MESON-NUCLEUS INTERACTION

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ABSTRACT

The total and elastic cross sections for pion-nucleus scattering at medium energies are known for several nuclei. There are mainly two theoretical approaches, the optical model potential and the Glauber approximation, which have been used to analyse the pion-nucleus scattering data. These theoretical calculations use some approximations such as the impulse approximation for the πN amplitude, the closure or the fixed scatterer approximation, the neglect of Pauli principle etc. The validity of these approximations cannot be taken for granted. In the 33-resonance energy region where the πN interaction is very strong, the validity of the impulse approximation, especially, is very dubious. In this thesis we examine the accuracy of some of these approximations, in particular that of the impulse approximation.

We evaluate the binding corrections to the impulse approximation by doing a model calculation for pion scattering from a bound nucleon at low and medium energies. We consider a nucleon bound in a shell model potential, which is taken to be a harmonic oscillator. The oscillator parameter is chosen such that the nuclear sizes are fitted. The nucleus under consideration thus contains only one nucleon. A separable πN interaction, which reproduces free πN scattering is assumed. Its parameters are determined such that at low energies it fits the scattering length and effective range of free πN scattering. At medium energies, on the other hand, the separable interaction is fitted to reproduce the 33-resonance. The nucleon recoil is taken into account in fitting the free πN scattering data.

Unlike the free πN scattering, the Schrödinger equation for π-bound nucleon scattering cannot be solved analytically. We obtain the

scattering amplitude in this case by means of the Padé approximant. We treat the binding potential exactly. We do not use the closure approximation to sum over the intermediate nuclear states; instead they are explicitly summed over. At low energies the binding correction is found to be negligible for s-wave πN scattering. At medium energies, however, the binding correction to the impulse approximation is found to be significantly large. There is a downward shift of the resonance energy due to binding. The total cross section at and near the resonance peak increases by about 60-70% compared to that in the impulse approximation. The differential cross section is more strongly forward peaked in comparison to that in the impulse approximation. The effect of the Pauli principle is also considered by excluding appropriate states which are already occupied by other nucleons. It pushes the resonance energy upwards, closer to that in the impulse approximation. Its significant effect is to further change the cross section. The implication of such large binding corrections to the impulse approximation for pion-nucleus scattering are also discussed for a simple model.

We also discuss the Pauli principle effects and the binding corrections to the impulse approximation for kaon-nucleus scattering at low energies. This is done for a model similar to the one used for π-bound nucleon scattering. The binding correction in this case is large compared to that for s-wave πN scattering. This is due to the fact that the s-wave $\bar{K}N$ interaction is more strong at low energies in comparison to the corresponding πN interaction.

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Chapter I

INTRODUCTION

The structure and properties of atomic nuclei have been the subject of intensive study ever since the nucleus was discovered by Rutherford in 1911. The scattering of electrons, which interact with the nucleons through electromagnetic interaction, has given accurate and detailed information about nuclear properties such as the charge distribution.[1] The muons, which behave essentially like heavy electrons, have been used to investigate nuclear structure through the study of muonic atoms. This has provided accurate determination of some nuclear properties like the nuclear size. The nucleons have also been used to probe nuclei. The mesons constitute another category of nuclear probes. Like the nucleons, the mesons interact through strong interaction in addition to the usual electromagnetic interaction. As we will describe later, the mesons, especially the pi-mesons, induce certain nuclear reactions which are not possible with electrons or nucleons as projectiles. However, in the past the use of pions in studying nuclei has been severely restricted because of the low intensity of pion beams and also their poor energy resolution. But the situation is changing now with the advent of the "meson factories" - the medium energy accelerators characterized by high beam intensities. The experiments planned with these intense pion beams include elastic and inelastic pion-nucleus scattering, charge exchange reactions, pion absorption, nuclear knock-out reactions and pionic atom experiments.[2,3,4]

The pion is the lightest strongly interacting particle. It is a short lived pseudoscalar boson with $J^P = 0^-$, where J and P denote its spin and parity, respectively. It has three charge states π^+, π^0 and π^-, which form an isospin triplet with isospin I = 1. Its average mass is 138.04 MeV. The exchange of pions generates the nuclear force. The pion-nucleon (πN) interaction at medium energies has several interesting properties that make the pion an important and a unique nuclear probe. For example, the lack of spin means a much simpler two-body amplitude. Double charge exchange reactions in pion-nucleus scattering are possible since pions form an isospin triplet and they can be absorbed or emitted by a nucleon in a nucleus. The incident pion is a distinct particle from the nucleus, so the antisymmetrization of wave functions with respect to the projectile is not required. Finally, the πN interaction at medium energies exhibits a strong energy dependence.[5] At low energies, pion kinetic energies $E_\pi \lesssim 100$ MeV, the πN interaction is very weak, an order of magnitude smaller than the nucleon-nucleon interaction at comparable energies. The most striking feature of the energy variation of the πN interaction is its domination by a resonance in p-wave, the 33-resonance (I=3/2,J=3/2), in the energy region $E_\pi \sim 100$-300 MeV. Beyond 300 MeV the πN interaction becomes weak again. This energy variation of the πN interaction is quite unlike that of the NN system, which shows no resonances.

The features particular to πN scattering have some interesting consequences in pion-nucleus (πA) scattering in the medium energy range. The π-nuclear reactions show energy variation which shows an effect of the πN resonance. A resonance is observed at an energy lower than the corresponding energy at which the 33-resonance occurs in πN scattering.

There are various theoretical approaches, based on the optical model potential and approximate multiple scattering methods such as the Glauber approximation, to reproduce the wealth of data on total and elastic cross sections for pion-nucleus scattering at medium energies.[6] Both the Glauber multiple scattering theory [7] and the lowest order optical potential [8] reproduce the peak of the total cross section and the elastic scattering data reasonably well for several nuclei.

The agreement between the experimental data and these theoretical calculations is surprizingly good in view of the many approximations made in such calculations. The approximations common to many of these calculations are the impulse approximation for the πN amplitude, the closure or the fixed scatterer approximation, the neglect of Pauli principle etc. The theoretical calculations describing the πA scattering obtain the πA scattering amplitude in terms of the πN scattering amplitude. This πN scattering amplitude should in principle be related to the t-matrix from a bound nucleon, but in practice the free πN t-matrix is usually used. This is known as the impulse approximation (IA). The IA has been used practically in all πA theoretical calculations. It is generally expected that the IA is valid if the energy of the projectile is much greater than the binding energy of the nucleon in the nucleus. But in the 33-resonance energy region where the πN interaction is very strong, the validity of the IA is very dubious. In fact it was pointed out by Watson and Goldberger [9] a long time ago that the binding corrections to the IA are very large. Nevertheless, the IA has been commonly used because it simplifies the πA calculations very much.

The corrections due to multiple scattering effects in πA scattering have been examined by a number of authors, especially for π-deuteron scattering. But very little attention has been paid to the problem of binding corrections to the IA.[4] Near the resonance the binding corrections to the IA can be so significant, that to perform πA calculations which include multiple scattering effects but neglect binding corrections may not be justified.

In this thesis an attempt has been made to examine the validity of some of the above mentioned approximations, in particular the impulse approximation. The motivation, therefore, is to analyse the more fundamental aspects of the meson-nucleus calculations rather than to fit the experimental data.

We evaluate the binding corrections to the IA by doing a model calculation for pion scattering from a bound nucleon at low and medium energies. We consider a nucleon bound in a shell model potential, which is taken to be a harmonic oscillator. The oscillator parameter is chosen such that the nuclear sizes are fitted. We assume a separable πN interaction. At low energies its parameters are determined such that the scattering length and effective range of free πN scattering are fitted. For the πN scattering at intermediate energies, the separable interaction is fitted to reproduce the free πN scattering data including the 33-resonance. We take account of nucleon recoil in free πN scattering.

The π-bound nucleon interaction is not separable, even though we have chosen a separable interaction for free πN scattering. So, unlike the free πN scattering, the Schrödinger equation for π-bound nucleon scattering cannot be solved analytically. We obtain the scattering amplitude in this case by the use of the Padé approximant. We do not make any expansion with respect to the binding potential; instead it is exactly treated. We do not use the closure approximation to sum over the intermediate nuclear states; instead they are explicitly summed over. At low energies it is found that the binding correction is negligible for s-wave πN scattering, there being a slight increase in the πN scattering length. For the πN scattering at medium energies the binding correction to the IA is found to be significantly large. It results in a downward shift of the resonance (in terms of the pion energy in the πA laboratory system), which is characterized by the vanishing of the real part of the forward πA scattering amplitude. The total cross section at and near the resonance increases by about 60-70% compared to that in the IA. The differential cross section is more strongly forward peaked in comparison to that in the IA. We also examine the effect of the Pauli principle by excluding the appropriate states which are already occupied by other nucleons. We find that it further increases the π-bound nucleon cross section and shifts the resonance energy upwards. The closure approximation, which is often used to sum over the intermediate nuclear states is also examined in this model. It leads to very different results which are not correct even qualitatively.

In addition to the pion-nucleus interaction some aspects of the kaon-nucleus interaction are briefly discussed in this thesis. Kaon (K) is another pseudoscalar meson with $J^P = 0^-$, that has occasionally been used as a nuclear probe. It exists in four states, which are classified into two isospin doublets with I = 1/2. The (K^+,K^0) doublet has strangeness S = 1, whereas the doublet $(\overline{K^0},K^-)$ has strangeness S= -1. In the following we will denote the latter collectively by $\overline{K}$. The average mass of $\overline{K}$ is 493.8 MeV. We are interested in $\overline{K}$ only since it forms the kaonic atoms. The $\overline{K}N$ interaction is dominantly s-wave at low energies with complex scattering length owing to absorption in the open $\pi\Lambda$ and $\pi\Sigma$ channels. We describe it by a separable interaction with a complex coupling constant. The binding correction, Pauli principle effects etc. are discussed for $\overline{K}$ scattering from a bound nucleon for a model similar to the one discussed above for the pion. The binding correction is large; it modifies the $\overline{K}N$ scattering length significantly.

The plan of the thesis is as follows. In the next chapter we review the meson-nucleon interactions. We discuss the effective πN interaction which describes the s-wave scattering lengths fairly well, and for the p-wave it gives the Chew-Low interaction [10]. The Chew-Low interaction is known to reproduce the qualitative features of the πN scattering data in the low and medium energy region, namely the 33-resonance in p-wave. We show that by including inelasticity, crossing and the nucleon recoil it gives an excellent quantitative fit to the data in the 33-resonance energy region. We also describe a separable πN interaction which is used later in πA calculations. Separable πN interactions [11] are commonly used as input in the πA calculations. We propose some justification for the equivalence of our separable interaction to the more basic field theoretic interaction for the binding problems. For the $\overline{K}N$ interaction also, a separable form is considered.

In chapter III we discuss the IA for meson scattering from a bound nucleon. The condition for the validity of the IA and different versions of the IA which are commonly used in literature are described. Chapter IV describes the meson scattering from a bound nucleon at low energies. Binding corrections to the IA for s-wave πN and $\bar{K}N$ scattering are discussed. In chapter V pion scattering from a bound nucleon at medium energies is considered. The validity of the IA, the closure approximation and the Pauli principle effects are discussed. Conclusions are presented in the last chapter. Some technical details are given in appendices A, B and C. In appendix D we briefly summarize the Glauber multiple scattering theory and give a brief description of a model calculation in which we have examined the reasons for the success of the Glauber theory at low and medium energies. Some interesting observations are listed in Appendix E.

Chapter II

MESON NUCLEON INTERACTION

1. Introduction

The meson-nucleon interaction is a basic ingredient in any theory for the meson-nucleus interaction. Hence, it is in order to review the former before embarking on the meson-nucleus problems. For the meson we first consider the pion in some detail, and later the kaon briefly.

Since we are interested in the medium energy phenomena, say upto about 300 MeV, we confine our discussion of the πN interaction to this energy range. The most striking feature of the πN scattering is that it is dominated by $\Delta(1236)$, a p-wave resonance in the $I=J=3/2$ state, other partial wave contributions being negligible except at very low energies where there are appreciable s-wave contributions. Any theory of the πN interaction should incorporate the above features of πN scattering.

Ever since Yukawa introduced the meson theory [12], the so-called Yukawa interaction has been thought of as a prototype of the meson-nucleon interaction. For the pion there are two types of the Yukawa interaction which are commonly considered, the pseudoscalar (ps) and pseudovector (pv) couplings. It was realized that these two different couplings lead to the same results for many problems, such as the one-pion exchange potential between nucleons, the low and medium energy photopion production, and the p-wave πN scattering. This is understood by reducing the Yukawa interaction to its nonrelativistic form by means of a Foldy-Wouthuysen type transformation. In the nonrelativistic limit one can separate the interaction into terms such that each of them corresponds to certain partial wave interaction. It can then be shown that for the p-wave part the ps and pv couplings are equivalent [13] provided that the coupling constants are appropriately related (equivalence theorem). As will be shown in section 3, the p-wave πN scattering can be satisfactorily described on the basis of the Yukawa interaction.

The situation of the s-wave is very different, however. The two couplings (ps and pv) give rise to very different s-wave interactions, and neither of them seems to be able to describe the s-wave πN scattering adequately. After very many attempts along this line it became plausible that perhaps it is wrong to take the Yukawa interaction literally as the basis interaction. A breakthrough came from a development in relation to the weak interactions. The concept of the partial conservation of axial-vector current (PCAC) [14] was successfully used by Nambu [15] to derive the Goldberger-Treiman relation, which relates the πN coupling constant, the axial coupling constant and pion weak decay constant. It was soon found that in combination with current algebra, the PCAC predicted the s-wave πN scattering lengths consistent with experimental data. [16]

In order to do any dynamical calculations such as for πA scattering, however, it is still convenient to have a Lagrangian or Hamiltonian which incorporates the PCAC and current algebra. Such an effective Lagrangian can indeed be constructed. This Lagrangian is required to satisfy the chiral SU(2) × SU(2) invariance which guarantees that the PCAC and current algebra or charge algebra are satisfied†.

† Strictly speaking the algebra of charges is satisfied but not necessarily the algebra of currents. But this is loosely referred to as current algebra.

The nonrelativistic limit of the Hamiltonian which follows from the chiral invariant Lagrangian has the s-wave and p-wave parts which can describe the experimental πN data very well.[17)] We will summarize this in section 2.

The effective πN interaction is now fairly well known. However, it is not easy to carry out more complicated calculations such as for π-nucleus, with it. We therefore, take the more pragmatic approach of choosing a phenomenological πN interaction which reproduces the scattering data. We consider πN separable interactions for s-wave and p-wave, which fit the experimental data well, in section 4. We give some justification for their equivalence to the more basic field theoretic interactions for the purpose of using these in evaluating binding corrections to the impulse approximation.

The $\overline{K}N$ interaction at low energies is dominated by s-wave scattering with negligible contributions of other partial waves. The scattering length is complex owing to strong absorption into the open $\pi\Sigma$ and $\pi\Lambda$ channels.[18)] These features should be included in a theory of the $\overline{K}N$ interaction. Now, unlike the πN interaction, the concept of PCAC is not very useful for the $\overline{K}N$ interaction. This is because the kaon mass is much larger compared to that of the pion, and the error caused by assuming M_K to vanish is probably large. So an effective Lagrangian, such as for πN, cannot be obtained for the $\overline{K}N$ system. We describe the $\overline{K}N$ scattering also by a phenomenological separable interaction, fitted to reproduce the low energy experimental data. This is discussed in section 5.

2. Effective Lagrangian for πN system

We start with the conventional Lagrangian for the πN system,

$$L = \bar{\psi}\,(i\gamma_\mu \partial_\mu - m)\,\psi + \frac{1}{2}\sum_\alpha (\partial_\mu \phi_\alpha \partial^\mu \phi_\alpha + \mu^2 \phi_\alpha{}^2) + L_{int} \tag{2.1}$$

where ψ and ϕ are the nucleon and pion fields; m and μ denote the nucleon and pion masses, respectively. α refers to the isospin of the pion, γ_μ are Dirac matrices with $\mu = 0,1,2,3$. Our units are such that $c = \hbar = 1$. Unless mentioned otherwise we will use the same units throughout this work. For the interaction we consider the ps πN coupling,

$$L_{int} = -ig\,\bar{\psi}\,\gamma_5\,\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\,\psi \tag{2.2}$$

Here $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, $\underset{\sim}{\tau}$ is the nucleon isospin operator and g is the ps coupling constant. The Hamiltonian describing the πN system can be derived from eq.(2.1) by a standard prescription.

It can be shown that, by a suitable unitary (Foldy-Wouthuysen type) transformation, the πN Hamiltonian in the nonrelativistic limit becomes [19]

$$H = H_N + H_\pi + H_I + H_{S1} + H_{S2} \tag{2.3}$$

where

$$H_N = \int d\underset{\sim}{r}\;\bar{\psi}\;(m+p^2/2m)\;\psi \tag{2.4}$$

$$H_\pi = \frac{1}{2}\sum_\alpha \int d\underset{\sim}{r}[\{\pi_\alpha(\underset{\sim}{r})\}^2 + \{\underset{\sim}{\nabla}\phi_\alpha(\underset{\sim}{r})\}^2 + \mu^2\phi_\alpha{}^2] \tag{2.5}$$

$$H_I = (g/2m)\int d\underset{\sim}{r}\;\rho(\underset{\sim}{r})\;\psi^*\;(\underset{\sim}{\sigma}.\underset{\sim}{\nabla})\;\underset{\sim}{\tau}.\underset{\sim}{\phi}\;\psi \tag{2.6}$$

$$H_{S1} = \lambda_1\int d\underset{\sim}{r}\;\bar{\psi}\;\psi\;\underset{\sim}{\phi}^2(r) \tag{2.7}$$

$$H_{S2} = \lambda_2 \int d\underset{\sim}{r}\ \psi^* \ \underset{\sim}{\pi}(\underset{\sim}{r}) \cdot \underset{\sim}{\tau} \times \underset{\sim}{\phi}(\underset{\sim}{r}) \tag{2.8}$$

Here terms upto $(\mu/m)^2$ have been retained. Note that ψ is a 2-component spinor in the above equations, the small components of the usual Dirac spinor being negligible in the nonrelativistic limit. H_N and H_π denote the nucleon and pion parts of the Hamiltonian, respectively. π_α is the momentum operator conjugate to ϕ_α. In the static limit, H_I gives rise to p-wave πN interaction only and it forms the basis of the Chew-Low (CL) theory. It is interesting to note that the pv interaction

$$L_{int} = -(f/\mu)\ \bar{\psi}\ \gamma_\mu \gamma_5\ \underset{\sim}{\tau} \cdot \partial_\mu \underset{\sim}{\phi}\ \psi \tag{2.9}$$

also gives rise to the same p-wave interaction H_I in the static limit provided that $f/\mu = g/2m$. The ps and pv interactions are thus equivalent so far as the p-wave interaction is concerned. As will be shown in the next section, it reproduces the πN scattering data very well at medium energies. In this section we consider only the s-wave parts of the Hamiltonian (2.3).

The last two terms (2.7) and (2.8) of the Hamiltonian give rise to s-wave πN interaction only. The coefficients λ_1 and λ_2 are given by

$$\lambda_1 = g^2/2m \quad ; \quad \lambda_2 = (g/2m)^2 \tag{2.10}$$

From eq. (2.7) it is obvious that H_{S1} is independent of isospin and gives equal s-wave πN scattering lengths a_1 and a_3 for $I = 1/2$ and $3/2$ states, respectively. The coefficient λ_2 of H_{S2} is much smaller than λ_1 (by a factor of 2m), so that H_{S1} gives the dominant contribution to s-wave πN scattering and hence $a_1 \approx a_3$. Now, the experimental values for a_1 and a_3 are known to be of opposite signs. The above Hamiltonian, therefore, does not reproduce even the qualitative results of s-wave πN scattering. The experimental values of a_1 and a_3 could be fitted if λ_1/λ_2 is about 1/10-th the ratio given by eq. (2.10). [20]

It is quite clear from the above discussion that the Yukawa interaction fails to describe the s-wave πN scattering. Various other theoretical approaches such as the dispersion relations [21] and current algebra [16] have been developed, which explain the s-wave scattering lengths rather well. Of central importance in obtaining these results by current algebra is the concept of a partially conserved axial-vector current (PCAC), also known as approximate chiral symmetry. [17] The notion of the chiral symmetry can be traced back to the V-A theory of the weak interaction, but we do not delve into this question in this thesis. According to the conserved vector current (CVC) hypothesis, the vector current j^μ is conserved , i.e.

$$\partial_\mu j^\mu = 0 \tag{2.11}$$

and there is no renormalization effect. It is identified with the isospin current. The axial-vector current j^5_μ , on the other hand, is known to be renormalized and is not conserved. The renormalisation effect, however, is small and it allows for the successful use of the PCAC. The PCAC relation between j^5_μ and the pion field ϕ is given by

$$\partial_\mu j^{5\mu}_{1+i2} = \sqrt{2}\, F_\pi\, \mu^2\, \phi \tag{2.12}$$

Here F_π is the pion decay constant, the suffix 1+i2 denotes a combination of the 1- and 2-components of the isospin. It follows from eq. (2.12) that the axial current is conserved in the limit of pion mass $\mu \to 0$. The pion being the lightest strongly interacting particle, setting its mass=0 is not too drastic an approximation. The PCAC is expected to, and it does give results to an accuracy of 10%. The s-wave πN scattering lengths derived by using the PCAC and the algebra of currents are in satisfactory agreement with the experimental results.

Although current algebra with the PCAC reproduces the correct s-wave scattering lengths, as noted above it would be more satisfactory to obtain the same from some kind of interaction Lagrangian. Such a Lagrangian can indeed be obtained by introducing the above notion of chiral symmetry in the strong interaction Lagrangians. [17] A Lagrangian L is said to be chiral invariant if

$$[Q^5_\alpha, L] = 0 \tag{2.13}$$

where the axial-vector charge Q^5_α is given by

$$Q^5_\alpha = \int d^3x \; j^5_{0\alpha}(x) \tag{2.13a}$$

It is obvious that eq. (2.13) is satisfied only in the limit of $\mu \to 0$, i.e. it is consistent with the PCAC.

Going back to the Lagrangian (2.1) we notice that it is invariant under the isospin transformation

$$\psi \to \exp[\tfrac{i}{2}\, \underset{\sim}{\tau} . \underset{\sim}{a}]\, \psi \tag{2.14a}$$

$$\underset{\sim}{\tau} . \underset{\sim}{\phi} \to \exp[\tfrac{i}{2}\, \underset{\sim}{\tau} . \underset{\sim}{a}]\; \underset{\sim}{\tau} . \underset{\sim}{\phi}\; \exp[-\tfrac{i}{2}\, \underset{\sim}{\tau} . \underset{\sim}{a}] \tag{2.14b}$$

where $\underset{\sim}{a}$ is a constant vector. This leads to the conserved vector isospin current

$$j_\mu = \bar{\psi}\, \gamma_\mu \frac{\tau}{2}\, \psi + \underset{\sim}{\phi} \times \partial_\mu \underset{\sim}{\phi} \tag{2.15}$$

Consider now the chiral transformation for the nucleon field defined by

$$\psi \to \exp[\tfrac{i}{2}\, \gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{a}]\, \psi \qquad (2.16)$$

It can be shown that the invariance of the Lagrangian under this transformation is equivalent to eq. (2.13), i.e. the Lagrangian is chiral invariant. We now want to test the invariance of the Lagrangian (2.1) under the above transformation (2.16). The first point to notice is that although the nucleon kinetic energy term is invariant under the chiral transformation, the nucleon mass term is not. This suggests that we attempt to cancel the variation of nucleon mass term by suitably defining the chiral transformation of the pion field such that the variation in πN interaction term cancels the variation in the nucleon mass term. It turns out that an infinite power series in $if\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{\phi}$ is required to obtain the invariance, and the result is a nonlinear chiral-invariant Lagrangian. Note that this chiral invariance is obtained in the limit of PCAC, i.e. $\mu \to 0$.

According to the general solution given by Gürsey,[22)] one writes the combined nucleon mass and πN interaction term in the form

$$L' = -m\bar{\psi}U(\underset{\sim}{\phi})\psi \qquad (2.17)$$

with U a matrix function of $if\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{\phi}$ alone with the following properties:

(i) $U = \sigma + 2if\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{\phi}\rho$

where σ and ρ are real functions of $(f\phi)^2$ alone, with $\sigma(0) = \rho(0) = 1$. The reality of σ and ρ follows from the Hermiticity of (2.17)

(ii) $UU^+ = U^+U = \sigma^2 + 4f^2\phi^2\rho^2 = 1$

This Unitarity condition follows from the demand that (2.17) be chiral invariant and that there exist a chiral transformation for the pion field which satisfies

$$U(\phi') = \exp[-\frac{i}{2}\gamma_5\tau\cdot a]\, U(\phi)\, \exp[-\frac{i}{2}\gamma_5\tau\cdot a] \tag{2.18}$$

The chiral invariant πN interactions are then determined by the forms of U. The best known example is the nonlinear σ model of Gell-Mann and Levy [22]

$$U = \sigma + 2if\gamma_5\tau\cdot\phi \quad , \quad \sigma = \sqrt{1 - 4f^2\phi^2} \tag{2.19}$$

Another well known model is due to Kramer et al. [22], where

$$U = \frac{1 + if\gamma_5\tau\cdot\phi}{1 - if\gamma_5\tau\cdot\phi} \tag{2.20}$$

Finally, the pion kinetic energy term is made chiral invariant by making the replacement

$$\frac{1}{2}(\partial_\mu\phi)^2 \to \frac{1}{64f^2}\,\mathrm{trace}\,(\partial_\mu U)(\partial_\mu U)^+$$

$$= \frac{1}{8f^2}(\partial_\mu\sigma)(\partial^\mu\sigma) + \frac{1}{2}\partial_\mu(\phi\rho)\partial^\mu(\phi\rho) \tag{2.21}$$

where the trace is taken with respect to the pion isospin. The conserved vector and axial-vector currents in this theory are given by

$$j_\mu = \frac{1}{2}\bar{\psi}\gamma_\mu\tau\psi + \phi\rho \times \partial_\mu\phi\rho \tag{2.22}$$

$$j_\mu^5 = \frac{1}{2}\bar{\psi}\gamma_\mu\gamma_5\tau\psi - \frac{1}{2f}[\sigma\partial_\mu\phi\rho - \phi\rho\partial_\mu\sigma] \tag{2.23}$$

which we identify with the weak interaction currents. In order that the axial vector current be renormalized to lowest order in f, another chiral invariant interaction is introduced into the original Lagrangian

$$L'' = -ig'f\bar{\psi}\gamma_\mu U\partial_\mu U^+\psi \tag{2.24}$$

This interaction changes the nucleon contribution to the axial current in eq. (2.23) into the form

$$\frac{1}{2}(1+g')\bar{\psi}\gamma_\mu\gamma_5\underset{\sim}{\tau}\psi \tag{2.25}$$

Here g' is adjusted to give the observed axial coupling renormalization $g_A|g_V$.

In this way we obtain the chiral invariant πN Lagrangian given by

$$L = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{64f^2}\,\mathrm{trace}(\partial_\mu U)(\partial_\mu U)^+ - m\bar{\psi}U(\underset{\sim}{\phi})\psi$$

$$-ig'f\bar{\psi}\,\gamma_\mu U\partial_\mu U^+\psi \tag{2.26}$$

If we now expand $\bar{\psi}U\psi$ to second order in the fields $f\underset{\sim}{\phi}$, we recover the conventional linear Lagrangian with terms which give rise to s-wave and p-wave πN interactions. The s-wave πN scattering lengths are well reproduced by performing calculations with this Lagrangian in the lowest order perturbation theory. It is found that $\lambda_1=0$ and λ_2 is consistent with the empirical data. The p-wave interaction is identical with the one obtained from the conventi:al Lagrangian with ps or pv coupling. Thus the chiral invariant effective Lagrangian reproduces all the features of low and medium energy πN scattering quite well. It is not immediately clear whether it can be used to carry out realistic calculations beyond the lowest order of perturbation theory. If only the lowest order terms are retained, however, the chiral invariant Lagrangian resembles the

the conventional linear one. In using such an approximate Lagrangian for higher order perturbation calculations, the neglect of higher order terms from the nonlinear effective Lagrangian may not be justified. So, although the effective interaction is now known, it is still not completely clear how realistic calculations can be carried out with it. However, for the p-wave interaction we take the approximate linear Lagrangian as the basis for our calculation, since it is known to reproduce the p-wave πN scattering data well. This is described in the next section.

3. Chew-Low Theory

The CL theory [10)] describes the p-wave πN interaction. In the last section we have seen that in the nonrelativistic limit both the ps and pv couplings lead to the same p-wave interaction, the CL interaction. The Hamiltonian for the πN system with p-wave interaction only is given by

$$H = H_N + H_\pi + H_I \tag{3.1}$$

where H_N, H_π and H_I are given by eqs. (2.4) (2.5) and (2.6), Relating the ps coupling constant g with the pv constant f through $g/2m = f/\mu$ we get for the interaction term

$$H_I = (f/\mu)\int d\underset{\sim}{r}\ \rho(\underset{\sim}{r})\psi^*(\underset{\sim}{\sigma}.\underset{\sim}{\nabla})\underset{\sim}{\tau}.\underset{\sim}{\phi}(\underset{\sim}{r})\psi \tag{3.2}$$

In the static limit, the nucleon kinetic energy term is zero, so that $H_N = m$. This will be suppressed because it is only a constant. In eq. (3.2) the source function $\rho(\underset{\sim}{r})$ describes the extent of the πN interaction region. It will be assumed spherically symmetric, i.e. $\rho(\underset{\sim}{r})=\rho(|\underset{\sim}{r}|)$, and real and normalized by $\int d\underset{\sim}{r}\rho(\underset{\sim}{r}) = 1$.

