RT MODELS
FOR TEMPORAL ORDER DISCRIMINATION

RESPONSE TIME MODELS
FOR TEMPORAL ORDER DISCRIMINATION

by

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ABSTRACT

The results of three experiments requiring the judgement of the temporal order of two continuous visual stimuli were reported. The stimuli were spatially separated and equal in intensity. Although the proportion of correct order judgements increased with the inter-stimulus-interval (ISI) in each experiment, there were differences in the response time (RT) results. When accuracy was stressed, mean RT increased as the ISI decreased. However, mean RT was relatively independent of ISI when RT deadlines were employed. As mean RT increased the slope of a linear relationship between the difference in mean RTs for the two order responses and a measure of performance accuracy decreased from positive to negative. Under accuracy conditions the mean RT for a correct response was less than the mean RT for an incorrect response by an amount which increased with an increase in ISI. When RT deadlines were imposed the mean RT for a correct response exceeded the mean RT for an incorrect response by an amount which increased with an increase in ISI.

The covariation between RT and response proportion measures was accounted for by a model which proposed that order discrimination involves three stages. In the stimulus information encoding stage, a sample of stimulus information is stored in a sensory storage buffer. In the decision stage the information obtained in the buffer drives a random walk process which generates quantitative predictions for response proportion and the mean RT for each order response. The response output stage contains processes such as motor actions which are assumed to contribute a constant component to the mean RT for each stimulus condition.

A model specifying that the order decision results from whichever stimulus is perceived first was shown to generate mean RT predictions which were not supported by the data obtained under accuracy conditions. It was concluded, therefore, that order discrimination involves a decision process which utilises information obtained from the stimulus pattern.

The use of a gap in one of the stimuli of a pair did not generate changes in response proportion which were predicted by a random walk decision process which was driven by information sampled directly from the stimulus pattern. However, the results of each experiment were consistent with the predictions of a model specifying that the parameters of the stored stimulus information vary with time since the commencement of the decision process.

When accuracy was stressed and the mean RT was long, the decision process was approximated by a terminal zero drift random walk. This model was shown to fit the data from Experiments I and III with a high degree of accuracy. Estimates of stimulus and response strategy parameters provided a means for assessing their separate contributions to order discrimination performance. However, this approximation failed to account for data obtained when RT deadlines were imposed.

The covariation between response time and response proportion measures was similar to that obtained in other psychophysical tasks. It was concluded, therefore, that the model proposed in this thesis has wide applicability to a variety of two-choice tasks.

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Finally I would like to dedicate this thesis to my parents for whose encouragement and sacrifices I am indebted.

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INTRODUCTION

A characteristic of human perceptual processing is the ability of various sensory receptor systems to code stimulation provided by physical stimulus energy. The physical energy generated by the stimulus may be described in terms of both physical location in space and the rate of change in the level of stimulus energy at each spatial location. Perception involves the coding of such spatio-temporal variations in stimulus energy or intensity in terms of corresponding changes in the activity of those neural structures which are activated by the transduced stimulus energy.

Suppose that the physical position and intensity of a stimulus are fixed and the stimulus, once turned on, remains on. In this case the stimulus is continuous. Under such conditions the sensory effect generated by the stimulus may vary as a function of time since stimulus onset. In order to track the time course of the sensation generated by a stimulus, an indirect psychophysical procedure is required.

In experiments described in this thesis two visual stimuli, equal in physical stimulus intensity and located in different fixed positions in the visual field, are illuminated sequentially. Both stimulus orders occur equally often during a block of experimental trials. In a simple psychophysical task using a set of such temporally separated stimulus pairs, the subject decides which light appeared to be illuminated first. Performance in this task can be described in terms of the percentage of correct order responses as a function of the time interval between the stimulus onsets. This <u>temporal order discrimination</u> task is a type of masking task since the order information provided by the stimulus pair is bounded by the times at which the two stimuli are detected by the visual system. For continuous stimuli, response accuracy in a temporal order discrimination task indicates the amount of order information available during the time interval between detection of the two stimuli.

A plausible mechanism underlying temporal order discrimination asserts that the perceived order of two stimulus events is governed by whichever stimulus is detected first. This mechanism is equivalent to a race between two stimulus dependent detection processes, the "winner" of the race being the process which first detects a change in the stimulus input. Accordingly, order inversion errors result from variability in the completion times for each process.

Alternatively, the relevant stimulus information for each stimulus is the amount of neural activity integrated over a sampling period of relatively fixed duration. If the sampling period is sufficiently long, the integrated neural activity depends on the pattern of stimulus events sampled. Hence order discrimination performance depends not only on the relative stimulus onset times but on all those changes in the stimulus input occurring during the sampling period which alter the neural activity in the visual system.

A quantitative account of order discrimination performance requires examination of decision processes as well as mechanisms for the accrual of stimulus information. A distinction can be made between decision mechanisms which utilise information obtained directly from the stimulus display during a sequence of observation intervals, and those which utilise the stored information obtained from a single observation of the stimulus display. In both cases, an evaluation of the separate contribution of stimulus information and response strategy influences on order discrimination performance is required.

In this thesis a random walk model for the decision process, together with suitable assumptions concerning the accrual of stimulus information, is proposed. The model generates independent estimates of stimulus information and response strategy parameters influencing order discrimination performance and accounts for the covariation between the relative proportion and the mean response time for correct and incorrect responses.

CHAPTER I

Temporal Order Discrimination and the Concept of Prior Entry

The historical antecedent of research on temporal order discrimination includes some of the earliest work in experimental psychology: the complication experiments (Boring, 1950). The term "complication" was originally employed by Herbart (1816) to refer to tasks involving the processing of stimuli from disparate sensory modalities, but the complication experiment evolved from problems astronomers often encountered while determining the precise time of a stellar transit. At the turn of the nineteenth century and before the introduction of mechanised methods of observation, the observer was required to synchronise the time at which a star passed a hairline meridian marker with an external time scale such as a click presented at one second intervals. If the stellar transit occurred between clicks the observer estimated the fraction of a second that had elapsed between the previous click and the sighting of the transit. When the transit occurred almost simultaneously with the click, variation between observers in the perceived order of the click and the transit was obtained (Sanford, 1888).

A commonly employed laboratory simulation of the stellar transit observation task consisted of a pendulum which was allowed to swing along a graduated scale in such a way that it rubbed against a metal bar at a predetermined point along the scale producing a click sound. The observer's task was to estimate the position of the pointer when the click was heard.

The position of the metal bar was varied from trial to trial so that there was a variable time interval between trial onset and presentation of the click. In some early experiments using this apparatus (Wündt, 1893) it was found that judgments of pointer position did not always correspond with the true pointer position. The direction of the error varied with the extent to which observers divided their attention between the click and pointer. Observers who attended to the click often reported the pointer position to be displaced to a location that it would have occupied if the click had occurred earlier. A displacement in the opposite direction was often reported by observers who tended to concentrate their attention on the pointer (Külpe, 1893).

A similar effect of attention was observed by Exner (1875) when observers were asked to discriminate the order of two rapidly successive stimuli from different sensory modalities. Exner demonstrated that under some circumstances a click would appear to occur before a light flash even though the light flash was actually presented first. He discovered that attending to the second stimulus in a click-flash pair made order inversion errors more frequent. In order to account for the effect of attention on order inversion errors in both the pendulum task and the click-flash experiments, Titchener (1899) proposed that directing one's attention towards a stimulus produces a more rapid rise of its representation in consciousness, so that its temporal location relative to a second stimulus is altered. This idea forms the basis for the concept of <u>prior entry</u>, suggested by von Tschisch (1885), which states that the stimulus towards which attention is directed is accorded more rapid processing by the sensory system stimulated.

A direct approach to the measurement of a prior entry effect using temporal order judgments was devised by Stone (1926). In an experiment employing auditory and cutaneous pulsed stimuli, Stone found that when subjects were asked to attend to the cutaneous stimulus and then report which stimulus appeared to occur first, the click could precede the cutaneous stimulus by 17 msec and still appear to occur simultaneously with it. When subjects were asked to attend to the click it was found that the click could be presented 23 msec after the cutaneous stimulus and still appear to occur simultaneously with it. Hence the effect of directing the subject's attention towards one of the two stimuli in a temporal order discrimination task was to generate a shift in the temporal separation of the stimuli necessary for the perception of apparent simultaneity, the point of subjective simultaneity (PSS).

Similar shifts in the PSS have been observed for visual stimuli when both relative retinal location and relative intensity have been varied. Rutschmann (1966) obtained temporal order judgments in a task employing equally intense monocular pairs of light flashes with duration 500 msec. The elements of a stimulus pair were presented at various stimulus onset asynchronies to the light adapted fovea and the peripheral retina. On each trial one light flash stimulated the fovea and the other light flash stimulated a point equally likely to be 30° of visual angle either to the left or to the right of the fixation point. The results indicated that for apparent simultaneity the peripheral stimulus preceded the foveal stimulus by 39 msec when presented nasally to the fovea, and by 46 msec when presented temporally to the fovea. Hence Rutschmann concluded that peripheral light flashes have a longer mean perception time than foveal light flashes. This result might be accounted for by assuming that during central fixation greater attention is paid to the foveal flash than to the peripheral flash. The foveal flash is granted "prior entry" and its perception time is thereby reduced.

Rutschmann (1973) presented pairs of 10 msec light flashes, one to the fovea and the other at various degrees of eccentricity along the horizontal meridian. The foveal flashes were binocular and the peripheral flashes were monocular. In a temporal order judgment task Rutschmann found that light flashes presented to the left visual field have a longer perception time as inferred from the PSS than light flashes presented at corresponding eccentricity to the right visual field. In a second experiment Rutschmann (1973) showed that the displacement of the PSS with increasing eccentricity of the peripheral stimulus is a linear function of visual angle. Hence the work of Rutschmann indicates a clear difference in relative perception times for stimuli presented at different positions in the visual field, when these differences are assumed to correspond with a displacement in the PSS for the psychometric functions obtained in a temporal order discrimination task.

Using a similar experimental design Gibbon and Rutschmann (1969) presented 10 msec light flashes, one to the foveal region of the left eye and the other to the nasal region of the retina of the right eye at a position 50° of visual angle along the horizontal meridian. The luminance of the foveal flash was varied between conditions so that its intensity was either higher than, equal to or lower than that of the peripheral light flash. In a temporal order discrimination task which used as stimuli pairs of foveal and peripheral light flashes, the PSS of the psychometric function for order discrimination shifted to a greater lag for the peripheral stimulus as the intensity of the foveal flash decreased. This result was interpreted to imply that the mean perception time for the foveal stimulus increased as its intensity decreased.

The use of the PSS of the psychometric function for temporal order discrimination as an estimate of the mean difference in perception times for the two stimuli is a variant of the "perceived-order" method which is commonly employed in visual psychophysics. In this procedure the subject sets the time interval between the stimuli so that critical stimulus events appear to occur simultaneously. It is then assumed that the temporal separation of the stimuli which corresponds to a judgment of simultaneity equals the difference in the mean perception times for these two stimuli.

The validity of this procedure relies on the assumption that in judgments of simultaneity the contribution of the decision process is independent of the temporal separation of the stimuli. Whereas the temporal order method is akin to the method of constant stimuli, the "perceived-order" method is a variant of the method of limits (Woodsworth & Schlosberg, 1951). Using a variety of visual stimulus configurations and the "perceived-order" method, Hansteen (1971), Lewis, Dunlap and Matteson (1972) and Walsh (1973) have all demonstrated that the mean perception time for stimulus offsets is less than the mean perception time for stimulus onsets.

A further application of the "perceived-order" method has been made by Efron (1973) and Haber and Standing (1970) who used clicks to be set by the subject so as to appear simultaneous with the apparent onset of a visual stimulus in one condition, and with the apparent offset of a visual stimulus with variable duration in another condition. Both studies reported a marked persistence of apparent stimulus duration beyond the actual stimulus duration for stimulus durations less than 125 msec.

Tasks employing judgments of temporal order and subjective simultaneity have shown that the distribution of attention between two stimuli and stimulus parameters such as the relative location of the stimuli in the visual field and their relative intensities influence the temporal separation of the stimuli necessary for the perception of apparent simultaneity. The studies by Stone and Rutschmann have shown that either focussing attention on one of the stimuli or increasing the relative intensity of one of the stimuli decreases the mean perception time for that stimulus relative to the mean perception time for the unattended or less intense stimulus. In addition, compared with foveally presented stimuli, equally intense stimuli presented peripherally have a longer mean perception time. With central fixation this latter effect is likely to be mediated by an attentional bias directed towards the foveal stimulus. So use of the PSS as an estimate of the difference in mean perception times for two stimuli has indicated that temporal order judgments are influenced by both subjective variables such as the allocation of attention and stimulus variables such as relative stimulus intensity. The displacement of the PSS as these independent variables are varied provides a measure of the prior entry effect.

Results analogous to those found in the literature on prior entry phenomena in vision also apply for the simple mean reaction time to visual stimuli. For example, Swift (1892) found that with a distracting auxiliary stimulus, the mean simple reaction time increased with attention to the disturbance. The classical results of Poffenberger (1912) demonstrated that simple reaction time increases as the stimulus is displaced further into the periphery of the visual field. Further, Cattell (1886) demonstrated that as the intensity of a light flash is increased the mean reaction time to that stimulus decreases.

What is the relationship between the difference in mean perception times for two stimuli inferred from the PSS which is obtained from judgments of temporal order or successiveness and from the difference in mean reaction times to the stimuli presented singly? Roufs (1963) used simultaneity judgments and simple RT (reaction time) to measure the relative perception times for two light flashes. In the simultaneity judgment task subjects set the time interval between the light flashes so that they appeared simultaneous. The corresponding measure for simple RT was the difference in mean RT to these stimuli presented separately. For one of his two subjects the correspondence between the two measures of relative perception times was high.

Using bisensory stimuli, Sanford (1971) failed to find a correspondence between reaction time and simultaneity judgment measures of the difference in perception times for the two stimuli. He used an apparatus similar to Wundt's complication pointer test. The stimuli used in the temporal order task were a click and the position of a pointer on a graduated scale. On each trial the subject located the position of the pointer as soon as he detected the click. The click intensity was varied between blocks of trials and the effect of intensity on perception time for the click relative to that for pointer position was measured in terms of changes in the apparent position of the pointer when the click was detected. In the RT task the same apparatus was used except that the pointer defined the duration of a variable foreperiod before the critical click stimulus was presented. The results showed that for the same decrease in click intensity the increase in mean RT exceeded the increase in mean difference in perception times. Hence reaction time measures tended to magnify the effect of changes in stimulus intensity on perception time.

Using cutaneous pulsed stimulus pairs, one to the index finger and the other to the contralateral second toe, Halliday and Mingay (1964) showed that a 20 msec difference in the onset of the cortically evoked responses to the two stimuli was not reflected in measures of the displacement of the PSS in a temporal order judgment task employing the same stimulus pair. Hence the temporal asynchrony required for subjective simultaneity in order judgments may not correspond exactly to the delay transmitted throughout the sensory and motor systems involved in a reaction time task.

In the Gibbon and Rutschmann (1969) experiment simple RT was measured for each light flash presented singly. Using the distributions of simple RT as estimates of the perception time distributions for each stimulus, predicted psychometric functions were obtained by convoluting appropriate pairs of RT distributions. As is customary in the "perceived-order" method, it was assumed that the subject's order response is generated by the first stimulus of a pair detected. The predicted psychometric functions in the Gibbon and Rutschmann study were quite close to the observed functions for one subject. For the other subject the predicted psychometric functions were displaced from the obtained functions so as to indicate that the mean perception time for the foveal stimulus was shorter than that for the peripheral stimulus to a greater extent in the temporal order task than in the simple RT task. This result suggests that attentional factors operating when two stimuli are presented in rapid succession may not operate in the same fashion when each stimulus is presented alone.

Although the qualitative effects of various independent variables such as relative stimulus location, relative stimulus attention and the allocation of attention have parallel effects on both the PSS in a temporal order discrimination task and the difference in mean simple RT to each stimulus presented alone, an exact quantitative correspondence between these measures does not always apply. The simple RT measures do not allow for possible interactions between the sensory effects generated by temporally continguous stimuli. In order to investigate the processes underlying temporal order discrimination a model describing a mechanism whereby stimulus information is sampled from the stimulus display and used in effecting an order decision is required.

CHAPTER II

Random Walk Models For Temporal Order Discrimination

Consider a pair of mutually discriminable stimuli S_1 and S_2 for which S_1 precedes S_2 by a time interval τ. For example, in the experiments to be described the stimuli are two equally intense lights situated equidistant above and below a centrally located fixation point. At the beginning of an experimental trial the fixation point is illuminated. T msec later S_1 (say) is illuminated, followed by the illumination of S_2 τ msec later. All three lights remain illuminated until the subject elicits a judgment of their order by depressing one of two response buttons indicating the response R_1 (S_1 first) and the response R_2 (S_2 first). From a series of experimental trials in which both stimulus order and inter-stimulus interval (ISI), τ , are varied the average data are represented by the relative proportion of each response and the mean response time for each response, conditioned upon both order and ISI. The aim of the models described in this chapter is to account quantitatively for the measured covariation between response proportion and mean response times.

Associated with each stimulus S_i ($i = 1,2$) is a set of n_i independent and mutually exclusive information sources or channels, C_i, which transmit neural activity generated by S_i. When the fixation point is illuminated at the beginning of a trial the subject (S) begins sampling neural activity in the two sets of channels. During the k^{th} observation period of duration Δt, the total activity sampled from channels C_i is represented by a random variable $A_{ik}, i = 1,2$.

The critical stimulus information producing an order decision is the difference in total activity sampled during the k^{th} observation period, $d_k = A_{1k} - A_{2k}$. Over a sequence of observation intervals S accumulates differences in total activity until one of two preset response threshold values for the accumulated stimulus difference is either reached or exceeded. An order response associated with this response threshold is then elicited. After n observations of stimulus information the accumulated activity difference is given by

$$D_n = D_o + \sum_{k=1}^{n} d_k$$

where D_o, the initial value of the sum is the starting point for the random walk. Since the sign of d_k is positive or negative depending on the relative values of A_{1k} and A_{2k}, D_n is represented by a positive or negative real number. Without loss of generality the response thresholds are set at A and -A for responses R_1 and R_2 respectively. The decision rule is then defined as:

Respond R_1 if $D_n \geq A$,

Respond R_2 if $D_n \leq -A$,

Otherwise sample another stimulus difference, d_{n+1}.

The mathematical representation of the decision process is in terms of a random walk of the random variable D_n along the real line constrained by absorbing barriers at A and -A. The expected value of D_o, $E(D_o)$, equals C, $-A \leq C \leq A$. The random walk decision process is similar to the sequential sampling scheme devised by Wald (1947) and applied to choice RT tasks by Stone (1960) and Laming (1968). In the sequential sampling model the random variable representing the step size to the random walk is the difference in the logarithms of the probability that the observed stimulus information arises from two hypothesised probability density functions (pdfs) with stimulus dependent parameters.

A problem with this representation is its assumption that the observer has a complete representation of the pdfs conditioned upon stimulus presented for the range of possible stimulus information values. Although this assumption might be appropriate in statistical hypothesis testing where known probability density functions are employed, it implies that S is capable of inferring the statistical properties of stimulus information variability. In contrast, the stimulus information accrual process considered in this chapter requires that S measure the difference between two stimulus dependent random variables. Knowledge of the form of the pdfs for these random variables is not necessary.

Properties of the Random Walk Model

The basic properties of the random walk decision process are represented in terms of both the probability of absorption and the mean number of steps to absorption at the two response thresholds. Predicted values for these measures in terms of A, C and parameters of the mgf for a stationary step size pdf are derived in Link and Heath (1975) and summarised in Link (1975) for the case in which the starting point for the random walk is fixed. An extension of these results in the presence of starting point variability is provided in the Appendix.

In a temporal order discrimination task a pair of temporally separated stimuli presented on a trial is defined as a stimulus pattern π_i, where the subscript $i = 1,2$, indexes the stimulus, S_i, which is illuminated first. Hence π_1 represents the pattern S_1 before S_2 and π_2 represents the pattern S_2 before S_1.

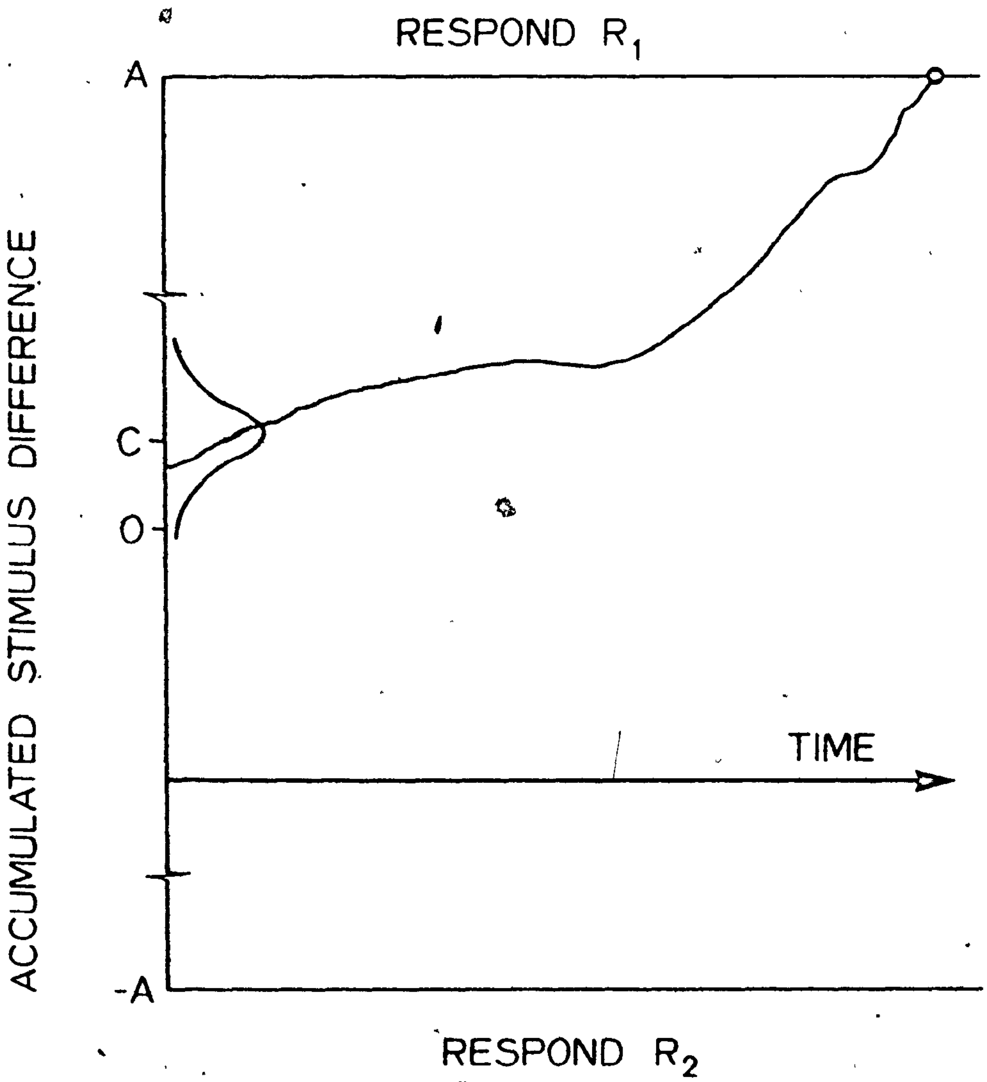
Two stimulus patterns π_1 and π_2 drive symmetric random walk decision processes if the step size distribution for π_2 is the mirror image of the step size distribution for π_1 about a value of zero (Link, 1975). For equally intense stimuli, patterns π_1 and π_2 drive symmetric random walk decision processes provided that the ISI is the same. Stimulus patterns which drive symmetric random walks are defined as symmetric stimulus patterns. Hence, the random walks generated by such patterns have mean step sizes which are equal but opposite in sign.

A pictorial representation of a random walk decision process is given in Figure 1. The vertical scale represents the accumulated stimulus difference and the horizontal scale represents time since the commencement of the process. Response thresholds are set at A and -A, and the distribution of starting points bounded by the interval [-A,A] has its mean value at C. A typical stochastic path for a random walk with positive drift is indicated. This path reaches the response threshold, A, and an R_1 response is elicited. Paths absorbing at -A generate an R_2 response. Since absorption is certain (Appendix p. 164) either R_1 or R_2 is elicited.

Response Probability Predictions

Let P_{ji} be the probability of response R_j to stimulus pattern π_i (i,j = 1,2). When there is starting point variability, P_{11} is given by Equation A9 in the Appendix. For normally distributed starting points with mean C, variance s_o^2 and a sufficiently small probability that D_o

Figure 1. The random walk decision process.


RESPOND R_1
A
C
O
-A
ACCUMULATED STIMULUS DIFFERENCE
TIME
RESPOND R_2

lies outside the interval [-A,A], equation A10 in the Appendix implies that

$$P_{11} = \frac{e^{\Theta_1(A+C)} - e^{\Theta_1 K}}{e^{\Theta_1(A+C)} - e^{-\Theta_1(A-C)}}, \quad -A \le C \le A, \quad K \le A + C \tag{1}$$

where $K = 0.5\, s_o^2 \Theta_1$, and Θ_1 is the non-zero root of $f^*(\Theta) = 1$. $f^*(\Theta)$ is the moment generating function (mgf) of the step size for the random walk, d, defined by

$$f^*(\Theta) = E[e^{-d\Theta}].$$

If the random walk always begins at C, the starting point variability, s_o^2, and hence the parameter K in equation (1), equals zero.

In the absence of starting point variability, Link (1975) has shown that data obtained using symmetric stimulus patterns provide estimates of $A\Theta_1$ and $C\Theta_1$. These estimates are given by

$$\widehat{A\Theta}_1 = \frac{1}{2} \ln\left[(P_{11}/P_{12})\cdot(P_{22}/P_{21})\right] \tag{2}$$

$$\widehat{C\Theta} = \frac{1}{2} \ln\left[(P_{22}/P_{21})\cdot(P_{12}/P_{11})\right] \tag{3}$$

When starting point variability cannot be ignored an estimate of $A\Theta_1$, $(\widehat{A\Theta}_1)'$, is given by

$$(\widehat{A\Theta}_1)' = \widehat{A\Theta}_1 - \frac{1}{2} \ln\left\{\left[\frac{e^{\Theta_1(A+C+K)} - 1}{e^{\Theta_1(A-C+K)} - 1}\right]\left[\frac{e^{\Theta_1(A-C)} - e^{\Theta_1 K}}{e^{\Theta_1(A+C)} - e^{\Theta_1 K}}\right]\right\} \tag{4}$$

where $\widehat{A\Theta}_1$ is given in equation (2). Since the second term in equation (4) vanishes when C equals zero, comparison with equation (2) reveals that $\hat{A}\Theta_1$ is unaffected by the presence of starting point variability when the mean starting point equals zero. For $\Theta_1 > 0$, the second term in equation (4) exceeds zero when $C < 0$, and is less than zero when $C > 0$. Hence in the presence of starting point variability, the estimated value of $A\Theta_1$ obtained from equation (2) exceeds its correct value when $C < 0$ and is less than the correct value when $C > 0$.

When the mean step size for the random walk is zero, the process is defined as having zero drift. In this case the stimulus pattern is represented as π_o. In the Appendix (Equation A11) it is shown that

$$P_{10} = \frac{1}{2}\,[1 + (C/A)], \quad -A \leq C \leq A. \tag{5}$$

For zero drift, the absorption probabilities do not depend on the variance of the starting point distribution. For A fixed, equation (5) generates the expected prediction that as C approaches A the probability of an R_1 response increases.

Decision Time Predictions

The decision time predictions of the random walk model are given by the number of steps to absorption multiplied by Δt, the time consumed by each step. If C is the mean starting point, m the mean step size and A the response threshold for a random walk with starting point variability, then it follows (Appendix p. 172) that the marginal mean number of steps to absorption conditioned upon the presentation of S_1 and pooled over both responses is given by

$$ED = \frac{A(2P_{11}-1) - C}{m} \tag{6}$$

Hence the marginal mean decision time equals (ED)Δt time units. As expected, an increase in A alone is accompanied by an increase in the marginal mean decision time.

Link (1975) has derived a test for detecting the presence of a random walk decision process based on a relationship between the marginal mean number of steps to absorption and the absorption probabilities for symmetric random walks. The test is valid when there is no starting point variability. The estimated marginal mean number of steps to absorption, is given by

$$\widehat{ED} = \hat{z}/m\Theta_1 \tag{7}$$

where $\hat{z} = A\hat{\Theta}_1(\hat{P}_{11}-\hat{P}_{12})$. Since $\hat{z}$ can be conveniently obtained from the data, equation (7) implies that for m and Θ_1 fixed, $\widehat{ED}$ is a linear function of $\hat{z}$.

When there is no starting point variability and C = 0, the difference in mean number of steps to absorption conditioned upon responses R_1 and R_2 respectively for the same stimulus pattern π_i, $EN_{1i} - EN_{2i}$, is given by

$$EN_{1i} - EN_{2i} = \frac{A}{m}\left(\frac{\gamma-1}{\gamma}\right) \tag{8}$$

(Link and Heath, 1975)

where m is the mean step size to the random walk, and γ is a parameter measuring the asymmetry of the step size mgf (Appendix, p. 175). Depending on the value of γ, the difference in response conditioned mean response times can be either positive, negative or zero.

For symmetric stimulus patterns generating symmetric random walks without starting point variability, the difference in the mean number of steps to absorption conditioned upon response R_1 for patterns π_1 and π_2, $EN_{11} - EN_{12}$, is given by

$$EN_{11} - EN_{12} = (A-C)\Theta_1 K' \quad (9)$$

where $K' = (\gamma-1)(\gamma m\Theta_1)^{-1}$ (Link, 1975).

The corresponding result for the difference in the mean number of steps to absorption conditioned upon response R_2 for patterns π_1 and π_2, $EN_{21} - EN_{22}$, is given by

$$EN_{21} - EN_{22} = (A+C)\Theta_1 K' \quad (10)$$

Estimates of $(A-C)\Theta_1$ and $(A+C)\Theta_1$ are given by

$$\widehat{(A-C)}\Theta_1 = \ln(\hat{P}_{11}/\hat{P}_{12}) \quad (11)$$

$$\widehat{(A+C)}\Theta_1 = \ln(\hat{P}_{22}/\hat{P}_{21}) \quad (12)$$

(Link, 1975), where the $\hat{P}_{ij}$, $i,j = 1,2$ are estimates of the corresponding P_{ij}.

For a random walk with starting point variability and zero drift the difference in the mean number of steps to absorption at A and −A is given by (Appendix, Equation A21)

$$EN_{10} - EN_{20} = \frac{-4AC}{3s^2} \quad (13)$$

where s^2 is the variance of the step size to the random walk and C is the mean starting point. This result does not depend on s_o^2, the variance of the starting point distribution.

Elimination of C from equations (5) and (13) yields a linear relationship between $EN_{10} - LN_{20}$ and $P_{10} - P_{20}$ for fixed A and s^2 since

$$EN_{10} - EN_{20} = \frac{-4A^2}{3s^2} (P_{10} - P_{20}) \qquad (14)$$

Equation (14) describes a relationship between the mean number of steps to absorption and the probability of absorption which is independent of the response bias parameter C. It provides a non-parametric test for detecting the presence of a zero drift random walk under conditions which maintain A and s^2 fixed.

The random walk model generates predictions for the probability of absorption and the mean number of steps to absorption at the two response thresholds. When starting point variability is negligible, equation (7) predicts a relationship between the marginal mean number of steps to absorption and probability measures for symmetric stimulus patterns. In the case of zero drift, equation (14) predicts a linear relationship between the difference in response conditioned mean number of steps to absorption and the difference in the probabilities of absorption at the two response thresholds. These results provide the basic mathematical predictions of the random walk decision process for temporal order discrimination. Specific predictions for the covariation between mean response time and relative response frequency in a binary choice task are also generated.

A Limited Observation Time (LOT) Model for Temporal Order Discrimination

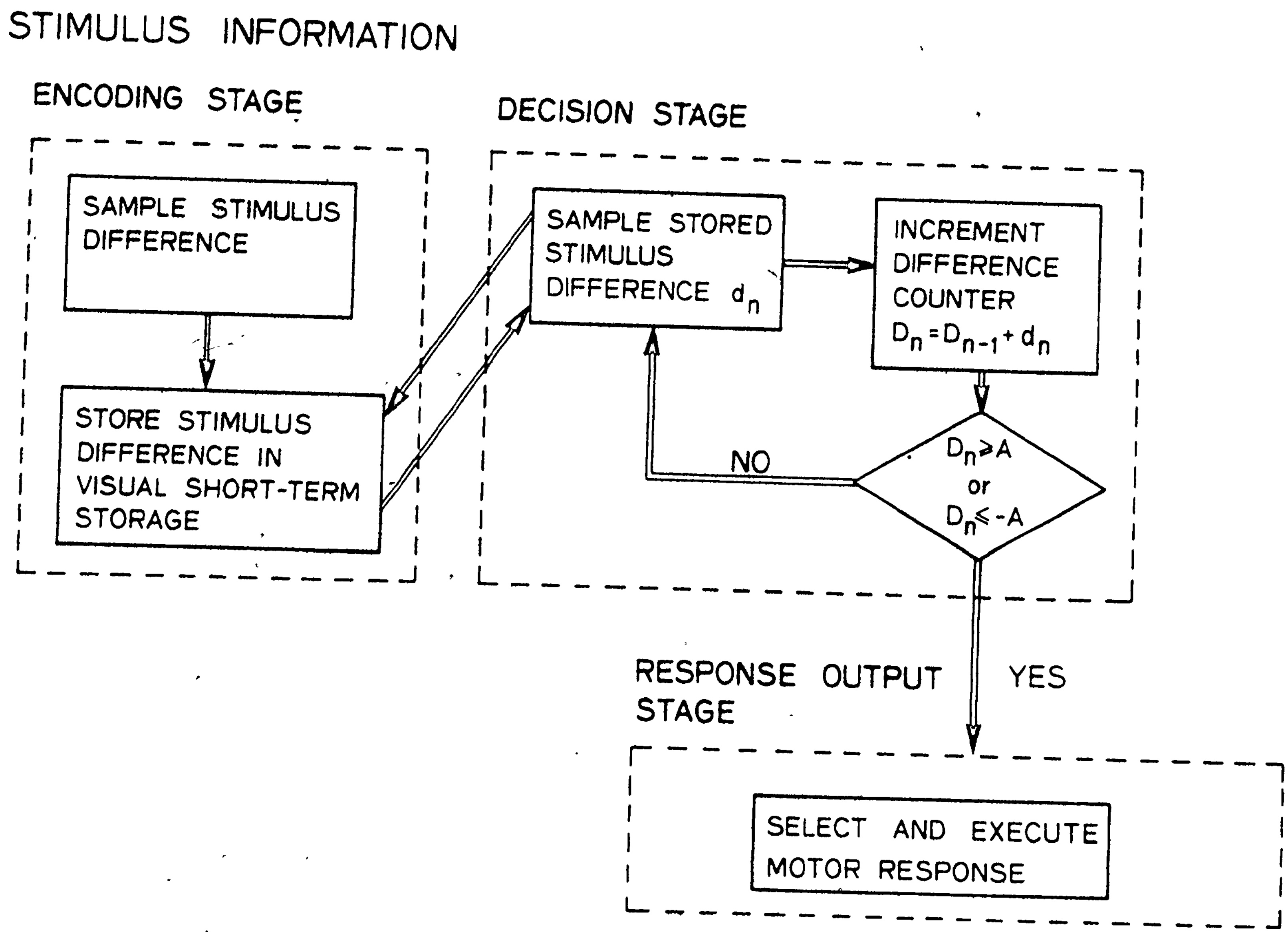
The LOT model for temporal order discrimination describes both the accrual of stimulus information and the manner in which a decision is reached. According to the pictorial representation of the model in Figure 2, temporal order discrimination involves three stages, a stimulus information encoding stage, a decision stage and a response output stage.

In the stimulus information encoding stage information from the transduced stimulus pattern presented on a trial is sampled for a finite sampling period T. The duration of the sampling period is either potentially modifiable by instructions, or constant, as is the perceptual moment (Stroud, 1955). The output from this stage is a difference in the summed sensory effects of the stimulation generated by the stimulus pattern during the sampling period T. This difference is stored in short-term memory and accessed during the decision stage. In the simplest version of the model it is assumed that the statistical properties of the stored representation of the stimulus pattern do not vary with time. Consequently the stored difference in stimulus information is a random variable with a stationary pdf. In a modification of the model the stationarity assumption is relaxed.

The decision stage depicted in Figure 1 consists of a random walk process driven by the stimulus difference information stored in sensory memory. During each arbitrarily small time interval, Δt, a sample of information is retrieved from storage and added to the accumulated stimulus difference. Depending on the value of this sum relative to the response thresholds set at A and -A either an order decision is executed or the sampling of stimulus difference information continues.

The starting point for the random walk decision process represents a response bias chosen by S on the basis of his expectations concerning

Figure 2. The LOT model for temporal order discrimination.


STIMULUS INFORMATION
ENCODING STAGE
SAMPLE STIMULUS DIFFERENCE
STORE STIMULUS DIFFERENCE IN VISUAL SHORT-TERM STORAGE
DECISION STAGE
SAMPLE STORED STIMULUS DIFFERENCE d_n
INCREMENT DIFFERENCE COUNTER $D_n = D_{n-1} + d_n$
$D_n \geq A$ or $D_n \leq -A$
NO
YES
RESPONSE OUTPUT STAGE
SELECT AND EXECUTE MOTOR RESPONSE

which response is likely to be required. Changes in expectancy over trials generate variability in the starting point for the decision process. As shown in the Appendix (p.166,174) the presence of starting point variability decreases both the probability of a correct R_1 response, P_{11}, and the marginal mean number of steps to absorption, ED.

The predicted decision time on each trial is the number of steps to absorption of the random walk process multiplied by the time consumed per step. The total response time (RT) consists of the decision time, the fixed stimulus sampling time T and the time required to select and execute a motor movement. The latter time is consumed in the response execution stage. The components of total RT which exclude the decision time are defined as the non-decision components. It is assumed that the mean non-decision component of total RT, ET_{ij}, for response R_i, EM_i, is additive with the decision component for that response, ED_{ij}, but independent of the stimulus pattern π_j presented. Hence

$$ET_{ij} = ED_{ij} + EM_i \tag{15}$$

For a pair of equiprobable symmetric stimulus patterns, equation (15) implies that the marginal mean RT is given by

$$ET = ED + \frac{1}{2}\left[EM_1+EM_2+(P_{11}-P_{22})(EM_1-EM_2)\right] \tag{16}$$

where ED is the marginal mean decision time and P_{ii}, $i = 1,2$ is the probability of a correct R_i response. For different symmetric stimulus pairs, the second term is constant provided that either EM_1 equals EM_2 or P_{11} equals P_{22}. Any problems introduced by a difference in EM_1 and EM_2 can be avoided by examining the effects of experimental variables on the mean RT for the same response.

Psychometric Function Predictions

The psychometric function for temporal order discrimination is a plot of the probability of an R_1 response for each stimulus pattern π_i. For normally distributed starting points to the random walk with mean C and variance s_o^2, the psychometric function for $\theta_1 > 0$ is given by equation (1). From the properties of the mgf (Cox and Miller, 1965, p. 48), $\theta_1 > 0$ when the mean step size to the random walk is positive, $\theta_1 = 0$ when the mean step size is zero and $\theta_1 < 0$ when the mean step size is negative. If equally intense stimuli S_1 and S_2 generate a positive mean step size when S_1 precedes S_2 and a negative mean step size when S_2 precedes S_1, then $\theta_1 > 0$ when S_1 precedes S_2 and $\theta_1 < 0$ when S_2 precedes S_1. When S_1 and S_2 are simultaneous, $\theta_1 = 0$. When S_2 precedes S_1, equation (1) can be modified by changing the sign of θ_1. From the definition of K, this does not change the value of the second term in the numerator. When S_1 and S_2 are simultaneous, K equals 0 and equation (5) applies. Hence the psychometric function is defined by

$$P_{11} = \frac{e^{\theta_1(A-C)}\left[e^{\theta_1(A+C)} - e^{\theta_1 K}\right]}{e^{2\theta_1 A} - 1}, \quad \theta_1 > 0 \tag{17}$$

$$P_{10} = \frac{1}{2}\left[1 + (C/A)\right], \quad \theta_1 = 0 \tag{18}$$

and

$$P_{12} = 1 - \frac{e^{\theta_1(A+C)}\left[e^{\theta_1(A-C)} - e^{\theta_1 K}\right]}{e^{2\theta_1 A} - 1}, \quad \theta_1 < 0 \tag{19}$$

For $A > C$ and $K << A$, P_{1i} $(i = 1,2)$ increase as Θ_1 increases. Hence the psychometric function is a monotonic increasing function of Θ_1. The intercept approaches 0.5 as A increases. For non-zero values of Θ_1, the proportion of correct order responses increases as A increases.

When starting point variability is negligible, estimates of $A\Theta_1$ and $C\Theta_1$ can be obtained from equations (2) and (3) for symmetric stimulus patterns. For equally intense continuous stimuli, symmetric stimulus patterns with the same ISI but different orders generate mean step sizes to the random walk which are equal but opposite in sign. In this case the values of Θ_1 are equal but opposite in sign.

Mean Response Time Predictions

For symmetric stimulus patterns equation (7) provides a test for the presence of a symmetric random walk process when starting point variability can be neglected. Provided that the mean non-decision times for each response are equal, equation (16) implies that these components of mean RT alter the intercept but not the slope of equation (7). Hence for a range of experimental conditions which do not cause $m\Theta_1$ to vary, equation (7) implies that the marginal mean RT is a linear function of $\hat{z}$ with slope $(m\Theta_1)^{-1}$ and intercept $\frac{1}{2}$ $(EM_1 + EM_2)$.

For the same response, equations (9) and (10) predict linear relationships between the difference between mean RT conditioned upon stimulus presented and estimates of $(A+C)\Theta_1$ and $(A-C)\Theta_1$ obtained from response probability data. In this case the predictions are unaffected by a difference in non-decision components of mean RT.

For both tests of the random walk model, linear relationships are predicted for a given pair of symmetric stimulus patterns when, under instructions generating an increased marginal mean RT, S increases the response threshold value, A. As implied by equation (6), an increase in A generates an increased marginal mean RT for the same stimulus condition.

The Real-Time Model For Temporal Order Discrimination

The real-time model for temporal order discrimination specifies that information is sampled from the stimulus pattern until sufficient information in favour of one of the response alternatives has been accumulated. The term real-time, borrowed from computer terminology, implies that the sampling of information from stimulus events and the decision process which is driven by this information operate in parallel. Unlike the LOT model, a sample of stimulus information is not placed in short-term storage and accessed later by a decision process. In this respect the real-time model is memoryless.

Sampling of stimulus information begins when the fixation point is detected. Thereafter, successive samples of stimulus information are obtained from the stimulus display during each sampling interval of duration Δ. The stimulus information driving the random walk decision process during the k^{th} sampling period is the difference, d_k, in the total neural activity in the sets of channels stimulated by each stimulus.

When no stimulus information has been sampled from the display the mean step size to the random walk is zero. This type of decision process occurs when no stimulus contingent neural activity has occurred before the first stimulus has been detected, and when the neural activity generated by each stimulus is the same. The latter condition holds for equally intense stimuli when both stimuli are continuously present.

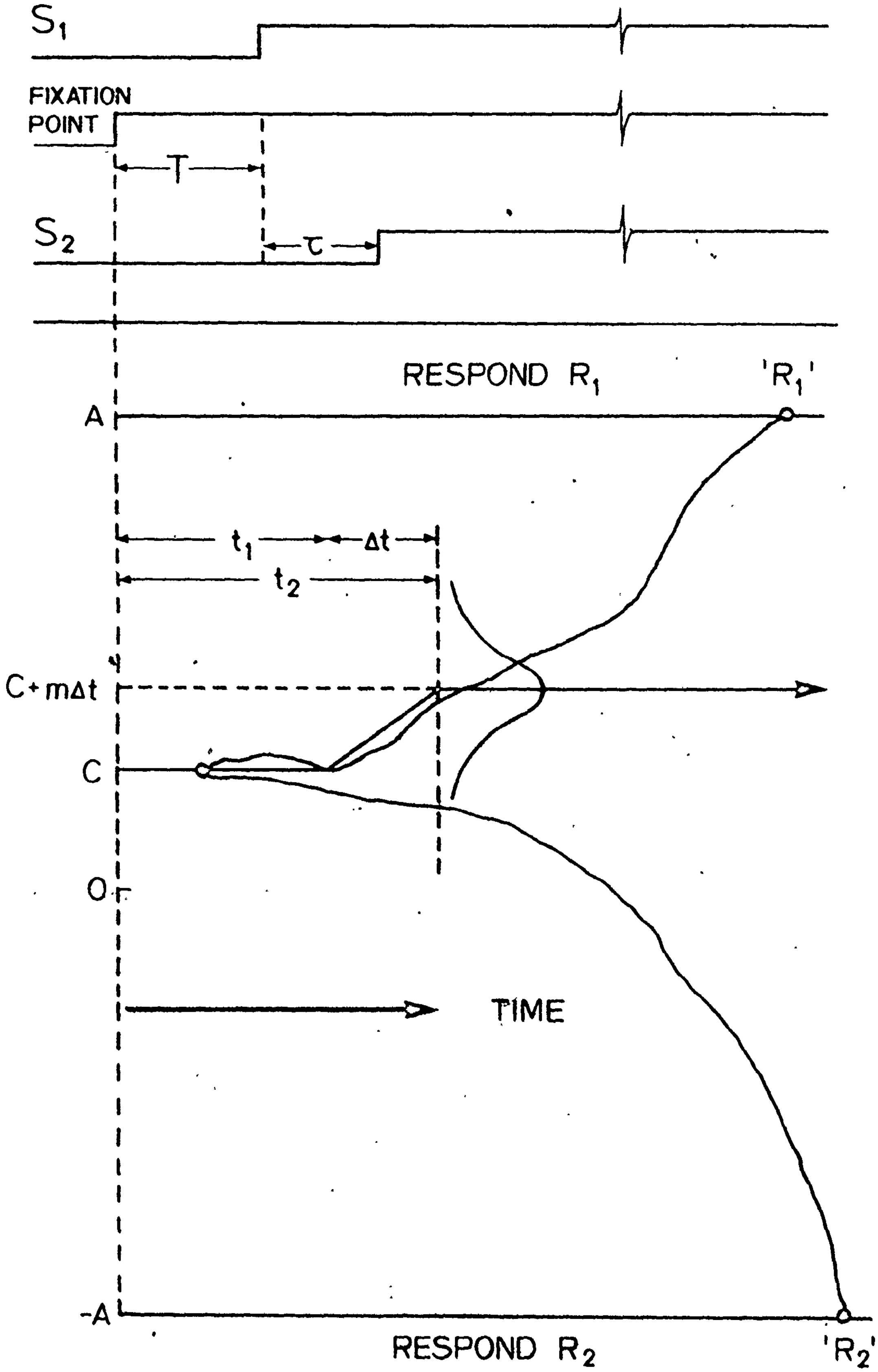
When more neural activity sampled during the sampling period emanates from S_1, the mean step size to the random walk is positive. When more neural activity emanates from S_2, the mean step size is negative. Hence the absolute value of the mean step size depends on the relative contribution of the neural activity obtained from each stimulus during the sampling period.

The random walk process corresponding to a stimulus pattern in which S_1 precedes S_2 by τ msec is depicted in Figure 3. If A is sufficiently large, the random walk process consists of three stages. The preparation stage with zero mean drift represents the effects of stimulus information accrued before neural activity generated by S_1 has occurred. Any increments to the random walk occurring during this stage result from spontaneous neural activity (Kuffler, FitzHugh & Barlow, 1957) with identical statistical properties for both sets of channels.

Following the preparation stage is the stimulus information stage during which a difference in neural activity generated by the two stimuli drives the random walk in a generally positive or negative direction. During this stage the order information present in the stimulus pattern shifts the mean displacement of the walk towards the response threshold associated with the correct order response. For example, if the response threshold at A corresponds to the response R_1 (S_1 first), then the stimulus pattern depicted in Figure 3 tends to displace the random walk decision process from a mean initial value at C towards A.

When no more order information is available from the stimulus pattern, the mean step size to the random walk reduces to zero. This terminal zero drift process continues in the third stage, the masking

Figure 3. The real-time model for temporal order discrimination.


S1
FIXATION
POINT
T
S2
τ
RESPOND R1
'R1'
A
t1
Δt
t2
C+mΔt
C
O
TIME
-A
RESPOND R2
'R2'

stage, until it absorbs at one of the response thresholds and the corresponding response is elicited. During this stage the order information provided by S_1 during the stimulus information stage is masked by the presence of S_2.

Provided that absorption of the decision process does not occur prior to the commencement of the masking stage, the decision process for the real-time model can be represented by a zero drift random walk with starting point variability. Variability in the starting point results from variability in the step sizes during the preparation and stimulus information stages. The mean starting point for the terminal zero drift process is a function of the order information contained in the stimulus pattern. In general, the larger the value of τ the greater the displacement of the mean starting point for the terminal zero drift process towards the correct response threshold and the more probable is the correct order response.

Psychometric Function and Mean Response Time Predictions

Equation (14) provides a test for a zero drift random walk decision process when A and s^2, the variance of the step size to the random walk, are fixed for various stimulus patterns. Under these conditions equation (14) predicts a linear relationship between the difference in response conditioned mean decision times and the difference between the proportion of R_1 and R_2 responses to the same stimulus pattern. The slope of this line is strictly negative. From equations (14) and (15) the difference in response conditioned mean RT is given by

$$T_1(\tau) - T_2(\tau) = \frac{-2A^2}{3s_1^2}(P_{10}-P_{20}) + EM_1 - EM_2 \qquad (20)$$

where $s^2 = 2s_1^2$.

Hence the intercept of equation (14) is non-zero when the mean non-decision times conditioned upon responses R_1 and R_2 differ.

When the sampling period, Δ, is arbitrarily small and the mean step size to the random walk during the stimulus information stage is constant and equal in absolute value for each stimulus, the duration of the stimulus information stage is bounded by the times at which the two stimuli generate neural activity. If these times, measured relative to the illumination of the fixation point, are denoted by t_i for stimulus S_i $(i = 1,2)$, then the mean displacement of the starting point for the terminal zero drift process is given by $m(t_1 - t_2)$ where m is the mean step size generated by S_1 alone.

If $\Delta t = t_1 - t_2$ is a random variable with pdf $f_{12}(\Delta t)$, then provided Δt is bounded so that absorption of the decision process does not occur prior to the beginning of the masking stage, the predicted psychometric function is given by

$$P_1(\tau) = \frac{1}{2}[1 + (C+m\tau)/A], \quad -A \leq C + m\tau \leq A \qquad (21)$$

where $\tau = E(\Delta t)$.

In this case $P_1(\tau)$ is a linear function of τ with slope $\frac{1}{2}(m/A)$ and intercept $\frac{1}{2}[1 + (C/A)]$. As A increases the slope of the psychometric function decreases, and the intercept approaches 0.5. Since Δt assumes both positive and negative values, the domain of equation (21) is a closed interval on the real-line containing zero.

Since s_1^2, the variance of step sizes to the terminal zero drift random walk process, is the sum of the variance of step sizes arising from independent neural processes generated by each stimulus, its value is independent of Δt. Consequently, the predicted difference in response conditioned mean RTs is given from equation (13) by

$$\Delta_1(\tau) \equiv T_1(\tau) - T_2(\tau) = \frac{-2A(C + m\tau)}{3s_1^2} + EM_1 - EM_2 \tag{22}$$

where $2s_1^2 = s^2$ and $\tau = L(\Delta t)$.

In this case $\Delta_1(\tau)$ is a linear function of τ with strictly negative slope.

Estimates of the parameters, A,C and m scaled by s_1 are obtained from

$$\hat{A} = \hat{m}/2\hat{p}_2 \tag{23}$$

$$\hat{C} = (2\hat{q}_2 - 1)\ \hat{m}/2\hat{p}_2 \tag{24}$$

$$\hat{m} = (-3\hat{p}_1\hat{p}_2/2)^{1/2} s_1 \tag{25}$$

where $\hat{p}_1$ is an estimate of the slope of the best-fitting least squares linear plot of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against τ, and $\hat{p}_2$ and $\hat{q}_2$ are estimates of the slope and intercept respectively of the best-fitting least squares plot of $\hat{P}_1(\tau)$ against τ.

An estimate of the difference in mean non-decision times for the two responses is obtained from the estimated intercept of the best-fitting least-squares linear fit of equation (20) to the data. The parameter estimates obtained from equations (23), (24) and (25) can then be used to predict the slope of equation (20). This procedure provides a consistency test for the model's predictions.

Comparison of the Predictions of the LOT and Real-time Models for Temporal Order Discrimination

The LOT and Real-time models specify that temporal order discrimination performance depends on three inter-related processes, a stimulus information accrual process, a decision process and a response execution process. Both models generate predictions concerning the form of the psychometric function, the trend in mean RT data, and the covariation between these performance measures for different stimulus patterns.

