

PARADOX AND THE PROBLEM OF CLARITY IN WITTGENSTEIN'S TLP

PARADOX
AND
THE DISSOLUTION OF THE PROBLEM OF CLARITY
IN
WITTGENSTEIN'S TRACTATUS LOGICO-PHILOSOPHICUS

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Abstract

The present thesis emphasizes one among many themes developed in Wittgenstein's TLP, namely, the elucidation of the symbolisms of colloquial languages and logically canonical notations. In accordance with this emphasis, I have read the Tractatus as providing initially a solution and ultimately an attempted dissolution of one key philosophical problem, what I have called the problem of clarity:

PC'

The problem as to how the symbols of a language must stand to one another and to the rest of reality so that what can be said can be said clearly.

The argument of the thesis, then, consists of two parts. The first shows how Wittgenstein's pictorial account of propositions proffers a solution to PC' under the constraint of the following three main assumption:

Realism Thesis (RT)

The meanings of symbols have being: they are either eternal, or temporal entities.

Independence Thesis (IT)

The being of entities does not - at least not without exception - depend upon being meant in language or thought.

A2 (Adequacy Thesis)

The canons of grammar and diction of colloquial languages are adequate for determining the canons of clarity (that is, the canons with respect to which what is said is counted as clearly or unclearly said), where these canons are directly

understood to be logical canons.

The second shows how through his criticisms - under RT and IT - of Russell's theory of types and theories of judgement, Wittgenstein commits himself and his sympathetic reader to an ultimate distinction between what can be said and what can be shown. This distinction directly gives rise to the paradox of TLP that according to the elucidation Wittgenstein gives of sentences with sense, the sentences of TLP must themselves lack sense. Accordingly, it is through his commitment to this paradox that Wittgenstein ceases to recognize PC', and other philosophical problems, as questions deserving of an answer in terms of propositions. This is how the problem of clarity purportedly receives its dissolution, at least when this problem is taken under the assumption of RT, IT and the assumption of the contingency of all facts.

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Introduction: The Problem of Clarity

0.0 In the Tractatus Logico-Philosophicus (TLP), Wittgenstein introduces a distinction between what can be said and what can be shown in language, a distinction according to which one cannot say or say anything about what the signs for thoughts can show. But, of course, in the process of introducing this distinction, what Wittgenstein writes in TLP are sentences - propositional signs of a language. So, Wittgenstein's very explanation of the meaning of sentences seems to lead directly into a paradox. It provokes a question as to how the sentences in TLP themselves show what, according to the preface, they are supposed to show, namely:

What can be said at all can be said clearly: and whereof one cannot speak thereof one must be silent.

For, if the distinction between saying and showing in TLP is supposed to be correct, then what the sentences in TLP are also supposed to show is that they do not say anything true (or false for that matter). Thus, the understanding of language and the world that TLP is supposed to evoke does not, contrary to appearances, consist in or depend upon an understanding of propositions, for by Wittgenstein's distinction there are no propositions in TLP to be understood. This consequence constitutes what I will call the paradox of

TLP, for it suggests a paradox that, according to TLP, no account can be provided of how the sentences in TLP show what they are supposed to show.

Though I think this result is unacceptable, I do not think that it directly presents a *reductio* of Wittgenstein's distinction between saying and showing, however paradoxical and ironical that distinction may thereby be made to appear. More explicitly, I do not think that one can reasonably avoid the result simply by rejecting Wittgenstein's distinction without appreciating how his explanation as a whole bears upon a set of problems for which the distinction - as introduced and elaborated in TLP - was taken to provide a solution or dissolution. For (1) only with respect to such an appreciation can one understand what remains open to question or how to ask those questions, and (2) once one appreciates this, one can better appreciate the assumptions with respect to which Wittgenstein's distinction appears compelling, even though one may ultimately come to understand that in many respects the distinction or the assumptions themselves are mistaken.

For the most part, the problems that Wittgenstein worked upon, through his Notebooks and the composition of TLP, to their 'dissolution' in the final sentence of the latter book were problems concerning the canons for clarity of assertion, whether assertion in language or thought. This concern is not

only evinced by what is quoted above from Wittgenstein's Preface but also by what Wittgenstein writes within TLP, for instance at 4.116. Though these problems arise in the works of Frege and Russell with an emphasis on epistemological as well as logical concerns, in Wittgenstein's work the emphasis is almost exclusively logical. Accordingly for Wittgenstein, as well as for Frege (and Russell), the canons with respect to which what is said is ultimately counted as clearly or unclearly asserted are logical canons.

Before Wittgenstein's pictorial account of propositions is examined, then, and before Wittgenstein's commitment to an ultimate rather than a relative distinction between saying and showing is exposed, the problems for which Wittgenstein's account was supposed to provide a solution ought to be discussed. This examination will be the task set for the present chapter. In the course of this task, one problem in particular will be examined, namely:

The Problem of Clarity (PC)

The problem as to what conditions a language must satisfy in order that what can be said can be said clearly within it.

For Wittgenstein, what counts as a clear assertion is so counted ultimately with respect to logical canons, which are supposed to express universal laws. These laws, moreover, are supposed to govern the ways in which the truth-value of *what*

is asserted directly depends upon the form of composition and the content of the assertion. But, as shall be seen momentarily, for Wittgenstein, universal rules which lack a foundation in that which has being independently of what is the case can only be accidentally universal and can only be nominally laws. Thus, Wittgenstein is committed to the following stronger version of the problem:

The Problem of Clarity' (PC*)

The problem as to how the symbols of a language must stand to one another and to the rest of reality so that what can be said can be clearly said within it.

A response that would resolve this problem would supposedly show -in the sense of elucidate - how a language could be devised and used so that no logical errors could be committed within it, if one understands or recognizes the canons of grammar and diction of that language. The symbols of such a language would supposedly stand to one another and the rest of reality in relations adequate for clear expression.

Now, reality, for Wittgenstein, consists of whatever has being, and what has being can be understood through the use of language. But language consists - roughly speaking - of signs and routines, routines the following of which results in the uttering of a combination of signs so that a truth or a falsehood is asserted. Since what is asserted or assertible is supposed to comply with essentially universal laws, the

universal validity of these laws must depend directly upon some unchanging reality that can be correctly understood through or in language. Accordingly, the above problem PC* arises subject to the constraint of the following assumptions:

Realism Thesis (RT)

The meanings of symbols have being: they either subsist or exist.¹

Independence Thesis (IT)

What has being does not - at least not without exception - depend for its being, or the ways in which it stands to other entities, upon its being meant by a symbol in language.

Apart from citing passages in TLP which explicitly commit Wittgenstein to both RT and IT (eg. 2.012 - 2.0121 and 2.021 - 2.0231), I argue that unless Wittgenstein were to accept both RT and IT, the universal validity of what the logical canons of a particular language are supposed to express would, for him, be put in question. That is, if IT, for instance, were not correct, then the way in which a propositional symbol depended for its truth-value would depend directly upon the logical canons of *its* language. Since languages develop

¹ In the course of this thesis, the term 'meaning' will be given a very special use. In order to have a neutral description of Wittgenstein's, Frege's and Russell's respective positions, the meanings of symbols will be supposed to have at most two components, sense and reference, and at least one, reference. However these components are conceived, the realist thesis will be held to apply to all components of meaning. That is, if a symbol is supposed to have both sense and reference, then by RT both of these will be supposed to have being, though one may subsist while the other exists.

contingently, the signs and routines of languages and so their logical canons may differ. Since, by the above assumption, canons express what is dependent upon them, there would be no necessity that, notwithstanding the differences in signs and routines, the logical canons of two disparate languages would express the same set of logical laws.

But even before this latter argument is presented, some more background to the problem needs to be developed. For these purposes, what would be especially revealing is a contrast between how PC arises for Frege and Russell and how it arises for Wittgenstein. What I hope to show is how for Frege and Russell the problem arises subject to considerations of the logical deficiencies of colloquial languages, while for Wittgenstein it is addressed subject to the conviction that colloquial languages are logically adequate. In this regard, one case of logical error that purportedly has language as its source will be outlined, namely, the case of the logical errors - whatever they might be - that occasion Russell's paradox. Wittgenstein will have to show how language can be understood as logically adequate, notwithstanding that such logical errors appear to arise within it.

0.1 Though many of the problems Wittgenstein inherited from Frege and Russell were problems of an epistemological character, PC presents a problem that gives some unity to the others. It is one that descends most especially from the

logical works of Frege and Russell.

Within their respective works, both Frege and Russell, sought to show how all truths expressed with an arithmetical or, more generally, a mathematical vocabulary can be expressed within a language consisting solely of a logical vocabulary. Such a language would be one within which whatever could be said about mathematical items could be said clearly and rigorously supported. Though the initial aims for the development of such a language was for the clear expression and rigorous support of, say, arithmetical truths, it was also supposed to set the canons for clarity and rigor within any language. The problem, PC*, arises then, from the need to vindicate these canons. Accordingly, it arises for both Frege and Russell subject to the rather stringent constraint imposed by the following assumption:

A1 It is ultimately with respect to the canons of classical logic that what is said is counted as clearly or as unclearly said.

Before continuing with the main discussion, the notion of a canon, whether of a logical or non-logical variety, deserves some explanation. For the purposes of this exposition, it is at least prudent to adopt an understanding of the orientation or force of utterances within a language which is at least compatible with Wittgenstein's and Frege's accounts of language. Unfortunately, both, especially the early

Wittgenstein, adopted a rather confining account of the varieties of orientation of our utterances which constitute a language or a community of speakers of one language. For both Frege and Wittgenstein, it was the orientation towards the assertion of truths which constituted a language. To this orientation, other moods such as the interrogative and hypothetical were accordingly supposed to be subordinated. Consider, for example, what Wittgenstein writes at 4.001:

The totality of propositions is language.

Here, Wittgenstein is betraying his bias in favour of the indicative mood of speech. For him, a language, though it may consist of other types of sentences, is dominated by utterances of indicative sentences. Frege exhibits similar commitments in his account of the content of judgments, assumptions and questions, for, according to that account, no cognitive content or value is introduced through assumptions or questions that cannot be expressed through assertion. What distinguishes assertion is what he called the movement towards a truth-value. Though in both Frege and Wittgenstein's cases the bias in favour of the assertoric is, I think, integral or akin to their respective biases in favour of what has being, that is their realism, I will be allowing in the following discussion that language, as Wittgenstein wants to elucidate it, is assertion or theory driven.

Given the above caveat, languages may be assumed to consist of signs and routines of sign combination the following of which results in an assertion or an array of assertions for speakers of that language. Here, by routine, I simply mean the sequence or array of types of events which result in the writing or utterance of marks or sounds. Descriptions of the types of events that constitute a routine may range from including descriptions of only the sequence of events that result in a type of sound to including descriptions of the utterers actions and interactions leading up to and following the utterance. This range of description is left open for the exposition of Wittgenstein's account of how the symbols of a language can result from our activity. What is at issue in allowing for a diversity of descriptions that may initially be involved in explaining symbolization is the account of the contexts with respect to which symbols and signs result from the combination of marks or sounds. More especially, the account of the events leading up to the production of linguistic symbols and signs - e.g. propositions and propositional signs - will be spoken of as the account of the background of the language - or of a set of propositions in the language.

Symbols taken with the routines which result in those symbols will be understood to exemplify rules for the reiteration of symbols which count as the same sign or as

instances of the same sign. Thus, when the background of a symbol is understood, a series or array of marks or sounds may be presented as exemplifying rules the following of which should result in an assertion or an argument in the language. Such an array will be called a paradigm of those rules, since it is with respect to that example that one comes to recognize those or other cases in which those rules can be or are applied. The canons of a language, then, are ordered pairs of rules and their respective paradigms, where the logical canons are of two sorts, one of which is subordinated to the other.

The first consists of sequences of symbols which exemplify rules the following which results in meaningful terms, adjectives, predicates and sentences within the language. Most, if not all, of the rules exemplified and formulated within these canons more directly constitute *some* of the grammatical canons in the language, and more especially concern how signs can be correctly formed within the language.

Since canons of grammar are usually established with respect to a background of meaningful occurrences of marks and sounds, grammatical canons are inseparable from canons which concern how such signs can be given a meaningful and not just grammatically correct occurrence within certain contexts (both linguistic and extra-linguistic contexts being included). These I will be calling canons of diction. Whether these canons may constitute an irreducible or ineliminable sort of

logical canon I will leave open. Moreover, it is not assumed, here, that all the canons of grammar (or diction) count as logical canons of the first sort. Rather, a canon of grammar within a language will count as a logical canon only if it provides for the formation of canons of the second sort, that is, only if it exemplifies rules presupposed in some logical canon of the second sort.

The second sort of canons consist of arrays of sentences which exemplify rules the following of which results - at best without exception - in the rigorous support of a proposition together with some formulations of those rules. The support of an assertion can, for present purposes, be said to be rigorous just in case every step in the routine directly counts as truth-preserving.²

Canons of grammar count as logical canons of the first sort only in so far as they adequately provide for canons of the second sort. In particular, a sentence with sense counts as logically canonical just in case the sentence *can* be a constituent of a logical canon of the second sort in the language of the sentence. The utterance of a logically canonical sentence is supposed to be able to clearly assert a truth (or falsehood), since the sentence's structure is

² The above distinction roughly corresponds to the distinction between formation rules within a language and transformation, or derivation, rules within the logical theory.

supposed to be such that it uniquely determines how what is said depends for its truth-value upon its content and the form of its composition. According to this account, standards of clarity are ultimately set or constrained by standards of rigour.

Though the above presents a precise and concise account of how the problem PC arises for Frege and Russell, it is important for the appreciation of Wittgenstein's response to Russell and Frege to understand more fully other assumptions from which it does arise. For these purposes, Frege's case is especially telling.

Within his Begriffsschrift, Frege sets the problem for the improvement of the routines the following of which ought to result in the recognition of what is true. Of these routines, the ones that were supposed to be essentially universal, since they were supposed to be determined by what Frege later called the laws of truth³, were logical methods. Moreover, these methods were supposedly in need of improvement

³ G. Frege, "Thoughts" Logical Investigations. (Library of Philosophy and Logic.) Edited with a Preface by P. Geach. Translated by P. Geach and R. H. Stoothoff. New Haven: Yale University Press, 1977, 1-30. The laws of truth are the laws of logic: more explicitly the laws with respect to which the truth-values of propositions depend upon the composition of their senses (the ways in which and the contents out of which the senses are composed). Since this applies to both simple and compound thoughts, both formation and transformation laws can be derived from these laws.

due to what Frege took to be the deficiencies of (colloquial) languages, especially as concerns the ambiguity and lack of explicitness of their respective grammatical rules. For instance, in "On the Scientific Justification of a Conceptual Notation," Frege writes as follows concerning these inadequacies:

Language is not governed by logical laws in such a way that mere adherence to grammar would guarantee the formal correctness of thought processes.⁴

To the lack of guarantees of formal correctness, Frege adds - one page later - the lack of explicitness as a further deficiency of (colloquial) languages. He writes:

In [ordinary] language, logical relations are almost always hinted at - left to guessing, not actually expressed.⁵

The remedy for these deficiencies might be initially thought to consist in the development of a separate notation for the expression of logical laws, a notation with respect to which arguments in colloquial language could be evaluated for their validity or invalidity. But, for Frege, such a remedy would not sufficiently redress the deficiencies inherent in

⁴ Frege, Gottlob. Conceptual Notation and Related Articles. Edited and Translated by T.W. Bynum. Oxford: Clarendon Press, 1972, 85.

⁵ Ibid, 86.

colloquial languages, for

Logical rules [externally applied] furnish little protection, as is shown by examples from disciplines in which the means of checking [for validity] fail.⁶

Rather, according to Frege, what was needed was 'a system of symbols {Ganzes von Zeichen} from which every ambiguity is banned, which has a strict logical form from which the content cannot escape.'⁷ This need, of course, itself presents the philosophical problem formulated as PC^{*}. For only with respect to an understanding of the logical requirements of clarity could one assure oneself that one's symbol system (or language) satisfies these requirements.

For the purpose of this section, the important point raised by Frege's case is that the problem, PC^{*}, arises for him subject to A1, and that it does so through his consideration of the deficiencies of colloquial languages. The recognition of these deficiencies commits him to the possibility of a language that was - in a rather narrow sense - logically canonical, that is, a language within which 'the mere adherence to grammar would guarantee the formal correctness of thought processes'.⁸ Though in Russell's case

⁶ Ibid, 86.

⁷ Ibid, 86.

⁸ Ibid, 85.

this point is less pronounced, a similar point still holds. For instance, in providing an overview of Principia, Russell writes in The Monist:

It [Principia] aims at being that sort of a language that, if you add a vocabulary, would be a logically perfect language.⁹

The reason that this point is so important is that though PC' is a problem for Wittgenstein and though it is so subject to the constraint of A1, Wittgenstein does not agree with either Frege or Russell concerning the deficiencies of ordinary language. Rather, PC' arises for Wittgenstein with respect to an opposing assumption, one for which his pictorial account of propositions can best be read as an attempted vindication; namely,

A2 Whatever counts as an assertion in language already stands in such a relation to reality that it complies with the logical requirements of clarity.

That is, that the marks (or sounds) that are written (or spoken) in accordance with the canons (grammar, diction, etc.) of particular languages can be written (or spoken) so as to say clearly what can be said.

In TLP, Wittgenstein expresses his commitment to A2 (and A1) in several sentences. Consider, for instance, what he

⁹ Bertrand Russell, 'The Philosophy of Logical Atomism', The Monist, 28 (1918), 520.

writes at 5.5563:

All propositions of our colloquial language are actually, just as they are, logically completely in order.

and earlier at 4.002:

Man possesses the capacity of constructing languages, in which every sense can be expressed, without knowing how and what each word means - just as one speaks without knowing how the single sounds are produced.

Both of these sentences (but especially the first) express the assumption that whatever counts as a language could not so count unless it were logically adequate, though not necessarily in the above or Frege's sense, logically canonical. One may have some doubts about whether the second sentence expresses such an assumption. The languages which Wittgenstein says man can construct and in which every sense can be expressed might be taken exclusively to be logically canonical. But, since Wittgenstein allows that these languages may be constructed by us without our knowing how each word means, it must be allowed that these languages need not be canonical, since in the case of such ignorance, the language would not guarantee the kind of explicit recognition of the true for which Frege thought a canonical language was designed.

The distinction between logical adequacy and canonicity

is one that will have to be elaborated later.¹⁰ But for present purposes, the distinction can be characterized roughly as follows:

Adequacy

A language is logically adequate if for whatever can be said there is a logically canonical expression in the language.

Canonicity

A language is logically canonical 1 if it is logically adequate and 2 if it is such that every propositional sign in the language is logically canonical.¹¹

According to these rough characterizations, the difference between adequacy and canonicity is the difference in the extent to which the rules of diction and grammar determine clarity of expression in a language. Canonicity is explicitly maximal in this respect. That is, a language is either canonical or it is not canonical - there are no degrees of canonicity. Adequacy is, if maximal, only implicitly so; that is, some further argumentation would be required to show that a language is adequate just in case it is logically canonical. But, given the earlier account of logical canons, what should be made clear, here, is that canonicity is a

¹⁰ See Appendix 1

¹¹ For Wittgenstein, the meaning consists of the sign's reference and sense. Of course, Frege similarly distinguishes between two components of the meaning of a symbol, sense and reference. But, for Frege, the reference of a propositional sign with meaning is a truth-value and the sense a composite of the senses of the symbols that occur in it.

requirement for the rigorous support of truths within a language, which passes as an epistemological as well as a logical concern¹². As we shall see in Chapter 4, for Wittgenstein, rigor is not what is required in languages.

According to the preceding exposition, the following contrast between Frege (and Russell) and Wittgenstein respectively on PC can now be more succinctly presented. For Frege (and Russell), since colloquial languages are not logically canonical, they are deficient for the purposes of rigor and so clarity. But, for Wittgenstein, notwithstanding that colloquial languages are not logically canonical, they are logically adequate and so sufficient for the purposes of clarity (if not rigor). Accordingly, in the case of Frege and Russell, the problem posed in PC is to show how, notwithstanding the deficiencies of colloquial languages, enough sentences do stand to reality in such a manner as to clearly express what they express, and so to exemplify the canons of classical logic. But, in the case of Wittgenstein, the problem is to show how, notwithstanding the ambiguity and lack of explicitness of the syntax of colloquial languages, languages do stand in such a relation to reality that whatever can be said can be clearly said within them.

¹² Of course, this distinction collapses for Frege and Russell in the case of *a priori* truths. But this is not so for Wittgenstein.

Now, in TLP, Wittgenstein most manifestly shows an interest in the problem, PC^{*}, (as understood above) at 5.511, where he asks:

How can the all-embracing logic which mirrors the world use such catches and manipulations ? [cf Notebooks 24.1.15]

But this question itself does not directly raise the problem, at least it does not raise PC subject to the constraint of A2, since *prima facie* this question deals only with the symbolism of pure logic and, in particular, the symbolism for denial. Consider, for instance, the passages surrounding this question in TLP and in the Notebooks. In 5.51, Wittgenstein explains his notation ' $N(\xi)$ ', where ' ξ ' stands for a class of propositions. According to that explanation, ' $N(\xi)$ ' expresses exactly what the conjunction of all the results of concatenating ' \sim ' with a proposition ' p ' in ξ expresses. In 5.512, Wittgenstein explains how ' $\sim p$ ' can express a true proposition. He explains:

That which denies in " $\sim p$ " is however not " \sim ", but that which all signs of this notation, which deny p , have in common.

Hence the common rule according to which " $\sim p$ ", " $\sim\sim p$ ", " $\sim p \vee \sim p$ ", " $\sim p \cdot \sim p$ ", etc, etc., (to infinity) are constructed. And this which is common to them all mirrors denial.

Within this context, the catches and manipulations about which Wittgenstein asks can only be taken to be the negation sign(s)

of a logically canonical notation and the rules with respect to which they in their respective notations can be concatenated with other signs to yield propositional signs. This interpretation is correct. But, firstly, the context in Notebooks where this question occurs suggests that the question applies not only to negation signs but other operation signs of a logically canonical notation¹³, and even in TLP, the discussion in 5.51-5.512 extends to 5.513 the explanations of negation signs to all operation signs. Secondly, consider sentences in TLP such as 5.5563 quoted above and

3.032 To present in language anything which "contradicts logic" is as impossible as in geometry to present by its co-ordinates a figure which contradicts the laws of space; ...

These suggest that the question Wittgenstein asks of logical symbolism in 5.511 of the signs of a logically canonical notation can without serious qualification also be extended to such signs as 'not' and 'and' etc, in colloquial languages. The question in the case of logical symbols simply deals with the symbolism stripped of any features in a colloquial language that are accidental to the problem of clarity of

¹³ The passage reads in 24.1.15:

The logical pseudo functions are operations.
Only operations can vanish!
The negative proposition excludes reality.

expression. This reading is corroborated and not undermined by what Wittgenstein further says at 4.002:

Language disguises [clothes] the thought; so that from the external form of the clothes one cannot infer the form of the thought they clothe, because the external form of the clothes is constructed with quite another object than to let the form of the body be recognized.

The silent adjustments to understand colloquial language are enormously complicated.

What Wittgenstein asks at 5.511 implicitly raises the question of how complexes of signs stand to that upon which the universality of the laws of truth ultimately depends so that they can comply with or express those laws, and so count also as expressions of what is susceptible of truth and falsehood. The 'silent adjustments' - or, as the later translation reads, the 'tacit conventions' - mentioned in 4.002 are what I have called the routines of writing, say, and reading with respect to which a sense is and can continue to be communicated. That is, they exhibit rules the following of which can result in the clear expression of sense, notwithstanding that they are silent - or in the background of expression. Due to their 'silence', as Wittgenstein puts it, or lack of explicit formulation, part of the problem posed here is to understand how these adjustments can be performed so as to result in the clear expression of sense. That these adjustments are part of the background of understanding what is said in colloquial

language is associated necessarily with the emergence of features in the symbolism - such as definite descriptions - which are not essential for clarity of expression. These features of the symbolism of colloquial languages may arise for other objectives than the clear expression of truths.¹⁴ Hence, there is no necessity that they show how propositions depend upon the meanings of their component symbols for their truth-value. Thus, according to Wittgenstein, they 'clothe' what is essential to clarity of assertion. Notwithstanding these observations, however, I do not think that Wittgenstein is rejecting at 4.002 that which he accepts at 5.5563 and 3.032. For he is still accepting that there are canons with respect to which one comes to understand what is said in colloquial languages. These may be especially complicated. They may, moreover, be left in the background of what is said but they are still governed by what, in Frege's phrase, points the way in logic, namely, truth. The difference is that only in the case of logics with logically canonical notations is truth the only object that points the way, in the sense that it is with respect to these canons that one explicitly comes to understand how assertions depend directly for their truth-value upon their form and content of composition.

With the relegation of some of the logical canons to the

¹⁴ Grammatical features such as tense and mood may be taken for example.

background, Wittgenstein further departs from Frege and Russell in his account of clarity, for he thereby resorts to the canons of diction and grammar of a language, or even a dialect, to be such as to detect and prevent the logical confusions, to prevent the utterance of nonsense. For such detection, these canons do not appear adequate. By 5.5563, what Wittgenstein must show - in order to respond to PC - is that this is only an appearance, one that disappears once one understands 'language and the world aright'.

This problem (the problem of showing the logical adequacy of languages) becomes especially pressing in the case of the linguistic and logical problems posed by Russell's paradox, for nothing in the canons of colloquial languages seems to prevent the paradox from arising nor serves to resolve it. Indeed, it is also difficult to avoid neatly in canonical notations, as can be testified by Russell's theory of types in Principia Mathematica (PM) and "Mathematical Logic." But let us consider how it can arise in ordinary languages.

The problem arises for a speaker of a language in a context with respect to which the following two signs count as symbols:

ϕ : 'is a member of itself'

\mathfrak{B} : 'the set of all items that are not members of themselves'

If ϕ counts as a predicate with sense then, by the principle of extensionality, \mathfrak{B} counts as a complete symbol. But then the result of applying \wedge to (ϕ, \mathfrak{B}) is a proposition, ρ .¹⁵ Then, in that case ρ is either true or false. But if true, then \mathfrak{B}^{\wedge} 'is a member of itself' is true, so \mathfrak{B}^{\wedge} 'is not a member of itself' is true. Similarly, if ρ is false, then ρ is true. So, ρ is neither true nor false. This, from a linguistic point of view, is how Russell's paradox can be understood to arise. However it may be resolved, the interest of this paradox for the better understanding of Wittgenstein's position is that it is not obvious that the English-mathematical canons of diction and grammar suffice to count ϕ as a meaningless predicate with respect to all contexts. For Frege and Russell, this certainly shows a logical deficiency in language. Not so, however, for Wittgenstein. As we shall see in chapter 4 when Wittgenstein's criticisms of Russell are examined, for Wittgenstein, it is supposed to show a deficiency in our understanding of language. It will be the burden of Wittgenstein's critique of Russell, then, to show how the error lies only in our understanding of languages.

0.2 Notwithstanding this problem and what Wittgenstein

¹⁵ ' \wedge ' indicates a mode of combination of signs within a language, where these signs are given within single quotation marks. Thus, we have, for instance, the following rule formulable within the language: If ' ζ ' and ' τ ' are signs, then ' $\zeta^{\wedge}\tau$ ' = ' ρ ' is a string of signs.

assumes at 4.002, one may continue to maintain what he assumes at 5.5563 for two very different sorts of reasons. Firstly, one may assume that no language is logically inadequate, since it is only the saying of sentences with respect to various routines that sets or establishes what Frege called the laws of truth. Accordingly, what would count as the logical canons in a language could not be assumed to express universal laws with respect to which what is asserted depends for its truth-value upon its composition. Secondly, one could admit that a language is logically adequate for the following reason: since the saying of sentences with respect to various canons of diction and grammar must yield logical canons of both sorts. But one could further assume that what the canons of language and diction must allow one to show is how the truth-value of any proposition depends upon what is the case, where what is the case is not without exception dependent upon its being represented in language. That is, one could assume that what Wittgenstein calls the logical form of a proposition is not language dependent, that the distinctions and connexions between function and argument, relations and relata is not ultimately introduced through language, but are distinctions which the canons of diction and grammar of every language must

respect.¹⁶

Though what Wittgenstein writes in TLP appears to support the first account (for instance, at 5.556), he is explicitly committed to the second in many key passages. For instance, according to what he writes at 5.552, logic is a *fortiori* prior to language since

Logic *precedes* every experience - that something is so.

This is re-affirmed - though not directly - by what he writes at 6.1231 and 6.1232:

The mark of logical propositions is not their general validity. To be general is only to be accidentally valid for all things. Logical general validity, we could call essential as opposed to accidental general validity.

Given this commitment to the a priority and essential universality of logical laws, Wittgenstein could not accept that any particular language establishes those laws. Nonetheless, each language must obey those laws. Thus, with respect to these considerations, PC* receives another formulation, namely:

¹⁶ According to this exposition, the difference between these two accounts is akin to the difference between saying, on the one hand, that for every language there is a set of logical laws which its canons express, since every language sets its own canons and so its own laws, and saying, on the other hand, that there is one set of logical laws that the canons of every language expresses, since the formation of canons which express those laws is a precondition for a language.

The problem as to how, notwithstanding the contingency of the origin and development of languages, the conventions with respect to which one utters sentences are adequate for the clear expression of what is truth-valuable.

This way of formulating the problem suggests that some conjoint commitment to what I have called the realism and independence theses. In particular, the following rationale for these assumptions becomes more compelling.

Given that the laws of logic are essentially universally valid and so followable in language, there must be some items upon whose subsistence (unchanging being) the universal validity of those laws depends and that can be meant by symbols in a language. This can be put another way. Not only are the laws of logic supposed to be essentially universal, they are also supposed to be regulative of how symbols mean within a language. This accords them a standing in reality. Hence, they must be grounded in what is real. Since moreover these laws are essentially universal, those items in whose being the validity of the laws is supposed to consist must abide or transcend all change. They must subsist, for otherwise the validity of the canons of logic could only be accidentally universal. This at least provides some rationale for some versions of RT and IT. What remains to be shown is not only that Wittgenstein is committed to some such rationale as the above, but that he is committed to RT and IT categorically. That Wittgenstein is committed to some such

rationale as the above, and so to some qualified version of RT and IT, is evidenced by his account of what he calls the substance of the world. That he is committed to RT and IT categorically, however, becomes evident only with respect to his acceptance of Russell's theory of definite descriptions (or, even more generally, of incomplete symbols) and his assumption that there are no propositions or thoughts true *a priori* (whether in the epistemological or metaphysical sense) of the world.

Though a demonstration of the latter point would be sufficient for the purposes of illustrating how Wittgenstein is committed to the above two theses, the former is especially pertinent to the concerns of the present chapter. Accordingly, the former point will be dealt with first and in greater detail than the latter. The passage in which Wittgenstein first suggests the logical importance of substance is that between 2.021 and 2.0231. There he explains the connexion between substance and the sense of propositions.

In particular, at 2.0211, Wittgenstein writes as follows:

If the world had no substance, then whether a proposition had sense would depend upon whether another proposition was true.

This is so for Wittgenstein because, by 2.021, the meanings of names, i.e. objects, are supposed to constitute the substance of the world. They are what are supposed to be denoted by the

terms of atomic propositions. So, if whether a term denotes is supposed to be independent of whether the proposition in which it occurs is true, then the meanings of terms of atomic propositions must have being independently of what is the case. They must subsist.

By the above analysis, the connexion between substance and sense is evident at 2.0211. But this is not to show that this connexion is based on prior logical and metaphysical commitments. This connexion, however, can be seen through a consideration of what Wittgenstein assumes in the following two sentences:

2.024 Substance is what exists independently of what is the case.

2.025 It is form and content.

What becomes evident here is that, for Wittgenstein, the subsistence of form, or its determination by what subsists, is what is essential for the validity of the canons of logic. This is corroborated by what he writes before and after these two sentences. For instance, at 2.023 and 2.0231, respectively, he writes:

2.023 It is clear that however different from the real world an imagined world may be, it must have something - a form - in common with the real world.

2.0231 This fixed form consists of objects.

And he writes elsewhere:

2.032 The way in which objects hang together in the atomic fact is the structure of the atomic fact.

2.0333 The form is the possibility of the structure.

2.15 That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another. This connexion of the elements of the picture is called the form of representation of the picture.

The earlier passages express how, for Wittgenstein, form is essential for representation. The later passages explain how form is realized in any instance of a representation. As will be seen, this becomes the basis for Wittgenstein's account of showing, where part of what is shown by a proposition is the way in which the truth or falsehood of the proposition depends upon the way in which the meanings of its terms stand to one another in the world. Neither of these passages, however, explicitly explains how the requirement of the subsistence of form and the grounding of form in the meanings of the terms of atomic propositions is, for Wittgenstein, a logical requirement. This point only becomes salient when these passages are taken in conjunction with what Wittgenstein writes much earlier. For instance, at 2.012, he writes:

In logic nothing is accidental: if a thing can occur in an atomic fact the possibility of that atomic fact must already be prejudged in the thing.

Form, logical form, is supposed to be that which constitutes the possibilities for objects to occur in states of affairs. We come to understand why some forms - the forms of atomic propositions - are said to be logical in the following sentence:

2.0121 ... (A logical entity cannot be merely possible. Logic treats of every possibility, and all possibilities are its facts.)

According to these sentences, the possibilities of which logic is supposed to treat are the possibilities of structure, namely, form. Thus, the logical entities of which he writes are supposed to be those upon whose being these possibilities directly depend, viz. objects.

Logical form is, for Wittgenstein, that with respect to which a proposition means a fact, since it is supposed to be that with respect to which the terms of the proposition are combined and with respect to which the meanings of those terms are supposed to be combined into the fact meant by the proposition, *if* the proposition is true. That is why, as I will discuss at length in the following chapter, the sense of an atomic proposition is supposed to consist only of the meanings of its terms and so a logical form. (It inherently has a positive orientation with respect to the true.) Thus, the form shown by a proposition determines how the proposition's truth or falsity depends upon the way in which

the objects meant by the proposition's terms stand to one another in the world.¹⁷ Accordingly, it is in virtue of showing logical form that a sentence says clearly what can be said and it is only with respect to the understanding of such a dependence that one recognizes valid and invalid arguments.

What the above exposition shows is how, for Wittgenstein, form must be grounded in what can be meant through, if not in, language without what is thereby meant being dependent for its being upon being meant. For, according to what is exposed in these passages, if this were not the case, then the validity of logical laws could only be accidentally universal.

Wittgenstein's commitments to RT and IT should now be quite apparent. I have also been urging that these commitments are prior in importance to what he assumes at 2.0211. The arguments for this latter point, however, do not make it quite as manifest as the weaker claim that has been made. The strongest argument that they ought to be read as more fundamental is that they are involved in the formulation of a problem for which the pictorial account of propositions is best read as a solution, namely, what I have called the problem of clarity. Some of the evidence for such an argument consists in what has already been discussed concerning

¹⁷ Moreover, this dependence is supposed to be such that the form does not predetermine how those objects stand to one another in the world.

Wittgenstein's, and Frege's and Russell's, conception of clarity of assertion. It might seem that the remaining evidence consists in indicating how Wittgenstein's pictorial account is compelling only if it is taken as a solution for PC^{*} under the constraint of RT and IT. Much of the evidence does, I think, consist in such indications. But if this were all the remaining evidence, then much of the prior rationale for reading TLP as a solution to such a version of the problem would be wanting. Fortunately for our purposes, Wittgenstein can be shown to express a prior, categorical commitment to RT and IT through his unquestioning acceptance of Russell's theory of definite descriptions.

The problem of definite descriptions arises with respect to the following considerations: That, according to the grammatical canons of most colloquial languages, definite descriptions occur directly in some propositions, even in cases where no real thing is meant by these descriptions. The problem is to determine which of the following cases ought to hold, and so set the logical canons with respect to which such propositions are understood:

Either i), in those cases in which no real item is meant (referred to and/or expressed) by the description, the definite description lacks a meaning (reference and/or sense), or ii), in such cases, it has a meaning and what it means has existence but does not satisfy the description or iii), in such cases, it has a meaning but what it means lacks being.¹⁸

According to Russell and Wittgenstein, only the first case, i), can be assumed to hold.¹⁹ That is, they both think that the meanings of symbols have being; in particular, that the meaning of a definite description, if it has one, consists in what it refers to, and what it refers to has being. Thus, for Russell and Wittgenstein, saying that a definite description lacks meaning is tantamount to saying that it does not refer

¹⁸ The word 'meaning' is being used, here, in a manner that departs both from the Russellian usage in "On Denoting", the usage in Geach's and Black's translations of Frege and Ogden's translations of Wittgenstein. 'Meaning' across these usages corresponded variously with Frege's own usage of 'Sinn' and 'Bedeutung'. I have followed Geach in translating 'Sinn' as 'sense'. I have followed Russell, however, in translating 'Bedeutung' as 'reference' or 'denotation'. Since I will be discussing all three of these philosophers, the first in the original and the other two in translation, a neutral term was needed to discuss and compare each of these philosophers on symbol significance. For these purposes, I have reserved the word 'meaning'. Accordingly, depending upon the philosopher discussed and the type of symbol cited, the meaning of a symbol may consist of at most two components, sense and reference, and at least one, either sense or reference.

¹⁹ Frege, of course, holds the second position. Unlike the third, which is essentially Meinong's, it does express a commitment to RT and IT. Accordingly, for Frege, in all such cases, the definite description would denote the null set. Admittedly, even given Frege's Platonism, the status of the null set as an entity is dubious. But since, for Frege, no such description is satisfied because no existing non-null item satisfies it, Frege is still committed to RT and IT.

to an item with being, even in cases where the description is allowed to occur as a term in a meaningful sentence.