The form of the CL interaction (3.2) can alternatively be determined by the requirement that the Hamiltonian be invariant under rotations in space and isospin space and under time and space inversions, together with the Yukawa assumption that pions are emitted singly in the elementary interaction process: $N \rightleftharpoons N+\pi$. The term $\underset{\sim}{\nabla}\phi$ occurs since ϕ is a pseudoscalar under spatial inversions, and the only true scalar under inversion which can be constructed from the operators $\phi, \psi^*\psi$ and $\psi^*\underset{\sim}{\sigma}\psi$ is $(\psi^*\underset{\sim}{\sigma}\psi).\underset{\sim}{\nabla}\phi$. The requirement that H_I be invariant under time inversion is responsible for the fact that no term in $\pi(\underset{\sim}{r})$ - the momentum canonically conjugate to $\phi(\underset{\sim}{r})$, occurs in H_I. The invariance of H under rotations in space and in isospin space implies the conservation of the total angular momentum $\underset{\sim}{J}$ and the isospin I, respectively.

That only p-wave pions are coupled to the nucleon in this theory can be seen from the following. The conservation of total angular momentum is each elementary process $N \rightleftharpoons N+\pi$ implies $\ell=0$ or 1, where ℓ is the orbital angular momentum. But parity conservation allows only $\ell=1$. This argument assumes a static nucleon, which does not recoil.

It is convenient to work in momentum space. In order to transform the Hamiltonian into momentum space, we expand the pion field operator in terms of plane waves (Note that we are working in Schrödinger representation, so that the operators are time-independent.)

$$\phi(\underset{\sim}{r}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2\omega}}(a_k\, e^{i\underset{\sim}{k}.\underset{\sim}{r}} + a_k^+\, e^{-i\underset{\sim}{k}.\underset{\sim}{r}}) \tag{3.3}$$

where the creation and annihilation operators of the pion satisfy the commutation relations

$$[a_k, a^+_{k'}] = \delta^{(3)}(\underset{\sim}{k}-\underset{\sim}{k}') \tag{3.4}$$

and $\omega = \sqrt{k^2+\mu^2}$ is the pion energy. With,

$$v(\underset{\sim}{k}) = \int e^{-i\underset{\sim}{k}.\underset{\sim}{r}}\rho(\underset{\sim}{r})d\underset{\sim}{r} \tag{3.5}$$

the Hamiltonian (3.1) is given by

$$H = \int d\underset{\sim}{k}\,\omega a^+_k a_k + \int d\underset{\sim}{k}(V_k a_k + V_k a^+_k) \tag{3.6}$$

$$V_k = i(4\pi)^{1/2}(f/\mu)\underset{\sim}{\sigma}.\underset{\sim}{k}\tau_k v(k)/(2\omega)^{1/2} \tag{3.7}$$

where the suffix k in τ_k stands for the kth component of the nucleon isotopic spin operator and it also describes the pion quantum numbers.

The Hamiltonian (3.6) has a complete set of eigenstates Ψ_n. These states include the four single-nucleon states Ψ_0, the one-meson states Ψ_k, two-meson states, etc. The T-matrix is given by

$$T_{k'}(n) = (\Psi^{(-)}_n, V_{k'}\Psi_0) \tag{3.8}$$

where $\Psi^{(-)}_n$ are the complete orthonormal set of incoming wave eigenstates. It can be shown that $T_{k'}(n)$ satisfies the Low equation

$$T_{k'}(k) = -\sum_n \left[\frac{T^+_k(n)T_{k'}(n)}{E_n - \omega - i\varepsilon} + \frac{T^+_{k'}(n)T_k(n)}{E_n + \omega} \right] \equiv T_{k'k}(\omega) \tag{3.9}$$

Here E_n is the exact intermediate state energy. The requirement that the scattering matrix be unitary is equivalent to the following statement:

$$T^+_k(k') - T_{k'}(k) = 2\pi i \sum_n \delta(E_n - \omega')T^+_k(n)T_{k'}(n) \tag{3.10}$$

when $\omega=\omega'$. The Low eq. (3.9) is clearly compatible with the unitarity condition. It is also crossing symmetric as can be seen by considering eq. (3.9) as a certain matrix function of a complex variable, z, defined by

$$t_{k'k}(z) = -\sum_n \left[\frac{T_k^+(n)T_{k'}(n)}{E_n - z} + \frac{T_{k'}^+(n)T_k(n)}{E_n + z}\right] \tag{3.11}$$

The limit of $t_{k'k}(z)$ as z approaches the positive real axis from above $(z \to \omega + i\varepsilon)$ is $T_{k'}(k)$. The crossing symmetry is expressed by the relation

$$t_{k'k}(z) = t_{kk'}(-z) \tag{3.12}$$

We now introduce the functions $h_\alpha(\omega)$, which are related to the phase shifts δ_α , through the following

$$T_{k'k}(\omega) = -v(k')v(k) \frac{4\pi}{(\omega'\omega)^{1/2}} \sum_{\alpha=1}^{4} P_\alpha(k,k')h_\alpha(\omega) \tag{3.13}$$

$$h_\alpha(\omega) = e^{i\delta_\alpha} \sin \delta_\alpha/\{(k^3/\mu^2)v^2(k)\} \tag{3.14}$$

where $\omega = (\mu^2+k^2)^{1/2}$, $\omega' = (\mu^2+k'^2)^{1/2}$ and the P_α's are projection operators for the four eigenstates of total angular momentum and isospin with subscript $\alpha = (2I,2J)$. These are given by

$$P_{11} = \frac{1}{3} \tau_k\tau_{k'}(\underset{\sim}{\sigma}.\underset{\sim}{k})(\underset{\sim}{\sigma}.\underset{\sim}{k}')/\mu^2$$

$$P_{13} = \frac{1}{3} \tau_k\tau_{k'}[3\underset{\sim}{k}.\underset{\sim}{k}' - (\underset{\sim}{\sigma}.\underset{\sim}{k})(\underset{\sim}{\sigma}.\underset{\sim}{k}')]/\mu^2$$

$$P_{31} = (\delta_{kk'} - \frac{1}{3} \tau_k\tau_{k'})(\underset{\sim}{\sigma}.\underset{\sim}{k})(\underset{\sim}{\sigma}.\underset{\sim}{k}')/\mu^2$$

$$P_{33} = (\delta_{kk'} - \frac{1}{3} \tau_k\tau_{k'})[3\underset{\sim}{k}.\underset{\sim}{k}'-(\underset{\sim}{\sigma}.\underset{\sim}{k})(\underset{\sim}{\sigma}.\underset{\sim}{k}')]/\mu^2 \tag{3.15}$$

The scattering phase shifts of the (1,3) and (3,1) states are equal in this theory, i.e. $h_{31} = h_{13}$. In the following we use the notations $h_1 = h_{11}$, $h_2 = h_{13} = h_{31}$ and $h_3 = h_{33}$. The functions $h_\alpha(\omega)$ satisfy the crossing relation

$$h_\alpha(\omega) = \sum_{\beta=1}^{3} A_{\alpha\beta}\, h_\beta(-\omega) \tag{3.16}$$

where the crossing matrix A is given by

$$A = \frac{1}{9}\begin{bmatrix} 1 & -8 & 16 \\ -2 & 7 & 4 \\ 4 & 4 & 1 \end{bmatrix} \tag{3.17}$$

The Low equations for the functions $h_\alpha(\omega)$ are given by

$$h_\alpha(\omega) = \frac{\lambda_\alpha}{\omega} + \frac{1}{\pi}\int_\mu^\infty d\omega' v^2(k')\left[\frac{\mathrm{Im}\; h_\alpha(\omega')}{\omega'-\omega-i\varepsilon} + \sum_\beta A_{\alpha\beta}\frac{\mathrm{Im}\; h_\beta(\omega')}{\omega'+\omega}\right] \tag{3.18}$$

where

$$\lambda_\alpha = \frac{2}{3}\;\frac{f^2}{\mu^2}\begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} \tag{3.19}$$

Here f^2 is the renormalized coupling constant. Note that the above Low equations are exact.

The Low equation for $h_\alpha(\omega)$ can be rewritten

$$h_\alpha(\omega) = \frac{\lambda_\alpha \mu^2}{\omega g_\alpha(\omega)} \tag{3.20}$$

with

$$g_\alpha(\omega) = 1 - \frac{\lambda_\alpha}{\pi}\,\omega\int_\mu^\infty d\omega'\;\frac{k'^3 v^2(k')}{\omega'^2}\left[\frac{F_\alpha(\omega')}{\omega'-\omega-i\varepsilon} + \frac{G_\alpha(\omega')}{\omega'+\omega}\right] \tag{3.21}$$

where $F_\alpha(\omega)$ and $G_\alpha(\omega)$ are weight functions defined for $\omega \geq \mu$. The term with $G_\alpha(\omega)$ is referred to as the crossing term. Using eqs. (3.14), (3.20) and (3.21) Re $g_\alpha(\omega)$ can be written in the form

$$\text{Re } g_\alpha(\omega) = \frac{\lambda_\alpha k^3 v^2(k)}{\omega} \cot \delta_\alpha(\omega) = 1 - \omega r_\alpha(\omega) \tag{3.22}$$

where $r_\alpha(\omega)$ is given by the principal part of the integral. The effective range approximation is based on the weak dependence of $r_\alpha(\omega)$ on the ω occuring in the denominators of the integrands in eq. (3.21). This seems reasonable for values of $\omega \ll \omega_c$, the maximum energy effectively allowed by the cutoff factor v(k). The complete neglect of the energy dependence of $r_\alpha(\omega)$ for $\omega \ll \omega_c$, i.e. $r_\alpha = r_\alpha(\omega)$ gives the effective range. Now $F_\alpha(\omega) > 0$, since it is related to the total cross section $\sigma_\alpha(\omega)$. But $G_\alpha(\omega)$, the crossing term is not necessarily positive for all α, owing to some negative elements in the crossing matrix A, eq. (3.17). However, if $G_\alpha(\omega)$ is assumed positive (it is indeed positive for $\alpha=3$), it follows from eqs. (3.21) and (3.19) that only r_3 is positive, i.e. that the 33 effective range is positive. With an appropriate choice of cutoff there will then be a resonance in the 33-state. The resonance will occur at $\omega_r = 1/r_3$ at which point $\delta_3 = 90^\circ$. The low energy p-wave phase shifts are thus known in terms of two parameters, the coupling constant f^2 and the effective range r_3 or equivalently the cutoff. So, the static CL theory reproduces the qualitative features of πN scattering data in the 33-resonance region.

Next, let us examine the extent to which the CL theory can reproduce the πN scattering data quantitatively. If the πN coupling constant is fixed to its experimental value ($f^2 \approx 0.08$), we are left with only one adjustable parameter, i.e. the form factor v(k) or, more simply the cutoff energy. The CL theory in its static form with only one adjustable parameter cannot fit the πN scattering data very well. For example, Eisenberg and Weber [23] used the CL theory in their πA calculations. They ignored the crossing term and did not take account of the inelasticity in fitting the πN scattering data. Although they fitted the 33-resonance energy, the calculated width was much too large. Also they had to use a large cutoff energy $\omega_c \sim 9.4\mu$, which is not very satisfactory because the CL theory is a nonrelativistic one. Dover and Lemmer's calculation [24] is very similar in this respect, i.e. they also neglect the crossing term and inelasticity effects. We have done a similar calculation which is described later in this section. Our results in table 1, with crossing and inelasticity ignored, show the same feature, viz. the width is very large and the cutoff energy is too high. Thus a quantitative fit to the πN scattering data is difficult to obtain with static CL theory when crossing and inelasticity are not taken account of. In fact we see from table 1 that even with crossing term and inelasticity, the static CL theory does not reproduce the πN scattering well. Now, from the viewpoint of particle physicists it may be sufficient that the qualitative features of πN scattering data are reproduced. But for intermediate energy nuclear physics the details of the experimental data are to be fitted.

In an attempt to fit the πN scattering data Dover et al. [25] do the following. They use the one-meson approximation, i.e. $F_3 = 1$ eq. (3.21). Then for a real coupling constant, the phase shifts δ_3

remain real beyond the π-production threshold. They take account of the inelastic channels by replacing the coupling constant λ_3 by an energy-dependent complex function $\lambda_3\gamma_3(\omega)$. They showed how $\gamma_3(\omega)$ and the form factor v(k) can be determined from given πN scattering data. In other words one can find $\gamma_3(\omega)$ such that the πN scattering data are exactly reproduced. However, the relation between $\gamma_3(\omega)$ and more basic πN interaction such as the CL interaction is not clear. In addition they ignore the crossing term. So their analysis is not very satisfactory.

In the following discussion we show that the CL theory can in fact fit the πN scattering data very well if the effects of nucleon recoil, crossing term and inelasticity are all included.[26] We show that the inelasticity can be taken care of without introducing an energy-dependent, complex coupling constant. The inelasticity is simply taken into account by $F_3(\omega)$, which is determined by,

$$F_3(\omega) = [\sigma_{tot}(\omega)/\sigma_{el}(\omega)]_3 = 1 + [\sigma_{inel}/\sigma_{el}]_3 \qquad (3.23)$$

In the one-meson approximation, $F_3 = 1$ at all energies, while we use F_3 determined by putting the experimental data for $\sigma_{inel}/\sigma_{el}$ into eq. (3.23). The phase shift δ_3 becomes complex when $F_3 > 1$. To determine the function G_3 in the crossing term we need the phase shifts in an unphysical region. It is not possible to obtain an exact analytic form for G_3 even in the one-meson approximation. CL[27], therefore, proposed an approximate solution in which $G_3 \approx 2$. But from a numerical iterative solution of Salzman [28], one can see tha $G_3 \approx 2$ is not a very accurate assumption. We will, however, still assume $G_3 = 2$. Moreover, when $F_3 \neq 1$ we further assume that

$$G_3(\omega) = 2\, F_3(\omega) \tag{3.24}$$

This is the only ambiguity in determining $h_3(\omega)$ in the static CL theory.

Next let us turn to the nucleon recoil effect which we have ignored so far. The πN total momentum defined by $\underset{\sim}{P} = \underset{\sim}{p} + \int d\underset{\sim}{k}\, \underset{\sim}{k}\, a_k^+ a_k$ is a constant of motion. In the centre of mass (CM) system, we put $\underset{\sim}{P}=0$ and hence $p = -\int d\underset{\sim}{k}\, \tilde{k}\, a_k^+ a_k$. Then $H_\pi + H_N$ becomes

$$H_\pi + H_N = \int d\underset{\sim}{k}\, \tilde{\omega}\, a_k^+ a_k + \int d\underset{\sim}{k}\, d\underset{\sim}{k}'\, a_k^+ a_{k'}^+ a_k a_{k'}/(2m) \tag{3.25}$$

where

$$\tilde{\omega} = \omega + k^2/(2m) \tag{3.26}$$

In our calculation we neglect the last term in eq. (3.25). The nucleon recoil is, then approximately treated by replacing ω in eq. (3.6) but not in eq. (3.7), by the CM energy $\tilde{\omega}$. As a further improvement we consider the relativistic recoil energy of the nucleon $E_N = \{(m^2+k^2)^{1/2}-m\}$ instead of the nonrelativistic energy $k^2/(2m)$. Then $\tilde{\omega} = \omega + E_N$. With nucleon recoil eq. (3.20) becomes

$$h_3(\tilde{\omega}) = \frac{\lambda_3 \mu^2}{\tilde{\omega} g_3(\tilde{\omega})} \tag{3.27}$$

where

$$g_3(\tilde{\omega}) = 1 - \frac{\lambda_3}{\pi}\, \tilde{\omega} \int_\mu^\infty d\omega'\, \frac{k'^3 v^2(k')}{\tilde{\omega}'^2} \left(\frac{F_3(\tilde{\omega}')}{\tilde{\omega}' - \tilde{\omega} - i\varepsilon} + \frac{G_3(\tilde{\omega}')}{\tilde{\omega}' + \tilde{\omega}} \right) \tag{3.28}$$

The integration variable $d\omega'$ has not been changed in eq. (3.28). We now introduce a new function $\bar{h}_3(\tilde{\omega})$, which is simply related to phase shifts. For the nonrelativistic nucleon recoil energy its relation with $h_3(\tilde{\omega})$ is given by the following.

$$\bar{h}_3(\tilde{\omega}) = h_3(\tilde{\omega}) \left(\frac{m}{m+\omega}\right) = \frac{e^{i\delta_3} \sin \delta_3}{(k^3/\mu^2)v^2(k)} \tag{3.29}$$

$$h_3(\tilde{\omega}) = \left(\frac{m+\omega}{m}\right) \frac{e^{i\delta_3} \sin \delta_3}{(k^3/\mu^2)v^2(k)} \tag{3.29a}$$

For relativistic nucleon recoil energy the above equation becomes

$$h_3(\tilde{\omega}) = \left(\frac{E_N+m+\omega}{E_N+m}\right) \frac{e^{i\delta_3} \sin \delta_3}{(k^3/\mu^2)v^2(k)} \tag{3.30}$$

Since we are mainly interested in the medium energy range where the 33-resonance dominates, let us put $h_3(\tilde{\omega})$ in the form of the Breit-Wigner formula. First the resonance energy is determined by $\mathrm{Re}[g_3(\tilde{\omega}_r)]=0$. Then we expand $\mathrm{Re}[g_3(\tilde{\omega})]$ around $\tilde{\omega}_r$:

$$\begin{aligned} g_3(\tilde{\omega}) &\approx (\tilde{\omega}-\tilde{\omega}_r)\, \mathrm{Re}[g_3'(\tilde{\omega}_r)] + i\, \mathrm{Im}[g_3(\tilde{\omega})] \\ &= \mathrm{Re}[g_3'(\tilde{\omega}_r)]\, \{\tilde{\omega}-\tilde{\omega}_r + \frac{i}{2}\, \Gamma(\tilde{\omega})\} \end{aligned} \tag{3.31}$$

where $g_3'(\tilde{\omega}) = dg_3(\tilde{\omega})/d\tilde{\omega}$ and $\Gamma(\tilde{\omega}) = 2\, \mathrm{Im}[g_3(\tilde{\omega})]/\mathrm{Re}[g_3(\tilde{\omega}_r)]$.

We have tried to fit the resonance energy $\tilde{\omega}_r$ and the width $\Gamma(\tilde{\omega}_r)$. For the experimental values, we took $\tilde{\omega}_r$ = 1233 MeV − m = 2.13μ, and Γ = 116 MeV = 0.84μ. For the masses we took μ = 138.0 MeV and m = 6.80μ. We considered three types of form factor $v(k)$: (i) square cutoff, $v(k) = \theta(k_c-k)$, (ii) Gaussian cutoff, $v(k) = \exp(-k^2/2k_c^2)$, and (iii) Yukawa cutoff, $v(k) = k_c^2/(k^2+k_c^2)$. Since the results are not sensitive to the

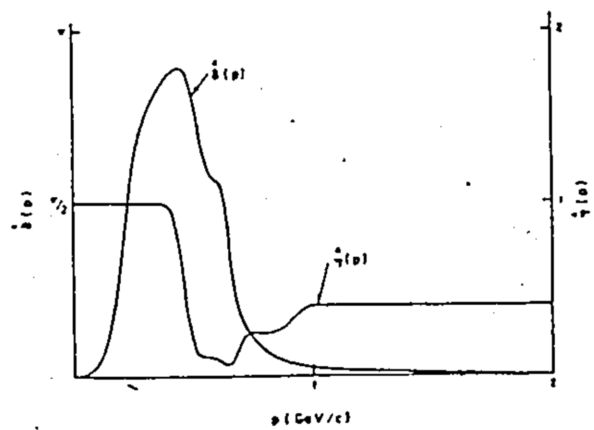

Fig. 1

This shows the inelasticity parameter $\hat{\eta}$ from which we have deduced the value for $[\sigma_{tot}/\sigma_{el}]_3$. The fig. has been taken from ref. 25.

choice of the form factor, we show only the results for the square cutoff. For the inelasticity we have taken the values from Dover et. al.[25) They plotted experimental values of $\hat{\eta}$ which are shown in fig. 1, from which one can see that $1/\hat{\eta}$ which corresponds to our $\sigma_{tot}/\sigma_{el} \simeq 1$ for $\tilde{\omega} < 4\mu$ and $1/\hat{\eta}$ increases rapidly for $4\mu \leq \tilde{\omega} \leq 5\mu$, and then it comes down to $1/\hat{\eta} \approx 2.5$. Our choice of inelasticity

$$\frac{\sigma_{tot}}{\sigma_{el}} = \begin{cases} 1 & \text{for } \tilde{\omega} < 4\mu \\ 2.5 & \text{for } \tilde{\omega} > 4\mu \end{cases} \tag{3.32}$$

is a reasonable assumption for σ_{tot}/σ_{el}, although it may somewhat underestimate the inelasticity for $4\mu \leq \tilde{\omega} \leq 7\mu$. We have considered cases with and without taking effects of nucleon recoil, the crossing term, and inelasticity.

We determined the cutoff energy $\omega_c = (\mu^2+k_c^2)^{1/2}$ such that, together with $\lambda_3 = \frac{4}{3} \times 0.08$, the resonance energy $\tilde{\omega}_r = 2.13\mu$ is fitted, and then we calculated the width $\Gamma(\omega_r)$. When the nucleon recoil is not included, we fitted $\omega_r = 2.13\mu$. The results are summarized in table 1. Almost a perfect agreement with experimental values is obtained in case D when the inelasticity is taken into account. It is gratifying that the cutoff energy is reasonable in this case. Other cases are not satisfactory since Γ is much too large and/or ω_c is much larger than m. We note that the inclusion of nucleon recoil is imperative for fitting the width of the resonance. It is clear that the effects of recoil, crossing term, and inelasticity are all important in improving the fit.

For the low energy region, the parameters of case D are found to give a good fit as can be seen by calculating the πN scattering length.

Table I

The cutoff energy ω_c which, together with $\lambda_3 = \frac{4}{3} \times 0.08$, fits the 33-resonance energy $\omega_r = 2.13\mu$. Γ is the calculated width (at the resonance energy), which is to be compared with the experimental value $\Gamma = 0.84\mu$. In case C, for example, the recoil effect is taken account of, but the crossing term is not.

Case	Recoil	Crossing term	Without inelasticity ω_c/μ	Without inelasticity Γ/μ	With inelasticity ω_c/μ	With inelasticity Γ/μ
A	No	No	10.51	1.31	6.28	1.14
B	No	Yes	6.39	1.75	4.94	1.61
C	Yes	No	55.30	0.80	14.84	0.68
D	Yes	Yes	15.10	0.92	6.79	0.83

$$\frac{1}{a} = (k^3/\mu\tilde{\omega}) \cot \delta_3\Big|_{k=0} = \frac{\mu}{\omega} \mathrm{Re}[\bar{h}_3(\tilde{\omega})]^{-1}_{k=0} \tag{3.33}$$

$$a^{-1} = \frac{m+\mu}{m} \left\{ \frac{1}{\mu\lambda_3} - \frac{1}{\pi} \int_\mu^\infty d\omega' \frac{k'^3 v^2(k')}{\tilde{\omega}'^2} \left(\frac{F_3(\tilde{\omega}')}{\tilde{\omega}'-\mu} + \frac{G_3(\tilde{\omega}')}{\tilde{\omega}'+\mu} \right) \right\} \tag{3.34}$$

This gives a = .17 μ^{-1} which is in good agreement with the experimental value of .215±.005 μ^{-1}.

Finally, we use this CL πN scattering amplitude to see how it is modified in a nuclear medium. The detailed calculation of pion-bound nucleon scattering amplitude and corrections to the impulse approximation will be examined in Chapter V. Here we just consider the qualitative effect based on a simple argument. Consider the scattering of a pion from a nucleon which is bound in an external potential, e.g. a harmonic oscillator potential. If we increase the strength of the potential, the nucleon becomes less and less mobile, and in the tight-binding limit the πN scattering amplitude is reduced to that in the static limit in which $m\to\infty$. Using the parameters of case D with inelasticity, we have examined how the 33-resonance varies as m varies from the actual nucleon mass to infinity. We have found that $\tilde{\omega}_r$ goes down to 1.12μ in the limit of $m\to\infty$. The binding effect thus seems to shift the resonance energy downwards.

4. Separable πN Interaction

We now know the πN interaction fairly well. The effective s-wave interaction reproduces the low energy data, whereas the CL theory describes the p-wave πN scattering at medium energies. But it is not easy to use these in π-nucleus calculations. There exist, however, a number of phenomenological interactions, which are simple to use in more complex calculations. Among these are several forms of separable interactions [11]. In this section we describe s-wave and p-wave separable πN interactions, which we will use in our π-nucleus calculations to estimate the binding corrections to the impulse approximation.

The separable πN interaction, unlike the effective s-wave interaction and the CL theory for p-wave scattering, has no solid theoretical basis. But as we shall show, the two are very similar in so far as the problem of examining the binding effect is concerned. The s-wave πN interaction H_{S1} of eq. (2.7) is quadratic in the pion field whereas the CL p-wave interaction, eq. (3.2), is linear in the pion creation and annihilation operators. The separable πN interaction given below, being quadratic in the pion field is obviously similar to H_{S1} for s-wave. But it is quadratic in the pion creation and annihilation operators for the p-wave also. We will show later in this section that this interaction is in fact a good simulation of the CL interaction for the binding problem.

The Hamiltonian for the πN system is given by eq. (3.1) with $H_\pi + H_N$ given by eq. (3.25). For the interaction H_I we consider the following separable form

$$H_I = -(\lambda/\mu)\int d\underline{r}_1 d\underline{r}_2 \; v(|\underline{r}-\underline{r}_1|)v(|\underline{r}-\underline{r}_2|)\phi(\underline{r}_1)\phi(\underline{r}_2) \qquad (4.1)$$

where $\underline{r}$ is the nucleon coordinate. The πN coupling constant λ is dimensionless and the form factor $v(\underline{r})$ is normalized by $\int d\underline{r} \; v(\underline{r}) = 1$ This interaction causes s-wave πN scattering in a given isospin channel

$I = 1/2$ or $3/2$, i.e., λ and $v(r)$ are determined for each isospin channel separately. The interaction (4.1) resembles the H_{S1} of eq. (2.7) except that the nucleon density $\bar{\psi}\psi$ has been replaced by the separable vertices $v(|\underline{r}-\underline{r}_1|)v(|\underline{r}-\underline{r}_2|)$. This suggests that H_{S1} can be effectively replaced by the separable interaction (4.1).

For the pion it is convenient to work in the momentum space. Hence we expand $\phi(\underline{r})$ in plane waves as before in eq. (3.3). With this H_π and H_I become

$$H_\pi = \int d\underline{k}\,\omega a_k^+ a_k \tag{4.2}$$

$$H_I = -(\lambda/\mu)(2\pi)^{-3}\int d\underline{k}d\underline{k}'(4\omega\omega')^{-1/2}g(k)g(k') \times (a_k e^{i\underline{k}.\underline{r}} + a_k^+ e^{-i\underline{k}.\underline{r}})(a_{k'} e^{i\underline{k}'.\underline{r}} + a_{k'}^+ e^{-i\underline{k}'.\underline{r}}) \tag{4.3}$$

where,

$$g(k) = \int d\underline{r}v(\underline{r})e^{i\underline{k}.\underline{r}} \tag{4.4}$$

We can rewrite eq. (4.3) as

$$H_I = -(\lambda/\mu)(2\pi)^{-3}\int d\underline{k}d\underline{k}'(\omega\omega')^{-1/2}g(k)g(k')a_k^+ a_{k'} e^{i(\underline{k}'-\underline{k}).\underline{r}} \tag{4.5}$$

In eq. (4.5) we have omitted terms of the form of $a_k a_{k'}$ and $a_k^+ a_{k'}^+$ in order to simplify the later calculations. For these two pion intermediate states, the energy denominator is much larger and we hope that their contribution will be small. The dominant contributions are due to one pion intermediate states. With the neglect of the two pion terms the interaction (4.5), however, is no longer crossing symmetric.

