR_t$

(6.1.20) $dH1_t = a_{22}dY2_t-a_{24}dR_t$

Adding (6.1.19) and (6.1.20), we have

(6.1.21) $dR_t = (a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24})$

Substituting (6.1.21) to (6.1.20), we have

(6.1.22) $dH1_t = a_{22}dY2_t - a_{24}(a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24})$

Substituting (6.1.21) and (6.1.22) to (6.1.16), we have

(6.1.23) $dY1_t = (\ a_{14}(1-u2_t-uF_t)dY2_t+((a_2-a_{10})(1-u1_t-uF_t)H1_t$

$-a_4-a_{16}-a_{12}-a_{14}(1-u2_t-uF_t)H1_t)(a_{18}dY1_t+a_{22}dY2_t)$

$/(a_{20}+a_{24})\ +\ ((a_2-a_{10}(1-u1_t-uF_t)R_t-a_{14}(1-u2_t-uF_t))*$

$(a_{22}dY2_t-a_{24}(a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24}))$

$+d_1d\overline{Y}+d_2d\overline{Y1}+d_3d\overline{Y2}\ )/(1-(a_2-a_{10})(1-u1_t-uF_t))$

Using the "e'i" to represent the above bracketed coefficients, we have

(6.1.24) $e_1 = 1-(a_2-a_{10})(1-u1_t-uF_t)$

(6.1.25) $e_2 = a_{14}(1-u2_t-uF_t)$

(6.1.26) $e_3 = (a_2-a_{10})(1-u1_t-uF_t)H1_t-a_4-a_{16}-a_{12}-a_{14}(1-u2_t-uF_tH1_t)$

(6.1.27) $e_4 = a_2-a_{10}(1-u1_t-uF_t)R_t-a_{14}(1-u2_t-uF_t)$

Substituting the above e'i coefficients into (6.1.23), we have

(6.1.28) $dY1_t = (e_2dY2_t+e_3(a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24})$
$+e_4(a_{22}dY2_t-a_{24}(a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24}))$
$+d_1d\overline{Y}+d_2d\overline{Y1}+d_3d\overline{Y2})/e_1$

Grouping the terms, we have

(6.1.29) $dY1_t = ((e_2+(e_3a_{22}-e_4a_{22}a_{24})/(a_{20}+a_{24}))dY2_t$
$+d_1d\overline{Y}+d_2d\overline{Y1}+d_3d\overline{Y2})/(e_1-(e_3a_{18}-e_4a_{18}a_{24})/(a_{20}+a_{24}))$

Using the "f'i" to represent the above bracketed coefficients, we have

(6.1.30) $f_1 = e_2 +(e_3a_{22}-e_4a_{22}a_{24})/(a_{20}+a_{24})$

(6.1.31) $f_2 = d_1/(e_1-(e_3a_{18}-e_4a_{18}a_{24})/(a_{20}+a_{24}))$

(6.1.32) $f_3 = d_2/(e_1-(e_3a_{18}-e_4a_{18}a_{24})/(a_{20}+a_{24}))$

(6.1.33) $f_4 = d_3/(e_1-(e_3a_{18}-e_4a_{18}a_{24})/(a_{20}+a_{24}))$

Sunstituting the above f's coefficients into (6.1.29), we have

(6.1.34) $dY1_t = f_1dY2_t+f_2d\overline{Y}+f_3d\overline{Y1}+f_4d\overline{Y2}$

For the second region, we have

(6.1.35) $dY2_t = (a_{10}(1-u1_t-uF_t)dY1_t-((a_6-a_{14})(1-u2_t-uF_t)H1_t$
$+a_8+a_{12}+a_{16}-a_{10}(1-u1_t-uF_t)H1_t)(a_{18}dY1_t+a_{22}dY2_t)$
$/(a_{20}+a_{24}) + (a_{10}(1-u1_t-uF_t)R_t-(a_6-a_{14})(1-u2_t-uF_t)R_t)*$
$(a_{22}dY2_t-a_{24}(a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24}))$
$+d_4d\overline{Y}+d_5d\overline{Y1}+d_6d\overline{Y2})/(1-(a_6-a_{14})(1-u2_t-uF_t))$

Using the "e'i" to represent the bracketed coefficients, we have

(6.1.36) $e_5 = 1-(a_6-a_{14})(1-u2_t-uF_t)$

(6.1.37) $e_6 = a_{10}(1-u1_t-uF_t)$

(6.1.38) $e_7 = (a_6-a_{14})(1-u2_t-uF_t)H1_t+a_8+a_{12}+a_{16}-a_{10}(1-u1_t-uF_t)H1_t$