Although both models incorporate similar random walk decision processes and response execution processes, they differ in the manner in which order information is obtained from the stimulus pattern. The LOT model specifies that a sample of information contained in the stimulus pattern is obtained during an observation period of finite duration. The stored information determines the sequence of increments to the random walk decision process, and no further information is sampled directly from the stimulus pattern. The real-time model, on the other hand, specifies that increments to the random walk decision process are generated by information sampled directly from the stimulus display. In this case no storage of stimulus information in memory is required.

For both models, parameters of the random walk decision process provide an assessment of the effects of response strategy and stimulus variables on performance. Both the value of the response threshold and the mean starting point for the random walk indicate the response strategy adopted by S. As Link (1975) has demonstrated, Ss performing a two-choice RT task select a starting point which is closer to the response threshold associated with the response corresponding to the more probable stimulus alternative. Also, changes in response threshold alter the average time consumed by the decision process.

Although the representation of response strategy is the same for both models, the models differ in their representation of the order information available in the stimulus pattern. If, for the LOT model there is no loss of order information in storage, at least for the duration of the decision process, then the order information is represented by the drift rate for the random walk. For the real-time model, on the other hand, order information is represented by a bias in the mean starting point for the random walk. In this case the random walk process is zero drift, provided that absorption does not occur during the stimulus information stage. Under these conditions, the predictions of the model are unaffected by starting point variability.

Psychometric Function Predictions

For continuous stimuli, both models predict that the psychometric function is a monotonic increasing function of the signed value of ISI (positive when S_1 precedes S_2 and negative when S_2 precedes S_1). Although generally curvilinear for the LOT model, the psychometric function is linear for the real-time model when the sampling time is arbitrarily small and the mean step size to the random walk generated by each stimulus is constant. In this case, the slope of the psychometric function decreases as the response threshold value increases. For simultaneous stimuli the intercept measures the response bias adopted by S. In contrast, an increase in response threshold value increases the mean slope of the psychometric function predicted by the LOT model.

Mean Response Time Predictions

Critical tests of the two models involve predicted linear relationships between mean response time measures and transformation of the response probability measures. In each case, the covariation between response time and response proportion measures depends on differences in the stimulus information sampling assumptions.

The models differ in the permissible relationships between response conditioned mean response times for each stimulus pattern. Whereas the predictions of the LOT model do not constrain the sign of the difference between response conditioned mean RTs, the real-time model predicts that response conditioned mean RT is less for the more probable response.

CHAPTER III

Temporal Order Discrimination Under Accuracy Instructions

In order to obtain reaction time data in a temporal order discrimination task three experiments which examined the effects of response and stimulus variables were run. Response proportion and mean response time data were collected to test the predictions of the random walk models. In Experiment I reaction time data were obtained under instructions emphasizing accuracy, and in Experiment II the speed-accuracy trade-off for order discrimination was examined using response time deadlines. Experiment III examined the effect of changes in stimulus parameters which have predictable effects on the psychometric function for order discrimination if the real-time model is correct.

Each experiment employed the same stimulus display. The successive illumination of two equally intense lights situated equidistant above and below a central fixation point served as the critical stimulus events. This vertical arrangement of the stimulus lights was chosen in order to eliminate the effects of a left-right visual scanning strategy which Sekuler et al (1973) have suggested operates in order discrimination tasks using visual stimuli. Except in Experiment III, once a stimulus light was illuminated it remained illuminated until the subject (S) responded. Hence there was only one order cue for each stimulus light. In Experiment III gaps were included in one of the stimulus lights, but the stimulus onsets were still the critical events to be ordered by S.

Experiment I

Temporal Order Discrimination Under Instructions Emphasizing Accuracy

Experiment I was performed in two phases, Experiment Ia preceding Experiment Ib. The same apparatus was employed in each experiment and the procedures used were similar.

METHOD

Subjects

Each phase of the experiment employed four university students as Ss. The Ss in Experiment Ia had had no previous experience in order discrimination tasks, whereas two of the Ss in Experiment Ib (NB and MG) had had previous training. Each S had satisfactory 20:20 corrected or uncorrected vision as measured by a Snellen eye-chart. They were paid $2 for each experimental session which lasted approximately one hour.

Apparatus

The Ss viewed three Ne51H neon bulbs, the image from which were projected on to the vertical plane at eye level by means of beam splitting prisms. The average rise time for each light was specified as approximately 25 μsec. Since all computer timing was achieved to within an accuracy of 100 μsec, the rise time for each light was instantaneous for all practical purposes. The intensities of the lights were measured by a Spectra photometer which was focussed on the images formed through the beam splitting prisms. The intensities of the top and bottom lights were each 5.5 ft/l and the intensity of the centre light was 0.03 ft/l.

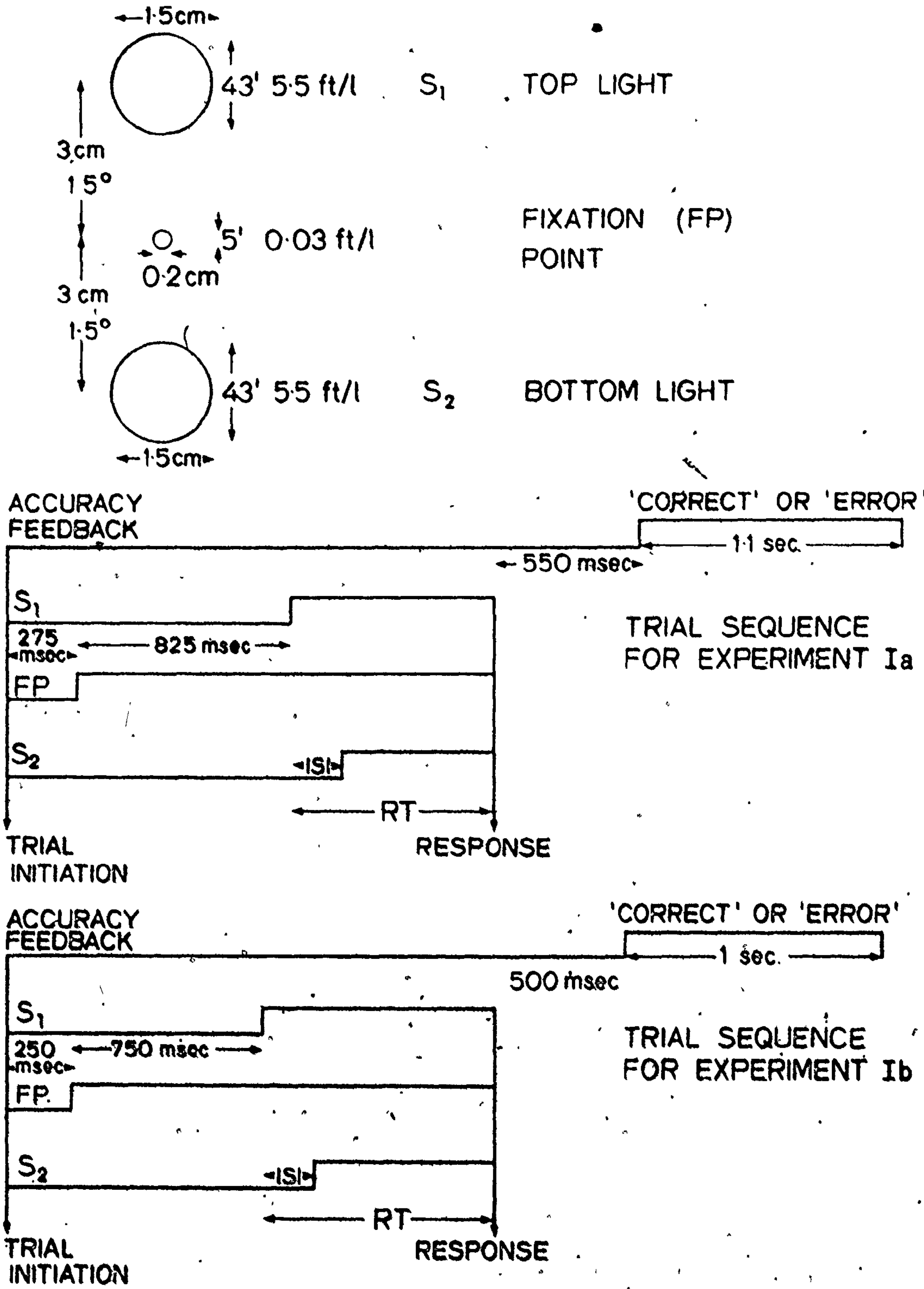
The visual display is depicted in Figure 4. The top and bottom lights were each 1.5 cm in diameter and shielded by a yellow faceted convex lens. The centre light was shielded by a red faceted convex lens so that the light transmitted through the lens had a diameter of 2 mm. The top and bottom lights were equidistant 3 cm above and below the central fixation light, as measured between their respective centres.

The S viewed the display from a chin rest at a distance of 120 cm so that each light was separated by 1.5° of visual angle. This degree of separation together with the presence of a centrally located fixation point was chosen so that interaction of the visual effects generated by the two stimulus lights would be reduced and the whole stimulus display could be viewed foveally. From the S's position, the stimulus and fixation lights subtended visual angles of 43' and 5' respectively. Ambient illumination was provided in order to reduce the effects on the S's adaptation level of a visually presented feedback display.

The response panel consisted of a box with dimensions 38 cm x 30 cm x 10 cm which contained two response buttons protruding 6 mm from the side of the box opposite S. The buttons were 3 cm apart and could be depressed by the forefinger of each hand. The S's arms rested gently on the upper surface of the box. For Ss RM, IK, MG and R the left button indicated the response "Top light first" (R_1) and the right button indicated the response "Bottom Light first" (R_2). For Ss, VY, CC NB and N the opposite response mapping was employed. Upon depression, each response button activated an 80 g. microswitch.

The sequence of trial events was controlled automatically by a PDP-8I computer which was situated in a different room to that occupied

Figure 4. Stimulus display and sequence of trial events for Experiment I.


1·5cm
43' 5·5 ft/l
S1
TOP LIGHT
3 cm
1·5°
5' 0·03 ft/l
0·2cm
FIXATION (FP)
POINT
3 cm
1·5°
43' 5·5 ft/l
S2
BOTTOM LIGHT
1·5cm
ACCURACY
FEEDBACK
'CORRECT' OR 'ERROR'
1·1 sec.
550 msec
S1
275
msec
825 msec
FP
S2
ISI
RT
TRIAL
INITIATION
RESPONSE
TRIAL SEQUENCE
FOR EXPERIMENT Ia
ACCURACY
FEEDBACK
'CORRECT' OR 'ERROR'
1 sec.
500 msec
S1
250
msec
750 msec
FP
S2
ISI
RT
TRIAL
INITIATION
RESPONSE
TRIAL SEQUENCE
FOR EXPERIMENT Ib

by the S. All timing of trial events was measured to within an accuracy of 100 μsec by a Type KW8/I real-time clock. Response accuracy feedback was provided by means of a calligraphic display (Tektronix 602, P4 phosphor) which was positioned approximately 10 cm below the level of the bottom stimulus light.

Procedure

Each experimental session in Experiment Ia contained three blocks of 235 trials. The first block was preceded by a five minute dark adaptation period and there was a two minute break between blocks of trials. Since the sequence of trial events was initiated by S, a rest could be taken at any time during the session. The first ten trials of each block contained at least one presentation of each trial type in a random order. They were treated as warm-up trials and ignored in subsequent data analysis. The remaining 225 trials contained presentations of each of the nine trial types in a random order. Each trial type was presented with equal frequency within the last 225 trials of each block.

For two of the Ss in Experiment Ib, N and R, who took considerably longer than the other Ss to complete each block of trials, each experimental session contained two blocks of trials. For the other two Ss each experimental session contained three blocks of trials. In order to ensure that runs of the same trial type were minimised the final 225 trials of a block consisted of a sequence of random permutations of each of the nine trial types.

The sequence of trial events for Experiment Ia is depicted in Figure 4. Each trial was initiated by S's depression and release of one of the response buttons. Following a 275 msec delay the centrally located fixation light was illuminated. After a delay of 825 msec the first stimulus light was illuminated. This was followed by one of five possible inter-stimulus intervals (ISIs) which was terminated by the illumination of the second stimulus light. For VY the ISI values were 0 msec, 5.5 msec, 11 msec, 16.5 msec and 22 msec and for the other three Ss the ISI values were 0 msec, 6.6 msec, 13.2 msec, 19.8 msec and 26.4 msec. All three lights remained illuminated until S responded by depressing one of the response buttons. If S depressed a response button prior to the illumination of the first stimulus light the same trial sequence was presented again.

As soon as S depressed a response button all three lights were extinguished and following a 550 msec delay, response accuracy feedback (CORRECT or ERROR) was written on the calligraphic display. On simultaneous (0 ISI) trials false response accuracy information was provided by displaying CORRECT when the number of msec recorded for the RT on that trial was odd and ERROR when the number of msec recorded for the RT on that trial was even. Following termination of the feedback display which lasted 1.1 sec, the S was free to initiate the next trial.

The sequence of trial events for Experiment Ib is depicted in Figure 4. Each trial was initiated by the S's simultaneous depression and release of both response buttons. All time intervals are 10% less than those employed in Experiment Ia.

The Ss were instructed to fixate the centre light once it was illuminated and to attend equally to the two stimulus lights. They were asked to obtain as many correct responses as possible. If they felt tired during a block of trials a short rest period was permitted.

Excluding practice sessions, data were analysed from 10 sessions for NB and MG, 11 sessions for RM and IK, 13 sessions for CC and VY and 15 sessions for N and R. A total of 58,500 observations were obtained for the eight Ss. RTs exceeding 4095 msec were not recorded.

RESULTS

The raw data obtained from each S are contained in Tables A1 and A2 for Experiments Ia and Ib respectively. The pooled data for Experiment Ib are contained in Table A3. The trends in both relative response frequency and mean RT as a function of the ISI (inter-stimulus interval) were examined.

Psychometric Function

The psychometric function for temporal order discrimination is a plot of estimates of the proportion of R_1 ("Top light first") responses as a function of τ, the ISI. τ is arbitrarily designated positive when S_1, the top stimulus light, precedes S_2, the bottom stimulus light, and negative when S_2 precedes S_1. The psychometric function, $\hat{P}_1(\tau)$, is then a function of τ.

Figure A1 contains plots of the psychometric function, $\hat{P}_1(\tau)$ for each S. Except for IK, $\hat{P}_1(\tau)$ was a monotonic increasing function as τ increased. For IK there was no improvement in accuracy with increased τ when S_1 preceded S_2.

Linear regression of $\hat{P}_1(\tau)$ against τ indicated that for each S a highly significant linear fit to the data points applied. This conclusion was substantiated by highly significant t-values for linearity (Hays, 1963, p521) ($p < .01$). So, in spite of an apparent curvilinear trend for some Ss, the hypothesis of linearity could not be rejected.

Table 1 contains estimates of the slope and intercept of the best fitting lines according to a least squares criterion, together with their respective 95% confidence intervals. Although there were individual differences in the estimate of the slope, the estimate of

TABLE 1

LINEAR REGRESSION FOR $\hat{P}_1(\tau)$ AND $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ AS A FUNCTION OF τ

		SLOPE	INTERCEPT	$t_{(7)}$ (linear)
CC	$\hat{P}_1(\tau)$	0.01556 ± 0.00145	0.501 ± 0.027	25.4 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-4.83 ± 0.92	46.2 ± 15.3	-12.5 ($p < .01$)
RM	$\hat{P}_1(\tau)$	0.01735 ± 0.00113	0.478 ± 0.019	36.5 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-5.12 ± 0.69	4.8 ± 11.7	-17.7 ($p < .01$)
IK	$\hat{P}_1(\tau)$	0.01260 ± 0.00574	0.516 ± 0.098	5.2 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-4.79 ± 2.03	17.4 ± 34.7	-5.6 ($p < .01$)
VY	$\hat{P}_1(\tau)$	0.02148 ± 0.00297	0.529 ± 0.042	17.1 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-5.57 ± 1.57	8.4 ± 22.3	-8.4 ($p < .01$)

TABLE 1

		SLOPE	INTERCEPT	$t_{(7)}$(linear)
NB	$\hat{P}_1(\tau)$	0.01658 ± 0.00184	0.583 ± 0.029	21.3 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-3.05 ± 0.95	-81.9 ± 14.7	-7.6 ($p < .01$)
R	$\hat{P}_1(\tau)$	0.01708 ± 0.00095	0.510 ± 0.015	42.6 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-7.18 ± 2.43	9.0 ± 37.6	-7.0 ($p < .01$)
MG	$\hat{P}_1(\tau)$	0.01975 ± 0.00290	0.504 ± 0.045	16.1 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-5.31 ± 1.92	-0.3 ± 29.8	-6.5 ($p < .01$)
N	$\hat{P}_1(\tau)$	0.02008 ± 0.00320	0.478 ± 0.050	14.8 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-23.55 ± 4.01	43.0 ± 62.1	-13.9 ($p < .01$)
POOLED	$\hat{P}_1(\tau)$	0.01839 ± 0.00191	0.519 ± 0.030	22.7 ($p < .01$)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-6.64 ± 1.82	-48.1 ± 28.3	-8.6 ($p < .01$)

the intercept of $\hat{P}_1(\tau)$ departed significantly from 0.5 for one S, NB ($p < .05$). For the other seven Ss, chance performance when $\tau = 0$ indicated that there was no response bias for order discrimination performance using equally intense visual stimuli.

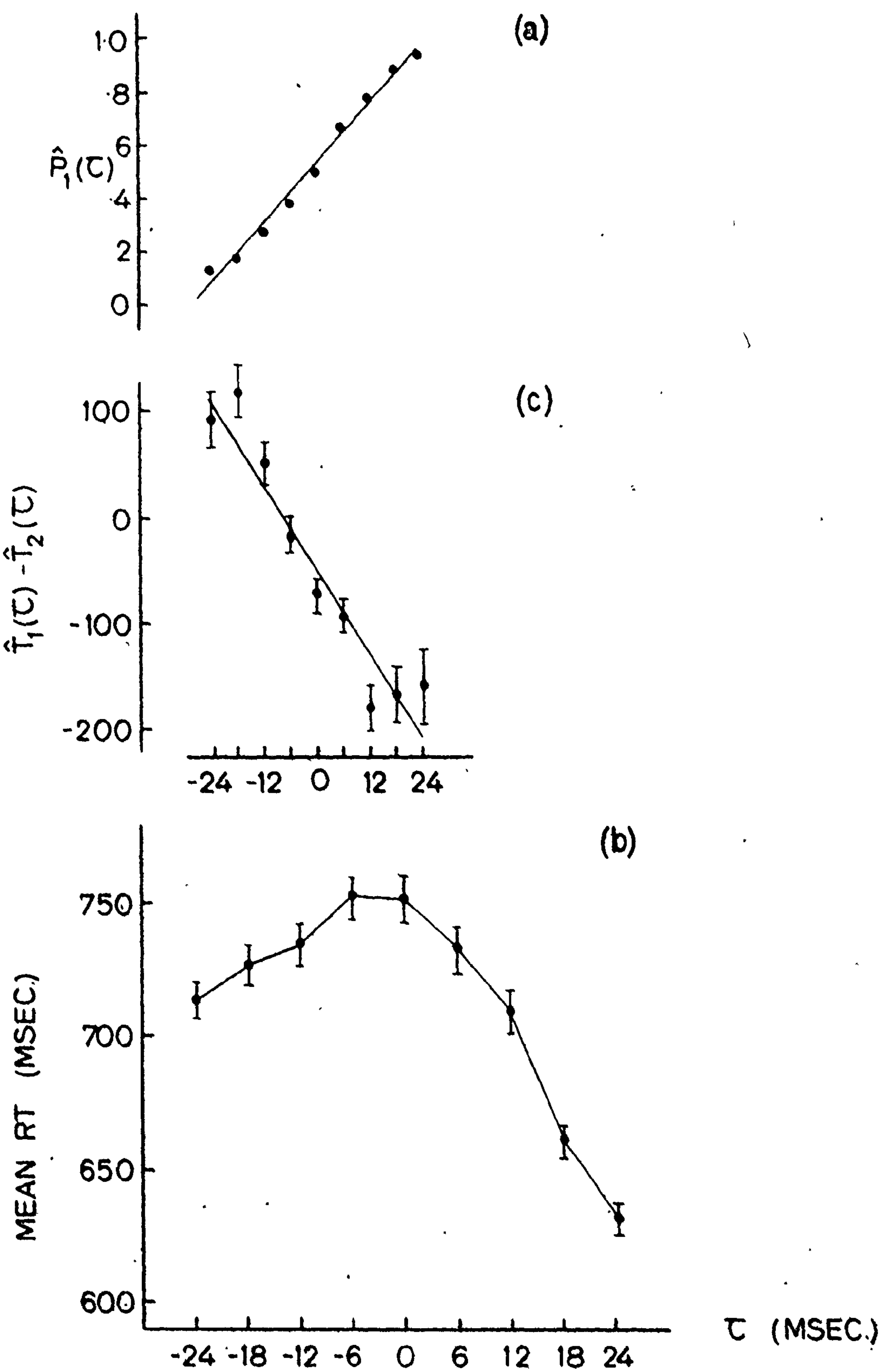
Figure 5(a) contains a plot of $\hat{P}_1(\tau)$ for data pooled across Ss in experiment Ib. $\hat{P}_1(\tau)$ was a monotonic increasing function of τ. The trend was significantly linear ($p<.01$) and the intercept did not deviate significantly from 0.5 ($p >.05$)

Marginal Mean RT

Marginal mean RT pooled across both responses is plotted as a function of τ in Figure A2 for each S. Marginal mean RT is indicated by the filled data points and the standard error of each estimated mean is shown. For six Ss the curve was convex upwards, the maximum mean RT corresponding to a value of τ close to zero. For five of these six Ss marginal mean RT decreased steadily as ISI increased. Consequently, the improvement in performance which resulted from an increase in ISI was accompanied by a decrease in marginal mean RT. The trend in the plot for these five Ss was also evident for data pooled over the four Ss in Experiment Ib. These data are plotted in Figure 5b.

For N a leveling off in the function relating marginal mean RT and τ occurred when S_2 preceded S_1 by a time interval exceeding 12 msec. In contrast to a steady decrease in marginal mean RT for values of τ less than -13.2 msec for the other Ss, there was an increase in marginal mean RT for positive values of τ for IK. The segment of the curve for positive τ corresponded

Figure 5. Psychometric function and mean RT data for Experiment Ib.


(a)
1·0
·8
·6
·4
·2
0
$\hat{P}_1(\tau)$
(c)
100
0
-100
-200
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$
-24 -12 0 12 24
(b)
750
700
650
600
MEAN RT (MSEC.)
-24 -18 -12 -6 0 6 12 18 24
τ (MSEC.)

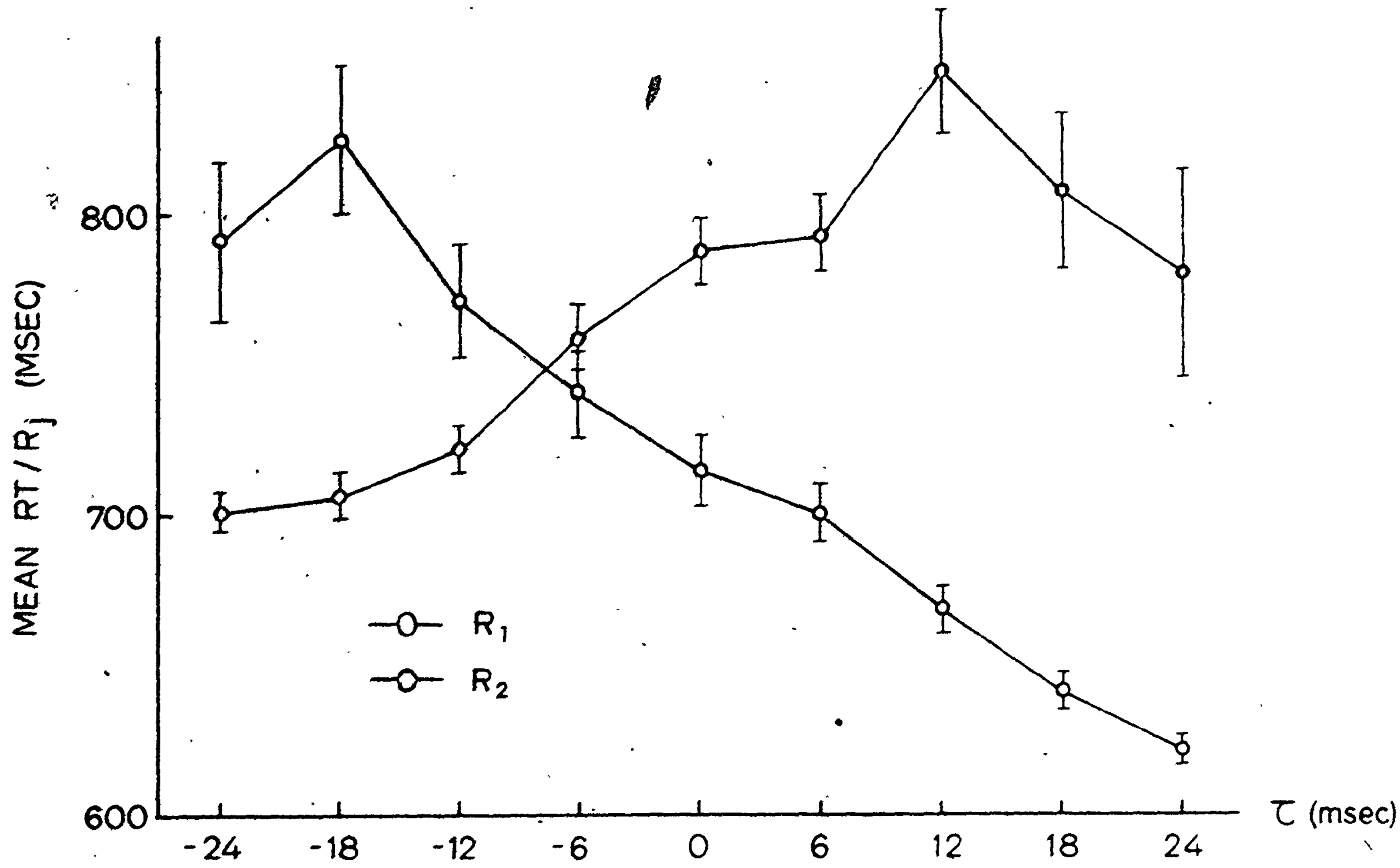
to the non-monotonic segment of the psychometric function. This result was consistent with the inverse relationship between marginal mean RT and the proportion of correct order responses obtained for the majority of the Ss. For NB a trend opposite to that observed for IK was obtained. For negative τ marginal mean RT increased with an increase in absolute value of τ until a maximum was reached when τ equalled -18 msec. The displacement of the maximum value of marginal mean RT towards a large negative value of τ resulted from a large mean difference between RT for responses R_1 and R_2 pooled over τ. For this S mean RTs for responses R_1 and R_2 were 447 msec and 535 msec respectively.

Response Conditioned Mean RT

Estimates of mean RT conditioned upon τ and the response elicited are plotted in Figure A3 for each S. Mean RTs for responses R_1 (top light first) and R_2 (bottom light first) are indicated by the unfilled and filled points respectively. The standard error of each mean is indicated. For all Ss except IK and R, mean RT conditioned upon R_1 decreased as τ increased. An inversion in the trend was observed for positive τ for IK, and for τ = -24 msec for R. For all Ss mean RT conditioned upon R_2 generally increased as τ increased. This trend was slight for NB and inversions in the trend occurred for positive τ for R and MG and for values of τ less than -12 msec for N.

A plot of response-conditioned mean RT as a function of τ for data pooled across the Ss in Experiment Ib is contained in Figure 6. Except for inversions in the trend for extreme values of τ, mean RT conditioned upon R_1 decreased, and mean RT conditioned upon R_2 increased with increasing τ. This trend was similar to that

Figure 6. Response conditioned mean RT as a function of ISI for Experiment Ib.


MEAN RT / R_j (MSEC)
800
700
600
-24
-18
-12
-6
0
6
12
18
24
τ (msec)
R_1
R_2

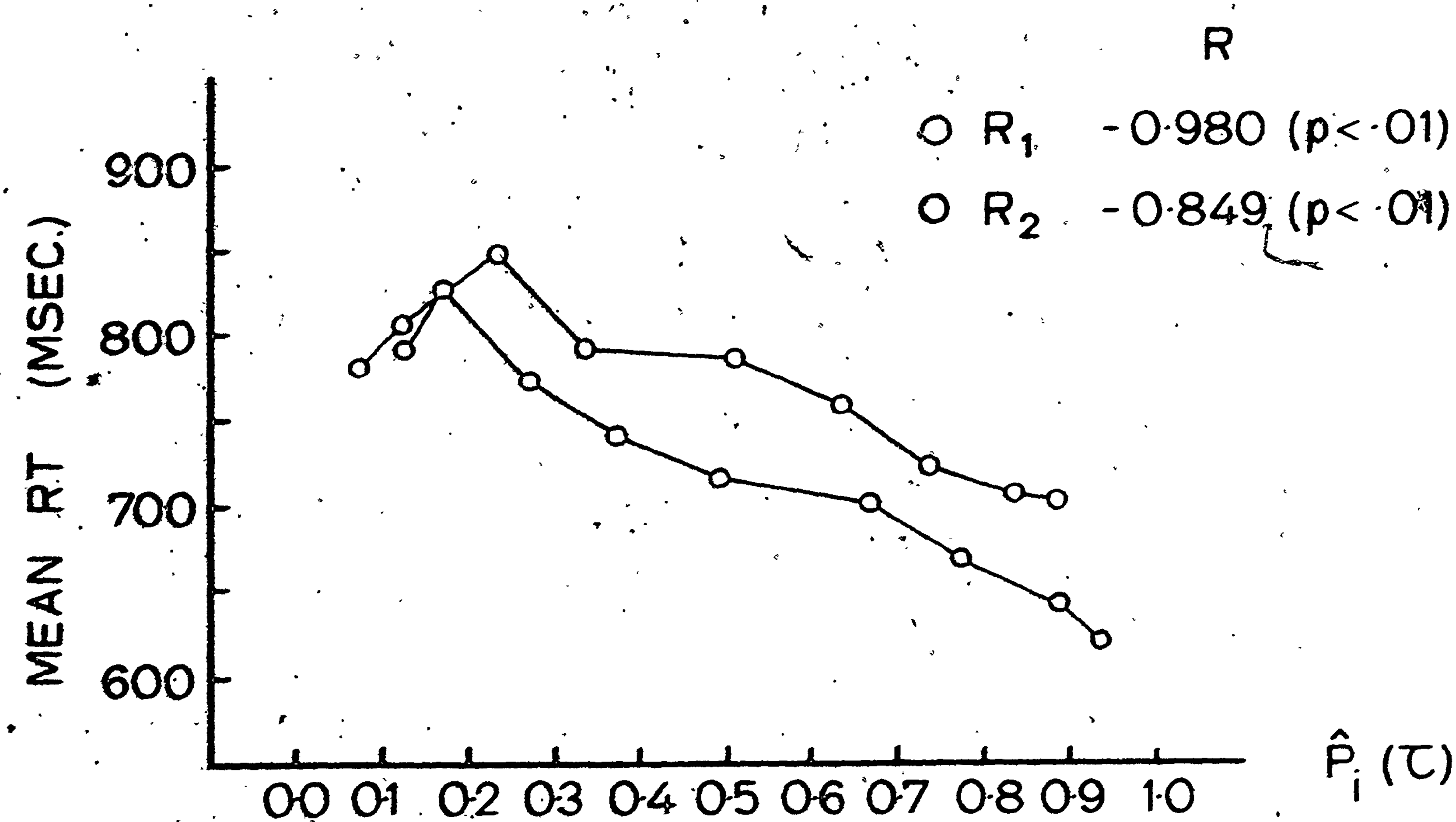
observed for most of the Ss.

Comparison of the trends observed for both the proportion of R_1 responses, and mean RT conditioned upon R_1 as τ increased indicated that there was an inverse relationship between response proportion and response conditioned mean RT. As response proportion increased mean RT decreased. A similar inverse relationship was noted for response R_2. Figure A4 depicts the relationship between response proportion and response conditioned mean RT for each S. For each S there was a statistically significant negative correlation between response proportion and response conditioned mean RT for both responses ($p<.05$). A similar trend for the data pooled across the Ss in Experiment Ib is shown in Figure 7.

The estimated difference in response conditioned mean RTs as a function of τ, $\hat{T}_1(\tau)-\hat{T}_2(\tau)$ is plotted in Figure A1 for each S. $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ decreased as τ increased. For NB, MG and R inversions in the trend occurred for the largest values of ISI. The trend for the data pooled across Ss in Experiment Ib is depicted in Figure 5c. Except for inversions in the trend for the largest values of ISI, $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ decreased as τ increased.

Linear regression of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against τ demonstrated that for each S and for the pooled data from Experiment Ib, a linear fit to the data was adequate. This conclusion was substantiated by the highly significant ($p<.01$) t values for linearity (Hays, 1963, p521) contained in Table 1. Table 1 contains estimates of the slope and intercept of the best-fitting linear relationship according to

Figure 7. Latency probability functions for Experiment Ib.


R
O R_1 -0·980 (p< ·01)
O R_2 -0·849 (p< ·01)
MEAN RT (MSEC.)
900
800
700
600
0·0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
$\hat{P}_i$ (τ)

a least squares criterion. Overlap of the 95% confidence intervals indicated that there were no significant individual differences in estimates of the slope for all but one of the eight Ss. The estimated slope for N was significantly more negative ($p<.05$) than the mean estimate of -5.12 obtained for the other Ss. The estimate of the intercept departed significantly from zero ($p<.05$) for two Ss, CC and NB.

Summary of the Results of Experiment I

The general trends observed in both response proportion and mean RT data for Experiments Ia and Ib were similar.

(a) Psychometric Function

For all Ss but one the psychometric function was a monotonic function of τ. In spite of an apparent curvilinearity in the psychometric function for four Ss no significant departure from a linear trend was obtained by regression analysis. Individual differences in the estimate of the slope of the best fitting linear psychometric function were slight compared with the 95% confidence interval for the estimate. For all Ss but one, the estimated intercept of the psychometric function for τ equal to zero did not depart significantly from 0.5. Hence for equally intense visual stimuli there was no overall tendency for a response bias effect.

(b) Mean RT

(i) Marginal Mean RT

Marginal mean RT as a function of τ was a convex upwards function of τ for six of the eight Ss. The maximum mean RT occurred at a value of τ close to zero. For these six Ss marginal mean RT was a decreasing function of ISI, the longer the ISI the shorter the mean RT. This result, in conjunction with the response proportion data, showed that the shorter the ISI, the more inaccurate was temporal order discrimination performance, and the greater was marginal mean RT pooled across stimulus order.

The estimated marginal mean RT pooled across stimulus order and ISI together with its corresponding standard error are given in Table 2. For each S except NB, mean RT exceeded 500 msec indicating that temporal order discrimination involves a lengthy decision process. The mean RT values approached those found in other discriminative RT tasks which employed instructions stressing accuracy, e.g. Pike and Ryder (1973), Carterette et al (1965). The mean RTs for IK and N exceeded the mean RTs for the other Ss, which were between 450 msec and 750 msec.

(ii) Response Conditioned Mean RT

In general the mean RT for response R_1 decreased as τ increased and a trend in the opposite direction was observed for response R_2. Except for IK, whose data showed an inversion in the trend for response R_1 for positive τ, these relationships hold for ISI values less than 18 msec. Outside of this range some inversions in the trends were found for some of the Ss in Experiment Ib.

The trend for response conditioned mean RT as a function of τ, together with the increase in the proportion of R_1 responses with increasing τ, implied that response conditioned mean RT was a decreasing function of the estimated response proportion for that response for each value of τ. Except for some inversions for low values of response proportion this negative correlation was found to be significant for each S. These plots are essentially the latency probability functions plotted by Pike (1971). A similar negative correlation between mean RT and response proportion was observed in one of Pike's experiments.

TABLE 2

OVERALL MEAN RT IN EXPERIMENTS Ia AND Ib

EXPERIMENT Ia	MEAN RT (msec)	S.E.
CC	603	3
RM	650	2
IK	941	5
VY	613	2
EXPERIMENT Ib		
NB	484	2
R	674	4
MG	620	2
N	1071	7
POOLED	712	3

For each $\underline{S}$, the difference in response conditioned mean RTs as a function of τ, $\hat{T}_1(\tau) - \hat{T}_2(\tau)$, decreased as τ increased. Regression analysis revealed that the trend could be accounted for by a linear function with negative slope. The estimated value of the slope was between -3.0 and -7.2 for all $\underline{S}$s except N. For this $\underline{S}$ the slope was considerably higher. The intercept was not significantly different from zero for six of the eight $\underline{S}$s.

The general accordance between the results for Experiments Ia and Ib implies that any difference in time intervals and the procedures for the two experiments did not alter the trends in the data to any significant extent.

CHAPTER IV

EXPERIMENT II

Temporal Order Discrimination Under Response Time (RT) Deadline Conditions.

RT deadlines have been employed to examine the speed-accuracy trade-off in simple psychophysical tasks (Fitts, 1966; Link & Tindall, 1971; Pachella & Pew, 1968). The technique involves an experimenter imposed limit on the time taken by S to effect a decision. Link (1971) has proposed that the use of RT deadlines as an independent variable provides information on the time course of the decision process by forcing S to terminate the accrual of stimulus information at times constrained by the RT deadline.

Experimental work using RT deadlines supports the general conclusion that the longer the RT deadline, the more accurate is task performance (Link & Tindall, 1971). In choice RT tasks, Yellott (1971) accounted for the improvement in performance with increased RT deadline by assuming that the RT distribution conditioned upon deadline for the same stimulus condition is a probability mixture of invariant RT distributions generated from hypothetical stimulus controlled and guessing states. Yellott did not test directly for the presence of a binary mixture in the RT distributions which were obtained under varying RT deadline conditions. Nevertheless, a model specifying that the improvement in performance with increased RT deadline resulted from a decreasing contribution of fast guess responses to the observed mean RT data, was supported by the data. The estimated mean RT conditioned upon responses generated from a stimulus controlled state was found to be invariant across changes in the RT deadline.

However, in a discriminative RT task involving the comparative judgment of line lengths, Link and Tindall (1971) found no support for the invariance of the RT distribution conditioned upon the response being generated by a stimulus controlled state as RT deadline was varied for a fixed stimulus difference. Changes in performance as a function of RT deadline could not be accounted for in terms of a mixture of stimulus controlled and fast-guess responses. Rather, there was evidence for such a mixture of state conditioned responses as stimulus discriminability was varied within an RT deadline condition. Moreover, in an extensive study of Green and Luce (1973) which employed a wide range of RT deadlines, the predictions of the Yellott (1971) fast-guess model were not supported, particularly when the RT deadline was long and when stimulus discriminability was low.

In discriminative RT tasks the improvement in performance with an increase in RT deadline cannot be interpreted as resulting solely from a decreased proportion of fast erroneous guess responses. It is conceivable, however, that changes in performance as a function of RT deadline result from the effects of a time limit on performance on the division of total RT between its several component times. If discrimination performance deteriorates as the RT deadline is shortened then the change in performance could result from a decrease in the amount of relevant stimulus information sampled, an aborted decision process, motor response interference, or any combination of these effects. Hence

RT deadlines may affect one or several stages of information processing in tasks requiring a difficult discrimination. It is worthwhile, therefore, to consider the effects of RT deadlines in terms of assumptions concerning stimulus information sampling and decision processes in psychophysical tasks.

A temporal order discrimination task differs from certain other psychophysical tasks since the critical information for the discrimination of order is available for a very short time. When both stimuli have been detected no further stimulus display information relevant to the order decision is available. This is particularly true when, as in the experiments described here, order information for each stimulus is confined to a single change in stimulus energy. By its very nature, the temporal order discrimination task imposes a limitation on the amount of stimulus information available for an order decision. When the ISI is reduced, order information is available for a shorter time and a decrease in performance accuracy is predicted. Similarly when the RT deadline is reduced, the sampling period during which order information is acquired might be shortened leading to a decrease in performance accuracy. Hence a decrease in both ISI and in RT deadline should have similar detrimental effects on order discrimination performance accuracy.

Experiment II examined the effect of RT deadlines on performance in a temporal order discrimination task. The experiment was run in two phases, Experiment IIa preceding Experiment IIb. The apparatus was identical for the two experiments but modifications were made in the procedure for Experiment IIb.

Link (1971) has shown that Ss can control their mean RT in accordance with the RT deadline condition as the latter varies from trial to trial. No sequential effects due to deadline were reported. Consequently, in Experiment II each independent variable, RT deadline, ISI and stimulus order was presented equally often within a block of trials.

METHOD

Subjects

Seven university students with no previous experience in order discrimination tasks served as Ss. Each S had satisfactory corrected or uncorrected 20:20 vision as measured by a Snellen eye-chart, and was paid $2.00 for each experimental session which lasted about one hour.

Apparatus

Except for the response panel, the apparatus was identical to that employed in Experiment I. The response panel consisted of a box containing three response buttons on its uppermost side. Each button had an 80 gm resistance to depression. The button closest to S was the trial initiation key (TIK), depression of which would initiate the sequence of trial events. The two response buttons were 3 cm apart and situated 8 cm in front of the TIK. For Ss DK, RR, MG and V the left button indicated the response "Top light first" (R_1) and the right button indicated the response "Bottom light first" (R_2). For JM, G and LN the opposite response mapping was employed.

Procedure

Each experimental session contained either one or two blocks of 298 trials for DK, RR and JM, two blocks for MG and one block for V and G.

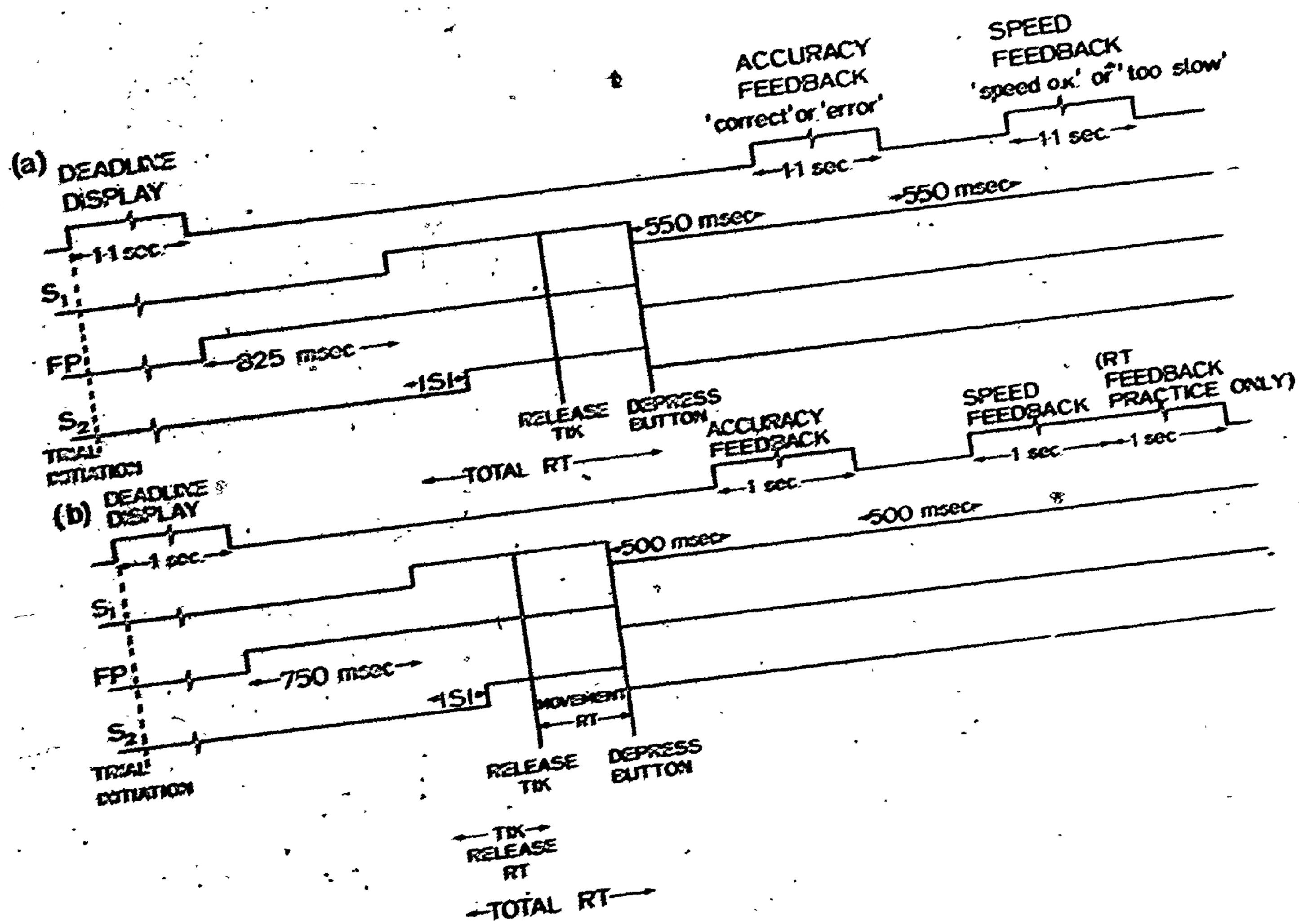
Depending on her state of fatigue LH completed two blocks of trials during some sessions. V and G reported considerable fatigue when more than one block of trials was completed within a single session and extending their endurance was avoided. The first block of each session was preceded by a five minute dark adaptation period and there was a two minute break between blocks of trials.

Since the sequence of events on each trial was initiated by S, a rest could be taken at any time during the session. The first ten trials of each block contained a random sequence of different trial types. These were treated as warm-up trials and ignored in subsequent data analysis. In Experiment IIa the remaining 288 trials contained presentations of each of the 24 trial types (2 orders x 3 deadlines x 4 ISIs) in a random order. In Experiment IIb runs of the same trial type were minimized by presenting a sequence of random permutations of each trial type. Each trial type occurred with equal frequency within the final 288 trials of a block.

The sequence of trial events is depicted in Figure 8. Each trial was initiated by <u>S</u>'s depression of the TIK with the right forefinger. Upon depression of the TIK, the RT deadline for that trial was displayed on the oscilloscope for 1.1 sec in Experiment IIa and 1 sec in Experiment IIb. The duration of the RT deadline was either 275 msec, 440 msec or 990 msec for DK; either 330 msec, 440 msec or 550 msec for RR and JM; and either 250 msec, 400 msec or 900 msec for the other four <u>S</u>s.

Immediately following termination of the deadline display, the fixation point was illuminated. Following a delay of 825 msec in Experiment IIa or 750 msec in Experiment IIb, the first stimulus light

Figure 8. Sequence of trial events for Experiment II.


(a) DEADLINE DISPLAY
1·1 sec.
S_1
FP
825 msec
ISI
S_2
TRIAL INITIATION
RELEASE TIK
DEPRESS BUTTON
TOTAL RT
550 msec
ACCURACY FEEDBACK
'correct' or 'error'
1·1 sec
550 msec
SPEED FEEDBACK
'speed o.k.' or 'too slow'
1·1 sec.
(b) DEADLINE DISPLAY
1 sec.
S_1
FP
750 msec
ISI
S_2
TRIAL INITIATION
MOVEMENT RT
RELEASE TIK
DEPRESS BUTTON
TIK RELEASE RT
TOTAL RT
500 msec
ACCURACY FEEDBACK
1 sec.
500 msec
SPEED FEEDBACK
1 sec
(RT FEEDBACK PRACTICE ONLY)
1 sec

was illuminated. After an ISI of either 5.5 msec, 11 msec, 22 msec or 44 msec for DK; either 5.5 msec, 16.5 msec, 27.5 msec or 44 msec for RR and JM; and either 5 msec, 10 msec, 20 msec and 40 msec for the other four Ss, the second stimulus light was illuminated.

S responded by releasing the TIK and depressing a response button with the same right forefinger. The trial was aborted and then reinitiated if S inadvertently depressed a response button without first releasing the TIK. Whenever S released the TIK before the first stimulus light was illuminated the sequence of trial events was repeated when S depressed the TIK.

Once S executed a response, the lights were extinguished and feedback was provided by means of the oscilloscope display. After a delay of 550 msec response accuracy feedback (CORRECT or ERROR) was displayed for 1.1 sec. 550 msec later response speed feedback was provided. If the RT for that trial was less than the RT deadline the message "SPEED OK" was displayed. Otherwise the message "TOO SLOW" was displayed. The message remained on the screen for 1.1 sec. S could then initiate the next trial by depressing the TIK. The 550 msec delay time in Experiment IIa was reduced to 500 msec in Experiment IIb. Similarly the 1.1 sec display time in Experiment IIa was reduced to 1 sec in Experiment IIb.

The Ss were instructed to fixate the centre light once it was illuminated and to attend equally to the two stimulus lights. They were instructed to respond as accurately as possible whilst at the same time attempting to beat the RT deadline for that trial. Ss were instructed to keep the TIK depressed until a decision had been made and then execute a movement from the TIK to the response button as rapidly as possible. Ss were given at least five practice sessions containing a total of 4190 trials. During the practice sessions Ss were informed of the statistics of their performance. In Experiment IIb response speed feedback was followed immediately by a display of the RT for that trial provided its value was less than 1000 msec. Otherwise the display remained blank during the 1 sec display period. This procedure was intended to facilitate S's adjustment of the mean RT in accordance with the varying RT deadline conditions. RT feedback was not provided during experimental sessions.

In Experiment IIa DK and RR served in 9 experimental sessions yielding 17 blocks of 288 trials each. JM served in 7 experimental sessions yielding 13 blocks of 288 trials each (for each S one of the sessions contained just one block of trials). In Experiment IIb MG served in 10 experimental sessions, LN in 13 experimental sessions and V and G each served in 20 experimental sessions. 46,944 observations were recorded for the seven Ss. RTs exceeding 4095 msec were not recorded and data obtained on such trials were excluded from the data analysis. For DK 6 trials were excluded for this reason. For the other Ss all RTs were less than 4095 msec.

RESULTS

The raw data obtained for each S are contained in Tables A4 and A5 for Experiments IIa and IIb respectively. Data pooled across the four Ss in Experiment IIb are contained in Table A6. The trends in both relative response frequency and mean RT were examined as ISI and RT deadline varied.

(a) Psychometric Functions

The psychometric functions for each S under the three RT deadline conditions are plotted in Figure A5. In each case $f_1(\tau)$ was a monotonic increasing function of τ. Except for the points for an ISI of 44 msec for RR and JM and for an ISI of 40 msec for LN, the psychometric functions were linear. Linear regression of $f_1(\tau)$ against τ indicated a highly significant linear trend for each S and each RT deadline condition. Highly significant t values for linearity (Hays, 1963, p. 521) ($p < .05$) indicated that the proportion of total variance accounted for by linear regression was high.

The slopes and intercepts of the best fitting lines together with their respective 95% confidence intervals are given in Table 3. In spite of a numerical increase in the estimates of the slopes for RR and JM as RT deadline increased, the overlap in the 95% confidence intervals for each pair of slopes indicated that this trend was not significant ($p > .05$). For DK and G, on the other hand, there was a highly significant increase in the slope of the psychometric function with an increase in RT deadline. Overlap of the 95% confidence intervals for the estimated slope of $f_1(\tau)$ for the two shortest deadline conditions for LN and MG, and for the two longest deadline conditions for V indicated that there was a non-significant change in estimated slope for these conditions as deadline varied ($p > .05$). All other comparisons for LN, MG and V yielded significant increases in the slope of $f_1(\tau)$ with increasing RT deadline ($p < .05$).

