GOVERNMENT DEFICITS AND DEBT IN A FEDERAL ECONOMY

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Abstract

In federal states such as Canada and the U.S., non-federal governments control a sizable fraction of total government revenues and expenditures. Despite this, the literature on the macroeconomic effect of government deficits and debt deals virtually exclusively with unitary states. Similarly, the literature which examines issues pertaining to non-federal governments ignores the issue of the effect their choosing to deficit finance might have on macroeconomic stability. This thesis represents an effort to bring together these two strands of research. The results indicate that many conclusions of the existing literature on the macroeconomic effect of government deficits and debt are either completely overturned or significantly modified when one considers a federal rather than a unitary state. In particular, we find that the condition(s) which must be satisfied for macroeconomic stability are made significantly more stringent when non-federal governments choose to deficit finance disturbances to their budget positions. We also find that the success of federal debt management policies are greatly influenced by the decision of non-federal governments to deficit finance. In conclusion then, we find that two issues which have played a prominent role in recent policy debates -- whether government deficit financing must eventually lead to large tax increases (or expenditure cuts), and the magnitude of deficit reductions necessary to maintain manageable levels of debt -- are both highly sensitive to decisions made not only at the federal government level but also at the non-federal level.

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Chapter One: Introduction

The purpose of this thesis is to bring together two areas of research which have developed separately from one another. The first has found particular interest in Canada as it examines the macro-economic impact of non-federal (municipal and provincial/state) governments. The second area of research examines the question of the stability of government deficit financing. What this thesis will endeavour to show is that the conclusions drawn from these heretofore separately pursued areas of research are in fact integrally related -- the stability of government deficit financing depends, in an important way, on the behaviour of both levels of government. This introduction is intended to briefly summarize these two literatures and suggest how they are inter-related.

The raison d'être of the literature examining the macroeconomic effect of non-federal governments is the fact that in federal economies (of which there are six among industrialized western economies) non-federal governments control a sizable fraction of total government revenue and expenditure (see Table 1). The existing literature on the economic effect of non-federal governments concentrates on basically two questions -- to what extent have non-federal governments in the past influenced macroeconomic variables and to what extent can they influence the macroeconomy in the future.

Table 1: Revenues and Expenditures of Non-Federal Governments as a Percentage of Total Government Revenues and Expenditures, 1982

	Austria	Australia	Canada	Germany	Switzerland	U.S.A.
Revenue	26.0	23.6	53.1	35.1	35.8	37.4
Revenue*	32.8	38.2	60.9	41.2	52.1	46.4
Expenditure	31.2	40.5	56.5	41.9	52.4	40.7

* includes transfers from the federal government.

Source: Calculated from data in Government Finance Statistics Yearbook 1984, International Monetary fund (Washington, 1984).

An early study using historical data to measure the influence of non-federal governments was Robinson and Courchene (1969). Using data covering the period 1952-66, Robinson and Courchene found that Canadian non-federal government budgets exerted a small but counter-cyclical effect. Lacroix and Rabeau (1981) updated the Robinson and Courchene study and found little net influence (either pro- or counter-cyclical) of non-federal government budgets on the economy over the period 1952-71. More recently, over the 1971-76 period, they found a small but counter-cyclical influence. A recent study by Curtis (1987) employs a different methodology but arrives at a similar conclusion -- although most of the counter-cyclical effect of government budgets was due to changes in the federal budget, the non-federal sector exerted a small but mainly counter-cyclical influence on the Canadian economy from 1970 to 1983. Winer (1979) uses a reduced form approach to measure the influence of the two levels of government using Canadian data covering the period 1947-73. Without attempting to conclude whether these influences were pro- or counter-cyclical, Winer concludes that over this period changes in non-federal government expenditures had a larger impact on nominal GNP than did changes in federal government expenditures.

A problem with these studies is that they cannot measure whether the influence exerted by non-federal government budget changes was due to discretionary policy or simply due to automatic stabilizers in their budgets. As a result, their conclusions offer little evidence on the question of whether non-federal governments can, by discretionary policy, influence macroeconomic variables. Auld (1975, 1987) attempts to address this problem by measuring the effect of changes in the Ontario government's full employment budget surplus (FEBS) on Ontario's GNP. A change in the FEBS indicates a discretionary change in expenditures and/or tax rates so that changes due to automatic stabilizers are omitted. Auld concludes that discretionary policies by the Ontario government have had their desired counter-cyclical effect. Wilson and Jump (1975) attempt a measure of the effect of a discretionary non-federal policy by choosing a period, the years 1975-76, during which the largest provinces (Ontario, Quebec, B.C., Alberta and Saskatchewan) simultaneously chose to initiate expansionary fiscal policies. The effect, they conclude, was strongly counter-cyclical.

There are two alternatives to using historical data in order to measure the impact of discretionary fiscal policy -- a theoretical model and a simulation model. The seminal paper by Oates (1968) offers a theoretical model of a regional economy in order to examine the impact on regional economic variables of a discretionary fiscal policy by the region's local government. Oates concludes that a discretionary policy, independently initiated by a non-federal government, will have little influence on income within its jurisdiction. Important to Oates' conclusion is his assumption of a high propensity to import on the part of both the regional private sector and the regional government. His paper prompted a number of simulation studies of varying degrees of sophistication designed to show that economically large non-federal jurisdictions have a small enough propensity to import that non-federal fiscal policies, even by the government of a single jurisdiction, can have a significant impact on macroeconomic variables. For example, Fortin (1982) uses Canadian data on inter-provincial trade to construct provincial propensities to import and uses these in conjunction with a simple reduced form Keynesian model to show that provincial expenditure multipliers resulting from independent provincial policy initiatives are between 76 and 89 percent (depending on the province) of the size of the multiplier due to a federally initiated, economy-wide expenditure change. In a somewhat more sophisticated model (structural equations for each province are estimated but the model is still of the fixed price, Keynesian variety) Miller and Wallace (1983) reach a similar conclusion. Finally, Wilson (1984) uses the FOCUS and PRISM macroeconometric models of the Canadian and Ontario economies to simulate the effect of fiscal policies initiated by the government of Ontario. Again the conclusion is that a change in expenditures by the government of Ontario will have nearly as large an impact on income in Ontario as an economy-wide change in federal government expenditures.

The other area of research in which we are interested is the literature which examines the question of the stability of government deficit financing. Like private economic agents, governments must satisfy a financing constraint equating sources and uses of funds. Thus the government budget constraint (GBC) requires that the per period flow of government expenditures (on goods and services and on interest on government debt) equals the flow of government revenue (from taxes and from bond sales to the public and the central bank). Beginning with the seminal paper by Blinder and Solow (1973), a large literature has developed which examines the conditions under which a government can deficit finance a disturbance to its GBC and expect that the GBC will eventually return to a position of equilibrium where the stock of bonds and/or the money supply are unchanging. In recent years, this question has come to the forefront of public policy debates as a past history of government bond financing has led to a growing public debt and a concern over whether its growth can be halted without resorting to large tax increases.

The existing literature on the question of the stability of government deficit financing deals exclusively with economies having only a single level of government -- a unitary state.[1] This is surprising since a number of major western economies are federal rather than unitary states and in a federal state the GBC's of each level of government must be interdependent. That is, any policy initiative by one level of government must, by affecting the private sector, change the level of national income, raise or lower interest rates, and/or change the rate of inflation. As changes in these macroeconomic variables will affect the GBC's of all other levels of government, the GBC's of all levels of government must be interdependent. How each of the other levels of government respond to this disturbance to its budget position will of course once again affect the private sector and hence each level of governments budget constraint. It would seem imperative then, in any discussion of the stability of deficit financing, to explicitly account for the reactions of all other levels of government since their reactions may significantly influence macroeconomic variables and hence the likelihood that a deficit caused by a policy initiative will eventually be closed.

Interestingly, the interdependence of GBC's in a federal economy is recognized in the literature on the macroeconomic impact of non-federal governments. Sheikh and Winer (1977) for example, note that an anti-inflation aggregate demand policy by the federal government may, if effective, increase the deficits of non-federal governments. This federal policy will therefore in whole or in part be offset by the response of non-federal governments to the change in their budget positions. Sheikh and Winer, however, do not investigate the implications of this for the stability of deficit financing. Instead they concentrate on the implications of this interdependence for the magnitude of policy multipliers. Similarly, Winer (1979) notes the interdependence of GBC's in a federation but proceeds to measure and compare the size of federal versus non-federal policy multipliers under the assumption that both levels of government change tax rates in response to budget disturbances. Hence the issue of the stability of deficit financing is avoided altogether. Auld (1980) also hints at the problem but he too fails to make the connection that the behaviour of one level of government may affect the stability of deficit financing by the other. Instead, his concern is that deficit financing by the provinces may increase crowding out and hence reduce the size of policy multipliers.

> The possibility of "crowding out" is particularly important in a federal state, provided the sub-federal levels of government have the power to incur deficits, which is certainly true in Canada. In attempting to conduct a national anti-recession fiscal policy, the federal government would face considerable uncertainty about the effects of a deficit (or increased deficit), unless it was aware of what ten other governments were planning. Bond sales to the public might be appropriate if provincial governments ... were not planning to increase their net debt. If they were planning to do so, then some monetarization of the federal deficit might be in order to reduce the pressure on interest rates.
>
> (Auld, 1980, p. 101)

Finally, empirical measures of the degree of budget interdependence between levels of government in a federation are available. Preston, Eyford and Saiyed (1981) conduct simulations using the CANDIDE macroeconometric model of Canada and find that if both levels of government are assumed to bond finance disturbances to their budgets, and if the federal government undertakes a policy initiative which creates a federal deficit, then the non-federal government sector realizes a smaller deficit (or a greater surplus) as a result. Similarly, Dungan and Wilson (1985) perform simulations using the FOCUS macroeconometric model and find that, again assuming both levels of government bond finance disturbances to their budgets (this is their "Alternative A"), a reduction in federal expenditures phased in gradually over a 10 year period would cause non-federal deficits to rise by approximately 40 percent of the resulting fall in the federal deficit. Neither of these simulation studies however, investigate how the behaviour of non-federal governments influence the likelihood of federal deficit financing being stable.

There would appear then, to be a significant new area of research which combines elements of two existing areas of research. In a federal economy, a heretofore ignored macroeconomic effect of non-federal governments is the effect they have on conclusions regarding the likelihood of deficit financing being stable. Similarly, in the literature investigating the conditions necessary for federal government deficit financing to be stable, it would seem important to model the response of non-federal governments to what they perceive to be an exogenous shock to their budget positions. This thesis represents a first step into this new area of research.

The thesis consists of three essays. In the first essay, chapter two of this thesis, we review the seminal paper by Blinder and Solow (1973) on the stability of deficit financing and extend it to the case of a federal economy. In the second essay, presented in chapter three, we follow Darby (1984) in relaxing the fixed price, zero growth assumptions of the Blinder and Solow model. Unlike Darby however, we allow for a variety of assumptions regarding the of late contentious issue of whether government bonds are net worth and we of course model a federal economy. In the third essay, presented in chapter four, we extend the model of essay two to the case of an open economy. The model in essay three is similar to the model in a recent paper by Scarth (1987a) except that unlike Scarth we again allow for any assumption regarding the issue of whether government bonds are net wealth, we include direct wealth effects, and of course we model a federal, rather than a unitary state. Finally, in chapter five, we summarize the three essays and review our conclusions.

FOOTNOTES

[1] There are two minor exceptions to this statement. Both Friedman (1948) and Christ (1979) recognize in their introductions that non-federal governments exist but both authors include them in the private sector and ignore the non-federal financing constraint thereafter. As we see in essay one below, this can only be legitimately done if one assumes non-federal governments never deficit finance -- an assumption which is far from true. It is interesting to note that Friedman stresses that it is desirable that the maximum amount of government activity be in the hands of state and local governments (see his first footnote). This preference, plus his proposal that government rates of tax and expenditure be unchanging and set so as to yield a balanced budget at full employment, implies that non-federal governments would have to deficit finance the effects of a business cycle on their budgets. It is somewhat surprising therefore that no attention has been given to establishing the consequences for economic stability of the existence of bond financing non-federal governments.

Chapter Two

Does Fiscal Policy Matter in a Federal Economy?

1. Introduction

In this first essay we review the results of the seminal paper by Blinder and Solow (1973) on the stability of government deficit financing and we extend this model to the case of a federal economy. We begin with this model despite its restrictive assumptions of fixed prices and zero growth both because it has retained its popularity even 14 years after its introduction (see, for example, the recent paper by Rau (1985) which offers a new interpretation) but more importantly because the major conclusions of the model -- that while money financing is stable, bond financing of a deficit may or may not be stable -- have proved to be robust to extensions to more sophisticated models. Before moving on to more elaborate models therefore, it is of interest to examine the implications for the conclusions of this relatively simple model of our extension to a federal economy. How the conclusions of this model are affected by our extension are not only of interest in themselves, but they may also indicate how the conclusions of more elaborate models may be affected by similar extensions.

2. The Model

We employ a simple demand determined model similar to that introduced by Blinder and Solow (1973) but adjusted to incorporate a second level of government. Prices and the stock of capital are assumed to be constant and we assume a closed economy. As our model has no supply side our findings should be interpreted as showing the impact of policy measures on an aggregate demand curve in price-quantity space. Whether this impact affects output, price, or some combination of the two depends on one's assumption regarding the shape of the aggregate supply curve. The model consists of the following six equations:

$$(1)\quad Y = C(Y^d, r, W) + G^f + G^n$$

$$(2)\quad Y^d = (1 - T^f - T^n)(Y + B^f + B^n)$$

$$(3)\quad W = M + (B^f + B^n)/r$$

$$(4)\quad M = L(Y, r, W)$$

$$(5)\quad \dot{M} + \frac{1}{r}\dot{B}^f = G^f + B^f - T^f(Y + B^f + B^n)$$

$$(6)\quad \frac{1}{r}\dot{B}^n = G^n + B^n - T^n(Y + B^f + B^n)$$

where

Y, Y^d = real income and real disposable income respectively

G^f, G^n = federal and non-federal government expenditures on goods and services respectively

T^f, T^n = federal and non-federal government proportional tax rates respectively where $0 < (T^f + T^n) < 1$

B^f, B^n = federal and non-federal government bonds outstanding (explained below)

M = real money supply

W = real private wealth

r = real interest rate

$C(\cdot)$ = private sector expenditures on goods and services

$L(\cdot)$ = demand for money.

Following convention time derivatives are denoted with a dot and subscripts denote partial derivatives (for notational ease we use C_y rather than C_{yd} to represent the propensity to consume out of disposable income.) The values of the partial derivatives are assumed to obey the following restrictions:

$$0 < C_y, C_w, L_w < 1 \qquad L_y > 0 \qquad C_r, L_r < 0.$$

Equations (1) and (2) describe equilibrium in the goods market. Private expenditures depend positively on disposable income and wealth but negatively on the interest rate. Income is subject to two tax rates, one for each level of government. National income consists of income from production (Y) plus interest paid on the public debt of each level of government ($B^f + B^n$). For simplicity we assume all bonds are consols each paying one dollar per period. Thus the symbol B represents both the number of bonds outstanding and the total current interest payments spent by that level of government. The price of such a bond is equal to the present value of this perpetual income stream and is thus $1/r$. We further assume that private agents regard federal and non-federal government bonds as perfect substitutes. There can occur therefore an exchange of federal for non-federal government bonds with no change in bond prices.

Equations (3) and (4) describe equilibrium in the financial sector. Private wealth is defined as consisting of the value of money holdings plus the value of bond holdings. We assume therefore that the full value of government bond holdings is viewed by private agents as net wealth. The demand for money depends positively on income and wealth but negatively on the rate of interest. The market clearing condition for the bond market is omitted via Walras' Law.

Equations (5) and (6) are the budget constraints faced by federal and non-federal governments respectively. Each requires that uses of funds equal sources of funds at every point in time. They differ only in that the federal government has available to it money creation as a source of funds whereas non-federal governments do not.[1] Note that since M represents the monetary base in (5) and the total money supply in (4) we are ignoring the existence of a banking sector. This is a conventional assumption in models employing a government budget constraint (GBC) and is equivalent to assuming a constant monetary base multiplier equal to unity. Note also that (6) represents the budget constraint for non-federal governments in aggregate. We assume therefore that all non-federal governments act in unison in that they all finance shocks to their budget positions in the same manner and all initiate the same type of fiscal policy when such policies are undertaken. In order to consider cases where non-federal governments do not act in unison, it would be necessary to define a separate GBC for each non-federal government. This would make it possible for some non-federal governments to tax finance while others bond finance. However, as each non-federal government collects tax revenue in a different sub-national economy, a different income variable would enter each non-federal GBC. We would therefore be required to model how each of these income variables is determined and this would necessitate that we model each sub-national economy as well as the trade and financial flows between them. The assumption that each non-federal government chooses the same policy variable to be endogenous enables us to aggregate non-federal GBC's into one and thereby enables us to greatly simplify our model.

The endogenous variables in the model are Y, Y^d, W, and r, plus one policy variable from each government budget constraint.

The model imposes constraints on the behaviour of all three sectors of the economy -- the federal and non-federal government sectors and the private sector. The constraints on government appear explicitly as equations (5) and (6) while the constraint on the private sector is implied by these equations and other relationships defined in the model. To see this note that the flow constraint on the private sector requires that changes in private wealth over time ($\dot{W}$) must equal private savings plus capital gains. Using the other relationships defined in the model this requirement can be expressed as,

$$\dot{M} + \frac{1}{r}(\dot{B}^f + \dot{B}^n) + \text{capital gains} = Y^d - C + \text{capital gains}$$

or,

$$\dot{M} + \frac{1}{r}(\dot{B}^f + \dot{B}^n) = (1 - T^f - T^n)(Y + B^f + B^n) - (Y - G^f - G^n)$$

or finally,

$$[G^f + B^f - T^f (Y + B^f + B^n) - \frac{1}{r}\dot{B}^f - \dot{M}]$$
$$+ [G^n + B^n - T^n(Y + B^f + B^n) - \frac{1}{r}\dot{B}^n] = 0.$$

Satisfaction of the two government budget constraints therefore ensures satisfaction of a private sector budget constraint so that the latter need not appear explicitly in the model. Note also that this derivation shows that in a federal economy it is not legitimate to simply ignore the non-federal GBC. If this is done then satisfaction of the federal GBC does not necessarily imply the private sector constraint is satisfied so it should enter the model as a separate constraint.

3. Policy Interdependence in a Federal Economy

Each level of government imposes a proportional income tax so that tax revenue is determined endogenously. Similarly, a component of each government's expenditures -- the service on its debt -- is in part a function of the interest rate and is therefore also endogenously determined. As a consequence of this each level of government must allow at least one policy variable -- its expenditures, tax rate, stock of bonds outstanding, or (for the federal government only) the stock of money -- to be determined endogenously in order to always obey its budget constraint. These considerations imply that in a federation there will be a significant degree of policy interdependence between levels of government. The actions of one level of government will, by affecting income (and consequently tax revenue) and/or the rate of interest, alter the budget position of the other level of government and thus initiate a change in the endogenous policy variable in that constraint. The response of the passive level of government to what it perceives as being an exogenous shock to its budget position may be a significant determinant of the impact of the active level of government's policy.

Such policy interdependence between levels of government can have two effects. First, it may impact upon the size of policy multipliers.

The multiplier associated with a federal policy initiative may significantly differ in magnitude depending on whether non-federal governments tax or bond finance shocks to their budget positions. Second, the stability of the model may depend critically upon the particular combination of policy variables chosen to be endogenous by the two levels of government.

The federal GBC contains four policy variables while the non-federal constraint contains three. Assuming for simplicity that each level of government allows only one policy variable to respond endogenously at any one time, there are therefore twelve possible combinations of endogenous policy variables. We have chosen the following four cases as being the most relevant:

- non-federal governments adjust tax rates residually while the federal government bond finances
- both levels of government bond finance
- non-federal governments adjust tax rates residually while the federal government money finances
- non-federal governments bond finance while the federal government money finances.

It is important to note that these cases do not simply represent examples of mixed financing by a government in a unitary state. Turnovsky (1977, pp. 68-85) considers this case by assuming a single level of government financing its budget deficit with <u>both</u> bond sales and money creation such that

$$\dot{M} = \delta \text{ (budget deficit)}$$

$$\dot{B} = r(1-\delta)\text{(budget deficit)}$$

where δ is a constant and $0 \leq \delta \leq 1$. If we restrict our attention for a moment to our case of non-federal bond financing and federal money financing one might suppose that we can equally well examine this case in the context of Turnovsky's model by assuming $\delta > 1$. That is, suppose the federal government increases its expenditures causing a federal deficit and a non-federal surplus (since non-federal tax revenues rise due to the policy induced increase in income). In response to the budget imbalances the federal government increases the money supply ($\dot{M} > 0$) while non-federal governments retire bonds ($\dot{B} < 0$). In the context of Turnovsky's model this set of responses implies $\delta > 1$. Thus it might appear our argument for the need to explicitly account for the actions of non-federal governments reduces to an argument that Turnovsky is wrong to restrict $0 \leq \delta \leq 1$ since $\delta > 1$ is a more relevant consideration in a federal economy.

This apparent equivalence of our model to Turnovsky's mixed financing case is however inaccurate. The crucial difference lies in the fact that $\dot{M}$ and $\dot{B}$ always respond to the same budget deficit in Turnovsky's model whereas in our model they respond to two different constraints. In Turnovsky's model the retirement of bonds causes the debt service to fall and thus the budget deficit to grow smaller (tax revenue falls as well but we assume this is dominated by the fall in the debt service). This causes the rates at which bonds are retired and money is issued to both decrease.[2] In our model the retirement of non-federal bonds reduces debt service in the non-federal constraint only. This has the effect of increasing the non-federal surplus causing the rate of bond retirement to increase. At the same time, the retirement of non-federal bonds causes federal tax revenue to fall thereby causing the federal deficit to grow and the rate of monetary expansion to increase as well. The two models therefore describe two far different responses to the same disturbance and are not equivalent. To properly derive the influence of non-federal governments in a federation one needs to model a separate budget constraint for each level of government. Their influence cannot be ascertained from a model of a unitary government which employs mixed financing.

4. Full and Quasi- Equilibrium

The static equations (1)-(4) are satisfied at each moment in time while dynamic equations (5) and (6) drive the model from one instantaneous equilibrium to another by changing stocks of bonds and/or money. If the model is stable, it will settle to a position of equilibrium after it has been disturbed. Define such an equilibrium as being a position where national income and the rate of interest are unchanging (i.e. $\dot{Y} = \dot{r} = 0$). If we take the time derivative of the static equations and impose our equilibrium condition, we can derive the conditions the model's remaining endogenous variables must satisfy in order for this equilibrium to be established. These stationarity conditions require

$$(7) \qquad \dot{W} = \dot{M} = (\dot{B}^f + \dot{B}^n) = 0.$$

Note in particular that when both levels of government bond finance the attainment of equilibrium does not require all flows to be zero. One level of government may continuously issue new bonds (and thus maintain a budget deficit) so long as the other level of government continuously retires bonds (and thus maintains a budget surplus) at an equal rate. As B^f and B^n are perfect substitutes such a situation leaves wealth, interest income, the rate of interest, and the level of national income all constant. The use of this type of equilibrium has been employed elsewhere in open economy models of unitary states and has been labelled a quasi-equilibrium.[3]

A quasi-equilibrium cannot persist indefinitely. To our case of two bond financing levels of goverment in a closed economy one can extend a criticism similar to that directed by Riley (1982) and Scarth (1984) toward open economy models employing the quasi-equilibrium concept. That is, such a situation leads to a growing debt-to-income ratio for that level of government which is continuously issuing bonds. This must eventually give rise to a confidence problem and a disinclination to purchase these bonds. When this occurs the quasi-equilibrium collapses and there must be further adjustment toward a full equilibrium where all flows are zero. In our discussion below we consider the case of a quasi-equilibrium when both levels of government bond finance not because we disagree with these criticisms -- on the contrary we find them persuasive -- but because as we'll see, the results of the literature can only be derived from our model if such a definition of equilibrium is used.

5. The Results

Case (a): Non-Federal Governments Adjust Tax Rates Residually while the Federal Government Bond Finances

With respect to the question of the stability of bond financing this case most closely resembles the situation examined by Blinder and Solow (1973, 1976a, 1976b). Wealth effects which give rise to the possibility of instability emanate from a single source -- the change in the stock of federal bonds outstanding. As the tax financing of non-federal governments gives rise to no further wealth effects we should expect that the condition for stability in this case should closely resemble that derived by Blinder and Solow.

Under this set of financing assumptions only the federal GBC is dynamic describing how the stock of federal bonds outstanding changes over time in response to changes in the size of the federal deficit. Taking the linear approximation of the model about a balanced federal budget yields,

$$(8) \qquad \dot{B}^f = r\left[1 - T^f - T^f\left(\frac{C_y\,(1-T^f) + \frac{1}{r}\,\beta}{1 - C_y(1-T^f) + \sigma L_y}\right)\right]\, dB^f$$

where dB^f = the deviation of B^f from its full equilibrium value

$$\sigma = \frac{C_r - ((B^f + B^n)/r^2)C_w}{L_r - ((B^f + B^n)/r^2)L_w} > 0$$

$$\beta = C_w - \sigma L_w \gtrless 0.$$

The model is stable only if the term in square brackets is negative. Rewriting this term stability can be seen to be an empirical question requiring

$$(9) \qquad \beta > r\left(\frac{1 - T^f}{T^f}\right)\,(1 - C_y + \sigma L_y)$$

which is essentially the result derived by Blinder and Solow except for the definition of the tax parameter involved.

