DISCRIMINATION OF BRIEF EMPTY TIME INTERVALS

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By

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This research deals with the coding of brief empty time intervals bounded by very brief auditory markers. From the results of two experiments it is concluded that the discrimination between two brief durations within the range .05 to .3 seconds is not based on the energy content of the stimulus pattern defining the durations. At the same time, this evidence supports the view that a central process codes time information, independently of the sensory events bounding the internal interval. This basic approach is involved in three mathematical models for duration discrimination which differ in their assumptions about the nature of this central process and in their predictions concerning the change in the discriminability of a pair of intervals T and T+ΔT as the base duration T increases. The data from four experiments are analyzed in detail with respect to each of these models. None is completely adequate in describing the functional relation between discriminability and base duration, but only one (Creelman's model) can be definitely rejected.

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I dedicate this thesis to my husband, Jules, without whose encouragement and constant support this work would not have been undertaken. I can now appreciate the pleasure and excitement he has experienced from his own research.

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I. INTRODUCTION

1.1 Categories of psychological time

The order of magnitude of a time interval should be an important consideration in speculation about the nature of psychological time. There are probably many ways in which an individual codes a time interval, depending on the size of the interval being dealt with, on how its boundaries are defined, and on what occurs during the interval. A number of authors have attempted to define categories of psychological time. Fraisse's review (1956) of 75 years previous work formed the basis of a classification scheme on which Michon (1967, 1970) and Ornstein (1969) later elaborated. These authors distinguished between

1. the experience of simultaneity and successiveness, including the temporal ordering of events,
2. the perception of short intervals of the order of a few seconds, including the perception of rhythmic patterns,
3. the experience of duration, or time passing, which involves the long term memory of events in the past,
4. the anticipation of a future, or a temporal perspective, which is socially and culturally determined.

Underlying attempts at categorization is the recognition that the variables influencing any one category may not be important for the others. Also, the methodology suitable for studying one category of psychological time may not be applicable to other categories.

The category dealt with in this thesis is that of very short intervals. According to the phenomenological evidence reviewed by Fraisse, the subjective experience of duration changes very markedly between .05 and 1 second. Michon (1967) found that around .5 to .7 sec, there occurs a doubling of the exponent of the power function which he and others have used to represent the relation between "real" time and subjective (estimated) time, for short time intervals. If the power function truly represents the correspondence between subjective and real time, this shift might indicate that a change in the judgment process is occurring around this point. Although where a change in process appears to occur may depend on the task and on how the time interval is defined, we will tentatively take the upper bound for intervals considered as being very short to be somewhere in the range .5 - .7 seconds.

There are two other lines of evidence which support the designation of a range of durations below .7 sec. as a category of psychological time distinct from the category of short durations of the order of several seconds. Much of this evidence is summarized in Woodrow (1951) and Fraisse (1956). First, with magnitude estimation procedures as well as with reproduction tasks, it is found that intervals less than some "indifference interval" are consistently overestimated while intervals longer than the indifference interval are underestimated. This indifference interval is often (but not always) found to be in the range .5 - .7 sec. Secondly, when discrimination thresholds (ΔT_{th}) are obtained, it is generally found that the Weber ratio $\Delta T_{th}/T$ is a decreasing function of T, for T less than .5 sec., and often reaches a minimum value in the region of .5 - .8 seconds.

1.2 General statement of the problem

In any study of psychological time, the basic problem concerns how individuals code a time interval defined by some pattern of stimulus events. In investigating how an observer discriminates between two very brief intervals in the range .05 - .3 sec., we can adopt procedures which may not be applicable to intervals of the order of a few seconds. An alternative to the scaling and reproduction procedures which have been widely used to study short intervals is the contemporary psychophysical approach used in the study of sensory processes, developed in connection with signal detection theory. This approach was used by Creelman (1962), whose experiments involved very short durations ranging from .04 to .8 sec. He incorporated a number of assumptions into a mathematical model which seemed to provide a good description of the data from five experiments. This was the first attempt to formulate and test a quantitative theory using a very specific assumption about how physical time is transformed into psychological time.

However, in Creelman's experiments there was a confounding of two stimulus variables: time and energy. His study used "filled" auditory intervals, so called because the time information is defined for the observer by the duration of an auditory pulse which remains present throughout the interval. Two filled intervals of different durations differ in the total amount of stimulus energy which they contain, as well as in their durations, and the discrimination between them may depend upon either time, or energy, or both.

Although time, like space, cannot be isolated from energy input, it does have a quantitative aspect which is independent of the energy conveying it. An interval bounded by two brief signals E_1 and E_2 is referred to as an empty interval, and two such patterns differing only in the time between the onsets of E_1 and E_2 differ in their time input but not in their energy content. Conversely, two patterns which differ only in the intensity of one or both the signals bounding the intervals, differ in their energy content but not in their time information. Hence using empty intervals would have the advantage that energy and time can be varied independently.

The goal towards which this thesis is directed is an adequate quantitative theory for the discrimination of very brief empty intervals. Incorporated into such a theory must be an assumption about how an observer codes the time information in an interval, as well as assumptions about how he combines the information from a pair of intervals when he must choose the longer one of the two. The experiments described here serve two functions. First, the data provide constraints on what assumptions can or need to be used. Secondly, the data can be analyzed in terms of those quantitative theories for duration discrimination which seem to be adequate in other contexts. Creelman's model is one of three such models.

1.3 A definition of psychological time

It is useful to set forth our assumptions regarding psychological time, as a context for a classification of models of the processes involved in the discrimination between brief time intervals. Any measure taken of time involves at least two steps. To present a time interval to an observer O, two external events E_1 and E_2 must occur in succession. We then obtain an "objective" measure T of the temporal separation between these two events by using some measuring device (a "clock") which assigns a number to this temporal separation. That is, some operation must be carried out which is initiated by the first event E_1 and terminated by the second E_2. The nature of this operation is not dependent on the nature of E_1 and E_2. Let time as measured by a clock external to the O be denoted by T_{ext} in the diagram below.

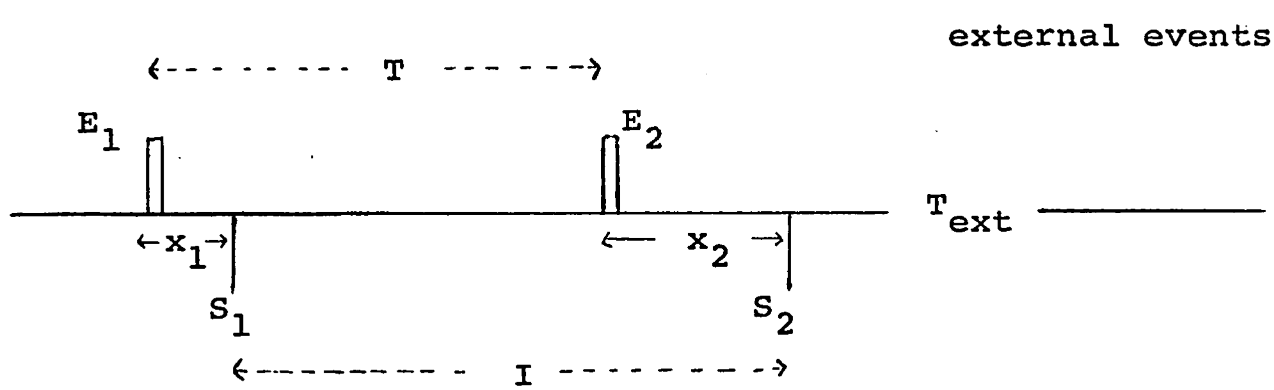

FIGURE 1: The relation between an external interval (T) and an internal interval (I).

Within the O, the psychological time T_ψ corresponding to T is determined by the temporal separation between two internal events s_1 and s_2 arising from the external events E_1 and E_2. That is, E_i (i=1,2) is mapped into a discrete internal event s_i, and it is the separation between these internal events which is assessed by the O. Let I represent the measure of this separation if it were possible to obtain it with the same device as is used in measuring T. However, the measure of the separation between s_1 and s_2 is achieved through some psychological process: T_ψ is considered to be a psychological coding of the separation I. An external interval of magnitude T corresponds to some internal interval of magnitude I (if measured in the same way as T) and this in turn has a psychological representation T_ψ. Hence when we consider the relation between T and T_ψ, for very short intervals, we may have to consider both the mapping of E_i into s_i and the way in which I is coded.

When T is repeatedly presented, variability in when s_i occurs with respect to E_i gives rise to variability in I, even when T is constant. From the diagram, the relation between T and I is given by

$$I = T + x_2 - x_1$$

where x_1 is the delay between the onset of E_1 and the onset s_1 of the internal interval I; x_2 is defined in a similar way. (We will refer to x_i as the latency of E_i). These delays are random variables with expected values denoted by $\varepsilon(x_i)$ and variance $var(x_i)$. If we assume that these delays are independent, then

$$\varepsilon(I) = \varepsilon(T) + \varepsilon(x_1) - \varepsilon(x_2) = T \quad \text{iff } \varepsilon(x_1) = \varepsilon(x_2)$$

and

$$var(I) = var(T) + var(x_1) + var(x_2) = 0$$

$$\text{iff } var(x_1) = var(x_2) = 0$$

since $var(x_i)$ is always a non-negative quantity, and T is a constant.

An assumption of major interest is that there exists a central process which measures or codes the temporal separation between the internal boundaries s_1 and s_2, this process being independent of how the internal boundaries are defined. This idea is central to Creelman's theory, and to a model proposed by Kristofferson (1965). However, the discrimination between T and T+ΔT could be influenced by energy dependent interactions between the boundaries - facilitation and inhibition as is inferred from masking phenomena. The type and amount of interaction would depend on the modality of the stimuli E_i, and on the size of the interval between them.

By choosing T large enough, or by having E_1 and E_2 in different modalities, we could minimize interactions between the boundaries. On the other hand, it is possible that a central timing mechanism might be used even when the discrimination between the stimulus patterns defining T and $T+\Delta T$ could be made on the basis of the total patterns of excitation. The interactions may instead influence the latency distributions of s_1 and s_2.

In relating different categories of temporal experience to the functional organization of memory, Michon (1970) suggested that "short intervals ... will fall within one single time 'chunk' of short term memory, while very brief intervals will be judged on the basis of a momentary 'iconic image' where we may expect interactions, not only with the information content of the intervals, but also with the energy distributions of the stimuli." (p. 256). The view that time information is obtained from a cognitive reconstruction of events occurring during that interval (Ornstein) could be adapted for very short empty intervals by assuming that the time information is inferred or reconstructed from the loss or decay of some type of information in the first boundary, with the amount of loss being some increasing function of T. The information subject to loss may or may not be energy dependent. Mathematical models assuming linear decay and exponential decay of excitation were derived and tested by McKee et al. (1970) and by Abel (1970), but these were inadequate in predicting the data from their experiments.

An alternative to the models above, is a class of models involving assumptions about the variance in x_1 and x_2 without including assumptions about further coding or measuring operations during I. This approach would be entailed in any theory assuming that a match or correlation of an internal memory standard is being made with the interval being presented. An example of this type of model is contained in a paper by Allan, Kristofferson and Weins (1971). Their "onset-offset delay" model could account adequately for the data they obtained using durations defined by continuous light flashes, of magnitudes ranging from 50 to 150 msec.

There are many possible approaches, then, for the construction of quantitative models. However, we can distinguish between several of these on the basis of the definition of psychological time that we have just considered. In particular, two interesting approaches can be distinguished according to whether the discrimination between T and T+ΔT depends on

1. information in the pattern, depending on the energy in the signals E_1 and E_2, or
2. information derived from events occurring between the boundaries of the interval, these events being independent of ongoing sensory events.

It is not claimed that these two classes exhaust all the possibilities. It may be that the discrimination depends critically on stimulus information in the intervals which is not a function of energy, provided sufficient energy is present to define this information. Or the discrimination might be achieved by some matching procedure.

1.4 Three quantitative models of time discrimination

In this section three models are presented in sufficient detail to make clear certain predictions which can be tested by the data from the experiments in this thesis. These three models are similar mainly in that they assume there is no energy dependent information in the E_1-E_2 pattern, which is used as a basis for time discrimination. In section 1.5, we review evidence which suggests that varying the energy in the stimulus pattern does not appreciably change performance provided there is no difficulty in detecting the onset and offset of the intervals to be discriminated. If this can be established for empty intervals as well, one approach to model building can be rejected tentatively, and it is then of considerable interest to use empty intervals in testing the following models.

A. Definition of notation and terms

We first define notation and terms which are used here and also in the description of the results of the experiments presented in part II. In these experiments, the O had a forced choice task: to indicate which is the longer interval in the pair $\{T_v, T_s\}$. T_s is the shorter interval in the pair and is called the standard or base duration. T_v differs from T_s by some amount ΔT and occurs first or second equally often. An S_1 pattern is a stimulus pattern in which the first interval is longer than the second: $S_1 = [T_v, T_s]$. An S_2 pattern is one in which the second is the longer interval: $S_2 = [T_s, T_v]$. The response alternatives are denoted by "1" or "2", indicating that the first, or the second, interval is chosen as the longer one. $\hat{P}(1|S_1)$ is the proportion of trials on which the S_1 patterns is presented and the first interval is correctly chosen as longer. It is an estimate of $P(1|S_1)$ the probability of being correct when the first interval is longer. $\hat{P}(2|S_2)$ is defined in a similar way.

One measure of the O's ability to discriminate between T_s and T_v is given by

$$\hat{P}(C) = \hat{P}(1|S_1) \cdot P(S_1) + \hat{P}(2|S_2) \cdot P(S_2)$$

$$= \frac{1}{2} [\hat{P}(1|S_1) + \hat{P}(2|S_2)]$$

when S_1 and S_2 occur equally often. Other performance measures are defined in terms of the specific model being considered, and are derived from the quantities $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ along with some probability density function specified by the model. Finally, we define the psychometric function as a plot of $\hat{P}(C)$, or some other performance measure, as a function of ΔT, with T_s as a parameter. It describes how performance improves in discriminating between T_s and $T_s + \Delta T$, as ΔT increases while T_s is kept fixed.

The measure $\hat{P}(C)$ has the advantage that it is not specific to any particular theory, as is the d' measure of signal detection theory. Further, $\hat{P}(C)$ in a symmetric $(P(S_1) = P(S_2) = \frac{1}{2})$ two-alternative forced choice (2AFC) design is independent of response bias, provided the response bias is defined on a "guessing" state in which the O does not have enough information on which to base his response, and the probability of this guessing state occurring is the same for S_1 and S_2.

B. Creelman's Theory

Among the few quantitative theories proposed for time discrimination is one formulated by Creelman (1962). It assumes a mechanism which counts the discharges of a very large number of independent units firing during the interval T to be measured. The probability density distribution for the number of counts from this random source can be approximated asymptotically by a normal distribution with mean and variance λT, when λT is large. The parameter λ is a constant reflecting the rate of firing of the pulse source. The response strategy is assumed to be one which associates the longer interval with the larger count. In order to obtain a predicted performance measure, consider the difference ΔN in counts obtained in the two intervals presented on each trial in a FC task. The mean of this difference distribution will be $\lambda \cdot \Delta T$ for the S_2 patterns $(T, T+\Delta T)$ and $-\lambda \cdot \Delta T$ for the S_1 patterns $(T+\Delta T, T)$. In both cases the variance is $\lambda \cdot (2T+\Delta T)$. When ΔN is greater than some criterion value, c, the decision is to choose the second interval as the longer in the pair. The corresponding predicted measure of sensitivity is the distance d' between the means of the difference distributions, expressed in standard deviation units:

$$d' = \frac{2\lambda \cdot \Delta T}{[\lambda(2T+\Delta T)]^{1/2}} \qquad (1\text{-}1)$$

Creelman modified this by including a factor $(1+KT)^{-1/2}$ to take into account memory loss of the count of the first interval while the count during the second is being obtained. He also included a term σ_v^2 in the variance to account for any uncertainty in the start and ending of the signals marking the durations. This parameter is an inverse function of the signal to noise ratio; $\sigma_v^2 = 0$ was used for signals "loud and clear" above the noise. The resulting expression for the detectability of a given duration difference ΔT is

$$d' = \frac{1}{(1+KT)^{1/2}} \cdot \frac{2\lambda^{1/2} \cdot \Delta T}{(2T+\Delta T+\sigma_v^2)^{1/2}} \qquad (1\text{-}2)$$

for the two interval forced choice situation. For equation (1-2), an estimate of d' is obtained from the data by using a table of areas under the normal curve, along with $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$. One prediction which follows immediately from this model is that d' will be a monotonic decreasing function of base duration T, when T is increased while ΔT is fixed. This prediction can be contrasted with those made by two other models.

C. Quantal counting models

Kristofferson (1965) has proposed a counting model of a somewhat different nature from Creelman's for duration discrimination. In three quite different types of experiments he has obtained behavioral constants of the order of 50 msec., highly correlated with each other and with the half-period of the alpha wave of the encephalogram (Kristofferson, 1967). These constants were interpreted within a theoretical framework which postulates a very stable internal periodic process, independent of ongoing sensory events. This periodic process, or "clock", is looked upon as generating time points which mark off equal units, or quanta, of time. The time points have an important role in controlling the gating of incoming sensory input and in regulating the flow of information through the central nervous system.

Kristofferson suggested (1965) that another possible function for this "clock" is in coding time information over some range of durations. An interval could be coded in terms of the number of time points occurring during it. This count depends only on when the interval begins with respect to the time base. The choice of which of two intervals is the longer one is made by comparing the number of time points occurring in each. In testing this model he used filled intervals defined by the time between the successive offsets of a visual and an auditory stimulus. No definite conclusions could be drawn about the adequacy of the model, but it did seem worth further consideration, both as a means of investigating duration discrimination, and as a possible extension of temporal quantum theory.

Kristofferson's model was elaborated and tested in a situation involving adjacent empty intervals defined by three very brief auditory signals chosen so as to be easily detected by the O (Carbotte & Kristofferson, 1971)*. If it is assumed that there is a fixed time base underlying the measurement process, the coding of the second interval will not be independent of the first and this dependence will be reflected in the shape of the predicted psychometric functions. The number of time points which will be contained within an interval depends on where the interval begins (or ends) with respect to the boundaries of an ongoing time quantum, i.e., on how far the onset (offset) of the interval is from the first (last) time point occurring within it. In the adjacent interval pattern, when a second interval immediately follows the first, where the first one ends with respect to

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* Hereafter referred to as C & K.

a time point determines where the second one begins, since the signal marking the end of the first interval also marks the beginning of the second interval. Thus for adjacent intervals the measure of the second interval can be considered as being dependent on the measure of the first. The original version of the counting model in Kristofferson (1965) assumed that the coding of the second interval in a FC task was independent of the first, but he did not speculate on how this independence might be achieved. A time base with a constant period plus no variability in the mapping of T into I would necessarily imply that there will be dependence between the coding of two intervals, no matter what the separation between them. It may be that one (or both) of these two assumptions - constant period, and no variability in I - will have to be modified. Nevertheless, it is of interest to compare the predictions of the model for separated intervals assuming complete independence, with the predictions made for adjacent intervals.

The FC psychometric functions for adjacent intervals obtained by C & K were not at all well described by a set of three straight line segments as is predicted by the counting model. However, a very simple modification of the model resulted in predicted psychometric functions which had many of the qualitative features seen in the empirical functions. In predicting the FC psychometric functions, it is assumed that whenever the difference in the number of time points in the two intervals is greater than zero, the O selects the longer interval without uncertainty (although his choice may be incorrect). However, when the count from the second interval equals the count from the first, the O is in a state of uncertainty as to which is the longer interval. The "two-look" version of the counting model assumes that the O has available a stored representation of the stimulus pattern which can be coded a second time, independently of the first coding. This assumption does not appear unreasonable; it might be associated with the strategy of auditory rehearsal assumed to be part of the sequence of processes involved in the perception and short term retention of verbal material (Norman, 1969).

In a second experiment in C & K, the O was to decide whether the members of a pair of adjacent intervals were the same, or different in duration. The stimulus alternatives were selected from the set

$$S_0 = (T, T)$$

$$S_1 = (T+\Delta T/2 \ , \ T-\Delta T/2)$$

$$S_2 = (T-\Delta T/2 \ , \ T+\Delta T/2)$$

with $P(S_0)$ always being 1/2. In one condition, the three alternatives were randomly intermixed within each session, while in another condition only S_1 or S_2 occurred within any session. However, in both conditions, there was a marked asymmetry in how S_1 and S_2 were treated by all three Os. The intervals were perceived as being equal as often in the S_1 pattern as in the S_0 pattern, whereas the probability of saying "same" when the S_2 pattern occurred was very much lower. This asymmetry suggested the inclusion of another parameter k into the counting model, incorporating the assumption that when the count from the first interval is only one more than the count from the second interval, the O may not always perceive the first interval as being longer than the second.

Figure 2 schematically represents the final version of the quantal counting model which was used in fitting both the adjacent interval psychometric functions and the psychometric functions for intervals separated by 2 seconds. It will be shown below that it is possible to fit the data by using only one free parameter - the period of the time base. The values of q used in predicting the individual psychometric functions and the data from the "same-different" experiment varied between Os but they were of the order of 25, 50 or 100 msec. Although no criterion of goodness of fit was used and the best estimates of q were not determined, the model did look interesting enough to merit further testing: it could account (quantitatively) for the results of the three experiments reasonably well and it appeared that the estimates of q for the different Os might be multiples of 25 msec.

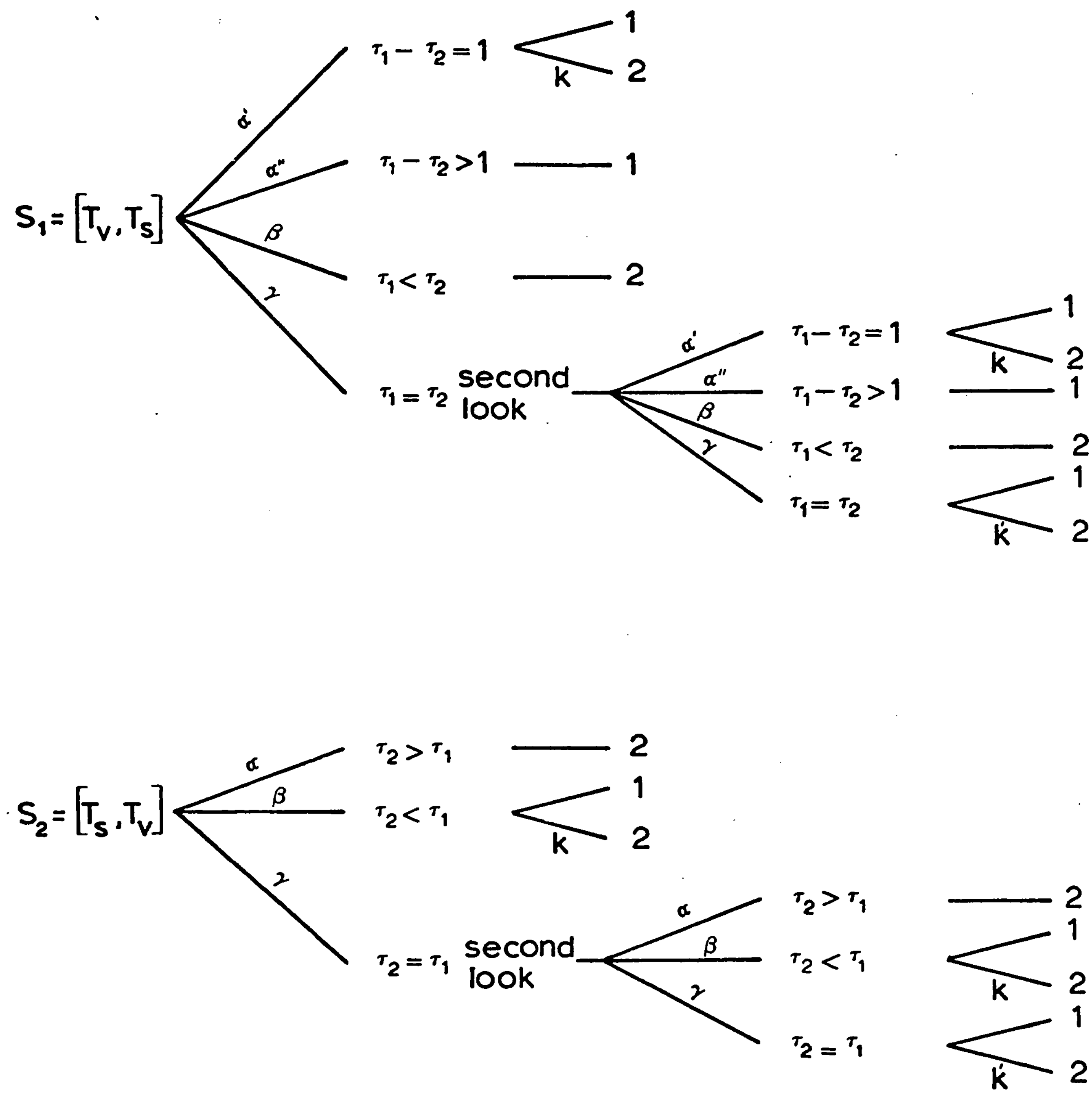

FIGURE 2: Schematic representation of the quantal counting model,

The tree diagram in figure 2 applies to the FC task for both dependent and independent intervals. Let τ_1 denote the count from the first interval to occur, and let τ_2 denote the count from the second. Then on each trial there are four possible outcomes for the coding of the pair of intervals which must be considered:

$$\tau_1 < \tau_2 \qquad \tau_1 = \tau_2 \qquad \tau_1 = \tau_2+1 \qquad \tau_1 > \tau_2+1$$

α denotes the proportion of trials on which $\tau_1 > \tau_2$ when the S_1 pattern occurs. This is the same as the proportion of trials on which $\tau_1 < \tau_2$ when the S_2 pattern occurs, for we can consider S_2 as being the reverse of the S_1 pattern for purposes of calculating $P(\tau_1 < \tau_2)$. (This does not imply at all that the O treats S_1 and S_2 symmetrically.)

γ denotes the proportion of trials in which $\tau_1 = \tau_2$. We write

$$\alpha = P(\tau_1 > \tau_2 | S_1) = P(\tau_2 > \tau_1 | S_2)$$

$$\gamma = P(\tau_1 = \tau_2) \qquad (1-3)$$

$$\beta = P(\tau_1 < \tau_2 | S_1)$$

and we can define a quantity α' in a similar way.

$$\alpha' = P(\tau_1 = \tau_2+1 | S_1)$$

The size of the quantities α, α' and γ depend on T_v and T_s, and on whether or not independence of τ_1 and τ_2

is assumed. Once q has been fixed at some value, we can write

$$T_s = (s+b_s)\cdot q \quad \text{where s is an integer and} \quad 0 \leq b_s < 1$$

$$T_v = (v+b_v)\cdot q \quad \text{where v is an integer and} \quad 0 \leq b_v < 1.$$

Table 1 shows α, α' and γ, as well as $\Delta T/q$ expressed in terms of b_v and b_s. The derivations are presented in Appendix I.

The decision process represented in figure 2 is summarized by

$$\begin{aligned} P(2|\tau_1 < \tau_2) &= 1 \\ P(2|\tau_1 = \tau_2+1) &= k \\ P(2|\tau_1 > \tau_2+1) &= 0 \\ P(2|\tau_1 = \tau_2) &= k' \end{aligned} \qquad (1\text{-}4)$$

This means that when $\tau_1 \neq \tau_2$, the O̲ chooses a response which is not necessarily the correct one, on the basis of the outcome of the initial coding. When the outcome of the initial coding is $\tau_1 = \tau_2$, the O makes his decision on the basis of the results of a recoding of the time information in the pattern. The probabilities associated with the outcomes of the second coding are assumed to be exactly the same as in the first. However, if with this "second look", the outcome is again $\tau_1 = \tau_2$, the O̲ chooses the second interval as being longer with probability k'.

Table 1

Quantities involved in the quantal counting model

Each entry in the second column is a range of $\Delta T/q$ for which the values of α, α' and γ are given by the expressions in that row. As ΔT increases, the values of α, α' and γ are determined from different expressions involving b_v and b_s.

$\{T_v, T_s\}$	range of $\Delta T/q$	range of b_v	α	α'	γ
τ_2 dependent on τ_1					
$\{s+b_v, s+b_s\}$	$0 \rightarrow (1-2b_s)$	$b_s \rightarrow (1-b_s)$	b_v	b_v	$1-b_v-b_s$
	$(1-2b_s) \rightarrow (1-b_s)$	$(1-b_s) \rightarrow 1$	$1-b_s$	$1-b_s$	b_v+b_s-1
$\{s+1+b_v, s+b_s\}$	$(1-b_s) \rightarrow 2(1-b_s)$	$0 \rightarrow (1-b_s)$	$1-b_s$	$1-b_v-b_s$	b_s
	$2(1-b_s) \rightarrow (2-b_s)$	$(1-b_s) \rightarrow 1$	b_v	b_v+b_s-1	$1-b_v$
$\{s+2+b_v, s+b_s\}$	$(2-b_s) \rightarrow (3-2b_s)$	$0 \rightarrow (1-b_s)$	1	b_s	0
	$(3-2b_s) \rightarrow (3-b_s)$	$(1-b_s) \rightarrow 1$	1	$1-b_v$	0
τ_2 independent of τ_1					
$\{s+b_v, s+b_s\}$	$0 \rightarrow (1-b_s)$	$b_s \rightarrow 1$	$b_v(1-b_s)$	$b_v(1-b_s)$	$(1-b_v)(1-b_s)+b_vb_s$
$\{s+1+b_v, s+b_s\}$	$(1-b_s) \rightarrow (2-b_s)$	$0 \rightarrow 1$	$1-b_s+b_vb_s$	$(1-b_v)(1-b_s)+b_vb_s$	$(1-b_v)b_s$
$\{s+2+b_v, s+b_s\}$	$(2-b_s) \rightarrow (3-b_s)$	$0 \rightarrow 1$	1	$(1-b_v)b_s$	0

From the diagram, we obtain (after simplification)

$$P(1|S_1) = \alpha + \alpha\gamma - \alpha' k(1+\gamma) + \gamma^2(1-k')$$

and

$$P(2|S_2) = \alpha + \alpha\gamma + \beta k(1+\gamma) + \gamma^2 k'$$

so that when $P(S_1) = P(S_2) = 1/2$ the expression for $P(C) = \frac{1}{2}\ [P(1|S_1) + P(2|S_2)]$ can be rewritten as

$$2P(C) = P - k(1+\gamma)(\alpha + \alpha' + \gamma - 1) \qquad (1\text{-}5)$$

where

$$P = 2\alpha + 2\alpha\gamma + \gamma^2 \ . \qquad (1\text{-}6)$$

Thus the predicted two-look psychometric function is given by a set of parametric equations for $P(C)$ and ΔT, involving b_v and b_s. Its explicit representation can be obtained by making the appropriate substitutions from Table 1, for α, α' and γ.

Equation (1-5) simplifies to

$$2P(C) = P(1-k) + k \qquad (1\text{-}7)$$

as long as $\Delta T/q$ is small enough so that $\alpha = \alpha'$. According to Table 1 this upper bound on $\Delta T/q$ is $1-b_s$. Equation (1-7) embodies a rather strong prediction. When q is varied over a wide range, we should find some value of q such that a plot of $2\hat{P}(C)$ as a function of P, for $0 < \Delta T/q \leq 1-b_s$, will be a straight line with the slope and intercept both having some value between 0 and 1, and the sum of these values adding to 1. This particular value of q could be used as $\hat{q}$ (an estimate of q) and $\hat{k}$ is determined from the slope and intercept; $\hat{q}$ and $\hat{k}$ can then be used to generate the entire psychometric function predicted by the model, fitting a particular set of values of $\hat{P}(C)$. Note that equation (1-7) thus gives us a procedure for obtaining $\hat{k}$, in which k is not varied independently of q; that is, k is not a free parameter as in the more conventional procedures of varying q and k independently in a search for that pair of values of q and k resulting in the psychometric function best fitting the data. However, we should be able to obtain approximately the same value for $\hat{q}$ from the two procedures.

The bias parameter k' does not enter into the relation (1-5); we need estimates only of q and k in fitting the model to $\hat{P}(C)$. However, we do need $\hat{k}'$ if we are interested in how well the model predicts $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$.

One basis of comparison between this model and Creelman's lies in their predictions about the effect on performance of changing base duration T_s while keeping ΔT fixed. A change in T_s changes b_s, and will also change b_v since $T_v = T_s + \Delta T$. Thus there will be a shift in all three quantities α, α' and γ but the magnitude of these changes could be such that P(C) either increases or decreases. Note, however, that if T_s is changed by an amount which is some integer multiple m of q, b_s and b_v are unchanged and the quantities α, α' and γ will be unchanged. Although the integers s and v are both increased by m, the probability that the difference between τ_1 and τ_2 is 0, 1 or 2 depends only on b_v and b_s. But P(C) also depends on k. If there is a change in k as base duration increases, there will be a change in P(C); if k decreases, an increase in P(C) is possible.

D. The quantal onset-offset model

In the quantal counting model, psychological time is dealt with as a discrete variable. Creelman's model treats it as a continuous variable, with a signal detection type of analysis for linking the time measurement process with the decision process, and then predicting performance. However, an alternative model in which the decision in a duration discrimination task is based on a continuous variable, has been proposed by Allan, Kristofferson and Weins (1971)*. The size of the internal interval I corresponding to a fixed interval T may be variable, but the range of values of I can be specified once we specify the probability distribution of the delays between the onset of T and the onset of I, and the offset of T and the offset of I. In the onset-offset model, it is assumed that these delays are

---

* Hereafter referred to as A&K&W.

independent uniformly distributed random variables such that

$$0 \leq d_{on}, d_{off} < q$$

where the delay in signaling the onset (offset) of the internal interval I is denoted by d_{on} (d_{off}). Thus the probability distribution for $I = T + d_{off} - d_{on}$ is triangular, defined over the range $T-q < I < T+q$ with expected value $E(I) = T$ and variance $q^2/6$. Let $f(I,T)$ denote this density function. No further assumptions are made about a measurement process occurring during the internal interval; the decision process involves only the random variable I.

Decision strategy I

One set of assumptions about how the decision is made in a FC task is the same as in the Creelman model. The choice of which is the longer interval is made by subtracting the size of the second interval I_2 from the first I_1, and comparing this difference ΔI with some criterion value c. The decision rule is of the form:

> if $\Delta I > c$, then choose the second as the longer interval; otherwise choose the first as the longer one.

Let $g_i(\Delta I)$ denote the probability density function of ΔI, conditional on the occurrence of the S_i pattern (i=1,2). A measure of the O's ability to discriminate between T and T+ΔT is given by the distance $d_{q,2}$ between the means of these two overlapping difference distributions expressed in units of q:

$$d_{q,2} = 2\Delta T/q \quad .$$

Let $I_1 = T_1 + x_2 - x_1$ and $I_2 = T_2 + x_4 - x_3$. Then

$$\Delta I = I_2 - I_1 = (T_2 + x_4 - x_3) - (T_1 + x_2 - x_1)$$

or

$$\Delta I = T_2 - T_1 + X \quad \text{with} \quad X = x_4 + x_1 - x_2 - x_3 \quad .$$

Hence the distributions for $g_i(\Delta I)$ can be specified once the probability density function of X, G(X), has been specified. These functions are given in Appendix IIa for both separated and adjacent intervals. Using the cumulative distribution of G(X), we obtain from $\hat{P}(1|S_1)$ an estimate of z_1 which is the distance of c from the mean of the distribution $g_1(\Delta I)$; and from $\hat{P}(2|S_2)$ we obtain an estimate of z_2, the distance of c to the mean of the distribution $g_2(\Delta I)$* (See figure 3a).

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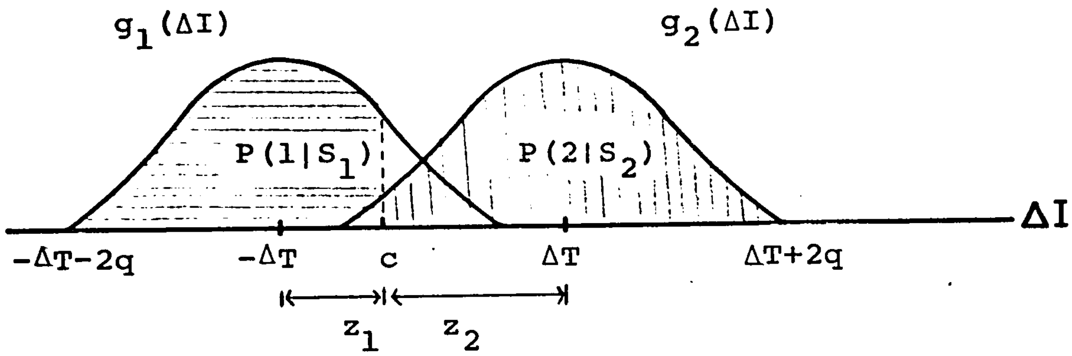
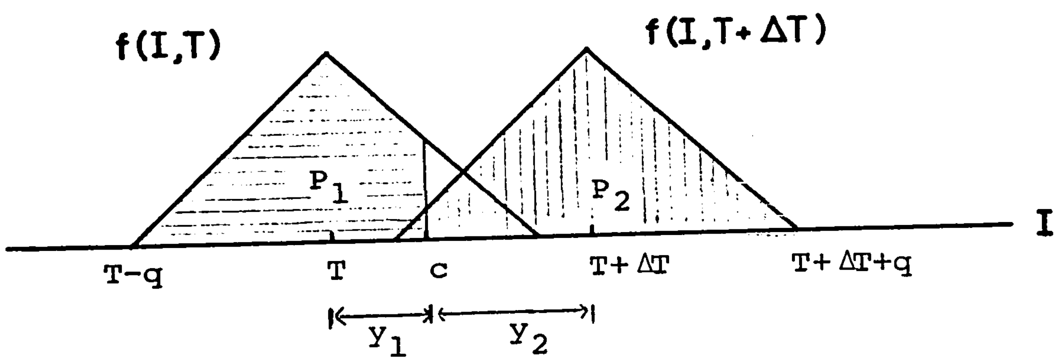
* This is completely analogous to how an estimate of d' is obtained when the distributions $g_i(\Delta I)$ are normal, as is the case in Creelman's model. The d' measure expresses the distance between the means in "standard deviation units", while the $d_{2,q}$ measure expresses the distance in "q-units".



(a) Decision strategy I: probability density functions of the random variable ΔI, conditional on $S_1=(T_v,T_s)$ or $S_2=(T_s,T_v)$ occurring. If $\Delta I>c$, the second interval is chosen as longer.

$$d_{q,2}= 2.\Delta T/q$$
$$\hat{d}_{q,2}= z_1 + z_2$$



(b) Decision strategy II: Probability density functions of the random variable I, conditional on the interval T or T+ΔT occurring. If $I>c$, the interval is "long".

$$d_{q,1} = \Delta T/q$$
$$\hat{d}_{q,1} = y_1 + y_2$$

FIGURE 3: PROBABILITY DENSITY FUNCTIONS FOR THE QUANTAL ONSET-OFFSET MODEL.

All distances are expressed in units of q.

The model predicts that the relation between the sensitivity measure $\hat{d}_{q,2}$ and ΔT should be a linear, zero intercept function, and the slope of this function gives an estimate of q. Moreover, note that $d_{q,2}$ does not depend on T. Hence changing the base duration T should not change performance in discriminating between T and $T+\Delta T$, when the performance measure is $d_{q,2}$.

Decision strategy II: a multiple decision strategy

A somewhat different type of decision strategy has been used in adapting the onset-offset model for a forced-choice task in successiveness discrimination (Kristofferson & Allan, 1971). It is assumed that a decision is made after each interval has been presented, this decision being that the interval is either "long" (L) or "short" (S) as compared to some criterion c. The choice of the longer interval is then made on the basis of one of four possible outcomes for the coding of the pair of intervals. If either (L,S) or (S,L) result, there is no uncertainty as to which interval to choose as the longer one in the pair. However, if the outcome is either (L,L) or (S,S), a guess has to be made. It can be shown (see Appendix IIb) that when S_1 and S_2 occur equally often,

$$P(C) = \frac{1}{2}(P_1 + P_2) \qquad (1\text{-}8)$$

where P_1 is the area under the distribution f(I,T) below c, and P_2 is the area under the distribution f(I,T+ΔT) above c (See figure 3b). An estimate of the quantities P_1 and P_2 are directly available in a single stimulus task (SS) where the O is presented with only one interval on each trial and is asked to identify it as either long or short. In the SS case, $\hat{P}_1$ is $\hat{P}$("short" / T) and $\hat{P}_2$ is $\hat{P}$("long" / T+ΔT). The relation between ΔT and $d_{q,1}$, the distance between the means of the distributions f(I,T) and f(I,T+ΔT) in units of q is

$$d_{q,1} = \Delta T/q \quad .$$

Again, note that the sensitivity measure $d_{q,1}$ is independent of the base duration T, and depends only on ΔT. P(C) depends on both ΔT and c, and so may change as T changes. An estimate $\hat{d}_{q,1}$ is easily obtained from $\hat{P}_1$ and $\hat{P}_2$ and a table of areas under the triangular distribution, or alternatively from the expressions

$$\begin{aligned} y_1 &= 1 - \sqrt{2 - 2\hat{P}_1} \\ y_2 &= 1 - \sqrt{2 - 2\hat{P}_2} \qquad (1\text{-}9) \\ \hat{d}_{q,1} &= y_1 + y_2 \quad . \end{aligned}$$

In a FC task, there is no simple correspondence between P_1 and P(1 / S_1). Nevertheless, the psychometric function is still predicted by equation (1-8), for T remaining constant. Moreover, with this decision strategy for the FC case, the predicted P(C) is the same as the predicted P(C) for the SS task, provided the criterion c is the same in the two situations. Since the derivation of the relation shown in (1-8) does not involve any specific probability density function for I, this prediction of no difference between P(C) from the SS and FC tasks would also be made by Creelman's model, if the decision strategy were as outlined above.

1.5 Varying the energy of the markers defining brief time intervals

In this section we briefly consider the time-intensity reciprocity obtained in many tasks involving stimulus durations less than certain critical values. This raises the question as to whether the coding of a very brief time interval might be based on the transformation of information along stimulus dimensions other than its duration. We then review the results of several duration discrimination studies in which the duration and/or the intensity of the stimuli defining the intervals were varied. Some of this evidence supports the conclusion that the psychological duration of even a very brief interval does not depend on intensity or total energy.

Any stimulus X can be defined by values along a number of dimensions

$$X = (x_1, x_2, x_3, \ldots, x_i) \quad .$$

For example, duration, frequency, and amplitude are a set of parameters needed to specify a simple auditory stimulus. Analogously, the psychological or sensory effect of this input might be specified by values along a set of psychological dimensions.

$$S = (s_1, s_2, \ldots, s_n) \quad ,$$

although these almost certainly do not correspond directly to the dimensions specifying the stimulus. That is, the value s_j along a psychological dimension may be a function of several of the stimulus dimensions x_k.