Russell's commitment is well known from 'On Denoting'. Wittgenstein most explicitly expresses this position at 3.24. Only Wittgenstein's case will be examined, here.

Wittgenstein, then, writes as follows concerning this question:

3.24 A proposition about a complex stands in an internal relation to a proposition about a constituent of the complex.

A complex can be given only by its description, which will be right or wrong. A proposition that mentions a complex will not be nonsensical, if the complex does not exist, but simply false.

What makes Wittgenstein's commitment to RT, here, opaque is the locution 'mentions a complex.....if the complex does not exist', which occurs in the second crucial passage. It seems to suggest that non-existent items can be meant. But, then, such a position would be interesting and, indeed, sound only if one could describe a non-existent complex rightly, which according to what he writes above cannot hold. For if one could rightly describe a complex, then in the case where the definite description occurs directly in a proposition, where the predicate of the proposition is the predicate of the description, the proposition ought to come out true. This is precisely what Wittgenstein denies in 3.24. Accordingly, Wittgenstein is committed categorically to RT. But, given his

prior commitment to the universality of the laws of truth, he is also committed to IT.

In the next four chapters, I will show how in TLP Wittgenstein initially attempts to answer explicitly the problems posed by PC*, and how with respect to the assumption of RT, IT and the contingency of the world, this attempt leads inexorably to the paradox. The perplexity that confounds the reader of TLP, at this point, however, will be shown to reveal not only a confusion in Wittgenstein's text, but in Frege's and Russell's as well.

Chapter 1: The Pictorial Account of Propositions

1.0 According to the previous chapter, the problem posed for Wittgenstein is to understand how whatever can be said can be said clearly in language, where what is clearly or unclearly said is so counted ultimately with respect to the canons of logic. This problem I have called the problem of clarity, (PC'). Henceforth, the problem will be understood to be subject to the constraints of the following assumptions: A1, A2, RT, and IT. The present chapter will provide an exposition of Wittgenstein's account of how an array of marks or sounds, say, can come to count as a proposition. According to what I will be arguing, here, Wittgenstein's account consists mostly in an elucidation of the background and routines with respect to which the array of marks - or a perceivable complex - can count as a proposition. In particular, I will show that Wittgenstein's elucidation of the background and routines is supposed to show how whatever counts correctly as a proposition with respect to them can also be counted as saying something clearly or unclearly. That is, according to Wittgenstein's pictorial account of propositions, the background and routines with respect to which an array can count as a propositional sign in language also ensure that the language is logically adequate. A

proposition is supposed to be a sign with sense. Thus, what will also need to be discussed in the course of the exposition is the question: in what is the sense of a proposition supposed to consist? Concerning this latter question, what I will be arguing is that, for Wittgenstein, the sense of a proposition is supposed to consist in an orientation of the meanings of the proposition's terms with respect to a form (or an ascent of forms topped with a (logical) form), ξ , and the truth-value, \uparrow , where only in the case of atomic propositions and their negations is ξ what Wittgenstein would count as a logical form.¹

1.1 Wittgenstein's account of elementary propositions ultimately derives from a metaphorical assimilation of propositions to pictures, where this assimilation is expressed

¹ As will be seen later in this chapter, the meanings of the terms of a proposition are taken to be components of the proposition's sense. The one sentence in TLP which supports this reading is the following:

2.222 In the agreement or disagreement of [the proposition's] sense with reality, its truth or falsity consists.

If the sense consisted merely in an orientation with respect to an ascent of forms and a truth-value (or, more faithfully, the two poles of a proposition, true and false), then since a true and a false proposition may show the same orientation of this sort, they may have the same sense. But in that case, they must both be true or both false, since the truth or falsity is supposed to consist in the agreement or disagreement in the proposition's sense with reality. The meanings of the terms of the two propositions are all that is left to distinguish between their respective senses.

explicitly through such sentences as the following:

4.01 The proposition is a picture of reality.

or even more explicitly:

4.03 ... The proposition asserts something, in so far as it is a picture.

4.06 Propositions can be true or false only by being pictures of reality.

Now, even in the case of atomic propositions, such an assimilation initially seems confused and, in so far as it is understandable, implausible. Usually propositions and pictures are distinguished with respect to the ways in which they respectively stand to reality. Indeed, if one follows a position akin to Frege's, then one will most likely think that the ways in which pictures and sentences stand to reality are so disparate that any attempt to elucidate the latter in terms of the former must result in a misunderstanding of both. For this reason, it is important to understand how Wittgenstein describes propositions as pictures in a sense of 'picture' that at once departs from and exploits the meaning given to the word in its application to drawings and photographs, say. Only with respect to such an understanding will Wittgenstein's account be found compelling. The novelty and import, I think, of Wittgenstein's assimilation can best be understood and appreciated through first considering such a position as

Frege's. More importantly, a consideration of such positions will introduce Wittgenstein's account of the background with respect to which the terms and predicates of simple sentences are supposed to mean. As I will be arguing, through his assimilation of propositions to pictures, Wittgenstein shows how because atomic propositions can mean only with respect to a background of picturing, a language that consisted only of atomic propositions could be logically adequate. [Wittgenstein imports from Frege the notion of the sense of a proposition and from Russell the notion of form - specifically, logical form. But, in doing so, he re-interprets them with respect to different assumptions about the way in which sense and form are expressed. According to this re-interpretation, both form and sense are shown by pictures, thoughts and propositions.]

1.11 According to Frege, a picture is a complex of objects which either corresponds or does not correspond to another complex of objects in the world. The correctness of a picture, moreover, is taken to be a measure of the accuracy with which it corresponds in the respects by means of which it and its corresponding complex of objects are respectively presented visually. Sentences, in contrast, do not

¹ "Thoughts" Logical Investigations. (Library of Philosophy and Logic.) Edited with a Preface by P.Geach. Translated by P.Geach and R.H.Stoothoff. New Haven: Yale University Press, 1977, 1-30.

correspond to any complexes of objects. Rather, they express senses, what Frege called thoughts, and denote objects, what he called truth-values. Indeed, under Frege's analysis of judgements, thoughts - which can, but need not, be contents of some person's judgement - are not even complexes of objects. Rather, they are composites, in the mathematical sense, of the senses of the terms and predicates of sentences (in this case, simple sentences) that express those thoughts.

This compositeness is one feature that reveals the functionality of Fregean sense. But this is already explained through Frege's elucidation of the senses of terms and predicates of propositions. Accordingly, Frege has the following analysis of the sense of propositions. Since the sense of a sentence gives the value true or the value false (depending on the way the world is) and is the composite of the sense of the predicate with the sense of the sequence of names given (in the same order) in the sentence, the sense of a predicate is a function from the class of sequences of objects to the set of truth-values and the sense of a term is a constant function from the class of objects (including the class of sequences of objects) to a single object.³ For

³ In 'Function and Concept' p.32-33, Frege suggests that a function ought to be defined for any argument. For instance, according to Geach's and Black's translation, he asserts:

It is thus necessary to lay down rules from which

Frege, the senses of both terms and propositions are constant functions, whose values are the *objects* they refer to. According to Frege, the meaning of every symbol consists of exactly two components, viz., sense and reference. As will be seen momentarily, the distinction between complete symbols and incomplete symbols in Frege's account amounts to a distinction between the dependent or independent being of their references. This distinction, moreover, is supposed to be reflected in any logically adequate symbolism.

This, in rough outline, is Frege's contrast between pictures and propositions. The most salient feature of this contrast consists in the following two observations:

it follows, e.g., what '0 + 1' is to mean, if '0' is to mean the Sun. (p.23)

The objections to which this position is susceptible are notorious. Most of these difficulties concern the apparent arbitrariness of the value, false, that relations give for apparently irrelevant arguments, as in the case of the thought purportedly expressed by 'The solar system is 0 + 1'. The only constraint of which I know that Frege imposes on concepts is that they take functions as arguments only if they do not take objects as arguments. This gives his distinction between first and second order concepts.

Very few have followed Frege on this point. Most logicians have at least introduced a map which determines the arity of any function. Other maps may be posited that more generally determine the source and target of any function. These would be the only universally defined maps.

1 Whereas the correctness of a painting consists in its correspondence with a complex of objects that the painting is about, the correctness of a proposition consists in the clarity with which it expresses a true thought, where the thought is the sense of the proposition and the true is a value that the thought gives for any argument given the way the world is as a whole. That is, it is not a property of the thought.⁴ [A corollary of this contrast is that senses are not objects and so are not constituents of the world. They cannot be the values of constant functions, whether simple or composite. That is, they cannot be named.⁵]

2 According to 1, pictures are not themselves true. Rather, a proposition that says that the picture corresponds to reality is true or false.

Supposedly, the canons of representational painting - especially those that give the theory of perspective - are not adequate to determine for a single painting whether or not it expresses a thought, let alone whether it expresses a true one. Moreover, even if they were, then the account of the truth or correctness of a painting would no longer be

⁴ The truth of the thought expressed by a proposition consists in the realization of the sense of the proposition's predicate by the sequence of objects denoted by the proposition's terms, in the order in which they occur in the proposition.

⁵ For Frege, the expression "the sense of 'is red'" is not a name for a concept. Even in the context of a sentence such as "The denotation of 'the red shoes' satisfies the sense of 'is red'", the denotation of "the sense of 'is red'" is some proxy or surrogate for a concept. If "the sense of 'is red'" denotes anything then it denotes the class of red objects. According to Frege, the sense of the expression "at least one object is such that ..." is a second order concept. Unlike the sense of 'is red', this concept takes the senses of predicates as arguments, not the denotations of names. Nonetheless, it still takes truth-values as values for different arguments. So, it is still a concept.

understood in terms of its correspondence with another object in the world; rather, its correctness would be understood in accordance with the assumptions upon which the correctness of a sentence is understood. This sort of assimilation is effected in both directions by Wittgenstein in TLP. In order to understand Wittgenstein's assimilation, one needs first to understand Frege's reasons for keeping the explanations of the correctness of pictures and of propositions separate. Though Wittgenstein's position does not result from a compelling criticism of Frege's explanations, it does result from a response to Frege's arguments for keeping those explanations separate.

According to Frege, all symbols in a language - even proper names and propositions - both express sense and refer (uniquely) to items. The contentious cases, for this account, are, of course, proper names and propositions. Propositions do not obviously refer to entities, and proper names do not obviously express sense. Moreover, it is not obvious, even if these points are granted, that the sense of a symbol is a function and not an object or, similarly, that the reference of a symbol is an object and not a function. Frege, however, offers a string of arguments for this position.

Frege begins his argument for the thesis that all symbols express sense and refer uniquely to entities is given early in his body of work, namely, in 'Sense and Reference'. On the

first page of that paper, Frege argues for the distinction between sense and reference by distinguishing between the cognitive and modal values of the following sentences:

i. $a = a$

ii. $a = b$

He argues that when one understands and says (i), one is making an *a priori* judgement, where what one judges - the content of the judgement - is (supposedly) necessarily true. He does not think that a similar analysis holds in the case of (ii). Rather, he argues that when one understands and says (ii), one is making an *a posteriori* judgement, where what one judges is contingently true - that is, true depending upon which way the world is. But if what (ii) says were true, then the reference of 'a' and 'b' could be the same. Since this is the case, the referents of 'a' and 'b' cannot be components of what (ii) says, for otherwise the modal, if not the cognitive, values of (i) and (ii) would be the same, which they are not. Thus, what must compose the content of the judgement that (i) is true must be items other than the references of 'a' and 'b'. These items are what Frege calls the senses of 'a' and 'b'. Though there are paths or routines the following or the performance of which results in the expression or grasp of the senses of 'a' and 'b', these do not, for Frege, constitute what Frege takes to be the sense of 'a' and 'b'. This is so

for Frege since for him 1) these routines and paths are contingent upon or immanent in the world, and 2) they are not functional or, if functional, they are not functional in the required meaning of the word.⁶ But, according to Frege, sense both transcends change and is functional. His arguments for these two positions provide the second part for the main argument.

Two assumptions, assumptions which impose further epistemological and logical requirements upon what counts as sense, provide the basis for the second part of Frege's argument. These are as follows:

1 If a symbol 'p' clearly expresses the content of a judgement, then that content is a composite of the senses of the symbols that occur in 'p'. (See Appendix)

2 If a symbol 'p' expresses the content of a judgement -

⁶ There are two types of meaningful occurrences of the English word, 'function', and correspondingly of the word, 'functional'. These two types are to be carefully distinguished for the course of this exposition. Whereas in an occurrence of the first type, 'a function' means a role that an item performs in a course of action, an example of the second type of occurrence is 'a function is an item with a source and target and, which for any item in the source, selects or otherwise determines exactly one item in the target'. Some of the confusion over the two types of occurrences derives from attempts to represent the meaning of 'function' in occurrences of the first type in terms of 'function' in occurrences of the second type. For Frege, the senses of linguistic symbols are correctly spoken of as functions in accordance with the second type of occurrences of 'function', and, accordingly, in at least some cases are not dependent upon the being of their arguments. Whether a distinction between the meaning, the extension or intension, of the term in its two occurrences can continue to be made intelligible is not at issue, here, so long as the meaning of function, according to Frege's account, remains intelligible.

that is, is a proposition - then at least one symbol occurs in 'p' that is incomplete or, in Frege's vocabulary, unsaturated. That symbol expresses a concept or a relation.

For Frege, functional symbols such as 'fx' count as the paradigms or canonical examples of incomplete symbols, and it is in comparison with them that all other incomplete symbols are understood. Their incompleteness is supposed to consist in the fact that what counts as their reference - whether this be a relation or the extension of a relation - depends for its being either upon there being objects or upon how objects stand to one another in reality. According to 1 and 2, then, since at least one component of a thought is a function and composes (but does not take or give as a value) the other components of a thought, all senses are functional.'

' Unfortunately, none of Frege's writings are especially clear as to which of the following alternatives should hold:

- 1 The reference of a predicate is a relation, that is, a function from (sequences of) objects to truth-values.
- 2 The reference of a predicate is the extension of a relation that the predicate expresses.

At least one of these positions must hold for Frege, since a relation must be one of the components of the meaning of a predicate. So, suppose that the relation R is a component of the meaning of the predicate 'P'. Then, if the sense of the predicate is the relation, then the reference of the predicate must be, not the truth-value, but the totality of arguments for which the relation gives the value, true, for otherwise the predicate would not necessarily refer to exactly one item. But this totality of arguments is the extension of the relation. This gives the second alternative, (2). On the other hand, if the sense of a predicate is not a relation, then the reference of the predicate is a relation, since, by

The first assumption follows from A1 [See Introduction], the requirement that the content of a judgment, especially

the initial supposition above, a relation is one of the components of the meaning of a predicate. This gives the first alternative, (1)

In 'Sense and Reference', Frege argues mostly about the senses of proper names, propositions and 'that'-clauses, not predicates. In 'On Concept and Object', however, Frege supports the position on relations for which he has become notorious, namely, the thesis that relations cannot be referred to by terms. He writes on p.46:

In logical discussions one quite often needs to say something about a concept and to express this in the form usual for such predications.....Consequently, one would expect that what is meant by the grammatical subject would be the concept; but the concept as such cannot play this part, in view of its predicative nature; it must first be converted into an object or, more precisely, an object must go proxy for it.

This would suggest that relations cannot be referred to at all, whether by terms or predicates. But this is not so for Frege. For he continues on p.47:

A concept is what is meant [referred to] by a predicate; an object is something that can never be the total meaning of a predicate, but can be what a subject means.

But despite the support this latter passage gives to the first alternative above, it conflicts with other theses of Frege's. For instance, by the thesis of compositionality, the sense of a proposition would not consist only of the senses of its component symbols, for the reference of the proposition's predicate would have to occur in the sense of the proposition. If the reference of a predicate is a relation, then the predicate's sense must be a constant or nullary function from objects to that relation. In that case, only the sense of a name in composition with the reference of a predicate gives a truth-value for any given argument, not the sense of a name and the sense of a predicate alone.

Whichever of these alternatives does in fact hold, one thesis is maintained for Frege: The reference of a predicate is dependent for its being upon the being of objects, and the sense of a predicate is a function. Accordingly, both alternatives commit Frege to a functional account of sense.

what Frege calls a scientific judgement, be susceptible of a clear expression. As we said in the Introduction, the criterion for clarity of expression is that the symbol be such that if one understands it, then one recognizes how what it expresses depends directly for its truth and falsity upon its form and content of composition and indirectly upon how objects stand to one another in the world. The first assumption above, 1, gives one way of fulfilling that requirement.

The second assumption follows from Frege's analysis of predicates. He observes, firstly, that since what counts as an element of the extension of a predicate depends upon the way objects stand to one another in the world, if the meaning of a predicate consisted only of its extension, then one could not understand a proposition unless one already knew that it was true. Since one must grasp some component of the meaning of a predicate in a proposition in order to understand a proposition, one could, prior to any observation, determine whether the reference of the sequence of names in that proposition are elements in the extension of the predicate. Accordingly, the meaning of a predicate consists of some other component than its extension, if its extension is a component of the predicate's meaning. Since, moreover, the referential component of a predicate is complete only if the world itself is complete, predicates are, even when they are primitive

ones, incomplete symbols. But a predicate expresses what is supposed to be asserted of objects in a judgement. That is, in the case of a verbal or literal proposition, they are supposed to signify how the objects denoted by the terms are supposed to stand to one another in the world, or rather how they are combined with one another.⁸

Given this account of sense, one can now see why Frege treats as disparate the respective ways in which pictures and propositions represent reality. According to Frege's analysis, any symbol for the content of a judgement - a thought - is supposed to have an incomplete symbol as a component, since only such a symbol reveals the functional feature of what (in the context of a proposition) that symbol is supposed to express. But a picture, for Frege, consists only of objects. Not even with respect to the perspectival canons of pictorial representation, can any of the components of a picture, then, count as an incomplete symbol. Moreover, if the components of the picture do not themselves correspond to any complexes of objects, then the picture itself would lack sense. Accordingly, it is difficult to see how a picture could represent unless it were true. That is, it is difficult

⁸ Note that if one takes the reference of the predicate to be a relation, and not the extension of a relation, then the sense of a name would still be a function (in the second sense discussed in fn.6), for only if the sense of the name were a function could it be directly composed with the reference of a predicate.

to see how one could understand a picture without recognizing whether it is correct or incorrect, unless one adjoined to it some proposition.

1.12 Wittgenstein's opposition to Frege's analysis of propositions consists both of a rejection of Frege's account of the meaning of linguistic symbols and in the development of an alternative account of sense. Concerning the first point, Wittgenstein follows Russell in rejecting Frege's assumptions that names express sense and propositions denote truth-values. Accordingly, like Russell, he treats every definite description as a symbol that depends for its meaning upon the truth of some existential proposition; that is, he treats definite descriptions as incomplete symbols.

Concerning the second point, Wittgenstein follows Frege in accepting that propositions express sense. Indeed, for Wittgenstein, they are the only symbols in language that do express sense. But, due to Wittgenstein's rejection of Frege's functional account of the meanings of terms and propositions, instead of characterizing the sense of a verbal proposition as a composite of functions, Wittgenstein characterizes it as consisting of the meanings of its terms and predicate. The problems with which this account had to deal concerned not only the problem of the meanings of predicates but also the problem as to how a proposition clearly expresses how the meanings of its constituent terms

stand to one another if the proposition is true.⁹ Both of these problems devolve from Russell's work on Theory of Knowledge. But, of course, they are also the very difficulties with pictorial representations that Frege cited as preventing pictures from expressing sense or at least from presenting a sense so that one who grasped it could clearly express it. Wittgenstein addressed both of these difficulties through his account of form and structure. Accordingly, it is precisely through his account of sense in terms of form that Wittgenstein assimilates propositions to pictures and so (supposedly) gets one to see how colloquial languages are logically adequate.

Neither of these responses to Frege's account is especially compelling on its own. Nonetheless, together they do provide a solution to the problem of clarity, at least as Wittgenstein understands it. That is, they provide a compelling account of the background and routines with respect to which a complex counts as a proposition in a language, an account which purports to show how with respect to such a background, language is logically adequate. Accordingly, both of the above parts of Wittgenstein's response to Frege will be outlined here.

1.13 Unfortunately for the sake of the first point, most of

⁹ This was the logical requirement for clarity of expression.

Wittgenstein's reasons for rejecting Frege's account are phrased in such a manner as to appeal either to his own assimilation of pictures and propositions or to Russell's criticisms of Frege, most of which are notoriously difficult and not obviously correct. Consider, for instance, what Wittgenstein says of Frege in the following sentences:

3.143 That the propositional sign is a fact is concealed by the ordinary form of expression, written or printed.
 (For in the printed proposition, for example, the sign of a proposition does not appear essentially different from a word. Thus, it was possible for Frege to call the proposition a compounded name.)

6.232 The identity of the meaning of two expressions cannot be asserted. For in order to be able to assert anything about their meaning, I must know their meaning, and if I know their meaning, I know whether they mean the same or something different.

In 3.143, Wittgenstein is already assuming that his pictorial account of what is truth-valuable - whether a thought or a proposition - is correct and then offering an explanation of how Frege could have failed to recognize what, according to that account, must hold: since a proposition is, like its constituent terms, written or spoken in colloquial languages, it is apt to be treated like a term. Not only does Wittgenstein not offer a criticism here, he has not even taken account of the functional feature of Fregean sense. But, according to the previous discussions, it is due to the functionality of the sense of the proposition's predicate that

the sense of a proposition is a function that gives exactly one truth-value for all legitimate arguments. Thus, what Wittgenstein says in 3.143 in no way points out a difficulty with Frege's account.

In 6.232, Wittgenstein is again presupposing a theory of denotation and knowledge of objects which simply departs from or opposes Frege's on the very point in question. Rather than arguing independently that the derivative cognitive or modal values of a proposition do not change when a name is substituted for another co-referential name in the proposition, Wittgenstein apparently appeals, here, only to the Russellian assumption that if one understands a proposition, then one knows - or is acquainted with - the denotations of its terms. Unless this assumption follows from some other independent consideration in TLP, then Wittgenstein's appeal here simply begs the question at issue.

The same sort of problems appear at other passages in which Wittgenstein discusses Frege's account. Notwithstanding these difficulties, some of these passages suggest more compelling objections. I will consider two. The first deals with Frege's assumption that propositions denote truth-values; the second, with Frege's assumption that even proper names express sense, where names are supposed to count as simple symbols.

In 4.431, Wittgenstein writes as follows concerning

Frege's account of truth-values and propositions:

Frege was therefore quite right to use [truth-conditions] as a starting point when he explained the signs of his conceptual notation. But the explanation of the concept of truth that he gives is mistaken: if 'the true' and 'the false' were really objects, and were the arguments in $\sim p$ etc, then Frege's method of determining the sense of $\sim p$ would leave it absolutely undetermined.

The criticisms of Frege that this passage is supposed to express are not easily interpreted and assessed, for, firstly, the passage does not obviously express a charitable reading of Frege on what ' $\sim p$ ' expresses and, secondly, if some version of Wittgenstein's criticism were to presuppose a charitable reading of Frege on what ' $\sim p$ ' expresses, then that version would not directly provide an objection that Frege's assumptions about 'the true' and 'the false' undermine his method of determining sense. Notwithstanding these confusions, however, what Wittgenstein writes at 4.431 can be better appreciated in light of one of Russell's objections in 'On Denoting'. Before this latter connection can be discerned, both of the above confusions will have to be cleared.

According to Frege, the true and the false are referents of propositions. So, Wittgenstein correctly attributes to Frege the assumption that the true and the false are objects. But this is not what, according to Wittgenstein's analysis, is obviously objectionable. Rather, it is some additional thesis

of Frege's that is supposed to be objectionable, though it remains unclear as to whether this thesis is supposed to constitute part of Frege's theory of sense. Which thesis is this? According to the remark quoted in 4.431, it is the thesis that the true and the false are arguments in $\sim p$ etc. and this thesis is supposed to be objectionable since it is supposed (perhaps in conjunction with the former assumption) to undermine 'Frege's method of determining the sense of $\sim p$ '. But, as we shall see, this criticism is flawed in its interpretation of Frege's account of the sense of ' $\sim p$ '.

What ' $\sim p$ ' is supposed to express in Wittgenstein's notation is not what ' $\neg x$ ' expresses in Frege's. According to 3.313, for Wittgenstein both ' $\sim p$ ' and the ' p ' in ' $\sim p$ ' are examples of apparent variables that give and take, respectively, propositions as values. Hence, according to Wittgenstein, ' $\sim p$ ' does not express an operation on the set of truth-values. But, for Frege, ' $\neg x$ ' expresses a function which when applied to the true results in the false, and when applied to anything else results in the true. Hence, for Frege, when ' $\top 2>3$ ' denotes the true, ' $2>3$ ' in ' $\top 2>3$ ' denotes the false.¹⁶ Accordingly, the latter expression does not in that context present a *judgement* about what ' 2 ' and ' 3 ' denote. That is, it does not canonically show that the writer

¹⁶ According to Frege, complete or unsaturated symbols denote the values given by their senses, for any arguments.

of the symbol ' $2 > 3$ ' takes and recommends its sense, the thought, as being true; rather, it canonically shows that, whether true or false, the writer is merely assuming the thought, presenting it for consideration. Frege writes '+ $2 > 3$ ' to present what according to Wittgenstein an inscription of ' $2 > 3$ ' already adequately expresses. This is a difference of which Wittgenstein himself was aware. Consider, for instance, what he writes at 4.063:

...the verb of the proposition is not "is true" or "is false" - as Frege thought - but that which "is true" must already contain the verb.

and at 4.442:

... Frege's assertion sign "+" is logically altogether meaningless; in Frege (and Russell) it only shows that these authors holds true the propositions marked in this way.

"," belongs therefore to the propositions no more than does the number of the proposition...

Since for Frege the ordinary idiom for what '+ $2 < 3$ ' presents is 'The thought that $2 < 3$ is true', what Wittgenstein's remarks quoted above amount to is the objection that within a canonical notation the sign '+' is superfluous.

Evidently, Wittgenstein's criticism mistakenly assimilates Frege's conceptual notation to Wittgenstein's interpretation of Russell's notation, and the last remark once again merely presumes the superfluity of the sign '+' within a canonical notation. Accordingly, some revision in Wittgenstein's argument needs to be effected if it is to express a more charitable reading of Frege.

Consider the following version of Wittgenstein's criticism that would result from a simple revision:

R If 'the true' and 'the false' denoted objects that were arguments of what ' $\top x$ ' expresses, then Frege's method of determining the sense of ' $\top 2 > 3$ ', for instance, would leave it absolutely undetermined.

This version is certainly more in keeping with Frege's understanding of his conceptual notation. But is this version of Wittgenstein's criticism more compelling? Certainly, it is in need of further elaboration before it can be taken seriously. Regrettably, Wittgenstein is not especially generous with explanations of his objections to other philosophers. Nor can one appeal to Wittgenstein's objections, for instance, that Frege failed to distinguish propositions from complex names or that Frege failed to distinguish between arguments and indices. For not only do these comments involve mistaken readings of Frege, they beg the question at issue. Moreover, Frege seems to have a rather

straightforward routine for determining the sense of ' $\vdash p$ ', ' $\vdash_q p$ ' etc. at least on condition of having determined the sense of ' p '. These routines would be governed by principles such as the following:

1 ' $\vdash p$ ' expresses a sense according to which ' $\top p$ ' denotes the true just in case ' $\neg p$ ' denotes the false.

2 ' $\vdash_q p$ ' expresses a sense according to which ' $\top_q p$ ' denotes the false just in case ' $\neg q$ ' denotes the true and ' $\neg p$ ' the false.

One apparent inadequacy with this account though is that it applies only on condition that the senses of other propositions or of their constituent symbols have been grasped. That is, in the case of atomic propositions (those such as do not occur in contexts that Frege schematizes as ' \vdash ' and ' \vdash_q ') it is unclear how their senses could be explicitly grasped if they also denote the true. This problem becomes even more troubling when one recalls that the distinction between sense and reference holds, for Frege, with respect to all symbols; for then the problem of explicitly grasping the sense of a symbol can only be shifted and not resolved if it is changed from the case of propositions to the case of constituent symbols of propositions. This becomes especially manifest in the case of proper names, for in the case of proper names, it is unlikely that one could grasp their senses without recognizing or knowing their respective

references. This commits Frege either to allowing that the references of some symbols determine their senses, and not conversely, or to assuming that the grasping of the sense of a primitive symbol and the recognition of its reference are interdependent, in which case, as will be seen, for Wittgenstein the grasping of the sense of a primitive symbol becomes superfluous.¹¹ Of course, if Frege were to accept the first disjunct, then this would considerably alter his account of sense. For Frege, the sense of a symbol determines its reference. Moreover, there is a further consideration which should also make the first disjunct unattractive for Frege. This consideration comes from Russell's argument in 'On Denoting' (page 49-50). Without entering into the details of that argument - which are notoriously difficult - one point that is relevant to Wittgenstein's criticisms is that, for Russell, 'there is no backward road from denotations to meanings [senses].'¹² This point could apply in Wittgenstein's criticism for if Russell's point were correct, then 'the true' or 'the false' would denote what some proposition denotes only if the specification of the truth-

¹¹ Another alternative is, of course, holism: the sense of a proposition is given with respect to its logical and other relations with other propositions.

¹² Russell, B.. "On Denoting", Logic and Knowledge: Essays, Edited by R.C. Marsh. London, Sydney and Wellington: Unwin and Hyman, 1956, 41-56. (Originally published in Mind, 14: 479-93.)

value of the proposition in no way determined its sense. But, for Wittgenstein, an atomic proposition is the only symbol in language that expresses sense, where, in this case, the sense is supposed to be such that whoever grasps it knows how things stand in the world, if it is true. So, Frege's explanation of 'the true' must, according to Wittgenstein, be mistaken.

Apart from Russell's detailed argument, there are considerations which put the second alternative above in question. These are presented by Wittgenstein in what he writes at 3.26 and 3.263. In the first sentence, Wittgenstein implicitly appeals to the distinction between proper and improper names. The logical difference between these names, for Frege, is that whereas the sense of a proper name is non-composite, the sense of an improper name is composite. Consider, for example, the difference between meaningful occurrences of demonstrative pronouns such as 'that' in 'That is white', and meaningful occurrences of names of past items such as 'Caesar' in 'Caesar defeated Pompeii'. Supposedly, whereas 'that' in 'That is white' refers independently of whether some proposition in which it occurs is true, 'Caesar' in a present occurrence of 'Caesar defeated Pompeii' could not denote unless it already occurred in several true propositions about its reference. Accordingly, for an improper name in a language, either it has been introduced in the context of a definition or some meaningful predicate occurs within it.

This is not the case with proper names. Indeed, according to Wittgenstein, one cannot come to know or recognize what a proper name denotes through grasping that something is true of it, even in the case of what is uniquely true of it. This is so because, in the case of proper names, sense would have to be simple. Even if some predicate happened to express a complete class of concepts uniquely satisfied by the denotation of a proper name, the denotation could only accidentally satisfy the concepts in that class, for otherwise at least one of those concepts would compose the sense of the proper name. But, in that case, the sense of a proper name would be composite, contrary to Frege's account of proper names. It thus becomes unclear as to whether two co-referential proper names must have the same sense, since if they could be different, then this difference could only consist in the accidental feature of the reference that it was referred to by different signs or by the same signs in different contexts. But all differences in sense between proper names essentially consists in differences in their common reference. Thus, it seems that two co-referential proper names must have the same sense, in which case the introduction of sense to account for the differences of reference of proper names becomes superfluous.

By the above considerations, not only is it superfluous to assume that propositions denote truth values and proper

names express senses, it is also implausible to assume that the sense of a proposition is a function. Thus, if propositions are still to be understood as expressing sense, then the sense of a proposition can no longer be understood as a function.

1.14 As suggested earlier, since, for Wittgenstein, propositions have sense and the meanings of the constituent terms of the proposition constitute the content of the proposition's sense, Wittgenstein's account has to deal with two problems viz.

1 The problem as to what and how predicates mean.

2 The problem as to how a proposition shows in which way the meanings of its constituent terms must stand to one another if the proposition is true.

Wittgenstein's solution to these problems consists in his characterization of the formal features of sense. Apart from having to allow that the sense of a proposition is composed of the meanings of the propositions's terms,¹³ Wittgenstein characterizes it as consisting in an orientation with respect to various forms - eg. space, time, colour, etc., and logical form. Since form is, for Wittgenstein, the possibility of structure and all complexes are structured, the adequate expression of a form is precisely its realization in a given

¹³ See footnote 1.

complex. That is, according to TLP, it is the structure of a complex that can show how objects other than the constituents of the complex can stand to one another in the world. The form that such a structure is supposed to realize is what Wittgenstein calls the form of representation of a complex, if that complex counts as a picture. The important point about Wittgenstein's account is that no complex could count as a picture unless it presented such a form, and so a sense. This is how in TLP Wittgenstein assimilates pictures to propositions. The reverse assimilation is effected through the introduction of what Wittgenstein calls logical form. This is what a complex must show if it is to count as clearly showing how things stand if it is true. The importance of logical form for Wittgenstein is that it is with respect to an understanding or grasp of it that one comes to distinguish explicitly between specific modes of combination, structures, and what is combined, objects. Or if one were to continue to accept Frege's account, logical form would be that which one must understand in order to distinguish between function and argument, concept and object. What I will be attempting to show in the remaining part of this section is how, for Wittgenstein, 1 an array of marks or sounds, say, could not count as an atomic proposition unless it showed a logical form, and 2 pictures show the 'meanings' of predicates, and they do so only through presenting a logical form. More

specifically, I will be arguing that, for Wittgenstein, an atomic proposition results directly from the projection of all but one of the constituents of a complex C_0 onto the constituents of another complex C_1 whose mode-of-combination is expressed by the remaining constituent of C_0 . This could not result except with respect to a logical form realized in the complex C_0 and already presented within a background of pictures. Accordingly, the pictorial account of propositions is supposed to show how the background and routines with respect to which an array of marks counts as a proposition in language are adequate for the clear expression of sense.

Wittgenstein first writes of the formal features of sense at 2.021-2.023. This section treats most explicitly of substance in its connection with sense. But since the substance of the world is supposed to consist of objects and these are further supposed to determine a form with respect to which complexes can be represented, this passage also treats implicitly of the connection between form and sense. Consider, for instance, what he writes at 2.0211:

If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.

The consequent of this conditional is supposed to be false (or otherwise mistaken) for if it were correct, then

2.0212 ...[it] would then be impossible to form a picture of the world (true or false).

That is, no sense could be formed. Wittgenstein then explains that what is required for a sense is not only a content but a form:

2.022 It is clear that however different from the real one an imagined world may be, it must have something - a form - in common with the real world.

2.0231 The substance of the world can only determine a form and not any material properties. For these are first presented by the propositions - first formed by the configuration of objects.

What is unclearly expressed in the second passage is Wittgenstein's distinction between a form and a material property (or relation). The difference is supposed to consist in the distinction between what is determined independently of what is the case and what is determined both by substance and by what is the case. Form falls on the former side of the distinction. Material properties on the latter. Though the distinction is still unclear, it receives some possible clarification in the following sentences:

2.0251 Space, time and colour (colouredness) are forms of objects.

2.0231 The way in which objects hang together in the atomic fact is the structure of the atomic fact.

2.033 The form is the possibility of the structure.

The first sentence lists a diversity of forms. The suggestion given by the subsequent sentences is that a material property (or relation) is determined by the realization of these forms in what is the case. The structures given by these determinations would induce various possible modes of combination of objects with respect to these forms. What makes this suggestion compelling is the terms with which Wittgenstein writes of space and time earlier in 2.0121. He writes there as follows:

Just as we cannot think of spatial objects at all apart from space, or temporal objects apart from time, so we cannot think of any object apart from the possibility of its connexion with other things.

The restrictions of the first remarks to spatial and temporal objects (rather than stating them categorically as in the case of the second) suggests that the forms of space, time and colour are less essential to objects than the possibility of entering into combination with one another, that is, less essential than the forms of which logic treats. Accordingly, as 2.0121 explains, these other forms are treated more specifically by mechanics and optics, say, than by logic. They, or their realizations, are what are presented by the propositions of those theories. Their relative accidental (or contingent) status could be summarized as follows: though it is essential that the forms of which logic treats be realized

in the world by what is the case, it is not essential that the forms of space, time and especially colour be realized also.

With respect to this account of form, structure, and material (or external) properties (or relations), Wittgenstein gives his account of pictorial representation, starting at 2.1. The first critical points of this account are as follows:

2.1 We make to ourselves pictures of facts.

2.141 The picture is a fact.

The first sentence indicates that the sense a picture presents is what we give to a complex. The second intimates that there are constraints inherent in the complex upon what sense can be given to it. The second point is what interests us most here. From what precedes and follows it in TLP, we understand that the constraint imposed is precisely that no sense can be given to a complex unless the contents of the sense and the complex are of the same multiplicity, and the form of the sense is realized in the structure of the complex. Consider, for instance, the sentence immediately preceding 2.141:

2.14 The picture consists in the fact that its elements are combined with one another in a definite way.

and, later,

2.15 That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another.

This connexion of the elements of the picture is called its structure, and the possibility of this structure is called the form of the representation of the picture.

