Modelling Risk Dependencies and Propagation in Supply Chains

MODELLING RISK DEPENDENCIES AND PROPAGATION IN SUPPLY CHAINS

By

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A Thesis
Submitted to the School of Computational Science & Engineering
and to the School of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of
Master of Applied Science

McMaster University

MASTER OF APPLIED SCIENCE (2013)	MCMASTER UNIVERSITY
(School of Computational Science & Engineering)	Hamilton, Ontario

TITLE:	**Modelling Risk Dependencies and Propagation in Supply Chains**
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NUMBER OF PAGES:	xii, 83

To my husband Ahmad

to my son Daniel

and to my parents and siblings

Abstract

Today's highly integrated supply chains are exposed to various types of risks which disrupt the normal flow of goods or services within a supply chain network. Since most of these individual risks are interconnected, a mitigation strategy to tackle one risk may result in the exacerbation of another.

Risk dependencies have been modelled using two approaches in the financial insurance literature : (i) random variables, and (ii) copulas. In this dissertation these studies are reviewed and extended. Also, applications for these models for different supply chain network configurations are presented. Then, a Poisson process model for risk propagation is proposed. Unlike the existing models, the transition rate of the proposed model not only expresses the time dependency, but also captures other possible dependencies in the network. Finally, the thesis is summarized and general directions and suggestions for future research on risk dependency and propagation modelling are provided.

Acknowledgements

All deepest thanks and gratitude are to Almighty God, the Most Merciful, the Most Compassionate.

I would like to express my honest thanks to my supervisor, Prof. Elkafi Hassini who gave me the opportunity to work on this very interesting research topic. I am really grateful for his encouragement and guidance throughout my research. Indeed, his careful supervision, patience and continuous support made it possible for me to accomplish this work.

I also would like to thank Dr. Kai Huang and Dr. Bartosz Protas for their valuable comments and suggestions which helped to improve the thesis.

During my Masters degree, I had many happy and joyful moments with Nooshin Tajik, Bahareh Mansouri, Sara Bakhtiar and Maryam Mohseni and I am greatly thankful for their friendship and support. Also, I would like to thank Ali Sahlodin and Hesam Shahriari who were helpful. Finally, I would like to thank my husband and my parents for their generous support and patience during my studies.

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Chapter 1

Introduction

1.1 Motivation

As the supply chain networks are becoming more connected and interdependent, a failure at any point of the network can result in major disruptions to the flow of products or services in the network. Furthermore, the increase in complexity and offshoring of products and services has increased the risk diversity within the supply networks (Harland *et al.*, 2003). The fact that supply chain operational models had traditionally emphasized economic efficiency, leading to supply chains operating on minimum inventories and facilities, is likely to delay the recovery from disruption (Jones, 2013). In addition, the use of just–in–time systems had meant that supply chains reduced their suppliers pool and in turn competing supply chains often share the same supplier (Jones, 2013). Thus a disruption in one supply point can lead to simultaneous disruptions in several supply chains. After the 2011 earthquake in Japan, the major auto manufacturers have started questioning the way they operate their supply chains and in particular how to incorporate risk effects in their design and operation. For example Aston Martin and Jaguar Land Rover have partnered with Toyota to share information about their supply chain network in the hope that they can anticipate risky events before they occur (Jones, 2013). This trend for collaboration was mostly driven by the realization that their supply networks are interconnected. As Guillaume Jacques, a purchasing manager at Toyota Motor Europe, said (Jones, 2013):

> Our supply chains are so interdependent that there is no point in Toyota trying to secure its supply chain on its own. Any manufacturer stopping production on a big scale would impact others within a very short time.

Thus most of the individual risks are interconnected, so a mitigation strategy to tackle one risk may result in exacerbation of another. Furthermore, the existing models in the literature, mainly from the finance and insurance fields, mostly focus on single risk events and when they consider multiple events they often assume that they are independent.

Since supply chain disruptions are unplanned and unanticipated, all supply chains are subject to various risks. Therefore, applying an appropriate mitigation tactic is so crucial to manage the risk of disruptions. To take some actions in advance of a disruption, it is necessary to predict the expected number of events and the probabilities of their occurrence to plan accordingly. By better understanding the mechanisms of risk propagation through the supply chain, we would be able to prevent the undesired effects of these events. Some existing models in the literature, from the social sciences, model the initiation and spread of some behaviours using a Poisson process. Using a time dependent transition rate for a Poisson process, social scientist would be able to describe a process where the probability first increases and then after reaching a maximum value decreases. We adopt a similar modelling framework to model risk propagation in supply networks by using a non-homogeneous Poisson process where the transition rate depends both on time and the number of events that occurred previously.

1.2 Contribution

We review the finance and insurance models on risk management with a focus on dependency modelling and show, when possible, how they can be applied in a supply chain context. We also outline the limitations of these models and present an extension that is relevant for supply chains' risk analysis.

As should be clear from the discussion in Chapter 3, the Poisson process is a proper model for risk propagation. We argue that the probability of a major supply chain risk event depends on the number of events already in existence and use a time and state dependent transition rate. We present the distribution functions of events' arrival time, total number of events observed during a period of time and cohort arrival counts; the chances of having a subset of the supply chain entities being affected by the major risk events.

1.3 Organization of this thesis

The remainder of this thesis is organized as follows:

In Chapter 2 we summarize the major modelling approaches for risk dependencies. Then we discuss risk in supply chains. We review the literature on risk dependency modelling, extend it, and outline how these models can be applied in a supply chain context.

In Chapter 3 we propose a model for risk propagation. Then we investigate the effect of the March 11, 2011 earthquake on Toyota and Honda supply chains in Chapter 4. We analyze the collected data in relation with the models reviewed in Chapter 2. At the end of this chapter we use the collected data as an empirical example for the proposed propagation model of Chapter 3.

Finally, we provide conclusions and suggestions for future research in Chapter 5.

Chapter 2

Risk Dependency Modelling and Supply Chain Applications

The outline of this chapter is as follows: First, we describe the required mathematical concepts in Section 1. Prevalent terms in the insurance context is also defined in this section. Types of supply chain risks as well as events and conditions that drive them are summarized in Section 2. We talk about the source of dependencies in supply chains at the end of Section 2. Risk dependencies have been modelled in the remainder of this chapter using two approaches: random variables and copulas. In each category (random variables and copulas) individual risks are either in a single class or divided in different classes. Furthermore, we outline the limitations of these models and present the applications of dependency models, when possible, in a supply chain context. In the last section of Chapter 2, we relax one of the critical assumptions of the existing models, that of individual risk independency, and present an extension, considering dependency among individual risk factors.

2.1 Risk dependency modelling terms and approaches

In this section, we define several terms and concepts that we will need later in this chapter. These definitions are taken from (Bauerle and Muller, 1998) and (Shaked and Shanthikumar, 1997).

Supermodular function: A function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be supermodular, if $f(x_1, \ldots, x_i + \epsilon, \ldots, x_j + \delta, \ldots, x_n) - f(x_1, \ldots, x_i + \epsilon, \ldots, x_j, \ldots, x_n) \geq$ $f(x_1, \ldots, x_i, \ldots, x_j + \delta, \ldots, x_n) - f(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ holds for all $x \in \mathbb{R}^n, 1 \leq i < j \leq n$ and all $\epsilon, \delta > 0$.

Symmetric function: A function $f : \mathbb{R}^n \to \mathbb{R}$ is called symmetric, if $f(x) = f(\pi x)$ for all permutations πx of x.

Supermodular ordering: A random vector $X = (X_1, \ldots, X_n)$ is said to be smaller than random vector $Y = (Y_1, \ldots, Y_n)$ in the supermodular ordering, written $X \leq_{sm} Y$, if $E[f(X)] \leq E[f(Y)]$ for all supermodular functions f assuming that the expectations exist.

Symmetric supermodular ordering: A random vector $X = (X_1, \ldots, X_n)$ is said to be smaller than the random vector $Y = (Y_1, \ldots, Y_n)$ in the symmetric supermodular ordering, written $X \leq_{symsm} Y$, if $E[f(X)] \leq E[f(Y)]$ for all symmetric supermodular functions f assuming that expectations exist.

Stop-loss transform: For arbitrary univariate random variables Y we denote the stop-loss transform $\pi_Y(t) = E(Y - t)^+ = \int_t^\infty \bar{F}_Y(x)dx, t \in \mathbb{R}$.

Stop-loss ordering: X precedes Y in stop-loss order, written $X \leq_{sl} Y$, if

$\pi_X(t) \leq \pi_Y(t)$ for all $t \in \mathbb{R}$.

Portfolio (in insurance): Is a collection of different risks caused by policy holders (insured). For example, risks like illnesses or accidents which threat individual life in life insurance.

Contract (in insurance): A legally binding agreement between insurer and insured that must be in writing to be enforceable.

Policy (in insurance): The formal contract issued by an insurer that contains terms and conditions of the insurance coverage and serves as its legal evidence.

Insurance premium: Insurance premium is the financial cost of an insurance policy, paid by policy holder either as a lump sum or in several instalments during the period covered by the policy. In case that the insurance premium is not paid when due, the insurance policy usually gets automatically cancelled.

Latent random variable: In statistics, latent variables are random variables that are not directly observed. Their properties must thus be inferred indirectly using a statistical model (latent variable model) connecting the latent (unobserved) variables to observed variables.

Mixture distribution: Any convex combination of probability density functions g_i defined as density function f is called a mixture distribution:

$$f(x) = \sum_{i=1}^{n} p_i g_i(x), \quad \sum_{i=1}^{n} p_i = 1, \quad n > 1.$$

In cases where each of the underlying distribution functions is continuous, the outcome mixture distribution function would be continuous as well. When g_i's are from a parametric family with unknown parameters θ_i's, the parametric mixture distribution is defined as $\sum_{i=1}^{n} p_i g(x|\theta_i)$.

Copula function: A copula function is a joint distribution function with marginal distribution functions as parameters. Therefore, properties of copulas are similar to those of joint distributions.

Frechet-Hoeffding copula bounds: The Frechet-Hoeffding Theorem states that for any copula $C : [0,1]^d \to [0,1]$ and any $(u_1, \ldots, u_d) \in [0,1]^d$ the following bounds hold:

$W(u_1, \ldots, u_d) \leq C(u_1, \ldots, u_d) \leq M(u_1, \ldots, u_d)$.

The function W is called lower Frechet-Hoeffding bound and is defined as $W(u_1, \ldots, u_d) = \max\left\{1 - d + \sum_{i=1}^{d} u_i, 0\right\}$.

The function M is called upper Frechet-Hoeffding bound and is defined as $M(u_1, \ldots, u_d) = \min\{u_1, \ldots, u_d\}$.

Dependence models based on copula: Models which use copula to describe the dependency between risks.

Single-class risk models: In these models, all risks of insurance portfolio are considered to belong to a single class.

Multi-class risk models: Models in which an insurance portfolio is generally divided into several classes. The insureds are classified according to the risk they represent for the insurer.

Prior: In Bayesian statistical inference, a prior probability distribution, often called simply the prior, of an uncertain quantity p is the probability distribution that would express one's uncertainty about p before the "data" is taken into account. It is meant to attribute uncertainty rather than randomness to the uncertain quantity. The unknown quantity may be a parameter or latent variable.

De Finettie's Theorem: To every infinite sequence of exchangeable random variables $\{X_n\}$ having values in $\{0, 1\}$, there corresponds a probability F over $[0, 1]$, such that:

$$p_{k,n} = P(X_1 = 1, \ldots, X_k = 1, \ldots, X_{k+1} = 0, X_n = 0) = \int_0^1 \theta^k (1-\theta)^{(n-k)} F(d\theta).$$

Two point distribution : If there are two possible values α_1 and α_2 in an experiment, then the probability distribution is:
$Pr(X = \alpha_1) = p$ and $Pr(X = \alpha_2) = 1 - p = q$ $p, q \geq 0$, $p + q = 1$.

Quasi- homogeneous portfolio: The portfolio is homogeneous with respect to claim amounts where all claim amounts have the same distribution for all the policyholders.

2.2 Risk in supply chains

2.2.1 Types of risks

As the vulnerability of a supply chain to disruption increases, it is important to identify and manage various types of risks to avoid any supply chain break down.

Supply chain risks can penetrate in every stage, not just the final stage of product/service delivery to customers. On the other hand, dependence within different business partners in each stage of production can worsen the impact of risk on supply chain. So, a way of reducing damages driven by risks is to determine factors which cause dependencies in a supply chain.

Supply chain risks can be divided into different types based on their realization impact on a business and its environment. A summary of the most important risks and their definitions can be found in Table 2.1. Some of these risks are mentioned by Chopra and Sodhi (2004) and Harland *et al.* (2003).

Supply chain risks can also be classified based on whether or not they impact strategic or operational supply chain decision making. In this context we define strategic supply chain risks as those the impact of which will be long-term causing the supply chain decision makers to change their strategics. For example, the earthquake in Japan has led major auto manufacturers, such as Toyota, to rethink their risk management strategies. On the other hand, an operational supply chain risk has a short-term impact on the supply chain operations. An example is a temporary machine breakdown or a limited disruption that may occur due to an employee leaving a company. In addition, supply chain risks can be classified based on their source, upstream, internal, or downstream . In Figure 2.1 we propose a strategy-source matrix for classifying the different supply chain risks.

The matrix in Figure 2.1 can be used to help in prioritizing risk planning and mitigation strategies. For example, we can see that Legal, IT, Quality and Reputation may involve all links in a supply chain. We would then expect these risks to be more dependent and their occurrence may cause a chain reaction. This then raises the important question of how to model risk dependencies in a supply chain and what factors are relevant in measuring supply chain dependencies. In the following section some factors which have an effect on the amount of dependence between supply chain

Table 2.1: Supply chain risks

Type of risk	**Examples**
Disruption	Natural disaster, labor strike, fires, and terrorism are examples of disruptions which can interrupt the flow of the material in a supplier or a manufacturer. Based on the effect, disruptions can be operational or strategic.
Quality	Poor quality in a supply source or in a manufacturer product or even any failure arising from customers can harm the quality of supply chain's products and/or services. Based on the duration of the impact, a quality risk can be operational or strategic.
Forecast	Long lead times, seasonality, product variety, short life cycles, small customer base, information falsification due to promotion, intensive, lack of supply-chain visibility and exaggeration of demand in times of product shortage, can cause an inaccurate forecast.
Legal	Any action arising from suppliers, customers, shareholders or employees which expose a firm to judicial process.
Reputation	This strategic risk is one of the most critical ones. Loss of confidence can destroy the whole value of a business financial.
Receivables	When a company is not able to collect the receivables, its performance will be affected.
IT	Any breakdown of information basis can destroy the highly networked environment of a company.
Capacity	Increasing or decreasing the capacity can be a strategic decision for the companies. Having too much of excess capacity can negatively affect a company's financial performance.
Market	Changes in demand of customers, sticking to a single marketing strategy, exchange rate risk and interest rate risk are examples of market risks.
Financial	Failure of debtors to meet their financial obligation and changes in financial markets can cause a financial loss for a company.
Competitive risk	An example is when a firm is not able to differ its products/services form the other competitors.
Human Resources	Businesses face a strategic risk when a key executive leaves the company.

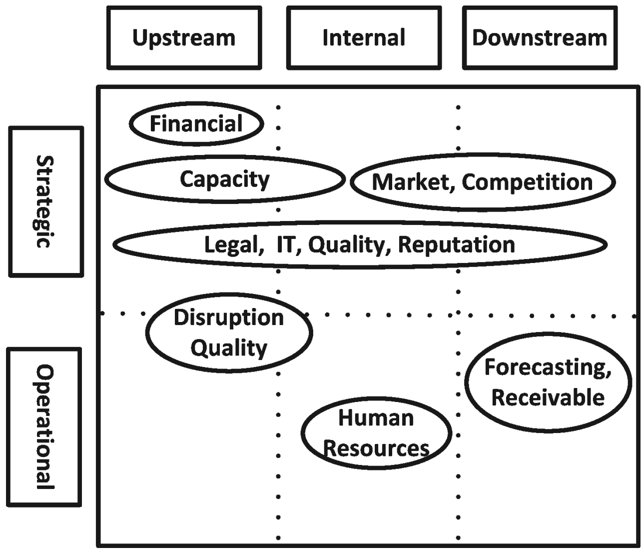

Figure 2.1: Strategy-source matrix for classifying risk.

links will be examined.

2.2.2 Types of supply chain dependencies

Dependence between links in a supply chain can be a function of different factors such as the type of relationship (transactional, preferential or strategic), the criticality of the item in the final product, spend value, number of suppliers, and location of suppliers. In Table 2.2 we list different factors that may cause supply chain dependencies together with their sources. Understanding these dependency factors and their sources would help us in building models for predicting and mitigating supply chain risks under such dependencies.

In the next section we focus on mathematical models for representing risk dependencies and indicate how they apply to supply chains.

Table 2.2: Sources of dependence in a supply chain.

Source of dependence	Examples
Sourcing	■ Number of suppliers ■ Location of supplier ■ Local or global sourcing ■ Supplier dependence ■ Supplier commitment to buyer ■ Supplier power ■ Financial strength ■ Type of contract
Distribution	■ Distribution network ■ Transportation network ■ Transportation modes ■ Rely on technologically advanced (key) suppliers
Customer	■ Volume ■ Loyalty ■ Customer-driven supply-chain
Information	■ Shared information ■ Push vs. pull ■ Information Technology service capabilities

2.3 Random variable-based dependent single-group risk models

In this section we look at two models that employ random variables to represent risk dependencies within a single group. The first model used a compound Poisson random variable and the second used a two point random variable.