For RR the intercepts of the best fitting linear psychometric functions at $\tau = 0$ departed significantly ($p < .05$) from 0.5 for the 330 msec and the 550 msec deadlines. There was no significant departure from an intercept of 0.5 for the 440 msec deadline. Although the estimate of the intercept approached 0.5 as RT deadline increased, the

TABLE 3

LINEAR REGRESSION FOR $\hat{P}_1(\tau)$ AND $\hat{T}_1(\tau) - \hat{T}_2(\tau)$

AS A FUNCTION OF τ

Deadline (msec)		SLOPE	INTERCEPT	t (linear) (df)
RR 330	$\hat{P}_1(\tau)$	0.01349 ± 0.00245	0.577 ± 0.046	15.2(4) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	0.04 ± 0.61	4.0 ± 11.4	0.2(4) ns
440	$\hat{P}_1(\tau)$	0.01418 ± 0.00305	0.533 ± 0.057	12.9(4) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	0.06 ± 0.97	-2.0 ± 18.1	0.2(4) ns
990	$\hat{P}_1(\tau)$	0.01613 ± 0.00072	0.522 ± 0.014	62.3(4) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	0.09 ± 0.43	-10.2 ± 8.2	0.5(4) ns
JM 330	$\hat{P}_1(\tau)$	0.01205 ± 0.00279	0.393 ± 0.052	12.0(4) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-0.26 ± 3.45	31.0 ± 64.7	-0.8(4) ns
440	$\hat{P}_1(\tau)$	0.01431 ± 0.00140	0.448 ± 0.026	20.3(4) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-0.97 ± 1.36	1.0 ± 25.6	2.0(4) ns
990	$\hat{P}_1(\tau)$	0.01605 ± 0.00198	0.453 ± 0.037	22.5(4) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-2.20 ± 2.15	15.5 ± 40.5	-2.9(4) (p < .05)
DK 275	$\hat{P}_1(\tau)$	0.00275 ± 0.00092	0.470 ± 0.023	7.3(6) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.60 ± 0.40	3.1 ± 10.2	10.2(6) (p < .01)
440	$\hat{P}_1(\tau)$	0.00496 ± 0.00101	0.403 ± 0.026	12.0(6) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.07 ± 0.51	3.4 ± 13.0	8.0(6) (p < .01)
990	$\hat{P}_1(\tau)$	0.00720 ± 0.00115	0.405 ± 0.029	15.4(6) (p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	2.83 ± 0.80	1.9 ± 20.4	6.8(6) (p < .01)

LINEAR REGRESSION FOR $\hat{P}_1(\tau)$ AND $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ AS A FUNCTION OF τ

	Deadline (msec)		SLOPE	INTERCEPT	t (linear)(df)
LN	250	$\hat{P}_1(\tau)$	0.00078 ± 0.00067	0.449 ± 0.015	2.8(6)($p < .05$)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	0.10 ± 0.61	4.0 ± 14.1	0.4(6) ns
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.09 ± 0.44	5.9 ± 10.3	0.5(6) ns
	400	$\hat{P}_1(\tau)$	0.00278 ± 0.00187	0.585 ± 0.045	3.6(6)($p < .05$)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.84 ± 0.67	16.1 ± 15.5	6.7(6)($p < .01$)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.71 ± 0.51	1.9 ± 11.7	3.4(6)($p < .05$)
	900	$\hat{P}_1(\tau)$	0.02019 ± 0.00381	0.577 ± 0.050	14.7(4)($p < .01$)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-0.05 ± 1.77	-62.3 ± 23.4	-0.1(4) ns
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	-1.46 ± 0.98	-40.2 ± 13.0	-4.1(4)($p < .05$)
V	250	$\hat{P}_1(\tau)$	0.00493 ± 0.00135	0.431 ± 0.031	8.9(6)($p < .01$)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	2.67 ± 1.45	25.3 ± 33.3	4.5(6)($p < .01$)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.85 ± 1.09	11.5 ± 25.1	1.9(6) ns
	400	$\hat{P}_1(\tau)$	0.00964 ± 0.00230	0.509 ± 0.053	10.3(6)($p < .01$)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	3.33 ± 0.48	33.9 ± 11.0	17.0(6)($p < .01$)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.85 ± 0.68	25.0 ± 15.7	3.0(6)($p < .05$)
	900	$\hat{P}_1(\tau)$	0.01171 ± 0.00202	0.474 ± 0.047	14.2(6)($p < .01$)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	-0.18 ± 3.30	25.1 ± 76.1	-0.1(6) ns
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	-1.19 ± 3.51	3.6 ± 80.0	-0.8(6) ns

Deadline (msec)			SLOPE	INTERCEPT	t (linear)(df)
MG	250	$\hat{P}_1(\tau)$	0.00255 ± 0.00160	0.396 ± 0.037	3.9(6)(p < .01)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	0.58 ± 0.50	6.6 ± 11.6	2.8(6)(p < .05)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.42 ± 0.27	6.6 ± 6.2	3.8(6)(p < .05)
	400	$\hat{P}_1(\tau)$	0.00594 ± 0.00183	0.470 ± 0.042	8.0(6)(p < .01)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.91 ± 0.40	18.0 ± 9.2	11.8(6)(p < .01)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.58 ± 0.75	18.4 ± 17.4	1.9(6) ns
	900	$\hat{P}_1(\tau)$	0.00936 ± 0.00164	0.506 ± 0.038	14.0(6)(p < .01)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	2.53 ± 0.60	7.3 ± 13.7	10.4(6)(p < .01)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.69 ± 0.34	12.6 ± 7.8	5.0(6)(p < .01)
G	250	$\hat{P}_1(\tau)$	0.00251 ± 0.00063	0.336 ± 0.015	9.7(6)(p < .01)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.37 ± 0.42	-29.0 ± 9.6	9.2(6)(p < .01)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.74 ± 0.33	-10.3 ± 7.5	5.6(6)(p < .01)
	400	$\hat{P}_1(\tau)$	0.00421 ± 0.00108	0.485 ± 0.025	9.5(6)(p < .01)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.82 ± 0.87	-42.0 ± 20.0	3.1(6)(p < .01)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.39 ± 0.47	-21.8 ± 10.8	3.1(6)(p < .05)
	900	$\hat{P}_1(\tau)$	0.00865 ± 0.00099	0.498 ± 0.023	21.4(6)(p < .01)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	0.37 ± 1.03	-72.3 ± 23.6	0.9(6) ns
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	-1.27 ± 0.82	-38.1 ± 18.9	-3.8(6)(p < .01)
POOLED	250	$\hat{P}_1(\tau)$	0.00256 ± 0.00062	0.454 ± 0.014	10.1(6)(p < .01)
		$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.35 ± 0.28	5.1 ± 6.6	11.9(6)(p < .01)
		$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.44 ± 0.24	4.9 ± 5.5	4.5(6)(p < .01)

Deadline (msec)		SLOPE	INTERCEPT	t (linear)(df)
400	$\hat{P}_1(\tau)$	0.00562 ± 0.00121	0.511 ± 0.028	11.3(6)(p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	2.33 ± 0.49	0.6 ± 11.3	11.6(6)(p < .01)
	$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	0.51 ± 0.37	2.4 ± 8.5	3.4(6)(p < .05)
900	$\hat{P}_1(\tau)$	0.01068 ± 0.00190	0.514 ± 0.044	13.7(6)(p < .01)
	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	1.56 ± 0.49	-24.8 ± 11.3	7.8(6)(p < .01)
	$\hat{M}_1(\tau) - \hat{M}_2(\tau)$	-0.50 ± 0.31	-7.9 ± 7.1	-4.0(6)(p < .01)

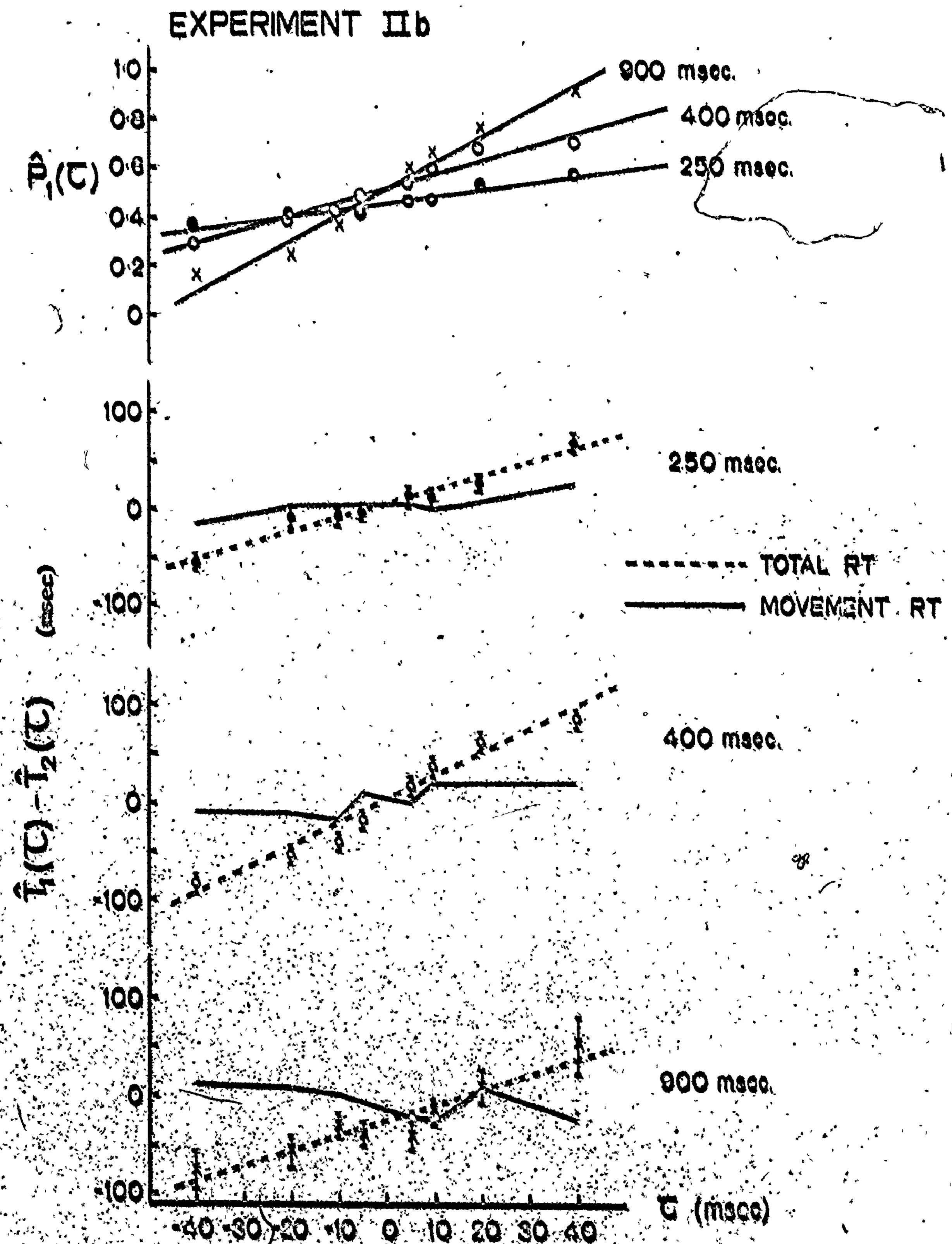
trend was not significant. A tendency for the estimates of the intercept of the best fitting linear psychometric functions to approach 0.5 as deadline increased was also not significant for JM and DK. Although the estimate of the intercept departed significantly from a value of 0.5 in each deadline condition for RR, a significant departure from a value of 0.5 was only observed for the shortest deadline condition for DK. For LN the estimated intercept of $\hat{P}_1(\tau)$ for $\tau = 0$ departed significantly from 0.5 for all deadline conditions ($p < .05$). For MG, V and G the estimated intercept departed significantly from 0.5 for the 250 msec deadline condition. For the longer deadline conditions the departure from 0.5 was not significant ($p > .05$).

The psychometric functions for each RT deadline condition obtained from data pooled across the four Ss in Experiment IIb are plotted in Figure 9. Each function was a monotonic increasing function of τ. Despite a tendency towards curvilinearity for the 400 msec and 900 msec deadline conditions, linear regression analysis indicated that the trend in the data was significantly linear ($p < .01$). Estimates of the slope of $\hat{P}_1(\tau)$ increased significantly as RT deadline increased. Although the estimated intercept of the psychometric function was significantly less than 0.5 ($p < .05$) for the 250 msec deadline condition, the estimated intercept did not deviate significantly from 0.5 in the two longer deadline conditions.

(b) Response Time Data

The total response time for each trial was defined as the time elapsing between the onset of the first stimulus light and the depression of a response button. The total RT was decomposed into two component times, the time elapsing between the onset of the first stimulus light

Figure 9. Psychometric functions and differences in response conditioned mean RT as a function of ISI for each RT deadline condition in Experiment IIb.



EXPERIMENT IIb
1·0
0·8
0·6
0·4
0·2
0
$\hat{P}_1(\tau)$
900 msec.
400 msec.
250 msec.
100
0
-100
250 msec.
TOTAL RT
MOVEMENT RT
(msec)
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$
400 msec.
900 msec.
-40 -30 -20 -10 0 10 20 30 40
τ (msec)

and the release of the TIK (TIK release time), and the time elapsing between the release of the TIK and the depression of a response button (movement time). Whereas both component times were measured in Experiment IIb, only total RT was measured in Experiment IIa. Changes in mean RT as a function of the level of the independent variables were examined. Corresponding changes in the higher moments of the RT distributions were not considered.

(i) Marginal Mean RT

Total mean RTs pooled over all independent variables within each RT deadline condition are given in Table 4. The standard error of estimate is indicated for each mean RT value. As RT deadline increased, marginal mean RT increased accordingly for each S and for data pooled across the four Ss in Experiment IIb. In general, the longer the deadline, the greater was the discrepancy between the mean RT and the deadline. Hence all Ss were able to control their overall mean RT in accordance with the changing RT deadline conditions.

Marginal mean RT pooled over the two responses R_1 and R_2 for each value of τ are plotted in Figure A6. In Experiment IIa mean total RT increased with an increase in RT deadline for all values of τ. Mean total RT was relatively independent of τ for each RT deadline. For Experiment IIb mean total RT together with the component TIK release and movement times are plotted. For the two shortest deadline conditions the trends indicated that total mean RT together with its measured components did not change appreciably as τ varied. There was a tendency towards a convex downwards function for total mean RT in the case of LN and V. Compared with the increase in mean TIK release time with increasing deadlines, the change in mean movement time with

TABLE 4

MARGINAL MEAN RT AS A FUNCTION OF RT DEADLINE

IN EXPERIMENT II

	Deadline (msec)	Mean RT (msec)
RR	330	333 ± 2
	440	405 ± 1
	550	428 ± 2
JM	330	349 ± 4
	440	409 ± 4
	550	450 ± 3
DK	275	265 ± 3
	440	287 ± 3
	990	325 ± 3

	Deadline (msec)	Mean RT (msec)
LN	250	197 ± 3
	400	255 ± 3
	900	319 ± 3
MG	250	212 ± 2
	400	309 ± 3
	900	368 ± 3
G	250	263 ± 3
	400	350 ± 4
	900	500 ± 5
V	250	252 ± 4
	400	360 ± 3
	900	455 ± 4
POOLED	250	231 ± 2
	400	319 ± 2
	900	460 ± 2

increasing deadline was slight.

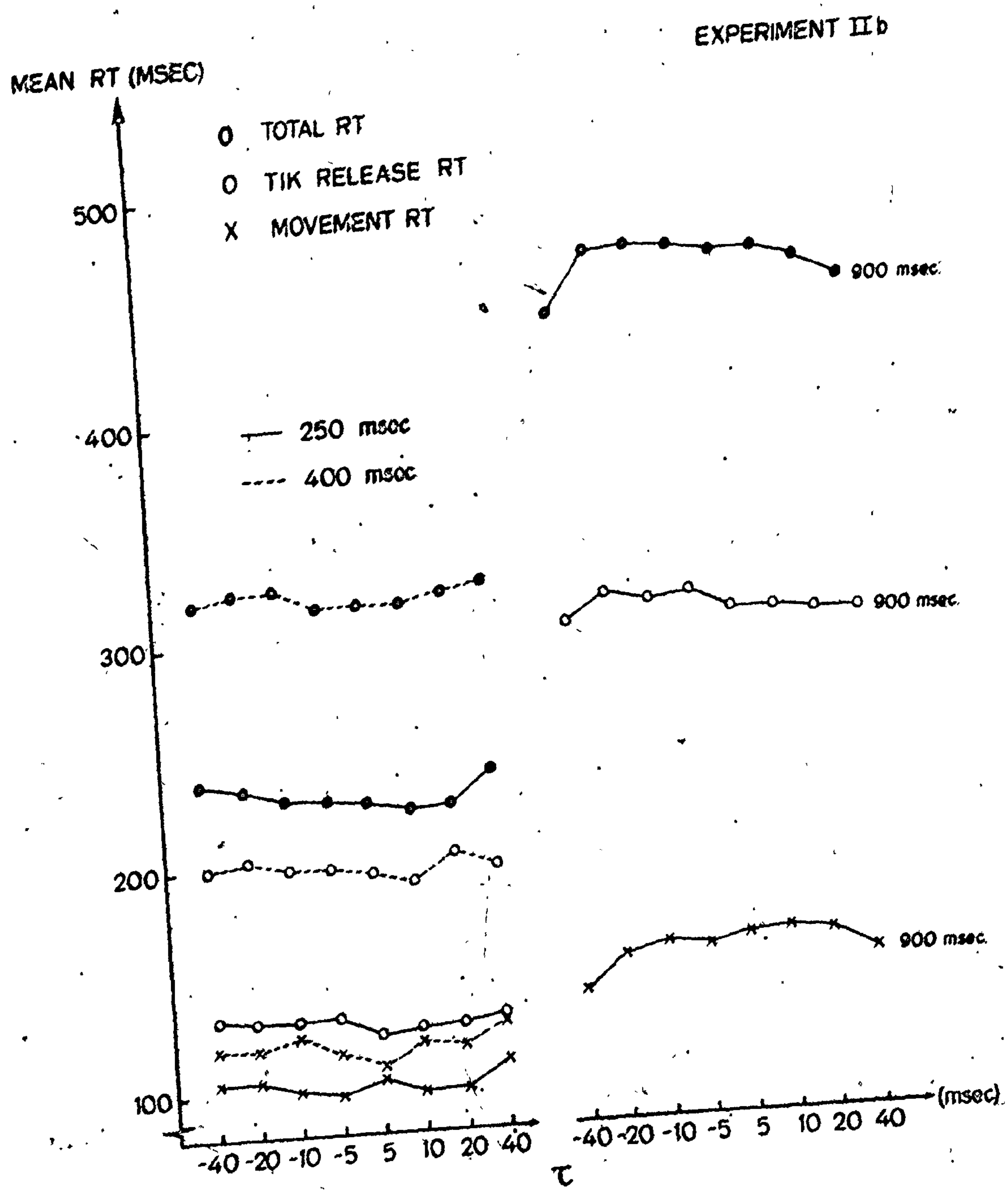
A corresponding plot of the marginal mean RT values as a function of τ for the pooled data of Experiment IIb in Figure 10 showed a trend similar to that observed for each S in that experiment. Except for a drop in mean RT for the longest value of ISI in the 900 msec deadline condition, the marginal mean RT for both total RT and its measured components did not vary with changes in τ.

(ii) Response Conditioned Mean RT

The trends in mean total RT conditioned upon response elicited, R_i, i = 1, 2, are represented in terms of the difference in these mean RTs conditioned upon responses R_1 and R_2 respectively, $\hat{T}_1(\tau) - \hat{T}_2(\tau)$, as τ varied within each RT deadline condition. The trends are depicted in Figure A5 for individual Ss and in Figure 9 for data pooled across the four Ss in Experiment IIb. In each figure the standard error of the estimate of each total mean RT is indicated.

The results of linear regression analysis of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ as a function of τ for each deadline condition are summarized in Table 3. A measure of the goodness-of-fit of a linear relationship is indicated by a t-value for linearity (Hays, 1963, p. 251). For RR at each RT deadline and for JM at the two shortest deadlines there was no evidence indicating a linear trend ($p > .05$). In each case the slope of the best-fitting line according to a least-squares criterion did not depart significantly from zero ($p > .05$). For JM in the 550 msec deadline condition there was a significant linear trend with negative slope ($p < .05$). For DK the trend at each deadline level was significantly linear with positive slope ($p < .01$). However, the slope did not increase significantly with an increase in RT deadline ($p > .05$).

Figure 10. Marginal mean RT as a function of ISI for each RT deadline condition in Experiment IIb.


EXPERIMENT II b
MEAN RT (MSEC)
O TOTAL RT
O TIK RELEASE RT
X MOVEMENT RT
500
400
300
200
100
250 msec
400 msec
900 msec
900 msec
900 msec
-40 -20 -10 -5 5 10 20 40
-40 -20 -10 -5 5 10 20 40
(msec)
τ

For the 250 msec deadline condition a significant linear relationship was found for three of the four $\underline{S}s$ in Experiment IIb ($p < .05$) and for the 900 msec deadline condition a significant linear relationship was found for only one $\underline{S}$, MG ($p < .01$). For the 400 msec deadline condition a significant linear relationship applied for each $\underline{S}$ ($p < .01$). Whenever a significant linear relationship was found the slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ was significantly positive. For MG, the estimate of the slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ increased significantly as the deadline increased ($p < .05$).

For data pooled across all four $\underline{S}s$ in Experiment IIb, the slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ increased significantly ($p < .05$), as the RT deadline increased from 250 msec to 400 msec. A decrease in slope with an increase in the RT deadline from 400 msec to 900 msec was found to be nonsignificant. In each case a highly significant linear trend was indicated ($p < .01$).

Associated with each plot of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ as a function of τ for Experiment IIb is a plot of the difference in estimated mean movement times for responses R_1 and R_2 respectively, $\hat{M}_1(\tau) - \hat{M}_2(\tau)$, for each value of τ. The results of linear regression analysis for the differences in mean movement times are shown in Table 3. Although a significant linear trend was present for each RT deadline condition for the pooled data, the estimates of the slopes differed significantly ($p < .05$) from the corresponding estimates for $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ for the same RT deadline condition. For the two shortest deadline conditions the estimated slopes of $\hat{M}_1(\tau) - \hat{M}_2(\tau)$ were significantly positive ($p < .05$) but not significantly different from each other. Although the direction of the trends in both mean total RTs and mean movements RTs was similar, the difference in mean movement times did not show the significant increase with an increase in RT deadline that was observed for the difference in mean total RTs. For the longest deadline condition the trends in the differences between mean total RT and mean movement RT were in opposite directions. This finding suggests that whereas movement RT and total RT are related when S is required to respond rapidly, this relationship is no longer apparent when a rapid response is not required. This finding for the pooled data was representative of the trends observed for each S.

The general positive slope of the relationship between $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ and τ indicated that the mean RT for correct responses exceeds the mean RT for an incorrect response by an amount which increases as ISI increases. Link and Tindall (1971) found a similar relationship between mean RT for correct responses and incorrect responses for their 460 msec deadline condition.

General Conclusions On The Effect of RT Deadlines on Temporal Order Discrimination

For all seven Ss, changes in RT deadlines generated a significant difference in mean RT conditioned upon RT deadline condition. These Ss demonstrated a capacity to control their mean RT for each RT deadline condition as the latter was varied over trials. Hence the use of RT deadlines was a convenient experimental procedure to investigate the speed-accuracy trade-off in a temporal order discrimination task.

(a) Psychometric Functions

For all seven Ss the psychometric functions were monotonic increasing functions of τ, the signed value of ISI. For all Ss the psychometric function for each RT deadline condition did not depart significantly from linearity. Except for RR and JM, and for LH and MG at the two shortest deadline conditions there was a significant increase in the slope of the psychometric function with increasing RT deadline. This trend was highly significant for the pooled data of Experiment IIb. This result was consistent with the hypothesis that accuracy in temporal order discrimination improves as the mean RT increases over the range 200 msec to 600 msec. For the pooled data in Experiment IIb the estimated intercepts of the psychometric functions approached 0.5 with an increase in the RT deadline from 250 msec to 400 msec. A similar trend was noted for each S in Experiment IIa.

(b) Response Time Data

Although the marginal mean RT increased with increasing deadline for all Ss, the absolute difference between this mean RT and the RT deadline increased as the RT deadline itself increased.

Marginal mean total RT pooled across both order responses was independent of τ for all Ss and for the two shortest deadline conditions. There was a tendency towards curvilinearity for two Ss in Experiment IIb at the longest deadline condition. This tendency was also apparent in the pooled data for this Experiment.

Separate analysis of TIK release and movement times in Experiment IIb indicated that compared with the large increase in marginal mean TIK release time with increasing RT deadline, the corresponding increase in marginal mean movement time was slight. There was an increase in the marginal mean movement time for the longest deadline indicating that the speed of finger movement was relaxed under reduced speed requirements.

(ii) Response Conditioned Mean RT

The difference in response conditioned mean total RT as a function of τ was significantly linear with positive slope for five of the seven Ss for the intermediate RT deadline condition. A similar trend was observed for four Ss at the shortest deadline condition and for two Ss at the longest deadline condition. For JM a significant linear trend with negative slope was obtained for a 550 msec deadline condition.

For the data from Experiment IIb pooled across all four Ss, the slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ as a function of τ increased significantly with an increase in RT deadline from 250 msec to 400 msec. In the 900 msec deadline condition, the estimated slope did not differ significantly from the value observed for the 400 msec deadline condition.

Comparison of the trends in the difference in response conditioned mean RTs for varying τ in Experiments I and II revealed a predominantly positive slope under speeded conditions and a negative slope under instructions emphasizing performance accuracy. In general, for the same stimulus condition, the mean RT for a correct response exceeded the mean RT for an incorrect response when response speed was emphasized. On the other hand, when performance accuracy was emphasized, the mean RT for a correct response was less than the mean RT for an incorrect response.

CHAPTER V

Interpretation of the Data in Experiments I and II in Terms of the Random Walk Models

The random walk models for temporal order discrimination generate testable predictions for the relationships between performance measures in order discrimination tasks. The success with which these models account for the relationships between these performance measures derived from the data of Experiments I and II was examined.

Real Time Model

The real time model represents the effect of a time difference between the two stimulus onsets in terms of a displacement of the mean starting point of a zero drift random walk decision process towards the correct response threshold for that stimulus order. Provided that the distribution of starting points for the terminal zero drift process is confined to lie between the response thresholds and the random walk generated by each stimulus has constant drift rate m, then the predicted psychometric function $\hat{P}_1(\tau)$ is a linear function of τ, the signed value of ISI, with positive slope. The predicted difference between the estimated mean response times conditioned upon responses R_1("Top light first") and R_2("Bottom light first") respectively, $\hat{T}_1(\tau) - \hat{T}_2(\tau)$, decreases linearly with τ.

Using estimates of the slope and intercept of $\hat{P}_1(\tau)$ and the slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$, estimates of the three parameters A, C and m can be obtained in terms of a scale factor s_1, representing the standard deviation of the step size to the random walk generated by each stimulus alone.

The parameters A and C provide estimates of the S's response bias. They provide an assessment of performance in a temporal order discrimination task which is independent of the contribution of stimulus factors. The latter are summarised by the parameter m.

A convenient test of the real-time model is provided by a predicted linear relationship between $T_1(\tau) - T_2(\tau)$ and $2P_1(\tau) - 1$. In practical applications of this test estimates $\hat{T}_1(\tau)$, $\hat{T}_2(\tau)$ and $\hat{P}_1(\tau)$ of $T_1(\tau)$, $T_2(\tau)$ and $P_1(\tau)$ respectively are employed. This linear relationship has strictly negative slope. If the mean non-decision component of RT does not depend on τ, then the intercept provides an estimate of the mean difference in the non-decision components of RT for responses R_1 and R_2.

A plot of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ for each S in Experiment I is depicted in Figure A7. For each S, $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ decreased as $2\hat{P}_1(\tau) - 1$ increased and an apparent linear trend provided substantial support for the real time model.

The results of a linear regression analysis of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ are given in Table 5. For all eight Ss the estimated slope was negative. The 95% confidence intervals for each estimate indicated that the individual differences in slope were non-significant ($p > .05$) for all Ss except NB and N. The estimates of the intercept differed significantly ($p < .05$) from zero for CC and NB.

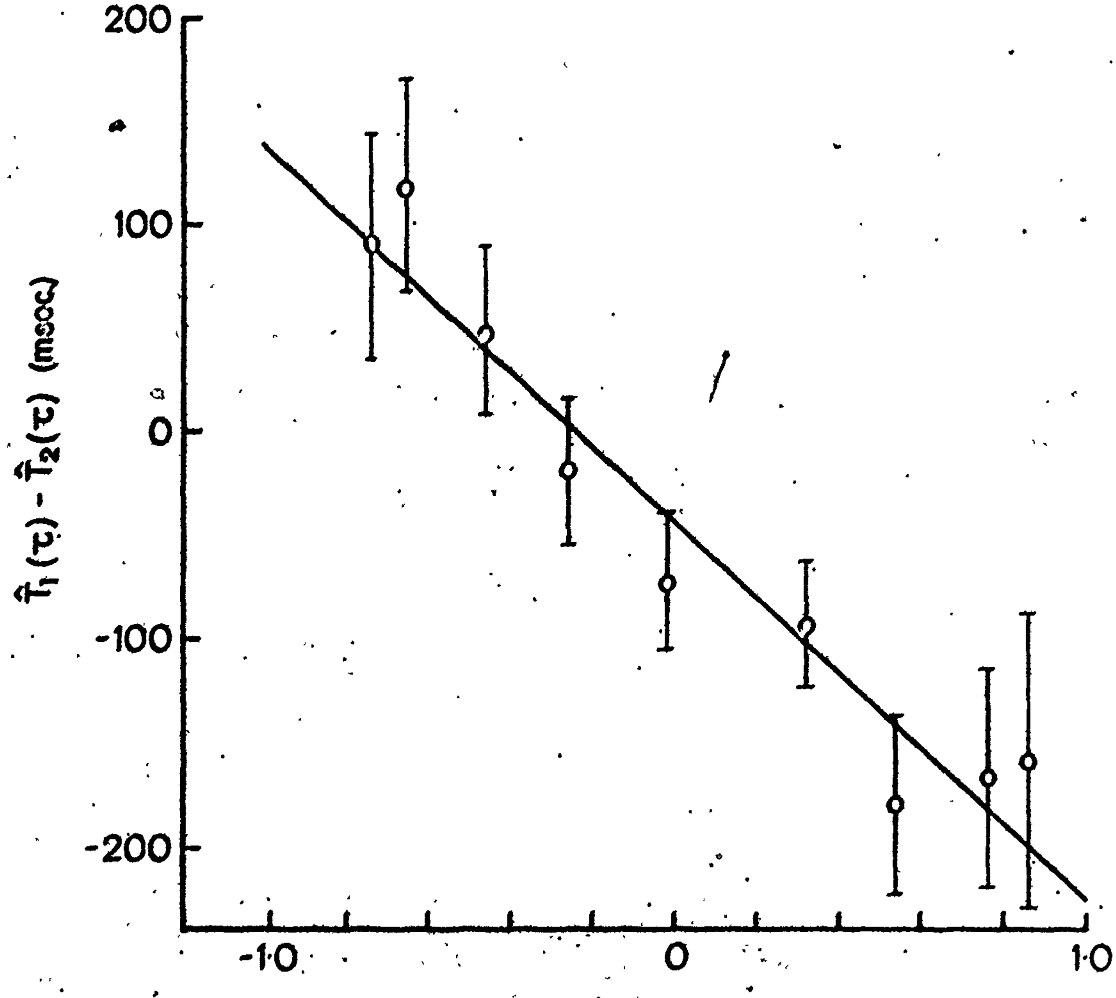
A plot of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ for data pooled across four Ss in Experiment Ib is depicted in Figure 11. The trend was similar to that observed for each S since $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ decreased as $2\hat{P}_1(\tau) - 1$ increased. The results of a linear regression analysis in Table 5 in-

TABLE 5

Linear Regression Analysis of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ Against $2\hat{P}_1(\tau) - 1$ in Experiment I

Experiment Ia	Slope	Intercept	R^2	t(linear) (df)	
CC	−152.2 ± 22.3	48.3 ± 11.9	0.974	−16.2 (7)	$p < .01$
RM	−146.5 ± 23.7	−1.7 ± 14.1	0.970	−14.6 (7)	$p < .01$
IK	−177.9 ± 52.9	23.0 ± 25.5	0.901	− 8.0 (7)	$p < .01$
VY	−128.7 ± 34.4	15.9 ± 21.4	0.918	− 8.8 (7)	$p < .01$
Experiment Ib					
NB	− 92.9 ± 19.9	−65.8 ± 11.0	0.945	−11.0 (7)	$p < .01$
R	−210.4 ± 71.3	13.4 ± 37.7	0.874	− 7.0 (7)	$p < .01$
MG	−135.3 ± 40.0	1.0 ± 24.9	0.901	− 8.0 (7)	$p < .01$
N	−583.7 ± 61.5	15.8 ± 39.0	0.986	−22.4 (7)	$p < .01$
POOLED	−181.4 ± 43.6	−41.7 ± 25.0	0.933	− 9.8 (7)	$p < .01$

Figure 11. $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ for Experiment Ib.



200
100
0
-100
-200
T̂1(τ) - T̂2(τ) (msec)
-1·0
0
1·0
2P̂1(τ)-1

dicated a highly significant linear trend with negative slope ($p < .01$). The finding that the estimated intercept differed significantly from zero ($p < .05$) implied that there was a difference in non-decision components of mean RTs for responses R_1 and R_2. The high values of the correlation coefficient squared, R^2, and highly significant t values for linearity ($p < .01$) indicated that a high proportion of the total variance was accounted for by a linear trend. An interesting discovery was that IK's data could be accounted for quite adequately by the real-time model. The relationship between $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ and $2\hat{P}_1(\tau) - 1$ does not depend on a substitution of a particular value of τ in order to examine the applicability of the model. It examines the relationship between response proportion and mean response time measures of performance independently of particular values of the stimulus parameters. Hence it was concluded that the anomolous data obtained from IK resulted from stimulus encoding processes and not from the response strategy chosen.

Estimates of the parameters A, C and m were obtained using equations (23), (24) and (25) in Chapter II. Estimates of the difference in mean non-decision times $\hat{M}_1 - \hat{M}_2$ were obtained from the estimate of the intercept of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ plotted against $2\hat{P}_1(\tau) - 1$ according to equation (20). A consistency test employed estimates of A and C scaled by s_1 (the standard deviation of a step size to the random walk process generated by each stimulus alone) and the estimated value of $\hat{M}_1 - \hat{M}_2$ to predict $\hat{I}$, the estimated intercept of equation (22), and $\hat{S}$, the estimated slope of equation (20).

The results of the parameter estimation procedure are presented in Table 6. The fit of the model to the data was quite satisfactory.

TABLE 6

Parameter Estimation for the Real-Time Model

for Temporal Order Discrimination

In Experiment I

	$\underline{S}$	$\frac{2\lambda}{\hat{m}}$	$\hat{A}$	$\hat{C}$	$\hat{m}$	$\hat{M}_1 - \hat{M}_2$	$\hat{I}$ obs.	$\hat{I}$ pred.	$\hat{S}$ obs.	$\hat{S}$ pred.
Experiment Ia	CC	64	15.3	0.0	0.48	48	46	48	-152	-155
	RM	57	14.9	-0.7	0.52	- 2	5	5	-147	-147
	IK	79	16.9	0.5	0.43	23	17	17	-178	-190
	VY	46	13.9	0.8	0.60	16	8	8	-129	-130
Experiment Ib	NP	60	11.7	1.9	0.39	-66	-82	-81	- 93	- 92
	R	58	17.8	0.4	0.61	13	9	9	-210	-210
	MG	51	14.2	0.1	0.56	1	0	0	-135	-134
	N	50	29.7	-1.3	1.19	16	43	42	-584	-589
	Pooled	54	16.5	0.6	0.61	-42	-48	-49	-181	-181

For each *S* and for the pooled data in Experiment Ib the correspondence between the observed and predicted values of $\hat{I}$ and $\hat{S}$ was close. This meant that the model provided a consistent account of the data.

The parameter estimates illustrated the efficiency with which the real-time model encompassed stimulus processing and response strategy variables in an order discrimination task. The estimate of m provided a measure of the mean step size to the random walk generated by each stimulus alone. Although the estimates of m were similar for some of the *Ss* differences between *Ss* would be expected if the value of the scale factor, s_1, varied. A similar discrepancy would be expected for the response strategy measures $\hat{A}$ and $\hat{C}$. A value of $\hat{C}$ close to 0 for six *Ss* indicated that there was no tendency towards a response bias. The estimates of the difference in mean residual RTs, $\hat{M}_1 - \hat{M}_2$, provided a measure of the relative effect of components of RT unrelated to the decision process. Hence the real-time model provided a measurement procedure for evaluating the separate effects of stimulus variables, response strategy and processes unrelated to the decision process in a temporal order discrimination task.

The ratio of the estimates of A and m provides a measure of performance which does not depend on the scale factor s_1. It can be shown that $2\hat{A}/\hat{m}$ gives an estimate of the time interval spanned by the linear psychometric function given by equation (21). The estimates of this ratio are provided in Table 6. Except for IK whose psychometric function was non-monotonic, these estimates lay between 46 msec and 64 msec. The mean value for seven *Ss* was 55 msec, a value close to that reported by Kristofferson (1967) for an estimate of a psychological unit of temporal processing. In terms of the real-time model $\hat{A}/\hat{m}$ provides an estimate of the minimum decision time for the correct detection of a stimulus prior to the presentation of a second masking stimulus. On the average, the data of Experiment I indicated that for visual stimuli this minimum time interval is about 27 msec.

Although the real-time model provided an adequate fit to data obtained in an order discrimination task stressing response accuracy, it did not fare so well when RT deadlines controlled performance. The results of a linear regression analysis of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ are given in Table 7. Associated with each estimate is its 95% confidence interval. For all the Ss in Experiment II the slope of the best-fitting linear plot of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ was either significantly positive, or zero. Only for the pooled data for JM and RR in the 550 msec deadline condition was there any evidence for a significantly negative slope ($p < .05$). Hence the real-time model failed to account for data obtained under RT deadline conditions.

The LOT Model

The data obtained in Experiment II can be used to test a prediction of the Limited Observation Time (LOT) model for temporal order discrimination under special conditions. There must be no starting point variability to the random walk decision process and the statistical properties of the step size to the random walk must be symmetrical when only the order of the stimuli is varied. This latter condition implies that the mean step size to the random walk for the order S_2 first is the negative of that for the order S_1 first. Under these conditions, equation (7) in Chapter II applies for changing RT deadline conditions provided it

TABLE 7

Linear Regression Analysis of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ Against $2\hat{P}_1(\tau) - 1$ For Experiment II

Deadline (msec)		Slope	Intercept	R^2	t(linear) (df)	
Experiment IIa						
JM	330	-46.2 ± 136.8	21.2 ± 69.3	0.180	-0.9 (4)	ns
	440	-32.3 ± 49.4	- 2.3 ± 27.1	0.452	-1.8 (4)	ns
	550	-63.9 ± 84.5	2.5 ± 51.8	0.524	-2.1 (4)	ns
RR	330	1.2 ± 22.3	3.8 ± 11.9	0.006	0.1 (4)	ns
	440	- 0.2 ± 35.1	- 2.5 ± 19.1	0.000	0.0 (4)	ns
	550	2.2 ± 13.6	-18.3 ± 8.3	0.048	0.4 (4)	ns
JM + RR	330	- 8.9 ± 17.4	7.1 ± 8.4	0.334	-1.4 (4)	ns
	440	- 5.3 ± 28.3	1.3 ± 15.2	0.064	-0.5 (4)	ns
	550	-20.3 ± 14.6	-11.7 ± 8.8	0.789	-3.9 (4)	$p < 0.05$
DK	275	284.8 ± 87.1	20.2 ± 13.8	0.914	8.0 (6)	$p < .01$
	440	161.7 ± 59.7	9.0 ± 15.5	0.880	6.6 (6)	$p < .01$
	990	159.1 ± 33.9	2.9 ± 12.6	0.956	11.5 (6)	$p < .01$

Deadline (msec)	Slope	Intercept	R^2	t(linear) (df)	
Experiment IIb					
LN 250	42.0 ± 330.2	83.1 ± 36.7	0.016	0.3 (6)	ns
400	242.6 ± 164.0	-25.1 ± 37.6	0.686	3.6 (6)	p < .05
900	- 4.8 ± 43.1	-61.6 ± 24.1	0.016	-0.3 (6)	ns
V 250	225.7 ± 184.4	57.4 ± 51.5	0.599	3.0 (6)	p < .05
400	166.7 ± 25.7	31.0 ± 11.8	0.976	15.8 (6)	p < .01
900	16.0 ± 142.6	14.7 ± 78.8	0.012	0.3 (6)	ns
G 250	302.7 ± 103.2	-51.7 ± 14.3	0.895	7.2 (6)	p < .01
400	219.3 ± 90.3	-35.4 ± 17.9	0.854	5.9 (6)	p < .01
900	20.8 ± 74.1	-64.6 ± 30.0	0.073	0.7 (6)	ns
MG 250	105.8 ± 65.4	28.8 ± 16.6	0.723	4.0 (6)	p < .05
400	148.7 ± 51.2	27.3 ± 15.0	0.893	7.1 (6)	p < .01
900	129.8 ± 45.0	5.8 ± 19.6	0.893	7.1 (6)	p < .01
Pooled 250	241.1 ± 91.2	28.0 ± 14.4	0.874	6.5 (6)	p < .01
400	206.1 ± 10.8	- 5.5 ± 2.9	0.998	46.5 (6)	p < .01
900	70.0 ± 27.4	26.3 ± 13.7	0.865	6.2 (6)	p < .01

is assumed that the statistical properties of the step size distribution remain invariant during the course of the decision process.

Changes in measures of order discrimination performance as RT deadline is increased may be due to an increase in A, the response threshold value, alone. In this case a plot of the estimated marginal mean RT, $\hat{T}$, against $\hat{z}$, as defined in equation (7), is a linear function for each value of ISI. The intercept of each function is an estimate of the mean marginal non-decision time and the slope is inversely related to the mean step size of the random walk process. Hence, as ISI increases, the slope should decrease monotonically but the intercept should remain invariant.

The relationship between marginal mean RT and $\hat{z}$ is illustrated in Figure A8. The upper graph shows the plot for Experiment IIa. Each line represents the relationship between marginal mean RT and $\hat{z}$ for a fixed value of ISI. The three points on each line represent the values obtained under increasing RT deadline conditions. Except for the data obtained for the 27.5 msec condition for pooled data obtained for RR and JM the relationships were approximately linear.

The estimates of the intercept, $\hat{M}$, of the best fitting line to each set of three points according to a least-squares criterion are shown in Table 8. For DK the intercepts were close for the two largest values of ISI. For the pooled data for RR and JM, the intercepts were the same for ISIs of 16.5 msec and 44 msec. Although the linear relationship was significant for only two lines ($p < .05$), the availability of just one degree of freedom for the t test for linearity (Hays, 1963, p. 521) required a close approximation to linearity for statistical significance to be achieved. Except for the largest value of ISI for RR and JM, the reci-

TABLE 8

Linear Regression For Marginal Mean RT

Against $\hat{z}$ In Experiment II

EXPERIMENT IIa	ISI msec	$\hat{m}_1$ [a]	$\hat{M}$ msec [b]	EXPERIMENT IIb	ISI msec	$\hat{m}_1$	$\hat{M}$ msec
DK	5.5	-0.0073	291 ns	Release RT Pooled Over Ss	5	0.0003	144 ns
	11.0	0.0003	211 ns		10	0.0010	127 $p < .05$
	22.0	0.0046	259 ns		20	0.0037	135 ns
	44.0	0.0140	261 $p < .05$		40	0.0097	141 ns
RR & JM	5.5	0.0008	343 ns	Total RT Pooled Over Ss	5	0.0002	246 ns
	16.5	0.0024	157 $p < .05$		10	0.0008	226 $p < .05$
	27.5	0.0128	265 ns		20	0.0028	235 ns
	44.0	0.0128	157 ns		40	0.0080	251 ns

a. $\hat{m}_1$ is the estimated slope of the least squares linear plot of marginal mean RT against z.

b. $\hat{M}$ is the estimated intercept of the least squares plot of marginal mean RT against z.

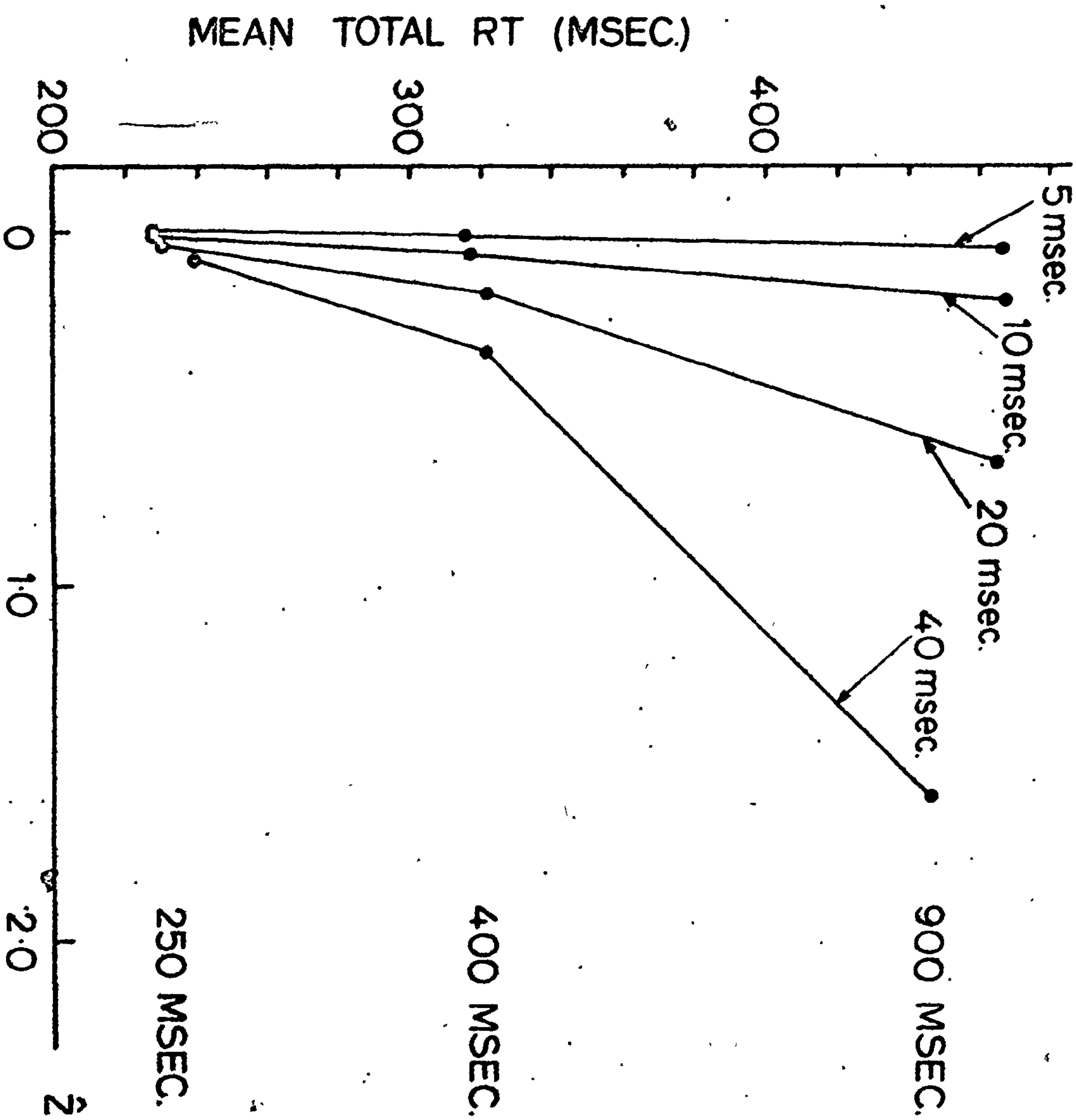
procal of the estimate of the slope of each line, which provides an estimate of $m\theta_1$, increased as ISI decreased. Hence the mean step size to the random walk increased as ISI increased.

The plot of marginal mean total RT as a function of $\hat{z}$ for the pooled data obtained in Experiment IIb is shown in Figure 12. Although there was a close approximation to linearity for the two shortest ISI values (for ISI = 10 msec the graph was significantly linear, $p < .05$), the slope decreased with increasing RT deadline. This result was consistent with the hypothesis that an increase in RT deadline causes an increase in $m\theta_1$ for a fixed ISI value. In spite of a departure from linearity for the longest ISIs, the estimated intercepts were close.

Laming (1968) suggested that Ss might commence sampling stimulus information before the registration of the first stimulus event. The effect of such pre-stimulus sampling is to introduce starting point variability in the random walk decision process. In the estimate of $\hat{z}$ in equation (7) it was assumed that starting point variability is negligible. However, it is clear from equation (4) that starting point variability modifies the estimate of $A\theta_1$, obtained from equation (2).

If the intercept of the best fitting linear psychometric function according to a least squares criterion represents the probability of an R_1 response when $\theta_1 = 0$, then the random walk decision process generating this value has zero drift. From equation (5), a significant departure of the intercept from a value of 0.5 indicates that C is non-zero. For the 250 msec RT deadline condition in Experiment IIb, the estimated intercept of the psychometric function was 0.45, a value significantly less than 0.5 ($p < .05$). Since the corresponding value of C is less than zero, the

Figure 12. Marginal mean RT as a function of $\hat{z}$ for Experiment IIb.

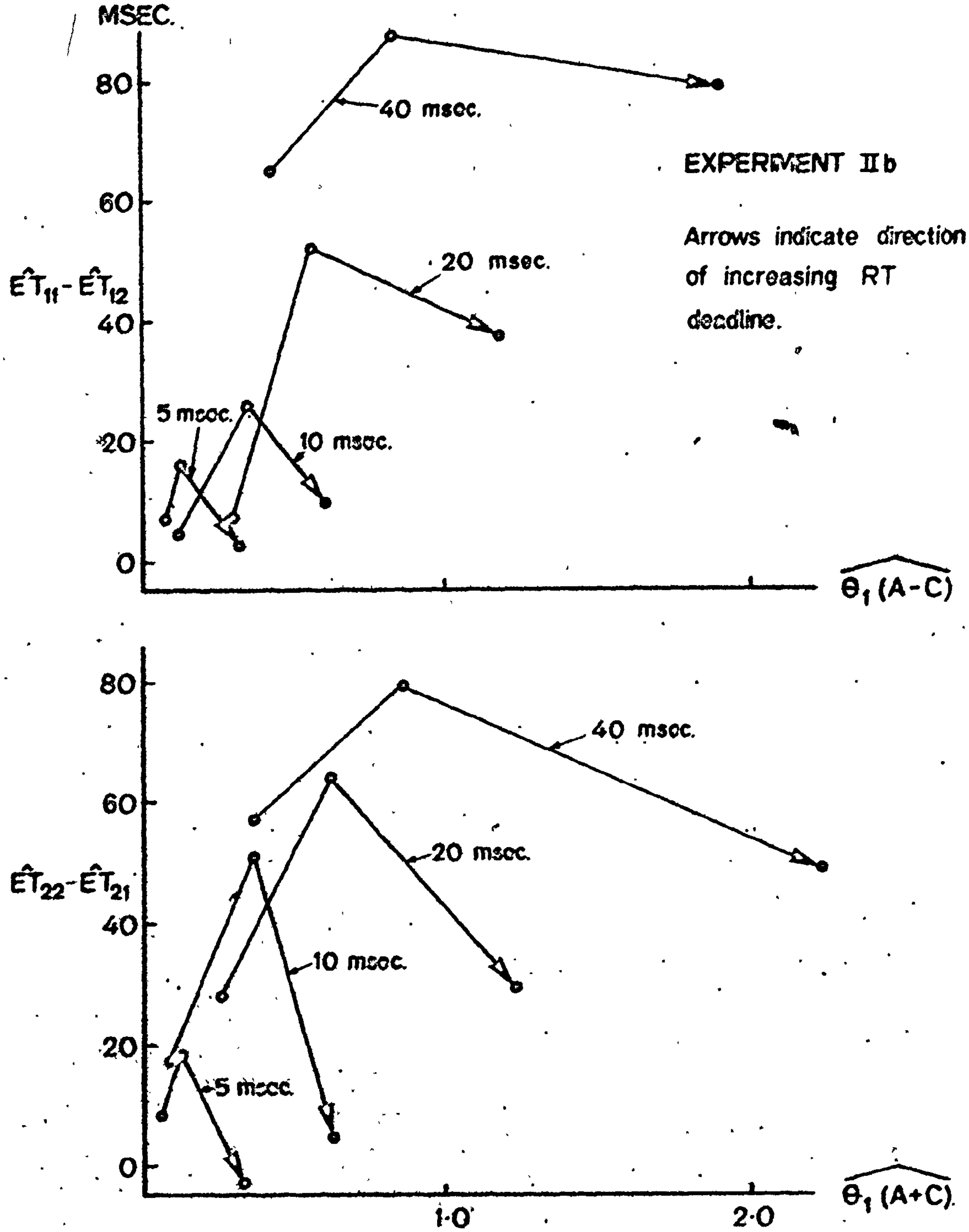

MEAN TOTAL RT (MSEC.)
200
300
400
0
1.0
2.0
Z
5 msec.
10 msec.
20 msec.
40 msec.
250 MSEC.
400 MSEC.
900 MSEC.

estimates of $A\theta_1$ obtained from equation (2) over-estimated the true value $A\theta_1$ when starting point variability was present (see Equation (4) and subsequent comments).

A reduction in $A\theta_1$ to allow for the effect of starting point variability results in a corresponding reduction in $\hat{z}$. Since $\hat{z}$ exceeds 0 for ISIs greater than 0, it is clear from Figure 12 that correcting $A\theta_1$ for starting point variability will not produce a linear relationship between marginal mean RT and $\hat{z}$ for the 20 msec and 40 msec ISIs. Hence the failure of a predicted linear relationship between mean RT and $\hat{z}$ to be supported by the pooled data from Experiment IIb could not be explained by assuming that there was starting point variability for the random walk decision process.

A plot of the difference in mean RT for response R_1 for stimulus patterns π_1 and π_2 respectively, $EN_{11} - EN_{12}$, as a function of the estimated value of $(A - C)\theta_1$ obtained from equation (11) is shown in Figure 13. Each curve is for a constant value of ISI and the direction of increasing RT deadline is indicated by the arrow. If increasing the RT deadline causes a change in the response strategy, e.g. an increase in A, and does not alter the stimulus dependent parameters, then each curve should be linear. This prediction was not supported by the data and it is concluded that the change in RT deadline alters the stimulus processing stages in temporal order discrimination. A similar conclusion was drawn from the plot of the difference in mean RT for response R_2 for stimulus patterns π_2 and π_1 respectively, $EN_{22} - EN_{21}$, as a function of the estimated value of $(A + C)\theta_1$ obtained from equation (12). These trends are also depicted in Figure 13.

