INNOVATION DIFFUSION AND URBAN GROWTH

RELATIONS BETWEEN THE DIFFUSION OF INNOVATIONS AND URBAN GROWTH

By

ALUN EDWARD JOSEPH, M.A.

A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfilment of the Requirements

For the Degree

Doctor of Philosophy

McMaster University

October, 1976

©

DOCTOR OF PHILOSOPHY (1976)
(Geography)

McMaster University
Hamilton, Ontario

TITLE: Relations between the Diffusion of Innovations and Urban Growth

AUTHOR: Alun Edward Joseph, B.A. (University of Liverpool)
, M.A. (Queen's University)

SUPERVISOR: Professor M.J. Webber

NUMBER OF PAGES: xii, 282

ABSTRACT

Interest in the processes of urban growth and innovation diffusion has not been restricted to geography, but has proliferated through related disciplines such as economics, sociology and regional science. However, it can be said that, both inside and outside geography, little in the way of detailed attention has been focussed upon the interface between the two processes; that is, the relations between innovation diffusion and urban growth. In this thesis, several aspects of these relations are examined.

The greater part of the analysis carried out in this thesis utilises hypothetical data, although real data for one diffusion process (Planned Regional Shopping Centres in Canada) are used in an attempt to ascertain the general viability of the spatial diffusion model as an integral part of an innovation - based growth model. The use of hypothetical data in the greater part of the analysis not only simplifies the analysis but also allows a degree of experimentation that would not be possible given the constraints often imposed upon the researcher by (real) data availability.

A spatial diffusion model is developed. In contrast to other spatial diffusion models, this model follows the diffusion of what is termed the 'general innovation', and its results are not specific to the diffusion of one innovation or group of innovations. Consequently, the model is able, both conceptually and practically, to take into account the diffusion of many innovations, whose origin may not be the same, within

the same system of cities over the same time period. Summary measures upon the general diffusion process are developed, and the diffusion process is simulated within hypothetical systems of cities with different parameters. Variation in the summary measures on the general diffusion process is shown to be an identifiable function of variation in the parameters of the system of cities in which diffusion occurs. This ability to comprehensively link the form of the diffusion process to that of the space in which diffusion occurs is an important contribution to existing knowledge of diffusion processes.

If the adoption of innovations by cities is equated with growth, or with growth at some rate above system norms, variation in the growth pattern for a system of cities can be intuitively linked to variation in the diffusion process within that system. To establish this relationship more precisely, a simple growth model using the results of the spatial diffusion model as input is developed, and the growth patterns produced in a hypothetical system of cities under different conditions observed. It is seen that several of these growth patterns resemble closely those identified for several real systems of cities over some time periods, which indicates that innovation may indeed provide a reasonable basis for modelling growth within a system of cities. The simple growth model is dynamic because diffusion influences urban growth and urban growth influences diffusion.

This research does not pretend to offer a solution to the urban growth problem. However, it does investigate some of the spatial properties of systems of cities, properties largely ignored in many other approaches to the urban growth problem, and the possible implications of these properties for urban growth.

ACKNOWLEDGEMENTS

I would like to acknowledge the help of my supervisor, Professor M.J. Webber, without whose continued help and encouragement the completion of this thesis would not have been possible. My thanks are extended also to the two other members of my supervisory committee, Professor F.L. Hall and Professor L.J. King, and to all members of the Geography department, past and present, who provided help or advice. Lastly, I wish to thank my patient wife, Gill, for typing various drafts of this thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

AIMS AND INTENTIONS

The aim of this thesis is to develop a model of growth within a system of cities, based upon the spatial diffusion of innovation. It is not maintained that innovation diffusion, or any other single factor, can produce a totally adequate explanation of urban growth. Instead, the intention is to demonstrate that a simple growth model based upon the spatial diffusion process is able to replicate some important regularities observed within the growth process, while at the same time yielding some insight into the nature of that process.

The problem posed is thus: given data on the location and size of cities, knowing how information flows depend on distance, given the number of innovations produced in a time period and the potential effect of any one innovation on the growth of a single city, estimate the pattern of growth rates within the system of cities. The problem is tackled in several stages. A model of the diffusion of innovations is developed in which it is assumed that the location pattern reflects the accessibility of cities to information about innovations, an assumption which is justified by some empirical analysis. It is possible to estimate from this model how the expected time taken for an innovation to reach a city (adoption time) depends on the access of that city to information about that innovation: because of the complexity of the model, this estimate is obtained by regression on many simulations of the diffusion process. Then the relationship between these adoption times and the system parameters (the location and size of cities, and the effect of distance on information flows) is estimated. At this point, the diffusion model is solved, and can be applied to the analysis of growth rates. Given the available data, a simple growth model is used to link adoption of innovations to growth and to examine the manner in which patterns of growth depend on the impact of individual innovations and the size of any one city. The aim of the thesis, of solving an innovation - growth model, has been accomplished: given system parameters, the rates of growth of cities in that system can be predicted by the model.

In terms of organisation, the research material is presented in three sections. The first section (Chapters 2 and 3) places the thesis in its general context and examines the major methodologies available to make the chosen model operational; the second section (Chapters 4 and 5) presents the diffusion model and investigates its properties; and the third section (Chapters 6 & 7) develops and evaluates a simple growth model and proposes some avenues for further research.

Chapter 2 embraces two important themes. Firstly, because it is desirable to keep the properties and implications of existing theory in mind (Richardson, 1973b), important elements of two approaches to the explanation of urban growth, the economic and the statistical, are examined. Throughout this partial review of existing theory, particular attention is paid to the manner in which space is treated, which has usually been inadequate. This abstract discussion is counterbalanced by a brief review of some of the major empirical 'system' regularities in the urban growth process, using published work on Central Canada as a source of data: for example, it is observed that the variance of city growth rates is negatively correlated with city size (Barber, 1972). Secondly, since the nature of the relationship between invention and innovation and urban growth has been subject to obfuscation and ambiguity (for example, consider the work of growth pole theorists such as Lasuén, 1969:1973), a detailed examination of this relationship is undertaken. It is observed that such a relationship does exist and that it provides a viable base for the development of a model of growth within a system of cities. The problem tackled in Chapter 3 is one of selecting the appropriate methodology by which to model the diffusion process. Two such methodologies, the economic and the spatial, are considered. Because evidence presented in the literature does not demonstrate the superiority of either methodology (Sheppard, 1974), the analysis of an available data set, that for the diffusion of Planned Regional Shopping Centres in Canada, is undertaken. The model developed to investigate the diffusion of Planned Regional Shopping Centres separates the effects of market size, information availability and demand for the innovation, since a failure to do so has handicapped previous efforts to model this and similar diffusion processes. The results of this analysis point toward the suitability of the spatial approach to diffusion modelling, and are considered important because this particular diffusion process has been interpreted as an aspatial, economically - determined one (Cohen, 1972). A brief review of the major elements within the spatial approach to diffusion modelling and of two works which antecede this one, Pedersen (1970) and Robson (1973), completes the first section of the thesis.

Given the viability of the spatial approach to diffusion modelling and keeping the shortcomings of previous modelling efforts in mind, a spatial diffusion model is proposed, and its properties investigated through the simulation of the diffusion process within a hypothetical system of cities (Chapter 4). However, although they yield considerable insight into the nature of the diffusion process, the results produced by such a simulation are specific to the system of cities in which diffusion occurs. Since the way in which the form of the diffusion process responds to changes in the parameters of the system of cities is of considerable relevance, both in the diffusion context and the growth context, it is examined in Chapter 5. This examination is carried out by repeatedly simulating the diffusion process within systems of cities incorporating different combinations of critical system parameters. The structure of each of these diffusion processes is summarised by a regression of two information potential measures on each city's mean adoption time. Variation in the coefficients of this regression is then linked to variation in the system parameters through regression. The regularity of these responses permits the estimation of city adoption times within any comparable system of cities in which the critical parameters and city information potential scores are known. This analysis of the properties of the diffusion model completes the second section of the thesis.

In the third and final section of the thesis, the reliability of the estimates of city adoption times and, consequently, of the analysis carried out in Chapter 5, is tested and a simple growth model is developed.

The properties of the simple growth model are investigated through simulation within a hypothetical system of cities. Given knowledge of the kinds of responses initiated in the form of the diffusion process by changes in the parameters of the system of cities (Chapter 5), in these simulations attention is focussed upon the nature of changes in the distribution of growth rates initiated by changes in the exogenously - determined parameters of the growth model. This simulation analysis is quite extensive, but at the same time limited; for example, only one form of the growth model is experimented with. Indeed, it becomes increasingly apparent that, although the objective set at the beginning of the thesis - of demonstrating that a simple growth model based upon the spatial diffusion process is able to produce reasonable distributions of city growth rates - is met, additional insight into the nature of the growth process might be obtained if further analysis using this model were to be undertaken. Some possibilities for further research are outlined and commented upon in Chapter 7.

In summary, this thesis produces three main results. Firstly, the analysis of the Canadian Planned Regional Shopping Centre diffusion demonstrates that it is possible to define the demand component within some diffusion processes, and that such definition permits a more reasonable testing of the economic and interaction hypotheses. Secondly, the investigation of the properties of the spatial diffusion model through simulation demonstrates the nature and properties of such a model in a manner more comprehensive than any that the author is familiar with. Thirdly, the development of a simple growth model based upon the diffusion of innovations, and the investigation of its properties through simulation, demonstrates that a growth model based upon the innovation diffusion process is able to produce sets of growth rates similar to those found within systems of cities.

The approach to analysis adopted in this thesis involves the handling of large amounts of hypothetical data. Where necessary to the understanding of results, summaries are included. Other data and/or the computer programmes used to generate them can be obtained from the author on request.

Equations are numbered consecutively in each chapter, but numbering is re-commenced at the start of each chapter. If, at any point, an equation from another chapter is referred to, it is preceded by the number of the origin chapter. For example, in Chapter 5, reference is made to equation (4.10).

The list that follows demonstrates the form of notation used in this thesis. Only frequently used terms are included. Exceptions to this form are indicated in the text.

S_i = Size of city i

P_i = Population of city i

d_{ij} = Distance between city i and city j

I_{ij} = Interaction between city i and city j

p_{ij} = Probability of a message being passed from city i to city j

A_i and B_i = Information potential measures for city i

$p_i^{(t)}$ = Probability of adoption by city i in time period t

λ	= The probability that contact between a non - knower and a knower leads to adoption
$\bar{t}_i$	= Mean adoption time for city i
s_i^2	= Variance of adoption time for city i
α, β, γ and δ	= Population coefficients (parameters)
a and b	= Sample coefficients
$\hat{a}$ and $\hat{b}$	= Estimates of sample coefficients
b	= Parameter on distance within the gravity model
q	= Slope of the rank size distribution
a_i	= Cumulative probability of adoption by city i at a given point in time
G_i	= Growth rate of city i
x	= The time period up to which adoption of an innovation contributes to urban growth
Q	= The form of the growth increment

CHAPTER 2

URBAN GROWTH

In order to place this work in context, some existing approaches to the urban growth problem are briefly reviewed. This review is divided into two major sections; the first deals with some of the major theoretical developments and the second with empirical analyses of a single urban system, that of Central Canada. The first of these sections, which is the more extensive, examines two important theoretical approaches to the explanation of urban growth, the economic and the statistical, and the nature of the relationship between innovation and urban growth and its possible use as the basis of a growth model.

2.1 Theoretical Explanations

Although this thesis is concerned specifically with the nature of the growth process within systems of cities, many urban growth models are developed within simpler spatial frameworks. Thus, some theories and models refer to growth within a single city or region , while others refer to growth within complex systems of cities or regions. In this first part of the chapter, the term 'regional growth' is used in both the urban and the regional contexts.

2.1.1 Economic Models

> "The existence of cities *per se* may be justified on social, political and cultural grounds, but these criteria would not seem to demand cities of *different*

> size. The most plausible systematic explanations of why there is a distribution of city sizes are based upon economic theory."(Richardson, 1973a, p. 157)

Richardson's viewpoint may be sound; nevertheless the majority of attempts to explain regional growth through economic arguments are less plausible. This point is illustrated by reference to four regional growth models which are prominent in the literature. These particular models are chosen not only for their prominence but also because they serve to illustrate the diversity of opinion within this single approach to the regional growth problem. Some reference is also made to econometric models.

The export base model has probably the longest history of any regional growth model (North, 1955). One concept is central to the model. Given that economic activity within a region can be divided into its export (basic) and residentiary (non-basic) components, it is proposed that growth in that region depends entirely upon that of the export sector. The relative performance of regions is thus linked to the relative performance of their export industries, a viewpoint which reflects the model's origins in trade theory (Richardson, 1973b). Such a monocausal explanation is an appealing one, particularly in light of the fact that empirical evidence for Stockholm (Artle, 1965) and for Philadelphia (Karaska, 1969) indicates that over 50% of the final demand for goods produced in large cities is external in origin. However, the disadvantages of the model are numerous and far outnumber its advantages. For example, the early work of Tiebout (1956) and of Blumenfeld (1955) shows that the model is sensitive to the size of the region and that it neglects the importance of a strong infrastructure (residentiary sector) in promoting growth. The model also supposes a free flow, and complete elasticity of supply, of the factors of production. Both Thompson (1968) and Richardson (1973b) point out the general problems associated with such assumptions while Borts and Stein (1964) and Lane (1966) focus upon the effects of inelasticity in the supply of labour upon the performance of the model.

Several attempts to overcome these problems have been made. For example, Paelinck (1970) estimates the industry by industry effects of increased demand for exports on growth. However, such efforts have not substantially reduced the disadvantages associated with the approach. This is evident in the recent work of Kottis (1972) and Mattilla (1973), both of whom work with Michigan data, where the problems of scale sensitivity (Tiebout, 1956) and definition of the export sector (Isard, 1960) again appear.

If the export base model represents an over - simplified approach to the regional growth problem, then the neoclassical model represents an inappropriate one. The works of Borts (1960) and Borts and Stein (1964) are the earliest attempts to incorporate aggregate neoclassical growth theory within a regional framework. But as Richardson (1973b) comments,

> "It began a borrowing from the literature on growth theory in general rather than working out an independent approach to the specific theoretical problems raised by the regional growth process." (Richardson, 1973b, p.2)

It is assumed that the relative growth of regions depends upon their labour and capital productivity rates; capital flows from high to low wage regions and labour in the opposite direction. The model therefore implies that the prosperity of regions converges over time. However, the inappropriateness of the neoclassical model, at the national as well as the regional level, has frequently been pointed out (Richardson, 1973b). At the national scale, the assumptions of perfect competition, full employment, a Cobb - Douglas production function, zero internal transport cost and uniform (in terms of age) capital stock have come under attack (Kaldor, 1966). At the regional scale, the opportunities for criticism increase. For example, it has been shown that the policies and problems existing at the national level, taxation and trade imbalance for instance, are not of the same importance at the regional level (Richardson,1973b). It is also obvious that the existence of space itself makes the assumption of perfect competition impractical, even in theory, while the assumption of full employment is just as weak given that unemployment continues to be one of the most pressing regional problems. However, despite these basic criticisms, the model still has many adherents (probably because it is still easier to borrow theory than to develop it) and it continues to yield neat and precise results (Kaldor, 1966).

The export base and neoclassical models represent orthodox macroeconomic approaches to the regional growth problem. The next two models discussed, cumulative causation and growth centre models, are more empirically - based. The neoclassical model predicts that the nature of capital and labour flows will, in the long term, lead to regional convergence although, as Richardson (1969) points out, such convergence will not occur if, for instance, the benefits of out - migration to poorer regions are outweighed by high rates of natural increase. On the other hand, the cumulative causation model, taking greater heed of empirical evidence, predicts regional divergence over time for some time intervals.

The principle of circular and cumulative causation was suggested by Myrdal (1957), who argued that "the play of forces in the market normally tends to increase rather than decrease the inequalities between regions" (Myrdal, 1957, p. 26). Regardless of the nature of the initial advantage, perhaps some resource or transportation factor, or simply chance, it is proposed that the build-up of agglomeration economies makes regional imbalance a self-sustaining phenomenom (Pred, 1966). Within such an imbalanced regional system, two types of effects are envisaged: the first or spread effects are favourable to poorer regions and include a growing demand for the primary products of poor regions, while the second or backwash effects are unfavourable to poorer regions and include the flow of labour, capital and goods and services from poor to rich regions. Hirschmann (1958) proposed that regional divergence might be halted by the onset of diseconomies of scale in the rich regions but this has yet to be empirically verified. A classic example of such a process in operation is contained in Wensley and Florence's (1940) analysis of the emergence of an industrial complex in the English Midlands. Both Rosenstein - Rodan (1943) and Thompson (1965) point out that continued investment of overhead capital in the rich (developed) regions constitutes a strong inertia effect which promotes continued regional imbalance. Webber (1972) presents a summary of the three major factors which determine the degree of polarisation within any regional system,

> "Polarisation is enhanced by extensive economies of agglomeration, a low marginal propensity to import the products of other regions and a low demand for unskilled immigrants, together with a relatively wide wage differential." (Webber, 1972, p. 83)

The major problem associated with Myrdal's framework is the very fact that it is a 'framework' rather than a model. Despite the efforts of Kaldor (1970) to formalise them, the hypotheses proposed by Myrdal remain untested in all but the most general of ways (Webber, 1972). Thus, whereas it was possible to level precise criticism at the export base model and the neoclassical model, criticism of the cumulative causation model is restricted to generalities. Indeed, generality (for instance, the precise nature of the flows within the system is not defined) constitutes the model's basic weakness. Developments upon the model, such as those initiated by Friedmann(1966), have augmented rather than reduced this over - generality.

To a considerable extent, the growth centre model suffers from this same shortcoming.

> "Despite the flimsy theoretical background and the lack of empirical verification, there is nonetheless a great deal of intuitive appeal in the notion of a growth centre in which economic and social development is initiated and transmitted to an area around it." (Darwent, 1969, p. 13)

This opinion is shared by others. In the Ontario context, the Hon. Charles McNaughton states that

> "By stimulating the outlying centres (in the Toronto Region) so that they can attract many more residents

and commuters, we can reduce the increasing congestion within, and now extending mainly westward from, Metropolitan Toronto. At the same time, we shall be laying the basic framework, for a carefully planned decentralised urban region of the future." (Ontario Department of Treasury and Economics: Regional Development Branch, 1970, p.18)

However, as in the majority of such proposals, the exact nature of these growth centres is not elaborated upon (King, 1974).

Growth centre theory originated in the work of a group of French practical economists led by Perroux, who first coined the phrase 'pole de croissance' in connection with economic development (Perroux, 1955). In a summary of his early work, Perroux (1970) observes that

"One of the aspects of structural change is the appearance and disappearance of industries, the variable proportion of the flow of total industrial production within these diverse industries in successive periods, and the varying rates of growth for different industries measured over one time period or over successive periods."

and consequently that

"...Growth does not appear everywhere at the same time; it becomes manifest at points or poles of growth, with variable intensity; it spreads through different channels, with variable terminal effects on the whole economy." (Perroux, 1970, p.9)

However, as originally conceived, these poles of growth were located in economic rather than physical space and thus identifiable through the analysis of input - output tables. It was those who developed Perroux's work who gave the concepts 'key industry', 'propulsive firm' and 'backward and forward linkages' a spatial interpretation (Hansen, 1971). For example, Nichols (1969) states that

"...A growth pole is an urban centre of economic activity which achieves self-sustaining growth to the point

> that growth is diffused outward into the pole regions." (Nichols, 1969, p.19),

although this assumes that polarised growth will be beneficial to those areas linked to the high growth region, whereas it has been suggested that the opposite may be true (Myrdal, 1957: Schultz, 1953). The degree of underdefinition within the growth centre framework is well illustrated by the failure of its proponents to agree even upon a minimum size for growth centres (Hansen, 1971: Hermansen, 1972).

These four models represent different approaches to the regional growth problem, each with its own strengths and weaknesses. However, they all share two major characteristics; a failure to adequately explain the origin of regional imbalance and an inadequate treatment of space. Consider the origin question first. Within the cumulative causation framework, Myrdal (1957) makes reference to factor endowment and advantageous location as possible explanations for the origin of regional imbalance, a view shared by Pred (1966). However, if the regions being considered are city regions or cities then, as Richardson (1973a) suggests, reference must be made to a myriad of social, political, cultural and economic factors to properly explain the origin of regional imbalance. Nevertheless, the failure to adequately explain the origin of regional imbalance is not a serious one if interest is focussed upon the behaviour of well-developed systems. It is therefore desirable to separately develop models to examine growth within well-developed systems and not to attempt to incorporate within them an explanation of the origins of such systems, which should be treated as a separate problem.

The second characteristic, an inadequate treatment of space, is of greater importance. For example, despite the fact that inter-regional factor flows form an integral part of the neoclassical world, the spatial framework within which flows are considered to occur is a grossly over - simplified one (Richardson, 1969). The assumption of zero transport cost also seems unreasonable, particularly since inter-regional variations in such costs might serve to partially explain regional growth. Similar criticisms can be levelled at the export base model, which reduces the complexity of a system of regions to the trivial dichotomy of the 'region' and the 'rest of the world'. Thus, although the importance of inter-regional linkages is stressed in the export base model, the precise nature of such linkages is difficult to define (Thompson, 1968). Like the neoclassical model, the export base model makes some very simple assumptions in connection with transport costs; for example, it is assumed that all export goods are transportable over the whole system (Webber, 1972). The cumulative causation model has only been formulated for the two or three region case and, like the export base model, concentrates usually upon the relationship between a single region and the rest of the system. This is even the case in growth centre theory, where the nature of spatial linkages is of considerable importance.

> "...to pass from the single pole to the system of poles it seems necessary to relate the framework of growth pole theory to those of central place theory and industrial structure analysis." (Lasuén, 1973, p. 164)

Lasuén recognises that a trivial treatment of space means that only very simple location theory is possible; but some of the problems associated with industrial structure analysis (for instance, its complexity) are evident in Paelinck's (1970) work and although central place theory provides some insight into the origin and nature of city systems, the lack of empirical verification makes it of doubtful value in the context proposed by Lasuén. It is clear though that, given the inadequate treatment of space within these regional growth models, it is desirable for any new theory to be formulated within a more fully-developed spatial framework.

Econometric models have been used to develop as well as to test theory. Excessive generality in the available theory has encouraged many researchers to develop their models with reference to data availability rather than to any preconceived causal structure. Econometric models have commonly been used to predict regional growth over the short term, usually through regression, and in this way may constitute as much of an explanation of regional growth as the theories discussed earlier. However, an important problem associated with the use of such models is that, in light of their results, many researchers are forced to postulate relationships that cannot be intuitively or theoretically justified.

> "The inadequate state of regional growth theory, and the contradictions and confusions that have not been sorted out because of limited testing, all means that each analyst has had a free rein to specify his own model without having to draw upon, or explain how he diverges from, any conventional wisdom." (Richardson, 1973b, p.36)

Where theory has been incorporated, it has frequently been of an outdated nature; for example, Bell's (1967) Massachusetts model utilises the export base concept.

Perhaps the most significant contribution of econometric models to the understanding of regional growth is represented by their continued attention to the nature of flows within systems of regions, although data limitations have caused attention to be directed mainly to inter-regional employment linkages; for example, see the work of Casetti, King and Jeffrey (1971) and Jeffrey (1974). More general analyses of inter-regional flows, such as migration studies, have frequently utilised the gravity framework (for example,see Preston, 1974), the utility of which will be explored in Chapter 4. The most sophisticated regional econometric model yet developed is that for Nova Scotia (Czamanski, 1968: 1972). Although the Nova Scotia model provides considerable insight into the nature of the regional economy and its behaviour over time, its performance is not completely reliable (Richardson, 1973b). For example, by manipulating the definition of industrial sectors, Czamanski (1972) is able to define three separate and different input - output tables for Nova Scotia. This may be symptomatic of the technical problems faced by workers in this area.

It has frequently been observed that firms and industries are linked within cities (Thompsón, 1965: Paelinck, 1970) and that such links extend, over space, to other cities and are of a complex nature (Pred, 1973:1975). The existence of inter-city linkages causes the symptoms of growth (for example, unemployment), and growth itself, to be spatially autocorrelated (Jeffrey, 1974: Bannister, 1975). It is therefore important to include spatial interdependence within any explanation of growth within a system of cities. Thus, although the four regional growth models reviewed provide considerable insight into the nature of growth, their inadequate treatment of space means that they do not provide a viable base for explanation within the system context. The econometric approach provides a partial solution to this spatial problem. Indeed, many econometric models are based upon inter-city linkages; for example, see the work of Jeffrey (1974) on unemployment impulses and Preston (1974) on migration. However, the paucity of theory and the fact that the phenomena modelled are the symptoms and not the causes of growth, limit the value of econometric growth models. It is proposed that an alternative path be explored, that of modelling the incidence and spread of a single growth - inducing phenomenon, innovation, within a system of cities. Although such a model could not provide a complete explanation of growth, it would provide considerable insight into the spatial nature of the growth process. The nature of the relationship between innovation and growth will be considered at a later point and, at this juncture, statistical explanations of growth, which consider growth in the system context, will be briefly reviewed.

2.1.2 Statistical Models

> "Since the size of a city is the net outcome of a multiplicity of forces the individual contributions to which are difficult to identify, there is a temptation to resort to stochastic models which treat urban growth determinants as proportional to city size."
> (Richardson, 1973a, p. 145)

Since the seminal works of Jefferson (1939) and Zipf (1949), students of urbanisation and economic development have shown continuing interest in the nature of city size distributions. Although static in nature, models developed to 'explain' city size distributions bear some relevance to the regional growth problem.

In any region, cities may be ranked in terms of population from the highest to the lowest, such that the largest city ranks one, the second two, and so on. When these ranks are plotted against city populations a regular relationship is observed. The distribution is seen to be skewed to the right, there being more small than large cities, and often to be adequately described by the relationship,

$$P_i r_i^{q} = K \qquad ,K = P_1 \tag{1}$$

where q and K are constants, r_i is the rank of the i^{th} city and P_i its population. Observed distributions are concave upwards and approximately linear when plotted on logarithmic axes (Berry and Garrison, 1958). If, instead of rank, the cumulative percentage of cities above some threshold size is considered, a number of distributions, including the Pareto and the Yule, may be fitted (Quandt, 1965). However, owing to the fact that the parameter q can be easily and meaningfully interpreted, the rank - size distribution has maintained some pre-eminence (Parr, 1970). If $q = 1$, the distribution of cities has been termed the rank - size rule (Zipf, 1949); if $q > 1$, there is a degree of metropolitan dominance; and if $q < 1$, large cities are relatively under - represented. The limiting cases of the distribution are $q = \infty$, when there is only one city and $q = 0$, when all cities are the same size. Berry (1961) presents many examples of city size distributions.

Several models have been developed to explain why such a distribution should describe city systems (Richardson, 1973c) but the most relevant to this work is that developed by Simon (1955). Simon postulates the operation of a law of proportionate effect, a law which states that the growth of any city is proportionate to its size. However, not every city need grow proportionately, provided that the probability of a growth increment is proportional to city size. It is observed that the distribution of city sizes produced by such a process conforms to the Yule distribution (Simon, 1955), which in turn conforms closely to the Pareto and Lognormal distributions. Similar arguments in connection with the growth of firms are presented by Simon and Bonini (1958). The proportionate effect relationship can be expressed in regression form,

$$S_i^t = bS_i^{t-1} + \varepsilon_i$$

where S_i^t is the size of city i at time t, b is a regression parameter, and ε_i is the error term (Robson, 1973). If b = 1, growth is a random increment independent of city size. Whereas, if $b \neq 1$, the growth increment is a linear function of city size, the coefficient of which is b - 1. If the law of proportionate effect holds, each of the n cities would have the same rate of growth up to a normally distributed error term.

Attempts to place an economic interpretation upon city size distributions have not been very successful. For example, Zipf (1949) considered lognormality to be symptomatic of development and Berry (1961) proposed that city size distributions become more lognormal as the system develops economically:

> "At the highest level of development a country will contain many specialised cities performing one or the other or several of these functions, and viewed in the aggregate a condition of entropy will obtain." (Berry, 1961, p. 584)

However, such arguments are mere conjecture hardly based on evidence and models of city size distribution begin to lose their attraction when it is remembered that the growth of individual cities has been shown to be the result of specific, and not stochastic, forces (Richardson, 1973a). Thus, one might agree with Fano (1969) and label this approach 'excessively random', although his proposal for constraining randomness within a Markovian framework would tend to bypass, rather than face, criticisms levelled by Mills (1972), Richardson (1973a), and others, regarding the failure of the statistical approach to consider internal, or other, limits on growth. Nevertheless, one might still cling to the statistical approach because it, unlike the economic approach outlined earlier, is able to operate easily within the city system context. However, this advantage is all but lost because city size distribution research has focussed upon aspatial city systems, the role of location as a growth determinent being consistently ignored.

The most useful result of the study of city size distributions is that growth can be statistically linked to city size. Such a link may be appropriately embodied within the law of proportionate effect.

Given the existence of this statistical link, it is valuable to examine the role of size per se in determining growth, particularly since the importance of size is stressed in such economic models as the growth centre model and cumulative causation model (Richardson, 1973b).

2.1.3 City Size, Innovation and Growth

> "City size may in at least some cases be what is called an intervening variable that forms a statistical link between one variable and another and may stand as a surrogate for one of them but is not the real independent variable itself." (Richardson, 1973a, p.8)

A number of reasons for the positive correlation between city size and growth have been suggested. For example, it is evident from the earlier discussion of the cumulative causation model that agglomeration economies associated with large cities play an important part in maintaining an already established imbalance between regions, although such economies are difficult to measure (Richardson, 1973a). Pred (1966) puts forward the related notion that every industry has a market threshold that must be met. Thus, owing to their size, certain cities may be precluded from obtaining certain industries, an argument similar to that put forward by Ullman (1962). However, one of the most persuasive lines of argument is that which maintains that cities tend to obtain growth impulses through the adoption of innovations and that the magnitude of such growth impulses is positively correlated with city size.

In a sense, the diffusion of all innovations, excepting those of a harmful nature such as disease, plays a part in the regional growth process. However, since growth is usually thought of in economic and aggregate terms such as total production,it is the diffusion of technological innovation which is of primary interest in the growth context.

Since the early work of Schumpeter (1928), many economists have stressed the importance of technological change within the growth process. For example, Cameron (1975) is of the opinion that

> "...the principal dynamic factor in the economic growth of the last two centuries has been the application of new technologies in agriculture, industry, transportation and communication and, indeed, in all aspects of economic activity." (Cameron, 1975, p. 217)

Although the impact of technological innovation upon growth has been well documented by economic historians (for example, see Hartwell, 1971 and Mathias, 1969) such documentation has been qualitative rather than quantitative in emphasis. Such an emphasis is dictated by the complex nature of the growth process. Indeed, Mansfield (1968) maintains that the complexity of the growth process makes it impossible to measure the precise contribution of any single factor such as innovation. Nevertheless, the weight of historical evidence points toward the importance of technological innovation in the growth process. For example, the economic history of nineteenth century Britain supplies numerous examples of rapid town or regional growth as a consequence of innovation within single firms or industries (Mathias, 1969). A similar relationship is equally important in the short - term and contemporary context.

> "Technical innovation is essential to corporate growth and is the principal means of corporate competition. Companies must innovate at an increasing rate. New products and processes have progressively shorter life cycles....

> a progressively larger share of corporate incomes comes from products introduced within the last ten years." (Schon, 1967, quoted by Pred, 1975, p. 253)

The tempo of innovation has increased because, as an economy becomes more developed, the number of innovations that become available within it increases dramatically (Schmoolker, 1966). Thus, the major difference between the contemporary and the historical situation is the number of innovations available within any system. Although a few examples of urban growth as a consequence of the adoption of specific innovations might still be found (Thomas and Le Heron, 1975), the number of innovations being adopted within the system means that the effect of a single innovation is difficult to detect (Lasuén, 1973), even in the rather gross manner practiced by economic historians. Nevertheless, innovation does occur because

> "Business firms are reasonably sensitive to variations in cost and profit because of competitive pressures. Should an opportunity for abnormally high profits arise because of the introduction of either a new product or a new, lower cost technique of production, existing firms will have both a positive incentive and a negative sanction - the threat of bankruptcy - to adopt it." (Cameron, 1975, p. 217-218)

If it is assumed, quite reasonably, that firms which innovate will grow, or grow faster than those that do not, it follows that the cities or regions in which they are located will benefit from their innovativeness:

> "...each adoption of a growth - inducing innovation by an organization...contributes to city - system development by the creation of jobs at the site of adoption." (Pred, 1975, p. 253)

Central to Schon's (1967) statement on the role of innovation in the growth process is the implicit assumption that, in connection with growth, it is not only the number of innovations adopted by the unit being considered (a city, a corporation or a firm) that is important but also the timing of such adoptions. Although an innovation might eventually be adopted by every firm in an industry or in every city in a system, the possibility for monopoly profit or competitive advantage afforded by early adoption causes the growth stimulated by adoption to decrease over time (Thompson, 1965). In the context both of the number of innovations adopted and the timing of adoption, the process by which innovations appear and then diffuse is clearly of considerable interest.