2.3.1 Compound Poisson approximation model (Goovaerts and Dhaene, 1996)

Consider a portfolio consisting of n dependent risks. With risk k we associate a claim amount X_k represented as $X_k = J_k B_k$, where J_k is a Bernoulli random variable which is equal to 1 if risk k causes at least one claim during the reference period with probability q_k, i.e., $Pr(J_k = 1) = q_k = 1 - Pr(J_k = 0)$, and B_k is the total claim amount produced by risk k. Dependence in this model is represented by the dependent indicators J_k. However, the conditional claim amounts B_k such that $J_k = 1$ (denoted by $B_k \mid J_k = 1$) are mutually independent.

The total claim of the portfolio during a fixed period of time is $S = X_1 + X_2 + ... + X_n$ and its cumulative distribution function is $F(s) = Pr(S \leq s)$. If we assume that all conditional claim amounts $B_k \mid J_k = 1$ have the same cumulative distribution F, the portfolio is quasi-homogeneous and thus F can be approximated with a compound Poisson with cumulative distribution function F^{cp} as follows:

$$F^{cp}(s) = \sum_{n=0}^{s} Pr(K = n) F^{n}(s) \qquad (s = 0, 1, ...)$$

where K is a poisson distribution random variable with parameter λ given by $\lambda = \sum_{k=1}^{n} q_k$ and $F(s)$ is the distribution given by

$$F(s) = \frac{1}{\lambda} \sum_{k=1}^{n} q_k Pr(B_k \leq s \mid J_k = 1).$$

It is worth noting that the above approximation does not hold when all conditional claim amounts $B_k \mid J_k = 1$'s are dependent.

Application of Model 2.3.1 in Supply Chains:

One possible application of this model is in serial supply chains (see Figure 2.2). Here we assume that $J_k = 1$ if risk k affects at least one of the supplier, manufacturer, distributer or retailer. The random variable B_k represents the total loss produced by risk k. Since in a serial supply chain each partner has at most one predecessor and one successor, the risk dependency is limited to such interactions. These supply chains exist where a business is dealing with simple products that do not require much value adding, except from inventory and distribution. An example can be found in produce supply chains where a farmer produces and packages fruits or vegetables and sells

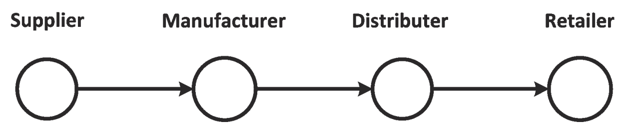

Figure 2.2: A serial supply chain.

them directly to consumers (pick your own) or through a distributor and/or retailer.

2.3.2 Two-point distribution model (Dhaene and Goovaerts, 1997)

Assume that the n risks $X_1, X_2, ..., X_n$ form a portfolio. In this model each risk $X_k(k = 1, ..., n)$ follows a two-point distribution in 0 and $\alpha_k \geq 0$:

$$Pr(X_k = 0) = p_k \quad \text{and} \quad Pr(X_k = \alpha_k) = 1 - p_k = q_k. \tag{2.1}$$

If random variables $X_1, ..., X_n$ are assumed mutually independent, cumulative distribution function of the total claims $S = X_1 + X_2 + ... + X_n$ is uniquely determined by the distributions Eq. (2.1) of the X_i's (using convolution of n probability mass functions). But, in this model we will not have the assumption of independency.

The expected aggregate claim will not be affected by the type of dependence between the individual risks because for each $S \in \Re$ (set $\Re$ is consisting of all random variables S that cab be written as $S = X_1 + X_2 + ... + X_n$) the mean value is calculated by:

$$E(S) = \sum_{k=1}^{n} q_k \alpha_k.$$

We will suppose that the individual risks are arranged in an increasing order $p_1 \leq p_2 \leq ... \leq p_n$, i.e., the risk with a lower subscript has a lower probability. The dependence between the individual risks is given by the following relation:

$$Pr(X_{k+1} = 0|X_k = 0) = 1, \quad k = 1, 2, ..., n-1. \tag{2.2}$$

It follows from Eq. (2.2) that:

$$\begin{aligned} Pr(X_{k+1} = 0) &= Pr(X_{k+1} = 0|X_k = 0)\, Pr(X_k = 0) + P(X_{k+1} = 0|X_k = \alpha_k)\, Pr(X_k = \alpha_k) \\ p_{k+1} &= 1 \times p_k + Pr(X_{k+1} = 0|X_k = \alpha_k) \times (1 - p_k) \end{aligned}$$

$$\Longrightarrow \qquad Pr(X_{k+1} = 0|X_k = \alpha_k) = \frac{p_{k+1} - p_k}{1 - p_k}$$

$$\begin{aligned} Pr(X_{k+1} = \alpha_{k+1}|X_k = 0) &= 0, \\ Pr(X_{k+1} = \alpha_{k+1}|X_k = \alpha_k) &= \frac{1 - p_{k+1}}{1 - p_k} \end{aligned} \tag{2.3}$$

This model has been developed for the analysis of life insurance claims. From (2.2) it follows that if person k stays alive then person $k+1$ stays alive, but if person $k+1$ stays alive then person $k+2$ stays alive, and so on. So we can conclude:

$$Pr(X_{k+j} = 0|X_k = 0) = 1, \quad k = 1, 2, ..., n-1;\ j = 1, ..., n-k.$$

This means that "if a person survives the exposure period, then all persons with greater survival probabilities will also survive." (Dhaene and Goovaerts, 1997)

Considering all former relations, possible outcome for S will be: $0, \alpha_1, \alpha_1+\alpha_2, \alpha_1+$

$\alpha_2 + \alpha_3, ..., \alpha_1 + ... + \alpha_n$, and we will have:

$$\begin{aligned}
Pr(S = 0) &= Pr(X_1 = 0; X_2 = 0, ..., X_n = 0) = Pr(X_1 = 0) = p_1, \\
Pr(S = \alpha_1 + \alpha_2 + ... + \alpha_k) &= Pr(X_1 = \alpha_1; X_2 = \alpha_2; ...; X_k = \alpha_k; X_{k+1} = 0; ...; X_n = 0) \\
&= Pr(X_k = \alpha_k; X_{k+1} = 0) \\
&= Pr(X_k = \alpha_k).Pr(X_{k+1} = 0|X_k = \alpha_k) \\
&= p_{k+1} - p_k, \quad k = 1, 2, ..., n-1, \\
Pr(S = \alpha_1 + ... + \alpha_n) &= Pr(X_1 = \alpha_1, ..., X_n = \alpha_n) = Pr(X_n = \alpha_n) = 1 - p_n.
\end{aligned}$$

Finally, the distribution function of S will be as follows:

$$F_S(s) = \begin{cases} p_1, & 0 \leq s \leq \alpha_1 \\ p_{k+1}, & \alpha_1 + ... + \alpha_k \leq s \leq \alpha_1 + ... + \alpha_{k+1}, \quad k = 1, 2, ..., n-1. \\ 1, & s \geq \alpha_1 + ... + \alpha_n. \end{cases}$$

Model 2.3.2 does not seem very practical compared to Model 2.3.1, since dependency between risks X_k ($k = 1, \ldots, n$), as expressed by (2.2), leads to the riskiest portfolio in the sense that it has the largest stop-loss premium. (Dhaene and Goovaerts, 1997).

Application of Model 2.3.2 in Supply Chains:

An assembly network would be a possible application of Model 2.3.2. In Figure 2.3 the outside supplier is a single source to the two downstream transmission plants. If a disruption happens in the outside supplier, even if partial, all downstream plants, including the assembly plants may be influenced to different degrees. The latter reflects the condition $Pr(X_{k+1} = 0 \mid X_k = 0) = 1$ in model 2.3.2.

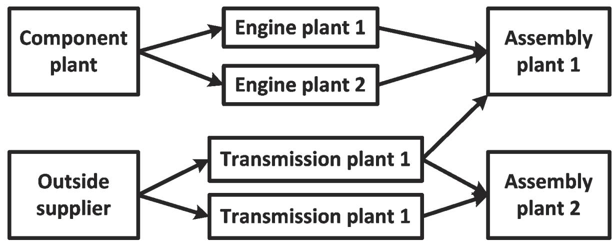

Figure 2.3: Schematic representation of Model 2.3.2 in supply-chain, (Deleris *et al.*, 2004).

2.4 Random variable-based dependent multi-group risk models

In this section we continue to discuss random variable-based models but for multi-group situations. We describe six models that incorporate risks due to different group factors.

2.4.1 Risk as a function of three risk factors

In the first model of Bauerle and Muller (1998), there is a strong dependence between members of one group of a portfolio, but the dependence between members of different groups is weaker.

Consider portfolio $X = (X_1, ..., X_n)$, consisting of n risks X_1, ..., X_n. Moreover, we assume that there exists an increasing function $g : \mathbb{R}^3 \to \mathbb{R}$ such that the k-th risk is given by X_k = g (Z_k, G_r, V) where k is in group r, V is an overall risk factor, G_r is a group-specific risk factor which impacts only the risks in group r and Z_k is an individual risk factor which indicates the individual share of risk X_k, $1 \leq k \leq n$. In general, comparing two risky portfolios with different sizes and number of groups is difficult. However, using Theorem 2.4.1.2 (that will be described later in this section) makes the comparison possible in some cases. In order to state the theorem, let L and L' be two n-dimensional vectors with

$L = (L_1, ..., L_r, 0, ..., 0), L' = (L'_1, ..., L'_l, 0, ..., 0)$
$1 \leq r, l \leq n, L_k, L'_k \in \mathbb{N}$ for all k and $\sum_{k=1}^{n} L_k = \sum_{k=1}^{n} L'_k = n$. Then, we consider two n-dimensional risky portfolios X and Y given by

$$
\begin{aligned}
X_1 &= g(Z_1, G_1, V) & Y_1 &= g(U_1, G_1, V) \\
&\vdots & &\vdots \\
X_{L_1} &= g(Z_{L_1}, G_1, V) & Y_{L'_1} &= g(U_{L'_1}, G_1, V) \\
X_{L_1+1} &= g(Z_{L_1+1}, G_2, V) & Y_{L'_1+1} &= g(U_{L'_1+1}, G_2, V) \\
&\vdots & &\vdots \\
X_{L_1+L_2} &= g(Z_{L_1+L_2}, G_2, V) & Y_{L'_1+L'_2} &= g(U_{L'_1+L'_2}, G_2, V) \\
&\vdots & &\vdots \\
X_n &= g(Z_n, G_r, V) & Y_n &= g(U_n, G_l, V)
\end{aligned}
$$

where the individual risk factors $Z_1, ..., Z_n, U_1, ..., U_n$ are *i.i.d.* random variables, the group specific risk factors $G_1, ..., G_{max(r,l)}$ are *i.i.d.* random variables and the environmental risk factor V is a random variable independent of Z_k, U_k and $G_{max(r,l)}$. Let $S = \sum_{k=1}^{n} X_k$ and $S' = \sum_{k=1}^{n} Y_k$.

The theorem uses majorization (Marshall and Olkin, 1979) for comparing the structure of vectors L and L'. The definition of majorization is as follows:

Definition 2.4.1.1 *let* X,$Y \in \mathbb{N}_0{}^n$ *and denote by* $X_{[1]} \geq ... \geq X_{[n]}$ *the decreasing rearrangement of* X*, and similarly for* Y*. We say that* Y *majorizes* X *(*$X \prec Y$*) if* *and only if*

$$\sum_{k=1}^{r} X_{[k]} \leq \sum_{k=1}^{r} Y_{[k]} \ , \ r = 1, ..., n-1, \qquad \textit{and} \qquad \sum_{k=1}^{n} X_{[k]} = \sum_{k=1}^{n} Y_{[k]}.$$

Intuitively speaking $X \prec Y$ means that in Y the groups are larger and/or more unequal.

The following theorem of Bauerle and Muller (1998) states the main result for this model:

Theorem 2.4.1.2 [*Bauerle and Muller (1998)*] *If* $L \prec L'$, *we get under the assumptions of Model 2.4.1:*

$$a) \qquad X \leq_{symsm} Y$$
$$b) \qquad S \leq_{sl} S'$$

where $\leq_{symsm}$ denotes symmetric supermodular ordering, and $\leq_{sl}$ denotes stop-loss ordering, as defined at the beginning of this chapter.

In this model we assume that all individual risk factors $Z_1, \ldots, Z_n, U_1, \ldots, U_n$ and all group risk factors are *i.i.d.*. In practice it is possible to have dependent individual risk factors or dependent group risk factors.

Application of Model 2.4.1 in Supply Chains

This model applies to supply chain networks that have unequal number of partners at each echelon and where one has a more extensive network than the other. In Figure 2.4 network 2 has larger and more unequal groups. So, based on the result of Model 2.4.1 network 2 is more risky than network 1. The situation in Figure 2.4 can result when looking at an alternative of a local manufacturing supply chain that is more

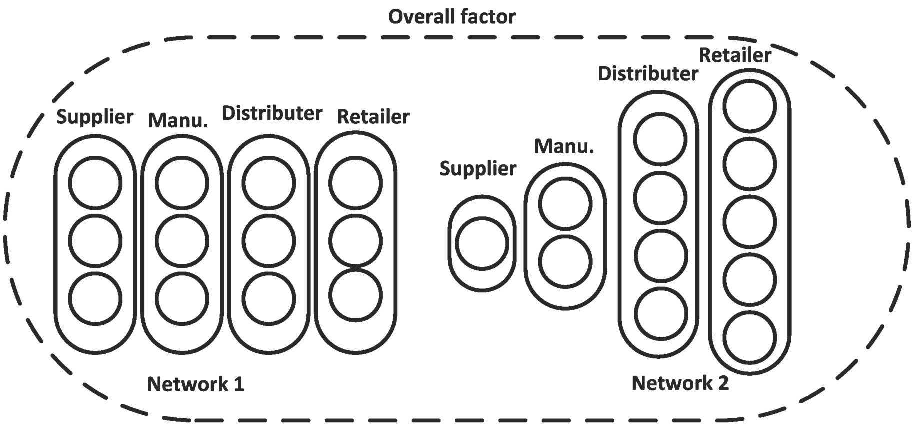

Figure 2.4: Schematic representation of an application of Model 2.4.1 in supply chain. Each circle represents a company/facility.

streamlined (e.g., Tesla electric cars) and comparing it to a global manufacturing supply chain that has a wide distribution network (e.g., Toyota).

2.4.2 Risk dependency through global system shocks (Genest *et al.*, 2003)

Consider a portfolio of $m \geq 1$ classes including $n_1, ..., n_m$ contracts, and X_{jk} is the risk related to the kth contract in the jth class. Therefore, the aggregate claim amount is given by:

$$S = \sum_{j=1}^{m} \sum_{k=1}^{n_j} X_{jk}.$$

A global shock, represented by an indicator random variable J_0, can impact whole parts of the portfolio. Risk in class j occurs with a probability that whether the overall (such as a disaster or catastrophe) has occurred ($J_0 = 1$) or not ($J_0 = 0$). Let $J_j^{(\alpha)}$ imply the presence or absence of a global shock in class j, given $J_0 = \alpha$, $\quad (\alpha = 1$ or $0)$. The risk related to the kth contract in the jth class for a fixed value of α and $\beta = J_j^{(\alpha)} \in [0, 1]$ is given by:

$$X_{jk}^{(\alpha\beta)} = J_{jk}^{(\alpha\beta)} B_{jk}^{(\alpha\beta)}$$

where occurrence Bernoulli random variables J_{jk} is one when at least one claim is filed, and the amount of total claim associated to the kth contract in the jth class is indicated by strictly positive random variable B_{jk}.

Let $\bar{I} = 1 - I$ for any probability or indicator function I. The risk X_{jk} may then be indicated in the form:

$$X_{jk} = J_0 \left(J_j^{(1)} X_{jk}^{(11)} + \bar{J}_j^{(1)} X_{jk}^{(10)} \right) + \bar{J}_0 \left(J_j^{(0)} X_{jk}^{(01)} + \bar{J}_j^{(0)} X_{jk}^{(00)} \right). \tag{2.4}$$

It sounds reasonable to assume that:

(a) J_0, the $J_j^{(\alpha)}$'s and the $J_{jk}^{(\alpha\beta)}$'s are mutually independent.

(b) $B_{jk}^{(\alpha\beta)}$'s are independent of each other and of all indicator random variables.

