

FREGE'S THEORY OF JUDGEMENT

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By

DAVID ANDREW BELL, B.A., M.A.

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AUTHOR: David Andrew Bell, B.A. (Trinity College, Dublin)
M.A. (McMaster University)

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ABSTRACT

This thesis attempts to examine and evaluate the contribution made by the German mathematician and philosopher Gottlob Frege (1848-1925) to the solution of certain epistemological problems which centre round the notion of a judgement. It is argued that it is in terms of such Fregean concepts as those of assertion, sense, reference, object and concept that an adequate account of the nature of human judgement is to be formulated, but that Frege's theory must undergo substantial modification before it is finally acceptable. The conclusion herein reached is that a judgement is an assertion that a given object falls under a certain concept (or that a given concept falls within a certain concept of higher order). Just this was, indeed, Frege's claim; but it is argued that certain key terms here cannot be understood as Frege originally intended; and an alternative interpretation is propounded and defended. This interpretation is specifically directed towards the notions of a sense-function and the unsaturatedness of concepts.

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But my greatest debts, which I gratefully acknowledge, are to Prof. N.L. Wilson who supervised this thesis with many incisive criticisms and helpful suggestions, and to Dr. Prof. Günther Patzig who is largely responsible for what understanding I have of Frege.

LIST OF ABBREVIATIONS

- A. Critique of Pure Reason, 1st edition, 1781. (Kant).
- ASB. "Ausführungen über Sinn und Bedeutung". (Frege).
- Ass. "Assertion". (Geach).
- B. Critique of Pure Reason, 2nd edition, 1787. (Kant).
- BoS. Bounds of Sense. (Strawson).
- Bs. Begriffsschrift ... etc. (Frege).
- BuG. "Begriff und Gegenstand". (Frege).
- Frege. Frege, Philosophy of Language. (Dummett).
- FuB. "Funktion und Begriff". (Frege).
- Funk. "Was ist eine Funktion?" (Frege).
- Furth. "Two Types of Denotation". (Furth).
- G&B Translations from ... Frege. (Geach and Black).
- Ged. "Der Gedanke". (Frege).
- Gef. "Gedankengefüge". (Frege).
- Gg. Grundgesetze der Arithmetik. (Frege).
- Gl. Grundlagen der Arithmetik. (Frege).
- IWT. Introduction to Wittgenstein's Tractatus. (Anscombe).
- KBS. "Kritische Beleuchtung ... E. Schröders ... etc". (Frege).
- Klemke. Essays on Frege. (Klemke).
- MN. Meaning and Necessity. (Carnap).
- Nach. Nachgelassene Schriften. (Frege).
- NB. Notebooks, 1914-1916. (Wittgenstein).

- Note. "Note: Frege on Functions". (Dummett).
PG. Philosophical Grammar. (Wittgenstein).
PI. Philosophical Investigations. (Wittgenstein).
PMa Principia Mathematica. (Russell).
PMs The Principles of Mathematics. (Russell).
SSP "Sentences, Statements and Propositions". (Lemmon).
SuB. "Über Sinn und Bedeutung". (Frege).
TLP. Tractatus Logico-Philosophicus. (Wittgenstein).
Vern. "Verneinung". (Frege).

Complete details of these works may be found in the Bibliography.

REFERENCES AND TRANSLATIONS

In order to reduce the number of footnotes to a minimum I have incorporated as many references as possible into the text. All unqualified numerical references are to page numbers (with the sole exception of references to Wittgenstein's Tractatus, which are by "Satz" number, as is conventional); and all page numbers are those of the original edition or publication of the work in question. Many of Frege's articles originally appeared in periodicals and journals not now easily available; but this creates no problem, as virtually all German reprints and English translations of these articles indicate the original pagination.

I have avoided the traditional but cumbersome method of referring to the Critique of Pure Reason by giving pagination in both first and second editions simultaneously (e.g. "A.25 = B.39") for only one of these is needed to locate the passage in question. Instead, I give the pagination in B., except, of course, for those passages which occur only in A.

Reliable translations into English exist of most of the German works quoted in this thesis, and, with only occasional modification, I have employed these where available. In those cases (most notably Frege's correspondence and Nachlass) where no published English translation exists, I have translated the German myself, but have thought it prudent to include the original as a footnote.

TABLE OF CONTENTS

DESCRIPTIVE NOTE 11

ABSTRACT 111

ACKNOWLEDGEMENTS iv

LIST OF ABBREVIATIONS v

REFERENCES AND TRANSLATIONS vii

TABLE OF CONTENTS viii

INTRODUCTION 1

CHAPTER I. FUNCTIONS AND REFERENCE

1. Functions and Ordinary Language 12

2. The Concept script 17

3. The Judgement Stroke 23

4. Reference 28

5. Some Objections 38

6. Reference Continued 46

7. Summary 57

CHAPTER II. FUNCTIONS AND SENSE

1. Introduction. 60

2. Some Problems 62

3. Sense in General 67

4. Proper Names 72

5. Sentences and Indirect Discourse	80
6. Functions	86
CHAPTER III. <u>ASSERTION</u>	
1. The <u>Begriffsschrift</u>	104
2. A Bogus Distinction	107
3. A Logical Distinction	109
4. A Syntactic Distinction	116
5. A Pragmatic Distinction	119
6. Judging and Wondering	125
7. The Modality of Judgement vs. the Judgement of Modality	129
8. Summary	133
CHAPTER IV. <u>THE THOUGHT</u>	
1. Introduction	138
2. " <u>Der Gedanke</u> "	139
3. Input Sense and Output Sense	145
4. Output Sense and Thought	154
5. "Concept Carried Connectedness"	162
6. Summary	175
NOTES	179
BIBLIOGRAPHY	191

INTRODUCTION

1. The philosophical problem which forms the background to, though not the explicit subject matter of the present investigation, made its first appearance in the literature in Plato's Sophist, where at one point the Stranger asserts: "the question at issue is whether all names may be connected with one another, or none, or only some of them".¹ To this Theaetetus responds that, clearly, the latter is the case. The Stranger's retort is that it is not only names or nouns, but verbs too, which are necessary if discourse is to be possible. But even granted this, he goes on, still not all combinations of nouns and verbs are meaningful:

[a speaker] not only names, but he does something by connecting verbs with nouns; and therefore we say that he discourses. ... And as there are certain things which fit one another, and other things which do not fit, so there are some vocal signs which do, and others which do not combine to form discourse.²

In other words, while

She it in from hid garden the him
is just a list or string of eight English words,

She hid it from him in the garden
is a sentence which says, or can be used to say, something. In Plato's metaphor, the words in the sentence "fit" one another, and so "combine"

to form a whole, a unity; the words in the list, on the other hand, remain just isolated words.

It is not only in the realm of words, however, that this phenomenon manifests itself; for just as a sentence possesses a unity quite absent from a mere list of words, so a thought (or judgement, or proposition) possesses a unity and completeness entirely absent from a mere medley or succession of images; impressions, ideas, concepts or meanings. For the Ancients this phenomenon was closely related to the problem of the One and the Many; Wittgenstein dubbed it "the essence of the propositional bond";³ Kant, "the synthetic unity of the manifold in [representations] in general";⁴ and Russell wrote about it as follows:

A proposition has a certain indefinable unity, in virtue of which it is an assertion; and this unity is so completely lost by analysis that no enumeration of constituents will restore it, even though it itself be mentioned as a constituent. There is, it must be confessed, a grave logical difficulty in this fact, for it is difficult not to believe that a whole must be constituted by its constituents.

(PMS.466).

And Frege, too, was clearly grappling with the same problem in the following passage from Über Begriff und Gegenstand:

... not all the parts of a thought can be complete; at least one must be "unsaturated" or predicative; otherwise they would not hold together. For example, the sense of the phrase "the number 2" does not hold together with that of the expression "the concept

prime number" without a link. We apply such a link in the sentence

The number 2 falls under the concept prime number; it is contained in the words "falls under", which need to be completed in two ways -- by a subject and an accusative; and only because their sense is thus unsaturated are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought. (BuG.205; G&B.54).

But it is not only in connexion with words and sentences, and with concepts and thoughts that this unity manifests itself. Taking up Frege's (and Plato's) metaphor of "linking" and "fitting", Wittgenstein asserted that:

In a state of affairs objects fit into one another like the links in a chain. (TLP.2.03).

There seem, in other words, to be three aspects to this phenomenon of propositional unity (as we can call it for the moment): there is the unity of sentences (linguistic), of thoughts and judgements (epistemological), and of states of affairs and facts (ontological).⁵ Now plainly these three aspects are very closely related one to another. If, following Russell, Frege and Wittgenstein, we identify the meaning of a sentence with the thought which it expresses and with the state of affairs which it asserts to exist, then the linguistic, epistemological and ontological aspects collapse into one another. But even if we do not subscribe to any such theory, still it is arguable in general that what ever it is that makes Caesar's loving Cleopatra a complete state of affairs, is so closely related to whatever

it is which makes "Caesar loves Cleopatra" a complete sentence, and to whatever it is that makes the thought that Caesar loves Cleopatra complete, that any explanatory appeal from one to the other must run the gravest risk of bankruptcy. If, as I believe, the linguistic, the epistemological and the ontological aspects are, indeed, but three different aspects of one and the same fundamental phenomenon, then no complete account of this phenomenon can be expected from within any one of these fields. The solution to the problem of the nature of the "propositional bond" must needs be a metaphysical solution. But metaphysical does not mean metaphorical; and some way must be found of translating all the talk of "bonds", "chains", "saturation", "fitting", "glue" and "synthesis" with which the literature on this topic abounds.

The present work does not claim to have offered any final and definitive account of the nature of propositional unity. It is, however, hoped that some light has been thrown on the problem, that some of the metaphors have been translated into more prosaic language, and that, programmatic though it may be, a possible solution has been at least adumbrated.

2. This thesis is also an extended attempt to answer the question: What is a judgement? And, roughly speaking, the answer will turn out to be that a judgement, at least in the simplest of cases, is an assertion that a given object falls under a particular concept. This, of course, is hardly illuminating until it be established what is an assertion, an object, a concept, and what is the nature of the relation

of falling under. A philosophical analysis of these concepts will comprise what we might call the logical topography of judgement.

This thesis is an essay in logical topography in this sense.

3. The notion of a "theory of judgement" no longer occupies the position of preëminence it once enjoyed, for example at the turn of the present century in the writings of Bradley, Frege, Russell, Moore and Wittgenstein.⁶ Indeed, with an exception to be noted in a moment, clearly neither the need for, nor the attraction of a theory of judgement is felt by contemporary philosophers. The exception to this rule is signaled by the large number of articles that have appeared over the last thirty years concerning what, following Church, has come to be called "the analysis of statements of assertion and belief". The philosophers with whom one associates this activity are, above all, Russell, Quine, Church, Carnap and Hintikka. And the problem to which they have addressed themselves is that of providing an adequate semantics for a class of sentences which are, paradigmatically, of the form:

(S) A f's that p.

The schema (S) represents the form of statements typically used (we are told) to ascribe a belief, judgement, doubt, hope, expectation, etc. to a person. In (S), "A" is to be replaced by the proper name of a person (or by a definite description, or personal pronoun); "f" is to be replaced by a verb such as "doubt", "hope", "believe", "judge" or "assert"; and "p" by any sentence. This will yield a sentence such as:

(1) John believes that all hope is lost.

Now it is not my intention to deny that such sentences as (1) present the linguist, the logician and the linguistic philosopher with problems and perplexities: on the contrary, I believe that such problems exist, and that they are both intriguing and important. My point is rather to explain why I have not concerned myself with such matters in this thesis, as this omission may well surprise those who believe that the analysis of statements of assertion and belief is the only legitimate heir to the activities once encompassed under the notion, now somewhat anachronistic, of a "theory of judgement".

There are, I believe, basically two considerations which together have led philosophers to place so much emphasis upon the analysis of statements of the form (S). The first has been expressed with admirable terseness by Wittgenstein in the Tractatus:

In the general propositional form propositions occur in other propositions only as the bases of truth operations

At first sight it looks as though it were also possible for one proposition to occur in another in a different way.

Particularly with certain forms of proposition in psychology, such as "A believes that p is the case" and "A has the thought that p", etc. (TLP. 5.54-5.541).

Wittgenstein's thought here is that while, on the one hand, in a truth-functional proposition such as "If p then q" or "p and q, but not-r", the truth-value of the whole is determined by the truth-values of the component propositions (a fact that is manifest in the conventional

procedure of laying out the truth conditions in a truth table), on the other hand, the truth value of (1) in no way depends upon the truth or falsity of the proposition which John believes or thinks. The proposition "p" as it occurs in the complex "A believes that p" behaves in a way which presents those who, like Carnap and Wittgenstein, would incline towards a purely extensional semantics with the gravest problems.

But if Wittgenstein and Carnap have interested contemporary philosophers in the purely semantic problems raised by the existence of statements of the form (S), the later Wittgenstein and such philosophers as Austin and Grice have been construed as providing another motive for our taking a closer look at statements of belief: they tell us a great deal about the phenomenon of belief. Now this is not the place for an examination of the fundamental methodological tenets of analytical or linguistic philosophy; though it is hardly contentious to claim that such philosophy typically proceeds to investigate a given phenomenon by examining the language, or linguistic behaviour, typically associated with it. "Essence is expressed by grammar", wrote Wittgenstein, "Grammar tells us what kind of object anything is." (PI.116). If the phenomenon under investigation is the nature of judgement, say, then the philosopher might proceed by asking such questions as: What are the typical means by which we ascribe a judgement to others and/or ourselves; and what are our criteria for so doing? What is the logical form of expressions of judgement? And so on. The answers provided to questions of this

sort by contemporary analytical philosophers have concerned, almost exclusively, expressions of the form (S). And this, I believe, is a mistake. The paradigmatic form for the expression of a judgement or belief that p is the case is not a sentence of the form "A f's that p", but rather one of the form: "p". To pass a judgement, or to express it, and to report or ascribe a judgement are not one and the same activity, any more than to make a promise and ~~to~~ report a promise are one and the same thing. Statements of the form (S) are primarily relevant to the ascription of judgements, to talk about judgements, and not to the formulation, assertion and expression of judgements. The philosophers with whom this thesis deals -- Frege, Wittgenstein and Kant -- have appreciated that it is to direct discourse, and not to oratio obliqua, that one must look for the key to the nature of human judgement. In a sense, a theory of judgement is a theory of the proposition; and while such a theory must, of course, accommodate the subclass of propositions which are of the form (S), it ought not to be based solely upon it.


4. As though to remind himself that the mental activities of judging and thinking are neither rare nor occult, Wittgenstein wrote: "Thought can only be something common-or-garden, and ordinary." (PG.108). In other words, thinking is a mundane, every day activity manifest by the vast majority of mankind during the greater part of their waking lives. The nature of judgement and thought is also, prima facie, a problem with which, of all the philosophical disciplines, epistemology

should deal. A word must be said here, therefore, about my use in the sequel of a series of rather esoteric logical symbols, and, indeed, about my having chosen to concentrate upon Frege -- a philosopher with whom one does not immediately associate a concern for matters epistemological. Rulon Wells has summarized the contributions upon which Frege's reputation rests as follows:

We remember Gottlob Frege (1848-1925) for three primary contributions. He made proof in mathematics and logic more precise and airtight, he showed how the basic notions of logic and mathematics could be assimilated to each other, and he produced a workable philosophy of mathematics.⁷

Now as an historical generalization, this is certainly correct. But throughout his career and especially in his later writings Frege was also concerned to produce a theory of discursive mental activity which deserves, what it has not yet received, the close attention of philosophers. That Frege's concern was prompted by his logico-mathematical investigations, and that he employed tools developed here to solve problems that are essentially epistemological, should not be allowed to detract from the value of his contribution to the latter subject.

As to the matter of Frege's logical symbolism: much of the body of this thesis is devoted precisely to showing why it constitutes a valuable adjunct to a theory of judgement. A few preliminary remarks, however, are here in order. Let us, then, examine briefly the complex sign:



(2) |— Δ

which is the basic form of all complete signs which may occur in the derivations and proofs of Frege's concept-script.⁸ This sign has three components. First there is the functional sign, concept-word or predicate:

(3) — ξ

and here the letter "ξ" is merely a place marker which marks the "gap" or "hole" into which the name of an object or of a truth-value may be inserted. The second element in (2) is

(4) Δ

which is just such a name. In the Begriffsschrift Frege stipulates that signs such as (4) must represent "the conceptual content of a possible judgement" (Bs.2), though in the Grundgesetze this doctrine was modified so that the functional sign (3) could take any referential name in its argument place. The third and final element in (2) is the small vertical stroke which occurs to the extreme left of the sign. This is the so called "judgement stroke" (Urteilstrich), and when it is added to a functional sign that has been completed by an object name in its argument place (i.e. a sign of the form: "— Δ"), the resulting sign (2) asserts that (4) is a name of the True. Needless to say, at this point the nature of, and justification for these distinctions must remain obscure. But even at this point we can note that Frege has provided a series of signs which enable us to represent an object ("Δ"); a concept ("— ξ"); an object's falling under a concept, i.e. a truth-value ("— Δ"); and an assertion ("|— Δ"). And

here we have the basic concepts needed in a theory of judgement. Moreover, in laying down the syntax and semantics for the concept-script, and in differentiating the latter from the language of common discourse, Frege is led to ask such questions as: What is a judgement? How does one assert something? What is it for an object to fall under a concept? When two people assert the same thing, what, if anything, do they have in common? How is understanding related to assertion and truth? Now just such questions as these, and Frege's answers to them, will be our concern in the present work.

CHAPTER I

Functions and Reference

(1) Functions and Ordinary Language

"It is, even now, not beyond all doubt what the word 'function' stands for ..." wrote Frege, not without a certain melancholy some eleven years after the publication of Funktion und Begriff and the Grundgesetze der Arithmetik. For it was in these works that Frege had outlined what he took to be a definitive solution to this problem. These works, however, fell dead-born from the press; and the article "Was ist eine Funktion?", of which the quoted observation constitutes the opening words, was a renewed attempt on Frege's part to set out clearly and cogently what might well be regarded as one of his most important doctrines: the "Ungesättigtheit der Begriffe", the unsaturatedness of concepts.

In the last analysis, however, the article was no more successful than its predecessors, and Frege eventually admits that although he is convinced of the truth of his doctrine of the unsaturatedness of concepts, still "no definition is here possible. I must confine myself to hinting at what I have in mind by means of a metaphorical expression; and here I rely on my reader's agreeing to meet me half way" (Funk.665).

Now, as if this were not trouble enough, the only argument

which Frege advances in this article for the conclusion that concepts are essentially "unsaturated" or "in need of completion" gives every appearance of being quite strikingly bad. Having so defined a function-name that it is essentially incomplete, by explaining that a function-name is what is left over after one or more occurrences of a singular term have been removed from a complex complete name, he then states:

The peculiarity of functional signs, which we here called 'unsaturatedness' has of course (natürlich) something corresponding to it in the functions themselves. They too may be called 'unsaturated'.

(Funk.665).

The aim of the present and immediately subsequent chapters is to offer an interpretation and, with some modification, a defense of the Fregean doctrine of the unsaturatedness of functions. It will be shown that Frege's theory constitutes an important contribution to the solution of the puzzles concerning the phenomenon of sentential or propositional unity.

The first task is to introduce some terminology in which to explain, and the theory which underlies, Frege's account of functions.¹ Let us begin, then, with the intuitive notion of a simple complete name, i.e. an expression which either names or purports to name a determinate object and which contains no proper part which fulfils or purports to fulfil this function. Expressions like "Socrates", "9" and "Pegasus" fall into this category. A complex complete name is an expression which names or purports to name a determinate object, but which contains as a proper part a complete name, e.g. "the wife of

Socrates", "g²" and "the stable of Pegasus". Usually a complex complete name will name an object distinct from that named by any component complete name; "the wife of Socrates", for example, names Xantippe. But this need not always be the case. "Socrates' son's father", for example, refers to Socrates. The relation in which a complete name stands to the object which bears it Frege calls "Bedeutung", a term which is variously translated as "nominatum", "denotation", "reference" "stands for" and "meaning".² With the exception of the first and last of these terms which I shall not use at all, I shall use all these terms interchangeably. Reference, denotation and standing for will always stand in sharp contrast to the sense (Sinn) which an expression expresses.³

At this point it is necessary to introduce two doctrines which Frege propounded concerning complex complete names. The first is this: the reference of a complex complete name is determined by the references of its component parts. We shall have cause to mention this principle for the determination of complex reference below, and can refer to it as PR1 for short. It is impossible to understand PR1 fully until the issue of the reference of incomplete expressions has been settled; still, one corollary of PR1 can be noted immediately: if a complex complete name contains a component name which lacks a reference, then the complete name in which it occurs will also be without reference. If "Pegasus" does not refer to anything, then neither does "the stable of Pegasus".

The second doctrinal assumption involved in Frege's account of

the nature of functions is this: sentences are to be construed as a species of complex complete name. Sentences, if they possess a reference, refer to their truth-values. Thus, just as "9" is a simple complete name of the number 9, and "9²" is a complex complete name of the number 81, so "9² = 81" is a complex complete name of the truth, and "9² = 80" a name of falsehood. (Frege says that these sentential names are names of the True and the False respectively).

In conjunction with the corollary to PR1, noted above, this rather implausible theory results in the more intuitively acceptable claim that any sentence which contains a complete name that lacks a reference will be without a determinate truth-value:

The sentence 'Odysseus was set ashore at Ithaca while sound asleep' obviously has a sense. But since it is doubtful whether the name 'Odysseus', occurring therein, has a reference, it is also doubtful whether the whole sentence has one. (Sub.32).

We can now turn to what is to be our central concern: incomplete expressions or function-names (Funktionsnamen). A function-name may be obtained by deleting or excising one or more complete component names from a complex complete name. The place or places formerly occupied by the excised names are marked either by pairs of empty brackets, or by use of the lower case Greek letters "ξ" and "ζ".⁴ Removal of the complete component name "Socrates" from the complex complete name "the wife of Socrates" leaves the incomplete expression "the wife of ξ" (or "the wife of ()"). Removal of the complete name "9" from the complex name "9²" leaves the incomplete expression "ξ²" (or "()²"). And, because sentences

have been construed as a species of complete referring expression, removal of the complete name "Aristotle" from "Aristotle was born in Stagira" leaves the incomplete expression " ξ was born in Stagira" (etc.).

Now Frege maintains that incomplete expressions generated in this way are also referring expressions. Although at this point it may seem an empty gesture, even granted that it is intelligible, I shall adopt the convention of designating that to which a given incomplete expression is supposed to refer by dropping the inverted commas and italicising the resulting expression. And so, analogously to saying

"Aristotle" refers to (the man) Aristotle

I shall say

" ξ^2 " refers to (the function) ξ^2

This latter statement, then, is a superficial and provisional gloss on the Fregean thesis that functions are unsaturated. Functions, just like function-names, have, in Miss Anscombe's happy phrase, "holes in them" (IWT.109). To say this is not to offer a solution to anything, however, but is merely to state the problem with which we must deal.

Finally, in the matter of terminology, the object referred to by the complete name which is inserted into the gap in a function-name is called the argument of the function; and the reference of the completed function-name is called the value of the function for that argument. And so the function ξ^2 yields the value 4 for the argument 2, and the value 81 for the argument 9. Functions, it is beginning to

emerge, perform the role of mapping objects (as arguments) onto objects (as values). It is also clear that those function-names which result from sentences map objects (as arguments) onto truth-values (as values). Such function-names would normally be called predicates and relational expressions. Frege calls that sub-class of function-names which take truth-values as their values "Begriffswörter" or predicates; and such expressions are said to refer to unsaturated entities called "Begriffe" or concepts.

(2) The Concept-script

All that has thus far been said about functions and concepts applies to the language of common discourse and to the ordinary formulae of arithmetic. Throughout his life, but especially at the time of writing the Grundgesetze, Frege manifested a profound mistrust of ordinary language and the linguistic habits of mathematicians. He did not, of course, deny that for everyday purposes everyday language was entirely adequate. He was determined, however, that for the expression of scientific truth, the vagueness, the ambiguity, and the contradictions which are present in ordinary language make it a most unsuitable medium. The first task in the construction of a rigorous science, he believed, was the formulation of an adequate notation, a Begriffsschrift or concept-script, from which all the infirmities of ordinary language had been removed.⁵ As Frege wrote in the Begriffsschrift:

In my formalized language... only that part of judgements which affects the possible inferences

is taken into consideration. Whatever is needed for a valid inference is fully expressed; what is not needed is, for the most part, not indicated either; no scope is left for conjecture. (Bs.3).

In particular, there are three desiderata which Frege tailored his concept-script to embody: (i) determinateness of sense for all signs; (ii) possession of reference by all signs (i.e. the possession of reference by all complex signs built up according to the rules of syntax out of simple signs which have reference); and (iii) perspicuity of logical syntax. It is generally agreed that Frege's ideography met these ideals as far as can reasonably be demanded. It was Frege's determined pursuit of these goals which lead to his invention of quantificational theory, of a truth-functional account of the logical connectives, of a theory of logical types, and of the need for a rigorous distinction between talking about a sign, what the sign means and the object, if any, to which it refers. History has, however judged that Frege's concept-script falls short of the ideal of perspicuity; it is now generally reckoned to be cumbersome, confusing and expensive to print. Russell, whose notation was to supplant Frege's, reacted in 1903 in a way which was to prove virtually universal: "His [Frege's] symbolism, though unfortunately so cumbrous as to be very difficult to employ in practice, is based upon an analysis of logical notions much more profound than Peano's, and is philosophically very superior to its more convenient rival." (PMs.501).

The element which, above all, gives Frege's concept-script its distinctive and rather bewildering appearance is the system of vertical

and horizontal lines which precede all well-formed formulae within it. We must examine two of these lines if we are to understand Frege's theory of functions. An example of a complex proposition from the concept-script would be as follows:

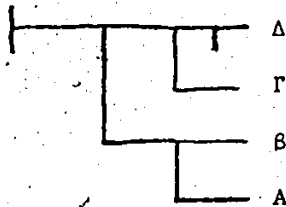


Fig 1⁶

Now all the horizontal lines in Fig 1, and in the concept-script as a whole, perform the same role. A horizontal line is a function-name whose argument place always occurs to its right, thus:

— ξ

The function thus denoted, Frege tells us⁷, takes as its value a truth-value for any object as argument. It is, therefore, a concept. More precisely, Frege stipulates that

— Δ

shall be a name of the True if " Δ " is a name of the True; in all other cases it shall be a name of the False. And so "we can say that

$\Delta = (\text{— } \Delta)$

is the truth-value of: Δ is a truth value. Thus — $\emptyset(\xi)$ is a concept, and — $\emptyset(\xi, \zeta)$ a relation, whether or not $\emptyset(\xi)$ is a concept or $\emptyset(\xi, \zeta)$ a relation." (Gg. 55).

This is, indeed, rather obscure. But there is a tendency amongst commentators to obscure matters still more by divorcing the horizontal

function-name from other predicates of a more usual sort, such as

ξ is now alive.

Dummett, for example, writes that the horizontal "in effect turns any singular term into a sentence" (Frege.315). And Thiel writes that it "makes possible the direct transition from objects, Δ , to truth-values, $\text{---} \Delta$ ".⁸ This makes the horizontal stroke appear to function like a magic wand which, merely by appearing before a name can transform it into a sentence. For while "Julius Caesar" is the name of a man,

--- Julius Caesar

is a name of the False!

In fact, the matter is by no means as obscure or implausible as this kind of account would make out. For, in a certain sense, all predicates transform names into sentences. And just as "Julius Caesar" is the name of a man, so

Julius Caesar is now alive

is a name of the False. In other words, the predicate " $\text{---} \xi$ " behaves in an exactly similar way to the predicate " ξ is now alive". There is, however, one difference between the horizontal stroke and an ordinary predicate like " ξ is now alive" which can be brought out by remarking that in the former but not in the latter case there is necessarily only one object which falls under the concept. In other words, while any number of objects fall under the concept ξ is now alive (e.g. Richard Nixon, Gerald Ford, and some 3000 million others), there is necessarily but one object which falls under the concept $\text{---} \xi$, viz: the True. The horizontal stroke must, therefore, be read as an identity

predicate, whose ordinary language equivalent, I suggest, would be:

ξ is identical to the True.

If this reading is accepted (and it will need only slight subsequent modification⁹), then the truth conditions of the function $\text{—}\xi$ can be seen to follow immediately and intuitively: the value of the function will be the True if and only if the argument is the True. The value will be the False, moreover, not only when the argument is the False but also when the argument is not a truth-value at all! And so it is now clearly and unproblematically false that Julius Caesar is identical to the True, i.e. that

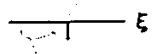
— Julius Caesar.

It is now possible to understand the force of Frege's remark that " $\Delta = (\text{—}\Delta)$ " is the truth-value of : Δ is a truth-value". The prefixing of the horizontal stroke to any name of a truth-value will in no way alter the reference of this name: any name of the True will remain a name of the True, and so, mutatis mutandis, for any name of the False. All other kinds of complete name, however, will be transformed into names of the False. And this means that it is impossible for a well formed concept-script formula to lack a reference (a truth-value) as long as the complete names inserted in the gap after the horizontal stroke have a reference, whatever that reference might be. In this way Frege banishes from the concept-script not only all propositions which do not possess a determinate truth-value, but also all possibility of a non-sentential name masquerading as a "possible content of judgement". Critics who have objected that Frege's notation allowed the formulation of nonsense

like "—2" and "—Julius Caesar" on the grounds that it is senseless to talk about "asserting that 2" or to dispute over "the truth of Julius Caesar" have failed to appreciate the ingenuity and the motivation for the horizontal stroke, which was introduced precisely to avoid the possibility of such syntactically mal-formed nonsense.

At this point, two more concept-script function-names can be introduced:

- (1) the small vertical stroke which hangs below a horizontal stroke, thus:

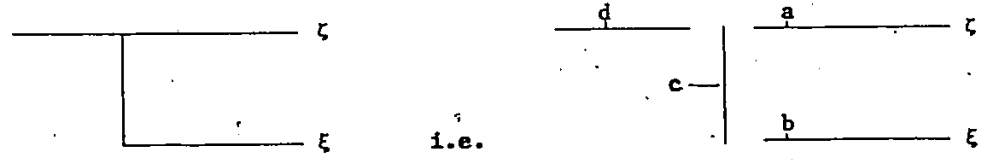


is so defined that it is to have the value False if the argument is the True, and in all other cases it is to have the value True. It clearly corresponds to the ordinary language predicate

ξ is not identical to the True.

This small hanging vertical stroke is Frege's negation sign.

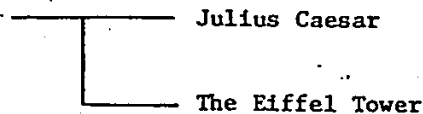
- (2) a long vertical line which joins two horizontals is called the conditional sign (c):



The conditional sign is so introduced that "the value of this function is to be the False if we take the True as the [ξ]-argument and at the same time take some other object as the [ζ]-argument; in all other cases it is to have the value the True." (FuB.28). In the "exploded" diagram, above right, the lines a and b are the horizontals which attach directly to whatever names are inserted into the argument places

" ξ " and " ζ ". It is to these horizontals that the vertical conditional stroke attaches (c). The horizontal stroke d then attaches to the whole thus formed. The whole complex sign illustrated above is, then, the equivalent concept-script representation of the ordinary incomplete expression "If ... then ...". (The place marked " ξ " is the antecedent, and that marked " ζ " the consequent). But there is one important difference between an ordinary language conditional schema, and the Fregean conditional function-name: the latter, like all function-names, is to be defined for all objects as argument; the former schema is, of course, not. And so while

If the Eiffel Tower, then Julius Caesar
is just nonsense, the complex sign



is actually true! This, as we have shown, is easily explained once the function-name " ξ " is read as " ξ is identical to the True".