It has been observed that inventive activity is concentrated in cities (Pred, 1966). Moreover, the amount of such activity is correlated with city size; for example, Thompson (1962) finds a correlation coefficient of 0.964 between the number of U.S. patents granted to individual states and the population size of the metropolises of those states. Why should this urban concentration of inventive activity occur? Pred (1966) proposes that invention is the product of the supply and demand conditions existing within industrialising centres and Schmoolker (1962) proposes that a 'snowball effect' operates; that is, a situation exists in which one invention makes others possible (for example, transistorisation in electronics). However, such hypotheses are difficult to test and the statistical link identified by Thompson (1962), that the number of innovations that originate in a region is proportional to its size, is the most critical result produced in the invention context. Of more importance in understanding the relationship between innovation and growth is the way in which growth - inducing innovations are diffused.

Unfortunately,

> "Relatively little is known about the various factors which influence the rate of diffusion and the way in which such innovations spread through the industrial structural fabric of the manufacturing sector." (Thomas and Le Heron, 1975, p. 231)

Although the innovative behaviour of single firms and of single industries has been subject to analysis by economists (Rosenberg, 1971) and the importance of factors such as the cost of applying the innovation, the expected rate of return, the compatability with existing production methods and the degree of obsolesence in existing capital stocks, demonstrated (Mansfield, 1962: Thomas and Le Heron, 1975), little research has been done on the process by which technological innovations appear at different locations at different times - the diffusion process. For example, Thomas and Le Heron (1975) find only two works, those of Hall and Johnson (1970) and Mansfield (1972), that provide comprehensive empirical evidence on the diffusion process within manufacturing industries. This paucity of empirical evidence contrasts sharply with that available for agricultural innovations (Rogers, 1962) and innovation in general (Gould, 1969: Hudson, 1972). Any general analysis of the link between innovation and growth must depend primarily therefore upon general knowledge of diffusion processes rather than upon knowledge of the diffusion of technological innovations.

To complete this discussion of the relationship between innovation and growth, it is useful to consider why innovation should be concentrated in cities and why larger cities should be more innovative than small ones. The reason for the first is rather obvious. The majority of innovations, and indeed of economic activities, demand the type of economic, social and organisational environment found only in cities. However, the preference for larger cities is more complex.

One of the major characteristics of technological innovation is the high degree of risk associated with it. A consumer who buys a newly-marketed commodity may risk only a few dollars, whereas an entrepreneur who adopts a new production process may well risk the financial well-being of his firm. Therefore, the degree of uncertainty on the part of an entrepreneur regarding a technological innovation will play an important part in deciding whether or not, when, or to what extent, adoption will occur. Since it has been shown that large cities tend to have better access to information than small cities (Brown, 1968: Hudson, 1972), the early appearance of innovative firms or industries in large cities might represent a growth advantage for areas with less uncertainty (Webber, 1972). The validity of this interpretation will be discussed in Chapter 3.

Thus, available evidence indicates that the diffusion of innovations, particularly those of a technological nature, plays an important part within the growth process. The most crucial aspects of this diffusion process are the number of innovations adopted and the timing of adoption. Given that the majority of these innovations are urban oriented, it would be useful to model the incidence and diffusion of technological innovation within a system of cities and to investigate the relationship between the form of this process and urban growth. Unfortunately, lack of sufficient information on the nature of technological diffusion processes would necessitate reliance upon knowledge of a more general nature concerning the form of the diffusion process.

2.2 Empirical Analysis of the Central Canadian Urban System

> "The rapid urbanisation in Canada during the last century and particularly during the post war years, has created an integrated and indentifiable system of cities. Increasingly, economic development is articulated through the urban system. Regional growth has become synonomous with the growth of cities in this system." (Bourne and Gad, 1972, p. 7)

Studies carried out on the Central Canadian urban system during the last ten years or so allow several general observations to be made:

(a) The Central Canadian system is composed of two distinct sub-systems, one centred upon Toronto (the Ontario sub-system) and the other upon Montreal (the Quebec sub-system) (Golant, 1972: King, 1967). These sub-systems tend to behave differently for it is observed that "...the urban centres of Ontario generally display a greater complexity and more continuous variance in structural character." (Golant, 1972, p.130). This observation illustrates the importance of properly defining the system of interest.

(b) Economic activity and, consequently, growth are concentrated in cities (Bourne and Gad, 1972). An approach that utilises the urban system as a spatial framework would therefore appear to be a very useful one.

(c) There exists a significant degree of spatial autocorrelation of urban growth. For example, in an analysis of the growth of small urban centres, Hodgson (1972) found a significant correlation between the growth rate of nearest neighbours (often up to the seventh order). This implies that spatial factors, or factors that operate with a spatial bias, are important within the growth process (see Bannister,1975 , for a fuller discussion).

(d) Econometric models incorporating economic variables yield satisfactory explanations of short-term growth within the system (Golant, 1972: Barber, 1972). This indicates that a basically economic approach to the explanation of growth is a valid one.

(e) Large cities experience steady growth rates and the variability of growth rates increases as city size decreases (Bourne and Gad, 1972: Barber, 1972). There is also evidence that, during some time periods, large cities grew at a faster rate than small ones (Davies, 1972).

These generalisations constitute the subjective interpretation and evaluation of a number of detailed empirical works and their brevity is in no way a reflection upon the quality of those works. Empirical analysis of urban systems is not easy given the paucity of relevant theory. For instance, Barber (1972) comments that

> "In the absence of a well - developed theory of urban and regional growth, it is...possible that several important growth determinents have been excluded from this enquiry." (Barber, 1972, p. 159)

Attempts to use inadequate theory (Davies, 1972: Hodge, 1972) or no theory at all (Golant, 1972: Barber, 1972) have tended to produce results of limited generality, although more recent work has been more productive. For example, Bannister (1975) comprehensively analyses the spatial and economic structure of a system and its effect upon growth. Nevertheless, beyond a set of specific findings, difficult to interpret out of context, this sort of empirical analysis produces a set of general observations that are by no means unique (Robson, 1973, presents similar findings for nineteenth century Britain); that growth is essentially an economic process, that it is spatially autocorrelated and that large cities sometimes grow faster, but always experience less variable growth rates, than small ones. It is necessary for any model of growth within a system of cities to incorporate these general observations.

2.3 Summary

This survey of regional growth has been both partial and simplistic in that not all theories about, or models of, regional growth have been reviewed, and of those considered the majority have received only limited attention. However, given the purpose of this chapter - to place this research in context - a partial and directed review such as this is justified. Although it is recognised that important approaches to the regional growth problem may have been omitted - for example, substantial insight into the nature of growth processes might be obtained through examination of the historical evidence presented by authors such as Nader (1975) - and others considered in too cavalier a manner - for example, closer examination of migration-based models (Shaw, 1974; Preston, 1974) and of recent published works on urban systems (Yeates, 1975) might have proved ~~fruitful - certain~~ general conclusions can still be made.

Economic models developed to explain regional growth offer only partial explanations and they are often internally weak - these are basic conclusions. Moreover, their generally superficial treatment of space and of spatial flows makes the majority of economic models particularly inappropriate for the examination of growth within systems of cities. Such a shortcoming is of considerable importance because it has been observed that growth is concentrated in cities. On the other hand, although statistical models offer an explanation of growth within a system context, the aspatial nature of that context precludes their consideration as a viable alternative to economic models. It therefore appears opportune to become less ambitious and to attempt a limited modelling of growth within a system of cities through examination of a single factor in the growth process, innovation. Although such a model could not provide a comprehensive explanation of city growth, it would provide considerable detailed insight into the behaviour of one growth determinant within a fully - developed spatial context, insight that might have some general relevance.

CHAPTER 3

MODELLING THE INNOVATION DIFFUSION PROCESS

In the first part of this chapter, the geographical approach to the modelling of the innovation diffusion process is evaluated by reference to theoretical and empirical work reported in the literature and by empirical analysis of one diffusion process, that of Planned Regional Shopping Centres in Canada. In the second part of this chapter, the contributions of two works related to this thesis topic, those of Pedersen (1970) and Robson (1973), are critically evaluated.

3.1 The Generality of the Geographical Approach to Diffusion Modelling

Innovation can take the form of new ideas, new organisations, or new products (Webber, 1972). The study of diffusion processes - how the adoption of an innovation progresses through a susceptible population - has been treated both in an aspatial, and in a spatial manner; the former predates the latter in the literature. The concern most evident in the early sociological literature is the nature of the

> "relationships between (i) an innovation or a collection of innovations and (ii) an adopter, emphasising in the study not only his personal and situational traits, which include (iii) his perception of the characteristics of the innovations, but also (iv) the process by which the innovation spreads, and (v) the functions of communication media and channels in the process." (Webber, 1972, p 222)

Study of these relationships was frequently carried out in a rural, agrarian context, but produced, for instance, insight of a general nature into the mechanics of the adoption process (Rogers, 1962). Various regularities in the innovation diffusion process were observed, of which the S-shaped curve of cumulative adoption over time has had the greatest impact upon subsequent studies of innovation diffusion. In contrast to the general interest in innovation diffusion displayed in the early sociological literature, the geographer's interest in the innovation diffusion problem has centred upon the nature of the communication processes that convert a potential adopter to an adopter,because of their obvious spatial implication. Indeed, complicating factors such as the nature of innovations and the personal characteristics of innovators have been largely ignored so as to facilitate the quest for spatial generality.

Early work in geography followed the inspiration and direction supplied by Hägerstrand's (1952) seminal work, concentrating upon the nature of the diffusion process at a local scale (Brown, 1968): see the examples given by Bowden (1965), Misra (1968), among others, and the theoretical work of Morrill (1968), Brown (1968) and Hudson (1969). However, Hägerstrand (1967) himself recognised that the existence of social and other hierarchies would serve to distort the diffusion process and thus prove his original framework too simplistic and scale - specific. Thus, geographers have become increasingly interested in the process of innovation diffusion within hierarchical, and particularly urban, systems (Hudson, 1972).

Hudson (1969; 1972), Pedersen (1970) and Berry (1972) have all examined diffusion within a hierarchical system. Of these three models, that developed by Hudson has the strongest 'hierarchical' flavour. Hudson (1969) attempts to model the diffusion process within a modified Christaller k = 3 system similar to that developed by Beckmann and McPherson (1970). However, Pred (1971) has pointed out several weaknesses caused by such an 'artificial' framework: in particular, information is only passed down the hierarchy. Despite the theoretical advances associated with Hudson's work, the approach developed by Pedersen (1970) and modified by Berry (1972) is more general.

The Pedersen model takes into account the fact that the urban system is a spatial as well as a functional one, such that models of flows within it may be based upon interaction rather than upon central place concepts. The simple gravity model

$$I_{ij} = k(P_i P_j)^{\alpha} d_{ij}{}^{\beta} \tag{1}$$

where P_i and P_j are the populations of cities i and j, d_{ij} is the distance between them, α and β are parameters, and k is a constant, demonstrates that if α tends to zero or if each city is the same size, the process is purely contagious, whereas if β tends to zero, while α is greater than zero, then the process is purely hierarchical. Since more generally, neither α nor β equals zero, the amount of information, I_j^T, received in any town j, by time T, from all towns i which adopted before it, is

$$I_j^T \propto \sum_{t=1}^{T} \sum_i I_{ij}^t \tag{2}$$

if information about the innovation flows between places according to (1). Both Pedersen (1970) and Berry (1972) attempt to link such information flows to the diffusion process within urban systems but the results obtained are not conclusive.

Hudson (1972) distinguishes between innovations which are adopted by a city as an organisational unit and those which are adopted by individuals within the city. The first of these is simpler to observe since one has merely to measure time of adoption in each city but the second is rather more complex since the growth of adoption within each city as well as within the system of cities has to be measured. The interaction diffusion model, (1) and (2), has been applied in the first context while the logistic curve model has often been applied in the second (Hudson, 1972). The nature of the logistic curve model has made this second type of study aspatial in the single city context while facilitating comparison between cities. This thesis focusses upon the diffusion of innovations between cities.

These interaction models are more complex than the earlier models of Hägerstrand (1952). Yet the geographer's spatial approach to diffusion remains predicated upon the assumption, more often implicit than explicit, that the availability of information among a population of potential adopters plays the decisive role in determining the sequence of adoption. However, there exists another approach which maintains that other factors, mainly of an economic character, associated with both the innovation and the population of adopters and non-adopters, are most important in determining the sequence of adoption. In the case of adoption within a system of cities, the spatial interaction approach isolates only one factor, information availability, while the economic approach also considers such factors as profitability and degree of entreprenurial ability (Griliches, 1957: Mansfield, 1961: Webber, 1972). Of these economic factors, profitability has often been considered to be pre-eminent (Mansfield, 1961). The debate between Griliches (1957: 1962) and Rogers and Havens (1961) is probably the best example of a clash between the two approaches and will serve to highlight some of the problems associated with attempts to resolve such a conflict.

Griliches (1957), after regressing logistic curves on adoption data, claimed that areal differences in adoption rates for hybrid corn were a function of differences in an economic variable, profitability. Rogers and Havens (1961) reacted by pointing out that profitability should be considered a pre-requisite rather than a sufficient condition for adoption and that information regarding profitability has itself to be diffused. The hybrid corn controversy could not be settled by empirical testing since the data were seen to fit both models and this itself may well constitute the most valuable contribution of the debate. Bonus (1973), in a study of the diffusion of consumer durables in West Germany, obtained a logistic curve of cumulative adoption of automobiles. Such a pattern of cumulative adoption over time has often been taken to indicate an information - based diffusion process but Bonus claimed that information concerning the automobile was widespread, if not universal, at the start of his study period and that the logistic growth curve resulted solely from an increase in income levels over time.

It is therefore necessary to examine more closely models of, and data about, the diffusion process. This is particularly necessary for entreprenurial innovations, i.e., those that demand large capital investment by the private sector. Information undoubtedly plays a considerable role in the diffusion of such innovations, particularly in reducing uncertainty (Webber, 1972), but other, specifically economic factors might also have to be taken into account. To examine the relative role of information and other, economic factors , the diffusion of Planned Regional Shopping Centres (PRSCs) in Canada is studied. A Planned Shopping Centre is defined by McKeever (1953):

> "...It refers to a group of commercial establishments planned, developed and managed as a unit, with offstreet parking provided on the property,...generally in an outlying, suburban area." (McKeever, 1953, p.7)

Following Sheppard (1974), the size threshold for a PRSC was taken to be 100,000 sq. feet of selling space.

The diffusion of PRSCs has been studied twice before. Cohen (1972) has analysed the process in the United States while Sheppard (1974) has examined Canadian data; their results constitute a valuable source for comparison.

3.1.1 The Diffusion of PRSCs

Cohen (1972) attempted to test a general hypothesis - that diffusion is not a purely informational process - by testing the specific hypothesis that the diffusion of PRSCs in the United States was partly an economic process. He proposed that the time and extent of adoption of this innovation was a function of profitability. Cohen examined the extent as well as the timing of adoption but in this study only the latter will be considered. The process was analysed in its entirety (1949-1968) and in four sub-periods (in order to gain some insight into the changing nature of the process over time). Variables were selected to represent what Cohen termed 'hierarchical' and 'market' factors within a regression model (Table 3:1). The 'hierarchical' variables relate to the size of a city and its market while the 'market' variables measure the structure of the market in a city.

Cohen's results are not startling, partly because the level of explanation (R^2) provided by the regression equations rarely exceeds 0.40. For the whole period, three size - related variables, retail sales volume (1948), disposable income (1948) and general merchandise sales (1948) proved to be significant: cities with a larger market obtained the innovation earlier than smaller cities. Of course, these three variables must be highly multicollinear. On analysing the process by sub-periods, it was found that the explanatory power of size characteristics declined over time, while market structure or affluence variables became more important. Thus, Cohen's analysis indicates that (market) size variables were dominant for the process as a whole but that in the later stages, when most large cities had already adopted, the structure of markets better serves to differentiate between cities. These results were seen by the author to substantially confirm the hypothesis that the diffusion of PRSCs was partly an economic process.

However, Cohen's analysis has several shortcomings. In a technical sense, the high levels of multicollinearity within the set of independent variables inflate standard errors and reduce the number of

Table 3:1

VARIABLES IN COHEN'S ANALYSIS

1.	Population.
2.	Total retail sales.
3.	Total general merchandise sales.
4.	Total disposable income.
5.	Per capita disposable income.
6.	The ratio of retail sales to disposable income.
7.	Per cent urban population.
8.	Relative strength of CBD.
9. & 10.	Income distribution variables.
11.	Per cent change in sales for the CBD.

Source: Cohen (1972, pp. 121-122).

significant variables; consequently it is difficult to interpret the results of the regressions. This technical problem arises partly from one of a conceptual nature. Cohen attempted to separate the hierarchical and market components within the diffusion process but the chosen variables were inadequate for this purpose. The appearance of (market) size variables in an equation was interpreted to indicate a 'hierarchical' process, while market structure variables were thought to indicate a 'market' process, but Table 3:1 indicates that the chosen variables cannot be interpreted in a such clearly distinctive manner. Furthermore, the observed pattern of adoption could result either if information flows (depending on size and location) were important or if entrepreneurs undertook a search procedure whereby larger markets, owing to their almost assured profitability, were chosen first. It is impossible within Cohen's framework to identify the individual influence of one or the other factor, although the lack of any distance variable in the analysis makes it a poor representation of the first interpretation. A similar ambiguity is present in Berry's (1972) work. The treatment of the market factor is also inadequate: the measures of the existing competition within centres are rather vague, their interpretation depending largely upon the sign associated with them in the regression equations. A more precise definition of the role of competition is needed if the economic nature of the process is to be properly investigated.

In his analysis of Canadian data, Sheppard (1974), profiting from Cohen's errors, tested more precisely the economic and interaction hypotheses. For instance, the importance of competition for consumer spending within a city is recognised. Sheppard refers to the Index of Saturation (Lalonde, 1967: Gist, 1968):

$$IRS_i = P_i PE_i / FS_i \tag{3}$$

where P_i = Population of city i,

PE_i = Per capita expenditure in i,

FS_i = Floor space of retailing in i,

IRS_i = Index of retail saturation in i.

However, owing to the lack of data for the supply (denominator) element in the equation, this index is not used. Instead, a less adequate index of competition is used:

$$C_i = TDI_i / RS_i \tag{4}$$

where TDI_i = Per capita disposable income in city i,

RS_i = Per capita retail sales in i,

C_i = Index of competition in i.

Such a measure is weak since it ignores the influence of population upon the demand for shops in a city. Sheppard also includes variables which reflect market size, market potential and growth potential in an economic model. Only the analysis of time of adoption is here considered.

Sheppard uses two separate models to compare the economic and interaction hypotheses. The economic model is developed within a linear multiple regression framework, although the version used is rather unusual. For each two - yearly time period, starting in 1952, a value of 1 is assigned to a city if it adopts the innovation and 0 if it does not. The set of independent variables (Table 3:2) is then regressed against this binary dependent variable. This process is repeated for each time period, previous adopters being omitted. The low R^2 values, ranging from 0.1234 to 0.5623, make interpretation of the results difficult.

The interaction model used is a variant of that developed by Pedersen (1970). For any period of time, T, the cumulative amount of information received by any city i, Y_i^T, is

$$Y_i^T = \sum_{t=1}^{T} \sum_{j \in C(t)} k(P_i P_j)^{\alpha} d_{ij}^{-\beta} \qquad (5)$$

where C(t) is the set of cities which have adopted by time t. Again, using a binary dependent variable (1 for adoption, 0 for non-adoption), estimates for α and β for each time period are obtained through application of a non-linear least squares algorithm. Despite the fact that levels of explanation are low, the contagion effect, which is reflected in the size of the β coefficient, becomes more important over time; this accords with other observations (Gould, 1969). The importance of the hierarchical effect decreases over time but is still significant even in the later phases of the process. Sheppard (1974) concedes that

> "Substantive interpretation of the results in terms of whether there is a contagion or hierarchical process operating is difficult without comparison with simulated results under controlled conditions."(Sheppard, 1974, p.69)

The economic model indicates that market size was the dominant factor throughout the period and that factors such as market structure.

Table 3:2

ECONOMIC VARIABLES IN SHEPPARD'S ANALYSIS

1. Population of the city.
2. Total disposable income per capita.
3. Total retail sales per capita.
4. Index of market quality.
5. Market potential of the city.
6. Population change since the previous time period as a fraction of the total population at that time.
7. Index of competition from other retailers.
8. The elasticity of expenditure with respect to income.
9. The change in incomes since the previous time period, adjusted by dividing by the total disposable income at that time.

<u>Source</u>: Sheppard (1974, p. 50a).

and competition were of distinctly lesser importance, results which conform quite closely to those of Cohen (1972).

The two models were compared for each time period and it was found that the interaction model provided a better fit during the early stages of the process, but was supplanted in later periods by the economic model. The results indicate that information flows were important in the early part of the process when uncertainty regarding the profitability of the innovation was widespread. As knowledge concerning the innovation approached ubiquity, uncertainty regarding profitability decreased and market size factors became dominant.

Both Cohen and Sheppard have failed to represent the economic factor adequately in their models and to distinguish it clearly from interaction effects. The main source of confusion, particularly in Cohen's work , is that a hierarchical sequence of adoption can arise out of radically different causal structures. The fact that larger markets obtain the innovation earlier does not prove or disprove the economic hypothesis. A better way of testing the hypothesis is to investigate whether or not PRSCs were first located in cities where the demand for them was greatest and, as will be shown, this demand factor is not synonomous with market size.

3.1.2 Derivation of the Model

The general hypothesis that arises out of this discussion is that PRSCs will locate earliest in those cities which have the largest markets. But this hypothesis encompasses two possibilities: that entrepreneurs perceive suitability for location to depend on market size or that information about the innovation is being diffused through the urban system in a manner which depends in part on the sizes of cities. Thus, these ideas are distinguished in the two alternative hypotheses: (i) that PRSCs will locate earliest in cities which have the greatest potential demand for them; and (ii) that PRSCs will locate earliest in cities which are most accessible to information.

Only three variables are used to test these hypotheses. They are:

M_i^t = Size of market at city i in time t,

D_i^t = Potential demand for shopping centres at city i in time t,

P_i^t = Information potential at city i in time t.

The first variable, M_i^t, is measured by the total disposable income in city i at time t, data for which are published. The other two variables are more complex and their derivation deserves some explanation.

The measure D_i^t is similar in nature to the Index of Saturation (LaLonde, 1967: Gist, 1968). Assume that the gross demand for retail services in any city i at any given time it is given by M_i^t (which is measured in dollars) and that the supply of retail services is represented by the number of retail employees in city i, RE_i^t. Thus, the level of service in relation to the demand for services in any city i at any time t is measured by the ratio RE_i^t/M_i^t (which has the dimensions of employees per dollar). The system average service ratio is $\sum_i RE_i^t/\sum_i M_i^t$ and the difference between this and city i's service ratio measures the divergence of i's level of service from the system average level. The product of this divergence and M_i^t is

$$D_i^t = (\sum_i RE_i^t / \sum_i M_i^t - RE_i^t/M_i^t)\ M_i^t \quad , \tag{6}$$

which measures the number of retail employees which must be added to city i in order to bring its level of service up to that of the system average. If D_i^t is positive, it represents a deficit of employees in city i at time t. Retail employment data were obtained from the Canadian Census: owing to the limited coverage of the 1951 census, only twenty-nine of the fifty-six adopters of PRSCs could be included in the study. Since the census data are not compiled annually, inter-censal estimates of the number of retail employees in each city were made with the compound interest formula, assuming that rates of growth within each period were stable. Thus

$$RE_i^{t+n} = RE_i^t e^{xn} \tag{7}$$

where x = rate of growth,
e = base of the natural logarithm
n = number of years since the last census at time t.

Annual estimates of M_i^t were obtained in the same way.

Consider now the derivation of P_i^t. Let p_{ij} be the likelihood that a given message concerning the innovation is sent from city i, which has the innovation, to city j, which does not. This is assumed to be proportional to the population of j, S_j, and the distance between them, d_{ij}, raised to some exponent, b:

$$p_{ij} \propto d_{ij}^{-b} S_j \tag{8}$$

Assume that each individual in the system is able to send only one message in a single time period; then p_{ij} is standardised to

$$p^*_{ij} = p_{ij}(S_i/\sum_j p_{ij}) \tag{9}$$

which measures the number of messages leaving i for city j in a time period. The information - receiving potential of any city, i, in the system is therefore

$$P_i = \sum_{j \in C(t)} p^*_{ji} \tag{10}$$

where $C(t)$ is the set of cities in which information about the innovation is available. $C(t)$ is regarded as the entire set of twenty-nine cities.

The 1952 values for each of the three variables are presented in Table 3:3.

3.1.3 Analysis

3.1.3.1 1952 - 1971

The actual date of adoption and rank by date of adoption for each city in the system is known. In an attempt to evaluate the role of each variable in the process, simulated sequences of adoption were generated on the basis of the city scores on each of the three variables. For example, consider the case of M_i^t. As was previously stated, application of a compound interest equation provides annual values for M_i^t. It is known that one city adopted the innovation in 1952, so the city with the highest M_i^{1952} value is allocated a PRSC and the rank 1 in the M_i^t

Table 3:3

1952 DATA VALUES, DATE OF ADOPTION,
RANK BY DATE OF ADOPTION AND SIMULATED RANKS, FOR TWENTY-NINE CITIES

City	Date of Adoption (1)	Rank by Date of Adoption	Market Size ($Millions) (2)	Rank by Market Size	Potential Demand (3)	Rank by Potential Demand	Information Potential (4)	Rank by Information Potential
Calgary	1958	12.0	128.05	7.5	-1058	23.5	13,026	18.0
Edmonton	1955	4.5	138.87	4.5	-2348	20.0	14,565	18.0
Victoria	1962	18.0	80.28	10.0	-504	10.0	24,543	14.5
Winnipeg	1959	13.0	277.52	4.5	-5932	29.0	21,532	16.0
St. John	1966	23.5	37.42	20.0	-1223	18.0	11,406	23.5
St. John's	1967	26.0	26.37	21.0	-2440	23.5	2,885	29.0
Halifax	1957	10.0	73.18	12.0	-2390	26.0	14,871	20.0
Sydney	1964	20.0	30.21	29.0	-76	27.5	7,061	27.5
Brantford	1962	18.0	40.38	23.5	272	7.5	32,959	10.0
Hamilton	1955	4.5	235.32	4.5	3863	4.5	204,076	4.5
Kingston	1957	10.0	31.91	23.5	-603	18.0	22,954	14.5
Kitchener-Waterloo	1955	4.5	56.99	14.5	833	21.0	63,075	4.5
London	1960	14.5	99.89	10.0	-656	23.5	48,790	7.5
Oshawa	1956	7.5	38.04	18.0	475	4.5	50,044	7.5
Ottawa	1955	4.5	197.68	4.5	809	4.5	97,415	4.5
Peterborough	1966	23.5	34.10	26.0	-272	14.5	21,499	18.0
St. Catharines	1957	10.0	45.19	16.0	196	7.5	36,972	10.0
Sarnia	1970	29.0	26.86	23.5	-167	10.0	12,663	23.5
Slt. Ste. Marie	1965	21.0	35.42	23.5	502	10.0	5,557	26.0
Sudbury	1956	7.5	45.13	18.0	97	14.5	12,991	21.0
Thunder Bay	1969	27.5	56.03	18.0	-25	13.0	6,042	27.5
Toronto	1952	1.0	1146.54	1.0	12583	1.0	433,917	1.0
Windsor	1962	18.0	127.67	7.5	1218	4.5	33,169	10.0
Montreal	1953	2.0	1051.09	2.0	8264	2.0	298,559	2.0
Quebec City	1960	14.5	90.77	10.0	-5772	27.5	69,222	4.5
Sherbrooke	1966	23.5	24.03	27.5	-917	16.0	26,055	13.0
Trois Rivieres	1969	27.5	18.63	27.5	-1780	23.5	30,743	12.0
Regina	1961	16.0	60.97	13.0	-1912	18.0	5,999	23.5
Saskatoon	1966	23.5	40.11	14.5	-829	12.0	4,770	23.5

Sources: 1. After Sheppard (1974, pp. 94-96)
2. Data from *Sales Management: The Magazine of Marketing*
3. Market size data from (2) and retail employment data from *The Census of Canada*, Ottawa: Statistics Canada (Dominion Bureau of Statistics).
4. Population data from *The Census of Canada* (3).

Note: All variables are defined in the text.

simulation. This city is omitted from further analysis. The next city to adopt did so in 1953, so the city with the highest M_i^{1953} score is allocated a PRSC and the rank 2. This city is also omitted from further analysis. This process continues until each city has been allotted a PRSC and a rank. Where more than one city adopts in a single year, for example 1955, the cities with the highest scores, that is those who will receive a PRSC, are allocated the appropriate average rank. This is consistent with the procedure used to rank the actual adoption sequence. Such a process is an advantageous one since it makes it unnecessary for the supply function and the threshold demand value for PRSCs, both of which probably change over time, to be estimated. This set of simulated ranks, together with similar sets for D_i^t and P_i^t, were compared with one another and with the actual adoption sequence using Spearman's rank correlation test (Table 3:4)

These results indicate that the diffusion process is a (market) size dependent one, the larger cities tending to adopt earlier than small cities. The relative failure of the potential demand hypothesis is also evident. The relatively large r_s coefficient between rank by date of adoption and rank by information potential makes the results more ambiguous because of the inter-correlation between market size and information potential. These results contain the same ambiguity as is present in the work of Cohen (1972) and Sheppard (1974), for the measure of market size reflects both demand and information, as Table 3:4 indicates. The effect of covariance among the simulated ranks can be removed from the r_s coefficients by calculating partial correlation coefficients

Table 3:4

THE 1952-1971 ADOPTION PROCESS:

SPEARMAN'S RANK CORRELATION (r_s) COEFFICIENTS (N=29)

		R_1	R_2	R_3	R_4
R_1	Rank by date of adoption	-	0.6952**	0.2695*	0.6610**
R_2	Rank by market size		-	0.2367	0.5195**
R_3	Rank by potential demand			-	0.3970*
R_4	Rank by information potential				-

* Denotes a t statistic significant at the .10 level.

** Denotes a t statistic significant at the .01 level.

(Conover , 1971). For example, the partial correlation coefficient $r_{s_{12.34}}$ is calculated as

$$r_{s_{12.34}} = \frac{r_{s_{12}} - r_{s_{13}} r_{s_{14}} r_{s_{23}} r_{s_{24}} r_{s_{34}}}{((1-r_{s_{13}})(1-r_{s_{14}})(1-r_{s_{23}})(1-r_{s_{24}})(1-r_{s_{34}}))^{1/2}} \qquad (11)$$

The values obtained are: $r_{s_{12.34}} = 0.6784$, $r_{s_{13.24}} = 0.2149$, and $r_{s_{14.23}} = 0.6437$. These coefficients have virtually the same values as the corresponding r_s coefficients shown in Table 3:4.

Since no R^2 value could be obtained from this non-parametric analysis, a stepwise multiple regression model using 1952 data was developed. The equation obtained is

$$\ln \hat{Y}_i = 7.59381 - 0.00116 \ln S_i - 0.00077 \ln P_i \qquad (12)$$

which has associated with it an R^2 value of 0.5419. Interpretation is made difficult by the presence of multicollinearity within the independent variable set: for example, the simple correlation between $\ln S_i$ and $\ln P_i$ is 0.689, but the structure of the regression model appears to confirm the earlier interpretation of the non-parametric analysis. However, the results of the non-parametric analysis are more reliable than the results of the regression analysis owing to the fact that the latter uses static (1952) values to describe a dynamic process extending over two decades.

Because of the experience of Cohen (1972) and Sheppard (1974), the process was divided into sub-periods for further analysis. The small size of the sample dictated a division into two sub-periods. 1961 was chosen as the dividing point since it was a Census year.