Genest *et al.* (2003) proposed the following method to find the distribution function $F_{X_{jk}}$ of X_{jk} from those of $X_{jk}^{(\alpha\beta)}$'s and $B_{jk}^{(\alpha\beta)}$'s. Define

$E(J_0) = r, \quad E(J_j^{(\alpha)}) = r_j^{(\alpha)}$ and $E(J_{jk}^{(\alpha\beta)}) = r_{jk}^{(\alpha\beta)}$ and similarly $\bar{r}, \bar{r}_j^{(\alpha)}$and $\bar{r}_{jk}^{(\alpha\beta)}$. Using conditions (a) and (b), for every choice of $1 \leq k \leq n_j$ and $1 \leq j \leq m$ we have:

$$\begin{aligned}
F_{X_{jk}}(x) &= r \left\{ r_j^{(1)} F_{X_{jk}^{(11)}}(x) + \bar{r}_j^{(1)} F_{X_{jk}^{(10)}}(x) \right\} + \bar{r} \left\{ r_j^{(0)} F_{X_{jk}^{(01)}}(x) + \bar{r}_j^{(0)} F_{X_{jk}^{(00)}}(x) \right\} \\
&= r \left[r_j^{(1)} \left\{ \bar{r}_{jk}^{(11)} + r_{jk}^{(11)} F_{B_{jk}^{(11)}}(x) \right\} + \bar{r}_j^{(1)} \left\{ \bar{r}_{jk}^{(10)} + r_{jk}^{(10)} F_{B_{jk}^{(10)}}(x) \right\} \right] \\
&+ \bar{r} \left[r_j^{(0)} \left\{ \bar{r}_{jk}^{(01)} + r_{jk}^{(01)} F_{B_{jk}^{(01)}}(x) \right\} + \bar{r}_j^{(0)} \left\{ \bar{r}_{jk}^{(00)} + r_{jk}^{(00)} F_{B_{jk}^{(00)}}(x) \right\} \right], x \geq 0
\end{aligned}$$

After a rearrangement of the terms we get:

$$\begin{aligned}
F_{X_{jk}}(x) &= \bar{q}_{jk} + r r_j^{(1)} r_{jk}^{(11)} F_{B_{jk}^{(11)}}(x) + r \bar{r}_j^{(1)} r_{jk}^{(10)} F_{B_{jk}^{(10)}}(x) \\
&+ \bar{r} r_j^{(0)} r_{jk}^{(01)} F_{B_{jk}^{(01)}}(x) + \bar{r} \bar{r}_j^{(0)} r_{jk}^{(00)} F_{B_{jk}^{(00)}}(x), x \geq 0
\end{aligned} \tag{2.5}$$

where

$$q_{jk} = rr_j^{(1)} r_{jk}^{(11)} + r\bar{r}_j^{(1)} r_{jk}^{(10)} + \bar{r} r_j^{(0)} r_{jk}^{(01)} + \bar{r}\bar{r}_j^{(0)} r_{jk}^{(00)} = P(X_{jk} \neq 0). \quad (2.6)$$

Accordingly, each risk $X_{jk} = J_{jk} B_{jk}$ may be denoted as it follows:

$$X_{jk} = \begin{cases} B_{jk}, & J_{jk} = 1 \\ 0, & J_{jk} = 0 \end{cases}$$

Based on the definition of indicator random variable J_{jk} with $P(J_{jk} = 1) = q_{jk}$ in Eq.(2.6),

$$\begin{aligned} F_{X_{jk}}(x) = Pr(X_{jk} \leq x) = Pr(B_{jk} J_{jk} \leq x) &= Pr\{B_{jk} J_{jk} \leq x | J_{jk} = 0\} Pr(J_{jk} = 0) \\ &+ Pr\{B_{jk} J_{jk} \leq x | J_{jk} = 1\} Pr(J_{jk} = 1) \\ &= (1 - q_{jk}) Pr\{x \geq 0\} + q_{jk} Pr\{B_{jk} \leq x\} \\ &= \bar{q}_{jk} + q_{jk} F_{B_{jk}}(x) \qquad (2.7) \end{aligned}$$

From (2.5) and (2.7), the distribution of the strictly positive random variable B_{jk} is a mixture distribution

$$\begin{aligned} F_{B_{jk}} &= \frac{rr_j^{(1)} r_{jk}^{(11)}}{q_{jk}} F_{B_{jk}{}^{(11)}} + \frac{r\bar{r}_j^{(1)} r_{jk}^{(10)}}{q_{jk}} F_{B_{jk}{}^{(10)}} \\ &+ \frac{\bar{r} r_j^{(0)} r_{jk}^{(01)}}{q_{jk}} F_{B_{jk}{}^{(01)}} + \frac{\bar{r}\bar{r}_j^{(0)} r_{jk}^{(00)}}{q_{jk}} F_{B_{jk}{}^{(00)}} \end{aligned}$$

As X_{jk}'s are dependent in this model, it is hard to calculate directly the distribution F_S of the total claim amount S of the portfolio. But, there is a handy way to avoid this problem. We can consider a mixture structure for S under conditions (a),

(b) and (2.4). F_S can then be formulated in the following form, where Θ is a latent random vector with distribution M:

$$F_S(x) = \int F_{S^{(\theta)}}(x) dM(\theta), \tag{2.8}$$

where $S^{(\theta)}$ is distributed as S given $\Theta = \theta$. To be clear, let $\Theta = (\Theta_0, ..., \Theta_m)$ have values in $\{0,1\}^{m+1}$ such that:

$$P(\Theta_0 = \alpha) = P(J_0 = \alpha) = r^{\alpha} \bar{r}^{(1-\alpha)}$$

and

$$\begin{aligned} P(\Theta_1 = \beta_1, ..., \Theta_m = \beta_m \big| \Theta_0 = \alpha) &= P(J_1^{(\alpha)} = \beta_1, ..., J_m^{(\alpha)} = \beta_m) \\ &= \prod_{j=1}^{m} \left(r_j{}^{(\alpha)}\right)^{\beta_j} \left(\bar{r}_j^{(\alpha)}\right)^{1-\beta_j} \end{aligned}$$

for all $\alpha, \beta_1, ..., \beta_m \in \{0,1\}$. For $\Theta = \theta$, write $X_{jk}^{(\theta)} = X_{jk}^{(\alpha\beta_j)}$ and let:

$$S_j^{(\theta)} = \sum_{k=1}^{n_j} X_{jk}^{(\theta)} \quad \text{and} \quad S^{(\theta)} = \sum_{j=1}^{m} S_j^{(\theta)} \tag{2.9}$$

So, based on Eq. (2.4) we will have:

$$\begin{aligned} F_s(x) &= P\big(\sum_{j=1}^{m}\sum_{k=1}^{n_j} X_{jk} \leq x\big) \\ &= \sum_{\alpha=0}^{1}\sum_{\beta_1=0}^{1} ... \sum_{\beta_m=0}^{1} P\big(\sum_{j=1}^{m}\sum_{k=1}^{n_j} X_{jk}^{(\alpha\beta_j)} \leq x\big) P\{\Theta = (\alpha, \beta_1, ..., \beta_m)\} \\ &= \sum_{\theta\in\{0,1\}^{m+1}} P\big(\sum_{j=1}^{m} S_j^{(\theta)} \leq x\big) P(\Theta = \theta) \\ &= \sum_{\theta\in\{0,1\}^{m+1}} F_{S^{(\theta)}}(x) P(\Theta = \theta), \end{aligned}$$

which is the same as Eq. (2.8) for Θ discrete.

As $S_j^{(\theta)}$'s are mutually independent and each of them is a finite sum of mutually independent random variables, we can take advantage of the former representation for finding the distribution function F_S for a given θ.

In this model using a mixture structure for S is a useful way to express F_s when the X_{jk} are dependent. But, we still do not know how to deal with cases in which the class and group risk factors and the B_{jk}'s are dependent.

Application of Model 2.4.2 in Supply Chains:

Figure 2.5 can be an example of Toyota's supply chain which operates its business worldwide. As explained in this model, companies (individual risk factors) that are in different regions (group risk factors) do not have any contribution with each other as they are assumed to be independent. But, a consequence of Toyota's excessive expansion was that it became increasingly dependent on suppliers outside Japan, or Toyota branches in other countries became dependent on suppliers in Japan. For example, Toyota gets only 15 percent of its parts including electronic and rubber components from Japan for cars and trucks built in North America. But, still it has

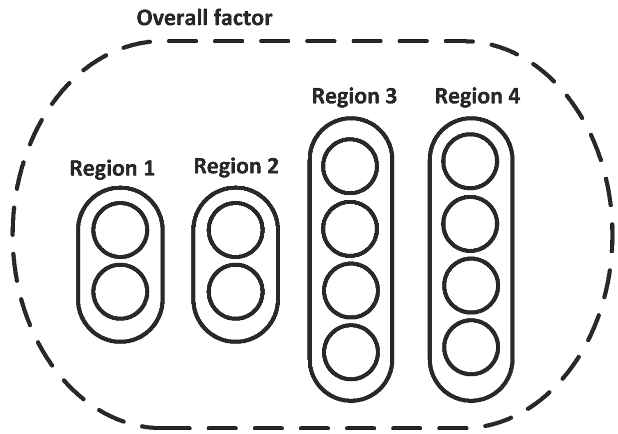

Figure 2.5: Schematic representation of Model 2.4.2 in supply chains. Each circle represents a company/facility.

to have all of them to build a vehicle. It means that a shut down of a part supplier in Japan can result in a shut down of an assembly plant in North America. If the model 2.4.2 is applied to a supply chain network, then we are able to figure out the distribution function of the total loss caused by risks on companies involved in the network.

2.4.3 Dependence via occurrence random variables

The Model proposed by Cossette *et al.* (2002) is a special case of Models 2.4.1 and 2.4.2. In this model dependence via occurrence random variables $I_{jk},\ j = 1, ..., m$ and $k = 1, ..., n_j$, causes dependence between risks $X_{jk},\ j = 1, ..., m$ and $k = 1, ..., n_j$, represented as $X_{jk} = I_{jk}B_{jk}$. We suppose that the occurrence of a claim for the kth policy in the jth class is a function of the individual, the class, and the global risk factors. Independent random variables J_{jk}, J_j and J_0 refer to these three risk factors, respectively. Random variable $I_{jk},\ j = 1, ..., m$, and $k = 1, ..., n_j$ is defined as:

$$I_{jk} = \min(J_{jk} + J_j + J_0, 1) \tag{2.10}$$

where J_{jk}, J_j, and J_0 are independent Bernoulli random variables with $P(J_{jk}=1)=\tilde{q}_{jk}$, and $P(J_{jk}=0)=\tilde{p}_{jk}=1-\tilde{q}_{jk}$, $P(J_j=1)=\tilde{q}_j$, $P(J_j=0)=\tilde{p}_j=1-\tilde{q}_j$, $P(J_0=1)=\tilde{q}_0$, and $P(J_0=0)=\tilde{p}_0=1-\tilde{q}_0$.

Based on Eq. (2.10) random variables I_{jk} are Bernoulli distributed and as a result the random vector $\underline{I}=(I_{11},\ldots,I_{1n_1},\ldots,I_{m1},\ldots,I_{mn_m})$ would have dependent components. The probability generation function (pgf) of I_{jk} is defined as follows:

$$\begin{aligned} P_{I_{jk}}(t) &= \sum_{I_{jk}=0}^{1} P(I_{jk})\, t^{I_{jk}} \\ &= p_{jk}+q_{jk}\, t, \end{aligned}$$

where $p_{jk}=\tilde{p}_0\,\tilde{p}_j\,\tilde{p}_{jk}$ and $q_{jk}=1-(1-\tilde{q}_0)(1-\tilde{q}_j)(1-\tilde{q}_{jk})$. If $\tilde{q}_j=\tilde{q}_0=0,\ j=1,...,m$, then $q_{jk}=\tilde{q}_{jk}$ which is a special case of the individual risk model. If $\tilde{q}_j=0$ then the portfolio will get only one class in this case. So, based on Eq. (2.10) of I_{jk}, the random vector $\underline{X}=(X_{11},...,X_{1n_1},...,X_{m1},...,X_{mn_m})$ has dependent components. The moment generating function (mgf) of each $X_{jk},\ j=1,...,m$ and $k=1,...,n_j$, is:

$$M_{X_{jk}}(t)=P_{I_{jk}}(M_{B_{jk}}(t))=p_{jk}+q_{jk}M_{B_{jk}}(t)$$

For finding moment generating function (mgf) of the aggregate claim amount S, we need to obtain the multivariate mgf of the random vector $\underline{X}$ which in turn is a function of the pgf of the random vector $\underline{I}$. The pgf of $\underline{I}$ is given by:

$$P_{\underline{I}}(\underline{t})=\tilde{p}_0\left[\prod_{j=1}^{m}\left(\tilde{q}_j\prod_{k=1}^{n_j}t_{jk}+\tilde{p}_j\prod_{k=1}^{n_j}P_{J_{jk}}\big(t_{jk}\big)\right)\right]+\tilde{q}_0\prod_{j=1}^{m}\prod_{k=1}^{n_j}t_{jk} \tag{2.11}$$

where $\underline{t}=(t_{11},...,t_{1n_1},...,t_{m1},...,t_{mn_m})$. The multivariate mgf of $\underline{X}$ using Eq. (2.11) is

$$M_{\underline{X}}(t) = P_{\underline{I}}\left(M_{B_{11}}(t_{11}), ..., M_{B_{1n_1}}(t_{1n_1}), ..., M_{B_{m1}}(t_{m1}), ..., M_{B_{mn_m}}(t_{mn_m})\right) \tag{2.12}$$

Given Eq.(2.12) we can use the following Lemma to find the mgf of S.

Lemma 2.4.3.1 [*Cossette* et al. *(2002)*]: *Let* $M_{Y_1,...,Y_n}(t_1, ..., t_n)$ *be the multivariate* mgf *of the vector* $(Y_1, ..., Y_n)$ *given by*

$$M_{Y_1,...,Y_n}(t_1, ..., t_n) = E\left[e^{t_1 Y_1} ... e^{t_n Y_n}\right]$$

Then, the mgf of $Z = Y_1 + ... + Y_n$ is

$$M_Z(t) = M_{Y_1,...,Y_n}(t, ..., t). \tag{2.13}$$

From Lemma 2.4.3.1 and Eq. (2.12), the mgf of S is:

$$\begin{aligned} M_S(t) &= \tilde{p}_0 \left[\prod_{j=1}^{m}\left(\tilde{q}_j \prod_{k=1}^{n_j} M_{B_{jk}}(t) + \tilde{p}_j \prod_{k=1}^{n_j} P_{J_{jk}} M_{B_{jk}}(t)\right)\right] \\ &+ \tilde{q}_0 \prod_{j=1}^{m}\prod_{k=1}^{n_j} M_{B_{jk}}(t) \end{aligned} \tag{2.14}$$

From Eq. (2.14), we can see that F_s is a convex combination of two cumulative distribution functions, F_U and F_V:

$$F_S(x) = \tilde{p}_0 F_U(x) + \tilde{q}_0 F_V(x), \quad x \geq 0, \tag{2.15}$$

where U and V are random variables with the following mgf:

$$\begin{aligned} M_U(t) &= \prod_{j=1}^{m}\Big(\tilde{q}_j\prod_{k=1}^{n_j}M_{B_{jk}}(t)+\tilde{p}_j\prod_{k=1}^{n_j}P_{J_{jk}}\big(M_{B_{jk}}(t)\big)\Big) \\ \text{and}\quad M_V(t) &= \prod_{j=1}^{m}\prod_{k=1}^{n_j}M_{B_{jk}}(t). \end{aligned}$$

The cumulative distribution functions of U and V are:

$$F_U = F_{C_1} * ...F_{C_m} \tag{2.16}$$

$$F_V = F_{B_{11}} * ... * F_{B_{1n_1}} * ... * F_{B_{m1}} * ... * F_{B_{mn_m}}, \tag{2.17}$$

where

$$\begin{aligned} F_{C_j} &= \tilde{q}_j(F_{B_{j1}} * ... * F_{B_{jn_j}})+\tilde{p}_j(F_{D_{j1}} * ... * F_{D_{jn_j}}), \quad j=1,...,m, \\ F_{D_{jk}} &= \tilde{p}_{jk}\triangle_0+\tilde{q}_{jk}F_{B_{jk}}, \quad k=1,...,n_j \end{aligned}$$

where symbol "$*$" denotes the convolution product between two cumulative distribution functions, and $\triangle_d$ is the Dirac function:

$$\triangle_d(x)=\begin{cases}1, & \text{if } x\geq d,\\ 0, & \text{otherwise}\end{cases}$$

In most cases like Model 2.4.3 we cannot find an explicit form for F_S. Therefore, we need to apply numerical approximation. In this model, given Eq. (2.15), two steps can be made to evaluate F_s numerically. First, with the appropriate formulas given in (2.16) and (2.17) F_U and F_V are computed. Then, F_s is calculated with (2.15).

In this model dependence between the occurrence random variables $I_{jk},\ j=1,\dots,m,$ and $k=1,\dots,n_j$, leads to dependent risks $X_{jk},\ j=1,\dots,m$, and $k=$

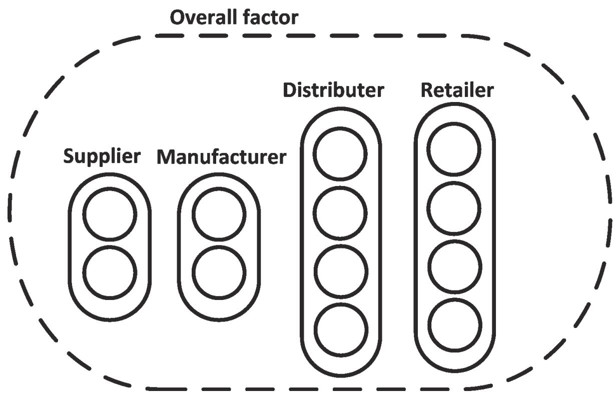

Figure 2.6: Schematic representation of Model 2.4.3 or 2.4.2 in supply-chain. Each circle represents a company/facility.