Blinder and Solow utilize only one GBC and define the tax parameter in that constraint (T') as the marginal propensity for federal, state, and local governments to tax and reduce income-conditioned transfer payments as GNP rises. To interpret T' in this manner however implies that their constraint is an aggregate constraint for all levels of government. When discussing money financing this implies that non-federal governments have access to money creation as a means of financing their deficits.[4] Alternatively we can interpret their model as simply wishing to ignore non-federal governments. If so then the proper assumption is to assume non-federal governments adjust their tax rates residually as their behaving in this manner does not alter the stability condition from what it would be if equation (6) was simply omitted from our model. Under this interpretation (9) shows that T' should be identified as the federal tax parameter only. This is an important consideration since a crucial element in judging the likelihood of (9) being satisfied is the magnitude of the tax parameter involved. Blinder and Solow have judged the magnitude of their T' to be quite large (in the vicinity of 0.5) making the satisfaction of (9) much more likely than if it were small.[5] Our result here indicates the relevant tax parameter is that of the federal government only thus making federal bond financing far less likely to be stable than Blinder and Solow have judged.[6]

If one accepts the argument of Christ (1979), this distinction regarding the proper tax parameter to enter (9) may seem somewhat irrelevant. That is, Christ (1979, p.533) suggests that casual empirical evidence indicates that aggregate demand is positively related to changes in the money supply. In the Blinder and Solow model (and ours) however, this relationship is satisfied under bond financing only if the parameters are such that the condition for bond finance stability fails. Thus Christ asserts bond financing must be unstable in that model. Recently however, Rose (1986) has stressed that if the government is bond financing, then there is no a priori reason to believe aggregate demand will rise as a result of a (bond financed) increase in the money supply. The monetary expansion creates a budgetary surplus initiating a retirement of outstanding bonds. If the Blinder and Solow model is stable, the net effect of the expansion of the money supply and the contraction of the number of bonds outstanding is to reduce aggregate demand in the long run. As Rose concludes, it is not clear why this result should be considered outrageous, thus leaving us with the conclusion that the stability of bond financing is an empirical question as Blinder and Solow claim.

Case (b): Both Levels of Government Bond Finance

The stationarity conditions show that under this set of financing assumptions a quasi-equilibrium is possible. This quasi-equilibrium requires only that the sum of the two government's budget imbalances be zero. The model is this case therefore reduces to equations (1)-(4) plus the sum of equations (5) and (6). Taking the linear approximation of this set of equations and substituting into the dynamic equation yields

$$(10) \qquad (\dot{B}^f + \dot{B}^n) = r[1 - (T^f + T^n)(1 + \lambda)]d(B^f + B^n)$$

where $\lambda = \dfrac{C_y(1-T^f-T^n) + \frac{1}{r}\beta}{1 - C_y(1-T^f - T^n) + \sigma L_y} \gtrless 0.$

This quasi-equilibrium is stable only if the sum of the two budget imbalances returns to zero after the model has been disturbed. Thus stability requires the term in square brackets to be negative which is equivalent to requiring

$$(11) \qquad \beta > r\left(\frac{1 - T^f - T^n}{T^f + T^n}\right)(1 - C_y + \sigma L_y).$$

Note that (11) is precisely the condition for stability discussed by Blinder and Solow as it involves the tax parameter of both levels of government. Therefore another way one could interpret their result is that they believe non-federal governments bond finance and they require only a quasi-equilibrium in the steady state. There is however no evidence to suggest they intended this interpretation.

Now suppose one agrees with criticisms of the use of the quasi-equilibrium concept and requires a full equilibrium instead. The conditions for stability in a full equilibrium are more stringent than those for a quasi-equilibrium because in a full equilibrium both budget deficits must be zero so that $\dot{B}^f = \dot{B}^n = 0$. Taking the linear approximation of (1)-(6) about balanced budgets and substituting the result into the dynamic equations yields

$$(12) \qquad \begin{bmatrix} \dot{B}^f \\ \dot{B}^n \end{bmatrix} = r\begin{bmatrix} 1 - T^f(1+\lambda) & -T^f(1+\lambda) \\ -T^n(1+\lambda) & 1 - T^n(1+\lambda) \end{bmatrix}\begin{bmatrix} dB^f \\ dB^n \end{bmatrix}.$$

The model is stable if the trace and determinant of the 2 x 2 matrix in (12) are negative and positive respectively. However, since

$$\text{trace} = 2 - (T^f + T^n)(1 + \lambda)$$

$$\text{determinant} = 1 - (T^f + T^n)(1 + \lambda)$$

satisfaction of both these conditions is impossible and the model is unambiguously unstable in a full equilibrium sense when both levels of government bond finance. The intuition behind this result is fairly straightforward. A federal deficit arising due to an expansionary federal fiscal policy causes income to rise and thus puts non-federal governments into a surplus position. The federal government is now issuing B^f while non-federal governments are retiring B^n. Every B^f issued raises wealth and income causing the non-federal surplus to increase. Similarly, every B^n retired reduces wealth and income causing the federal deficit to grow. The two levels of government are therefore acting at cross purposes each making it more difficult for the other to return to a balanced budget position.

Case (c): Non-Federal Governments Adjust Tax Rates Residually while the Federal Government Money Finances

In this case bond stocks are unchanging and the only dynamic equation is (6) describing changes in the stock of money over time as a function of the size of the federal deficit. By assuming non-federal governments are avoiding deficit financing we should again derive a stability condition similar to that derived by Blinder and Solow. Taking the linear approximation of equations (1)-(6) and substituting into the dynamic equation yields

$$(13) \qquad \dot{M} = -T^f \left[\frac{\alpha}{1 - C_y(1-T^f) + \sigma L_y} \right] dM$$

where $\alpha = C_w + (1 - L_w)\sigma > 0$.

Stability requires the term in square brackets to be positive, a condition which is unambiguously satisfied and identical to that derived by Blinder and Solow if their T' is interpreted as the federal tax parameter only.

As stability is guaranteed in this case it is relevant to examine the full equilibrium policy multipliers. The most interesting of these are

$$dY/dG^f = 1/T^f$$

and

$$dY/dG^n = 0.$$

The federal expenditure multiplier is a familiar result. Interestingly however, the effect on income of a change in non-federal expenditures is zero. Any expansionary non-federal policy produces a federal surplus (by raising federal tax revenue) which initiates a contraction of the money supply. With expenditures constant the federal surplus will disappear only when tax revenues return to their initial level. Thus in equilibrium the level of national income must return to its initial level and non-federal policy initiatives are consequently ineffective.

Case (d): Non-Federal Governments Bond Finance while the Federal Government Money Finances

Under this set of assumptions both government budget constraints are dynamic but inspection of the stationarity conditions shows that no quasi-equilibrium is possible. Thus both budget deficits must be zero in equilibrium. Taking the linear approximation of equations (1)-(6) and substituting into the dynamic equations yields

$$\text{(14)} \quad \begin{bmatrix} \dot{B}^n \\ \dot{M} \end{bmatrix} = \begin{bmatrix} 1 - T^n(1+\lambda) & -T^n\theta \\ -T^f(1+\lambda) & -T^f\theta \end{bmatrix} \begin{bmatrix} dB^n \\ dM \end{bmatrix}$$

where $\theta = \dfrac{\alpha}{1 - C_y(1-T^f-T^n) + \sigma L_y} > 0.$

Stability requires that the trace and determinant of the 2 x 2 matrix in (14) to be negative and positive respectively. The determinant however is unambiguously negative so that the model is unstable under this set of financing assumptions.

The intuition behind this result is again fairly straightforward and was outlined earlier in section 3. A policy initiative creating a deficit for that level of government raises income and thereby puts the other level of government into surplus. The financing responses of the two levels of government are now at cross purposes with one another. An increase in the money supply in response to a federal deficit increases the non-federal surplus and hence the rate at which B^n are retired. But an increased rate of retirement for B^n widens the federal deficit calling forth an increased rate of monetary expansion. Clearly it is impossible for both levels of government to return to balanced budgets and the model must be unstable.

6. An Alternative Definition of the Fiscal Policy Variable

In the previous section we assumed governments used their expenditures on goods and services (G) as the exogenous fiscal policy variable. Tobin and Buiter (1976), Christ (1979), and Cohen and deLeeuw (1980) propose a number of alternative definitions of what might describe a government's fiscal policy variable. On the basis that it seems to receive the explicit attention of policy-makers, total government expenditures (G plus gross interest payments on government debt, B) would appear to be a reasonable choice. Christ finds that in the Blinder and Solow (1973) model this definition of the fiscal policy variable makes bond financing stable so long as one assumes that a change in the money supply has a positive effect on aggregate demand in the long run. In Christ's view then, this alternative definition of the fiscal policy variable offers a solution to the problem of bond finance instability -- so long as government defines fiscal policy as maintaining total expenditures above (or below) some previous level, bond financing will be stable.

As noted earlier, we concur with Rose (1986) that there is no a priori reason to believe a bond financed change in the money supply will have a positive effect on aggregate demand in the long run. It is easy to show that in our model, as in the Blinder and Solow model, if the relationship is a negative one then bond financing is necessarily unstable using this alternative definition of the fiscal policy variable whereas it remained an empirical question when the fiscal policy variable was simply G. In our view then, this alternative definition of the fiscal policy variable may in fact increase the likelihood of bond finance instability. In what follows however, we will assume Christ is right -- that is, a bond financed change in the money supply will have a positive effect on aggregate demand in the long run. Even with this assumption, we will show that in a federal economy this alternative definition of the fiscal policy variable does not guarantee bond finance stability.

To examine the effect of this redefinition of the fiscal policy variable on our model of a federal economy, simply define $G^{f*} = G^f + B^f$ and $G^{n*} = G^n + B^n$ and substitute for G^f and G^n respectively in equations (1)-(6). As expected, given Christ's assumption case (a) is now stable. However, if we require a full equilibrium of case (b), the model remains unstable despite Christ's assumption. We derive the same result for case (d) so that stability of money financing continues to require that non-federal governments avoid deficit financing.[7]

In a federal economy therefore, even given Christ's assumption regarding the long run effect of a bond financed change in the money supply on aggregate demand, this alternative definition of the fiscal policy variable fails to solve the problem of bond finance instability except for the unrealistic case of tax financing non-federal governments.

7. Summary and Conclusions

The purpose of this essay was to extend the seminal work of Blinder and Solow (1973) to the case of a federal economy and to examine how that model's conclusions regarding the stability of deficit financing were affected by that extension. Our results suggest that concern should be expressed over any heavy reliance on bond financing and that this concern should be directed toward both levels of government.

Consider first the results from section 5. Our discussion of cases (a) and (b) indicates that the result of Blinder and Solow (1973) that the stability of bond financing is an empirical question can only be derived if non-federal governments tax finance (in which case Blinder and Solow interpret their T' incorrectly) or if non-federal governments bond finance and we accept quasi-equilibrium as a proper definition of a steady state. This surely shows that their result rests on a shaky foundation as quasi-equilibrium is simply not an attractive concept and non-federal governments do not typically maintain balanced budgets.[8] When we make the more realistic assumption that non-federal governments deficit finance and require a full equilibrium we find the proper conclusion is that bond financing is unambiguously unstable and is not an empirical question at all.

The federal government has the alternative of abandoning the monetary growth rule and instead rely on changes in the money supply to finance federal deficits. Cases (c) and (d) however, show that money financing by the federal government is only stable if non-federal governments avoid bond financing. Thus the general perception that money financing produces a stable equilibrium (see for example Blinder and Solow (1973), Christ (1979), Rau (1985)) is seen to rest heavily on an implicit assumption that non-federal governments adjust tax rates residually so as to always maintain zero budget deficits. Our concern over a reliance on bond financing by non-federal governments is therefore most strongly expressed here. Bond financing by non-federal governments makes unstable the federal government's only realistic alternative to bond financing.

In Section 6 we examined an alternative definition of the fiscal policy variable. We showed that this re-definition of the fiscal policy variable solved the problem of bond finance instability only if one accepts Christ's assumption regarding the long run impact of a bond financed change in the money supply on aggregate demand <u>and</u> if one assumes non-federal governments never deficit finance. Dropping either of these assumptions again leaves us with our conclusion that bond financing in a federal economy generates macroeconomic instability.

Finally, our results are indicative of the significance of policy interdependence between levels of government in a federation. Success of policy initiatives by one level of government depend to a significant degree on the policy choices made by the other. This is reflected in our stability analyses where we found successful stabilization efforts required that <u>both</u> levels of government avoid heavy reliance on bond financing. It is reflected also in the result that a fully coordinated tax financed change in expenditures by all non-federal governments has no affect on national income if the federal government chooses to money finance shocks to its budget position.

FOOTNOTES

[1] On this see Courchene (1986, pp.54-55). In return for being exempted from the 1963 U.S. Interest Equalization Charge, Canada was required to ensure that its foreign exchange reserves not exceed $2.6 billion. By the mid-1960s however, they were surpassing this level. A possible solution to the problem was for the Bank of Canada to use these "excess" reserves to retire outstanding Canadian debt held abroad. However, as most of this debt was of provincial issue, the Bank refrained from purchasing this debt for fear that it might lead to future demands that it monetarize provincial debt. Thus while provincial debt could conceivably be monetarized, the Bank of Canada has studiously avoided doing so even indirectly.

[2] In Turnovsky's model the assumption of $\delta > 1$ makes it unambiguously stable.

[3] For a discussion of full versus quasi-equilibrium see Swoboda (1972) and Fratianni (1976). Models employing quasi-equilibriums include Turnovsky (1976), Witte (1979), Murray (1980), Riley (1982).

[4] To avoid this problem Christ (1979) makes explicit the assumption that his GBC represents that of the federal government only and he assumes non-federal governments are included in the private sector. As our discussion in section 2 shows however, this is not a legitimate method of dealing with the existence of non-federal governments.

[5] In their 1973 paper Blinder and Solow judge $T' > 0.50$. In their 1976b paper T' is no longer viewed as being in excess of one-half but is still defined as the tax parameter in an aggregate GBC and is thus still viewed as being quite large.

[6] For example, suppose the tax parameter for all governments combined (Blinder and Solow's T') is 0.5. but that of the federal government alone (our T^f) is only 0.25. Then stability is three times less likely when the proper tax parameter is used.

[7] In both case (d) and in case (b) when a full equilibrium is required, there are two dynamic equations and stability requires satisfaction of trace and determinant conditions. In each case trace $\gtrless 0$ and determinant = 0.

[8] From the table below we see that in Canada and the U.S. non-federal governments do not typically maintain balanced budgets and in fact often realize imbalances which are large even relative to those of the federal government.

Deficits (-) and Surpluses (+) by Level of Government, Selected Years

	Canada*		U.S.**	
	Federal	Non-Federal	Federal	Non-Federal
1973	0.4	-0.8	- 14.9	14.9
1976	- 3.3	-2.4	- 73.7	6.8
1979	- 9.4	1.1	- 40.2	26.5
1982	-25.1	-4.7	-127.9	34.8
1983	-20.4	-4.9	-207.8	43.7
1984	-30.5	-1.9	-185.3	63.9
1985	-32.3	-2.2	-212.3	60.9

* billions of Canadian dollars
** billions of U.S. dollars

Sources: Canada: National Income and Expenditure Accounts, 13-001.
U.S.: Budget of the U.S. Government, Fiscal Year 1988, Historical Tables, U.S. Gov't. Printing Office, 1987.

Chapter Three

The Macroeconomic Stability of Alternative Monetary Policies in a Federal Economy with Non-Zero Growth and Flexible Prices

1. Introduction

In this essay we relax the restrictive assumptions of fixed prices and zero economic growth imposed on the model discussed in chapter two. In doing so, we will first construct a model with zero growth but flexible prices so as to better compare the specification of our model with many of those appearing in the literature. After making this comparison, we will add economic growth to give us the final form of our model. Growth will be assumed to be constant and exogenously determined. In this we follow the methodology of a number of recent studies designed to investigate the conditions under which deficit financing leads to a stable equilibrium. These include the theoretical models of Sargent and Wallace (1981), McCallum (1981), Darby (1984), and Scarth (1987a, 1987b) and the simulation models of Fortin (1985) and Duguay and Rabeau (1987). Also, we will construct our model so as to allow us to investigate the implications for stability of a variety of assumptions regarding whether or not government bonds are net wealth. This remains a highly contentious issue and since stability conditions are sensitive to the particular assumption one imposes it would seem important to allow for this flexibility. Finally, we will model the non-federal government sector so as to investigate its role in determining conditions for macroeconomic stability.

2. The Model

In this section we develop a deterministic, flexible price IS-LM model of a closed federal economy with a constant, exogenously determined rate of growth and rational economic agents. Our goal is to specify a model flexible enough that we will be able to examine the implications of as many of the specifications appearing in the literature as possible. In order to better illustrate how we have done so, we begin by specifying a model with zero economic growth. This model consists of the following eight equations.

$$(1) \quad Y = C(Y^d, r-\pi, W) + (1-\alpha)(G^f + G^s)$$

$$(2) \quad W = M/P + \beta\ (B^f/P + B^s/P)$$

$$(3) \quad Y^d = Y + r\ (B^f/P + B^s/P) - (t^f + t^s)(Y + rB^f/P + rB^s/P)$$
$$- \pi W + \alpha(G^f + G^s) - (1-\beta)(1/P)(\dot{B}^f + \dot{B}^s)$$

$$(4) \quad M/P = L(Y, r, W)$$

$$(5) \quad p = \theta(Y - \bar{Y}) + \pi$$

$$(6) \quad \pi = p$$

$$(7) \quad (1/P)(\dot{M} + \dot{B}^f) = G^f + rB^f/P - t^f(Y + rB^f/P + rB^s/P)$$

$$(8) \quad (1/P)\dot{B}^s = G^s + rB^s/P - t^s(Y + rB^f/P + rB^s/P)$$

where: $Y, \bar{Y}$ = real output and real capacity output

r = nominal interest rate

π = expected rate of inflation

P = price level

p = $\dot{P}/P$ = rate of inflation

$r-\pi$ = expected real interest rate

B^f, B^s = nominal value of outstanding federal and non-federal bond stocks respectively

rB^f, rB^s = nominal interest paid on federal and non-federal bonds respectively

M = nominal money supply

Y^d = real private disposable income

G^f, G^s = real federal and non-federal government expenditures on goods and services respectively

t^f, t^s = federal and non-federal tax rates

β = the fraction of government bonds which private agents perceive to be net wealth

α = the fraction of G^f and G^s which private agents perceive to be equivalent to private expenditure

$L(\cdot)$ = real demand for money balances

$C(\cdot)$ = real expenditure enjoyed by private agents

θ = rate of which inflation rises due to excess demand in the product market ($\theta > 0$)

W = private agents subjective value of real net wealth.

Following convention we denote time derivatives with a dot and partial derivatives with subscripts. We assume partial derivatives have the usual signs;

$0 < C_{yd} < 1$ $\qquad C_{r-\pi} < 0$ $\qquad C_w > 0$

$L_y > 0$ $\qquad L_r < 0$ $\qquad 0 < L_w < 1$

Equation (1) is the GNP identity ($Y = C + G^f + G^s$) after substituting the following equation for C;

$$(9) \quad C = C(Y^d, r-\pi, W) - \alpha(G^f + G^s)$$

where C = real market purchases by private agents.

This specification follows Hodrick (1980) and Barro (1981). It reflects an assumption that the public is likely to perceive at least some fraction of government expenditure as being a substitute for private expenditure (for example, expenditures on non-public goods such as education, fire protection, school lunches, etc.). Thus real expenditures enjoyed by private agents (defined by $C(\cdot)$) is given by the sum of real market purchases by private agents (C) and real expenditures by government that the public perceives to be a substitute for private expenditures ($\alpha(G^f + G^s)$). For simplicity, we assume the same fraction of federal and non-federal government expenditures are perceived to be substitutes for private expenditure and we also assume α is a constant and $0 \leq \alpha \leq 1$.[1] Substitute (9) into the GNP identity and we have equation (1).

Equation (2) defines net wealth. If $\beta = 0$ we impose what has been variously referred to as the Ricardian equivalence, the Ricardian non-equivalence, and the pre-Ricardian equivalence theorem or proposition.[2] This proposition, which we will refer to as simply the equivalence hypothesis, states that government bonds are not perceived to be net wealth since they entitle agents to both interest payments and a future tax liability with the same present value. If we let $\beta = 1$ we adopt the assumption implicitly imposed in our first essay -- that agents fail to associate a bond issue with a future tax liability and hence perceive the full value of government bonds to be a component of net wealth. McCallum (1984) notes that the equivalence hypothesis is merely a particular application of the rational expectations hypothesis and suggests that since the latter has been shown to have considerable merit the former should also be adopted. The equivalence hypothesis however, requires the satisfaction of assumptions other than that of rational price expectations so that adoption of this assumption does not necessarily require one to assume $\beta = 0$. For example, a necessary assumption for the $\beta = 0$ assumption to be relevant is that private agents have infinite lives or satisfy a bequest motive strong enough to cause them to behave as if they have infinite lives.[3] Empirical studies have provided support both for and against acceptance of the equivalence hypothesis. Seater (1985) offers a review of these empirical studies and notes that a serious shortcoming is that non-federal government debt is ignored despite the fact that it comprises a large part of total U.S. government debt. Thus the opposing conclusions of two recent studies using Canadian data may be due to their choices of whether to include non-federal debt in their calculations. Johnson (1986) finds little support for the equivalance hypothesis but includes only federal debt in his calculations. Katsaitis (1987), on the other hand, includes non-federal debt in his calculations and finds evidence supporting the equivalence hypothesis. In recognition of the controversy regarding the equivalence hypothesis we have constructed our model so as to be able to accommodate any assumed value for β. Thus we assume $0 \leq \beta \leq 1$.[4]

Equation (3) defines real disposable income as income from production plus interest income earned on government bonds, minus taxes (which includes the inflation tax on wealth), plus the value private agents place on government expenditures, minus the savings perceived as being necessary to finance the implied future tax liability inherent in new issues of government bonds.[5] This specification is a generalized version of closed economy definitions of disposable income defined elsewhere. That is, let $\beta = 1$ and $\alpha = 0$ and we have the definition employed by Turnovsky (1979) and by Stemp and Turnovsky (1984) for a unitary state. Let $\beta = 1$, $\alpha = 0$ and also assume prices are fixed and we are left with the definition employed by Blinder and Solow (1973). Still assuming fixed prices and $\alpha = 0$, if we let $0 \leq \beta \leq 1$ we have disposable income as defined by Bruce (1977). If we allow prices to be flexible and assume $\alpha = \beta = 0$, it is easy to show that our equation (3) is equivalent to the definition of disposable income employed by McCallum (1978).[6] Finally, with flexible prices, $\alpha = 0$, and $0 < \beta < 1$, we have the definition employed by Duguay and Rabeau (1987).

Equation (4) is the LM equation requiring that the real money supply equal real demand for money balances. The determinants of money demand are standard ones with the usual signs.

Equation (5) is an expectations augmented Phillips curve where the unity coefficient on the expected inflation term indicates acceptance of the "natural rate" hypothesis.

Equation (6) imposes the assumption of perfect foresight -- the deterministic analogue of rational expectations. Taylor (1985) notes that while rational expectations may not be an appropriate assumption if one is interested in the short-run impact of a change in policy regimes (since agents may require a period of time in which to learn about the new policy and hence may fail to anticipate policy initiatives), it is appropriate if one measures long-run effects. As our concern will be to measure only the long-run effects of policy, we have adopted the assumption of rational expectations. Together, equations (5) and (6) imply output is pegged at capacity output $\bar{Y}$.

Equations (7) and (8) are the government budget constraints (GBC's) of the federal and non-federal government sectors respectively. Both require that uses of funds equal sources of funds at all times. They differ only in that the federal government has a revenue source not available to non-federal governments -- that of money creation. Note that the variable M enters the federal GBC and thus represents high-powered money and also enters the wealth identity (2) and the LM equation (4) so that it also represents the total money supply. This is the usual assumption in GBC models and it implies either that there is no banking system or that the money multplier is unity.