(3) The Judgement or Assertion stroke.

In order to complete our account of the nature of function-names in the concept-script, it is necessary to introduce another special Fregean symbol, but one that is not, and cannot be used to form, a function-name. This sign Frege calls the judgement stroke (Urteilstrich), and it consists in a small vertical line which occurs to the left of the horizontal:

There are three things to note in connexion with this sign: (i) it can only occur to the left of the dominant horizontal in a complex sign. It cannot, in other words, occur more than once in any complete sign, and cannot be prefixed by any sign whatever. (ii) it cannot occur in a function-name; and, because of the way in which function-names were introduced, this means that any sign in which it occurs cannot be a name. (iii) its presence indicates that the signs which follow it are employed assertively. Signs of the form " $\vdash\Delta$ " cannot be proper names, Frege maintains, and hence cannot be used to form function-names because they signify assertions; and clearly it is impossible to assert either a proper name or a function-name. The sign " $\vdash\xi$ ", which would result from treating " $\vdash\Delta$ " as a proper name capable of generating functional expressions, would have to signify (per impossibile) that " ξ " was being asserted. But " ξ " is merely a gap holder in an incomplete expression, and is not the sort of thing that can possibly be asserted. (Also see below, pp. 122-124). All this Frege summarizes in a troublesome footnote, to which we shall have cause to return on a number of occasions:

The judgement stroke cannot be used to construct a functional expression; for it does not serve, in conjunction with other signs to designate an object. " $\vdash 2 + 3 = 5$ " does not designate anything; it asserts something. (FuB.22n).

What, then is the difference between the two signs

(1) $\vdash 2 + 3 = 5,$

and

(2) $\vdash 2 + 3 = 5?$

It would appear, if we take literally Frege's contention that (1) is a name of an object (the True) while (2) is an assertion that something is the case (that $2 + 3 = 5$), that the difference between (1) and (2) is analogous to that between, say,

(3) Dr. Smith

and

(4) Smith is a doctor.

(1) and (3), we are tempted to say, are names; (2) and (4) are assertions. Now this comes, indeed, close to the truth. There is also, however, a sense in which (1) is not a name,¹¹ and (4) is not an assertion. According to the analysis given above (1) will be equivalent to " $2 + 3 = 5$ is identical to the True"; and this certainly does not look like a name, but rather a sentence. But then, if (1) is a name, but somehow at the same time a sentence, why should not the same thing also hold of (4)? In which case the intended contrast between names and assertions collapses. The way out of this difficulty is to amend the reading of the horizontal stroke so that the intended contrast becomes clear.¹²

Now, for Frege (1) is a name; we can be sure of that. It is the name of a truth-value which necessarily results when the function-name "—ξ" is completed by any sign that has reference, in this case: " $2 + 3 = 5$ ". The reference of the completed function-name is, however, precisely a truth-value, and this would indicate that its nature is, in some way, sentential. But at the same time it is entirely lacking in assertive force. Now there exists in ordinary language a type of

expression which conforms exactly to these requirements. The nominalization of an indicative sentence is (usually¹³) without assertive force, and yet is "in some way sentential" in that a simple grammatical transformation can render its sentential form explicit. Consequently the function-name

—ξ

ought to be interpreted, not as the identity sentence mentioned above, but rather as the corresponding complex noun phrase:

ξ's being identical to the True.

In this form, to repeat, it is clear (i) that a nominal rôle is intended, (ii) that assertive force is absent, and yet (iii) that a simple grammatical operation will always yield a sentence. (Eg. by the addition of the phrase "... is a fact"). This account, then, will destroy the temptation to say that if (1) is a name then (4) must be a name too. They are obviously of quite different form.

The foregoing remarks were suggested by the following passage from an article in which Frege compares his concept-script with that of Peano's:

In the formula

$$(2 > 3) \supset (7^2 = 0)$$

a sense of strangeness is at first felt, due to the unusual employment of the signs ">" and "=". For usually such a sign serves two distinct purposes: on the one hand it is meant to designate a relation, while on the other hand it is meant to assert the holding of this relation between certain objects.

Accordingly it looks as though something false
 $(2 > 3, 7 = 0)$ is being asserted in that formula--
 which is not the case at all. That is to say, we
 must deprive the relational sign of the assertive
force with which it has been unintentionally in-
 vested.¹⁴

Now the obvious way to divest a relational sign of its assertive
 force is to employ a noun phrase (R's obtaining) and not a senten-
 tial form (R obtains). It is important to note, however, that such
 grammatical transformations are not possible in the concept-script
 or in the formulae of logic or arithmetic. In arithmetic the expres-
 sion

$$(5) \quad 2 + 3 = 5$$

must double not only for

$$(6) \quad 2 \text{ plus } 3\text{'s equalling } 5$$

but also for

$$(7) \quad 2 \text{ plus } 3 \text{ does equal } 5.$$

Frege, then, introduces the two expressions

$$(8) \quad \text{---}2 + 3 = 5$$

and

$$(9) \quad \text{┆---}2 + 3 = 5$$

to correspond to (6) and (7) respectively. "This is a manifestation",
 wrote Frege, "of my desire to have every objective distinction reflected
 in my symbolism". (loc. cit.)

My intention at this point is not to embark upon a detailed examina-
 tion of the concept assertion, nor of the uses to which Frege's judgement-

stroke can be put. These topics were introduced here only in order to show that a nominalized reading of the horizontal better suits Frege's intentions than one that is overtly sentential. In so far, therefore, as there are other uses to which the judgement stroke can be put, and other distinctions which the two sign types " $\text{---}\Delta$ " and " $\text{---}\Delta$ " can be used to mark and which are not mentioned here, Frege's motivation must needs remain obscure. This matter will be taken up again in Chapter III below.

(4) Reference

The procedure which Frege typically employed to isolate those concepts necessary for an adequate semantics was what I shall call the procedure of noting invariance under substitution. The classical example of this method is to be found in the opening paragraphs of the essay "Über Sinn und Bedeutung", where the reference of an expression is distinguished from its sense by noting that there are two things that may change (or remain the same) when different terms are substituted in an identity statement. One such variant is the truth-value of the sentence, and it appears that the truth-value of an identity statement will not vary as long as co-referential terms are substituted. But in the process of substituting co-referential terms, clearly something else is changed. This is especially clear in the case where the identity statement is of the form " $a = a$ ", and one term with the same reference as " a " is substituted to yield a statement of the form " $a = b$ ". Although " $a = a$ " and " $a = b$ " will have the

same truth-value, they are "obviously statements of differing cognitive value" (SuB.25). While the former is merely an instance of the principle of identity and can be known a priori, the latter may well contain "substantial information". If, however, terms are substituted which have the same cognitive value, then the cognitive value of the whole will be unchanged. This cognitive value is called by Frege the sense (Sinn) of an expression. The sense of a sentence Frege calls a Thought (Gedanke).¹⁵

Now the notion of the reference of an expression has thus far been treated as having a sufficiently clear intuitive content to enable us to proceed without scrutinizing this content too closely in its own right. And surely this assumption is justified when it is the reference of singular terms that is at issue. The reference of a singular term is that object which is picked out or named by the expression. But Frege so extended the notion of reference that for him it makes sense to ask of any expression of a given grammatical type, "What is its reference?" Not only proper names, but predicates, sentences and even logical connectives were also construed by Frege as referring expressions. Here one's pre-theoretical intuitions are of no avail; we need a semantical theory in terms of which we can understand such questions as: "To what does the word 'and' refer?" and "What is the reference of the sentence 'Julius Caesar is dead'?"

From the point of view of Fregean scholarship the matter has another problematic aspect. In the final works which Frege published during his lifetime (the three parts of the Logische Untersuchungen¹⁶)

we find, surprisingly, no mention of the doctrines, once so central, that sentences refer to their truth-values and function-names refer to unsaturated entities called functions or concepts. This is enough to raise the suspicion that these doctrines are not ultimately as important as Frege made out in the middle period; for Frege is able, in the Logische Untersuchungen, to propound most of his important theories without any recourse or reference to them. The only passage in which Frege even hints that he continued to maintain this doctrine concerning reference in the works published after 1906 appears in "Der Gedanke", where Frege writes, somewhat enigmatically:

The meaning of the word "true" seems altogether unique. May we not here be dealing with something which cannot, in the ordinary sense, be called a property at all? (Ged. 61-2)

However, he then immediately adds:

In spite of this doubt I want first to express myself in accordance with ordinary usage, as if Truth were a quality, until something more to the point can be found. (loc. cit.)

Nothing "more to the point" is to be found, however, in the remainder of the essay, nor in the subsequent parts of the Logische Untersuchungen, published or unpublished.

But perhaps Frege is being disingenuous. For the documents which compose the Nachlass reveal that as late as 1919 he had not abandoned the doctrine:

On the other hand, not only a sentence part, but also a complete sentence whose sense is a Thought can have

a reference. All sentences which express a true Thought have the same reference; and all sentences which express a false Thought have the same reference (the True, and the False).¹⁷

Now this makes it plausible to suppose that Frege did not change his mind on this central question, but merely refrained from mentioning the theory in the works which he published after 1906. To what can this reticence be attributed? Dummett has suggested that Frege's move here was a "strategic one", and that he was "leaving the notion of reference for later introduction". (Frege.660-1). Frege was well aware, Dummett argues, that his theories of sentential and functional reference were very far from plausible, and consequently likely to alienate those who might otherwise take an interest in his work. Now such speculation about Frege's psychological motivation is, I suggest, profitless. Such speculation tells us nothing about Frege's philosophical theories; and it remains, moreover, ultimately unsubstantiated guesswork. There is, however, another way in which Frege's reticence in this matter can be interpreted; one which throws some light on his philosophical motivation.

In the Begriffsschrift and in the works which comprise the middle-period Frege is almost exclusively concerned with the pure, formal sciences (logic, arithmetic) and with the development of a concept-script that would adequately embody their truths.¹⁸ The works published after 1906 are of quite a different character. No longer is the emphasis upon "pure thought" and the concept-script, but rather upon scientific thought and language in general. And this includes both pure and applied science. Now, by a scientific language—or better: a scientific use of language—

Frege means simply one which is "directed towards the truth, and only the truth". (Ged.63). This, therefore, includes not only the tautologies of logic and the analytic truths of arithmetic (if such they be) but, indeed, all sentences that are not fictitious, poetical, fanciful or meant dishonestly. Frege mentions explicitly in this context: mathematics, physics, chemistry and history.

(loc. cit.).

Now, if we take seriously Frege's claim that:

It is just as important to neglect distinctions that do not touch the heart of the matter as to make distinctions which concern what is essential. But what is essential depends on one's purpose... (Ged.64).

then I think that it is possible to construe the absence of the theory of the Bedeutung of sentences (and function-names) as motivated by methodological considerations, determined by the increase in the scope of the investigation. The doctrine that a sentence denotes its truth-value assumed a central role in the attempt to axiomatize arithmetic, where, as Dummett has remarked, it "permitted a great economy in Frege's classification of the denizens of his ontological universe: concepts ... appeared as merely a special case of unary functions, and relations as a special case of binary ones; a concept is just a function whose value, for any argument, is a truth-value. This effected a certain streamlining in the formal system of Grundgesetze as compared with that of Begriffsschrift." (Frege.644). But when Frege turned to consider the language of general science, this theory was no longer needed to account for the phenomena under investigation. And for the same reason a special

sign "┆—" to mark the possession of assertive force is no longer required. For in ordinary, non-formal language, as Frege says, "the real assertive force lies ... in the form of an indicative sentence" (Ged.63).

It is plausible, then, to maintain that there was no doctrinal change on Frege's part, but merely a change in tactic. This was not, however, dictated by external considerations (eg. a desire not to confuse or repel his readers) but resulted from the change in the character of the material upon which he was working.

All this, regrettably, tells us nothing as to the truth of the doctrine that sentences denote their truth-values. And we are left in the position of being committed to a doctrine which is obscure, highly implausible and, moreover, possibly unnecessary. The following considerations will, I hope, show that this doctrine is not an inescapable consequence of accepting a Fregean account of sense, reference and judgement etc. In other words, the theory that sentences and functional expressions are a type of name can be shown to be, in Wittgenstein's metaphor, a wheel which turns freely and is without connexion with other parts of the mechanism. This conclusion is, indeed, suggested by the fact that Frege can say so much in his later works without any recourse to this doctrine. But to remain satisfied with this suggestion would be to leave open the very real possibility that Frege's closet contains a skeleton which might ultimately make the whole house uninhabitable.

Let us begin, then with an examination of the Fregean doctrine that functional expressions, and predicates, refer to or name an

unsaturated entity called a function, or concept.

For the sake of brevity I shall restrict my discussion to those function-names which have only one argument place, and whose values are always truth-values. In other words, I shall talk only about predicates and concepts; though what I say is easily extended to cover, on the one side, relations, and on the other side, functions which are non-sentential. A predicate will always be a form of words of some sort, as will a name and a sentence. Concepts, relations, functions, extensions, Thoughts and truth-values will never be words of any sort. We must now try to ascertain what these latter are, and how they are related to various forms of words.

Frege's doctrine concerning the relations which obtain between a predicate, its sense, its reference and its extension is certainly the most obscure and puzzling area in Fregean semantics. The visible tip of this iceberg of obscurity is the seemingly paradoxical assertion that

the three words "the concept horse" designate an object, but on that very account they do not designate a concept (BuG.195)

and hence that

the concept horse is not a concept. (BuG.196)

At least part of the thought which receives such paradoxical expression is this: if one defines a proper name (Eigennamen) as "a sign for an object" (BuG.198n), and one defines an object as "whatever is not a function" (Gg.7), then it will follow, of course, that there can be no such thing as a proper name of a concept. The motive which

prompts this set of definitions is that objects are complete in themselves, saturated or self-subsistent, while concepts are, in contrast, incomplete, unsaturated and unable to stand by themselves. What does this mean? It means, in part, the indisputable truth that an object can never be predicated of an object. It also means that a list of names cannot assert a Thought, i.e. anything that could possess a truth-value. And it also means that a concept cannot fall under a concept of the same level. We must examine these claims in detail.

We can begin by taking the predicate

(1) () is drunk.¹⁹

as an example. Suppose that now a second predicate is introduced, such that it applies to all and only those objects to which (1) applies. So any completion of this predicate, viz:

(2) () is \emptyset

by a referring expression will result in a sentence of the same truth-value as the sentence which results from the completion of (1) by that same expression. (1) and (2) are clearly co-extensive. Perhaps, it might be thought, (1) and (2) both denote the same concept. But it is just this suggestion which Frege denies is even a meaningful hypothesis. For it to be meaningful, he argues, it must be possible to formulate an identity statement of the form

(3) () = ()

and in which the gaps are occupied by terms denoting concepts. But this cannot be done; for

(4) () is drunk = () is \emptyset

is nonsense. And

(5) "() is drunk" = "() is \emptyset "

is simply false. Indeed, expressions of the same form as (5) will be false for any two typographically differing expressions as terms in the identity statement. "Identity, properly speaking, does not apply to concepts" concludes Frege. But he goes on to add: "coincidence in extension is a necessary and sufficient condition for the occurrence between concepts of the relation corresponding to identity between objects" (Rev. Huss.).²⁰ But if this is the case, why not take the simplifying step of identifying a predicate's reference with its extension? There would then be no need to resort to the strange notion of an un-nameable but objectively real unsaturated entity, or "concept", which a predicate denotes.

In the first place, it is clear that the extension of a concept-word or predicate is not its sense, for when co-extensional predicates are interchanged in a sentential context the Thought expressed by the sentence may change. Thus, "() is a chimera" and "() is a unicorn" share the same extension, though they do not yield sentences expressing the same Thought when completed in the same way. (Not that there is the slightest intuitive reason to think that they would or should).

An argument of the same form is not, however, available to show that the reference and extension of a predicate are different. For, at least in Frege's extensional system it is the case that co-extensional concepts can be interchanged salva veritate. How, then, does Frege think that he can prise apart the notions of reference and extension?

In the realm of reference, what a name denotes and what a predicate denotes must always and absolutely be distinguished: "I do not want to say it is false to assert of an object what is ... asserted of a concept; I want to say it is impossible, senseless to do 'so.'" (BuG.200). This senselessness is manifest in the syntactical malformedness of the (putative) sentences which result from an attempt to predicate a concept of a concept of the same level; i.e. to treat a concept-word as a proper name. We noticed this malformity in connexion with expressions like (4) above. The reason for this malformity is simply that English sentences do not have holes in them, and Fregean predicates do. Two alternatives suggest themselves at this point: either the hole can be rendered harmless, by enclosing the expression in inverted commas; or the holes can be filled in. The first alternative does not enable us to treat a concept as an object, for we end up referring to an expression, and not to the concept we intended (as in (5) above). The second alternative, filling in the hole, looks at first sight more promising. Is not the predicate (1) just one way of referring to that concept which is also denoted by the expressions "drunkenness" and "the concept drunk"? For Frege, as we have already seen, the possibility of substitution salva veritate is the criterion for ascribing sameness of reference to two or more expressions. But neither of these two alternative expressions can be substituted for (1) as it occurs in, say,

(6) Julius Caesar is drunk.

For the expressions which result:

(7) Julius Caesar drunkenness

and

(8) Julius Caesar the concept drunk

are nonsense. No such complete names can be substituted for an incomplete predicate, because the result is a list and not a sentence.

(5) Some Objections.

There are a number of objections to the foregoing reconstruction of some Fregean theses concerning concepts which may well have occurred to the reader, and which it will be profitable to deal with at this point. In the first place, as Dummett has remarked,²¹ it might well be thought that Frege has confused talk about signs with talk about what those signs refer to. Once Frege's paradox about the concept horse is framed in the formal and not the material mode it ceases to be problematic:

There is no more paradox in the fact that the expression "the grammatical predicate 'is red'" is not a grammatical predicate, than there is in the fact that the phrase "the city of Berlin" is not a city. In the material mode of speech Frege was forced into such at least superficially contradictory expressions as "the concept horse is not a concept" ... but when we are talking about expressions then we have no motive for denying the obvious fact that "is a horse" is a predicate, nor for affirming the obvious falsehood

that the phrase "the predicate 'is a horse'" is a predicate.

In other words, (W. Marshall's words): "Frege has taken a linguistic difference to be a rift in nature".²²

Another response is that the whole question of "unsaturatedness" would not arise if the convention of marking the "gaps" in predicates were not adopted, or, again, if nominalized transformations were allowed to be co-referential with the paradigm predicates from which they result. In other words, it can appear tendentious to claim that the predicate which occurs in the sentence "Julius Caesar is drunk" is this:

(9) () is drunk.

For one thing, the expression (9) does not even occur in the quoted sentence. And, finally, even granted that such expressions as (9) are in order, this hardly entails that such formulations are the only ones possible. There are occasions (so the objection runs) when we want to assert something of a concept, and here the concept must needs appear as subject and in a nominalized form, as eg. in the sentence "Drunkenness is undesirable".

Objections such as these manifest a misunderstanding of Frege's theory of functions: of his distinction between marks (Merkmale) and qualities (Eigenschaften); between concepts of first and higher levels; between an object's falling under a concept (and the analogous subter relation), the subordination (Unterordnung) of one concept and another (and the analogous sub relation), and the falling of one concept within another concept of higher level. The following account of these Fregean

doctrines is meant to be only as complete as is necessary in order to deal with the forementioned objections. Much remains to be said about the finer points of Frege's theory that is, however, not germane to the present task.

There are three distinct and dominant, and two subsidiary and derivative, relations which together constitute the formal nucleus of Frege's theory of functions and concepts:

- (I) ξ falls under ζ (ξ = object; ζ = concept, level-1).
 (II) ξ is subordinate to ζ (ξ = concept; ζ = concept of same level).
 (III) ξ falls within ζ (ζ = concept of higher level than ξ).

and

- (i) ξ subter ζ (ξ = object; ζ = class).
 (ii) ξ sub ζ (ξ = class; ζ = class).

Relation I is that relation in which an object stands to a concept when the concept maps that object onto the True. Julius Caesar falls under the concept () is now dead, for example. Relation (i) is the extensional form of I, holding between an object and a class rather than between an object and a concept. "x subter α " would today be written as " $x \in \alpha$ ". Relation II obtains between like-levelled concepts, e.g., when one is a mark of the other. A mark, ψ , of a concept, θ , is best²³ construed as a concept under which an object must fall if that object is to fall under the concept θ . The concept θ will then be subordinate to the concept ψ , (cf. BuG.201-2). In the symbolism current today, if $(x)(\theta x \supset \psi x)$ then θ is subordinate to ψ . Relation (ii) is the extensional parallel of II: if a is the class of objects which fall under the concept θ , and b is the class of objects falling under ψ , then "a sub b"

can be expanded to read " $(x)(x \in a \supset x \in b)$ ". Relation III is one of the most important tools in the implementation of Frege's logistic programme. The relation is that between a concept, \emptyset , and another concept, Ω , such that \emptyset is of level-n, Ω is of level-n+1, and " Ω " signifies a property possessed by \emptyset . Cardinality and existence, for example, are concepts of second-level within which fall other first-level concepts. "The fundamental part of my results" wrote Frege in the Introduction to the Grundgesetze, "is expressed ... by saying that assignment of a number involves an assertion about a concept; upon this my present work is based", (Gg.ix). And so, to say that there are three coins in the fountain is to assert of the concept

() is a coin in the fountain

that it has this property (falls within this second-level concept): has three instances. And to assert that there are no square circles is to assert of the concept

(^o) is a square circle

that the number of objects falling under it = 0.

Brief though they are, these remarks enable us to see that certain of the foregoing objections are without substance. Kerry's objection, for example, that a concept can be an object, the object referred to by a nominalised predicate expression occurring as the subject of a sentence such as

(10) Drunkenness is undesirable,

is now seen to involve a confusion of logical type. For if the subject term is to be construed as referring to a concept, then the sentence

must be construed as possessing the logical form of Relation III above.

In which case (10) may be expanded, albeit somewhat picturesquely, to read: "() is drunk is a concept which ought to have no instances".

And here the unsaturatedness of the subject concept is made manifest, the temptation to construe "drunkenness" as the name of an object is destroyed, and

() is a concept which ought to have no instances is clearly a second-level concept which takes first-level concepts as its arguments. If, on the other hand, the term "drunkenness" in (10) is taken to refer to an abstract object, then () is undesirable must be construed as a first-level concept. In other words, sentence (10) may be construed as an example either of Relation III, or of Relation I. And Kerry's objection is only plausible when these two relations are confused one with the other. And parenthetically one might mention here the obvious relevance of these considerations for Ramsey's famous puzzle; for there is no reason why we should not construe such a sentence as

(11) Socrates is wise

as being either an example of Relation I, in which case the object Socrates is said to fall under the first-level concept () is wise; or as an example of Relation III, in which case the first-level concept:

() is wise

is said to fall within the second-level concept:

What Socrates is ()

Here a simple monogrammatical union will yield the sentence:

What Socrates is is wise,

which is just another way of saying that Socrates is wise. But this by no means tends to show, as Ramsey thought, that there is no difference between objects and concepts. For, to repeat, Relation III is not a relation between an object and a concept, but between two concepts.

It was suggested above that Frege's struggles with the concept could simply be avoided by formulating the theory in the formal and not the material mode. If this were done, it was suggested, it would be nonsense to say that "the concept horse is not a concept"; but what this remark is trying to say could be formulated thus: "the predicate 'is a horse'" is not a predicate; and this is certainly true. Now this objection is one which will have considerable force for semanticists and logicians who, following Carnap and Tarski, find the distinction between object-language and meta-language(s) clear, helpful and in some sense inescapable. Frege, however, like Wittgenstein, made no radical distinction between talking about extra linguistic entities and talk about languages. They did not, of course, confuse talking about the weather with talking about "the weather". But this involved merely a difference in subject matter, and not a radical difference in the type of language being employed. Both philosophers, moreover, denied that ultimately any problem could be solved by appeal to a meta-language which possessed, say, a richer vocabulary than, or different semantic rules from, the object-language under discussion. In the

case before us, i.e. the nature of predication, whatever problems arise over the predicate "() is a horse" will also arise over the meta-language predicate "() is a predicate". (Does it denote a concept? Is it defined for all objects? Is it unsaturated? etc.)

The problem confronting Frege concerns the nature of predication; if the meta-language contains predication then merely stating everything in the formal mode will not solve the problem, for statements in the formal mode will manifest the phenomenon to be explained just as much as those in the material mode. An infinite hierarchy of meta-languages would be a precondition of a solution; and this is impossible. And if, on the other hand, the meta-language did not contain predication (though it is difficult to know what this would involve), then, again, its explanatory power would be nullified. As van Heijenoort has shown,²⁴ Frege did not follow Boole and De Morgan in restricting his concept-script to an arbitrary universe of discourse: "The universe of discourse comprehends only what we agree to consider at a certain time, in a certain context. For Frege it cannot be a question of changing universes. One could not even say that he restricts himself to one universe ... Frege's universe consists of all that there is, and it is fixed." But then expressions are just as much objects in the world, objects possessing properties and entering into relations, as are tables and chairs. To say that "() is drunk" is a predicate is to assert of the object (which happens to be an expression): "() is drunk" that it falls under the concept:

() is a predicate.

If our objective is to understand the relations which obtain between objects and concepts, then an appeal to a hierarchy of languages will be of no avail. Frege would have agreed with Wittgenstein that "the calculus is, as it were, autonomous--Language must speak for itself" (PG.63).

Now, finally, it was objected that to ask what is the sense or the reference of the expression "() is drunk" is a futile exercise; for this expression is not an expression of English, and certainly does not occur in the sentence "Julius Caesar is drunk". Now the easy way out here is to say that normally people have no need to refer to predicates in isolation, and so natural languages tend to lack any clear conventional way of doing this. We are free, therefore, to introduce our own conventions here; and the convention adopted has the built-in heuristic device that it indicates the place where a name can be inserted so as to yield a sentence. But this is too easy to have any explanatory power. More light is shed on the nature of predicate expressions and predication by the following suggestions by Geach, which, though not based on any explicit pronouncement of Frege, are entirely Fregean in character. Geach argues that a predicate is, strictly speaking, not an expression which can be displayed by itself within quotes, but rather "a common property of expressions":

... when I say that "Booth shot Lincoln" and "Booth shot Booth" contain the common predicate "Booth shot--", I do not mean that the expression last quoted occurs in both sentences; plainly it does not, for neither

sentence contains a dash! What I mean is that the two sentences have the common property of being related in the same way to the expression "Booth shot--"; viz: each of them is obtained by substituting a uniquely referring name for the dash in this expression; and this common property is the common predicate. ²⁵

This last statement is rather obscure. Perhaps we can modify it somewhat as follows, while still keeping hold of Geach's central insight.

The two sentences "Julius Caesar is drunk" and "Cleopatra is drunk" have something in common. But this is not just the presence of the two words "is drunk", but the property of being composed of a singular term immediately followed by "is drunk". (This property is not, for example, possessed by the sentences "There is, drunk or sober, no man can beat an Irishman" and "Why is drunk a bad thing to be?"). We can say a predicate is an expression which maps singular terms onto sentences; and as such it is uncontentious that it carry with it an indication of the place where the singular term is to be placed.

There is much more to be said about the nature of predicates and their "unsaturatedness", but this must be postponed for the moment. ²⁶

(6) Reference continued.

We must now attempt an answer to the question: Why ought predicates

to be construed as a species of referring expression?

Frege's procedure for assigning references to sentences is, as we have seen, to look for that which remains invariant when co-referential terms are interchanged. This--or at least, one such--invariant turns out to be the truth-value of the sentence. The justification of this procedure is the basic principle for the determination of complex reference (PR1) that the reference of a complex complete name is determined by the references of its component parts.

Now this principle works well when the component parts whose reference is in question are singular terms. So it is that Frege can claim with great plausibility that the reason that the sentence "Odysseus was set ashore at Ithaca while sound asleep" is without a reference (a truth-value) is that at least one of its parts ("Odysseus") is without reference, (SuB.32). Of course the sentence is by no means meaningless or useless; it expresses an intelligible sense. Frege then asks why it is that the sense is not enough for us; and he answers: "Because, and to the extent that, we are concerned with ... truth-value. It is the striving for truth that drives us always from the sense to the reference" (SuB.33). Now if "reference" is to mean for predicates even approximately what it means for singular terms, then there must be some property which a predicate may possess, and the absence of which will deprive any sentence in which it occurs of a determinate truth-value. But it is difficult to conceive what this property might be, over and above the possession by the predicate of

an unambiguous sense. To the question: What would it be like for a predicate to possess a sense but lack a reference? there seems intuitively to be no answer. For certainly it cannot be replied that a predicate's reference is the conceptual extension: empty concepts can with perfect propriety be used in the most rigorous of scientific languages. Frege himself insisted on this very point. In the posthumously published fragment "Ausführungen über Sinn und Bedeutung", where, having argued that proper names which do not refer to anything must be banished from a scientific language, he writes:

Concept words without a reference must also be rejected though this does not exile those that are self-contradictory--for a concept can legitimately be quite empty--but, rather, those whose borders are vague. (ASB. in Nach.133)

Concepts whose borders are vague, Frege maintains, are without reference. This, then is the work expected of the notion reference as it applies to predicates.

It is beginning to emerge that Frege has not one, but two notions of reference. These notions hang together so well in the case of singular terms that they are hard to distinguish in this context. In the case of predicates, however, they are not only distinguishable, they are difficult to reconcile! One notion is this: the reference of an expression is that extra-linguistic entity with which the expression has been correlated or which it picks out. The other notion of reference is that it is a property which an expression must possess if that expression is to be truth-valuable (to coin a phrase). By truth-valuable, I

mean such that it either possesses a truth-value, or is capable of being used (and not just mentioned) in a sentence which possesses a truth-value. We have seen that vacuous singular terms are not truth-valuable. Indeed being truth-valuable in the case of proper names just is possessing reference in the other sense, i.e. "being directed to an object". Precisely what is implausible in Frege's theory of predicate reference is the idea that there is something to which predicates refer, and not that there should be some property which they may possess and in virtue of which they are truth-valuable. Frege never clearly distinguished these two notions either for proper names (which is understandable) or for predicates (which is puzzling). He wrote as though being truth-valuable was a clear indication that an expression was correlated with some extra-linguistic entity. We shall have to see if these two strands in Frege's thought are related in the way Frege obviously believed them to be, and if not, whether either is worth salvaging.

This is how Frege specifies the property which a predicate, or indeed any functional sign, must possess if it is to be truth-valuable:

A name of a first-level function of one argument has reference ... if the proper name that results from the filling of the argument place of this function-name always has reference, just as long as the name substituted has reference. (Gg.I,46).

Or, again,

It must be certain, for every object, whether or not it falls under the concept; and any concept-word that fails to meet this condition of

referentiality is referenceless. (ASB. in Nach.133).

This doctrine is very far from intuitively obvious. How is this notion of a concept's having sharp borders related to the more usual notion of reference? It is, I believe, possible to show that these two notions are linked by an important analogy which Frege can take credit for discovering. It is to be doubted, however, that this analogy is strong enough to justify extension of the term reference so as to cover predicates and other incomplete expressions.

