HUME'S PHILOSOPHY OF GEOMETRY

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By

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ABSTRACT

Hume's writings with respect to geometry represent one of the least studied and least understood parts of his philosophy. Commentators have been inclined to view Hume's analysis of geometric knowledge, particularly that in the Treatise, as confused and ill-conceived. However, as we shall see, if we take care to situate Hume's philosophy of geometry within the framework of his empiricism and familiarize ourselves with its main concerns and course of reasoning, we will be led to a much more positive view and evaluation of his efforts.

As Hume saw it, the ancient view of geometry as a perfectly precise and certain science lay at the heart of the excesses which plagued both scepticism and rationalism. In the Treatise he applied his copy principle to the fundamentals of this subject and concluded that this view of geometry was based on a confusion. In one fell swoop Hume believed that he had preserved the foundations of geometry against the attack of scepticism, undermined some of the most important doctrines of rationalism, and removed what had long been viewed as an insurmountable obstacle to an empiricist epistemology.

Incredibly, when we turn to the Enquiry, we find Hume embracing geometry as a perfectly precise and certain science. Little is known as to the reason behind this rather remarkable shift in view and there is considerable controversy as to just how sharp a break from the Treatise Hume intended. Hume himself is virtually silent on the topic. However, by laying the proper groundwork, we shall be able to understand why Hume found it necessary to break from the Treatise and why he pursued the direction he did. Overall, our findings will reveal that Hume's struggles with geometry, in spite of their shortcomings, clearly bear the mark of his unique genius.

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ABBREVIATIONS

For the readers convenience, references to certain of Hume's and Berkeley's works are cited in parentheses immediately following quotations, the abbreviations being used appear in the table below.

T. A Treatise of Human Nature. ed. by L. A. Selby-Bigge. Oxford, 1888.

E. An Enquiry Concerning Human Understanding from Enquiries Concerning the Human Understanding and Concerning the Principles of Morals. ed. by L. A. Selby-Bigge. 2nd.ed. Oxford, 1902.

P. A Treatise Concerning the Principles of Human Knowledge. from The Works of Bishop Berkeley. ed. by A.A. Luce and T.E. Jessop. vol.II Edinburgh: Nelson, 1949. (References to this work will be of the form (P par. 6) , where 'par' designates the paragraph number cited in the text.

CHAPTER 1

INTRODUCTION

Since ancient times the study of geometry has proved to be fertile ground for the production and growth of philosophic thought. Nowhere has this been more evident than in the philosophy of space. The fact that geometry had traditionally been viewed as the science of space guaranteed that these two subjects would become deeply intertwined early on in the history of philosophy. These closely connected philosophies came to play an increasingly dominant role in shaping the metaphysical systems of the leading thinkers of the seventeenth and eighteenth centuries, a movement which reached its greatest expression in the critical philosophy of Immanuel Kant. In the present work we shall look at the treatment these twin subjects received at the hands of the eighteenth century philosopher widely acknowledged to be the greatest of the classical empiricists; namely, David Hume.

The primary focus of our attention will be on Hume's philosophy of geometry and we shall concern ourselves with his philosophy of space only in so far as is necessary for achieving a critical understanding of his writings on geometry. Our general objective will be to present an exposition of Hume's writings which will (a) situate his views on geometry and (in so far as is necessary) space within their proper historical context, (b) place them in their strongest and most favourable light within the framework of his empiricism, and (c) enable us to assess their principal strengths and weaknesses.

Generally speaking, the value of an inquiry which aims to shed some new light on the teachings of so important and influential a philosopher as Hume speaks for itself and requires little by way of justification. However, with respect to the present topic the reader might well ask, why bother? After all, Hume's writings on space and geometry represents one of, if not the, most harshly criticized and curtly dismissed parts of his philosophy. Kemp Smith, in what continues to be one of the most thorough examinations of this part of Hume's philosophy, writes:

> Hume's own positive teaching, that space and time consist of physical points is, I think we must agree, one of the least satisfactory parts of his philosophy, as he himself later seems to have recognized...[1]

A more recent commentator, James Noxon, expresses similar misgivings.

> Hume's unavailing struggles with space and time, his attempt to derive their ideas from experience conformably with his first principle that all ideas copy sense impressions and to elucidate geometry as the science of spatial extension yielded the least-admired part of the Treatise...[2]

C.D. Broad uses much stronger language in expressing his dissatisfaction.

> To conclude, there seems to be nothing whatever in Hume's doctrine of Space except a great deal of ingenuity wasted in recommending and defending palpable nonsense...[3]

John Laird is a bit more gentle in his criticism. He begins by noting that Hume's commentators "often say that his talents were not suited to an adequate discussion of space, time and mathematics in their relations to physics, and it is true that he was not a mathematician".[4] He grants that Hume's discussion in this Part of the Treatise (i.e. Part II) "deserves some of the very hard things that have been said of it".[5] He even goes so far as to maintain that many of the arguments Hume uses in defense of his analysis of the idea of space so essential to geometry "would not deceive a child".[6] Nonetheless, he takes due notice of the fact that Hume's analysis of space and the foundations of geometry "was not altogether unworthy of the attention of mathematicians".[7]

This brief review of the literature suggests that Anthony Flew is on safe ground when he states that Hume's doctrine of space "is by common consent, one of the least satisfactory Parts of the Treatise".[8] Given this general attitude, it is hardly surprising that Hume's writings with respect to space and geometry represent one of the least studied parts of the Treatise. As late as the mid-sixties one commentator felt it safe to write, "hardly anyone is familiar with Hume's work on space and time".[9] Flew himself believed that these words "strike near the mark"[10] and consequently set out to contribute to the mastery of this part of Hume's philosophy. He acknowledged that "it might easily be argued that there is no very good reason why anyone should struggle to gain such familiarity -- except, of course, simply in order to better understand Hume and to appreciate his weaknesses as well as his strengths."[11] Flew, however, wisely points out that there will be plenty of time to ponder the profits of this exercise "when we have first mastered what Hume's problems were and how he believed he had solved them."[12] Though Flew could not, in a brief essay, offer a thorough treatment of this topic, his conclusion that the philosophy of geometry which underlies so much of Hume's philosophy of space is bizarre suggests that little of real value is to be gained from the struggle.

Given the low esteem in which Hume's writings on space and geometry are held and the fact that they have exerted virtually no influence, historical or otherwise, upon the study of these subjects, we must take seriously the question as to why we are bothering with this aspect of Hume's thought. It will take considerable effort to deal with the logical gaps, apparent inconsistencies, and elliptical and ambiguous statements which have frustrated commentators and led to the harsh criticisms and general neglect noted above. The reader might well ask why she should consider joining us in this struggle, and we need to offer a better reason than simply to gain a deeper understanding of Hume at his absolute worst and most inconsequential. Our situation, unlike Flew's, demands that we, at the very outset, ponder the profits of undertaking this exercise. Thus before we move on and present a general outline of our course of study, it will be necessary to indicate why we believe this study is in order.

One reason for undertaking a serious study into Hume's notorious writings on geometry is suggested by the renewal of interest over the last few decades in the relation between contemporary empiricism and the philosophy of geometry. The argument here starts with the well-known fact that Hume's philososphy has exerted a strong influence on the various schools of empiricist thought in the present century. As one Hume scholar puts it,

> David Hume is the most influential precursor of modern empiricism. By modern empiricism, I intend a belief that all cognitive conflicts can be resolved, in principle, by either appeals to matters of fact, via scientific procedure, or by appeal to some sets of natural or conventional standards, whether linguistic, mathematical, aesthetic or political. This belief itself, is a consequent of an apprehension that all synthetic knowledge is based on experience, and that the rest can be reduced to a set of self-evident truths. In this broad sense, Modern Empiricism encompasses classes such as Logical Empiricism, Logical Atomism, and Philosophical Analysis, and unique

individuals, such as Russell and Moore...[13]

Using the term 'contemporary empiricism' in the same broad sense, Harold Morick argues that

> Because the fundamental doctrines of contemporary empiricism are essentially refinements and modifications of David Hume's basic tenets in the theory of knowledge, the best entrance to an understanding of the foundations of contemporary empiricism is an understanding of the epistemological tenets of this great eighteenth century thinker...[14]

For Morick the study of Hume is of particular significance given that one of "the main themes of philosophy and theory of science in the last twenty years is the critical assessment of the foundations of contemporary empiricism."[15] In fact, he goes so far as to say that the "most important single development in philosophy and theory of science since World War II"[16] has been "the emergence of a full blown criticism of the foundations of empiricism by scientists, historians of science, and science-oriented philosophers."[17] It is in light of this criticism that Morick finds it important to "turn to the groundwork that Hume laid for contemporary empiricism."[18]

With respect to the theory of science and mathematics logical positivism has been far and away the most dominant school of contemporary empiricist thought and the primary target of the criticism noted by Morick. In fact, the position of positivism in these areas not only survived well after logical positivism was rejected as a general epistemology, but was so dominant and widespread that it came to be referred to as the 'Received View.'[19] We would expect, then, that much of the interest in contemporary empiricism of which Morick speaks would be concerned with the critical assessment of logical positivism. Michael Friedman observes that this has indeed been the case. We are informed that there has been "in recent years a veritable flowering of historically oriented reconsiderations of logical positivism."[20] Friedman himself is of the opinion that

> ...achieving a better understanding of the background, development, and actual philosophical content of logical positivism is not merely of historical interest. For the fact remains that our present situation evolves directly - for better or for worse - from the rise and fall of positivism, and what I want to suggest is that we will never successfully move beyond our present philosophical situation until we attain a properly self-conscious appreciation of our own immediate philosophical background.[21]

With respect to the current situation in the philosophy of science Friedmain argues that we cannot make real progress unless

> ...we grant the positivists their due and consider their arguments on their own terms. If we simply reject all their basic distinctions out of hand, we shall never learn from their mistakes.[22]

Lawrence Sklar argues the same general point with respect to the philosophy of geometry.

> ...Nearly all of the interesting work on the epistemology of geometry which exercises contemporary philosophers starts out with a rather strong sympathy for empiricist positions but forthrightly faces up to the difficulties facing empiricism when its claims are examined honestly and in detail...[23]

In light of our discussion thus far we would expect to find Sklar's analysis of the difficulties facing empiricist positions to center largely around the main features of the Received View. And we do. In fact much of his discussion of empiricist positions centers around the writings of Hans Reichenbach. Reichenbach, of course, was not only a leading member of the positivist movement but also one of the most influential figures in the philosophy of space and geometry in this century.[24] The dominant role Reichenbach has played in the philosophy of geometry in this century goes a long way toward explaining why empiricism in the philosophy of geometry has become so strongly identified with the main features of the Received View.

If the present situation in the philosophy of geometry is such as to make an historically oriented reconsideration of the Received View a necessity, it might well seem that the natural place to begin is with a serious study of the writings of the most influential precursor to logical positivism - David Hume. Though the argument here might seem straightforward enough, the fact remains that Hume's writings on geometry have been ignored even by those who are otherwise in agreement with Friedman and Sklar. Sklar himself makes only a passing reference to Hume in this context. He merely states that "Hume reflected somewhat upon the epistemic justifications of geometric beliefs and offered an empiricist account."[25] He does not bother to examine Hume's account, but proceeds, instead, to face up to the difficulties confronting contemporary empiricism when its claims regarding geometry "are examined honestly and in detail." A similar treatment of Hume's views is simply not judged to be relevant for this purpose.

Hume, however, did far more than simply "reflect somewhat upon the epistemic justifications of geometric beliefs." He devoted a significant portion of _A Treatise of Human Nature_ to a discussion of geometry and took up the topic again, though much more briefly, in _An Enquiry Concerning Human Understanding_. He later wrote an essay dedicated solely to the foundations of geometry, but unfortunately it was never published. Hume, as we shall see, sought to come to grips with some of the most basic and long-standing problems in the history of the philosophy of geometry, problems which had widely been regarded as posing obvious and insurmountable obstacles to an empiricist epistemology. These problems concerned (a) the paradoxes surrounding the principle of infinite divisiblity of space (extension), (b) the self-evident nature of the most basic propositions of geometry,and (c) the exactness of the propositions and ideas of geometry and the inexactness of sense experience

Now the immediate question at issue is not whether Hume was able to succeed in this task, and thereby advance the cause of empiricism. The question, rather, is whether it is reasonable to assume that Hume's attempt to come to grips with some of the most basic and historically important problems in the philosophy of geometry and analyze them within the framework of an empiricist epistemology was so badly misconstrued and out of keeping with his philosophical acumen that it contains little, if anything, of value? Are we justified in dismissing out of hand the possibility that a detailed study of Hume's writings may prove to be an important source of insight into the epistemological problems which geomtry presents to the philosophy of empiricism? Are we to treat Hume's philosophy of geometry in a manner which is exactly opposite to that in which we are asked to treat contemporary empiricist positions; that is, simply to dismiss it without first achieving a solid understanding of its background, development, and philosophical content?

It is worth noting that Reichenbach too was of the opinion that there is little of value in Hume's writings on geometry. He has argued that Hume's interpretation of geometry "is none too well founded"[26] and that "he had no good argument for his conceptions."[27] In general, he believed that there was little, if anything, in Hume's writings on mathematics which could be taken as a token of his genius. But let us ask, on what basis was this conclusion reached? Certainly, it was not the result of anything approaching a serious study of Hume's writings. Rather, it was based on the belief that Hume could not possibly have come to grips with the aforementioned problems and meet the arguments of the rationalists because he had no way of anticipating the revolutionary discoveries of non-Euclidean geometries and mathematical logic.[28]

Reichenbach could readily concede that Hume was fully aware of the rationalists' view that geometry stands as an insurmountable obstacle to an empiricist epistemology. And he could certainly grant that Hume's faith in the fundamental soundness of his empiricism led him to revolt against this long-standing point of view and attempt to analyze the fundamentals of the geometry of his day within the framework of an empiricist epistemology. However, he would not be inclined to take Hume's efforts seriously, but rather as so much flailing about in the dark. Hume, we would be told, was doubly handicapped. Firstly, he lacked the mathematical talents to take on the likes of a Descartes or a Leibniz. Secondly, even if he had the mathematical talent, the stage of the development of geometry in his day was not sufficiently advanced to give him the insight necessary to present good arguments in favour of an empiricist position. To the contrary, it appeared to give the rationalist a clear upper hand, and there was little an empiricist could do, even one as astute as Hume, to strip geometry of its rationalist disguise.

The working assumption of the positivists was that these developments so revolutionized our understanding of the nature of geometry that all previous attempts at analysing the foundations of this discipline are to be viewed as being little more than historical curiosities. The problems which the thinkers of the past took to be so essential to the philosophy of geometry are no longer thought to be the same problems which are of concern to the contemporary philosopher working in this field. Reichenbach himself was led to make a sharp distinction between the old and the new philosophy of geometry. Those philosophers who work in the new philosphy and who must deal with the reality of non-Euclidean geometries "do not look back: their work would not profit from historical considerations."[30]

Reichenbach's position was by no means peculiar to himself, or to positivism, or even, more broadly, to contemporary empiricism. It represents what has been a very dominant trend in the philosophy of mathematics in this century. The philosophy of mathematics became identified with the analysis of the foundations of mathematics with the primary tool of analysis being mathematical logic. This effectively removed the philosophy of mathematics from the domain of general accessibility and turned it into a specialized field open only to experts. As Thomas Tymoczko remarks a "typical intelligent philosopher, versed in general mathematics, will feel that he does not know enough mathematical logic to comprehend the philosophy of mathematics."[30] Patrick Suppes takes a favourable view of this trend toward specialization and observes that

> Beginning in the latter part of the nineteenth century with the work of Frege and others, the philosophy of mathematics... became a more technical and more deeply developed and specialized subject. Today, neither philosophers nor mathematicians who do not have some special interest in foundations of mathematics attempt to contribute to the large and continually developing literature...[31]

So long as the critics of positivism operated within this framework and continued to emphasize the importance of technical issues in the foundations of mathematics, there was little reason for them to turn back and re-examine the tradition of classical empiricism. And certainly there was no reason for them to study so minor a figure in the history of the philosophy of mathematics as Hume. The positivist philosophy of mathematics may well deserve to be a subject of continuing interest in spite of its shortcomings, but it would be argued that we have moved too far beyond the broad general tradition that once characterized this branch of philosophy to benefit from historical considerations. Thus, even if it is granted that the study of Hume's philosphy is the best entrance to an understanding of the basic tenets logical positivism construed as a general theory of knowledge, it will be denied that it could be of similar relevance for an understanding of positivism construed primarily as a philosophy of mathematics and science.

However, in more recent times there has been a marked shift away from this approach to the philosophy of mathematics. A.D. Irvine informs us that "the past decade and a half has witnessed a renaissance in the philosophy of mathematics not seen since the days of Hilbert, Russell and Brouwer in the early part of this century."[32] To a large extent this rebirth of interest has been in response to what Tymockzo sees as "a growing dissatisfaction with the foundations approaches to mathematics"[33] and to the growing recognition that "one difficulty with the positivists' approach is that, instead of really dissolving the problems of the philosophy of mathematics, they merely turn away from them."[34] Thus, as we might expect, with this revival of interest in the philosophy of mathematics has come a renewed interest of sorts in some of the more important traditional themes in the philosophy of mathematics which were largely brushed aside by the positivists and their early critics.

Instances are not hard to find. The positivists, for example, believed that with the development of mathematical logic we could now see that the age-old controversy surrounding the nature of mathematical intuition is a matter which really belongs to psychology.[35] However, we now find critics of positivism arguing that "the basic problem of mathematical epistemology"[36] is "to account for the phenomenon of intuitive knowledge in mathematics, to make it intelligible."[37] The re-emergence of this ancient and controversial topic is in part a consequence of the fact that the

> ... tenor of much recent work in the philosophy of mathematics has been dictated by the popular assumption that Platonism is defunct. Some embrace that assumption and look for alternatives, others deny it and attempt to revive Platonism, but either way it is the starting point...[38]

According to James Brown, the "modern brand of mathematical platonism is not the same as Plato's, but it has much in common with it, and it has to face many of the same objections."[39] Brown himself takes one of the more traditional Platonistic positions in that he holds that the abstract objects of mathematics exist independently of us, ouside of space and time, and that the mind somehow has the ability to intuit at least some of these objects and grasp certain truths about them. Michael Resnick adopts a somewhat similar Platonist position in that he holds "that mathematical objects are causally inert and exist independently of us and our mental lives."[40]

Penelope Maddy raises the familiar objection which faces these Plationist positions. If mathematics "is the study of objective, ideal entities without position in space or time, and if humans beings have the sort of down-to-earth cognitive capabilities we think they do, how is it that we manage to know any mathematics at all?"[41] For Maddy, the dilemna confronting us in the philosophy of mathematics today is that we "seem forced to choose between denying that mathematics is about numbers, functions, and sets, as these are usually understood, and affirming the existence of some heretofore unheard of, probably unnatural form of, human information-gathering."[42] This, of course, is a problem which had occupied philosophers from Plato through Kant. The positivists, with their interpretation of pure mathematics as an uninterpreted formal system, believed that they could easily brush this problem aside. This, however, is no longer considered acceptable. As Maddy sees it, the principal challenge confronting the philosophy of mathematics in the 1990s is to meet this dilemma head-on and "find a way out."[43]

Maddy's proposal is to adopt a less traditional form of Platonism. She retains an ontological commitment to mathematical objects, such as numbers and functions, but attempts to "bring them into our familiar space-time context"[44] where, she argues, "they are accessible to our ordinary perception."[45] It is here that we can begin to see a similarity between some of the current interests in the philosophy of mathematics and Hume's philosophy. Hume, as we shall see, was aware that some form of Platonism, complete with a belief in some pure and intellectual mode of perception "of which the superior faculties of the soul are alone capable" (T.72), was the tacit working philosophy of most of the philosophers and mathematicians of his day. Unlike his contemporary counterparts, Hume could not invoke a purely logical interpretation of the concepts and propositions of geometry and simply brush this ancient and difficult problem aside. Instead, he had to meet the problem head-on. Hume, no less than Maddy and other critics of the more traditional forms of Platonism in mathematics, was convinced that the epistemological starting point for a sound philosophy of mathematics is the assumption that the most basic objects of geometry are accessible to ordinary perception.

It is not difficult to find other instances of ancient problems in the philosophy of geometry which Hume attempted to meet head-on but which were all too cavalierly dismissed by the positivists (and contemporary empiricists in general). Consider, for example, the problem of the exactness of mathematics and the inexactness of ordinary sense perception. This problem is clearly related to the problem concerning the nature of mathematical intuition and, as we have noted, was a matter of primary concern for Hume. Naturally, the positivists were of the opinion that the advances made in modern mathematical logic enabled them to explain the exactness of mathematics in a manner which is consistent with an empiricist epistemology and theory of perception. However, Bertrand Russell, whose work in mathematical logic had done much to make this view part of the conventional wisdom in the philosophy of geometry, believed that philosophers had yet to come to grips with the problem.

> This problem of the exactness of mathematics and the inexactness of sense is an ancient one, which Plato solved by the fantastic hypothesis of reminiscence. In modern times, like some other unsolved problems, it has been forgotten through familiarity, like a bad smell which you no longer notice because you have lived with it for so long...[46]

In light of these examples, we can begin to see that Hume's writings on geometry cannot be so easily dismissed on the pretext that he, like all the great historical figures, lacked the expertise necessary to identify, much less deal with, those issues in the philosophy of mathematics which are of any real significance. Certainly, those who make a doctrine of ordinary perception central to their mathematical epistemology or speculate as to how we acquire beliefs about causally inert mathematical objects bring the philosophy of mathematics back into the main body of philosophy. It can no longer be construed as an area of philosophy best left to those with a technical competence in mathematical logic and "some special interest in foundations of mathematics."

Hume set out to show that in so far as our geometrical ideas are rooted in everyday life and can be comprehended by our ordinary faculties, they can be analyzed in accordance with an empiricist doctrine of perception. Thus, a careful study of Hume's analysis would seem to be an ideal starting point for the type of critical inquiry into the epistemology of geometry which Sklar has advocated. The task of analysing the most basic and intuitively straightforward ideas of geometry in terms of an empiricist doctrine of ordinary perception is a task for which Hume was amply qualified, whatever his mathematical shortcomings may have been. Indeed, this is an area where we would expect Hume's genius to exhibit itself.

Lest we begin to give a false or inflated view of what we hope to achieve with our study, let us restate that our primary aim is to gain a critical understanding of Hume's struggles with geometry. Our reason for relating the study of Hume to some of the trends in the philosophy of mathematics in the last few decades was to make good on our claim that a lengthy study one of the least-admired parts of Hume's philosophy is very much in order and has an appeal which goes well beyond a narrowly construed interest in Hume scholarship.

If our inquiry proves successful we will have taken one of the steps necessary for "attaining a properly self-conscious appreciation of our own immediate philosophical background" with respect to the epistemology of geometry. Of course, the actual value of our inquiry remains to be seen and must ultimately speak for itself. We may conclude at this point that there is ample ground for saying that Hume's philosophy of geometry deserves no less a sympathetic and critical study than that of logical positivism or of contemporary empiricism in general.

Given that we shall be dealing with one of the least-admired parts of Hume's philosophy, it is not suprising that we should find considerable controversy surrounding the interpretation of his position even with respect to the most basic and seemingly straightforward of issues. For example, commentators disagree as to whether Hume's position in the Treatise is best classified as stating that the propositions of geometry are synthetic a priori or synthetic a posteriori. With respect to his position in the Enquiry there is disagreement as to whether his position is best classified as holding that these propositions are synthetic a priori or analytic. Needless to say, there is even disagreement over so basic a question as to whether the positions of the Treatise and the Enquiry are in harmony or at odds with one another.

It might seem that if Hume's writings have generated such controversy then they must contain considerable confusion and ambiguity - so much so as to cast a large shadow of doubt over the possibility of presenting a coherent interpretation. However, the situation is far from what it might seem and from what it is generally be held to be. As we shall see, the controversy we find stems in large part from the failure of commentators to familiarize themselves with the historical context in which Hume's views on geometry were developed and to see how they fit in as an integral part of his overall philosophy. Our first point of business, therefore, will be to present the background necessary for situating Hume's views wihin the framework of his empiricism. We shall take up this task in the following chapter. The main topics of discussion will be (a) the all important controversy surrounding the principle of infinite divisibility, (b) the influence of Berkeley, (c) scepticism, and (d) rationalism. Though all of these issues are important for an understanding of Hume's philosophy of geometry, we shall here make a few brief comments about two of them. Firstly, it is well known that Hume's analysis of the abstract idea of space was motivated mainly out of a desire to put an end to all those ancient paradoxes concerning the divisibility of extension which continued to plague geometry and which, in Hume's eyes, served as an open invitation to scepticism. Robert Fogelin's observation that the " philosophers of the seventeeth- and eighteenth-centuries were perplexed and fascinated with the fascinated with the notion of infinite divisibility"[47] applies to Hume as well as anyone. In the course of our study we shall attempt to show how Hume's attempt to deal with the isue of infinite divisibility formed an integral part of his philosophy of geometry. We shall pay particular attention to the close relationship between his views concerning infinite divisibility and the exactness and certainty of geometry. A failure to appreciate the full impact of this relation is one main reason why commentators have failed to appreciate the logical coherence of Hume's teachings with respect to geometry in the _Treatise_, as well as the significant break from these teachings we find in the _Enquiry_.

Secondly, there is the important contrast between empiricism and rationalism. In an important sense, the philosophy of geometry represents the first line of battle between these two opposing philosophies, and we shall attempt to make clear some of the basic warring issues. This will help set the stage for a more detailed comparison in later chapters. Though Hume does not make any explicit references to rationalism in his discussion of geometry, it is doubtful that the rationalists' position could have been very far from his thoughts. In fact, there are places where his writings seem clearly directed against it and openly invite comparison. Interestingly, more often than not, it is Leibniz's writings which can most easily and most fruitfully be brought into contrast with Hume's views. These comparisons will help place us in a position to assess the strengths and weaknesses of Hume's position.

In chapter three we shall turn our attention to the task of presenting an exposition of the philosophy of geometry of the Treatise. Our main challenge will be to deal with the logical gaps, ambiguities, and apparent inconsistencies in this least-admired and heavily criticized part of the Treatise. We shall attempt to put forward an interpretation which confers the maximum possible coherence on Hume's writings and which is in keeping with the fact that Hume was was making a serious effort at coming to grips with the leading problems which geometry was seen as presenting to the philosophy of empiricism. In essence, Hume was charting the general line of reasoning he believed an empiricist philosophy of geometry must follow. Our objective in this chapter will be to present a coherent overview of the general features of this program.

Once we have laid out the program in the philosophy of geometry Hume is advocating in the Treatise, we shall proceed to the task of critically assessing its strengths and weaknesses. This will be the subject of chapter four. In order to determine the key strengths of Hume's position, we shall adopt the method of rational reconstruction. We shall attempt to show how, based on the arguments of the Treatise, Hume could have attacked some of the most fundamental tenets of the rationalist philosophy of geometry. In particular, we shall see that it takes little effort to read into Hume's arguments some powerful criticisms of the rationalists' doctrines of analyticity and intellectual intuition.

On the other side of the coin, we shall see that one of the weak links in Hume's analysis is that he failed to offer a plausible account of the psychological origin of the belief in the exactness of the ideas of geometry and the self-evident nature of its axioms. As we shall see, this failure is of first importance and openly invites the criticism that Hume's analysis ultimately failed to capture the essence of our ordinary understanding of the ideas and propositions most essential to geometry.

In chapters five and six we shall shift our attention to the Enquiry. In the first of these chapters we shall focus on a passage (E.25) which has been the source of ongoing controversy among commentators. The main point at issue is whether this passage represents a sharp break from the Treatise regarding the epistemological status of the propositions of geometry. The standard interpretation is that it represents a radical break as Hume is now seen as arguing that the propositions of geometry are analytic. This is the view which most contemporary empiricists seem to have taken to be Hume's more considered opinion on the subject. In the last few decades, though, a number of philosophers have challenged this view and have argued that Hume's position in the Treatise and the Enquiry are in basic harmony and that in both he held the propositions of geometry to be synthetic a priori. Our own position will lie somewhat between these two extremes. We shall argue that the Enquiry represents a sharp break from some of the main teachings of the Treatise with respect to the status of the ideas and propositions of geometry. However, it shall be our contention that while one of the main elements of this break is a shift in view regarding the certainty of the propositions most essential to geometry, there is no corresponding shift to the view that these propositions are analytic. We shall maintain that if one wishes to categorize Hume's position in terms of the analytic - synthetic distinction, a distinction which he himself did not explicitly employ, then it is most correct to say that in both the Treatise and the Enquiry he held the propositions of geometry to be synthetic.

Hume's shift in view concerning the status of the propositions of geometry brought about a further shift in view concerning the exactness of the ideas of geometry, a shift which has not been fully appreciated. As we shall see, this break from the position of the Treatise is in sharp conflict with the copy principle and thus represents a serious problem for the position of the Enquiry. To bring this chapter to a conclusion, we shall speculate as to why Hume felt it necessary to break so radically with his position in the Treatise and adopt a position which places great strain upon his philosophy, if not causing it to collapse altogether.

One of the constants in both the Treatise and the Enquiry is Hume's opposition to the principle of infinite divisibility.In chapter six we shall re-examine Hume's position regarding this principle in light of his break from the Treatise regarding the status of geometry. We shall argue that Hume's shift in view created profound difficulties for him as he was forced to abandon the line of attack he employed in the Treatise against the principle of infinite divisibility. The task which faced Hume in the Enquiry was to show that in spite of his break from the Treatise, he could still mount a successful attack against the principle of infinite divisibility. It was a task at which he did not, and could not, succeed.

Our final chapter shall contain a brief summary of our findings. We shall conclude with a few remarks concerning the relevance of our inquiry from both an historical and a contemporary perspective.

CHAPTER 1

NOTES

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19). Fredrick Suppe ed. The Structure of Scientific Theories (Chicago: University of Illinois Press, 1974) p. 3 Suppe's Introduction to this work provides one of the most thorough accounts to date of the historical development of the Received View and the criticism which led to its demise. According to Suppe, the name 'the Received View' was first introduced by Putnam in "What THeories Are Not."(See Logic, Methodology, and Philosophy of Science, (Stanford: Stanford University Press, 1962).

20). Michael Friedman, "The Re-evaluatuion of Positivism," Journal of Philosophy, LXXXVIII, No. 10, (1991), p.505

21). Michael Friedman, "The Re-evaluation of Positivism," p.505

22). Michael Friedman, Foundations of Space-Time Theories (Princeton: Princeton University Press, 1983), p.31

23).Lawrence Sklar, Space,Time, and Spacetime (Berkeley: University of California Press, 1974), p.86

24). Sklar, for example, maintains that Reichenbach "is most responsible for the philosophical study of space and time in its current form." ibid., p.iii

25). Lawrence Sklar, p.85

26). Hans Reichenbach, The Rise of Scientific Philosophy, (University of California Press, 1968), p.86

27). ibid., pp.86-87

28). See, for example, Hans Reichenbach, The Rise of Scientific Philosophy, pp. 86 and 141-142.

29). Hans Reichenbach, p.325 Reichenbach is here speaking about philosophy in general but there is no question but that he saw this assessment as hold especially true with respect to the philosophy of science and mathematics.

30). Thomas Tymockzo, "Challenging Foundations," in New Directions in the Philosophy of Mathematics ed. Thomas Tymockzo (Birkhauser Boston,inc., 1986) p.2

31). Patrick Suppes,ed., Space, Time, and Geometry (Boston: D. Reidel Publishing Company, 1973) p.IX Suppe himself speaks quite approvingly of this trend toward specialization.

32). A.D Irvine, "Nominalism, Realism, and Physicalism in Mathematics," in Physicalism in Mathematics ed. A.D. Irvine (Kluwer Academic Publishers, 1990) p.IX

33) Thomas Tymockzo, New Directions in Mathematics p.XV

34). Stephen Barker, "Logical Positivism and the Philosophy of Mathematics," in The Legacy of Logical Positivism, ed. by Peter Achinstein and Stephen Barker (Baltimore: The John's Hopkins Press, 1969), p.240

35). Gian-Carlo Rota makes the general observation that those philosophers who seek to apply the techniques of mathematical logic to philosophical problems tend to "cut all ties to the past on the claim that the messages of the past philosophers are now "obsolete"." He maintains that they "justify their neglect of the old and substantial questions of philosophy" by "resorting to the ruse of claiming that many question formerly thought to be philosophical are instead "purely psychological" and should be dealt with in the psychology department." (see Gian-Carlo Rota, "Mathematics and Philosophy: The Story of a Misunderstanding," Review of Metaphysics, XLIV,No.2 (Dec. 1990) In this respect these philosophers are continuing in the tradition of positivism however critical of it they might otherwise be. It is worth noting that we find one of Hume's more sympathetic commentators objecting to the "logical positivist's treatment of Hume as a superannuated precursor who muddled psychology with logic." (James Noxon, Hume's Philosophical Developoment p.150)

36). Philip Davis and Reuben Hersh, The Mathematical Experience p.397

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38). Penelope Maddy, "Mathematical Epistemology: What is the Question?", Monist, 67, (1984), p.46

39). James Robert Brown, "Pi in the Sky," in Physicalism in Mathematics ed. A.D. Irvine, p.107

40). Michael D. Resnik, "Beliefs About Mathematical Objects," in Mathematical Physicalism ed. A.D. Irvine, p.41

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44). ibid., p.156

45). ibid., p.156

46). Bertrand Russell, Human Knowledge: Its Scopes and Limits (New York: Simon and Schuster, 1948) p.238

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Chapter 2

Philosophical Background

I

Introduction

In the present chapter we shall take the first step toward gaining a critical understanding of Hume's writings on space and geometry by attempting to make clear precisely how they fit into the general framework of his empiricism and form an essential part thereof. We shall attempt to make clear why Hume was drawn to the topic of space and geometry in the first place and how his discussion of it relates to his basic opposition to rationalism and Pyrrhonian (excessive) scepticism. This background will yield us the perspective we need to follow up on Flew's advice and gain insight into "what Hume's problems were and how he believed he had solved them."

II

Berkeley, Space, and Geometry

John Wright has argued that Hume's analyses of the ideas of space and time, causality and matter "become more intelligible when they are seen against the background of the analyses of these ideas presented by Bayle, Descartes, Berkeley and Malebranche."[1] It is well known that Hume's analysis of the idea of space, especially as it related to the principle of infinite divisibility, borrowed heavily from the writings of Bayle (principally, from the article 'Zenon' in his Dictionary) and Berkeley. However, our immediate concern here is not with the actual analysis of this idea but with the basic philosophical framework in which this analysis was carried out. Here, Berkeley was without question the dominant figure of influence and Hume himself left no question as to the high esteem in which he held his teachings. It was Hume's belief that the "writings of that most ingenious author form the best lessons of skepticism, which are to be found among the ancient or modern philosophers, Bayle not excepted" (E. 155n). To be sure, Hume saw certain defects in Berkeley's teachings regarding scepticism, defects which he hoped to overcome.