(6.1.39) $e_8 = a_{10}(1-u1_t-uF_t)R_t-(a_6-a_{14})(1-u2_t-uF_t)R_t$

Substituting the above e's coefficients into (6.1.35), we have

(6.1.40) $dY2_t = (e_6dY1_t+e_7(a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24})$

$+e_8(a_{22}dY2_t-a_{24}(a_{18}dY1_t+a_{22}dY2_t)/(a_{20}+a_{24}))$

$+d_4d\overline{Y}+d_5d\overline{Y1}+d_6d\overline{Y2}\ )/e_5$

Grouping the terms, we have

(6.1.41) $dY2_t = (\ (e_6+(e_7a_{18}-e_8a_{18}e_{24})/(a_{20}+a_{24}))dY1_t$

$+d_4d\overline{Y}+d_5d\overline{Y1}+d_6d\overline{Y2}\ )/(e_5+(e_7a_{22}+e_8a_{20}a_{22})/(a_{20}+a_{24})\ )$

Using the "f'i" to represent the above coefficients, we have

(6.1.42) $f_5 = e_6+(e_7a_{18}-e_8a_{18}e_{24})/(a_{20}+a_{24})$

(6.1.43) $f_6 = d_4/(e_5+(e_7a_{22}+e_8a_{20}a_{22})/(a_{20}+a_{24}))$

(6.1.44) $f_7 = d_5/(e_5+(e_7a_{22}+e_8a_{20}a_{22})/(a_{20}+a_{24}))$

(6.1.45) $f_8 = d_6/(e_5+(e_7a_{22}+e_8a_{20}a_{22})/(a_{20}+a_{24}))$

Substituting the above f's coefficents into (6.1.40), we have

(6.1.46) $dY2_t = f_5dY1_t+f_6d\overline{Y}+f_7d\overline{Y1}+f_8d\overline{Y2}$

From (6.1.34) and (6.1.41), we have

(6.1.47) $f_5dY1_t = f_1f_5dY2_t+f_2f_5d\overline{Y}+f_3f_5d\overline{Y1}+f_4f_5d\overline{Y2}$

(6.1.48) $f_1dY2_t = f_1f_5dY1_t+f_1f_6d\overline{Y}+f_1f_7d\overline{Y1}+f_1f_8d\overline{Y2}$

Adding (6.1.41) and (6.1.46), we have

(6.1.49) $(1-f_1f_5)dY2_t = (f_6+f_2f_5)d\overline{Y}+(f_7+f_3f_5)d\overline{Y1}+(f_8+f_4f_5)d\overline{Y2}$

Therefore, we have

(6.1.50) $dY2_t/d\overline{Y} = (\partial Y2_t/\partial GF_t)(\partial GF_t/\partial\overline{Y}) = (f_6+f_2f_5)/(1-f_1f_5)$

(6.1.51) $dY2_t/d\overline{Y1}= (\partial Y2_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})= (f_7+f_3f_5)/(1-f_1f_5)$

(6.1.52) $dY2_t/d\overline{Y2}= (\partial Y2_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})= (f_8+f_4f_5)/(1-f_1f_5)$

Adding (6.1 26) and (6.1.31), we have

(6.1.53) $dY1_t/d\overline{Y} = (\partial Y1_t/\partial GF_t)(\partial GF_t/\partial\overline{Y}) = (f_2+f_1f_6)/(1-f_1f_5)$

(6.1.54) $dY1_t/d\overline{Y1}= (\partial Y1_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})= (f_3+f_1f_7)/(1-f_1f_5)$

(6.1.55) $dY1_t/d\overline{Y2}= (\partial Y1_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})= (f_4+f_1f_8)/(1-f_1f_5)$

Substituting the above partial multipliers into (6.1.21), we have

(6.1.56) $dR_t/d\overline{Y} = (\partial R_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

$= ((a_{18}(f_2+f_1f_6)+a_{22}(f_6+f_2f_5))/(a_{20}+a_{24})(1-f_1f_5)$

(6.1.57) $dR_t/d\overline{Y1} = (\partial R_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$

$= ((a_{18}(f_3+f_1f_7)+a_{22}(f_7+f_3f_5))/(a_{20}+a_{24})(1-f_1f_5)$

(6.1.58) $dR_t/d\overline{Y2} = (\partial R_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})$

$= ((a_{18}(f_4+f_1f_8)+a_{22}(f_8+f_4f_5))/(a_{20}+a_{24})(1-f_1f_5)$

Substituting the above partial multipliers into (6.1.22), we have

(6.1.59) $dH1_t/d\overline{Y} = (\partial H1_t/\partial GF_t)(\partial GF_t/\partial\overline{Y})$

$= ((a_{20}a_{22}(f_6+f_2f_5)-a_{18}a_{24}(f_2+f_1f_6))/(a_{20}+a_{24})(1-f_1f_5)$