As we saw earlier, Frege's notion of a concept is introduced and explained via the operation of removing from an assertive sentence one or more occurrences of a singular term. In this way the notion of a predicate's being true of an object is explicated in terms of the truth of the sentence which results from the completion of the predicate by any name of that object. Sentences refer (if they refer) to their truth-values; and Frege never tires of insisting that there are two and only two of these; tertium non datur. Sentences which refer neither to the True nor to the False do not, therefore, refer at all. But now, if concepts are allowed which do not meet the condition of sharpness, this whole doctrine is threatened. For if a given concept, F() is undefined for an object, a, then the sentence "F(a)" will not have determinate truth conditions and will not succeed in referring. This state of affairs directly threatens the principle of reference PR1, whereby the reference of a complex expression is a function of the references of its component expressions. For that the function-name "F()" has both a sense and a reference is proved by there being

just one true or false sentence in which an object is said to satisfy it. If, say, "F(b)" is true (or false), and the proper name "a" has a reference, then "F(a)" must also have a reference. And so predicates must be everywhere defined if they are anywhere defined, (cf. Furth.30). The analogy, mentioned above, is that a predicate which is everywhere defined (if it is anywhere defined) is truth-valuable in just the way in which a proper name is truth-valuable if it is "directed towards an object". That predicates must have sharp boundaries is an especially important demand for a concept-script which allows of universal quantification; if the predicate "F()" is undefined for just one object, a, then the universal statement " $(x) F(x)$ " will also be without a determinate truth-value.

Is the subsumption of predicates under the category of referring expressions, then, just a peculiar demand of the concept-script, with little or no relevance to everyday thought and language? One thing is clear: the vast majority, perhaps even all, of the predicates employed in the language of common discourse lack the absolutely sharp limits which Frege demands.

Throughout Frege's works there are general and more or less dogmatic assertions of the need for all concept-words to be defined for all objects as arguments. There is, however, so far as I know, only one place at which he subjects this doctrine to the scrutiny which it obviously deserves. The examination occurs in the Grundgesetze (§§ 62-5), and it is fair to say that the arguments advanced by Frege in favour of his conjecture are almost entirely negligible.

Frege approaches the issue via the suggestion that the range of function-names might be restricted, so that functions would not need to be defined for all objects. The function

$$(12) \quad () + () = 1$$

for example, need not possess a value for the arguments: the sun and the moon. One could surely restrict substitution in (12) to the natural numbers. To this suggestion Frege has a number of objections. In the first place he argues that before substitution could be thus restricted, we should first have to be in possession of a watertight definition of the concept number: "We may indeed specify that only numbers can stand in our relation ... But with that would have to go a complete definition of the word 'number', and that is just what is most lacking". Basically Frege's objection is not that such a definition is theoretically impossible, but merely that it would be rather difficult to formulate:

If people would actually try to lay down laws that stopped the formation of such concept names as this [i.e. "The sun + the moon = 1"] which, though linguistically possible, are inadmissible, they would soon find the task exceedingly difficult, and probably impracticable. (Gg.62).

Frege's argument seems to be an argument from laziness: we ought to define all concepts for all objects because this is less of a task than formulating the restrictions that would otherwise be necessary. But this is false. It is Frege who owes us an explanation of what the sense and the reference of such an inappropriately completed

function-name is. Of course it is possible to stipulate ad hoc what Quine calls a "don't care" value for inappropriate substitutions. It is not generally realized, however, that this stipulation must accord with and be based upon the prior and clear distinction between what is and what is not an appropriate substitution. In the example we have been using, before it can be stipulated that for non-numerical substitutions the function (12) is to have the value the False, or Quine, or the null set, or whatever, we must be in possession of a clear distinction between what is and what is not a number. And if such a distinction is already available then there is no reason why, pace Frege, it cannot be employed to rule out non-numerical substitutions altogether. Frege's weak argument to the effect that formulating restrictions on substitution instances is an "exceedingly difficult task" is one which seems to have gained wide acceptance. Geach, for example, writes as follows:

"What is $(\langle + 2 \rangle$ " [where " \langle " refers to the Moon]; the answer is a matter of arbitrary stipulation; 'the only point of a rule to this effect is that there should be a rule'. ... For Frege, every complex designation must have a reference, if it is to be well formed; so we can deny a reference to designations like " $\langle + 2 \rangle$ " only if we are going to have formation rules that exclude them from our language. The framing of such rules in a watertight way is a much heavier business than stipulations which would supply a reference for this sort

of designation -- say, the stipulation that when the sign preceding and following the plus sign do not both stand for numbers [sic] the whole expression has the same reference as the sign preceding the plus sign, so that "(+ 2" would designate the Moon, and "2 + (" , the number 2. (Three Philosophers. p. 148; my italics).

Now such ad hoc stipulations are not so easy to produce as Geach seems to think. In the first place, under the token stipulation given in the last sentence of this passage, it would no longer be true that, eg.

$$(x) (y) (x + y = y + x).$$

And if this proposition is to be true, then we must have a rider to the effect that substitutions must be numerical; but the argument was designed to show that such restrictions are impracticable. But this is a small point. Much more important is Geach's remark (immediately preceding "[sic]") that stipulation is only necessary when non-numerical signs are substituted in an arithmetical function-name. Now, no stipulation is possible when the names substituted in "() + ()" are "2" and "3" respectively. For the value of "2 + 3" is 5. If I "stipulate" that it shall be 3 then I am simply making a mistake, and introducing an element of anarchy into arithmetic. Now if anarchy is to be avoided, there must be a fool-proof way of distinguishing those cases in which ad hoc stipulation is necessary, from those where it is not allowed. But if such a criterion is already available, there can be no objection to employing it to outlaw expressions of the former type, instead of forcing an interpretation on them.

To outlaw such expressions, as meaningless is much more in keeping with the way we normally use language; for an expression like " $(+2 = \text{Geach})$ " ought to turn out to be senseless on any account! If it is senseless then it cannot have a reference (a truth-value), and merely stipulating a reference does not thereby provide an expression with a sense.

In brief, then, there is no substance to this objection to restricting possible arguments of functions. Apart from anything else, this objection relies on the validity of a distinction which it is the very purpose of the argument to deny.

Frege has one more argument to show that any restriction on the domain of substitution is impossible. This argument is independent of the one examined above, and runs as follows:

Let us suppose for once that the concept number has been sharply defined; let it be laid down that italic letters are to indicate only numbers ... By a well known law of logic, the proposition

if a is a number and b is a number, then $a+b = b+a$ can be transformed into the proposition

if $a+b \neq b+a$ and a is a number, then b is not a number and here it is impossible to maintain the restriction to the domain of numbers. (Gg. 565).

The conclusion which Frege draws is that "The force of the situation works irresistably towards the breaking down of such restrictions". If the situation has any force, then it is considerably stronger than Frege realizes; for it works towards the destruction of all categorical boundaries whatsoever, including those which Frege would

wish to keep intact (eg. the distinction between objects and concepts). Consider, for example, the exactly parallel argument in which it is assumed that italic letters range over all objects unrestrictedly. It will then be true that

if a is an object and b is an object then a+b = b+a;

but by a well known law of logic, this can be contraposed to yield

if a+b ≠ b+a and a is an object, then b is not an object.

This ought to show that the categorical difference between objects and functions is untenable. But there is something radically wrong with a statement like "b is not an object": it breaks the syntactical rules under which we are working. It cannot "turn out" that b is not an object; "b" is a sign for an object. It is evident that something is here amiss; if we accept Frege's mode of argument, it will commit us to a type-free and absolutely homogeneous syntax and ontology. I do not know if this idea is intelligible, but it was certainly far from Frege's mind. Just what has gone wrong here Wittgenstein was one of the first to diagnose, in the Tractatus. He argued that paradox and nonsense results when formal concepts (object, function, number, concept) are treated as common or garden material concepts. If the concept object is employed to mark a syntactical type, then the putative predicate "() is an object" will not function like a material concept, say, "() is an elephant"; for the former will be true of everything that can meaningfully be substituted in it. Any attempt to apply the predicate "() is an object" to what is not an object will result not in a false sentence, but in a

syntactically mal-formed expression. Wittgenstein argued that in a consistent and perspicuous language, that something was an object, a function, a number etc. was shown in the symbolism employed, and could not be said. Frege adopted this view as far as concerns functions: that something is a function is shown by the incompleteness of the sign used to refer to it; but "() is a function" is an impossible sign combination. Frege, however, never extended this doctrine to include "() is an object", nor did he acknowledge the possibility that there might be other formal concepts. There is, I have suggested, no good reason why he should not do either of these things.

(7) Summary.

The gist of these observations is that there seem to be no good reasons for demanding, with Frege, that function-names and predicates have a determinate value for all objects as arguments. But, as this is a necessary and sufficient condition for their being said to refer, these considerations also tell against the doctrine that function-names are a species of referring expression.

The tenor of these remarks has not, however, been entirely negative; for the crucial importance of the notion of sense is beginning to emerge. Quine, who believes that attempts to restrict quantification are misconceived, being based on accidents of language rather than on any purely logical considerations, would represent the Fregean statement that, for all objects whatsoever,

$$\underline{a} + \underline{b} = \underline{b} + \underline{a}$$

(x)(y) (If x is a natural number, and y is a natural number, then $x + y = y + x$)²⁷

And Quine would say that the quantifiers here range over all objects, the necessary restriction being brought off internally, so to speak, by the antecedent of the conditional. And in the case in which x is Socrates, say, and y is the Moon, Quine would say we have a "don't care" case, in which the antecedent is simply false and so constitutes no exception to the generalization. But this is a little slick. Quine owes us an account of how a senseless sentence can possess a truth-value. And if the expression "If Socrates is a natural number and the Moon is a natural number, then Socrates plus the Moon is equal to the Moon plus Socrates" is not senseless, then Quine owes us an account of what its sense is. For, in Frege's words, a sign without a sense is "just an empty series of sounds"; and the act of assigning a truth-value to an expression does not thereby give it a sense.

Frege is not consistent on this issue; but in what, it could be argued, are his more profound moments, he tends to emphasize that possession of a clear sense is the sine qua non of an expression's being used in a language of any sort:

A proper name must at least have a sense ... otherwise it would merely be a series of empty sounds and wrongly called a name. A proper name stands for an object through the mediation of its sense and through this alone.

A concept-word must also have a sense ... and this applies, indeed, to every sign or complex of signs ...
(ASB. in Nach. 135)

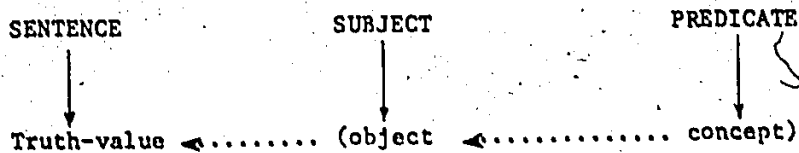
Now this line of thought runs counter to that which claims that we can stipulate a reference for a nonsensical expression, and thereby introduce the expression unexceptionably into a language. But before we can fully appreciate the relative merits of these two approaches, we must examine the notion of sense.

CHAPTER II

Functions and Sense

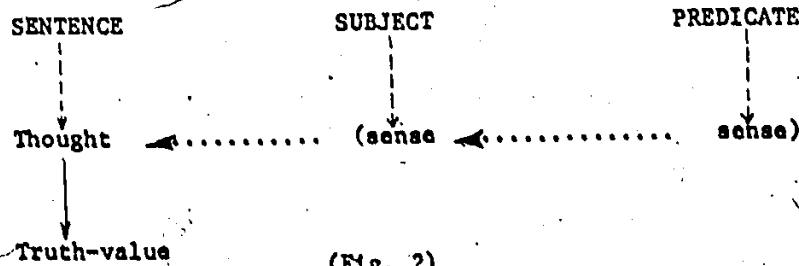
(1) Introduction.

The principle for the determination of complex reference examined above (PR1 for short), viz: that the reference of a complex expression is determined by the references of the component parts, both complete and incomplete, can be represented diagrammatically as follows:



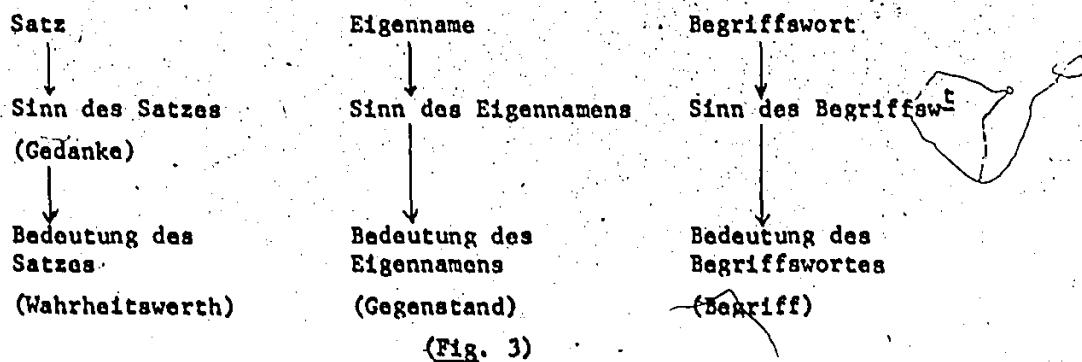
(Fig. 1)

PR1 is the principle upon which Frege primarily relies when he is examining scientific language and the semantics for the concept-script. Elsewhere in his writings, however, he seems to invoke a quite different principle for the determination of reference; we can call it PR2. PR2 is this: (a) the reference of any expression is determined by the sense of that expression; (b) the sense of a complex expression is determined by the senses of its component parts. PR2 can be represented as follows:



(Fig. 2)

Frege never clearly distinguished these two principles, and so never felt the need to reconcile them. In general, however, the emphasis was on PR1 when he talked about formal language and the concept-script, while he seemed to rely primarily on PR2 when examining ordinary modes of discourse. That Frege did not clearly distinguish these two principles is shown by the following diagram, taken from a letter which Frege wrote to Husserl (May 1891), and which is an incomplete amalgamation of Figs 1 and 2 above:



In Fig. 3 the reference of the sentence is determined vertically by the sense of the sentence, according to PR2(a). It is certain from many other passages in Frege's writings that he believed the reference of a sentence to be determined also by the references of the component expressions. Plainly the reference of complex expressions (both sentential and sub-sentential) is over-determined. In the previous chapter we advanced reasons for rejecting PR1, as either unilluminating (when it demands that predicates denote strange unsaturated entities) or as false, at least for natural languages, (when it demands that predicates and functions be defined for all objects). We must now examine PR2, and

ascertain whether or not it fares better than its predecessor.

It can be noted immediately that PR2 is, intuitively, not only clearer than PR1, it is more plausible as well. That the truth-value of a sentence, or the reference of a complex expression such as a definite description, is determined by the sense of that sentence or expression, and that this, in its turn, is determined by the senses of the words which go to make it up, these are theses which seem scarcely disputable. They have, however, been disputed. The next section is an attempt to outline some of the problems which face any prospective theory of sense.

(2) Some Problems

Frege begins the essay "Über Sinn und Bedeutung", in which the distinction between an expression's sense and its reference was first expounded, with the observation that "a = a" and "a = b" are obviously statements of differing cognitive value (Erkenntniswerte). The opening paragraphs of this essay constitute, in effect, a distinction between the cognitive-value and the truth-value of a sentence. Thus, if "a" and "b" are two different names for one and the same object, then both "a = a" and "a = b" will be true; and they will be true for the same reason: the two terms flanking the identity sign stand for the same thing. The first identity statement is, however trivially analytic, while the second "may contain very valuable extensions of our knowledge" (SuB.25). It seems that the notion of sense, as it is introduced by Frege, is to be an epistemological notion. The sense of a declarative sentence will

be, roughly, that which we understand when we understand the sentence. This is the interpretation which Dummett has recently given. "What can a model of sense be" he asks "but a model of what it is to grasp a sense?" (Frege. 293). "The notion of sense was required in the first place in order to explain how our sentences come to have the cognitive value which they have for us" (op.cit.240).

Early in the article under discussion, however, Frege introduces a slightly different notion of sense: "I should like to call the sense of a sign [that] wherein the mode of presentation (Art des Gegebenseins) is contained" (SuB.26). What is "presented" is, of course, the reference of the expression. Now, regardless of how justifiable an explanation of sense along such lines may ultimately be, this notion cannot be used to introduce the concept of sense in general, i.e. for expressions of any type; for it has not yet been established what is to count as the reference of expressions other than proper names, if anything at all. Because the notions of predicate- and sentence-reference are quite without intuitive, pre-theoretical content, such notions cannot be used to introduce a theory which will then, in turn, explain them.

Nor is the language of cognitive value free of difficulties. In the first place it only seems appropriate to talk about the cognitive value of a sentence; for only a sentence can be understood, can contain information, can express knowledge and so on. This is a point which Wittgenstein has expressed with rhetorical force as follows: "Doesn't understanding only start with a proposition, with a whole proposition? Can you understand half a proposition?" (PG.39). And yet Frege clearly

intends that the notion of sense should apply univocally to proper names and predicates as well as to sentences. (The possession by predicate expressions of either sense or reference is nowhere mentioned in SuR. Frege was obviously reserving treatment of this topic for a second article, "Ausführungen über Sinn und Bedeutung", which, however, he never published.)¹

Neither the account in terms of cognitive value, nor that in terms of the mode of presentation of a reference will suffice unamended to introduce and explain the general notion of the sense of an expression. For sub-sentential expressions do not possess cognitive value; and expressions other than proper names do not clearly possess reference.

There is another, more fundamental difficulty confronting any attempt to formulate a coherent theory of sense, however: it is strangely difficult to say what the sense of an expression is. "On hearing the assertion 'This sentence makes sense' you cannot really ask 'What sense?'" wrote Wittgenstein, "Just as on hearing the assertion 'This combination of words is a sentence' you cannot ask 'What sentence?'" (PG.51). And in an analogous way, the question "What is the sense of the word 'Aristotle?'" seems to defy explicit and precise answer. Some have thought this state of affairs evidence for the fact that there are no such things as senses. Russell wrote, in "On Denoting":

The difficulty in speaking about the meaning [sense] of a denoting complex may be stated thus: The moment we put the complex in a proposition, the proposition is about the denotation..."

And here I must quote Searle's delightfully graphic account of how this

is supposed to happen:

Imagine a game where marbles are dropped into bowls through pipes. This act is called referring. Pipes (senses) lead to bowls (references). It is a rule of the game that anything can be referred to. The difficulty is though that we cannot live up to this rule because we cannot refer to a pipe. Every time a marble drops into a pipe it goes through to the corresponding bowl.²

Now, as in fact Searle argues against Russell, this is not quite right. For we can refer to senses, and we do this in the way in which Frege indicated: by employing an expression of the form: the sense of the expression " ... ". It was just confusion on Russell's part which led him to deny this. But this does not clear up the difficulty, not mentioned by Searle, that we cannot say what the sense of an expression is. The closest we may approach to this is to say that the sense of a given expression E_1 is the same as the sense of another expression, E_2 . There seems to be an interesting analogy here with certain difficulties concerning Fregean concepts: concept-words refer, but we cannot stipulate what it is they refer to. I shall return to this point shortly.

In addition to saying that E_1 has the same sense as E_2 , we can also hint at or indicate what the sense of an expression is. "Aristotle" we might say to someone who did not understand our use of the word, "was a Stagirite philosopher taught by Plato". And in so saying we would give our hearer some idea of what the sense of the word "Aristotle" is. But it would be a grave mistake to construe such elucidations³ as precise stipulations of sense. For one thing, if "Aristotle" is to have

the same sense as one or more identifying definite descriptions of Aristotle, then such obviously contingent statements as "Aristotle was taught by Plato" or "Aristotle was born in Stagira" will turn out to be classically analytic: true solely in virtue of the meanings of the terms involved. Russell and Bradley subscribed to versions of this peculiar theory; Frege did not.⁴ And yet there are those who would foist it upon Frege⁵, claiming that for him, as for Russell, a proper name is somehow "equivalent to" or "an abbreviation for" a definite description. This ascription is based solely on a footnote to "Über Sinn und Bedeutung", where Frege writes:

In the case of an actual proper name such as "Aristotle" opinions as to the sense may differ. It might, for instance, be taken to be the following: the pupil of Plato and the teacher of Alexander the Great. Anybody who does this will attach another sense to the sentence "Aristotle was born in Stagira" than will a man who takes as the sense of the name: the teacher of Alexander the Great who was born in Stagira. So long as the reference remains the same, such variations in sense may be tolerated. (SuB.27n).

It must be admitted that in this passage, confused and confusing though it is, Frege does seem to be giving the sense of a proper name by saying that it is some definite description. What Frege is actually doing, of course, is giving the reference of the name "Aristotle". It is Aristotle himself, and not the sense of the name "Aristotle", who was born in Stagira, was taught by Plato and so on. Frege is mistaken here;

and doubtless just this mistake was the origin of Russell's pseudo-problem about referring to the reference of an expression when what was intended was the sense. But although Frege is confused here, he is not just confused; he has run afoul of a real problem: one cannot say what the sense of a proper name is. What Frege is actually doing is indicating or elucidating the sense of the name "Aristotle" by giving the reference. In the quoted footnote he mistakes this for giving the sense. Generally, however, Frege was quite clear about the difference. If sense is the "mode of presentation" of a referring expression, then it is impossible for a proper name to have the same sense as a definite description, precisely because these two types of expression present their references in quite different ways: a name is correlated with an object, while a description is true of it. Frege is quite adamant that from sameness of reference, sameness of sense can never be inferred.

(3) Sense in general.

As a start towards the construction of a Fregean theory of sense, we can, somewhat negatively, begin by saying what sense isn't; i.e. by distinguishing sense from other ingredients in what might, with intentional vagueness, be called the "meaning" of an expression. The categorization of semantically significant factors present in any language (of reasonable complexity) capable of expressing truths is effected, by Frege, via the following five key concepts: sense,

reference, colouration, indication and force.

(i) The first thesis concerning senses is that they are objective. And this alone is sufficient to distinguish the notion of sense from what Frege calls "colouration" (Färbung, Beleuchtung). Frege distinguishes two types of colouration; one consists in the subjective images, memories and associations which a given expression may on a particular occasion cause to come before someone's mind:

The ... sense of a sign is to be distinguished from the associated idea ... The same sense is not always connected, even in the same man, with the same idea. The idea is subjective: one man's idea is not that of another ... This constitutes the essential difference between the idea and the sign's sense. (SuB.29).

To take an extreme example, the name "Venice" may remind me of my last year's holiday. Only one who thought the sense of a sign is something mental and subjective would be tempted to claim that such imagistic associations were a part of an expression's meaning.

A second phenomenon which Frege includes within the notion of an expression's colouration is what today would be called its connotation. Although this concept is rather difficult to define exactly (Dummett has called it "a rag-bag of a concept") it is easily recognized on an intuitive level. It would generally be admitted that from the following pairs of terms either might be used in place of the other without appreciable change in meaning: "Dead" or "deceased", "old" or "aged", "bastard progeny" or "illegitimate offspring", "perspiring" or "sweating". What little difference in meaning there may

be between the members of these pairs, it is not enough to alter the statement which one might make in employing one rather than the other. The differences are rather in nuance, suggestiveness or sound. Frege is wrong to class this as a subjective phenomenon: the connotations of some terms are as objective and public as their senses. In fact, however, Frege does not need to call them subjective, because he has a more adequate and sharper criterion for distinguishing sense from colouration/connotation: the sense of an expression is that part of its meaning which is not its force, but which is what we earlier called "truth-valuable". In other words, all differences between two expressions which are too slight, too subjective, too vague to affect the truth-value of any sentence in which they occur are to be excluded from the sense of the expressions, and lumped under the term "colouration".

(ii) The notion of force (Kraft) applies only to sentences and sentential phenomena, and about this notion I shall have a great deal more to say in Chapter III. The remarks in the present section are intended merely as preliminary and provisional means of distinguishing between the sense and the force of an expression. Roughly speaking, then, it is in terms of the force with which it is used or uttered that we would distinguish between a sentence's being a question, an assertion, a promise, a command, the expression of a wish, or an example of a grammatical point. One and the same spoken sentence, say, "I promise to make you pay the money", might possess any of these forces; tone of voice and context of utterance would normally indicate which

force was intended. Sense, on the other hand is a notion which attaches to sentences per se, and the sense of the sentence "I promise to make you pay the money" will remain constant throughout such changes in force. This is because the force of a sentence is something which is only present when the sentence is used, uttered or written on a particular occasion. To use a well worn but still serviceable distinction: the sense of an expression is the business of semantics, its force the business of pragmatics.⁶

(iii) The term "andeuten", usually translated as "indicate", applies to the way in which the variables of logic and arithmetic function. Variables are introduced in order to achieve generality, and in the equation " $a+b = b+a$ " the letters " a " and " b " stand indeterminately for any natural numbers. Frege introduced the technical term "andeuten" in order to combat the prevalent belief that variables refer to (bedeuten) a strange entity called a variable number.⁷ He was combatting the view that the two signs " 2 " and " n^2 " function in the same way by denoting numbers, only that while " 2 " denoted a determinate number, " n^2 " denoted an indeterminate number. "We cannot say that " n " denotes (bedeutet) one indefinite number; but we can say that it indicates (andeutet) numbers indefinitely." (Funk. 659). Frege points out that the indefiniteness attaches adverbially to the verb "to indicate" and not adjectivally to the number(s) indicated. Signs which, like variables, only indicate but do not refer have, according to Frege, neither reference or sense, outside a sentential context. "We sometimes use the expression 'the number n ', eg. 'If the number n

is even then $\cos \pi = 1$ '. Here only the whole has a sense" (Funk.659).

"Now what does ' $x^2 + 3x$ ' denote? Strictly speaking, nothing at all; for the letter ' x ' only indicates numbers and does not refer to them".

(Funk.663). Clearly Frege construed variables as syncategoremata, incomplete symbols which are only meaningful when at work within a sentence; and in a sentence they only indicate and do not refer to their values.

(iv) These notions of colouration, force and indication, as well as the notion of sense, can all be roughly classed as aspects of, or distinctions within the notion of an expression's meaning. We come now to the notion of reference, and as Dummett has observed with considerable insight: "Reference, as Frege understood it, is not an ingredient in meaning at all: someone who does not know the reference of an expression does not thereby show that he does not understand, or only partially understands, the expression", (Frege.84). Dummett then adds that, for Frege, reference "is a notion required in a theory of meaning--in the general account of how language functions--just as the notion of truth is so required: but the reference of a term is no more part of what is ordinarily understood as its meaning than the truth-value of a sentence is." (loc.cit). This contention is surely incontestable in the case of sentences and their truth-values: I can fully and perfectly understand the sentence "The Empire State Building has 150 storeys", even though I am in total ignorance of its truth or falsity. But Dummett's contention does not so obviously apply to proper names.

(4) Proper Names.

Can we say that I fully understand the name "Julius Caesar" even if I do not know to whom or to what it refers? Here we must tread carefully; for although I believe the matter is quite simple, the history of recent philosophy is not without examples of those who have gone adrift over just this point, the most spectacular being Russell. Now, Russell notwithstanding, the verb "to know" as it is used in the phrase "to know the reference of an expression" cannot be taken to mean to be acquainted with. For then the ridiculous conclusion would be that, being unacquainted with Julius Caesar, I could not understand such a sentence as "Julius Caesar is now dead". And, again Russell notwithstanding, the person Julius Caesar is no part to the meaning of the expression "Julius Caesar", nor of any expression in which this occurs. For as Wittgenstein has shown, if he were then it would be the case that part of the meaning of the sentence "Julius Caesar is dead" used to be bald, crossed the Rubicon and is buried in Rome. But meanings do not cross rivers or lose their hair. Frege makes just this point in an unpublished letter to Jourdain (January 1914):

When we find the same word, say "Etna", in two sentences we realize that there is also something common to the two Thoughts [thus expressed], something which corresponds to this word. ... Now this part of the Thought which corresponds to the name "Etna" cannot be the reference of this name. For if it were, of course, every lump of solidified lava that

is part of Etna would also be part of the Thought that Etna is higher than Vesuvius. But it seems absurd to me that lumps of lava, and indeed other such things of which I have no knowledge, should be parts of a Thought.⁸

The object referred to by a proper name is no part of the meaning or sense of that name.

On the other hand, however, it must be admitted that there is a strong temptation to claim that in order to understand the proper name "Julius Caesar"--or, better, in order to understand sentences in which that name is used--one must know who Julius Caesar is. This temptation has its origins in the intuition that if "Julius Caesar" occupies the subject position in a sentence, that sentence will be about Julius Caesar. But if one does not know who Julius Caesar is, but uses such a sentence, then, literally, one will not know what one is talking about. And so, the argument goes, the answer to the question: Can one be said to understand the sentence "Julius Caesar is dead" if one does not know who Julius Caesar is? ought to be "No". And the result of this negative answer is the attempt to enrich the notion of the sense of a proper name, so that the sense specifies who or what its reference is, and any one who understands the sense thereby knows who or what is referred to. This, as we saw above, was the approach adopted at one point by Frege: someone who knows the sense of the name "Aristotle" thereby knows that Aristotle was the pupil of Plato and the teacher of Alexander the Great, or some such uniquely identifying description.

But this cannot be right. For if we say that the sense of the name "Aristotle" is: the pupil of Plato...etc., then we are identifying the sense with the reference of the name. But if we adopt the alternative formulation, whereby the sense of the name "Aristotle" is the same as the sense of the expression "the pupil of Plato ... etc." then this too is false. These two expressions do not have the same sense. Moreover I can well understand sentences in which the name "Aristotle" occurs without knowing the truth of just this definite description; and if the requirements are weakened so that any true definite description will here suffice, then the requirements no longer correspond to the intuitions which prompted them. For knowing just one obscure fact which happens to be true of Aristotle and nobody else hardly constitutes "knowing who Aristotle is". And, finally, there are the difficulties noted above which concern the truth conditions for sentences containing the word "Aristotle". Searle has expressed these difficulties succinctly as follows:

If it is asserted that a proper name is a kind of shorthand description then we ought to be able to present the description in place of the proper name. But how are we to proceed with this? If we try to present a complete description of the object as the sense of the proper name, odd consequences would ensue, e.g. that any true statement about the object using the name as subject would be analytic, any false statement self-contradictory, that the meaning of the name (and perhaps the identity of the object) would change every time there was a change at all in the object, that the name have different meanings for different people, etc. (Proper Names, 169)

I can see no way to overcome the difficulties engendered by a negative answer to the question: Can one be said to know the sense of the sentence "Julius Caesar is dead" if one does not know who Julius Caesar is? And as the temptation to answer no is not irresistible, we ought to examine the alternative.

In the first place an affirmative answer is not without its intuitive appeal. We would not say, for example, that a professor of Roman political history had a better understanding of the sentence "Julius Caesar is dead" than a native speaker of English who knew nothing more about Julius Caesar than that he was a Roman general. Knowledge of facts about Julius Caesar is, in other words, of strictly limited relevance to the understanding of sentences which contain his name. Is it, then, of any relevance at all? The following considerations might seem to show that it is not. The opening sentence of D.H. Lawrence's short story "The Fox" is this: "The two girls were usually known by their surnames, Banford and March".⁹ Now, native speakers of English do not react to this sentence with puzzlement; they have no difficulty in understanding its sense. And yet the sum-total of our knowledge of the two girls is that they have the names "Banford" and "March", which (as Searle, Strawson, Kripke and others have pointed out) is second only to knowing nothing about them whatsoever. This lack of knowledge has, however, no effect upon our understanding of the meaning of the sentence, which neither increases nor decreases as we read the rest of the story. This would seem to indicate that as long as we understand that a proper name is a proper name, we can understand any sentence in which

it occurs. In response to the assertion "Clara is arriving early to-night" I can reply with perfect propriety that I understand exactly what this sentence means, though I do not happen to know who Clara is. Indeed my grasp of the meaning of the sentence is manifest, and not belied, by my saying in response "I don't know who Clara is".