Figure 13. Differences in response conditioned mean RT as a function of estimates of $(A-C)\theta_1$ and $(A+C)\theta_1$ for Experiment IIb.



MSEC.
80
60
40
20
0
$\hat{ET}_{11} - \hat{ET}_{12}$
40 msec.
20 msec.
10 msec.
5 msec.
EXPERIMENT IIb
Arrows indicate direction of increasing RT deadline.
$\widehat{\theta_1 (A-C)}$
80
60
40
20
0
$\hat{ET}_{22} - \hat{ET}_{21}$
40 msec.
20 msec.
10 msec.
5 msec.
1·0
2·0
$\widehat{\theta_1 (A+C)}$

General Conclusions

The real-time model provided an adequate account of the covariation between response time and response proportion measures in a temporal order discrimination task employing continuous visual stimuli and instructions stressing accuracy. The model failed to account for the direction of RT results obtained under instructions emphasising response speed.

The LOT model, which could encompass the direction of RT results obtained under both speed and accuracy conditions, was unable to account for the covariation of RT and response proportion data under the assumption that changes in instructions altered Ss response strategy. It was concluded that the decrease in response speed generated by RT deadline instructions altered the stimulus information processing aspects of temporal order discrimination. In Chapter VII it is proposed that the results obtained in Experiments I and II can be accounted for by the LOT model with a non-stationary random walk decision process.

CHAPTER VI

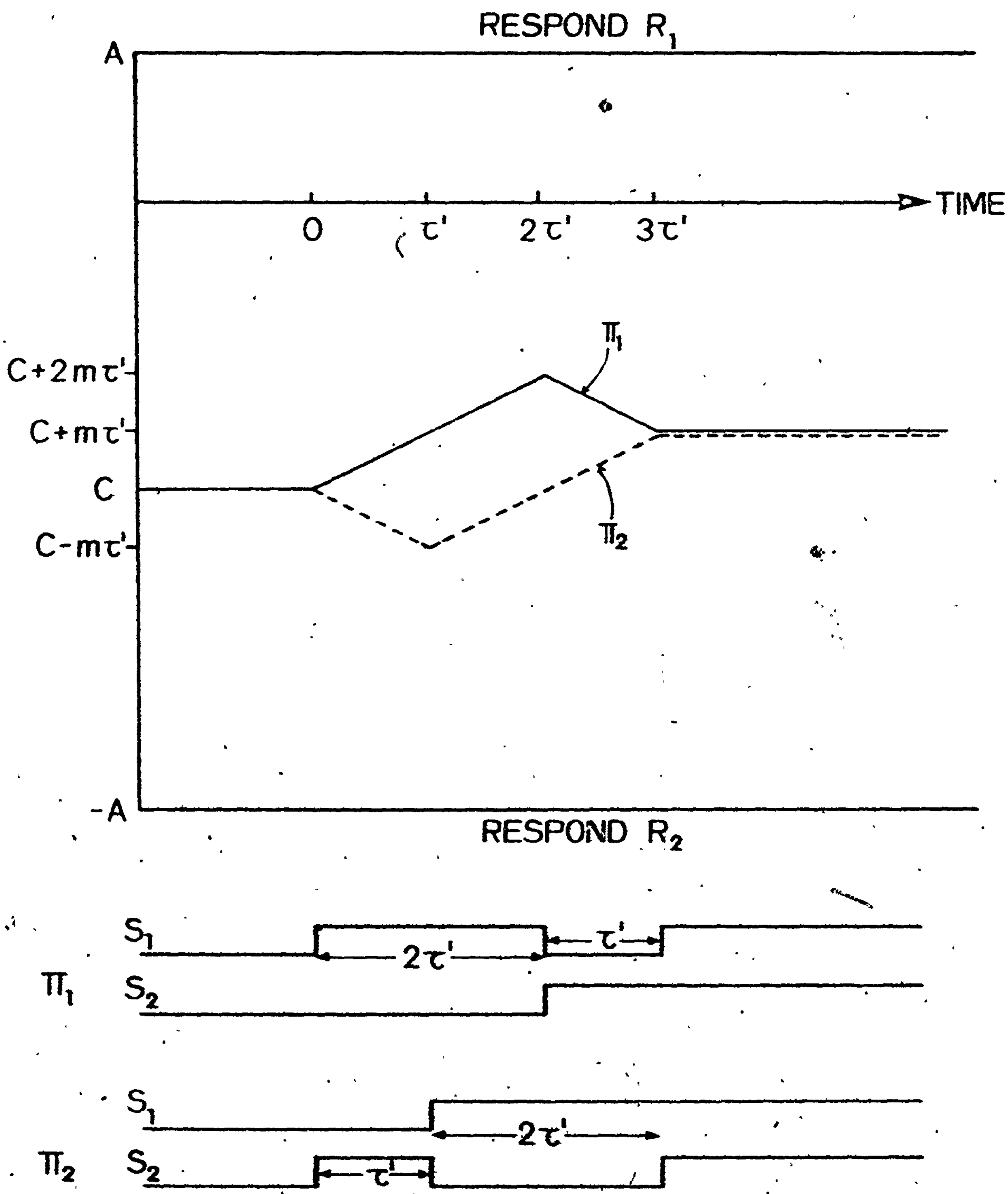
EXPERIMENT III: A Test of the Real-time Model for Temporal Order Discrimination.

The real-time model for temporal order discrimination is based on the assumption that S samples stimulus information continuously over time until sufficient evidence to generate an order response is obtained. In the simplest version of the model, the decision process is driven by the information sampled from the two stimuli. Hence any changes in the parameters of the stimuli should modify the rate of accrual of stimulus information, and alter the course of the decision process accordingly.

Figure 14 depicts the expected sample paths followed by the decision process generated by stimulus patterns π_1 and π_2. For π_1, stimulus S_1 precedes S_2 by $2\tau'$ msec. When S_2 is turned on, S_1 is extinguished for τ' msec. Thereafter, S_1 is turned on again and both stimuli remain illuminated. For π_2, stimulus S_2 precedes S_1 by τ' msec. When S_1 is turned on, S_2 is turned off for $2\tau'$ msec. Thereafter, S_2 is turned on again and both stimuli remain illuminated. S_1 and S_2 have the same intensity and generate the same constant mean step size for the random walk. The sampling period is assumed to be arbitrarily small.

In the random walk representation of the decision process for judging the onset order of the two stimuli, the process begins at C when time, t, equals 0. The response thresholds at A and -A represent responses R_1 (S_1 first) and R_2 (S_2 first) respectively. Time is represented in the horizontal direction.

Figure 14. Expected sample paths predicted by the real-time model for stimulus patterns π_1 and π_2.


RESPOND R_1
A
TIME
0
τ'
$2\tau'$
$3\tau'$
Π_1
$C+2m\tau'$
$C+m\tau'$
C
$C-m\tau'$
Π_2
-A
RESPOND R_2
S_1
$2\tau'$
τ'
Π_1
S_2
S_1
$2\tau'$
Π_2
S_2
τ'

The sensory effect of S_1 generates a mean step size to the random walk equal to m, and the sensory effect of S_2 generates a mean step size of −m. When both stimuli are present, the mean step size to the random walk is zero. For stimulus pattern π_1 the expected position after $3\tau'$ msec equals $C + 2m\tau' - m\tau' = C + m\tau'$. Similarly the expected position after $3\tau'$ msec for stimulus pattern π_2 is $C - m\tau' + 2m\tau' = C + m\tau'$. Hence the expected positions for sample paths of the random walk process generated by both stimulus patterns after $3\tau'$ msec are the same. In addition, if the variance of the step size distribution is the same for each stimulus, the variance of starting points to the zero drift process at time $3\tau'$ msec is identical for both stimulus patterns.

Since stimulus patterns π_1 and π_2 generate the same distribution of starting points for the terminal zero drift random walk decision process, the results derived in Chapter II for the real-time model imply that the predicted proportion of R_1 responses and the difference between response conditioned mean RTs should be the same for both stimulus patterns. Hence use of patterned stimuli provides a critical test of the real-time model.

Experiment III tested the validity of the stimulus information sampling assumptions of the real-time model when response accuracy was stressed. Order discrimination performance was examined using stimulus patterns π_1 and π_2 as test conditions for τ' equal to 10 msec. Together with the test patterns generated by reversing the order of S_1 and S_2, these four test cases are shown in Table 9 as trial types 8 to 11. Except for the ISI values, trial types 1 to 7 were those used in Experiment I. The values of ISI were 0, 10, 20 and 30 msec. For each trial type both stimulus lights were continuously illuminated 30 msec following the onset of the first stimulus light.

The predicted proportions of R_1 responses, $P_i(R_1)$, as a function of trial type i are given in Table 9. According to the real-time model the estimated values of $P_i(R_1)$ should not be significantly different for trial types 3, 8 and 10, and for 5, 9 and 11. If the gaps have any effect on temporal order discrimination then $\hat{P}_3(R_1)$ should not be significantly different from $\hat{P}_8(R_1)$ and $\hat{P}_{10}(R_1)$, and $\hat{P}_5(R_1)$ should not be significantly different from $\hat{P}_9(R_1)$ and $\hat{P}_{11}(R_1)$.

METHOD

Subjects

Four university students, three of whom had had previous training in temporal order discrimination tasks (V, R and RH), were employed. Each S had satisfactory 20:20 corrected or uncorrected vision as measured by a Snellen eye-chart. All except RH, the author, were paid $2 per experimental session which lasted approximately forty minutes.

Apparatus

The apparatus was the same as that used in Experiments I and II. The response panel was that used in Experiment II. For S's V and R, the left button indicated the reponse "Top light first" and the right button indicated the response "Bottom light first". For JO and RH the opposite response mapping was employed.

TABLE 9

PREDICTED VALUES OF $P_i(R_1)$ FOR EACH

TRIAL TYPE USED IN EXPERIMENT III

Condition	Trial Type		$P_i(R_1)$
Control	1		(A + C + 30m)/2A
	2		(A + C + 20m)/2A
	3		(A + C + 10m)/2A
	4		(A + C)/2A
	5		(A + C − 10m)/2A
	6		(A + C − 20m)/2A
	7		(A + C − 30m)/2A
Test	8	(π_1, S_1 1st)	(A + C + 10m)/2A
	9	(π_2, S_1 1st)	(A + C − 10m)/2A
	10	(π_2, S_2 1st)	(A + C + 10m)/2A
	11	(π_1, S_2 1st)	(A + C − 10m)/2A

Procedure

Each experimental session contained three blocks of 230 trials for all Ss except V. For this S, who took longer to perform the task than the other Ss, only two blocks of trials could be completed within a session. The first block was preceded by a five minute dark adaptation period and there was a two minute break between blocks of trials. Since the sequence of trial events was initiated by S, a rest could be taken at any time during the session. The first ten trials of each block contained a random presentation of different trial types. They were treated as warm-up trials and ignored in subsequent data analysis. The remaining 220 trials consisted of a sequence of random permutations of each of the eleven trial types. In this way runs of the same trial type were minimised and each trial type was counter-balanced for position with the block of trials. Each trial type occurred with equal frequency within the final 220 trials of a block.

The trial initiation key (TIK) was not employed in this experiment and Ss were requested not to depress it. Each trial was initiated by depressing and releasing both response buttons with the two forefingers. 250 msec after trial initiation the central fixation light was illuminated and the sequence of trial events was presented 750 msec later.

All three lights remained illuminated until S responded. The lights were then turned off and S was free to initiate the next trial. No response accuracy feedback was provided since an illusory effect of the presence of gaps was being investigated. If S responded before the onset of the first stimulus light the trial was aborted and presented again.

The Ss were instructed to fixate the centre light once it was illuminated and to attend equally to the two stimulus lights. They were requested to perform as accurately as possible. If they felt tired during a block of trials, a short rest period was permitted.

Excluding practice sessions, 30 blocks of experimental data were analysed for each S. The data were obtained from 10 experimental sessions for RH, JO and R and 15 experimental sessions for V. For all Ss there was a total of 26,400 observations. For V 7 trials generating RTs exceeding 4095 msec were excluded from the data analysis. For the other Ss data obtained on all trials were analysed.

RESULTS

The raw data obtained from each S are given in Table A7. The data pooled across all four Ss are contained in Table A8. Differences in performance under test and control conditions were examined in terms of the data pooled across all four Ss.

(a) Psychometric Function

The psychometric function is depicted in Figure 15. The control conditions are represented by filled points and the test conditions are represented by unfilled points. Each point is based on 2160 observations and the standard error of estimate does not exceed the unit of measurement, 0.01. Although monotonic throughout its range there was a distinct tendency for the psychometric function to be curvilinear.

A comparison of the estimated proportion of R_1 responses, $\hat{P}_1(\tau)$, obtained for the value of τ in the test and control conditions is presented in Table 10. A z test applied to the significance of the difference between two independent proportions (Ferguson, 1959, p. 146) revealed that $\hat{P}_1(\tau)$

Figure 15. Psychometric function and response conditioned mean RT for each stimulus pattern in Experiment III.

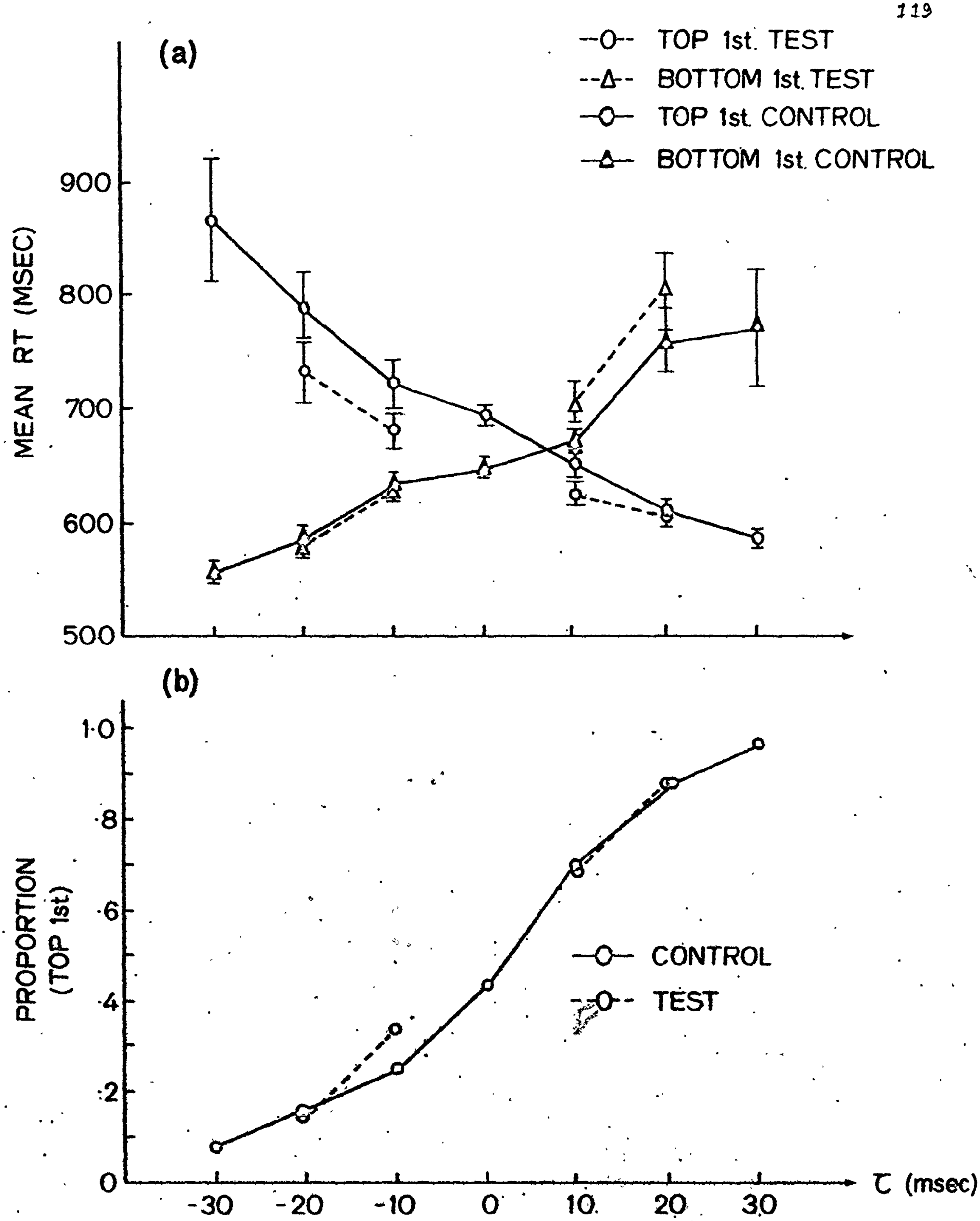

(a)
--O-- TOP 1st. TEST
--Δ-- BOTTOM 1st. TEST
—O— TOP 1st. CONTROL
—Δ— BOTTOM 1st. CONTROL
MEAN RT (MSEC)
900
800
700
600
500
(b)
PROPORTION (TOP 1st)
1·0
·8
·6
·4
·2
0
—O— CONTROL
--O-- TEST
-30
-20
-10
0
10
20
30
τ (msec)

TABLE 10

Proportion of R_1 Responses Under Test

And Control Conditions in Experiment III

	τ(msec)	Test	Control	Z	
V	20	0.82 ± 0.03	0.80 ± 0.03	0.84	ns
	10	0.57 ± 0.04	0.66 ± 0.04	-3.04	$p \leq .01$
	-10	0.31 ± 0.04	0.22 ± 0.04	3.34	$p < .01$
	-20	0.14 ± 0.03	0.17 ± 0.03	-1.36	ns
RH	20	0.93 ± 0.02	0.93 ± 0.02	0	ns
	10	0.77 ± 0.04	0.77 ± 0.04	0	ns
	-10	0.29 ± 0.04	0.21 ± 0.03	3.04	$p < .01$
	-20	0.06 ± 0.02	0.06 ± 0.02	0	ns
R	20	0.91 ± 0.02	0.90 ± 0.03	0.56	ns
	10	0.74 ± 0.04	0.71 ± 0.04	1.10	ns
	-10	0.32 ± 0.04	0.23 ± 0.04	3.31	$p < .01$
	-20	0.11 ± 0.03	0.12 ± 0.03	-0.52	ns
JO	20	0.87 ± 0.03	0.88 ± 0.03	0.50	ns
	10	0.64 ± 0.04	0.65 ± 0.04	-0.34	ns
	-10	0.43 ± 0.04	0.35 ± 0.04	2.70	$p < .01$
	-20	0.23 ± 0.04	0.25 ± 0.04	-0.77	ns
Pooled	20	0.88 ± 0.01	0.88 ± 0.01	0	ns
	10	0.68 ± 0.02	0.70 ± 0.02	-1.42	ns
	-10	0.34 ± 0.02	0.25 ± 0.02	6.48	$p \leq .01$
	-20	0.14 ± 0.01	0.15 ± 0.02	-0.93	ns

was significantly less in the test condition compared with the corresponding control condition when τ equalled -10 msec ($p < .01$). All other comparisons were not significant ($p > .05$). Hence the gap only affected temporal order discrimination when S_2 preceded S_1 by 10 msec. In this case the error proportion increased significantly in the test condition.

If the stimulus information sampling assumptions of the real-time model are correct then the proportion of incorrect order responses for stimulus pattern 10, in which S_2 preceded S_1 by 10 msec, should equal 0.70, the proportion of correct order responses for pattern 3. Clearly the increase in order error rate generated by the gap in stimulus pattern 10 was significantly less than that predicted by the real-time model. Moreover the failure of the model's predictions for the other test conditions to be verified experimentally questioned the validity of the stimulus information sampling assumptions.

(b) Response Time Data

(i) Marginal Mean RT

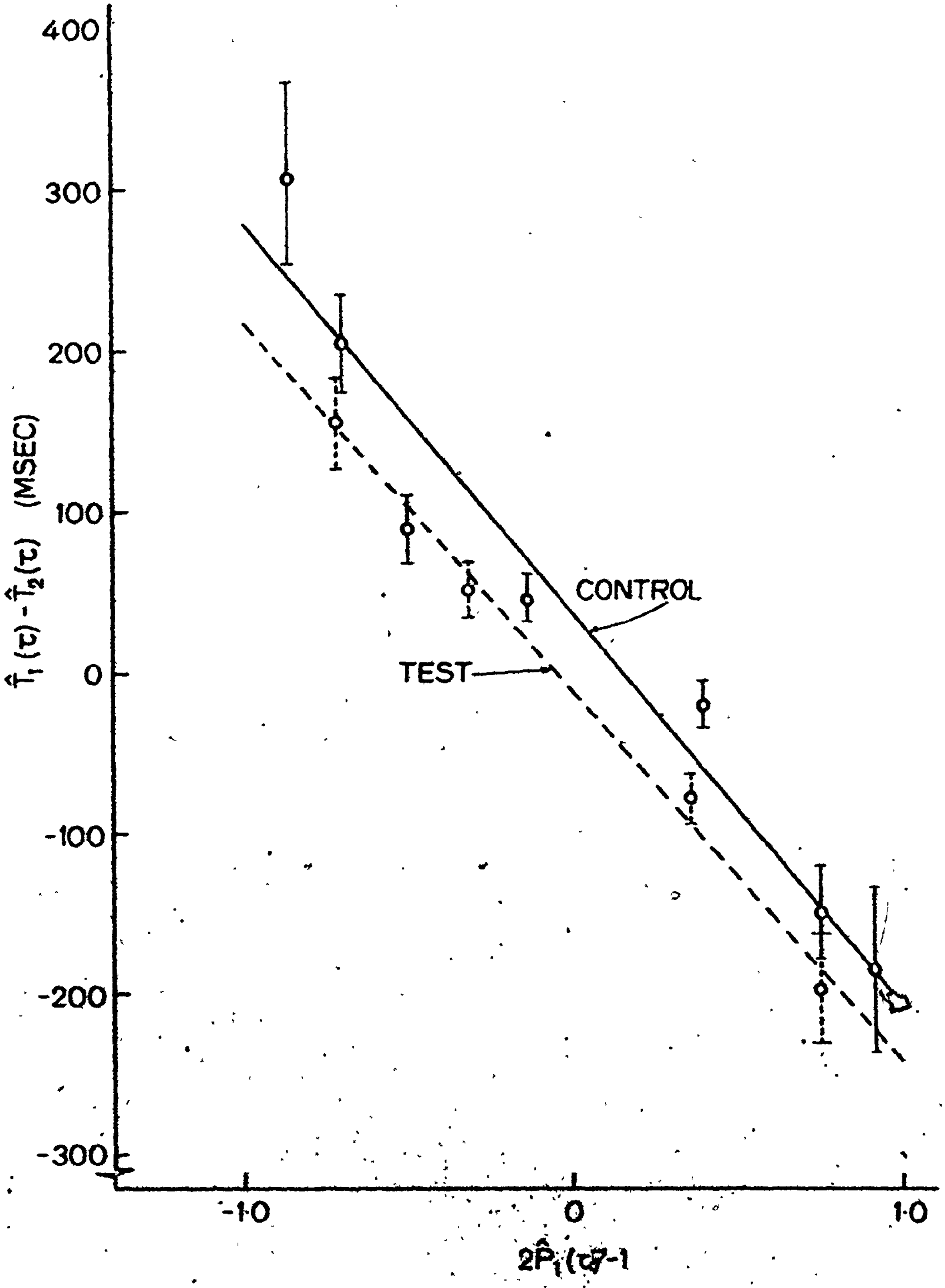
Marginal mean RT pooled over both responses for each stimulus pattern is depicted in Figure A9. The 95% confidence interval for the estimate of each marginal mean RT is indicated. Control conditions are indicated by filled points and test conditions are represented by unfilled points. The overlap of confidence intervals for each comparison of test and control conditions at each value of τ indicated that marginal mean RT did not differ significantly between conditions. The shape of the function relating marginal mean RT and τ was similar for the control conditions in this experiment and for the data of Experiment I.

(ii) Response Conditioned Mean RT

A plot of the relationship between τ and mean RT conditioned upon response R_1 and R_2 is presented in Figure 15. The trend in the data for the control conditions was similar to that observed in Experiment I. Mean RT for response R_1 decreased monotonically as τ increased. Conversely, mean RT for response R_2 increased monotonically as τ increased. Associated with each point in Figure 15 is its standard error of the mean.

Figure 16 contains a plot of the estimated difference in response conditioned mean RT against $2\hat{P}_1(\tau) - 1$ for each stimulus pattern employed in both control and test conditions. Linear regression of these estimates indicated that there was a highly significant linear trend for both the control and test conditions ($p < .01$). The correlation coefficient squared was 0.94 for the control conditions and 0.99 for the test conditions. The 95% confidence intervals for the slopes of the best-fitting lines according a least squares criterion were -243 ± 62 and -229 ± 59 for the control and test conditions respectively. The 95% confidence intervals for the intercepts were 38 ± 42 and -13 ± 34 respectively. Hence overlap of the confidence intervals indicated that there were no significant differences ($p > .05$) in the slope and intercepts of the best fitting lines for the control and test conditions. Although the negative slope of the plot of $T_1(\tau) - T_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ indicates that the data can be accounted for by a real-time model, the failure of the predicted correspondences between response probability measures for test and control conditions indicates that the stimulus sampling assumptions of the model require modification.

Figure 16. $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against $2\hat{P}_1(\tau) - 1$ for Control and Test conditions in Experiment III.


400
300
200
100
0
-100
-200
-300
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (MSEC)
CONTROL
TEST
-1·0
0
1·0
$2\hat{P}_1(\tau)-1$

The real-time model may still apply if the assumption that the mean drift rate can be predicted directly from the stimulus pattern is relaxed. It is not valid to conclude that the mean drift rate for the random walk is constant and that the sampling period is arbitrarily small. The finding that the gap had no significant effect on order discrimination performance for an ISI equal to 20 msec indicates that the gap is not included in the neural activity sampled. This could occur because the visual system is not sensitive to 10 msec gaps in a continuous stimulus.

<u>General Conclusions</u>

The results of Experiment III indicated that the simplest assumptions of the real-time model regarding the sampling of stimulus information in a temporal order discrimination task are wrong. The predicted equivalence in performance for certain control and test stimulus patterns when the mean drift rate generated by each stimulus was assumed to be constant was not supported by the data.

A gap in one of the stimuli in test patterns caused a significant change in performance for one pattern, namely that in which the bottom light preceded the top light by 10 msec. This result was found for all four <u>Ss</u>, and provided evidence for an asymmetry in the sensitivity of the visual system to gaps in continuous stimuli as a function of their position in the visual field.

For the control conditions both response proportion and mean response time data varied with ISI in a similar fashion to that observed in Experiment I. The psychometric function increased monotonically with τ and marginal mean RT was a convex upwards function of τ attaining its maximum value at τ equal to 0. As ISI increased for each stimulus order, marginal mean RT decreased monotonically. Mean RT conditioned upon response R_1 decreased monotonically and mean RT conditioned upon response R_2 increased monotonically as τ increased. Plots of the difference in response conditioned mean RT as a function of $2\hat{P}_1(\tau) - 1$ for control and test conditions indicated that in both cases the data could be fit by lines with negative slope. The real-time model could account quite adequately for the covariation in mean RT and response proportion provided that it is assumed that the changes in mean drift rate of the random walk during the stimulus information stage cannot be predicted directly from physical changes in the stimulus pattern.

CHAPTER VII

A Generalisation of the LOT Model For Temporal Order Discrimination.

The data obtained in Experiment III failed to support the simplest stimulus information sampling assumptions of the real time model for temporal order discrimination. The RT predictions of the model were not supported by the data obtained in Experiment II. However the results of Experiments I and III under accuracy conditions provided substantial support for the assumption of a terminal zero drift masking stage.

If the LOT model is correct and the concept of a limited observation period is valid, then the mean RT results of Experiment I pose no problem for the model. Equation (8) accommodates the general result that mean RT conditioned upon a correct response was less than the mean RT conditioned upon an incorrect response by allowing γ, the asymmetry parameter of the step size distribution of the random walk, to be less than one. However, the difference between response conditioned mean RTs increased as ISI increased. Hence the absolute value of $(\gamma-1)/\gamma m$ must increase with ISI. This constraint restricts the class of step size distributions that can account for the data.

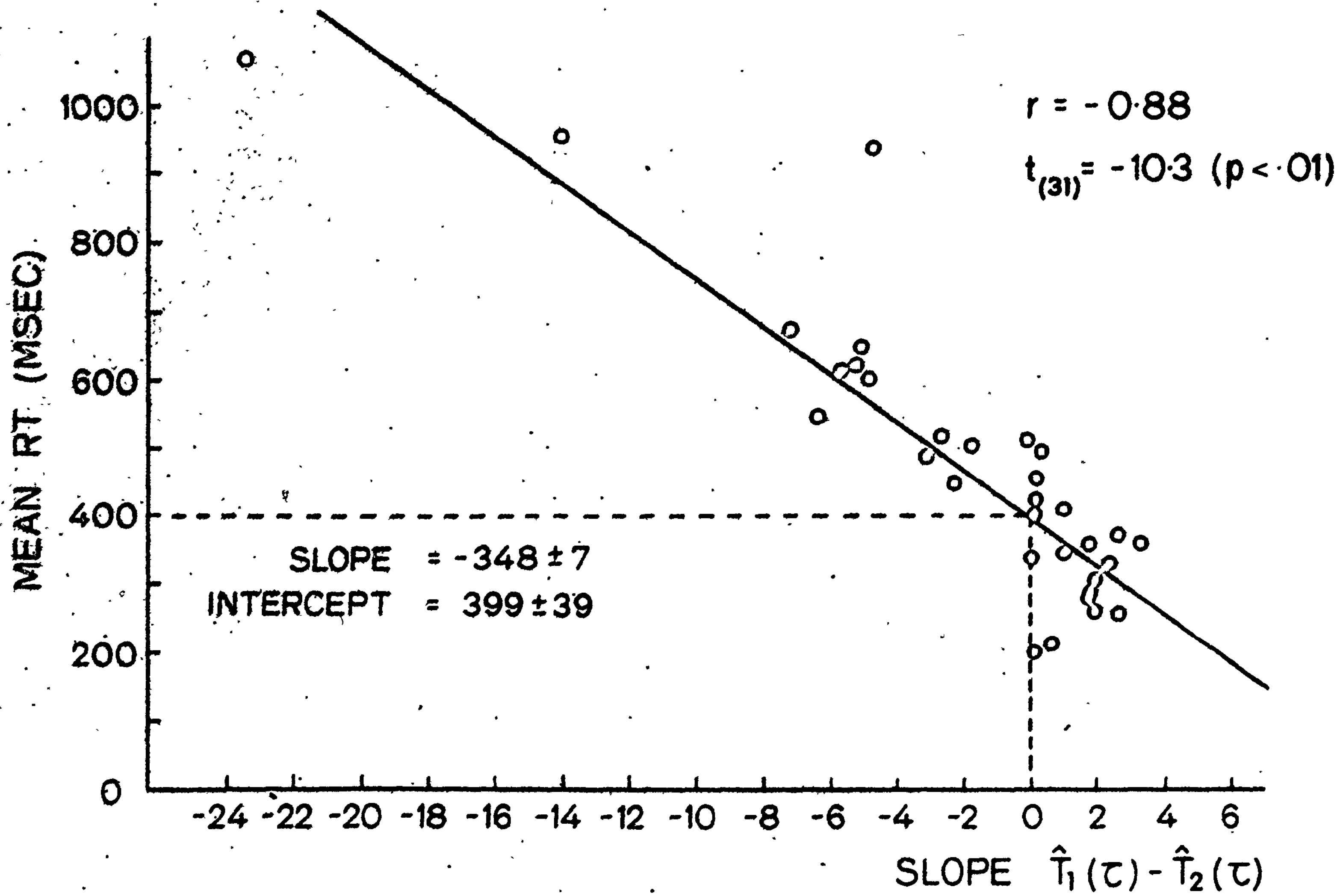
Similarly the LOT model can account for the result of Experiment IIb that mean RT conditioned upon a correct response was greater than mean RT conditioned upon an incorrect response, for values of γ greater than one. The tendency for the difference between response conditioned mean RTs to increase as ISI increased requires a constraint on the class of step size distributions similar to that needed for the data of Experiment I.

The pattern of results obtained in the three experiments showed that as performance accuracy increases the slope of the plot of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against τ changes from positive to negative. In Figure 17 marginal mean RT pooled across both responses and ISI is plotted against the slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$. The points plotted in the figure were derived from the individual subject data for all three experiments. For Experiment II a separate point was plotted for each RT deadline condition. The correlation was significantly negative ($p < 0.01$) as expected, and the estimated intercept of the best fitting line was 399 ± 39 msec. This value represents the marginal mean RT associated with equal response conditioned mean RTs for all values of τ.

The expression for the marginal mean RT for the LOT model (equation A16) does not depend on the asymmetry parameter γ. This result applies because the expression for the probability of absorption at A, P_A, does not depend on γ (equation A10). Hence the LOT model with a stationary asymmetrical step size distribution cannot account for the negative correlation between marginal mean RT and the slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ as a function of τ.

It is most unlikely that across a range of marginal mean RT values from 200 msec to over one second the stored representation of the stimulus difference sampled on a trial has stationary properties. If a rapid response is required, the time spent in stimulus processing may not be sufficient to generate a stored representation with high

Figure 17. Marginal mean RT as a function of the estimated slope of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ against τ for each S in the three experiments.


r = -0·88
$t_{(31)}$ = -10·3 (p < ·01)
SLOPE = -348 ± 7
INTERCEPT = 399 ± 39
MEAN RT (MSEC)
0
200
400
600
800
1000
-24 -22 -20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6
SLOPE $\hat{T}_1(\tau) - \hat{T}_2(\tau)$

fidelity. Hence the decision process will operate with a noisy stimulus image in short-term visual storage and performance accuracy will suffer accordingly. On the other hand, when the mean RT is long, the fidelity of the stored stimulus image may decline gradually over time if there is no further information in the stimulus display to update it. Hence it is hypothesised that the initial increase and gradual decline of the fidelity of the stored representation of the stimulus difference sampled in an order discrimination task generate a non-stationarity in the distribution of the step size for the random walk decision process.

A pictorial representation of the modified LOT model for temporal order discrimination is presented in Figure 2. According to this representation temporal order discrimination involves three stages, a <u>stimulus information encoding stage</u>, a <u>decision stage</u> and a <u>response output stage</u>.

In the stimulus information encoding stage a sample of stimulus information is obtained during a finite sampling period, T. The sampling period may be under the control of the <u>S</u> and therefore susceptible to the influence of RT deadlines. Alternatively, it may be a fixed time interval like the <u>perceptual moment</u> (Stroud, 1955). In the context of temporal order discrimination it will be assumed that a difference in information derived from each stimulus during the ith observation interval d_i is the critical information accessed by the decision process.

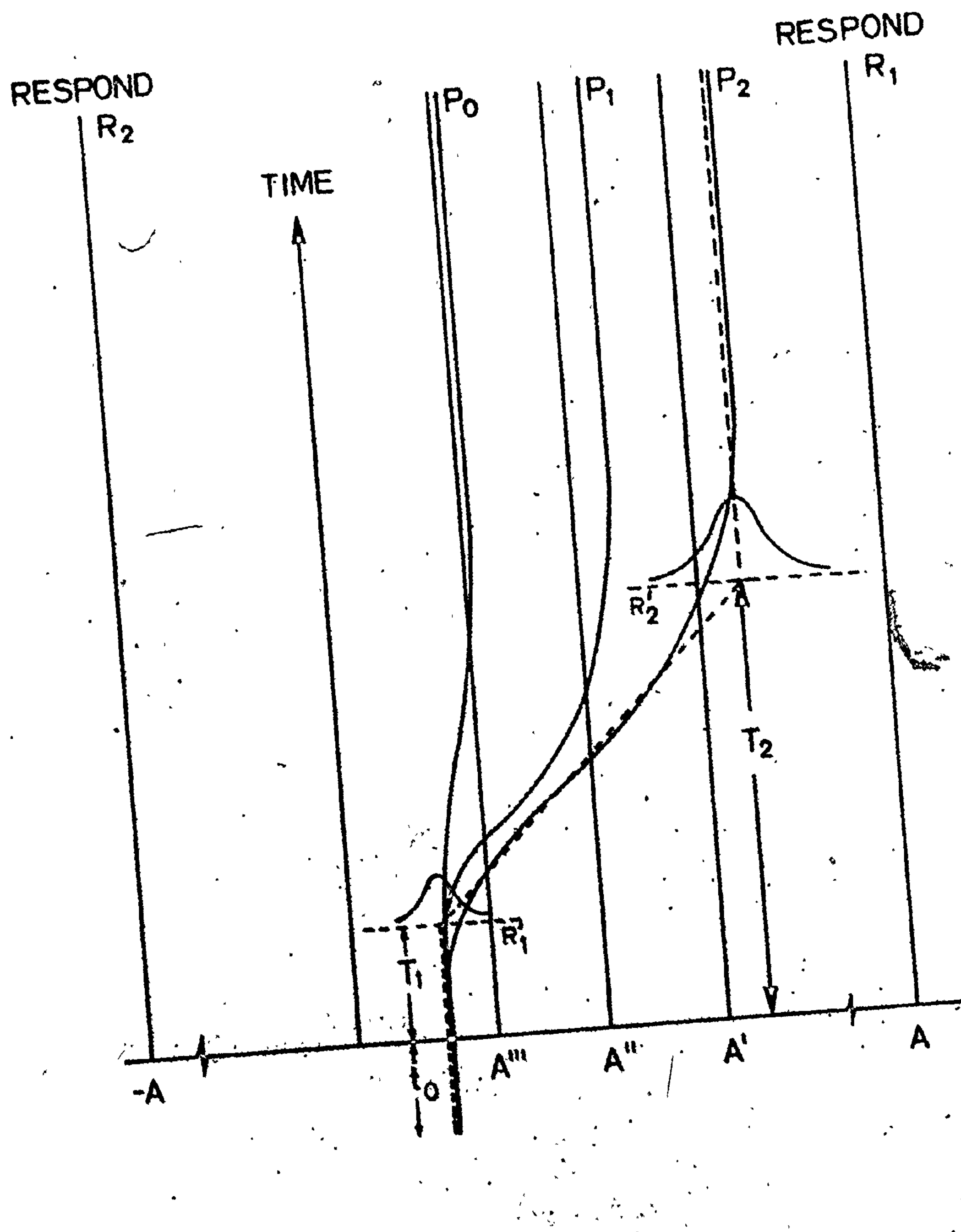
Each stimulus difference is stored in a short-term visual storage register. Initially the fidelity of the representation is improved by various signal averaging and filtering techniques such as those proposed by Shallice (1967). By these means a clear image of the stimulus information is built up in short-term storage. If no update of stored information occurs as the result of another stimulus difference sampled from the display, the fidelity of the stored representation decays spontaneously.

The assumption of a decay of information in short-term visual storage has been verified in studies by Sperling (1960) and others. Vanthoor and Eijkman (1973) measured the memory strength of visual stimuli as a function of the delay of a partial report indicator cue. From the change in performance as a function of the cue delay an approximation to the form of the iconic memory signal was obtained. This function reached a peak value about 125 msec after presentation of the stimulus and then declined rapidly over the following 300 msec. It is hypothesised that a similar time course for the fidelity of the sampled stimulus difference occurs in the present modol.

In the decision stage the random walk process is similar to that described for the LOT model, except that the distribution of step sizes to the random walk is no longer stationary. While an image of the stimulus difference is being built up in short-term visual storage the mean step size to the random walk increases, whereas the mean step size gradually decreases as the fidelity of the stored stimulus difference declines.

Figure 18 depicts the mean sample paths for two realisations of a non-stationary random walk decision process. For both paths P_1 and P_2 the drift towards the response threshold at A increases to a maximum value and then decreases to an asymptotic value of zero. The possibility of pre-stimulus sampling is allowed for by assuming that

Figure 18. The tandem random walk decision process for the modified LOT model for temporal order discrimination.


RESPOND
R2
TIME
P0
P1
P2
RESPOND
R1
R2'
T2
T1
R1'
-A
0
A'''
A''
A'
A

the decision process begins t_a msec before the presentation of the stimulus. During this period the process has a zero drift rate.

A possible means of examining the non-stationarity property of the decision process is to force the S to vary the position of the response threshold values. When the response threshold is large (say at A) then a considerable amount of the decision process is controlled by the terminal zero drift, whereas at A"' close to the origin, the process is governed by the initial non-zero drift section. At intermediate values, such as at A' and A", the process is governed to various extents by both an increasing and decreasing drift rate.

It is assumed that the larger the ISI, the greater is the mean displacement of the random walk process prior to the terminal zero drift segment. Hence path P_2 arises from a larger ISI value than does path P_1. For intermediate values of the response threshold such as at A', the dominant trend of the random walk process depends on the value of ISI. For path P_2 the non-zero drift segment dominates whereas for path P_1 there is a significant contribution from the terminal zero drift segment. Hence the characteristics of the decision process depend not only on the position of the response threshold but on the range of ISI values used.

Since it is difficult to derive simple analytic expressions for the predicted statistics for a non-stationary random walk process, the non-stationarity will be approximated by a tandem random walk process. The approximate tandem process corresponding to path P_2 consists of three segments. The first segment is a zero drift process terminating at the point R_1, T_1 msec after the onset of the first stimulus. This is followed by a non-zero drift process terminating at R_2', T_2 msec after the onset of the first stimulus. The final segment is a terminal zero drift process.

For the response thresholds at A" and A"', the non-stationary process can be represented approximately by a non-zero drift process with starting point variability beginning at R_1' at time T_1. On the other hand, for the response threshold at A, the non-stationary random walk process can be represented approximately by a zero drift process with starting point variability, beginning at R_2' at time T_2. In such cases the decision process for the modified LOT model can be approximated by a process which is conceptually similar to that described for the real-time model.

In order to calculate the probability of absorption at A and the mean number of steps to absorption at A and -A respectively, we can use the results derived in the Appendix. These results apply to a range of ISIs for which the probability of absorption before the beginning of the terminal tandem process is negligible. Using the assumption of normally distributed steps to the random walk an expression for the difference in mean number of steps to absorption at A and -A is given by equation A20. The sign of this difference is determined by the sign of $\bar{K}$, a coefficient proportional to the mean difference in step size for the two processes contributing to the tandem process. The probability of absorption at A is given by equation A10.

When the response threshold is sufficiently close to the origin so that the contribution of the terminal zero drift process is negligible, the decision process can be represented in terms of a stationary random walk process with starting point variability. In this case the coefficient K in equation (A20) can be represented by $\frac{1}{2}s_o^2\theta_1 - C$ where C is the mean starting point of the process and s_o^2 is the starting point variance.

When θ_1 is small so that terms of order θ_1^2 and higher can be ignored, approximate expressions for the probability of absorption at A, $P_1(\theta_1)$, and the difference between the mean number of steps to absorption conditioned upon absorption at A and -A respectively, $T_1(\theta_1) - T_2(\theta_1)$, can be obtained.

$$P_1(\theta_1) \simeq \frac{A+C}{2A} + \left(\frac{A^2 - C^2 - s_o^2}{4A}\right)\theta_1, \quad s_o^2 < A \tag{26}$$

$$T_1(\theta_1) - T_2(\theta_1) \simeq \frac{-4AC}{3s_1^2} + \left(\frac{2As_o^2}{3s_1^2}\right)\theta_1, \quad s_o^2 < A \tag{27}$$

where s_1^2 is the variance of the step size to the non-zero drift process.

When θ_1 is proportional to τ, the ISI, so that $\theta_1 = \alpha\tau$ then $P_1(\tau)$ is a linear function of τ with intercept (A+C)/2A and positive slope $\alpha(A^2-C^2-s_o^2)/4A$. As A increases, the intercept approaches 0.5 and the slope increases. An increase in starting point variability, s_o^2, causes a decrease in the slope.

Similarly $T_1(\tau) - T_2(\tau)$ is a linear function of τ with intercept $-4AC/3s_1^2$ and positive slope $2\alpha As_o^2/3s_1^2$. As both A and S_o^2 increase the slope increases.

When A is large so that the process is dominated by the terminal zero drift process, we obtain the psychometric function and difference between mean response conditioned RTs predicted by the real-time model. In this case the mean starting point for the terminal zero drift process is given by $(T_2-T_1)m(\tau)^{+C}$, where $m(\tau)=\alpha\tau$ if the mean step size for the non-zero drift process is a linear function of τ. Hence the predicted results are those derived for the real-time model in Chapter II.

In general, the predicted results are determined by the value of $K = \alpha\theta_1$ where $\alpha = \frac{1}{2}s_o^2[1 - (\theta_o/\theta_1)]$. The sign of α, and hence K, is determined by the ratio of the values of θ for the two constituent processes. If $\theta_o < \theta_1$, $K > 0$ and the results are similar to those obtained in the presence of starting point variability. When $\theta_o > \theta_1$, $K < 0$ and the results are similar to those obtained for the real-time model. When $\theta_o = \theta_1$, $K = 0$ and the tandem process degenerates into a single process with a stationary step size distribution.

For the general case, equation (26) becomes:

$$P_1(\theta_1) \simeq \frac{1}{2} + \left(\frac{A^2 - 2\alpha}{6A}\right)\theta_1 \qquad (28)$$

and equation (27) becomes:

$$T_1(\theta_1) - T_2(\theta_1) \simeq \left(\frac{4A\alpha}{3s_1^2}\right)\theta_1 \qquad (29)$$

Eliminating θ_1 from equations (28) and (29) we obtain:

$$T_1(\theta_1) - T_2(\theta_1) = b\,[2P_1(\theta_1) - 1] \qquad (30)$$

where

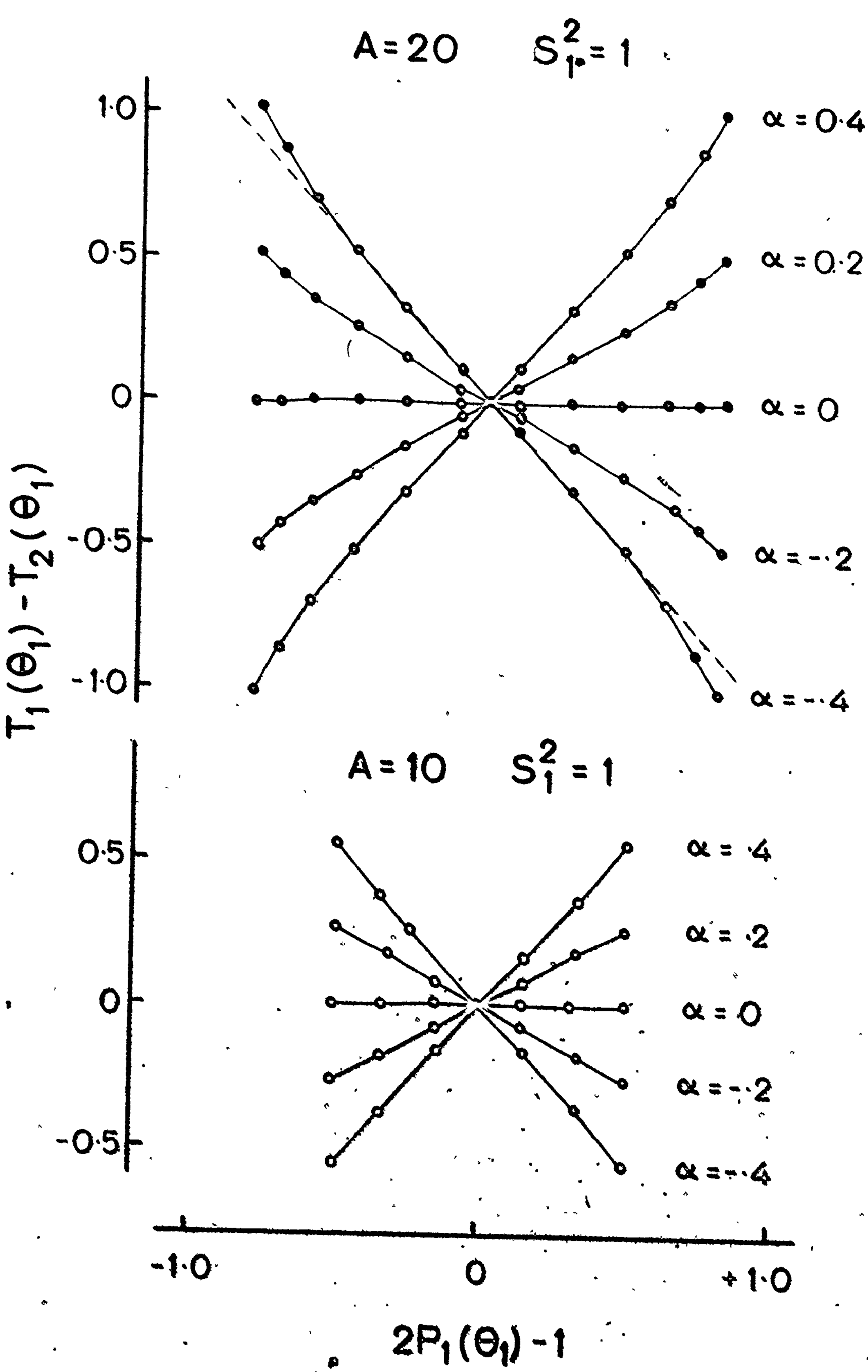
$$b = \frac{8A^2\alpha}{3(A^2 - 2\alpha)S_1^2}$$

Since $\alpha < A$, the sign of b is the sign of α. Hence when $\theta_o < \theta_1$, $\alpha > 0$ and b is positive. When $\theta_o > \theta_1$, $\alpha < 0$ and b is negative. Finally when $\theta_o = \theta_1$, $\alpha = 0$ and b is zero. So the slope of equation (30) provides a measure of the direction of the non-stationarity in the decision process.

Figure 19 shows plots of $T_1(\theta_1) - T(\theta_1)$ against $2P_1(\theta_1) - 1$ using equations (A20) and (A10) with C = 0 and $K = \frac{1}{2}s_o^2\theta_1$. Two values of A were used and α was varied, where $\alpha = \frac{1}{2}s_o^2[1 - (\theta_o/\theta_1)]$. Each plot was approximately linear indicating that equation (30) provides a good approximation to the predictions of the model. A positive correlation between α and the slope of the plot was evident.

Figure A10 shows a plot of estimates of $A\theta_1$ obtained from equation (2) for the LOT model with a stationary step size distribution as a function of τ for pooled data from Experiments Ib and III. In each case the estimate of $A\theta_1$ was a linear function of the ISI, τ. Clearly for $\theta_1 = \beta\tau$, where β is a constant, the linear relationship of equation (30) also applies for a plot of $\tilde{P}_1(\tau)$ against $\hat{T}_1(\tau) - \hat{T}_2(\tau)$.

Figure 19. Predictions of the tandem random walk decision process for the relationship between $T_1(\Theta_1) - T_2(\Theta_1)$ and $2P_1(\Theta_1) - 1$.

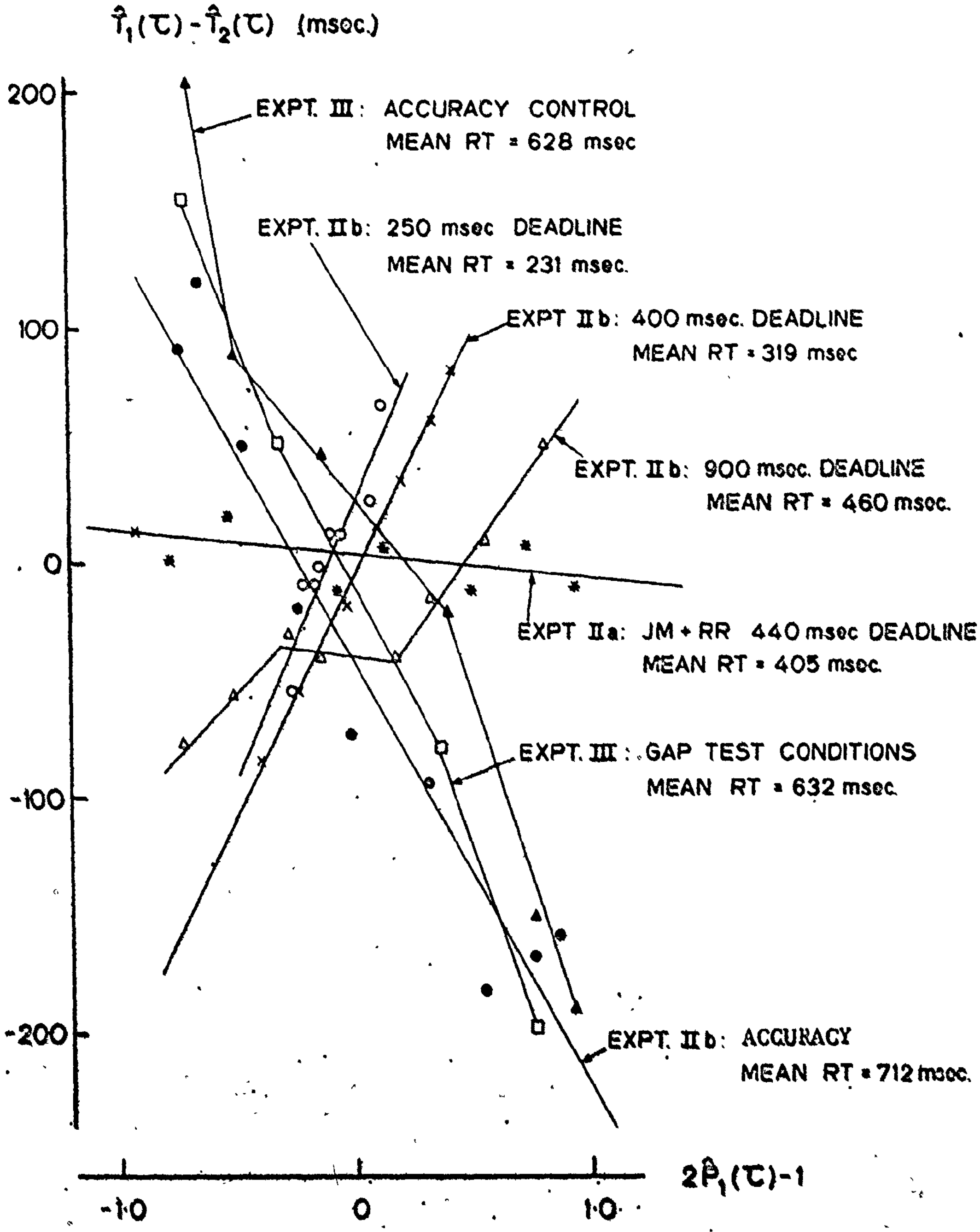

A=20 $S_1^2=1$
1·0
0·5
0
-0·5
-1·0
α = 0·4
α = 0·2
α = 0
α = -·2
α = -·4
$T_1(\theta_1) - T_2(\theta_1)$
A = 10 $S_1^2 = 1$
0·5
0
-0·5
α = ·4
α = ·2
α = 0
α = -·2
α = -·4
-1·0
0
+1·0
$2P_1(\theta_1) - 1$

In this case the intercept of $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ includes a difference $\hat{M}_1 - \hat{M}_2$ in mean non-decision times resulting from the response output stage depicted in Figure 2. We maintain the assumption that $\hat{M}_1 - \hat{M}_2$ is constant for all values of τ.