3.1.3.2 1952 - 1961

A non-parametric analysis identical to that performed for the period as a whole was first carried out (Table 3:5). The results of this analysis make the overall picture a little clearer. In comparison with the analysis of the entire process, the importance of the market size variable is decreased while that of potential demand is increased. This result indicates that the potential demand in a city plays an important part in determining the city's position in the adoption sequence.

The relatively poor performance of the market size variable is consistent with hypothesis (i), while the relatively high r_s coefficient (0.5759) associated with information potential also indicates the importance of information flows during the early part of the process. This latter result is consistent with Sheppard (1974). More precise interpretation of these results is made difficult by the high r_s coefficient (0.6007) between information potential and potential demand. The high correlation between two of the simulated rankings causes the partial correlation coefficients to differ from the sample r_s coefficients. The value obtained for $r_{s_{12.34}}$ is 0.3767, for $r_{s_{13.24}}$ is 0.7192, and for $r_{s_{14.23}}$ is 0.5225. Each partial correlation coefficient is smaller than the corresponding r_s coefficient but their importance relative to one another remains the same. Thus, although both potential demand and information potential play important roles in the first half of the

Table 3:5

THE 1952-1961 ADOPTION PROCESS:

SPEARMAN'S RANK CORRELATION (r_s) COEFFICIENTS (N=16)

		R_1	R_2	R_3	R_4
R_1	Rank by date of adoption	-	0.4511*	0.7522**	0.5759**
R_2	Rank by market size		-	0.2322	0.4535*
R_3	Rank by potential demand			-	0.6007**
R_4	Rank by information potential				-

* Denotes a t statistic significant at the .10 level.

** Denotes a t statistic significant at the .01 level.

adoption process, their precise contribution cannot be determined within this data set.

A stepwise multiple regression model was also developed (subject to the limitations noted before). The equation obtained is

$$\hat{Y}_i = 1956.79364 - 0.00043D_i \quad . \qquad (13)$$

which has associated with it $R^2 = 0.6305$, markedly higher than that for the first (1952-1971) model. Such an R^2 value, particularly in the context of the relatively poor results of Cohen (1972) and Sheppard (1974), appears to confirm the interpretation of the non-parametric analysis.

3.1.3.3 <u>1962 - 1971</u>

A non-parametric analysis was first performed (Table 3:6). The partial correlation coefficients obtained from this matrix of r_s coefficients correspond almost exactly with the simple r_s coefficients. The values obtained are $r_{s_{12.34}} = 0.2574$, $r_{s_{13.24}} = 0.1786$, and $r_{s_{14.23}} = 0.3905$. From these results, it appears as if the process after 1962 is either a random one or one that is influenced by factors not represented in this analysis. The only moderately significant r_s coefficient, that associated with information potential, indicates the role of location relative to existing adopters, but this result is only tentative. No significant multiple regression equation was obtained for this period. The similarity of the sample cities in this late adoption group in terms of their scores on all three variables makes adoption within this model appear equally likely in each of them (Table 3:7).

Table 3:6

THE 1962-1971 ADOPTION PROCESS:

SPEARMAN'S RANK CORRELATION (r_s) COEFFICIENTS (N=13)

		R_1	R_2	R_3	R_4
R_1	Rank by date of adoption	-	0.2611	0.1840	0.3929*
R_2	Rank by market size		-	0.6511**	0.1635
R_3	Rank by potential demand				0.3104
R_4	Rank by information potential				-

* Denotes a t statistic significant at the .10 level.

** Denotes a t statistic significant at the .01 level.

Table 3:7

MEANS AND VARIANCES OF THE

THREE INDEPENDENT VARIABLES IN 1952 (N=16) and 1962 (N=13)

	1952		1962	
	MEAN	VARIANCE	MEAN	VARIANCE
Market size (S_i)	232×10^6	112×10^9	142×10^6	8.1×10^9
Potential demand (D_i)	390	20×10^6	-530	1.4×10^6
Information Potential (P_i)	88×10^3	$13{,}342 \times 10^6$	14.7×10^3	106×10^6

The actual adoption time may be a function of the availability of local capital, suitable sites and planning permission - factors that it was not possible to include in an analysis of this type - rather than variations in information and demand.

3.1.4. Assessment of Results

This analysis demonstrates that, for the first half of the process at least, a specifically economic variable operated; this itself constitutes an important result. However, this conclusion does not imply that the process is an aspatial one because the importance of the information potential variable, particularly during the first ten years of the diffusion process, justifies to some extent the interaction view of diffusion, though failure of information potential to dominate the process shows that for the diffusion of some innovations other factors are important too.

Therefore, the information - based approach is able to provide some explanation of the diffusion of even condition - sensitive innovations like PRSCs, although it is recognised that the generality of the information - based approach might partly be the result of other factors such as market size and demand, whose influence it is difficult to disentangle from that of information availability. Given that no operational framework of equivalent generality is available for the diffusion of technological innovations, the information - based approach to the modelling of diffusion processes is adopted in this thesis.

3.2 Spatial Diffusion Methodology

It has been noted that interest in the innovation diffusion process has not been restricted to geographers (Brown, 1968). However, excepting that advanced by some economists (Griliches,1957), the approach adopted by other workers in the area (for example,by sociologists, rural sociologists and epidemiologists) is similar to that adopted by geographers. This common approach is based upon the assumption that adoption does not occur until sufficient interaction has occurred between a non-adopter and an adopter (Rogers, 1962: Brown, 1968: Bailey, 1957). Nevertheless, despite the valuable contributions produced in these disciplines, work in geography is of most relevance to the development of a model for the diffusion of innovation within a system of cities.

The development of innovation diffusion modelling in geography is well documented (Brown, 1968: Hudson, 1972), but it is important to consider more closely the nature of information flows within a system of cities.

3.2.1 Information Flows Within a System of Cities

In the distant past, when spatial organisation was simple in nature, the diffusion process was dominated by contagion effects; that is, it was one in which the frictional effect of distance was of paramount importance. However, as a cause or consequence of increasing urbanisation, communication systems have increased in sophistication and efficiency over time. The role of urbanisation in changing the nature of the diffusion process has been demonstrated for a specific innovation by Pyle (1969) and more generally by Pred (1966; 1971). These studies indicate that the structure of the diffusion space plays an important part in shaping the diffusion process.

Hudson (1972) suggests that information flows can be divided into local, regional and international components, each with its own structural characteristics. Since this thesis is concerned with the diffusion process within a system of cities, regional flows are of greatest relevance. It has been frequently observed that the structure of the inter-city information network is stable over substantial periods of time (Bogue, 1949: Hudson, 1972). Since the diffusion of innovation has been linked to information flows, it follows that the inter-city diffusion process can also be assumed to be stable over substantial periods of time. Given this relationship, it is reasonable to contend that all innovations that diffuse within the same system of cities within the same time period will do so in a similar manner (Lasuén, 1973). This observed stability in the inter-city information network and diffusion process is of considerable importance since it permits the development of spatial models of the innovation diffusion process based upon the gravity concept (Hudson, 1972). The use of the gravity concept in the modelling of diffusion processes has already been considered (section 3.1.1), but it will now be re-examined in connection with two antecedent works.

3.2.2 Two Gravity- based Models of the Inter-city Diffusion Process

Pedersen's (1970) model is based upon the assumption that, for any innovation , the likelihood of adoption by any city is a function of information flowing into that city from existing adopters. Since personal contact is important within the adoption process even when mass media facilities are available (Coleman, Katz and Menzel, 1957), Pedersen considered the diffusion process in terms of a series of interactions between adopters and non-adopters and characterised these interactions by equation (1). This interaction model has the advantage of containing the pure contagion and pure hierarchical diffusion processes as limiting cases. Within Pedersen's framework, assuming that the innovation is profitable there, a city, i, will adopt when the information it receives from existing adopters concerning the innovation exceeds some threshold, F, which is known, so that the moment of adoption for the i^{th} city, t_i, can be derived from the equation

$$F = \sum_{j=1}^{i-1} (k\ (P_i P_j)\ d_{ij}^{-\beta})\ (t_i - t_j) \tag{14}$$

where P_i is the population of city i and P_j that of a city, j, which adopted at time t_j.

Pedersen (1970) used this model to examine the diffusion of innovation within a linearised k=3 central place system in which the innovation originates in the largest centre. Through simulation, a number of regularities in the diffusion process were observed. For example, the distance decay parameter, β, determines the magnitude of the hierarchical and contagion effects within the process and also serves to speed up or slow down the overall process.

Pedersen's (1970) work possesses several shortcomings. For example, like Hudson (1969), Pedersen tests the spatial diffusion model within a central place framework. The results are consequently constrained by the over - structured nature of the diffusion space (Pred, 1971). Criticisms may also be levelled at the diffusion process assumed; for example, only the diffusion of innovations originating in the largest city in the system are modelled. In general, although the details of the simulation procedure are not presented, Pedersen's conclusions seem far more comprehensive than the extent of experimentation justifies.

The most comprehensive effort to model the inter-city diffusion process and its relationship with growth is that of Robson (1973). Robson prefaced his diffusion analysis with a study of nineteenth century urban growth in England and Wales. From this study, certain regularities within the growth process were apparent. For instance,

> "In virtually every case, the standard deviations and the coefficients of variation grow progressively smaller for the larger size groups." (Robson, 1973, p. 83)

Robson attempted to explain this observation in terms of the diffusion of innovations. As a preliminary to the development of a spatial diffusion model,data on the diffusion of several innovations were examined. For example, in relation to Starr - Bowkett Building Societies it was observed that

> "Spread occurs first by the filtering of the societies to the regional centres or largest towns within regions and subsequently societies spread out to other, and smaller, places within the region.(Robson, 1973, p. 165)

Similarly strong size effects were evident within the diffusion of telephone exchanges, gasworks and street lighting. However, instead of considering the diffusion of these specific innovations, Robson proposed that "we can...think of the whole set of the innovations as averaging out into a 'composite' general innovation..." (Robson, 1973, p.187)

A model similar to Pedersen's (1970) was used as the basis for simulating the diffusion process. Assume that there is a system of cities in which some cities have adopted the composite innovation; each city, i, has a population, P_i, $i = 1,2.....,N$. The probability, Pr_i, of city i, which has not yet adopted, receiving a message in any time period will depend upon its size, its distance from cities which have already adopted and the number of messages emitted by the adopter cities. Thus,

$$Pr_i = k \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} \left(P_i^{\alpha} d_{ij}^{\beta} S_j \right) \right) \tag{15}$$

where S_j is the number of messages sent by the j^{th} adopter in a time period and k is a weight which converts the sum of the expressions for all places to 1.0 and individual city scores to an adoption probability. If the number of messages emitted by existing adopters and the parameters of the gravity model, α and β, are given exogenously, application of equation (15) produces an adoption probability for each city for each time period. Given these adoption probabilities, allocation of messages is made on a probablistic, Monte Carlo, basis. If exogenously determined resistence levels have been overcome, a message received by a non-adopter causes it to adopt the composite innovation, while a message received by an adopter stimulates further growth.

Since it attempts to model, over a long period to time, the diffusion of many innovations in terms of a composite whole, Robson's model is a very simple one. Thus, in connection with the growth that results from adoption, it was proposed that

> "For any one of the innovations, the effect on growth would almost certainly decline over time: both in terms of the fact that economic returns may be lower for later adopters and that, within a given town, one innovation would eventually be supercseeded by others. But, in effect, the simulation pools together the effects of all these innovations by assuming a continuing stream of inventiveness and innovation over time." (Robson, 1973, p. 190)

Like Pedersen, Robson reduces the generality of his results by testing the model within a constrained central place framework. Moreover, the model contains a basic flaw. At the start of a simulation, it is arbitrarily decided that certain cities have the innovation which, on the basis of empirical evidence (Chapter 2, section 2.1.3), is clearly a restrictive and unreasonable assumption. The process by which the composite innovation spreads from the original adopter cities is then determined through the application of equation (15). Given exogenously determined emission rates and adoption thresholds, the key variable in the diffusion process is the amount of information received in a city from existing adopters of the composite innovation. As time goes on, an increasing number of cities will have adopted the composite innovation and the probability of a city receiving a message becomes dependent only upon its population potential. Therefore, after the first time period, Robson's diffusion model degenerates into a potential model. Since the composite innovation is made up of a stream of innovations, the set of existing adopters should be re-determined at various points within the simulation process. As the model stands, in later time periods, even very small cities are considered to be existing adopters of the innovations then being diffused, simply because they adopted the composite innovation at some earlier point in time. Consequently, Robson's model is only accurate if one innovation is being diffused or if growth itself is considered an innovation.

3.2.2 Conclusions and Summary

It appears that observed stability in inter-city information flows permits the development of spatial models of the inter-city diffusion process based upon the gravity concept. However, examination of two previous attempts to model the inter-city diffusion process in this way indicated that such a model has not yet been properly formulated. It is possible though to list the desirable general characteristics of such a model. For example, it is necessary to determine more reasonably the origin of innovations and to permit the diffusion of many innovations at the same time. A model incorporating general characteristics such as these is presented in the next chapter.

CHAPTER 4

A MODEL FOR THE DIFFUSION OF INNOVATION WITHIN A SYSTEM OF CITIES

Innovation may provide a suitable basis for modelling growth within a system of cities. This is a conclusion of Chapter 2. Two important results emerged from the discussion of diffusion processes in Chapter 3: firstly, theoretical and empirical evidence in the literature and the analysis of the diffusion of PRSCs demonstrate that, in terms of the types of innovation whose diffusion can be modelled, the spatial approach has considerable generality and, secondly, that two previous efforts to model the inter-city diffusion process using an interaction model produce results of only limited generality and reliability. Keeping the shortcomings of these previous works in mind, a model for the diffusion of innovation within a system of cities will now be presented. Since it is difficult to model interaction within, and between, cities within the same framework, following Hudson (1972), the model only examines the adoption of innovation by cities. Although some of its implications for growth are considered in Chapter 6, the diffusion of innovation within cities is a separate problem and is not examined in this thesis.

4.1 Some Basic Assumptions and Propositions

Certain assumptions are made in connection with the nature of the diffusion process:

(i) the city as an entity adopts an innovation,

(ii) the probability of a city adopting a given innovation at some point in time is a positive function of the information available to it concerning the innovation, and

(iii) the amount of information available to any city within a system depends upon the spatial and hierarchical structure of that system.

Now consider the diffusion of innovation within a system of cities each with a given size, S_i, $i=1,2,\ldots\ldots,N$, and location with respect to each other, d_{ij}, i, $j=1,2,\ldots\ldots,N$. On the basis of available evidence (Thompson, 1962), it can be assumed that, over any period of time, the number of inventions produced in any one city, X_i, is proportional to its size, S_i:

$$X_i \propto S_i \qquad (1)$$

By extension, the probability of any given innovation originating (being invented) in city $i, p_i^{(1)}$, is a function of its size, S_i, relative to that of all cities in the system, $\sum_i S_i$. That is,

$$p_i^{(1)} = S_i / \sum_i S_i \qquad (2)$$

In one time period, the relative likelihood of a message being transmitted from any particular city, j, p_{ij}, is proportional to the gross interaction between the cities, I_{ij}, which is, in turn, a function of the size and spacing of cities within the system. Thus,

$$p_{ij} \propto I_{ij} = S_i S_j d_{ij}^{-\beta} \tag{3}$$

In contrast with Pedersen's formulation (equation 3.2), this interaction model is simplified by the assmption that the parameter on city size, α in equation (3.2), is equal to 1. Consequently, the relative strength of the hierarchical and neighbourhood components within the set of inter-city interactions depends only upon the magnitude of the parameter on distance, β.

If $P_i^{(t)}$ is taken to represent the cumulative probability that city i has adopted the innovation at time t, where

$$P_i^{(t)} = \sum_{j=1}^{t} p_i^{(j)} \; ; \; P_i^{(1)} = p_i^{(1)} \tag{4}$$

then the probability that it adopts the innovation within any time period of length dt, $dp_i^{(t)}$, is

$$dp_i^{(t)} = \lambda(1 - P_i^{(t-1)}) \sum_{\substack{j \\ j \neq i}} (P_j^{(t-1)} p_{ji})dt \tag{5}$$

Since λ, the diffusion constant, is the same for all cities, relative to that of other cities, city i's adoption probability is a function of

(i) $\sum_{\substack{j \\ j \neq i}} (P_j^{(t-1)} p_{ji})$, the sum of its interaction with existing information sources, and

(ii) $1 - P_i^{(t-1)}$, the probability that it has not yet adopted.

In (i), within - city interaction is discounted because, given assumption (i), only information from outside a city can influence city i's probability of adoption. Element (ii) has a similar effect to the retardation mechanism often incorporated into diffusion models (Hudson, 1972) and implies that the probability of any given piece of information prompting adoption by city i during the time period ending at t decreases as the probability that city i has already adopted increases.

Before proceeding with a more detailed exposition of the model, an important characteristic of this model vis a vis others in the field should be noted. Both Pedersen's (1970) model and Robson's (1973) model demand that the origin of an innovation be known. To make their models operational, they both make the assumption that all innovations originate in the system's largest city. However, in this model it is possible for an innovation to originate in any city, the probability of such an event being a function of city size (equation 2). In light of Thompson's (1962) and Pred's (1966) work, this is a more reasonable way of modelling the invention process than that employed by Pedersen (1970) and Robson (1973). Given the nature of the invention process, the model follows the diffusion of what might be termed the 'general' innovation, a process which incorporates within itself the whole spectrum of diffusions that can occur within a specified system of cities. Therefore, the mean and variance, or any other summary measures, of the city adoption probabilities produced by equation (5) would constitute summary measures upon the system's total innovation diffusion process.

Since the complexity of the inter-city diffusion process makes it difficult to obtain analytical solutions for the model embodied in equation (5) (see Preston, 1974, for examples of the limitations of an analytic approach to a similar problem, that of inter-city migration), simulation is used to examine the model's properties. This examination of the model's properties centres upon identifying the nature and extent of the relationship between the form of the diffusion process and the parameters of the system of cities in which it takes place. The diffusion process is simulated within hypothetical systems of cities. Real systems are not used because the location pattern is one of the parameters whose relationship with the form of the diffusion process is examined. However, in terms of the inter-city diffusion process, the properties of the hypothetical system of cities are no different from those of a real system of cities.

4.2 Diffusion Within a Hypothetical System of Cities

4.2.1 The Nature of the System

The hypothetical system of cities is composed of N centres whose location within the system space is determined through the random generation of co-ordinates. A random location pattern such as this is more in accord with reality than the central place patterns used by Pedersen (1970) and Robson (1973). The set of inter-city distances, d_{ij}, is determined from the cartesian co-ordinates and the size of cities follows a rank - size equation. There are two reasons for using a rank - size distribution of city sizes. Firstly, the rank - size distribution is a very useful one because, if the size of the largest city in the system is constant, the city size distribution can be summarised by a single parameter, the slope, q, of the rank - size line. Secondly, the rank - size distribution has been seen to fit many systems of cities, both across space and over time (Berry, 1961) and although the reason for such a regularity has not been adequately explained (Dziewonski, 1972), it is hard to ignore it. If the size of the largest city in the system, S_1, is given, the size of any city of rank r, S_r, can be determined through application of the rank - size equation,

$$S_r = S_1 r^{-q} \tag{6}$$

The limits of this distribution are obtained when q is 0.0 or ∞. In the first case, every city in the system has the same size and, in the second, there is only one city.

4.2.2 Information Flows

Given a set of cities with fixed size and location, the number of messages being transmitted from any particular city i to any particular city j, x_{ij}, in any time period, is proportional to the size of the destination, S_j, and its distance from i, d_{ij}, raised to some power, b. Thus,

$$x_{ij} = S_j d_{ij}^{-b} \tag{7}$$

which is derived from the general gravity equation (equation 3). It is assumed that, during the time it takes for one innovation to diffuse, city growth is not large enough to seriously distort the interaction pattern and x_{ij} values are not subscripted for time. This assumption is necessary to the model's operation. Thus, S_j is the size of city j at the start of the diffusion process and the same set of values is used throughout.

It is assumed that each individual sends only one message in each time period and therefore x_{ij} is standardised,

$$x^*_{ij} = x_{ij}\frac{S_i}{\sum_j x_{ij}} \tag{8}$$

Furthermore, if it is assumed that only one message is transmitted within the whole system in any one time period, x^*_{ij} can be transformed into the probability, p_{ij}, that the message is passed from any city i to any city j:

$$p_{ij} = \frac{x^*_{ij}}{\sum_i\sum_j x^*_{ij}}$$

$$= \frac{x^*_{ij}}{\sum_i\sum_j \left(x_{ij}\frac{S_i}{\sum_j x_{ij}}\right)}$$

$$= \frac{x^*_{ij}}{\sum_i S_i \sum_j \left(\frac{x_{ij}}{\sum_j x_{ij}}\right)}$$

$$p_{ij} = \frac{x^*_{ij}}{\sum_i S_i} \quad (9)$$

Two summary measures on inter-city information flows, Potential, A_i, and S Potential, B_i, are calculated for each of the N cities. Potential is the basic measure of information potential within the system and is the standardised number of messages that a city receives from the rest of the system in a single time period. Thus,

$$A_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} x^*_{ji} \quad (10)$$

S Potential, reflects accessibility to information from larger cities within the system,

$$B_i = \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} (x^*_{ji} S_j)\right) / \sum_{j=1}^{N} S_j \quad (11)$$

S Potential is calculated because overall accessibility to information from the rest of the system, as represented by Potential, is not an adequate indicator of accessibility to information concerning the innovation in early time periods. This point can be clarified by reference to equation (5). Element (i) in that equation represents interaction with existing adopters. As a consequence of the assumed form of the invention process (equation 2), in earlier time periods, the probability of any city, i, having already adopted the innovation, $P_i^{(t-1)}$, is largely a function of that city's size and consequently S Potential, which measures accessibility to information from larger cities, is a better indicator of the path of diffusion in these earlier time periods than is Potential.

4.2.3 The Adoption Process

The probability of an innovation originating in city i, $p_i^{(1)}$, is proportional to its size, S_i, and can be derived from equation (2). In subsequent time periods, each city's adoption probability, $p_i^{(t)}$, can be obtained from a version of equation (5),

$$p_i^{(t)} = \lambda(1 - P_i^{(t-1)}) \sum_{\substack{j=1 \\ j \neq i}}^{N} (P_j^{(t-1)} p_{ji}), \quad t=1,2,\ldots n. \qquad (12)$$

The diffusion parameter, λ, is a constant which serves to slow down, or quicken up, the diffusion process. Because the length of a time period is not specified, the value of λ has only limited theoretical significance.

The cumulative probability of adoption for any one of the N cities, $P_i^{(t)}$, is the sum of its $p_i^{(t)}$ values for all previous time periods. Thus,

$$P_i^{(t)} = \sum_{k=1}^{t} p_i^{(k)} \qquad (13)$$

A city, i, is certain to have adopted the innovation when its cumulative probability of adoption, $P_i^{(t)}$, reaches unity. Theoretically, this occurs only when $t = \infty$. From the $p_i^{(t)}$ values, it is possible also to calculate the mean adoption probability for the system, $p^{(t)}$, for each time period, t, where

$$p^{(t)} = \frac{1}{N} \sum_{i=1}^{N} p_i^{(t)}, \tag{14}$$

the variance of adoption probabilities for the system, $\sigma^2, p^{(t)}$, for each time period t, where

$$\sigma^2, p^{(t)} = \frac{1}{N} \sum_{i=1}^{N} (p_i^{(t)})^2 - (p^{(t)})^2 \tag{15}$$

and the cumulative mean adoption probability for the system, $\bar{P}^{(t)}$, for each time period, t, where

$$P^{(t)} = \sum_{j=1}^{t} p^{(j)} \tag{16}$$

To facilitate analysis of the diffusion process, the mean and variance of each city adoption probability distribution is calculated. Mean adoption time for city i, $\bar{t}_i$, is

$$\bar{t}_i = \sum_{t=1}^{\infty} (t p_i^{(t)}) \tag{17}$$

and its variance of adoption time, s_i^2, is

$$s_i^2 = \sum_{t=1}^{\infty} (t^2 p_i^{(t)}) - \bar{t}_i^2 \tag{18}$$

To aid clarification and to facilitate the introduction of fresh propositions, a numerical application of the simulation model will be presented and commented upon.

4.3 A Numerical Application

4.3.1 Description

The parameters of the system of cities in which the diffusion process is simulated are as follows:

Number of cities = 20
Size of largest city = 100,000
Diffusion constant (λ) = 50
City size parameter (q) = 1.0
Distance decay parameter (b) = 2.0
Location pattern = Figure 4:1

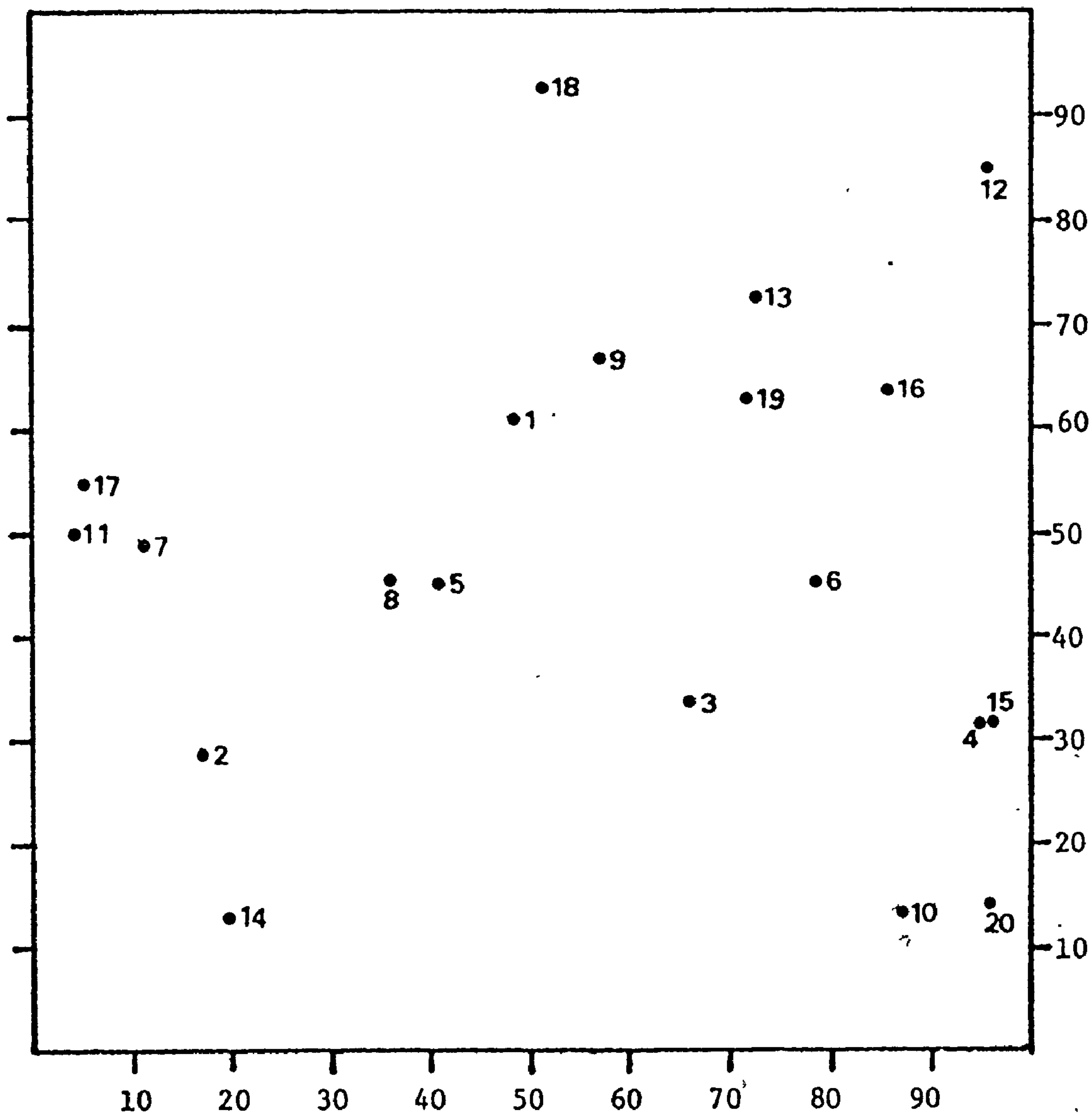
Given the location of cities (Figure 4:1), the matrix of inter-city distances, d_{ij}, can be calculated. Although it is the inter-city diffusion process which is under study, the existence of intra-city information flows is not neglected. Within any system of cities, a certain proportion of the flow of information is within, rather than between, cities. Since the balance between these proportions is a function of the structure of the system, it is unreasonable to assume that it is constant for all systems. Assuming that there is a frictional effect on the flow of information within cities, to take this flow into account it is necessary to measure the average distance between individuals within a city. A number of measures have been suggested (Isard, 1960) and in this analysis the following relationship is used to estimate average intra-city distances (Webber, 1974),

$$d_{ii} = 0.126\ S_i^{0.4} \qquad (19)$$

Using equations (10) and (11), the two information potential

Figure 4:1

LOCATION OF CITIES



measures, A_i and B_i, are calculated for each of the N cities (Table 4:1). On examination of these data, it is apparent that the total Potential, $\sum_{i=1}^{N} A_i$, within the system of cities is less than its total population, $\sum_{i=1}^{N} S_i$. This is because A_i does not include self-Potential; that is, Potential derived from within - city interaction. Given the generally small size of intra-city, relative to inter-city, distances, the calculation of intra-city information flows deprives the system as a whole of a substantial portion of its total possible information potential. Nevertheless, since intra-city information flows do exists, such a method of calculation is a reasonable one and ensures that both A_i and B_i accurately reflect the innovative potential of the system's cities. The normalised interaction probabilities, p_{ij}, (Table 4:2) also reflect the importance of making a clear dichotomy between inter and intra-city interaction. For example, note that p_{11}, the propensity of city 1 to interact with itself, constitutes 18.48% of the total interaction within the system. Omission of internal propensities would increase the interaction between large cities and small cities, which would have considerable effects upon the diffusion process.

Spatial autocorrelation measures are also computed. Over the last ten years, the theoretical and practical importance of spatial autocorrelation has achieved some prominence in the geographic literature (King, 1969: Cliff and Ord, 1971). The importance of proximity within the diffusion model makes the existence of spatial autocorrelation within the diffusion process probable. It is necessary to establish to what degree spatial autocorrelation exists, within the system of cities

Table 4:1

THE PRE-DIFFUSION STATUS OF THE SYSTEM

i	S_i	A_i	B_i	Autocorrelation of S_i
1	100000	28568	1537	1.0000
2	50000	10365	992	-.1029
3	33333	9457	1208	.0706
4	25000	12304	601	-.1879
5	20000	16980	2406	-.2179
6	16666	5958	699	-.2344
7	14285	7413	664	-.2086
8	12500	12862	1501	.2511
9	11111	12447	2905	-.0043
10	10000	2305	167	-.1709
11	9090	6544	416	-.1008
12	8333	1010	112	-.1687
13	7692	2986	365	.2274
14	7142	2530	341	.1335
15	6666	24228	1703	-.1749
16	6250	1918	189	-.0196
17	5882	4214	261	-.1765
18	5555	1023	174	-.1793
19	5263	2838	343	.0580
20	5000	1518	89	-.1349

Table 4:2

A PART OF THE NORMALISED INTERACTION MATRIX

i \ j	1	2	3	4	5	6	7	8	9	10
1	.18484	.00694	.00882	.00229	.01834	.00402	.00264	.00863	.02787	.02787
2	.00877	.10165	.00245	.00072	.00429	.00073	.00555	.00356	.00064	.00034
3	.01056	.00232	.05919	.00324	.00285	.00623	.00049	.00132	.00105	.00127
4	.00029	.00007	.00035	.00453	.00006	.00033	.00001	.00003	.00004	.00002
5	.01197	.00221	.00156	.00029	.01748	.00042	.00060	.01914	.00055	.00011
6	.00476	.00068	.00617	.00298	.00076	.02549	.00017	.00037	.00066	.00049
7	.00267	.00445	.00042	.00014	.00093	.00015	.01813	.00083	.00019	.00005
8	.00497	.00163	.00064	.00013	.01691	.00018	.00047	.00878	.00024	.00054
9	.01817	.00033	.00057	.00019	.00055	.00037	.00012	.00027	.00863	.00005
10	.00108	.00039	.00156	.00255	.00025	.00062	.00008	.00014	.00012	.01721

as well as the diffusion process. Compared with that of Cliff and Ord (1971), who were concerned with the 'areal' as opposed to the 'punctiform' problem, the spatial autocorrelation measure used in this thesis is a relatively simple one. For example, the spatial autocorrelation statistic for city size is calculated as follows: the relationship between the size of each of the N cities and that of its nearest neighbour is estimated through simple correlation, the relationship with the second nearest neighbour is similarly estimated, and so on, up to the nineteenth nearest neighbour. In each case, r, the correlation coefficient, constitutes a spatial autocorrelation statistic whose significance can be tested using Student's t test. The nearest neighbour spatial autocorrelation coefficients for city size are presented in Table 4:1. It should be noted that the first entry in that table, 1.000, does not mean that there is a perfect correlation between the size of each city and that of its nearest neighbour. This perfect correlation occurs because, for the calculation of all spatial autocorrelation statistics, a city always has itself as its nearest neighbour. Consequently, it is the second value in the table which is referred to as the first nearest neighbour spatial autocorrelation coefficient, and so on. The absence of significant spatial autocorrelation for city size reflects the random way in which cities of given size were located.