Values along many (if not all) sensory dimensions depend on the duration of the stimulus. There is abundant evidence that variations in stimulus duration have considerable influence on detection and discrimination performance. A time-intensity reciprocity holds for visual and auditory thresholds, provided the stimulus duration is less than some critical value (Bartlett, 1962; Green et al., 1957). For auditory thresholds, the critical duration depends on the frequency of the tone used, and can vary anywhere from 30 to 70 ms for high frequencies, to 125 to 175 ms for low frequencies (Watson, 1969). Amplitude and frequency discrimination of suprathreshold stimuli depend on the duration of the stimulus; the frequency or intensity difference required for $\hat{P}(C) = .75$ reaches an asymptotic value with some critical stimulus duration which is frequency dependent, being 100 ms or more for low frequency (250 cps) tones, but 25 ms or less for high frequency (4000 cps) tones (Henning, 1970). When intensity is kept constant, judgments of brightness, loudness or vibrotactile intensity increase with increasing stimulus duration up to some critical value beyond which these judgments are independent of duration (Stevens & Hall, 1966; Berglund et al., 1967). Using magnitude estimation procedures, Stevens and Hall found that the critical duration in brightness judgments depended on the intensity, varying from 5 ms for high intensities to 150 ms for low intensities. The critical duration for loudness judgments was about 150 ms, no matter what the intensity level. However, Small, Brand and Cox (1962) found that in a loudness matching paradim, the critical durations for apparent loudness were intensity dependent, varying from 15 to 50 ms. Critical durations seem to depend on many factors; modality, task, response measure and other stimulus parameters (Norman & Kahneman, 1968; Grossberg, 1968).

The question could now be asked whether psychological duration, when it is of the order of magnitude of the "critical durations" referred to above, is a function of stimulus parameters other than the time information x_t defined by the stimulus. In view of the time-intensity reciprocity in vision and audition, it is of particular interest to determine the effect on discrimination of brief time intervals, of manipulating intensity and total energy.

Zacks (1970) has reported a study indicating that even though a time-intensity tradeoff holds in terms of detectability of brief flashes of light, this does not necessarily imply that the time or intensity information become individually unavailable. A 4 ms flash was shown to be as (imperfectly) detectable as an 81 ms flash of the same total energy; the increase in detectability appeared to be the same for both, as total energy was increased above threshold level. However, even for these energy levels just above threshold, the short flash could be discriminated (imperfectly) from the longer one. On the other hand, although the task was defined in terms of duration, no conclusions can be drawn about what dimension was used in making the discrimination since the flashes differed in both intensity and duration.

Nilsson (1969) obtained difference thresholds for empty intervals bounded by 1 ms light flashes, the standard T ranging from 0 to 75 ms. These are intervals within which energy summation occurs for threshold intensity flashes (Clark, 1958), and both summation and inhibition has been observed at suprathreshold intensities (Ikeda , 1965). In Nilsson's study, the observers had a 3-alternative forced choice task, to select the interval of magnitude $T+\Delta T$ which was different from the two other intervals of magnitude T. The luminance values used were 50, 200 and 2000 ml; however, the duration difference thresholds were not significantly affected by the luminance differences, and an analysis of variance indicated no statistically significant interaction between luminance and base duration.

There is other evidence suggesting that the processes involved in judging the duration of brief light flashes do not use the total energy contained in the flash. Allan et al. (1971) report that changing the luminance difference between two flashes, which were to be discriminated on the basis of duration, did not affect the level of performance. Observers were to discriminate between two light flashes S_0 = 100 ms and S_1 = 120 ms, and were not informed about a luminance difference; S_0 was fixed at 15 ft-l while that of S_1 could be either 15, 13 or 11 ft-l. The discriminability of the 20 ms duration difference was the same for the various luminance differences. On the other hand, performance did vary with the size of these luminance differences when Os were asked to discriminate on the basis of luminance and ΔT was reduced to zero.

Creelman (1961) examined the effect of signal voltage on the discrimination of a pair of filled auditory intervals, of duration 100 and 130 ms. The signal voltages were chosen to cover a wide range of detectability of the duration difference. Performance improved rapidly with signal voltage at low signal to noise ratios. At the higher voltage levels (above .025 volts) there seemed to be a levelling off of performance, but this asymptote was not completely defined - it is only suggested. The interpretation of these results was that the effect of increased intensity is to reduce uncertainty in the onset and offset of the intervals to be measured; at the low intensities there may be difficulty in detecting the presence of the signal against the noise background. In another experiment, he studied duration discrimination as a function of base duration, at two intensity levels. ΔT was fixed at 40 ms. With the signal intensities at .010 v and .042 v, for the 4 observers, there appeared to be an interaction between base duration and intensity in their effects on duration discrimination. That is, intensity differences resulted in a larger difference in the sensitivity measure d' at the smaller base duration (80 ms) than at the larger base duration (320 ms), and this difference appeared to decrease monotonically as base duration increased. However, if we translate his d' values back into $\hat{P}(C)$ values (by using Table II in the appendix of Swets (1964)) we find that the intensity difference results in about the same difference in $\hat{P}(C)$ at T = 320 as it does at T = 80 for 3 of 4 Os, and is of the order of about .15. This is true as well for the d' values from a replication of the experiment using 3 different Os, and a slightly different set of base durations. At T = 50, the difference between $\hat{P}(C)$ at the two intensity levels is about the same as the difference at T = 400 ms. Hence if $\hat{P}(C)$ were used as the dependent variable, one might have to conclude that there is no interaction between the base duration and intensity.

In the discrimination of even shorter empty intervals bounded by brief auditory pulses (2000 cps tones), it has been found that changing the duration of the pulses has no effect (Abel, 1970). The first (or second) pulse defining the interval was kept fixed at 10 ms, while the second (first) was varied from 4 to 16 ms. The intervals to be discriminated were 25 and 30 ms for 3 Os, and 15 and 20 ms for a fourth. The level of performance did not change systematically as the duration of either pulse was varied.

In another study, Abel (1972) investigated the effect of varying the duration and the intensity of noise burst markers bounding empty intervals ranging in base duration from .1 to 640 msec. Three conditions were run successively, which allowed for comparison of performance at two intensity levels and two values of the total energy in the markers; the parameters of the noise bursts were (1) 10 ms, 85 db; (2) 300 ms, 70 db and (3) 10 ms, 70 db. The dependent variable was that value of ΔT necessary for $\hat{P}(C) = .75$ in a two alternative forced choice task. It was found that $\Delta T_{.75}$ was consistently less for the 85 db markers, over the entire range. On the other hand, the $\Delta T_{.75}$ vs T functions for the equal intensity conditions (i.e., conditions (2) and (3)) overlap each other, when plotted on log-log scales. The intensity of the markers seemed to be a more important stimulus parameter for the discrimination than was their duration, for empty intervals of less than 160 ms. But it should be noted that using a log-log plot obscures the fact that the difference in $\Delta T_{.75}$ at the two intensities is 6 ms at T = 40 and 5 ms at T = 160, while it is 10 ms at T = 80. Hence it does not appear that the difference in absolute values of $\Delta T_{.75}$ is a monotonic function of base duration over the range 40-160 ms. This point becomes even more interesting in the light of the results of a later experiment (Abel, 1972b) in which she investigated the discrimination of filled auditory intervals: $\Delta T_{.75}$ was independent of the amplitude of the noise bursts. For filled intervals, the $\Delta T_{.75}$ vs T functions overlap, regardless of whether the noise burst is low or high pass filtered, its intensity is reduced, or whether a sinusoid replaces the noise burst.

The implication of the evidence summarized above seems to be that variations along stimulus dimensions which depend on the amplitude and the total energy of the signals defining the time interval are relatively unimportant for duration discrimination, and that this is true even for very brief intervals, as long as the onsets and offsets of the intervals are clearly defined. This conclusion is suggested for both filled and empty intervals, and in both the visual and auditory modalities. If this result were found as well with empty intervals bounded by brief auditory signals, we would have additional support for the approach involved in the theoretical models already presented. This approach is based on the assumption of some central mechanism coding the time information in a stimulus pattern - a mechanism which does not use information from the sensory events defining an interval, other than time information.

1.6 Are both intervals used in the FC task?

In deriving predictions about performance in the 2AFC task from the models in section 1.4, it is implicitly assumed that the O always analyzes both intervals which are presented on every trial. If this were not actually the case, then the inadequacy of a particular model in describing the data from a FC task might be due to incorrect assumptions about how the information in a pair of intervals is used to arrive at a correct response, even though the assumption about how the duration of a single interval is coded may be valid. Hence if we are going to test models with data from FC tasks, it would be highly desirable to select certain stimulus alternatives in such a way that we can know whether the O is using both intervals.

The possibility that both intervals might not be used on each and every trial was raised by McKee et al. (1970) in discussing the results of a FC duration discrimination study in which they varied the interstimulus interval (ISI). The time intervals were defined by brief visual dark flashes (the time between the offset and onset of a light). They found that varying the ISI from 1/2 to 2 sec. had no effect on performance in discriminating duration differences of 10 or 30 msec. The base duration was 50 msec. This result is in marked contrast to the effect of ISI in certain

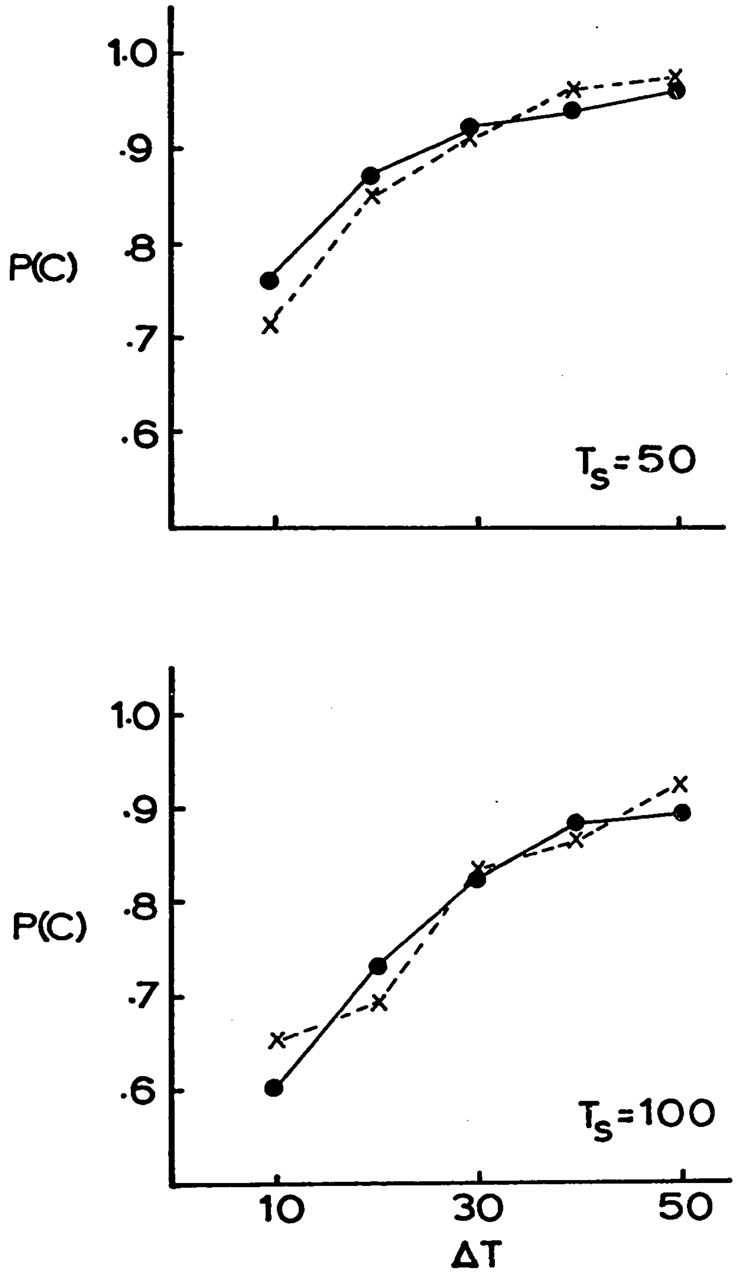


FIGURE 4: FC and SS psychometric functions, for visual dark flashes. Data averaged over 3 Os. Base durations 50 and 100 msec.

FC - [•——•]
SS - [×- - -×]

visual and auditory discriminations reported by Kinchla and Smyzer (1967). The ISI variable may have no effect, according to McKee et al., because the response may be based on the time information in only one interval; i.e., one interval is consistently ignored. The O would be reducing the FC task to a SS task if he used only the first, or only the second interval throughout the experiment.

The close similarity between the averaged FC and SS psychometric functions obtained in another study by McKee et al. would also be expected if it were the case that either the first or the second interval is consistently ignored in the FC task. In obtaining these functions, T was always the shortest interval within a session. One group of Os was given the FC task, and another group the SS task. In both cases, only one value of T and ΔT were used within any one session, selected from 2 values of T and 5 of ΔT. The averaged data is shown in Figure 4. At T=50 msec. the FC procedure yields a psychometric function slightly above that from the SS procedure, but the maximum difference is only .06 (at ΔT=10 msec.), and this difference progressively decreases as ΔT increases. On the other hand, at T=100 msec., the functions from the two conditions overlap along the entire range of ΔT.

Another interpretation of both the similarity of FC and SS performance, and the insensitivity of performance to changes in ISI as seen by McKee et al. can be based on the assumption that in the FC task a decision is made after each interval is presented as to whether it is "long" or "short"; this is the decision strategy assumed in the second version of the onset-offset delay model, which predicts that P(C) from the FC and SS tasks can be the same*. It could also be that the decision about the first interval is not sensitive to the delay between the two intervals.

Even though we can propose an alternate interpretation of the results which originally raised the question as to whether both intervals are used in the FC task, we are still faced with the important problem of establishing that the O does use both intervals on each trial.

How might we arrange the stimulus alternatives so that we can know whether the O is consistently ignoring the first or the second interval? Consider the following set of stimulus alternatives, occurring with equal frequency:

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* If we assume that in the FC situation, two SS responses are made and these are combined to give the decision as to which is the longer interval, then it is predicted that P(C) will be the same for SS and FC provided that the same criterion is used in the two tasks when deciding whether a given interval is short or long. That is, in the FC task, the SS response "short" to an interval, when it is short, is the same as it would be if that interval were presented alone in a SS task. This prediction holds regardless of the response biases in the FC task.

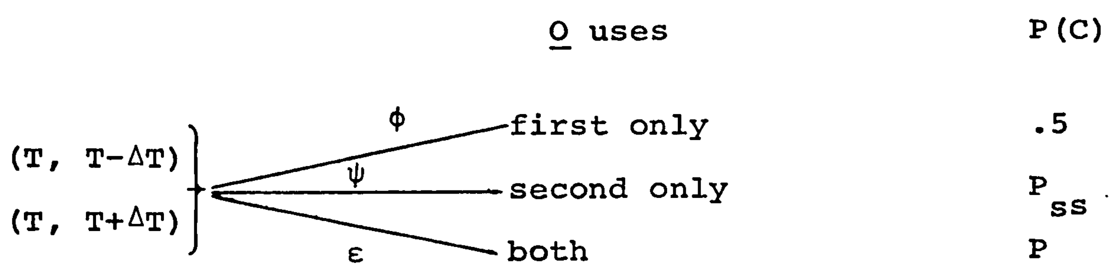
stimulus	correct response
$S_1' = (T, T-\Delta T)$	1
$S_2' = (T-\Delta T, T)$	2
$S_1 = (T+\Delta T, T)$	1
$S_2 = (T, T+\Delta T)$	2

(1-10)

When the stimulus alternatives are only S_1 and S_2, T is always identifiable as the "short" interval. Within this larger set, however, the identity of T as "short" or "long" now depends on whether $T+\Delta T$ or $T-\Delta T$ occurs with it. If the decision as to which is the longest interval in the pattern were being based only on the first interval, $\hat{P}(C)$ for patterns S_1' and S_2 where T occurs first could only be approximately .5. Similarly, if the decision were being based only on the second interval, $\hat{P}(C)$ for the S_1 and S_2' patterns would be at chance. Hence if only the first or only the second interval were being used, this would be immediately apparent from the data.

What pattern of results might we expect to see if on some proportion ϕ of the trials only the first interval were used, on some proportion ψ only the second interval were used, and on all the remaining trials both intervals were used? The tree diagram below represents the outcomes of this "looking strategy" when it is applied to those trials where

T occurs first. P is the probability of a correct response on those trials where T occurs first and <u>both</u> intervals are used in making the decision. When the O uses the first interval only, and T occurs first, the probability of being correct in choosing the longer interval of the pair will be at chance level since T can be either longer or shorter than the interval which follows it. On those trials where only the second interval is used, and either T-ΔT or T+ΔT occur second, it is assumed that the probability of being correct in choosing the longer interval will be P_{ss}, the proportion of



correct responses in identifying T-ΔT as "short" and T+ΔT as "long" when given a SS task involving this pair of intervals as the stimulus alternatives. From the above diagram we can write

$$P(C|T\ \text{first}) = \phi(.5) + \psi P_{ss} + \varepsilon P \quad . \qquad (1\text{-}11)$$

From a similar diagram applying to the trials where T occurs second, we can write

$$(P|T \text{ second}) = (P_{ss}) + (.5) + P' \qquad (1\text{-}12)$$

provided the "looking strategy" is independent of whether T occurs first or second. (This assumption will be discussed later.) P' is the probability of a correct response on those trials where T occurs second, and both intervals are used. Subtracting (1-12) from (1-11), we obtain

$$\Delta P = P(C|T \text{ first}) - P(C|T \text{ second})$$

$$= (\psi-\phi)(P_{ss} - .5) + \varepsilon(P-P') \qquad (1\text{-}13)$$

and combining (1-11) and (1-12), we obtain

$$P(C|T) = P(C|T \text{ first})\cdot P(T \text{ first})$$

$$+ P(C|T \text{ second})\cdot P(T \text{ second})$$

$$= \frac{1}{2}[(\phi+\psi)(P_{ss} + .5) + \varepsilon(P+P')]$$

$$= \frac{1}{2}[(1-\varepsilon)(P_{ss} + .5) + \varepsilon(P+P')] \qquad (1\text{-}14)$$

when T occurs first or second equally often.

When we have ϕ or $\psi = 1$, we have the situation in which the O is consistently ignoring the first or the second interval throughout the experiment. As has already been pointed out, whether or not this is the case is easily determined by seeing whether $\hat{P}(C|T \text{ first})$ or $\hat{P}(C|T \text{ second})$ is near .5. If the data indicate that both $\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$ are above chance, it may still be true that $\varepsilon=0$, while now $\phi+\psi = 1$; on some trials only the first interval is used, and on all the remaining trials only the second is used. The discrepancy between $\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$ could take on some large value depending on the discrepancy between ϕ and ψ. If the decision as to which is the longer interval relies mainly on the first interval without also consistently using the second ($\phi>>\psi$), $\hat{P}(C|T \text{ first})$ should be much smaller than $\hat{P}(C|T \text{ second})$ If the decision were based mainly on the second interval ($\psi>>\phi$), the reverse inequality should hold.

On the other hand, equation (1-13) allows for non-zero values of ΔP even when $\phi = \psi = 0$ and $\varepsilon=1$. There is no a priori reason to expect that the term (P-P') should be zero. Hence the quantity ΔP in itself gives us no information about the discrepancy between ϕ and ψ, or whether $\varepsilon=1$.

From equation (1-14) we can easily obtain a lower bound for ε even though we cannot calculate its actual value. (P and P' are unknown.) Since each of P_{ss}, P and P' is less than or equal to 1, we can rewrite (1-14) as

$$P(C|T) \le \frac{1}{2}\,[(1-\varepsilon)(1 + .5) + \varepsilon(2)]$$

which can be solved for ε:

$$\varepsilon \ge \frac{P(C|T) - .75}{.25} \quad . \qquad (1\text{-}15)$$

Whenever $\hat{P}(C|T)$ is greater than .75, for at least one set of alternatives as in (1-10), we know that $\varepsilon \neq 0$. If we wish to establish that ε is 1, the stimulus alternatives should be chosen so as to maximize $\hat{P}(C|T)$ for at least one value of T; we should include a value of ΔT large enough so that $\hat{P}(C|T)$ is ≈ 1.

It is important to keep in mind that the above analysis is concerned with the effects of strategies or factors determining the selection of which interval is coded, which are not influenced by the outcome of the processing of the first interval. That is, it is implicitly assumed that whether or not the second interval is coded does not depend on any decision about the first interval. Alternative models could be formulated, using the assumption that one of three decisions is made after each interval: "short", "long" or "uncertain". When the decision after the first interval is "uncertain" the information from the second interval would be needed. Thus both intervals might be used much more often when T is first, since an uncertain decision could occur much more often after T has been presented than after T-ΔT or T+ΔT. This type of model for FC data could be elaborated once we have a model which can account adequately for SS data from duration discrimination situations involving empty auditory intervals of the same order of magnitude as in the FC situation.

1.7 Summary of the experiments

The question with which we are concerned is, how does the observer make a discrimination between two very brief time intervals when these are in the range of approximately 50 to 300 msec. The intervals are empty; that is, they are defined by the temporal separation between two very brief auditory signals, so that the energy input can be kept constant while the time information is varied. Hence a change in duration is not associated with a change in total energy of the pattern defining this duration.

There are two approaches in dealing with this question. One is to establish what stimulus parameters are important - or not important-in making the discrimination, and from this information to make inferences as to the nature of the processes involved in the discrimination. Another approach is to test quantitative models making specific assumptions about the processes involved, and to determine to what extent these models can describe experimental results. Both approaches are used in this thesis. The experiments described here were not designed to test any specific model, although the paradigm did originate in experimental tests (described in Carbotte and Kristofferson, 1971) of the quantal counting model.

In the four experiments described in chapter II, the problem as defined to the observer O is the same throughout.

On each of a large number of trials, the O is presented with a pair of empty intervals of duration T and T+ΔT. The O's task is to indicate whether the longer interval occurred as first or second in the pair. In the first experiment, the intensity of the pulses was varied over a wide range to determine the effect on performance when there is a large change in the energy of the signals defining the onset and offset of each interval. The second experiment was done as a preliminary to the third, as a check on whether the O consistently ignores the first or the second interval when he has a two alternative forced choice task. Although we can conclude only that the O uses both intervals on some non-zero proportion of the trials, the results are more useful in other respects, since the base duration T was varied from 50 to 275 msec. In the third experiment, the separation between the two intervals was varied systematically. ΔT was constant for each O, while T varied from 115 to 250 msec. For T=150 and T=250, a sample of discrimination performance in a single stimulus task was also obtained. In experiment 4, psychometric functions were obtained for T=100 and T=200 msec.

All four experiments provide evidence on how the ability to discriminate between T and T+ΔT changes as T increases. Any adequate model for duration discrimination of very short intervals will have to account for the nature of these changes; in chapter III the data from these four experiments are analyzed in detail with respect to the models presented in 1.4.

II. EXPERIMENTS

2.1 Apparatus and general procedure

Within each experiment, at least one O was experimentally naive, while the others had participated in one or more previous duration discrimination experiments involving the same type of stimuli. The Os were all student volunteers, paid $2.00 per session.

An experimental session lasted for approximately an hour, and generally there was only one session per day per O. Each session was divided into either 3 or 4 blocks of trials, with rest periods of several minutes between the blocks. Os were allowed to take longer breaks whenever they requested. A block lasted for 12 to 15 minutes, depending on the stimulus parameters and on the number of trials per block.

Os were seated in a sound attenuated cubicle, isolated from the experimental control room. An intercom was available to the O throughout the session, for communication between the cubicle and the control room.

The general sequence of events was the same on every trial of all the experiments, except for the third experiment in which a number of single stimulus sessions were run. All events were controlled by a PDP-8S computer interfaced to an electronic switch (Model 929E, Grason-Stadler, West Concord, Mass.). A 250 msec. visual warning signal was followed 2 seconds later by a pattern of 4 very brief auditory pulses. The time between the onsets of the first and second pulses defined a time interval labeled as T_1 while the time between the onsets of the third and fourth pulses was labeled as T_2. The interval between the onsets of pulses 2 and 3 was designated as the ISI, or interstimulus interval. The O was instructed to indicate whether T_1 or T_2 was the longer interval by pressing one of two microswitches interfaced with the computer. He was told that the longer interval was as likely to be first as second. The time for a response was limited to 4 seconds, and if a correct response was made in that time, feedback was given in the form of two 125 msec. light flashes. The next trial began 1.5 seconds later. At the end of each session the O was given a summary of his performance.

The electronic switch gated a sine wave at zero crossings in its cycle for presentation of square wave pulses. The sine wave (of frequency 2000 ± 10 Hz) was delivered to the switch by an audio oscillator (Model 201C, Hewlett-Packard, Toronto, Ontario). The rise-decay times for the pulses were set at 1 msec. The amplitude in all the experiments except the first was set at .4 rms volts, as measured and continuously monitored at the switch by a voltmeter; this amplitude gave a clearly audible signal. The intensities of the four pulses defining the two time intervals were identical. The auditory signals were presented to the O binaurally over earphones.

All durations were controlled by the computer. The duration of each stimulus pulse was programmed for 10 msec. A check on the durations T_1 and T_2, and on the durations of the auditory stimuli, was made periodically by using an electronic counter.

2.2 Experiment 1: Varying the intensity of the boundaries

Most of the evidence reviewed in section 1.5 supported the conclusion that the total energy in the stimulus pattern defining a time interval is not an important parameter in duration discrimination, even for very short intervals of the order of 50 msec. or less (Allan et al., 1971; Nilsson, 1969; Abel, 1970; Abel, 1972b). However, from other experiments on duration discrimination in which varying the intensity did significantly influence the dependent variable, it appeared that the effects decreased as the base duration increased (Creelman, 1962; Abel, 1972a). When time intervals are defined by auditory stimuli, there may be some critical value of T_s within the range 50-300 msec., below which the intensity of the stimuli can provide an important cue for discriminating between the two patterns defining the intervals T_s and $T_s+\Delta T$. If this is the case, then when performance is plotted as a function of base duration T_s with intensity as a parameter, the separation between these curves at various intensities should decrease as T_s increases, up to the critical value of T_s beyond which the separation between these curves is independent of base duration. Hence the main purpose of this experiment was to determine whether for a given fixed value of ΔT, the difference in $\hat{P}(C)$ when using high as compared to moderate or low intensity markers would diminish as the base duration T_s is increased.

Procedure

In version A, each session consisted of three blocks of 90 trials. Within each block, the intensity of the signals bounding the intervals was kept constant, but this intensity was changed from block to block. The intensity readings as measured at the switch were .08, .3 and 6 rms. volts, and the corresponding sound pressure level of continuous tones as measured at the earphones were 61, 72 and 98 db. With no signal being presented the sound level reading was 54 db., because of ambient noise from a fan ventilating the booth. These levels were obtained with a sound level meter (model 1551-C, General Radio) calibrated with a standard source (sound level calibrator, type 1562A, General Radio).

Three values of base duration (50, 150 and 250 msec.) and one value of ΔT (10 msec.) were used. One value of T_s was used over three consecutive sessions, then a second, and finally the third. The order of presentation of T_s and the three intensity levels were counterbalanced among three Os.

Two of the Os had participated in a previous experiment, but the third (JT) was naive; V and RM had taken part in at least 32 sessions in experiment 4. All three were given two practice sessions in which it was established that ΔT = 10 msec. would give a value for $\hat{P}(C)$ of at least .70 at each of the three base durations, at the medium intensity. The order of presentation of T_s for each O was:

V : 50, 150, 250
RM: 150, 250, 50
JT: 250, 50, 150 .

Version B was essentially a replication of version A, with all three values of T_s randomly intermixed within all blocks. Hence from trial to trial, the observers were uncertain as to the order of magnitude of the intervals which would occur. Only the high and low intensity levels were used; these alternated from block to block for 6 sessions for V, and for 10 sessions for JT and RM.

Results

Tables 2 and 3 give the data from versions A and B respectively. Figure 5 shows the main effect of changing the intensity of the boundaries on the proportion of correct responses. Although there is an overall improvement in performance as intensity increases in both versions, the effect of an increase in intensity of almost 40 db. is an increase of .04 in $P(C)_{av}$. In Figure 5 the data has been averaged over the 3 values of T_s and over the three Os.

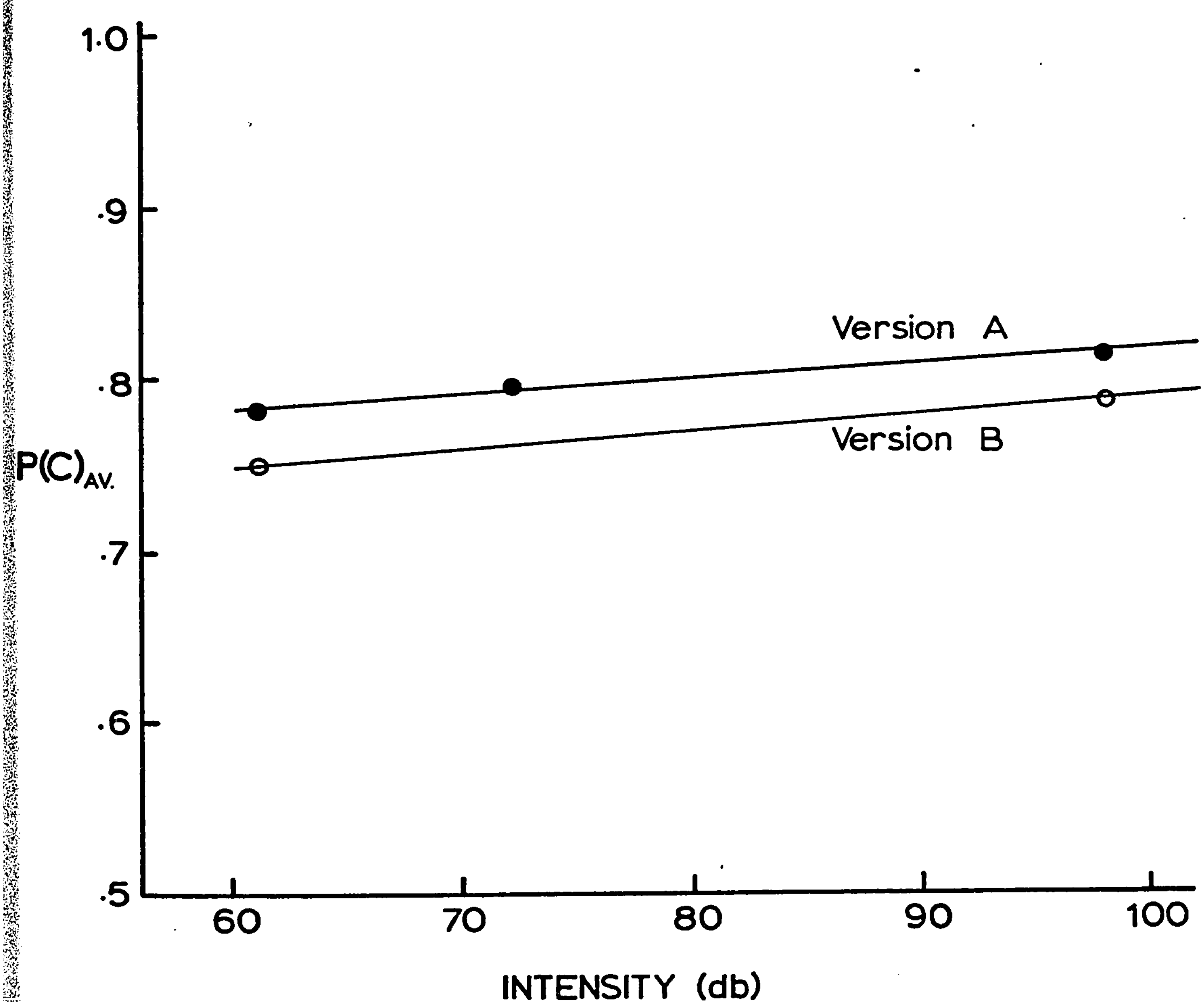


FIGURE 5: Overall effect of increasing the intensity of the auditory stimuli, on the proportion of correct responses. Data averaged over 3 Os and 3 base durations.

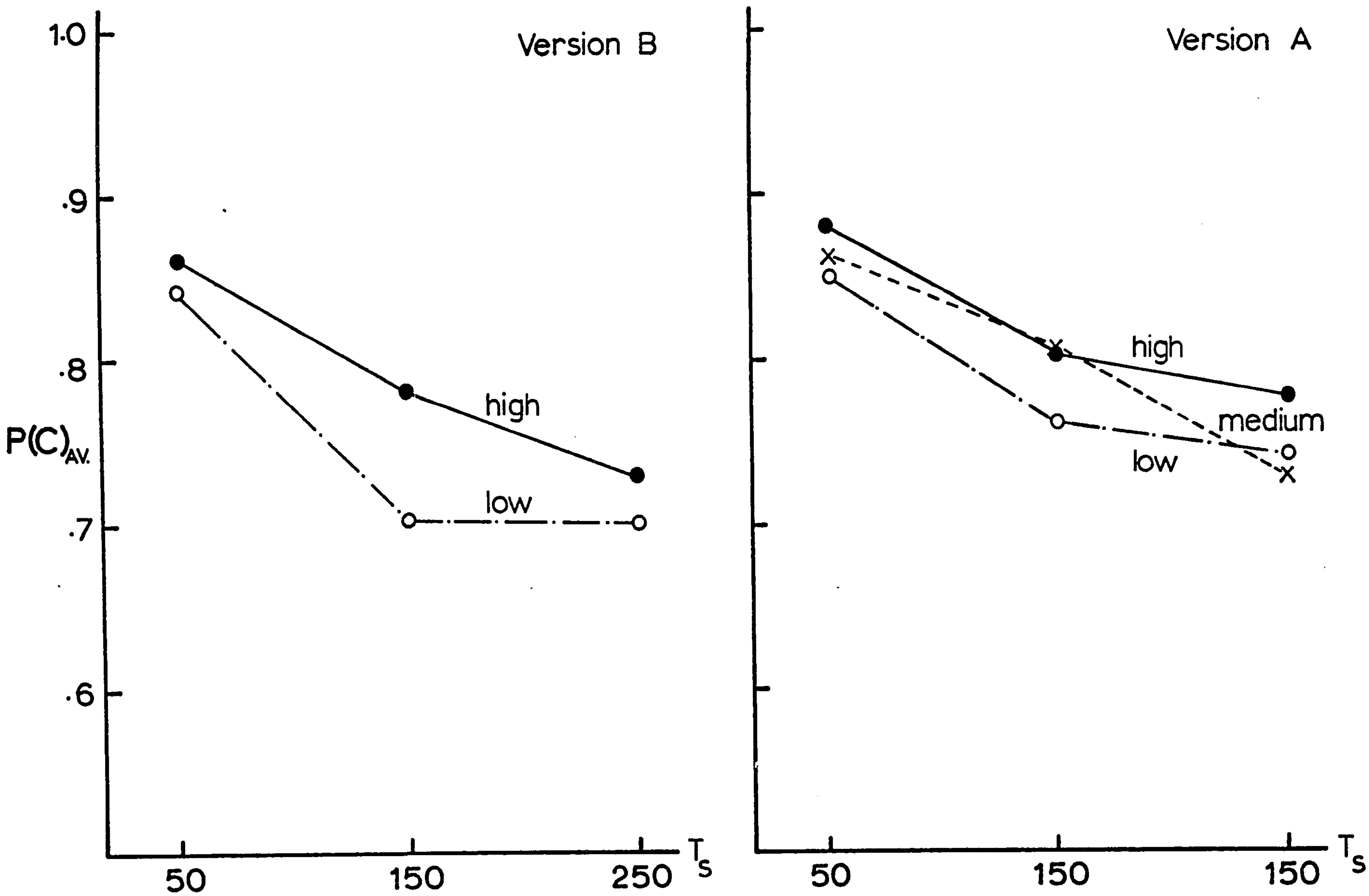


FIGURE 6: $\hat{P}(C)_{av}$ as a function of base duration, averaged over the 3 Qs. Boundary intensity is the parameter. 810 trials per point in version A; 1150 trials per point in version B.

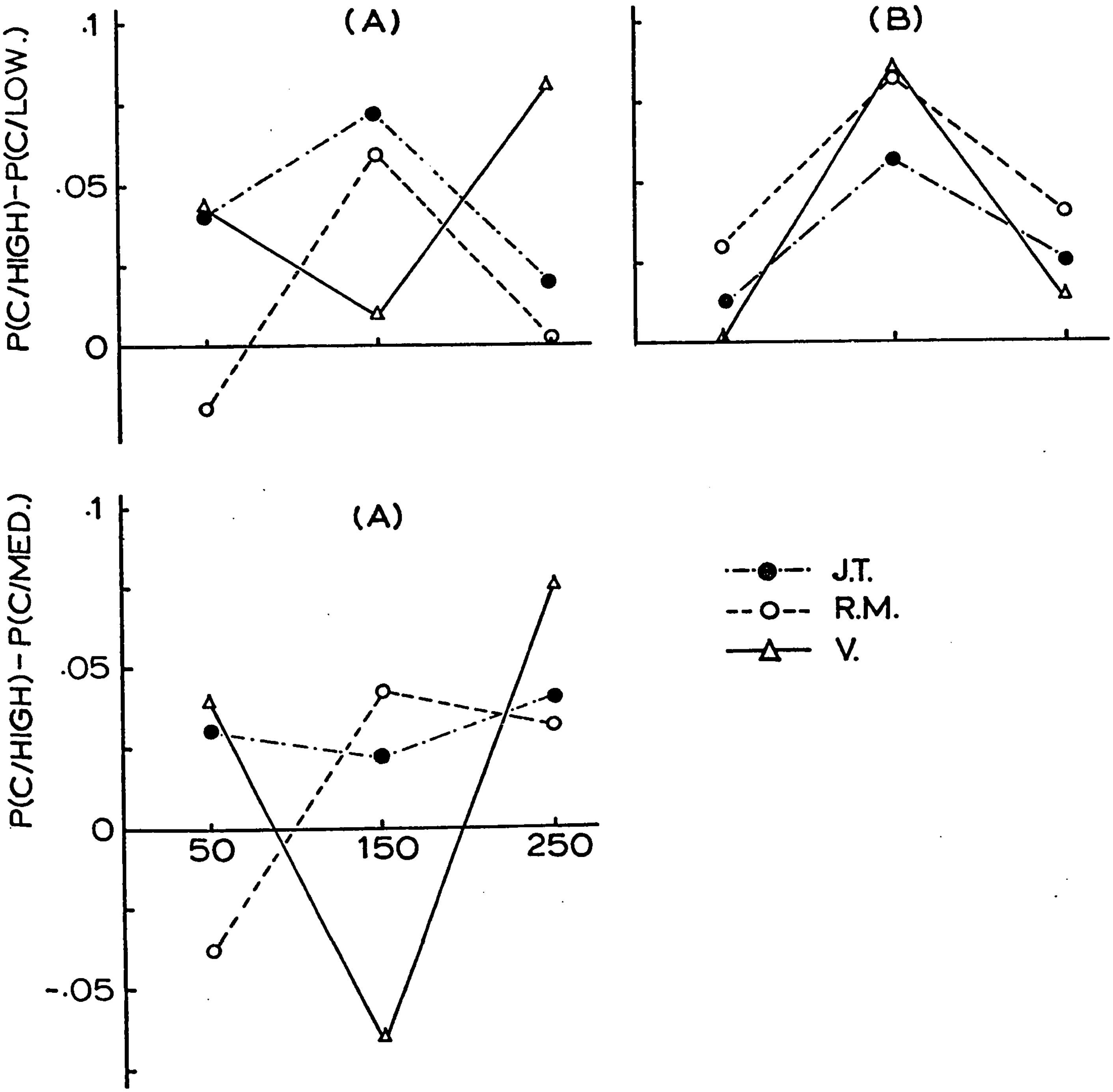


FIGURE 7: The change in $\hat{P}(C)$ at each base duration, when there is a change in the intensity of the signals bounding the intervals. Individual data.

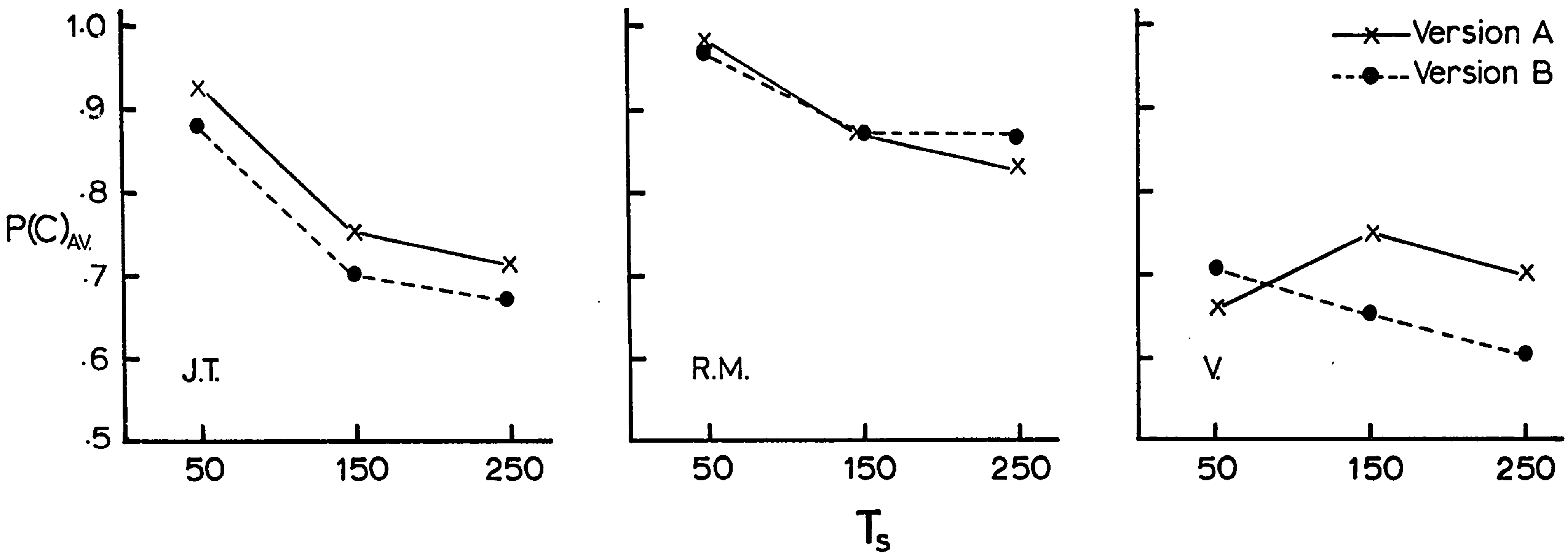


FIGURE 8: $\hat{P}(C)_{av}$ as a function of base duration, for each O. Data averaged over all intensity conditions, within each version.