> ...He [Berkeley] professes, however, in his title page (and undoubtedly with great truth) to have composed his book [A Treatise Concerning the Principles of Human Knowledge] against the skeptics as well as against the atheists and the free thinkers. But that all his arguments, though otherwise intended, are in reality, merely skeptical, appears from this, that they admit of no answer and produce no conviction. Their only effect is to cause that momentary amazement and

> irresolution and confusion, which is the result of skepticism. (E.155n)

In spite of the serious shortcomings Hume saw in Berkeley's philosophy, he believed that it held the key to real philosophical advancement. This, we shall see, he believed to be particularly true with respect to how it dealt with certain basic issues concerning the fundamental principles of geometry which were of first importance for the philosophy of space. It will thus be very much to our advantage to dedicate the remainder of this section to a brief study of Berkeley's philosophy. For our purposes, it will suffice to restrict ourselves to Berkeley's <u>A Treatise Concerning the Principles of Knowledge</u>.

Berkeley took a strong sceptical position regarding the rationalists' claim that we can acquire real knowledge of things as they exist apart from their sensible manifestations. To be sure, he had no difficulty conceding to the rationalists that in order to gain such knowledge the mind must possess innate ideas and a special faculty (eg., the light of reason) through which they can be apprehended clearly and distinctly. However, Berkeley was philosophically committed to the view that all our ideas have their origin in the senses and imagination alone and thus he denied the reality of these innate metaphysical ideas.

There is, of course, more to Berkeley's rejection of the rationalists' doctrine of innate ideas than a dogmatic adherence to empiricism. Since the earliest days of philosophy the more rationally inclined thinkers had tried to present a clear and distinct picture of a purely intelligible realm of forms (things-in-themselves). In Berkeley's estimation, little had been accomplished. In spite of the tremendous effort expended over the centuries, the discipline of metaphysics had not even begun to approximate a science. It was not simply that throughout history there had been ongoing debates and controversies with little having been resolved. For Berkeley the main problem was that metaphysicians were continually putting forward doctrines which ended in contradiction and absurdity. This state of affairs had led many learned men to adopt what Berkeley regarded as a most extreme and destructive form of scepticism. Berkeley set out to rectify this situation and announced that it was his aim

> ...to try if I can discover what those principles are, which have introduced all that doubtfulness and uncertainty, those absurdities and contradictions into the several sects of philosophy insomuch as the wisest of men have thought our ignorance incurable, conceiving it to arise from the natural dulness and limitations of our faculties...(P. par. 4)

As Berkeley saw it, one of the main difficulties facing the metaphysician of his day was that the moment he began to reflect abstractly on any matter, even those which seemed so perfectly clear to him in everyday life, he was inexorably drawn into a labyrinth of absurdities and contradictions.

> ...no sooner do we depart from sense and instinct to follow the light of a superior principle, to reason, meditate, and reflect on the nature of things, but a thousand scruples spring up in our minds, concerning those things which before we seemed fully to comprehend. Prejudices and errors of sense do from all parts discover themselves to our view; and endeavouring to correct these by reason, we are sensibly drawn into uncouth paradoxes, difficulties and inconsistencies, which multiply and grow upon us as we advance in speculation; till at length, having wandered thro' many intricate mazes, we find our selves just where we were, or, which is worse, sit down in forlorn scepticism (P. par. 1).

Those who sat down in forlorn scepticism were those who believed that the ultimate cause of all these absurdities and paradoxes was grounded in either "the obscurity of things, or the natural weakness and imperfection of our understanding" (P. par. 2). For them, it was simply not possible for reason to find its way through the twists and turns of the labyrinth in which it had become entangled. It is important that we bear in mind that the sceptic Berkeley had in mind did not maintain that it is only when we attempt to reason abstractly with respect to the more obscure and intricate notions of metaphysics and seek to gain knowledge of things as they "really" are that we become lost in a maze of contradictions and absurdities. If this were all he wished to argue, Berkeley would have found little room for disagreement. The sceptic, however, went further and maintained that all of our attempts at gaining a rational understanding of the world around us, as well as of ourselves, were just so many paths leading into the labyrinth. Sextus Empiricus and Pierre Bayle were two of the more prominent proponents of this extreme form of scepticism. Richard Popkin gives the following account of Bayle, the most influential sceptic of the seventeenth century.

> Bayle repeatedly showed that the many attempts by human beings to explain or understand their world were all just "high roads to Pyrrhonism", since they only made every supposition more perplexing, absurd, and dubious. Rational activity, no matter what problem it is directed at, leads to complete skepticism, since reason invariably leads us astray. In his article "Acosta" Bayle compared reason to a corrosive powder that first eats up errors, but then goes on to eat up truths. "When it is left on its own, it goes so far that it no longer knows where it is, and can find no stopping place."[2]

For sceptics such as Bayle even the most basic and straightforward abstract reasonings of geometry were seen as ending in conclusions which were big with contradiction and absurdity. The notorious principle of infinite divisibility represented the clearest example of this phenomenon. Starting from geometrical ideas and principles which seemed perfectly clear and incontestable and following a chain of reasoning which seemed logically impeccable, philosophers and mathematicians alike were led to certain conclusions concerning the divisibility and composition of extension that struck many, including Berkeley, as being ridiculously unreasonable. In placing the cause of the difficulties surrounding the principle of infinite divisibility in "the natural weakness and imperfection of our understanding," the sceptic called into doubt the soundness of abstract reasoning even when restricted to the most basic and elementary part of geometry. If we cannot depend on rational activity to expand the domain of our knowledge in this discipline without falling into absurdity and contradiction, where can we depend on it? If we cannot ultimately resolve the paradoxes surrounding the principle of infinite divisibility (the type of paradoxes Zeno delighted in producing) and show that our geometrical reasoning do not support conclusions which are big with contradiction and absurdity, what alternative do we have but to sit down in forlorn scepticism?

Berkeley believed that the many attempts made by philosophers and mathematicians to rid geometry of these paradoxes simply led them deeper and deeper into the labyrinth and thus to conclusions which only added to the embarrassment of philosophy and geometry. The attempt to explain the infinite divisibility of finite extension by postulating the existence of infinitesimal quantities was, for Berkeley, a case in point. And yet, he could hardly accept the sceptic's contention that the source of all our troubles lies either in an inherent defect in our rational faculty or in some hidden inconsistency in the seemingly clear and straightforward abstract ideas of geometry. In order to silence the sceptic, Berkeley set out to identify what he believed to be the real source of our errors and show how we could rid ourselves of all those absurdities and contradictions which seem to accompany our abstract reflections, including those which are most essential to mathematics and science.

It was Berkeley's belief that the aforementioned difficulties, which had caused so much embarrassment for philosophy and led some of the most able thinkers down the path of Pyrrhonism, had their origin in a false doctrine of ideas. The conventional wisdom of the day maintained that "the mind hath a power of framing abstract ideas or notions of things" (P. par. 6), and this opinion, Berkeley believed, played "a chief part in rendering speculation intricate and perplexed, and to have occasioned innumerable errors and difficulties in almost all parts of knowledge" (P. par. 6). Not surprisingly, the mathematical ideas were offered as incontestable evidence that the mind has the power to frame for itself such abstractions. For example, it was very much a part of the established view of the day that the mind can readily frame for itself the clear and distinct idea of a triangle in general, a triangle which is neither isosceles, scalene, nor right. Similarly, it was assumed that the mind possesses an abstract idea of extension in general, an idea of extension which neither contains nor is limited by any particular degree of magnitude. Berkeley was convinced, however, that once we realize that the mind has no such powers of conception, we shall be able to rid philosophy of those difficulties which have embarrassed it since its earliest days and have thrown all rational speculation into doubt.

In order to avoid any misunderstanding regarding Berkeley's doctrine, let us make clear that he was objecting to abstract general ideas, not abstract ideas per se. Berkeley maintained that all our ideas, even those we call abstract, are really particular in nature and thus serve to represent some particular object, relation, or quality. This, of course, is not to deny that an abstract idea may be considered to be general or universal in the sense that it represents many other ideas which bear some particular relation to it. Berkeley fully acknowledged that "an idea, which considered in itself is particular, becomes general, by being made to represent a standard for all particular ideas of the same sort" (P. par. 12). He felt it necessary to deny only that the mind possessed ideas which are in themselves general and which thus serve to represent the abstract form of some such standard.

To illustrate his position, Berkeley offered the following example. A geometer may draw for himself a black straight line which is one inch in length. This line is in itself something particular, and thus is fixed in both its degree of quality and quantity. However, this particular line may become general in its signification in that it may come to represent other particular straight lines regardless of their colour and length. In this sense one may speak of this particular line as representing a straight line in general, and whatsoever the geometer demonstrates of the one holds equally well for the other (assuming, of course, that he makes no special reference to the particular length or colour of the original line). What Berkeley wished to emphasize with this example is that the particular line "owes its generality not to its being the sign of an abstract or general line, but of all particular right lines that may possibly exist..." (P. par. 12).

What has been said above concerning the line is intended to hold equally well for our idea of it. According to Berkeley's doctrine the abstract idea of a straight line represented a standard which is fixed in its degrees of quantity and quality. In so far as it served to denote other ideas to which it was related, it functioned as a general idea. This, however, was not to suggest that the mind has the power of framing for itself the idea of a form of a straight line in general. Such an idea would not only be general in its signification but also in the mind's conception of it. It would represent the very essence of a straight line, a perfection which a particular line could only more or less approximate. For Berkeley, such an abstract notion was to be dismissed as a mere fiction of the mind, altogether lacking in any meaning and significance.

Though Berkeley denied that the mind has the power of framing and comprehending ideas which are in themselves something general, he acknowledged that nothing is more natural for the mind than to view its abstract ideas as if they represented something which is in itself of a general nature. Once it has fallen victim to this error, the mind quickly comes to believe falsely that it can conceive the essence of things as they exist apart from all relation to the senses. This confusion, he believed, takes place even with respect to those ideas which are most clearly rooted in ordinary life, such as our ideas of space, time and motion, and was seen as the source of all those embarrassing contradictions which plague our abstract reasoning. By keeping in mind that all our ideas are particular, Berkeley believed he could chart a course between the extremes of forlorn scepticism and dogmatic rationalism. He was convinced, for example, that by keeping in mind that our abstract idea of space (extension) is really particular in the mind's conception of it, we shall be able to see quite clearly that any given finite extension is only finitely divisible. By arguing that the principle of infinite divisibility has its origin in a false doctrine of ideas, Berkeley believed that he could banish it from the foundations of geometry and thereby put to rest all those absurdities and paradoxes which had so delighted the sceptic.

Berkeley, then, did not attempt to rid philosophy of its absurdities and contradictions by guiding reason through the labyrinth, as the rationalists (most notably, Leibniz) sought to do, but by keeping it from entering the labyrinth in the first place. By restricting all rational activity to the contemplation of ideas which are particular he had hoped to put an end to all those speculative flights of fancy which invited so much contempt and ridicule from the sceptic. Contrary to the conventional wisdom of his day, Berkeley did not believe that the discipline of mathematics (or physics) presupposed general ideas. He was convinced that in so far as these disciplines purport to say anything meaningful and significant about the world, they can be shown to be founded upon ideas which are in strict accordance with his doctrine. He thus hoped to show that these disciplines do not serve as gateways leading into the metaphysical realm of the purely intelligible. Of course, if it should happen that geometry cannot be founded upon an idea of space which is in itself something particular, as Descartes and Leibniz would have argued, then Berkeley's philosophy cannot even get off the ground. It would not succeed in keeping anyone from sitting down in forlorn scepticism.

III

<u>The Challenge of Rationalism</u>

With this brief sketch of Berkeley's position before us, let us proceed with our discussion of Hume. As we have already noted, Hume held Berkeley's teachings regarding scepticism in high esteem. It is not surprising, then, that we should find Hume fully embracing Berkeley's doctrine of abstract ideas.

> A very material question has been started concerning <u>abstract</u> or <u>general</u> ideas, <u>whether they be general or particular in the mind's conception of them</u>. A great philosopher [Berkeley] has disputed the receiv'd opinion in this particular, and has asserted, that all general ideas are nothing but particular ones, annexed to a certain term, which gives them a more extensive signification, and makes us recall upon occasion other individuals, which are similar to them. As I look upon this to be one of the greatest and most valuable discoveries that has been made of late years in the republic of letters, I shall here endeavour to confirm it by some arguments, which I hope will put it beyond all doubt and controversy.(T.17, My brackets)

For our purposes it will not be necessary to examine Hume's arguments in any detail. It will suffice to note that Berkeley's doctrine is in complete accordance with Hume's most basic epistemological principle, the so-called "copy principle". According to this principle, "<u>all our simple ideas</u>

in their first appearances are deriv'd from simple impressions, which are correspondent to them, and which they exactly represent" (T.4). Since "no impression can be present to the mind without being determin'd in its degree of both quantity and quality" (T.19), it follows that our ideas, which are but copies of these impressions, must also be so determined. In other words, our ideas must be particular in the mind's conception of them. Like Berkeley, Hume was quick to acknowledge that an idea which is in itself something particular may nonetheless be general (universal) in its signification.

> ...some ideas are particular in their nature but general in their representation. A particular idea becomes general by being annex'd to a general term; that is, to a term, which from a customary conjunction has a relation to many other particular ideas, and readily recalls them in imagination (T. 22).

Hume, of course, did not accept Berkeley's doctrine in isolation from his overall philosophical perspective regarding scepticism. In an important respect Hume's philosophy can be viewed as a serious attempt to continue with and render consistent the program initiated by Berkeley. Hume believed that Berkeley's doctrine of abstract ideas held the key for charting a course of analysis which would avoid the extremes of dogmatic rationalism and radical (Pyrrhonian) scepticism. Nowhere was this more clear than with respect to the philosophy of space and geometry. As Hume was only too aware, some of the strongest arguments put forward by the rationalists and the radical sceptics in favour of their basic points of view centered around the analysis of the idea of space and the central role it played in geometry.

In this section we shall focus on the rationalist position and the challenge it presented to Hume. One of the most significant points of disagreements between Hume and the rationalists centered around the nature and origin of our ideas. The following passage from Leibniz expresses most succinctly the basics of the rationalist position.

> There are thus three levels of concepts: those which are sensible only, which are the objects produced by each sense in particular; those which are sensible and intelligible, which belong to the common sense; and those which are intelligible only, which belong to the understanding. The first and second together are imaginable, but the third lie beyond the imagination. The second and third are intelligible and distinct, but the first are confused, although they may be recognizable.[3]

At one end of the spectrum we have the purely sensible ideas (concepts) which are in complete agreement with Hume's copy principle. At the other end we have the purely intelligible ideas, ideas which could not be more in conflict with Hume's first principle and, consequently, Berkeley's doctrine of abstract ideas. Since the intelligible ideas cannot be of a sensible origin, they are not fixed or determined by any sensible limits concerning quality or quantity. These ideas were construed as representing things (relations, qualities) as they exist apart from all such sensible determinations and thus as representing something which is of a purely general nature. The rationalists held the ideas of self, substance, causality, and God as prime examples of metaphysical ideas. Through the analysis of these ideas the rationalists sought to attain knowledge of the intelligible realm of things-in-themselves.

Hume took a hard sceptical line against the possibility of attaining such knowledge. He maintained that these metaphysical flights into the ethereal heights of human reason invariably end in conclusions which are big with contradiction and absurdity. Following closely in the footsteps of Berkeley, Hume was not about to place the cause of these difficulties in a defect in our intellectual faculties to reason consistently or in inconsistencies holding among our abstract ideas. Like Berkeley, he believed that most, if not all, of our philosophical difficulties are deeply rooted in a strong and natural propensity of the mind to view certain abstract ideas as if they were general in the mind's conception of them. Hume adopted Berkeley's basic point of view and believed that once we realize that even our most abstract ideas have their origin in sense experience, our flights into the realm of dogmatic metaphysics will come to an end. As a result, we will no longer encounter the paradoxes and absurdities which only serve as so many invitations to sit down in forlorn scepticism.

The contrast between Hume and Leibniz is particularly interesting for our purposes since Leibniz, more than any other rationalist, sought to give the senses their proper due. He acknowledged, for example, that even "our most abstract thoughts are in need of sense perception".[4] He further granted that "in our present state the external senses are necessary for our thinking and that if we had none we would not think".[5] He conceded to the empiricist that "we never have thoughts so abstract that something is not mixed with them from the sense".[6] Nonetheless, he insisted that what is necessary for our thinking "need not make up its essence"[7] and that "reasoning demands more than what is sensible".[8]

In the tradition of modern rationalism, Leibniz maintained that the intelligible or metaphysical ideas are, in themselves, both clear and distinct. However, owing to the influence of the senses, a certain confusion is generated and we fail to comprehend what is distinct in these ideas. In other words, our ordinary awareness of these ideas is at best clear but confused. For Leibniz, this confusion generated by the senses was the real source of those absurdities and contradictions which proved to be such an embarrassment to philosophy. Like all rationalists, he regarded it as a principal task of philosophy to abstract away that which is sensible and the source of so much confusion and reveal that which is truly distinct in our metaphysical ideas of self, substance, God, causality, etc. In other words, he believed that philosophical analysis can lead us through the labyrinth of darkness and into the light of a priori knowledge.

It is understandable that commentators have focused much attention on Hume's attempt to show that our ideas of self, substance, and causality can be exhaustively analyzed in accordance with his first principles. However, they have all too often overlooked the important and pivotal role which geometry played in this controversy concerning the origin of ideas. For this, we have to go back and take a closer look at what Leibniz deemed to be the second level of ideas. What he had in mind here were the mathematical ideas, with the abstract idea of space (or extension) being a prime example. The rationalists were most insistent in arguing that there is more in our everyday idea of space than can be explained in terms of sense experience. They regarded it as being evident to anyone who would reflect on the matter that the abstract idea of space which is so essential to geometry, an idea which is rooted in everyday experience, contains an intelligible as well as an obvious sensible element. This intelligible element, they argued, consisted in the representation of the pure form of extension in general.

The rationalists had strong reason to make so bold a claim. Mathematicians had long taken it to be self-evident that they could exhibit, in thought, the essence or form of the most basic ideas of geometry. For example, they took it to be self-evident that we can readily frame for ourselves a clear and distinct idea of the form of a perfectly straight line in general. This abstract idea was seen as being absolutely essential to our understanding of the most basic and straightforward principles of geometry, and it was scarcely deemed necessary to argue that no empirical construct could be adequate to this idea. For the rationalists, the existence of geometry was seen as offering conclusive evidence that we can conceive of space in a manner which enables us to render clear and distinct certain general abstract ideas such as that of perfect straightness. Geometry, then, was seen as offering indisputable evidence that we possess ideas which contain more than the senses and imagination alone can furnish. As Leibniz put it,

> ... in order to conceive numbers and even shapes distinctly and to build sciences from them, we must reach something which sense cannot furnish but which understanding adds to it.[9]

In general, the rationalists maintained that the mathematical ideas represented perfectly clear and straightforward examples of ideas which contain more than what comes from the empirical faculties and which can be rendered distinct in the mind's conception of them. They regarded it as readily evident to anyone who would give the matter a moment of careful attention that these ideas contain some nonsensible element which constitutes their essence and makes it possible for us to define them. From here it was only a short step to the realm of metaphysics and the purely intelligible. In order to close the door on rationalism and its doctrine of ideas, Hume needed to show conclusively that the ideas which are most essential to geometry (and mathematics in general) can be analyzed in accordance with Berkeley's doctrine of abstract ideas without, in the process, destroying anything which forms an integral part of the discipline. It was here, then, at the level of the mathematical ideas, that one of the most important battle lines between empiricism and rationalism had been drawn. And, as we have already noted, Hume was only too aware that the mathematician had long proved to be a most powerful ally of the rationalists in these matters.

> ...'tis usual with mathematicians, to pretend, that those ideas, which are their objects, are of so refin'd and spiritual a nature, that they fall not under the conception of the fancy, but must be comprehended by a pure and intellectual view, of which the superior faculties of the soul are alone capable. The same notion runs thro' most parts of philosophy, and is principally made use of to explain our abstract ideas, and to show how we can form an idea of a triangle, for instance, which shall neither be an isosceles nor scalenum, nor be confin'd

> to any particular length and proportion of sides. 'Tis easy to see, why philosophers are so fond of this notion of some spiritual and refin'd perceptions; since by that means they cover many of their absurdities, and may refuse to submit to the decisions of clear ideas, by appealing to such as are obscure and uncertain. But to destroy this artifice, we need but reflect on that principle so oft insisted on, that all our ideas are copy'd from our impressions...(T.72)

Hume speculated, in the above, as to why philosophers found this notion of some refined and spiritual perceptions (intellectual intuitions) attractive, but the immediate question at issue is, why did the mathematician find it so attractive? Why did the mathematician believe that the most basic ideas of his discipline, ideas which are among our most clear and easily formed, are of so refined and spiritual a nature that they can only be comprehended by an intellectual mode of intuition? Surely, it was not the mathematician's intention to cloud his science in obscurity and refuse to submit to the decisions of clear ideas. Rather, the mathematician found it difficult, if not impossible, to discount the received view among philosophers; namely, that anyone who cared to reflect upon the basic ideas of geometry would readily see that they contain more than what can be derived from sense and imagination alone. However, in spite of the fact that the leading mathematicians of the day fully supported the rationalists on this important point, Hume remained convinced that the ideas of geometry, and mathematics in general, could be analyzed in accordance with his copy principle and Berkeley's doctrine. Even as late as the Enquiry we find Hume reaffirming his commitment to these principles.

> ... The idea of extension is entirely acquired from the senses of sight and feeling ... An extension, that is neither tangible nor visible, cannot possibly be conceived: and a tangible or visible extension, which is neither hard nor soft, black nor white is equally beyond the reach of human conception. Let any man try to conceive a triangle in general, which is neither Isosceles or Scalenum, nor has any particular length or proportion of sides; and he will soon perceive the absurdity of all scholastic notions with regard to abstraction and general ideas (E.154-55).

As Hume was aware, the idea of space, or extension, (like Descartes, Hume virtually identifies the two) is the principal object of geometry. Therefore, if he is to get his empiricism off the ground, Hume must show, and not merely assert, that the idea of space which is so essential to geometry can be explained in accordance with his first principles. He must show that nothing which is truly essential to geometry requires us to assume that the idea of space is of an intrinsically general nature. If he is to give a proper defense of his first principles, he must come to grips with the arguments of the rationalists, and their mathematician allies, to the contrary. He must show that these arguments ultimately rest on a confusion which has entered into their thinking with respect to those ideas and principles which, to all outward appearances, seemed so clear and evident. Needless to say, the burden of proof rests heavily upon Hume's shoulders.

IV

The Challenge of Scepticism

Rosemary Newman has stated that "Hume's primary motive for discussing space and time in the Treatise, and that which gave direction to this discussion, was his concern for mathematical knowledge, especially geometry".[10] From our discussion thus far we can readily see why this concern should have proved to be a primary motive for Hume's analysis of space. However, in order to set the whole picture before us, we must bring the sceptic back into the discussion. As we have already seen, the sceptic's contention that even the abstract reasoning of geometry supported conclusions which were big with contradiction and absurdity. They were paraded about by the sceptic as proof that abstract rational activity, no matter at what problem it is directed, is an unreliable source of knowledge. After all, if we cannot trust even our most simple and straightforward geometrical reasoning about space to be free from contradiction and absurdity, how can we rely on any mode of abstract reasoning, least of all the metaphysical mode, to extend our knowledge of ourselves and the world around us? As Hume observed,

> The chief objection against all abstract reasoning is derived from the ideas of space and time; ideas, which in common life and to a careless view, are very clear and intelligible, but when they pass through the scrutiny of the profound sciences (and they are the chief objects of these sciences) afford principles, which seem full of absurdity and contradiction...(E.156)

As Hume saw it, the sceptic was on safe ground in arguing that some of the principles embraced by geometers as being fundamental to their science yield conclusions which are big with contradiction and absurdity. Like Berkeley, Hume regarded the principle of infinite divisibility as the prime example. According to this principle a finite space can be divided _ad infinitum_, thus implying that there is no smallest part of extension. Since analysis seemed to require that whatever is extended must be thought of as containing parts into which it can be further divided, it seemed clear to Hume that those who accept the infinite divisibility of extension are forced to conclude that all finitely extended things must be composed of an infinite number of parts. And, more often than not, this was indeed the case. As Kant observed, the dogmatic metaphysician concluded that if extension is infinitely divisible, "it consists of an infinite multitude of parts; for a whole must in advance already contain within itself all the parts in their entirety into which it can be divided."[11]

For Hume, however, this was preposterous and shocking to common sense. What could be more obvious than that an infinite number of parts, no matter how small, must yield a line which is infinite in extension? To postulate, as did many leading mathematicians, the existence of infinitesimal quantities - quantities which are greater than zero but smaller than any determinate magnitude, and whose infinite sum yields a finite extension - was, for Hume, totally absurd. This assumption, he believed, "shocks the clearest and most natural principles of human reason" (E.156), and ultimately plays right into the hands of the sceptic.

What Hume found so extraordinary and perplexing about all this was that "these seemingly absurd opinions are supported by a chain of reasoning, the clearest and most natural; nor is it possible to allow the premises without admitting the consequences" (E156). Herein lay the strength of the sceptic's argument. The same principles and logic which lead us to conclude that the sum of the angles of a triangle equals two right angles seem to lead us, no less straightforwardly, to the conclusion that a finite extension is infinitely divisible. It seemed scarcely possible to accept the former conclusions without accepting the latter. Hume assessed the situation as follows.

> ... Reason here seems thrown into a kind of amazement and suspense, which, without the suggestions of any sceptic, gives her a diffidence of herself, and of the ground on which she treads. She sees a full light, which illuminates certain place; but that light borders upon the most profound darkness. And between these she is so dazzled and confounded, that she scarcely can pronounce with certainty and assurance concerning any one object. (E.157)

Hume, of course, was not about to sit down in forlorn scepticism. He sought instead to turn the tables on the sceptic and show that the sceptic's position is no less absurd than any of the consequences of the principle of infinite divisibility. Hume had no trouble accepting the claim that the so-called 'intelligible' ideas, of which metaphysicians were so fond, contain circumstances which are contradictory to themselves and other such ideas. It was quite easy for him to accept that our abstract reasonings involving such vague and obscure notions should be big with contradiction and absurdity. However, he found it impossible to believe that this could hold true with respect to the idea of space or any other idea which is essential to geometry. On Hume's reckoning, these latter ideas are firmly rooted in everyday life and are derived from some of our most immediate and least deceitful sense perceptions. He found it absolutely incomprehensible that these ideas should contain contradictory circumstances.

> Yet still reason must remain restless, and unquiet, even with regard to that scepticism, to which she is driven by these seeming absurdities and contradictions. How any clear, distinct idea can contain circumstances, contradictory to itself, or to any other clear, distinct idea, is absolutely incomprehensible; and is, perhaps, as absurd as any proposition, which can be formed. So that nothing can be more sceptical, or more full of doubt and hesitation, than this scepticism itself, which arises from some of the paradoxical conclusions of geometry or the science of quantity (E.p. 157-8).

Leibniz also regarded the resolution of the difficulties surrounding the principle of infinite divisibility to be one of the central tasks confronting philosophy. However, unlike Berkeley and Hume, he was of the opinion that this principle was absolutely essential to mathematics. Consequently, he saw no alternative but to work his through the labyrinth of paradox and confusion into which we are drawn by some of our most straightforward geometrical reasonings. Following in the rationalist, Leibniz sought to achieve this end by first attaining a clear and distinct understanding of the intelligible element which constitutes the essence of our idea of space (extension). He was ultimately led to the conclusion that a proper analysis of the metaphysical idea of a simple substance (monad) held the key to resolving the long standing difficulties surrounding the principle of the infinite divisibility of extension.

Needless to say, Hume believed that the metaphysical approach had very little to offer. He was convinced that it would serve only to embroil us in endless speculations about the nature of things-in-themselves and thereby lead us into an endless maze of paradox and confusion. In Hume's estimation Berkeley, who was himself fairly well-versed in the mathematical sciences, was the first really to advance the cause of philosophy in this matter. Under Berkeley's tutelage, Hume traced the source of all the paradoxes and absurdities surrounding our geometrical reasoning concerning the divisibility of extension to a natural propensity of the mind to view the idea of extension (space) as if it was in itself the representation of something general. He believed that the thinking of the mathematician, no less than that of the philosopher, was infected with a false doctrine of ideas. Hume fully agreed with Berkeley that it is only by guarding against this error and keeping in mind that even the most basic ideas of geometry are really particular in the mind's conception of them that we shall be able to secure the foundations of geometry against the attack of the sceptic.

In essence, Hume accepted Berkeley's view that the principle of infinite divisibility was really a false metaphysical doctrine which, owing to a natural propensity of the mind for confusion, had come to be regarded as a fundamental tenet of mathematics. As extreme and absurd as this may have appeared to many, Hume found it quite a reasonable supposition. After all, it was a natural consequence of Berkeley's overall program for dealing with metaphysical controversies which held such great promise. The assumption that the abstract reasoning of geometry were not immune from the infectious error which had contaminated so much of metaphysics did not seem unreasonable. From Hume's perspective, the fact that Berkeley's doctrine could so neatly exorcise a problem which threatened to undermine the foundations of geometry, foundations he was committed to preserving, was a powerful argument in its favour. It was thus to become a cornerstone of his philosophy of geometry.

V

Conclusion

Everything in our discussion thus far points to the conclusion that the analysis of the basic ideas of geometry, especially that of space, was of first importance for Hume and formed an integral part of his philosophy. This, however, is not the view which most commentators have embraced. John Passmore, for example, has recently stated that though Hume was "interested in mathematics, especially in geometry"[12] and "the first lengthy discussion in the Treatise is about Space and Time",[13] these concerns "had a subordinate role in his great enterprise of constructing an adequate theory of the human mind and human society."[14] The point being argued here is that while Hume's lengthy discussion of space and geometry had a special purpose which may be of interest in itself, it had little bearing on the central concerns and matters of Book I of the Treatise.

George Pappas offers what may be a major reason why commentators have failed to appreciate the full significance of Hume's lengthy discussion of space and geometry and the main thrust of his most important arguments. Though it is well-known that Hume followed Berkeley in rejecting abstract general ideas, Pappas maintains that it has not been clear to commentators "why each were so opposed to abstraction and abstract ideas."[15] He goes so far as to claim that "it has seemed to many that Berkeley's rejection of such ideas is representative of merely a local dispute between him and Locke, while the case of Hume, so it seemed, verged on total mystery."[16] It is Pappas's contention that for both Berkeley and Hume the rejection of abstract ideas which are intrinsically general "lies at the very heart of their respective philosophical doctrines and theories."[17]

There is an obvious and basic point of agreement between our position in this matter and that of Pappas. However, there is a difference in our methods of inquiry and perspective that is worth noting. Pappas makes clear that he is not interested in examining "the psychological question, what were Berkeley's and Hume's intentions when they were moved to reject abstraction and abstract general ideas?"[18] Though he concedes that this is a question of considerable interest and importance, he has chosen to focus on the question concerning the philosophical role "which the rejection of abstraction and abstract general ideas play in the philosophies of Berkeley and Hume."[19] It is his opinion that the answer to the second question will shed important light on the first.

Our approach to the general question concerning Berkeley's and Hume's rejection of abstract general ideas has been just the opposite. In order to become clear as to why Berkeley and Hume were so radically opposed to abstract general ideas, we first noted that it was the intention of both men to rid philosophy of all those embarrassing paradoxes and absurdities which they believed accompanied even its most basic abstract reasoning and left it vulnerable to the attack of a most extreme and destructive form of scepticism. They were convinced that these difficulties were grounded in the prevailing doctrine of general ideas, a doctrine which was fundamental to the philosophy of rationalism. They saw no option but to reject this doctrine and replace it with a doctrine of abstract ideas which would enable them to chart a safe passage between the extremes of dogmatic metaphysics and excessive scepticism.

In becoming clear as to the intentions of Berkeley and

Hume we have placed ourselves in a position to appreciate the role which the rejection of abstract general ideas played in their respective philosophies. More to the point for our purposes, we have placed ourselves in a position to appreciate why Hume found it necessary to devote a significant portion of Book I of the Treatise to a discussion of geometry and the idea of space. Hume's belief that all our abstract ideas are intrinsically particular was not only central to his critique of dogmatic metaphysics and excessive scepticism, it was also in complete conflict with what had long been the received view among philosophers and mathematicians regarding the nature and origin of the most basic and straightforward ideas of mathematics. Nowhere was this more painfully obvious than in the case of geometry. As we have seen, Hume himself was all too aware that the received view had long held it to be self-evident that the ideas which are most essential to geometry are of such an abstract and general nature that they could not possibly have their origin in the empirical faculties of sense and imagination. Hume had no choice but to throw down the gauntlet and prepare to do battle. He had to show that there are no serious philosophical obstacles standing in the way of interpreting the fundamentals of geometry in accordance with Berkeley's doctrine of abstract ideas.

Geometry, of course, is a human creation and the mode of thinking which is essential to it is carved out by human minds. Any philosophy which finds itself unable to do justice to the mode of thought in which human beings engage when they are actually doing geometry (as opposed to theorizing about doing geometry) is fundamentally defective. The task confronting Hume, then, is to show that his first principles are capable of doing justice to this particular abstract mode of thinking. Given the historical climate in which Hume was writing and the hostile reception that awaited any attempt at an empiricist analysis of the ideas of geometry, it is hardly surprising that the first lengthy discussion of the *Treatise* should represent a serious attempt at doing precisely that. It is to this that we now turn.

Chapter 2

Notes

1). John P. Wright, The Sceptical Realism of David Hume, (Minneapolis: Univerity of Minnesota Press, 1983), p.4

2). Richard Popkin, "Pierre Bayle," Encyclopedia of Philosophy.(1967; rpt. New York: Macmillan Publishing Company, 1972).

3). Gottfried Leibniz, "On What is Independent of Sense and of Matter," Gottfried Leibniz: Philosophical Letters and Papers, ed. Leroy E. Loemker, 2nd ed., (Boston: D. Reidel Publishing Company, 1976), p.549

4). Gottfried Leibniz, "Reply to the Thoughts on the System of Preestablished Harmony contained in the Second Edition of Mr. Bayle's Critical Dictionary, Article Rorarius," Philosophical Papers and Letters, p.580

5). Gottfried Leibniz, "On What is Independent of Thought and Matter," p.551

6). ibid., P.551

7). ibid., p.551

8). ibid., p.551

9). ibid., p.548

10). Rosemary Newman, "Hume on Space and Geometry," Hume Studies, VII, No.1 (1981), p.20

11). Immanuel Kant, Metaphysical Foundations of Natural Science, trans. James Ellington (New York: The Bobbs-Merrill Co., 1970), p.53 Kant too was of the opinion that in reflecting on the controversy surrounding the principle of infinite divisibility, particularly as it related to the divisibility of matter, the philosopher "ventures into a labyrinth out of which it becomes difficult for him to find his way, even in questions immediately concerning him." (ibid., p.53) However, Kant, like Leibniz, saw no option but to forge ahead and find a path through this labyrinth. To attempt to block the entrance way into this maze of difficulties by banishing the principle of infinite divisibilty from geometry was as unthinkable for Kant as it was for Leibniz.

12). "Hume: Dialogue with John Passmore," The Great Philosophers, ed. Bryan Magee, (Oxford: Oxford University Press, 1988), p.156

13). ibid., p.156

14). ibid., p.157

15). George S. Pappas, "Abstract General Ideas in Hume," Hume Studies, XV, No.2 (1989), p.339. Michael

Williams also draws our attention to the fact that "Hume's views about abstract ideas have not loomed large in recent critical discussion of his theory of the understanding" in spite of the fact that "it is clear that Hume himself sets great store by them." (See, Michael Williams, "Hume's Criterion of Significance," Canadian Journal of Philosophy, vol., 15, No. 2 (June 1985), p.291).