The model we have described above assumes zero economic growth. We have presented this model in order to better illustrate the implications of allowing for the possibilities of $0 \leq \beta,\ \alpha \leq 1$ for a standard GBC macroeconomic model. However, McCallum (1981) has noted that if one assumes Ricardian equivalence (that is, $\beta = 0$) then a necessary condition for macroeconomic stability under a monetary rule (defined below) is a positive rate of economic growth. Since the $\beta = 0$ assumption has been a popular one recently, we wish to examine it for the case of a federal economy. In order to make this examination worthwhile however, McCallum's result suggests we must modify our model to allow for a positive rate of economic growth. To do so, we will follow

McCallum (1981), Sargent and Wallace (1981), Darby (1984), Fortin (1985) and Scarth (1987b) in assuming a constant exogenously determined rate of growth in output. To incorporate this assumption we simply divide through the model by Y. Further simplifying by substituting equations (4)-(6) into the rest of the model and assuming the private expenditure and money demand functions are linear, our model becomes:

$$(10) \quad 1 = C_1 y^d + C_2(r-p) + C_3(m + \beta(b^f + b^s)) + (1-\alpha)(g^f + g^s)$$

$$(11) \quad y^d = (1 + rb^f + rb^s)(1-t^f-t^s) - p(m + b^f + b^s) + \alpha(g^f + g^s) - (1-\beta)(\dot{b}^f + \dot{b}^s) - n(b^f + b^s)(1-\beta)$$

$$(12) \quad m = L_1 + L_2 r + L_3(m + \beta(b^f + b^s))$$

$$(13) \quad \dot{m} + \dot{b}^f = g^f + rb^f - t^f(1 + rb^f + rb^s) - (m + b^f)(n + p)$$

$$(14) \quad \dot{b}^s = g^s + rb^s - t^s(1 + rb^f + rb^s) - b^s(n + p)$$

where;[7]

$y^d = Y^d/Y$		$C_1 = C_{yd}$
$g^f = G^f/Y$		$C_2 = C_{r-\pi}/Y$
$g^s = G^s/Y$		$C_3 = C_W$
$m = M/PY$		$L_1 = L_Y$
$b^f = B^f/PY$		$L_2 = L_r/Y$
$b^s = B^s/PY$		$L_3 = L_W$

$n = \dot{Y}/Y$ = exogenous constant

and where we have made use of the following relationships;

$$\dot{b}^f = (1/PY)\ \dot{B}^f - b^f(n + p)$$

$$\dot{b}^s = (1/PY)\ \dot{B}^s - b^s(n + p)$$

$$\dot{m} = (1/PY)\ \dot{M} - m\ (n + p).$$

Equations (10)-(14) now describe a fairly general model of a closed federal economy with rational economic agents and a constant, exogenously determined rate of growth in real output (n). The endogenous variables in the model are y^d, r, p, one of b^f, m, t^f, g^f, and one of b^s, t^s, g^s. This categorization of endogenous variables reflects the fact that at least one variable in each GBC must be endogenous so as to ensure satisfaction of these constraints at all times and the assumption that both levels of government choose to allow only one variable in their GBC to be endogenously determined. Thus we do not consider here more complex modes of financing where two or more policy variables are allowed to vary in certain proportions.[8]

3. Alternative Monetary Policies

The list of endogenous variables defined above indicates that even given our assumption that only one policy variable in each GBC is endogenous at any one time, there are twelve possible combinations of endogenous policy variables between the two levels of government. However, we ignore as being unrealistic those cases involving expenditure financing (g^f and g^s endogenous) and cases involving federal tax financing (t^f endogenous). This leaves us with just two financing options available to each level of government.

One option available to the federal government is to adopt a monetary rule such that $\dot{m} = 0$. Under such a rule the real money supply is allowed to grow but only at a rate sufficient to keep the ratio of money to output constant. Underlying this form of the monetary rule is the notion of an "accommodating" monetary policy in which the monetary authorities increase the nominal money supply to meet transactions needs which grow at a rate equal to the sum of the rates of output growth and inflation. This particular form of the monetary growth rule has been widely used in the literature having been adopted by Sargent (1977), Turnovsky (1979), McCallum (1981), Nguyen and Turnovsky (1983) and Scarth (1987b). An implication of this rule of course is that b^f is endogenous (i.e. $\dot{b}^f \neq 0$). Thus following such a rule implies the federal government has chosen to bond finance federal deficits.

The second option open to the federal government is to adopt an accommodating bond growth policy such that $\dot{b}^f = 0$. Here the stock of federal bonds is allowed to vary so as to maintain a constant federal bond to output ratio. An implication of this policy is that m is endogenous (i.e. $\dot{m} \neq 0$) so that adoption of this policy implies the federal government has chosen to money finance federal deficits.

There are also two financing options open to the non-federal government sector. One option we will consider involves non-federal governments adopting an accommodating bond growth policy such that $\dot{b}^s = 0$. The implication of this policy of course is that non-federal governments tax finance (t^s endogenous) deficits. A major reason for our interest in this case is that as we'll show below it is an implicit assumption of those who model unitary states (i.e. only consider one level of government) but offer policy advice for federal states. In order to discuss their results therefore, we need to consider this case.

The second financing option available to the non-federal government sector is that of deficit (bond) financing so that b^s is endogenous ($\dot{b}^s \neq 0$) and the non-federal tax rate is given. The macroeconomic effect of this option, which is easily the most common choice of non-federal governments, has not been examined in the literature.

4. Macroeconomic Stability under a Monetary Growth Rule

In this section we assume the federal government has adopted the monetary rule we described above. Setting $\dot{m} = 0$ and substituting (13) and (14) for $\dot{b}^f$ and $\dot{b}^s$ respectively, equation (11) becomes,

$$
\begin{aligned}
(15)\quad y^d = 1 - (1-\alpha)(g^f + g^s) + mn(1-\beta) \\
+ \beta[g^f + g^s + r(b^f + b^s) - (t^f + t^s)(1 + rb^f + rb^s) \\
- (m + b^f + b^s)p].
\end{aligned}
$$

The model we consider in this section therefore consists of static equations (10), (12), and (15), and asset accumulation identities (13) (with $\dot{m} = 0$) and (14).[9] Note that if we assume Ricardian equivalence ($\beta = 0$) the model dichotomizes into two blocks -- the static equations on the one hand and the GBC's on the other -- since variables t^f, t^s,

b^f, and b^s appear only in the latter. This is precisely the result of McCallum (1978, 1981) and it allows one to specify only GBC's in order to examine the conditions necessary for convergence under a monetary rule.

The static equations can be solved for r and p as functions of b^f, b^s, and exogenous variables;

$$r = F(b^f,\ b^s,\ \text{exogenous variables})$$

$$p = H(b^f,\ b^s,\ \text{exogenous variables})$$

where the partial derivatives of these functions are;

$$F_{b^f} = F_{b^s} \equiv F_b = -\beta L_3/L_2 \geq 0$$

$$H_{b^f} = H_{b^s} \equiv H_b =$$

$$= \beta\ \frac{L_2(C_3+C_1(r(1-t^f-t^s)-p))-L_3(C_2+\beta C_1(1-t^f-t^s)(b^f+b^s))}{L_2(C_2\ +\ \beta C_1(m+b^f+b^s))} \gtrless 0$$

The ambiguity of the sign of H_b is due to the fact that a ceteris paribus increase in b^f and/or b^s causes both a rightward shift in the IS curve due to the implied increase in personal income and the increase in net wealth, and a leftward shift in the LM curve also due to the increase in net wealth (for a further discussion of these effects, see Rau (1985)). All these influences require that $\beta \neq 0$ so that $H_b = 0$ otherwise.

(i) Non-Federal Tax Financing ($\dot{b}^s = 0$):

We consider first the case where non-federal governments tax finance (i.e. maintain a constant non-federal debt to output ratio) in the face of a federal monetary growth rule. Our interest in this case stems mainly from the fact that the results of the literature can only be derived for the case of a federal economy if one makes this assumption. The purpose of this sub-section is therefore to prove this assertion and to review the literature on the question of the stability of a monetary rule.

With $\dot{b}^s = 0$, equation (14) becomes a static equation and (13) remains as the only dynamic equation.[10] Differentiating (13) we see that convergence to a constant federal debt to output ratio requires

$$(16)\quad \frac{\partial \dot{b}^f}{\partial b^f} = -[n - (r(1-t^f) - p)] + [b^f(1-t^f) - t^f b^s]F_b - (m+b^f)H_b < 0.$$

If one assumes an equivalence between bonded debt and taxes (i.e. $\beta = 0$), then $F_b = H_b = 0$ and the necessary and sufficient condition for convergence becomes simply

$$(17)\quad n > r(1 - t^f) - p.$$

This is precisely the result of McCallum (1981) and Darby (1984). This is an important result as it implies that if (17) is not satisfied, then any federal deficit must eventually be monetarized and bond financing is simply not a feasible long-term solution to the financing requirements of the (federal) government.

Concern over whether or not the condition that the real growth rate exceeds the real after-tax interest rate is satisfied has become widespread even though it is not always made clear by those expressing this concern that the condition is only necessary and sufficient for convergence if one makes the restrictive assumption that $\beta = 0$. To see this, let $\beta > 0$ so that at least a fraction of government debt is perceived as net wealth. Assuming that $[b^f(1-t^f) - t^f b^s] > 0$ (if we let $t^f = 0.25$, then this condition is satisfied so long as $b^s < 3b^f$ -- a condition we assume is easily satisfied in federal economies), inspection of (16) shows that if $H_b > 0$ then (17) is neither a necessary nor a sufficient condition for convergence.

The intuition behind this result is as follows. Convergence requires that tax revenues rise more quickly than interest payments on bonded debt, otherwise a bond financed deficit will be explosive. There are two sources of new tax revenue available to the federal government -- a growing tax base and inflation. The assumption of the equivalence hypothesis implies tax revenue from inflation is unaffected by federal debt policy (i.e. $H_b = 0$). Thus the growth in the tax base alone must be sufficient to cause convergence. This condition is given in (17). If, however, $\beta > 0$, then the inflation tax is now sensitive to debt policy. It is now possible that even if (17) fails so that n alone is not sufficiently large to cause convergence, this plus the inflation tax may be sufficient. Such a possibility is described by (16) and requires $H_b > 0$.

This result is consistent with the analytical results of those who employ zero growth models (see for example, Turnovsky (1979), Stemp and Turnovsky (1984), Rose (1986), and for an open economy Basevi and Giavazzi (1986)) since with $n = 0$ these models necessarily violate (17) and yet these researchers maintain convergence remains a possibility. In each case the necessary condition for convergence is that $H_b > 0$ (which of course requires $\beta > 0$).[11] It is also consistent with the simulation results of Turnovsky and Nguyen (1980) who find that a monetary rule is stable for 59% of their parameter sets despite their assumption of zero growth. Finally, we interpret this result as being consistent with Miller and Sargent (1984) who assume a positive rate of growth. They indicate that the assumption that r is independent of budget policy should not be taken seriously and that in models where r is in fact dependent on budget policy there is much more involved than simply comparing growth rates and after-tax real interest rates. From our expression for F_b above, $\beta > 0$ is necessary for what Miller and Sargent identify as the more realistic case of $F_b > 0$. Scarth (1987b) comes to a similar conclusion using a model with a positive rate of growth and $\beta > 0$. That is, he finds the condition in (17) to be helpful for establishing convergence but that it is not sufficient.

Our results derived here under the assumption that non-federal governments tax finance are therefore consistent with the existing literature which assumes a unitary state. It is therefore apparent that since the literature offers policy advice to governments of federal states, it implicitly assumes non-federal governments choose to tax finance all disturbances to their budget positions.

(ii) Non-Federal Bond Financing ($\dot{b}^s \neq 0$):

In this sub-section we make a more realistic assumption regarding the behaviour of non-federal governments. That is, we assume non-federal governments allow the non-federal debt to output ratio to be endogenously determined by bond financing disturbances to their budget positions.

As we discussed in our first essay we can define two conditions for stability given this set of financing assumptions. One definition simply requires that the sum of the federal and the non-federal budget deficits equals zero in equilibrium. As federal and non-federal bonds are perfect substitutes for one another, if one level of government continuously issues new bonds while the other continuously retires old bonds at an equal rate, then no other macroeconomic variables will be affected and the economy settles to a "quasi-equilibrium" where only b^f and b^s continue to vary. To examine this case add (14) to (13) and define $b = b^f + b^s$. Convergence of the sum of b^f and b^s to a new steady state level requires that

$$\frac{\partial \dot{b}}{\partial b} = -A < 0$$

where $A \equiv [n - (r(1-t^f-t^s)-p)] - b(1-t^f-t^s)F_b + (m+b)H_b \gtrless 0.$

Here we get a result very similar to that discussed in part (i) above. That is, if $\beta = 0$ then convergence requires $n > r(1-t^f-t^s)-p$ which is a somewhat easier condition to satisfy than (17) due to the extra tax parameter. If $\beta > 0$ however, this condition is neither necessary nor sufficient for convergence.

This result is of limited usefulness because the quasi-equilibrium one obtains should A be positive is likely to be short-lived. That is, eventually the debt to output ratio of that level of government which is continuously issuing new bonds must reach a level such that a crisis of confidence develops causing private economic agents to stop their purchases of these bonds. When this occurs there must occur a further adjustment to a full equilibrium where both levels of government maintain a constant debt to output ratio.

Such a full equilibrium is the second definition of equilibrium possible, given this set of financing assumptions. Here we require that both levels of government adjust to a constant debt to output ratio in equilibrium. In order to examine the question of stability we therefore must use the two dynamic equations separately. The characteristic equation of this system of dynamic equations can be written as

$$(18)\qquad \begin{bmatrix} \partial\dot{b}^f/\partial b^f - \lambda & \partial\dot{b}^f/\partial b^s \\ \partial\dot{b}^s/\partial b^f & \partial\dot{b}^s/\partial b^s - \lambda \end{bmatrix} = 0$$

where λ = a characteristic root of the characteristic equation, and where differentiation of (13) and (14) yields $\partial\dot{b}^f/\partial b^f$ as defined in (16) and;

$$\partial\dot{b}^f/\partial b^s = -rt^f + [b^f(1-t^f) - t^f b^s]F_b - (m+b^f)H_b$$

$$\partial\dot{b}^s/\partial b^f = -rt^s + [b^s(1-t^s) - t^s b^f]F_b - b^s H_b$$

$$\partial\dot{b}^s/\partial b^f = -[n - (r(1-t^s)-p] + [b^s(1-t^s) - t^s b^f]F_b - b^s H_b.$$

Expansion of (18) and use of the Routh Theorem derives the following two necessary and sufficient conditions for convergence;

(19) $[n - (r-p)]\ A > 0$

(20) $[n - (r-p)] + A > 0.$

Satisfaction of (19) requires that A and $[n - (r-p)]$ be of the same sign. Satisfaction of (20) requires that these terms both be positive. Thus convergence to a full equilibrium requires $A > 0$ and

(21) $n > r-p.$

There are two important conclusions to be drawn from this result. First, (21) is a necessary condition for convergence <u>regardless of the value of β</u>. If $\beta = 0$, (21) is necessary and sufficient since then $A = n - (r(1-t^f-t^s)-p)$ and (21) is an unambiguously more stringent condition than $A > 0$. If $\beta > 0$, (21) is only a necessary condition since $A > 0$ is also required for convergence.

We view the result that (21) is necessary for convergence regardless of the value of β as an important result for the following reason. McCallum (1984) identifies the monetarist hypothesis as saying bond financed deficits have no effect on aggregate demand. As McCallum notes, the main intellectual support for this proposition is the equivalence hypothesis. Thus, McCallum follows Bruce (1977) and Tobin (1980) in identifying the monetarist hypothesis with the assumption that $\beta = 0$. Adopting this criterion for judging whether one is a monetarist or a non-monetarist, we see the importance of the result that (21) is a necessary condition for convergence regardless of the value of β. That is, when non-federal governments tax finance (or are implicitly assumed to be tax financing), the discussion of Darby (1984) regarding the likelihood of (17) being satisfied might appear rather uninteresting to non-monetarists as (17) is neither necessary nor sufficient for convergence when $\beta > 0$ as non-monetarists believe. If, however, non-federal governments bond finance, monetarists and non-monetarists alike must judge the more stringent condition in (21) to be of crucial importance in establishing the likelihood of convergence under a monetary rule. In discussions of a federal economy with deficit financing non-federal governments one need not therefore, be a monetarist to be concerned about the relative magnitudes of real growth rates and real interest rates.

The second important conclusion to be drawn from the result that (21) is necessary for convergence is that (21) is a significantly more stringent condition that is (17). The intuition behind this finding is fairly straightforward. Suppose $\beta = 0$ so that (17) and (21) are necessary and sufficient conditions for convergence for the cases of non-federal tax and bond financing respectively. Now suppose both levels of government are bond financing and the federal government is running a deficit. With $\beta = 0$, $\partial\dot{b}^S/\partial b^f < 0$ so that every bond issued by the federal government reduces the deficit (or adds to the surplus) of non-federal governments causing them to issue fewer bonds (or retire more bonds) than they would otherwise.[12] Since b^S enters the federal GBC as a revenue source, we see that the decision of non-federal governments to bond rather than tax finance imposes a drag on federal revenues which would otherwise not exist. Consequently, the condition for convergence is much more stringent when non-federal governments choose to bond finance.

How much more stringent is (21) than (17)? One way to answer this is to adopt the reasoning of McCallum (1981). McCallum notes that since (r-p) should be close in magnitude to the steady-state value of n, then with $0 < t < 1$ the condition for convergence described in (17) will likely be satisfied. If we now note that if non-federal governments bond finance (21) is necessary for convergence, we see that by McCallum's reasoning convergence is now problematic. Another way of comparing (17) and (21) is to consider some reasonable parameter values. For example, the following values would seem to fairly represent conditions of the last few years; $n = 0.03$, $r = 0.08$, $p = 0.04$, and $t^f = 0.20$. Using these values, (17) is satisfied but (21) is not by a significant margin. Those who assume either implicitly or explicitly that non-federal governments tax finance when in fact they bond finance therefore significantly overstate the likelihood of convergence in a federal economy with a federal government obeying a monetary rule.

5. Macroeconomic Stability under Money Financing

In this section we assume the federal government has chosen to maintain a constant debt to output ratio ($\dot{b}^f = 0$) and has therefore decided to money finance any disturbances to its budget position. Setting $\dot{b}^f = 0$ and substituting (14) into (11) for $\dot{b}^s$, we can re-write the definition of disposable income as;

$$(22) \quad y^d = 1 + \alpha g^f - (1-\alpha)g^s + rb^f - t^f(1 + rb^f + rb^s) - p(m + b^f) - nb^f(1-\beta) + \beta(g^s + rb^s - t^s(1 + rb^f + rb^s) - pb^s).$$

The model we consider in this section therefore consists of static equations (10), (12), and (22) and asset accumulation identities (13) and (14).[13] The static equations can be solved for r and p as functions of b^S, m, and exogenous variables;

$$r = J(b^S,\ m,\ \text{exogenous variables})$$

$$p = K(b^S,\ m,\ \text{exogenous variables})$$

where the partial derivatives of these functions include;

$$J_m = (1 - L_3)/L_2 < 0$$

$$J_{b^S} \equiv J_b = -\beta L_3/L_2 \geq 0$$

$$K_m = \frac{L_2(C_3-pC_1)+(1-L_3)(C_2+C_1[b^f(1-t^f)-t^f b^S]+\beta C_1[b^S(1-t^S)-t^S b^f])}{L_2(C_2+C_1(m+b^f+\beta b^S))} < 0$$

$$K_{b^S} \equiv K_b = \beta\frac{L_2(C_3+C_1(r(1-t^S)-p))-L_3(C_2+C_1[b^f(1-t^f)-t^f b^S]+\beta C_1[b^S(1-t^S)-t^S b^f])}{L_2(C_2+C_1(m+b^f+\beta b^S))}$$

$$-\frac{rt^f C_1}{C_2+C_1(m+b^f+\beta b^S)} \gtrless 0.$$

We make the assumption that K_m is negative based on the following considerations.[14] From the definition of expenditures enjoyed by the private sector we have;

$$\partial C/\partial p = -[C_2 + C_1(m + b^f + \beta b^S)]$$

$$\partial C/\partial r = C_2 + C_1[b^f(1-t^f)-t^f b^S] + \beta C_1[b^S(1-t^S)-t^S b^f]$$

$$\partial C/\partial m = C_3 - pC_1.$$

Changes in r and p impart both an income and a substitution effect on private expenditures and these effects are of opposite sign. We assume the substitution effect dominates in each case so that $\partial C/\partial p > 0$ and $\partial C/\partial r < 0$. Similarly, a change in wealth in the form of money has both an income and a wealth effect on private expenditure. Here we assume the wealth effect dominates so that $\partial C/\partial m > 0$. Taken together these assumptions imply $K_m < 0$ but leave $K_b \gtrless 0$.[15]

The reasons for the sign ambiguity of K_b are the same as those discussed in section 4 with respect to the sign of H_b. That is, a ceteris paribus increase in b^S gives rise to wealth induced shifts of IS (to the right) and LM (to the left) which are offsetting to some degree. If $\beta = 0$ so that b^S is not a component of net wealth, these wealth induced shifts fail to arise. Note however that even if $\beta = 0$, $K_b > 0$ whereas $H_b = 0$. The difference is due to the fact that in the present case the federal government is money financing. As a result, an increase in b^S (which ceteris paribus adds to the surplus -- or reduces the deficit -- of the federal government by increasing federal tax revenue) causes a decrease in the money supply and hence an increase in aggregate demand (given $K_m < 0$).

(i) Non-Federal Tax Financing ($\dot{b}^S = 0$):

As in section 4 we begin by first considering the case where non-federal governments maintain a constant non-federal debt to output ratio and hence allow their tax rate to vary endogenously. Again our interest lies mainly in showing that the results of the literature can only be derived for the case of a federal economy if one assumes non-federal governments choose to tax finance in the face of federal money financing.

With $\dot{b}^s = 0$, equation (14) becomes a static equation and (13) remains as the only dynamic equation.[16] Differentiating (13) we see that convergence to a constant money to output ratio requires

$$(23)\quad \partial\dot{m}/\partial m = -(n+p) + [b^f(1-t^f)-t^f b^s]\, J_m - (m+b^f)K_m < 0.$$

With $K_m < 0$ and $J_m < 0$ we see that convergence is not guaranteed. This is consistent with the results of Turnovsky (1979) and Stemp and Turnovsky (1984) who assume zero growth, and with Scarth (1987b) who assumes positive, exogenous growth. Scarth concludes that for reasonable parameter values convergence is not a serious issue in this case. A simulation model with economic growth by Nguyen and Turnovsky (1983) also indicates that money financing is highly stable, being so for 83% of the parameter sets they consider.

By assuming non-federal governments tax finance when the federal government money finances, we therefore derive the results found in the literature where unitary states are assumed. Those seeking policy advice from the literature for the case of federal economies should therefore be aware of this rather strong implicit assumption of these studies. Our strategy will be to follow the conclusion of these studies that $\partial\dot{m}/\partial m < 0$ is true for this case and simply note the effect altering the assumption regarding the financing behaviour of non-federal governments to the more realistic case of bond financing has on the likelihood of convergence being attained.

Note that the magnitude of β is not an issue under money financing when non-federal governments are assumed to tax finance as β enters neither J_m nor K_m (see footnote 16).

(ii) Non-Federal Bond Financing ($\dot{b}^s \neq 0$):

Assuming now that non-federal governments are bond financing disturbances to their budget positions, we have two dynamic equations ((13) and (14)). The characteristic equation of this system of dynamic equations can be written as

$$(24)\quad \begin{bmatrix} \partial\dot{m}/\partial m - \lambda & \partial\dot{m}/\partial b^s \\ \partial\dot{b}^s/\partial m & \partial\dot{b}^s/\partial b^s - \lambda \end{bmatrix} = 0$$

where λ = a characteristic root of the characteristic equation, and where differentiation of (13) and (14) yields $\partial\dot{m}/\partial m$ as defined in (23) and;

$$\partial\dot{m}/\partial b^s = -rt^f + [b^f(1-t^f) - t^f b^s]\ J_b - (m+b^f)\ K_b$$

$$\partial\dot{b}^s/\partial m = [b^s(1-t^s) - t^s b^f]\ J_m - b^s\ K_m$$

$$\partial\dot{b}^s/\partial b^s = -\ [n - (r(1-t^s)-p)] + [b^s(1-t^s) - t^s b^f]\ J_b - b^s\ K_b.$$

The necessary and sufficient conditions for convergence given this set of financing assumptions are therefore

$$(25)\quad \partial\dot{m}/\partial m + \partial\dot{b}^s/\partial b^s < 0$$

and

$$(26)\quad (\partial\dot{m}/\partial m)(\partial\dot{b}^s/\partial b^s) - (\partial\dot{m}/\partial b^s)(\partial\dot{b}^s/\partial m) > 0.$$

By assumption we have $\partial\dot{m}/\partial m < 0$ so that money financing is stable if non-federal governments tax finance. Expansion shows that $\partial\dot{b}^s/\partial m > 0$ regardless of the value of β given our assumptions regarding the signs of the partial derivatives of the private expenditure function made earlier.[17] If we make the monetarist assumption that $\beta = 0$, a number of terms in (25) and (26) simplify. Expansion shows that if $\beta = 0$, then

$$\frac{\partial\dot{m}}{\partial b^s} = \frac{-rt^f C_2}{C_2 + C_1(m + b^f)} < 0,$$

$$\frac{\partial\dot{b}^s}{\partial b^s} = -[n - (r(1-t^s)-p)] + \frac{b^s rt^f C_1}{C_2 + C_1(m + b^f)} \gtrless 0,$$

and (26) can be re-written as requiring

$$(26') \quad \frac{\partial\dot{m}}{\partial m}[n-(r(1-t^s)-p)] + \frac{rt^f[b^s L_2(C_3+nC_1)+t^s(b^f+b^s)(1-L_3)C_2]}{L_2(C_2+C_1(m+b^f))} < 0.$$

Note first of all that unlike the cases where the federal government bond finances, a condition requiring the growth rate to be greater than an after tax real interest rate is not necessary for convergence. Inspection of (25) and (26') shows that $n > r(1-t^s)-p$ is only sufficient, not necessary for convergence. It is interesting to note however that if we follow Turnovsky (1979) and Stemp and Turnovsky (1984) in assuming lump sum taxes ($t^f = t^s = 0$), then $n > r-p$ is a necessary and sufficient condition (given our assumption that $\partial\dot{m}/\partial m < 0$) for convergence. Thus the claim of Turnovsky (1979, p. 37) that his assumption of lump sum taxes changes nothing of substance is incorrect for the case of a federal economy. With lump sum taxes and the monetarist assumption that $\beta = 0$, the growth rate must exceed the real interest rate regardless of how the federal government chooses to finance its deficits if non-federal governments choose to deficit finance. Such a result would of course have very serious consequences for a federal economy. If the growth rate was <u>less</u> than the real interest rate and if the monetarist assumption that $\beta = 0$ was true, then convergence under federal money financing would require that non-federal governments avoid deficit financing. Otherwise, convergence would be impossible regardless of how the federal government chose to finance its deficits.