The s-wave πN scattering amplitude is obtained by solving the Schrödinger equation with interaction (4.5). Before doing that we note that the nucleon recoil has been taken account of in $H_\pi + H_N$ of eq. (3.25) where ω has been replaced by $\tilde{\omega}=\omega+k^2/(2m)$ in H_π. The Schrödinger equation for the pion wave function then becomes

$$(\tilde{\omega}'-\bar{\omega})\phi(\underline{k}') = \frac{\lambda}{(2\pi)^3\mu}\int d\underline{k}'' \frac{g(k')g(k'')}{(\omega'\omega'')^{1/2}}\phi(\underline{k}'') \tag{4.6}$$

where $\bar{\omega}$ is the total πN energy (excluding the nucleon rest mass) in the centre of mass (CM) system. Eq. (4.6) can be rewritten as

$$\phi(\underline{k}') = \delta(\underline{k}'-\underline{k}) + \frac{\lambda}{(2\pi)^3\mu}\frac{g(k')C}{\sqrt{\omega'}(\tilde{\omega}'-\bar{\omega}-i\varepsilon)} \tag{4.7}$$

where

$$C = \int d\underline{k}'' \frac{g(k'')\phi(\underline{k}'')}{\sqrt{\omega''}} \tag{4.8}$$

Multiplying both sides of eq. (4.7) by $g(k')/\sqrt{\omega'}$ and integrating over $d\underline{k}'$ we obtain

$$C = \frac{g(k)}{\sqrt{\omega}} + C\frac{\lambda}{(2\pi)^3\mu}\int d\underline{k}' \frac{g^2(k')}{\omega'(\tilde{\omega}'-\bar{\omega}-i\varepsilon)} \tag{4.9}$$

This can be solved for C to give

$$C = \frac{g(k)}{\sqrt{\omega}}\left[1 - \frac{\lambda}{2\pi^2\mu}\int_0^\infty dk' \frac{k'^2 g^2(k')}{\omega'(\tilde{\omega}'-\bar{\omega}-i\varepsilon)}\right]^{-1} \tag{4.9a}$$

where we have done the angular integration over $d\hat{k}'$. Substituting the value of C into eq. (4.7) we obtain

$$\phi(\underset{\sim}{k}') = \delta(\underset{\sim}{k}'-\underset{\sim}{k}) + \frac{\lambda}{(2\pi)^3\mu}\ \frac{g(k')g(k)}{(\omega'\omega)^{1/2}(\tilde{\omega}'-\tilde{\omega}-i\varepsilon)}\left[1 - \frac{\lambda}{2\pi^2\mu}\int_0^\infty dk'\ \frac{k'^2 g^2(k')}{\omega'(\omega'-\omega-i\varepsilon)}\right]^{-1}.$$

(4.10)

The wave function $\psi(\underset{\sim}{r})$ in the configuration space is given by the Fourier transform of $\phi(\underset{\sim}{k})$.

$$\psi(\underset{\sim}{r}) = \int d\underset{\sim}{k}\,\phi(\underset{\sim}{k})e^{i\underset{\sim}{k}.\underset{\sim}{r}}$$

$$= e^{i\underset{\sim}{k}.\underset{\sim}{r}} + f(\underset{\sim}{k})\ \frac{e^{ikr}}{r} \qquad (4.11)$$

where $f(\underset{\sim}{k})$ is the s-wave πN scattering amplitude. Taking the Fourier transform of eq. (4.7) or (4.10) and comparing it with eq. (4.11) we get for the s-wave πN scattering amplitude,

$$f_s(\underset{\sim}{k}) = \frac{e^{i\delta}\sin\delta}{k} = \frac{m}{m+\omega}\ \frac{\lambda g^2(k)}{2\pi\mu D_s(k)} \qquad (4.12)$$

$$D_s(k) = 1-\frac{\lambda}{2\pi^2\mu}\int_0^\infty dk'\ \frac{k'^2 g^2(k')}{\omega'(\omega'-\omega-i\varepsilon)} \qquad (4.13)$$

and δ is the s-wave πN phase shift in a given isospin channel. Note that eq. (4.12) represents the exact solution of the Schrödinger equation for the interaction (4.5).

The scattering length a and effective range r which are defined by

$$k\cot\delta = a^{-1} + \frac{1}{2}rk^2 \qquad (4.14)$$

are determined by

$$\frac{1}{a} = \frac{m+\mu}{m} \frac{2\pi\mu D_s(0)}{\lambda} = \frac{m+\mu}{m} \left\{ \frac{2\pi\mu}{\lambda} - \frac{1}{\pi} \int_0^\infty dk \frac{k^2 g^2(k)}{\omega(\omega-\mu)} \right\} \tag{4.15}$$

and

$$\frac{1}{2} r k^2 = \frac{m+\omega}{m} \frac{2\pi\mu \mathrm{Re} D_s(k)}{\lambda g^2(k)} - \frac{1}{a} \tag{4.16}$$

The parameters of the separable interaction, λ and $g(k)$ can now be determined for each isospin channel by fitting a and r. The πN interaction is attractive for I=1/2 and repulsive for I=3/2. For the latter, the interaction is much weaker. We have considered the I=1/2 channel and fitted [29] $a = 0.178\ \mu^{-1}$, $r = 2.295\ \mu^{-1}$. With straight cutoff $g(k) = \theta(k_c - k)$ we found it impossible to fit r. The form factor of the Yamaguchi type

$$g(k) = \beta^2/(\beta^2+k^2)\ , \quad \beta = 4.90\mu \tag{4.17}$$

together with $\lambda = 0.993$ reproduces a and r.

So far we have only considered the s-wave πN separable interaction. We now describe the p-wave πN interaction. In order to get p-wave πN scattering we modify eq. (4.5) by introducing the angular momentum projection operator $P(\underset{\sim}{k},\underset{\sim}{k}')$ defined by

$$P(\underset{\sim}{k},\underset{\sim}{k}') = (3\underset{\sim}{k}.\underset{\sim}{k}' - \underset{\sim}{\sigma}.\underset{\sim}{k}\underset{\sim}{\sigma}.\underset{\sim}{k}')/\mu^2 \tag{4.18}$$

which causes interaction in the J = 3/2 state only. We supress the isospin part of the projection operator P of eq. (4.18). It should be understood that H_I defined below acts only in the I = 3/2 state.

$$H_I = -(\lambda/\mu)(2\pi)^{-3}\int d\underset{\sim}{k}\,d\underset{\sim}{k}'(\omega\omega')^{-1/2}g(k)g(k')P(\underset{\sim}{k},\underset{\sim}{k}')\, a^{+}_{k}a_{k'}\exp\{i(\underset{\sim}{k}'-\underset{\sim}{k}).\underset{\sim}{r}\} \tag{4.19}$$

Schrödinger equation for the pion wave function now becomes

$$(\bar{\omega}'-\bar{\omega})\phi(\underset{\sim}{k}') = \frac{\lambda}{(2\pi)^3\mu}\int d\underset{\sim}{k}''\,\frac{g(k')g(k'')}{(\omega'\omega'')^{1/2}}P(\underset{\sim}{k}',\underset{\sim}{k}'')\phi(\underset{\sim}{k}'') \tag{4.20}$$

This can be solved exactly as before. The only additional relation required is the following

$$\int d\hat{k}'\; P(\underset{\sim}{k},\underset{\sim}{k}')P(\underset{\sim}{k}',\underset{\sim}{k}'') = 4\pi(k'/\mu)^2P(\underset{\sim}{k},\underset{\sim}{k}'') \tag{4.21}$$

The resulting p-wave πN scattering amplitude for the 33-state is then given by

$$f_p(\underset{\sim}{k}) = \frac{e^{i\delta_3}\sin\delta_3}{k}\,\frac{P(\underset{\sim}{k}',\underset{\sim}{k})}{(k/\mu)^2} = \frac{m}{m+\omega}\,\frac{\lambda g^2(k)P(\underset{\sim}{k}',\underset{\sim}{k})}{2\pi\mu D_p(k)} \tag{4.22}$$

where

$$D_p(k) = 1 - \frac{\lambda}{2\pi^2\mu^3}\int_0^\infty dk'\,\frac{k'^4 g^2(k')}{\omega'(\bar{\omega}'-\omega-i\varepsilon)} \tag{4.23}$$

and δ_3 is the 33-phase shift. For the interaction that we have chosen, the πN phase shifts in all other partial waves vanish. The total cross section for π^+p scattering from an unpolarized proton target is given by the optical theorem

$$\sigma(\pi^+p) = (4\pi/k)\ \mathrm{Im}\ f(\underset{\sim}{k}) \tag{4.24}$$

We now want to detmine λ and $g(k)$ by fitting the energy and width of the 33-resonance. The resonance energy $\tilde{\omega}_r$ is determined by $\mathrm{Re}\ D(\tilde{\omega}_r) = 0$ as in the case of CL interaction. The width Γ is then determined by expanding $\mathrm{Re}\ D(\tilde{\omega})$ around $\tilde{\omega}_r$,

$$D(\tilde{\omega}) = (\tilde{\omega}-\tilde{\omega}_r)\ \mathrm{Re}\ D'(\tilde{\omega}_r) + i\ \mathrm{Im}\ D(\tilde{\omega})$$

$$= \mathrm{Re}\ D'(\tilde{\omega}_r)[\tilde{\omega}-\tilde{\omega}_r + i(\Gamma/2)] \tag{4.25}$$

where $D'(\tilde{\omega}) = d\ D(\tilde{\omega})/d\tilde{\omega}$ and $\Gamma = 2\ \mathrm{Im}\ D(\tilde{\omega}_r)/\mathrm{Re}\ D'(\tilde{\omega}_r)$. For the energy and the width of the resonance, we take $\tilde{\omega}_r = 2.13\mu$ and $\Gamma = .84\mu$. We choose the square cutoff $g(k) = \theta(k_c-k)$ for the nucleon form factor. We tried other forms such as the Gaussian, but the results are quite insensitive to the details of the form factor. The resonance energy $\tilde{\omega}_r$ and the width Γ are fitted by taking the following values for λ and $\omega_c = (k_c^2+\mu^2)^{1/2}$;

$$\lambda = .1536\ ,\quad \omega_c = 7.21\mu \tag{4.26}$$

The 33 πN scattering length a_3 is given by

$$1/a_3 = (k^3/\mu^2)\ \cot\delta_3\Big|_{k=0} = \frac{m+\mu}{m}\,\frac{2\pi\mu}{\lambda}\,D(0) \tag{4.27}$$

With the parameters of eq. (4.26) it is found to be, $a_3 = 0.10\mu^{-1}$, whereas the experimental value is $a_3 = 0.21\mu^{-1}$. It is possible to improve the fit by choosing an appropriate form of $g(k)$, but since we are mainly interested in the 33-resonance energy region the scattering amplitude obtained above is satisfactory. In fact the effective range plot for this

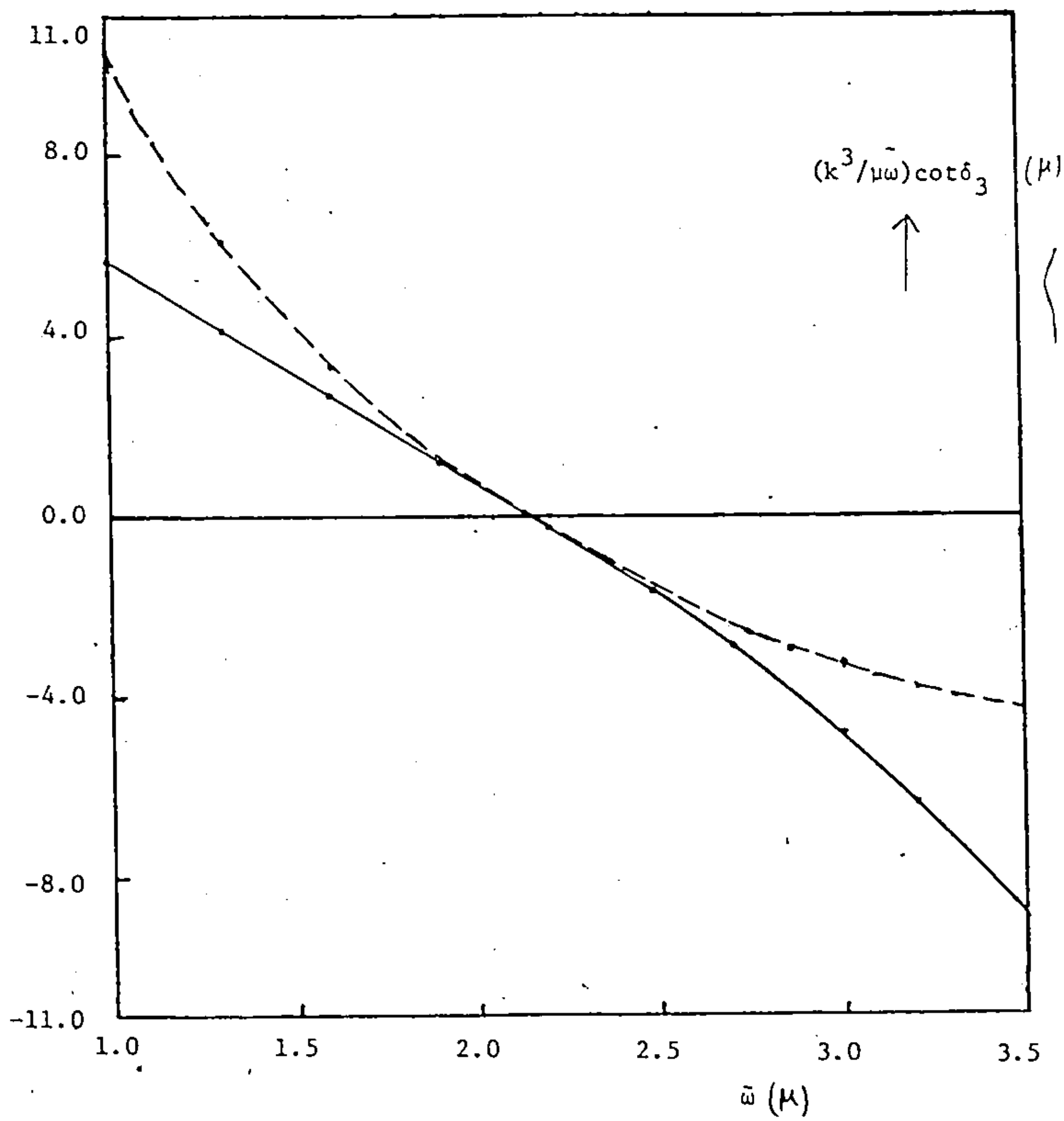

Fig. 2

The effective range plot for both the CL and the separable interactions. $(k^3/\mu\tilde{\omega})\cot\delta_3$ versus $\tilde{\omega}$ is shown. The solid curve corresponds to the CL interaction and the dashed curve represents the separable interaction.

amplitude, as shown in fig. 2 is very similar to that of the CL theory around the resonance energy. In this figure we have plotted $(k^3/\mu\bar{\omega})\cot\delta_3$ versus $\bar{\omega}$ for the separable as well as the CL interaction.

This separable model of the p-wave πN interaction is used later on in this thesis to examine the binding effects for pion scattering from a bound nucleon. Although it is a phenomenological model, we believe that for the problem under consideration it is in fact a good simulation of the CL model. Let us compare some typical diagrams for πN scattering in the two models shown in fig. 3. The binding effect for π-bound nucleon scattering is due to the change in the nucleon propagator in the intermediate states. Then diagram A' in fig. 3 is modified but A is not. We show in Appendix A, however, that the binding effect on A', or more generally on a similar part (with two-pion intermediate states) of a larger diagram, is quite negligible and the main contribution stems from the one-pion intermediate state in which the energy denominator can vanish. Hence it is a good approximation to replace A' by A, and the circled part of B' by the corresponding part of B in fig. 3.

We also examine the Pauli principle effect in πA calculations with this separable πN interaction. However, unlike for binding effects, this interaction is not similar to the CL interaction for the Pauli principle effect. This is because the Pauli principle affects the lowest order diagram A' for the CL interaction but the corresponding diagram A for the separable interaction is unaffected. The effects due to the Pauli principle are thus expected to be more pronounced for the CL interaction than for the separable interaction. So, our results with Pauli principle

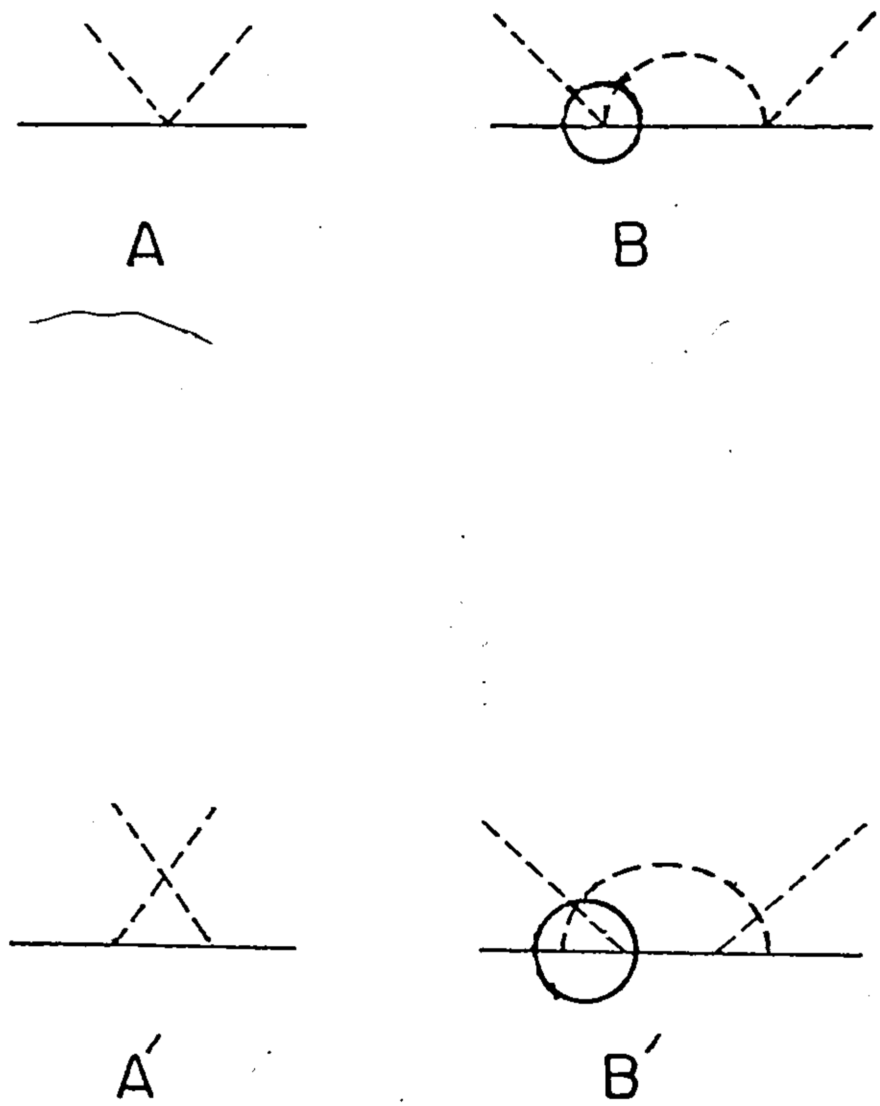

Fig. 3

Diagrams A and B represent the first and second order πN scattering for the separable interaction. Diagrams A' and B' are the corresponding ones for the Chew-Low theory.

effects should be taken with this point in mind.

5. $\bar{K}N$ Interaction

The $\bar{K}N$ interaction is dominantly in the s-wave at low energies. The s-wave scattering length in both the isospin channels I=0 and I=1 is complex due to strong absorption into the open $\pi\Lambda$ and $\pi\Sigma$ channels. The $\bar{K}N$ scattering in the I=0 channel is much stronger because of the resonance $\Lambda(1405)$, with spin J=1/2, which can be regarded as a quasi-bound state of the particles $\bar{K}$ and N with the possibility of decay into the $\pi\Sigma$ channel. There are a variety of phenomenological models which fit the empirical data fairly well. We assume a separable model for $\bar{K}N$ interaction, like the one introduced in the last section for s-wave πN interaction. The $\bar{K}N$ interaction is then given by eq. (4.5), where the $\bar{K}N$ coupling constant λ is complex now because of the complex scattering length. The scattering amplitude is given by eq. (4.12). λ and $g(k)$ are obtained by fitting the complex a and the real part of r. The data for the $\bar{K}N$ scattering parameters are summarized by (in units of fm) [29]

$$a = \begin{cases} -1.66 + 0.69i \\ -0.09 + 0.54i \end{cases} , \quad r = \begin{cases} 0. \\ 0.34 \end{cases} \text{for I} = \begin{cases} 0 \\ 1 \end{cases} \tag{5.1}$$

Although the low energy $\bar{K}N$ data indicate that the interaction is dominantly s-wave, the partial wave analysis is complicated by the multichannel nature of the $\bar{K}N$ interaction; inelastic channels $\pi\Lambda, \pi\Sigma$ and even $\pi\pi\Lambda$ are open at the $\bar{K}N$ threshold energy. Also the $\bar{K}N$ data are not accurate enough to provide reliable effective ranges.

We used a square form factor $g(k)=\theta(k_c-k)$ for simplicity. With this simple form of g(k) we found it difficult to fit r=0 for I=0, and hence we decided to take Re(r) = 0.34fm for both I=0 and I=1 states. We used μ = 493.8 MeV and m = 1.903μ. The values of λ and k_c together with r fitted are shown in table 2. The function $D_s(k)$ of eq. (4.13) has a zero for a complex value of k, which corresponds to a quasi-bound state with binding energy 15 MeV and width 35 MeV. The agreement with the corresponding figures of Λ(1405) is not very satisfactory. However, we are mainly interested in seeing how the $\bar{K}N$ interaction is modified by binding, i.e. the binding effect on s-wave scattering length. This separable model for $\bar{K}N$ interaction is useful for making a qualitative estimate.

Table 2

Parameters for the $\bar{K}N$ interaction. The cutoff momentum k_c, and complex coupling constant λ are shown, which fit the scattering length a of eq. (4.15) and the effective range r(fm) shown in the table.

I	k_c/μ	$Re(\lambda)$	$Im(\lambda)$	Re(r)	Im(r)
0	1.964	11.80	1.25	0.34	-0.012
1	2.083	6.43	4.97	0.34	-0.099

Chapter III

THE IMPULSE APPROXIMATION

The scattering of a particle from a composite system such as the nucleus, is generally described in terms of approximate multiple scattering methods such as the Glauber approximation or by relatively simple optical model potentials. In these calculations the particle-nucleus scattering amplitude is obtained in terms of the particle-nucleon scattering amplitude. This scattering amplitude should in principle be related to the t-matrix for a bound nucleon, but in practice the free nucleon-particle t-matrix is usually used. This is known as the impulse approximation (IA). It is generally expected that the IA is valid if the energy of the projectile is much larger than the binding energy of the nucleon in the target.

In the case of πA scattering at medium energies, the IA has been used in most of the theoretical calculations. In this energy region where the πN interaction is dominated by the 33-resonance, the validity of the IA is quite dubious. In the next two chapters we will estimate the binding corrections to the IA for πN scattering at low and medium energies and for $\bar{K}N$ scattering at low energies, by doing a model calculation in which the nucleon is bound in a shell model potential. In sect. 1 of this chapter we derive the relation between the t-matrix for a bound nucleon and that for a free nucleon.

In $MN_f^{\dagger}$ scattering the nucleon momentum is precisely known. In

† We denote a free nucleon by N_f and a bound nucleon by N_b.

the IA, on the other hand, although the bound nucleon is considered as free, it does not have a definite momentum. This is due to Fermi motion. The IA takes different forms depending upon the choice of nucleon momentum. In sect. 2 we consider a fixed nucleus with nucleons moving in it with all possible momenta. The IA in this case corresponds to obtaining the MN_f amplitude for a given nucleon momentum and then averaging it with certain momentum distribution function. Many authors, on the other hand, consider the motion of the nucleus as a whole, with nucleons "frozen" in it, which gives a fixed value of the nucleon momentum. In this case, however, the MN scattering does not conserve momentum. The difficulties associated with the IA of this form are discussed in sect. 3. The results of the IA described here will be presented in Chapter V along with the π-bound nucleon results.

1. Relation between MN_b and MN_f t-matrices

Consider a nucleon bound in a potential $V(\underset{\sim}{r})$ centred at $\underset{\sim}{r} = 0$. The nucleon will be treated nonrelativistically with kinetic energy operator K_N. The nuclear Hamiltonian is given by

$$H_A = K_N + V \tag{1.1}$$

In the initial bound state the wave function $g_0(\underset{\sim}{r},s)$ satisfies the Schrödinger equation

$$H_A g_0 = W_0 g_0 \tag{1.2}$$

where W_0 is the energy of the nucleon and s is its spin.

Let us consider the scattering of a meson from this bound nucleon [9]. The meson is assumed to interact with the nucleon through a potential v.

The Hamiltonian of the whole system is given by

$$H = H_A + K_M + v = H_0 + v \tag{1.3}$$

where K_M is the meson energy operator; the meson is treated relativistically. For elastic MN_b scattering the nucleon remains in the initial bound state. Both the initial and final wave functions are given by,

$$\chi_a = (2\pi)^{-3/2} e^{i\underline{k}.\underline{r}} g_0 \tag{1.4}$$

This satisfies the Schrodinger equation

$$(K_M+K_N+V)\chi_a = (\omega+W_0)\chi_a = E_a\chi_a \tag{1.4a}$$

where $\omega = (\mu^2+k^2)^{1/2}$ is the meson energy.

Using the techniques of "scattering by two potentials", the steady state wave function for the complete scattering process is determined from

$$\psi_a^+ = \chi_a + \frac{1}{E_a+i\eta-H_0} \tau(E_a)\chi_a \tag{1.5}$$

The MN_b t-matrix $\tau(E_a)$ satisfies the following equation

$$\tau(E_a) = v + v \frac{1}{E_a+i\eta-H} v \tag{1.6a}$$

$$= v + v \frac{1}{E_a+i\eta-H_0} \tau(E_a) \tag{1.6b}$$

$$= v + \tau(E_a) \frac{1}{E_a+i\eta-H_0} v \tag{1.6c}$$

This t-matrix is a many-body operator as a result of the nuclear

Hamiltonian H_A in the propagator, which represents the effect of binding.

The scattering matrix $t(E_a^0)$ representing the MN_f t-matrix via the potential v is given by

$$t(E_a^0) = v + v \frac{1}{E_a^0+i\eta-K_M-K_N-v} v \tag{1.7a}$$

$$= v + v \frac{1}{E_a^0+i\eta-K_M-K_N} t(E_a^0) \tag{1.7b}$$

$$= v + t(E_a^0) \frac{1}{E_a^0+i\eta-K_m-K_N} v \tag{1.7c}$$

Here the energy

$$E_a^0 = \omega + E_{N,P} \tag{1.8}$$

corresponds to a free nucleon with nucleon momentum $\underset{\sim}{P}$ and $E_{N,P} = P^2/2M$; for small momenta $\underset{\sim}{P}, E_a$ and E_a^0 differ roughly by the binding energy of the nucleon. This collision matrix $t(E_a^0)$ is a two-body operator.

The IA consists of taking $\tau(E_a)$ to be the MN_f scattering matrix $t(E_a^0)$, i.e.

$$\tau(E_a) = t(E_a^0) \tag{1.9}$$

The accuracy of the IA can be assessed by finding a relation between τ and t. This can easily be done by eliminating v between eqs. (1.6) and (1.7). Premultiply eq. (1.6b) by

$$\{1 + t(E_a^0) \frac{1}{E_a^0+i\eta-K_M-K_N}\}$$

and use eq. (1.7c) to obtain

$$\left(1 + t(E_a^0) \frac{1}{E_a^0+i\eta-K_M-K_N}\right)\tau(E_a) = t(E_a^0)\left(1 + \frac{1}{E_a+i\eta-K_M-H_A}\tau(E_a)\right) \tag{1.10}$$

This can be rewritten as

$$\tau(E_a) = t(E_a^0) + t(E_a^0)\left[\frac{1}{E_a+i\eta-K_M-H_A} - \frac{1}{E_a^0+i\eta-K_M-K_N}\right]\tau(E_a) \tag{1.11}$$

The error involved in the IA is of the order of

$$\Delta = \tau(E_a) - t(E_a^0) \approx t(E_a^0\left[\frac{1}{E_a+i\eta-K_M-H_A} - \frac{1}{E_a^0+i\eta-K_M-K_N}\right] t(E_a^0) \tag{1.12}$$

If,

$$E_a = E_a^0 + R_a \tag{1.13}$$

where R_a is an appropriate level shift - roughly equal to the binding potential in perturbation theory, relating the energies $E_{N,P}$ and W_0, then Δ is approximately given as,

$$\Delta = R_a t(E_a^0) \frac{1}{(E_a^0+i\eta-K_M-K_N)^2} t(E_a^0) \tag{1.14}$$

The importance of Δ can be estimated by taking a matrix element of Δ.

$$\langle \underset{\sim}{k},\underset{\sim}{Q}|\Delta|\underset{\sim}{p},\underset{\sim}{P}\rangle = \delta(\underset{\sim}{k}+\underset{\sim}{Q}-\underset{\sim}{p}-\underset{\sim}{P})R_a\int d^3p'd^3P'\langle \underset{\sim}{k},\underset{\sim}{Q}|t|\underset{\sim}{p}',\underset{\sim}{P}'\rangle$$

$$\times \frac{\delta(\underset{\sim}{p}'+\underset{\sim}{P}'-\underset{\sim}{p}-\underset{\sim}{P})\langle \underset{\sim}{p}',\underset{\sim}{P}'|t|\underset{\sim}{p},\underset{\sim}{P}\rangle}{(\omega_p+E_{N,P}+i\eta-\omega_{p'}-E_{N,P'})^2} \tag{1.15}$$

An order of magnitude estimate of (1.15) can be done, by assuming that for very large ω_p, the principal contribution to the integration comes from the domains near the singularity of the denominator, so that the two matrices t are restricted to the energy shell. Then ignoring the angular dependence of the t's (Goldberger and Watson) [9] it is found that

$$\langle \underset{\sim}{k},\underset{\sim}{Q}|\Delta|\underset{\sim}{p},\underset{\sim}{P}\rangle = t\left(\frac{f}{\lambda\!\!\!^{-}}\,\frac{R_a T}{\hbar}\right) \qquad (1.16)$$

where $\lambda\!\!\!^{-} = \hbar/p$, f is the MN_f scattering amplitude and $T = \hbar/\omega_p + Q$ is the time duration of collison, Q being the time delay. With this eq. (1.11) becomes

$$\tau(E_a) = t(E_a^0)\left[1 + 0\left(\left|\frac{f}{\lambda\!\!\!^{-}}\,\frac{R_a T}{\hbar}\right|\right)\right] \qquad (1.17)\,.$$

First of all we note that the IA is valid when the first Born approximation may be used. This follows from eqs. (1.6) and (1.7), since then $\tau(E_a) = v = t(E_a^0)$. In the weak binding limit, $V/\omega << 1$, the IA is a good approximation. The correction term in eq. (1.17) is rather small in the absence of resonant scattering and in the weak binding limit. Near the 33-resonance in πN scattering, however, the IA for πA scattering seems to fail as can be seen by evaluating the correction term in eq. (1.17). We take R_a = 7 MeV and the time delay $Q = (\Gamma/2)$, where Γ = 120 MeV is the resonance width, $T = \hbar/\omega + Q$. The scattering amplitude is $f \simeq 4$ fm, while $\lambda\!\!\!^{-} \simeq 1.4$ fm. The correction term in eq. (1.17) is then found to be

$$\left|\frac{f}{\lambda\!\!\!^{-}}\,\frac{R_a T}{\hbar}\right| \simeq .4 \qquad (1.18)$$

which is insufficient to imply the validity of the IA.