(6.1.60) $dH1_t/d\overline{Y1} = (\partial H1_t/\partial G1_t)(\partial G1_t/\partial\overline{Y1})$

$= ((a_{20}a_{24}(f_7+f_3f_5)-a_{18}a_{24}(f_3+f_1f_7))/(a_{20}+a_{24})(1-f_1f_5)$

(6.1.61) $dH1_t/d\overline{Y2} = (\partial H1_t/\partial G2_t)(\partial G2_t/\partial\overline{Y2})$

$= ((a_{20}a_{24}(f_8+f_4f_5)-a_{18}a_{24}(f_4+f_1f_8))/(a_{20}+a_{24})(1-f_1f_5)$

Bibliography

Auld D. "Counter-Cyclical Budget Effects in Ontario:Some Preliminary Evidence", Canadian Tax Journal , XXVIII (1975), 173-183.

Auld D. "Decentralized Fiscal Policy", paper read at Canadian Economics Association Meeting, Quebec City,1976.

Auld D. "Decentralizing Fiscal Policy" in Canadian Confederation at the Crossroads . edited by M. Walker, Vancouver: The Fraser Institute, 1979, 269-283.

Barber C. Theory of Fiscal Policy as Applied to a Province . Toronto: Queen's Printer, 1966.

Barth H. "Potential-Oriented Debt, The Concept of the German Council of Economic Experts", German Economic Review , (1977), 12-22.

Blinder A. and Solow R. "Does Fiscal Policy Matter?", Journal of Public Economics , II (1973), 319-337.

Blinder A. and Solow R. "Analytical Foundations of Fiscal Policy", in The Economics of Public Finance . Washington, D.C.: The Brookings Institute, 1974, 3-118.

Blinder A. and Solow R. "Does Fiscal Policy Matter? A Correction", Journal of Public Economics , V (1976), 183-184.

Buchanan J. "Federalism and Fiscal Equity", American Economic Review , XL (1950), 583-599.

Buiter W. "Capacity Constraints, Government Financing and the Short Run and Long Run Effects of Fiscal Policy", mimeo, Princetion University (1976).

Buiter W. "An Integration of Short Run Neo-Keynesian Analysis and Growth Theory", De Economist , Vol.125, No.3 (1977), 340-359.

Chand S. "Summary Measures of Fiscal Influences", Staff Papers, International Monetary Fund , XXIV, No.2 (1977), 405-449.

Chow G. "On the Long-Run and Short-Run Demand for Money", Journal of Political Economy , LXXIV (1966), 111-131.

Christ C. "A Short-Run Aggregate Demand Model of the Interdependence and Effects of Monetary and Fiscal Policies With Keyensian and Classical Interest Elasticies", American Economic Review , LVII (1967), 434-443.

Christ C. "A Simple Macroeconomic Model With a Government Budget Restraint", Journal of Political Economy , LXXVI (1968), 53-67.

Christ C. "Some Dynamic Theory of Macroeconomic Policy Effects on Income and Prices Under the Government Budget Restraint", Journal of Monetary Economics , IV (1978), 45-70.

Christ C. "On Fiscal and Monetary Policies and the Government Budget Restraint", American Economic Review , LXIX (1979), 526-538.

Christofides L. "The Federal Government Budget Contraints, 1955-1975", Canadian Public Policy , III (1977), 291-298.

Courchene T. "The Transfer System and Regional Disparities", in Canadian Confederation at the Crossroads , edited by Walker M., Vancouver: The Fraser Institute, 1979, 145-186.

Dernburg T. F. and Dernburg J. D. Macroeconomic Analysis: An Introduction to Comparative Statics and Dynamics , Reading, Mass: Addison-Welsey, 1969.

Dernburg T. F. "Fiscal Analysis in the Federal Republic of Germany: The Cyclical Neutral Budget", Staff Papers, International Monetary Fund , XXII (1975), 825-857.

Dolde W. "Monetary and Fiscal Policy in a Consistent Dynamic Model-The Intermediate Run", Board of Governors of the Federal Reserve System, Washington: 1975.

Engerman S. "Regional Aspects of Stabilization Policy", in Essays in Fiscal Federalism , R. Musgrave (editor), Washington: The Brookings Institute, 1966, 7-63.

Foot D., Pesando J., Sawyer J., and Winder J. The Ontario Economy 1977-1987 . Toronto: Ontario Economic Council, 1977, 39-40.

Hansen A. and Perloff H. State and Local Finance in the National Economy New York: Norton, 1944.

Holbrook R. "Optimal Economic Policy and the Problem of Instrument Instability", American Economic Review , LXII (1972), 57-65.

Infante E. and Stein J. "Does Fiscal Policy Matter?", Journal of Monetary Economics , II (1976), 473-500.