According to these points, a picture results from the co-ordination of the constituents of a complex C_0 with the constituents of some other complex C_1 with respect to a form realized by a structure of C_0 . The form with respect to which the constituents of a complex are co-ordinated with those of another is, of course, the form of representation. But the role of logical form becomes especially apparent in the consideration that a picture can, for Wittgenstein, be false, for it then becomes evident that not only is the complex C_0 structured in accordance with logical form, so is C_1 .¹⁴ If this were not so, then the constituents of C_0 could not be co-ordinated with those of C_1 for lack of agreement in multiplicity of those complexes. This Wittgenstein emphasizes in the following three sentences:

¹⁴ The form of representation could not guarantee this since in the case of incorrect pictures, if both C_0 and C_1 realized the form of representation, then the picture would be correct after all.

2.18 What every picture, of whatever form, must have in common with reality in order to be able to represent at all - rightly or falsely - is the logical form, that is, the form of reality.

2.181 If the form of representation is the logical form, then the picture is called a logical picture.

2.182 Every picture is also a logical picture. (On the other hand, for example, not every picture is spatial.)

Each of these sentences expresses, for Wittgenstein, what he took to be the fundamental role of logical form, not only for pictorial representation but for representation in thought and language. This is so precisely because, for Wittgenstein, logical form consists in the very possibility of objects entering into combination with one another.

The connexion of logical form with the distinction between function and argument, however, is not explicitly elucidated until much later in the Tractatus. For instance, at 5.47, he writes:

Where there is composition [combination], there is argument and function, and where these are, all logical constraints already are.

One could say: the one logical constraint is that which all propositions, according to their nature, have in common with one another.

That however is the general form of proposition.

That the general form of proposition consists in or depends upon logical form is not evident from the context of this passage. But this point does become evident through a consideration of an early letter to Russell. In particular,

consider what Wittgenstein writes in his letter of 16 January, 1913:

... I now think that qualities, relations (like love) etc. are all copulae! That means I for instance analyze a subject-predicate proposition, say, "Socrates is human" into "Socrates" and "something is human," (which I think is not complex).

Wittgenstein, here, already specifies the status of qualities and relations as modes-of-combination or, as he phrases it, copulae. They are, accordingly, not objects. What he writes directly below this passage, however, relates this distinction to what Wittgenstein, and then Russell, briefly took to be the symbol for a logical form. He writes:

... I want a theory of types to tell me that 'Mortality is Socrates' is nonsensical because if I treat 'Mortality' as a proper name (as I did) there is nothing to prevent me to make the substitutions the wrong way round. But if I analyze (it) (as I now do) into Socrates and $(\exists x) (x \text{ is mortal})$ or more generally into x and $(\exists x) \phi(x)$ it becomes impossible to substitute the wrong way round because the two symbols are now of a different kind themselves.

Though Wittgenstein would later revise his account about the symbolism of logical forms, the parallel between that distinction Wittgenstein (circa 1913) expected " $(\exists x)\phi(x)$ " to express and the distinction Wittgenstein in TLP expected some structures to show is exact: it is the distinction between the mode-of-combination of complexes and objects. Accordingly, a picture which effects this distinction is a

logical picture, a thought. This leaves us with the last point of this section. That concerning the routines the following of which results in an atomic proposition in language.

Apart from the above considerations concerning the distinction between modes of combination of objects and objects, there are several sentences which suggest that propositions must result from the following of a routine different from that followed in the case of other pictures. Consider, for instance, the following sentences:

3.1432 We must not say, "The complex sign 'aRb' says 'a stands in relation R to b';" but we must say, "That 'a' stands in a certain relation to 'b' says that aRb."

Wittgenstein is making two points in this sentence. The first says that all propositional signs express sense through the way in which the constituent symbols stand to one another in the sign. The second concerns what Wittgenstein calls [3.11] the method of projection. According to Wittgenstein's elucidation of this method, a proposition results from the projection of the signs in a propositional sign onto the constituents of some complex. Accordingly, what Wittgenstein explains in 3.142 is that it is only the names of an atomic proposition that are projected one to one and onto the constituents of some complex. The predicate already expresses the mode of combination asserted of the denotations of the

specific terms of the proposition. The form is what is shown by the structure of the propositional signs.

From the foregoing sections, one should be able to understand how for Wittgenstein the background and routines with respect to which an array of marks can count as a proposition in a language are adequate for the clear expression of sense. For Wittgenstein, adequacy is a requirement for languagehood. The sense of a proposition is supposed to be such that one could not fail to be capable of expressing it, if one understands one's language aright. Or as Wittgenstein himself cryptically remarks:

5.4732 We cannot give a sign the wrong sense.

The rationale for this largely consists in other observations. For instance, what he writes concerning what goes unexpressed in language:

3.262 What does not get expressed in the sign is shown by its application. What the signs conceal, their application declares.

4.002 ... Colloquial language is a part of the human organism and is not less complicated than it.

Supposedly, the background and routines of language are supposed to be in no need of alteration. Though they are tacit, concealed, clothed, they are also supposed to be such that if one is rightly oriented to them, then one understands

how language is logically adequate.

Molecular and General Propositions

2.0 In the previous chapter, I examined Wittgenstein's account of how pictures can express sense through presenting form. This examination consisted mostly of an outline of Frege's functional account of sense and of Wittgenstein's reasons for departing from Frege's account. The last section, however, dealt explicitly with Wittgenstein's account of form. According to this examination, Wittgenstein's account is supposed to reveal how the background and routines with respect to which the propositional sign is given a sense provide for the logical adequacy of languages. For Wittgenstein, no sense could or can be given to an array of marks or sounds, say, unless that array could be counted as presenting what Wittgenstein calls a logical form, where logical form is that which must be shown if the distinction between object and mode-of-combination, function and argument is to be presented. Thus, for Wittgenstein (in the case of atomic propositions), the routines which are followed in producing a proposition are adequate for the clear expression of sense. But this has yet to be shown in the case of molecular and general propositions. With respect to these cases, two problems arise. The first is presented as follows:

1 The problem as to how signs such as 'and', 'or' and 'not' and 'At least one item is such that' compound the senses of propositions or propositional functions with which they are combined.

This is the explicit problem of how the meanings of molecular and general propositions depend upon the meanings of atomic propositions, or such propositions whose terms are directly projected onto the constituents of some complex. The focus set, here, is upon how the symbolism of molecular and general propositions depends upon the symbolism of atomic propositions and pictures. But this manner of formulating the problem raises a much more crucial problem for Wittgenstein's account of general propositions, namely:

2 The problem of what the sense of a proposition that mentions an existent complex consists in.

As we shall see, Wittgenstein's solution to this problem commits him to the assumption that propositions purportedly about complexes count as general propositions, even in cases where complexes exist which satisfy those descriptions. But this solution fails to deal satisfactorily with cases in which the terms of a proposition appear to denote complexes and in which they have been directly projected onto the constituents of some other complex. Given Wittgenstein's pictorial account of atomic propositions and his account of general and molecular propositions, these cases threaten to undermine

Wittgenstein's assumption that colloquial languages are logically adequate.

Accordingly, the answers Wittgenstein gives to the first and second problems above show how he must deal with the cases of molecular and general propositions, given the assumptions with which he accepts the problem of clarity.¹ I will be arguing that whereas his answer to the first problem further shows the adequacy of languages, his answer to the second introduces several difficulties. In particular, I argue that either his answer to the second problem does not give an adequate and coherent account of the routines the following of which results in propositions about complexes or the answer he gives does not show the adequacy of ordinary language for the expression of the senses of such propositions. In his answer, Wittgenstein falters especially on the question as to whether a complex can be named and described. I will suggest, however, two ways in which Wittgenstein's account can be remedied.

Once these cases have been settled, the problem as to how language and theory are, for Wittgenstein, respectively distinct from a logically canonical notation and logic may be more explicitly examined.

¹ In particular, the realism and independence assumptions as well as the assumption that languages are logically adequate.

or 'The stabbing of Caesar by Brutus'.

The second sentence which gives Wittgenstein's position on the understanding of signs such as 'not', 'and' etc. supplements the negative point in 4.0312 with a positive one. It is as follows:

5.2341 The sense of a truth-function of p is a function of the sense of p .

Denial, logical addition, logical multiplication etc., etc., are operations.

(Denial reverses the sense of a proposition.)

According to Wittgenstein, a truth-function is a proposition that results from the combination of a sign ' p ' with propositional signs ' p_0 ' ... ' p_n ', where the truth-value of that result is specified as depending upon the truth-values of the propositions ' p_0 ' ... ' p_n '. For instance, (2) is supposed to result from (1) through a concatenation of the signs in (1) with 'not'. With respect to the canons of English grammar and diction, this result - (2) - is supposed to be true just in case (1) is false. Accordingly, the sense of (2) and of any proposition that results from the performance of such a routine is supposed to be determined by specifying the conditions of truth or falsehood of the original propositional sign upon which the truth-value of the result is supposed to depend. Thus, by these remarks, the sign 'not' as it enters into 'a is not F' or 'a is not R b' where 'is F' and 'is R' are English predicates, expresses how the truth-value of 'a is

not F' depends upon the truth-value of 'a is F'. Similarly, the other conjunctions (both subordinating and co-ordinating) are supposed to show how the truth-value of the propositions in which they enter depend for their truth-value upon the propositions which - indirectly - occur in them. Accordingly, the logical constants do characterize the senses of the propositions in which they occur. But, by 4.0312, they do not characterize the sense of a proposition in the way that a name or a predicate characterizes the sense of a proposition. In particular, they do not mean either an object or a mode-of-combination of a complex with which that sense may agree. Since this is the case, the problem arises for Wittgenstein as to what feature of the sense of a proposition these signs can determine. Wittgenstein, rather cryptically answers this question in 5.2341, where he explains that denial reverses the sense given by a proposition. What this suggests is that, at least in the case of 'not' the feature of sense that is determined is the orientation of the sense of a proposition with respect to the truth-value, T. For instance, if the sense of (1) may be assumed to have a positive orientation with respect to T, then (2) must be assumed to have a negative orientation with respect to T. Or more explicitly, the sense of a proposition may be assumed to consist in the proposition's expression of a positive or negative orientation of the meanings of its names to the meaning of its predicate,

2.1 The crux of Wittgenstein's account of the sense of molecular propositions is given in two sentences. The first is as follows:

4.0312 The possibility of propositions is based upon the principle of the representation of objects by signs.

My fundamental thought is that the "logical constants" do not represent. That the logic of facts cannot be represented.

Following Russell, Wittgenstein calls terms such as 'not', 'and' and 'or' logical constants. These are terms which when combined (or concatenated) with propositions or with the constituent symbols of propositions result in new propositions. For instance, in English, the result of recombining the signs of the proposition (1) 'Caesar was stabbed by Antony' with 'not' so that the proposition (2) 'Caesar was not stabbed by Antony' results. With respect to this example, what Wittgenstein is asserting in 4.0312 is that though 'not' is written in the writing of (2) and (2) is a proposition, 'not' does not mean any item that may be counted as a constituent or as a mode-of-combination of a complex. That is, even though (2) is true, there is no complex in whose being the truth of (2) consists. Thus, by 4.0312, there is no complex which has as a mode-of-combination or constituent a meaning of 'not', though, of course, there can be a complex which has as a mode of combination the meaning of 'stabbed' or of 'before' or has as a constituent the meanings of 'Caesar'

where this orientation is given with respect to a logical form shown by the proposition and the truth-value T. The senses of conjunctions of propositions may then be assumed to consist in a positive orientation given to the set of senses of the conjoined propositions. In this manner, given a set of propositions ξ , all the senses that depend upon the senses of the propositions in ξ are predetermined even before some person utters a proposition that expresses such a sense.

One interesting feature of this account is that, for Wittgenstein, a picture can express a negative sense (a sense with a negative orientation) even though it lacks such a sign as '~'. The orientation of sense can be given (and expressed) without the uses of a sign such as 'not' or '~'. The manner or context in which one presents a picture may itself determine the orientation of the picture's or proposition's sense.'

Given this analysis, one can better appreciate how, for Wittgenstein, even in the case of the senses of molecular propositions, the routines the following of which results in the expression of sense are logically adequate. For instance, the routines the following of which results in negations (propositions which express negative senses) determine how

² In the case of propositions written ironically, as in Swift's, A Tale of a Tub, a negative sense is expressed without any occurrence of a negation sign.

some sign, say, '~', can be used to express a negative sense, in the language of those routines. According to Wittgenstein, confusion can arise with respect to the understanding of such signs not through the following of those routines but through some such assumption as that all the signs in a symbol refer to objects. Not only do the routines not give rise to such an assumption, once one comes to understand these routines, for Wittgenstein, one ceases to accept similarly mistaken assumptions about how constituent signs of a proposition characterize the proposition's sense. This is so since ultimately these routines are those the following of which is supposed to result in the depiction of complexes. Since that which explicitly means is distinguished from that which (tacitly) expresses in a picture, all routines which are supposed to result in depiction must comply with or respect this distinction if they are to be consistently successful. Thus, the canons of grammar and diction within a language must be adequate for the distinguishing of signs which can denote complexes or objects and those which can express relations from signs which express the orientation of combinations of senses.

2.20 The case of general propositions is a little more complicated than that of molecular propositions. For with respect to that case functional expressions such as 'x is to the left of y' are introduced, and with these further

propositional signs may also result, though, as we shall see, not necessarily new propositions. ³ Following Russell, Wittgenstein treats these as propositional functions, where 'x' and 'y' behave like pronouns. Once 'x' and 'y' are replaced by names or descriptions, a proposition results. But this is not the only way in which a proposition can result from some routines performed with such functional expressions (propositional functions). Another important routine the following of which results in the expression of sense from some propositional function is quantifier introduction. This routine consists in the concatenation of expressions such as 'At least one x is such that' or 'For some x, ...' with a propositional function ' ϕ ' in which 'x' enters as a variable. With respect to the results of the performances of such routines, distinctions arise between variables of a proposition that occur in that proposition within the scope of a quantificational sign and those that do not. Besides deciding whether or not the variable that occur in the propositional function also occur in the quantificational sign, the scope of a quantificational sign is decided with respect to canons of grammar, in particular canons of punctuation within a language. Consider, for instance, the following proposition:

³ These result from the substitution of variables (or pronouns) for terms in a proposition.

(3) 'At least one x is such that x is black and for any x , if x is black, then x is extended'.

In (3), the scope of 'At least one x is such that' is ' x is black' and not ' x is black and for any x , if x is black, then x is extended'. This is clear, notwithstanding the double occurrence of ' x ' in (3), since it is tacitly understood that the scope of a quantificational expression extends no further in a sign than is necessary for the whole sign to count as a proposition. Since, in (3), 'if x is black, then x is extended' is prefixed with the quantificational sign 'for any x ', it would be superfluous to extend the scope of 'At least one x is such that' through the rest of the sign. Moreover, even if the above scope-rule were not tacitly understood, the scope of the quantificational expression could still be reasonably counted as excluding 'and for any x , if x is black, then x is extended' for if it did not, then no proposition would unambiguously result. It would be unclear, for instance, whether or not the proposition that results would be false if 'At least one x is such that if x is black, then x is extended' were false. Of course, present canons of English or of any language may not be adequate for the disambiguation of scope within signs that result from quantification introduction but nothing within colloquial languages prevents the establishment of such canons. It is for this reason that colloquial languages may still be counted as logically

adequate. They can be extended and refined so as to accommodate the expression of new senses and they can do so only with respect to an extended background of picturing.

Besides this distinction between free and bound occurrences of variables, another distinction, one more pertinent to the two problems discussed earlier about general propositions, becomes manifest, namely, Wittgenstein's distinction between formal and non-formal concepts. Before this distinction is invoked in the special case of general propositions, it should be discussed in general.

2.21 Wittgenstein explains the difference between formal and non-formal concepts at 4.122 - 4.126, 4.12721. This is combined with a discussion of internal relations and properties. Since he also speaks of internal relations as formal relations, as relations of structures, if there is a distinction, for Wittgenstein, between formal relations and formal concepts, then this distinction is difficult to discern within TLP. Accordingly, I will treat these terms, 'relation' and 'concept', as covering the same items, for Wittgenstein.⁴

⁴ Of course, whereas Wittgenstein speaks of formal relations as holding between objects, he speaks of concepts as having objects fall under them. Moreover, whereas he appears to be more concerned with how we can think of formal relations, he appears to be more concerned with how formal concepts can be expressed. If Wittgenstein meant to express a distinction, here, then it would consist in the difference that whereas concepts have being only for expression in language, relations do not. Concepts, then, would count as items which mediate between representations of how things

According to the passage cited above, a formal concept under which an item falls is one with respect to which it is unthinkable that that item does *not* fall under it. For instance, he writes in TLP as follows:

4.123 A property is internal if it is unthinkable that its object does not possess it.

4.12721 The formal concept is already given with an object, which falls under it.

Wittgenstein further explains that formal concepts are expressed in ordinary languages by terms such as 'object', 'complex', 'fact' and 'function', and within canonical notations are correctly expressed through what he calls the features of the notation. For instance, the variables of a

stand to one another and the reality of how things stand to one another, namely, the properties and relations. But still no such distinction has been made evident, here. For, firstly, Wittgenstein is not consistent in distinguishing his descriptions of relations and concepts, at least not in translation, (see, for instance, 4.124 where Wittgenstein speaks of formal properties as expressible or not expressible and other passages in which he speaks of properties and concepts as existing). Secondly, even if his descriptions of properties and concepts were kept distinct, this would not argue in itself a difference in what items the term 'properties' or 'relations' covers and what the term 'concepts' covers, since the difference may simply consist in a difference in what is being explained about what both types of terms cover. Lastly, in 4.126, Wittgenstein explains that he introduces the expression 'formal concept' "in order to make clear the confusion of formal concepts with proper concepts which runs through the whole of the old logic." This suggests that the change in terminology is not in a change of the items or matters of which he has previously been discussing but a change for the purposes of clarification of what he has already discussed.

canonical notation are supposed to exhibit features according to which they express formal concepts - 'x' expresses the formal concept expressed by 'object', a concept under which the meaning of any name in the notation substitutable for 'x' is supposed to necessarily fall.

The importance of this distinction between formal and informal concepts in Wittgenstein's account of general propositions consists in the tensions between his truth-functional account of propositions and his account of the meaning or significance of quantificational signs such as ' $\exists x$ '. I will be arguing that according to other theses in TLP, signs such as ' $\exists x$ ' or 'At least one item is such that...' directly or explicitly express formal concepts not expressed directly in truth-functions. Consider the case of primitive predicates. Their meanings are supposed to be modes-of-combination shown by pictures. Since they are shown, some objects must be combined by them. Thus, for every meaningful predicate, it is unthinkable that its meaning not fall under the concept expressed by ' $\exists x$ '.⁹

⁹ Of course, this conclusion does not also hold generally of derived predicates or propositional functions. Nonetheless, contrast this with the case of primitive names. For Wittgenstein, there is no one propositional function which expresses a concept under which the meaning of any primitive name must fall. Whether there is a concept under which an object can fall depends upon whether some fact and which fact is the case, even though the being of objects does not depend upon what is the case or even upon whether some fact is the case. By contrast, for Wittgenstein, there are no objects, or

Much of the above discussion is not yet to the point concerning Wittgenstein's account of the sense of general propositions, but it is a necessary preliminary since general propositions are the results of concatenations of 'At least one x is such that' with a propositional function ' ϕ ' with ' x ' entering ' ϕ ' outside the scope of any quantificational signs in ' ϕ '.⁶

2.22 Concerning the sense of general propositions, Wittgenstein has principally two observations to make. The first is - with a slight alteration in expression - as follows:

If ξ is the totality of propositions resulting from the substitution of names, $\bar{\quad}$ for free occurrences of ' x ' in ' ϕ ', then $\forall\{P:P\in\xi\}$ is true just in case 'At least one x is such that ϕ ' is true. [See 5.54 and 5.52]

This asserts the truth-conditional equivalence of general propositions with certain forms of disjunctions. According to Wittgenstein's analysis of the sense of molecular propositions

first order concepts (concepts under which objects fall) unless they fall under formal relations. In the case of proper concepts, it is essential that they fall under the concept expressed by 'At least one x is such that..', for, as argued in the previous chapter, the being of primitive relations and concepts does depend upon what is the case.

⁶ In such a case, ' x ' will be said to occur freely in ' ϕ '; otherwise, unfreely.

⁷ Assuming that there are as many names as objects combinable by the mode-of-combination expressed by ' f '.

in terms of truth-conditions, this ought to argue for the equivalence in sense of $\forall P$ and 'At least one x is such that ϕ '. But by Wittgenstein's second observation, this is just not so. He writes immediately afterwards the following sentences:

5.521 I separate the concept *all* from the truth-function.

Frege and Russell have introduced generality in connexion with the logical product or the logical sum. Then it would be difficult to understand the propositions " $(\exists x).fx$ " and " $(x).fx$ " in which both ideas lie concealed.

5.522 That which is peculiar to the "symbolism of generality" is firstly, that it refers to a logical proto-type, and secondly, that it makes constants prominent.

According to 5.521, the quantificational expression 'For every x ' expresses a concept. But unless this were just a manner of speaking, he would have to concede that the sign 'is a totality', to which it is closely allied, counts as a genuine predicate.⁸ But what Wittgenstein writes elsewhere concerning the impossibility of speaking of totalities precludes Wittgenstein from allowing 'is a totality', and so 'For every x ', to express non-formal concepts. Accordingly, what keeps " $(\exists x).fx$ " from expressing a sense that is equivalent to what some disjunction ' $\vee\{fa:Fa\in\xi\}$ ' expresses is that whereas the former contains a sign that expresses a formal concept, the

⁸ By 5.52, 'For every x , fx ' is true just in case ' ξ is a totality of values of ' fx ' and ' $\wedge\{P:P\in\xi\}$ ' are true.

latter does not. (But, then, strictly speaking, for Wittgenstein, 'At least one x is such that fx ' lacks sense.) These remarks can be seen through a consideration of the following alternative.

Either Wittgenstein allows that the totality of objects of which the meaning of 'f' is predicable in a world is independent of what is the case, or he allows that 'f' may mean the same with respect to different facts being the case while 'At least one x is such that fx ' expresses a different sense.

In order to see why this is so, suppose that the first disjunct does not hold. Then there may be two worlds - two totalities of facts - W_0 and W_1 with respect to which the totality of objects of which the meaning of 'f' is predicable differs. Suppose that in each of those worlds every object in the respective totalities of predicables is named. Let ξ_{w_0} and ξ_{w_1} be the totalities of propositions that result from the substitution of the names in the respective totalities for ' x ' in ' fx '. Then 'At least one x is such that fx ' expresses in W_0 what $\vee\{P: P \in \xi_{w_0}\}$ expresses in W_0 . Similarly, 'At least one x is such that fx ' expresses in W_1 what $\vee\{P: P \in \xi_{w_1}\}$ expresses in W_1 . But $\vee\{P: P \in \xi_{w_0}\}$ expresses a different sense than $\vee\{P: P \in \xi_{w_1}\}$. So, 'At least one x is such that fx ' expresses a sense in W_0 that is different from the sense it expresses in W_1 .

Now, neither of these alternatives is acceptable for Wittgenstein. By 3.3, no object is nameable unless it is a

constituent of a complex. Thus, with respect to a world W , only such objects as occur in existing complexes can be said to bear properties or stand in relations to other objects. But which way the world is is entirely contingent. Thus, the totality of objects of which a property is predicable depends upon what is the case in the world in which the predication is made. The other disjunct is unacceptable, also, since it violates compositionality. The predicate and the quantificational expression are allowed, here, to continue to express the same concepts while the proposition resulting from their combination is allowed to change. This problem becomes even more obvious if one allows that every fact in W_0 is a fact in W_1 , but not conversely. Then $\forall \xi_{W_0}$ expresses a sense expressible in W but distinct from the sense expressed by $\forall \xi_{W_1}$ while 'At least one x is such that fx ' does not.

The only way for Wittgenstein to avoid this dilemma is to assume that 'At least one x is such that ...' expresses a formal concept not explicitly expressible by any truth-function, even one with infinitely many propositions as bases. As observed earlier, this is contradicted by Wittgenstein at such propositions as 5, where he explains that all propositions are essentially truth-functions of elementary propositions. But this is corroborated by what Wittgenstein assumes at 5.552. In the former he suggests not only that every proposition is, depending upon which way the world is,

truth-conditionally equivalent to some truth-function of elementary propositions, but that it has the same sense as such a truth-function. By the above considerations, this position is untenable. Fortunately, 5.552 suggests an alternative account. There he says that what distinguishes a general proposition from a truth-function is that 'it refers to a proto-type'. Though Wittgenstein's notion of a proto-type is no more clear than his account of general propositions, it does clarify the formality of the concepts expressed by quantificational expressions.

At 3.315, Wittgenstein explains that a logical proto-type results from the replacement of all constituent symbols in an atomic proposition by variables, where, in general, there is a distinction between term and predicate variables. For instance, given 'a is black', the sign ' ϕx ' counts as its logical proto-type. Corresponding to this proto-type are both i) a class of propositions from which through like substitution such a proto-type results, and ii) a logical form shown by each of the propositions in that class of propositions and supposedly by ' ϕx ' as well. In so far as a general proposition such as ' $\exists x(fx)$ ' refers to or accentuates the proto-type ' ϕx ', it differs from such a proposition as may be expressed as

' $fa_0 \vee fa_1 \vee \dots$ '

for this latter expression is less articulate in the respect of expressing what is generalized, that is, in expressing what counts as the name and what as the predicate. This inarticulateness is compensated for only if one rewrites the expression as follows:

$$' \forall \{fa_x : fa^k \in \xi\} '$$

where one stipulates that ξ is the totality of values of 'fx'. Without such a stipulation, of course, one would not know that 'At least one x is such that fx' is false if one knew that each 'fa_x' in ξ is false. But this very stipulation re-introduces the expression of a formal concept, namely, the concept of a totality.

Given the above account of Wittgenstein's analysis of general propositions, it may seem especially unlikely that the routines with respect to which propositions are introduced are adequate for the clear expression of senses of general propositions. But, for Wittgenstein, the contrary is in fact the case. This can be appreciated through reconsidering Wittgenstein's account of formal concepts. They are supposed to be such that they are already given with items which fall under them. For Wittgenstein, one could not be presented with a mode-of-combination (primitive relation) unless some complex existed whose constituents were combined (related) by that mode-of-combination (relation), nor could one be presented with a concept unless some complex existed one of whose

constituents fell under that concept. This is a corollary of Wittgenstein's characterization of the background of language as consisting of pictures, where a picture is a complex whose structure shows a pictorial form. The pictorial form was supposed to constitute a (non-logical) mode-of-combination of objects. Thus, no primitive concept or relation could be presented by a picture unless the formal concept expressed by the quantifier ' $\exists x$ ' were also expressed. That is, one could not understand a picture, or proposition, for that matter, unless one grasped the formal concept expressed by ' $\exists x$ '.

Thus, regarding the routines with respect to which general propositions are introduced, Wittgenstein explains that these routines could be successful only in so far as their results satisfied the following conditions:

- i) They have variables occurring in term positions of propositions from which they result.
- ii) They show what is generalized and the scope of the generalization in the proposition.

But, as will be presently discussed, many expressions that, according to Wittgenstein's account of sense, count as general propositions do not satisfy these conditions. In particular, many expressions in ordinary language express the senses of general propositions even though they only have term signs and not variables occurring in term positions. For example, according to both Russell and Wittgenstein, the proposition

'The first Emperor of Rome was assassinated on the ides of March' is supposed to be equivalent in meaning to the proposition 'There is one man who was first emperor of Rome and he was assassinated on the ides of March', even though the first apparently has and the second lacks an occurrence of a denotative phrase.

2.30 This brings us to the second problem mentioned in the introduction of this chapter, namely:

The problem of showing what the sense of a proposition that mentions an existent complex consists in.

Though Wittgenstein's analysis of general propositions was shown to introduce some difficulties for his account of propositions, none of those difficulties is peculiar to Wittgenstein's analysis. By contrast, his answer to the above problem raises objections that are unique for his account of sense and of the expression of sense in ordinary language. Most of these problems devolve from Wittgenstein's assumption that complexes cannot be named. In particular, this assumption seems to undermine Wittgenstein's account of the routines with respect to which a complex counts as a proposition in a language, for since most of what we speak would have to count as about complexes, it is unlikely that a proper name could occur in whatever counts as a proposition in a colloquial language. In that case, only the forms of

general propositions would be presented in ordinary language and presented by propositions which are structured like logical pictures of atomic facts. But, as I have argued in Chapters 0 and 1, Wittgenstein's account of the canons of grammar and diction in the background of a proposition in a language was supposed to show how languages are logically adequate. It seems, now, that no canons in the background of a proposition - not even one from a logically canonical notation - could be adequate for the clear expression of sense.⁹

Considering this problem, one can appreciate how it is important for Wittgenstein to give an account of the sense of a proposition that mentions a complex. Unless he has, in conjunction with his account of the sense of propositions about complexes, an account of how, notwithstanding their structures, many of those propositions clearly express sense, his pictorial account of propositions would cease to support

⁹ Though a logically canonical notation could express the sense of a general proposition clearly, it could not express the sense of an atomic proposition. Moreover, a propositional sign in a language could, for Wittgenstein, express a general sense only if the values of its variables have been stipulated and if its predicates have been given a meaning. Neither of these routines could be performed except with respect to a background of logical pictures, i.e. thoughts. But Wittgenstein's account of sense makes it unlikely that even thoughts - which count as psychological facts - could express the sense of an atomic proposition. Most of our thoughts are shown to be about complexes, not simples (eg. chairs, tables, people).

the assumption that languages are or can be logically adequate. The only account of the latter sort that Wittgenstein provides in TLP is in terms of the determination of form through the routines of assertion. These - the routines followed and the background setting a context of assertion - are supposed to show the sense of what is said when one follows these routines. This position Wittgenstein articulates at 3.26-3.263 and at 3.24, 3.262 and 4.002. It is at these and other passages that Wittgenstein provides for a response to the above difficulties in terms of his pictorial account of the sense of propositions.

Each of these points, however, can be illustrated in detail only after his account of the sense of propositions about complexes has been clarified. Accordingly, an exposition of this position is, now, in order.

2.31 Wittgenstein first writes explicitly of propositions that mention complexes at 3.24. He writes as follows:

A proposition about a complex stands in an internal relation to the proposition about its constituent part.¹⁰ A complex can only be given by its description, and this will either be right or wrong ...

¹⁰ This translation is a bit misleading, since it suggests that Wittgenstein is assuming something in 3.24 only of complexes with exactly one constituent. In contrast, Pears and McGuinness translate 3.24 as follows:

A proposition about a complex stands in an internal relation to a proposition about a constituent of the complex.

Though Wittgenstein unequivocally expresses his position, here, one question concerning that position is insufficiently explained in TLP, namely, the question as to what terms and predicates enter into a definite description that denotes an existent complex. Some of what needs to be elaborated in the case of this point consists in a discussion of Russell's work in Theory of Knowledge on the specification of complexes, in particular, the specification of what Russell calls permutive complexes. From Wittgenstein's Notebooks and letters to Russell, we know that Russell frequently showed Wittgenstein his work on these problems and in this passage, Wittgenstein is strongly relying upon Russell's account of definite descriptions. This, of course, is not sufficient evidence for reading what Wittgenstein writes above along the lines of Russell's analysis. Nonetheless, whether or not Wittgenstein has Russell's analysis of definite descriptions of complexes in mind, Russell's account presents a clearly articulated solution which Wittgenstein's remarks here may be taken to follow or depart from. Accordingly, some illumination of what Wittgenstein assumes above may result from a brief exposition of Russell's analysis.

The problem of the specification of complexes arises for Russell with respect to his assumption that the truth of a judgement consists in the existence of a complex corresponding

to that judgement. The problem for Russell was to provide a rule with respect to which one could give for any accurate specification of a judgement, a specification of what must exist if the judgement is true. Consider, for instance, Russell's case of someone's judgement that a is before b. If this judgement were true, then some complex consisting of a and b and whose mode of combination is the relation /before/ exists. But, of course, two complexes can exist that satisfy this description, one in whose existence the truth of the judgement that a is before b consists, another in whose existence the truth of the judgement that b is before a consists. Since, moreover, these complexes are not compossible (cannot both exist), the lack of a definite description of the latter complex would seriously impede Russell's theory of the truth of judgements. In order to overcome this problem, Russell postulated that for each relating relation R, there is a set $\{C_R^k: k < \alpha(R)\}$ ¹¹ of binary relations such that for all $x_0, \dots, x_{\alpha(R)}$ and every judgement that $Rx_0, \dots, x_{\alpha(R)}$, that judgement is true just in case exactly one γ exists such that $x_0 C_R^0 \gamma, x_1 C_R^1 \gamma, \dots, x_{\alpha(R)} C_R^{\alpha(R)} \gamma$.¹² With respect to the above case, this postulate enabled Russell to uniquely specify the complex in whose existence the truth

¹¹ $\alpha(R)$ is the arity of the relation, R.

¹² In cases where R is permutable, some of the relations C_R^k are equivalent.

of the judgement that a is before b consists. For instance, it could then be specified as the complex γ such that a is the earlier constituent of γ and b is the later constituent of γ .

Suppose, now, that in 3.24 Wittgenstein is presuming Russell's analysis of definite descriptions. Then some clarification of what Wittgenstein assumes at the first and second sentences in that entry can be provided. But, as will be seen, a deeper confusion is introduced into Wittgenstein's account.

According to what Wittgenstein assumes at the second sentence in 3.24, if 'R' is a meaningful predicate, no true proposition could result from the substitution of a proper name '**b**' for the variable ' γ ' in the induced propositional function ' $\wedge a_k C_R^k \gamma$ '. In order to understand Wittgenstein's rationale for this assumption, suppose that the contrary assumption is correct. Then since, by 3.3, a name means only in the context of a proposition, there is some atomic proposition 'p' in which '**b**' occurs as a term. By the above assumption, 'p' is truth-conditionally equivalent to the result of substituting ' $(\iota \gamma) (\wedge a_k C_R^k)$ ' for '**b**' in 'p'. But this result is truth-conditionally equivalent to the proposition 'q' = ' $\exists \gamma (\forall \gamma_1 (\wedge a_k C_R^k \gamma_1 \leftrightarrow \gamma = \gamma_1) \wedge p[\gamma/\mathbf{b}])$ ', by Russell's theory of definite descriptions. Now either truth-conditional equivalence suffices for sameness of sense or it does not. In the former case, 'p' may count both as an elementary and as a

general proposition, which is in violation of Wittgenstein's account of atomic and general propositions. In the latter case, since, for Wittgenstein, the meaning of a term, if it has any meaning, consists solely in its reference, the only way in which the sense of 'p' could be distinct from the sense of 'q' is if ' $\exists\gamma(\forall\gamma_1(\wedge a_k C_R^k \gamma_1 \leftrightarrow \gamma=\gamma_1))$ ' is true. This is so, for '**6**' is a proper name and is supposed to refer to what satisfies but is not specified in ' $\exists\gamma(\forall\gamma_1(\wedge a_k C_R^k \gamma_1 \leftrightarrow \gamma=\gamma_1))$ '. Thus, whether 'p' has the sense that it does depends upon whether ' $\exists\gamma(\forall\gamma_1(\wedge a_k C_R^k \gamma_1 \leftrightarrow \gamma=\gamma_1))$ ' counts as true. This is not supposed to be allowed for in Wittgenstein's pictorial account of propositions. In particular, it conflicts with what he assumes at 2.0211.

2.0211 If the world had no substance, then whether a proposition had sense would depend upon whether another proposition was true.

Since Wittgenstein, here, is supposed to be showing that the world has substance, what he is assuming at 2.0211 is that whether a proposition has sense cannot without exception depend upon whether another proposition is true. According to Wittgenstein's pictorial account, atomic propositions - at least- are supposed to be such exceptions. Thus, by that account ' $\wedge a_k C_R^k \mathbf{6}$ ' cannot correctly count as a true proposition if '**6**' correctly counts as a proper name.

The first sentence quoted above from 3.24 indirectly

tells us about the possible meanings of the binary predicates ' C_R^k '. According to what Wittgenstein assumes at the first sentence in 3.24, the proposition ' $Ra_0, \dots a_{\alpha(R)}$ ' stands in an internal relation to ' $F(\iota\gamma)(\wedge a_k C_R^k \gamma)$ '. This is so, for Wittgenstein, for principally one reason, namely, because ' $Ra_0, \dots a_{\alpha(R)}$ ' is true if ' $F(\iota\gamma)(\wedge a_k C_R^k \gamma)$ ' is true, even though the latter is not the result of an application of some truth-operation upon the former. That is, neither the meaning of ' F ' nor the sense of ' $Ra_0, \dots a_{\alpha(R)}$ ' determine a sense for ' $F(\iota\gamma)(\wedge a_k C_R^k \gamma)$ '. Rather, the sense of ' $F(\iota\gamma)(\wedge a_k C_R^k \gamma)$ ' is only determined through the contextual definition of the predicates ' C_R^k '. Since, for Wittgenstein, the propositions ' $Ra_0, \dots, a_{\alpha(R)}$ ' and ' $F(\iota\gamma)(\wedge a_k C_R^k \gamma)$ ' are supposed to be internally related to one another, an initially plausible response would be that the meaning of ' C_R^k ' consists only in a formal relation between the meanings of ' $a_0, \dots, a_{\alpha(R)}$ ' and the fact that would make ' $Ra_0, \dots a_{\alpha(R)}$ ' true. Further support for this reading derives from a consideration of what Wittgenstein assumes at the second sentence. For if ' C_R^k ', say, expressed a formal relation, then for some proper names ' $\mathfrak{a}_0, \mathfrak{a}_1, \mathfrak{a}_{\alpha(R)}$ ' could express a sense of which the meaning of ' C_R^k ' was a formal feature. But, by the above analysis of what Wittgenstein assumes at the second sentence, no such sense could be expressed by ' C_R^k '.

