THE DILEMMA OF PARTICIPATION
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A COMMENTARY ON PLATO'S *PARMENIDES* 131A9-E3

By

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Abstract

In separating the Form from its particulars, Plato is left with the task of describing the way in which they are related to one another. One possible way of construing this relation is to suppose that particulars receive a share of the Form. The discussion between Parmenides and Socrates, in the *Parmenides* 131a9-e3, interprets this sharing in a material sense: either the whole of the Form is received by each particular, or part of the Form is received by each particular. This disjunction turns out to be a destructive dilemma - the so-called Dilemma of Participation.

The three main sections of this work study in detail the Dilemma of Participation, as it is presented in the *Parmenides*. The first section considers the disjunct that the whole of the Form is received by each particular (131a9-b2). By using a system of classical extensional mereology, it is demonstrated that Parmenides' *reductio ad absurdum* of this disjunct is deductively valid. The second section deals with Socrates' objection to this argument (which he makes in the guise of the Day Analogy), and Parmenides' response to the objection (which he makes in the guise of the Sail Analogy) (131b3-c4). The validity of Parmenides' response depends on the sense of "day" Socrates intends in the Day Analogy. It is argued (against S. Panagiotou) that there is a sense of "day" that makes Parmenides' response invalid. The third section considers the disjunct that part of the Form is received by each particular (131c5-e3). Two current interpretations of this disjunct (that of T. Scaltsas and R. E. Allen) are recounted and critiqued, and a new interpretation is proposed (an interpretation based partially on that of Proclus, and under which Parmenides' argument against this disjunct is valid).
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As the writing of this thesis is set against the backdrop of the greatest failure of my life, I should like to thank Father Tom Wilding and David Little, without whose constant support I should never have written a word. Furthermore, I should like to thank James Steeves, who has seemingly never run out of patience listening to the trials of these past months. No less gratitude is due to my family and friends, each of whom have played a rôle in supporting me: Justin Busch, Ravi Danesh, Jason Griffiths, Brian Hendrix, David Hitchcock, Chris Richinson, Charissa Varma, my mother and father, my sister and brother, and my maternal grandmother.

I dedicate this work to the memory of my maternal grandfather, Frederick Charles Whitbourn (1922-1996).


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<td>&lt;</td>
<td>part</td>
<td>( a &lt; b \rightarrow a \preceq b \lor a = b ) [(SD1)]</td>
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<td>◢</td>
<td>present in</td>
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<td>◣</td>
<td>named</td>
<td>primitive</td>
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* Constants: \( a, b, c, d, e, \chi \) | Variables: \( \mu, \nu \) | Predicates: \( \lambda \)
List of Theorems

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<td>$T\beta$</td>
<td>$(\forall x)(\forall y)(\forall u)(\forall v)[(\neg u \land y&lt;v \land u \lor v) \implies x\neq y]$</td>
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<td>$T\gamma$</td>
<td>$(\forall x)(\forall y)(\forall u)(\forall v)[(\neg u \land y&lt;v \land u \lor v) \implies x\neq y]$</td>
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<tr>
<td>F_</td>
<td>_ is a Form</td>
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</tr>
<tr>
<td>I_,,</td>
<td>_ is present in _</td>
<td>12</td>
</tr>
<tr>
<td>I_,,,</td>
<td>_ is present in _ at time _</td>
<td>16</td>
</tr>
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<td>M_,,</td>
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<td>N_</td>
<td>_ is a name</td>
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<td>P_</td>
<td>_ is a particular</td>
<td>11</td>
</tr>
<tr>
<td>Q_,,</td>
<td>_ participates in _</td>
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<td>S_,,</td>
<td>_ is separate from _</td>
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<td>S_,,,</td>
<td>_ is separate from _ at time _</td>
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</tr>
<tr>
<td>T_</td>
<td>_ is a time</td>
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<tr>
<td>W_,,</td>
<td>_ is the whole of _</td>
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Introduction

“This problem will most decidedly never cease to exist, and nor has it arrived on the scene just now.”¹

The problem of the one and the many endures because it is at once necessary to distinguish the one from the many, in the sense that the one is separate from the many that it qualifies, and necessary to relate the one to the many, in the sense that the many are manifestations of the one. Helen and Penelope are both ‘beautiful’; but it is equally unpalatable to suppose that either one is the same as ‘beauty,’ as to suppose that they do not have ‘beauty’ in common: in the former case, to be ‘beautiful’ would literally be to be Helen or Penelope; in the latter case, Helen’s being ‘beautiful’ would be have nothing in common with Penelope’s being ‘beautiful.’ Hence, the one and the many are neither strictly the same nor strictly different: they are at once intimately different and diversely the same. Enter the problem of participation.

The relation between the one and the many is described in the Phaedo as one of participation (100c4-6): “If some other thing is beautiful other than Beauty itself, it is beautiful on account of nothing other than that it participates (μετέχει) in Beauty itself; and I speak thus for all.” We may distinguish three elements involved in this relation: the Form (Beauty); the character (being beautiful); and the particular (that which is beautiful). The Form is the same for all the particulars that participate in it, but the character that each

¹Socrates, on the problem of the one and the many (Philebus 15d4-6). All translations are my own, unless otherwise noted.
particular has in virtue of this participation is peculiar to the particular in question: Helen and Penelope participate in the same Form Beauty, but Helen’s beauty is not the same as Penelope’s beauty (cf. 102b8-103a10). Aside from pointing out that particulars get the name of the Form by participating in the Form (102a10-b2), Phaedo relates nothing else about the nature of participation. We have little more than a name.

Participation is a relation that reappears in the Parmenides, this time as Socrates’ response to Zeno’s paradox that a given particular is unqualifiedly both like and unlike (127e1-4). Socrates solves the paradox by proposing that the qualifications of likeness and unlikeness are separate from that which they qualify (128e6-29a2). Now that he has separated the Form from its particulars to solve Zeno’s paradox, Socrates is left with the difficulty of binding them together again. He does this with participation. The particular acquires the character of the Form (that is, becomes like the Form) in the respect that and to the degree that it participates (μετάλαμβάνειν) in the Form (129a2-6). Again, we are told little about the nature of participation up to this point; but we do have a name.

**The Names of Participation**

Parmenides commences his attack on participation by assuming that the names used to describe the relation, “μετέχειν” and “μετάλαμβάνειν” were not chosen arbitrarily, but give a clue as to the nature of participation. The verbs “μεταλαμβάνειν” and “μετέχειν” are

---

2 Note that line numbers for the Parmenides follow Diès’ text.

3 This move is seconded by Aristotle’s diagnosis of the problem (Physics I, 186a24-34), for Aristotle supposes that the monism Zeno is defending arises from the failure to make this distinction (Allen 1997, 92).
commonly used as verbs of sharing, and so they invite a particular interpretation because of the grammatical structure of such verbs. Verbs of sharing take the partitive genitive for the object in which the subject participates, and the part received stands in the accusative (Smyth 1956, §§1343-4). For example, in "μεταλαμβάνει τὸ πέμπτον μέρος τῶν ψήφων" (cf. Apology, 36a7-b2), the subject receives a fifth part (accusative) of the votes (partitive genitive). Hence, when a particular participates in a Form, it is natural to ask what part of the Form the particular receives.

**The Dilemma of Participation**

What part of the Form does the particular receive? The possible responses to this question may be formulated as an exclusive disjunction: either the particular receives the whole Form, or the particular receives a part of the Form. Socrates commits himself to this disjunction by admitting that these are the only two ways of construing participation (131a6-8). We may represent the disjunction as follows, employing the language of presence in, with $F$ being the Form, and $u$ being the particular of the same name as the Form:

If $u$ participates in $F$, then

\[
\begin{align*}
&\text{(a) the whole of $F$ is present in $u$;} \\
&\quad \text{or} \\
&\text{(b) a part of $F$ is present in $u$.}
\end{align*}
\]

This disjunction earns the name "Dilemma of Participation,"\(^4\) because Parmenides contends (and Socrates is obliged to accept) that participation necessarily entails this exclusive disjunction, and both its disjuncts lead headlong into absurdity. The disjuncts

\(^4\)This is the name Allen gives to 131a9-c3 (1997, 8). We borrow this name, but apply it to 131a9-e3.
become Horns - (α) the First, and (β) the Second - and taken together they are wielded as a wedge, splitting "the claim that Ideas exist from the claim that other things partake of them and are named after them" (Allen 1997, 193). So either the Form is strictly the same as particulars, eliminating the need for participation but committing us to Eleatic monism, or the Form is strictly different from particulars, not related to them in any way. As we have noted above, both of these alternatives have undesirable consequences.

Hence, the Dilemma is presented as being fatal to participation; but as necessary as it is to come up with a solution, evidence suggests that this will not be easy. First, the Dilemma is the centrepiece of the *Parmenides*, and many of Parmenides' other criticisms depend to some degree or another on the Dilemma. If the Dilemma is easy to solve, then the *Parmenides* becomes little more than an elaborate collection of puzzles that seem more difficult than they actually are. Second, the Dilemma is the only one of Parmenides' criticisms that Plato returns to in a later dialogue. In the *Philebus*, Socrates recounts a version of the Dilemma, and suggests that it is one of those questions that causes all manner of dissatisfaction if not properly settled (Philebus 15b1-c3; cf. Robinson 1953, 231-2). It is clear that Plato thought the problem difficult, but worthy of serious consideration.

Despite the difficulty of the problem, Plato never suggests that it is insoluble. When Parmenides has thoroughly trounced into the ground the arguments of a wet-behind-the-ears Socrates, he does not suggest that Socrates embrace Zeno's paradox, but rather that it will be necessary to solve the Dilemma if the possibility of discourse is to be preserved (135b6-c3). Furthermore, a more mature Socrates in the *Philebus* suggests that looking for number
in the many is the correct way to deal with the problem of the one and the many (17c11-e6). Therefore, the Dilemma of Participation is a problem that is difficult, but one that has a solution.

**The Relation of Separation**

Any analysis of the relation between Forms and particulars must give attention to the relation of separation, for it is through the proposal that Forms are separate that participation becomes a consideration. The adverb χωρίς is used to describe the relation between Forms and particulars (130b1-3), between Forms and the character of a particular (130b3-5), between Forms and other Forms (129d7-e4) and between particulars and other particulars (131b1-2). F. M. Cornford suggests that the separation of Forms from particulars is tied to the Doctrine of Reminiscence: the soul apprehends the Forms in a fundamentally different way than that in which it apprehends particulars, and so the two are said to be separate (1957, 74-5). This may be true of the relation between Forms and particulars, but it cannot be applied to all the other pairs said to be separate. For example, particulars are separate from other particulars, but they are both apprehended in the same way.

The more basic sense of "separation" seems to be the sense in which Parmenides uses it, when he asks Socrates if there is any sort of participation separate (χωρίς) from that in which the whole Form is present in the particular, and that in which part of the Form is present in the particular (131a5-7). Here, Parmenides is asking if there are sorts of participation other than the two already mentioned; that is, if there are sorts of participation not identical to either of these two sorts. Thus, whatever else it might come to mean in other
contexts, if \( x \) is separate from \( y \), then we may at least assert that \( x \) is not identical to \( y \) (and from this it follows that \( y \) is not identical to \( x \)).

**The Relation between Characters, Names and Forms**

The three aforementioned elements of participation that appear in the *Phaedo* reappear in the *Parmenides*. As pointed out already, when describing his solution to Zeno’s Paradox, Socrates says that a particular acquires the character of the Form (that is, become like the Form) in the respect that and to the degree that it participates in the Form (128e12-29a6).\(^5\) For example, if Socrates participates in the Form Pale, he acquires the character of paleness. However, when we turn to the Dilemma, there is no mention of characters; the only thing Parmenides says is that participants get the same name as the Form (130e5-31a1). Characters and Forms on one hand, and names and Forms on the other, are clearly related: by participation in the Form, the particular (a) acquires the character corresponding to the Form, and (b) gets the same name as the Form. But the question must be raised, is there a connexion between the character and the name, or are they merely coincidental upon the particular’s participation in the Form.

That the character and the name are coincidental seems doubtful, but the relation between the two is not made explicit in the *Parmenides*, and so to the *Phaedo* we must turn. That particulars get the same name as the Form (b) is confirmed in the *Phaedo* (78d10-e1; 102b2), but what is more important is that Socrates explains why. Simmias has the names

\(^5\)Characters are explicitly distinguished from Forms in the *Parmenides* at 129b1-c1.
“small” and “large” because he is between the largeness of Phaedo and the smallness of Socrates (102c10-d2). That is, Simmias has the character of small (with respect to Phaedo), and the character of large (with respect to Socrates). It is from the characters of small and large that Simmias gets the names “small” and “large.”

A little further on, we find more evidence that the particular gets its name from the character, as Socrates overturns an objection by declaring that (103b6-c1),

Then (α) we were talking about things having opposites, calling them by the name of those [opposites], but now (β) we are talking about those things themselves from whom the [particulars] named after them have their names.

The opposites that are referred to at (α) recall the earlier argument that advances the proposition that that which comes to be does so from its opposite (70e1-71a11). Whatever the merits or difficulties of this argument might be, it is clear that it is characters and not Forms that are being considered. In this way, when Socrates says that the things that have opposites (that is, particulars) are called by the name of those opposites, he is meaning that particulars are called by the name of their characters. At (β), the “things themselves” are clearly the Forms, his point being that the Forms are unlike characters in that they do not come to be (cf. Gallop 1975, 196-7). Therefore, we have two claims being made by Socrates: at (α), the particular is called by the name of the character (c), and at (β), the particular gets the name of the Form (b).

We now have three propositions to work with: (a) the particular acquires the character corresponding to the Form; (b) the particular gets the same name as the Form; and (c) the particular gets the name of its character. By simply conjoining these propositions, we run
into difficulties. First, (a) appears to be prior by nature to (c), because the particular gets the name of the character only after it acquires the character. Second, (c) makes (b) redundant, because the particular already gets its name from the character, and so there is no need to suppose that it gets its name from the Form.

The solution is to take (a) as being prior by nature to (c), and (c) as being prior by nature to (b). Thus, the particular has the character it has in virtue of the Form (a), and so the particular gets the name of the character (c); but if the particular gets the name of the character, and the character corresponds to the Form, then the particular gets the name of the Form (b). For example, if Socrates has the name “pale,” it is because he has the character of paleness; but he has the character of paleness in virtue of his participation in the Pale. Hence, it seems that the particular getting the name of the Form is shorthand for the particular getting the name of the character it has, and having the character in virtue of the Form. Note that this solution avoids the two difficulties raised above: first, (a) is prior by nature to (c); second, (b) is explained in terms of a dependence on (c), and so (b) is not made redundant in the face of (c).

**The Structure of the Commentary**

The Dilemma of Participation consists of three sections: the First Horn, the Day and the Sail Analogies, and the Second Horn. Accordingly, our commentary shall fall into three sections, each corresponding to a section of the Dilemma. The first section of our commentary shall examine the premisses that belong to both Horns of the Dilemma, and then show that the argument for the absurdity of the First Horn is deductively valid. In the second
section, we shall turn to a consideration of the Day and Sail Analogies. There we shall ask whether or not Parmenides' substitution of the Sail Analogy for the Day Analogy is legitimate. In the third section, we shall interrogate two proposed interpretations of the Second Horn, and end up proposing a new interpretation. Under this interpretation, Parmenides' argument for the Second Horn is valid, and so the Dilemma is destructive if we accept the assumptions with which Parmenides and Socrates proceed.
A. The First Horn (131a9-b2)

The First Horn of the Dilemma of Participation is a *reductio ad absurdum*. The assumption on which the absurdity turns is that the particular participates in the whole of the Form, such that the whole Form is present in each of the many particulars of the same name as the Form (131a9-10). The conclusion, that the Form is separate from itself (131b1-2), does not oppugn the existence of Forms, but demonstrates that particulars do not participate by receiving the whole of the Form.

*Translation into First-Order Logic*

Symbolising the First Horn, and proving that the resulting sequent is valid, is a laborious task, but it can be done. It is useful to translate the argument into a symbolic argument-form, because such an exercise draws out the ambiguities in the language of the argument, and applies a fixed interpretation. Nonetheless, if we are going to apply a fixed interpretation to that which is inherently unfixed, we must consider several possible translations. The first translation shall be a preliminary translation into classical first-order logic; this will be the foundation from which our other translations are developed.

A brief note about this translation, and translations to follow: In translating arguments, we will be careful to preserve the various nuances of the argument, a measure that will lead to some apparent redundancy in the argument-form. For example, we shall eventually translate the Form being “present as a whole in” as “the fusion of the parts of the
Form is present in,” which is the same as saying that “the whole Form is present in.” The reference to fusion is redundant in a sense, because it is merely a qualification made explicit: if the Form is present in, it is assumed that the whole Form is present in unless otherwise stated. But fusion plays an important rôle in one of the interpretations of the Second Horn, and so it is easier to see the correlation if all the qualifications are made explicit.

The first premiss of the argument is simply the acknowledgment by Socrates that “there are certain Forms, in which these others participate and after which [these others] get their names” (130e5-31a1). This means that some Forms are participated in by some particulars, not necessarily that all Forms are participated in; for it is conceivable that there are Forms in which no particulars participate. In this way, the claim is existential, not universal: there is at least one Form and at least one particular, and the particular participates in the Form:

\[
\begin{align*}
(1.a) & \quad \text{There is at least one Form and at least one particular, and the particular participates in the Form.} \\
(1.b) & \quad \text{There is at least one } x, \text{ and at least one } u, \text{ such that } x \text{ is a Form, and } u \text{ is a particular, and } u \text{ participates in } x. \\
(1.c) & \quad (\exists x)(\exists u)(F_x \& P_u \& Q_u)
\end{align*}
\]

The remaining claim, that the particular gets its name from the Form, is not directly relevant to the proof of the First Horn. It is separate and will be dealt with when the discussion turns to the Second Horn.

\[\text{Cf. Appendix B: The Second Disjunct.}\]

\[\text{The following abbreviations are used: } F_\_ : \_ \text{ is a Form; } P_\_ : \_ \text{ is a particular; } Q_\_, : \_ \text{ participates in } \_.\]
The second premiss of the First Horn proceeds directly from the disjunction that generates the Horns of the Dilemma. Parmenides asks Socrates if “each participant participates in the whole of the Form, or in part of the Form” (131a5). If the particular participates in the whole of the Form, that which is present in the particular is the whole of the Form; if the particular participates in part of the Form, that which is present in the particular is part of the Form. One might suppose that because “Form” is preceded by the definite article, Parmenides is referring not to all Forms, but to a specific Form. But the context makes it clear that the Form considered here is just an arbitrary Form in which the particular participates. Thus, the disjuncts themselves are universals:8

(2.a) Each participant participates in the whole of the Form, or in part of the Form.
(2.b) If a particular participates in the Form, then the whole of the Form is present in the particular; or part of the Form is present in the particular.
(2.c) For any x, and for any u, such that x is a Form, and u is a particular, and u participates in x, then there exists some z such that z is present in u, and z is the whole of x; or for any x, and for any u such that x is a Form, and u is a particular, and u participates in x, then there exists some z such that z is present in u, and z is a part of x.
(2.d) \{(\forall x)(\forall u)[(Fx \& Pu \& Qux) \rightarrow (\exists z)(Izu \& Wzx)]\} \lor \{(\forall x)(\forall u)[(Fx \& Pu \& Qux) \rightarrow (\exists z)(Izu \& Mzx)]\}

The consequent of the left conditional expresses that the whole of the Form is present in the particular, and is in contrast to that of the right conditional, which expresses that part of the Form is present in the particular. The device of supposing that there is some z that is either the whole or a part of the Form avoids the difficulty of saying that the Form itself is

8The following abbreviations are introduced: I__:_ is present in_; M__:_ is a part of_; W__:_ is the whole of_.

a whole or a part. That is, if we say that the Form is present in the particular, and ascribe being either a whole or a part to that which is present in the particular with "Iṣu" and then specify that x is either a whole or a part, we confuse the whole Form with that which is present in the particular: they are not necessarily identical. Presumably the Form is a whole in each case, and it is that which is present in the particular that is a whole or a part. (That the Form itself is not a whole is an additional consideration, and is a possible consequence of the Second Horn, but such a possibility is not intended by the First Horn.)