Now normally this response would be in order. But suppose that, in fact, "Clara" is the name of a hurricane: It is then at least arguably the case that I have not understood the meaning of the sentence; that I have positively misunderstood it. (Generally speaking we can say that if a person has failed to understand a sentence his reaction will be one of puzzlement, whereas if he has misunderstood it his reaction will be an inappropriate one). And if I respond to the news of the imminent arrival of Clara (the hurricane) by saying "Oh good. Does she play bridge?" and by setting an extra place for dinner, then this is *prima facie* evidence for the conclusion that I have misunderstood the sentence in question.

If these observations are to the point, then a necessary condition of a person's being truly said to understand a proper name (or, better: to understand sentences containing that proper name) is that he knows what sort or type of object the name refers to.¹⁰ The considerations adduced in the preceding paragraphs are, of course, inconclusive. They are intended to show merely that the conclusion reached is neither implausible nor recondite, conforming as it does to the things we would normally say about someone's use and understanding of proper names. Nonetheless, such arguments from plausibility are here secondary to more

directly philosophical considerations. For to accept the conclusion that the sense of a proper name is its referring to a determinate object of a given sort (or at least purporting to do so) is both to avoid the difficulties inherent in the barren Russellian notion of a logically proper name, and the difficulties inherent in the enriched notion of a proper name as shorthand for one or more definite descriptions. This leaves open, however, the possibility of accepting two important theses concerning proper names: we are free to accept with Frege that the sense of a proper name is usually indicated (but not stated) by identifying the reference, usually by employing a definite description; and with Strawson that it is a necessary condition for the possibility of our employing proper names at all that there be "a backdrop of uniquely individuating descriptions" which pick out the object referred to by a name, independently of its being referred to by that name.¹¹ Neither of these theses entail that such descriptions are part of the sense of the proper name.

The force of Frege's analogy between a sentence and its truth-value, and a proper name and its reference can now be elucidated. Just as one does not have to know whether a sentence is true or false in order to be able to understand it, so one does not need to know what object a name denotes in order to understand the sense of the name. And yet, (1) before one can be said to understand a sentence one must recognise that it is an expression which purports to be either-true-or-false (though one need not know which), so before one can be said to understand a proper name, one must recognise that it is an expression which purports

to refer to an object of a certain sort (though one need not know to which object). (ii) the concepts of truth-value and name reference are linked in that a necessary (but not sufficient) condition for a sentence's being either true or false is that any proper name used therein should denote an object. As Frege observed, anyone who took a sentence of the form "N.N is such and such" to be true (or false) would thereby be according a reference to the name "N.N."

To go this far with Frege does not, however, commit us to the further conclusion that the relation between a sentence and its truth-value is the same as that between a name and the object to which it refers. For while names are correlated conventionally with their references, sentences are manifestly not correlated conventionally with their truth-values.

The sense of a proper name, then, is that it purports to refer to a determinate object of a given sort with which it has been conventionally correlated. I understand the sense of "Julius Caesar", say, when I know this: "Julius Caesar" is a proper name of Julius Caesar; and Julius Caesar is a man. On this analysis, of course, it turns out that "Julius Caesar is a man" is analytic; but this is a form of essentialism to which I have no objections. It will likewise be analytic that any given object falls under the sortal concept of largest scope under which it does in fact fall: Thus that 2 is a number, that love is an emotion, pain a sensation, Clara a hurricane and Aristotle a man, all these will be analytic. (It will also be impossible that Aristotle was a hurricane or 2 an emotion). This incorporation of a sortal

specification into the sense of a proper name has, as we shall see, very great advantages for the theory of functions.

Now it might be argued that this move, which allows the analyticity of the statement that Aristotle is a man, say, creates more difficulties than it solves. For if it is analytic that Aristotle is a man, and it follows that, therefore, something is a man, then this too must be analytic. But how, the objection runs, can it be analytic that there exists at least one man? Now this unacceptable consequence is avoided by denying that the move from

Aristotle is a man

to

Something is a man

is valid. And we are, in fact, committed to denying this by the foregoing theory of sense; for it was maintained that the sense of a proper name is that which one understands when one is able to use it correctly. And it is no part of one's understanding of a proper name that its purported bearer actually exists. It is part of the sense of such names as "Aristotle", "Pegasus", "Odysseus" etc. that they purport to refer, but not that they actually succeed in referring. To deny this would commit us to one of two unacceptable consequences: either it would have to be denied that there can be an intelligible sentence which contains a vacuous proper name; or it would have to be asserted that all singular terms are successfully referential. Although it might be possible to lay down such rules for the manipulation of signs within a calculus, both alternatives are simply false with respect to the understanding we have of

natural language. (The importance of the distinction between purported reference and successful reference is further discussed in Chapter IV, section (3) below). But in this case the move from (1) to (2) is as invalid, without the further premiss that Aristotle exists, as is the move from

Pegasus was a horse

to

Something was a horse.

(5) Sentences and Indirect Discourse.

The sense which an indicative sentence expresses Frege calls a Thought (Gedanke), and his most extended treatment of this notion is in the article (1918) of the same name. As Chapter IV of the present thesis comprises a detailed and critical examination of the Fregean notion of Thought, the present section will constitute merely the briefest introduction. The following, then, are some of the most important theses concerning Thoughts.

(1) Thoughts are objective. "By a Thought I understand not the subjective performance of thinking but its objective content, which is capable of being the common property of several thinkers" (SuB. 32n). Frege distinguishes the Thought, which is the truth-valuable component of a sentence's meaning, from the subjective contents of consciousness like ideas sensations and images. "We are not the bearers of Thoughts as we are the bearers of ideas" (Ged. 74). The aim of the sciences is to establish which Thoughts are true and which false; and Frege argues that

such sciences as mathematics and physics, say, deal with an objective and impersonal realm of facts, and are not merely branches of psychology dealing with peoples' ideas about mathematics and physics respectively.

But not only are Thoughts, for Frege, objective entities in that their existence is not dependent upon their being grasped or thought by any person; they are also objective in that their internal make-up may contain only that which is in theory truth-valuable: "I call a Thought something for which the question of truth arises" (Ged. 60). In other words, the difference in meaning between the two sentences

Alas, John Smith has lost a leg

and

John Smith has lost a leg

do not concern the sense, the Thought expressed by the sentences.¹²

"What is called mood, fragrance, illumination in a poem, what is portrayed by cadence and rhythm, does not belong to the Thought" (Ged. 63). Consequently Frege distinguishes between the content (Inhalt) of a sentence and the Thought (Gedanke) which it expresses: the content may include the Thought, along with other elements of the sentence's overall meaning, for instance its connotations, emotive elements, presuppositions, emphases and the like. "Thus the contents of a sentence often go beyond the Thought expressed by it" (Ged. 63).

(ii) Thoughts are complete. If the content of a sentence may extend further than the Thought expressed by it, it may also happen, as Frege points out, that a sentence possesses an intelligible content but yet

expresses no Thought. Sentences which contain irreducible indexical and token-reflexive elements will be of this sort, as will those which contain a tensed verb without a precise time indication, or those which are too vague to possess a determinate truth-value.

The sentence

I am drunk today

for example, does not express a Fregean Thought. In the first place, the personal pronoun only picks out a person when the sentence is considered along with its context of utterance. In everyday speech it is usually clear from the context of utterance who is the subject of an assertion expressed by a sentence whose grammatical subject is a personal pronoun. But in such a case it is not the sentence, but the sentence along with its contextual determinants, which expresses the Thought. Secondly, the above sentence fails to express a Thought because it contains no indication of the time of utterance; though again this is something that is usually supplied by the context in which it is spoken or written. "If a time indication is needed by the present tense one must know when the sentence was uttered in order to apprehend the Thought correctly. Therefore the time of utterance is part of the expression of the Thought" (Ged.64). Now if such indeterminacies are resolved, not by the context, but by explicit elements within the sentence, then the sentence itself will express a Thought. In the present case the sentence would have to be of the form:

I (so and so) am drunk today (the such and such, at time t).

But if all indeterminacies of sense are thus resolved internally, the Thought expressed by this sentence will possess a truth-value which may

never change, regardless of who utters the sentence, or when, or into which language the sentence is translated, or what change of sense the words in it may undergo. Moreover the Thought expressed by this sentence would be true (or false) regardless of whether anyone ever uttered it, or entertained it. It is such considerations as these which lead Frege to assert that Thoughts are eternal and immutable, and that

(iii) Thoughts are the primary bearers of truth-values. "When we say a sentence is true, we really mean its sense is". For the moment we can ignore the more contentious aspects of this doctrine, and concentrate upon its plausible elements. Thus it is that, speaking common sensically, truth (or falsity) is ascribed to what a person says, the statements that he makes, and not to the words or sentences which he uses. In certain circumstances, for instance, a person with a knowledge of a number of European languages would be able to express the Thought that it is raining by saying "Il pleut", "Il piove", "Es regnet" or "It is raining". The Thought (at least when completed by appropriate time and place indicators) would be the same in each case; and it would presumably be the speaker's intention to assert this Thought (rather than, say, utter just these words) in the belief that in so doing, what he was asserting was true. Although there are limits to the amount of work that can be demanded of this vehicle/content model, the intuitions from which it springs are surely unexceptionable.

(iv) Thoughts are unassertive. In his early and middle period works Frege identified a Thought with the truth-valuable meaning of an

indicative sentence, and maintained that interrogative and imperative sentences expressed, not Thoughts, but questions and commands respectively. In the Logische Untersuchungen, however, he modified this doctrine so as to take account of the fact that certain questions can be answered either "Yes" or "No". (Frege calls such questions propositional-questions, Satzfragen). Such a question, Frege argues, must express a Thought -- a Thought which an affirmative answer asserts to be true, and a negative answer, false. It is the Thought itself which is either true or false; but the Thought itself does not say whether it is True or False either as expressed by an indicative sentence or by a propositional question. For if it did, a propositional question would contain within itself its own answer. The Thought itself must therefore be without assertive force (cf. GeD.62; Vern.145; GeF.34). As Wittgenstein wrote in the Tractatus, where his use of the term proposition (Satz) corresponds closely to Frege's use of the term "Thought": "It is quite impossible for a proposition to state that it itself is true" (4.442). Wittgenstein directed this remark at both Frege and Russell; but in the former case at least the criticism is misplaced. For Frege maintained that assertive force must be clearly distinguished from the Thought expressed by a sentence, and introduced the special sign "—" in order to indicate when assertive force was present. But while this force attaches to the Thought, it is, pace Wittgenstein, no part of the Thought. If it were, all propositional questions would be begged.

(v) Normal and indirect sense must be distinguished. The four theses

examined above concern what Frege called the normal sense (Gewöhnlichen Sinn) of an indicative sentence or propositional question. The two sentences "Julius Caesar is dead" and "Is Julius Caesar dead?" both, as they stand, express the same Thought, and the former refers to its truth-value. There are certain contexts and constrictions which, however, will alter this state of affairs: quotation, for example: "In direct quotation a sentence designates another sentence, and in indirect quotation a Thought" (Sub. 36).

If words are used in the ordinary way, what one intends to speak of is their reference. It can also happen, however, that one wishes to speak about the words themselves, or about their sense. This happens, for instance, when the words of another are (directly) quoted. One's words then first designate words of the other speaker, and only the latter have their normal reference.... In reported speech one talks about the sense of another person's remarks... In reported speech words are used indirectly or have their indirect reference. We distinguish accordingly the normal reference from the indirect reference of a word; and its normal from its indirect sense. The indirect reference of a word is accordingly its normal sense. (Sub. 28)

In other words, if I utter the sentence "Julius Caesar is dead" in a normal conversational situation, the words which I utter have their normal sense and reference: "Julius Caesar" refers to the man of that name, and the sentence as a whole expresses the Thought that Julius Caesar is dead, and refers to the True. But if I now say that the sentence "Julius Caesar is dead" occurs in the preceding sentence on this page, then the words in the last quoted sentence refer to words in the previous sentence. And

finally, if I say "Shakespeare said that Julius Caesar is dead", the subordinate clause "that Caesar is dead" refers to the sense of the sentence "Julius Caesar is dead", i.e. to the Thought thus expressed. And so, for Frege, in ascribing a statement or a belief to someone, I neither refer to the words (if any) which he uttered or entertained, nor to the truth-value (if any) of what he asserted or believed, but only to the Thought which he expressed or entertained.

(6) Functions.

After the lengthy digression of the last three sections we can now return to the problem which constitutes the primary subject matter of the present chapter: Can a more satisfactory account of predicates and predication be provided by employing the notion of unsaturatedness of sense than was possible, following Frege, by appeal to the notion of reference?

P.F. Strawson has summarised the Kantian thesis that "Thoughts without content are empty and intuitions without concepts are blind" (B. 75) as follows: "The duality of intuitions and concepts is, in fact, just one form or aspect of a duality which must be recognized in any philosophy which is seriously concerned with human knowledge, its objects, or its expression and communication" (Bounds of Sense. 47). Within the latter realm of expression and communication, Strawson adds, "we must recognize the need for such linguistic or other devices as will enable us both to classify or describe in general terms and to indicate to what particular cases our classifications or descriptions are being

applied" (Loc.cit.). Now, although Kant was primarily exercised by the problem of establishing and clarifying this duality within an epistemological context, there is an important strand in Kantian thought with which Frege would have been in full accord: that experience is "anything to us" depends upon our ability to subsume objects under concepts. And this ability is both manifest in and dependent upon our ability to formulate and express assertoric judgements: "The only use which the understanding can make of concepts is to judge by means of them" (B. 93). It is this thesis which justifies the use which Kant makes of the traditional classification of the "functions of judgement" as the means of isolating and identifying those concepts which are necessary to our having the experiences we do have. But what Kant endeavoured to establish by appeal to the Table of Judgements, Frege attempted by appeal to the semantic and syntactic properties of the expressions we use in formulating judgements. Frege, indeed, translated the transcendental-psychological investigation of Kant into a semantico-ontological one. Which is to say that where Kant urges the absolute irredicibility of intuitions and concepts, but yet the necessity of their co-operation in any possible human experience, Frege urges the absolute irredicibility of objects and concepts (or functions), and of proper names and functional signs, but yet maintains that no meaningful Thought can be formulated without the participation within it of elements corresponding to both types. And for both philosophers the reason is the same: if this were not the case then thoughts would not possess the unity or completeness which is their essential and defining characteristic. "By a 'function'" wrote Kant, "I mean the unity

of the act of bringing various representations under one representation" (B. 93). Frege makes much the same point in the following passages, albeit in a somewhat different terminology. I have quoted these passages in full, in order to combat the widespread, almost universally held belief amongst critics of, and commentators on, Frege that he nowhere explicitly deals with, or even hints at the unsaturatedness of functional sense.¹³ The following passages show that not only did Frege maintain that predicates and other expressions possess a sense as well as a reference (this has been doubted¹⁴), but also that both are essentially incomplete, and, moreover, it is predicate sense that is the prior, more basic concept:

Statements in general ... can be imagined to be split up into two parts; one complete in itself, and the other in need of supplementation or 'unsaturated'. Thus, e.g. we split up the sentence

'Caesar conquered Gaul'

into 'Caesar' and 'conquered Gaul'. The second part is 'unsaturated'--it contains an empty place; only when this place is filled up with a proper name does a complete sense appear. (FuB. 17).

If then we look upon Thoughts as composed of simple parts and take these, in turn, to correspond to the simple parts of sentences...the question now arises how the Thought comes to be constructed, and how its parts are so combined together that the whole amounts to something more than the parts taken separately....The whole owes its unity to the fact that the Thought saturates the unsaturated part or, as we might also say, completes the part in need of completion. ...A Thought is saturated and needs no completion. (Gef. 36-7).

The words 'unsaturated' and 'predicative' seem to suit the sense better than the reference... (ASB. in Nach. 129).

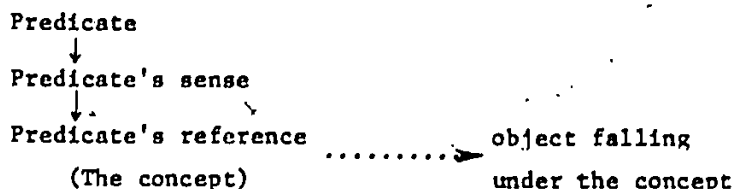
Frege did, of course, on many occasions allude to the incompleteness of predicate reference, of the concept. But what has gone entirely unnoticed is that he justified this doctrine by appeal to the prior notion of the unsaturatedness of predicate sense. And this is nowhere clearer than in the following passage, where, having stated that objects and concepts (references) must be absolutely distinguished, he then justifies this claim as follows:

For not all the parts of a Thought may be complete; at least one must be 'unsaturated' or predicative; otherwise they would not hold together. For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept prime number' without a link. We apply such a link in the sentence 'the number 2 falls under the concept prime number'; the link is contained in the words 'falls under', which need to be completed in two ways: by a subject and an accusative. And only because their sense is unsaturated are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a Thought. (BuG. 205).

Frege also insists that "the assertion that suits the concept does not suit the object" (BuG. 200); but assertive force attaches to the Thought expressed by a declarative sentence: one can assert a Thought, one cannot assert an object or a truth-value. And so, again, it is ultimately in the realm of sense that the irreducible difference between concepts and objects must be located. The senselessness of attempting to assert

of a concept what should be asserted of an object lies in the fact that what results is not a sentence but an incomplete expression; while attempting to assert of an object what is appropriate to a concept results in a list. So it is that, respectively, "Conquered Gaul conquered Britain" is an incomplete expression, while "Julius Caesar, the conqueror of Gaul" is just a list containing two names. (This matter was treated in more detail above Ch. I. § 2).

At this point we can take up again the question of predicate reference from where we left it at the end of Chapter I. We argued that Frege had not one, but two notions of reference: the transitive notion, whereby having a reference involves being correlated with some entity in the real world; and the intransitive notion, whereby referring involved no more than possessing some (unspecified) property in virtue of which the expression in question could be used in sentences possessing a determinate truth-value. This latter we called the property of being truth-valuable. It was argued, further, that Frege's account of what it is for a predicate to be truth-valuable, namely its being defined for all objects whatever as argument, is mistaken -- at least if it is intended to apply to the language of common discourse. But what of the claim that predicates refer transitively, i.e. that, besides expressing a sense, a predicate also refers to a concept, on the model which Frege presented in his letter to Husserl:



(Fig. 4)

What useful work is the notion of transitive reference doing here? Why ought we to include the notion of a concept here at all? Frege's argument is a weak one. In the letter to Husserl,¹⁵ immediately following the diagram of which Fig. 4 is a part, Frege wrote: "I have written the last step from concept to object sideways in order to indicate that it belongs to the same level, that objects and concepts have the same objectivity". His thought here¹⁶ is this: the objects referred to by proper names, and the truth-values referred to by sentences are objective entities, existing in the real world. But truth-values are determined (by the principle PR1) by the coming together of an object and a concept or other function. It is clear to Frege that objects, truth-values and concepts must be entities which exist in the same realm, the real world; otherwise they would not be able to "come together". Now, even if we ignore the metaphorical nature of all this, and even if we overlook the naive assumption that there is a realm of sense (Reiche der Sinne) quite distinct from the realm of reference (Reiche der Bedeutung) which is the real, objective world (this distinction will not hold even within Frege's system; for one can refer to senses which must therefore have the same objectivity as objects etc.) -- even if we ignore such matters, Frege's argument carries no conviction. In the first place, it is based on the highly questionable claim that sentences refer to their truth-values; and in the second place, it is based on the claim, which Frege eventually repudiated, that the reference of a complex referring expression is composed of the references of its component parts. Frege eventually realized that, of course, Denmark is not a part of Copenhagen, even though

Copenhagen is the reference of the complex complete name "The capital of Denmark" in which the name "Denmark" occurs.¹⁷ (In examining Frege's principle for the determination of complex reference, PRI, I have interpreted Frege's theory, charitably, as saying that the references of the component expressions determine, but do not comprise, the reference of the complex whole). The argument for the transitive reference of predicates and other incomplete expressions offered by Frege is a tissue of muddles, mistakes and metaphors which will not bear scrutiny.

We asked earlier: What useful work does the Fregean notion of a concept, as the transitive reference of a predicate, perform? And the answer is, surely, none. We can see that this is the case if we ask: What would it be like for a predicate to lack a reference, though it expressed a clear sense? This would involve the predicate's being able to participate in sentences expressing a clear and univocal Thought, but which could not have a truth-value, either true or false. This, we saw earlier, is a conceivable state of affairs with respect to proper names (like "Odysseus") which have a sense but lack a reference. Is it, however, conceivable with respect to predicate expressions? If "a" is a proper name which successfully refers, and "F()" is a predicate with a clear sense, under what conditions would the sentence "F(a)" express a Thought, but one which was neither true or false? There are only two types of case in which this state of affairs is even prima facie plausible: (i) the case in which, although "F()" has a clear sense for a certain range of objects as arguments, it is not defined for objects of the type to which a belongs. For example, "F()" might be "() is prime", and "a" might be

a non-numerical name, say, "Aristotle". (ii), there is the case in which the predicate itself contains a proper name which lacks a reference, as does, for instance, the predicate "() is a brother of Odysseus" or "() saw a moon of Vulcan".

In cases of type-(ii), it follows from the Fregean principle for the determination of reference, P_{RI}, that any sentence which contains the predicate "() saw a moon of Vulcan" will be without a truth-value; for such a sentence will contain a proper name which lacks a reference: "Vulcan". But suppose, for example, a philosopher-astronomer, say, Hegel, claims in seriousness to have sighted one of the moons of Vulcan. Our reaction would be that he is mistaken, that what he says is false. For there is no planet Vulcan, and hence no moons of Vulcan which can be seen. And if it is impossible to do something, then any claim to have succeeded in doing that thing must be false. And, contrarilywise, the sentence

It is not the case that Hegel saw a moon of Vulcan
is true.

In cases like (i), in which the predicate has a clear sense but is completed by a name of an object belonging to a type to which the predicate does not apply, again, Frege's theory yields the most implausible results. If any sentence in which the predicate "() is a prime number" occurs is to have a truth-value, then every sentence in which it occurs must have a truth-value (assuming that the name which occurs in the argument place has a reference). But here are included such sentences as "Aristotle is a prime number", "The square root of Aristotle is a prime number", "The steam from my porridge plus the square root of Aristotle is a prime number" and so on. We noted above (Chapter I, section 6) that

there are insuperable difficulties facing any attempt either to construe such nonsensical sentences as possessing a truth-value in virtue of the sense (?) which they express, or to stipulate a truth-value for them ad hoc. Now, the account we have given of the notion of sense as it applies to proper names (Chapter II, section 4) can be employed to construct a more feasible theory. The sense of a proper name contains a stipulation or indication of the type of object to which it refers. In other words, anyone who understands or knows how to use a given proper name thereby knows at least one sortal concept under which the object to which it refers falls. A sortal concept is a concept which tells us what sort or type of thing an object is; more precisely, a sortal concept provides us with an independent principle of counting.¹⁸ In Strawson's words: "A sortal [concept] supplies a principle for distinguishing and counting individual particulars which presupposes no antecedent principle, or method, of individuating the particulars it collects" (Individuals, 168-9). Such concepts as man, number, word, sensation or (moment in) time are sortal concepts; wise, prime, efficient, English and old, on the other hand, are not. I shall call such non-sortal concepts attributive. As Strawson has remarked, attributive concepts, even though they may supply "principles of grouping, even of counting, particulars, supply such principles only for particulars already distinguished or distinguishable, in accordance with some antecedent principle or method" (Loc. cit.). The suggestion advanced in Chapter II, Section 4, then, boils down to this: the only concept or concepts which we necessarily know an object to fall under when we know the sense of that object's name are pure sortal concepts. I

say "pure" because, of course, there are sortal concepts which contain explicit attributive elements. One can, for example, individuate and count bald left-handed men, and bald left-handed man is technically a sortal concept. It is, however, a mixed sortal, containing within it, besides the pure sortal concept man, the attributive elements bald and left-handed. And so to understand the meaning of a sentence in which the name "Aristotle" is used, I must at least know one pure sortal concept under which Aristotle falls; I do not need to know, of any attributive concept, whether or not Aristotle falls under it. This means that I need to know rather little about Aristotle in order to understand sentences in which his name occurs; but then names are rather uninformative.

Our contention, then, is this: a name manifests, through its sense, a certain sortal physiognomy, which is recognized by anyone who knows how to use that name. Now, if it can be shown that (again metaphorically) the argument place in a predicate also possesses a similar sortal physiognomy, determined by its sense, then we shall have the beginnings of a modal of sentence sense which will clearly outlaw such inappropriate acts of predication as those instanced above (type-(1)).

But before proceeding to the defence of a particular theory of predicate sense, it will be as well if we pause briefly here and ask: What is to be expected of such a theory; what requirements ought it to fulfil? Sense, we have already said, is a notion needed in epistemology, in order to explain how we learn, understand, and communicate by means of language. So our account must conform to the phenomenological data (with which we, as native language users, are fully familiar) concerning what it is to

understand a predicate (or an act of predication, or a sentence in which a given predicate is used). Secondly, as a basically Fregean position is being defended here, we must endeavour to provide an interpretation and justification of his doctrine that concepts are a form of function, and that functions are essentially unsaturated. We must also show, thirdly, that this theory is possible and plausible in the absence of Frege's notion of predicate reference. Fourthly, we need an account of what is amiss with such sentences as "Green ideas sleep furiously" and "Three was the colour of the pain in my chair", where the predicates are clearly not defined for all objects as arguments, and where, as a result, Frege's theory does not apply. And finally, our account of predicate sense must combine with that provided above of the sense of proper names, so as to constitute at least the bare bones of a theory of sentence sense, of the Thought.

What, then, is the sense of a predicate? Given the preceding accounts of what sense in general is,¹⁹ and of what a predicate is,²⁰ this question is tantamount to asking: When a person understands a sentence, say: "A is drunk", what is it that he understands that will enable him also and therewith to understand the sentences "B is drunk", "C is drunk" and so on? What does that person know when he knows how to use the predicate "() is drunk"? The answer towards which we are working is that such a person knows a rule or principle for the collection of objects of a certain sort of type. Although this is an answer that is not without its contentious and philosophical aspects, we can treat it for the moment as simply an attempt to embody this uncontrovertable observation: If some

one is unable to identify or pick out drunken beings, if he repeatedly predicates "() is drunk" of sober individuals, or of objects which are of the wrong type (e.g. trees, tables and clouds), then that person would not be said to understand what the predicate "() is drunk" means. And this is true even should the person be able to define the predicate by saying it means "being intoxicated or under the influence of alcohol". For if he then persisted in making either predominantly false, or predominantly inappropriate judgements of drunkenness, this would merely show that he did not understand the definition either. A person's ability to formulate appropriate and largely true judgements employing a given concept is a more basic criterion for the ascription to him of a grasp of that concept, than an ability to define, or say what that concept means. This is especially clear in the case of those concepts which, like colour concepts, cannot be defined verbally.

But in what way are we now using the word "concept"? We have argued that the Fregean notion of a concept (as the reference of a predicate) is, if not actually mistaken, then at least otiose. Is there, then, a more respectable use for the term? It was employed in the last paragraph as synonymous with the sense of a predicate; and it is thus that it will be employed henceforward, for the remainder of the present work.

The provisional result of the investigation thus far is that a concept is what a predicate expresses, the sense of a predicate; and this has been glossed as being a rule for the collection of objects of a certain sort or type. A person's grasp of such a rule will be manifest in his ability to pick out objects to which the predicate applies; and one, but

by no means the only way, of doing this is by asserting true judgements about such objects. That is, picking out the objects in language, by referring to them, and asserting truly that they fall under the concept in question.

All this sounds rather un-Fregean. And the reason is that we have introduced what might be called a transcendental element into our account. Frege's approach, by contrast, was essentially immanent, involving the immediate extrapolation from syntax and semantics to ontology, from the nature of language to the nature of the world. Typically Frege ascribes a denotation ("in the world") to all semantically primitive elements: proper names, predicates and functional signs, sentences, subordinate clauses and even logical connectives. And such primitive elements are isolated by procedures which appeal to the needs of logic and mathematics, and the possibilities of substitution salva veritate and salva congruitate. In this general approach Frege has set the tone, the aims and the methods of virtually all subsequent work in philosophical logic and semantics, at least in the English speaking world. Russell and the early Wittgenstein, and later Quine, Tarski, Carnap, Church and Davidson, with only minor deviations, have trodden the Fregean path. The only major exception has been the author of the Philosophical Investigations (though one ought to mention here those who, like Grice, Austin and Strawson, have been strongly influenced by him).²¹ Wittgenstein realized in his later work that only a transcendental approach to the study of language is able to establish conclusions of the type in which we are here interested. And he showed that such an approach must needs assume that there is a community

of language users whose linguistic activities are not cut off from other, non-linguistic habits, desires, and activities. Language, Wittgenstein maintained, is not a formal calculus, but a human tool whose construction and function becomes incomprehensible when it is divorced from the "forms of life" in which it is used. It is indeed precisely to restore the notion of a language to the context in which it belongs, from which it has been removed by philosophers, and wherein only it can be adequately understood, that Wittgenstein employs the concept of a "language-game".

In a recently published book²² Hintikka has complained that the "transcendental point of view", "which focuses on the human activities which are basically involved in our obtaining whatever information we have, is notoriously absent from recent philosophising". Hintikka, though he claims he is not overlooking the "rich Wittgensteinian literature on 'language-games'"; still feels that

... the study of the relations of our language -- or anybody's language, for that matter -- to the reality which it speaks of has either been left unattended, or else has been discussed only in terms of unanalysed 'interpretations', 'valuations' ... or comparable unanalysed static ties between language and the world, ... even though it is obvious that these are not natural relations but are only created by and sustained through certain human activities and human institutions.²³

One can only agree. And the responsibility for this neglect lies, more than with any other single philosopher, with Gottlob Frege. The point is not, however, that Frege's conclusions were false, or his method misconceived; on the contrary, it was because his method was so apt and his

conclusions, on the whole, so correct that this deficiency passed unnoticed for so long. And the deficiency is this: the immanent methodology is insufficiently powerful to establish the conclusions which Frege wishes, about, say, concepts, judgement, assertion and so on. Frege's intuitions were, on the whole, remarkably sound. But, as he was time and again forced to concede, the method of direct extrapolation could only indicate, and not establish, the soundness of these intuitions: "I can here only give hints" he wrote on more than one occasion, "and here a metaphorical account must suffice", "the reader must agree to meet me half way" and "not begrudge a pinch of salt". In the case at hand, the nature of predicates and concepts, (which, as a matter of fact, it was in connexion with, that most of the foregoing disclaimers were issued) the examination of substitution possibilities salva veritate and salva congruitate will not suffice to establish the results Frege desires.

If I have understood him aright, it is a transcendental element which Dummett attempts to introduce into his account of why Frege was right to deny that a concept and a concept-correlate²⁴ cannot be one and the same thing. Contesting the un-Fregean assertion that the two expressions "the relation of difference" and "() is different from ()" can stand for the same thing, Dummett writes:

We cannot say this, because we come by the notion of a relation via the distinction between relational expressions and others, and "the relation of difference" is not a relational expression. A relation, that is, is explained as being that for which a relational expression stands, and hence if we allow that an expression

of a different kind can stand for a relation, the whole explanation of what a relation is falls to the ground.

("Frege on Functions: a Reply", Klemke.282).