2. Impulse Approximation : nucleon motion

In this section we consider two possible forms of the IA for the case with fixed nucleus, taking account of the nucleon motion. Our model calculation in the following chapters assumes a fixed nucleus but takes account of the nucleon motion. We estimate the binding effects by comparing the results of those calculations with the IA described in this section. We will use this IA for π-nucleus scattering at medium energies. In what follows we consider a pion for the incident particle, although the results are quite general and are valid for any projectile.

Consider scattering of a pion from a nucleon bound in a potential, which is fixed at the origin. In the IA the pion interacts with N_b as if it were a free nucleon. However, unlike N_f, the nucleon inside the potential well does not have a fixed momentum. We take account of the nucleon motion by averaging the πN_f t-matrix with appropriate nucleon momentum distribution.[30)]

Let $\underset{\sim}{k}$,p be the momenta of the pion and the nucleon respectively in the πA CM system in which the nuclear centre is fixed at the origin. Actually, since the nucleon is fixed and its recoil is ignored, there is no distinction between the CM and the laboratory (lab) systems for πA scattering. However, in our πN_b calculation in Chapter V, the pion energies before and after the scattering are the same in an elastic scattering process, which is the case in πA CM system. Hence we interpret it to be in the πA CM system. For the sake of uniformity we use the same frame of reference in discussing the IA also. The πN_f t-matrix $t(\omega_0)$ is required in the πA CM system. It is related to the t-matrix in πN CM system, $\tilde{t}(W)$ by [31)]

$$\langle \underset{\sim}{k}', \underset{\sim}{p}' | t(\omega_0) | \underset{\sim}{k}, \underset{\sim}{p} \rangle = \gamma \langle \underset{\sim}{\kappa}' | \tilde{t}(W) | \underset{\sim}{\kappa} \rangle \tag{2.1}$$

where ω_0 is the πN energy, $\underset{\sim}{k}', \underset{\sim}{p}'$ are the final momenta of the pion and the nucleon in πA CM and $\underset{\sim}{\kappa}, \underset{\sim}{\kappa}'$ are the initial and final pion momenta and W is the πN energy in the πN CM. The factor γ is given by

$$\gamma = \left[\frac{E_\pi(\kappa)E_\pi(\kappa')E_N(\kappa)E_N(\kappa')}{E_\pi(k)E_\pi(k')E_N(k)E_N(k')} \right]^{1/2} \tag{2.2}$$

The pion momentum $\underset{\sim}{\kappa}$ and energy $\varepsilon(\kappa) = (\underset{\sim}{\kappa}^2+\mu^2)^{1/2}$ in πN CM can be determined from the Lorentz invariance of the Mandelstam variable $s = (k+p)^2$. This implies the following relation for $\varepsilon(\underset{\sim}{\kappa})$

$$\varepsilon(\underset{\sim}{\kappa}) = \frac{E_\pi E_N + \mu^2 - pk\cos\theta_p}{\{2E_\pi E_N + m^2 + \mu^2 - 2pk\cos\theta_p\}^{1/2}} \tag{2.3}$$

where E_π, E_N are the pion and nucleon energies, respectively in πA CM and θ_p is the angle of nucleon momentum with respect to that of the incoming pion. With this we obtain t as a function of $(k,p,\cos\theta_p)$. This is averaged over the nucleon momentum distribution to obtain $t_{av}(\underset{\sim}{k})$,

$$t_{av}(\underset{\sim}{k}) = \int d^3p\, \rho(p)\, t(\underset{\sim}{k},p,\cos\theta_p) \tag{2.4}$$

where $\rho(p)$ is the nucleon momentum distribution. We will consider the case in which the nucleon is bound in the s-state of a harmonic oscillator potential. The momentum distribution in this case is given by

$$\rho(p) = (\pi/b^3)^{-1/2} \exp(-p^2b^2) \tag{2.5}$$

where $\int \rho(p)d^3p = 1$ and the HO parameter b^2, for a nucleus of mass number A, is given by [32)]

$$b^2 = 1/(m\eta) \qquad (2.6)$$

$$\eta = 41\ A^{-1/3}\ \text{MeV} \qquad (2.7)$$

The scattering amplitude is related to the on-shell t-matrix through

$$f = -4\pi^2\omega t \qquad (2.8)$$

for t we use the averaged t_{av} to obtain the πN scattering amplitude in the IA. We will refer to this form of IA as IA(A).

Note that we have obtained the forward scattering amplitude only. For any finite momentum transfer, the initial and final wave functions in momentum space are different. In such a case the nucleon momentum distribution with which the t-matrix should be averaged is not uniquely known. So, we do not present the results for angular distributions in the IA(A).

There is another form of the IA for a fixed nucleus, which takes account of the nucleon momentum distribution. This is due to Chew.[33)] We will follow the description given by Mott and Massey.[33)] Consider two distinguishable particles 1 and 2 of equal mass m, with particle 2 bound to an infinitely massive centre under the action of an attractive potential $\hbar^2 W(r_2)/2m$, where r_2 is measured from the centre. If the particle 1 is incident with wave vector $\underset{\sim}{k}_0$ on particle 2 bound in its ground state with energy $\hbar^2\varepsilon_0/2m$, then the full wave equation for the entire system is

$$[\nabla_1^2 + \nabla_2^2 + k_0^2 + \varepsilon_0 - U(r_{12}) - W(r_2)]\Psi = 0 \qquad (2.10)$$

where $\hbar^2 U(r_{12})/2m$ is the interaction energy between particles 1 and 2. According to the Born approximation, the amplitude for scattering of particle 1 into the solid angle $d\Omega$ leaving the particle 2 in an excited state of energy ε_n is given by

$$f^b_{on} = -\frac{1}{4\pi}\int U(r_{12})\, e^{i(\underset{\sim}{k}_0-\underset{\sim}{k}_n)\cdot\underset{\sim}{r}_1}\,\psi_0(r_2)\psi^*_n(r_2)d\underset{\sim}{r}_1 d\underset{\sim}{r}_2 \qquad (2.11)$$

where ψ_0,ψ_n are the wave functions for the initial and final states of particle 2 and $\underset{\sim}{k}_0,\underset{\sim}{k}_n$ are the initial and final wave vectors of particle 2.

Consider the momentum transform of the wave function $\psi_n(\underset{\sim}{r})$,

$$g_n(\kappa) = (2\pi)^{-3/2}\int e^{-i\underset{\sim}{\kappa}\cdot\underset{\sim}{r}}\,\psi_n(\underset{\sim}{r})d\underset{\sim}{r} \qquad (2.12)$$

with inverse

$$\psi_n(\underset{\sim}{r}) = (2\pi)^{-3/2}\int e^{i\underset{\sim}{\kappa}\cdot\underset{\sim}{r}}\,g_n(\underset{\sim}{\kappa})d\underset{\sim}{\kappa} \qquad (2.13)$$

With these eq. (2.11) becomes

$$f^b_{on} = -\frac{1}{2(2\pi)^4}\int e^{i(\underset{\sim}{k}_0-\underset{\sim}{k}_n)\cdot\underset{\sim}{r}_1+i(\underset{\sim}{\kappa}_0-\underset{\sim}{\kappa}_n)\cdot\underset{\sim}{r}_2}\,U(\underset{\sim}{r}_{12})\,\times$$

$$\times\, g^*_n(\kappa_n)g_0(\kappa_0)d\underset{\sim}{\kappa}_n d\underset{\sim}{\kappa}_0 d\underset{\sim}{r}_1 d\underset{\sim}{r}_2 \qquad (2.14)$$

Transforming to the CM and relative variables $R = \frac{1}{2}(\underset{\sim}{r}_1+\underset{\sim}{r}_2)$, $r = \underset{\sim}{r}_1-\underset{\sim}{r}_2$, eq. (2.14) reduces to

$$f^b_{on} = -\frac{1}{2(2\pi)^4}\int \exp\{i(\underset{\sim}{k}_0-\underset{\sim}{k}_n+\underset{\sim}{\kappa}_0-\underset{\sim}{\kappa}_n)\cdot R + \frac{1}{2}i(k_0-k_n-\kappa_0+\kappa_n)\cdot r\}\,\times$$

$$U(r)g^*_n(\kappa_n)g_0(\kappa_0)d\underset{\sim}{\kappa}_n d\underset{\sim}{\kappa}_0 d\underset{\sim}{R}d\underset{\sim}{r} \qquad (2.15)$$

$$= -\frac{1}{4\pi}\int \delta(\underset{\sim}{k}_0+\underset{\sim}{\kappa}_0-\underset{\sim}{k}_n-\underset{\sim}{\kappa}_n)\ \exp\{\frac{1}{2}\ i(\underset{\sim}{k}_0-\underset{\sim}{k}_n-\underset{\sim}{\kappa}_0+\underset{\sim}{\kappa}_n)\cdot\underset{\sim}{r}\}\ \times$$

$$U(r)g_n^*(\kappa_n)g_0(\kappa_0)d\kappa_n d\underset{\sim}{\kappa}_0 d\underset{\sim}{r} \qquad (2.16)$$

$$f_{on}^b = 2\int f_b\{\frac{1}{2}(k_0-\kappa_0),\frac{1}{2}(\underset{\sim}{k}_n-\underset{\sim}{\kappa}_n)\}\delta(\underset{\sim}{k}_0+\underset{\sim}{\kappa}_0-\underset{\sim}{k}_n-\underset{\sim}{\kappa}_n)\ \times$$

$$g_n^*(\underset{\sim}{\kappa}_n)g_0(\underset{\sim}{\kappa}_0)d\underset{\sim}{\kappa}_n d\underset{\sim}{\kappa}_0 \qquad (2.17)$$

where

$$f_b\left\{\frac{1}{2}(\underset{\sim}{k}_0-\underset{\sim}{\kappa}_0),\ \frac{1}{2}(\underset{\sim}{k}_n-\underset{\sim}{\kappa}_n)\right\} = -\frac{1}{8\pi}\int \exp\left\{\frac{1}{2}\ i(\underset{\sim}{k}_0-\underset{\sim}{k}_n-\underset{\sim}{\kappa}_0+\underset{\sim}{\kappa}_n)\cdot\underset{\sim}{r}\right\}U(\underset{\sim}{r})d\underset{\sim}{r} \qquad (2.18)$$

is the amplitude in the Born approximation, for scattering of particle 1 by 2 in which the relative wave vector changes from $\frac{1}{2}(\underset{\sim}{k}_0-\underset{\sim}{\kappa}_0)$ to $\frac{1}{2}(\underset{\sim}{k}_n-\underset{\sim}{\kappa}_n)$. The extra factor of 1/2 in eq. (2.18) arises because in the amplitude for the collision between two particles, reduced mass m/2 replaces the mass m which is appropriate for a collision of a particle with an infinitely massive core. In eq. (2.17) this scattered amplitude is averaged over the initial and final momentum distributions of the bound particle. The additional factor of δ-function guarantees that momentum is conserved in the collisions between the particles.

From eq. (2.17) it follows that an improved approximation for the scattering amplitude from a bound particle might be obtained by replacing the Born amplitude for the free-free collision between particles 1 and 2 by the exact amplitude for such collisions, leaving the form factor unaltered. We thus obtain

$$f_{on} = 2f\left\{\frac{1}{2}(k_0-\kappa_0),\frac{1}{2}(k_n-\kappa_n)\right\} F_{on}(q) \tag{2.19}$$

where the form factor $F_{on}(q)$ is given by

$$F_{on}(q) = \int g_n^*(\kappa_n) g_0(k_n+\kappa_n-k_0)d\kappa_n$$
$$= \int \psi_n^*(r)\psi_0(r)\, e^{i(k_0-k_n)\cdot r}\, dr \tag{2.20}$$

If k_0, k_n are large compared with the effective values of κ_0 and κ_n, the amplitude f will depend strongly on the momentum transfer $q = k_n - k_0$ but not so strongly on the initial or final relative momenta, i.e.,

$$f\left\{\frac{1}{2}(k_0-\kappa_0),\frac{1}{2}(k_n-\kappa_n)\right\} \approx f(\frac{1}{2}\, k_0, k_n - \frac{1}{2}\, k_0) \tag{2.21}$$

With this,

$$f_{on} = 2f(\frac{1}{2}\, k_0, k_n - \frac{1}{2}\, k_0)\; F_{on}(q) \tag{2.22}$$

If the masses of the incident and bound particles are m_1 and m_2 respectively, k_0 is the incident wave number and q the momentum transfer, the scattered amplitude in the IA is given by

$$f_{on} = \frac{m_1+m_2}{m_2}\, f\left\{\frac{m_2}{m_1+m_2}\, k_0\ ,\ \frac{m_2}{m_1+m_2}\, k_0+q\right\} F_{on}(q) \tag{2.23}$$

This is valid for nonrelativistic particles, whereas in case of pion scattering from a bound nucleon the former is relativistic and is described by Klein-Gordon equation. In addition the πN interaction is non-local in contrast to the local potential assumed above. But we assume the above form for IA with one replacement, that of pion mass by its energy ω.

$$f_{on}(\pi N_b) = \frac{m+\omega}{m} f_{\pi N_f}\left\{\frac{m}{m+\omega} \underset{\sim}{k}_0 \, , \, \frac{m}{m+\omega} \underset{\sim}{k}_0 + \underset{\sim}{q}\right\} \tag{2.24}$$

It is not clear how the factor γ should be incorporated in the relativistic version of the above form of the IA and hence we do not include this factor. We will call this IA by IA(B).

3. Impulse Approximation : "frozen" nucleon

In the last section we have described the IA for πA scattering in which the nucleus is fixed but the motion of the bound nucleon is taken into account. In this section we consider the IA when the nucleons are "frozen" but the nuclear motion as a whole is taken into account. Our model calculation for πN_b scattering in the following chapters assumes a fixed nucleus, so that the results of the IA described in this section cannot be compared directly with it. Nevertheless it is an interesting comparison.

The matrix elements of the πN_f t-matrix, $\langle \underset{\sim}{k}', \underset{\sim}{p}' | t(E_a^0) | \underset{\sim}{k}, \underset{\sim}{p} \rangle$ depend on the initial and final pion momenta $\underset{\sim}{k}', \underset{\sim}{k}$; the nucleon momenta $\underset{\sim}{p}', \underset{\sim}{p}$ and an energy variable E_a^0. We choose the value of this energy variable, called ω_0, such that it represents the collision energy of the πN collision as seen in the πA CM system. For a given value of the nucleon momentum $\underset{\sim}{p}$, that collision energy is

$$\omega_0 = (k_0^2+\mu^2)^{1/2} + (p^2+m^2)^{1/2} \tag{3.1}$$

It is hoped that this choice ω_0 for E_a^0 makes the correction to the IA negligible. [8a)]

Various authors make different choices for $\underset{\sim}{p}$. We will consider two such cases, one due to Tabakin et. al [8a)] and the other due to Landau [34)], with the corresponding forms of the IA referred to as IA(C) and IA(D), respectively. According to the former, the nucleons are initially "frozen" in the moving target nucleus, so that each nucleon has a momentum $\underset{\sim}{p} = -\underset{\sim}{k}_0/A$ where $\underset{\sim}{k}_0$ is the on-shell πA CM momentum. After the collision again the nucleon is assumed to be "frozen", so that its final momentum is $\underset{\sim}{p}' = -\underset{0}{k'}/A$ where $\underset{0}{k'}$ is the final πA CM momentum. Note that the momentum is not conserved for πN collision even for energy conserving πA scattering ($k_0 = k_0'$), since the initial and final total πN momenta are $\underset{\sim}{k}_0(1-1/A)$ and $k_0'(1-1/A)$. This is due to the "frozen" nucleon approximation and is a major drawback of the IA resulting from this.

With this choice of the nucleon momentum $\underset{\sim}{p}$, the energy W of the πN system in its CM frame, which is different from that in the πA CM, can be obtained by using the Lorentz invariance of $s = (p_\pi + p_N)^2$.

$$W^2 = s = [E_\pi(\kappa) + E_N(\kappa)]^2$$
$$= [E_\pi(k_0) + E_N(k_0/A)]^2 - k_0^2(1-A^{-1})^2 \tag{3.2}$$

where $\underset{\sim}{\kappa}$ in the πN CM momentum. The t-matrix $t(\omega_0)$ in πA CM is related to t(W) in πN CM according to the relation (2.1).

The relation between the scattering angle $\theta_{\pi A}$, in πA CM and $\theta_{\pi N}$ in πN CM, can be obtained by using the invariance of the Mandelstam variable $t = (p_\pi^{\text{initial}} - p_\pi^{\text{final}})^2$.

$$\cos\theta_{\pi N} = (E_\pi(\kappa)E_\pi(\kappa') - E_\pi(k)E_\pi(k') + kk' \cos\theta_{\pi A})/\kappa\kappa' \tag{3.3}$$

This angle transformation leads to unphysical angles $\theta_{\pi N}$ for large angle scattering in πA CM, even for energy conserving scattering, i.e. $k = k'$. in πA CM. As we have noted earlier that πN momentum is not conserved, although πN energy is conserved for on-shell πA collisions. This difficulty of unphysical angles $\theta_{\pi N}$ is a consequence of the nonconservation of πN momentum.

The nonconservation of πN momentum in the "frozen" nucleon approximation can be avoided by choosing an "optimal" value for the nucleon momentum $\underset{\sim}{p}$. According to Landau[34] this optimal choice corresponds to the nucleon momentum at which the nuclear form factor $F(\underset{\sim}{p}-\underset{\sim}{q},\underset{\sim}{p})$ peaks. It turns out that the peaks occur at two values of momenta $\underset{\sim}{p}_1,\underset{\sim}{p}_2$ given by

$$\underset{\sim}{p}_1 = -\underset{\sim}{k}_0/A \tag{3.4}$$

$$\underset{\sim}{p}_2 = -\underset{\sim}{k}_0/A + \underset{\sim}{q}(A-1)/A \tag{3.5}$$

where $\underset{\sim}{q}$ is the momentum transfer in πA CM. Landau's "optimal" choice is an average of these two values, viz.

$$\underset{\sim}{p} = -\underset{\sim}{k}_0/A + \frac{(A-1)}{2A}\underset{\sim}{q} \tag{3.6}$$

This choice of $\underset{\sim}{p}$ gives on-energy shell πN scattering for on-energy shell πA scattering. The Lorentz invariance of s implies the following the relation for πN energy W instead of (3.2)

$$W^2 \quad [E_\pi(\kappa)+E_N(\kappa)]^2 = [E_\pi(k_0)+E_N(p)]^2 - \frac{1}{2}k_0^2(1-A^{-1})^2(1+\cos\theta_{\pi A}) \tag{3.7}$$

This implies that κ,κ' depend upon the scattering angle $\theta_{\pi A}$. This is a result of the $\underset{\sim}{q}$-dependent choice of $\underset{\sim}{p}$. For energy conserving πA scattering only physical values of $\cos\theta_{\pi N}$ occur in eq. (3.3).

Chapter IV

MESON-NUCLEUS SCATTERING AT LOW ENERGIES

For meson-nucleus scattering the impulse approximation (IA) is extensively used. The IA, as we have noted in the last chapter, consists in replacing the MN_b t-matrix τ by the MN_f t-matrix t, i.e. $\tau = t$, and it is expected to be valid if the energy of the projectile is much larger than the binding energy of the target nucleus, but is questionable at low energies. Nevertheless, the IA is commonly used and is in fact often successful for low energy scatterings. For example, the optical potential for the meson-nucleus interaction which is constructed on the basis of the IA works quite well for the pion [35] but fails for the kaon [36]. The real part of the $\bar{K}$-nucleus optical potential is predicted to be repulsive, whereas the experimental results indicate an attractive potential. There is thus a qualitative disagreement between the optical potential based on the IA and the empirical data.

In this chapter we examine the validity of the IA, or the binding effect on MN scattering at very low energies. We consider a model in which a meson (π or $\bar{K}$) is scattered from a nucleon which is bound in a harmonic oscillator potential. We assume separable MN interactions, introduced in the last chapter, which reproduce the s-wave MN_f scattering data. We then obtain the MN_b scattering amplitude in a perturbation series, which is unitarized by means of the Padé approximant. The intermediate states are explicitly summed over, in contrast to the so-called closure approximation (CA) in which the intermediate excitation energy is ignored. The binding effect is found to be small for the pion, but is quite large for the kaon. [37)]

1. The Model and MN_f scattering

We consider a "nucleus" which consists of a single nucleon bound in a "shell-model potential" V(r). The nucleon also interacts with a meson field through a separable interaction. The Hamiltonian for the MN system is given by

$$H = H_N + H_M + H_I \tag{1.1}$$

where H_N and H_M are given in standard notations by

$$H_N = p^2/(2m) + V(r) \tag{1.2}$$

$$H_M = \frac{1}{2} \sum_\alpha \int d\underset{\sim}{r} \, \{\pi_\alpha^2 + (\nabla\phi_\alpha)^2 + \mu^2\phi_\alpha^2\} \tag{1.3}$$

The suffix α refers to the charge of the meson of mass μ. ϕ_α represents the meson field and the π_α its conjugate momentum. For the interaction H_I we assume the separable form, eq. (4.5) given in Chapter II, i.e.

$$H_I = -(\lambda/\mu)(2\pi)^{-3}\int d\underset{\sim}{k}\, d\underset{\sim}{k}'(\omega\omega')^{-1/2}\, g(k)g(k')a_k^+ a_{k'} e^{i(\underset{\sim}{k}'-\underset{\sim}{k})\cdot\underset{\sim}{r}} \tag{1.4}$$

Here λ is the MN coupling constant and g(k) is the form factor normalized such that g(0) = 1. The pion energy $\omega = (k^2+\mu^2)^{1/2}$. We have suppressed isospin indices in H_I but it is understood that H_I or λ and g(k) are specified for each isospin state [I = 0 or 1 for $\bar{K}N$, and I = 1/2 or 3/2 for πN]. The interaction (1.4) leads to s-wave MN scattering. Note that eq. (1.4) has been obtained in the one meson approximation; the terms of the form $a_k a_{k'}$ and $a_k^+ a_{k'}^+$ have been omitted. As noted in the last chapter the two-meson intermediate state is expected to be small due to the large energy denominator.

As shown in Chapter II, the Schrödinger equation can be solved exactly for this separable interaction. The MN_f scattering amplitude is given by, eq. (4.12) of Chapter II,

$$f(k) = \frac{e^{i\delta}\sin\delta}{k} = \frac{m}{(m+\omega)} \frac{\lambda g^2(k)}{2\pi\mu D(k)} \tag{1.5}$$

where δ is the s-wave phase shift, and

$$D(k) = 1 - \frac{\lambda}{2\pi^2\mu} \int_0^\infty dk' \frac{k'^2 g^2(k')}{\omega'(\omega'-\tilde{\omega}-i\varepsilon)} \tag{1.6}$$

$$\tilde{\omega} = \omega + k^2/(2m) \tag{1.7}$$

Here $k^2/2m$ denotes the nucleon recoil energy. Note that the on-energy-shell t-matrix is related to the scattering amplitude through

$$f = -4\pi^2\omega\{m/(m+\omega)\}t \tag{1.8}$$

The scattering length a and effective range r are given by

$$\frac{1}{a} = \frac{m+\mu}{m}\left\{\frac{2\pi\mu}{\lambda} - \frac{1}{\pi}\int_0^\infty dk \frac{k^2 g^2(k)}{\omega(\omega-\mu)}\right\} \tag{1.9}$$

and

$$\frac{1}{2}rk^2 = \frac{m+\omega}{m} \frac{2\pi\mu \operatorname{Re} D(k)}{\lambda g^2(k)} - \frac{1}{a} \tag{1.10}$$

In the last chapter we have determined λ and $g(k)$ by fitting a and r to the MN_f values.

2. MN_b Scattering

Unlike MN_f scattering, the Schrodinger equation for MN_b scattering cannot be solved analytically. This is because, although the MN interaction is separable, the M-nucleus interaction is not. Therefore we obtain the MN_b scattering amplitude upto second order with respect to λ and then unitarize it by means of the Padé approximant. [38] Note that the Padé approximant gives the exact result for MN_f scattering.