$1, \ldots, n_j$. But, Cossette *et al.* (2002) assumed that all three risk factors (J_{jk}, J_j, $and\, J_0$) used in the definition of I_{jk} are independent Bernoulli random variables which can be considered as a restriction to this model. In addition, there isn't any explicit form for F_s and we have to use a numerical approximation to calculate F_s.

Application of Model 2.4.3 in Supply Chains

One possible example of this model is the supply chain of Toyota for car model "Prius" in which the supplier, manufacturer, retailer, and distributor are all in Japan. All four links considered as group risk factors in this model, must have a close relationship to manufacture a vehicle and send it to customers. So, dependence between these business partners is required. In addition, collaboration and shared principles within companies (individual risk factors) in each group, like suppliers, is necessary to implement various activities.

2.4.4 Common mixture model

In a second model of Bauerle and Muller (1998) we compare the portfolios based on the number of external mechanisms that influence them. Assume that there are two n dimensional random vectors X and Y with the structure:

$$\begin{aligned} (X_1, ..., X_n) &= (g_1(Z_1, W), ..., g_n(Z_n, W)) \\ (Y_1, ..., Y_n) &= (\tilde{g}_1(U_1, V, W), ..., \tilde{g}_n(U_n, V, W)) \end{aligned}$$

where $Z_1, ..., Z_n, U_1, ..., U_n$ are $i.i.d.$ random variables and (V, W) is a random vector independent of Z_k and U_k. Also, $g : \mathbb{R}^2 \to \mathbb{R}$ and $\tilde{g} : \mathbb{R}^3 \to \mathbb{R}$ are such that for every fixed W and all $k = 1, ..., n$, we have:

$$g_k(Z_k, W) \stackrel{d}{=} \tilde{g}_k(U_k, V, W),$$

i.e., they have the same distribution.

Let $S = \sum_{k=1}^{n} X_k$ and $S' = \sum_{k=1}^{n} Y_k$. In the following theorem Bauerle and Muller (1998) show that given some conditions on functions $\tilde{g}_i$, the portfolio Y is more risky that the portfolio X.

Theorem 2.4.4.1 [*Bauerle and Muller (1998)*] *If the functions $\tilde{g}_i$ is increasing in the second argument, then:*

$$\begin{aligned} &a) \qquad X \leq_{symsm} Y \\ &b) \qquad S \leq_{sl} S'. \end{aligned} \tag{2.18}$$

There is more dependence in portfolio Y than in X due to the extra environmental variable V. So, the external mechanism V, which has a common influence on all risks in portfolio Y is an important risk factor.

As a special case if we assume that W is constant then $Y_k = \tilde{g}_k(U_k, V)$ and $X_k = g_k(Z_k)$. Then, Bauerle and Muller (1998) obtain the following corollary of theorem 2.4.4.1.

Corollary 2.4.4.2 [*Bauerle and Muller (1998)*] *Let V be any random variable and let $Y = Y_1, \ldots, Y_n$ be a random vector such that $Y_1, \ldots, Y_n$ are conditionally independent given $V = v$ and such that the conditional distributions $P(Y_k \mid V = v)$ are stochastically increasing in v for all $k = 1, \ldots, n$. Moreover, let $X = (X_1, \ldots, X_n)$ be a random vector of independent random variables with the same marginal distribution as Y. Then*

$$X \leq_{sm} Y \qquad \text{and} \qquad S = \sum_{k=1}^{n} X_k \leq_{sm} S' = \sum_{k=1}^{n} Y_k. \tag{2.19}$$

In the following theorem Shaked and Shanthikumar (1997) consider the same assumptions as Corollary 2.4.4.2.

Theorem 2.4.4.3 [*Shaked and Shanthikumar (1997)*] *Let $X = (X_1, X_2, \ldots, X_n)$ be a conditionally increasing in sequence random vector and let $Y = (Y_1, Y_2, \ldots, Y_n)$ be a random vector of independent random variables such that $X_k =_{st} Y_k, k = 1, \ldots, n$. Then $Y \leq_{sm} X$.*

In contrast to Model 2.4.1, in this model, the comparison of two portfolios is based on the number of external mechanisms which affect the portfolios. In this model we assume that all individual random variables $Z_1, ..., Z_n$ and $U_1, ..., U_n$ are independent. Also, (V, W) is a random vector independent of Z_i and U_i. But, the productivity of a network is based on the contribution of individual factors in each group and collaboration of business partners. Therefore, the assumption of independence can be considered as a restriction for this model.

Application of Model 2.4.4 in Supply Chains:

In figure 2.7 Network 2 is affected by external mechanisms. So, the dependence between risks in Network 2 is more than that in Network 1. As a result we can

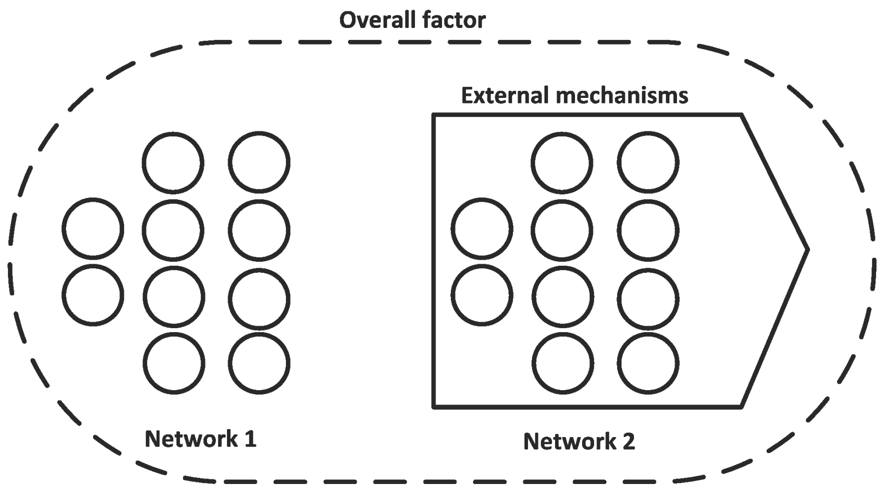

Figure 2.7: Schematic representation of Model 2.4.4 in supply-chain. Each circle represents a company/facility.

conclude that Network 2 is more risky than Network 1. If we assume the supply chain of Toyota as Network 2 and the supply chain of Honda as Network 1. The severe natural disaster that occurred in northeastern Japan on March 11, 2011 hit both supply chains. But, the powerful tsunami triggered by earthquake had very bad effect on some plants of Toyota in Miyako. So, we can say that the supply chain of Toyota is more risky than that of Honda because of the geographic location of its plants.

2.4.5 Two-point distribution model

2.4.5.1 Indistinguishable individuals(Bauerle and Muller, 1998)

When permutation does not have any effect on the joint distribution of the random vector of n risks of a portfolio, we can say that these individual risks are indistinguishable individuals. This implies that the marginal distribution is the same for all risks, i.e., there is a $p \in (0, 1)$ and some $\alpha > 0$ such that $P(X_k = 0) = p = 1 - P(X_k = \alpha)$ for all $k = 1, ..., n$. In probability theory a sequence of such random variables is said to be exchangeable (or interchangeable). Suppose that $\alpha = 1$ then random variables $X_1, X_2, ..., X_n$ establish a sequence of exchangeable Bernoulli variables.

Hence if we assume S_n to be the total amount of claims in a portfolio of n risks, which occur from a sequence of exchangeable Bernoulli variables then based on De Finetti theorem, S_n is a mixture of binomial distributions, i.e.,

$$P(S_n = K) = \int_0^1 \binom{n}{k} \gamma^k (1-\gamma)^{n-k} F(d\gamma).$$

where γ is a dependence parameter, which continuously varies between independence and maximal dependence and F is the probability distribution of γ over $[0, 1]$. In fact, F is a prior for random vector γ given $\Gamma = \gamma$. Bauerle and Muller (1998) in the following theorem show how the riskiness of the portfolio (S_n) is influenced by mixing distribution F.

Theorem 2.4.5.1 [*Bauerle and Muller (1998)*] *Let $S_n(S'_n)$ be the total claim amount of a portfolio of n risks, which stem from a sequence of exchangeable Bernoulli variables with mixing distribution $F(F')$. Then $F \leq_{sl} F'$ implies $S_n \leq_{sl} S'$.*

In contrast to Models 2.4.1 and 2.4.4 of Bauerle and Muller (1998), all risks should have the same marginal distribution here. The total claim S_n in this case is a mixture of binomial distributions. But, we did not talk about the distribution of S in the former models.

Application of Model 2.4.5.1 in Supply Chains

Consider Figure 2.8 consisting of Network 1 and Network 2. In this model we assume that individual risks (shown with circles in Network 1 and with diamonds in Network 2) are a sequence of exchangeable Bernoulli variables. In Network 1 (Network 2) each risk factor is 1 if it produces any loss with probability of p (or p' for Network 2) otherwise is 0 with probability of $1-p$ (or $1-p'$ for Network 2). But, the

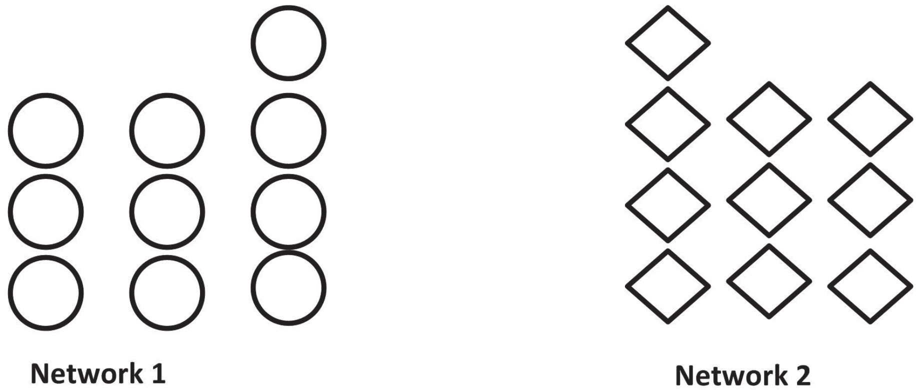

Figure 2.8: Schematic representation of Model 2.4.5.1 in a supply chain. Each circle represents a company and each diamond indicates a facility. If we assume that the prior probability of dependence factor in Network 2 is greater than Network 1, then Network 2 becomes more risky.

quantities p and p' are unknown. If $F(F')$ expresses the probability distribution about uncertainty of $p(p')$ then based on the result of Theorem 2.4.5.1 we can conclude that Network 2 is more risky than Network 1 if the prior of p' (i.e., F') is greater than the prior of p (i.e., F) in stop-loss order.

2.4.5.2 Distinguishable individuals

The additive damage model proposed by Bauerle and Muller (1998) is a well known model in probability theory. In this model we are dealing with two sources which produce some normally distributed damage. One source has the same effect on all individuals, while the impact of the other one depends on the individual behaviour of each individual. We will have the claim amount of β, if the sum of these two damages surpasses some level z_k.

The distribution function of the distinguishable individuals model which is constructed based on model 2.4.4 assumes only two values. Let $N(\mu, \sigma^2)$ denote a univariate normal distribution with mean μ and variance $\sigma^2 > 0$. $N(\mu, 0)$ indicates a one-point distribution in μ. If $X \sim N(0, 1)$, then assuming $P(X \leq z_p) = p$, z_p would be the p-quantile of the standard normal distribution. Now we consider model 2.4.4 with $W \sim N(0, \sigma^2), V \sim N(0, \tau^2 - \sigma^2), Z_k \sim N(0, 1 - \sigma^2)$ and $U_k \sim N(0, 1 - \tau^2)$ when $0 \leq \sigma^2 < \tau^2 \leq 1$. All random variables should be independent in this model. We define:

$$g_k(z, w) = \begin{cases} \beta_k, & z + w \geq z_{p_k} \\ 0, & \text{else} \end{cases}$$

and

$$\tilde{g}_k(u, v, w) = \begin{cases} \beta_k, & u + v + w \geq z_{p_k} \\ 0, & \text{else} \end{cases}$$

Recall from model 2.4.4 that $X_k = g_k(Z_k, W)$ and $Y_k = \tilde{g}_k(U_k, V, W)$ for $k = 1, ..., n$. It would be easy to show that all conditions in Model 2.4.4 are held for this model, so we can say that $X \leq_{sl} Y$ and therefore X is less risky than Y. This model is an extension of model 2.4.4. But, distributions and functions of all risks assume only two values. Also, all random variables Z, U, W, V are normally distributed and the effect of additive damage depends on the behaviour of each individual.

Application of model 2.4.5.2 in supply chains

Assume that in Network 1 each risk is a function of an individual risk factor (company) and an overall risk factor (natural disaster). See Fig. 2.9. The first source of damage (natural disaster) influences all companies in the same manner. But, occurrence of the second source of risk, like inventory decline depends on the individual behaviour of each company, such as how fast it recovers from a disaster. So, the loss amount of β_k in company k occurs if the sum of these damages (natural disaster and inventory decline) get larger than a specific amount. In Network 2 each risk is a function of an individual risk factor (company), an external mechanism (demand change) and an overall risk factor (natural disaster). The effect of the natural disaster and the demand change are the same for all companies. But, each company faces an inventory decline, if it cannot manage the crisis properly . If a total damages arising from these sources exceed some level z_{p_k}, then company tolerates a loss of β_k. Since, the construction of this model is based on Model 2.4.4, we can apply the result of Theorem 2.4.4.1 to conclude that Network 2 is more risky than Network 1 due to the additive damage.

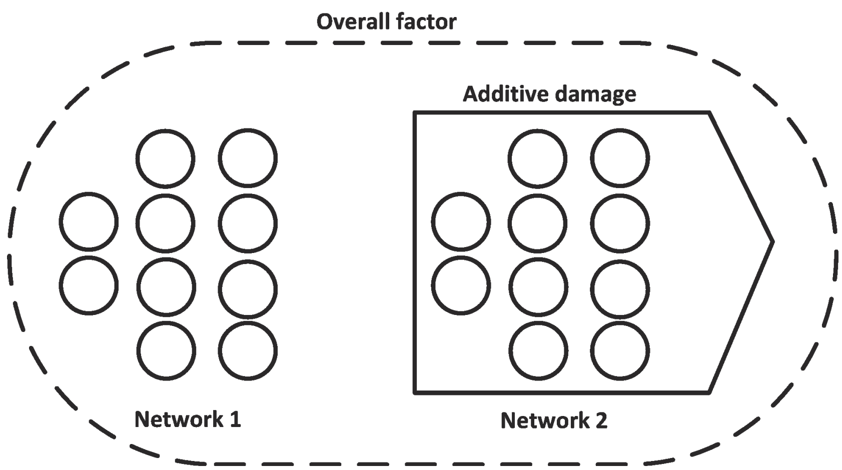

Figure 2.9: Schematic representation of Model 2.4.5.2 in a supply chain. Each circle is a representation of a Bernoulli random variable that is associated with a company/facility.

2.5 Distribution function (copula) based dependency models

In this section we consider two copula-based models; one for a single group and the other for multiple groups.

2.5.1 Copula based dependent single-group risks model (Genest *et al.*, 2003)

In the single-class portfolio problem, where each individual risk may be represented in the form $X_k = J_k B_k$ with indicator Bernoulli random variable J_k with expectation q_k, using copula is another way of modelling dependence among the components of the vector $J = (J_1, ..., J_n)$. We can write the joint cumulative distribution function of J as follows:

$$F_J(j_1, ..., j_n) = C\{F_{J_1}(j_1), ..., F_{J_n}(j_n)\}, \quad j_k = 0 \text{ or } 1, \quad k = 1, ..., n \tag{2.20}$$

Where, $F_{J_k}(t) = P(J_k \leq t)$ is the marginal cumulative distribution function of J_k with $F_{J_k} = 0$, for $j_k \leq 0$. $C : [0,1]^n \to [0,1]$ is a copula. All marginals F_{J_k}'s of a cumulative distribution function are uniformly distributed on the unit interval. We can say that copula is the distribution function of a random vector with uniform marginals. The advantage of writing F_J as in (2.20) is that it allows us to separate the definition of the marginals $F_{J_1}, F_{J_2}, ..., F_{J_n}$ and the definition of the dependence which is constructed through the copula $C(u_1, ..., u_n)$. Various copulas can be found in the literature. The simplest one is the independent copula:

$$C^{Ind}(u_1, ..., u_n) = u_1 \times \ldots \times u_n.$$

See Genest *et al.* (2003) and Cossette *et al.* (2002) for some examples of copulas.