Insofar as the Dilemma is a disjunction, and insofar as we are currently considering the First Horn to the exclusion of the Second, the right disjunct, that part of the Form is present in the particular, may be deleted from our symbolisation of the second premiss (we shall return to it later). This yields the following simplification of the second premiss, within the context of the First Horn:

\[(2.e) \quad (\forall x)(\forall y)((F_x & P_u & Q_u) \rightarrow (\exists z)(I_zu & W_zx))]\]

Parmenides is now ready to formulate explicitly the argument of the First Horn. This formulation has two functions: the first is to establish the logical structure of the *reductio*; the second is to make explicit two unexpressed qualifications, with respect to the Forms, of (1.c) and (2.e). The argument appears in the dialogue as follows (131a9-b2):

**Parmenides:** Does it seem to you that the whole Form is in each of the many, being one, or how? [a9-10]

**Socrates:** What prevents it, Parmenides, <from being one>? [a11-12]

**Parmenides:** Then being one and the same, it will, at one and the same time, be present as a whole in things that are many yet separate; and thus it itself would be separate from itself. [b1-2]
The first question (a9-10) lays the groundwork for the contradiction, pitting the presence of the whole Form in each of the many particulars against the Form being one. That is, he presents two propositions that are incompatible, resulting from the whole Form being present in each particular: (a) the whole Form is present in each of the many, (b) the Form is one. Socrates appears not to see any difficulty with these two propositions coinciding (a11-12), and so Parmenides draws out the difficulty for him by expanding on them (b1-2). He asserts that (c) the Form is present in each of the things that are many and separate at one and the same time, and that (d) the Form is one and the same.

Propositions (c) and (d) are expanded versions of (a) and (b) respectively. First, (a) states that the whole Form is present in each of the many, and (c) reiterates this, making the additional claims that the many are also separate, and that the Form is present in each of the many at one and the same time. Second, (b) supposes that the Form is one, and (d) reiterates this, adding that the Form is also the same. Because (c) and (d) reiterate the claims already made by (a) and (b), it is clear that in symbolising the First Horn, (a) and (b) may be ignored in favour of (c) and (d). Nevertheless, neither (c) nor (d) are additional premisses of the First Horn; instead, they introduce two additional qualifications: one numeric, and one temporal. These qualifications are made with respect to the Forms of the first and second premisses, and so what was implicit there becomes explicit here. We shall make them explicit in our symbolisation.⁹

⁹Proclus would approve (cf. 1987, IV, §§860-1, trans. G. Morrow): “Notice the precision of [Plato’s] terms. He is not content to say ‘(everywhere) one,’ but adds ‘and the
The numeric qualification is that the Form is "one and the same ... in things that are many yet separate" (b1). Here, Parmenides provides additional information about the Form to which the first premiss refers. The first premiss states that there is at least one Form and at least one particular, and the particular participates in and gets its name from the Form. The numeric qualification adds that the Form is one and the same, and the particulars that participate in the Form are many yet separate. The first premiss ought to be emended in accordance with these additional stipulations. This may be accomplished by introducing another Form y and another particular v, such that v participates in y. "One and the same" can then be construed by equating x with y, and "in things that are many yet separate" can be construed by specifying that u is separate from v:10

\[(\exists x)(\exists y)(\exists u)(\exists v)(Fx \& Fy \& Pu \& Pv \& Qux \& Quy \& x=y \& Suv)\]

The temporal qualification is that the Form is present in the particulars "at one and the same time" (b1). This complicates the formulae of the First Horn, but it is a significant contribution to the argument, and so must be made explicit. Without this qualification, the relation between the Form and the particular might be some time-sharing arrangement: the whole Form might be present in particular u at some time, withdraw and be present in particular v at another time, and then withdraw and return to u. Meanwhile, the names of u and v would convulse wildly: even Heraclitus would hesitate to embrace this theory of flux!

10The following abbreviation is introduced: S__:_ is separate from _.\]
The temporal qualification may be made explicit by introducing two times, \( s \) and \( t \), and equating them. This also means that a temporal qualification has to be added to the relations of participation and separation, and to the present-in relation. We may accomplish this by turning these relations into three-place relations:\(^{11}\)

\[(1.e)\ (\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t)(F_x & F_y & P_u & P_v & T_s & T_t & Q_u x_s & Q_v y_t & x = y & S_u v_s & S_u v_t & s = t)\]

\[(2.f)\ (\forall x)(\forall u)(\forall s)[(F_x & P_u & T_s & Q_u x_s) \rightarrow (\exists z)(I_z u_s & W_z x)]\]

The conclusion of the First Horn is that the Form is "separate from itself" (b2). Although Parmenides does not make it explicit, presumably the relation of separation does not shed its temporal qualification. Multiple separate particulars participate in the Form at the same time, and so the Form is separate from itself at the time in question. In this way the conclusion may be symbolised as follows:

\[(C.a)\ \text{The Form is separate from itself.}\]
\[(C.b)\ \text{There is at least one Form such that the Form is separate from itself at some time.}\]
\[(C.c)\ \text{There exists an } x \text{ such that } x \text{ is a Form, and an } s \text{ such that } s \text{ is a time, and } x \text{ is separate from itself at time } s.\]
\[(C.d)\ (\exists x)(\exists s)(F_x & T_s & S_x x_s)\]

The above considerations yield the following argument-form for the First Horn:

\[(\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t)(F_x & F_y & P_u & P_v & T_s & T_t & Q_u x_s & Q_v y_t & x = y & S_u v_s & S_u v_t & s = t)\]

\[(\forall x)(\forall u)(\forall s)[(F_x & P_u & T_s & Q_u x_s) \rightarrow (\exists z)(I_z u_s & W_z x)]\]

\[\therefore (\exists x)(\exists s)(F_x & T_s & S_x x_s)\]

\(^{11}\)The predicates I\( _{\_\_} \), Q\( _{\_\_} \), and S\( _{\_\_} \) are temporally qualified as follows: I\( _{\_\_} \), _\_ is present in _ at time _; Q\( _{\_\_} \), _\_ participates in _ at time _; S\( _{\_\_} \), _\_ is separate from _ at time _. In addition, the following abbreviation is introduced: T\( _{\_\_} \), _\_ is a time.
The difficulty with this translation is that there is no sequence of valid inferences that allows the conclusion to be inferred from the premiss.\textsuperscript{12} There is no additional premiss implicitly expressed by the argument of the First Horn; the problem is that first-order logic is not powerful enough to express completely the argument-form. The two relations that are essential to an understanding of the First Horn are the present-in relation and the separation relation. They are both translated as uninterpreted constants, and so the logical significance proper to these relations is nullified in the current argument-form. We shall now turn to a discussion of these relations, with a view to rendering an argument-form that takes their meanings into account.

\textit{The Present-In Relation}

The present-in relation, in the present context, is nothing less than the relation of universals to particulars, and so it is not surprising that Parmenides leaves it undefined. Commentators take, if at all, a material model for this relation, and derive a contradiction from there (Allen 1997 135; Scalsas 1989, 77; Panagiotou 1987, 21). For example, suppose that Socrates and Theaetetus sit down to a freshly baked olive pie. If Socrates eats the whole of the pie, then the whole of the pie is present in Socrates’ stomach; it would be contradictory to assert that the whole of the pie is also present in Theaetetus’ stomach. In a similar way,

\textsuperscript{12}That is, $(\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t)(Fx \land Fy \land Pu \land Pv \land Ts \land Tt \land Quxs \land Qvyt \land x=y \land Suvx \land Suvt \land s=t), (\forall x)(\forall u)(\forall s)(Fx \land Pu \land Ts \land Quxs) \land (\exists z)(Izus \land Wzx) \neq (\exists x)(\exists s)(Fx \land Ts \land Sxxs)$. Here is a counterexample: Domain = \{α,β\}; Ext(F)=Ext(P)=Ext(T)=\{*\}; Ext(W)=\{*\}; Ext(I)=Ext(Q)=\{*\}; Ext(S) = \{<α,β,α>, <β,α,β>\}; (* = all permutations).
if the Form is present in a particular, it would be contradictory to assert that it is present in a separate particular.

In *Physics* IV, 3, Aristotle discusses several senses of the present-in relation, but the primary sense, from which all the others are derived, is the sense in which content is present in a vessel (210a23-4; cf. *Metaphysics*, Α23). When the vessel is considered together with its content, both the vessel and the content are said to be parts of the whole (*Physics*, IV, 3, 210a27-9). Thus, it is only by extension that the content is said to be present in the vessel: properly speaking, the content is present in the vessel-and-content considered as a whole. For example, a draught of wine is said to be present in a glass; but if the draught of wine were actually present in the glass, it would occupy the same place as the glass, which is impossible. Rather, the draught of wine is present in the glass-of-wine considered as a whole.

That the Form is a part of the particular-and-Form considered as a whole, is the sense that modern commentators tend to ascribe to the Form being present in the particular (Allen 1997, 135; Scaltsas 1989, 75).\(^{13}\) Indeed, this inference seems to be supported by Plato himself: Parmenides says that everything is related to everything else as sameness, difference, part to whole, or whole to part (146b2-5; cf. Allen 1997, 138). Since everything is related to everything else in one of these four ways, it may be supposed that the Form is related to its participating particular in one of these ways. Now, in the *Euthydemus*, Socrates

\(^{13}\)This also follows naturally from the grammar of sharing: that which shares gets that which is shared (cf. p. 2; Cornford 1957, 85; Allen 1997, 135-6).
balks at saying that Beauty is strictly the same or entirely different from the beautiful particular (Euthydemus, 301a3-4; cf. Sprague 1967, 92-3): "[beautiful particulars] are different from Beauty itself; however, some Beauty is present in each one of them." Beauty is different from its particulars in that it can be distinguished from its particulars, but not so different that it is prevented from being present in its particulars. Thus, when Socrates says Beauty is different from its particulars, he means different in the sense of not being identical to its particulars.\(^{14}\) In the context of the Parmenides, this means that Beauty is related to the beautiful particular as part to whole or whole to part; but it must be the former: Lotte is a vessel, and Beauty is her content, not \textit{vice versa}. Therefore, Plato seems to be suggesting that the Form's presence in the particular is its being a part of the particular-and-Form considered as a whole.

P. Simons presents a system of classical extensional mereology called SC (1987, 9-41), and with this system it is possible to express the relation between the Form and the particular, if they are taken to be analogous to content being present in a vessel as parts of the content-and-vessel whole. This sense of "present in" corresponds to the most basic mereological concept, that of the proper part. "Proper part" is a technical term in mereology, which corresponds to the everyday sense of "part," and the sense in which content is part of the vessel-and-content whole. In this way, when the Form is present in the particular, it is

\footnote{Using terminology that we shall develop presently, the Form is different from the particular not in the sense of being disjoint (which is the sense Socrates is thinking of when he says beautiful things are different from Beauty), but in the sense of being non-identical.}
a proper part of the Form-and-particular whole. To express symbolically that \( a \) is a proper part of \( b \), we write \( \text{\texttt{a\ll b}} \). A proper part may be contrasted with a part, because while a proper part may not be identical to that of which it is a proper part, if \( a \) is a part of \( b \), written \( \text{\texttt{a\ll b}} \), then \( a \) is either a proper part of or identical to \( b \). Thus, we may define a part in terms of being either a proper part or an identity, such that \( \text{\texttt{a\ll b}} \rightarrow \text{\texttt{a\ll b}} \lor \text{\texttt{a\ll b}} \) (SD1) (Simons 1987, 37).

The present-in relation is a three-place predicate, because \( \text{I}_{ab\chi} \) is taken to mean that \( a \) is present in \( b \) at time \( \chi \). We can express this by writing \( \chi \) as a subscript: \( \text{\texttt{a\ll b}} \). The addition of the subscript does not change any of the definitions, axioms or theorems involving proper parts as long as \( \chi \) does not vary. For example, proper parts are transitive, so that since \( \text{\texttt{(a\ll b} \& \text{\texttt{b\ll c}}) \rightarrow \text{\texttt{a\ll c}}} \) (Simons 1987, 37), we may also assert that \( \text{\texttt{(a\ll b} \& \text{\texttt{b\ll c}}) \rightarrow \text{\texttt{a\ll c}}} \). For example, imagine a triangle ABC, drawn in the sand, whose side AB is bisected by point D. In this case, line segment AD is a proper part of line segment AB, and line segment AB is a proper part of triangle ABC. Therefore line segment AD is a proper part of triangle ABC. By adding the temporal qualification “at daybreak,” we do not change the truth of this statement: if line segment AD is a proper part at daybreak of line segment AB, and line segment AB is a proper part at daybreak of triangle ABC, then line segment AD is a proper part at daybreak of triangle ABC. However, if we change the temporal qualifications, we are no longer assured of the truth of our statement: just because line segment AD is a proper part of line segment AB at daybreak does not mean that it will be a proper part of triangle ABC at dusk. Line segments AC and BC might have been erased.
so that AB is a proper part at dusk of pentagon ABEFG. Substituting $[a \ll b]$ for $l_{ab} \chi$ in the second premiss of the First Horn generates the following:

$$(2.g) \quad (\forall x)(\forall u)(\forall s)[(Fx \& Pu \& Ts \& Quxs) \rightarrow (\exists z)(z \ll u \& Wzx)]$$

**The Separation Relation**

Two additional mereological concepts are required to symbolise the separation relation. First, two individuals overlap if and only if they have some part in common. Imagine a triangle having vertices A, B and C. AB and BC overlap, because they both have point B in common. If $a$ and $b$ overlap, written $[a \vartriangle b]$, the relation may be defined in terms of parts: $[a \vartriangle b] \leftrightarrow (\exists \mu)(\mu \prec a \& \mu \prec b)$ (SD2) (Simons 1987, 37). Second, two individuals are disjoint from each other, written $[a \upharpoonright b]$, if and only if they do not overlap: $[a \upharpoonright b] \leftrightarrow (\sim(a \circ b))$ (SD3) (Simons 1987, 37). Points A, B and C are all disjoint (even though they are all connected by line segments) because they have no part in common with each other. There is an important difference between $a$ being disjoint from $b$, and $a$ not being identical to $b$, because while $[a \upharpoonright b \rightarrow a \ast b]$, it is not the case that $[a \ast b \rightarrow a \upharpoonright b]$. AB and BC are not identical to each other, and yet they are not disjoint (for they have point B in common). In contrast, it follows from A being disjoint from B that A is not identical to B.

As discussed previously (p. 5), the basic sense of separation is that of non-identity, and so we may convey this by substituting $[a \ast \chi]$, for $Sa \chi b$, in the argument-form, leading to the following translations of the first premiss and the conclusion:
At first glance, this looks like a fine translation: if we can infer (C.e) from (1.f) and (2.g), it will turn out that the Form is not identical to itself, which will be the reductio to which we have been looking forward. But (C.e) cannot be inferred from (1.f) and (2.g);\textsuperscript{15} this is because our system of classical extensional mereology is not sufficiently fine-grained to express the First Horn. For example, suppose that two lines overlap at point A. Both lines have the name of “straight,” and so the Straight is present in each line as a whole; but if the Straight is a present in the line, it is a proper part of the line. Hence, if the Straight is a proper part of point A, it need not be separate from itself, but may still be a proper part of both lines. Generalising from this case, our system is not able to account for situations where particulars overlap such that the Form is present in the overlap.\textsuperscript{16}

The solution to this problem is to render the separation of two particulars not as two particulars that are non-identical, but as two particulars that have no part in common; that is, two particulars that are disjoint. This narrows the scope of the symbolisation of the First Horn, but not drastically. Presumably the Dilemma is meant to apply with equal force to

\textsuperscript{15}That is, $(\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t)(Fx \& Fy \& Pu \& Pv \& Ts \& Tt \& Quxs \& Qvyt \& x=y \& u^xv \& u^yv \& s=t), (\forall x)(\forall u)(\forall s)[(Fx \& Pu \& Ts \& Quxs) \rightarrow (\exists z)(z \triangleleft u \& Wzx)] \neq (\exists x)(\exists s)(Fx \& Ts \& x^x).) \text{ Here is a counterexample: Domain } = \{\alpha, \beta, \gamma\}; \text{ Ext}(F)=\text{Ext}(P)=\text{Ext}(T)=\{\ast\}; \text{ Ext}(W)=\{\ast\}; \text{ Ext}(Q)=\{\ast\}; \text{ Ext}(\langle a \rangle)=\text{Ext}(\langle \beta \rangle)=\text{Ext}(\langle \gamma \rangle)=\{\langle a, \beta \rangle, \langle a, \gamma \rangle, \langle \beta, \gamma \rangle\}; \{\ast\} = \text{ all permutations}.

\textsuperscript{16}Using mereological terminology, when the Form is a proper part of the binary product (Simons 1987, 13-4).
cases where the particulars overlap. For example, if the two lines that intersect at point A are the only two straight particulars that exist, there is no indication that the Dilemma does not apply equally well here (at least, when Socrates poses his objection in the form of the Day Analogy, it is not this one). Therefore, by making the two particulars disjoint in the premiss, the argument-form becomes a special case of the First Horn. Nevertheless, we can say that the scope has not been narrowed drastically, because the vast majority of cases of participation will have at least two particulars that are disjoint. The case of two lines that intersect at point A may not be covered by our translation, but they are not the only particulars in which the Form of Straight is present. Thus, we may replace \( a \neq b \) with \( a \parallel b \) in the first premiss, understanding that the resulting formula is a special case of the First Horn:

\[
(1.g) \quad (\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t)(Fx \& Fy \& Pu \& Pv \& Ts \& Tt \& Quxs \& Qvyt \& x=y \& u \parallel v \& u \parallel v \& s=t)
\]

**Present-in as a Whole**

One last substitution needs to be made, in order to make our translation as close as possible to the argument-form of which the First Horn is an instance. We might be tempted

\[17\text{As we shall see, the Day Analogy is a counterexample of the First Horn. If the Dilemma does not apply to the case of two intersecting lines, one might expect Socrates to raise this as a counterexample instead of the Day Analogy, since it does not depend on an analogy but is a direct objection to Parmenides' line of argument.}

\[18\text{Note that we could make this substitution in (C.e) as well, but two terms being disjoint is a stronger claim than two terms being non-identical (for while the former implies the latter, the latter does not imply the former). It is not necessary to make the stronger claim to demonstrate the absurdity of the conclusion.} \]
to substitute for "Wzx" in the second premiss "(∀z)(z≺x x − z≺u)," thereby expressing that every part of x is present in u at time s. However, this substitution would see the elimination of "x≺u" that appears later in the formula, because "(∀z)(z≺x x − z≺u)" is the same as stating "x≺u" when z is the same as x. "x≺u" would not merely be redundant; it would be subsumed by another conjunct. Since our purpose in symbolising the argument is to draw out distinctions, and not to obscure them, it is preferable to formulate a translation that does not subsume the Form being present in a particular under the Form being a whole.

There are two additional mereological concepts that are required to express the Form being a whole. The first is that of a binary sum. The binary sum of a and b, written \( |a+b| \), is identified with that which something overlaps if and only if it overlaps at least one of a and b: \( |a+b| = \mu(\forall u)[(\mu^a \cdot (\mu^b)] \) (Simons 1987, 32). For example, a broom is the sum of its head and handle, or a fortnight is the sum of the first and the second week. We may also nest binary sums in such a way that a sum can have more than two constituents. For example, suppose that a wagon’s five constituent parts are its wheels, axle, body, rails and yoke (cf. Theaetetus, 207a6-7; cf. Burnyeat 1990, 191-4). The sum of these parts, called a wagonsum, may be described by nesting binary sums in the following manner: wagonsum = \{((wheels+axle)+(body+rails))+yoke\}; but the more parts we have, the more tedious this becomes.