For each of the cities, invention probabilities are determined by equation (2) and subsequent adoption probabilities by successive application of equation (12). Cumulative probabilities of adoption are calculated by equation (13) and system means, variances and cumulative means by equations (14), (15) and (16). The probability distributions for cities 1, 10 and 20 are graphed in Figure 4:2. $\bar{t}_i$ and s_i^2, are calculated by equations (17) and (18) and are listed in Table 4:3. Inspection of these values suggests that $\bar{t}_i$ and s_i^2 are positively correlated: see also Figure 4:3. Simple linear regression is used to estimate the exact form of the relationship. The best - fit equation is

$$\ln s_i^2 = \gamma + \delta \ln \bar{t}_i \qquad (20a)$$

where $\gamma = -3.2103$ and $\delta = 2.8763$, which implies that

$$s_i^2 = e^{-3.2103}\, \bar{t}_i^{\,2.8763} \qquad (20b)$$

The coefficient of determination, r^2, associated with the equation is .9638, and indicates that $\bar{t}_i$ alone can be considered an adequate summery measure upon city adoption probability distributions. Nearest Neighbour spatial autocorrelation coefficients for $\bar{t}_i$ are calculated in a manner identical to that outlined in connection with city size and are listed in Table 4:3. The first nearest neighbour autocorrelation coefficient is statistically significant at the .05 level. The degree to which spatial autocorrelation is general for this type of diffusion process will be investigated in Chapter 5.

Figure 4:2

CITIES 1, 10 AND 20: PROBABILITIES OF ADOPTION OVER TIME

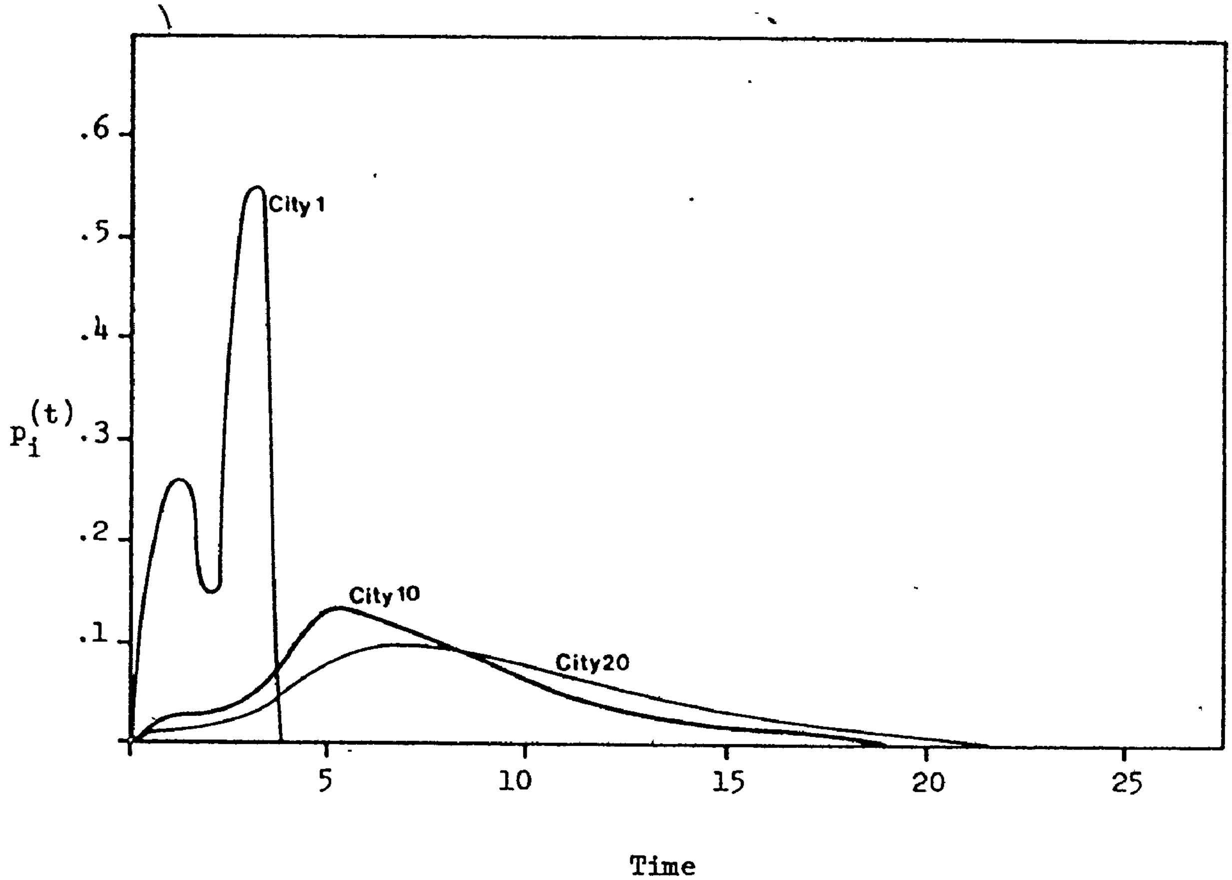


Figure 4:3

MEAN ADOPTION TIME ($\overline{t}_i$) AGAINST VARIANCE OF ADOPTION TIME (s_i^2)

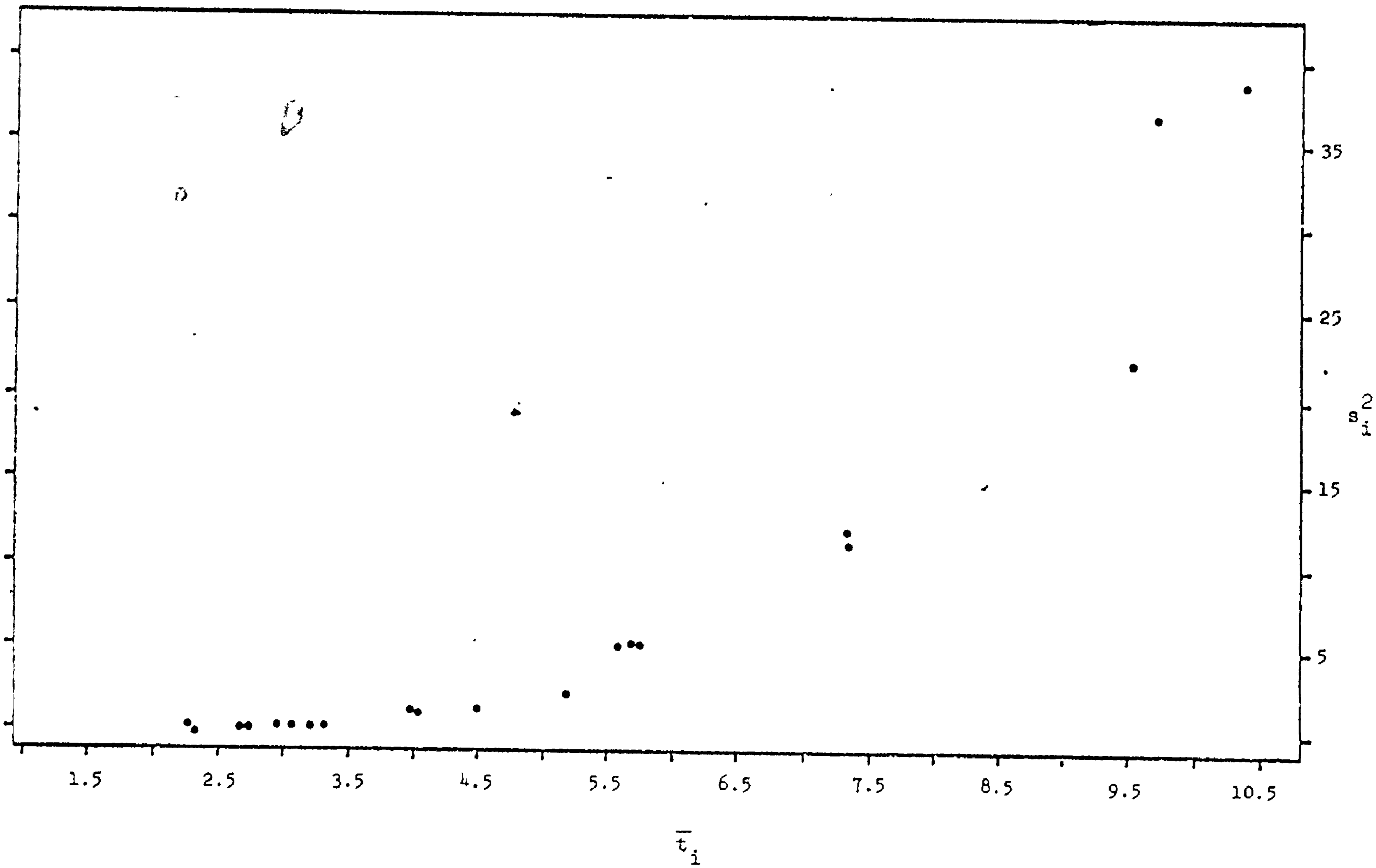


Table 4:3

A SUMMARY OF THE DIFFUSION PROCESS

i	$\bar{t}_i$	s_i^2	$\hat{s}_i^2$	$s_i^2 - \hat{s}_i^2$	Autocorrelation of $\bar{t}_i$
1	2.2598	.8082	.4209	.3873	1.0000
2	3.2069	1.6220	1.1518	.4701	.5996
3	3.2426	1.3978	1.1892	.2086	.0158
4	3.3321	1.0215	1.2860	-.2645	-.1890
5	2.7109	.7126	.7104	.0022	-.0358
6	4.0011	2.0841	2.1766	-.0925	.2514
7	4.0302	1.8739	2.2224	-.3486	.3212
8	2.9771	.7079	.9300	-.2221	.0275
9	2.7078	.7103	.7080	.0023	-.2034
10	7.2763	12.4072	12.1575	.2498	-.1412
11	4.5024	2.1110	3.0567	-.9457	-.3985
12	10.3859	40.4722	33.8313	6.6409	-.3132
13	5.5875	6.1976	5.6880	.5097	.2069
14	5.7190	6.3710	6.0816	.2894	.1531
15	3.0631	.7551	1.0095	-.2543	-.5474
16	7.2945	13.1576	12.2451	.9125	-.0226
17	5.2426	3.1821	4.7356	-1.5535	-.0230
18	9.6767	38.0575	27.6039	10.4536	.1026
19	5.7197	6.3384	6.0836	.2549	-.3821
20	9.5159	22.5638	26.3046	-3.7408	-.2372

4.3.2 Some Further Comments

There is considerable variation within the relationship between city size and city Potential (Table 4:1). This variation arises out of the use of an interaction model to calculate x^*_{ij} values. Within such a model, large cities tend to achieve high Potentials because they 'attract' more interaction but this is to some extent offset by the fact that their own population masses are not sources of information for them though they are for other cities. Also, owing to the relatively high frictional effect of distance on information flows within this system (b=2), the effect of location upon Potential is considerable. For example, note the relatively high A_i values for cities 5,8 and 9, and particularly that for city 15 (Table 4:1, Figure 4:4). Reference to Figure 4:1 indicates that these high A_i values are a consequence of location. These cities also have relatively high B_i values (Figure 4:5). Therefore, there is considerable variation within the size - potential relationships because, owing to their proximity, certain cities are able to draw excessively upon the population masses of larger cities and because cities are not allowed to interact with themselves.

Ignoring for the moment the unusual configuration of the adoption probability distribution for city 1 (see Chapter 5, section 5.1.1 for a detailed discussion), the city adoption probabilities are of the form expected; larger and more accessible cities have a more negatively skewed distribution of adoption probabilities than smaller and less accessible cities (Table 4:3, Figure 4:2). On examination of Tables 4:1 and 4:3, it is apparent that each city's $\bar{t}_i$ is linked to

Figure 4:4

CITY SIZE (S_i) AGAINST CITY INFORMATION POTENTIAL (A_i)

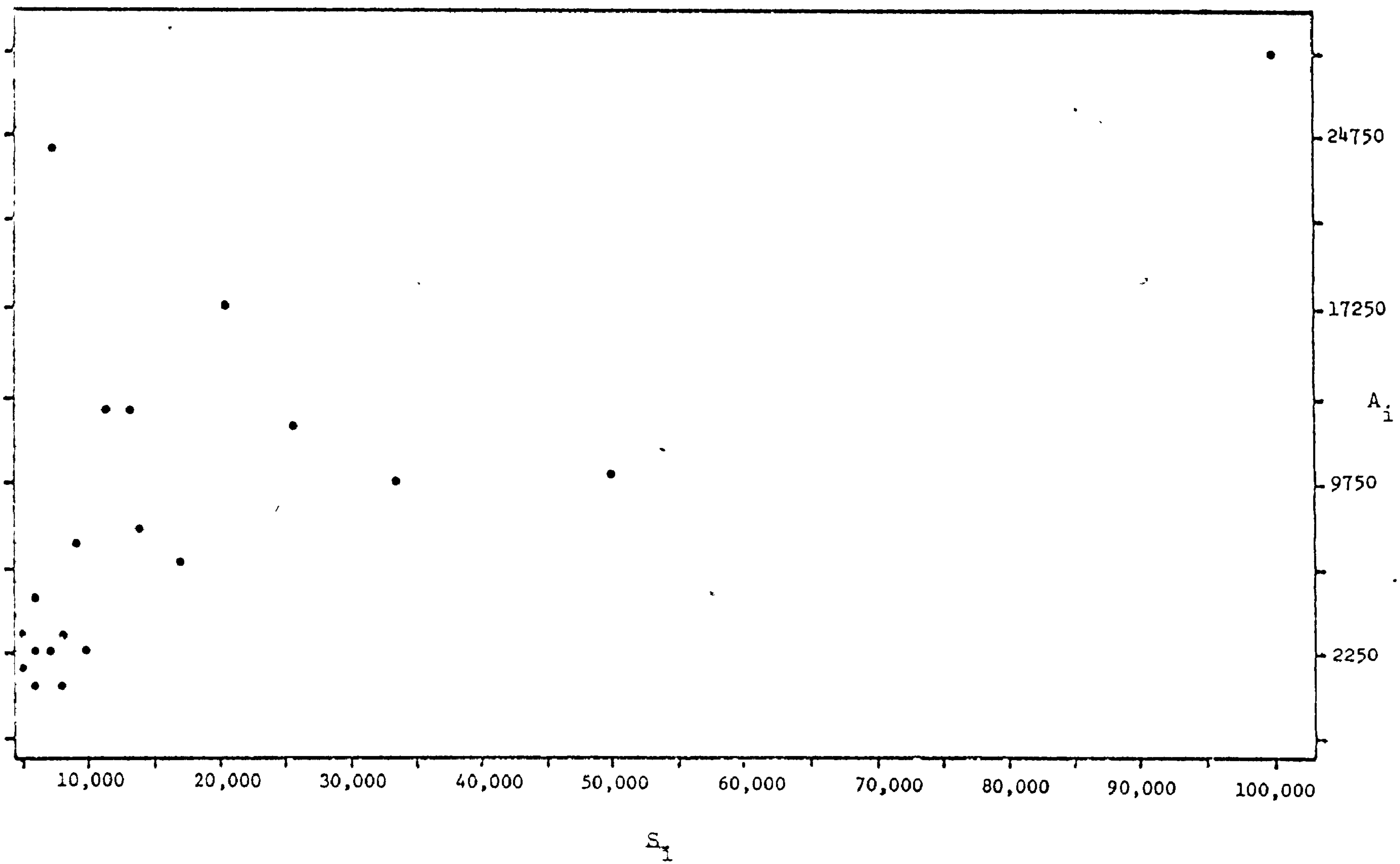
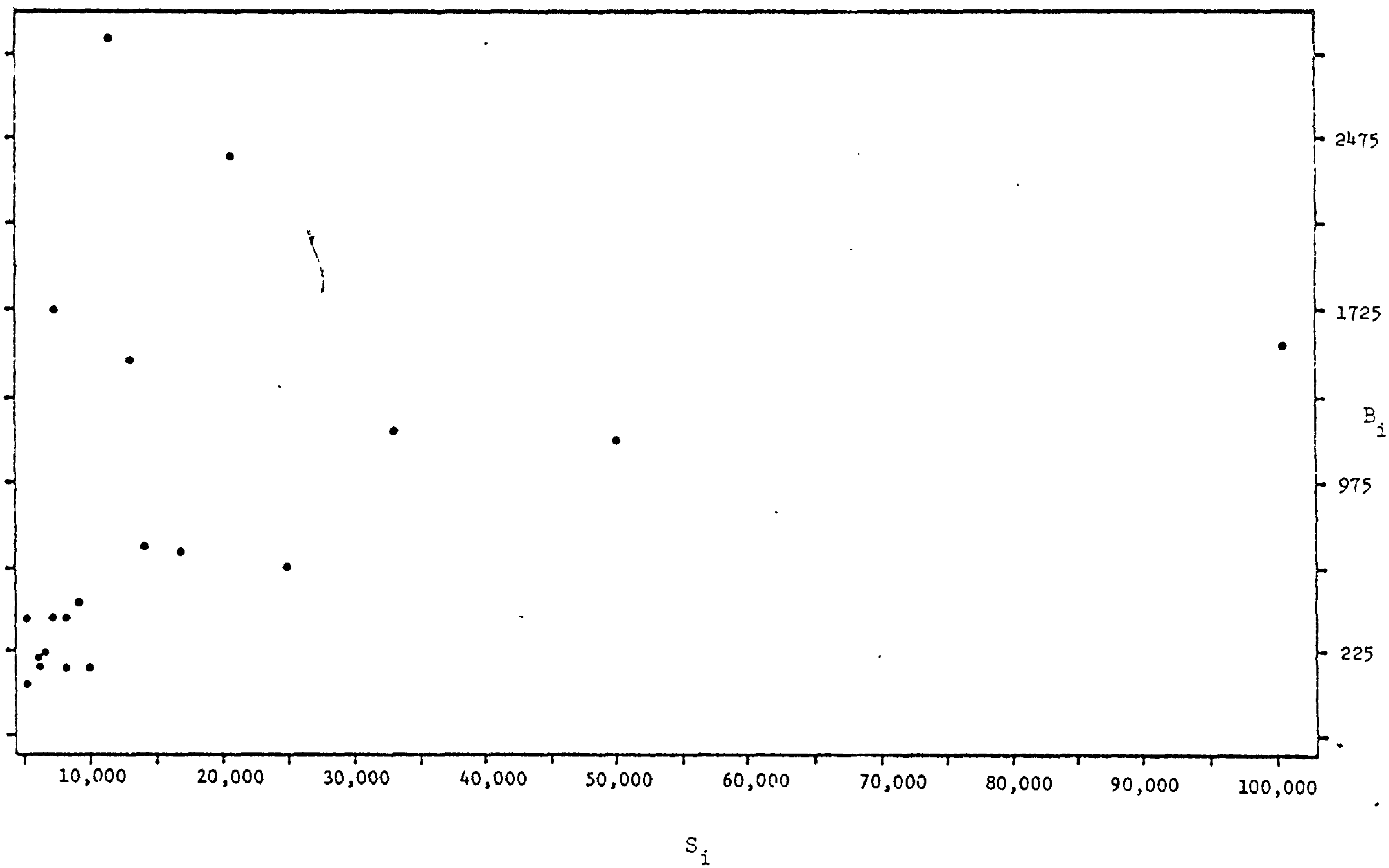


Figure 4:5

CITY SIZE (S_i) AGAINST CITY INFORMATION POTENTIAL (B_i)



its A_i and B_i (Figures 4:6 and 4:7). The precise form of this relationship is estimated through regression of A_i and B_i on $\bar{t}_i$. The best fit equation is of the form

$$\ln\bar{t}_i = \alpha - \beta_1 \ln A_i - \beta_2 \ln B_i \qquad (21a)$$

where $\alpha = 5.0739$, $\beta_1 = -.2896$ and $\beta_2 = -.1691$, which implies that

$$\bar{t}_i = e^{5.0739} A_i^{-.2896} B_i^{-.1691} \qquad (21b)$$

This equation has associated with it an R^2 value of .9764,and its significance for the modelling of the diffusion process becomes apparent in Chapter 5.

4.4 Some Conclusions

It has been demonstrated that it is possible to develop a model for the diffusion of innovation within a system of cities and to investigate its properties through simulation. Unlike that of Pedersen (1970) or Robson (1973), this model incorporates the possibility for multiple origin points and examines a general diffusion process rather than one associated with a single innovation. There are other differences between this and previous diffusion models. For example, the ability to summarise the relationship between information potential and mean adoption time (equation 21) provides a basis for investigating the relationship between the parameters of the system of cities and the form of the diffusion process.

Figure 4:6

CITY INFORMATION POTENTIAL (A_i) AGAINST MEAN ADOPTION TIME ($\bar{t}_i$)

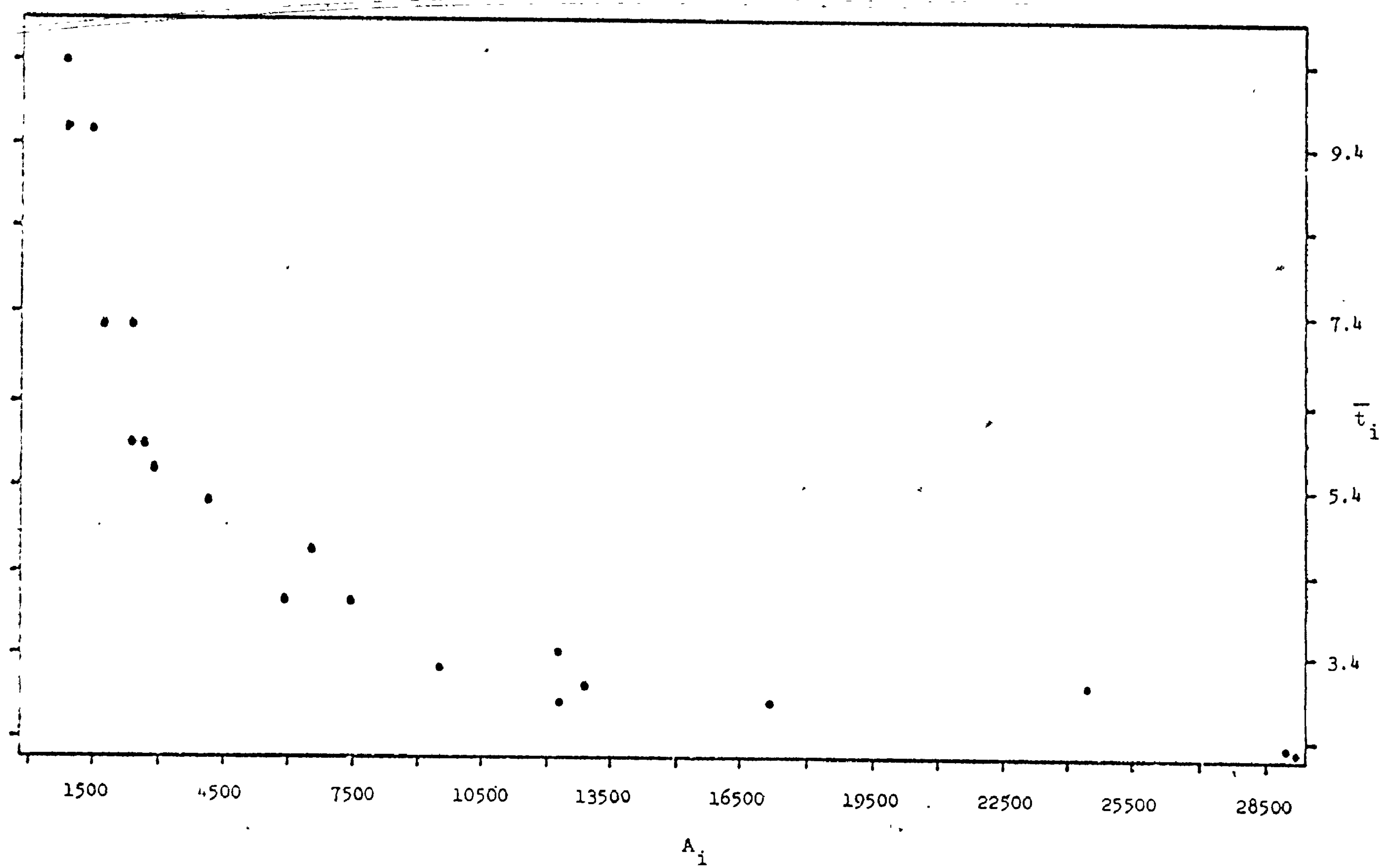
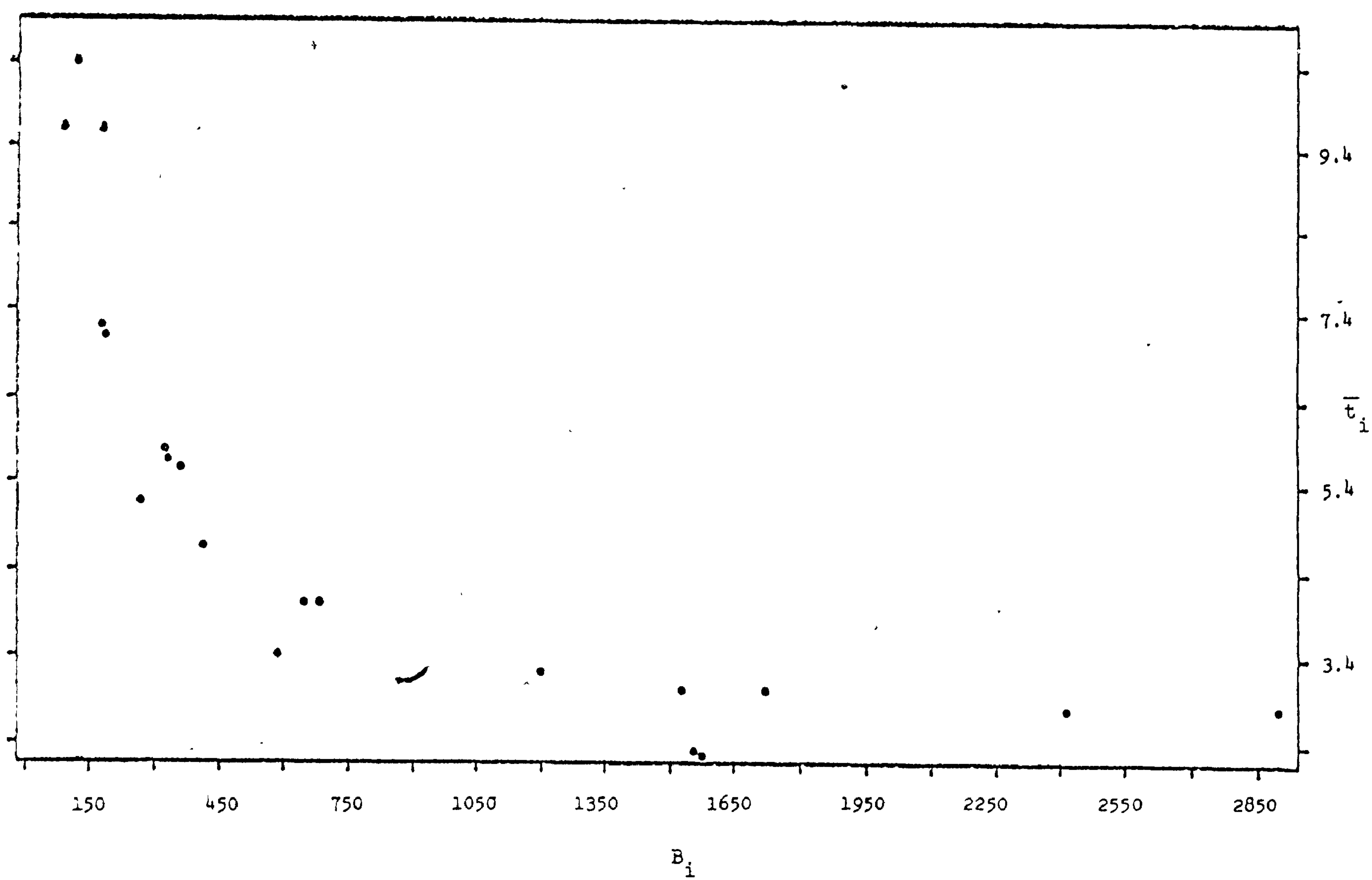


Figure 4:7

CITY INFORMATION POTENTIAL (B_i) AGAINST MEAN ADOPTION TIME ($\bar{t}_i$)



CHAPTER 5

PROPERTIES OF THE MODEL

In Chapter 4 a model for the diffusion of innovation within a system of cities was developed, and its nature was demonstrated by simulating the diffusion process within a hypothetical system of cities: Although this simulation analysis produced a number of useful insights into the nature of the process modelled, any conclusions made are specific to that particular system of cities because of the model's sensitivity to the structure of the diffusion space. Consequently, to explore the relations between innovation diffusion and urban growth, it is necessary first to investigate comprehensively the relations between the form of the diffusion process and that of system of cities within which it occurs. Although some qualitative evaluation is performed, by far the greater part of this investigation depends upon regression analysis.

On the assumption that the regression of city information potentials (A_i and B_i) on city mean adoption time ($\bar{t}_i$) summarises adequately the form of the diffusion process, the effects of changes in the parameters of the diffusion space (first b and q, and then Location) on the form of the diffusion process are estimated through examination of the variation in the coefficients of the summary regression that results from simulating the diffusion process within systems of cities with different parameters. This analysis is comprehensive enough to permit the prediction of city adoption times within comparable systems of cities.

Three of the system parameters, the number of cities in the system, the size of the largest city and the diffusion constant (λ), are held constant while the city size parameter, q, the frictional effect of distance on information flows, b, and Location are permitted to vary. Of the three parameters held constant, variation in λ and the size of the largest city does not affect the inter-city structure of the diffusion process but variation in the number of cities in the system does. If diffusion occurred within a frictionless environment, the addition of an extra city to the system would not affect the inter-city structure of the diffusion process. However, since diffusion does not occur within a frictionless environment, ignoring the effect of changing the number of cities in the system on the diffusion process constitutes a simplification, although a necessary one.

5.1 Some Qualitative Observations on the Effects of Changing b,q, and Location

The results presented and discussed at the end of Chapter 4 are specific to the diffusion process within a system of cities with a given pattern of location (Figure 4:1), a q value of 1.0 and b value of 2.0. This will be referred to as System 1. To ascertain the nature of the diffusion processes that occur within systems of cities with different parameters, the diffusion process is simulated within systems with the following parameters:

System 2 Number of cities = 20
Size of largest city = 100,000
Diffusion constant (λ) = 50.0
City size parameter (q) = 1.0
Distance decay parameter (b) = 1.0
Location pattern = Figure 4:1

System 3 Number of cities = 20
Size of largest city = 100,000
Diffusion constant (λ) = 50.0
City size parameter (q) = 2.0
Distance decay parameter (b) = 1.0
Location pattern = Figure 4:1

System 4 Number of cities = 20
Size of largest city = 100,000
Diffusion constant (λ) = 50.0
City size parameter (q) = 1.0
Distance decay parameter(b) = 1.0
Location pattern = Figure 5:1

Some of the results of these simulations will now be summarised. Sample graphs are presented in Appendix 1.

5.1.1 Changing b (System 2)

As a consequence of the closer correlation between city size and A_i and B_i, which is reflected in the distribution of $\bar{t}_i$ and s_i^2, the form of the diffusion process is more hierarchical than that observed for System 1 and, as a consequence of the decreased propensity for intra-city interaction, it is more rapid. This increased rapidity is reflected in a decrease in mean $\bar{t}_i$ from 5.1226 in System 1 to 3.5029 in this one (Table 5:1). Similarly, the variance of $\bar{t}_i$ decreases from 5.9988 to .7923. $\ln\bar{t}_i$ is again strongly correlated with $\ln s_i^2$, r^2 = .8857. Furthermore, $\bar{t}_i$ can once more be accurately estimated from A_i and B_i, although the coefficients of the regression are different from those obtained for System 1. These results are not counter - intuitive and correspond with other observations on the response of the diffusion process to changes in the interaction pattern (Hudson, 1972: Pedersen, 1970). Before discussing the effects of changing q and Location, one more result produced in the System 2 simulation will be considered.