We introduce a "single particle wave function" ψ_ν for the nucleon by

$$H_N \psi_\nu = E_N \psi_\nu \tag{2.1}$$

where ν stands for the set of quantum numbers (n,ℓ,m), and also we set $E_0 = 0$ for the ground state. In order to evaluate the matrix element $\langle\nu'|H_I|\nu\rangle$, it is convenient to write ψ_ν as

$$\psi_\nu(\underset{\sim}{r}) = f_{n\ell}(r)\, Y_{\ell m}(\hat{r}) \;, \quad \hat{\underset{\sim}{r}} = (\theta,\phi) \tag{2.2}$$

Then we obtain

$$\langle \underset{\sim}{k}',\nu'|H_I|\underset{\sim}{k},\nu\rangle = \frac{-\lambda}{(2\pi)^3\mu}\,\frac{g(k')g(k)}{(\omega'\omega)^{1/2}}\,F_{\nu'\nu}(\underset{\sim}{q}) \tag{2.3}$$

where $|\underset{\sim}{k},\nu\rangle$ stands for the state such that the meson momentum is $\underset{\sim}{k}$ and the nucleon state is ν. Also, $\underset{\sim}{q} = \underset{\sim}{k}'-\underset{\sim}{k}$ and

$$F_{\nu'\nu}(\underset{\sim}{q}) = \int d\underset{\sim}{r}\; \psi^*_{\nu'}(\underset{\sim}{r})\psi_\nu(\underset{\sim}{r})\exp(i\underset{\sim}{q}\cdot\underset{\sim}{r}) \tag{2.4}$$

We first consider the case where the nucleon is in the ground state before and after the collision. The lowest order t-matrix is given by

$$t^{(1)} = \langle \underset{\sim}{k}',0|H_I|\underset{\sim}{k},0\rangle = \frac{-\lambda}{(2\pi)^3\mu} \frac{g(k')g(k)}{(\omega'\omega)^{1/2}} F_{00}(\underset{\sim}{q}) \qquad (2.5)$$

The second order term is

$$t^{(2)} = -\sum_\nu \int d\underset{\sim}{k}'' \frac{\langle \underset{\sim}{k}',0|H_I|\underset{\sim}{k}'',\nu\rangle\langle \underset{\sim}{k}'',\nu|H_I|\underset{\sim}{k},0\rangle}{E_\nu+\omega''-\omega-i\varepsilon}$$

$$= -\left\{\frac{\lambda}{(2\pi)^3\mu}\right\}^2 \frac{g(k')g(k)}{(\omega'\omega)^{1/2}} \sum_\nu \int d\underset{\sim}{k}'' \frac{F^*_{\nu 0}(\underset{\sim}{k}''-\underset{\sim}{k}')F_{\nu 0}(\underset{\sim}{k}''-\underset{\sim}{k})g^2(k'')}{\omega''(E_\nu+\omega''-\omega-i\varepsilon)}$$

(2.6)

In the following we confine ourselves to scattering at zero energy, i.e. $k=k'=0$, which is sufficient for evaluating the MN_b scattering length. Note that $g(0)=F_{00}(0) = 1$ and hence $t^{(1)} = -\lambda[(2\pi)^3\mu^2]^{-1}$. The angular integration with respect to $\underset{\sim}{k}''$ in $t^{(2)}$ can be done by using

$$F_{\nu 0}(\underset{\sim}{q}) = \sqrt{4\pi}\, i^\ell\, G_\nu(q) Y^*_{\ell m}(\hat{\underset{\sim}{q}}) \qquad (2.7)$$

where

$$G_\nu(q) = \int_0^\infty dr\, r^2 f_\nu(r) f_0(r) j_\ell(qr) \qquad (2.8)$$

After doing the m-summation with

$$\sum_m Y^*_{\ell m}(\widehat{\underset{\sim}{k}''-\underset{\sim}{k}})Y_{\ell m}(\widehat{\underset{\sim}{k}''-\underset{\sim}{k}'}) = \{(2\ell+1)/4\pi\}P_\ell(\widehat{\underset{\sim}{k}',\underset{\sim}{k}}) \qquad (2.9)$$

we obtain for $t^{(2)}$

$$t^{(2)} = \frac{-4\pi\lambda^2}{(2\pi)^6\mu^3} \sum_{n,\ell} (2\ell+1)\int_0^\infty dk \frac{k^2 G_\nu^2(k)}{\omega(E_\nu+\omega-\mu-i\epsilon)} \tag{2.10}$$

At finite energies $t^{(1)}$ and $t^{(2)}$ consist of all partial waves, but only the s-wave contributes at zero energy. The Padé approximant is to construct the t-matrix by

$$t = t^{(1)}/[1-t^{(2)}/t^{(1)}] \tag{2.11}$$

For a fixed target nucleus the relation between the MN_b scattering amplitude F and the t-matrix, eq. (1.8) reduces to $F = -4\pi^2\omega t$. Then F is given by

$$F(k) = \frac{\lambda}{2\pi\mu D_b(k)} \tag{2.12}$$

where

$$D_b(k) = 1 - \frac{\lambda}{2\pi^2\mu} \sum_{n,\ell} (2\ell+1)\int_0^\infty dk \frac{k^2 G_\nu^2(k)}{\omega(E_\nu+\omega-\mu-i\epsilon)} \tag{2.13}$$

Using eq. (1.10), the MN_b scattering length A is determined by

$$\frac{1}{A} = \frac{2\pi\mu}{\lambda} - \frac{1}{\pi} \sum_{n,\ell} (2\ell+1)\int_0^\infty dk \frac{k^2 G_\nu^2(k)}{\omega(E_\nu+\omega-\mu)} \tag{2.14}$$

We have not specified the "shell-model potential" V(r) so far, and all the above results are quite general. We now assume a harmonic oscillator (HO) potential for V(r), i.e.

$$V(r) = \frac{1}{2} m\eta^2 r^2 \quad , \quad E_\nu = E_n = n\eta \tag{2.15}$$

For η we assume [32)]

$$\eta = 41\ A^{-1/3}\ \mathrm{MeV} \tag{2.16}$$

which gives a reasonable nuclear size for the nucleus of mass number A. For the HO potential the radial part of ψ_ν is given by [39)]

$$f_{n\ell}(r) = b^{-3/2}(2b/r)^{1/2}\ \Lambda_\kappa^{\ell+1/2}(r^2/b^2) \tag{2.17a}$$

$$\Lambda_\kappa^\alpha(t) = \left[\Gamma(\alpha+1)\binom{\kappa+\alpha}{\kappa}\right]^{-1/2} e^{-t/2}\ t^{\alpha/2}\ L_\kappa^\alpha(t) \tag{2.17b}$$

where $\kappa = \frac{1}{2}(n-\ell)>0$ and $b^2 = 1/(m\eta)$. With this, G_ν in eq. (2.8) becomes

$$G_\nu(q) = C_\nu(bq)^n\ \exp(-b^2q^2/4) \tag{2.18}$$

where the coefficients C_ν are given by

$$C_\nu = 2^{-(n+1/2)}[\kappa!(\kappa+\ell+1/2)!]^{-1/2} \tag{2.19}$$

The ℓ-summation in eq.(2.14), for a fixed value of n, can be done by using the relation

$$\sum_\ell (2\ell+1)\ C_\nu^2 = 1/(2^n n!) \tag{2.20}$$

It is in fact unnecessary to know C_ν explicitly to derive the above relation (see Appendix B). With this eq. (2.14) becomes

$$\frac{1}{A} = \frac{2\pi\mu}{\lambda} - \frac{1}{\pi}\sum_{n=0}\frac{1}{n!}\int_0^\infty dk\ \frac{k^2g^2(k)(b^2k^2/2)^n}{\omega(\omega-\mu+n\eta)}\ \exp(-b^2k^2/2) \tag{2.21}$$

The loose binding ($\eta\to0$) and tight-binding ($\eta\to\infty$) limits are found to be

$$\frac{1}{A_L} = \frac{2\pi\mu}{\lambda} - \frac{1}{\pi}\int_0^\infty dk\ \frac{k^2g^2(k)}{\omega(\omega-\mu)} \tag{2.22}$$

and

$$\frac{1}{A_T} = \frac{2\pi\mu}{\lambda} - \frac{1}{\pi}\int_0^\infty dk\, \frac{k^2 g^2(k)}{\omega(\omega-\mu)} \tag{2.23}$$

respectively. Comparison of eqs. (1.10) and (2.22) shows that the t-matrices for MN_f and MN_b scatterings agree in the loose binding limit as expected intuitively, and hence the IA is exact in this limit. On the other hand A_T agrees with a if $m\to\infty$. This is natural because the nucleon is held fixed in the tight-binding limit. The so-called closure approximation (discussed more fully in the next chapter) in which the excitation energy E_ν is ignored agrees with the tight-binding limit. We will see that A_T is very different from A and A_L.

The above results are for a nucleon bound in the ground state (n=0) of the HO potential well. We now describe the case in which the nucleon is bound in the n=1 state (p-state) before and after the collision. In the lowest order the t-matrix element is given by

$$t'^{(1)} = \langle \underset{\sim}{k}',1|H_I|\underset{\sim}{k},1\rangle = \frac{-\lambda}{(2\pi)^3\mu}\,\frac{g(k')g(k)}{(\omega'\omega)^{1/2}}\,F_{11}(\underset{\sim}{q}) \tag{2.24}$$

where we have used a prime to distinguish $t'^{(1)}$ from $t^{(1)}$ of eq. (2.5). For $k'=k=0$, $t'^{(1)} = -\lambda[(2\pi)^3\mu^2]^{-1}$. The second order term $t'^{(2)}$ is obtained from $t^{(2)}$ of eq. (2.6) by replacing $F_{\nu 0}$ by $F_{\nu 1}$ and E_ν by $E_\nu - E_1$, i.e.

$$t'^{(2)} = -\left\{\frac{\lambda}{(2\pi)^3\mu}\right\}^2 \frac{g(k')g(k)}{(\omega'\omega)^{1/2}} \sum_\nu \int d\underset{\sim}{k}''\, \frac{F^*_{\nu 1}(\underset{\sim}{k}''-\underset{\sim}{k}')F_{\nu 1}(\underset{\sim}{k}''-\underset{\sim}{k})g^2(k'')}{\omega''(E_\nu - E_1 + \omega'' - \omega - i\epsilon)} \tag{2.25}$$

In order to handle $F^*_{\nu 1}F_{\nu 1}$ we have to examine ψ_ν for $\nu = (1,1,m)$. First note that for $(\ell,m) = (1,m)$

$$\psi_{1m} = \sqrt{2}\ \psi_{00}\ r_{m/b} \quad , \quad r_m = \sqrt{4\pi/3}\ \ r\ Y_{1m} \tag{2.26}$$

Then for any quantity Q, which is a function of $\underset{\sim}{r}$, we have

$$\langle 1,1|Q|1,1\rangle = \langle 1,-1|Q|1,-1\rangle = b^{-2}\langle 0|(x^2+y^2)Q|0\rangle \tag{2.27a}$$

and

$$\langle 1,0|Q|1,0\rangle = 2\ b^{-2}\langle 0|z^2Q|0\rangle \tag{2.27b}$$

so that the average with respect to m can be done as

$$\begin{aligned}\langle 1|Q|1\rangle &= \frac{1}{3}\sum_m \langle 1m|Q|1m\rangle \\ &= (2/3b^2)\langle 0|r^2Q|0\rangle \end{aligned}\tag{2.28}$$

For the specific case when $Q = e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}$, eq. (2.28) becomes

$$\begin{aligned}\langle 1|e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}|1\rangle &= (2/3b^2)\ \langle 0|r^2e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}|0\rangle \\ &= (2/3b^2)(\underset{\sim}{\nabla}\cdot\underset{\sim}{\nabla}')\sum_\nu\langle 0|e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}|\nu\rangle\langle\nu|e^{-i\underset{\sim}{k}'\cdot\underset{\sim}{r}}|0\rangle\end{aligned}\tag{2.29}$$

where $\underset{\sim}{q} = \underset{\sim}{k}-\underset{\sim}{k}'$, $\underset{\sim}{\nabla} = \nabla_{\underset{\sim}{k}}$ and $\underset{\sim}{\nabla}' = \nabla_{k'}$. Eq. (2.29) is equivalent to

$$\sum_\nu F^*_{\nu 1}(\underset{\sim}{k}')F_{\nu 1}(\underset{\sim}{k}) = (2/3b^2)(\underset{\sim}{\nabla}\cdot\underset{\sim}{\nabla}')\sum_\nu F^*_{\nu 0}(\underset{\sim}{k}')F_{\nu 0}(\underset{\sim}{k}) \tag{2.30}$$

This can be readily generalized to $\sum_{\nu} F^{*}_{\nu 1} F_{\nu 1}$ × (m-independent factor) so that eq. (2.25) becomes

$$t'^{(2)} = (2/3b^2)(\underline{\nabla}.\underline{\nabla}')\, t^{(2)} \text{ \{with } E_\nu \text{ replaced by } E_\nu - E_1\} \qquad (2.31)$$

From $t'^{(1)}$ and $t'^{(2)}$ we can construct the t-matrix by Padé approximant (2.11) and obtain the scattering amplitude. The ℓ-summation can be done as before. The resulting MN_b scattering length A' for this case is then given by

$$\frac{1}{A'} = \frac{2\pi\mu}{\lambda} - \frac{1}{3\pi} \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^\infty dk \, \frac{k^2 g^2(k)(b^2k^2/2)^n}{\omega(\omega-\mu-\eta+n\eta)} \times$$

$$\times \{n(n+2) - nb^2k^2 + (b^2k^2/2)^2\} \exp(-b^2k^2/2) \qquad (2.32)$$

3. $\bar{K}N_b$ Scattering

We have fitted a and r for $\bar{K}N_f$ scattering in the isospin channels I=0 and I=1. The parameters λ and the cutoff k_c are shown in table 2, Chapter II. We now consider two cases wich nucleon bound in a shell-model potential appropriate to ^4He and ^{16}O, i.e. with A=4 and A=16, respectively and evaluate the $\bar{K}N_b$ scattering length for both I=0 and I=1 separately.

Before calculating the scattering length A, it would be instructive to examine the integrals which appear in eqs. (2.21∿23). For the form factor g(k), we use $g(k) = \theta(k_c - k)$, since this form was used in

$\bar{K}N_f$ scattering. Table 1 shows the values of the following integrals

$$J_L = \int_o^{k_c} \frac{k^2 dk}{\omega(\omega-\mu)} \quad J_T = \int_o^{k_c} \frac{k^2 dk}{\omega(\omega-\mu)}$$

$$J = \sum_{n=0}^{\infty} J_n \ , \ J_n = \frac{1}{n!} \int_o^{k_c} dk \, \frac{k^2 (b^2 k^2/2)^n}{\omega(\omega-\mu+n\eta)} \exp(-b^2 k^2/2) \qquad (3.1)$$

It is seen that $J_L < J < J_T$, and J is much closer to J_L than J_T , $(J-J_L)/J_L$ being of the order of 10%. The latter indicates the extent to which the integral J is modified by the binding effect. The breakdown of J into J_n shows the contribution to J from lower intermediate states.

The results for A_L, A and A_T are summarized in table 2. The difference $(A-A_L)$ represents the binding effect. Although the binding effect on J is only about 10%, the effect on A can be much more significant depending on the way the two terms in eqs. (2.21~23) add up. Note that $|\text{Re } A|$ decreases due to binding in I=0 state but increases in the I=1 state. The scattering length is thus seen to be considerably modified by the binding effect. This implies that the IA which is commonly used in constructing the $\bar{K}$-nucleus optical potential is of dubious validity.

As we mentioned before, it is well known that the optical potential obtained with the IA gives $\bar{K}$-nuclear scattering lengths of sign opposite to the observed scattering length. The binding correction to the IA is a step in the right direction, but it does not resolve the difficulty completely. Seki [40] also obtained a binding correction of the same sign as ours, using multiple scattering theory. The $\bar{K}N_b$ t-matrix τ is related to the $\bar{K}N_f$ t-matrix t through the relation

$$\tau = t + t \, \bar{G}_\alpha \, \tau \qquad (3.2)$$

Table 1

Values of the integrals defined by eq. (3.1) in units of kaon mass. The mass number 4, for example, means that the harmonic oscillator constant η of eq. (2.16) with A = 4 is used.

I	J_L	J_T	mass number	J	J_0	J_1	J_2
0	2.08	3.39	4	2.31	0.77	0.24	0.17
			16	2.27	0.62	0.19	0.14
1	2.17	3.56	4	2.41	0.77	0.24	0.17
			16	2.36	0.62	0.19	0.14

Table 2

The $\bar{K}N_b$ scattering length in fm : A_L, A_T and A are given by eqs. (2.22), (2.23) and (2.21), respectively, while A' is obtained from eq. (2.32) by dropping the n=0 term. A_Q is the "quenched" one explained in sect. 3. The upper and lower numbers in parentheses are the real and imaginary parts, respectively.

I	A_L	A_T	mass number	A	A_Q	A'
0	$\begin{bmatrix}-2.53\\1.05\end{bmatrix}$	$\begin{bmatrix}-0.72\\0.07\end{bmatrix}$	4	$\begin{bmatrix}-1.78\\0.48\end{bmatrix}$	$\begin{bmatrix}-3.27\\2.13\end{bmatrix}$	
			16	$\begin{bmatrix}-1.90\\0.55\end{bmatrix}$	$\begin{bmatrix}-3.21\\5.12\end{bmatrix}$	$\begin{bmatrix}-2.20\\0.76\end{bmatrix}$
1	$\begin{bmatrix}-0.14\\0.82\end{bmatrix}$	$\begin{bmatrix}-0.42\\0.38\end{bmatrix}$	4	$\begin{bmatrix}-0.25\\0.77\end{bmatrix}$	$\begin{bmatrix}-0.25\\0.77\end{bmatrix}$	
			16	$\begin{bmatrix}-0.23\\0.78\end{bmatrix}$	$\begin{bmatrix}-0.14\\0.82\end{bmatrix}$	$\begin{bmatrix}-0.18\\0.81\end{bmatrix}$

where,

$$\bar{G}_\alpha = (E^+ - H_0)^{-1}(E_\alpha - E + U)(E_\alpha^+ - K)^{-1} \tag{3.3}$$

with $E_\alpha^+ = E_\alpha + i\varepsilon$ $(\varepsilon \to 0^+)$ being the eigenvalue of the total kinetic energy operator K and $H_0 = H_N + H_M$. Seki makes the approximation

$$\langle \bar{G}_\alpha \rangle_0 = \langle G \rangle_0 \tag{3.4}$$

where $G = (E^+ - H_0)^{-1}$ and $\langle\ \rangle_0$ implies the expectation value of an operator taken between the states of the nucleus in the ground state and the free $\bar{K}$-meson, which are eigenstates of H_0. Then neglecting the effect of charge exchange reaction $K^- + p \to \bar{K}^0 + n$ and the influence of the mass difference in the non-charge-exchange amplitude, he obtains

$$A = a\{1 + (m+\mu)/m[3/(\pi \langle r^2 \rangle)]^{1/2} a\}^{-1} \tag{3.5}$$

The resulting scattering length A is considerably modified and is similar to our result. Myhrer's [41] calculation assumes that the $\Lambda(1405)$ resonance dominates the $\bar{K}N$ system at threshold. In a three-body treatment of the $\bar{K}d$ system he points out that in order to get the correct sign of the real part of the $\bar{K}$-nucleus optical potential, we must use an off-shell $\bar{K}N$ amplitude and that higher $\bar{K}N$ multiple scattering including intermediate nuclear excitations are important. In contrast, our model does not have any multiple scattering effects, there being only one nucleon in the "nucleus". But we take account of intermediate nuclear excitations. Our results are only qualitative in nature and they exhibit the importance of the binding effect. To do a more realistic analysis of the binding effect one has to include inelastic channels such as the $\pi\Sigma$ into interaction explicitly. [42] There exist several different approaches of getting the right sign of the real part of $\bar{K}$-nucleus optical potential. [43]

We have also examined the quenching effect due to the Pauli principle. If there are other nucleons those intermediate states which are already occupied by them have to be excluded in the summation of eq. (2.21). We do not consider scattering from different nucleons. In the I=1 case, the neutron (of K^-n) remains neutron throughout the scattering process, and hence there is no quenching for ^4He while the n=1(p-state) has to be excluded for ^{16}O. In the I=0 case, since the two transitions $\bar{K}p\to\bar{K}p$ and $\bar{K}p\to\bar{K}^0n$ take place with equal probability, one half of the n=0 contribution is suppressed for both ^4He and ^{16}O, and of course n=1 is suppressed for ^{16}O. The scattering length obtained in this way is shown under A_Q in table 2. For I=0, A_Q is very different from A.

The last column of table 2 shows A', the $\bar{K}N_b$ scattering length when the nucleon is bound in p-state (n=1). This was calculated by dropping the n=0 term in the summation in eq. (2.32). Hence A' is for a p-state nucleon when the n=0 state is fully occupied by other nucleons. Note that A' is not very different from A_L.

4. πN_b Scattering

For πN_f scattering a and r have been fitted for I=1/2 channel using a form factor of the Yamaguchi type. The πN interaction is attractive for I=1/2 and repulsive for I=3/2. The latter is much weaker. We find that $A_L = ((m+\mu)/m)\ a = 0.204\mu^{-1}$ and $A_T = 0.221\mu^{-1}$ and

$$A = \begin{cases} 0.207\mu^{-1} \\ 0.206\mu^{-1} \end{cases} \text{for} \quad A = \begin{cases} 4 \\ 16 \end{cases} \tag{4.1}$$

The binding effect $A-A_L$ is only 1~2%. The reason why the effect is much smaller than that for the kaon is that the πN interaction is much weaker at low energies than the $\bar{K}N$ interaction at similar energies. Our binding correction is of the same sign but somewhat smaller than that recently estimated by Myhrer [44] by means of the multiple scattering theory. The correction to the IA due to nuclear binding, for πN interaction at low energies, was first examined by Moyer and Koltun [44a].

Chapter V

MESON-NUCLEUS SCATTERING AT MEDIUM ENERGIES

1. Introduction

The meson-nucleus interactions have been studied since the first meson - the pion was discovered almost three decades ago. But due to experimental limitations, the theory of meson interactions with nuclei was not tested beyond its qualitative features [2]. However, in the recent past several important pion-nucleus experiments have been performed. And now with the advent of meson factories, which produce very intense pion beams, a variety of experiments at medium energies are expected to yield extensive new data on meson-nucleus interactions. A considerable amount of information has already been accumulated for pion-nucleus scattering at medium energies. [6] For example the total and elastic cross sections for several nuclei ranging from ^{4}He to ^{32}S are quite accurately known. Other processes have also been investigated to some extent; e.g. inelastic scattering, knock-out reactions, single and double charge exchange photo-production of pions on the nucleus, pion absorption and pion production. For the kaon-nucleus scattering, on the other hand, very little is known at medium energies. We have already discussed the kaon-nucleus scattering at low energies in the last chapter. In this chapter we only consider the elastic pion-nucleus scattering at medium energies.

There has been a great deal of theoretical work to understand the observed total and elastic cross sections. It has been found that these cross sections can be reproduced reasonably well by means of the optical model [8] and the multiple scattering theories such as the Glauber theory. [7] In these calculations the pion-nucleus (πA) scattering amplitude is obtained in terms of the πN scattering amplitude. This πN scattering amplitude should in principle be related to the t-matrix from a bound nucleon, but in practice the free πN t-matrix is usually used. This, as we have seen, is the IA. The IA is the basic ingredient of almost all πA calculations done so far. We know that the IA is a good approximation if the projectile energy is much larger than the binding energy of the nucleon in the target. However, near the πN resonance, the relatively long πN interaction time enhances the probability of interaction of the πN system with the surrounding nuclear medium. The binding corrections to the IA are thus expected to be quite appreciable at energies in the 33-resonance region. We have already pointed this out in our discussion on the IA in chapter II. In fact, this observation was made by Goldberger and Watson [9] a long time ago. The use of the IA for πA scattering at energies near the 33-resonance is, therefore, questionable. Nevertheless, the IA is almost invariably used since it simplifies the calculations a great deal.

The multiple scattering corrections for πA scattering have been examined by very many authors. But little attention has been paid to the problem of binding corrections to the IA. [4] Since the binding corrections near the resonance are expected to be large, the πA calculations which include multiple scattering effects but ignore the binding effects may not be justified. Now there are two corrections to the IA. N_b is described by a bound state wave function whereas N_f by a plane wave. The other correction is due to the Pauli principle which prevents N_b from being excited into states which are occupied by other nucleons. These corrections have been examined by some authors. Kohmura [45] and Schmit [8b,46] estimated the binding effect for a nucleon bound in an HO potential. The only significant effect they found is the shift of the resonance energy. However, the shift is upwards in ref. 45 but downwards in ref. 8b. For deuteron Julius [47] and Myhrer and Koltun [48] have estimated the binding correction in the resonance energy region. The former finds an increase in the elastic cross section of up to 40% for backward angles, while the latter obtain an upward shift of the 33-resonance by about 15 MeV in the πd CM system. Some other authors have examined the effect of the Pauli principle by considering nuclear matter. [49,23,24] More recently Julius and Rogers [50] have estimated the binding corrections to the optical potential by including the first order correction to the IA in eq. (1.17) of Chapter III viz.

$$\tau = t[1+O(|(f/\lambda\!\!\!^{-})(TR_a/\hbar)|)] \qquad (1.1)$$

They find that the resulting correction to the potential is of the order of 50 - 100% near the resonance, with an increase in the πN_b total cross section by the same amount. This treatment of binding effect is different from that in ref. 8b, where the influence of a potential which binds the target nucleon is simulated by shifting the energy argument in the πN t-matrix from E to E-U. Here U is the binding potential of the target nucleon treated as constant. This is the so-called "quasiclassical" approximation.

In this chapter we examine the validity of the IA for the πN interaction in the 33-resonance energy region [51] for a model similar to the one used in the last chapter for meson-nucleus scattering at low energies.

We consider pion scattering from a nucleon bound in an HO potential. For the πN interaction we assume a separable type one introduced in Chapter II, which reproduces the πN_f scattering at intermediate energies including the 33-resonance. The πN_b scattering amplitude is obtained by means of the Padé approximant.[38] We do not make any expansion with respect to the binding potential; instead it is exactly treated in our model. This is in contrast to ref. 50, where an expansion with respect to potential is made and only second order term is retained. Unlike some previous calculations we do not use the closure approximation, but do the summation over intermediate states explicitly. Our results are very different from those of refs. 45,46 and 8b, but are similar to those of Julius and Rogers.[50] We find that the 33-resonance energy is shifted downwards (in terms of the pion energy in the πA laboratory system) by the binding effect. Crucial differences between our analysis and thos of refs. 45,8b lie in the treatment of the nucleon recoil as well as in the summation for intermediate nuclear states. The main result of our calculation is that the total cross section for πN_b scattering increases by about 60-70% compared with the IA near the resonance, and the πN_b elastic cross section is more strongly forward peaked.[51]

We also examine the effect of the Pauli principle in our model calculation. We assume that some of the levels in the HO potential binding the nucleon, are filled by spectator nucleons. The exclusion of these occupied intermediate states from the summation gives the Pauli principle effect. The result is an upward shift of the resonance energy and an increase of the total cross section at energies above the resonance.

The validity of the closure approximation is also examined in this model.

2. The Model and πN_f Scattering

The model is the same as used in the last chapter except that the πN interaction now causes πN_f scattering only in the 33-state. The Hamiltonian is

$$H = H_N + H_\pi + H_I \tag{2.1}$$

where

$$H_N = p^2/(2m)+V(r) \tag{2.2}$$

$$H_\pi = \int d\underset{\sim}{k}\; \omega a^+_k a_k \tag{2.3}$$

and for H_I we choose the separable form, eq. (4.19) given in Chapter II, i.e.

$$H_I = \frac{-\lambda}{(2\pi)^3\mu}\int d\underset{\sim}{k}\, d\underset{\sim}{k}'\, \frac{g(k)g(k')}{(\omega\omega')^{1/2}}\, P(\underset{\sim}{k},\underset{\sim}{k}')a^+_k a_{k'}\, e^{i(\underset{\sim}{k}'-\underset{\sim}{k})\cdot\underset{\sim}{r}} \tag{2.4}$$

The projection operator $P(\underset{\sim}{k},\underset{\sim}{k}')$ defined by eq. (4.18), Chapter II, causes interaction in the J=3/2 state only. The isospin part of P is suppressed; it should be understood that H_I acts only in the I=3/2 state.

The πN_f scattering amplitude is given by eq. (4.22), Chapter II

$$f(k,0) = \frac{e^{i\delta}\sin\delta}{k}\; \frac{P(\underset{\sim}{k}',\underset{\sim}{k})}{(k/\mu)^2} = \frac{m}{m+\omega}\; \frac{\lambda g^2(k)P(\underset{\sim}{k}',\underset{\sim}{k})}{2\pi\mu D(k)} \tag{2.5}$$

where δ is the 33-phase shift, $\underset{\sim}{k}$ and $\underset{\sim}{k}'$ are the initial and final momenta in the πN CM system, θ is the scattering angle, and

$$D(k) = 1 - \frac{\lambda}{2\pi^2\mu^3}\int_0^\infty dk' \frac{k'^4 g^2(k')}{\omega'(\omega-\omega-i\epsilon)} \tag{2.6}$$

Because of the projection operator in H_I, phase shifts for partial waves other than the 33-state vanish. The total cross section for π^+p scattering from an unpolarized proton target is given by

$$\sigma_{f,tot} = (8\pi/k^2)\sin^2\delta = (4\pi/k)\mathrm{Im}\, f(k,0) \tag{2.7}$$

where the suffix f refers to πN_f scattering. Note that $P(k,k) = 2(k/\mu)^2$. The angular distribution is of the form of $(1+3\cos^2\theta)$.

The parameters λ and $g(k)$ have been determined in Chpater II by fitting the energy and width of the 33-resonance. With the square cut-off $g(k) = \theta(k_c-k)$ for the form factor,

$$\lambda = 0.1536 \quad , \quad \omega_c = 7.21\mu \tag{2.8}$$

reproduce the πN_f scattering data in the medium energy range.