The second mereological concept, that of fusion, can be used to avoid this sort of tedium. A fusion is the sum of all objects satisfying a certain predicate. We may symbolically represent fusion as \( σd(λd) \), such that σ is a variable-binding operator and d is
the sum of all objects that are \( \lambda \), and \( \lambda d(\lambda d) = \mu(\forall \mu)[\mu \circ \lambda \rightarrow (\exists \nu)(\lambda \nu \& \nu \circ \mu)] \) (Simons 1987, 15; 37). Thus, we can use the concept of fusion to describe a wagon sum with ease: \( \text{wagon sum} = \sigma x(x \lt \text{wagon}) = \{(\text{wheels+axle}) + (\text{body+rails}) + \text{yoke}\} \). That is, a wagon sum is the sum of every \( x \) such that \( x \) is a part of a wagon. Generally, \( \sigma d(d \lt e) \) is the fusion of all that is a part of \( e \), and whatever constant is identified with this fusion will be the sum of those parts (in other words, the whole).\(^{19}\) This may be applied to (2.g) by substituting "\( z = \sigma p(p \lt x) \)" for "\( Wzx, \)" which allows the claim that the Form is whole is expressed without eliminating "\( x \in u \)" in the process:

\[
(\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t) (Fx \& Fy \& Pu \& Pv \& Ts \& Tt \& Quxs \& Qvyt \& x = y \& u \ Subseteq v \& u \ Subseteq v \& s = t) \\
(\forall x)(\forall u)(\forall s) [(Fx \& Pu \& Ts \& Quxs) \rightarrow (\exists z)(z \in u \& z = \sigma p(p \lt x))] \\
\therefore (\exists x)(\exists s)(Fx \& Ts \& x \neq x)
\]

Difficulties with the Proper-part Relation: Points and Elephants

The First Horn seems to lead nicely to a reductio on this interpretation,\(^{20}\) but there are serious difficulties that stem from construing the present-in relation as a proper part. Hitherto, we have taken the Form’s presence in the particular as a necessary condition of the particular’s having the same name as the Form. This is supported by the Phaedo. There, we find Socrates saying that beautiful things are beautiful in virtue of Beauty (\( \kappa \alpha \lambda \delta \) ) (100c2-3), large things are large in virtue of the Large (\( \mu e \gamma \theta e t \) ) (100e5-6) and ten exceeds eight in virtue of Two (\( \delta \nu o i \nu \) ) (101b4). We may formulate this as a general principle. A particular

\(^{19}\)This will be employed as Theorem Ta (cf. Simons 1987, 40, SCT63): (\( \forall x)[x = \sigma y(y \lt x)]\).

\(^{20}\)The corresponding sequent is proved in Appendix A.
has a certain name in virtue of participation in the corresponding Form, and so under our current interpretation,

(a) If the particular has a certain name, the Form of the same name is a proper part of the particular.

This principle (a) is at odds with the presence of the Form in a particular that has no parts. Consider, for example, Euclid’s first definition in his *Elements*, that “a point is that which has no part” (I, Def. 1, trans. T. Heath).\(^{21}\) By (a), since the point has the name of “one,” the One is a proper part of the point. But if the One is a proper part of the point, the point has parts and so ceases to be one (and moreover by the definition is no longer a point).

Our difficulties multiply when we consider that the Form’s presence in the particular is also the sufficient condition of the particular’s having the same name as the Form. Let us state from the outset that by taking the Form’s presence as a sufficient condition, we do not intend to advance the claim that the Form is the efficient cause of the particular’s having the same name as the Form.\(^{22}\) Instead, we merely wish to suggest that, given the Form’s presence in a particular, we can infer that the particular has the same name as the Form.

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\(^{21}\) Plato does not give a definition of a point, but according to Aristotle, Plato said that the point was the principle of the line (*Metaphysics*, A9, 922a21-2). In the *Topics*, Aristotle states that “it is said that a point is the limit of a line” (VI, 4, 141b21-2, trans. T. Irwin), but whether he had Plato in mind when he said this is not immediately clear from the text. See Heath’s commentary on the first definition for more information on pre-Euclidean definitions of the point (1956, 155-6).

\(^{22}\) G. Vlastos has put to rest the view that the Forms are simply efficient causes (1969, 291-325, esp. 303-5); and while there are those who think (justifiably in my view) that Vlastos goes too far in stripping the Forms of all causal efficacy (Sweeney 1988, 126-9), this is a debate into which we shall not step.
We find this principle supported in both the *Phaedo* and the *Parmenides*. In the former, Beauty makes (ποιεῖ) all beautiful things beautiful (100d4-5; cf. Vlastos 1969, 305). In the latter, things that get a share of the Like become (γίγνεσθαι) like in the degree and to the respect that they get a share (129a3-5). This is picked up by Parmenides when he asks Socrates "if things getting a share of the Like become (γίγνεσθαι) like, of the Large become (γίγνεσθαι) large, and of Beauty and the Just become (γίγνεσθαι) beautiful and just" (131a1-3). Note that the use of γίγνεσθαι as opposed to εἶναι implies motion as opposed to immediacy. We might draw on this, as well as the use of ποιεῖν, as evidence that the Forms have some causal efficacy, but we need not make our claim this strong. We only need to hold that the particular’s participation in a given Form is sufficient to infer the particular’s having the same name as the Form; in other words,

(b) If the Form is a proper part of the particular, the particular has the same name as the Form.

Beyond the textual evidence cited above, we find Vlastos’ submitting that "x [a particular] is F [a character] (that is, x has the character, F) if, and only if, x participates in Φ [the Form]" (1969, 301). That is, the Form’s presence in the particular is the sufficient and necessary condition for the particular’s having the name of the Form. Again, this is not to take the Form as the efficient cause of the corresponding character, for Vlastos vehemently rejects this. Rather, Vlastos likens the relation between the presence of the Form in the

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23 Such an argument would be far from simple, given Vlastos’ objections to supposing that ποιεῖν indicates causal efficacy (1969, 305-6).
particular, and the particular’s having the corresponding character, to the relation between
premises and their semantically entailed conclusion: "... our assurance of the truth of the
former warrants equal assurance of the truth of the latter" (1969, 296). One would not say
that the premises are the cause of the conclusion (in the sense that the premises are self-
movers), but rather that given the premises, one may infer the conclusion.

Now that we have established principle (b), there is a serious difficulty that finds its
origin in the transitivity of the proper-part relation, and the apparent non-transitivity of the
present-in relation: when Forms are considered to be proper parts of participants, it is
possible for the same thing to be qualified by opposites in the same respect. In his solution
to Zeno’s Paradox, Socrates suggests that there is nothing strange in particulars being
qualified by opposites (129c4-d5); but the inference is that they are qualified by opposites
in different respects: Socrates is one with respect to being a man, but many with respect to
the parts of his body (cf. 129a4-7). By construing the present-in relation as the proper-part
relation, we allow the possibility that the same particular is qualified by opposites in the
same respect. Now, imagine that we have a large elephant with a small ear (an Indian
Elephant). Since the ear is small, we may infer by (a) that the ear has the name “small” in
virtue of Small being a proper part of the ear. Due to the transitivity of the proper-part
relation, if Small is a proper part of the ear, and the ear is a proper part of the whole elephant,
it may be concluded that Small is a proper part of the whole elephant. Moreover, Large is
a proper part of the whole elephant by (a), because the whole elephant has the name “large.”
Hence, both Large and Small are proper parts of the whole elephant, and by (b), the whole
elephant has the names "large" and "small," and is qualified by these opposites in the same respect. This difficulty seems to have its origin in the transitivity of the proper-part relation, a property that is not shared by the present-in relation.

One might object to these considerations, because we are appealing to two different senses of "present in" identified by Aristotle. The first is the primary sense, in which content is said to be present in a vessel (Physics, IV, 3, 210a27-9); the second is a derivative sense, in which a part is said to be present in the whole (Physics, IV, 3, 210a15-6). This objection has some validity, because there is clearly a distinction between a draught of wine being a proper part of a glass-of-wine, and an ear being a proper part of an elephant; for the former is present-in as content is present in a vessel, and the latter is present-in as a body part is present in the whole body. Mereology obscures this distinction. Nevertheless, it makes no difference to the problem of the present-in relation being non-transitive: similar difficulties may be exhibited by putting a small grey mouse in a large white bathtub. In this case, the bathtub will have both Large and Small, and both White and Grey, as proper parts, each of which is present in the bathtub in the primary sense of "present in."

These considerations mean that the present-in relation is not adequately expressed by the relation between parts and wholes; but in what other way could the Form be present in the particular? Commentators are silent. It is possible, of course, to say that the relation between Forms and particulars is unique, and that the Dilemma turns on an abuse of the metaphor of participation. Once this is admitted, the First Horn is easily dismissed, and the Dilemma collapses: there is no longer any reason that the Form should not be present as a
whole in many particulars at one and the same time. Of course, we are no closer to understanding what the present-in relation entails.

Aristotle distinguishes a sense of “present in” in which form is present in matter (Physics, IV, 3, 210a20-2). For example, greyness is present in the surface of the elephant’s ear. Being present in the surface, greyness is part of the surface insofar as it may be distinguished from the surface (for the surface is also extended in space, and has length and width), but the greyness may not be separated from the surface. This means that the relation of greyness to the surface is not transitive: just because greyness is present in the ear does not mean that it is a proper part of the elephant of which the ear is a proper part. That is, the fact that Grey is present in the ear does not imply that Grey is present in the whole elephant. Unfortunately, Aristotle does not go into detail about this relation, but we may represent that \( a \) is present in \( b \) by writing \( \text{ap}_x \). Again, we may use a subscript \( \gamma \) as a temporal qualification, and replace all occurrences of \( \text{ap}_x \) in our symbolic argument-form with \( \text{ap}_x \):

\[
(\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t)(Fx \& Fy \& Pu \& Pv \& Ts \& Tt \& Qvxs \& Qvyt \& x=y \& u \frac{1}{2} v \& u \frac{1}{2} v \& s=t) \\
(\forall x)(\forall u)(\forall s)((Fx \& Pu \& Ts \& Qvxs) \rightarrow (\exists z)(z \& u \& z=\text{ap}(p \& x))) \\
\therefore (\exists x)(\exists s)(Fx \& Ts \& x \& x)
\]

Of course, the question of what \( \text{ap}_x \) means still remains. Without axioms to disclose the rôle of \( \text{ap}_x \), the present-in relation has the status of an uninterpreted three-place predicate. This is equivalent to the claim that the present-in relation is unique, and it neatly prevents the proof of the First Horn.
The Present-in Relation Revisited

If the present-in relation is not unique, there must be some relation to which it may be likened. So far we have only demonstrated that it cannot be likened to the proper-part relation of classical extensional mereology. It is beyond the scope of this paper to undertake a thorough examination of the present-in relation, which is fortunate, because such a task is beyond the power of this writer. Nevertheless, we have caught a glimpse of what a more involved description of the present-in relation might involve.

We can begin by stating that the present-in relation is not unique. If the present-in relation were unique, one would expect Socrates to claim that it is peculiar to Forms alone (cf. Allen 1997, 136). Instead, Socrates proposes the Day Analogy (131b3-6), which suggests that the present-in relation is intelligible in terms of a relation that we can grasp and define. We may take the same line against those who believe that the Dilemma turns on an abuse of metaphor. To submit that the present-in relation is a metaphor, is to invite the question, to what does the metaphor refer. If the metaphor refers (or indicates a similarity) to something, then we have only delayed defining that something; if it refers (or indicates a similarity) to nothing, then we are simply asserting that the present-in relation is unique.

Moreover, that to which the present-in relation is likened should not offend either of the two ways that the relation has been described. The first way the relation has been described is by the word “ένειν.” Our difficulties do not turn on an abuse of metaphor, but nor did Plato arbitrarily select the word used to describe the relation between Forms and particulars. Of course, there is much room for interpretation as to what “ένειν” means; but
the word ought not to be ignored completely. It would be difficult, for example, given two disjoint solids, to suppose that the one was present in the other. The second way that the relation has been described is by analogy to the day (b3-6). If the Day Analogy is Socrates' solution to the difficulties of the First Horn, then the relation between the day and its many places must have some resemblance to that between the Form and its many participating particulars. Even if the Day Analogy turns out to be a false cast, it is not unreasonable to suppose that there is something to Socrates' suggestion: the analogy is (almost certainly) more than a random utterance.

From our discussion of the proper-part relation, it is clear that the present-in relation is different; but one of our prime motivations for construing the present-in relation as the proper part relation was Parmenides' claim that everything is related to everything else as sameness, difference, part to whole, or whole to part (cf. p. 18). As long as we accept this, and as long as we consider the part Parmenides is talking about to be the proper part of classical extensional mereology, these difficulties will not absent themselves. But suppose we take a lesson in spelling from the *Theaetetus* (203c4-6):

Look here, what do we mean by 'the syllable': both letters (and if there should be more than two, all the letters), or some single form (ιδεαν) arising from their combination?

The latter alternative suggests that the syllable is not an aggregate of its letters, but that which arises from a certain ordered combination of letters. Considering the first syllable of Socrates' name, it is clear that it is not merely an aggregate of 'S' and 'O,' otherwise "Socrates" would be the same as "Oscrates." But if 'S' and 'O' are the constituent parts of
the syllable in question, the arrangement is something in addition to these parts, and yet it itself is not a part.

In this way, there might be an alternative to the Form being related to the particular as a part to a whole: the Form may be present in the particular as a arrangement of certain constituent parts of the particular. Consider Euclid’s definition of an equilateral triangle: an equilateral triangle is that figure which is contained by three equal straight lines (Elements I, Def. 14, Def. 19, Def. 20). 24 Euclid specifies the constituent parts of the triangle, namely three equal straight lines, and the arrangement of these parts, namely that they be ordered end to end in such a way that they bound the figure in question.

Construing the present-in function is fraught with difficulties, some of which are raised in the Theaetetus (203c8 ff.). For this reason, we shall not pursue it further here. However, Theaetetus and Socrates have suggested an alternative mode of attack on the Dilemma: if we cannot crumple either of the Horns, perhaps we may leap through them.

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24For “figure” in Plato, see the Meno: “I say that a figure is that in which the solid terminates; or if I might speak briefly, a figure is the boundary of the solid” (76a5-7); for “line,” see the Parmenides: “But it would be straight, if the middle should run to both ends” (137e3-5). See also Heath’s commentary on Euclid’s definitions (1956, 165-6, 182-3, 187-8).
B. The Day and Sail Analogies (131b3-c4)

The Day Analogy has been praised for being the single spot of colour on the *Parmenides*’ otherwise grey canvass; but its genius lies less in the illustration chosen, and more in the transition effected between the Horns of the Dilemma. Parmenides has just demonstrated that the premisses of the First Horn lead to a contradiction, but this conclusion depends on the truth of Theorem $T_\gamma$:

$$(T_\gamma): \ (\forall x)(\forall y)(\exists u)(\forall v)[(x \neq u \ & \ y \neq v \ & \ u \neq v) \rightarrow x \neq y]$$

Another way of expressing this theorem (and adding a temporal qualification) is that one and the same thing cannot be present, at one and the same time, in many separate things.

Socrates makes an objection to $T_\gamma$ in the guise of the Day Analogy: he provides a counterexample. He claims that one and the same day is, at one and the same time, in many (presumably separate) places, and yet is not separate from itself (b3-5). Parmenides responds to this counterexample by replacing it with the Sail Analogy, a move that serves two purposes. First, the Sail Analogy neutralises the force of Socrates’ objection, because the sail provides obvious support for $T_\gamma$. Second, the sail is an excellent illustration of the thesis of the Second Horn (that part of the Form is present in each particular), whereby prompting a transition to this thesis. Here are the Analogies as they appear in the text (131b3-c4):
**Socrates:** [The Form] would not [be separate from itself], at least if it were like the day (ἡμέρα), which, while being one and the same, is in many places at once, and is nevertheless not separate from itself. If it were like this, each of the Forms, [being] one and the same, would be present in all [its participants] at one and the same time. [b3-6]

**Parmenides:** Very clever, Socrates. You make one and the same thing be in many places at one and the same time, just as if (ὁδοὺ ἐν), having spread a sail over many men, you said that one whole was over many; or is this not the sort of thing you mean to say? [b7-10]

**Socrates:** Perhaps. [c1]

**Parmenides:** So would the whole sail be over each, or would [part of it be over one], and another part of it be over another? [c2-3]

**Socrates:** Part. [c4]

The First Horn depends on the validity of Τγ, and so Socrates’ instincts are right in challenging it: to deny Τγ is to crumple the First Horn. Parmenides avoids this prospect not by attacking the Day Analogy directly, but by replacing it with the Sail Analogy - a replacement Socrates reluctantly accepts (c1). In making this substitution, Parmenides is making two claims about it. The first is that the Sail Analogy supports Τγ. We may commonly say that one and the same sail covers many men, in the same way that one and the same roof covers many men (cf. Panagiotou 1987, 19).25 But if we were to speak precisely, we would say that part of the sail covers one man, and another part of the sail covers another man. Thus, the Sail Analogy supports Τγ.

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25David Hitchcock raises an interesting question: why a sail? Is there something in Plato’s experience that would lead him to an analogy of casting a sail over a number of men? The translation of “στηλώ” as “sail” seems to be standard in English scholarship; however, A. Diès translates the word as “un voile,” meaning a veil or web, as opposed to “une voile,” meaning a sail.
The second claim Parmenides makes is that the Sail Analogy is equivalent to the Day Analogy, a claim that “οὐλ οἱ” (b8) makes explicit. Much depends on whether the Sail Analogy is equivalent to the Day Analogy; because if it is, we are logically committed to the Second Horn (unless there is some other counterexample for Tγ). However, if the Sail Analogy is not equivalent to the Day Analogy, it may be that the Day Analogy is a valid counterexample of Tγ. Thus, if Tγ fails to hold for the day, it may also fail to hold for the Forms.

*The Validity of Parmenides’ Substitution*

There is a surfeit of recent literature concerning the validity of Parmenides’ substitution of the Sail Analogy for the Day Analogy, and opinions diverge. On one hand, K. Sayre and K. Dorer hold that a young Socrates is cowed into accepting Parmenides’ substitution, a substitution that should never have been accepted (1996, 76-7; 1994, 28-9). On the other hand, S. Panagiotou and R. E. Allen advance the claim that Socrates is on the wrong track, and Parmenides is right to make the substitution (1987, 12; 1997, 131-3).26 To decide who is correct is to decide whether Parmenides’ substitution is valid; but validity is not the same as equivalence. Parmenides makes the claim that the analogies are equivalent, and in this case he would certainly be justified in making the substitution. But even if they are not equivalent, there may be other justifications for making the substitution. (Note that

26Panagiotou’s discussion of the analogies in his article “The Day and the Sail Analogies in Plato’s *Parmenides*” is the only thorough treatment of this passage that I know of (since Proclus). The other commentators mentioned present their views, but do not go into much detail.
if the substitution is justified in some other way than by equivalence, we may still be suspicious of our findings, because Parmenides explicitly makes the claim of equivalence.)

Hence, we may articulate the criterion for the validity of Parmenides’ substitution. The Sail Analogy may be validly substituted for the Day Analogy if and only if one (or more) of the following conditions are met:

(a) the Sail Analogy is equivalent to the Day Analogy;
(b) the Day Analogy is not a counterexample of \( \Theta \gamma \); or
(c) the Day Analogy begs the question.

The first condition (a) under which the substitution is valid is if the Sail Analogy is indeed equivalent to the Day Analogy (as Parmenides claims). When a sail is spread over many men, it was noted above that the whole sail does not cover each man, but part of the sail covers one, and another part of the sail covers another. This is in accordance with a material model of the Forms. In contrast, Socrates claims that one and the same \( \eta \mu \epsilon \rho \alpha \) is, at one and the same time, in many places, while not being separate from itself. If it turns out that Socrates’ assertion is not the case, but rather that part of \( \eta \mu \epsilon \rho \alpha \) is in one place, and another part of \( \eta \mu \epsilon \rho \alpha \) is in another, then the two analogies are equivalent. Moreover, the Sail Analogy has the virtue of providing a more concrete analogue than the Day Analogy, and so Parmenides would be justified in making the substitution.