2.5.2 Copula based dependent multi-group risks model (Genest *et al.*, 2003)

This section is a multi-class extension of the Archimedean model of previous section. Assume that the joint distribution function of $(n_1+...+n_m)$-dimensional vector $J=(J_1',...,J_m')$ of occurrence random variables is written as:

$$F_J(j_1',...,j_m') = \int_0^\infty ... \int_0^\infty \prod_{j=1}^{m}\prod_{k=1}^{n_j}\{F^*_{J_{jk}}(j_{jk}')\}^{\theta_j} dM(\theta_1,...,\theta_m),$$

Where M is the m-variate distribution function of the latent vector $\Theta=(\theta_1,...,\theta_m)\in[0,\infty)$ and $F^*_{J_{jk}}=\exp[-\phi_j\{F_{J_{jk}}\}]$ is the cumulative distribution function of Bernoulli random variable J_{jk} with expectation $1-\exp\{-\phi_j(1-q_{jk})\}$, $1\le k\le n_j$ and $1\le j\le m$. The J_{jk}'s are mutually independent Bernoulli random variables, conditioned on the value of Θ with

$$E(J_{jk}|\Theta=\theta)=E(J_{jk}|\Theta_j=\theta_j)=1-\exp\{-\theta_j\phi_j(1-q_{jk})\}=q_{jk\theta_j}.$$

We assume as before that the B_{jk}'s are independent among themselves and from all indicators. Let $J_{jk}^{(\theta)}$ be mutually independent indicator random variables with mean $q_{jk\theta_j}$ for all $1\le k\le n_j$ and $1\le j\le m$. Considering Eq. (2.21), the sums $S_j^{(\theta)}$ and $S^{(\theta)}$ is defined as in Eq. (2.9).

$$X_{jk}^{(\theta)}=X_{jk}^{(\theta_j)}=J_{jk}^{(\theta_j)}B_{jk}\quad (Given\,\Theta=\theta)$$

As S^θ are composed of mutually independent terms, the total claim distribution F_S may again be calculated in the form, Eq.(2.8).

In this model as in model 2.4.3 we assume that occurrence random variables J_{jk}, $j=1,...,m$ and $k=1,...,n_j$, are dependent. But, we use copula to separate the definition of the dependence between components of $\underline{J} = (J_{11}, ..., J_{1n_1}, ..., J_{mn_1}, ..., J_{mn_m})$ and definition of the marginals $F_{J_{jk}}$.

Even though the models reviewed in Chapter 2 provide valuable insight, some assumptions they require do not necessarily fit with real life supply chains. One of the most strongest assumptions may be the independence between the individual risks. Therefore, in the next section we extended the existing models and remove the assumption of individual risks independence.

2.6 Extension: Dependent individual risk factors

Lemma 2.6.0.1 [*Tchen (1980)*]: *Suppose there are two n-dimensional random vectors Z and U where the distribution function of U is Frechet upper bound. Then $Z \leq_{sm} U$. (Proof is in Theorem 5.(a) of Tchen (1980))*

Theorem 2.6.0.2 *Consider n-dimensional random vectors Z and Frechet upper bound distributed U such that $Z \leq_{sm} U$ (Lemma 2.6.0.1) and let W be a m-dimensional vector, which is independent from Z and U. Then,*

$$\begin{aligned} &Z \leq_{sm} U \\ &\Rightarrow (g_1(Z_1, W), g_2(Z_2, W), ..., g_n(Z_n, W)) \leq_{sm} (g_1(U_1, W), g_2(U_2, W), ..., g_n(U_n, W)) \end{aligned}$$

where $g_k(z, w)'s, k = 1, 2, ..., n,$ are all monotone in z for every w.
If we suppose the following constructions for X and Y

$$\begin{aligned} X &= (g_1(Z_1, W), g_2(Z_2, W), ..., g_n(Z_n, W)) \\ Y &= (g_1(U_1, W), g_2(U_2, W), ..., g_n(U_n, W)) \end{aligned}$$

Then $X \leq_{sm} Y$ results in $S \leq_{sl} S'$ (Theorem 2.6 of Bauerle and Muller (1998)).

Proof: Since $Z \leq_{sm} U$ and $g_k(z,w), k = 1,\ldots,n$ are monotone functions in z for every w, from Theorem 2.2(a) of Shaked and Shanthikumar (1997) it follows that

$$\begin{aligned}
&(Z_1,\ldots,Z_n) \leq_{sm} (U_1,\ldots,U_n)\\
&\Rightarrow (g_1(Z_1,W|W=w),\ldots,g_n(Z_n,W|W=w)) \leq_{sm}\\
&\qquad\qquad (g_1(U_1,W|W=w),\ldots,g_n(U_n,W|W=w))
\end{aligned}$$

The super modular stochastic order is closed under mixtures (Theorem 2.2(d) of Shaked and Shanthikumar (1997)). Hence

$$(g_1(Z_1,W),\ldots,g_n(Z_n,W)) \leq (g_1(U_1,W),\ldots,g_n(U_n,W)) \Rightarrow X \leq_{sm} Y.$$

Application of model 2.6 in supply chains

The most important aspect of Figure 2.10 compared to other figures is the connection between companies (individual risks factors). It means that in this model we relax the assumption of independence between individual risks. Since cycles in Networks (1 and 2) of Figure 2.10 are not important we can call this structure a tree structure. Considering all assumptions of the last model, we can conclude that Network 1 is less risky than Network 2 due to the existence of less risky companies in Network 1 compared to Network 2.

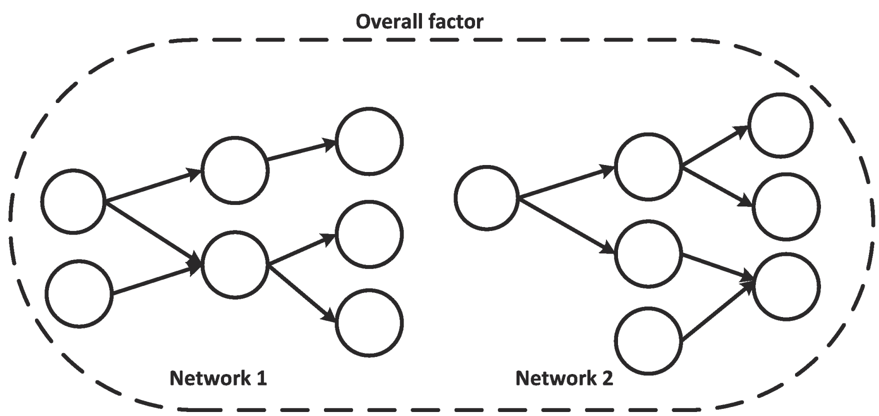

Figure 2.10: Schematic Representation of model 2.6 in a supply chain. Each circle represents a company/facility, and connections denotes dependence.

2.7 Conclusion

In this chapter we have reviewed and extended the literature on risk dependency modelling. All models of dependencies reviewed in this chapter are summarized in Table 2.3.

Table 2.3: Comparison of reviewed risk dependency models.

Type of dependence	**Reference**
Multi-groups Dependence between groups	2.4.1: Bauerle and Muller (1998), 2.4.3: Cossette *et al.* (2002), 2.4.2: Genest *et al.* (2003), 2.5.2: Genest *et al.* (2003), 2.4.4: Bauerle and Muller (1998), 2.4.5.2: Bauerle and Muller (1998)
Multi-groups Dependence within groups	Removing overall risk factor V in 2.4.1: Bauerle and Muller (1998), Let $\tilde{q} = 0$ in 2.4.3: Cossette *et al.* (2002), Let J = 0 in 2.4.2: Albers (1999)
Single-group Dependent risks	Let $\tilde{q}_j = 0$ in 2.4.3: Cossette *et al.* (2002), 2.3.1: Goovaerts and Dhaene (1996), 2.3.2: Dhaene and Goovaerts (1997), 2.5.1: Genest *et al.* (2003)
Dependent individual risk factors	2.6: Leila M. Beigi (2013)

We have also showed how these models can be applied in a supply chain environment. We have indicated the shortcomings of these models in relation to supply chain environments. These present possible future research venues in this area.

Chapter 3

A risk propagation model

In Chapter 3 we model risk propagation in supply chains that accounts for dependencies in time and network structure.

3.1 Literature review

Since Chapter 2 includes a review on risk dependence modelling, the literature review in this chapter focuses on risk propagation. Several mechanisms of risk propagation or transmission have already been studied. Within the social science literature, Diekmann and Mitter (1982) proposed a time dependent Poisson process to describe the dynamic aspect of some social behaviours like occupational mobility and deviant behaviour. A general contagion model was proposed by Hamilton and Hamilton (1981). They modelled the intensity as a product of two functions. The first function expressed the contagion and the second one indicated the time dependency. We borrowed some of our assumptions from these references. Coleman (1964) presented a simple time-homogeneous contagion model in which the expected number of events per period grows linearly with the cumulative number of events already observed. Diekman (1979) modelled a contagion influence similar to that used by Coleman, adding an inhibiting mechanism through the time dependent function.

In the finance literature, Bardos and Stili (2007) identified patterns necessary for risk contagion and bankruptcy to occur and found that risk transmission occurs when receivables show a significant part of total assets.

In the supply chain literature, Serrano *et al.* (2010) built a supply chain model based on empirical findings from the financial literature to show how the variability of payments to suppliers propagates upstream, which has a major impact on risk. The extended Petri net-based modelling presented by Wu *et al.* (2007) using a "Disruption Analysis Network" (DA-NET) approach models how changes propagate through a supply chain. The states that are reachable from a given initial marking in a supply chain network are determined by this approach in order to calculate the impact of the attributes. Bilsel (2009) used a Markov chain based approach and employed 'mean first passage time' to model the probability that a risk will propagate from its place of occurrence to another part in the network.

3.2 Model assumptions

In this section, the assumptions underlying the risk propagation model will be examined. Since the probability of an event changes over time, a time dependent stochastic process where the transition rate (or expected number of events during a unit of time) is a function of time would be a suitable model. Also, models like 'Weibull process' (Mann *et al.*, 1974) in which the probability of an event is a monotone function of time would not be a realistic model. Because, this model cannot explain a process where the probability initially increases, eventually reaches a maximum value, and finally decreases to zero asymptotically. The shape of time dependent function of our transition rate is like a sickle (inverted U)(Diekmann and Mitter, 1982). Other than the time dependency, transition rate governing our model needs to take into account the contagion. So, an ordinary time dependent Poison process will not be very applicable in modelling risk propagation. Once an event occurs for an element like a country, then the probability of a second element being affected will be changed. Or when an element is affected, its probability of getting affected again is changed. Therefore, we will present a simple time-inhomogeneous contagion model in which the transition rate is a product of two functions, as follows.

$$\alpha_k(t) = \alpha_k \, w(t) \tag{3.1}$$

The first term on the right, α_k, describes contagion, while the second term, $w(t)$, expresses time dependence.

According to Hamilton and Hamilton (1981), and based on Diekman (1979) and Coleman (1964), the following assumptions and modifications of the Poisson process are used to model the system described above.

Assumption 1. The probability of exactly one event in a small time interval $(t, t+\Delta t)$ is asymptotically proportional to the length of the interval.

$Pr[1 \text{ event in } (t, t+\Delta t)] = [\alpha_k \, w(t)].\Delta t + O(\Delta t)$

where $O(\Delta t)$ is a function such that $\lim_{\Delta t \to 0} \frac{O(\Delta t)}{\Delta t} = 0$.

Assumption 2. The probability of more than one event in a small interval is insignificant: $\sum_{n>1} P[n \text{ events in } (t, t+\Delta t)] = O(\Delta t)$

Assumption 3. Once one has allowed for any contagious impact through the function α_k, the occurrence of events in disjoint intervals of time are statistically independent.

The above three assumptions are in fact an extension of four properties of the non-homogeneous Poisson process (Ross, 2010) but replacing the time-dependent Poisson rate $\alpha(t)$ with $\alpha_k \, w(t)$.

To describe the above assumptions in an example, here we consider the study on Chapter 10, page 302 of the book by Coleman (1964). Let's consider the number of phonograph records bought by girls in a specific period of time, i.e., a fixed t equal to one month. The probability that one girl bought just one phonograph record was proportional to the length of time under study. It would be more likely to buy one phonograph record in one month rather than one week. The probability that a person bought a record in just one second was almost insignificant. Every girl had only a small likelihood of buying a record, but having bought one, she was more likely to buy a second. Having bought the second, she was even more likely to buy a third, and so on. Using a contagion model as $\alpha_k = a + k\,b$, the initial transition rate of buying a record was considered to be a and each record she buys increases this rate by an amount b. Coleman (1964) applied the contagion Poisson process model for the data on one month only and obtained a better performance compared to a regular Poisson process model, meaning that contagion Poisson is a good fit to this data.

All assumptions employed by Diekman (1979) and Coleman (1964) in developing their respective models of contagion, are formalized by the above framework. But, Hamilton and Hamilton (1981) claimed that Assumptions 1-3 also have an immediately useful implication. Let t_k stands for the time of the kth event and define Y_k to be the length of time between kth and ($k+1$)th event:

$$Y_k = t_{k+1} - t_k.$$

Since the occurrence of the kth event is a random variable, Y_k itself would be a random variable, so, assumptions 1-3 are restated based on the probability law for Y_k, as follows.

Theorem 3.2.0.3 [*Hamilton and Hamilton (1981)*] *Under Assumptions 1-3, the length of time Y_k separating the kth and $k+1$th events has the density*

$$f_{Y_k}(t - t_k) = \alpha_k\, w(t)\, \exp\{-\alpha_k \int_{t_k}^{t} w(s)ds\}$$

where $\exp(x)$ indicates the base for natural logarithms e raised to x power. For $k \neq j$, Y_k and Y_j are independent.

Based on the Theorem 3.2.0.3 intervals between events have exponential densities when the process is a time-homogeneous process (i.e., $w(t) = 1$).

$$f_{Y_k}(y) = \alpha_k \, e^{-\alpha_k y}.$$

The reciprocal of the expected number of daily events, $1/\alpha_k$, is the expected waiting time between events. Therefore, when α_k is an increasing function of k then with each new event the waiting time between events becomes shorter.

Another quantity of interest is length of T_n , elapsed time between the 0th and $n+1$th events. It is easily seen that

$$T_n = Y_0 + Y_1 + \ldots + Y_n.$$

If $\alpha_k \neq \alpha_j$ for all $k \neq j$, then the sum of independent variables with individual densities denoted in Theorem 3.2.0.3 would have a continuous Chiang distribution.

Theorem 3.2.0.4 [*Chiang (1980)*]: *The length of time T_n separating the 0th and the $n+1$th event has the density*

$$\begin{aligned} f_{T_n}(t) &= [(-1)^n \alpha_0 \alpha_1 \ldots \alpha_n w(t)] \\ &\quad \times \sum_{k=0}^{n} \left\{ \frac{\exp[-\alpha_k \int_0^t w(s)ds]}{\prod\limits_{\substack{j=0 \\ j \neq k}}^{n} (\alpha_k - \alpha_j)} \right\}. \end{aligned} \tag{3.2}$$

A version of the Chiang's proof is provided in Hamilton and Hamilton (1981). It is almost straightforward to go from Theorem 3.2.0.4 (where the time interval between events is taken as random variables and the occurrence of events are fixed) to Theorem 3.2.0.5, in which the number of events occurring during the time interval is considered as a random variable and the time interval of interest is fixed.

Theorem 3.2.0.5 [*Chiang (1980)*]: *The total number of events X_t observed during $(0, t)$ has a discrete Chiang distribution:*

$$\begin{aligned} P[X_t = n] &= [(-1)^n \alpha_0 \alpha_1 \ldots \alpha_{n-1}] \\ &\quad \times \sum_{k=0}^{n} \left\{ \frac{\exp[-\alpha_k \int_0^t w(s)ds]}{\prod_{\substack{j=0 \\ j \neq k}}^{n} (\alpha_k - \alpha_j)} \right\}. \end{aligned} \tag{3.3}$$

Proof of Theorem 3.2.0.5 can be found in Hamilton and Hamilton (1981).

3.3 A time-inhomogeneous contagion model of risk propagation

As explained in the previous section, the transition rate of our model is a product of two functions, the first function indicates the contagion effect and the second one would be a time-dependent function.

The simplest generalization of the poisson process to take into account contagion is to assume a linear contagion as a first function of transition rate.

We set up the following model of contagion effect:

1. Each element has a given transition rate a of becoming affected.

2. Each affected element has an impact on each of those that are not, adding an increment b to the transition rate of each element that is not affected.

3. The number of affected elements is k.

Given these assumptions, the transition rate for each element when there are already k affected elements is a linear format of k as follows, similar to that used by Coleman (1964):

$$\alpha_k = a + k\,b \tag{3.4}$$

If we assume sickle-type time dependency with $w(t) = c\,t\,exp(-t/\lambda)$, the transition rate $\alpha_k(t)$ takes the following form:

$$\alpha_k(t) = \alpha_k\,w(t) = (a + k\,b)\,(c\,t\,exp(-t/\lambda)). \tag{3.5}$$

Note that $a \geq 0$, $b \geq 0$, $k \geq 0$, $t \geq 0$, $\lambda > 0$, $c > 0$ which results in $\alpha_k(t) > 0$.

Now that we have fully described our transition rate, we can use it to answer questions such as: When are events expected to occur? How many events are expected to happen in a given time interval? And, what are the chances that a certain group, out of the population studied, will be affected? The letter is referred to as the cohort effect in the literature (Diekmann and Mitter, 1982). There are important questions for understanding and managing risk propagation in a supply chain. We will answer them in the next three sections.