Even returning to the case where taxes are not lump sum, the sign of $[n-(r(1-t^S)-p)]$ plays in important role. That is, if $n > r(1-t^S)-p$, then $\partial\dot{b}^S/\partial b^S < 0$ and both (25) and (26') are satisfied given our assumption that $\partial\dot{m}/\partial m < 0$. What is important to note here is that the decision of non-federal governments to bond rather than tax finance does not lessen the likelihood of convergence. Indeed, if $\partial\dot{m}/\partial m > 0$ so that money financing is unstable when non-federal governments tax finance (a result Nguyen and Turnovsky (1983) found in 17% of their cases), then a switch by non-federal governments to bond financing actually increases the likelihood of convergence. That is, if $\partial\dot{m}/\partial m > 0$ then $\partial\dot{b}^S/\partial b^S < 0$ may be sufficient to cause (25) to be satisfied and in (26') since $[n-(r(1-t^S)-p)]$ is a fraction and the last term is negative, then (26') may also be satisfied despite $\partial\dot{m}/\partial m > 0$. Therefore, if $\beta = 0$ and $n > r(r-t^S)-p$, non-federal bond financing actually increases the likelihood of convergence when the federal government money finances.

The intuition behind this result is as follows. Suppose the federal government suffers a deficit requiring that it cause the money supply to grow. Given the signs of K_m and J_m, the impact effect of this is to decrease both the rate of inflation (and hence inflation tax revenue) and the nominal interest rate (thus reducing interest payments on outstanding debt). If inflation tax revenue falls more quickly than interest payments on outstanding debt, the federal deficit grows and we have instability ($\partial\dot{m}/\partial m > 0$). Now suppose we have bond financing non-federal governments. The increase in the money supply puts non-federal governments in deficit (recall that $\partial\dot{b}^s/\partial m > 0$) causing them to issue bonds. The increase in b^s causes federal tax revenues to grow for two reasons. First, b^s is a component of personal income so that the tax base grows. Second, an increase in b^s causes the rate of inflation to increase ($K_b > 0$ when $\beta = 0$) so that federal revenue from the inflation tax also grows. Both these effects therefore help to close the federal deficit ($\partial\dot{m}/\partial b^s <$ 0 if $\beta = 0$). If as well $n > r(1-t^s)-p$, then the non-federal deficit also closes ($\partial\dot{b}^s/\partial b^s < 0$ if this is true). Thus if money financing was previously unstable, it may become stable if non-federal governments choose to bond rather than tax finance.

If however $n < r(1-t^s)-p$, then $\partial\dot{b}^s/\partial b^s \gtrless 0$ and both (25) and (26') may fail despite assuming $\partial\dot{m}/\partial m < 0$. Here the decision of non-federal governments to bond finance lessens the likelihood of convergence because the growth rate is possibly insufficient to cause the non-federal deficit to close once b^s is increased.

If therefore, one imposes the monetarist assumption that $\beta = 0$ and assumes non-federal governments bond finance, then the question of whether or not the growth rate exceeds the real after tax interest rate becomes an important one even if the federal government money finances. If taxes are lump sum then $n > r-p$ is necessary for convergence. If taxes are not lump sum and $n > r(1-t^S)-p$ then non-federal bond financing increases the likelihood of convergence. If however $n < r(1-t^S)-p$ then convergence under money financing may require that non-federal governments avoid bond financing.

These conclusions change dramatically when we drop the monetarist assumption that $\beta = 0$ and assume instead that at least some fraction of the value of government bonds is perceived as net wealth by the public. That is, with $\beta > 0$ the sign of $[n-(r(1-t^S)-p)]$ looses much of its significance. It is helpful for convergence if $n > r(1-t^S)-p$ but this condition is not necessary and it is not even sufficient (as it is when $\beta = 0$). To see why this is so, consider again our example where the federal budget has fallen into deficit requiring an expansion of the money supply. Once again non-federal governments are pushed into deficit due to the expansion of the money supply (recall that $\partial\dot{b}^S/\partial m > 0$ regardless of the value of β) causing them to issue bonds. The increase in b^S adds to the tax base and thereby increases federal tax revenues. However, the impact effect of the increase in b^S on inflation is now ambiguous ($K_b \gtrless 0$ when $\beta > 0$) so that one influence helpful in closing the federal deficit (an increase in the inflation tax) is no longer necessarily present when $\beta > 0$. Further, with $\beta > 0$ we now have $J_b > 0$ so that the increase in b^S

also increases the rate of interest and hence the interest payments due on outstanding federal debt. Thus, with $\beta > 0$ we have a new influence due to the bond financing behaviour of the non-federal governments which acts to <u>widen</u> the federal deficit. When we assume $\beta > 0$ therefore, non-federal bond financing no longer has an unambiguously positive effect on reducing the federal deficit ($\partial\dot{m}/\partial b^S \gtrless 0$ if $\beta > 0$). Thus even if $n > r(1-t^S)-p$ so that the non-federal deficit tends to close (though not unambiguously so since $K_b \gtrless 0$), the federal deficit may widen due to the influence of the increase in b^S so that the economy fails to converge. Relative to the case where $\beta = 0$ therefore, there seems to be a tendency toward instability when $\beta > 0$ and non-federal governments bond finance.

6. The Burden of Government Debt in a Federal Economy

Economic agents are said to suffer a burden imposed by government bonded (as opposed to monetarized) debt if consumption possibilities are reduced by the decision to bond rather than tax finance. Whether or not economic agents suffer such a burden has long been, and continues to be, the subject of debate. In his review of his debate, Modigliani (1983) identifies the "super-sophisticated no burden view" as that consistent with the assumption of the equivalence hypothesis. The equivalence hypothesis states that private economic agents perceive a government bond issue as a future tax liability and as a result increase current savings in order to prepare for future taxes. Thus the stimulative effect on aggregate demand of a higher deficit is exactly offset by the depressive effect of increased personal savings. As a result, it does not matter whether government finances increased expenditures by raising taxes now or by selling bonds now and raising taxes later -- the effect on aggregate demand is the same regardless of which scheme is adopted and hence no burden is imposed by the decision to bond finance.

This view of the burden of bonded government debt has been stated solely for the case of a unitary state. In this section we show that although imposition of the equivalence hypothesis implies federal government debt imposes no burden, it does not imply the same for non-federal government debt.

In order to better illustrate how the existence of non-federal governments affects the "super-sophisticated no burden view", it will be useful to first derive the results for a unitary state. To do so, omit the non-federal GBC (equation (14)) from our model and let $g^s = b^s = t^s = 0$. Now let $\beta = 0$ so as to impose the equivalence hypothesis. Assuming first of all that the (federal) government obeys a monetary rule so that it bond finances disturbances to its budget position (m exogeneous, b^f endogeneous), we calculate the following full equilibrium ($\dot{b}^f = 0$) comparative static results;

(27) $dy^d/dt^f = 0$

(28) $dy^d/dg^f = -(1-\alpha) < 0.$

These are the standard results of the super-sophisticated no burden view. The result in (27) indicates that a re-financing of any portion of a given budget by bonded debt rather than taxes has no influence on the consumption possibilities of the private sector. From (28) we obtain the result that a bond financed change in government expenditures only reduces consumption possibilities to the extent that such expenditures are not viewed as equivalent to private expenditures by the public.

Before leaving this review of the results for a unitary state it will be useful to calculate an equilibrium comparative static result for the case of a money financing (m endogeneous, b^f exogeneous) government. We find that,

$$(29)\quad dy^d/dt^f = -(1 + rb^f)\ nC_2L_2/\Delta_1$$

where $\Delta_1 = (n+p)\ C_2L_2 + (C_3 + nC_1)(m + b^f)L_2 + C_2(1 - L_3)(m + t^fb^f) \gtrless 0$. Expansion of (23) -- the necessary and sufficient condition for convergence under money financing -- indicates that the sign of Δ_1 determines whether or not convergence is attained given money financing.[18] For comparative static results to be useful one must assume convergence and this requires $\Delta_1 > 0$. Thus, $dy^d/dt^f < 0$, indicating that re-financing any portion of a given budget increases the consumption possibilities of the private sector. This result is not unexpected as it simply indicates that even given the equivalence hypothesis the choice between money and tax financing remains important.

Now consider a federal economy so that (14) is again a part of our model and consider a bond financing federal government. Calculation of full equilibrium comparative static multipliers again generates the results given in (27) and (28) regardless of the financing behaviour of non-federal governments. This is not an unexpected result. At the non-federal level, the change in the magnitude of b^f occasioned by the federal switch of bond for tax revenue causes a budget imbalance (since b^f enters the tax base). Whether this causes a change in the non-federal tax rate or a change in the magnitude of b^s makes no difference -- the effect on y^d is the same due to the imposition of the equivalence hypothesis. The same results are of course obtained if it is the non-federal government sector which initiates a switch of bond for tax revenue. Thus the super-sophisticated no burden view is robust to our extension to a federal economy at least with respect to the case of a bond financing federal government.

When the federal government money finances however, non-federal debt can no longer be judged to be neutral with respect to the consumption possibilities of the private sector. To see this, consider first of all the full equilibrium comparative static effect of a re-financing of part of a given non-federal government sector budget;

$$(30) \quad dy^d/dt^s = (1 + rb^f + rb^s)\ rt^f nC_2L_2/\Delta_2$$

where $\Delta_2 = [n-(r(1-t^s)-p)]\Delta_1{}^* - rt^f[b^sL_2(C_3+nC_1)+C_2(1-L_3)t^s(b^f+b^s)] \gtrless 0$. Expansion of (26') -- a necessary condition for convergence given this combination of government financing choices -- shows that $\Delta_2 > 0$ is necessary for convergence.[19] For comparative static results to be useful we must assume convergence and hence that Δ_2 is positive. As a result, $dy^d/dt^s > 0$ indicating the private economic agents suffer a burden due to non-federal refinancing in favour of greater bond revenue <u>despite</u> the equivalence hypothesis. Similarly, even assuming $\alpha=1$, we obtain;

$$dy^d/dg^s = -rt^f nC_2L_2/\Delta_2 < 0$$

so that a bond financed increase in non-federal government expenditures reduces consumption possibilities. It is interesting to note that both these results become larger in absolute magnitude the smaller is the value of the term $[n-(r(1-t^s)-p)]$.

The reason for these results is due to the interdependence of GBC's in a federal economy. Any change in the stock of non-federal bonds must affect the budget position of the federal government (since b^s enters the tax base) and thus cause a change in the money supply -- something which affects consumption possibilities with or without the equivalence hypothesis. That the magnitude of the term $[n-(r(1-t^s)-p)]$ is important in determining the magnitude of the burden of non-federal debt is not surprising. The smaller is this term, ceteris paribus, the more slowly will any bond financed deficit close and therefore the greater will be the total change in the money supply and hence the greater will be the reduction in consumption possibilities.

Another way of looking at this issue is to compare the effect on consumption possibilities of a switch of tax revenue for monetarized debt in a unitary state with that in a federal state with bond financing non-federal governments. The former effect is given in (29) above and it shows that increasing the proportion of the federal budget financed by monetarized debt (by reducing the proportion financed by tax revenue) increases the consumption possibilities of the private sector. If we now allow for the existence of deficit financing non-federal governments, the effect of this same switch away from tax revenue in favour of monetarized debt causes;

$$(31)\ dy^d/dt^f = -\ (1+rb^f+rb^s)nC_2L_2[n-(r(1-t^s)-p)]/\Delta_2$$

which is of ambiguous sign due to the ambiguity of the sign of $[n-(r(1-t^s)-p)]$. The difference between (29) and (31) measures the effect on consumption possibilities of there being bond financing non-federal governments and hence measures the burden of non-federal debt.

Note that if $n < r(1-t^s)-p$, then (31) is positive indicating that if the federal government increases the proportion of its budget financed by monetarized debt, this would <u>reduce</u> consumption possibilities. This is opposite to our result from (29) indicating that the issue of non-federal debt caused by the federal re-financing (recall that $\partial\dot{b}^s/\partial m > 0$) unambiguously imposes a burden on private economic agents. If however $n > r(1-t^s)-p$, then both (29) and (31) are negative. It is easily shown however that in this case (29) is unambiguously smaller than (31) indicating that the amount by which consumption possibilities are increased by the federal re-financing in favour of monetarized debt is reduced due to the issue of non-federal bonds.[20] Thus non-federal bonds impose a burden on the private sector regardless of the sign of $[n-(r(1-t^s)-p)]$. As was the case in our discussion of (30) however, this burden is greater the smaller is this term.

7. Summary and Conclusions

We have endeavoured in this essay to construct a model which was general enough in its specification that we could investigate the implications of explicitly modelling a non-federal government sector for as many of the models appearing in the literature as possible. Thus we allowed for a zero or a positive rate of growth, lump sum or proportional taxes, and either the Keynesian or the monetarist view (or, indeed, anything in between) regarding the question of whether government bonds are net wealth. The main results of our analysis can be summarized as the following.

First, we showed that results found in the literature can only be derived if a unitary state is assumed or if non-federal governments are assumed to tax finance disturbances to their budget positions. Consequently, policy-makers in federal states should be aware of this strong, and totally unrealistic assumption regarding the behaviour of non-federal governments which is implicit in existing models in the literature.

Second, if the federal government has chosen to obey a monetary growth rule, and if non-federal governments bond finance disturbances to their budget positions, then $n > r-p$ is a necessary condition for convergence regardless of the value of β. This is a crucial result since in a unitary state (or a federal state with tax financing non-federal governments) the similar, but much less stringent condition $n > r(1-t^f)-p$ is necessary for convergence only if $\beta = 0$. If non-federal governments deficit finance therefore, then regardless of the degree to which private agents view government bonds as net wealth, a monetary growth rule is a feasible long-term policy for the federal government only if the real rate of growth exceeds the real, before tax, interest rate.

Third, if non-federal governments deficit finance then the magnitude of the real growth rate and real after tax interest rates plays a role in determining convergence even if the federal government money finances deficits. For example, if $\beta = 0$, then $n > r(1-t^f)-p$ is sufficient, though not necessary, for convergence. If however, we further assume lump sum taxes as is often done in the literature, then $n > r-p$ is necessary for convergence. Relaxing the restrictive assumptions of the equivalence hypothesis (so that $\beta > 0$) makes $n > r(1-t^f)-p$ helpful, but neither necessary nor sufficient for convergence. In this case then, the relative magnitudes of real growth rates and real interest rates takes on the same importance for convergence as is the case in a unitary state when the (federal) government obeys a monetary rule.

Finally, our analysis shows that explicitly modelling a deficit financing non-federal government sector has important implications for the burden of government debt. That is, despite assuming $\beta = 0$, if the federal government money finances, then private agents suffer a burden due to the issue of bonds by non-federal governments. Further, this burden grows the smaller is the term $[n-(r(1-t^S)-p)]$ so that the magnitude of this term has important consequences in a federal economy not only for convergence but also for the burden of government debt.

FOOTNOTES

[1] Hodrick (1980) notes that α need not be a constant as the public may periodically reassess their perception of the value of government expenditures. He suggests that public scandals might cause α to change in value and thus consequently affect macroeconomic variables.

[2] See O'Driscoll (1977) for a discussion of the evolution of this terminology.

[3] For a full discussion of the Ricardian equivalence hypothesis and the assumptions necessary for it to hold, see Barro (1974). For criticisms of the Ricardian equivalence hypothesis, see Tobin (1980) and more recently Bruce and Purvis (1986). Modigliani (1986) compares the life-cycle hypothesis (LCH) to the Ricardian equivalence hypothesis as alternative views regarding the determinants of savings. He notes that if the LCH is an accurate model of reality then private saving should be largely invariant to the size of government debt -- an implication opposite to that suggested by the Ricardian equivalence hypothesis.

[4] Barro (1974) notes that $\beta < 0$ is a possibility due to uncertainty regarding the distribution of taxes in the future. We ignore this possibility.

[5] The inflation tax on wealth is written as πW rather than pW because disposable income is supposed to be an "expected" concept when used in the expenditure function. Thus only the expected depreciation of financial assets due to inflation is subtracted from disposable income. See Sargent (1979, pp. 15-17).

[6] Let $\alpha = \beta = 0$ and use equations (7) and (8) to substitute for $r(B^f/P + B^s/P) - (t^f + t^s)(Y + rB^f/P + rB^s/P)$ in equation (3). The result is

$$Y^d = Y - G^f - G^s + (1/P)\dot{M} - (M/P)\pi$$

which is the federal economy version of the definition employed by McCallum (1978).

[7] Note that it is necessary to assume the rate of growth in output is equal to that of population in order that C_2 and L_2 be constants. That is, in a model with growth, $C_{r-\pi}$ and L_r will increase in value at a rate proportional to the rate of population growth. From our definitions of C_2 and L_2, we see they are constant only if the rate of population growth is equal to the rate of growth in output. This assumption is made explicit in Sargent and Wallace (1981) and is implicit in McCallum (1981) and Darby (1984).

[8] For examples of such an analysis see the discussion of "mixed financing" in Turnovsky (1977, pp. 68-85) for the case of a fixed price model and Stemp and Turnovsky (1984) for the case of a variable price model.

[9] We are assuming for the moment that non-federal governments are bond financing so that $\dot{b}^s \neq 0$.

[10] The fact that (13) is now a static equation implies it should be used along with (10), (12) and (15) to solve for F_b and H_b. This adjustment leaves F_b unaffected but causes H_b to become;

$$H_b = \beta \frac{L_2(C_3+C_1(r(1-t^f)-p)) - L_3(C_2+\beta C_1[b^f(1-t^f)-t^f b^s])}{L_2(C_2+\beta C_1(m+b^f))}$$

The magnitude of H_b is affected but the sign ambiguity remains.

[11] See for example Turnovsky (1979) who models a unitary state with $\beta = 1$, $t = 0$, and $n = 0$. Using his equation (12), convergence in his model requires

$$\partial\dot{b}/\partial b = (r-p) + (W - \bar{m})\ F_b - WH_b < 0$$

where his notation is similar to ours ($W = m + b$). From his table 1C, $F_b > 0$ and $H_b \gtrless 0$, and $(r-p) > 0$ by assumption. Thus convergence requires $H_b > 0$. For the purpose of examining an explosive economy, Basevi and Giavazzi (1986) assume not only n=0 but also that H_b is insufficiently large to cause convergence.

[12] Recall from our discussion in the introduction to the thesis that empirical evidence suggests this type of budget interdependence is quite significant. Hence, the magnitude of this "feedback" from the non-federal sector is not likely to be small.

[13] Again, we are assuming for the moment that non-federal governments are bond financing.

[14] The following discussion follows closely that of Turnovsky (1979, p. 37).

[15] Another derivative of this private expenditure function is $\partial C/\partial b^s = \beta C_3 + \beta C_1(r(1-t^s)-p) - rC_1 t^f \gtrless 0$. The first term defines the wealth effect due to an increase in b^s. The second term defines one part of the income effect of an increase in b^s. It shows the increase in income net of non-federal taxes when new non-federal bonds are issued. The last term shows the decrease in income due to an increase in federal taxes when b^s increases. If $\beta = 1$, $\partial C/\partial b^s > 0$ as Turnovsky (1979) discusses. If however, $\beta = 0$, then the first two terms disappear reflecting the fact that with Ricardian equivalence an increase in holdings of non-federal bonds does not add to net wealth and they entitle agents to an increase in non-federal taxes equal in present value to the increase in interest income. The third term remains however as federal taxes increase but no offsetting interest payments are forthcoming. As a result $\partial C/\partial b^s < 0$ when $\beta = 0$.

[16] With (14) being a static equation it should be used in the calculation of J_m, J_b, K_m, and K_b. The only one of these partial derivatives affected by this consideration is K_m. Omit terms multiplied by β from the definition of K_m given in the text and we have the proper definition of K_m when $\dot{b}^S = 0$. The sign of K_m is unaffected.

[17] Expansion allows us to re-write the definition of $\frac{\partial \dot{b}^S}{\partial m}$ as

$$\frac{-b^S L_2(C_3-pC_1)+ C_1(1-L_3)b^S(m+t^f(b^f+b^S))-t^S(b^f+b^S)(1-L_3)(C_2+C_1(m+b^f))}{L_2(C_2 + C_1(m + b^f+\beta b^S))}$$

which is positive given our previous assumptions.

[18] To be precise, we should note that the expression in (23) is derived for the case of a federal economy with tax financing non-federal governments while (29) is derived for a unitary state. As a result, expansion of (23) in fact yields $\Delta_1{}^* = \Delta_1 + C_2(1-L_3)t^f b^S$ as the expression the sign of which determines whether or not convergence is attained when the federal government money finances and non-federal governments tax finance. Convergence requires $\Delta_1{}^* > 0$ which in turn requires Δ_1 -- the corresponding expression for a unitary state (where $b^S= 0$) -- be positive.

[19] Recall that $\Delta_1{}^* > 0$ is necessary for convergence when non-federal governments tax finance (see footnote 18). In Section 5 we simply assumed $\Delta_1{}^* > 0$ and examined how non-federal bond financing affected the likelihood of convergence. Thus, we assume $\Delta_1{}^* > 0$ here as well when examining comparative static results. The sign of Δ_2 remains ambiguous however since $[n-(r(1-t^S)-p)] \gtrless 0$. A sufficient but not necessary condition for convergence is $[n-(r(1-t^S)-p)] > 0$.

[20] To see this, re-write (31) as;

$$\text{(31')} \quad dy^d/dt^f = -(1+rb^f+rb^S)nC_2L_2/\Delta_2[n-(r(1-t^S)-p)]^{-1}.$$

From the definitions of Δ_2 and $\Delta_1{}^*$, the denominator of the above expression can be alternatively written as:

$$\Delta_1{}^* - rt^f[b^S L_2(C_3+nC_1) + C_2(1-L_3)t^S(b^f+ b^S)]/[n-(r(1-t^S)-p)]$$

or finally,

$$\Delta_1 - rt^f\, b^S\, L_2(C_3+nC_1)/[n-(r(1-t^S)-p)]$$

$$- C_2(1-L_3)\{t^S(b^f+ b^S)/[n-(r(1-t^S)-p)] - t^f b^S\}.$$

Given the assumption that $[n-(r(1-t^S)-p)] > 0$, the second term is positive. The sign of the third term depends on the sign of the term in curved brackets. For any reasonable value of n, the term $[n-(r(1-t^S)-p)]$ is a fraction so that the term in curved brackets is positive for reasonable values of t^f and t^S. Hence the third term is also positive. Clearly then, $\Delta_2[n-(r(1-t^S)-p)]^{-1} > \Delta_1$. Noting now that the numerator of (31') is unambiguously smaller than that of (29), we have shown the expression in (29) to be smaller than that in (31') (and hence (31)).

Chapter Four

Government Deficits and Conditions for Macroeconomic Stability in a Small, Open, Federal Economy

1. Introduction

By explicitly modelling a non-federal government sector we have, in the previous two essays, moved some of the more important models used in the literature to investigate the stability of deficit financing a significant step closer toward a more accurate description of a federal economy. For many federal economies however (our particular interest is Canada) a further refinement in specification is required before these models adequately reflect the true macroeconomic structure of these economies. This further refinement is necessary to model an open rather than a closed economy as we have done so far.

We will follow the same procedure adopted in the second essay in that we will first construct a flexible price open economy model with zero growth in order to facilitate a comparison of our model's specification with the various specifications appearing in the literature. Again our goal will be to construct a model with a very general specification so as to enable us to show, for as many existing models as possible, the implications of explicitly recognizing the existence of non-federal government. We will then, following the methodology recently adopted by Sargent and Wallace (1981), McCallum (1981), Darby (1984) and Scarth (1987a, 1987b), assume a constant exogenously determined rate of growth and add this to our model. The result will be a model similar in some respects to the recent model employed in Scarth

(1987a) but with a number of modifications. First, we will allow for direct wealth effects whereas Scarth does not. Second, we will allow for the imposition of any assumption regarding the contentious issue of whether private agents view government bonds as net wealth whereas Scarth imposes the equivalence hypothesis. Finally, we will model a non-federal government sector whereas Scarth assumes a unitary state.

2. The Model

The model we consider in this essay is the open economy analogue to the closed economy model employed in essay two. It is a deterministic model of a small, open, federal economy with an endogenous price level, exogenously determined growth, a flexible exchange rate, and perfect foresight. As was the case in essay two, we allow for the possibility that private economic agents may not perceive government bonds to be a part of net wealth.

The model contains three asset accumulation identities -- a government budget constraint (GBC) for each level of government and a balance of payments constraint. In order to keep what will be seen to be a fairly unwieldy model tractable, we will make the simplifying assumptions that government is the only issuer of internationally traded debt, and that this debt is denominated in domestic currency. As Dornbusch and Fischer (1980) note, it is inevitable that a special case be chosen when defining these assumptions. This particular set of assumptions has however proved fairly popular having been adopted by Scarth (1975), Turnovsky (1976), Kingston and Turnovsky (1978), and Camilleri, Nguyen, and Campbell (1984).