The second condition (b) under which Parmenides’ substitution is valid is if the Day Analogy is not a counterexample of \( \Theta \gamma \) in the way Socrates is proposing.\(^{27}\) According to our

\(^{27}\)We add “in the way Socrates is proposing,” because there are certain senses of “\( \eta \mu \epsilon \rho \alpha \)” which look like promising counterexamples of \( \Theta \gamma \), but cannot be employed because
current interpretation, ἡμέρα is Socrates’ proposed counterexample of Ῥγ: it is one and the same, and yet it is in many places at once. But if the Day Analogy does not function as a valid counterexample of Ῥγ, Socrates’ proposal falls flat. Parmenides is justified in replacing the day with the sail, for the support of the latter for Ῥγ is eminently clear, and it gets the argument back on track with little ado. Of course, we may add the caveat that there might be some other counterexample for Ῥγ that has not been considered. In this case, Parmenides is certainly not at fault for reeling in Socrates’ false cast, for there may be cases for which Ῥγ does not hold.

The third condition (c) under which Parmenides’ substitution is valid is if the Day Analogy begs the question. The Dilemma attacks the possibility that many particulars can get the same name. According to the Day Analogy, ἡμέρα is one and the same in many separate places. But if ἡμέρα is thought to be one and the same merely because the same name is being given to that which is in many separate places, then the Day Analogy attempts to solve the Dilemma by assuming that many particulars can get the same name. If Socrates’ proposed counterexample begs the question in this way, then Parmenides’ substitution is not merely valid - he is showing off his dialectical prowess. In one fell swoop, Parmenides has

they do not seem to fit within the (admittedly scanty) context of the Day Analogy. For example, when Teiresias says to Oedipus, “I bid you to that proclamation/ Which you decreed to hold fast, and from this day/ Forth (καὶ ἡμέρα τῆς νόμον) address neither these men nor me, since you are the accursed pollution of this land” (Oedipus Rex, 350-3). Clearly “ἡμέρα” refers to a moment in time; but if Socrates is referring to a point in time in the Day Analogy, there seems to be no reason for him not to state this explicitly.
replaced the day with an example that does not beg the question (thus setting aside the fallacy) and moreover supports the theorem under attack.

**Testing for Validity**

Whether Parmenides' substitution is valid depends on the sense of "ημέρα" in question, for the substitution might be valid for some senses, and invalid for other senses. Hence, we may test the validity of Parmenides' substitution by analysing different senses of "ημέρα." If there is a sense that does not meet the criterion for validity, then Socrates has been, as Crombie fears, "railroaded into accepting" the Sail Analogy (1963, II, 331). If we can find no such sense, then it is probable that the substitution is valid (and it is only probable, because even if the criterion for validity is met by all the senses we analyse, there still might be some sense that we have not analysed that does not meet the criterion).

The task of analysing the different senses of "ημέρα" is greatly simplified by the fact that its senses are largely coincidental with the senses of the English word "day." Furthermore, we need not interrogate every sense of "day" (which are numerous), but only those that appear to be a candidate for not fulfilling the criterion of validity. Here are three different senses of "day" that we shall analyse:\textsuperscript{28}

(I) And he blushed- for already a streak of day (ημερας) appeared, so that it might be clearly seen - and said "if this is in some way like the previous [cases, the reason] is clear: to become a sophist."

(II) And now, Ares, driven to madness,

\textsuperscript{28}The selected passages are: (I) Plato's *Protagoras* 312a2-4 (cf. Panagiotou 1987, 18); (II) Sophocles' *Trachiniae* 653-4; and (III) Jane Austen's *Pride and Prejudice* I, 7, 36.
Drive away her troublous day (ἁμέραν).

(III) That she should have walked three miles so early in the day, in such dirty weather, and by herself, was almost incredible to Mrs Hurst and Miss Bingley; and Elizabeth was convinced that they held her in contempt for it.  

(I) In the first sense of “day,” the day is that which allows Socrates to see Hippocrates’ colour; and so it must refer to daylight. Since the source of the daylight that shines on Socrates and Hippocrates is the same, one might erroneously infer that one and the same daylight shines on both; an erroneous inference because part of the daylight shines on Socrates, and another part on Hippocrates. Day in this sense is just like the sail; for just as part of the daylight shines on one and another part shines on another, so too part of the sail covers one and another part covers another. This reading makes the Day Analogy equivalent to the Sail Analogy, and so the first condition (a) is met. If this is the sense of “day” Socrates intends, Parmenides is justified in substituting the Sail Analogy for the Day Analogy.

(II) The second sense of “day” refers to a period of Deianira’s life during which she is suffering, for she is faithfully and apprehensively awaiting the return of her husband. Because the extent of this period is relative to Deianira’s suffering (for it is called her

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29Cf. *Iliad*, I, 601-2, for the same sense in classical Greek: “Thus, thereafter they feasted all day (ἡμαρη) until the sun went/ Under, nor was the hunger of anyone denied a fair portion.”

30Rossavaer seems to make this error (1983, 20): “The Day in this sense is something that embraces all the separate things, the day shines over them all.” But since he goes on to say “this is what relates the visible things to one another; but the light of day itself is not like another visible thing,” he may have the sun of the *Republic* in mind (cf. 508b1-10). While the Day Analogy certainly recalls the sun of the *Republic*, it is wrong to suppose that the latter is directly applicable here.
“ἐπίσοπον ἀμέραν”), it is doubtful that this sense of day will function as a counterexample of Τγ. First, it does not seem to make sense to speak of this day being in many places; and if it be insisted that it is in a place, it most likely is only in one place - that which Deianira’s heart occupies. Second, it seems that this period of suffering is a unity for no other reason than that the chorus has called all the moments that make up this period by the same name: Deianira’s “ἐπίσοπον ἀμέραν.” For these reasons, the second condition (b) and the third condition (c) are met.

(III) In the third sense of “day,” the day is a time interval during the early part of which Elizabeth walked three miles to Netherfield. Because her journey spans the early part of this interval, and it is unseemly for a young lady to be traipsing around the countryside before the sun has risen, we may safely conclude that the time interval in question is that between sunrise and sunset. If this sense of “day” is not to be immediately convicted of meeting condition (b), we must accommodate Socrates’ claim that the time interval in question is in many places (πολλαχοῦ) (b4) - for time intervals are not usually thought of as being in a place.

We might be tempted to argue that this day is in many places at once from the fact that Elizabeth begins the day at Longbourn, and Mr Darcy begins the day at Netherfield, and yet the day, for both of them, is the same. Nevertheless, we are not entitled to infer that the day is in many places at once, given that the day is the same for both of them; for it might
be that Elizabeth and Mr Darcy are in two places, and that they both share in the same day,\textsuperscript{31} but not that the day itself is in two places. The distinction is subtle but necessary. If it is those that participate in the day, and not the day itself, that are in places, then we meet condition (b), because Socrates plainly intends πολλάχρονο to modify ἡμέρα (b3-4). Furthermore, by assuming that Elizabeth and Mr Darcy have a share of the same day we are begging the question; for such a sharing is prohibited as long as the Dilemma stands.

What we need, to legitimately claim that one and the same day is in many places, is for the day to be the same for Elizabeth and Mr Darcy because one and the same day is present in their respective locations. As we have mentioned, time intervals are not usually thought to be in places; but since we are dealing with a time interval that is relative to the light of the sun, there is a sense in which this time interval is in a place. After all, if the time interval begins when the sun rises and ends when the sun sets, that time interval will be the same for Longbourn and Netherfield (which are only three miles apart). But this interval is not coincident with its namesake (according to its date) in Athens; for in Athens, it began and ended two hours earlier. Therefore, this time interval is in a place, because while it is in many places at once, it is in some places and not others.\textsuperscript{32}

\textsuperscript{31}Parmenides will make this claim presently (151e7-2a2): “But is ‘to be’ something other than sharing in being with the present time, just as ‘the was’ has being in common with the past time, and ‘the will be’ has being in common with the time to come?”

\textsuperscript{32}Panagiotou also arrives at the conclusion that there is a sense in which time is in place (1987, 18-9): “No one would dispute that a given moment of a day is just that moment of day, and that, in this sense, the same moment is over many places at once.” Hence, if every moment is in many places at once, the entire day (as the succession of moments between sunrise and sunset) must also be in many places at once.
It has been noted previously that the present-in relation is based on a material model (p. 17), and that it is this model that validates $\Upsilon$: the whole pie cannot be in Socrates’ stomach and in Theaetetus’ stomach at one and the same time. If our reading is correct, then the Day Analogy is challenging $\Upsilon$ by overturning the material model, and suggesting a temporal model in its place (cf. Sayre 1996, 76): a temporal interval is not bound by the same restrictions as is a material object, and so the unity of a temporal interval is not compromised by its being in many places at once.

At this point, it becomes clear that Parmenides’ claim of equivalence is false for this sense of the “day” (cf. p. 35): the analogies are not equivalent, because it is not the case that part of the day is in one place, and another part of the day is in another place.33 Therefore, condition (a) is not met. Moreover, the time interval in question is not a unity merely because the succession of moments of the interval are being given the same name; its unity is based on a natural astronomical movement. Therefore, the third condition (c) is not met.

It remains to be seen, however, whether in this sense of “day” the day is a counterexample for $\Upsilon$. It might seem patently obvious that it is; after all, $\Upsilon$ asserts that one and the same thing cannot be present, at one and the same time, in many separate things. And here we have a time interval that is an exception - condition (b) is therefore not met. However, we cannot forget a claim we made previously; that is, to say that the Form is

33Cf. Dorter 1994, 29: “To be sure, it has been pointed out since antiquity that a day can be divided, in the sense that the sun reaches some places later than others. But this would not be true within the same city (or along any longitude), so Socrates’ example is valid.”
present, without qualification, in the particular, is the same as saying that the whole Form is present in the particular (cf. p. 25). Therefore, if the day is, without qualification, in many places at once, then it is the whole day that is present.\textsuperscript{34}

Panagiotou raises this very issue, only to conclude that it is absurd that the whole day should be in many places at once (1987, 18; 19):

One may suggest that the point at issue is not whether or not a time interval is simultaneous with itself at any given moment, but rather whether or not “any given moment” of the time interval is simultaneous and, hence, identical with the \textit{whole} interval ... However, the question is whether or not that moment is the same as the \textit{whole} day. A positive answer would, absurdly, collapse the \textit{whole} day into any given \textit{moment} of it.

That is, Socrates’ counterexample is invalid, because he needs not only to show that one and the same day is in many places at once, but that one and the same day - as a whole - is in many places at once. If we accept Panagiotou’s view (and since it follows from Theorem T\textalpha, there is good reason to accept it), we meet the second condition (b); for the whole time interval is not in many places at once the way a whole Form is supposed to be in the First Horn. What, if anything, has gone wrong?

Given the material model of the Forms that Parmenides’ argument assumes (cf. p. 17), if the whole Form is present in a particular, then no part of the Form is not a part of the particular in question: if the whole of the Small is present in the elephant’s ear, no part of the Small is not part of the ear. Parmenides is careful to introduce the temporal qualification that

\textsuperscript{34}This move (cf. Theorem T\textalpha) creates some unnecessary difficulties for our interpretation; but the consideration of these difficulties serves to draw out hitherto concealed features of the temporal model.
the whole Form is present in many particulars "at one and the same time" (b1; cf. p. 15); and he is right to do so. But in so doing, he has predisposed us to think of the Form as being present in many particulars - at some given moment (we get a static "snapshot" of the present-in relation, so to speak). There is no difficulty in thinking of the Form present in a single particular at a given moment; for the wholeness of the Form does not conflict with its being present in the particular at some given moment; and when we consider the Form present in many particulars at a given moment, the contradictions of the First Horn are sprung upon us as expected. Thus, considering the Form's presence in many particulars - at a given moment - is the natural way to consider the issue under the material model. But when this propensity to consider participation at some given moment is transferred to the temporal model of the Forms, we arrive at Panagiotou's position. According to Socrates, the day is present in many places; and so the claim becomes that the whole day is present in many places at some given moment.

We must now consider whether the day under the temporal model is bound by the same stipulations as Forms under the material model. There are two good reasons to think not. The first is that the propensity to consider participation at some given moment is nowhere made an explicit requirement of participation. To be sure, it works admirably well with the material model; but it is not necessary to apply this mode of analysis to the temporal

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35 Note that "at one and the same time" is different from "at some given moment"; for it is "at some given moment" that the Form is present in many particulars "at one and the same time", not *vice versa*. 
model. Temporal intervals simply call for a different mode of analysis. Second, wholeness with respect to temporal intervals is different from wholeness with respect to material objects. A temporal interval may very well be in a given place as a whole, but only if every successive moment of the interval is in that place. In our example, the whole day is present in many places at once if every successive moment of the day, from sunrise to sunset, is both in Longbourn and in Netherfield. In contrast, a material object is in a place as a whole at any given moment that it is in that place as a whole.

Therefore, it seems that Panagitou’s objection is not applicable to the current sense of “day,” as long as it be admitted that temporal intervals are not bound by the same restrictions as material objects. Condition (b) is not met, and so if Socrates intended this sense of “day” in the Day Analogy, Parmenides’ substitution is not valid.36

**Conflicting Senses**

If the ἰμέρα of the Day Analogy has the sense in (I), then Parmenides’ substitution is valid; if it has the sense in (III), then the substitution is invalid. Unfortunately, the context of the Day Analogy is of little aid in determining which sense Socrates had in mind: “πολλαχοῖ” (b4) while not offensive to the sense in (III), fits more naturally with the sense in (I); but many commentators have thought Socrates’ hesitant “ἒσως” (c1) indicates that

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36Sayre adds two additional reasons to take the temporal model seriously: it reappears later in the *Parmenides* at the third deduction of the first hypothesis, and also when Socrates restates the Dilemma of Participation in the *Philebus* (cf. 1996, 77).
Socrates knows that something has gone wrong, but is unable to put his finger on it.\textsuperscript{37} How do we decide which sense of “day” has tumbled out of Socrates’ mouth?

In the absence of decisive contextual evidence, a compelling reason to suppose that Socrates means both the sense in (I) and the sense in (III) at once, is the suspicion that Plato himself is inviting us to participate in the dialogue.\textsuperscript{38} If Parmenides takes a theory of Forms largely similar to that presented in the \textit{Phaedo} and in the \textit{Republic} and impales it on the First Horn, it is natural to suppose that we are being challenged: how can this conclusion be avoided? We have been given a hint - the temporal model. The sense in (III) is a valid counterexample of \(\Upsilon\), and so the relation between the Form and its many particulars is somehow like the relation between the day as a temporal interval and its many places; but the question is - how?

\hspace{1cm}

\textsuperscript{37}See Panagiotou (1987, 20), for a dissenting opinion, and Dorter (1994, 23, 29-30), for the rôle of hesitations throughout this part of the dialogue.

\textsuperscript{38}To suggest that both senses are intended at once is not to say that Plato conflates the senses, as Panagiotou suggests (1987, 17). Although Plato may do so at \textit{Timaeus} 45b5, the fact that daylight is meant at \textit{Protagoras} 312a2-3, and a time interval is meant at \textit{Timaeus} 37e1-3, suggests that it is quite possible for the senses to be held apart in the Day Analogy.
C. The Second Horn (131c5-e3)

The Second Horn is a *reductio ad absurdum*, which makes the Dilemma of Participation a destructive dilemma. The absurdity of the First Horn issues out of its premiss: it is simply not the case that the Form is at one and the same time present as a whole in more than one separate particular. In contrast, the Second Horn supposes that part of the Form is present in the particular; but this is not intrinsically contradictory; some additional understanding must be supplied. Here is the Second Horn, as it is appears in the dialogue (131c5-e3).39

*Parmenides:* And so, Socrates, the Forms themselves are divisible, and the participants of the Forms would participate in part [of the Forms]; and no longer would the whole [Form] be present in each, but part of each [Form] would be [present in each]. [c5-7]

*Socrates:* So it appears. [c8]

*Parmenides:* And so, will you be willing, Socrates, to say that the one Form, in truth, is divided for us, and will it still be one? [c9-10]

*Socrates:* In no wise. [c11]

*Parmenides:* For consider: if you divide the Large itself and each of the many large things are large by a part of the Large smaller than the Large itself, will this not seem unreasonable? [c12-d2]

*Socrates:* Certainly. [d3]

*Parmenides:* Well, will each [particular] have some small given part of the equal by which, though [this part] is less than the Equal itself, the possessor will be equal to something? [d4-5]

*Socrates:* Impossible. [d6]

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39For ease of reference, the three paradoxes (c12-e3) shall be called the “Paradoxes of Divisibility,” a variation on Allen’s division (1997, 9).
Parmenides: But suppose that one of us has a part of the Small: the Small will be larger than that [part] of it, inasmuch as it is part of [the Small], and in this way the Small itself will be larger; and that to which the subtracted part is added will not be larger but smaller than before.

d7-e2

Socrates: Surely this could not arise. [e3]

The main interpretive problem in the Second Horn is whether the Second Horn is stated completely prior to the Paradoxes of Divisibility (c5-11), as Allen maintains (the Opposites Interpretation), or whether T. Scaltsas is correct in according to the Paradoxes of Divisibility a more prominent rôle (the Disjunction Interpretation) (Allen 1997, 145; Scaltsas 1989, 77; cf. Cornford 1957, 85). The latter interpretation is quite innovative, but also (as we shall show) quite incorrect: it simply cannot be borne out by the text. Nevertheless, we shall discuss the Disjunction Interpretation, not only to see where Scaltsas goes wrong and what his objections to more traditional interpretations are, but also to note the contribution he makes to the interpretation of the first two Paradoxes of Divisibility. Then, we shall turn to a discussion of the Second Horn under the Opposites Interpretation.

*The Disjunction Interpretation (Scaltsas)*

The Disjunction Interpretation takes the Second Horn itself to be a disjunction that proceeds from the assumption that if any particular participates in the Form, then part of the Form is present in the particular (cf. pp. 12, 51). Scaltsas states the disjunction as follows (1987, 77):

**EITHER** [each part of the Form will be what the Form is, i.e. the realised sufficient condition for being $f$, and therefore] there will be many Forms $F$
OR [each part of the Form will not be what the Form is, and therefore] having a part of the Form will not make a thing.

The portions of the disjunction that Scaltsas supplies are bracketed, and so the unbracketed portions must be supplied by Plato. This means that for the first disjunct, Parmenides’ question whether, and Socrates’ denial that, if the Form is divided it will still be one (e9-11), are the same as “there will be many Forms F.” For the second disjunct, “having a part of the Form will not make a thing” is represented by the Paradoxes of Divisibility, which illustrate contradictions that arise when part of the Large, and then part of the Equal, are present in a particular. σμικρότερο (d1) in the First Paradox, and ἐλάττων (d5) in the Second Paradox are given a qualitative sense, that part of the Form being present in the particular is insufficient for the particular’s having the same name as the Form (1987, 77-9).\textsuperscript{40} Under the Disjunction Interpretation, the Third Paradox becomes an orphan: σμικρότερον (e1) is not used qualitatively but quantitatively, and so is not part of the second disjunct (1987, 80).\textsuperscript{41} It is included because of its similarity, quantitatively speaking, to the other two Paradoxes: it is an absurdity “which Plato could not resist” (1987, 81).

\textsuperscript{40}Why does Plato choose the irregular form σμικρότερο, as opposed to the regular ἐλάττω (d1), when he uses ἐλάττων in the next paradox (d5)? Furthermore, why not use the Attic τὸ μικρὸν as the name of the Form rather than the Ionic τὸ σμικρὸν?