3.3.1 Arrival times

Based on Assumption 1 (of Poisson process) the probability of no event within time t would be (Diekmann and Mitter, 1982):

$$\begin{aligned} P_0(t) \quad &= \exp[-\int_0^t \alpha_k(\tau)d\tau] = exp[-\alpha_k \int_0^t w(\tau)d\tau] \\ &= \exp[-(a+k\,b)\int_0^t c\,t\,\exp(-t/\lambda)] \\ &= \exp\{-(a+k\,b)\,\lambda\,c\,[\lambda-(t+\lambda)\,\exp(-t/\lambda)]\} \end{aligned} \tag{3.6}$$

The cumulative distribution function $F(t)$ of the duration in the state of safety (i.e. the arrival time t of an event) is simply the probability of an event's occurrence within time t:

$$\begin{aligned} F(t) = 1 - P_0(t) \quad &= \quad 1 - exp[-\int_0^t \alpha_k(\tau)d\tau] \\ &= 1 - exp[-\alpha_k \int_0^t w(\tau)d\tau] \\ &= 1 - exp\{-(a+k\,b)\,\lambda\,c[\lambda-(t+\lambda)\,exp(-t/\lambda)]\} \end{aligned} \tag{3.7}$$

The density of arrival time t is therefore

$$\begin{aligned} f(t) \quad &= \alpha_k(t)\,exp[-\int_0^t \alpha_k(\tau)d\tau] \\ &= (a+k\,b)\,c\,t\,\exp(-t/\lambda)\exp[-(a+k\,b)\int_0^t c\,t\,\exp(-t/\lambda)] \\ &= (a+k\,b)\,c\,t\,\exp(-t/\lambda)\exp\{-(a+k\,b)\,\lambda\,c\,[\lambda-(t+\lambda)\exp(-t/\lambda)]\} \end{aligned} \tag{3.8}$$

Using 3.7 and 3.8, a simple expression for $\alpha_k(t)$ would be obtained:

$$\alpha_k(t) = \frac{f(t)}{1-F(t)}. \tag{3.9}$$

This is known as the failure rate. We note that unlike the exponential failure distribution, $\alpha_k(t)$ is not constant in time; it increase to a maximum and then decreases, approaching zero in the limit.

3.3.2 Event count

Under our hypothesized transition rate, the density function for the total number of events occurring over $(0, t)$ can be found by substituting functions $\alpha_k = a + k\,b$ and $w(t) = c\,t\,exp(-t/\lambda)$ into Theorem 3.2.0.5, as follows.

$$
\begin{aligned}
P[X_t = n] &= [(-1)^n a(a+b)\dots(a+(n-1)b)] \\
&\quad \times \sum_{k=0}^{n} \frac{\exp[-(a+k\,b)\int_0^t c\,\tau\ \exp(-\tau/\lambda)d\tau]}{[(a+k\,b)-(a+0\,b)][(a+k\,b)-(a+1\,b)]\dots[(a+k\,b)-(a+n\,b)]} \\
&= [(-1)^n a(a+b)\dots(a+(n-1)b)] \\
&\quad \times \frac{1}{b^n}\sum_{k=0}^{n} \frac{\exp[-(a+k\,b)\,\lambda\,c[\lambda-(t+\lambda)\exp(-t/\lambda)]]}{(-1)^{(n-k)}k!(n-k)!} \\
&= [(-1)^n a(a+b)\dots(a+(n-1)\,b)] \times \frac{e^{-a\,\lambda\,c[\lambda-(t+\lambda)\ \exp(-t/\lambda)]}}{b^n} \\
&\quad \times \underbrace{\sum_{k=0}^{n} \frac{e^{-k\,b\,\lambda\,c[\lambda-(t+\lambda)\exp(-t/\lambda)]}}{(-1)^{(n-k)}k!(n-k)!}}_{\mathrm{B}}
\end{aligned}
$$

we define $-\lambda\,c[\lambda-(t+\lambda)\ \exp(-t/\lambda)] = \gamma$ and $e^{(\gamma b)} = z$. We have

$$
\begin{aligned}
B &= \sum_{k=0}^{n} \frac{z^k}{(-1)^{(n-k)}\,k!\,(n-k)!} \\
&= \frac{1}{(-1)^n}\sum_{k=0}^{n} \frac{(-1)^k\,z^k}{k!\,(n-k)!} = \frac{(1-z)^n}{(-1)^n\,n!}
\end{aligned}
$$

Therefore, we will have

$$P[X_t = n] = [a(a+b)(a+2b)\dots(a+(n-1)b]\,\frac{e^{a\gamma}(1-e^{b\gamma})^n}{n!} \tag{3.10}$$

The expected value of the random variable 'number of events' $X(t)$ would be:

$$\begin{aligned} E[X(t)] &= \int_0^t \alpha_k(\tau)d\tau = \int_0^t \alpha_k\, w(\tau)d\tau = \alpha_k \int_0^t c\,\tau\, \exp(-\tau/\lambda)d\tau \\ &= (a+k\,b)\lambda\, c[\lambda - (t+\lambda)\, \exp(-t/\lambda)] \end{aligned} \tag{3.11}$$

3.3.3 Cohort arrival counts

According to the following reasons the probability of occurrence of n events out of N events follows the binomial distribution with mean number of $1 - P_0(t)$.

1. Based on the Assumption 3 the occurrence of events in disjoint intervals of time would be stochastically independent.
2. There are $\binom{N}{n}$ combinations of arrivers with the arrival time probability of $1 - P_0(t)$ and survivors with the survival probability of $P_0(t)$.

$$\begin{aligned} P(n) &= \binom{N}{n}[1 - P_0(t)]^n[P_0(t)]^{(N-n)} \\ &= \binom{N}{n}[1 - \exp\{-(a+k\,b)\,\lambda\, c[\lambda - (t-\lambda)\, \exp(-t/\lambda)]\}]^n \\ &\quad \times[\exp\{-(a+k\,b)\,\lambda\, c[\lambda - (t-\lambda)\, \exp(-t/\lambda)]\}]^{(N-n)} \end{aligned}$$

3.4 Conclusion

We have presented a model for risk propagation in supply chains. In the next chapter, as a support to the concepts proposed in Chapter 2 and 3, the propagation of risk in Toyota and Honda's supply chains after the March 11, 2011 earthquake will be analyzed.

Our model can be extended in different ways. For example, more complex dependencies could be incorporated in $W(t)$. Also, it would be interesting to include explicit dependencies such as geographic correlations.

Chapter 4

A case study

4.1 Discussion of the case study

A powerful earthquake of nine magnitude hit the North-eastern Japan on March 11, 2011. The earthquake triggered gigantic tsunami waves which shook Japan and damaged the nearby coastline. This earthquake is said to be one of the five most powerful earthquakes the world has ever witnessed. This earthquake was so powerful that it moved the Japanese coast by 8 meters and shifted the Earth's axis. It is well known that Japan is situated on the Pacific 'Ring of Fire' which is prone to large scale volcanic eruptions and earthquakes. March 11 earthquake left almost 15,883 deaths, 6,149 injured, and 2,652 people missing in Japan. Four blasts occurred in all four reactors of the 40-years old Fukushima Daiichi nuclear plant, situated 240-km from Tokyo. Due to probable radiation threat, twenty kilometres of area around the plant has been evacuated. Many countries fearing contamination, stopped importing food items from Japan. Since Japan itself is witness to the biggest ever nuclear disasters of Hiroshima and Nagasaki, this radiation threat cannot be undermined.

The Japanese economy was badly affected by this deadly combo of earthquake and tsunami. Economic impacts of the crisis felt across the globe. Asian stock markets dropped down in the aftermath of this disaster. Japan's multinational auto industry like Toyota, Honda and Nissan had to shut down many of their plants. After Japan's destructive earthquake and tsunami hit auto parts supply chains, Toyota faced the biggest crisis ever. Toyota as the world's largest auto-maker since 2008, was threatened to fall to third place globally. Toyota's production dropped by almost two-thirds in March and its Japanese factories were expected to work at 50 per cent of capacity in May and June.

In this chapter we focus on mapping out the propagation of risk in the supply chain of Toyota. Since, some of reviewed models in Chapter 2 take advantage of statistical tools to determine the riskiest network, other than that of Toyota, we needed to elect another company to construct a relevant empirical example. Among all the other auto-maker in Japan, the type of Honda's supply chain (will be discussed in detail later) made it the most suitable one for being the second network in our empirical example.

4.1.1 Toyota Motor Corporation

The Toyota Motor corporation describes its production strategy as a 'lean manufacturing system' or a 'Just-in-Time' (JIT) system. The goal is to deliver vehicles as quickly and efficiently as possible to the customer. The 'Jidoka' and JIT are two foundations of the Toyota production system. 'Jidoka' means that the occurrence of a problem causes the immediate stop of a production equipment to prevent the production of a defective product. The second concept 'Just in Time' means that in a consecutive production flow each process produces what is needed by the next process.

Toyota reduced the number of its suppliers and awarded long-term contracts to the remaining. In addition, in order to be able to control the lower tiers, Toyota encouraged top-tier vendors. Toyota built a deep relationship with its suppliers and

Table 4.4: Number of overseas manufacturing plants of Toyota (as of Dec. 2012)

Region	**Manufacturing companies**
North America	11
Latin America	14
Europe	8
Africa	3
Asia(excluding Japan)	24
Oceania	1
Midddle east	1
Overseas total	52

adapted a partnership model. Toyota's Japanese companies worked very close with its suppliers in Canada, the United States, and Mexico. The main focus of Toyota was on manufacturing while Honda was mostly a producer of vehicles (Akimova, 2011).

As of December 2012, Toyota had 52 overseas manufacturing companies (see Figure 4.1) in 27 countries and regions. It has 16 plants (4 of them are 100% subsidiaries plants) in Japan. Toyota's vehicles are sold in more than 160 countries and regions. Table 4.4 shows the number of manufacturing plants of Toyota in 7 main regions.

Figure 4.1: Worldwide operations of Toyota. Source: Toyota Motor Corporation.

Table 4.5: Number of overseas manufacturing plants of Honda

Region	**Automobile Manufacturing companies**
North America	6
Asia / Oceania	8
Europe / middle east / Africa	2
South America	2
China	4
Overseas total	22

4.1.2 Honda Motor Corporation, Ltd.

Honda has a tiered approach to its supply chain. With a goal of building major subsystems, to a few of key suppliers, Honda reduced the number of suppliers that it directly supervised (Choi and Linton, 2011). Although, the initial selection of second-tier suppliers was done by Honda, the top-tier suppliers have been delegated too much power to manage the second-tier. In contrast to Toyota which built a long-term contract with its suppliers, Honda kept a flexible working relationship with its suppliers. 'There is no other contract' with the suppliers, Honda manager said. Honda achieved the most supplier self reliance approaches. US-owned companies which are identified as strong and component actors in Honda supply chain, tried to reduce the dependence on Honda by adapting the new knowledge quickly (MacDuffie and Helper, 1997).

'Honda has approximately 400 core suppliers which do not include the indirect suppliers (e.g. repair parts, operating tools, et cetera)' (Choi and Hong, 2002). Honda operates 22 automobile manufacturing plants (see the map in Fig. 4.2) in different regions outside Japan and 6 automobile factories in Japan. Table 4.5 gives the number of Honda's plants in 5 main regions outside Japan.

Figure 4.2: Overseas automobile manufacturing plants of Honda, (Source: Honda worldwide).

4.1.3 Data collection

Since we needed a multi-disciplinary database that provided access to archived news and comprehensive data for an in-depth analysis, we chose Factiva. Factiva contained content from more than 31,000 sources from 200 countries in 26 languages. This comprehensive data combined with robust search features made Factiva an appropriate database for our study, specifically since we have no access to (possibly classified) actual Toyota's and Honda's supply chain and production information to analyze the effect of March 11, 2011 earthquake.

In Factiva, for a targeted and relevant search we entered a date range in Data option in its main search screen. Based on the news from the major news sources like New York Times and Wall Street Journal, Toyota and Honda entered a production recovery phase in September. So, we decided to limit our period of study from March 11, 2011 to September 30, 2011. Using the combination of keywords in the Free Text box we obtained our search results. Our search in Factiva was done in several steps.

Articles that contained keywords ‘shut down’, ‘closure’, ‘shutdown’ and ‘bankruptcy’ could indicate the most costly consequences of earthquake on supply chains of Toyota and Honda. However, we noticed that lots of news resulted from entering ‘shutdown’ or ‘closure’ as keywords were repeted in news that contained the keyword ‘shut down’. Also, news found from entering the search keyword ‘bankruptcy’ were not related to Toyota or Honda, but majority of them were about the Chrysler and GM companies which went through the bankruptcy in 2009.

Initially, we looked for full articles that contained all three terms ‘earthquake’ and ‘Toyota’ and ‘shut down’ (using ‘and’ to identify articles that contained all three words) and we retrieved 1200 (287 duplicates) results for Toyota and using the same keywords for Honda, we got 719 (193 duplicates) items. As reading this amount of articles was quite time-consuming, we decided to use a search operator to limit the search results. ‘Earthquake /N100/ Toyota /N100/ shut down’ found articles containing earthquake within 100 words of Toyota and Toyota within 100 words from shut down. Using the search operator of /N100/ limited the search to 504 (130 duplicates) articles for Toyota and 248 (69 duplicates) for Honda. In the second part of news search I entered ‘earthquake /N100/ Toyota or Honda /N100/ closure not shut down’ and Factiva provided us with the news that contained earthquake within 100 words from Toyota or Honda within 100 words from closure. Using ‘not’ allowed us to exclude news having shut down. We retrieved 97 (29 duplicates) articles from this search. We continued our news search by entering ‘earthquake /N100/ Toyota or Honda /N100/ shutdown not shut down not closure’ and we got 213 (47 duplicates) results. Totally, we found 594 news for Toyota (excluding the duplicates and those that were related to Honda) and 379 news for Honda (excluding duplications and those which were related to Toyota). When we went through the news details we noticed that some of them were identical and talked about the same things. After subtracting these similar news we found 307 out of 594 news for Toyota and 82 out of 379 news for Honda. This still contained a large number of news and there was the possibility of having some identical items. So, we browsed and investigated all collected news again. Finally, we obtained 210 news for Toyota and 68 news for Honda.

To analyze the collected data from the previous steps we needed to classify them into distinct groups. Based on what resulted from the news search, the impact of earthquake could be shut down of a plant, halt production, production cut, financial impact and so on. Words like ‘shut down’, ‘halt’ and ‘cut’ might be considered as the same class. But, the news containing these words were not informative enough to make such a decision, so for an easier analysis we decided to consider different classes for each of these keywords. To achieve a consistency in data analysis we used almost the same classifications of earthquake’s impact on both Toyota’ supply chain and Honda’s supply chain. The effect classes used for both Toyota and Honda were: ‘shut down’, ‘cut’, ‘halt’, ‘suspension’, ‘financial’, ‘shortage’, ‘price increase’, ‘lost production’, ‘damage’, ‘cut down’, ‘delay’. Two additional classes were used for Toyota including ‘reputation’ and ‘decline’.

For seven classes including ‘shut down’, ‘cut’, ‘cut down’, ‘halt’, ‘suspension’, ‘shortage’, ‘delay’ we tried to pick the exact word used in the news as a class. But, for the following terms we made a decision based on our understanding:

- Decline: referring to a low inventory level.
- Shortage: referring to a deficiency in a quantity of something like pigment.
- Price increase: referring to a lost discount or a price increase.
- Financial: referring to a lost sale or a lost market share.

- Reputation: referring to a downgrade title for Toyota from the largest to the third car-maker in the world.

- Lost production: referring to a fewer production of vehicles.

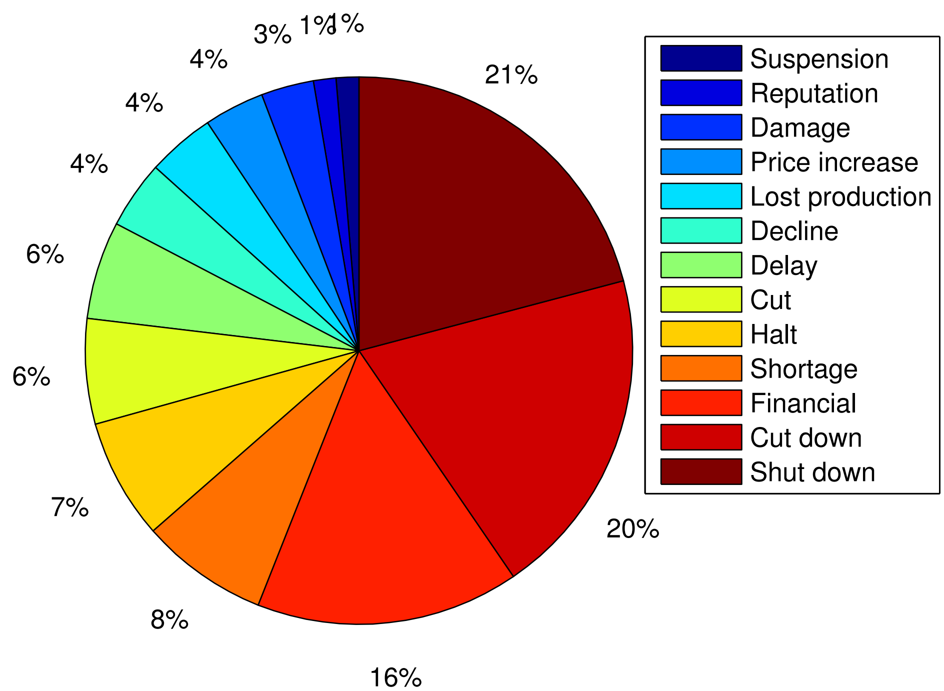

Figure 4.3: Toyota: percentage of each event in all countries affected by Japan's 2011 earthquake.