Table 2

Summary of results for each intensity condition in experiment 1(A)

Each P(C) is based on approximately 270 trials. (*) indicates where $\hat{P}(C/high)-\hat{P}(C/low)$ is significant. One base duration is used for 3 successive sessions.

		HIGH			MEDIUM			LOW		
Obs	T	$P(1/S_1)$	$P(2/S_2)$	P(C)	$P(1/S_1)$	$P(2/S_2)$	P(C)	$P(1/S_1)$	$P(2/S_2)$	P(C)
JT	50	.969	.933	.951	.948	.896	.922	.926	.896	.911
	150	.809	.754	.781	.772	.746	.759	.677	.741	.709
	250	.835	.625	.729	.687	.689	.688	.719	.701	.710
RM	50	.970	.953	.962	1.00	1.00	1.00	.978	.985	.982
	* 150	.954	.854	.904	.885	.838	.862	.850	.849	.845
	250	.865	.825	.845	.838	.788	.812	.831	.855	.843
V	50	.528	.852	.690	.532	.768	.650	.530	.763	.650
	150	.715	.746	.731	.711	.881	.786	.634	.807	.721
	* 250	.821	.676	.750	.674	.674	.674	.664	.672	.670

Table 3: Summary of results of experiment 1(B)

Two intensity levels; three base durations randomly intermixed within each session. N is the number of observations on which each P(C) is based. * indicates where the difference $P(C|high) - P(C|low)$ is significant.

		HIGH			LOW		
Obs	T	$P(1\|S_1)$	$P(2\|S_2)$	P(C)	$P(1\|S_1)$	$P(2\|S_2)$	P(C)
JT	50	.891	.879	.885	.893	.851	.872
(N = 440)	150	.832	.620	.726	.743	.596	.669
	250	.538	.829	.683	.480	.839	.659
RM	* 50	.961	1.00	.981	.931	.968	.950
(N = 420)	*150	.853	.966	.909	.794	.858	.826
	250	.874	.903	.889	.848	.845	.848
V	50	.752	.662	.706	.751	.675	.704
(N = 300)	*150	.533	.867	.700	.534	.693	.614
	250	.530	.689	.609	.586	.604	.595

In this experiment, interest lies not as much in the overall effect on $\hat{P}(C)$ of varying the intensity, as in whether performance with the short base duration is more sensitive to a change in intensity than performance with much larger base durations. Figure 6 shows $\hat{P}(C)_{av}$ as a function of base duration, with intensity as a parameter. The data is averaged over the 3 Os. In both versions, at each base duration, $\hat{P}(C)_{av}$ with the high intensity markers is greater than $\hat{P}(C)_{av}$ with the low intensity markers. However, this difference is not a decreasing function of base duration since it is the same at T=50 as at T=250 (approximately .02), and is twice as large at T=150.

Figure 7 shows the difference between $\hat{P}(C)$ at the high intensity and $\hat{P}(C)$ at the low intensity

$$\hat{P}(C|high) - \hat{P}(C|low)$$

plotted as a function of base duration for each observer. A larger difference at T=150 than at 50 or 250 is seen for all three Os in version B, and for two of the three Os in version A. On the other hand, we do not find a similar pattern in the plot of $[\hat{P}(C|high) - \hat{P}(C|medium)]$ from version A; here there is no tendency for the difference to be somewhat larger at T=150 than at T=50 or T=250.

Figure 8 shows the change in $\hat{P}(C)$ as base duration increases, for each observer; the data are averaged over all intensities. The monotonic decrease in $\hat{P}(C)$ as a function of base duration which is seen in Fig. 6 at each of the intensities, is obtained for 2 of the 3 Os in version A and is seen for all three Os in version B. For 2 Os, the change in $\hat{P}(C)$ as T_s increases from 150 to 250 is much smaller than the change when T_s increases from 50 to 150. At the low intensity, there is no change in $\hat{P}(C)$ when base duration increases from 150 to 250 (Figure 6). This result holds for each of the individual Os (Tables 2 and 3)

There is a close similarity in the results from versions A and B, both in the effect on $\hat{P}(C)$ of increasing the intensity of the stimuli bounding the intervals, and in the change in $\hat{P}(C)$ as base duration increases (see Figures 6, 7 and 8). For the 3 Os, when all three base durations are randomly intermixed, $\hat{P}(C)$ at each base duration is somewhat smaller than the corresponding values of $\hat{P}(C)$ obtained by using one base duration over three consecutive sessions. In Table 4 we compare the performance levels obtained from the two versions, at each base duration; the high and low intensity conditions are considered separately since they were run in separate blocks. For JT and RM, the effect of uncertainty as to the size of the intervals which will occur on a trial is approximately the same at all base durations, and is small compared to the effect on $\hat{P}(C)$ of increasing the base duration from 50 to 150 msec. (see Figure 8). For the third O (V) whose averaged performance as shown in Figure 8 is considerably influenced by this uncertainty, we find in

Table 4

Change in P(C) at each base duration, when 3 base durations occur per session (A) as compared to one base duration per session (B)

		Base Duration		
Intensity	Obs	50	150	250
	JT	.06	.05	.05
HIGH	RM	-.02	-.01	-.04
	V	.02	.03	.14
	JT	.04	.04	.05
LOW	RM	.03	.02	.01
	V	-.05	.11	.07

Table 4 that performance at only <u>one</u> base duration within each intensity condition is appreciably affected, not all three.

Discussion

The highest intensity level used in this experiment produced a very loud sound, but it was not uncomfortable, and did not cause startle. The lowest intensity level produced a very faint sound which was detectable on all trials, but could be missed if, for example, the O swallowed during its presentation. The intensity at which these 10 msec. pulses were less than fully detectable was less than .03 rms. volts. This was determined informally with the experimenter acting as an O for blocks of 70 trials. With a yes-no detection procedure (i.e., with the signal absent on half the trials) the signal was fully detectable at each of several intensities between .03 and .08.

To assess the magnitude of the range over which intensities were varied in this experiment, we note that Green and Luce (1971) varied the intensities of their auditory signals over a range of 40 db., and obtained a decrease in mean reaction time (RT) to the onsets of these signals from 2.9 to .26 sec., under conditions requiring a low false alarm rate. On the other hand, Murray (1970) varied intensity over a 60 db. range, but obtained a decrease in mean

RT of at most .1 sec.; the longest mean RT at the lowest intensity was about .5 sec. The magnitude of the overall effect of changing intensity on RT would seem to depend strongly on what the lowest level is, relative to threshold, rather than on the size of the range over which the intensity is varied. This point will be considered further when we discuss the effect of intensity on duration discrimination obtained by Creelman.

Our results indicate that although there is a consistent improvement in performance as marker intensity is increased over a fairly wide range, the change in $\hat{P}(C)$ is small. A binomial test was used to determine whether the differences between the proportions $\hat{P}(C|\text{high})$ and $\hat{P}(C|\text{low})$ at each base duration were large enough to be significant for individual Os (McNemar, 1962). At both T=50 and T=250, these differences failed to reach significance at the .05 level in 5 instances out of 6 (2 versions x 3 Os). At T=150, three of the six differences are significant at the .05 level. The asterisks in Tables 2 and 3 indicate which of the differences are significant. No conclusion is being drawn on the basis of the results of these tests as to whether varying the intensity has a significant effect on $\hat{P}(C)$, at any one of the 3 base durations; we are interested only in assessing the magnitude of these differences. When k independent tests are carried out on the same set of data with the significance level for each test set at $\alpha = .05$, the probability that one or more of these yield a spuriously significant result is $1 - (1-\alpha)^k \cong \alpha \cdot k$, for small α (Hays, 1963; p. 376). Hence some of the five differences which were significant according to the test used, may be significant by chance alone.

The effect of intensity obtained here is small compared to the effects obtained by Creelman in his third experiment where an increase in intensity of about 12 db. (corresponding to an increase in signal amplitude from .010 v. to .042 v.)* resulted in an increase in $\hat{P}(C)$ of approximately .15, with base durations ranging from 80 to 320 msec. However, the lower signal intensity (.010 v.) was at the same voltage level as the background noise added at the earphones. Hence, as Creelman suggests, there may well have been considerable difficulty in detecting the onsets and offsets of the signals in the noise; there may have been difficulty in detecting the presence of the signals on some trials. In his first experiment, a signal to noise ratio of about 10 db. was needed for asymptotic performance in discriminating between durations of 100 and 130

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* An increase in signal voltage from v_1 to v_2 corresponds to an increase in sound pressure level of $20 \log_{10}(v_2/v_1)$.

msec. Any large improvement in performance could reasonably be attributed to an increase in the number of trials on which the signals were detectable.

In this experiment, the low intensity was chosen so as to have signals which would be detected by the *O* on each presentation; it was intended to avoid problems with detectability of the boundaries of the intervals. Nevertheless, the possibility cannot be discounted that on some trials the *O* might have missed one or more of the boundaries of the two intervals, when the boundaries were faint. On the other hand, there is also some improvement in performance as the intensity is increased from a moderate (.3 v.) to a high (6 v.) level. In this case it is very much less likely that the performance difference can be attributed to less difficulty in perceiving the high intensity markers than the moderate ones.

An alternative interpretation of the small effect of intensity is that increasing the intensity of the brief signals bounding the external intervals decreases the variability in the latencies of the events bounding the internal intervals, resulting in a small increase in the detectability of a difference in duration between the two intervals. This interpretation seems plausible, when we take into account the finding that both the mean and the variance of simple reaction times to auditory stimuli is decreased as stimulus intensity is increased (Green and Luce, 1971; Murray, 1970).

Evidence from this experiment makes it seem quite unlikely that the discrimination between two brief intervals is based on a code which uses the energy in the stimulus pattern defining the time intervals. There is no a priori reason to expect that with a 250 msec. separation between two brief auditory pulses there is no longer any interaction between the sensory effects of these pulses. Plomp (1964) has found that it takes from 200 to 300 msec. for the sensation due to an auditory pulse to decay to its threshold value, and Massaro (1971) has found that a masking tone interferes with the identification of the pitch of a brief preceding test tone until the interval between the test and masking tones is of the order of 250 msec. On the other hand, if an interaction were being used as the basis for coding the time interval, it should be much less effective as a cue for duration discrimination when T=250 than when T=50 or 150. But we have found that with the low intensity boundaries there was no change in $\hat{P}(C)$ as the base duration increased from 150 to 250. Moreover, decreasing the intensity of the pulses should systematically diminish the discriminability of a difference in the temporal distance between the pulses, but we find no such systematic change at T=250; $\hat{P}(C)$ with the medium intensity markers is slightly less than with the low intensity markers. Finally, there is no indication that when the base duration is 50 msec., the code is intensity dependent; the effect on $\hat{P}(C)$ of changing the intensity of the boundaries is the same when T=50 as when T=250. Hence these results reinforce the view that duration discrimination is based on an internal time code which is independent of energy effects, except perhaps secondarily in that the variability of the latencies of the events bounding the internal intervals can be influenced by the intensity of the signals defining the external time intervals.

2.3 Experiment 2 : Are both intervals used?

The original intention of this experiment was simply to test the hypothesis that the O uses only one interval when he has a FC task. McKee et al. (1970) advanced this hypothesis in discussing their finding that varying the ISI in a FC duration discrimination task had no effect on performance; the time intervals to be discriminated were defined by brief dark flashes of 50 msec. base duration. If an attempt were made to replicate this result of McKee et al. with very brief empty auditory intervals, it would be highly desirable to arrange the stimulus alternatives so that the O must use the information from both intervals if performance is to be above chance level.

The following set of alternatives (already considered in 1.6) can provide information as to whether only the first or only the second interval is being used:

stimulus	correct response
$S_1' = [T, T-\Delta T]$	1
$S_2' = [T-\Delta T, T]$	2
$S_1 = [T+\Delta T, T]$	1
$S_2 = [T, T+\Delta T]$	2

(I)

When T occurs first (in the S_1' and S_2 patterns) the correct response can be either "1" or "2", since T can be either longer or shorter than the interval which follows it. Hence if only the first interval is being used throughout the experiment, performance with these two patterns should be at chance; i.e.,

$$\hat{P}(C|T \text{ first}) = \frac{1}{2}\,[\hat{P}(1|T, T-\Delta T) + \hat{P}(2|T, T+\Delta T)]$$

should be approximately .5, while

$$\hat{P}(C|T \text{ second}) = \frac{1}{2}\,[\hat{P}(1|T+\Delta T, T) + \hat{P}(2|T-\Delta T, T)]$$

should be at about the same level as SS performance in discriminating between $T+\Delta T$ and $T-\Delta T$. On the other hand, if only the second interval is being used throughout the experiment, $\hat{P}(C|T \text{ second})$ should be approximately .5.

The second aim of this experiment was to determine whether there is a difference between $\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$ which is independent of T. That is, if we randomly intermix two sets of alternatives as in (I), generated by using two values of T and one of ΔT, are non-zero values of

$$\Delta P = \hat{P}(C|T \text{ first}) - \hat{P}(C|T \text{ second})$$

seen at both values of T? If the first interval is used most of the time without consistently taking into account the information from the second interval, then $\hat{P}(C|T \text{ first})$ should be much smaller than $\hat{P}(C|T \text{ second})$, at both values of T. Similarly, if the second interval is used without consistently taking into account the information from the first, we should find that $\hat{P}(C|T \text{ second})$ is less than $\hat{P}(C|T \text{ first})$, again at both values of T. For if there are factors influencing the selection of which interval is used, and if these factors are not dependent on the outcome of the processing of the first interval, then they should affect performance at two values of T to the same degree.

However, it was shown in 1.6 that ΔP might be a function of more than the discrepancy between how often only the first and only the second is used. The expression (1-13) for ΔP also involves a term which might be non-zero even when both intervals are always being used. If it were in fact the case that the O is always using both intervals, can the models of 1.4 allow for differences between $\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$ if and when they are obtained?

Unfortunately, the analysis given in 1.6 was not derived until some time after this experiment was completed. A lower bound on ε, the proportion of trials on which both intervals are used in making the decision, is given by

$$\varepsilon \geq [P(C|T) - .75]/.25 \quad .$$

The experiment would have been much more informative if it had been designed to take into account the considerations of 1.6, by including a very large value of ΔT so as to obtain a value for $P(C|T)$ as close as possible to 1, and thereby obtain a lower bound on ε of nearly 1.0.

A third aim of this experiment is to explore further the change in performance as base duration T_s is increased by smaller amounts than the 100 msec. increments used in experiment 1. We define $P(C)_-$ and $P(C)_+$ for the set (I) by

$$P(C)_- = \frac{1}{2}\,[P(1|T,\ T-\Delta T) + P(2|T-\Delta T,\ T)]$$

$$P(C)_+ = \frac{1}{2}\,[P(1|T+\Delta T,\ T) + P(2|T,\ T+\Delta T)] \quad .$$

$P(C)_-$ gives performance in discriminating between T_s and $T_s+\Delta T$ when the base duration T_s is $T-\Delta T$. $P(C)_+$ gives performance at base duration T. Hence when 2 sets of alternatives (involving 2 values of T: T_1 and T_2) are randomly intermixed, we can consider the two sets of alternatives as containing four base durations:

$$T_1-\Delta T\ ,\ T_1\ ,\ T_2-\Delta T\ ,\ T_2 \quad .$$

Any empirical statement about the change in performance as base duration increases may have to be conditionalized on the procedure used. Moreover, the values of ΔP will be closely linked to changes in the conditional probabilities of a correct response, $P(1|S_1)$ and $P(2|S_2)$, as the base duration increases from $T-\Delta T$ to T. This is easily seen from the following inequalities. Whenever $\Delta P \neq 0$, we have

$$P(C|T \text{ first}) \neq P(C|T \text{ second})$$

or

$$P(1|S_1') + P(2|S_2) \neq P(1|S_1) + P(2|S_2')$$

which can be rewritten as

either $P(2|S_2) - P(1|S_1) \neq P(2|S_2') - P(1|S_1')$

or as $P(2|S_2) - P(2|S_2') \neq P(1|S_1) - P(1|S_1')$.

If we have a model which predicts changes in the conditional probabilities $P(2|S_2)$ when the base duration increases by a small amount ΔT, which are unequal to the changes in $P(1|S_1)$, we will at the same time have a model predicting a difference between $P(C|T \text{ first})$ and $P(C|T \text{ second})$, even though both intervals are being used on every trial.

Procedure

Each session consisted of four blocks of 80 trials. Within each session, eight different pairs of intervals were randomly intermixed and presented equally often. These were

T, T-ΔT	T, T+ΔT
T-ΔT, T	T+ΔT, T

for two values of T. ΔT was chosen so as to have $\hat{P}$(C) about .85 in discriminating between 150 and 150+ΔT. After running four sessions with two values of T, a second pair of values for T was selected, and four more sessions were run. The ranges of base durations covered by these two sets of base durations intervals were non-overlapping, except for one O (HL) who had the largest value of ΔT.

The table below is a summary of the values of T and ΔT used for each O in the two sets of sessions.

Observer	ΔT	T (sessions 1-4)	T (sessions 5-8)
MD	25	75 , 150	200 , 275
V	25	75 , 150	200 , 275
HL	50	100 , 200	200 , 300
DQ	25	200 , 275	75 , 150
IR	25	200 , 275	75 , 150
RM	10	200 , 275	75 , 150

One group of Os began with the shorter base durations, and then had the set of longer base durations. A second group of 3 Os began with the longer base durations, and then were presented with the shorter ones. Of the six Os, only two were naive - DQ and IR of the second group. The Os in the first group had participated for several weeks in another experiment involving base durations 50 and 150 msec. Each of the 3 Os in the second group had one practice session; as no systematic change was seen over the succeeding 4 sessions, the data from these 4 sessions was retained.

Results and discussion

Tables 5 and 6 summarize the results for the two groups of Os. The probability estimates $\hat{P}(C|T \text{ first})$, $\hat{P}(C|T \text{ second})$, $\hat{P}(C)_+$, and $\hat{P}(C)_-$ are each based on 320 trials ($\sigma < .028$). In each set of sessions, there are eight different pairs of intervals randomly intermixed. The entries in the first two rows and four columns of Table 5 give the proportion of trials on which the O correctly selected the longer interval, for each of the 8 pairs in the first four sessions. For example, for MD, the stimulus alternatives in sessions 1-4 and the corresponding proportions of correct responses (reading from left to right in row 1, and then in row 2) are the following:

75 , 50	.630
50 , 75	.788
100 , 75	.484
75 , 100	.748
150 , 125	.671
125 , 150	.800
175 , 150	.491
150 , 175	.931

(a) Are both intervals used?

Table 6 shows $\hat{P}(C|T \text{ first})$, $\hat{P}(C|T \text{ second})$ and $\hat{P}(C|T)$. For each of the six Os, neither of the quantities $\hat{P}(C|T \text{ first})$

Table 5: Summary of results from experiment 3

Each P(C) is based on 320 trials. The first two values of T were used in sessions 1-4, and the second two values of T were used in sessions 5-8.

$$P(C)_- = \frac{1}{2}[P(1\ T,T-\Delta T) + P(2\ T-\Delta T,T)]$$

$$P(C)_+ = \frac{1}{2}[P(1\ T+\Delta T,T) + P(2\ T,T+\Delta T)]$$

	T	$P(1\|T,T-\Delta T)$	$P(2\|T-\Delta T,T)$	$P(1\|T+\Delta T,T)$	$P(2\|T,T+\Delta T)$	$P(C)_-$	$P(C)_+$
MD	75	.630	.788	.484	.748	.708	.616
(ΔT = 25)	150	.671	.800	.491	.931	.736	.710
	200	.826	.791	.805	.745	.809	.775
	275	.506	.831	.506	.836	.672	.684
V	75	.873	.949	.647	.936	.911	.792
(ΔT = 25)	150	.906	.835	.892	.849	.870	.871
	200	.879	.925	.760	.955	.902	.859
	275	.716	.914	.684	.884	.814	.784
HL	75	.758	.949	.617	.810	.856	.715
(ΔT = 50)	150	.608	.834	.564	.899	.725	.733
	200	.823	.801	.766	.776	.812	.771
	275	.647	.747	.627	.758	.697	.694

Table 5 - continued

	T	$P(1\mid T,T-\Delta T)$	$P(2\mid T-\Delta T,T)$	$P(2\mid T,T+\Delta T)$	$P(1\mid T+\Delta T,T)$	$P(C)_-$	$P(C)_+$
RM	275	.780	.869	.883	.660	.824	.773
(ΔT = 10)	200	.758	.867	.863	.780	.813	.821
	150	.887	.792	.852	.893	.840	.872
	75	.932	.881	.962	.805	.907	.884
IR	275	.791	.813	.847	.614	.802	.730
(ΔT = 25)	200	.904	.772	.769	.873	.837	.822
	150	.724	.819	.944	.719	.772	.831
	75	.908	.850	.925	.879	.879	.902
DQ	275	.753	.865	.799	.827	.809	.813
(ΔT = 25)	200	.865	.865	.956	.818	.865	.889
	150	.935	.880	.925	.791	.907	.859
	75	.975	.994	.981	.949	.984	.965

or $\hat{P}(C|T$ second) is at chance level for any value of T. Hence we can reject the possibility that only the first or only the second interval is being used throughout any series of sessions. Moreover, the lower bound on ε is greater than zero, since for at least one value of T in each series, $\hat{P}(C|T)$ is greater than .75; this statement holds in 11 out of 12 instances. (6 Os x 2 sets of intervals.) Hence according to the analysis of 1.6, the Os are using both intervals on some non-zero proportion of the trials, and we can reject the possibility that the Os reduce the FC task to a SS task, by always using only one of the two intervals on each trial.

(b) Is $\hat{P}(C|T$ first) different from $\hat{P}(C|T$ second)?

In Table 6, $\hat{P}(C|T$ first) is greater than or equal to $\hat{P}(C|T$ second) in 22 out of 24 cases. (6 Os x 4 values of T.) The quantity $\Delta P = \hat{P}(C|T \text{ first}) - \hat{P}(C|T \text{ second})$ is shown as a function of T in Figure 9, for the individual Os. The top half of the figure is for the group of 3 Os who had the set of shorter base durations first. It appears that within each set of intervals containing 2 values of T, ΔP is generally greater at one value of T than at the other. A value of ΔP is taken to be non-zero if the corresponding value of $|z|$ in Table 6 is greater than the criterion value $z_{.975} = 1.96$ (a normal deviate). The values of z are calculated by using a binomial test for determining whether two independent proportions are significantly different (McNemar, p. 60). Since

Table 6

P(C|T first) and P(C|T second), derived from the data in Table 5, N=320 trials for each proportion ($\sigma < .028$). Where $|z|$ is greater than $z_{.975} = 1.96$, ΔP is taken to be significantly different from zero.

Obs.	T	P(C|T first)	P(C|T second)	ΔP	z
MD	75	.689	.636	.05	2.84
	150	.801	.645	.15	8.79
	200	.786	.798	-.01	- .78
	250	.671	.669	.00	.14
V	75	.905	.798	.11	8.71
	150	.878	.863	.02	1.06
	200	.917	.843	.08	5.80
	250	.800	.799	.00	.06
HL	100	.784	.783	.00	.06
	200	.754	.699	.05	3.09
	200	.800	.784	.02	1.00
	300	.702	.687	.01	.85
RM	75	.947	.843	.10	8.58
	150	.870	.843	.03	1.95
	200	.811	.824	-.01	- .85
	275	.832	.765	.06	4.22
IR	75	.917	.865	.05	4.21
	150	.834	.769	.06	4.12
	200	.837	.823	.02	.94
	275	.819	.714	.11	6.31
DQ	75	.978	.972	.01	1.05
	150	.930	.836	.09	7.43
	200	.911	.842	.07	5.30
	275	.776	.846	-.07	-4.52

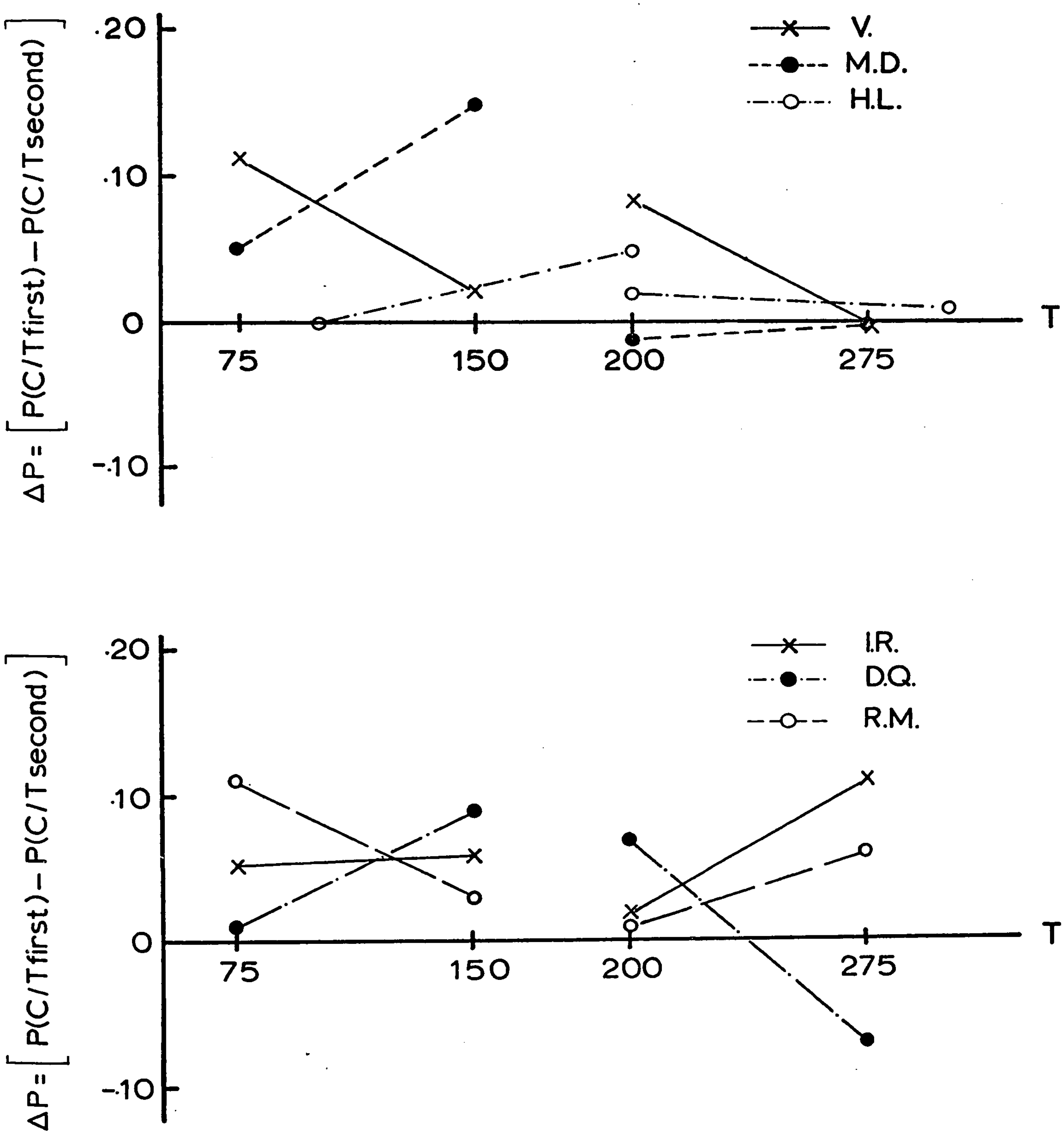


FIGURE 9: $[\hat{P}(C \mid T \text{ first}) - \hat{P}(C \mid T \text{ second})]$ as a function of T for individual Os. 640 trials per point

the proportions $p_1 = \hat{P}(C|T \text{ first})$ and $p_2 = \hat{P}(C|T \text{ second})$ are based on equal numbers of trials, we use

$$z = \frac{p_1 - p_2}{\sigma} \quad \text{with} \quad \sigma = \left[\frac{p(1-p)}{2N}\right]^{1/2} \quad \text{and}$$

$$p = \frac{p_1 + p_2}{2} .$$

Of the 24 values of ΔP given in Table 6, 13 of these are non-zero according to the above criterion. However, for each O, there is generally only one value of T within a series for which there is a large (significant) difference between $\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$. Whatever factors may be operative in producing non-zero values of ΔP, these do not appear to affect performance at both values of T within a series to the same degree.

(c) Performance as a function of base duration

The stimulus alternatives within each session can be considered as involving 4 base durations T_s and one (positive) value of ΔT. Hence there is a total of 8 base durations for each O. In Table 5, $\hat{P}(C)_-$ shows performance at base duration $T-\Delta T$, and $\hat{P}(C)_+$ shows performance at base duration T. Thus for MD, the entries in the column $\hat{P}(C)_-$ gives $\hat{P}(C)$ for base durations (reading down) 50, 125, 175 and 250, when ΔT is fixed at 25 msec. For the second group

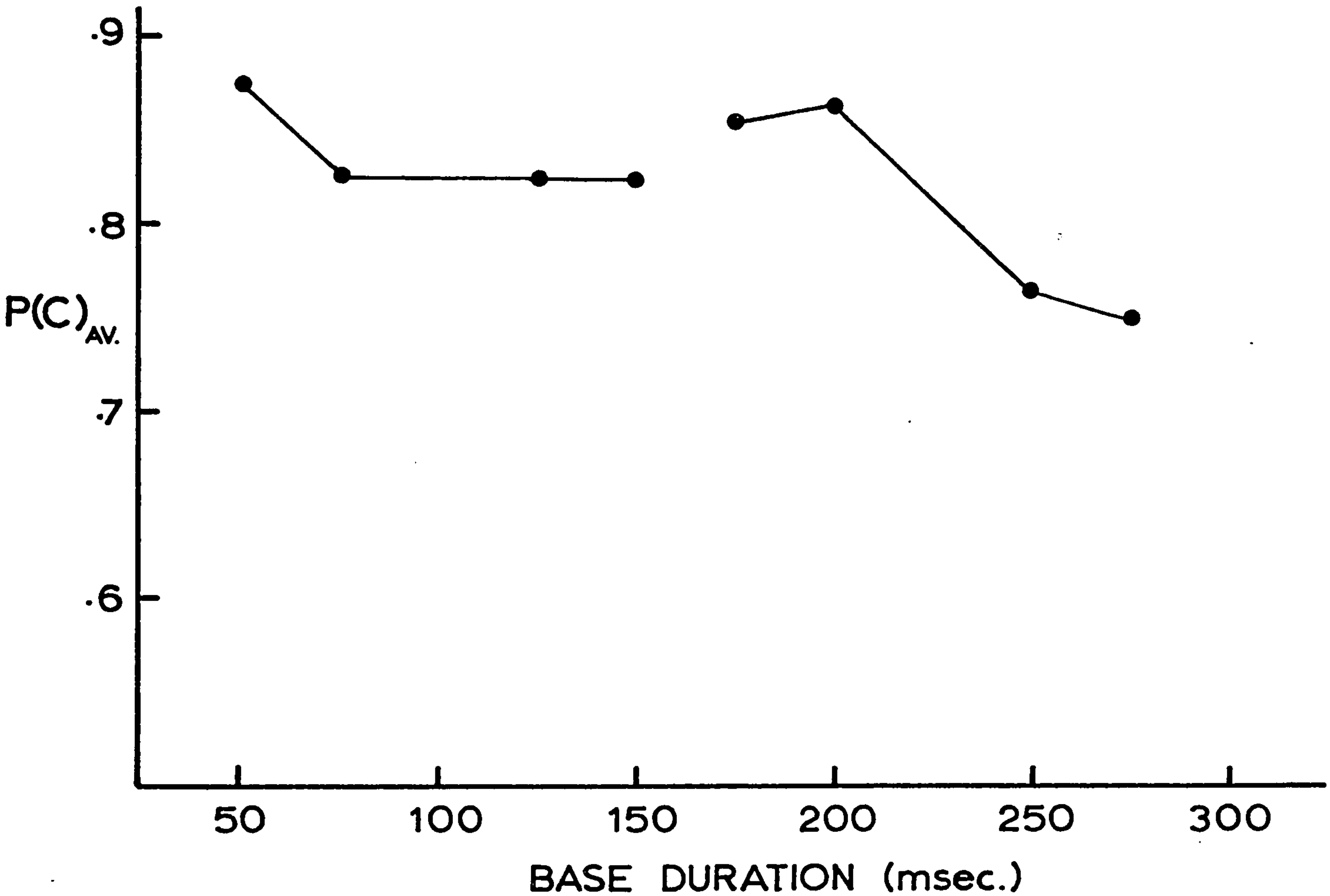


FIGURE 10: $\hat{P}(C)$ as a function of base duration, averaged over 4 Os. ΔT=25 msec. The ranges 50-150 and 175-275 are covered in different series of sessions.

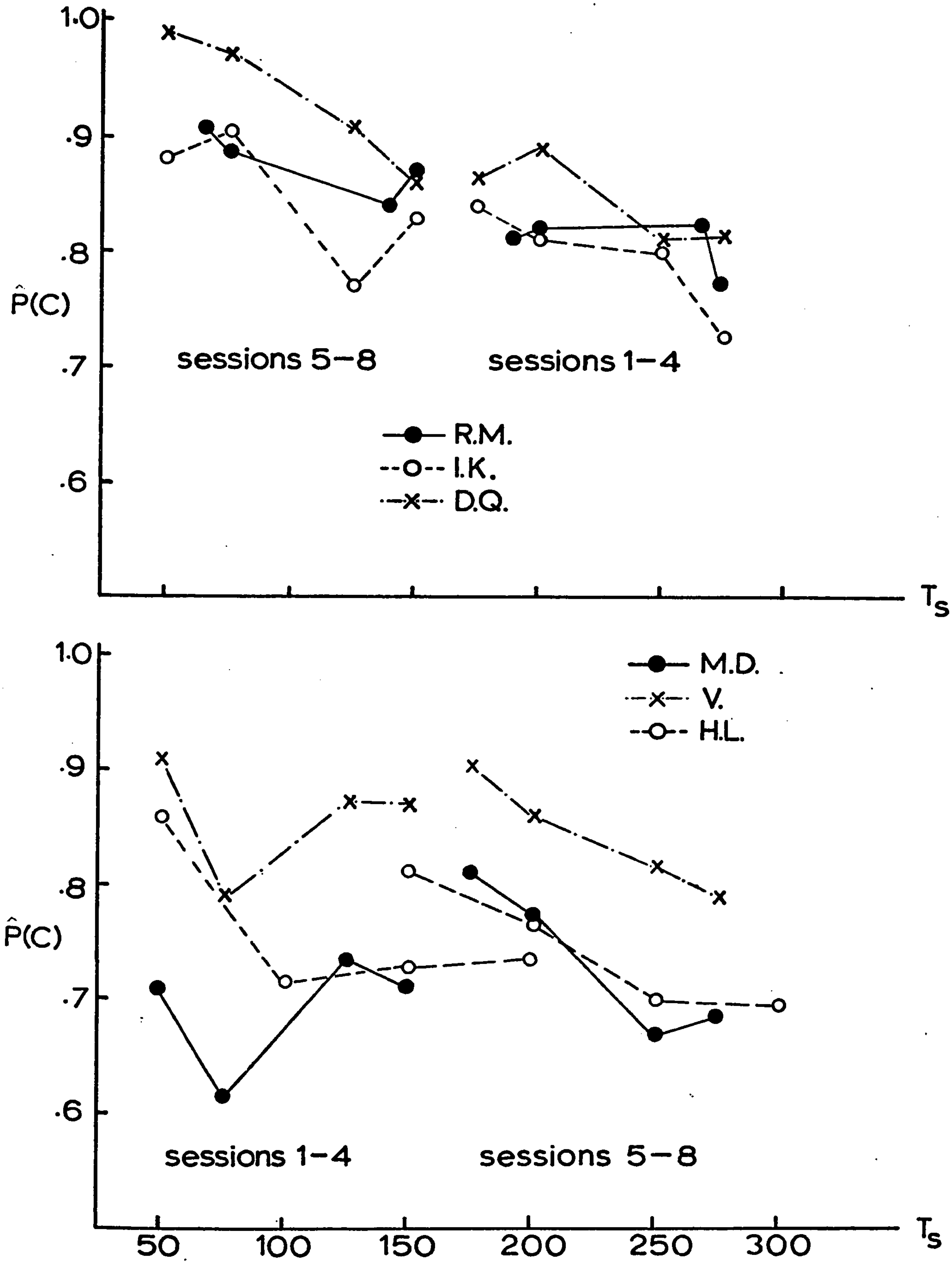


FIGURE 11: $\hat{P}(C)$ as a function of base duration, for individual Os.

of Os, base durations decrease in magnitude as one reads down the columns. These quantities are plotted for individual Os in Figure 11.

For 4 Os, the two sets of intervals cover a range from 50 to 275 msec. We can get some idea of how the discriminability of a fixed duration difference ΔT depends on the base duration with which it is associated. It must be kept in mind that there is a "break" in the function; 175 is the longest interval occurring in one series, but it is the shortest interval occurring in the other series. The averaged data for four Os is shown in Figure 10. In this averaged function, $\hat{P}(C)$ changes least across the 50-150 msec. range; in fact it is constant from 75 to 150 msec. This can be contrasted to a relatively large decrease of .10 in $\hat{P}(C)$ as T_s increases from 200 to 250 msec. In most cases in the individual data shown in Figure 11 there is relatively little difference in $\hat{P}(C)$ between T_s=125 and T_s=200, although there is a tendency for performance at the smallest base duration within each set to be somewhat better than performance with the largest.

On the basis of the results from experiment 1, and from those reported by Creelman and by Abel (1971), it might have been predicted that when ΔT is fixed we should see a continuous decrease in performance in discriminating between T_s and $T_s+\Delta T$ as the base duration T_s increases. In Abel's data (averaged over 3 Os), $\Delta T_{.75}$ is an increasing function of T_s, for T_s ranging from 40 to 640 msec. In Creelman's second experiment, the plot of d' as a function of T_s over the range from 20 to 320 msec. is a decreasing function for each of his Os. In this experiment, a decrease in averaged performance is not seen over a range of at least 125 msec. It may be that the function relating $\hat{P}(C)$ to base duration is a step function. However, in Creelman's third experiment there was a large drop in d' between T=80 and T=160; both these values are in the range where $\hat{P}(C)_{av}$ is constant here. Thus the discrepancy between these results and Creelman's cannot be explained in terms of Creelman having sampled performance at some point along each step of the function. The shape of the relation between $\hat{P}(C)$ and T_s may depend on the way in which it is obtained...on whether or not a small portion of the base duration range is used over a number of sessions, and on whether or not the T_s values occur randomly intermixed, so as to have some uncertainty as to the order of magnitude of the intervals which will occur on any trial.

In exploring the change in performance as base duration increases by small amounts, we can also examine the conditional probabilities of a correct response, $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$; since $\hat{P}(C)$ is the average of these two terms, we can ask whether they vary as a function of T_s in the same way as does the marginal probability of being correct, $\hat{P}(C)$. The first two columns of Table 5 give $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ for base durations T-ΔT; the next two columns give these quantities for base durations T. Figure 12 shows $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ plotted as a function of T_s, averaged over the four Os who had the same value of ΔT. Figure 13 shows the same quantities, plotted for individual Os (160 trials per point, $\sigma < .04$).

One immediate feature of the plots of $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ shown in Figures 12 and 13 is that $\hat{P}(2|S_2)$ is often much larger than $\hat{P}(1|S_1)$. There is also a tendency in all the individual data (except for DQ) for $\hat{P}(2|S_2)$ to vary within a much narrower range than does $\hat{P}(1|S_1)$, over the set of larger base durations. In some cases, $\hat{P}(2|S_2)$ increases as T_s increases, while $\hat{P}(1|S_1)$ decreases. In the region of transition from one set of T_s values to the next, the position of $\hat{P}(2|S_2)$ relative to $\hat{P}(1|S_1)$ often reverses. It would be of interest to know whether this "reversal" would be seen if the set of T_s values in this transition range (125-250 msec.) were all randomly intermixed; this reversal might be a result of having T_s long in one series and short in another, relative to other base durations also occurring.

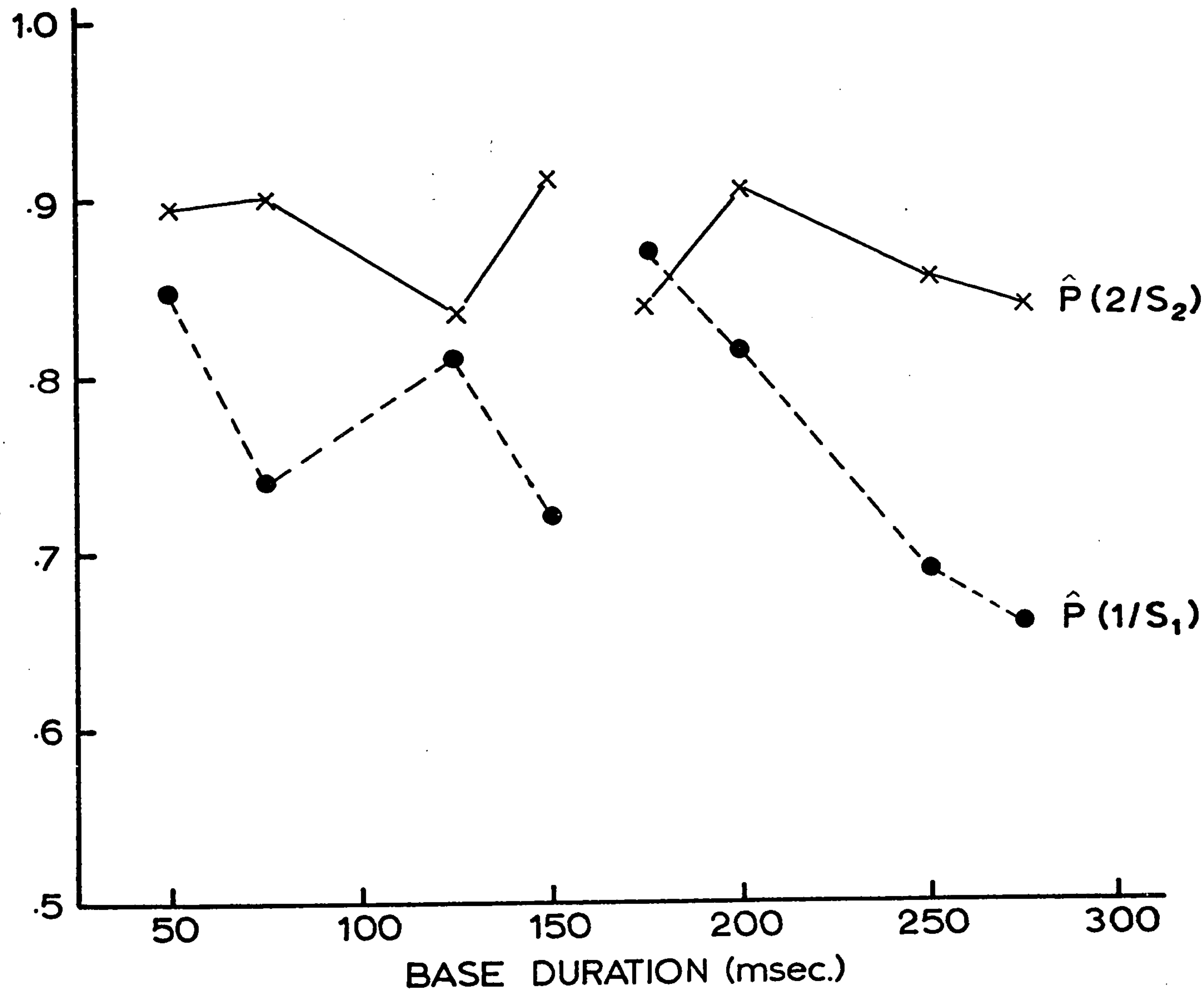


FIGURE 12: Conditional probabilities of being correct $\hat{P}(1|S_1)$ •·······• and $\hat{P}(2|S_2)$ x———x as a function of base duration. Data averaged over 4 Os who had ΔT=25.