The difficulty with this reading of 3.24 is that since,

for Wittgenstein, formal concepts and relations are not correctly expressed by predicates, if ' C_R^k ' merely expressed a formal relation, then it is difficult to see in what the truth of ' $F(\iota\gamma) (\wedge a_k C_R^k \gamma)$ ' consists.¹³ With respect to the above reading, it is difficult to understand how, for Wittgenstein, ' $F(\iota\gamma) (\wedge a_k C_R^k \gamma)$ ' can express sense, even if ' F ' has already been given a meaning. But, as Wittgenstein also explains in 3.24, the occurrence of a definite description as a term in a sentence is not supposed to make the sentence senseless.

Now, the above reading is premised upon the assumption that Wittgenstein follows Russell's analysis of definite descriptions of complexes. Can a different, more promising, reading result from the assumption that Wittgenstein departs from Russell's analysis? Perhaps. But the only path one can follow in order to satisfactorily answer this question is to determine in what respect Wittgenstein's analysis must depart from Russell's. What led the above reading into difficulties was the assumption that predicates such as ' C_R^k ' must occur in the definite description of a complex. According to the manner in which these are introduced in a language, they could not be such as occur in atomic or molecular propositions.

¹³ Is ' $F(\iota\gamma) (\wedge a_k C_R^k \gamma)$ ' true, for instance, just in case ' $Ra_0, \dots, a_{\alpha(R)} \wedge \forall a_k Fa_k$ ' or ' $Ra_0, \dots, a_{\alpha(R)} \wedge (F \text{ holds almost everywhere in } \{a_1, \dots, a_{\alpha(R)}\})$ ' is true.

Thus, their significance could be allowed to consist solely in some formal relations or in some use. In neither case would their significance be such as to allow ' $F(\iota\gamma) (\wedge a_x C_R^k \gamma)$ ' to express a sense. Evidently, the respect in which Wittgenstein's analysis must depart from Russell's is in assuming that - in some cases - the predicates that ultimately occur in the definite description of an existent complex mean what can be meant by predicates in atomic propositions.

Given this reading of Wittgenstein's analysis, his response to the problem set at the beginning of this chapter must be that the sense of a proposition that mentions an existent complex is the sense of some existential proposition. In particular, a proposition ' $F[(\iota x) (\phi)]$ ' has the same sense as the proposition ' $\exists y (\phi[y/x] \wedge Fy) \wedge \sim \exists x \exists y (\phi \wedge \phi[y/x])$ ', where the primitive predicates that occur in the propositional function ' ϕ ' mean what can be presented by the structure of some picture.

Now, if Wittgenstein were to allow that ' $\exists y (\phi[y/x] \wedge Fy) \wedge \sim \exists x \exists y (\phi \wedge \phi[y/x])$ ' expressed the same sense as some truth-function of the results of substituting names into ' $\phi[y/x] \wedge Fy$ ' and ' $\phi \wedge \phi[y/x]$ ', then Wittgenstein would have to allow that the complex denoted by ' $(\iota x) (\phi)$ ' is nameable after all. Accordingly, he must assume that the sense of ' $\exists y (\phi[y/x] \wedge \sim \exists x \exists y (\phi \wedge \phi[y/x]))$ ' is distinct from the sense of such truth-functions. In particular, its sense cannot consist of any

specified objects, whether simple or complex.

The singular oddity of Wittgenstein's position is that, according to it, though a proposition may be about an existent complex, that complex is not a constituent of the sense of that proposition. Whether or not this oddity constitutes an outright contradiction remains to be seen. I will be returning to it in the conclusion of this thesis. But at least a single reading of Wittgenstein's account of the sense of propositions about complexes has been settled upon. Accordingly, I may proceed to explain more fully how what Wittgenstein writes in 3.26-3.263 and 3.24, 3.262 and 4.002 provides for an account of how propositions such as 'That is red' clearly express sense, even in contexts where 'That' means a complex.

2.32 The first passage (3.26-3.263), in particular, suggests that since some terms are introduced in the context of definitions involving definite descriptions,

e.g. 'Fa' is true just in case 'F(ιx)(ϕ)' is true

the propositions in which they occur indirectly express through that introduction the sense of propositions that result from the substitution of the definite description for that term, where this result is understood to be equivalent to some general proposition. The clarity of expression - the showing of form - consists in the understanding of the

routines of introducing definite descriptions and terms into propositions. But not all terms that mean complexes come to occur in propositions through the performance of such routines. Some seem to be projected directly onto complex constituents of facts - e.g. the demonstrative in 'That is a chair and that is red'. According to Wittgenstein, this, of course, cannot be so. Notwithstanding the initial implausibility of Wittgenstein's position, some of what Wittgenstein remarks in 3.24, 3.262, 4.002 suggests how these cases can be clearly distinguished from such cases in which a simple object is so denoted. In 3.24, Wittgenstein writes,

That a propositional element signifies a complex can be seen from an indeterminateness in the proposition in which it occurs. We know that everything is not yet determined by this proposition. (The notation for generality contains a prototype.)

Though this passage deals expressly with the first case above, the point it expresses can be adapted to some of what Wittgenstein writes at 3.262 and 4.002 so that it applies also to the second. In 3.262, Wittgenstein remarks that 'what does not get expressed in the sign is shown by its application', and in 4.002,

Man possesses the capacity of constructing languages, in which every sense can be expressed, without having an idea how and what each word means ... Colloquial language is a part of the human organism and is not less complicated than it. From it it is humanly impossible to

gather *immediately*¹⁴ the logic of language.

In the last sentence quoted from 4.002, notwithstanding that Wittgenstein assumes that it is difficult to understand clearly propositions such as 'That is red', where 'that' mentions a complex, he does at least assume that they show a form, that they are susceptible to a periphrasis into a logically canonical proposition. Wittgenstein explains in 3.262 that we must look to not only the sign but the background and routines with respect to which it counts as meaningful in a given context. Supposedly, the background and routines with respect to which a term such as 'that' means a complex must properly include and involve the background and routines with respect to which it can mean a simple. Accordingly, in lieu of the notations for generality, one can clearly say something about a complex, if one understands the background with respect to which the proposition says something of the complex. The background and not the notation shows how a proposition purportedly about a complex expresses what is canonically expressed by a general proposition. Since 'that' is a demonstrative pronoun, there is some indeterminateness in the proposition in which it occurs. What 'that' refers to cannot be determined except with respect to antecedent occurrences of 'red' in propositions and with

¹⁴ My italics.

respect to antecedent occurrences of 'that' or some other element of a logical, atomic picture, for otherwise one's pointing within one's field of vision while uttering 'that' in 'That is red' would not determine a complex rather than simple red item in that field.

In the case of terms that have entered into propositions via definitions, Wittgenstein has a fairly straight-forward analysis of how the propositions in which they occur can be clearly understood. But, in some cases, a term occurs in a proposition and means a complex even though it did not enter into that proposition via some definition. It is cases such as these that present difficulties for Wittgenstein's pictorial account of propositions, for, according to the above analysis, these propositions express the senses of general propositions. Unless the background and non-definitional routines with respect to which such arrays of signs count as propositions are also such that they show those propositions to be about complexes, it would seem that account of the sense of propositions about complexes undermines his assumption that languages are logically adequate. The above considerations suggest how, for Wittgenstein, these difficulties may be overcome. But unless these considerations can receive clear illustration within contexts of colloquial speech and writing, they will not remain very compelling. Unfortunately, Wittgenstein is not especially forthcoming with illustrations

and it is unlikely that any decisive cases could be provided.

2.4 The preceding two chapters have presented Wittgenstein's pictorial account of propositions as providing for a solution to what I have called the problem of clarity. It has been supposed to be an explanation the understanding of which gets the reader into the position of accepting the logical adequacy of language, not necessarily as it is written and spoken but as it can be if written or spoken carefully. The above difficulties, however, raise doubts concerning the correctness of this explanation - at least as a solution to PC*. Accordingly, the previous discussion of these difficulties provides an appropriate point of departure for a reading of TLP as providing for a dissolution of the problem. The assumption of this latter point is that these difficulties are not peculiar to Wittgenstein's position, but must arise with respect to any attempt to explain how what can be said can be said clearly, whether this be argued in the case of languages or logically canonical notations. The next chapter, in particular, will introduce Wittgenstein's distinction between saying and showing. I will attempt to show how many of Wittgenstein's criticisms of Russell (and Frege) can best be understood as objections that Russell (and Frege) failed to distinguish between the ways in which languages and logically canonical notations properly mean through misunderstanding the distinction between what can be said and what can be shown.

The last chapter will show how with respect to the assumptions Wittgenstein accepts in dealing with PC* the distinction between what can be said and what can be shown must be taken as an ultimate rather than a relative one.

Chapter 3: Russell's Paradox and the Theory of Types

3.0 In the previous two chapters, Wittgenstein's pictorial account of propositions has been read as an attempt to show how colloquial languages are logically adequate. While the first chapter argues that, for Wittgenstein, the background and routines with respect to which a complex counts as an atomic proposition provide for the clear expression (in the language of that proposition) of what can be said, the second argues that the routines with respect to which a complex of marks, say, counts as a molecular or general proposition preserve the adequacy of the language of elementary propositions, while extending its expressiveness. Both chapters argue that, for Wittgenstein, a language could not arise except with respect to a background in which and routines the performance of which sufficed for the showing of what Wittgenstein called logical form. Since, for Wittgenstein, the recognition logical form was essential for the understanding of propositions, no complex of signs in a language could count as a proposition unless it showed - whether directly or indirectly - how to say clearly in its language what it says. Accordingly, a spoken or written language could not arise unless the routines the following of which results in the propositions of that language are

adequate for the clear expression of what can be expressed.

In the course of the exposition in the second chapter, many propositions in colloquial language have been described as results of routines which do not exemplify previously formulated, agreed upon or set rules and yet which express the senses of general propositions within language. For instance, it often seems in colloquial language that the terms in a proposition denote complex constituents of a fact even though those terms have not entered into that proposition through some definition. Apart from some of the cases considered in the last chapter, one need only consider occurrences of demonstrative pronouns in questions such as 'What is that?' and in the sentences uttered in response to them. If such a question presupposed an understanding of some definite description which the reference of 'that' satisfied, then there would apparently be no need for asking the question in the first place. In many cases, though, what the correct response to such a question tells us is that the reference of 'that' is complex. Accordingly, one might mistake such occurrences of demonstrative pronouns as occurrences of proper names. Moreover, the language is lacking in any explicit syntactic rules which could detect or prevent the mistake. This suggests a logical deficiency of colloquial languages. The routines of the language may be followed by both utterer and auditor without any guarantee that their results clearly

communicate sense. Wittgenstein's response, of course, is that since a term can count as a proper name only if it occurs in a proposition all of whose terms have - in the context of that proposition - been directly co-ordinated with the simple constituents of a complex (which is shown by the background with respect to which a term counts as meaningful), the background of language compensates for the lack of syntactic rules. In particular, a term could not mean a complex in the context of a proposition except with respect to a background in which one term means a constituent of that complex. Accordingly, one could not understand the proposition about the complex unless one understood a proposition about some of its constituents.

However unconvincing such an appeal may be, it can be the only way in which Wittgenstein's pictorial account can address this problem, at least while this account is supposed to show how whatever can be said can be clearly said, whether in a language or a logical notation. Accordingly, these difficulties with Wittgenstein's account of propositions about complexes threaten to undermine not only Wittgenstein's solution of the problem PC* but the very assumptions under which Wittgenstein attempts to solve that problem, namely, the thesis of realism, independence and contingency. This becomes manifest, however, only through a consideration of Wittgenstein's criticisms of Russell.

These criticisms can best be read as objections to Russell's introduction of verbal distinctions, routines and signs for saying that which, according to Wittgenstein, lacks sense. In particular, Wittgenstein attempts to explain how such verbal distinctions as Russell's distinction between logical types of objects (orders of propositions) are obstructive or superfluous in a canonical notation for the showing of what can be shown. Since these signs and distinctions were introduced for the clear saying of something that was supposed to be only unclearly sayable in colloquial languages, these criticisms indicate the logical adequacy of ordinary language. In particular, if these criticisms were sound, then it could no longer count as a deficiency in language that it only shows what some signs distinguished according to types or orders were supposed to say clearly, for, by those criticisms, these latter signs must lack sense.

A distinction that becomes prominent through these criticisms is that between what one can say or say something about and what one can show, whether in thought or in language. Though Wittgenstein's criticisms of Russell's theory of types need not be phrased in terms of this distinction, his criticisms can consistently, if only retroactively, be read as objections that Russell (and Frege) introduce signs and distinctions in their (logically) canonical notations for the saying of what can only be shown

and of what must already be shown in colloquial languages. Accordingly, the distinction eventually gets invoked as an ultimate rather than a relative one. This, of course, leads directly to the paradox of TLP and the purported dissolution of the problem of clarity of expression, PC*.

By the above analysis, two examinations of TLP need to be provided in order to show how this purported dissolution of the problem of clarity can arise. The first consists in an exposition of Wittgenstein's criticisms of Russell's theory of types and of Wittgenstein's own solution of Russell's, and similar, paradoxes. The second consists in an examination of how with respect to the basic assumptions of PC* Wittgenstein's distinction between saying and showing can only be taken as an ultimate one. The first will be the concern of the present and following chapters.

3.1 Wittgenstein's criticisms of Russell's theory of types and orders come in two instalments. One instalment presents difficulties for that theory in conjunction with Russell's theory of judgement. Another presents objections to the theory of types and orders alone. But since the objections presented in the latter treat that theory as an attempt to say what can only be shown, the latter instalment depends upon the former for its cogency, for it is only with respect to the difficulties presented for Russell's theories of types and judgements that Wittgenstein's distinction between saying and

showing becomes compelling. Accordingly, I will deal first with some of the difficulties presented for Russell's adjunction of his theory of judgement to the theory of types and orders.

The adjunction of a theory of judgement to Russell's logic was supposed to provide an account of the entities directly constituted in accordance or compliance with the laws of that logic, the entities which directly count as true or false, namely, judgements. Given Russell's logicist and realist assumptions (for evidence of which see Principles of Mathematics page 43 (46-47), Principia Mathematica 14.21, and Theory of Knowledge page 48), none of these propositions of logic could be accepted as expressing universally valid laws unless they governed entities of some sort. More explicitly, if no adequate account of such entities could be forthcoming, then Russell's logic itself would lack a realist interpretation, which lack would threaten to undermine the superordination of logic over other more specific and purportedly contingently true (or false) theories.

The theory of judgement Russell himself settled upon (the multiple relation theory) also gave the promise of reducing the ontological commitments of his logic, for, as we will momentarily see, this theory replaced Russell's earlier theory of propositions (as the subsistent entities in whose being the validity of the laws of logic partially consisted). Without

such a replacement, Russell's theory of orders would have burdened his logic with commitments to a rather ornate manifold of entities.

From the above considerations, one can appreciate the importance of Russell's multiple relation theory of judgments for Russell's logical theory. Of course, Russell came to accept his theory of judgement through considerations more relevant than the above to the problem of elucidating the nature of truth and of what is properly true. But these considerations count as substantive reasons only with respect to the background of Russell's logical work, in particular, his treatment of the 'logical' paradoxes. Accordingly, this treatment should, first, be examined.

A linguistic version of Russell's paradox was already presented in the introduction. This version will be re-examined later in this chapter.¹ But it is not as a linguistic but as a logical problem that Russell presented and addressed his paradox. So, it is as a logical problem that it should be examined here in this exposition of Russell's theory of types.

From the Principles of Mathematics through Principia

¹ Recall: A linguistic version was given to suggest how the paradox revealed a logical inadequacy of colloquial languages - the background and routines of colloquial languages are not themselves adequate for the detection and prevention of such paradoxes as Russell's paradox.

Mathematica, Russell sought in his logical work to secure for mathematics a foundation in logic. In Principia, in particular, he and Whitehead hoped to show - as rigorously as possible - how any proposition in mathematics could be expressed and supported in purely logical terms, i.e. by propositions consisting solely of such signs as ' \sim ', ' \rightarrow ', ' \exists ' and predicates whose meanings - in that context - need not be specified. Since a crucial part of that demonstration consisted in showing how propositions purportedly about numbers could be replaced by propositions about equivalence classes of extensions of propositional functions, the set theoretical paradoxes, especially Russell's, presented difficulties not just for a special new theory of mathematics - the theory of aggregates - but induced several problems for the most general theory of all, logic. Even if - as was attempted in the Principles of Mathematics - propositions purportedly about sets were themselves ultimately to be replaced by statements that two propositional functions are formally equivalent,² the set theoretical problem would not, of course, disappear with those propositions about sets but reappear as a paradox concerning propositional functions.

In apparently set theoretical but ultimately logical terms, then, the following presents Russell's paradox:

² This is licensed in PM by the axiom of reducibility.

1.0 Suppose that there is a set of all sets that are not members of themselves.

1.1 By the axiom of extensionality, there is exactly one such set.

1.2 Call this one set A.

1.3 Then either A is a member of itself or A is not a member of itself. That is, either A is a member of A or A is not a member of A (by law of excluded middle). But if A is a member of itself, then A does not satisfy the defining propositional functions of A and so is not a member of A (by the axiom of membership).³ Moreover, if A is not a member of itself, then A does satisfy the defining propositional function of A and so is a member of itself. Thus, A is neither a member of itself nor not a member of itself, which is a contradiction.

Of course, one may refuse to treat this result as constituting a logical paradox - a paralogism - and treat it rather as a reductio of the initial assumption. This is, in fact, a position adopted by Zermelo and later von Neumann. But it could not be an option for Russell (or, for that matter, Frege). If Russell were to deny the initial assumption, rather than simply rejecting the sentence as senseless, then Russell would have to accept that some propositional function lacked an extension, even though it was introduced in purely logical terms. In that case, Russell would have to abandon his logicism. Accordingly, Russell must diagnose and treat the problem quite differently. He must show not that some false assumption about extensions of propositional functions

³ $a \in \mathfrak{B}$ just in case $\exists ! \phi (\forall x) ((\phi ! x = x \in \mathfrak{B}) \wedge \phi ! a)$

has been introduced where set theoretical terms have been introduced. Rather, he must show how some logical error has already been committed in taking 'the supposition that there is a set of all sets that are not members of themselves' as a symbol, even if it is only taken as what Russell called an incomplete symbol.

Russell attempted to show this through the elaboration of his vicious circle principle and the theory of types and orders. While the first principle dealt directly with the detection and prevention of the logical error committed in the above argument, the theory of types and orders purported to present the background of meaning with respect to which that principle could be understood to correctly detect and present similar logical errors.

In Chapter I and II of Principia, Russell presents his vicious circle principle. He writes,

Whatever involves *all* of a collection must not be one of the collection.⁴

The vagueness of this formulation consists in the vagueness of the term 'involves'. Russell uses it in order that the principle, so formulated, covers many cases - not only the case of Russell's paradox but also the liar's paradox, for instance.

⁴ See page 37, PM.

In the case of the liar's paradox, that which would both involve *all* of a collection and be one of the collection would be the following sentence, if it were a proposition:

(1) The proposition (1) is false.

The difficulty with (1) is that it could not count as a proposition unless it indirectly mentioned a totality of propositions of which it would itself count as a member. The phrase 'the proposition (1)' is a definite description where '(1)' functions as an index. The common noun 'proposition' functions as the name of a totality of which (1) is indicated to be a member. Unless this phrase mentioned such a totality, the sentence would lack a meaning. In this manner, the vicious circle principle has been violated.

In the case of Russell's paradox, the pronoun 'it' in 'It is a member of itself' could not be given an antecedent unless that antecedent mentioned a totality which was itself a member of itself. Thus, by the vicious circle principle, the string of signs 'It is a member of itself' cannot express (or itself count as) a propositional function. But since no proposition could result from giving an antecedent for 'it' in 'It is a member of itself' and every proposition is such that it is possible for some sentence to express its negation, no proposition could result from giving an antecedent for 'it' in

'It is not a member of itself'.⁵ Thus, since this proposition⁶ occurs in the sentence 'There is a set A such that for every item, it is a member of A just in case it is not a member of itself', the sentence purportedly expressing the initial supposition indirectly has an occurrence of a phrase which cannot be interpreted in compliance with Russell's vicious circle principle. Thus, there can be no such supposition expressed by 1.0.

A special but fundamental violation of the vicious circle principle occurs when one takes the extension or a value of a propositional function as an argument of that function. For, as Russell himself remarks in Principia,

This form of the fallacy is very instructive, and its avoidance leads, as we shall see, to the hierarchy of types.'

This becomes apparent with Russell's following definition of a logical type:

'Of course, both of these expressions violate the vicious circle principle, but the second violates it because the first occurs within the second and the first directly violates the vicious circle principle.

⁶ A sentence with pronouns but no antecedents for those pronouns.

'Whitehead, Alfred North, and Bertrand Russell. Principia Mathematica to *56. Cambridge: Cambridge University Press, 1962, 39.

A type is a totality of items which can occur as legitimate arguments for a given propositional function.⁸

The connection between this definition and the vicious circle principle is that since all purported propositions about extensions of propositional functions can be replaced by propositions consisting only of those propositional functions, all the set theoretic vicious circle fallacies arise through taking as argument for a function some item not of the appropriate type for that function.

This may briefly explain the connection between the vicious circle principle and the theory of types. But I have also urged that the theory is supposed to provide the background with respect to which the principle can be assumed to correctly detect and prevent logical errors. This is so since the theory of types - unlike the vicious circle principle - provided a general licence for introducing set-theoretic terms and a general recipe for doing so without incurring set-theoretic paradoxes. Such terms, as discussed earlier, were essential to Russell's attempt in Principia to provide the truths of mathematics with a purely logical formulation. But, more obviously in the Principles of

⁸ "Mathematical Logic as Based on the Theory of Types", Logic and Knowledge: Essays, Edited by R.C. Marsh. London, Sydney and Wellington: Unwin and Hyman, 1956. (Originally published in American Journal of Mathematics. 1908)

Mathematics, given Russell's commitments to propositions and propositional functions as subsistent entities, the theory of logical types also committed Russell to *a priori* assumptions concerning how entities can and must stand to one another in reality. Such commitments made logical errors such as those purportedly committed in the case of Russell's paradox not just violations of grammar and diction, say, but evidence of deep confusion over how things stand to one another in the world. Conversely, the vicious circle principle is not just a linguistic rule, one whose validity consists in and depends solely upon the canons of grammar and diction of a particular language, whether a purportedly canonical or uncanonical language. Rather, it is a principle which is supposed to be in agreement with the *a priori* order of entities, or at least this is what the theory of types was supposed to corroborate if not explain. I stress this role of the theory of types in Russell's work even though it was not emphasized by Russell and he accepted the theory only with recurrent reservations, because it is with regard to this role that Wittgenstein critiques the theory. [It is important to remember, here, that Russell recommended the theory only in so far as it dealt satisfactorily with the paradoxes and appealed to common sense observations about what counts as meaningless.]

3.20 Two aspects of Russell's theory of types remain to be discussed: firstly, Russell's detailed account of the

hierarchies⁹ of types and orders; secondly, Russell's notion of typical ambiguity.

Though it is unclear even in Principia whether or not Russell allowed propositional functions only a linguistic reality, it is clear that his explanation of the hierarchies of types is best explained and illustrated in linguistic terms. In particular, the bottom type of a hierarchy is best explained in terms of Russell's account of proper names, for the meanings of such terms are what are supposed to constitute the bottom type. According to Russell's account, a term 'a' counts as a proper name with respect to a given context C just in case, with respect to C, for every propositional function 'p' in which 'a' occurs the following holds:

$$\text{PN } \exists x (\forall y (a'(y) \leftrightarrow y=x) \wedge \sim p[x/a]) \leftrightarrow \sim \exists x (\forall y (a^*(y) \leftrightarrow y=x) \wedge p[x/a]), \text{ where } 'a'(y)' = 'a=y' \text{ if } 'a' \text{ is a single undefined word or } '\phi(y)' \text{ if } 'a' = '\iota x(\phi)'$$

This would, if it were true, count as a presupposition of any judgement in that context. Once the bottom type has been determined, a hierarchy of types with respect to C can be determined as follows by induction on n:

1 The denotations of the terms that count as proper names with respect to C constitute a zero level C-type. The names themselves will be said to be C-type symbols.

⁹ I will be using the account given in Mathematical Logic.

n+1 Propositional symbols consisting only of k'th level C-type symbols generate n+1'th level C-type symbols subject to the following, where for all k, $k < n+1$ and for some k, $k = n$:

1 If 'p' is a propositional symbol consisting only of k'th level C-type terms, where $k < n+1$ and some constituent term is of level n, then 'p' is an n+1'th level C-type symbol.

2 If ' p_0 ', ... ' p_n ' are n+1'th level C-type symbols and 'p' is a truth-function of ' p_0 ', ... ' p_n ', then 'p' is an n+1'th level C-type symbol.

3 If 'p' is a n+1'th level C-type symbol and 'a' is a k'th level C-type term occurring in 'p', then ' $\exists x (p[x/a])$ ' is a n+1'th level C-type symbol.

4 Nothing is an n+1'th level C-type symbol unless it can be obtained by repeated application of (1) - (3).

Of course, type distinctions more refined than the above can be introduced. For instance, if instead of treating types as cumulative, that is, as including members of lower types, they were assumed to have members of exactly one level, then the condition n+1 could be replaced by the following:

n+1 Propositional symbols consisting only of n'th level C-type symbols generate n+1'th level symbols subject to the following:

1 If 'p' is a propositional symbol consisting only of n'th level C-type terms, then 'p' is a n+1'th level C-type symbol.

2 If ' p_0 ', ... ' p_n ' are n+1'th level C-type symbols and 'p' is a truth-function of ' p_0 ', ... ' p_n ', then 'p' is an n+1'th

level C-type symbol.

3 If 'p' is a n+1'th level C-type symbol and 'a' is an n'th level C-type symbol occurring in 'p', then ' $\exists x (p[x/a])$ ' is a n+1'th level C-type symbol.

4 Nothing is an n+1'th level C-type symbol unless it can be obtained by repeated application of (1) - (3).

In order to cover all the symbols typed in the first formulation, however, two further conditions must be given the following formulations:

K1 If ' p_0 ', ... ' p_n ' are respectively k_0 , ... k_n 'th level C-type symbols and 'p' is a truth-function of ' p_0 ', ... ' p_n ', then 'p' is a (k_0, \dots, k_n) level C-type symbol, where (k_0, k_1, \dots, k_n) indexes a type sequence. (Type sequences, according to this formulation, also count as types.)

K2 If 'p' is a (k_0, \dots, k_n) C-type symbol and 'a' is a k_j 'th C-type symbol occurring in 'p', then ' $\exists x (p[x/a])$ ' is a (k_0, \dots, k_n) C-type symbol.

Both of these formulations are legitimate. But whereas the advantage of the former is simplicity, the advantage of the latter formulation is the nicety of its type and order distinctions. In the case of propositions purportedly about complexes, the refined indexing of types and orders will be of more help than the simpler indexing. Accordingly, only the latter formulation will be invoked in the following discussion. Of course, all comments and conclusions expressed in terms of the latter formulation have dual, if not

equivalent, expressions in terms of the former.

This hierarchy of C-types induces a hierarchy of orders of propositional function symbols. Propositional function symbols are determined according to their arity, the type of their (complexes of) legitimate arguments and the type (or orders) of their values. For instance, if ' ϕ ' is a propositional function symbol with ' x ' occurring freely in ' ϕ ', ' a ' is a legitimate argument of ' ϕ ' and ' p ' results from the substitution of ' a ' for ' x ' in ' ϕ ', ' ϕ ' could be unambiguously indexed according to its order by ' B^A ', if ' a ' and ' p ' were indexed according to type by ' A ' and ' B ', respectively.¹⁰

The above does not cover the full complexity and abstractness of Russell's theory of types. Once functions from types to types have been formulated and indexed, functions from orders to types, types to orders, and orders to orders can be formulated and indexed - in full compliance with the vicious circle principle.¹¹ But the display of such aspects of Russell's theory is not entirely germane to the

¹⁰ If ' ϕ ' is a propositional function symbol with free variables ' x ', ' y ', ' a ' and ' b ' can legitimately be substituted for ' x ' and ' y ', respectively and ' p ' = ' $\phi[a/x][b/y]$ ', then ' ϕ ' could be indexed by $B^{(A,C)}$ where ' A ' is the index of ' a ', ' B ' of ' p ' and ' C ' of ' b '.

¹¹ For instance, in Russell's notation, $[a/z](\phi!z)$ is a function from $B^A \times A$ to B^B where B is the type of the values of $f!z$ and A the type of arguments of $f!z$, where f is a legitimate argument for ϕ .

purposes of the present chapter. Rather, all that remains to be discussed is Russell's notion of typical ambiguity, especially in so far as this applies to Russell's axiom of reducibility. Two points need to be discussed in order to examine this notion; namely, Russell's distinction between apparent and real variables and his definition of predicative propositional function symbols. But before either of these points can be addressed some remarks concerning the indexing of predicates and definite descriptions should be presented, for the typical status of their meanings, we shall see, will be largely in question in the context of Wittgenstein's criticisms.

3.21 From the above outline of Russell's type theory in Principia Mathematica and Mathematical Logic, one cannot tell whether such symbols as the following are determinately indexed according to their type or order:

1 ' R ', where ' R ' is a primitive predicate with respect to C .

2 ' $\iota \gamma (a_0 C_r \gamma \wedge \dots \wedge a_{\alpha(R)} C_r^{\alpha(R)} \gamma)$ ', where ' a_k ', $k < \alpha(R)$, is a proper name and $\{C_r^k, k < \alpha(R)\}$ is the set of position predicates induced by ' R '.

3 ' $x(Rx)$ ', where this is an abbreviation for ' $x_0 \dots x_n (R(x_0 \dots x_n))$ '.

In Mathematical Logic and Principia, Russell explicitly presents the typing only of proper names, propositional

symbols and propositional function symbols.¹² Thus, the indexing of the above symbols is to be determined according to some assimilation of predicates and definite descriptions to proper names, propositions and propositional functions. Given Russell's routines for replacing statements purportedly about extensions of propositional functions by statements consisting only of those propositional functions,¹³ a term ' $x(Rx)$ ' for the extension of a propositional function symbol ' Rx ' might be reasonably indexed according to the order-index of ' Rx ', even though the former is an object-term and not, like the latter, a function-term. The implicit conflation of objects with functions, here, would not be benign if, in Principia, Russell accepted that there were such entities as classes or if he lacked routines for reducing purported talk about classes into purely logical terms. Since he, however, treated class terms as incomplete symbols (that is, as symbols that characterize the meanings of propositions in which they occur but which do not refer to any entities at all), the indexing of class terms according to order, rather than type, would not incur any logical confusion. Moreover, even if a logical confusion were introduced through such an indexing, it could, after all, be avoided by indexing a class term ' $x(\phi)$ ' accordingly to the

¹² In Mathematical Logic, see pages 75-80, and in Principia, see pages 48-55.

¹³ In Principia, see pages 74-80.

type of the value of ' ϕ ' for some substitution of possible arguments for ' x '. The main type theoretical constraint is, however, that a class-term ' $x(\phi)$ ' is indexed, the indexing must show ' $x(\phi)$ ' to be of higher type or order than any term ' a ' for which ' $\phi[a/x]$ ' is a proposition.

The cases of predicates and definite descriptions of complexes is less easily settled than that of class terms. For instance, on the one hand, since ' R ' is a primitive symbol, it ought to count as a proper name. In that case, it would be indexed according to the index of ' a_k ', $k < \alpha(R)$, even though its meaning would relate the constituents of a complex denoted by ' $\iota \gamma (a_0 C_R^0 \gamma \wedge \dots a_{\alpha(R)} C_R^{\alpha(R)} \gamma)$ ' and not be one of the constituents of that complex. This is Russell's position in the Principles of Mathematics. For instance, in Appendix B of the Principles, he writes:

It would seem that all objects designated by single words, whether things or concepts, are of this type. Thus e.g. the relations that occur in actual propositions are of the same type as things, though relations in extension ... are of a different type.¹⁴

On the other hand, since ' R ' directly corresponds to a propositional function symbol ' Rx ', it might seem much more

¹⁴ Russell, Bertrand. Principles of Mathematics. 2nd ed. New York and London: W.W. Norton and Company, 1937, 523. Notice that, as stated, this passage introduces contradiction. Russell recognizes this in PM on p.60, with his example (3).

appropriate to index it according to the index of 'Rx'. In that case, it would count as a first order propositional function symbol. This alternative, of course, only arose for Russell once he elaborated his theory of orders of propositional functions, a theory not elaborated until after writing the Appendix on types in the Principles. As will be seen, this ambiguity of indexing of predicates according to type or order is preserved rather than eliminated in Russell's theories of belief - both early and late. For in both the early and late theories, Russell allows the meanings of predicates such as 'R' to occur as constituents of those complexes which are supposed - with respect to each theory - to count as directly true or false. [In the early theory, meanings of such predicates occur as constituents of subsistent entities, namely, propositions. In the later theory, they occur as constituents of existent complexes, namely, judgements.]¹⁵ The distinctions between the meanings

¹⁵ Russell does not use the two words 'subsistence' and 'existence' to distinguish between two modes of being. Wittgenstein, however, does. According to his distinction, whereas that which subsists has being independently of what is the case, what (strictly) exists has being but not necessarily independently of what is the case. For instance, some items may have being only as complexes or by being constituents of some complex. But since the being of a complex, for Wittgenstein and Russell, always coincides with some (positive) fact being the case, both would be said to exist. The point upon which Wittgenstein and Russell disagree is that for Russell, if certain abstract complexes, mathematical structures, for instance, were allowed to have being, then, notwithstanding that they would be eternal, their being would

of predicates and meanings of names would be that only the former could occur, even if only by proxy, as a mode-of-combination of some complex.¹⁶

The indexing of descriptions of complexes seems susceptible to similar difficulties. On the one hand, there seem to be cases in which ' $\iota\gamma(a_0C_R^0\gamma \wedge \dots a_{\alpha(R)}C_R^{\alpha(R)}\gamma)$ ' can be legitimately substituted for some ' a ', where this name occurs in an elementary proposition. Consider, for instance, the predicate 'is before'. This predicate induces the predicate 'is the earlier constituent of' and 'is the later constituent of'. Given these predicates, we have the following complex terms and propositions:

also coincide with some (general) facts being the case. Accordingly, the distinction as Wittgenstein draws it would collapse for Russell. The two words will be used, here, simply to distinguish between eternal and temporal being.

¹⁶ Russell distinguishes between relations in extension and relations in intension. With relations in intension, he further distinguishes between relations which relate constituents of a complex and those that occur as constituents. The former relations in intension, Russell calls relating relations and the latter may be called universal relations. Every relating relation is or corresponds to exactly one universal relation. The difference between a universal relation and a relating relation, if there is one, is that whereas a universal relation can (if only by proxy) correspond to many complexes whose constituents are related by the relation, a relating relation can correspond to only one such complex. Thus, whereas a universal relation corresponds to many relating relations, a relating relation corresponds to exactly one universal relation. The meaning of 'R' must, of course, be a universal relation. Nonetheless, the meaning of 'R' is distinct from the meaning of 'a' in that it can correspond to many relating relations. This makes their status as individuals, if not objects, dubious.

T0 'The γ such that a_0 is the earlier and b_0 the later constituent of γ , where a_0 and b_0 are the only constituents of γ .'

T1 'The γ such that a_1 is the earlier and b_1 the later constituent of γ , where a_1 and b_1 are the only constituents of γ .'

P0 ' a_0 is before b_0 '

P1 ' b_0 is before a_1 '

P2 ' γ_0 is before γ_1 ', where ' γ_0 ' = T0 and ' γ_1 ' = T1

Now, it is not at all obvious that the meanings given to or selected for 'is before' in P2 must depend upon some meaning that can be selected for 'is before' in P0, if these meanings are assumed to be distinct. More explicitly, even if the two occurrences of 'is before' counted as different in meaning, it is neither obvious that the being of what 'is before' in P2 means depends upon the being of what 'is before' in P0 means. So, 'is before' may be assumed to occur in both P0 and P2 as a primitive symbol and with the same meaning. Of course, ' a_0 is before γ_0 ' cannot count as true, since ' a_0 ' is supposed to denote a constituent of the meaning of ' γ_0 ', and constituents of complexes do not occur before the complexes of which they are constituents. But it is not obvious as to whether ' a_0 is before γ_0 ' should count as necessarily false or as nonsense. Only if the latter were the case would 'is before' be typically ambiguous through its occurrences in P0 and P2 or P1 and P2.

On the other hand, the predicate 'is a constituent of' is quite analogous to the set-theoretic predicate 'is a member of'. Indeed, as in the case of the analogous set-theoretic supposition, the supposition that there is a complex γ such that every complex γ_0 which is not a constituent of itself is a constituent of γ that leads to a contradiction, that is, if no type restrictions are involved.¹⁷ The only difference between the case of talk about sets and the case of talk about complexes is that in the former but not in the latter the supposition may simply be assumed to be false without directly undermining Russell's logicism.¹⁸

The decision as to which of these alternatives holds must ultimately appeal to the vicious circle principle. If the substitution of ' γ ' for ' a_0 ' or ' b_0 ' violated the vicious circle, then ' γ_0 ' and ' a_0 ' ought to be indexed in such a manner as to show that the meaning of ' γ_0 ' is of next higher type than the meaning of ' a_0 '. But would such a substitution violate the vicious circle principle? That is, does ' γ_0 '

¹⁷ If γ is a constituent of itself, then γ is not a constituent of itself. If γ is not a constituent of itself, then γ is a constituent of itself.