With these assumptions we can write the two GBC's as;

(1) $(1/P)(\dot{M} + \dot{B}^f) = G^f + rB^f/P - t^f(Y + rB^f/P + rB^s/P - rB^*/P) - h^f rB^*/P$

(2) $(1/P)\dot{B}^s = G^s + rB^s/P - t^s(Y + rB^f/P + rB^s/P - rB^*/P) - h^s rB^*/P$

where Y = real national income

r = nominal interest rate

P = domestic price index

M = nominal domestic money supply

B^f, B^s = outstanding stocks of federal and non-federal government bonds, respectively

t^f, t^s = federal and non-federal government tax rates

h^f, h^s = federal and non-federal government withholding tax rates levied on foreign income earned within the domestic economy

G^f, G^s = real federal and non-federal government expenditures on goods and services, respectively

B^* = outstanding stock of government (both federal and non-federal) bonds held by foreign residents

and where a dot over a variable denotes the time derivative of that variable.

These constraints are essentially the same as those defined for our closed economy model. They differ from their closed economy counterparts due to the existence of B^*. Income tax rates t^f and t^s are applied only against income earned by domestic residents -- thus real interest payments paid to foreign residents holding domestic government bonds (rB^*/P) must be netted out of the tax base against which t^f and t^s are applied. We assume a withholding tax is applied by each level of government against these interest payments flowing to non-residents. As is typically done in government budget constraint models, we abstract from a fractional reserve banking system. Thus M denotes both the monetary base and the total money supply. Equation (1) is the GBC of the federal government while (2) defines the GBC of the aggregated non-federal government sector. These constraints differ only in that we assume the central bank chooses to monetarize only the debt of the federal government.

The third asset accumulation identity in the model is the balance of payments constraint;

(3) $$(1/P)\dot{B}^* = -X + (1-h^f-h^s)\ rB^*/P$$

where X = net exports.

The right hand side of (3) defines the current account whereas the left hand side defines the capital account. With a flexible exchange rate the deficit (surplus) in the current account must offset the surplus (deficit) in the capital account. The capital account defines net movements of internationally traded assets. As Canada is clearly a net borrower on international credit markets, we have for simplicity, assumed domestic residents cannot hold foreign issued bonds so that $(1/P)\dot{B}^*$ is the correct definition of the capital account.

The three asset accumulation identities drive the model from one instantaneous equilibrium to another by changing stocks of money and bonds. At every point in time, each of these instantaneous equilibriums is described by the following static equations.

(4) $$Y = C(Y^d,\ r-\pi,\ W) + (1-\alpha)(G^f + G^s) + X$$

(5) $$W = M/P + \beta(B^f/P + B^s/P) - (1-h^f-h^s)B^*/P$$

(6) $$Y^d = Y + r/P(B^f + B^s - B^*) - (t^f + t^s)(Y + rB^f/P + rB^s/P - rB^*/P) - \pi W + \alpha(G^f + G^s) - (1-\beta)(1/P)(\dot{B}^f + \dot{B}^s) + (h^f + h^s)(1/P)\dot{B}^*$$

(7) $$M/P = L(Y,\ r,\ W)$$

(8) $$\pi = p$$

(9) $$p = p^* + \dot{E}/E$$

(10) $$r = r^* + \dot{E}/E$$

where E = the price in domestic currency of one unit of foreign currency (the exchange rate)

W = the subjective value of real net wealth

Y^d = real disposable income

p = the rate of domestic inflation ($\dot{P}/P$)

π = the expected rate of domestic inflation

r^* = the nominal foreign interest rate

p^* = the rate of foreign inflation (we assume $r^* - p^* > 0$)

α = the fraction of government expenditure on goods and services private agents view as being equivalent to private expenditure

β = the fraction of the value of government bond holdings private agents judged to be a component of net wealth.

Equation (4) is derived in the same manner as in essay two. That is, total real consumption enjoyed by private agents consists of private consumption expenditures, C, plus that fraction α of government expenditures which private agents view as being equivalent to C. This latter component is due to a recognition that some government expenditures are substitutes for private expenditures (for example, police, education, school lunches, etc.). Following our practice from essay two, we assume $0 \leq \alpha \leq 1$ and that α is constant in value. We also assume for simplicity that the same fractions of G^f and G^s are judged equivalent to private expenditures. Total private consumption is assumed to be positively related to disposable income ($0 < C_{yd} < 1$) and wealth ($0 < L_w < 1$) but negatively related to the expected real interest rate ($C_{r-\pi} < 0$). Thus we have;

$$C + \alpha(G^f + G^s) = C(Y^d, r-\pi, W).$$

Adding this relationship to the GNP identity for an open economy ($Y = C + G^f + G^s + X$) yields (4).

Equation (5) defines the subjective value of real net wealth. If $\beta=0$, we are imposing the equivalence hypothesis that private agents do not perceive government bonds to be a part of net wealth due to the tax liability they imply for the future. If $\beta=1$, we are imposing the "Keynesian" assumption that private agents do not associate a government bond issue with a future tax liability and hence view the full value of government bonds to be net wealth. Our specification that $0 \leq \beta \leq 1$ allows for both these assumptions as well as for the assumption that there is a partial discounting for future taxes when evaluating net wealth. Our specification of wealth is the same as that for our closed economy model except for the fact that we must subtract the value of domestic bonds (net of withholding taxes) held by foreign residents in order to define the net wealth of domestic residents.

Equation (6) defines real disposable income of domestic residents to be equal to income from production, plus interest income earned by domestic residents, minus taxes, minus the expected depreciation of financial assets due to inflation (the inflation tax), plus the value of government expenditures viewed as being equivalent to private expenditures, minus the income domestic residents perceive as being necessary to save in order to finance future tax liabilities incurred due to current government bond financing, plus the income domestic residents do not have to save in anticipation of future taxes because of the withholding taxes paid by foreign residents. This specification of disposable income is a generalized version of specifications appearing in the literature for the case of a unitary state. That is, if we omit withholding taxes we are left with the specification employed by Hodrick (1980). If we also let $\alpha=\beta=0$, we are left with the definition used by Boyer and Hodrick (1982). Still ignoring withholding taxes, if we let $\alpha=0$, $\beta=1$, we have the specification employed by Kingston and Turnovsky (1978), Riley (1982), Camilleri, Nguyen, and Campbell (1984), and Kawai (1985).[1] If we further assume fixed prices for domestically produced goods (so that as a consequence, $\pi=0$), we are left with the specifications of Scarth (1975), Turnovsky (1976), Allen (1977), and Rodriquez (1979). Finally, if we re-instate withholding taxes and flexible prices and let $\alpha>0$, $\beta=0$, we can use other equations in our model to re-write (6) as;

$$Y^d = Y - (1-\alpha)(G^f + G^s) + (\dot{M}/P) - (1-h^f - h^s)[r(1-h^f - h^s)-p]B^*/P$$
$$- (h^f + h^s)X$$

which is the federal economy version of the specification employed by Scarth (1987a) when he assumes domestic bonds held by foreign residents are denominated in domestic currency.[2] Note that except for the last two terms the definition of disposable income written in this form is equivalent to that defined by McCallum (1978) for the case of a closed economy. As the last two terms are negative, this indicates that disposable income is less in an open economy since domestic residents must finance their debt owed to foreigners. Our specification of disposable income is therefore a generalized form of specifications appearing elsewhere in the literature with the added complication of allowing for a second level of government.

Equation (7) describes equilibrium in the market for money. The demand for money function is standard with money demand positively related to income ($L_y > 0$) and wealth ($0 < L_w < 1$) but negatively related to the nominal interest rate ($L_r < 0$).

Equation (8) imposes the assumption of perfect foresight on the part of private economic agents. This follows our practice in essay number two where we argued that since our goal is to examine the long-run consequences of government debt policy this seems an appropriate assumption.

A concentration on long-term effects is also our justification for assuming purchasing power parity (PPP) -- defined in equation (9) -- is satisfied. The assumption of perfect foresight explains the fact that the rate of change in the actual rather than the expected exchange rate enters (9). We assume the domestic economy is a price-taker in world commodity trade so that p^* is exogenously determined.

Finally, we assume efficient world financial markets so that perfect capital mobility prevails. As a result, we assume covered interest parity described in (10). The domestic economy is also assumed to be small on world financial markets so that r^* is also exogenously determined.

It should be noted here that as well as imposing a balance of payments constraint and constraints on the financing behaviour of both levels of government, our model implicitly satisfies a budget constraint for private economic agents. This constraint requires that in each period savings (resulting in a change in wealth over time, $\dot{W}$) be equal to the actual disposable income of households (that is, Y^d as defined above minus expenditures made by government that households view as being equivalent to household consumption, $\alpha(G^f + G^s)$) minus actual private consumption expenditures, C.[3] Thus, we require that;

$$\dot{W} = Y^d - \alpha(G^f + G^s) - C.$$

Substituting (6) for Y^d, the time derivative of (5) for $\dot{W}$, using the GNP identity to replace C with $(Y - G^f - G^s - X)$, and imposing the assumption of perfect foresight, enables one to re-write this constraint in the following manner;

$$0 = \{G^f + rB^f/P - t^f(Y + rB^f/P + rB^s/P - rB^*/P) - h^f rB^*/P - (1/P)(\dot{M} + \dot{B}^f)\}$$
$$+ \{G^s + rB^s/P - t^s(Y + rB^f/P + rB^s/P - rB^*/P) - h^s rB^*/P - (1/P)\dot{B}^s\}$$
$$+ \{(1/P)\dot{B}^* + X - (1 - h^f - h^s)\ rB^*/P\}.$$

The terms grouped into curved brackets are respectively, the federal GBC, the non-federal GBC, and the balance of payments constraint. As our model requires satisfaction of each of these constraints it also satisfies the private sector constraint so that this latter constraint need not appear explicitly in the model.

The model we have described above contains no economic growth. There are two reasons why we wish to adjust the above model so as to allow for economic growth. First, our concentration will be on deriving conditions necessary for convergence to a long-run steady state so that allowing for growth would seem a necessary pre-requisite. Second, the recent popularity of the equivalence hypothesis makes it important for us to examine conditions for convergence given this assumption. Just as McCallum (1981) found for the closed economy case, Scarth (1987a) finds that given the equivalence hypothesis, a positive rate of economic growth is a necessary condition for convergence in an open economy. In order to make our examination of this case worthwhile therefore, it may require that we allow for economic growth.

Following common practice (see for example, McCallum (1981), Sargent and Wallace (1981), Darby (1984), and Scarth (1987a, 1987b)) we will assume an exogenously determined constant rate of growth in output and population (n). In order to add growth to our model, divide equations (1) - (7) by Y and use the following relationships;

$$(1/PY)\dot{B}^f = \dot{b}^f + b^f(n+p)$$

$$(1/PY)\dot{B}^s = \dot{b}^s + b^s(n+p)$$

$$(1/PY)\dot{B}^* = \dot{b}^* + b^*(n+p)$$

$$(1/PY)\dot{M} = \dot{m} + m(n+p)$$

where $n = \dot{Y}/Y$ $\qquad b^* = B^*/PY$

$b^f = B^f/PY$ $\qquad m = M/PY$

$b^s = B^s/PY.$

The model can be further simplified by substituting (9) into (10), (3) into (6), equations (5) and (8) into the rest of the model, and finally by assuming linear money demand and private expenditure functions. With all these adjustments, our model can now be written as the following set of equations.

$$(11) \quad 1 = C_1 y^d + C_2(r-p) + C_3[m + \beta(b^f + b^s) - b^*(1-h^f - h^s)] + (1-\alpha)(g^f + g^s) + x$$

$$(12) \quad y^d = (1 + rb^f + rb^s - rb^*)(1 - t^f - t^s) + \alpha(g^f + g^s) - (h^f + h^s)\, x - p(m + b^f + b^s - b^*) + (h^f + h^s)b^*[r(1-h^f - h^s) - p] - n(b^f + b^s)(1-\beta) - (\dot{b}^f + \dot{b}^s)(1-\beta)$$

$$(13) \quad m = L_1 + L_2 r + L_3[m + \beta(b^f + b^s) - b^*(1-h^f - h^s)]$$

$$(14) \quad r-p = r^* - p^*$$

$$(15) \quad \dot{m} + \dot{b}^f = g^f + rb^f - t^f(1 + rb^f + rb^s - rb^*) - (n+p)(m+b^f) - h^f rb^*$$

$$(16) \quad \dot{b}^s = g^s + rb^s - t^s(1 + rb^f + rb^s - rb^*) - (n+p)b^s - h^s rb^*$$

$$(17) \quad \dot{b}^* = -x - b^*[n - (r(1-h^f - h^s)-p)]$$

where $y^d = Y^d/Y$ $\qquad C_2 = C_{r-\pi}/Y$

$g^f = G^f/Y$ $\qquad C_3 = C_w$

$g^s = G^s/Y$ $\qquad L_1 = L_y$

$x = X/Y$ $\qquad L_2 = L_r/Y$

$C_1 = C_{yd}$ $\qquad L_3 = L_w$

Note that in a growing economy $C_{r-\pi}$ and L_r will change in value at a rate proportional to the rate of growth in population. Thus our assumption that the rate of growth in output is equal to the rate of growth in population implies C_2 and L_2 are constants. Also note that the endogenous variables in the model are y^d, r, p, b*, x, and one policy variable from each GBC. By assuming only one policy variable from each GBC can be endogenously determined at any one time, we omit consideration of more complex modes of government finance where two or more policy variables may vary in pre-determined proportions.

The model described by equations (11) - (17) is a generalized version of other models of small, open economies with flexible prices and a flexible exchange rate appearing in the literature. Except for Hodrick (1980) who, like us, allows for $0 \le \beta \le 1$, other models appearing in the literature assume that either $\beta=0$ -- an assumption Bruce (1977), Tobin (1980), and McCallum (1984) identify with the monetarist position that bond financial deficits can have no effect on aggregate demand -- or make the "Keynesian" assumption that $\beta=1$. Those models employing the Keynesian assumption that $\beta=1$ also make the restrictive assumption that economic growth (n) is zero (see Kingston and Turnovsky

(1978), Riley (1982), Camilleri, et. al. (1984), Kawai (1985), Basevi and Giavazzi (1986)). Models employing the monetarist assumption that $\beta=0$ assume both zero growth (see Boyer and Hodrick (1982) and Kimbrough (1985)) and an exogenously determined positive rate of growth (see Scarth (1987a)). Another characteristic which plays an important role in determining the conclusions of models appearing in the literature is whether or not wealth effects appear in the money demand and private expenditure functions. Kingston and Turnovsky (1978), Dornbusch and Fischer (1980), and Kawai (1985) all note that instability results in their theoretical models in the absence of direct wealth effects. Similarly, Camilleri, et. al. (1984) note that there is an increased tendency toward instability in their simulation model when direct wealth effects are ignored. Given our interest in deriving conditions for convergence, we have therefore followed Kingston and Turnovsky (1978), Hodrick (1980), Boyer and Hodrick (1982), Riley (1982), Camilleri, et. al. (1984), and Kawai (1985) in specifying direct wealth effects in the money demand and private expenditure functions. Dornbusch and Fisher (1980) and Kimbrough (1985) both set $L_3=0$ while Scarth (1987a) specifies no direct wealth effects, setting $C_3=L_3=0$.

An assumption common to all the studies referred to above is that they assume a unitary system of government. By assuming a federal system of government our model therefore differs in an important way from all previous models appearing in the literature.

3. Alternative Government Financing Options

Following our practice in essay two, we will limit our attention to just two financing policies for the federal government. One option is to adopt a monetary rule which we will define as requiring $\dot{m}=0$. This form of monetary rule is often identified as an "accommodating" monetary policy in that the nominal money supply is allowed to grow endogenously at a rate just sufficient to satisfy transaction needs which in turn grow at a rate equal to the sum of the rates of inflation and output growth. An alternative form of a monetary rule is to assume the nominal money supply grows at an exogenously determined constant rate. There are three reasons for our choosing to adopt the "accommodating" form of the monetary rule. First, its use is widespread having been adopted by Sargent (1977), Turnovsky (1979), McCallum (1981), and Scarth (1987b) in closed economy models, and recently by Kimbrough (1985) in an open economy model. Second, adoption of the alternative form of the monetary rule would result in a fourth differential equation in our stability analyses making it extremely difficult to derive analytical results of any easy to interpret form. Finally, in a simulation study examining the dynamic effects of these alternative monetary rules, Nguyen and Turnovsky (1983) have found that the dynamic behaviour of their model is in general quite similar under these two alternative definitions. Thus, we would argue little is lost by choosing one form of the monetary rule over the other. Note that an implication of this rule is that b^f is the endogenous variable in the federal GBC so the federal government can be said to be bond financing disturbances to its budget position.

The second financing option available to the federal government is to adopt an accommodating bond growth policy such that $\dot{b}^f = 0$. Analogous to the monetary rule option, the stock of federal government bonds is here allowed to grow endogenously at a rate sufficient to maintain at a constant level the ratio of the real value of federal bonds to output. An implication of this policy is that m is the endogenous variable in the federal GBC so the federal government can be said to be money financing disturbances to its budget position.

The two financing options available to the non-federal government sector are also the same as those considered in essay two. The first is an accommodating bond growth policy where $\dot{b}^s=0$ so that t^s is endogenous in the non-federal GBC and non-federal governments can be said to be tax financing disturbances to their budgets. The second option is that of deficit (bond) financing so that b^s is endogenously determined.

In consideration of the fact that in this essay we are dealing with an open economy, there is one further set of financing assumptions we need to consider in addition to the four described above -- that of tax financing by both levels of government (t^f and t^s endogenous; m, b^f and b^s exogenous). The reason for our interest in this case is not that we feel it is empirically relevant -- it is clearly not -- but is rather due to the following consideration. In an open economy we have an asset accumulation identity (the balance of payments constraint) not present in our closed economy model. This implies there will always be an additional condition for convergence over and above what we found in our closed economy model. This additional condition will define the circumstances under which b^* will converge to a steady-state value. A number of works in the literature have made the simplifying assumption that government tax finances disturbances to its budget and hence have isolated the conditions for convergence to that which causes the balance of payments constraint to converge (see Rodriquez (1979), Hodrick (1980), and Kawai (1985)). In each of these studies, convergence is by no means assured so that even in the absence of government deficit financing the economy may be inherently unstable. Our strategy will be to derive this condition for convergence in the absence of government deficit financing and simply assume it is satisfied. This will allow us to proceed with our examination of the implications for convergence of various government deficit policies within the framework of an economy which is stable in the absence of these policies. Only in this way can we clearly identify any additional conditions for convergence caused by various combinations of government deficit financing choices.

4. Conditions for Convergence under Pure Tax Financing

In this section we make the unrealistic assumption that both levels of government tax finance (i.e., adjust their tax rates endogenously holding m, b^f and b^s constant) disturbances to their budget positions. This isolates the balance of payments constraint (defined in (17)) as the only dynamic equation in our model. This equation defines how the model moves from one instantaneous equilibrium to another by changing the stock of bonds held by foreign residents. Convergence requires simply that $\partial \dot{b}^*/\partial b^* < 0$. Differentiating (17) we find;

$$(18) \qquad \partial\dot{b}^*/\partial b^* = -[n-(r(1-h^f- h^s)-p)] - \partial x/\partial b^* + b^*(1-h^f- h^s)\partial r/\partial b^* - b^*\partial p/\partial b^*$$

The rest of the model (equations (11) - (16)) consists of static equations describing each of these instantaneous equilibriums. Solving these six static equations we derive the solutions for the short-run multipliers $\partial x/\partial b^*$, $\partial r/\partial b^*$, and $\partial p/\partial b^*$ appearing in (18) (in the short run, endogenous variables are y^d, r, p, x, t^f, and t^s while b^* is exogenous since it takes on a given value in each instantaneous equilibrium and changes in value only between such equilibriums). Using Cramer's rule we solve for;

$$\partial r/\partial b^* = \partial p/\partial b^* = (1-h^f- h^s)\ L_3/L_2 < 0$$

$$\frac{\partial x}{\partial b^*} = \frac{(1-h^f-h^s)\{C_3+C_1[r(1-h^f-h^s)-p]-C_1(h^f+h^s)b^*(1-h^f-h^s)L_3/L_2\}}{1 - C_1(h^f+ h^s)} > 0.$$

Substituting these expressions into (18) we can re-write the condition for convergence as;

$$\partial\dot{b}^*/\partial b^* = -D$$

where

$$(19) \qquad D = n-(1-C_1)[r(1-h^f-h^s)-p]/[1-C_1(h^f+h^s)] + (1-h^f-h^s)\gamma$$

and

$$\gamma = \{C_3 + (h^f+ h^s)b^*(1-C_1)L_3/L_2\}/[1-C_1(h^f+ h^s)] \gtrless 0.$$

Convergence therefore requires $D>0$ but inspection of (19) shows that D is of ambiguous sign. Thus convergence is by no means assured even in the absence of government deficit financing. This result follows

Rodriquez (1979), Hodrick (1980), and Kawai (1985) all of whom assume government tax financing. Note that if n=0, then the direct wealth effect in the private expenditure function ($C_3>0$) is necessary for convergence -- a result derived also by Kawai (1985) who assumes zero growth.[4]

In the analyses of stability which follow, we will therefore simply assume D>0 so that in the absence of government deficit financing the domestic economy is stable. The strategy in what follows then, will be to identify any conditions in addition to D>0 which must be satisfied to ensure convergence under alternative government deficit financing schemes.

5. Conditions for Convergence under a Monetary Rule

In this section we assume the federal government has adopted the monetary rule we described above. Setting $\dot{m}=0$ and substituting (15) and (16) into (12) for $\dot{b}^f$ and $\dot{b}^s$ respectively, equation (12) becomes:

$$(20)\qquad y^d = 1 - (1-\alpha)(g^f+ g^s) + (1-\beta)mn - b^*(1-h^f- h^s)[r(1-h^f- h^s)-p]$$

$$+ (h^f+ h^s)x + \beta\{g^f+ g^s+ r(b^f+ b^s)-(t^f+ t^s)(1+rb^f+ rb^s- rb^*)$$

$$- p(m + b^f+ b^s) - r(h^f+ h^s)b^*\}.$$

The model we consider in this section therefore consists of static equations (11), (13), (14), and (20), and asset accumulation identities (15) (with $\dot{m}=0$), (16), and (17).

The asset accumulation identities drive the model from one instantaneous equilibrium to another by changing the values of b^f, b^s and b^*. The static equations describe each of these instantaneous equilibriums within which values of b^f, b^s and b^* are given. Thus we can solve the static equations for values of variables endogenous in the instantaneous equilibriums (y^d, r, p, and x) in terms of b^f, b^s, b^* and other exogenous variables. The partial derivatives of these reduced forms therefore describe short run multipliers. Some of these partial derivatives are given by the following.

(21) $\partial r/\partial b^f = \partial r/\partial b^s = \partial p/\partial b^f = \partial p/\partial b^s = -\beta L_3/L_2 \geq 0$

(22) $\partial r/\partial b^* = \partial p/\partial b^* = (1-h^f- h^s)L_3/L_2 < 0$

(23) $\partial x/\partial b^f = \partial x/\partial b^s = -\beta\{\epsilon + \beta C_1\mu\} \gtrless 0$

(24) $\partial x/\partial b^* = (1-h^f- h^s)\{\tilde{\epsilon} + \beta C_1\mu\} - \beta\omega \gtrless 0$

where

$$\mu \equiv [m + (t^f+ t^s)(b^f+ b^s- b^*) + (h^f+ h^s)b^*](L_3/L_2)/[1-C_1(h^f+ h^s)] < 0$$

$$\epsilon \equiv [C_3+C_1(r(1-t^f-t^s)-p)-C_1(1-h^f-h^s)b^*(h^f+h^s)L_3/L_2]/[1-C_1(h^f+h^s)]>0$$

$$\tilde{\epsilon} \equiv [C_3+C_1(r(1-h^f-h^s)-p)-C_1(1-h^f-h^s)b^*(h^f+h^s)L_3/L_2]/[1-C_1(h^f+h^s)]>0$$

$$\omega \equiv C_1 \ (t^f+ t^s- h^f- h^s)r/[1-C_1(h^f+ h^s)] \gtrless 0.$$

The fact that any exogenous shock has the same effect on r as on p is as expected given (14) and our assumption that r^* and p^* are exogenously determined. If β=0 so that the equivalence hypothesis is imposed, then changes in b^f and b^s can have no effect on aggregate demand. Not surprisingly then, the expressions in (21) and (23) are zero given this assumption. If however $\beta>0$, then an increase in b^f or b^s causes an increase in the interest income and wealth of domestic residents. This causes a rightward shift in IS, a leftward shift in LM, and an exchange rate depreciation. As a result, the domestic interest rate (and hence p) must rise if $\beta > 0$ while the effect on the ratio of net exports to output remains ambiguous since the increases in interest income and wealth influence net exports in the opposite direction from the effect due to the exchange rate depreciation. An increase in foreign holdings of domestic bonds (b^*) causes a reduction in domestic wealth regardless of the value of β. As a result an increase in b^* causes a leftward shift in IS, a rightward shift in LM and an exchange rate appreciation. As a result, $\partial r/\partial b^* = \partial p/\partial b^* < 0$ regardless of the value of β. Once again, however, the effect on net exports is ambiguous as the reduction in wealth and interest income affects net exports in the opposite direction from the effect due to the exchange rate appreciation.