\textsuperscript{41}Scaltsas’ innovation is an ontological variation (apparently arrived at independently) on a point made by Panagiotou several years earlier. There, Panagiotou argues that if we suppose the Form being present in the particular to have an explanatory function, part of the Form is insufficient to explain the particular’s having the same name. Like Scaltsas, Panagiotou is unable to avoid orphaning the Third Paradox (1987, 51-2). The reading applied by Scaltsas and Panagiotou seems to have been originally suggested (although not followed through) by Proclus (1987, §865, trans. G. Morrow): “If each of us has a part [of Man] and not the whole, how could he be called a man and not part of a man?”
We begin our formal examination of Scaltsas’ argument by recalling the first two premisses of the First Horn, since they are premisses not of the First Horn alone, but of the whole Dilemma of Participation:

\[(1.\text{h}) \quad (\exists x)(\exists y)(\exists u)(\exists v)(\exists s)(\exists t) (Fx & Fy & Pu & Pv & Ts & Tt & Quxs & Qvyt & x=y & u \mid v & u \mid v & s=t)\]
\[(2.\text{d}) \quad \{(\forall x)(\forall u) [(Fx & Pu & Qux) \rightarrow (\exists z)(Izu & Wxz)] \} \lor \{(\forall x)(\forall u) [(Fx & Pu & Qux) \rightarrow (\exists z)(Izu & Mzx)] \}\]

The left disjunct of the second premiss may be deleted, since it is proper to the First Horn; and we may update the right disjunct with the now-familiar device of temporal qualification, as well as using the new present-in symbol for “Izu”:

\[(2.\text{h}) \quad (\forall x)(\forall u)(\forall s) [(Fx & Pu & Ts & Quxs) \rightarrow (\exists z)(z_{s_1}u & Mzx)]\]

Furthermore, “Mzx” may be expressed by the proper-part relation. If part of the Form is present in the particular, it is understood from the context of the Dilemma that the whole Form is not present in the particular (that is, the Horns are exclusive). This may be expressed by supposing that the \(z\) that is present in \(u\) at time \(s\) is a proper part of \(x\) at time \(s\); but also that there is some other \(z\) such that \(z\) is a proper part of \(x\), but that \(z\) is not present in \(u\) at time \(s\):

\[(2.i) \quad (\forall x)(\forall u)(\forall s) [(Fx & Pu & Ts & Quxs) \rightarrow ((\exists z)(z_{s_1}u & z_{s_2}x) \& (\exists z)(\neg z_{s_1}u & z_{s_2}x))]\]

**The First Disjunct**

What interpretation of Socrates’ admission that, if the Form is divided it will not be one, will yield the reading of the Disjunction Interpretation, that there are many Forms? If “\(\exists v\)” (c10) is taken in the sense of singularity, the negation would be that the Form is not a
singularity; that is, the Form is not one but many. In Scaltsas' language, this means that the Form is "cloned" (1987, 77). Presumably the Form being cloned means that there are at least two Forms of the same sort that are not identical; but it is no easy matter to represent this symbolically, because we currently have no device to determine what sort of Form a given Form is.

Fortunately, something Parmenides says earlier suggests a solution (130e5-31a1): "There are certain Forms, in which these others participate and after which [these others] get their names." We avoided symbolising the claim about names earlier, because it was not relevant to the First Horn (cf. p. 11); but it has now become relevant. The first part of this statement was symbolised earlier, and appears in our current considerations as premiss (1.h). Now the second part, that particulars get their names from the Forms in which they participate, comes to the fore.

Let us begin by assuming that the name the particular gets from the Form is the same name as the Form: if Socrates participates in a Form that has the name "Pare," he does not get the name "dark" or "sallow," but "pale." But further, he gets the name "pale" because he has the character of paleness; and he has this character in virtue of his participation in the Form. As pointed out previously, the particular "getting the name of the Form" is shorthand for the particular getting the name the character it has, and it has the character in virtue of the Form (p. 6).

If Socrates gets the name "pale" from participation in Form x, and also gets the name "pale" from participation in Form y, then x and y are Forms of the same sort, because the
particular gets the same name from each of them. Further, Forms \( x \) and \( y \) must have the same name, because the particular gets the same name as the Form. This consideration allows us to define what it means for two Forms to be clones: two Forms are clones if and only if they have the same name (that is, if and only if they are of the same sort), and are not identical. Given this, it seems simple enough to represent the conclusion of the first disjunct.\(^4\)

\[
\begin{align*}
(D.a) & \quad \text{There are at least two Forms, } x \text{ and } y, \text{ such that they have the same name but are not identical.} \\
(D.b) & \quad \text{There exists an } x, \text{ and there exists a } y, \text{ and there exists an } r, \text{ and there exists an } s, \text{ such that } x \text{ is a Form, and } y \text{ is a Form, and } r \text{ is a name, and } s \text{ is a time; and } x \text{ has the name } r, \text{ and } y \text{ has the name } r, \text{ and } x \text{ is not identical to } y \text{ at time } s. \\
(D.c) & \quad (\exists x)(\exists y)(\exists r)(\exists s)(Fx \& Fy \& Nr \& Ts \& x \succ r \& y \succ r \& x \equiv_{s} y)
\end{align*}
\]

The semantics of \( a \succ b \) must be defined, because as it stands in the conclusion, the Forms \( x \) and \( y \) merely have some name \( r \). As a preliminary definition, we may say that the Form has the same name as that name the particular gets when it participates in the Form (this definition is based partially on Parmenides' assertion that the particular gets the name of the Form). There are three difficulties that arise from this definition of the name of the Form, and they will now be examined in turn.

(I) The first difficulty is a remnant of the \textit{Phaedo}, and it is raised at this point, not because the \textit{Parmenides} alludes to it, but because a case can be made that the theory of Forms here presented is identical to that put forward in the \textit{Phaedo} (Cornford 1957, 70). In a passage of the \textit{Phaedo}, Socrates suggests that the Forms are interrelated (104e8-10):

\[
\text{__________________________}
\]

\(^4\) The following abbreviation and symbol are introduced: \( N : _a \) is a name; \( _a \succ _b \) has the name \( _b \).
Now the three, not being opposite to the Even, nevertheless does not receive it, for [the three] always brings the opposite to [the Even]; and the two [always brings the opposite to] the Odd.

This passage, and the discussion that follows it, is controversial, in part because it is unclear from the context whether “the three” and “the two” refer to things that are three and two respectively, or whether Socrates means the Forms (Gallop 1975, 205-6).\textsuperscript{43} But if we think that “the three” and “the two” do refer to Forms, it is no longer clear what name is proper to the Form. If the Three brings along the opposite of the Even, which is the Odd, then that which participates in the Three will have the name “three”; but it will also have the name “odd.” This creates some doubt as to whether, by our current definition, the Form Three has the name “Three” or “Odd.”

A response to this problem can be formulated by carefully noting Socrates’ distinction between the Form and that which the Form brings along. The Three brings along the Odd, but this is not to say that the Three is the Odd; and so even though the particular that has the name “three” also has the name “odd,” we should say that it has the name “three” in virtue of Three, but other names in virtue of something other than Three. Thus, we may transfer Socrates’ distinction to names: the name that the particular has in virtue of the Form is different from other names that the particular has in virtue of other Forms that the Form brings along. And so the Form has the same name as the particular would get in virtue of

\textsuperscript{43}For this reason, neither “three” nor “two” have been capitalised in the translation.
the Form should it participate in the Form (and not any other names that the particular has in virtue of the other Forms that are brought along with the Form).

(II) The second difficulty focusses on what we have been calling the name of the Form; for it appears that Forms may have many names. The Pale, to be sure, is called by the name “Pale.” But aside from having this name, it is also called “Form” (128e6-29a2) and “separate” (130b1-30) in the Parmenides. Elsewhere, Forms are called “unchangeable” (Phaedo, 78d1-3; 8-9), “eternal,” “immovable” and “intelligible” (Timaeus, 29b5-7). Turning again to the Phaedo for help, we find that Socrates says that the Form has “its own name for all time” (103e2-4); and this name is none other than the name the particular gets in virtue of the Form should it participate in the Form. For example, when Socrates participates in the Form Pale, even though the Pale might have many other names, Socrates does not get these names by reason of his participation in the Pale. (That is, Socrates is not also called “Form” or “immovable.”) This name we shall call the primary name of the Form, and all other names shall be called secondary.

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44 Note also that if our analysis of the relation of names to Forms is correct, Socrates’ saying that it is the Form’s “own name” suggests that there is only one Form of each name, and so only one Form of each sort.

45 D. Keyt distinguishes between the ideal attribute, which is the attribute “whose absence from a thing entails that the thing is not a Platonic Idea,” and a proper attribute, which is the attribute “whose absence from a thing entails that the thing is not an instance of the given Form” (1969, 12; 13). For example, eternality is an ideal attribute, for without this Beauty would not be a Form; but beauty is a proper attribute, for without this Beauty would not be the Form Beauty. What is meant by “primary name” is narrower than the name of a proper attribute, for a Form may have many proper attributes, but only one primary name. For example, man and animal are proper attributes of the Form of Man (1969, 13), but “Man” is the only primary name of the same.
(III) The third difficulty is that some names are ambiguous. For example, a measuring rod sixty-six inches in length has the name “perch,” as does a spiny-finned freshwater fish. According to the definition, the Form in virtue of which the rod and the fish are what they are is the Form Perch. But it would be absurd to suppose that the Form in which the rod participates is the same as the Form in which the fish participates, even though both Forms have the same name (lest we have a very unhappy rod, or a very inactive fish). The solution to this problem is easy to state, but it would be very troublesome to work out in detail. “Perch” meaning rod and “perch” meaning fish are different, even though they have the same spelling. The name of the rod comes from pertica, meaning pole, whereas the name of the fish comes from περκή; it is an accident of our language that both names have the same spelling. Since this ambiguity is relative to English, we should not expect the names of Forms to share in this ambiguity (or that of any other language).

Now we are prepared to define the semantics of \( a > b \), which we have been reading as “\( a \) has the name of \( b \)”\footnote{“Name” is being used in a broad sense, comprehending any kind or characteristic that can be said of the particular. For example, “perch” is a name of the perch, but so is “fish” and “spiny-finned.”}: (a) if \( a \) is a Form, then \( a > b \) is true if \( b \) is the primary name of the Form, which is to say that name that a particular would get in virtue of the Form should it participate in the Form, otherwise it is false; or (b) if \( a \) is a particular, then \( a > b \) is true if \( b \) is a name of the particular in question,\footnote{“Name” is being used in a broad sense, comprehending any kind or characteristic that can be said of the particular. For example, “perch” is a name of the perch, but so is “fish” and “spiny-finned.”} otherwise it is false. We may add a temporal qualification in the usual manner. Note that although it is not contradictory, it would seem
odd to apply a temporal qualification if \( a \) is a Form, because presumably Forms do not change their names.

In the light of these considerations with respect to the conclusion of the first disjunct, it is desirable to revise premiss (1.h). The first premiss asserts (among other things) that there exists a Form \( x \) and a Form \( y \), and that \( x \) is identical to \( y \). In order to make the premiss relevant to the first disjunct, we must also give \( x \) and \( y \) names. It will be added that there exists some name \( r \), and \( r \) is the name of \( x \) and \( y \). Here is the revised translation of the first premiss:

\[(1.i) \quad (\exists x)(\exists y)(\exists r)(\exists u)(\exists v)(\exists s)(\exists t)(Fx \land Fy \land Nr \land x > r \land y > r \land Pu \land Pv \land Ts \land Tt \land Quxs \land Qvvt \land x = y \land u \downarrow s, v \land u \downarrow t, v \land s = t)\]

In order to derive the conclusion, the first disjunct requires a third and a fourth premiss over and above the two we already have. The third premiss is Scaltsas’ claim that “the difficulty in the first disjunct was caused by the implicit assumption that each part of the Form can perform the function of the Form” (1989, 78). Since Form \( F \) is the source of \( f \)-ness in particulars, the function of the Form is to make particulars \( f \) (1989, 70); and if the particular is \( f \), it will have the name \( f \). And so the claim becomes: if part of the Form is present in the particular, the particular gets the name of the Form:

\[(3.a) \quad (\forall x)(\forall u)(\forall s)(\forall r)(\forall z)(Fx \land Pu \land Ts \land Nr \land x > r \land z \downarrow s, u \land z \downarrow s, x) \rightarrow u > r, r] \]
The fourth premiss is discussed by Scaltsas when he describes what he calls the contagion model of explanation: "What the things acquire from the source is what it takes to be $f$. Minimally, what it takes to be $f$ is the necessary and sufficient condition for being $f$" (1987, 69). That is, what things acquire from the source of $f$-ness is the necessary and sufficient condition for being $f$. In the first disjunct, the things in question are particulars, and the source of $f$-ness is the Form $F$. The Form $F$ is therefore the necessary and sufficient condition for the particular being $f$, or in the language that has been used hitherto, the whole Form being present in the particular is the necessary and sufficient condition for the particular's having the same name as the Form:

(4.a) The whole Form being present in the particular is the necessary and sufficient condition for the particular's having the same name as the Form.

(4.b) For any $x$, for any $u$, for any $s$ and for any $r$, if $x$ is a Form, and $u$ is a particular, and $s$ is a time, and $r$ is a name, and the primary name of $x$ is $r$; then for some $z$, if $z$ is present in $u$ at time $s$, and $z$ is the whole of $x$, if and only if $u$ has the name $r$ at time $s$.

(4.c) $(\forall x)(\forall u)(\forall s)(\forall r)((Fx \& Pu \& Ts \& Nr \& x\rightarrow r) \rightarrow [\exists z](z \subset u \& z=\sigma p(p\lt x)) \leftarrow (u\rightarrow r)]

Armed with these two additional premisses, the argument of the first disjunct may be put as follows; and we do indeed ferret out the conclusion (which is an absurdity, and sufficient evidence to reject the first disjunct).\textsuperscript{47}

\textsuperscript{47}The corresponding sequent is proved in Appendix B: The First Disjunct.
(∃x)(∃y)(∃r)(∃u)(∃v)(∃s)(∃t)(Fx & Fy & Nr & x>r & y>r & Pu & Pv & Ts & Tt & Quxs & Qvyt & x=y & u\v & u\v & s=t)
(∀x)(∀u)(∀v)(Fx & Pu & Ts & Quxs) → ((∃z)(z₃,u & z₅,x) & (∃z)(~z₃,u & z₅,x))
(∀x)(∀u)(∀v)(Fx & Pu & Ts & Nr & x>r & z₃,u & z₅,x) → u\r
(∀x)(∀u)(∀v)(Fx & Pu & Ts & Nr & x>r) → ((∃z)(z₃,u & z=op(p<q)) → (u\r))
(∃x)(∃y)(∃r)(∃s)(Fx & Fy & Nr & Ts & x>r & y>r & x\y)

By the first two premises, we may suppose that two separate particulars participate in the same Form, and so part of the Form is present in particular u, and part of the Form is present in particular v. The part is the sufficient condition for the particular’s having the same name as the Form by the third premise, and so both u and v have the same name as the Form. By the fourth premise, the particular’s having a name of which there is a Form is the sufficient condition for the whole Form, of the same name as the particular, being present in the particular. Hence, the whole Form is present in u and the whole Form is present in v; we may call these wholes x and y respectively (with x and y both having the same name, because their respective particulars both have the same name). Now, if x has the same name as y, x and y are the same sort of Form; but they cannot be identical because the particulars u and v are disjoint by the first premise. Therefore, x and y are clones, being separate yet being the same sort of Form. Since cloning itself conflicts with the assumed singularity of the Form, we have successfully fashioned the absurdity that justifies Socrates saying that if the Form is divided it cannot be one (c11).

The Second Disjunct

Since the first disjunct has led to absurdity, we are logically committed to the second disjunct: the part of the Form present in the particular is not sufficient for the particular’s
having the same name as Form. This disjunct is illustrated by the first two Paradoxes of Divisibility: if you divide the Large itself, each of the many large things will be large by a part of the Large smaller than the Large itself (c12-d2); if you divide the Equal itself, that which is equal will be equal by that which is less that the Equal itself (d4-5). The Disjunction Interpretation takes "σμικροτέρον" (d1) and "ὁλάττονι" (d5) in a qualitative sense, as opposed to a quantitative sense. That is, the comparative is used in each case not to indicate that the paradox is generated from the part being of lesser size, but rather that the part is inadequate to fulfill the rôle of the Form. Scaltsas describes this by analogy: just as “a car is not only smaller than a truck, but also it cannot carry the load a truck can,” so too the part of the Form cannot do the work of the whole Form (Scaltsas 1989, 78-9). Note that this interpretation adheres closely to a material sense of parts: it is as if only part of the wine required to make a pleasant drink has been poured into the bowl, and so the mixture is too watery.

The conclusion of the second disjunct is that “each part [of the Form] is not what the Form is” (Scaltsas 1989, 78). It bears a certain resemblance to premiss (3.c) of the first disjunct; but it is not enough to assert the denial of (3.c) like so:

(E.a) \(~(\forall x)(\forall u)(\forall s)(\forall r)(\forall z)(F x & Pu & Ts & Nr & x > r & z > u & z \ll x \rightarrow u > r)\)

This means that there is at least one case where part of the Form being present in the particular does not issue in the particular’s having the name of the Form. Although Scaltsas does not make his denial universal, it clearly has a universal meaning: for any part of any
Form, that part is not what the Form is. This may be reflected in our symbolisation by moving the negation in (E.a) after the arrow:

\[(E.b) \ (\forall x)(\forall u)(\forall s)(\forall r)(\forall \forall z)[(Fx \ & Pu \ & Ts \ & Nr \ & x > r \ & z_s u \ & x \ll x \ \rightarrow \ \sim u > r]\]

However, we are still not in the clear, because when the whole Form is present in the particular, it is also the case that part of the Form is present in the particular. Thus, we may borrow from the consequent of (2.i) to arrive at,

\[(E.c) \ (\forall x)(\forall u)(\forall s)(\forall r)[(Fx \ & Pu \ & Ts \ & Nr \ & x > r \ & (\exists z)(z_s u \ & z \ll x) \ & (\exists z)(\sim z_s u \ & z \ll x)) \ \rightarrow \ \sim u > r]\]

In other words, if it is only part of the Form that is present in the particular, the particular does not have the same name as the Form. 48

In premiss (4.c), we said that the whole Form being present in the particular is the necessary and sufficient condition for the particular’s having the same name as the Form (p. 58). Once this has been admitted, and premisses (1.i) and (2.i) have been recalled (for they are premisses of both disjuncts), the conclusion (E.c) is not difficult to derive. After all, if

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48 The conclusion may be translated directly (according to our established pattern) from the assertion that “if it is only part of the Form that is present in the particular, the particular does not have the same name as the Form,” as follows:

\[(E.d) \ If \ only \ part \ of \ the \ Form \ is \ present \ in \ the \ particular, \ it \ is \ not \ the \ case \ that \ the \ particular \ has \ the \ name \ of \ the \ Form.\]

\[(E.c) \ For \ any \ x, \ for \ any \ u, \ for \ any \ s \ and \ for \ any \ r, \ if \ x \ is \ a \ Form, \ and \ u \ is \ a \ particular, \ and \ s \ is \ a \ time \ and \ r \ is \ a \ name, \ and \ the \ primary \ name \ of \ x \ is \ r, \ and \ for \ some \ z, \ z \ is \ present \ in \ u \ at \ time \ s, \ and \ z \ is \ a \ proper \ part \ of \ x \ at \ time \ s, \ and \ for \ some \ z, \ it \ is \ not \ the \ case \ that \ z \ is \ present \ in \ u \ at \ time \ s, \ and \ z \ is \ a \ proper \ part \ of \ x \ at \ time \ s; \ then \ it \ is \ not \ the \ case \ that \ u \ has \ the \ name \ r \ at \ time \ s.\]

\[(E.f) \ (\forall x)(\forall u)(\forall s)(\forall r)[(Fx \ & Pu \ & Ts \ & Nr \ & x > r \ & (\exists z)(z_s u \ & z \ll x) \ & (\exists z)(\sim z_s u \ & z \ll x)) \ \rightarrow \ \sim u > r]\]
only part of the Form is present in the particular, and the whole Form being present in the
particular is the necessary and sufficient condition for the particular getting the name of the
Form, it is obvious that the particular will not get the name of the Form. The symbolic
argument-form of the Second Disjunct is as follows:

\[(\exists x)(\exists y)(\exists z)(\exists u)(\exists v)(\exists s)(\exists t)(Fx \& Fy \& Nr \& x \succ r \& y \succ r \& Pu \& Pv \& Ts \& Tt \&
Quxs \& Qvyt \& x = y \& u \| v \& u \| v \& s = t)
(\forall x)(\forall u)(\forall s)[(Fx \& Pu \& Ts \& Quxs) \rightarrow ((\exists z)(z \succ u \& z \ll x) \& (\exists z)(\neg z \succ u \& z \ll x))]\]
(\forall x)(\forall u)(\forall s)(\forall r)[(Fx \& Pu \& Ts \& Nr \& x \succ r) \rightarrow [(\exists z)(z \succ u \& z \ll p < x)) \rightarrow (u \succ r)]\]
(\forall x)(\forall u)(\forall s)(\forall r)[(Fx \& Pu \& Ts \& Nr \& x \succ r \& (\exists z)(z \succ u \& z \ll x) \& (\exists z)(\neg z \succ u \& z \ll x)) \rightarrow (u \succ r)]\]