¹⁸ Whereas Russell needs to assume that for every propositional function symbol ' ϕ ' which preserves equivalence of reference from arguments to values (that is, is extensional), ' $E!(x(\phi))$ ' is true with respect to talk about sets, he need not assume that for any extensional description ' η ' of a complex ' $E!\eta$ ' is true with respect to talk about complexes. His reduction of mathematics to logic does not require the latter assumption.

involve or presuppose a totality of which any legitimate argument for 'x is before b₁' is a member? Unfortunately, answering this question is no easy matter. The only totality that 'γ₀' seems to presuppose is that consisting exactly of the meanings of 'a₀', 'b₀', 'is before', 'is the earlier constituent of', etc., not the totality of arguments for 'x is before b₁'.¹⁹ But on page 47 in Principia, Russell writes:

The considerations so far adduced in favour of the view that a function cannot significantly have as argument anything defined in terms of the function itself have been more or less indirect.

Of course, the meaning of 'ιγ (∧ a_xC_R^kγ)' can only be given indirectly in terms of 'rx'. But if this were to preclude 'ιγ (∧ a_xC_R^kγ)' from counting as a possible substitute in 'Rx', then it would also preclude 'ιx(φ)' from being a substitute in 'φ', which would be absurd. For in that case, 'the first son of Henry VIII' would not denote an object which could be said to be a first son of Henry VIII. Thus, although what Russell writes on page 47 seems more specific than the original statement of the vicious circle principle, it cannot be

¹⁹ Suppose that 'is before' were defined as follows: for all γ₀ and γ₁, γ₁ is before γ₀ just in case no constituent of γ₁ is before a constituent of γ₀. Though the meaning of 'is before' is defined in terms of 'is before', the definition does not presuppose the totality of legitimate arguments for 'x is before a₀', no more than the proposition 'For all x, y, if x is before y, then y is not before x'. Moreover, if 'a' and 'b' denote non-complexes, then 'a is before γ₀' may simply count as false and not as nonsense.

correct. Accordingly, it would seem that there is no violation of the vicious circle principle. This analysis recommends Russell's earlier position in the Principles, according to which what Russell called the class as one²⁰ counts as 'an individual, provided its members are individuals'.²¹ This position respects the common sense view that 'the objects of daily life, persons, tables, chairs, apples, etc.,' are individuals, since as Russell says these are all 'classes as one'.

Notwithstanding this appeal, according to Russell, there still seems to be some confusion in treating terms for complexes as semantically and logically similar to terms for their constituents.²² This will become clearer once we turn to Russell's theory of judgement, for, as we shall see then, in the course of elaborating that theory, Russell comes to accept that only some complexes can be properly named, namely, those whose constituents cannot be permuted so as to determine another complex with the same relating relation. These complexes, however, have a very unusual status.

²⁰ See Chapter VI, number 74. A class is one in so far as it is a term. In Russell's Principles, classes can have an ambiguous status analogous to relations in his later logic and epistemology.

²¹ Page 523 in Principles.

²² This is so especially considering Russell's non-realism with respect to class-terms. At least in Theory of Knowledge, Russell remained a realist with respect to some complex-terms.

The foregoing remarks disclose several difficulties for Russell's theory of types. The main difficulty - from a linguistic point of view - is that that theory leaves the indexing of symbols such as 'R' and ' $\iota\gamma (a_0C_R^0\gamma \wedge \dots a_nC_R^n\gamma)$ ' according to type indeterminate. Though Wittgenstein's criticisms of Russell's theory do not directly present or deal with these difficulties, these difficulties do give Wittgenstein's criticisms a cogency they would otherwise lack.

3.22 The doctrine of systematic ambiguity is one which bears upon many logical assumptions. But it obtains no greater importance than it obtains in connection with the axiom of reducibility. This axiom is not only crucial in Principia for the formulation of a single definition of identity, it is also crucial for the replacement of set-theoretical statements of mathematical assertions by purely logical statements, as Russell explains in Chapter II, section vi and Chapter III, page 76. Russell explicitly invokes the doctrine of systematic ambiguity in the formulation of the axiom of reducibility. In order to treat of Russell's doctrine of systematic ambiguity in connection with the axiom of reducibility, two points need to be addressed, namely, Russell's distinction between real and apparent variables and his definition of predicative functions.

The distinction between real and apparent variables is roughly akin to the distinction between variables that do not

occur within the scope of a quantifier and those that do. This latter distinction is, of course, the presently more familiar distinction between free and bound variables. But this latter distinction is not exactly Russell's. Though the difference between Russell's and the present standard distinction ultimately consists in a difference in the account of variables and constants as symbols in a language, the novel import of Russell's distinction is most clearly explained in terms of his distinction between '(1) asserting a propositional function, and (2) asserting that the function is always true ...''

Suppose, then, for the purpose of this explanation, that ' ϕ ' is a propositional function in which ' x ' alone occurs freely. Then the following, according to Russell, express distinct judgements:

$$\underline{1} \quad \text{'}\vdash\phi\text{'}$$

$$\underline{2} \quad \text{'}\vdash\forall x(\phi)\text{'}$$

Though from both 1 and 2, the judgement expressed by ' $\vdash\phi[\eta/x]$ ' directly follows, where ' η ' can be legitimately substituted for ' x ' in ' ϕ ', the difference between 1 and 2 is that whereas 1 presents a judgement which asserts separately every possible result of legitimate substitution of a term for ' x ' in ' ϕ ', 2

²³ Page 65 'Mathematical Logic' in Logic and Knowledge.

presents a judgement which asserts collectively all those results of substitution. That is, whereas $\underline{2}$ involves some mention of one type of the values presentable through legitimate substitution of a term for 'x' in ' ϕ ', $\underline{1}$ does not involve such mention.

The curiosity of this distinction is that, for Russell, such occurrences of a propositional function as ' ϕ ' in ' $\vdash\phi$ ' are legitimate only if any result of legitimate substitution of a term for the free variable in the propositional function is true or some such result is false. This suggests that only the syntax of the language to which the propositional function symbols belong can intrinsically show that some ' $\vdash\phi$ ' correctly counts as an assertion, for, by the above, the judgement that ' $\vdash\phi$ ' cannot be deduced from any propositions of the form of ' $\vdash p$ ', where ' p ' lacks free variables. Accordingly, for all ' ϕ ', ' $\vdash\phi$ ' is supposed to count as a judgement only if ' $\vdash\phi$ ' counts as a primitive logical assertion or any that may follow from such assertions. Thus, ' $\vdash p \vee \sim p$ ', ' $\vdash Fx \vee \sim Fx$ ' are obvious examples of symbols with free variables that express judgements.

One important feature of this distinction between real and apparent variables is that it is supposed to licence, for Russell, the assertion of logical truths of diverse types in a single formulation. That is, it is supposed to systematize and exploit the ambiguity of some symbols in the language of

Principia for the singular expression of any logical and so any mathematical truth. Without the distinction, Russell's type theory would undermine his attempt in Principia to show that any mathematical judgement can be given a purely logical formulation.

In this connection between Russell's doctrine of systematic ambiguity and the axiom of logic, Russell's distinction between predicative and impredicative propositional functions plays an important role.

Russell's theory of types allows that two propositional functions may, for the same argument, give values of different type or order. But that theory would, if symbols for types could occur as terms and not just indices, also count the following as a definition of a partial order on types:

Def1 A type A is lower than a type B just in case for every \mathfrak{B} in B there is a propositional function ϕ such that i) the value of ϕ at some value η in A is \mathfrak{B} , and ii) ϕ does not presuppose for its evaluation the evaluation of some propositional function Ψ which takes arguments from \mathfrak{B} .

Def2 A type A is covered by a type B just in case A is lower than B and there is no type C such that C is higher than A and lower than B.

The necessity of condition i) in Def1 should be fairly obvious. Unless B consisted of the values of ϕ for the arguments in A, the members of B could not presuppose the type A. But this condition is not sufficient, since, by the axiom

of choice, many functions give individuals as values for sets as arguments.²⁴ In those cases, the function requires for its evaluation the evaluation of propositional functions whose extensions those sets are. The second condition ii) excludes these cases. Thus, both of these conditions ought to ensure that Def1 defines a strict partial order on types.

The second definition formulates the notion of consecutive types or orders. Given this latter definition, we have the following definition of a predicative propositional function:

Def3 A propositional function ϕ is predicative just in case its totality of arguments is covered by the type order of its values.

Now, what the axiom of reducibility asserts is that for any function ϕ which gives propositions as values there is a predicative function Ψ which can take the same arguments as ϕ and which, for each of those arguments, gives a truth just in case ϕ gives a truth. This Russell formulates in Principia as follows:

$$' \vdash : (\exists \Psi) : \phi x \rightarrow \Psi ! x . '$$

The sign ' ϕ ' in this context is, of course, supposed to count

²⁴ The axiom of choice asserts that for any family of disjoint sets Λ there is a function f from Λ to $\bigvee \Lambda$ such that for all $A \in \Lambda$, $f(A) \in A$.

as a free variable, and so is supposed to be ambiguous as a free variable. So it is also supposed to be ambiguous as to its type. The sole restriction is that within a single context of deduction, if ' η ' is indexable as the same or higher type as some result of substitution into ' $(\exists\Psi):\phi_{x \rightarrow x}\Psi!x$ ', then ' η ' cannot be substituted for ' ϕ '. Thus, whereas ' $(\exists\Psi):\phi_{x \rightarrow x}\Psi!x$ ' is indeterminate in type or order in ' $\vdash:(\exists\Psi):\phi_{x \leftrightarrow x}\Psi!x$ ', it is determinate in type or order in ' $\vdash:(\exists\Psi):\phi_{x \leftrightarrow x}\Psi!x, \vdash(\exists\Psi):fx \leftrightarrow x\Psi!x[f/\phi]$ '. In the latter context, for instance, one cannot substitute ' $(\exists\Psi):fx \leftrightarrow x\Psi!x$ ' for ' ϕ ', for then ' Ψ ' and ' x ' would have to be indexed according to different types within one context. This would not be an instance of typical ambiguity but of typical confusion.

Chapter 4: The Problem of Relations in Russell's Theory of Judgement and Wittgenstein's Distinction between Saying and Showing

4.0 In the foregoing chapter, Russell's type theory has been presented in mostly linguistic terms. But Russell himself was not content to give only a linguistic account. Due to what he saw to be the logical inadequacy of most languages, that which directly counted as true or false in accordance with universal laws could not likely be any such entity as a sentence of a colloquial language. Thus, for Russell, if the formulae of Principia were supposed to express universal laws, then there must be some entities which are governed by those laws besides sentences, for, otherwise, he could not give a satisfactory realist interpretation of his logic in Principia.

In the course of his logical and epistemological work from the Principles to Principia, Russell elaborated essentially two types of theories of judgement which provided a realist interpretation for his logic. The first, a dual relation theory of belief, belongs to the work of the Principles. The second belongs to the work of Principia and Theory of Knowledge, and presents what has come to be called Russell's multiple relation theory of judgement. Though, as we shall see, the change in theory was due to several logical difficulties, rather than unwanted ontological commitments, the change - with the development of the theory of incomplete

symbols - brought with it the opportunity to eliminate some rather extravagant ontological commitments his type theory would otherwise have introduced.

What is of crucial importance in the exposition of the later theory of judgement is his account of logical form. It was, in that context, supposed to cover or deal with any items the understanding of which was necessary for the understanding of some class of judgments. More especially, it was patently introduced for the solution of several problems concerning the analysis of the truth and falsehood of judgements. Among these problems, only one is directly relevant to Wittgenstein's criticisms of Russell's account of logical form, namely, what has come to be called the wide direction problem. What I will attempt to show in this chapter is that, for Wittgenstein, Russell's theory of judgement could not provide for a realist interpretation of his logic. In particular, appealing to some results in Nicholas Griffin's 1985 paper, 'Russell's Multiple Relation Theory of Judgment'¹, what I mean to show is that Wittgenstein's criticisms amount to the objection that Russell's conditions for a complex to count as a belief are insufficient for counting as beliefs only those complexes that induce or otherwise ground propositions of some determinate order of Russell's type

¹ *Philosophical Studies* 47 (1985) pages 213-247.

theory. For Wittgenstein, this objection does not merely undermine the adjunction of Russell's theory of judgment to his theory of types and orders. It undermines both theories, for it points to a flaw common to both, namely, the assumption that the meanings of predicates can occur as constituents of complexes that directly count as exclusively true or false, whether those complexes are taken to be beliefs or propositions. This assumption itself is what provides for distinctions between types and orders, since, by that assumption, such meanings may count as arguments of higher order functions. The assumption also requires the introduction of a further constituent of beliefs, namely, logical form, and a further necessary condition of a complex to count as a belief, namely, the condition that the subject of the belief complex understand the logical form. Unless these requirements were fulfilled, the theory would lack conditions sufficient for excluding a complex from counting as a belief that, say, a relation R stands in the relation R to some other entities.

These objections are echoed in many of Wittgenstein's comments about what can be shown and what said or referred to in language or thought. What Wittgenstein retains in Russell's account is the notion of logical form, though not without some modification. But what he rejects is the type theory and its basic assumption that the meanings of

predicates are in some sense objects. In particular, what Wittgenstein eschews is the assumption that ' $\exists\phi\exists x(\phi x)$ ' or ' ϕx ' is a symbol that denotes a logical form. These, if they were, in Wittgenstein's sense, propositions, would allow one to assert or assert something about logical form. Given his criticisms of Russell's theories of types and judgements, however, this is not something that Wittgenstein could reasonably accept. Logical form can only be shown. Accordingly, in the context of logic, where no values for ' ϕ ' and ' x ' have been stipulated, the symbols ' $\exists\phi\exists x(\phi x)$ ' and ' ϕ ' do not say anything, they only show more perspicuously a logical form. This they can do, moreover, only with respect to a background in which the symbols of the canonical notation have been shown to regiment in a logically perspicuous manner the symbols in a language.

Once each of these points has been made, some of Wittgenstein's more direct criticisms of Russell's theory of logical types can be shown to be compelling. They will also be shown to corroborate many other passages in TLP where Wittgenstein emphasizes what he takes to be the logical adequacy of languages.

But before any of these points can be raised, some of the background of Russell's theory of judgement and the theory itself need to be examined and explained. Given such an examination and explanation, one will better see how

Wittgenstein's version of the wide direction problem devolves from the problem of distinguishing modes-of-combination from constituents of complexes.

4.1 According to Russell's early theory of belief (prior to 1906) a belief of some subject was a complex consisting of the subject and a proposition both of which were related by the relation of belief. (Henceforth, verbs and adjectives presented in italics and slashes -e.g. /belief/ - will be assumed to denote relations.) Since Russell treated propositions as the meanings of assertoric sentences, his account of propositions was expressed in some of the same terms as he spoke of other types of complexes of objects or of wholes. Unfortunately, given the lack of a theory of orders in the Principles, Russell had no place for them in his early theory of types. The important difference for Russell's account of propositions was that it always had to come to terms with the peculiar unity or being of *all* propositions, notwithstanding their differences with respect to their truth or falsity. Of course, as I have remarked earlier, though in Russell's later type theory he treats more explicitly of propositions than of other complexes, once he had developed his multiple relation theory of judgement and incomplete symbols, no such entities as subsistent falsehoods needed to be assigned to an order. For any of the following may have been assumed to count as what is directly true or false:

1 A judgement complex.

2 A proposition induced or abstracted or otherwise obtained from some person's judgement(s).

3 An asserted sentence.

Analogous alternatives also would hold for propositional functions. This change in Russell's position is so crucial for many of the problems to be discussed that it is important to illustrate and elaborate upon it further.

Consider the following case:

(1) Jones believes the proposition that Brutus stabbed Caesar.

According to Russell, the constituents of (1) are Jones, the proposition that Brutus stabbed Caesar, and the relation */believes/*. The second constituent,

(2) The proposition that Brutus stabbed Caesar,

itself, however, was also supposed to have constituents, namely, Brutus, Caesar and the relation */stabbed/*. But, of course, the proposition is not determined by its constituents, for otherwise it would not be distinguishable from the following:

(3) The proposition that Caesar stabbed Brutus.

and (2) and (3) are distinct propositions. Accordingly, there

must be some mode in which the constituents of (2) and (3) respectively are combined that determines different positions for those constituents to occur in (2) and (3). The unity of these constituents in (2) and (3) under this mode-of-combination is precisely what sets the problem for Russell's early theory of belief. Since every proposition, whether true or false, is a unity consisting of terms rightly combined with one another in a determinate order, Russell was confronted with the problem of explaining in what the falsity of (3), say, was supposed to consist. This problem was especially exacerbated by Russell's (and Moore's) account of a verbal noun.

According to Chapter IV of the Principles, though a verb and its corresponding verbal-noun - (a noun obtained via some conjugation of the verb eg. gerunds) - are grammatically different symbols, they nonetheless mean the same item, namely, a relation. This assumption is of a piece with Russell's early thesis that verbs and adjectives as well as nouns denote constituents of the proposition meant by the sentences in which they themselves occur as symbols. For instance, if the verb 'stabbed' denoted a relation in 'Brutus stabbed Caesar', then 'stabbing' in 'The stabbing of Caesar by Brutus was revenged by Antony' denoted the same relation. As this example already suggests, Russell's notion of a verbal noun allows for the possibility that a proposition can be

denoted. For instance, following that example, 'The stabbing of Brutus by Caesar' denotes the proposition that Caesar stabbed Brutus.²

Due to this analysis of the meaning of verbal nouns, Russell could not account for the falsity of (3). According to that analysis, since the proposition is one believed by Jones, its constituents must be united in the way in which Jones believes them to be. But, in that case, the meaning of 'the stabbing of Brutus by Caesar' has being, so that (3) should be true after all. Moreover, even if, contrary to his analysis of verbal nouns, Russell were to distinguish between what 'the stabbing of Brutus by Caesar' means from what 'the proposition that Caesar stabbed Brutus' means on the basis that their modes-of-combination are different, Russell is still left with a considerable problem.

The suggestion I am making is that whereas the mode-of-combination of what 'the stabbing of Caesar by Brutus' denotes is the relation */stabbed/*, this relation is a constituent of what 'the proposition that Brutus stabbed Caesar' denotes.

² In this analysis, I have been following R. Cartwright's analysis in 'A Neglected Theory of Truth' in Philosophical Essays. The neglected theory of which Cartwright treats is the theory that a true proposition is a fact, where its truth is a simple property. Cartwright argues that there was more promise to this Frege-like thesis than Russell supposed and that Russell failed to recognize this due to a failure to count 'the stabbing of Caesar by Brutus' and 'the proposition that Brutus stabbed Caesar' as different in meaning.

Since, according to this account - one, by the way, that Russell was to approximate after the Principles³ -, the mode-of-combination of the constituents of a complex cannot itself be a constituent of the complex, the two phrases would denote distinct complexes or wholes. The problem would be to understand how to distinguish or separate the mode-of-combination of the proposition from that of the corresponding complex, so that the proposition has being whether or not its constituents are combined by the relation occurring in the proposition. If no distinction could be intelligibly maintained, then it would be difficult to understand how for this theory a proposition could be false.

These and other difficulties with Russell's early account of propositions led him to abandon that account and to elaborate a different analysis of judgements. In order to grasp his later analysis, let us return to our earlier example given at (1). According to Russell's later analysis, then Jones' belief given in (1) consists of Jones, Brutus, Caesar and the relation */stabbed/*. These are, moreover, all related by the relation */belief/*. Since this latter relation - in contrast to the relation */stabbed/* - is not a constituent of the complex denoted by (1) - that is, the complex of Jones' belief - it is said to be the relating relation of that

³ See the manuscript On Functions, 27 Oct. 1904 in the Russell archives page 1.

complex. (The relation /stabbed/ will be said to be a constituent relation of the complex).

An interesting linguistic corollary of the analysis is that whereas (1) can mean independently of its direct occurrence in another symbol, 'the proposition that Brutus stabbed Caesar' can mean explicitly only in the context of a symbol such as (1). This is why, for Russell, the 'the proposition that Brutus stabbed Caesar' is an incomplete symbol.⁴ What may be inferred from this is that, for Russell,

⁴ Of course, in Principia Mathematica page 44, Russell says, "... when I judge 'Socrates is human', the meaning is completed by the act of judging, and we no longer have an incomplete symbol." Gregory Landini, in 'A New Interpretation of Russell's Multiple Relation Theory of Judgement' page 41, cites passages such as 40 - 46 in Principia to support his account that in the case of non-atomic propositions what Russell has in mind are simply sentences in a language that happen to be asserted or uttered under appropriate conditions, not a complex with a subject as a constituent and a cognitive relation as the relating relation. But if page 40 supported Landini's interpretation, then any symbol which meant only in the context of assertion would count as incomplete. Since, however, in Russell's and even Wittgenstein's account, all symbols require such contexts, every symbol would count as incomplete. This is not a position Russell, Wittgenstein or Frege would have found acceptable. Moreover, Landini's interpretation is not compatible with what Russell and Whitehead write on page 40 in Principia. There they assert that the totality of values of propositional functions ought to be determined intensionally. The intension which determines which substitutions into a propositional function symbol are legitimate would not be, in all cases of higher order propositional function symbols, just another symbol, but some term the propositional function means. Given the dependent status of such meanings, there are problems for Russell in allowing them to occur as arguments of functions, a position he eventually abandoned with the thesis that propositional functions occur through their values. But it should be clear that the hierarchies of types and orders do

either there is no entity that such incomplete symbols mean, or they mean only on condition that symbols such as (1) in which they occur mean. In either case, what follows from this is that Russell's later theory of judgement is supposed to take the place his earlier theory of propositions could have fulfilled in his interpretation of his type theory. (See previous footnote for more on this).

Now, in Theory of Knowledge, Russell's analysis consists of two approaches to the problem of individuating judgements. The first approach is epistemological and concerns the determination of the other cognitive relations the subject of a belief must stand in to the other constituents of the belief-complex, if such a complex is to exist. The second approach is logical and concerns the determination of what the truth or falsity of a belief consists in. Though it is this latter approach which most concerns us, here, the former also needs to be discussed in order to fully present Russell's difficulties. Since Russell's theory of judgement in Theory of Knowledge did not advance much further than the case of atomic judgments and these are supposed to induce the propositions from which all first order propositions are

have an extra-linguistic status. The only important change from the Principles to Principia and then the works on the theory of judgement is that they are no longer subsistent. They, rather, consist of concrete items - judgements of various orders.

supposed to be generated, nothing is lost in our discussion if we restrict our exposition of Russell to his treatment of this case.

According to Russell, an atomic judgement is a judgement consisting of exactly one constituent relation. It is a judgement that does not depend for its truth or falsity upon any other judgements. Rather, its truth (or falsity) is supposed to consist independently or directly in the existence (or non-existence) of a complex that consists exactly of its non-subjective, non-relational constituents and whose relating relation is or corresponds to the constituent relation of the judgement. For instance, returning to our earlier example, if we allowed (1) to report an atomic belief, then its truth would directly consist in the existence of a complex consisting of Brutus and Caesar, where these are related by the relation */stabbed/* in an order corresponding to the order in which they are mentioned. This analysis already tacitly raises what has been called the narrow direction problem.⁵ That is, the problem of distinguishing between judgements such as

- (3) A's judgement that Brutus stabbed Caesar.
- & (4) A's judgement that Caesar stabbed Brutus.

⁵ N. Griffin, 'Russell's Multiple Relation Theory of Judgement'.

on the basis of specifying which complex must exist so that (3) is true and (4) false, and which complex must exist so that (4) is true and (3) false. As we saw earlier, Russell dealt with this problem through positing for each universal relation, R , a set of associated relations $\{C_R^k : k < a(R)\}$ which would determine positions within any complex whose constituents were related by R (or some instance of R).

Russell's dominant logical condition upon whether a complex counts as a judgement is that it must be exclusively true or false. In the case of atomic judgement-complexes, either there is exactly one complex that corresponds to the judgement or there is not exactly one. This condition, however, also sets constraints on Russell's epistemological conditions. In particular, it determines that the epistemological or cognitive conditions must be such that it is impossible that these be fulfilled and the logical conditions be unfulfilled.

The overall weakness of Russell's theory consists in both i) its requirement that the epistemological or cognitive conditions be sufficient for the logical ones, and ii) its essential lack of any specification of such epistemological conditions. His sole requirement was that the subject of a belief be acquainted with (or understand) the other constituents of the belief; that, for instance, if (1) counted as an atomic judgment, then Jones is acquainted with Caesar,

Brutus and the universal relation */stabbed/*. Since, for Russell, a subject must be capable of being acquainted with items, even relations, independently of whether the object of his acquaintance is the constituent of some belief or assumption of his, Russell's conditions are not sufficient to determine that the believer, Jones, for instance, takes the objects of his acquaintance in a logically correct manner. In particular, since, for Russell, the objects of acquaintance can be distinct in type or order, Russell's conditions do not preclude cases where one believes a relation to hold between objects of the wrong type or number to be relatable by the relation.

Consider, for instance, the following cases:

(5) Dr. Fischer believes that greed is greedy.

(6) Jones believes that evil is (identical to) Dr. Fischer.

If we assume that 'greed' in (5) denotes a universal with which Dr. Fischer is acquainted, then, by Russell's assumption about predicates the meaning of 'is greedy' is either the same as the meaning of 'greed' or is a propositional function determined by the meaning of 'greed'. Accordingly, since Russell's theory does not discount (5) from denoting a belief-complex, it allows that some propositional function occurs in

itself as argument, either directly or by proxy.⁶ But however the meaning of 'is greedy' occurs as its own argument, this presents serious difficulties for Russell's theory of judgement. For if the meaning of 'is greedy' occurs directly as its own argument, then (5) gives a direct violation of the vicious circle principle and Russell's theory of types. This becomes manifest if instead of considering the meaning of 'is greedy' we consider the extension of the meaning. Then we see that, by (5), some set is a member of itself. Even if the meaning of 'is greedy' were only allowed to occur as its own argument by proxy of the meaning of 'greed', where this is, by the occurrence of 'greed' in (5), supposed to be different from the former meaning, Russell's theory of types and orders fails to count a meaning of 'greed is greedy' in (5) as of exactly one order. Accordingly, the belief supposedly denoted by (5) would lack a truth-value. A similar analysis holds for (6).

As Griffin has pointed out in his paper 'Russell's Multiple Relation Theory', the above difficulties are difficulties which Wittgenstein presented to Russell in

⁶ It would occur by proxy, if Russell does distinguish between the propositional function and the universal.. Then the universal would stand as proxy for the function since Russell has a no-class theory in Principia. That is, assuming the axiom of reducibility, he replaces occurrences of ' $\exists x(\phi)$ ' and ' $x(\phi)=y(\psi)$ ' by ' $\exists \psi(\phi \leftrightarrow_x \psi!)$ ' and ' $\phi \leftrightarrow_x \psi$ ' and purports to thereby preserve all proofs of mathematical propositions.

correspondence. In particular, Griffin cites Wittgenstein's letter to Russell on January 1913 as raising the wide direction problem in connection with the difficulty of treating relations as constituents of judgement-complexes.⁷

The passage cited in the letter reads as follows:

I have changed my views on 'atomic' complexes: I now think that Qualities, Relations (like Love), etc. are all copulae! That means I for instance analyze a subject-predicate prop[osition], say, 'Socrates is human' into 'Socrates' and 'Something is human' (which I think is not a complex). The reason for this is a very fundamental one: I think that there cannot be different types of things! In other words whatever can be symbolized by a simple proper name must belong to one type. And further: every theory of types must be rendered superfluous by a proper theory of the symbolism: For instance if I analyze the proposition Socrates is mortal into Socrates, Mortality and $(\exists x, y) \epsilon_1(x, y)$, I want a theory of types to tell me that 'Mortality' as a proper name (as I did) there is *nothing to prevent me to make the substitutions the wrong way round*.⁸ But if I analyze [it] (as I now do) into Socrates and $(\exists x) x$ is mortal, or generally into x and $(\exists x)\phi(x)$ it becomes impossible to substitute the wrong way round because the two symbols are now of a different kind themselves.

Wittgenstein is presenting many points in this passage, most having to do with his own analysis of propositions as symbols. But the objections he has to Russell's theory of types are quite evident. Consider, for instance, his remark that 'Something is human' is simple. The import of this must be that the proposition lacks any constituent that is a complete

⁷ See page 229-231 and 237 of that paper.

⁸ The italics are mine.

symbol. Both the quantifier 'Something' and 'is human' do not mean anything that can have being independently. The predicate 'is human' is likened to the copula 'is'. It is supposed to be a mode-of-combination sign, one that in concatenation with names results in a complete symbol. Accordingly, what it means is also a mode-of-combination determined by some complex, and so cannot occur, on Wittgenstein's view, as a constituent of a complex.

This shows where Wittgenstein disagrees with Russell in his account of the meanings of predicates and so his account of relations. For Wittgenstein, relations cannot double as constituents and modes-of-combinations, whether directly or by proxy. For Russell, however, they can. That Wittgenstein thought Russell's position involved a fundamental mistake, one that was responsible for his type theory, is suggested by his remark that if 'Mortality' is supposed to mean a constituent of what 'Socrates is mortal' means, then only a type theory could tell me that 'Mortality' cannot be legitimately substituted for 'Socrates' in that sentence. But as he remarks, 'a proper theory of symbolism' must render a type theory superfluous. This is not just a remark that in a canonical notation type distinctions are superfluous, as his last remarks suggest. For if this were the only point that he was making, then he would not have spoken of a theory of symbolism! He must be further suggesting that the symbols of

colloquial languages must already show the illegitimacy of such substitutions, and this is what a proper 'theory' should make plain.⁹

As a reply to objections of this sort, Russell admitted another entity as a necessary constituent of a belief, namely, what he called a logical form.¹⁰ These entities were, for Russell, simple, general facts the understanding of which was supposed to enable a subject to distinguish from among his objects of acquaintance those which are relations and, having distinguished these, to take the right numbers of the right types of objects to be relatable by those relations.

Russell gave two accounts of logical form, and, incidently or indirectly, two accounts of the symbolism of logical forms. Most of Wittgenstein's criticisms directly concern Russell's symbolism for logical forms, but indirectly apply to Russell's theory of logical types. The force of these criticisms consists in showing the difficulties Russell would have in saying within his symbolism that various objects

⁹ The one discrepancy between this position and Wittgenstein's position in TLP is that here he analyzes the existential proposition 'Something is mortal' out of 'Socrates is mortal'. According to his later analysis of existential propositions as truth-functionally equivalent to disjunctions of values of propositional functions, this earlier position must be mistaken.

¹⁰ Indeed, as Griffin argues in 'Russell's Multiple Relation Theory', it is quite likely that Russell introduced this constituent as a direct response to Wittgenstein's criticisms. See p. 229-235 of that article.

are of a certain type. Even with the introduction of a logical form as a constituent of a belief, Russell could not explain how an understanding of logical form could enable one to distinguish from among the objects of one's acquaintance propositional functions and their arguments - at least not unless he supposed that such a distinction presupposed judgements that the items were functions and objects of appropriate orders and types, respectively, to determine a complex. But not only would this supposition be in violation of his assumption that atomic judgements do not presuppose other judgements, it would be in violation of his theory of types, since the judgements presupposed would be of a higher order.¹¹ Accordingly, even with the introduction of logical form as a constituent of a belief, Russell's theory of judgement does not exclude complexes from counting as judgements of no determinate order.

According to Russell's first account of logical form, the expression of a logical form results from uniform substitution of variables for the terms and predicates of a linguistic proposition, called the expression's proto-type.¹² It is supposed to present a logical situation, a complex of relative

¹¹ They would induce propositions of higher order than the propositions induced by the atomic judgement.

¹² In Ogden's translation of TLP, the word 'proto-type' has a very different use. It applies to the expression with no constants, except logical ones.

logical positions into which objects of appropriate relative types can be placed so that a complex results. Thus, in the case of Jones' belief that Brutus stabbed Caesar, not only is Jones supposed by Russell to be acquainted with Brutus, Caesar and the relation /stabbed/, he is also supposed to stand in a dual cognitive relation¹³ with something expressed or denoted by ' $\phi(x,y)$ ', where ' x ', ' y ', and ' ϕ ' count as term and predicate variables, respectively. These are all also supposed to be constituents of Jones' belief. That is, if, following Russell's account of logical form, such colloquial idioms as 'A believes that a stands in the relation R to b' are in general to be regimented in Russell's canonical notation as ' $B(A, a, b, R, \phi(x,y))$ ', then ' A ', ' a ', ' b ', ' R ' and ' $x\phi y$ ' are supposed to denote or express constituents of what ' $B(A, a, b, R, \phi(x,y))$ ' denotes.

Russell's rationale for this account consists of two considerations. The first he states on page 97 of Theory of Knowledge:

A given dual relation is still of a class of more or less similar entities, namely dual relations; but "dual relation" itself, although it might seem to be one of a class whose other members would be "triple relation," etc., is really, in a very important sense, unique, and not a member of any class containing any terms other than itself. Every logical notion, in a very important sense,

¹³ Either acquaintance or understanding. Russell does not conclusively prefer one relation over the other in his account.

is or involves a *summum genus*, and results from a process of generalization which has been carried to its utmost limit.

The procedure of uniform substitution of variables for terms and predicates is one which results in an expression which lacks any but logical constants. One which would supposedly be indexed as of maximal order once the variables have been indexed. As Russell remarks in the sentence immediately following this passage, such expressions are the only ones that could count as purely logical expressions, for

This [maximal generality] is a peculiarity of logic, and a touchstone by which logical propositions may be distinguished from all others.

The second consists in the requirement that a logical form be, like a propositional function, something in need of completion. It must consist not of objects but of positions into which objects can be placed to determine a complex object. Whatever the status of these positions, they cannot be objects that can be recombined to give different forms, even though the variables in their expressions can be recombined to yield expressions for different forms, for otherwise the dual form denoted by ' $\phi(x,y)$ ' would not be maximal and could not be directly understood by someone who believes that a dual complex exists.

One of Wittgenstein's principal objections to this

account is that it presupposes that the complete symbols of a canonical notation can contain real variables. One of Russell's reasons for accepting this assumption is (via the doctrine of systematic ambiguity) to give the laws of logic a single and comprehensive formulation, one that, as in ' $\vdash p \vee \sim p$ ', covers all orders of propositions. This rationale is, as we have seen, especially manifest in Russell's formulation of his axiom of reducibility:

$$\vdash : (\exists f) : \phi x \rightarrow_x f!x.$$

Wittgenstein writes in his letter of 22.7.13¹⁴ concerning this formulation that it is nonsense only if it says what some closure of ' $(\exists f) : \phi x \rightarrow_x f!x.$ ' says, for if it does not 'no general laws can ever follow from (the) axiom'. This presents the argument for the position Wittgenstein expresses a year earlier in his letter of 22 June 1912 that,

... the prop[osition]s of Logic contain only apparent variables.

Clearly, it is the doctrine of systematic ambiguity that is ultimately in question here. But what Wittgenstein says tells us very little about what is wrong. His comment that 'no general laws can follow' from Russell's formulation of the

¹⁴ Pages 122 and 123 of his Notebooks.

axiom is hardly explicit enough to communicate to us the objection he is attempting to express. Russell could simply reply that any inference from Russell's axiom to a proposition expressed by a universal closure of its formula $(\exists f:\phi_x \rightarrow_x f!x)$ is permitted under the proviso that such an inference sets a context within which other occurrences of the variable ' ϕ ' are determinately indexed according to order or type. In that case, general laws could follow from Russell's axiom. The only reason that Wittgenstein could not be satisfied with this response is that in the context of Russell's theory of judgement such symbols as ' $\phi(x,y)$ ' are supposed to denote items the understanding of which supposedly enables a subject to discriminate propositional functions from objects and recognize their respective arguments, but the items which they denote are either indeterminate as to their own order or which order of propositional function they denote is determined only in contexts such as ' $B(A, a, b, R, \phi(x,y))$ '. But, whichever of these latter alternatives holds, they are not compatible with Russell's account of logical form, for, in the first case, logical forms are not assigned a place by Russell's type theory in Principia, and, in the second, the logical form no longer shows the order or types of the relations and objects of a subject's acquaintance prior to belief. The problems Russell encountered with his treatment of relations as constituents of belief-complexes merely gets exacerbated and

not resolved by positing a denotation of ' $\phi(x,y)$ ', say, as a constituent.¹⁵

According to Russell's second account of logical form, the expression is supposed to result from a complete existential closure of a result of uniform substitution of variables for terms and predicates in a simple sentence. Russell supposed this expression to present a general, simple fact.¹⁶ For instance, whereas according to the first account ' $\phi(x,y)$ ' present a dual logical form (the logical form of all complexes with exactly two constituents) according to this account such a form is presented by ' $\exists x \exists y (\phi(x,y))$ ' or in English, 'Something stands in some relation to something'. This account, as Griffin argues¹⁷, was most likely developed as a response to the criticisms Wittgenstein made to Russell in his correspondences of January 1913 and 22.7.1913. In particular, the move to an existentially quantified expression for a logical form *seems* to take care of Wittgenstein's objection immediately above, and the claim that a logical form and its expression are simple seems to follow Wittgenstein's marginal comments in his letter of January 1913 that the sign

¹⁵ Note: Russell had his own reasons for rejecting the above account. See the regress argument on page 113 in Theory of Knowledge.

¹⁶ See pages 113-114 in Theory of Knowledge.

¹⁷ See pages 232-235 of his article.