4.1.4 Data analysis

After data cleaning and classification of effects, I used Mathworks Matlab software to perform the numerical analysis and to plot the pie charts. For simplicity, Microsoft Powerpoint is used to present the maps and propagation of risk through all countries from March 11, 2011 to September 30, 2011. See Figures 4.7 to 4.13.

These presentations allow us to extract some useful information and to identify the most vulnerable part of Toyota and Honda's supply chains.

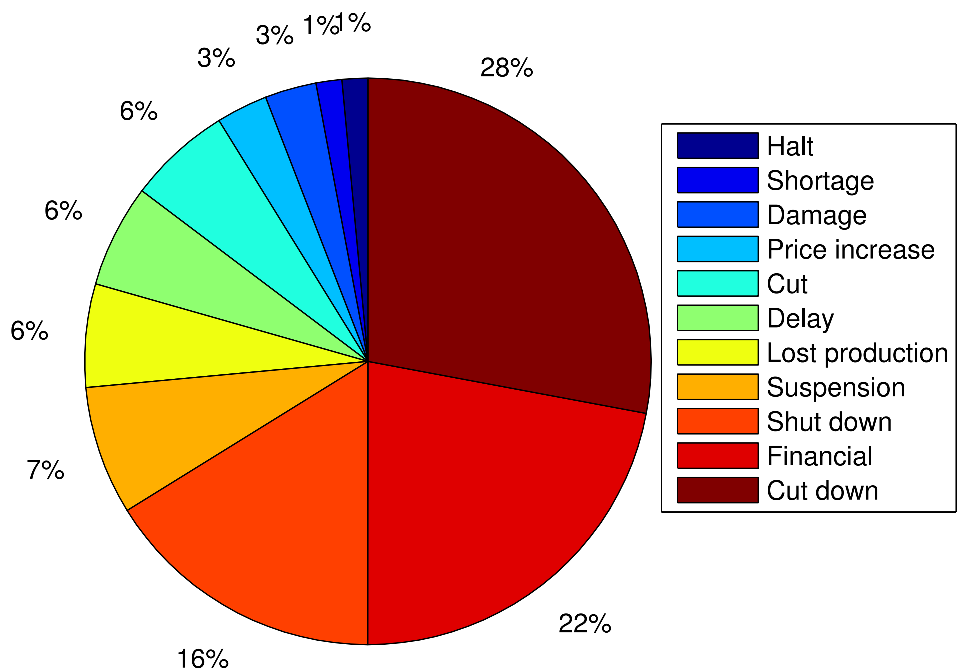

Figure 4.4: Honda: Percentage of each event in all countries affected by Japan's 2011 earthquake.

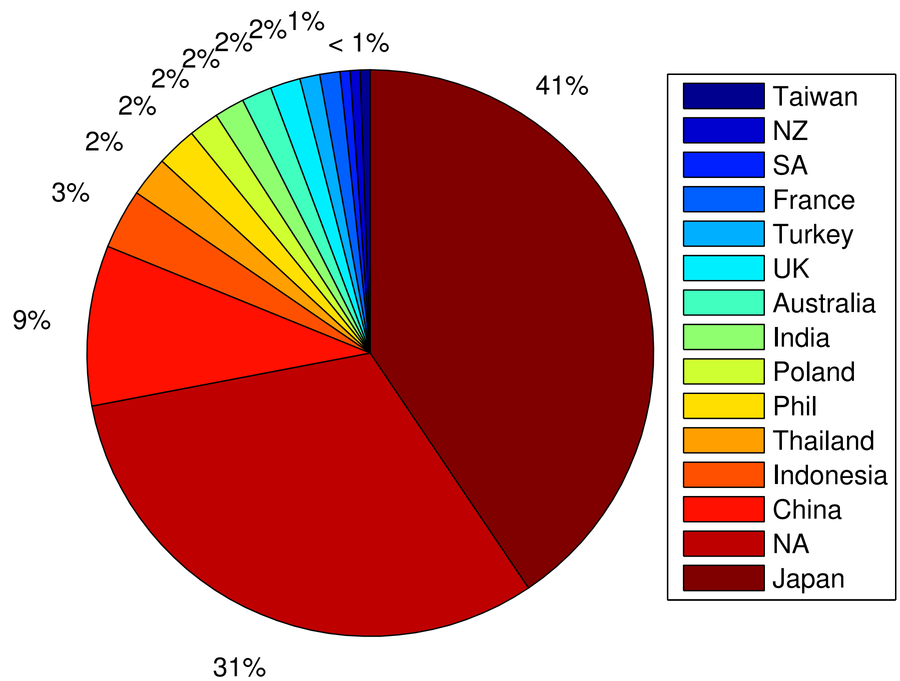

Figure 4.5: Toyota: Number of events in each country. NZ stands for New Zealand, NA stands for North America, SA stands for South Africa, and Phil stands for Philippine.

The piecharts for Figures 4.3 and 4.4 show the percentage of each class of event in all affected countries in which Toyota and Honda have facilities. Overall, it can be seen that 'shut down', 'cut down' and 'financial' accounted for three most important events that affected Toyota and Honda's plants after March, 2011 earthquake in Japan. 'Shut down' was the most frequent event affecting Toyota's plants accounting for 20%, while 'cut down' was the most important event influencing Honda's plants accounting for 28% of cases. 'Cut down' was the second largest event affecting Toyota's plants. Similarly, with regard to Honda's plants, 'financial' accounted for 22% of events. Another major event influencing Toyota's plants was 'financial ' with about 16% and

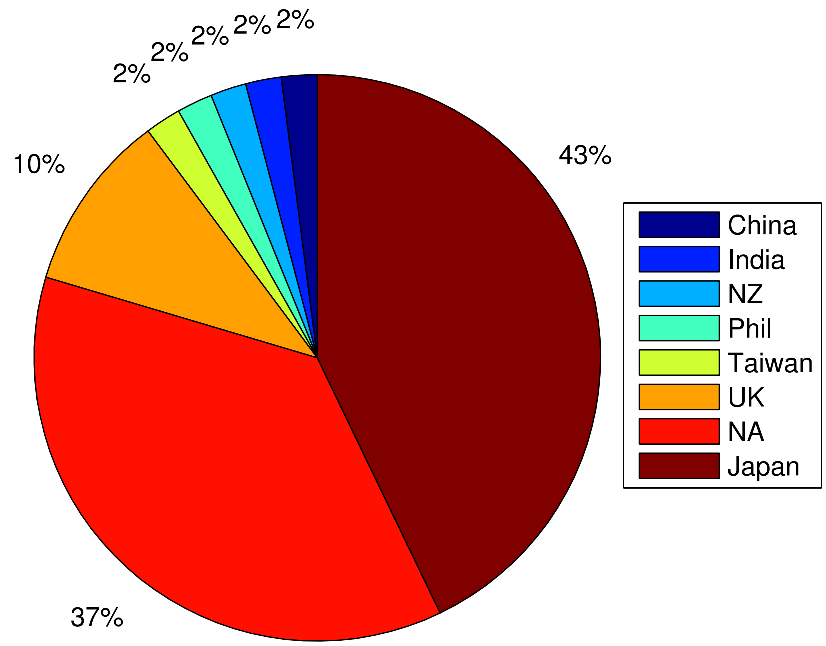

Figure 4.6: Honda: Number of events in each country. NZ stands for New Zealand, NA stands for North America, and Phil stands for Philippine.

'shut down' with the same percentage was the third largest proportion event affecting Honda's plants. The other categories were smaller. 'shortage' brought in 8% of overall events for Toyota's facilities, and this was followed by 'halt' at 7%. 'Cut', 'delay', 'decline', 'lost production', 'price increase', 'damage', 'reputation', 'suspension' were other small classes of major events accounting for 29% combined. 'Suspension' made up approximately 7% of events influencing Honda's plants, with 'lost production', 'delay', 'cut', 'price increase', 'damage', 'shortage', 'halt', making up the remaining 26%.

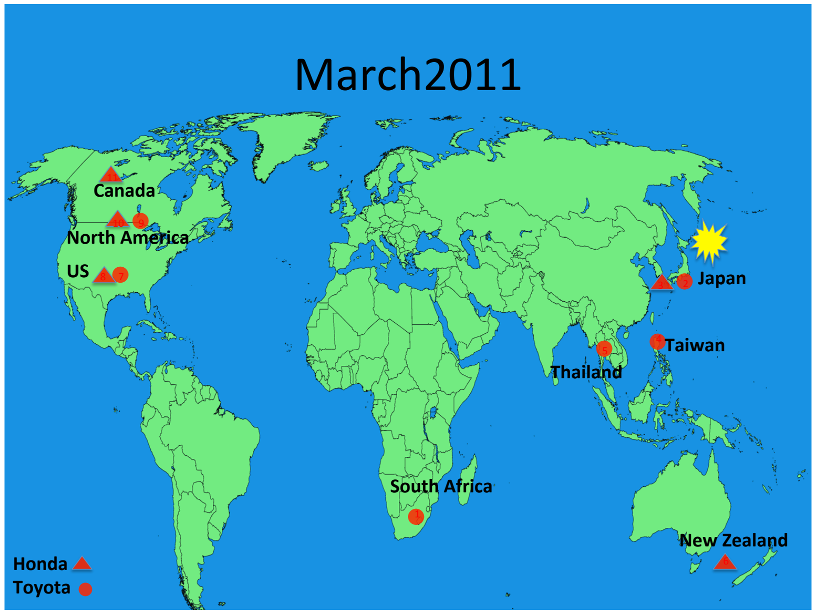

Figure 4.7: Risk propagation in March 2011. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

Figure 4.8: Risk propagation in April 2011. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

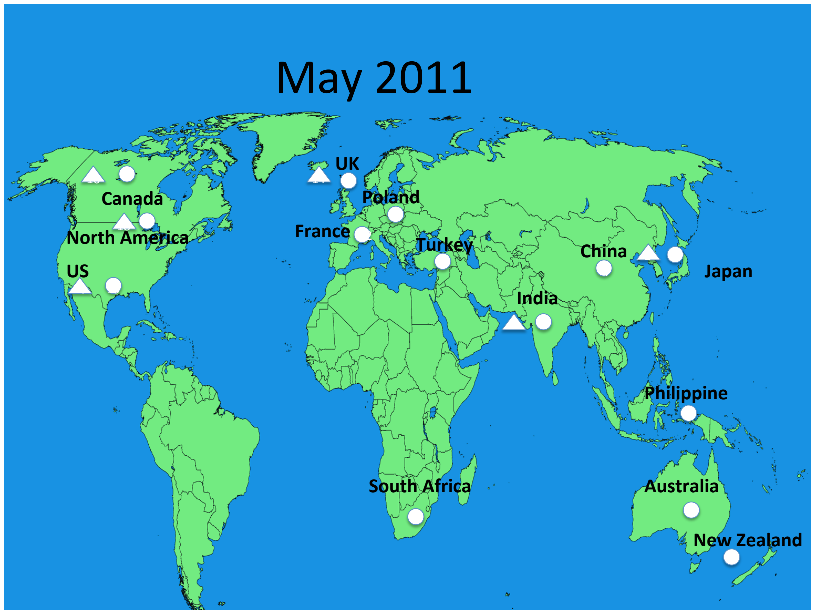

Figure 4.9: Risk propagation in May 2011. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

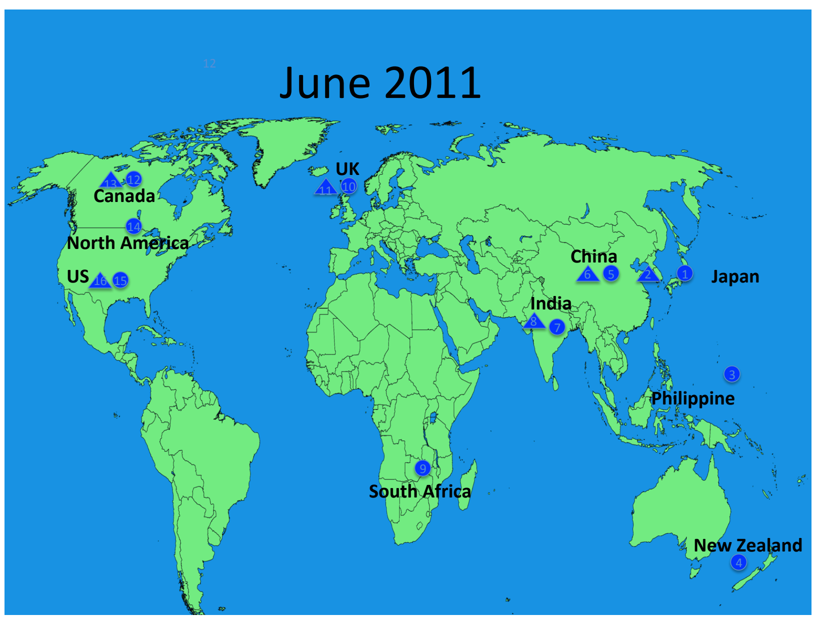

Figure 4.10: Risk propagation in June 2011. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

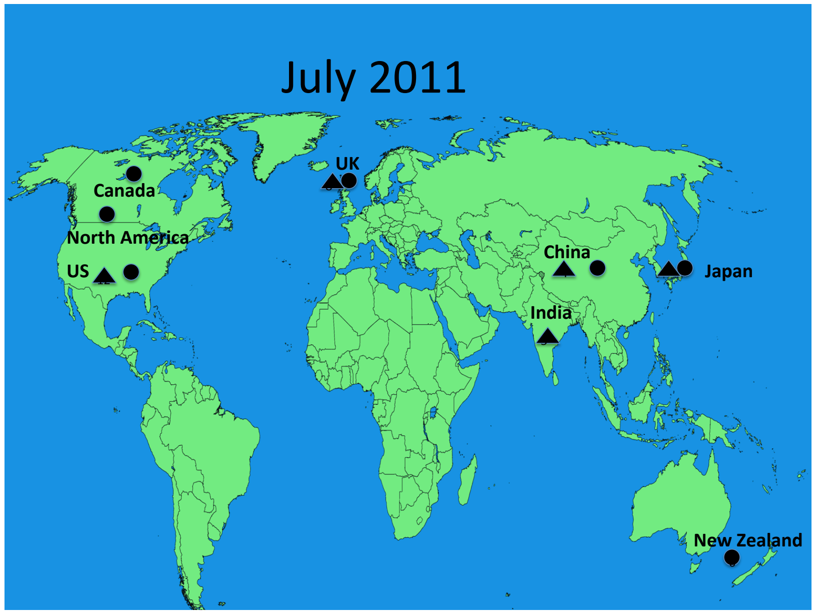

Figure 4.11: Risk propagation in July 2011. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

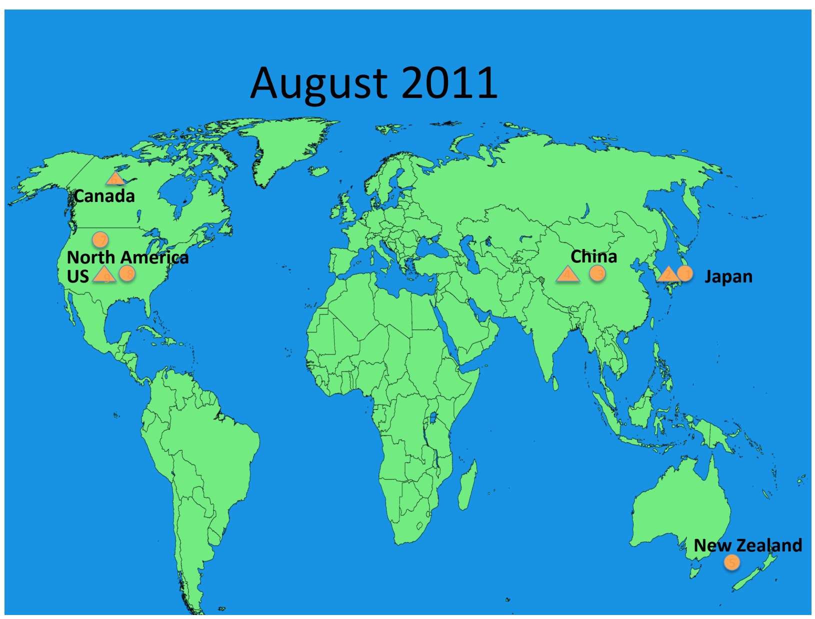

Figure 4.12: Risk propagation in August 2011. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

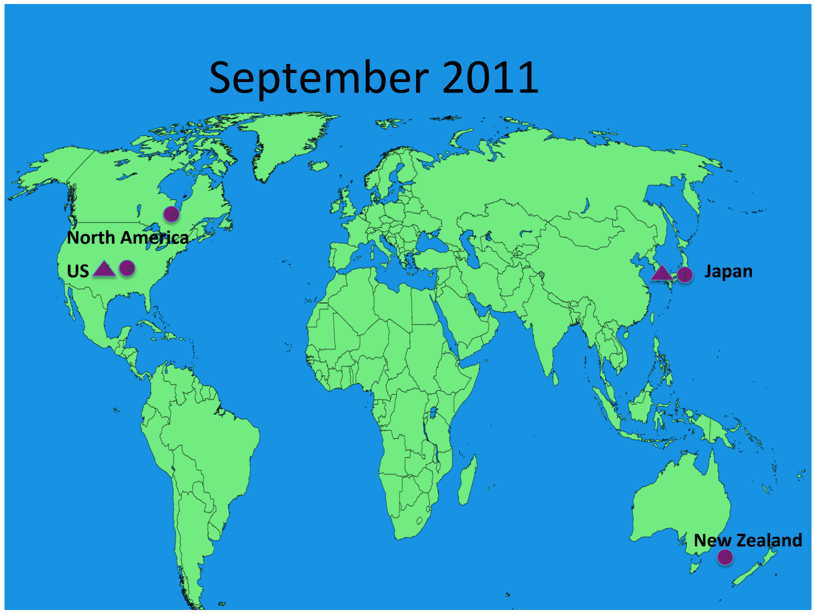

Figure 4.13: Risk propagation in September 2011. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

Piecharts 4.5 and 4.6 show all the countries where Toyota and Honda have production facilities and were affected by the massive earthquake and its aftermath. The key information that stood out from the comparison of the two piecharts was that both companies of Toyota and Honda in 'Japan' and 'North America' were affected significantly. As Toyota and Honda had the greatest number of manufacturing plants in these regions, it seemed reasonable that they tolerated the most significant impact. Toyota had 9 plants in China and after 'Japan' and 'North America', 'China' was the the most vulnerable country to earthquake while 4 automobile factories of Honda were in China and it has encountered just one event. The reason could be the availability of stored parts for a few months to meet the production needs on time. The company of Honda in UK was the third susceptible company being affected. The amount of destruction caused by earthquake in the other countries could be justified in terms of the number of plants in them.