(i) Non-Federal Tax Financing ($\dot{b}^s = 0$):

In this sub-section we assume the non-federal government sector is tax financing disturbances to its budget position (i.e., we assume it is maintaining a constant ratio of non-federal bonds to output). Our interest in this case is two-fold. First, it is an assumption implicitly made by the literature and we wish to review the results of the literature. Second, only by deriving results for this case can we identify any new conditions for convergence which might arise due to the more realistic case of non-federal deficit financing.

Before proceeding with our examination of this case, we should note that the short run multipliers derived above assumed non-federal governments were bond financing (b^S endogenous). When non-federal governments tax finance, the non-federal GBC (equation (16)) becomes a static equation and as a result the short run multipliers should be calculated using this additional equation. Only the definitions of $\partial x/\partial b^f$ and $\partial x/\partial b^*$ are affected (for our analysis of stability in this sub-section we do not need to know $\partial x/\partial b^S$) and these become;

$$(23') \quad \partial x/\partial b^f = -\beta\{\epsilon + \beta C_1\tilde{\mu} + rC_1t^S/[1-C_1(h^f + h^S)]\} \gtrless 0$$

$$(24') \quad \partial x/\partial b^* = (1-h^f - h^S)\{\tilde{\epsilon} + \beta C_1\tilde{\mu}\} - \beta\,\tilde{\omega} \gtrless 0$$

where $\tilde{\mu} \equiv [m + t^f(b^f + b^S - b^*) + h^f b^*](L_3/L_2)/[1-C_1(h^f + h^S)] < 0$

$$\tilde{\omega} \equiv C_1(t^f - h^f)r/[1-C_1(h^f + h^S)] \gtrless 0.$$

These expressions differ only slightly in magnitude as compared to their previous definitions in (23) and (24). The sign ambiguities remain however, for the reasons discussed above.

When non-federal government tax finance, our model contains just two asset accumulation identities -- the federal GBC and the balance of payments constraint. The characteristic equation of this system of dynamic equations can be written in matrix form as;

$$(25) \qquad \begin{bmatrix} \partial\dot{b}^*/\partial b^* - \lambda & \partial\dot{b}^*/\partial b^f \\ \partial\dot{b}^f/\partial b^* & \partial\dot{b}^f/\partial b^f - \lambda \end{bmatrix} = 0$$

where λ is a characteristic root.

Differentiation of dynamic equations (15) and (17) yields the following expressions for the terms in (25). (These have been simplified using the fact that $\partial p/\partial b^f = \partial r/\partial b^f$ and $\partial p/\partial b^* = \partial r/\partial b^*$).

$$\partial\dot{b}^*/\partial b^* = - H - \partial x/\partial b^* - b^*(h^f + h^s)\partial r/\partial b^*$$

$$(26) \qquad \partial\dot{b}^*/\partial b^f = -\partial x/\partial b^f - b^*(h^f+h^s)\partial r/\partial b^f$$

$$\partial\dot{b}^f/\partial b^* = r(t^f-h^f) - [m + t^f(b^f + b^s - b^*) + h^f b^*]\partial r/\partial b^*$$

$$\partial\dot{b}^f/\partial b^f = - T - [m + t^f(b^f+b^s-b^*) + h^f b^*]\partial r/\partial b^f$$

where $T \equiv [n - (r(1-t^f)-p]$

$H \equiv [n - (r(1-h^f - h^s)-p]$.

The Routh-Hurwitz theorem requires that the following two necessary and sufficient conditions be satisfied for the model to coverage to a new equilibrium after having been disturbed;

$$(27) \qquad (\partial\dot{b}^*/\partial b^*) + (\partial\dot{b}^f/\partial b^f) < 0$$

$$(28) \qquad (\partial\dot{b}^*/\partial b^*)(\partial\dot{b}^f/\partial b^f) - (\partial\dot{b}^*/\partial b^f)(\partial\dot{b}^f/\partial b^*) > 0.$$

Substituting (21), (22), (23') and (24') into the expressions defined in (26) enables us to re-write (27) and (28) as requiring;

$$(29) \qquad D + T - \beta\{\tilde{\omega} + \tilde{\mu}(1-C_1)\} > 0$$

and

(30) $\quad DT - \beta\{n\tilde{\omega} + r(t^f - h^f)\gamma + \tilde{\mu}H(1-C_1)\} > 0.$

Due to the ambiguous signs of many of the terms in (29) and (30), little can be said about the likelihood of these conditions being satisfied without some further assumptions. In the literature, a number of assumptions have been made to simplify (29) and (30) and thus allow for an unambiguous answer to the question of whether the model is stable. The early fixed price models of Scarth (1975, 1977) and Allen (1977) simplifed these expressions by assuming β=1, n=0, with h^f=0 (and, because these were models of unitary states, h^s=b^s=0). With these assumptions (29) and (30) become, respectively,

$$-(r-p) - \{(1-C_1)[r(1-t^f)-p] - C_3 + (1-C_1)[m+t^f(b^f-b^*)]L_3/L_2\} > 0$$

and

$$(r-p)\{(1-C_1)[r(1-t^f)-p] - C_3 + (1-C_1)[m+t^f(b^f-b^*)]L_3/L_2\} > 0.$$

Given an assumption that (r-p)>0, one of these conditions must fail so that the model is unambiguously unstable -- a result derived by Scarth and by Allen. This instability result carries over to later models with flexible prices and less than perfect foresight, when the assumptions of β=1, n=0, and h^f=0 are retained. For example, when Kawai (1985) considers a bond financing government, he finds a strong tendency towards instability as a small nominal interest rate is necessary to satisfy one stability condition but results in a tendency for the other condition to fail. Finally, there are those models which assume perfect foresight and impose the assumptions of β=1, n=0, and h^f=0. Basevi and Giavazzi (1986) conclude that their model is unstable for positive real interest rates and use this feature of their model to investigate stabilization policies in an explosive open economy. In a simulation study, Camilleri, et. al. (1984) adopt a different monetary growth rule from ours (they assume the monetary authority restricts the nominal money supply to grow at a constant rate) and as a result must deal with an extra differential equation. Their conclusions regarding stability are basically the same however -- given these assumptions it is not possible for all of their conditions for stability to be satisfied simultaneously. As the unstable root of their system of dynamic equations contains variables endogenous to the model, they assume these variables "jump" to steady-state values necessary to eliminate the influence of the unstable root. With respect to models with zero growth and the Keynesian assumption of $\beta=1$ therefore, our model generates results consistent with those found in the literature.

Another way of simplifying the conditions in (29) and (30) is to impose the equivalence hypothesis by letting $\beta=0$. By inspection, satisfaction of (30) new requires that the terms D and T have the same sign and satisfaction of (29) requires that both these terms be positive. It is interesting to note that Kimbrough (1985) investigates the long-run comparative statics of a model employing the assumption of $\beta=0$ and an assumption of zero economic growth ($n=0$) without analyzing the stability of properties of his model. From our results above, if $n=0$ then T is clearly negative causing one to question the validity of comparative static results. This result that the model is unambiguously unstable if one assumes $\beta=0$ and $n=0$ is exactly the result McCallum (1981) derives for the case of a closed economy (also see essay two above) so that we see here that positive economic growth is also a necessary condition for stability in an open economy when $\beta=0$. Scarth (1987a) derives this same result in his open economy model.

If we follow Scarth (1987a) in assuming $C_3=L_3=0$ (as well as $\beta=0$) then $\gamma=0$ and our conditions for stability become $T > 0$ and

$$D' = n - (1-C_1)[r(1-h^f-h^s)-p]/[1-C_1(h^f+h^s)] > 0$$

which are precisely the conditions derived by Scarth.[5] Our discussion in section 4 indicates that $D'>0$ is a condition which must be satisfied even if both levels of government tax finance and this is a result one can derive as well from Scarth's model if it is assumed the government in his model of a unitary state tax finances disturbances to its budget position.[6] The stability condition which must be satisfied as a consequence of the federal government's decision to obey a monetary rule is therefore only $T>0$ and not, as a reading of Scarth might lead one to conclude, $D'>0$ as well -- the latter condition must be satisfied whether or not a monetary rule is obeyed. Which of these two conditions is the more stringent therefore determines whether the governments decision to obey a monetary rule makes stability more difficult. If $D'>0$ is the more stringent condition, then the decision to obey a monetary rule causes <u>no</u> problem for stability in an open economy -- a dramatically different result than that derived for a closed economy. As D' and T contain only terms whose value we know with a good degree of certainty, we can easily make this comparison. $D'>0$ is the more stringent condition for stability if

$$(1-C_1)[r(1-h^f- h^s)-p]/[1-C_1(h^f+ h^s)] > [r(1-t^f)-p].$$

For reasonable values of other parameters (C_1 = 0.9, t^f = 0.25, r = 0.08, p = 0.04) it is easy to show that D'>0 is the more stringent condition only if (h^f+h^s) > 1.6 so that we concur with Scarth that given these assumptions, T>0 is the more stringent condition.[7] Note that relaxing Scarth's assumption that C_3=L_3=0 (so that γ=0) would be unlikely to influence this conclusion. That is, in a survey of the literature, Laidler (1977, chapter 7) found that most empirical studies estimate the interest rate elasticity of money demand to be approximately -0.5. Thus our L_2 = -0.5 M/rPY. Using r = 0.08 and recent Canadian data, L_2 = -0.43.[8] Even assuming low estimates of L_2(-0.3), and C_1(0.7) and high estimates of L_3(0.2), b*(0.4) and h^f+h^s(0.5), the term $(h^f+h^s)b^*(1-C_1)L_3/L_2$ is equal to only -0.04. As the value of C_3 is typically judged to be in excess of this value (Nguyen and Turnovsky (1983) for example judge C_3 = 0.05 and the recent study by Duguay and Rabeau (1987) sets C_3 = 0.045), it seems safe to assume γ > 0. This assumption makes D>0 a less stringent condition than D'>0 so that T>0 remains the more stringent condition for convergence whether or not one assumes direct wealth effects.

Now let us return to the general form of the stability conditions described in (29) and (30) where $0<\beta<1$ and n, L_3, C_3 > 0 and let us assume γ>0 for the reasons discussed above. Further, we assume D>0 so that the economy is stable in the absence of government deficit financing. An important determinant of whether these conditions are satisfied is the size of the federal withholding tax rate (h^f) relative to the federal income tax rate (t^f). If $t^f \geq h^f$, then $\tilde{\omega} \geq 0$ and the term containing γ is ≥ 0. In this case, a necessary condition for (30) to be satisfied is that at least one of T and H be positive -- if both these terms are negative, the condition in (30) cannot be satisfied.[9] If however, $t^f < h^f$, then $\tilde{\omega} < 0$, as is the term containing γ. It is now neither necessary nor sufficient for T or H to be positive -- both can be negative and (29) and (30) can still be satisfied. Therefore, if $t^f \geq h^f$ we derive an important result for the open economy which was not obtained for this set of government financing choices in the closed economy. That is, so long as $t^f \geq h^f$, then either T or H must be positive even though $\beta > 0$.

The intuition of why the relative magnitudes of t^f and h^f plays an important role in determining the likelihood of convergence is as follows. An increase in foreign purchases of domestic bonds (i.e. an increase in b*) has a conflicting effect on total tax revenue. By reducing the interest income earned by domestic residents, the income tax base is reduced thus reducing income tax revenue. At the same time, however, an increase in b* increases the tax base against which the withholding tax is applied. If $t^f > h^f$, the net effect of an increase in b* is therefore to decrease total tax revenue. If the federal government is bond financing therefore, any increase in b* due to a current account deficit causes a reduction in tax revenue and consequently makes it more difficult to close a given federal government deficit. If however, $t^f < h^f$, then an increase in b* causes a net increase in tax revenue making it somewhat easier to close a given federal deficit. Thus convergence conditions are more stringent of $t^f > h^f$. This effect, which of course is not present in a closed economy, is strong enough to require that the real growth rate exceed a real after tax interest rate (either T or H must be positive) regardless of the value of β -- a condition necessary for convergence in the closed economy only under the extreme assumption that $\beta=0$.

This is an important result as it implies that in an open economy with a federal government which has chosen to satisfy a monetary growth rule, one need not accept the extreme assumption of the equivalence hypothesis to be concerned about the relative magnitudes of real growth rates and real after tax rates of return. These magnitudes must be of concern to all economists regardless of whether they accept the Keynesian view that $\beta=1$, the monetarist view that $\beta=0$, or whether they believe $0<\beta<1$.

(ii) Non-Federal Bond Financing ($\dot{b}^S \neq 0$):

The results from the previous sub-section require that one assume non-federal governments adjust their income tax rates endogenously in response to disturbances to their budget positions. In this sub-section we assume non-federal governments bond finance any change to their budget positions. This is the empirically more relevant assumption but one that has not been investigated in the literature.

If non-federal governments bond finance, the non-federal GBC (equation (16)) becomes a dynamic equation along with the federal GBC (15) and the balance of payments constraint (17). The short-run comparative static results in (21)-(24) are derived using static equations (11), (13), (14), and (20) as described above. With an additional dynamic asset accumulation identity, the characteristic equation of this system of equations becomes;

$$(31)\quad \begin{bmatrix} \partial\dot{b}^*/\partial b^* - \lambda & \partial\dot{b}^*/\partial b^f & \partial\dot{b}^*/\partial b^s \\ \partial\dot{b}^f/\partial b^* & \partial\dot{b}^f/\partial b^f - \lambda & \partial\dot{b}^f/\partial b^s \\ \partial\dot{b}^s/\partial b^* & \partial\dot{b}^s/\partial b^f & \partial\dot{b}^s/\partial b^s - \lambda \end{bmatrix} = 0$$

where λ is a characteristic root.

Differentiation of dynamic equations (15)-(17) yields expressions for the terms in (31). These expressions are described in (26) above and by the following (again, these have been simplified using the fact that $\partial p/\partial b^s = \partial r/\partial b^s$).

$$\partial\dot{b}^*/\partial b^s = -\partial x/\partial b^s - b^*(h^f+h^s)\partial r/\partial b^s$$

$$(32)\quad \partial\dot{b}^f/\partial b^s = -\, rt^f - [m + t^f(b^f+b^s-b^*) + h^f b^*)]\partial r/\partial b^s$$

$$\partial\dot{b}^s/\partial b^f = -\, rt^s - [t^s(b^f+b^s-b^*) + h^s b^*)]\partial r/\partial b^f$$

$$\partial\dot{b}^s/\partial b^s = -[n - (r(1-t^s)-p)] - [t^s(b^f+b^s-b^*) + h^s b^*]\partial r/\partial b^s$$

$$\partial\dot{b}^s/\partial b^* = r(t^s-h^s) - [t^s(b^f+b^s-b^*) + h^s b^*]\partial r/\partial b^*$$

Expanding (31) gives us the following form of the characteristic equation;[10]

$$(33)\quad \lambda^3 + \lambda^2\{N - [K+J]\} + \lambda\{[KJ-RV] - N[K+J]\} + N[KJ-RV] = 0$$

where $N \equiv [n-(r-p)]$

$K \equiv \partial\dot{b}^*/\partial b^*$

$J \equiv [\partial\dot{b}^f/\partial b^s + \partial\dot{b}^s/\partial b^s]$

$R \equiv \partial\dot{b}^*/\partial b^s$

$V \equiv [\partial\dot{b}^f/\partial b^* + \partial\dot{b}^s/\partial b^*]$.

From the Routh-Hurwitz theorem, our model will converge to a steady-state only if the following necessary and sufficient conditions are satisfied;

(a) $N - [K+J] > 0$

(b) $[KJ-RV] - N[K+J] > 0$

(c) $N[KJ-RV] > 0$

(d) $-[K+J]\{N^2 + [KJ-RV] - N(K+J)\} > 0.$[11]

By inspection of the conditions in (a)-(d), we can simplify the necessary and sufficient conditions for convergence. That is, satisfaction of (c) requires that N and [KJ-RV] have the same sign. Assuming (b) is satisfied, satisfaction of (d) requires $-[K+J] > 0$. This, plus the condition that N and [KJ-RV] have the same sign, and the requirement that (b) be positive, implies N and [KJ-RV] both be positive. Therefore, the conditions $N > 0$, $-[K+J] > 0$, and $[KJ-RV] > 0$ are all necessary for convergence and by inspection of (a)-(d), they are also sufficient conditions.

Substituting (26) and (32) into our definitions of $-[K+J]$ and $[KJ-RV]$ and using the short-run comparative static results in (21)-(24) to simplify the result enables us to show that;

$$-[K+J] = D + [n-(r(1-t^f-t^s)-p)] - \beta\{\omega + \mu(1-C_1)\}$$

$$[KJ-RV] = D[n-(r(1-t^f-t^s)-p)] - \beta\{n\omega + r(t^f+t^s-h^f-h^s)\gamma + \mu H(1-C_1)\}.$$

Except for differences in the definitions of μ versus $\tilde{\mu}$, ω versus $\tilde{\omega}$ and T versus $[n-(r(1-t^f-t^s)-p]$, these terms are exactly the conditions for convergence defined in (29) and (30) for the case of tax financing non-federal governments. Assuming these differences have an insignificant impact on the sign of these terms, it can therefore be argued that the decision of non-federal governments to bond rather than tax finance does not significantly alter our perception of the likelihood of these conditions being satisfied.

The extra condition which must be satisfied when non-federal governments bond finance is therefore simply that N be positive. There are two important points to note about this condition. First, it is a condition which must be satisfied regardless of the value of β and the relative magnitudes of the federal income and withholding tax rates, and second, it is a significantly more stringent condition than either $T>0$ or $H>0$. Recall that if non-federal governments tax finance, one of the weaker conditions that T or H be positive was necessary only if $t^f \geq h^f$. When non-federal governments bond finance, whatever the values of income and withholding tax rates, convergence requires that N be positive.

This result, that when non-federal governments deficit finance $N > 0$ is necessary for convergence regardless of the value of β, parallels our findings for a closed economy discussed in our second essay. Its implication is that concerns regarding the relative magnitudes of growth rates and real before tax interest rates should be independent of whether one is a monetarist (and hence believes $\beta = 0$) or a non-monetarist ($0 < \beta < 1$) and that this is true regardless of whether the economy is open or closed.

6. Conditions for Convergence Under Money Financing

In this section we assume the federal government has adopted a policy of maintaining a constant debt/GNP ratio ($\dot{b}^f = 0$). As a consequence of this policy, any disturbance to the federal GBC is financed by a change in the money supply ($\dot{m} \neq 0$) -- hence the federal government may be said to be money financing. Assuming for the moment that non-federal governments are bond financing ($\dot{b}^s \neq 0$), we can substitute (15) and (16) into (12) for $\dot{m}$ and $\dot{b}^s$ respectively. This enables us to re-write the definition of real disposable income as;

$$
\begin{aligned}
(34) \qquad y^d = {} & 1 - (1-\alpha)g^s + \alpha g^f + b^f[r(1-t^f)-p] - b^f(1-\beta)n \\
& - t^f(1 + rb^s) - (h^f + h^s)x - pm \\
& - b^*\{(1-h^f-h^s)[r(1-h^f-h^s)-p] - r(t^f-h^f)\} \\
& + \beta\{g^s + b^s[r(1-t^s)-p] - t^s(1 + rb^f) + b^*(t^s- h^s)\ r\}.
\end{aligned}
$$

The model we consider in this section therefore consists of static equations (11), (13), (14) and (34), and asset accumulation identities (15), (16) and (17). The static equations can be solved for values of variables endogenous in the short run (y^d, r, p, x) in terms of variables exogenous during this period but endogenous between each short run period (m, b^s, b^*). The partial derivatives of these reduced forms therefore describe short run multipliers. Some of these are given by the following;

$$(35) \quad \partial r/\partial m = \partial p/\partial m = (1-L_3)/L_2 \quad < \quad 0$$

$$(36) \quad \partial r/\partial b^S = \partial p/\partial b^S = -\beta L_3/L_2 \quad \geq \quad 0$$

$$(37) \quad \partial r/\partial b^* = \partial p/\partial b^* = (1-h^f-h^S)\ L_3/L_2 \quad < \quad 0$$

$$(38) \quad \partial x/\partial b^S = -\beta\{C_1\delta L_3/L_2+\theta\} + rC_1(t^f+\beta t^S)/[1-C_1(h^f+h^S)] \gtrless 0$$

$$(39) \quad \frac{\partial x}{\partial b^*} = (1-h^f-h^S)\{C_1\delta L_3/L_2+\theta\} - \frac{rC_1[t^f+\beta t^S+h^S(1-\beta)-(h^f+h^S)(h^f+h^S)]}{1-C_1(h^f+h^S)} \gtrless 0$$

$$(40) \quad \partial x/\partial m = C_1\delta(1-L_3)/L_2 - [C_3-pC_1]/[1-C_1(h^f+h^S)] \gtrless 0$$

$$\text{where } \delta \equiv \frac{m + (t^f+\beta t^S)(b^f+b^S-b^*) + b^*(h^f+h^S)(h^f+h^S) - b^*(1-\beta)h^S}{1-C_1(h^f+h^S)} \gtrless 0$$

$$\theta \equiv [C_3 + C_1(r-p)]/[1-C_1(h^f+h^S)] > 0.$$

The expressions in (36) and (37) are the same as those derived in section 5 (see (21) and (22)) and their signs are due to the reasons discussed there. Similarly, the ambiguity of the signs of (38) and (39) are due to reasons discussed in section 5. Finally, the ambiguity of the sign of (40) is due to the fact that if m increases, this not only increases wealth but also causes an exchange rate depreciation by causing a rise in the nominal interest rate (see (35)). As these influences have opposite affects on net exports, the sign of (40) is ambiguous.

(i) Non-Federal Tax Financing ($\dot{b}^s = 0$):

As was the case in section 5, we begin our analysis of convergence conditions under the assumption of a money financing federal government by first assuming non-federal governments tax finance disturbances to their budget positions. As always, the conditions for convergence we derive under this assumption are equivalent to those one would derive if a unitary state was assumed. Since the literature assumes a unitary state, our interest in this case is therefore mainly to review the literature. It will also be necessary to examine this case so as to identify any additional stability conditions which arise due to the existence of bond financing non-federal governments.

As always, the assumption of tax financing non-federal governments implies the non-federal GBC becomes a static equation to be used in the calculation of short run multipliers. The multipliers in (35) - (40) assumed non-federal bond financing so that the non-federal GBC was not used in their calculation. When non-federal governments tax finance, only the expressions in (39) and (40) are affected and these become;

$$(39') \quad \partial x/\partial b^* = (1-h^f-h^s)\{C_1 \emptyset L_3/L_2+\theta\} - rC_1[t^f+h^s-(h^f+h^s)(h^f+h^s)]/[1-C_1(h^f+h^s)] \gtrless 0$$

$$(40') \quad \partial x/\partial m = C_1\emptyset(1-L_3)/L_2 - [C_3-pC_1]/[1-C_1(h^f+h^s)] \gtrless 0$$

$$\text{where } \emptyset = \frac{m + t^f(b^f+b^s-b^*) + b^*(h^f+h^s)(h^f+h^s) - b^*h^s}{1 - C_1(h^f+h^s)} \gtrless 0.$$

The magnitude of these short run comparative static results change slightly from their values in (39) and (40), but the sign ambiguity remains.

With only two asset accumulation identities -- the federal GBC and the balance of payments constraint -- the characteristic equation of this system of dynamic equations can be written in matrix form as;

$$(41)\qquad \begin{vmatrix} \partial\dot{m}/\partial m - \lambda & \partial\dot{m}/\partial b^* \\ \partial\dot{b}^*/\partial m & \partial\dot{b}^*/\partial b^* - \lambda \end{vmatrix} = 0$$

where λ is a characteristic root.

Differentiation of the dynamic equations (15) and (17) yields the following expressions for the terms in (41) (these have been simplified by using the fact that $\partial r/\partial m = \partial p/\partial m$, $\partial r/\partial b^S = \partial p/\partial b^S$, and $\partial r/\partial b^* = \partial p/\partial b^*$).

$$(42)\qquad \begin{aligned} \partial\dot{m}/\partial m &= -(n + p) - [m + t^f(b^f+b^S-b^*) + h^f b^*]\partial r/\partial m \gtrless 0 \\ \partial\dot{m}/\partial b^* &= r(t^f-h^f) - [m + t^f(b^f+b^S-b^*) + h^f b^*]\partial r/\partial b^* \gtrless 0 \\ \partial\dot{b}^*/\partial m &= -\partial x/\partial m - b^*(h^f+h^S)\partial r/\partial m \gtrless 0 \\ \partial\dot{b}^*/\partial b^* &= -H - \partial x/\partial b^* - b^*(h^f+h^S)\partial r/\partial b^* \gtrless 0. \end{aligned}$$

From the Routh-Horowitz theorem, the following two conditions constitute necessary and sufficient conditions for convergence:

$$(43)\qquad (\partial\dot{m}/\partial m) + (\partial\dot{b}^*/\partial b^*) < 0$$

$$(44)\qquad (\partial\dot{m}/\partial m)(\partial\dot{b}^*/\partial b^*) - (\partial\dot{m}/\partial b^*)(\partial\dot{b}^*/\partial m) > 0.$$

As each of the terms in (42) is of ambiguous sign, it is difficult to say if the conditions in (43) and (44) are satisfied. Note that assuming $\beta = 0$ does not enable us to sign any of the terms in (42) even though a number of terms in the definitions of $\partial x/\partial b^*$ and $\partial x/\partial m$ disappear. Despite the ambiguous signs of the terms in (43) and (44), it is possible to derive some useful information from these conditions.

