# Computationally Efficient Blind-Adaptive Algorithms for Multi-Antenna Systems

# COMPUTATIONALLY EFFICIENT BLIND-ADAPTIVE ALGORITHMS FOR MULTI-ANTENNA SYSTEMS

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A Thesis

Submitted to the School of Graduate Studies

in partial fulfilment of the requirements

for the degree

Doctor of Philosophy

McMaster University

DOCTOR OF PHILOSOPHY (2008) MCMASTER UNIVERSITY

(Electrical and Computer Engineering) Hamilton, Ontario, Canada

TITLE: **Computationally Efficient Blind-Adaptive Algorithms for Multi-Antenna Systems**

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NUMBER OF PAGES: xxii, 165

*To my wife Thevika*

*and*

*our dearest son Athavan*

# Abstract

Multi-input multi-output (MIMO) systems are expected to play a crucial role in future wireless communications and a significant increase of interest in all aspects of MIMO system design has been seen in the past decade. The primary interest of this thesis is in the receiver part of the MIMO system. In this area, continuous interest has been shown in developing blind-adaptive decoding algorithms. While blind decoding algorithms improve data throughput by enabling the system designer to replace training symbols with data, they also tend to perform robustly against any environment disturbances, compared to their training-based counterparts. On the other hand, considering the fact that the wireless end user environment is becoming increasingly mobile, adaptive algorithms have the ability to improve the performance of a system regardless of whether it is a blind system or a training-based one. The primary difficulty faced by blind and adaptive algorithms is that they generally are computationally intense. In this thesis, we develop semi-blind and blind decoding algorithms that are adaptive in nature as well as computationally efficient for multi-antenna systems.

First, we consider the problem of channel tracking for MIMO communication systems where the MIMO channel is time-varying. We consider a class of MIMO systems where

orthogonal space-time block codes (OSTBCs) are used as the underlying space-time coding schemes. For a general MIMO system with any number of transmitting and receiving antenna combinations, a two-step MIMO channel tracking algorithm is proposed. As the first step, Kalman filtering is used to obtain an initial channel estimate for the current block based on the channel estimates obtained for previous blocks. Then, in the second step, the so-obtained initial channel estimate is refined using a decision-directed iterative method. We show that, due to specific properties of orthogonal space-time block codes, both the Kalman filter and the decision-directed algorithm can be significantly simplified. Then, we extend the above receiver for MIMO-OFDM systems and propose a computationally efficient semi-blind receiver for MIMO systems in frequency-selective channels. Further, for the proposed receivers, we have derived theoretical performance analysis in terms of probability of error. Assuming the knowledge of the transmitted symbols for the first block, we have derived the instantaneous signal to interference and noise ratio (SINR) for consecutive transmission blocks in the absence of training, by exploiting Kalman filtering to track the channel in a decision-directed mode. Later, we extend the the theoretical performance limit comparisons for time-domain vs. frequency-domain channel tracking for MIMO-OFDM systems. Further, we study the advantage of adaptive channel tracking algorithms in comp-type pilot aided channel estimation schemes in practical MIMO-OFDM systems.

After that, an efficient sequential Monte-Carlo (SMC) algorithm is developed for blind detection in MIMO systems where OSTBCs are used as the underlying space–time coding scheme. The proposed algorithm employs Rao-Blackwellization strategy to marginalize out the (unwanted) channel coefficients and uses optimal importance function to generate samples to propagate the posterior distribution. The algorithm is blind in the sense that, unlike the earlier ones, the transmission of training symbols is not required by this scheme.

The marginalization involves the computation of (hundreds of) Kalman filters running in parallel resulting in intense computer requirement. We show that, the marginalization step can be significantly simplified for the specified problem under no additional assumptions — resulting in huge computational savings. Further, we extend this result to frequency selective channels and propose a novel and efficient SMC receiver for MIMO-OFDM systems.

Finally, a novel adaptive algorithm is presented for directional MIMO systems. Specifically, the problem of direction of arrival (DOA) tracking of an unknown time-varying number of mobile sources is considered. The challenging part of the problem is the unknown, time-varying number of sources that demand a combination of source enumeration techniques and sequential state estimation methods to track the time-varying number of DOAs. In this thesis, we transform the problem into a novel state-space model, and, by employing probability hypothesis density (PHD) filtering technique, propose a simple algorithm that is able to track the number of sources as well as the corresponding directions of arrivals. In addition to the fact that the proposed algorithm is simple and easier to implement, simulation results show that, the PHD implementation yields superior performance over competing schemes in tracking rapidly varying number of targets.

# Acknowledgements

First of all, I would like to express my deep sense of gratitude to my supervisor Dr. Kirubarajan, who offered me an opportunity to be a graduate student and conduct research under his guidance. I thank Dr. Kirubarajan for spending countless hours discussing my work, my research papers and this thesis.

I owe a great deal to Dr. Shahbazpanahi for his collaborative work with us. I am also grateful to Prof. Reilly and Dr. Sinha for taking time to share their knowledge and expertise in their research fields in our collaborative research efforts. Further, I thank Prof. Reilly and Dr. Hranilovic for being in my supervisory committee and for giving valuable suggestions on my research.

I am grateful to all the staff members of our department for their kind support. Especially, I thank Cheryl Gies and Helen Jachna for their constant help and support. Further, I thank Terry Greenlay and Cosmin Coriou who solved all computer related problems. I am grateful to my fellow graduate students of estimation, tracking and fusion laboratory (ETFLab) for their support during my studies. Especially, I acknowledge Sathyan and Nanthan for typing my comprehensive exam answers and Punithan for spending time with me in discussions about PHD filters. I am grateful to my cubicle neighbour, Michael Botros, for being helpful and supportive.

My life at McMaster has been very productive. During the past four years, my wife Thevika and I both worked towards our PhD's. Above all, during this period, we had our dearest son, Athavan, who has already hinted that he might be much more smarter than both of us. However, it is unfair to claim those as entirely our efforts without acknowledging my supervisor for being very understanding about all the no show-ups in the office and late submissions of papers. For the same reason, I thank Dr. Dmitri V. Malakhov, my wife's supervisor, who has been very flexible in fixing meeting times with her.

I thank my parents and siblings for their unconditional support of me during these four years and beyond.

None of this would have been possible without the love and support I received from Thevika and our wonderful Athavan. I have been learning a lot from Athavan. How to achieve what I want? How to love and to be loved? How to learn and how to teach? And, of course, how to speak good English. I thank him further for his help in whitening my (blue) coloured days into bright ones just by smiling at me. After all, that is why we named him after *The Sun.*

Finally, but most importantly, I would like to thank this wonderful country, Canada, for giving me the opportunity to have much of what I have. *O Canada! How shall I pay you back!*

# List of Acronyms

| | |
|---|---|
| AR1 | First order Auto-Regressive |
| AR2 | Second order Auto-Regressive |
| BER | Bit Error Rate |
| CP | Cyclic Prefix |
| CSI | Channel State Information |
| DDKF | Decision Directed Kalman Filter |
| DFT | Discrete Fourier Transform |
| DOA | Direction of Arrival |
| FFT | Fast Fourier Transform |
| IDDKF | Iterative Decision Directed Kalman Filter |
| IDFT | Inverse Discrete Fourier Transform |
| IFFT | Inverse Fast Fourier Transform |
| KF | Kalman Filter |
| MAN | Metropolitan Area Networks |
| MAP | Maximum a posteriori |
| MIMO | Multi-input Multi-output |
| MMSE | Minimum Mean Square Error |
| NE | Normalized Error |
| NMSE | Normalized Mean Square Error |

| | |
|---|---|
| OFDM | Orthogonal Frequency Division Multiplexing |
| OSTBC | Orthogonal Space-Time Block Codes |
| PF | Particle Filter |
| PHD | Probability Hypothesis Density |
| RADAR | RAdio Detection And Ranging |
| Rx | Receiver |
| SER | Symbol Error Rate |
| SINR | Signal to Interference and Noise Ratio |
| SISO | Single Input Single Output |
| SMC | Sequential Monte-Carlo |
| SNR | Signal to Ratio |
| SONAR | SOund NAvigation and Ranging |
| SS | Steady-State |
| Tx | Transmitter |
| WiMAX | Worldwide Interoperability for Microwave Access |
| WLAN | Wireless Local Area Network |

# List of Notations

| | |
|---|---|
| $\mathbf{x}$ | a vector |
| $\mathbf{X}$ | a matrix |
| $\mathrm{X}$ | a set |
| $j$ | $\sqrt{-1}$ |
| $(.)^T$ | the transpose of a vector or a matrix |
| $(.)^*$ | the conjugate of the argument |
| $(.)^H$ | the Hermitian transpose of a vector or a matrix |
| $(.)^{-1}$ | the inverse of a matrix |
| $\|\|.\|\|$ | the Euclidean norm of a vector |
| $\|.\|$ | absolute value of a scalar |
| $\mathbf{x}(n\|m)$ | the state of the variable $\mathbf{x}$ at time $n$ given all the information up to time $m$ |
| $\mathbf{I}_M$ | the identity matrix of dimension $M \times M$ |
| $\otimes$ | the Kronecker matrix product |
| $\mathrm{Re}\{.\}$ | the real part of the complex argument |
| $\mathrm{Im}\{.\}$ | the imaginary part of the complex argument |
| $\mathbf{e}_k$ | the $k$th column of identity matrix |
| $\mathrm{vec}\{.\}$ | the vectorization operator of a matrix stacking all columns one below the other |
| $\underline{\mathbf{X}}$ | the underline operator of matrix $\mathbf{X}$ – stacking real and imaginary parts of vec$\mathbf{X}$ one (former) below the other (later) |

| | |
|---|---|
| $N$ | number of transmitting antennas |
| $M$ | number of receiving antennas |
| $N_s$ | number of samples in particle filter implementation |
| $L$ | number sub-carriers in OFDM |
| $P$ | number of channel taps |
| $T$ | number of time blocks in OSTBC |
| $\mathbf{y}[n]$ | receiver observation at $n$th time interval |
| $A(w)$ | fast Fourier transform of the received signal $\mathbf{y}(n)$ |
| $\mathbf{G}[n]$ | channel matrix in time domain |
| $\mathbf{H}_i[n]$ | frequency response of the channel matrix at the $i$th sub-carrier |

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# Chapter 1

# Introduction

## 1.1 Multi Antenna Communication Systems

The objective of fourth generation (4G) communication systems is to achieve broadband connectivity *anytime, anywhere* between *anything* [27,40,48]. During the next two decades, this objective is likely to be achieved by employing multiple antennas at the base station (service provider) as well as at the end-user devices. In this regard, the past decade has seen an explosion of interest in multi-antenna systems, especially, multi-input multi-output (MIMO) communication systems.

MIMO communication systems provide a promising approach to deliver higher data throughput without the need for increased power and bandwidth. Space-time coding adds another dimension for the diversity space, in addition to the spatial diversity achieved by multiple antennas in the MIMO system. Among different space-time coding schemes presented in the literature, orthogonal space-time block codes (OSTBCs) [3], [109] are of particular interest because they achieve full diversity at a low receiver complexity.

OFDM is an attractive technique due to its capability to transform a frequency selective channel into a set of parallel flat fading channels [14, 26, 52, 70, 122]. Hence, MIMO systems combined with OFDM, known as MIMO-OFDM in the literature, is playing an important role in current wireless communications. For example, broadband wireless LAN technology, standardized as IEEE 802.11n, and worldwide inter-operability for microwave access (WiMAX) technology [33, 34, 45, 64, 73], standardized as IEEE 802.16, deploy OSTBCs for MIMO-OFDM communications.

Beamforming [112] is a spatial filtering technique in which antenna arrays are used to extract signals from the desired direction and to avoid receiving spatially interfering signals. Direction of arrival estimation and beamforming are complementary techniques in the sense that a beamformer requires the DOA of the desired user or the interferer in order to perform spatial filtering. Even though the terms spatial diversity and spatial filtering (separation) may imply different interests they can be combined to achieve higher data transmission rates. Directional MIMO antennas that are able to enjoy spatial multiplexing while avoiding co-channel interference are expected to provide higher data throughput compared to their omnidirectional MIMO counterparts [54, 60].

## 1.2 Overview of the Thesis

### 1.2.1 Motivation and objectives of the thesis

Conventional communication system designs spend up to to 20% of their available bandwidth on training which is better utilized for data transfer itself. It has always been the interest of academic and industrial communities to reduce the amount of training, and numerous approaches have already been proposed for blind decoding in MIMO systems. However, there are challenges remaining still for blind implementations. One of them is the fact that blind decoding algorithms are computationally intense, requiring additional hardware and power at tiny mobile terminals.

Further, most of the practical communication system designs and implementations assume that the statistical properties of the transmission medium is stationary or slow-varying. Even though the stationary assumption is good enough to achieve acceptable data throughput, it should be noted that whenever such assumption is violated, the data throughput decreases due to the increase in retransmission requests. Considering the increasingly mobile nature of today's wireless applications, non-stationarity is a reality that is hard to ignore. Another interesting feature of adaptive algorithms is that they have the capability to improve the performance of the training-based receivers under non-stationary conditions. Hence, finding a computationally efficient alternative to the stationary assumption is important. Indeed, there have been considerable interest shown on developing adaptive receiver algorithms for non-stationary environments.

Interestingly enough, it is realized that the successful implementation of both of the above approaches (adaptivity and blindness) have one common bottleneck, namely, computational complexity. Hence, it is highly important to be mindful of the fact that, any blind/adaptive decoding algorithm should be computationally efficient before they can be considered for implementation.

Motivated by the above facts, in this thesis, we present some blind adaptive decoding algorithms that are able to perform adequately under non-stationary environments in a computationally efficient manner.

### 1.2.2 Thesis outline

- Chapter 1 gives an introduction to multi-antenna communication systems and describes its potential to become part of future communication systems for years to come. Further, a brief discussion on currents challenges of MIMO systems is presented and the motivation for current and future research is argued.

- Chapter 2 provides a brief introduction to MIMO systems. Then, the concept of space-time coding is discussed and mathematical representation of orthogonal space-time block codes (OSTBC) are presented. Next, some special properties of OSTBCs are developed. After that, the flat-fading MIMO channel model is introduced and then the frequency selective channel model is developed and the relationship between time-domain and frequency-domain channel is derived. Finally, the MIMO-OFDM system is discussed.

- Chapter 3 gives some background information on Bayesian sequential state estimation. Starting from Bayes theorem, the sequential state estimation schemes for linear Gaussian case and non-linear non-Gaussian case, Kalman filter and particle-filter, respectively, are discussed. Then, a first moment approximation of the particle filter, the PHD filter, is discussed.

- Chapter 4 presents a semi-blind Kalman filter based decision-directed algorithm for MIMO systems under flat-fading channels. First, a literature search result on similar receivers is presented. Next the Kalman filter based conditional channel tracking scheme is presented. After that, a two step iterative scheme is proposed to improve the performance of the algorithm. Further, a novel scheme to reduce the computational complexity is also presented. Finally, simulation examples are illustrated regarding the performance of the proposed scheme.

- Chapter 5 presents a semi-blind Kalman filter based decision-directed algorithm for MIMO systems under frequency-selective fading channels. First a revision on previous MIMO-OFDM receivers is presented. Then, the conditional channel tracking scheme based on Kalman filter is presented. After that assuming constant modulus constellations, a novel way to simplify the algorithm and an iterative scheme to improve the performance have been presented. Finally, the performance of the algorithm and computational savings have been discussed through examples.

- Chapter 6 provides a theoretical performance analysis of the algorithms presented in Chapter4 and Chapter 5. First a background study on the related area is summarized. Then, an instantaneous signal to interference and noise ratio (SINR) in the decision directed mode has been derived and using this expression, analytical expression for BER is derived. Further, the performance related to different modes of the Kalman filtering channel estimation scheme, such as the predicted, updated and steady-state channel estimates have been discussed. Finally, based on the derivations, the performance of the receivers for various options are analyzed.

- Chapter 7 presents an efficient blind decoding scheme for MIMO systems employing OSTBCs. First, a literature review on SMC receivers for MIMO systems is presented. Next, a computationally efficient SMC receiver for MIMO systems under flat-fading channels is presented and using the simplification structure proposed in Chapter 4 the SMC receiver is simplified. After that, a novel tone-by-tone receiver is proposed for frequency selective fading channels and some methods to simplify it are presented. Finally, the computational savings achieved by the proposed receiver is discussed.

- Chapter 8 presents a new state-space approach to track the mobile users from the base-station in directional MIMO systems. First, literature study of the adaptive array processing techniques is presented. Then, the proposed state-space model is derived. Then, based on PHD filtering method, a novel DOA tracking algorithm that is able to track the DOAs of (time-varying) unknown number of targets using one snapshot of array observation. Finally, the results of the simulation studies are presented.

- Chapter 9 provides some concluding remarks on the thesis and provides possible future extensions to the research and discusses the future of the research in the related area.

### 1.2.3 Contributions

The scientific contribution of the thesis can be summarized under the following three areas.

- *Blind decoding schemes:*

  Starting with MIMO systems for flat-fading channels, we have developed a semi-blind decoding scheme that can be used for any number of transmitting/receiving antenna combinations. The scheme is called semi-blind because it requires the transmission of pilot symbols once in a while in order to re-initialize the channel estimates. Then, this result was extended for frequency-selective channels using OFDM technology. Further, we have proposed an efficient blind SMC receiver for MIMO systems under both flat-fading and frequency-selective channels.

- *Adaptive signal processing techniques:*

  The proposed receivers above are all adaptive under block-fading assumptions, i.e., they assume channel variations within one block of space-time transmission as negligible. The proposed receivers model the channel process as first order AR process and use Kalman filtering to track the channel variations. Further, a novel state-space approach is proposed for DOA tracking of unknown varying number of targets using passive sensor arrays.

- *Computation reduction techniques:*
  Firstly, we have extensively studied the nature of the signal models of the MIMO systems and OSTBCs and came up with a way to analytically simplify the above receivers. It is shown that some distinct properties of the OSTBCs enable us to achieve significant computational saving through simplification.
  Secondly, we have studied the use of approximation techniques in adaptive algorithms. Specifically, we have proposed the use of tracking approximate posterior for DOA tracking problem.

Specifically, the following scientific contribution has been achieved and reported in this thesis.

1. A novel decision-directed algorithm for channel tracking and data detection for MIMO systems employing generally defined OSTBCs.

2. An new way to exploit the special properties of the OSTBCs in diagonalizing the Kalman filter algorithm.

3. An efficient semi-blind receiver for MIMO-OFDM systems.

4. An efficient blind Sequential Monte carlo (SMC) receiver for MIMO systems employing OSTBCs in flat-fading channels.

5. A novel blind receiver for MIMO-OFDM systems based on Sequential Monte carlo (SMC) methods.

6. A theoretical performance analysis of the effect of time-selective channel.

7. A new state-space approach for computationally efficient adaptive array processing.

### 1.2.4 Related publications

Some selected contents of this thesis are published or submitted or for publication in the following journals and conferences.

**Journals articles:**

[1] B. Balakumar, S. Shahbazpanahi and T. Kirubarajan, "Joint MIMO Channel Tracking and Symbol Decoding Using Kalman Filtering", in press, *IEEE Transactions on Signal Processing.*

[2] B. Balakumar, S. Shahbazpanahi and T. Kirubarajan, " Simplified Kalman Filter for Tracking Doubly Selective Channels in Space-Time Coded MIMO-OFDM Systems: Algorithm and Performance Study", *to be submitted to IEEE Transactions on Wireless Communications.*

[3] B. Balakumar, T. Kirubarajan and J. P. Reilly, "Computationally Efficient Blind Sequential Monte-Carlo Receivers for MIMO Systems", *to be submitted to IEEE Transactions on Signal Processing.*

[4] B. Balakumar, T. Kirubarajan, A. Sinha, and J. P. Reilly "On the Use of PHD Filtering in Adaptive Array Processing", *to be submitted to IEEE Transactions on Signal Processing.*

**Conference papers:**

[5] B. Balakumar, S. Shahbazpanahi and T. Kirubarajan, "Joint MIMO channel tracking and symbol decoding for orthogonal space–time block codes", *14th European Signal Processing Conference*, Sept. 2006.

[6] B. Balakumar, S. Shahbazpanahi and T. Kirubarajan, "A Kalman filtering approach to joint MIMO channel tracking and symbol decoding for orthogonal space time block codes", *Fourth IEEE Workshop on sensor array and multi-channel processing*, pp. 244–248, July 2006.

[7] B. Balakumar, A. Sinha, T. Kirubarajan and J. P. Reilly, "PHD filtering for tracking an unknown number of sources using an array of sensors", *13th IEEE Workshop on Statistical Signal Processing*, pp. 43–48, July 2005.

# Chapter 2

# Multi-Input Multi-Output Systems

## 2.1 MIMO Systems and Space-Time Coding

Consider a MIMO system with $N$ transmit and $M$ receive antennas. In a time-varying flat-fading channel scenario, the signal received by the $p$th receive antenna at time $t$, $y_p(t)$ is given by

$$y_p(t) = \sum_{q=1}^{N} g_{pq}(t)x_q(t) + v_p(t) \tag{2.1}$$

where $x_q(t)$ is the signal transmitted from the $q$th antenna at time $t$, $g_{pq}(t)$ is the time-varying channel coefficient between the $p$th transmit antenna and the $q$th receive antenna, and $v_p(t)$ is the noise measured at the $p$th receive antennas. The noise $v_p(t)$ is assumed to be zero-mean complex Gaussian and spatio-temporally white with variance $\sigma_v^2/2$ per real dimension.

In discrete-time domain the received signal (2.1) can be written in vector format as

$$\mathbf{y}(n) = \mathbf{x}(n)\mathbf{G}(n) + \mathbf{v}(n) \tag{2.2}$$

where $\mathbf{G}(n)$ is the $N \times M$ channel matrix with its $(p, q)$ element equal to $g_{pq}(n)$, and

$$\mathbf{y}(n) = [y_1(n)\ y_2(n)\ \ldots\ y_M(n)] \tag{2.3}$$

$$\mathbf{x}(n) = [x_1(n)\ x_2(n)\ \ldots\ x_N(n)] \tag{2.4}$$

$$\mathbf{v}(n) = [v_1(n)\ v_2(n)\ \ldots\ v_M(n)] \tag{2.5}$$

are the row-vectors of the received signals, transmitted signals, and noise, respectively.

We consider a block transmission scheme and assume that within the block period $T$ the channel is fixed, i.e., the channel is assumed to be *quasi-static*. However, between different blocks the channel can change. Based on such an assumption, the $n$th received block can be written as

$$\mathbf{Y}(n) = \mathbf{X}(n)\mathbf{G}(n) + \mathbf{V}(n) \tag{2.6}$$

where $\mathbf{Y}(n)$ is the $T \times M$ matrix of the received signals, $\mathbf{X}(n)$ is the $T \times N$ matrix of transmitted signals, $\mathbf{V}(n)$ is the $T \times M$ matrix of noise, and $\mathbf{G}(m)$ is the $N \times M$ channel matrix during the $n$th block period. The noise $\mathbf{V}(n)$ is assumed to be zero-mean complex Gaussian and both spatially and temporally white with variance $\sigma_v^2/2$ per real dimension.

## 2.2 Orthogonal Space-Time Block Codes

### 2.2.1 General definition of OSTBCs

In space-time block coding, the matrix $\mathbf{X}(n)$ is a mapping that transforms a block of complex symbols to a $T \times N$ complex matrix. Hence, we hereafter replace $\mathbf{X}(n)$ with $\mathbf{X}(\mathrm{s}(n))$ where $\mathrm{s}(n)$ is the $n$th symbol vector of length $K$. Let us define $\mathrm{s}(n)$ as $\mathrm{s}(n) = [s_1(n)\ s_2(n)\ \cdots\ s_K(n)]^T$where $(\cdot)^T$ denotes the transpose operator. The $T \times N$ matrix $\mathbf{X}(\mathrm{s}(n))$ is called an OSTBC [3], [109] if i) all elements of $\mathbf{X}(\mathrm{s}(n))$ are linear functions of the $K$ complex variables $s_1(n), s_2(n), \ldots, s_K(n)$ and their complex conjugates, ii) and if, for any arbitrary s, $\mathbf{X}(\mathrm{s}(n))$ satisfies $\mathbf{X}^H(\mathrm{s}(n))\mathbf{X}(\mathrm{s}(n)) = \|\mathrm{s}(n)\|^2\mathbf{I}_N$ where $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\|\cdot\|$ is the Euclidean norm, and $(\cdot)^H$ denotes Hermitian transpose.

It follows from the definition of OSTBC that matrix $\mathbf{X}(\mathrm{s}(n))$ can be written as

$$\mathbf{X}(\mathrm{s}(n)) = \sum_{k=1}^{K}(\mathbf{C}_k\mathrm{Re}\{s_k(n)\} + \mathbf{D}_k\mathrm{Im}\{s_k(n)\}) \tag{2.7}$$

where $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real and imaginary parts, respectively, and $\mathbf{C}_k$ and $\mathbf{D}_k$ matrices are defined as[1]

$$\mathbf{C}_k = \mathbf{X}(\mathbf{u}_k) \tag{2.8}$$

$$\mathbf{D}_k = \mathbf{X}(j\mathbf{u}_k) \tag{2.9}$$

[1]In fact, any OSTBC is completely defined by the set of matrices $\{\mathbf{C}_k, \mathbf{D}_k\}_{k=1}^{K}$.

where $\mathbf{u}_k$ is the $k$th column of identity matrix $\mathbf{I}_K$ and $j = \sqrt{-1}$. Let us define the "underline" operator for a matrix $\mathbf{P}$ as

$$\underline{\mathbf{P}} \triangleq \begin{bmatrix} \text{vec}\{\text{Re}(\mathbf{P})\} \\ \text{vec}\{\text{Im}(\mathbf{P})\} \end{bmatrix} \tag{2.10}$$

where vec$\{\cdot\}$ refers to the vectorization operator stacking all the columns of a matrix on top of each other. Using (2.7) and (2.28), one can re-write (7.186) as [31]

$$\tilde{\mathbf{y}}(n) \triangleq \underline{\mathbf{Y}(n)} = \mathbf{A}\big(\mathbf{G}(n)\big)\,\tilde{\mathbf{s}}_n + \tilde{\mathbf{v}}_n \tag{2.11}$$

where $\tilde{\mathbf{s}}_n \triangleq \underline{\mathbf{s}(n)}$, $\tilde{\mathbf{v}}_n \triangleq \underline{\mathbf{V}(n)}$ and the $2MT \times 2K$ *real* matrix $\mathbf{A}\big(\mathbf{G}(n)\big)$ is given by

$$\mathbf{A}\big(\mathbf{G}(n)\big) = [\underline{\mathbf{C}_1\mathbf{G}(n)}\ \dots\ \underline{\mathbf{C}_K\mathbf{G}(n)}\ \underline{\mathbf{D}_1\mathbf{G}(n)}\ \dots\ \underline{\mathbf{D}_K\mathbf{G}(n)}]. \tag{2.12}$$

It has been shown that for any channel matrix $\mathbf{G}(n)$, the matrix $\mathbf{A}\big(\mathbf{G}(n)\big)$ satisfies the so-called *decoupling* property, i.e., its columns are orthogonal to each other and have identical norms [53]. More specifically, it satisfies

$$\mathbf{A}^T\big(\mathbf{G}(n)\big)\,\mathbf{A}\big(\mathbf{G}(n)\big) = \|\mathbf{G}(n)\|_F^2\mathbf{I}_{2K} \tag{2.13}$$

where $\|.\|_F$ denotes the Frobenius norm. Let us define the $2MN \times 1$ *time-varying channel vector* $\mathbf{g}(n)$ as $\mathbf{h(n)} \triangleq \underline{\mathbf{G}(n)}$. With a small abuse of notation, we hereafter replace $\mathbf{A}\big(\mathbf{G}(n)\big)$ with $\mathbf{A}\big(\mathbf{g}(n)\big)$. Therefore, we rewrite (2.13) as

$$\mathbf{A}^T\big(\mathbf{g}(n)\big)\,\mathbf{A}\big(\mathbf{g}(n)\big) = \|\mathbf{g}(n)\|^2\mathbf{I}_{2K} \tag{2.14}$$

Since $\mathbf{A}(\mathrm{g}(n))$ is linear in $\mathrm{g}(n)$, there exists a unique $4KMT \times 2MN$ matrix $\mathbf{\Phi}$ such that $\mathrm{vec}\{\mathbf{A}(\mathrm{g}(n))\} = \mathbf{\Phi}\mathrm{g}(n)$ where $\mathbf{\Phi}$ is a $4KMT \times 2MN$ matrix whose $k$th column is given by

$$[\mathbf{\Phi}]_k = \mathrm{vec}\{\mathbf{A}(\mathrm{e}_k)\} \,. \tag{2.15}$$

Here, $[\cdot]_k$ denotes the $k$th column of a matrix and $\mathrm{e}_k$ is the $k$th column of identity matrix $\mathrm{I}_{2MN}$. Note that matrix $\mathbf{\Phi}$ can be written as $\mathbf{\Phi} = [\mathbf{\Phi}_1^T \ \mathbf{\Phi}_2^T \ \cdots \ \mathbf{\Phi}_{2K}^T]^T$ where each sub-matrix $\mathbf{\Phi}_k$ $(k = 1, \ldots 2K)$ describes the linear relationship between the $k$th column of $\mathbf{A}(\mathrm{g}(n))$ and $\mathrm{g}(n)$, i.e., $[\mathbf{A}(\mathrm{g})]_k = \mathbf{\Phi}_k\mathrm{g}$.

### 2.2.2 ML decoding of OSTBCs

Given the channel vector $\mathrm{g}(n)$, the optimal ML decoder for OSTBCs consists of a linear receiver followed by a symbol-by-symbol decoder [28]. Indeed, the linear receiver computes $\hat{\tilde{\mathrm{s}}}_n$, the estimate of $\tilde{\mathrm{s}}_n$ as

$$\hat{\tilde{\mathrm{s}}}_n = \frac{1}{\| \mathrm{g}(n) \|^2}\mathbf{A}^T(\mathrm{g}(n))\, \tilde{\mathrm{y}}_n. \tag{2.16}$$

Then, the symbol-by-symbol decoder builds the estimate $\hat{\mathrm{s}}(n)$, of vector $\mathrm{s}(n)$ as $\hat{\mathrm{s}}(n) = [\mathbf{I}_K \ j\mathbf{I}_K]\hat{\tilde{\mathrm{s}}}_n$. The $k$th element of $\hat{\mathrm{s}}(n)$ is compared with all points in the constellation corresponding to $s_k(n)$ and the closest point in this constellation to the $k$th element of $\hat{\mathrm{s}}(n)$ is accepted as the $k$th decoded symbol.

Note however that implementation of the ML decoder requires the knowledge of the time-varying channel. If the channel is fixed, one can use training to estimate the channel in a non-blind fashion. However, in practice, the channel is time-varying, and hence tracking of the MIMO channel is required. Recently, blind channel estimation has been studied in the literature (see for example [99]). The blind channel estimation of [99] is based on the assumption that the channel is fixed, and hence, it is not applicable to time-varying channels.

## 2.3 MIMO Channel

### 2.3.1 Flat-fading MIMO channel

Without assuming any model for the MIMO channel, the problem of joint channel tracking and symbol detection is ill-posed. Fortunately, in many practical scenarios, the wireless channels can be modeled with a few parameters. It has been shown in [120] that the first-order AR model can be used as a sufficiently precise method to describe the time-varying behavior of wireless channels. Based on this model, we assume that the channel variation between adjacent blocks is modeled as a first order autoregressive (AR) model, i.e.,

$$\mathbf{G}(n) = \alpha \mathbf{G}(n-1) + \mathbf{W}(n) \tag{2.17}$$

where $\mathbf{W}(n)$ is an $N \times M$ noise matrix that is assumed to be zero-mean complex Gaussian with independent entries and variance of $\sigma_w^2/2$ per real dimension. This implies that $\mathbf{W}(n)$, and consequently $\mathbf{G}(n)$, are zero-mean wide-sense stationary processes. The parameter $\alpha$ is a complex scalar that can be estimated using the method of [111], and hence, it is herein assumed to be known. The noise variance $\sigma_w^2$ and $\alpha$ are related as $\sigma_w^2 = \sigma_g^2(1 - |\alpha|^2)$ where $\sigma_g^2$ is the variance of each element of $\mathbf{G}(n)$ and $|\cdot|$ denotes the amplitude of a complex number.

### 2.3.2 Frequency-selective fading MIMO channel

In frequency-selective environment, the channel at different taps will contribute to a certain received data. Lets assume the $N \times M$ channel matrix at the $p$th tap as $\mathbf{G}_p[n]$. For each tap $p$, we model the channel variation between adjacent OFDM words as a first order autoregressive (AR) model. Hence,

$$\mathbf{G}_p[n+1] = \alpha_p \mathbf{G}_p[n] + \mathbf{U}_p[n] \quad p = 0, \ldots, P-1 \tag{2.18}$$

where elements of $\mathbf{U}_p[n]$ are i.i.d Gaussian with zero mean and variance $\sigma_p^2 = \sigma_g^2(1 - |\alpha_p|^2)$ and $\sigma_g^2$ is the variance of the entries of $\mathbf{G}_p[n]$. The complex parameters $\alpha_p$, $p = 0, 1, \ldots, P-1$ can be estimated [111] and hence are assumed known here.

As noted in (2.23), the frequency response of the channel for the $i$th sub-carrier, $\mathbf{h}_i[n]$, is a function of the time domain MIMO channel impulse response $\{\mathbf{G}_p[n]\}_{p=0}^{P-1}$. Usually, the channel taps are much smaller in length compared to the number of tones. Hence, it is desirable to track the channel in time domain for better performance.

We now use the relationship between the channel frequency response and the channel impulse response to represent the observed data in terms of the channel impulse response. Later, this model will be used to derive the Kalman filtering based tracking scheme for the channel coefficients.

Using the Fourier relationship between the channel frequency response and channel impulse, $\mathbf{h}_i[n]$ can be written as

$$\mathbf{h}_i[n] = \mathbf{W}_i \mathbf{g}[n] \tag{2.19}$$

where

$$\mathbf{W}_i = \begin{bmatrix} \Re\{\mathbf{w}_i^T\} \otimes \mathbf{I}_{MN} & -\Im\{\mathbf{w}_i^T\} \otimes \mathbf{I}_{MN} \\ \Im\{\mathbf{w}_i^T\} \otimes \mathbf{I}_{MN} & \Re\{\mathbf{w}_i^T\} \otimes \mathbf{I}_{MN} \end{bmatrix}. \tag{2.20}$$

Here, $\otimes$ is the Kronecker product, $\mathbf{g}[n] = \underline{\mathbf{G}[n]}$, $\mathbf{G}[n] = (\mathbf{g}_0[n] \quad \ldots \quad \mathbf{g}_{P-1}[n])$, $\mathbf{g}_p[n] = \mathrm{vec}\{\mathbf{G}_p[n]\}$, $\mathbf{w}_i = [1, e^{j2\pi i/L}, \ldots, e^{j2\pi i(P-1)/L}]^{\mathrm{H}}$. Also, we can write

$$\mathbf{h}[n] = \mathbf{W}\mathbf{g}[n] \tag{2.21}$$

where $\mathbf{W} = \left[\mathbf{W}_0^T, \ldots, \mathbf{W}_{L-1}^T\right]^T$.