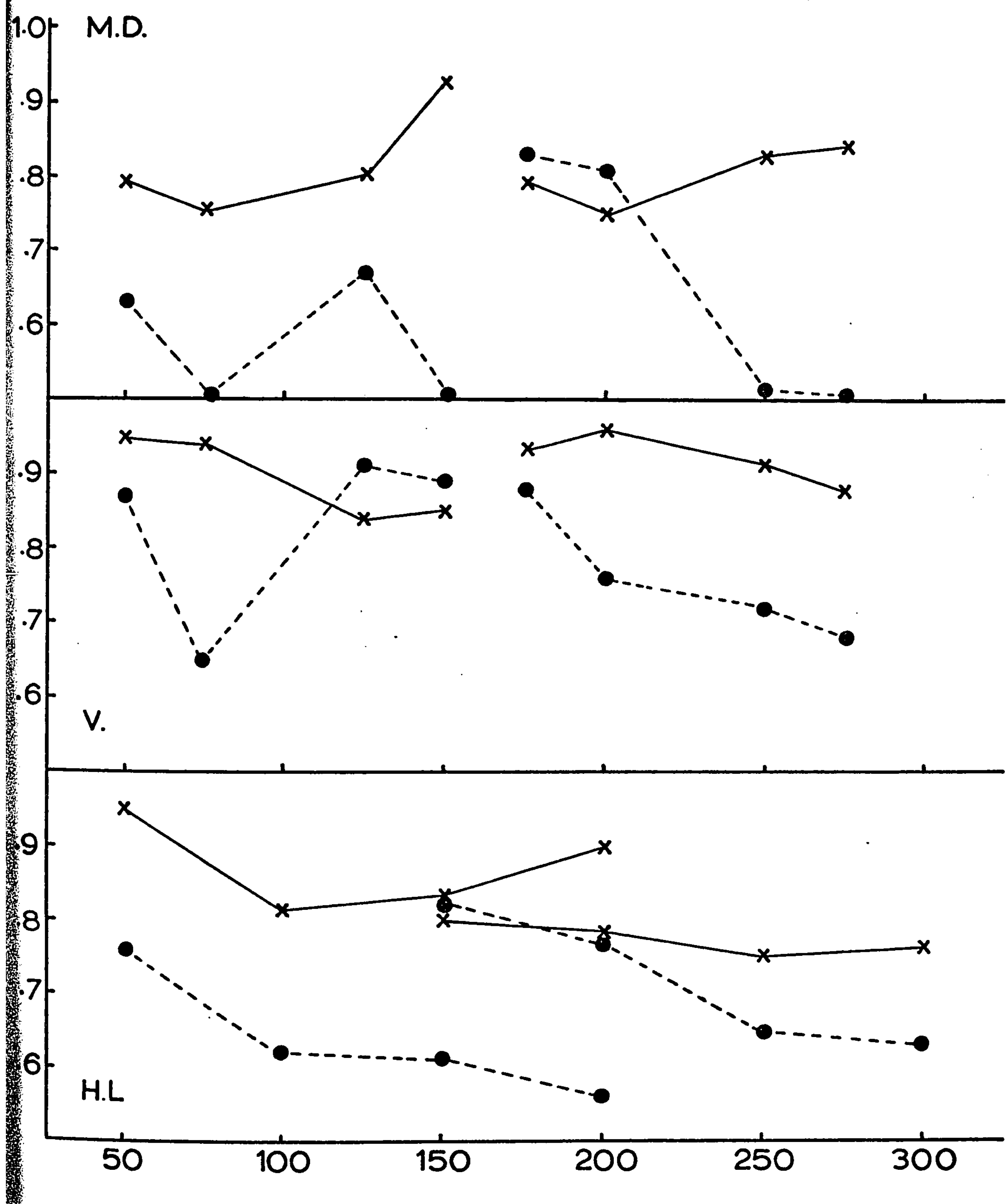


FIGURE 13: Conditional probabilities of being correct $\hat{P}(1|S_1)$ •-----• and $\hat{P}(2|S_2)$ x——x as a function of base duration. Individual data.

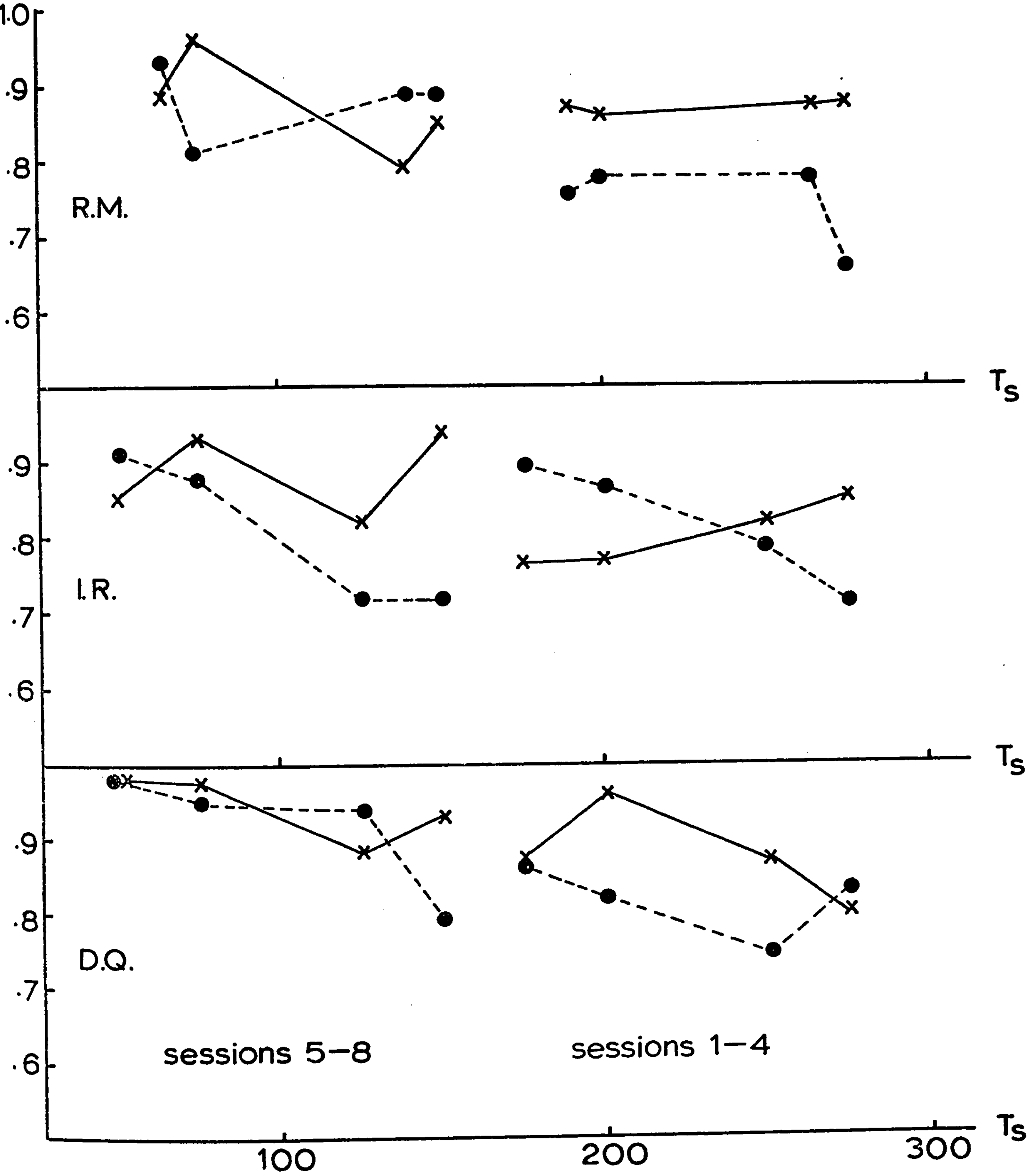


FIGURE 13: Conditional probabilities of being correct $\hat{P}(1|S_1)$ •- - - - - - -• and $\hat{P}(2|S_2)$ x———x as a function of base duration. Individual data.

In Figure 13, the differences between the values of $\hat{P}(2|S_2)$ and $\hat{P}(1|S_1)$ at each value of T_s do not seem to be constant, within any one set of four base durations. In the introduction to this experiment, it was indicated that where $\hat{P}(C|T \text{ first})$ is different from $\hat{P}(C|T \text{ second})$, there will be changes in the difference $\hat{P}(2|S_2) - \hat{P}(1|S_1)$ as the base duration increases from $T-\Delta T$ to T. We will define the <u>response preference</u> at base duration T_s as the difference between how often the second interval is chosen as the longer one, and how often the first is chosen as longer, conditional on the occurrence of the stimulus patterns S_1 and S_2. The response preference is given by

$$P(2|S_1 \text{ or } S_2) - P(1|S_1 \text{ or } S_2)$$

$$= [P(2|S_1)\cdot P(S_1) + P(2|S_2)\cdot P(S_2)]$$

$$- [P(1|S_1)\cdot P(S_1) + P(1|S_2)\cdot P(S_2)]$$

which reduces to

$$P(2|S_2) - P(1|S_1)$$

since $P(S_2) = P(S_1) = \frac{1}{2}$, $P(1|S_2) = 1 - P(2|S_2)$,

and $P(2|S_1) = 1 - P(1|S_1)$.

Any model of the coding process which will allow for stimulus dependent changes in the response preference $\hat{P}(2|S_2) - \hat{P}(1|S_1)$ will also predict differences between $\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$, depending on exactly what the value of T or ΔT happens to be. In the theoretical analysis (Chapter III), it will be determined to what extent each of the models of 1.4 can provide for the type of variation in $\hat{P}(2|S_2)$ and $\hat{P}(1|S_1)$ which has been seen in this experiment; using the assumption that both intervals are being used on each trial.

2.4 Experiment 3: Varying the interstimulus interval

In many studies concerned with very short term auditory perceptual memory, a decrement in performance has been obtained as the interstimulus interval (ISI) is increased. In pitch discrimination, a decline in performance is obtained with increasing ISI, provided that a roving standard is used (Harris, 1952; Wickelgren, 1967; Massaro, 1969; Bull & Cuddy, 1972). The use of a roving standard supposedly precludes the reliance on a long term memory standard with which to compare each alternative within a trial. Tanner (1962) reported results from a study of amplitude discrimination in which ISI varied from 0 to 10 seconds. A measure of "efficiency" in performance reached a maximum at 1/2 sec. separation between the two signals, and then slowly decreased as the ISI was further increased. Kinchla and Smyzer (1967) have obtained data showing a monotonic decrease in performance in an amplitude discrimination task where the ISI was varied from 0 to 2 seconds.

On the other hand, the time information contained in very brief intervals may not be susceptible to the perceptual memory loss which has been inferred from studies like the ones mentioned above. The results of an experiment done by Small and Campbell (1962) suggest that some minimal separation between time intervals may be needed for optimal performance in duration discrimination, after which further increases in ISI up to 3.2 sec. do not result in a decrease in the dependent variable. In this study, Os decided whether the second (variable) member of a pair of filled auditory intervals was longer or shorter than the first (the standard). The time intervals were defined by noise bursts, or by pure tones of either 250 or 5000 cps. On each trial, the variable interval could have any one of 7 durations, ranging from .25 to 2.5 times the standard. The dependent variable was a difference threshold, $\Delta T_{.5}$ or that ΔT required for 50% "variable longer" judgments. The separations between the intervals were .05, .2, .8 and 3.2 seconds. For a 400 msec. standard, $\Delta T_{.5}$ was the same at all of these ISIs, and for the three different types of auditory signals. For 40 msec. pure tone standards, $\Delta T_{.5}$ reached a minimum (asymptotic) value when the ISI was between .2 and .8 sec.; but for the 40 msec. noise burst standard, $\Delta T_{.5}$ did not change when the ISI was increased beyond .2 seconds. These results are consistent with the finding of McKee et al. (1970) that when the ISI was increased over the range from 1/2 to 2 seconds, there was no change in the discriminability of a difference in duration between two very brief visual dark flashes.

One aim of this experiment was to replicate the results of McKee et al. (already summarized in 1.6), using empty intervals bounded by brief auditory signals, with the stimulus alternatives chosen in such a way that we could determine whether the O is using both intervals on each trial in the FC task. If changing the separation between the intervals in a FC task has no effect on $\hat{P}(C)$, it would be highly desirable to rule out the possibility (as suggested by McKee et al.) that only one interval on each trial is being used. This problem was the motivation for experiment 2, and the stimulus alternatives were chosen for this experiment in the same way as they were there.

In this experiment, we also explore the change in performance as ISI increases from 0 to 1/2 seconds. With very short ISIs, performance may be disrupted because the time information from the second interval is no longer independent of the information from the first. Both the quantal counting model and the first version of the quantal onset-offset model can make predictions differentiating between performance conditional on having the intervals effectively independent, and performance in the situation where the intervals are adjacent (ISI=0) and a specific type of dependence between the intervals is involved. Hence it is of interest to determine whether such a difference in performance with adjacent and with widely separated intervals can be consistently obtained.

One might consider the first interval in the FC task as a "standard" with which the O can compare the second interval. This "standard" has to be retained only during the ISI, until the second interval has been presented. This view of the FC situation can be contrasted to the situation in the SS paradigm, where the O has to compare the single interval on each trial with some standard held in long term memory. A second aim of this experiment was to obtain a sample of single stimulus performance from each observer in discriminating between T and T+ΔT, after conditions which would seem to be unfavorable to the formation of a stable internal standard - i.e., immediately after the O has had much experience with a FC task involving several standards randomly intermixed.

Procedure

On any trial, any one of eight pairs of intervals could occur. These were the pairs

T, T+ΔT	T+ΔT, T
T, T-ΔT	T-ΔT, T

for two values of T: 150 and 250 msec. ΔT was constant for each O, and was chosen so as to have a performance level of around .85 in discriminating between 150 and 150+ΔT, when the separation between the intervals was 1 second. ΔT=35 msec. was used for two Os (V and MD) who had taken part in experiment 2; ΔT=25 was used for a third, naive O.

Each session consisted of 4 blocks of 80 trials. Within any one session, the ISI was fixed at one of six values: 0, 1/8, 1/4, 1/2, 1 or 2 seconds. The ISI varied from session to session, with each member of the set being used once before the set was cycled through again, in another order. This was repeated four times.

Once within each cycle, a single stimulus session was run. In two sessions, the stimulus alternatives were 150 and 150+ΔT, and in another two sessions the alternatives were 250 and 250+ΔT. The *O* was presented with only one interval on each trial, and he had two response alternatives: the interval was to be identified as either "short" or "long". The boundaries of each interval were identical to those in the FC task. Feedback was stimulus dependent, occurring on only those trials on which the interval presented was of magnitude T. Each SS session consisted of either 3 or 4 blocks of 100 trials.

Results and discussion

Table 7 lists $\hat{P}(1|S_1)$, $\hat{P}(2|S_2)$ and $\hat{P}(C)$ for the 3 *Os*, at the different ISI values in the FC procedure. These proportions are presented for what can be considered as four base durations: 150-ΔT, 150, 250-ΔT and 250. Each value for $\hat{P}(C)$ is based on 240 trials; the data from the first cycle through the set of ISIs is not included here.

Table 8 summarizes SS performance. $\hat{P}(\text{short}|T)$ is the proportion of trials on which T was presented and the response was "short". $\hat{P}(\text{long}|T+\Delta T)$ is defined in a similar way. Performance for the first 200 trials and the last 200

Table 7

Summary of results from experiment 3; FC procedure, ISI varied from 0 to 2 sec.

		ISI = 0			ISI = 1/8			ISI = 1/4		
Obs	$T, T+\Delta T$	$\hat{P}(1\|S_1)$	$\hat{P}(2\|S_2)$	$\hat{P}(C)$	$\hat{P}(1\|S_1)$	$\hat{P}(2\|S_2)$	$\hat{P}(C)$	$\hat{P}(1\|S_1)$	$\hat{P}(2\|S_2)$	$\hat{P}(C)$
	115,150	.858	.817	.838	.725	.508	.618	.810	.626	.715
MD	150,185	.892	.667	.781	.669	.701	.685	.723	.791	.757
	215,250	.831	.576	.703	.576	.771	.674	.600	.867	.733
	250,285	.773	.597	.685	.381	.835	.611	.555	.907	.730
	115,150	.620	.967	.793	.908	.678	.794	.521	.958	.742
V	150,185	.538	.950	.749	.783	.933	.858	.595	.983	.788
	215,250	.800	.792	.796	.824	.908	.873	.739	.833	.787
	250,285	.731	.731	.731	.767	.775	.771	.815	.824	.819
	125,150	.487	.814	.648	.766	.867	.818	.784	.864	.825
DH	150,175	.681	.746	.713	.965	.500	.738	.648	.958	.810
	225,250	.930	.292	.612	.610	.514	.561	.832	.468	.658
	250,275	.832	.418	.628	.519	.620	.567	.883	.451	.665

Table 7 - continued

Obs	T,T+ΔT	ISI = 1/2 $\hat{P}(1\|S_1)$	ISI = 1/2 $\hat{P}(2\|S_2)$	ISI = 1/2 $\hat{P}(C)$	ISI = 1 $\hat{P}(1\|S_1)$	ISI = 1 $\hat{P}(2\|S_2)$	ISI = 1 $\hat{P}(C)$	ISI = 2 $\hat{P}(1\|S_1)$	ISI = 2 $\hat{P}(2\|S_2)$	ISI = 2 $\hat{P}(C)$
	115,150	.933	.683	.808	.925	.763	.845	.950	.798	.874
MD	150,185	.849	.861	.855	.884	.822	.843	.819	.720	.769
	215,250	.597	.807	.702	.814	.862	.838	.930	.769	.849
	250,285	.546	.924	.734	.703	.842	.773	.787	.771	.779
	115,150	.625	.933	.779	.633	.926	.780	.583	.952	.770
V	150,185	.705	.984	.844	.630	.992	.811	.644	.983	.814
	215,250	.890	.932	.911	.822	.958	.924	.655	.958	.807
	250,285	.700	.857	.879	.811	.934	.873	.664	.908	.785
	125,150	.974	.951	.962	.975	.942	.959	.918	.966	.942
DH	150,175	.931	.983	.958	.932	.974	.953	.929	.983	.957
	225,250	.856	.743	.802	.913	.761	.838	.949	.670	.810
	250,275	.836	.781	.810	.877	.722	.800	.897	.726	.812

Table 8: Single Stimulus Performance (experiment 3)

	T = 150			T = 250		
MD: ΔT=35	P(short|T)	P(long|T+ΔT)	P(C)	P(short|T)	P(long|T+ΔT)	P(C)
first 200	.85	.72	.79	* .85	.84	.85
last 200	.84	.84	.84	.76	.82	.79
total	.85	.83	.84	.80	.82	.81
V : ΔT=35						
first 200	.78	.89	.83	* .74	.72	.73
last 200	.82	.81	.82	.70	.81	.76
total	.80	.85	.82	.72	.77	.74
DH: ΔT=25						
first 200	* .91	.91	.91	.78	.81	.80
last 200	.86	.94	.90	.87	.80	.84
total	.88	.92	.90	.82	.81	.81

Total N=600 for each O; * indicates performance from the first block of trials in which the O had the SS task.

trials are shown, as well as proportions which include all the data collected under this procedure. In this table, $\hat{P}(C)$ is defined by

$$\hat{P}(C) = \frac{1}{2}\,(\hat{P}(\text{short}|T) + \hat{P}(\text{long}|T+\Delta T))$$

(a) Performance as a function of ISI

A large improvement in performance occurs as the ISI increases from 0 to 1/2 seconds, but further increases in the ISI up to its maximum value of 2 seconds result in relatively little further change in $\hat{P}(C)$. Figure 14 shows the overall effect of ISI upon $\hat{P}(C)$, averaged over the 3 Qs and the 4 base durations. This averaging is intended only to show the important features of the data. Any conclusions drawn from this figure will have to be qualified by looking at the individual data. Averaging over Qs is justified if we can be reasonably confident that the data reflect the same processes in those Qs. In Figure 14, $\hat{P}(C)_{av}$ reaches its maximum value when the ISI is 1 sec.; this maximum differs from the values of $\hat{P}(C)_{av}$ at 1/2 and 2 sec. by .02 and .025 respectively. In contrast to these changes, we have an increase of .12 in $\hat{P}(C)_{av}$ as the ISI increases from 0 to 1/2 seconds.

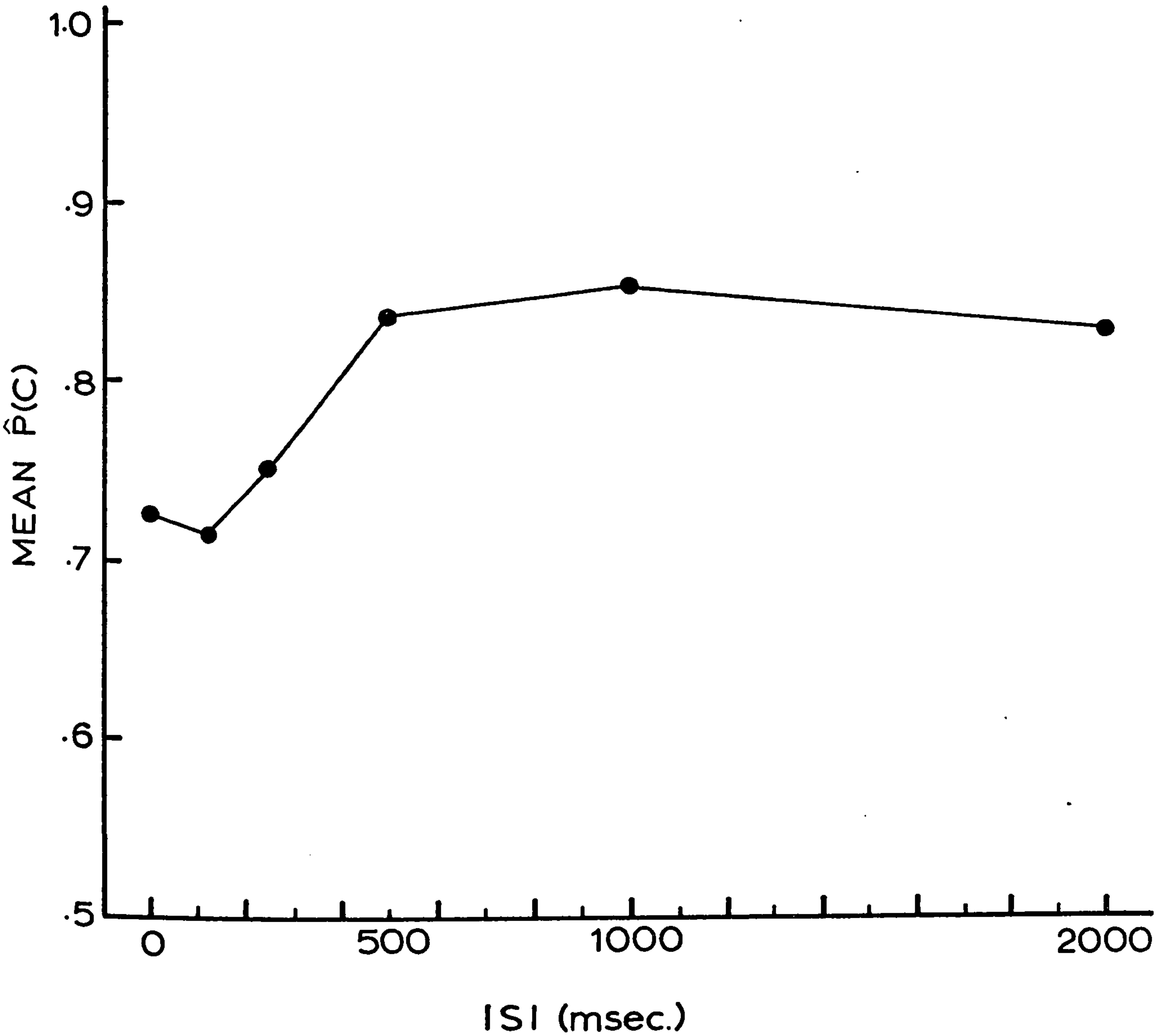


FIGURE 14: Overall effect of interstimulus interval upon $\hat{P}(C)$ Data is averaged over 3 Qs and 4 values of base duration.

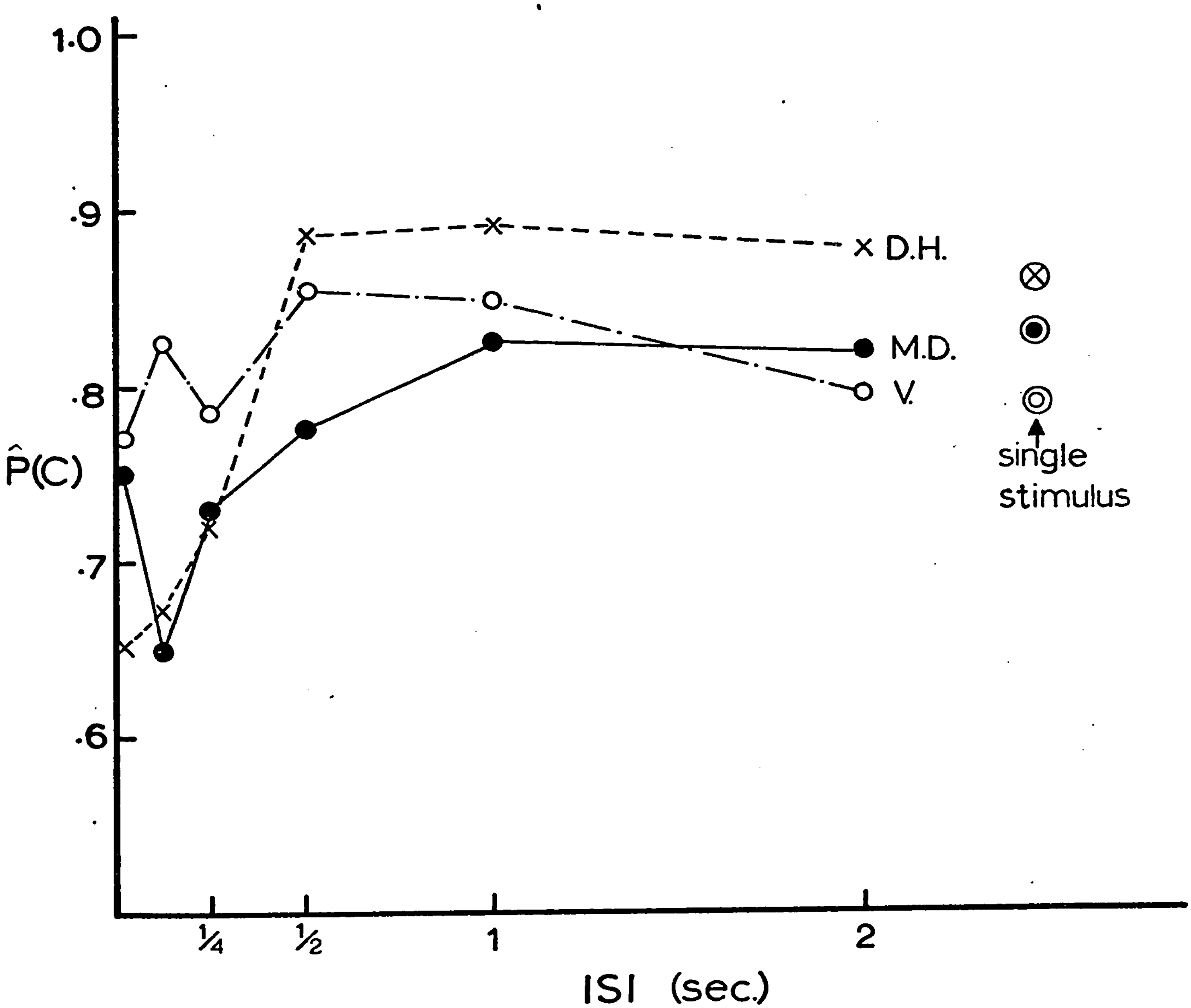
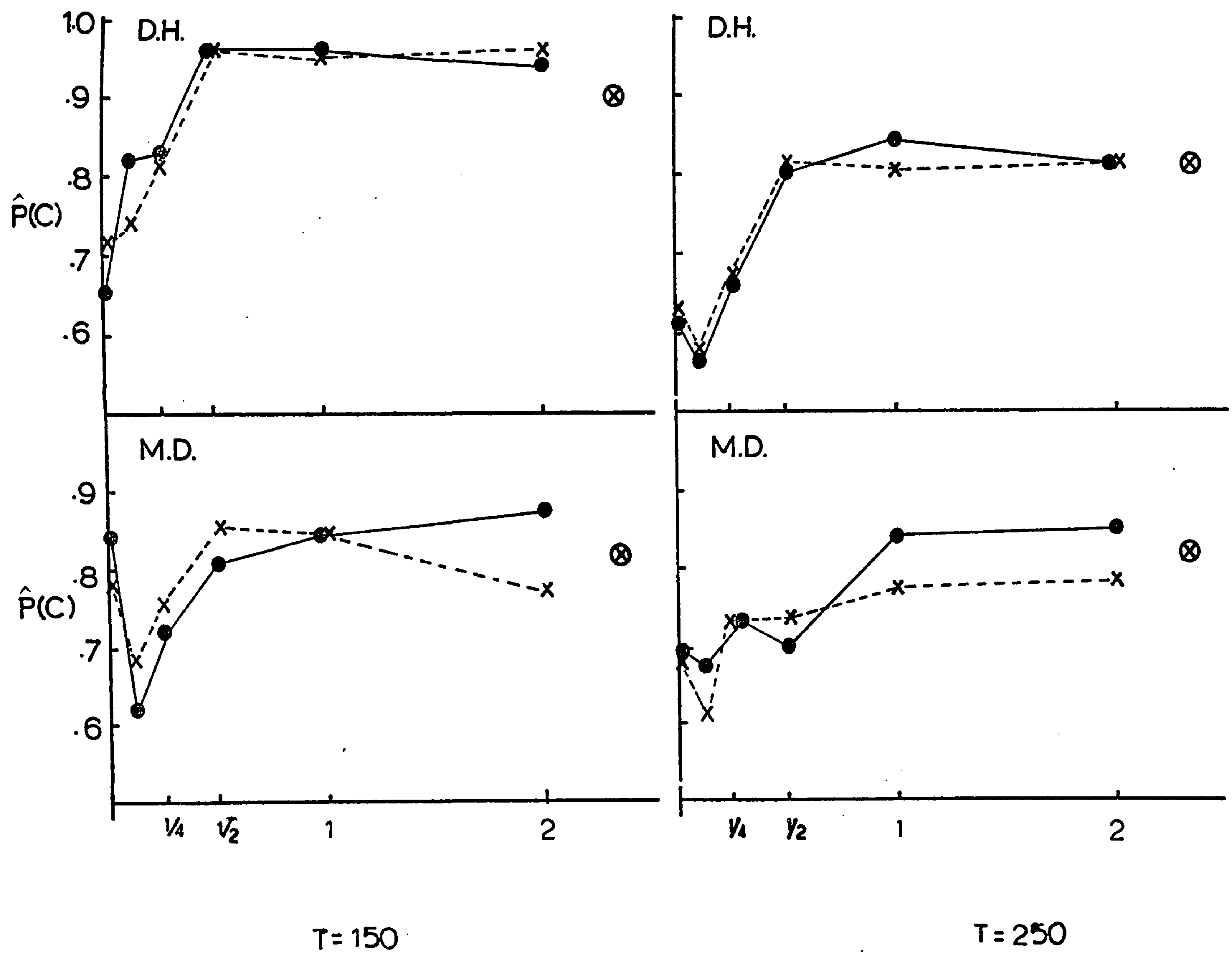


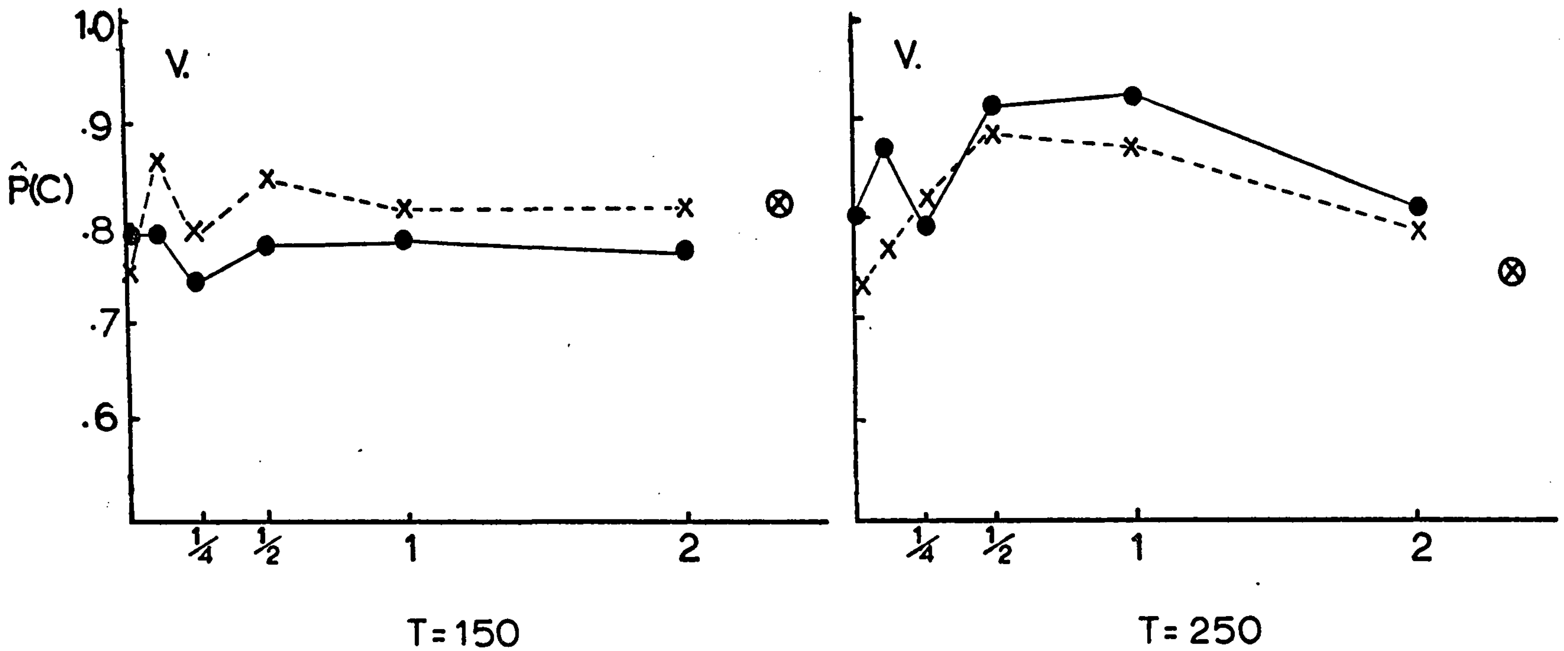
FIGURE 15: Effect of ISI upon $\hat{P}(C)$, for individual Os. [N=960 trials per point.] Data averaged over 4 base durations. The circled data points represent performance from the SS sessions, averaged over two base durations (150 and 250).

FIGURE 16: The effect of ISI upon $\hat{P}(C)$, at each of four base durations. Individual data.

The panel on the right shows $\hat{P}(C)$ at base durations
(250-ΔT) •———• and 250 x- - - - - - - -x
The panel on the left shows $\hat{P}(C)$ at base durations
(150-ΔT) •———• and 150 x- - - - - - - -x

The circled data points show performance from sessions with the SS procedure.


D.H.
M.D.
P̂(C)
1.0
.9
.8
.7
.6
1/4
1/2
1
2
T = 150
T = 250



1.0
.9
.8
.7
.6
$\hat{P}(C)$
V.
1/4
1/2
1
2
T=150
V.
1/4
1/2
1
2
T= 250

Figure 15 shows the effect of ISI upon $\hat{P}(C)$ for the individual Os; the data is averaged over the 4 base durations. Averaging over base durations, within an O, is justified if the processes used at the 4 base durations is the same, and this may be a reasonable assumption since all the base durations were randomly intermixed within each session. In Figure 15, an increase in $\hat{P}(C)_{av}$ as ISI increases from 1/2 to 1 sec. is seen only with one O, as is the decrease in $\hat{P}(C)_{av}$ as ISI goes from 1 to 2 seconds.

Figure 16 shows the individual data from Table 7; base duration is the parameter (240 trials per point; $\sigma<.032$). For each O, the functional relation between $\hat{P}(C)$ and ISI is determined once at each of four base durations; we shall consider this functional relation to be replicated four times within each O. A decrease in $\hat{P}(C)$ as ISI increases from 1 to 2 seconds is seen in only 3 of the 12 replications shown in Figure 16. Also, the increase in $\hat{P}(C)$ as ISI goes from 1/2 to 1 sec. is seen in only two of the 12 replications. Hence we draw the conclusion that performance reaches an asymptotic value when the interval between the durations to be discriminated is 1/2 sec., and this level is maintained over the range of ISIs from 1/2 to 2 seconds.

When performance becomes independent of ISI, it is not because only one interval is being used on each trial. Table 9 shows $\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$ for T=150 and

Table 9

$\hat{P}(C|T \text{ first})$ and $\hat{P}(C|T \text{ second})$ at the different ISIs in experiment 3

		T = 150		T = 250	
Obs.	ISI	$\hat{P}(C\|T \text{ first})$	$\hat{P}(C\|T \text{ second})$	$\hat{P}(C\|T \text{ first})$	$\hat{P}(C\|T \text{ second})$
MD	0	.764	.854	.675	.713
	1/8	.713	.589	.707	.576
	1/4	.801	.674	.752	.711
	1/2	.896	.766	.759	.676
	1	.867	.824	.828	.782
	2	.836	.809	.850	.778
V	0	.785	.755	.766	.762
	1/8	.921	.731	.799	.838
	1/4	.775	.758	.781	.824
	1/2	.806	.818	.873	.916
	1	.812	.779	.879	.883
	2	.782	.802	.782	.809
DH	0	.615	.747	.679	.562
	1/8	.633	.915	.615	.516
	1/4	.871	.761	.647	.677
	1/2	.975	.941	.821	.791
	1	.975	.937	.817	.819
	2	.950	.948	.839	.784

T=250, at each ISI. Neither of these quantities is at chance level for ISIs greater than 1/4 second. Moreover, for all Os, $\hat{P}(C|T)$ is greater than .75 for at least one value of T, with ISIs of 0, 1/2, 1 and 2 seconds. Hence according to the analysis presented in 1.6, the decision as to which is the longer interval is being based on both intervals on some non-zero proportion of the trials.

In general, performance is disrupted when the ISI is made very short. In Figure 15, all 3 Os show a decrease in $\hat{P}(C)$ when the ISI is reduced from 1/2 second, although for one O there is little difference between performance with adjacent intervals (ISI=0) and with intervals separated by 2 seconds. On the other hand, in the individual data shown in Figure 16 there seems to be no consistent pattern in the shape of the functional relation between $\hat{P}(C)$ and ISI over the range from 0 to 1/2 second. There may be a variety of ways in which an O can deal with very short ISIs.

The results of varying the ISI in this experiment indicate that whatever information from the first interval is being retained over the ISI, it is not susceptible to the same kind of decay over a few seconds as is often seen in certain other auditory discriminations in which two observations are to be compared. The finding in this experiment that $\hat{P}(C)$ in a FC duration discrimination task does not change as the ISI increases from 1/2 to 2 seconds is consistent with the results of McKee et al. obtained with empty time intervals defined by visual dark flashes. Our results are also consistent with those obtained by Small and Campbell, who used durations defined by filled auditory intervals separated by ISIs of .8 and 3.2 seconds. However, in their study, the ISI at which asymptotic values for the dependent variable were obtained depended on the order of magnitude of the standard, while this was not the case in this experiment. Intervals of the order of 40 and 400 msec. may well be qualitatively different, whereas intervals in the range 115-285 msec. may be dealt with in the same way, especially if there is trial to trial uncertainty as to which base duration will occur.

We still cannot make definite statements in interpreting these results. It may be that the time information obtained from the first interval is not susceptible to the perceptual memory loss seen with other types of sensory information. Or it may be that a decision about the first interval is compared or combined with a decision about the second interval, and this decision is more durable than the sensory information in a stimulus. This latter interpretation, based on the assumption that a decision is made after each interval, seems plausible in the light of the disruption in performance occurring with very short ISIs. Moreover, a similarity of FC and SS performance is predicted by certain models assuming that a decision occurs after each interval, and we find that the performance levels from the two tasks are very close in this experiment. This is the next feature of the data that we consider.

(b) SS performance compared to FC performance

Table 10 shows FC performance at the three largest interstimulus intervals compared to single stimulus performance, at the two base durations T=150 and T=250. The data is averaged over the 3 Os. In this table, SS performance in discriminating between T and $T+\Delta T$ is the same as FC performance when the intervals are separated by an ISI of 2 seconds. In the individual data shown in Figures 15 and 16, there is some variability in the level of SS performance as compared to FC performance, but there is no tendency for $\hat{P}(C)$ from the FC task to be consistently larger than $\hat{P}(C)$ from the SS task.