**Objections to the Disjunction Interpretation**

There are four objections that may be made against the Disjunction Interpretation.
The first has already been commented upon, and that is the orphaned Third Paradox of
Divisibility. The qualitative reading cannot be applied to the Third Paradox: if the Small is
divided, the Small is greater than that which is a part of it (d7-9). By analogy, in comparison
to a slice, the whole of the olive pie is greater. Furthermore, Socrates denies that that to
which part of the Small is added will be smaller than before (d9-e2). Again by analogy, if
we add the pie to the box, so that part of the pie is present in the box, the box becomes
greater (in this case, greater with respect to weight). Clearly “σμικρότερον” (e1) is intended
in a quantitative sense; but if this is the case, it has no direct relation to the other two
Paradoxes. Scaltsas realises this, and relegates the Third Paradox to the status of a “further
impossibility” (1989, 80-1); but his explanation for the inconsistency he attributes to Plato,

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49The corresponding sequent is proved in Appendix B: The Second Disjunct.
that Plato simply could not resist mentioning the absurdity, while possible, is not entirely convincing. If Plato is using σμκρότερο (d1) and ἑλάττων (d5) qualitatively, why would he then use σμκρότερον (e1), a word with merely a different inflection than that used in the First Paradox (d1), in a significantly different sense? If the irregular σμκρότερος were used strictly qualitatively, and ἑλάττων strictly quantitatively (or vice versa), the argument of the Disjunction Interpretation for reading the first two Paradoxes exclusively in a qualitative sense might have more weight. At the same time, we ought to take note of Scaltsas’ reading; and so it is best to say that while Scaltsas is not wrong in attributing a qualitative sense to the comparatives in the first two Paradoxes, nothing prevents all three Paradoxes from being read in the quantitative sense as well.50

The second objection has to do with the third premiss of the first disjunct. The third premiss states that the particular has the same name as the Form if and only if the whole Form is present in the particular. But the Second Horn proceeds on the assumption that part of the Form is present in the particular, not the whole; and so if the particular has the same name as the Form, it is reasonable to attribute this to part of the Form being present in the particular (and not to the whole of the Form). Hence, why should one assume that, if the particular has the name of the Form, that that which is present in the particular is the whole Form? It seems more reasonable to hold that if the particular has the same name as the Form, part of the Form is present in the particular. Given that the third premiss conflicts

50 As Allen points out, “given the aporetic character of Parmenides’ objections, ... perplexities may be read at many levels” (1997, 136).
with the initial assumption of the Second Horn, using it to support the conclusion that the Form is cloned is a questionable move; but it is necessary to support the Disjunction Interpretation.

The sense of “ἕν” (c10) in Parmenides’ statement is the third objection. Scaltsas assumes that Socrates’ declaration that it is impossible for the Form to be one (c9-11) means that the Form is cloned (1987, 77-8). Indeed, taking “ἕν” in the sense of singularity, so that Socrates issues a denial on account of an anticipated cloning of Forms, is one possibility. But “ἕν” may also have the sense of indivisibility, so that Socrates’ denial would mean that the Form is no longer one, because the particulars are separate from each other, and parts of the Form are present in the particulars. That is, the indivisibility of the Form is compromised when parts of the Form are present in separate particulars. This latter sense of “ἕν” seems to be the more natural reading for two reasons. First, it is unlikely that Plato was being so laconic that he thought it unnecessary to clarify the absurdity that the Form is no longer one because it is being cloned. Second, the inference that the Form is not indivisible because its parts are separate, is perfectly in line with the thesis of the Second Horn: the Form has parts present in separate particulars, and so the Form cannot be one.

The fourth and most serious objection attacks the very possibility of reading the Second Horn as a disjunction, and has its foundation in one word: “γέρ” (c12). There seems to be no good reason not to construe this word in its usual sense, as a causal conjunction: such a conjunction serves “to confirm the truth of a previous statement” or it “refers to a thought implied in what has preceded” (Smyth 1956, §2810). There is no sense of “γέρ” that
supports the interpretation that Plato is introducing the second half of a disjunction, as the
Disjunction Interpretation would have us believe. Instead, we should expect Parmenides to
explicate what is meant by Socrates’ unwillingness to say that “one Form, in truth is divided
for us” and that it will “still be one” (c9-10).

These four objections, when taken together, suggest that the text itself ultimately
resists the Disjunction Interpretation; but our examination of the interpretation is not
fruitless. First, we have established that the sense of “ἐν” is not that of singularity, but most
likely that of indivisibility. Second, since the Second Horn is completely stated just prior
to the Paradoxes of Divisibility, the Paradoxes are illustrations of the Second Horn (cf.
Cornford 1957, 85). Third, the “σμικρότερο” (d1) and “ἔλαττον” (d5) of the first two
Paradoxes may be interpreted qualitatively, but “σμικρότερον” (e1) admits only of a
quantitative sense. Finally, there are two charges that Scaltsas makes against more
traditional interpretations that should be answered, if possible: (i) the Paradoxes are
implausible but not logical impossibilities,\footnote{This depends on Scaltsas’ reading of Socrates’ “ἄδονατον” (d6) as indicating a logical impossibility. This is certainly one sense of the word, and it seems plausible (although not necessary) that Socrates is using it in that sense here.} and (ii) the Paradoxes are not generalisable,
although the results are stated generally (131e4-6).\footnote{If the results are not generalisable, the Paradoxes are special cases, and it is possible that participation for most Forms occurs by part of the Form being present in the particular. However, it would also be possible to argue that the Paradoxes are counterexamples intended to show that participation in part of the Form is untenable.}
The Opposites Interpretation (Allen)53

The Opposites Interpretation is so called because it purports to find a qualification by opposites in the four consequences that follow from the thesis of the Second Horn, that if particulars participate in the Form, part of the Form is present in each particular. The first consequence is Socrates’ granting that if the Form is divided it cannot be one (e9-10), and the remaining three consequences are the three Paradoxes of Divisibility (c12-d3; d4-d6; d7-e3).

The impetus for grouping these four consequences together comes from Allen noting on one hand that the modality of the Second Horn is very different from that of the First, and on the other hand holding that the Second Horn is a reductio (1997, 133; 152). The thesis of the First Horn contradicts an axiom of classical extensional mereology; namely, that one and the same thing cannot be, at one and the same time, a part of more than one disjoint particular (cf. Tβ, p. 88). Its modality is that of impossibility. In contrast, the thesis of the Second Horn is not intrinsically contradictory; it is merely implausible. Should we be presented with the theses of the Dilemma, we are led to conclude that there is either “no participation [by the First Horn], or participation in parts [by the Second Horn]” (because the First Horn is impossible, and the Second Horn is merely implausible) (Allen 1997, 133). The four consequences are grouped together to justify the claim that the Second Horn is a

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53The Opposites Interpretation is my synthesis of two separate sections of Allen’s commentary (1997, 131, 145-52). While I have reworked these sections into a single interpretation, I have tried to preserve Allen’s meaning. I have altered slightly the structure of Allen’s interpretation as it appears in his commentary for the sake of a unified explication.
reductio: even if the thesis of the Second Horn is not absurd, the consequences to which it leads are unacceptable.

The first consequence of the Second Horn follows directly from the Sail Analogy (Allen 1997, 131). From the standpoint of the Sail Analogy, the Form is qualified by being one and divided at the same time. Recall that the upshot of the Sail Analogy is that it is not the whole sail that is over each man, but a part of the sail (131c2-4). Suppose that we have a sail with an area of sixteen square feet under which we compel to stand sixteen men, four abreast and four deep, arranged in such a way that each stands under his own square foot of sail. By analogy to the Form and its participants, each man participates in the sail. Because we are employing a verb of sharing (that is, μεταλαμβάνειν; cf. p. 5), it is incumbent upon us to determine what part of the sail each man gets. In this case, each man gets the square foot of sail directly above him. But this means that the sail is in pieces and is no longer one: each part of the sail is characterised, at least in part, by being unique to the man that has it. For example, the first man has the first square foot of sail, and not any other square foot.

By analogy, the Form cannot be one when it is divided amongst its participants, because its parts are characterised by being unique to the participants that receive those parts. Thus, by supposing that the particular receives a part of the Form, the Form is discovered to have the characters of one and divided at the same time. Both “one” and “divided” are taken in a material sense that follows from the Sail Analogy. These terms are opposites, because “one” is said of the Form as if it were a physical contiguous unity, and “divided” is said of such a whole when its parts are disjoint from each other. Cornford points out, “this is to
understand 'part' and 'whole' in the most gross and material sense” (1957, 85); but this is exactly what transpires should the parts of the Form be present in separate particulars.

The remaining consequences of the Second Horn are contained in the Paradoxes of Divisibility, and all three are accounted for similarly. We begin with the challenge that Socrates make when he is presenting his solution to Zeno’s Paradox (129d7-e4):

I should be filled with wonder, Zeno, if someone should distinguish separately those Forms that I just mentioned - such as likeness, and unlikeness, and many and the one, and rest and motion, and all such things - and then show that amongst themselves these things are able to be combined and separated.

Socrates would be loath to accept the Like being unlike, or the Large being small. The Opposites Interpretation construes the Paradoxes in such a way that this comes about. This is accomplished by first showing that the Form and its parts have the character that correspond to the Form’s primary name (the so-called self-predication of the Form) (cf. p. 55). Then we shall show that either the Form or its parts have the opposite of that primary name.

In order to justify the Form having the character that corresponds to the primary name, we turn to the Phaedo. There, we note that Socrates talks about Beauty as if it were beautiful (100c4-6): “If some other thing is beautiful other than Beauty itself, it is beautiful on account of nothing other than that it participates in Beauty itself; and I speak thus for all.” That is, Form F is f(a); and something is f in virtue of Form F. It is (a) that is important for our current argument; and so by (a), the Large itself is large, the Equal itself is equal, and the Small itself is small.
It is not sufficient, as will become clear presently, to say that the Form F is \( f \); the claim that each part of the Form F is \( f \) must also be substantiated. With this in mind, we turn to the discussion Parmenides has with Aristotle about the nature of the Small (cf. Allen 1997, 146, n. 24). Parmenides advances the claim that if the Small is equal to or larger than that in which it is present, it does the work (\( \piρ\alphaττειν \)) of the Equal and the Large respectively (150a7-b1). If the Small is equal, it does the work of the Equal, and if the Small is large, it does the work of the Large; and so if the Small is to do the work of the Small, we should expect it to be small. The general principle seems to be that if a particular is \( f \) in virtue of a Form, then that Form itself is \( f \). By applying this principle to the parts of Form F, if a particular is \( f \) in virtue of a part of Form F, then that part should also be \( f \) (b). Therefore, by (b), parts of the Large are large, parts of the Equal are equal, and parts of the Small are small.

At this point, the Opposites Interpretation now appeals to the mathematical axiom that the whole is greater than its parts, and that its parts are smaller than the whole (Allen 1997, 146; cf. Elements I, Axiom 5). By this axiom, a whole Form is larger (c), and part of the Form is smaller (or less) (d). Here are the four relevant principles that the Opposites Interpretation employs for its reading of the Paradoxes of Divisibility, in summary:

(a) Form F is \( f \);
(b) The parts of Form F are \( f \);
(c) The whole Form is larger [than its parts];
(d) Part of the Form is smaller (or less) [than the whole].

The Paradoxes of Divisibility may now be generated. According to the First Paradox, "each of the many large things will be large by a part of the Large smaller than the Large
itself” (c12-d2). Socrates rejects this as being unreasonable (d3), because we have predicated both large (b), and small (d), of a part of the Large. In the Second Paradox, Parmenides suggests that each particular will “have some small given part of the equal by which, though [this part] is less than the Equal itself, the possessor will be equal to something” (d4-5). This is also rejected by Socrates (d6), because equal (b), and less (d), have both been predicated of a part of the Equal. Finally, the Third Paradox states that “the Small itself will be larger than that [part] of it, inasmuch as it is part of [the Small], and in this way the Small itself will be larger” (d7-9). This predicates both larger (c), and small (a), of the Small; but Socrates cannot accept this (e3). According to the Opposites Interpretation, this brings to pass that which Socrates thought would be a portent: the Forms have been qualified by their opposites.

Allen thinks that the Opposites Interpretation accommodates both of Scaltsas’ objections (cf. p. 65). Recall that the first objection was that more traditional interpretations find implausibilities in the Paradoxes of Divisibility, but not impossibilities, and so they are unable to explain Socrates’ “ἀδόλυτον” (d6). A defender of the Opposites Interpretation would reply that impossibilities have been generated, at least in the First and Third of the Paradoxes. In the First Paradox, the part of the Large is both large and small, and in the Third Paradox, the Form Small is both small and large. Since these two Paradoxes proceed from the thesis of the Second Horn, at the very least we have on our hands two counterexamples that give us reason to reject the thesis that parts of the Form are present in the particular.
Nevertheless, in order to address Scaltsas' second objection, the Paradoxes must not merely be counterexamples; they must be generalisable. Allen answers that although all Forms, or parts of Forms, are not qualified by opposites, every Form that is participated in has parts. From this we may infer that “every idea is therefore qualified by largeness as a whole, and by Smallness with respect to its parts” (1997, 148). If every Form is qualified by largeness, every Form participates in the Large; and if every part is qualified by smallness, every part participates in the Small. But participation in the Large and the Small, by the thesis of the Second Horn, has been shown to be absurd. Therefore, this “absurdity extends to every Idea that is participated in” (Allen 1997, 148).

A Confusion over Relations

The Opposites Interpretation seems to depend on a confusion over relations. The confusion over relations has its origin in the Phaedo, where Socrates supposes that the character and the corresponding comparative may be explained by participation in the same Form (100e5):

By the Large, large things are large, and larger things are larger, and by the Small, smaller things are smaller.

Socrates is making two claims: (a) a particular gets the character of the same name as the Form in virtue of the Form, and (b) a particular gets the comparative of the name corresponding to the Form in virtue of the Form. The first claim (a) is nothing out of the ordinary, and has been advanced all along: the Colossus of Rhodes is large in virtue of the
Large. It is the second claim (b) that presents difficulties, because it seems to treat relations in the same way it treats characters: the Colossus is larger in virtue of the Large.

If Plato treats characters and relations in the same way, and is not clear about their differences, there is the distinct possibility that he confuses them, or at the very least is loose in his treatment of characters and their corresponding comparatives.\textsuperscript{54} One might expect this confusion, or looseness, to be made manifest by a sliding between relation and the character: that which is larger is sometimes said to be large, and that which is smaller is sometimes said to be small, and so on.\textsuperscript{55} This understanding of the confusion about relations seems to be reflected in Allen’s analysis of the Paradoxes.

In Allen’s analysis of the three Paradoxes, the qualification by opposites is brought about as follows (1997, 146):

If $a$ is large, it will be large by a part of Largeness smaller than largeness itself, and will therefore be large by what is small; and since Largeness is larger than any part of itself, Largeness is large. If $a$ is equal to something, it will be equal by a part of Equality smaller than Equality itself, and will therefore be equal by what is small. If $a$ is small, it will be small by a part of Smallness smaller than Smallness itself, and therefore Smallness is large.

In this explanation, we can observe sliding from the comparative to the corresponding character in each case, in order to admit a qualification by opposites. For the First Paradox,

\textsuperscript{54}Such looseness can be observed in the \textit{Phaedo} (70e4-71a11), where Socrates seems to think that the relation between relations holds between characters; but this is surely false. One possible explanation for Socrates’ conclusion is that relations are being taken to be the same as their corresponding characters (cf. Gallop 1975, 108).

\textsuperscript{55}Henceforth it is this confusion, in particular, that we mean when we talk about the confusion over relations.
recall that it is part of the Large that is both large and small. The part is large because it is
that in virtue of which something is large; but Allen clearly says that the large particular is
"large by a part of Largeness smaller than largeness itself ... and will therefore be large by
what is small." In other words, the part of Largeness is smaller, and so it is small. The same
situation arises in the Second Paradox, for an equal particular is "equal by a part of Equality
smaller than Equality itself, and will therefore be equal by what is small." Again, the part
is smaller than the whole, and so it is small. The Third Paradox changes the pattern
somewhat; this time, it is the Form Small that is both small and large. The Small is small
because participation in the Small is that in virtue of which small particulars are small. But
because a small particular "will be small by a part of Smallness smaller than Smallness itself
... Smallness is large." That is, part of the Small is smaller than the Small, and so the Small
itself is larger than its parts; and in this way the Small is large.\footnote{Sayre makes a similar slip in his description of the Third Paradox (1996, 77, my
italics): "On one hand, if things become small by participating in only part of Smallness, the
Form itself will be larger than the parts in which things participate. A consequence is that
Smallness itself will be large ..."}

So long as it is admitted that a confusion over relations reigns, the Opposites
Interpretation seems eminently plausible. Unfortunately, that there is confusion over
relations is called into question from many quarters. We shall begin by probing the supposed
origin of the confusion: Plato treats characters and relations in the same way, but he should
treat them differently. If this is true, it behoves us to discover exactly which difference
between characters and relations is being neglected. But this is not enough. There is nothing
vicious in neglecting a difference that does not lead us into error in the course of our current considerations. As such, not only must Plato neglect a difference between characters and relations, but that difference must clearly allow a slide from a relation to its corresponding comparative (of the sort that is exhibited by the Opposites Interpretation).

For our present purposes, there are two important differences between the character and the relation. These differences, while related, are not the same; and to be confused about the one does not entail confusion about the other. The first is that the character is asserted of the particular of which it is a character, whereas the relation is asserted both of the particular and of that to which the particular is related. For example, if the Colossus of Rhodes is large, large is a character that belongs to the Colossus. But if the Colossus of Rhodes is larger, one immediately wonders what it is that the Colossus of Rhodes is larger than (although perhaps due to its size one would not need to wonder long). The point is that the comparative must make reference both to the participant and to that which the participant is related by means of the comparative.

Plato does not neglect this first difference, as is made clear by passages in the *Charmides*, the *Republic* and the *Sophist* (cf. Allen 1997, 87). In the *Charmides*, Socrates advances the claim that if something is greater, it is greater than something else or itself (168b5-c2). This sentiment is echoed in the *Republic* (IV, 438b4-5): “Do you not understand, I said, that the greater is such as to be greater than something?” Furthermore, in the *Sophist*, the Stranger asserts that what is different is always so called with reference to another thing (255d1-2). If Plato is aware of this difference between characters and
relations, it is unlikely that he would slide between the relation and the character. He recognises the relation to be said of the participant only in reference to that to which the relation relates the particular. Hence, any argument for Plato’s confusing characters and relations cannot be traced to Plato’s neglecting this first difference.

The second difference between the character and the relation is that a character is thought to belong to the particular, but a relation subsists between its terms. For example, if Simmias is tall, tallness is the character that belongs to Simmias. But if Simmias is taller than Socrates, taller is not a relation that belongs to Simmias; it subsists between Simmias and Socrates, belonging not more to the one than to the other. This is reflected in symbolic logic as the difference between monadic predicates and polyadic predicates. According to Cornford and Allen, Plato takes the relation to belong to the participant, and not to subsist between the participant and that to which the participant is related (Cornford 1957, 78, n. 1; Allen 1997, 86). In other words, Plato fails to recognise the second difference between the character and the relation.