Figure 5:1

LOCATION OF CITIES

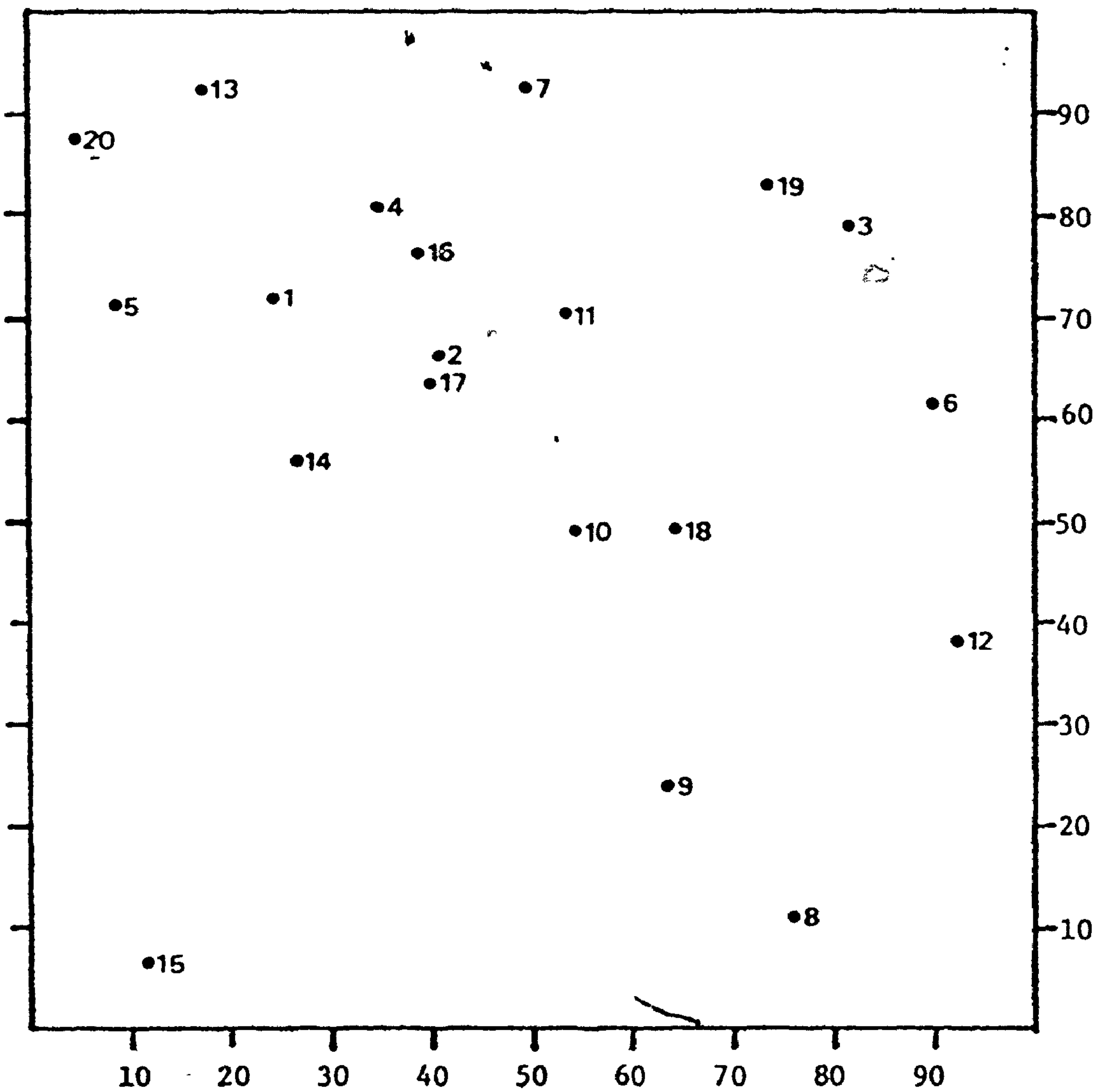


Table 5:1

SUMMARY RESULTS FOR TWENTY SIMULATIONS

Simulation No.	q	b	Mean A_i	Variance of $A_i (10^3)$
1	.5	1.00	30906	236954
2	.5	1.25	29379	232368
3	.5	1.50	27775	227410
4	.5	1.75	26158	220901
5	.5	2.00	24606	215239
6	1.0	1.00	12427	151623
7	1.0	1.25	11390	117628
8	1.0	1.50	10340	90622
9	1.0	1.75	9318	70780
10	1.0	2.00	8373	57277
11	1.5	1.00	5583	75432
12	1.5	1.25	4899	52841
13	1.5	1.50	4247	34975
14	1.5	1.75	3654	22699
15	1.5	2.00	3142	15259
16	2.0	1.00	2788	33091
17	2.0	1.25	2343	22064
18	2.0	1.50	1931	13419
19	2.0	1.75	1573	6317
20	2.0	2.00	1246	2313

Table 5:1 (cont'd)

Coeff. of Variation of $A_i(10^3)$	Mean B_i	Variance of $B_i(10^3)$	Coeff. of Variation of $B_i(10^3)$	Mean $\overline{t}_i$
7.67	1865	706	.38	3.39
7.91	1770	765	.43	3.52
8.19	1671	833	.50	3.67
8.45	1574	895	.57	3.87
8.75	1481	952	.64	4.09
12.20	1268	878	.69	3.50
10.33	1145	734	.64	3.76
8.76	1030	651	.63	4.11
7.60	925	616	.67	4.54
6.84	834	615	.74	5.12
13.51	962	1023	1.06	4.75
10.79	821	714	.87	5.40
8.24	701	516	.74	6.31
6.21	601	400	.67	7.56
4.86	519	340	.66	9.30
11.87	675	897	1.33	10.08
9.42	552	553	1.00	11.72
6.95	450	345	.77	13.51
4.02	342	241	.70	15.40
2.26	269	173	.64	17.28

Table 5:1 (cont'd)

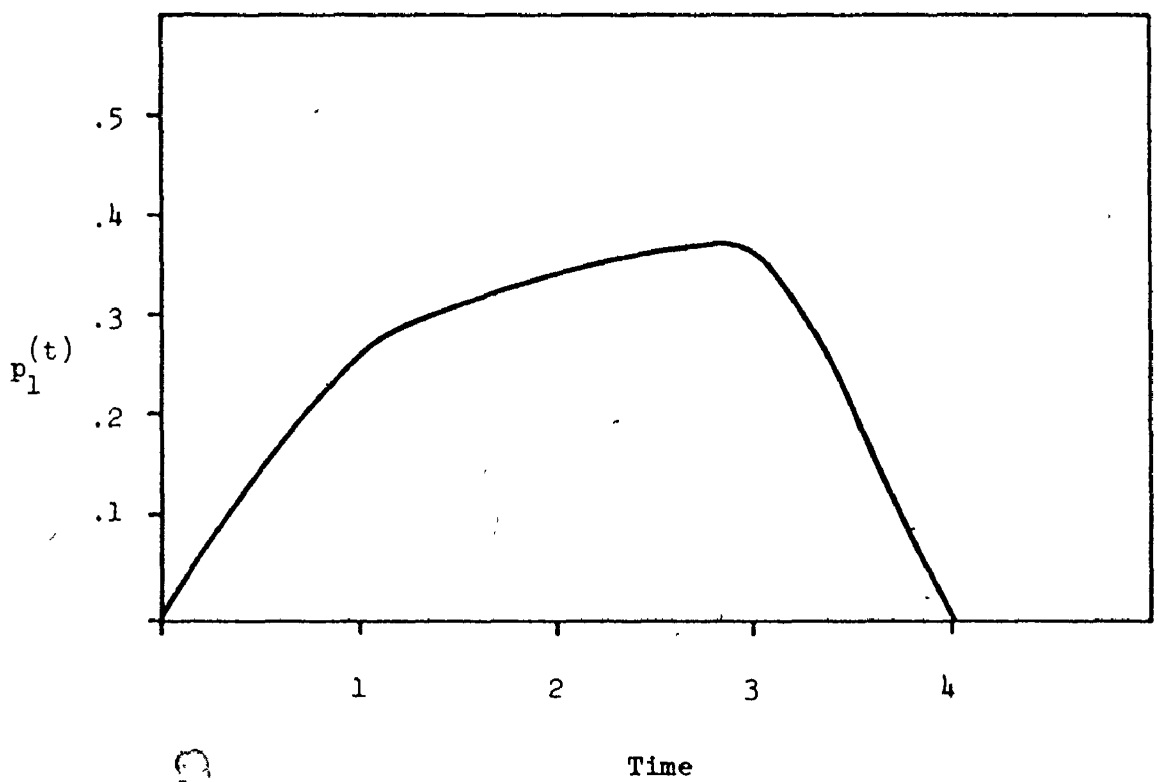
Variance of $\bar{t}_i$	α	β_1	β_2	R^2
.18	3.6363	-.15718	-.10966	.9943
.28	3.7785	-.15583	-.12917	.9939
.45	3.9897	-.16764	-.13970	.9911
.71	4.2123	-.18061	-.15019	.9891
1.12	4.4629	-.20457	-.14972	.9840
.79	3.8144	-.13361	-.20119	.9936
1.24	4.0717	-.15726	-.20464	.9900
2.03	4.3736	-.19703	-.19292	.9865
3.41	4.7138	-.24238	-.18064	.9818
6.00	5.0739	-.28957	-.16957	.9764
5.67	4.7426	-.25390	-.20796	.9753
9.10	5.0402	-.27332	-.22682	.9820
15.59	5.3481	-.29877	-.24058	.9853
28.31	5.6606	-.33743	-.23939	.9846
54.09	5.9766	-.38432	-.22981	.9812
63.09	5.7959	-.38864	-.22562	.9729
87.96	6.0158	-.44684	-.19371	.9809
115.29	6.1054	-.45877	-.19557	.9875
143.63	5.9812	-.44321	-.20012	.9713
189.28	5.9416	-.44212	-.19338	.9601

In the System 1 simulation, city 1's probability of adoption increases, than decreases, and then increases again, from time periods (1) to (3) (Figure 4:2). However, this anomaly is not present in the System 2 simulation (Figure 5:2). This seemingly erratic behaviour results from the inclusion of an invention period within the model. In the first time period (the invention period), city adoption probabilities are a function only of city size (equation 4.2) whereas, in subsequent time periods, city adoption probabilities are a function of location as well as city size (equation 4.12). Consequently, although the invention probability for city 1 is identical in both systems, namely .2780, in System 1, city 1's probability of adoption in the second time period is less than .2780 while, in System 2, it is larger. This reflects the increase in the positive correlation between city size and A_i and B_i in System 2 caused by the decrease in the frictional effect of distance. The adoption probability anomaly observed in System 1 is therefore a consequence of the difference between the method of calculating adoption probabilities in time period (1) and that in subsequent time periods, and it will only be apparent in systems in which the frictional effect of distance is relatively strong and the correlation between city size and A_i and B_i relatively weak.

The fact that, for certain cities within some systems, the probability of adopting the general innovation decreases from time period (1) to time period (2) does not imply that the same would be true if the diffusion of a single innovation within the same system were being considered. Consider again city 1's adoption behaviour in

Figure 5:2

CITY 1 : PROBABILITIES OF ADOPTION OVER TIME



System 1. That city would either invent a specific innovation or it would receive it sometime after time period (1) but it could not do both. Therefore, this anomaly is technical rather than conceptual in origin and is a consequence of the model's generality. Such an anomaly has not been observed in the diffusion literature because other students of the innovation diffusion process have considered only the diffusion of single innovations, which always have a unique origin. However, it follows that any extrapolation based upon a single diffusion process to account for multiple diffusions would carry the implicit assumption that all innovation originate within the same city. Indeed, such an assumption is a very convenient one, for it becomes increasingly apparent in the remainder of this thesis that the possibility for multiple invention sites incorporated into the diffusion model makes the modelling and interpretation of the diffusion process far more complex than it would otherwise be.

5.1.2 Changing q (System 3)

As a consequence of the increased polarisation of city size, the propensity for intra-city interaction among the larger cities in the system is greatly increased. For example, p_{11}, city 1's propensity to interact with itself, constitutes 51.82% of the total interaction within this system, as opposed to 13.93% in System 2. The structure of inter-city information flows, which determines the form of the A_i and B_i distributions, causes the diffusion process to be dominated by hierarchical effects. Together with the increased propensity for intra-city interaction, this polarised distribution of information potential causes the diffusion process for the whole system to be less rapid than that for System 2. This conclusion is highlighted by a mean $\overline{t}_i$ of 10.0810 and a variance of 63.0933. The simple correlation between $\ln\overline{t}_i$ and $\ln s_i^2$ is .9901. However, there is some marked deviation at the lower ($\overline{t}_i$) end of the distribution between predicted and actual s_i^2 which will be discussed at the end of this chapter. $\overline{t}_i$ can once again be accurately estimated from A_i and B_i.

5.1.3 Changing Location (System 4)

In essence, the change in Location substantially alters the city by city form of the diffusion process but not its general structure. Since, given a b value of 0.0, the diffusion process within this system would be identical to that in System 2, it is clear that the effect of Location upon the form of the diffusion process is intimately connected with that of distance. Given the same distribution of city sizes, the location of any city determines the way in which its information potential and, consequently, its adoption behaviour deviates from that which would occur if information potential and diffusion were purely size - dependent.

5.2 Characterising the Link between b,q and Location and the Form of the Diffusion Process

The results produced by simulating the diffusion process within different systems indicate that changes in the parameters of the system of cities affect the form of the diffusion process. By taking the regression of A_i and B_i on $\overline{t}_i$ (equation 4:21) as a summary of the form of the diffusion process, changes in the form of the diffusion process can be linked to the parameters of the system of cities through the coefficients, α, β_1 and β_2, of that regression equation. In the remainder of this chapter, methods for characterising this link will be outlined. Although the model was run for twenty location patterns, it is convenient to consider first the response of the diffusion process to changes in b and q within a single location pattern, that depicted in Figure 4:1.

5.2.1 b and q

The model is run for twenty combinations of b and q. The q values selected are 0.5, 1.0, 1.5 and 2.0, which encompasses those generally found for systems of cities (Berry, 1961). The set of b values combined in turn with each q value is 1.00, 1.25,1.50, 1.75 and 2.00, which also encompasses those generally found for systems of cities (Haggett, 1972).

The amount of data that would have to be presented makes it impossible to describe these twenty runs of the model in detail and therefore only a limited amount of summary information for each of the runs is presented (Table 5:1). The first eight columns of Table 5:1 contain summary data on information potential and mean adoption time.

The mean of A_i, variance of A_i and mean of B_i vary in a regular fashion in response to changes in b and q, the responses to the latter being of a greater apparent magnitude than those to the former. On the other hand, the variance of B_i, the coefficient of variation of A_i and the coefficient of variation of B_i display less regular behaviour. This complexity in the behaviour of the summary measures on information potential results from the complexity of inter-city interaction. For example, as a consequence of the way in which intra-city interaction is calculated, change in the variance of A_i is dependent upon the pattern of location as well as the total inter-city interaction within the system. Similarly, the behaviour of the variance of B_i is also dependent upon locational characteristics. Moreover, since B_i reflects the distribution of pair - wise interactions within the system, it is particularly sensitive to changes in the system parameters.

The more complex relationships between b and q and the summary measures on information potential are restricted to less - polarised city size distributions. This is because the polarised distribution of city size in high q value systems places a severe constraint upon changes in the distribution of information potential associated with changes in b. Such changes in the distribution of information potential occur within high q value systems but they have little effect upon the order in which cities adopt the innovation, which is determined primarily by city size.

Both the mean and variance of $\bar{t}_i$ are positively correlated with b and q. Moreover, the mean and variance of $\bar{t}_i$ are highly correlated, r = 0.9849, which permits consideration of mean $\bar{t}_i$ as an adequate single measure upon the diffusion process in each system. A regression of b and q on mean $\bar{t}_i$ produces the best - fit equation,

$$\text{mean } \hat{\bar{t}}_{i_k} = -2.7389 + 1.8667 b_k + 5.1667 q_k \quad , \qquad (1)$$

which has associated with it an R^2 value of .7784, which is quite high considering that the relationship between b and q and mean $\bar{t}_i$ is not direct but effected through an interaction model in which not all relationships are linear. The relatively poor performance of b, whose introduction into the equation only increases the R^2 value by .0438, is partly a consequence of the fact that a change in b tends to cause a change in the rank ordering of city adoption times within a given city size distribution and the impact of such re-ordering is not necessarily reflected in changes in the mean and variance of $\bar{t}_i$, and partly of the fact that the very magnitude of the variation caused by changes in q overpowers that caused by changes in b. Consequently, the poor performance of b in this regression should not be taken to mean that the structure of inter-city interaction plays an unimportant part in the diffusion process.

The data presented in the last four columns of Table 5:1 are a summary of the regression of A_i and B_i on $\bar{t}_i$ for each of the twenty systems. However, owing to a technical problem, the validity of the partial regression coefficients which constitute the core of these summary data is in doubt.

> "Multicollinearity constitutes a threat and often a very serious threat both to the proper specification and the effective estimation of the type of structural relationship commonly sought through the use of regression." (Farrar and Glauber, 1967, p. 93)

The three major consequences of multicollinearity are listed by Johnston (1963):

1) "The precision of estimation falls so that it becomes very difficult if not impossible to disentangle the relative influences of the various X variables. This loss of precision has 3 aspects: Specific estimates may have large errors; these estimates may be highly correlated, one with another; and the sampling variances of the coefficients may be very large."

2) "Investigators are sometimes led to drop variables incorrectly from an analysis because their coefficients are not significantly

different from zero, but the true situation may not be that a variable has no effect but simply that the set of sample data has not enabled us to pick it up."

3) "Estimates of coefficients become very sensitive to particular sets of sample data, and the addition of a few more observations can sometimes produce dramatic shifts in some of the coefficients." (Johnston, 1963, p. 160)

The existence of such consequences makes it evident that the interpretation of partial regression coefficients within a highly collinear set of independent variables (see Table 5:2) is not a simple task.

Since only two independent variables, A_i and B_i, are used, a relatively simple solution to the multicollinearity problem can be employed. Recall the structure of the two information potential measures. The Potential of a city (A_i) is the sum of the standardised information flows that it attracts from other cities in the system (equation 4.10) and its S Potential (B_i) is the sum of each of these standardised information flows weighted by the size of the origin, divided by the total population of the system, a constant (equation 4.11). Thus, the high correlation between A_i and B_i occurs because the second is a weighted version of the first. Because it is the unique part of B_i which is of interest, it is desirable to separate it from that part which is shared with A_i. $\ln B_i$ can be reduced to its unique element by regressing $\ln A_i$ upon it,

$$\ln \hat{B}_i = a + b \ln A_i \qquad (2)$$

and calculating the residual, R_i, between the actual and predicted values,

$$R_i = \ln B_i - \ln \hat{B}_i \qquad (3)$$

Table 5:2

THE SIMPLE CORRELATION BETWEEN ln A_i AND ln B_i IN TWENTY SIMULATIONS

Simulation No.	r	Simulation No.	r
1	.977	11	.948
2	.971	12	.937
3	.965	13	.928
4	.961	14	.920
5	.958	15	.913
6	.957	16	.952
7	.947	17	.936
8	.937	18	.925
9	.927	19	.918
10	.919	20	.915

The R_i values produced for System 2 are listed in Table 5:3. A negative R_i value means that a city has a substantial amount of its interaction with cities of relatively small size, whereas a positive R_i value means that a city has a substantial amount of its interaction with cities of relatively large size and thus benefits from interaction with likely sources of the innovation. R_i is calculated for all twenty systems and regressions of $\ln A_i$ and R_i on $\ln \bar{t}_i$ performed (Table 5:4).

The degree of explanation (R^2) furnished by the regressions is uniformly high, which reflects the importance of information availability in the diffusion process, although, in the q = 0.5 and q = 1.0 systems, there appears to be a tendency for the level of explanation to decrease as the frictional effect of distance upon interaction increases. It is probable that these, and other, small deviations originate from two sources. The first of these is rounding error, which can be assumed to have a random effect and does not explain systematic variation, and the second and more likely is that, owing to the complexity of their relationship with $\bar{t}_i$, the efficiency of A_i and R_i as explanatory variables varies from system to system.

The partial regression coefficients indicate that A_i is always the more important explanatory variable. The A_i coefficient becomes larger as both b and q increase, reflecting a change in the scale and distribution of A_i values and, through the operation of the model, that of $\bar{t}_i$. The R_i coefficient is both less important and less predictable. The unpredictable behaviour of R_i's coefficient reflects the complex nature both of the measure and of its relationship with

Table 5:3

R_i VALUES FOR SYSTEM 2

City No.	$\ln A_i$	$\ln B_i$	$\ln \hat{B}_i$	R_i
1	10.9595	8.1224	8.5590	-.4366
2	10.1494	7.8968	7.8305	.0664
3	9.9884	7.8679	7.6857	.1822
4	9.8154	7.2828	7.5301	-.2473
5	9.8911	7.9291	7.5981	.3310
6	9.3977	7.2442	7.1544	.0898
7	9.2803	7.0975	7.0488	.0487
8	9.4689	7.4343	7.2184	.2159
9	9.3899	7.6578	7.1474	.5104
10	8.6036	6.3008	6.4401	-.1393
11	8.8485	6.5596	6.6604	-.1008
12	8.1659	6.0113	6.0465	-.0353
13	8.6054	6.5236	6.4418	.0818
14	8.4031	6.3190	6.2599	.0591
15	9.4948	6.9451	7.2417	-.2966
16	8.3129	6.1137	6.1787	-.0650
17	8.4222	6.1203	6.2770	-.1567
18	7.9711	5.9940	5.8713	.1227
19	8.3887	6.2989	6.2469	.0521
20	7.9610	5.5797	5.8622	-.2825

Table 5:4

REGRESSION OF $\ln A_i$ AND R_i ON $\ln \bar{t}_i$

Simulation No.	α	β_1	β_2	R^2	Explanation Added by R_i
1	3.8812	-.26080	-.10967	.9943	.0075
2	4.0912	-.28011	-.12923	.9939	.0119
3	4.3508	-.30416	-.13976	.9911	.0144
4	4.6177	-.32890	-.15024	.9890	.0164
5	4.8785	-.35337	-.14977	.9840	.0155
6	4.0754	-.31456	-.20118	.9936	.0294
7	4.3569	-.34257	-.20464	.9900	.0325
8	4.6680	-.37373	-.19291	.9865	.0299
9	5.0118	-.40952	-.18066	.9818	.0260
10	5.3679	-.44706	-.16917	.9764	.0219
11	4.9014	-.44000	-.20794	.9753	.0191
12	5.2125	-.47440	-.22685	.9820	.0240
13	5.5372	-.51110	-.24055	.9853	.0267
14	5.8634	-.54898	-.23935	.9846	.0259
15	6.1842	-.5879	-.22984	.9812	.0229
16	5.9093	-.59285	-.22572	.9729	.0119
17	6.1109	-.61958	-.19369	.9809	.0106
18	6.1955	-.63037	-.19559	.9875	.0122
19	6.1398	-.62284	-.20053	.9828	.0140
20	5.8899	-.58789	-.18840	.9547	.0140

the form of the diffusion process, and its small importance (Table 5:4) indicates that, in this location pattern at least, gross accessibility to information from the rest of system, as measured by A_i, is more important in determining time of adoption than accessibility to information from larger cities, as measured by R_i.

The fact that the partial regression coefficients for A_i in simulations 19 and 20 do not fit the orderly sequence observed in the other eighteen simulations is an unfortunate, but easily explained, anomaly. Reference to the city adoption probability distributions in these two systems revealed that several small cities failed to attain a cumulative adoption probability of 1.0 during the one hundred time periods over which the diffusion process was simulated because of their almost negligible information potential scores. Consequently, the $\bar{t}_i$s calculated for these cities are unreliable and the last two simulations are omitted from further analysis; reducing the number of simulations in the sample from twenty to eighteen per location pattern.

It is possible to estimate the exact form of the relationship between b and q and the coefficients of the equation $\ln\hat{\bar{t}}_i = \alpha - \beta_1 \ln A_i - \beta_2 R_i$ through regression (Table 5:5). These regressions provide considerable insight into the nature of the relationship between the form of the diffusion process and the parameters of the system of cities in which diffusion occurs. The limited importance of R_i in the summary regressions on the diffusion process (Table 5:4) permits attention to be focussed upon the regressions for α and β_1. Within both sets of regressions, the signs associated with the relevant partial regression coefficients

Table 5:5

REGRESSIONS OF b AND q ON α, β_1 AND β_2

	a	$b_1(b)$	$b_2(q)$	R^2
α	1.89780	1.10995 (.51351)	1.32938 (.93657)	.9578
β_1	-.00211	-.11577 (-.33482)	-.22586 (-.99473)	.9747
β_1	-.09681	-.01571 (-.14237)	-.05866 (-.80949)	.6316

Note: Standardised partial regression coefficients in parenthesis.

indicate that an increase in q or b causes an increase in the size of α and β_1. These relationships reflect the fact that changes in the distribution of A_i cause changes in the distribution of $\bar{t}_i$. In connection with q, changes in the distribution of A_i are the result of the substantial re-structuring of the city size distribution that occurs when q becomes larger and have already been commented upon (section 5.1.2). The relationship between q and α and β_1 is expected, but a similar relationship between b and α and β_1 indicates that, although an increase in the frictional effect of distance benefits some fortunately - located cities, it also causes changes in the distribution of A_i within a system of cities with fixed size and location by reducing inter-city interaction. The larger standardised partial regression coefficient associated with q than with b in each of the regressions indicates that the re-structuring of the city size distribution has a greater impact upon the form of the diffusion process than the re-structuring of inter-city interaction.

Given the number of calculations and complex relationships that lie between the setting of the system parameters and the estimation of the regression coefficients, the R^2 values associated with the α and β_1 equations are relatively high (Table 5:5). The smaller R^2 value associated with the β_2 equation is a function of the complexity of the effect that R_i represents, and the fact that it is a residual. However, given the small amount of explanation associated with R_i in the summary regressions on the diffusion process, such a level of explanation is considered adequate.

Consider now the spatial autocorrelation of values of $\bar{t}_i$. Only the first and most important nearest neighbour spatial autocorrelation statistic is discussed (Table 5:6). These results indicate that there exists a substantial amount of spatial autocorrelation within the diffusion process. Moreover, there exists a correlation between the degree of spatial autocorrelation prevalent within a system of cities and the parameters b and q of that system. In fact, the simple correlation between the spatial autocorrelation statistic and b is .739 and that between it and q is -.743. On the basis of earlier results and of knowledge of the diffusion process, such correlations, positive with b and negative with q, seem reasonable but, given the complexity of the diffusion process and the limited amount of information provided by simulation within a single location pattern, it would be unwise to discount the possible existence of other forms of the relationship. Regression of b and q on the nearest neighbour spatial autocorrelation statistics (N_k) produces the equation,

$$\hat{N}_k = 0.3746 + \underset{(.6205)}{.1708b_k} - \underset{(-.6253)}{.1130q_k} \qquad (4)$$

The magnitude of the standardised partial regression coefficients (in parenthesis) demonstrates the almost equal importance of b and q in determining the degree of spatial autocorrelation within these diffusion processes. Again, given the large number of intervening relationships between the setting of b and q and the calculation of the spatial autocorrelation statistic, the R^2 value associated with this

Table 5:6

FIRST NEAREST NEIGHBOUR SPATIAL AUTOCORRELATION STATISTICS
FOR EIGHTEEN SYSTEMS OF CITIES

System No.	$\hat{N}_k$		
1	.4895	10	.5596*
2	.5576*	11	.3533
3	.5927*	12	.4039
4	.6158*	13	.4493
5	.6078*	14	.4950*
6	.4266	15	.5410*
7	.4894	16	.2726
8	.5406*	17	.3697
9	.5769*	18	.4719

Note: * denotes statistical significance at the .05 level.

regression, .9236, is high.

5.2.2 Location

Using the same eighteen combinations of b and q, the model was run for another nineteen location patterns (Appendix 2), making 360 simulations in all.

Consider the modelling framework developed up to this point. It has been demonstrated that

$$\ln \hat{t}_i = \alpha - \beta_1 \ln A_i - \beta_2 R_i \qquad (5)$$

and, on the basis of results from one location pattern, that

$$\hat{\alpha}_k = a^1 + b_1^1 b_k + b_2^1 q_k \qquad (6)$$

$$\hat{\beta}_{1k} = -a^2 - b_1^2 b_k - b_2^2 q_k \qquad (7)$$

$$\hat{\beta}_{2k} = -a^3 - b_1^3 b_k - b_2^3 q_k \qquad (8)$$

Data on α, β_1 and β_2 are given in Appendix 3. Therefore, to establish the link between Location and the form of the diffusion process, it is necessary to be able to estimate the coefficients of equations (6), (7) and (8). Before considering ways of doing this, some discussion of the structure of the α (Table 5:7), β_1 (Table 5:8) and β_2 (Table 5:9) regressions produced for each location pattern is necessary.