This is rather obscure, and just as it stands is, I believe, false; though this may be merely the result of an unfortunate choice of words on Dummett's part. What a relation is, Dummett says, is explained as that for which a relational expression stands; and likewise for concepts with respect to predicates. This is the way in which they are introduced to us. This seems to be right. But it hardly entails that, once introduced, once explained, there can subsequently be no alternative way of referring to a relation, without, that is, imperiling the original explanation. The possibility of the subsequent employment of a non-relational expression to refer to a relation surely does not entail the demise of "the whole explanation of what a relation is".

But there is an ambiguity here; and this means that there are two problems of "explaining what a relation is". One is the problem of explaining what a relation is per se (i.e. as distinct from an object or a concept or an event, say); the other is the problem of explaining what a particular, but as yet unspecified relation is (e.g. the relation of difference). Let us call explanations of the former type generic explanations, and those of the latter type, specific. The specific case is of interest to philosophy, if at all, only in so far as that any generic explanation must not run counter to the specific ways in which relations are actually introduced and explained. But it is just this which Dummett seems to do. There is no reason why one would not introduce a specific relation (the

relation of Xity) by means of entirely non-relational expressions. But of course it is possible to do this only if one already has the generic notion of a relation. And it is this latter that cannot possibly be introduced or explained by any other means than via the common properties of relational expressions. And relational expressions are themselves abstractions from sentences.²⁵ If a semantic model is to be adequate to the language we speak, then it must allow that that language is accessible to human beings, that it is learnable. The nature and function of concepts (and relations) in human thought and communication cannot be divorced from our ability to apply them, with a considerable degree of success, to the objects we encounter. This ability is manifest in the making of true assertions, in the uttering of true sentences; and this is something we learn to do.

But does not this constitute a rejection, lock stock and barrel, of the Fregean notion of a concept? I do not think so. On the contrary, it constitutes a reinterpretation of the notion of unsaturatedness, a reaffirmation of the Fregean account of what a predicate is, and a development, along Fregean lines, of the notion of sense as it applies to incomplete expressions. This, it is true, nullifies Frege's platonism, according to which there exists something to which a predicate stands in the same relation as a proper name stands to its bearer; but it leaves the most important of Frege's insights intact, indeed, reinforced.

The concept is essentially predicative. A name of an object, on the other hand, is quite incapable of being used as a grammatical predicate. (BuG.193)

Concept-words must possess a sense ... otherwise they are just an empty series of sounds. (ASB)

With this we can agree, if we add that (i) possession of a univocal sense is all that a predicate need possess in order to be truth-valuable, and (ii) this notion of sense is one that cannot be divorced from the other linguistic activities of a community of language users. But this is, precisely, to interpret Frege's insight that a predicate results from the deletion of a singular term from an assertive sentence, from a transcendental point of view. Before we turn to an explicit examination of the transcendental aspects of the notion of sense. However, we must pause to consider certain problems associated with the nature of assertion.

CHAPTER III

Assertion

The purpose of the present chapter is to examine the notion of assertion or assertive force: to examine critically Frege's theory concerning this, and to establish an account that will constitute a useful element in our analysis of judgement.

(1) The Begriffsschrift

In section 3 of chapter I, we to some extent anticipated the concerns of the present chapter when we subjected Frege's use of the two concept-script signs "┆" and "—" to a brief scrutiny. We there, however, examined only his use of these signs in the Grundgesetze of 1893. To understand Frege's theory fully we must now return to his first published work, the Begriffsschrift of 1879, and to the first doctrine propounded in that work:

A judgement is always to be expressed by means of the sign

┆

This stands to the left of the sign or complex of signs in which the content of the judgement is given. (Bs.1)

And a few lines later he specifies that

As a constituent of the sign ┆ the horizontal stroke

combines the symbols following into a whole; assertion, which is expressed by the vertical stroke, relates to the whole thus formed. (Bs.2)

Already, then, a number of important distinctions have been introduced with a disconcerting brevity. On the one hand there is the representation, as it appears in the concept-script, of a complete judgement that something is the case:

(a) $\vdash \Delta$

If the vertical stroke to the left of this complex is omitted, there remains the symbol

(b) $\text{---} \Delta$

which, Frege tells us, represents "a thought", "a proposition" or "a mere complex of ideas". If (a) represents the judgement that "unlike magnetic poles attract one another", Frege says, then the sign " $\text{---} \Delta$ ", "will not express this judgement; it will be intended merely to produce in the reader the idea of the mutual attraction of unlike magnetic poles" (Bs.2). Finally, there is the sign

(c) Δ

which represents the "conceptual content" of (a) and (b). " Δ " must always represent a "possible content of judgement"; it cannot, therefore, represent an object or, indeed, anything that is not fundamentally sentential or propositional.

In the opening sections of the Begriffsschrift, then, Frege establishes the following terminology, to which I shall endeavour to adhere, at least until something more appropriate can be established:

(a) represents a judgement; (b) represents a proposition; (c) represents a conceptual content of a possible judgement. The sign "┆" is called the judgement sign, and it is composed of two parts: the horizontal or content stroke, and the vertical assertion stroke.

So much, then, for the broad outlines of what is surely a somewhat recondite series of distinctions. Why, we might well ask, did Frege think it so important to have the difference (whatever it might exactly be) between asserted and unasserted contents of judgement clearly marked in the concept-script? The distinction is one that went unmarked in the history of logic before Frege; and it has gone largely unmarked since.¹ Presumably, however, Frege thought that to overlook this distinction would be to jeopardize the validity of proofs formulated within the concept-script: "In my formalized language ... only that part of judgements which affects possible inferences is taken into consideration. Whatever is needed for valid inference is fully expressed; what is not needed is for the most part not indicated either; no scope is left for conjecture" (Bs.3). In what way, then, would the validity of inference be vitiated or threatened by a failure to be clear about the presence or absence of assertive force? This is one question which we must answer. Another, more basic one is this: just what did Frege think that assertive force was, and to what did he think it attached? Roughly speaking, the answers to these questions will be as follows. On the one hand, Frege's intuitions concerning the rôle of assertion in inference are well founded: assertive force is something which can and does affect the validity of proofs. On the other hand, however, Frege was confused about the nature of this force; sometimes

he thought it a psychological phenomenon attaching to mental acts; and sometimes a purely logical matter concerning the position of propositions within a proof; and yet again, a matter of syntax concerning the difference between a sentence and the corresponding nominalization. But even here Frege's intuitions are not untoward; for the two signs "—" and "┌—" can be used to mark important distinctions in all these areas. Frege was to write in the Grundgesetze that

the same thing may never be defined twice, because it would then remain in doubt whether these definitions were consistent with one another. (Gg. §33)

Now although Frege nowhere in the Begriffsschrift defines the terms "proposition", "judgement" or "assertion", or the signs "—" or "┌—", exactly the same point can be made about the partial explanations which he there offers: it is not clear whether they are consistent with one another. To these explanations I now turn.

(2) A Bogus Distinction

One of the distinctions considered essential by most traditional theories of judgement was that between the subject and the predicate of a judgement. This distinction, Frege tells us somewhat surprisingly "finds no place in my way of representing a judgement" (Bs.2). What Frege means here, however, is not that he repudiates in its entirety any distinction between subject and predicate, but rather that he rejects one, traditional way of representing this distinction.

The Greek capital "Δ", we have said, is to represent the conceptual

content of a judgement; and some idea of what this latter is, is provided by Frege's remark that the following two sentences have the same conceptual content: "The Greeks defeated the Persians at Plataea" and "The Persians were defeated by the Greeks at Plataea". The criterion for ascribing sameness of conceptual content here is that "all inferences which can be drawn from the first judgement when combined with other ones can be drawn from the second when combined with the same other judgements" (Bs.3). Clearly, then, the difference between the two sentences, which consists primarily in a difference in subject/predicate construction, is logically immaterial and would not be represented in a perspicuous notation. What it is that the two sentences express can, without equivocation, be represented by the single sign " Δ ". Now here, I believe, long before the introduction of the distinction between sense and reference we can witness the first emergence of a train of thought which was eventually to lead to the construing of sentences as a species of name, referring to their truth-values. For, having decided to symbolize the conceptual content of a judgement in a way which transcends internal subject/predicate analysis, Frège feels compelled to introduce a predicate of which this conceptual content itself is the subject. Already " Δ " is beginning to look rather like a singular term! The predicate which Frege then introduces is the sign " \vdash ":

Our symbolic language is a language of this sort: the symbol \vdash is the common predicate of all judgements ... Such a language would have only a single predicate, viz: "is a fact". (Bs.4)

In other words, ordinary judgements of the form: S is P will be translated

into those of the form: that S is P is a fact.² Such, then, is the role of the sign " \vdash " in the concept-script. It is to be the "common predicate of all judgements".

There is little to be said in favour of this move. It introduces neither clarity nor economy nor elegance, and is indeed a retrograde step as far as the attainment of such virtues is concerned. In the first place, that S is P is a fact (or: $\vdash \Delta$) is considerably more cumbersome than S is P (or: Δ). And secondly, it would generally be accepted that understanding the latter was a prerequisite of understanding the former. The judgement that S is P, that is to say, actually occurs within the judgement that that S is P is a fact. It appears that Frege's radical alternative of a language with only one predicate is no more than ordinary language, with an additional non-functional decoration in the form of That ... is a fact, ($\vdash \dots$).

But though we can reject out of hand the interpretation of the judgement sign as a universal predicate, we ought not thereby to affirm that Frege's claim to have found a universal property of judgements is equally empty. For Frege had other motives for introducing the judgement sign into the concept-script; and, moreover, continued to employ the sign long after he had realized that neither assertion nor truth can be a predicate, and that the phenomenon he wished to indicate with the sign " \vdash " cannot be a part of a judgement.

(3) A Logical Distinction

The judgement sign can only occur at the beginning, i.e. to the left of

a sign or complex of signs representing a conceptual content. The vertical stroke which comprises one of its two parts signifies the presence of assertive force. These two characteristics enable the judgement stroke to mark a valid and important distinction: between assertive and unassertive occurrences of one and the same proposition.

In the following inference schema (modus ponens), the propositional variable "p" occurs twice:

- (i) If p then q
- (ii) But p,
- (iii) Therefore q.

Now, roughly speaking to begin with, there is a sense in which both occurrences of "p" must stand for one and the same thing: otherwise the argument is vitiated by equivocation. On the other hand, however, there must be a sense in which the two occurrences of "p" are quite different: if "p" in (i) is assertive, then premiss (ii) is redundant; but if "p" in (ii) is not assertive, then the argument will not go through.

Russell seems to have grasped just half of this argument in the following passage from The Principles of Mathematics:

It is plain that, if I may be allowed to use the word assertion in a non-psychological sense, the proposition "p implies q" asserts an implication, though it does not assert p or q. (PMS.35).

This is quite right; but then, inexplicably, Russell goes on to add that

The p and q which enter into this proposition are not strictly the same as the p and q which are separate propositions, at least if they are true.

Now this would imply that either all inferences of the form modus ponens

(to take but one example) are invalid, or, at least, that all those with either a true antecedent or a true consequent in the conditional premiss are invalid. This is, of course, quite unacceptable.

Let us examine first the occurrence of "p" in premiss (i). It will be admitted that there is a marked difference in the truth-conditions between sentences of the form "If p then q" and those of the form "Because p, therefore q". If someone were to utter the sentence "Because it is raining, (therefore) you will stay indoors", he would be understood as having asserted that it is raining, and hence that the person addressed will stay indoors. This stands in contrast to the utterance of the sentence "If it is raining, then you will stay indoors", in which neither the antecedent nor the consequent are asserted. In other words, "Because p, therefore q, and not-p" is a contradiction; "If p then q, and not-p" is not. Our grounds for saying that "p" is not assertive as it occurs in the antecedent (or the consequent) of a hypothetical proposition are (a) it cannot be exported, and (b) the conjunction of "not-p" to the hypothetical proposition does not yield a contradiction. In both these cases an "If ... then ..." sentence stands contrasted with one of the form "Because ... therefore ...". And it is precisely because "p" is unasserted in premiss (i), that premiss (ii) is needed; for premiss (ii) simply asserts that p is true.

The upshot of this is that we are compelled to recognise that one and the same proposition may occur now assertively, now unassertively. Geach has called this "the Frege point"³, and seems to consider it the only point of substance which can be extracted from Frege's analysis of

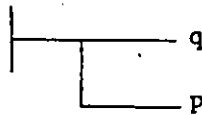
assertion, of his use of the judgement sign. As we shall see, there are indeed other "Frege points", but this is not to deny either the validity or the importance of this one.

But perhaps it will be argued that, inescapable though the distinction made here undoubtedly is, it is surely so obvious and incontrovertible as to hardly warrant the space here devoted to it. Yet the list of philosophers who have denied it is long and distinguished, and includes, for example, Ryle, Cook Wilson, Strawson and Wittgenstein.⁴ These philosophers have argued that the variables "p", "q" etc., as they appear in a schema such as that for modus ponens illustrated above, range over propositions or statements. They have maintained, further, that it is a necessary condition for something's being a proposition or statement that it propose, state, or assert that something is the case. But if this were the case then modus tollens would be a logical contradiction; for it contains within it an occurrence of "p" and an occurrence of "not-p":

If q then p
But not-p
therefore not-q.

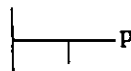
Ryle takes the heroic course of denying, therefore, that "p" and "q" as they occur within "If q then p" are in fact statements or propositions. And this entails denying that the traditional truth-functional account of such conditional statements is accurate. Ryle suggests that we either treat "If-p" and "then-q" as indissoluble wholes, or we treat "If p then q" as a "licence" to perform certain logical operations, and

not as a statement or proposition. But such heroism is surely misplaced. Frege's argument shows that the actual possession or non-possession of assertive force is without bearing on questions of propositional identity. And his notation makes it quite clear when a proposition possesses assertive force. The proposition "If p then q" is written in the concept-script as



and this reveals that it is the complex as a whole to which assertive force attaches, and not to either "p" or "q" individually.⁵

It is not, of course, solely on account of the existence of conditional statements that we must recognise that propositional identity is unaffected by assertive force; for they are in principle eliminable. In the disjunction "either p or q" neither disjunct is asserted, and, more clearly still, "p" is not asserted in or by "not-p". Here there is another pitfall which Frege's perspicuous notation neatly avoids. For even though "p" is not asserted in or by "not-p", this does not entail that "not-p" does not make an assertion. Negation does not serve to cancel assertive force, but, as it were, to reverse it. In other words, the opposite of assertion is not denial, for a denial is an assertion. "not-p" does not make no claim, it makes the claim that "p" is false. Frege's sign for "not-p" is



and the presence of the assertion stroke indicates, again, that the whole is asserted. The small vertical negation stroke which is attached to the

content stroke indicates that "negation is a mark of the content" of an assertion and in no way modifies the assertive force itself.⁶

In the case of sub-sentential, truth functional contexts, then, Frege's use of the judgement stroke marks a valid and useful distinction, which enables us to avoid certain types of philosophical confusion. There is another type of context, however, in which a proposition may occur unassertively, and for which Frege failed to account. This is puzzling as there seems to be no reason why this should be so.

Consider the following proof by reductio that

	$p \supset (q \supset p):$	
1.	$\sim (p \supset (q \supset p))$	Hypothesis.
2.	$p \ \& \ \sim (q \supset p)$	\supset & Trans. (1)
3.	p	& Elim. (2)
4.	$\sim (q \supset p)$	& Elim. (2)
5.	$q \ \& \ \sim p$	\supset & Trans. (4)
6.	$\sim p$	& Elim. (6)
7.	$p \ \& \ \sim p$	Conj. (3), (6)
8.	$\sim \sim (p \supset (q \supset p))$	Reductio, (1), (2-7)
9.	$p \supset (q \supset p)$	Double Neg. (Q.E.D.).

In this proof, premiss (1) is not something given as true; indeed it is known to be false. It is, rather, something assumed for the sake of the argument, an hypothesis or supposition. It cannot, therefore, be the case that premiss (1) as a whole possesses assertive force, and the conventions governing the layout of indirect proofs are designed precisely to indicate this fact. What falls outside the vertical bracket is to possess assertive force, as normal; but propositions which fall within this bracket occur non-assertively.

Now this case differs somewhat from the sub-sentential case examined earlier. For here it is not a propositional component that is devoid of assertive force, even though the whole proposition within which it occurs possesses this. Rather, it is a free standing proposition as a whole, a premiss in an argument, which is without assertive force. This cannot be construed as a special case of the former type, by construing an argument as a proposition comprising the conjunction of its different steps; for this way lies the vicious infinite regress of Carroll's "What the Tortoise said to Achilles".

It is difficult to know what Frege's attitude to indirect proofs was. Certainly he did not allow them to be formulated within the concept-script, as every step in a valid proof must be preceded by the assertion sign. (This is so because, for Frege, expressions preceded only by the horizontal stroke are mere names; and nothing can be validly and formally inferred from names). Now if Frege had been unaware of the possibility of indirect proofs, then his omission would at least have been comprehensible. And if, on the other hand, Frege had denied that indirect proofs were possible, this would have been a mistake. But the situation is more puzzling than that. On the one hand he clearly acknowledged the possibility of valid indirect proofs: "In an indirect proof", he wrote in Verneinung, "knowledge of the truth is attained precisely through our grasping a false thought" (Vern.145). And in the Begriffsschrift too, he had written of indirect proofs as though they were entirely unobjectionable (see Bs.4). And yet Frege wrote in the comments he sent to Jourdain, on the latter's article

"The Development of the theories of Mathematical Logic ...":

A mere thought which is not recognized as true cannot be a premiss. Only after a thought has been recognized by me [sic] as true can it be a premiss for me. Mere hypotheses cannot be used as a premiss.⁸

This is a peculiar remark. Even more peculiar is Frege's statement, in a private letter to Jourdain some two years later:

If a sentence which expresses a false thought is uttered with assertive force, it is logically useless and, precisely speaking, unintelligible.⁹

As against this one can only insist with Wittgenstein that "One can draw inferences from a false proposition" (TLP.4.023). Conditional proofs and other forms of indirect proof depend upon the very possibility which Frege is here denying, namely, the possibility of discharging an hypothesis. Although I do not believe that Frege's attitude towards indirect proof can be made entirely comprehensible, a further attempt is made below, section (5).

All this does not in any way, however, weaken his claim that propositional identity survives changes in assertiveness. Indeed, it is precisely this theory which must be understood if indirect proofs are to be intelligible. For to discharge an hypothesis is, precisely, to take a proposition (or its negation) from a context in which it is not asserted, and place it in a context in which it is.

(4) A Syntactic Distinction

I wish now to take up the discussion of Frege's doctrine of assertion

and his use of the two signs " \vdash " and " \dashv " during the period 1891-1917, from the point at which we left it at the end of section 3, Chapter I. To summarize briefly: the introduction of the distinction between sense and reference (not formulated in the Begriffsschrift) and its application to sentences led to the claim that sentences were a species of complex complete name, which referred to their truth-values. But even granted that an indicative sentence refers to its truth-value, it is clear, and was clear to Frege, that it may also do more than this: it may assert that something is the case. In order to distinguish those cases in which a sentence merely refers to a truth-value from those cases in which it asserts something (and we have seen in the preceding section that there is need for some such distinction) Frege introduces, or re-introduces, the two signs " \dashv " and " \vdash ". The explanation which he gives of them, however, is quite different from that offered in the Begriffsschrift. The sign

— ξ

is a function-name denoting a function under which one and only one object falls: the True. We attempted to make sense of this doctrine by offering

ξ 's being identical to the True

as a reading of this function-name. This reading has the merit of making clear that (i) assertive force is absent; (ii) a nominal role is intended; (iii) that only the True can fall under the concept thus denoted, and hence that the truth conditions for " $\dashv\xi$ " follow intuitively; and (iv), as Frege demands, that an expression which is the name of a truth-value

always results when a referring proper name is substituted for "ξ".
 The possible arguments for this function do not, that is, have to be restricted to "possible contents of judgement". In the Begriffsschrift theory, that is,

— Julius Caesar

would be malformed: Julius Caesar, being non-propositional, is not a "possible content of judgement," (CF. Bs.2-4). In the later theory, however, the function

—ξ

was, like all functions, to be defined for any and every object (including Julius Caesar) as argument.

Now if I am right, the two signs "┆" and "—" can be employed to mark a syntactical distinction. On this reading the difference between

(1) ┆ Δ

and

(2) — Δ

where "Δ" represents the content of the sentence "This book is red", say will represent the difference between the sentence

(3) This book is red

and the complex noun phrase

(4) This book's being red

or, equally:

That this book is red

The circumstance of this book's being red

Red's being the colour of this book, etc.

Plainly the signs in question are not needed in ordinary language, where the possibility of such grammatical transformations exists. But in logic, or arithmetic, or in any field in which the possibility of such transformations is absent, this convention might well be adopted.

In the Philosophical Investigations, in a series of remarks addressed to the problem of assertion, Wittgenstein wrote:

Imagine a picture representing a boxer in a particular stance. Now this picture can be used to tell someone how he should stand, should hold himself, or how a certain man did stand in such and such a place; and so on. One might (using the language of chemistry) call this picture a sentence radical. This will be how Frege thought of the assumption (Annahme). (PI.12e).¹⁰

The Thought expressed by an indicative sentence (which Wittgenstein here mistakenly calls the Annahme) can also be expressed by the corresponding question, and by the corresponding nominalization. If this Thought is represented by a unary variable like "p" or "A", then (1) above will represent an indicative sentence, and (2) will represent the corresponding sentence radical. For the most appropriate form which we can give to a sentence radical is that of a complex noun phrase.

(5) The Grundgesetze: A Pragmatic Distinction

The syntactical distinction between an indicative sentence and its nominalization, we have seen, is one which, in a perspicuous notation, the two signs " |—" and "—" might be employed to mark. We have also

seen that Frege used them for this purpose. But not only for this purpose: the horizontal stroke and the judgement sign are also used, and at the same time, to mark a different distinction, and one that is rather more difficult to characterize. What we have to accomplish now is not just the elucidation of a Fregean doctrine, but the disentanglement of a piece of confusion.

In a footnote to FuB., Frege wrote:

The judgement sign [—] cannot be used to construct a functional expression; for it does not serve, in conjunction with other signs, to designate an object. (FuB.22n).

Now it follows from the account which Frege gives of the generation of function-names that predicates are function-names denoting functions which map objects onto truth-values. Predicates are obtained by the removal of one or more occurrences of a singular term from an indicative sentence. But if, as the above quoted footnote asserts, the sign "—" cannot be used to construct a function-name, and if any complex sign in which it occurs is not a denoting sign, it follows that any complex expression containing the judgement sign cannot be a sentence. For we know that, for Frege, sentences are names and can be used to construct function-names, viz: predicates.

If expressions preceded by the judgement stroke are not sentences, what are they? What contrast, over and above those noted in the immediately preceding sections, did Frege wish to draw by means of the judgement sign and the horizontal? To answer these questions we must examine a train of thought which, despite his avowed intent

to rid logic of its psychological appurtenances, Frege never managed to transcend. It is particularly obvious in the following passages, which span the whole of Frege's creative life:

No judgement is performed if the assertion stroke is absent.

— A

merely requires the formulation of the idea that A is not the case, without expressing whether this idea is true. (Bs.10).

If we write down an equation or inequality, eg. $5 > 4$, we ordinarily wish at the same time to express a judgement; in our example we want to assert that 5 is greater than 4. According to the view I am here presenting, " $5 > 4$ " and " $1+3=5$ " just give us expressions for truth-values, without making any assertion. This separation of the act from the subject matter of judgement seems to me indispensable; for otherwise we could not express a mere supposition—the putting of a case, without the simultaneous judgement as to its arising or not. Thus we need a special sign in order to assert something. (FuB.21-2).

It must be pointed out once more, to grasp a thought is not yet to judge. (Vern.152).

We are not, it appears, distinguishing here between assertive and unassertive occurrences of propositions, nor between sentences and sentence radicals. Rather the distinction seems to be between acts of different sorts (supposing vs. judging); or perhaps between an act and its content or object (my judging vs. what I judge).

The following, I believe, is as close an approximation as is possible to Frege's intention in using the judgement sign and the horizontal in the Grundgesetze and in Funktion und Begriff.¹¹

The interpretation of the horizontal is unproblematic: it is a function-name which denotes a concept under which only the True falls, and for reasons already given is best read as a complex noun clause. Expressions which are preceded by the judgement stroke, on the other hand, are to be regarded as sentences-which-are-actually-asserted-to-be-true. The action of placing the vertical assertion stroke before the horizontal effects two results: it transforms a noun clause into a sentence and it asserts the Thought expressed by that sentence. And so, in conformity with the original aims of the concept-script, Frege has taken what he believes to be a functionally ambiguous expression type (the sentence) and has replaced it with two expression types ($\vdash\Delta$ and $\text{---}\Delta$) which together perform all the roles of the sentence (or so Frege believes), but each one of which is unambiguous and univocal. A sentence can be used merely to present an hypothesis, to express a Thought; or it can be used to assert that a given Thought is true. The presenting of an hypothesis is performed by use of the referring sign " $\text{---}\Delta$ ", while the assertion that the hypothesis is true is performed by signs of the form " $\vdash\Delta$ ", and these latter, as Frege says "do not serve to denote anything; they assert something", (FuB.22n).

If this interpretation is accepted, a number of otherwise perplexing characteristics of Frege's system become comprehensible. Take for example the matter of indirect proofs. We earlier found it difficult

to understand why Frege outlawed such proofs from the Begriffsschrift. But with the theory modified as it was after 1891, this becomes understandable (though not therewith acceptable). Put briefly the matter is thus: indirect proofs depend upon the possibility of an expression, or its negation, being used now as an hypothesis, and now as an assertion. But in construing this very possibility as a form of functional ambiguity, and in so rigorously pursuing the idea "one job: one expression", Frege made such proofs impossible within the concept-script. In other words, nothing can be inferred, formally, from an expression of the form " $\neg A$ ", because this is a name and does not say anything.¹² But on the other hand we cannot symbolize what is either known to be false, or what is not known to be true, by expressions of the form " $\neg A$ ", because such expressions are assertions, and so to do would be to lie! We can now, moreover, understand some of the strangely egocentric remarks which Frege makes about validity in proofs, eg. "Only when a thought has been recognized by me as true can it be a premiss for me". That this should be the case follows from the fact that a premiss must be preceded by the judgement stroke, and this indicates that what follows it has been asserted by someone. The propositions preceded by the judgement stroke in the Begriffsschrift, for example, were asserted by Frege. "We are probably best in accord with ordinary usage" Frege wrote, though in a slightly different connexion, "if we take judgement to be an act of judging, as a leap is an act of leaping... If the judgement is an act, it happens at a certain time and thereafter belongs to the past. With an act there also belongs an agent,

and we do not know the act completely if we do not know the agent" (Vern.151n). The expression "┆—" stands, in use, as a record of an act of judgement performed by the person who wrote the sign.

One misconception must be dealt with here. The judgement sign cannot be translated into any ordinary language phrase or sentence. "┆—" is not synonymous with "it is asserted that", "I assert", "Gottlob Frege asserts that"... or indeed by any other such expression. This is because any such translation will materially alter the truth conditions of any expression in which it occurs. Thus if we read "┆— 2+2=4" as "It is asserted that 2+2=4", the latter but not the former will depend, not upon whether 2+2=4, but upon whether this is asserted. In writing "┆—" before a sentence one is asserting it, one is not claiming to assert it. The judgement stroke is, so to speak a pure performative operator.¹³ Let us compare it with another performative operator, though not a pure one: "I promise...". It is generally acknowledged that someone who, under normal circumstances, utters the sentence "I promise to come tomorrow" would therewith have performed an act of promising. And in uttering such a sentence one is not reporting or describing an act of promising, but, rather, promising. This is not the case, however, when the verb to promise is used in other tenses than the present and in other persons than the first person singular or plural. "He promised to come" is not a promise, and neither is "I promised to come". "I promise" is a performative verb, which can also be used in non-performative ways, eg. to describe, report and so on. In contrast,

the judgement sign is used to bring off the act of assertion, but cannot be used to report or describe acts of assertion. And the reason for this is that otherwise the judgement sign would contribute materially to the truth conditions of expressions in which it occurred, by transforming extensional into intensional contexts. And even if we interpreted the judgement sign as, say, "I assert" and stipulated that it was to "suffer change neither of tense nor of subject", still this would be inadequate. For the judgement sign, if it is to perform the function for which it was intended, namely the distinguishing between assertive and unassertive occurrences of propositions,¹⁴ must be incapable of appearing within a subordinate clause. And, of course, "I assert that p" can occur as a subordinate clause, say as the antecedent of a conditional statement: "If I assert that p, then". And here "I assert that p" is not asserted.

Such, then, is the conclusion to which we are led by taking to its logical conclusion Frege's Grundgesetze account of the judgement sign. The entities with which Logic deals are neither sentences nor propositions, but bear a striking resemblance to what have lately come to be known as speech acts, assertive speech acts. I shall not concern myself with the merits of this doctrine as far as concerns logic, but will examine some of the interesting extensions that can be made to it in the analysis of the notion of judgement.

(6) Judging and Wondering

We can distinguish between assertive and unassertive mental acts in a

way directly analogous to the way in which we distinguished between assertive and unassertive occurrences of propositions. We can distinguish, that is, between mental acts, or states of mind, which involve the agent or possessor in a commitment to the truth of some claim, for those which involve no such commitment. Consider, for example, the following two lists:

GROUP A

wondering
 thinking about
 pondering
 examining
 considering
 imagining
 etc.

GROUP B

judging
 thinking that
 believing
 acknowledging
 denying
 agreeing
 remembering that... etc.

Group A comprises those acts and attitudes which involve the possessor in no commitment to the truth of some claim that such and such is the case. Thus, when I wonder if it is raining I neither assert nor judge nor deny that it is, or is not, raining. One might say that, intentionally or unintentionally, I am keeping an open mind on the matter. The acts and attitudes which fall within Group B, on the other hand, do commit the agent or possessor to the truth of the corresponding claim; this is evident in that my judgement, my belief, etc. can be either correct or incorrect, true or false. One cannot, that is, say "He wondered if it was raining, and what he wondered was true", though one can make the corresponding statement when "judged" is substituted for "wondered".

Both types of act may have what Frege called the same conceptual

content. That which is the object of my wondering may also (though not at one and the same time) be the object of my act of judgement. After wondering if it is raining I may go to the window and ascertain that, in fact, it is raining. What I previously wondered about, and what I subsequently judge to be the case are one and the same; only my attitude has undergone a change. If this were not the case, then, for example, an experiment could not confirm an hypothesis; for what is confirmed as being the case would be different from what was previously embodied in the hypothesis. Frege's notation can, therefore, be used to mark this distinction: wondering whether Δ will be represented as

— Δ

while judging that Δ will become

| — Δ

Some such distinction, between types of internal act, seems to lie behind the Begriffsschrift use of the horizontal stroke and the judgement sign:

If we omit the little vertical stroke (...) the judgement is to be transformed into a mere complex of ideas; the author is not expressing his recognition or non-recognition of the truth of this. (Bs.2).

At this point it will be well to get clear about expressions.

People are said to express not only judgements but feelings and thoughts and sentences as well; and sentences are also said to express thoughts as well as truths. In the first place, then, we can distinguish between internal and external acts of judgement. An internal act of judgement

is a mental act which (like deciding, day dreaming or wanting a cigarette) may on any given occasion on which it occurs not result in any overt physical behaviour whatsoever.¹⁵ An external act of judgement is an overt, physical act of some sort. External acts of judgement may be either linguistic or non-linguistic. I might express my judgement of a particular theatrical production, for instance, by saying "That was appalling" (a linguistic act) or by throwing garden produce at the performers (a non-linguistic act). A linguistic external act of judgement may be either sentential or non-sentential.