Comparison of performance levels from two different tasks (FC and SS) is meaningful only inasmuch as the results allow us to test and perhaps rule out certain models. Even though the time information in a single interval may be coded in the same way in the two tasks, a difference in performance may be expected from differences in how the choice of a response is made on the basis of the coded information. With an adequate model of the processes involved in the discrimination of very brief time intervals we should be able to relate performance under the two conditions. That

Table 10

Forced-choice performance at the three largest interstimulus intervals compared to single stimulus performance at two base durations.

	Forced-Choice (N = 720 each)			Single Stimulus (N = 900 each)
Base Duration	ISI=1/2	ISI = 1	ISI = 2	
150	.886	.869	.847	.853
250	.808	.815	.792	.787

is, by applying a model to the data from one task, we should be able to obtain estimates of parameters of the model with which we can then predict performance in the other task. In the theoretical analysis we attempt to do this with the quantal onset-offset model, one version of which predicts that FC and SS performance in discriminating between two intervals will be the same. Creelman's model predicts that FC performance will be better than SS, while the quantal counting model has not yet been successfully adapted for the SS situation.

(c) The effect of base duration on performance

In the FC task, the stimulus alternatives cover a range of base durations from 115 to 250 msec. which overlaps the two ranges covered in experiment 2. It is of interest to determine whether the functional relation between $\hat{P}(C)$ and base duration obtained here is consistent with that obtained in experiment 2. Figure 17 shows $\hat{P}(C)$ as a function of T_s, with the data averaged over all 3 Os and the 3 longest ISIs. Figure 18 shows the same function for individual Os. In Figure 17, between $T_s = 150$ and $T_s = 250$ there is a monotonic decrease in $\hat{P}(C)$. However, in the individual data one of the three Os shows an increase in $\hat{P}(C)$ as T_s increases from 115 to 215, while for the two others $\hat{P}(C)$ decreases monotonically over the same range.

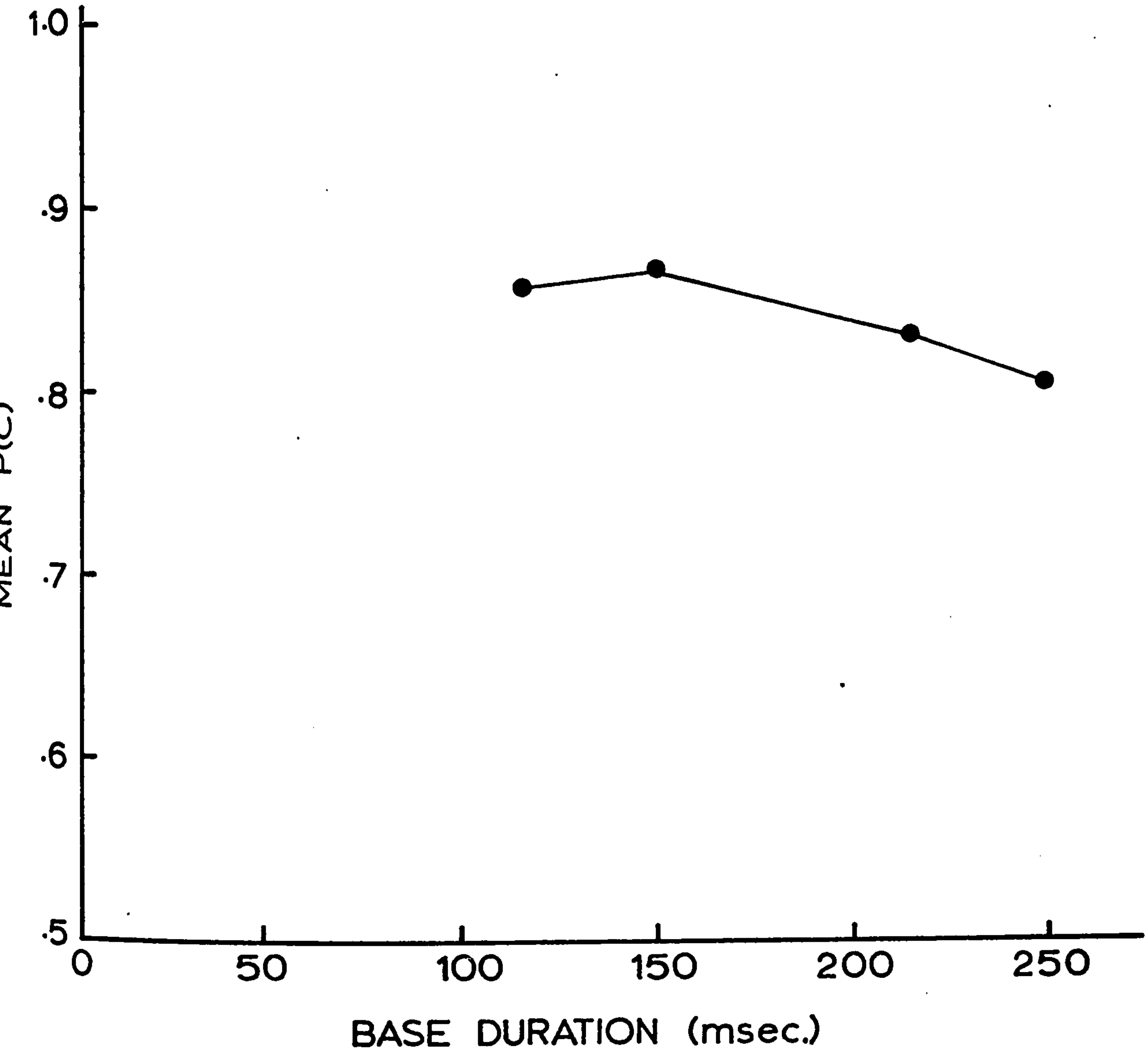


FIGURE 17: Overall effect of base duration upon performance at the three longest ISIs combined. Data averaged over 3 Os.

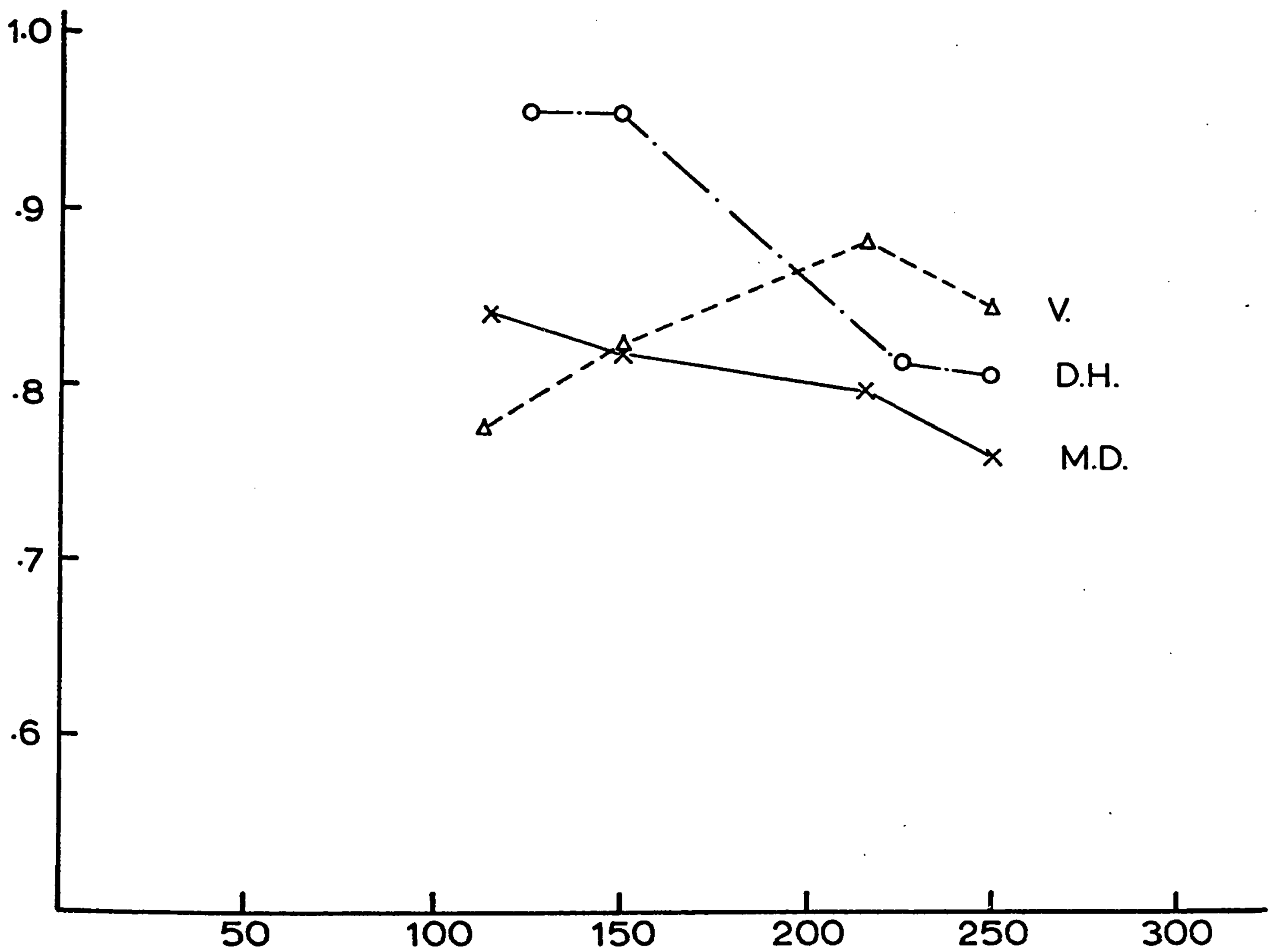


FIGURE 18: $\hat{P}(C)$ as a function of base duration, for individual Os. Data from the three longest ISIs combined. N=720 trials per point.

The effect on performance of increasing the base duration in this experiment, together with similar results obtained in experiment 2, suggest that the effect of base duration on performance can vary between individual Os, and may depend on what portion of the base duration range is being considered. In experiment 2 a monotonic decreasing function was obtained from several Os, but there were others for whom this was not true. When the data from a number of Os is averaged, $\hat{P}(C)_{av}$ may be constant over some range of base durations, but this is not necessarily true of each observer.

2.5 Experiment 4: Psychometric functions

In this experiment, psychometric functions were obtained at two base durations, $T_s = 100$ and $T_s = 200$. The original intention was to use the data to test the quantal counting model, but the data is also analyzed in considerable detail in the theoretical analysis section with respect to each of the other models presented in 1.4. In addition, we can now also compare these results with those from two other series of experiments. One interesting comparison which can be made is between FC and SS psychometric functions. Do the FC and SS psychometric functions for empty auditory intervals coincide, as did the psychometric functions for visual dark flashes when the base duration was 100 msec.? The rationale for making this comparison has already been outlined in the discussion of experiment 3. The second comparison we make is with the results from the first three experiments presented in this chapter. We have now examined the influence of base duration on performance in three experiments where one value of ΔT was used with one or more base durations. However, the discrimination between T_s and $T_s+\Delta T$ may be influenced by the set of stimulus alternatives within which these intervals occur, and this factor may have an effect on the functional relation between $\hat{P}(C)$ and T_s.

Procedure

Within each session, T_s was fixed. Five values of ΔT were used. On any trial either $S_1 = \{T_{vi}, T_s\}$ or $S_2 = \{T_s, T_{vi}\}$ could occur, with T_{vi} taking on any one of 5 values. The maximum ΔT was 30 msec. for 4 Os, and 50 msec. for a fifth. The set of ΔT were chosen after two practice sessions in which some idea was obtained of how large ΔT would have to be in order that $\hat{P}(C) \cong .9$ when $T_s = 100$. Sixteen sessions were run at each base duration, divided into eight with $T_s = 200$, sixteen with $T_s = 100$ and a final set of eight with $T_s = 200$. Two Os (PL, KL) did not complete the entire series, but both functions are based on at least 300 trials per point, ($\sigma < .015$). For the other 3 Os, the functions are based on nearly 800 trials per point, ($\sigma < .009$). PL and V were naive Os; the other three had participated in experiments on duration discrimination involving adjacent empty intervals. The boundaries of the intervals were 6 msec. pulses, and the intervals on each trial were separated by an ISI of 2 seconds.

Results

Table 11 shows $\hat{P}(1|S_1)$, $\hat{P}(2|S_2)$ and $\hat{P}(C)$ at each base duration. Appendix 3 gives the data over blocks of 8 sessions. The psychometric functions, $\hat{P}(C)$ as a function of

Table 11

Summary of performance - Experiment 4

	$T_s = 100$						$T_s = 200$				
	ΔT	$P(1\|S_1)$	$P(2\|S_2)$	P(C)	d'		ΔT	$P(1\|S_1)$	$P(2\|S_2)$	P(C)	d'
RM	6	.849	.855	.85	2.12		6	.801	.713	.76	1.40
	12	.960	.942	.95	3.30		12	.903	.857	.88	2.36
(N=750)	18	.994	.991	.99	4.64	(N=800)	18	.967	.962	.96	3.80
	24	1.	1.	1.	-		24	.975	.977	.98	4.10
	30	1.	1.	1.	-		30	.987	.989	.99	4.64
HS	6	.674	.708	.69	1.00		6	.621	.635	.63	.66
	12	.827	.890	.86	2.18		12	.730	.721	.73	1.19
(N=800)	18	.920	.961	.94	3.15	(N=800)	18	.775	.835	.80	1.76
	24	.939	.982	.96	3.60		24	.820	.912	.87	2.26
	30	.968	.998	.98	4.64		30	.876	.932	.90	2.64
V	6	.520	.639	.58	.41		6	.698	.650	.67	.91
	12	.659	.832	.75	1.36		12	.790	.777	.78	1.58
(N=850)	18	.815	.907	.86	2.26	(N=800)	18	.816	.889	.85	2.14
	24	.882	.962	.92	2.92		24	.839	.929	.88	2.46
	30	.922	.988	.95	3.72		30	.887	.938	.92	2.78
PL	6	.722	.671	.70	1.02		6	.782	.553	.66	.9
	12	.917	.819	.87	2.32		12	.929	.743	.84	2.11
(N=290)	18	.923	.928	.93	2.87	(N=360)	18	.912	.867	.89	2.47
	24	.979	.972	.98	3.93		24	.944	.945	.94	3.19
	30	.986	1.	1.	-		30	.956	.972	.96	3.63
KL	10	.471	.694	.58	.43		10	.411	.705	.56	.32
	20	.469	.815	.64	.50		20	.460	.804	.63	.74
(N=770)	30	.615	.868	.74	1.44	(N=400)	30	.638	.851	.74	1.40
	40	.702	.915	.81	1.92		40	.613	.893	.75	1.51
	50	.775	.945	.86	2.41		50	.643	.977	.85	2.41

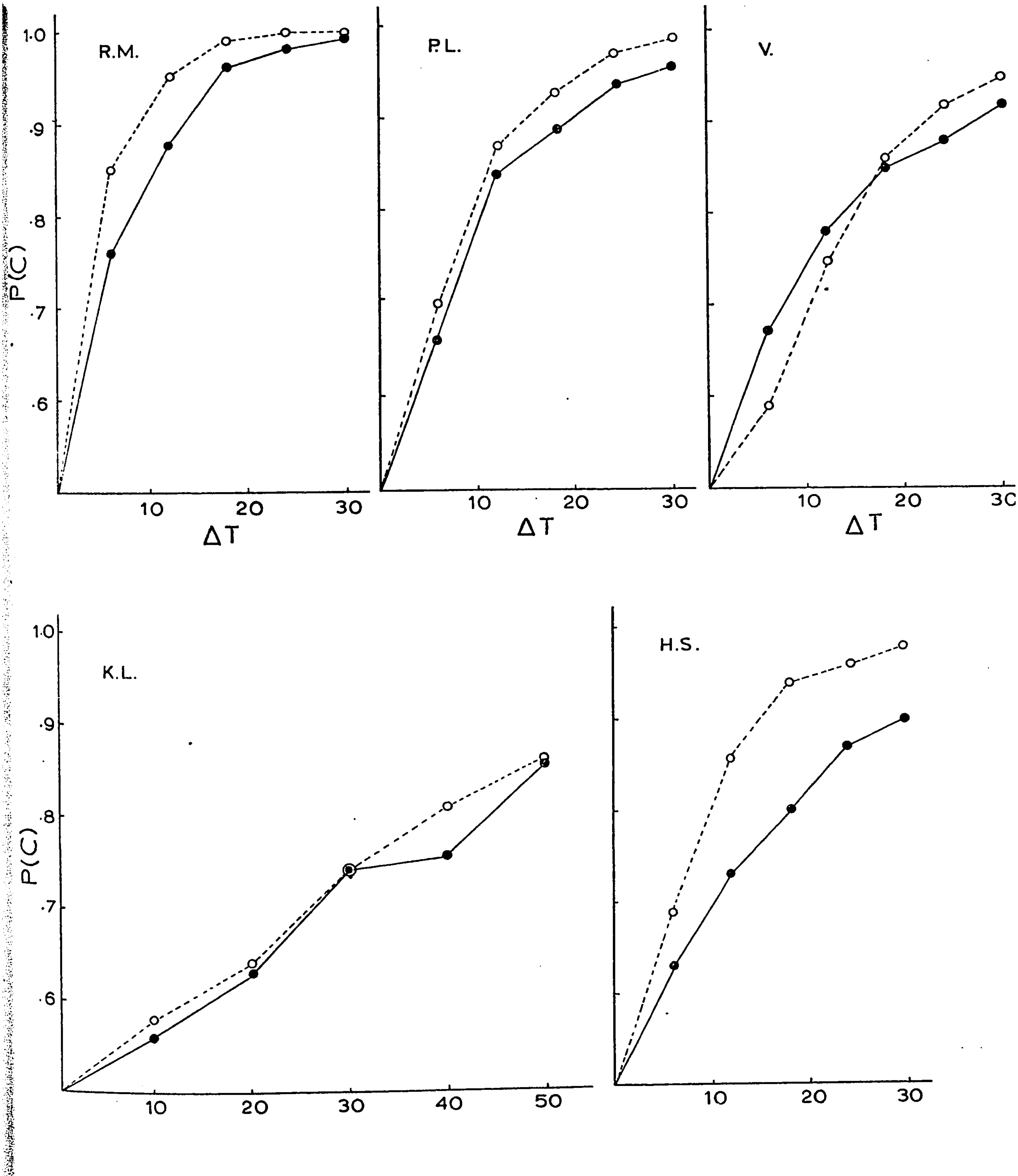
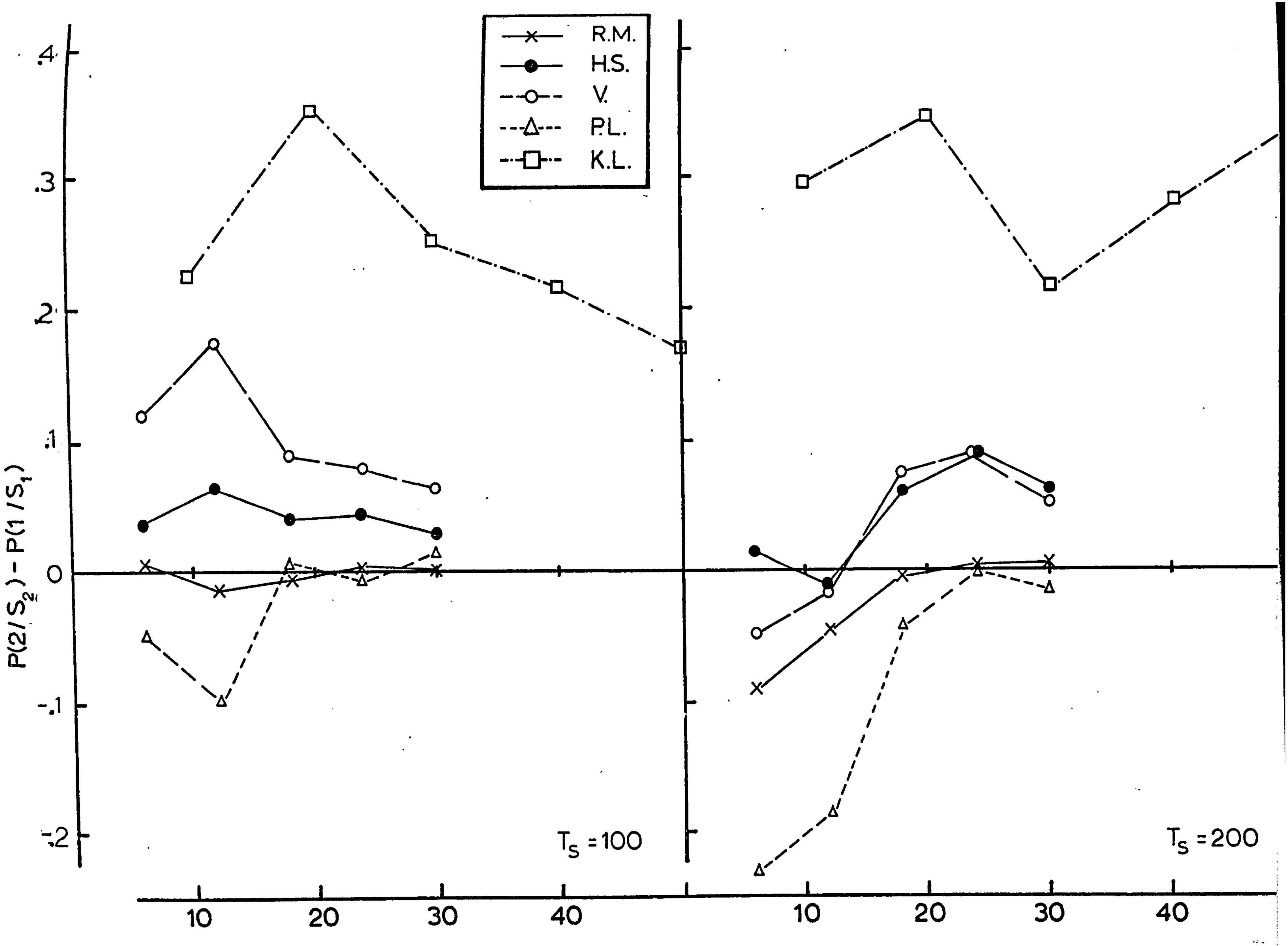


FIGURE 19: Duration discrimination functions for empty auditory intervals. T_s=100 [o----o] and T_s=200 [●——●].

FIGURE 20: The difference between the conditional probabilities of a correct response, $[\hat{P}(2|S_2) - \hat{P}(1|S_1)]$ at each value of ΔT. Individual data for each of two base durations.



R.M.
H.S.
V.
P.L.
K.L.
.4
.3
.2
.1
0
-.1
-.2
P(2/S2) - P(1/S1)
Ts =100
Ts =200
10
20
30
40
10
20
30
40

ΔT, are shown in Figure 19. These are monotonic increasing functions, concave downward for four of the five Os. For one O (KL), performance could be described by a zero intercept straight line.

The effect of a change in base duration is not consistent over all Os. For one (KL) there is essentially no difference in performance at the two base durations. For another (V) performance with $T_s = 200$ is better than at $T_s = 100$, for $T \leq 18$ msec. For the remaining three Os, $\hat{P}(C)$ is larger at 100 than at 200 for all values of ΔT.

Comparing $\hat{P}(1|S_1)$ with $\hat{P}(2|S_2)$, definite response preferences are seen in several, but not all, instances. The difference $[\hat{P}(2|S_2)-\hat{P}(1|S_1)]$ as a function of ΔT is shown in Figure 20, for each O at the two base durations. When the proportions $\hat{P}(2|S_2)$ and $\hat{P}(1|S_1)$ are each based on 400 trials, a maximum estimate of the standard deviation of their difference is .018. Hence for 3 Os (RM, HS and V) any difference larger than .035 is significant at the .05 level.* For KL at T_s=100, and PL at both base durations, N>150 and the criterion which is applied is .057. According to the above criteria, then, $\hat{P}(2|S_2)$ is greater than

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* Any difference which is greater than $(z_{.975})\cdot(\hat{\sigma}_{max})$ will be significant at the .05 level; there may be others less than this value which are significant as well.

$\hat{P}(1|S_1)$ over most of the range of ΔT in at least 6 out of 10 cases. For one O, this preference is reversed; when $\Delta T < 15$, $\hat{P}(2|S_2)$ is less than $\hat{P}(1|S_1)$ for both base durations. A very striking response preference is seen with KL, for whom it was also found that $\hat{P}(1|S_1)$ was less than .5 until $\Delta T > 20$ msec.

Discussion

(a) Comparing FC and SS performance

Figure 21 shows averaged psychometric functions, $\hat{P}(C)_{av}$ as a function of ΔT; the data is averaged over the 4 Os who had the same set of ΔT. For comparison, in the same figure are shown averaged psychometric functions obtained from a study using a SS task, for one group of 3 Os with base durations 250 and 300 msec., and another group of 5 Os with base durations 50 and 100 msec. (Kristofferson and Allan, 1971). In the SS task, only one value of T_s and ΔT were used within each session, selected from 2 values of T_s and 4 of ΔT. The interval markers were the same as those used in this experiment, except that the duration of the auditory pulse was 10 msec. instead of 6 msec. In the FC task used in this experiment, 5 values of ΔT are randomly intermixed within each session, but only one base duration occurs over a series of sessions. Hence T is always the short interval in each pair.

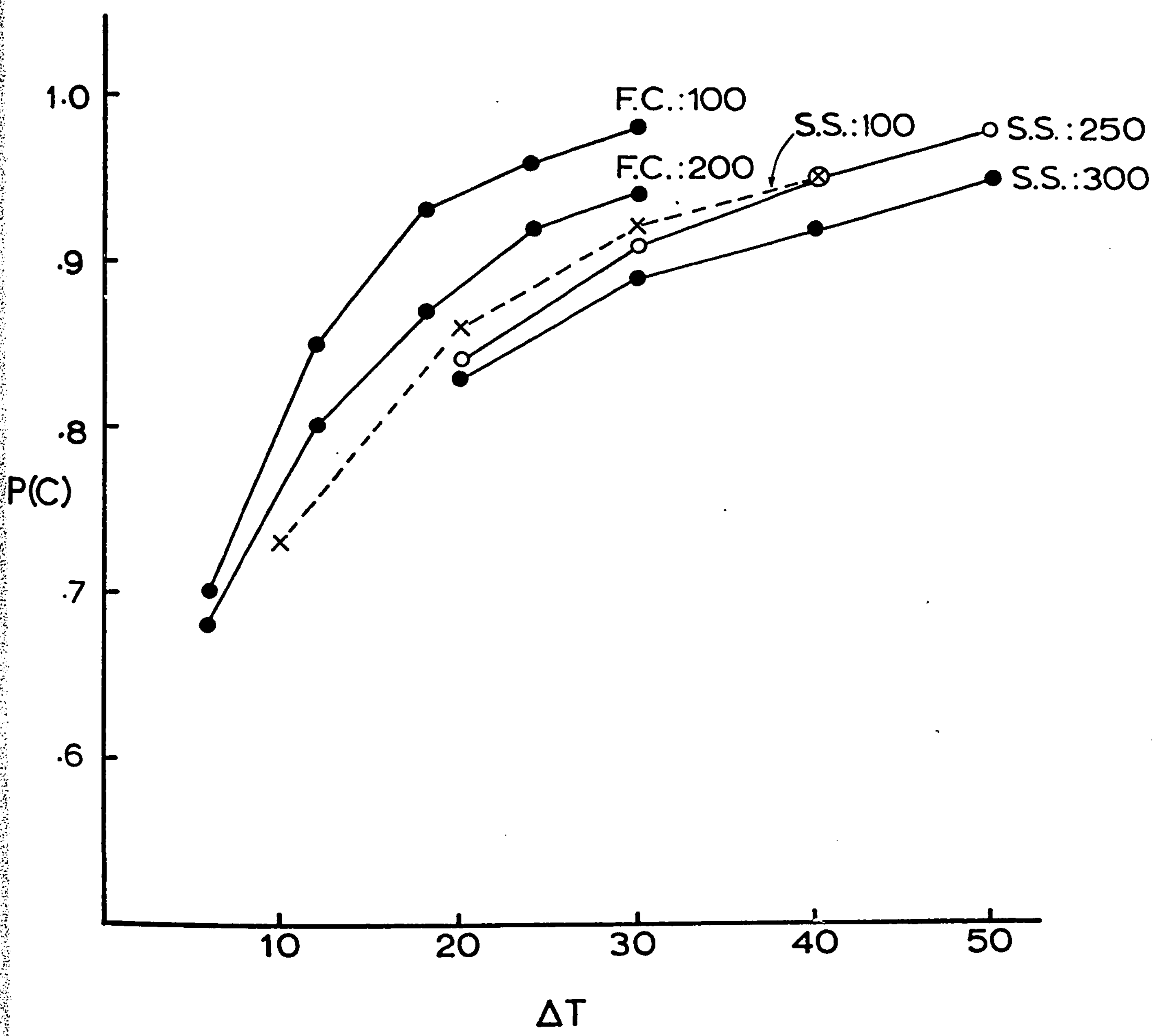

FIGURE 21: Averaged psychometric functions from the FC and SS procedures for empty intervals bounded by brief auditory markers. The base duration is indicated on each function.

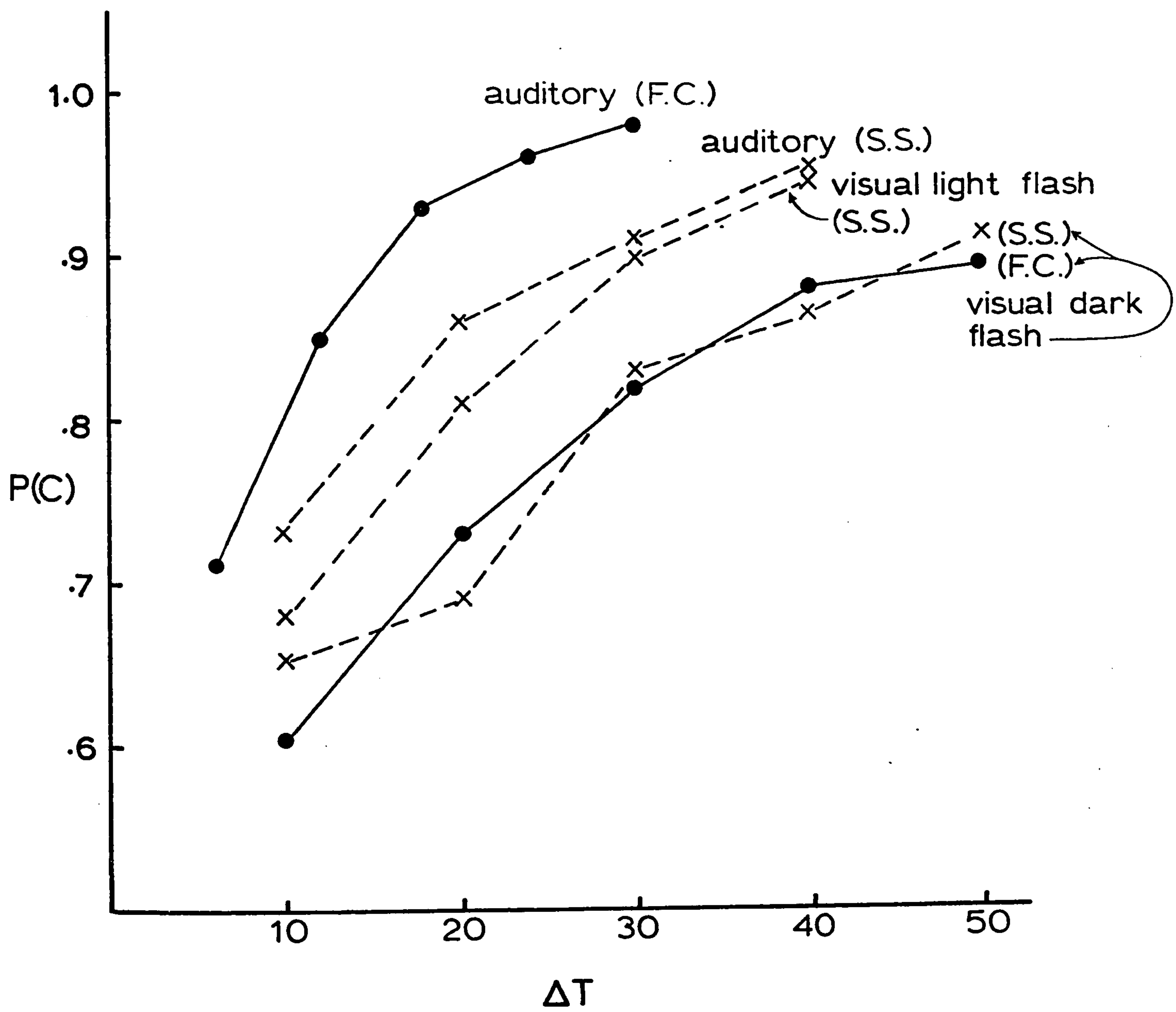

FIGURE 22: Averaged psychometric functions for base duration T=100, for intervals defined by visual and auditory markers. [•———•] FC procedure
[x-----x] SS procedure

The same situation held in the SS case; T was always designated as the short interval.

In Figure 21, the upper portions of the SS psychometric functions look similar to the FC functions, in that for any particular value of T_s they are monotonic functions concave downwards, and as T_s increases they appear to shift slowly downward. Moreover, the psychometric functions for empty auditory intervals are higher than those obtained when the time intervals are defined in the visual modality, for the same base duration. In Figure 22 there is a large discrepancy between the FC and SS functions at T=100 which can be contrasted to the close similarity between the FC and SS functions found by McKee et al. for visual dark flashes. We cannot draw any firm conclusion as to whether the FC and SS functions at T_s=200 would coincide, although it seems possible that they would. For ΔT ranging from 20 to 30 msec., the difference between $P(C)_{av}$ at T_s=200 with the FC task, and $P(C)_{av}$ at T_s=250 with a SS task is only about .04*.

What are the implications of a substantial discrepancy between the FC and SS psychometric functions at T_s=100, but not at T_s=200? The use of a single base duration over

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* Note that in the FC case we have not included the psychometric function of KL, whose performance is very much lower than that from the four other Os over the range 6-30 msec. We could include his function by reading off from Figure 19 the values of $\hat{P}(C)$ corresponding to the values of ΔT used for the other Os. However, this has not been done since his performance seems to be qualitatively different from the performance of the four other Os.

many successive sessions in the FC task may be the reason for this outcome. In experiment 3 we found a close similarity in performance from the SS and FC procedures at T_s=150 and T_s=250, even though the Os had very little experience with the SS task as compared with the FC. This raises the question as to whether the discrimination between T_s and $T_s+\Delta T$ when T_s=100 can be made on a different basis than that involved when T_s=200, under the procedure used in this experiment; it may be that randomizing several base durations within a session is a necessary experimental condition for obtaining data which reflect the same processes at different base durations, when auditory intervals of base durations of the order of 100 msec. or less are involved. We consider this possibility in more detail in the next section, after comparing performance from experiments 1-3 with the averaged psychometric functions obtained from this experiment.

(b) FC performance as a function of the set of stimulus alternatives

Although the task in experiments 1 to 4 has always been defined to the O in the same way (except for the few SS sessions in experiment 3), the sets of stimulus alternatives within a session differ in several respects. There are strong indications that the discrimination between T and $T+\Delta T$ is influenced by the composition of the set of stimulus alternatives within which T and $T+\Delta T$ occur. In this experiment, one value of T occurs over many sessions, while in experiments 1B, 2 and 3, several values of T are intermixed. Also, in this experiment and in 1A, T is the shortest interval which occurs over a number of sessions, while in experiments 2 and 3, T can be longer or shorter than the interval occurring with it on any trial. When we compare performance from experiments 1, 2 and 3 with the averaged psychometric functions, we find that choosing the stimulus alternatives such that T can be either longer or shorter than the interval accompanying it on a trial may have a substantial decremental effect on the value of $\hat{P}(C)$ in discriminating between T and T+ΔT; this decremental effect does not appear to be due simply to having a number of different base durations present within a session.

The comparisons we wish to consider are illustrated in Figure 23, where the solid curves are the averaged FC psychometric functions of Figure 21, obtained from 4 of the 5 Os in this experiment. The points in Figure 23b represent individual data from experiment 3, with the solid (open) symbols representing performance on the SS (FC) task. The abscissa for each point is the value of ΔT for that O, and the base duration 150 (250) is symbolized by a triangle (circle). In Figure 23a, the points represent data averaged over a number of Os; the number beside each point indicates the base duration to which ΔT was added.

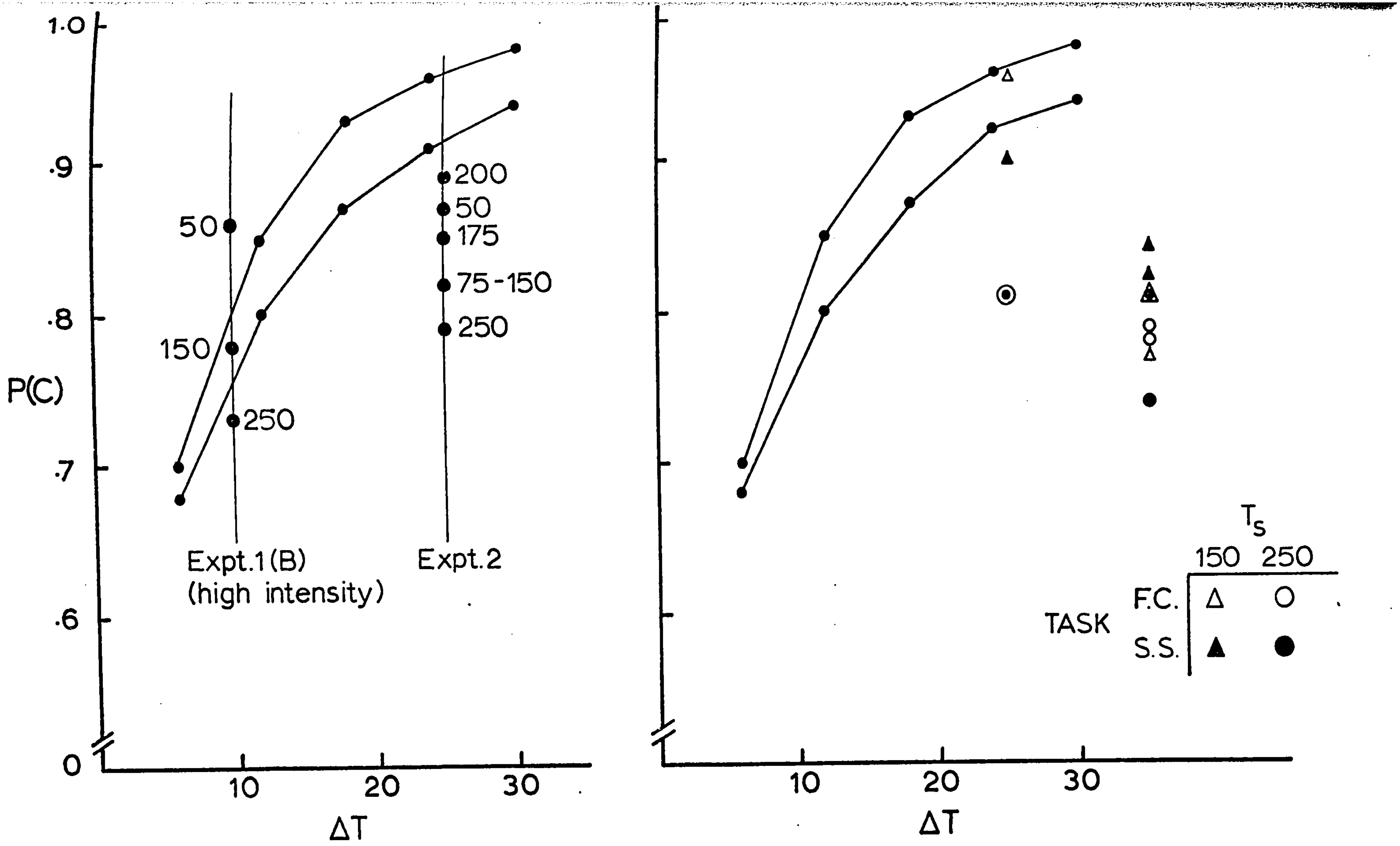

FIGURE 23: (a) Averaged data from experiments 1 and 2 compared to averaged psychometric functions from experiment 4. The numbers beside each point indicate the base duration.

(b) Individual data from experiment 3 compared to the averaged psychometric functions. $\Delta T=25$ for one O; $\Delta T=35$ for the other two Os. The task and base duration for each data point is indicated by the legend.

In all cases in experiment 3, except for DH at T=150, SS performance was less than that obtained from Kristofferson's Os who had had only the SS task over many sessions. (The ordinates in Figure 23b can be compared to the SS psychometric functions in Figure 21.) On the other hand, FC performance for T_s=150 and T_s=250 was also less than what would be expected on the basis of the averaged FC psychometric functions (Figure 23b), and this discrepancy cannot be reasonably attributed to differences in the amount of experience with the FC task in the two experiments. In experiment 2, we have the same kind of discrepancy between $\hat{P}(C)_{av}$ and the performance which could be expected from the psychometric functions; for the four Os who had ΔT = 25 msec., $\hat{P}(C)_{av}$ is .82 for $75 \leq T_s \leq 150$, which is .10 less than the corresponding value of $\hat{P}(C)$ on the FC psychometric function for T_s=200. Moreover, this discrepancy is unlikely to be due to the fact that there are several base durations used within a session, as compared to only one over several sessions. For we can look at where the averaged $\hat{P}(C)$ values from experiment 1(B) occur with respect to the FC psychometric functions. The base durations 50, 150 and 250 are randomly intermixed within each session, and ΔT is fixed at 10 msec. In Figure 23a, the values of $\hat{P}(C)_{av}$ from the high intensity condition lie just about where one would expect to find them, if they were points on psychometric functions for T_s=50, 150 and 250, obtained in the same way as those we have for T_s=100 and 200. However, the ordering of the values for $\hat{P}(C)_{av}$ from experiment 2, for base durations $50 \leq T_s \leq 275$, does not behave in any systematic way.

From the above considerations we tentatively conclude that choosing the stimulus alternatives such that T can be either shorter or longer than the interval accompanying it can have a considerable decremental effect on the ability to discriminate between T and T+ΔT*. It would be of interest to know why this decrement occurs. Why should having to discriminate between T and T-ΔT influence the discrimination between T and T+ΔT? Are we discouraging the use of an alternative - or an additional - cue, which, as is suggested in Kristofferson and Allan (1971), may be available for very short durations defined by auditory stimuli? Are we forcing the O into another mode of operation by using the pair of variable intervals T-ΔT and T+ΔT with the standard interval T? Or does this simply reflect a property of the processes involved? For example, changing the makeup of the set of stimulus alternatives may involve a change in the number or location of criteria on the "internal time" dimension, as

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* Note that this decremental effect does not invariably occur. For example, it does not occur with DH in experiment 3, when T=150, while it does for T=250. In experiment 2, it does not occur at all for RM, or for DQ and IR (both naive Os) when T=75.

might be assumed in the second version of the onset-offset delay model. This possibility is explored in the theoretical analysis.