16). ibid., p.350

17). ibid., p.350

18). ibid., p.339

19). ibid., p.339

Chapter 3

The Position of the *Treatise* : An Overview

I

Introduction

Until the latter half of the nineteenth century the conventional wisdom among philosophers and mathematicians had been that the most basic propositions of Euclidean geometry (with the possible exception of the parallel axiom) were self-evident a priori truths about physical space; that is, the space in which planets, stars, and animals exist and move around. Physical space thus construed is the space of ordinary experience. The philosophical implications of this belief were far reaching. Understandably, the self-evident nature of these propositions was taken as immediately ruling out the possibility of developing an acceptable philosophy of space within the framework of empiricism. It was taken as patently obvious that geometry could not be the source of any real certainty regarding the essential structure of space if its most basic principles and ideas were derived solely from sense experience. The thesis here, of course, is that if geometry is construed as being grounded upon a purely empirical idea or doctrine of space then the axioms would have to be construed as being empirical propositions. So interpreted, the axioms of Euclidean geometry, taken in their full universality, could not lay any legitimate claim to being self-evident propositions about the world. In trying to account for the acclaimed intuitive certainty of the axioms, the more rationally-minded philosophers were led to conclude that the ideas which are most essential to geometry contain an intelligible, as well as a sensible, element. According to this view, our geometrical ideas contain an element which is intrinsically general in the mind's conception of it and apprehended through an intellectual mode of intuition.

Hume, of course, was well aware of the insurmountable obstacle the supposed self-evident certainty of the axioms was seen as posing for the philosophy of empiricism, and we would naturally expect him to have confronted this issue head-on. As we shall see, this is indeed one of the principal objectives of Hume's discussion of geometry in the Treatise. However, it is important that we take due heed of what lies before us and proceed with the appropriate caution. There is serious disagreement among commentators as to where Hume stood, in this most notorious part of the Treatise, with respect even to the most basic and epistemologically relevant issues in the philosophy of geometry of his day. For example, Wright expresses the general consensus among commentators when he states that in the Treatise Hume argued that the "fundamental propositions of geometry are merely contingent,"[1] just the view we would expect an empiricist to hold. But others, such as Atkinson, Newman, and Steiner, argue that it is much closer to the truth to say that Hume, like Kant, held the fundamental propositions of Euclidean geometry to be synthetic a priori.

The first challenge we must face is to become clear just where Hume stood regarding the epistemological status of the axioms of Euclidean geometry. Newman is not far off when she states that "the task of interpretation here is made difficult by the paucity of detailed comment by Hume."[2] However, our task is by no means as difficult as we might be led to think. By reading Hume's comments in the light of our discussion in the previous chapter and keeping in mind his strong commitment to Berkeley's doctrine of abstract ideas and his opposition to the doctrine of infinite divisibility, we shall be able to construct for ourselves an interpretation which is consistent with the spirit of the Treatise and does justice to Hume's philosophical acumen.

It is important that we bear in mind throughout our inquiry that the axioms of Euclidean geometry were traditionally understood as being universal and self-evident propositions about the space of everyday experience. We must be careful not to interpret the traditional view in light of certain twentieth century distinctions between mathematical and physical space or between pure and physical geometry.

According to the traditional view, the abstract idea of space which is so essential to geometry is derived from our everyday conception of space. Indeed, Hume himself believed that the abstract idea of space which is so essential to geometry has its origins in everyday sense experience. There would seem, therefore, to be only two general lines of argument open to Hume. He can argue against the traditional view and maintain that the axioms are not universal and self-evident truths about that which we, in ordinary life, designate as 'space', or he can argue that they are such truths and that this poses no grave difficulty for his empiricism. The latter option would involve arguing that an idea of space which has its origin in the empirical faculties of sense and imagination alone can be the source of self-evident and universal truths about space. Our findings will suggest that in the *Treatise* Hume clearly embraced the former line of reasoning.

II

Precision, Certainty and the Axioms

We shall begin our study of the *Treatise* by considering some passages in which Hume plainly seems to be rejecting the received view of his day

> ...geometry, or the *art*, by which we fix proportions of figures; tho' it much excels, both in universality and exactness, the loose judgements of the senses and imagination; yet never attains a perfect precision and

> exactness. Its first principles are still drawn from the general appearance of the objects; and that appearance can never afford us any security, when we examine the prodigious minuteness of which nature is susceptible. Our ideas seem to give a perfect assurance, that no two right lines can have a common segment; but if we consider these ideas, we shall see that they always suppose a sensible inclination of the two lines, and that where the angle they form is extremely small, we have no standard of a right line so precise as to assure us of the truth of this proposition. 'Tis the same case with most of the primary decisions of the mathematics. (T.70-71)

It was Hume's contention that geometry is "built on ideas which are not exact, and maxims, which are not precisely true" (T. 45). He thus maintained that when

> ...geometry decides anything concerning the proportion of quantity, we ought not to look for the utmost <u>precision</u> and exactness. None of its proofs extend so far. It takes the dimension and proportions of figures justly; but with some liberty. Its errors are never considerable; nor would it err at all, if it did not aspire to such an absolute perfection. (T. 45)

The general thesis Hume wished to put forward seems clear enough. Geometry is built upon ideas which are not exact and this lack of exactness keeps us from possessing full certainty with respect to the truth of even the most basic propositions of geometry, including those which appear to be perfectly self-evident. This point is further argued in a well-known passage in which Hume contrasts geometry with arithmetic and algebra.

> There remain, therefore, algebra and arithmetic as the only sciences, in which we can carry on a chain of reasoning to any degree of intricacy, and yet preserve a perfect exactness and certainty. We are possest of a precise standard, by which we can judge of the equality and proportion of numbers; and according as they correspond or not to that standard, we determine their relations, without any possibility of error. When two numbers are so combin'd as that the one has always an unite answering to every unite of the other, we pronounce them equal; and 'tis for want of such a standard of equality in extension, that geometry can scarce be esteem'd a perfect and infallible science. (T. 71)

We shall not concern ourselves with whether Hume can attribute perfect exactness and certainty to the propositions of arithmetic and still remain true to his empiricism. Suffice to say, the crucial question here is whether the idea of a numerical unit which is so essential to arithmetic can be derived from the senses and imagination alone. As far back as Plato, rationalist philosophers had answered this in the negative. However, we shall put this matter aside. Our interest is Hume's views regarding geometry, and the central message regarding this discipline comes across loud and clear. Geometry, we are told, is not, as was traditionally believed, an infallible science.

While this reading of Hume's comments may seem straightforward enough, Atkinson suggests that it rests on a confusion. He would dismiss our reading of the above passage as resulting from a failure to appreciate the difference between denying that a proposition is precise and exact and denying that it is certain and necessary. As Atkinson sees it, Hume has put most of his emphasis "on the imperfect precision and exactness of geometry."[3] He interprets the passage presently under discussion as asserting that what

> ...geometry lacks is not certainty and necessity, but precision and exactness. Hume's idea seems simply to be that whilst a quick look will give us all the assurance we can possibly have that two (small) collections are equal in number, a quick look will assure us only that two geometrical figures are _roughly_ equal in area, a longer look will enable us to make a more exact judgement, but however long we look, whatever procedures of juxtaposition, etc., we may carry out in thought or in fact, we shall only make our judgements more, never completely, exact.[4]

It is Atkinson's contention that the difference Hume sees between arithmetic and geometry is not that the former is certain and infallible while the latter is not. Rather, it is that the propositions of arithmetic are both certain and exact, whereas the propositions of geometry cannot be both certain and exact. According to Atkinson, Hume's position amounts to saying that if "geometrical propositions are construed as precise they lack full certainty, if certain they lack full precision."[5] There is an important element of truth in what Atkinson has to say here. However, this element gets somewhat distorted, and as a result Atkinson ends up giving a rather misleading interpretation of Hume's position in the _Treatise_.

Consider, for example, that in the paragraph immediately following Hume's comparison of arithmetic and geometry, a paragraph which Atkinson ignores, Hume tells us that geometry "falls short of that precision and certainty, which are peculiar to arithmetic and algebra" (T. 71). He tells us that the reason he imputes " any defect to geometry is, because its original and fundamental principles are derived merely from appearances" (T.71). He goes on to add that "this defect so far attends it, as to keep it from ever aspiring to a full certainty" (T. 71). It would certainly appear, then, that in some philosophically important sense Hume wished to deny to geometry the full certainty, as well as the perfect exactness, of arithmetic and algebra.

Lest we start to get too far ahead of ourselves, let us go back and spell out more fully what the connection is between Hume's belief that our geometrical ideas are lacking in perfect precision and exactness and his belief that geometry cannot aspire to the full certainty it was commonly thought to possess. It is here that we shall be able to make clear the basics of our differences with Atkinson as well as appreciate the important element of truth in his thesis. The first point to consider is that, as Atkinson has noted, Hume repeatedly emphasized in the Treatise that none of our geometrical ideas are perfectly precise and exact. The question we must ask is, why did Hume find it necessary to emphasize this point so often? What was the motivation behind this?

To answer these questions, let us first recall that Hume's copy principle brought him into sharp conflict with the conventional wisdom of his day regarding the nature and origin of our geometrical ideas. The conventional wisdom, as we have seen, maintained that the ideas which are most essential to geometry "are of so refined and spiritual a nature, that they fall not under the conception of the fancy, but must be comprehended by a pure and intellectual view, of which the superior faculties of the soul are alone capable." In other words, the conventional wisdom held it to be self-evident that the ideas of geometry can be comprehended in a manner which is both clear and exact (or, as the rationalists would say, 'clear and distinct'). To repeat a previous example, it was held to be self-evident that the mind, taken in its common situation, can frame for itself the idea of a straight line which is so clear and exact (distinct) that it cannot be conceived as containing within itself the slightest degree of curvature. The rationalists would have certainly challenged Hume to show them an impression of a line which presents itself in so clear and exact a manner that it cannot be conceived as containing even the slightest possible degree of curvature.

Hume agreed with the rationalists that the empirical faculties of sense and imagination were not in themselves capable of rendering the impression, or corresponding idea, of a straight line so precise and exact that it could never be conceived as containing the least degree of curvature. However, he was not about to follow in the footsteps of the rationalist and their mathematician allies and allow that some superior faculty of the soul could come to our aid and enable us to render our geometrical ideas perfectly precise. He saw no option but to take strong exception to the conventional wisdom and insist

> ... that the ideas which are most essential to geometry, viz. those of equality and inequality, of a right line and a plain surface, are far from being exact and determinate, according to our common method of conceiving them. Not only we are incapable of telling, if the case be in any degree doubtful, when such particular figures are equal; when such a line is a right one, and such a surface a plain one; but we can form no idea of that proportion, or these figures, which is firm and invariable ... In vain shou'd we have recourse to the common topic, and employ the supposition of a deity, whose omnipotence may enable him to form a perfect geometrical figure, and describe a right line without any curve or inflexion. As the ultimate standard of these figures is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of; since the true perfection of any thing consists in its conformity to its standard. (T.50-51)

Needless to say, it was only because mathematicians and philosophers understood the most basic propositions of geometry in terms of ideas which they regarded as clear and exact that they came to receive them, in their full universality, as self-evident certainties. Given what they took to be an intuitively straightforward idea of perfect or absolute straightness, these thinkers maintained that it was immediately self-evident that no two straight lines, no matter how minute their angle of inclination, could be made to share a common segment. In their estimation, anyone who cared to reflect upon this idea would readily see that it is absurd to deny the truth of this proposition.

Hume, however, remained committed to the view that the ideas most essential to geometry are really particular in the mind's conception of them and thus lack the precision and exactness required to confer full certainty on the propositions of geometry. Given the profound and far-reaching implications of the issue at hand, Hume proceeded to spell out his opposition to the conventional wisdom.

> Now since these ideas are so loose and uncertain, I wou'd fain ask any mathematician what infallible assurance he has, not only of the more intricate and obscure propositions of his science, but of the most vulgar and obvious principles? How can he prove to me, for instance, that two right lines cannot have one common segment? Or that 'tis impossible to draw more than one right line betwixt any two points? Shou'd he tell me, that these opinions are obviously absurd, and repugnant to our clear ideas; I wou'd answer, that I do not deny, where two right lines incline upon each other with a sensible angle, but 'tis absurd to imagine them to have a common segment. But supposing these two lines to approach at the rate of an inch in twenty leagues, I perceive no absurdity in asserting, that upon their contact they become one. For, I beseech you, by what rule or standard do you judge, when you assert, that the line, in which I have

> suppos'd then to concur, cannot make the same right line with those two, that form so small an angle betwixt them? ... The original standard of a right line is in reality nothing but a certain general appearance; and 'tis evident right lines may be made to concur with each other, and yet correspond to this standard, tho' corrected by all the means either practicable or imaginable (T. 51-52).

There can be no question but that Hume has rejected the traditional view concerning the certainty of the propositions of geometry. He has told us in no uncertain terms that owing to the inexactness of the ideas most essential to geometry, the mathematician can have no infallible assurance as to the truth even of those propositions which he took as most self-evident. He readily acknowledged, of course, that our ideas do indeed seem to give us a perfect assurance as to the truth of these propositions. After all, Hume could hardly have argued that it was out of blind stupidity that the most able mathematicians and philosophers were led to receive these propositions as truths whose self-evidence was beyond question. What he wished to argue, however, was that once we see through our natural propensity for confusion in this matter and recognize the inexactness and lack of precision inherent in these ideas, we immediately realize that we must place serious qualifications on the assurance under discussion, qualifications which have important and far-reaching epistemological implications.

We can readily appreciate the connection between

Hume's claim that the ideas of geometry are not perfectly precise and exact and his claim that even the most vulgar and obvious propositions of the discipline lack full certainty. We must not, however, brush aside too hastily the fact that Hume was willing to grant a necessity of sorts to these propositions so long as they were assigned a far more restricted interpretation than mathematicians were accustomed to giving them. Hume saw no problem in meeting the conventional wisdom part way and granting that it is absurd and repugnant to our clearest geometric ideas to suggest that straight lines which incline upon each other at a sensible angle can, upon meeting, become one line. He saw nothing epistemologically problematic in granting both certainty and necessity[6] to the axioms of geometry once they were interpreted within empirical limits. But contrary to what had for so long been the received view among philosophers, Hume was convinced that it was neither absurd nor contradictory to our clearest geometric ideas to assume that straight lines which incline upon each other at very minute angles may, upon meeting, become one. As we shall see, a failure to keep clearly in mind Hume's differences with the traditional view regarding the certainty and necessity of the propositions of geometry has been the source of some confusion, and has led to some misleading comparisons with Kant.

III

Precision, Certainty and Infinite Divisibility

In the "Appendix" of the Treatise Hume stated that if mathematicians employ the inaccurate standard of equality (and presumably, also that of a straight line) then "their first principles, tho' certain and infallible, are too coarse to afford any such subtle inferences as they commonly draw from them" (T.638). This might seem to support Atkinson's claim that Hume wished only to deny to geometry the perfect exactness and precision of arithmetic and thus to be at odds with our discussion in the preceding section. Though we shall not yet speak specifically in terms of Atkinson's position, we shall endeavour to show that Hume's remark can quite easily be rendered consistent with our reading of the Treatise thus far. In the process we shall gain a deeper understanding as to why Hume found it so necessary to emphasize the inexactness of the abstract ideas so essential to geometry.

The fact that Hume stated that he wished the passage containing these remarks to be inserted back into the main body of his discussion on geometry, immediately following the passage at T.51-52 which we have been discussing, makes it clear that Hume was not signalling a shift in his position. Most definitely, he was not suggesting that he is now of the opinion that we can know with certainty that no two straight lines can share a common segment. His position remained that the idea of straight line is only exact and precise enough to assure us that this proposition holds true within certain empirical limits. In general, it is safe to say that Hume continued to be of the opinion that if the mathematician is willing to interpret the axioms of geometry as holding only within certain sensible limits then the resulting propositions will indeed be self-evident truths. They will, of course, lack the precision and universality of the axioms of Euclidean geometry as they have been understood down through the ages.

To help avoid any confusion, let us designate the result of interpreting the propositions of Euclidean geometry in accordance with Berkeley's doctrine of abstract ideas and Hume's copy principle as 'geometry$_H$'. The first principles of geometry$_H$ (axioms$_H$) are interpreted as holding only within certain sensible limits as dictated by the degree of inexactness of their ideas and thus as lacking the full precision of the axioms of Euclidean geometry. In this sense, axioms$_H$ represent a watered-down version of the original axioms. On our reading, then, it was clearly the principles of geometry$_H$ that Hume had in mind at T.638 as being certain and infallible.

While there are obvious differences between the most basic propositions of Euclidean geometry and geometry$_H$ which are of theoretical and philosophical significance, Hume did not see any considerable difference between the two when judged from a practical level. At such a level, defined within certain sensible limits, the two geometries were judged to be virtually identical. When, for example, geometry$_H$ asserts that the sum of the angles of a triangle is equal to 180 degrees, it does so within a restricted domain of observation and measurement and in terms of an idea of equality that is not perfectly exact. But as far as the practical affairs of mankind were concerned, Hume saw this deviation from the traditional interpretation of the content of this theorem of Euclidean geometry as a mere subtlety which was of little practical relevance.

We now come to the central point which Hume wished to make in the "Appendix" and have inserted back into the main body of his discussion. Simply put, he wished to argue that if we interpret the principles of geometry in a manner which renders them truly self-evident, we arrive at propositions which are too coarse to support so subtle an inference as that concerning infinite divisibility. In the paragraph immediately following the insert from the "Appendix" we are told that there is no geometrical demonstration for the infinite divisibility of extension and that we must "regard all the mathematical arguments for infinite divisibility as utterly sophistical"(T.52). These arguments were considered sophistical because they, like so many of the arguments of metaphysics, were the result of that infectious error which deceives us into viewing our abstract ideas as if they were of a general nature and can be rendered precise and exact. Hume, in other words, was developing further his claim that none of the demonstrations of geometry

> ... can have sufficient weight to establish such a principle, as this of infinite divisibility; and that because with regard to such minute objects, they are not properly demonstrations, being built on ideas, which are not exact, and maxims, which are not precisely true... (T.44-45)

Given the philosophical importance which Hume attached to resolving the controversies surrounding the principle of infinite divisibility, it is apropos that we follow up on this connection and determine precisely how it ties in with our present discussion concerning the status of the axioms of geometry. Fortunately, the connection itself is quite straightforward. We have already seen that, unlike his rationalist counterparts, Hume did not believe that it was possible to resolve the difficulties surrounding the principle of infinite divisibility in a way which would allow this principle to remain a part of geometry, let alone an integral part. Rather, he believed that the principle itself had to be eliminated. However, from the mathematicians' point of view, this principle was so inseparably bound up with the other principles of geometry that it could not be banished without destroying the foundations of the discipline. The telling point here is that if we concede to the mathematician that the ideas of equality, straight line, etc. are so clear and determinate that they afford us a self-evident certainty of the truth of the axioms of Euclidean geometry as they were traditionally understood, he will have no difficulty demonstrating that the principle of infinite divisibility follows as an immediate consequence.

Fogelin notes that "since the mathematical proofs were well known to the writers of the seventeenth- and eighteenth-centuries, they often alluded to them in a general way without spelling them out in detail."[7] The philosophical controversy surrounding the principle of infinite divisibility was of such importance to the philosophers of this period that these proofs were collected in various works. Fogelin mentions two such works - The Port-Royal Logic and Isaac Barrow's Lectiones Mathematicae. With respect to the former work Fogelin maintains that "it seems reasonable that writers interested in this topic would be familiar with this important work."[8] While it would certainly seem reasonable to include Hume among those familiar with this work, Fogelin only explicitly claims that he was familiar with the latter.[9] We shall take a closer look at Hume's dealings with these proofs in chapter 6. For now, let us simply bear in mind that Hume was aware of the geometrical proofs in favour of the infinite divisibility of extension and that these proofs were themselves widely received as being as straightforward and incontestable as any in geometry.

Whatever critics may say of his mathematical talents, Hume was more than capable of understanding the geometric demonstrations in favour of infinite divisibility. Thus, he was not about to argue that the mathematician was mistaken in thinking that the principle of infinite divisibility follows demonstrably from the manner in which the ideas and axioms of geometry were traditionally understood. Nor, as we have seen, was he willing to follow the sceptic and see this as proof that even our most straightforward mathematical reasonings end in conclusions which are big with contradiction. Following in Berkeley's footsteps he reasoned that the source of the error in these demonstrations lay in a false doctrine of ideas which infected the mathematicians' thinking and led them to attribute a greater exactness and generality to their ideas than was warranted. So long as this error infected their thinking, the mathematicians would be lead by a seemingly straightforward and impeccable line of reasoning to the conclusion that extension is infinitely divisible.

Hume realized that if he were to concede that our geometric ideas are precise enough to assure us of the precise truth of the axioms then the mathematician would be able to lead him to the conclusion that extension is infinitely divisible. Given how central it was to Hume's philosophy that the principle of infinite divisibility be banished from geometry, we can readily appreciate why Hume repeatedly emphasized that the ideas of geometry are not as exact and precise as mathematicians and philosophers had taken them to be.

Needless to say, to the overwhelming majority of philosophers and mathematicians, including the noted philosopher-mathematicians Descartes and Leibniz, Hume's interpretation of the ideas and principles of geometry would have appeared utter nonsense. Denying the exact and general nature of our geometric ideas and the full certainty of the axioms would have struck these thinkers as being no less absurd and ridiculous than any of the paradoxes associated with the principle of infinite divisibility. Hume, after all, was denying something which these thinkers held to be so intuitively straightforward as to be absolutely certain to anyone who cared to give the matter a moments reflection. Furthermore, from their point of view Hume's analysis of the ideas and axioms of geometry amounted to nothing less than the immediate and complete destruction of geometry. In short, they would have objected that in trying to rid geometry of its most embarrassing paradoxes, Hume had in effect thrown out the baby with the bath and unwittingly given himself over to scepticism.

Hume, however, remained confident that once the initial shock of his account had worn off, his analysis would be seen in a more positive light. It would be seen, for example, that if we are prepared to water down the original principles of geometry and interpret them in light of certain sensible limits, we shall arrive at propositions which are as self-evident and certain as we could ever hope to attain. It would be further seen that these propositions can serve as the first principles of a geometry (i.e., geometry$_H$) which is, for most intents and purposes, virtually identical to the results of Euclidean geometry. As far as Hume could see, the only truly significant difference between the two geometries existed at the theoretical level. The embarrassing principle of infinite divisibility is replaced by a principle of finite divisibility, and this, Hume believed, was a powerful point in favour of his analysis.

IV

Atkinson's Thesis and Its Defenders

With this general overview of Hume's position before us, let us proceed to examine more closely Atkinson's thesis that the least objectionable way to classify Hume's position in the *Treatise* is to say that he, like Kant, held that the "propositions of geometry are synthetic *a priori*; i.e. necessarily true but to be established not by the analysis of concepts but by an appeal to intuition."[10] Clearly, we have no quarrel with the claim that Hume held the propositions of geometry to be synthetic; a point we shall discuss more fully in the next chapter. But what are we to make of the claim that Hume held the propositions of geometry to be necessary truths? If Atkinson merely means to suggest that Hume believed it was possible to interpret the most basic propositions of geometry in a manner which would render their falsity absurd and contradictory to what is most clearly exhibited in our geometric ideas and reasonings, we are in basic agreement. We can readily grant that Hume believed that if we interpret these propositions within certain sensible limits and thereby render them far less precise than they were traditionally understood to be, then we may hold them to be certain and necessary. In other words, if Atkinson wishes only to argue that Hume held the propositions of geometry$_H$ to be synthetic and certain, we have no reason to quarrel with him.

Since Atkinson interprets the position of the Treatise to be that "what geometry lacks is not certainty and necessity, but precision and exactness," this might well seem to be all that he wishes to claim. However, in that case it is rather misleading for him to bring Kant into the picture and associate his position regarding the synthetic a priori nature of the propositions of geometry with that of Hume. This move tends to suggest that Hume and Kant were speaking about the same geometrical propositions, and this, we have seen, is simply not so. Unlike Kant, Hume did not hold the traditional propositions of Euclidean geometry, not even the most "vulgar" and obvious ones, to be synthetic a priori. Epistemologically speaking, there is a world of difference between asserting that the propositions of Euclidean geometry are synthetic and necessary and asserting that the propositions of geometry$_H$ are synthetic and necessary and Atkinson does not appear to have taken proper notice of this. Consider, for example, Atkinson's claim that while both Hume and Kant held that the most basic propositions of geometry can only be established as true by an appeal to intuition, Kant is

> ...undoubtedly the more thorough and self-consistent in working out this view. He saw that if geometrical propositions were a priori then the intuition in question must be a pure intuition, and that space must be a pure (form of) intuition. Hume, on the other hand, is committed to the view that space and time are empirical ideas, and some of his observations... can be read as direct replies to some of Kant's arguments for the priority of space and time. Hume's "official" view of space and time is thus diametrically opposed to Kant's...[11]

Atkinson writes as if it were a straightforward matter that Hume's position regarding the certainty of the axioms requires that our idea of space be grounded in a pure, as opposed to an empirical (sensible), intuition. This would indeed be the case had Hume, like Kant, granted a priori necessity to the axioms as they were traditionally understood in their full exactness. Hume, however, was far too astute to have failed to realize this. Indeed, as we have argued, this is precisely why he rejected the traditional view regarding the certainty of the axioms of geometry. As we have seen, Hume was only too aware that if he assigned full certainty to the proposition that no two straight lines can, upon meeting, share a common segment, he would have had to ground this certainty in an idea of a straight line which is of so refined and spiritual a nature that it could only be "comprehended by a pure and intellectual view, of which the superior faculties of the soul are alone capable." Hume, in other words, was aware that he could not accept even the most "vulgar" and obvious propositions of the geometry of his day as being fully certain without being forced into embracing a doctrine of pure intuition.

What Hume accepted as being certain was the proposition that no two straight lines which approach each other at a sensible angle can, upon meeting, become one. He considered this proposition as certain in the sense that "where two right lines incline upon each other with a sensible angle, but 'tis absurd to imagine them to have a common segment" (T.51). Hume was convinced that this level of certainty did not presuppose any spiritual and refined idea of a straight line. He firmly believed that his empiricist epistemology could easily account for the origin of the certainty of geometry$_H$. What Atkinson needs to do is show that Hume is wrong on this point and that even here he needs to have recourse to a doctrine of pure intuition. This, however, is something he does not do.

Atkinson is not alone in comparing Hume's philosophy of geometry in the Treatise to that of Kant's. Newman and Steiner have argued along similar lines. It is Steiner's contention that "with respect to the characterization of mathematics as synthetic a priori knowledge, there is no difference between Kant and Hume."[12] However, having said this, he goes on to acknowledge that there is "a difference between them concerning exact truth of geometric statements."[13] According to Steiner:

> ...Hume's real doctrine is that the theorems of geometry are not strictly true, a point of view which antedates Einstein... Note, however, Hume is still entitled to his view... that mathematics is a priori. For even on his view... that geometry is not strictly true, the propositions of geometry could be rephrased in a way which would make them strictly true. One could simply say, as Einstein said: the sum of the angles of a triangle is closer and closer to 180 degrees, the smaller the triangle is. And this proposition is true, and - since it is arrived at by comparing ideas - a priori.[14]

Though Steiner acknowledges that there is a difference between Hume and Kant regarding the exact truth of the propositions of geometry, he does not follow through and discuss how profound and far reaching this difference is. He does not drive home the point that Hume's first principles forced him to out and out reject the traditional view regarding the general nature and exactness of the ideas most essential to the geometry of his day. Consequently, he does not pay due heed to the fact that Hume was strongly opposed to assigning certainty even to the most basic and "vulgar" propositions of Euclidean geometry. The belief that the most basic propositions of Euclidean geometry are a priori truths has played a most fundamental role in the history of philosophy and in Kant's philosophy in particular. Steiner passes over the fact that in the Treatise the only way Hume could accept the axioms of geometry as certain was to restrict significantly their precision and thus alter their meaning - so much so that mathematicians would have seen his interpretation as destroying the very foundations of geometry.

Steiner, we must acknowledge, has taken notice of the critically important passage at T.51 in which Hume challenged the mathematician regarding the certainty of even his most basic and straightforward propositions. But in commenting on this, he states only that Hume's position is that "the theorems of geometry are not strictly true." He makes no explicit reference to the axioms themselves and how radically different Kant's position regarding their epistemological status is from Hume's position in the Treatise. The reader may be inclined to dismiss this as somewhat of a carping criticism, thinking that this difference is surely implicit in Steiner's comments above. Let us, therefore, look more closely at his claim that though Hume denied that the theorems of geometry are strictly true, he is still entitled to the view that geometry is a priori. Hume, we are told, could rephrase the theorems in a way which would make them strictly true. Though Hume could not say that it is strictly true that the sum of the angles of any given triangle is equal to 180 degrees, Steiner believes that he could say that we can know for certain that the smaller the triangle is the closer and closer is the sum of its angles to 180 degrees.

We can readily understand why Hume could not allow the geometer to claim we can know with certainty that the original theorem of Euclidean geometry is strictly true. Firstly, the idea of equality which is so essential to geometry is not sufficiently precise and exact to render this theorem strictly true. Secondly, the demonstration which the geometer offers for this theorem presupposes the abstract idea of a perfectly straight line, and Hume insists that the mind cannot frame for itself any such idea. Now, if the abstract notion of perfect equality and perfect straightness are really fictions which must be banished from our geometrical reasonings, how can we claim to know with certainty that the smaller and smaller a triangle becomes, the closer and closer is the sum of its angles to 180 degrees?

Steiner must assume that Hume's position in the Treatise is consistent with the claim that we can know with certainty that as a triangle becomes smaller and smaller its sides approach the Euclidean ideal of perfect straight lines.

And yet, how can this possibly be the case? If we assume with Hume that the abstract idea of a straight line cannot even assure us that two straight lines can never enclose a space, how can it afford us any certainty regarding the limit which the sum of the angles of a triangle must approach as its sides get straighter and straighter? How can we know a priori that the limit is not slightly more or less than 180 degrees? The rationalists and Kant claimed that we can conceive a priori the clear and exact representation of the ideal limit toward which physical lines converge as they become straighter and straighter. This limit, they claimed, is the perfectly straight line of Euclidean geometry.

Hume, however, cannot allow for any such a priori conception. The most he can concede is that sound reasoning convinces us that there must be a limit which lines approach as they become straighter and straighter. But the idea which stands at the limit must itself be of a sensible nature, and thus be in basic accordance with the copy principle. Thus, it is Kant and the rationalists, not Hume, who can claim that we can know a priori that 180 degrees is the limit which the sum of the angles of a triangle approaches as its sides become straighter and straighter.

Steiner's interpretation is also at odds with Hume's claim concerning "the fallacy of geometrical demonstrations, when carry'd beyond a certain degree of minuteness" (T.53). The principles of geometry, Hume tells us, "can never afford us any security, when we examine the prodigious minuteness of which nature is susceptible" (T.71). On our reading of the Treatise the most Hume can claim to follow from the demonstration under discussion is that for triangles which fall with certain sensible limits, the sum of the angles is equal, more or less, to 180 degrees.

In fairness to Steiner, we must bear in mind that his is a fairly short article and that the main point he wishes to argue is that in the Prolegomena Kant misrepresented Hume's philosophy of mathematics - a misrepresentation which Steiner believes "has coloured all subsequent interpretations of the latter's work."[15] Kant, having taken notice of certain comments in the Enquiry, was led to believe that Hume held the propositions of mathematics, including those of geometry, to be a priori. However, Kant was also led by these comments to believe that Hume held that these propositions were grounded in the law of contradiction alone and were thus analytic. Steiner's argument is that Kant failed to realize that Hume, like himself, considered the a priori propositions of geometry to be synthetic. His point, then, is that Kant was correct to read Hume as arguing that the propositions of geometry are a priori but wrong to read him as arguing that these propositions are analytic.

Steiner is aware that his argument turns on a controversial point. Kant was referring to Hume's philosophy of geometry as it was presented in the Enquiry, and Steiner himself cites only from the Enquiry in support of his claim that Hume considered geometrical knowledge to be a priori. However, when seeking to establish that Hume held the propositions of geometry to be synthetic, he cites from the Treatise. The controversy here lies in the fact that the majority of Hume scholars are of the opinion that the philosophy of geometry set forth in the Enquiry contradicts that of the Treatise. Steiner's response is simply to state that he does not believe "that there is any contradiction between the philosophy of mathematics taught in the Treatise and that taught in the Enquiry."[16]

While this assumption is vital to Steiner's interpretation, we cannot discuss it here without getting too far ahead of ourselves. We shall take up Hume's position in the Enquiry in chapters 5 and 6 and for now simply note that our findings will strongly suggest that the philosophy of geometry presented in this later work represents a sharp and philosophically significant break from that of the Treatise. However, all this aside, we can still insist that whatever merits there may be to Steiner's position, it is very misleading to adopt the Kantian notion of the a priori when describing Hume's view of geometry in the Treatise. For Kant, necessity and strict universality were the sure criteria of all a priori knowledge, and he held these criteria to be inseparable from one another.[17] Kant held the proposition that we can construct one and only one straight line between two points to be an a priori truth because he held this proposition, in all its precision and universality, to be a necessary truth.

The fact that Hume may well have been willing to concede that this proposition can be rendered certain if it is interpreted in a manner which is sensitive to empirical limitations imposed by the inexactness of our faculties of sense and imagination, is all quite beside the point. The philosophically significant point which needs to be highlighted is that Hume flatly rejected that the axioms of geometry, as they were actually understood by mathematicians, were truths whose falsity was absurd and inconceivable. This most central part of Hume's philosophy of geometry is being buried by misleading comparisons with Kant, whatever element of truth there may otherwise be. The Kantian notion of a priori truth cannot be used to characterize a position which only accepts the propositions of geometry as holding true within certain empirical limits. Correctly or incorrectly, Kant assumed that Hume's position in the Enquiry was that we can know with perfect certainty that no two straight lines, no matter how minute their angle of inclination, can, upon contact, become one. It was only because he interpreted Hume thusly, that Kant took him to be of the opinion that the propositions of geometry are a priori truths.

Newman offers a more explicit and in depth defense of Atkinson's thesis. Like Atkinson (and Steiner), she maintains that Hume held that the propositions of geometry "are necessary synthetic truths"[18] and believes that in this regard he "approaches the position later assumed by Kant".[19] And like Atkinson, she maintains that the resemblance between the two is limited. Newman is quick to point out that though Hume believed that geometry is a body of synthetic necessary truth, he "makes no attempt to secure its necessary status with the Kantian formula."[20] For Hume, we are told, "geometry remains all along an empirical a posteriori science."[21] Needless to say, Newman is drawn to a conclusion similar to that of Atkinson's; namely, that Hume failed "to account for and secure the certainty of geometrical knowledge without relinquishing his belief in its empirical status".[22] However, Newman offers a defense of her interpretation that goes well beyond a mere echoing of Atkinson's arguments. She believes that implicit in Hume's writings on space and geometry are two very important distinctions - that between theoretical and practical geometry and that between physical and perceptual space.