## 2.4 MIMO-OFDM System

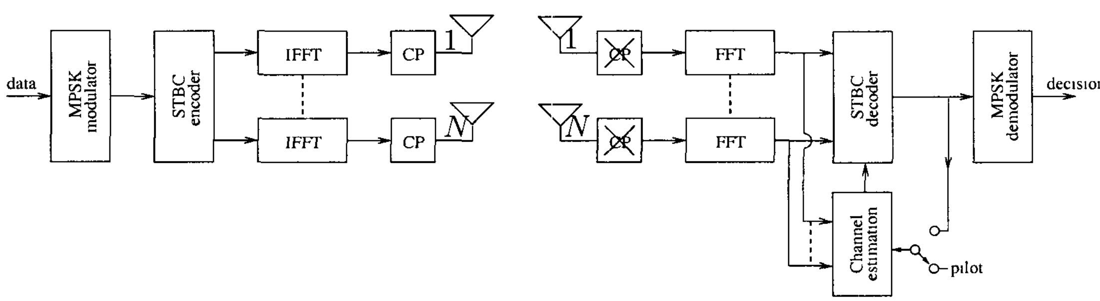


Figure 2.1: Block diagram of the MIMO-OFDM system

Consider a MIMO-OFDM system with $L$ sub-carriers, $N$ transmit antennas and $M$ receive antennas. Using vector notations, the received signal vector for the $i$th tone during the $n$th OFDM word can be written as [129]

$$\mathbf{y}_i[n] = \mathbf{x}_i[n]\mathbb{H}_i[n] + \mathbf{v}_i[n] \tag{2.22}$$

where $\mathbf{y}_i[n]$ is the $1 \times M$ vector of received signals, $\mathbf{x}_i[n]$ is the $1 \times N$ vector of transmitted signals, and $\mathbf{v}_i[n]$ is the $1 \times M$ vector of noise at the $i$th tone. The noise is assumed to be uncorrelated zero-mean complex Gaussian with variance $\sigma_v^2/2$ per real dimension. The $N \times M$ complex matrix $\mathbf{H}_i[n]$ is the frequency response of the MIMO channel at the $i$th sub-carrier and can be written, in terms of the time-domain channel impulse response, as

$$\mathbf{H}_i[n] = \sum_{p=0}^{P-1} \mathbf{G}_p[n] e^{-j2\pi ip/L} \tag{2.23}$$

where $j = \sqrt{-1}$, the $(r, q)$th element of $\mathbf{G}_p[n]$ is the $p$th tap of the time-domain channel impulse response from the $r$th transmitter to the $q$th receiver, and $P$ is the maximum delay spread of the channel. We assume that $P < L$. We consider a block transmission scheme with block length $T$ and assume that within the OFDM word length $PT$, the channel frequency response is fixed. However, between different OFDM words the channel frequency response can change. Based on such an assumption, the $n$th received block for the $i$th sub-carrier can be written as

$$\mathbf{Y}_i[n] = \mathbf{X}_i[n]\mathbf{H}_i[n] + \mathbf{V}_i[n] \tag{2.24}$$

where

$$\begin{aligned}
\mathbf{Y}_i[n] &= \left[\mathbf{y}_i^T[nT - T + 1], \cdots, \mathbf{y}_i^T[nT]\right]^T \\
\mathbf{X}_i[n] &= \left[\mathbf{x}_i^T[nT - T + 1], \cdots, \mathbf{x}_i^T[nT]\right]^T \\
\mathbf{V}_i[n] &= \left[\mathbf{v}_i^T[nT - T + 1], \cdots, \mathbf{v}_i^T[nT]\right]^T
\end{aligned}$$

denote the received data, the transmitted data, and the measurement noise matrices, respectively, and $(\cdot)^T$ represents the transpose operator.

Now, the received blocks due to all the sub-carriers, the $n$th received OFDM word, can be written as

$$\mathbb{Y}[n] = \mathbb{X}[n]\mathbb{H}[n] + \mathbb{V}[n] \tag{2.25}$$

where

$$\mathbb{Y}[n] = \begin{pmatrix} \mathbf{Y}_0[n] \\ \vdots \\ \mathbf{Y}_{L-1}[n] \end{pmatrix}, \mathbb{X}[n] = \begin{pmatrix} \mathbf{X}_0[n] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{X}_{L-1}[n] \end{pmatrix}$$

$$\mathbb{H}[n] = \begin{pmatrix} \mathbf{H}_0[n] \\ \vdots \\ \mathbf{H}_{L-1}[n] \end{pmatrix}, \mathbb{V}[n] = \begin{pmatrix} \mathbf{V}_0[n] \\ \vdots \\ \mathbf{V}_{L-1}[n] \end{pmatrix} \tag{2.26}$$

In space-time block coding, the matrix $\mathbf{X}_i[n]$ is a mapping that transforms a block of complex symbols into a $T \times N$ complex matrix. Hence, we hereafter replace $\mathbf{X}_i[n]$ with $\mathbf{X}(\mathrm{s}_i[n])$ where $\mathrm{s}_i[n]$ of length $K$ is the $i$th transmitted symbol vector of the $n$th OFDM word. Let us define $\mathrm{s}_i[n]$ as $\mathrm{s}_i[n] = (s_{1,i}[n] \; s_{2,i}[n] \; \cdots \; s_{K,i}[n])^T$. By selecting $\mathbf{X}(\mathrm{s}_i[n])$ as an OSTBC [53, 109], we can write the observed data of the $i$th sub-carrier as [53]

$$\tilde{\mathbf{y}}_{n,i} \triangleq \underline{\mathbf{Y}_i[n]} = \mathbf{A}\big(\mathbb{h}_i[n]\big)\, \tilde{\mathbf{s}}_{n,i} + \tilde{\mathbf{v}}_{n,i} \tag{2.27}$$

where $\mathbf{h}_i[n] = \underline{\mathbf{H}_i[n]}, \tilde{\mathbf{s}}_{n,i} = \underline{\mathbf{s}_i[n]}, \tilde{\mathbf{v}}_{n,i} = \underline{\mathbf{V}_i[n]}$, the "underline" operator for a matrix $\mathbf{P}$ is defined as

$$\underline{\mathbf{P}} \triangleq \begin{bmatrix} \text{vec}\{\Re(\mathbf{P})\} \\ \text{vec}\{\Im(\mathbf{P})\} \end{bmatrix} \tag{2.28}$$

and vec$\{\cdot\}$ refers to the vectorization operator stacking all the columns of a matrix on top of each other, and $\Re(\cdot)$ and $\Im(\cdot)$ represent the real and imaginary parts, respectively. The matrix $\mathbf{A}(\cdot)$ is the so-called vitual channel matrix and for any real $2MN \times 1$ vector $\mathbf{h}_i[n] = \underline{\mathbf{H}_i[n]}$ is given by [31]

$$\mathbf{A}(\mathbf{h}_i[n]) = [\underline{\mathbf{X}(\mathbf{e}_1)\mathbf{H}_i[n]} \; \ldots \; \underline{\mathbf{X}(\mathbf{e}_K)\mathbf{H}_i[n]} \; \underline{\mathbf{X}(j\mathbf{e}_1)\mathbf{H}_i[n]} \; \ldots \; \underline{\mathbf{X}(j\mathbf{e}_K)\mathbf{H}_i[n]}] \tag{2.29}$$

where $\mathbf{e}_k$ is the $k$th column of the $K \times K$ identity matrix $\mathbf{I}_K$.

# Chapter 3

# Bayesian Filtering Theory

## 3.1 Bayesian Theorem

Consider a random parameter $\mathbf{x}(n)$ and its observation $\mathbf{y}(n)$. Bayes theorem enables us to write the joint probability of $\mathbf{x}(n)$ and $\mathbf{y}(n)$ as follows.

$$p(\,\mathbf{x}(n)|\mathbf{y}(n)\,) = p(\,\mathbf{x}(n)|\mathbf{y}(n)\,)p(\,\mathbf{y}(n)\,) = p(\,\mathbf{y}(n)|\mathbf{x}(n)\,)p(\,\mathbf{x}(n)\,) \tag{3.30}$$

where $p(\mathbf{x}(n))$ and $p(\mathbf{y}(n))$ are the prior densities $p(\mathbf{y}(n)|\mathbf{x}(n))$ is the likelihood and $p(\mathbf{x}(n)|\mathbf{y}(n))$ is called the posterior distribution.

The posterior distribution contains all the statistical information about the unknown parameter $p(\mathbf{x}(n))$ from time 1 to time $n$ given the corresponding observations $p(\mathbf{y}(1))$ to $p(\mathbf{y}(n))$. The posterior distribution can be used to get an estimate of the unknown parameter at a certain time instance. For example, a maximum a posteriori (MAP) estimate of the parameter is obtained as

$$\hat{\mathbf{x}}^{\mathrm{MAP}} = \arg\max_{\mathbf{x}(n)} p(\mathbf{x}(n)|\mathbf{y}(n)) \tag{3.31}$$

## 3.2 Bayesian Filtering

### 3.2.1 Bayesian filtering of single target

First, for simplicity, consider the following single target versions of the state and observation equations

$$\begin{aligned} \mathbf{x}(n) &= \mathbf{f}(\mathbf{x}(n-1), \mathbf{v}(n)) && (3.32) \\ \mathbf{y}(n) &= \mathbf{h}(\mathbf{x}(n), \mathbf{w}(n)). && (3.33) \end{aligned}$$

Bayesian approach tends to track the *posterior density* of the current state

$$p(\mathbf{x}(n)|\mathbf{y}(n)) \tag{3.34}$$

assuming that the initial density $p(\mathbf{x}(0)|\mathbf{y}(0))$ or the *prior* is available.

Given the posterior density at time $n$, $p(\mathbf{x}(n)|\mathbf{y}(n))$, and the observation at time $n+1$, $\mathbf{y}(n+1)$, the posterior at time $n+1$, $p(\mathbf{x}(n+1)|\mathbf{y}(n+1))$, is constructed in the following two steps

[1] *Prediction*

$$p(\mathbf{x}(n+1)|\mathbf{y}(n)) = \int p(\mathbf{x}(n+1)|\mathbf{x}(n))p(\mathbf{x}(n)|\mathbf{y}(n))d\mathbf{x}(n) \tag{3.35}$$

[2] *Update*

$$p(\mathbf{x}(n+1)|\mathbf{y}(n+1)) = \frac{p(\mathbf{y}(n+1)|\mathbf{x}(n+1))p(x(n+1)|\mathbf{y}(n))}{p(\mathbf{y}(n+1)|\mathbf{y}(n))} \tag{3.36}$$

where

- $p(\mathbf{x}(n+1)|\mathbf{y}(n))$ is the time prediction of the posterior $p(\mathbf{x}(n)|\mathbf{y}(n))$ to the next time step.
- $p(\mathbf{x}(n+1)|\mathbf{x}(n))$ is the Markov transition density that models the motion of the target from time $n$ to $n+1$.
- $p(\mathbf{y}(n+1)|\mathbf{x}(n+1))$ is the sensor likelihood function.
- The normalizing constant $p(\mathbf{y}(n+1)|\mathbf{y}(n))$ is given by

$$p(\mathbf{y}(n+1)|\mathbf{y}(n)) = \int p(\mathbf{y}(n+1)|\mathbf{x}(n+1))p(\mathbf{x}(n+1)|\mathbf{y}(n))d\mathbf{x}(n+1) \tag{3.37}$$

### 3.2.2 Bayesian filtering of multiple targets

Similar to (3.35) & (3.36), the multi-target Bayesian filtering can be summarized as follows

$$p(X(n+1)|Y(n)) = \int p(X(n+1)|X(n))p(X(n)|Y(n))dX(n) \tag{3.38}$$

$$p(X(n+1)|Y(n+1)) = \frac{p(Y(n+1)|X(n+1))p(x(n+1)|Y(n))}{p(Y(n+1)|Y(n))} \tag{3.39}$$

where $X(n)$ and $Y(n)$ are multi–target state state set and observation set, respectively, i.e.,

$$X(n) = \Big\{\{\}, \{\mathbf{x}_1(n)\}, \{\mathbf{x}_1(n), \mathbf{x}_2(n)\}, \ldots, \{\mathbf{x}_1(n), \ldots, \mathbf{x}_{n_X}(n)\}\Big\} \tag{3.40}$$

$$Y(n) = \Big\{\{\}, \{\mathbf{y}_1(n)\}, \{\mathbf{y}_1(n), \mathbf{y}_2(n)\}, \ldots, \{\mathbf{y}_1(n), \ldots, \mathbf{y}_{n_Y}(n)\}\Big\} \tag{3.41}$$

where the null set denotes the possibility of the presence of no target/observation at a certain instant.

Computing (3.38) & (3.39) can be done by exploiting the knowledge of finite–set statistics (FISST) [68]. Further, even when the number of targets is small, the multi–target Bayesian filtering above may not be a desired approach due to its demand for high computational cost. Hence, alternate methods were sought and as a result, PHD filtering [68], tracking the first moment of the posterior rather than the full posterior itself, was found to be a reasonable alternative in terms of computational requirement and performance.

## 3.3 Bayesian Filtering for Linear Gaussian Systems

In a linear Gaussian system, i.e., $\mathcal{H}(\mathbf{x})$ is now written as $\mathbf{H}\mathbf{x}$, propagating the full posterior is equivalent to propagating the first two moments given by

$$\mathbf{x}(n|n) = \int \mathbf{x}(n) p(\mathbf{x}(n)|\mathbf{y}(n)) d\mathbf{x}(n) \tag{3.42}$$

$$\mathbf{C}(n|n) = \int \mathbf{x}(n)\mathbf{x}(n)^T p(\mathbf{x}(n)|\mathbf{y}(n)) d\mathbf{x}(n). \tag{3.43}$$

Hence, Bayesian tracking or propagating the full posterior for single targets under linear

Gaussian assumption is summarized as

$$\begin{array}{ccccc} p(\mathbf{x}(n)|\mathbf{y}(n)) & \stackrel{prediction}{\longrightarrow} & p(\mathbf{x}(n+1)|\mathbf{y}(n)) & \stackrel{update}{\longrightarrow} & p(\mathbf{x}(n+1)|\mathbf{y}(n+1)) \\ \left\{ \begin{array}{c} \mathbf{x}(n|n) \\ \mathbf{P}(n|n) \end{array} \right\} & & \left\{ \begin{array}{c} \mathbf{x}(n+1|n) \\ \mathbf{P}(n+1|n) \end{array} \right\} & & \left\{ \begin{array}{c} \mathbf{x}(n+1|n+1) \\ \mathbf{P}(n+1|n+1) \end{array} \right\} \end{array} \quad (3.44)$$

where

$$\mathbf{P}(n|n) = \mathbf{C}(n|n) - \mathbf{x}(n|n)\mathbf{x}^T(n|n) \quad (3.45)$$

$$\mathbf{x}(n+1|n) = \mathbf{F}\mathbf{x}(n|n) \quad (3.46)$$

$$\mathbf{P}(n+1|n) = \mathbf{F}\mathbf{P}(n|n)\mathbf{F}^T + \mathbf{Q} \quad (3.47)$$

$$\mathbf{x}(n+1|n+1) = \mathbf{x}(n+1|n) + \mathbf{G}(n+1)\boldsymbol{\nu}(n+1) \quad (3.48)$$

$$\mathbf{P}(n+1|n+1) = \mathbf{P}(n+1|n) - \mathbf{G}(n+1)\mathbf{P}_{\nu}(n+1)\mathbf{G}^T(n+1) \quad (3.49)$$

and $\boldsymbol{\nu}(n+1)$, $\mathbf{P}_{\boldsymbol{\nu}}(n+1)$ and $\mathbf{G}(n+1)$ are the innovation, innovation–covariance and the Kalman gain, respectively [8].

## 3.4 Approximate Bayesian Filtering

Unlike the linear Gaussian case, the analytical solution for the propagation of posterior over time is not feasible for a general case. Often, the posterior distributions are propagated in the form of a histogram by employing sequential Monte-Carlo (SMC) methods [25]. While this methods provides an attractive solution for many of the problems for which any analytical form solution is unthinkable, the computational requirement is also relatively high. Hence, huge interest was shown in the past on producing computationally feasible implementations of SMC methods.

In this section we discuss an approximate implementation schemes for SMC methods. First, we discuss a relatively known case, the first moment approximate Bayesian filtering in linear Gaussian case, known as $\alpha,\ \beta,\ \gamma$ *filter* in the literature [8]. Then, we discuss the general case of the first moment filter, the PHD filter.

### 3.4.1 Approximation for linear Gaussian systems

The approximate filtering for linear Gaussian systems can be summarized as follows.

$$\begin{array}{ccccc} \tilde{p}(\mathbf{x}(n)|\mathbf{y}(n)) & \stackrel{prediction}{\longrightarrow} & \tilde{p}(\mathbf{x}(n+1)|\mathbf{y}(n)) & \stackrel{update}{\longrightarrow} & \tilde{p}(\mathbf{x}(n+1)|\mathbf{y}(n+1)) \\ \left\{\ \mathbf{x}(n|n)\ \right\} & & \left\{\ \mathbf{x}(n+1|n)\ \right\} & & \left\{\ \mathbf{x}(n+1|n+1)\ \right\} \end{array} \quad (3.50)$$

where

$$\mathbf{x}(n+1|n) = \mathbf{F}\mathbf{x}(n|n) \quad (3.51)$$

$$\mathbf{x}(n+1|n+1) = \mathbf{x}(n+1|n) + \mathbf{G}\boldsymbol{\nu}(n+1). \quad (3.52)$$

and the expression $\tilde{p}(.)$ denotes that the distribution is only approximate.

Note that, the Kalman gain above is not a function of $n$, rather, it is a matrix of constants. Examples of constant gain Kalman filter include the $\alpha, \beta$ and the $\alpha, \beta, \gamma$ filters [8].

### 3.4.2 Approximation for general systems

The first moment of the posterior density (3.34), also known as the PHD, is defined as follows [68]

$$\mathrm{D}(n|n) = \mathrm{D}(\mathbf{x}(n)|Y(n)) = \int p(\{\mathbf{x}(n)\} \cup Z|Y(n))dZ \tag{3.53}$$

PHD holds a unique property that its integral on any region $S$ of the state space is the expected number of targets contained in $S$, i.e.,

$$N(n|n) = \int_S \mathrm{D}(\mathbf{x}(n)|Y(n:n))d\mathbf{x}(n). \tag{3.54}$$

Hence, the above property could be considered as an alternate definition for the PHD.

The propagation of PHD over time is summarized as [68]

$$\begin{array}{ccccc} p(X(n)|Y(n)) & \stackrel{prediction}{\longrightarrow} & p(X(n+1)|Y(n)) & \stackrel{update}{\longrightarrow} & p(X(n+1)|Y(n+1)) \\ \downarrow & & \downarrow & & \downarrow \\ \mathrm{D}(n|n) & \stackrel{prediction}{\longrightarrow} & \mathrm{D}(n+1|n) & \stackrel{update}{\longrightarrow} & \mathrm{D}(n+1|n+1). \end{array} \tag{3.55}$$

where the down arrow specifies the first moment of the distribution shown above it.

It is now is remaining to define the prediction and the update operators of the PHD in (3.55).

#### 3.4.2.1 The prediction operator of the PHD

In a general scenario assuming target disappearance, targets spawning and entry of new targets, the predicted PHD is given by [68]

$$\begin{aligned} \mathrm{D}(n+1|n) &= \mathrm{D}(\mathbf{x}(n+1)|Y(n)) \\ &= \mathrm{D}_b(\mathbf{x}(n+1)) + \int \Phi(\mathbf{x}(n+1)|\mathbf{x}(n))\mathrm{D}(\mathbf{x}(n)|Y(n))d\mathbf{x}(n) \end{aligned} \quad (3.56)$$

where

$$\Phi(\mathbf{x}(n+1)|\mathbf{x}(n)) = \left[e_{t+1|t}(\mathbf{x}(n))p(\mathbf{x}(n+1)|\mathbf{x}(n)) + \mathrm{D}_s(\mathbf{x}(n+1)|\mathbf{x}(n))\right] \quad (3.57)$$

and

- $\mathrm{D}_b(\mathbf{x}(n+1))$ – PHD of newborn spontaneous targets at time step $n+1$.
- $e_{t+1|t}(\mathbf{x}(n))$ – probability that a target with state $\mathbf{x}(n)$ at time step $n$ will survive at time step $n+1$.
- $\mathrm{D}_s(\mathbf{x}(n+1)|\mathbf{x}(n))$ – PHD of spawned targets at time step $n+1$ from a target with state $\mathbf{x}(n)$.

#### 3.4.2.2 The update operator of the PHD

Given the observations at time–step $n+1$, $Y(n+1)$, The predicted PHD above is updated as follows

$$\begin{aligned} \mathrm{D}(n+1|n+1) &= \mathrm{D}(\mathbf{x}(n+1)|Y(n+1)) \\ &= \Bigg[ \sum_{y(n)\in Y(n+1)} \frac{p_D(\mathbf{x}(n+1))\mathbf{p}(\mathbf{y}(n+1)|\mathbf{x}(n+1))}{\lambda(n+1)c(\mathbf{y}(n+1)) + \psi(\mathbf{y}(n+1)|Y(n+1))} + \\ & \qquad 1 - p_D(\mathbf{x}(n+1)) \Bigg] \mathrm{D}(n+1|n) \end{aligned} \tag{3.58}$$

where

$$\psi(\mathbf{y}(n+1)|Y(n+1)) = \int p_D(\mathbf{x}(n+1))p(\mathbf{y}(n+1)|\mathbf{x}(n+1))\mathrm{D}(n+1|n)d\mathbf{x}(n+1) \tag{3.59}$$

and

- $p_D(\mathbf{x}(n+1))$ – detection probability of a target with state $\mathbf{x}(n+1)$ at time step $n+1$.
- $p(\mathbf{y}(n+1)|\mathbf{x}(n+1))$ – single target likelihood function.
- $\lambda(n+1)$ – poisson distributed false alarm per scan.
- $c(\mathbf{y}(n+1))$ – clutter probability density.

# Chapter 4

# Joint MIMO Channel Tracking and Symbol Decoding

## 4.1 Introduction

It has recently been shown in [99] that for a majority of OSTBCs, the MIMO channel is blindly identifiable. This interesting property of OSTBCs is based on the assumption that the channel is fixed during a long enough time interval. However, the channel may be time-varying in practice due to the mobility of the transmitter and/or receiver, as well as due to the carrier frequency mismatch between the transmitter and receiver. Therefore, channel tracking is essential in these cases.

In [61], Kalman filtering has been studied in application to channel tracking for MIMO communication systems. The method of [61] is based on two assumptions. First, the underlying space-time coding scheme is based on Alamouti code [3], and therefore its application is limited to the case of two transmit antennas. Second, the channel is assumed to be time-varying during the transmission of each block. The latter assumption implies that the linear ML receiver is optimal in a mean sense [61].

Kalman filtering has been applied to the problem of MIMO channel tracking in several other research reports [50]. Also, in [22], a frequency domain equalization method has been proposed for single carrier MIMO systems. Particle filtering has also been used in [37] for MIMO channel tracking. However, neither of the methods of [50] and [37] has been developed for OSTBCs. In [2], a Kalman filtering approach has been used in the maximization step of an expectation-maximization (EM) method to track the frequency selective MIMO channel when the underlying code is an OSTBC and when an orthogonal frequency division multiplexing (OFDM) is used. However, this method does not make use of the structure of the underlying OSTBC to simplify Kalman filtering.

In this chapter, we extend the result of [61] for any type of OSTBCs and show that Kalman filtering can be significantly simplified due to the specific structure of OSTBCs. Unlike [61], we assume that the channel is fixed during the transmission of each block of data, and it can only change between blocks. Based on such an assumption, we develop a two-step channel tracking algorithm. In the first step, Kalman filtering is used at the beginning of each block to obtain an initial channel estimate for that block based on the channel estimate obtained for previous block. In the second step, to improve the quality of the channel estimate obtained by Kalman filtering, we propose a simple iterative channel estimation technique. This iterative method is in fact a decision-directed algorithm and it consists of sequential use of a linear receiver and a linear channel estimator.

We should mention that the idea of iterating the MIMO channel and symbol estimates is very common in the literature (see for example [44]). However, this chapter presents two contributions to the field: First, we extend the results in [61] to any type of OSTBCs.

Second, we use interesting properties of OSTBCs to show that the Kalman filtering based channel tracking can be significantly simplified.

The rest of this Chapter is organized as follows. In Section 4.2 we present our semi-blind channel tracking and data decoding algorithm and derive the simplified version. In section 4.3 simulation results are presented to demonstrate the performance of the proposed algorithm and in Section 4.4 the Chapter is concluded.

## 4.2 Kalman Filter Based Channel Tracking

In this Section, we study the problem of channel tracking via Kalman filtering. We propose a two-step channel tracking algorithm. In the first step of this algorithm, Kalman filtering is used to obtain an initial channel estimate for each block based on the channel estimates obtained for the previous blocks. In the second step, the so-obtained initial channel estimate is refined using an iterative decision-directed technique, which involves a linear ML channel estimator based on the decoded symbols. In fact, the linearity of such an ML channel estimator follows from the interesting properties of OSTBCs. We will also show that due to the specific structure of OSTBCs, Kalman filtering based channel tracking can be significantly simplified.

### 4.2.1 Conditional channel estimate

In this section, we derive the ML channel estimate of the MIMO system given the knowledge of the transmitted symbols. First, we rewrite (2.11) as

$$\tilde{\mathrm{y}}_n = \mathrm{B}(\tilde{\mathrm{s}}_n)\mathrm{g}(n) + \tilde{\mathrm{v}}_n \tag{4.60}$$

where the $2MT \times 2MN$ real matrix $\mathbf{B}(\tilde{\mathbf{s}}_n)$ is defined as

$$\mathbf{B}(\tilde{\mathbf{s}}_n) \triangleq [\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_n \quad \mathbf{A}(\mathbf{e}_2)\tilde{\mathbf{s}}_n \ \ldots \ \mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_n] \tag{4.61}$$

and $\mathbf{e}_k$, as defined earlier, is the $k$th column of the identity matrix $\mathbf{I}_{2MN}$. The following Lemma plays an important role in simplifying the forthcoming Kalman filtering algorithm.

**Lemma 4.1** *The matrix* $\mathbf{B}(\tilde{\mathbf{s}}_n)$ *satisfies*

$$\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{B}(\tilde{\mathbf{s}}_n) = \|\mathbf{s}(n)\|^2\mathbf{I}_{2MN} \tag{4.62}$$

*Proof:* We first show that the sub-matrices $\{\mathbf{\Phi}_k\}_{k=1}^{2K}$ satisfy the following equations:

$$\mathbf{\Phi}_l^T\mathbf{\Phi}_m = \begin{cases} \mathbf{I}_{2MN} & \text{if } l = m, \\ -\mathbf{\Phi}_m^T\mathbf{\Phi}_l & \text{if } l \neq m. \end{cases} \tag{4.63}$$

To prove (4.63), we use the decoupling property in (2.14). Indeed, for any channel vector $\mathbf{g}$, the decoupling property in (2.14) implies that

$$[\mathbf{A}(\mathbf{g})]_l^T[\mathbf{A}(\mathbf{g})]_l = \|\mathbf{g}\|^2 \tag{4.64}$$

or

$$\mathbf{g}^T\mathbf{\Phi}_l^T\mathbf{\Phi}_l\mathbf{g} = \mathbf{g}^T\mathbf{g} \tag{4.65}$$

Since (4.65) holds true for any $\mathbf{g}$ and because $\mathbf{\Phi}_l^T\mathbf{\Phi}_l$ is a symmetric matrix, we conclude that $\mathbf{\Phi}_l^T\mathbf{\Phi}_l = \mathbf{I}_{2MN}$. To prove the second part of (4.63), based on the fact that different columns of $\mathbf{A}(\mathbf{g})$ are orthogonal to each other, we can write

$$\left.\begin{array}{rcccl} [\mathbf{A}(\mathbf{g})]_l^T[\mathbf{A}(\mathbf{g})]_m & = & \mathbf{g}^T\mathbf{\Phi}_l^T\mathbf{\Phi}_m\mathbf{g} & = & 0 \\ [\mathbf{A}(\mathbf{g})]_m^T[\mathbf{A}(\mathbf{g})]_l & = & \mathbf{g}^T\mathbf{\Phi}_m^T\mathbf{\Phi}_l\mathbf{g} & = & 0 \end{array}\right\} \Rightarrow \mathbf{g}^T(\mathbf{\Phi}_l^T\mathbf{\Phi}_m + \mathbf{\Phi}_m^T\mathbf{\Phi}_l)\mathbf{g} = 0 \quad (4.66)$$

Since (4.66) holds true for any vector $\mathbf{g}$ and since $\mathbf{\Phi}_l^T\mathbf{\Phi}_m + \mathbf{\Phi}_m^T\mathbf{\Phi}_l$ is a symmetric matrix, we conclude that $\mathbf{\Phi}_l^T\mathbf{\Phi}_m + \mathbf{\Phi}_m^T\mathbf{\Phi}_l = 0$. This completes the proof of (4.63).

We now use (4.63) to prove (4.62). To do so, we note that

$$\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{B}(\tilde{\mathbf{s}}_n) =$$
$$\begin{bmatrix} \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_1)\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_n & \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_1)\mathbf{A}(\mathbf{e}_2)\tilde{\mathbf{s}}_n & \cdots & \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_1)\mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_n \\ \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_2)\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_n & \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_2)\mathbf{A}(\mathbf{e}_2)\tilde{\mathbf{s}}_n & \cdots & \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_2)\mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_n \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_{2MN})\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_n & \tilde{\mathbf{s}}_n^T\mathbf{A}^T(\mathbf{e}_{2MN})\mathbf{A}(\mathbf{e}_2)\tilde{\mathbf{s}}_n & \cdots & \tilde{\mathbf{s}}_n\mathbf{A}^T(\mathbf{e}_{2MN})\mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_n \end{bmatrix} \quad (4.67)$$

where we have used (4.61). Note also that

$$\tilde{\mathbf{s}}_n^T \underbrace{\mathbf{A}^T(\mathbf{e}_l)\mathbf{A}(\mathbf{e}_l)}_{\|\mathbf{e}_l\|^2\mathbf{I}_{2K}} \tilde{\mathbf{s}}_n = \|\tilde{\mathbf{s}}_n^T\|^2 \quad (4.68)$$

which follows from the decoupling property. For $l \neq m$, the following set of equalities holds true:

$$\begin{aligned}
\tilde{\mathbf{s}}_n^T \mathbf{A}^T(\mathbf{e}_l)\mathbf{A}(\mathbf{e}_m)\tilde{\mathbf{s}}_n &= \sum_{r=1}^{2K}\sum_{s=1}^{2K} \tilde{s}_{n,r}[\mathbf{A}(\mathbf{e}_l)]_r^T[\mathbf{A}(\mathbf{e}_m)]_s\tilde{s}_{n,s} \\
&= \sum_{r=1}^{2K}\sum_{s=1,s\neq r}^{2K} \tilde{s}_{n,r}\left(\mathbf{e}_l^T\mathbf{\Phi}_r^T\mathbf{\Phi}_s\mathbf{e}_m\right)\tilde{s}_{n,s} + \sum_{r=1}^{2K}\tilde{s}_{n,r}\big(\mathbf{e}_l^T\overbrace{\mathbf{\Phi}_r^T\mathbf{\Phi}_r}^{\mathbf{I}_{2MN}}\mathbf{e}_m\big)\tilde{s}_{n,r} \\
&= \sum_{r=1}^{2K}\sum_{s=1,s\neq r}^{2K} \tilde{s}_{n,r}\left(\mathbf{e}_l^T\mathbf{\Phi}_r^T\mathbf{\Phi}_s\mathbf{e}_m\right)\tilde{s}_{n,s} + \sum_{r=1}^{2K}\tilde{s}_{n,r}\overbrace{\mathbf{e}_l^T\mathbf{e}_m}^{0}\tilde{s}_{n,r} \\
&= -\sum_{r=1}^{2K}\sum_{s=1,s\neq r}^{2K} \tilde{s}_{n,s}\left(\mathbf{e}_l^T\mathbf{\Phi}_s^T\mathbf{\Phi}_r\mathbf{e}_m\right)\tilde{s}_{n,r} + 0 \\
&= -\sum_{s=1}^{2K}\sum_{r=1,r\neq s}^{2K} \tilde{s}_{n,s}\left(\mathbf{e}_l^T\mathbf{\Phi}_s^T\mathbf{\Phi}_r\mathbf{e}_m\right)\tilde{s}_{n,r} = -\tilde{\mathbf{s}}_n^T \mathbf{A}^T(\mathbf{e}_l)\mathbf{A}(\mathbf{e}_m)\tilde{\mathbf{s}}_n
\end{aligned}$$

where $\tilde{s}_{n,r}$ is the $r$th element of $\tilde{\mathbf{s}}_n$. Therefore, we obtain that, for $l \neq m$

$$\tilde{\mathbf{s}}_n^T \mathbf{A}^T(\mathbf{e}_l)\mathbf{A}(\mathbf{e}_m)\tilde{\mathbf{s}}_n = 0 \tag{4.69}$$

It follows from (4.68) and (4.69) that $\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{B}(\tilde{\mathbf{s}}_n) = \|\tilde{\mathbf{s}}_n\|^2\mathbf{I}_{2MN} = \|\mathbf{s}(n)\|^2\mathbf{I}_{2MN}$ and the proof is complete. ■

It follows from (4.60) and (4.62) that given $\tilde{\mathbf{s}}_n$, the ML estimate of the channel vector $\mathbf{g}(n)$ can be obtained as

$$\hat{\mathbf{g}}_{\mathrm{ML}}(n) = \frac{1}{\|\mathbf{s}(n)\|^2}\mathbf{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}_n. \tag{4.70}$$

Therefore, if the information symbols were available, the optimal ML channel estimation would involve a linear estimator as in (4.70). However, in practice, the information symbols are not available and they have to be estimated. To overcome this problem, one can use an iterative decision-directed channel estimation scheme. That is, given an initial channel estimate for the $n$th block, say $\hat{\mathrm{g}}^{(0)}(n)$, one can replace $\mathrm{g}(n)$ in (2.16) with $\hat{\mathrm{g}}^{(0)}(n)$ and obtain an estimate for $\tilde{\mathrm{s}}_n$, say $\hat{\tilde{\mathrm{s}}}_n^{(0)}$. This estimate of $\tilde{\mathrm{s}}_n$ will, in turn, be used in (4.70) instead of $\tilde{\mathrm{s}}(n)$ to obtain a new estimate for $\mathrm{g}(n)$, say $\hat{\mathrm{g}}^{(1)}(n)$. This new channel estimate will again be used in (2.16) instead of $\mathrm{g}(n)$ to obtain a new estimate of $\mathrm{s}(n)$. This procedure is repeated until the normalized difference between two consecutive channel estimates is negligible. The accuracy of this iterative decision-directed channel estimation scheme depends on the availability of a precise enough initial channel vector estimate $\hat{\mathrm{g}}^{(0)}(n)$. We propose to use Kalman filtering to obtain the initial channel estimate, $\hat{\mathrm{g}}^{(0)}(n)$, based on the channel estimates obtained for the previous blocks as well as the $n$th block received data.

### 4.2.2 Conditional channel tracking

In this section, we detail the time-varying channel tracking problem using Kalman filtering given the transmitted data symbols. We also show that using Lemma 1, the Kalman filter can be simplified significantly. To show this, we use (4.60) as the observation model of the Kalman filter [8]. Note that the data model in (4.60) is real-valued. To obtain a real-valued state transition equation, we can rewrite (2.17) as

$$\mathrm{g}(n) = \mathbf{F}\mathrm{g}(n-1) + \mathbf{w}(n) \tag{4.71}$$

where

$$\mathbf{F} \triangleq \begin{bmatrix} \mathrm{Re}(\alpha)\mathbf{I}_{MN} & -\mathrm{Im}(\alpha)\mathbf{I}_{MN} \\ \mathrm{Im}(\alpha)\mathbf{I}_{MN} & \mathrm{Re}(\alpha)\mathbf{I}_{MN} \end{bmatrix}$$

and $\mathbf{w}(n) = \underline{\mathbf{W}(n)}$ is the real-valued process noise with covariance matrix $\mathbf{Q} = (\sigma_w^2/2)\mathbf{I}_{2MN}$. We can use (4.71) as the real-valued state transition equation required for Kalman filtering.

The Kalman filtering problem for channel tracking in OSTBC-based MIMO communication system can now be formally stated as follows: Given the measurement-to-state matrix $\mathbf{B}(\tilde{\mathbf{s}}_n)$, use the observed data $\tilde{\mathbf{y}}_n$ to find the minimum mean squared error (MMSE) estimate of the components of the state vector $\mathbf{g}(n)$ for each $n \geq 1$.