If we were to entertain the notion that more than one process is involved for intervals in the range from approximately 50 to 300 msec., we would need to establish criteria by which we could reliably distinguish between different modes of operation. These criteria might be obtained by determining whether, for T=50 as compared to T=300, we find consistently that

1. the presence of $(T-\Delta T, T)$ influences the discrimination between T and $T+\Delta T$,
2. there is a large response preference,
3. performance on SS and FC tasks are at the same level.

Other criteria might be used as well, such as whether $\hat{P}(C)$ is sensitive to a small (50 msec.) change in base duration, when ΔT is fixed.

2.6 Summary of the results

This section contains a review of what are considered to be the more important experimental results; in some cases the implication of these results are briefly mentioned.

1. Varying the intensity of the auditory pulses bounding brief empty intervals has a small effect on $\hat{P}(C)$, which does not decrease as the order of magnitude of the intervals increases. The effect as measured by the difference $[\hat{P}(C|\text{high})-\hat{P}(C|\text{low})]$ is the same for the smallest (50,60 msec.) intervals as for the largest (250,260 msec.), and a change in intensity of 37 db results in a change of about .03 in the averaged $\hat{P}(C)$ at these two base durations.

Although $[\hat{P}(C|\text{high})-\hat{P}(C|\text{low})]$ is consistently larger at T=150 than at T=50 or T=250, the difference $[\hat{P}(C|\text{high})-\hat{P}(C|\text{medium})]$ does not show this feature; when this latter difference is plotted as a function of base duration, no consistent pattern is seen among the 3 Qs.

The discrimination between T and T+ΔT is not sensitive to large changes in the intensity of the auditory signals defining these intervals. We conclude that energy dependent cues are not important in coding the duration of a brief interval.

2. $\hat{P}(C)$ in discriminating between T and T+ΔT, as T increases over the range 50-300 msec., does not necessarily decrease monotonically, as would be predicted from other experiments using the FC procedure and auditory stimuli (Abel, Creelman). The shape of the function relating P(C) and T may depend on whether or not several base durations are randomly intermixed, and on whether or not the interval T can be long or short with respect to the other interval occurring with it on a trial.

3. When several base durations are intermixed within a session, as compared to having only one per session, the effect of uncertainty as to which T will occur on a trial is generally small, and not specific to any one base duration (experiment 1). This is true in particular for those Os for whom $\hat{P}(C)$ at T_s=50 is much larger than $\hat{P}(C)$ at T_s=150 and 250.

If a large difference in performance between T=50 and T=250 is due to the availability of cues at T=50 in addition to, or more effective than, those used at T=250, then uncertainty as to the order of magnitude of the intervals occurring on a trial does not selectively disrupt the use of these cues.

4. Effect of varying the ISI from 0 to 2 seconds:

(a) $\hat{P}(C)$ increases as ISI increases from 0 to 1/2 sec. Some minimal separation is needed for asymptotic, optimal performance. This result is consistent with three interpretations:

i. at small ISIs, the measurement or observation of the second interval is not independent of that of the first, as would be predicted by the quantal counting model, or it may be that

ii. some time is needed to make and store a decision about the first interval, as would be predicted by the multiple decision strategy version of the onset-offset delay model, or

iii. if the first interval is used as a criterion against which to compare the second interval, some time is necessary to prepare the criterion for use.

(b) At ISIs greater than 1/2 sec., after asymptotic performance is obtained, there is no decrease in $\hat{P}(C)$ even though both intervals are being used on at least some non-zero proportion of the trials.

There is no loss of the information in the first interval during ISIs of a magnitude equal to those used in other experiments involving intensity and frequency discrimination, where a monotonic decrease in $\hat{P}(C)$ has been obtained.

5. Under certain conditions, forced choice $\hat{P}(C)$ is at the same level as single stimulus $\hat{P}(C)$ (experiment 3). The

implication of this result is that any model assuming that T and T+ΔT give rise to random variables with the same variance, cannot also assume that the decision is based on a "difference" between two internal measures, as is often done in SDT models for FC discrimination tasks. If the variance increases with increasing base duration (as in Creelman's model), no general statement can be made.

6. Response preferences:

For a given pair of intervals, T and T+ΔT, $P(S_1) = P(S_2) = 1/2$ so that the responses "1" and "2" are called for equally often. If there is no response bias, $P(1) = P(2)$. Since $P(2) - P(1) = P(2|S_2) - P(1|S_1)$, this is equivalent to saying that the probability of being correct when the second interval is longer should be the same as the probability of being correct when the first is longer. However, $\hat{P}(2|S_2) - \hat{P}(1|S_1)$ often assumes very large values. There are several interesting features of this bias, which strongly suggest that it is stimulus dependent, and it is for this reason that the term "preference" is used instead of "bias".

(a) The preference seems to depend on the size of T_s. In experiments 2 and 3, $\hat{P}(2|S_2)$ is constant or increases while $\hat{P}(1|S_1)$ decreases, often by much larger amounts. $\hat{P}(2|S_2)$ varies over a much smaller range than does $\hat{P}(1|S_1)$

(b) The preference is seen in experiment 4, where only one base duration is used over many sessions, and several values of ΔT are randomly intermixed.

(c) In experiment 3, there is no indication that this preference is less with the longest ISI, or at the largest base duration.

(d) The preference is seen in experiment 1, where there is one ΔT and one T_s over 3 sessions. When changes are made in the intensity of the boundaries, $\hat{P}(1|S_1)$ seems to be less stable than $\hat{P}(2|S_2)$.

Any adequate quantitative model for duration discrimination of very brief intervals, with a FC task, will have to allow for differences in the probabilities of a correct response conditional on whether the longer interval occurs first or second. A difference in the conditional probabilities of a correct response may reflect more than "response bias"; it may also reflect properties of the process by which the time intervals are coded.

III. THEORETICAL ANALYSIS

3.1 The quantal counting model

(a) Fitting the psychometric functions

Estimates of three parameters - q, k and k' - involved in the quantal counting model can be obtained by fitting the model to the data from experiment 4. These parameters were determined by minimizing a chi-square variable defined in terms of the two quantities $P(1|S_1)$ and $P(2|S_2)$ which are predicted by the equations

$$P(1|S_1) = (\alpha+\gamma\alpha) - \alpha' k(1+\gamma) + \gamma^2(1-k')$$

$$P(2|S_2) = (\alpha+\gamma\alpha) + \beta k(1+\gamma) + \gamma^2 k' \quad . \tag{3-1}$$

For q fixed at some value, we can find values for k and k' minimizing the sum of the two chi-squared variables

$$\chi^2(P1) = n \cdot \sum_{i=1}^{5} [P1_i - \hat{P}_i(1|S_1)]^2/[P1_i \cdot (1-P1_i)]$$

and (3-2)

$$\chi^2(P2) = n \cdot \sum_{i=1}^{5} [P2_i - \hat{P}_i(2|S_2)]^2/[P2_i \cdot (1-P2_i)]$$

where $\hat{P}_i(1|S_1)$ is $\hat{P}(1|S_1)$ obtained at the i'th value of ΔT.

$P1_i$ is $P(1|S_1)$ predicted by using equation (3-1), at the i'th value of ΔT, and n is the number of observations on which $\hat{P}(1|S_1)$ is based. $\hat{P}_i(2|S_2)$ and $P2_i$ are defined in a similar way.

The parameter q was varied over a wide range and a pair of values for k and k' minimizing the sum $\chi^2 = \chi^2(P1) + \chi^2(P2)$ was obtained at each q. The value of q giving an overall minimum for the sum was taken as $\hat{q}$, and the corresponding values of k and k' are $\hat{k}$ and $\hat{k}'$. These are shown in Table 12 when the data combined over 16 sessions for each base duration T_s are used in the estimation procedure. In Table 13, the data from sets of 8 sessions (Appendix 3) have been used.

Assuming that the χ^2 sum has 10-3=7 degrees of freedom, (since three parameters are varied independently in determining the set giving the minimum value for this sum), χ^2 must be greater than 14.1 to be significant at the .05 level. Hence the total deviation of the predicted quantities $P(1|S_1)$ and $P(2|S_2)$ from those observed in experiment 4 is not significant at both base durations, for 3 of the 5 Os.

Table 14 gives the psychometric functions predicted by the model when q and k have the values given in Table 12. The fit to the data from experiment 4 looks good. $\chi^2(P(C))$ can be calculated, but we cannot assess its significance since the degrees of freedom cannot be specified. The sum of the squared deviations $\sum_{i=1}^{5} [P_i(C) - \hat{P}(C)]^2$ of the predicted values from those obtained at each ΔT is listed in the last column of Table 12 ; we will be able to compare the fit of these predicted functions to the data with the fit of predicted functions when $\hat{q}$ and $\hat{k}$ are obtained in another estimation procedure which in effect treats the model as a one parameter model.

We are primarily interested in the values of $\hat{q}$ which in the model reflects a central periodic process. Are these estimates of q the same at both base durations for a given individual, and is there any consistency between Os in these estimates? In Table 12 there is a tendency for the values of $\hat{q}$ to cluster in the range 19-31 msec., although the minimum is 15 and the maximum is 82. There are 6 out of 10 instances (5 Os x 2 base durations) where $\hat{q}$ is between 19 and 31 msec.; where $\hat{q}$ is at the extremes of this range, (for RM and V), χ^2 has its largest, significant values.

Within an O, $\hat{q}$ tends to be larger at T=200 than at T=100, although in Table 12 the estimates are within 5 msec. of each other for 3 of the 5 Os. This is small compared to the discrepancy seen with HS, for whom q at T=200 is nearly twice what it is at T=100. For the fifth O (KL) the estimates differ by 20 msec., even though it was with this O that the psychometric functions obtained at the two base durations differed the least. For this O there was a large range of q - about 20 msec. - where χ^2(P1) and χ^2(P2) were both less than 5; although when T=100 the minimum value

Table 12

Estimates of q, k and k' obtained by fitting the quantal counting model to $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ from experiment 4. (Data from all sessions at each base duration are combined, for each O.) SS is the sum of squared deviations of the obtained $\hat{P}(C)$ values from those predicted by using $\hat{q}$ and $\hat{k}$. These values are to be compared with the fit of the psychometric functions predicted when we use the values of $\hat{q}$ and $\hat{k}$ given in Table 15. The astericks indicate where the sum $[\chi^2(P1) + \chi^2(P2)]$ is significant at the .05 level.

Obs.	T_s	$\hat{q}$	$\hat{k}$	$\hat{k}'$	$\chi^2(P1)$	$\chi^2(P2)$	SS
PL	100	22	.02	.35	2.2	.8	.00011
	200	26.3	.05	.20	3.7	2.6	.00055
HS	100	23	.05	.55	1.9	2.6	.00036
	200	41.2	.10	.50	4.5	4.3	.00091
V *	100	30.5	.10	.55	12.4	7.1	.00298
*	200	27	.20	.15	7.5	13.7	.00222
KL	100	66.5	.25	.85	4.8	4.1	.00135
	200	82.5	.24	.75	7.6	3.2	.00282
RM *	100	15	0	.5	8.8	8.9	.00211
*	200	19.5	.05	.05	2.1	22.5	.00210

Table 13

Estimates of q, k and k' obtained by fitting the quantal counting model to $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ from experiment 4. For those Os who had 16 sessions at each base duration, the data is divided into two sets (A and B) at each T_s, and $\hat{q}$, $\hat{k}$ and $\hat{k}'$ are obtained separately for each set of data.

Obs.		T_s	$\hat{q}$	$\hat{k}$	$\hat{k}'$	χ^2(P1)	χ^2(P2)
HS		100A	29	.10	.60	3.4	5.3
		100B	19	.04	.40	4.1	2.3
		200A	42	.10	.45	2.1	2.6
		200B	31.6	.20	.15	2.0	4.0
V	*	100A	31.9	.10	.60	10.1	15.9
		100B	30	.05	.65	4.2	2.4
	*	200A	26.6	.15	.30	9.9	5.2
		200B	31.2	.15	.20	4.7	5.9
RM		100A	17.5	0	.50	9.1	4.1
		100B	12.1	0	.68	11.2	2.8
	*	200A	23.8	.02	.35	11.0	13.4
		200B	19.7	.05	.10	5.7	1.6
KL		100A	64.5	.30	.75	7.9	3.9
		100B	82.5	.08	.65	2.8	1.2

Table 14

Psychometric functions predicted by the quantal counting model, using the values of $\hat{q}$ and $\hat{k}$ given in Table 12. $\hat{P}(C)$ is from the data listed in Table 11 of experiment 4. P(C) gives the predicted values.

	$T_s=100$			$T_s=200$		
Obs.	ΔT	$\hat{P}(C)$	P(C)	ΔT	$\hat{P}(C)$	P(C)
PL	6	.70	.704	6	.66	.670
	12	.87	.870	12	.84	.823
	18	.93	.933	18	.89	.891
	24	.98	.974	24	.94	.941
	30	1.0	.993	30	.96	.972
HS	6	.69	.682	6	.63	.622
	12	.86	.844	12	.73	.712
	18	.94	.934	18	.80	.789
	24	.96	.962	24	.87	.851
	30	.98	.980	30	.90	.899
V	6	.58	.634	6	.67	.631
	12	.75	.752	12	.78	.755
	18	.86	.855	18	.85	.849
	24	.92	.923	24	.88	.890
	30	.95	.942	30	.92	.921
KL	10	.58	.585	10	.56	.569
	20	.64	.670	20	.63	.636
	30	.74	.754	30	.74	.701
	40	.81	.808	40	.75	.765
	50	.86	.844	50	.85	.819
RM	6	.85	.807	6	.76	.715
	12	.95	.937	12	.88	.886
	18	.99	.996	18	.96	.958
	24	1.00	1.00	24	.98	.977
	30	1.00	1.00	30	.99	.989

for the sum occurred at q=62, the sum was just slightly larger and still not significant at q=82. In all the other cases the minimum value for χ^2 was very much better defined.

Using Table 13, we can evaluate the consistency of the estimates of q within 3 Os when estimates are obtained from successive blocks of 8 sessions. For 2 of these 3 Os, the two estimates at each base duration are again within 5 msec. of each other.

The mean of the 8 estimates from the four Os in experiment 4 who had the same set of values for ΔT is 25.6. This mean value is half as large as the mean estimated value of the parameter M in Kristofferson's two state version of his attention switching model for successiveness discrimination (1967). M is the interval between the time points at which attention can switch from one input channel to another. A second behavioral parameter taken to reflect this time base (Q) is obtained from modeling reaction time distributions. From the earliest experiments, the mean Q was about 50 msec., but from the analysis of further experiments (1969) he has obtained "small quanta" with an average value of 24 msec. He has not yet reported any attempt to account for the small and large quanta within the same theoretical framework. Nevertheless, it is of interest that both "small" and "large" quanta have been obtained in applying the onset-offset delay model to duration discrimination. The delays in the onset and offset of the internal interval are assumed to be independent uniformly distributed random variables, ranging from 0 to q msec. With light flashes, the mean value for $\hat{q}$ is about 24 msec. (A&K&W). With empty auditory intervals, the model begins to hold when the base duration is 450 msec., and the mean $\hat{q}$ is now 50 msec. (K&A, 1971).

In fitting the model to the psychometric functions of experiment 4, there is a procedure for obtaining estimates of two parameters q and k, in which $\hat{k}$ depends on $\hat{q}$. This procedure is based on a prediction derived in section 1.4, and restated here. The relation between P(C) and P (the combination $2\alpha + 2\alpha\gamma + \gamma^2$) reduces to

$$2P(C) = P(1-k) + k \tag{3-3}$$

as long as $\Delta T \leq (1-b_s)\cdot q$. (3-4)

Equation (3-3) implies that if we vary q over a wide range, and for each value of q we find the slope and intercept of the best fitting straight line for the plot of those points $(2\hat{P}(C), P)$ for which (3-4) holds, we should find some value of q such that two conditions are satisfied:

i. the sum of the slope (A) and the intercept (B) of this best fitting straight line is close to 1, and

ii. both A and B have values between 0 and 1.

This particular value of q is taken as $\hat{q}$, and $\hat{k}$ is determined from the slope and intercept; $\hat{q}$ and $\hat{k}$ can then be used to generate the entire predicted psychometric function, P(C) as a function of ΔT.

In experiment 4, five values of ΔT were used, so there can be 0 to 5 points satisfying the restriction (3-4) on ΔT. Since we need at least two points to determine a line, there are some values of q for which A and B cannot be determined. However, an indication of whether a good estimate of q for that O might be in this region was determined by examining the values obtained for A and B when q was just above and below this region. It usually was the case that $A+B \neq 1$.

Table 15 shows values of $\hat{q}$, $\hat{k}$ and $\sum_{i=1}^{5} [P(C) - \hat{P}(C)]^2$ which were obtained by the above procedure. A value of q was selected from the set of values at each base duration such that conditions i and ii were satisfied, and these two values were as close as possible to each other in magnitude. Table 15 indicates that we can find such a pair differing by at most 2 msec. The values of A and B from which $\hat{k}$ is obtained are also shown.

Estimates for q and k could not be determined by this procedure for RM at both base durations, and for V at T_s=100. For V there was no value of q such that B was

Table 15

Estimates of q and k obtained by treating the quantal counting model as if q were the only free parameter. SS is the sum of squared deviations of the obtained $\hat{P}(C)$ values from those predicted by using $\hat{q}$ and $\hat{k}$.

Obs.	$T_s = 100$					$T_s = 200$				
	$\hat{q}$	B	A	$\hat{k}$	SS	$\hat{q}$	B	A	$\hat{k}$	SS
V						25	.305	.727	.27	.00331
HS	25	.041	.952	.05	.00050	24.6	.340	.663	.34	.00101
PL	23	.022	.996	.00	.00079	24.6	.072	.917	.08	.00901
KL	82	-.002	.960	.04	.00390	82	.213	.780	.22	.00242

positive, and at the same time A was less than 1. (If the slope A is greater than 1, performance is better than the model predicts, even with k=0). For RM, q was varied upwards from 10 msec. There were only a few values of q for which A and B could be calculated in the range of these small q. In those cases, either A+B >> 1, or both A and B fell outside the bounds from 0 to 1. For this O we would need to determine $\hat{P}(C)$ for several values of ΔT below 12 msec., in order to have at least two values such that $\Delta T < (1-b_s)q$ when q<12.

In a way, the failure in the case of V to obtain an estimate of k and q at T_s=100 from the above analysis enhances our confidence in the estimates obtained for the other Os. (The failure in the case of RM is inconclusive.) The fact that we can obtain values for A and B between 0 and 1, such that A+B is very close to 1, is then not due to some formal property of the analysis, but because the data do conform to the predictions of the model.

The values of $\hat{q}$ shown in Table 15 do compare well with the estimates shown in Table 12 , except for HS at T_s=200 and KL at T_s=100. The advantage of the second procedure is that it does in effect treat the quantal counting model as a one parameter model. However, in the procedure first described we use all the data in estimating q and k - $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ at all five values of ΔT - and we obtain predicted psychometric functions giving a better fit to the experimental data than does the second procedure, where we use $\hat{P}(C)$ for as few as two values of ΔT. The values of $SS = \sum_{i=1}^{5} (P_i(C) - \hat{P}_i(C))^2$ shown in Table 12 are smaller than the corresponding values shown in Table 15.

(b) Accounting for the changes in performance as base duration changes:

We now determine how well the model can predict the conditional probabilities of a correct response $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$, from experiments 2 and 3, where four base durations were randomly intermixed over a series of sessions while ΔT was kept constant. Table 16 shows the values of q, k and k' giving an overall minimum for $\chi^2 = \chi^2(P1) + \chi^2(P2)$ for each set of base durations, for each O. This overall minimum χ^2 is obtained by fitting eight data points; it has 8-3 = 5 degrees of freedom so whenever it is greater than 11.1 the predicted values of $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ differ significantly (at the .05 level) from those obtained. The minimum values of χ^2 in Table 16 are very large and significant for all Os, except for RM for the set of larger T_s. Hence the model definitely does not provide a good description of the data from experiments 2 and 3 when we assume that q and k are the same at each of the four base durations within a session.

Table 16

Values of q, k and k' minimizing the sum $[\chi^2(P1) + \chi^2(P2)]$ for the conditional probabilities of a correct response from 4 base durations.

Experiment 2

	T_s range	q	k	k'	$\chi^2(P1)$	$\chi^2(P2)$
MD	50-150	64.5	.40	.45	5.8	18.9
	175-275	61.5	.25	.45	65.6	8.0
V	50-150	33.5	.25	0	25.5	11.4
	175-275	44.5	.15	.60	16.8	4.8
IR	50-150	36.5	.25	0	40.4	6.6
	175-275	44.5	.25	0	26.0	4.5
DQ	50-150	39.5	0	.6	26.8	21.2
	175-275	51.5	0	.6	4.9	13.5
HL	50-200	95.5	.35	.45	12.0	17.7
	150-300	99.5	.40	0	20.5	2.7
RM	65-150	18	.05	.30	7.7	19.9
	190-275	23.5	0	.75	3.6	2.9
Experiment 3 (ISI = 1 sec.)						
MD	115-250	61	.20	0	13.1	5.4
V	115-250	45.5	.30	.60	18.6	3.6
DH	125-250	48.5	0	.75	5.17	43.10

Differences in the psychometric functions at two different base durations can be accounted for by a change in one or both of the parameters q and k; in Table 12, $\hat{q}$ and $\hat{k}$ tended to be larger for T_s=200 than for T_s=100. In using the model to predict the data from sessions where several base durations are intermixed, we would have to assume that the period of the time base used in coding the intervals is the same for all base durations within the session but we can explore the possibility that k is not constant across base duration. This dependence of k on base duration may underlie the poor fit of the model to the data from experiments 2 and 3, compared to the relatively good prediction of the data from experiment 4. How well can the model fit the data from experiments 2 and 3 if we retain the requirement that q be the same for all base durations within a session, but relax this assumption for k and k'?

Table 17 shows parameter estimates obtained by using a modified version of the procedure used in fitting the psychometric functions. Using only four data points ($\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ from two base durations), we can determine which set of parameters (q, k, k') give an overall minimum for $\chi^2 = [\chi^2(P1) + \chi^2(P2)]$ which is now a sum of four terms, and has one degree of freedom. With one degree of freedom, χ^2 is significant at the .05 (.01) level when it is greater than 3.84 (6.63). We use this criterion to assess

Table 17

Estimates of q, k and k' obtained by relaxing the assumption that k and k' are the same for all base durations within a session. (Experiment 2) * indicates that χ^2 is large and significant with all q, for that pair of base durations.

Obs.		T_s	q	k	k'	χ^2(P1)	χ^2(P2)
MD	*	50, 75	35.5	.50	0	1.05	76.92
		125,150	35.5	.50	0	.87	1.86
		175,200	67.5	0	.45	.34	.93
		250,275	67.5	.40	.60	.37	.47
V	*	50, 75	43.5	.10	.75	17.06	1.74
		125,150	43.5	.05	.30	.39	.49
		175,200	42.8	.00	.75	3.18	.87
		250,275	42.8	.30	.45	.38	3.20
DQ	*	50, 75	27	.00	.75	.54	.56
		125,150	27	.25	.00	14.35	3.99
		175,200	32.1	.20	.00	.09	.52
		250,275	32.1	.20	.00	1.45	5.96
IR		50, 75	35.5	.10	0	.48	.83
		125,150	35.5	.35	0	3.51	1.89
		175,200	44.5	.15	0	.70	.20
		250,275	44.5	.35	.15	2.61	1.08
HL	*	50,100	93.5	.25	.45	5.28	16.86
		150,200	93.5	.40	.45	.14	.66
		150,200	99.1	.20	.15	1.30	.65
		250,300	99.1	.45	.00	.68	.13
RM	*	65, 75	17.9	.05	.45	9.89	12.5
		140,150	17.9	.05	.30	.27	.84
		190,200	17.9	.15	.45	1.18	.54
		265,275	17.9	.25	.45	3.20	.30

Table 17 - continued

Experiment 3: data from conditions with ISI = 1 sec. and ISI = 0.

Obs.	T_s	q	k	k'	χ^2(P1)	χ^2(P2)
MD	115,150	61.5	.15	0	.95	.11
ISI=1	215,250	61.5	.25	.15	1.51	1.68
ISI=0	115,150	55.5	.05	.75	2.85	.16
	215,250	55.5	.65	.15	1.22	.26
V	115,150	45.9	.40	.60	.60	2.51
ISI=1	215,250	45.9	.15	.60	.08	1.18
ISI=0	115,150	55.9	.30	.90	.77	1.33
	215,250	55.9	.40	.0	.19	4.43
DH	125,150	30.2	.05	0	2.76	1.18
ISI=1	225,250	30.2	.35	0	8.18	.17
ISI=0	125,150	68.5	0	.3	2.11	.10
	225,250	68.5	.60	.60	3.13	.11

<u>Note</u>: For DH, program uses $k = P(1|\tau_2=\tau_1+1)$ since response preference is $\hat{P}(1|S_1) >> \hat{P}(2|S_2)$ for this <u>O</u>.

the goodness of fit of the model to the conditional probabilities of a correct response at two base durations.

The values of q, k and k' shown in Table 17 are not necessarily those giving the overall minimum value for χ^2, for that pair of base durations; these sets of parameter estimates are chosen according to other criteria. Let the four base durations within a session be denoted by T_i (i=1, 4). For the two smaller base durations, T_1 and T_2, q was varied over a wide range and values of k and k' minimizing χ^2 were determined for each q. This procedure was repeated for the two larger base durations, T_3 and T_4. Then sets of parameter values (q_1, k_1, k_1') and (q_1, k_2, k_2') for the smaller and larger base durations respectively were selected such that

i. when using (q_1, k_1, k_1') in the model, χ^2 for the data from T_1 and T_2 was nonsignificant (with one d.f. for χ^2) and close to its minimum value, and

ii. when using (q_1, k_2, k_2') in the model, χ^2 for the data from T_3 and T_4 was nonsignificant (with two d.f. for χ^2) and as small as possible.

The astericks in Table 17 indicate that χ^2 was large and significant for all q for that pair of base durations within the set of four. This was the case for several Os when the smallest T_s in the set was 50 (for V, MD and HL) or 65 (for RM). For these Os, the set of parameters

(q_1, k_2, k_2') minimizes χ^2 for the two larger base durations.

There are several points to be noted about the estimates shown in Table 17:

1. When we relax the assumption that the parameters k and k' (defined on the decision process) are the same for all base durations within a session, we find that we can generally get a good fit of the data (i.e., a nonsignificant χ^2) by this procedure. However, k is always larger for the two longer base durations within each set of four. (There are also changes in k', but there is no consistent pattern.)

2. In the discussion of experiment 4, it was pointed out that the averaged $\hat{P}(C)$ values from experiments 2 and 3 were much less than would be expected on the basis of the averaged psychometric functions from experiment 4. It was suggested there that the processes involved in coding the intervals might not be the same in the two situations. On the other hand, it is quite possible to account for the lower performance levels with larger values for both of the parameters q and k, relative to the estimates obtained by fitting the model to the data from experiment 4. In Table 17, q_1 varies over the range 17-99 msec., and is greater than 30 for most Os. Two of the 6 Os in experiment 2 also participated in experiment 4; we can compare the estimates of q from the two situations. For RM, q is approximately the same (about 18); but for V, q_1 is at least 10 msec. larger than $\hat{q}$ in Table 12.

3. For each of the two Os who participated in both experiments 2 and 3, the value of q_1 for the range 175-275 is approximately the same (±6 msec.) as the value for the range 115-250. For these Os, the model can also predict performance with adjacent intervals (ISI=0; experiment 3), but not necessarily with the same value of q_1 as is obtained from the performance levels when ISI = 1 sec.

The value of q_1 satisfying conditions (i) and (ii) for a set of 4 base durations is not necessarily unique. However, we know that we can find values of q_1 such that the model fits the data from each O, for those sets of base durations excluding the smallest (50, 65 msec.). The implication of this failure of the model to fit the data from the smallest base durations may be that there is an additional - or an alternative - cue available for the small base durations*. This additional cue may be the reason for the relatively poor fit of the model to the psychometric function from 16 sessions at T=100 for RM, for whom we also obtained the smallest estimate of q (15).

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* Note also that those Os for whom the model does fit at the smallest T_s are the two naive Os in experiment 2 who had the set of longer base durations first...suggesting that they did not have enough experience with the shortest base durations to establish the use of this alternative process.

We also consider the possibility that q_1 for a set of four base durations is not a good estimate of the period of the time base. For two Os now, there are indications that q may double in value when the smallest duration in a series of sessions is increased by at least 100 msec. (see MD in Table 17, and HS in Table 12). If the distance between the time points counted during an interval can be either q or 2q (depending on the order of magnitude of the intervals), then it is possible that either q or 2q can be used within a session, with a resulting discrepancy between q and the estimate of q which we obtain by fitting the model to the data. Moreover, this may account for the result that $\hat{q}$ and q_1 often increase with an increase in base duration; for as base duration increases, the proportion of trials on which 2q is used may increase*.

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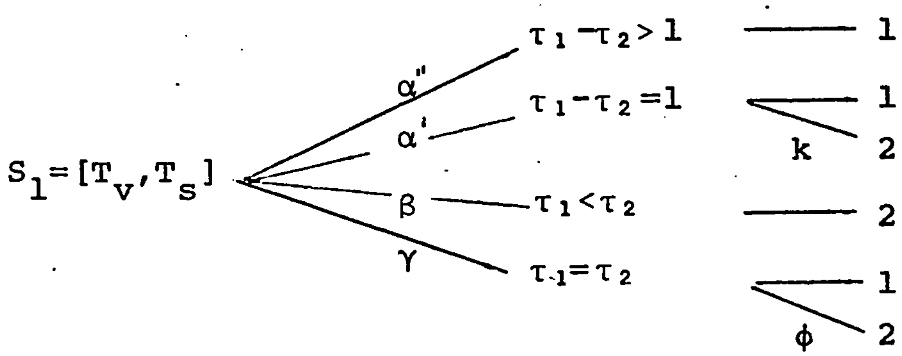
* These speculations are motivated by Kristofferson's two-state model for successiveness discrimination (1967).

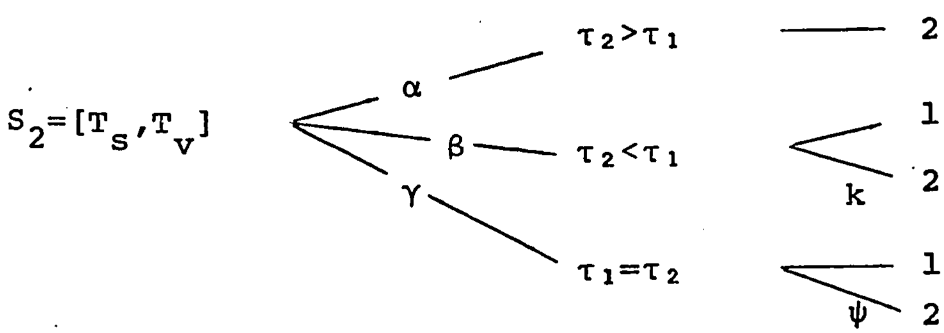
What is k?

A bias parameter can be thought of as describing the relative occurrence of the response alternatives, when the O is in a guessing state where he has not obtained enough information on which to base his response. In the quantal counting model, the state $\tau_1 = \tau_2$ occurs with equal probabilities for the S_1 and S_2 patterns, and k' is a conventional bias parameter summarizing the decision process if a second look has already been taken: the response "1" or "2" is selected with a probability which is independent of the stimulus pattern which has been presented. Hence the probability of a correct response is independent of k', whereas it is not independent of k which has been defined in a different way.

We have defined k as the probability that the incorrect response will be made when the S_1 pattern is presented, and the measurement process has resulted in a count of one more in the first interval than in the second. This state occurs with different probabilities for the S_1 and S_2 patterns. If S_1 and S_2 were treated symmetrically, this difference of one between the counts would be enough to make a correct decision. However, it is as if there is some probability of losing this information before a response is selected.

The parameter k might be taken to summarize more than the selection of a response. To see this, consider the following analogy. Suppose we collapsed the tree diagram of Figure 2, to represent only the outcomes of the initial counting and the eventual response selection conditional on these outcomes. We would then have the tree diagrams below.





The state $\tau_1 - \tau_2 = 1$ is characterized by uncertainty, and the O will not always be correct in choosing the longer interval. Let $\phi = P(2|\tau_1=\tau_2)$ for the S_1 pattern, and $\psi = P(2|\tau_1=\tau_2)$ for the S_2 pattern. Referring back to the tree diagram in Figure 2, we can write

$$\phi = \alpha' k + \beta + \gamma k'$$

$$\psi = \alpha + \beta k + \gamma k' \quad .$$

ϕ is not a conventional bias parameter. Since the response selection is preceded by another process, the second look, we have $\phi \neq \psi$. Moreover, ϕ and ψ will be systematically related to the stimulus parameters T_s and ΔT, since α, β and γ are dependent on T_s and T_v.

On the basis of the above considerations, then, we do not reject the model if k is not constant across all base durations within a session. It may be considered as a parameter which summarizes the results of some other process intervening between the outcome $\tau_1 - \tau_2 = 1$ and the final selection of a response, in the same way as ϕ and ψ summarize the results of a second look intervening between the outcome $\tau_1 = \tau_2$ and the final selection of a response.

3.2 Creelman's model

(a) The psychometric functions

In testing Creelman's model, σ_v^2 is assumed to be zero since the signals marking the onset and offset of the intervals were clear and very easily detectable. Creelman assumed $\sigma_v^2 = 0$ for $v_s = .08$ rms volts, in his second experiment where the filled intervals ranged from 20 to 360 ms and the intensity of a noise background was .01 rms volts. In experiments 2, 3 and 4 here, the intensity of the 10 ms pulses was .4 rms volts.

Let u represent the quantity $\Delta T/(2T+\Delta T)^{1/2}$. Then the relation

$$d' = (1+KT)^{-1/2} \cdot 2\lambda^{1/2} \cdot \Delta T/(2T+\Delta T)^{1/2}$$

can be written as

$$d' = (1+KT)^{-1/2} \cdot c \cdot u \qquad (3\text{-}8)$$

where $c = 2\lambda^{1/2}$. When T is constant, we have that a plot of d' vs u should be a zero intercept straight line, with slope $c(1+KT)^{-1/2}$. As the base duration increases from T_1 to T_2, the slope of this line should decrease, or remain constant if K=0. Moreover, an estimate of K can be obtained from the ratio r of these slopes. For if

$$r = c(1+KT_1)^{-1/2}/c(1+KT_2)^{-1/2}$$

then $r^2 = (1+KT_2)/(1+KT_1)$

and $K = (1-r^2)/(r^2T_1-T_2)$. (3-9)

Hence when r=1, K=0.

In figure 24, the plots of d' vs u are shown for each O in experiment 4. The lines shown there were fit by eye. In only one case (for HS) is the slope of the plot for T_s = 200 clearly less than the slope for T_s = 100. In two other cases, a single line could fit the points from both base durations (RM, PL). However, for the two remaining Os, (KL and V) the points for T_s= 200 all lie above the line for T_s= 100, and r is less than 1. Hence K would be negative, which is inconsistent with the sense in which this parameter was introduced. That is, either there is memory loss during the second interval of the count from the first (K>0) or there is not (K=0). If K is negative but (1+KT) is > 0 we have $1/(1+KT)^{1/2}$ greater than 1, implying an enhancement of d' due to "memory loss"! Table 18 shows the slopes of the "least squares fit" lines for these plots. Here r is less than 1 in 3/5 cases. Where r is greater than 1, the corresponding value of K is of the same order as those Creelman

Table 18

Creelman's model: Estimate of K obtained from the ratios (r) of slopes (m) of the best fitting zero intercept lines for the plots of d' vs $u = \Delta T/(2T+\Delta T)^{1/2}$ (experiment 4)

	$T_s = 100$	$T_s = 200$		
Obs	m_1	m_2	$r = m_1/m_2$	K
Rm	3.96	3.57	1.109	2.98
V	1.82	2.14	.852	- 2.16
PL	2.50	2.71	.920	- 1.6
HS	2.38	1.91	1.247	12.5
KL	.72	.92	.781	- 2.81

Table 19

Creelman's model: estimates of K for experiment 2 from ratios of d' at two different base durations

		75-ΔT,150-ΔT	75,150	200-ΔT,275-ΔT	200,275
Obs	ΔT	K_1	K_2	K_3	K_4
RM	10	-2.19	-3.0	-2.0	0
V	25	-3.9	-5.0	9.7	-11.5
MD	25	-6.9	-6.3	-8.5	- 2.5
DQ	25	.45	6.9	2.38	-12.3
IR	25	2.13	-2.0	.47	-22.3

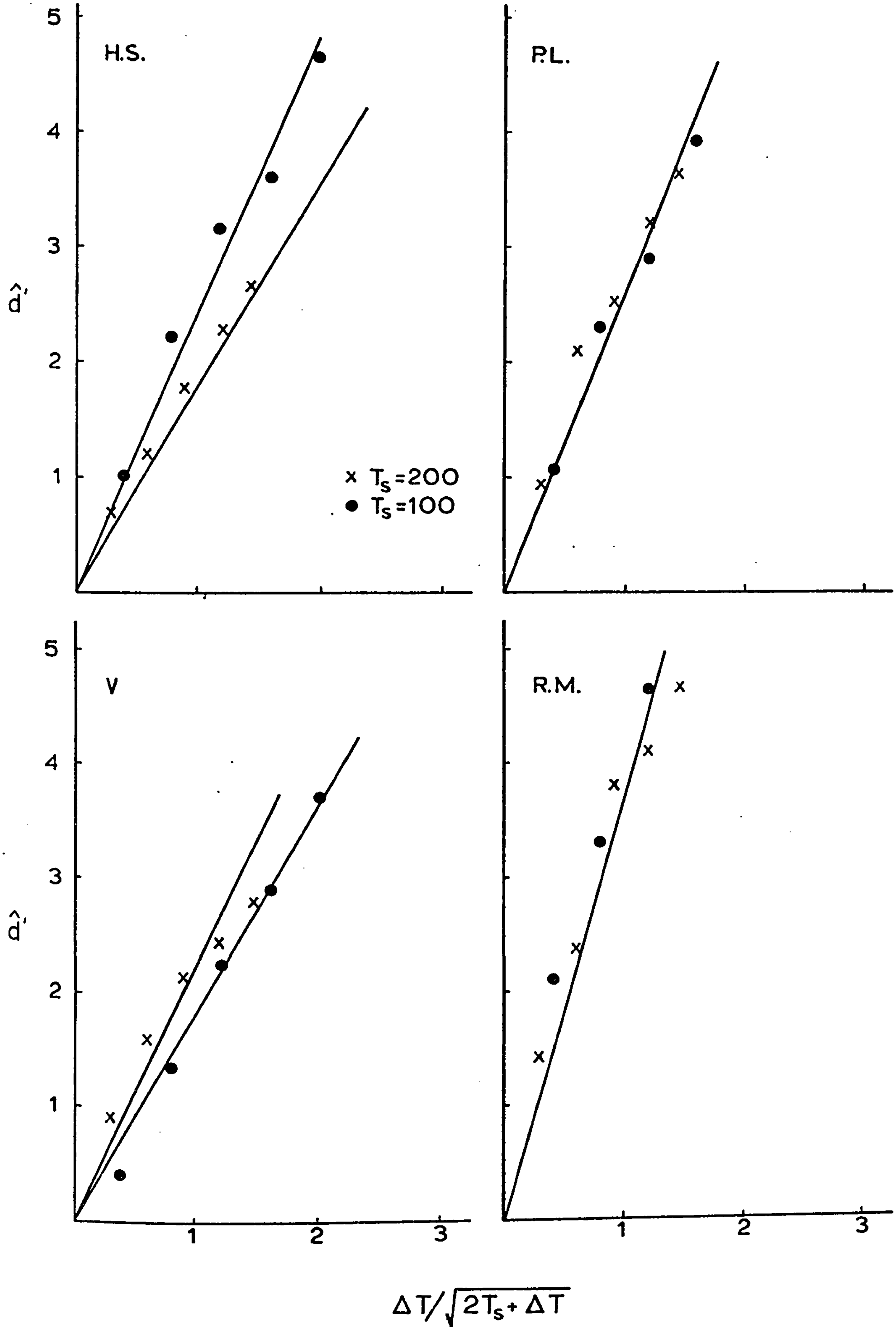


FIGURE 24: Creelman's model: d' as a function of $u = [\Delta T/(2T+\Delta T)^{1/2}]$ for the data from experiment 4; T=100 [•] and T=200 [×].

used, which were in the range $3 \leq K \leq 20$. The unit of measurement for T is taken to be in sec. It should be noted that our estimates of d' are obtained from $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$ in exactly the same way as Creelman obtained his (Swets, p. 270).

(b) d' as a function of base duration

In experiments 1, 2 and 3, it was often seen that the relation between $\hat{d}'$ and T_s was not a monotonic decreasing function, as predicted by the model. However, the non-monotonicity may simply be due to variability in the data. An interesting test of the model can be made by again looking at the plots of $\hat{d}'$ vs u. If K is zero, this plot should be fit by the same zero intercept line as is obtained when T_s is fixed and ΔT is varied. Note that as T increases, u decreases. If K is greater than 0, we want to know something about the way in which the slope of the function (3-8) changes as u varies - is the relation concave up or down, how much deviation can we expect from linearity, and how does this deviation depend on the particular range of T_s and the size of K?

The slope M at any point along the function is given by obtaining the derivative of the function with respect to u

$$M = \frac{d}{du}\left[(1+KT)^{-1/2}\cdot cu\right] \quad .$$

and expressing the result in terms of T:

$$M = \frac{Kc(2T+\ T)}{2(1+KT)^{3/2}} + \frac{c}{(1+KT)^{1/2}} \quad . \tag{3-10}$$

When K=0, we have M=c for all T, as must be the case when d' = cu; the slope of this linear function is simply c. Setting c=1, T = .025, and fixing K at various values, M was computed as T varied from .04 to .3 sec in steps of .01. This covers the range in which we are interested in these experiments. K was varied from 2 to 10, and from .2 to 1. The results of these calculations showed that the slope M is a monotonic decreasing function as T increases within this range, when K is anything larger than approximately 3. Stated in terms of u, the slope M <u>increases</u> as u increases, so the plot of d' vs u should be <u>concave upward</u>. However, the change in M may be so small as not to be apparent from the data. Taking c=1 as the reference slope (for K=0), the slope when K=4 varies only from .983 (at T = 40 ms) to .977 (at T = 300 ms). That is, the plot of d' vs u would essentially appear linear. This is true also for any value of K smaller than 4. On the other hand, when K = 10, the slope changes from .95 to .81; and for K = 20, the change is even more pronounced: from .89 to .65. Hence if for any <u>O</u> the plot of d' vs u has a concave upwards tendency, K may be fairly large.