> ...Whatever may be the position in the Enquiry, in the Treatise Hume cannot disregard the contrast between a theoretical geometry held to be exactly descriptive of physical space, and a practical geometry which holds good for perceptual (sensible)

> space but which may remain only inexactly descriptive of the properties of physical space. And it is with this contrast in mind that Hume's attribution of imperfection to geometry at T.45 and T.71 must be understood...[23]

On Newman's reading of the Treatise, Hume is seen as classifying practical geometry alongside arithmetic and algebra as being an object of knowledge and certainty. She argues that when Hume speaks of geometry as being an inexact mathematics, he is really contrasting practical geometry "with the theoretical possibility of an exact spatial arithmetic based on the concept of a mathematical point as a unit and employing definitions of its basic concepts."[24]

> ...Practical geometry is to be denied scientific status only when it is viewed against the background of a purely theoretical and ideal conception of geometry as exactly descriptive of physical space, where the nature of the space is only incompletely accessible to the senses. Such a theoretical geometry is for Hume an ideal which the imagination and reason working together, construct on the basis of the imagination's minimal idea of a part of extension...[25]

Newman's basic thesis is that Hume held the axioms of geometry to be synthetic necessary propositions when viewed as descriptive of perceptual space. The question is whether she is referring here to geometry as it was understood by the mathematicians of the day or to something along the lines of what we have designated as 'geometry$_H$'- which was really a program for re-interpreting the foundations of geometry in accordance with specific philosophical doctrines and not an actually existing branch of mathematics. If she means that Hume held something like the latter to be the geometry which is descriptive of perceptual space then she may well have a case. It would appear, however, that what Newman wishes to argue is that Hume, like Kant, took the actually existing geometry of the day (i.e. Euclidean geometry) as being a body of synthetic necessary propositions which are descriptive of perceptual space. Newman does not make any distinction between the actual geometry of the day, in its full exactness, and something along the lines of geometry$_H$. The only distinction she makes is between geometry as descriptive of physical space (theoretical geometry) and geometry as descriptive of perceptual space (practical geometry). On Newman's reading Hume only wished to deny to geometry the full precision and certainty of arithmetic when it is viewed as descriptive of physical space. The implication is that Hume accepted the conventional wisdom with respect to geometry being a body of precise and certain knowledge about what we in everyday life call 'space'.

To clarify matters, let us make clear in what this distinction between perceptual and physical space consists. Clearly, the distinction is not to be thought in terms of space as it is perceived and space as it exists in itself apart from any relation to sense perception. Hume could never allow for the idea of space as it exists in itself. Rather, the distinction is to be understood as follows. Perceptual space is the space we perceive in everyday life, the space in which trees and the distant planets and stars are perceived to exist. This perception we call 'space' is itself a complex perception. However, owing to the inexactness of the senses we do not perceive distinctly the simple impressions which actually constitute this complex perception. All that is given to us in sense experience is the united appearance. Physical space, then, is simply that order of simple impressions that is actually present to the mind, and perceptual space is the united appearance that we perceive owing to the inexactness of the senses.

On Hume's analysis, if we possessed a clear and distinct idea of space (extension), as the rationalists had maintained, then we would be able to represent to ourselves this order of minimal sensibles and construct for ourselves a spatial arithmetic - a geometry that would possess the same precision and exactness as arithmetic and algebra. Hume took it to be self-evident, though, that any such clear and distinct representation is far removed from the manner in which the idea of space actually presents itself to the mind. He believed that all that analysis can do is inform us that since our idea of space is a complex idea, it must consist in a certain order of minimal sensibles. He manifestly did not believe that the understanding could come to the aid of the senses and render this idea perfectly precise and exact. The most he would allow was that analysis can present us with a distant notion of an order of indivisibles which the senses, owing to their lack of exactness, represent as a united appearance. As far as Hume was concerned, this far and distant notion was useless and played no role in the foundation of geometry.

It follows that Hume would have classified Euclidean geometry as practical geometry and, on Newman's interpretation, was in basic agreement with the traditional view concerning the certainty of the propositions of Euclidean geometry. Like Atkinson, Newman sees Hume's differences with the traditional view to center primarily around the question of the nature of this necessity and whether it could be grounded in an empirical idea of space. Unfortunately, she makes no reference to the passage at T.51 in which Hume explicitly denied that the geometer can have any infallible assurance regarding even the most basic and straightforward principles of geometry. She would have no alternative here but to argue that what Hume meant to say was that the mathematician can have an infallible assurance that these principles are exactly true of perceptual, but not of physical, space.

This interpretation does not seem to hold up to close scrutiny. In the first place, if the distinctions between physical and perceptual space and practical and theoretical geometry were central to Hume's philosophy of geometry, we would assume that Hume would have made an explicit reference to them, especially when making the argument at T.51. More to the point, we can only wonder why Hume would challenge the mathematician and argue that he can have no infallible assurance that the principles of geometry are true of physical space when he knew full well that the mathematician did not conceive of space as an order of indivisible points nor geometry as a spatial arithmetic. Hume was fully aware that there "are few or no mathematicians who defend the hypothesis of indivisible points" (T.45). Perhaps the most glaring weakness in this interpretation becomes apparent when we consider again Hume's claim that the mathematician cannot know with certainty that no two straight lines can be made to concur, a point which he repeatedly emphasized.

> ...The original standard of a right line is in reality nothing but a certain general appearance; and 'tis evident right lines may be made to concur with each other, and yet correspond to this standard, tho' corrected by all the means practicable or imaginable. (T.52)

Can there be any question but that when Hume speaks of the possibility of two straight lines which incline upon each other at the rate of one inch in twenty leagues may, upon contact, become one, he is speaking about lines in perceptual space? These lines are clearly descriptive of perceptual space. Hume's point is that our standard of a straight line, a standard which is derived from the general appearance of things and not from some underlying order of indivisible points, is not so precise and exact as to afford us any certain assurance that these two lines will not, upon contact, become one - not even on the assumption that they continue to conform exactly to the standard throughout their extension. He explicitly denied that there is anything we can do to render this idea so precise and exact as to afford us any certainty concerning the truth of the proposition in question.

Newman, it would seem, would have us believe that Hume agreed that we can indeed know with full certainty and exactness that in perceptual space no two straight lines can be made to enclose a space. On our reading, Hume was only too aware that such knowledge presupposes an idea of a straight line which is too refined and precise to be in accordance with his first principle and Berkeley's doctrine of abstract ideas. Rather than belabour this point further, let us take the next step and examine the principal passage from the _Treatise_ which Atkinson and Newman believe supports their view that Hume, like Kant, held the axioms of Euclidean geometry to be necessary truths. Both view Hume's opening comments in Part III section I of the _Treatise_ as offering solid support for this interpretation. Hume's discussion here concerned what he regarded as the different kinds of philosophical relations: resemblance, identity, relations of time and place, proportion in quantity or number, degrees in any quality, contrariety, and causation. He divided these relations into two categories;

"into such as depend entirely upon ideas, which we compare together, and such as may be chang'd without any change in the ideas" (T.69). It was Hume's opinion that only the relations of resemblance, contrariety, degrees in quality, and proportions in quantity or number "which depending solely upon ideas, can be the objects of knowledge and certainty" (T.70). He maintained that the first three of these relations "are discoverable at first sight, and fall more properly under the province of intuition than demonstration" (T.70). He tells us, for example,

> ...tho' it be impossible to judge exactly of the degrees of any quality, such as colour, taste, heat, cold when the difference betwixt them is very small; yet 'tis easy to decide, that any of them is superior of inferior to another, when their difference is considerable. And this decision we always pronounce at first sight, without any enquiry or reasoning. (T.70)

As Atkinson observes[26], Hume goes on to state that we also "might at one view observe a superiority or inferiority betwixt any numbers, or figures; especially where the difference is very great and remarkable" (T.70). In all other cases involving proportions of quantity or number we are told that we "must settle the proportions with some liberty or proceed in a more <u>artificial</u> manner" (T.70). Atkinson interprets this last statement to mean that "we must employ mathematics, which comprises geometry, arithmetic and algebra"[27]. He takes it to be clear that in these passages "Hume regards mathematical propositions [including those of geometry] as necessary and a priori."[28] Newman draws a similar conclusion. Commenting on Hume's division of relations into two categories she writes,

> ...In as far as the division corresponds to any epistemological distinction currently recognized, it would seem to differentiate truths which are in some sense necessary from those which are contingent. From Hume's inclusion of geometry within the former category it emerges that he considers some type of necessity attends geometrical axioms and inferences. Just what kind of necessity this is remains unclear, bearing in mind his assertion that the first principles of this science are drawn from the senses.[29]

Newman argues that since "the axioms of geometry are said to be provided by the senses, any necessity belonging to them cannot be conceptual or analytic even in a broad sense".[30] She reasons that this necessity must be synthetic and that "the synthetic relation must itself be based upon an intuitive comparison of certain sensible ideas".[31] However, both Newman and Atkinson takes some highly questionable leaps in their arguments. To be sure, Hume includes geometrical relations within that category of relations which can be objects of knowledge and certainty. But he only states that relations concerning proportions in quantity or number can be objects of knowledge and certainty, not, as the conventional wisdom would have had it, that they always are. In an earlier passage, for example, Hume stated that though our decisions concerning the proportions of greater, less, and equal "be sometimes infallible, they are not always so, nor are judgments of this kind more exempt from doubt and error, than those on any other subject"(T.47). Hume maintained further (at T.49) that this point holds equally well for the judgments we make concerning curved and straight lines and then, to remove any question as to his intentions, went on to assert (at T.51) that we cannot have any infallible assurance regarding those supposedly self-evident judgments about straight lines that mathematicians had regarded as the first principles of geometry.

On our reading, Hume was far too astute to make the error which Atkinson and Newman seem to attribute to him; the error of believing he could grant a real certainty to the axioms of Euclidean geometry as descriptive of the space of ordinary experience without relinquishing his belief that the idea of space is derived from impressions and is thus particular in the mind's conception of it. There is nothing in the passages cited at T.70 to suggest otherwise.

V

Demonstrative Certainty and the Theorems

A critic might object that we have conveniently ignored the fact that immediately after dividing the seven philosophical relations into two classes, Hume went on to remark that " 'tis from the idea of a triangle, that we discover the relation of equality, which its three angles bear to two right ones; and this relation is invariable, as long as our idea remains the same" (T.69). He later went on to assert that

> ...the necessity which makes two times two equal to four, or three angles of a triangle equal to two right ones, lies only in an act of the understanding by which we consider and compare these ideas... (T.166)

At first glance it might seem that Hume is placing geometry and arithmetic on an equal footing. He appears to be saying that we can know with full certainty that one of the most basic and important theorems of Euclidean geometry is necessarily true, and this hardly squares with what we have been arguing. If anything, it tends to square with the reading originally endorsed by Atkinson and developed by Newman. How, after all, can Hume possibly claim to know that the above theorem of geometry is necessarily true unless he assumes that we can know with full certainty that the axioms of Euclidean geometry are true?

Hume's position would definitely prove problematic for us if he intended the above to assert that we can know with full certainty that the sum of the angles of any given triangle, no matter how small or large, equals exactly the sum of two right angles. But this is so radically at odds with his repeated emphasis upon the lack of precision and exactness of our geometric ideas that it is extremely doubtful that this could have been his intention. However, if Hume intended only to say that through an act of the understanding we can consider and compare our geometric ideas in a manner which enables us to know with certainty that the sum of the angles of those triangles which fall within the limits of observation and measurement is more or less equal to the sum of the angles of two right angles then it is by no means evident that the above passage poses any difficulty for us. Hume most certainly could not follow in the footsteps of the rationalists and allow that through an act of the understanding we can come to know with full certainty that the sum of the angles of any given triangle, no matter how large or small, is equal precisely to precisely the sum of two right angles. But he did believe that through an act of the understanding we can come to know that the corresponding proposition of geometry$_H$ is certain.

Hume has argued all along, even in the "Appendix", that because the first principles of geometry are founded on the senses and imagination alone, they can never possess any necessity and exactness beyond what these faculties alone can judge. If these faculties are not precise and exact enough to enable us to determine absolutely that no two straight lines can possibly concur then we cannot assign full certainty to the corresponding principle of geometry. However, Hume acknowledged that there is an important respect in which it may be said that geometry "much excels, both in universality and exactness, the loose judgments of the senses and imagination" (T.70-71). In order to obviate any difficulty the reader may have in understanding his claim "that tho' geometry falls short of that perfection and certainty, which are peculiar to arithmetic and algebra, yet it excels the imperfect judgments of our senses and imagination" (T. 71), Hume offered the following explanation.

> ...The reason I impute any defect to geometry, is, because its original and fundamental principles are deriv'd merely from appearances; and it may perhaps be imagin'd, that this defect must always attend it, and keep it from ever reaching a greater exactness in the comparison of objects or ideas, than what our eye or imagination is alone able to attain. I own that this defect so far attends it, as to keep it from ever aspiring to a full certainty: But since these fundamental principles depend on the easiest and least deceitful appearances, they bestow on their consequences a degree of exactness, of which these consequences are singly incapable. 'Tis impossible for the eye to determine the angles of a chiliagon to be equal to 1996 right angles, or make any conjecture, that approaches this proportion; but when it determines, that right lines cannot concur; that we cannot draw more than one right line between two points; its mistakes can never be of any consequence. And this is the nature and use of geometry, to run us up to such appearances, as, by reason of their simplicity, cannot lead us into any considerable error. (T.71-72).

Hume knew that the senses and imagination could not in themselves even begin to account for the degree of exactness and certainty which geometry assigns to its theorems. He knew full well that anyone who understood the proof that the sum of the angles of a chiliagon equals 1996 right angles (or, for that matter that the sum of the angles of a triangle equals two right angles) would readily realize that geometry assigns to this proposition a degree of exactness and certainty that rivals that of its most obvious and vulgar propositions. In order to explain how, in this instance, geometry is able to exceed by far the limits of what the senses and imagination are alone capable of achieving, Hume had to bring the understanding into the picture. In short, he had to allow that the understanding is here capable of coming to our aid and enabling us, in a sense, to move beyond the limits of these two faculties.

Hume's position seems straightforward enough. Through an act of the understanding, whereby we consider and compare our most straightforward and easily formed geometrical ideas, we discover that certain obvious propositions concerning these ideas cannot be denied without contradicting what is most clearly evident in these ideas. By considering and comparing our ideas in light of these principles, we discover that other, less obvious, propositions, such as that concerning the sum of the angles of a triangle, can be demonstrated. Hume saw no difficulty in granting that the mode of reasoning which geometers employed in demonstrating their theorems allowed them to bestow upon these propositions a far greater degree of exactness and certainty than can be derived from the senses and imagination alone. But he could not accept rationalists' contention that through an act of the understanding we may come to know with full certainty that no two straight lines can ever share a common segment. This would presuppose that the idea of a straight line contains more than what can be derived from sense impressions - an element which the understanding can render perfectly precise and exact.

To avoid any confusion, let us note that Hume is not suggesting that we can do nothing to render the most basic ideas of geometry more precise and exact than the first appearances from which they originally were derived. He granted, for example, that though our idea of a straight line is originally derived from a particular appearance, we can correct "the first appearance by a more accurate consideration, and by a comparison with some rule, of whose rectitude from repeated trials we have a greater assurance" (T.49). Nonetheless, he insisted that since the ultimate standard of a straight line "is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of" (T.51). The understanding may help us improve upon the loose judgments of the senses and imagination but it can never, Hume believed, transcend the imperfections inherent in these faculties and attain true precision and exactness. He saw no way to grant such a power to the understanding (or any other of the "higher" faculties of the mind) without abandoning his first principles and crossing over into rationalism.

Section VI

Summary and Conclusion

Hume was in agreement with the traditional view in that he accepted that any certainty and necessity we attribute to the propositions of geometry would have to have its origin in an act of the understanding by which we consider and compare our abstract geometric ideas. Hume also agreed that in order for the understanding to discover any necessary connections holding among these ideas which would render the propositions of Euclidean geometry fully certain, the ideas themselves would have to be of so refined and general a nature that they could not possibly have their origin in the faculties of sense and imagination. Since his empiricist epistemology would not allow him to accept the reality of abstract general ideas, he had no option save to take the radical step of out and out rejecting the traditional view regarding the full certainty of the propositions of geometry, including those whose certainty was considered self-evident and beyond any reasonable level of doubt.

We have seen that Hume was able to temper somewhat his seemingly radical departure from one of the most widely accepted and influential doctrines in the history of philosophy. While his empiricism would not allow him to accept the traditional view concerning the perfectly precise and exact nature of our geometric ideas, it did not prevent him from conceding that these abstract ideas are among our clearest and most easily formed. There is a sensible difference between a curve and a straight line, and though it cannot be rendered so perfectly precise and exact as to enable us to comprehend a distinct boundary between the two, it can, within limits, be clearly perceived. The clarity of this perceived difference is such that insofar as we are clearly imagining two lines as being straight, we cannot imagine them to concur or enclose a space. If we imagine two lines possessing a common segment or enclosing a space then we must imagine at least one of them as being curved. Within limits of what can be clearly sensed and imagined, Hume was willing to grant that it is inconceivable that the principles of geometry could be false.

For Hume, then, so long as we are willing to stay within the limits of what is clearly evident to the senses and imagination, we can know with certainty that no two straight lines can enclose a space or become one. However, as Hume was quick to realize, this is a far cry from what mathematicians and philosophers claimed we can know with certainty. These thinkers believed that the nature of their geometric ideas was such that they could abstract beyond the limits of what was clearly evident to the senses and imagination and, through an a priori act of the intellect (the so-called 'light of reason'), come to know that no two straight lines can, under any circumstances, be made to concur or enclose a space. It was Hume's conviction that if we properly analyze our ideas according to sound philosophical principles and do not allow ourselves to be seduced by the mind's propensity for generalization in its conceptions, we are led to conclude that there is no real ground for this claim of certainty. On Hume's analysis there is nothing absurd or contradictory in assuming that the propositions of Euclidean geometry might actually be false. It is not repugnant to our clear idea of a straight line to assume that two straight lines may, if extended far enough, enclose a space.

It was also necessary for Hume to reject the traditional view regarding the certainty of the axioms because of the position he adopted with respect to the controversial principle of infinite divisibility. He was convinced that the general position advanced by Berkeley was correct and that the principle of infinite divisibility was really a piece of sophistry masquerading as a principle of mathematics. If banishing it from geometry required Hume to take the view that the basic propositions of that science lack the precision and full certainty which philosophers and mathematicians attributed to them and took to be self-evident then so be it. So certain was Hume with respect to the fundamental soundness of Berkeley's approach to the paradoxes surrounding the principle in question, he was willing to assume that the mathematician, no less than the dogmatic metaphysician, had become the unwitting victim of a false doctrine of ideas which so infected his thinking that he became confused even regarding that in his subject which appeared most clear and incontestable. Banishing the principle of infinite divisibility went hand in hand with banishing the supposedly confused notions of perfect straightness and equality in extension and the false sense of precision and certainty they engendered.

The fact Hume was willing to concede that the propositions of geometry can be made certain if they are interpreted in a manner which is consistent with Berkeley's doctrine of abstract ideas and sensitive to the inexact nature of our empirical faculties, is not something to be glossed over. It represents an important element of Hume's philosophy of geometry. What he believed he was offering here was a program for re-interpreting the fundamentals of geometry which would banish the offending doctrine of infinite divisibility and yet preserve everything in the discipline which was of any practical consequence. He viewed the lack of precision and universality in the resulting geometry, geometry$_H$, as a small price to pay for securing the certainty of the geometrical mode of abstract reasoning against sceptical attack. However, we ought not allow the certainty which Hume was willing to attribute to the propositions of geometry$_H$ to lead us into any misleading comparisons with Kant or cause us to downplay the radical nature of his break from the traditional view concerning the precision and certainty of geometry.

Chapter 3

Notes

1). John P. Wright, The Sceptical Realism of David Hume, (Manchester: Manchester University Press, 1983), p. 99

2). Rosemary Newman, " Hume on Space and Geometry," p.20

3). R. H. Atkinson, " Hume on Mathematics," Philosophical Quarterly, no. 10 (1960), p.131

4). ibid., p.130

5). ibid., p.130 fn.7

6). For Hume, "necessity is something, that exists in the mind, not in objects"(T.165) or in their corresponding ideas. As we shall see in the next chapter, Hume opposed the rationalists' assumption that the necessity of a geometric proposition can have its origin in the analysis of an idea. For example, he would reject the possibility that an analysis of our idea of a straight line reveals that it is necessarily the case that no two straight lines can enclose a space. However, as is clear from the passage at T.51-2, Hume did believe that if we consider and compare our geometric ideas, and thereby go outside what is contained in any one of them, we can indeed discover that, within limits, it is absurd and contradictory to assume that two straight lines can enclose a space. The sense of contradiction here, of course, is not, as in the case of the rationalists, that of a formal contradiction. It is a contradiction in the sense that it contradicts what the mind clearly and readily perceives to be the case when it considers and compares its various ideas. In other words, while Hume can say that it is a contradiction to maintain that two straight lines which approach each other at a sensible angle can concur, he cannot say that it is a self-contradiction evident to anyone who comprehends what is contained in the idea of a straight line. The point to bear in mind here is that there is a definite sense in which it may be said that the certainty Hume was willing to attribute to the propositions of geometry, once they were suitably restricted, has its origin in their necessity.

7). Robert Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility," p.48

8). ibid., p.48

9). ibid., p.48

10). ibid., p.132

11). ibid., p.132

12). Mark Steiner, Kant's Misrepresentation of Hume's

Philosophy in the Prolegomena," Hume Studies, XIII, No.2 (1987), p.405

13). ibid., p.405
14). ibid., pp. 404-45
15). ibid., p.400
16). ibid., p.404
17). ibid., p.404
18). Rosemary Newman, p.26
19). ibid., p.26
20). ibid., p.26
21). ibid., p.26
22). ibid., p. 27
23). ibid., pp. 22-23
24). ibid., p.22
25). ibid., p.22-23
26). R. F. Atkinson, p.128
27). ibid., p.128
28). ibid., p.128
29). Rosemary Newman, p.24
30). ibid., p.25
31). ibid., p.25

Chapter 4

The Position of the Treatise: A Critical Evaluation

Introduction

Hume has offered a general program for interpreting the ideas and propositions of geometry in accordance with the logic of his empiricism. However, there is a large difference between presenting the general features of a program and demonstrating that it can actually be executed. While Hume was confident that the program we have identified as 'geometry$_H$' could be carried out and would preserve everything in Euclidean geometry which was of practical value, he himself made no moves in that direction. He did not, for example, actually show that we can replace the geometer's conception of a perfectly straight line with a less exact idea and still be able to demonstrate, within certain limits of precision, that the sum of the angles of a triangle is equal to 180 degrees. And it was far from obvious that any such demonstration could be carried out.

It is understandable that Hume would have left the task of dealing with such technical matters to those more skilled and talented in mathematics. However, given what is at issue, it will not suffice for Hume merely to present a general outline of a program for rendering geometry consistent with the demands of an empiricist epistemology. Rationalist philosophers had long argued that there were insurmountable philosophical, as well as mathematical, obstacles which stood in the way of carrying out any such program and their basic point of view continued to function as an integral part of the conventional wisdom of the seventeenth and eighteenth centuries. It was thus incumbent upon Hume to make a serious effort at coming to grips with the rationalists' position.

In the present chapter we shall endeavour to identify, develop, and critically assess that in the Treatise which can serve as a retort to the basic rationalist position (and hence, to the conventional wisdom of the day) with respect to the nature and origin of the ideas and principles most essential to geometry. We shall begin with a discussion of Hume's philosophy of geometry as it stands in relation to the rationalists' doctrine of clear and distinct ideas and then, in section III, consider it as it stands in relation to their doctrine of analytic truth. Since these twin doctrines were at the heart of the rationalists' philosophy of geometry and were diametrically opposed to the central teachings of the Treatise, we would naturally expect Hume to have made them the target of some of his sharpest criticisms. Unfortunately, he did not address these doctrines in the direct and explicit manner we would expect. As a consequence, this aspect of his philosophy of geometry has gone largely unnoticed. However, as we shall see, it will not prove too difficult to extract from Hume's writings criticisms which strike at the very heart of these historically significant doctrines.

In order to complete his critique of the rationalists and their mathematician allies and pave the way for a proper and convincing defense of his own controversial analysis, Hume must explain how it is that these otherwise astute and critical thinkers came to deceive themselves into believing that the ideas which are most essential to geometry can easily be rendered exact and determinate. All along Hume has assumed that he could explain away this pervasive belief in terms of a confusion which naturally infects our thinking, and at some point he must supply that explanation. To his credit, he sought to do precisely that. In section IV we shall attempt to spell out and assess critically the essentials of his argument. It is here that we shall discover a weak link in Hume's chain of reasoning, a weakness which seriously threatens to undermine some of the most important teachings of the Treatise with respect to the nature of our geometric ideas.

II

A Critique of the Doctrine of Clear and Distinct Ideas

The doctrine of clear and distinct geometric ideas played an absolutely central role in the rationalist philosophies of the seventeenth and eighteenth centuries. Though Hume made no explicit reference to this important doctrine, it was clearly what he had in mind when he criticized mathematicians for thinking "that those ideas, which are their objects, are of so refin'd and spiritual a nature, that they ... must be comprehended by a pure intellectual view, of which the superior faculties of the soul are alone capable" (T.72). In assuming that the ideas of geometry are not perfectly precise, exact, and determinate Hume was, in essence, rejecting the rationalist claim that these ideas are distinct. In the previous chapter we worked through the general implications of this assumption for Hume's philosophy of geometry. In order to take the next step and subject Hume's position to a critical evaluation, we need to spell out in more detail Hume's differences with the rationalists. We shall begin with Hume's observation that

> ...Nothing is more apparent to the senses, than the distinction betwixt a curve and a right line; nor are there any ideas we more easily form than the ideas of these objects. But however easily we may form these ideas, 'tis impossible to produce any definition of them, which will fix precise boundaries betwixt them... (T.49)

In allowing that the difference between a straight line and a curve is readily apparent to the senses, Hume was conceding to the rationalists that the corresponding geometric ideas are indeed clear. To say that these ideas are clear is to say that we can readily recognize instances of them. Since we, in ordinary life, can observe instances where it is plain that a particular line looks straight or looks curved, Hume accepted the clearness of the corresponding ideas as incontestable. However, to go further and maintain that these ideas are perfectly exact or distinct, we would have to hold that everything in them is so clear and evident that we can readily recognize a precise boundary between them. We would not only be able to recognize a clear difference between lines which look straight and look curved but we would also be able to recognize, if only at a conceptual level, a clear difference between lines which merely appear to the senses as straight and lines which are truly straight. We would be able to frame for ourselves the idea of a line which is so perfect that we clearly recognize it as being without the slightest degree of curvature.

Understandably, Hume believed he was on safe ground in denying that we can render the difference between a straight line and a curve so clear and precise as to be able to recognize a sharp boundary between them. After all, what could be more obvious than Hume's claim that these appearances run insensibly into one another and that there are plenty of instances where we are incapable of telling, by use of the senses alone, if a particular line is best classified as being straight or curved? And even though we can use art and instrument to aid us in determining more precisely the boundary between a curve and a straight line, is it any less obvious that there is nothing we can do to overcome completely the defect inherent in our sense perceptions and describe a clear and perfectly precise boundary separating these two ideas? Employing the language of the rationalists, we would characterize Hume's position as being that our original idea of a straight line is clear but somewhat confused and that though we can strive to render this idea more and more exact and determinate, we can never render it perfectly so (i.e. distinct)[1].

Since the rationalists accepted the basic empiricist position with respect to the nature of sense perception, they readily agreed that there is nothing we can do to render our sensible ideas distinct. However, because they held it to be self-evident that our geometric ideas present themselves to the mind in a manner which is both clear and distinct, they postulated a nonsensible element which the mind, <u>taken in its common situation</u>, can readily intuit. This aspect of rationalism was nicely illustrated by Plato in the <u>Meno</u>. In this dialogue Socrates sought to demonstrate that a slave-boy, who had not received any formal education and was totally unschooled in mathematics, was quite capable of apprehending the most basic ideas and truths of geometry once he was made to reflect on them. Leibniz regarded this as a most beautiful illustration himself said of these truths that "one can lead a child to them by simple questions in the Socratic manner, without telling him anything: and without having him experiment at all about the truth of what is asked him."[2] This led Leibniz, and the rationalists in general, to conclude that "there is a light which is born with us,"[3] the so-called 'light of reason', through which we can readily intuit that which is intelligible and distinct in our geometric ideas and apprehend the certainty of those propositions which are most basic to geometry.

What Hume needs to show is that in so far as we are concerned with presenting an analysis of that which is implicit in our everyday thinking with respect to the ideas and propositions of geometry, the doctrine of clear and distinct ideas is a non-starter. The elements for constructing such an argument are suggested in the following.

> ...When we draw lines upon paper or any continu'd surface, there is a certain order, by which the lines run along from one point to another, that they may produce the entire impression of a curve or a right line; but this order is perfectly unknown, and nothing is observ'd but the united appearance... (T.49).

Contrary to the Cartesians, Hume held that the unity we ascribe to extension is a collective, and not a real or substantial, unity. He believed that what analysis reveals is that the idea of extension is itself a complex idea - the image of a finite number of indivisible points situated in a particular manner. However, because these individual points, which alone possess real unity, are crowded together and cannot be discerned by the senses, we perceive only the united appearance. Accordingly, our natural tendency is to ascribe a unity to the complex (i.e. the collection of individual points) and view it as a single thing[4]. But this unity, Hume argues, is a mere fiction. It follows that if we were capable of intuiting a priori that which is truly exact and precise in our idea of a straight line, we would intuit that order of points which underlies the "united" appearance of a straight line. We would be able to comprehend clearly and distinctly that order of points which constitutes the essence of our geometric idea of <u>straight line</u> and be able to draw a clear and precise boundary between it and the idea of <u>curve</u>.

Naturally, Hume held that the faculties of sense and imagination are too gross to afford us a clear and distinct idea of any such order of points, and he insisted that there is nothing the understanding can do to overcome this defect completely. While rational analysis can reveal to us that our idea of a straight line is really the idea of a particular order of indivisible points, it can do nothing to render this idea clear and exact and allow us to comprehend that order which constitutes its essence. Thus, this idea of an order of indivisible impressions is a far and distant notion and is clearly "not the standard from which we form the idea of a right line" (T.52). This complex order presents itself to us as a united appearance, and it is this appearance, with its lack of precision and exactness, which serves as our original standard of straightness. And as Hume has argued, there is no absurdity in imagining that two lines may conform perfectly to this standard, or any improved version thereof, and still, if extended far enough, enclose a space.

Again, we can bring Hume's position into contrast with that of Leibniz. Like Hume, Leibniz was critical of the Cartesian view that extension is itself something substantial, and argued instead that it must be conceived in terms of a certain order of indivisible points. For both Hume and Leibniz, the idea of unity which underlies the appearance of extension is the idea of an indivisible (unextended) point. However, unlike Hume, Leibniz conceived of these points as logical, not perceptual, simples. These simples were taken to be simple immaterial substances (monads) and thus were not to be thought of as being spatially situated (i.e., as being near or far apart[5]). Within the framework of Leibniz's metaphysics, the idea of a perfect straight line would have to be analyzed in terms of a logical (internal) order of simple substances.

This order represented the distinct intelligible element which Leibniz believed formed an essential part of our everyday idea of a straight line.

Even with so brief a sketch before us, we can easily conjecture how Hume would have responded to this rationalist doctrine. No doubt, he would have argued that this idea of a logical order of simple substances is even further removed from the common understanding of mankind than is the order of simple impressions to which he referred. He would have dismissed as absurd the suggestion that the idea of so abstract and obscure an ordering is easily intuited by the vulgar and mathematician alike and is the reason they held the basic propositions of geometry to be certain. From an epistemological point of view, it is clear that for whatever reason the common understanding of mankind is induced to ascribe certainty to the more obvious propositions of Euclidean geometry, it is not because of any clear and distinct understanding regarding a logical order of simple substances.

In a later part of Book I of the _Treatise_, Hume inquired into the causes which induce us to believe in the existence of an external world. He believed that "whoever wou'd explain the origin of the _common_ opinion concerning the continu'd and distinct existence of body, must take the mind in its _common_ situation" (T.213) and "must entirely conform [themselves] to [the vulgar's] manner of thinking and expressing themselves" (T. 202). The same point holds equally well here. The rationalists were convinced that the certainty which the common understanding attributes to the more obvious propositions is real, and not a mere figment of the imagination. This was one of the cornerstones of their philosophy of geometry, and it required an explanation as to how the "vulgar" could come to possess such knowledge. This entailed explaining how it is possible for the mind, taken in its common situation, to conceive for itself, at a most basic and intuitive level, the clear and distinct ideas of straight line, equality, etc. and how, when it reflects upon these ideas, it can readily intuit that certain universal and precise propositions about space are true beyond all possible doubt.

Now the rationalists may wish to argue that this is possible only if one assumes that our geometrical ideas contain an intelligible element and that the mind possesses a faculty of non-sensible intuition through which it comprehends clearly and distinctly the essence of this element. However, it would be of little value to postulate an intelligible element which is of such a nature that it satisfies all the purely logical demands which are placed upon it but which is far removed from the common understanding of mankind. Such an element could never be the source of the certainty which the common understanding was thought to discover and would be without any epistemological significance or justification.

It is unfortunate that Hume did not focus more sharply on the rationalists' doctrine of ideas and the unique nature they assigned to the ideas of mathematics. If he had, he could have exposed a fundamental weakness in their analysis. We may recall from our discussion in chapter 2 that the rationalists viewed the mathematical ideas as occupying a middle ground between the empirical and metaphysical ideas. Like the metaphysical ideas, they were thought to contain an intelligible element which enabled them to be rendered distinct. Like the empirical ideas, they were thought to contain a sensible element which enabled them to be imaged. The crucial point here is that the intelligible and sensible elements were conceived as being homogeneous. This was regarded as the reason why the basic abstract ideas of mathematics are so much more accessible and comprehensible to our everyday mode of thinking than those of metaphysics. The sensible appearance we call 'straight line' was conceived as more or less resembling the pure form of a straight line in general. The straighter the line the more closely it resembled or approximated the form. From the point of view of our everyday manner of thinking, this was certainly the most natural way to view the relationship and, not surprisingly, was the way mathematicians conceived it. Kant took particular notice of this point.

> ...the empirical concept of a <u>plate</u> is homogeneous with the pure geometrical concept

> of a circle. The roundness which is thought in the latter can be intuited in the former.
>
> But the pure concepts of understanding being quite heterogeneous with empirical intuitions, and indeed from all sensible intuitions, can never be met in any intuition.[6]

The dilemma facing the rationalists is plain to see. How can the pure form of a straight line or a circle be any more homogeneous with a sensible appearance than can the pure form of causality or substance? In both cases our awareness of the form is grounded in an intellectual mode of intuition, a mode of intuition which is completely distinct from the sensible mode. If the intelligible object and the sensible object are given through two distinct and heterogeneous modes of intuition then these two objects must themselves be heterogeneous. If that which is distinct and determinate in our idea of a straight line is apprehended through an intellectual intuition then it cannot resemble that in our idea which is apprehended through a sensible perception. No matter how easy we assume it to be for the mind to intuit that which is distinct in our idea of a straight line, the fact that this distinct element is of an intelligible nature means that it cannot bear the least resemblance to anything given to us through the senses.