Given the estimate of the state at time $n-1$, i.e., $\mathbf{g}(n-1|n-1)$, and the associated error covariance matrix $\mathbf{P}(n-1|n-1)$, the Kalman filter [8] is used to obtain the estimate of the state at time $n$, i.e., $\mathbf{g}(n|n)$ and the associated error covariance matrix $\mathbf{P}(n|n)$. The Kalman filtering algorithm can be summarized as follows:

$$\mathbf{g}(n|n-1) = \mathbf{F}\mathbf{g}(n-1|n-1) \tag{4.72}$$

$$\mathbf{P}(n|n-1) = \mathbf{F}\mathbf{P}(n-1|n-1)\mathbf{F}^T + \mathbf{Q} \tag{4.73}$$

$$\hat{\tilde{\mathbf{y}}}_n = \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{g}(n|n-1) \tag{4.74}$$

$$\boldsymbol{\nu}(n) = \tilde{\mathbf{y}}_n - \hat{\tilde{\mathbf{y}}}_n \tag{4.75}$$

$$\mathbf{P}_\nu(n) = \mathbf{R} + \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{P}(n|n-1)\mathbf{B}^T(\tilde{\mathbf{s}}_n) \tag{4.76}$$

$$\mathbf{G}_{\text{KF}}(n) = \mathbf{P}(n|n-1)\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{P}_\nu^{-1}(n) \tag{4.77}$$

$$\mathbf{g}(n|n) = \mathbf{g}(n|n-1) + \mathbf{G}_{\text{KF}}(n)\boldsymbol{\nu}(n) \tag{4.78}$$

$$\mathbf{P}(n|n) = \mathbf{P}(n|n-1) - \mathbf{G}_{\text{KF}}(n)\mathbf{P}_\nu(n)\mathbf{G}^T(n) \tag{4.79}$$

where $\mathbf{g}(n|n-1)$ is the predicted state, $\mathbf{P}(n|n-1)$ is the covariance matrix of the predicted state, $\hat{\tilde{\mathbf{y}}}_n$ is the predicted observation, $\boldsymbol{\nu}(n)$ is the innovation process, $\mathbf{P}_\nu(n)$ is the innovation covariance matrix, $\mathbf{G}_{\text{KF}}(n)$ is the Kalman gain [8], and $\mathbf{R} = E\{\tilde{\mathbf{v}}_n\tilde{\mathbf{v}}_n^T\}$ is the covariance matrix of the measurement noise $\tilde{\mathbf{v}}_n$. As we assumed that the measurement noise is spatio-temporally white with a variance of $\sigma_v^2/2$ per real dimension, $\mathbf{R} = (\sigma_v^2/2)\mathbf{I}_{2MT}$ holds true.

### 4.2.3 Simplified algorithm

The following Lemma uses the result of Lemma 1 to reduce the computational complexity of finding $\mathbf{P}_\nu^{-1}(n)$ in (4.77).

**Lemma 4.2** *If* $\mathbf{P}(n-1|n-1)$ *is a diagonal matrix, then,* $\mathbf{P}(n|n-1)$ *in (4.73) and* $\mathbf{P}(n|n)$ *in (4.79) are also diagonal, i.e., if*

$$\mathbf{P}(n-1|n-1) = \delta_{n-1}\mathbf{I}_{2MN} \tag{4.80}$$

*then*

$$\mathbf{P}(n|n-1) = \beta_n \mathbf{I}_{2MN} \tag{4.81}$$

$$\mathbf{P}(n|n) = \delta_n \mathbf{I}_{2MN} \tag{4.82}$$

*where*

$$\beta_n = \delta_{n-1}|\alpha|^2 + \frac{\sigma_w^2}{2} \qquad \textit{and} \qquad \delta_n = \frac{\sigma_v^2 \beta_n}{2\|s(n)\|^2 \beta_n + \sigma_v^2}. \tag{4.83}$$

*Proof:* Substituting (4.80) into the predicted state in (4.73), we can rewrite it as

$$\mathbf{P}(n|n-1) = \delta_{n-1}\mathbf{F}\mathbf{F}^T + \mathbf{Q} = |\alpha|^2\delta_{n-1}\mathbf{I}_{2MN} + \mathbf{Q} = \underbrace{\left(|\alpha|\delta_{n-1}^2 + \frac{\sigma_w^2}{2}\right)}_{\beta_n}\mathbf{I}_{2MN}. \tag{4.84}$$

Inserting (4.84) into (4.76) and using matrix inversion lemma, $\mathbf{P}_\nu^{-1}(n)$ in (4.76) can be written as

$$\begin{aligned}\mathbf{P}_\nu^{-1}(n) &= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbf{B}(\tilde{\mathbf{s}}_n)\Big(\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\mathbf{B}(\tilde{\mathbf{s}}_n) + \mathbf{P}^{-1}(n|n-1)\Big)^{-1}\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1} \\ &= \frac{2}{\sigma_v^2}\mathbf{I}_{2MT} - \frac{4}{\sigma_v^4}\mathbf{B}(\tilde{\mathbf{s}}_n)\left(\frac{2}{\sigma_v^2}\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{B}(\tilde{\mathbf{s}}_n) + \frac{1}{\beta_n}\mathbf{I}_{2MN}\right)^{-1}\mathbf{B}^T(\tilde{\mathbf{s}}_n) \\ &= \frac{2}{\sigma_v^2}\mathbf{I}_{2MT} - \left(\frac{4\beta_n}{2\|\mathbf{s}(n)\|^2\beta_n\sigma_v^2 + \sigma_v^4}\right)\mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{B}^T(\tilde{\mathbf{s}}_n) \end{aligned} \tag{4.85}$$

where the fact that $\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{B}(\tilde{\mathbf{s}}_n) = \|\mathbf{s}(n)\|^2\mathbf{I}_{2MN}$ has been used.

Using (4.77) and (4.85), we rewrite (4.79) as

$$\begin{aligned}\mathbf{P}(n|n) &= \mathbf{P}(n|n-1) \\ &- \mathbf{P}(n|n-1)\mathbf{B}^T(\tilde{\mathbf{s}}_n)\left(\frac{2}{\sigma_v^2}\mathbf{I}_{2MT} - \left(\frac{4\beta_n}{2\|\mathbf{s}(n)\|^2\beta_n\sigma_v^2 + \sigma_v^4}\right)\mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{B}^T(\tilde{\mathbf{s}}_n)\right) \\ &\qquad\qquad \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{P}(n|n-1) \\ &= \left(\frac{\sigma_v^2\beta_n}{2\|s(n)\|^2\beta_n + \sigma_v^2}\right)\mathbf{I}_{2MN}\,. \end{aligned} \tag{4.86}$$

The proof is complete. ■

Based on Lemma 4.2, if $\mathbf{P}(0|0)$ is initialized as a diagonal matrix, $\mathbf{P}(n|n-1)$ and $\mathbf{P}(n|n)$ always take the form of (4.81) and (4.82), respectively. Hence, $\mathbf{P}_\nu^{-1}$ in (4.77) is simplified as in (4.85). It is also noteworthy that using (4.84) and (4.85) the Kalman filter gain $\mathbf{G}_{\mathrm{KF}}(n)$ in (4.77) can be written as

$$\mathbf{G}_{\mathrm{KF}}(n) = \underbrace{\beta_n\left(\frac{2}{\sigma_v^2} - \frac{4\beta_n\|\mathbf{s}(n)\|^2}{2\|\mathbf{s}(n)\|^2\beta_n\sigma_v^2 + \sigma_v^4}\right)}_{\triangleq\mu_n}\mathbf{B}^T(\tilde{\mathbf{s}}_n)\,. \tag{4.87}$$

Using (4.74), (4.75) and (4.87), we can simplify (4.78) as

$$\begin{aligned} \mathbf{g}(n|n) &= \mathbf{g}(n|n-1) + \mu_n \mathbf{B}^T(\tilde{\mathbf{s}}_n)\Big(\tilde{\mathbf{y}}(n) - \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{g}(n|n-1)\Big) \\ &= (1 - \mu_n \|\mathbf{s}(n)\|^2)\mathbf{g}(n|n-1) + \mu_n \mathbf{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}(n)\,. \end{aligned} \tag{4.88}$$

Therefore, the Kalman filtering algorithm presented in (4.72)–(4.79) can be simplified as it follows:

$$\mathbf{g}(n|n-1) = \mathbf{F}\mathbf{g}(n-1|n-1) \tag{4.89}$$

$$\beta_n = \delta_{n-1}\|\alpha\|^2 + \frac{\sigma_w^2}{2}, \tag{4.90}$$

$$\mu_n = \beta_n\left(\frac{2}{\sigma_v^2} - \frac{4\beta_n\|\mathbf{s}(n)\|^2}{2|\,\mathbf{s}(n)\|^2\beta_n\sigma_v^2 + \sigma_v^4}\right) \tag{4.91}$$

$$\hat{\mathbf{g}}^{(0)}(n) = \mathbf{g}(n|n) = (1 - \mu_n\|\mathbf{s}(n)\|^2)\mathbf{g}(n|n-1) + \mu_n \mathbf{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}(n) \tag{4.92}$$

$$\delta_n = \frac{\sigma_v^2\beta_n}{2\|s(n)\|^2\beta_n - \sigma_v^2}\,. \tag{4.93}$$

We then use the so-obtained $\hat{\mathbf{g}}^{(0)}(n)$ in the aforementioned iterative procedure to improve its accuracy.

*Remark 1:* Note that the simplified Kalman filter requires the knowledge of the symbol vector $\tilde{\mathbf{s}}_n$ (or $\mathbf{s}(n)$). However, the primary objective is to decode $\mathbf{s}(n)$. To overcome this obstacle, we propose to replace $\tilde{\mathbf{s}}_n$ in the Kalman filter equations (4.89)–(4.93), by its estimate, which is obtained by replacing the true channel vector in (2.16) by the predicted channel vector $\mathbf{g}(n|n-1)$ as

$$\hat{\tilde{\mathbf{s}}}_n = \frac{1}{\|\mathbf{g}(n|n-1)\|^2}\mathbf{A}^T\big(\mathbf{g}(n|n-1)\big)\tilde{\mathbf{y}}_n\,. \tag{4.94}$$

Note that given the predicted channel vector $\mathbf{g}(n|n-1)$, the symbol estimate in (4.94) is optimal in the ML sense.

*Remark 2:* To initiate the whole process, we also need to obtain an accurate enough channel estimate $\hat{\mathbf{g}}(0)$ as well as its initial covariance $\delta_0 \mathbf{I}_{2MN}$. To obtain such an initial channel estimate, one can use a training block $\mathbf{s}(0)$, which is known at the receiver. At the beginning of the tracking process, the receiver can then use (4.70) to obtain the ML estimate of $\mathbf{g}(0)$ as

$$\hat{\mathbf{g}}(0) = \frac{1}{\|\mathbf{s}(0)\|^2} \mathbf{B}^T(\tilde{\mathbf{s}}_0)\tilde{\mathbf{y}}_0 \tag{4.95}$$

where $\tilde{\mathbf{s}}_0 = \underline{\mathbf{s}(0)}$ is defined.

To find $\delta_0$, we note that

$$\begin{aligned} \delta_0 \mathbf{I}_{2MN} &= E\left\{ (\hat{\mathbf{g}}(0) - \mathbf{g}(0)) (\hat{\mathbf{g}}(0) - \mathbf{g}(0))^T \right\} \\ &= E\left\{ \hat{\mathbf{g}}(0)\hat{\mathbf{g}}^T(0) \right\} + E\left\{ \mathbf{g}(0)\mathbf{g}^T(0) \right\} - 2E\left\{ \hat{\mathbf{g}}(0)\mathbf{g}^T(0) \right\} \end{aligned} \tag{4.96}$$

where channel is assumed zero-mean and

$$\begin{aligned} E\left\{ \hat{\mathbf{g}}(0)\hat{\mathbf{g}}^T(0) \right\} &= \frac{1}{\|\mathbf{s}(0)\|^4} \mathbf{B}^T(\tilde{\mathbf{s}}_0) E\left\{ \tilde{\mathbf{y}}_0 \tilde{\mathbf{y}}_0^T \right\} \mathbf{B}(\tilde{\mathbf{s}}_0) \\ &= \frac{1}{\|\mathbf{s}(0)\|^4} \mathbf{B}^T(\tilde{\mathbf{s}}_0) \left( \mathbf{B}(\tilde{\mathbf{s}}_0) \underbrace{E\{\mathbf{g}(0)\mathbf{g}^T(0)\}}_{(\sigma_h^2/2)\mathbf{I}_{2MN}} \mathbf{B}^T(\tilde{\mathbf{s}}_0) + (\sigma_v^2/2)\mathbf{I}_{2MT} \right) \mathbf{B}(\tilde{\mathbf{s}}_0) \\ &= \left( \frac{\sigma_h^2}{2} + \frac{\sigma_v^2}{2\|\mathbf{s}(0)\|^2} \right) \mathbf{I}_{2MN} \\ E\left\{ \hat{\mathbf{g}}(0)\mathbf{g}^T(0) \right\} &= \frac{1}{\|\mathbf{s}(0)\|^2} \mathbf{B}^T(\tilde{\mathbf{s}}_0) E\left\{ \left( \check{\mathbf{B}}(\tilde{\mathbf{s}}_0)\mathbf{g}(0) + \tilde{\mathbf{v}}_0 \right) \mathbf{g}^T(0) \right\} = \frac{\sigma_h^2}{2} \mathbf{I}_{2MN} \end{aligned}$$

Therefore, one can obtain $\delta_0$ as

$$\delta_0 = \frac{1}{2}\frac{\sigma_v^2}{\|\mathbf{s}(0)\|^2}. \tag{4.97}$$

*Remark 3:* To avoid error propagation, we need to repeat training once in a while. The training repetition period (TRP) determines the bandwidth efficiency of the system and it is defined as the distance, in terms of number of blocks, between two consecutive training blocks.

### 4.2.4 Computational savings

In terms of computational complexity, the proposed channel tracking method enjoys the low computational complexity of linear processing. More specifically, the first step requires the computation of $\mathbf{B}^T(\hat{\tilde{\mathbf{s}}}_n)\tilde{\mathbf{y}}(n)$, and therefore, $2MT$ real multiplications are required for computation of each entry of $\mathbf{g}(n|n)$. Taking into account that $\mathbf{g}(n|n)$ is of length $2MN$, the total computational complexity of the first step is of order $\mathcal{O}(M^2NT)$. The second step is indeed an iterative algorithm. In each iteration, we need to compute four quantities: $\|\mathbf{g}^{(k-1)}(n)\|^2$, $\mathbf{A}^T(\mathbf{g}^{(k-1)}(n))\tilde{\mathbf{y}}_n$, $\|\hat{\tilde{\mathbf{s}}}_n^k\|^2$, and $\mathbf{B}^T(\hat{\tilde{\mathbf{s}}}_n^{(k)})\tilde{\mathbf{y}}_n$. Computing these four quantities requires $2MN$, $4KMT$, $2K$, and $4M^2NT$ real multiplications, respectively. Therefore, the computational complexity of the second step is of the order $\mathcal{O}(M^2NT)$ per iteration of the first step. The traditional Kalman filtering method involves the computation of $\mathbf{P}_\nu^{-1}(n)$. This amounts to a computational complexity of the order $\mathcal{O}(M^3T^3)$ per iteration. Therefore, the proposed method significantly reduces the computational complexity of the traditional Kalman filtering.

Table 4.1: Standard KF based MIMO receiver complexity

| | Number of floating point operations |
|---|---|
| (4.72) | $\mathcal{O}(M^2N^2)$ |
| (4.73) | $\mathcal{O}(M^3N^3)$ |
| (4.74) | $\mathcal{O}(M^2NT)$ |
| (4.75) | $\mathcal{O}(MT)$ |
| (4.76) | $\mathcal{O}(M^3N^2T)$ |
| (4.77) | $\mathcal{O}(M^3N^2T)$ |
| (4.78) | $\mathcal{O}(M^2N)$ |
| (4.79) | $\mathcal{O}(M^3NT^2)$ |
| Complexity order | $\mathcal{O}(M^3NT^2)$ |

Table 4.2: Simplified KF based MIMO receiver complexity

| | Number of floating point operations |
|---|---|
| (4.89) | $\mathcal{O}(M^2N^2)$ |
| (4.90) | $C$ |
| (4.91) | $C$ |
| (4.92) | $\mathcal{O}(M^2T^2)$ |
| (4.93) | $C$ |
| Complexity order | $\mathcal{O}(M^2NT^2)$ |

## 4.3 Simulation Results

In our numerical example, we consider the 3/4 rate code of [53] with $N = M = T = 4$, and $K = 3$. The SNR is defined as $\sigma_h^2/\sigma_v^2$. In each simulation run, the elements of $\mathbf{H}(n)$ are generated according to Jakes model [43] corresponding to $F_mT_s = 0.0045$ where $F_m$ is the doppler frequency and $T_s$ is the sampling time. This results in $\alpha = J_0(0.2\pi F_mT_s)e^{j2\pi F_oT_s} = 0.9998e^{j0.0283}$ where $J_0(\cdot)$ is the zeroth order Bessel function

Table 4.3: MIMO receiver complexity comparison

| Standard KF | Simplified KF |
|---|---|
| $\mathcal{O}(M^3NT^2)$ | $\mathcal{O}(M^2NT^2)$ |

of first kind. In terms of channel estimation accuracy, we compare our Kalman filtering based channel tracking technique with the online implementation of the technique developed in [99]. In order to implement the method of [99] in an online manner, we have used the subspace tracking approach proposed in section III.G of [99]. In our comparison, we use normalized mean squared error (NMSE) of the channel estimates defined as

$$\text{NMSE} = E\left\{\frac{||\mathbf{H}(n) - \hat{\mathbf{H}}(n)||^2}{||\mathbf{H}(n)||^2}\right\}. \tag{4.98}$$

In terms of symbol error rate (SER), we compare our method not only with the method of [99] but also with the differential space-time coding scheme [53]. The latter scheme does not require the channel to be estimated. It should be mentioned that the two methods with which the proposed method is compared do not require regular transmission of pilots, whereas the proposed method does.

### 4.3.1 Channel tracking performance

In this section we experiment the channel tracking performance of the proposed algorithm. Figure 4.2 shows the real parts of instantaneous channel estimation value along with true channel value and Figure 4.3 shows the corresponding imaginary part.

Figure 4.4 shows the NMSE of the channel estimates versus the block index $n$, for different methods and for two different values of SNR. In this figure, TRP = 10 blocks is chosen. As can be seen from this figure, compared to the method of [99], the proposed channel tracking scheme has a lower NMSE as it tracks the channel between every two pilots.

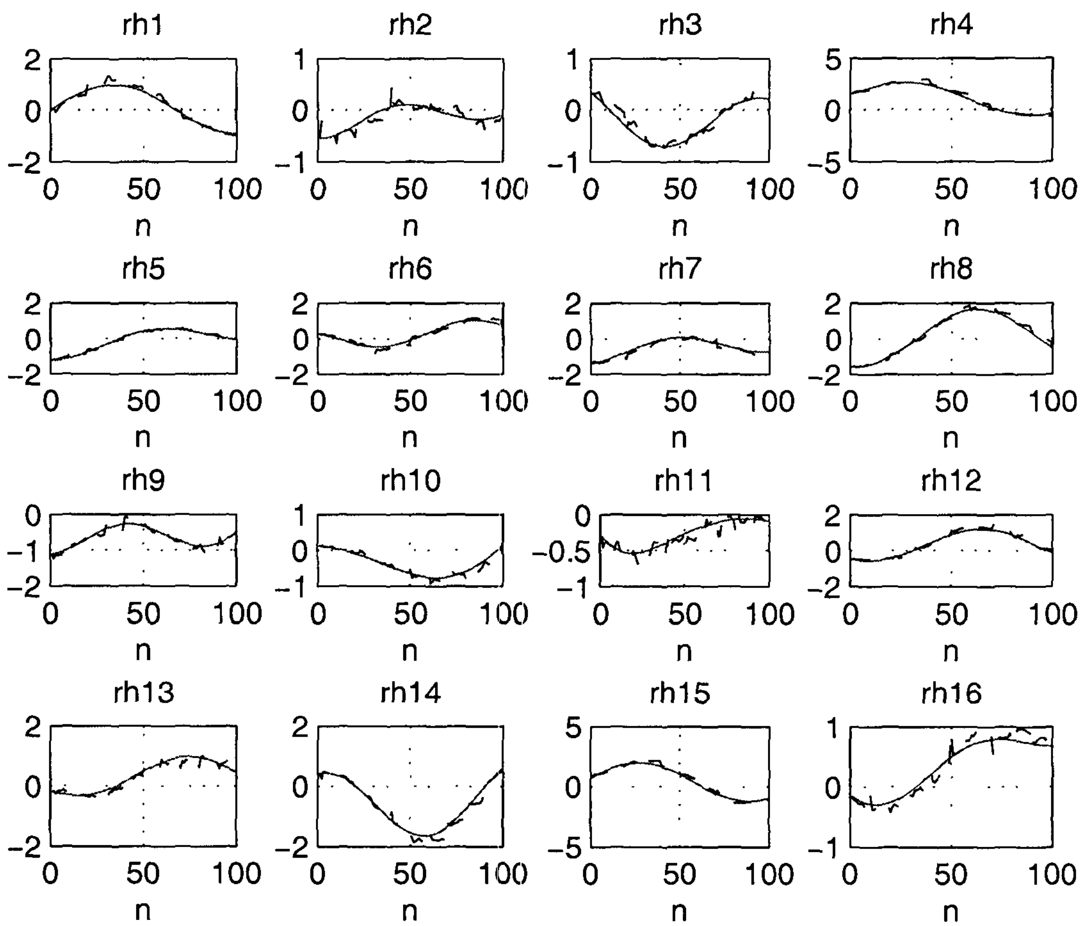


Figure 4.2: True vs. estimated channel (real part)

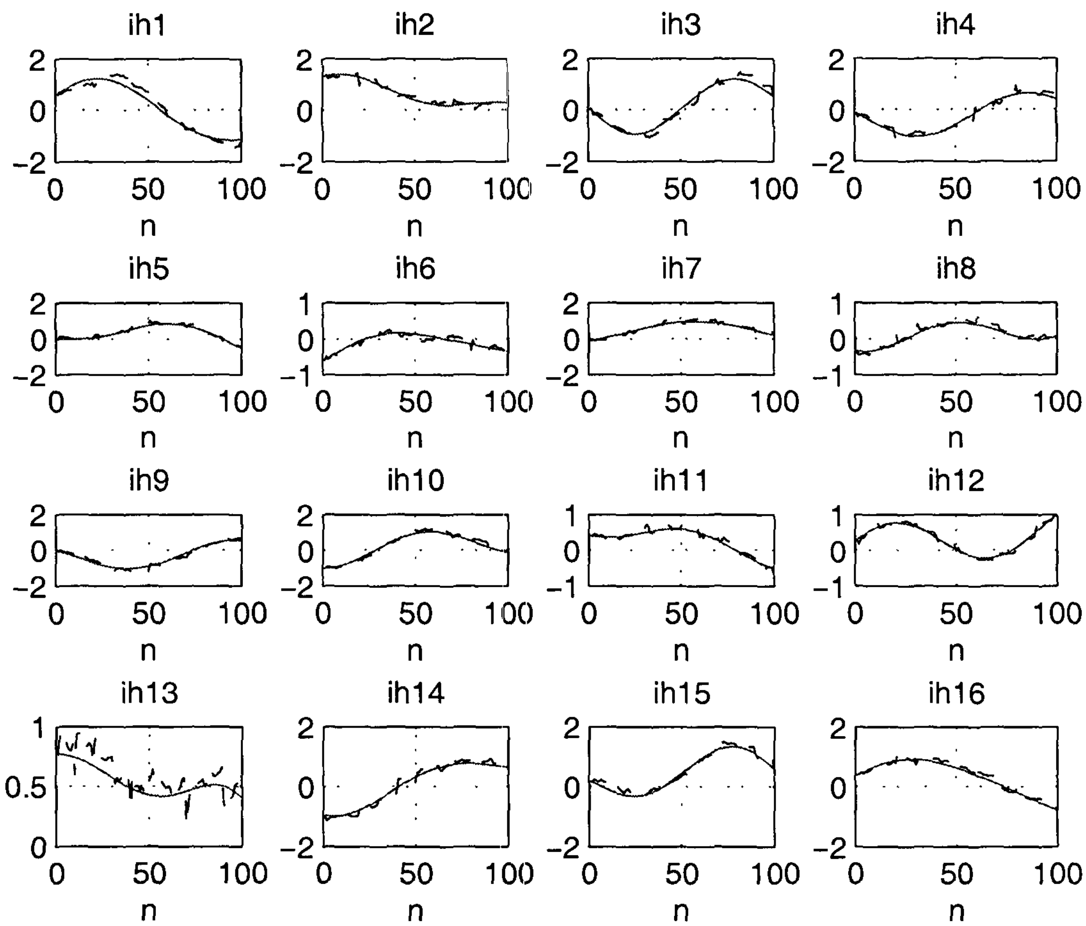


Figure 4.3: True vs. estimated channel (imginary part)

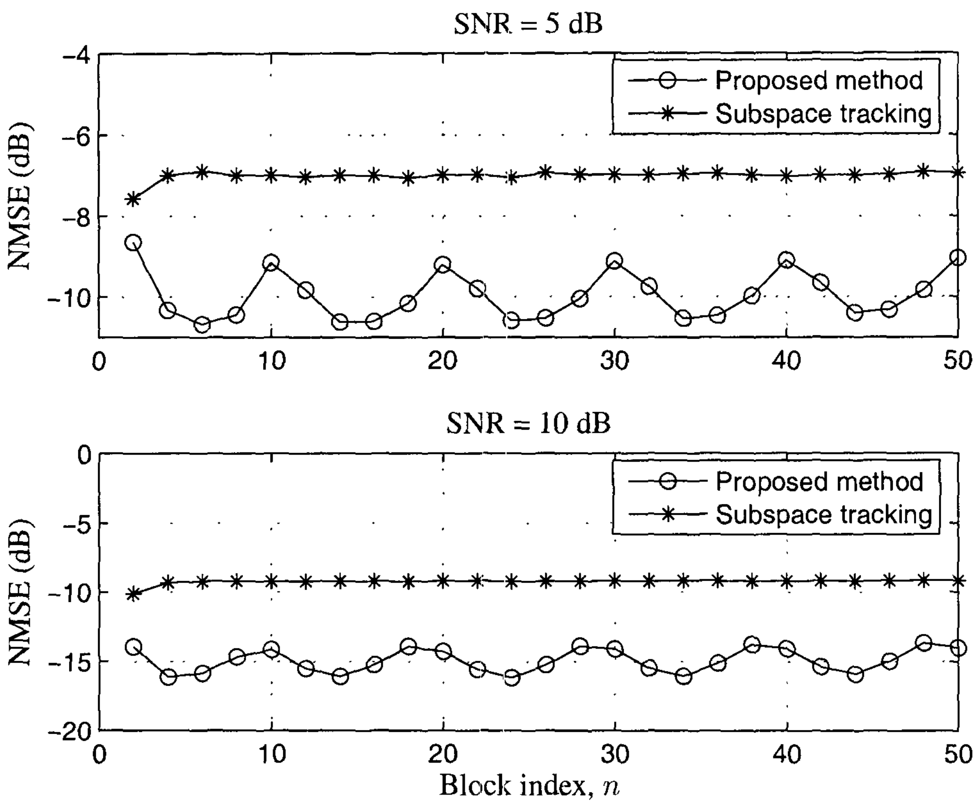


Figure 4.4: The NMSE of channel estimates versus block index $n$

### 4.3.2 SER vs. SNR

Figure 4.5 illustrates the SERs of different methods, versus SNR, for TRP = 10. In this figure, we have also plotted the SER for the (clairvoyant) coherent ML receiver that is aware of the time-varying channel. It is noteworthy that the latter receiver does not correspond to any practical application and it is herein considered only for the sake of comparison. We have also plotted the performance of a differential coding scheme which uses the same OTSTBC which we have used in our method. As can be seen from this figure, for TRP = 10, our Kalman filtering based technique outperforms the differential space-time coding

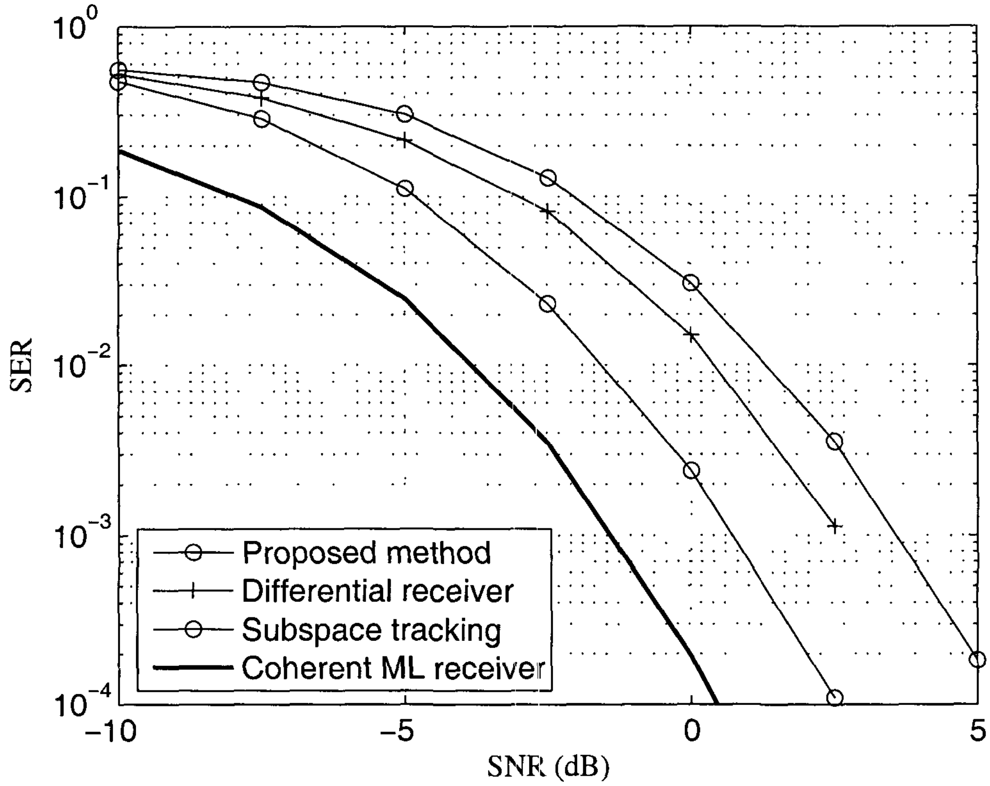


Figure 4.5: The SERs versus SNR for different OSTBC decoding schemes

scheme by 1 dB.

It is interesting to observe that our technique outperforms the technique of [99] by more than 2 dB. In fact, when applied to track a time-varying MIMO channel, the algorithm of [99] performs even worse than the differential scheme. This is not surprising as [99] assumes that the MIMO channel is fixed within the observation interval. Therefore, the method of [99] is not applicable whenever the MIMO channel variations are fairly fast.

It is worth mentioning that each training block, the performance of our technique in terms of the NMSE degrades until the next training block is received. This explains the "periodic" behavior of our algorithm.

### 4.3.3 Continuous vs. block fading channel

In this example, we compare the performance of the receiver for two different channel assumptions, namely, continuous fading channels and block fading channels.

First, we compare the channel tracking performance for both of the channel models. For the comparison, the channel between first transmitting antenna and first receiving antenna, $\mathbf{H}_{1,1}(n)$ is considered. Figure 4.6 shows the real parts of the true channel along with the estimated channel as if the true channel is transmitted in block fading manner. In the same plot, we also plot the estimated channel if the same true channel is transmitted in a continuous manner. Figure 4.7 shows the imaginary parts of such comparison. Both of the figures suggest that, the channel tracking performance is better for block fading assumptions. However, it should be noted that the channel estimation for continuous fading channel is also performed reasonably well. This claim is supported in the following figures.

Next, we compare the NMSE of both of the assumptions. Figure 4.8 shows the NMSE values vs. block index for the channel tracking schemes assuming block-fading and continuous fading, respectively. As expected, the block fading assumption is reasonably good even for continuous fading channels. In quantitative terms, on average, the block-fading assumption is able to perform with 2 dB loss in terms of NMSE. It is also worth mentioning that, typically, the NMSE curve is expected to be in a saw-tooth manner, however, due to the complex structure of the decision-directed receiver algorithm, the NMSE curves are wavy in nature.

Finally, the SER vs. SNR comparison is given if figure 4.9 for both of the channel assumption. As the figure suggests, the block fading assumption costs less than 1 dB in performance in terms of SER for an $fT$ value of 0.0045.

## 4.4 Conclusions

In this Chapter, we have investigated the problem of channel tracking for MIMO communication systems where the MIMO channel is time-varying. We considered MIMO systems where orthogonal space-time block codes are used to encode the information symbols. For such systems, we presented a two-step MIMO channel tracking algorithm. As the first step, Kalman filtering is used to obtain an initial channel estimate for the current block based on the channel estimates obtained for previous blocks. Then, in the second step, the so-obtained initial channel estimate is refined using a decision-directed iterative method. We have shown that due to the interesting properties of orthogonal space-time block codes, both the Kalman filter and the decision-directed algorithm can be significantly simplified. To initiate this method and to avoid error propagation, one needs to use a training block once in a while. The number of information carrying blocks between two consecutive training blocks (called training repetition period) is a measure of bandwidth efficiency of our channel tracking scheme. Our simulation results show that with a training repetition period of 10 blocks, our channel tracking method can have a performance, in terms of symbol error rate, within 2 dB from the coherent ML receiver.