Consider, first of all, the term $\partial\dot{m}/\partial m$ and recall that in the context of closed economy models there seems to be a good deal of agreement that this term is negative for reasonable parameter values (see for example, the discussion of Christ (1979) and Scarth (1987b)). If we refer back to the definition of $\partial\dot{m}/\partial m$ derived in our closed economy model of essay number two (see (23) in essay two), we can expand and re-write that definition in the following way;

$$\frac{\partial\dot{m}}{\partial m} = -(n + p) - [m + t^f(b^f+b^s-b^*) + h^f b^*]\partial r/\partial m$$

$$- \frac{(m+b^f)\{(C_3-pC_1)-C_1[m+t^f(b^f+b^s)](1-L_3)/L_2\}}{C_2 + C_1(m+b^f)} - b^*(t^f-h^f)(1-L_3)/L_2$$

where, following Turnovsky (1979), it was assumed that $(C_3-pC_1) > 0$ and $[C_2 + C_1(m+b^f)] < 0$. The first two terms in the above expression define the value of $\partial\dot{m}/\partial m$ in our open economy model (see (42)). The third term is positive while the sign of the last term is positive if $t^f > h^f$ but negative if $t^f < h^f$. If we assume $t^f \geq h^f$, then given the literature's belief that the closed economy definition of $\partial\dot{m}/\partial m$ is negative we can therefore conclude the literature would also unambiguously view the open economy definition of $\partial\dot{m}/\partial m$ is negative in both the open and closed economy versions of our model. Note that by assuming $t^f \geq h^f$, we can now sign $\partial\dot{m}/\partial b^* > 0$.

With the assumption of $\partial\dot{m}/\partial m < 0$ in mind, it is now useful to expand the stability condition in (44) using the expressions in (42) and the short run comparative static results in (35), (37), (39') and (40'). The condition in (44) now requires

$$(44')\quad -D\partial\dot{m}/\partial m - \{r(t^f-h^f)-[m+t^f(b^f+b^s-b^*)+h^f b^*](1-h^f-h^s)L_3/L_2\}\{nC_1 + C_3 - (h^f+h^s)b^*(1-C_1)(1-L_3)/L_2\}/[1-C_1(h^f+h^s)] > 0.$$

The terms in the first set of curved brackets are positive given our assumption that $t^f \geq h^f$, and the terms in the second set of curved brackets are unambiguously positive. Thus convergence requires $-D\partial\dot{m}/\partial m > 0$. As our discussion above indicates, it is a strongly held view of the literature that $\partial\dot{m}/\partial m$ is negative so that convergence requires simply that D be positive. Recall however, that $D > 0$ is the condition necessary for convergence when all governments tax finance -- a condition we assume is satisfied so that the economy is inherently stable in the absence of government deficit financing. With all these assumptions therefore, we conclude that the necessary condition for convergence defined in (44') (and hence the condition in (44)) is satisfied. Critical assumptions however are that the economy is stable in the absence of government deficit financing (so that $D > 0$) and that money financing is stable in a closed economy (so that $\partial\dot{m}/\partial m < 0$). Note that this result indicates that had Scarth (1987a) examined the conditions for convergence given money financing he would have found $D > 0$ was a necessary condition -- a condition he found was also necessary given bond financing.[12] Thus in a unitary state, $D > 0$ is a necessary condition for convergence regardless of the financing choice made by the (federal) government.

While $D > 0$ and $(\partial\dot{m}/\partial m) < 0$ are necessary for convergence, they are not sufficient. To see this, we expand (43) so as to re-write the other necessary condition for convergence as requiring

$$(43') \quad \partial\dot{m}/\partial m - D + \{\tilde{\omega} - C_1(1-h^f-h^s)\tilde{\mu}\} < 0$$

where $\tilde{\omega} \geq 0$ if $t^f \geq h^f$ so that the term in curved brackets is positive. Thus our assumptions $\partial\dot{m}/\partial m < 0$ and $D > 0$ are only necessary, not sufficient conditions for convergence.

An interesting question at this point is whether convergence is more or less likely when the federal government money rather than bond finances (assuming in both cases that non-federal governments are tax financing). Recall from section 5(i) that when the federal government bond finances, if we assume $D > 0$ and $t^f > h^f$ then a necessary condition for convergence is that either T or H must be positive. Our results above indicate that if the federal government money finances and if we assume $D > 0$ and $t^f > h^f$, then neither T nor H need be positive for convergence. Thus we derive a result consistent with those of Blinder and Solow (1973, 1976), and Christ (1979) who assume closed economies, as well as those of Scarth (1977) and Kawai (1985) who assume open economies -- that is, the economy is more likely to be unstable when the federal government bond finances than when it money finances.

In part (ii) of this section to be discussed below, we relax the assumption implicitly made in the literature that non-federal governments either do not exist or, if they are assumed to exist, they are assumed to tax finance disturbances to their budgets. Thus in subsection (ii) below, we assume non-federal governments are bond financing. In analyzing conditions for convergence we will assume the stability conditions defined in (43) and (44) are satisfied so that money financing in the absence of non-federal deficit financing is stable. This parallels the approach in our second essay and it enables us to isolate any additional stability conditions due solely to the decision of non-federal governments to bond rather than tax finance disturbances to their budget positions.

(ii) Non-Federal Bond Financing ($\dot{b}^S \neq 0$):

The analysis of convergence becomes significantly more complicated when we examine the more relevant case of bond financing non-federal governments. The non-federal GBC now becomes a dynamic equation along with the federal GBC and the balance of payments constraint. The characteristic equation of this system of dynamic equations can be written in matrix form as;

$$(45) \qquad \begin{bmatrix} \partial\dot{m}/\partial m - \lambda & \partial\dot{m}/\partial b^* & \partial\dot{m}/\partial b^S \\ \partial\dot{b}^*/\partial m & \partial\dot{b}^*/\partial b^* - \lambda & \partial\dot{b}^*/\partial b^S \\ \partial\dot{b}^S/\partial m & \partial\dot{b}^S/\partial b^* & \partial\dot{b}^S/\partial b^S - \lambda \end{bmatrix} = 0$$

where λ is a characteristic root.

Differentiation of the two GBC's and the balance of payments constraint yields the expression defined in (42) plus the following:

$$\partial\dot{b}^s/\partial m = -[t^s(b^f+b^s-b^*) + h^s b^*]\partial r/\partial m > 0$$

$$\partial\dot{b}^s/\partial b^* = r(t^s-h^s) - [t^s(b^f+b^s-b^*) + h^s b^*]\partial r/\partial b^* > 0$$

(46) $$\partial\dot{b}^s/\partial b^s = -[n-(r(1-t^s)-p)] - [t^s(b^f+b^s-b^*) + h^s b^*]\partial r/\partial b^s \gtrless 0$$

$$\partial\dot{m}/\partial b^s = -rt^f - [m + t^f(b^f+b^s-b^*) + h^f b^*]\partial r/\partial b^s < 0$$

$$\partial\dot{b}^*/\partial b^s = -\partial x/\partial b^s - b^*(h^f+h^s)\partial r/\partial b^s \gtrless 0$$

where the sign of $\partial\dot{b}^s/\partial b^*$ reflects our assumption that $t^s \geq h^s$.

By expanding (45), we can derive the necessary and sufficient conditions for convergence given this set of financing assumptions. However, such an expansion shows that even knowing the signs of a number of the elements in (45) and even assuming the conditions in (43) and (44) are satisfied, little can be said about the likelihood of these conditions being satisfied. The reason for this is that the specification of our model is very general. Most models in the GBC literature are special cases differing from ours by a simplifying assumption in specification. Our strategy then, will be to simplify our characteristic equation by imposing those assumptions which have proved to be popular in the literature. By doing so, we will be able to examine whether the conclusions found in the literature for the case of a unitary state are robust to our extension to a federal state.

An especially popular assumption in the literature is that of lump sum taxes ($t^f = t^s = h^f = h^s = 0$) having been adopted by Boyer and Hodrick (1982), Kawai (1985), and Kimbrough (1985) in their GBC models of open economies. This assumption greatly simplifies (45) as it implies $\partial\dot{b}^s/\partial m = \partial\dot{b}^s/\partial b^* = 0$ and $\partial\dot{b}^s/\partial b^s = -N$. With this assumption, expansion of (45) yields the following necessary and sufficient conditions for convergence;

(a) $N - A > 0$

(b) $F - NA > 0$

(c) $NF > 0$

(d) $-A\{N^2 + [F - NA]\} > 0$

where $A \equiv [(\partial\dot{m}/\partial m) + (\partial\dot{b}^*/\partial b^*)]$

$F \equiv [(\partial\dot{m}/\partial m)(\partial\dot{b}^*/\partial b^*) - (\partial\dot{m}/\partial b^*)(\partial\dot{b}^*/\partial m)]$

$N \equiv n - (r-p)$

and where we assume federal money financing is stable in the absence of non-federal deficit financing so that $A < 0$ and $F > 0$. Satisfaction of the condition in (c) requires that N be positive and by inspection we see that $N > 0$ is a sufficient as well as a necessary condition.

There are two important points to be made regarding this result. First, it is a condition which must be satisfied regardless of the value of β. Second, the requirement that the real growth rate be greater than the real interest rate (i.e. that $N > 0$) is the same condition which is necessary for convergence when both levels of government bond finance. Thus, if one assumes lump sum taxes, an insufficiently large growth rate relative to the real interest rate not only causes bond financing to be unstable but also causes money financing to be unstable. In such a situation then, federal deficit financing of either type causes government debt to explode. This conclusion, which is relevant for the case of a federal economy with deficit financing non-federal governments, is in sharp contrast to the finding of Kawai (1985) who models a unitary state and judges convergence to be feasible despite his assumption of zero output growth (n=0). Clearly, the results of Kawai's model are not robust to an extension to a federal economy.[13]

Another way of simplifying (45) is to follow Scarth (1975, 1977) and Allen (1977) -- both of whom assume fixed prices -- and Camilleri, et al. (1984) -- who assume flexible prices and perfect foresight -- in specifying non-lump sum income tax rates but no withholding taxes ($h^f = h^s = 0$) and who make the Keynesian assumption that $\beta = 1$. With these assumptions, expansion of (45) yields the following necessary and sufficient conditions for convergence:[14]

(a) $N + [(\partial \dot{b}^s/\partial b^s) - A] > 0$

(b) $N[(\partial \dot{b}^s/\partial b^s) - A] + [F - Z] > 0$

(c) $N[F - Z] > 0$

(d) $[(\partial \dot{b}^s/\partial b^s) - A]\{N^2 + N[(\partial \dot{b}^s/\partial b^s) - A] + [F - Z]\} > 0$

where $Z \equiv (\partial \dot{m}/\partial m)(\partial \dot{b}^s/\partial b^*) - (\partial \dot{m}/\partial b^*)(\partial \dot{b}^s/\partial m) < 0$

$$\frac{\partial \dot{b}^s}{\partial b^s} \equiv -[n - (r(1-t^s)-p)] + t^s(b^f+b^s-b^*)L_3/L_2 \gtrless 0.$$

The sign of Z is determined by our assumption that $(\partial\dot{m}/\partial m)$ is negative and by the known signs of $(\partial\dot{b}^S/\partial b^*)$, $(\partial\dot{m}/\partial b^*)$, and $(\partial\dot{b}^S/\partial m)$ given in (42) and (46). Assuming convergence in the absence of non-federal bond financing, we assume $F > 0$ (and $A < 0$) so that we can sign $[F - Z] > 0$. Satisfaction of the condition in (c) therefore requires that N be positive. Assuming the condition in (b) is satisfied, the term in curved brackets in (d) is positive so that (d) is satisfied only if $[(\partial\dot{b}^S/\partial b^S) - A]$ is positive -- a condition which may or may not be satisfied since if $N > 0$, then $(\partial\dot{b}^S/\partial b^S) < 0$. Therefore, $N > 0$ is only a necessary, not a sufficient, condition for convergence when non-federal governments deficit finance.

Once again, therefore, convergence requires that the real growth rate exceed the real, before tax interest rate even when the federal government money finances disturbances to its budget position. As this was not necessary for convergence in the absence of non-federal bond financing, we see that once again such behaviour by non-federal governments significantly reduces the likelihood of convergence given federal money financing. Note also that this result indicates that the conclusion of Scarth (1977) and Allen (1977) for the case of unitary states that convergence is at least a possibility when the (federal) government money finances would not appear to be robust to an extension to a federal economy with deficit financing non-federal governments. That is, both Scarth and Allen assume zero economic growth causing N to be negative.[15] Similarly, although Camilleri, et. al. (1984) do not investigate the likelihood of convergence when their (federal) government money finances, our result indicates their model would fail to converge if this option was investigated due to their assumption of zero growth.

A final simplification of (45) worth considering is to assume $L_3 = 0$ -- an assumption employed in the recent paper by Scarth (1987a). To simplify our analysis still further, we will assume income and withholding tax rates are equal (an assumption which seems realistic -- see footnote 7). Together, these assumptions imply $\partial\dot{m}/\partial b^* = \partial\dot{b}^S/\partial b^* = 0$, $\partial\dot{b}^*/\partial b^* = -D$, and $\partial\dot{b}^S/\partial b^S = -[n - r(1-t^S)-p)]$ so that expansion of (45) now yields the following necessary and sufficient conditions for convergence;

(a) $D - (\partial\dot{m}/\partial m) + Q > 0$

(b) $Q[D - (\partial\dot{m}/\partial m)] - D(\partial\dot{m}/\partial m) - (\partial\dot{m}/\partial b^S)(\partial\dot{b}^S/\partial m) > 0$

(c) $-D\{Q(\partial\dot{m}/\partial m) + (\partial\dot{m}/\partial b^S)(\partial\dot{b}^S/\partial m)\} > 0$

(d) $[Q-(\partial\dot{m}/\partial m)]\{D^2 + Q[D-(\partial\dot{m}/\partial m)] - D(\partial\dot{m}/\partial m)-(\partial\dot{m}/\partial b^S)(\partial\dot{b}^S/\partial m)\} > 0$

where $Q \equiv [n - (r(1-t^S)-p]$.

Given that $(\partial\dot{m}/\partial b^S) < 0$ and $(\partial\dot{b}^S/\partial m) > 0$ unambiguously, and given our assumptions $D > 0$ and $(\partial\dot{m}/\partial m) < 0$, we see by inspection that $Q > 0$ is a sufficient, but not necessary, condition for convergence. This result is of interest since with these same assumptions, the conditions $D > 0$ and $(\partial\dot{m}/\partial m) < 0$ are both necessary and sufficient for convergence when non-federal governments tax finance (see (43') and (44') and let $L_3 = 0$ and $t^f = h^f$). Here we see that if non-federal governments choose to deficit finance, $D > 0$ and $(\partial\dot{m}/\partial m) < 0$ are no longer sufficient so that convergence is somewhat more problematic.

An examination of the conditions for convergence when non-federal governments choose to deficit finance in the face of federal money financing is therefore extremely difficult as it involves an examination of a 3x3 characteristic matrix. In order to derive some useful analytical results, we have imposed some of the specification simplifications which have proved popular in the literature. These results are suggestive that when non-federal governments choose to deficit finance they significantly reduce the likelihood of convergence when the federal government money finances changes to its budget position. In fact, it would appear that in a federal economy with deficit financing non-federal governments, concern over the relative magnitudes of growth rates and real interest rates should be expressed when the federal government money finances as well as when it bond finances.

7. The Interdependence of Government Debt Management Policies

Due to the interdependence of GBC's in a federation we have noted that a policy initiative by one level of government should significantly influence the budget position of the other. In sections 5 and 6 above we confirmed that one effect of such policy interdependence is that stability conditions are sensitive in an important way to the particular financing decisions of both levels of government. Another effect of budget interdependence is that the ultimate outcome of any policy initiative will depend upon the response of the other level of government to the imbalance in its budget position caused by that initiative. Recently the federal government has been concerned with policies to reduce the federal debt/GNP ratio (i.e., to reduce the size of b^f). This creates a problem recently recognized by the Macdonald Commission:

> A potential source of significant tension in federal-provincial fiscal relations in the present context concerns the cutting and trimming of programs for financial reasons. Again, the problem flows from interdependence and from the likelihood that in the foreseeable future, all governments will be anxious to restrain expenditures and to manage their finances. The efforts of one order of government to do so will inevitably affect the budgetary situation of the other.[16]

In this section we examine this issue more closely by deriving the full equilibrium comparative static effects on the debt/GNP ratio of each level of government caused by one level of government's efforts to reduce expenditures on goods and services. We assume both levels of government bond finance budget disturbances (hence the federal government is following a monetary rule) as this assumption seems consistent with recent experience in Canada. To make comparative static results relevant, we must assume convergence. Thus we assume the necessary and sufficient conditions for convergence given these financing assumptions -- $N > 0$, $[K + J] < 0$, and $[KJ - RV] > 0$ -- are all satisfied. Finally, for simplicity we assume private economic agents view all government expenditures on goods and services as being equivalent to private expenditures so that we set $\alpha = 1$.

In full equilibrium, $\dot{b}^f = \dot{b}^s = \dot{b}^* = 0$ and endogenous variables are y^d, r, p, x, b^*, b^f, and b^s. Imposing these restrictions on our model (equations (11)-(17)) we can use Cramer's rule to solve for comparative static results. Consider first the effect on debt/GNP ratios of an effort by non-federal governments to reduce their expenditures on goods and services;

$$(47)\qquad \frac{db^S}{dg^S} = \frac{DT - \beta\{n\tilde{\omega} + r(t^f-h^f)\gamma + \tilde{\mu}H(1-C_1)\}}{[KJ - RV]N}$$

$$= \frac{D}{[KJ - RV]} + \sigma$$

$$(48)\qquad \frac{db^f}{dg^S} = -\sigma$$

$$\text{where } \sigma \equiv \frac{rt^fD - \beta\{n\tilde{\omega} + r(t^f-h^f)\gamma + \tilde{\mu}H(1-C_1)\}}{[KJ - RV]N} \gtrless 0.$$

The numerator of db^S/dg^S is exactly one of the necessary conditions for convergence we derive when non-federal governments are assumed to tax finance and the federal government bond finances (see (30)). Recall that in our discussion of the conditions for convergence when both levels of government bond finance (see section 5(ii)) we noted that this expression is very similar to [KJ - RV] in magnitude and that if one is positive the other is also most likely to be positive. Since we assume [KJ - RV] > 0, we should therefore sign $db^S/dg^S > 0$.

In (47) we re-write db^S/dg^S as consisting of two parts -- D/[KJ-RV] which is unambiguously positive given our assumption of convergence, and σ which is of ambiguous sign. It is useful to re-write db^S/dg^S in this manner since in (48) we note that the effect on the federal debt/GNP ratio due to the change in non-federal expenditures is exactly $-\sigma$. This is an important result as it illustrates that part of the effort by non-federal governments to reduce their debt/GNP ratio by reducing g^s is reflected in a "debt transfer effect" (σ). If σ is positive, the federal debt/GNP ratio (b^f) rises due to a decrease in g^s while the non-federal debt/GNP ratio (b^s) falls by an amount equal to not only $D/[KJ - RV]$ but by an amount σ as well. Thus, an amount σ is simply transferred from b^s to b^f due to the reduction in g^s. If however σ is negative, b^f falls due to a decrease in g^s and the reduction in b^s is made smaller since an amount σ is now transferred from b^f to b^s.

Now consider the effect on debt/GNP ratios of an effort by the federal government to reduce its expenditures on goods and services;

$$(49)\qquad \frac{db^f}{dg^f} = \frac{DQ - \beta\{n\bar{\omega} + r(t^s-h^s)\gamma + \bar{\mu}H(1-C_1)\}}{[KJ - RV]N}$$

$$= \frac{D}{[KJ - RV]} + \tilde{\sigma}$$

$$(50)\qquad \frac{db^s}{dg^f} = -\tilde{\sigma}$$

$$\text{where } \tilde{\sigma} \equiv \frac{rt^sD - \beta\{n\bar{\omega} + r(t^s-h^s)\gamma + \bar{\mu}H(1-C_1)\}}{[KJ - RV]N} \gtrless 0$$

$$\bar{\omega} \equiv C_1(t^s-h^s)r/[1-C_1(h^f+h^s)] \gtrless 0$$

$$\bar{\mu} \equiv [t^s(b^f+b^s-b^*) + h^sb^*](L_3/L_2)/[1-C_1(h^f+h^s)] < 0.$$

Except for the slight differences in the definitions of T versus Q, $\tilde{\omega}$ versus $\bar{\omega}$, and $\tilde{\mu}$ versus $\bar{\mu}$, the numerator of (49) is the same as the numerator of (47). As the latter was judged to be positive given convergence, so too should we sign the numerator of (49) positive. Hence like db^s/dg^s, we sign db^f/dg^f positive so that a reduction in federal expenditures will, after full adjustment to long run equilibrium, lead to a reduction in the federal debt/GNP ratio. Again it is useful to re-write (49) as consisting of two parts -- $D/[KJ - RV]$ which is unambiguously positive and $\tilde{\sigma}$ which is of ambiguous sign. The effect on the non-federal debt/GNP ratio of a decrease in federal expenditures is simply $-\tilde{\sigma}$ indicating that, as was the case with g^s, part of the effect of a reduction in g^f is reflected in a "debt transfer" from one level of government to the other. Again, if $\tilde{\sigma}$ is positive, this debt transfer aids the federal government's efforts to reduce b^f by reducing g^f but only at the expense of increasing b^s. If $\tilde{\sigma}$ is negative, b^s is reduced by federal expenditure cuts.

By inspection, σ and $\tilde{\sigma}$ are very similar in magnitude so that it is reasonable to suppose they have the same sign. Note that if $t^f \leq h^f$ and $t^s \leq h^s$, then it is unambiguously true that σ and $\tilde{\sigma}$ are positive so that a reduction in expenditures by one level of government causes a debt transfer onto the other level of government. Recalling the fact that in Canada withholding tax rates are approximately 25% (see footnote 7), this result is not unlikely. Another especially interesting case to consider is to assume the equivalence hypothesis is valid so that $\beta=0$. If this is so, then $[KJ - RV] = D[n-(r(1-t^f-t^s)-p)]$ and we can re-write (47)-(50) as;

(47') $db^s/dg^s = 1/[n-(r(1-t^f-t^s)-p)] + \sigma$

(48') $db^f/dg^s = -\sigma$

(49') $db^f/dg^f = 1/[n - (r(1-t^f-t^s)-p)] + \tilde{\sigma}$

(50') $db^s/dg^f = -\tilde{\sigma}$

where now $\sigma = (rt^f/N)/[n-(r(1-t^f-t^s)-p)]$

$\tilde{\sigma} = (rt^s/N)/[n-(r(1-t^f-t^s)-p)]$.

Note that σ and $\tilde{\sigma}$ are unambiguously positive so that the efforts of one level of government to reduce its debt/GNP ratio by cutting expenditures has the effect of increasing the debt/GNP ratio of the other level of government. But note also that if $(rt/N) > 1$ then more than half of any reduction in a level of government's debt/GNP ratio caused by a reduction in its level of expenditures is due to a debt transfer onto the other level of government. To get an idea of the magnitude of (rt/N), assume $t = 0.20$, $n = 0.045$, $r = 0.09$, and $p = 0.05$. These values are chosen so as to ensure that N is positive (a necessary condition for convergence) and to ensure real after interest rates -- the smallest of which is $[r(1-t^f-t^s)-p)]$ -- are positive. With these values, $(rt/N) = 3.6$ so that the debt transfer effect makes up fully 78% of the total effect! Another way of looking at this is to note that if <u>both</u> levels of government reduce their expenditures in an effort to lower their debt/GNP ratios, then the total effect this will have will be only 22% of what each government could expect if it had acted alone due to the offsetting debt transfer effects.

As a final comment on the problem of managing government debt in a federal economy, it is important to note that for at least two reasons those who attempt to simulate the effect on the federal debt/GNP ratio of various federal expenditure policies while ignoring the existence of non-federal governments (see for example, Fortin (1985) and Tobin (1986)) provide quite misleading results. The first reason why this is so is that such attempts ignore the fact that the same economic conditions which cause the federal debt/GNP ratio to increase will also cause the non-federal debt/GNP ratio to rise. It is likely therefore, that if the federal government reacts to this development by reducing its expenditures then non-federal governments will do so as well giving rise to a debt transfer effect which may significantly reduce the response of b^f to a reduction in g^f. By ignoring the likely response of non-federal governments therefore, such studies provide what may be a seriously misleading indication of the effectiveness of expenditures cuts in reducing the federal debt/GNP ratio. A second reason why studies which ignore the existence of non-federal governments when simulating the effect of federal expenditure policies on the federal debt/GNP ratio provide misleading results can be seen by noting that if one ignores non-federal governments and hence let $t^s = h^s = b^s = 0$, then $\tilde{\sigma} = 0$. As a consequence, a potentially large fraction of db^f/dg^f is ignored.

8. The Burden of Government Debt and Government Expenditure in an Open, Federal Economy

In essay two we examined the question of the burden of government debt in a closed federal economy when the equivalence hypothesis (β=0) was assumed to hold. There we found that so long as both levels of government bond finance the conclusion in the literature that there is no burden to government debt is robust to our extension to a federal economy. However, should the federal government money finance disturbances to its budget position, then we found that the decision of non-federal governments to bond finance <u>did</u> impose a burden in inverse proportion to the magnitude to the term $[n-(r(1-t^s)-p)]$. Finally, we also derived the standard closed economy result that bond financed government expenditures reduce consumption possibilities only to the extent that private economic agents view these expenditures as not being equivalent to private expenditures. This result held so long as the "passive" level of government (i.e., that level of government not initiating the change in expenditures) did not choose to money finance the resulting disturbance to its budget position.