Nevertheless, this is not enough to convict Plato of being confused about relations. Leibniz distinguished three ways that two lines may be related (Leibniz 1973, 232; cf. Allen 1997, 87, trans. M. Morris):

As a ratio of the greater L to the smaller M, as a ratio of the smaller M to the greater L, and lastly as something abstracted from both of them, that is to say as the relation between L and M, without considering which is the anterior and which the posterior, which the subject and which the object.
The first two ways described by Leibniz seem to treat relations as characters, in that greater belong to \( L \), and smaller belongs to \( M \); and no errors are made, for in each case, greater is related to \( M \) and smaller to \( L \). Moreover, should Leibniz not recognise the third way, it does not follow that he would slide between a comparative and its corresponding character; and in not recognising the third way, Plato is not committed to such a slide. If we do not vigorously hold apart the first and second differences, we might think that Plato’s failing to recognise this second difference (which is Leibniz’ third way) is sufficient to justify sliding between a comparative and its corresponding character. However, in this case the slide would not be Plato’s, but ours.

The final blow to the thesis that Plato is confused over relations is that Allen himself does not think that Plato is confused (1997, 85-9). But considering this, what other way is there to explain Allen’s interpretation that since part of the Large is smaller than the Large, the large particular is large by that which is small? It is not reasonable to admit that just because something is smaller that it is small;\(^{57}\) the thumb of the Colossus of Rhodes was smaller than the whole statue, but this does not make it small.\(^{58}\) Gallop points out that things are never absolutely large (1975, 184),\(^{59}\) and we may extend this to other qualitative

\(^{57}\) A world in which “\((\forall x)(\forall y)[x < y \rightarrow \text{Small}(x)]\)” were true would be quite upsetting.

\(^{58}\) “Pauci pollicem eius amplectuntur, maiores sunt digitii quam pleraeque statuae” (Histoire Naturelle, xxxiv.41.6-7).

\(^{59}\) Cornford seems to have a similar point in mind when he says that Plato “thinks of tallness as an internal property on the same footing as ‘hot’ or ‘white’” (1935, 44). Presumably, Cornford is thinking that things are never absolutely tall; but one may also raise the question of whether anything is absolutely hot or white. Hera may have white arms, but
characters. For example, if Simmias is tall, one understands implicitly that he is tall in comparison with other men, not in comparison with the Colossus. Hence, some (if not all) qualitative characters appear to be implicitly relative, for when we say that something is tall, we mean that it is tall in relation to some other. Nevertheless, even if we accept this reasoning, if this is what Allen has in mind, one would expect him to argue for this point explicitly; but he does not.

*Further Objections to the Opposites Interpretation*

There are two additional objections that can be raised against the Opposites Interpretation, both of which are rooted in textual complications of the Paradoxes. The first is a conflict between Parmenides’ statement of the Paradoxes, and Allen’s reading of the same. Taking the First Paradox as representative of the other two, the Opposites Interpretation holds that part of the Large is qualified by largeness and smallness at the same time. This does not seem to be supported by the text. Parmenides states that large particulars are “large by a part of the Large smaller than the Large itself” (c12-d2). If Plato had in mind that part of the Large is qualified by largeness and smallness at the same time, why would he not say that large particulars are large by a part of the Large that is itself small? In each of the three Paradoxes, the comparative is used in preference to the positive, we would think her ill, and not beautiful, should her arms be as white as the paper on which this is written: her arms are only white in comparison to the shades of white we expect beautiful arms to be. Therefore, perhaps it is not that tallness is on the same footing as hot or white, but that hot or white are on the same footing as tallness.
but it is the positive form of the adjective that is required by the Opposites Interpretation: it is small, not smaller, that is the opposite of large.

The second objection is that the Second Paradox does not seem to point to a qualification by opposites. There, Parmenides contends that the equal particular is equal by a part of the Equal "less than the Equal itself" (d4-5). According to Allen, the second Paradox implies that part of the Equal is unequal (1997, 146), not that it is less; and although inequality might be a consequence of the second Paradox, it is part of the Equal being less that Socrates takes to be impossible (d6). Perhaps a case may be made for holding that from the part being less one may infer that the part is unequal (cf. 140b7-c4; Allen 1997, 145), but the Opposites Interpretation would be better supported had Parmenides explicitly claimed that the part of the Equal in question was unequal (ἀπόφως or ἀνισόφως) as opposed to less (ἐλάττων).

Further, we may enumerate two additional difficulties. The first difficulty is parasitic upon the objection that, according to the text, part of the Large is qualified by being both large and smaller, not large and small (c12-d2). If part of the large is qualified by being both large and smaller, we are unable to explain Socrates' "ἀδύνατον" (d6), because something being both large and smaller is not a logical absurdity. There is nothing unreasonable in holding that the thumb of the Colossus of Rhodes is large, but that it is nevertheless smaller than the whole statue.

The second difficulty takes issue with Allen's generalisation of the Paradoxes. Recall that, according to Allen, since every Form is a whole of parts, every Form participates in the
Large, and every part participates in the Small (1997, 148; cf. p. 70). While this is an admittedly ingenious way of generalising the Paradoxes, the Dilemma does not make reference to participation of Forms in Forms. This is not to say that Allen’s generalisation is invalid, but merely to point out that it does not seem to be supported by the text.

The Justificatory Interpretation

The Disjunction Interpretation and the Opposites Interpretation are both of them quite innovative, but neither fully explains the Second Horn of the Dilemma. Therefore, we shall present the Justificatory Interpretation, so called because it takes the Paradoxes of Divisibility as justifying the claim that the Form is not divided (c9-11). The interpretation has two main parts, the first being an analysis of that which is justified by the Paradoxes, and the second being the justifications themselves.

After accepting that part of the Form is present in each particular (c5-7), Parmenides asks Socrates, “will you be willing, Socrates, to say that the one Form, in truth, is divided for us, and will it still be one” (c9-10). Although Socrates denies this as if it were a single question (c11), Parmenides has asked two questions: (a) is the one Form divided, and (b) will the divided Form be one. When these two questions are considered by themselves, Socrates’ denial rings true for both. If we interpret “ἐν” (c9) as “indivisibility,” then that indivisibility

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60 This interpretation uses principles already introduced by Scaltsas and by Allen; but it is Proclus to whom we are indebted for the main structure of this interpretation. It is he who first proposed that the Paradoxes are justificatory (Proclus 1987, IV, §866, trans. G. Morrow): “to show that it is absurd to suppose ideal being as divisible, he [Plato] builds his arguments upon Greatness, Equality and Smallness, since each of these is manifestly concerned with magnitude.”
is compromised when the Form is divided. This follows from the material model of the
Forms that has been re-established by the Sail Analogy: if a sail is divided, it is no longer a
physically contiguous unity; its indivisibility is compromised (cf. p. 58). Under this
assumption, Socrates responds correctly (a) that the one Form is not divided, and (b) that the
divided Form will not be one.

It may appear that there is nothing to be gained by dividing question (a) from
question (b); but this is because we have only considered the questions by themselves. When
we consider these questions in a broader context of the Justificatory Interpretation, we are
presented with the problem of what "γάρ" (c12) is intended to justify (cf. p. 64). If we do
not distinguish between these two questions, the most natural reading is that it justifies why
the Form will not be one. But if the two questions are separated, there are two consequences
that "γάρ" could justify: (a) why the one Form is not divided, or (b) why the Form will not
be one. As it turns out, it is necessary to distinguish between these two questions, because
the Paradoxes of Divisibility do not show why the Form will not be one, but instead show
why the one Form is not divisible. Moreover, it is appropriate to demonstrate that the one
Form is not divided, as opposed to the Form not being one, because the Second Horn is
explicitly concerned with the possibility of dividing Forms (that is, of the Forms having
parts), and only implicitly concerned with the singularity of divided Forms.

*Homoeomereity and the Paradoxes*

Something is homoeomereous if the whole does not differ in kind from its proper parts
(cf. Allen 1997, 148). For example, wine in a mixing bowl is homoeomereous, because the
wine that is drawn into a cup does not differ in kind from that in the bowl. In contrast, a wagon is not homoeomorous, because its parts (e.g., wheels, axle, body, rail and yoke) each differ in kind from the whole wagon. There are two principles that show a divided Form would be homoeomorous (cf. p. 69):

(a) Form F is f;
(b) The parts of Form F are f.

In this way, since the Large is itself large by (a), and the parts of the Large are themselves large by (b), the Large is homoeomorous; this should hold for all Forms.

The rôle of the Paradoxes of Divisibility is to impugn this supposed homoeomereity on two levels. (I) At the level of a Form’s character (that is, the sense in which the Large and parts of the Large are large), the Paradoxes function merely as counterexamples. They are merely counterexamples, because such arguments only apply to Forms that involve magnitude. (II) At the level of a Form’s function (that is, the sense in which the particular is large by participating in the Large, and hence part of the Large is present in the particular), the Paradoxes function as general arguments against the divisibility of Forms. When these attacks on the homoeomereity of Forms are considered together, we are forced to conclude that Forms are not divisible.

(I) If a Form is homoeomorous with respect to character, then parts of the Form should have the same character as the whole Form. This is clearly not true in the case of magnitudes, as long as we are under the spell of the Sail Analogy, which employs the material model of the Forms. For example, a perch is a sixty-six inch length; but it would
be absurd to suppose that such a magnitude is homoeomerous, because any part of that length will necessarily be shorter in length than the whole. When we move to a consideration of the Large, the Equal and the Small, similar absurdities are encountered. Parts of the Large are smaller than the Large itself (c12-d2); parts of the Equal are smaller than the Equal itself (d4-5), and the Small is larger than parts of the Small (d7-e2). That is, because the parts of these magnitudes are not equal to the magnitudes themselves, the Large, the Equal and the Small are all nonhomoeomerous. These counterexamples, taken alone, would be enough to cast into doubt the thesis of the Second Horn, that parts of the Form are present in particulars; at the very least, the Second Horn would not be applicable to the Forms of magnitudes. The Paradoxes are most obvious when dealing with magnitudes; but to stop reading the Paradoxes at the level of a Form's character is to deny them their universality - for at the level of a Form's function the paradoxes are applicable to all Forms.

(II) If a Form is homoeomerous with respect to function, then parts of the Form will have the same function as the whole Form. Within the context of the Dilemma, the function of a Form is to be that in virtue of which a particular gets the same name as the Form.61 This reading follows Scaltsas' reading of the Second Disjunct for the first two Paradoxes (cf. p. 59): a part of the Form cannot do the work that the whole Form can. This is indicated by

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61As mentioned previously, Proclus seems to suggest this reading, but he does not follow through on it (1987, §865, trans. G. Morrow): “if each of us has a part [of Man] and not the whole, how could he be called a man and not a part of man?”
taking “σμικρότερος” (d1) and “ἐλάττων” (d5) in a qualitative sense. However, we must be careful not to orphan the Third Paradox in our interpretation (cf. p. 62).

The first part of the Third Paradox states that “the Small will be larger than that [part] of it, inasmuch as it is part of [the Small], and in this way (οὐτο δῆ) the Small itself will be larger” (d7-9). (This is the part that is relevant to our current interpretation; the second part of the Third Paradox shall be dealt with presently.) It is unlikely that “οὐτο δῆ” (d8-9) has an inferential force. Earlier, we ruled out the possibility that Plato is confused about relations (p. 74 ff.); and so an inferential reading of “οὐτο δῆ” would not mean that, because the Small is larger than a given part of it, we may infer that the Small is larger without qualification. Rather, it would mean that, because the Small is larger than a given part of it, we may infer the Small is larger than a given part of it - a redundant inference. It is more likely that “οὐτο δῆ” is intensive (cf. Denniston 1954, p. 209), serving to emphasise the result that the Small is larger than its parts. Should the Small be larger than its parts, the same difficulty that appears in the previous two Paradoxes appears here: the whole Form (in this case, the Small) is not present in the particular, for the Form is larger than its parts; and so the particular does not get the name of the Form. One might object that if this is what Plato had intended, he could have used “σμικρότερος,” or “ἐλάττων,” to express a part of the Small not being able to do the work of the whole Form, in the same way that he did with the Large and the Equal. But this invites confusion when we read the Third Paradox at the level of the Form’s character; for then both the Small and its parts would have the character of
smallness, which suggests homoeomereity. By pointing out that the Small is larger, this obscurity is avoided.

There are two loose ends that need to be tied up, given this two-level reading of the Paradoxes. The first is the remaining part of the Third Paradox (d9-e2): “that to which the subtracted part [of the Small] is added will not be larger but smaller than before.” This is not part of the Paradoxes proper, in that it does not follow, on either level, the pattern established by the three Paradoxes. However, it is a further consequence of the material model of the Forms, and functions as a counterexample by suggesting that the Small is not divisible. Plato adds it here because it is peculiar to the Small.

The second loose end is to note that this interpretation answers Scaltsas’ charges against other interpretations of the Paradoxes. Scaltsas charged that other readings could not (i) account for the Paradoxes as logical impossibilities, as indicated by Socrates’ “ἀδύνατον” (d6), and (ii) account for the results of the Paradoxes being stated generally (131e4-6). Both of these charges are accommodated, because at both levels the Paradoxes are absurdities with respect to the assumed homoeomereity of Forms, and at the second level the Paradoxes need not be limited to Forms of magnitude.
Conclusion

"These problems about the one and the many... cause all manner of perplexity if they are not suitably answered, but enlightenment if resolved."\footnote{62}

The exclusive disjunction with which we started, that either the whole Form is present in the particular, or part of the Form is present in the particular, is revealed as a destructive dilemma. The thesis of the first disjunct is a redactio on the assumption that nothing can be present in more than one place at any given time (Theorem T\gamma). The thesis of the second disjunct, while not itself a contradiction, leads to unacceptable results that are recounted in the Paradoxes of Divisibility. For the time being, the Dilemma has succeeded in splitting the claim that the Forms exist from the claim that particulars are named after the Forms.

Moreover, while the Dilemma of the Parmenides is applied to particulars participating in Forms, it may be extended to include Forms participating in Forms (cf. Allen 1997, 141-2). Parmenides infers that the One participates in Being from the fact that it is said to be, but its being signifies something other than its nature (142b5-7). Hence, any Form that is said to be, but whose being signifies something other than its nature, may be understood to participate in Being. But if a Form participates in Being, then it receives the whole of Being, or a part of Being; thus, the Dilemma obtains for Forms participating in Forms as well.

\footnote{62}Socrates, on the problem of the one and the many (Philebus 15b8-c3).
Despite the destructive force of the Dilemma, we find Socrates suggesting in the *Philebus* that it can be solved (17c11-e6), and so we are left with a question: what has gone wrong? It is quite likely, in a game of spot-the-guilty-assumption, that we would be right to finger the material model of the Forms. After all, the Dilemma is supported by the material model on four counts: first, that the singularity of the Form is incompatible with the Form being in many separate places at once (the First Horn); second, that the indivisibility of the Form is incompatible with parts of the Form being in many separate places at once (the Second Horn); third, that if something shares in the Form, it receives either the whole of the Form or part of the Form (the exclusive disjunction); and fourth, that the Form is present in the particular as Jack is present in the box (cf. Allen 1997, 135) - it is a proper part of the particular. Dismiss the material model, and the Dilemma collapses: it is no longer absurd to claim that one and the same Form is present in many separate places at once, the First Horn crumples, and we are home free - sort of.

*Beyond the Material Model*

Overthrowing the material model allows us to escape the Dilemma, but it still leaves us with the problem of supplying an account of participation. Fortunately, Plato has dropped a number of clues in later dialogues that suggest ways to move beyond the material model (a journey that will, most assuredly, not be simple):

1. A temporal model of the Forms is suggested by the Day Analogy as a replacement for the material model. By taking "day" in the sense of a time interval between sunrise and sunset, we have hit upon that which is in many places at once, and is not separate from itself.
The challenge is to raise the Analogy from the status of a counterexample to that of a model of participation. That temporal intervals are present in many places at once is apparent; but one needs to describe the way in which this happens, and then apply it to the Forms. The Day Analogy asserts that a time interval is in many places at once, but articulating how this comes about is another matter entirely. According to Sayre (1996, 77), Socrates sketches the temporal model as a solution to the Dilemma in the *Philebus* (17c11-e6).

(II) The Dream in the *Theaetetus* suggests that particulars are not merely an aggregate of constituent parts, but a certain arrangement of those parts (203e2-5); and so the Form may be present in the particular as the arrangement of these parts (cf. p. 32ff.). Such a Form would not be present in the particular as a physical part, and so it would not be bound by the restrictions of the material model. Of course, if one follows this suggestion, it becomes necessary to describe the nature of a constituent part and its own relation to the Forms. Furthermore, in the Dream, constituent parts are plaited together (205b2-5) - an image that returns in the *Sophist* (262b9-d7). The art of plaiting is also discussed in the *Politicus*, first as a mercantile art (279c7-83a9), and then as a kingly art (305e ff.). Detailing what this plaiting involves is bound to be an onerous chore, but there is little doubt that it is essential to the structure of particulars.

(III) The *Timaeus* describes participation in terms of the imitation of paradigms (27d5-29d3; 30c2-31b3) Again, we avoid the restrictions of the material model, because if the particular shares in the Form by being like the Form, it does not seem necessary for the Form to be a proper part of the particular. Parmenides overturns this model of participation
in the *Parmenides* when Socrates proposes it (132c12-33a7), and so there are clearly difficulties with this view; but its reëmergence in the *Timaeus* suggests that those difficulties are not fatal.

(IV) Finally, two examples constantly recur in close proximity to discussions about plaiting and participation; as such, it would be foolish to ignore them in any account of participation. The first is the example of letters and syllables, which appears in the *Theaetetus* (202c6-203e7, 207c7-208b7), the *Sophist* (253a4-12), the *Politicus* (277e1-78d7) and the *Philebus* (17a8-b10). The second is the example of musical intervals and rhythms, which appears in the *Sophist* (253b1-4) and in the *Philebus* (17b11-c6) - and both of these times it follows a discussion of letters and syllables.

Given the diversity of these suggestions, it is probable that they can be used in combination to move beyond the material model of the Forms. Taken together, they seem to indicate a direction, but one we are bound to follow lest we lose the power of discourse as Parmenides warns (135b6-c3):

But indeed, Socrates, if someone will not allow that there are Forms of things that are (having looked at all these difficulties just now discussed and others of such a sort), and will not distinguish some Form for each individual thing, he will not have anything to which to turn his mind, since he will not allow one Idea, ever the same, of each of the things that are; and thus he will utterly destroy the power of discourse.
Appendix A (The First Horn)

From the stated premisses of the First Horn (p. 25), a contradiction can be derived; and on the basis of this contradiction, by *ex falso quodlibet*, the conclusion may be asserted. However, in order to follow more closely the proof as it is presented in the dialogue, we shall begin by demonstrating the theorem of classical extensional mereology on which the thesis of the First Horn runs aground. The thesis of the First Horn is that one and the same Form is present in many separate particulars; but this is contrary to the theorem that the parts of disjoint wholes cannot be identical. Symbolically, the theorem may be represented as follows:

(Tβ.a) Parts of disjoint wholes cannot be identical.
(Tβ.b) For any x, for any y, for any u and for any v; if x is a part of u, and y is a part of v, and u is disjoint from v, then x is not identical to y.
(Tβ.c) \((\forall x)(\forall y)(\forall u)(\forall v)[(x<u \& y<v \& u\{v\}) \to x\neq y]\)

Although our target formula is a universal, it is convenient to start with arbitrarily selected constants and formulate the conditional before universalising:

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<td>1</td>
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<td>2</td>
<td>Assumption</td>
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<td>1</td>
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<td>1</td>
<td>1 &amp;E</td>
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<td>4 def SD3</td>
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<td>1</td>
<td>5 def SD2</td>
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Theorem Tβ may be temporally qualified in the usual manner, by supposing that there is some time χ which temporally qualifies the four relations of the theorem.