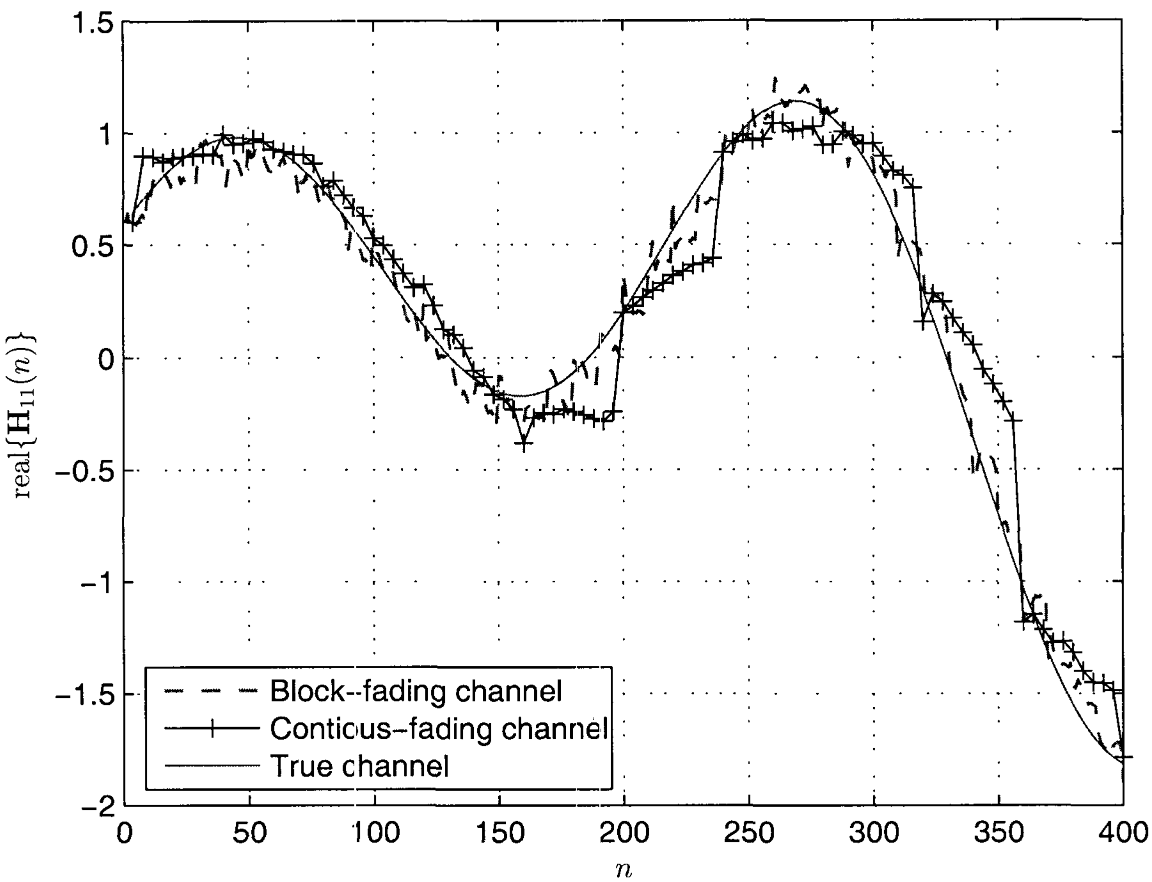


Figure 4.6: Continuous vs. block fading channels - real part

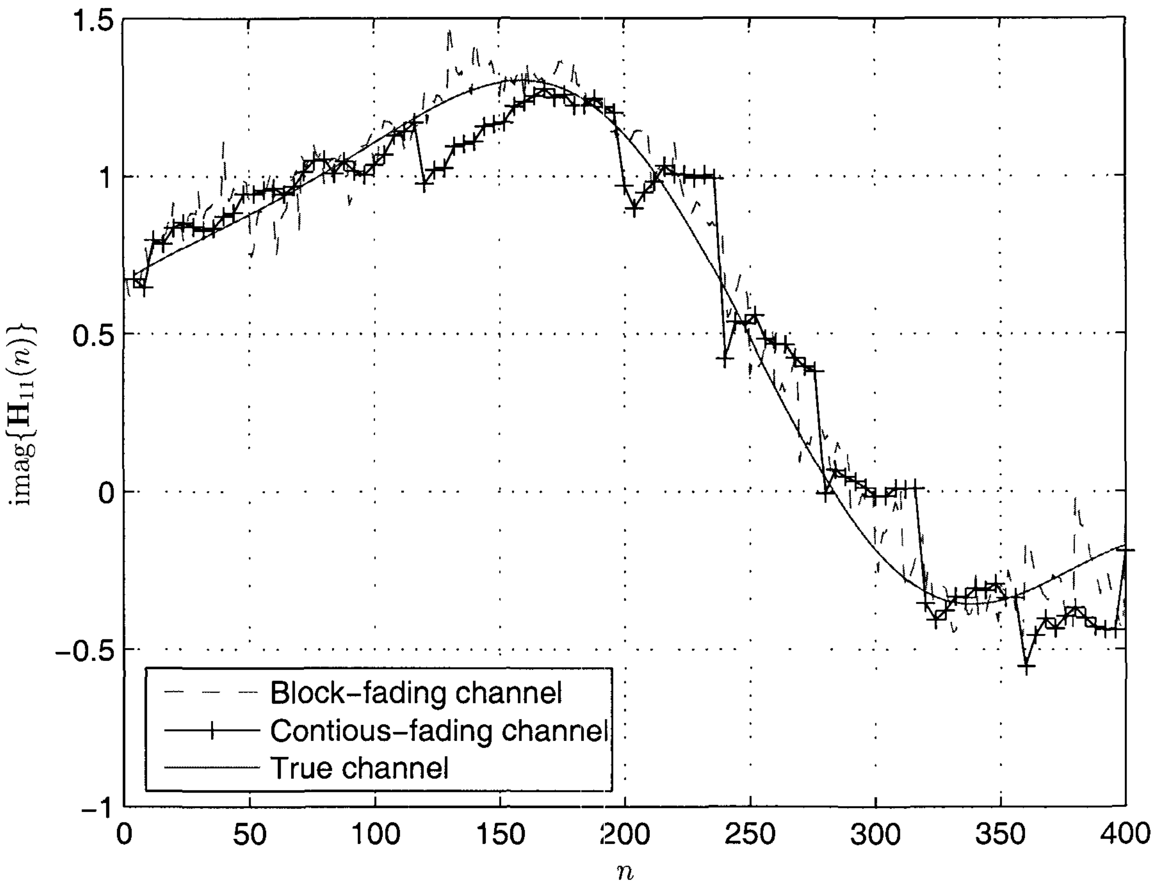


Figure 4.7: Continuous vs. block fading channels - imaginary part

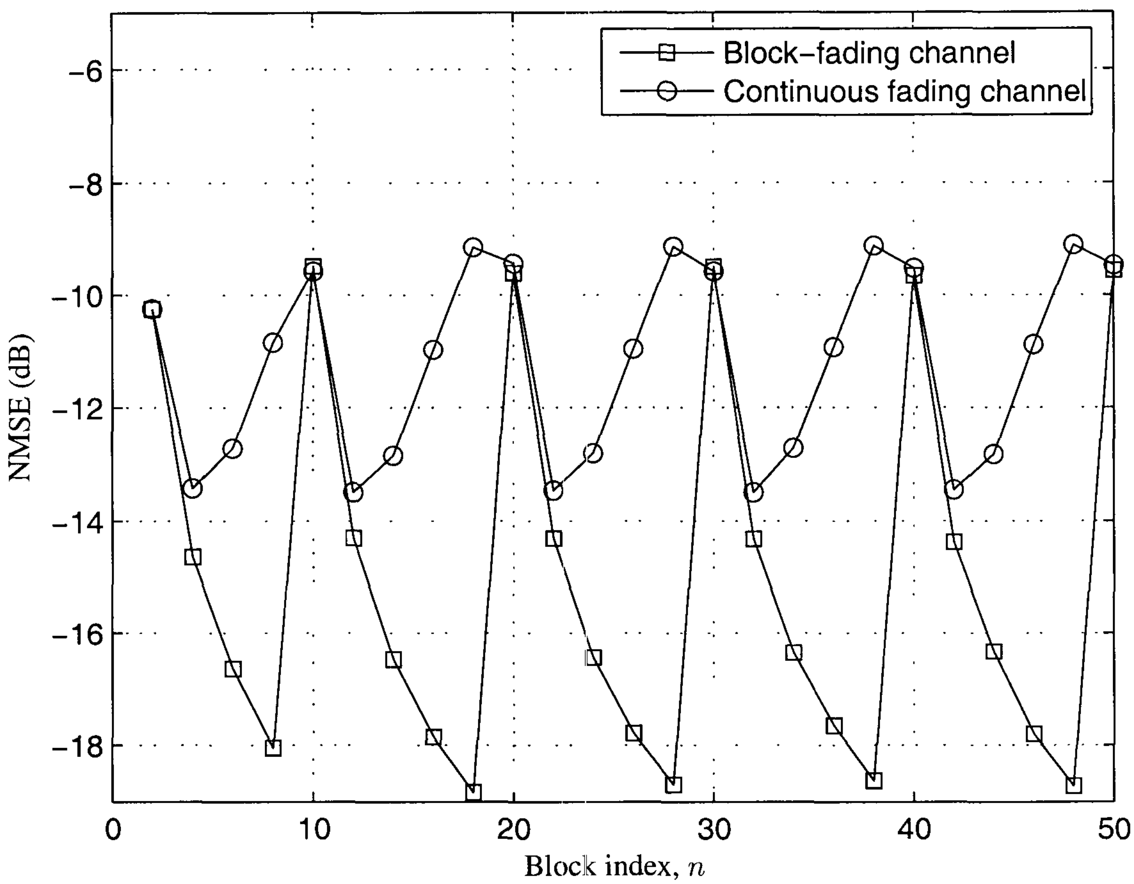


Figure 4.8: Continuous vs. block fading channels - NMSE

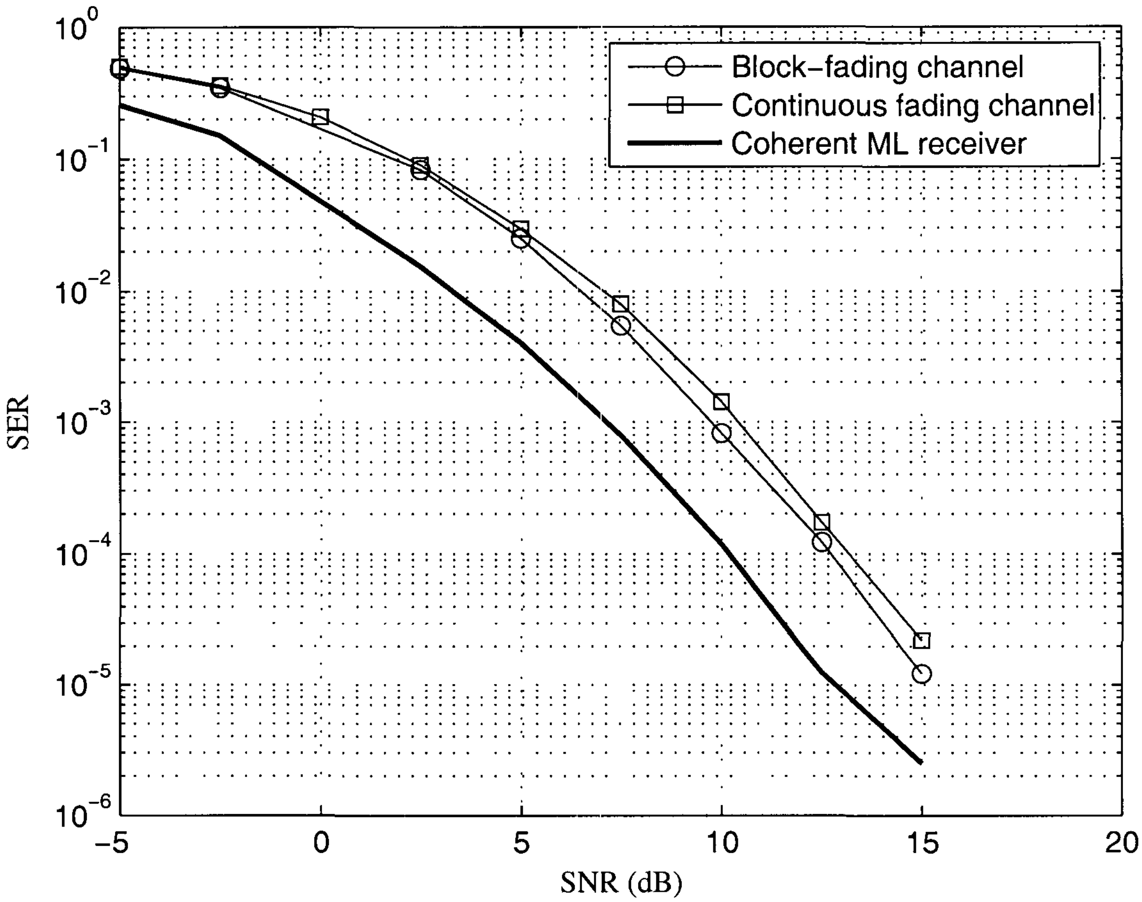


Figure 4.9: Continuous vs. block fading channels - SER

# Chapter 5

# Computationally Efficient Receivers for MIMO-OFDM Systems

## 5.1 Introduction

Time selectivity has become an increasingly important problem as MIMO-OFDM systems are being considered in a wide variety of mobile communication applications. A distinguished feature of OFDM is that the time varying channel can be estimated using some of the tones as pilot tones in every block of transmission. However, this leads to a significant reduction in data throughput. Hence, channel tracking algorithms for MIMO-OFDM systems can offer improvements in data throughput. In addition to that, some prior knowledge of the channel dynamics could also be exploited in performing a better channel estimation through channel tracking techniques. Many schemes for both channel estimation and channel tracking techniques have been explored by researchers in the past decade.

The use of least squares (LS) estimation and minimum mean square error (MMSE) estimation algorithms for channel estimation in MIMO-OFDM systems has been extensively studied in the literature. In [103] the time varying channel is approximated as polynomials and then estimated. In [15], channel estimation based on subspace tracking and LS estimation is presented. LS and MMSE channel estimators for MIMO-OFDM based on BLAST are reported in [6]. Blind channel estimation for linearly pre-coded MIMO-OFDM systems with multi-user interference was proposed in [125]. Channel estimation for MIMO-OFDM using a special structure of training sequence (cyclic comp type training sequence) was proposed in [100]. A minimum mean square error (MMSE) channel estimator for OFDM systems was proposed in [56]. Some modified versions of [56] were reported in [9, 55, 57] and in [7]. In [65] a MIMO-OFDM channel estimation scheme was developed based on training symbol arrangements. In [86] a MIMO-OFDM channel estimation scheme was developed based on LS estimation. In [69] a MMSE channel estimation scheme was derived for MIMO-OFDM system with spatial correlations. In [38] Optimal pilot sequence selection for MMSE channel estimation for MIMO-OFDM systems is presented. In [76] an LMMSE channel estimator was derived for MIMO-OFDM systems.

Some further notable channel estimation schemes include the iterative receivers based on the expectation maximization algorithm [2, 62], channel estimator based on the probabilistic data association (PDA) technique based on the time of arrivals (TOAs) estimation [121], subspace based techniques [127, 130], and the fast fourier transform (FFT) based techniques [85]. In general the above methods are able to perform well with time-varying channels provided that the channel variations are negligible during the transmission of one block of data. However, when some additional channel statistics are available, these methods are unable to utilize them to improve the performance of the receiver.

Kalman filtering has been previously studied in application to channel tracking for MIMO-OFDM systems with different space-time coding schemes. In [2], the Kalman filter has been used in the maximization step of the EM algorithm to track the frequency selective MIMO channel when the underlying code is an OSTBC and when OFDM is used. In [97], an extended Kalman filter (EKF) based channel tracking scheme was developed for MIMO-OFDM systems. A decision directed Kalman filter based receiver is proposed in [92] for MIMO-OFDM systems and a soft–Kalman filter based receiver for MIMO–OFDM systems was reported in [49]. Although these techniques can be applied to OSTBCs, they have not been designed to exploit interesting properties of OSTBCs to the full extent.

In Chapter 4, we have shown that in scenarios with flat fading channels, symbol detection performance improvements and computational complexity reduction can be achieved by exploiting the special properties of OSTBCs. In this Chapter, this result and propose an iterative decision-directed Kalman filter (IDDKF) approach for channel tracking and data decoding in MIMO-OFDM systems. We assume that the channel is quasi stationary and that the synchronization is perfect. Our IDDKF algorithm is similar to the one reported in [92]. However, this Chapter presents two contributions to the field. First, we propose a simple decision-directed iterative technique to improve the channel tracking performance. Second, noticing that the dominant computational complexity comes from the implementation of the Kalman filter, we use special properties of OSTBCs to significantly simplify the Kalman filtering algorithm. This results in remarkable computational savings. We also show that our technique outperforms the method of [92] in terms of symbol error rate.

The rest of the Chapter is organized as follows. In the following section, the MIMO-OFDM system model is briefly discussed. In Section 5.2, the simplified Kalman filtering based channel tracking scheme is presented. Simulation results are provided in Section 5.3 and conclusions are drawn in Section 5.4.

## 5.2 Channel Tracking and Symbol Decoding Algorithm

### 5.2.1 Conditional ML data decoding

Using (2.27), the $n$th received OFDM word can be written as

$$\tilde{\mathbf{y}}_n = \mathbb{A}(\mathbf{h}[n])\tilde{\mathbf{s}}_n + \tilde{\mathbf{v}}_n \tag{5.99}$$

where

$$\begin{aligned}
\tilde{\mathbf{y}}_n &= \left[\tilde{\mathbf{y}}_{n,0}^{\mathrm{T}}, \ldots, \tilde{y}_{n,L-1}^{\mathrm{T}}\right]^{\mathrm{T}} \\
\tilde{\mathbf{s}}_n &= \left[\tilde{\mathbf{s}}_{n,0}^{\mathrm{T}} \ldots \tilde{\mathbf{s}}_{n,L-1}^{\mathrm{T}}\right]^{T} \\
\mathbb{A}(\mathbf{h}[n]) &= \text{blockdiag}\left\{\mathbf{A}(\mathbf{h}_0[n]), \ldots, \mathbf{A}(\mathbf{h}_{L-1}[n])\right\} \\
\mathbf{h}[n] &= \left[\mathbf{h}_0^T[n], \ldots, \mathbb{h}_L[n]^T\right]^T \\
\tilde{\mathbf{v}}_n &= \left(\tilde{\mathbf{v}}_{n,0}^{\mathrm{T}} \cdots \tilde{\mathbf{v}}_{n,L-1}^{\mathrm{T}}\right)
\end{aligned}$$

Given the channel vector $\mathbf{h}_i[n]$, the optimal maximum likelihood (ML) detection for OSTBC based MIMO-OFDM system can be done in a tone-by-tone basis. The linear receiver computes $\hat{\tilde{\mathbf{s}}}_{n,i}$, the estimate of $\tilde{\mathbf{s}}_{n,i}$, as

$$\hat{\tilde{\mathbf{s}}}_{n,i} = \frac{1}{\|\mathbf{h}_i[n]\|^2}\mathbf{A}^T(\mathbf{h}_i[n])\tilde{\mathbf{y}}_{n,i}. \tag{5.100}$$

The symbol-by-symbol decoder then builds $\hat{\mathbf{s}}_i[n]$, the estimate of vector $\mathbf{s}_i[n]$, as $\hat{\mathbf{s}}_i[n] = [\mathbf{I}_K \; j\mathbf{I}_K]\hat{\tilde{\mathbf{s}}}_{n,i}$. The $k$th element of $\hat{\mathbf{s}}_i[n]$ is compared with all the points in the constellation corresponding to $s_{k,i}[n]$ and the closest point to the $k$th element of $\hat{\mathbf{s}}_i[n]$ is accepted as the $k$th decoded symbol.

Note that implementation of the ML decoder requires the knowledge of the time-varying channel. If the channel is fixed, one can use training to estimate the channel. However, in practice, the channel is time-varying, and hence tracking of the MIMO channel is required.

### 5.2.2 Conditional ML channel estimation

To mathematically derive the two-step channel tracking algorithm, we substitute (2.19) into (2.27) and re-write it as

$$\tilde{\mathbf{y}}_{n,i} = \mathbf{B}(\tilde{\mathbf{s}}_{n,i})\mathbf{h}_i[n] + \tilde{\mathbf{v}}_{n,i} = \mathbf{B}(\tilde{\mathbf{s}}_{n,i})\mathbf{W}_i\mathbf{g}[n] + \tilde{\mathbf{v}}_{n,i} \tag{5.101}$$

where the $2MT \times 2MN$ real matrix $\mathbf{B}(\tilde{\mathbf{s}}_{n,i})$ is defined as

$$\mathbf{B}(\tilde{\mathbf{s}}_{n,i}) \triangleq [\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_{n,i} \quad \mathbf{A}(\mathbf{e}_2)\tilde{\mathbf{s}}_{n,i} \; \ldots \; \mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_{n,i}] \tag{5.102}$$

and $\mathbf{e}_k$ is the $k$th column of the $2MN \times 2MN$ identity matrix $\mathbf{I}_{2MN}$. It has been shown in Section 4.2.2 that $\mathbf{B}(\tilde{\mathbf{s}}_{n,i})$ has orthogonal columns and the norm of each column is equal to $||\mathbf{s}_i[n]||^2$, i.e., it satisfies

$$\mathbf{B}^T(\tilde{\mathbf{s}}_{n,i})\mathbf{B}(\tilde{\mathbf{s}}_{n,i}) = ||\mathbf{s}_i[n]||^2\mathbf{I}_{2MN}. \tag{5.103}$$

Now, (5.99) can be written as

$$\tilde{\mathbf{y}}_n = \mathbb{B}(\tilde{\mathbf{s}}_n)\mathbf{W}\mathbf{g}[n] + \tilde{\mathbf{v}}_n \tag{5.104}$$

where $\mathbb{B}(\tilde{\mathbf{s}}_n) = \text{blockdiag}\,\{\mathbf{B}(\tilde{\mathbf{s}}_{n,0}), \ldots, \mathbf{B}(\tilde{\mathbf{s}}_{n,L-1})\}$. Using (5.103), it can be shown that

$$\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbb{B}(\tilde{\mathbf{s}}_n) = \mathbf{D}_n \otimes \mathbf{I}_{2MN} \tag{5.105}$$

where $\mathbf{D}_n = \text{diag}\{\|\mathbf{s}_i[n]\|^2, \ldots, \|\mathbf{s}_{L-1}[n]\|^2\}$. It follows from (5.104) and (5.105) that given $\tilde{\mathbf{s}}_n$, the ML estimate of the channel vector $\mathbf{g}[n]$ can be obtained as

$$\hat{\mathbf{g}}_{\text{ML}}[n] = \left(\mathbf{W}^T\mathbf{D}_n \otimes \mathbf{I}_{2MN}\mathbf{W}\right)^{-1}\mathbf{W}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}_n. \tag{5.106}$$

### 5.2.3 Channel tracking via Kalman filtering

From (2.18), the channel dynamics can be written as

$$\mathbf{g}[n+1] = \mathbb{F}\mathbf{g}[n] + \mathbf{u}[n] \tag{5.107}$$

where

$$\mathbf{F} = \begin{bmatrix} \Re\{\boldsymbol{\Gamma}\} \otimes \mathbf{I}_{MN} & -\Im\{\boldsymbol{\Gamma}\} \otimes \mathbf{I}_{MN} \\ \Im\{\boldsymbol{\Gamma}\} \otimes \mathbf{I}_{MN} & \Re\{\boldsymbol{\Gamma}\} \otimes \mathbf{I}_{MN} \end{bmatrix}, \tag{5.108}$$

$\mathbf{u}[n] = \underline{\mathbf{U}[n]}$ , $\boldsymbol{\Gamma} = \text{diag}\{\alpha_0, \ldots, \alpha_{P-1}\}$ and $\mathbf{U}[n] = [\mathbf{U}_0[n], \ldots, \mathbf{U}_{P-1}[n]]$. Further, it can be noted that $\mathbf{u}[n]$ is an i.i.d. Gaussian noise vector with zero mean and covariance $\mathbf{Q} = \mathbf{I}_2 \otimes \boldsymbol{\Sigma} \otimes \mathbf{I}_{MN}$ where $\boldsymbol{\Sigma} = \text{diag}\{\sigma_0, \ldots, \sigma_{P-1}\}$.

The Kalman filtering problem for channel tracking in OSTBC-based MIMO-OFDM communication system can now be formally stated as follows: Given the measurement-to-state matrix $\mathbb{B}(\tilde{\mathbf{s}}_n)$, use the observed data $\tilde{\mathbf{y}}_n$ to find the minimum mean squared error (MMSE) estimate of the components of the state vector $\mathbf{g}[n]$ for each $n \geq 1$.

Given the estimate of the state at time $n-1$, $\mathbf{g}[n-1|n-1]$ and the associated error covariance matrix $\mathbf{P}[n-1|n-1]$, the Kalman filter is used to obtain the estimate of the state at time $n$, i.e., $\mathbf{g}[n|n]$ and the associated error covariance matrix $\mathbf{P}[n|n]$. The Kalman filtering algorithm can be summarized as it follows:

$$\mathbf{g}[n|n-1] = \mathbf{F}\mathbf{g}[n-1|n-1] \tag{5.109}$$

$$\mathbf{P}[n|n-1] = \mathbf{F}\mathbf{P}[n-1|n-1]\mathbf{F}^T + \mathbf{Q} \tag{5.110}$$

$$\hat{\tilde{\mathbf{y}}}[n] = \mathbb{B}(\tilde{\mathbf{s}}_n)\mathbf{W}\mathbf{g}[n|n-1] \tag{5.111}$$

$$\boldsymbol{\nu}[n] = \tilde{\mathbf{y}}[n] - \hat{\tilde{\mathbf{y}}}[n] \tag{5.112}$$

$$\mathbf{P}_\nu[n] = \mathbf{R} + \mathbb{B}(\tilde{\mathbf{s}}_n)\mathbf{W}\mathbf{P}[n|n-1]\mathbf{W}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n) \tag{5.113}$$

$$\mathbf{G}_{\mathrm{K}}[n] = \mathbf{P}[n|n-1]\mathbf{W}^T\mathbb{B}^T(\tilde{s}_n)\mathbf{P}_\nu^{-1}[n] \tag{5.114}$$

$$\mathbf{g}[n|n] = \mathbf{g}[n|n-1] + \mathbf{G}_{\mathrm{K}}[n]\boldsymbol{\nu}[n] \tag{5.115}$$

$$\mathbf{P}[n|n] = \mathbf{P}[n|n-1] - \mathbf{G}_{\mathrm{K}}[n]\mathbf{P}_\nu[n]\mathbf{G}_{\mathrm{K}}^T[n] \tag{5.116}$$

where $\mathbf{g}[n|n-1]$ is the predicted state, $\mathbf{P}[n|n-1]$ is the covariance matrix of the predicted state, $\hat{\tilde{\mathbf{y}}}[n]$ is the predicted observation, $\boldsymbol{\nu}[n]$ is the innovation process, $\mathbf{P}_\nu[n]$ is the innovation covariance matrix, $\mathbf{G}_{\mathrm{K}}[n]$ is the Kalman gain, and $\mathbf{R} = E\{\tilde{\mathbf{v}}[n]\tilde{\mathbf{v}}[n]^T\}$ is the covariance matrix of the measurement noise $\tilde{\mathbf{v}}[n]$. As we assumed that the measurement noise is white with variance $\sigma_v^2/2$ per real dimension, $\mathbf{R} = (\sigma_v^2/2)\mathbf{I}_{2MTL}$ holds true.

### 5.2.4 Simplified Kalman filter

Now, assuming constant modulus signals, we show that the Kalman filtering algorithm can be significantly simplified using the special properties of the OSTBCs introduced earlier. The following Lemma is introduced to reduce the computational complexity of the Kalman filtering algorithm in (5.109)–(5.116).

**Lemma 5.1** *For constant modulus signals, if* $\mathbf{P}[n-1|n-1]$ *is a block diagonal matrix, then* $\mathbf{P}[n|n-1]$ *in (5.110) and* $\mathbf{P}[n|n]$ *in (5.116) are also block diagonal. More specifically, if*

$$\mathbf{P}[n-1|n-1] = \mathbb{I}_2 \otimes \mathbf{\Phi}_{n-1} \otimes \mathbf{I}_{MN} \tag{5.117}$$

*then*

$$\begin{aligned} \mathbf{P}[n|n-1] &= \mathbf{I}_2 \otimes \mathbf{\Psi}_n \otimes \mathbf{I}_{MN} &(5.118)\\ \mathbf{P}[n|n] &= \mathbf{I}_2 \otimes \mathbf{\Phi}_n \otimes \mathbf{I}_{MN} &(5.119) \end{aligned}$$

*where*

$$\begin{aligned} \mathbf{\Phi}_n &= \text{diag}\left\{\delta_n^0, \ldots, \delta_n^{P-1}\right\} &(5.120)\\ \mathbf{\Psi}_n &= \text{diag}\left\{\beta_n^0, \ldots, \beta_n^{P-1}\right\} &(5.121) \end{aligned}$$

*and*

$$\begin{aligned} \beta_n^p &= \delta_{n-1}^p |\alpha_p|^2 + \frac{{\sigma_p}^2}{2} &(5.122)\\ \delta_n^p &= \frac{\beta_n^p \sigma_v^2}{2s^2 L \beta_n^p + \sigma_v^2} &(5.123) \end{aligned}$$

*Proof:* See Appendix A. ■

*Proposition I: Using the above Lemma, the Kalman updated channel estimate in (5.115) is simplified as*

$$\begin{aligned}\mathbf{g}[n|n] &= (\mathbf{I}_2 \otimes \boldsymbol{\Delta}_n \otimes \mathbf{I}_{MN})\, \mathbf{g}[n|n-1] \\ &\quad + (\mathbf{I}_2 \otimes \boldsymbol{\Lambda}_n \otimes \mathbf{I}_{MN})\, \mathbf{W}^T \mathbb{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}[n] \qquad (5.124)\end{aligned}$$

*where* $\boldsymbol{\Delta}_n = \mathrm{diag}\left\{\tilde{\mu}_n^0, \ldots, \tilde{\mu}_n^{P-1}\right\}$, $\tilde{\mu}_n^p = 1 - s^2 L \mu_n^p$ *and*

$$\begin{aligned}\mu_n^p &= \frac{2\beta_n^p}{2s^2 L \beta_n^p + \sigma_v^2} \\ \tilde{\mu}_n^p &= 1 - s^2 L \mu_n^p\end{aligned}$$

*proof:* Using $(A.245)$ and $(A.246)$ the Kalman gain in (5.114) can be written as

$$\begin{aligned}\mathbf{G}_{\mathrm{K}}[n] &= \mathbf{P}[n|n-1]\tilde{\mathbf{W}}^T \mathbb{B}^T(\tilde{s}_n) \\ &\quad \left(\mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}}\left(\mathbf{I}_2 \otimes \mathrm{diag}\left\{\gamma_n^0, \ldots, \gamma_n^{P-1}\right\} \otimes \mathbf{I}_{MN}\right)\tilde{\mathbf{W}}^T \mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\right) \\ &= \left(\mathbf{I}_2 \otimes \mathrm{diag}\left\{\frac{2\beta_n^0}{2s^2 L \beta_n^0 + \sigma_v^2}, \ldots, \frac{2\beta_n^{P-1}}{2s^2 L \beta_n^{P-1} + \sigma_v^2}\right\} \otimes \mathbf{I}_{MN}\right) \tilde{\mathbf{W}}^T \mathbb{B}^T(\tilde{\mathbf{s}}_n) \\ &= (\mathbf{I}_2 \otimes \boldsymbol{\Lambda}_n \otimes \mathbf{I}_{MN})\, \mathbf{W}^T \mathbb{B}^T(\tilde{\mathbf{s}}_n) \qquad (5.125)\end{aligned}$$

where $\boldsymbol{\Lambda}_n = \mathrm{diag}\left\{\mu_n^0, \ldots, \mu_n^{P-1}\right\}$ and $\mu_n^p = \frac{2\beta_n^p}{2s^2 L \beta_n^p + \sigma_v^2}$.

Using (5.111), (5.112),and (5.125), we can simplify (5.115) as

$$\begin{aligned}\mathbf{g}[n|n] &= (\mathbf{I}_2 \otimes \boldsymbol{\Delta}_n \otimes \mathbf{I}_{MN})\, \mathbf{g}[n|n-1] \\ &\quad + (\mathbf{I}_2 \otimes \boldsymbol{\Lambda}_n \otimes \mathbf{I}_{MN})\, \mathbf{W}^T \mathbb{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}[n] \qquad (5.126)\end{aligned}$$

where $\mathbf{\Delta}_n = \text{diag}\left\{\tilde{\mu}_n^0, \ldots, \tilde{\mu}_n^{P-1}\right\}$ and $\tilde{\mu}_n^p = 1 - s^2 L \mu_n^p$.

.

Therefore, the Kalman filtering algorithm presented in (5.109)–(5.116) can be simplified as it follows:

$$
\begin{aligned}
\mathbf{g}[n|n-1] &= \mathbf{F}\mathbf{g}[n-1|n-1] && (5.127)\\
\beta_n^p &= \delta_{n-1}^p |\alpha_p|^2 + \frac{\sigma_p^2}{2} && (5.128)\\
\mu_n^p &= \frac{2\beta_n^p}{2s^2 L \beta_n^p + \sigma_v^2} && (5.129)\\
\tilde{\mu}_n^p &= 1 - s^2 L \mu_n^p && (5.130)\\
\delta_n^p &= \frac{\sigma_v^2 \beta_n^0}{2s^2 L \beta_n^0 + \sigma_v^2} && (5.131)\\
\mathbf{g}[n|n] &= \left(\mathbf{I}_2 \otimes \mathbf{\Delta}_n \otimes \mathbf{I}_{MN}\right) \mathbf{g}[n|n-1] \\
&\quad + \left(\mathbf{I}_2 \otimes \mathbf{\Lambda}_n \otimes \mathbf{I}_{MN}\right) \mathbf{W}^T \mathbb{B}^T(\tilde{\mathbf{s}}_n) \tilde{\mathbf{y}}[n] && (5.132)
\end{aligned}
$$

### 5.2.5 Initializing the Kalman filter:

To initiate the whole process, we also need to obtain an accurate enough channel estimate $\hat{\mathbf{g}}[0]$ as well as its initial covariance $\mathbf{I}_2 \otimes \mathbb{D}_{\delta_0} \otimes \mathbf{I}_{MN}$. To obtain such an initial channel estimate, one can use a training block $\mathbf{s}[0]$, which is known at the receiver. At the beginning of the tracking process, the receiver can then use (5.104) to obtain the ML estimate of $\mathbf{g}[0]$ as

$$\hat{\mathbf{g}}[0] = \frac{1}{s^2 L} \ddot{\mathbf{W}}^T \mathbb{B}^T(\tilde{\mathbf{s}}_0) \tilde{\mathbf{y}}_0 \quad (5.133)$$

where $\tilde{\mathbf{s}}_0 = \underline{\mathbf{s}[0]}$ is defined.

To find $\mathbb{D}_{\delta_0}$, we note that

$$\begin{aligned}\mathbf{I}_2 \otimes \mathbf{\Phi}_0 \otimes \mathbf{I}_{MN} &= E\left\{(\hat{\mathbf{g}}[0]-\mathbf{g}[0])(\hat{\mathbf{g}}[0]-\mathbf{g}[0])^T\right\} \\ &= E\left\{\hat{\mathbf{g}}[0]\hat{\mathbf{g}}^T[0]\right\} + E\left\{\mathbf{g}[0]\mathbf{g}^T[0]\right\} - E\left\{\hat{\mathbf{g}}[0]\mathbf{g}^T[0]\right\} - E\left\{\mathbf{g}[0]\hat{\mathbf{g}}^T[0]\right\} \\ &= \frac{\sigma_v^2}{2s^2L}\mathbf{I}_{2MNP} \qquad (5.134)\end{aligned}$$

where

$$\begin{aligned}\mathbf{I}_2 \otimes \mathbb{D}_{\delta_0} \otimes \mathbf{I}_{MN} &= E\left\{\hat{\mathbf{g}}[0]\hat{\mathbf{g}}^T[0]\right\} \\ &= \frac{1}{s^4L^2}\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_0)E\left\{\tilde{\mathbf{y}}_0\tilde{\mathbf{y}}_0^T\right\}\mathbb{B}(\tilde{\mathbf{s}}_0)\tilde{\mathbf{W}} \\ &= \frac{1}{s^4L^2}\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_0) \\ &\quad \left(\mathbb{B}(\tilde{\mathbf{s}}_0)\tilde{\mathbf{W}}\underbrace{E\{\mathbf{g}[0]\mathbf{g}^T[0]\}}_{\mathbf{I}_2\otimes\mathbb{D}_{\sigma_g^2/2}\otimes\mathbf{I}_{2MNP}}\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_0) + (\sigma_v^2/2)\mathbf{I}_{2MT}\right)\mathbb{B}(\tilde{\mathbf{s}}_0)\tilde{\mathbf{W}} \\ &= \mathbf{I}_2 \otimes \mathbb{D}_{\sigma_g^2/2} \otimes \mathbf{I}_{MN} + \frac{\sigma_v^2}{2s^2L}\mathbf{I}_{2MNP} \\ &= \mathbf{I}_2 \otimes \mathbb{D}_{\delta_0} \otimes \mathbf{I}_{MN} + \frac{\sigma_v^2}{2s^2L}\mathbf{I}_{2MNP} \\ &= \mathbf{I}_2 \otimes \operatorname{diag}\left\{\frac{1}{2}\left(\frac{\sigma_{w,0}^2}{1-|\alpha_0|^2}+\frac{\sigma_v^2}{s^2L}\right), \ldots \right. \\ &\qquad \left. ,\frac{1}{2}\left(\frac{\sigma_{w,P-1}^2}{1-|\alpha_{P-1}|^2}+\frac{\sigma_v^2}{s^2L}\right)\right\} \otimes \mathbf{I}_{MN} \qquad (5.135)\end{aligned}$$

Therefore, using the knowledge of $\alpha$, $\sigma_v^2$, $\sigma_w^2$, and $\mathbf{s}[0]$, one can obtain $\mathbb{D}_{\delta_0}$ as

$$\mathbb{D}_{\delta_0} = \operatorname{diag}\left\{\frac{1}{2}\left(\frac{\sigma_{w,0}^2}{1-|\alpha_0|^2}+\frac{\sigma_v^2}{s^2L}\right), \ldots, \frac{1}{2}\left(\frac{\sigma_{w,P-1}^2}{1-|\alpha_{P-1}|^2}+\frac{\sigma_v^2}{s^2L}\right)\right\}. \qquad (5.136)$$

### 5.2.6 Channel tracking in training based systems

The simplified Kalman filter in the previous section can be used with slight modification for channel tracking in comp-type [71] training based MIMO-OFDM systems, where, at each time block, a number of equally spaced tones are used to send pilot symbols enabling the receiver to track the channel coefficients, and the rest of the tones are used to send the data. According to this scheme, the CSI values obtained from the training are then used to decode the data. It is discussed in [71] that the sufficient number of tones required for accurate channel estimation is equal to the channel length. As we have discussed in Chapter 1, we will use the simplified Kalman filter algorithm above to show in Section 5.3 that the system performance can be improved by employing the proposed adaptive algorithm.

### 5.2.7 Summary of the algorithm

The ML decoding discussed in Section 5.2.1 and the channel tracking scheme discussed in Section 5.2.3 are conditioned on the true channel transmitted data, respectively. In the following, we propose a decision directed iterative algorithm to track the channel and decode the data in a blind fashion between two training blocks.