'Something is Mortal' is not complex. Assuming the simplicity of logical form, moreover, probably seemed to block any chance of a regress. Since Russell supposed it to be simple, that is, to lack constituents, he was not thereby committed to the subsistence of another form with respect to which it is configured. Hence, unlike his first account, he was not committed to an unbounded hierarchy of forms with respect to which a single atomic fact is supposed or judged to be composed.

But Wittgenstein could not have been satisfied with this account either. For the existential closure of a formula such as ' $\exists\phi\exists x(\phi x)$ ' would have to be indexed as belonging to exactly one order. In that case, it would lack the generality which Russell required for his theory of judgement. For instance, no cognitive relation short of an understanding that $\exists\phi\exists x\exists y(\phi(x,y))$ is of the right order to be the logical form of a complex consisting of a, b, say, could be adequate to determine a belief about a and b.¹⁸

¹⁸ Though in Principia page 132, Russell defines an individual as anything that is neither a proposition nor a function, his explicit formulation of types does not prevent there being complexes of different types and his theory of definite descriptions almost makes distinctions between types of complexes a requirement. See also p.76 of 'Mathematical Logic', where Russell writes, 'We may define an individual as something destitute of complexity.'

This points to a deeper problem in Russell's account of symbolism. ' $\exists\phi\exists x\exists y(\phi(x,y))$ ' is supposedly a propositional symbol. But, then, given what ' $\exists\phi\exists x\exists y(\phi(x,y))$ ' is supposed to mean, in some sense, it must be an exception to Russell's

According to these objections, Russell's introduction of logical form could not resolve the family of problems associated with the wide-direction problem. Moreover, as Griffin convincingly argues,¹⁹ they provide a compelling argument for one of Wittgenstein's more cryptic comments in TLP on Russell's theory of judgement, namely:

5.5422 The correct explanation of the form of the proposition "A judges p" must show that it is impossible to judge a nonsense. (Russell's theory does not satisfy this condition.)

By Wittgenstein's objections above, Russell's analysis fails to count such sentences as "Anthony believes that $a \in a$ " as meaningless; since even after his introduction of logical form as a constituent of the belief, Russell could not explain how our understanding of a logical form could enable a subject to discriminate among the objects of his acquaintance a universal

thesis that all such symbols are incomplete. That is, if the canons of grammar within the language of ' $\exists \phi \exists x \exists y (\phi(x,y))$ ' are understood, then with respect to that background understanding, ' $\exists \phi \exists x \exists y (\phi(x,y))$ ' can mean independently of its occurrences in contexts such as 'A understands that ...' (see the footnote on Landini). An important turn in Wittgenstein's work is that, in contrast to Russell, Wittgenstein took propositional symbols and names as constituting the two fundamental and essentially different types of complete symbols in a language. This was to make of the symbols in logic a different sort of exception. For as we shall see, according to this account, such logical symbols, not merely those that show the forms of tautologies and contradictions, do not say anything. They merely show what other symbols already, but perhaps less clearly, show.

¹⁹ In his article, see pages 237-239.

relation (or propositional function) and recognise its relata (or arguments) - at least not without presupposing higher order judgements. That the above objections do provide the rationale for 5.5422 is corroborated by earlier versions of the criticisms - for example, Wittgenstein's letter to Russell (dated by Russell as "June 1913"):

I can now express my objection to your theory of judgement exactly: I believe it is obvious that, from the prop[osition] 'A judges that (say) a is in the Rel[ation] R to b', if correctly analyzed, the pro[osition] 'aRb \vee ~aRb' must follow directly without the use of any other premises. This condition is not fulfilled by your theory.²⁰

Now, the problem with allowing 'A judges a ϵ a' to denote a judgement is that if such is the case, then, according to Russell's theory of propositions as based on his theory of judgement, in 'A judges a ϵ a', 'a ϵ a' denotes a proposition of no determinate order. Accordingly, the theory of types and orders lacks the realist interpretation that the theory of judgement was supposed to provide. Indeed, for Wittgenstein, it is an assumption common to both that seems to be the source of the difficulties that arise in the adjunction of Russell's theory of judgement to his theory of types and orders, namely, the assumption that the relating relation of a complex corresponding to a true atomic judgement is, whether directly

²⁰ Wittgenstein, Letters to Russell, Keynes and Moore ed. by G.H. von Wright (Blackwell, Oxford) [R12].

or by proxy, a relata of the main relation of the judgement complex. It is this assumption that gives to Russell's later theory of types its characteristic form, one in which there are orders of propositions and propositional functions. But the assumption is not itself responsible for the paradoxes that theory was supposed to prevent, for even for Frege, who maintains a sharp distinction between objects and relations, Russell's paradox arises. Evidently, the paradoxes have other assumptions as sources, for instance, Russell and Frege's logicism.

4.2 It is well known that Wittgenstein also espoused a rather stringent version of logicism. That is, he followed Russell and Frege at least in so far as to assume that nothing could be shown in mathematical notation that could not be shown in purely logical notation. (The difference between Wittgenstein's position and Russell's and Frege's positions, however, is that he, unlike Russell and Frege, did not think that anything is ever said in pure logic. According to Wittgenstein, mathematics and logic only show things. Moreover, due to his abandonment in TLP of a theory of classes, all that remains to be shown in mathematics for Wittgenstein comprises little more than what has passed for natural arithmetic.) Considering Wittgenstein's criticisms of Russell's theory of judgements and the logicist and realist requirements of Russell's theory of types that the theory of

judgements was supposed to fulfil, Wittgenstein's alternative account of logical and extra-logical symbolism would provide a natural successor to Russell's logicism, if Wittgenstein's account could avoid Russell's and the other paradoxes. The theory of types would then be shown to be superfluous. This is the point that Wittgenstein put to Russell in his letter of 16 January 1913, where he wrote that 'every theory of types must be rendered superfluous by a proper theory of symbolism.'

As we saw earlier, Wittgenstein presents there his own solution to Russell's paradox. He explains:

But if I analyse [the proposition that 'Socrates is Mortal'] into Socrates and $(\exists x) x$ is mortal, or generally into x and $(\exists x)\phi(x)$ it becomes impossible to substitute the wrong way round because the two symbols are now of a different kind themselves.

Since Wittgenstein, even more so than Russell, advocates a no-class theory,²¹ all the paradoxes for Wittgenstein arise from confluences of the (formal) concepts of function and argument. In TLP, Wittgenstein returns to this position in 3.332-3.333. This passage in TLP follows upon some crucial numbers in TLP, where Wittgenstein discusses the substitutional errors that some speakers (for Wittgenstein, traditional philosophers) can make or be committed to making, and where he criticizes Russell's theory of types. The importance of the sections

²¹ See 6.031

dealing with these matters - at least for one of the arguments of this thesis - is that they corroborate many of the passages in TLP where Wittgenstein seems to accept that colloquial languages are logically adequate. In particular, Wittgenstein's criticisms of Russell's type theory and his own purported dissolution of Russell's paradox argue that type distinctions are superfluous within a canonical notation, so that the mere lack of them within an ordinary language does not directly undermine that language's adequacy. Accordingly, for Wittgenstein, if ordinary language has resources other than a refined notation for clearly showing the distinction between function and argument, then its adequacy is thereby vindicated. But, as has already been argued about the pictorial theory of propositions and as will be discussed presently, Wittgenstein accepted the antecedent of this conditional, and so was committed to the consequent.

Those passages in which Wittgenstein speaks of the substitutional errors which are not *prevented* by the grammar and diction of colloquial languages, however, can be read as undermining the assumption of the logical adequacy of languages. Nonetheless, I think that they, in conjunction with other similar passages in TLP, are not only compatible with this assumption, but support it. Before turning to the final chapter, I will attempt to substantiate both of these points.

In 3.3 and 3.331, Wittgenstein argues against Russell that his (i.e. Russell's) formal syntax is not logical, not canonical, since (1) in his theory of types, Russell must mention the meaning of signs to determine their syntactic role, and (2) '(a logical syntax) must admit of being established without mention being thereby made of the *meaning* of a sign'. Due to the brevity and abstractness of Wittgenstein's remarks, here, neither the force nor the exact target of this objection is entirely manifest, even when taken in the context in which it occurs. Rather, in order to appreciate what Wittgenstein writes here, one must return to the passages in Principia and 'Mathematical Logic' for which Wittgenstein's remarks may serve as marginalia. Fortunately, these passages have already been summarized in the previous chapter. According to the exposition given there of Russell's type theory, the only point at which Russell must mention the meanings of signs in his account of types is in his definition of a type as a range of significance of a propositional function (symbol). The rest of the account consists in the formulation of rules for the indexing of signs according to the ways in which they are derived from propositions consisting only of proper names. Russell even has a purely syntactic criterion for proper names or, even more generally, for primitive symbols. Since the indexing is purely a matter of syntax, the (context relative) types and orders of the

meanings of symbols in a language can be determined without *prima facie* mentioning any one of those meanings. These rules themselves would be objectionable, then, only if their application presupposed that the phrase 'is a range of significance of a propositional function' expressed a (non-formal) concept, for, as explained in the section in Chapter 2 on generality, such a phrase, for Wittgenstein, could only express a formal concept. Of course, Russell would never have allowed any symbol for a type to occur as a term in his logic and the above rules could be followed without invoking or requiring this presupposition. But, then, the indexing would have none but the negative and syntactic use of preventing substitutions of lower indexed by 'higher' indexed symbols. Clearly, the indexing acquires a positive use only if it is assumed that it shows how different classes of entities are ordered according to their abstractness and complexity. But this is tantamount to assuming that the phrase 'is a range of significance of a propositional function (sign)' expressed a concept that some entity could fall under. This point becomes more evident in Wittgenstein's own solution of the problem, for it suggests that what the indexing of symbols according to those rules is supposed to show could be shown without

assuming that there are distinct types of entities.²²

In 3.332, Wittgenstein formulates what he takes to be the essential point of the theory of types:

3.332 No proposition can say anything about itself, because the propositional sign cannot be contained in itself (that is the "whole theory of types").

This, of course, *prima facie* deals only with paradoxes such as the liar. Moreover, the argument comes a little quickly to its conclusion, since it fails to deal with such cases as,

The proposition (1) is false. (1)

which is not obviously different from the following:

A is red. (2)

The proposition (2) is false. (3)

From a simple description of the signs, it does not seem obvious that just as the meaning of '(2)' is invariant across its occurrence beside 'A is red' and its occurrence in 'the proposition (2) is false', the meaning of '(1)' is invariant across its occurrence beside 'The proposition (1) is false' and in 'The proposition (1) is false'. Wittgenstein's full

²² It is no accident that these criticisms directly follow Wittgenstein's formulation of Occam's razor: If a sign is not necessary then it is meaningless (3.328). Since type indices are superfluous in a canonical notation and every symbol in a canonical notation ought to be essential, type distinctions, for Wittgenstein, are meaningless.

explanation of why the two cases are disanalogous is given only in terms of his treatment at 3.333 of another of the paradoxes. The rationale given there gets extended to the above case through Wittgenstein's earlier comments in 3.26 and 3.327.

In 3.333, Wittgenstein deals with the paradox resulting from the substitution of a functional sign for the variable that shows the place for any name 'a' of an argument for the meaning of 'fx'. He writes at first:

3.333 A function cannot be its own argument, because the functional sign already contains the prototype of its own argument and it cannot contain itself.

According to Wittgenstein, a prototype is a result of complete substitution of variables for terms and predicates in an atomic proposition.²³ Wittgenstein explains further:

If, for example, we suppose that the function $F(fx)$ could be its own argument, then there would be a proposition " $F(fx)$ ", and in this the outer function and the inner function F must have different meanings; for the inner has the form $\phi(fx)$, the outer the form $\Psi(\phi(fx))$. Common to both functions is only the letter " F ", which by itself signifies nothing.

This is at once clear, if instead of " $F(F(u))$ " we write " $(\exists\phi):F(\phi u). \phi u = Fu$ ".

Herewith Russell's paradox vanishes.

²³ See 3.315 in TLP. Since prototypes correspond to logical forms and there are no objects corresponding to ' \wedge ' and ' \sim ' (i.e. logical forms are simple), ' $\phi x \wedge \sim \psi y$ ' is not a prototype.

The tacit assumption here is that the uniform substitution that is to be effected proceeds from those expressions which occur in a symbol but which contain no occurrences of other symbols and then from symbols which directly occur within a symbol to the symbol in which they occur, where no single variable(-symbol) occurs twice within the resulting expression unless in both those occurrences it occurs directly in one symbol. This explains why the result of uniform substitution of variables for terms and predicates in 'F(a,a)' is ' $\phi(x,x)$ ' and the result of uniform substitution in 'f(f(x))' is ' $\Psi(\phi(x))$ '. The difference in variable is supposed to express - with the difference in the order of occurrence in the sign - an essential difference in the meanings of symbols that could be substituted for ' Ψ ' and ' ϕ '.

Wittgenstein's rationale for this assumption is given earlier at 3.315 and 3.317. In 3.315, Wittgenstein explains that corresponding to a prototype is a class of propositional signs which are determined to count as the values of that prototype. He writes:

If we change a constituent part of a proposition into a variable, there is a class of propositions which are the values of the resulting variable proposition. This class in general still depends on what, by arbitrary agreement, we mean by parts of that proposition. *But if we change all those signs, whose meaning was arbitrarily determined into variables, there always remains such a class. But this is now no longer dependent on any agreement, it depends only on the nature of the proposition. It corresponds to a logical form, to a logical prototype.*

What the assumption above is supposed to respect, according to this passage, is that a propositional function symbol can occur within another symbol only once its class of values has been determined, for otherwise the meaning of the sign in which it occurs could not be fully determined. Moreover, the meaning of the symbol in which it occurs is not determined by the occurrence of the other symbol within it, even if, as in the case of ' $f(f(x))$ ', the predicate sign of both symbols is the same. Rather, this is only fully determined by the stipulation of values of ' $f(f(x))$ ', where this is constrained by the class of values of ' $f(x)$ '.

In 3.317, Wittgenstein stresses that this method of symbolizing consists *essentially* only in the description of the symbols, not in any characterization of what they mean. This is why in a logical symbolism the syntax ought not to presuppose any mention of the meanings of symbols, for, in the context of a logic, the symbols of a canonical notation ought only to reflect that which is essential to any symbolism. If it did presuppose any mention of the meanings of signs, then all it could show would depend upon or be within the scope of the language within which those meanings are mentioned. This would contravene, for Wittgenstein, the essential universality of logic.

According to this analysis, the occurrences of '(1)' in

The proposition (1) is false. (1)

could not mean the same, for if they could, then the values of '(x) is false' could be determined independently of the values of '(x)'.²⁴

This in essentials looks no different from the Russellian method of indexing symbols according to type and order, and really it isn't. It might be thought, then, that the only difference between Wittgenstein's and Russell's solution of the paradoxes is that whereas Russell mentions types and orders, Wittgenstein does not. Since, in the above, Wittgenstein allows function symbols to occur as arguments, even if not their own arguments, within propositional function symbols, Wittgenstein nonetheless seems to be committed to Russell's distinctions between types and orders, but refuses to acknowledge this commitment. But this is not the case. Firstly, the point of this discussion in 3.332-

²⁴ Another problem with 'The proposition (1) is false'⁽¹⁾ for Wittgenstein is that there is no proposition in which the '(1)' beside 'The proposition (1) is false' can be specified to occur. But if '(1)' were a name of the proposition 'The proposition (1) is false', then it would have to occur in the context of a proposition, (by 3.3). The problem is not merely that '(1)' does not occur in its second occurrence as a term in a propositional sign. Rather, the problem is that '(1)' occurs as a name or term beside 'The proposition (1) is false' just in case it occurs as a name in '(1)'. What prevents either of these constituents of the bi-conditionals from holding, for Wittgenstein, is not just that a contradiction would result if they did hold, but that it does not comply with the methods of symbolization Wittgenstein sets out in 3.315 and 3.317.

3.333 is to show that the paradoxes can be prevented without explicitly involving distinctions between types of things and orders of propositions and propositional functions. Secondly, unlike for Russell, for Wittgenstein, there is no distinction between a propositional symbol and a proposition. Consequently, the values of which Wittgenstein speaks are all symbols and, as we have seen, all propositional symbols, for Wittgenstein, are facts. There is no distinction between different types or orders of facts. Both of these points are corroborated elsewhere in TLP where Wittgenstein mentions both formal concepts and Russell's type theory. Consider, for instance, what he writes in 4.126:

Formal concepts cannot, like proper concepts be presented by a function.

As we have seen earlier, among the more important formal concept expressions, for Wittgenstein, is the sign ' $\exists x$ '. In Principia Mathematica, Russell has ' $f!(\phi(z))$ ' count as a symbol for a propositional function of two arguments, one of which is supposed to be a function expressed by ' ϕ '.²⁵ Among the functions that he equates with the propositional function expressed by ' $f!(\phi(z))$ ' is the propositional function expressed by ' $\exists z(\phi z)$ ' or even ' $\exists p(z)(\phi z \rightarrow p!z)$ '. Accordingly, Russell is

²⁵ See, for instance, the bottom of p.52, Principia Mathematica.

committed, here, to an assumption that Wittgenstein repudiates, namely, that formal concepts can be consistently presented by function symbols.

That Wittgenstein means to treat formally that which Russell and Whitehead treat functionally in the theory of types is further evinced by what he writes at 5.251 and 5.252:

5.251 A function cannot be its own argument, but the result of an operation can be its own basis.

5.252 Only in this way [that is, through the application of operations] is the progress from term to term in a formal series possible (from type to type in the hierarchy of Russell and Whitehead). ..

The terms in a formal series themselves are not supposed to denote objects.²⁶ The formal series is only supposed to show the internal (whether logical or extra-logical) relations between facts of varying complexity. These are the internal relations that Wittgenstein supposed to hold between, for instance, a complex and its constituents.

According to the above analysis, then, what Wittgenstein is supposed to be showing in 3.332 - 3.333 is not only the superfluity of explicit distinctions between types of objects. He is also attempting to show what his criticisms of Russell's

²⁶ Eg. ' $aRb, \exists x(aRx.xRb), \exists x\exists y(aRx.xRy.yRb) \dots$ ' is supposed to show a formal series. The corresponding operation is presented by ' $[a, x, xR']$ ', where ' xR' ' is supposed to denote what 'the y such that xRy ' denotes, given a proper substitution of co-referential names ' a ' and ' b ' for ' x ' in the respective expressions.

theory of judgements show, namely that such distinctions commit one to a confused analysis of language. The crucial point for our purposes is that, for Wittgenstein, it is a method of symbolization which is essential to meaning within language which, when understood aright, prevents substitutional errors; for it suggests (1) that colloquial languages are already equipped with canons of grammar and diction adequate for the resolution of confluences of the formal concepts, function and argument, object and fact, and (2) the thesis that in logic propositional symbols do not say anything, but only clearly show how to say what can be clearly said. This role of logic is fulfilled for Wittgenstein not only in a canonical notation but also in a language. The difference is that only in the former can this be shown systematically or rigorously.²⁷

²⁷ It has often been suggested that, for Wittgenstein, the signs of a canonical notation are grammatical rules. But this cannot be correct, for grammatical rules explicitly stipulate how from a given set of signs of a language various sentences can be constructed. But grammatical rules are specific to given languages. They are not universal. Nor can they be logical rules either in the sense that they explicitly stipulate how from any set of symbols from any language various propositions can be constructed or in the sense that they explicitly stipulate how from any set of signs from the canonical notation various sentences can be constructed. The former case does not hold, for Wittgenstein, since it is also a matter of use as to whether a string of symbols is a symbol (Cf. Wittgenstein's considerations of 'Socrates is identical' 5.473 -5.4733). The latter does not hold since the signs are themselves the result of an application of formal rules, the rules according to which the notation and the logic presented in terms of the signs of the notation are explained.

In some of the passages preceding 3.33, Wittgenstein makes remarks which upon initial consideration seem incompatible with the above reading. For instance, in 3.323, he writes:

In the language of everyday life it very often happens that the same word signifies in two different ways - and therefore belongs to two different symbols - or that two words, which signify in different ways are apparently applied in the same way in the proposition.

He continues after giving some illustrations:

3.324 Thus there easily arise the most fundamental confusions (of which the whole of philosophy is full).

3.325 In order to avoid these errors, we must employ a symbolism which excludes them, by not applying the same sign in different symbols and by not applying signs in the same way which signify in different ways.

These passages suggest that, for Wittgenstein, colloquial languages are inadequate since some speakers and writers in the language can utter nonsense within it. This calls for a brief comment before this chapter is brought to a conclusion.

Firstly, it is important to consider some of the illustrations Wittgenstein presents in 3.323 of the cases of

The only alternative left is that the signs of the canonical notation show clearly what canonical expressions of any language show, namely, logical form and internal relations determined by logical forms. But 1 they do so systematically, and 2 only once they have been correlated with such canonical expressions. Then they show within certain parameters how to say propositions clearly within the language.

signs that mean differently in different contexts, especially in connection with some of the surrounding passages. Upon such consideration, it will become apparent that the errors to which he alludes in 3.324 are supposed to arise not for lack of logically adequate grammatical canons within the language but due to a failure on the part of the speakers to understand those canons.

Consider, then, the examples Wittgenstein cites. Among these one should be sufficient to show what we need to show here, namely, the one he presents at the end of 3.323:

(In the proposition "Green is green" - where the first word is a proper name and the last an adjective - these words have not merely different meanings but they are *different symbols*.)

Given Wittgenstein's account of the method of symbolization, the only way this sentence could count as a proposition in English is if the occurrence of "Green" in the term position meant an object and the occurrence of "is green" meant a property. The word 'is' here may occur in "Green is green" as a copula or as an identity sign, but given 1 Wittgenstein's account of proper names, and 2 the prior determination of 'green' as an adjective, only if 'is' occurred as a copula in "Green is green" could "Green is green" count as a proposition. This is supposed to be determined by the methods of symbolization within any language, for, as he writes in

3.326,

In order to recognize the symbol in the sign we must consider the significant use.

The uses which we must consider are, I have suggested, given in those contexts in which the following of certain routines of sign combination have yielded the canon of grammar and diction within the language.

Secondly, it is important to consider that the speakers to whom he attributes the fundamental confusions about language are philosophers. This suggests that the canonical symbolism of which he writes in 3.325 is recommended, especially for the resolution of philosophical confusions. This is corroborated by what Wittgenstein writes in 4.003:

Most propositions and questions, that have been written about philosophical matters, are not false, but senseless. ...

Most questions and propositions of the philosophers result from the fact that we do not understand the logic of *our language*.²⁸

(They are of the same kind as the question whether the Good is more or less identical to the Beautiful.)

And so it is not to be wondered at that the deepest problems are really no problems.

That he cites as an example of philosophical nonsense the interrogative sentence 'Is the Good more or less identical than the Beautiful' is telling, for it bears a striking

²⁸ My italics.

resemblance to the example 'Green is green'. The difference between the two is that whereas the words 'the Good' and 'is identical' in 'Is the Good more or less identical to the Beautiful' are assumed to have meanings which have important logical relations to the meaning of 'is good' in 'A is good' and 'A is identical to B', the word 'Green' in both its occurrences need not be assumed to have the same or internally related meanings. To be sure, philosophers can be committed to these fundamental confusions through mistaken analyses of 'Green is green', but this is why, for Wittgenstein, the sole activity of philosophers is to show that these analyses are mistaken through devising and elucidating a canonical notation and logic which can show how to say what can be clearly said. This is, after all, the view of philosophy for which Wittgenstein has (justly, I think) become notorious in TLP, namely, the one formulated at the following sentences:

4.112 The object of philosophy is the logical clarification of thoughts.

Philosophy is not a theory but an activity.

At this point, Wittgenstein at once attempts to set the standards for philosophy and for clarity of thought. The irony and paradox, as we shall more clearly see in the next chapter, is that these are standards with which the main text of TLP does not directly comply. Indeed, the preface comes closer to compliance than the text. But it is all too obvious that if

that is all that is permitted by those standards, then the works that comply with those standards must be less philosophically satisfying than those that do not. This is how Wittgenstein purportedly dissolves the problem of clarity, at least when taken with the realist, independence and contingency assumptions, or so I shall be arguing in the conclusion of this thesis.

In Wittgenstein's notes and letters, as well as in TLP, one finds that what Wittgenstein retained from Russell's theory of judgement was precisely what Wittgenstein had introduced, namely, the notion of logical form. Logical forms, as we saw in the introduction and the first chapter of this thesis, became, for Wittgenstein, exactly the entities with which logic was supposed to deal. Accordingly, given that, for Wittgenstein, Russell's conjunction of both the theory of judgement and the theory of types showed the faulty account of the symbolism for logical forms to which the latter of these theories committed Russell, Wittgenstein had to present an alternative account of symbolism especially in the case of a logically canonical notation. This is where, as we shall see, the distinction between saying and showing becomes crucial. For the thesis that Wittgenstein articulates is that the 'propositions' of logic only show what they in some sense mean. They do not say anything. This holds, however, not only in the cases of what Wittgenstein calls tautologies and

contradictions, but also in the case of symbols that show the forms of contingent propositions, provided these symbols occur within a purely logical context such as a derivation of one tautology from another (see 6.031-6.111).

The importance of this remark for the argument of this thesis is that it corroborates many of the passages in TLP where Wittgenstein seems to accept that colloquial languages are logically adequate. For, according to the above remark, all that a canonical notation must show is logical form and the logical relations between propositions within colloquial languages. But as the first part of this thesis has attempted to show, for Wittgenstein, this is exactly what the canons of grammar and diction within ordinary languages enable their propositions to show. In particular, this is what Wittgenstein's pictorial account is supposed to elucidate.

This leaves some questions open, however. For even if the distinction between saying and showing were found to be a sound one in some respects, it is not immediately evident that it must be an ultimate one. Considering the paradox of TLP, one might think that all that can and should be accepted is a relative distinction. Accordingly, one might further suppose that no serious changes would be required in Russell's type theory to accommodate this distinction. But as I shall be arguing in the next chapter this is not entirely correct.

Conclusion: The Paradox of TLP

5.1 In the previous two chapters, I have argued that Wittgenstein's criticisms of Russell's type theory amount to the objection that Russell's attempts to distinguish between different types of objects or orders of propositions presuppose a confused analysis of language. This confusion was supposed to consist in Russell's failure to distinguish between what can be shown and what can be said in language. Though Wittgenstein's most compelling objections were seen to treat of the inconsistency in Russell's adjunction of his multiple relation theory of judgement to the theory of types and orders, where the former was supposed to provide for a realist interpretation of the latter, they were also seen to be implicit critiques of Russell's theory of symbolism. A proper elucidation of symbolism seemed, therefore, to be in order, one which dealt adequately with the set theoretical and other paradoxes by showing how they arise from a misunderstanding of language. For Wittgenstein, an essential facet of any proper elucidation of symbolism would consist in the elucidation of a distinction between saying and showing. I have urged throughout this thesis that, for Wittgenstein, once one has understood this distinction, one understands how what can be said can be said clearly in colloquial languages

and how canonical notations - at least in the context of logic - can only show clearly what is already shown within colloquial languages. Thus, Wittgenstein's distinction also plays an essential role in Wittgenstein's purported solution of the problem of clarity, at least when that problem is taken with the assumptions of realism, independence, and, as I will argue in this chapter, the assumption of the contingency of all facts.

But while playing an essential role in Wittgenstein's solution of the problem of clarity, Wittgenstein's distinction between saying and showing also plays a pivotal role in the dissolution of the problem under those assumptions. According to Wittgenstein's elucidation of the logic of our language, propositions are complete symbols in a language which 'mean' in two (essentially) irreducible ways: they express assertions or thoughts,¹ and they show the sense of what they say - in the case of atomic thoughts and their negations, the orientation of their content with respect to a logical form and the two poles, the true and the false. Consider, for instance, what he writes at 4.022:

The proposition shows its sense.

¹ Which among the many thoughts with the same sense is expressed depends upon the context.

The proposition shows how things stand, if it is true.
And it says, that they so stand.

But according to the full elucidation of the distinction in TLP, no proposition can assert or say anything about what one proposition shows. The assumption to the contrary has already been mentioned as a flaw in Russell's symbolism for logical form, for, if you may recall, he counted such signs as ' $\exists\phi\exists x(\phi x)$ ' as expressing propositions in logic. For Wittgenstein, if this were the case, then they would by his analysis say what they show. Moreover, since, for Russell, the meanings of propositional symbols (propositions, for Wittgenstein) can occur as arguments in 'x is true' and 'y is a constituent of A's judgement', something could also be said about logical form.

Though Wittgenstein accepts this analysis, he explicitly commits himself to it only at a few sentences in TLP. One in particular is notorious, namely:

4.1212 What can be shown cannot be said.

Now, it is precisely at such a sentence that the paradox of TLP is explicitly introduced. For according to it, none of the sentences in TLP can say anything true that would support what 4.1212 is supposed to show. Thus, if Wittgenstein's elucidation shows what 4.1212 in particular is supposed to show, then it cannot have solved the problem of clarity

through saying anything true. But, then, the problem is no longer a propositional problem, one that can be answered through propositions. Supposedly, this is how the problem of clarity, finally, gets dissolved through a careful reading of TLP. Once one understands the difference between saying and showing, between language and theory, and canonical notations and logic, one recognizes how the elucidation of these differences said nothing. Accordingly, elucidation of the difference and even the devising of canonical notations for this elucidation becomes superfluous within such an understanding.

But what remains unacceptable about this result is that Wittgenstein can provide no satisfactory account of how the sentences of TLP elucidate what they are supposed to elucidate. Through this elucidation, what they are supposed also to show is that they lack sense. But since the sentences of TLP are sentences in a language, they cannot show this directly in the way that signs in a canonical notation clearly show form. The problem is to understand how the sentences of TLP can be understood to elucidate what they are supposed to elucidate while acknowledging that they are senseless. The only alternative, I think, is to take the sentences of TLP as showing to the reader (1) how the attempt to say something about what is supposed to be essential to some saying of a proposition introduces confusion, and (2) how to understand

the distinction between saying and showing.

Both of these points TLP can only show through illustration. In TLP, Wittgenstein fully elucidates only two ways in which something can be shown: through a valid sequence of propositions and through instantiating logical form. The sentences of TLP are, then, supposed to illustrate attempts to say something true about what is supposed to be essential to the saying of a proposition. Following this example, the reader is committed to the distinction between saying and showing. With respect to this distinction, one comes to understand how the attempt is futile or superfluous, either directly through understanding the distinction or eventually through one's attempt to evaluate TLP in accordance with its own distinction. In the course of the latter route, a canonical notation and logic may be devised in accordance with Wittgenstein's distinction. The canonical notation would then show how the sentences of TLP lack the form of contingent, sensible sentences.

If the distinction is itself not supposed to be merely a heuristic device for the dissolution of the problem of clarity, but is one the understanding of which, say, properly orients Wittgenstein and his careful reader to the understanding of language, thought and the world, then TLP could itself succeed as an invitation to such an orientation, not as an illustration of its advantages, but as an

illustration of the disadvantages (inconsistencies, for instance) of an alternative orientation, namely, the orientation he adopts in TLP for the purposes of the dissolution of the realism and independence assumptions. The preface of TLP counts as a better illustration of the orientation recommended by passages such as 6.24-7. It better complies with the standards for philosophy that Wittgenstein sets within TLP at

7 Whereof one cannot speak, thereof one must be silent.

since it is only in the preface that Wittgenstein refuses to speak explicitly about the matters he has purportedly shown to transcend speech.

The problem is that this is not an illustration of a philosophical orientation that can ultimately be found philosophically satisfying, at least not in comparison with others. This is not simply because from this orientation one ceases to ask such questions as, say,

Q1 In what does the truth, sense or justification of a proposition consist?

with an expectation of propositional understanding. Rather, the dissatisfaction derives from the renewal of arguments over the questions to which Wittgenstein allows true or false answers. Invariably, these arguments eventually turn upon

questions such as Q1. From the orientation Wittgenstein recommends - once one assumes that orientation - there can be no recognition of commitments to different propositional answers to Q1. This would be comforting only if this nonetheless allowed us to come to some understanding with respect to the original questions in dispute. But this seems unlikely.

None of the above, of course, can count as a refutation of the orientation the Tractatus recommends, for there can be no refutation of an understanding which ceases to assert anything concerning a purported question. But it should help to make clear what questions and options are open. Before any open alternatives to the Tractarian orientation towards the understanding of language, thought and the world can be seriously considered, one first needs to recognize what options it closes. Most of the remaining sections of this chapter will deal with the assumptions that the sentences of TLP show to be inconsistent. Through the course of those sections it will be important to be mindful of what Wittgenstein assumes at 6.54:

My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.)

In order to show how the sentences in TLP show certain

assumptions to be mistaken, one must assume, contrary to Wittgenstein's gloss of TLP in 6.54, that the sentences in TLP are allowed to count as propositions. Eventually, however, one must attempt to read, or assess whether one can consistently read, TLP in compliance with what Wittgenstein assumes at 6.54.

Supposing, then, that the sentences of TLP count as propositions - in the sense which Wittgenstein elucidates - propositions such as 4.1212 explicitly introduce paradox into TLP.² But what is not made so explicit is how Wittgenstein's introduction of the distinction between saying and showing leads inexorably to this paradox. Rather, this is left unsaid. That is, Wittgenstein does not attempt to show these connections in the form of an argument. Granting that within the context of a single sentence nothing is and nothing can be said about what is shown, it does not follow that there is no context within which something can be said about something that is shown. Accordingly, one might suppose, for instance, that Wittgenstein's distinction - when introduced - could be combined with a revised theory of orders, where the order of a proposition is a function of the abstractness and complexity

² The paradox is that 4.1212 itself cannot be allowed to say anything. For suppose that it did say something, then what it says is the proposition that, for any p, what p shows no q can say or say anything about. But 'what p shows' can denote something about which some instantiation of 4.1212 says something, contrary to what 4.1212 says.

of its constituents or components. By this supposition, one might further suggest that 4.1212 could be replaced without loss of consistency or elucidatory value by the following:

4.1212' What one proposition can show no proposition of the same order can say or say something about, or

4.1212'' What one proposition can show only a proposition of higher order can say or say something about.

A similar elimination of the paradox could be effected through the elaboration of a theory of contexts of utterance. With respect to such a theory, 4.1212 could be replaced by the following:

4.1212*** Within any context of assertion something is shown about which nothing is said and which is itself left unsaid.

These sentences could at least allow the other sentences in TLP to say something about the way in which language and thought are so related to the world that what can be said or thought can be clearly said or thought. For instance, according to those sentences, what sentences like 'That is white' say, when 'That' is co-ordinated with some object is the proposition that the object is white, where part of what it shows is its logical form. If replacement of 4.1212 by 4.1212' preserved the consistency and elucidatory value of TLP, then, according to that replacement, the sentence, "The logical form of the proposition that the object is white is

the form of what it depicts," could count as a true proposition. The paradox could thereby be avoided.

But notwithstanding Wittgenstein's lack of explicit or direct argument³ for 4.1212, such a consideration of replacement, if maintained, would betray a considerable misunderstanding of the pictorial account of propositions. Indeed, the replacement of 4.1212 by 4.1212* or 4.1212** would be taken to re-introduce one of the very perplexities that Wittgenstein's pictorial account was supposed to dissolve, namely, the perplexity that, according to Wittgenstein, would arise once one assumed that form could be said or denoted. As we saw in the previous chapter, for Wittgenstein, the assumption committed Russell in his logical-cognitive analyses of judgements to first his type theory and ultimately to the violation of his type theory. But this perplexity, this inconsistency, would not have been counted as peculiar to Russell's theory of types and orders,⁴ for revision would be

³ If Wittgenstein gives one, then it is probably an attempt at an indirect argument, most likely a reductio to the absurdity of an infinite regress of contexts of assertion or orders of propositions. But this is an absurdity only if it is assumed that there is a maximal class of propositions or a maximum context of meaningful assertion, namely, one totality of facts.

⁴ Russell accused Wittgenstein's account of logical form and meaning of being mystical, since the account prohibits saying anything about logical form and meaning. Russell thought that one could say something about the logical form and meaning of expressions in a language within a higher order of language. This could be continued, so he (and later

effected, as we shall see, only at the cost of some fundamental assumptions.

This fundamental perplexity is most directly manifest in TLP through the contradiction of an assumption expressed at 2.0211. Wittgenstein writes:

If the world had no substance, then whether a proposition had sense would depend upon whether another proposition was true.

Since Wittgenstein is here attempting to show that the world

Tarski) proposed, indefinitely without any threat of contradiction. Rather, it could be continued with the promise of increased understanding or improved explanation. But 1 if contradiction can be avoided through such an ascent of languages, then it can only be avoided through the employment of rather arbitrary devices or rules, and 2 such an ascent does not promise either a termination or an improvement in understanding. It cannot show any more clearly what the canonical notation whose signs are mapped onto canonical signs in a language shows. This is especially evident once one recognizes that the so called languages in the ascent must, if the procedure is to be canonical, be canonical notations themselves.

Evidently some confusion over the distinction between canonical notations and languages is involved here. One does not use a canonical notation in order to better show the sense and forms of just another canonical notation. If the canonical hierarchy of canonical notation is to more clearly show the sense and form of the signs of any language, then the signs of each of the canonical notations must be mapped onto canonical symbols of languages in a hierarchy based on that language. But languages are not spoken or written as meta-languages of others. Rather, they are translated into each other via direct and indirect quotation or direct quotation and disquotation through truth-valuation and decomposition. Thus, the Wittgensteinian reply to the Russellian complaint against the mysticism of his account is to show how Russell's and Frege's (even Tarski's) alternatives are confused or obscure.

has substance, he is tacitly assuming the negation of the consequent of 2.0211, namely, that whether a proposition showed sense or more exactly form could not depend upon whether or not some proposition was true. [The sentences immediately following 2.0211 are,

2.0212 It would then be impossible to form a picture of the world (true or false).

2.022 It is clear that however different from the real one an imagined world may be, it must have something - a form - in common with the real world.