In order to re-examine these questions for the case of an open federal economy, we let $\dot{m} = \dot{b}^* = \dot{b}^f = \dot{b}^s = 0$ so as to calculate full equilibrium comparative static results and we let $\beta=0$ so as to impose the equivalence hypothesis. Finally, we assume the relevant stability conditions are satisfied in order to make relevant our discussion of full equilibrium comparative static results. Consider first of all the question of the burden of bonded debt. If we omit the non-federal GBC from our model and let $b^s = t^s = 0$, then we have a model of a unitary state. Assuming the (federal) government bond finances (so that b^f is endogenous and m is exogenous) we calculate

(51) $dy^d/dt^f = 0$

which indicates that a re-financing of any portion of a given budget by bonded debt rather than taxes has no effect on the consumption possibilities of private economic agents. Thus bonded debt in an open unitary state imposes no burden as was the case in the closed economy. Mutoh (1985) and Purvis (1985) derive this same result, both under the assumption of a unitary state. If now we assume a federal state, this same conclusion results -- a re-financing of any portion of a given budget by taxes rather than bonds imposes no burden regardless of whether the other level of government tax or bond finances disturbances to its budget position.

What however, is the burden of non-federal bonded debt if the federal government is money financing? Recalling the difficulty we had in determining the sign of the stability conditions for this set of financing assumptions it is not surprising that few useful inferences can be drawn from comparative static results without some simplifying assumptions. A useful set of assumptions is to assume $L_3 = 0$ and that income and withholding tax rates are equal.

Recalling our discussion in essay two, there are two ways of answering the question of the burden of non-federal debt when the federal government is money financing. The first is to simply calculate the effect on consumption possibilities of a re-financing of part of the non-federal budget by bonds rather than taxes. Restricting L_3 to be zero and income and withholding tax rates to be equal, this yields

$$(52) \qquad \frac{dy^d}{dt^s} = \frac{-[1+r(b^f+b^s-b^*)](n+C_3)rt^f\{[n-(r(1-t^f-t^s)-p)]+(1-t^f-t^s)b^*(t^f+t^s)(1/L_2)\}}{D\{Q(\partial\dot{m}/\partial m) + (\partial\dot{m}/\partial b^s)(\partial\dot{b}^s/\partial m)\}}$$

where, recalling our calculations in section 6, convergence requires the denominator to be negative. The numerator is of ambiguous sign due to the ambiguity of the sign of the term in curved brackets. Therefore, it is uncertain whether non-federal debt imposes a burden ($dy^d/dt^s > 0$) or actually increases consumption possibilities ($dy^d/dt^s < 0$). This is quite unlike our closed economy result where it was the case that non-federal debt unambiguously reduced consumption possibilities. Therefore, contrary to the unitary state results of Mutoh (1985), in a federal economy one's conclusion regarding the burden of government debt can be sensitive to whether the economy is open or closed.

The second way of looking at this issue is to calculate the effect on consumption possibilities of re-financing a part of the federal budget by monetarized debt rather than taxes for a unitary state and for a federal state with bond financing non-federal governments. Consumption possibilities will be affected in both cases but the difference between them will indicate the effect on consumption possibilities due to the existence of bond financing non-federal governments. In the closed economy, this exercise showed that there was a burden associated with non-federal debt. In an open economy, this conclusion changes somewhat. That is, still assuming $L_3 = 0$ and that income and withholding taxes are equal, we calculate the effect on consumption possibilities of a federal re-financing from tax revenue to monetarized debt to be

$$(53) \qquad \frac{dy^d}{dt^f} = \frac{[1+r(b^f-b^*)](n+C_3)\{[n-(r(1-t^f)-p)]+(1-t^f)b^*t^f(1/L_2)\}}{D(\partial\dot{m}/\partial m)}$$

for a unitary state, and

(54) $$\frac{dy^d}{dt^f} =$$

$$\frac{[1+r(b^f+b^s-b^*)](n+C_3)Q\{[n-(r(1-t^f-t^s)-p)]+(1-t^f-t^s)b^*(t^f+t^s)(1/L_2)\}}{D\{Q(\partial\dot{m}/\partial m) + (\partial\dot{b}^s/\partial m)(\partial\dot{m}/\partial b^s)\}}$$

for a federal state with bond financing non-federal governments.

Convergence requires the denominator in both these expressions to be negative. The numerators are of ambiguous sign however due to the ambiguity of the sign of the terms in curved brackets and, for (54), the sign of Q. Furthermore, without knowledge of the precise magnitudes of the various parameters, it is difficult to ascertain the relative magnitude of these expressions. Once again therefore, it is uncertain whether or not non-federal debt imposes a burden, a result quite unlike that derived for a closed economy.

Another interesting difference in comparative static results between our closed and open economy model has to do with the question of the "burden" of government expenditures. In the closed economy we found that for a bond financing government (assuming the other level of government does not money finance),

(55) $$\frac{dy^d}{dg^f} = \frac{dy^d}{dg^s} = -(1-\alpha) < 0.$$

That is, government expenditures reduce consumption possibilities only to the extent that private economic agents view these expenditures as not being equivalent to private expenditures -- a result also derived by Scarth (1987a). Now let us assume an open economy where both levels of government are bond financing. We now find that

(56) $$\frac{dy^d}{dg^f} = \frac{dy^d}{dg^s} = -(1-\alpha)U < 0.$$

where $U = (1-h^f-h^s)(n+C_3)/[1-C_1(h^f+h^s)]\ D > 0$

and where the signs are due to the fact that $D > 0$ is necessary for convergence. Once again government expenditures do not reduce consumption possibilities if these expenditures are judged to be equivalent to private expenditures (i.e., if $\alpha = 1$). However, if $\alpha < 1$ then government expenditure can be seen to reduce consumption possibilities in an open economy either more or less than is the case in a closed economy depending on whether U is greater or less than unity. This same result is derived by Scarth (1987a) who however assumes $C_3 = L_3 = 0$ and of course assumes a unitary state so that $h^s = 0$.[17]

As Scarth suggests, we can get an idea of the magnitude of U by employing some reasonable parameter values. By doing so, it turns out to be the case that once again it matters whether one is considering a unitary or a federal state.

As a first step, and in order to compare our results with Scarth's most directly, we follow his specification and assume $C_3 = L_3 = 0$. For reasonable parameter values, Scarth chooses $h^f = t^f = 0.3$, $C_1 = 0.9$, $r = 0.08$, $p = 0.03$ and $n = 0.03$. These values are chosen so as to satisfy the conditions for convergence in a unitary state with a bond financing government. Recalling our discussion in section 5, this requires $T \equiv [n-(r(1-t^f)-p)] > 0$. With these values U = 1.09 for a unitary state and Scarth concludes that consumption possibilities are reduced by nine percent more in an open economy than in a closed economy. Further, he notes that the value of U can only be less than unity if $n > 0.087$ which is an unreasonably large value. Thus he judges his result to be robust for all reasonable values of n. Now consider a federal state so that $h^S \neq 0$ in our definition of U. It is important to note that Scarth's reasonable parameter values are no longer adequate since now convergence requires satisfaction of a more stringent condition -- that $N \equiv [n-(r-p)]$ be positive. In a federal state then, more reasonable values are $t^f = t^S = h^f = h^S = 0.2$, $C_1 = 0.9$, $r = 0.09$, $p = 0.05$ and $n = 0.045$. These values are chosen so as to ensure $N > 0$ and to ensure that after tax real interest rates -- the smallest of which is $[r(1-t^f-t^S)-p]$ -- are positive. With these values, $U = 0.951$ in a federal state so that we obtain a result opposite to that derived by Scarth -- government expenditures reduce consumption possibilities by five percent <u>less</u> in an open economy than in a closed economy.[18] Further, since the value of U increases only if the value of n falls, and since convergence requires that n not fall below 0.041 (limiting ourselves to three decimal places), then the <u>maximum</u> value U can take is 0.952. In a federal state therefore, the social cost of government spending is <u>less</u> in an open economy than a closed economy whereas the opposite is true in a unitary state.

Our discussion of this issue so far has imposed Scarth's assumption that $C_3 = L_3 = 0$. If we relax this assumption we need values not only for C_3 and L_3 but also for L_2 and b* since these variables appear in D when $L_3 \neq 0$. In section 5 above we found justification in the literature for assuming $C_3 = 0.05$ and $L_2 = -0.43$. We further assume $L_3 =$

0.10 and $b^* = 0.2$ are reasonable. With these values, $U = 1.03$ in a unitary state and U can be less than unity only if $n > 0.147$. As a 14.7% real growth rate is certainly unlikely, Scarth's conclusion that the social cost of government spending is greater in an open economy seems robust to a relaxation of his assumption of zero direct wealth effects (note however that the magnitude of U is significantly reduced). In a federal state, $U = 0.995$ so that the social cost of government spending is approximately equal in open and closed economies when direct wealth effects exist.

9. Summary and Conclusions

Of the three models in this thesis, the model in this third essay best represents the Canadian economy as it assumes not only a federal economy but also a small, open economy. The results derived from our examination of this model pertain to three areas; the stability of deficit financing, the problem of debt management, and the question of the burden of government debt.

Consider first our findings relevant to the question of the stability of government deficit financing. In section 4 we derived the condition for convergence when both levels of government avoid deficit financing altogether and simply adjust their tax rates in response to disturbances to their budget positions. We confirmed the results of Rodriquez (1979), Hodrick (1980), and Kawai (1985) that even given an absence of deficit financing, convergence is problematic requiring $D > 0$. In order to concentrate on deriving conditions for convergence due solely to the decisions of governments to deficit finance, we therefore assumed $D > 0$ in later sections. In section 5 we examined conditions for convergence given that the federal government has chosen to obey a monetary growth rule. If we assume non-federal governments tax finance, we derive results one would also derive had a unitary state been assumed. If, given tax financing non-federal governments, we imposed the equivalence hypothesis, we derive results similar to those derived by Scarth (1987a) who assumes $\beta=0$. These results indicate that $n > r(1-t^f)-p$ is necessary for convergence as is the condition $D > 0$. Thus in an open economy, the decision by the federal government to obey a monetary rule requires that the same condition for convergence be satisfied as has to be satisfied in a closed economy. If however, $\beta>0$ so that the equivalence hypothesis is no longer imposed, then the conditions for convergence are somewhat more complicated. If the federal income tax rate (t^f) exceeds, or is equal to, the federal withholding tax rate (h^f) then either $[n-(r(1-t^f)-p)]$ or $[n-(r(1-h^f)-p)]$ must be positive for convergence and this is true regardless of the value of β. This is an important result as it indicates that if $t^f \geq h^f$, then Scarth's result does not require his restrictive assumption that $\beta=0$. In an open, unitary state then, one needs to be concerned about the relative magnitudes of real growth rates and after tax real interest rates regardless of whether one is a monetarist (and hence believe $\beta=0$) or a Keynesian (and hence believe $\beta=1$). If however $t^f < h^f$, then growth rates in excess of real after tax interest rates are only helpful, not necessary, for convergence, the same result derived for a closed unitary state.

Still assuming the federal government is following a monetary growth rule, we found that deficit financing by non-federal governments causes convergence to require $n > r-p$ regardless of the relative magnitudes of income and withholding tax rates and regardless of the value of β. The decision of non-federal governments to deficit finance therefore makes the conditions for convergence significantly more stringent.

If the federal government chooses to abandon the monetary growth rule and instead money finance its deficits, and if non-federal governments are assumed to tax finance disturbances to their budget positions, then convergence again requires $D > 0$ plus a condition the literature has traditionally assumed is satisfied. If however, non-federal governments choose to deficit finance, then convergence is far more problematic. Interestingly, if we impose some simplifying assumptions which have appeared in the literature -- that is, lump sum taxes or the combination of $\beta=1$ and no withholding taxes -- then convergence requires $n > r-p$ and this is true regardless of values of β. Thus even if the federal government chooses to money finance its deficits, the decision of non-federal governments to deficit finance means concern should be expressed over the relative magnitudes of real growth rates and real interest rates.

In section 7 we assumed both levels of government have chosen to bond finance (as this seems most consistent with recent behaviour in Canada) and investigated the effects of efforts to reduce debt/GNP ratios by reducing government expenditures. We found that after full adjustment to long run equilibrium, the effect of one level of government reducing its expenditures was to not only reduce its own debt/GNP ratio but also to transfer a part of this debt onto the other level of government. It was found that this debt transfer effect was potentially large indicating that in a federal economy effective debt management may require a good deal of inter-governmental cooperation.

In section 8 we examined two issues both of which required that we impose the equivalence hypothesis and derive full equilibrium comparative static results. The first issue dealt with the question of the burden of bonded debt. We found that so long as both levels of government bond finance, the result of Mutoh (1985) and Purvis (1985) that there is no burden associated with bonded debt was robust to our extension to a federal economy. If, however, the federal government was assumed to money finance then the bonded debt of non-federal governments was no longer found to be neutral in its impact on private sector consumption possibilities. Contrary to our result from the closed economy case however, we were unable to ascertain unambiguously whether non-federal debt increased or reduced consumption possibilities.

The other issue we investigated in section 8 was the question of whether bond financed government expenditures increased or decreased consumption possibilities relative to the closed economy case. Scarth (1987a) concludes that for reasonable parameter values, consumption possibilities are reduced by approximately 9 percent more in an open than in a closed economy. We conclude that due to the fact that stability conditions in a federal state are more stringent than those in a unitary state, the opposite is true -- consumption possibilities are reduced less in an open economy than a closed economy. If we assume like Scarth that direct wealth effects are zero, then for reasonable parameter values government expenditures reduced consumption possibilities by 5 percent less in an open than in a closed economy. If direct wealth effects are not zero, we found that consumption possibilities were virtually unaffected by the choice of open or closed economy.

Footnotes

[1] Although Kawai (1985) notes the existence of the inflation tax, he omits this term from his definition of disposable income. This is justified by assuming capital gains or losses are fully saved and have no impact on expenditure decisions. See Kimbrough (1985) for another study which ignores the inflation tax.

[2] To re-write (6) in this form we need to substitute the two GBC's for $r/P(B^f + B^s) - (t^f + t^s)(Y + rB^f/P + rB^s/P - rB^*/P)$, the balance of payments constraint for $(1/P)\dot{B}^*$, equation (5) for W, and we need to use equation (8) (discussed below) and the relationship $(1/P)\dot{M} - (M/P)p = (\dot{M/P})$.

[3] Actual capital gains (losses) are both income and savings and thus cancel from this constraint.

[4] This assumes $[r(1-h^f-h^s)-p] > 0$, an assumption we made above in signing $\partial x/\partial b^*$. Note also that since they ignore the government sector, Dornbusch and Fischer (1980) can also be grouped with Rodriquez, Hodrick, and Kawai as a study which abstracts from asset accumulation via the GBC. Like Kawai, they also assume $n = 0$ and they also find that $C_3 > 0$ is a necessary condition for convergence.

[5] Scarth (1987a) derives two sets of stability conditions. The differences between these conditions depends on whether one assumes consumption depends on current disposable income or permanent disposable income (where steady-state values of m, r, p, b^f, and x enter the definition of y^d). Our stability conditions are directly comparable to Scarth's second set of conditions derived under the assumption that consumption depends on current disposable income. The slight difference which remains between our D' and Scarth's H' (replace r with r* and p with p* and our D' becomes Scarth's H') is due to Scarth's assumption that interest obligations owed to foreign residents are denominated in foreign currency.

[6] Using that version of Scarth's model most compatible with ours -- i.e., the model described by his equations (4), (7), (11a) and (10) -- we let his $\dot{b} = 0$ and make his income tax parameter (t) endogenous. This leaves the balance of payments constraint (7) as the only dynamic equation. Re-writing this constraint by substituting (10) into (11a) for y^d and the result into (7) for (x-im) we have

$$\dot{b}^f = \{[r(1-w)-p](1-c)/[1-cw]-n\}b^f + \text{exogenous variables}$$

where Scarth's b^f is our b^*, his c is our C_1, and w is his (federal) governments withholding tax rate. Therefore, even in the absence of deficit financing convergence requires that the term in curved brackets be negative -- a condition equivalent to our $D' > 0$.

[7] In Canada, the general rate of withholding tax on investment income paid to non-residents is 25% at the federal level. Source: The National Finances, 1985-86, Canadian Tax Foundation.

[8] In 1985, nominal GNP (PY) was 453,754 while the nominal money supply (M1) was 31,489 (both figures measured in millions of dollars). Source: Bank of Canada Review.

[9] Like the condition T>0, the requirement H>0 is also more stringent than the condition D>0. Thus both T and H can be negative without violating our assumption that D>0.

[10] Fortunately, the rather formidable task of expanding (31) can be simplified by row reduction. That is, subtract column 3 from column 2 and note that $[\partial\dot{b}^*/\partial b^f - \partial\dot{b}^*/\partial b^s] = 0$, $[\partial\dot{b}^f/\partial b^f - \partial\dot{b}^f/\partial b^s] = -[n-(r-p)]$, and $[\partial\dot{b}^s/\partial b^f - \partial\dot{b}^s/\partial b^s] = -[n-(r-p)]$. Then add row 3 to row 2 and we have, using the definitions defined in the text,

$$\begin{bmatrix} K - \lambda & 0 & R \\ V & 0 & J - \lambda \\ \partial\dot{b}^s/\partial b^* & N + \lambda & \partial\dot{b}^s/\partial b^s - \lambda \end{bmatrix} = 0.$$

[11] Stability requires that the real parts of all of the roots of the n^{th} degree polynomial equation

$$a_0 \lambda^n + a_1 \lambda^{n-1} + \dots + a_{n-1} \lambda + a_n = 0$$

be negative. The Routh-Hurwitz theorem states that the necessary and sufficient conditions for this to be true are that $a_1, a_2, \dots, a_{n-1}, a_n > 0$ and that the first n of the following sequence of Routh determinants

$$|a_1|, \quad \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}, \quad \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}, \dots$$

all be positive. In the text, the conditions in (a), (b) and (c) correspond to the requirement that a_1, a_2 and a_3 be positive. Since in a 3x3 system $a_4 = a_5 = \dots = 0$, the only additional requirement necessary to satisfy the sign requirements of the Routh determinants is $a_1 a_2 - a_3 > 0$. This condition is written (after simplification) as the condition in (d).

[12] Scarth's assumptions that $C_3 = L_3 = 0$ would not affect the sign of (44').

[13] Neither Boyer and Holdrick (1982) nor Kimbrough (1985) who also examine open economy GBC models with lump sum taxes, investigate stability under money financing.

[14] It is helpful in expanding (45) to note that given the assumption of $\beta=1$ and $h^f = h^s = 0$, the following are true: $(\partial\dot{m}/\partial b^s)+(\partial\dot{m}/\partial b^*) = 0$, $(\partial\dot{b}^*/\partial b^s)+(\partial\dot{b}^*/\partial b^*) = -N$, and $(\partial\dot{b}^s/\partial b^s)+(\partial\dot{b}^s/\partial b^*) = -N$.

[15] Recall that we obtained this same result for the case of a closed fixed price economy with $\beta=1$ and $n=0$ in essay number one. There we found that the result of Blinder and Solow (1973) that money financing is stable is not robust to our extension to a federal economy with bond financing non-federal governments. Thus with respect to Keynesian ($\beta=1$) models with fixed prices and zero growth, non-federal bond financing causes federal money financing to become unstable regardless of whether the economy is open or closed.

[16] Report of the Royal Commission on the Economic Union and Development Prospects for Canada, Volume 3, pp. 247-8. This royal commission is popularity referred to as the Macdonald Commission.

[17] Scarth writes U in a slightly different way (calling it (1+A)) but it is easy to show our definitions are equivalent. That is, setting $C_3 = L_3 = 0$ and assuming a unitary state so as to follow Scarth's specification, our U becomes

$$U = (1-h^f)n/[1-C_1h^f]\{n-(1-C_1)[r(1-h^f)-p]/[1-C_1h^f]\}$$

$$= -(1-h^f)n/\{[r(1-h^f)-p](1-C_1)-n(1-C_1h^f)\}$$

$$= 1 - \frac{\{[r(1-h^f)-p] - nh^f\}(1-C_1)}{[r(1-h^f)-p](1-C_1) - n(1-C_1h^f)}$$

which is Scarth's (1+A) term.

[18] Note that our parameter values also satisfy Scarth's stability conditions. If he had used these values, Scarth would have calculated U=1.04 and that a real growth rate in excess of 11% would be necessary to cause U to be less than one. Scarth's general conclusions regarding the social cost of government expenditures in a unitary state would therefore not be affected by our change in parameter values.

Chapter Five: Summary and Conclusions

A number of important western economies are federal states where non-federal governments control a large fraction of total government revenue and expenditure. Despite this, recent research investigating the conditions under which government deficit financing can lead to a stable equilibrium has assumed a unitary state. A major aim of this thesis has been to investigate how the existence of a non-federal government sector affects the conclusions of the existing literature on the stability of deficit financing. We have as well investigated how conclusions regarding the burden of government debt and how debt management policies are affected by an extension to a federal state.

With respect to the question of the stability of deficit financing we found that our extension to a federal economy produced results dramatically different from those found in the literature all of which are applicable only to a unitary state. The well-known results of Blinder and Solow (1973) -- that while money financing is stable, the stability of bond financing is an empirical question -- are completely overturned if deficit financing non-federal governments are assumed to exist. In this case, both modes of federal deficit financing are unambiguously unstable so that convergence in that model is incompatible with the existence of deficit financing non-federal governments. In our second essay we investigated a flexible price model with non-zero growth and allowed for a variety of assumptions regarding the recently contentious issue of whether or not private economic agents view government bonds as net wealth. There we found that if the federal government obeys a monetary growth rule and non-federal governments choose to deficit finance disturbances to their budget positions, then a necessary condition for convergence is that the real growth rate exceed the real before tax interest rate and this is true regardless of one's view about the degree to which government bonds are judged to be the net wealth. This result is critical for two reasons. First, based on the literature's results derived from models of unitary states, non-monetarists need not have been concerned about the relative sizes of growth rates and real interest rates as convergence required the former to be greater than the latter only if the equivalence hypothesis was imposed. Our result indicates that in a federal state monetarists and non-monetarists alike must view this condition as being critical for convergence. The second important point to be made about our result is that the requirement that the real growth rate exceed the real before tax interest rate is a significantly more stringent condition than the requirement that the real growth rate exceed the after tax real interest rate. Thus in a federal state with deficit financing non-federal governments and a federal government choosing to obey a monetary growth rule, the conditions under which convergence can be realized are much more restrictive than is typically judged to be the case in the literature. If instead of a monetary rule the federal government chooses to money finance its deficits, the existence of non-federal governments again causes the conditions for convergence to be more stringent than is the case in a unitary state. Indeed, if we impose some of the simplifying assumptions often found in the literature, convergence requires that the real growth rate exceed the real interest rate when the federal government money finances as well as when it obeys a monetary growth rule. Finally, in essay three we investigated a model most relevant to the Canadian case by modelling not only a federal state but an open economy as well. The existence of deficit financing non-federal states was again found to have a critical impact on conditions for stability with many of the important results from the closed economy proving to be robust to our extension to an open economy.

With respect to the question of macroeconomic stability therefore, we conclude that conditions for convergence are significantly more stringent in a federal than a unitary state so that policy-makers in federal states should be wary in drawing inferences from models of unitary states. We might also note that our results indicate that a positive rate of economic growth is often critical for convergence so that is is not in general possible to examine convergence in a federal economy using a model assuming zero growth as is often done for unitary states.

In examining the question of the burden of government debt we found that contrary to the results in the literature, there is a case where bonded debt <u>does</u> impose a burden on private agents despite the imposition of the equivalence hypothesis. This occurs if, in a closed economy, the federal government money finances its deficit while non-federal governments deficit finance. In an open economy, we found that in this same situation non-federal bonded debt again influenced private consumption possibilities but in an ambiguous direction. We also investigated an issue raised by Scarth (1987a) that government expenditures would lower consumption possibilities to a greater extent in an open than a closed economy. When a federal economy is considered, we found the opposite was true if we followed Scarth's assumption of zero direct wealth effects, but that consumption possibilities were largely insensitive to the switch from a closed to an open economy when non-zero direct wealth effects were assumed.

Finally, in essay three we examined how the existence of non-federal governments influences efforts by the federal government to lower its debt/GNP ratio. This question is of interest as recent federal government policy in Canada has been directed toward lowering the federal debt/GNP ratio. We found that the efforts of one level of government to reduce its debt/GNP ratio by reducing expenditures gives rise to a "debt transfer effect" whereby a potentially large fraction of the reduction in its own debt/GNP ratio is gained at the expense of the other level of government whose debt/GNP ratio rises as a result of the other level of government's policy. Effective debt management in a federal economy would therefore seem to require a good deal of inter-governmental cooperation and planning in order to avoid the largely offsetting debt transfer effects which arise if, as the Macdonald Commission has recently warned will likely occur, all levels of government choose to simultaneously try to lower their debt/GNP ratios by cutting expenditures.

In conclusion then, the findings of this thesis suggest that it is seriously misleading to draw inferences, to be applied to a federal state, from economic models which assume a unitary state. Unfortunately for policy-makers in federal states, the existing literature deals exclusively with models of unitary states. This thesis represents a first effort in modelling a federal state.

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