[1] Use a block of symbols to obtain the initial channel estimate for $\mathrm{g}[0]$ as

$$\hat{\mathrm{g}}[0] = \frac{1}{s^2 L}\tilde{\mathbf{W}}^T \mathbb{B}^T(\tilde{\mathrm{s}}_0)\tilde{\mathrm{y}}_0 \tag{5.137}$$

[2] The channel estimate covariance is initialized as

$$\mathbf{P}[0] = \mathbf{I}_2 \otimes \mathbb{D}_{\delta_0} \otimes \mathbf{I}_{MN} \tag{5.138}$$

[3] Set $n = 1$

[4] Given $\tilde{y}[n]$ and $g[n-1|n-1] = g[n-1]$, use the IDDKF as shown in Figure 5.10 to get the updated channel estimate $g[n|n]$. Note that the normalized channel estimation error is defined as

$$\text{NE} = \frac{\|g^1[n] - g^0[n]\|^2}{\|g^0[n]\|^2} \tag{5.139}$$

[5] Set $\hat{g}[n] = \hat{g}[n|n]$ and obtain the following estimate of $\tilde{s}_n$ as

$$\hat{\tilde{s}}_n = \frac{1}{\| L\tilde{\mathbf{W}}\hat{g}[n] \|^2} \mathbb{A}^T \left(\tilde{\mathbf{W}}\hat{g}[n]\right) \tilde{y}_n \tag{5.140}$$

[6] Compute the estimate of the $s(n)$ as

$$\hat{s}(n) = [\mathbf{I}_K \ j\mathbf{I}_k]\hat{\tilde{s}}_n \,. \tag{5.141}$$

[7] Compare the $k$th element of $\hat{s}(n)$ with all the points in the constellation corresponding to $s_k(n)$ and introduce the closest point of this constellation to the $k$th element of $\hat{s}(n)$ as the $k$th decoded symbol.

[8] Set $n \leftarrow n + 1$. If $n <$ TRP go to 4, otherwise go to 1.

### 5.2.8 Computational savings

The computational complexity of the simplified Kalman filter is of order $\mathcal{O}(M^2NTLP)$. The second step is an iterative algorithm. In each iteration, we need to compute four quantities: $\|h^{(k-1)}(n)\|^2$, $\mathbf{A}^T\left(h^{(k-1)}(n)\right)\tilde{y}_n$, $\|\hat{\tilde{s}}_n^k\|^2$, and $\mathbf{B}^T(\hat{\tilde{s}}_n^{(k)})\tilde{y}_n$. Computing these four

Table 5.4: Standard KF based MIMO-OFDM receiver complexity

| | Number of floating point operations |
|---|---|
| (5.109) | $\mathcal{O}(M^2N^2P^2)$ |
| (5.110) | $\mathcal{O}(M^3N^3P^3)$ |
| (5.111) | $\mathcal{O}(M^2NPTL)$ |
| (5.112) | $\mathcal{O}(MTL)$ |
| (5.113) | $\mathcal{O}(M^3N^2P^2TL)$ |
| (5.114) | $\mathcal{O}(M^3N^2P^2TL)$ |
| (5.115) | $\mathcal{O}(M^2NPTL)$ |
| (5.116) | $\mathcal{O}(M^3NPT^2L^2)$ |
| Complexity order | $\mathcal{O}(M^3NPT^2L^2)$ |

Table 5.5: Simplified KF based MIMO-OFDM receiver complexity

| | Number of floating point operations |
|---|---|
| (5.127) | $\mathcal{O}(M^2N^2P^2)$ |
| (5.128) | $\mathcal{O}(P^2)$ |
| (5.129) | $\mathcal{O}(P)$ |
| (5.130) | $\mathcal{O}(P)$ |
| (5.132) | $\mathcal{O}(M^2T^2L^2)$ |
| (5.131) | $\mathcal{O}(P)$ |
| Complexity order | $\mathcal{O}(M^2T^2L^2)$ |

quantities requires $2MNLP$, $4KMTL$, $2KL$, and $4M^2NTLP$ real multiplications per iteration, respectively. Therefore, the computational complexity of the second step is of the order $\mathcal{O}(M^2NTLP)$ per iteration. In the algorithm presented in [2] the maximization step involves Kalman filter which requires the computation of $\mathbf{P}_{\nu}^{-1}(n)$. This amount to a computational complexity of the order $\mathcal{O}(M^3T^3L^3)$ per iteration. Hence, the proposed algorithm clearly enjoys computational savings over [2].

Table 5.6: MIMO-OFDM receiver complexity comparison

| Standard KF | Simplified KF |
|---|---|
| $\mathcal{O}(M^3NPT^2L^2)$ | $\mathcal{O}(M^2T^2L^2)$ |

## 5.3 Simulation Results

In this section, we provide computer simulation results of the proposed scheme. The time domain MIMO wireless channel between each transmit and receive antenna is generated according to Jake's model [43, 107] with $P = 8$ independent Raleigh fading paths of equal powers. The MIMO channel at each path is generated according to Jake's model with the same value of $fT$ where $f$ is the doppler frequency and $T$ is the sampling time. The received signal power is normalized to 1 at each receiving antenna and hence the signal to noise ratio (SNR) is defined as

$$\mathrm{SNR} = \frac{1}{NP\sigma_v^2} \tag{5.142}$$

The channel tracking performance of the proposed scheme is measured in terms of the normalized mean squared error (NMSE) of the channel estimates defined as

$$\mathrm{NMSE} = E\left\{\frac{\|\mathbf{H}(n) - \hat{\mathbf{H}}(n)\|^2}{\|\mathbf{H}(n)\|^2}\right\}. \tag{5.143}$$

We select a $4 \times 4$ MIMO system and $L = 64$ sub-carriers. The underlying OSTBC scheme is considered to be the 3/4 rate code of [53, eq.(7.4.10)] with $N = M = T = 4$, and $K = 3$.

### 5.3.1 Channel tracking performance

In this example, we plot NMSE versus block number for different number of sub-carriers in Figure 5.11. As expected from example 2, this example confirms that as the number of sub-carriers increase, the channel tracking performance moves towards the steady-state performance of the KF channel tracking scheme. Further, this example shows that when $L$ is large enough it also helps to reduce the error propagation in the absence of training blocks. Indeed, as this figure suggests, when we use large enough $L$ such as $L = 64$ in this case, the receiver is able to operate without the training blocks.

### 5.3.2 Symbol error rate studies

In this example we show compare the theoretical curves of SER to that from the simulation. Figure 5.12 shows the simulated SER of the proposed Kalman filter detector and that of the coherent receiver. The corresponding SER values derived theoretically in Chapter 6 are also plotted for comparison.

### 5.3.3 Effect of the iteration

In this example, we study the effect of iteration on the performance. Figure 5.13 (top) shows the SER vs. EPS value and the figure in the bottom of the same figure the corresponding iteration numbers taken.

### 5.3.4 Performance against Doppler frequency

In this example we study the performance of the proposed receiver against doppler frequency. Figure 5.14 shows the SER vs. $fT$ values for the proposed scheme as well as for coherent ML receiver and the DDKF scheme of [92]. The figure shows that the proposed iteration scheme is able to perform better that that of [92] up to $fT = 0.1$.

## 5.4 Conclusions

A computationally efficient time-domain channel tracking technique for the OSTBC based MIMO-OFDM has been proposed. For such systems, a two-step channel tracking algorithm was proposed. As the first step, Kalman filtering is used to obtain an initial channel estimate, for each OFDM word, based on the channel estimates obtained for the previous words. In the second step, an iterative decision-directed method is used to refine the initial channel estimate obtained in the first step. It is shown that due to specific structure of OSTBC, both steps can be significantly simplified. A theoretical performance analysis of the proposed algorithm was also presented.

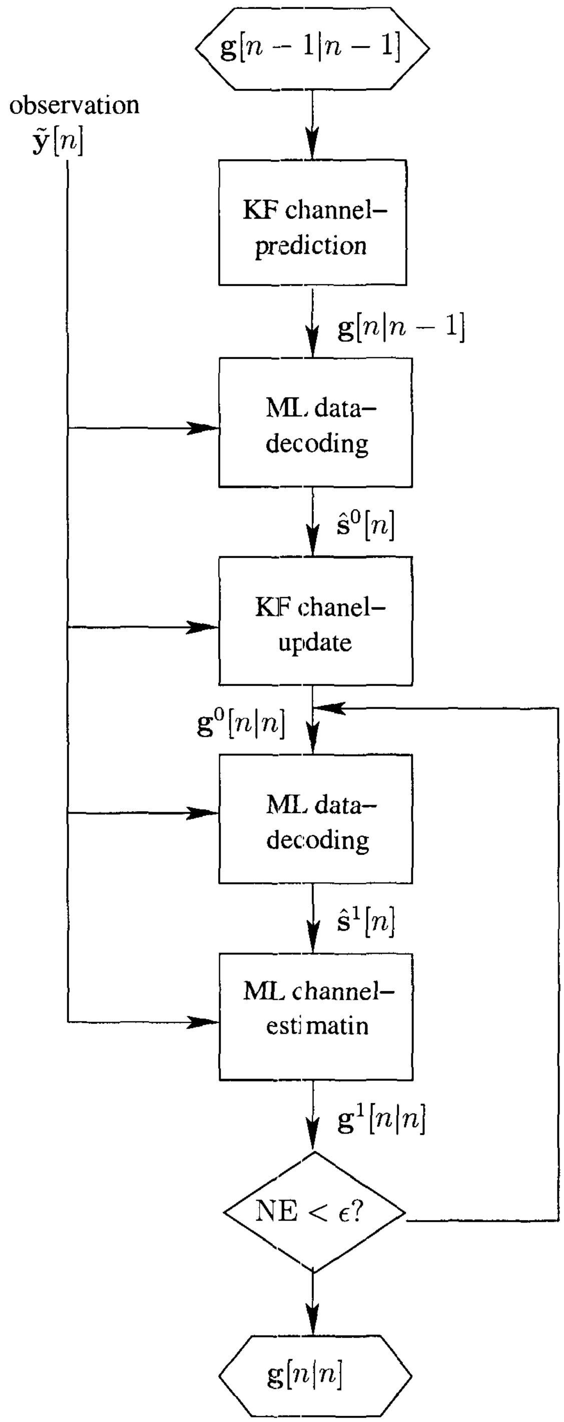


Figure 5.10: Proposed receiver

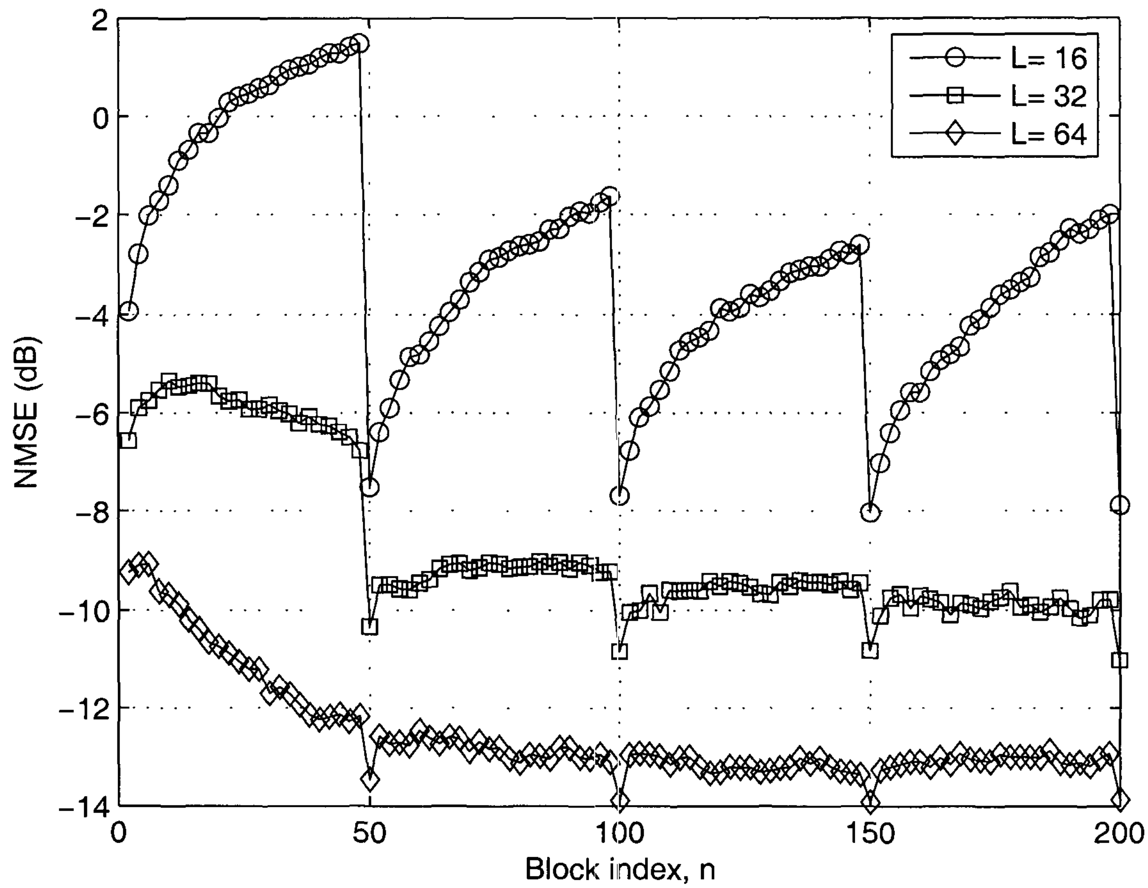


Figure 5.11: Channel tracking performance

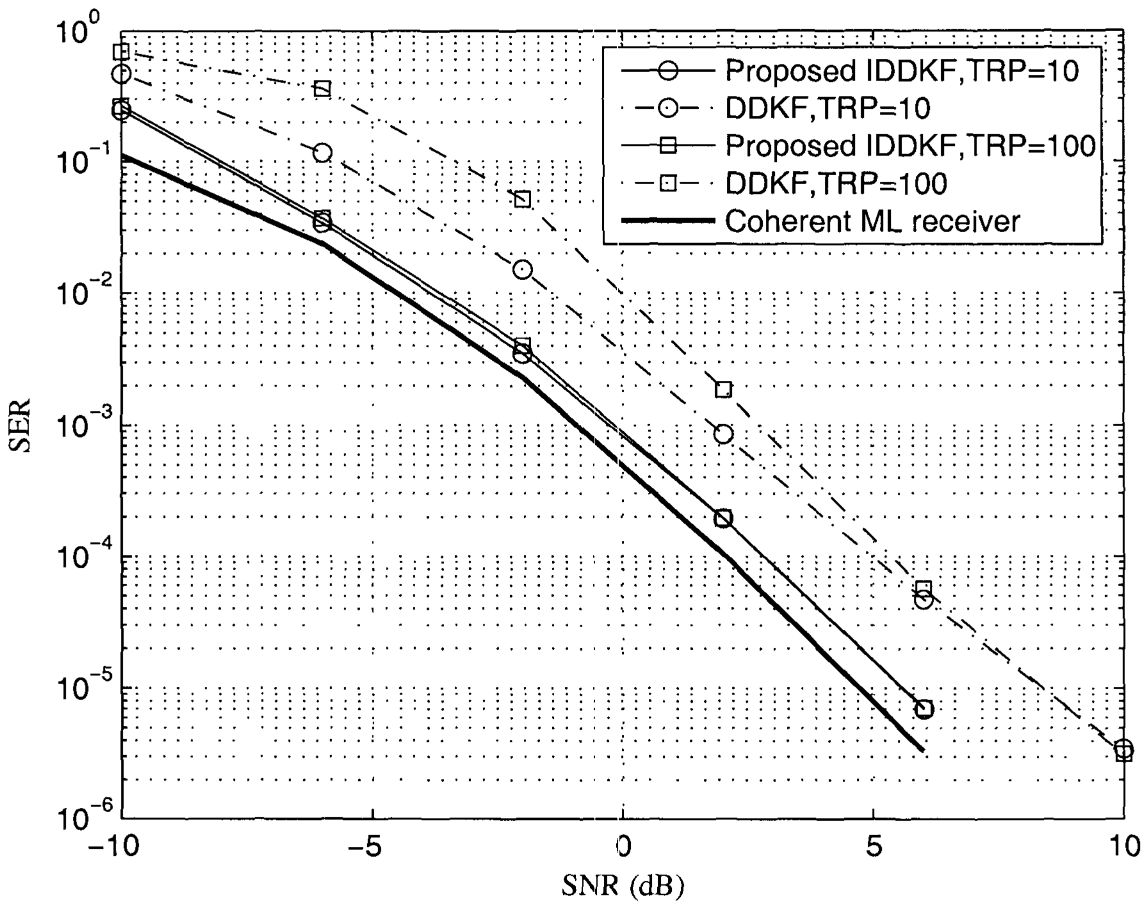


Figure 5.12: Symbol error rate studies

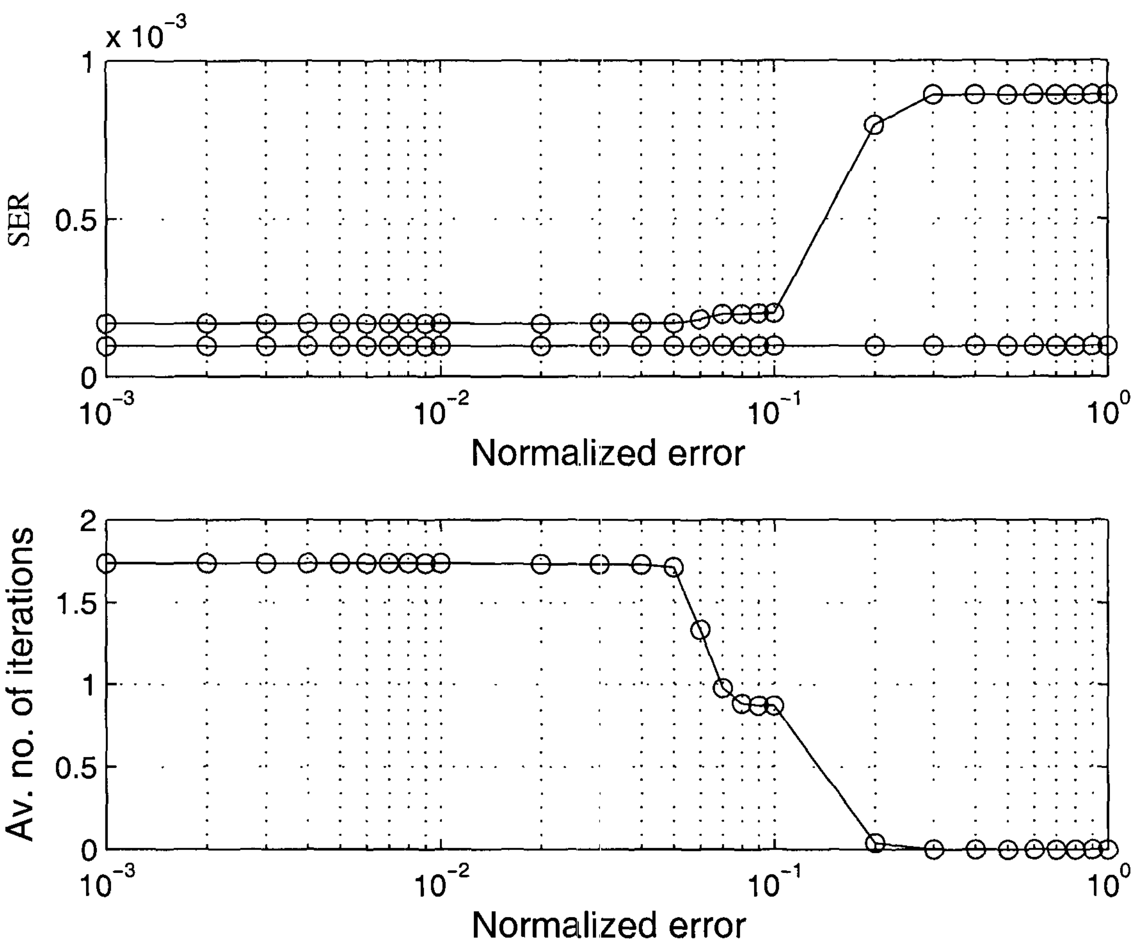


Figure 5.13: Effect of the iteration

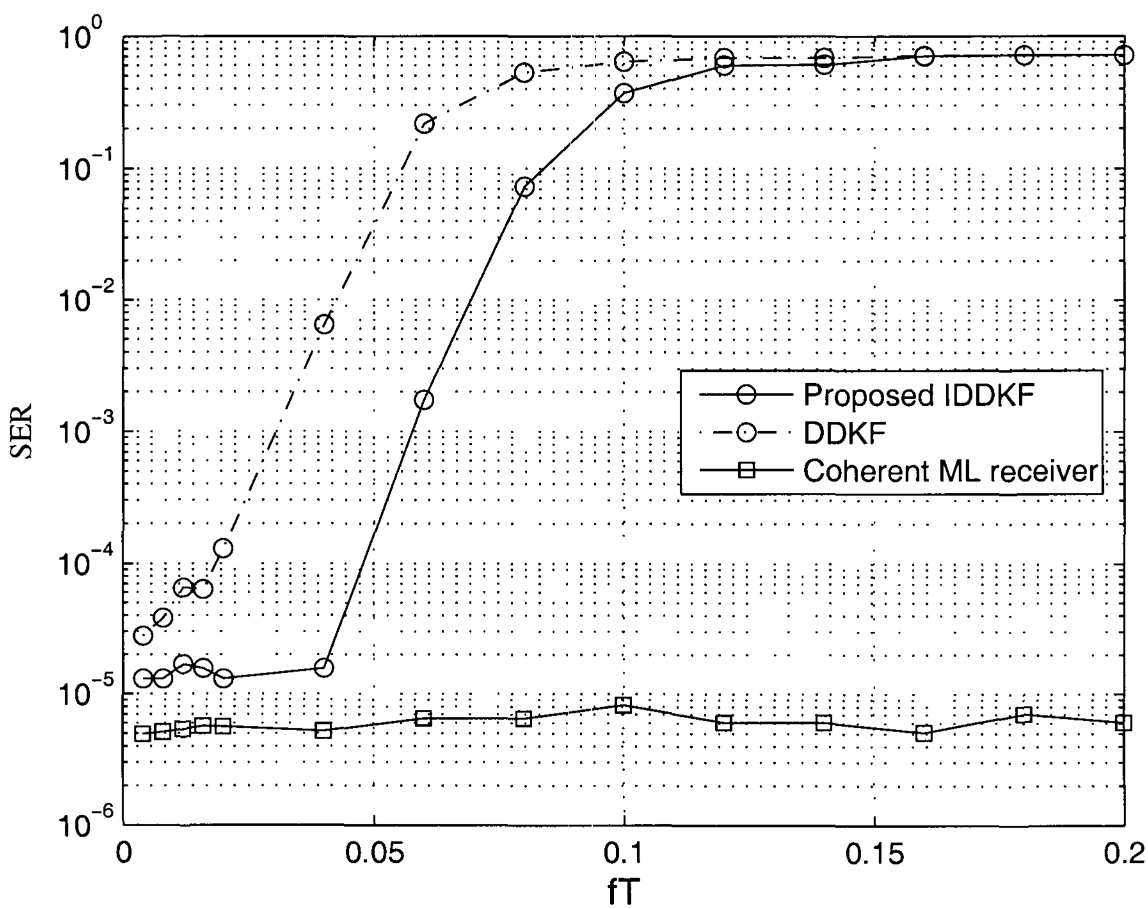


Figure 5.14: Performance against Doppler frequency

# Chapter 6

# Performance Analysis of MIMO Receivers

## 6.1 Introduction

Theoretical performance analysis is a part of communication system design. In this chapter, we develop theoretical performance limits of the receivers proposed in the preceding two Chapters.

Performance analysis of MIMO systems and space-time coding has been considerably attended by the academic community in the past decade. Firstly, an approximate expression for the bit- error probability for OSTBC's was derived in [28]. An upper bound on the error probability was reported in [94]. Then, an exact expression was derived in [32] and later an extended version considering multi-dimensional constellations was reported in [30]. All these derivations assumed coherent detection, i.e., they assumed the knowledge of the channel at the receiver.

Some previous works on performance evaluation for space-time codes under imperfect channel knowledge include probability of error derivation for MMSE estimated channels [84]. The work reported in [82] analyzes the effects of time selectivity in differential modulated systems. A lower bound on probability of error of Alamouti St-Coded OFDM Systems with frequency offset was derived in [5]. The impact of time-selective fading on orthogonal space-time block coding was derived in [74]. All of the above derivations are approximate ones considering a specific receiver structure. The performance of Kalman filter based decision-directed receiver for MIMO-OFDM system was never studied before. Further, in this Chapter, we derive general expressions regardless of the number of Tx/Rx antennas.

In this Chapter, we derive approximate BER expressions for the proposed MIMO receivers for flat fading as well as frequency-selective channels. Assuming the ML channel estimates for the first (training) block, we have derived the instantaneous signal to interference and noise ratio (SINR) for consecutive transmission blocks in the absence of training, by exploiting Kalman filtering to track the channel in a decision-directed mode. The derivation is general for any number of transmitting and receiving antennas. We study the effect in performance for different stages of the channel estimation, i.e., the Kalman predicted channel, Kalman updated channel and the steady-state channel estimate, and conclude that for MIMO-OFDM systems, the steady state performance is accelerated with the number of sub-carriers. This observation lead us for the derivation of the theoretical performance limits for time-domain and frequency-domain channel tracking for MIMO-OFDM systems. As expected, the theoretical and simulated BER curves match and confirm that the proposed time-domain channel tracking is superior in performance to that of frequency domain one.

The rest of this Chapter is organized as follows. In Section 6.2.1 we have derived an approximate value of the SNR value at each block in the absence of training. In Section 6.2.2 the averaged probability for different cases of the decision-directed Kalman filter estimation and their suitability is discussed. In Section 6.3, some numerical examples are provided and in Section 6.4 some conclusions are drawn.

## 6.2 Decision-Directed Kalman Filter Receiver

### 6.2.1 Instantaneous SNR

Let us assume that we have used pilot symbols at block $n-1$ to get the ML channel estimate $\mathrm{g}[n-1|n-1]$ and the corresponding estimation error covariance (see (5.106) and (5.135))

$$\mathbf{g}[n-1|n-1] = \left(\mathbf{W}^T\mathbf{D}_{n-1} \otimes \mathbf{I}_{2MN}\mathbf{W}\right)^{-1} \mathbf{W}^T\mathbb{B}^T(\tilde{\mathbf{s}}_{n-1})\tilde{\mathbf{y}}_{n-1} \quad (6.144)$$

$$\mathbf{P}[n-1|n-1] = \delta^{\mathrm{g}}_{n-1}\mathbf{I}_{2MNP} \quad (6.145)$$

where $\delta^{\mathrm{g}}_{n-1} = \sigma_v^2/(2s^2L)$, and for simplicity, it is assumed that the channel statistics remain the same at each taps, i.e., $\alpha_0 = \alpha_1 =, \dots, = \alpha_{P-1} = \alpha$.

Pre-multiplying (6.144) by $\mathbf{W}_i$ we get the initial estimated frequency response at the $i$th sub-carrier as

$$\mathbf{h}_i[n-1|n-1] = \mathbf{W}_i\mathbf{g}[n-1] \quad (6.146)$$

The corresponding error covariance can be written as

$$\mathbf{P}_{\mathrm{h}}[n-1|n-1] = \mathbf{W}_i\mathbf{P}[n-1|n-1]\mathbf{W}_i^T \quad (6.147)$$

$$= \delta_{n-1}^{\mathrm{h}}\mathbf{I}_{2MNP}. \quad (6.148)$$

where $\delta_{n-1}^{\mathrm{h}} = P\delta_{n-1}^{\mathrm{g}}$.

With no pilot available at block $n$, the Kalman prediction of the channel at block $n$ is written as

$$\mathbf{g}[n|n-1] = \mathbf{F}\mathbf{g}[n-1|n-1] \quad (6.149)$$

We can write the above channel estimate as

$$\mathbf{g}[n|n-1] = \mathbf{g}[n] + \varepsilon[n|n-1] \quad (6.150)$$

where $\mathbf{g}[n]$ is the true channel and $\varepsilon[n|n-1]$ is a $2MN \times 1$ vector that accounts for the channel estimation errors at block $n$. It can be verified that $\varepsilon[n|n-1]$ has zero mean and covariance matrix

$$\mathbf{P}_{\varepsilon}[n|n-1] = \mathbf{F}\mathbf{P}[n-1|n-1]\mathbf{F}^T + \mathbf{Q}$$

$$= \beta_n^{\mathrm{g}}\mathbf{I}_{2MNP} \quad (6.151)$$

where $\beta_n^{\mathrm{g}} = \delta_{n-1}^{\mathrm{g}}|\alpha|^2 + \sigma_u^2/2$.

Pre-multiplying (6.150) by $\mathbf{W}_i$ we get the estimated frequency response at the $i$th sub-carrier as

$$\mathbf{h}_i[n|n-1] = \mathbf{h}_i[n] + \xi_i[n|n-1] \quad (6.152)$$

where $\mathbf{h}_i[n]$ is the true response and $\xi_i[n|n-1]$ is the error associated to $\varepsilon[n|n-1]$. It can be noticed that $\xi_i[n|n-1]$ has zero mean and covariance matrix

$$\begin{aligned} \mathbf{P}_{\xi}[n|n-1] &= \mathbf{W}_i \mathbf{P}_{\varepsilon}[n|n-1]\mathbf{W}_i^T \\ &= \beta_n^{\mathrm{h}} \mathbf{I}_{2MN} \end{aligned} \tag{6.153}$$

where $\beta_n^{\mathrm{h}} = P\beta_n^{\mathrm{g}}$.

Assuming that the data was decoded error free using the predicted channel $\mathbf{h}_i[n|n-1]$, the updated channel is given as

$$\mathbf{g}[n|n] = \mathbf{g}[n|n-1] + \mathbf{G}_{\mathrm{K}}\nu[n] \tag{6.154}$$

where $\nu[n]$ is the innovation process that is zero-men with covariance matrix $\mathbf{P}_{\nu}[n]$ (see 5.2). Similar to (6.150), we can write the above channel estimate as

$$\mathbf{g}[n|n] = \mathbf{g}[n] + \varepsilon[n|n] \tag{6.155}$$

The corresponding error covariance can be written as

$$\mathbf{P}_{\varepsilon}[n|n] = \delta_n^{\mathrm{g}} \mathbf{I}_{2MNP} \tag{6.156}$$

where

$$\delta_n^{\mathrm{g}} = \frac{\sigma_v^2 \beta_n^{\mathrm{g}}}{2s^2 L \beta_n^{\mathrm{g}} + \sigma_v^2} \tag{6.157}$$

It is worth mentioning that $\mathbf{h}_i[n|n-1]$ is zero mean with covariance

$$\begin{aligned}\mathbf{P}_h[n|n-1] &= E\Big\{\mathbf{h}_i[n|n-1]\mathbf{h}_i^T[n|n-1]\Big\} \\ &= \mathbf{W}_i\mathbf{F}E\Big\{\mathbf{g}[n|n-1]\mathbf{g}^T[n|n-1]\Big\}\mathbf{F}^T\mathbf{W}_i^T \\ &= \rho_n\mathbf{I}_{2MN}\end{aligned} \tag{6.158}$$

where

$$\rho_n = P|\alpha|^2\left(\frac{\sigma_g^2}{2}+\frac{\sigma_v^2}{2s^2L}\right). \tag{6.159}$$

Assuming the channel estimate at block $n$ as $\hat{\mathbf{h}}_i[n] = \mathbf{h}_i[n|n-1]$ the linear receiver that computes $\hat{\tilde{\mathbf{s}}}_{n,i}$, the estimate of $\tilde{\mathbf{s}}_{n,i}$, is written as

$$\begin{aligned}\hat{\tilde{\mathbf{s}}}_{n,i} &= \frac{1}{\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\tilde{\mathbf{y}}_{n,i} \\ &= \frac{1}{\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\Big(\mathbf{A}\big(\hat{\mathbf{h}}_i[n]-\xi_i[n]\big)\tilde{\mathbf{s}}_i[n]+\tilde{\mathbf{v}}_{n,i}\Big) \\ &= \frac{1}{\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\Big(\mathbf{A}(\hat{\mathbf{h}}_i[n])\tilde{\mathbf{s}}_i[n]-\mathbf{A}(\xi_i[n])\tilde{\mathbf{s}}_i[n]+\tilde{\mathbf{v}}_{n,i}\Big) \\ &= \tilde{\mathbf{s}}_i[n]-\frac{1}{\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\mathbf{A}(\xi_i[n])\tilde{\mathbf{s}}_i[n]+\frac{1}{\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\tilde{\mathbf{v}}_{i,n} \\ &= \tilde{\mathbf{s}}_i[n]-\frac{1}{\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\mathbf{B}(\tilde{\mathbf{s}}_i[n])\xi_i[n]+\frac{1}{\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\tilde{\mathbf{v}}_{i,n} \\ &= \tilde{\mathbf{s}}_i[n]-\tilde{v}_{i,n}+\tilde{\mathbf{z}}_{i,n}\end{aligned} \tag{6.160}$$

where the noise term $\tilde{\mathbf{z}}_{i,n}$ is uncorrelated and zero mean with covariance matrix

$$\mathbf{R}_{\tilde{\mathbf{z}}} = \frac{\sigma_v^2}{2\|\hat{\mathbf{h}}_i[n]\|^2}\mathbf{I}_{2K}, \tag{6.161}$$

and the vector $\tilde{\upsilon}_{i,n}$ zero mean with covariance matrix

$$\begin{aligned}\mathbf{R}_{\tilde{\upsilon}} &= \frac{1}{\|\hat{\mathbf{h}}_i[n]\|^4} E\Big\{\mathbf{A}^T(\hat{\mathbf{h}}_i[n])\mathbf{B}(\tilde{\mathbf{s}}_i[n])\xi_i[n]\xi_i^T[n]\mathbf{B}^T(\tilde{\mathbf{s}}_i[n])\mathbf{A}(\hat{\mathbf{h}}_i[n])\Big\} \\ &= \frac{1}{\|\hat{\mathbf{h}}_i[n]\|^4}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])E_{\tilde{\mathbf{s}}_i[n]}\left\{\mathbf{B}(\hat{\mathbf{s}}_i[n])E_{\xi_i[n]}\Big\{\xi_i[n]\xi_i^T[n]\Big\}\mathbf{B}^T(\tilde{\mathbf{s}}_i[n])\right\}\mathbf{A}(\hat{\mathbf{h}}_i[n]) \\ &= \frac{\beta_n^{\mathrm{h}}}{\|\hat{\mathbf{h}}_i[n]\|^4}\mathbf{A}^T(\hat{\mathbf{h}}_i[n])E_{\tilde{\mathbf{s}}_i[n]}\Big\{\mathbf{B}(\tilde{\mathbf{s}}_i[n])\mathbf{B}^T(\tilde{\mathbf{s}}_i[n])\Big\}\mathbf{A}(\hat{\mathbf{h}}_i[n]) \qquad (6.162)\end{aligned}$$

where $E_x(.)$ denotes the expectation of the argument with respect to $x$.

The following lemma derives a property of the OSTBCs, which will be useful to simplify (6.162).

**Lemma 6.1** *The matrix* $\mathbf{B}(\tilde{\mathbf{s}}_n)$ *satisfies*

$$\mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{B}^T(\tilde{\mathbf{s}}_n) = \frac{(T-d)N}{T}\sigma_s^2\mathbf{I}_{2MT} \qquad (6.163)$$

*where $d$ is the number of zeros contained in each column of* $\mathbf{X}(\mathbf{s}[n])$.

*proof:* See Appendix B. ■

Using 6.1, (6.162) is simplified as follows

$$\begin{aligned}\mathbf{R}_{\tilde{\upsilon}} &= \left(\frac{\beta_n^{\mathrm{h}}\sigma_s^2}{\|\hat{\mathbf{h}}_i[n]\|^2}\right)\left(\frac{T-d}{T}\right)N\mathbf{I}_{2K} \\ &= \left(\frac{\tilde{\beta}_n^{\mathrm{h}}\sigma_s^2}{\|\hat{\mathbf{h}}_i[n]\|^2}\right)\mathbf{I}_{2K} \qquad (6.164)\end{aligned}$$

where $\tilde{\beta}_n^{\mathrm{h}} = \beta_n^{\mathrm{h}}\left(\frac{T-d}{T}\right)N$.