The relationships between b and q and the α coefficient are always positive, and always strong; the R^2 value associated with the equation never falling below .9000 (Table 5:7). It is also notable that,

Table 5:7

REGRESSIONS OF b AND q ON a

Loc. Pat. No.	a	b_1(b)	b_2(q)	R^2	Stand.b_1	Stand.b_2	Stand.b_1/Stand.b_2
1	1.81290	1.35190	1.26261	.9684	.62465	.88853	.70301
2	2.19450	1.21265	.96963	.9415	.68205	.83145	.82127
3	1.69847	1.09508	1.47395	.9065	.46759	.95839	.40789
4	1.90703	.74763	1.89153	.9782	.37601	1.44867	.25956
5	2.77428	.95845	1.03518	.9684	.56002	.92238	.60601
6	1.72067	1.11208	1.40997	.9667	.49054	.94710	.51794
7	1.84473	1.56906	1.10912	.9514	.74263	.78932	.94085
8	1.89760	1.10995	1.32938	.9575	.51351	.93657	.54029
9	2.16781	1.17456	1.17153	.9597	.59306	.90076	.65838
10	2.26341	1.13970	1.09965	.9577	.60690	.69167	.66048
11	2.12879	1.16846	1.16059	.9490	.59155	.89474	.66114
12	1.43246	1.38105	1.49469	.9741	.56151	.92555	.60068
13	1.63299	1.02400	1.43071	.9542	.44978	.95695	.47001
14	1.90301	1.24556	1.22435	.9398	.59321	.68795	.66606
15	2.04468	.93962	1.21276	.9027	.46637	.92857	.50679
16	1.69152	1.48392	1.22795	.9480	.67056	.84501	.79357
17	1.64378	1.05270	1.50317	.9604	.44270	.96262	.45969
18	2.97372	1.02065	.58257	.9040	.79666	.69232	1.15072
19	1.78115	1.11387	1.34002	.9685	.51451	.94256	.54586
20	2.24856	1.00933	1.20675	.9385	.50907	.92683	.54925

Table 5:8

REGRESSIONS OF b AND q ON β_1

Loc. Pat. No.	a	b_1(b)	b_2(q)	R^2	Stand. b_1	Stand. b_2	Stand. b_1/Stand. b_2
1	.01091	-.14575	-.21510	.9788	-.43404	-.97546	.44496
2	-.03425	-.13152	-.17933	.9769	-.46362	-.96560	.48014
3	.01259	-.11057	-.23647	.9756	-.30678	-.99910	.30706
4	-.03014	-.07579	-.20532	.9866	-.24516	-1.01136	.24240
5	-.11677	-.08724	-.18352	.9865	-.31374	-1.00503	.31217
6	.02062	-.12191	-.23115	.9741	-.34387	-.99267	.34634
7	.01694	-.17223	-.20844	.9680	-.51163	-.94328	.54261
8	-.00211	-.11577	-.22586	.9747	-.33462	-.99473	.33060
9	-.03542	-.12211	-.20469	.9778	-.38650	-.96075	.39175
10	-.04260	-.12091	-.19557	.9767	-.39919	-.98325	.40599
11	-.02279	-.12690	-.20436	.9699	-.39942	-.97950	.40778
12	.04631	-.14408	-.24394	.9744	-.36233	-.98574	.36786
13	.02373	-.10744	-.23339	.9672	-.30069	-.99532	.30230
14	.00260	-.13434	-.21368	.9659	-.40310	-.97037	.41266
15	-.00123	-.11004	-.21562	.9341	-.32642	-.97399	.33513
16	.02794	-.15657	-.21636	.9672	-.46191	-.96063	.48084
17	.03058	-.11631	-.24300	.9715	-.31310	-.99612	.31432
18	-.16147	-.08566	-.10812	.9272	-.48370	-.92971	.52027
19	.00801	-.11791	-.22249	.9797	-.34644	-.99549	.34801
20	-.05124	-.09755	-.20638	.9600	-.30593	-.99516	.30742

Table 5:9

REGRESSIONS OF b AND q ON β_2

Loc. Pat. No.	a	$b_1(b)$	$b_2(q)$	R^2	Stand. b_1	Stand b_2	Stand. b_1/Stand. b_2
1	-.16275	.00541	-.02206	.5256	.11195	-.69517	-.16105
2	-.13097	-.05736	-.02037	.6528	-.77497	-.41909	1.84916
3	-.13734	.02820	-.05749	.5266	.21108	-.65529	-.32212
4	-.15963	-.01997	-.02164	.0307	-.45156	-.74514	.00601
5	-.25489	-.00867	.03733	.6408	-.11740	.76978	-.15252
6	-.16373	-.00139	-.06366	.7311	-.01626	-.89266	.02046
7	-.20766	.02585	-.02187	.2366	.27522	-.35458	-.77619
8	-.09681	-.01571	-.05666	.6316	-.14237	-.80949	.17587
9	-.23679	-.00635	-.03116	.1769	-.05440	-.40673	.13374
10	-.12148	-.02739	-.06427	.6618	-.20179	-.94543	.21344
11	-.11778	-.03077	-.11359	.9234	-.19061	-1.07155	.17789
12	-.17015	.02257	-.04496	.3792	.18273	-.55429	-.32966
13	-.16038	.02426	-.04417	.7119	.27047	-.74929	-.36096
14	-.12269	-.02238	-.06806	.6037	-.17911	-.82946	.21594
15	-.21363	.06849	-.06418	.5985	.40892	-.58352	-.70078
16	-.16664	.00842	-.03355	.5306	.11502	-.69794	-.16481
17	-.15578	.02151	-.11526	.9440	.11556	-.94296	-.12255
18	-.33262	.10171	.09235	.6070	.56162	.80418	.72324
19	-.17155	.04426	-.00598	.2958	.51378	-.10571	-4.86035
20	-.24293	.07067	-.03369	.8325	.67848	-.49407	-1.37324

except for Location Pattern No. 18, the standardised partial regression coefficient on q is always larger than that on b. This substantiates earlier observations on the comparative importance of b and q in determining the form of the diffusion process. Because it is not possible to compare the standardised partial regression coefficients from one location pattern with those from another, a ratio (b_1/b_2) of the two was calculated for each location pattern (Table 5:7). A value of unity for this ratio indicates a perfect balance between the influence of b and of q, while one of less than unity indicates an imbalance in favour of q. The range of values for the ratio, .25956 for Location Pattern No. 4 to 1.15072 for Location Pattern No. 18, demonstrates the degree of variability in the form of the diffusion process that is attributable to the location pattern.

In the β_1 regressions (Table 5:8), the relationship between b and q and β_1 is always negative. However, since β_1 itself extends over a negative range, this implies that β_1 becomes larger (negatively) as b and q become larger. Standardisation of the partial regression coefficients indicates the pre-eminence of q in determining the size of β_1. The ratios (b_1/b_2) of the standardised partial regression coefficients in these regressions are highly correlated with those in the α regressions, r = .9334. These results indicate the importance of q as a constraint upon the form of the diffusion process.

The structure of the β_2 regressions (Table 5:9) is more complex than that of the α or the β_1 regressions. Firstly, in the α and β_1 regressions, the R^2 value does not fall below .9000 but, in the β_2

regressions, there is a considerable variation, .1769 to .9440, in explanatory power. However, it can be demonstrated that this variation in explanatory power has little effect upon estimates of $\bar{t}_i$. Recall that R_i plays a subordinate role to A_i in the determination of $\bar{t}_i$ (Table 5:4 and Appendix 3) and consider the set of actual and estimated β_2 coefficients for Location Pattern No. 9 (Table 5:10) which has the lowest R^2 value in the set. These data indicate that, in terms of explanation, the contribution of R_i to the summary regressions is generally low. Moreover, the residuals between actual and estimated β_2 are relatively small. Taken together, this means that the possible effects of mis-estimating the β_2 coefficient on the estimation of $\bar{t}_i$ are quite small, even with a regression of low explanatory power such as this.

Of greater interest in connection with the β_2 regressions is the variation in the sign associated with the partial regression coefficients b_1^3 and b_2^3 . Except for Location Patterns 5 and 18, the partial regression coefficient associated with q is always negative. This implies that the magnitude of the (negative) β_2 coefficient increases as q becomes larger. However, the small size of these negative coefficients means that the increase in the size of β_2 caused by an increase in q is quite small. Indeed, data on the structure of the summary regression equations (Table 5:4 and Appendix 3) indicate that, relative to A_i, R_i's role in the determination of $\bar{t}_i$ tends to decrease in importance as q becomes larger. Consequently, the positive partial regression coefficients in the regressions for Location Patterns

Table 5:10

LOCATION PATTERN NO. 9: A COMPARISON OF ACTUAL AND ESTIMATED β_2 COEFFICIENTS

Simulation No.	β_2	$\hat{\beta}_2$	$\beta_2 - \hat{\beta}_2$	Explanation Added by R_i
1	-.1913	-.2295	.0382	.0294
2	-.2148	-.2311	.0163	.0408
3	-.2500	-.2327	-.0173	.0542
4	-.2747	-.2343	-.0404	.0585
5	-.3059	-.2359	-.0700	.0626
6	-.1851	-.2140	.0289	.0303
7	-.1962	-.2155	.0193	.0370
8	-.2067	-.2171	.0104	.0406
9	-.2171	-.2187	.0016	.0414
10	-.2233	-.2203	-.0030	.0395
11	-.2158	-.1984	-.0.74	.0194
12	-.1903	-.2000	.0097	.0167
13	-.1711	-.2015	.0304	.0141
14	-.1591	-.2031	.0440	.0119
15	-.1652	-.2047	.0395	.0125
16	-.2720	-.1828	.0892	.0141
17	-.2121	-.1844	.0277	.0101
18	-.1586	-.1859	.0273	.0068

5 and 18 indicate a more pronounced decline in R_i's role within the diffusion process as q becomes larger and nothing more. Thus, in terms of the relative importance of A_i and R_i, Location Pattern No. 18 is the most responsive to change in q, while Location Pattern No. 17 is the least responsive. The partial regression coefficients associated with b in the β_2 regressions display even greater variation than those associated with q, although their generally small size makes this variation of less importance than it might otherwise be.

Both the variation in sign and the smallness of the coefficients in the β_2' regressions might be the result of the complexity of the relationship between R_i and b and q. The complexity of R_i's relationship with the system parameters makes it difficult to both estimate and interpret, although it is interesting to note that, within some location patterns, the dominance of either b or q forces some uniformity (as indicated by the size of R^2 value) upon the behaviour of the β_2 coefficient.

Methods by which the coefficients of equations (6), (7) and(8), and thus the effect of Location upon the diffusion process, can be estimated will now be discussed. Two point pattern measures - first order nearest neighbour distance and spatial entropy (Curry, 1972) - were calculated for each location pattern (Table 5:11), but were found to be poor predictors of the coefficients of equations (6), (7) and (8). Therefore, another approach to explaining inter-location pattern variation in the form of the diffusion process had to be utilised.

There exists a distinct pattern in the response of the system summary measures on information potential to changes in b and q (section

Table 5:11

NEAREST NEIGHBOUR AND SPATIAL ENTROPY SCORES
FOR TWENTY LOCATION PATTERNS

	First order Nearest Neighbour			
Loc. Pat. No.	X_{13} Mean	X_{14} Variance	X_{15} Coefficient of Variation	X_{16} Spatial Entropy
1	11.15	162.20	14.54	3.1295
2	14.71	298.39	20.28	3.3159
3	14.92	267.67	17.93	3.1802
4	5.06	30.20	5.96	3.1466
5	3.78	17.62	4.65	3.2351
6	12.53	195.27	15.58	3.2035
7	14.31	238.89	16.69	3.2818
8	12.34	171.25	13.87	3.1673
9	14.00	227.94	16.27	3.2237
10	13.23	229.27	17.33	3.2394
11	10.19	110.05	10.80	2.9910
12	14.11	234.29	16.60	3.2229
13	11.43	155.47	13.50	3.2302
14	15.87	283.03	17.83	3.2271
15	12.25	164.45	13.42	3.1298
16	11.14	141.16	12.67	3.2237
17	12.55	166.55	13.27	3.2635
18	10.31	133.14	14.85	3.0849
19	12.03	186.01	15.45	3.0550
20	12.51	205.34	16.41	2.9410

Note: The coefficient of variation is defined as variance/mean

5.2.1), and this pattern of response varies from location pattern to location pattern (see Appendix 3). Taken together, differences in these responses may help to explain inter-location pattern variation in the coefficients of equations (6), (7) and (8). In other words, being a function of Location, inter-location pattern variation among the simple correlations between the summary measures on A_i and B_i and b and q (Table 5:12) is considered a surrogate for that variable. However, there exists a considerable degree of correlation within the set of simple correlation variables (Table 5:13). For example, the simple correlation between q and mean B_i is highly correlated with that between q and mean A_i. Consequently, use of these simple correlations as explanatory variables in a regression model might give rise to the sort of multicollinearity problem discussed in section 5.2.1. Moreover, since variation in the simple correlation variables is partially determined by the point pattern characteristics of the location pattern, the simple correlation variables are also correlated with the point pattern measures, nearest neighbour and spatial entropy (Table 5:14)

These data provide some insight into the nature of inter-location pattern variation in the simple correlation variables. For example, the correlations for X_{13} (mean first order nearest neighbour distance) with the simple correlation variables are markedly higher than those for X_{16} (spatial entropy), which reflects the importance of distance relationships in determining the form of the diffusion process.

The high level of intercorrelation among the explanatory variables evident in Tables 5:13 and 5:14 means that the development of

Table 5:12

THE SIMPLE CORRELATION BETWEEN THE SYSTEM SUMMARY MEASURES ON INFORMATION POTENTIAL AND b AND q

Loc. Pat. No.	X_1 b:Mean B_i	X_2 b:Var. B_i	X_3 b:C. of V. B_i	X_4 q:Mean B_i	X_5 q:Var. B_i	X_6 q:C. of V. B_i
1	-.173	-.398	-.313	-.910	-.453	.789
2	-.194	-.352	-.180	-.902	.992	.988
3	-.266	-.744	-.621	-.857	-.192	.690
4	-.029	-.359	-.269	-.940	.959	.982
5	.489	.392	.152	-.889	.787	.927
6	-.263	-.644	-.364	-.867	.758	.963
7	-.216	-.178	-.216	-.880	-.660	.657
8	-.188	-.609	-.400	-.897	-.119	.852
9	-.160	.617	.506	-.916	.224	.598
10	-.178	-.377	-.144	-.910	.769	.980
11	-.127	-.048	-.031	-.924	.844	.965
12	-.273	-.730	-.570	-.853	-.074	.740
13	-.188	-.628	-.514	-.900	-.034	.730
14	-.260	.085	.292	-.875	.304	.772
15	-.105	.749	.624	-.943	-.079	.430
16	-.172	-.232	-.198	-.906	-.660	.685
17	-.247	-.809	-.154	-.875	.507	.885
18	.080	.128	.117	.698	.885	.919
19	-.133	.294	.284	-.924	-.118	.750
20	.062	.819	.764	-.979	-.179	.142

Table 5:12 (Cont'd)

X_7 b:Mean A_i	X_8 b:Var. A_i	X_9 b:C. of V. A_i	X_{10} q:Mean A_i	X_{11} q:Var. A_i	X_{12} q:C. of V. A_i
-.014	-.104	-.733	-.911	-.905	.153
-.029	-.042	-.488	-.910	-.929	.233
-.081	-.129	-.678	-.889	-.877	.115
.082	-.139	-.582	-.926	-.866	.811
.142	.289	.073	-.938	-.892	.242
-.051	-.444	-.696	-.899	-.780	.678
-.068	-.147	-.725	-.894	-.870	.038
-.044	-.239	-.809	-.901	-.869	.317
-.055	-.041	-.098	-.902	-.945	.232
-.030	-.109	-.581	-.907	-.926	.364
-.011	-.176	-.580	-.910	-.907	.700
-.084	-.223	-.728	-.886	-.841	.412
-.033	-.312	-.803	-.903	-.840	.423
-.087	-.235	-.517	-.819	-.897	.390
-.027	-.041	-.457	-.909	-.932	-.007
-.044	-.264	-.745	-.900	-.865	.314
-.051	-.410	-.774	-.899	-.801	.572
.074	.363	.439	-.921	-.957	.679
-.008	-.110	-.726	-.912	-.909	.221
.017	.244	.432	-.920	-.894	-.455

Table 5:13

INTERCORRELATION AMONG THE SIMPLE CORRELATION VARIABLES

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
X_1	1.0	.543	.435	.246	.307	-.017	.913	.783	.700	-.977	-.313	-.121
X_2		1.0	.976	.070	-.032	-.567	.379	.671	.696	-.493	-.646	-.496
X_3			1.0	.055	.022	-.558	.289	.593	.675	-.426	-.620	-.775
X_4				1.0	.282	.202	.337	.493	.524	.213	.328	.318
X_5					1.0	.607	.468	.179	.296	-.452	-.111	.608
X_6						1.0	.198	-.246	-.323	-.107	.141	.820
X_7							1.0	.689	.596	-.962	-.276	.137
X_8								1.0	.882	-.691	-.703	-.368
X_9									1.0	-.599	-.519	-.217
X_{10}										1.0	.371	-.006
X_{11}											1.0	.282
X_{12}												1.0

Note: All variables are defined in Table 5:12

Table 5:14

CORRELATION BETWEEN SIMPLE CORRELATION VARIABLES AND POINT PATTERN MEASURES

	X_{13}	X_{14}	X_{15}	X_{16}
X_1	-.777	-.666	-.658	-.264
X_2	-.157	-.130	-.119	-.470
X_3	-.044	-.022	-.007	-.476
X_4	-.101	-.066	.047	-.163
X_5	-.359	-.246	-.255	.041
X_6	-.294	-.233	-.243	.384
X_7	-.893	-.780	-.732	-.317
X_8	-.355	-.209	-.159	-.386
X_9	-.258	-.150	-.101	-.411
X_{10}	.821	.711	.649	.380
X_{11}	-.013	-.101	-.142	.271
X_{12}	-.317	-.355	-.350	.179

Note: See Tables 5:11 and 5:12 for definitions of variables.

regression models to estimate the coefficients of equations (6),(7) and (8) is not a simple procedure. To alleviate the problem of multicollinearity, it is necessary to reduce the number of independent variables in some way. Such a reduction can be accomplished in a number of ways. For example, the artificial orthogonalisation (Riddell, 1970) or parsimony (Johnston, 1963) solutions could be utilised. However, the use of either of these methods creates fresh interpretation problems and the solution to the problem of multicollinearity adopted in this part of the analysis is a very pragmatic one.

Using selected combinations from the sixteen independent variables, satisfactory estimating equations can be produced within which all partial correlation coefficients are statistically significant (Table 5:15). However, because their composition cannot be considered unique (a similar level of explanation might be obtained with an alternate mix of independent variables); their interpretation is not straightforward.

The R^2 value associated with each of the nine estimating equations is satisfactorily high, particularly since each dependent variable is itself an estimate. However, before making some general comments in connection with the structure of the models, it is useful to place these results in context. It has been shown that

$$\ln \bar{t}_i = \alpha - \beta_1 \ln A_i + \beta_2 R_i \qquad \text{(Equation 5)}$$

and that, for each location pattern, variation in α, β_1 and β_2, which

Table 5:15

ESTIMATING EQUATIONS FOR THE COEFFICIENTS OF EQUATIONS (6), (7) AND (8)

	Equation	R^2
α	$\hat{a}^1 = 2.0977 + .25117X_5 + 1.4456X_8$	(.8745)
	$\hat{b}_1^1 = 11.39150 - .22532X_4 - .39713X_5 + 1.08915X_6 + .52479X_9 + 12.05464X_{10}$	(.7144)
	$\hat{b}_2^1 = 2.62814 - .45607X_4 - .62614X_6 + 1.65869X_{11} + .63751X_{12}$	(.6932)
β_1	$\hat{a}^2 = -.01896 - .25022X_8 - .07493X_{12}$	(.9216)
	$\hat{b}_1^2 = -.05756 - .01934X_2 + .02864X_5 - .08539X_6 + .29633X_7$	(.7301)
	$\hat{b}_2^2 = -1.01069 + .04991X_4 - .71428X_{10} - .21982X_{11}$	(.8696)
β_2	$\hat{a}^3 = -.33020 - .06091X_7 + .03110X_5 - .09158X_9 + .00347X_{15}$	(.7436)
	$\hat{b}_1^3 = .39988 + .08359X_4 - .14002X_6 + .2388X_{11}$	(.8135)
	$\hat{b}_2^3 = -.37972 + .06327X_4 - .02270X_5 + .11647X_8 - .00474X_{13} + .14735X_{10}$	(.7555)

Note: All partial regression coefficients are significant at the .05 level.

summarise the form of the diffusion process, can be linked to changes in the system parameters, b and q,

$$\hat{\alpha}_k = +a^1 + b_1^1 b_k + b_2^1 q_k \qquad \text{(Equation 6)}$$

$$\hat{\beta}_{1_k} = \pm a^2 - b_1^2 b_k - b_2^2 q_k \qquad \text{(Equation 7)}$$

$$\hat{\beta}_{2_k} = -a^3 \pm b_1^3 b_k \pm b_2^3 q_k \qquad \text{(Equation 8)}$$

In turn, the coefficients of these equations can be estimated using the equations presented in Table 5:15. To establish the connection between the relationships evident within these estimating equations and the nature of the diffusion process, it is necessary to refer back, through the second level estimating equations (6,7 and 8), to the original summary equation (Equation 5). For example, consider the case of variable X_4 in the b_2^2 equation. The partial regression coefficient associated with X_4 (the simple correlation between mean B_i and q) in that equation is a positive one. This implies that b_2^2 is larger in location patterns in which mean B_i is more strongly correlated with q. In turn, this implies that the importance of q in determining β_1 and, consequently, the structure of the diffusion process, is greater in location patterns in which variation in mean B_i is more closely tied to variation in q. Such a result is in not counter - intuitive.

Thus, although inter-location pattern variation in the form of the diffusion process, as reflected in the variation of the coefficients of equations (6), (7) and (8), is complex, it can be identified and estimated. The fact that this identification and estimation is made possible only by the use of surrogate variables for Location constitutes a result of some importance - to estimate the role of inter-location pattern variation in a process as complex as inter-city diffusion, complex measures upon the system of cities in which diffusion occurs have to be used. The simple correlation variables used as a surrogate for Location within this analysis are essentially measures upon the information exchange characteristics of a given location pattern under different city size and distance decay conditions. The further development of such measures may provide a useful basis for research into the role of Location within interaction processes.

Some further discussion of spatial autocorrelation is necessary. In section 5.2.1, it was shown that city adoption times are spatially autocorrelated and that, given a fixed location pattern, the level of spatial autocorrelation is dependent upon the magnitude of the parameters b and q (Table 5:6). Examination of data for the other nineteen location patterns (Appendix 4) indicates that inter-location pattern variation in the relationship between the spatial autocorrelation of $\bar{t}_i$ and b and q is pronounced. Regression is used to characterise this relationship and to demonstrate the degree of inter-location pattern variation (Table 5:16).

The R^2 values associated with these regressions are generally high, although those for Location Patterns 5 and 18 are conspicuously low. Reference to the appropriate Tables (Appendix 4) reveals that, in Location Patterns 5 and 18, the relationship between the spatial autocorrelation of $\bar{t}_i$ and the distance decay parameter is irregular in form; that is, it increases and decreases within the same q set, and varies between q sets. This is symptomatic of the type of complex relationship between system parameters evident at other points in this analysis. The fact that such complexity is not particularly noticeable in the other eighteen location patterns is attributable to the existence of a basic structural relationship between the level of spatial autocorrelation prevalent within a diffusion process and the parameters of the system in which it occurs. The form of this basic relationship, a positive correlation with b and a negative correlation with q, is apparent from the partial regression coefficients presented in Table 5:16.

Considering the importance of distance relationships in the determination of inter-city interaction within systems in which b is high, it is not surprising that $\bar{t}_i$ becomes more spatially autocorrelated as the distance decay parameter increases. The negative correlation between the level of spatial autocorrelation and q demonstrates once again that the constraint imposed upon the form of the diffusion process by the city size distribution becomes more pronounced as the distribution becomes more polarised. Spatial autocorrelation is decreased because, as q is increased, the diffusion process becomes more hierarchical in form and less sensitive to locational factors.

5.3 The Link Between $\bar{t}_i$ and s_i^2

Throughout this investigation of the properties of the diffusion model, $\bar{t}_i$ has been assumed to be an adequate summary measure on the distribution of city adoption probabilities. Particularly important in this respect is the fact that s_i^2 was seen to be highly correlated with $\bar{t}_i$ on numerous occasions (for example, see Table 4:3). However, although the $\bar{t}_i : s_i^2$ relationship is always of the general form

Table 5:16

REGRESSIONS OF b AND q ON THE FIRST NEAREST NEIGHBOUR AUTOCORRELATION STATISTICS FOR TWENTY LOCATION PATTERNS

Loc. Pat. No.	R^2	a	b_1(b)	b_2(q)
1	.9236	.37465	.17077	-.11301
2	.9366	.56325	.09267	-.27870
3	.8918	-.19292	.40285	-.17295
4	.6349	-.35598	.32477	.02624
5	.3205	.83607	.06729	-.00919
6	.8505	-.11304	.31434	-.06186
7	.8957	.49591	.23182	.01302
8	.8993	.22802	.43318	-.23997
9	.9265	.56272	.15096	-.11808
10	.6679	.56351	.09199	-.19198
11	.9414	.25872	.34482	-.28275
12	.9895	-.19706	.60251	-.35146
13	.9743	-.15306	.46448	-.16435
14	.9914	.66085	.15210	-.29132
15	.9745	.45525	.19483	-.23612
16	.9654	.26759	.36838	-.22224
17	.9011	-.18228	.37037	.04079
18	.2076	.78123	-.11288	.00616
19	.9739	.12479	.31684	-.25428
20	.8199	.56242	.08646	-.04190

$$\ln s_i^2 = \gamma + \delta \ln \bar{t}_i \quad \text{(Equation 4.20a)}$$

there exist variations in its precise form within, and between, location patterns (Appendix 4). First, consider the pattern of variation within the $\bar{t}_i : s_i^2$ regressions for Location Pattern No. 1 (Table 5:17).

The generally high level of explanation (R^2) associated with these regressions justifies the assumption that $\bar{t}_i$ alone can be considered an adequate measure on a city's distribution of adoption probabilities. However, at the same time, it is evident that the strength of the relationship between $\bar{t}_i$ and s_i^2 varies from system to system within the same location pattern. This variation occurs because the effect of the probability distribution anomaly (see section 5.1.1) is greater within less polarised city size distributions, in which the range of $\bar{t}_i$ is relatively small, than within more polarised city size distributions, in which the range of $\bar{t}_i$ is relatively large. Of considerably greater interest is the evident variation in the relationship between the coefficients of equation (4.20a) and the system parameters, b and q.

It is apparent from Table 5:17 that δ and γ are both sensitive to changes in b and q. However, although within the q = 0.5 and q = 1.0 systems, δ and γ both increase in size as b and q increase, within the q = 1.5 and q = 2.0 systems, they both decrease in size as b and q increase. The reason for this change of behaviour is technical in nature and can be traced to the city adoption probability distributions from which $\bar{t}_i$ and s_i^2 are derived.

The effect of the adoption probability distribution anomaly on

Table 5:17

REGRESSIONS OF $\ln\bar{t}_i$ ON $\ln s_i^2$ FOR LOCATION PATTERN NO. 1.

Simulation No.	γ	δ	r^2
1	-1.9510	1.5834	.8465
2	-2.3065	1.9802	.6805
3	-2.7321	2.2994	.9033
4	-3.0176	2.5545	.9273
5	-3.2288	2.7303	.9470
6	-3.0390	2.6245	.8857
7	-3.1634	2.7728	.9112
8	-3.1919	2.8364	.9348
9	-3.2416	2.8856	.9536
10	-3.2103	2.8703	.9638
11	-3.3232	3.0968	.9526
12	-3.2448	3.0322	.9582
13	-3.0735	2.9223	.9671
14	-2.9056	2.8178	.9743
15	-2.7517	2.7252	.9790
16	-2.7852	2.6115	.9603
17	-2.5902	2.7046	.9839
18	-2.4095	2.6152	.9861

s_i^2 values is a function of the difference between the relevant city adoption probabilities in the first, and those in subsequent, time periods (section 5.1.1). Within the q = 0.5 and q = 1.0 systems, the city size distribution is such that invention probabilities are relatively well dispersed and, consequently, the anomaly does not have a strong effect on the structure of the $\bar{t}_i : s_i^2$ relationship. Within the q = 0.5 and q = 1.0 systems, the variance, as well as the mean, of adoption times for large cities decreases as the proportion of total information potential concentrated in certain (early adopter) cities increases; large $\bar{t}_i$s are associated with proportionately larger s_i^2s and small $\bar{t}_i$s with proportionately smaller s_i^2s, On the other hand, within the q = 1.5 and q = 2.0 systems, the distribution of city sizes and invention probabilities is such that the effect of the probability distribution anomaly on the form of the $\bar{t}_i$: s_i^2 relationship is stronger. Thus, although the ratio between the $\bar{t}_i$ and s_i^2 values of low potential cities is still increasing as the proportion of total information potential concentrated in certain (early adopter) cities increases, the ratio between the $\bar{t}_i$ and s_i^2 values for high potential cities is not decreasing but increasing because the probability distribution anomaly inflates the s_i^2 values of large cities. Therefore, it is variation in the effect of the adoption probability distribution anomaly that gives rise to the dichotomy in the relationship between $\bar{t}_i$ and s_i^2 and b and q.

The generality of this dichotomy in the structure of the $\bar{t}_i$: s_i^2 relationship is verified on the inspection of data on δ and γ for the other nineteen location patterns (Appendix 4). Within each of these location patterns, the relationship between δ and γ and b and q can be estimated through regression. Since, within each location pattern, the response of δ and γ to changes in b and q occurs in two distinct phases, the estimation procedure is undertaken twice, once for the q = 0.5 and q = 1.0 systems and once again for the q = 1.5 and q = 2.0 systems (Tables 5:18 to 5:21). These regressions indicate that, for the q = 0.5 and q = 1.0 systems,

$$\hat{\gamma}_{1_k} = a^1 - b_1^1 b_k - b_2^1 q_k \quad (9)$$

$$\hat{\delta}_{1_k} = a^2 + b_1^2 b_k + b_2^2 q_k \quad (10)$$

and that, for the q = 1.5 and q = 2.0 systems,

$$\hat{\gamma}_{2_k} = a^3 + b_1^3 b_k + b_2^3 q_k \quad (11)$$

$$\hat{\delta}_{2_k} = a^4 - b_1^4 b_k - b_2^4 q_k \quad (12)$$

Given a fixed location pattern (Figure 4:1), equations (9), (10), (11) and (12) can be used to estimate the coefficients γ and δ of the $\bar{t}_i : s_i^2$ relationship, and thereby constitute an explanation of variation in the form of that relationship within Location Pattern No. 1. To establish the nature of inter-location pattern variation in the form of the relationship (see Appendix 4), it is necessary to be able to estimate the coefficients of equations (9), (10), (11) and (12). The procedure used to develop these estimating equations (Table 5:22) is

Table 5:18

$\bar{t}_i$: s_i^2: REGRESSION OF b AND q ON γ (q = .5 and q = 1.0)

Loc. Pat. No.	R^2	a	$b_1(b)$	$b_2(q)$
1	.7600	-1.07417	-.72171	-1.01253
2	.4833	-2.38462	-.46542	-.15538
3	.6843	-.60414	-1.10686	-.37376
4	.8256	.08541	-1.55274	-.47043
5	.7807	-2.08125	-.41375	-.52803
6	.7308	-.71393	-.96533	-.61349
7	.6006	-1.59521	-.50130	-.65451
8	.6714	-1.03381	-.74274	-.88376
9	.8576	-.97800	-.82765	-.92552
10	.6557	-1.76191	-.53960	-.42501
11	.8248	-1.06778	-.65690	-.90122
12	.5842	-1.07061	-.96531	.18864
13	.8110	-.02132	-1.03428	-.99434
14	.8738	-1.06393	-.58641	-.77624
15	.9624	-.36951	-1.28656	-.84398
16	.7375	-.63507	-.75990	-.83922
17	.8829	-.43527	-.53027	-1.51559
18	.5228	-2.22657	-.49584	-.14345
19	.7981	-.77321	-1.03669	-.92069
20	.8717	-1.18405	-.82392	-1.03688

Table 5:19

$\overline{t}_i : s_i^2$: REGRESSION OF b AND q ON δ (q = .5 AND q = 1.0)

Loc. Pat. No.	R^2	a	$b_1(b)$	$b_2(q)$
1	.8352	.61474	.69683	1.13912
2	.7191	1.60251	.49937	.00646
3	.8007	.14971	1.04803	.67748
4	.8394	-.29612	1.37719	.44966
5	.7738	1.62923	.36925	.47154
6	.8239	.25086	.92601	.81962
7	.7685	1.01774	.55119	.81790
8	.7752	.58579	.72063	.98998
9	.8833	.58953	.74128	1.04545
10	.8152	1.11137	.56408	.70006
11	.8927	.58383	.63936	1.02367
12	.7572	.44641	.98199	.23929
13	.8876	-.33449	1.01497	1.11285
14	.9494	.44853	.66710	1.03671
15	.9667	.06747	1.11363	.99215
16	.8509	.17896	.76924	1.02733
17	.9330	-.03041	.61272	1.55542
18	.6029	1.72383	.42101	.35974
19	.8373	.38891	.94238	1.00657
20	.8740	.84230	.71641	1.03639

Table 5:20

$\overline{t}_i$: s_i^2 : REGRESSION OF b AND q ON γ (q = 1.5 AND q = 2.0)

Loc. Pat. No.	R^2	a	$b_1(b)$	$b_2(q)$
1	.9922	-5.84735	.61923	1.23915
2	.9891	-5.24651	.42498	.98653
3	.9932	-5.68108	.94742	1.09032
4	.9631	-5.15534	.39551	.90824
5	.7845	-5.14351	.04547	.92093
6	.9941	-6.35750	.97543	1.35737
7	.9902	-5.75133	.78006	1.27227
8	.9880	-5.24589	.67824	1.00600
9	.9955	-5.53626	.68265	1.01788
10	.9667	-4.65987	.38562	.73774
11	.9728	-5.95048	.47410	1.46455
12	.9895	-5.67245	1.20096	.98115
13	.9707	-5.75250	.89487	1.16066
14	.9963	-5.26816	.70502	.83427
15	.9847	-5.68150	.37636	1.23500
16	.9890	-5.56331	.84079	1.10788
17	.9868	-5.78645	.43812	1.07797
18	.9068	-4.37404	.29311	.78966
19	.9076	-5.08468	.19252	1.06683
20	.9014	-6.23005	.42464	1.46556

Table 5:21

$\bar{t}_i$: s_i^2 : REGRESSION OF b AND q ON δ (q = 1.5 AND q = 2.0)

Loc. Pat. No.	R^2	a	$b_1(b)$	$b_2(q)$
1	.9969	4.40960	-.38466	-.60917
2	.9887	4.18963	-.30916	-.50639
3	.9956	4.04255	-.48123	-.42220
4	.9423	3.72786	-.33734	-.23998
5	.8224	4.14404	-.29020	-.45495
6	.9657	4.54385	-.52297	-.62553
7	.9926	3.98510	-.41825	-.44502
8	.9752	3.83239	-.39202	-.36457
9	.9865	4.25940	-.43154	-.50691
10	.9520	3.79735	-.28034	-.34905
11	.9762	4.17316	-.29059	-.58376
12	.9921	3.86201	-.57949	-.28839
13	.9815	4.02367	-.47215	-.40056
14	.9939	4.19695	-.42289	-.44819
15	.9840	4.46361	-.26930	-.65691
16	.9943	3.98461	-.45811	-.40982
17	.9675	4.22599	-.51258	-.45550
18	.8714	3.44587	-.27753	-.24531
19	.9383	4.02446	-.19309	-.50534
20	.9180	4.81196	-.36518	-.80398

Table 5:22

ESTIMATING EQUATIONS FOR THE COEFFICIENTS OF EQUATIONS (9), (10) (11) AND (12)

(q = .5 and 1.0)

	$\hat{a}^1 = 5.64988 - 1.53186X_6 - 2.82967X_8 - .08345X_{15} - 1.46019X_{16}$	(R^2 = .7763)
γ_1	$\hat{b}_1^1 = -1.38108 + 2.93371X_6 - 9.75000X_7 + 1.14325X_9 - 1.52750X_{12} - .00457X_{14}$	(R^2 = .6720)
	$\hat{b}_2^1 = 54.74935 - .78716X_4 + 3.00140X_8 + 60.49309X_{10} + 1.16259X_{12} - .41463X_{13} + .25365X_{15}$	(R^2 = .7826)
	$\hat{a}^2 = -2.51601 + 2.82859X_1 + 1.72153X_6 + 1.54356X_8 + .19287X_{13}$	(R^2 = .8539)
δ_1	$\hat{b}_1^2 = 3.22332 - 2.38646X_6 - .66538X_9 + 1.26780X_{12} - .25313X_{13} + .009202X_{14}$	(R^2 = .7939)
	$\hat{b}_2^2 = -62.35970 - .40305X_3 - .37243X_5 - 2.14064X_8 + .60770X_9 - 67.19746X_{10} + .47196X_{13} - .22361X_{15}$	(R^2 = .8387)

Table 5:22 (cont'd)

(q = 1.5 and 2.0)

	$\hat{a}^3 = -11.00288 + .37539X_4 + 1.03247X_6 - 5.66613X_{11}$	$(R^2 = .6442)$
γ_2	$\hat{b}_1^3 = 17.75411 + 16.25613X_{10} + 2.71811X_{12}$	$(R^2 = .8757)$
	$\hat{b}_2^3 = -22.52987 + 1.09325X_2 - 1.29296X_3 - 8.65614X_7 - 33.41165X_{10} + 3.22913X_{11} - 1.23541X_{16}$	$(R^2 = .8286)$
	$\hat{a}^4 = 4.49886 - .42856X_4 + .22585X_5 - 1.02915X_6$	$(R^2 = .5641)$
δ_2	$\hat{b}_1^4 = -6.60992 - .10322X_9 - 5.32356X_{10} - 1.52498X_{11}$	$(R^2 = .8188)$
	$\hat{b}_2^4 = -2.84158X_5 + .42325X_8 + .52856X_{12} + .72043X_{16}$	$(R^2 = .6835)$

Note: All partial regression coefficients are significant at the .05 level.

identical to that outlined earlier in connection with the summary regressions on the diffusion process (section 5.2.2). The ability to estimate these coefficients and, through them, the form of the $\bar{t}_i : s_i^2$ relationship, serves to reinforce earlier conclusions (section 5.2.2) regarding the predictability of the diffusion process

5.4 Summary and Conclusions

By allowing the distribution of city sizes and the location of cities, as well as the structure of inter-city interaction, to vary, the properties of a spatial diffusion model have been investigated in a manner more comprehensive than any undertaken elsewhere. Although qualitative comparison of the diffusion processes that occur within different systems of cities provided a useful base for further analysis (section 5.1), the major part of this investigation was dependent upon regression, both to summarise the form of the diffusion process and to establish the link between this form and the parameters of the system of cities in which diffusion occurs. The results produced by this analysis indicate that the form of the diffusion process is more sensitive to the distribution of city sizes than it is to the structure of inter-city interaction. If early adoption is equated with growth or with faster growth (which will be discussed in Chapter 6), this result is in accord with that produced by the cumulative causation and growth centre models (Chapter 2), that city size and its distribution plays an important part in determining the pattern of growth. Hierarchical effects dominate the diffusion process, even within less polarised city size distributions. Although a high frictional effect of distance on information flows increases the information potential of some fortunately-located small cities (for example, see Table 4:1), it has the overall effect of decreasing the flow of information out of large cities, which are the major sources of information about innovations, thereby increasing the range of adoption times for most cities.