4.2 Risk assessment

Based on the data we collected, the supply chain of Toyota seems to be more risky than Honda, as the total number of risk events occurred in Toyota's plants (and particularly in Japanese plants) was much more than that of Honda. Based on models discussed in Chapter 2, the factors that could be used as justifications of excess number of events affecting Toyota's supply chain are denoted as follows:

1. The number of manufacturing plants of Toyota was more than Honda's plants, so the exposer of Toyota's supply chain to risk was more.

2. Looking at the map of Toyota and Honda manufacturing plants in Japan (see Fig. 4.14) shows that unequal number of plants were located at each echelon of Toyota supply chain and one of the echelons has a large number of plants. A total of 12 plants were located in and around Toyota city and other areas of the Aichi prefecture. Toyota has established three other domestic manufacturing companies outside of Aichi, in Kyush, Hokkaide and Tohoku. So, based on the result of Model 2.4.1 of chapter 2, Toyota' supply chain would be more risky than Honda' supply supply chain.

3. The type of Toyota's supply chain (explained in 4.1.1) implies that the supply chain network was more dependent in Toyota. Toyota took care of all its suppliers and each supplier was regarded as a 'member of a family'. But, Honda picked up some key suppliers and had a good relationship with them. Therefore, Toyota encountered more difficulties after the earthquake.

4. Model 2.4.4 of Chapter 2 could be applied to Toyota's supply chain network where the shut down of two nuclear power plants would be considered as an external mechanism. Temporary shutdown (starting May 9 for 2 years) of three reactors of Hamaoka nuclear power plants run by Chubu Electric Power, which powered the bulk of Toyota motor group factories in central Japan, hurt the production of Toyota. Also,

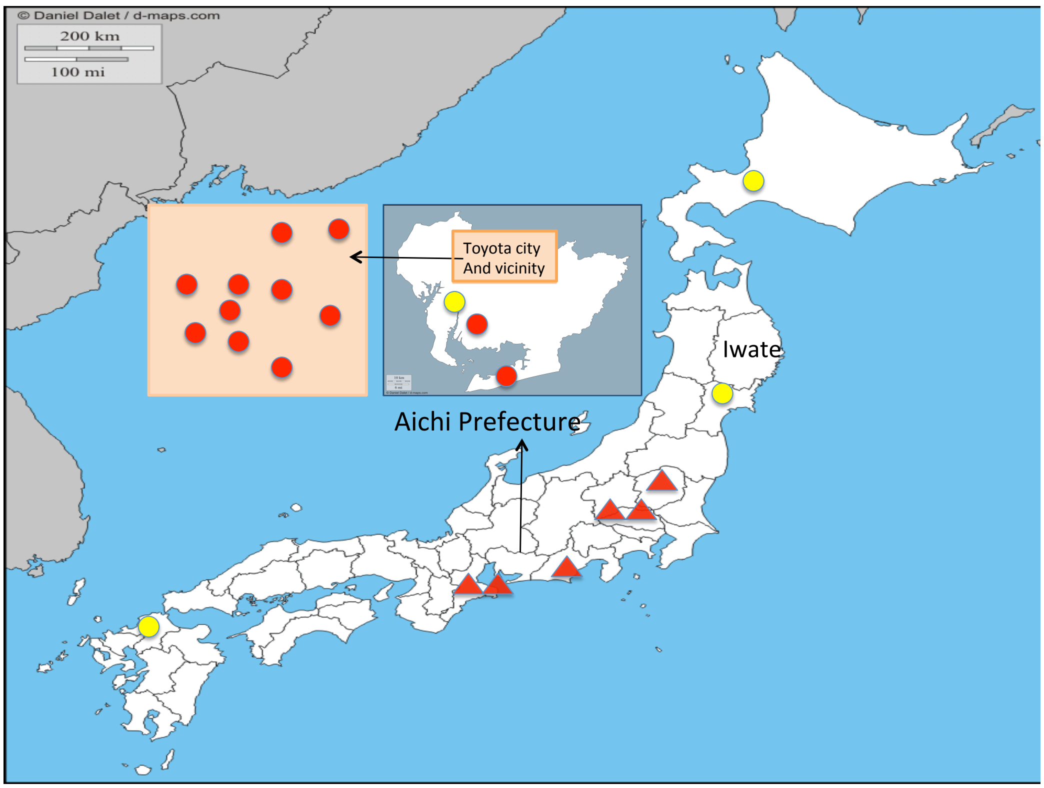

Figure 4.14: Automobile manufacturing plants of Toyota and Honda in Japan. Each circle represents a plant of Toyota and each triangle represents a plant of Honda.

a meltdown at the Fukushima-Daiichi nuclear plant that supplied power to Toyota's headquarter forced Toyota to shut down its Japanese production facilities. However, Honda was affected by the shut down of Hamaoka plant too, but the impact of closure on Honda's supply chain was not as significant as that on Toyota. Hence, because of the effect of the external mechanism, Toyota's supply chain would be more risky than Honda's supply chain.

4.3 Numerical results

We assumed that the number of events affecting Toyota and Honda supply chains after March 11, 2011 earthquake in Japan follows a time-inhomogeneous contagion Poisson process model. The probability that exactly one event occurred was proportional to the length of time. Also, the probability of having more than one event in a very small period of time like one second is almost negligible. Once a country is affected, the probability of getting affected again will be increased. An affected country influences other countries due to the contagion impact. For a fixed t, using a contagion model as $\alpha_k = a + k\,b$, the initial transition rate (i.e., for $k = 0$) of getting affected is considered to be $a\,w(t)$ and when a country becomes affected the transition rate will increase by an amount $b\,w(t)$. Since the contagion effect is considered within a fixed period of time the number of events happening in non-overlapping intervals (i.e., two different months) is assumed to be independent.

4.3.1 Contagion Poisson process

Based on the data we gathered so far, for the whole six-month period following the earthquake, we fitted the contagion part, ($\alpha_k = a + k\,b$), of our Poisson process model to the collected data. Using the estimation method in Coleman (1964), we found the estimates for a and b. We also compared the results with the estimates

Table 4.6: Honda: Number of countries having k events in the period of six months. RMSE stands for root mean square error.

k	n_k	**Contagion Poisson** $a = 1.73,\ b = 0.29$	**Poisson** $a = 2$
1	10	8.92	5.68
2	9	11.21	11.37
3	7	9.81	11.37
4	7	6.54	7.58
5	5	3.68	3.79
6 or more	4	1.84	1.52
RMSE		1.85	2.92

Table 4.7: Toyota: Number of countries having k events in the period of six months. RMSE stands for root mean square error.

k	n_k	**Contagion Poisson** $a = 1.73,\ b = 0.29$	**Poisson** $a = 2.97$
1	17	14.4	5.13
2	16	18.32	15.24
3	14	19.95	22.63
4	13	17.15	22.4
5	11	12.79	16.63
6	11	8.66	9.88
7	11	5.46	4.89
8 ore more	7	3.27	2.08
RMSE		3.84	7.06

using a simple Poisson process model without contagion. The estimation results of contagion model are shown in Table 4.6 for Honda's events, and in Table 4.7 for Toyota's events along with the actual distributions. On the last row of each table the root mean square error (RMSE) for each estimation is written. To be a more fair, in the numerical analysis in this section, an event that lasted for M months in a country, was considered as M events. For both Honda and Toyota data, the RMSE for the contagious model fitting are lower than the RMSE for the simple Poisson model.

4.3.2 A time-inhomogeneous contagion model

A non-linear least-squares (LS) estimation method in Mathworks Matlab software (function: lsqnonlin.m) was used to find the unknown parameters in time-inhomogeneous contagion (TIC) model of Eq. (3.10). The LS estimation algorithm in Matlab function lsqnonlin.m performs an optimization procedure to start from the initial guess and then finds the sub-optimum solution using an iterative large-scale trust-region reflective Newton method, (Boyd and Vandenberghe, 2004; Coleman and Li, 1996, 1994). The unknown parameters in model (3.10) are: a, b, c, λ. We used the initial guess of [1, 1, 1, 1] for these four parameters, and used the lower bound of zero for all four parameters in the optimization procedure.

To implement this procedure in our case study, the time t corresponds to the month number ($t = 1, 2, \ldots, 6$) and we count the number of events, k, in the interval $(0, t)$, where $k = 1, 2, \ldots, 6$ for the Honda case and $k = 1, 2, \ldots, 8$ for the Toyota case. The results for the Honda data are shown in Table 4.8. The unknown parameters were estimated using the collection of data in all the six months, resulting in $a = 1.17$, $b = 1.22$, $c = 6.76$, and $\lambda = 0.37$. On the last row of the table the root mean square error (RMSE) of the TIC estimation for each month is written. Similarly the results for the Toyota case are shown in Table 4.9 and the LS estimate for the unknown parameters are $a = 1.34$, $b = 1.11$ $c = 4.55$, $\lambda = 0.54$. The actual data, Poisson, contagion Poisson and TIC model estimates are compared in Figures 4.15 and 4.16. As can be seen in both figures, the RMSE for TIC model is the lowest, meaning that TIC is the best model to fit the data.

Table 4.8: Honda: Number of countries having k events in each month. (a = 1.17, b = 1.22, c = 6.76, λ = 0.37). TIC denotes the 'time-inhomogeneous contagion model' estimate. RMSE stands for root mean square error.

month, t	1		2		3		4		5		6	
k	n_k	TIC	n_k	TIC	n_k	TIC	n_k	TIC	n_k	TIC	n_k	TIC
1	8	5.91	9	8.63	10	9.57	10	10.35	10	10.88	10	11.15
2	4	4.06	7	6.91	8	7.77	9	8.42	9	8.85	9	9.07
3	2	2.80	5	5.56	5	6.35	6	6.89	7	7.25	7	7.42
4	2	1.94	4	4.5	5	5.21	5	5.66	6	5.95	7	6.1
5	2	1.35	4	3.64	5	4.28	5	4.66	5	4.90	5	5.02
6 or more	2	0.94	3	2.95	3	3.53	4	3.84	4	4.04	4	4.14
RMSE		1.04		0.37		0.7		0.55		0.38		0.62

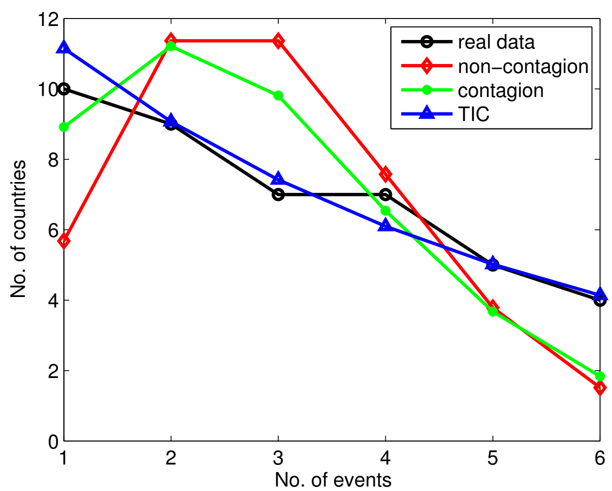

Figure 4.15: Honda case: comparison of actual data, Poisson, contagion Poisson and 'time-inhomogeneous contagion' (TIC) model estimates.

Table 4.9: Toyota: Number of countries having k events in each month. (a = 1.34, b = 1.11 c = 4.55, λ = 0.54). TIC denotes the 'time-inhomogeneous contagion model' estimate.

month, t	1		2		3		4		5		6	
k	n_k	TIC	n_k	TIC	n_k	TIC	n_k	TIC	n_k	TIC	n_k	TIC
1	14	10.57	17	15.82	17	17.24	17	17.28	17	17.18	17	17.16
2	6	7.20	15	14.12	16	16.11	16	16.3	16	16.24	16	16.22
3	5	4.76	12	12.21	14	14.59	14	14.9	14	14.87	14	14.86
4	4	3.09	10	10.39	13	13	13	13.4	13	13.39	13	13.39
5	3	1.99	8	8.75	11	11.47	11	11.93	11	11.95	11	11.95
6	2	1.27	6	7.32	11	10.05	11	10.55	11	10.59	11	10.6
7	2	0.81	6	6.1	10	8.76	11	9.29	11	9.34	11	9.35
8 ore more	2	0.51	5	5.06	5	7.62	7	8.15	7	8.21	7	8.22
RMSE		1.56		0.77		1.11		0.9		0.89		0.88

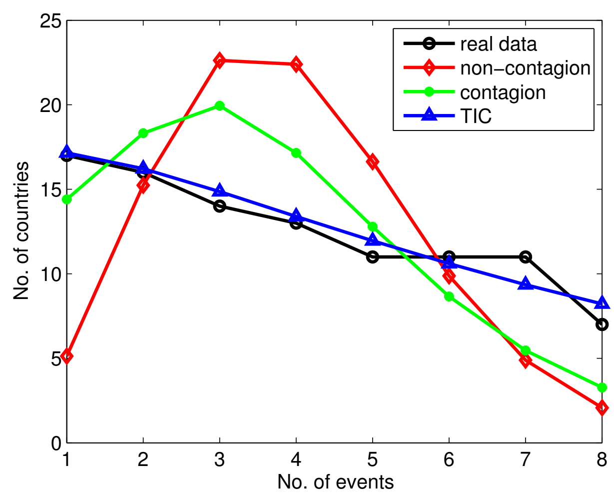

Figure 4.16: Toyota case: comparison of actual data, Poisson, contagion Poisson and 'time-inhomogeneous contagion' (TIC) model estimates.

4.4 Conclusion

We have studied the catastrophic earthquake that hit Japan in March 2011 and how it has impacted two well known automobile supply chains: Toyota and Honda. Based on models from Chapter 2 and 3 we tried to analyse the risk dependencies and how they have impacted both companies differently.

In addition to applying the model to a more rich set of data, it will be worthwhile applying our model on a real-time bases to validate its ability to predict event occurrences and dependencies in a supply chain after a major catastrophic events.

Chapter 5

Conclusion

5.1 Summary

In this thesis we have reviewed risk dependency models in Chapter 2. We then extended the existing models by relaxing one of the assumptions and presented a multi-class dependency model where there could be a dependency between individual risks. In addition, applications of these models in a supply chain context were outlined.

In Chapter 3 we proposed a Poisson process model for risk propagation. One of the most important findings to emerge from this study is to show that the time dependent intensity rate of the Poisson process can take into account the contagion factor.

In Chapter 4 we have looked at a case study of two global auto manufacturing. We used results from Chapter 2 and 3 to analyze how these two companies are impacted by the major earthquake that hit Japan in 2011.

5.2 Future work

For future research, this study identified several new opportunities. Specifically, opportunities exist to study the modelling of risk dependencies where group specific risk factors in a multi-class dependency model are dependent. Also, development of our risk propagation model that follow our requirements in this study is an opportunity to present a new model. Other than dependence on time and state k, the transition rate can be a function of other factors like distance from the source of risk. It would be worthwhile to observe the propagation of risk through the time and geographic location, incorporating correlation factors.

Another potential research area related to this work and particularly to our case studies and numerical example is to investigate the robustness as well as the sensitivity of the contagion risk dependency models. This means measuring how the modelling error changes by perturbations and random changes in input data. In our case study of the impact of Japan's March 2011 earthquake on Honda and Toyota supply chain networks, we estimated the parameters for contagion Poisson models and then measured the modelling performance by RMSE, comparing it with input real data (the event count data). We need to investigate how sensitive these parameter estimates (and as a result the modelling performance) are when the event count data changes slightly.

In our case study, we collected data on the number of events (like shut down, cut down, price increase, damage, lost production, etc.) occurring in each country for a period of six months. A particular event could be more important and critical than the other ones based on the scale of the damage, the effect and the importance of the manufacturing facility on the Toyota and Honda companies worldwide. For example, a lost production at a very large manufacturing facility could be more important than a shut down of a small facility. We did not consider the severity or importance of each event because we did not have access to such detailed information. An extension of our work could be to add the severity information and then construct a revised risk propagation model.

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