Neither of these sentences provides non-question-begging support for the above assumption, since they fail to explain fully why the consequent of 2.0211 would imply 2.0212 or the negation of 2.022. Accordingly, the explanation must consist in some assumptions prior to these in TLP.]

Now if one assumed that the sense of a proposition could be denoted, then, since terms denote only in the context of propositions, one would allow that whether or not that proposition showed sense would depend upon whether or not the following propositions were true:

The proposition that the proposition had that sense.

The proposition that there is such a sense to be shown.

Or, more explicitly, one would allow that, without exception whether the sentence 'S' says the proposition that S depends

upon whether 'Some item is such that it is the sense of the proposition that S' says something true.

According to this analysis, one cannot consistently assent to 2.0211 and assume that sense or form can be denoted. Thus, replacement of 4.1212 by 4.1212* or by 4.1212** would not preserve the consistency of Wittgenstein's pictorial account of propositions.

Though this analysis is, I think, correct, a consideration of the following point should show how the analysis requires further elucidation.

Even though the perplexity outlined above may be directly manifest in the contradiction of what is assumed at 2.0211, the perplexity must itself consist in the violation of some prior constraints on Wittgenstein's account of sense. For suppose that this were not the case. Then, as in the case of 4.1212, no prior constraint for Wittgenstein's account of sense could be provided against the replacement of what he tacitly assumes at 2.0211 by the following:

Whether a proposition has a sense cannot depend upon whether some proposition of the same or lower order is true.

But, then once again the distinction between saying and showing could be relativised to orders of propositions or contexts of assertions, contrary to the standing assumption of this analysis.

Accordingly, some prior constraints upon Wittgenstein's account of sense must explain his commitments to what he assumes at 2.0211 and 4.1212. As I have argued in the introduction of this thesis, the prior constraints upon Wittgenstein's account of sense are those he inherited from Russell and Frege through his acceptance of the problem of clarity. This problem, it was argued, was supposed to be solved subject to the following two fundamental assumptions:

RT The meaning (or the components of the meaning) of every symbol has being.

IT That which has being does not - at least not without exceptions - depend for its being, or for the ways in which it stands to other entities, upon its being meant by a symbol in language.

I will try to show in the remainder of this section that these are the assumptions that commit Wittgenstein to what he further assumes at 2.0211 and so at 4.1212. If my argument is correct, then this will give another version of how a particular formulation of the problem of clarity has been dissolved in TLP. But, given this version, acquiescence in the paradox of TLP - as this has been expounded in the first section of this chapter - will no longer be the only realist option left once one has encountered the paradox of TLP.

A further assumption of TLP will be invoked in the following argument, namely,

CT Without exception, whether an atomic fact is the case is entirely contingent.

Before turning to the main argument, some explanation of CT and of how (or at which sentences in TLP) Wittgenstein is committed to CT needs to be provided, for this assumption has hitherto only been briefly mentioned and discussed.

In the case of CT, then, Wittgenstein's commitment arises from three assumptions in TLP:

- (1) No proposition is a priori true.
- (2) Atomic facts are independent, that is, whether one atomic fact is the case does not depend upon whether some other fact is the case.
- (3) That there is a world is mysterious.

Wittgenstein expresses (1) quite explicitly in the following passages:

2.22 The picture represents what it represents, independently of its truth or falsehood, through the form of representation.

2.221 What the picture represents is its sense.

2.222 In the agreement or disagreement of its sense with reality, its truth or falsity consists.

2.225 There is no picture which is a priori true.

And (2) at:

2.06 Atomic facts are independent of one another.

2.062 From the existence or non-existence of an atomic fact we cannot infer the existence or non-existence of another.

And, lastly, (3) at:

6.44 Not how the world is, is the mystical, but that it is.

Together these assumptions give the strongest possible version of CT, what might be called a thesis of radical contingency. While the first two argue simply for the contingency of which atomic facts exist and which do not, the second argues further for the contingency of the world. The mysteriousness of there being some fact consists in the essential lack of any propositions that could explain why something is the case, rather than not. But where reasons are lacking, so is necessity.

Having dealt with Wittgenstein's commitments to CT, we may now return to what Wittgenstein assumes at 2.0211.

To some extent, what Wittgenstein says in TLP at 2.0211 is a bit misleading, since it combines a linguistic with a cosmological point. Moreover, it is vague. The linguistic point is that whether some state of affairs counts as a proposition cannot, without exception, depend upon whether some other state of affairs counts as a true proposition. Or, to put it more formally,

Ind For some propositional sign 'p', whether it has sense in C cannot depend upon whether for some context D and some propositional sign 'q', 'q' counts as a proposition in D and 'q' is true.

The cosmological point is that what has being cannot without exception depend upon whether something is the case, that is, something subsists. This 'something' is, as we have seen in the introduction, what Wittgenstein calls the substance of the world. The connection between these two points consists in Wittgenstein's account of proper names and atomic propositions. Atomic propositions are what are supposed to be complete symbols which contain no occurrences of other complete symbols but names. Thus, they express sense directly. By some of the assumptions that support the contingency assumption, it must be possible for an atomic proposition to be false. Now, though, for Wittgenstein, there are negative facts,⁵ since he denies that there are objects corresponding to '~', '^', etc., he denies that there is any such thing as a complex corresponding to a negative fact. Thus, the names occurring in an atomic propositions must name what has being independently of what is the case.

One can already recognize in this analysis Wittgenstein's prior commitments to RT and IT, but only in a rather indirect manner. In order to more clearly discern Wittgenstein's

⁵ Facts which make atomic propositions false and their respective negations true.

commitments in 2.0211, suppose the contrary. That is, suppose that whether some proposition P had sense depended upon whether some other proposition P^1 was true. Then there is a sequence P, P^1, \dots of propositions such that for each term X^k in the sequence whether X^k has sense depends upon whether X^{k+1} is true and every such sequence lacks a minimum and lacks recurrences.⁶ Then, for some proposition P , there would be an unbounded sequence of propositions P, P^1, \dots such that either all of them lack sense or P has sense and every proposition afterward is true. Since no proposition lacks sense, the first case is absurd. In the second case, the following must hold:

$$\begin{aligned} & P^k \rightarrow P^{k+1} \text{ and } \sim P^k \rightarrow \sim P^{k+1} \\ \text{so that } & \sim P^{k+1} \rightarrow \sim P^k \\ \text{Hence, } & \sim P^{k+1} \rightarrow \perp \end{aligned}$$

So, $\tau \rightarrow P^{k+1}$, that is, the sense of P^{k+1} necessarily agrees with reality.

If this argument were to count as a sound reductio, then it must be false that the sense of some proposition necessarily agrees with reality. That CT is presupposed should be quite evident, for if some sense necessarily agreed with reality, then some fact, whose constituents were the

⁶ For Wittgenstein, this proposition holds, since whether a proposition has sense is independent of which truth-value it takes.

contents of the sense, would necessarily be the case. But the contingency thesis itself, though it is a fundamental metaphysical assumption is not directly relevant to the logical problem with which TLP is dealing. In particular, one of the problems associated with the problem of clarity is to provide an account of the essential universality of the laws of logic notwithstanding that what, according to TLP, is not evaluable with respect to those laws is contingent, namely, propositions. This problem would not be complicated if, instead of CT, the proposition that some picture is a priori true and some fact is necessarily the case were assumed. For then the laws of logic would count as universally valid since they are prescribed by whatever grasps or projects a sense, e.g. a subject. Since, by the above quotations, some fact could be necessary only if some sense necessarily agreed with reality, some sense would have to be assumed to subsist. That is, some sense would have to be assumed to be eternal. Thus, since all sense has being for expression in some language (even if only a mental one), whatever such a subsistent sense determined to be the case must eventually be represented in thought or language. This would be contrary to both the

⁷ Of course, it does not follow from the proposition that sense can be expressed that, at some context, it is expressed. But saying that sense is for expression is not the same as saying merely that it can be expressed. Rather, it presupposes that there is something with respect to which sense is what it is, namely, a language. Sense is not needed

independence and realism theses, for by the independence thesis, for some fact, it is not essential to say of it that it is represented - it is only essential that the fact be representable. But the realism thesis was presupposed from the very outset in Wittgenstein's account of the meanings of proper names. So, it is the realism and independence theses that commit Wittgenstein to CT.

If this analysis is correct, then it follows that what TLP shows is that the assumptions RT, IT, and CT of the problem of clarity are not consistent with the presumption that the sentences of TLP are propositions, for together these assumptions commit Wittgenstein to an ultimate distinction between saying and showing, and therewith the assumption that the sentences of TLP do not say anything. Two options seem to be open, then. Either the assumptions RT, IT, and CT are not entirely correct or the assumption that the sentences of TLP are propositions (truth-valuable) is mistaken. In either case, the problem of clarity, at least as it was originally formulated, can no longer be supposed to be resolved in terms

for objects to stand in determinate relations to one another. The objects and the forms they determine, for Wittgenstein, are supposed to be sufficient for the constitution of atomic facts and so the world. Wittgenstein himself articulates this position at 2.0231. Sense, then, is needed only for representation. It is what we project. [2.1] Thus, if there is sense, then there must be a language which makes for the possibility of its expression; otherwise, as a subsistent entity, it would be superfluous.

of propositions. In the first case, the assumptions with respect to which that problem was taken would no longer be supposed to hold and, in the second case, no propositions could be assumed to answer it.

Of course, Wittgenstein adopts the latter alternative. For instance, he writes:

6.54 My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.)

He must surmount these propositions; then he sees the world rightly.

I have already given some suggestion as to why this alternative is not especially satisfactory. To provide an assessment of Wittgenstein's elucidations in TLP, I will present considerations of 2.0211 which recommend some revision in the assumptions with respect to which the problem of clarity has been taken. In particular, I will deal with one corollary of what Wittgenstein assumes at 2.0211, namely:

RT' The meaning of a proper name has being independently of what is the case.

This corollary is responsible for many of the more curious positions of Wittgenstein's. For instance, the assumption that the proposition ' $\exists x(x=a)$ ' is senseless for any proper

name 'a'.⁸ But it is also the source of many difficulties for

⁸ For, Frege all names, whether proper or improper, express sense. Their senses are constant functions. The logical difference for Frege between a proper and an improper name is that the sense of a proper name is indecomposable and that of an improper name is composite, is composed of the references (or senses) of different predicates. Nonetheless, depending upon which way the world is as a whole, the references of a proper and an improper name may coincide in one object. Thus, for Frege, the sentence ' $\exists x(x=a)$ ' is not a piece of nonsense.

For Russell, no name expresses sense. Though proper names, unlike improper names, without exception have meaning, what a proper name names necessarily satisfies a predicative proposition. (by the axiom of reducibility.) With respect to these assumptions, Russell can further assume that ' $\exists x(x=a)$ ' does say an assertion, even in the context of logic, for according to those assumptions, for any two names 'a' and 'b', 'a=b' says that a and b satisfy the same predicative propositional functions.

Wittgenstein, of course, accepts neither of these positions. By RT', the reference of a proper name need not have occurred in any of the states of affairs in which it occurs. Thus, none of the concepts or propositional functions which it satisfied in those states of affairs hold essentially of it. But, for Wittgenstein, if terms expressed sense then the sense of a proper name would correspond to an informal concept that the reference of the name essentially satisfies. So, though it is by no means inconsistent to assume that proper names express sense - as Frege does - for Wittgenstein such an assumption is superfluous, both in the sense that Wittgenstein is not committed to any such assumption and in the sense that Wittgenstein does not need the assumption in order to elucidate how proper names mean. Accordingly, for Wittgenstein, if ' $\exists x(x=a)$ ' expresses sense, this is not because 'a' expresses sense.

Wittgenstein's problem with Russell's account is not very different from his problem with Frege's. By RT', when considered independently of what is the case, several objects may be able to occur in the same positions within the same situations, even though the objects are different. Hence, for Wittgenstein, Russell's interpretation of 'a=b' is false. This Wittgenstein articulates at several sentences in TLP, for instance, at 2.0233 & 2.02331, but nowhere more explicitly than at 5.5302:

Russell's definition of '=' won't do; because according

Wittgenstein's account of propositions about complexes. One of these we have already dealt with in the chapter on molecular propositions, namely, the assumption that complexes cannot be properly named. This follows from RT', since complexes depend for their being upon what is the case - for instance, if a negative fact is the case then no corresponding complex exists.

This is unacceptable not just because it flaunts the common unquestioned assumptions that the everyday objects of our lives, the only ones with which we are familiar, can be named properly. Rather, it is unacceptable because it ceases to treat complexes as objects. Since all propositions purportedly about complexes, for Wittgenstein, get replaced by

to it one cannot say that two objects have all their properties in common. (Even if this proposition is never true, it is nevertheless *significant*.)

Incidentally, this position also accounts for the consistency of 2.0211 and RT. An apparent problem with 2.0211 is that it conflicts with a version of the realism assumption, according to which the meanings of symbols have being:

RT* If 'a' is a proper name, then 'Fa' has sense only if there is some true existential proposition satisfied by the meaning of 'a'.

Given the above account of the senselessness of ' $\exists x(x=a)$ ' and Wittgenstein's account of logical symbols, one can appreciate why no such existential proposition as the consequent posits could be formulated, for such a symbol could be only of one of the following two forms:

$$\begin{aligned} &\exists x(x=a) \\ &\exists \phi \exists x(\phi x) \end{aligned}$$

disjunctions of elementary propositions, none of whose names name complexes, it is plain that Wittgenstein's language-independent ontology consists only of simple objects, the forms they determine and atomic facts. Complexes are evidently constructions of our language. If the independence assumption is maintained, which in contrast to the other two I think it should, then it follows that complexes lack being.

Given such consequences as these, the orientation of our understanding which the last sections of TLP and the preface recommend seems irredeemably bleak. Some alternative, which takes the problem of elucidating the symbolism of logic and with it the problem of clarity, seems to be in demand. Considering the importance of respecting the universal validity of logic, though not necessarily classical logic, what ought to be preserved in some form is the independence assumption. In contrast, the contingency thesis and the realism assumption have seemed to be the source of many problems. Accordingly, the appropriateness of these assumptions should be seriously reconsidered.

Many of the problems that, according to the above discussion, have arisen from the assumption of RT, IT, and CT, have previously been shown to be especially vexatious over the question as to whether or not complex entities can be denoted directly. In particular, in Chapter 2, I have argued that Wittgenstein's commitment to the problematic assumption that

complexes cannot be denoted by proper names arises from such assumptions as the following:

W1 A proper name occurs as a term in an atomic proposition.

W2 The meaning of a proper name has being independently of what is the case.

Suppose that these two sentences expressed a consistent set of assumptions. Then, of course, 'atomic proposition' must be interpreted in such a manner that W1 is consistent with W2. That is, a proposition would have to be counted as atomic only if when true, the fact in whose existence its truth consists did not depend upon some other fact being the case. Thus, together these assumptions imply the following further assumption:

W3 The meaning of a proper name is not (in any way) complex.

For if the meaning of a proper name were complex, that is, if it had constituents, then its being would depend upon how those constituents stood to one another in the world.

As discussed earlier in Chapter 2 and just previously, this latter assumption is objectionable for two reasons. Firstly, it is contrary to and fails to account for abundant linguistic evidence to the effect that we directly denote complex entities, whether ostensively or otherwise. Secondly, and more importantly, it fails to conform with the more

substantive realist assumption that not only simple, or primitive, objects but complexes have being independently of whether they are represented in language or thought. This latter objection arises from the observation that if complexes can only be described, or otherwise indirectly denoted,⁹ then, for Wittgenstein, the existential proposition that there is exactly one thing satisfying the predicate, 'P', say, in the description of a complex cannot, if true, be truth-conditionally equivalent to the disjunction of all the possible results of correctly concatenating a proper name with 'P'. This suggests that complexes cannot directly count as entities in the world, or as having being independently of being represented, but must, rather, be constructed from such entities that do directly count as entities in the world.¹⁰

⁹ Note that in observing a distinction between direct and indirect denotation, one is in no way committed to the Russellian account of the meaning of terms, namely, that their meaning, if any, consists only of their references. In particular, one may still follow Frege in accepting that the meaning, if any, of a term consists of a sense and a reference. A term, then, would be said to directly denote an object just in case its sense is simple, or non-composite, and indirectly otherwise.

¹⁰ This is a consequence which Wittgenstein seems quite willing to accept in many passages in TLP. See especially those passages in which he discusses how much truth there is in solipsism (5.6-5.63), and the relation of logic to Newtonian mechanics (6.341-6.342). It is not altogether clear as to whether they support or deny the assumption that which complexes there are depends not only upon which atomic facts are the case but also upon the canons of grammar and diction within a language.

The source of the difficulties in this case is, I think, the second sentence, W2, for it is only when W1 is taken as consistent with W2 that the phrase 'atomic proposition' receives its problematic elucidation, and it is only the conjunction of these sentences, so understood, that implies W3. W1 alone does not imply W3. For if W2 were not accepted, a simple sentence would count as an atomic proposition just in case according to the canons of grammar and diction no symbol occurs in any of its terms or its predicate, and neither any of its terms nor its predicate inherits its meaning from any other symbol in the language, whether by explicit or implicit definition in terms of propositions in which the term or predicate does not occur. Accordingly, a proper name or a primitive predicate would be a simple sign which means independently of what other symbols mean, where these are not combined with it in its context of meaning. None of this implies that the meaning of a simple symbol is simple itself. In this case, the requirement that the meaning of a simple symbol be simple comes only from W2. Moreover, Wittgenstein's commitment to W2 itself arises from his assumption of another version of CT, for it is only with respect to his assumption that whether an atomic proposition is true cannot depend upon whether another atomic proposition is true that W2 at all appears compelling. One important point to notice here is that W2 may be rejected while still adopting a specific version of

the realist assumption, namely:

W2' The meaning of a proper name or a primitive predicate has being.

If W1 and W2' were maintained and some proper names were allowed to mean complexes, then some propositions could count as *a posteriori* necessary. For instance, if '**6**' and ' a_0 ', ' a_1 ' ... ' a_n ' were to count as proper names, where the existence of the reference of '**6**' depended upon the references of ' a_0 ', ..., ' a_n ' standing in the relation meant by ' R ', then the truth of ' $a_k C_R^k \mathbf{6}$ ' would depend upon, even follow from, the truth of ' $R(a_0, a_1, \dots, a_n)$ ', even in contexts in which ' $\mathbf{6} = (\exists \gamma) (\wedge a_k C_P^k \gamma)$ ' has not been established either by implicit or explicit definition.¹¹

The realism assumption, however, need not be extended to all symbols. That is, even though it might be reasonable to assume that the references of proper names have being, it does not follow that the references of all nouns have being. Accordingly, contrary to what Russell, Frege and Wittgenstein each assume, some distinction may be assumed to apply between saying that exactly one thing has a property, P , and

¹¹ Though in the case of proper names that mean (existing) complexes, the propositions in which they occur depend for their meaning upon the truth of other propositions, this is not generally the case. For instance, though ' $F(\mathbf{6})$ ' has meaning only if ' $R(a_0, a_1, \dots, a_n)$ ' is true, ' $R(a_0, \dots, a_n)$ ' may have meaning independently of whether another proposition is true.

saying that the item that is P has being. Though the former follows from the latter, the latter does not follow from the former. They are not logically equivalent statements. This recommends that a predicate 'being', 'actual' or even 'eternal' can be introduced into a theory as a primitive symbol, so that a theory may support truths about items that lack being, what I will call non-entities. One problem would be to present a schema with respect to which one could correctly formulate within a theory sufficient conditions for being. Another equally pressing problem would be to distinguish such ontological predicates as 'B!'='has being' from other meaningful predicates in the language so that the following does not count as a valid schema:

Schema A

$$\begin{array}{l} \underline{1} \ a = (\exists x) (Fx \wedge B!x) \\ \hline \underline{2} \ B!a \end{array}$$

or

Schema A*

$$\begin{array}{l} \exists y (y = (\exists x) (Fx \wedge B!x)) \\ \hline B![(\exists x) (Fx \wedge B!x)] \end{array}$$

even though, for propositional function symbols in which only predicates relevantly distinct from "B!" occur, the following counts as valid:

Schema B

$$\begin{array}{l} \underline{1} \ a = (\exists x) (\phi) \\ \hline \underline{2} \ \phi(a) \end{array}$$

or

Schema B*

$$\begin{array}{l} \exists y (y = (\exists x) (\phi)) \\ \hline \phi[(\exists x) (\phi)] \end{array}$$

The problem with Schema A is that it would allow one to infer from the description of an item that it has being, even in cases where no such item has or could have being. Consider, for example, the instance of the above schema that would result, if 'is a round square' were substituted for 'F'. No being could be both round and square. Such an item as a round square is impossible. *A fortiori*, even if there are items that are round and square, there is no item that is round and square and has being. Unless non-arbitrary, non-circular restrictions upon substitutes for ' ϕ ' can be provided which exclude Schema A from counting as a special case of Schema B, the introduction of 'B!' as a primitive would threaten to incur contradiction. Part of the difficulty of this problem is that one must distinguish occurrences of 'B!' from such occurrences of predicates which validate Schema B without undermining 'B!''s status as a primitive predicate.

Both of these problem I think are capable of solution, but the first I think is more especially a problem for each theory. Each theory may have its own conditions or schemas for the conditions necessary and sufficient for the being, becoming and perishing of entities. My preference is for a set of inductive conditions, conditions, however, which also provide for, or at least do not preclude, the formulation of the becoming and perishing of entities. In particular, an inductive schema for conditions necessary and sufficient for

the combination of entities or constituents of entities into a single actual entity, where the former come to occur in the past of the latter, would be especially appealing. But appeal aside this is a problem for which no solution can be adequately introduced and explained, here. Accordingly, I will treat briefly only of the second problem. But a solution to this problem can be forthcoming only once one has established distinctions between different modes of being, and three sets of distinctions are crucial for the position which I would tentatively recommend, here.

The first provides a rather rough criteria for an item to count as an item with being, that is, an entity, and, is adapted from some of the work of W. James. In many works, but especially in Varieties of Religious Experience, James speaks of our sense or feeling of reality.¹² As he characterized it, the feeling is a feeling of some difference having been made to our experience, or practice. Part of his concern was to account for our discrimination of perceptions which disclose real items and those which do not. Since most of his emphasis was upon how a community comes to settle upon what counts as real, what is real for James is what makes a difference to the practices and experiences of the members of that community. It is because of James' addition of this last

¹² See 'Lecture 3', p. 62.

clause that it is not exactly the distinction that I require, here. The clause imposes a condition which seems to make the being of an entity, and not just our recognition of the entity, dependent upon the practices of perceivers. However broadly one construes the term 'practice' or 'perception', this condition can only count as premature. The notion that entities, but not non-entities, make a difference, however, is, I think, an important and promising one. Since what needs to be accounted for among items, here, is their countability, the distinction between entities and non-entities, items that lack being, initially appears to be simply that whereas entities make a difference to *what* is counted among items, non-entities do not. Since, moreover, two minimal, necessary conditions for countability are having a determinate composition and having determinable occurrences in other items, this initial formulation can be further specified as follows:

Whereas entities make a difference to the composition *and* occurrences of other items, non-entities do not.

The composition of an item would be allowed to be a matter of what properties it bears, what items occur in it and which relations or functions in virtue of which those items occur in it. Thus, any item directly in virtue of which another item results or endures through any change in composition will thus

count as an entity.¹³

According to the second distinction, there are temporal, eternal and sempiternal entities. Among temporal entities, only past items and entities which are present, or complex items which endure into the present, have being, where present entities or entities which endure into the present will all be called actual entities. Items in the future of present entities, if there are any, will not be assumed to have being. Accordingly, one point that needs to be explained is the asymmetry in the account of items in the past and future of present entities. Whereas past entities are assumed to have being, future ones are not. One rationale for this distinction is that whereas the items in the past of some actual entities occur in fixed relations to one another as constituents of that actual entity, items in the future do not. But another rationale derives directly from the above distinction between entities and non-entities. Future items, unlike past items, are not items directly in virtue of which a present entity is composed. Even in the case of a future item which is planned, expected, or anticipated it is not the item planned, expected or anticipated that makes a difference to what happens, but the events of the planning etc..

¹³ This is not meant to imply that, for any item, all of its constituents, or all of the relations that its constituents stand in to one another, are entities, since some non-entities have non-entities as constituents.

Sempiternal entities are any items which have an eternal entity as a constituent yet depend for their composition, if not their being, upon what is the case. An obvious case of a sempiternal entity would be a complex which has a temporal entity or a non-entity as a direct constituent or component. Other cases arise, however, if, following Wittgenstein in TLP, we allow that some modes-of-combination count among items only with respect to a background consisting of facts. This may be assumed to apply to how the eternal entities, such as some concepts and relations, come to occur in the meanings of symbols, in which case all composite meanings of symbols would be assumed to be sempiternal. Moreover, if, following Frege, one distinguishes between two components in the meaning of linguistic symbols, namely, sense and reference, then all meanings of symbols are sempiternal, even though, as in the case of some predicates, their reference (or sense) is eternal.¹⁴ Indeed, all composite senses of symbols, especially the senses of terms which refer to entities that

¹⁴ Recall that if the reference of a predicate counts as a relation, then the sense of a sentence is a composite of the senses of terms in the sentence with the reference of the predicate. Note also that if the reference of a predicate is a relation, then, for Frege, its sense is some item the grasping of which would suffice for the recognition of whether or not a given sequence of objects stand in that relation to one another. Such an item seems to depend not only upon how the world is, but since it mediates between the concept and our grasp of it, also upon our cognitive capacities and background.

depend for their being upon our grasp of senses - eg. tables, chairs - count as sempiternal.

A final distinction arises, now, between discriminating and non-discriminating properties (or discriminants and non-discriminants). Roughly, the distinction derives from the following condition:

P is a property in virtue of which what has P and what lacks P count as different items independently of the context in which these items occur.

Whereas discriminants do satisfy this condition, non-discriminants do not. Since the same item that is present may be past, past, present, future and so actual and non-actual; being and non-being count as non-discriminants, moreover, any properties which an entity may inherit in relation to what is actual will count as a non-discriminant. This will become clearer with respect to the following definition:

An item, a, is complete just in case for every discriminant, P, either a has P or there are some discriminants which a has that preclude a from having P, or from having P or any contrary of P.

Completeness is assumed to be a necessary but not a sufficient condition for an item to have being, irrespective of which mode of being it has. Thus, past entities may be assumed to be complete, even though, since there are always new actualities, they stand in new relations to what is actual.

This last distinction provides, at least tentatively, the following solution to the above problem with schemas A and B. The problem was to provide a non-circular and non-arbitrary condition upon the predicates that occur within a propositional function symbol ' ϕ ' so that Schema A does not count as a valid instance of Schema B. This condition can now be given in the following principle:

If, for all χ occurring in ϕ with x occurring freely, χ is a discriminant predicate, that is, that is, refers to a discriminant, then $\exists y (y = (\lambda x) (\phi)) = \phi[(\lambda x) (\phi)]$ is a valid schema.

Since 'is actual', 'is an entity' do not count as discriminant predicates, Schema A cannot, by this principle, count as a valid instance of Schema B.¹⁵

These remarks suggest some way in which Wittgenstein's paradox can be overcome. In particular, since the meaning of the predicate 'correctly depicts' would not change or constitute the items which it happens to relate, it would, of course, count as a non-discriminant predicate. Thus, following Wittgenstein's analysis of representations, suppose

¹⁵ The distinction between discriminants and non-discriminants also provides for an improved formulation of the distinction between entities and non-entities. For instance, since non-entities can be constituents of entities, e.g. beliefs and propositions, one may allow that non-entities do make differences to entities. Nonetheless, one may still deny that non-entities make discriminating differences to other entities. This would make the distinction much more reasonable.

that one's sensation depicts some complex. Then, though the sensation itself can only show that it stands in this relation, since the sensation may come to occur as a constituent of complexes consisting of the sensation and the depicted complex, there can eventually arise a proposition that the sensation depicted some item. Of course, whether the sensation expresses a sense determines whether that proposition is true, contrary to what Wittgenstein assumes at 2.0211. But since the proposition itself is not eternal and the component relation /depicts/ is a non-discriminant, the actuality of the depicted complex need not be assumed to depend upon its being depicted. Thus, the account outlined above not only allows that one can eventually assert that a given complex occurs as a constituent of another complex, it also allows for the continuable extension of contexts of assertion so that for every complex one can eventually assert something of the mode in which its constituents have been combined and represented. Thus, in some important respects, one could, contrary to what Wittgenstein tells us, be allowed to eventually say something true about what according to Wittgenstein can only be shown.

Accordingly, not only do the problems of clarity and the elucidation of logic remain open problems, but a solution to these problems under a qualified version of realism also remains an open option.

Appendix 1

In the following two appendices, I will be attempting to rigorously formulate some of the distinctions and assumptions introduced and elucidated briefly in the body of the thesis. Two distinctions, in particular, will be emphasized in the first appendix:

- i) The distinction between mode-of-combination signs, terms and strict terms.
- ii) The distinction between a logically canonical sign (or symbol) and a logically uncanonical sign (or symbol).

Both of these distinctions occur at critical points in the exegesis and appraisal of Frege's, Wittgenstein's and Russell's respective philosophies of language, logic and meaning. The first distinction is drawn in the course of comparing these authors on their respective accounts of the semantic status of copulae, adjectives, prepositions (mode-of-combination signs) and nouns (strict terms) and the ontological status of the meanings of such signs (if they are assumed to have any meanings). The second distinction is drawn in giving an account of the logical adequacy or canonicity of a language or notation. More especially, it is required in giving a neutral formulation of Russell's distinction between terms which are complete signs and terms

which are incomplete.

The second appendix supplements the first with a more rigorous characterization of what I have called the background of a symbol. This will provide for a more explicit formulation of the theses that Wittgenstein assumes at 2.0211 and at 3 in TLP.

One term of central importance to this exposition is the verb 'occurs'. Signs and symbols will be spoken of as arrays of characters (marks or sounds) and as such will be said to character a sign if they are written in the writing of that sign. Accordingly, every sign that occurs within another sign will be said to character that sign. The converse implication will not be assumed to hold. An elucidation of the verb 'occurs' will be attempted according to which terms, strict terms and mode-of-combination signs can be spoken of determinately with regards to the roles they respectively play in how or whether one sign occurs within another a sign which the terms or mode-of-combination signs character. In particular, I will say that one sign occurs within another sign just in case the former characters the latter and the composition of the meaning (if any) of the latter depends upon the meaning (if any) of the former. Since one of the points of contention through the work of Frege, Russell and Wittgenstein is over the interpretation of predicates, prepositions, conjunctions etc., mode-of-combination signs

cannot be distinguished according to whether they have meaning or whether their reference or sense has a dependent mode-of-being. The elucidation of 'occurs' which I propose, here, is meant to comply with this constraint. Though this term will be defined in the following discussion, the definition will make sense only with respect to a brief characterization of a (colloquial) language. For only with respect to such an elucidation can one come to understand the relation of the relation which the term 'occurs' is supposed to denote or express, namely, signs and symbols.

A language is assumed to consist of types of arrays of characters and routines of writing them in order to express assertions, present hypotheses or ask questions. Not every one of these routines, though, is assumed to be such that if one follows or performs it, then the performance directly results in an assertion, hypothesis or question, for many routines, if followed, directly result only in the writing of various well-formed parts of speech, parts of speech, moreover, which may well lack meanings. Following Frege's and Wittgenstein's emphasis in the characterization of language, I will also be assuming that it is from the reiteration of signs which occur in propositions that other meaningful occurrences of signs can obtain. Accordingly, for Frege and Wittgenstein, languages are inseparable from theories, where a theory is assumed to consist of symbols of a language and

(in addition to the above routines) methods or routines for the construction of truth-preserving or truth-supporting sequences of assertions of the language. Though every theory has a language, many theories, differing not only in what propositions they support but also in their vocabularies, may derive from a common language.

Two problems that remain to be discussed are i) the problem of distinguishing between characters, signs and symbols and ii) the problem of equating arrays of characters which count as or present the same sign. For the solution of these two problems, a formulation of sign and symbol decompositions will be presented. Given these formulations, I will propose and defend definitions of sameness of sign and sameness of symbol type. In particular, I will be attempting to show that two symbols are of the same type just in case they present the same sign and there is a meaning preserving isomorphism between their respective decompositions. The sameness of type between two series of marks or sounds will be invoked in this context as primitive, even though it is allowed that whether two marks or sounds are of the same type may have to be decided within some theory. A comprehensive theory of symbolism and signs, evidently, not only presupposes other theories as data but also presupposes that other theories provide *prima facie* support for some assumptions about some of the data, for instance, assumptions about

characters, marks and sounds.

The difference between a sign and a symbol should first receive some elucidation. What this difference is supposed to consist in is that whereas with respect to some set of routines the latter counts as having a meaning, the former does not. More accurately, a symbol in a language is a sign in a determinate context, where in that context various rules are exemplified according to which the sign counts as having a meaning. Consider the following case:

Caesar was the first Emperor of Rome. He defeated Pompeii and was slain by Brutus. He was identical.

Given the canons of English diction from English histories of Rome, 'Caesar' counts not only as a sign, a noun, it counts as a meaningful noun. Given the canons of English grammar and the position and ending of 'identical' in this context, it is a sign, in particular, an adjective, but it lacks meaning. Of course, 'identical' has other occurrences in which it prima-facie has a meaning, for instance

Cicero is identical to Tully.
Cicero is self-identical.

But, even if, contrary to Wittgenstein's assumptions, these sentences expressed sense, the meaningful occurrence of 'identical' in these sentences would not bequeath some meaning for 'identical' above. Though 'He was identical' may be read

as an ellipsis for 'He was self-identical' nothing in English grammar or diction determines that it should be so read. Rather, only rules exemplified in the context of utterance of that sentence could determine it, and no such rules are exemplified in this context.¹ Thus, an adjectival meaning remains to be given to 'identical' and this may be given without invoking any antecedent meaningful occurrences of 'identical'.

The difference between a sign and an array of marks or sounds is that whereas with respect to some set of routines the former is essentially counted as grammatically related to some symbols - since, for instance, it consists of items which occur in symbols and which are combined together in accordance with the way in which those symbols are composed - the latter is not.

Consider, for instance, the following context:

Proxical Tam wob pally. Ob ~~X~~ V

Given the canons of English Orthography and grammar, 'Proxical Tam wob pally' counts as an English sentence if

'proxical' counts as an adjective
'Tam' counts as a noun

¹ All contexts of utterance which result in assertion exemplify routines of symbolization. This does not entail that such rules will be exemplified or ought to be exemplified in other contexts.

'wob' counts as a verb
& 'pally' counts as an adverb

which, given the endings, they may readily so count. But 'xv' can only count as a series of marks, since the constituent marks do not even count as English characters. 'xv' could of course be adjoined to English in a theory and so count as a sign but only if their occurrences as symbols in that theory were determined. This is not determined in the above context.²

Whereas a mark which correctly counts as a sign is so counted in connexion with a set of routines for its concatenation with other marks or for the selection and concatenation of its constituent marks so that a symbol results, a mark which fails to count as a sign is not counted in connexion with such routines. Thus, written signs are taken as exemplifying both orthographic and syntactic properties. For instance, whereas it makes sense to ask how a word is spelled or written, or whether it is a noun or verb, it would not make sense to ask how the marks are spelled or

² This distinction appears to be less tenable in cases where symbols from different languages or symbol systems (eg. stop-signs, arrows etc.) are combined with one another. In such cases, though no antecedent meaningful occurrences of the signs bequeath a meaning for the symbols in such a novel combination, the meanings can be adopted. Nonetheless, when a meaning for the combination is decided with respect to such a background this exemplifies rules which may be followed in other cases.

whether it is a noun or verb. Having said this, it is important to reiterate, here, that though a set of routines may be exemplified in a context and subsequently followed, the same marks may count as different signs within a language, not just as different symbols, since the marks may in different contexts count as different parts of speech. For instance, 'white' can occur both as an adjective and as a noun, and so may also count as different symbols with different types of meanings. It might be thought that if two symbols are spelled the same way, then they ought to count as the same sign. Given a distinction between types and tokens of signs and symbols, then, different symbols spelled in the same way would count as the tokens of one type of sign, they may not count as the same type of symbol. Symbol types instead of signs would then be assumed to be taken with syntactic properties. Though this is a viable alternative, I would rather say, for instance, that different signs may be spelled the same way. Saying that a mark counts as a sign, I think, commits one at most to the being of the mark, or of a symbol that coincides with exactly one mark. Signs, however, may count as either non-entities or sempiternal entities, even though, as we shall see, they are allowed to stand in determinate relations to one another and to be denoted by other symbols.