In Figure 20 $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ is plotted as a function of $2\hat{P}_1(\tau) - 1$, where $\hat{T}_1(\tau)$, $\hat{T}_2(\tau)$ and $\hat{P}_1(\tau)$ are the estimated values of $T_1(\tau)$, $T_2(\tau)$ and $P_1(\tau)$ respectively. For the accuracy conditions of Experiments Ib and III the plots had negative slope indicating that $\theta_1 < \theta_o$ for the non-stationary tandem random walk decision process. For the control conditions of Experiment III, the slope increased for the longer values of $2\hat{P}_1(\tau) - 1$. Such a departure is likely to occur when τ, and hence θ_1, is large. In this case the linearity of the approximation equations (28) and (29) is no longer valid. A tendency for the slope to increase with increasing θ_1 is revealed by observing the trend in the plot for $\alpha = -.4$ in the $A = 20$ condition in Figure 19.

For the RT deadline conditions in Experiment IIb each plot had positive slope indicating that $\alpha > 0$ and $\theta_o < \theta_1$. In the 900 msec deadline condition the slope was close to zero for small values of τ. This indicated that $\alpha \approx 0$ so that for small τ the decision process was essentially stationary. When τ was large there was clear evidence for a departure from stationarity. Such a trend might be expected if the response threshold is positioned at A' in Figure 18. For path P_o generated by a short value of τ the overall change in the rate of drift is slight so that any measure of a departure from a stationary process will be close to 0. For larger values of τ generating the path P_2 the departure from stationarity is more pronounced.

Figure 20. Relationships between $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ and $2\hat{P}_1(\tau) - 1$ for each experiment.



$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec.)
200
100
0
-100
-200
EXPT. III : ACCURACY CONTROL
MEAN RT = 628 msec
EXPT. IIb: 250 msec DEADLINE
MEAN RT = 231 msec.
EXPT IIb: 400 msec. DEADLINE
MEAN RT = 319 msec
EXPT. IIb: 900 msec. DEADLINE
MEAN RT = 460 msec.
EXPT IIa: JM + RR 440 msec DEADLINE
MEAN RT = 405 msec.
EXPT. III : GAP TEST CONDITIONS
MEAN RT = 632 msec.
EXPT. IIb: ACCURACY
MEAN RT = 712 msec.
-1·0
0
1·0
$2\hat{P}_1(\tau)-1$

Data pooled across Ss JM and RR for the 440 msec deadline in Experiment IIa indicated a stationary decision process since the slope was close to zero. These data exemplified a transition case between the generally positive slope for the RT deadline conditions of Experiment IIb and the negative slope for the accuracy conditions of Experiments Ib and III.

If it is assumed that relaxation of the speed constraint on RT performance allows the S to increase A, the response threshold, then these results imply that there is a change in the non-stationarity of the decision process as A increases. For small values of A generated by short RT deadline conditions the increase in the drift rate dominates so that α is greater than zero and the slope of equation (30) is positive. As A increases further, the non-stationarity parameter reduces to zero. In this case the drift rate is relatively constant over a large proportion of the duration of the process. For the largest values of A, the terminal zero drift process dominates so that α is less than zero and the slope of equation (30) is negative.

The data obtained in the three experiments on temporal order discrimination provided support for the notion of a stored stimulus difference with time varying fidelity. Initially an image of the stimulus difference is constructed in short-term visual storage. Unless further information can be sampled from the stimulus display to reconstruct the image, the image decays spontaneously.

The decay of the image is important in an order discrimination task employing stimulus onsets as critical events to be ordered. If the sampling interval T is large relative to the range of ISIs, no order information can be obtained from subsequent samples. Hence under no RT constraints, a terminal zero drift process is likely to dominate and the covariation between RT and response proportion measures observed in Experiments I and III would be predicted.

CHAPTER VIII

DISCUSSION

A tandem random walk model for temporal order discrimination has been proposed. According to this model, order discrimination involves three distinct stages, a stimulus information encoding stage, a decision stage and a response output stage.

In the stimulus information encoding stage the difference in stimulus information sampled from the two temporally asynchronous stimuli during an observation interval is stored in a short-term visual register. As a result of further information processing of the stored stimulus difference, the fidelity of the image improves. Following an increase in clarity of the stored information there is a process of gradual decay of the image in short-term storage. Unless the fidelity of the image can be restored by another observation of the stimulus display there is a gradual loss of stimulus difference information over time. In a temporal order task in which the critical events are the stimulus onsets, all of the relevant order information is contained in the initial observations of the display. Hence loss of order information from the short-term visual storage register is to be expected.

The decision process is represented as a random walk between two absorbing barriers defining response thresholds. The step size distribution is determined by the statistical properties of the stimulus difference information stored in the short-term storage register. Owing to changes in the fidelity of the stored image, the step size distribution for the random walk is assumed to be non-stationary. The extent of non-stationarity of the decision process is represented by a tandem random walk process. This representation generates approximate analytic expressions for the probabilities of absorption and the mean number of steps to absorption at the two response thresholds.

In the response output stage the motor response associated with the response threshold which is reached first during the decision stage is selected and executed. It is assumed that the non-decision components of mean RT that are consumed in this stage are additive with the response conditioned mean decision time and are independent of the temporal asynchrony of the stimuli.

The results of three experiments involving the discrimination of the order of illumination of two rapidly successive visual stimuli provided support for the tandem random walk model. Under instructions emphasizing response accuracy a terminal random walk decision process with zero drift provided a close approximation to the obtained response proportion and response conditioned mean RT data. A version of the tandem random walk model which assumed that stimulus information was transferred continuously into the decision process without being temporarily stored was invalidated by the data obtained in the test conditions of Experiment III. In these conditions a gap in one of the stimuli did not alter performance to the extent predicted by this version of the model.

A comparison of data obtained from Experiment II which used RT deadlines to control _Ss_' mean RT, with the data obtained under accuracy conditions indicated that the statistical properties of the stimulus information utilised by the decision process are not time invariant. In the RT deadline conditions the decision process utilised information obtained during the build-up of an image of the stimulus difference in short-term visual storage.

The concept of prior entry suggests that order discrimination involves a race between the processing of the information provided by two temporally asynchronous stimuli. The stimulus registered first is the "winner" and is reported as occurring first. The prior entry concept was originally employed to account for the somewhat puzzling effects of attention on order inversion errors (Von Tchisch, 1885). It forms the theoretical basis for the commonly employed "perceived-order" method in visual psychophysics. The predictions of these models have been examined in the literature. For example, Gibbon and Rutschmann, (1969) showed that the prior entry concept could account for the temporal order discrimination performance of one S, as the relative intensity of the two stimuli was varied.

For two equally intense stimuli S_1 and S_2 with S_1 preceding S_2 by δ msecs, the prior entry concept implies that order discrimination results from a race between the perception times for S_1 and S_2. Variability of order discrimination performance and the occurrence of order inversion errors are accommodated by assuming that the perception times for both stimuli are identically and independently distributed random variables. If the marginal mean RT for a fixed value of δ is equal to the mean winning time for a race between these random variables plus a constant mean motor time, then it can be shown that the marginal mean RT decreases monotonically as δ decreases (Appendix p 180). However the data obtained under accuracy conditions in Experiments I and III did not uphold this prediction. Rather, marginal mean RT increased as δ decreased.

The concept of prior entry provides a theoretical account of order errors in terms of the variability in perception times. The failure of the prediction of this model in terms of marginal mean RT suggests that the decision process plays an important role in order discrimination tasks. It has been shown that a tendency for marginal mean RT to increase as the ISI decreases results from particular assumptions concerning the decision process in order discrimination.

The random walk decision process offers a convenient means for measuring the relative contributions to performance of stimulus factors and response bias factors. Under accuracy conditions it was suggested that an estimate of the mean drift rate during the non-zero drift stage of the process was related to the information accrued from the stimulus pair. Estimates of the response thresholds provided an independent measure of response bias in an order discrimination task.

An evaluation of response bias in order discrimination tasks aids in the interpretation of the effects of various independent variables on performance. For example, the displacement of the psychometric function for order discrimination observed in Stone's (1926) experiment on the effect of attention on "prior entry" could be accounted for by a change in C, the mean starting point for the random walk. If greater attention to the click causes an increase in C and greater attention to the tap causes a decrease in C from an unbiased value of 0, then according to equation (21), the intercept of the psychometric function, $(A + C)/2A$, should change with the focus of attention. Attention to the click should displace the psychometric function upwards relative to that obtained when attention is focussed on the tap.

Sternberg and Knoll's (1973, p. 661) replotting of Stone's data indicates that the effect of attention is indeed a response bias effect. They noted a distinct change in the intercept of the psychometric function as the focus of attention was varied. The shape of the psychometric function was relatively unaltered by attention. This prediction would apply if the total distance between the response thresholds, 2A, was not altered with focussed attention. According to equation (21) the slope of the psychometric function, m/2A, would then be constant, provided that focussed attention does not affect the mean drift rate, m, generated by each stimulus alone.

The assumption that the S samples information from the stimulus display during an observation period is similar to the concept of the perceptual moment. Stroud (1955) hypothesized that temporal resolution is constrained by an upper limit to the rate at which order information can be extracted from successive inputs. The perceptual moment defines a time interval during which no order information can be extracted from inputs to the sensory systems. Stroud suggested that the perceptual moment operates independently of the times of arrival of input stimulation.

The results of Experiment III indicated that Ss might sample stimulus information for a short period of time. The finding that the presence of a gap in one of the stimuli 20 msec after the onset of the first stimulus of a pair had no effect on order discrimination performance suggests that an average observation interval of about 20 msec might suffice for adequate performance in an order discrimination task employing visual stimuli.

The non-stationarity assumption for the statistical properties of the information utilised in making an order decision allows for the effect of memory processes in the discrimination of order. Both the rejection of a prior-entry "race" model, and the transitory nature of the order information provided by the illumination of two lights in rapid succession suggest that the time course of the representation of information in short-term visual storage is reflected in order discrimination performance.

The representation of order discrimination performance under accuracy conditions in terms of a zero drift random walk process is reminiscent of Kinchla and Smyzer's (1967) diffusion model for perceptual memory. According to this model, the decay of information in short-term storage is represented by a linear increase in noise variance with time. In terms of the random walk model discussed in this thesis, such an increase in noise variance is equivalent to the effect of a random walk process with zero drift. In fact, the non-stationary random walk process is equivalent to a process in which the noise variance decreases as a result of a filtering process and then increases as the fidelity of the stored image decays. Hence the signal-to-noise ratio of the stored stimulus difference is assumed to vary over time. For example, for normally distributed increments to the random walk, a change in θ_1 over time can result from either a change in the mean step size, m, or the step size variance, s_1^2, since $\theta_1 = 2m/s_1^2$ in this case.

The pattern of response conditioned mean RT data indicated that the direction of the difference between mean RT for correct and incorrect responses depends on the value of the marginal mean RT. Under accuracy conditions, order discrimination performance was superior, marginal mean RT was long and incorrect responses were longer than correct responses. Under RT deadline conditions the opposite trend was noted. A decline in performance accuracy accompanied both a decreased marginal mean RT and relatively faster mean RTs for incorrect responses.

The pattern of RT results observed in the three experiments has been reported in previous work on discriminative RT. In the line length discrimination experiments of Henmon (1911) two of the S's who exhibited long marginal mean RTs also had longer mean RTs for incorrect responses. Kellogg (1931) measured RT in a brightness discrimination employing a range of differences in stimulus intensity. A similar pattern of RT results to that observed in Experiment I was obtained. Marginal mean RT decreased as the stimulus difference increased and mean RT for incorrect responses exceeded mean RT for correct responses for the same stimulus difference by an amount which increased with the stimulus difference. Perhaps, then, the tandem random walk model proposed in this thesis has applicability to a variety of discriminative RT tasks.

Sekuler (1965, 1966) investigated the relationship between correct and incorrect mean RTs in a backward masking task using visual stimuli. In each experiment, a test flash was followed 40 msec later by a 100 msec masking flash. The consistent finding was that the mean RT for a correct response was less than the mean RT for an incorrect response, a trend similar to that found in Experiments I and III.

A tendency for marginal mean RT to decrease as the stimulus difference increased was observed by Vickers, Nettelbeck and Willson(1972).

The task involved a discrimination of line lengths which were masked after various time intervals by extending both comparison lines until they were equal in length. Vickers *et al* observed both an increase in the probability of a correct response and a decrease in marginal mean RT as the delay of the masking stimulus was increased. These results are similar to those reported in this thesis for the effect of increasing the ISI in a temporal order discrimination task. The studies by Sekuler and Vickers *et al* which employed a backward masking procedure with an effect similar to that generated when both visual stimuli in an order discrimination task have been illuminated, revealed a pattern of RT results similar to that observed for the accuracy conditions of Experiment I.

The tendency for incorrect responses to be faster than correct responses under RT deadline conditions in Experiment II was similar to the trends noted by Link and Tindall (1971) for the effect of RT deadlines on comparative judgments of line length. In this study the mean RT for correct responses was greater than the mean RT for incorrect responses in a 460 msec deadline condition. This difference increased with an increase in the stimulus difference. A similar effect was observed in Experiment IIb reported in this thesis for each RT deadline condition.

The approximation of a non-stationary random walk decision process by a tandem random walk process offers an additional technique to that proposed by Link and Heath (1975) for resolving a problem with the original formulation of a random walk decision process for psychophysical tasks by Stone (1960). In Stone's model mean RTs for correct and incorrect responses for the same stimulus difference are predicted to be equal. In the Link and Heath (1975) model, differences in response conditioned mean RTs result from an asymmetry in the mgf of the step size for the random walk. In the tandem random walk model proposed in this thesis, these differences result from a non-stationary random walk decision process.

The tendency for mean RTs for incorrect responses to exceed mean RTs for correct responses results from a decrease in θ_1 over time. For normally distributed steps this decrease in θ_1 may result from an increase in the variance of a step with time.

The accumulator model assumes that information sampled from the stimulus display generates an increment in one or the other of two independent counters with pre-set response thresholds. When the count recorded by one counter reaches its threshold value the response associated with that counter is elicited. This model generates mean RTs for correct responses which are less than mean RTs for incorrect responses. Vickers, Caudrey and Willson(1971) have suggested that a form of the accumulator model with normally distributed increments can account for data obtained in a variety of discriminative RT tasks emphasising accuracy.

The representation of temporal order discrimination in terms of the general tandem random walk model provides a framework for examining the effects of both stimulus and response strategy variables. The results of two experiments using visual stimuli have shown that when response accuracy is stressed, the covariation between response time and response proportion measures can be accounted for by a simple linear relationship. This relationship does not depend on specific assumptions concerning the manner in which the visual information is represented. The results of a third experiment, which forced <u>S</u>'s to vary their marginal mean RT by imposing RT deadlines, indicated that the rate at which information is acquired from a stimulus varies as a function of time since stimulus onset.

The theoretical account of the data obtained in the experiments described in this thesis has implications for the interpretation of data obtained in other discriminative RT tasks in which the difficulty of the discrimination is varied by altering both the amount of stimulus information displayed and the time during which the information is available. In particular, the random walk decision model may provide an adequate account of data obtained from experiments on short-term sensory memory. It would provide an extension to the commonly employed signal detection procedures (Murdock, 1974) by elucidating the covariation between response time and response accuracy measures obtained in such tasks. Furthermore, the tandem random walk model enables the experimenter to determine the time course of the build-up and decay of the sensations evoked by a stimulus by observing changes in direction of the dominant component of a non-stationary random walk decision process driven by the transduced stimulus information.

REFERENCES

Boring, E. G. A history of experimental psychology. New York: Appleton, 1950.

Carterette, E. C., Friedman, M. P. and Cosmides, R. Reaction-time distributions in the detection of weak signals in noise. J. Acoust. Soc. Am., 1965, 38, 531-542.

Cattell, J. M. The influence of the intensity of the stimulus on the length of the reaction time. Brain, 1886, 8, 512-515.

Cox, D. R. and Miller, H. D. The theory of stochastic processes. London: Methuen, 1965.

Efron, R. An invariant characteristic of perceptual systems in the time domain. In: S. Kornblum (Ed.) Attention and performance IV. New York: Academic Press, 1973, 713-736.

Exner, S. Untersuchungen über die einfachsten psychische Prozesse. Pflügers Arch. ges. Physiol., 1875, II, IV, Abhandlung.

Ferguson, G. A. Statistical analysis in psychology and education. New York: McGraw-Hill, 1959.

Fitts, P. M. Cognitive aspects of information processing: III. Set for speed versus accuracy. J. exp. Psychol., 1966, 71, 843-857.

Gibbon, J. and Rutschmann, R. Temporal order judgement and reaction time. Science, 1969, 165, 413-415.

Green, D. M. and Luce, R. D. Speed accuracy trade off in auditory detection. In: S. Kornblum (Ed.) Attention and performance IV. New York: Academic Press, 1973, 547-569.

Haber, R. N. and Standing, L. G. Direct estimates of the apparent duration of a flash. Canad. J. Psychol., 1970, 24, 216-229.

Halliday, A. M. and Mingay, R. On the resolution of small time intervals and the effect of conduction delays on the judgement of simultaneity. Quart. J. Psychol., 1964, 16, 35-46.

Hansteen, R. W. Visual latency as a function of stimulus onset, offset and background luminance. J. Opt. Soc. Amer., 1971, 61, 1190-1195.

Hays, W. L. Statistics. New York: Holt, Rinehart & Winston, 1963.

Henmon, V. A. C. The relation of the time of a judgement to its accuracy. Psychol. Rev., 1911, 18, 186-201.

Herbart, J. F. Lehrbuch zur Psychologie. 1816.

Kellogg, W. N. Time of judgement in psychometric measures. Amer. J. Psychol., 1931, 43, 65-86.

Kinchla, R. A. and Smyzer, F. A diffusion model of perceptual memory. Perception and Psychophysics, 1967, 2, 219-229.

Kristofferson, A. B. Attention and psychophysical time. Acta Psychol., 1967, 27, 93-100.

Kuffler, S. W., FitzHugh, R. and Barlow, H. B. Maintained activity in the cat's retina in light and darkness. J. Gen. Physiol., 1957, 40, 683-702.

Külpe, O. Outlines of psychology. London: Allan & Unwin, 1893.

Laming, D. R. J. Information theory of choice reaction times. New York: Academic Press, 1968.

Lewis, J. H., Dunlap, W. P. and Matteson, H. H. Perceptual latency as a function of stimulus onset and offset and retinal location. Vision Res., 1972, 12, 1725-1731.

Link, S. W. Applying RT deadlines to discrimination reaction time. Psychon. Sci., 1971, 25, 355-358.

Link, S. W. The relative judgement theory for two choice response time. J. Math. Psychol., 1975, 12, 114-135.

Link, S. W. and Heath, R. A. A sequential theory of psychological discrimination. Psychometrika, 1975, 40, 77-105.

Link, S. W. and Tindall, A. D. Speed versus accuracy in comparative judgements of line length. Perception & Psychophysics, 1971, 9, 284-288.

Murdock, B. B. Jr. Human memory: theory and data. New York: Wiley, 1974.

Pachella, R. G. and Pew, R. W. Speed accuracy tradeoff in reaction time: effect of discrete criterion times. J. exp. Psychol., 1968, 76, 19-24.

Pike, A. R. The latencies of correct and incorrect responses in discrimination and detection tasks: Their interpretation in terms of a model based on simple counting. Perception & Psychophysics, 1971, 9, 455-460.

Pike, A. R. and Ryder, P. Response latencies in the yes/no detection task: An assessment of two basic models. Perception and Psychophysics, 1973, 13, 224-232.

Poffenberger, A. T. Reaction time to retinal stimulation, with special reference to the time lost in conduction through nerve centers. Arch. Psychol., 1912, 3, 1-73.

Roufs, J. A. J. Perception lag as a function of stimulus luminance. Vision Res., 1963, 3, 81-91.

Rutschmann, R. Perception of temporal order and relative visual latency. Science, 1966, 152, 1099-1101.

Rutschmann, R. Visual perception of temporal order. In: S. Kornblum (Ed.) Attention and performance IV. New York: Academic Press, 1973, 687-701.

Sanford, A. J. Effects of changes in the intensity of white noise on simultaneity judgements and simple reaction time. Quart. J. exp. Psychol., 1971, 23, 296-303.

Sanford, E. C. The personal equation. Amer. J. Psychol., 1888, 2, 3-38, 271-298, 403-430.

Sekuler, R. W. Signal detection, choice response times, and visual backward masking. Canad. J. Psychol., 1965, 19, 118-132.

Sekuler, R. W. Choice times and detection with backward visual masking. Canad. J. Psychol., 1966, 20, 34-42.

Sekuler, R. W., Tynan, P. and Levinson, E. Visual temporal order: a new illusion. Science, 1973, 180, 210-212.

Shallice, T. Temporal summation and absolute brightness thresholds. Brit. J. Math. Stat. Psychol., 1967, 20, 129-162.

Sperling, G. The information available in brief visual presentations. Psychol. Monogr., 1960, 74, 1-29.

Sternberg, S. and Knoll, R. L. The perception of temporal order: fundamental issues and a general model. In S. Kornblum (Ed.) Attention and performance IV. New York: Academic Press, 1973, 629-685.

Stone, M. Models for choice reaction time. Psychometrika, 1960, 25, 251-260.

Stone, S. A. Prior entry in the auditory-tactual complication. Amer. J. Psychol., 1926, 37, 284-287.

Stroud, J. M. The fine structure of psychological time. In Quastler, H. (Ed.) Information theory in psychology. Glencoe, Ill: Free Press, 1955, 174-205.

Swift, E. J. Disturbance of the attention during simple mental processes. Amer. J. Psychol., 1892, 5, 1-19.

Titchener, E. B. An outline of psychology. New York: McMillan, 1899.

von Tschisch, W. Ueber die Zeitverhältnisse der Apperception. Phil. Studien, 1885, 2, 603.

Vanthoor, F. L. J. and Eijkman, E. G. J. Time course of the iconic memory signal. Acta Psychol., 1973, 37, 79-85.

Vickers, D., Caudrey, D., and Willson, R. J. Discriminating between the frequency of occurrence of two alternative events. Acta Psychol., 1971, 35, 151-172.

Vickers, D., Nettelbeck, T. and Willson, R. J. Perceptual indices of performance: the measurement of 'inspection time' and 'noise' in the visual system. Perception, 1972, 1, 263-295.

Wald, A. Sequential analysis. New York: Wiley, 1947.

Walsh, T. Visual onset and offset latencies. Quart. J. exp. Psychol., 1973, 25, 154-162.

Woodsworth, R. S. and Schlosberg, H. Experimental psychology. New York: Holt, Rinehart and Winston, 1954.

Wundt, W. Grundzüge der physiologischen Psychologie. Leibzig: Engelmann, 1893.

Yellott, J. I. Correction for guessing and the speed-accuracy trade-off in choice reaction time. J. Math. Psychol., 1971, 8, 159-199.

APPENDIX I: Theoretical development of the models.

APPENDIX

Wald's Identity for Random Walks with Starting Point Variability

Let X_i represent the i^{th} value in a sequence of n independent observations of a stationary unidimensional real-valued random variable X. Let Z_o be the initial value of the sum

$$Z_n = Z_o + \sum_{i=1}^{n} X_i \qquad \text{(A1)}$$

Then Z_n is defined as a <u>random walk process</u> along the real line. Let f(x) and $f^*(\theta)$ be the pdf and mgf respectively for the stationary random variable X, and $f_o(x)$ and $f_o^*(\theta)$ be the pdf and mgf respectively for the initial value Z_o. The mgfs $f^*(\theta)$ and $f_o^*(\theta)$ are defined by

$$f^*(\theta) = E(e^{-\theta Z})$$

and

$$f_o^*(\theta) = E(e^{-\theta Z_o}), \text{ respectively.}$$

When absorbing barriers for the bounded random walk process are set at A and -A along the real line, theorem A1 applies.

<u>Theorem A1 (Wald's Identity)</u>

Let $Z_n = Z_o + X_1 + X_2 + \cdots + X_n$ be the sum of a random variable Z_o with pdf $f_o(x)$ and mgf $f_o^*(\theta)$, and a sequence of n identically and independently distributed random variables X_i, i = 1, n, with stationary pdfs f(x) and identical mgfs $f^*(\theta)$. Then, for $A > 0$ satisfying $\Pr(|Z_o| > A) = 0$ and for $f^*(\theta) = 1$,

$$E\left\{\exp(-\theta Z_N)[f^*(\theta)]^{-N}\right\} = f_o^*(\theta) \qquad \text{(A2)}$$

where n=N is the step upon which absorption occurs.

Proof:

The proof follows that given in Cox and Miller (1965, Ch. 2) for the derivation of Wald's Identity for a fixed starting value.

Let Λ be a real positive number and define

$$f_n(x)\,dx = \Pr(-\Lambda < Z_o, Z_1, Z_2, \ldots, Z_{n-1} < \Lambda,\ x < Z_n < x + dx)$$

$$n = 1, 2, \ldots \quad ; \quad -\infty < x < \infty.$$

Assume that $Z_{n-1} = y$ so that if $Z_n = x$, the increment $X_n = x-y$. A recurrence relation for the $f_n(x)$ can be derived as follows:

$$\Pr(x < Z_n < x + dx \mid Z_{n-1} = y) = \Pr(x-y < X_n < x-y + dx)$$
$$= f(x-y)dx$$

Hence

$$f_n(x)dx = \int_{-\Lambda}^{\Lambda} \Pr(x < Z_n < x + dx \mid Z_{n-1} = y) f_{n-1}(y)dy$$
$$= \left\{\int_{-\Lambda}^{\Lambda} f(x-y)f_{n-1}(y)dy\right\} dx$$
$$f_n(x) = \int_{-\Lambda}^{\Lambda} f(x-y)f_{n-1}(y)dy, \quad n = 1, 2, \ldots \qquad \text{(A3)}$$

Taking Laplace transforms of both sides of equation (A3) we obtain

$$\int_{-\infty}^{\infty} e^{-\theta x} f_n(x)dx = \int_{-\infty}^{\infty} e^{-\theta x}\left\{\int_{-\Lambda}^{\Lambda} f_{n-1}(y)\ f(x-y)dy\right\} dx$$
$$= \int_{-\Lambda}^{\Lambda} e^{-\theta y} f_{n-1}(y)\left\{\int_{-\infty}^{\infty} e^{-\theta(x-y)} f(x-y)dx\right\} dy$$
$$= f^*(\theta)\int_{-\Lambda}^{\Lambda} e^{-\theta y} f_{n-1}(y)dy.$$
$$= f^*(\theta)\ \phi_{n-1}(\theta) \qquad \text{(A4)}$$

where $\phi_n(\theta) = \int_{-\Lambda}^{\Lambda} e^{-\theta y} f_n(y)dy.$

Equation (A4) is a restatement of the fact that the sum Z_n consists of independently and identically distributed increments X_i provided that absorption has not occurred at the (n-1)th step so that $-\Lambda < Z_{n-1} < \Lambda$.

We now use the definition of equation (65) derived by Cox and Miller (1965, p. 53) for the mgf with respect to n and with respect to x over the absorbing states,

$$E(e^{-\theta Z_N}s^N) = \sum_{n=1}^{\infty} s^n \left\{ \int_{-\infty}^{-\Lambda} e^{-\theta x} f_n(x)\,dx + \int_{\Lambda}^{\infty} e^{-\theta x} f_n(x)\,dx \right\}$$

$$= \sum_{n=1}^{\infty} s^n \left\{ \int_{-\infty}^{\infty} e^{-\theta x} f_n(x)\,dx - \phi_n(\theta) \right\} \qquad (A5)$$

Using equations (A4) and (A5),

$$E(e^{-\theta Z_N}s^N) = \sum_{n=1}^{\infty} s^n f^*(\theta)\ \phi_{n-1}(\theta) - \sum_{n=1}^{\infty} s^n \phi_n(\theta)$$

$$= sf^*(\theta) \sum_{n=0}^{\infty} s^n \phi_n(\theta) - \sum_{n=0}^{\infty} s^n \phi_n(\theta) + \phi_0(\theta)$$

$$= \phi_0(\theta) - [1 - sf^*(\theta)] \sum_{n=0}^{\infty} s^n \phi_n(\theta)$$

By assumption $\int_{-\Lambda}^{\Lambda} f_0(x)dx = 1$ so that $\phi_0(\theta) = f_0^*(\theta)$.

Hence

$$E(e^{-\theta Z_N}s^N) = f_0^*(\theta) - [1 - sf^*(\theta)] \sum_{n=0}^{\infty} s^n \phi_n(\theta)$$

When $sf^*(\theta) = 1$, we obtain

$$E(e^{-\theta Z_N}[f^*(\theta)]^{-N}) = f_0^*(\theta)$$

The method employed to obtain expressions for both the probabilities of absorption and the moments of the number of steps to absorption conditioned upon absorption at Λ and $-\Lambda$ is similar to that described in Cox and Miller (1965, p. 56), and in Link and Heath (1975).

Let P_Λ be the probability of absorption at Λ. Wald (1947, p. 157) has shown that in the two absorbing barrier case, eventual absorption at one of the barriers occurs with probability one. Hence

$$P_\Lambda + P_{-\Lambda} = 1$$

where $P_{-\Lambda}$ is the probability of absorption at $-\Lambda$. Let $E_\Lambda(.)$ and $E_{-\Lambda}(.)$

denote expectations taken conditional upon absorption at A and $-A$ respectively. Suppose that the overshoot of Z_N across the absorbing barriers at A and $-A$ when absorption occurs is negligible, so that $Z_N = A$ when $Z_N \geq A$ and $Z_N = -A$ when $Z_N \leq -A$. Then Wald's Identity (Equation A2) can be rewritten as

$$P_A e^{-\theta A} E_A\{[f^*(\theta)]^{-N}\} + P_{-A} e^{\theta A} E_{-A}\{[f^*(\theta)]^{-N}\} = f_o^*(\theta) \qquad \text{(A8)}$$

(i) Absorption probabilities

Let $f^*(\theta_1) = 1$ for $\theta_1 \neq 0$. Then equation (A8) becomes

$$P_A e^{-\theta_1 A} + P_{-A} e^{\theta_1 A} = f_o^*(\theta_1)$$

Since $P_A + P_{-A} = 1$, we obtain

$$P_A = \frac{e^{\theta_1 A} - f_o^*(\theta_1)}{e^{\theta_1 A} - e^{-\theta_1 A}} \qquad \text{(A9)}$$

If $f_o(x)$ is a truncated normal pdf with parameters C and s_o^2, defined on $[-A, A]$ then

$$f_o^*(\theta) = \left\{\frac{\Phi_A(\theta) - \Phi_{-A}(\theta)}{\Phi_A(0) - \Phi_{-A}(0)}\right\} \exp[-C\theta + \tfrac{1}{2}s_o^2\theta^2]$$

where

$$\Phi_p(\theta) = \int_{-\infty}^{p} \frac{1}{\sqrt{2\pi}s_o} \exp - \tfrac{1}{2}\left[\frac{x + (\theta s_o^2 - C)}{s_o}\right]^2 dx.$$

When A is sufficiently large such that $\Phi_A(\theta) \doteq 1$ and $\Phi_{-A}(\theta) \doteq 0$, there exist values of θ such that

$$f_o^*(\theta) \doteq \exp[-C\theta + \tfrac{1}{2}s_o^2\theta^2]$$

Substituting this expression for $f_o^*(\theta)$ into equation (A9) yields

$$P_A = \frac{e^{\theta_1(A+C)} - e^{\frac{1}{2}s_o^2\theta_1^2}}{e^{\theta_1(A+C)} - e^{-\theta_1(A-C)}} \quad , \quad -A \leq C \leq A \qquad \text{(A10)}$$

Suppose $\theta_1 > 0$. Then for A and C fixed, P_A decreases as s_o^2 increases. When $\theta_1 < 0$,

$$P_A = \frac{e^{\frac{1}{2}s_o^2\theta_1^2} - e^{-\theta_1(A+C)}}{e^{\theta_1(A-C)} - e^{-\theta_1(A+C)}} \quad , \quad -A \leq C \leq A$$

Hence P_A increases as s_o^2 increases for A and C fixed. So an increase in the starting point variability, s_o^2, for the random walk process increases the error rate. This result was also proven by Laming (1968, p. 141).

In the limit as $\theta_1 \to 0$, the probability of absorption at A becomes

$$P_A = \frac{A+C}{2A} \quad , \qquad -A \leq C \leq A \qquad \text{(A11)}$$

a result that does not depend on s_o^2.

(ii) <u>Mean number of steps to absorption.</u>

$E_A[\{f^*(\theta)\}^{-N}]$ and $E_{-A}[\{f^*(\theta)\}^{-N}]$ are the pgfs for the number of steps to absorption at A and -A respectively. Successive differentiation of these expressions and setting the value of $s = [f^*(\theta)]^{-1} = 1$ yields the moments of the conditional distributions of the number of steps to absorption of A and $-A$. The marginal moments can also be obtained.

Following Cox and Miller (1965, p. 57) let $r_1(s)$ and $r_2(s)$ be two real roots of the equation $s = [f^*(\theta)]^{-1}$, for $1 \leq s \leq [\min(f^*(\theta))]^{-1}$. Provided that $s > \min[f^*(\theta)]$, such roots exist by virtue of the convexity property of the mgf (Cox and Miller, 1965, p. 48).

Substituting $\theta = r_i(s)$, $i = 1,2$, successively into equation (A8) we obtain the set of simultaneous equations:

$$P_A e^{-r_i(s)A} E_A(s^N) + P_{-A} e^{r_i(s)A} E_{-A}(s^N) = u_i(s), \quad i = 1,2,$$

where

$$u_i(s) = f_o^*(r_i(s)), \quad i = 1,2.$$

Solving for $E_A(s^N)$ and $E_{-A}(s^N)$, we obtain

$$E_A(s^N) = \frac{Y}{P_A H} \quad ; \quad E_{-A}(s^N) = \frac{W}{P_{-A} H}$$

where $Y = u_1(s)e^{r_2(s)A} - u_2(s)e^{r_1(s)A}$,

$$W = u_2(s)e^{-r_1(s)A} - u_1(s)e^{-r_2(s)A},$$

and $H = e^{(r_2(s) - r_1(s))A} - e^{-(r_2(s) - r_1(s))A}$

Setting $s = e^{-\theta}$, we obtain the cumulant generating functions (cgfs)

$$K_A(\theta) = \ln\left(\frac{Y}{P_A H}\right) \quad \text{and} \quad K_{-A}(\theta) = \ln\left(\frac{W}{P_{-A} H}\right) \qquad \text{(A12)}$$

corresponding to $E_A(s^N)$ and $E_{-A}(s^N)$ respectively. Although we will only derive expressions for the first moments of the distributions of the number of steps to absorption, the use of the cgf greatly facilitates calculations of the higher moments.

Differentiating equations (A12) with respect to θ, we obtain

$$\frac{\partial K_A(\theta)}{\partial\theta} = \frac{Y'}{Y} - \frac{H'}{H}$$

and

$$\frac{\partial K_{-A}(\theta)}{\partial\theta} = \frac{W'}{W} - \frac{H'}{H} .$$

Let $g_{ij}(A) = u_i(s)e^{r_j(s)A}$, $i,j = 1,2$.

Hence $Y = g_{12}(A) - g_{21}(A)$ and $W = g_{21}(-A) - g_{12}(-A)$.

Now

$$\frac{dg_{ij}(A)}{d\theta} = \frac{du_i(s)}{ds}\frac{ds}{d\theta}e^{r_j(s)A} + u_i(s)A\frac{dr_j(s)}{ds}\frac{ds}{d\theta}e^{r_j(s)A}$$

$$= -e^{-\theta}\left\{\frac{du_i(s)}{ds} + A\,u_i(s)\frac{dr_j(s)}{ds}\right\}e^{r_j(s)A}$$

Now $\dfrac{du_i(s)}{ds} = \dfrac{dr_i(s)}{ds}\dfrac{d\,f_o^*(r_i(s))}{dr_i(s)}$

When θ = 0, s = 1, so that

$$u_i' = \frac{du_i(s)}{ds}\Big|_{s=1} = \frac{dr_i(s)}{ds}\Big|_{s=1}\frac{df_o^*(r_i(s))}{dr_i(s)}\Big|_{s=1}$$

$$= \frac{dr_i(s)}{ds}\Big|_{s=1} n_i^{(1)} = r_i' n_i^{(1)}$$

where $\quad m_i^{(i)} = \left. \dfrac{df_o^*(r_i(s))}{dr_i(s)} \right|_{s=1}$

and $\quad r_i{}' = \left. \dfrac{dr_i(s)}{ds} \right|_{s=1}$

Differentiating $sf^*(\theta) = sf^*(r_i(s)) = 1$ with respect to s we obtain

$$f^*(r_i(s)) + s \frac{df^*(r_i(s))}{dr_i(s)} \frac{dr_i(s)}{ds} = 0$$

$$\frac{dr_i(s)}{ds} = - \frac{f^*(r_i(s))}{s \left[\dfrac{df^*(r_i(s))}{dr_i(s)} \right]}$$

$$r_1{}' = \frac{1}{m} \quad \text{and} \quad r_2{}' = \frac{-1}{\gamma m}$$

where $\quad \left. \dfrac{df^*(r_1(s))}{dr_1(s)} \right|_{s=1} = -m$

and $\quad \left. \dfrac{df^*(r_2(s))}{dr_2(s)} \right|_{s=1} = \gamma m.$

Since $f^*(r_i(1)) = 1$ by the definition of $r_i(s)$, $i = 1, 2$, we let $r_1(1) = 0$ and $r_2(1) = \theta_1$ be the roots of $f^*(\theta) = 1$. γ is the absolute value of the ratio of the slopes of the mgf at the roots of $f^*(\theta) = 1$. Link and

Heath (1975) discuss the importance of this parameter in some detail.

From the expressions for W, we obtain

$$\frac{W'}{W} = \frac{-[u_2'(s) - Au_2(s)r_1'(s)]e^{-r_1(s)A} + [u_1'(s) - Au_1(s)r_2'(s)]e^{-r_2(s)A}}{u_2(s)e^{-r_1(s)A} - u_1(s)e^{-r_2(s)A}}$$

We calculate the value of this expression when $s = 1$, using the expressions

$$u_1' \equiv u_1'(s)\Big|_{s=1} = r_1'n_1^{(1)} = n_1^{(1)}/m$$

$$u_2' \equiv u_2'(s)\Big|_{s=1} = r_2'n_1^{(2)} = \frac{-n_1^{(2)}}{\gamma m}$$

$$u_1 \equiv u_1(s)\Big|_{s=1} = f_0^*(0) = 1$$

$$u_2 \equiv u_2(s)\Big|_{s=1} = f_0^*(\theta_1)$$

$$r_1(s)\Big|_{s=1} = 0\ , \qquad r_2(s)\Big|_{s=1} = \theta_1\ .$$

$$\frac{W'}{W}\Big|_{s=1} = \frac{[n_1^{(2)} + A\gamma f_0^*(\theta_1)] + [n_1^{(1)}\gamma + A]e^{-\theta_1 A}}{\gamma[Af_0^*(0) - e^{-\theta_1 A}]}$$

Similarly,

$$\left.\frac{Y'}{Y}\right|_{s=1} = - \frac{(n_1^{(1)}\gamma - A)e^{\theta_1 A} + [n_1^{(2)} - A\gamma f_0^*(\theta_1)]}{\gamma m[e^{\theta_1 A} - f_0^*(\theta_1)]}$$

Let $h(A) = e^{[r_1(s) - r_2(s)]A}$

Then $h'(A) = A[r_1'(s) - r_2'(s)]h(A)$

Since $H = h(-A) - h(A)$, then

$$\frac{H'}{H} = \frac{h'(-A) - h'(A)}{h(-A) - h(A)}$$

Hence

$$\left.\frac{H'}{H}\right|_{s=1} = \frac{-A\left(\frac{1}{m} + \frac{1}{\gamma m}\right) e^{\theta_1 A} - A\left(\frac{1}{m} + \frac{1}{\gamma m}\right) e^{-\theta_1 A}}{e^{\theta_1 A} - e^{-\theta_1 A}}$$

$$= -\frac{A}{m}\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{e^{2\theta_1 A} + 1}{e^{2\theta_1 A} - 1}\right)$$

The mean number of steps to absorption conditioned upon absorption at A and $-A$ are given by $k_A = \left.\frac{\partial K_A}{\partial \theta}\right|_{\theta=0}$ and $k_{-A} = \left.\frac{\partial K_{-A}}{\partial \theta}\right|_{\theta=0}$ respectively.

Since $s = e^{-\theta}$, $\theta = 0$ implies that $s = 1$. Hence

$$k_A = \frac{Y'}{Y}\bigg|_{s=1} - \frac{H'}{H}\bigg|_{s=1}$$

$$= \frac{A}{m}\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{e^{2\theta_1 A}+1}{e^{2\theta_1 A}-1}\right) - \frac{(A-n_1^{(1)}\gamma)e^{\theta_1 A} + [A\gamma f_o^*(\theta_1) - n_1^{(2)}]}{\gamma m[e^{\theta_1 A} - f_o^*(\theta_1)]} \qquad \text{(A13)}$$

and $k_{-A} = \frac{W'}{W}\bigg|_{s=1} - \frac{H'}{H}\bigg|_{s=1}$

$$= \frac{A}{m}\left(\frac{1+\gamma}{\gamma}\right)\left(\frac{e^{2\theta_1 A}+1}{e^{2\theta_1 A}-1}\right) - \frac{[A\gamma f_o^*(\theta_1) + n_1^{(2)}] + (A + n_1^{(1)}\gamma)e^{-\theta_1 A}}{\gamma m[f_o^*(\theta_1) - e^{-\theta_1 A}]} \qquad \text{(A14)}.$$

<u>Theorem A2</u>

The marginal mean number of steps to absorption at A and $-A$ for a random walk process with independent and identically distributed steps and starting point variability is given by

$$D = \frac{[A - E(Z_o)]P_A - [A + E(Z_o)]P_{-A}}{m} \qquad \text{(A15)}$$

where m is the mean step size and $E(Z_o)$ is the expected starting point.

<u>Proof</u>

From equation (A9)

$$P_A = \frac{e^{\theta_1 A}(e^{\theta_1 A} - f_o^*(\theta_1))}{e^{2\theta_1 A} - 1}, \qquad P_{-A} = \frac{f_o^*(\theta_1)e^{\theta_1 A} - 1}{e^{2\theta_1 A} - 1}$$

$$P_\Lambda k_\Lambda = \frac{\Lambda}{m}\left(\frac{1+\gamma}{\gamma}\right)\left[\frac{e^{\theta_1\Lambda}(e^{\theta_1\Lambda}-f_o^*(\theta_1))(e^{2\theta_1\Lambda}+1)}{(e^{2\theta_1\Lambda}-1)^2}\right] - \left(\frac{e^{\theta_1\Lambda}}{e^{2\theta_1\Lambda}-1}\right)\left[\frac{(\Lambda-n_1^{(1)}\gamma)e^{\theta_1\Lambda}+[\Lambda\gamma f_o^*(\theta_1)-n_1^{(2)}]}{\gamma m}\right]$$

$$= \frac{\Lambda(1+\gamma)e^{\theta_1\Lambda}(e^{\theta_1\Lambda}-f_o^*(\theta_1))(e^{2\theta_1\Lambda}+1) - e^{\theta_1\Lambda}(e^{2\theta_1\Lambda}-1)[(\Lambda-n_1^{(1)}\gamma)e^{\theta_1\Lambda}+[\Lambda\gamma f_o^*(\theta_1)-n_1^{(2)}]]}{\gamma m(e^{2\theta_1\Lambda}-1)^2}$$

Similarly,

$$P_{-\Lambda}k_{-\Lambda} = \frac{\Lambda(1+\gamma)(f_o^*(\theta_1)e^{\theta_1\Lambda}-1)(e^{2\theta_1\Lambda}+1) - \left\{[\Lambda\gamma f_o^*(\theta_1)+n_1^{(2)}]e^{\theta_1\Lambda}+(\Lambda+n_1^{(1)}\gamma)\right\}(e^{2\theta_1\Lambda}-1)}{\gamma m(e^{2\theta_1\Lambda}-1)^2}$$

$$D = P_\Lambda k_\Lambda + P_{-\Lambda}k_{-\Lambda}$$

$$= \frac{1}{\gamma m}\left\{\Lambda(1+\gamma)\left(\frac{e^{2\theta_1\Lambda}+1}{e^{2\theta_1\Lambda}-1}\right) - \frac{[(\Lambda-n_1^{(1)}\gamma)e^{2\theta_1\Lambda} + 2\Lambda\gamma f_o^*(\theta_1)e^{\theta_1\Lambda} + \Lambda+n_1^{(1)}\gamma]}{e^{2\theta_1\Lambda}-1}\right\}$$

$$= \frac{1}{\gamma m(e^{2\theta_1\Lambda}-1)}\{\gamma(\Lambda+n_1^{(1)})e^{2\theta_1\Lambda} - 2\Lambda\gamma f_o^*(\theta_1)e^{\theta_1\Lambda} + \gamma(\Lambda-n_1^{(1)})\}$$

$$= \frac{1}{m}\left\{\frac{e^{\theta_1\Lambda}[(\Lambda+n_1^{(1)})e^{\theta_1\Lambda} - \Lambda f_o^*(\theta)]}{e^{2\theta_1\Lambda}-1} - \frac{(\Lambda f_o^* e^{\theta_1\Lambda} - \Lambda + n_1^{(1)})}{e^{2\theta_1\Lambda}-1}\right\}$$

$$= \frac{1}{m}\{\Lambda P_\Lambda - \Lambda P_{-\Lambda} + n_1^{(1)}\}$$

But $n_1^{(1)} = -E(Z_0)$ so that

$$D = \frac{[A - E(Z_0)]P_A - [A + E(Z_0)]P_{-A}}{m}$$

The result of Theorem A2 is equivalent to equation (86) of Cox and Miller (1965, p. 58). It expresses the simple relationship that the marginal mean number of steps to absorption is equal to the expected displacement, $[A - E(Z_0)]P_A - [A + E(Z_0)]P_{-A}$, divided by the average displacement per unit time, m.

Rewriting equation (A15) as

$$D = \frac{2AP_A - [A + E(Z_0)]}{m} \qquad \text{(A16)}$$

we find that for A, $E(Z_0)$ and m fixed, the relationships resulting from changes in D and P_A to D′ and P_A' respectively are given by

$$P_A \gtreqless P_A' \Rightarrow D \gtreqless D' \quad \text{if } \theta_1 > 0$$

and

$$P_A \gtreqless P_A' \Rightarrow D \lesseqgtr D' \quad \text{if } \theta_1 < 0 \qquad \text{(A17)}$$

where $D' = \frac{2AP_A' - [A + E(Z_0)]}{m}$.

For $\theta_1 > 0$ and $f_0(x)$ normally distributed we discovered that as s_0^2 increases, P_A decreases. Hence (A17) implies a decrease in D, the marginal mean number of steps to absorption as s_0^2 increases.

For $\theta_1 < 0$ (and $m < 0$), P_A increases as s_0^2 increases so that D decreases as s_0^2 increases. Hence an increase in starting point variability causes a decrease in the marginal mean number of steps to absorption.

(iii) Difference in Mean Number of Steps to Absorption Conditioned Upon Absorbing Barrier.

From equations (A13) and (A14),

$$k_\Lambda - k_{-\Lambda} = \frac{[\Lambda\gamma f_0^*(\theta_1)+n_1^{(2)}] + [\Lambda+n_1^{(1)}\gamma]e^{-\theta_1\Lambda}}{\gamma m[f_0^*(\theta_1) - e^{-\theta_1\Lambda}]} - \frac{(\Lambda-n_1^{(1)}\gamma)e^{\theta_1\Lambda}+[\Lambda\gamma f_0^*(\theta_1)-n_1^{(2)}]}{\gamma m[e^{\theta_1\Lambda} - f_0^*(\theta_1)]} \qquad \text{(A18)}$$

Case 1 No Starting Point Variability.

In this case $f_0^*(\theta) = e^{-\theta C}$ for a starting point at C defined by a Dirac delta function at C.

Now

$$n_1^{(1)} = \frac{d}{d\theta} f_0^*(\theta)\Big|_{\theta=0} = -C$$

$$n_1^{(2)} = \frac{d}{d\theta} f_0^*(\theta)\Big|_{\theta=\theta_1} = -Ce^{-\theta_1 C}$$

$$k_\Lambda - k_{-\Lambda} = \frac{(\Lambda\gamma-C)e^{\theta_1(\Lambda-C)} + (\Lambda-C\gamma)}{\gamma m(e^{\theta_1(\Lambda-C)} - 1)} - \frac{(\Lambda+C\gamma)e^{\theta_1(\Lambda+C)} + (\Lambda\gamma+C)}{\gamma m(e^{\theta_1(\Lambda+C)} - 1)}$$

When C = 0

$$k_\Lambda - k_{-\Lambda} = \frac{\Lambda(\gamma e^{\theta_1\Lambda} + 1)}{\gamma m(e^{\theta_1\Lambda} - 1)} - \frac{\Lambda(e^{\theta_1\Lambda} + \gamma)}{\gamma m(e^{\theta_1\Lambda} - 1)}$$

$$= \frac{\Lambda}{m}\left(\frac{\gamma-1}{\gamma}\right)$$

Hence

$$k_\Lambda \gtreqless k_{-\Lambda} \quad \text{iff} \quad \gamma \gtreqless 1, \quad m > 0$$

$$k_\Lambda \gtreqless k_{-\Lambda} \quad \text{iff} \quad \gamma \lesseqgtr 1, \quad m < 0.$$

These relationships were derived in Link and Heath (1975). They imply that the direction of the difference in the conditional mean number of steps to absorption at Λ and $-\Lambda$ for a fixed starting point at 0 depends on the value of the asymmetry parameter γ. In particular when $\gamma = 1$, $k_\Lambda = k_{-\Lambda}$ irrespective of the value of m.

<u>Case 2</u> <u>Starting Point Variability and Normally Distributed Step Sizes to the Random Walk</u>

Let $f(x)$ be a normal pdf with mean m and variance s^2 and let $f_o(x)$ be a truncated normal pdf with parameters m_o and s_o^2, defined on the interval $[-\Lambda, \Lambda]$. Then the corresponding mgfs are

$$f^*(\theta) = \exp\,(-m\theta + \tfrac{1}{2}s^2\theta^2)$$

and
$$f_o^*(\theta) = \phi(\theta;\Lambda)\,\exp\,(-m_o\theta + \tfrac{1}{2}s_o^2\theta^2),\ \text{respectively,}$$

where $\phi(\theta;\Lambda)$ is a function of θ and Λ (see p. 165).

If $\Lambda > 0$ is sufficiently large then there exists a range of values of θ for which $\phi(\theta;\Lambda) \doteq 1$. Hence

$$f_o^*(\theta) \doteq \exp\,(-m_o\theta + \tfrac{1}{2}s_o^2\theta^2)$$

When
$$f^*(\theta_1) = 1,\ \theta_1 = \frac{2m}{s^2}$$

and when
$$f_o^*(\theta_o) = 1,\ \theta_o = \frac{2m_o}{s_o^2}$$

When
$$f^*(\theta) = 1/t,$$

$$r_1(t) = \frac{m + (m^2 - 2s^2 \ln t)^{1/2}}{s^2}$$

and
$$r_1' = \left.\frac{dr_1(t)}{dt}\right|_{t=1} = -\frac{1}{m}$$

Similarly $r_2' \equiv \left.\frac{dr_2(t)}{dt}\right|_{t=1} = \frac{-1}{m}$

Hence for a normal pdf, $\gamma = 1$ owing to the symmetry of the mgf about its minimum value.

$$n_1^{(1)} = \left.\frac{df_o^*(\theta)}{d\theta}\right|_{\theta=0} = -m_o$$

$$n_1^{(2)} = \left.\frac{df_o^*(\theta)}{d\theta}\right|_{\theta=\theta_1} = (-m_o + s_o^2\theta_1)\exp(-m_o\theta_1 + \tfrac{1}{2}s_o^2\theta_1^2)$$

$$= (K + \frac{s_o^2\theta_1}{2})e^{K\theta_1}$$

where $\quad K = \frac{s_o^2}{2}(\theta_1 - \theta_o)$.