3. πN_b Scattering

The calculation of πN_b scattering is the same as in the last chapter except for a complication concerning partial waves. At zero energy we only had the s-wave contributing to πN_b scattering. But all partial waves contribute at nonzero energies. We consider the case in which the nucleon is bound in the ground state before and after the collision. The t-matrix elements in the first and second order with respect to λ are given respectively by

$$t^{(1)} = \frac{-\lambda}{(2\pi)^3\mu} \frac{g(k')g(k)}{(\omega'\omega)^{1/2}} F_{00}(q)P(\underset{\sim}{k}',\underset{\sim}{k}) \tag{3.1}$$

where $\underset{\sim}{q} = \underset{\sim}{k}'-\underset{\sim}{k}$ and $F_{00}(q) = \exp(-b^2q^2/4)$, and

$$t^{(2)} = -\left\{\frac{\lambda}{(2\mu)^3\mu}\right\}^2 \frac{g(k')g(k)}{(\omega'\omega)^{1/2}}$$

$$\times \sum_{\nu} \int d\underset{\sim}{k}'' \frac{g^2(k'')P(\underset{\sim}{k}',\underset{\sim}{k}'')P(\underset{\sim}{k}'',\underset{\sim}{k})F_{0\nu}(\underset{\sim}{k}''-\underset{\sim}{k}')F_{\nu 0}(\underset{\sim}{k}-\underset{\sim}{k}'')}{\omega''(\omega''-\omega+E_{\nu}-i\epsilon)} \tag{3.2}$$

Using the relation (2.7) for $F_{\nu 0}(\underset{\sim}{q})$ and carrying out the m-summation with the help of eq. (2.9) of Chapter II, the above equation can be simplified to

$$t^{(2)} = -\left\{\frac{\lambda}{(2\pi)^3\mu}\right\}^2 \frac{g(k')g(k)}{(\omega'\omega)^{1/2}}$$

$$\times \sum_{n,\ell} (2\ell+1)\int d\underset{\sim}{k}'' \frac{g^2(k'')P(\underset{\sim}{k}',\underset{\sim}{k}'')P(\underset{\sim}{k}'',\underset{\sim}{k})G_{\nu}(\underset{\sim}{k}''-\underset{\sim}{k}')G_{\nu}(\underset{\sim}{k}-\underset{\sim}{k}'')P_{\ell}(\hat{k}'\hat{k})}{\omega''(\omega''-\omega+E_{\nu}-i\epsilon)} \tag{3.3}$$

Now for the HO potential, the functions $G_{\nu}(q)$ are given by eq. (2.18) of Chapter II. Using this and the following relation for the ℓ-summation (derived in Appendix B)

$$\sum_{\ell} (2\ell+1)C_{n\ell}^2 P_{\ell}(\hat{k}'\hat{k}) = (2^n.n!)^{-1} \cos^n\theta \tag{3.4}$$

eq. (3.3), for elastic scattering k'=k, can be rewritten as

$$t^{(2)} = -\left\{\frac{\lambda}{(2\pi)^3\mu}\right\}^2 \frac{g^2(k)}{\omega} \sum_{n=0}^{\infty} \frac{1}{n!} \int d\underset{\sim}{k}'' \frac{g^2(k'')P(\underset{\sim}{k}',\underset{\sim}{k}'')P(\underset{\sim}{k}'',\underset{\sim}{k})}{\omega''(\omega''-\omega+n\eta-i\epsilon)}$$

$$\times [b^2(\underset{\sim}{k}''-\underset{\sim}{k}')\cdot(\underset{\sim}{k}''-\underset{\sim}{k})/2]^n \exp[-b^2\{|\underset{\sim}{k}''-\underset{\sim}{k}'|^2+|\underset{\sim}{k}''-\underset{\sim}{k}|\}/4] \tag{3.5}$$

Here we have put $E_\nu = n\eta$, the excitation energy of the nucleon to the n^{th} level in the HO potential. Note that b^2 is the HO parameter related to η through [32]

$$b^2 = 1/(m\eta) \tag{3.6}$$

Although only the $p_{3/2}$ state contributes to πN_f scattering, all partial waves enter for πN_b scattering. This is different from the situation of zero energy πN_b scattering in the last chapter. Before applying the Padé approximant we have to decompose $t^{(1)}$ and $t^{(2)}$ into partial waves. This can be done as follows. Since the nucleon is bound in an s-state our "nucleus" has spin 1/2. The πN_b scattering amplitude (and hence the on-shell t-matrix) takes the form [52]

$$F(\theta) = f(\theta) + i\underset{\sim}{\sigma}.\underset{\sim}{n}g(\theta) \tag{3.7}$$

where θ is the scattering angle and the unit vector $\underset{\sim}{n}$ is defined by $\underset{\sim}{n} = \hat{\underset{\sim}{k}}'\times\hat{\underset{\sim}{k}}$. It is understood that F, f and g depend on k also. The spin-nonflip and spin-flip amplitudes f and g are related to partial wave amplitudes $f_{\ell\pm}$ as follows:

$$f(\theta) = \sum_{\ell=0}^{\infty}\{\ell f_{\ell-}+(\ell+1)f_{\ell+}\}P_\ell(\cos\theta) \tag{3.8}$$

$$g(\theta) = \sum_{\ell=0}^{\infty} \{f_{\ell-} - f_{\ell+}\} P_\ell'(\cos\theta) \tag{3.9}$$

Here $\ell\pm$ correspond to $J=\ell\pm\frac{1}{2}$. Eqs. (3.8) and (3.9) can be inverted to give

$$f_{\ell\pm} = \frac{1}{2}\int_{-1}^{1} d(\cos\theta)\{f(\theta)P_\ell(\cos\theta) - [\cos\theta P_\ell(\cos\theta) - P_{\ell\pm1}(\cos\theta)]\, g(\theta)\} \tag{3.10}$$

In order to obtain the spin-nonflip (NF) and spin-flip (F) parts of $t^{(1)}$, note that for elastic scattering (k=k')

$$P(\underset{\sim}{k}',\underset{\sim}{k}) = (k/\mu)^2(2\cos\theta - i\underset{\sim}{\sigma}.\underset{\sim}{n}) \tag{3.11}$$

$$q^2 = (\underset{\sim}{k}'-\underset{\sim}{k})^2 = 2k^2(1-\cos\theta) \tag{3.12}$$

Using these we obtain from eq. (3.1)

$$t_{NF}^{(1)} = -\frac{\lambda}{(2\pi)^3\mu^3}\ \frac{k^2 g^2(k)}{\omega}\, 2\cos\theta\ \exp\{-b^2k^2(1-\cos\theta)/2\} \tag{3.13}$$

$$t_{F}^{(1)} = \frac{\lambda}{(2\pi)^3\mu^3}\ \frac{k^2 g^2(k)}{\omega}\ \exp\{-b^2k^2(1-\cos\theta)/2\} \tag{3.14}$$

We substitute eqs. (3.13) and (3.14) into eq. (3.10) to obtain the first order t-matrix $t_{\ell\pm}^{(1)}$ for the ℓ-th partial wave. For the second order t-matrix element $t^{(2)}$, using eqs. (C.5) and (C.6) for the product $P(\underset{\sim}{k}',\underset{\sim}{k}'')P(\underset{\sim}{k}'',\underset{\sim}{k})$

(see Appendix C) in eq. (3.5), we obtain the spin-nonflip and spin-flip parts as follows:

$$t^{(2)}_{NF} = -\left\{\frac{\lambda}{(2\pi)^3\mu^3}\right\}^2 \frac{k^2 g^2(k)}{\omega} \sum_{n=0}^{\infty} \frac{1}{n!} \int d\underset{\sim}{k}'' \frac{[k'' g(k'')]^2}{\omega''(\omega''-\omega+n\eta-i\varepsilon)}$$
$$\times \{3(\cos^2\frac{\theta}{2}\cos^2\alpha - \sin^2\frac{\theta}{2}\sin^2\alpha\cos^2\beta) + \cos\theta\}$$
$$\times [b^2(k''^2-2k''k\cos\frac{\theta}{2}\cos\alpha+k^2\cos\theta)/2]^n \exp[-b^2(k''^2-2k''k\cos\frac{\theta}{2}\cos\alpha+k^2)/2]$$

(3.15)

$$t^{(2)}_{F} = -\left\{\frac{\lambda}{(2\pi)^3\mu^3}\right\}^2 \frac{k^2 g^2(k)}{\omega} \sum_{n=0}^{\infty} \frac{1}{n!} \int d\underset{\sim}{k}'' \frac{[k'' g(k'')]^2}{\omega''(\omega''-\omega+n\eta-i\varepsilon)}$$
$$\times \{1-3[\sin\alpha\cos\beta)^2+\cos^2\alpha]\}$$
$$\times [b^2(k''^2-2k''k\cos\frac{\theta}{2}\cos\alpha+k^2\cos\theta)/2]^n \exp[-b^2(k''^2-2k''k\cos\frac{\theta}{2}\cos\alpha+k^2)/2]$$

(3.16)

The angles α and β specify the orientation of vector $\underset{\sim}{k}''$ as defined in Appendix C. Again using eq. (3.10) together with $t^{(2)}_{NF}$ and $t^{(2)}_{F}$, we obtain $t^{(2)}_{\ell\pm}$. We then construct the partial wave t-matrix $t_{\ell\pm}$ by means of the Padé approximant

$$t_{\ell\pm} = t^{(1)}_{\ell\pm}/[1 - t^{(2)}_{\ell\pm}/t^{(1)}_{\ell\pm}] \qquad (3.17)$$

Knowing $t_{\ell\pm}$ we can obtain the total πN_b t-matrix from eqs. (3.7~9). The πN_b elastic scattering amplitude F is related to the on-shell t-matrix element by

$$F(k,\theta) = -4\pi^2\omega t(k,\theta) \qquad (3.18)$$

The total and elastic (differential) cross sections for π^+p scattering are given by

$$\sigma_{b,tot} = 4\pi/k \text{ Im } F(k,0) \tag{3.19}$$

$$\sigma_{b,el}(\theta) = |F(k,\theta)|^2 \tag{3.20}$$

where the suffix b refers to the πN_b scattering.

As we shall see the πN_b total cross section $\sigma_{b,tot}$ as a function of energy shows a broad peak. This peak is often referred to as the "33-resonance", although unlike in πN_f scattering partial waves other than $p_{3/2}$ are also contributing in πN_b scattering. Since the peak is broad the resonance energy becomes quite ambiguous. We define the resonance energy by the condition ReF(k,0)=0. Then the energy for the peak of the cross section will be considerably lower than the resonance energy.

Since the centre of the "nucleus" is fixed, there is no distinction between the πA CM system and πA lab system in our model. It is however more natural to interpret our system in which the nucleus is fixed as the πA CM system. This is because for elastic scattering such that k=k', we have the same initial and final pion energies, which is the case in the πA CM system. We therefore interpret that all the above formulae for πN_b scattering are for the πA CM system. When we compare various results in the following, however, we give the cross section etc. as a function of the pion lab energy. For example for the case which simulates ^{16}O, first we obtain the cross section in the πA CM system and then transform the CM energy to the πA lab energy using the "nuclear mass" 16m. If M_A

is the mass of the target nucleus with mass number A and E_π^{CM} the pion CM energy, then the lab energy E_π^L is given by

$$E_\pi^L = \frac{1}{M_A}\left[((E_\pi^{CM})^2-1)+(((E_\pi^{CM})^2-1)^2+((1+M_A^2)(E_\pi^{CM})^2-1))^{1/2}\right] \qquad (3.21)$$

For heavier nuclei, however, there is not much difference between the lab and CM frames, i.e. between E_π^L and E_π^{CM}.

We now present the results of our πN_b calculation for two cases corresponding to N_b in ^{16}O and ^4He. Alongside the πN_b results we will also show the results obtained in the impulse approximation for our model. Recall the IA discussed in Chapter II. We have discussed the IA for two different cases, one corresponding to the nucleus being fixed with nucleons moving inside it, the other is the commonly used "frozen" nucleon approximation with motion of the nucleus considered as a whole. In the former category we discussed two different forms which we referred to as the IA(A) and IA(B). The latter also has different forms, of which we have considered only two and referred to them as the IA(C) and IA(D). Note that in our model of πN_b scattering, the nucleus is fixed whereas the motion of the nucleon inside it has been taken care of. The comparison of our πN_b results with IA(A) and IA(B) is, therefore, more appropriate. Nevertheless we give the results of IA(C) and IA(D) also just to illustrate the differences between various forms of the IA's. These IA's are commonly used in most πA calculations. As we shall see, however, within the IA different forms show some variations from one another.

In figs. 1 and 2 we have plotted ReF(k,0) versus the pion lab energy, with nucleon bound in an HO potential appropriate to ^{16}O and

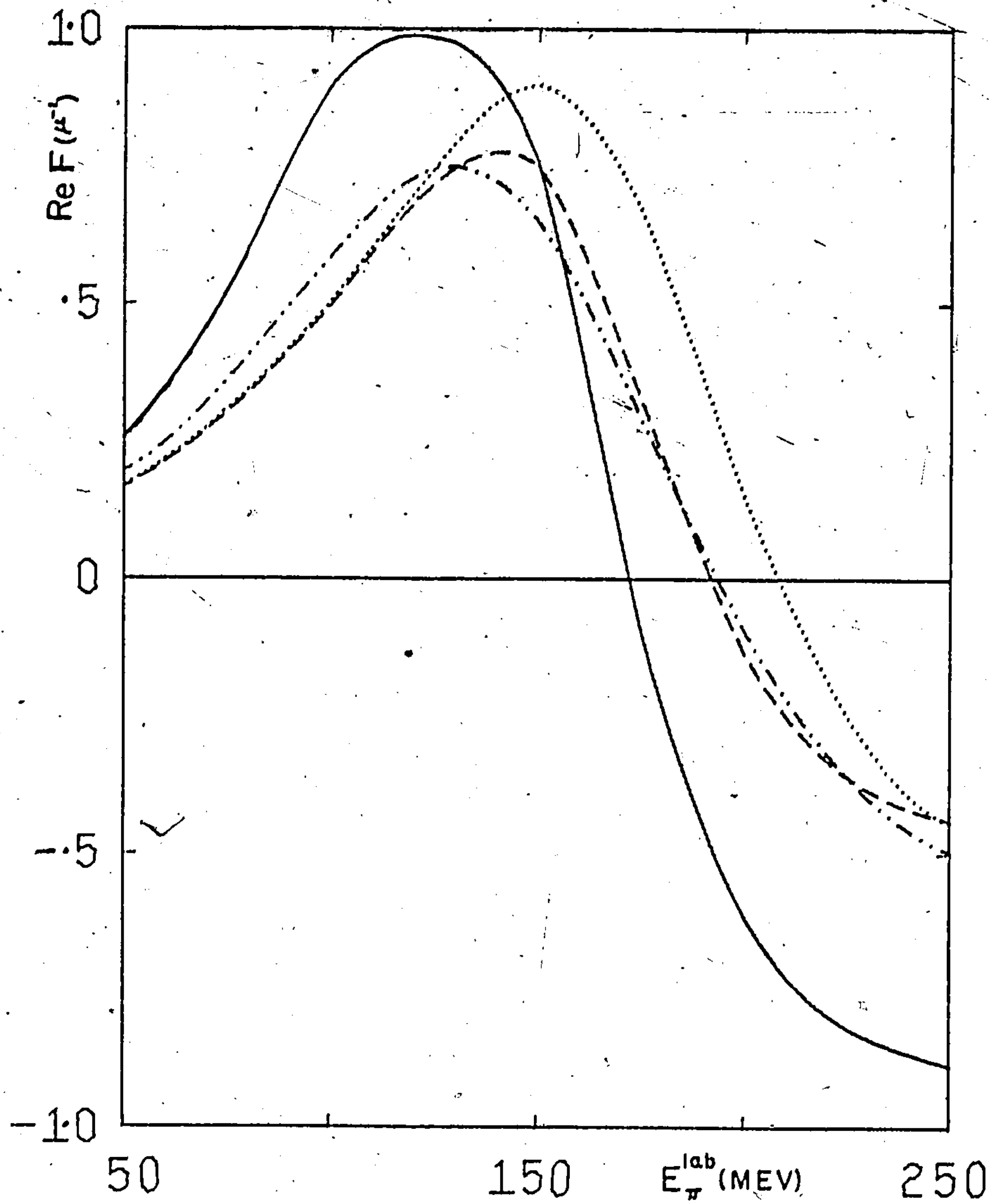

Fig. 1 The real part of the πN_b forward scattering amplitude, Re(F) in units of μ^{-1}, as a function of pion lab energy (MeV). This is for the nucleon bound in an HO potential well appropriate to ^{16}O. The solid curve represents the πN_b calculation, the other curves correspond to various forms of the IA. The dotted curve shows the IA(B), the dashed curve IA(C), the dash-dot IA(D) and the dash-dot-dot shows the IA(A). The meaning of the IA's is explained in Chapter III.

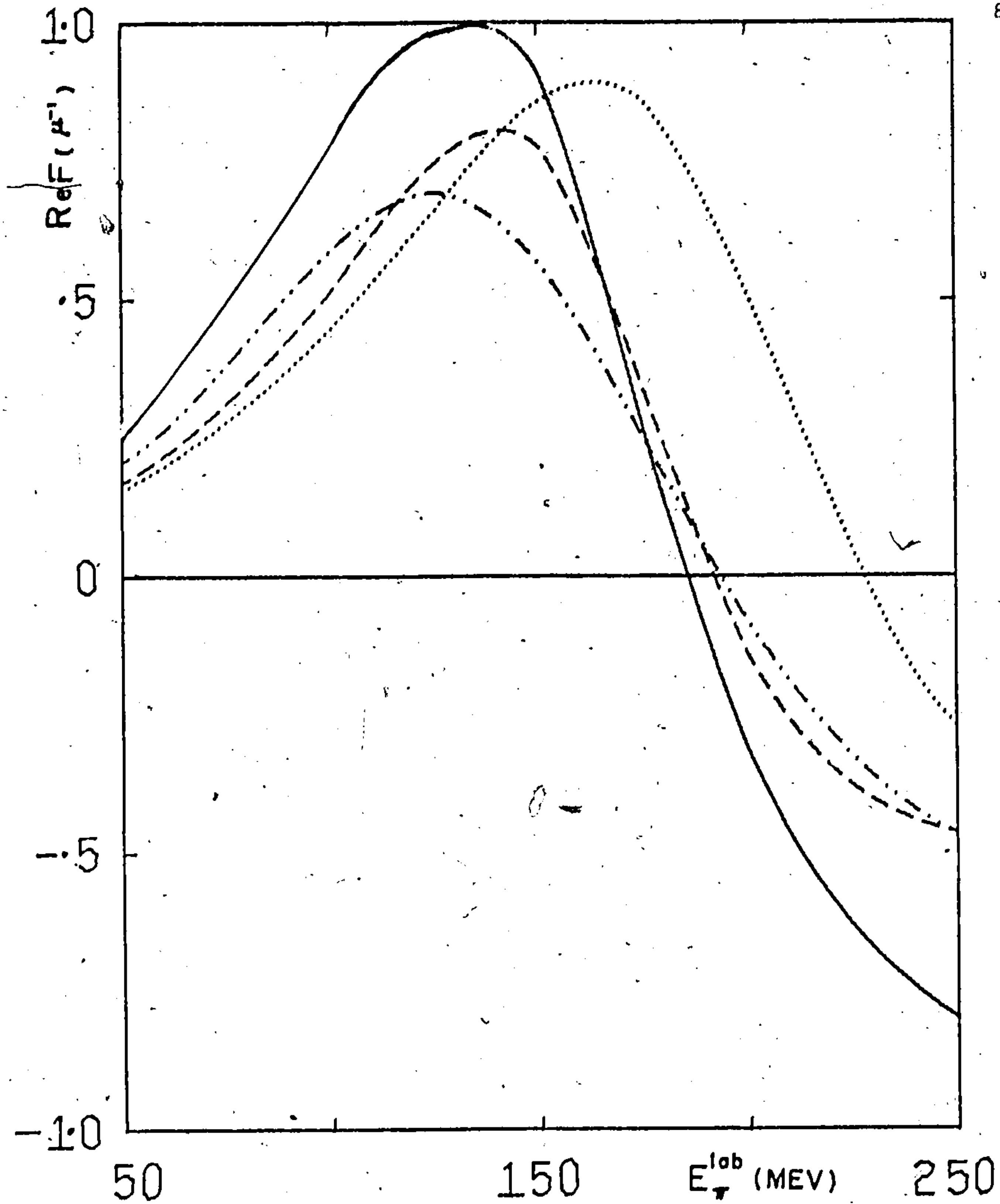

Fig. 2

The same as in Fig. 1, but for N_b in ^4He.

^{4}He nuclei respectively. The solid curves show the results of the πN_b calculation. Whereas the other curves are for different forms of the IA which are explained in the figure captions. The resonance energy is found to be 174 MeV and 186 MeV for ^{16}O and ^{4}He respectively. There is thus a downward shift in the resonance energy as compared with the πN_f resonance energy of 192 MeV in the πN_f lab system. The results of the IA differ from one another. For example the resonance energy is 194 MeV and 208 MeV for IA(A) and IA(B) respectively, for the case of ^{16}O. The IA(C) and IA(D) corresponding to the "frozen" nucleon approximation exhibit the resonance at about 192 MeV for ^{16}O. Note that the scattering amplitude F here contains all partial waves with significant contributions coming from 5 to 10 partial waves depending upon the incident pion energy. The p-wave alone in the πN_b scattering exhibits a resonance at an energy lower than that of the total F. For example for ^{16}O it is at 163 MeV.

Next we have shown the total cross section $\sigma_b(\pi^+ p)$ from a nucleon bound in the 1s state of the HO potential, against the pion lab energy. Again the solid curves in figs. 3 and 4 correspond to the πN_b scattering for ^{16}O and ^{4}He respectively. We see that the πN_b cross section is about 290 mb for ^{16}O at the peak, which is about 50% higher than that of the corresponding πN_f peak value. The different IA results are also shown. For example, the total cross section for ^{16}O is 176 mb in IA(A) and 182 mb in IA(B) at the resonance peak. We thus see that the IA results differ from one another by about 5-10%. But the πN_b total cross section is about 60-70% greater than any of the IA cross sections. So, the ambiguity in the IA does not in any way obscure the binding correction.

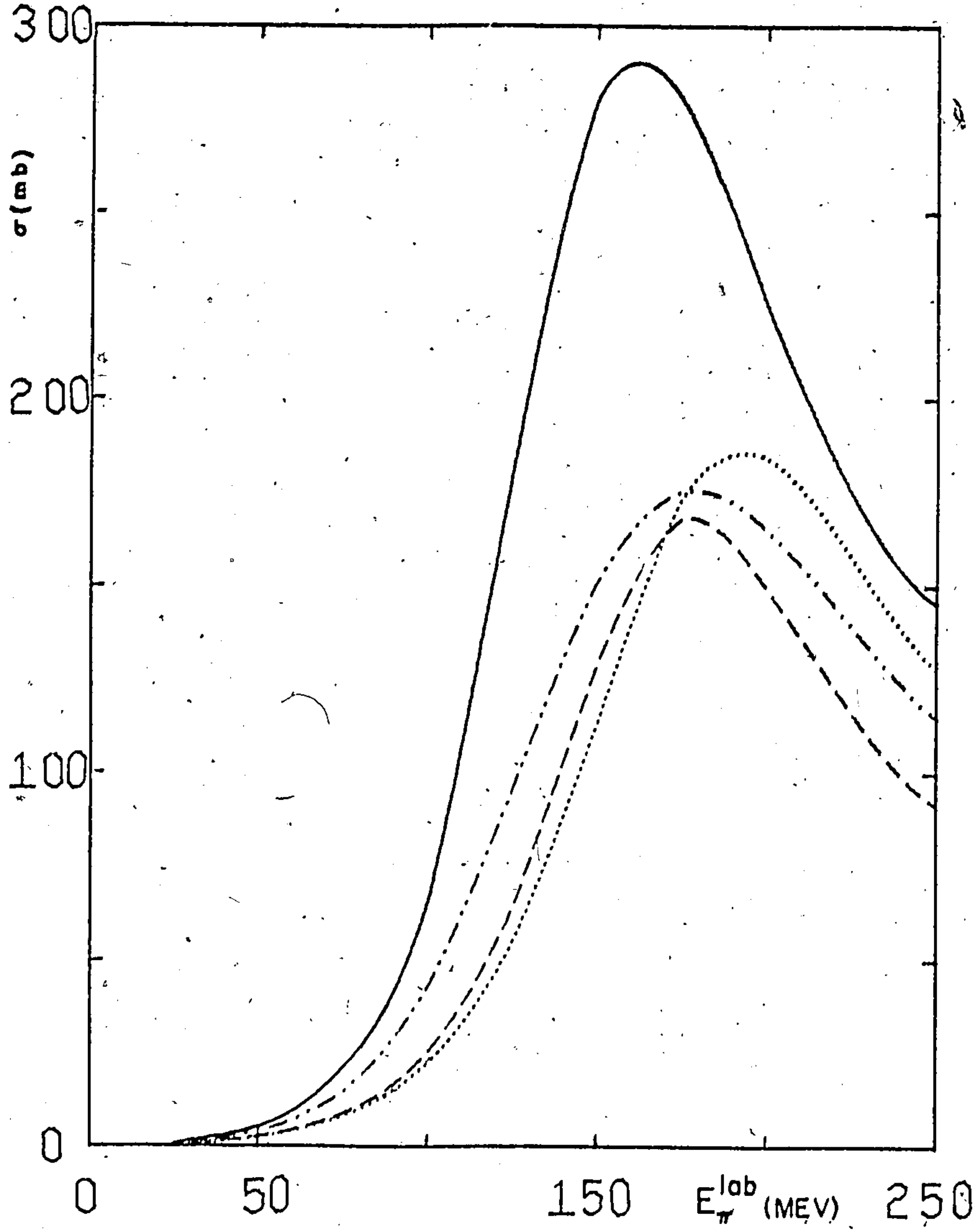

Fig. 3

The total cross section $\sigma_{b,tot}$ in mb, as a function of pion lab energy (MeV). This is for the nucleon bound in an HO potential well appropriate to ^{16}O. The curves are as explained in fig. 1.

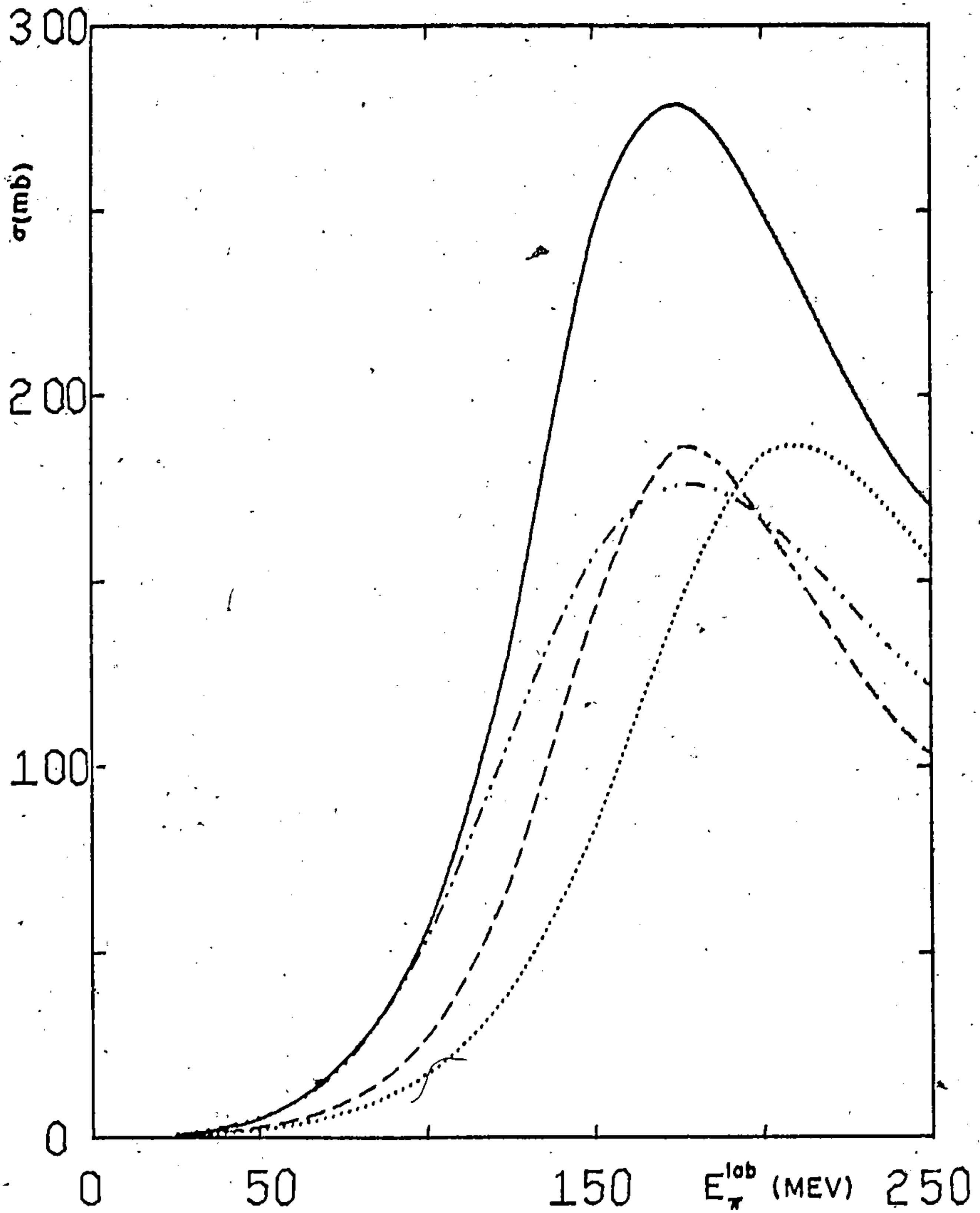

Fig. 4

The same as in fig. 3, but for N_b in ^4He.

The significant contributions to the πN_b scattering come from the first 5 to 10 partial waves as pointed out above, whereas πN_f scattering takes place only through p-wave. The p-wave scattering in πN_b is suppressed by binding to about half that in the free case. The overall increase in the πN_b total cross section is the effect of binding, with contributions from a large number of partial waves. The contributions of various partial waves are shown in table 1 for πN_b calculation and the IA(B) for comparison. It is clear that the binding effect increases the contributions of different partial waves.

The πN_b elastic cross section, divided by $F_{00}^2(q)$, for ^{16}O at three different pion lab energies of 150,200, 250 MeV is shown in figs. 5, 6 and 7 respectively. The curves other than the solid ones show the results of the IA. Notice that the angular dependence of πN_b is very different from $(1+3\cos^2\theta)$ for πN_f ~~scattering~~. The πN_b cross section near the resonance is strongly forward - peaked compared to any of the IA elastic cross section. The πN_b forward cross section is much greater than that in the IA at all energies considered. The backward cross section, on the other hand, is much smaller than that in the IA at and above the resonance energy.

4. Validity of the Closure Approximation

In the last section we have done the summation over intermediate nuclear state explicitly. In order to examine the validity of the CA we perform closure on intermediate states,[53] i.e. use the relation

$$\sum_{\nu} F_{0\nu}(\underset{\sim}{k}''-\underset{\sim}{k}')F_{\nu 0}(\underset{\sim}{k}-\underset{\sim}{k}'') = F_{00}(\underset{\sim}{k}-\underset{\sim}{k}') \qquad (4.1)$$

Table 1

Partial wave contributions to the total cross section in mb at various energies. The πN_b calculation and the IA(B) are shown for a comparison. This is for the nucleon bound in an HO potential appropriate to ^{16}O.