I shall use the phrase an S-expression of a judgement in this sense: "it is raining", "es regnet" and "il pleut" are all S-expressions for the judgement that it is raining. And so an S-expression, as I shall use the phrase, must always be a meaningful sentence in the indicative mood. An A-expression of a judgement, on the other hand, is an action, and as such must be a dateable performance by an agent. Plainly no S-expression can be an A-expression: no sentence can be an action. Yet, equally plainly, there is the closest relation between these categories; for it is by uttering a sentence that one paradigmatically expresses a judgement. To utter the sentence "it is raining" is to A-express the judgement which is S-expressed by "es regnet". Though here we have an ambiguity in the term "judgement" as well. This can be resolved, provisionally, by stipulating that the terms related by the two expressive relations are as follows: a sentence-type S-expresses a proposition; a person (or an act) A-expresses a judgement.

The relation A-expresses, then, relates internal and external acts of judgement: The following diagram makes clear the distinctions and terminology I have introduced; though it must be emphasized here that these do not, and are not meant to constitute even the beginnings of a theory of judgement. This marks merely the establishment of a provisional terminology based on distinctions which have been taken over from both common sense and from the history of philosophy, but into whose import and appropriateness we have yet to enquire.

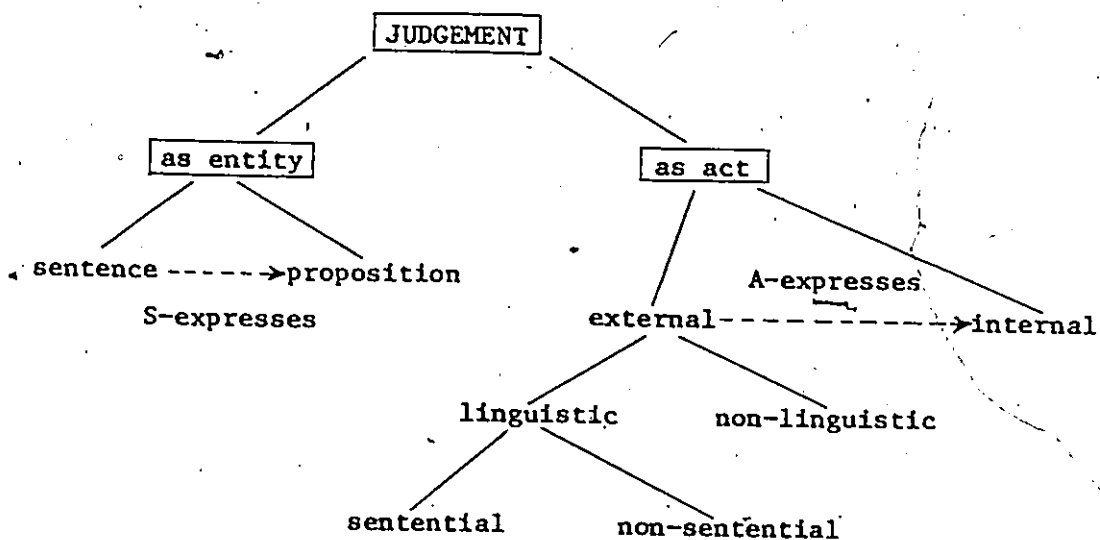


Fig. 5

(7) The Modality of Judgement vs. the Judgement of Modality

In the preceding section we distinguished briefly those psychological attitudes and acts which do, from those which do not, involve a commitment, on the part of the person who has or does them, to the truth or

falsity of a claim that something is the case. We took as paradigmatic of the former sort judging, and of the latter sort of wondering. In section (5) we saw that the sign " \vdash " could be used as a pure performative operator, used in performing the act of assertion. We must now ask whether these two uses are compatible, and whether there is need for a corresponding performative operator for the act of wondering.

Let us examine the following sentences:

- (1) It is raining.
- (2) I wonder whether it is raining.
- (3) Perhaps it is raining.
- (4) It is possible that it is raining.

To utter sentence (1) would constitute an adequate A-expression of a judgement,¹⁶ while sentence (2) might well be employed to express a wondering. Given that the object of the judgement expressed by (1) is the same as that of the wondering expressed by (2), we might symbolize these two cases as " $\vdash \Delta$ " and " $\text{---} \Delta$ " respectively. But the following considerations show that only internal acts may be thus represented. There seems to be an asymmetry between the ways in which we may represent internal and external acts of wondering.

The distinguishing feature of acts of wondering is that they are neither true nor false. In wondering whether it is raining I neither affirm nor deny that it is raining. Yet the sentence (2) which one might employ to express a wondering is clearly capable of possessing a truth-value, and could be uttered with assertive force. This would be the case; for instance, if I were asked what I do when

I hear a tapping noise on the roof, and I answer "I wonder whether it is raining". My reply asserts that I perform a certain act, and is true if in fact I do perform this act and otherwise false. It appears, then, that there is an asymmetry between internal and external acts of wondering which is absent from the activity of judging. For an interior act of wondering is, ex hypothesi, without assertive force, and can be represented by the horizontal stroke; whereas the sign "—" can be used to represent either an interior or an exterior act of judging (both are assertive) or an exterior act of wondering. Now this asymmetry might well be taken as indicating that there is some private or ineffable aspect to wondering which is absent from judging. In my opinion much that is unacceptable in Frege's account of judgement stems from some such misconstrual. Notice, for instance, how he omits from the following list any external linguistic act corresponding to the interior act of wondering. "We may distinguish", he writes:

- (i) the apprehension of a thought. [interior act of wondering]
- (ii) the recognition of the truth of a thought. [interior act of judging]
- (iii) the manifestation of this judgement [exterior act of judging].

Time and again Frege's attitude seems to be that, while judging and asserting are activities which may occur in the objective, intersubjective world, such activities as wondering or grasping a thought are ineluctably interior: private to the person who performs them. And so, while a judgement can be a truth, a wondering can be only a "mere complex of ideas" (cf. Bs.2-4). But we cannot allow that there

are public judgements with private contents; some way must be found of representing wonderings without giving them assertive force, on the one hand, or relegating them to some shadowy realm of the purely subjective, on the other.

It will be helpful if we first examine sentences (3) and (4), as the issues are somewhat more clearly defined here. In the language of common discourse both these sentences can be used to make an assertion (about a possibility) or to express a wondering (about the weather). Possibilities, just as well as actualities and necessities, may constitute the subject matter of assertions. But one can also hedge one's bets and refuse to commit oneself to the truth or falsity of a given proposition, by saying "Perhaps that's the case (and perhaps it isn't)". Frege makes just this point when he writes:

If a proposition is presented as possible, then either the speaker is refraining from judgement...or else he is saying that in general the negation of the proposition is false. In the latter case we have what is usually termed a particular affirmative judgement. (Bs.5).

"It is possible that Δ" (or more colloquially: "perhaps Δ") can be symbolized as either

(5) $\vdash (\Diamond \Delta)^{18}$

or

(6) $\text{---} \Delta$

(5) is a judgement of possibility; (6) is a possible judgement, or wondering.

Now if this is right, then there is no longer any reason to

deny that Frege's threefold division can be extended so as to cover all four parts of the two intersecting distinctions between internal/external and assertive/non-assertive acts as follows:

- (i) Wondering (internal non-assertive act)
- (ii) Judging (internal assertive act)
- (iii) Expressing a wondering (non-assertive locutionary act)
- (iv) Expressing a judgement (assertive locutionary act).¹⁹

This fourfold division is again examined, below, in Chapter IV.

(8) Summary

At this point we have mentioned seven prima facie distinct uses to which the concept-script signs "—" and "—" might have been put by Frege; and, with one exception, we have seen that there is indeed a valid distinction that can be marked in these cases. But what are we to say of Frege's philosophical motivation, over and above noting that it was somewhat confused? Can a univocal reading be given to the so-called judgement sign and the horizontal stroke? I think the answer is no. But we can make considerable progress in the direction of reducing the chaos which Frege bequeathed.

In the first place, there are two fundamental notions of assertion or assertive force. One notion, the more basic for Frege, is the "logical" notion examined in section (3) above. The test for deciding whether a given proposition occurs assertively within a given context is the test of exportation: if the proposition can be removed from that context without further ado, and displayed on a line in a proof

by itself, then that proposition is asserted in the original context.²⁰ Thus "p" occurs assertively in "p & q" but not in either "if p then q" or "not-p". As a limiting case, of course, "p" occurs assertively in "p", and "if p then q" occurs assertively in "if p then q". Which is to say that any free standing, unembedded proposition possesses assertive force, as far as concerns the needs of logic. But this test explains nothing: it is exportation that is explained and justified by the possession by a proposition of assertive force, and not the reverse! Still, this logical notion of assertion is one which Frege is right to indicate in his concept-script; for it is an objective phenomenon which must be recognized if validity is not to be vitiated, and which should, therefore, be embodied in a perspicuous notation. This is not to deny that the possession or absence of assertive force is usually so obvious intuitively that it can remain unmarked without danger; but a perspicuous notation sets itself the aim of making explicit such hidden but effective forces, no matter how obvious they may be.

The second, different though closely analogous notion of assertion attaches to speech acts and to their interior, mental counterparts. Here assertion is a human action; it is something which a certain person does with a certain proposition at a certain time. This is done when a person commits himself to the truth of that proposition, and this he may do either out loud, or to himself. The paradigmatic form of an exterior assertion is the uttering of an indicative sentence. As Frege rightly insisted the "assertion sign" of ordinary language, so to speak, lies in the indicative mood of the verb. Yet there are contexts of

utterance (and tones of voice, etc.) which will rob an utterance of assertive force. For instance, I may say firmly and with conviction "Someone has just stolen your car", but if I am taking part in an elocution class, or a theatrical performance, I will not be taken to have asserted anything.

We can now, I believe, introduce a modification into Frege's account(s) of the nature of assertion, and the use(s) of the sign " \vdash " which will enable us to preserve the most important insights established by Frege while controlling (but not resolving) the ambiguity of his presentation. The sign " \vdash " can be employed within logic to mark assertive from unassertive occurrences of propositions when the sign is followed by a sentential variable ("p", "q" etc.).

Thus modus ponens will be written:

$$\begin{array}{l} \vdash \text{ (if p then q)} \\ \vdash p \\ \hline \vdash q \end{array}$$

And this shows that only one occurrence of "p" is assertive.

We can introduce a third element here, corresponding to the horizontal stroke: when the horizontal (without the vertical assertion stroke) occurs before a sentential variable, and the whole sign is to represent the nominalization of that sentence. Thus " $\text{—}p$ " will represent the nominalization of "p", i.e. "that p". This corresponds closely with Frege's intentions in the Grundgesetze, as established in section (4) above.

When the signs " \vdash " and " — " are employed with a different

style of variable, the capital Greek letters " Δ " and " Γ ", say, they are to have a different meaning from the foregoing. In other words, the judgement sign and the horizontal stroke are systematically ambiguous syncategoremata: they have no independent meaning outside contexts of certain types, and within such contexts their ambiguity is to be resolved by the style of variable to which they are attached. " $\vdash \Delta$ " is to represent the judgement that Δ ; and " $\text{---} \Delta$ " is to represent the content of this act: that Δ .

Now, so far, this accords well with the intentions which Frege embodied in his mature works. So well, in fact, that we are now able to accomplish something which Frege also wished to accomplish, and was unable. We wish to use the signs " \vdash " and " --- " to distinguish between certain kinds of mental acts, judging and wondering. But if these signs are employed to mark the distinction between an act of judgement and its content or object, it cannot be simultaneously used to distinguish between types of acts. For otherwise " $\text{---} \Delta$ " would have to represent both the content of the judgement that Δ , and the act of wondering whether Δ . And this is a nonsensical assimilation. As was pointed out (above, §7) it is at this point that Frege's doctrine is left incomplete, so as to avoid explicit contradiction. But we can, following a hint of Frege's, introduce a new sign to represent the act of wondering. We perform an act of wondering, Frege says, "when we formulate a propositional question".²¹ And so if " $\vdash \Delta$ " represents the judgement that Δ , " $? \text{---} \Delta$ " will represent the act of wondering whether Δ . Both acts have the same content: $\text{---} \Delta$.

We have, then, introduced a systematic ambiguity into the judgement sign and the horizontal stroke, albeit one which is easily resolved in use; and we have introduced, what Frege never did, a special sign for the act of wondering, or formulating an hypothesis. We have, however, preserved Frege's important insights that there are the closest analogies between the logical and the pragmatic aspects of assertion, between sentences and their nominalizations and propositional attitudes and the propositions which are their objects.

We must now discover in greater detail what kind of entity "Δ" stands for; what a Fregean Thought or proposition is.

CHAPTER IV

The Thought

(1) Introduction.

At this point we have already provided an account of the notion of sense as it applies to proper names (above, Ch.II,§4), and a preliminary account of how it applies to predicate expressions (above, Ch.II,§7). It remains to be shown how these two theories come together to provide an account of sentence sense, and how this notion (or, rather, these notions, as it will transpire) of sentence sense relate to the act of judgement.

The title of the present chapter is, however, intentionally ambiguous our subject matter will comprise not only an examination of "The Thought", Frege's term for the sense of a declarative sentence, but also of "The Thought" ("Der Gedanke"), Frege's essay of 1918 in which he propounded his final theory of judgement.¹ This final theory differs somewhat from that offered in the earlier works, and we must therefore devote some time to establishing in what ways his theory changed and whether these changes are acceptable. The following section comprises a brief examination of some of the less desirable elements in Frege's later theory; the reader may omit this section without loss of continuity. The section is mostly exegetical, and the main argument is taken up again in section (3).

(2) "Der Gedanke"

As with so much of Frege's work, the essay of 1918 was motivated in large part by the desire to combat the then widespread importation of psychological and merely subjective elements into logic and the foundations of mathematics. Frege's aim was to "prevent the blurring of the boundaries between psychology and logic" (Ged.59). And in this respect "Der Gedanke" takes its place alongside the other major works in the mainstream of Fregean thought. In most other respects, however, the essay is exceptional.

Perhaps the first unusual characteristic to strike the reader familiar only with Frege's earlier works is the style and tone in which it is written. One notices, for example, the complete absence of any special symbolism or formalization; the appeal to examples drawn from history, literature and everyday situations, rather than from arithmetic and the formal sciences; and, in general, the shift in emphasis from the esoteric to the quotidian. If the origins of the formal approach to language and philosophy (as exemplified later by Church, Quine and Carnap, say) can be located in the earlier works of Frege like the Begriffsschrift and Grundgesetze, then surely "Der Gedanke" constitutes one of the earliest examples of what has more recently come to be known as the philosophy of ordinary language. The changed style and attitudes embodied in the later works can, I believe, be traced to two sources. One is the disenchantment which Frege doubtless felt with the purely formal approach to semantics, after the discovery by Russell in 1902 of the paradox to which his system

gives rise. We can also speculate that the visits paid to Frege by the young Wittgenstein after about 1911 were influential in this respect.²

The difference which resulted can best be summarized by saying that in "Der Gedanke" Frege is primarily interested in the possibility of human communication (albeit the communication of scientific and factual information) and not, as previously, in the construction of a perspicuous notation for the expression of "truths of reason", i.e. of logic and arithmetic.

Be that as it may; what chiefly concerns us here are the philosophical doctrines concerning thought, judgement, sense, etc. which Frege propounded after 1918, and the differences between these and those of the earlier works. Two such differences have already been noted (see above, Ch.I, §4): (i) in addition to indicative sentences, certain types of interrogative sentences are now admitted to be capable of expressing a Thought; (ii) no appeal is now made to the once central doctrines that a sentence refers to its truth-value, and that function-names refer to functions and predicates to concepts. The emphasis has shifted completely from the "realm of reference" to the "realm of sense" (Reich des Sinnes).

Even a cursory reading of "Der Gedanke" reveals that Frege is there relying upon a definite and very traditional theory of human nature, human learning and human judgement. The theory runs, with depressing familiarity, as follows. Man's mental powers are to be divided into those cognitive and those volitional; the cognitive faculties include Perception and Reason, while the volitional faculty is the will. The will is the agent which translates the activities of the cognitive

faculties into action. "How does a Thought act?" asks Frege, "By being apprehended and taken to be true. This is a process in the inner world of a thinker which can have further consequences in this inner world and which, encroaching on the sphere of the will, can make itself noticeable in the outer world" (Ged.76). Thoughts, Frege agrees with Plato, are eternal, immutable essences which are neither created, nor sustained, nor in any way altered by any human activity; nor are they perceivable by any human sense. And so, Frege concludes, "Thoughts are neither things in the outside world nor ideas. A third realm must be recognized" (Ged.69).³ But having once posited this "third realm" whose denizens are Thoughts, Frege must allow that there is the possibility of some sort of human contact with this realm -- otherwise thinking would be impossible. He is led, therefore, to posit the existence of a special faculty whose function is the contemplation of the timeless inhabitants of the "realm of sense":

We are not the bearers of Thoughts as we are the bearers of our own ideas ... So it is advisable to choose a special expression for the "perception" of a Thought, and the word "apprehend" (fassen) suggests itself for the purpose. A particular mental capacity, the power of thought, must correspond to the apprehension of a Thought. (Ged.74).

As in the platonic tradition, then, only reason has access to truth. Knowledge is a two termed relation subsisting between an individual's faculty of reason, and an eternal and immutable object called a true Thought, whereby reason perceives or apprehends the Thought and judges it to be true. "But when a Thought is apprehended, it at first only brings

about changes in the inner world of the apprehender. Yet it remains untouched in its true essence; since the changes it undergoes involve only its inessential properties" (Ged.76). (This last claim is, incidentally, quite empty; for only a few lines earlier Frege had stipulatively defined a Thought's inessential properties as follows: "A property of a Thought will be called inessential which consists in or follows from its being apprehended by a thinker". Clearly, under this stipulation it will indeed follow that no essential properties of a Thought are changed by its being apprehended!)

Alongside, and partly intermingled with the foregoing platonic theory of the human faculties, their functions and their objects, there runs an equally crude and anachronistic associationist theory of meaning-intention. Because Thoughts are immutable and atemporal, existing prior to and independently of their being grasped or judged by any person, Frege is forced to construe the relations between a person and a Thought, and also between a Thought and the language in which it may be expressed and communicated, as being arbitrary and non-complex. It is as though a person were possessed of a stock of sentences which, as the need arose, he could associate with Thoughts he had apprehended and which he wished to communicate. By uttering such a sentence he then tries to induce the same Thought to enter another's consciousness:

One communicates a Thought. How does this happen? One brings about changes in the common outside world which, perceived by another person, are supposed to induce him to apprehend a Thought. (Ged.77, my italics).

The difficulties inherent in this model are brought to the fore by the

italicised phrases in this last quotation. When a speaker utters a sentence with the intention of communicating a Thought, his words are not "supposed to induce" the hearer to apprehend some Thought. In the first place, causally to induce a Thought to enter someone's mind is not an activity which we would normally call a form of communication. If, for example, I possessed a drug which, when administered, brought it about that the recipient thought that snow is white, it would be wrong to say that in administering this drug I was communicating the Thought that snow is white. But when I say that snow is white, on the other hand, I am not performing an act with which I hope my hearer will associate a Thought (preferable, of course, the thought that snow is white). On the contrary, I know that if my hearer speaks English he will understand my words, and in so doing will understand the Thought I wish to communicate. The notion of an act of association fulfils no function here; and this is reason enough to dispense with it. The inappropriateness of this naive associationist model has, however, further, more serious consequences for Frege's theory of judgement. For it leads, as Wittgenstein has taught us to expect, to a radical linguistic scepticism. "Yet there is a doubt" confesses Frege, "is it at all the same Thought which first one person expresses and now another?" (Ged.65). There is no possibility of answering this question by appeal to the associationist model for we never have access to the other person's Thoughts, but only to the signs he uses. But this way of construing the communication situation results, not in a mild form of problematic scepticism with respect to the possibility of successful communication, but rather in a fundamental

incoherence, a private language:

If you say: "How can I know what he means when I see only the signs he gives?" then I say: "How is he to know what he means, when he has nothing but the signs either?" (PG.40).

If the whole notion of a Thought with which a person associates a sentence is to have any explanatory power, then there must be some way in which the Thought can be identified independently of the language with which it is associated. Now clearly a person cannot do this when the Thought is not his own, and hence the act of association is not his own either; a fact which Frege recognized and accepted; a fact which makes a problematic scepticism with respect to the possibility of successful communication unavoidable. What Frege did not realize, however, and what Wittgenstein has shown, is that a person cannot identify a Thought independently of the language in which it may be expressed, even in his own case. The associationist model depends upon the speaker's making the right association between sentence and Thought. "But in the present case I have no criterion for correctness. One wants to say: whatever is going to seem right to me is right. And this only means that here we can't talk about 'right'", (PI.92).

Now my primary concern here is not the refutation of what is objectionable in Frege's theory, but rather the justification and expansion of what is defensible and illuminating. Accordingly I shall henceforth ignore his crude associationism and the faculty psychology with which it is buttressed, and pass to more interesting matters.

(3) Input Sense and Output Sense

Let us for a moment examine the two Fregean principles:

(PS1) The sense of a sentence is determined by the senses of its component parts,

and

(PR2) The truth-value of a sentence is determined by its sense. [And how things stand].

These two principles⁴ depend for their usefulness and plausibility upon there being a sense of "sense" which remains constant throughout.

Now the first principle (PS1) might be thought false on the following grounds. The sense of a sentence is not exhaustively determined by the senses of the component expressions because, in many cases, the context of utterance is also a crucially important factor. This would seem to be the case most clearly when the sentence in question contains token reflexive or indexical elements. Thus the sense expressed by the sentence "I ate plum-pudding today", in so far as this sense is capable of determining a truth-value via (PR2), must itself be determined in part by contextual factors (who says it, and when). Frege allows that this is, indeed, the case, but insists that this is an aspect of the language of common discourse which is neither essential nor desirable. Any information which, either from laziness or convenience, we allow to remain tacitly contained in the context of utterance could, he maintains, be explicitly stated in the sentence. (It is just this claim which I shall contest in the sequel). The more nearly a language approaches the ideals of clarity, objectivity, precision and universality, Frege argues, the less will the truth conditions of its sentences depend upon accidents

of their utterances. Contextually dependent items must, therefore, be expanded and rendered explicit so that their contextual dependence is severed. Then it will be the sentence per se which expresses a sense capable of being either true or false. And so, rather than say "I ate plum-pudding today", one ought to say something of the form: "I, so and so, ate plum-pudding today, the such and such". In this way one will have said something which is either true or false, and not merely implied or suggested it. We will return to this matter in a moment.

A second objection to (PS1) might run as follows: it is not merely the senses of the component expressions which determine the sense of a sentence, because the order in which the expressions occur is also relevant. "Caesar loved Cleopatra" and "Cleopatra loved Caesar" are two sentences, the objection runs, with the same components, but which clearly express different senses. And this difference in sense seems to be a function of the different word orders. We have already argued against this position (Ch. II, §7); and here it is sufficient to observe that this objection depends upon our construing the elements of a sentence as being the words which compose it. But Frege's categorial grammar does not analyse these two sentences into the constituents "Caesar", "Cleopatra" and "loved"; but rather into the two proper names "Caesar" and "Cleopatra" and the two-placed predicate "() loved ()". But then, if the sense of an expression is what one understands when one is able to use that expression correctly, the order of insertion of singular terms into the argument-places of this predicate is clearly a part of the sense of that predicate. In other words, if someone did not know that insertion in the

order: [a loved b] expresses a different sense from that in the order: [b loved a], then we could not say of that person that they understood the predicate "() loved ()".

The principle (PS1), I shall argue, demands a notion of sense which is, pace Frege, different from that demanded by (PR2). In other words, just as we earlier discovered that implicit in Frege's semantical theory were two notions of reference, so we can also discover two distinct notions of sense. We can discover in the Fregean notion of sense two conflicting elements which, following a suggestion of Wiggins, I shall call the "input sense" and the "output sense" of an expression.⁵

We can begin at an intuitive level by observing the peculiarity of Frege's claim that it is the sense or meaning of a sentence which is primarily either true or false: "When we say a sentence is true", he writes, "we really mean its sense is" (Ged.60). To this one wants to respond that what a sentence says, or is used to say, may well be true; but its meaning cannot be either true or false. Equally peculiar, and for much the same reasons, is the claim that the objects of thought are meanings or senses. To say that I believe that I shall have plum-pudding tomorrow does not, *prima facie*, seem to mean the same as saying that I believe the meaning of the sentence "I shall have plum-pudding tomorrow". It is difficult to conceive what it might be to "believe a meaning" or "believe a sense". My intuition is that there is here some categorial transgression analogous to that involved in talk of "asserting an object", "hearing a number" or "kicking an emotion" (when these phrases are intended non-metaphorically). We must see whether this intuition can be substantiated.

Let us examine the two sentences:

(1) I ate plum-pudding today

and

(2) Today I ate plum-pudding.

Although these sentences belong to two different types, they can be admitted to express the same meaning, in some sense of that term. It seems clear, too, that this meaning is determined by the senses of the component elements, in conformity with (PS1). It will be this meaning, moreover, which is preserved by any correct translation of these sentences into another language. I shall call the concept thus partially isolated the input sense or, simply, the meaning, of an expression. Synonymy, then, is the relation of identity for input senses or meanings. Expressions which express an input sense are not necessarily sentential; for otherwise (i) (PS1) would lose all plausibility, and (ii) we should not be able to say of two words or phrases that they were synonymous.

Now while the input sense of (1) is the same as that of (2), it is different from that of

(3) Yesterday you ate plum-pudding.

This is so because "today" does not mean the same as "yesterday", "I" does not mean the same as "you". But if a person A said sentence (1) or (2) on a given Monday, and on the following Tuesday another person B said sentence (3) to A, then there would be a sense in which they had both said the same thing, viz: that A ate plum-pudding that Monday. We have a choice of locutions here: we can say that A and B both express the same proposition, that they make the same statement or claim, that they assert

the same thing, or express the same thought. But choice of any of the terms "statement", "thought", "assertion", "proposition" etc. might seem in this context to be the tacit invocation of some philosophical theory: fewer questions will be begged, and will seem to have been begged, if we coin a special term for what it is that A's utterance of (1) and B's utterance of (3) have in common. I shall call this the output sense of the expressions. Unlike input sense, output sense is essentially sentential.

The need for some such distinction as this has been argued on a number of occasions by contemporary philosophers.⁶ But, to the best of my knowledge, all have maintained that the essential difference between a sentence's meaning and its output sense is that while the truth-value of the former may change, the truth-value of the latter is fixed and invariant for all time. Frege too hints at this doctrine in "Der Gedanke" (cf. Ged. 64-5). The clearest statement of this view, however, is provided by E.J. Lemmon:

Sentences will vary their truth-values ... from context to context; so for that matter will propositions, as the senses of sentences. By contrast, statements are true or false once and for all. The statement that Brutus killed Caesar is true for all time. (SSB.97).

Lemmon's argument for the need to distinguish between input senses and output senses (he calls them "propositions" and "statements" respectively) is grounded in the need to resolve the "deep seated ambiguity in the notion of saying the same thing":

If I say "I am hot" and you say to me "You are hot" then in one sense we have said the same thing (made the same statement). If I say "I am hot" and you say "I am hot",

then in a different sense we have said the same thing (uttered the same sentence). If I say "I am hot" and you, being French, say "J'ai chaud", then we have neither uttered the same sentence, nor made the same statement; but there is still a sense in which we have said the same thing, namely expressed the same sense. (SSP.98).

With this I am in complete accord. But nothing here commits one to the position that input senses may be true or false. The crucial difference between meanings and output senses is not that the truth-values of the former may change while those of the latter are invariant, but that the former do not possess truth-values at all.

The substantial issues concerning reference (by which I mean whether or not an expression actually succeeds in referring, and if so, to what) do not concern the notion of an expression's input sense. (This was discussed in more detail above, Ch.II,§5). Such sentences as "The present King of France is bald", "Odysseus was set ashore at Ithaca while sound asleep" and the like, express a sense irrespective of whether or not the singular terms occurring within them actually refer. This, of course, is a central Fregean doctrine: "If it were only a question of the sense of the sentence, the Thought, it would be unnecessary to bother with the reference of a part of the sentence; only the sense, not the reference, of the part is relevant to the sense of the whole" (SuB.33). Now this doctrine can only apply to the input sense of an expression; for as we have seen, sub-sentential expressions do not have output sense. On the other hand, however, the substantial issues concerning reference do affect the output sense, if any, of a sentence. If a singular term which does not refer, in fact, to anything occurs as the subject term of a

declarative sentence, then that sentence will not express an output sense. For in (putatively) asserting of the present king of France that he is bald, I am not in fact asserting baldness of anything: there is no present king of France. There is here only pseudo-assertion just as, and just because, there is here only pseudo-reference.

If the substantial issues concerning reference do not affect the input sense or meaning of a sentence, the formal or grammatical issues concerning reference do. The input sense of the sentence "Odysseus was set ashore at Ithaca while sound asleep", for example, is in part determined by the input sense of the proper name "Odysseus". And as we have already seen, it is part of the latter that the name purports to refer to a man called Odysseus.

These considerations lend weight to the claim that (i) input sense and output sense are to be distinguished, and (ii) input senses are never true or false. For if what is primarily true or false is the assertion, of some object, that it possesses a given characteristic, then the possession of reference by the subject term of the sentence employed to make this claim will be a necessary condition of its being either true or false. Sentential input sense, on the other hand, is insensitive to success or failure of reference. One can significantly assert that the input sense of the two sentences "I am hot" and "J'ai chaud" are the same; but one cannot ask whether it is true. As a matter of fact, both Frege and Lemmon would agree that such a sentence as "I am hot" does not express a meaning which is either true or false. But they would deny this on the grounds that the meaning of this sentence is too indeterminate to

possess a truth-value, not that meanings are not the sort of thing that can be said to be true or false. And here we must take up the discussion of the relevance of the context of utterance, mentioned at the beginning of this section. Lemmon repeats the Fregean point that certain sentences only express a meaning which is truth-valuable "when taken in conjunction with their context of utterance"; for it is only there that indeterminacies of sentence meaning are resolved. That Lemmon believes that the distinction between sentence meaning and sentence output sense is only superficial is shown by his claim that "the distinction is only worth upholding ... in connexion with those sentences whose truth-value is contextually dependent" (loc. cit.)

To begin with, we can ask: on what grounds, and with what justification do such philosophers say the meaning of a sentence like "I am hot" is indeterminate? Certainly the sentence can be understood by any native speaker of English; it can be translated into other languages -- and translated, moreover, without any suspicion that there is here such vagueness, ambiguity, or incompleteness of meaning as would render such a translation a mere approximation or guess. Of course, the reference of its putatively referring part, and hence its truth-value are undetermined; but then these are no part of its sense! And, indeed, anyone who understands the sentence thereby knows that the sentence does not even purport to say who is hot, nor when, nor where, nor how hot, etc. As Wittgenstein rightly observed: "A proposition may well be an incomplete picture of a certain situation, but it is always a complete picture of something" (TLP.5.156, cf. PI.599). What might with greater propriety

be called "indeterminateness of sense" is manifest by sentences which are ambiguous, sentences whose meaning (or meanings) is unclear, and not by sentences which do not, "by themselves," determine a truth-value. The meaning of a sentence is determined by the meanings of its component expressions (PS1), and it is this which makes the meaning or input sense of a sentence independent of contextual factors. If we make the harmless but simplifying assumption that meanings do not change with time, then we can say that the meaning of the sentence "I am hot" was the same yesterday as it is today, and as it was last century, irrespective of who uttered it, and where. On the other hand, the output senses which this sentence was or might have been intended to express will have differed on almost every occasion of its use. But what of such a sentence as, to use Lemmon's example, "Brutus killed Caesar"? This sentence contains no token-reflexive elements, and therefore might seem to be "true for all time". Strangely enough, Lemmon himself provides the answer to this question, even though he contradicts himself in so doing. The sentence "Brutus killed Caesar", he says, is to be considered true for all time only because "most contemporary contexts would disclose that the reference of the two proper names 'Brutus' and 'Caesar' were well known Romans of that name" (SSP.99). What Lemmon is alluding to here is the fact that there is no general linguistic rule to the effect that one and only one object shall be assigned a given proper name. But this means that the context is always relevant to the determination of output sense: and this is enough to establish the universal validity of the distinction between input and output sense, i.e. between what a sentence means, and

what it says.