The SNR of the decision variable can now be written as

$$\mathrm{SNR} = \frac{\sigma_s^2\|\hat{\mathbf{h}}_i[n]\|^2}{2\tilde{\beta}_n^{\mathrm{h}} + \sigma_v^2} \tag{6.165}$$

### 6.2.2 Averaged probability of error

Considering $m$-ary square QAM, the probability of error can now be approximately written as [12]

$$P_{e,n} = \frac{2(\sqrt{m}-1)}{\sqrt{m}\log_2\sqrt{m}} Q\left(\sqrt{\frac{3\mathrm{SNR}}{m-1}}\right) = c\, Q(a_n\sqrt{\varsigma}) \tag{6.166}$$

where

$$c = \frac{2(\sqrt{m}-1)}{\sqrt{m}\log_2\sqrt{m}}, \tag{6.167}$$

$$a_n^2 = \frac{\sqrt{3}}{\sqrt{m-1}}\left(\frac{\rho_n\sigma_s^2}{2\tilde{\beta}_n^{\mathrm{h}} + \sigma_v^2}\right), \tag{6.168}$$

$\varsigma = \frac{\|\hat{h}[n]\|^2}{\rho_n}$ and $Q(x)$ is the complementary error function defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt. \tag{6.169}$$

The averaged BER is written as

$$\begin{aligned} \bar{P}_{e,n} &= c\, E\{Q(a\sqrt{\varsigma})\} \\ &= c\int_0^\infty Q(a\sqrt{\varsigma})p_\varsigma(\varsigma)d\varsigma \end{aligned} \tag{6.170}$$

The integration (6.170) can be analyzed under two different cases.

**Case I:** $P = 1$

In this case we have $\hat{\mathbf{h}}_i[n] = \mathbf{I}_{2MN}\hat{\mathbf{g}}[n]$ where $\hat{\mathbf{g}}[n]$ is i.i.d Gaussian with zero mean and variance $\rho_n$. The probability distribution of $\varsigma = \|\mathbf{g}[n]\|^2/\rho_n$ is chi-squared with $k = 2MNP$ degrees of freedom, i.e.,

$$p_\varsigma(\varsigma) = \frac{(1/2)^{k/2}}{\Gamma(k/2)}\varsigma^{k/2-1}e^{-\varsigma/2} \tag{6.171}$$

the moment generating function of the above pdf is given as $(1-2s)^{-k/2}$. From [101, pp. 124], the integration of (6.170) reduces to

$$\bar{P}_{e,n} = c\,\frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{a_n^2+\sin^2\theta}\right)^m d\theta \tag{6.172}$$

where $m = MNP$.

Finally, the average bit error rate is written as

$$\bar{P}_{e,n} = c\,\frac{1}{2}\left[1-\mu_n\sum_{k=0}^{m-1}\begin{pmatrix}2k\\k\end{pmatrix}\left(\frac{1-\mu_n^2}{4}\right)^k\right] \tag{6.173}$$

where

$$\mu_n = \sqrt{\frac{a_n^2}{1+a_n^2}}. \tag{6.174}$$

**Case II:** $P > 1$

In this case, the elements of $\hat{\mathbf{h}}_i[n] = \mathbf{W}_i\hat{\mathbf{g}}[n]$ are correlated and hence the distribution of $\varsigma$ is complicated and finding a closed form solution for (6.170) is very challenging. Luckily, the samples of $\varsigma$ can be easily generated and hence the following monte carlo integration can be applied to find the averaged BER

$$\bar{P}_{e,n} = \sum_{j=1}^{N_s} Q(\varsigma_j) \tag{6.175}$$

where $\varsigma_j \sim p(\varsigma)$ and $N_s$ is the number of samples taken in the integration.

Let us assume that, starting from the $n$th block, there are no pilot symbols for $L_t$ blocks. Hence, from $n$th block to $(n-1+L_t)$th block, i.e., for $L_t$ stages, the receiver operates in a blind fashion as proposed in Section 5.2. During these stages, the BER could be analyzed under the following three channel estimation scenarios.

a) *Predicted Channel:* In the absence of pilot symbols, the best possible channel estimate at any of the $L_t$ stages will be the forward predicted channel from the pilot-aided channel estimate obtained during the $n-1$th block. Hence, at the $L_t$th stage, the channel estimate will be

$$\hat{\mathbf{g}}[n-1+l_t|n-1] = \mathbf{F}^{l_t}\mathbf{g}[n-1|n-1] \;\; l_t = 1, \ldots L_t \tag{6.176}$$

the error covariance corresponding to the above estimate is given as

$$\mathbf{P}_\epsilon[n-1+l_t|n-1] \tag{6.177}$$

b) *KF updated channel:* Let us assume that using the predicted channel $\mathbf{g}[n|n-1]$ error free decoding of the transmitted data was achieved at 1st stage, i.e. in the $n$th block, then using observation $\tilde{\mathbf{y}}_n$ and the decoded data $\hat{\bar{\mathbf{s}}}_n$ a KF updated channel $\mathbf{g}[n|n]$ could be obtained. Since it is already assumed that the the transmitted data was decoded errorfree at the $n$th block, the KF update channel $\mathbf{g}[n|n]$, even though it is more accurate estimate than the predicted channel estimate $\mathrm{g}[n|n-1]$, will not be useful for the current block. However, in the absence of training, the KF updated channel estimate will be useful in the next stage.

Now, assuming that in the above fashion error-free decoding occurred for $l_t$ stages, the KF updated channel estimate and the corresponding estimation error covariance could be obtained using the Kalman filter algorithm presented in Section 5.2.

c) *KF updated channel at the steady state:* Assuming that the error free decoding was achieved for for long enough that the KF based channel tracking scheme achieved steady state, the steady state error covariance can be obtained by solving the *matrix Riccati equation*

$$\mathbf{P}[n|n-1] = \mathbf{F}\mathbf{P}[n|n]\mathbf{F}^T + \mathbf{Q} \tag{6.178}$$

simplifying (6.178) results in the following

$$\tilde{\beta}_n = |\alpha|^2 \left( \frac{\sigma_v^2 \tilde{\beta}_n}{2s^2 L \tilde{\beta}_n + \sigma_v^2} \right) + \sigma_u^2 \tag{6.179}$$

Let $\hat{\tilde{\beta}}$ be the solution of the above equation, then the average Probability at steady state can be obtained by replacing $\tilde{\beta}_n^g$ in (6.151) by $\hat{\tilde{\beta}}$.

Now, for the first two cases above, assuming TRP $= L_t$, the average probability over the $L_t$ blocks without pilot symbols is obtained as

$$\bar{P}_e = \frac{1}{L} \sum_{l=1}^{L_t} \bar{P}_{e,n+l-1} \tag{6.180}$$

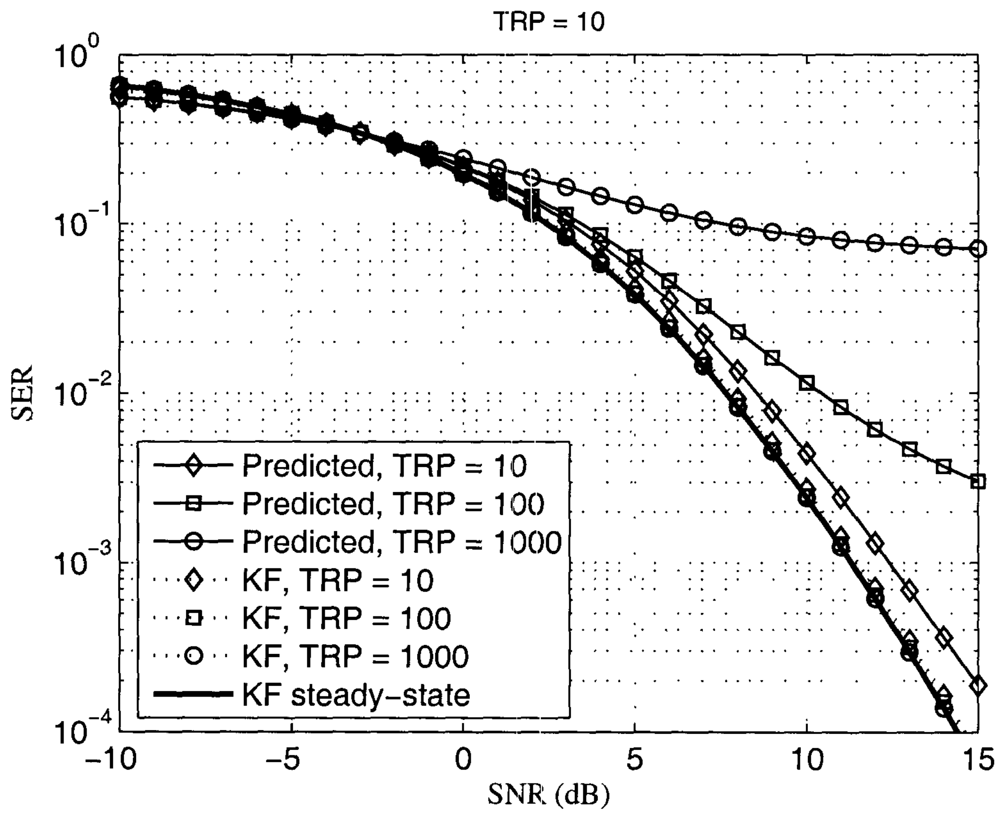


Figure 6.15: Example 1 – Effect of TRP

## 6.3 Numerical Studies

### 6.3.1 Effect of TRP

In this example, we plot the symbol error rate (SER) versus SNR for different $L_t$ values. For the predicted channel, the performance degrades as $L_t$ increases. This is behaviour is justifiable since the predicted channel uses no other information than the presumed channel model in (2.18) to predict the channel value, the chnnel estimation error increases with $L_t$.

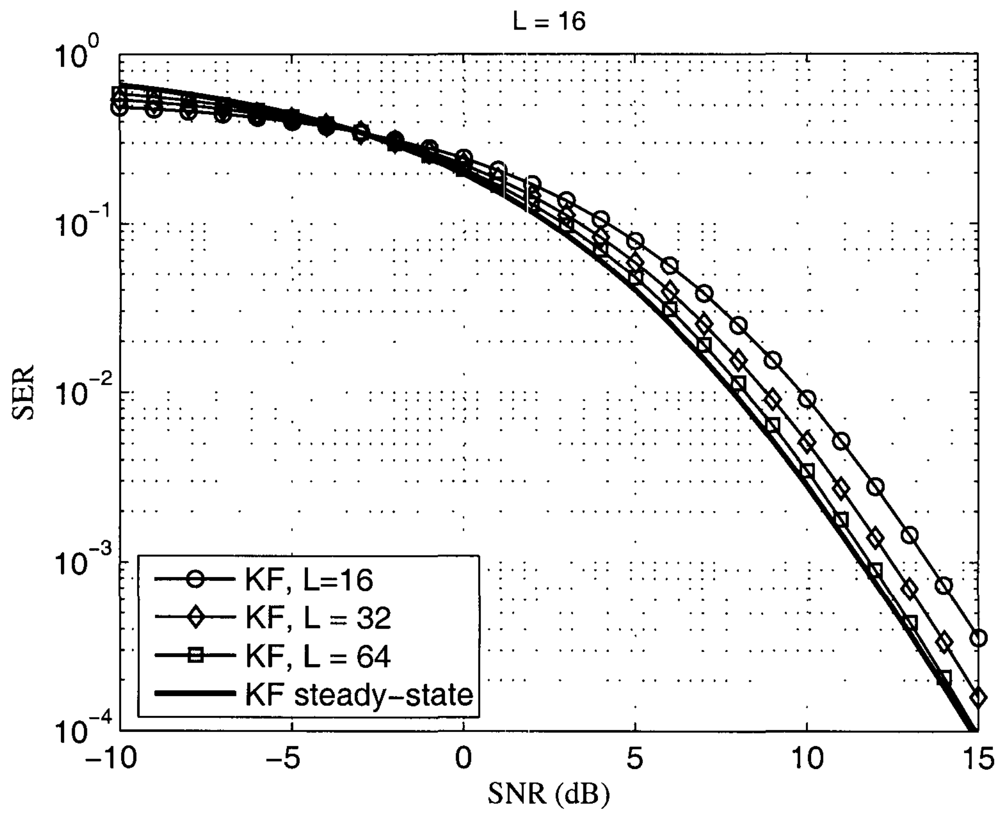


Figure 6.16: Example 2 – Effect of the number of sub-carriers

### 6.3.2 Effect of the number of sub-carriers

In this example we show that the stead-state KF performance can be achieved for smaller $L_t$ values by increasing the number of sub-carriers. Fig. 6.16 shows that for a given $L_t = 10$, as the number of sub-carriers increases, the KF performance approaches the steady-state performance. This fact is further confirmed in the simulation studies as explained in example 4.

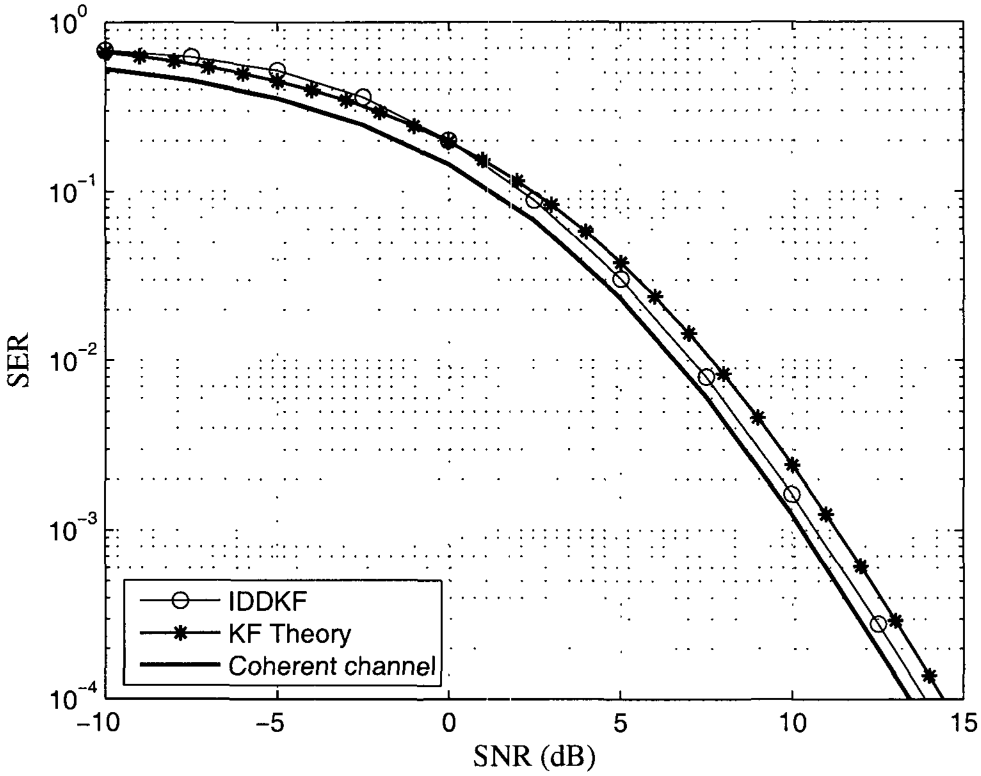


Figure 6.17: Example 3 - Theory versus simulation

### 6.3.3 Theory vs. simulation

In this example we show compare the theoretical curves of SER to that from the simulation. Fig. 6.17 shows the simulated SER of the proposed Kalman filter detector and that of the coherent receiver. The theoretically derived corresponding SER values are also plotted for comparison.

## 6.4 Conclusions

In this Chapter, we have derived a theoretical performance analysis scheme for the proposed MIMO receivers in under frequency-selective fading channel conditions. The result for flat-fading channels can also be obtained by setting the number of sub-carriers to one. Assuming the knowledge of training symbols for the first block, we have derived the instantaneous signal to interference and noise ratio (SINR) for consecutive transmission blocks in the absence of training, by exploiting Kalman filtering to track the channel in a decision-directed mode, for a general MIMO system consisting of any number of Tx/Rx antennas. We have studied the effect in performance for different stages of the channel estimation, i.e., the Kalman predicted channel, Kalman updated channel and the steady-state channel estimate, and conclude that for MIMO-OFDM systems, the steady state performance is accelerated with the number of sub-carriers.

# Chapter 7

# Blind Sequential Monte-Carlo Receivers for MIMO Systems

## 7.1 Introduction

The receivers we have proposed in Chapter 4 and Chapter 5 fall in the category of semi-blind receivers in the sense that they need the knowledge of training symbols once in a while in order to avoid error propagation. It is quite intuitive that the semi-blind receivers will loose a considerable portion of their bandwidth in training. Having this fact in mind, some blind receivers, which do not require any training, have been reported in the literature. Among them, the blind blind receivers for unknown stationary channels and that of fading channels are some primary categories under which numerous proposals have been already made.

The use of Sequential Monte-Carlo methods for symbol detection in flat fading channels was first reported in [39]. Later, a blind receiver for MIMO systems based on sequential Monte-Carlo methods was reported in [36]. Based on this, some modified versions of the receivers were reported in [24, 113]. Blind Monte-Carlo receivers for data decoding in OFDM systems were reported in [63, 123, 124, 126]. These receivers are particularly of interest here because, given adequate number of particles (in other words, if we assume that computational complexity is not a constraint), these receivers promise the performance of semi-blind receivers proposed in Chapter 4 & 5 without the use of training. However, considering the computational requirements of these algorithms, these receivers loose the attention for practical applications. Further, as shown later in this Chapter, when MIMO channel becomes frequency selective, these receivers become virtually unusable as the computational requirement exponentially increases with the number of sub-carriers. Some attempts to reduce the computational complexity of the receiver was reported in [1].

In this Chapter, we propose a method to reduce the computational complexity of the above SMC receiver for flat-fading channels. First, we address the case where the MIMO channel is frequency flat. As in the case of [36], the proposed algorithm also employs Rao-Blackwellization strategy to marginalize out the (unwanted) channel coefficients and uses optimal importance function to generate samples to propagate the posterior distribution. The algorithm is blind in the sense that unlike other competing schemes, the knowledge of the channel coefficients are not assumed by the receiver. The marginalization involves the computation of (hundreds of) Kalman filters running in parallel. From the simplification results reported in Chapter 4 we show that, the marginalization step can be significantly simplified if the underlining space-time coding scheme is OSTBC under no additional assumption — resulting in huge computational savings.

Later, we extend this result to MIMO-OFDM systems and propose a novel tone-by-tone receiver for frequency selective channels. As discussed in detail in the following sections, the marginalization of the channel parameter involves the computation of a certain likelihood function over all possibilities of the transmitted symbol values. While a $4 \times 4$ MIMO systems could have a possible $2^4$ values, a MIMO-OFDM system with 256 sub-carriers will have $2^{1024}$ ones, making it impractical to implement. We propose to implement the SMC receiver in a tone-by-tone basis by modeling the necessary channel dynamics in frequency domain. This significantly reduces the the computational.

The rest of this chapter is organized as follows. Having the proposed SMC receiver in mind, a MIMO signal model that is slightly different the one introduced in Chapter 2 is presented in Section 7.2.1. The SMC detector is summarized in Section 7.2.2 and the proposed simplification is detailed in Section 7.2.3. A summary of the proposed algorithm is presented in Section 7.2.4.

## 7.2 Efficient Implementation of SMC Receiver for OSTBCs

### 7.2.1 Signal model

In this section, we present a signal model that is slightly different to the one presented in Chapter 2. Consider a MIMO system with $N$ transmit and $M$ receive antennas. In a time-varying flat-fading channel scenario, the signal received by the $p$th receive antenna at time

$t$, $y_p(t)$ is given by

$$y_p(t) = \sum_{q=1}^{N} h_{pq}(t)x_q(t) + v_p(t) \tag{7.181}$$

where $x_q(t)$ is the signal transmitted from the $q$th antenna at time $t$, $h_{pq}(t)$ is the time-varying channel coefficient between the $p$th transmit antenna and the $q$th receive antenna, and $v_p(t)$ is the noise measured at the $p$th receive antennas. The noise $v_p(t)$ is assumed to be zero-mean complex Gaussian and spatio-temporally white with variance $\sigma_v^2/2$ per real dimension.

Using vector notations, the received signal vector can be written as

$$\mathbf{y}(t) = \mathbf{x}(t)\mathbf{H}(t) + \mathbf{v}(t) \tag{7.182}$$

where $\mathbf{H}(t)$ is the $N \times M$ channel matrix with its $(p, q)$ element equal to $h_{pq}(t)$, and

$$\mathbf{y}(t) = [y_1(t)\ y_2(t)\ \ldots\ y_M(t)] \tag{7.183}$$

$$\mathbf{x}(t) = [x_1(t)\ x_2(t)\ \ldots\ x_N(t)] \tag{7.184}$$

$$\mathbf{v}(t) = [v_1(t)\ v_2(t)\ \ldots\ v_M(t)] \tag{7.185}$$

are the row-vectors of the received signals, transmitted signals, and noise, respectively.

We consider a block transmission scheme and assume that within the block period $T$, the channel is fixed, i.e., the channel is assumed to be *quasi-static*. However, between different blocks the channel can change. Based on such an assumption, the $n$th received block can be written as

$$\mathbf{Y}(n) = \mathbf{X}(n)\mathbb{H}(n) + \mathbf{V}(n) \tag{7.186}$$

where

$$\mathbf{Y}(n) = \begin{bmatrix} \mathbf{y}(nT - T + 1) \\ \mathbf{y}(nT - T + 2) \\ \vdots \\ \mathbf{y}(nT) \end{bmatrix}, \qquad \mathbf{X}(n) = \begin{bmatrix} \mathbf{x}(nT - T + 1) \\ \mathbf{x}(nT - T + 2) \\ \vdots \\ \mathbf{x}(nT) \end{bmatrix},$$

and

$$\mathbf{V}(n) = \begin{bmatrix} \mathbf{v}(nT - T + 1) \\ \mathbf{v}(nT - T + 2) \\ \vdots \\ \mathbf{v}(nT) \end{bmatrix}$$

denote the received data, the transmitted data, and the measurement noise matrices, respectively.

According to the definition of OSTBCs in (2.7), let us re-write the $n$th received block (7.186) in the following format

$$\mathbf{y}_n = \mathbf{B}(\mathbf{s}_n)\mathbb{h}_n + \mathbf{v}_n \tag{7.187}$$

where

$$\mathbf{y}_n = \text{vec}\{\mathbf{Y}(n)\}, \tag{7.188}$$

$$\mathbf{B}(\mathbf{s}_n) = \mathbf{X}(\mathbf{s}_n) \otimes \mathbf{I}_M, \tag{7.189}$$

$$\mathbf{h}_n = \text{vec}\{\mathbf{H}(n)\}, \tag{7.190}$$

$$\mathbf{v}_n = \text{vec}\{\mathbf{V}(n)\}, \tag{7.191}$$

$\mathbf{v}_n$ is the i.i.d Gaussian noise vector that has zero mean and covariance $\Sigma_v = \sigma_v^2 \mathbf{I}_{MT}$, $\otimes$ refers to the kronecker product and vec$\{\cdot\}$ refers to the vectorization operator which stacks all the columns of a matrix on top of each other.

It can be easily verified that the matrix $\mathbf{B}(\mathbf{s}_n)$ satisfies

$$\mathbf{B}^{\mathrm{T}}(\mathbf{s}_n)\mathbf{B}(\mathbf{s}_n) = \|\mathbf{s}\|^2 \mathbf{I}_{MN}. \tag{7.192}$$

Now, assuming first order AR model for the channel, the channel dynamics can be written as

$$\mathbf{h}_n = \mathbf{F}\mathbf{h}_{n-1} + \mathbf{w}_n \tag{7.193}$$

where, $\mathbf{w}_n$ is the i.i.d Gaussian noise vector that has zero mean and covariance $\Sigma_w = \sigma_w^2 \mathbf{I}_{MN}$.

### 7.2.2 The blind SMC detector

We apply Bayes' rule to factorize the posterior distribution of the unknown parameters as follows

$$p(\mathbf{s}_{1:n}, \mathbf{h}_{1:n}|\mathbf{y}_{1:n}) = p(\mathbf{h}_{1:n}|\mathbf{s}_{1:n}, \mathbf{y}_{1:n})p(\mathbf{s}_{1:n}|\mathbf{y}_{1:n}) \tag{7.194}$$

where $p(\mathbf{s}_{1:n}|\mathbf{y}_{1:n})$ is the *marginalized posterior distribution*. Our interest from now on will be focused on how to propagate the marginalized posterior distribution over time.

Now, let us approximate the marginal posterior distribution as

$$p(\mathbf{s}_{1:n}|\mathbf{y}_{1:n}) \approx \sum_{i=1}^{N_s} w_n^i \delta(\mathbf{s}_{1:n} - \mathbf{s}_{1:n}^i) \tag{7.195}$$

where $N_s$ is the number of samples and $w_n^i$ are the importance weights written as

$$w_n^i = \frac{p(\mathbf{s}_{1:n}^i|\mathbf{y}_{1:n})}{q(\mathbf{s}_{1:n}^i|\mathbf{y}_{1:n})} \tag{7.196}$$

where $q(.)$ is the importance density.

By choosing the importance density to factorize as

$$q(\mathbf{s}_{1:n}|\mathbf{y}_{1:n}) = q(\mathbf{s}_n|\mathbf{s}_{1:n-1}, \mathbf{y}_{1:n})q(\mathbf{s}_{1:n-1}|\mathbf{y}_{1:n-1}) \tag{7.197}$$

the importance weights can be recursively updated as follows

$$w_n^i \propto w_{n-1}^i \frac{p(\mathbf{y}_n|\mathbf{s}_n^i)p(\mathbf{s}_n^i|\mathbf{s}_{n-1}^i)}{q(\mathbf{s}_n^i|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n})} \tag{7.198}$$

By choosing the optimal importance density as

$$q(\mathbf{s}_n^i|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n}) = p(\mathbf{s}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n}) \tag{7.199}$$

the weight update (7.198) can be written as

$$w_n^i = w_{n-1}^i p(\mathbf{y}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n-1}) \tag{7.200}$$

where

$$p(\mathbf{y}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n-1}) = \sum_{\mathbf{a}_j \in A} p(\mathbf{y}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n-1}, \mathbf{s}_n = \mathbf{a}_j) p(\mathbf{s}_n = \mathbf{a}_j) \tag{7.201}$$

Now, the first term in (7.201) can be written as

$$p(\mathbf{y}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n-1}, \mathbf{s}_n = \mathbf{a}_j) = \int \underbrace{p(\mathbf{y}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n-1}, \mathbf{s}_n = \mathbf{a}_j, \mathbf{h}_n)}_{\mathrm{D_I}} \underbrace{p(\mathbf{h}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n-1})}_{\mathrm{D_{II}}} d\mathbf{h}_n \tag{7.202}$$

In the rest of this section, the distributions $\mathrm{D_I}$ and $\mathrm{D_{II}}$ are derived and the integration in (7.202) is evaluated.

First, from (7.187), the distribution $\mathrm{D_I}$ is derived as

$$\begin{aligned} p(\mathbf{y}_n|\mathbf{s}_{1:n-1}^i, \mathbf{y}_{1:n-1}, \tilde{\mathbf{s}}_n = \mathbf{a}_j, \mathbf{h}_n) &= p(\mathbf{y}_n|\tilde{\mathbf{s}}_n = \mathbf{a}_j, \mathbf{h}_n) \\ &= \mathcal{N}_c(\mathbf{y}_n; \mathbf{B}(\mathbf{a}_j)\mathbf{h}_n, \mathbf{\Sigma}_w) \\ &= \frac{1}{(\pi)^{2MT}|\mathbf{\Sigma}_w|} \exp\Big[-(\mathbf{y}_n - \mathbf{B}(\mathbf{a}_j)\mathbf{h}_n)^{\mathrm{H}} \mathbf{\Sigma}_w^{-1} \\ &\qquad (\mathbf{y}_n - \mathbf{B}(\mathbf{a}_j)\mathbf{h}_n)\Big] \end{aligned} \tag{7.203}$$

Now, consider the distribution $D_{II}$. From (7.187) and (7.193) we notice that

$$p(\mathbf{h}_n|\mathbf{s}^i_{1:n}, \mathbf{y}_{1:n}) \sim \mathcal{N}_c\left(\mathbf{h}^i_{n|n}, \mathbf{P}^i_{n|n}\right) \tag{7.204}$$

where, given $\mathbf{h}^i_{n-1|n-1}$ and $\mathbf{P}^i_{n-1|n-1}$, Kalman filter [8] can be used as follows to get $\mathbf{h}^i_{n|n}$ and $\mathbf{P}^i_{n|n}$

$$\mathbf{h}^i_{n|n-1} = \mathbf{F}\mathbf{h}^i_{n-1|n-1} \tag{7.205}$$

$$\mathbf{P}^i_{n|n-1} = \mathbf{F}\mathbf{P}^i_{n-1|n-1}\mathbf{F}^{\mathrm{H}} + \mathbf{\Sigma}_w \tag{7.206}$$

$$\hat{\mathbf{y}}^i_n = \mathbf{B}(\mathbf{s}^i_n)\mathbf{h}^i_{n|n-1} \tag{7.207}$$

$$\boldsymbol{\nu}^i_n = \mathbf{y}_n - \hat{\mathbf{y}}^i_n \tag{7.208}$$

$$\mathbf{P}^i_{\nu,n} = \mathbf{\Sigma}_v + \mathbf{B}(\mathbf{s}^i_n)\mathbf{P}^i_{n|n-1}\mathbf{B}^{\mathrm{H}}(\mathbf{s}^i_n) \tag{7.209}$$

$$\mathbf{G}^i_n = \mathbf{P}^i_{n|n-1}\mathbf{B}^{\mathrm{H}}(\mathbf{s}^i_n)\mathbf{P}^{-1(j)}_{\nu,n} \tag{7.210}$$

$$\mathbf{h}^i_{n|n} = \mathbf{h}^i_{n|n-1} + \mathbf{G}^i_n\boldsymbol{\nu}^i_n \tag{7.211}$$

$$\mathbf{P}^i_{n|n} = \mathbf{P}^i_{n|n-1} - \mathbf{G}^i_n\mathbf{P}^i_{\nu,n}\mathbf{G}^{\mathrm{H}(i)}_n \tag{7.212}$$

where $\mathbf{h}^i_{n|n-1}$ is the predicted state, $\mathbf{P}^i_{n|n-1}$ is the covariance matrix of the predicted state, $\hat{\mathbf{y}}^i_n$ is the predicted observation, $\boldsymbol{\nu}^i_n$ is the innovation process, $\mathbf{P}^i_{\nu,n}$ is the innovation covariance matrix, and $\mathbf{G}^i_n$ is the Kalman gain [8].

Also note from the above that the Kalman predictor (7.205)–(7.206) can be used to find the distribution

$$p(\mathbf{h}_{n+1}|\mathbf{s}^i_{1:n}, \mathbf{y}_{1:n}) \sim \mathcal{N}_c\left(\mathbf{h}^i_{n+1|n}, \mathbf{P}^i_{n+1|n}\right) \tag{7.213}$$

where $\mathbf{h}^i_{n+1|n} = \mathbf{F}\mathbf{h}^i_{n|n}$ and $\mathbf{P}^i_{n+1|n} = \mathbf{F}\mathbf{P}^i_{n|n}\mathbf{F}^{\mathrm{H}} + \mathbf{Q}$.

Hence, the distribution $D_{II}$ is obtained as

$$\begin{aligned} p(\mathbf{h}_n|\mathbf{s}^i_{1:n-1}, \mathbf{y}_{1:n-1}) &= \mathcal{N}_c\left(\mathbf{h}_n; \mathbf{h}^i_{n|n-1}, \mathbf{P}^i_{n|n-1}\right) \\ &= \frac{1}{(\pi)^{2MN}|\mathbf{P}^i_{n|n-1}|} \exp\left[-\left(\mathbf{h}_n - \mathbf{h}^i_{n|n-1}\right)^{\mathrm{H}}\right. \\ &\qquad \left.\mathbf{P}^{-1(i)}_{n|n-1}\left(\mathbf{h}_n - \mathbf{h}^i_{n|n-1}\right)\right] \end{aligned} \tag{7.214}$$

where $|\mathbf{X}|$ is the determinant of the matrix $\mathbf{X}$ and $\mathcal{N}_c(\mathbf{x}, \bar{\mathbf{x}}, \Sigma_x)$ denotes normally distributed probability distribution of complex varible $\mathbf{x}$ with mean $\bar{\mathbf{x}}$ and covariance $\Sigma_x$.

Substituting (7.203) and (7.214)into (7.202) we get [23]

$$p(\mathbf{y}_n|\mathbf{s}^i_{1:n-1}, \mathbf{y}_{1:n-1}, \mathbf{s}_n = \mathbf{a}_j) = \mathcal{N}_c\left(\mathbf{y}_n; \mathbf{B}(\mathbf{a}_j)\mathbf{h}^i_{n|n-1}, \mathbf{B}(\mathbf{a}_j)\mathbf{P}^i_{n|n-1}\mathbf{B}^{\mathrm{H}}(\mathbf{a}_j) + \Sigma_w\right) \tag{7.215}$$

### 7.2.3 The simplified receiver

It should be realised that, at a certain block $n$, the procedure (7.205)–(7.212) should be repeated for each sample $i$ separately resulting in very high computational cost. Hence, we can simplify (7.205)–(7.212) as it follows (see Chapter 4).

If $\mathbf{P}^i_{n-1|n-1}$ is a diagonal matrix, then, $\mathbf{P}^i_{n|n-1}$ in (7.206) and $\mathbf{P}^i_{n|n}$ in (7.212) are also diagonal, i.e., if

$$\mathbf{P}^i_{n-1|n-1} = \delta^i_{n-1} I_{MN} \tag{7.216}$$

then

$$\begin{aligned} \mathbf{P}^i_{n|n-1} &= \beta^i_n \mathbf{I}_{MN} \quad &(7.217) \\ \mathbf{P}^i_{n|n} &= \delta^i_n \mathbf{I}_{MN} \quad &(7.218) \end{aligned}$$

where

$$\beta_n^i = \delta_{n-1}^i|\alpha|^2 + \sigma_w^2 \tag{7.219}$$

and

$$\delta_n^i = \frac{\sigma_v^2\beta_n^i}{\|s_n^i\|^2\beta_n^i + \sigma_v^2}. \tag{7.220}$$

It can also be proved that (see Chapter 4), $\mathbf{h}_{n|n}^i$ in (7.211) can be simplified as

$$\mathbf{h}_{n|n}^i = (1 - \mu_n^i\|\mathbf{s}_n^i\|^2)\mathbf{h}_{n|n-1}^i + \mu_n^i\mathbf{B}^{\mathrm{H}}(\mathbf{s}_n^i)\mathbf{y}_n \tag{7.221}$$

where

$$\mu_n^i = \frac{\beta_n^i}{\|\mathbf{s}_n^i\|^2\beta_n^i + \sigma_v^2} \tag{7.222}$$

Therefore, (7.205) – (7.212) can be simplified as it follows:

$$\mathbf{h}_{n|n-1}^i = \mathbf{F}\mathbf{h}_{n-1|n-1}^i \tag{7.223}$$

$$\beta_n^i = \delta_{n-1}^i\|\alpha\|^2 + \frac{\sigma_w^2}{2} \tag{7.224}$$

$$\mu_n^i = \frac{\beta_n^i}{\|\mathbf{s}_n^i\|^2\beta_n^i + \sigma_v^2} \tag{7.225}$$

$$\mathbf{h}_{n|n}^i = (1 - \mu_n^i\|\mathbf{s}_n^i\|^2)\mathbf{h}_{n|n-1}^i + \mu_n^i\mathbf{B}^{\mathrm{H}}(\mathbf{s}_n^i)\tilde{\mathbf{y}}_n \tag{7.226}$$

$$\delta_n^i = \frac{\sigma_v^2\beta_n^i}{\|s_n^i\|^2\beta_n^i + \sigma_v^2}. \tag{7.227}$$

### 7.2.4 Summary of the algorithm

The proposed blind detection algorithm is summarized as below.