In figure 25, the plot of d' vs u is shown for data from experiment 1 (B) where all base durations are randomly intermixed. Base durations T_s are indicated below the u-axis. At the high intensity, the plots for JT and RM seem reasonably well described by zero intercept straight lines, but this is definitely not so for V, for whom we had K=0 in experiment 4. The quantity m indicates the slope of the best fit straight line. In the low intensity condition, for RM the point corresponding to T = 250 lies considerably above the line through 150 and 50; that is, d' is better than the model predicts on the basis of performance at 50 and 150, with K=0. The slopes in this condition are less for all Os than in the high intensity condition. This result could be accounted for if λ were a function of intensity. However, Creelman assumes that the effect of increasing intensity is to decrease σ_v^2 and not to increase λ. Note that for RM the slope of the best fitting line corresponds well with the average of the slopes obtained from the psychometric functions: 3.77 (see Table 18). This is certainly not true for V.

Looking at the plots for RM from experiment 2 (figure 26), we see that performance at the small T_s (65, 75 ms) is somewhat less than predicted by a zero intercept line of slope 3.77. Otherwise the straight line appears to

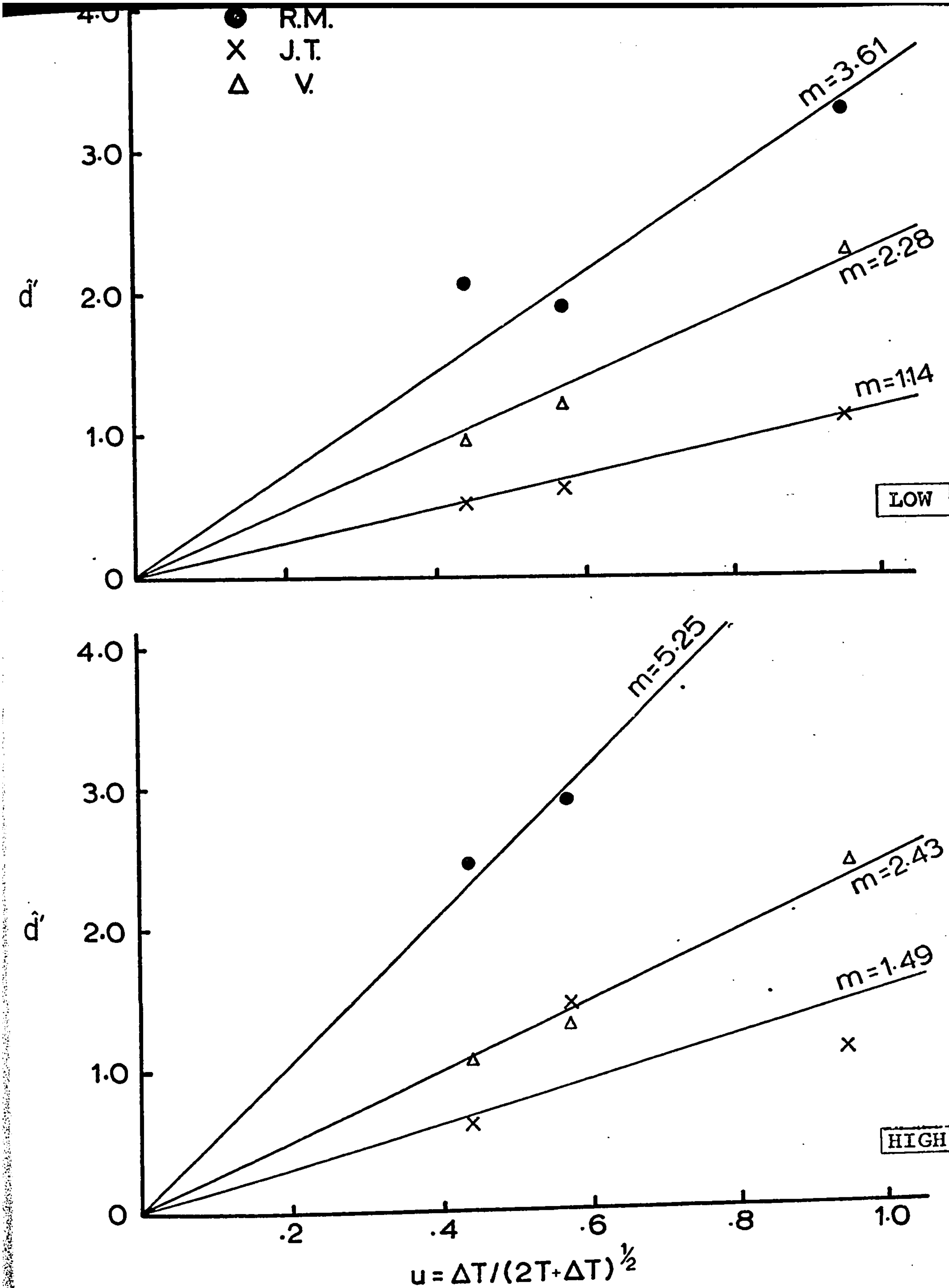


FIGURE 25: Creelman's model: $\hat{d}'$ as a function of u, for individual Os in experiment 1B. High and low intensity conditions are shown in separate plots.

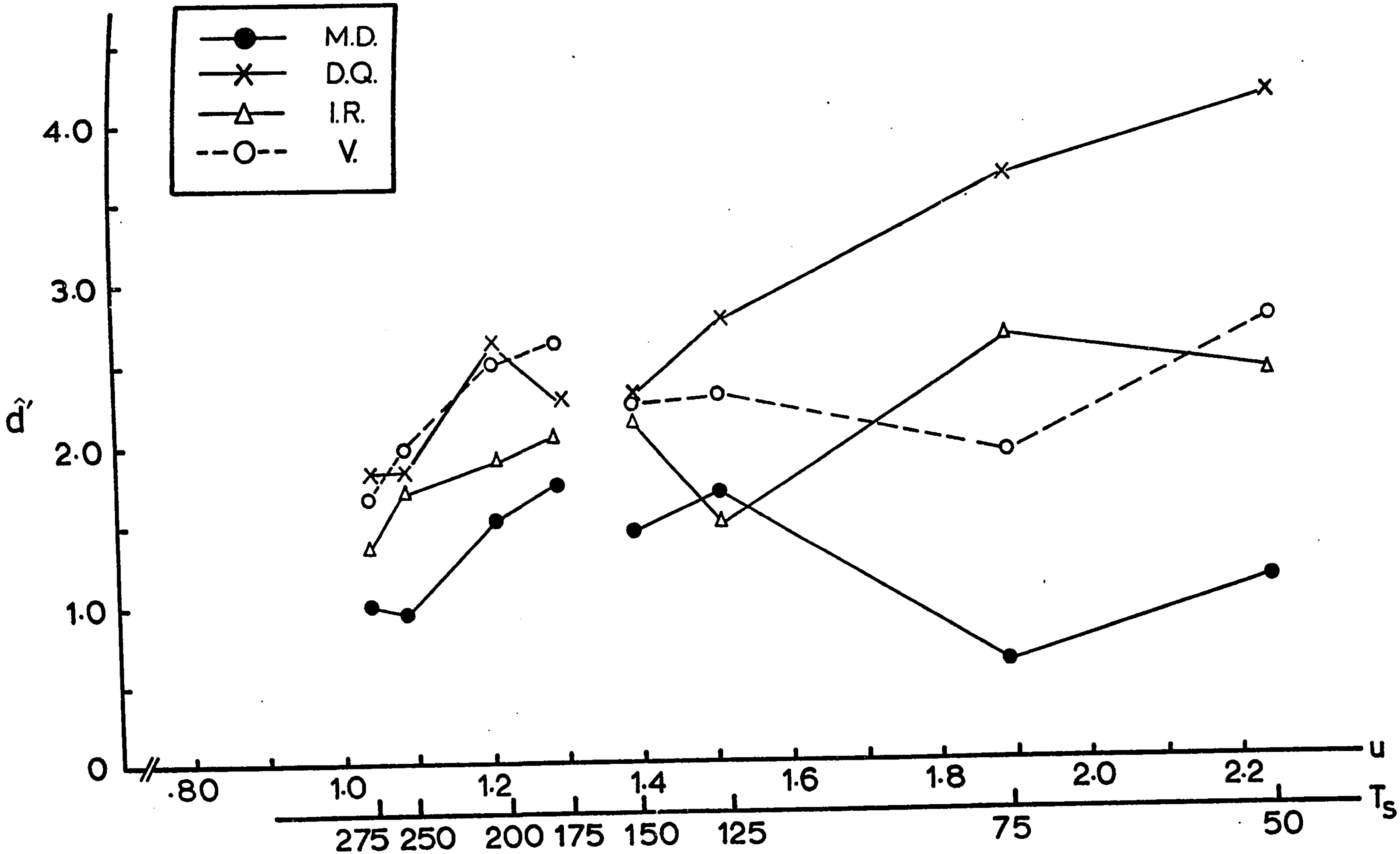
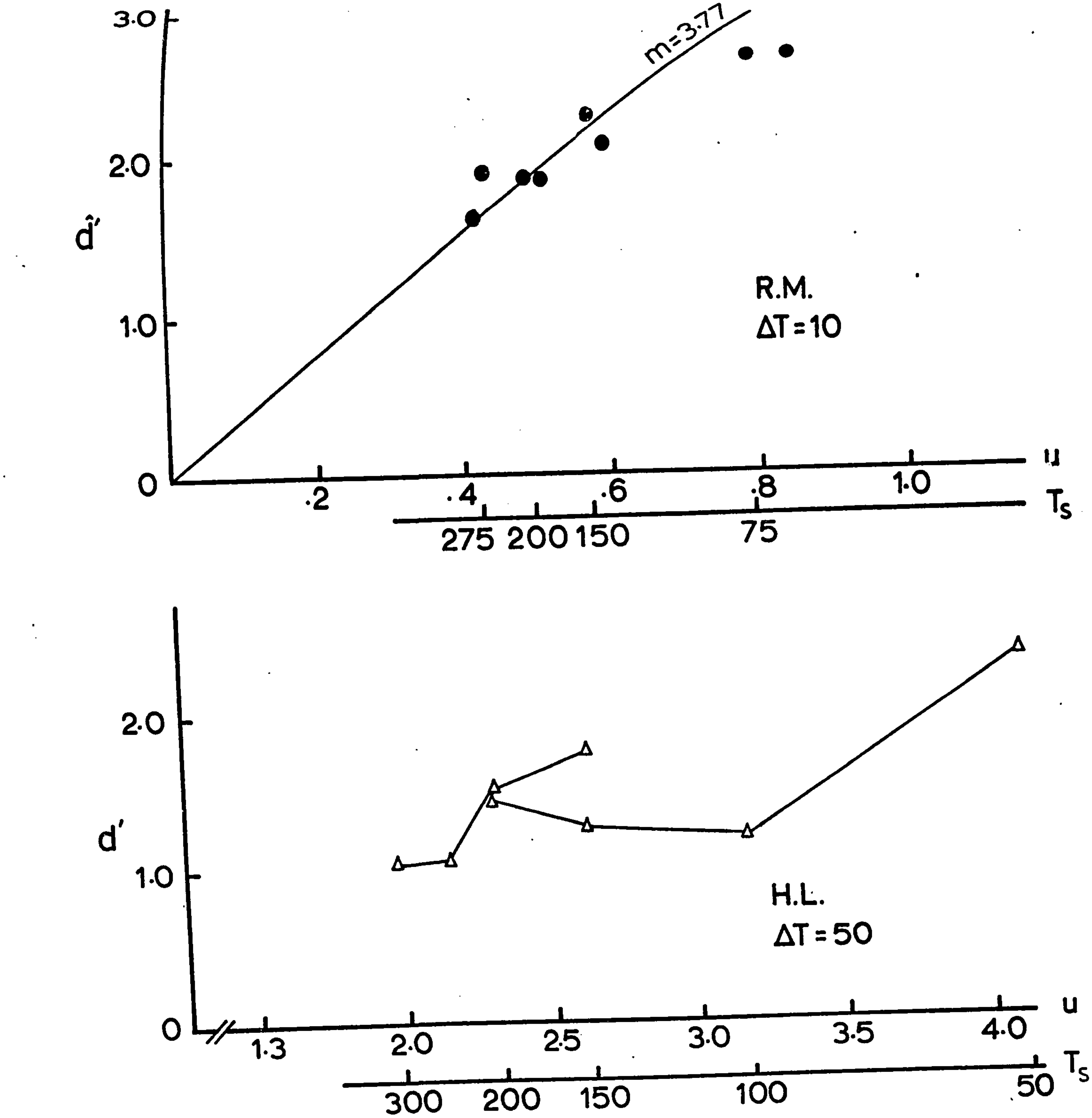


FIGURE 26: Creelman's model: $\hat{d}'$ as a function of u, for individual Os in experiment 2. Increasing u corresponds to decreasing T_s, shown on a separate axis below the u-axis.


3.0
2.0
1.0
0
d̂′
m=3.77
R.M.
ΔT=10
u
.2
.4
.6
.8
1.0
Ts
275
200
150
75
2.0
1.0
0
d′
H.L.
ΔT=50
u
1.3
2.0
2.5
3.0
3.5
4.0
Ts
300
200
150
100
50

provide a quite good description of the plot. The picture is quite different for the set of four Os who had ΔT = 25 ms. There is only one (DQ) for whom a zero intercept straight line might be considered. Moreover, the tendency seems to be for the slope to decrease, as u increases, for all Os.

The inadequacy of Creelman's model in dealing with the data of experiment 2 is further established by estimating K from pairs of values of $\hat{d}'$, for base durations which are separated by 75 ms.
For each O we can get 4 estimates of K, by using the following relation:

$$R = (\hat{d}'_1/\hat{d}'_2)^2 = \frac{(1+KT_2)}{(1+KT_1)} \cdot \frac{(2T_2+\Delta T)}{(2T_1+\Delta T)}$$

which can be solved for K:

$$K = (1-RB)/(RBT_1-T_2) \quad \text{where} \quad B = (2T_1+\Delta T)/(2T_2+\Delta T) \tag{3-11}$$

These estimates of K are shown in Table 19. They vary over a large range and are consistently negative for several Os. For RM they are negative in 3 out of 4 instances, but at least (1+KT) is always positive, so that the square root is a real number. The interpretation of these negative values for K is that as T increases, $\hat{d}'$ is not decreasing as fast as the model would predict, even with K=0. Note that in terms of actual $\hat{d}'$ levels, $\hat{d}'$ at T = 75 ms in several instances is not much higher than d' at T = 200.

(c) Response preferences

The conclusion from the above analysis is that Creelman's model is quite inadequate for the data from these experiments, even for those Os for whom performance at T_s= 50 is much better than at T_s= 150. The one possible exception is RM. Furthermore, additional assumptions would be required to account for the kind of response preferences often seen. One would have to make some assumption about a dependence of the criterion on a base duration, which is not implied in the way the model is formulated. In deriving the predicted relation between d' and T_s, it is assumed that in the forced choice task the O is comparing the measures of two intervals, and this is the only basis for his decision. Even when all T_s occur randomly intermixed, the O would also have to obtain on each trial some information as to the absolute magnitude of the interval being presented, if the criterion is a function of T_s. With the assumption of a systematic criterion shift, we could replicate the effect of having $P(2|S_2)$ approximately constant while $P(1|S_1)$ decreases, only if the variances of the underlying distributions increase as T_s increases, while the distance between the means is independent of T_s. In this case, if the O adjusts his criterion in such a way that $P(2|S_2)$ is constant, then $P(1|S_1)$ will necessarily have to decrease.

3.3 Quantal onset-offset model (I)

(a) The estimates of $d_{q,2}$ for the FC task are obtained directly by using $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$, and a table of areas under the distribution specified in section 1.4. Figure 27 shows the resulting plots of $\hat{d}_{q,2}$ vs ΔT, obtained from the data of experiment 4. At $T_s = 100$, the relations appear to be reasonably well described by zero intercept straight lines for all 5 Os with $\hat{q}$ varying from 12 to 75 ms. For 3 Os, $\hat{q}$ is between 20 and 30 ms. However, at $T_s = 200$, for 4 Os the points deviate systematically from the straight line fitting the points at $T_s = 100$, whereas the model predicts that the same straight line should fit both sets of points. It may be that $T_s = 200$ is above the range in which this model applies.

(b) If there is a limited range over which the model holds, we may be able to determine this by looking at $\hat{d}_{q,2}$ obtained from the experiments in which ΔT is fixed and T_s is varied. For the range within which the onset-offset model holds, $\hat{d}_q$ should be constant as T_s changes. However, we find that we cannot consistently locate a range within which $\hat{d}_q$ is constant. Plots of $\hat{d}_{q,2}$ vs ΔT for experiments 2 and 3 are shown in figures 28 and 29. For two Os in

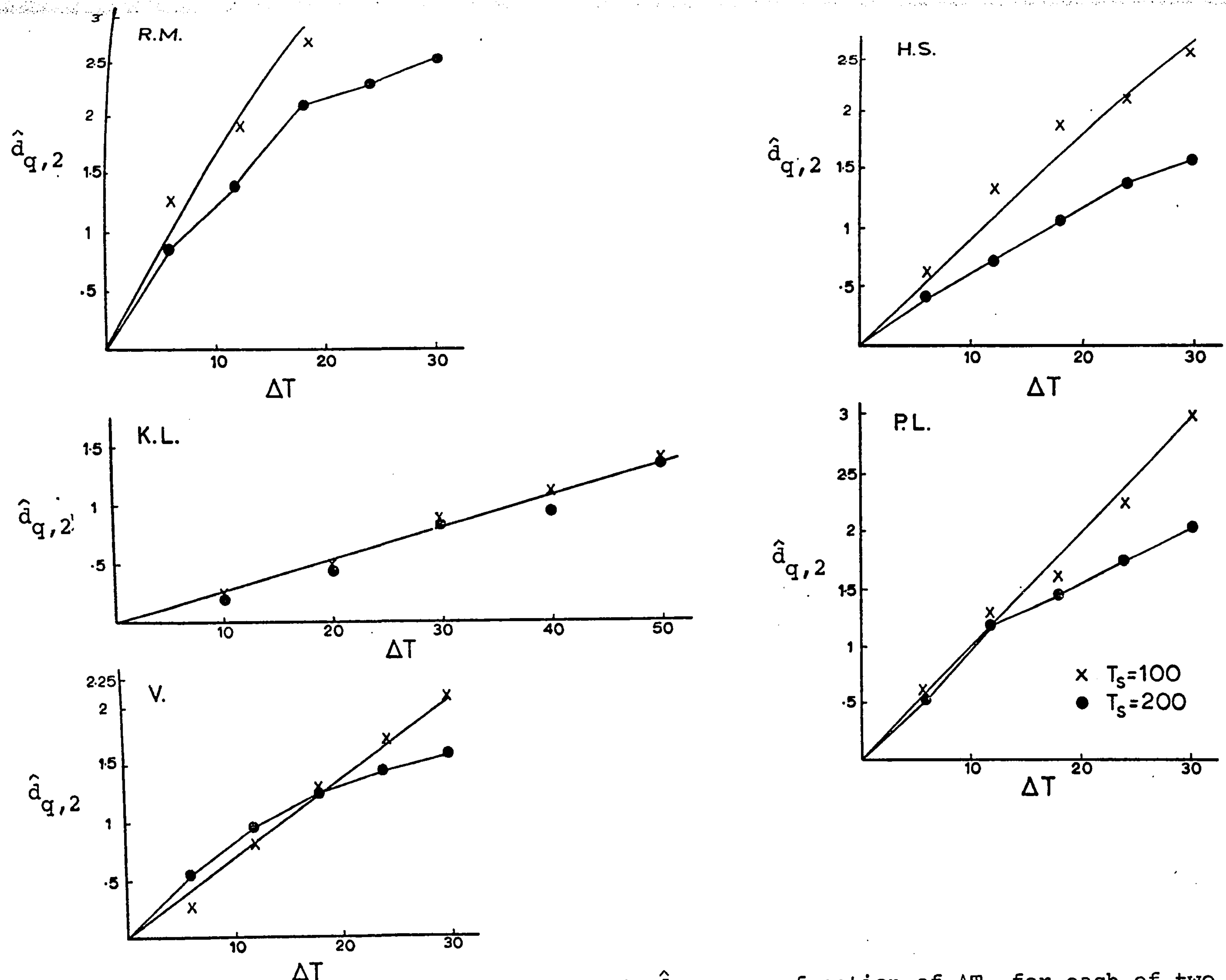


FIGURE 27: $\hat{d}_{q,2}$ as a function of ΔT, for each of two base durations. Data from experiment 4.

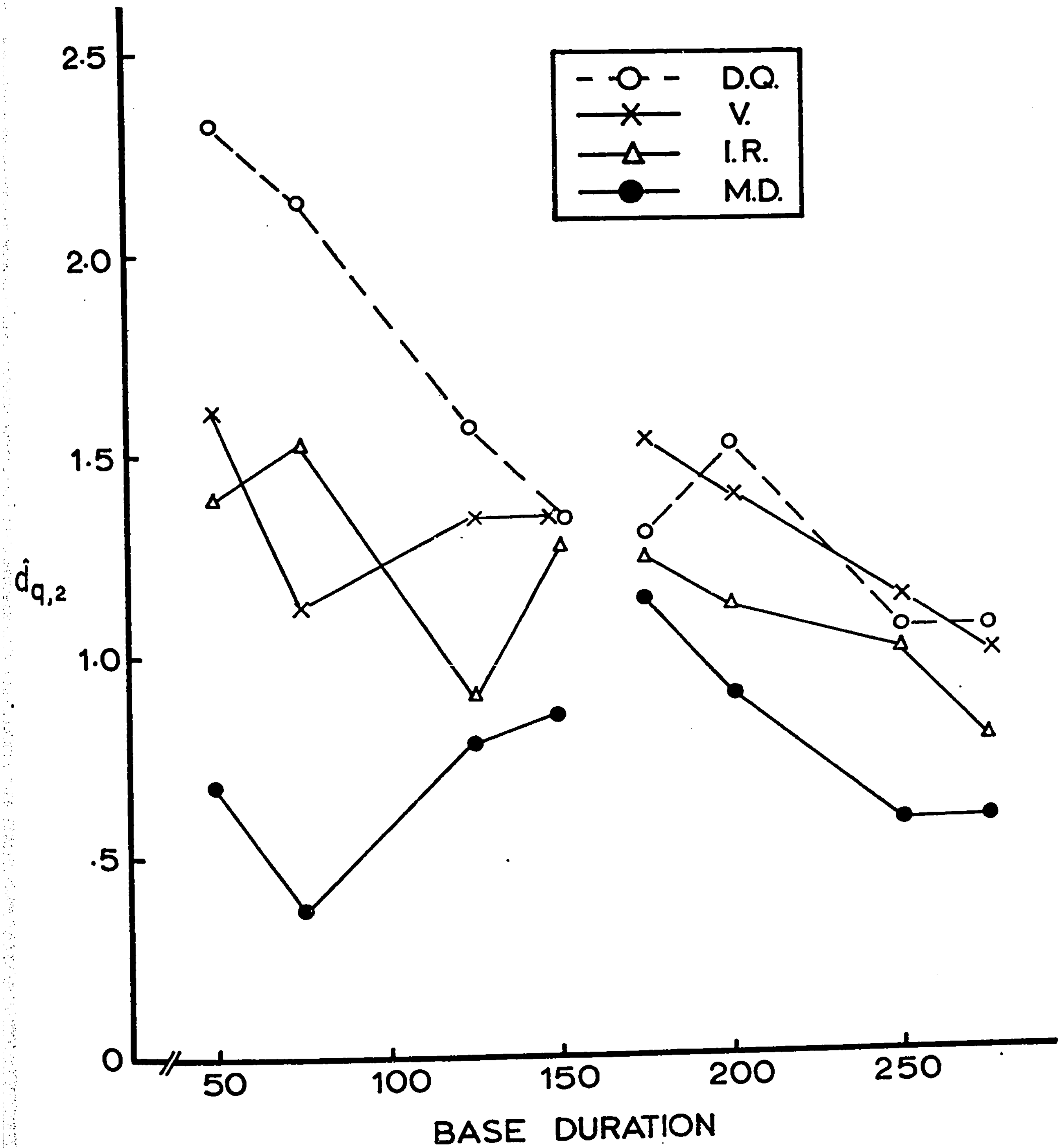


FIGURE 28: $\hat{d}_{q,2}$ as a function of base duration, for 4 Os from experiment 2. ΔT=25 msec.

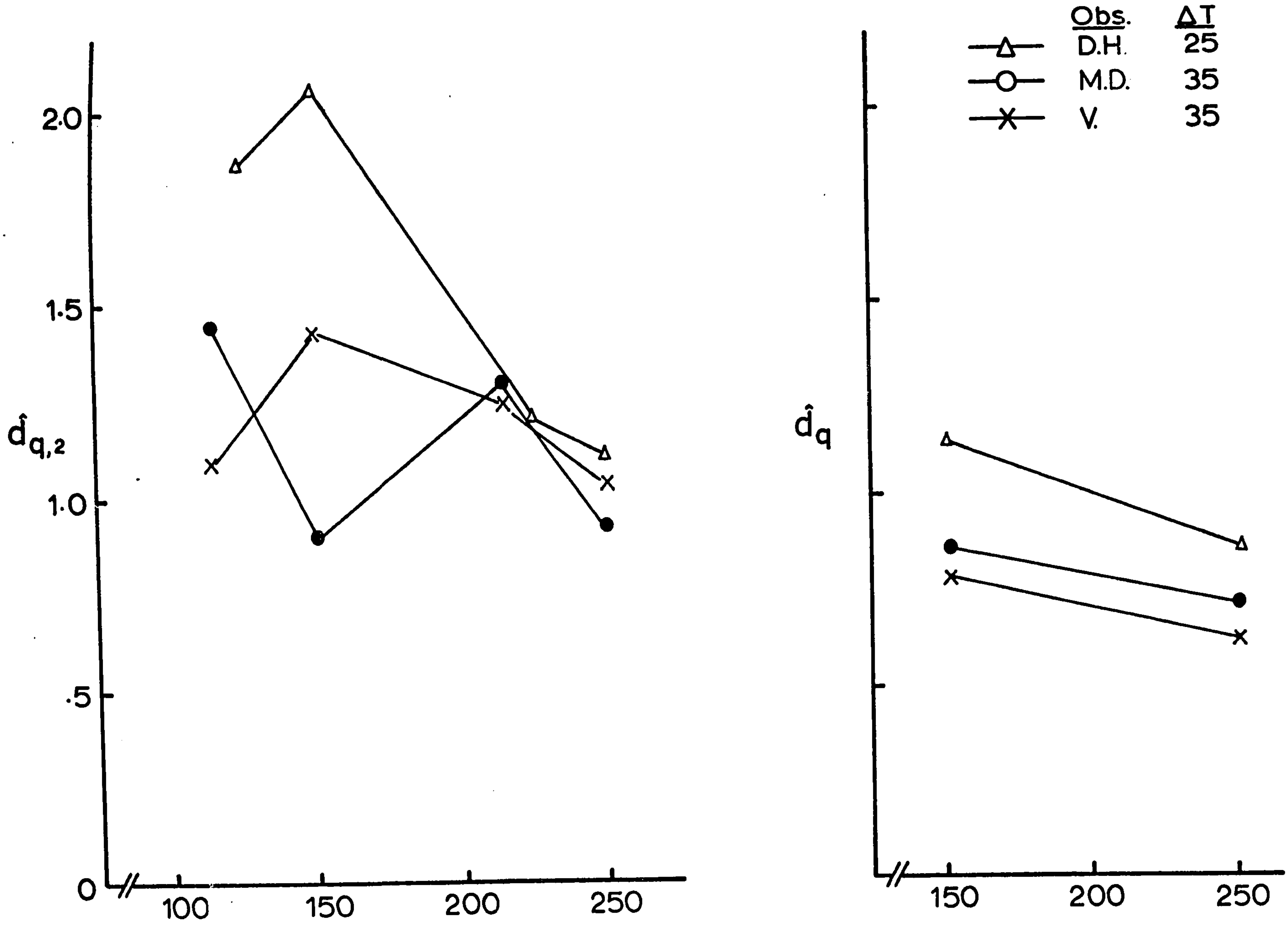


FIGURE 29: $\hat{d}_{q,2}$ from FC data, and $\hat{d}_q$ from SS data, plotted as a function of base duration. 3 Os in experiment 3.

Table 20

Estimates of $d_{q,2}$ and q, using version I of the onset-offset model where $d_q = 2\Delta T/q$ Data from the high intensity condition of experiment 1.

		Version A		Version B	
Obs	T_s	$\hat{d}_{q,2}$	$\hat{q}$	$\hat{d}_{q,2}$	$\hat{q}$
V	50	.66	30	.66	30
	150	.74	27	.71	28
	250	.82	24	.34	59
RM	50	2.05	9.8	*	
	150	1.60	12.5	1.67	12.0
	250	1.21	16.5	1.44	13.9
JT	50	1.94	10.3	1.42	17.9
	150	.93	21.5	.75	26.7
	250	.77	26	.62	32.3

* indicates $\hat{d}_{q,2}$ cannot be obtained, since $\hat{P}(2|S_2) = 1$.

experiment 3, (MD and V) we might consider $\hat{d}_{q,2}$ as fluctuating around the value 1.2, for T_s ranging from 115 to 250 ms. Variability in $\hat{P}(C)$ will affect the values of $\hat{d}_{q,2}$, but it has not been assessed how much fluctuation in $\hat{d}_{q,2}$ we can expect on the basis of sampling variability in $\hat{P}(C)$. On the other hand, for both these Os $\hat{d}_q$ from the SS task is somewhat less at 250 than at 150.

For the third O (DH) in experiment 3, there is a systematic decrease in $\hat{d}_{q,2}$ as T_s increases from 125 to 250. This decrease is also seen in the SS condition. And again in experiment 2, we find a monotonic decrease in $\hat{d}_{q,2}$ over the range 175-275 ms, for the 4 Os who have $\Delta T = 25$ ms.

Finally, we look at the values of $\hat{d}_{q,2}$ from experiment 1. For two Os, $\hat{d}_{q,2}$ is much higher at $T_s = 50$ than elsewhere, although it is approximately the same at 150 as at 250. Table 20 shows $\hat{d}_{q,2}$ and the corresponding values of $\hat{q}$, for data from the high intensity conditions. For RM, $\hat{q}$ varies between 10 and 17 ms. For JT, $\hat{q}$ is in the 20-30 ms range, except for $T_s = 50$. For V, estimates of q are also in the 20-30 ms range, but with one awkward exception. For $T_s = 250$ in the condition where the three base durations were intermixed, $\hat{q} = 59$. All estimates of q should be of approximately the same size in this condition, if the model were valid. Uncertainty with respect to the size of the interval which will occur on any trial should affect the onset latencies of all the intervals, irrespective of their order of magnitude. This result, taken together with the finding that we cannot consistently locate a range of base durations over which $\hat{d}_{q,2}$ is constant for a fixed ΔT, allows us to reject the quantal onset-offset model assuming that the choice of the longer interval is based on the difference between the magnitudes of two internal intervals.

Quantal onset-offset model (II)

(a) If we are to obtain a plot of $\hat{d}_q$ vs ΔT from the psychometric functions of experiment 4, we have to find some way of obtaining estimates of P_1 and P_2 from $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$. Once $\hat{c}$ and $\hat{q}$ are determined, $\hat{P}_1$ is fixed for all ΔT, for a given base duration. To obtain $\hat{c}$ and $\hat{q}$, the parameters c and q were varied independently to find that pair minimizing the sum of squared deviations

$$\sum_{1}^{5} [P(C) - \hat{P}(C)]^2 \quad .$$

These are shown in Table 21. In order to obtain $\hat{P}_2$, we use the relations

$$\hat{P}(1/S_1) = (P_1+P_2)(1-b) + P_1P_2(1-2b) \qquad (3\text{-}12)$$

$$\hat{P}(2/S_2) = (P_1+P_2)b \qquad + P_1P_2(2b-1) \qquad (3\text{-}13)$$

in which the simplifying assumption has been used that

$$P(1/L,L) = P(1/S,S) = b \quad .$$

To obtain an estimate of b, we use one value of ΔT, and calculate P_2 in the same way as P_1 was calculated, using the location of the criterion with respect to the mean of the

Table 21

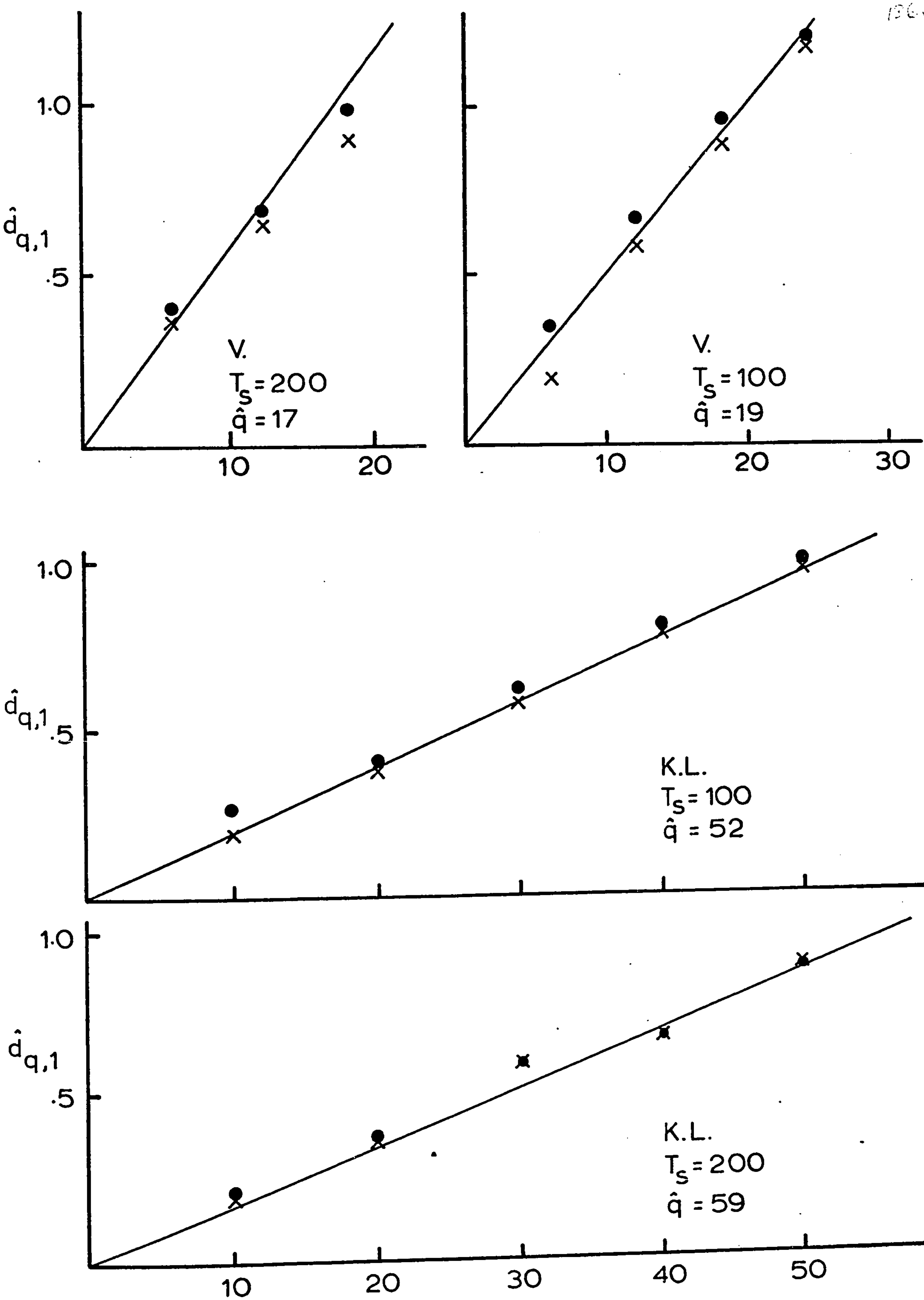
Parameter estimates for the quantal onset-offset model (II) obtained by fitting the model to the data from experiment 4. The bias parameter $\hat{b}$ which is needed to calculate the values of $\hat{d}_{q,1}$ plotted in Figure 25, can be calculated from any one of the 5 values of ΔT used within a session.

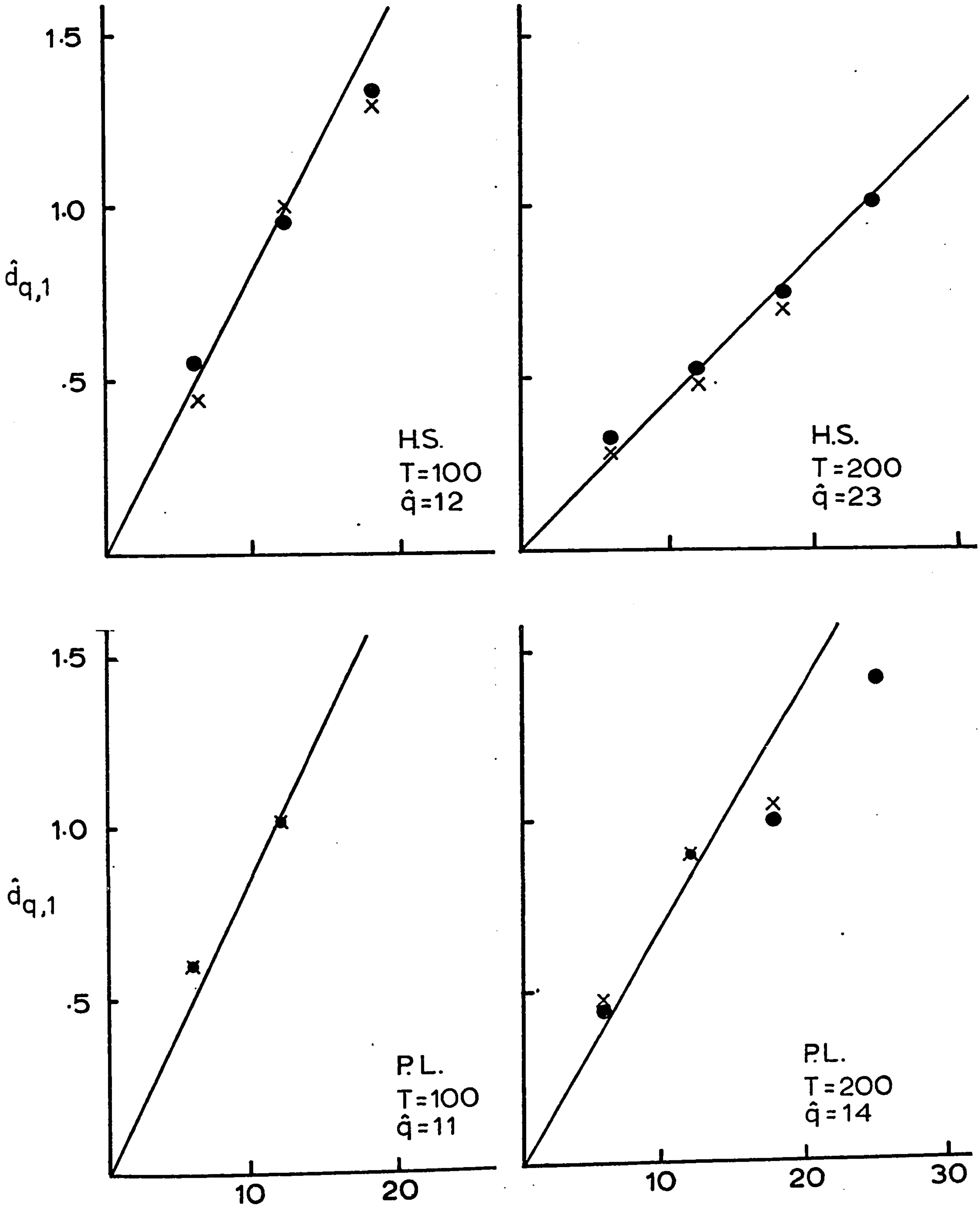
	$T_s = 100$				$T_s = 200$			
Obs.	$\hat{q}$	$\hat{c}$	$\hat{b}$ (ΔT=6)	$\hat{b}$ (ΔT=24)	$\hat{q}$	$\hat{c}$	$\hat{b}$ (ΔT=6)	$\hat{b}$ (ΔT=24)
HS	12	107	.47	.25	23	208	.487	.308
V	19	111	.414	.192	17	205	.551	.319
PL	11	107	.544	.553	14	207	.708	.658*
RM	10	105			10	205		
**KL	52	121	.321	.182	58	222	.261	.274

* b is calculated at ΔT=6 and 18

** b is calculated at ΔT=10 and 30

FIGURE 30: $\hat{d}_{q,1}$ plotted as a function of ΔT, for 4 of the 5 Qs in experiment 4. The points • (×) correspond to values of $\hat{d}_{q,1}$ obtained when $\hat{b}$ is calculated from a small (larger) value of ΔT used with each base duration. The parameters $\hat{q}, \hat{c}$, and $\hat{b}$ used to calculate $\hat{d}_{q,1}$ at each base duration are listed in Table 21. The zero intercept straight line of slope $\hat{q}$ is predicted by the quantal onset-offset model (II).


1.0
$\hat{d}_{q,1}$
.5
V.
$T_S = 200$
$\hat{q} = 17$
10
20
V.
$T_S = 100$
$\hat{q} = 19$
10
20
30
1.0
$\hat{d}_{q,1}$
.5
K.L.
$T_S = 100$
$\hat{q} = 52$
1.0
$\hat{d}_{q,1}$
.5
K.L.
$T_S = 200$
$\hat{q} = 59$
10
20
30
40
50


1.5
1.0
.5
$\hat{d}_{q,1}$
10
20
30
H.S.
T=100
$\hat{q}$=12
H.S.
T=200
$\hat{q}$=23
P.L.
T=100
$\hat{q}$=11
P.L.
T=200
$\hat{q}$=14

distribution $f(I, T+\Delta T)$. Now in (3-12) and (3-13), the only unknown is b: we take $\hat{b}$ as the average of the solutions of these two equations. Using $\hat{b}$ and $\hat{P}_1$, for each value of ΔT we solve both (3-12) and (3-13) for P_2; $\hat{P}_2$ is the average of these solutions. $\hat{d}_q$ is obtained from equation (1-9) as described in 1.4.