Jianakoplos N. "The Growing Link Between the Federal Government and State and Local Government Financing", Federal Reserve Bank of St. Louis , LIX (1977), 2-8.

Jones B., Bardecki N., and Hull B. "Regional Stabilization Policy in Canada-The Ontario Record", mimeo, Taxation and Fiscal Policy Branch, Queen's Park, Toronto: 1977.

McKeough D. Ontario Budget 1978 , Toronto: Ministry of Treasury, Economics and Intergovernmental Affairs, Fiscal Policy Division, 1978, 25-27.

McKinnon R. and Oates W. "The Implications of International Economic Integration for Monetary, Fiscal, and Exchange Rate Policy", Princetion Studies in International Finance No.16 , 1966.

Musgrave R. The Theory of Public Finance . New York: McGraw-Hill, 1959.

Musgrave R. Essays in Fiscal Federalism . (editor) Washington: The Brookings Institute, 1966.

Nguyen D. and Turnovsky S. "Monetary and Fiscal Policies in An Inflationary Economy, A Simulation Approach", Journal of Macroeconomics , I (1979), 259-283.

Oates W. "The Theory of Public Finance in a Federal Setting", Canadian Journal of Economics , I (1968), 37-54.

Oates W. "Budget Balance and Equilibrium Income: A Comment of the Efficiency of Fiscal and Monetary Policy in an Open Economy", Journal of Finance , XXI (1966), 489-498.

Oates W. Fiscal Federalism . New York: Harcourt Brace Jovanovich, 1972.

Ott D. and Ott A. "Budget Balance and Equilibrium Income", Journal of Finance , XX (1965), 71-77.

Pyle D. and Turnovsky S. "The Dynamics of Government Policy in an Inflationary Economy", Journal of Money, Credit, and Banking , VIII (1976), 411-437.

Rafuse Jr. "Cyclical Behavior of State-Local Finances", in Essays in Fiscal Federalism , R. Musgrave (editor), Washington: The Brookings Institute, 1966, 64-115.

Robinson T. and Courchene T. "Fiscal Federalism and Economic Stability, An Examination of Multi-Level Public Finance in Canada, 1952-1965", Canadian Journal of Economics , II (1969), 165-189.

Scarth W. "The Financing of Stabilization Policies: Evidence for the Canadian Economy", Canadian Journal of Economics , VI (1973), 301-318.

Scarth W. "Fiscal Policy and the Government Budget Constrant under Alternative Exchange Rate Systems", Oxford Economic Papers , XXVII (1975), 10-20.975).

Scarth W. "A Note on the 'Crowding Out of Private Expenditures by Bond-Financing Increases in Government Spending", Journal of Public Economics , V (1976), 385-387.

Scarth W. "The Government Budget Constraint in An Open Economy: A Further Comment", Oxford Economic Papers , XXIX (1977), 145-151.

Scarth W. "Bond-Financed Fiscal Policy and the Problem of Instrument Instability", Journal of Macroeconomics , I (1979), 107-117.

Statistics Canada. System of National Accounts, National Income and Expenditure Accounts , Ottawa: Catalogue 13-201 Annual, 1962-1976.

Swan N. "Theory of Fiscal Policy in a Region", mimeo, Ottawa: Department of Regional Economic Expansion, June, 1977.

Tobin J. "A General Equilibrium Approach to Monetary Theory", Journal of Money, Credit and Banking , I (1969), 15-29.

Tobin J. and Buiter W. "Long-Run Effects of Fiscal and Monetary Policy on Aggregate Demand", in J. Stein (editor), Monetarism , Amsterdam: North-Holland, 1974.

Trebilcock M., Gordon K., and Prichard J. "Restrictions on the Interprovincial Mobility of Resources: Goods, Capital and Labour, an Illustrative Survey", in Intergovernmental Relations , Toronto: Ontario Economic Council, 1979, 101-122.

Turnovsky S. Macroeconomic Analysis and Stabilization Policy . Cambridge: University Press, 1977.

Turnovsky S. "Alternative Passive Monetary Policies in An Inflationary Economy", Journal of Macroeconomics , I (1979), 33-63.

Wilson T. "The Province and Stabilization Policy", in Intergovernmental Relations , Ontario Economic Council, 1977, 123-136.

Winer S. and Sheikh M. "Stabilization and Nonfederal Behavior in an Open Federal State: An Econometric Study of the Fixed Exchange Rate, Canadian Case", Empirical Economics , II (1977), 195-211.

Winer S. "Short Run Monetary-Fiscal Influences in a Federal State: With Applications to the Canadian Economy, 1947 to 1973", Public Finance Quarterly , VII (1979), 395-424.

Yamane T. Mathematics For Economists . Englewood Cliffs., New Jersey: Prentice-Hall, 1962.