---

a) Initialization: For $i = 1, \ldots, N_s$

1.a) Set $\delta_0^i = \epsilon_i$

1.b) Set $\mathbf{h}_{0|0}^i \sim \mathcal{N}_c(\mathbf{0}_{MN\times 1}, \delta_0^i \mathbf{I}_{MN})$

1.c) Set $w_0^i = 1$

1.d) Set $\mathbf{s}_0^i \sim p(\mathbf{s}_0)$

b) Set $n = 1$

c) For $i = 1, \ldots, N_s$

3.a) Draw a sample $\mathbf{s}_n^i$ from the set $A^K$

3.b) Compute $\mathbf{h}_{n|n-1}^i$ and $\mathbf{P}_{n|n-1}^i$ as

$$\mathbf{h}_{n|n-1}^i = \mathbf{F}\mathbf{h}_{n-1|n-1}^i$$
$$\mathbf{P}_{n|n-1}^i = \mathbf{F}\mathbf{P}_{n-1|n-1}^i\mathbf{F}^{\mathrm{H}} + \mathbf{Q}$$

3.c) Set $l_i = 0$

3.d) For $j = 1, \ldots, K$

$$l_i = l_i + \mathcal{N}_c\left(\mathbf{y}_n; \mathbf{B}(\mathbf{a}_j)\mathbf{h}_{n|n-1}^i, \mathbf{B}(\mathbf{a}_j)\mathbf{P}_{n|n-1}^i\mathbf{B}^{\mathrm{H}}(\mathbf{a}_j) + \mathbf{\Sigma}_w\right)$$

3.e) Compute the updated weights as $\hat{w}_n^i = w_{n-1}^i l_i$

3.f) Normalize the importnce weights as $w_n^i = \frac{\hat{w}_n^i}{\sum_{p=1}^{N_s} \hat{w}_n^p}$

3.g) Compute the state and covariance update of the Kalman filter as

$$\beta_n^i = \delta_{n-1}^i \|\alpha\|^2 + \sigma_w^2$$
$$\mu_n^i = \frac{\beta_n^i}{\|\mathbf{s}_n^i\|^2 \beta_n^i + \sigma_v^2}$$
$$\mathbf{h}_{n|n}^i = (1 - \mu_n^i \|\mathbf{s}_n^i\|^2)\mathbf{h}_{n|n-1}^i + \mu_n^i \mathbf{B}^{\mathrm{H}}(\mathbf{s}_n^i)\tilde{\mathbf{y}}_n$$
$$\delta_n^i = \frac{\sigma_v^2 \beta_n^i}{\|s_n^i\|^2 \beta_n^i + \sigma_v^2}$$

```
d) Compute the a posteriori symbol estimte as in (3.31)

e) Perform resampling as
```
$\left[\{\mathbf{x}_n^i, w_n^i\}_{i=1}^{N_s}\right] = \text{RESAMPLE}\left[\{\mathbf{x}_n^i, w_n^i\}_{i=1}^{N_s}\right]$

where $\mathbf{x}_n^i = \{\mathbf{s}_n^i, \mathbf{h}_{n|n}^i, \mathbf{P}_{n|n}^i\}$

f) Set $n \leftarrow n+1$ and go to 3)

---

## 7.3 Simulation results

In our first numerical example, we consider the Alamouti scheme with two transmitting antennas and two receiving antennas. In each simulation run, the elements of $\mathbf{H}(n)$ are generated according to Jakes model [43] corresponding to $F_m T_s = 0.0045$ where $F_m$ is the doppler frequency and $T_s$ is the sampling time. This results in $\alpha = J_0(0.2\pi F_m T_s)e^{j2\pi F_o T_s} = 0.9998e^{j0.0283}$ where $J_0(\cdot)$ is the zeroth order Bessel function of first kind. The number of samples used in the simulation is $N_s = 100$.

Throughout the simulations, the SNR is defined as

$$\text{SNR} = \frac{\sigma_h^2}{N\sigma_v^2} \tag{7.228}$$

where the term $N$ in the denominator appears to normalize the received power to one.

Figure 7.18 shows the symbol error rate performance of the SMC receiver. The performance of the coherent ML receiver that assumes perfect CSI is also plotted for comparison.

During the progress of the algorithm, it will quite interesting to see the how the the marginalization of the channel is performing. Hence, at each data block $n$, the best channel

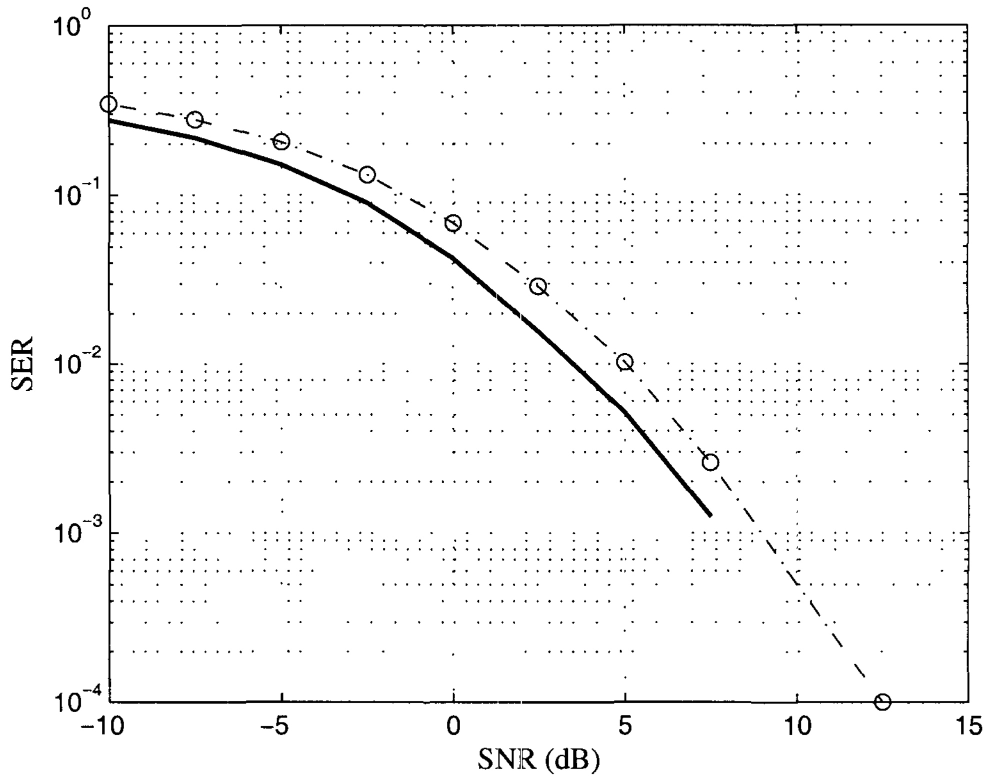


Figure 7.18: The SERs versus SNR using the SMC receiver

sample value out of all $N_s$ samples is plotted in figure 7.19 and 7.19.

## 7.4 Conclusions

In this Chapter, we have introduced an an efficient way to reduce the computational complexity of the sequential Monte-Carlo based algorithms for symbol detection in MIMO systems having OSTBCs as their underlying space-time coding scheme. Further, we have extended this result to frequency selective channels and proposed a novel SMC detector for

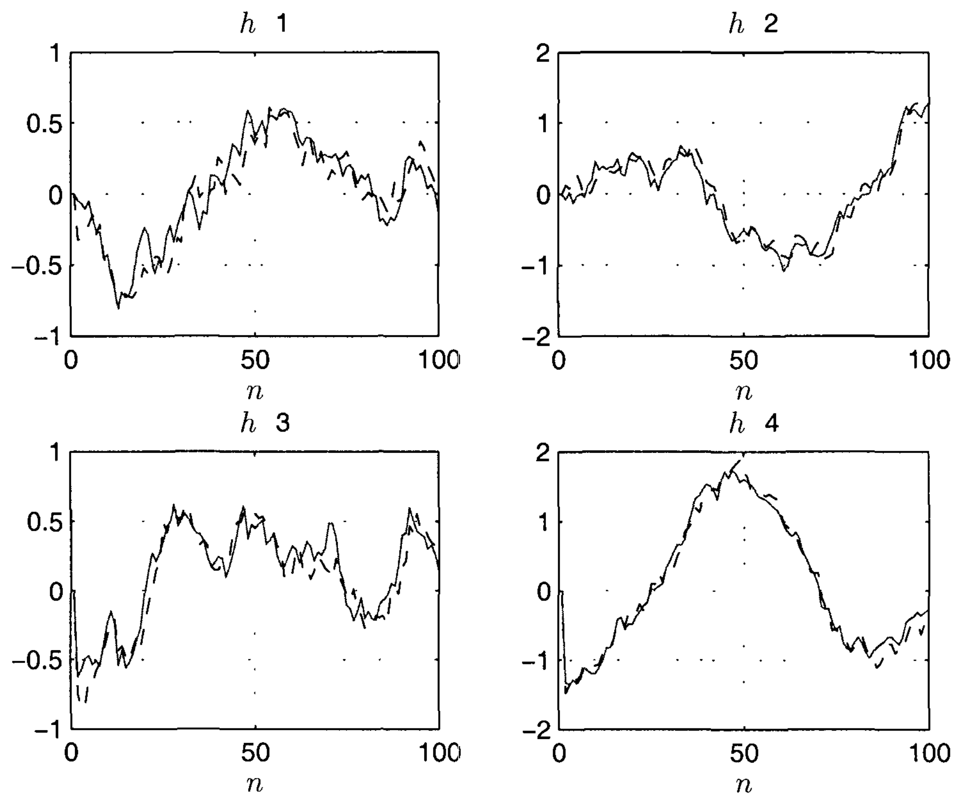


Figure 7.19: The best channel value at each iteration by the SMC receiver - real part

MIMO-OFDM systems.

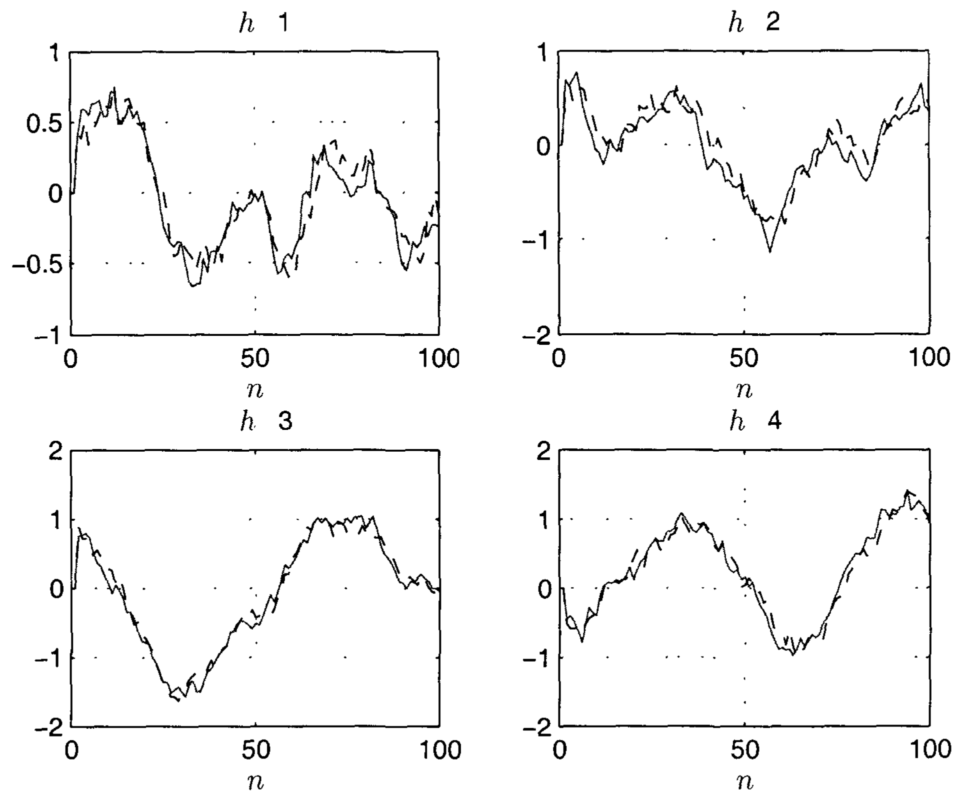


Figure 7.20: The best channel value at each iteration by the SMC receiver - imaginary part

# Chapter 8

# A New State-Space Approach to Adaptive Array Processing

## 8.1 Introduction

It is known that the capacity of MIMO systems can be increased by employing spatial separation techniques [10, 104, 106]. Some beamforming based MIMO systems that improve the data data transmission rate are reported in [13, 58, 77–79, 83, 93, 128]. In this Chapter, we present a novel DOA tracking technique that is useful in directional MIMO systems under highly mobile environments.

Numerous DOA estimation techniques that focus on high resolution techniques, such as the beamforming techniques [11,46], subspace-based techniques [98, 114], and maximum likelihood (ML) techniques [42], can be found in the literature. The accuracy of such techniques depends on the accuracy of the temporal averages taken over the samples, hence, these methods tend to fail in the presence moving targets. Further, the aforementioned methods assume that the number of targets is either known or fixed whereas in practice many applications require the DOA tracking of an unknown number of dynamic sources where the temporal averages are no longer accurate.

Recently, several new approaches have been developed for the DOA tracking of (a known number of) moving sources [29, 47, 80, 88, 89, 95, 96, 108]. These methods, in general, assume that the targets are piece-wise stationary and make use of conventional high resolution techniques during the stationary-assumed period. These methods are expected to perform poorly for rapidly moving targets. In [131], an ML based technique that does not assume the piece-wise stationarity is presented, however, this technique assumes that the number of targets are known at the receiver.

In recent years, Bayesian filtering techniques, which make use of the Bayes theorem to propagate the posterior distribution of the unknown states (usually) in the form of a histogram, have been drawing much attention in the field of target tracking and signal processing. In addition to that, in array signal processing, the nature of the array observation equation is highly non-linear which makes Bayesian techniques/particle filtering a natural candidate of choice. Indeed, a particle filtering approach was proposed in [75, 87] to track the DPAs of a known number of moving sources. However, when the number of targets (dimension of the unknown states) changes, the Bayesian techniques cannot be directly used to propagate the posterior. Rather, they require additional steps to update the model order and re-initialize the filter. Such a technique for tracking DOA of unknown number of sources is presented in [72], where, Metropolis Hastings (MH) algorithm is exploited for source enumeration.

The problem of data association as well as that of the unknown number of sources are addressed together by using the PHD filter. It propagates the first moment of the multi-target posterior, known as the probability hypothesis density (PHD), which is defined over the state space of one target. The PHD has local maxima at the expected locations of the target states. Hence, target locations can be obtained from the PHD. Further, the PHD holds a nice property that the integral of the PHD over the state space is the expected number of targets. Hence, the PHD filter [68] is found to be a computationally efficient alternative to the FISST method of propagating the full posterior.

In recent years, the PHD filtering techniques have gained considerable attention from researches from various areas. Some extensive theoretical Particle filter implementation are reported in [20, 81, 116, 118, 119]. The convergence analysis of the PHD filter is presented in [17, 19] and an analysis of data association for PHD filter is reported in [18]. PHD filtering technique finds its primary application in target tracking area. The use of PHD filtering for joint detection and tracking of maneuvering targets is analyzed in [115]. A multi target tracking algorithm using PHD filters with passive radar observations is presented in [110]. Applications of PHD filtering in multi-sensor vehicle tracking and track labeling are analyzed in [66] and [59], respectively. The application of PHD filtering techniques in video processing is analyzed in [16] that in image processing is reported in [21, 41, 67].

In this chapter we present a novel state-space approach for DOA tracking problem using single snapshot observations. We transform DOA tracking into a multi-target tracking problem where observations from the targets are independent of each other. We propose the use of fast Fourier transform (DFT) and inverse fast Fourier transform (IFFT) techniques to find a coarse estimate of the array observation due to individual targets and to use PHD filtering to track the DOAs as well as the number of targets.

The rest of this chapter is organized as follows. Section 8.2 formulates the DOA tracking problem. Section 8.3 introduces the state space model used in our proposed method. The proposed PhD filter based algorithm is presented in Section 8.4. Simulation examples of the proposed algorithm are given in Section 8.5 and the chapter is concluded in Section 8.6.

## 8.2 Problem Formulation

Consider a uniform linear array of $M$ sensors. At time $n$, let $\phi_i(n), i = 1, 2, \ldots, K_n$, be the directions of arrival of $K_n$ *narrow band* sources that are in the *far-field* of the array and let the corresponding amplitudes of the sources be $a_i(n)$, $i = 1, 2, \ldots, K_n$. The transmission medium is assumed to be *isotropic* and *non-dispersive.*

The $(M \times 1)$ array observation vector $\mathbf{y}(n)$, which is the superposition of the incident signals from the $K_n$ distinct sources embedded in Gaussian noise, is given by

$$\mathbf{y}(n) = \sum_{i=1}^{K_n} \mathbf{s}(\phi_i(n))a_i(n) + \mathbf{w}(n) \tag{8.229}$$

where,

$$\mathbf{s}(\phi_i(n)) = [1, e^{-j(d_0 w_0/c_0)\sin\phi_i(n)}, \ldots, e^{-j((M-1)d_0 w_0/c_0)\sin\phi_i(n)}]^T$$

is the *steering vector* corresponding to the $k$th source, $\mathbf{w}(n)$ is an additive white Gaussian noise (AWGN) vector with zero mean and variance $\sigma_w^2$, $c_0$ is the velocity of propagation, and $w_0$ is the center frequency of the narrow band sources.

In vector format, the array observation in (8.229) can be written as

$$\mathbf{y}(n) = \mathbf{S}(\mathbf{\Phi}(n))\mathbf{a}(n) + \mathbf{w}(n) \tag{8.230}$$

where

$$\begin{aligned}
\mathbf{S}(\mathbf{\Phi}(n)) &= [\mathbf{s}(\phi_1(n)), \ldots, \mathbf{s}(\phi_{K_n}(n))] \\
\mathbf{\Phi}(n) &= [\phi_1(n), \ldots, \phi_{K_n}(n)]^T \\
\mathbf{a}(n) &= [a_1(n), \ldots, a_{K_n}(n)]^T
\end{aligned}$$

The states are assumed to evolve as follows

$$\phi_i(n) = \phi_i(n-1) + v_i(n); \tag{8.231}$$

$$a_i(n) \sim \mathcal{N}_c(\mathbf{0}, \sigma^2_{a,i}) \tag{8.232}$$

where $v_i(n)$ is an i.i.d Gaussian noise with zero mean and covariance $\sigma^2_{v,i}$, $\sigma^2_{a,i}$ is the variance of the received signals and $\mathcal{N}_c$ denotes Gaussian distribution for complex variables.

The objective of the problem is to estimate the number of sources at time $n$, $K_n$, and their corresponding DOAs $\phi_i(n)$ given the array observations $\mathbf{y}(n)$ in the absence of the knowledge of the amplitude of the incident waveforms $a_i(n)$.

## 8.3 New State-Space Model

Since our interest is in tracking directions of arrivals only, the following lemma is useful to formulate our new state-space model.

**Lemma 8.1** *The conditional probability of the array observation due to the ith incident waveform, $\mathbf{y}_i(n)$, given the DOA of the corresponding source $\phi_i(n)$ is given by*

$$p(\mathbf{y}_i(n)|\phi_i(n)) \simeq \left(\frac{\pi}{2\sigma_a^2}\right)^{1/2}\left(\frac{1}{\pi\sigma_w^2}\right)^{M} \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2\right\}\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\exp\left\{\frac{\beta^2}{8\alpha}\right\}\mathrm{I}_0\left(\frac{\beta^2}{8\alpha}\right) \quad (8.233)$$

*where*

$$\alpha = \left(\frac{M}{\sigma_w^2}+\frac{1}{2\sigma_a^2}\right),$$
$$\beta = \frac{2M}{\sigma_w^2}|A(\tilde{w}_i)|,$$

*$y_{m,i}$ is the ith incident waveform at the mth array element, $A(\tilde{w}_i(n))$ is the discrete Fourier transform (DFT) of the array observation due to the ith source, i.e.,*

$$A(\tilde{w}_i(n)) = \frac{1}{M}\sum_{m=1}^{M} y_{i,m}(n)\exp(-j\tilde{w}_i m(n)). \quad (8.234)$$

*proof:* See Appendix C. ■

Hence, by the use of the above lemma, the state–space equations given in (8.229)–(8.232) can be written as

$$\phi_i(n) = \phi_i(n-1)+v_i(n)$$
$$\mathbf{y}_i(n) = \mathcal{H}(\phi_i(n),\mathbf{w}(n)) \quad (8.235)$$

where $\mathcal{H}(.)$ is a non–linear function that is defined such that the conditional probability $p(\mathbf{y}_i(n)|\phi_i(n))$ is given as in (8.233).

## 8.4 Sequential Monte Carlo Implementation

### 8.4.1 The prior

It is assumed that the prior is uniformly distributed, i.e.

$$\phi(0) \sim \mathcal{U}\left[\phi_{\min}, \phi_{\max}\right] \tag{8.236}$$

where $\mathcal{U}$ denotes uniform distribution.

### 8.4.2 Prediction operator

Let us consider that we have the posterior PHD at time $n$ as,

$$\mathrm{D}(n|n) = \sum_{i=1}^{N_n^{\phi} N_s} w_t^i \delta(\phi(n) - \phi^i(n)) \tag{8.237}$$

then, the predicted PHD at time step $(n+1)$ in (3.56) could be written as

$$\mathrm{D}(n+1|n) = \mathrm{D}_b(\phi(n+1)) + \sum_{i=1}^{N_n^{\phi} N_s} w_t^i \Phi(\phi^i(n+1), \phi^i(n)) \tag{8.238}$$

A particle approximation of the above could be obtained from sequential important sampling technique [4]. Let us draw samples from two different proposal densities as

$$\phi^i(n) = \begin{cases} q_\Phi(\phi^i(n+1)|\phi^i(n), Y(n+1)) & i = 1, \dots, N_n^\phi N_s{}^1 \\ q_b(\phi^i(n+1)|Y(n+1)) & i = N_n^\phi N_s + 1, \dots, (N_n^\phi + N_n^b)N_s \end{cases} \tag{8.239}$$

The predicted PHD is now written as

$$\bar{\mathrm{D}}(n+1|n) = \sum_{i=1}^{(N_n^\phi + N_n^b)N_s} w_{n+1|n}^i \delta(\phi(n+1|n) - \phi^i(n+1|n)) \tag{8.240}$$

where

$$w_{n+1|n}^i = \begin{cases} \frac{\Phi(\phi^i(n+1)|\phi^i(n))}{q_\Phi(\phi^i(n+1)|\phi^i(n), Y(n+1))} & i = 1, \dots, N_n^\phi N_s \\ \frac{\mathrm{D}_b(\phi^i(n+1))}{q_b(\phi^i(n+1), Y(n+1))} & i = N_n^\phi N_s + 1, \dots, (N_n^\phi + N_n^b)N_s \end{cases} \tag{8.241}$$

### 8.4.3 Update operator

Assuming that we have the predicted particles as in (8.240) and by considering the update operator in (3.58), particle representation of the updated PHD is now written as [117]

$$\mathrm{D}(n+1|n+1) = \sum_{i=1}^{(N_n^\phi + N_n^b)N_s} w_{n+1}^i \delta(\phi(n+1) - \phi^i(n+1)) \tag{8.242}$$

where

$$w_{n+1}^i = \left[1 - p_D^i(\phi^i(n+1)) + \sum_{\mathbf{y}(n) \in Y(n+1)} \frac{p_D(\phi^i(n+1)) p(\mathbf{y}(n+1)|\phi^i(n+1))}{\lambda(n+1)C(\mathbf{y}(n+1)) + c_{n+1}(\mathbf{y}(n+1))}\right] \tag{8.243}$$

and

$$c_{n+1}(\mathbf{y}(n+1)) = \sum_{i=1}^{(N_n^{\phi}+N_n^{b})N_s} p_D(\phi^i(n+1))p(\mathbf{y}(n+1)|\phi^i(n+1))w_{n+1|n}^i \quad (8.244)$$

### 8.4.4 Resampling

The SMC implementation of the PHD filter suffers from the commonly known degeneracy problem in which after a few steps importance weight of one particle approaches to one as that of all the other particles approach to zero. The resampling step is introduced to reduce the degeneracy problem. In resampling, the particles with larger weights are duplicated while particles with smaller weights are removed.

### 8.4.5 State estimation

The updated PHD represents the first moment of the multi-target posterior. Several methods can be found in the literature to estimate the state from the PHD. A simple but computationally efficient method is to use clustering. By clustering technique, $N_{n+1}^{\phi}N_s$ particles are clustered into $N_{n+1}^{\phi}$ groups. Then, mean value of this cluster is considered as the state estimate of the target at time $n+1$.

### 8.4.6 Summary of the algorithm

This section gives a summary of the proposed DOA tracking scheme.

---

a) Initialize the prior PHD at $n=0$ as $\{\xi_0^i, w_{0|0}^i\}_{i=1}^{N_0^\phi N_s}$, where

$$\begin{aligned}\xi_0^i &\sim \mathcal{U}\left[\phi_{\min}, \phi_{\max}\right] \\ w_{0|0}^i &= \frac{1}{N_s}\end{aligned}$$

b) Set $n = n+1$

c) Prediction

3.a) sample predicted particles and their weights as $\{\xi_{n+1|n}^i, w_{n+1|n}^i\}_{i=1}^{N_n^\phi N_s}$, where

$$\begin{aligned}\xi_{n+1|n}^i &\sim q_\Phi(\xi_{n+1}^i|\xi_n^i, Y_n) \\ w_{n+1|n}^i &= \frac{\Phi(\xi_{n+1}^i, \xi_n^i)}{q_\Phi(\xi_{n+1}^i|\xi_n^i, Y_n)} w_{n|n}^i\end{aligned}$$

3.b) sample newborn particles and their weights as $\{\xi_{n+1|n}^i, w_{n+1|n}^i\}_{i=N_n^\phi N_s+1}^{(N_n^\phi+N_0^b)N_s}$, where

$$\begin{aligned}\xi_{n+1|n}^i &\sim q_b(\xi_{n+1}^i|Y_n) \\ w_{n+1|n}^i &= \frac{\mathrm{D}_b(\xi_{n+1}^i)}{q_b(\xi_{n+1}^i|Y_n)} w_{n|n}^i\end{aligned}$$

d) Obtain the DFT of the array observation and obtain the number of potential targets $N_{n+1}^{\mathbf{y}}$.

e) Find the updated particles and their weights as

$\{\xi^i_{n+1} = \xi^i_{n+1|n}, w^i_{n+1|n+1}\}_{i=N^\phi_n N_s+1}^{(N^\phi_n+N^b_0)N_s}$, where

$$w^i_{n+1|n+1} = \left[1 - p_D(\xi^i_{n+1}) + \sum_{j=1}^{N^{\mathbf{y}}_{n+1}} \frac{p_D(\xi^i_{n+1})p(\mathbf{y}^j_{n+1}|\xi^i_{n+1|n+1})}{\lambda_{n+1}C(\mathbf{y}^j_{n+1}) + c_{n+1}(\mathbf{y}^j_{n+1})}\right] w^i_{n+1|n}$$

$$c_{n+1}(\mathbf{y}^j_{n+1}) = \sum_{i=1}^{(N^\phi_n+N^b_n)N_s} p_D(\xi^i_{n+1})p(\mathbf{y}^j_{n+1}|\xi^i_{n+1})w^i_{n+1|n}$$

and the likelihood $p(\mathbf{y}^j_{n+1}|\xi^i_{n+1})$ is computed as

$$p(\mathbf{y}^j_{n+1}|\xi^i_{n+1}) = \left(\frac{1}{\pi\sigma^2_w}\right)^M \exp\left\{-\frac{M}{\sigma^2_w}\left(\frac{1}{MK}\sum_{m=1}^{M} {\mathbf{y}^i_{n+1}}^2(m) - 2\bar{b}^i_{n+1}\mathrm{Re}\left[\mathrm{e}^{-\jmath\bar{\theta}^i_{n+1}}A(\bar{\bar{w}}^i_{n+1})\right] + (\bar{b}^i_{n+1})^2\right)\right\}$$

where

$$\bar{b}^i_{n+1} = |\xi^i_{n+1}(2)|$$
$$\bar{\theta}^i_{n+1} = \angle\{\xi^i_{n+1}(2)\}$$
$$\bar{\bar{w}}^i_{n+1} = (w_0 d_0/c_0)\sin\xi^i_{n+1}(1)A(\bar{\bar{w}}^i_{n+1})$$

f) Compute the expected number of targets

$$N^\phi_{n+1} = \mathrm{round}\left\{\sum_{i=1}^{(N^\phi_n+N^b_n)N_s} w^i_{n+1|n+1}\right\}$$

where round$\{.\}$ is the nearest integer of the argument.

g) Re-sample the updated particles as

$$\left[\{\xi^i_{n+1}, w^i_{n+1|n+1}\}_{i=1}^{N^\phi_{n+1}N_s}\right] = \text{RESAMPLE}\left[\{\xi^i_{n+1}, w^i_{n+1|n+1}\}_{i=1}^{(N^\phi_n+N^b_0)N_s}\right]$$

h) Go to 2)

i) Obtain the state estimation at time $n+1$

by the use of clustering algorithm as

$$[\hat{\phi}(n+1)] = \text{CLUSTER}\left[\{\xi^i_{n+1}, w^i_{n+1|n+1}\}_{i=1}^{N^\phi_{n+1}N_s}\right]$$

---

## 8.5 Simulation Results

In this section, a simulation examples are presented to demonstrate the performance of the proposed algorithm. The true DOAs are generated using a first order random walk model with a variance of $\sigma_v^2$. Table 8.7 gives the values of the parameters used in the simulation. The receiver array is uniform linear and consists of $M = 8$ elements.

Table 8.7: Parameters used in the simulation

| $\sigma_v^2$ | $\sigma_w^2$ | $\phi(1)$ | $\sigma_a^2$ |
|---|---|---|---|
| 5deg.$^2$ | 0.01 | $[-20^0, 40^0]^T$ | 0.0707 |

### 8.5.1 Tracking fixed number of sources

In this example, the number of sources are kept fixed at 2 while the DOAs were varied with time. In each figures the continuous line shows the true value and the dots show the estimated value.

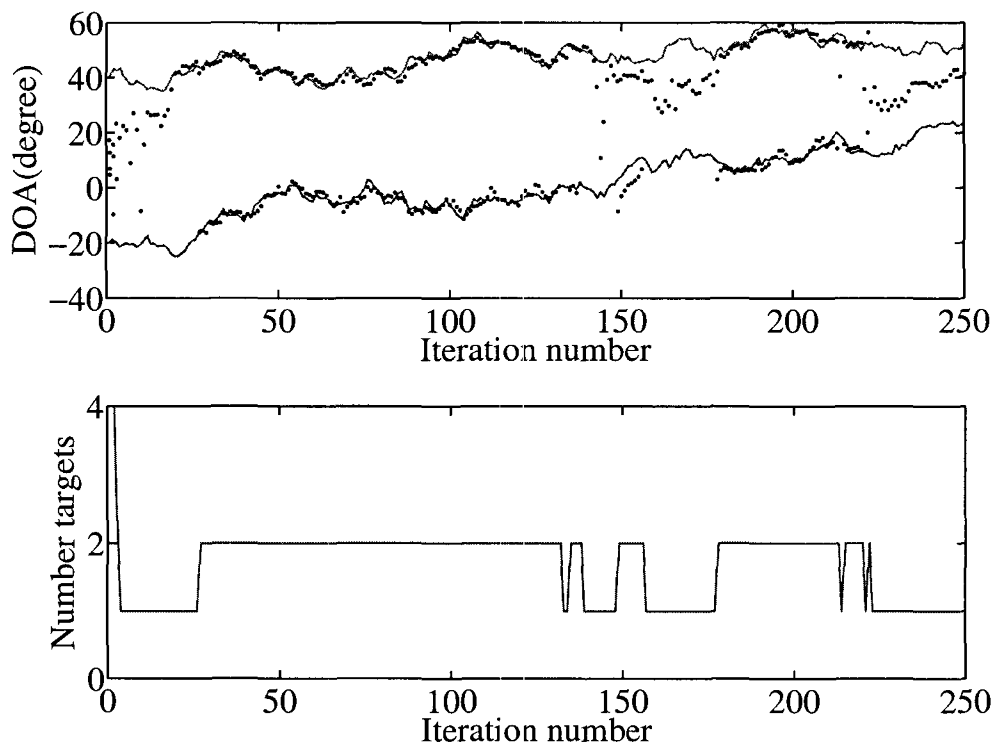


Figure 8.21: Tracking fixed number of moving sources by PF

Figure 8.21 (top) shows the estimated DOAs of the RJMCMC method against the iteration number. This method uses $N_s = 100$ particles and the number of targets was randomly initialized. As shown in the bottom of the figure, at start, it takes almost 25 iterations to converge to the correct number of targets. It can be observed from the figure that when the targets get closer, the estimated number of targets falls to one and the DOA estimation becomes inaccurate. Which means, the targets are unresolved.

Figure 8.22 (top) shows the estimated DOAs of the PHD filter against the iteration number. The PHD filter also uses 100 particles. As shown in the bottom of the figure, the PHD filter instantly estimates the number of targets correctly and it also can separate

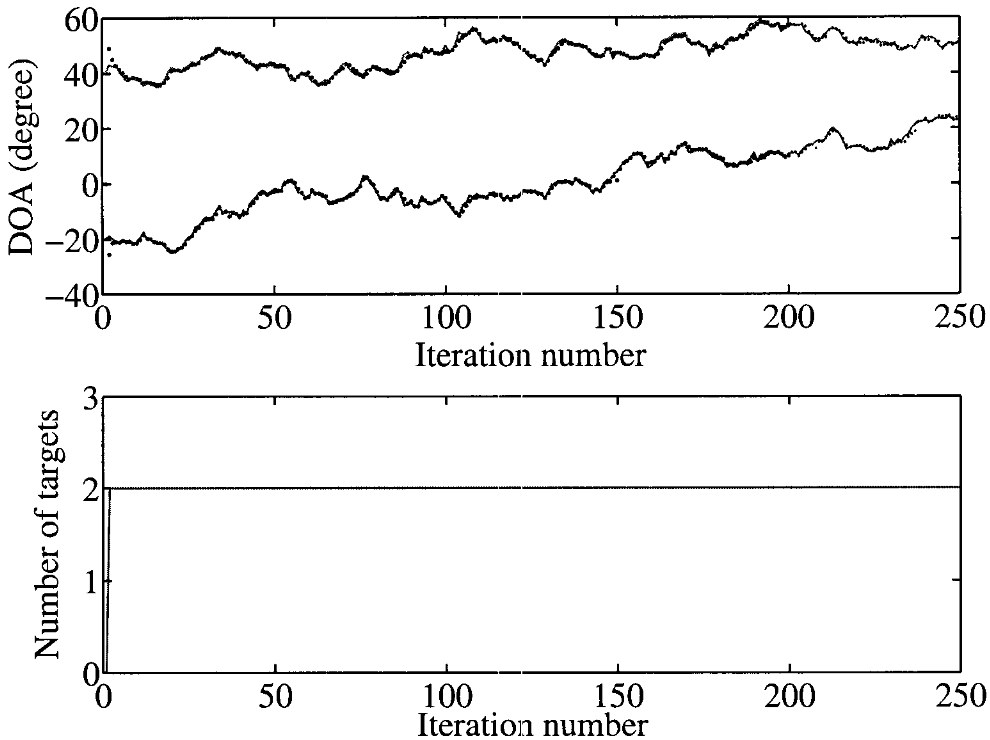


Figure 8.22: Tracking fixed number of moving sources by PHD filter

tracks of very closely spaced targets when the other filter fails. Further, in contrast to the RJMCMC method, PHD filter does not need any initialization for the number of targets.

### 8.5.2 Tracking varying number of sources

In this example, there is only one target from the start to iteration 50. Another target enters at iteration 51 and the first one leaves at iteration 151.