Consider again the logical order of simple indivisible substances which Leibniz believed constituted the form of a straight line. It is difficult enough to comprehend in what this non-spatial internal order consists, let alone understand how it can resemble, or be in any way homogeneous with, some particular spatial shape. Indeed, it is difficult to see how this logical form can be any more homogeneous with the spatial shape we, in everyday life, call 'straight line' than with the spatial shape we call 'circle' or 'triangle'. In order to explain how it is that ideas which have their origin a priori in the intellect can have application to the sensible world and be the source of necessary knowledge, Leibniz postulated a doctrine of pre-established harmony. According to this doctrine, God has placed the sensible and intellectual realms in such harmony that what we must necessarily think through the latter we find exhibited, if only confusedly and after some effort, in the former. Whatever may be said in favour of this doctrine, it provides nothing by way of explaining how the intelligible and sensible form of a straight line can be so harmonized and united together that the latter may be seen as an approximate image of the former. Instead of a single idea of a straight line containing an intelligible and sensible element, we end up with two completely separate ideas of a straight line. Mathematical thinking thus becomes totally divorced from our everyday mode of thinking, and we are left without an account of the full certainty which the common understanding was assumed to have discovered.

Had Hume pursued this line of criticism and carried it to its completion, he would have been in a position to reveal a fundamental weakness in the conventional wisdom of his day and place his own highly controversial analysis in a more favourable light. He could have certainly made a strong case that the abstract idea of a straight line which is essential to geometry is, according to our common mode of conceiving it, homogeneous with those sensible appearances which we, in everyday life, call 'straight lines.' He could have made an equally strong argument that the only way this relationship of resemblance can exist between the idea and its object is if both have their origin in the same mode of intuition. Since the appearance is obviously grounded in a sensible mode of intuition, it then follows that so must the idea. It would have hardly been considered ridiculous to argue that a sensible idea of a straight line, no matter how clear and easy it is for the mind to conceive, can never be rendered perfectly distinct. Indeed, the rationalists themselves took this to be a truism.

III

A Critique of the Rationalists' Doctrine of Analytic Truth

The doctrine of clear and distinct ideas was intended to function as an integral part of a rational justification for the widely held belief in the full certainty of the propositions of geometry. Based on what they assumed to be a clear understanding of that which is distinct in our idea of a straight line, the rationalists maintained that it can be seen to be a self-contradiction to assert that two straight lines can enclose a space or concur or that there exists a shorter distance between two points than a straight line. These propositions were held to be linguistic expressions (or definitions) of that which we readily intuit as clear and distinct in our geometric ideas. In short, the rationalists maintained that the certainty which the common understanding assigns to these propositions is grounded in the fact that they are analytic truths. Hume was a bit more direct in arguing against this aspect of the rationalist philosophy of geometry.

> ...'Tis true, mathematicians pretend they give an exact definition of a right line, when they say, _it is the shortest way betwixt two points_. But in the first place, I observe, that this is more properly the discovery of one of the properties of a right line, than a just definition of it. For I ask anyone, if upon the mention of a right line he thinks immediately on such a particular appearance, and if 'tis not by accident only that he considers this property? A right line can be comprehended alone; but this definition is unintelligible without a comparison with other lines, which we conceive to be more extended. In common life 'tis established as a maxim, that the straightest way is always the shortest; which would be as absurd to say, the shortest way is always the shortest, if our ideas of a right line was not different from that of the shortest way betwixt two points. (T.49-50)

As Atkinson, Steiner, and Newman have observed, Hume is in essence arguing that _the shortest distance between two_ points is a straight line (and by implication, the other "self-evident" propositions of geometry) is (are) not analytic. Here, they are correct in drawing a strong comparison between the position of Hume and Kant. In fact, part of Hume's argument is virtually identical to an argument Kant was later to give in favour of the view that the above proposition (hereafter, we shall refer to this proposition as the 'principle of straightness') is really synthetic.

> ...just as little is any principle of geometry analytical. That a straight line is the shortest distance between two points is a synthetical proposition. For my concept of straight line contains nothing of quantity, but only a quality. The concept "shortest distance" is therefore altogether additional and cannot be obtained by any analysis of the concept of 'straight line'...[7]

Like Kant, Hume argues that we can comprehend the idea of straight line and understand what it means to say that a line is straight without being led by any necessity of thought to a quantitative comparison with other lines. For both Hume and Kant straightness represented a sensible quality that a line possesses, and no matter how much we focus on this quality alone, we will not be able to derive any information as to whether it represents the shortest distance between two points. Both argued that in order to gain any knowledge of such a quantitative determination, we must go outside our idea of that sensible figure we call 'straight line' and consider lines of different figure.

The rationalists would have had no difficulty agreeing that insofar as we confine ourselves to that which is sensible in our idea of a straight line, Hume's analysis is essentially correct. But again, they would have taken the certainty of the axioms to be such as to offer indisputable evidence that this line of analysis is necessarily incomplete. In their view this certainty went well beyond a mere practical or moral certainty and was rooted in the logical necessity of the propositions[8]. It was thus thought to have its origin in an understanding of that which the light of reason reveals as being distinct in our geometric ideas.

Now, if we assume that the principle of straightness follows from our understanding of that which is contained in the idea of a straight line (and thus, from an understanding of what it means for a line to be conceived as straight) then the principle itself is really a definition of our idea of a straight line. It would clearly involve a logical contradiction to say that a straight line does not represent the shortest possible distance between two points. Hume's objection is that if we accept this analysis, we are led to the absurdity of stating that the principle of straightness, as it is actually understood according to our everyday manner of thinking, says no more about the world than that the shortest distance between two points is the shortest distance between two points. With this, Hume hoped to turn the tables on the rationalists and show that it is their analysis, not his, which is at odds with what is most clear and evident in our actual understanding of the ideas and propositions most essential to geometry.

Let us again emphasize that the rationalists maintained that the full certainty of the axioms is readily apparent to anyone of average intelligence once they are led to reflect upon them. Hume's argument is that in order to accept these truths and their alleged certainty as being of a purely rational or logical nature, we must assume that the axioms of geometry are themselves either statements of identity or can be reduced to statements of identity. This, Hume maintains, is so absurd and contrary to our actual understanding of these propositions that we have no alternative but to reject the rationalists' doctrine of analyticity - at least as it pertains to the propositions of geometry.

Let us make clear that the criticism we are extracting from Hume's writings is not based on a perverse understanding of the rationalist position. Hume was by no means attacking a straw man. No less a figure than Leibniz had argued that the propositions of geometry are grounded in an a priori understanding of their terms and that this implies that the propositions themselves must be reducible to identities.

> ...every axiom, once its terms are understood may be reduced to the principle of contradiction... This is the only, and highest criterion of truth in abstract things, that is, things which do not depend on experience - that it must either be an identity or be reducible to identities...[9]

> ...The great foundation of mathematics is the <u>principle of contradiction</u> or identity, that is, that a proposition cannot be true and false at the same time and that therefore A is A and cannot be non-A. This single principle is sufficient to demonstrate every part of arithmetic and geometry, that is, all mathematical principles.[10]

> ...the primary necessity in propositions is this: A is A. Hence, only identities are indemonstrable, but all axioms are demonstrable, even though they are mostly so clear and easy that they do not need demonstration; nevertheless, they are demonstrable in the sense that if their terms are understood (i.e. by substituting the definitions for the term defined), it becomes clear that they are necessary or that their contrary implies a contradiction in terms...[11]

For Leibniz, and the rationalists in general, the principles of mathematics were thought to be derived from a clear and distinct intellectual intuition "which Plato called an idea <u>and which, when expressed in words is the same as a definition</u>"[12]. This clear and distinct intuition was construed as the non-sensible element implicit in our everyday understanding of the ideas of geometry and the reason why we take it to be fully certain that a straight line represents the shortest distance between two points (and that no two straight lines can concur or enclose a space). Hume raised the seemingly obvious point that for whatever reason the common understanding comes to view the most "vulgar" and evident propositions of geometry as certain, it is not because it clearly understands them to be mere definitions or reducible to identities. A doctrine which is so at odds with the manner in which these propositions are actually comprehended in everyday life could never serve as the rational ground for any certainty the common understanding of mankind assigns to them.

Now it might seem that Leibniz could easily concede that such a straightforward proposition of geometry as the principle of straightness is not readily understood as being reducible to a statement of identity, at least not according to our ordinary understanding of it. Leibniz, after all, acknowledged that though the axioms can be reduced to statements of identity, and are thus demonstrable, they "are so clear and easy that they do not need a demonstration." The implication here, it might be thought, is that one can easily apprehend the certainty of the axioms without being able to demonstrate it in any formal sense.

It might be argued, for example, that because we can clearly and distinctly intuit that one geometrical idea is inseparably bound up with, or contained in, another, we readily comprehend that the corresponding axiom is fully certain. However, because we rarely engage in the difficult task of determining precisely in what this certainty consists, we fail to realize that these propositions are founded upon definitions and can be reduced to statements of the form "A is A". In other words, the certainty which the common understanding finds to hold here is grounded in a clear, but somewhat confused, intuition of a necessary connection holding between the ideas of straight line and shortest distance.

This response, however, fails to come to grips with the main thrust of Hume's argument. There is more to Hume's objection than the simple claim that the most obvious and "vulgar" propositions of geometry are not commonly understood to be mere statements of identity. The crux of his argument is that these propositions cannot possibly be reduced to statements of identity without being totally divorced from the manner in which they are actually understood in everyday life. Hume could have challenged Leibniz to explain how the propositions of geometry can be of any practical value and relate to the world of sense experience if they ultimately say no more than A is A. Again, we are led to conclude that the idea of an intelligible form of perfect straightness lies beyond our common powers and faculties of conception and plays no role in our actual understanding of the propositions of geometry.

IV

The Final Hurdle

Hume had chastised philosophers for greedily embracing "whatever has the air of a paradox, and is contrary to the first and most unprejudic'd, notions of mankind" (T.26). Far from "shewing the superiority of their science, which cou'd discover opinions so remote from vulgar conception" (T.26), Hume believed that they only open themselves to the ridicule of the sceptic. However, he was aware that the natural inclination of mankind was such that the glitter of these speculations proved most seductive.

> ...any thing propos'd to us, which causes suprize and admiration, gives such a satisfaction to the mind, that it indulges in those agreeable emotions, and it will never be perswaded that its pleasure is entirely without foundation. From these dispositions in philosophers and their disciples arises that mutual complaisance betwixt them; while the former provide such plenty of strange and unaccountable opinions, and the latter so readily believe. Of this mutual complaisance I cannot give a more evident instance than in the doctrine of infinite divisibility... (T.26)

Most assuredly, Hume would have placed Leibniz's view that the essence of our understanding of the principles of geometry can be reduced to propositions of the form "A is A" as being high on the list of strange and unaccountable opinions which are far removed from vulgar conception. It is no less certain that Leibniz would have raised the same criticism against Hume's claim that we cannot conceive for ourselves an idea of perfect straightness nor know with certainty that two straight lines can never concur or enclose a space. It was taken to be patently obvious that the vulgar, no less than the mathematician and the philosopher, can easily abstract beyond that which is given to them in sense experience and conceive for themselves the clear and exact idea of a perfectly straight line.

Hume, of course, was of the opinion that the mind has a natural propensity for confusion even when it restricts itself to matters which appear most clear and straightforward. He believed that the conventional wisdom regarding the exact and determinate nature of our geometric ideas, which virtually all mathematicians accepted as incontestable, was an example of how infectious and insidious this error can be. However, in order to provide a solid defense for his own controversial position, Hume must go well beyond making general comments regarding the nature of this error and its relation to Berkeley's doctrine of general ideas. He must spell out how it is that the mind is so thoroughly deceived into believing that its most basic and intuitively straightforward geometric ideas are exact and determinate according to its common manner of conceiving them. If Hume can present such an explanation, he will have taken a major step toward removing one of the most historically significant obstacles confronting empiricism. His argument turns on the claim

> ...that the imagination, when set into any train of thinking, is apt to continue, even when its object fails it, and like a galley put in motion by oars, carries on its course

> without any new impulse. This I have assign'd for the reason, why, after considering several loose standards of equality, and correcting them by each other, we proceed to imagine so correct and exact a standard of that relation, as is not liable to the least error or variation... (T.198)

The earlier argument to which Hume refers concerned the fact that the judgements we originally make concerning the relations of greater, less, and equal are, in many instances, open to correction.

> ...We frequently correct our first opinion by a review and reflection; and pronounce those objects to be equal, which at first we esteem'd unequal; and regard an object as less, tho' before it appear'd greater than another. Nor is this the only correction, which these judgements of the senses undergo; but we often discover our error by a juxtaposition of the objects; or where that is impracticable, by the use of some common and invariable measure, which being successfully appl'd to each, informs us of their different proportions. And even this correction is susceptible of a new correction, and of different degrees of exactness according to the nature of the instrument by which we measure the bodies, and the care which we employ in the comparison. (T.47)

After subjecting the original judgement concerning equality to correction and recognizing that this process can be repeated again and again, the natural tendency of the imagination is to carry on in this train of thinking and feign for itself a series of ever more precise and exact standards of equality - standards which exceed what it presently has art and instrument to describe. The reason why the imagination behaves like a galley which continues to move even after its oars have ceased is that "we clearly perceive, that we are not possessed of any art of measuring, which can secure us from all error and uncertainty" (T.48). Since each correction, whether real or imagined, must be viewed as possibly open to further correction, we too hastily conclude that we are justified in imagining that all empirical standards are open to correction. As a result, the imagination feigns for itself an unending series of correction after correction.

Let us again recall that Hume maintained that a finite length is really composed of a finite number of indivisible points (minimal sensibles) into which it can be divided. Accordingly, the most precise and exact standard of equality of extension to which we can, in principle, aspire is one which would allow us to judge this equality on the basis of the number of indivisible points and the manner in which they are disposed. However, this idea of equality must remain a far and distant notion.

> ...For as the points, which enter into the composition of any line or surface, whether perceiv'd by sight or touch, are so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number, such a computation will never afford us a standard, by which we may judge of proportions...(T.45)

Through careful observation we can perceive that a minute difference exists between two lines which formerly, to a more casual observation, appeared equal. But however much care we take when focusing on this minute difference, we can never gain any real assurance that it represents the minimal difference (impression) which can exist. Hume fully granted that there is a "natural infirmity and unsteadiness both of our imagination and senses, when employed on such minute objects" (T.41-42) and, as a consequence, these faculties "give us disproportion'd images of things and represent as minute and uncompounded what is really great and composed of a vast number of parts" (T.28). The telling point here is that Hume granted to the conventional wisdom that "sound reason convinces us that there are bodies vastly more minute than those, which appear to the senses" (T.48). From this, he alleges, we conclude too hastily that the most minute difference in length which our present instruments and art of measuring allow us to detect is itself composed of a vast number of parts. As a consequence, we are led to carry on in our train of thinking and feign correction after correction as if extension were infinitely divisible.

In order to satisfy its rational pretensions and bring its activity to a logical completion, the imagination contrives for itself the abstract ideal of an infinitely precise standard of equality. This fictitious standard is assumed to be of so refined and spiritual a nature that any two lines which are defined as being equal relative to it cannot be conceived as containing any difference in length, no matter how small. As Hume well knew, it was relative to this perfect and general standard that the geometer claimed to know with full certainty that equals added to equals must always yield equals. And, of course, it was his contention that

> ...This standard is plainly imaginary. For as the very idea of equality is that of such a particular appearance corrected by juxta-position or a common measure, the notion of any correction beyond what we have instruments and art to make, is a mere fiction of the mind, and useless as well as incomprehensible. But tho' this standard be only imaginary, the fiction however is very natural; nor is anything more usual, than for the mind to proceed after this manner with any action, even after the reason has ceas'd, which first determin'd it to begin... (T.48)

Naturally, Hume believed that we

> ...may apply the same reasoning to CURVE and RIGHT lines... tho' we can give no perfect definition of these lines, nor produce any very exact method of distinguishing the one from the other; yet this hinders us not from correcting the first appearance by a more accurate consideration, and by a comparison with some rule, of whose rectitude from repeated trials we have a greater assurance. And 'tis from these corrections, and by carrying on the same action of the mind, even when its reason fails us, that we form the loose idea of a perfect standard to these figures, without being able to explain or comprehend it. (T.49)

Just as the faculties of sense and imagination may represent as simple and uncompounded what is really great and composed of a vast number of parts, so too they can represent as straight that which contains some degree of curvature.

Sound reason convinces us that there may exist a vast number of degrees of curvature which are less than that which is described by our original standard of straightness. Since "we are not possessed of any art or measuring, which can secure us from all error and uncertainty," we cannot say of any impression (or hence, any idea) that it definitely represents a line which is devoid of all curvature and is not open to further correction. Again, it is assumed that because we do not recognize our latest correction as a limit, the imagination continues in its train of thinking and contrives a series of ever straighter lines even though there is no empirical basis for any of them. It abstracts away all sensible limitations and proceeds as if it were a pure faculty of the intellect. In order to bring its train of thought to an end, the imagination feigns for itself the figment of an ideal standard of perfect straightness, a fiction which it falsely imagines to be of so refined and spiritual a nature that it cannot be conceived as containing even the slightest degree of curvature. This fiction would thus be conceived as the abstract form which any given empirical standard of straightness more or less approximates, and as the limit toward which the feigned series converges.

On the face of it, Hume's account appears hopelessly naive. We are asked to believe that otherwise astute and critical thinkers have been pulled along in their thinking by a rather simplistic flight of fancy to feign for themselves distant and incomprehensible fictions. Specifically, we are asked to believe that, throughout history, mathematicians and philosophers were deceived by this flight of fancy into receiving obscure and incomprehensible fictions as being so clear and exact as to be the source of an intuitive and full certainty with respect to the most basic propositions of geometry. If all this is nothing but an illusion then Hume owes us a plausible explanation of this illusion. What he is asking us to accept is that, in this instance, philosophers and mathematicians were victims of blind stupidity, and this is far from plausible.

It is one thing to suggest that the metaphysician Descartes was the victim of self-deception when, after long periods of meditation, he believed that he had discovered within himself a clear and distinct idea of God. However, it is quite another to say that the mathematician Descartes was the victim of self-deception when he believed that he possessed a clear and distinct idea of straight line. In the latter case we are not dealing with an idea which required considerable analysis before it could be comprehended in any but a confused manner. Rather, we are dealing with an idea which philosophers and mathematicians generally agreed was implicit in our everyday way of conceiving straightness and could be easily comprehended by the common understanding of mankind. In fact, Descartes would have insisted that it is easier to doubt the existence of the external world than to believe that this idea which we take to be so intuitively clear and straightforward is really some obscure and distant figment of the imagination. If this be all an illusion then it is a fantastic one and what Hume has offered thus far hardly seems to do it justice.

Given what is at issue here, let us move beyond these largely psychological considerations and take a closer look at the logic of Hume's argument. Of particular significance here is the assumption that we may, at the very outset, conceive our original standard of a straight line (equality) and its corrections as describing lines which may really be curved (unequal). Unless we were already capable of conceiving things in this way and could be persuaded that there are degrees of curvature (and parts of extension) which are vastly more minute than what is discernible to the senses, Hume would be hard put to explain how it is that we are so naturally inclined to continue in this train of thinking and feign correction after correction without any empirically definable limit in sight. Hume, of course, insisted that there is a finite limit as to how far we can go in correcting our standard of a straight line and that the idea which stands at the finite limit represents the idea of the straightest possible line. However, if we accept the train of thinking by which we correct our standards as basically sound, as Hume most certainly does, then we must also accept that we may legitimately conceive the possibility that any given standard we may possess describes a line which is really curved and may, at some future date, be replaced by a standard which describes an even straighter line.

To be sure, Hume would insist that we cannot actually frame for ourselves the idea of a straighter line than that which stands at the finite limit. This is an immediate consequence of the logic of his empiricism. Nonetheless, there is nothing in the idea or its corresponding impression which can afford us any certainty that it represents the idea at the limit and that no further correction is possible. We cannot render this idea so exact and determinate as to comprehend that it stands at the finite limit and is not open to further correction. For the very reason that we may legitimately conceive the possibility that our other standards may describe lines which possess some degree of curvature, we may also conceive the possibility that this standard describes a line which is ever-so-slightly curved. If this were not the case, Hume would be hard put to offer a plausible explanation as to why an intelligent person would assume that the series of corrections can be imagined as extended indefinitely. This manner of thinking, however, does not seem consistent with Hume's claim that our idea of a straight line is nothing but the copy of an impression. Thus, Hume is faced with the criticism that the train of thinking he employs to generate the necessary series of corrections itself presupposes that there is more to our ordinary conception of a straight line than can be analyzed in accordance with the copy principle[13]. He has opened himself to the charge that the train of thinking he employs presupposes the very "fiction" whose origin he is trying to explain.

E.W. Van Steenburgh, one of the few philosophers to focus on this matter, attempts to defend Hume against the charge that the "process of correction presupposes use of the very idea...which it is alleged to generate."[14] He maintains, for example, that we can generate a series of circles according to a comparison of roundness without ever presupposing the abstract idea of a perfect circle.

> ...let us grant, for the sake of argument, that circles do "correct" one another relative to a standard circle. It not only does not follow, it is just false, that the standard must be a perfect circle, i.e., a circle satisfying the geometer's definition of 'circle'. Thus, I may have no idea of a perfect circle. Yet given three different circles, none perfect, I compare them and discover that the first is more like and the second is less like the third... By expanding to new items and repeating comparisons, the series is enlarged. None of the items in the enlarged series, however, is necessarily the circle specified by the geometer's definition of 'circle' and, more to the point, one could know the items in the series and know how the series is generated without having or using the idea of a perfect circle.[15]

From this he concludes that it

> ...does not follow from the fact that one can make corrections, that one must correct for a perfect standard. One could "correct" up to a

> sensible limit without having the idea of a perfect standard...[16]

As we would expect, Van Steenburgh maintains that "the forgoing argument holds for any geometric notion susceptible to degrees."[17] He notes, for example, that we do say "that one line is straighter than another - showing that lines may be compared for degrees of straightness."[18] Thus, he would argue that we can generate a finite series of lines according to a judgement as to which is straighter than which without, in the process, employing the geometric ideal of a perfect standard of straight line. Indeed, he would argue that this series generates, rather than presupposes, the idea of a perfect standard of straight line. However, in assessing this purported defense of Hume, it is important that we bear in mind that the critical issue before us is not whether Hume can generate, in a noncircular way, a series of corrections up to some finite limit. Rather, it is whether Hume can use this series of corrections to explain how the mathematician comes so naturally to feign for himself the abstract and general notion of a standard of perfect straightness and deceive himself into believing that this fiction is so clear and determinate that it affords him an intuitive certainty with respect to the axioms of geometry.

Assume, for example, that we have ordered a series of lines according to which appears to be visually straighter than which. Clearly, we can generate this series without presupposing any idea of perfect straightness. (Indeed, we could train a chimpanzee to order lines according to this criterion.) Let us further assume that a more careful observation and comparison allows us to observe a sensible difference between the line we originally designated as 'straightest' and some new line such that we plainly see the new line to be straighter. This being the case, we have corrected our original standard without in any way presupposing an ideal standard of perfect straightness. We did not judge the one standard to be straighter than the other because one more closely resembled or approximated some ideal standard of perfect straightness. Rather, we did it on the basis of a perceived sensible difference according to which line is observed to be straighter and thus, in seeming accordance with Hume's empiricism. In thinking the possibility that our present standard may be open to correction, we are only thinking the possibility that we may someday actually see that some particular appearance represents a straighter line.

We can easily concede that the series of corrections which can be generated in accordance with this visual criterion is finite and that we can, in principle, correct up to this finite limit without presupposing the offending conception of perfect straightness. However, it is difficult to see how anyone of intelligence would be induced by the momentum of this train of thinking to imagine correction after correction as if the senses were capable of infinite precision. And, of course, Hume himself would have dismissed any such suggestion as nonsense. As we have seen, Hume allowed that sound reasoning convinces us that there are differences in curvature which are too minute to be discerned clearly by the unaided senses. It was essential to Hume's account that art and instrument be able to come to the aid of the senses and enable us to extend the series of corrections far beyond what the faculties of sense and imagination alone are capable. It was this which gave the imagination the momentum to continue on its own and feign correction after correction. The real question at issue is whether <u>this</u> train of thinking, which employs art and instrument, "presupposes use of the very idea ... which it is alleged to generate."

It might seem that both Hume and Van Steenburgh are on solid ground in maintaining that it does not. If the purely visual criterion poses no problems then why should a criterion which allows for the art of measurement to come to the aid of the senses and considerably extend the series of corrections? After all, it is by means of instruments and measurements, not pure forms, that we render our sensible appearances of straightness, equality, etc., more precise and exact. We can understand why Hume believed that everything here was in basic accordance with the demands of his empiricism. Of course, we are still left wondering how able philosophers and mathematicians could have been mindlessly pulled along by this train of thinking and fallen victim to the error Hume alleges. However, this aside, there is a serious problem. The very instruments and techniques of measurement we employ are constructed and "read" in accordance with sciences, specifically Newtonian physics, which themselves presuppose, at a most fundamental level, the results of Euclidean geometry.

Consider, for example, Hume's own claim that

> ...'Tis not for want of rays of light striking on our eyes, that the minute parts of distant bodies convey not any sensible impression; but because they are remov'd beyond that distance, at which their impressions were reduc'd to a minimum, and were incapable of any farther diminution. A microscope or telescope, which renders them visible, produces not any new rays of light, but only spreads those, which always flowed from them; and by that means both gives parts to impressions, which to the naked eye appear simple and uncompounded, and advances to a minimum, what was formerly imperceptible. (T. 27-28)

Hume interprets the result of looking into a microscope as demonstrating that the senses give us "disproprtion'd images of things, and represent as minute and uncompounded what is really great and compos'd of a vast number of parts" (T.28). However, in order to draw this conclusion, Hume must know something about the physics of the instrument (or rely on the testimony of others who do). The microscope itself does not give us an image which is more minute than that given to the unaided eye. When we look into the microscope, we see an image which is considerably larger than what originally appeared to the eye. It requires a chain of reasoning to conclude that what we are really seeing through the microscope is a true magnified image of the impression which appeared simple and uncompounded to the eye and that the parts we see are parts of the original impression (and not creations of the instrument). This, however, is a chain of reasoning which itself presupposes classical physics (eg., optics), and Euclidean geometry forms an integral part of this theory.

We cannot read the instruments and measurements we employ to correct our present standards as if we were reading names from a phone directory. In order, for example, to infer from the reading of an instrument or the taking of a measurement that one line is straighter than another, a chain of reasoning involving theoretical considerations must be employed. This chain of reasoning will naturally presuppose geometry. Hume would be hard put to argue that the essence of the geometrical reasoning which is employed here can be captured by geometry$_H$ with its limited precision and certainty. If the ideas most essential to geometry$_H$ are nothing but the copies of impressions and cannot be the source of any truly precise and exact judgements about straightness and equality of extension then it is difficult to see how they can be employed as part of a chain of reasoning which leads us to correct these impression.

We can illustrate the difficulty here with a simple example. Let us assume that we possess two standards, X_1 and X_2, which describe lines which appear to our most careful observations as being equally straight. Let us further assume that we become persuaded by our most accurate measurements that X_2 describes a shorter distance between two points than does X_1. Are we to conclude from this that X2 describes a straighter line than does X1? This is certainly what would seem most natural according to our common way of thinking. However, in order to draw this conclusion, we must assume that we can know with full certainty that a straight line <u>always</u> represents the shortest distance between any two points in space. If we can have no such perfect assurance then we must remain open to the possibility that X_1 describes the straighter line.

Now, Hume did indeed grant that in "common life 'tis establish'd as a maxim, that the streightest way is always the shortest" (T. 50). But he did not intend this to represent a break from his basic tenet that geometry is "built on ideas, which are not exact, and maxims, which are not precisely true" (T.45). In referring to the principle of straightness as a maxim of common life, Hume was only stating that where we clearly perceive that a difference in curvature exists between two lines, we can be certain that the straighter line represents the shorter distance. However, since the idea of straight line which serves us well in common life is not so perfect as to enable us to conceive a clear and precise boundary separating curved from straight lines, we cannot go further and claim to know with full certainty that the straighter line always represents the shorter distance. We must remain open to the possibility that where the difference in curvature is too minute to be the object of a clear sense perception, as in the case of X_1 and X_2, we cannot be certain that the straighter of the two lines represents the shorter distance. Hence, in order to interpret the results of our measurement as demonstrating that X_2 is straighter than X_1, Hume would have to employ reasoning which already presupposes the very "fiction" of perfect straightness he is trying to explain away.

If Hume cannot use a measurement of length to come to the aid of the senses and correct our original standard of straightness, it is difficult to see what measurement or instrument he could employ.[19] No matter what reasoning and instrument he employs to correct a given standard of straightness, he will have to employ geometric maxims which are more precise and certain than the teachings of the Treatise can allow.

V

Summary and Conclusion

Bertrand Russell has argued that the "problem of the exactness of mathematics and the inexactness of sense is an ancient one ... [which] in modern times ... has been forgotten through familiarity, like a bad smell which you no longer notice because you have lived with it for so long."[20] As long as we reside "in the region of mathematical formulas, everything appears precise, but when we seek to interpret them it turns out that the precision is partly illusory."[21] The rationalists, of course, tended to remain in the region of mathematics and consequently, focused on the precision and exactness (distinctness) of mathematics. This precision, they maintained, was implicit in our everyday mode of thinking and could be readily comprehended by the common understanding of mankind. The fundamental philosophical challenge was to explain the nature and origin of this precision, and the doctrines of innate ideas and intellectual intuition (light of reason) formed the basis of their analysis. However, since they construed this precision to be essentially rational in character, they were unable to offer a plausible account of how the ideas of mathematics can be given a sensible interpretation and be applied to experience. Mathematical thinking was divorced from our everyday mode of thinking and the objects of geometry could no longer be conceived as bearing the slightest resemblance to objects of sense experience.

Hume, we have argued, was aware of this defect in the rationalists' philosophy. The problem, he was convinced, lay not in an erroneous analysis of the precision and exactness which mathematicians assigned to geometry but in the view that geometry is in fact a precise and exact science. In order to render our geometrical judgements consistent with what he regarded as the ordinary powers and faculties of mankind and make intelligible their application to experience, Hume took the radical step of denying that the ideas and propositions of geometry possess anything like the precision and exactness mathematicians assumed them to have. As a consequence, he was led to deny that these possess anything like the full certainty mathematicians ascribed to them.

As radical as Hume's step was, he felt it was amply justified - and for reasons which went well beyond a mere dogmatic adherence to his copy principle. He agreed that in order for the ideas of geometry to possess the precision and exactness they were alleged to have, they would have to have their origin in some pure and intellectual view. But this assumption, he argued, was open to serious criticism and, in general, led to a dead end. The only option he could see was to place the origin of these ideas in sense experience and deny them a perfect exactness and precision. This position, he believed, was consistent with the fact that (a) geometry has an obvious application to the world of sense experience, (b) we conceive of our geometrical ideas as more or less resembling appearances, and (c) the study of geometry is accessible to people of ordinary faculties and powers. Furthermore, he believed that it would enable him to put an end to all those embarrassing paradoxes surrounding infinite divisibility.

What posed a problem for Hume was that virtually all mathematicians sided with the rationalists in holding geometry to be an exact and precise subject - no less so than arithmetic or algebra. They agreed with the rationalists that the precision and exactness of the ideas most essential to geometry were so clear and evident as to be beyond all reasonable of doubt. The conviction here was grounded in what was taken as a clear, immediate, and incontestable intuition. The difficulty which faced Hume was that he had no option but to argue that what these astute thinkers take to be so intuitively straightforward is really an incomprehensible figment of the imagination.

Now, it was very much a part of the general program Hume inherited from Berkeley that the mind has a natural propensity for confusion in matters relating to its abstract ideas and is easily deceived into conceiving its particular and inexact abstract ideas as if they were of a general and exact nature. However, it is surprising that Hume should have taken the difficult challenge before him so lightly and dismissed it with what came to little more than a few hand-waving comments. It is incredible that Hume believed that a rather simplistic flight of fancy could have been the source of so remarkable and deep-seated an illusion.

Upon closer examination we found that Hume's account of this illusion came to little more than an exercise in question-begging. As a consequence, he was unable to defend his own analysis against the charge that it is contrary to the first and most unprejudiced notions of mankind and shocking to common sense. In the final analysis, Hume was unable to remove the obstacle which the precision and certainty of geometry had placed in the path of empiricism and overcome one of the greatest challenges facing his copy principle.

Chapter 4

Notes

1). To further clarify this point, let us consider the idea of blood. The idea is clear in that it suffices for recognizing instances of this idea when they are presented to us. However, the idea is not distinct in that we do not see clearly in what this idea consists and thus cannot define criteria for identifying blood. If we look under a microscope, we gain a better understanding of what blood is and can now define some criteria for identifying something as blood. In this case our idea of blood has been rendered more distinct. However, it is not perfectly distinct as there is still some confusion in it. For example, we know that blood contains cells but we do not know in what these cells consist. A more powerful microscope may afford us a clearer understanding of the structure of these cells and thereby render the idea even more distinct. However, some confusion still remains for we do not see clearly in what these structures within structures consist. Since we must rely upon experience to advance our knowledge in this regard, it is safe to assume that we shall never be able to render everything about the structure and make-up of blood perfectly clear and thereby render the idea perfectly distinct. For the rationalists, the situation was entirely different with respect to our mathematical ideas. These ideas were thought to contain something intelligible and when the light of reason shines on this element the result is like looking through an ideal microscope. Everything regarding this element can be made perfectly clear and we can comprehend the smallest differences which separate one idea (eg. straight line) from another (eg. curve) which might closely resemble it. The situation with the metaphysical ideas is somewhat similar except that here things are not so intuitive and straightforward. There is greater darkness (confusion) and more effort is required in order to bring everything out into the full light of reason.

2). Gottfried Leibniz, "On What is Independent of Sense and Matter," p. 551

3). ibid., p. 551

4). Hume no doubt believed that by allowing this fiction to infect their thinking, the Cartesians were easily led to conclude that every part of extension is itself extended and thus, that finite extension is infinitely divisible.

5). Unlike Hume, Leibniz did not conceive of these indivisible substances as compositional elements. The appearance of extension was not a confused appearance of a number of monads crowded closely together.

6). Immanuel Kant, Critique of Pure Reason, A137 = B176

7). Immanuel Kant, Prolegomena to Any Future Metaphysics, trans. Peter Lucas, (Manchester: Manchester University Press, 1953), p.20. See also Critique of Pure Reason, B17

8). Moral certainty refers to that species of certainty which experience can afford us. It is the certainty we can have regarding matters of fact. This certainty is based on criteria which possess the greatest probability. Leibniz, for example, maintains that we can know with moral certainty that certain phenomena are real, and not mere figments of our imagination (as in the case of dreams). Metaphysical certainty is the certainty which results from the analysis and connection of ideas such that to affirm the contrary would involve a logical contradiction. For Leibniz, metaphysical certainty presupposes necessity whereas moral certainty does not. Accordingly, he would classify the certainty Hume attributes to geometry$_H$ as moral certainty.