The relationship between the form of the diffusion process and the city size distribution and the structure of inter-city interaction was found to be dependent upon the location of cities (section 5.2.2). In essence, the location of cities determines the way in which their adoption behaviour responds to change in the structure of inter-city interaction. The effect of Location upon the form of the diffusion process is thus intertwined with that of b and q, as well as vice versa. The complexity of the relationship between Location and the form of the diffusion process is reflected in the need to use complex surrogate variables to explain inter-location pattern variation in the form of the diffusion process.

As well as producing insight into the nature of the relationship between the form of the diffusion process and the parameters of the system of cities in which it occurs, this analysis permits estimation of city adoption times without actually running the diffusion model. Given the general nature of the diffusion process modelled, these equations represent a means of characterising the inter-city diffusion process within any system of (twenty) cities and they will be used to provide input to a growth model in the next chapter. The ability to develop equations such as these demonstrates that it is possible to comprehensively characterise the nature of interaction - based processes within systems of cities.

It should be noted that formal questions of statistical inference have not been posed consistently during this investigation of the properties of the diffusion model because analysis is either carried out on a population - for example, the regression of city information potentials (A_i and B_i) on city mean adoption time ($\bar{t}_i$) - or it is carried out in a context in which prediction and not inference is the primary concern - for example, the regression of b and q on the coefficients of equation 5. Consequently, significance tests are performed only at certain points in the analysis, such as the estimation of spatial autocorrelation within the diffusion process, where the identification of significant or non-significant statistics aids in interpretation.

CHAPTER 6

A SIMPLE GROWTH MODEL

6.1 The Antecedents

Two previous attempts to model the link between innovation diffusion and growth, those of Pedersen (1970) and Robson (1973), will be examined. First, consider the work of Pedersen (1970). Pedersen (1970) proposed that

> "When a town adopts the innovation, it starts to grow at a fixed rate (or increases its growth rate), Given two towns originally of the same size, the one that adopts first will thus achieve the greatest size." (Pedersen, 1970, p.227)

However, Pedersen's results are based upon the diffusion of a single innovation and his proposal for linking diffusion and growth is more suited to his diffusion model than to the more complex one outlined in Chapter 4. Pedersen's consideration of only a single innovation means that estimated growth rates are dependent solely upon the adoption time for that one innovation. Since empirical evidence (Fred, 1966: Robson, 1973: Lasuén, 1973) indicates that innovations do not diffuse singly, the growth patterns generated by Pedersen's model are of limited generality. However, Robson's (1973) model does attempt to cater for the diffusion of more than a single innovation and his proposals for linking innovation diffusion and growth are worthy of close scrutiny.

Robson (1973) experimented with several procedures for estimating the growth that results from adoption of a composite innovation (see chapter 3, section 3.2.2 for a discussion of Robson's diffusion model). For example, in one formulation, it was proposed that the growth generated in town j on reception of a message concerning the composite innovation is $\sqrt[4]{P_j}$, where P_j is the population of city j, in the time period in which the message is received and $\sqrt[4]{P_j}/2$ in the following time period. Thus, for any given time period, the growth of city j is given by

$$N_j^t \sqrt[4]{P_j^t} + N_j^{t-1} \sqrt[4]{P_j^{t-1}}/2 + \sqrt[4]{P_j^t} \qquad (1)$$

where N_j^t is the number of messages received by city j in time period t and N_j^{t-1} is the number received by that city in the previous time period. Robson proposed that the inclusion of the $\sqrt[4]{P_j^t}$ term in each time period, regardless of the number of messages received, represents self-generated growth within the city.

Through numerical simulation within a k = 7 central place system, Robson concluded that the size - dependent formulation embodied in equation (1) over-emphasises the role of city size in the growth process and proposed instead that each message yields a fixed increment of population to the receiving city:

> "This increment would then represent say, the addition of a factory or a unit of tertiary employment which would be the same size irrespective of the size of town to which it was added." (Robson, 1973, p. 195)

By assuming that each message generates an additional population of 100 for the receiving city in the time period in which it is received, equation (1) is replaced by

$$N_j^t 100 + N_j^{t-1} 50 + 100 \qquad (2)$$

This formulation was used as the basis for numerous simulations of the growth process, again within a k = 7 central place system. By varying the simulation rules (for example, adoption thresholds and the frictional effect of distance on information flows), Robson was able to produce growth rates similar to those for nineteenth century Britain.

Robson's (1973) method for linking the results of the diffusion process and growth is more sophisticated than Pedersen's but the basic error in his diffusion model, that is, the failure to separate the adoption of earlier innovations from that of later innovations (see Chapter 3, section 3.2.2), reduces the credibility of his results. Nevertheless, it is useful to consider some of the implications of Robson's method for linking the results of the diffusion process and growth. Two alternative frameworks are proposed: firstly, one in which the growth resulting from the reception of a stimulus is proportional to the size of the receiving city (equation 1); and, secondly, one in which it is uniform for all cities regardless of their size (equation 2). Note that the inclusion of a constant to represent self-generating growth serves to distort the results produced by both frameworks. On examination of the growth patterns produced by a simulation model incorporating equation (1), Robson concluded that the size - dependent procedure over-emphasises the role of city size in the growth process. This conclusion is only partly correct.

Assume that innovations in the secondary and tertiary sectors of industry are being considered. An innovation in the tertiary sector, such as a Planned Regional Shopping Centre, will generate employment in relation to its own size, and the size and number of such facilities located in a city has been seen to be related to city size (Cohen, 1972: Sheppard, 1974). In general, it is reasonable to assume that the employment increase generated by the adoption of an innovation in the tertiary sector is related to the size of the adopting city. This in itself is considerable evidence for the importance of city size in determining growth increments and a similar case can be made in connection with innovations in the secondary sector.

In the theoretical context, Richardson (1973a) points out that agglomeration economies cause the optimum size of a factory to be correlated with city size, and, in the empirical context, Robson (1973) presents indirect evidence from nineteenth century Britain. For example, Robson (1973) examines the diffusion of gasworks in nineteenth century Britain and it is reasonable to assume that the employment generated in a city by the adoption of an innovation such as a gasworks is a positive function of the demand in that city for gas, which is, in turn, a positive function of city size. Moreover, the growth generated by innovation within existing industries, which constitutes a large proportion of total innovation within the secondary sector (Mansfield, 1968), tends to be a positive function of the size of the adopting unit (Richardson, 1973a: Thompson, 1965). However, despite the plausibility of these arguments for a size - related increment, there may be only a limited elasticity of firm size even within the secondary sector: for example, see Sheppard's (1974) data of PRSCs in Metropolitan Toronto. Therefore, the growth model developed to link the results of the diffusion process to growth investigates the implications of both fixed and size - related growth increments.

The general diffusion model developed in this thesis is for the inter-city diffusion of innovations but, in the discussion of the possible form of the growth increment, reference was made to the number of units of a given innovation that would appear in a city. This implies that for some innovations there exists an intra-city diffusion process. However, because it is not possible to model such a process in this thesis, it is assumed that the form of the intra-city diffusion process is uniform throughout the system and that for any city its results are the same as those produced by some size - related increment.

6.2 A Simple Growth Model

The procedure outlined in Chapter 5 permits the estimation of city means ($\bar{t}_i$s) and variances (s_i^2s) of adoption time for the diffusion of a general innovation within any system of twenty cities with given size, location and propensity for information exchange. How can these measures on the diffusion process be translated into growth rates?

Consider a system of cities with known size distribution (q), location relative to one another, and propensity for interaction (b). Within this system, city information potentials are calculated using equations 4.10 and 4.11 and surrogate variables for Location corresponding to those presented in Tables 5:11 and 5:12 are obtained. The surrogate variables for Location are, in turn, used to estimate the coefficients of equations 5.6, 5.7 and 5.8 (Table 5:15), which, in their turn, provide estimates of the coefficients of equation 5.5. Given the appropriate city potential values, $\bar{t}_i$ values can be estimated from equation 5.5. Estimates of s_i^2 are obtained from these $\hat{\bar{t}}_i$ values using the data and equations presented in Chapter 5, section 5.3. To estimate the probability, a_i, that the i^{th} city has adopted the general innovation by some point in time, $t^{(x)}$, it is necessary to re-create each of the city adoption probability distributions from $\hat{\bar{t}}_i$ and $\hat{s}_i^2$. If the city adoption probability distributions were normal in form, a_is could be estimated by integrating the area under the normal curves but, because the city adoption probability distributions are skewed (Figure 4:2), it is necessary to use the gamma distribution. Thus,

$$\hat{a}_i = \int_o^x \frac{1}{\Gamma(\alpha + 1)\beta^{(\alpha + 1)}} x^{\alpha} e^{-x/\beta} dx \qquad (3)$$

where x is that point in time up to which the area under the curve is integrated and α and β are the parameters of the gamma distribution, α and β can be estimated using the first moment around zero, μ, and the second moment around the mean, σ^2. Since these moments are known to be

$$\mu = \beta(\alpha + 1) \qquad (4)$$

$$\sigma^2 = \beta^2(\alpha + 1) \qquad (5)$$

and since estimates of the means and the variances are available, it is possible to calculate $\hat{\alpha}$ and $\hat{\beta}$ for each of the city adoption probability distributions. From equation (4),

$$\hat{\beta} = \frac{\hat{\bar{t}}_i}{(\hat{\alpha} + 1)}$$

and, substituting for $\hat{\beta}$ in equation (5),

$$\hat{s}_i^2 = \frac{\hat{\bar{t}}_i}{(\hat{\alpha} + 1)} \quad \frac{\hat{\bar{t}}_i}{(\hat{\alpha} + 1)} \quad (\hat{\alpha} + 1)$$

$$\hat{s}_i^2 = \frac{\hat{\bar{t}}_i^{\,2}}{(\hat{\alpha} + 1)}$$

Therefore,

$$\hat{\alpha} = \frac{\hat{\bar{t}}_i^{\,2}}{\hat{s}_i^2} - 1 \qquad (6)$$

By substituting back into equation (4),

$$\hat{\bar{t}}_i = \hat{\beta} \left(\frac{\hat{\bar{t}}_i^{\,2}}{\hat{s}_i^2} \right)$$

$$\hat{\beta} = \frac{\hat{s}_i^2}{\hat{\bar{t}}_i} \qquad (7)$$

Using the appropriate $\hat{\alpha}$ value, $\Gamma(\hat{\alpha} + 1)$ can be calculated,

$$\Gamma(\hat{\alpha} + 1) = \int_0^\infty x^{\hat{\alpha}} e^{-x} dx \qquad (8)$$

and $\hat{a}_i$ calculated from equation (3).

It is reasonable to assume that a number of innovations, N, are available for adoption within the period $t^{(0)}$ to $t^{(x)}$. If it is assumed that each of these N innovations becomes available to the system at time $t^{(0)}$, it follows that the diffusion of each of them is governed by the same set of system parameters, hence $\hat{\bar{t}}_i$ and $\hat{s}_i^2$ can be taken to be measures upon the mean diffusion paths of each of the N innovations. If adoptions are independent, the probability of any city, i, adopting b of the N innovations, $F(b)_i$, is given by the binomial:

$$F(b)_i = \hat{a}_i{}^b(1 - \hat{a}_i)^{N-b}\binom{N}{b} \qquad (9)$$

The mean of this distribution is $N\hat{a}_i$ and its variance is $N\hat{a}_i(1 - \hat{a}_i)$. Furthermore, if it is assumed that the adoption of each of the N innovations yields an identical growth increment, Q, to each adopting city, city i's expected percentage growth rate in the period $t^{(0)}$ to $t^{(x)}$ is G_i:

$$G_i = \frac{QN\hat{a}_i}{S_i} \quad 100\% \qquad (10)$$

where S_i is the size of city i at time $t^{(0)}$. By superscripting each variable for time, the growth model can be expressed within a dynamic context. Thus, $S_i^{(1)}$, the size of city i at the end of the first growth period, $t^{(0)}$ to $t^{(x)}$, is equal to $S_i^{(0)} + QN\hat{a}_1^{(1)}$, and is used as the basis for the calculation of $\hat{\bar{t}}_i^{(2)}$ and $\hat{s}_i^{2(2)}$, which are, in turn, used to estimate the second expected growth increment, $QN\hat{a}_i^{(2)}$, and so on.

6.3 The Short - term Simulation of City Growth

6.3.1 The Accuracy of $\bar{t}_i$ and s_i^2 Estimates

Before investigating the properties of the simple growth model within a hypothetical system of cities, it is necessary to ascertain the accuracy of the $\bar{t}_i$ and s_i^2 estimates used as input to the growth model. The test procedure used is a simple one. $\bar{t}_i$ and s_i^2 values for System 1 are estimated using the equations developed in Chapter 5 (Table 6:1). These estimates are compared with actual $\bar{t}_i$s and s_i^2s (Table 4:3) through regression of $\hat{\bar{t}}_i$ on $\bar{t}_i$ and $\hat{s}_i^2$ on s_i^2. If predicted and actual values are identical, the slope of these regressions, b, will be 1.0. The equation for $\bar{t}_i$ is

$$\bar{t}_i = .0889 + .9317\hat{\bar{t}}_i \quad (r^2 = .9856,\ t=.1301,\ s.e. = .5251) \quad (11)$$

the b coefficient of which is not significantly different from 1.0 at the .05 level according to a t test (Hoel, 1962). This result attests to the reliability of the estimation procedure for $\bar{t}_i$ outlined in Chapter 5. The equation for s_i^2 is

$$s_i^2 = 1.1618 + .6371\hat{s}_i^2 \quad (r^2 = .9239,\ t = .2480,\ s.e.=1.4633) \quad (12)$$

the b coefficient of which is also not significantly different from 1.0 at the .05 level. However, on reference to the appropriate tables (4:3 and 6:1), it is apparent that there is some considerable deviation between

Table 6:1

ESTIMATES OF $\bar{t}_i$ AND s_i^2
(LOCATION PATTERN NO. 1, b = 2.0 AND q = 1.0)

City No.	$\hat{t}_i$	$\hat{s}_i^2$
1	2.1895	.3710
2	3.1754	1.0950
3	3.1262	1.0464
4	3.3591	1.2899
5	2.3042	.4305
6	3.9395	2.1126
7	3.7860	1.8271
8	2.7443	.7161
9	2.4147	.4934
10	6.9694	10.7964
11	4.3188	2.6807
12	9.5061	26.6512
13	5.5130	5.4563
14	5.8560	6.5044
15	2.2434	.3983
16	7.1472	11.6179
17	5.3744	5.0665
18	8.6528	20.2678
19	5.6639	5.9025
20	8.9098	22.0705

actual and estimated s_i^2 at the extremities of the s_i^2 distribution. These deviations, particularly those at the upper limit of the distribution, inflate the standard error around the estimate of b and guarantee acceptance of the null hypothesis that the slope of the regression is not significantly different from unity, thereby distorting the results of the test. Therefore, although estimates of $\bar{t}_i$ are very reliable, those of s_i^2 are less so. However, it will be shown (section 6.4) that the under - prediction of large s_i^2 values has only a limited effect upon growth estimates.

6.3.2 Designing the Simulation Procedure

Within the simple framework outlined in section 6.2, three variables determine the distribution of city growth rates:

(i) the distribution of $\hat{\bar{t}}_i$ and $\hat{s}_i^2$,

(ii) the length of the time period, $t^{(0)}$ to $t^{(x)}$, over which $\hat{a}_i$ is calculated, and

(iii) the nature of the growth increment, Q, which a city obtains on adopting an innovation.

The number of innovations, N, available within the system during a given period of time has no effect upon the distribution of growth rates and is treated as a constant.

Although the inter-city structure of $\hat{\bar{t}}_i$ and $\hat{s}_i^2$ is determined by the system parameters, their actual magnitude is dependent upon λ, the diffusion constant. The inclusion of a constant in the diffusion equation (equation 4.12) is permissible because the actual length of a time period need not be defined within this theoretical analysis. $\hat{\bar{t}}_i$ and $\hat{s}_i^2$ are therefore relative, and not absolute, measures on city adoption behaviour, their distribution constituting a measure upon the diffusion characteristics of the system. Since the behaviour of $\bar{t}_i$ and s_i^2 in response to changes in the parameters of the system of cities has already been investigated, it is possible to extrapolate this known diffusion behaviour to anticipate the relationship between the parameters of the system of cities and the distribution of growth rates. For example, since the location of a city determines how its adoption behaviour deviates from that which would occur if the diffusion process was dependent only upon city size, it follows that deviation from size - expected growth patterns would be greater within systems of cities in which distance has a high frictional effect on information flows than within systems of cities in which it has a low frictional effect. In systems in which a high frictional effect of distance on information flows is prevalent, the growth potential of smaller cities would be closely tied to their location. By slowing down the diffusion process, a high frictional effect of distance on information flows would also cause a reduction of the total growth within the system of cities.

The results of the diffusion analysis also indicate that the distribution of city size would constrain the relationship between the structure of inter-city interaction and growth. Since the diffusion process within a system of cities of uniform size is totally location - dependent, the steepening of the city size distribution would cause the diffusion process and related growth effects to become increasingly size - dependent and location - independent. This means that, in polarised city size distributions, deviations from the size - dependent mode would be rare and really only possible for small cities in very close proximity to large cities. These are a few of the implications of the diffusion results presented in Chapter 5; their generality permits attention to be focussed upon the effects on the pattern of growth of variation in variables (ii) and (iii).

Although the length of a diffusion time period is not defined, the specification of the time period $t^{(0)}$ to $t^{(x)}$ has a considerable influence upon the distribution of $\hat{a}_i$ values and hence of growth rates. In terms of the growth process, $t^{(x)}$ is that point in time up to which a city obtains a growth stimulus from the adoption of an innovation originating in the system at time $t^{(0)}$. If x is large, every city may have an $\hat{a}_i$ value of 1.0 but, as x becomes smaller, the relative advantage of early adopters in the growth process is increased. In theory, it is desirable to test the model for a large number of x values but, in practice, a small number of x values suffices to demonstrate the effects of its variation.

The third variable, the form of the growth increment, Q, has already been discussed in general terms (section 6.1). In short, a Q value dependent only upon city size favours larger cities, while one that is completely independent of city size favours smaller cities. Both these extremes, and three composites, are experimented with. Simulations of the growth process are carried out within a single hypothetical system of cities, System 1. Within this system of cities, growth rates are estimated using x values of 2,3 and 4 in combination with each of five Q formulations. The Q formulations used are:

$Q_1 = \hat{Na}_i 10$

$Q_2 = \hat{Na}_i .0005S_i$

$Q_3 = (\hat{Na}_i 10 + \hat{Na}_i .0005S_i)/2$

$Q_4 = (\hat{Na}_i 10 + \hat{Na}_i .0010S_i)/2$

$Q_5 = (\hat{Na}_i 10 + \hat{Na}_i .0015S_i)/2$

Within each of the fifteen simulations, 100 innovations are available for adoption in each of five growth periods. City size is incremented at the end of each growth period and city growth rates are calculated at the end of the fifth time period. For x = 2, the period over which growth is simulated spans 10 diffusion time periods, for x = 3, 15 diffusion time periods and for x = 4, 20 diffusion time periods. Consequently, it is the growth that results from the diffusion of 500 innovations and not growth over a fixed period of time that is examined.

The growth model is designed to predict relative, rather than absolute, growth rates (the possibility of predicting the latter will be examined in Chapter 7). To this end, the total population within the system of cities is held constant throughout the simulation period. At the end of each growth period, the city growth increments, $Q\hat{Na}_i$ are summed, $\sum_i Q\hat{Na}_i$, and the incremented population of each city reduced by the proportion $\sum_i S_i/(\sum_i S_i + \sum_i Q\hat{Na}_i)$. That is,

$$S_i^{(1)} = (S_i^{(0)} + Q\hat{Na}_i^{(1)}) \sum_i S_i^{(0)}/(\sum_i S_i^{(0)} + \sum_i Q\hat{Na}_i^{(1)}), \qquad (13)$$

where $S_i^{(1)}$ is the normalised size of city i at the end of growth period (1). The percentage growth rate for each city in the system at the end of the simulation period, G_i, is calculated,

$$G_i = ((S_i^{(5)} - S_i^{(0)})/S_i^{(0)})100\% \qquad (14)$$

The mean and variance of city growth rates for various size classes are calculated. These summary results act as a monitor on the effect of changing x and Q on the distribution of growth rates and allow comparison of the model's results with an empirically observed growth regularity - that the mean of growth rates is uniform for different size classes or weakly (positively) correlated with city size, but that the variance of growth rates decreases with increasing city size (see Chapter 2, section 2.2). The small number of cities in the hypothetical system dictates division into only four size classes. The first size class is composed of the three cities with over 30,000 inhabitants, the second of the four cities with between 13,500 and 30,000 inhabitants, the third of the six cities with between 7,5000 and 13,500 inhabitants, and the fourth of the seven cities with fewer than 7,500 inhabitants.

6.3.3 Numerical Simulation

The five growth simulations with x = 4 are first discussed and then contrasts produced by changing x outlined and evaluated.

The city growth rates produced in each of the five x = 4 simulations are listed in Table 6:2 and class means and variances in Table 6:3. Graphs of city growth rates against initial size are given in Appendix 5. Setting x = 4 produces a distribution of $\hat{a}_i$ values which favours medium - sized and small cities (Table 6:4 and Figure 6:1). This bias is reflected in the Q_1 growth rates; the class variance of growth

Table 6:2

CITY GROWTH RATES: x = 4

City No.	Q_1	Q_2	Q_3	Q_4	Q_5
1	-9.15	7.81	-.95	2.79	6.49
2	-6.55	.95	-2.88	-2.47	-2.10
3	-2.74	.62	-1.05	-.79	-.59
4	.80	-1.46	-.10	-.55	-1.14
5	8.94	3.61	6.53	8.27	9.91
6	1.36	-5.47	-1.91	-4.65	-7.43
7	6.16	-4.57	.94	-1.40	-3.85
8	20.71	1.89	11.76	12.74	13.62
9	26.81	3.01	15.53	17.00	18.36
10	-5.68	-12.43	-9.01	-14.78	-20.19
11	11.30	-7.44	1.80	-2.09	-6.03
12	-7.54	-13.48	10.45	-16.54	-22.17
13	3.16	-10.65	-3.74	-8.91	-13.85
14	2.25	-11.27	-4.52	-10.00	-15.22
15	54.52	2.94	29.90	31.17	32.28
16	-1.19	-12.56	-6.88	-12.80	-18.34
17	10.97	-10.53	-.18	-5.56	-10.75
18	-3.29	-13.24	-8.21	-14.30	-19.95
19	10.23	-10.95	-.53	-6.01	-11.22
20	-2.51	-13.39	-7.86	-14.21	-20.21

Table 6:3

MEAN AND VARIANCE OF CLASS GROWTH RATES FOR FIFTEEN SIMULATIONS

Simulation No.	x	Q	Class 1 μ	Class 1 σ²
1	4	1	-6.14	6.93
2	4	2	3.13	10.99
3	4	3	-1.63	.79
4	4	4	-.15	4.82
5	4	5	1.27	14.00
6	3	1	-4.31	1.3[illegible]
7	3	2	1.93	32.[illegible]
8	3	3	-1.26	6.[illegible]
9	3	4	-.41	[illegible]
10	3	5	.39	[illegible]
11	2	1	-1.62	[illegible]
12	2	2	.4[illegible]	[illegible]
13	2	3	-.5[illegible]	[illegible]
14	2	4	-.[illegible]	[illegible]
15	2	5	-.[illegible]	[illegible]

Table 6:3 (cont'd)

Class 2 μ	Class 2 σ^2	Class 3 μ	Class 3 σ^2	Class 4 μ	Class 4 σ^2
4.32	11.46	8.13	163.08	10.14	357.36
1.97	12.62	-6.52	43.78	-9.80	28.36
1.36	9.93	.98	96.89	.23	156.05
.41	22.91	-2.10	166.74	-4.53	223.60
-.63	41.99	-5.04	249.55	-9.03	297.04
2.26	27.03	5.16	148.31	5.91	383.01
-3.04	24.96	-5.72	35.36	-8.17	27.40
-.27	26.16	-.17	84.58	-1.03	159.78
-1.74	58.36	-2.81	149.08	-5.01	226.68
-3.26	103.64	-5.35	228.99	-8.86	297.56
.45	8.62	1.43	22.16	2.49	84.87
-1.71	6.63	-2.60	5.01	-3.24	5.75
.60	7.64	-.57	12.15	-.36	33.62
1.44	16.87	-1.82	21.96	-2.02	47.61
2.29	30.10	-3.07	34.95	-3.69	62.80

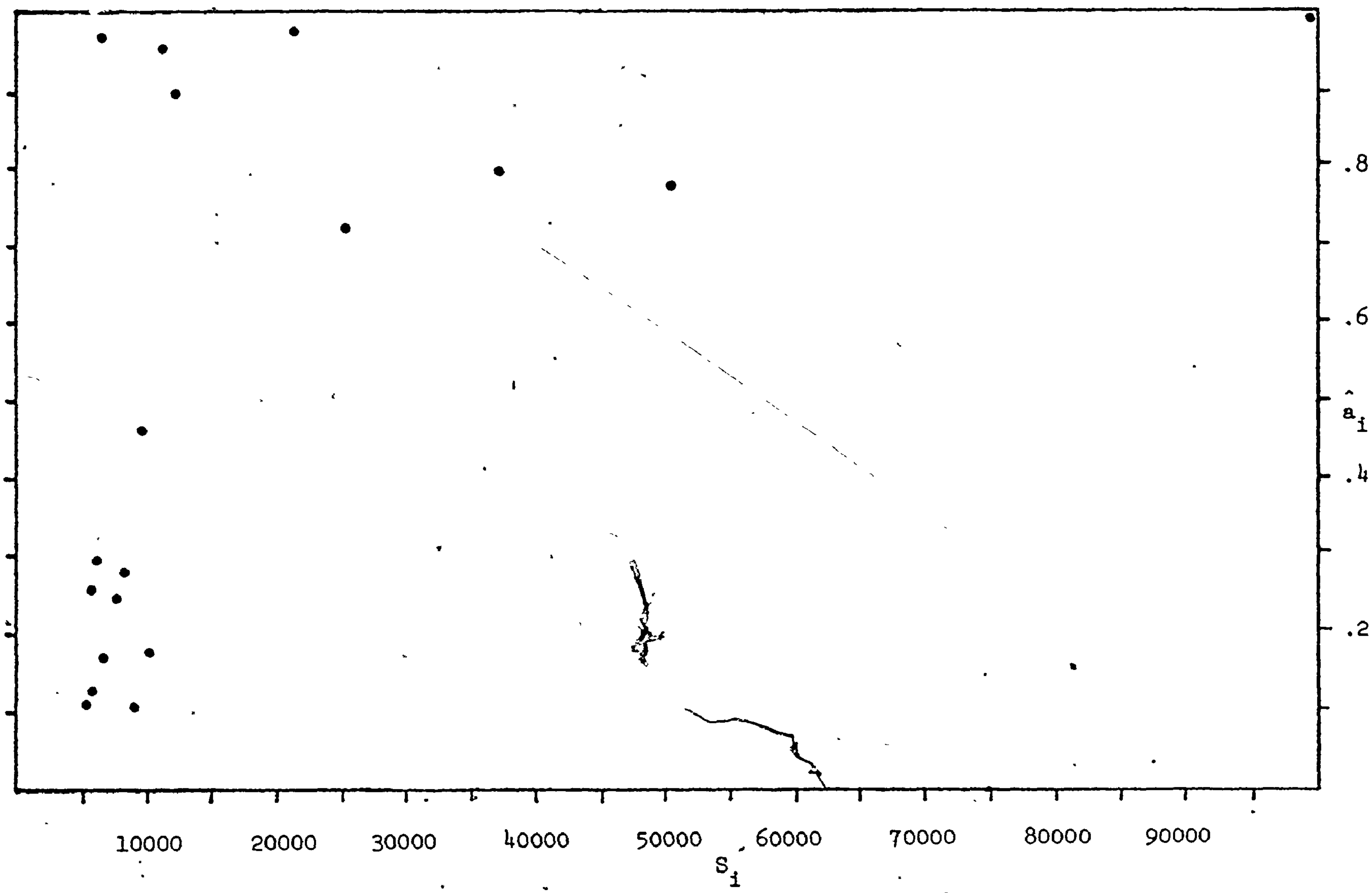
Table 6:4

ESTIMATES OF BINOMIAL PROBABILITIES ($\hat{a}_i$)

City	x=2	x=3	x=4
1	.4102	.9006	.9937
2	.1173	.4760	.7896
3	.1235	.4939	.8139
4	.0977	.4152	.7400
5	.3484	.8559	.9869
6	.0591	.2696	.5541
7	.0680	.3065	.6081
8	.1924	.6538	.9197
9	.2982	.8077	.9765
10	.0214	.0805	.1804
11	.0478	.2189	.4712
12	.0185	.0561	.1148
13	.0289	.1238	.2831
14	.0263	.1095	.2505
15	.3799	.8804	.9910
16	.0210	.0775	.1727
17	.0301	.1307	.2985
18	.0188	.0611	.1290
19	.0277	.1170	.2679
20	.0187	.0594	.1242

Figure 6:1

INITIAL CITY SIZE (S_i) AGAINST ESTIMATED CUMULATIVE PROBABILITY OF ADOPTION ($\hat{a}_i$), x = 4



rates increases as city size decreases but the class mean also increases. However, abandonment of the absolute growth increment, Q_1, in favour of the size - dependent one, Q_2, more than compensates for the bias toward medium - sized and small cities in the $\hat{a}_i$ distribution; the class mean, as well as the class variance, of growth rates decreases with decreasing size. As might be anticipated, the composite growth increment, Q_3, produces a combination of the above effects; the class variance of growth rates increases as city size decreases and, although by no means uniform, the class mean displays less variation than in the Q_1 and Q_2 simulations. As the importance of the size element within the composite growth increment is increased (Q_4 and Q_5), the advantage afforded to larger cities is increased and the distribution of growth rates is changed. For example, in the Q_3 simulation, city 7 has a higher growth rate than city 2, whereas, in the Q_5 simulation, the reverse is true. However, only in the Q_5 simulation is the size effect strong enough to compensate for the bias toward medium - sized and small cities in the $\hat{a}_i$ distribution; the first size class has a higher mean growth rate than the second size class.