Setting $\gamma = 1$ in equation (A18) and substituting for $n_1^{(1)}$, $n_1^{(2)}$ and $f_o^*(\theta_1)$ we obtain

$$k_A - k_{-A} = \left(\frac{2AK\theta_1}{m}\right)\left\{\frac{\left(\frac{\sinh A\theta_1}{A\theta_1}\right) - \left(\frac{\sinh K\theta_1}{K\theta_1}\right)}{\cosh A\theta_1 - \cosh K\theta_1}\right\} \qquad \text{(A20)}$$

Now $\frac{d}{dA}\cosh A\theta_1 = \theta_1 \sinh A\theta_1 > 0$ if $\theta_1 > 0$

and $\frac{d}{dA}\cosh A\theta_1 > 0$ if $\theta_1 < 0$.

Hence for $|A| > |K|$, $\theta_1 > 0$

$\cosh A\theta_1 > \cosh K\theta_1 > 0$.

Similarly for $|A| > |K|$, $\theta_1 < 0$,

$$\cosh A\theta_1 > \cosh K\theta_1 > 0.$$

Since $\frac{\sinh x}{x} = 1 + \frac{x^2}{3!} + \frac{x^4}{5!} + \cdots$

$$\frac{d}{dx}\left(\frac{\sinh x}{x}\right) = \frac{2x}{3!} + \frac{4x^3}{5!} + \cdots$$

$$> 0 \text{ if } x > 0$$

$$< 0 \text{ if } x < 0.$$

For $|A| > |K|$, $\theta_1 > 0$

$$\frac{\sinh A\theta_1}{A\theta_1} > \frac{\sinh K\theta_1}{K\theta_1}$$

and for $|A| > |K|$, $\theta_1 < 0$

$$\frac{\sinh A\theta_1}{A\theta_1} < \frac{\sinh K\theta_1}{K\theta_1}$$

When $\theta_1 > 0$,

$k_A - k_{-A} > 0$ if $K > 0$

$k_A - k_{-A} < 0$ if $K < 0$.

When $\theta_1 < 0$,

$k_A - k_{-A} < 0$ if $K > 0$

$k_A - k_{-A} > 0$ if $K < 0$.

Hence the sign of the difference in mean number of steps to absorption conditioned upon absorption barrier depends on both the sign of θ_1 and on the sign of $\theta_1 - \theta_0$.

In the case of starting point variability with a constant mean starting point at 0, $\theta_0 = 0$ and $\theta_1 > \theta_0$ if $\theta_1 > 0$. Hence $K > 0$ and $k_A > k_{-A}$, a result obtained by Laming (1968, p. 142). If θ_1 is small and fixed, then for $\theta_0 > \theta_1 > 0$, $k_A < k_{-A}$. Hence the result in equation (A20) generates all orders of k_A and k_{-A} and is a generalisation of the starting point variability result obtained by Laming (1968).

When $\theta_1 = 0$, an expression for $k_A - k_{-A}$ can be obtained by taking limits as θ_1 approaches 0. In this case

$$k_A - k_{-A} = \frac{-4Am_0}{3s^2} \qquad \text{(A21)}$$

independent of $s_0{}^2$.

Proof That the Mean Marginal Response Time for the Race Model for Temporal Order Discrimination Increases as the ISI Increases.

Consider two stimuli S_1 and S_2 for which S_1 precedes S_2 by δ msec. Let the perception time lags conditioned upon stimulus presented be denoted by the random variable T_i for stimulus S_i, $i = 1,2$, and assume that the density functions for these random variables are identically and independently distributed as $f(t)$, $t \geq 0$.

We are interested in the distribution of the first arriving stimulus conditional upon S_1 being presented δ msec before S_2. In other words we are interested in the random variable $T = \min(T_1,T_2)$.

Now

$$\Pr(T=t) = f(t) \int_t^\infty f(u-\delta)du + f(t-\delta) \int_t^\infty f(u)du, \quad t \geq \delta$$

$$= f(t) \qquad\qquad , \quad 0 \leq t < \delta.$$

So for the expected value of T, $E_\delta(T)$, we have

$$E_\delta(T) = \int_0^\infty t \Pr(T=t)dt$$

$$= \int_0^\delta tf(t)dt + \int_\delta^\infty t\{f(t)[1-F(t-\delta)] + f(t-\delta)[1-F(t)]\} dt$$

where $F(x) = \int_0^x f(t)dt$ so that $\frac{dF(x)}{dx} = f(x)$.

Suppose $\delta' = \delta + \delta'' > 0$, and let $\delta'' > 0$ be so small that $(\delta'')^2$ is negligible.

Then

$$E_{\delta'}(T) = \int_0^{\delta'} tf(t)dt + \int_{\delta'}^{\infty} t(f(t)[1-F(t-\delta')] + f(t-\delta')[1-F(t)])dt$$

$$E_{\delta}(T) = \int_0^{\delta} tf(t)dt + \int_{\delta}^{\infty} t(f(t)[1-F(t-\delta)] + f(t-\delta)[1-F(t)])dt$$

$$E_{\delta'}(T) - E_{\delta}(T) = \int_{\delta}^{\delta'} tf(t)dt + \int_{\delta'}^{\infty} t(f(t)[1-F(t-\delta')] + f(t-\delta')[1-F(t)])dt$$

$$- \int_{\delta}^{\infty} t(f(t)[1-F(t-\delta)] + f(t-\delta)[1-F(t)])dt.$$

By Taylor's Theorem:

$$F(t-\delta') = F(t-\delta) + (\delta-\delta')f(t-\delta) + O(\delta-\delta')^2$$

$$= F(t-\delta) - \delta'' f(t-\delta)$$

and $$f(t-\delta') = f(t-\delta) - \delta'' f'(t-\delta)$$

$$\int_{\delta'}^{\infty} t(f(t)[1-F(t-\delta')] + f(t-\delta')[1-F(t)])dt$$

$$= \int_{\delta'}^{\infty} t(f(t)[1-F(t-\delta) + \delta'' f(t-\delta)] + [f(t-\delta) - \delta'' f'(t-\delta)][1-F(t)])dt$$

$$= \int_{\delta'}^{\infty} t(f(t)[1-F(t-\delta)] + f(t-\delta)[1-F(t)])dt + \int_{\delta'}^{\infty} \delta'' t(f(t)f(t-\delta) - f'(t-\delta)[1-F(t)])dt$$

$$E_{\delta'}(T) - E_{\delta}(T) = \int_{\delta}^{\delta'} tf(t)dt - \int_{\delta}^{\delta'} t(f(t)[1-F(t-\delta)] + f(t-\delta)[1-F(t)])dt$$

$$+ \delta'' \int_{\delta'}^{\infty} t(f(t)f(t-\delta) - f'(t-\delta)[1-F(t)])dt.$$

Now the integral between δ and δ' becomes

$$\int_{\delta}^{\delta'} \{tf(t)F(t-\delta) - tf(t-\delta)[1-F(t)]\}dt$$

$$\simeq \left(\frac{\delta'-\delta}{2}\right) \{-\delta' f(\delta'-\delta)[1-F(\delta')] + \delta f(0)[1-F(\delta)]\}$$

$$\simeq \frac{\delta''}{2} \{-\delta' f(\delta'')[1-F(\delta')] + \delta f(0)[1-F(\delta)]\} \text{ where } F(0) = 0, \quad F(\delta'') = 0$$

$$\simeq \frac{\delta''}{2} [\delta f(0) - \delta' f(\delta'') + \delta' f(\delta'')F(\delta') - \delta f(0)F(\delta)]$$

If $f(\delta'') \simeq f(0)$ as δ'' is small,

$$\text{then LHS} \simeq \frac{\delta''}{2} [(\delta-\delta')f(0) + f(0)\{\delta' F(\delta') - \delta F(\delta)\}]$$

As $\delta' F(\delta') \simeq \delta'\{F(\delta) + \delta'' f(\delta)\}$ by Taylor's Theorem,

$$\text{then} \quad \delta' F(\delta') - \delta F(\delta) \simeq \delta'' F(\delta) + \delta'\delta'' f(\delta)$$
$$= \delta''[F(\delta) + \delta' f(\delta)]$$

$$\text{LHS} \simeq \frac{(\delta'')^2}{2} [f(0) + f(0)\{F(\delta) + \delta' f(\delta)\}] = O(\delta''^2) = 0$$

$$E_{\delta'}(T) - E_{\delta}(T) \simeq \delta'' \int_{\delta'}^{\infty} t\{f(t)f(t-\delta) - f'(t-\delta)[1-F(t)]\}dt.$$

Integrating the integral on the RHS by parts and noting that

$$-\frac{d}{dt} \{f(t-\delta)[1-F(t)]\} = f(t)f(t-\delta) - f'(t-\delta)[1-F(t)]$$

we have

$$E_{\delta'}(T) - E_{\delta}(T) \simeq \delta'' \left[t\{f(t-\delta)[1-F(t)]\}\Big|_{\infty}^{\delta'} + \int_{\delta'}^{\infty} f(t-\delta)[1-F(t)]dt \right]$$

Since f(t) is a pdf assume that $F(\infty) = 1$, $f(\infty) = 0$.

$$\text{So} \quad E_{\delta'}(T) - E_{\delta}(T) \doteq \delta'' \{\delta' f(\delta'-\delta)[1-F(\delta)] + \int_{\delta'}^{\infty} f(t-\delta)[1-F(t)]dt\}$$

$$> 0 \text{ as } f(t-\delta)[1-F(t)] \geq 0, \text{ for all } t.$$

$E_{\delta'}(T) > E_{\delta}(T)$

$\delta' > \delta \Rightarrow E_{\delta'}(T) > E_{\delta}(T)$

As δ increases $E_{\delta}(T)$ increases monotonically.

Appendix II: Raw data

TABLE A1

DATA FOR EXPERIMENT Ia

R_1 = "Top 1st"

R_2 = "Bottom 1st"

$\hat{T}_{12}(\tau) = \hat{T}_1(\tau) - \hat{T}_2(\tau)$

	ISI (msec)	RESPONSE	$\hat{P}_1(\tau)$	$\overline{RT}$ (msec)	S.E.	N	MARGINAL MEAN RT	S.E.	$\hat{T}_{12}(\tau)$	S.E.
CC	26.4	R_1	0.87	600	7	808	606	7	-52	22
		R_2		652	21	117				
	19.8	R_1	0.80	600	6	734	608	6	-41	19
		R_2		641	18	189				
	13.2	R_1	0.75	615	8	691	620	7	-21	17
		R_2		636	15	232				
	6.6	R_1	0.65	629	9	601	636	7	-19	15
		R_2		648	12	322				
	0	R_1	0.48	645	10	479	631	8	30	16
		R_2		615	13	445				
	-6.6	R_1	0.41	671	15	379	616	8	93	17
		R_2		578	8	546				
	-13.2	R_1	0.28	685	17	256	600	8	118	19
		R_2		567	8	669				
	-19.8	R_1	0.18	697	20	170	577	7	147	21
		R_2		550	7	755				
	-26.4	R_1	0.09	697	30	85	536	6	177	31
		R_2		520	6	840				
						overall Mean RT 603±2				
RH	26.4	R_1	0.91	625	6	793	636	6	-122	25
		R_2		747	24	82				
	19.8	R_1	0.83	629	6	722	648	6	-109	19
		R_2		738	18	153				

	ISI (msec)	RESPONSE	$\hat{P}_1(\tau)$	$\overline{RT}$ (msec)	S.E.	N	MARGINAL $\overline{RT}$	S.E.	$\hat{T}_{12}(\tau)$	S.E.
	13.2	R_1	0.71	647	7	621	662	6	-52	14
		R_2		699	12	252				
	6.6	R_1	0.62	661	7	542	673	6	-32	11
		R_2		693	9	333				
	0	R_1	0.50	684	9	434	687	7	-5	13
		R_2		689	10	439				
	-6.6	R_1	0.36	715	12	312	678	7	57	14
		R_2		658	8	563				
	-13.2	R_1	0.21	679	14	182	636	6	54	15
		R_2		625	6	693				
	-19.8	R_1	0.12	711	19	102	627	7	95	21
		R_2		616	8	773				
	-26.4	R_1	0.04	749	40	37	599	6	157	40
		R_2		592	6	838				
						overall mean RT 605±2				
IK	26.4	R_1	0.65	1002	14	505	1023	13	-61	29
		R_2		1063	25	270				
	19.8	R_1	0.74	930	16	577	947	14	-68	31
		R_2		998	27	198				
	13.2	R_1	0.75	947	17	585	955	15	-32	33
		R_2		979	29	190				
	6.6	R_1	0.75	924	17	579	938	15	-54	36
		R_2		978	32	196				
	0	R_1	0.65	914	10	501	922	9	-22	21
		R_2		936	18	270				
	-6.6	R_1	0.51	959	16	398	957	14	4	29
		R_2		955	24	375				

	ISI (msec)	RESPONSE	$\hat{P}_1(\tau)$	$\overline{RT}$ (msec)	S.E.	N	MARGINAL $\overline{RT}$	S.E.	$\hat{T}_{12}(\tau)$	S.E.
	-13.2	R_1	0.31	1001	32	238	959	16	60	36
		R_2		941	18	537				
	-19.8	R_1	0.17	1002	42	131	908	14	113	44
		R_2		889	14	644				
	-26.4	R_1	0.11	1053	57	83	859	19	217	60
		R_2		836	20	691				
						overall Mean RT		941±5		
VY	22	R_1	0.93	585	7	907	592	7	-96	35
		R_2		681	34	67				
	16.5	R_1	0.87	602	6	850	608	6	-47	24
		R_2		649	23	124				
	11	R_1	0.80	602	7	777	619	7	-82	19
		R_2		684	18	197				
	5.5	R_1	0.73	613	9	715	625	8	-44	17
		R_2		657	14	259				
	0	R_1	0.58	636	9	566	640	7	-9	13
		R_2		645	10	405				
	-5.5	R_1	0.39	655	11	381	636	7	31	14
		R_2		624	9	593				
	-11	R_1	0.27	674	15	263	624	8	69	17
		R_2		605	9	714				
	-16.5	R_1	0.12	660	23	116	590	6	80	24
		R_2		580	6	859				
	-22	R_1	0.07	747	44	64	584	6	174	44
		R_2		573	5	911				
							overall Mean RT	613±2		

TABLE A2

DATA FOR EXPERIMENT Ib

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	ISI (MSEC)	RESPONSE	$\hat{P}_1(\tau)$	$\overline{RT}$ (MSEC)	S.E.	N	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	S.E.
NB	24	R_1	0.92	426	4	688	436	4	-121	22
		R_2		547	22	62				
	18	R_1	0.89	426	4	668	441	4	-139	18
		R_2		565	18	82				
	12	R_1	0.81	434	5	608	461	7	-140	28
		R_2		574	28	142				
	6	R_1	0.73	437	5	543	468	5	-114	13
		R_2		551	12	206				
	0	R_1	0.60	448	6	448	483	6	- 87	13
		R_2		535	12	302				
	- 6	R_1	0.49	461	7	368	501	7	- 79	13
		R_2		540	11	381				
	-12	R_1	0.38	490	15	281	519	8	- 47	17
		R_2		537	9	469				
	-18	R_1	0.24	518	20	179	530	10	- 16	23
		R_2		534	12	571				
	-24	R_1	0.19	519	19	143	514	7	6	21
		R_2		513	8	606				
							overall MEAN RT	484 $\pm$2		
R	24	R_1	0.91	603	11	679	624	12	-219	70
		R_2		822	69	71				
	18	R_1	0.83	628	11	621	636	11	- 46	34
		R_2		674	32	129				
	12	R_1	0.72	660	13	538	686	14	- 91	37
		R_2		751	35	210				
	6	R_1	0.63	688	17	473	696	13	- 21	25
		R_2		709	19	277				
	0	R_1	0.50	718	19	377	726	14	- 16	28
		R_2		734	21	373				

TABLE A2

DATA FOR EXPERIMENT 1b

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	ISI (MSEC)	RESPONSE	$\hat{P}_1(\tau)$	$\overline{RT}$ (MSEC)	S.E.	N	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	S.E.
R	-6	R_1	0.40	741	24	303	717	14	41	30
		R_2		700	18	447				
	-12	R_1	0.28	770	29	210	695	13	104	32
		R_2		666	14	539				
	-18	R_1	0.19	822	40	141	661	11	198	41
		R_2		624	10	609				
	-24	R_1	0.13	742	38	99	628	11	131	40
		R_2		611	11	651				
							overall MEAN RT 674 ±4			
KG	24	R_1	0.94	590	5	704	593	5	- 56	31
		R_2		646	31	46				
	18	R_1	0.90	600	5	678	614	5	-150	25
		R_2		750	25	72				
	12	R_1	0.79	609	6	594	625	6	- 75	16
		R_2		684	15	156				
	6	R_1	0.67	604	6	502	625	6	- 64	13
		R_2		668	12	248				
	0	R_1	0.45	623	8	340	627	6	- 8	11
		R_2		631	8	410				
	- 6	R_1	0.33	665	12	244	639	6	39	13
		R_2		626	6	506				
	-12	R_1	0.22	688	16	167	633	5	71	17
		R_2		617	5	583				
	-18	R_1	0.13	723	21	96	619	6	119	22
		R_2		604	6	654				
	-24	R_1	0.11	715	21	82	607	4	121	21
		R_2		594	4	668				
							overall MEAN RT 620 ±2			
N	24	R_1	0.96	852	14	721	872	15	-530	110
		R_2		1382	109	29				

TABLE A2

DATA FOR EXPERIMENT Ib

R_1 = "Top 1st"

R_2 = "Bottom 1st"

ISI (MSEC)	RESPONSE	$\hat{P}_1(\tau)$	$\overline{RT}$ (MSEC)	S.E.	N	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	S.E.
18	R_1	0.90	905	18	672	950	19	-450	80
	R_2		1355	78	75				
12	R_1	0.74	990	23	554	1067	21	-295	52
	R_2		1285	47	194	1080	15		
6	R_1	0.61	1133	30	451	1138	22	- 12	43
	R_2		1145	31	295				
0	R_1	0.41	1205	40	303	1169	23	60	49
			1145	28	444				
- 6	R_1	0.27	1336	50	202	1151	21	254	55
	R_2		1082	22	544				
-12	R_1	0.19	1432	68	141	1094	22	417	71
	R_2		1015	20	602				
-18	R_1	0.13	1491	77	98	1096	21	455	80
	R_2		1036	20	650				
-24	R_1	0.09	1549	91	66	1104	20	488	93
	R_2		1061	19	679				

overall MEAN RT 1071 ± 7

TABLE A3

POOLED DATA FOR EXPERIMENT Ib

R_1 = "Top 1st"

R_2 = "Bottom 1st"

ISI (MSEC)	RESPONSE	$\hat{P}_1(\tau)$	$\overline{RT}$ (MSEC)	S.E.	N	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau) - \hat{T}_2(\tau)$	S.E.
24	R_1	0.93	620	6	2792	631	6	-159	36
	R_2		779	35	208				
18	R_1	0.88	640	6	2639	660	6	-167	27
	R_2		807	26	358				
12	R_1	0.77	667	8	2294	709	8	-181	22
	R_2		848	21	702				
6	R_1	0.66	700	10	1969	732	8	- 93	16
	R_2		793	13	1026				
0	R_1	0.49	714	12	1468	751	9	- 73	17
	R_2		787	12	1529				
- 6	R_1	0.37	740	15	1117	752	8	- 19	18
	R_2		759	10	1878				
-12	R_1	0.27	771	19	799	734	8	50	21
	R_2		721	8	2193				
-18	R_1	0.17	825	25	514	726	8	119	26
	R_2		706	8	2484				
-24	R_1	0.13	791	27	390	713	7	90	28
	R_2		701	7	2604				

overall MEAN RT 712 ± 3

TABLE A4 EXPERIMENT IIa

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT	S.E.
RR	330	5.5	R_1	139	0.68	328	4	− 1	12	328	4
		5.5	R_2	65		329	11				
		−5.5	R_1	109	0.52	330	6	8	7	326	4
		−5.5	R_2	97		322	4				
	330	16.5	R_1	168	0.82	338	6	16	11	335	5
		16.5	R_2	36		322	9				
		−16.5	R_1	77	0.38	319	5	− 9	7	325	4
		−16.5	R_2	127		328	5				
	330	27.5	R_1	178	0.90	334	4	0	10	334	4
		27.5	R_2	20		334	9				
		−27.5	R_1	33	0.16	344	20	10	20	336	5
		−27.5	R_2	174		334	4				
	330	44	R_1	193	0.95	338	5	19	11	337	5
		44	R_2	11		319	10				
		−44	R_1	21	0.10	331	9	−11	11	341	6
		−44	R_2	183		342	7				
								overall MEAN RT 333 ± 2			
	440	5.5	R_1	120	0.59	404	5	− 1	8	404	4
		5.5	R_2	85		405	6				
		−5.5	R_1	110	0.54	396	5	−14	9	404	4
		−5.5	R_2	94		413	7				
	440	16.5	R_1	158	0.77	398	3	−20	14	403	4
		16.5	R_2	46		418	14				
		−16.5	R_1	51	0.25	416	11	13	12	406	4
		−16.5	R_2	153		403	4				

TABLE A4 . EXPERIMENT IIa

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\bar{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT	S.E.
440	27.5	R_1	186	0.91	413	5	16	11	412	5
	27.5	R_2	19		397	10				
	-27.5	R_1	28	0.14	397	7	- 6	8	402	3
	-27.5	R_2	176		403	3				
440	44	R_1	197	0.97	404	4	18	14	403	4
	44	R_2	7		386	13				
	-44	R_1	11	0.05	417	11	11	12	407	5
	-44	R_2	193		406	5				
							overall MEAN RT 405 ±1			
550	5.5	R_1	125	0.61	418	5	-22	9	427	4
	5.5	R_2	79		440	7				
	-5.5	R_1	92	0.45	416	6	-28	9	431	5
	-5.5	R_2	112		444	7				
550	16.5	R_1	161	0.79	429	6	-15	10	432	5
	16.5	R_2	43		444	8				
	-16.5	R_1	49	0.24	418	6	-10	8	426	4
	-16.5	R_2	155		428	5				
550	27.5	R_1	196	0.96	427	4	-13	21	428	4
	27.5	R_2	9		440	21				
	-27.5	R_1	17	0.08	406	10	-21	11	425	4
	-27.5	R_2	187		427	4				
550	44	R_1	202	0.99	424	4	60	23	423	4
	44	R_2	2		364	23				
	-44	R_1	8	0.04	418	25	-11	25	429	5
	-44	R_2	197		429	5				
							overall MEAN RT 428 ±2			

TABLE A4 EXPERIMENT IIa

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT	S.E.
JM	330	5.5	R_1	61	0.39	361	11	20	14	349	7
		5.5	R_2	95		341	9				
		-5.5	R_1	51	0.33	359	15	19	20	346	10
		-5.5	R_2	105		340	13				
	330	16.5	R_1	99	0.63	347	8	- 2	14	348	7
		16.5	R_2	57		349	12				
		-16.5	R_1	26	0.17	492	165	141	165	375	29
		-16.5	R_2	130		351	10				
	330	27.5	R_1	112	0.74	352	6	10	13	349	5
		27.5	R_2	40		342	12				
		-27.5	R_1	16	0.10	330	13	- 2	15	332	6
		-27.5	R_2	141		332	7				
	330	44	R_1	147	0.94	355	9	21	50	354	9
		44	R_2	9		334	49				
		-44	R_1	4	0.03	340	59	- 3	60	343	8
		-44	R_2	152		343	8				
								overall MEAN RT 349 $\pm$4			
	440	5.5	R_1	82	0.53	415	15	17	17	407	9
		5.5	R_2	74		398	7				
		-5.5	R_1	55	0.35	414	11	- 1	15	415	8
		-5.5	R_2	101		415	10				
	440	16.5	R_1	112	0.72	430	12	2	23	429	10
		16.5	R_2	44		428	20				
		-16.5	R_1	32	0.21	425	20	31	22	400	9
		-16.5	R_2	124		394	10				

TABLE A4 EXPERIMENT IIa

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\bar{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT	S.E.
440	27.5	R_1	128	0.82	419	13	-52	77	429	18
	27.5	R_2	29		471	76				
	-27.5	R_1	9	0.06	406	20	9	22	398	9
	-27.5	R_2	147		397	9				
440	44	R_1	152	0.97	397	6	-58	69	398	6
	44	R_2	4		455	69				
	-44	R_1	1	0.01	395	-	- 2	-	397	10
	-44	R_2	155		397	10				
						overall MEAN RT	409 ± 4			
550	5.5	R_1	84	0.54	458	9	- 3	13	459	7
	5.5	R_2	72		461	10				
	-5.5	R_1	49	0.31	469	13	21	15	455	7
	-5.5	R_2	107		448	8				
550	16.5	R_1	116	0.74	444	9	-43	25	455	9
	16.5	R_2	40		487	23				
	-16.5	R_1	29	0.19	472	17	12	19	462	8
	-16.5	R_2	127		460	9				
550	27.5	R_1	140	0.90	449	11	-16	28	451	10
	27.5	R_2	16		465	26				
	-27.5	R_1	6	0.04	561	50	122	50	444	7
	-27.5	R_2	150		439	7				
550	44	R_1	152	0.97	455	11	56	15	454	11
	44	R_2	4		399	10				
	-44	R_1	5	0.03	417	41	- 7	41	424	6
	-44	R_2	153		424	6				
						overall MEAN RT	450 ± 3			

TABLE A4

INDIVIDUAL SUBJECT DATA

EXPERIMENT IIa

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT	S.E.
DK	275	5.5	R_1	100	0.49	283	13	31	16	267	8
		5.5	R_2	104		252	9				
		-5.5	R_1	97	0.48	253	7	-15	14	261	7
		-5.5	R_2	105		268	12				
	275	11	R_1	100	0.49	273	10	19	13	263	6
		11	R_2	104		254	8				
		-11	R_1	80	0.39	245	9	-33	14	265	8
		-11	R_2	124		278	11				
	275	22	R_1	112	0.56	290	18	42	21	271	11
		22	R_2	89		248	11				
		-22	R_1	84	0.41	245	10	-25	14	260	7
		-22	R_2	122		270	10				
	275	44	R_1	118	0.58	298	12	75	14	266	8
		44	R_2	86		223	7				
		-44	R_1	73	0.36	223	7	-69	14	267	8
		-44	R_2	131		292	12				
								overall MEAN RT 265 $\pm$3			
	440	5.5	R_1	114	0.56	293	12	18	16	285	8
		5.5	R_2	89		275	11				
		-5.5	R_1	86	0.42	265	8	-23	12	278	6
		-5.5	R_2	118		288	9				
	440	11	R_1	108	0.53	297	10	19	15	288	7
		11	R_2	96		278	11				
		-11	R_1	89	0.44	268	10	-17	14	278	7
		-11	R_2	115		285	10				

TABLE A4 — EXPERIMENT IIa

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\bar{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	$\hat{T}_1(\tau)\cdot\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT	S.E.
440	22	R_1	124	0.61	307	11	20	20	299	9
	22	R_2	80		287	17				
	-22	R_1	70	0.35	268	12	-30	15	288	7
	-22	R_2	131		298	9				
440	44	R_1	138	0.68	322	12	98	13	290	9
	44	R_2	66		224	5				
	-44	R_1	55	0.27	246	10	-58	13	288	7
	-44	R_2	149		304	9				

overall MEAN RT 287 ±3

DEADLINE	τ	RESPONSE	N	$\bar{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	$\hat{T}_1(\tau)\cdot\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT	S.E.
990	5.5	R_1	107	0.52	336	11	5	17	334	8
	5.5	R_2	97		331	13				
	-5.5	R_1	95	0.47	312	10	3	15	310	7
	-5.5	R_2	109		309	11				
990	11	R_1	108	0.53	312	13	4	17	310	9
	11	R_2	96		308	11				
	-11	R_1	83	0.41	320	14	-28	18	337	9
	-11	R_2	121		348	11				
990	22	R_1	142	0.69	349	15	66	21	329	11
	22	R_2	63		283	14				
	-22	R_1	57	0.28	275	12	-91	18	341	10
	-22	R_2	147		366	13				
990	44	R_1	161	0.79	334	8	100	12	313	7
	44	R_2	43		234	9				
	-44	R_1	39	0.19	268	16	-74	18	328	8
	-44	R_2	165		342	9				

overall MEAN RT 325 ±3

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	TIK RELEASE RT MEAN	TIK RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
LN 250	5	R_1	111	0.46	212	14	86	7	27	15	197	7
	5	R_2	129		185	6	80	4				
	-5	R_1	106	0.44	201	10	89	7	10	13	195	6
	-5	R_2	134		191	8	82	4				
250	10	R_1	103	0.43	180	10	73	5	-10	14	186	7
	10	R_2	137		190	10	83	6				
	-10	R_1	104	0.43	214	18	85	7	23	19	201	9
	-10	R_2	136		191	6	87	5				
250	20	R_1	114	0.48	190	9	78	5	- 9	14	195	7
	20	R_2	126		199	11	88	6				
	-20	R_1	111	0.46	199	11	87	7	- 3	18	201	9
	-20	R_2	129		202	14	86	6				
250	40	R_1	114	0.48	201	14	78	5	7	17	197	9
	40	R_2	126		194	10	77	5				
	-40	R_1	99	0.41	193	14	82	6	-13	18	201	9
	-40	R_2	141		206	12	90	6				
								overall MEAN RT 197 $\pm$3				
400	5	R_1	145	0.60	252	9	140	6	24	13	243	7
	5	R_2	95		228	9	121	7				
	-5	R_1	158	0.66	259	10	140	6	1	21	259	9
	-5	R_2	82		258	18	127	8				
400	10	R_1	150	0.63	272	14	139	7	60	16	250	9
	10	R_2	90		212	7	109	7				
	-10	R_1	125	0.52	249	9	140	8	-23	17	260	9
	-10	R_2	115		272	15	139	8				

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	T1K RELEASE RT MEAN	T1K RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
400	20	R_1	161	0.67	281	10	159	7	68	12	259	7
	20	R_2	79		213	7	110	6				
	-20	R_1	125	0.52	231	7	124	7	-35	15	248	7
	-20	R_2	115		266	13	142	8				
400	40	R_1	153	0.64	286	14	150	7	77	15	258	9
	40	R_2	87		209	6	101	6				
	-40	R_1	106	0.44	242	14	119	6	-43	20	266	10
	-40	R_2	134		285	14	148	8				
							overall MEAN RT 255 ±3					
900	5	R_1	175	0.73	502	11	325	7	-85	25	525	10
	5	R_2	65		587	23	374	15				
	-5	R_1	107	0.45	505	15	346	11	-66	19	542	10
	-5	R_2	133		571	12	372	8				
900	10	R_1	196	0.82	502	9	337	7	-79	25	516	9
	10	R_2	44		581	23	373	17				
	-10	R_1	86	0.36	502	15	339	11	-41	19	528	9
	-10	R_2	154		543	12	356	8				
900	20	R_1	223	0.93	506	8	335	6	-41	81	509	9
	20	R_2	17		547	81	293	38				
	-20	R_1	46	0.17	496	35	325	26	-62	37	546	11
	-20	R_2	194		558	11	372	7				
900	40	R_1	236	0.98	492	7	331	6	271	22	487	7
	40	R_2	4		221	21	93	28				
	-40	R_1	29	0.12	374	37	222	26	-139	38	496	9
	-40	R_2	211		513	8	348	6				
							overall MEAN RT 519 ±3					

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	TIK RELEASE RT MEAN	TIK RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
MG	250	5	R_1	90	0.38	221	12	100	6	8	18	216	10
		5	R_2	150		213	14	102	4				
		-5	R_1	78	0.33	203	7	100	6	-7	9	208	4
		-5	R_2	162		210	5	106	4				
	250	10	R_1	96	0.40	219	8	109	6	14	12	211	6
		10	R_2	144		205	9	101	4				
		-10	R_1	86	0.36	201	6	96	5	-23	11	216	6
		-10	R_2	154		224	9	110	5				
	250	20	R_1	121	0.50	221	8	106	5	23	9	210	5
		20	R_2	119		198	4	103	4				
		-20	R_1	79	0.33	216	12	101	5	- 1	14	217	7
		-20	R_2	161		217	8	103	4				
	250	40	R_1	125	0.52	228	10	105	5	39	11	209	6
		40	R_2	115		189	4	94	4				
		-40	R_1	83	0.35	212	7	107	5	0	9	212	5
		-40	R_2	157		212	6	105	4				
										overall MEAN RT 212 ±2			
	400	5	R_1	107	0.45	331	11	184	5	28	16	315	8
		5	R_2	133		303	11	167	5				
		-5	R_1	95	0.40	316	10	179	6	21	12	303	6
		-5	R_2	145		295	7	181	5				
	400	10	R_1	143	0.60	313	10	175	5	37	14	298	7
		10	R_2	97		276	10	159	5				
		-10	R_1	87	0.36	300	13	160	7	-11	16	307	8
		-10	R_2	153		311	10	178	5				

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	TIK RELEASE RT MEAN	TIK RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
400	20	R_1	145	0.60	343	8	199	5	65	10	317	6
	20	R_2	95		278	6	166	5				
	-20	R_1	83	0.35	284	10	169	7	-36	15	308	8
	-20	R_2	157		320	11	178	5				
400	40	R_1	172	0.72	351	8	189	5	90	11	326	7
	40	R_2	68		261	7	154	7				
	-40	R_1	67	0.28	264	8	155	8	- 50	12	300	7
	-40	R_2	173		314	9	180	5				
								overall MEAN RT	309 ± 3			
900	5	R_1	132	0.55	368	10	214	5	- 6	15	371	7
	5	R_2	108		374	11	225	6				
	- 5	R_1	99	0.41	364	12	211	7	- 8	15	369	7
	- 5	R_2	141		372	9	230	6				
900	10	R_1	145	0.60	395	10	232	6	31	16	383	8
	10	R_2	95		364	13	216	8				
	-10	R_1	84	0.35	358	15	210	7	- 1	17	359	7
	-10	R_2	156		359	8	225	6				
900	20	R_1	174	0.73	389	9	227	5	59	17	373	8
	20	R_2	66		330	15	192	10				
	-20	R_1	76	0.32	342	14	198	8	- 24	16	358	7
	-20	R_2	164		366	7	234	5				
900	40	R_1	215	0.90	387	7	232	5	113	18	375	7
	40	R_2	25		274	17	163	16				
	-40	R_1	45	0.19	271	10	160	10	-106	12	352	6
	-40	R_2	195		371	7	236	5				
								overall MEAN RT	368 ± 3			

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	TIK RELEASE RT MEAN	TIK RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
250	5	R_1	130	0.54	253	12	136	7	- 18	18	261	9
	5	R_2	110		271	14	149	8				
	-5	R_1	128	0.53	225	8	126	7	- 55	14	251	7
	-5	R_2	112		280	11	158	7				
250	10	R_1	126	0.53	237	10	131	8	- 26	14	249	7
	10	R_2	113		263	10	150	8				
	-10	R_1	128	0.53	237	10	132	8	- 42	16	257	8
	-10	R_2	112		279	12	154	8				
250	20	R_1	144	0.60	273	13	149	8	6	17	271	9
	20	R_2	96		267	11	148	7				
	-20	R_1	118	0.49	230	9	130	7	- 60	15	261	8
	-20	R_2	122		290	12	156	7				
250	40	R_1	153	0.64	301	13	157	7	44	17	285	9
	40	R_2	87		257	11	141	8				
	-40	R_1	104	0.43	223	8	125	7	- 81	14	269	8
	-40	R_2	136		304	11	180	9				
							overall MEAN RT		263 $\pm$3			
400	5	R_1	126	0.53	333	14	199	8	- 18	23	342	11
	5	R_2	114		351	18	197	8				
	-5	R_1	104	0.43	299	12	181	8	- 65	18	336	9
	-5	R_2	136		364	13	217	8				
400	10	R_1	131	0.55	330	13	190	7	- 19	20	339	10
	10	R_2	109		349	15	204	10				
	-10	R_1	97	0.41	303	13	180	9	-100	19	362	10
	-10	R_2	142		403	14	234	9				

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	TIK RELEASE RT MEAN	TIK RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
400	20	R_1	140	0.58	344	11	206	8	4	19	342	9
	20	R_2	100		340	15	204	9				
	-20	R_1	92	0.38	334	18	202	12	- 46	21	362	10
	-20	R_2	148		380	11	229	7				
400	40	R_1	155	0.65	377	15	222	9	25	27	368	12
	40	R_2	85		352	22	190	10				
	-40	R_1	85	0.35	273	13	165	9	-117	17	349	9
	-40	R_2	155		390	11	233	7				
									overall MEAN RT 350 $\pm$4			
900	5	R_1	127	0.53	439	17	286	10	-103	28	487	14
	5	R_2	113		542	22	321	11				
	-5	R_1	106	0.44	450	22	282	11	- 76	27	492	13
	-5	R_2	134		526	15	333	10				
900	10	R_1	137	0.57	472	16	300	9	- 54	32	495	15
	10	R_2	103		526	28	296	12				
	-10	R_1	101	0.42	496	26	292	13	- 53	30	527	14
	-10	R_2	139		549	14	350	11				
900	20	R_1	171	0.71	491	16	316	9	- 29	32	499	14
	20	R_2	69		520	28	310	19				
	-20	R_1	69	0.29	425	28	258	12	-111	31	504	13
	-20	R_2	171		536	14	341	9				
900	40	R_1	201	0.84	494	12	330	8	- 75	54	506	13
	40	R_2	39		569	53	302	27				
	-40	R_1	42	0.18	419	38	238	19	- 77	40	483	11
	-40	R_2	198		496	11	322	8				
									overall MEAN RT 500 $\pm$5			

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	TIK RELEASE RT MEAN	TIK RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
V	250	5	R_1	107	0.45	245	16	182	12	18	26	235	13
		5	R_2	133		227	20	164	12				
		-5	R_1	91	0.38	282	16	200	12	33	21	262	10
		-5	R_2	149		249	13	196	10				
	250	10	R_1	126	0.53	268	14	199	10	47	21	246	11
		10	R_2	114		221	16	160	10				
		-10	R_1	83	0.35	244	14	189	12	-7	20	249	10
		-10	R_2	157		251	14	190	10				
	250	20	R_1	134	0.56	252	12	193	10	62	20	225	10
		20	R_2	106		190	16	138	9				
		-20	R_1	71	0.30	295	31	190	14	45	34	263	13
		-20	R_2	169		250	13	191	10				
	250	40	R_1	147	0.61	319	15	226	9	134	19	263	11
		40	R_2	93		175	12	141	9				
		-40	R_1	64	0.27	176	14	130	10	-130	21	271	13
		-40	R_2	176		306	16	209	9				
									overall MEAN RT	252 $\pm$4			
	400	5	R_1	144	0.60	378	13	289	9	45	22	360	11
		5	R_2	96		333	18	266	12				
		-5	R_1	104	0.43	375	15	269	8	22	20	363	10
		-5	R_2	136		353	13	286	9				
	400	10	R_1	151	0.63	384	13	283	6	72	21	357	11
		10	R_2	89		312	17	250	12				
		-10	R_1	95	0.40	357	11	275	7	-10	19	363	10
		-10	R_2	145		367	15	272	9				

TABLE A5 EXPERIMENT IIb

INDIVIDUAL SUBJECT DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	T1K RELEASE RT MEAN	T1K RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
400	20	R_1	192	0.80	386	11	287	5	121	20	362	10
	20	R_2	48		265	17	230	15				
	−20	R_1	64	0.27	336	20	248	10	−46	24	370	11
	−20	R_2	176		382	13	298	7				
400	40	R_1	195	0.81	369	8	284	5	157	17	340	8
	40	R_2	45		212	15	174	14				
	−40	R_1	31	0.13	282	29	207	18	− 90	31	360	10
	−40	R_2	209		372	10	295	6				
							overall MEAN RT		360 ± 3			
900	5	R_1	129	0.54	462	14	354	9	− 19	22	471	11
	5	R_2	111		481	17	366	11				
	−5	R_1	98	0.41	453	17	333	11	− 19	22	465	11
	−5	R_2	142		474	14	378	9				
900	10	R_1	155	0.65	457	13	345	8	− 25	28	466	12
	10	R_2	85		482	25	352	15				
	−10	R_1	72	0.30	430	22	328	14	− 48	27	464	13
	−10	R_2	168		478	16	359	8				
900	20	R_1	180	0.75	452	12	336	7	− 31	34	460	12
	20	R_2	60		483	32	362	20				
	−20	R_1	40	0.17	498	40	322	25	− 46	41	460	10
	−20	R_2	200		452	10	358	7				
900	40	R_1	219	0.91	440	8	339	5	156	51	426	9
	40	R_2	21		284	50	212	34				
	−40	R_1	15	0.06	563	87	405	49	143	87	429	10
	−40	R_2	225		420	9	345	6				
							overall MEAN RT		455 ± 4			

TABLE A6

EXPERIMENT IIb - POOLED DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	T1K RELEASE RT MEAN	T1K RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
250	5	R_1	438	0.46	234	7	127	4	12	10	227	5
		R_2	522		222	7	122	4				
	-5	R_1	403	0.42	227	6	128	4	-3	8	229	4
		R_2	557		230	5	135	3				
250	10	R_1	451	0.47	229	6	132	4	12	8	223	4
		R_2	508		217	6	120	3				
	-10	R_1	401	0.42	225	7	124	4	-9	9	230	5
		R_2	559		234	6	136	4				
250	20	R_1	513	0.53	237	6	135	4	26	8	225	4
		R_2	447		211	6	117	3				
	-20	R_1	379	0.40	230	8	123	4	-9	10	235	5
		R_2	581		239	6	136	1				
250	40	R_1	539	0.56	268	7	147	4	67	9	239	5
		R_2	421		201	5	109	3				
	-40	R_1	350	0.37	203	6	109	4	-55	8	238	4
		R_2	610		258	6	148	4				
					Overall Mean RT				231 ± 2			
400	5	R_1	522	0.54	322	6	204	4	16	9	315	5
		R_2	438		306	7	187	4				
	-5	R_1	461	0.48	306	6	186	4	-18	8	315	4
		R_2	499		324	6	211	4				
400	10	R_1	575	0.60	325	6	197	3	35	9	311	5
		R_2	385		290	7	181	5				
	-10	R_1	404	0.42	299	6	186	4	-42	9	323	5
		R_2	555		341	7	209	4				

TABLE A6

EXPERIMENT IIb POOLED DATA

R_1 = "Top 1st"

R_2 = "Bottom 1st"

DEADLINE	τ	RESPONSE	N	$\hat{P}_1(\tau)$	TOTAL RT MEAN	TOTAL RT S.E.	TIK RELEASE RT MEAN	TIK RELEASE RT S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.	MARGINAL MEAN RT (TOTAL)	S.E.
400	20	R_1	638	0.67	340	5	217	3	61	8	320	4
		R_2	322		270	6	174	4				
	-20	R_1	364	0.38	288	7	176	5	-55	9	322	5
		R_2	596		343	6	219	4				
400	40	R_1	675	0.70	347	6	215	4	82	10	323	5
		R_2	285		265	8	152	5				
	-40	R_1	289	0.30	260	7	150	4	-84	9	319	5
		R_2	671		344	6	222	3				
					Overall Mean RT				319 ± 2			
900	5	R_1	563	0.59	447	7	297	4	-40	12	464	6
		R_2	397		487	10	316	6				
	-5	R_1	410	0.43	444	9	294	5	-40	11	467	6
		R_2	550		484	7	328	4				
900	10	R_1	633	0.66	460	6	307	4	-15	14	465	6
		R_2	327		475	13	298	7				
	-10	R_1	343	0.36	450	11	291	6	-30	13	469	6
		R_2	617		480	7	322	4				
900	20	R_1	748	0.78	462	6	306	3	10	17	460	6
		R_2	212		452	16	287	10				
	-20	R_1	231	0.24	424	15	263	8	-57	16	467	6
		R_2	729		481	6	330	4				
900	40	R_1	871	0.91	454	5	303	3	51	30	449	5
		R_2	89		403	30	232	15				
	-40	R_1	131	0.14	375	19	227	11	-77	20	441	5
		R_2	829		452	5	315	3				
					Overall Mean RT				460 ± 2			

TABLE A7 EXPERIMENT III

INDIVIDUAL SUBJECT DATA

V

C = CONTROL

T = GAP TEST

R_1 = "Top 1st"

R_2 = "Bottom 1st"

τ	RESPONSE	N	$\hat{P}_1(\tau)$	MEAN RT	S.E.	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.
C 30	R_1	505	0.94	963	18	974	18	-162	95
	R_2	35		1125	93				
-30	R_1	46	0.09	1593	104	833	18	831	105
	R_2	494		762	14				
20	R_1	432	0.80	1004	21	1011	20	-37	58
	R_2	108	$\pm$0.007	1041	54				
-20	R_1	93	0.17	1315	67	895	19	507	69
	R_2	447	$\pm$0.006	808	15				
10	R_1	357	0.66	1043	27	991	21	154	40
	R_2	183	$\pm$0.010	889	29				
-10	R_1	118	0.22	1341	57	972	22	473	60
	R_2	420	$\pm$0.007	868	20				
0	R_1	217	0.40	1161	40	986	22	293	47
	R_2	323		868	24				
						overall MEAN RT	952 $\pm$ 8		
T 20	R_1	440	0.82	986	20	1018	21	-172	70
	R_2	99	$\pm$0.006	1158	67				
-20	R_1	75	0.14	1274	59	844	16	500	61
	R_2	463	$\pm$0.005	774	14				
10	R_1	308	0.57	1053	25	1026	21	62	43
	R_2	232	$\pm$0.011	991	35				
-10	R_1	167	0.31	1183	42	984	22	289	49
	R_2	371	$\pm$0.009	894	25				

TABLE A7 EXPERIMENT III

INDIVIDUAL SUBJECT DATA

RH

C = CONTROL

T = GAP TEST

R_1 = "Top 1st"

R_2 = "Bottom 1st"

τ	RESPONSE	N	$\hat{P}_1(\tau)$	MEAN RT	S.E.	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.
C 30	R_1	524	0.97	475	4	476	4	-35	37
	R_2	16		510	37				
-30	R_1	11	0.02	531	54	483	3	49	54
	R_2	529		482	3				
20	R_1	503	0.93	490	4	492	4	-27	22
	R_2	37	±0.003	517	22				
-20	R_1	31	0.06	578	26	504	4	79	26
	R_2	509	±0.002	499	4				
10	R_1	418	0.77	533	6	537	5	-16	11
	R_2	122	±0.008	549	9				
-10	R_1	112	0.21	563	11	532	6	39	12
	R_2	428	±0.007	524	5				
0	R_1	228	0.42	555	7	549	5	11	9
	R_2	312		544	6				
						overall MEAN RT	511 ±2		
T 20	R_1	503	0.93	499	4	504	4	-73	22
	R_2	37	±0.003	572	22				
-20	R_1	33	0.06	560	21	504	4	60	21
	R_2	507	±0.002	500	4				
10	R_1	413	0.77	523	8	531	7	-33	14
	R_2	127	±0.008	556	11				
-10	R_1	157	0.29	540	9	527	4	19	10
	R_2	383	±0.009	521	5				

TABLE A7 EXPERIMENT III

INDIVIDUAL SUBJECT DATA

R

C = CONTROL

T = GAP TEST

R_1 = "Top 1st"

R_2 = "Bottom 1st"

	τ	RESPONSE	N	$\hat{P}_1(\tau)$	MEAN RT	S.E.	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.
C	30	R_1	529	0.98	457	6	462	7	-258	126
		R_2	11		715	126				
	-30	R_1	21	0.04	695	127	500	7	203	127
		R_2	519		492	5				
	20	R_1	487	0.90	511	8	521	8	-101	33
		R_2	53	± 0.004	612	32				
	-20	R_1	64	0.12	674	45	546	9	145	46
		R_2	476	± 0.005	529	8				
	10	R_1	385	0.71	557	11	584	9	- 94	20
		R_2	155	± 0.009	651	17				
	-10	R_1	126	0.23	641	28	600	10	53	30
		R_2	414	± 0.008	588	10				
	0	R_1	249	0.46	596	14	619	10	- 43	21
		R_2	291		639	15				
							overall MEAN RT	548 ± 3		
T	20	R_1	490	0.91	489	8	501	9	-134	51
		R_2	50	± 0.004	623	50				
	-20	R_1	61	0.11	605	33	545	7	68	34
		R_2	479	± 0.004	537	6				
	10	R_1	399	0.74	539	13	559	11	- 75	24
		R_2	141	± 0.008	614	20				
	-10	R_1	174	0.32	570	19	572	10	- 3	22
		R_2	366	± 0.009	573	11				

TABLE A7 EXPERIMENT III

INDIVIDUAL SUBJECT DATA

JO

C = CONTROL

T = GAP TEST

R_1 = "Top 1st"

R_2 = "Bottom 1st"

τ	RESPONSE	N	$\hat{P}_1(\tau)$	MEAN RT	S.E.	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.
C 30	R_1	513	0.95	461	6	462	6	-28	36
	R_2	27		489	35				
-30	R_1	72	0.13	499	24	496	6	4	25
	R_2	468		495	6				
20	R_1	476	0.88	482	6	490	6	-66	30
	R_2	64	±0.005	548	29				
-20	R_1	133	0.25	522	13	506	6	21	14
	R_2	407	±0.008	501	6				
10	R_1	349	0.65	500	9	521	7	-60	15
	R_2	191	±0.010	560	12				
-10	R_1	189	0.35	485	9	518	7	-51	13
	R_2	351	±0.010	536	9				
0	R_1	237	0.44	503	9	518	6	-26	12
	R_2	303		529	8				
						overall MEAN RT 502	±2		
T 20	R_1	469	0.87	481	7	490	7	-71	21
	R_2	71	±0.005	552	20				
-20	R_1	126	0.23	520	30	509	8	14	31
	R_2	414	±0.008	506	6				
10	R_1	347	0.64	473	7	492	6	-53	13
	R_2	193	±0.010	526	11				
-10	R_1	231	0.43	495	9	505	6	-18	12
	R_2	309	±0.011	513	8				

TABLE A8

EXPERIMENT III POOLED DATA

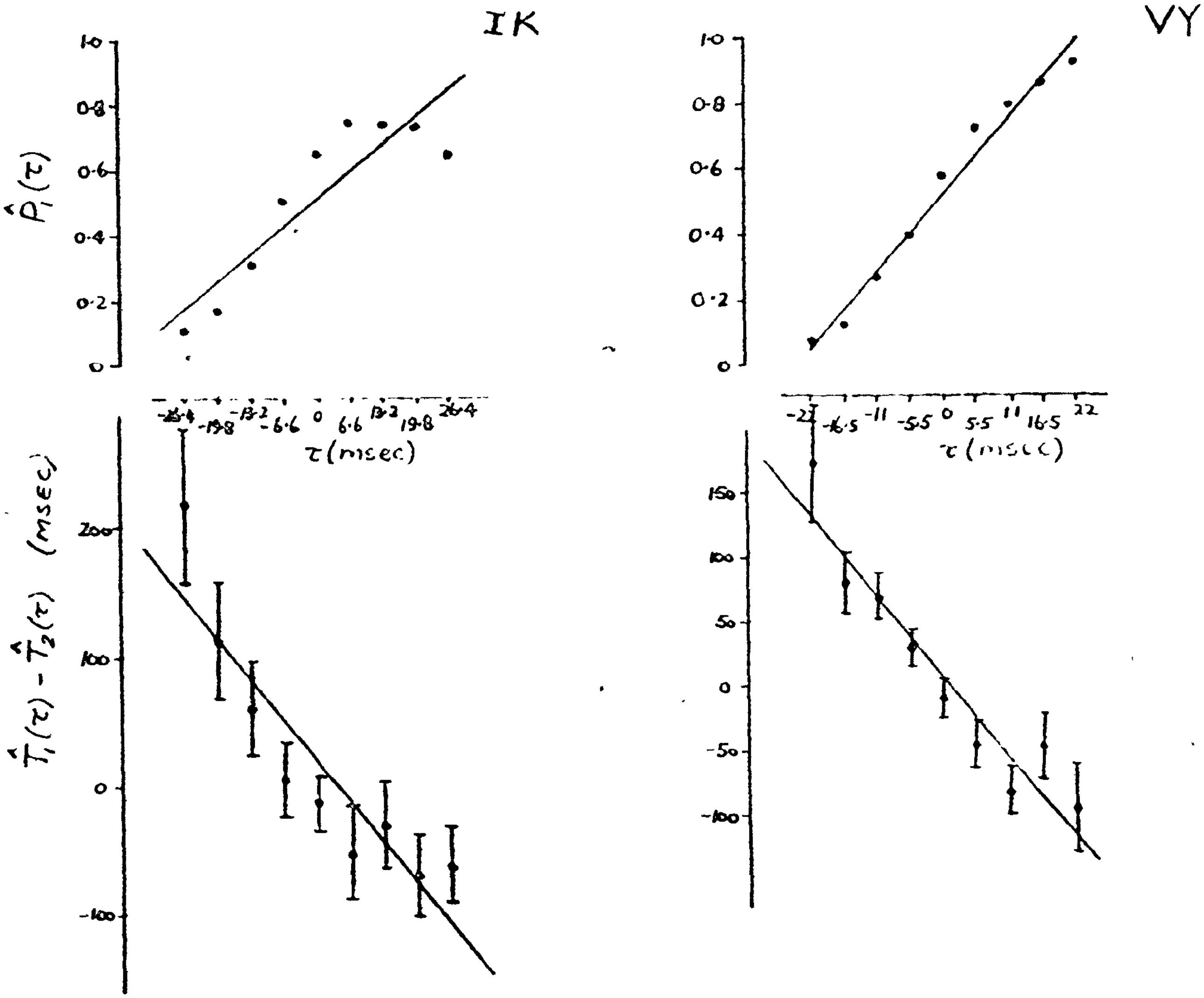
C = CONTROL
T = GAP TEST

R_1 = "Top 1st"
R_2 = "Bottom 1st"