The proof of the First Horn shall focus on deriving "x<u & y<v & u\v" from the premisses. By theorem Tβ, we are then able to infer "x*y"; and since "x=y" is a conjunction of the first premiss, identity elimination is used to infer the "x*y" that is required for the conclusion. Here is the proof:

1  (1) \((\forall x)(\forall y)(\exists u)(\forall v)(\exists s)(\exists t)(Fx & Fy & Pu & Pv & Ts & Tt & Quxs & Qvyt & x=y & u\v & u\v & s=t)\)
   Premiss

2  (2) \((\forall x)(\forall\forall)(\forall s)(Fx & Pu & Ts & Quxs) \rightarrow
   (\exists z)(x\ll u & s=\sigma p(p\ll x))\)
   Premiss

3  (3) Fx & Fy & Pu & Pv & Ts & Tt & Quxs & Qvty & x=y & u\v & u\v & s=t
   Assumption

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63 Quantifier Shift (Forbes 1994, 192).

64 Here and elsewhere, free variables are used for instantiations to indicate an x that has various values, as opposed to a particular x (cf. Church 1956, I, 11; I, 11, n. 28).

65 For convenience, we employ a slightly more flexible =E rule than Forbes, by adding the following to his definition of Identity Elimination (cf. 1994, 269): Or for any sentence \(q\alpha t\) and \(t_1\ll t_2\) of LFOL, where \(t_1\) and \(t_2\) are individual constants, if \(t_1\ll t_2\) occurs at a line j in a proof, and \(q\alpha t\) occurs at line k, then at a line m we may infer \(q\alpha t_1\), writing on the left of m the numbers which appear on the left of j and k. \(q\alpha t\) results from \(q\alpha t_2\) by replacing at least one occurrence of \(t_2\) in \(q\alpha t_2\) by \(t_1\).
At this point, we have derived the formula \(a \neq b\), but what we want is the formula \(x \neq y\).

This is obtained by employing Theorem Ta (p. 25), which states that the whole is identical to the fusion of its parts. By the right conjunct of (12), \(a\) is identical to the fusion of the parts of \(x\), and by the right conjunct of (16), \(b\) is identical to the fusion of the parts of \(y\):

\[
(\forall x)[x = \sigma y(y < x)]
\]

12 (26) \(a = \sigma p(p < x)\) 12 \&E
12 (27) \(x = \sigma p(p < x)\) 27 \&E
12 (28) \(a = x\) 27, 28 =E

2 (4) \((Fx \& Pu \& Ts \& Quxs) \rightarrow (\exists z)(z < u \& z = \sigma p(p < x))\) 2 \&E (3 times)
3 (5) \(Fx \& Pu \& Ts \& Quxs\) 3 \&E (10 times)
2, 3 (6) \((\exists z)(z < u \& z = \sigma p(p < x))\) 4, 5 \&E
2 (7) \((Fy \& Pv \& Tt \& Qvyt) \rightarrow (\exists z)(z < v \& z = \sigma p(p < y))\) 2 \&E (3 times)
3 (8) \((Fy \& Pv \& Tt \& Qvyt)\) 3 \&E (10 times)
2, 3 (9) \((\exists z)(z < v \& z = \sigma p(p < y))\) 7, 8 \&E
2 (10) \(s = t\) 2 \&E
2, 3 (11) \((\exists z)(z < v \& z = \sigma p(p < y))\) 10, 9 \&E
12 (12) \(a < u \& a = \sigma p(p < x)\) Assumption
12 (13) \(a < u\)
12 (14) \(a < u \& a = u\)
12 (15) \(a < u\) 13 \&I
16 (16) \(b < v \& b = \sigma p(p < y)\) Assumption
16 (17) \(b < v\)
16 (18) \(b < v \& b = _1 v\)
16 (19) \(b < v\) 17 \&I
12, 16 (20) \(a < u \& b < _1 v\) 18 \&I
3 (21) \(_1 v\) 12, 16 \&I
3, 12, 16 (22) \(a < u \& b < _1 v \& u_1 v\)
12 (23) \((\forall x)(\forall y)(\forall u)(\forall v)(\forall s)[(x < _1 u \& y < _1 v \& u_1 v) \rightarrow x \neq _1 y]\) TI (Tβ)
24 (24) \((a < _1 u \& b < _1 v \& u_1 v) \rightarrow a \neq _1 b\) 23 \&E (4 times)
3, 12, 16 (25) \(a \neq _1 b\) 24, 22 \&E
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<td>16</td>
<td>(31)</td>
<td>( y = \sigma p(p &lt; y) )</td>
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<td>( x #_s b )</td>
<td>29, 25 = E</td>
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<td>( x #_s y )</td>
<td>32, 33 = E</td>
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<td>2, 3, 12</td>
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<td>11, 16, 34 ( \exists E )</td>
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<td>( x #_s y )</td>
<td>6, 12, 35 ( \exists E )</td>
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<td>(37)</td>
<td>( Fx &amp; Ts )</td>
<td>3 &amp; E (12 times)</td>
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<td>2, 3</td>
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<td>( Fx &amp; Ts &amp; x #_s y )</td>
<td>37, 36 &amp; I</td>
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<tr>
<td>3</td>
<td>(39)</td>
<td>( x = y )</td>
<td>3 &amp; E (13 times)</td>
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<td>2, 3</td>
<td>(40)</td>
<td>( Fx &amp; Ts &amp; x #_s x )</td>
<td>38, 39 = E</td>
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<tr>
<td>2, 3</td>
<td>(41)</td>
<td>( (\exists x)(\exists s)(Fx &amp; Ts &amp; x #_s x) )</td>
<td>40 ( \exists I ) (2 times)</td>
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<tr>
<td>2</td>
<td>(42)</td>
<td>( (\exists x)(\exists s)(Fx &amp; Ts &amp; x #_s x) )</td>
<td>1, 3, 41 ( \exists E ) (6 times) ♦</td>
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Note we are not violating the second restriction of existential elimination in the final line of the proof, the restriction being that the constant assumed in the assumption cannot appear in the inference (Forbes 1994, 199). Although both "x" and "s" appear in both the assumption and the inference, those that appear at (3) are arbitrarily selected constants, while those that appear at (41) are bound variables.
Appendix B (The Disjunction Interpretation)

The First Disjunct

The initial strategy employed to prove the sequent corresponding to the argument-form of the first disjunct (p. 58) is to show that because part of the Form being present in the particular is the sufficient condition for the particular’s having the same name as the Form, there are two separate particulars, u and v, that both have the same name at the same time:

1 (1) \((\exists x)(\exists y)(\exists r)(\exists u)(\exists v)(\exists s)(\exists t)(F_x \& F_y \& N_r \& x \supset r \& y \supset r \& P_u \& P_v \& T_s \& T_t \& Q_u \& Q_v \& 0t \supset y \& u \supset v \& u \supset s = t)\) & Premiss

2 (2) \((\forall x)(\forall u)(\forall s)(\forall r)(\forall z)(F_x \& P_u \& T_s \& Q_u \& S_z(A \supset r \& z \supset x) \supset u \supset r)\) & Premiss

3 (3) \((\forall x)(\forall u)(\forall s)(\forall r)(\forall z)(\forall x)(\forall z)(F_x \& P_u \& T_s \& Q_u \& S_z(A \supset r \& z \supset x) \supset u \supset r)\) & Premiss

4 (4) \((\forall x)(\forall u)(\forall s)(\forall r)(\forall z)(F_x \& P_u \& T_s \& Q_u \& S_z(A \supset r \& z \supset x) \supset u \supset r)\) & Premiss

5 (5) \(F_x \& F_y \& N_r \& x \supset r \& y \supset r \& P_u \& P_v \& T_s \& T_t \& Q_u \& Q_v \& 0t \supset y \& u \supset v \& u \supset s = t\) & Assumption

2 (6) \((F_x \& P_u \& T_s \& Q_u) \supset ((\exists z)(z \supset u \& z \supset x) \& (\exists z)(\neg z \supset u \& z \supset x))\) & 2 V(E (3 times)

5 (7) \(F_x \& P_u \& T_s \& Q_u\) & 5 &E (11 times)

2, 5 (8) \((\exists z)(z \supset u \& z \supset x) \& (\exists z)(\neg z \supset u \& z \supset x)\) & 6, 7 \-E

2, 5 (9) \((\exists z)(z \supset u \& z \supset x)\) & 8 &E

3 (10) \((F_x \& P_u \& T_s \& N_r \& x \supset r \& a \supset u \& a \supset x) \supset u \supset r\) & 3 V(E (4 times)

5 (11) \(F_x \& P_u \& T_s \& N_r \& x \supset r\) & 5 &E (10 times)

12 (12) \(a \supset u \& a \supset x\) & Assumption

5, 12 (13) \((F_x \& P_u \& T_s \& N_r \& x \supset r \& a \supset u \& a \supset x\) & 11, 12 &I
According to (4), the particular’s having the name of the Form is the necessary and sufficient condition for the Form being present in the particular as a whole; and so this premiss shall be used to show that c and d are present in two separate particulars, u and v:

\[
\begin{align*}
3, 5, 12 & \quad (14) \quad u_\rightarrow, r \\
2 & \quad (15) \quad (Fy \land Pv \land Tt \land Qvty) \rightarrow ((\exists z)(z_\rightarrow, v \land z_\leftarrow, y) \land (\exists z)(\neg z_\rightarrow, v \land z_\leftarrow, y)) \\
5 & \quad (16) \quad Py \land Pv \land Tt \land Qvty \\
2, 5 & \quad (17) \quad (\exists z)(z_\rightarrow, v \land z_\leftarrow, y) \land (\exists z)(\neg z_\rightarrow, v \land z_\leftarrow, y) \\
2, 5 & \quad (18) \quad (\exists z)(z_\rightarrow, v \land z_\leftarrow, y) \\
3 & \quad (19) \quad (Py \land Pv \land Tt \land Nr \land y_\rightarrow, r \land b_\rightarrow, v \land a_\leftarrow, y) \\
& \quad \rightarrow v_\rightarrow, r \\
5 & \quad (20) \quad Py \land Pv \land Tt \land Nr \land y_\rightarrow, r \\
21 & \quad (21) \quad b_\rightarrow, v \land b_\leftarrow, y \\
5, 21 & \quad (22) \quad Py \land Pv \land Tt \land Nr \land y_\rightarrow, r \land b_\rightarrow, v \land b_\leftarrow, y \\
3, 5, 21 & \quad (23) \quad v_\rightarrow, r 
\end{align*}
\]

\[
\begin{align*}
4 & \quad (24) \quad (Fx \land Pu \land Ts \land Nr \land x_\rightarrow, r) \rightarrow [(\exists z)(z_\rightarrow, s, u \land z=\sigma p(p_\lt, x)) \rightarrow (u_\rightarrow, r)] \\
4, 5 & \quad (25) \quad (\exists z)(z_\rightarrow, s, u \land z=\sigma p(p_\lt, x)) \rightarrow (u_\rightarrow, r) \\
3, 4, 5, & \quad (26) \quad (\exists z)(z_\rightarrow, s, u \land z=\sigma p(p_\lt, x)) \\
12 & \quad (27) \quad (Py \land Pv \land Tt \land Nr \land y_\rightarrow, r) \rightarrow [(\exists z)(z_\rightarrow, v \land z=\sigma p(p_\lt, y)) \rightarrow (v_\rightarrow, r)] \\
4, 5 & \quad (28) \quad (\exists z)(z_\rightarrow, v \land z=\sigma p(p_\lt, y)) \rightarrow (v_\rightarrow, r) \\
3, 4, 5, & \quad (29) \quad (\exists z)(z_\rightarrow, v \land z=\sigma p(p_\lt, y)) \\
21 & \quad (30) \quad c_\rightarrow, u \land c=\sigma p(p_\lt, x) \\
30 & \quad (31) \quad c_\rightarrow, u \\
32 & \quad (32) \quad d_\rightarrow, v \land d=\sigma p(p_\lt, y) \\
32 & \quad (33) \quad d_\rightarrow, v \\
5 & \quad (34) \quad s=t \\
5, 32 & \quad (35) \quad d_\rightarrow, v \\
5, 30, 32 & \quad (36) \quad c_\rightarrow, u \land d_\rightarrow, v \\
5 & \quad (37) \quad u_\rightarrow, v \\
5, 30, 32 & \quad (38) \quad c_\rightarrow, u \land d_\rightarrow, v \land u_\rightarrow, y \\
36, 37 & \quad \text{&I}
\end{align*}
\]

\[66\text{Forbes 1994, 117.}\]
It was pointed out earlier that modern commentators, including Scaltsas, consider the Form’s presence in the particular as its being a part of the particular-and-Form considered as a whole (p. 18). This means that the Form’s presence in the particular is bound by the same restriction that gave rise to theorem $T\beta$. There it was submitted that parts of disjoint wholes cannot be identical. Here we transform the theorem for our present purposes, but still in keeping with Scaltsas’ assumptions:

$(T\gamma.a)$ That which is present in disjoint particulars cannot be identical.

$(T\gamma.b)$ For any $x$, for any $y$, for any $u$ and for any $v$, if $x$ is present in $u$, and $y$ is present in $v$, and $u$ is disjoint from $v$, then $x$ is not identical to $y$.

$(T\gamma.c)$ $(\forall x)(\forall y)(\forall u)(\forall v)[(x \in u \land y \in v \land u \not\subset v) \rightarrow x \neq y]$

The proof for $T\gamma$ (not presented here) follows the same pattern as the proof for $T\beta$ (p. 88), because for Scaltsas, the present-in relation is taken to be inferentially identical to the proper-part relation. As with $T\beta$, $T\gamma$ may be temporally qualified by supposing that there is some time $\chi$ which temporally qualifies the four relations of the theorem.

(39) $(\forall x)(\forall y)(\forall u)(\forall v)(x \in u \land y \in v \land u \not\subset v) \rightarrow x \neq y$

$(T\gamma)$

(39) $(\forall x)(\forall y)(\forall u)(\forall v)(x \in u \land y \in v \land u \not\subset v) \rightarrow x \neq y$

(40) $(c \in u \land d \in v \land u \not\subset v) \rightarrow c \neq d$

$T\gamma$ (4 times)

(41) $c \neq d$

(42) $(\forall x)[x = \sigma y (y < x)]$

$T\alpha$

(43) $x = \sigma y (y < x)$

$\forall E$

(44) $c = \sigma y (y < x)$

$\forall E$

(45) $x = c$

$= E$

(46) $x \neq d$

$= E$

(47) $y = \sigma y (p < y)$

$\forall E$

(48) $d = \sigma y (p < y)$

$\forall E$

(49) $y = d$

$= E$

(50) $x \neq y$

$= E$

(51) $Fx \land FY \land Nr \land Ts \land x \succ r \land y \succ r$

$5 \land E$ (9 times)

(52) $Fx \land FY \land Nr \land Ts \land x \succ r \land y \succ r \land x \neq y$

$51, 52 \land I$
The Second Disjunct

The argument-form that corresponds to the second disjunct (p. 62) may be proved by a shorter sequence of inferences than that of the first disjunct. The overall strategy of the proof is to show that, given that only part of the Form is present in the particular, and that the whole Form's being present in the particular is the necessary and sufficient condition for the particular's having the same name as the Form, it is impossible that the particular should have the name of the Form:

1  (1)  (∃x)(∃y)(∃r)(∃u)(∃v)(∃s)(∃t)(Fx & Fy & Nr & x>r & y>r & Pu & Pv & Ts & Tt & Quxs & Qvyt & x=y & u₁,v & u₁,v & s=t)  Premiss
2  (2)  (∀x)(∀u)(∀s)[(Fx & Pu & Ts & Quxs) → ((∃z)(z<s,u & z<x) & (∃z)(~z<s,u & z<x))]  Premiss
3  (3)  (∀x)(∀u)(∀s)(∀r){(Fx & Pu & Ts & Nr & x>r) → [(∃z)(z<s,u & z=σp(p<x) . . . (u₁,r)]}  Premiss
4  (4)  u₁,r  Assumption
5  (5)  Fx & Fy & Nr & x>r & y>r & Pu & Pv & Ts & Tt & Quxs & Qvyt & x=y & u₁,v & u₁,v & s=t  Assumption
At this point, we reach an impasse in our proof unless we realise that if the whole Form is present in the particular, as (12) suggests, then every part of the Form must be present in the particular. We may state this as a new theorem (the proof of which is not presented here):

(T6.a) If the whole Form is present in the particular, then every part of the Form is present in the particular.

(T6.b) For any $x$, and for any $y$, and for any $u$ and for any $s$, if $x$ is present in $u$ at time $s$, and $y$ is a proper part of $x$ at time $s$, then $y$ is present in $u$ at time $s$.

(T6.c) $(\forall x)(\forall y)(\forall u)(\forall s)[(x \in u \& y \subset x) \rightarrow y \in u]$

This may now be used to generate the contradiction that part of the Form cannot be not present in the particular when the whole Form is present in the particular:

<table>
<thead>
<tr>
<th>Step</th>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>(\forall x)(\forall y)(\forall u)(\forall s)[(x \in u &amp; y \subset x) \rightarrow y \in u]</td>
<td>TI (T6)</td>
</tr>
<tr>
<td>14</td>
<td>(a \in u &amp; b \subset a) \rightarrow b \in u</td>
<td>Assumption</td>
</tr>
<tr>
<td>15</td>
<td>a \in u</td>
<td>Assumption</td>
</tr>
<tr>
<td>16</td>
<td>a \in u</td>
<td>Assumption</td>
</tr>
<tr>
<td>17</td>
<td>a \in \sigma(p &lt; x)</td>
<td>Assumption</td>
</tr>
<tr>
<td>18</td>
<td>(\forall x)[x = \sigma(y &lt; x)]</td>
<td>TI (Tα)</td>
</tr>
<tr>
<td>19</td>
<td>x = \sigma(y &lt; x)</td>
<td>TI (Tα)</td>
</tr>
<tr>
<td>20</td>
<td>a = x</td>
<td>17, 19 =e</td>
</tr>
<tr>
<td>21</td>
<td>(\exists z)(\neg z \in u &amp; z \subset x)</td>
<td>Assumption</td>
</tr>
<tr>
<td>22</td>
<td>\neg b \in u &amp; b \subset x</td>
<td>Assumption</td>
</tr>
<tr>
<td>23</td>
<td>b \subset x</td>
<td>Assumption</td>
</tr>
<tr>
<td>24</td>
<td>b \subset a</td>
<td>Assumption</td>
</tr>
<tr>
<td>25</td>
<td>a \in u &amp; b \subset a</td>
<td>Assumption</td>
</tr>
<tr>
<td>26</td>
<td>b \in u</td>
<td>Assumption</td>
</tr>
<tr>
<td>27</td>
<td>\neg b \in u</td>
<td>Assumption</td>
</tr>
</tbody>
</table>
Now to complete the proof, we shall extricate ourselves from the last three assumptions made by means of existential eliminations; and we shall discharge the first assumption by negation introduction. That is, it is not the case (given our premisses) that the particular has the name of the Form. Having derived the consequent of the conclusion, it is a simple matter to insert the antecedent and use universal introduction to secure the target formula:

\[\begin{align*}
2, 5, 15 & \quad (28) \quad \land & & 21, 22, 28 \exists E \\
2, 3, 4, 5 & \quad (29) \quad \land & & 12, 15, 28 \exists E \\
1, 2, 3, 4 & \quad (30) \quad \land & & 1, 5, 29 \exists E \\
1, 2, 3 & \quad (31) \quad \sim u \rightarrow r & & 4, 30 \sim I \\
1, 2, 3 & \quad (32) \quad (Fx \& Pu \& Ts \& Nr \& x \rightarrow r \& (\exists z)(z \prec u \& z \prec x) \& (\exists z)(\sim z \prec u \& z \prec x)) \rightarrow \sim u \rightarrow r & & 31 SI (PMI)^{67} \\
1, 2, 3 & \quad (33) \quad (\forall x)(\forall u)(\forall s)(\forall r)[(Fx \& Pu \& Ts \& Nr \& x \rightarrow r \& (\exists z)(z \prec u \& z \prec x) \& (\exists z)(\sim z \prec u \& z \prec x)) \rightarrow \sim u \rightarrow r] & & 32 \forall I (4 \text{ times}) \dagger \\
\end{align*}\]

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^{67}\text{Paradox of Material Implication (Forbes 1994, 123).}
Literature Cited