Summary: The distinction between the notion of input sense and that of output sense has been grounded in the following four asymmetries:

- (1) Output sense is essentially sentential, whereas input sense (as employed, e.g. in (PS1)) is not.
- (2) Output sense is sensitive to failure of reference, whereas input sense is not. (Hence input sense alone is incapable of determining a truth-value).
- (3) The notion of indeterminacy does not apply to input sense in the way in which it applies to output sense. In the former case it is equivalent to ambiguity.
- (4) Input sense is not contextually dependent; output sense is.

(4) Output Sense and Thought

In the light of the foregoing, we may now establish certain conclusions concerning the two principles (PS1) and (PR2). (PS1) is now seen to be true only if the word "sense", in both its occurrences, is construed as signifying input sense. For it is true neither that sentence input sense nor that sentence output sense is determined by component output sense (there is no such thing as "component output sense"); nor that sentence output sense is determined by component input sense (as the context of utterance is also always a crucially determining factor). The second principle (PR2), on the other hand, must be read as relating the concepts truth-value and output sense; for as we saw above, mere input sense is incapable of either possessing or determining a truth-value.

In short: there is no univocal sense of "sense" which will satisfy both Frege's principles.

How serious a modification does this necessitate to Frege's theory of judgement? For Frege the objects of thought as well as the primary bearers of truth-value are Gedanken. And Gedanken are the senses of indicative sentences. But in what sense of "sense" can this doctrine still be maintained? It seems clear that, whatever else the objects of thought are, (i) they are possible bearers of truth-value (otherwise we could not talk, as we wish to, of "true beliefs", "false judgements", "the confirmation of suspicion" etc.); (ii) they are essentially expressible and communicable in language, and (iii) they are not, we have suggested, meanings or input senses, but claims or statements.⁷ The notion of output sense fulfils all three requirements. But before we may presume that we are possessed of a cogent and useful modification of Frege's theory, we must examine in greater detail the notion of output sense and discover if it will do the work required of it.

First, we might note, briefly, that the distinction between input sense and output sense provides us with a nice justification of the famous Fregean principle that a name only has reference in the context of a sentence.⁸ Sub-sentential expressions, we have said, cannot express an output sense; but since input sense alone is incapable of determining a reference it follows that sub-sentential expressions, in and of themselves, cannot be said to refer. They can, of course, be said to refer when they are used in a given proposition; but in this case it is their contribution to the total output sense which enables them to do this.

At one point in "Der Gedanke" Frege denies that any special sign marking the presence of assertive force is necessary in the language of common discourse:

We declare the recognition of the truth in the form of a declarative sentence. We do not have to use the word "true" for this. And even when we do use it, the real assertive force lies not in it, but in the form of the indicative sentence, and where this loses its assertive force, the word "true" cannot put it back again. (Ged. 62-3).

Now this would seem to imply that the utterance of a sentence in the indicative mood is, so to speak, naturally assertive: unless something intervenes to prevent an assertion from being made, then by simply uttering an indicative sentence one will be asserting that something is the case. This seems to be right. Not only are unqualified utterances considered assertive (pragmatically), but unembedded propositions in direct proofs in logic are considered assertive (logically). There are, however, as Frege points out, a number of types of occasion where this normal assertive force is cancelled. For example, an actor who, as part of a play said "This building is on fire", would not be taken to have asserted that the theatre is on fire. Nor would this have been asserted by someone who said "If this building is on fire, I shall leave". As Wiggins has observed: "One might say that 'p' automatically says that p unless you obstruct it from doing so".⁹ He further remarks that "the common propositional content 'that p' must really be got by subtraction from the assertion of 'p', rather than from the latter by the addition

of '—' to 'p'.¹⁰ And this is, indeed, exactly the way in which Frege proceeded in the Begriffsschrift, (cf. Bs.2-3). Now this is a deceptively important point. Formalists will perhaps argue that it makes little difference, if any, whether we regard assertive force as something which attaches to propositions or thoughts or, conversely, propositions and thoughts as abstractions from assertions. And as far as concerns the laying down of consistent rules for the use of the sign "—" in a formal calculus, this is no doubt correct. On the other hand, if one is concerned with the philosophical analysis of the notions of judgement, thought, assertion, and the typical expressions which such acts receive in ordinary situations, then the differences between these two approaches become significant. Frege's naive realism with respect to the entities he calls "Thoughts", for example, can be traced to the Grundgesetze theory wherein "—ξ" denotes a concept, and "—Δ" an object to which assertive force may then be attached. In the Begriffsschrift, however, the notion of assertion was taken as primitive, and the content of an assertion (—Δ) was obtained from this by subtraction. But this latter approach, by emphasizing that assertion is prior and that assertion is a human act, robs of plausibility the claim that there exist Thoughts prior to and independent of acts of thinking and judging. It is this that has enabled Thiel to call Frege's Begriffsschrift theory "entirely Kantian". The analysis of judgement according to the schema:

agent + act + independent object

is, in the Begriffsschrift, by no means inescapable or even attractive. Neither smiling nor walking, for example, are human acts which essentially

involve independently existing objects (smiles and walks). We can give an account of smiling, say, without claiming that there exist smiles independently of the things people do with their faces. And likewise, the doctrine that the act of assertion is prior to that of the common content of various assertions destroys the temptation, to which Frege succumbed fully in the later works, to say that such contents are independently existing objects.

Historical Note: Considerations of the kind adduced in the last paragraph are usually associated with the later writings of Ludwig Wittgenstein; for it was he that gave most forceful expression to the theory that certain problems in philosophy are created, ex nihilo, so to speak, by our failure to perceive properly the way in which language works. One of the strongest temptations here, the one to which in his middle period works Frege submitted fully, is to posit an entity corresponding to any and every substantive expression. One talks naturally of "the meaning of the expression ..." or "the number of ..." or "the concept ..." and so on. Frege himself denied that expressions of the form "the concept ..." refer to concepts, though he believed that they refer to objects. In general, however, throughout the works which he published after 1891, Frege never hesitated to posit an entity corresponding to substantive expressions: even the True and the False were objects. It is surprising therefore to learn that in the last years of his life Frege came to accept a position not dissimilar to that of Wittgenstein. In the private diary which Frege kept during 1924, his thoughts turned to the analysis of arithmetical statements which he had

first propounded thirty years earlier, and which Russell had shown to be untenable. And Frege concluded that he had been led astray by the surface forms of language:

My efforts to get clear about what one wants to call a number have ended in failure. One is far too easily misled by language ... The sentence "six is an even number" ... seems comparable to "Sirius is a fixed star" ... in which an object should be represented as falling under a concept. And so the words "six", "five" etc. seem to be proper names of objects ... Thus it appears that the problem is to produce the object which appears to be signified by numerical words and signs."

And on the following day he wrote:

From the first lesson on, we are so used to employing the word "number" and the numerals that we don't really consider any justification of these uses necessary ... But with longer concern with this matter one comes to suspect that it is the use of language which leads one astray, that the numerals are absolutely not proper names of objects; and words like "number", "square root" and the like are not concept-words. So that a sentence like "four is a square number" does not express the subsumption of an object under a concept. And so this sentence cannot at all be construed like "Sirius is a fixed star". But then, how?¹¹

Unfortunately Frege never answered this question; nor did he, so far as we know, apply this line of reasoning to other forms of expression of interest to the philosopher. Had he done so, many of the criticisms of his account of judgement offered in the preceding pages would have been

rendered superfluous.

Now the theses that (i) sentence utterances are naturally assertive (i.e. assertive until proved otherwise) and that (ii) thoughts are not independent objects to which assertive force may be attached, these theses have interesting analogies in the phenomenological sphere. One such has been mentioned by Geach. "It is possible" he writes, "that a thought is assertoric in character unless it loses this character by occurring only as an element in a more complicated thought".¹² And he adds: "This would be a neat solution to the problem of how thought is related to judgement". Now Geach is here maintaining the validity of a distinction between assertive and unassertive occurrences of thoughts (mental acts) directly analogous to the distinction between assertive and unassertive occurrences of propositions in logic, (see above, Ch.III,53). The convention governing propositions which occur within a direct proof is that all free standing unembedded propositions are assertive; embedded propositions, on the other hand, (with the exception of the conjuncts of a conjunction) occur without assertive force. In our earlier examination, however, we decided that, pace Geach, this is not the only way in which linguistic phenomena may manifest or be without that force. A proposition which occurs as a step in an indirect proof, for example, does not occur assertively; though, as Lewis Carroll's paradox shows, the occurrence of the proposition within that proof cannot be construed as a form of propositional embedding. There is, moreover, a pragmatic dimension to assertion which does not correspond to the former logical dimension. These facts count against Geach's assimilation of the mental to the logical, and

his conclusion that "a thought is assertoric in character, unless ... it occurs only as an element in more complicated thought". I believe we can approach closer to the phenomenological facts, while still retaining Geach's partial insight, if we say that thoughts naturally occur assertively when they do not occur as part of a more complicated thought. In other words it is not inevitable that a thought should occur assertively if it is unembedded, but it is usual, or natural, that it should. And by "natural" I mean that an additional act is necessary if its assertiveness is to be cancelled. In other words, we construe judgements as phenomenologically basic, and then regard the formation of an hypothesis, or the interior act of wondering, as the suspension of judgement as to the truth of a given claim. And suspending judgement, as Husserl clearly saw,¹³ is an act which occurs rarely and is difficult to perform. And this is the case because we are more receptive to, and practiced in dealing with, thoughts in whose truth (or falsity) we have some reason or motive to be interested. The thoughts that normally occur to us concern matters which we hope or fear, expect or intend, infer or believe; and rarely occur to a truly disinterested mind prepared to abstain from judgement as to their truth. As, indeed, Frege maintained in the Begriffsschrift, the act of judgement is prior to the act of thought: "If we omit the little vertical stroke at the end of the horizontal stroke, then the judgement is to be transformed into a mere complex of ideas; the author is not expressing his recognition or non-recognition of the truth of this." (Bs2-3).

The thesis that thoughts naturally occur assertively, and that an additional act, the suspension of judgement, is necessary if this force

is to be cancelled seems, then, to be confirmed by the available phenomenological and psychological data. There is, however, a logical reason why this should be so. If thoughts naturally occurred or were apprehended unassertively, an extra act of assertion or affirmation would be necessary in order to transform the thought into a judgement. But this act of assertion would itself be a judgement that the thought was true (or false.) But if thoughts occur unassertively, the thought or the truth (or falsity) of the first thought would also occur unassertively, and would itself need to be judged to be true...and so on ad infinitum. If judgement is to be possible, then thoughts must be able to occur assertively; and, phenomenologically speaking, we can say that normally they do so.

(5) "Concept carried connectedness"

I have chosen as the title of this penultimate section a phrase used by P.F. Strawson in talking about Kant's theory of concepts and judgements.¹⁴ I have done so because it seems to me that the phrase highlights one of the more fundamental philosophical motives which have led philosophers to take an interest in the phenomenon of judgement. It is, to put it crudely, in and through our ability to judge, to subsume objects under concepts, to assert that things are, or are not, thus and so, that we make sense of the world. It is in judgement that order is brought to chaos and unity is imposed upon the disparate and divergent elements of experience. This is at least the case in the Kantian tradition. But as Kant clearly saw, and as Strawson has recently emphasized, this unity is not brought primarily through judgements of unity (i.e. judgements employing the concept () is a unity) but through the

unity of judgement: the puzzling phenomenon we mentioned in the Introduction to this thesis. It was in connexion with this fundamental phenomenon that Frege introduced and, even in the face of the gravest difficulties, continued to insist upon the "unsaturated" nature of concepts. It is, he claimed, because concepts are essentially incomplete that they can combine with objects so as to yield something which is a unity — a thought or a proposition — and not a mere aggregate of atomic elements. And although this doctrine did not enable Frege to provide an account of the unity of judgement, at the very least it enabled him to take account of it, and thereby to escape the logical tangles which enveloped, amongst others, Russell and Moore. Russell found that his analysis of propositions could not account for their unity:

A proposition has a certain indefinable unity, in virtue of which it is an assertion; and this unity is so completely lost by analysis that no enumeration of constituents will restore it, even though it itself be mentioned as a constituent. There is, it must be confessed, a grave logical difficulty in this fact, for it is difficult not to believe that a whole must be constituted by its constituents. (PMs, 466).

And Moore too found himself in difficulties over this very point in his article "On the Nature of Judgement".¹⁵ Having asserted, like Frege, that a concept is neither a material object nor a mental or subjective idea, and that a proposition is composed of concepts, Moore is then led to ask "wherein a proposition differs from a concept" (p.179). And his answer is:

The difference between a concept and a proposition in

virtue of which the latter alone can be called true or false, would seem to lie merely in the simplicity of the former. (p.180).

But if concepts and propositions are entities belonging to the same logical type and differing only in degree of complexity, one must ask how it can be that a "simple" proposition like "This rose is red" can be true or false, while a "complex" concept, like that of being the dog that bit the cat that killed the rat that ate the grain that fed the mouse that lived in the house that Jack built, cannot possibly be either true or false. Something, one wants to say, is missing from the concept (however "complex") which is present necessarily in the proposition (however "simple"); and the nearest we have come so far to identifying this something is to say that it consists in a certain completeness or saturation. Wittgenstein was exercised by this problem in the Tractatus, and an examination of his views will prove instructive.

The problem as Wittgenstein saw it was to explain why "A proposition is not a blend of words":

A proposition is articulate. / Only facts can express a sense, a set of names cannot. / Although a propositional sign is a fact, this is obscured by the usual form of expression in writing or print. / For in a printed proposition, for example, no essential difference is apparent between a propositional sign and a word.

(TLP: 3.14-3.143).

In Frege's concept-script, the essential difference between a sign for an object and a sign for a concept was shown by the concept-word's carrying with it an empty set of brackets. But Frege did not, as Wittgenstein here advises, make any such typographical distinction between names of

objects and sentences. Indeed, the sign "A" could stand equally for Julius Caesar or for the thought that Julius Caesar is dead. Wittgenstein's proposed perspicuous notation, on the other hand, would make such syntactical ambiguity impossible. Now the concept-script or perspicuous notation proposed in the Tractatus is considerably more complex and sophisticated than Frege's. In the first place it depends upon the programme of logical analysis whereby the propositions of ordinary language would be reduced to so called "elementary propositions". "An elementary proposition consists of names. It is a nexus, a concatenation of names" (4.22).¹⁶

Now the manifold and profound difficulties which are contained in Wittgenstein's analytic programme, and hence in his conception of an object and of a simple name, do not need to be explored here. Wittgenstein has presented us, I shall argue, with the tools for the solution of the problem of propositional unity; and we need only know that a Tractarian name is a simple sign which signifies by standing for a non-complex object, and that an elementary proposition is composed entirely and exclusively of such names. For now we can state the problem in the strongest possible terms: How can an elementary proposition, a mere nexus of names, ever say anything? Why is an elementary proposition not a list? This way of posing the problem is useful in that it blocks any immediate and non-explanatory appeal to the unifying role performed by predicates and relational expressions (i.e. functions) within the proposition. This was Frege's tactic. Functions do, indeed, play a crucial unifying role within the proposition; but merely to assert this, as Frege does, is not to solve the problem but to state it.

It would appear, then, that a Wittgensteinian elementary proposition contains no functional signs but only names. And this makes the explanation of the possibility of propositional unity all the more imperative. Wittgenstein admitted as much; for immediately after introducing the concept of an elementary proposition he remarked: "Es frägt sich hier, wie kommt der Satzverband zustande", "Here the problem is, how does the propositional bond come about?" (4.22). That Wittgenstein will eventually appeal to a Fregean analysis in terms of function and object is, however, indicated by his remark at 5.47 that "Wherever there is composition, argument and function are present". And an elementary proposition is necessarily composite, being a concatenation of names.

Frege, we have seen, maintains that if one occurrence of a name is removed from a sentence there remains a functional expression, and this is not a name. In the following passage Wittgenstein takes the Fregean programme a step further:

If we turn a constituent of a proposition into a variable, there is a class of propositions all of which are values of the resulting variable proposition. In general, this class too will be dependent upon the meaning that our arbitrary conventions have given to parts of the original proposition. But if all the signs in it that have arbitrarily determined meanings are turned into variables, we shall still get a class of this kind. This one, however, is not dependent on any convention, but solely on the nature of the proposition. It corresponds to a logical form -- a logical proto-picture (einem logischen Urbild). (3.315).

Now it is not clear from the text whether Wittgenstein is here talking

about ordinary, unanalysed propositions, or about elementary propositions: let us assume that he had in mind the latter. If all the names which compose an elementary proposition are transformed into variables (or, as I shall prefer, into Fregean "holes"), then we can say that the result will be an elementary Urbild. What might an elementary Urbild look like? If we allow that the following is an elementary proposition in which are concatenated three object names:

(1) a b c

then, by extracting one of these names, we can generate something which corresponds closely to a Fregean concept-word:

(2) a () c

and by removing all the names we generate the elementary Urbild:

(3) [] () { }

The different styles of bracketing are here employed to show that different names, i.e. names of different objects, are to be inserted into the gaps.

All this, as yet, brings us no nearer to understanding why (1) above is not, after all, a list. Wittgenstein suggests why this is not so in the following passage, one of the more obscure in the Tractatus:

Instead of, 'The complex sign "aRb" says that a stands to b in the relation R', we ought to put, 'That "a" stands to "b" in a certain relation says that aRb'.
(3.1432).

Now even granted that "a" and "b" are here names of simple objects, still the propositional sign "aRb" cannot be elementary because it contains the sign "R" which is not a name of an object. Nonetheless what Wittgenstein is here saying is that the significant element in such a proposition, the

element which enables the proposition to say that a and b are thus related is that in the propositional sign itself "a" and "b" are related in such and such a way, i.e. they occur to the left and the right, respectively, of the sign "R". That "a" occurs to the left, and "b" to the right of the sign "R" is what enables the propositional sign "aRb" to say that a and b are related in some specific way. In other words the relations in which the signs stand to one another within the propositional sign can, via a rule of projection, show the relations between the objects in the state of affairs which the proposition asserts to obtain. And the function of the Urbild is, precisely, to capture or show these relations in which the signs (names) in an elementary proposition stand to one another. The Urbild shows the form of an elementary proposition, and Wittgenstein is claiming that this form has a significant (i.e. a signifying) role to play. In fact, the notion of the form of an elementary proposition has replaced the Fregean notion of a propositional function or concept. The Urbild (3) is generated in exactly the way in which are generated Fregean function-names: by the successive removal from an indicative sentence of the names which occur therein. Only in the case of an elementary propositional sign, all the elements are names. What Wittgenstein has shown is that even in this limiting case, a sentence is not a list of names, and that even here "function and argument are present". The Urbild, in representing the form of a proposition, is fulfilling the same role as the Fregean concept-word or predicate. Can we reverse this and say that a Fregean concept-word represents the form of a proposition?

We can; indeed this is just the account provided of the logical form of predicates in Chapter II above, where it was argued that a predicate is not, strictly speaking, an expression but a common property of a class of sentences. What is common to the sentences "A is drunk", "B is drunk", "C is drunk" etc. is not that they all contain the expression "is drunk", but that they are all built up by insertion of a proper name into the gap in the predicate "() is drunk". And what enables the sentence "A is drunk" to say that A is drunk is that the proper name "A" occurs to the immediate left of the expression "is drunk". What Wittgenstein has shown is that neither the words "is drunk" nor, indeed, any other predicative expression is logically necessary for the statement that A is drunk. (Such predicative expressions may, however, be practically valuable, even indispensable). What gets expressed by the prefixing of a proper name to the words "is drunk" could also be expressed by the concatenation of various names; and the concatenation would be represented by an Urbild. It must be emphasized, however, that the Urbild is not a part of the proposition. An elementary Urbild merely shows the relations in which the names stand to one another within a given class of propositions. An example will make this clear. If "[] ()" is an elementary Urbild, it might, under a certain rule of projection, indicate that the object referred to by the name which is substituted in the left-hand bracket is being said to be to the left of the object referred to by the name substituted in the right-hand bracket. More simply: if "a" and "b" are two simple names, then "a b" will say that a is to the left of b. Clearly the relations in which "[]" and "()" stand to one another in the Urbild "[] ()" are performing the

same function as the Fregean function-name "[] is to the left of ()". This is so, at least, given that rule of projection concerning that Urbild.¹⁷

All our problems concerning the unity of judgement are now centred in the notion of a rule of projection. For a rule of projection is that which enables "a b" (or "a is to the left of b") to say that a is to the left of b. But according to our modified Fregean principles, this is nothing more than the sense of such expressions. If we grant that the way in which the terms "a" and "b" contribute to this sense, by standing proxy for the two individuals a and b, is relatively unproblematic, then we are here dealing again with the notion of predicate sense, i.e. the concept. Now one is tempted to say that the sense of the predicate "() is to the left of ()", or the rule of projection for the Urbild "[] ()", is this: that whatever is taken as the first argument (in left-to-right order) is being claimed to be to the left of whatever is taken as the second argument. But this is strangely self-stultifying. For one thing, the explanation of what the predicate means employs the predicate itself. Secondly, we need here a general account of predicate sense, and not a (putative) definition of one particular predicate. But is such a general account even remotely possible? Is it not the case that any such account must suffer from a more general form of the circularity manifest by the (putative) particular definition given above? For any account of what a predicate is will, presumably, employ one or more predicates, and will therefore needs assume what it is trying to provide, namely an understanding of predication. Kant

recognized this clearly; he concluded that the activities of judgement and predication could not be taught:

If understanding in general is to be viewed as the faculty of rules, judgement will be the faculty of subsuming under rules; that is, of distinguishing whether something does or does not fall under a given rule. ... If it is sought to give general instructions how we are to subsume under these rules, that is, to distinguish whether something does or does not come under them, that could only be by means of another rule. This in turn, for the very reason that it is a rule, again demands guidance from judgement. And thus it appears that, though understanding is capable of being instructed, and of being equipped with rules, judgement is a peculiar talent which can be practiced only, and not taught. (B.172).

All such talk of the unteachable and the ineffable brings with it an atmosphere of paradox; but there is, strictly speaking, no paradox, no contradiction here. People do make judgements and communicate them to others, and moreover, they learn to do this. The problem is that the skills which are acquired in learning to judge and to speak are so basic, that any attempt to elucidate them in prosaic and superficial terms will run a considerable risk of being paradoxical, circular or self-contradictory. For example, if we elucidate what it is to follow a given rule in terms of another rule of higher order (and this is possible) then the temptation is to generalise this account (and this is impossible): for learning how to follow a rule in general cannot possibly depend upon our already being able to follow a rule! The hierarchy of rules must come to an end, and it does so, not in a rule of the highest level of generality,

but on the contrary, in actual practice:

"How am I able to obey a rule?" -- if this is not a question about causes, then it is about the justification for my following the rule in the way I do.

If I have exhausted the justifications I have reached bedrock, and my spade is turned. Then I am inclined to say: "This is simply what I do". (PI. §217).

Wittgenstein's point can be expanded as follows. When we ask of a given act, say the shaking of hands, how it can have the significance which it has for us, there are a number of different types of answers possible. The answer that seems to be most philosophically pertinent is that the action is conventional, that there are certain institutions and practices in the community in which the action occurs which render such an act an expression of friendship or trust or agreement. We say that such conventions or rules endow the movement of proffering and shaking another's hand with significance. But the rule is not a law which issues in imperatives, nor is it an empirical generalisation about the behaviour of individuals. For there are many regularities which are causally determined (like sneezing in the presence of pepper) which one would not describe as conventional. The rule does not make obligatory, nor does it even encourage hand shaking; and it does not describe the activity of hand shaking. It intimates that if one shakes hands with another under relatively normal circumstances, then that act may be construed by others as an expression of friendship etc. The rule is simply that people do this as a rule. But this is a fact.

If we ask why it is a fact, then we might be asking for a causal

explanation of why people shake hands, and the causal explanation might take many forms: historical, sociological, physiological, psychological and so on. But it is no part of the philosophical enterprise to provide such causal accounts of why we behave as we do. Nor would such a causal account explain the significance of such acts. To the question: Why do people do this? There is, secondly, the response in terms of their justification for so doing. Such justifications may be couched in two general sorts of terms. Either one can refer to peoples' motives and intentions in so acting, or one can talk about the objective justifications for following such a rule, the consequences and implications of so doing. But, again, these matters are matters of empirical fact, and as such are not the specific concern of the philosopher. There seems, in fact, to be no philosophical problem here. A rule is a conventional practice to which the actions of the members of a given community conform, other things being equal. Whatever more we want to say about rules, if what we say is to be informative and not circular or empty, must be about the facts. We can say that shaking hands is an expression (normally) of friendship or trust; but we cannot then ask how shaking hands is an expression of this, in a philosophical sense. For the only possibility would be to explain the act's significance in terms of its conformity to the rule, and to explain the rule in terms of the act's significance. This, however, is not to explain anything. The only thing one can say here is that "this is what we do": our philosophical problems have become factual problems. "You can't get behind the rules, because there isn't any behind". (PG.244)

What has been said above about such non-linguistic conventions, such as hand shaking, also applies to the conventions within which language users work. The rule for the use of the predicate expression "is red" can be stated thus: when this expression is prefixed by the name of an object, the result of uttering the whole expression is, normally, to say that that object is red. This tells us nothing, of course, though it does, in Wittgenstein's words, "give expression to the fact that we look to the rule for instruction and do something, without appealing to anything else for guidance. ... The rule is the last arbiter for the way I am to go" (PI.86). It also gives expression to the fact that rules are not linguistic entities, nor are they necessarily stateable in language, but are, rather, conventional practices amongst a community of people. If the rule were not the last arbiter, then Kant's vicious infinite regress would make the whole concept of a rule unintelligible. The concept of a rule or convention is not however unintelligible: it is merely that point in analysis at which certain philosophical problems are reduced to problems of fact.

The unity of judgement, then, is the unity of an act which conforms to a rule. The complex series of movements, raising one's right hand, extending it, grasping another's hand and shaking it up and down is only construable as one act when it is seen as conforming to the conventions of hand shaking. Otherwise it is merely a temporally related series of bodily movements. In the same way, conjoining the words "is dead" to the words "Julius Caesar" in the absence of the conventions governing the use of names and predicates in general and these expressions

In particular, is not an accomplishment to which we would ascribe the unity of the judgement that Caesar is dead. But the conventions governing the use of names and predicates in general is nothing but the syntax of the language (or at least a central part of it); and the conventions governing the use of the above mentioned expressions are what we have called their input sense.

(6) Summary

In the present thesis we have examined certain problems which arise in connexion with the notion of a judgement, and with the notion of a theory of judgement. Such problems concern the nature of assertion and predication; the unity of judgement; the relation between, on the one hand, concepts and propositions, and, on the other hand, between concepts and objects. It has been argued that Frege's works contain an analysis of concepts such as these which is, though only with modification and amplification, philosophically defensible.

In Chapter One the Fregean notion of a concept, as the reference of a function-name whose value for any object as argument is a truth-value, was examined. Frege's theory that a concept is the reference of an incomplete expression depends upon (i) his construing truth-values as objects referred to by sentences and (ii) his theory that concepts must yield a (truth-)value for any object whatsoever as argument. Neither of these positions, it was argued, are desirable or illuminating when the language under examination is the language of common discourse and not, say, the perspicuous notation of the concept-script. In which case,

the theory that a concept is the reference of a predicate is otiose, and must fall to Occam's Razor. But this leaves it incumbent upon us to provide an alternative theory of the semantic rôle performed by predicates and incomplete expressions. This task was begun in Chapter Two, and completed in the present chapter. It was argued that a concept is best elucidated in terms of an expression's sense rather than of its reference; for this is systematically more consistent, ontologically more economical and phenomenologically closer to linguistic experience. The sense of a proper name, we proposed, was its purporting to refer to a particular object of a given sort which bears that name. A predicate, on the other hand, is not strictly speaking an expression, but is rather the common form of certain sentences. And the sense of a predicate is, accordingly, as Frege said about its reference, "incomplete". This incompleteness is manifest by the empty pair of brackets which it always carries with it, and which, it was suggested, possess a given sortal shape. This sortal shape prevents the substitution of any object name in the argument place, and hence by-passes the Fregean demand that a truth-value be found, or stipulated, for any sentence which results from the insertion of a successfully referring name in the argument place of a predicate which is anywhere well-defined.

In Chapter Three we examined the development of Frege's views about assertion, and of his differing uses and explanations of the two concept-script signs "—|—" and "—|". We decided that there was little merit in the Begriffsschrift theory that every judgement has the same predicate, namely "() is a fact". But what this could be construed as

saying, namely that judgements have a common form, a complete and an incomplete part which together are potentially assertive, is philosophically interesting. It was also argued that there is no univocal sense of "assertion" of which the conflicting claims Frege makes about the meaning of the sign "—|—" are all true. "Assertion", that is, stands for a family of different but related phenomena which can be classified roughly as logical, pragmatic, psychological and syntactic. The signs "—|—" and "—|" can also be employed to mark the difference between an act of judgement and the content or object of that act.

In Chapter Four we examined in greater detail the related notions of predication, assertion and sense, which are the crucial ingredients in Frege's account of the Thought (Der Gedanke). At this point, however it was necessary to appeal to a tradition in philosophy to which Frege does not belong: the tradition represented by Kant and Wittgenstein. Only by appeal to such phenomena as the way language is used and learned and the way that thoughts are communicated could certain theses about sense and predication be substantiated. The substantiation took an essentially Kantian form: indeed, the final theory of judgement here defended is that which Kant adumbrated in the following passages:

All knowledge demands a concept ... But a concept is always, as regards its form, something universal which serves as a rule. (A.106).

Concepts rest on functions. By "function" I mean the unity of the act of bringing various representations under one representation. ... Now the only use which the understanding can make of these concepts is to

judge by means of them. ... Accordingly, all judgements are functions of unity among our representations. (B.93).

What Kant attempted to capture in his theory of the spontaneity of the activity of synthesis, what motivated Wittgenstein's Tractarian theory of the concatenation function shown by an Urbild, and what, with charity, we can say lay behind Frege's insistence upon the "unsaturatedness of concepts" was one and the same fundamental intuition: a judgement is an act, the act of putting together elements (whether psychical or physical or linguistic) in conformity with certain rules. It is in terms of these rules that the significance and unity of the act must be explained, and it is in the notion of a rule that the philosophical account of judgement comes to an end:

If I have exhausted the justifications I have reached bedrock, and my spade is turned. Then I am inclined to say: "This is simply what we do". (PI. §217).