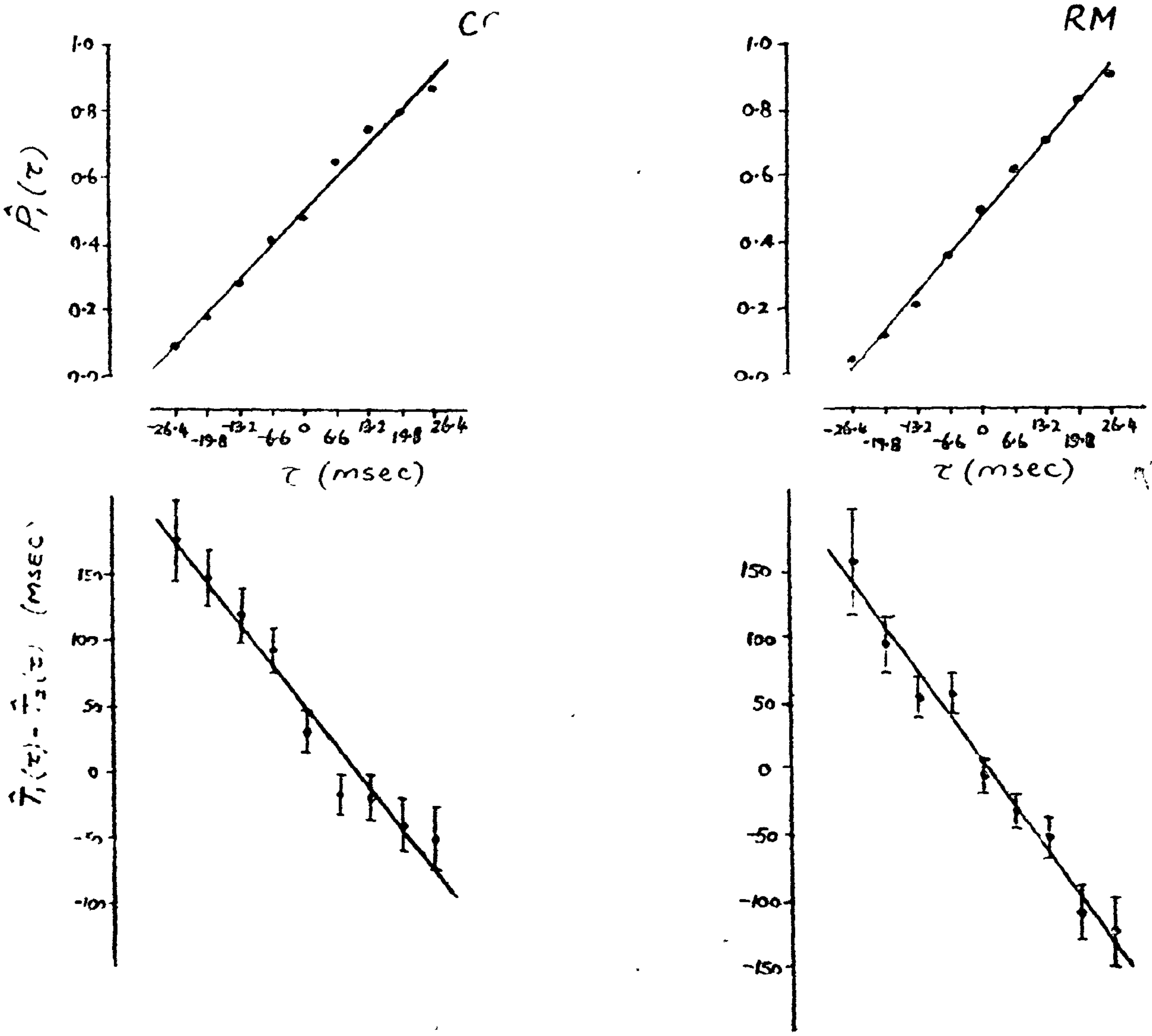
τ	RESPONSE	N	$\hat{P}_1(\tau)$	S.E.	MEAN RT	S.E.	MARGINAL MEAN RT	S.E.	$\hat{T}_1(\tau)-\hat{T}_2(\tau)$	S.E.
C 30	R_1	2071	0.96		586	7	594	7	-185	51
	R_2	89			771	51				
-30	R_1	150	0.07		865	55	577	6	309	55
	R_2	2010			556	5				
+20	R_1	1898	0.88	0.002	610	7	628	7	-150	29
	R_2	262			760	28				
-20	R_1	321	0.15	0.003	787	29	612	7	205	30
	R_2	1839			582	6				
+10	R_1	1509	0.70	0.005	652	9	658	7	- 20	14
	R_2	651			672	11				
-10	R_1	545	0.25	0.004	722	20	655	7	89	21
	R_2	1613			633	7				
0	R_1	931	0.43		694	13	668	8	46	16
	R_2	1229			648	9				
							overall MEAN RT	628 ± 3		
T +20	R_1	1902	0.88	0.002	605	7	628	7	-197	34
	R_2	257			802	33				
-20	R_1	295	0.14	0.003	734	28	600	6	155	28
	R_2	1863			579	5				
+10	R_1	1467	0.68	0.005	627	9	652	8	- 78	17
	R_2	693			705	15				
-10	R_1	729	0.34	0.005	680	15	646	7	51	17
	R_2	1429			629	8				

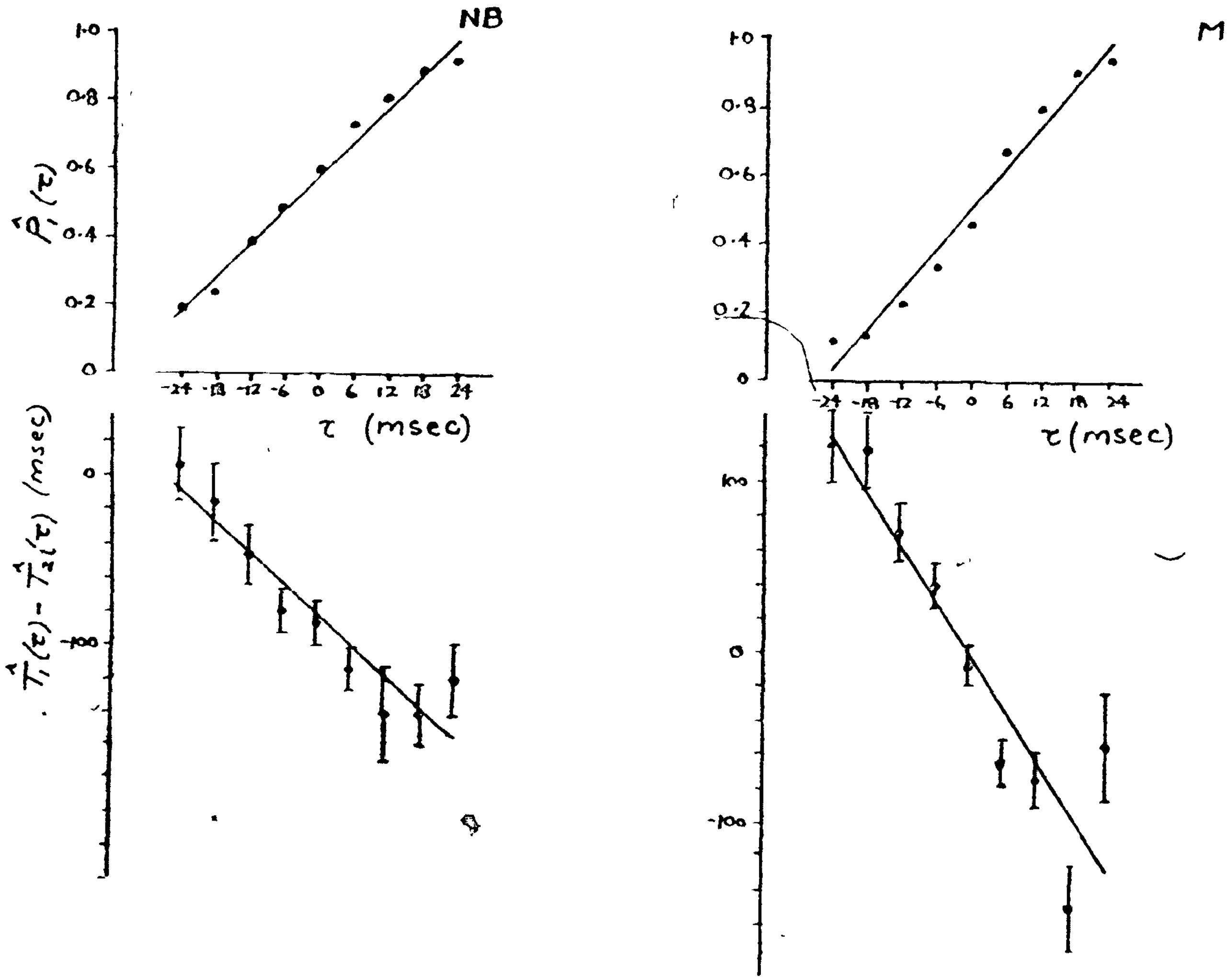
Appendix III: Figures for individual Ss.

Figure A1. Psychometric functions and the difference in response conditioned mean RT as a function of ISI for each S in Experiment I.



IK
VY
$\hat{P}_1(\tau)$
1·0
0·8
0·6
0·4
0·2
0
-26·4
-19·8
-13·2
-6·6
0
6·6
13·2
19·8
26·4
τ (msec)
-22
-16·5
-11
-5·5
0
5·5
11
16·5
22
τ (msec)
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)
200
100
0
-100
150
100
50
0
-50
-100


RM
$\hat{P}_1(\tau)$
τ (msec)
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (MSEC)


NB
MG
1.0
0.8
0.6
0.4
0.2
0
-24 -18 -12 -6 0 6 12 18 24
τ (msec)
0
-100
100

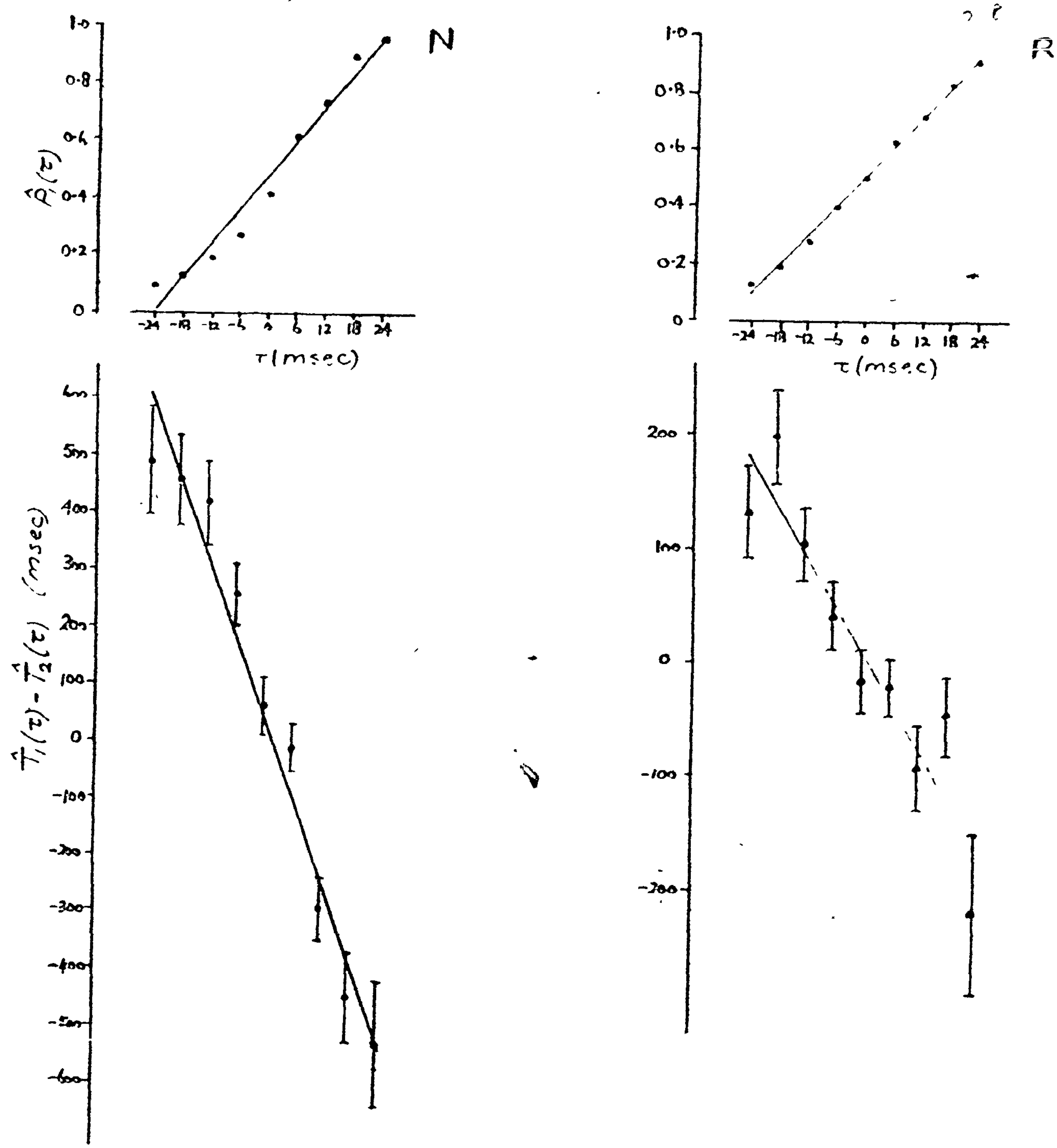
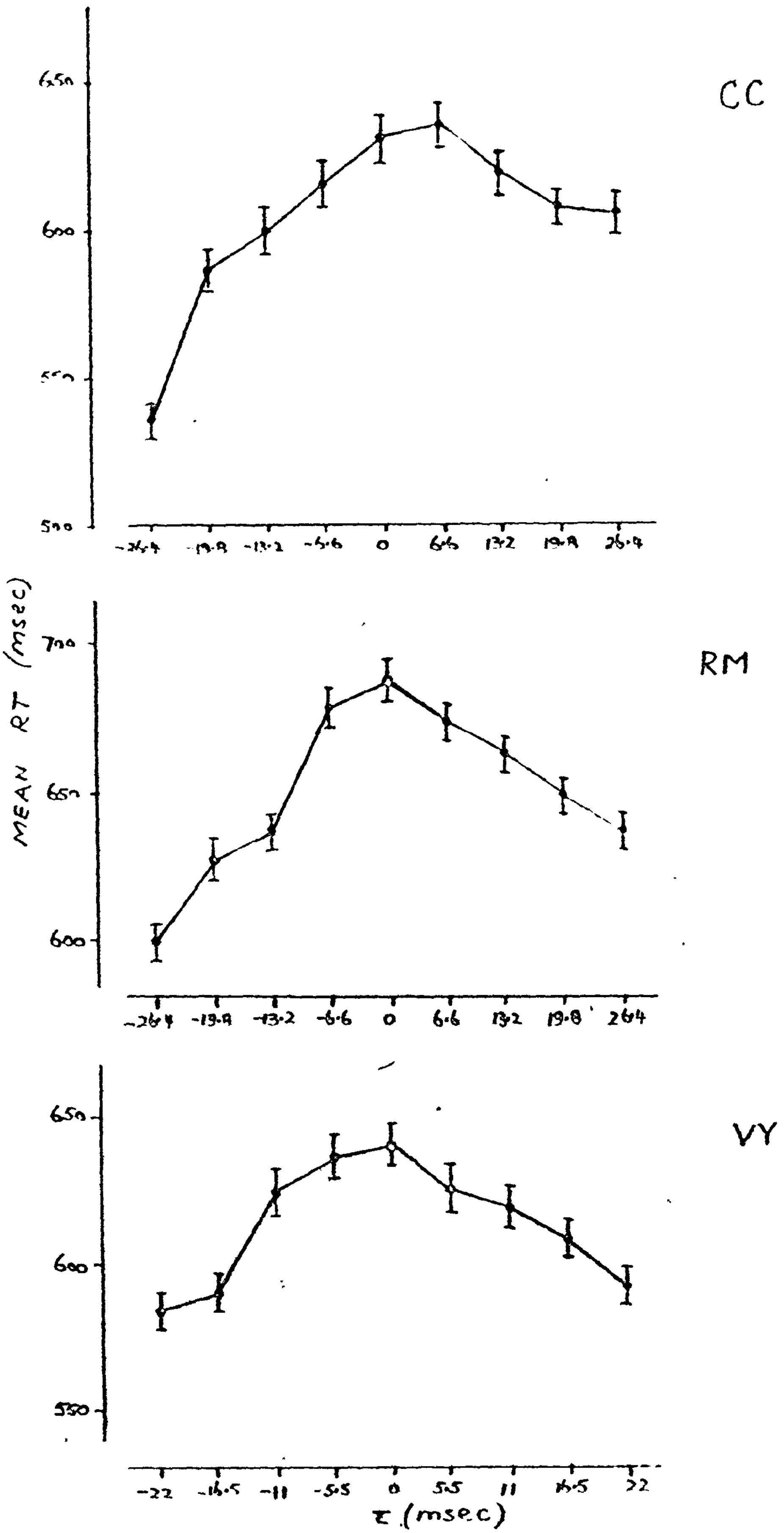
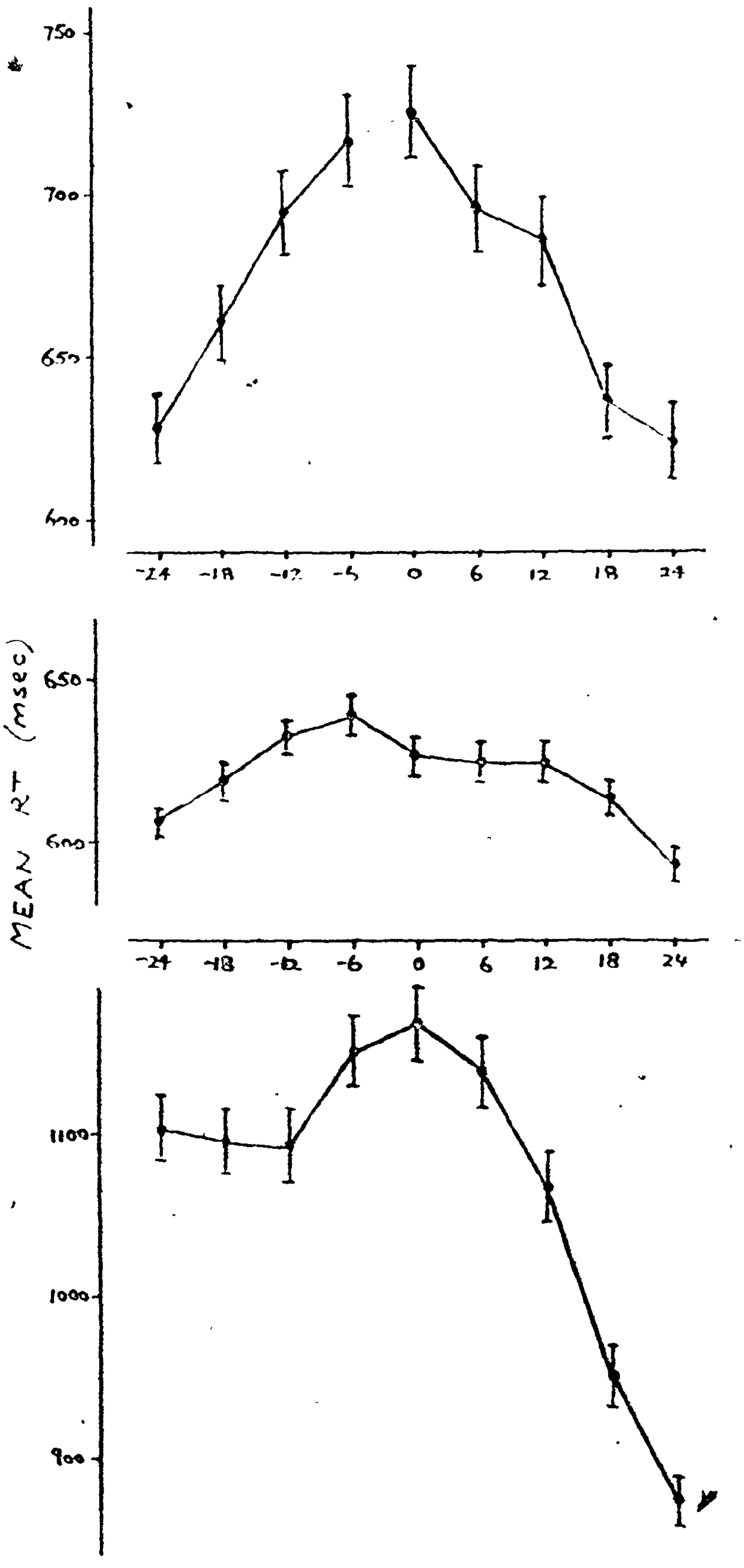
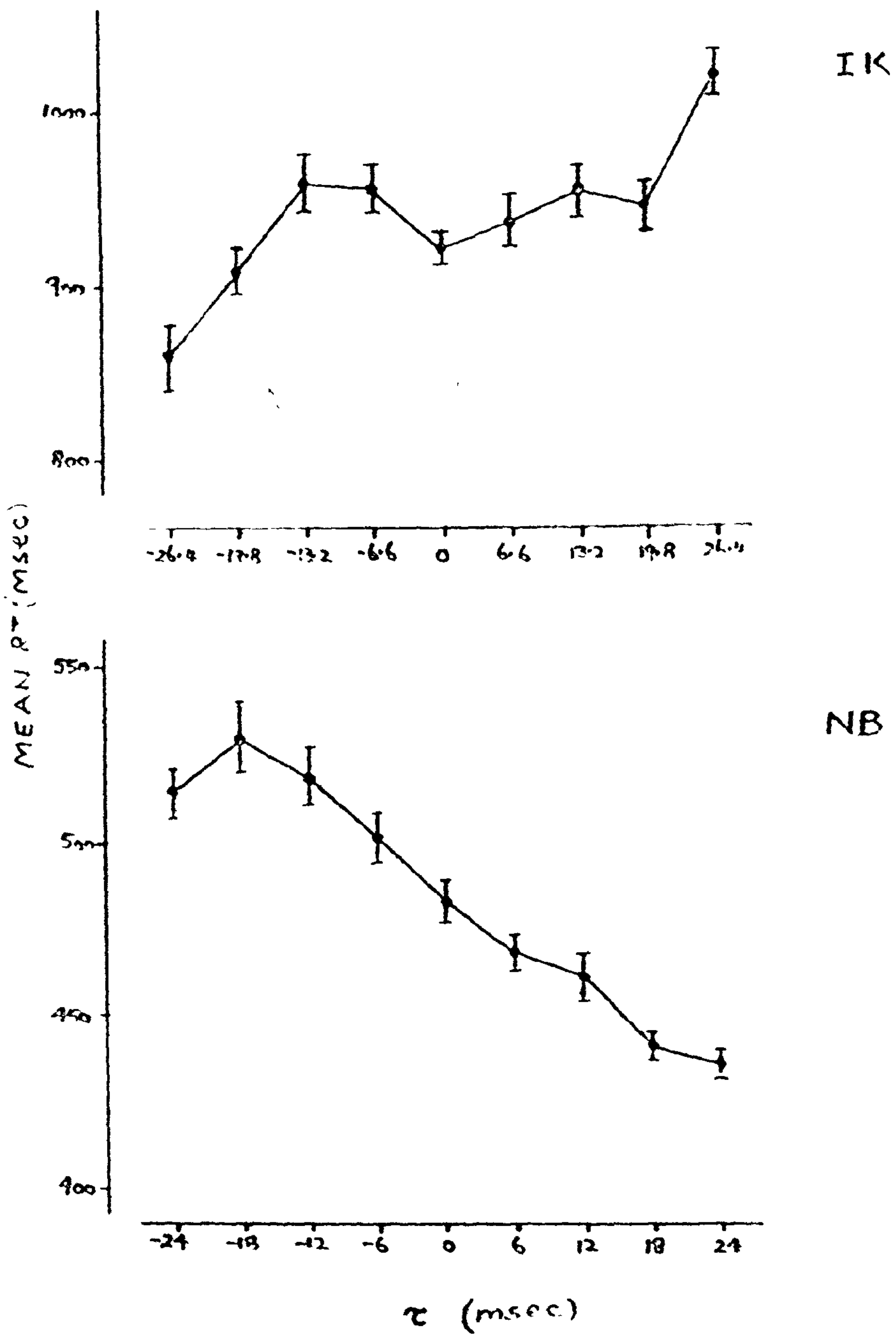

N
R
$\hat{\rho}(\tau)$
τ (msec)
τ (msec)
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)

Figure A2. Marginal mean RT as a function of ISI for each S in Experiment I.

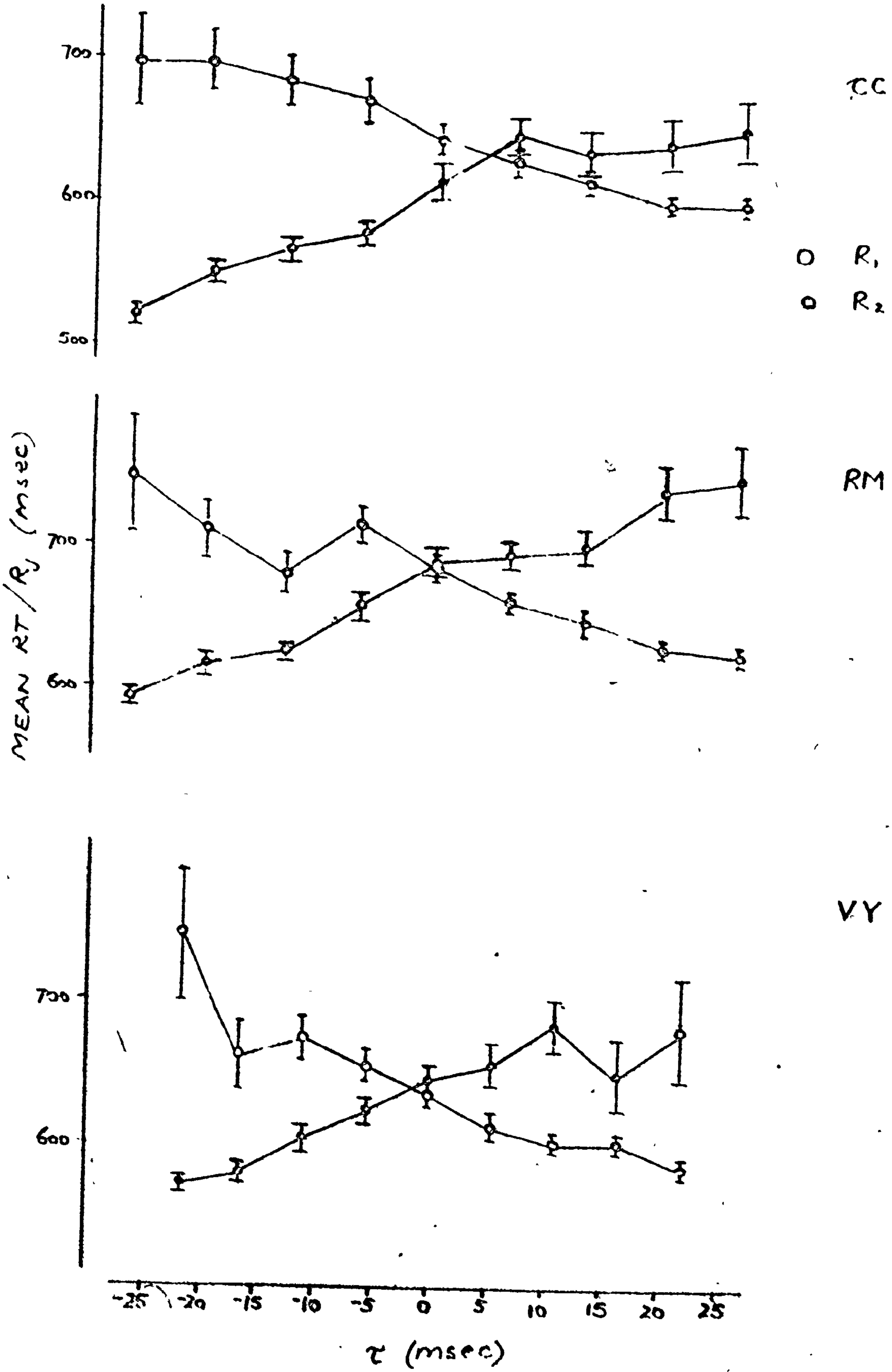

CC
RM
VY
MEAN RT (msec)
τ (msec)


R
MG
N
MEAN RT (msec)
τ (msec)
750
700
650
600
1100
1000
900
-24 -18 -12 -6 0 6 12 18 24

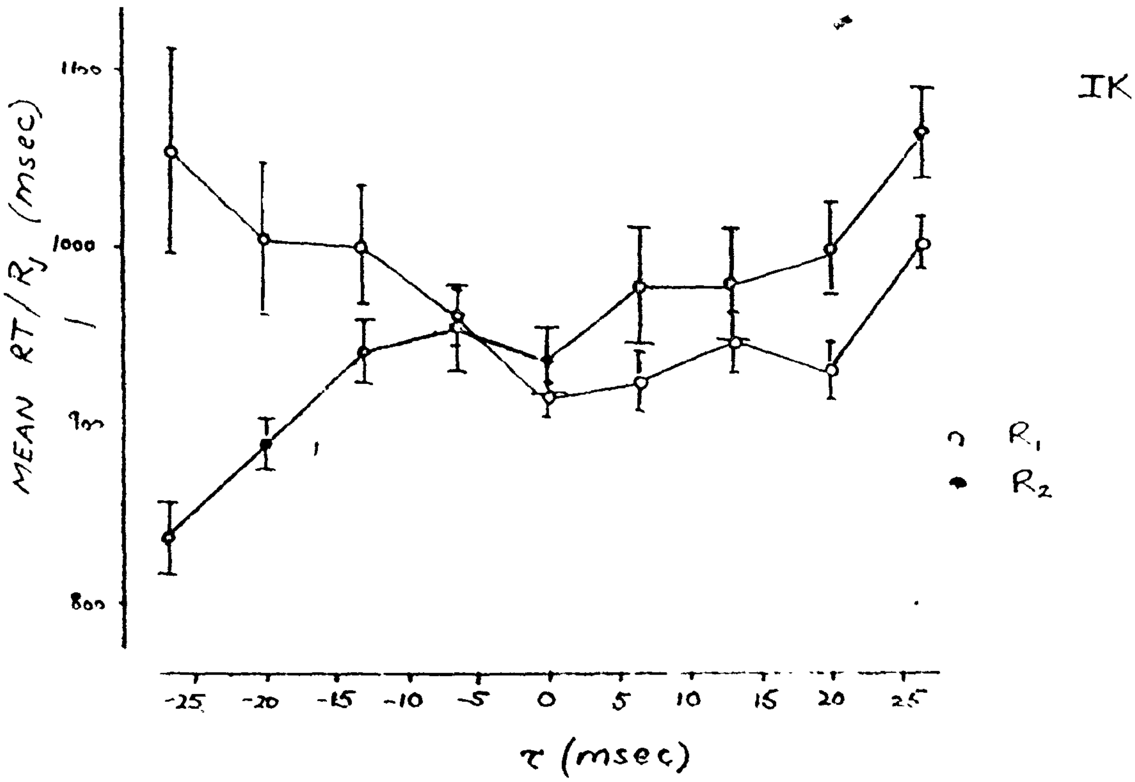


IK
1000
900
800
-26.4
-19.8
-13.2
-6.6
0
6.6
13.2
19.8
26.4
MEAN RT (msec)
NB
550
500
450
400
-24
-18
-12
-6
0
6
12
18
24
τ (msec)

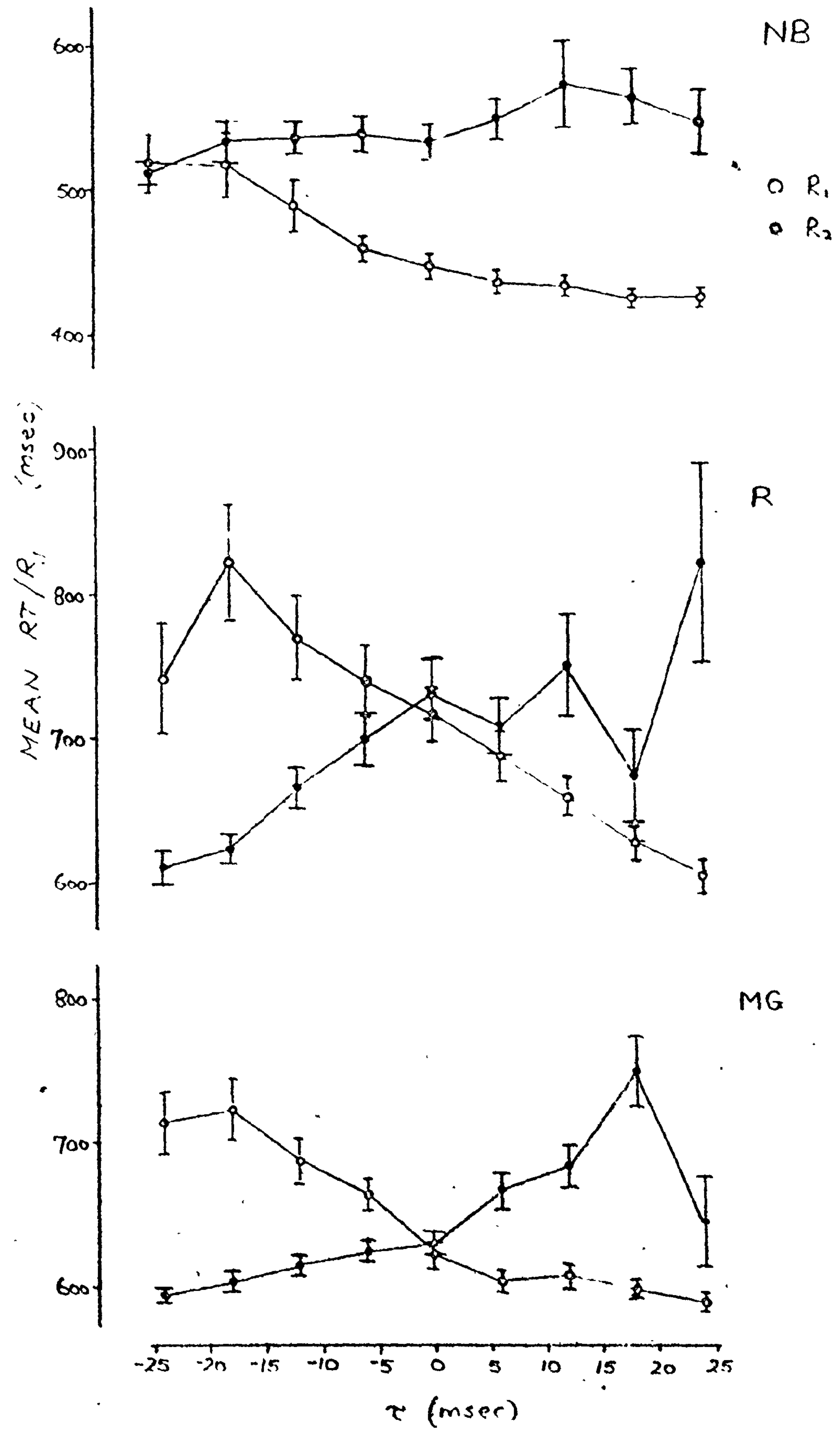
Figure A3. Response conditioned mean RT as a function of ISI for each $\underline{S}$ in Experiment I.

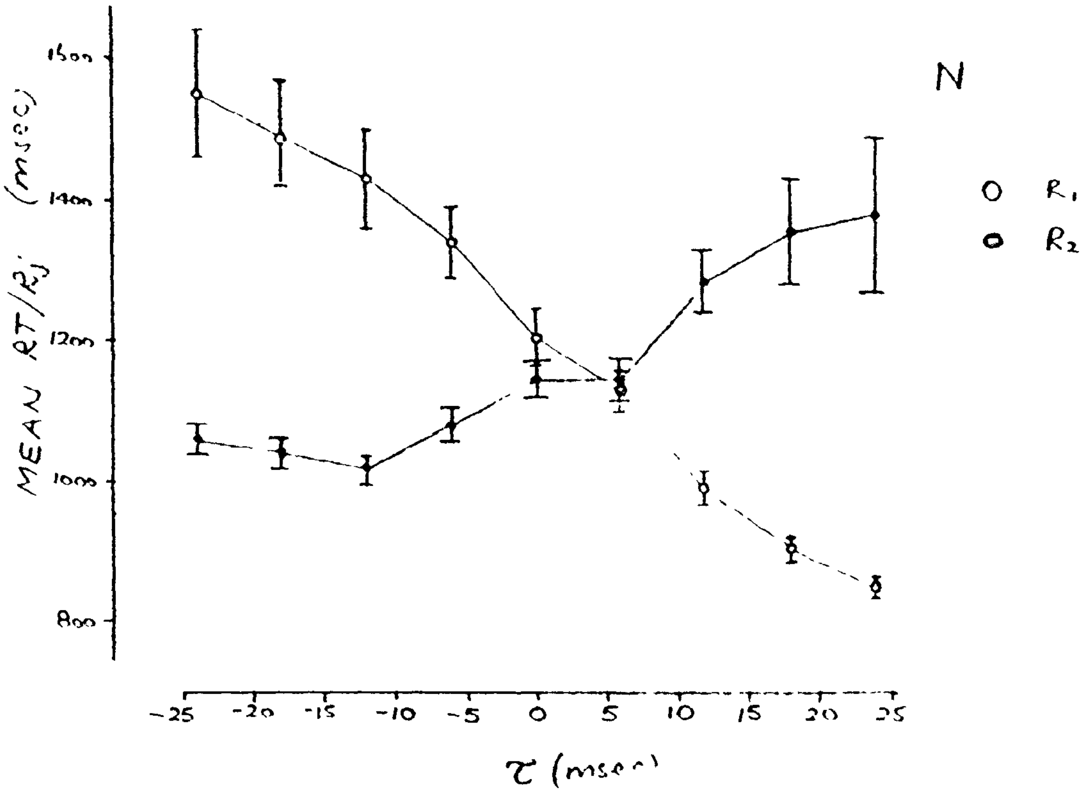


TC
RM
VY
O R1
o R2
MEAN RT/RJ (msec)
700
600
500
-25 -20 -15 -10 -5 0 5 10 15 20 25
τ (msec)



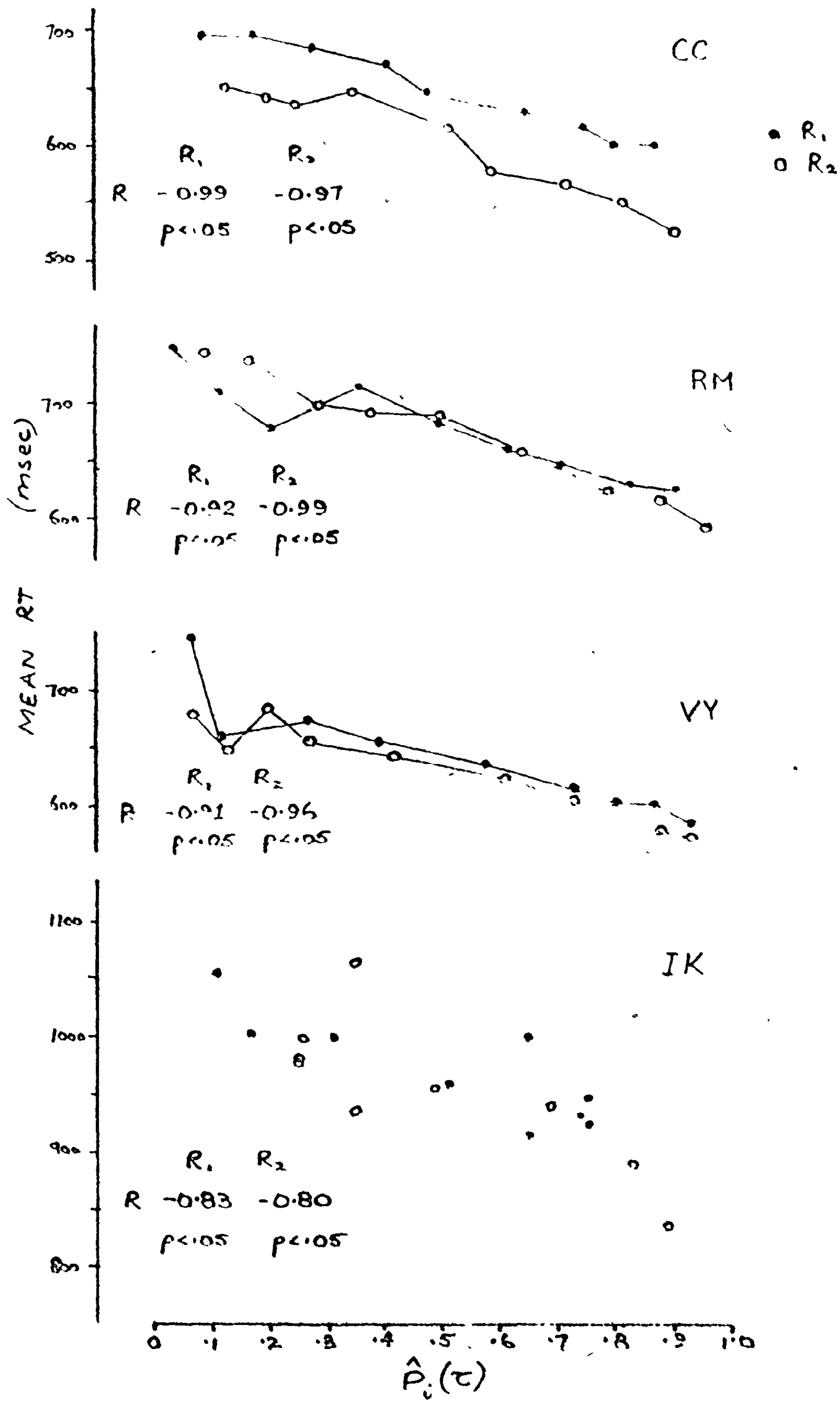
IK
MEAN RT/R_J (msec)
1100
1000
900
800
-25
-20
-15
-10
-5
0
5
10
15
20
25
τ (msec)
R_1
R_2


NB
R
MG
O R_1
● R_2
MEAN RT/R_1 (msec)
600
500
400
900
800
700
600
800
700
600
-25 -20 -15 -10 -5 0 5 10 15 20 25
τ (msec)

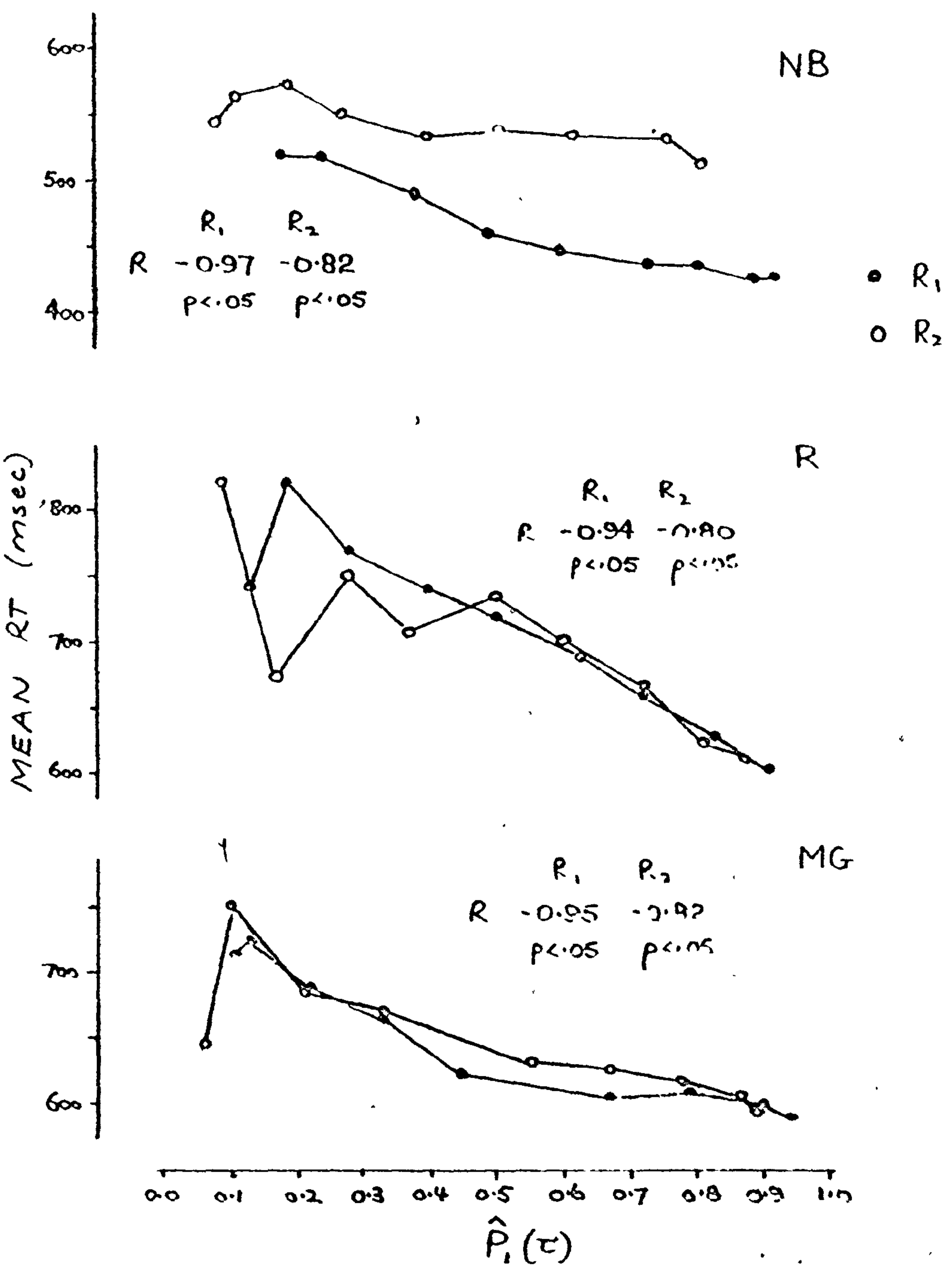


N
R1
R2
MEAN RT/Rj (msec)
1600
1400
1200
1000
800
-25
-20
-15
-10
-5
0
5
10
15
20
25
τ (mse

Figure A4. Latency probability functions for each S in Experiment I.