E^{lab}(MeV)	50		100		150		200		250	
ℓ	IA(B)	πN_b	IA(B)	πN_b	IA(B)	πN_b	IA(B)	πN_b	IA(B)	πN_b
0	-0.2	0.4	2.9	9.0	15.7	39.4	25.4	21.8	16.7	11.8
1	1.9	4.4	12.6	37.0	50.8	99.6	70.2	54.7	43.0	30.1
2	0.4	0.7	5.8	16.1	33.2	88.8	55.8	61.2	38.5	36.7
3	0.1		1.3	3.3	11.0	40.5	24.1	45.9	20.0	29.9
4			0.2	0.3	2.4	10.7	7.0	25.7	7.2	19.5
5					0.4	1.8	1.5	11.1	1.9	10.4
6					0.1	0.2	0.3	3.7	0.5	4.8
7								0.9	0.1	1.9
8								0.1		0.7
9										0.2
10										0.1

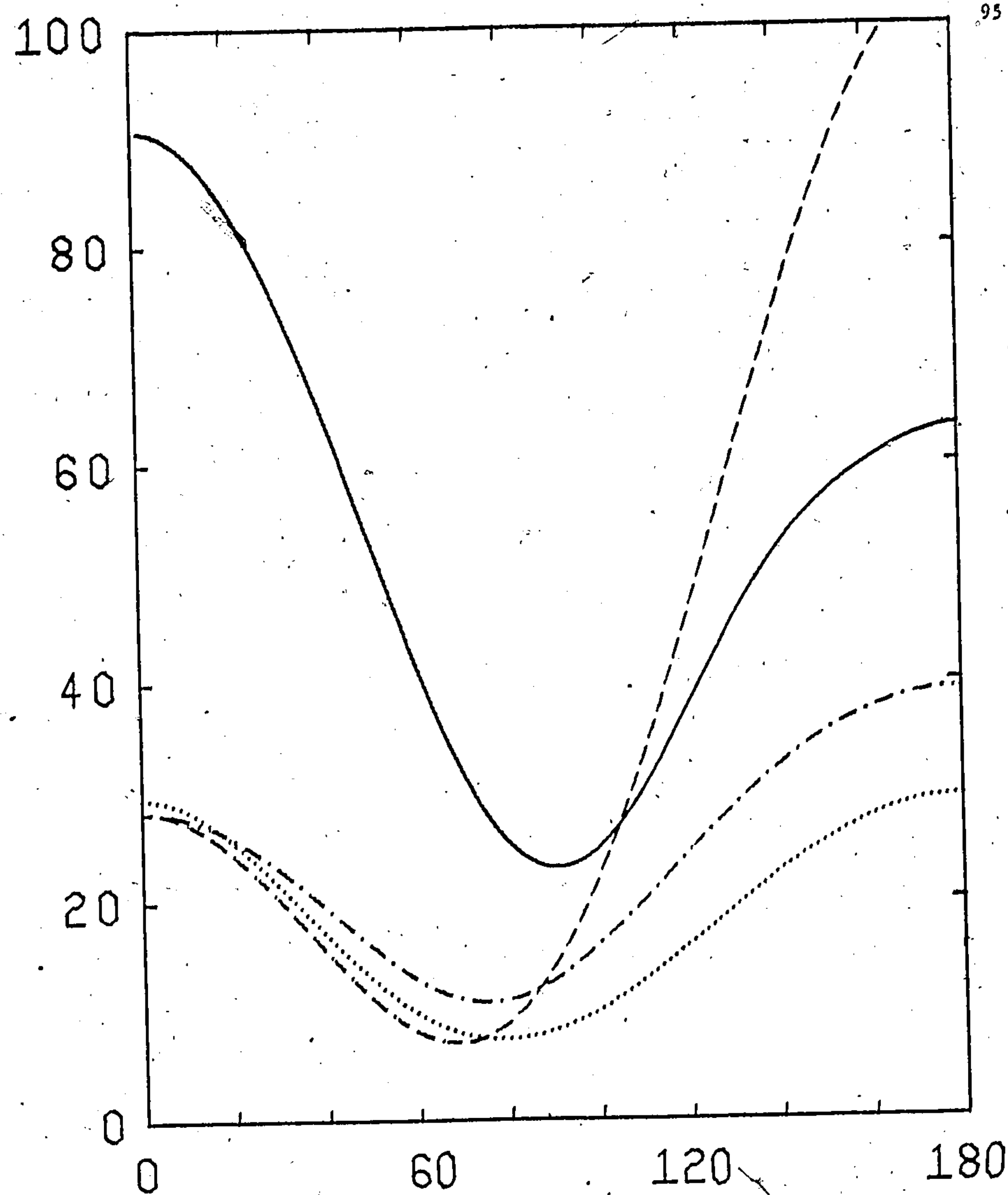

Fig. 5

The differential cross section divided by $F_{00}^2(q)$ for πN_b scattering (mb) as a function of the πA CM scattering angle. This is for the nucleon bound in an HO potential well appropriate to ^{16}O. Pion lab energy = 150 MeV. The curves are as explained as in fig. 1.

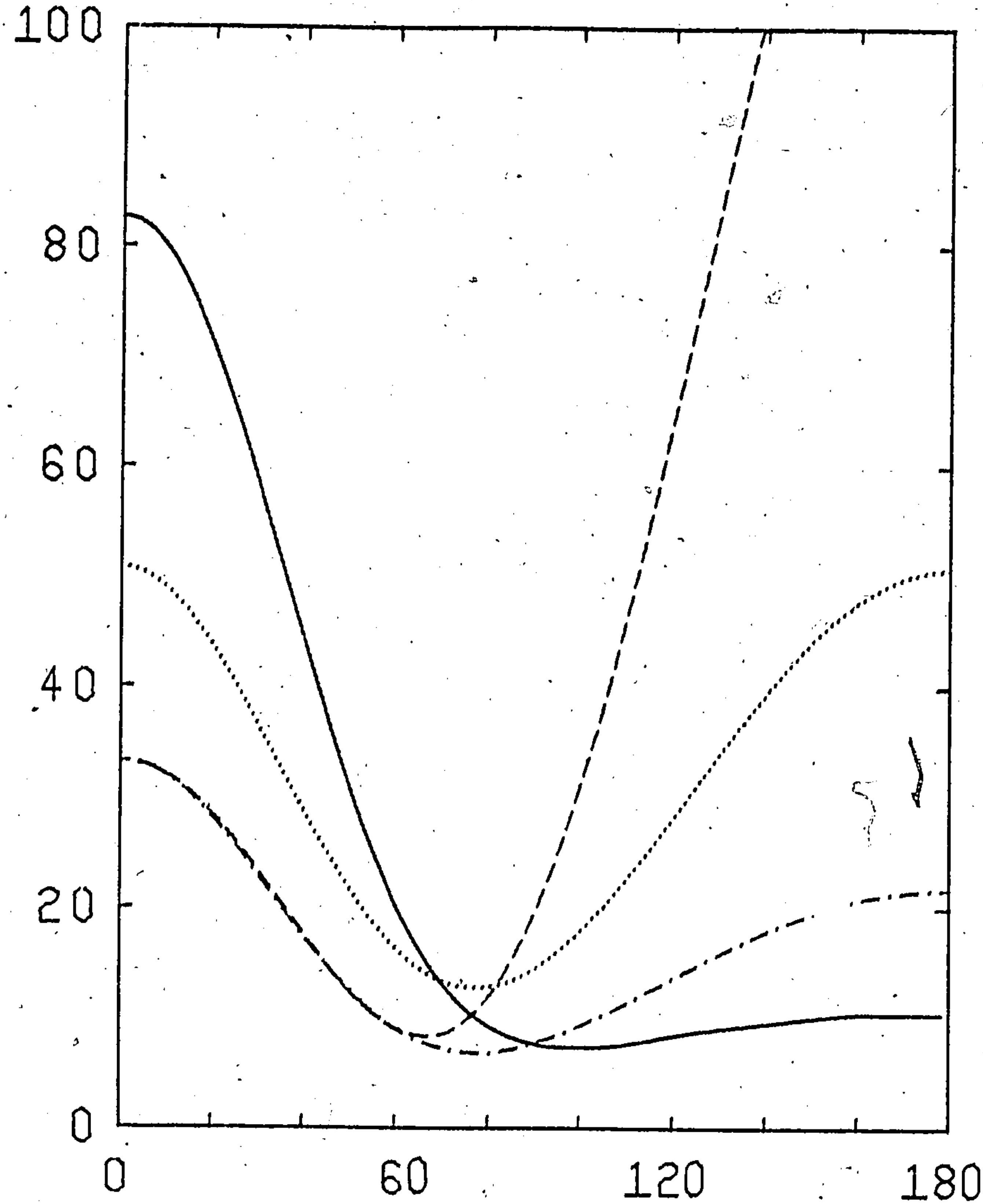

Fig. 6

The same as in fig. 5, but for pion lab energy = 200 MeV.

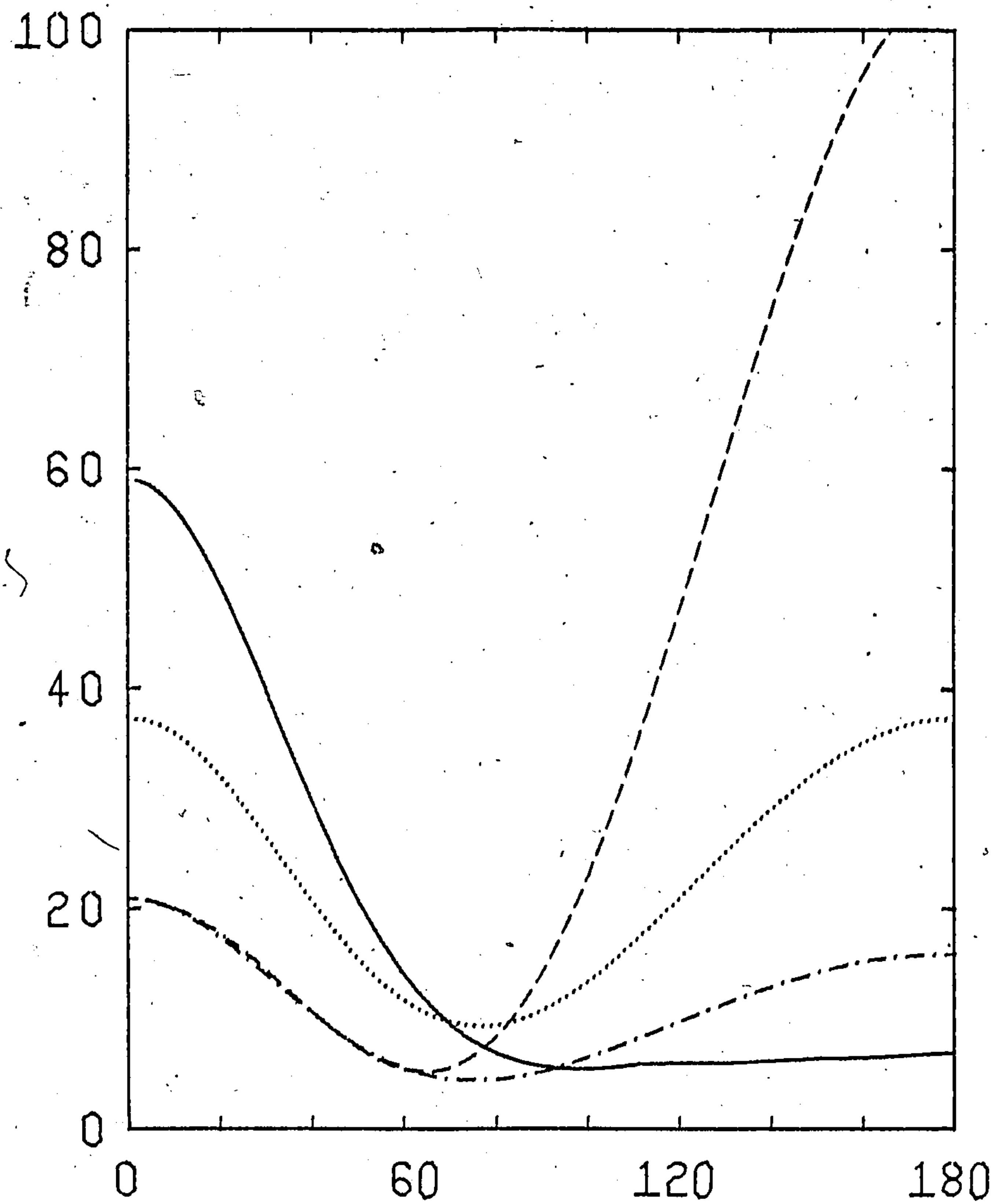

Fig. 7

The same as in fig. 5, but for pion lab energy = 250 MeV.

With this we get the following expression for the second order t-matrix element $t^{(2)}$

$$t^{(2)}_{CA} = -\left\{\frac{\lambda}{(2\pi)^3\mu}\right\}^2 \frac{g(k')g(k)}{(\omega'\omega)^{1/2}} F_{00}(\underset{\sim}{k}-\underset{\sim}{k}')$$

$$\times \int d\underset{\sim}{k}'' \frac{g^2(k'')F(\underset{\sim}{k}',\underset{\sim}{k}'')F(\underset{\sim}{k}'',\underset{\sim}{k})}{\omega''(\omega''-\omega+\bar{n}\eta-i\varepsilon)} \tag{4.2}$$

where $\bar{n}\eta$ is the mean excitation energy of the nucleon. This term is usually set equal to zero in the CA. It is argued that only low lying states will contribute significantly. With $\bar{n}\eta=0$ and eqs. (C.5) and (C.6) (derived in Appendix C) we obtain the spin-nonflip and spin-flip parts of $t^{(2)}_{CA}$

$$t^{(2)}_{CA(NF)} = -\left\{\frac{\lambda}{(2\pi)^3\mu^3}\right\}^2 \frac{k^2g^2(k)}{\omega} \exp[-b^2k^2(1-\cos\theta)/2]$$

$$\times (8\pi\cos\theta) \int_0^\infty dk' \frac{k'^4g^2(k')}{\omega'(\omega'-\omega-i\varepsilon)} \tag{4.3}$$

$$t^{(2)}_{CA(F)} = -\left\{\frac{\lambda}{(2\pi)^3\mu^3}\right\}^2 \frac{k^2g^2(k)}{\omega} \exp[-b^2k^2(1-\cos\theta)/2]$$

$$\times (-4\pi) \int_0^\infty dk' \frac{k'^4g^2(k')}{\omega'(\omega'-\omega-i\varepsilon)} \tag{4.4}$$

The angular dependences of $t^{(2)}_{CA}$ and $t^{(1)}$ are the same, so that their ratio is independent of angles. The use of the Padé approximant to obtain the

πN_b scattering amplitude in the CA then leads to a denominator that has no ℓ-dependence. The πN_b scattering amplitude in the CA becomes

$$F_{CA} = \frac{\lambda g^2(k)}{2\pi\mu} \frac{P(\underset{\sim}{k}',\underset{\sim}{k})}{D_{CA}(k)} \tag{4.5}$$

where,

$$D_{CA}(k) = 1 - \frac{\lambda}{2\pi^2\mu^3}\int_0^\infty dk' \frac{k'^4 g^2(k')}{\omega'(\omega'-\omega-i\epsilon)} \tag{4.6}$$

We note immediately that D_{CA} is the same as D, eq. (2.6), for πN_f scattering except that there is no recoil term in D_{CA}. Thus the πN_b scattering amplitude in the CA reduces to the πN_f amplitude in which the nucleon is fixed. With the parameters λ, ω_C of the πN interaction given by eq. (2.8), it turns out that the πN_b scattering amplitude (4.5) exhibits no resonance at any incident pion energy. Instead there is a bound state with the binding energy of 69 MeV. The CA in our model of πN_b scattering, therefore, leads to results which are qualitatively different from those obtained in the last section where intermediate nuclear states were summed over explicitly.

We conclude this section by discussing the limiting cases of nucleon binding. In the limit of infinite binding $\eta \to \infty$, only the $n=0$ term contributes in eqs. (3.15) and (3.16) and hence the πN_b scattering amplitude of sect. 3 reduces to F_{CA} of eq. (4.5). As we have pointed out above it is very different from the πN_f amplitude. In the weak binding limit $\eta \to 0$, on the other hand, we would expect that the πN_f scattering amplitude is recovered. This can be easily verified for zero energy pion, but we have not been able to show this for any pion energy.

5. Pauli Principle Effect

So far we have considered a "nucleus" which consists of only one nucleon, and have allowed the nucleon to be excited to all levels in the intermediate state. Now let us suppose that a few lowest levels are occupied by other spectator nucleons. According to the Pauli principle, the nucleon which is struck by the incident pion cannot go to the levels already occupied. For example in π-^{16}O scattering, the n=1 level is forbidden for the s-state nucleon as an intermediate state. Examination of eqs. (3.15) and (3.16) for $t^{(2)}$ together with the Padé approximant (3.17) reveals that this should push the resonance energy upwards. This is indeed the case. In this particular case of π-^{16}O scattering, the πN_b resonance is found to occur at pion lab energy of 200 MeV. This is to be compared with 194 MeV in the IA(A) and 174 MeV in πN_b scattering with binding correction alone. When the Pauli principle effect is combined with binding correction, the resonance energy becomes close to that found in the IA(A). For the cross section its effect is to decrease the πN_b total cross section below the resonance energy, whereas there is a significant increase above the resonance. For example at pion lab energies of 150 and 200 MeV, the πN_b total cross sections without and with the Pauli principle effect are 201, 192mb and 225, 322mb respectively.

As we noted in Chapter II, the separable πN interaction is not similar to the more basic CL interaction so far as the Pauli principle effect is concerned. The lowest order diagram in CL interaction is modified by the Pauli principle whereas the corresponding diagram for the separable interaction is not. Hence the effect is expected to be more important for the CL interaction. Our result should, therefore, be taken qualitatively only.

6. Discussion

It is clear from our analysis that the IA is a rather poor approximation in the neighbourhood of the 33-resonance. The binding corrections to the IA are indeed quite significant. The binding effect pushes the resonance energy downwards compared with that in the IA. The πN_b total cross section near the resonance is about 60-70% greater than the corresponding IA cross section and the elastic cross section shows a much stronger forward peak than for πN_f scattering. For instance, for N_b corresponding to ^{16}O HO parameter, the πN_b resonance energy is found to be 174 MeV with the total cross section at the resonance peak of about 290mb, where as the corresponding figures in the IA(A) are 194 MeV and 176mb, respectively. Unlike in πN_f scattering, the πN_b scattering is not confined to p-wave only, but several partial waves contribute. Significant contributions come from the first 5 to 10 partial waves depending upon the incident pion energy. In fact the contributions of d- and f-waves are as important as that of the p-wave. For the πN_b elastic cross section the superposition of a large number of partial waves leads to a strong forward peak, with strong cancellations in the backward direction. Finally, the effect of the Pauli principle is to shift the resonance energy higher, for example from 174 MeV to 200 MeV for ^{16}O. It results in a decrease in the cross section below the resonance energy (corresponding to πN_f scattering), whereas there is an increase above the resonance.

In our model we have only one nucleon bound in a potential well. In a real nucleus, of course, there are many nucleons. As was discussed in Chapter I, the πA scattering amplitude can then be constructed in terms of τ, the πN_b scattering matrix, which satisfies the following equation

$$\tau = v + v(E+i\eta - H_0)^{-1} \tau \qquad (6.1)$$

Here v is the interaction between the incident pion and N_b. The incident pion scatters repeatedly from this bound nucleon. The most important intermediate states which contribute to τ in eq. (6.1) are probably those in which the struck nucleon is excited while all other nucleons remain undisturbed. This means that all nucleons except the struck one are spectator nucleons, and so far as the binding effect on τ is concerned they simply generate the potential which binds the struck nucleon. This is essentially the underlying idea of our calculation.

The basic mechanism for the downward shift of the resonance energy is that the nucleon mass is effectively increased by the binding effect. When the strength of the potential which binds the nucleon is increased, the nucleon becomes less and less mobile and finally in the tight binding limit the πN scattering amplitude reduces to that in the static limit in which $m \to \infty$. This is indeed what we have shown for pion scattering from a nucleon bound in an HO potential, with separable πN interaction. In this limit we found that the resonance disappears; instead a πN bound state emerges. For the CL interaction the effect of the increasing mass is somewhat less dramatic. As we showed in Chapter II, the resonance remains in the tight binding limit, but at 17 MeV.[26] Of course for a realistic strength of the HO potential the effect is much more moderate.

In addition to the binding effect, we simulated the effect of the Pauli principle by excluding some lower intermediate states. This effectively weakens the πN_b interaction and hence the resonance energy is shifted upwards. Note that the effective strength of the πN interaction increases with increase in pion energy (because of the momentum factor in the interaction). The reason why the Pauli principle increases the cross section above the resonance energy is not clear. In discussing the Pauli principle effects in πA scattering Landau and McMillan [54] found that it decreased the πA total cross section around the resonance energy (corresponding to πN_f scattering), whereas above the resonance energy their calculation shows an increase. They have not emphasized this point. In their model they have also used a separable πN interaction. We pointed out at the end of sect. 5 that the Pauli principle effect is probably more important for the CL interaction than for the separable πN interaction.

We now want to comment on some related works and compare our results with them. First we consider Kohmura's [45] calculation. He has evaluated the binding correction to the IA for a model similar to ours. But he assumed the πA interaction to be separable, used the CA to sum over the intermediate nuclear states and neglected the nucleon recoil in fitting the πN_f scattering. Moreover, the IA is not clearly described; in fact the IA and the πN_f scattering amplitude are interchangeably used, which is not correct. Because of the CA it is not possible to consider the Pauli principle effect in ref. 45. Kohmura finds that the binding effect shifts the resonance energy upwards by a slight amount. He does not discuss any changes in the magnitude of cross section due to binding. His conclusion is that the binding corrections to the IA are not so large as were estimated by Goldberger and Watson.[9] In our calculation we do not make use of any of the above approximations. Our results, as we have pointed out above, indicate that the IA is a rather poor approximation in the 33-resonance energy region and that the binding corrections are quite large. The CA is found to be a very bad approximation in our model.

Schmit[46,8b] considers a model much the same as Kohmura's. He has also used the CA and has neglected the nucleon recoil in πN_f scattering. As we have noted earlier, his treatment of the binding effect is in the so-called "quasiclassical" approximation, which amounts to shifting the energy argument of the πN t-matrix from E to $E'=E-U$, U being the binding potential of the target nucleon which is negative. As this effective energy E' is larger than the free-particle energy by about 20-30 MeV, there is a downward shift in the resonance energy by the corresponding amount. The Pauli principle effect cannot be taken account of in this treatment. Note that our πN_b calculation does not make any quasiclassical approximation.

Myhrer and Koltun[48] have studied the πd scattering as a three-body problem in the 33-resonance energy region. They examine the effect of dynamical excitations of the target as well as of multiple scattering and nucleon motion. They find that the resonance occurs at p=215 MeV/C in the πd CM system, whereas the 33-resonance momentum in the IA(C) with nucleons frozen in the deuteron is 205 MeV/C in the πd CM. Their conclusion then is that the binding effect results in an upward shift of the resonance energy. The multiple scattering effects, which contribute a downward shift are small for the deuteron, and hence the net effect is an upward shift in the resonance energy in the πd CM system due to binding. Their calculation is exact. They explain this result in the following way. The energy argument of the πN scattering amplitude is assumed to be shifted due to binding from $\bar{E}$ to E'

$$E' = E + U_{NA} \tag{6.2}$$

where U_{NA} is the average potential in the initial state which binds the nucleon. In the intermediate states the binding potential is small and is neglected, so that the replacement (6.2) implies an upward shift by $-U_{NA}$ MeV, of the resonance in the nuclear medium. For the deuteron this shift is about 15 MeV. Hüfner's [4] interpretation of the binding effect is similar. (Incidentally Schmit's choice of energy replacement is E'=E-U and not eq. (6.2). This changes even the qualitative nature of the binding effect. There is some confusion on this point in the literature.) Now, our calculation of the binding effect is different. The nucleon is bound in the intermediate states as well as in the initial state, unlike the above argument where the effect of binding potential in the intermediate states is neglected. So it is not possible to predict the sign of the shift just by looking at the energy denominator.

Hüfner's [4] analysis or conjecture of the binding effect agrees with our basic idea which we have described earlier, i.e. the pion scatters repeatedly from one bound nucleon, which is excited while all other nucleons are spectator nucleons. But for the binding correction in the intermediate state, he argues that it is unimportant. It is not obvious whether the neglect of binding in the intermediate states is apriori justified. Moreover, how well can the binding effect be simulated simply by replacing E by an effective energy E' as the argument of the πN scattering amplitude, is also not clear.

Another comment which we would like to make on Myhrer and Koltun's [48] work is related to the nonrelativistic treatment of the pion and the suppression of spin in their calculation. Because of the neglect of spin they cannot make quantitative predictions of cross section. In order to illustrate the differences between the nonrelativistic and relativistic treatment of the pion we have shown in table 2 the resonance energies in both cases for πd and $\pi-{}^{16}O$ CM systems in the IA(C). In the πd CM the resonance enrgy is 164 MeV for nonrelativistic kinematics but 172 MeV if relativistic energies are used for π and N. Similarly we see a difference of 10 MeV for the resonance energy in the $\pi-{}^{16}O$ CM. This effect is purely coming from the relativistic treatment of the pion and the nucleon. What effect would the relativistic pion energy have on exact πd calculation, if such a calculation were possible, is not clear. The effect, however, may not be negligible. In our calculation, on the other hand, we treat the pion relativistically and take account of the nuclear spin in addition to including the effects of dynamical excitations of the target and nucleon motion. Note, however, that there are no multiple scatterings from other nucleons in our model since the nucleus has only one nucleon.

For the sake of completeness we briefly summarize the results of various calculations which have examined the effect of the Pauli principle by considering nuclear matter. [23,24,49] In such calculations the initial and final wave functions are taken to be plane waves. Unlike our calculation, where the potential binding the nucleon modifies the bound state wave function, there is no binding effect in nuclear matter. Only the Pauli principle modifies the πN scattering amplitude in nuclear matter. Kimura

Table 2

The resonance energies (MeV) in the πA CM and Lab systems are shown in the IA(C) for deuteron and ^{16}O. Two cases corresponding to the non-relativistic and relativistic treatment of pion and nucleon are shown.

	πd		π-^{16}O	
Case	CM	Lab	CM	Lab
NonRelativistic	164	176	179	180
Relativistic	172	192	189	192

and Nagashima's [49b] calculation exhibits a very large upward shift of the resonance energy, by about 200 MeV for the CL amplitude. Bethe's [49a] result on the other hand shows a downward shift of the resonace to $\omega_r = 1.15\mu$. There are some differences in the two approaches, but they both consider the CL πN amplitude. Dover and Lemmer [24] predict an upward shift of the resonance energy due to the action of the Pauli principle which reduces the πN coupling constant. In their most recent calculation, Weber and Eisenberg [23C] claim that the Pauli blocking in πA scattering has little effect at high energies; the main blocking effect occurs below the resonance energy. It is obvious from these diverse results that the situation is far from having been resolved about the Pauli principle effects for nuclear matter. It is difficult to compare these results with our model calculation since the former completely ignore the binding effect.

Finally we would like to mention Julius and Rogers' [50] calculation. They include the first order binding correction to the πN_f t-matrix by retaining the second term in eq. (1.1). The resulting corrections to the optical potential are 50-100% near the resonance and fall to negligible values at energies less than 120 MeV and greater than 450 MeV. Their fig. 5 in which they plot the binding correction to the optical potential for π-^{12}C (Re $V_{opt}(r)$), indicates that Re $V_{opt}(r)$ vanishes at an energy slightly lower than that for the optical potential without the binding effect. Unlike ref. 50, where in the expansion with respect to the binding potential only the second order correction is taken account of, we do not make an expansion with respect to the binding potential, instead it is treated exactly. We have used the Padé approximant to obtain the

πN_b amplitude. The accuracy of the Padé approximant for various scattering problems is known to be very good.[38] This is the only approximation in our calculation. For the nucleon bound in an HO potential it is difficult to estimate its binding energy. We are aware of the unrealistic nature of this potential. But we believe that for the problem under consideration it is a reasonable binding potential. It has been used quite extensively in literature for similar problems. Within these limitations of our model we have done all the calculations consistently.

Before concluding this chapter we discuss the significance of such large binding corrections to the IA, for πA calculations by examining an extremely simple picture of πA scattering.[50] Consider the simple optical potential given by the product of the forward πN amplitude and the nuclear density. We then neglect the real part of the πN amplitude and take the nucleus to be a uniform sphere of radius R. On solving the Klein-Gordon equation in the eikonal approximation, Glauber obtains the total πA cross section which is given by

$$\sigma_{\pi A} = 2\pi R^2 [1 - \frac{2}{x^2} + \frac{2e^{-x}}{x} (1 + \frac{1}{x})] \tag{6.3}$$

where

$$x = 3A\sigma_{\pi N}/4\pi R^2 \tag{6.4}$$

and $\sigma_{\pi N}$ is the isospin averaged πN total cross section. The sensitivity of $\sigma_{\pi A}$ to the two parameters R and $\sigma_{\pi N}$ can be determined from $d\sigma_{\pi A}/dR^2$ and $d\sigma_{\pi A}/d\sigma_{\pi N}$, respectively. The former indicates the sensitivity to nuclear structure, the latter to the dynamical modifications of the optical potential, for example through the binding correction. The two quantities $d\sigma_{\pi A}/dR^2$ and $d\sigma_{\pi A}/d\sigma_{\pi N}$ are shown as functions of pion lab energy in fig. 8. We see

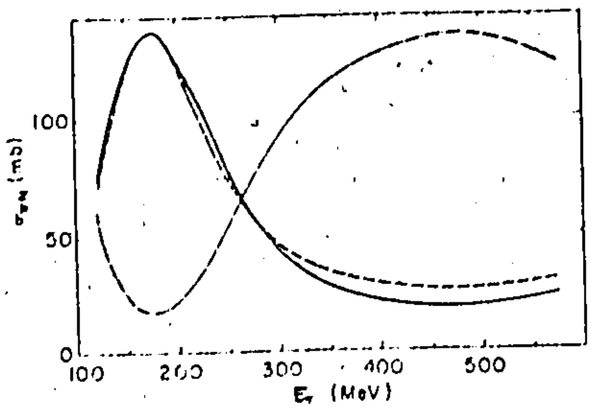

Fig 8

This fig. has been taken from ref. 50. It shows the "sensitivity" functions $d\sigma_{\pi A}/dR^2$ (solid curve) and $d\sigma_{\pi A}/d\sigma_{\pi N}$ (dash-dot-curve) in arbitrary units as functions of pion lab energy. The dashed curve is the average total cross section $\frac{1}{2}$ $(\sigma_{\pi p}+\sigma_{\pi n})$.

that sensitivity to R^2, the nuclear structure effects, is maximal at the resonance peak. The modifications of $\sigma_{\pi N}$, on the other hand, result in minimal effects on $\sigma_{\pi A}$ at the resonance peak. For example doubling the πN cross section at the peak increases $\sigma_{\pi A}$ by $\lesssim$ 10% at the resonance energy. Thus our binding corrections to the IA, though quite large, will only increase the πA cross section by $\lesssim$ 10% at the peak. But the effect of corrections will be maximum at energies away from the resonance peak. As we have seen above the binding corrections to $\sigma_{\pi N}$ are negligible at energies $\lesssim$ 100 MeV so that the πA total cross section is unaffected at such energies. At energies $\gtrsim$ 200 MeV the binding effect increases $\sigma_{\pi N}$ by 20∿25% and then decreases to negligible effect at energies > 400 MeV. The πA cross section will thus increase by about 20% at energies above 200 MeV, i.e. on the tail of the peak. It is in this energy region that the experimental values of πA cross section are greater than what most πA calculations using the IA predict. Our binding correction will thus tend to increase these πA cross sections resulting in better agreement with experimental data.