NOTES

INTRODUCTION

1. Plato, The Sophist, translated by B. Jowett, The Dialogues of Plato, vol. III, p. 530.
2. Op. cit., p. 531.
3. TLP, 4.221. Cf. NB, 5, 37, 41.
4. B.105.
5. These terms are merely convenient labels and should not be interpreted literally.
6. See, e.g. F.H. Bradley, The Principles of Logic, 2nd edition, 1928, passim; Appearance and Reality, 1893, passim. G.E. Moore, "The Nature of Judgement", Mind, VII, (1899), 176-93. B. Russell, The Principles of Mathematics, 1903, 465f. (with A.N. Whitehead) Principia Mathematica, 2nd edition, Cambridge, 1925, Vol. 1, 43.
7. R.S. Wells, "Frege's Ontology", Review of Metaphysics, IV, (1951), 537-573.
8. I refer to Frege's booklet of 1879 as "the Begriffsschrift", and to the perspicuous notation which is outlined as the "concept-script".

CHAPTER I

1. The terminology is, on the whole, Fregean. I have, however preferred the term "predicate" to the cumbersome "concept-word" (Begriffswort), and have borrowed the terminology of simple and

and complex complete names from Furth.

The doctrinal assumptions which underlie Frege's account of functions and concepts (many of which are quite implausible) have been, in the first instance, merely expounded without criticism or comment. These assumptions and theories are subject to scrutiny in their own right, however, at a more appropriate place.

2. I learn from Mr. M.A.E. Dummett that the English translators of Frege's works have recently agreed upon a standard rendering for Frege's key terms. Most surprisingly they have chosen "meaning" as the translation of "Bedeutung". Although there are indeed strong arguments in favour of this move, I have thought it best to employ the translation which has been traditional for some time now.
3. The notion of sense is examined in detail below, in Chapter II.
4. Frege employs the lower case Greek letters " ξ " and " ζ " as gap-holders or place markers. They are not variables, and as signs without either sense or reference they cannot occur within the concept-script, but only in the informal explanation of the latter.
The capital letters " Δ " and " Γ " are used as if they stood for some determinate object, without its being stated which object this is. And so while " ξ^2 " is an incomplete expression, " Δ^2 " is a complete expression, though it is not known what object it refers to.
5. I shall refer to Frege's booklet of 1879 as the Begriffsschrift and the formal language which it outlined as the concept-script.
6. Fig.1 is the concept-script representation of what would today be written as " $(A \supset B) \supset (\Gamma \supset \sim \Delta)$ ".
7. Vida. e.g. Gg.15 and FuB.21-3.

8. C. Thiel, Sense and Reference in Frege's Logic, Dordrecht, 1968, 53.
9. Vide text corresponding to footnote 12 below.
10. We cannot here employ the gap-holder " ξ " to write " $|\text{---}\xi$ " because this would be to construe the judgement stroke as a function-name with an argument-place. Not only does Frege expressly forbid this, but, as we shall see in Chapter III, the judgement sign possesses semantical properties which make its use in the absence of a content impossible.
11. At least, this is so according to the account given above; which is why it must be modified.
12. This is the modification promised in the text corresponding to footnote 9.
13. As Geach has noted, there are exceptions to the rule that complex noun phrases cannot possess assertive force, eg. some containing the phrase "the fact that" like "John's being aware of the fact that $2+2=4$ ", which seems to contain the covert assertion that $2+2$ does = 4.
P.T. Geach, "Assertion", Philosophical Review, LXXIV, (1965).
14. G. Frege, "Über die Begriffsschrift des Herrn Peano und meine Eigene", 1897, p. 377.
Reprinted in I Angelelli (ed.) Kleine Schriften, p. 232.
15. These are only preliminary remarks; see below, Chapter II for more detailed treatment of the notion of sense.
16. The Logische Untersuchungen comprise the three published essays: "Der Gedanke" (1918) "Verneinung" (1918) and "Gedankengeflüge" (1923) and also the posthumously published "Logische Allgemeinheit" which was written in 1923 and published in Nach. in 1969.

17. Doch nicht nur ein Satzteil, auch ein ganzer Satz, dessen Sinn einer Gedanke ist, kann eine Bedeutung haben. Alle Sätze, die einen wahren Gedanken ausdrücken, haben dieselbe Bedeutung, und alle Sätze, die einen falschen Gedanken ausdrücken, haben dieselbe Bedeutung (das Wahre und das Falsche).
 "Aufzeichnung für Ludwig Darmstaedter", (1919). Repr: Nach.276.
18. Frege's creative life divides naturally into three periods:
 (1) Early period: until 1890
 (2) Middle period: 1890 to 1906
 (3) Late period: 1906 to 1923.
19. (a) When not examining the concept-script I shall employ the alternative convention of marking the argument place in a predicate expression with an empty pair of brackets. There is, of course, no significant difference between "() is drunk" and "ξ is drunk".
 (b) Strictly speaking "() is drunk" is not a fully fledged Fregean predicate; for the truth conditions of the sentence which results from its completion by a proper name are indeterminate because (i) there is no time indication, (ii) the concept of drunkenness is vague, and (iii) it is not, in ordinary language, defined for such objects as numbers, say, or pieces of furniture. These requirements are dealt with in the present section.
20. In more technical terminology: () = () is a first-level relation which can only take objects as arguments. The "analogous" relation for concepts Frege writes thus: $\lambda \xi \chi(\xi) = \lambda \xi \psi(\xi)$ or (ii) $\lambda \xi \chi(\xi) = \lambda \xi \psi(\xi)$.
 Expression (ii) would today be written as (iii) $(x)(\chi(x) \equiv \psi(x))$.
 Expression (ii) asserts the identity of the two Wertverlaufe of the functions $\chi(\)$ and $\psi(\)$.
 The quotation is from Frege's Review of Husserl's Philosophie der Arithmetik, translated in Mind, LXXXI, July 1972, 327.

21. M.A.E. Dummett, "Frege on functions; A Reply", Philosophical Review, LXIV, (1955), 96-107.
22. W. Marshall, "Frege's Theory of Functions and Objects", Philosophical Review, LXII, (1953), 374-390. Reprinted in Klemke; quotation on p. 267.
23. Frege has a disconcerting tendency to explicate the relation between Merkmale and Begriffe in terms of the relation of a whole to its parts. I have thought it best to avoid this usage which seems to me misleading. Perhaps Frege's clearest statement is this:
 "In my way of speaking, a thing can be at once a property and a mark, but not of the same thing. I call the concepts under which an object falls its properties... If the object x has the properties ϕ , χ , and ψ , I may combine them into Ω I then call ϕ , χ , and ψ marks of the concept Ω ." (BuG, 201).
24. J. van Heijenoort, "Logic as Calculus and Logic as Language", Synthese, XVII, (1967), 324-30.
25. P.T. Geach, "Quine on Classes and Properties", p. 410. Reprinted in Klemke p. 481.
26. This discussion is taken up again in Chapter II below.
27. Vide, eg. W.v.O. Quine, Elementary Logic, Harper and Row, N.Y., 1965, 1132-36.

CHAPTER II

1. Cf. the editor's comments in Nach.128. The article was written at approximately the same time as BuB.
2. J.R. Searle, "Russell's Objections to Frege's Theory of Sense and Reference", Analysis, XVIII, (1957), 137-143. Reprinted in Klemke, 337-345. The quotation appears on p.341 of the latter work.

3. The term "elucidation" is Wittgenstein's (cf. TLP.3.263 and 6.54) and will be discussed, along with the problem of saying what sense an expression expresses, in Chapter IV of the present thesis.
4. Cf. Russell, "The Philosophy of Logical Atomism" in (ed.) R.C. Marsh, Logic and Knowledge, p. 323. And F.H. Bradley, Principles of Logic, passim.
5. For example, P.T. Geach in Three Philosophers, p. 137; and F. Zabeh, What is in a Name? pp. 13-15.
6. Although the remarks in this section are an adequate statement of Frege's position, they will need to be modified in the light of the discussion below, Chapter III, passim.
7. Cf. Russell's remark in "On Denoting":
 "'a man' denotes not many men, but an ambiguous man."
 Frege would no more countenance "ambiguous men" than he would "variable numbers".
8. Wenn wir in zwei Sätzen dasselbe Wort, z.B. "Aetna", finden, so erkennen wir auch in den entsprechenden Gedanken etwas Gemeinsames, das diesem Worte entspricht. ...Nun, dieser Teil des Gedankens, der den Namen "Aetna" entspricht, kann nicht die Bedeutung dieses Namens sein. Dann wäre ja auch jedes einzelne Stück erstarrter Lava, das ein Teil des Aetna ist, auch Teil des Gedankens, dass der Aetna höher ist als Vesuv. Es scheint mir aber ungerneimt, dass Stücke Lava und zwar auch solche, von denen ich keine Kenntnis habe, Teile eines Gedankens sein sollen.
9. D.H. Lawrence, The Short Novels, Vol. I, London, 1956, Chapter III, p. 3.
10. The notion of a sortal concept, to which tacit appeal is here made, is further elucidated below, Chapter II, §4. It has been brought to my attention, in private correspondence with Prof. T. Chapman, that the doctrine concerning the sense of proper names defended in

the text corresponding to this footnote is similar to that propounded by P.T. Geach in Reference and Generality. Geach's "nominal essence" and my "sortal physiognomy" (see also below, Chapter II, §5) are, indeed, attempts to capture the same phenomenon; though Geach's approach is predominantly logical, mine predominantly epistemological.

11. Vide P.F. Strawson, Individuals, Chapter 6.
12. Cf. Semantic Syntax, (ed.) P.A.M. Seuren, Oxford, 1974, 101.
13. W. Marshall, "Frege's Theory of Functions and Objects", Philosophical Review, LXII, (1953), 374-390. (Also in Klamke, 249-267).
R. Grossmann, "Frege's Ontology", Philosophical Review, LXX, (1961), 23-40.
14. W. Marshall, "Sense and reference: a Reply", Philosophical Review, LXV, (1956), 342-361.
15. The letter is dated "Jena, den 24. Mai, 1891".
16. For further elucidation of Frege's rather obscure reasoning on this issue, see Frege's "Einleitung in der Logik" and "Logik in der Mathematik" (repr. in Nach pp. 201-212 and 219-270, respectively.)
Cf. also Dummett's Frege, 206-7.
17. I have ignored this whole trend in Frege's thought. It seems to me indefensible, and was, indeed, eventually repudiated by Frege himself. There is little to add to the clarification achieved by the two articles:
H. Jackson, "Frege on Sense Functions" Analysis, XXIII, (1962),
and M.A.E. Dummett, "Note: Frege on Functions", Philosophical Review, LXV, (1956).
18. In addition to the work of Strawson, see also D. Wiggins, Identity and Spatio-Temporal Continuity, Oxford (1967), pp. 1-3.

19. See above, Chapter II, §3.
20. See above, Chapter I, §5.
21. Cf. Strawson's remarks on this topic in Meaning and Truth, Oxford, 1970, pp. 4-5.
22. J. Hintikka, Logic, Language Games and Information, Oxford, 1973.
23. Op. cit., p. 99.
24. "Concept correlate" is R.S. Wells' name for the type of object which a nominalised predicate (e.g. "drunkenness") denotes. See R.S. Wells, "Frege's Ontology", Review of Metaphysics, IV, (1951), 537-573. See also Frege, BuG. 197-8, (Trans. in G&H. 46-47).
25. See above Chapter I, §5.

CHAPTER III

1. So far as I am aware there exist only two detailed treatments of the notion of assertion published in English: P.T. Geach, "Assertion", Philosophical Review, LXXIV, (1965), 448-54; M.A.E. Dummett, Frege, Chapter 10. Although the sign "—" has passed into general usage, it is not today employed with the meaning which Frege attached to it. It is now considered a unitary symbol which indicates that a proposition is a theorem, provable in a given system.
2. That S is P is a fact is the form in which Frege presents a singular judgement in the Begriffsschrift. In fact there are a number of different forms to which "...is a fact" could be attached, e.g.
 - The circumstance that S is P...
 - S's being P...
 - S's P-ing...and so on.

3. P.T. Geach, "Assertion", p. 452.
4. G. Kyle, "'If', 'So' and 'Because'", in (ed.) M. Black, Philosophical Analysis, Cornell, 1950.
 J.C. Wilson, Statement and Inference, Oxford, 1926.
 P.F. Strawson, Introduction to Logical Theory, London, 1952.
 L. Wittgenstein, Tractatus Logico-Philosophicus, London, 1922.
5. Cf. the explanation of the elements comprising the sign
 above, pp. 18-24.
6. See Begriffsschrift, pp. 3-4.
7. Lewis Carroll, "What the Tortoise said to Achilles", Mind, IV, (1895), 278-80.
8. P.E.B. Jourdain, "The Development of the Theories of Mathematica Logic and the Principles of Mathematics: Gottlob Frege", Quarterly Journal of Applied Mathematics XLIII (1912), 237-69.
9. Letter from Frege to Jourdain, undated but probably written in January 1914:
 Wenn ein Satz, der mit behauptender Kraft ausgesprochen wird, einem falschen Gedanken ausdrückt, so ist er logisch unbrauchbar und, genau genommen, unverständlich.
10. (a) For the genesis and development of Russell's and Wittgenstein's confusion over the (Meinongian) notion of an assumption (Annahme), see G.E.M. Anscombe, Introduction to Wittgenstein's Tractatus, pp. 105-6.
 These confusions need not concern us here: we can read "Thought" for "assumption" in the passage quoted from the Investigations.
 (b) Miss Anscombe translates "Satzradical" as "proposition radical". Although this may well be the best translation in the context in which it occurs, the use to which I wish to put the term makes "sentence radical" a better translation.

11. I do not mean to imply that what follows was Fraga's explicit and conscious intention, but merely that this interpretation conforms with the greater part of the textual evidence. It is almost certain that Fraga did not clearly distinguish the various elements he lumped together under the heading "assertive force".
12. One can of course infer things informally from a name. I know from the name "Dr. J. Smith", for example, that its bearer is a doctor. But this is a form of inference in which Fraga, and most logicians, are uninterested. It is clear, however, that "Dr. J. Smith" does not say anything, does not make a claim that could be either true or false. And for this reason it cannot constitute a premiss of an argument; no name can constitute a premiss, and this is as true of "—A" as it is of "Dr. J. Smith".
13. The phrase is not Dummett's, but on the whole the argument is. See Fraga, pp. 333-336.
14. See Chapter III section 3 above.
15. This is not to say that it might always be the case that such internal acts were without any corresponding overt physical manifestation.
16. I will not usually indicate whether it is an A-expression or an S-expression that is in question; for this ambiguity is usually resolved by the context. Throughout the present section, for example, it is with S-expressions that we are concerned.
17. Ced.62. Translated in Klemke.513.
18. " \Diamond " is here employed in the standard way, as the modal operator for possibility.
19. This division is further discussed in Chapter IV below.
20. This rule stands even in the case of exportation within an indirect proof; for in this case the indirect proof itself must be regarded as the "context" within which the proposition occurs.

21. Ged. 62. A propositional question is one that may be answered either "Yes" or "No".

CHAPTER IV

1. Though the present chapter concentrates upon Frege's 1918 essay, "Der Gedanke", these remarks must not be taken to imply that the other parts of the Logische Untersuchungen (see below) are exceptions to the generalizations made about Frege's philosophical position after 1918. The other parts of the Logische Untersuchungen are:
"Die Verneinung. Eine logische Untersuchung", (1918),
"Logische Untersuchungen. Dritter Teil: Gedankengefüge". (1923),
 and the posthumously published essay on Generality.
2. Dummett (Frege. 661) has suggested that Wittgenstein and Frege met early in 1914; but it is clear from the correspondence between Wittgenstein and Russell (on deposit in the Bertrand Russell Archives at McMaster University) that they had already met in 1912.
3. Nineteen years earlier G.E. Moore had produced virtually the same theory, in virtually the same words:

A proposition is composed not of words, nor yet of thoughts, but of concepts. Concepts are possible objects of thought...It is indifferent to their nature whether anybody thinks them or not. They are incapable of change; and the relation into which they enter with the knowing subject implies no action or reaction.

See "The Nature of Judgement", Mind, VII, (1899), *passim*.
4. These two principles were first introduced and discussed above, Chapter II, §1.

5. See D. Wiggins, "On Sentence Sense, Word Sense and Difference of Word Sense:." in (eds.) D.A. Steinberg and L.A. Jacobovits, Semantics. An Interdisciplinary Reader, Cambridge, 1971.
6. See, for example:
 - J. Teichman, "Propositions", Philosophical Review, LXX (1961), 500-17.
 - R. Cartwright, "Propositions" in (ed.) R.J. Butler, Analytical Philosophy, (1st series), Oxford, 1962, 81-103.
 - E.J. Lemmon, "Sentences, Statements and Propositions", in (eds.) B. Williams and A. Montefiore, British Analytical Philosophy, London, 1966.
7. But cf. the subsequent remarks about the relation of assertion to output sense. To say that the object of thought is a claim is not to say that it is necessarily assertive.
8. G. Frege, Die Grundlagen der Arithmetik, p. 48 Cf. Dummett, Frege pp. 192, and Wittgenstein, TLP.3.3.
9. D. Wiggins, op.cit.
10. Op. cit.
11. Diary entries for 23.11.1924 and 24.11.1924. (My translation).
12. P.T. Geach, "Assertion", p.457.
13. E. Husserl, Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie, 1913; translated by W.R. Boyce Gibson as Ideas: General Introduction to Pure Phenomenology. See especially Section II, Chapter I, §§30-2 concerning the possibility of the Epoché or phenomenological reduction.
14. P.F. Strawson, The Bounds of Sense, pp. 110 and 117.
15. G.E. Moore, "The Nature of Judgement", p. 180.
16. I have henceforth omitted the abbreviation "TLP" from references to Wittgenstein's Tractatus. The decimal numbers are indication enough of which work is being referred to.

17. It was, unfortunately, only after the completion of this thesis that I first read M.D. Resnik's articles "Frege's Theory of Incomplete Entities" (Philosophy of Science vol. 32, July-October 1965, pp. 329-341) and "Frege's Context Principle Revisited" (forthcoming). These articles contain an interesting discussion of some of the theses concerning sense, reference and "unsaturatedness" defended herein.

BIBLIOGRAPHY

The following abbreviations are employed in the Bibliography:

ASP: Archiv für systematische Philosophie.

BPI: Beiträge zur Philosophie des Deutschen Idealismus.

BVK: Berichte über die Verhandlungen der Königlich Sächsischen
Gesellschaft der Wissenschaften zu Leipzig.

J. of Phil.: The Journal of Philosophy

PAS: Proceedings of the Aristotelian Society.

PAS (Supp): Proceedings of the Aristotelian Society, Supplementary
series.

Ph. of Sc.: Philosophy of Science.

Philos.: Philosophy.

Phil. Rev.: The Philosophical Review.

Phil. Studs.: Philosophical Studies.

QJFAM.: The Quarterly Journal of Pure and Applied Mathematics.

Rev. Met.: The Review of Metaphysics.

VWP: Vierteljahresschrift für wissenschaftliche Philosophie.

ZPK: Zeitschrift für Philosophie und philosophische Kritik.

BIBLIOGRAPHY

- Ajdukiewicz, K. "On Syntactical Coherence", Rev. Mat. XX (1967),
635-47. (Transl. by P.T. Geach).
- Angelielli, I. Studies on Gottlob Frege and Traditional Philosophy,
Dordrecht, 1967.
- Anscombe, G.E.M. Intention, Oxford, 1957.
- -----

An Introduction to Wittgenstein's Tractatus, London,
1959.
- -----

Three Philosophers, (with P.T. Geach), Oxford, 1961.
- Austin, J. Philosophical Papers, Oxford, 1961.
- -----

How to do things with Words, Oxford, 1962.
- Bartlett, J.A. "On Questioning the Validity of Frege's Concept of
a Function", J. of Phil., LXI, (1964), 203. Reprinted
in Klemke. 407-8.
- Black, M. Problems of Analysis; Philosophical Essays, Ithaca,
1954.
- -----

Philosophy in America, London, 1965.
- -----

"Frege on Functions", in Problems of Analysis; Reprinted
in Klemke. 223-48.
- Bradley, F.H. Appearance and Reality, London, 1893.
- -----
The Principles of Logic (2nd ed.), London, 1928.

- Dummett, M.A.E. "Frege on Functions; a Reply", Phil. Rev., LXIV, (1955), 96-107.
- "Note, Frege on Functions", Phil. Rev., LXV, (1956), 229-30.
- "Nominalism", Phil. Rev., LXV, (1956), 491-505.
- "Truth", PAS, LIX, (1958), 141-62.
- Frege, The Philosophy of Language, London, 1973.
- (Ed., with J.N. Crossley), Formal Systems and Recursive Functions, Amsterdam, 1965.
- Feldman, P. "Sortal Predicates", Nous, VII, (1973), 268-282.
- Frege, G. Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens, Halle a. S., 1879.
- Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl, Breslau, 1884.
- Function und Begriff, Jena, 1891.
- "Über Sinn und Bedeutung", in ZPK, (1892), 25-50.
- "Über Begriff und Gegenstand", in VWP, XVI, (1892), 192-205.
- Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet, I, Jena, 1893.
- Review of E.G. Husserl, Philosophie der Arithmetik, I, in ZPK, (1894), 313-32.

- Frege, G. "Kritische Beleuchtung einiger Punkte in E. Schröders Vorlesungen über die Algebra der Logik", in ASP, I, (1895), 433-56.
- "Über die Begriffsschrift des Herrn Peano und meine eigene", in BVK, XLVIII (1897), 361-78.
- Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet, II, Jena, 1903.
- "Was ist eine Funktion?" in Festschrift Ludwig Boltzmann gewidmet zum sechzigsten Geburtstage, 20. Februar 1904, Leipzig, 1904, 656-66.
- Notes to P.E.B. Jourdain on "The Development of the Theories of Mathematical Logic and the Principles of Mathematics: Gottlob Frege", in OJPAM, XLIII, (1912), 237-69.
- "Der Gedanke", BPI, I, (1918), 58-77.
- "Die Verneinung. Eine logische Untersuchung", BPI, I, (1918), 143-57.
- "Logische Untersuchungen. Dritter Teil: Gedankengefüge", BPI, III, (1923), 36-51.
- Nachgelassene Schriften, (ed.) H. Hermes, F. Kambartel and F. Kaulbach, Hamburg, 1969.
- Furth, M. "Two Types of Denotation", in (ed.) N. Rescher, Studies in Logical Theory, Oxford, 1968, 9-45.
- Geach, P.T. "Quine on Classes and Properties", Phil. Rev., LXII, (1953), 409-12.

- Coach, P.T. "Class and Concept", Phil. Rev., LXIV, (1955), 561-70.
- Mental Acts, London, 1957.
- "Assertion", Phil. Rev. LXXIV, (1965), 448-54.
- Reference and Generality, emmended edition, Ithaca, 1968.
- (with G.E.M. Anscombe), Three Philosophers, Oxford, 1961.
- Logic Matters, Oxford, 1972.
- Griffin, J. Wittgenstein's Logical Atomism, Oxford, 1964.
- Haas, W. "The Theory of Translation", Philos. 37, (1962), 208-28.
- Harman, G. Thought, Princeton, 1973.
- (ed., with D. Davidson), Semantics of Natural Language, Dordrecht, 1972.
- Heinz, J. Subjects and Predicables, the Hague, 1973.
- Hintikka, J. Knowledge and Belief, Ithaca, 1962.
- Models for Modalities, Dordrecht, 1969.
- Jackson, H. "Frege on Sense Functions", Analysis, XXIII, (1962), 84-87.
- "Frege's Ontology", Phil. Rev., LXIX, (1960), 394-5.
- Jones, E.E.C. "Mr. Russell's Objections to Frege's Analysis of Propositions", Mind, XIX, (1910), 379-386.

- Jourdain, P. "The Development of the Theories of Mathematical Logic and the principles of Mathematics: Gottlob Frege", in QJPAM, XLIII, (1912), 237-69.
- Kant, I. The Critique of Pure Reason, transl. N. Kemp Smith, London, 1929.
- Kenny, A. Action, Emotion and Will, London, 1963.
- Klemke, E.D. (ed.) Essays on Frege, Urbana, 1968.
- Kneale, W. and M. The Development of Logic, Oxford, 1962.
- Kripke, S. "Naming and Necessity", in (eds.) D. Davidson and J. Hintikka, Semantics of Natural Language, Dordrecht, 1972.
- Lemmon, E.J. "Sentences, Statements and Propositions", in (eds.) B. Williams and A. Montefiore, British Analytical Philosophy, London, 1966.
- Long, P. "Are Predicates and Relational Expressions Incomplete?", Phil. Rev., LXVII, (1969), 90-98.
- Marshall, W. "Frege's Theory of Functions and Objects", Phil. Rev., LXII, (1953), 347-90.
- "Sense and Reference, a Reply", Phil. Rev., LXV, (1956), 342-61.
- Moore, G.E. "The Nature of Judgement", Mind, VII, (1899), 176-93.
- "Structure in Sentence and in Fact", Ph. of Sc., V, (1938).
- Some Main Problems of Philosophy, London, 1953.

- Patzig, G. Sprache und Logik, Göttingen, 1970
- Pitcher, G. (Ed.) Truth, Englewood Cliffs, 1964.
- (Ed.) Ryle, New York, 1970.
- Prior, A.N. "Oratio Obliqua", PAS (Supp.), XXXVII, (1963),
 115-26.
- "Indirect Speech and Extensionality", Phil. Studs.,
 15, (1964), 35-8.
- Objects of Thought, (eds. P.T. Geach and A. Kenny),
 Oxford, 1971.
- Quine, W.V.O. Elementary Logic, New York, 1965.
- "Designation and Existence", in J. of Phil., XXXVI,
 (1939); repr. in (eds.) H. Feigl and W. Sellars,
Readings in Philosophical Analysis, New York, 1949.
- Ramsey, F.P. "Universals", Mind, XXXIV, (1925), 401-7; repr. in.
- Foundations of Mathematics and Other Logical Essays,
 (ed.) R.B. Brathwaite, London, 1931.
- Russell, B. "On Denoting", Mind, XIV, (1905), 479-93.
- The Principles of Mathematics, London, 1903.
- Principia Mathematica, (with A.N. Whitehead), Cambridge,
 1910-3; 2nd ed. 1925.
- "The Philosophy of Logical Atomism" in (ed.) R.C. Marsh,
Logic and Knowledge, London, 1956, 175-282.
- Ryle, G. "Systematically Misleading Expressions", repr. in
Logic and Language, (ed.) A. Flew, 1st Series, Oxford,
 1952.

- Ryle, G. "If', 'So' and 'Because'" repr. in (ed.) M. Black, Philosophical Analysis, Cornell, 1950, 323-40.
- "Are There Propositions?", PAS., XXX, (1925), 401-7.
- Schwydar, D. "Uses of Language and Uses of Words", Theoria, XXVI, (1960), 31-43.
- Searle, J. Speech Acts; an Essay in the Philosophy of Language, London, 1970.
- "Russell's Objections to Frege's Theory of Sense and Reference", Analysis, XVII, (1957), 137-143.
- Seuren, P. (ed.) Semantic Syntax, Oxford, 1974.
- Sloman, A. "Functions and Rogators", in (eds.) J.N. Crossley and M.A.E. Dummett, Formal Systems and Recursive Functions, Amsterdam, 1965, 156-175.
- Strawson, P. "Truth", Analysis, IX, (1949), 83-97.
- "On Referring", Mind, LIX, (1950), 320-44.
- Introduction to Logical Theory, London, 1952.
- Individuals; an Essay in Descriptive Metaphysics, London, 1959.
- "Singular Terms and Predication", J. of Phil., LVII, (1961), 393-412.
- The Bounds of Sense, London, 1966.
- (ed.) Philosophical Logic, Oxford, 1967.
- Meaning and Truth, Oxford, 1970.

- Strawson, P. Subject and Predicate in Logic and Language, London, 1974.
- Summers, F. "Predicability", in (ed.) M. Black, Philosophy in America, London, 1965.
- Teichman, J. "Propositions", Phil. Rev., LXX, (1961), 500-517.
- Thiel, C. Sinn und Bedeutung in der Logik Gottlob Freges, Meisenheim Glan, 1967. English transl. by J. Binkley, Sense and Reference in Frege's Logic, Dordrecht, 1968.
- Walker, J. A Study of Frege, Oxford, 1965. *nd.*
- Wells, R.S. "Frege's Ontology", Rev. Met., IV, (1951), 537-573.
- Whitehead, A. See Russell, B.
- Wiggins, D. Identity and Spatio-Temporal Continuity, Oxford, 1967.
- "On sentence sense, word sense and difference of word sense. Towards a philosophical theory of dictionaries", in (eds.) D.D. Steinberg and L.A. Jacobovits, Semantics. An Interdisciplinary Reader, Cambridge, 1971.
- Wilson, J.C. Statement and Inference, Oxford, 1926.
- Wilson, N.L. The Concept of Language, 2nd ed., Toronto, 1959.
- "The Trouble with Meanings", Dialogue, IV, (1964), 52-64.
- Wittgenstein, L. Tractatus Logico-Philosophicus, trans. D.F. Pears and B.F. McGuinness, London, 1961.
- Philosophical Investigations, (ed.) G.E.M. Anscombe, G.H. von Wright and R. Rhees; trans. G.E.M. Anscombe, Oxford, 1953, 2nd ed. 1958.

Wittgenstein, L. Remarks on the Foundations of Mathematics, (ed.)
G.E.M. Anscombe, G.H. von Wright and R. Rhees; trans.
G.E.M. Anscombe, Oxford, 1956.

----- Notebooks, 1914-1916, (ed.) G.H. von Wright and G.E.M.
Anscombe; trans. G.E.M. Anscombe, Oxford, 1961.

----- Philosophical Grammar, (ed.) R. Rhees; trans. A. Kenny,
Oxford, 1974.

BIBLIOGRAPHYAPPENDIX A: Reprints of Frege's Works

Begriffsschrift und andere Aufsätze, (ed.) I. Angelelli, Darmstadt and Hildesheim, 2nd edition, 1964.

Kleine Schriften, (ed.) I. Angelelli, Darmstadt and Hildesheim, 1967.

Funktion, Begriff und Gegenstand. Fünf logische Studien, (ed.) G. Patzig, Göttingen, 2nd edition, 1966.

Logische Untersuchungen, (ed.) G. Patzig, Göttingen, 1966.

The Foundations of Arithmetic. A Logico-Mathematical Enquiry into the Concept of Number, (ed.) J.L. Austin, Oxford, 2nd edition, 1953.

Die Grundlagen der Arithmetik, Breslau, 1934; and Darmstadt and Hildesheim, 1962.

Grundgesetze der Arithmetik, Darmstadt and Hildesheim, 1962.

APPENDIX B: English Translations of Frege's Works

The Foundations of Arithmetic. A Logico-Mathematical Enquiry into the Concept of Number, (transl.) J.L. Austin, Oxford, 1953.

Translations from the Philosophical Writings of Gottlob Frege, (transl.) P.T. Geach and M. Black, Oxford, 1960.

The Basic Laws of Arithmetic, (transl.) M. Furth, Berkeley and Los Angeles, 1964.

"The Thought: a Logical Enquiry", (transl.) A. and M. Quinton, Mind, LXV, (1956), 289-311. Repr. in (ed.) E.D. Klemke, Essays on Frege, 507-35; and in (ed.) P.F. Strawson, Philosophical Logic, 17-38.

"Compound Thoughts", (transl.) R.H. Stoothoff, Mind, LXXII, (1963), 1-17. Repr. in (ed.) E.D. Klemke, Essays on Frege, 537-58.

Conceptual Notation and Related Articles, (ed. and transl.) T.W. Bynum, Oxford, 1972.