CC
R₁
R₂
R R₁ R₂
-0·99 -0·97
p<·05 p<·05
RM
R -0·92 -0·99
p<·05 p<·05
VY
R -0·91 -0·96
p<·05 p<·05
IK
R -0·83 -0·80
p<·05 p<·05
MEAN RT (msec)
P̂ᵢ(τ)


NB
R1 R2
R -0.97 -0.82
p<.05 p<.05
• R1
o R2
R
R1 R2
R -0.94 -0.90
p<.05 p<.05
MG
R1 R2
R -0.95 -0.92
p<.05 p<.05
MEAN RT (msec)
$\hat{P}_1(\tau)$

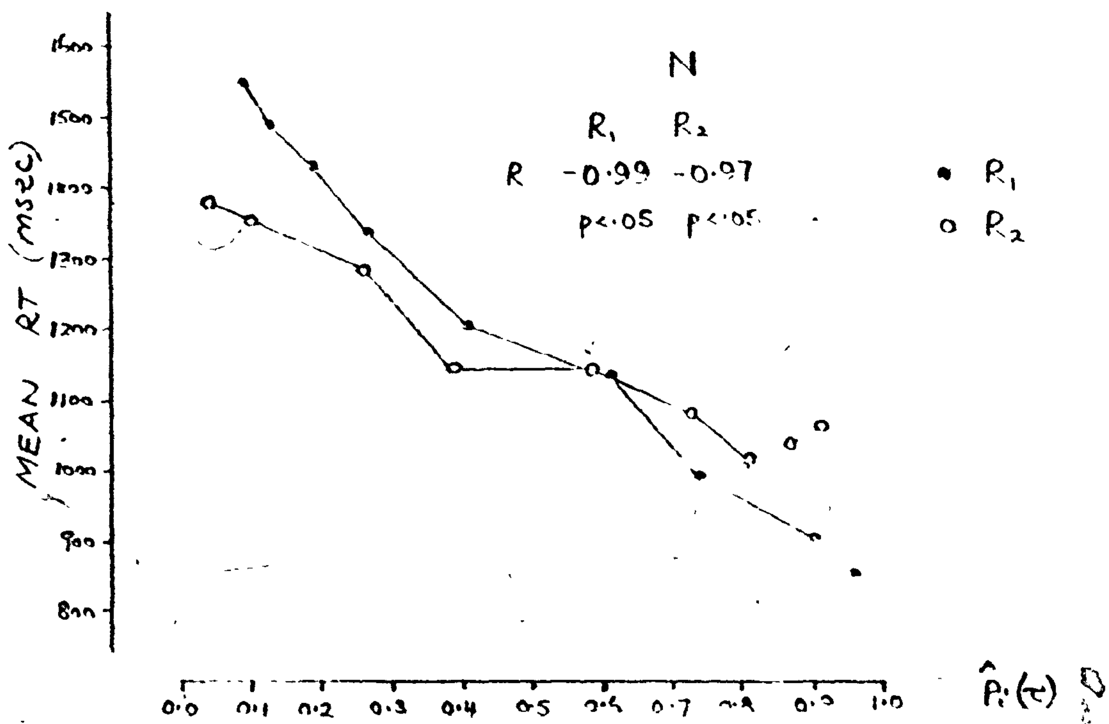
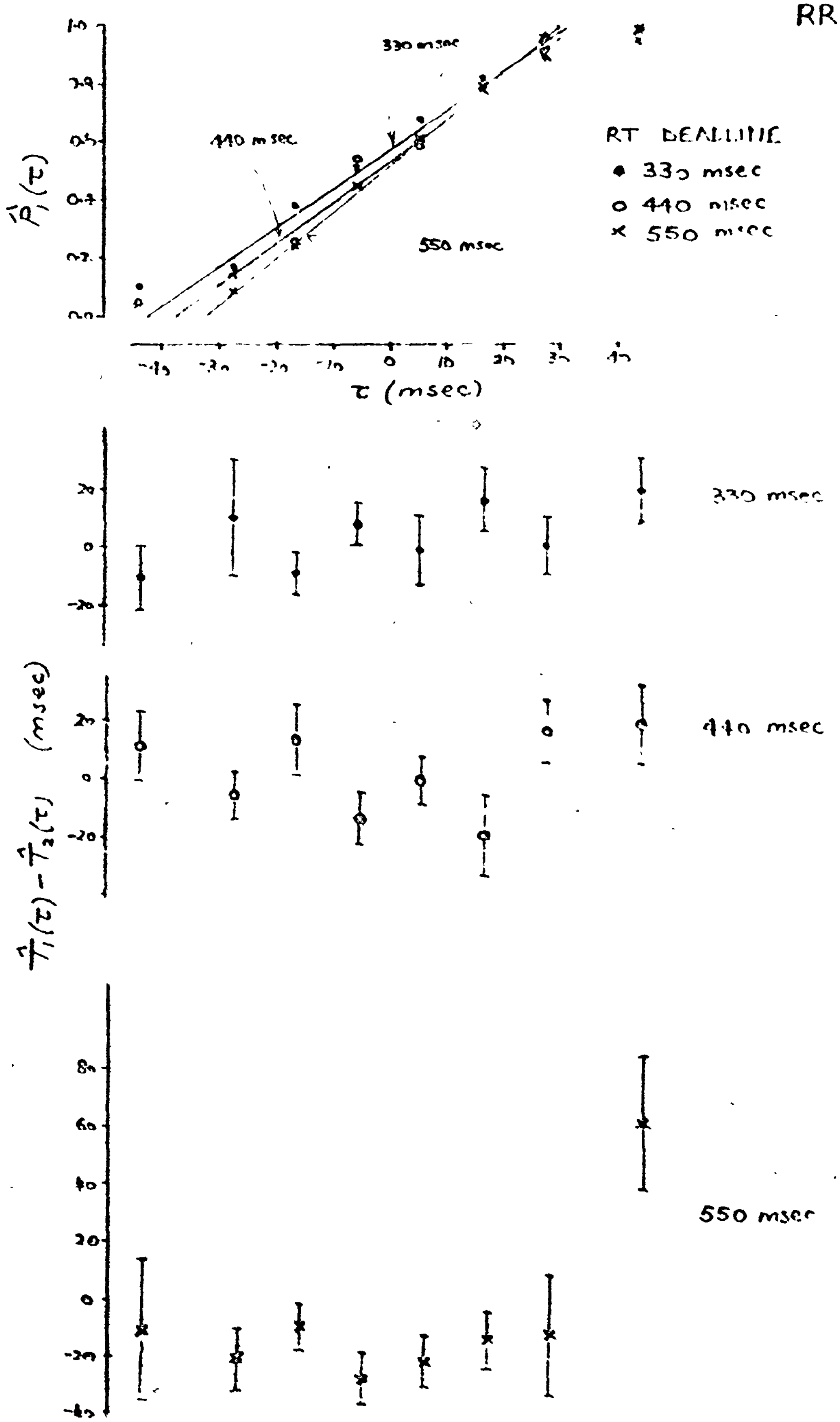
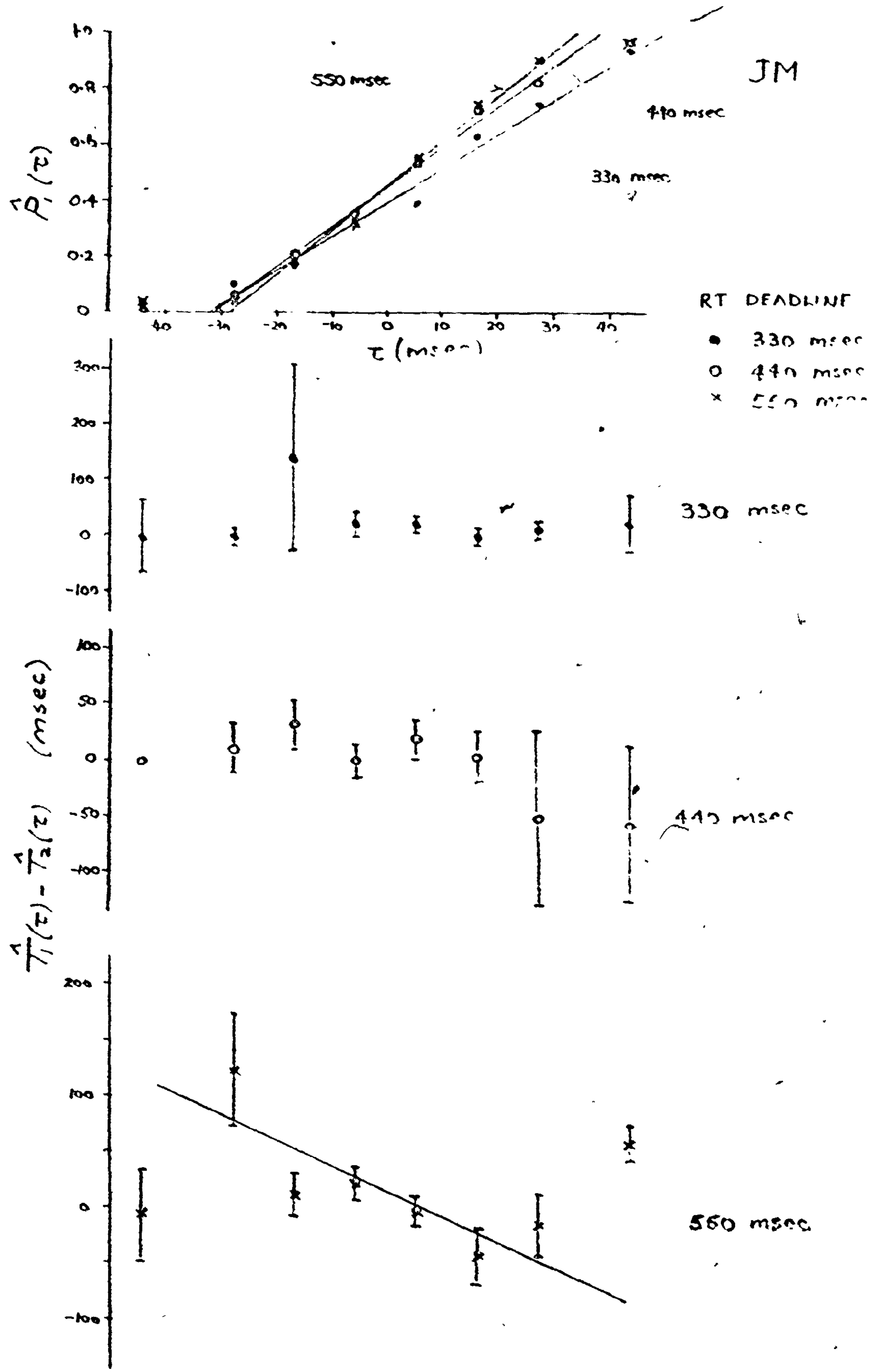
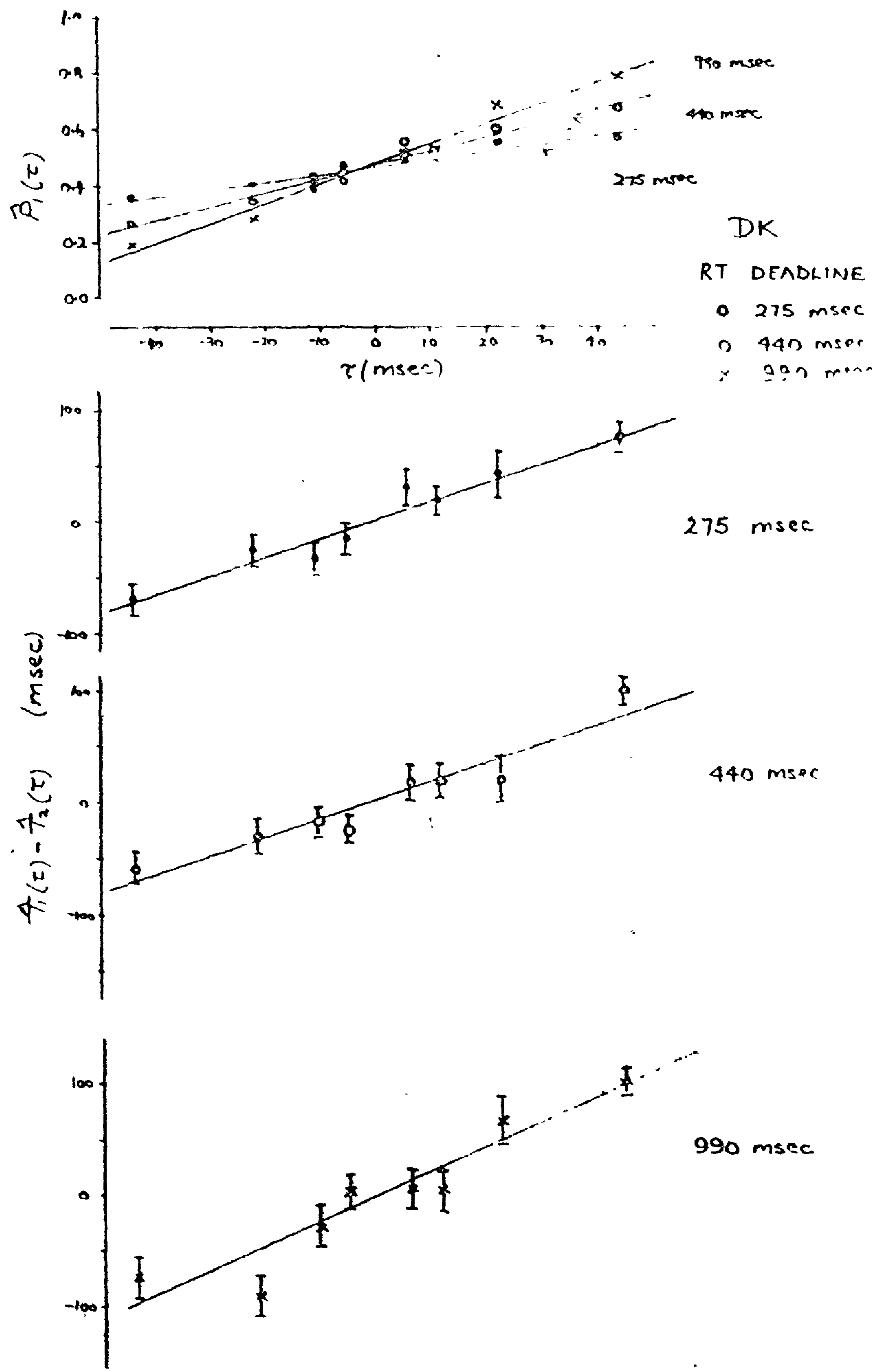

N
R₁ R₂
R -0·99 -0·97
p<·05 p<·05
• R₁
o R₂
MEAN RT (msec)
1600
1500
1400
1300
1200
1100
1000
900
800
0·0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
P̂ᵢ(τ)

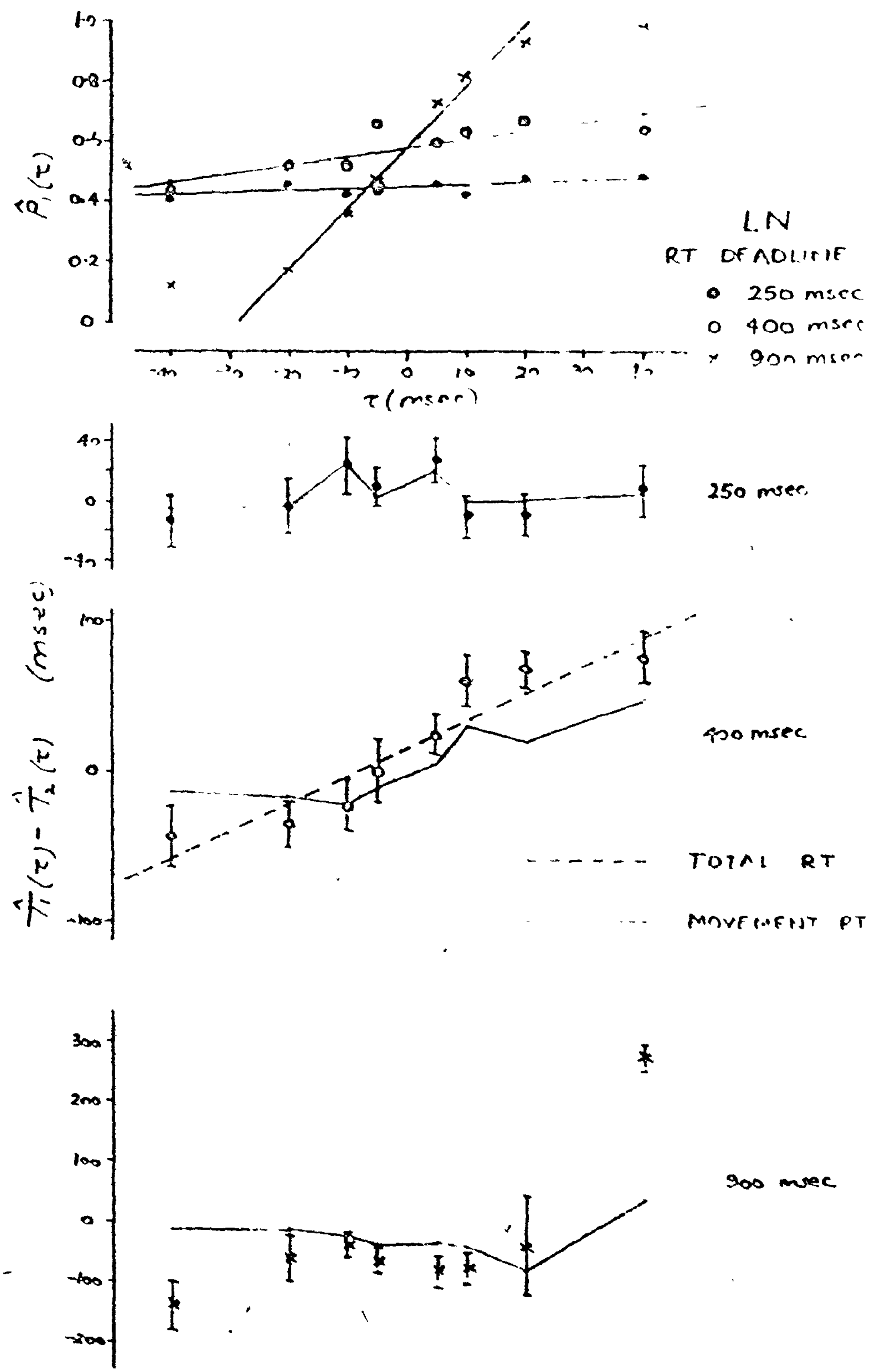
Figure A5. Psychometric functions and the differences in response conditioned mean RTs as a function of ISI for each RT deadline condition and for each S in Experiment II.

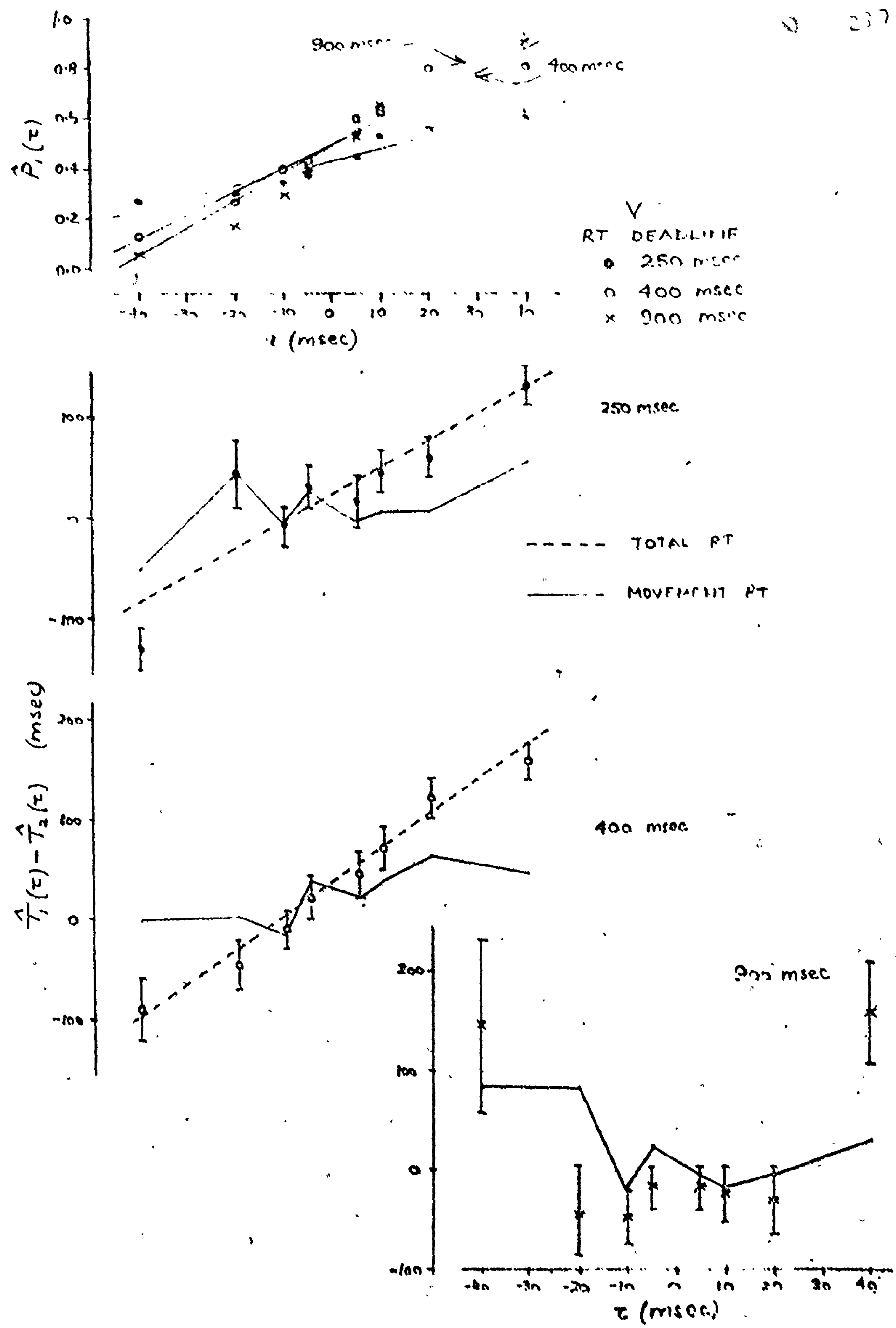

RR
RT DEADLINE
• 330 msec
o 440 msec
× 550 msec
330 msec
440 msec
550 msec
τ (msec)
330 msec
440 msec
550 msec
(msec)



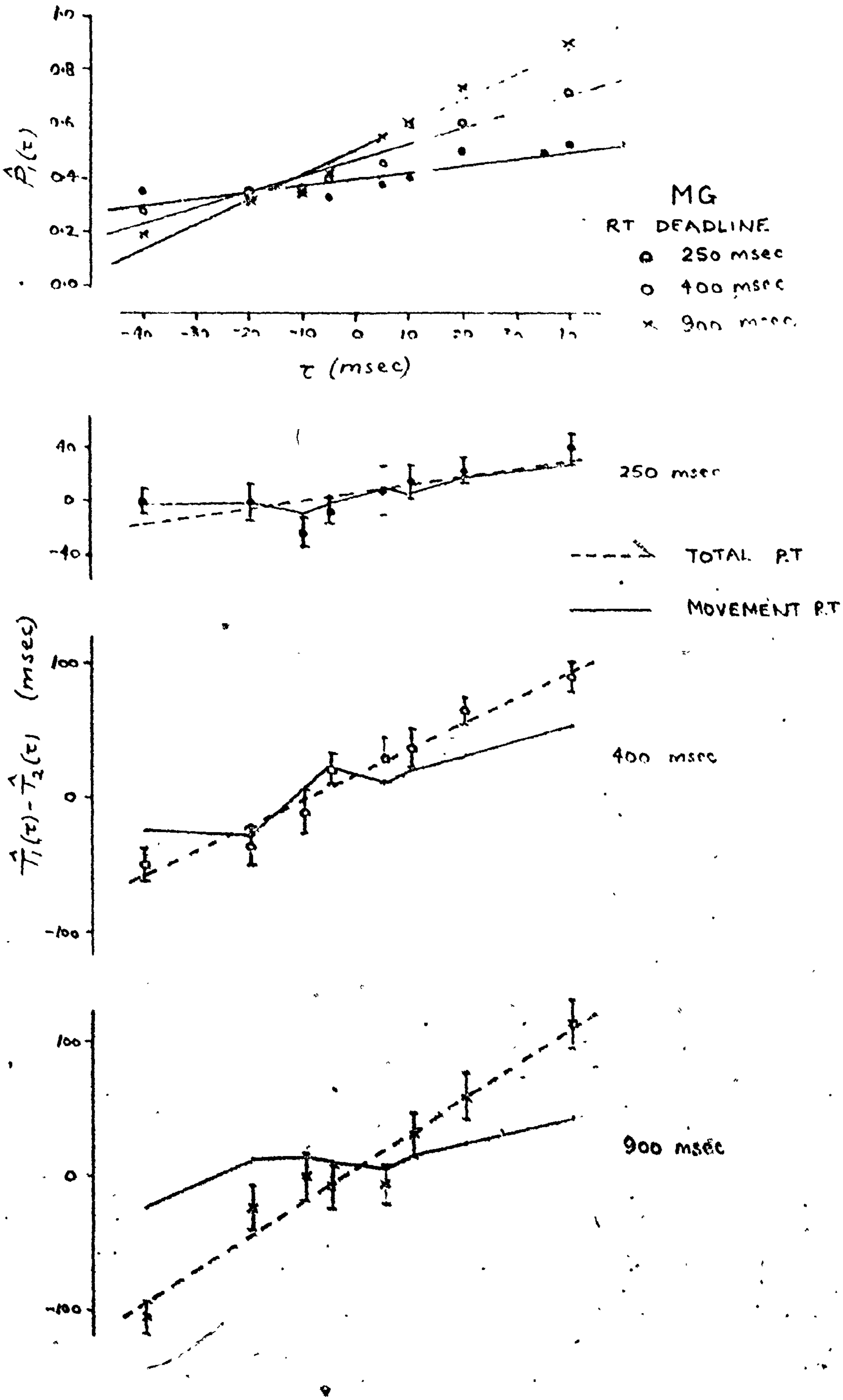
JM
550 msec
440 msec
330 msec
RT DEADLINE
330 msec
440 msec
330 msec
440 msec
560 msec


990 msec
440 msec
275 msec
DK
RT DEADLINE
275 msec
440 msec
τ (msec)
275 msec
440 msec
990 msec
(msec)

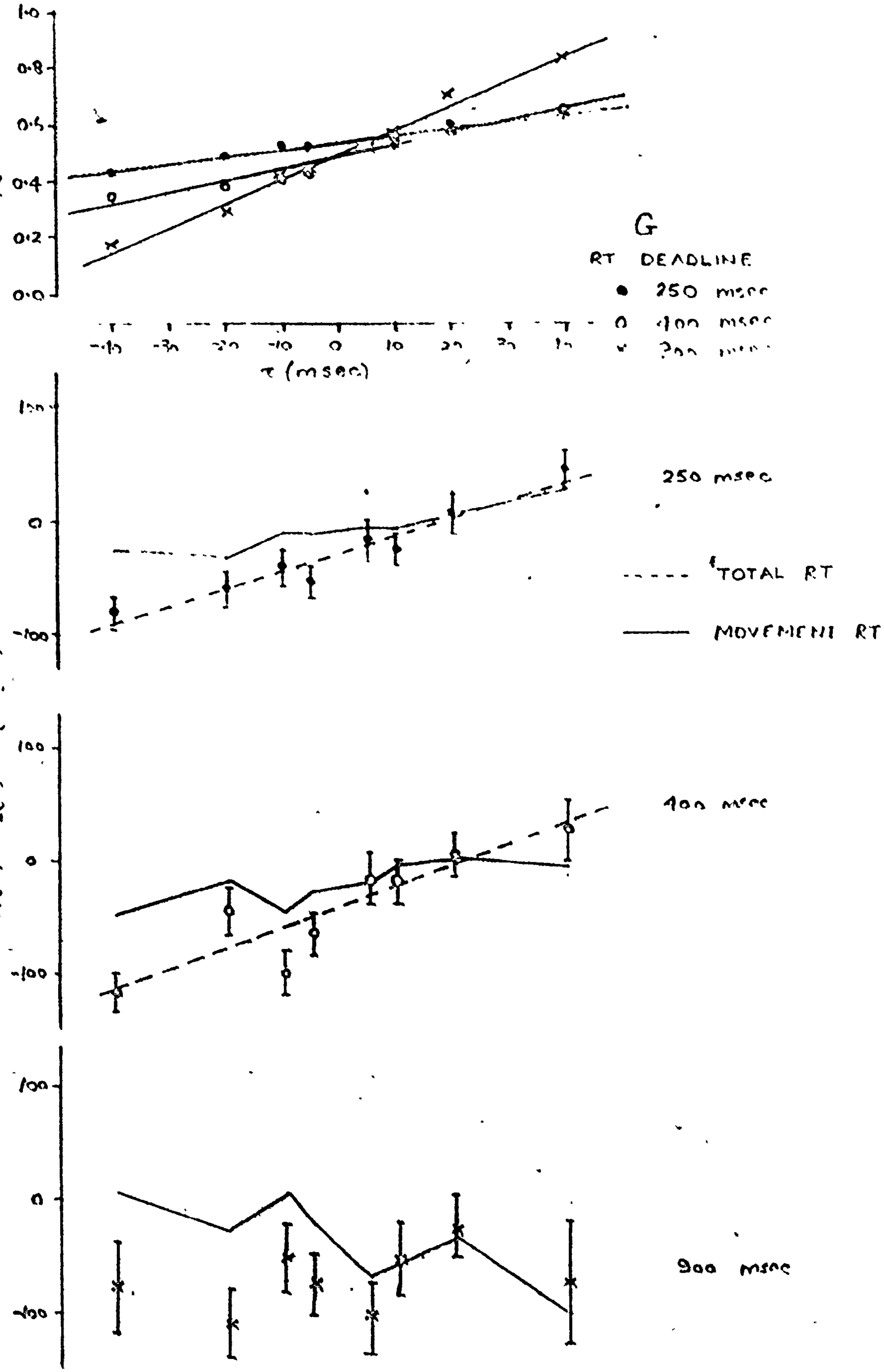

LN
RT DEADLINE
250 msec
400 msec
900 msec
$\hat{P}_1(\tau)$
τ (msec)
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)
250 msec
400 msec
TOTAL RT
MOVEMENT RT
900 msec



$\hat{P}_1(\tau)$
900 msec
400 msec
RT DEADLINE
250 msec
400 msec
900 msec
τ (msec)
250 msec
TOTAL RT
MOVEMENT RT
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)
400 msec
900 msec
τ (msec)

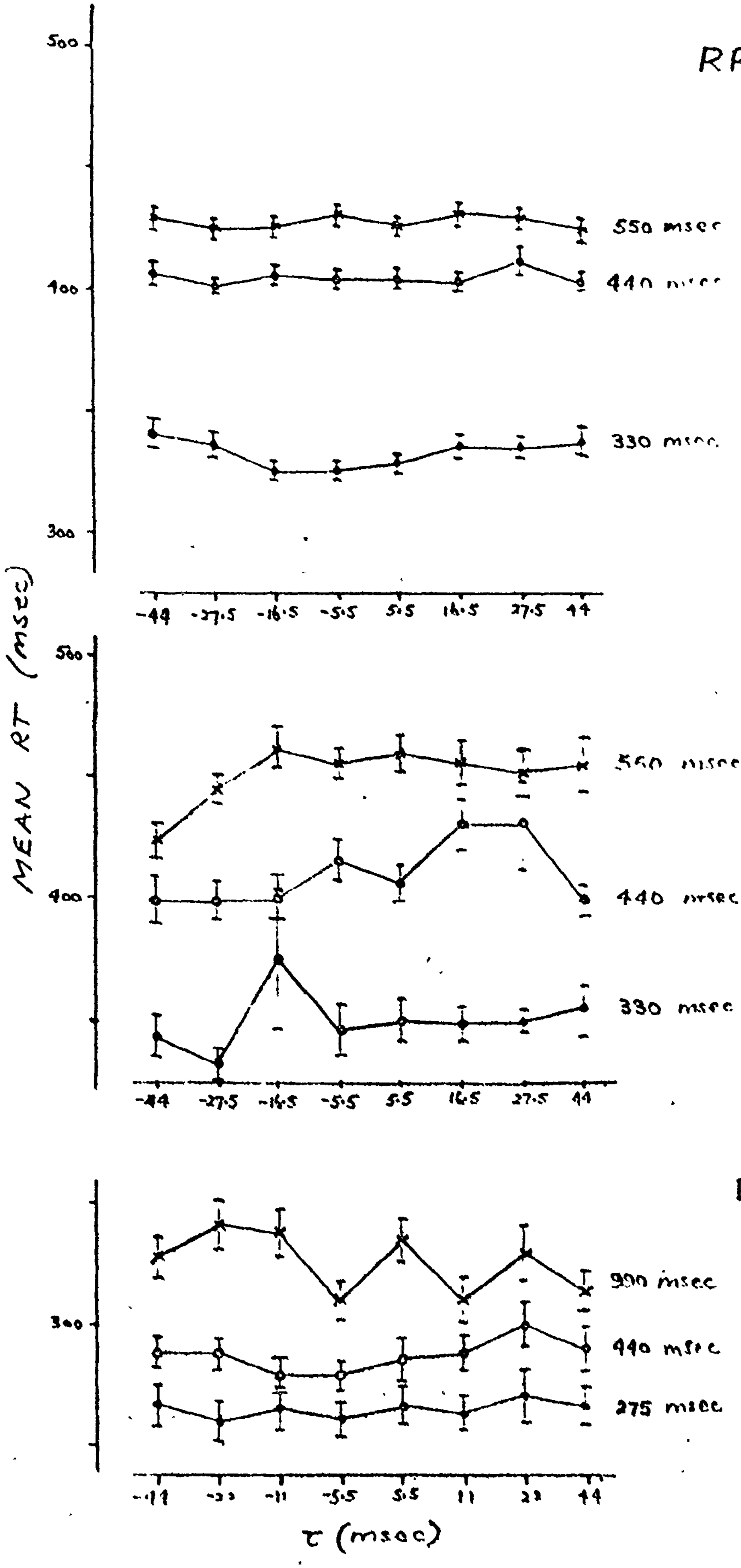


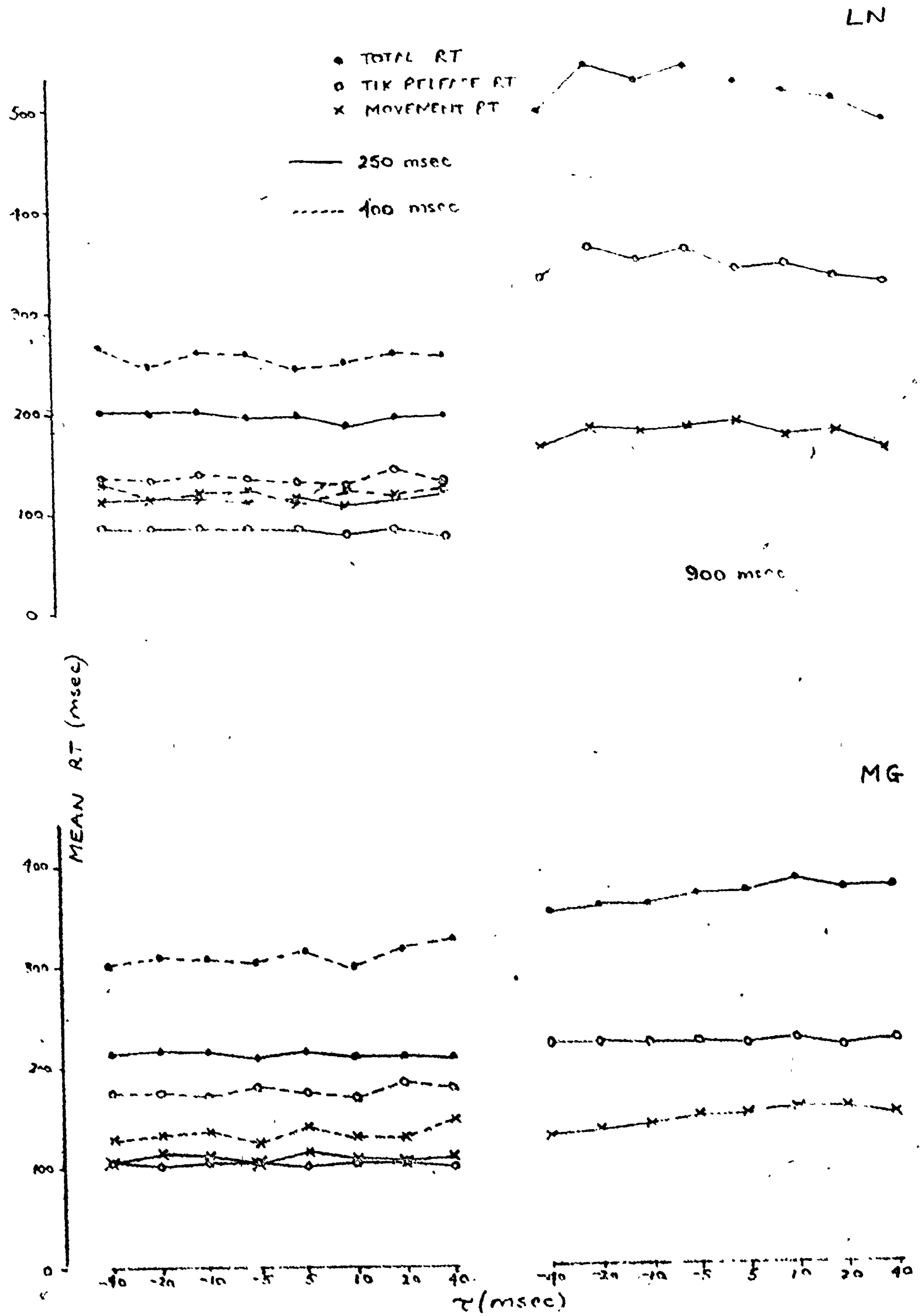
1.0
0.8
0.6
0.4
0.2
0.0
$\hat{p}_1(\tau)$
-40
-30
-20
-10
0
10
20
30
40
τ (msec)
MG
RT DEADLINE
250 msec
400 msec
900 msec
40
0
-40
250 msec
TOTAL RT
MOVEMENT RT
100
0
-100
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)
400 msec
100
0
-100
900 msec



1.0
0.8
0.6
0.4
0.2
0.0
$\hat{P}_1(\tau)$
G
RT DEADLINE
250 msec
400 msec
900 msec
τ (msec)
100
0
-100
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)
250 msec
TOTAL RT
MOVEMENT RT
400 msec
900 msec

Figure A6. Marginal mean RT as a function of ISI for each RT deadline condition and for each S in Experiment II.


RR
500
400
300
550 msec
440 msec
330 msec
-44 -27.5 -16.5 -5.5 5.5 16.5 27.5 44
MEAN RT (msec)
JM
500
400
550 msec
440 msec
330 msec
-44 -27.5 -16.5 -5.5 5.5 16.5 27.5 44
DK
300
990 msec
440 msec
275 msec
-44 -33 -11 -5.5 5.5 11 33 44
τ (msec)


LN
• TOTAL RT
o TIK RELEASE RT
x MOVEMENT RT
—— 250 msec
------ 400 msec
900 msec
MEAN RT (msec)
500
400
300
200
100
0
MG
400
300
200
100
0
-40 -20 -10 -5 5 10 20 40
-40 -20 -10 -5 5 10 20 40
τ (msec)

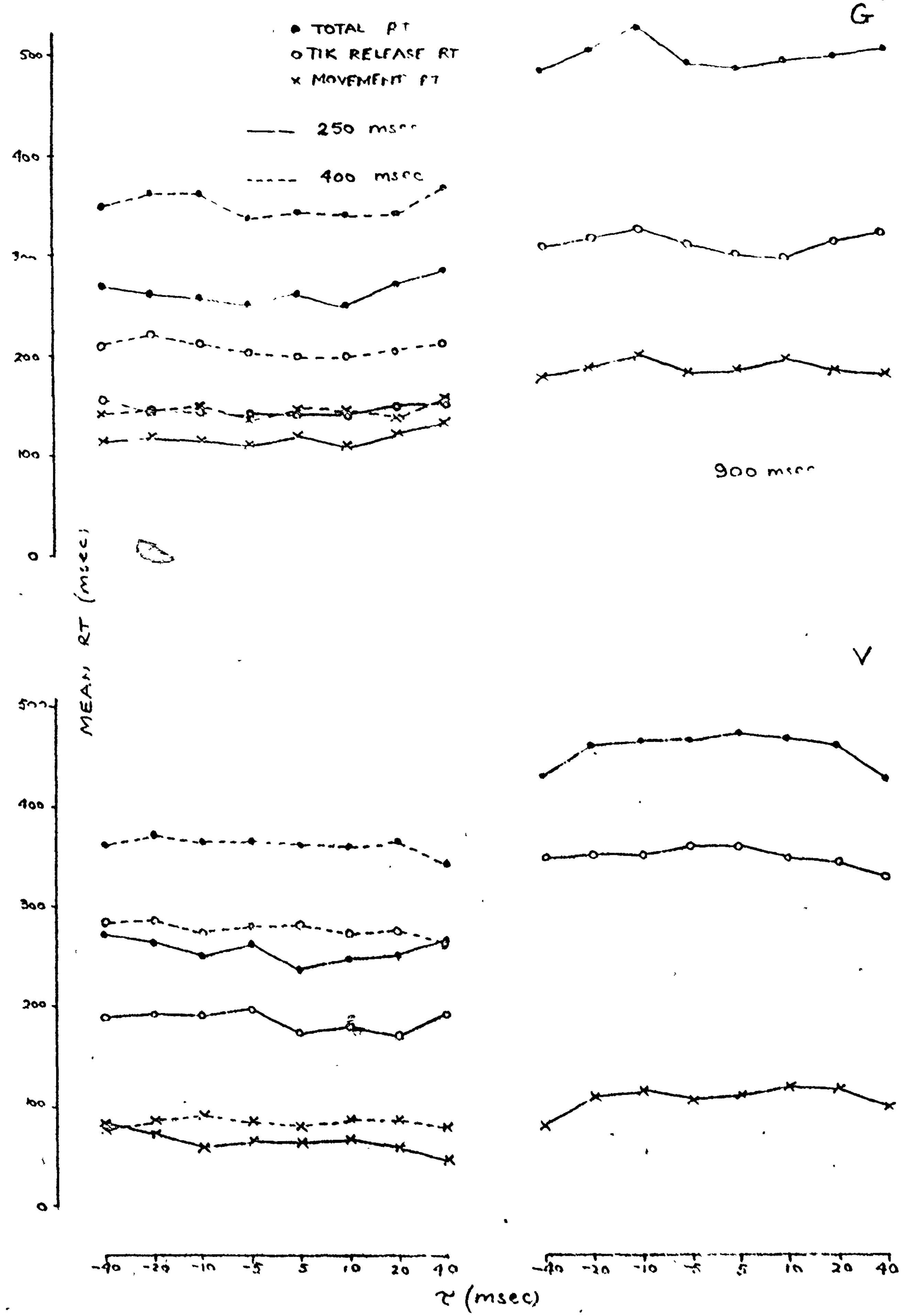
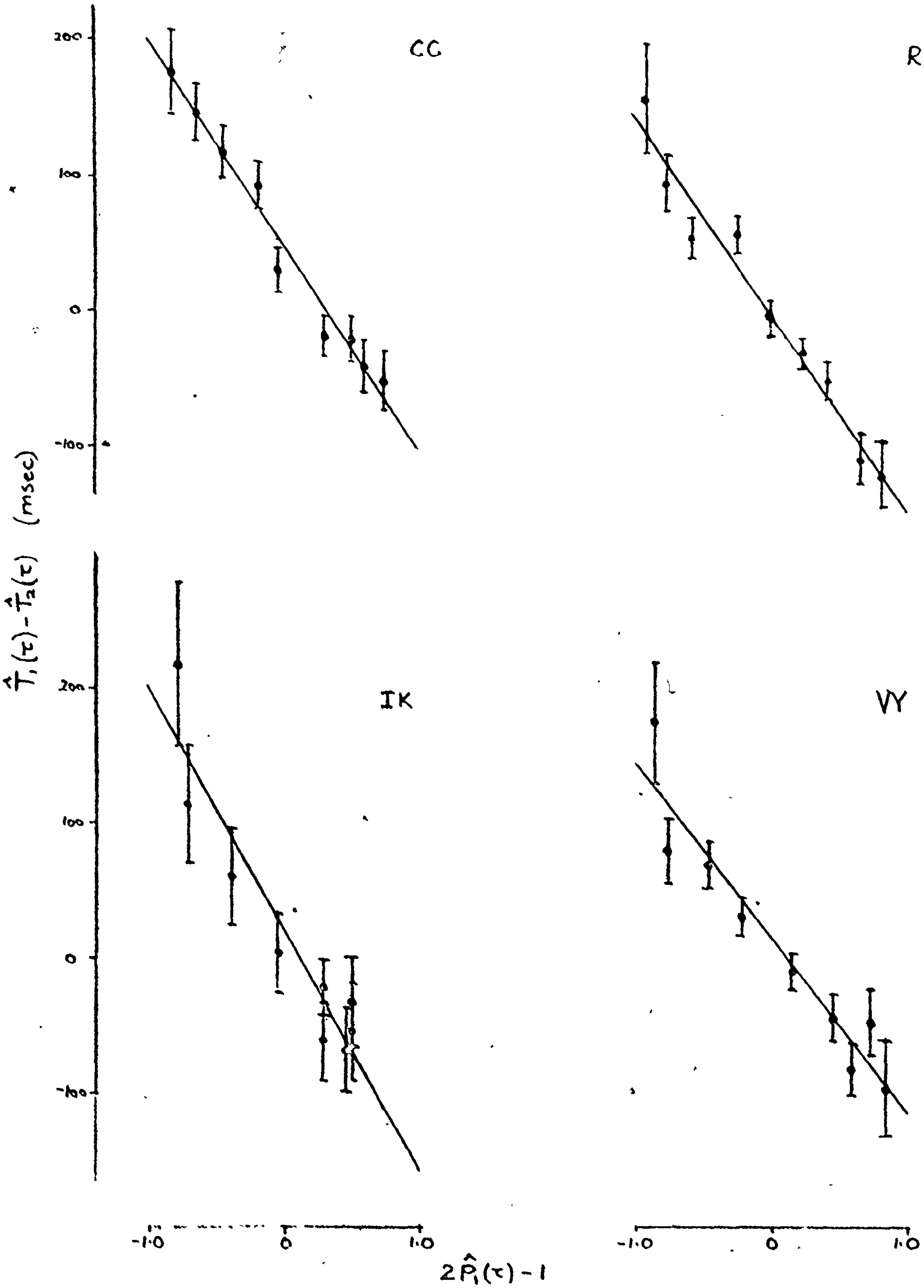
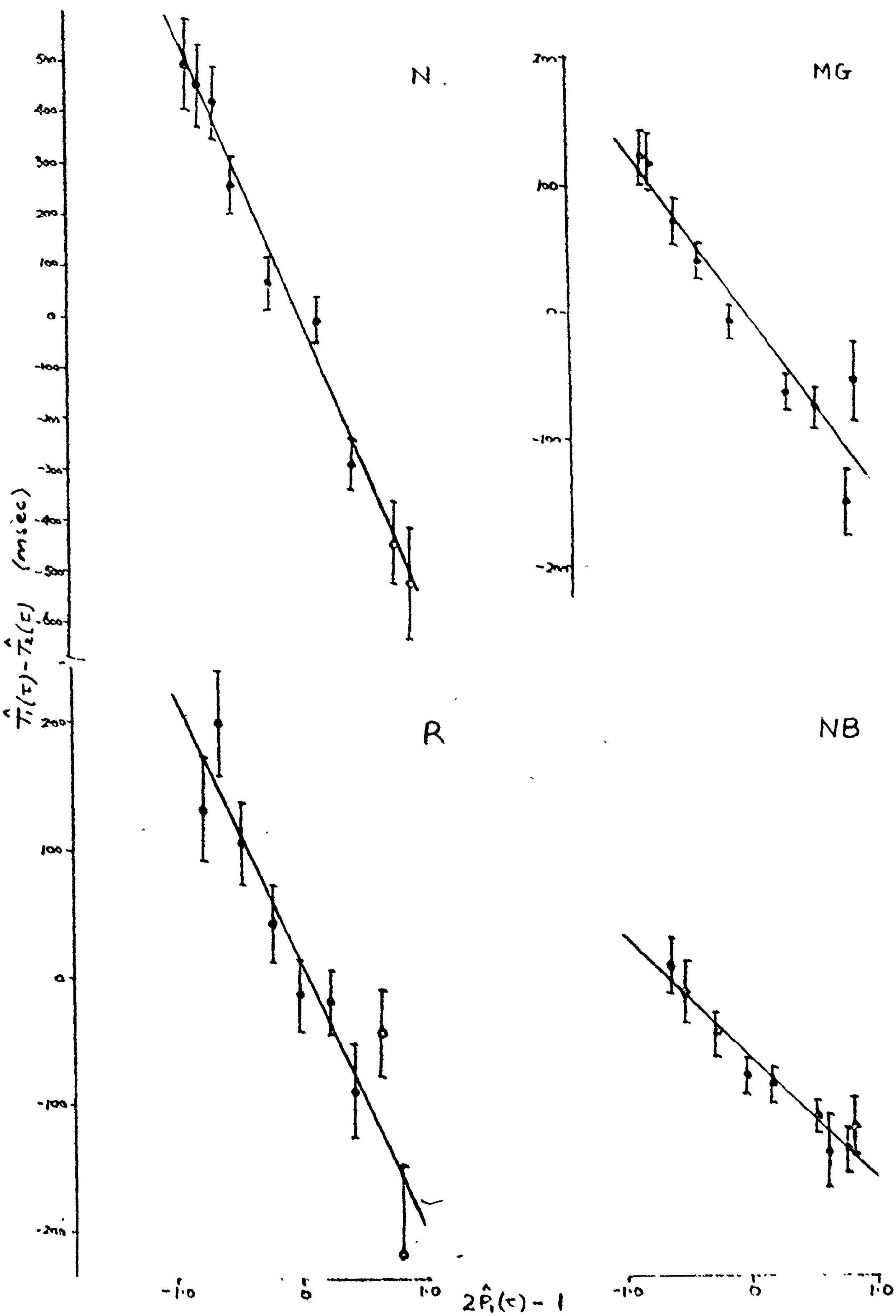

G
• TOTAL RT
o TIK RELEASE RT
x MOVEMENT RT
250 msec
400 msec
900 msec
V
MEAN RT (msec)
τ (msec)
-40 -20 -10 -5 5 10 20 40

Figure A7. Relationship between $\hat{T}_1(\tau) - \hat{T}_2(\tau)$ and $2\hat{P}_1.(\tau) - 1$ for each S in Experiment I.


CC
RM
IK
VY
200
100
0
-100
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)
-1.0
0
1.0
$2\hat{P}_1(\tau) - 1$



N.
MG
R
NB
$\hat{T}_1(\tau) - \hat{T}_2(\tau)$ (msec)
$2\hat{P}_1(\tau) - 1$
-1.0
0
1.0

Figure A8. Relationship between marginal mean total RT and $\hat{z}$ in Experiment IIa.

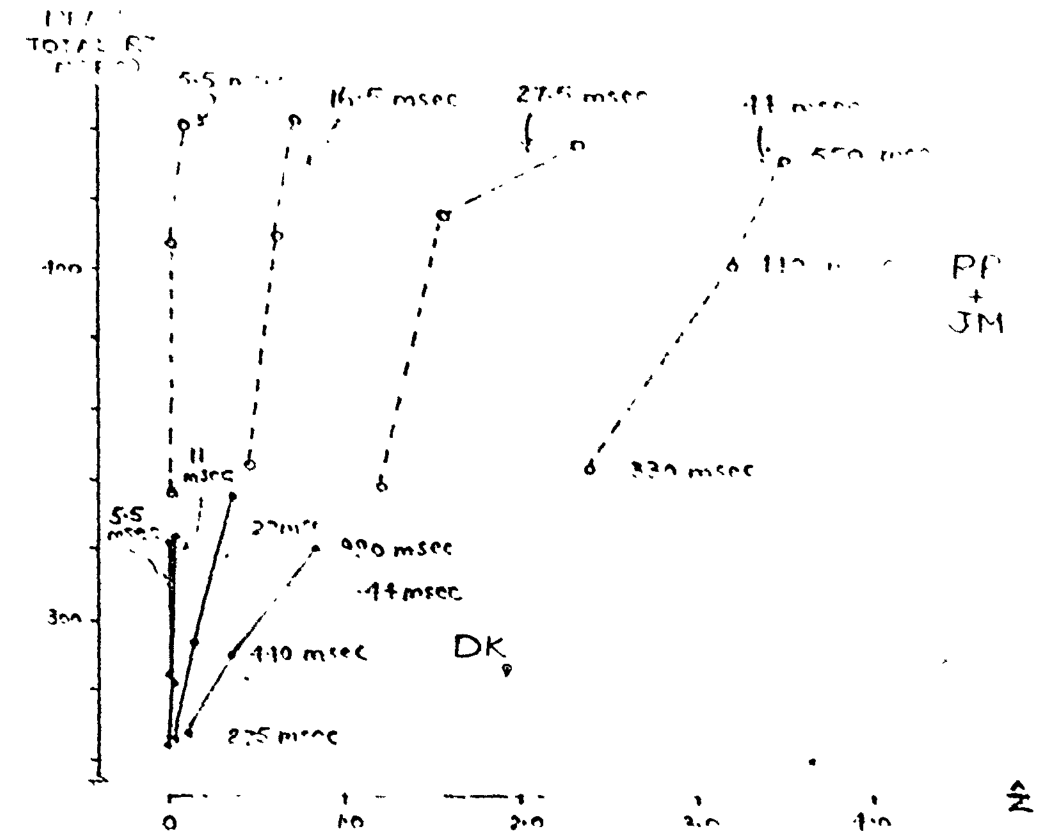
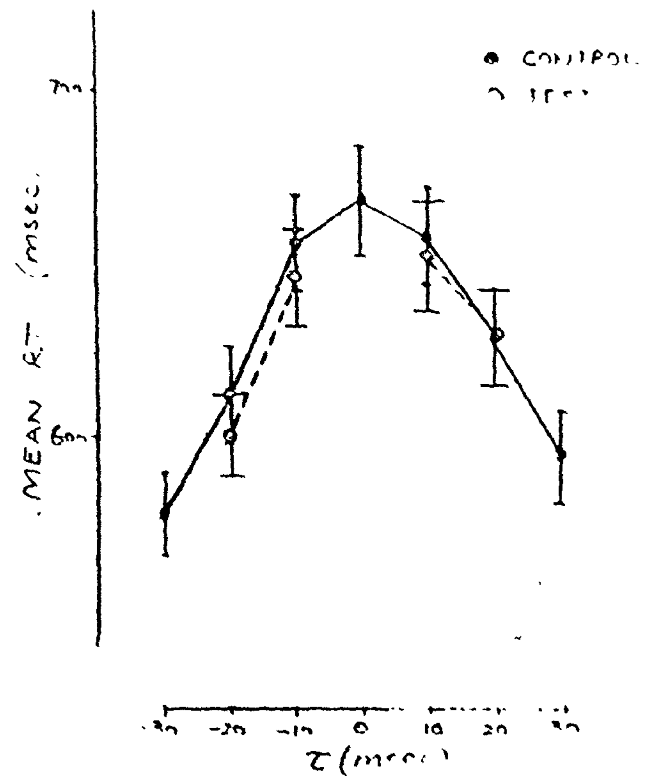
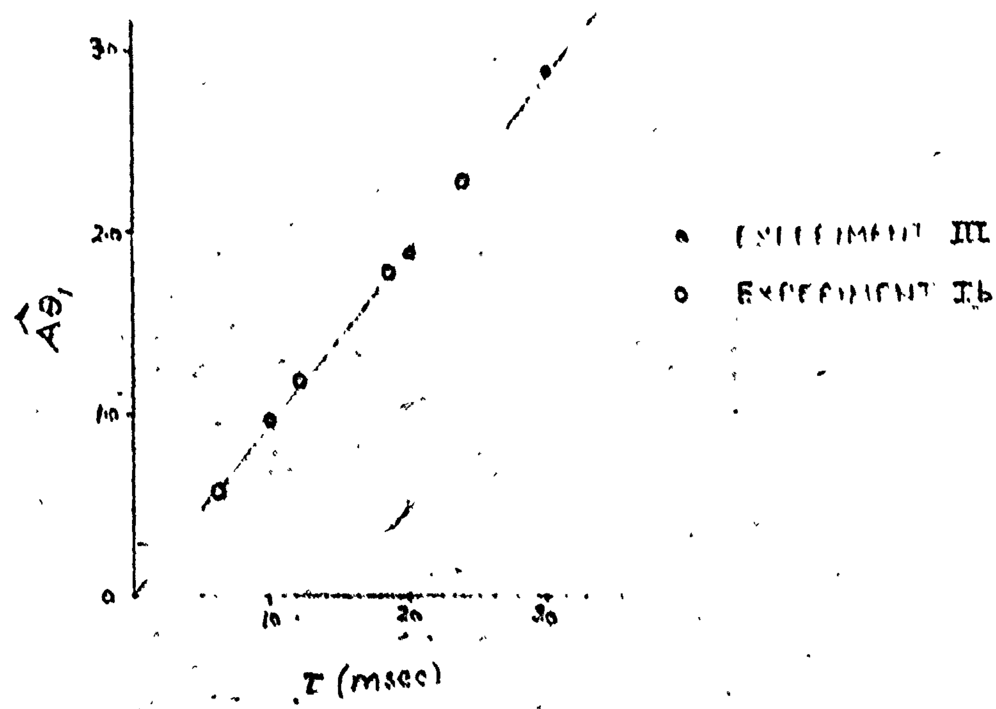

TOTAL RT
5.5 msec
16.5 msec
27.5 msec
44 msec
550 msec
110 msec
PF
+
JM
330 msec
11
msec
5.5
msec
22 msec
990 msec
44 msec
400
300
110 msec
DK
275 msec
0
1.0
2.0
3.0
4.0
Ẑ

Figure A9. Marginal mean RT as a function of ISI for Control and Test conditions of Experiment III.



CONTROL
MEAN RT (msec)
700
600
-20
-10
0
10
20

Figure A10. Relationship between $\widehat{\lambda\Theta}_1$ and τ in Experiments Ib and III.


3.0
2.0
1.0
0
10
20
30
$\hat{A}_{\theta_1}$
τ (msec)
• EXPERIMENT III
○ EXPERIMENT IIb