9). Gottfried Leibniz, " On Universal Synthesis and Analysis, Or the Art of Discovery and Judgement," in Philosophical Letters and Papers, p. 232

10). Gottfried Leibniz, Leibniz's Second Letter to Samuel Clarke, in Philosophical Papers and Letters, p.677

11). Gottfried Leibniz, " Letter to Herman Conring," in Philosophical Papers and Letters, p.187

12). Gottfried Leibniz, " Elements of Natural Law," in philosophical Papers and Letters, p.133

13). It is worth noting that Hume himself recognized that there are some exceptions to his copy principles, phenomena "which may prove, that 'tis not absolutely impossible for ideas to go before their corresponding impressions" (T. 55). To illustrate this point, he imagined

> ...a person to have enjoyed his sight for thirty years, and to have become perfectly well acquainted with colours of all kinds, excepting one particular shade of blue, for instance, which it never has been his fortune to meet with. Let all the different shades of that colour, except that single one be placed before him, descending gradually from the deepest to their lightest; tis plain, that he will perceive a blank, where that shade is wanting, and will be sensible, that there is a greater distance in that place

> betwixt the contiguous colours, than in any other. Now I ask, whether tis possible for him, from his own imagination, to supply this deficiency, and raise up to himself the idea of that particular shade, 'tho it had never been conveyed to him by his senses? I believe there are few but will be of opinion that he can; and this may serve as a proof, that the simple ideas are not always derived from the correspondent impression; tho the instance is so particular and singular, that tis scarce worth our observing, and does not merit that for it alone we should alter our general maxim. (T.6)

Naturally, Hume intended these remarks to hold equally well for other intensive magnitudes, such as sounds. However, the instance here is not quite as singular as he suggests, for the point he raises holds equally well for degrees of curvature and straightness. Imagine, for example, a series of curves passing through two points arranged in the order of their curvature. If we then remove some of the curves and show them to the person in Hume's example, will he here not also be sensible that there is a greater distance between some of the curves and will he not be able to raise up to himself the ideas of those particular curves which are missing? By the same token, it would seem equally clear that a person who has not been raised in so neatly a carpentered environment as our own could, from his own imagination, raise up to himself an idea of a line which he judges straighter than any appearance which has heretofore been given to him in sense experience. In fact, if we (in the carpentered environment) may conceive the possibility that our original idea of a straight line is not only lacking in precision but may actually be rendered more exact, then Hume cannot rule out the possibility of someone (say an artist) raising up to himself, from his own imagination, an idea of a line which is straighter than any he has sensed. Had Hume taken proper notice of this, he would have realized that he could not so easily bring the ideas of geometry under his copy principle.

14). E. W. Van Steenburgh, "Hume's Geometric Objects," Hume Studies, VI, No. 1 (April, 1980), p. 65

15). ibid., p. 65-66

16). ibid., p. 66

Van Steenburgh goes on to add, "Upon reaching the limit, that item at the limit is assigned perfect status." As he sees it, Hume's position is that corrections "up to a sensible limit yield perfect geometric ideas" (p. 64). These comments relate to what Van Steenburgh sees as a shift in view in the Enquiry concerning the nature of geometric ideas and we shall have to wait before we can properly address his argument. However, as we shall presently argue, Hume cannot consider the idea of straight line which stands at the limit as being perfect in the sense of being so clear and determinate that we cannot conceive of it as containing any curvature. That is, he cannot consider this idea as perfect in the same sense that the mathematician considered his "idea" of a straight line as perfect.

17). ibid., p. 66

18). ibid., p. 66

19). These difficulties become all the more apparent once we realize the presuppositions which enter into the measurement of length. Firstly, we must presuppose a criterion of straightness for we must measure along a straight line. Secondly, we must presuppose that our measuring rod is rigid; that is, that it remains equal to itself as it is transported through space. Aside from the physics involved, we are presupposing the geometric concept of equality. It should not be thought that Hume's problems can easily be resolved in light of the fact that the general theory of relativity does not presuppose that space is Euclidean. The instruments which were used to test the general theory were themselves constructed and interpreted in accordance with theories which presupposed the concepts of Euclidean geometry. It would be absurd to suggest that the results of these measurements (concerning the curvature of light) demonstrate that the Euclidean concept of a perfect straight line is a meaningless fiction.

20). Bertrand Russell, Human Knowledge : Its Scope and Limits, p. 238. See p. 22 of this work.

21). ibid., p. 242

Chapter 5

The *Enquiry*: A Shift in View

I

Introduction

It is noteworthy that Kant had a completely different understanding of Hume's philosophy of geometry than what we have presented to be the case.

> Hume being prompted to cast his eye over the whole field of *a priori* cognitions in which human understanding claims such mighty possessions (a calling he felt worthy of a philosopher) needlessly severed from it a whole, and indeed its most valuable, province, namely, pure mathematics; for he imagined its nature or, so to speak, the state constitution of this empire depended on totally different principles, namely, on the law of contradiction alone; and although he did not divide judgements in this manner formally and universally as I have done here, what he said was equivalent to this: that mathematics contains only analytical, but metaphysics synthetical, *a priori* propositions...[1]

On Kant's reading, Hume not only put all mathematics on an equal footing but held the axioms of that discipline, including those of geometry, to be a priori truths which could be derived from the principle of contradiction alone. While our differences with Kant seem quite significant, they do not in themselves suggest that either our or Kant's understanding of Hume must be mistaken. It is widely accepted that Kant's account is based completely on Hume's treatment of mathematics in the Enquiry. In fact, the consensus is that Kant was largely, if not wholly, ignorant of the particulars of Hume's treatment of this subject in the Treatise. We are told that "only the Enquiry, not the Treatise, was available to [Kant] in its entirety."[2] The fact that in the passage cited above Kant went on to insist that "subjecting the axiom's of mathematics... to experience [was] a thing [Hume] was far too acute to do"[3] suggests that if Part II Book I of the Treatise was available to him, he never read it.

Granted that Kant was not speaking about the treatment of geometry in the Treatise, we are left wondering whether there could possibly be anything to his account of its treatment in the Enquiry. Could it be that Hume underwent anything like this radical and far-reaching shift in his thinking about geometry? This question has attracted the attention of commentators and, not surprisingly, has created considerable controversy. The problem is that the comments in the Enquiry which led Kant to his assessment are extremely brief and have lent themselves to wildly diverse interpretations. Those who follow Atkinson in holding that the "treatment of mathematics in the Enquiry is simply a shortened and simplified version of that in the Treatise"[4] dismiss Kant's account as a misreading caused largely by his ignorance of the more developed discussion in the Treatise. Steiner, as we have seen, views Kant's reading of the Enquiry as a fundamental misrepresentation of Hume which has "colored all subsequent interpretations of the latter's work."[5] He maintains that there is nothing in the Enquiry which lends "any credence to Kant's assertion that Hume's view is that mathematical propositions are analytic."[6] In general, he denies that "there is any contradiction between the philosophy of mathematics taught in the Treatise and that taught in the Enquiry."[7]

On the other hand, those who see the treatment of geometry in the Enquiry as representing a sharp break from its treatment in the Treatise often interpret the Enquiry along Kantian lines. Noxon, for example, maintains that in the Treatise "Hume was taking a very empirical view of geometry regarding it as a factual description of physical space"[8] and was indeed "making precisely the mistake which Kant supposed 'he had too much insight' to make, viz. submitting 'the axioms of pure mathematics to experience'."[9] Moving to the Enquiry, he finds it significant that Hume here "dropped his attempt to distinguish the logical standing of the propositions of geometry from those of algebra and arithmetic; all alike express analytic truths which rest upon the law of noncontradiction."[10]

In the present chapter we shall focus on those remarks in the Enquiry which lie at the center of this controversy and attempt to determine to what extent they represent a definite and significant shift in view away from the position of the Treatise. But before we begin our inquiry, let us take further notice of the rather striking difference between the Treatise and the Enquiry concerning the depth of their treatment of geometry. In the Enquiry there is nothing like the forty odd pages in the Treatise dedicated to the philosophy of space and geometry. Hume's discussion of these topics in the Enquiry occupies only five or so pages and most of this concerns the principle of infinite divisibility. His discussion of the status of the ideas and propositions of geometry which has generated so much controversy comes to little more than a few isolated remarks which are scattered about. This striking paucity of discussion should not be taken to suggest that Hume had lost interest in the philosophy of geometry or that he was by and large content with what he had written in the Treatise. Passmore[11] suggests that the reason Hume devoted so little discussion to the philosophy of geometry in the Enquiry is that he intended to dedicate a whole separate work to the topic. In the Enquiry Hume seemed satisfied simply to drop a few comments, suggestions, and hints regarding his more considered views on geometry, and left the detailed discussion to a separate work.

Hume actually completed this work rather late in his philosophical career, but unfortunately it was never published. The details surrounding Hume's failure to publish this work are discussed in a letter to William Strahan. In this letter Hume expressed his concern that a bookseller in London had advertised a new book

> ...containing among other things, two of my suppress'd Essays. These I suppose are two Essays of mine, one on Suicide another on the Immortality of the Soul, which were printed by Andrew Millar about seventeen Years ago, and which from my abundant Prudence I suppress'd and would not wish now to be reviv'd. I know not if you were acquainted with this Transaction. It was this: I intended to print four Dissertations, the natural history of religion, on the Passions, on Tragedy, and on the metaphysical Principles of Geometry. I sent them to Mr. Millar; but before the last was printed, I happened to meet Lord Stanhope, who was in the Country, and he convinced me, that either there was some defect in the argument or in its perspicuity; I forget which; and I wrote to Mr. Millar, that I would not print that Essay; but upon his remonstrating that the other essays would not make a Volume, I sent him up these two, which I had never intended to publish. They were printed; but it was no sooner done than I repented; and Mr. Millar and I agreed to suppress them at common Charges, and I wrote a new Essay on the Standard of Taste, to supply their place...[12]

Lord Stanhope was an amateur mathematician of some distinction and it is clear that Hume found his criticism quite devastating. Though Hume made only a passing reference to Stanhope's criticism and claimed he could not even remember what it concerned, the fact remains that he not only withdrew the essay from publication, but was also willing to substitute for it two other essays which he never intended to publish.[13] It is most unfortunate that we do not know the precise subject matter of the essay, whether it dealt primarily with the epistemological status of the propositions of geometry or with the principle of infinite divisibility. In the course of our inquiry we shall attempt to identify and assess those difficulties which are all too quickly brushed aside in the Enquiry and which would have been proper subject matter for a later study dedicated to working out details.

II

The Analytic - Synthetic Controversy

The first, most famous, and most controversial aspect of Hume's discussion of geometry in the Enquiry consists in just a few brief remarks.

> All objects of human reason or inquiry may naturally be divided into two kinds, to wit, Relations of Ideas, and Matters of Fact. Of the first kind are the sciences of Geometry, Algebra, and Arithmetic; and in short, every affirmation which is either intuitively or demonstrably certain. That the square of the hypotenuse is equal to the square of the two sides, is a proposition which expresses a relation between these figures. That three times five is equal to the half of thirty of, expresses

> a relation between these numbers. Propositions of this kind are discoverable by the mere operation of thought, without dependence of what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would forever retain their certainty and evidence. (E.25)

It is easy to understand why many philosophers have followed Kant in reading these remarks as definitely implying that the truths of geometry are analytic. Geometry is now viewed as a science on a par with arithmetic and the certainty of its propositions is said to be discovered "by the mere operation of thought, without dependence on what is anywhere existent in the universe." It sounds as if Hume is saying that the certainty of these propositions depends solely upon the analysis of their ideas, and thus upon the principle of contradiction. Kant probably found this all too suggestive of Leibniz's distinction between truths of reason and truths of fact and Hume appears to give added weight to such a reading in the very next paragraph.

> Matters of fact, which are the second objects of human reason, are not ascertained in the same manner; nor is our evidence of their truth, however great, of a like nature with the foregoing. The contrary of every matter of fact is still possible; because it can never imply a contradiction, and is conceived with the same facility and distinctness, as if ever so conformable to reality. _That the sun will not rise tomorrow_ is no less intelligible a proposition and implies no more contradiction than the affirmation _that it will rise_. We should in vain, therefore, attempt to demonstrate its

> falsehood. Were it demonstratively false, it should imply a contradiction, and could never be distinctly conceived by the mind. (E. 25-26)

It is hardly surprising that this passage is often taken as conclusive evidence that Hume held the propositions of geometry, and mathematics in general, to be analytic. Flew captured what many have taken to be its essence when he states that the "differential between 'Relations of Ideas' and 'Matters of Fact' are: that whereas the former can be known apriori and cannot be denied without self-contradiction; the latter can be denied without self-contradiction, and can be known only aposteriori."[14] He fails to find in the Enquiry "any sufficient reason for suggesting that geometry consists in a system of synthetic apriori truths."[15] This reading of the Enquiry is reflected in Reichenbach's assertion that Hume "arrives at the result that all knowledge is either analytic or derived from experience: mathematics and logic are analytic; all synthetic knowledge is derived from experience"[16] as well as Putnam's contention that the "standpoint of Hume was that ... geometric theory was analytic."[17]

It is interesting that some twenty-five years earlier Flew was not nearly so sure what to make of Hume's position in the Enquiry. He was inclined to dismiss a positivist reading, such as Reichenbach's, as an anachronism and found it "doubtful whether it is really correct to attribute to Hume the view that mathematics contains no synthetic elements."[18] He was "by no means certain how far Hume was committed to the denial that, with minor exceptions, mathematics is analytic."[19] Flew argued that Hume's remarks on mathematics at E.25 were "presented as an incidental illustration of the meaning and value of his fundamental distinction between propositions expressing the relations of ideas and propositions expressing matters of fact"[20] and cautioned us "not to try to squeeze out from it more than Hume himself put in."[21] However, as we have seen, Flew came to a complete turnabout in his opinion. Whereas he formerly shied away from assigning to Hume Leibniz's view that the propositions of mathematics are analytic in "the sense that they are 'reducible to statements of identity - obvious tautologies - with the help only of mathematically acceptable definitions,"[22] he is now willing to claim that Hume came to see that the train of reasoning and inquiry needed to demonstrate the truths of arithmetic, algebra and geometry are, in effect, nothing but devices for "'recognizing tautology when it is complicated.' "[23]

Those who believe that Hume has continued to adhere to the position of the Treatise believe that there are comments in the Enquiry which most definitely require us to think differently from the later Flew. For one, Hume continued to hold that "all the ideas of quantity, upon which mathematicians reason, are nothing but particular, and such as are suggested by the senses"(E.158n). He dismissed the Cartesian doctrine that the idea of extension is derived by abstraction from all sensible qualities as "an opinion which, if we examine it accurately, we shall find to be unintelligible and even absurd" (E. 154). Here, no less than in the Treatise, we are reminded that "the idea of extension is entirely acquired from the senses of sight and feeling" (E. 154). The fact that Hume continued to emphasize the sensible nature of the ideas of geometry certainly adds an element of credence to the view that his position cannot be so easily characterized as holding the propositions of geometry to be analytic.

Newman makes much of this. She argues that there is "no suggestion present anywhere in the Enquiry that Hume wishes to retract the earlier view that the basic concepts and principles of geometry are all derived from appearances,"[24] and she maintains that the controversial passage at E.25 "is far from being conclusive evidence of a change in Hume's opinion of the epistemological status of geometrical propositions."[25] Because she believes that Hume's position in the Enquiry continues to be that the axioms of geometry are derived from appearances, Newman is led to conclude that Hume's position is best characterized as holding that "any necessity belonging to them cannot be conceptual or analytic even in a broad sense; it must be synthetic; and the synthetic relation must itself be based upon an intuitive comparison of certain sensible ideas."[26]

Even apart from Newman's reasoning, it is difficult to believe that Hume would have turned his back on his argument in the Treatise and taken the position that the axioms of geometry are reducible to statements of identity by substituting the definitions for the terms defined. He had already exposed the utter futility of this move in the Treatise, arguing that it brings us into sharp conflict with the manner in which the axioms are actually understood. It is difficult to believe that Hume would, on the one hand, emphasize the sensible nature of the ideas of geometry and then, on the other, insist that a proposition like the axiom of straightness says no more than that the shortest way between two points is the shortest way. Furthermore, as Zabeeh observes, "Hume regards geometry (in both the Treatise and Enquiries) to be a science which is conversant with the properties of physical space."[27] Hume could hardly have wished to argue that Euclidean geometry is conversant with the properties of physical space even though its most fundamental propositions are mere definitions and tautologies.

One of the central messages of the Treatise and the Enquiry was that only mathematics allows of demonstrations. In both works Hume insisted that the train of reasoning we employ to demonstrate the theorems of geometry cannot be reduced to a mere logical manipulation of definitions. We may recall from chapter 3 that Hume allowed that "the necessity, which makes ... three angles of a triangle equal to two right ones, lies only in an act of the understanding, by which we consider and compare these ideas" (T.166). In the Treatise the operation of thought (act of the understanding) we employ to demonstrate such theorems was wedded to a train of reasoning and inquiry which went well beyond the mere logical manipulation of definitions to a consideration and comparison of the ideas (figures) themselves. We can easily see that Hume is committed to the same point of view in the Enquiry.

> ... That the square of the hypotenuse is equal to the squares of the other two sides, cannot be known let the terms be ever so exactly defined, without a train of reasoning and enquiry. But to convince us of this proposition, that where there is no property, there can be no injustice, it is only necessary to define the terms, and explain injustice as a violation of property ... It is the same case with all those pretended syllogistic reasonings, which may be founded in every other branch of learning, except the sciences of quantity and number... (E.163)

Since we are dealing with a matter of considerable controversy, let us make clear the strong link connecting the two works as far as concerns the non-analytic nature of geometry. As we have seen, the Enquiry shares the Treatise's commitment to Berkeley's doctrine of abstract general ideas. Hume continued to insist, for example, that the general term 'straight line' refers only to particular ideas which bear a certain resemblance with respect to the figures they represent. Each of these particular ideas is viewed as a separate existence which may be comprehended alone. Hence, no matter how carefully we analyze a given particular idea of straight line, we will never discover any knowledge of properties or relations concerning those other particular ideas we designate by the same term. The conclusion of the Treatise, we have seen, was that idea-analysis alone could never reveal to us whether two straight lines can be made to concur or enclose a space. Hume maintained that in order to gain any such knowledge, we would have to go outside the contents of the particular ideas themselves and seek to discover what external connections exist among these ideas.

In the Treatise this operation of thought included as a necessary element, an activity of the imagination through which the particular ideas (images) were viewed in relation to one another. If Hume is now of the opinion that the propositions of geometry are analytic then he must also be of the opinion that the idea of a straight line is a general idea which contains within itself the essentials for determining that no two straight line can concur. The various particular ideas which are designated by the term 'straight line' would now have to be viewed as instances of a more general idea.

Hume was as aware as anyone that the rationalist doctrine of analytic truth presupposed a doctrine of general ideas. If we assume that the Enquiry represents a shift in view which is best categorized as holding that the axioms of geometry are analytic truths then we would have to assume that Hume intended the Enquiry to represent a break from Berkeley's doctrine of general ideas and the general philosophical framework of the Treatise. Most assuredly, he had no such intention. Of course, it may well be the case that in the Enquiry Hume unwittingly adopted a philosophy of geometry which ultimately cannot be reconciled with his continued acceptance of the copy principle and Berkeley's doctrine. In fact, we shall argue that this turns out to be the case. However, it is a wholly different matter to suggest that Hume adopted a new position with respect to the certainty of the propositions of geometry which he knew full well could not be rendered consistent with his two most important doctrines. And yet, this is exactly what we would have to conclude if we were to maintain that Hume abandoned the position of the Treatise and conceded to the conventional wisdom of his day that the certainty of the principles of geometry is grounded solely in the analysis of ideas and the principle of contradiction.

But what, it may be asked, are we to make of the fact that Hume has clearly stated that the truths of geometry cannot be denied without contradiction? This poses a problem only if we follow Kant and Flew and interpret Hume to mean by 'contradiction' a formal or self-contradiction. Arthur Pap, for one, does not believe that we must take this step. Commenting on Kant's reading of Hume he writes,

> ...Kant must have been misled by Hume's use of the word "contradiction." From the fact that Hume says that p cannot be denied without contradiction one cannot infer that Hume holds p to be analytic in Kant's sense; for Hume used "contradiction" in a wider - or perhaps we should say looser-sense than "formal contradiction." Whatever is inconceivable is contradictory, in Hume's terminology...[28]

Everything turns on how we are to understand the term 'conceivable'. As Pap notes, the term "is sometimes used in the sense of 'logically conceivable' (self-consistent) and sometimes in the narrower sense of 'intuitively conceivable'."[29] He assumes that Hume is using the term in the latter sense, and he justifies this by arguing that Hume "identifies (unlike, for example, Leibniz) the <u>conceivable</u> with the <u>imaginable</u>; 'idea' is in Hume's usage synonymous with 'mental image'."[30] There can be no question but that Hume continued to view the ideas of geometry as images and that we cannot conceive these ideas apart from our capacity to form images of them. As one commentator has put it,

> Hume accepted Berkeley's sensory criterion of conceivability ... For both Hume and Berkeley, what we can conceive or imagine is no different from what we sense. All ideas, as Hume frequently reminds us, are derived from corresponding impressions which they resemble...[31]

In the <u>Treatise</u> Hume acknowledged that we make both intuitively and demonstratively certain judgements in geometry. He allowed, for example, that our idea of a straight line is clear and determinate enough to assure us that two straight lines which incline upon each other at a sensible angle cannot, upon meeting, become one. Though he did not believe that the denial of this proposition leads to a formal contradiction of the idea of a straight line with itself, he certainly did believe that it is "obviously absurd and repugnant to our clear ideas" (T. 51). More precisely, he held it to be repugnant and contradictory to what we plainly discover to be the case when we consider and compare our clearest geometric ideas in the imagination. We have been told that "where two right lines incline upon each other with a sensible angle, <u>but 'tis absurd to imagine them to have a common segment</u>" (T.51, my emphasis). The denial of this proposition is inconceivable in that we easily discover that we cannot imagine two such lines concurring. It contradicts what we plainly discover to be the case when we consider and compare our ideas in the imagination. The denial of this proposition is repugnant to our clear ideas in a manner and depth that the denial of the proposition, <u>the sun will rise tomorrow</u>, is not.

If we follow Pap's lead, it becomes clear that there is no reason whatsoever to assume that Hume has turned his back on his argument in the <u>Treatise</u> regarding the synthetic nature of the propositions of geometry. If we interpret the troublesome word 'contradiction' in a looser sense than 'formal contradiction,' taking it to refer to that which is repugnant to what we discover to be the case with respect to our clearest geometric ideas, then the positions of the Treatise and Enquiry can be seen to go hand in hand as far as this particular issue is concerned. There is, of course, a profoundly significant point of difference in that in the Enquiry Hume seems to be conceding to the conventional wisdom that the axioms of Euclidean geometry possess full certainty. Even so, this hardly warrants that we now assume that Hume views these propositions as analytic. He would surely still wish to argue that the idea of a straight line is the idea of a particular figure or quality of a line and that no matter how long we reflect upon the idea of this particular figure, we shall not find the least reason for assuming that straight lines cannot concur.

There appears, then, little to recommend the view that Hume's position in the Enquiry is that the denial of an axiom or theorem of Euclidean geometry entails a formal contradiction. If we insist on interpreting Hume's position in accordance with the Kantian distinction between analytic and synthetic propositions then the least objectionable way to do this is to continue to assign to Hume the view that the proposition of geometry are synthetic. As we have seen, this reading can easily be rendered consistent with the spirit of the text and enables us to preserve some continuity with the Treatise on a matter of profound significance for Hume's philosophy.

III

Geometry, Certainty, and Precision

At this point we may begin to wonder whether the brief remarks at E.25 must be interpreted as signalling a significant break from the position of the Treatise. Though it is commonly accepted that Hume intended to restore full certainty to the propositions of geometry, there is room for controversy. For one thing, the grouping of geometry, algebra, and arithmetic under the category of Relations of Ideas does not in itself signal a shift in view away from the Treatise. The distinction in the Enquiry between Relations of ideas and Matters of Fact corresponds to the distinction in the Treatise between those relations "such as depend entirely on the ideas, which we compare together [Relations of Ideas], and such as may be chang'd without any change in the ideas [Matters of Fact]" (T.69). In both works geometry, arithmetic, and algebra are placed under that category of relations which can be the object of knowledge and certainty. Hume, we have argued, allows that the inexact propositions of geometry$_H$ are no less certain than the exact propositions of arithmetic. Only if we assume that Hume is now assigning both precision and certainty to the propositions of geometry as they were traditionally conceived, can we conclude that his comment here represents a definite break from the position of the Treatise. The problem is that Hume's remarks at E.25 are somewhat ambiguous on this point.[32]

Hume's claim that the truths demonstrated by Euclid do not depend upon what is anywhere existent in the universe is often taken as a sure sign that E.25 represents a significant shift in his thinking about geometry. Wright, for example, sees this as showing that

> ...Hume clearly abandoned the view put forward in the Treatise that the certainty of geometrical propositions depends upon the real existence of their objects: but he did not deny that these propositions actually do apply to the real world of objects. He merely reverted to the view, held by many philosophers of his day, that geometrical propositions are necessary, and that their necessity does not depend upon the existence of geometrical objects. Nevertheless, he assumed that Euclidean geometric relations apply to all actually extended objets...[33]

It is interesting that Wright should read all this into Hume's comment, given that he is otherwise of the opinion that "Hume's remarks on geometry in the Enquiry are so brief that it is difficult to tell what he believed."[34] While there is much to be said in favor of the interpretation here advanced, we cannot yet claim to have before us a clear and unambiguous statement of a significant break from the position of the Treatise. Did Hume wish to say that even the certainty of the axioms does not depend on the existence in the universe of objects which correspond exactly to their ideas? Or did he only mean to say that the theorems which we demonstrate about circles, triangles, etc., do not depend on their objects existing anywhere outside the realm of ideas? He only explicitly mentions the latter, and this is what Newman takes to be his intention. Consequently, she sees nothing here which signals a clear shift away from the view of the Treatise. She correctly observes that Hume's position regarding the sensible origin of our geometric ideas "in no way commits him to holding all geometrical concepts to be exemplified in nature."[35] The geometer "can construct further concepts as figures in perceptual space, utilising those primary concepts which are empirically given."[36] The claim that the truths which geometers demonstrate do not require that these ideas are exemplified anywhere in the universe is perfectly consistent with the teachings of the Treatise.

While we can agree that the language of E.25 is such as to suggest that Hume has indeed moved away from some of the teachings of the Treatise, the fact remains that it is somewhat ambiguous as to the precise nature of this shift in view. What we need is a clear statement of Hume's intentions which can serve as a foothold for interpreting E.25. Fortunately, such a statement can be found.

> The Great Advantage of the mathematical sciences above the moral consists in this; that the ideas of the former, being sensible, are always clear and determinate, the smallest distinction between

> them is immediately perceptible, and the same terms are still expressive of the same ideas without ambiguity or variation. An oval is never mistaken for a circle, nor an hyperbola for an ellipsis. The isosceles and scalenum are distinguished by boundaries more exact than vice and virtue, right and wrong. If any term be defined in geometry, the mind readily, of itself substitutes on all occasions the definitions for the term defined, or even when no definition is employed, the object itself may be presented to the senses, and by that means be steadily and clearly apprehended...(E.60)

Hume's position here could hardly stand in greater conflict with that of the Treatise, save for the fact that he continues to view the ideas of geometry as being of a wholly sensible nature. Incredibly, we are told that ideas of geometry are as precise as those of arithmetic. Whereas we were formerly told that "we are incapable of telling, if the cases be in any degree doubtful, when such particular figures are equal; when such a line is a right one, and such a surface a plain one" (T.51), we are now told that the smallest distinction between the ideas of these figures is immediately perceptible. Instead of telling us that "we can form no idea of ... these figures , which is firm and invariable" (T.51), we are informed that "the same term is still expressive of the same ideas, without ambiguity or variation."

Hume was so emphatic in the Treatise in denying that the ideas of geometry are determinate (precise or exact), according to our actual method of conceiving them, that his comments at E.60 can hardly be dismissed as a mere slip of the pen. Of course, the remarks at E.60 could easily be rendered consistent with the teachings of the Treatise if it could be argued that the only mathematical sciences Hume had in mind here were algebra and arithmetic. However, the only examples of clear and determinate mathematical ideas which Hume gives us are taken from geometry. What clearer sign of a shift in view could we ask for?

Hume, though, does tend to downplay things a bit. He does not mention examples which would immediately catch the eye of the reader who is familiar with the discussion of geometry in the Treatise. He does not, for example, state that the ideas of straight line and curve are so clear and determinate that the least difference between them is immediately perceptible - though that is clearly what he is implying. Instead of telling us that an isosceles triangle is never mistaken for a scalene triangle, he states only that the "two are distinguished by boundaries more exact than vice and virtue, right and wrong." But Hume is committed to a much stronger thesis; namely, that the smallest differences in length are immediately perceptible. Stated thusly, it becomes immediately evident that a clear and determinate standard of equality is being presupposed.

The conclusion seems unavoidable. In an astonishing about face, Hume has conceded to the conventional wisdom that the ideas which are most essential to geometry, including those of straight line and equality, are clear and determinate and afford us an immediate and full certainty regarding the truth of the axioms. The relevance of this remarkable shift in view for the interpretation of Hume's intentions at E.25 is obvious. This is precisely the shift Hume would have had to make if his intention was to break from the Treatise and place geometry on an equal footing with the exact science of arithmetic. Here, then, we have the foothold we were seeking and solid evidence that the philosophy of geometry briefly outlined at E.25 represents a radical break from the position of the Treatise. Geometry is no longer described as "the art, by which we fix the proportions of figure"(T.71), but as a science alongside of arithmetic in its precision and certainty.

Surprisingly, commentators have by and large paid scant attention to this shift in view to clear and determinate ideas. As a consequence, little attention has been paid to an important source of tension in the Enquiry between Hume's view of geometric ideas as clear and determinate and his continued acceptance of the copy principle as a fundamental principle of philosophy. This tension becomes evident the moment we challenge Hume to show us the impression (appearance) of a straight line which can be comprehended in so clear and determinate a manner that we can immediately perceive that it is without any ambiguity or variation, an appearance which we immediately perceive to be without the least possible curvature. Hume cannot just turn his back on his argument in the Treatise and now casually state, as if it were a matter of little controversy, that our faculty of sense is able to perceive clearly even the smallest degree of difference between a straight line and a curve, and that the imagination can readily frame for itself the clear and exact image of a line devoid of all possible curvature.

Nor can Hume simply set aside his view in the Treatise that the empirical faculties of sense and imagination suffer the defect of giving us a disproportioned image of things and represent as small and uncompounded what really contains a vast number of parts. Indeed, it is difficult, if not impossible, to believe that Hume intended to break from this position and accept that these faculties are so perfectly precise and exact that they can clearly and immediately detect the smallest possible difference between the impressions of a curve and a straight line, a circle and an oval, and an isosceles and a scalene triangle. Such a thesis would openly invite ridicule. As far as concerns the actual impressions of these figures, Hume surely still wished to hold that the faculties of sense and imagination are incapable of achieving any such precision and exactness. What he no longer wished to hold was the conclusion he formerly believed to follow immediately from this; namely, that the ideas of geometry must themselves be too loose and indeterminate to afford as a full certainty regarding the propositions of that discipline.

Hume really had no alternative but to continue to hold that even if there existed in nature a perfect Euclidean straight line, our senses, though aided by whatever art and instrument available, could never reveal this fact to us. In this respect it is not surprising that we find him suggesting, at E.25, that the ideas of geometry do not depend upon what is anywhere existent in the universe. To be sure, Hume must still hold that clear and determinate geometric ideas, including those which are primary, are somehow traceable back to sensible appearances. But none of these ideas presuppose that there actually exists in nature any figure or proportion which exactly corresponds to them and from which they are precisely copied.

In keeping with the spirit of his former work, Hume boldly challenged those who dismissed his copy principle to produce

> ...that idea which, in their opinion, is not derived from this source. It will be incumbent on us, if we would maintain our doctrine, to produce the impression, or lively perception, which corresponds to it. (E.19-20)

Hume may have been confident that his philosophy could stand up to any such challenge and shoulder the requisite burden of proof. But the fact remains that he did not take up the mantle and produce the impression which corresponds to the intuitively clear and determinate geometric idea of a perfect straight line or of perfect equality. And yet, as Hume well knew, the conventional wisdom would have offered these ideas as obvious counter-examples. One would certainly think that if Hume could have brought these ideas into harmony with his copy principle, he would have done so. After all, in so doing he would have robbed the rationalist critics of empiricism of one of their most powerful objections, to say nothing of correcting the erroneous view of the Treatise. It is significant that he does not so much as drop a hint.

In the Enquiry Hume repeated his claim that there is "one contradictory phenomenon, which may prove that it is not absolutely impossible for ideas to arise, independent of their impressions" (E.20). He was, of course, referring to the example of the missing shade of colour, an example he continued to view as "so singular, that it is scarcely worth our observing, and does not merit that for it alone we should alter our general maxim" (E.21). In spite of this acknowledged counter-example, we are again told that by applying the copy principle to all those disputes involving abstract ideas, we bring these ideas "into so clear a light we may reasonably hope to remove all dispute which may arise, concerning their nature and reality" (E.22). With the rationalist philosophy firmly in mind, Hume argues that if we understand by innate "what is original or copied from no precedent perception, then we may assert that all our impressions are innate, and our ideas are not innate" (E.22 fn). However, Hume has yet to bring the geometric ideas into so clear a light that we can understand how he hoped to hold them to be determinate and exact enough to yield us a full certainty regarding the axioms of geometry and yet avoid the received view of his day that they are innate. Suggesting, as he does at E.25, that the geometric ideas do not depend upon what is anywhere existent in the universe, does more to cast shadows than light.

Hume can hardly just accept the geometric ideas as a contradictory phenomenon which is "so singular, that it is scarcely worth our observing, and does not merit for it alone we should alter our general maxim." It might seem rather surprising, then, that Hume says nothing regarding how he proposed to render his shift in view regarding the nature of these ideas consistent with his first principle. Perhaps this was one of those problems Hume believed would be best dealt with in a separate work dedicated solely to the philosophy of geometry. Of course, we can only speculate as to whether he actually took up this task in his later work, and even if he did, whether it was criticism of his attempted solution that convinced him not to let this work on geometry see the light of day. However, one point seems quite clear. In breaking from his view in the Treatise and arguing that the ideas of geometry are so clear and determinate as to afford us an infallible assurance of the axioms, Hume has painted himself into a corner and there appears to be no ready exit in sight.

IV

E.25 Revisited

Given that the passage at E.25 has been at the center of most discussions concerning the philosophy of geometry outlined in the Enquiry, it should come as no surprise that it has been subject to various interpretations. For the most part these interpretations are based on a reading of the Enquiry which either largely glosses over or completely ignores the far-reaching significance of the shift to clear and determinate ideas. To further clarify our own position in relation to this shift in view, we shall consider briefly the positions of three philosophers, Zabeeh, Newman, and Van Steenburgh, with respect to the controversial passage in question.