I have thus far attempted to elucidate the differences between characters, signs and symbols. In order to further

refine these elucidations and prepare for some definitions, the following quotation conventions need to be introduced:

i) ' ζ ', stands for a (series of) mark(s) or sound(s).

ii) ' ζ ' stands for a sign. ' ζ '^c is a sign in a given context.

iii) $/\zeta/$ stands for a symbol.

iv) $\Theta[/math> $/\zeta/$ $]$ stands for the class of symbols of a language with the same meaning as $/\zeta/$, where i) the meaning of a symbol will have at most two components, viz., sense and reference, and ii) ' $/\zeta/\Theta/\eta/$ ' means that $/\zeta/$ and $/\eta/$ have the same meaning. (Thus, Θ counts as an equivalence relation.)$

v) A combination of marks, signs or symbols will be expressed by ' \wedge '. That is, if η_0, η_1 are marks, signs, or symbols, then $\eta_0 \wedge \eta_1$ counts either as a mark, a sign or a symbol, depending upon the occasional and background routines with respect to which they were combined.³

These devices enable us within the language to make different assertions about marks, signs and symbols, respectively. The requirement for such devices becomes especially evident in the case of marks and signs that also count in some contexts as propositions. For if $/\zeta/$ counts as a symbol, then the

³ This allows that in the absence of any routines with respect to which a combination of symbols may count as a symbol the combination may count only as a sign. But combinations of signs cannot degenerate into marks. It should also be noted that ' \wedge ', may indicate a space in a sign and may be concatenated with other signs or symbols. Punctuation devices may also count as marks or signs.

following *directly* count as propositions:

- 1 A assumed $/\zeta/$ and denied $/\eta/$, even though $/\zeta/ \Theta / \eta/$.
- 2 ' ζ ' says $/\zeta/$ or says *that* ζ .
- 3 $/\zeta/$ refers to ζ .
- 4 $/\zeta/$ is true just in case ζ .⁴
- 5 A marked ' ζ ,.
- 6 A wrote ' ζ '

but the following two do not:

- 7 A assumed ' ζ ,.

⁴. 3 and 4 in this list give two disquotation idioms, namely, 'refer to' and 'is true just in case'. These idioms allow one to shift *within* a language or between languages from talk about the symbolism to either the reference to or assertion of part of what the symbols mean (the reference in the first case, the sense in the latter. Note that if $/\zeta/ \Theta / \eta/$, then the following also count as symbols:

$/\zeta/$ refers to η

$/\zeta/$ is true just in case η

Similarly, $/\text{The reference of } / \zeta / \text{ is } \eta /$ also counts as a symbol, so that 'The reference of $/ \dots /$ is \dots ' also counts as a disquotation idiom. The Fregean thesis that sense of a term cannot be referred to by what we will call later a strict term commits him to the assumption that the only sign that can occur at the empty place in $/\text{The sense of } / \zeta / \text{ is } \dots /$ is one that involves some quotation, in particular, $/\text{The sense of } / \eta //$. The tracing of Frege's commitment to this assumption is, I think, one result of Russell's argument in 'On Denoting'. Whether this commitment is itself objectionable depends upon whether one insists that a meaning is something about which one can directly assert a truth or falsehood. For Frege, since sense cannot be talked about except in terms of quoted expressions and sense is a component of the meaning of a symbol, the meaning of symbol cannot be talked about.

8 A denied 'ζ'

Rather, if the marks and signs indicated in 7 and 8 were indexed to contexts within which they counted also as symbols or coincided with symbols, then 7 and 8 would through that coincidence also count as propositions.

This brings us to our first set of definitions.

Def. 1 One sign 'ζ' is said to character another sign 'η' just in case with respect to the routines of the writing of 'η', 'ζ' is written in the writing of 'η'

Def. 2 One sign 'ζ' is said to occur in another sign 'η' just in case with respect to the routines of the writing of 'η', 'ζ' characters 'η' and the meaning (if any) of 'η' depends for its composition upon the meaning (if any) of 'ζ', but not conversely.⁵

⁵ That is, if and when 'η' means, what 'η' means depends upon whether or not 'ζ' means and when both 'η' and 'ζ' mean, what 'η' means depends upon what 'ζ' means, but not conversely.

The rationale of this formulation becomes clear if one considers how, for Russell, definite descriptions mean and occur in propositions. A definite description may lack a meaning. This may determine what the proposition in which it occurs means but not whether it means. For if the definite description had a meaning and occurred within a true proposition, then the proposition would not just present an existential judgement. It would correspond to a complex.

Since the meaning of a symbol may have two direct components, viz. sense and reference, some further explanation concerning the dependence of the composition of one meaning upon the composition of another is required. For, according to Frege, whereas the sense of a symbol can only subsist, the reference either exists, subsists or has some derivative mode of being, depending upon whether, for Frege, the reference of a predicate is a relation or the extension of a relation.

For Frege, then, the composition of one meaning will be said to depend upon the composition of another just in case the composition of the sense in the former meaning depends

Def.3 One sign 'ζ' is said to directly occur in another sign 'η' just in case 'ζ' occurs in 'η' and there is no sign 'ξ' such that 'ζ' occurs in 'ξ' and 'ξ' occurs in 'η'.

The rationale for the latter two definitions consists in the assumption that signs need not be bound to a single context and so may count as meaningful in some contexts and meaningless in others. According to this assumption, whether a sign occurs within another sign depends upon whether, according to background canons of grammar and diction, the former can count as a component symbol of the latter sign, when the latter counts as a symbol. Consider, for example, the following cases in English:

The sign 'France' occurs directly within 'the King of France'

The sign 'the King of France' occurs within 'The King of France betrayed Joan of Arc.'

With respect to the canons of English diction and grammar, 'the King of France' can count as a symbol only if 'France' can count as a component symbol. The signs 'the', 'of' and even the signs 'King' and 'betrayed', as we shall see, occupy peculiar and important positions within other signs. It is not especially clear whether or not they should be counted as occurring in a proposition. The first two are what will be

upon the composition of the sense in the latter meaning.

called (strict) mode-of-combination signs, the latter two will be treated as ambiguous between mode-of-combination signs and what will be called terms. Rather than defining 'occurrence' and 'mode-of-combination sign' in such a manner that precludes the assumption that a given mode-of-combination sign occurs in another sign, or that it can be equivalent in meaning in some context with a term, mode-of-combination signs will simply be counted according to the way in which their status as signs does or does not depend upon their combination with other signs to form a sign.

Def. 4 A sign ' ρ ' is said to be a mode-of-combination-sign of arity $n+1$ just in case there are signs ' χ_0 ', ' χ_1 ', ..., ' χ_n ' and a sign ' ψ ' such that ' ψ ' results from some concatenation of ' ρ ' with ' χ_0 ', ' χ_1 ', ..., ' χ_n ', where ' χ_0 ', ..., ' χ_n ' are, thereby, counted as directly occurring in ' ψ ' and for all k and all signs ' χ_0 ', ' χ_1 ', ..., ' χ_k ', if there is a sign ' ψ ' that results from some concatenation of ' ρ ' with ' χ_0 ', ' χ_1 ', ..., ' χ_k ', then $k=n$.

Def. 5 A sign will be called a strict term if it can directly occur within signs of diverse complexity, that is, within signs that consist of different numbers of signs. It will simply be called a term if it can directly occur within a sign.

The second part of Def. 4 calls for some comment. It is supposed to formulate the following distinguishing feature of mode-of-combination signs: their unsaturatedness. Whether the signs ' χ_k ' count as nouns or pronouns, terms or variables (pro-terms), what is singled out is that mode-of-combination signs can correctly character another sign only if they

concatenate with a definite number of other signs, whether terms or variables, so that these directly occur in the same sign. Though, of course, nouns occur in sentences only if they concatenate with predicates, and sentences occur in other sentences only if they concatenate with conjunctions, nouns can concatenate with predicates of any arity and sentences with conjunctions of any arity. Thus, they can occur directly in sentences which consist of different numbers of signs. This is why they are allowed to count as strict terms.

None of the above comments presupposes that mode-of-combination signs do occur in other signs, or are terms. They are simply compatible with the Fregean, and Russellian assumption that some mode-of-combination signs double as terms. Two Russellian assumptions that distinguish his analysis from Frege's (and Wittgenstein's) are the assumption that the meaning of some mode-of-combination signs, namely, predicates, can be the same as the meanings of terms, and the assumption that the meaning of a symbol, if it has any, consists simply in its reference to some entity. For Frege, not only are predicates terms that are distinct in meaning from strict terms, the meaning of any term is supposed to consist in both a reference and a sense, the latter counting as an object or a function and the former always as a

function.⁶ These contrasts between Wittgenstein, Russell and Frege will be discussed further once the definition of a sign and symbol decomposition will be presented.

Several other definitions, however, will have to be formulated before the definition of a sign decomposition can be fully introduced. In particular, a definition of a structure will be needed which will provide for a graphical separation of terms and mode-of-combination signs in the decomposition of a strict term, while showing the order of occurrence of constituent signs of a given sign. For this purpose we have the following definition of trees, and their constituents, lines and branches:

Def 6. Lines and branches are determined or defined as follows:

i) A line l is determined by two points, the bottom, $\perp(l)$, and the top, $\top(l)$, of the line.

ii) Lines are *connectable* under an operation called *gluing*. A line l will be said to be *glued* to another line m if $\top(l)=\top(m)$, $\perp(l)=\perp(m)$, $\top(l)=\perp(m)$ or $\perp(l)=\top(m)$, that is, if they share an end-point. The result of such a gluing is a figure. (Notice that if the two end-points of a line are identical, then the resulting figure is a point.) Figures can also be glued at the end-points of their

⁶ As was emphasized in the first chapter, it is not clear whether or not the reference of a term must be an object. In the case of predicates, for instance, it is unclear whether or not, for Frege, the reference of a predicate is a relation or its extension, the former of which counts as a function from (sequences of) objects to truth-values and the latter as an object. If the former were the case, then predicates could be further distinguished from strict terms according to their reference: only predicates, not strict terms, refer to functions.

constituent lines. Two lines (and, similarly, two figures) are said to be separate if they are not glued.

iii) Any figure which results from gluing n lines, $n > 0$, is a branch if the tops of the n lines share or coincide in a single point. The point which all the lines of a branch, \mathcal{B} , share is the node, $n(\mathcal{B})$, of that branch.

iv) A figure, A , is topped just in case there is exactly one line, l , in A such that for every line, m , in A , $\perp(m) \neq \tau(l)$. The top of such a line in a topped figure, A , is called the top of the figure, $\tau(A)$.

Trees are topped figures defined recursively as follows:

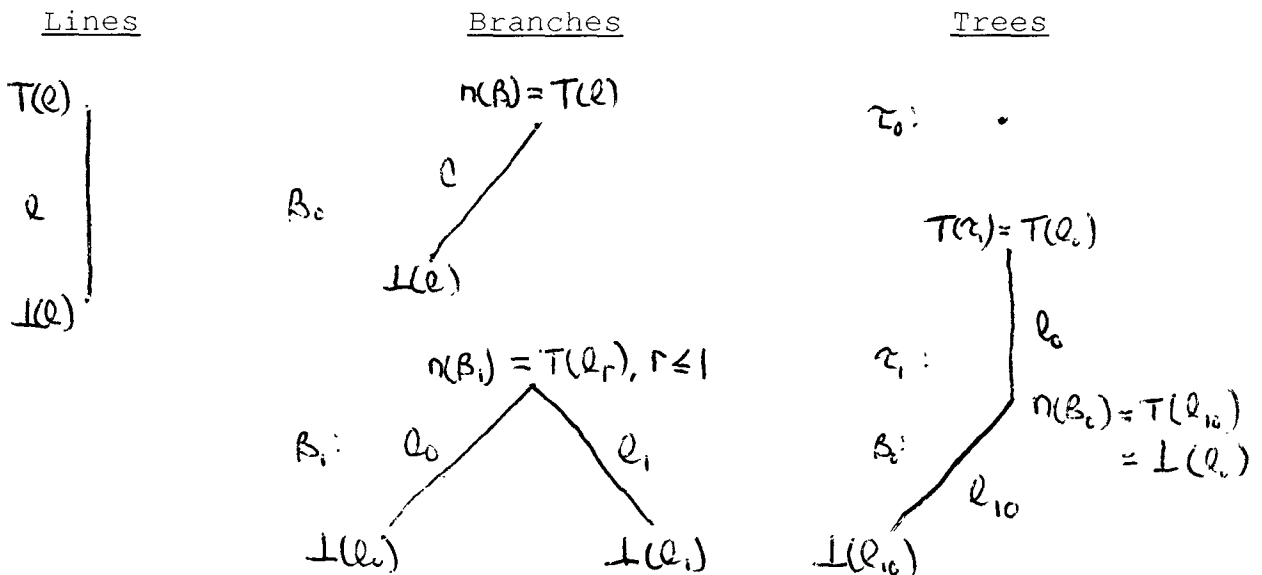
A1 A single point is a tree of rank 0.

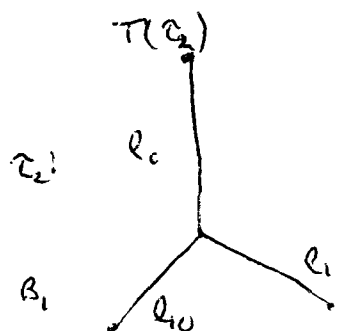
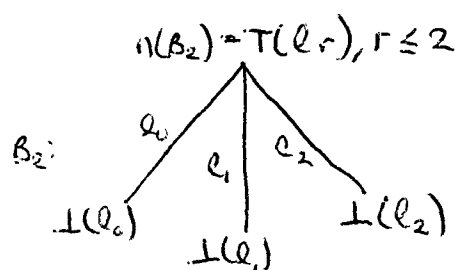
A2 The result of gluing the bottom of a single line to the node of a branch is a tree of rank 1.

A3 If A and B are trees of rank $0 < k < n$, then, for any line l in A , if there is no line m in A such that $\perp(l) = \tau(m)$, then the result of gluing A and B such that $\perp(l) = \tau(B)$ is a tree, C , of rank $n+1$.

A4 That's all.

Since lines are oriented, lines branches and trees have the following obvious graphical representations.





The set of nodes of a tree, $N(A)$, consists exactly of the points that are nodes of branches in the tree. That is, $N(A) = \{n(\mathbf{B}) : \mathbf{B} \text{ is a branch in } A\}$. A trunk is a line in a tree A whose top is the top of some tree glued together with some other tree in A . The top of a trunk will be called the tip of the trunk, and the set of tips of a tree consists exactly of the tips of trunks in the tree. Accordingly, in the graph of a tree, a tip is either the top of a tree or the bottom of a line in a branch. This gives us the appropriate separation of tips from nodes in a tree, so that we have a definition of a symbol decomposition which separates strict terms from mode-of-combination signs:

Def.6 A decoration of a tree results from assigning to each node and to each tip in the tree exactly

one sign.

Def.7 A decorated tree is a sign-decomposition just in case 1 at the bottom of each line in a branch a sign which directly occurs in the sign at the tip of the trunk of the branch and 2 at the node of each branch is a mode-of-combination-sign in virtue of which the signs at the bottom tips of the branch directly occur in the sign at the tip of the trunk.'

A decomposition is said to have a subordinate decomposition if some terms occur in one of the mode-of-combination signs assigned to a node. Such a decomposition would have to be adjoined to the main decomposition at the node, since it would not be shown in the main decomposition itself. But the same principle is followed in the case of subordinate decompositions, or decompositions of mode-of-combination signs, as is adopted in the case of decompositions of strict terms. One feature of a sign which is not illuminated in this formulation of a sign decomposition is the order, if any, in which direct constituents of a sign are presented in the

⁷ The terms /tree/ and /decoration/ were suggested through a reading of Peter Aszel's Non-well-founded Sets, CSLI series. He uses these terms in his definition and representation of sets.

One assumption that will be supposed to be a constraint upon what counts as a language is that for each symbol within the language, there is exactly one decomposition. A set of signs and routines which do not support this assumption will not be counted as determining a language. This constraint upon languages is supposed to be weaker than the assumption that all symbols are canonical, since the routines with respect to which a series of marks may count as a symbol in the language might be context specific. Thus, the same marks may count as different signs and the same signs as different symbols within a language.

characters which present the sign. For instance, according to the way in which English signs are read, 'this' is presented before 'that' in 'This is to the left of that', even though they both directly occur in this sentence. But since trees have tops, various types of orientations can be induced on them depending upon whether their lines are embedded onto the same or different surfaces of an oriented k -dimensional figure. If each surface is oriented and the surfaces of k -dimensional figures are partially ordered, then the required orientation of the tips in the tree can be provided. In the case of European languages, the only orientation required is that determined by the top of the tree, where the tree has been embedded in a flat surface (such as is presented by a sheet of paper on which the above tree graphs have been written). This provides a distinction between left and right nodes and tips in a tree, so that, for English signs, the following rule induces an orientation on branch parts of sign decompositions:

R If one sign occurs to the left of another sign in a branch of a tree embedded in a flat surface, then the one sign is presented before the other sign in any array of characters which presents a sign in which they directly occur.

In this context, a decomposition will be said to be fully oriented if it is the result of a decoration of a tree in accordance with the above definition of a decomposition and in

accordance with this rule. As remarked above, for other cases a full orientation might require a multi-linear rather than a uni-linear order of the tips of a decomposition. Nonetheless, the above linear ordering provides for the following principle of sameness of sign:

P1 Two arrays of characters present the same simple sign just in case they present the same type of array of characters and if both present mode-of-combination signs of the same arity or both present strict terms.

P2 Two arrays of characters present the same complex sign just in case there is an order and orientation preserving bijection, f , from one decomposition to the other and for any character, c , if c occurs at the bottom of the source decomposition, or at the bottom of one of its subordinate decompositions, then c and $f(c)$ present the same simple sign.

A few differences between Wittgenstein, on the one hand, and Frege and Russell, on the other, is that, for Wittgenstein but not for the others, the following hold:

Signs which occur at the tips of sign decompositions are strict terms, and these are the only terms which can refer and can occur in other signs.

Some mode-of-combination signs, such as verbs and adjectives, are assumed to mean non-formal concepts, while others, such as variables and quantificational idioms, express only formal concepts.

Some mode-of-combination signs, such as conjunctions, express no concepts at all.

For Frege, all mode-of-combination signs are terms and refer to entities and express senses. Moreover, quantificational expressions such as 'at least one item is such that' have concepts as components of their meanings, what Frege called second order concepts just as expressions such as 'it is white' have concepts as components of their meanings. For Frege, the difference between these two concepts was that whereas the concept expressed by 'it is white' takes objects for arguments, the concept expressed by 'at least one item is such that..' takes concepts for arguments.

If there is one feature that distinguishes Russell's from Frege's and Wittgenstein's positions it is his assumption that the meanings of adjectives, verbs, conjunctions and logical signs such as ' $\exists x(\phi)$ ', where ' ϕ ' lacks occurrences of symbols, denote entities of increasing abstractness, entities which, moreover, can occur as constituents of complexes. Moreover, according to Russell, the meanings of these symbols are or can be the references of strict terms.

These differences between these authors can be illuminated with respect to an analysis of the graphs of some symbol decompositions. Two, in particular, will be especially illuminating, namely, the graphs of the fully oriented symbol decompositions of the following two signs:

E 'That is white and it is not to the left of this.'

E 'At least on item is such that it is to the left of this.'

But before turning to the analysis of these two graphs a definition of a fully oriented symbol decomposition has to be provided. Such a definition is given with the following:

Def. 7 A symbol decomposition with respect to a context C is a sign decomposition in which all and only the signs of the tree which have meaning in that context are quoted as symbols.

Def. 8 A fully oriented symbol decomposition is a symbol decomposition which results from a fully oriented sign decomposition.

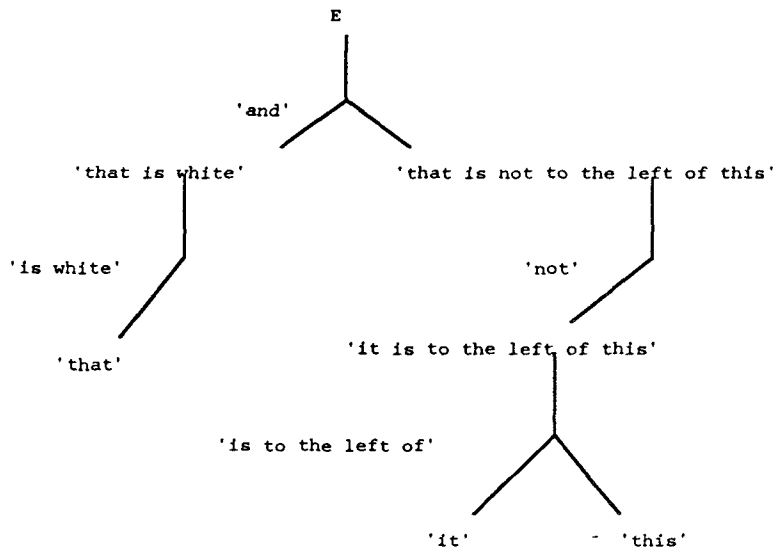
We also have the following definition of sameness of type of symbol.

Def. 9 Two symbols are of the same type just in case they present the same signs and there is an order, orientation and meaning preserving bijection from the decomposition of one of the symbols to the decomposition of the other.

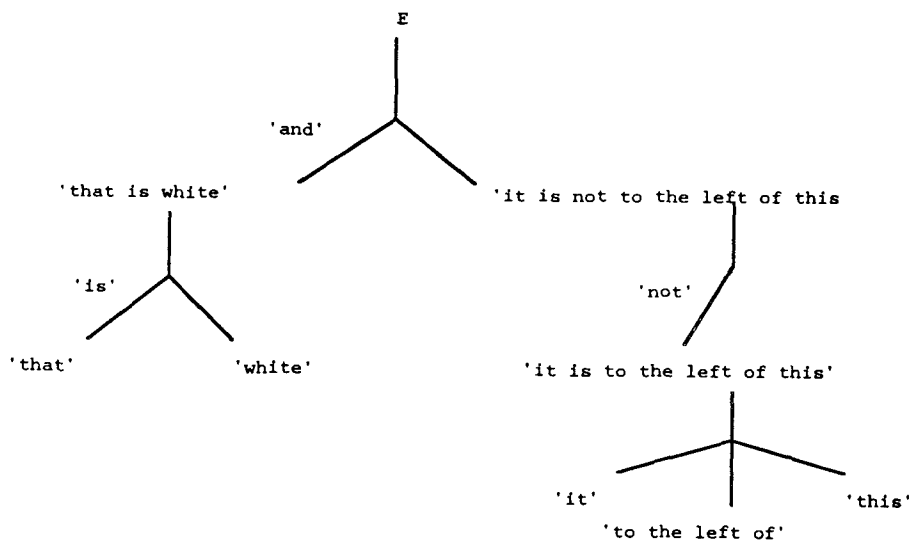
Though this definition will not be immediately needed, it will be invoked in some of the final discussion on logically canonical signs, and their meanings.

Consider, then, for the purposes of illustrating the foregoing definitions and discussions, the following plausible alternative graphs of the decomposition of E. Whereas the first is Wittgensteinian, the second is loosely Russellian:

Graph 1

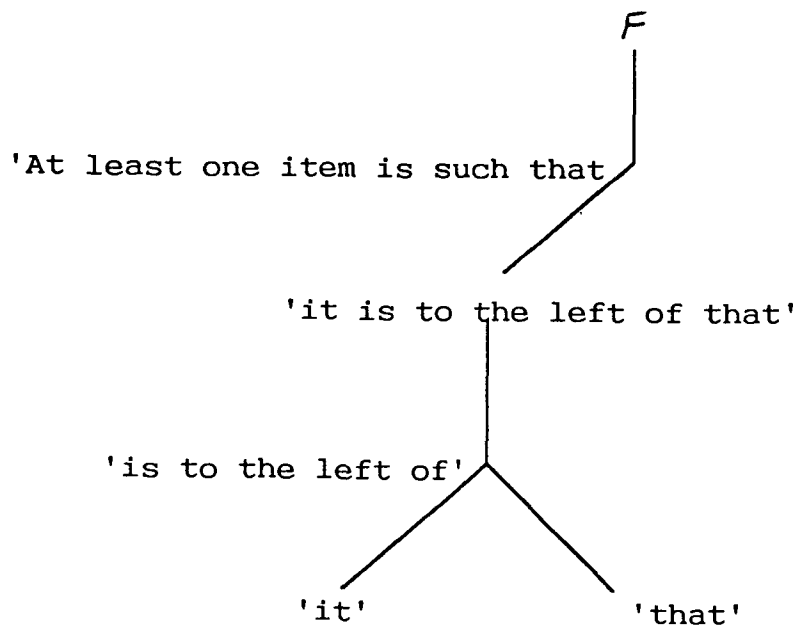


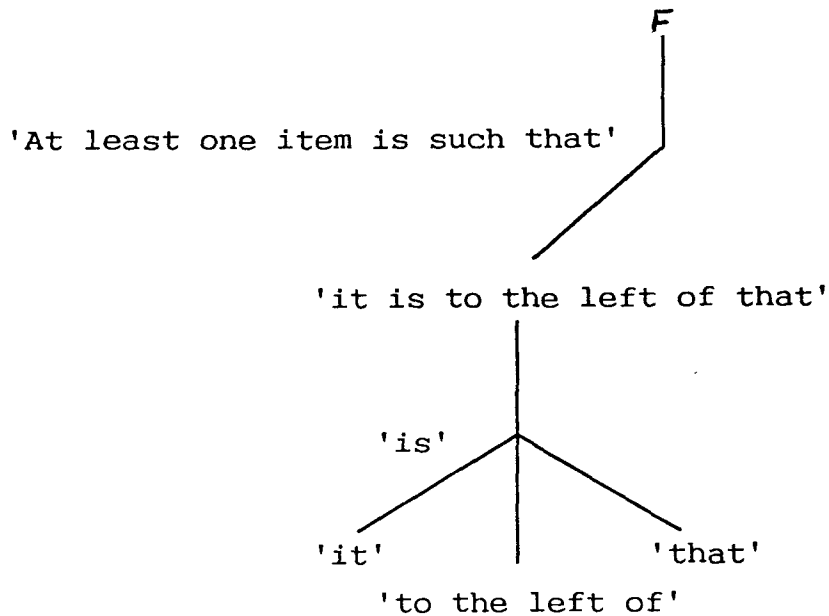
Graph 2



Note that in both graph 1 and 2, /it/Θ/that/. In these cases, the pronoun 'it' counts as a term, since it has a meaningful antecedent. In the symbol decomposition of F, however, the status of the pronoun is quite different. This is reflected in both the first, Wittgensteinian graph and in the Second Russellian graph below:

Graph 3



Graph 4

The difference between 1 and 2 and 3 and 4 is that, according to 3 and 4, the predicate 'it is to the left of' occurs as a term, and 'it' either as a mode-of-combination sign or as an essential part of a term. As was discussed in Chapter 2, for Wittgenstein, this analysis committed him to a treatment of the quantificational expression and the pronoun as both expressing formal concepts. One respect in which this graphing may conflict with his pictorial account of propositions is if it commits him to the assumption that the meanings of predicates, that is, what he calls concepts, are eternal entities. According to Chapter 1, Wittgenstein's pictorial account of propositions treats only the meanings of names as subsistent.

What remains unclear, here, is whether or not 'white'

should count as essentially part of a mode-of-combination sign 'is white' or as a strict term. That 'white' may occur as a strict term in 'White is a colour' does not decide the issue, since the same marks may have been written but with different signs occurring in this and the above context (in virtue of counting as distinct kinds of symbols), especially since in English one would count '*This shade of white* is a colour' as more precise. If 'white' occurs as a strict term above, then only 'is' occurs as the mode-of-combination sign in 'That is white'. Similar remarks could be made about 'to the left of', except that the prepositions prevent this sign from occurring correctly as a strict term, given the grammatical canons of English. Russell, of course, would have allowed both 'white' and 'to the left of' to denote the same entities as some strict terms, if they did not count as strict terms themselves. As mentioned in the previous paragraph, the decomposition of F provides some evidence for this position.

Some of these remarks can be illuminated further through considering the following context for E:

Thomas says, "That is white and it is not to the left of this," while pointing successively at two regions. And Paul, looking to where Thomas is pointing, says, "That is true."

According to Russell, within this context, every sign counts as a term. The only contentious cases are 'it' and 'is to the

left of'. In the case of 'it', however, 'That' counts as the antecedent of 'it' in this context, so that it counts as having the same meaning as 'that'. If this were not the case, then E would not count as a propositional symbol, but as a propositional function symbol. In that case, 'it' would occur indirectly within E, but would lack meaning. So that leaves the latter sign to be accounted for.

For Russell, the predicate, 'is to the left of', means what 'Being to the left of' means, and this meaning is a constituent of what the top sign means. Thus, in the proposition /Being-to-the-left-of is a relation/ which, according to Russell, decomposes as

/Being-to-the-left-of is a relation/

'is a relation'

/Being-to-the-left-of/

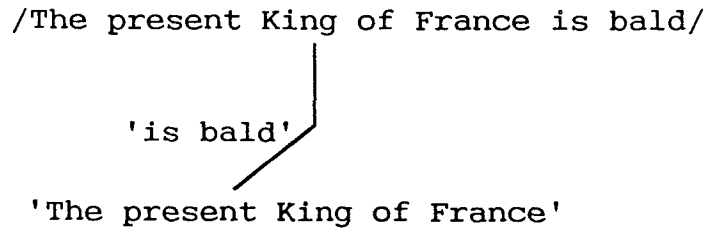
/Being-to-the-left-of/ means what /is to the left of/ means, even though the latter is assigned to a node and the former to a tip in their respective symbol decompositions. As has been discussed, in Chapter 1 and Chapter 4, this is an assumption which both Frege and Wittgenstein rejected. For Frege, though all symbols have sense and reference, no term can mean what a mode-of-combination symbol can mean. Depending upon whether, for Frege, the reference of a predicate is a relation or an extension of a relation, either the reference of a term is

distinct from the reference of any predicate or the reference of a term is distinct from the sense of a predicate, that is, the reference of a term is not a function, but an object. For Wittgenstein, all verbs, adjectives, prepositions count as mode-of-combination signs. Thus, in combination with auxiliary verbs such as 'is', adjectives count as essentially part of mode-of-combination signs and in combination with nouns (as in 'white rabbit') they count as mode-combination-signs through which the nouns directly occur in other terms.

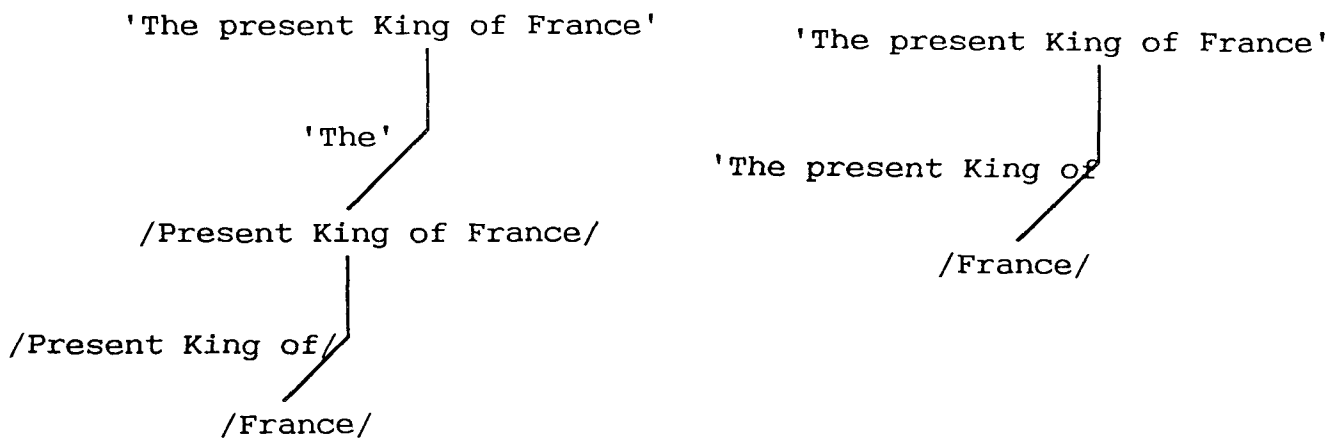
The above set of definitions affords us the following formulation of canonicity:

LC A propositional sign, p , is logically canonical just in case with respect to any context, C , all the terms in its sign-decomposition count as symbols, if all the bottom ones in the decomposition do and/or all the terms in the subordinate decompositions do.

According to this definition, for Russell, all sentences in which definite descriptions occur as terms will not count as canonical, since a definite description may lack reference to an entity, even though it occurs in a meaningful sentence and all the terms which occur in the definite description have meaning. Consider, for instance, the following graph of the decomposition of 'The present King of France is bald':



The decomposition of /The present King of France/ may receive one the following alternative graphical representations, depending upon which mode-of-combination sign is taken as superordinate.



The symbol, 'The present King of France is bald', according to

Russell, is false, since there is no present King of France. Since there is no present King of France, 'the present King of France' does not refer to an entity that stands to the reference of /France/ in the relation referred to by /is a king of/ (or, equivalently, /has inherited the rule of/). According to Russell, since there is no such entity, the definite description lacks a reference, even though its component signs in the present context have meaning. Thus, for Russell, this definite description makes the above meaningful sentence, /The present King of France is bald/, uncanonical. Since, moreover, it is in virtue of the structure of the definite description that this sentence is uncanonical, all definite descriptions, whether they refer to entities or not, make the sentences in which they occur uncanonical. Accordingly, any term which makes a sentence uncanonical will be called an incomplete term, even in contexts where it counts as meaningful.

A notation will be said to be logically canonical if all of its propositional signs, that is, signs which can directly count as true or as false, are canonical. A notation will count as a logically adequate language if for any proposition whose constituents can be recognized or grasped by the speakers of that language there is a logically canonical propositional symbol which expresses that proposition. Whereas not all logically adequate languages can canonically

express the same propositions, a logically canonical notation is a notation in which one *could* canonically express any proposition, provided, whether through translation or with respect to a developed background of canons of diction, its constituent signs could be given the requisite meanings. It is this last proviso, as well as the many paradoxes, which suggest that no language could be devised such that there is a single interpretation of its primitive signs according to which any possible propositions could receive a canonical expression through some propositional sign that results from the concatenation of those signs. There are always new entities and non-entities to refer to. Given a requirement of a background for a language, these observations provide a theoretical and not just a practical constraint on the expressive scope of any language. Languages must continually be refined and extended, they must borrow or introduce new canons of diction and, perhaps, even grammar, in order to express new propositions. A further constraint on language - oddly enough - is corrigibility, the refinability and extendibility of its canons.

Given the above account of canonicity, two logically canonical symbols which are equivalent in meaning ought to count as having isomorphic symbol decompositions, where an isomorphism is an order, orientation and meaning preserving bijection. (Note: it does not follow from this that they are

the same signs, since their characters may be different.) Since, even for Wittgenstein, truth-functionally equivalent canonical symbols may have non-isomorphic decompositions, truth-conditional equivalence cannot give sameness of meaning (or, in Wittgenstein's account, sameness of sense). This suggests that the logical conjunctions 'and' and 'or' etc. do have meanings, namely, they mean functions between propositions and propositional functions.

Appendix 2

The previous appendix provided a relatively formal account of sign decompositions and of incomplete terms. On the basis of these accounts, the present appendix will briefly formulate the notion of a symbol ancestry. This notion will enable us to give a rigorous analysis of the background with respect to which a mark or sound counts as meaningful. The principal definition upon which this analysis is developed is that of inheritance, though even this definition will have to presuppose the notion of a background of contexts ordered temporally.

Def. 1 ' η '^c inherits meaning from ' θ '^d just in case (1) ' η ' is the same type of mark as ' θ ', (2) ' η '^c Θ ' θ '^d and (3) d temporally precedes c.

Def. 2 ' η '^c inherits structure from ' θ '^d just in case the symbol decomposition of ' η ' with respect to c is isomorphic to the symbol decomposition of ' θ ' with respect to d and d temporally precedes c.

The definition of direct inheritance should be fairly obvious in both cases.

Def. 3 An ancestry A of a symbol θ is an ordered pair $\langle B_0, B_1 \rangle$ of chains of symbols, B_0 and B_1 , ordered according to direct inheritance of meaning and structure, respectively.

Suppose ' θ '^a is such that for no context d, ' θ '^c inherits meaning directly from ' θ '^a. Then ' θ '^c is a minimal instance of the signs ' θ '^a that count as members of $\Theta['\theta^c]$.

Nonetheless, ' \mathfrak{G} '^c may inherit structure from some other symbol ' η '^d and its constituent symbols ' \mathfrak{G}_0 '^c, ' \mathfrak{G}_1 '^c, ' \mathfrak{G}_n '^c may have inherited meaning from ' \mathfrak{G}_0 '^{f0}, ' \mathfrak{G}_n '^{fn}. Suppose, then, that ' \mathfrak{G} '^c is a minimal instance and its constituent symbols are minimal instances as well. Then it stands directly in some depictive or projective relation with a complex, though it need not be true. These minimal instances set the canons, both grammatical and dictive, with respect to which other marks count as signs and symbols. Supposedly, once one understands that the background of a language must consist of these canons the problem concerning the logical adequacy of languages disappears. Put more succinctly, the background of a language consists of the ancestries of its constituent signs, so that, provided the language is still carefully spoken and written, an understanding of what the canons express must be a continuing legacy for speakers of the language.

Final Aside

The above account provides for the following explicitly linguistic reformulations of what Wittgenstein assumes at 2.0211 and 2.22 and at 3.3:

2.0211' For some propositional symbol ' p '^c, whether it has sense with respect to C cannot depend upon whether for some context D and some mark ' q ', ' q ' counts as a true proposition with respect to D.

3.3* If 'a' counts as a name in C, then there is a sign 'p' such that 'p' counts as a proposition in C and 'a' counts as a term in the symbol decomposition of 'p' with respect to C.

Given this reformulation, what Wittgenstein can be read as assuming at 2.0211 is that there is some minimal instance in an ancestry and that such an instance can be false. Accordingly, the sense of propositions, for Wittgenstein, is not deferred indefinitely, so that there is some determinacy in the truth or falsity of propositions, and the ways in which objects stand to one another in the world are not without exception dependent upon our representations of them.

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