Chapter VI

CONCLUDING REMARKS

There are mainly two theoretical approaches that have been used to analyse the pion-nucleus scattering data at medium energies. These are the optical model potential and approximate multiple scattering methods such as the Glauber approximation. The approximations underlying these calculations are the IA, the closure approximation, the neglect of the Pauli principle etc. The validity of all these approximations becomes questionable at medium energies where the πN interaction becomes very strong due to the 33-resonance. The main theme of this thesis has been to examine the validity of some of these approximations, in particular that of the impulse approximation.

We have considered the binding corrections to the IA and the Pauli principle effects for πN interaction at low and medium energies and for $\bar{K}N$ interaction at low energies. These were discussed in detail in chapters IV and V and the conclusions were presented there. Here we will briefly summarize the main points. At low energies there is a negligible change in the s-wave πN scattering length, whereas the $\bar{K}N$ scattering length is significantly modified due to the binding effect.

We have seen that the IA is a poor approximation for πN interaction at medium energies. The binding corrections to the IA are found to be very large, in agreement with the qualitative estimation by Goldberger and Watson.[9] The πN_b total cross section at the resonance peak is found to be 60-70% greater than that in the IA. The elastic cross section is more strongly forward peaked compared to that in the IA. The resonance energy shifts downwards due to the binding effect. The effect of the Pauli principle is to push the resonance energy upwards. The binding correction together with the Pauli principle effect does not displace the resonance energy appreciably from that in the IA(A). The only significant effect is the change in the cross section; a decrease below the resonance energy while there is an increase at energies above the resonance.

We also discussed the implications of such large corrections to the IA for pion-nucleus scattering by considering a simple model proposed in ref. 50. We noted that the πA cross section is modified by $\lesssim 10\%$ at the resonance peak, while at energies away from the peak the πA cross section increases nearly by the same amount as the correction to the IA. The binding correction to the IA increases the πA total cross section at energies greater than the peak energy, improving the agreement of theoretical calculations with the data in that region.

We have also examined the validity of the closure approximation in our model and found that it fails to reproduce even the qualitative features of the more exact calculation in which we have summed over the intermediate nuclear states explicitly.

Our model calculation shows how large the corrections to the IA are. This suggests that the other approximations which are commonly used in πA calculations cannot be taken for granted in the 33-resonance energy region. There is a need to examine the validity of approximations such as the multiple scattering approximation and the factorization approximation for the optical potential. For example, the lowest order correction to the multiple scattering approximation gives rise to a term that is proportional to the two-body nuclear correlations. Some work has already been done in this direction.[55] These corrections seem to have non-negligible effects on the lowest order optical potential. This may enable us to extract information about the nucleon-nucleon correlation in nuclei.

The pion-nucleus physics has held out the promise of new information about nuclear structure for over two decades now. However, it has not been fulfilled so far. There is a gap of six years between the review on this subject by Koltun[2] and the very recent one by Hüfner.[4] The quality of experiments has improved during this period. Several new interesting features like the downward shift of the resonance energy in πA scattering have been observed. However, one still finds reasonable agreement between the data and the lowest order optical potential or the Glauber approximation. The gross features of the data are reproduced by the above two approaches. The success of these calculations seems to indicate that only gross nuclear properties, such as the nuclear size, and only general features of the πN interaction are apparently involved in the reaction. One might conclude from the present state of our understanding of pion-nucleus physics that new information about nuclear structure, such as the N-N correlation, or about πN interaction cannot be obtained from πA scattering.

We believe, however, that before reaching these rather negative conclusions there is a great deal to be clarified theoretically. First of all, why do these simple theories like the first order optical potential and the Glauber approximation work so well for πA scattering in this energy domain? It is especially puzzling in view of our findings that the IA is a rather poor approximation. It is important to understand the reasons of the success of simple theories before we can expect to learn any new nuclear physics from pion-nucleus interaction. The examination of the more fundamental aspects of the pion-nucleus interaction and of nucleus itself is probably necessary. For example the possibility of $\Delta(1236)$ being a particle inside the nucleus is completely ignored in such conventional multiple scattering theories. If Δ were a constituent particle of the nucleus, in addition to the nucleons, it would have some important consequences. For example, the effect due to the Pauli principle will be suppressed.

It is quite possible that these multiple scattering theories are not the last word on the subject of pion-nucleus interaction, and that these may not lead to any new information on nuclear structure. There may have to be a complete breakaway from this approach. As an analogy we can look into the problem of the s-wave πN scattering lengths. These could not be explained by the Yukawa interaction, which was thought of as the basis interaction for a long time. It was only after some new theoretical approaches, such as the dispersion relations and current algebra were developed along altogether different lines, that the problem was resolved.

For the pion-nucleus interaction the investigations to date perhaps constitute a beginning. In fact the physicists are not really sure whether any important new things about nuclear structure can be learned with pion probes. I want to substantiate this by quoting the title of a proposed panel discussion "Can one learn important new things about nuclear structure with pions" in the forthcoming International Conference on Meson Nuclear Physics to be held at Carnegie-Mellon University in May, 1976.

APPENDIX A

We will evaluate the lowest order diagram for πN scattering in the Chew-Low theory when the nucleon is bound in an harmonic oscillator potential. The binding effect manifests itself through the modification of the propagator and the vertices have extra factors coming from the bound state wave functions. Apart from the vertex factors which are common to free and bound cases we have

$$I_b = \sum_{\nu} \frac{F_{o\nu}(\underset{\sim}{k})F^{*}_{\nu o}(\underset{\sim}{k}')}{\omega-(\omega+\omega'+n\eta)} \quad , \tag{A.1}$$

where $F_{o\nu}(\underset{\sim}{k})$ is defined by eq. (2.7) of Chapter IV, and the energy in the intermediate state is $\omega+\omega'+n\eta$. Using eqs. (2.9) and (2.20) of Chapter IV, eq. (A.1) can be reduced to

$$I_b = \exp\{-\frac{b^2(k^2+k'^2)}{4}\} \sum_n \frac{1}{n!} \frac{1}{\omega'+n\eta} \left(\frac{b^2\underset{\sim}{k}.\underset{\sim}{k}'}{2}\right)^n$$

$$= \int_o^\infty d\xi \, \exp[\frac{1}{2} b^2 k.k'(e^{-\xi\eta}-1)-\xi\omega] \quad . \tag{A.2}$$

In the tight binding limit $\eta\to\infty$, eq. (A.2) reduces to $(1/\omega)$ which is the propagator for the Chew-Low interaction with a static nucleon. In the weak binding limit $\eta\to o$, retaining terms up to ξ in (A.2) we obtain

$$I(\eta\to o) = \int_o^\infty d\xi \, \exp[-\xi(\omega+K')]/(\omega+K') \tag{A.3}$$

where $K' = \underset{\sim}{k}.\underset{\sim}{k}'/(2m)$. For finite but weak binding, retaining terms up to second order in ξ we obtain

$$I_b = \int_0^\infty d\lambda \left(1 + \frac{1}{2}\lambda^2 K'\eta\right) \exp\left[\geq \frac{\lambda}{2}(\omega+K')\right]$$

$$= \frac{1}{\omega+K'}\left[1 + \frac{K'\eta}{(\omega+K')^2}\right] \quad . \tag{A.4}$$

The second term in the bracket is the binding correction which satisfies

$$\frac{K'\eta}{(\omega+K')^2} < \frac{1}{\omega+K'} < \frac{\eta}{u} \gtrsim 0.1 \quad . \tag{A.5}$$

APPENDIX B

To begin with let us recall the following eqs. (2.7) and (2.18) from Chapter IV

$$F_{\nu o}(\underset{\sim}{q}) = \sqrt{4\pi}\; i^{\ell}\; G_{\nu}(q) Y^{*}_{\ell m}(\hat{\underset{\sim}{q}}) \tag{B.1}$$

where

$$G_{\nu}(q) = c_{\nu}\; \exp(-b^2 q^2/4)(bq)^n \quad . \tag{B.2}$$

The coefficient c_{ν} is known but we do not need its explicit expression.

In order to derive the following relation

$$\sum_{\ell} (2\ell+1) C^2_{\nu}\; P_{\ell}(\cos\theta) = (2^n . n!)^{-1} \cos^n\theta \tag{B.3}$$

we start with the form factor for the 1 s-state $F_{oo}(q)$.

$$F_{oo}(q) = \langle o | e^{i\underset{\sim}{q}.\underset{\sim}{r}} | o \rangle = \exp(-b^2 q^2/4)$$

$$= \exp[-\frac{b^2}{4}(k^2+k'^2)] \sum_{n=0}^{\infty} \frac{1}{n!} (\frac{b^2 \underset{\sim}{k}.\underset{\sim}{k'}}{2})^n \quad . \tag{B.4}$$

Using eqs. (B.1) and B.2) this can be rewritten as

$$F_{oo}(q) = \Sigma_{\nu} \langle o | e^{i\underset{\sim}{k'}.\underset{\sim}{r}} | \nu \rangle \langle \nu | e^{-i\underset{\sim}{k}.\underset{\sim}{r}} | o \rangle$$

$$= 4\pi \sum_{\nu} G_{\nu}(k') G_{\nu}(k) Y^{*}_{\ell m}(\hat{\underset{\sim}{k'}}) Y_{\ell m}(\hat{\underset{\sim}{k}})$$

$$= 4\pi \sum_{n,\ell} (2\ell+1) P_{\ell}(\cos\theta) G_{\nu}(k') G_{\nu}(k)$$

$$= \exp[-\frac{b^2}{4}(k^2+k'^2)] \sum_{n=0}^{\infty} (b^2 \underset{\sim}{k}\; \underset{\sim}{k'})^n \sum_{\ell} (2\ell+1) c^2_{n\ell} P_{\ell}(\cos\theta) \quad . \tag{B.5}$$

Here and also in eq. (3.3), ℓ takes all values that are allowed for a given value of n. The comparison of eqs. (B.4) and (B.5) leads to eq. (B.3).

APPENDIX C

Consider the product of projection operators $P(\underset{\sim}{k}',\underset{\sim}{k}'')P(\underset{\sim}{k}'',\underset{\sim}{k})$

$$P(\underset{\sim}{k}',\underset{\sim}{k}'')P(\underset{\sim}{k}'',\underset{\sim}{k}) = [9(\underset{\sim}{k}'.\underset{\sim}{k}'')(\underset{\sim}{k}''.\underset{\sim}{k}) - 3(\underset{\sim}{k}'.\underset{\sim}{k}'')(\underset{\sim}{\sigma}.\underset{\sim}{k}'')(\underset{\sim}{\sigma}.\underset{\sim}{k}) -$$

$$- 3(\underset{\sim}{\sigma}.\underset{\sim}{k}')(\underset{\sim}{\sigma}.\underset{\sim}{k}'')(\underset{\sim}{k}''.\underset{\sim}{k}) + (\underset{\sim}{\sigma}.\underset{\sim}{k}')k''^2(\underset{\sim}{\sigma}.\underset{\sim}{k})]/\mu^4 \qquad (C.1)$$

$$= [\{3(\underset{\sim}{k}'.\underset{\sim}{k}'')(\underset{\sim}{k}''.\underset{\sim}{k}) + (\underset{\sim}{k}'.\underset{\sim}{k})k''^2\} + i\underset{\sim}{\sigma}.\{k''^2(\underset{\sim}{k}'\times\underset{\sim}{k}) -$$

$$- 3(\underset{\sim}{k}'.\underset{\sim}{k}'')(\underset{\sim}{k}''\times\underset{\sim}{k}) - 3(\underset{\sim}{k}'\times\underset{\sim}{k}'')(\underset{\sim}{k}''.\underset{\sim}{k})\}]/\mu^4 . \qquad (C.2)$$

Let us choose the coordinates as follows

$$\underset{\sim}{k} = k(\sin\frac{\theta}{2}\ ,\ 0,\ \cos\frac{\theta}{2})$$

$$\underset{\sim}{k}' = k(-\sin\frac{\theta}{2},\ 0,\ \cos\frac{\theta}{2})\ ,$$

$$\underset{\sim}{k}'' = k''\ (\sin\alpha\ \cos\beta,\ \sin\alpha\ \sin\beta,\ \cos\alpha)\ . \qquad (C.3)$$

Then we obtain

$$\underset{\sim}{k}'.\underset{\sim}{k} = k^2\cos\theta$$

$$\underset{\sim}{k}''.\underset{\sim}{k} = k''k(\sin\frac{\theta}{2}\ \sin\alpha\ \cos\beta + \cos\frac{\theta}{2}\ \cos\alpha),$$

$$\underset{\sim}{k}''.\underset{\sim}{k}' = k''k(-\sin\frac{\theta}{2}\ \sin\alpha\ \cos\beta + \cos\frac{\theta}{2}\ \cos\alpha). \qquad (C.4)$$

With these, the spin-nonflip and spin-flip terms in eq. (C.2) become

$$\mu^{-4}(k''k)^2[3(\cos^2\frac{\theta}{2}\ \cos^2\alpha - \sin^2\frac{\theta}{2}\ \sin^2\alpha\ \cos^2\beta) + \cos\theta]\ , \qquad (C.5)$$

$$i\underset{\sim}{\sigma}.(\hat{\underset{\sim}{k}}'\times\hat{\underset{\sim}{k}})\mu^{-4}(k''k)^2[1 - 3\{(\sin\alpha\ \cos\beta)^2 + \cos^2\alpha\}]\ . \qquad (C.6)$$

APPENDIX D

ON THE VALIDITY OF THE GLAUBER APPROXIMATION

We have noted in Chapter I that the Glauber approximation (GA) is one of the most widely used multiple scattering approaches for analysing the πA scattering data. It has been quite successful, as has been the optical model potential, in reproducing the data of πA total and elastic cross sections for several nuclei. The GA is expected to describe the hadron-nucleus scattering at very high energies only, since the assumptions underlying it are justified in this energy region. The success of the GA in describing the πA scattering in the 33-resonance energy region is rather surprising in view of the fact that some of these assumptions are violated. In this appendix we briefly describe the GA and the assumptions on which it is based. At the end we give a brief description of a model calculation in which we examine the reasons for the success of the GA at low and medium energies.

The Glauber's multiple scattering formalism relates the particle-nucleus scattering amplitude to the particle-nucleon scattering amplitude.[7] It is a semi-classical approximation. It makes use of the fact that the high energy scattering of strongly interacting particles has a very strong forward diffraction peak. The description of this phenomenon on the basis of a partial wave expansion would require a large number of terms, which involve delicate cancellations among themselves. In the semi-classical picture of scattering, on the other hand, this feature occurs quite naturally. In this description we consider the particle trajectory passing through an absorbing sphere and giving rise to a diffraction pattern (this makes use of the concepts of both geometrical and physical optics). The trajectory is specified completely by the classical impact parameter b.

Let us consider πA scattering. We can write down a representation for the πN scattering amplitude in terms of the impact parameter:

$$f(q) = \frac{ik}{2\pi}\int d^2b\ e^{i\underset{\sim}{q}.\underset{\sim}{b}}\ (1-e^{2i\chi(\underset{\sim}{b})}) \tag{D.1}$$

where the integration is over a plane perpendicular to the incident beam direction. If we assume that the elastic πN amplitude is spin independent, this implies that the phase shift $\chi(b)$ depends only on the magnitude of $\underset{\sim}{b}$, so that

$$f(q) = ik\int_0^{\infty} bdb\ J_0(qb)(1 - e^{2i\chi(b)}) \tag{D.2}$$

This description can be obtained by writing down the partial wave expansion of the πN scattering amplitude in the CM frame

$$f(\theta) = \frac{1}{2ik}\sum_{\ell}\ (2\ell+1)(e^{2i\delta_\ell}-1)P_\ell(\cos\theta) \tag{D.3}$$

and identifying the impact parameter in terms of the angular momentum by

$$b = (\ell + \frac{1}{2})/k \tag{D.4}$$

The impact parameter phase shift $\chi(\underset{\sim}{b})$ is related to the partial wave phase shifts δ_ℓ by

$$\exp[2i\chi(\underset{\sim}{b})] = \sum_{\ell=0}^{\infty}\ 2(2\ell+1)[J_{2\ell+1}(2kb)/(2kb)]\exp(2i\delta_\ell) \tag{D.5}$$

Now we need to calculate the impact parameter phase shifts $\chi(b)$. Just as in classical optics this can be computed by adding up the infinitesimal phase changes $\delta\chi$ along an element $\delta\underset{\sim}{r}$ of the trajectory $\underset{\sim}{r}$ using

$$\delta\chi = n(\underset{\sim}{r})k(\underset{\sim}{r}).\delta\underset{\sim}{r} \tag{D.6}$$

where $n(\underset{\sim}{r})$ is the local refractive index and $k(\underset{\sim}{r})$ is the local wave number. The whole crux of Glauber's theory is in the simplifying assumption that a good approximation to the phase shift can be obtained, not by adding up the elemental phase shifts along the true trajectory, but rather along the z-axis, i.e. along the undeviated beam direction. For the scattering of particles from a spherically symmetric potential $V(\underset{\sim}{r})$, the Schrödinger equation leads to the refractive index

$$n = \frac{1}{k}\sqrt{k^2 - V(r)} \tag{D.7}$$

This implies the following expression for the phase shift

$$\chi(b) = -\frac{1}{4k}\int_{-\infty}^{\infty} V(b^2+z^2)\,dz \tag{D.8}$$

In the general case when the potential is not spherically symmetric

$$\chi(\underset{\sim}{b}) = -\frac{1}{2k}\int_{-\infty}^{\infty} V(\underset{\sim}{b},z)\,dz \tag{D.9}$$

Eqs. (D.8) and (D.9) show that if the potential $V(\underset{\sim}{r})$ is a sum of number of potentials centered at $\underset{\sim}{r}_j$, then the phase shift of the scattered particle from the system is the sum of the phase shifts from the individual potentials at the appropriate impact parameters, i.e.

$$\chi(\underset{\sim}{b}) = \sum_j \chi_j(\underset{\sim}{b}-\underset{\sim}{s}_j) \tag{D.10}$$

where $\underset{\sim}{s}_j$ is the projection of $\underset{\sim}{r}_j$ in the x-y plane. This additivity of the phase shifts forms the basis of all dynamical calculations.

In the GA then the πA scattering amplitude is given by

$$F(q) = \frac{ik}{2\pi} \int d^2b \; e^{i\underset{\sim}{q}.\underset{\sim}{b}} (1 - e^{2i\chi_N(b)}) \tag{D.11}$$

where the phase shift $\chi_N(b)$ suffered by the particle is given by the additivity of phase shifts (D.10), $\chi_j(b)$ being the phase shifts by the individual nucleons. There is, however, one big difference between a nucleus and the superpositions of potentials for which (D.10) is valid. The potentials are fixed in space whereas the nucleons in the nucleus are randomly distributed and are free to move about. If R is a typical size of the nucleus, the momentum spread of the nucleon due to Fermi motion is $\Delta k \sim 1/R$. The condition that the Fermi motion be small compared to the incident momentum k is equivalent to $kR >> 1$. Providing this condition is satisfied, during the passage of the projectile we can take the nucleons as being fixed in their positions. The observed πA amplitude is F(q) averaged over the position of the nucleons in the nucleus, which with the use of eqs. (D.10) and (D.11) becomes

$$F_{fi}(q) = \frac{ik}{2\pi} \int d^2b \; e^{i\underset{\sim}{q}.\underset{\sim}{b}} <f|[1 - \prod_{j=1}^{A} e^{2i\chi_j(b-s_j)}]|i> \tag{D.12}$$

Here, use has been made of the so-called fixed scatterer approximation (FSA).

Now eq. (D.1) can be approximately inverted to give the phase shifts $\chi(b)$ in terms of the πN amplitudes

$$e^{2i\chi(b)} = 1 - \frac{1}{2\pi ik} \int d^2q \; f(\underset{\sim}{q}) e^{-i\underset{\sim}{q}.\underset{\sim}{b}} \tag{D.13}$$

Note that for the $\pi\check{N}$ amplitudes we have used the IA. In principle, however, the πN_b amplitudes should have been used. With this and the internal nuclear wave functions $\phi(\underset{\sim}{r}_1,..\underset{\sim}{r}_A)$ the πA scattering amplitude can be written as

$$F_{fi}(\underset{\sim}{q}) = \frac{ik}{2\pi}\int e^{i\underset{\sim}{q}.\underset{\sim}{b}} d^2b \int d^3\underset{\sim}{r}_1 \ldots d^3\underset{\sim}{r}_A\ \phi_f^*(\underset{\sim}{r}_1\ldots\underset{\sim}{r}_A)\phi_i(\underset{\sim}{r}_1\ldots\underset{\sim}{r}_A)\delta^{(3)}(A^{-1}\Sigma\ \underset{\sim}{r}_j)\ \times$$

$$\times\left[1 - \prod_{j=1}^{A}(1 - \frac{1}{2\pi ik}\int e^{-i\underset{\sim}{q}'.(\underset{\sim}{b}-\underset{\sim}{s}_j)} f_j(\underset{\sim}{q}')d^2q')\right] \qquad (D.14)$$

If we expand out the product over the nucleons j in eq. (D.14), then we see that F will be represented as a polynomial in f. This can be interpreted as multiple scattering expansion; the lowest order is single scattering, second order is double scattering etc. The order of polynomial being A, it restricts the multiple scatterings to A. Physically this is due to the assumed predominance of forward scattering in πN amplitudes. The GA neglects any large angle scatterings.

For elastic scattering the lowest order term of single scattering in eq. (D.14) gives

$$F_{fi}(q) = \sum_{j=1}^{A} f_j(q)\int e^{i\underset{\sim}{q}.\underset{\sim}{s}_j}\rho(\underset{\sim}{r}_j)d^3\underset{\sim}{r}_j \qquad (D.15)$$

where $\rho(\underset{\sim}{r}_j)$ are one-particle densities and only elastic scattering is considered. For the general πA scattering, let us assume the following form for high energy πN amplitude

$$f(q) = f(0)\ \exp(-\frac{1}{2}\beta^2 q^2) \qquad (D.16)$$

where q is the momentum transfer and β is a parameter which is adjusted such that the forward peak in elastic πN scattering is fitted. For elastic scattering we need only the nuclear density, which we assume factorizable, i.e.

$$\rho(\underset{\sim}{r}_1 \ldots \underset{\sim}{r}_A) = \tilde{\rho}(\underset{\sim}{r} \quad \ldots \tilde{\rho}(\underset{\sim}{r}_A) \tag{D.17}$$

For harmonic oscillator

$$\tilde{\rho}(\underset{\sim}{r}_j) = (\alpha^2/\pi)^{3/2} \exp(-\alpha^2 r_j^2) \tag{D.18}$$

With these, eq. (D.14) yields for the πA elastic scattering amplitude

$$F(q) = f(0)\exp(q^2/4A\alpha^2) \sum_{n=1}^{A} \binom{A}{n} \frac{1}{n} \left[\frac{2i\alpha^2 f(0)}{k(1+2\alpha^2\beta^2)}\right]^{n-1} \times$$

$$\times \exp\left[-\frac{(1+2\alpha^2\beta^2)}{4n\alpha^2} \; q^2\right] \tag{D.19}$$

Given the πN amplitude parameter β^2 and the harmonic oscillator size parameter α^2 for the nucleus, the πA scattering amplitude can be calculated in the GA by eq. (D.19).

We will now enumerate the assumptions on which the GA has been derived. These are the following:

a) Validity of the impulse approximation.

b) Predominance of forward scattering in the elementary πN amplitude i.e. it is strongly forward peaked so that a semi-classical description of the scattering can be used.

c) Additivity of impact parameter phase shifts from individual nucleons.

d) The validity of the FSA.

All these assumptions are probably justified at high energies. But at low and medium energies, especially in the 33-resonance energy region, some of these assumptions for πN scattering amplitude are quite questionable.

For example, in this energy region the πN scattering is dominated by p-wave and the differential cross section is of the form of $(1+3\cos^2\theta)$, so that the backward scattering is just as important as the forward one. Also as we have seen earlier, the IA is a very poor approximation at these energies. In view of these the success of the GA in reproducing the πA scattering data at intermediate energies is very strange. The reasons for this success can be analysed by examining a soluble model for which some of the assumptions underlying the GA are not satisfied. We have done such a calculation for Brueckner's model [56] and also for a one dimensional model which simulates nucleon-nucleus scattering. [57,58] We give a very brief description of this work.

We first describe a calculation for πd scattering using Brueckner's model for scattering from two non-overlapping potentials. [57] For simplicity we assume that there is only elastic s-wave πN scattering. With nucleons fixed at two points we can write down the exact scattering amplitude from the two nucleon system in terms of the individual free πN amplitudes (The validity of the IA is assumed). This amplitude is averaged with an appropriate density function to get the πd scattering amplitude. This is the FSA. We have evaluated the πd scattering amplitude in the double scattering approximation (DSA) and also in the GA. Both these are found to work well at low and medium energies. Note that the πN scattering in the present model is isotropic in contradiction with one of the basic assumptions in the GA. The reason why the GA works well is clearly seen to be the suppression of the double scattering term and the higher order scatterings in the FSA, due to the averaging with the density distribution. After having done this calculation we realized that similar but more realistic calculations (including p-wave πN interaction) have been done yielding similar results.[59] However, for this model the exact πd scattering amplitude is not known; instead the FSA is taken to be the 'exact' result with which the GA is compared. The validity of the FSA, therefore, cannot be examined in this model. In order to test the validity of the FSA, we have examined a one dimensional model.

We considered a one dimensional, three body problem in which a particle is scattered from a two particle bound system, the interactions between the three particles being δ function potentials. Parameters were chosen such that the model simulated Nd scattering. The exact scattering amplitude is known for this model.[60] The Nd scattering amplitude in the FSA is obtained by averaging the exact scattering amplitude from two fixed nucleons, using an appropriate density distribution. The GA is obtained from the FSA by neglecting the contribution due to backward NN scattering. Now, for the δ function potential in one dimension which simulates the NN scattering, the backscattering cross section is equal to the forward scattering cross section. For Nd scattering the GA is found to work well at energies as low as 15∿20 MeV. The reason for the success of the GA at such low energies is the suppression of the backscattering in Nd scattering by the averaging with respect to nucleon distribution in the deuteron. This model clearly shows that averaging is crucial for the success of the GA. The FSA is found to be a better approximation than the GA. However, it is not clear in this model as to why the FSA is so good an approximation. We have also extended this analysis to N-nucleus scattering when the target contains more than 2 nucleons. Again both the FSA and GA are very good at energies $\geq$ the binding energy of the target.

In the above approximations we have assumed the validity of the IA. Both the FSA and GA are found to be fairly good approximations at low energies. In view of our findings in this thesis that the IA is a rather poor approximation for πN amplitude at medium energies, this result is puzzling. It is possible that this is a peculiar feature of this one dimensional model or the Brueckner model.

APPENDIX E

SOME INTERESTING OBSERVATIONS

During the course of this work on the investigation of the IA we came across several interesting features in the literature; at times very puzzling. For example, Schmit's calculation is widely referred to. But the sign of shift of resonance energy is either quoted positive or negative, as we noted in Chapter V. Sometimes even the form of the IA being used is not clearly described. And as we have seen in Chapter III there are several versions for it. It is possible that all these are known to experts. Nevertheless we give below a compilation of some interesting observations, which might be useful.

1. Goldberger and Watson [9)] estimated in 1964 that the binding corrections to the IA could be of the order of 50∿100%. Nevertheless the IA has been extensively used.
2. Kimura and Nagashima [49b)] to our knowledge, were the first to incorporate the nuclear many body effects by applying the Pauli principle. They examined the modification of the CL πN amplitude in the nuclear medium and found a very large effect; the resonance energy shifted upwards by more than 200 MeV.
3. The above significant result of Kimura and Nagashima has been almost completely ignored. Various other authors examining more recently the modification of the CL amplitude in nuclear medium have failed to mention this ref. (with the exception of Schmit[46]).
4. We have already noted the diverse results of Bethe; Weber and Eisenberg; Dover and Lemmer on essentially similar problem, in Chapter V. There is

no definitive result on this question yet; rather it is more appropriate to say that the situation is quite confusing.

5. The problem of binding corrections to the IA is in no better state of understanding. In the initial stages of work the effort has been only to see the binding effect on the shift of resonance energy. As we have seen in Chapter V the situation is far from clear in this respect. Even the sign of shift of the resonance energy does not seem to have been settled. Julius and Rogers' calculation and our results show that perhaps the major binding effect is the significant rise of total cross section.

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