Zabeeh has placed considerable importance on rendering the latter half of E.25 consistent with Hume's general empiricist outlook. His analysis takes place in the context of a broader work which aims to

> ...reveal the main difficulty, not only with Hume's general principles, but also with any theory which endeavors to provide a true account of the meaning of the host of concepts and ideas which can not, without much sacrifice, be reduced to simple data of sensory experience.[37]

For Zabeeh, the mathematical sciences represent one such class of ideas. He believes the greatest difficulty facing those who seek to understand Hume's treatment of mathematics, including geometry, is the "vagueness and vacillating position of [his] account."[38] This circumstance, he tells us, has been the cause of "some conflicting opinions of Hume's critics concerning his theory of analytic-synthetic, and his view on the nature of arithmetic and geometry."[39] To illustrate his point, he focuses on a conflict of opinion between two German commentators, J.J. Bauman and Rudolph Metz, concerning the interpretation of E.25. Bauman argues that Hume cannot mean that the idea of a triangle is "a pure conception of the mind, an idea without impression or sensation, since that is downright ruled out by his uppermost principle."[40] Accordingly, Bauman reasons that Hume must either mean "that mathematics will remain an eternal truth even if all triangles, after having given the sensory impression of the idea, disappeared from the world"[41] or "that once we presuppose the idea as copied from the perception, we don't have to worry about the sensation or impression any more."[42]

Metz, on the other hand, found the various attempts to bring the problematic claim at E.25 "in agreement with Hume's empiricism and thus to forgo an acceptance of a shift of viewpoint from the earlier to the later work ... too artificial and too little based on Hume's text."[43] He believed that this passage represents an important place in Hume's system where "the sensualistic impressionism is in fact broken and relinquished and Hume makes no attempt to link this part of his teaching, even if only externally, to the whole."[44]

It is Zabeeh's contention that the situation here is such as to call "for a thorough investigation of Hume's position."[45] He goes so far as to maintain "that the settling of this issue is germane to a clear understanding of Hume's entire theory of knowledge."[46] Near the end of his work, after giving a brief overview of Hume's philosophy of geometry, Zabeeh believed himself to be in a position to "safely interpret Hume's ambiguous assertions in the Enquiries which produced divergent opinions among his critics."[47] He is most concerned with interpreting the ambiguous assertion that the truths of Euclid are discovered without dependence on what is anywhere existent in the universe.

> By making this assertion, he does not want to imply that "though there never were a circle or triangle even for Euclid" the geometrical truths discovered by him would forever retain their certainty and evidence, but that once Euclid observed a geometrical figure and discovered certain relations, "the conclusions which [he] draws from considering one circle are the same which [he] would form upon surveying all the circles in the universe."(E.43)[48]

Zabeeh has presented an interpretation which renders the latter half of E.25 consistent with the letter of the copy principle. However, his reading appears to fall squarely under that category of interpretation which is "too artificial and too little based on Hume's text." And, in fact, Zabeeh has been so criticized. One critic has objected that Zabeeh has presented "a tortured interpretation of a key Passage in the Enquiry in order to render Hume consistent."[49] Another has objected that "whenever possible Hume should be taken at his word: observation of figures is not a truth condition for properties of geometry."[50] One is naturally inclined to think that if Zabeeh's interpretation were what Hume really wished to say, he would have said it and not left himself open to an almost unavoidable misunderstanding on so important a matter. As Wright observes,[51] Hume has all but adopted verbatim the language Descartes employed in his fifth Meditation when discussing essentially the same basic issue.

> ...When I imagine a triangle, although there may nowhere in the world be such a figure outside my thought, or ever have been, there is nevertheless in this figure a certain determinate nature, form, or essence which is immutable and eternal ... as appears from the fact that diverse properties of that triangle can be demonstrated, viz. that its three angles are equal to two right ones...[52]

To Wright's observation we might add that Hume has adopted language which bears a marked resemblance to that employed by Locke in discussing this matter.

> ... The mathematician considers the truth and properties belonging to a rectangle or circle only as they are in idea in his own mind. For

> it is possible he never found either of them existing mathematically, i.e. precisely true, in his life. But yet the knowledge he has of any truths or properties belonging to a circle, or any other mathematical figure, are nevertheless true and certain, even of real things existing: because real things are no further concerned, nor intended to be meant by any such propositions, than as things really agree to those archetypes in his mind.[53]

He went on to add,

> ...All the discourse of the mathematicians about the squaring of a circle, conic sections, or any other part of mathematics, concern not the existence of any of those figures: but their demonstrations, which depend on their ideas, are the same, whether there be any square or circle existing in the world or no...[54]

Zabeeh would have us believe that Hume was so careless in expressing his position on a matter of such fundamental importance for his philosophy that he chose words which the mathematicians and philosophers (both rationalists and empiricists) of his day would have interpreted to mean something wholly different from what he intended to say. The more plausible interpretation would be that Hume found it necessary to concede this point to the conventional wisdom, and, like Locke, believed that it could somehow be rendered consistent with the demands of empiricism. However, as, tortured as Zabeeh's interpretation may seem, we must bear in mind that it is arrived at in a serious attempt to render

Hume's remarks in the latter half of E.25 consistent with his continued emphasis upon the copy principle and Berkeley's doctrine. Zabeeh might well ask his critics how Hume could have possibly hoped to argue that the ideas most essential to geometry are sensible and in basic agreement with his first principles if they do not depend upon what is anywhere existent in the universe. It might be argued that a tortured interpretation is worth pursuing if it can help bring Hume's new position in the Enquiry into harmony with his first principles.

Unfortunately, Zabeeh's interpretation does not take us very far and in the final analysis has little to recommend it. Zabeeh recognizes that the passage at E.25 represents a break from the Treatise in that geometry is now placed on an equal footing with arithmetic and its "relations, if true, are exact and universal."[55] However, he has failed to follow through and recognize the shift in view to clear and determinate ideas which underlies the move to include geometry as one of the sciences "in which we can carry on a chain of reasoning to any degree of intricacy, and yet preserve a perfect exactness and certainty" (T.71). Though he does take notice of the relevant passage at E.60, Zabeeh makes no mention of how strongly it contrasts with the central teaching of the Treatise. He simply observes that Hume here continues to hold to the view of the Treatise that the definitions of geometrical terms "are not verbal definitions; they are rather descriptions of physical space."[56] As a consequence, he fails to appreciate the full extent of Hume's break from the Treatise.

Zabeeh sees Hume's isolated remark that "The conclusions which [reason] draws from considering one circle are the same which it would form upon surveying all the circles in the universe" (E.43) as holding the key to understanding the last two sentences of E.25. What Hume meant to say here, we are told, is that the truths which Euclid demonstrates to hold for a particular existing circle hold for all circles which may exist. The demonstration itself presupposes only the existence of a particular impression (idea), and thus remains valid even if there does not exist anywhere in the universe another circle. Given what he takes to be Hume's intentions at E.25, Zabeeh moves on to consider how he would respond to an obvious criticism.

> ...Suppose in observing a circle we discover that a point in the circumference of that circle is not exactly the same distance from the centre as the rest of the points in the circumference of the same circle: Is that peculiar relation true about all circles? But if someone objects that this particular is not a true circle, he already presupposes a definition of a circle which is not necessarily a description of any observed circle...[57]

Zabeeh's failure to recognize the shift to clear and determinate geometric ideas is evident. Viewed as an isolated remark, Hume's comment at E.43 can easily be rendered consistent with the Treatise. We would simply interpret E.43 as stating that the conclusions which reason draws from considering the idea of a particular circle are the same it would form, within certain limits of precision, upon surveying all the particular ideas which are designated by the general term 'circle'. In the Treatise the demonstrations of geometry were viewed as being "built on ideas, which are not exact, and maxims, which are not precisely true" (T.45). However, as Zabeeh has fully recognized, in the Enquiry the propositions of geometry are received as precise and universal maxims. What is being stated at E.43 is that the precise truths which reason draws from considering one clear and determinate idea of a particular circle are the exact same that it would form from examining any other clear and determinate idea which we designate by the general term 'circle'. Since we are dealing with clear and determinate ideas, we can immediately perceive the smallest difference between them. Unlike in the Treatise, the general term 'circle' "is always expressive of [exactly] the same ideas without ambiguity or variation." Thus, the position of the Enquiry is that the idea of a figure with all but one of its points an exactly equal distance from the center is never mistaken, by the geometer, for the idea of a circle. The criticism which Zabeeh envisages is based on a misunderstanding of the position of the Enquiry.

By failing to take proper notice of Hume's shift to clear and determinate ideas, Zabeeh is forced to add more twists and turns to his interpretation. He believes that

> In answer to these objections, no doubt Hume resort to his principle of analyticity ... The real reason for Hume's belief in the apodeictic certainty of mathematical truths, viz., that "the truths demonstrated by Euclid for ever retain their certainty and evidence," even if "there never were a circle or a triangle in nature," is that he appears to assume that we never allow such truths to be controverted by empirical evidence. That is to say, if perchance we find, by measurement, that the sum of the angles of a Euclidean triangle does not equal 180 degrees, either we say that we measured wrongly or we say that the triangle we have been measuring is not Euclidean.[58]

Even apart from our discussion in section II we may question whether it is reasonable to assume that this is anything like what Hume has in mind in the Enquiry. Is it really reasonable to assume that Hume now wishes to assert that a geometer can, in principle, demonstrate with full certainty that the sum of the angles of some actual triangle (appearance) is equal to 180 degrees and that we must necessarily assume that any measurement to the contrary is incorrect? Clearly not, unless we are prepared to assume that Hume completely lost his good sense. Yet, on Zabeeh's reading what alternative do we have? If Hume would have been willing to concede that some future measurement may force us to conclude that the sum of the angles of this actually existing triangle (or the copy we make of it in the imagination) is somewhat more or less than 180 degrees then we are back to square one. We would in effect be assuming that the idea of a triangle which is essential to the demonstration of the theorem under discussion is not dependent upon there actually existing such a figure (impression). Rather than pursue such a unflattering and tortured interpretation, let us acknowledge the shift to determinate ideas and simply take Hume at his word. Geometric ideas, though sensible, do not depend upon the existence of any object (impression) which exactly corresponds to them.

Newman believes that if we take Hume at his word at E.25, we fail to find any significant break from the teachings of the Treatise. She finds it noteworthy, though, that "there is no reference in the Enquiry to the absence of an exact standard of equality in geometry"[59] and grants that we may take this to mean that geometry is now "classed as a science alongside arithmetic and algebra."[60] She does not, however, believe that this carries anything like the significance it is generally accorded and most definitely does not signal that Hume intended any break from his earlier teachings regarding the status of the ideas and propositions of geometry.

Insofar as she argues that this shift in view does not represent any substantial shift away from the view of the Treatise regarding the synthetic nature of the truths of geometry, we are in agreement. However, she does not consider the shift to determinate ideas which the new standing of geometry demands. The reason, I believe, is rooted in her reading of the Treatise; specifically, her virtual identification of Hume's position with Kant's view of the propositions of geometry as synthetic a priori truths. This reading makes the move to place geometry on an equal footing with arithmetic seem a far less dramatic break from the Treatise than does our reading. She suggests that "the lack of any reference to the imprecision of geometry compared to arithmetic and algebra reflects Hume's relinquishment of the Treatise conception of a possible ideal precise geometry with which practical geometry stands contrasted."[61] This shift in view away from the theoretical/practical distinction is not taken as a matter of deep significance as far as the main teachings of the Treatise are concerned.

The relevant points of our disagreement with her reading of the Treatise (including the theoretical\practical distinction) have been discussed in detail in chapter 3, and we shall add nothing more here. We shall simply observe that the propositions of geometry which the Enquiry holds to be synthetic and certain are the exact and universal propositions of Euclidean geometry, and this is a far cry from the position of the Treatise.

Because she ignores the shift to clear and determinate ideas, Newman sees no problem in rendering "Hume's obscure comment that intuitively or demonstratively certain propositions 'are discoverable by the mere operation of thought, without dependence of what is anywhere existent in the universe'"[62] consistent with the Treatise. As we have seen, she believes that all Hume is committed to here is maintaining that the ideas of triangle, circle, etc., can be constructed out of the primary ideas which, she assumes, possess the same status as in the Treatise. She proceeds to argue that

> ... Such constructs would represent possible forms of natural objects even if no natural examples of them exist, and would serve to furnish information about the properties of sensible (perceptual)space...[63]

The problem with this interpretation is that it glosses over the fact that we are talking about clear and determinate forms of natural objects, and these were most emphatically dismissed in the Treatise as being mere figments of our imagination. Furthermore, Newman is assuming that Hume is keeping to the view of the Treatise and insisting that the primary ideas of geometry are nothing but copies of precedent impressions. However, Hume could just as well have stated that though there never were a perfect straight line in nature, the truths intuited by Euclid (e.g. two straight lines cannot be made to concur or enclose a space) would forever retain their certainty or evidence. The shift to determinate ideas required

Hume to concede to the conventional wisdom that the ideas most essential to geometry are archetypes which do not depend upon what is anywhere existent in the universe.

Van Steenburgh is one philosopher who has taken due notice of "Hume's shift in view in the Enquiries to clear, determinate ideas." [64] He argues that it allows us to read into Hume's later work a distinction between pure geometry (which deals exclusively with clear and determinate ideas and is a branch of mathematics) and applied geometry (which includes clear but indeterminate ideas and is a branch of empirical science). He maintains that a proper understanding of the significance of the shift to clear and determinate ideas allows us to see how the truths of geometry "remain truths whether or not anything in nature corresponds to them, i.e., geometry is not merely the science of measurement [as it was in the Treatise]."[65] In other words, Van Steenburgh is of the opinion that if we interpret E.25 in terms of clear and determinate ideas and the pure/applied geometry distinction which follows from it, we can offer a straightforward interpretation which enables us to take Hume at his word and avoid the extremes to which Zabeeh is driven.[66]

We can readily accept Van Steenburgh's claim that a distinction between pure and applied geometry is implicit in the shift to determinate ideas and that we must interpret E.25 as being about pure geometry. Nonetheless, our views are not in complete harmony. Van Steenburgh maintains that in the Treatise Hume accepted the view that we can frame for ourselves determinate ideas of straight line, circle, etc., but did not believe they were of any relevance to geometry. Thus, on his reading, the shift to clear and determinate geometric ideas does not represent anything like the radical shift in view we have depicted. He sees it, rather, as representing only a shift in view regarding the question as to whether clear and determinate ideas are of any relevance for geometry.

> Even in the Treatise, Hume argues for ultimate, *i.e.*, perfect sensible geometric standards ... However, in the Treatise Hume is still spellbound by the picture of geometry as a quasi-empirical science. Holding that where there are no differences or distinctions, determinate ideas cannot be of any use in drawing them, he continues to insist on generic ideas and the weakness and fallibility of geometry. The change in his view in the Enquiries consists in moving to clear and determinate ideas and reconstructing geometric propositions as true of the world just to the extent the world matches these clear and determinate ideas. Whether or not there is a match, the propositions remain certain.[67]

According to Van Steenburgh, implicit in the teachings of the Treatise is the view that "corrections up to a sensible limit yield perfect ideas."[68] It is Van Steenburgh's contention that in the Treatise Hume held that we may, through a process of correction, render our idea of a straight line truly clear and determinate. However, this reading seems very much at odds with his own dictum that "whenever possible Hume should be taken at his word." Hume has clearly stated that "an exact idea can never be built on such as are loose and indeterminate" (T.50) and that "we are not possess'd of any instrument or art of reasoning, which can secure us from all error or uncertainty" (T.48). Though Hume allowed that we can correct our impressions of a straight line and, with the aid of art and instrument, improve upon our method of distinguishing a straight line from a curve, he insisted that "we can give no perfect definition of these lines, nor produce any very exact method of distinguishing the one from the other"(T.49). This is a far cry from the view of the Enquiry that our geometric ideas are so clear and determinate that we can immediately perceive the smallest differences between them. To say the least, it would have been rather strange of Hume to concede to the conventional wisdom that we can indeed conceive for ourselves the idea of a perfect straight line which can afford us an infallible assurance that no two straight lines can concur or enclose a space and then turn around and argue that this idea is totally useless as far as geometry is concerned.

Contrary to Van Steenburgh, we have argued that the reason Hume introduced this talk of employing art and instrument to correct our original standards was to explain how mathematicians and philosophers came to deceive themselves into believing that they possessed clear and determinate ideas of a perfect straight line, circle, etc. However, there is no need to dwell any further upon our differences with Van Steenburgh with respect to the Treatise. In the Enquiry Hume has definitely made the shift to clear and determinate geometric ideas, and we may well wonder if he is now prepared to argue that these ideas have their origin in the process of correction described in the Treatise. The answer to this seems a clear no. We are told that the precision of our geometric ideas is easily and clearly intuited by the mind, and this is hardly characteristic of ideas which are generated by a fairly intricate process of corections.

If we were to assume that these ideas have their origin in this process then we would have to assume that only those who possessed the necessary art and instruments to bring the series of corrections to completion would be able to frame for themselves clear and determinate geometric ideas and reason accordingly. This most certainly would have excluded Euclid, and it is obvious that this is not what Hume had in mind. The only way he could avoid this would be to argue that the geometers of his day were in fact in possession of instruments and techniques of measuring which could secure them from all error and uncertainty. Surely, Hume was not about to embrace anything like this.

Van Steenburgh is correct in thinking that Hume must continue to conceive of the ultimate standard of straight line in terms of a finite sensible limit which our various corrections approach. What he has failed to appreciate is that, in the Enquiry, not only can we now conceive this idea

idea in a clear and determinate manner but also that this idea is comprehended independently of the actual series of corrections. The mind, Hume is now telling us, has the power of intuiting a priori the particular idea of a straight line which stands at the limit of our powers of sensibility and imagination. Hume can still maintain that since the ultimate standard of a geometrical figure "is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of" (T.51), but only if he is prepared to grant that these faculties are capable of determining far greater perfection than what he was willing to grant in the *Treatise*, a perfection which lies beyond what art and instrument can determine. The problem, of course, is that any such capability seems to be at complete odds with the atomistic philosophy of sense perception which lies at the heart of Hume's empiricism. The irony is that this has nowhere been more clearly and forcibly argued than in the *Treatise.*

V

Conclusion

We have yet to confront the question which has been leaping out at us for some time now. Why did Hume find it necessary to break from the position of the *Treatise* and concede to the critics of empiricism that the ideas most essential to geometry are exact and determinate according to our common method of conceiving them? Given the highly problematic nature of this radical shift in view, we can only assume that, upon reflection, Hume discovered that his position in the Treatise was fatally flawed, and in spite of its otherwise strong appeal, had to be abandoned. The question is, what was the flaw?

The answer, I believe, is not difficult to find. The argument which led from the copy principle to the indeterminate nature of our geometric conceptions was, in itself, impeccable. However, in order to defend his controversial analysis against the conventional wisdom of his day, Hume had to take the extraordinarily bold step of arguing that, throughout history, even the most astute mathematicians and philosophers were fundamentally confused in their thinking with respect to the ideas most essential to geometry. Specifically, he found it necessary to argue that these thinkers were guilty of mistaking certain distant, obscure and incomprehensible figments of their imagination for intuitively clear and determinate ideas which were absolutely essential to geometry. Ultimately, everything depended upon Hume being able to offer a plausible account of this most incredible error.

In our analysis, Hume failed miserably. Given the weak and superficial nature of his account, it is not unreasonable to assume that Hume, upon a more strict review of his argument, found his reasoning very defective. This being the case, Hume would have been forced to conclude that he had failed to bring the ideas of geometry into harmony with his copy principle and remove what had long been viewed as one of the most formidable obstacles confronting the philosophy of empiricism. We may well suppose that Hume tried desperately to salvage the basics of his position in the Treatise but found that he could not find a plausible argument which would allow him to dismiss the seemingly intuitive and incontestable conceptions of geometry as obscure and useless fictions. So long as there was the slightest hope of keeping to the view that the ideas of geometry are essentially indeterminate in the mind's conception of them, Hume would surely have clung to it. Understandably, he came to see the situation as hopeless.

In spite of his best efforts, Hume found it impossible to deny the determinate nature of our geometric ideas. He no doubt found that the longer he subjected his idea of a straight line to inspection, the more absurd the position of the Treatise appeared. The rationalists had long insisted that the idea of a perfectly straight line was so intuitive and immediate that it could be readily comprehended by the common understanding of mankind. This, they maintained, was so evident as to be beyond any shadow of doubt. By the time of the Enquiry, Hume was prepared to concede this point. He was willing to concede that the mind can frame for itself an idea of a straight line which is so perfect that it cannot be conceived as containing within itself the least degree of curvature. Having made this move, he was no longer in a position to challenge the mathematician as to how he could know with certainty that even the most obvious and "vulgar" propositions of geometry are precisely true. He was not about to argue that the mathematician was fundamentally confused in thinking that this idea could give them a perfect assurance that no two straight lines can concur or enclose a space. He thus found himself forced to concede that geometry was no less a precise and certain science than arithmetic and that its ideas are not mere copies of impressions. No doubt, Hume was beginning to appreciate why geometry had traditionally been viewed as posing such an insurmountable obstacle for empiricism.

Chapter 6

NOTES

1). Immanuel Kant, Prolegomena to Any Future Metaphysics, p.17
2). John P. Wright, The Sceptical Realism of David Hume, p.99
3). Immanuel Kant, p.17
4). R.H. Atkinson, "Hume on Mathematics", p.133
5) James Steiner, "Kant's Misrepresentations of Hume's Philosophy of Mathematics in the Prolegomena", Hume's Studies, p.400
6). ibid., p.403
7). ibid., p.404
8). James Noxon, Hume's Philosophical Development, p. 114
9). ibid., p.114
10). ibid., p.115
11). John Passmore, Hume's Intentions, rev. ed. (New York: Basic Books, Inc., 1968), p.16
12). Letters of David Hume, ed. J. Y. Greig (Oxford: Clarendon Press, 1932), II, Letter 465, p.464-65
13). It was fear of prosecution for blasphemy that motivated Hume to withdraw these two essay from publication.
14). Anthony Flew, David Hume, Philosopher of Moral Science, (New York: Basil Blackwill, 1986), p.46
15). ibid., p.47
16). Hans Reichenbach, The Rise of Scientific Philosophy, p.86
17). Hilary Putnam, "The Analytic and the Synthetic," Minnesota Studies in the Philosophy of Science, eds. Herbert Feigel and Grover Maxwell (University of Minnesota Press,1962), p.376
18). Anthony Flew, Hume's Philosophy of Belief, 3rd imp. (New York: Humanities Press, 1969), p.64
19). ibid., p.65
20). ibid., p.66
21). ibid., p.66
22). ibid., p.66
23). Anthony Flew, David Hume, Philosopher of Moral Science, p.47
24). Rosemary Newman, " Hume on Space and Geometry," p.28
25). ibid., p.28
26). ibid., p.25
27). 22). Farhang Zabeeh, Hume: Precursor of Modern Empiricism, p.141
28). Arthur Pap, Semantics and Necessary Truth (New Haven: Yale University Press, 1958), p. 70-71
29). ibid., p.75
30). ibid., p.75

31).John P. Wright, Hume's Sceptical Realism, p.94
32).The fact that Hume refers to the Pythagorean Theorem as a demonstratively certain proposition does not itself signal a break from the Treatise. We would first have to know if he is claiming that this proposition is precisely true. Hume could classify this proposition as a theorem of geometry$_H$ and still refer to it as demonstratively certain. Similarly, the fact that Hume speaks of the "truths demonstrated by Euclid" does not decide the issue. In the Treatise Hume could argue that, strictly speaking, the truths which Euclid demonstrated are the truths of geometry$_H$. It was Hume's opinion that Euclid, like virtually all geometers, mistakenly believed that the truths which he demonstrated were precise as well as certain. We are looking for a clear sign that Hume wishes to place geometry on an equal standing with arithmetic with respect to precision as well as certainty.
33). John P. Wright, p.99
34). ibid., p.99
35). Rosemary Newman, p.29
36). ibid., p.29
37). Fahrang Zabeeh, p.23
38). ibid., p.21
39). ibid., p.21
40). ibid., p.21
41). ibid., p.21
42). ibid., p.21
43). ibid., p.22
44). ibid., p.22
45). ibid., p.22
46). ibid., p.22
47). ibid., p.142
48). ibid., p.142
49). Robert A. Imlay, " Hume on Intuitive and Demonstrative Inference," Hume Studies, I, (Nov.1975), p.38
50). E. W. Van Steenburgh, " Hume's Geometric Objects," p.62
51). John P. Wright, p.98
52). Rene Descartes, The Philosophical Works of Descartes, trans. by Elizabeth S. Haldane and G.R.T. Ross, (Cambridge: Cambridge University Press, 1975), vol. I, p.180
53). John Locke, An Essay Concerning Human Understanding, annotated and collated by Alexander Campbell Fraser (New York: Dover Publications,inc., 1959), II, chp. 4, no.6 (p.231)
54). ibid., chp.4, no.6 (p.233)
55). Fahrang Zabeeh, p.140
56). ibid., p.142
57). ibid., p.143
58). ibid., p.143
59). Rosemary Newman, p.28
60). ibid., p.27
61). ibid., p.28

62). ibid., p.27-28
63). ibid., p.29
64). E.W. Van Steenburgh, p.63
65). ibid., p.63
66). As Van Steenburgh notes, Zabeeh does not believe that Hume intended any distinction between pure and applied geometry.(See
Van Steenburgh p.63 and Zabeeh p.)
67). ibid., p.67
In defense of his claim that,in the Treatise, Hume argues for ultimate, and hence, perfect, sensible geometric standards Van Steenburgh cites the following passage: "As the ultimate standard of these figures is deriv'd from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of; since the true perfection of anything consists in its conformity to its standard."(T.51) However, on our reading of the Treatise it is clear that Hume is here arguing against perfect standards. He is arguing that since our sensible faculties cannot be the source of any true perfection in these matters, the ultimate standards we may frame for ourselves will always be somewhat indeterminate.
68). ibid., p.64

Chapter 6

Infinite Divisibility Revisited

I

Introduction

Like virtually all commentators, Atkinson acknowledges that there are some obvious differences between the Treatise and Enquiry with respect to their treatment of geometry. Specifically, he notes that "no distinction in point of precision and exactness is allowed between arithmetic and algebra and geometry, and the last is now alleged to be a science."[1] We have argued that this represents a significant break from the view of geometry presented in the Treatise. Atkinson, however, insists that "Hume betrays no awareness that the view of mathematics in the Enquiry is significantly different from that in the Treatise."[2] In defense of his reading he points to the fact that the "doctrine of infinite divisibility is attacked in both works."[3]

Curiously, Atkinson makes no attempt to move beyond this remark and spell out how it supports his rather controversial reading of the Enquiry. However, it is not difficult to see what his rationale must be. As Newman correctly observes, there is "a close relationship between Hume's approach to geometry and his concern with refuting the doctrine of infinite divisibility."[4] In fact, the relationship here is so close that a significant shift in Hume's thinking with respect to the status of geometry, such as we have suggested, would clearly necessitate an equally significant shift in his attack against the doctrine of infinite divisibility. Atkinson seems to believe that since Hume continued to attack the principle of infinite divisibility in the Enquiry, he could not have considered his shift in view regarding the status of geometry to represent a significant break from the position of the Treatise.

In order to argue this point, Atkinson must show that there is no significant difference between the line of attack employed in the Treatise and that employed in the Enquiry. This he does not do. He merely assumes that Hume's position in the Enquiry concerning the refutation of infinite divisibility "is simply a shortened and simplified version of that in the Treatise."[5] This, however, is not something which can be gleaned straightaway from the text. As Newman herself notes, "although Hume retains his Treatise antagonism to infinite divisibility, he does not repeat any of his earlier arguments against the thesis."[6]

Fogelin too recognizes a difference between the two works in their treatment of infinite divisibility. He informs us that "Hume's fascination with the problem of infinite divisibility is carried over to the Enquiry, where the discussion is curious and, in fact, not altogether forthcoming."[7] But why is there this difference between the two works? Why doesn't Hume repeat any of his earlier arguments against infinite divisibility? Could it be that he no longer considers these arguments to be valid? Could it be that the shift in view at E.25 regarding the status of geometry necessitated a break from the line of attack employed in the Treatise against the doctrine of infinite divisibility? These are important questions but Atkinson, like most commentators[8], fails to give them due consideration.

In order to complete our analysis of the Enquiry, we shall focus our attention on Hume's all too brief discussion of infinite divisibility and attempt to assess it in light of our findings in chapters 3 and 5. We shall argue that there is a very definite connection between Hume's change of opinion regarding the status of geometry as a perfect and infallible science and the fact that the discussion of infinite divisibility in the Enquiry does not appear forthcoming and does not repeat the line of attack employed in the Treatise. In short, we shall argue that, in placing geometry on an equal standing with arithmetic, Hume was forced to reject the strategy he adopted in the Treatise for refuting the doctrine of infinite divisibility. Though the logic of his empiricism would not allow him to abandon his crusade against this doctrine, he was forced by his shift in view at E.25 to go back and consider the task of refuting it anew.

Hume, unfortunately, had very little to say with respect to the new line of attack he proposed to adopt. In fact, he did not go beyond dropping a rather brief and vague hint as to how he proposed to deal with the difficulties surrounding the controversial doctrine. This, however, will prove no great obstacle. It shall be our argument that there was really only one option open for Hume to pursue if he wished his attack to be consistent with his new view of geometry. In short, we shall argue that he had little choice but to embrace as the centerpiece of his philosophy of geometry the very standard of equality which he dismissed in the _Treatise_ as useless and incomprehensible. As we shall see, in the end this move accomplished little except to create further tension between Hume's philosophy of geometry and his copy principle.

II

A New Standard of Equality

As Hume well knew, the conventional wisdom among philosophers and mathematicians had long held that the infinite divisibility of extension was supported by some of our most straightforward geometric reasoning. Accordingly, it was held to be as firmly demonstrated as a truth of geometry could possibly be. What Hume required was a philosophy of geometry which could explain "why geometry fails of evidence in this single point, while all its other reasoning command our fullest assent and approbation" (T.52). In the _Treatise_, of course, Hume believed he had hit upon a solution which would allow him to assent to the basic soundness of our geometric mode of reasoning and yet refute the mathematical arguments for infinite divisibility.

> ...none of these demonstrations can have sufficient weight to establish such a principle, as this of infinite divisibility; and that because with regard to such minute objects, they are not properly demonstrations, being built on ideas, which are not exact, and maxims, which are not precisely true. When geometry decides any thing concerning the proportions of quantity, we ought not to look for the utmost _precision_ and exactness. None of its proofs extends so far... (T 44-45).

Though Hume continued, in the _Enquiry_, to hold to the view that the doctrine of infinite divisibility is a metaphysical pretension which has no place in mathematics, his shift in view regarding the precision of geometry forced him to abandon this line of attack. The demonstrations of geometry were now said to be based on ideas and truths which are no less certain and precise than those of arithmetic. In granting that the mathematician can demonstrate that the sum of the angles of a triangle is _precisely_ equal to the sum of two exact right angles, the _Treatise_ refutation of the arguments in support of infinite divisibility was effectively undermined. Naturally, this point was not lost on Hume.

> ...But what renders the matter more extraordinary, is, that these seemingly absurd opinions are supported by a chain of reasoning, the clearest and most natural; nor is it possible for us to allow the premises without admitting the consequences. Nothing can be more convincing and satisfactory than all the conclusions concerning the properties of circles and triangles; and yet, when these are once received, how can we deny, that the angle of contact between a circle and its tangent is infinitely less than any rectilinear angle, that as you may increase the diameter of the circle _in infinitum_, this angle of contact becomes still less, even _in infinitum_... The demonstration of these principles seems as unexceptionable as that which proves the three angles of a triangle to be equal to two right ones, though the latter opinion be natural and easy, and the former big with contradiction and absurdity. Reason here seems to be thrown into a kind of amazement and suspense, which, without the suggestions of any sceptic, gives her a diffidence of herself, and of the ground on which she treads. She sees a full light, which illuminates certain places; but that light borders upon the most profound darkness. And between these she is so dazzled and confounded, that she scarcely can pronounce with certainty and assurance concerning any one object.(E.156-57)

We do not find anything quite like this in the _Treatise_. In fact, we find just the opposite. In the _Treatise_ Hume was quite confident that he could easily expose the error in the geometric demonstrations which purported to establish the infinite divisibility of extension; including those "which are deriv'd from the <u>point of contact</u>" (T.53). He challenged the mathematician "to form as accurately as possible, the ideas of a circle and a right line" (T.53) and then asked, "if upon the conception of their contact he can conceive them as touching in a mathematical point, or he must necessarily imagine them to concur for some space" (T.53). Hume was confident that no matter which way he chooses, the mathematician "runs himself into equal difficulties" (T.53) and is forced to conclude a finite extension is only finitely divisible.

Suppose, for example, that the mathematician "affirms, that in tracing these figures in his imagination, he can imagine them to touch only in a point" (T.53). In that case, Hume reasoned, he must allow that extension is composed of distinguishable indivisible parts, and this, he insisted, implies the finite divisibility of extension. But, as Hume well knew, "there are few or no mathematicians who defend the hypothesis of indivisible points" (T.45). Since Hume was not about to let the mathematician claim that he can actually conceive of the circle and its tangent touching in some infinitesimally small space, the only alternative he saw open to the mathematician was for him to own up to the fact that he must necessarily imagine the two to concur. However, if this be the case then clearly the angle of contact cannot be less than any rectilinear angle.

In the <u>Treatise</u> Hume was free to argue that our ideas of straight line and circle are not determinate enough to afford us an infallible assurance that a straight line can be made to intersect a circle in exactly one point or that only one straight line can be tangent to a circle at a given point. In the Treatise, then, Hume was able to dismiss those demonstrations for the infinite divisibility of extension which are derived from the point of contact between a circle and its tangent on the grounds that they are "built on ideas, which are not exact, and maxims, which are not precisely true."

But as we have seen, by the time of the Enquiry Hume came to see things much differently. The arguments of the Treatise, which rested so heavily upon the view "that geometry can scarce be esteem'd a perfect and infallible science," were no longer acceptable. What Hume desperately needed was a new strategy for attacking the mathematical demonstrations in favour of infinite divisibility. Not surprisingly, he continued to believe that Berkeley's doctrine of general ideas held the key to finding a solution which would enable him to banish this offensive doctrine and yet preserve the validity and soundness of all our other geometric reasoning.

> It seems to me not impossible to avoid these absurdities and contradictions, if it be admitted, that there is no such thing as abstract or general ideas, properly speaking; but that all general ideas are, in reality, particular ones, attached to a general term, which recalls, upon occasion, other particular ones, that resemble, in

> certain circumstances, the idea, present to the mind... If this be admitted (as seems reasonable) it follows that all the ideas of quantity, upon which the mathematicians reason, are nothing but particular, and such as are suggested by the senses and imagination, and consequently, cannot be infinitely divisible. It is sufficient to have dropped this hint at present, without prosecuting it any farther. It certainly concerns all lovers of science not to expose themselves to the ridicule and contempt of the ignorant by their conclusions; and this seems the readiest solution of these difficulties. (E.158, fn.1)

In two earlier editions, K and L, Hume expanded somewhat upon his rather meager hint. Following the words "and consequently cannot be infinitely divisible," he wrote:

> In general, we may pronounce that the ideas of "greater," "less," or "equal," which are the chief objects of geometry, are far from being so exact or determinate as to be the foundation of such extraordinary inferences. Ask a mathematician what he means when he pronounces two quantities to be equal, and he must say that the idea of "equality" is one of those which cannot be defined, and that it is sufficient to place two equal quantities before anyone, in order to suggest it. Now this is an appeal to the general appearances of objects to the imagination or senses, and consequently can never afford conclusions so directly contrary to these faculties.[9]

It may well be that Hume was capable of prosecuting this hint further but felt it best to leave this difficult task to the later work he intended to dedicate solely to geometry. This, however, is of little help to us. We are simply given the same general strategy employed in the _Treatise_, but without the slightest indication of the new direction we are to pursue in order to refute the mathematical arguments in question. However, the task of pursuing Hume's hint is far from hopeless. By reviewing how this hint was prosecuted in the _Treatise_, we shall be able to construct the general features of the only line of attack which was really open to Hume in the _Enquiry_.