In Figure 30 we show the resulting values of $\hat{d}_{q,1}$, at each value of ΔT and for the two base durations, for four of the five Os in experiment 4. The zero intercept straight line of slope $\hat{q}$ in each plot is the one predicted by the model. In each plot there are two sets of points: (•) corresponds to the values of $\hat{d}_{q,1}$ obtained when the bias parameter $\hat{b}$ is calculated from a small value of ΔT, and (×) represents the values of $\hat{d}_{q,1}$ obtained when $\hat{b}$ is obtained at a larger value of ΔT. Table 21 shows the values of $\hat{q}$, $\hat{c}$ and the two values of $\hat{b}$ which are involved in each plot.

Generally, the points in Figure 30 do conform to the straight lines predicted. However, for 3 of the 5 Os, the values of $\hat{q}$ in Table 21 are less than those obtained by A&K&W, where the mean value of $\hat{q}$ was 24 msec. For one O, $\hat{q}$ is 23, but only at T=200. For a fifth O, $\hat{q}$ is between 50 and 60 msec. In 3 of the 5 cases, $\hat{q}$ at the longer base duration is greater than $\hat{q}$ at $T_s = 100$. Finally, it should be noted that in Table 21, the bias parameter $\hat{b}$ as estimated from a smaller ΔT is usually greater than the value of $\hat{b}$ obtained from the larger ΔT, whereas $\hat{b}$ should be approximately the same no matter which ΔT is used in calculating it since all the values of ΔT were randomized within a session.

(b) Predicting FC results by using SS data

The above estimation procedures needed to obtain values of $\hat{d}_{q,1}$ from the FC data of experiment 4 are very roundabout compared to the direct way in which $\hat{d}_q$ can be obtained from SS data. However, it is relatively simple to predict certain FC results from experiment 3 by using the SS data to obtain estimates of P_1 and P_2 for equations (3-12) and (3-13).

The model assumes that when a decision is made after each interval in the FC task as to whether it is "long" or "short", the underlying distributions on which these decisions are based are the same as in the SS task. We also assume the same criterion is used in both tasks, so that $\hat{P}(S|T)$ is an estimate of P_1 for the FC task, and $\hat{P}(L|T+\Delta T)$ is an estimate of P_2. Then for equations (3-12) and (3-13) the only unknown is b; $\hat{b}$ is obtained by using $\hat{P}(1|S_1)$ and solving (3-12) for b. We then see how close we can come to predicting $\hat{P}(2|S_2)$ by using (3-13). The first 200 and the last 200 trials in the SS condition are analyzed separately, and the results are shown in Table 22. Except for MD and DH at T=150, we can predict to within .06 the values of $\hat{P}(2|S_2)$ in the FC condition by using the last 200 trials of the SS data. For these two Os it is at T=150 that we have the largest discrepancy between $\hat{P}(C)$ from the two tasks...of the order of .07. The criterion location can have a considerable effect on the values of P_1 and P_2, and also on $\hat{P}(C)$. Hence it could be argued that the criterion is not the same in the two tasks at $T_s = 150$ for these Os, and this is the reason for the larger differences in $\hat{P}(C)$, and the discrepancy between $\hat{P}(2|S_2)$ and the predictions of $\hat{P}(2|S_2)$ shown in Table 22.

Table 22

Predictions of $\hat{P}(2|S_2)$ from the FC condition obtained by using data from the SS condition (experiment 3). For each O, separate estimates of $\hat{d}_q$, and the other quantities derived from this measure, are given for the first 200 and the last 200 trials of the SS condition, at each base duration.

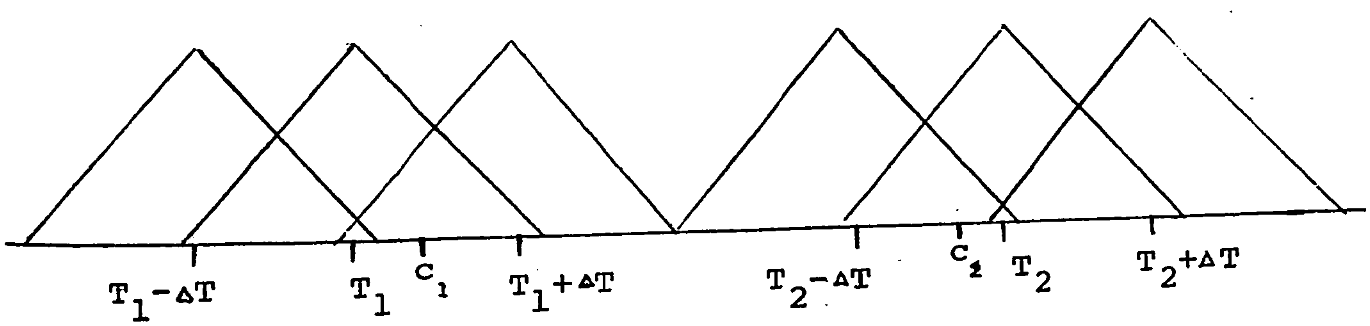
	$T_s = 150$					$T_s = 250$				
Obs.	$\hat{P}(2\|S_2)$	$\hat{d}_q$ (SS)	$\hat{q}$	$\hat{b}$	$P(2\|S_2)$	$\hat{P}(2\|S_2)$	$\hat{d}_q$ (SS)	$\hat{q}$	$\hat{b}$	$P(2\|S_2)$
V	.98	.89		-.19	1.03*	.91	.51		.32	.80
(ΔT=35)		.78	45	-.08	.99		.61	57	.35	.85
MD	.72	,70		.61	.75	.77	.88		.29	.90
(ΔT=35)		.86	41	.42	.86		.71	49	.50	.79
DH	.98	1.16		.62	.84	.73	.73		.82	.69
(ΔT=25)		1.14	22	.66	.87		.86	29	.73	.77

* taking $\hat{b}$=0, $P(2|S_2)$ is .97 .

(c) $\hat{P}(C)$ and response preferences as a function of base duration

An attempt was made to simulate the type of "response preference" seen in experiments 2 and 3, where the $T \pm \Delta T$ design was used. Recall that with the two values of T used within a series of sessions, we have essentially 4 base durations. The O would need at least two criteria, in the FC task; $T_2+\Delta T$ is "long" with respect to one of these (c_2), and $T_1+\Delta T$ is "long" with respect to a second (c_1), even though it is shorter than $T_2+\Delta T$.

Suppose c shifts in its position with respect to T, as T increases from T_1 to T_2. That is, suppose $c_1 > T_1$ and $c_2 < T_2$. The diagram below represents the situation in mind.



We assume that the same criterion (c_i) is involved in discriminating between T_i and $T_i+\Delta T$, as between T_i and $T_i-\Delta T$ (i=1,2). Also, b is taken to be the same for all base durations with a session. Using these assumptions, we obtain tables of values of $P(1|S_1)$, $P(2|S_2)$ and $P(C)$ (as a function of T_s) in which we see the following features resembling some of the results of experiments 2 and 3. If $b < .5$:

1. $P(1|S_1)$ is always less than $P(2|S_2)$, no matter where c is,
2. the difference $P(2|S_2) - P(1|S_1)$ changes very markedly as T_s changes,
3. $P(2|S_2)$ varies over a much narrower range than does $P(1|S_1)$, as T_s increases,
4. $P(C)$ as a function of T_s is nonmonotonic.

If $b > .5$, the same statements hold, but with the roles of $P(1|S_1)$ and $P(2|S_2)$ reversed. An example of the type of results generated by using the above assumptions with respect to c and b is given in the following table.

	$b = .3$	$q = 24$	$\Delta T = 25$	
criterion	T_s	$P(1\|S_1)$	$P(2\|S_2)$	$P(C)$
152	125	.594	.826	.710
152	150	.706	.874	.790
246	225	.703	.893	.823
246	250	.543	.804	.674

However, the simulation fails to imitate the data in several crucial respects.

1. $P(1|S_1)$ does not appear to be a monotonic function

of T_s, as is often seen with $\hat{P}(1|S_1)$,

2. $P(2|S_2)$ does not increase while $P(1|S_1)$ decreases; they increase or decrease together,

3. It can be shown that $P(C)_- = P(C)_+^*$ if c=T. But the maximum P(C) possible is .75, in this case, regardless of how big ΔT may be.

If one of $P(C)_+$ or $P(C)_-$ is greater than .75, the other quantity will necessarily be less than .75. But examining the data from experiments 2 and 3, we find that both $\hat{P}(C)_+$ and $\hat{P}(C)_-$ are greater than .75, for both values of T occurring over a series of sessions, for all Os except MD and HL in experiment 2.

The implication of this discrepancy between the data and the predictions is that we would have to modify the model again by assuming that two criteria are involved in discriminating between T and $T-\Delta T$, and T and $T+\Delta T$. This would make a total of 4 criteria to be held at one time when two values of T are used over a series of sessions. It could be assumed that the internal interval is compared with 4 criteria in succession. This kind of notion underlies the generation of ROC curves by using confidence ratings (Green & Swets).

The onset-offset model of A&K&W, as elaborated here, is generally unsuccessful in dealing with our data from

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* $P(C)_-$ denotes P(C) for base duration $T_s = T-\Delta T$

$P(C)_+$ denotes P(C) for base duration $T_s = T$.

experiments involving empty auditory intervals. The simplest, most direct test of the basic model is made by obtaining $\hat{d}_q$ from a SS task, and this has been done with auditory intervals ranging from 50 to 500 ms (Kristofferson & Allan, 1971)* Plots of $\hat{d}_q$ vs ΔT are shown, with base duration as a parameter. The values of T_s used were 50, 100, 250, 300 and 450 ms. The plots are non-linear, concave downward, and shift systematically downward as T_s increases, until at $T_s = 450$ the linear zero intercept function predicted by the onset-offset model is obtained. Further data (Kristofferson, unpublished) indicate that at $T_s = 600$, the same zero intercept linear function is obtained as for $T_s = 450$. Another way of stating these results is that with ΔT fixed, $\hat{d}_q$ decreases as T_s increases up to some critical value for T_s after which $\hat{d}_q$ is constant. The decrease in $\hat{d}_q$ obtained in experiment 3 here with the SS task, as T_s increases from 150 to 250 is in line with the decreases shown in Kristofferson & Allan. However, for 2 <u>O</u>s $\hat{d}_q$ at $T = 250$ is half as large as would be predicted from the averaged data shown in K&A. If we estimate q, we get values of approximately 50 ms (see Table 22). In K&A, the slope of the $\hat{d}_q$ vs ΔT plot, for T_s=450, is such that $\hat{q} = 50$ ms. What this correspondence in the values of $\hat{q}$ implies is not at all clear.

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* referred to as K&A.

If this model applies at base durations above some critical value, it would be interesting to determine whether the critical value for T_s depends on practice with the SS task, and whether it is influenced by extensive prior experience with the FC task. The Os in experiment 3 were relatively inexperienced with the SS task, but highly familiar with the FC task; the Os of K&A were highly practiced with the SS task. It would also be interesting to determine how well SS results can predict FC data when the base durations are above the critical value. For three Os in experiment 3, we can come quite close to predicting $\hat{P}(2|S_2)$ at T=250.

IV. SUMMARY AND CONCLUSIONS

The problem dealt with in this thesis was formulated as a rather broad question: when an O discriminates between two very short intervals bounded by brief auditory signals, on what basis is he making his decision? The largest interval used has been about 300 msec., which is still small as compared to the size of the intervals used in most previous investigations of "short" intervals; sensory interactions between the boundaries of the interval may be important for intervals in the range of 50-300 msec. However, the results of manipulating two variables (the intensity of the signals, and the interstimulus interval (ISI) in a two-alternative forced choice task) suggest that the information on which the discrimination is based differs considerably from the sensory information used in certain other auditory discriminations. Although performance is slightly better with high amplitude boundaries, energy dependent cues do not seem to be important. Once the boundaries are clearly detectable, performance is not appreciably changed by changes in the energy of the signals bounding the intervals, even for pairs of intervals (50-60 msec.) whose durations are less than the critical duration for energy summation inferred in auditory detection tasks. Further, the information from the first interval can be retained without loss over ISIs of at least

2 seconds. When the $\underline{O}$ has to choose which is the larger of two separated intervals, performance is the same with an ISI of 2 seconds as with an ISI of 1/2 second. On the other hand, there is a marked increase in the performance measure $\hat{P}(C)$ when the separation between the intervals is increased from 0 to 1/2 seconds. These results can be contrasted to results from frequency and amplitude discrimination where performance decreases continuously as ISI increases from zero.

It is reasonable, then, to assume that a central process is involved in the coding of a time interval, a process which is independent of the sensory events defining the time information. Such an assumption is involved in three different theoretical accounts (formulated as mathematical models) of what may be happening in duration discrimination.

Of the three models, Creelman's can be definitely rejected as an adequate description of the data presented in this thesis. For a forced choice (FC) task, it assumes a "Poisson clock" to measure the internal intervals, loss of the time information from the first interval while the second is being measured, and a decision based on the difference between the measures of the 2 intervals. The model predicts a monotonic decrease in d' as base duration increases, but it is rejected because d' does not decrease as fast as is predicted, even with no memory decay of the measure of the first interval.

The quantal onset-offset model assumed that there is variability in the internal interval I corresponding to a given external interval T only because of variability in the latencies of the events bounding the internal interval; these delays (uniformly distributed random variables) are independent of the order of magnitude of the interval. This model was originally formulated to account for single stimulus (SS) data. Here, the FC task was used throughout except for a sampling of SS performance in experiment 3 where it was found that the performance measure $\hat{d}_q$ was not independent of base duration, contrary to what the model predicts. Since conditions were not arranged to obtain optimal SS performance, this sample is not an adequate basis for rejecting the model. For the FC task, there are two versions of the model which can be considered. The first assumes that a decision is based on the difference between two measures. This version can be rejected at once because it is one of a class of models which is ruled out by the finding that SS performance is at the same level as FC performance, even with no previous practice in the SS condition (experiment 3). Moreover, $\hat{d}_{q,2}$ is often not independent of base duration, contrary to what is predicted. A second version is not so easily rejected. It assumes that after each interval occurs, a decision is made as to whether the interval is "long" or "short" with respect to some criterion. It predicts that $\hat{d}_{q,1}$ from the SS and FC conditions will be equal, or that $\hat{P}(C)$ will be the same provided the same criterion is used in the two tasks. Using the model and the SS data, certain FC results can be quite well predicted. A second test involves performance as a function of base duration. Empirically, $\hat{P}(C)$ as a function of base duration is not always a monotonic decreasing relation. However, it is possible to have different predicted values of P(C) as base duration increases; P(C) depends on the discrepancy between the magnitudes of T_s and an internal criterion interval used in making the discrimination between T_s and $T_s+\Delta T$, and this discrepancy may change with increasing base duration. The predicted performance measure $d_{q,1}$ is invariant with respect to the size of the criterion interval, and is also independent of T_s. Unfortunately, the estimation of $d_{q,1}$ from the FC data is indirect, and hence this test of the model cannot be made with the FC data.

The quantal counting model, in its present form, has to be rejected as well. Like Creelman's model, it assumes no variability in the latencies of the internal events bounding the internal interval; all variability in the mapping of T into its psychological representative T_ψ is due to the nature of the process by which the internal interval I is measured. It is assumed that a very stable periodic process generating "time points" is involved in coding an interval. The measure of a particular interval T is the number of time points occurring within the boundaries of I, and this measure can have one of two integer values. When an O has to indicate which of two intervals is the longer one, he compares two integer measures; if they have the same value, he is in a state of uncertainty. In this case it is assumed that the decision is based on a recoding of the time information - a "second look". Estimates of q, the period of the time base, and two other parameters were obtained by fitting predicted functions to the conditional probabilities of being correct, $\hat{P}(1|S_1)$ and $\hat{P}(2|S_2)$. The psychometric functions could then be well predicted by the model. There was an interesting tendency for the estimates of q to cluster in the range 19-31 msec., although the total range spanned by the estimates from the 10 psychometric functions fitted was 15-82 msec. Since q is a behavioral parameter reflecting a central periodic process, the degree of consistency in the estimates of q within and between Os is an important criterion in assessing the success of the model in accounting for the data. According to this criterion, the model may deserve further exploration.

However, certain problems with the model remain unresolved. One is the interpretation of a "bias" parameter defined in the decision process. This parameter was originally introduced because of evidence that when Os were to discriminate between adjacent empty intervals, the S_1 and S_2

stimulus patterns (longer interval first, or longer interval second) were not treated symetrically. It is assumed that when the count from the second interval is one more than the count from the first, the second interval is always chosen as the longer one. However, if the count from the _first_ interval is only one more than the count from the second, the O does _not_ consistently choose the first interval as the longer one, but selects the second interval with probability k. It appears that k may be related to base duration, implying that it is not a conventional bias parameter.

A second problem concerns the assumption of independence of the measures of two intervals. Experimentally, optimal performance can occur with as little as 500 msec. separation between the intervals. The best predicted performance is obtained by assuming independence of the measures of the two intervals. How can the coding of the second interval be independent of that of the first, if there is no variability in the period of the time base underlying the coding of an internal interval I, and no variability in the mapping of the external interval T into the internal one? One approach to this problem would be to explore, theoretically, the effect of having some variability in the spacing of the time points, and/or variability in the mapping of T into I. The latter source of variability is especially worth exploring if we are to develop a theoretical framework within which we can account for duration discrimination for intervals defined by various types of events. Evidence is accumulating that duration discrimination functions do depend on how the durations are defined for very brief intervals. Discrimination is easier if the durations are defined by auditory signals than by visual or bimodal signals (K&A). This does not necessarily imply that different processes are involved in coding the duration of the internal interval I. It may be instead that the mapping of T into I can have more or less variability, according to the modality through which the time input occurs. Another source for the difference could be a rehearsal strategy (the "second look") available only when the durations are defined in the auditory modality; examining response latencies might be informative with respect to this point.

To tackle the problem of variability in the mapping of an external interval into its internal representative, a model for the perception of temporal order proposed by Sternberg and Knoll (1971) might provide a fruitful approach. Temporal order, successiveness, and duration discrimination can all be viewed within the same paradigm, for these represent three questions we can ask about the temporal relation between two events, A and B. In the Sternberg and Knoll model, the psychometric function for temporal order is an estimate of the cumulative distribution of two random variables. One represents the difference in the latencies of certain internal events corresponding to A and B, their "arrival time" difference. In duration discrimination, we are interested in this same random variable, since it describes the mapping of the external interval between A and B into an internal one. A second random variable in the temporal order model is involved in the decision process; that is, it summarizes how the decision "A before B" is made, on the basis of the arrival time differences. In duration discrimination, the second random variable we are concerned with is the measure of the internal interval. A decision is then made on the basis of this second random variable, if such a measure is taken.

The quantal counting model represents an attempt to account quantitatively for the discrimination of brief empty intervals by postulating the same time base as is involved in Kristofferson's theory of psychophysical time. A measure of success is whether, in fitting the model to the data, we obtain estimates of the period q of the time base which agree with the estimates obtained by Kristofferson. There may be a systematic correspondence, in that for some Os $\hat{q}$ is approximately half as large as the values he has obtained by applying a two-state version of his attention switching model to successiveness discrimination.

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APPENDIX I

DERIVATIONS FOR THE QUANTAL COUNTING MODEL

(a) Coding of a single interval

It is assumed that the brief successive signals bounding the empty intervals each define a point in time for the O in such a way that the internal interval I which is produced is equal to the corresponding external interval T on every presentation. This very strong assumption is used to simplify the initial derivations, but it is likely to require revision.

The interval I is considered as being superimposed on a stable time base which is not affected by ongoing sensory events, with the time points occurring at equal intervals of q ms. If we express the duration I as a multiple of q, we can write

$$I = mq + e \qquad \text{with} \qquad 0 \leq e < q$$

or

$$I = (m+b)q \qquad \text{with} \qquad 0 \leq b < 1 \quad .$$

Let τ represent the number of time points occurring during I. Then on each trial or presentation of T, τ takes on one of two integer values m or m+1 (see figure A-1), with

$$P(\tau = m) = 1 - e/q$$

$$P(\tau = m+1) = e/q \quad .$$

To show this, we let ΔK_1 be the time between the onset of I and the first time point to occur within it. ΔK_2 is the time between the offset of I and the last time point occurring within it. If the onset of I can occur anywhere with respect to the time base, ΔK_1 is a uniformly distributed random variable such that $0 \leq K_1 < q$, and with cumulative distribution $F_x(\Delta K_1) = P(\Delta K_1 \leq x) = x/q$. The magnitude of ΔK_2 depends on what ΔK_1 happens to be, but we must always have $0 \leq \Delta K_2 < q$. When we have m complete quanta lying within I (see figure A-1) bounded by m+1 time points, we can write

$$I = mq + \Delta K_1 + \Delta K_2 \qquad \text{with} \quad \Delta K_1 + \Delta K_2 = e$$

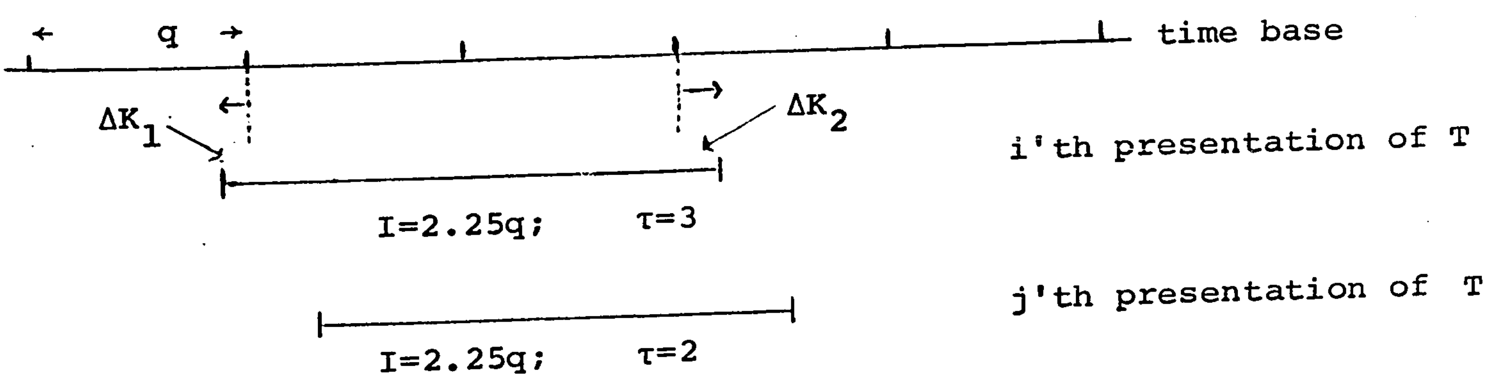


Figure A-1: The onset of I can occur anywhere with respect to a time point. Therefore ΔK_1 is a uniformly distributed random variable with $0 \leq \Delta K_1 < q$.

Thus we have $\tau = m+1$ when ever $\Delta K_1 \leq e$.

That is, $P(\tau=m+1 \mid \Delta K_1 \leq e) = 1$.

Hence $P(\tau = m+1) = P(\tau=m+1 \mid \Delta K_1 \leq e) \cdot P(\Delta K_1 \leq e)$

$= e/q$.

On the other hand, when $e < \Delta K_1 < q$, we have $\tau=m$. In this case there are m-1 complete quanta within I, and we write

$$I = (m-1)q + \Delta K_1 + \Delta K_2 \quad \text{with} \quad \Delta K_1 + \Delta K_2 = q+e$$

Hence

$$P(\tau = m) = P(\tau=m \mid e < \Delta K_1 < q) \cdot P(e < \Delta K_1 < q)$$

$$= (q-e)/q = 1-e/q \quad .$$

(b) Coding of pairs of intervals

We can express the durations T_s and T_v ($T_s \leq T_v$) in units of q:

$$T_s = (s + b_s) \cdot q$$

$$T_v = (v + b_v) \cdot q$$

where s and v are integers and $0 \leq b_s, b_v < 1$. Given the pattern $S_1 = (T_v, T_s)$, the outcome of coding the interval

T_v followed by T_s will be one of the four pairs of integers

$$v,\ s$$
$$v,\ s+1$$
$$v+1,\ s$$
$$v+1,\ s+1\ .$$

The quantities α, α' and γ are defined in terms of the difference between the members of these four pairs. We need to calculate the probabilities associated with each of these pairs.

Independent intervals

Let P(i&j) denote the probability of the occurrence of i time points in the first interval and j time points in the second. If it is assumed that the outcome of the coding of the second interval is independent of the coding of the first, we can use the relation

$$P(i\&j) = P(\tau_1=i)\cdot P(\tau_2=j)$$

and we associate the following probabilities to each of the four possible outcomes.

$$P(v\&s) = (1-b_v)\cdot(1-b_s)$$

$$P(v\&s+1) = (1-b_v)\cdot b_s$$

$$P(v+1\&s) = b_v\cdot(1-b_s) \qquad \text{(A-1)}$$

$$P(v+1\&s+1) = b_v\cdot b_s \quad .$$

When ΔT is small enough so that v=s in (A-1), (i.e., $T_v = (s+b_v)\cdot q$) we have

$$\alpha = P(\tau_1>\tau_2|S_1) = P(s+1\&s) = b_v\cdot(1-b_s)$$

$$\gamma = P(\tau_1=\tau_2) = P(s\&s) + P(s+1\&s+1)$$

$$= (1-b_v)(1-b_s) + b_v b_s$$

and $\alpha' = \alpha$ since the maximum difference between τ_1 and τ_2 is 1.

When ΔT is such that $T_v = (s+1+b_v)q$, (i.e., $v = s+1$), the possible outcomes are

(s+1, s)

(s+1, s+1)

(s+2, s)

(s+2, s+1)

Hence $\alpha = P(s+1\&s) + P(s+2\&s) + P(s+2\&s+1)$

i.e., $\alpha = (1-b_v)\cdot(1-b_s) + b_v\cdot(1-b_s) + b_v b_s$

$$\gamma = P(s+1\&s+1) = (1-b_v)\cdot b_s$$

and $\alpha' = (1-b_v)(1-b_s) + b_v\cdot b_s$.

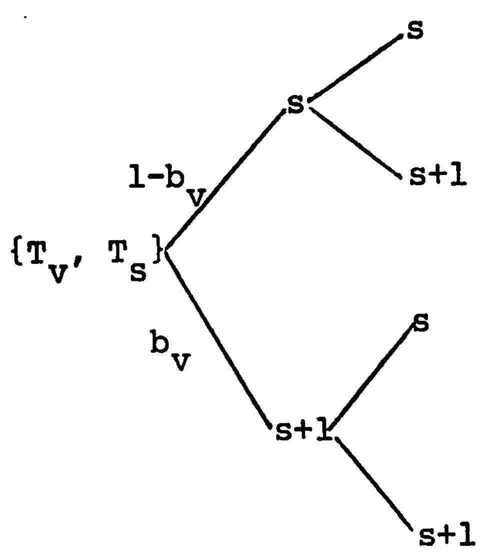
The final entry of table 1 for $T_v = (s+2+b_v)q$ is obtained in a similar way.

Adjacent intervals

If the coding of the second interval depends on the first, it is no longer true that, when S_1 occurs, $P(\tau_2 = s+1) = b_s$. In order to calculate the probabilities of the four possible outcomes in (A-1) for adjacent intervals, we consider the number of time points τ_D in the interval

$$T_D = T_v + T_s \quad .$$

The total number of time points in T_D, together with the number occurring in the first interval, constrains the number assigned to the second interval. T_D can be written as



I		II
2s	$P(\tau_d=2s) = 0$	(1) $P(s\&s) = 0$
2s+1, 2s+1	$P(\tau_d=2s+1)=1-b_d$	(2) $P(s\&s+1) = 1-b_v$
		(3) $P(s+1\&s) = (1-b_d)-(1-b_v) = b_v - b_d$
2s+2	$P(\tau_d=2s+2)=b_d$	(4) $P(s+1\&s+1) = b_d$

outcome of coding T_1 — outcome of coding T_2 — total number of time points in T_D — possible outcomes of the coding process

Figure A2: A schematic interpretation of equations (A-2) to (A-8) in Appendix I. τ_1, τ_2, τ_d represent the number of time points in T_1, T_2, T_D respectively. When $T_v = (s+b_v)q$, $T_s = (s+b_s)q$ and $b_s+b_v \geq 1$, we have $d=2s+1$ and $b_d = b_v+b_s-1$. The sum of outcomes (1) and (2) is equal to the probability that $\tau_1=s$. Similarly, the sum of outcomes (3) and (4) is the probability that $\tau_1=s+1$. These relations, when combined with the relations in column I, yield the probabilities in column II.

$$T_D = (v+s+b_v+b_s)\cdot q$$

or as

$$T_D = (d+b_d)q \qquad \text{with} \qquad 0 \le b_d < 1$$

If $b_v + b_s < 1$ we have $d = s+v$ and $b_d = b_s + b_v$. In this case $\tau_D = s+v$ or $s+v+1$. On the other hand, if $b_v + b_s \ge 1$, we have $\tau_D = s + v + 1$ or $s + v + 2$ since the integer d equals $s + v + 1$, and $b_d = b_v + b_s - 1$. There will be a total of six-cases to consider: $v=s$, $v=s+1$, $v=s+2$, each with two subconditions $b_v + b_s < 1$ and $b_v + b_s \ge 1$.

As an example, take $v=s$, and $b_v + b_s \ge 1$. Figure A-2 represents the various outcomes possible on any trial and their associated probabilities. Since $d = 2s + 1$,

$$P(s\&s) = 0 = P(\tau_D = 2s) \tag{A-2}$$

$$P(s+1\&s+1) = b_d = P(\tau_D = 2s+2) \tag{A-3}$$

$$P(s+1\&s) + P(s\&s+1) = 1 - b_d = P(\tau_D = 2s+1) \quad . \tag{A-4}$$

But by applying the relation

$$P(i\&j) = P(\tau_2=j \mid \tau_1=i)\cdot P(\tau_1 = i) \quad .$$

We can write

$$P(s\&s) + P(s\&s{+}1) = P(\tau_1 = s) = 1 - b_v \qquad \text{(A-5)}$$

$$P(s{+}1\&s) + P(s{+}1\&s{+}1) = P(\tau_1 = s{+}1) = b_v \;. \qquad \text{(A-6)}$$

From (A-2) and (A-5) we obtain

$$P(s\&s{+}1) = 1 - b_v \qquad \text{(A-7)}$$

and from (A-3) and (A-6) we have

$$P(s{+}1\&s) = b_v - b_d \;. \qquad \text{(A-8)}$$

Substituting $b_v + b_s - 1$ for b_d, equations (A-2), (A-3), (A-7) and (A-8) can now be used to write out the expressions for α, α' and γ in exactly the same way as was done for the independent interval case, when v=s, yielding

$$\alpha = P(s{+}1\&s) = (1-b_d) - (1-b_v) = b_v - b_d = 1 - b_s$$

$$\gamma = P(s\&s) + P(s{+}1\&s{+}1) = 0 + b_d = b_v + b_s - 1$$

$$\alpha' = \alpha$$

for $\Delta T = (b_v - b_s)q$ provided $b_v + b_s \geq 1$ and $b_v < 1$, i.e., $1 - b_s \leq b_v < 1$. The other entries in Table 1 are obtained in a similar way.

APPENDIX II

ONSET-OFFSET DELAY MODEL

(a) Probability density functions, for decision strategy I

On each trial the O is assumed to make a decision on the basis of the random variable ΔI defined as the difference between the two quantities

$$I_1 = T_1 + x_2 - x_1 \qquad I_2 = T_2 + x_4 - x_3$$

where T_1 and T_2 are constants, but x_i $(i=1,4)$ are independent uniformly distributed random variables such that $0 \leq x_i < q$. We write

$$\Delta I = T_2 - T_1 + X \tag{A-9}$$

with $X = x_4 + x_1 - x_3 - x_2$.

We require the probability density function of ΔI, conditional on $S_1 = \{T_v, T_s\}$ or $S_2 = \{T_s, T_v\}$ occurring. Once we have $G(X)$, the probability density function of X, we obtain $g_1(\Delta I)$ by substituting $\Delta I + \Delta T$ for X in $G(X)$; since for S_1, $T_2 - T_1 = T_s - T_v = -\Delta T$ and solving (A-9) for X gives $X = \Delta I + \Delta T$. Similarly, $g_2(\Delta I)$ is obtained by substituting $\Delta I - \Delta T$ for X in $G(X)$, since for S_2, $T_2 - T_1 = T_v - T_s = \Delta T$ and solving (A-9) for X gives $X = \Delta I - \Delta T$.

The derivation of G(X) is given in McKee et al (Appendix I). It essentially involves the convolution of two triangular distributions. The result expressed in the notation used here is given by the following set of four equations

$$G(X) = \begin{cases} (2q+X)^3/6q^4 & \text{for} \quad -2q \le X \le -q \\ (4q^3-6qX^2-3X^3)/6q^4 & \text{for} \quad -q \le X < 0 \\ (4q^3-6qX^2+3X^3)/6q^4 & \text{for} \quad 0 \le X \le q \\ (2q-X)^3/6q^4 & \text{for} \quad q \le X \le 2q \end{cases} \qquad \text{(A-10)}$$

The cumulative distribution for X is obtained by making the transformation $y = X/q$ in the integral $\int_{-\infty}^{c} G(X)\,dX$; in effect all distances along the ΔI axis are expressed in units of q. The following equations are used to set up a table of areas under the G(X) distribution, which can then be used in the same way as a table of areas under the normal distribution.

$$\int_{-\infty}^{c} G(X)\,dX = \begin{cases} \frac{1}{24}(2+c)^4 & \text{for } -2 \le c' \le -1 \\ \frac{1}{2} + \frac{1}{6}(4c'-2c'^3-\frac{3c'^4}{4}) & \text{for } -1 \le c' \le 0 \\ \frac{1}{2} + \frac{1}{6}(4c'-2c'^3+\frac{3c'^4}{4}) & \text{for } 0 \le c' \le 1 \\ 1 - \frac{1}{24}(2-c')^4 & \text{for } 1 \le c' \le 2 \end{cases} \quad \text{(A-11)}$$

Adjacent intervals

Let T_1 and T_2 be a pair of adjacent intervals defined by the time between three successive pulses. In this case, the delay in the onset of the second interval is assumed to be equal to the delay in the offset of the first interval. Hence the random variable whose probability distribution is required is

$$Y = x_1 + x_2 - 2x_3$$

where x_i (i=1,3) are independent and uniformly distributed, such that $0 \le x_i < q$. The probability distribution for Y is derived in Appendix II of C & K and is given by

$$G_a(Y) = \begin{cases} (2q+Y)^2/4q^3 & \text{for} \quad -2q \le Y \le -q \\ (2q^2-Y^2)/4q^3 & \quad -q \le Y \le q \\ (2q-Y)^2/4q^3 & \quad q \le Y \le 2q \end{cases} \qquad \text{(A-12)}$$

The cumulative distribution for Y and the probability density functions of ΔI conditional on S_1 or S_2 occurring are obtained in exactly the same way as in the case where the delay in the onset of I_2 is completely independent of the offset of I_1. The analog of equation (A-11) is

$$\int_{-\infty}^{c} G_a(Y)\,dY = \begin{cases} (2+c')^3/12 & \text{for} \quad -2 \le c' \le -1 \\ \frac{1}{2} + (2c'-c'^3/3)/4 & \text{for} \quad -1 \le c' \le 1 \\ 1 - (2-c')^3/12 & \text{for} \quad 1 \le c' \le 2 \end{cases} \qquad \text{(A-13)}$$

(b) Decision strategy II

Figure A3 schematically represents the "multiple decision" strategy involved in the second version of the quantal onset-offset model, adapted from A&K&W for the FC procedure. On each trial there are 4 possible outcomes for the coding of the pair of intervals: [L,L], [L,S], [S,L] and [S,S]. The quantities P_1 and P_2 are defined by

$$P_1 = P(S|T)$$

and

$$P_2 = P(L|T+\Delta T) \quad .$$

Figure 3 in section 1-4 illustrates the relation between P_1, P_2 and a criterion. The S_1 pattern is the pair of intervals $(T+\Delta T,\ T)$ and the S_2 pattern is the pair $(T,\ T+\Delta T)$. Hence

$$P([L,S]|S_1) = P(L|T+\Delta T) \cdot P(S|T)$$

$$= P_2 \cdot P_1 \quad ,$$

if we assume that the coding of the second interval is independent of that of the first. The probability of each of the outcomes, conditional on whether S_1 or S_2 has been presented, is obtained in a similar way. The final decision process is summarized by

$$P(2|[L,L]) = b$$
$$P(2|[L,S]) = 0$$
$$P(2|[S,L]) = 1$$
$$P(2|[S,S]) = b' \quad .$$

We can now write $P(1|S_1)$ and $P(2|S_2)$ in terms of P_1, P_2, b and b':

$$P(1|S_1) = P([L,S]|S_1) + P([L,L]|S_1)\cdot P(1|[L,L])$$
$$+ P([S,S]|S_1)\cdot P(1|[S,S])$$
$$= P_2\cdot P_1 + P_2\cdot(1-P_1)\cdot(1-b) + (1-P_2)\cdot P_1(1-b') \tag{14}$$

and in a similar way we obtain

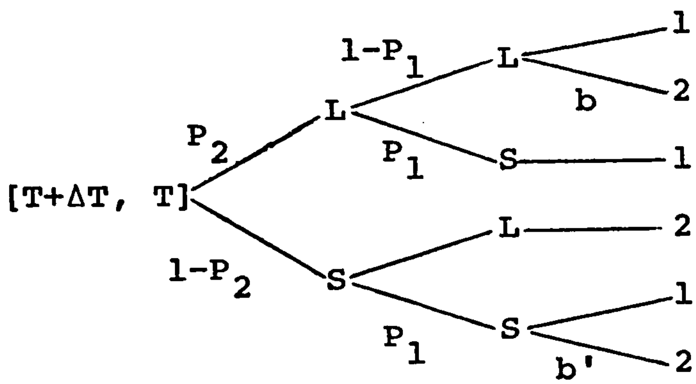
$$P(2|S_2) = P_1\cdot P_2 + (1-P_1)\cdot P_2\cdot b + P_1(1-P_2)\cdot b' \tag{15}$$

Combining and simplifying (14) and (15), we obtain

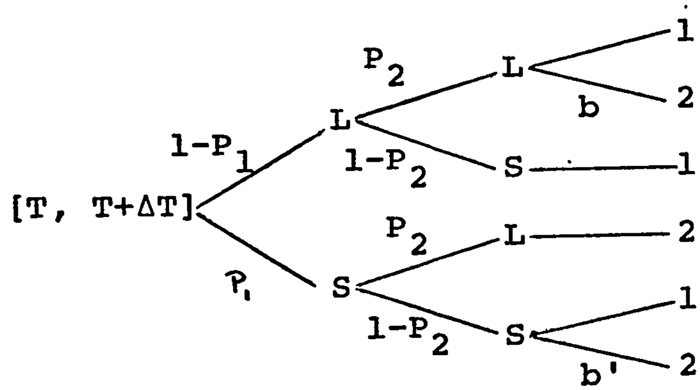
$$P(C) = \frac{1}{2}\,[P(1|S_1) + P(2|S_2)]$$
$$= \frac{1}{2}\,[P_1 + P_2] \quad .$$

$$P_2 = P(L|T+\Delta T)$$

$$P_1 = P(S|T)$$



$$P(1|S_1) = P_2(1-P_1)(1-b) + P_1P_2 + (1-P_2)P_1(1-b')$$



$$P(2|S_2) = (1-P_1)P_2(b) + P_1P_2 + P_1(1-P_2)b'$$

Figure A3: Decision strategy II for the quantal onset-offset model. Each interval is coded as either "S" or "L", and the choice of the longer interval is then made on the basis of one of four outcomes: [L,L], [L,S], [S,L] or [S,S].

Appendix 3

The following table contains the data from experiment 4, divided into blocks of approximately 8 sessions each for those Os who completed 16 sessions at each base duration. When T_s=200, 200(A) refers to a block of 8 sessions, and 200(B) refers to a second block which was run after the 16 sessions with base duration T_s=100. For one O (HS) there appeared to be a drop in the overall daily error rate after about 6 sessions at T_s=100; hence for this O the data at T_s=100 is divided into one block with N = 340 trials and a second block with N = 650.

	ΔT	$\hat{P}(1\|S_1)$	$\hat{P}(2\|S_2)$	$\hat{P}(C)$		ΔT	$\hat{P}(1\|S_1)$	$\hat{P}(2\|S_2)$	$\hat{P}(C)$
HS	6	.523	.652	.588	HS	6	.755	.738	.747
	12	.713	.867	.790		12	.887	.902	.895
100(A)	18	.829	.924	.877	100(B)	18	.968	.981	.975
	24	.872	.960	.916		24	.975	.994	.985
N=340	30	.931	.990	.960	N=650	30	.987	1.00	.993
HS	6	.615	.642	.629	HS	6	.626	.628	.627
	12	.722	.694	.708		12	.739	.746	.743
200(A)	18	.763	.818	.791	200(B)	18	.788	.853	.821
	24	.827	.870	.849		24	.813	.954	.884
N=400	30	.894	.925	.910	N=400	30	.858	.939	.899
RM	6	.823	.808	.816	RM	6	.876	.902	.889
	12	.931	.903	.917		12	.989	.983	.986
100(A)	18	1.00	.982	.991	100(B)	18	.989	1.00	.995
	24	1.00	1.00	1.00		24	1.00	1.00	1.00
N=350	30	.999	1.00	1.00	N=400	30	1.00	1.00	1.00
RM	6	.794	.772	.783	RM	6	.808	.656	.732
	12	.890	.839	.864		12	.915	.870	.893
200(A)	18	.970	.944	.957	200(B)	18	.965	.980	.972
	24	.965	.965	.965		24	.985	.990	.988
N=400	30	.977	.983	.980	N=400	30	.995	.995	.995

Appendix 3 - continued

	ΔT	$\hat{P}(1\|S_1)$	$\hat{P}(2\|S_2)$	$\hat{P}(C)$		ΔT	$\hat{P}(1\|S_1)$	$\hat{P}(2\|S_2)$	$\hat{P}(C)$
V	6	.711	.680	.695	V	6	.685	.621	.653
	12	.830	.780	.805		12	.750	.774	.762
200(A)	18	.810	.915	.863	200(B)	18	.822	.864	.843
	24	.844	.939	.891		24	.833	.919	.876
N=400	30	.894	.963	.928	N=400	30	.879	.915	.897
V	6	.505	.610	.558	V	6	.533	.665	.600
	12	.623	.844	.734		12	.691	.821	.756
100(A)	18	.790	.867	.828	100(B)	18	.837	.943	.889
	24	.854	.960	.907		24	.906	.964	.935
N=450	30	.925	1.00	.963	N=400	30	.919	.977	.948
KL	10	.419	.733	.58	KL	10	.530	.651	.59
	20	.393	.875	.63		20	.554	.750	.65
100(A)	30	.574	.903	.74	100(B)	30	.661	.828	.74
	40	.650	.946	.80		40	.757	.879	.82
N=400	50	.720	.970	.84	N=370	50	.837	.917	.88