Figure 8.23 (top) shows the estimated DOAs of the RJMCMC method against the iteration number. The number of targets was randomly initialized. As shown in the bottom of

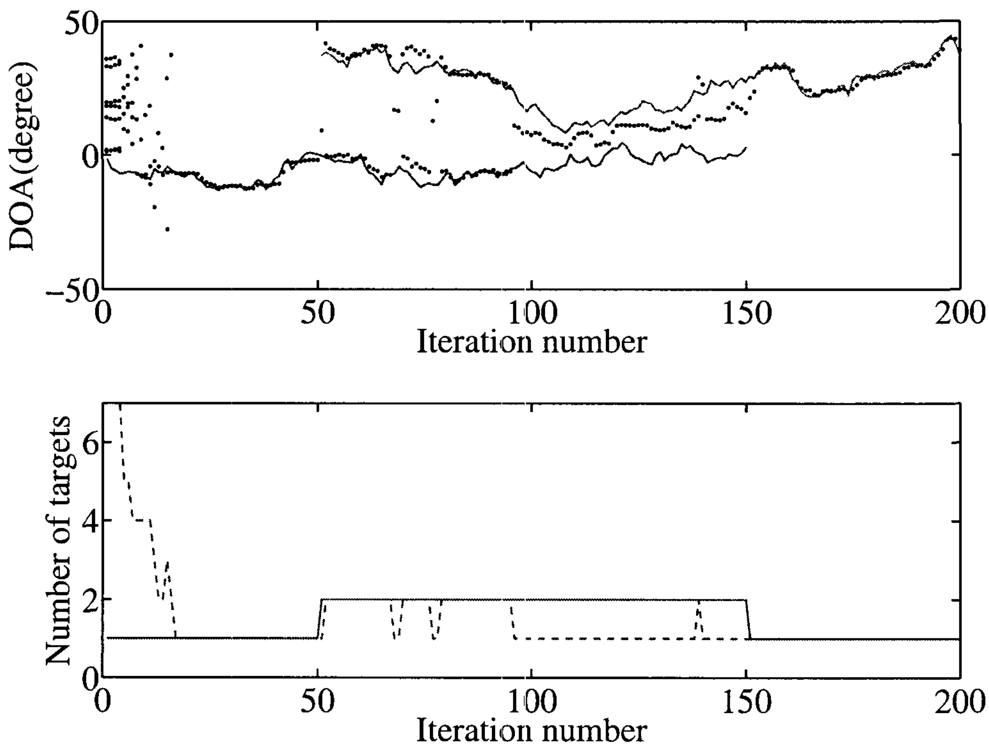


Figure 8.23: Tracking varying number of moving sources by PF

the figure, it takes almost 25 iterations to converge to the correct number of targets. When the second target enters, it is quickly identified. However, when the targets becomes closer, the estimated number of targets becomes one and the DOA estimation becomes inaccurate.

Figure 8.24 (top) shows the estimated DOAs of the PHD filter against the iteration number. The bottom of the figure shows that the proposed method instantly tracks the number of targets initially as well as when the number of targets changes.

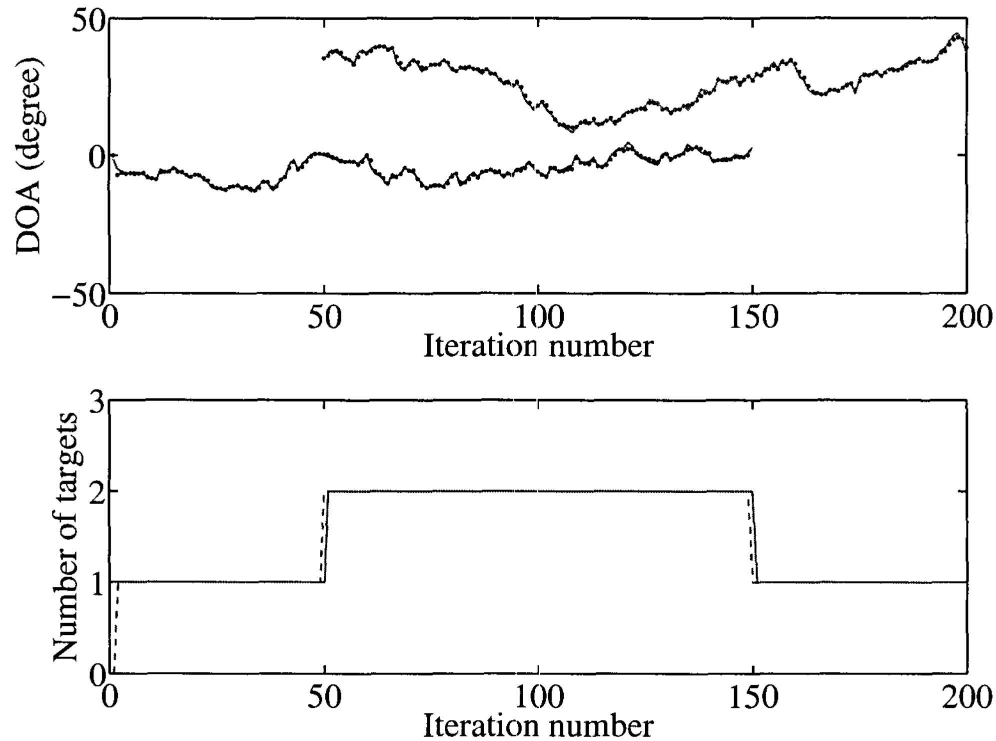


Figure 8.24: Tracking varying number of moving sources by PHD filter

## 8.6 Conclusions

In this chapter, a PHD filter based algorithm has been proposed for tracking the DOAs of time-varying number of targets by converting the original tracking problem into a multi-target tracking framework. The challenge in such a transformation is that, unlike multi-target tracking with active sensors, observations due to individual targets does not exist, rather, the (passive) array observation is the superposition of the signals received from all the targets. We derive an approximate observation model for the observations due to individual targets by deriving an approximate likelihood function. Then, the PHD filtering algorithm is exploited to track unknown number of sources as well as their corresponding DOAs. Simulation results show that, as expected, the proposed method is able to track the number of sources as well as their corresponding directions of arrivals over highly non-stationary environments.

# Chapter 9

# Concluding Remarks and Future Directions

## 9.1 Concluding Remarks

In this thesis, we have developed several blind and semi-blind decoding algorithms and an array processing algorithm, which are adaptive in nature as well as computationally efficient, for multi-antenna systems.

In Chapter 4, we have investigated the problem of channel tracking for MIMO communication systems where the MIMO channel is time-varying. Considering OSTBCs as the underlying space-time coding scheme, we have presented a two-step MIMO channel tracking algorithm for flat-fading channels. As the first step, Kalman filtering is used to obtain an initial channel estimate for the current block based on the channel estimates obtained for previous blocks. Then, in the second step, the so-obtained initial channel estimate is refined using a decision-directed iterative method. We have shown that due to the interesting properties of orthogonal space-time block codes, both the Kalman filter and the decision-directed algorithm can be significantly simplified. The proposed algorithm is semi-blind in the sense that training data is needed to be sent once in a while in order to prevent error propagation. The number of information carrying blocks between two consecutive training blocks, denoted as training repetition period (TRP), is used as a measure of bandwidth efficiency of our channel tracking scheme. Simulation results show that with a TRP of 10 blocks, our channel tracking method can have a performance, in terms of symbol error rate, within 2 dB from the coherent ML receiver.

In Chapter 5, we have developed a computationally efficient time-domain channel tracking technique for MIMO systems under frequency selective fading. By employing OFDM technique which decouples the channel into parallel flat fading channels, we extend the receiver in Chapter 4 for MIMO-OFDM systems. Assuming constant modulus constellations, we have significantly simplified the proposed algorithm.

In Chapter 6, we have derived a theoretical performance analysis scheme for the proposed MIMO receivers in Chapters 4 & 5 respectively. Assuming the knowledge of training symbols for the first block, we have derived the instantaneous signal to interference and noise ratio for consecutive transmission blocks in the absence of training, by exploiting Kalman filtering to track the channel in a decision-directed mode, for a general MIMO system consisting of any number of Tx/Rx antennas. We have studied the effect in performance for different stages of the channel estimation, i.e., the Kalman-predicted channel, Kalman-updated channel and the steady-state channel estimate, and conclude that for MIMO-OFDM systems, the steady state performance is accelerated with the number of sub-carriers.

In Chapter 7, we have proposed an efficient Sequential Monte Carlo algorithm for blind detection in MIMO systems. The proposed algorithm employs Rao-Blackwellization strategy to marginalize the channel coefficients and uses optimal importance function to generate samples to propagate the posterior distribution. We have applied the simplification strategies developed in earlier chapters to significantly reduce the computational complexity of the proposed algorithm. Later, we have extended this result to propose a novel and efficient tone-by-tone receiver for MIMO-OFDM system.

In Chapter 8, a PHD filter based algorithm has been proposed for tracking the DOAs of time-varying number of mobile users in directional MIMO systems. We have transformed the problem into a new state space approach and employed PHD filtering algorithm to track the DOAs of unknown number of sources. The proposed algorithm is computationally efficient and simpler to implement compared to its particle filtering counterpart. Simulation results show that, the the proposed scheme yields superior performance over competing particle filter based schemes in tracking rapidly varying number of targets.

## 9.2 Possible Extensions

In this section, we propose some possible extensions of the algorithms developed in Chapters 4 – 8 along with some proposals to solve them.

### 9.2.1 Blind SMC receiver for MIMO-OFDM with CFO estimation

The MIMO-OFDM receiver proposed in Chapter 5 assumes that the career frequencies are synchronized at the receiver. However, in practice, OFDM is highly sensitivity to the carrier frequency offset (CFO) errors caused by the misalignment in carrier frequencies at the receiver due to fluctuations in receiver RF oscillators or channel Doppler frequency.

Hence, a more realistic receiver will include a mechanism to estimate/track the CFO as well.

### 9.2.2 Relaxing quasi-time-varying assumptions

The channel tracking and data decoding algorithms presented in Chapter 4 and Chapter 5 assume that the time selective channel remains stationary during the transmission of a block period. Even though this is a reasonable assumption at low Doppler frequencies, a more realistic approach will be to consider a continuous time-varying channel. Relaxing the quasi-time-varying assumption makes the vector form input output MIMO relationship of Chapter 2 unusable. Further, under these assumption, the ML data decoding of OSTBC is no longer linear as derived in Chapter 2. For this problem, smoothing techniques available in the target tracking literature might provide an effective solution.

### 9.2.3 Wideband DOA tracking

The proposed DOA tracking algorithm in Chapter 8 is only applicable to narrow-band sources. However, many practical applications may require DOA tracking of wide-band sources. For instance, as discussed in Chapter 1, an exciting future application for wide-band DOA tracking will be the use of beamforming to enhance data transmission rates through spatial filtering in mobile WiMAX systems. For such applications, a PHD filter based wide-band DOA tracking scheme will be a computationally feasible solution to provide the directions of arrivals of signals from moving WiMAX (mobile) stations, in order to perform spatial filtering and achieve higher data throughput.

### 9.2.4 Rao-Blackwellized PHD filter

In Bayesian state estimation, it is important to use every opportunity to reduce the state space analytically in order to reduce the variance of the particles. In this regard, Rao-Blackwellization is employed in certain type of state space models, as discussed in Chapter 7. However, this strategy in its current implementation is only applicable in tracking full posteriors. It will be an interesting problem to device a strategy to apply Rao-Blackwellization on first moment tracking of the posteriors, i.e., PHD filters.

### 9.2.5 Novel hybrid filtering algorithm

The PHD filtering algorithm proposed for narrow-band DOA tracking in Chapter 8 is best suited for reasonably separated sources. When the sources stay closer for long, the performance of PhD filter is expected to degrade. However, during this period, tracking the full posterior might give a more precise density of the posterior at the cost of additional computation. In other words, while it is inefficient to track the full posterior using particle filter of well separated targets, it is better to track the full posterior when the sources are closer for longer. Hence, a natural solution to this scenario is to use both of the methods in a switching manner as appropriate. The challenging parts of the problem are the construction of the full posterior from the PHD and the development of a switching scenario.

## 9.3 Future Directions

In this section, we discuss some future directions, that must be explored in order to achieve higher data transmission rates in future wireless communication systems.

### 9.3.1 Blind adaptive MIMO receiver design

Adaptive blind receivers for MIMO systems needs further study in many directions before it gets any chance to enter into standards and eventually in practical communication systems. Some important directions may include

- The development of blind adaptive receivers for various channel models: The proposed channel tracking schemes assumed in Chapters 4 and 5 approximated the channel as AR1 model. Receivers considering other channel models should be developed and their performance must be theoretically analyzed.

- Newer frequency bands, such as the one in 60GHz range [102], are becoming stronger candidates of indoor wireless applications. For such bands, various studies are needed to be done on time-varying channel modelling.

- Any assumption on the channel over indefinite periods of time is not valid. For instance, when we assume a highly non-stationary channel model at the receiver indefinitely, there is a good possibility that the channel be stationary for considerable interval. Hence, employing computationally intense, power hungry receivers for relatively stationary channel is not an efficient solution. Hence, cross layer designs [51, 105], that pass the information on the receiver performance to other layers to better optimize the receiver operation should be extensively studied. For example, the decision-directed algorithms proposed in Chapters 4 and 5 iterate between channel estimate and data estimate values. If the information from higher layers are available about the correctness of the decoding, such information could be used to switch the receiver between decision-directed mode and continuous blind detection mode in which no iteration is needed since the decoded data is errorfree.

### 9.3.2 Computational reduction

Computational cost is continuing to be a bottleneck for implementing many blind adaptive algorithms developed for achieving higher data rates. In this thesis, we proposed two distinct approaches towards achieving computational savings, namely, Computational savings through simplification (Chapters 4–7) and computational savings through approximation (Chapter 8). While, these approaches should be further explored, different new techniques should be analyzed as well.

### 9.3.3 Performance analysis

It is well known that developing a theoretical performance analysis for the SMC detector is difficult, even though the asymptotic performance is intuitive, i.e., when the number of samples are higher enough, the performance of the SMC detector will approach that of the coherent receiver. In general, closed form performance analysis in terms of the number of particles is a relatively difficult problem. However, since the proposed SMC detector in Chapter 7 samples the particles from a discrete finite set, a closed form theoretical analysis that gives the probability of error performance in terms of the number of particles, might be possible. Such theoretical performance analysis is a crucial factor in analyzing the feasibility of such blind receivers for practical systems.

### 9.3.4 Adaptive array processing

A more exciting and cost effective approach will be to use the existing MIMO antennas to track the DOAs. However, when the number of antenna array elements decrease, the performance of any DOA tracking scheme is expected to decrease. Hence, using the MIMO antennas for DOA tracking might be approached by exploring advanced signal processing techniques.

# Appendix A

# Proof of Lemma 5.1

Substituting (5.117) into (5.110), we can rewrite it as

$$
\begin{aligned}
&\mathbf{P}[n|n-1] \\
&= \begin{pmatrix} \mathrm{diag}\left\{\delta_{n-1}^{0}|\alpha_0|^2,\ldots,\delta_{n-1}^{P-1}|\alpha_{P-1}|^2\right\} & \mathbf{0} \\ \mathbf{0} & \mathrm{diag}\left\{\delta_{n-1}^{0}|\alpha_0|^2,\ldots,\delta_{n-1}^{P-1}|\alpha_{P-1}|^2\right\} \end{pmatrix} \\
&\otimes \mathbf{I}_{MN} + \mathbf{Q} \\
&= \mathbf{I}_2 \otimes \mathrm{diag}\left\{\delta_{n-1}^{0}|\alpha_0|^2 + \frac{\sigma_0^2}{2},\ldots,\delta_{n-1}^{P-1}|\alpha_{P-1}|^2 + \frac{\sigma_{P-1}^2}{2}\right\} \otimes \mathbf{I}_{MN} \\
&= \mathbf{I}_2 \otimes \mathbf{\Psi}_n \otimes \mathbf{I}_{MN} \qquad \text{(A.245)}
\end{aligned}
$$

Now, inserting (A.245) into (5.113) and using matrix inversion lemma, $\mathbf{P}_{\nu}^{-1}[n]$ in (5.113) can be written as

$$\begin{aligned}
\mathbf{P}_{\nu}^{-1}[n] &= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}} \\
&\quad \left(\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}} + \mathbf{P}^{-1}[n|n-1]\right)^{-1}\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1} \\
&= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}} \\
&\quad \left(\frac{2}{\sigma_v^2}\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}} + \mathbf{P}^{-1}[n|n-1]\right)^{-1}\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1} \\
&= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}} \\
&\quad \left(\frac{2L}{\sigma_v^2 s^2}\mathbf{I}_{2MNP} + \mathbf{I}_2 \otimes \operatorname{diag}\left\{\frac{1}{\beta_n^0}, \dots, \frac{1}{\beta_n^{P-1}}\right\} \otimes \mathbf{I}_{MN}\right)^{-1}\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1} \\
&= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}} \\
&\quad \left(\mathbf{I}_2 \otimes \operatorname{diag}\left\{\frac{\sigma_v^2\beta_n^0}{2s^2L\beta_n^0 + \sigma_v^2}, \dots, \frac{\sigma_v^2\beta_n^{P-1}}{2s^2L\beta_n^{P-1} + \sigma_v^2}\right\} \otimes \mathbf{I}_{MN}\right)\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1} \\
&= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}} \\
&\quad \left(\mathbf{I}_2 \otimes \operatorname{diag}\left\{\gamma_n^0, \dots, \gamma_n^{P-1}\right\} \otimes \mathbf{I}_{MN}\right)\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1} \qquad \text{(A.246)}
\end{aligned}$$

where $\gamma_n^p = \frac{\sigma_v^2\beta_n^p}{2s^2L\beta_n^p+\sigma_v^2}$. Note that in deriving (A.246), we have taken into account that for constant modulus signals, i.e., for $\|\mathbf{s}_{n,0}\| =, \dots, = \|\mathbf{s}_{n,L-1}\| = s^2$, (5.105) can be written as

$$\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbb{B}(\tilde{\mathbf{s}}_n) = s^2\mathbf{I}_{2MNL}\,. \qquad \text{(A.247)}$$

Using (5.114) and (A.246), we can write (5.116) as

$$
\begin{aligned}
\mathbf{P}[n|n] &= \mathbf{P}[n|n-1] - \mathbf{P}[n|n-1]\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{P}_\nu^{-1}[n]\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}}\mathbf{P}[n|n-1] \\
&= \mathbf{P}[n|n-1] - \mathbf{P}[n|n-1]\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n) \\
&\quad \left(\mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}}\left(\mathbf{I}_2 \otimes \operatorname{diag}\left\{\gamma_n^0, \ldots, \gamma_n^{P-1}\right\} \otimes \mathbf{I}_{MN}\right)\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\right) \\
&\quad \mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}}\mathbf{P}[n|n-1] \\
&= \mathbf{P}[n|n-1] - \mathbf{P}[n|n-1]\tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}}\mathbf{P}[n|n-1] + \mathbf{P}[n|n-1] \\
&\quad \tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}}\left(\mathbf{I}_2 \otimes \operatorname{diag}\left\{\gamma_n^0, \ldots, \gamma_n^{P-1}\right\} \otimes \mathbf{I}_{MN}\right) \\
&\quad \tilde{\mathbf{W}}^T\mathbb{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\mathbb{B}(\tilde{\mathbf{s}}_n)\tilde{\mathbf{W}}\mathbf{P}[n|n-1] \\
&= \mathbf{P}[n|n-1] - \left(\frac{2s^2L}{\sigma_v^2}\right)\mathbf{P}^2[n|n-1] \\
&\quad + \left(\frac{2s^2L}{\sigma_v^2}\right)\left(\mathbf{I}_2 \otimes \operatorname{diag}\left\{\gamma_n^0, \ldots, \gamma_n^{P-1}\right\} \otimes \mathbf{I}_{MN}\right)\left(\frac{2s^2L}{\sigma_v^2}\right)\mathbf{P}^2[n|n-1] \\
&= \mathbf{I}_2 \otimes \operatorname{diag}\left\{\frac{\sigma_v^2\beta_n^0}{2s^2L\beta_n^0 + \sigma_v^2}, \ldots, \frac{\sigma_v^2\beta_n^{P-1}}{2s^2L\beta_n^{P-1} + \sigma_v^2}\right\} \otimes \mathbf{I}_{MN} \\
&= \mathbf{I}_2 \otimes \mathbb{D}_{\delta_n} \otimes \mathbf{I}_{MN} \qquad \text{(A.248)}
\end{aligned}
$$

The proof is complete.

# Appendix B

# Proof of Lemma 6.1

Consider the required expectation

$$\begin{aligned}
E_{\tilde{\mathbf{s}}_n}\left\{\mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{B}^T(\tilde{\mathbf{s}}_n)\right\} &= \left[\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_n \ldots \mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_n\right]\left[\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_n \ldots \mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_n\right]^T \\
&= \mathbf{A}(\mathbf{e}_1)E_{\tilde{\mathbf{s}}_n}\left\{\tilde{\mathbf{s}}_n\tilde{\mathbf{s}}_n^T\right\}\mathbf{A}^T(\mathbf{e}_1) + \ldots \\
&\quad + \mathbf{A}(\mathbf{e}_{2MN})E_{\tilde{\mathbf{s}}_n}\left\{\tilde{\mathbf{s}}_n\tilde{\mathbf{s}}_n^T\right\}\mathbf{A}^T(\mathbf{e}_{2MN}) \\
&= \sigma_s^2 \sum_{i=1}^{2MN} \mathbf{A}(\mathbf{e}_i)\mathbf{A}^T(\mathbf{e}_i) \qquad \text{(B.249)}
\end{aligned}$$

Now, it is left to find the expression $\sum_{i=1}^{2MN} \mathbf{A}(\mathbf{e}_i)\mathbf{A}^T(\mathbf{e}_i)$.

Let us write $\mathbf{C}_k$ and $\mathbf{D}_k$ of 2.2.1 as

$$\mathbf{C}_k = [\mathbf{c}_k^1 \ \mathbf{c}_k^2 \ \ldots \ \mathbf{c}_k^N] \qquad \text{(B.250)}$$

$$\mathbf{D}_k = [\mathbf{d}_k^1 \ \mathbf{d}_k^2 \ \ldots \ \mathbf{d}_k^N] \qquad \text{(B.251)}$$

It can be verified that $\mathbf{c}_k^i$ and $\mathbf{d}_k^i$ hold the following properties for $i = 1, \ldots, N$.

$$\sum_{k=1}^{K} \mathbf{c}_k^i \mathbf{c}_k^{i\,T} = \tilde{\mathbf{I}}_T^i \tag{B.252}$$

$$\sum_{k=1}^{K} \mathbf{d}_k^i \mathbf{d}_k^{i\,T} = -\tilde{\mathbf{I}}_T^i \tag{B.253}$$

where $\mathbf{I}_T^i$ is an identity matrix with $d$ number of diagonal elements made zero. The position of those $d$ diagonal elements are determined by he type of OSTBC. Further, $d = 0$ is also a possible value. Further, the following property is satisfied by $\mathbf{I}_T^i$

$$\sum_{i=1}^{N} \mathbf{I}_T^i = \frac{(T - d)N}{T} \mathbf{I}_T \tag{B.254}$$

The structure of $\mathbf{A}(\mathbf{e}_i)$ is given for some values of $i$ below.

$$\mathbf{A}(\mathbf{e}_1) = \begin{bmatrix} \mathbf{c}_1^1 & \dots & \mathbf{c}_k^1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.255}$$

$$\mathbf{A}(\mathbf{e}_2) = \begin{bmatrix} \mathbf{c}_1^2 & \dots & \mathbf{c}_k^2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.256}$$

$$\vdots$$

$$\mathbf{A}(\mathbf{e}_N) = \begin{bmatrix} \mathbf{c}_1^N & \dots & \mathbf{c}_k^N & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.257}$$

$$\mathbf{A}(\mathbf{e}_{N+1}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{c}_1^1 & \dots & \mathbf{c}_k^1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.258}$$

$$\vdots$$

$$\mathbf{A}(\mathbf{e}_{N+N}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{c}_1^N & \dots & \mathbf{c}_k^N & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.259}$$

$$\vdots$$

$$\mathbf{A}(\mathbf{e}_{MN}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{c}_1^N & \dots & \mathbf{c}_k^N & 0 & \dots & 0 \end{bmatrix} \tag{B.260}$$

$$\mathbf{A}(\mathbf{e}_{MN+1}) = \begin{bmatrix} 0 & \dots & 0 & -\mathbf{d}_1^1 & \dots & -\mathbf{d}_k^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.261}$$

$$\mathbf{A}(\mathbf{e}_{MN+2}) = \begin{bmatrix} 0 & \dots & 0 & -\mathbf{d}_1^2 & \dots & -\mathbf{d}_k^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.262}$$

$$\vdots$$

$$\mathbf{A}(\mathbf{e}_{MN+N}) = \begin{bmatrix} 0 & \dots & 0 & -\mathbf{d}_1^N & \dots & -\mathbf{d}_k^N \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.263}$$

$$\mathbf{A}(\mathbf{e}_{MN+N+1}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \dots & 0 & -\mathbf{d}_1^1 & \dots & -\mathbf{d}_k^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.264}$$

$$\vdots$$

$$\mathbf{A}(\mathbf{e}_{MN+2N}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \dots & 0 & -\mathbf{d}_1^N & \dots & -\mathbf{d}_k^N \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{B.265}$$

$$\vdots$$

$$\mathbf{A}(\mathbf{e}_{2MN}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & -\mathbf{d}_1^N & \dots & \mathbf{d}_k^N \end{bmatrix} \tag{B.266}$$

Now, it can be easily verified that

$$\sum_{i=1}^{2MN} \mathbf{A}(\mathbf{e}_i)\mathbf{A}^T(\mathbf{e}_i) = \frac{(T-d)N}{T}\mathbf{I}_{2MT} \tag{B.267}$$

Hence, the required expectation becomes

$$E_{\tilde{\mathbf{s}}_n}\left\{\mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{B}^T(\tilde{\mathbf{s}}_n)\right\} = \sigma_s^2\frac{(T-d)}{T}N\mathbf{I}_{2MT} \tag{B.268}$$

# Appendix C

# Proof of Lemma 8.1

The unknown parameters of all $K_t$ sources can be considered as the following matrix

$$\mathbf{X}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \ldots, \mathbf{x}_{K_t}(t)] = \begin{bmatrix} \phi_1(t) & \ldots & \phi_{K_t}(t) \\ a_1(t) & \ldots & a_{K_t}(t) \end{bmatrix} \quad \text{(C.269)}$$

Consider one snapshot of the array observation

$$\mathbf{y}(t) = \boldsymbol{\alpha}(t) + j\boldsymbol{\beta}(t) \quad \text{(C.270)}$$

where

$$\begin{aligned} \mathbf{y}(t) &= [y_1(t), y_2(t), \ldots, y_M(t)]^T \\ \alpha(t) &= [\alpha_1(t), \alpha_2(t), \ldots, \alpha_M(t)]^T \\ \beta(t) &= [\beta_1(t), \beta_2(t), \ldots, \beta_M(t)]^T \end{aligned}$$

The joint probability density function of $\mathbf{y}(t)$, given the unknown parameters $\mathbf{X}(t)$, is given by[1]

$$f(\mathbf{y}|\mathbf{X}) = \left(\frac{1}{\pi\sigma_w}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M}\left((\alpha_m - \mu_m)^2 + (\beta_m - \nu_m)^2\right)\right\} \tag{C.271}$$

where

$$\begin{aligned} \mu_m &= \sum_{i=1}^{K} b_i \cos(\tilde{w}_i m + \theta_i) \\ \nu_m &= \sum_{i=1}^{K} b_i \sin(\tilde{w}_i m + \theta_i) \end{aligned}$$

and $b_i = |a_i|$ and $\theta_i = \angle\{a_i\}$.

# Single-target case

Consider the case where only one target is present. Then, the likelihood function becomes

$$f(\mathbf{y}|\mathbf{X}) = f(\mathbf{y}|\mathbf{x}) = \left(\frac{1}{\pi\sigma_w}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M}\left((\alpha_m - \mu_m)^2 + (\beta_m - \nu_m)^2\right)\right\} \tag{C.272}$$

where

$$\begin{aligned} \mu_m &= b\cos(\tilde{w}m + \theta) \\ \nu_m &= b\sin(\tilde{w}m + \theta) \end{aligned}$$

[1]From here on, the time index $t$ is suppressed to simplify the notation.

Re-arranging the above equation, we get [90]

$$f(\mathbf{y}|\mathbf{x}) = \left(\frac{1}{\pi\sigma_w}\right)^M \exp\left\{-\frac{M}{\sigma_w^2}\left(\frac{1}{M}\sum_{m=1}^{M} y_m^2 - \frac{2}{M}\sum_{m=1}^{M}(\alpha_m\mu_m + \beta_m\nu_m) + b^2\right)\right\} \tag{C.273}$$

After some more re-arranging

$$f(\mathbf{y}|\phi, b, \theta) = \left(\frac{1}{\pi\sigma_w}\right)^M \exp\left\{-\frac{M}{\sigma_w^2}\left(\frac{1}{M}\sum_{m=1}^{M} y_m^2 - 2b\mathrm{Re}\left[\mathrm{e}^{-j\theta}A(\tilde{w})\right] + b^2\right)\right\} \tag{C.274}$$

where $A(\tilde{w})$ is the discrete Fourier transform of the array observation given by

$$A(\tilde{w}) = \frac{1}{M}\sum_{m=1}^{M} y_m \exp(-j\tilde{w}m)$$

# Multi-target case

Let us simplify the multi-target likelihood function in (C.271) (see also [91])

$$f(\mathbf{y}|\mathbf{X}) = \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{M}{\sigma_w^2}\left(\frac{1}{M}\sum_{m=1}^{M} y_m^2 - \sum_{i=1}^{K}\{2b_i\mathrm{Re}\left[A(\tilde{w}_i)\right]\} + a^2 + \frac{1}{M}\sum_{i\neq k}\sum_{k} b_i b_k \sum_{m}\cos\left(m(\tilde{w}_i - \tilde{w}_k)\right)\right)\right\} \tag{C.275}$$

Since we are interested in $f(\mathbf{y}_i|\mathbf{x}_i)$, where $i = 1, \ldots, K$, the probability in (C.275) should be marginalized with respect to $\mathbf{y}_i$ and $\mathbf{x}_i$ , $i = 1, \ldots, i-1, i+1, \ldots, K$, to get the required likelihood. i.e.,

$$f(\mathbf{y}_i|\mathbf{x}_i) = \int\int\cdots\int f(\mathbf{y}|\mathbf{X})dy_1\ldots dy_{i-1}dy_{i+1}\ldots dy_K dx_1\ldots dx_{i-1}dx_{i+1}\ldots dx_K \tag{C.276}$$

However, such marginalization will require huge computation. Hence, it is desired to look for some approximations.

By neglecting the last term inside the exponential in (C.275), we get an approximate likelihood function given by [91]

$$\bar{f}(\mathbf{y}|\mathbf{X}) = \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{M}{\sigma_w^2}\left(\frac{1}{M}\sum_{m=1}^{M} y_m^2 - \sum_{i=1}^{K}\left\{2b_i\mathrm{Re}\left[\mathrm{e}^{-j\theta_i}A(\tilde{w}_i)\right] - b_i^2\right\}\right)\right\} \tag{C.277}$$

We notice that the above equation can be written as

$$\begin{aligned}
\bar{f}(\mathbf{y}|\mathbf{X}) &= \left(\frac{1}{\pi\sigma_w^2}\right)^M \\
&\quad \exp\left\{-\frac{M}{\sigma_w^2}\left(\sum_{i=1}^{K}\left(\frac{1}{M}\sum_{m=1}^{M} y_{m,i}^2\right) - \sum_{i=1}^{K}\left\{2b_i\mathrm{Re}\left[\mathrm{e}^{-j\theta_i}A(\tilde{w}_i)\right] - b_i^2\right\}\right)\right\} \\
&= \left(\pi\sigma_w^2\right)^{(K-1)M} \\
&\quad \prod_{i=1}^{K}\left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{M}{\sigma_w^2}\left(\frac{1}{M}\sum_{m=1}^{M} y_{m,i}^2 - 2b_i\mathrm{Re}\left[\mathrm{e}^{-j\theta_i}A(\tilde{w}_i)\right] + b_i^2\right)\right\} \\
&\propto \prod_{i=1}^{K} f(\mathbf{y}_i|\mathbf{x}_i)
\end{aligned} \tag{C.278}$$

where, $y_{m,i}$ is the observation at the $m$th sensor due to the $i$th source. Assuming equal power of the sources, $y_{m,i}^2$ is approximately given as $y_{m,i}^2 = (y_m^2/K)$.

Hence, the likelihood function becomes

$$L(\mathbf{y}_i|\phi_i, b_i, \theta_i) = \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{M}{\sigma_w^2}\left(\frac{1}{MK}\sum_{m=1}^{M} y_m^2 - 2b_i\mathrm{Re}\left[\mathrm{e}^{-\jmath\theta_i}A(\tilde{w}_i)\right] + b_i^2\right)\right\} \tag{C.279}$$

## Marginalization and the required likelihood

In this section we employ marginalization techniques on (C.279) to find the required probability density function.

$$
\begin{aligned}
L(\mathbf{y}_i|\phi_i) &= \int L(\mathbf{y}_i|\phi_i, b_i, \theta_i)p(\theta_i)p(b_i)d\theta_i db_i \\
&= \int_0^\infty \int_{-\pi/2}^{\pi/2} \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2 - \frac{M}{\sigma_w^2}b_i^2\right. \\
&\quad \left. +\frac{2Mb_i}{\sigma_w^2}\mathrm{Re}\left[\mathrm{e}^{-j\theta_i}A(\tilde{\omega}_i)\right]\right\} p(\theta_i)d\theta_i p(b_i)db_i \\
&= \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2\right\} \\
&\quad \int_0^\infty \exp\left\{-\frac{M}{\sigma_w^2}b_i^2\right\}\int_{-\pi/2}^{\pi/2}\exp\left\{c\cos\theta_i + d\sin\theta_i\right\}p(\theta_i)d\theta_i p(b_i)db_i \\
&= \left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2\right\}\int_0^\infty \exp\left\{-\frac{M}{\sigma_w^2}b_i^2\right\}\pi\mathbf{I}_0(\sqrt{c^2+d^2})p(b_i)db_i \\
&= \pi\left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2\right\}\int_0^\infty \exp\left\{-\frac{M}{\sigma_w^2}b_i^2\right\}\mathbf{I}_0(\beta b_i)p(b_i)db_i \\
&= \left(\frac{\pi}{2\sigma_a^2}\right)^{1/2}\left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2\right\} \\
&\quad \int_0^\infty \exp\left\{-\left(\frac{M}{\sigma_w^2}+\frac{1}{2\sigma_a^2}\right)b_i^2\right\}\mathbf{I}_0(\beta b_i)db_i \\
&= \left(\frac{\pi}{2\sigma_a^2}\right)^{1/2}\left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2\right\}\int_0^\infty \exp\left\{-\alpha b_i^2\right\}\mathbf{I}_0(\beta b_i)db_i \\
&= \left(\frac{\pi}{2\sigma_a^2}\right)^{1/2}\left(\frac{1}{\pi\sigma_w^2}\right)^M \exp\left\{-\frac{1}{\sigma_w^2}\sum_{m=1}^{M} y_{m,i}^2\right\}\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\exp\left\{\frac{\beta^2}{8\alpha}\right\}\mathrm{I}_0\left(\frac{\beta^2}{8\alpha}\right)
\end{aligned}
\tag{C.280}
$$

where

$$
\begin{aligned}
c &= \frac{2Mb_i}{\sigma_w^2}\mathrm{Re}[A(\tilde{w}_i)] \\
d &= \frac{2Mb_i}{\sigma_w^2}\mathrm{Im}[A(\tilde{w}_i)] \\
\alpha &= \left(\frac{M}{\sigma_w^2} + \frac{1}{2\sigma_a^2}\right) \\
\beta &= \frac{2M}{\sigma_w^2}|A(\tilde{w}_i)| \\
p(\theta_i) &\sim \mathcal{U}[-\pi/2, \pi/2] \\
p(b_i) &\sim \frac{1}{\sqrt{2\pi\sigma_a^2}}\exp\left\{-\frac{b_i^2}{2\sigma_a^2}\right\}
\end{aligned}
$$

and $\mathbf{I}_0$ is the modified bessel function of the first kind.

Note that the following definite integrals from [35] were exploited in arriving to (C.280)

$$\int_{-\pi/2}^{\pi/2} \exp\{c\cos\theta + d\sin\theta\}\, p(\theta)d\theta = \pi\mathbf{I}_0(\sqrt{c^2+d^2}) \tag{C.281}$$

$$\int_0^{\infty} \exp\{-\alpha x^2\}\, \mathbf{I}_\nu(\beta x)dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\exp\left\{\frac{\beta^2}{8\alpha}\right\} \mathrm{I}_{\frac{1}{2}\nu}\left(\frac{\beta^2}{8\alpha}\right) \tag{C.282}$$

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