The main point to bear in mind is that the analysis of the idea of equality was at the center of the _Treatise_ attack against the doctrine of infinite divisibility. In _An Abstract of a Treatise of Human Nature_ Hume summarized the basics of his argument against "those mathematical arguments which have been adduced for it."[10]

> ...All geometry is founded on the notions of equality and inequality, and therefore, according as we have or have not an exact standard of that relation, the science itself will or will not admit of great exactness. Now there is an exact standard of equality, if we suppose that quantity is composed of indivisible points. Two lines are equal when the numbers of the points that compose them are equal, and when there is a point in one corresponding to a point in the other. But though this standard be exact, it is useless, since we can never compute the number of points in any line. It is, besides, founded on the supposition of finite divisibility, and therefore can never afford any conclusion against it...[11]

Hume was convinced that since we must reject the standard of equality based on the enumeration of indivisible points as useless, "we have none that has any pretensions to exactness."[12] However, he was aware that both philosophers and mathematicians believed that they possessed a perfectly exact standard of equality which was not based on the supposition that extension is composed of indivisible points. These thinkers, he tells us,

> ...when asked what they mean by "equality," say that the word admits of no definition, and that it is sufficient to place before us two equal bodies, such as two diameters of a circle, to make us understand that term. Now this is taking the general appearance of the objects for the standard of that proportion, and renders our imagination and senses the ultimate judges of it. But such a standard admits of no exactness, and can never afford any conclusion contrary to the imagination and senses...[13]

The fact that the above argument is virtually identical to the passage found in the two earlier editions of the Enquiry suggests an important link connecting Hume's earlier and later positions regarding the refutation of infinite divisibility. It is here that we may begin to prosecute Hume's hint concerning Berkeley's doctrine of ideas. As Hume saw it, philosophers and mathematicians were deceived into thinking that their various particular ideas of extension, which were derived solely from the faculties of sense and imagination, were really instances of a more abstract and general idea. This general idea of extension was taken by these thinkers to be of so perfect and spiritual a nature that it could only be "comprehended by a pure and intellectual view, of which the superior faculties of the soul are alone capable." Corresponding to this fictitious notion of extension was the equally fictitious notion of a perfect and spiritual standard of equality in extension. The various particular standards of equality, which were likewise derived from the faculties of sense and imagination and corrected by whatever art and instrument we may possess, were conceived as mere approximations of this intellectual and perfect standard of equality in extension.

It was Hume's contention that these fictitious notions of extension and equality infected the geometrical reasoning of even the most astute and philosophically minded mathematicians, including the likes of Euclid, Descartes, Leibniz, and Newton, and led them to conclude from the logic of their demonstrations that extension is infinitely divisible. In both the Treatise and the Enquiry Hume believed that the proper antidote to this confusion in our geometric thinking was to realize that the idea of equality in extension is particular in the mind's conception of it.

In the Treatise Hume was of the opinion that the only idea of equality in extension we can clearly comprehend, and which is of any use in geometry, is that which is copied from the general appearance of objects. This idea, he reasoned, cannot be rendered exact enough to support so subtle a conclusion as that of infinite divisibility. Nor could it be rendered so exact as to place geometry on an equal footing with arithmetic regarding the precision of its most fundamental truths. However, he eventually came to see this latter consequence as an unacceptable price to pay for ridding geometry of its paradoxical doctrine. Thus, the principal task which faced his later philosophy of geometry was to define a standard of equality in extension which is consistent with Berkeley's doctrine of ideas and the finite divisibility of extension and yet clear and exact enough to restore geometry to its rightful place alongside arithmetic and algebra.

Hume, of course, does not so much as drop a hint as to the standard of equality which he believed could meet these demands. Clearly, though, he could not turn his back on the argument of the Treatise and the Abstract and try to argue that an idea of extension derived from a sensible comparison of the general appearances of objets could afford us a standard of equality exact enough to place geometry on an equal standing with arithmetic. Seemingly then, the only real option open to Hume was to break from his position in the Treatise and the Abstract and accept as useful the standard of equality defined in terms of a one-to-one correspondence of indivisible points. This move would not only enable him to define an exact standard of equality in extension which is consistent with Berkeley's doctrine, but would also directly respond to the main argument of the Treatise for denying full precision to geometry. Hume, we may recall, argued that in arithmetic we

> ...are possest of a precise standard, by which we can judge of the equality and proportion of numbers;... When two numbers are so combin'd, as that the one has always an unite answering to every unite of the other, we pronounce them equal; and 'tis for want of such a standard of equality in extension, that geometry can scarce be esteem'd a perfect and infallible science.(T.71)

Clearly, if Hume is to place geometry on an equal standing with arithmetic, he must be prepared to concede that this science possesses a standard of equality in extension which allows us to pronounce lines or surfaces equal whenever the one has always an indivisible point (unit) answering to every indivisible point (unit) of the other. Of course, this shift in view presupposes the distinction between pure geometry (determinate ideas) and applied geometry (generic ideas) noted by van Steenburgh. In so far as we are dealing with the application of geometry to experience, we must concern ourselves with judgements of equality based on measurement and comparison. In this domain Hume must still keep to the analysis of the _Treatise_. He could hardly ignore his earlier observation that

> ...as the points, which enter into the composition of any line or surface,...are so minute and so confounded with each other,...'tis utterly impossible for the mind to compute their number,...No one will ever be able to determine by an exact enumeration, that an inch has fewer points than a foot, or a foot fewer than an ell or any greater measure; for which reason we seldom or never consider this as the standard of equality or inequality.(T.45)

Hume could thus still dismiss the exact standard as useless for the practical purposes of judging equality in extension. But what he found he could no longer do was dismiss this standard of equality in extension as useless for the purposes of mathematical thinking and demonstration. To the contrary, it is now to function as the cornerstone of his philosophy of geometry and his attack against the mathematical arguments for infinite divisibility. Hume, it would appear, is now of the opinion that the fictitious notion of an intellectual standard of perfect equality can be replaced by a sensible standard of equality based on an exact enumeration of indivisible points without any loss regarding the certainty and precision of geometry in comparison to arithmetic. The only difference he believed the geometer would find is that he is no longer led by the logic of his demonstrations to conclude that extension is infinitely divisible. Such, at least, seems to be the position to which Hume is driven by the logic of his shift in view at E.25.

III

Philosophical Difficulties

Assuming we have accurately captured the essence of Hume's position, we must wonder why he didn't expand upon the hint he offered in the Enquiry and give the reader a more solid understanding of his proposed solution. Why devote five paragraphs to a discussion of the perplexity of the problem of infinite divisibility, and then skimp on so important a matter as outlining for the reader the key element of the proposed solution? At the very least, Hume could have expanded upon his reference to equality in editions K and L and informed his reader that he, unlike the vast majority of philosophers and mathematicians of his day, was of the opinion that the idea of equality so essential to geometry is to be defined in terms of an enumeration of indivisible points. He chose, instead, to delete all reference to the idea of equality. But why? Why not clearly state his new position regarding equality? We are, after all, only talking about adding a few sentences to his brief hint. Hume could have still left the task of prosecuting the details to a later work.

Most likely, Hume recognized that even the slightest hint of his new position with respect to the idea of equality in extension would serve only to generate unwanted controversy regarding the soundness of his philosophy. Hume, we are told, hoped that the Enquiry "would provide the right approach and introduction to the essentials of his philosophy."[14] Opening the door to controversy here would serve only to raise doubts about the soundness of these essentials, and this would hardly serve Hume's purposes.

It is not difficult to see that Hume had good reason to exercise caution. The fact is that the position which Hume must embrace in the Enquiry with respect to the idea of equality poses enormous difficulties. He tells us in his hint that "all ideas of quantity upon which mathematicians reason...are such as are suggested by the senses and the imagination." He was surely aware, though, that a precise perception of extension as a contiguous order of distinct indivisible points is not something which is clearly suggested by the senses. The logic of Hume's empiricism may dictate that a given appearance of extension is composed of an exact number of contiguous indivisibles, but the fact remains that "nothing is observ'd but the united appearance" (T.49). The senses are not refined or precise enough to present this order to us in any but a confused manner. Since Hume cannot allow that we possess an innate idea of the order of indivisible points which underlies the united appearance, he must conclude that we cannot frame any clear and exact idea of it.

It was considerations such as the above which originally led Hume to dismiss the idea of a standard of equality in extension based on an enumeration of indivisible points as useless. He reasoned that though the standard may be in itself exact, we can only frame for ourselves a loose and distant notion of it. How, then, can he now hope to argue that the senses can suggest to us a standard of equality which is so exact that it enables us to assign a perfect certainty and precision to the truths of geometry? Are we to assume that the copy can be rendered more exact than the impression?

It is difficult to exaggerate the enormity of the task facing Hume. Consider, for example, that he must assume that the ideas of equality and inequality are "always clear and determinate, the smallest distinction between them is immediately perceptible" (E.60). This means that the clear and determinate idea of an indivisible point must be construed as being immediately perceptible. If this were not the case, the standard of equality in question would hardly be able to function as a clear and exact idea of geometry. Now a clear and exact idea of an indivisible point is the idea of a part of extension which we immediately comprehend as being absolutely without parts. The challenge confronting Hume is to show us the impression of a minute part of extension which presents itself to us in so clear and determinate a manner that we can immediately perceive that it is absolutely without parts. It is difficult to see how he could possibly hope to show us such an impression.

Hume must surely continue to hold to the view that our senses are not perfect and all too often "represent as uncompounded what is really great and compos'd of a vast number of parts" (T.28). The addition or subtraction of a single indivisible part of extension, he must still believe, "is not discernible either in the appearance or measuring" (T.48). But if it is not discernible, how can we form a clear and exact idea of it? How can Hume possibly hope to avoid concluding that the notion of any correction beyond what we have art and instrument to make is "useless as well as incomprehensible" (T.48)? In short, how can he hope to render his dramatic shift in view with respect to the idea of equality in extension consistent with the demands of his copy principle?

The difficulties here relate back to the general problem of explaining how the shift to clear and determinate geometric ideas can be rendered consistent with the copy principle. The present example simply reinforces our view in the previous chapter that it cannot. However, Hume's difficulties do not stop here. The conceptions of extension and equality which Hume now assumes to be clear and exact and fundamental to geometry were conceptions which most mathematicians and philosophers dismissed as contrary to reason. Leibniz, for example, was so convinced of this that he took it to be certain "certain that the continuum cannot be compounded of points."[15] It was Leibniz's opinion that the difficulties surrounding the doctrine of infinite divisibility could never be resolved on the assumption that extension is composed of indivisible parts. Similarly, Kant rejected as absurd the conception of points which are "simple, and yet as having the characteristic of being able, as parts of space, to fill space through their mere aggregation."[16]

To understand the reasoning which led the overwhelming majority of philosophers to reject the doctrine of indivisible parts of extension, we need only inquire as to how these indivisible elements are to be conceived. It is perhaps tempting to think of them as being analogous to any other part of extension, only smaller. This, however, would constitute a serious error. For one thing, points which must be conceived as indivisible must be conceived as absolutely devoid of parts and, thus, as unextended. For another, an indivisible point must be conceived as being devoid of figure or shape. If we were to ascribe to them a particular figure, say a that of a circle or triangle, we would have to conceive of them as possessing distinguishable parts.[17] The question is, how can points which are absolutely devoid of extension and figure combine together and form an extended figure? How can points lie alongside one another and form a continuous figure if they do not possess external parts which can serve as their points of contact? Passmore, like Leibniz and Kant, argues that they cannot.

> ...how can such points lie alongside one another in such a way as to make up a continuous extension?... Two points can never lie contiguous to one another, because to be contiguous they would have to touch only at a certain point; and a point cannot itself touch at a point except by being that point. Hume cannot satisfactorily answer this objection.[18]

Hume, of course, was aware of this line of criticism, and in the Treatise he tried to answer it. He focused on the prevailing view of his day which held that the exact and indivisible points of geometer were properly to be thought of as modalities and not as real parts of extension.

> ...It has often been maintain'd in the schools, that extension must be divisible, in infinitum, because the system of mathematical points is absurd; and that system is absurd, because a mathematical point is a non-entity, and consequently can never by its conjunction with others form a real existence. This wou'd be perfectly decisive, were there no medium betwixt the infinite divisibility of matter, and the non-entity of mathematical points. But there is evidently a medium, viz. the bestowing a colour or solidity on these points; and the absurdity of both extremes is a demonstration of the truth and reality of this medium...(T.40)

There is little, if anything, positive to say about Hume's proposed solution. Firstly, as one commentator points out, the "residual problem of how sensible qualities could be bestowed upon an unextended point is, understandably, never resolved by Hume."[19] Secondly, Passmore is surely correct when he argues that "even supposing that points can be solid or coloured without being divisible, the problem still remains."[20] Whether the points be solid or coloured or whatever, we are still left trying to comprehend how indivisible points can lie contiguous with each other and form a continuous extended figure. Supposing that the unextended shapeless points are red does not in the least help us remove the conceptual difficulties noted above.

Again, we can appreciate why Hume would not have wished to open the door to a discussion of the problems involved in defining the idea of equality in terms of indivisible points. But we are left with a nagging question. How could he have ever believed that he could reconcile this shift in view regarding the nature of the idea of equality with his continued adherence to the copy principle?

IV

A Criticism from Pythagoras

Thus far we have spoken primarily of the conceptual difficulties surrounding Hume's analysis of extension as a conjunction of indivisible points. It remains to consider, albeit briefly, the challenge which the demonstrations of geometry presented for his analysis. We shall center our discussion around one of Hume's favourite examples of a demonstratively certain proposition of Euclid, the Pythagorean theorem. For simplicity, let us assume that we are dealing with an isosceles right triangle. From the Pythagorean theorem we know that the length of the hypotenuse is exactly equal to the length of one of the equal sides times the square root of two. From Hume's perspective, this was equivalent to stating that the geometer can demonstrate with full certainty and precision that the number of indivisible points which make up the length of the hypotenuse is equal to the square root of two times the number of points which make up the length of one of the equal sides. But this means that the number of points which constitute the length of the hypotenuse cannot be expressed in terms of a whole number. And yet, it is absurd to speak in terms of a fraction of an indivisible point.

Since the square root of two is an irrational number (i.e. cannot be expressed as the ratio of two integers), it immediately follows that the sides of the isosceles right triangle are incommensurable with the hypotenuse. In other words, if we assume that the length of one of the sides is divided into _k_ equal parts (where '_k_' is an integer) then it will always be the case that, no matter how large _k_ is, there will be an integer _n_ such that the addition of _n_ of these parts will be less than the length of the hypotenuse and the addition of _n_ + 1 of these parts will be greater than the length of the hypotenuse. In short, if Hume assumes that the Pythagorean theorem is a precise and certain truth, as he clearly does, then he cannot conceive of the length of each side of an isosceles right triangle as nothing but the conjunction or addition of a finite number of indivisible parts.

V

Conclusion

We opened this chapter with Atkinson's claim that Hume's continued attack against the doctrine of infinite divisibility suggests that he did not intend to break, in any significant way, from the philosophy of geometry of the Treatise. We have attempted to show not only that our claim of a significant shift in Hume's thinking with respect to geometry can be rendered consistent with Hume's continued criticism of the principle of infinite divisibility, but also that it leads to some important insights into Hume's discussion of this controversial principle in the Enquiry. Hume, we have argued, was driven by the logic of the philosophy of geometry outlined at E.25 to accept that geometry was founded on a clear and exact standard of equality. Only by embracing such a standard could he move away from the position of the Treatise, and accept that geometry, like arithmetic and algebra, is a perfect and infallible science.

Though he was understandably reluctant to discuss the nature of this standard, it was not difficult for us to determine what his position had to be. He had only one option open to him, and that was to accept as clear and determinate the standard of equality which he formerly dismissed as useless and incomprehensible. It was no doubt one of the principal aims of his later work on geometry to show that this standard of equality in extension is sufficient for the purposes of (a) securing the perfect exactness and certainty of the truths of Euclidean geometry and (b) showing the error in all those demonstrations which were thought to prove the infinite divisibility of extension. As we have seen, Lord Stanhope would have had plenty of opportunity for criticism, and it can hardly be surprising that Hume would have found at least some of his arguments quite devastating. Indeed, it would have been remarkable if he could not have been persuaded to refrain from publishing his geometry of indivisible points.

Chapter 6

Notes

1). R.F. Atkinson, "Hume on Mathematics," p.133
2). ibid., p.133
3). ibid., p.133
4). Rosemary Newman, "Hume on Space and Geometry," p.29
5). R.F. Atkinson, p.133
6). Rosemary Newman, p.27
7). Robert Fogelin, "Hume and Berkeley on the Proofs of Infinite Divisibility," p.57
8). For example, neither Newman nor Fogelin really go much beyond their initial observations.
9). David Hume, An Inquiry Concerning Human Understanding, ed. with an introduction by Charles W. Hendel (New York: The Bobbs-Merrill Company, Inc., 1955), p.166 fn.2
An Enquiry Concerning Human Understanding was first published as Philosophical Essays Concerning Human Understanding. It is in the latter that we find the aforementioned editions, K and L.
10). David Hume, An Abstract of a Treatise of Human Nature, in An Inquiry Concerning Human Understanding, ed. by Charles W. Hendel p.195
11). ibid., p.195
12). ibid., p.195
13). ibid., p.195
14). See Hendel, An Inquiry Concerning Human Understanding, p.vii
15). Gottfried Leibniz, Philosophical Papers and Letters, no.47, p.454
16). Immanuel Kant, Critique of Pure Reason, ed. by Kemp Smith, A439 = B467
17). Hence, we cannot conceive of indivisible points as being tiny dots since that would be to ascribe to them the figure of a circle. Hume,I believe, is guilty of this error at T. 41.
18). John Passmore, Hume's Intentions, p.111-12
19). James Noxon, Hume's philosophical Development, p.115
20). John Passmore, p.111

Chapter 7

Summary and Conclusion

Our primary aim in the present work has been to examine Hume's writings on geometry in a manner which would enable us to (a) confer upon them the greatest possible consistency and coherency and (b) assess both their strengths and weaknesses. Owing to the nature of the controversy, criticism, and general confusion which Hume's writings on geometry have generated, we sought first to situate his teachings within the overall framework of his empiricism. This was the task we set for ourselves in chapter two. We discovered that certain epistemological considerations regarding geometry played a far more integral and important role in the development of Hume's philosophy than generally has been recognized. Based on our findings, it would be no exaggeration to say that the philosophy of geometry was one of the major battlefronts in Hume's war against scepticism and rationalism.

In order to refute the sceptic, Hume believed that it was necessary to show that the infamous doctrine of infinite divisibility was not a product of sound geometrical reasoning. In order to put an end to the dogmatic metaphysical speculascience. Though Hume did not explicitly state that he was breaking from his position in the Treatise, the shift in view is so clearly evident that it has caught the attention of virtually every commentator. Few, however, have focused on the fundamentals of this rather remarkable shift in Hume's thinking. All too often the issue has been discussed solely in terms of the analytic/synthetic distinction, a topic which itself has become a matter of growing controversy. For the record, our findings led us to conclude that the most accurate way to describe Hume's shift in view is to say that in the Treatise he held the propositions of Euclidean geometry (as opposed to geometry$_H$) to be synthetic a posteriori whereas in the Enquiry he held them to synthetic a priori. Consequently, we have been led to conclude that the position of the Enquiry regarding the status of geometry does indeed represent a philosophically significant shift away from the view of the Treatise.

However, becoming clear as how best to classify the positions of the Treatise and the Enquiry does not in itself take us very far toward understanding the rationale behind this rather remarkable shift in Hume's thinking. What needs to be determined is why he found it necessary to break from the position of the Treatise regarding the status of geometry. This, however, is an issue which is rarely, if ever, discussed.

In the Treatise we were told repeatedly that the reason we must reject the conventional wisdom regarding the perfect certainty of even the most evident of our geometric principles was that our geometric ideas are lacking true precision and exactness. We must assume, then, that Hume had come to accept either (a) that inexact and indeterminate geometric ideas can, after all, afford us a perfect assurance that the principles of Euclidean geometry are precisely true or (b) that the ideas most essential to geometry are, in fact, exact and determinate. Since alternative (a) was not an option Hume could seriously entertain, he had little choice but to opt for (b). In an often overlooked passage in the <u>Enquiry</u> (E.60) we found Hume conceding not only that the ideas of geometry are always clear and determinate but also that the smallest difference between them is <u>immediately</u> perceptible. This concession to the conventional wisdom of the day was so radically at odds with what was argued in the <u>Treatise</u> that it can hardly be viewed as a mere oversight. It represents, rather, a fundamental point of departure from the philosophy of geometry of the <u>Treatise</u>.

In spite of this shift in his thinking, Hume continued to adhere to the copy principle and to Berkeley's doctrine of general ideas. And, of course, he continued to attack the doctrine of infinite divisibility as a metaphysical pretension which has absolutely no place in mathematics. Hume, in other words, was still committed to the program initiated by Berkeley for closing the door on the sceptic and the rationalist. The problem was that he now embraced a view of geometry which seemed to be at complete odds with his copy principle. Perfectly determinate geometric ideas can hardly be derived from the inexact impressions of the senses, even when these impressions are corrected by whatever art and instrument available.

It would be a mistake to assume that Hume did not appreciate the enormous difficulty which this shift in view presented for his empiricism. He was only too aware that the exactness of the ideas of geometry had long been viewed as a major obstacle standing in the way of an empiricist epistemology. As we have seen, in the Treatise Hume railed against the conventional wisdom regarding the exactness of the ideas of geometry precisely because it could not be rendered consistent with his copy principle. He knew full well that he was in a no position to argue, as he did in the example of the missing shade of blue, that what we have here is an exception to the copy principle "which is so singular, that it is scarcely worth our observing, and does merit for it alone we should alter our general maxim."[1] Hume knew as well as any one that if the ideas most essential to geometry proved an exception to his copy principle, his empiricism would have been dealt a fatal blow.

With the shift to clear and determinate ideas Hume was forced to consider anew the geometric arguments in favour of infinite divisibility. The line of attack employed in the

Treatise was based on the assumption that geometry is built on ideas which are not truly determinate and on axioms which are not precisely true. As such, he could simply rule out the possibility that the ideas and first principles of geometry could support a chain of reasoning subtle enough to demonstrate the infinite divisibility of extension. However, in the Enquiry Hume was forced to confront the mathematician on his own ground. He had to argue that even granting that geometry, like arithmetic, is a perfect precise and certain science, it cannot be demonstrated that extension is infinitely divisible. This, of course, was a battle which Hume simply could not win. The mathematical arguments for infinite divisibility cannot, as Hume believed, be explained away.

Why, then, did Hume undergo this shift in his thinking? Why did he abandon a philosophy of geometry which was in harmony with his copy principle and embrace one which was hopelessly at odds with it? Though Hume did not inform us as to the reason for the shift in his thinking, we have argued that it is not too difficult to determine what must have caused him to abandon the position of the Treatise. It was one thing for Hume to present an analysis of the ideas of geometry which was consistent with the logic of his empiricism and quite another for him to defend it. The fact that this analysis enabled him to make quick work of the doctrine of infinite divisibility no doubt convinced Hume that it held real promise and was pointed in the right direction.

However, this alone could hardly serve as a defense of his analysis against the criticism of the conventional wisdom. The strength of the conventional wisdom rested on a seemingly incontestable claim of self-evidence. It was claimed by philosophers and mathematicians alike that the ideas of perfect straightness and perfect equality in extension were so clear and evident to the common understanding of mankind as to be beyond all question of doubt. The rationalists, of course, regarded these ideas as the clearest example of ideas which are too general and too precise to be of a sensible origin. These ideas, they insisted, must have their origin in some purely intelligible mode of intuition. This view so dominated the philosophical landscape of the seventeenth and eighteenth centuries that Hume had little choice save to meet it head on. The only option open to him in the Treatise was to argue that those who embraced the conventional wisdom in this matter were victims of an illusion.

In chapter four we examined Hume's argument and found it seriously defective. At the end of chapter five, we suggested that by the time of the Enquiry Hume had become aware of just how defective and implausible the argument of the Treatise was.[2] It is reasonable, we maintained, to assume that Hume found himself at an impasse and was forced to conclude that it was foolhardy for him to continue to dismiss the ideas of perfect equality and perfect straightness as obscure and incomprehensible fictions.

Of course, Hume continued to accept that in so far as we are dealing with actual impressions, the argument of the Treatise clearly holds. Our senses and measuring instruments could never afford us any absolute assurance that no two actually existing impressions of a straight line can ever concur or enclose a space. What Hume could no longer accept was that the analysis of the Treatise had actually captured the essence of our everyday idea of a straight line. Thus, in addition to the empirical idea of a straight line we find in the Treatise, Hume had to introduce an intuitive notion of a perfectly straight line which, though sensible, was not derived from what is anywhere existent in the universe.[3] And once he gave up trying to deny the reality of the intuitive idea of a perfectly straight line, he saw no way that he could deny the certainty of the axioms of Euclidean geometry.

We were able to understand, then, why Hume broke from the position of the Treatise and accepted that the ideas which are most essential to geometry are so clear and determinate, according to our common method of conceiving them, that the smallest difference between them is immediately perceptible. And we were certainly able to understand why, once he underwent this shift in his thinking, he found it necessary to concede to the conventional wisdom that we can know with true certainty that no two straight lines, even if they incline upon each other at the rate of one inch in twenty leagues, can be made to concur. What we were not able to understand, was how Hume could possibly have hoped to render this shift in his thinking consistent with his copy principle. His greatest challenge here was to overcome the argument of the Treatise and render his new position consistent with the logic of his empiricism. As our findings in chapters five strongly suggest, he was not at all sure how to do this. In the Enquiry Hume chose simply to ignore the issue, even when it was all but begging to be addressed.

We may safely conclude, then, that Hume failed to defend his copy principle against what had historically been viewed as a philosophically significant counter-example; namely, the precise ideas of geometry. Indeed, we may go so far as to claim that there is nothing which Hume could have done to defend his copy principle against the charge that it is an inadequate tool for the analysis of these ideas. Thus, his philosophy of geometry cannot get off the ground and his empiricism is dealt a fatal blow. With this, it is tempting to conclude that our study is now complete. However, this would be to overlook some of the most important accomplishments of Hume's analysis. In taking his empiricism to its proper conclusions, Hume not only exposed the inherent weakness of his own philosophy for dealing with the analysis of geometric knowledge but also that of the rationalists, and in particular Leibniz, as well.

As we have seen, both Hume and the rationalists took as the starting point of their philosophy of geometry the task of analyzing the nature of the everyday thinking which underlies our mathematical conceptions.[4] The rationalists, however, were convinced that the certainty and precision which the common understanding of mankind assigns to the most basic propositions of Euclidean geometry could not be analyzed in accordance with an empiricist doctrine of ideas and perception. In their view a proper analysis of the nature and origin of our everyday understanding with respect to the ideas and propositions most essential to geometry leads straightaway to their doctrines of innate ideas, intellectual intuition, and analytic truth. Hume, however, was able to show that if we interpret the ideas and principles of geometry in strict accordance with these doctrines, we end up with an interpretation which is sharply at odds with the manner in which they are actually understood in everyday life.

There is little need to review the details our discussion of Hume's arguments. Suffice to say, Hume was on solid ground in arguing that if we set out to explain the certainty which the common understanding of mankind attributes to the axioms then we must give ourselves over to our everyday manner of thinking. The problem with the rationalists' doctrine of analyticity is that it must assume that the certainty which the "vulgar" assign to the axioms when they have occasion to reflect upon them is rooted in an understanding of the axioms as mere statements of identity or logical truths. This, we have seen, was the position which Leibniz believed was a necessary consequence of the doctrine of analyticity to which rationalism was committed. Hume, however, had no difficulty demonstrating that this leads to the absurd conclusion that the essence of the axioms of geometry, as they are comprehended by the common understanding of mankind, can be expressed by a proposition of the form 'A is A.' And this, he correctly argued, is absurd.

Similarly, we found that Hume was able to raise a number of arguments to the effect that the rationalists' doctrine of intellectual intuition was incapable of coming to grips with the manner in which the ideas most essential to geometry are actually understood. For example, the rationalists acknowledged that the idea of a perfectly straight line stands in a relation of resemblance to objects given to us in ordinary sense experience. That is, they acknowledged that certain objects of sense experience are conceived as approximations or images of the perfectly straight line of geometry. However, as we argued in chapter four, if the form and its object belong to two totally different modes of intuition then we can hardly view the one has somehow resembling the other.

Hume, we may safely assume, was keenly aware of this inconsistency implicit in the rationalists' view of the nature and origin of the ideas of geometry. Thus, when he made the shift to determinate geometric ideas and embraced an intuitive conception of perfect straightness, equality, etc., he continued to hold to the view that these ideas are of a sensible nature. The problem was that he could not reconcile these intuitive sensible ideas, which do not depend upon what is anywhere existent in the universe, with his copy principle. Nor could he explain how they could be immediately perceptible.

We have observed that, in the Treatise, Hume clearly anticipated some of Kant's most important arguments in support of the view that geometry is a body of synthetic truths. We further noted that a strong case can be made for claiming that, in the Enquiry, Hume anticipated Kant's position regarding the synthetic a priori nature of geometry. It does not appear, though, that Hume came close to anticipating Kant's doctrine of pure sensible intuition. And yet, the shift in view at E.25 seems to cry out for precisely some such doctrine. If Hume had taken this step and argued that the idea of a perfect straight line is the copy of a pure sensible form, he would have been in a position to explain how such an idea could be clear and determinate and not depend on what is anywhere existent in the universe.

Kant, as is well known, held Hume philosophical acumen in high esteem and, in general, was of the opinion that it would "repay us to make clear to ourselves, ..., the course of the reasoning, and the errors, of so acute and estimable a man." Of course, he clearly did not have Hume's reasoning with

with respect to geometry in mind here. Like most philosophers, Kant was inclined to dismiss these reasonings as an exception to this rule. Nonetheless, it should be evident that Kant's words ring true even with respect to this most notorious part of Hume's philosophy. To be sure, Hume was never able to present a philosophy of geometry consistent with his empiricism, and his opposition to the doctrine of infinite divisibility was wrong-headed from the start. But that is not what is at issue here. The question is whether we gain anything of value after we have struggled to make clear to ourselves the course of Hume's reasonings with respect to gometry and identify the source of his errors. From the perspective of the history of ideas, there can be little doubt but that we have been amply repaid for our efforts.[5]

In our introductory chapter we observed that there has been of late a growing interest in "historically oriented reconsiderations of logical positivism." We noted that these reconsiderations are claimed to be necessary if we are to move beyond our present situation in the philosophy of geometry. Since Hume is generally considered to be the most influential precursor of logical positivism, we assumed that an inquiry into his writings might well prove to be an ideal starting point for an historical reconsideration of the positivists' position regading the philosophy of geometry. The fact that our study has taken us some distance toward gaining a fresh and critical perspective into those problems in the history of the philosophy of geometry which had long been viewed as posing insurmountable difficulties for empiricism, problems which critics now claim the positivists tended to gloss over and dismiss as matters of psychology[6], adds some weight to this argument.

Given the often harsh criticisms which have been levelled against Hume's struggles with the philosophy of geometry, it is worthwhile pointing out the valuable historical perspective our study has afforded us and its relevance for the study of topics of current interest to philosophers. However, let us make clear that we are not suggesting that the primary value of our study is that it holds promise for gaining a deeper understanding of Kant and positivism. That it should have relevance for such inquiries is not suprising once we realize that, contrary to the prevailing view, Hume's writings on geometry are clearly representative of his unique genius. The most important conclusion of our study is that Hume's struggles to come to grips with some of the most important problems in the history of the philosophy of geometry represented a serious effort on the part of a gifted thinker well-suited for this task. The study of these struggles, like the study of all such efforts of intellect, is of value in itself. That it should serve to stimulate further thought and point beyond itself is merely a manifestation of this value.

Chapter 7

Notes

1). See p. 166 note 13.

2). The positivists believed that they had succeeded in explaining away the ancient belief in the necessity of the axioms of Euclidean geometry in terms of a rather simplistic confusion regarding our concepts. (See, for example, Rudolph Carnap, Philosophical Foundations of Physics, (New York: Basic Books, Inc., 1966),pp.182-83. Putnam, however, has dismissed these attempts as absurd. (See, Hilary Putnam, Mathematics Matter and Method: Philosophical Parers, 2nd. ed. (Cambridge: Cambridge University Press, 1979), pp. ix-x.

3). Alan Hausman conjectures that Reichenbach too was guilty of smuggling into his philosophy of geometry an intuitive notion of the concept of an Euclidean straight line which has no physically existing counterpart. (Alan Hausman, "Non-Euclidean Geometry and Relative Consistecy Proofs," in Motion and Time, Space and Matter, ed. by Peter Machamer and Robert Turnbull, (Columbus: Ohio State University Press.1976), p.433

4). Albert Einstein captured the spirit and importance of this task when he remarked,

> The whole of science is nothing more than the refinement of everyday thinking. It is for this reason that the critical thinking of the physicist cannot be restricted to the examination of the concepts of his own specific field. He cannot proceed without considering critically a much more difficult problem, the problem of analysing the nature of everyday thinking. (Albert Einstein, "Physics and Reality," in Ideas and Opinions, ed, by Carl Seelig, (New York: Crown Publishers Inc., 1982), p.290)

Of course, it was part of the legacy of positivism that geometry was to be viewed as an uninterpreted formal system. Thus, the problem of the analysis of our ordinary concepts of straight line and equality was no longer thought to be relevant to the philosophy of geometry. Einstein himself was a strong advocate of the positivist position (see Albert Einstein, "Geometry and Experience," in Ideas _and Opinions_, pp. 232-46). Like the positivists he held that mathematics "cannot predicate anything about the objects of our intuition" (p.234) and that that which content to our everyday concepts of straight line, point, etc. "is not relevant to mathematics" (234). It is noteworthy that with the demise of the Received View, we find philosophers again reminding us of the importance which our everyday understanding of these concepts has for the philosophy of geometry. See for example, Stephen Barker, _Philosophy of Mathematics_, (New Jersey: Prentice-Hall, Inc., 1964), pp.48-55. See also Michael Friedman, "Grunbaum on the Conventionality of Geometry" in _Space, Time, and Geometry_, ed. by Patrick Suppes.)

5). It is not difficult to see that our study has brought us to the very doorstep of Kant's critical philosophy of geometry. Unfortunately, it lies well beyond the scope of our study to pursue this topic further.

6). The positivists, of course, dismissed the epistemological reflections of the philosophers of the past, including Hume, as the product of a confused mixture of psychological and logical investigations. They maintained that epistemology was really nothing but logical analysis. Though this view was not well received as a general philosophy, it dominated the philosophy of geometry. Thus, that in the philosophy of geometry which could not be subject to logical analysis was conveniently dismissed as a matter of psychology. However, as we observed in our introductory chapter and in the above notes, these problems are beginning to resurface. The fact which must be faced is that it was Hume, and not his contemporary counterparts, who struggled to solve them within the framework of an empiricist epistemology.

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