These simulations demonstrate that, although the diffusion results impose a constraint upon the form of the growth pattern through the distribution of $\hat{a}_i$, the manipulation of one of the growth parameters , Q, also has substantial effects upon the distribution of growth rates. To investigate further the relationship between the diffusion results and the parameters of the growth model, similar sets of Q simulations for $x = 3$ and $x = 2$ are examined.

The binomial probabilities, $\hat{a}_i$ estimated for $x = 3$ and $x = 2$ are listed in Table 6:4 and graphed against city size in Figures 6:2 and 6:3. Comparisons with the probabilities for x = 4 indicate that the growth advantage afforded to early adopters increases as x decreases. For example, consider the values of $\hat{a}_1$ and $\hat{a}_2$ for each of the x values. For x = 4, their values are .9937 and .7896 respectively, for x = 3, .9006 and .4760 respectively and, for x = 2, .4102 and .1173 respectively, which demonstrates the re-structuring effect associated with changing x. This re-structuring effect is particularly evident in the Q_1 growth rates for both x = 3(Table 6:5) and x = 4 (Table 6:6).

A comparison of the three sets of Q_1 growth rates indicates that, under size - independent growth conditions, lowering x reduces the growth potential of medium - sized cities. For example, consider the growth rates for cities 1 and 7. In the x = 4 simulation, their growth rates are -9.15% and 6.16% respectively, in the x = 3 simulation, -5.17% and .61% respectively and, in the x = 2 simulation, -1.23% and -.96% respectively (see also the appropriate graphs in Appendix 5). However, the re-structuring of $\hat{a}_i$ values in favour of early adopters caused by lowering x does not mean that the effect of using a size - independent growth increment is nullified. This is evident from the class mean and class variance data (Table 6:3).

Comparison of the growth rates produced by the size - dependent growth increment, Q_2, indicates a similar change in the fortune of medium - sized cities. For example, city 8 has a growth rate of 1.89% in the x = 4 simulation but one of only -.81% in the x = 2 simulation. Since its x = 3 value, .75%, lies between the x = 4 and x = 2 values,

Figure 6.2

INITIAL CITY SIZE (S_i) AGAINST ESTIMATED CUMULATIVE PROBABILITY OF ADOPTION ($\hat{a}_i$), x = 3

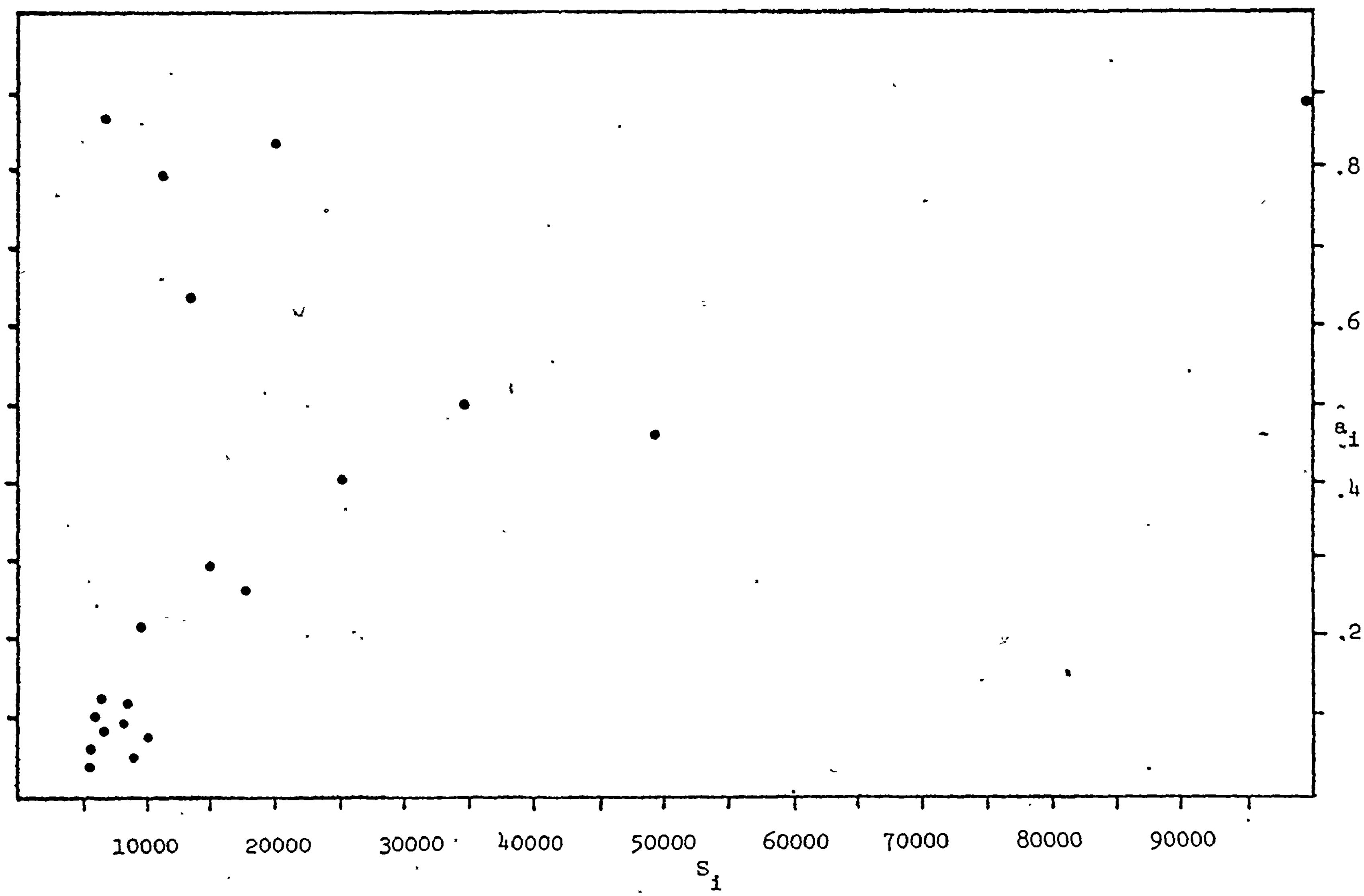


Figure 6:3

INITIAL CITY SIZE (S_i) AGAINST ESTIMATED CUMULATIVE PROBABILITY OF ADOPTION ($\hat{a}_i$), x = 2

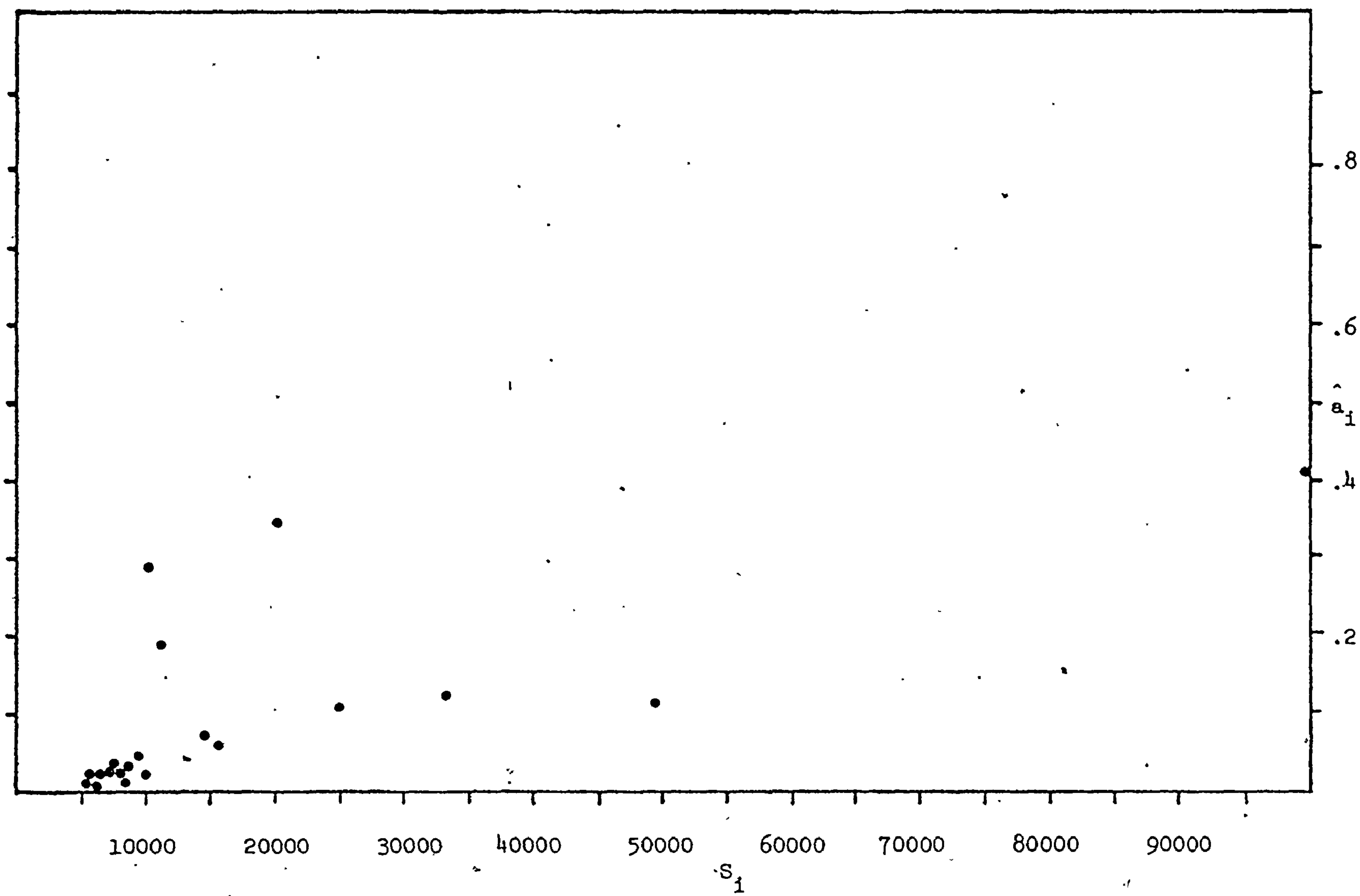


Table 6:5

CITY GROWTH RATES : x = 3

City	Q_1	Q_2	Q_3	Q_4	Q_5
1	-5.17	9.95	2.18	7.01	11.84
2	-5.09	-2.09	-3.60	-4.74	-5.96
3	-2.67	-2.08	-2.35	-3.49	-4.71
4	-.66	-4.03	-2.22	-4.02	-5.94
5	11.12	5.39	8.47	11.26	14.03
6	-2.02	-7.04	-4.47	-8.00	-11.52
7	.61	-6.46	-2.88	-6.21	-9.60
8	16.71	.75	8.90	9.46	9.94
9	26.26	4.22	15.68	18.21	20.77
10	-5.74	-10.50	-8.08	-13.04	-17.78
11	2.00	-8.17	-3.15	-7.34	-11.49
12	-6.30	-10.89	-8.56	-13.65	-18.47
13	-1.98	-9.73	-5.83	-10.53	-15.04
14	-2.36	-10.03	-6.19	-11.06	-15.74
15	53.57	4.61	29.78	31.73	33.25
16	-3.69	-10.55	-7.10	-12.12	-16.90
17	1.11	-9.69	-4.38	-9.20	-13.86
18	-4.32	-10.81	-7.53	-12.62	-17.45
19	.98	-9.86	-4.46	-9.28	-13.91
20	-3.92	-10.86	-7.38	-12.53	-17.61

Table 6:6

CITY GROWTH RATES: x = 2

City	Q_1	Q_2	Q_3	Q_4	Q_5
1	-1.23	5.64	2.16	4.98	7.83
2	-2.13	-2.14	-2.13	-3.23	-4.38
3	-1.48	-2.10	-1.78	-2.87	-3.99
4	-1.22	-2.70	-1.94	-3.28	-4.65
5	5.52	2.71	4.16	5.64	7.17
6	-1.56	-3.51	-2.52	-4.29	-6.08
7	-.96	-3.36	-2.15	-3.86	-5.60
8	4.57	-.81	1.89	1.49	1.06
9	10.59	1.59	6.15	7.21	8.37
10	-2.22	-4.25	-3.22	-5.34	-7.44
11	-.71	-3.77	-2.23	-4.14	-6.06
12	-2.18	-4.31	-3.23	-5.36	-7.49
13	-1.44	-4.11	-2.76	-4.81	-6.86
14	-1.47	-4.17	-2.81	-4.90	-6.98
15	25.04	2.64	13.83	14.87	15.71
16	-1.63	-4.26	-2.93	-5.05	-7.17
17	-.76	-4.10	-2.43	-4.49	-6.54
18	-1.60	-4.30	-2.94	-5.08	-7.20
19	-.70	-4.14	-2.41	-4.48	-6.54
20	-1.43	-4.31	-2.86	-5.00	-7.13

the behaviour of city 8's growth rate in response to changes in x is approximately linear. However, in other instances, such linearity is not present; for example, city 1's growth rate is highest in the x = 3 simulation. Such complex behaviour occurs because the effect of changing x varies from city to city. Thus, city 1's growth rate is highest in the x = 3 simulation because it is under these growth conditions that its advantage relative to the rest of the system is greatest.

As was true for x = 4, the composite growth increments (Q_3, Q_4 and Q_5) for x = 3 and x = 2 produce interesting results. For example, for Q_5, a decrease in x causes a decrease in the relative growth of medium - sized cities. For all three values of x, increasing the magnitude of the size element within the growth increment causes a relative increase in the growth rate of certain, but not all, large cities. For example, for all three values of x, increasing the magnitude of the size element within the composite growth increment causes city 1's growth rate to increase, but city 2's to decrease. This is symptomatic of the inter-dependencies that exist among city growth rates and among the growth parameters. Although city 2's growth increases in absolute terms as the magnitude of the size element within the composite growth increment is increases, this absolute increase does not compensate for the loss in relative advantage (over city 1) caused by the reduction in the importance of the absolute element within the composite growth increment. Thus, the response of a city's growth rate to changes in a growth parameter not only depends upon the prevalent value for the other growth parameter but also upon the responses of the other cities in the system to changes in the growth parameters; responses which are determined by the results of the diffusion process.

Comparing these results with Canadian data (see Chapter 2, section 2.2), indicates:(i) that the time period during which cities obtain a growth increment from adoption should be considered to be relatively short, otherwise the predicted growth of medium - sized cities is larger than reality;and (ii) that the growth increment that a city obtains from the adoption of an innovation should be considered some function of its own size, otherwise the predicted growth of small cities is larger than reality.

This simulation - based analysis of short - term growth indicates that, under certain conditions, the simple growth model produces reasonable distributions of growth rates. Each of the parameters of the growth model and the distribution of adoption times has a different relationship with the pattern of growth within a hypothetical system of cities. The effect of Q is to increase or decrease the advantage afforded to cities of large size; the results presented in Table 6:3 substantiate this conclusion. The parameter x has a more complex role. The magnitude of x determines the precise effect of the distribution of adoption times upon growth rates. Although early adopters have a consistent advantage over late adopters, owing to the complex nature of the city adoption probability distributions, the distribution of $\hat{a}_i$ is sensitive also to changes in x.

6.3.4 The Spatial Autocorrelation of Growth Rates

The first nearest neighbour spatial autocorrelation statistic for growth rates for each of the fifteen growth simulations is presented in Table 6:7. It has been noted that, city adoption behaviour, as summarised by $\bar{t}_i$, is spatially autocorrelated (Chapter 5, section 5.2). Indeed, the first nearest neighbour spatial autocorrelation statistic for $\bar{t}_i$ in the system of cities used in the growth simulations is .5996. However, the data presented in Table 6:7 indicate that there exists considerable variation in the effect of this spatial autocorrelation of adoption time upon growth rates.

The first conclusion that can be drawn from the data presented in Table 6:7 is that variation in Q has a more pronounced effect upon the spatial autocorrelation of growth rates than variation in x, primarily because possible variation in the $\hat{a}_i$ distribution in response to a change in x is constrained by the fixed distribution of adoption times. Consequently, although a change in x causes some changes in the distribution of growth rates, it does not substantially alter the basic pattern of growth over space. However, changes in Q have a more pronounced effect upon the pattern of growth over space.

The absence of spatial autocorrelation of growth rates in the Q_1 simulations occurs because the absolute growth increment causes small cities, which tend to be late adopters, to have relatively high growth rates. Thus, the absolute growth increment, which nullifies the growth advantage that earlier adopters obtain from adopting more innovations within the time period over which $\hat{a}_i$ is calculated, also

Table 6:7

SPATIAL AUTOCORRELATION OF GROWTH RATES

	x=4	x=3	x=2
Q_1	-.1834	-.0863	-.0847
Q_2	.6016	.6135	.5344
Q_3	.1319	.1890	.1516
Q_4	.3391	.3526	.2958
Q_5	.4487	.4444	.3818

nullifies the effect of spatially - a[illegible] spatial pattern of growth. Owing to the [illegible] the determination of city adoption time, the [illegible] growth rates increases as the importance [illegible] increment increases. This result is a very im[illegible] that an absolute growth increment, by making the [illegible] times unimportant in the determination of growth rates, [illegible] the model's conceptual validity. Consequently, a comp[illegible] increment, which keeps this important result of the diffusion process intact, and which also produces intuitively reasonable distrib[illegible] growth rates by size class, appears to be the most reasonable form of the growth increment.

The spatial autocorrelation of city size is also monitored in each of the growth simulations but, owing to the relatively short period over which the growth process is simulated, even within the most spatially autocorrelated growth environment, the x = 3, Q_4 system, the spatial autocorrelation of city size increases only from -.1029 to -.0469. Consequently, the results produced by this short - term simulation of growth are of only limited usefulness in testing for the increasing spatial autocorrelation of city size over time and therefore the possibility for simulating growth over the long term is examined.

6.4 The Long Term Simulation of City Growth

The procedure used to estimate $\bar{t}_i$ and s_i^2 , the summary measures on the diffusion process and input to the growth model, is unstable over the long term. This instability occurs because the dynamic nature of the growth model causes the city size distribution to change in form over time. The initial distribution of city sizes is rank - size, with slope q = 1.0. This q value is used not only to determine city sizes but also as input into the estimating equations for $\bar{t}_i$ and s_i^2 (chapter 5, section 5.2). As the growth model is iterated during the short - term simulation of growth, the estimates of locational variables and city information potentials, which are also inputs into the estimating equations for $\bar{t}_i$ and s_i^2, respond to changes in city size. However, both b and q are assumed to remain constant. In the case of b, there is little to indicate why such an assumption could not be extended over relatively long time periods but, in the case of q, no such extension is possible.

The stability in city size distributions reported in the literature (Madden, 1956) is the consequence of an overall stability in the factors which determine growth (Richardson, 1973a). Although individual cities may experience unusually high, or unusually low, growth rates over short periods of time, the behaviour of the system as a whole displays considerable stability because the factors that determine growth are essentially those that determine the size of cities. Such stability allows statistical models based upon concepts such as 'proportionate effect' (Simon, 1955) to have some credibility. The importance of locational effects within this process has been well documented by economists as well as geographers (Webber, 1972) and has been observed to take the form of spatial autocorrelation of phenomena such as growth rates, unemployment and city size. However, within the hypothetical system of cities in which growth is simulated, city size is not spatially autocorrelated because cities are located at random. The existence of strong locational effects within the diffusion process causes considerable instability within the city size distribution as the system attempts to adjust itself to the determined growth conditions. This instability causes deviation from the rank - size distribution, which means that estimates of $\bar{t}_i$ and s_i^2 are only reliable over the short term or as long as the city size distribution maintains a rank - size profile.

To investigate the growth process over longer periods of time, several simulations are carried out using the diffusion model to directly calculate the binomial probabilities, a_i. For computational convenience, the number of innovations available to the system within each growth period is raised from 100 to 1000, which has the effect of accelerating any tendency toward increased spatial autocorrelation.

Before examining the long - term growth pattern from System 1, with x = 3 and using the Q_3 growth increment, the binomial probabilities obtained directly from the model (Table 6:8) are compared with those estimated using the procedure outlined in section 6.3.2 (Table 6:4). A regression test identical to that performed on estimated and actual $\bar{t}_i$s and s_i^2 s is used to facilitate this comparison. For x = 2, the regression of $\hat{a}_i$ on a_i produces the equation,

$$a_i = .0504 + .9144\hat{a}_i (r^2 = .8359, t = .3733, s.e. = .2293) \quad (15)$$

for x = 3,

Table 6:8

BINOMIAL PROBABILITIES CALCULATED DIRECTLY FROM THE DIFFUSION MODEL

City	x=2	x=3	x=4
1	.4323	.9900	.9900
2	.2577	.4898	.8567
3	.2450	.4964	.8733
4	.1473	.4646	.9365
5	.3715	.8220	.9900
6	.1391	.3151	.6191
7	.1285	.2963	.5939
8	.2362	.7120	.9900
9	.4222	.7992	.9900
10	.0504	.0978	.1957
11	.0817	.2972	.4491
12	.0384	.0685	.1332
13	.0711	.1655	.3536
14	.0663	.1517	.3073
15	.2509	.6274	.9900
16	.0432	.0955	.2054
17	.0521	.1349	.3074
18	.0393	.0821	.1735
19	.0617	.1520	.3331
20	.0261	.0509	.1028

$$a_i = .0317 + .9282\hat{a}_i (r^2 = .9530,\ t = .2880,\ s.e. = .2493) \quad (16)$$

and, for x = 4,

$$a_i = .0311 + 1.0097\hat{a}_i (r^2 = .9744,\ t = .1925,\ s.e. = .0504) \quad (17)$$

Using a t test, the b coefficient in each of the regressions was found to be insignificantly different from 1.0 at the 0.5 level. However, as was the case in the regressions of $\hat{\bar{t}}_i$ on $\bar{t}_i$ and $\hat{s}_i^2$ on s_i^2 (section 6.2), the t values are influenced strongly by the standard errors. Indeed, closer examination of the data (Tables 6:4 and 6:8) suggests that estimates of $\hat{a}_i$ are more reliable for x=.3 and x= 4 than for x =2. The divergence between a_i and $\hat{a}_i$ for x = 2 is a consequence of the inclusion of a size - dependent invention period within the diffusion model. The anomalies in the city adoption probability distributions that arise from the inclusion of a size - dependent invention period within the diffusion model (for example, see Chapter 5, section 5.3) are smoothed out when a_i is estimated but not when the probabilities are directly calculated from the diffusion model. As x increases, the distorting effects associated with the inclusion of an invention period are lessened and estimated probabilities more closely approximate those generated by the diffusion model.

The long - term growth rates produced within the x = 3, Q_3 system are presented in Table 6:9 and graphs of growth rates against city size in Appendix 5. Several results are apparent. Firstly, growth is concentrated in a small number of cities, which is caused by the importance of city size, both within the diffusion model and the growth model. Secondly, the constraint imposed upon the growth pattern by the initial distribution of adoption times (that is, the results of the diffusion process) is reflected in the fact that city 15, which benefits from its close proximity to city 4 (Figure 4:1), is the only city with a population of less than 10,000 which attains a positive growth rate. Thirdly, developments are cumulative. As the growth model is iterated, the population of early adopters increases and their position within the diffusion process improves and, consequently, population becomes concentrated within a small number of cities. This is an important result and indicates that at least one factor in the growth process exerts a polarising influence on growth patterns. The distribution of city populations at the end of the twenty-fifth iteration (Table 6:10) reflects this propensity for population to concentrate within a few cities. These populations can be compared with those at the start of the growth simulation (Table 4:1). This comparison indicates that because of the mutually reinforcing effects of growth and early adoption, repeated application of the growth model destroys the symmetry of the original city size distribution.

Consider the evidence concerning spatial autocorrelation produced by the long - term simulation of growth (Table 6:11). These results

Table 6:9

LONG - RUN GROWTH RATES (PERCENTAGES, BASED UPON INITIAL SIZE)

	Iteration No.				
City	5	10	15	20	25
1	10.37	7.95	.31	-8.11	-15.74
2	-26.27	-57.86	-79.75	-91.56	-96.79
3	-18.40	-45.48	-68.84	-84.57	-93.13
4	-3.28	37.12	73.76	97.08	111.44
5	63.24	121.17	155.62	175.14	185.51
6	-26.65	-59.99	-81.52	-92.22	-96.88
7	-27.12	-62.76	-83.91	-93.59	-97.55
8	97.63	201.78	268.77	310.09	334.61
9	118.84	236.63	312.66	359.69	387.71
10	-49.08	-82.06	-94.24	-98.22	-99.46
11	-31.34	-69.20	-88.43	-95.94	-98.62
12	-51.06	-82.81	-94.38	-98.22	-99.45
13	-29.40	-55.85	-73.30	-84.16	-90.74
14	-36.19	-73.50	-90.73	-96.97	-99.04
15	113.18	305.60	467.77	576.22	646.60
16	-42.67	-75.35	-90.20	-96.24	-98.58
17	-34.77	-71.59	-89.37	-96.25	-98.71
18	-42.97	-74.52	-89.27	-95.62	-98.25
19	-20.24	-40.41	-56.18	-68.04	-76.99
20	-50.98	-83.40	-94.81	-98.43	-99.53

Table 6:10

CITY POPULATIONS AFTER 25 ITERATIONS

City	S_i	City	S_i
1	84,258	11	125
2	1,606	12	45
3	2,888	13	712
4	52,859	14	68
5	57,101	15	40,773
6	519	16	88
7	349	17	75
8	54,326	18	97
9	54,190	19	1,210
10	54	20	23

Table 6:11

CHANGING SPATIAL AUTOCORRELATION OF CITY SIZE, $\bar{t}_i$ AND GROWTH RATES OVER TIME

Iteration No.	S_i	$\bar{t}_i$	G_i
0	-.1029	-----	-----
5	.2674	.6381	.3859
10	.5644	.6218	.3915
15	.7329	.5380	.3815
20	.8161	.5462	.3684
25	.8565	.5237	.3569

indicate that city size becomes more spatially autocorrelated over time. However, over the same time period, the spatial autocorrelation of $\overline{t}_i$ increases and then decreases, and that of growth rates remains fairly constant. The behaviour of the city size autocorrelation is not unexpected because growth rates are autocorrelated. The behaviour of the $\overline{t}_i$ autocorrelation indicates that the system attains its peak of spatial dependency somewhere between the fifth and the tenth time period . Since $\overline{t}_i$ is a function of location relative to other cities in the system as well as city size, the divergence between the level of its spatial autocorrelation and that of city size is to be expected. The relative stability in the spatial autocorrelation of growth rates reflects the extent of the constraint imposed upon the growth pattern by the initial distribution of adoption times and its generally low level is a consequence of a growth process which causes growth to be concentrated in a few cities.

6.5 Some Conclusions

Thus, a simple growth model based upon the spatial diffusion process produces reasonable patterns of growth within a hypothetical system of cities. However, the equations developed to characterise the diffusion process can only be used to estimate adoption times in the short-term, a consequence of simulating the growth process within what is an unstable environment. To test the model in the long - term and to investigate its effect upon the spatial autocorrelation of city size, it is necessary to calculate adoption probabilities directly from the diffusion model.

The results produced by the analysis of the growth process are less general than those produced by the analysis of the diffusion process. The diffusion results represent measures upon the diffusion potential of a given system of cities and do not relate to any specific diffusion process or processes; they are general because variation in the inter-city structure of the diffusion process can be linked to the parameters of the system in which it occurs. The growth results are less general because the nature of the relationships among its parameters has only been partially investigated. Although the results of simulating growth within a hypothetical system of cities demonstrate that a simple model based upon spatial diffusion is able to produce reasonable patterns of growth, the conditions that determine whether or not a given set of adoption times produces a reasonable pattern of growth have not been comprehensively investigated. Consequently, although the feasibility of developing a model for growth within a system of cities, based upon spatial diffusion, has been established, significant avenues for further research are apparent, some of the most important of which are discussed in the next chapter.

CHAPTER 7

SOME FUTURE DIRECTIONS

Evidence presented in the literature (for examples, see Thomas and Le Heron, 1975) and the results of the PRSC analysis indicate that the diffusion of some innovations is influenced by factors other than information availability. However, the lack of substantial empirical evidence on, and theory concerning, the nature of such diffusion processes, imply that the development of a diffusion model incorporating factors such as the demand for an innovation, its compatability with existing production processes and the age of capital stock (Day, 1970: Webber, 1972) is not feasible at the present time. Empirical analysis of, and the development of theory concerning, the diffusion of technological and organisational innovations that contribute substantially to urban growth constitutes a first avenue for further research.

It is primarily because of the lack of a feasible alternative that the spatial framework is the most suitable for modelling the diffusion process within a system of cities, and although a model of the inter-city diffusion process based upon information availability could not satisfactorily explain all diffusion processes, the results of the PRSC analysis demonstrate that it provides some explanations of the diffusion of most innovations. However, the importance of city size within the spatial diffusion model causes some confusion in the model's interpretation because city size may be a surrogate for variables other than information availability (Richardson, 1973a), including market size and economies of scale. Analysis of diffusions in which these variables are involved might indicate their importance in the diffusion process, particularly in relation to that of information availability.

The results presented in Chapter 5 concerning the relationship between the form of the diffusion process and the parameters of the system of cities in which it occurs yield considerable insight into the possible pattern of growth, as well as of diffusion. For example, the constraint upon the form of the diffusion process imposed by the distribution of city sizes has considerable implications for growth centre and related policies; the diffusion results indicate that growth centres located in peripheral areas would have a low expectation for success. Indeed, the results of the diffusion analysis indicate that, although the improvement of communication systems and the consequent increase of total inter-city interaction increases the growth potential of a system of cities, the effect of this improvement upon the distribution of growth rates depends primarily upon the distribution of city sizes. This implies that the imposition of checks on the size of large cities might be a suitable policy for spreading growth within more-primate city systems. Such policies are in accord with those recommended by advocates of the cumulative causation model.

The results produced by the diffusion model can be interpreted as the diffusion characteristics, or diffusion potential, of a system of cities with given parameters, and are not specific to the diffusion of any single innovation or group of innovations. However, the growth results are less general because the properties of the growth model were not investigated as comprehensively as those of the diffusion model. This could be remedied by undertaking a regression - based investigation of properties similar to that outlined in Chapter 5. This constitutes a second avenue for further research.

Consider the following as an example of what might be attempted in connection with this second avenue for further research. The city growth rates, G_i, produced by an innovation - based growth model with exogenously determined parameters, x and Q, are some function of city mean adoption times, $\bar{t}_i$,

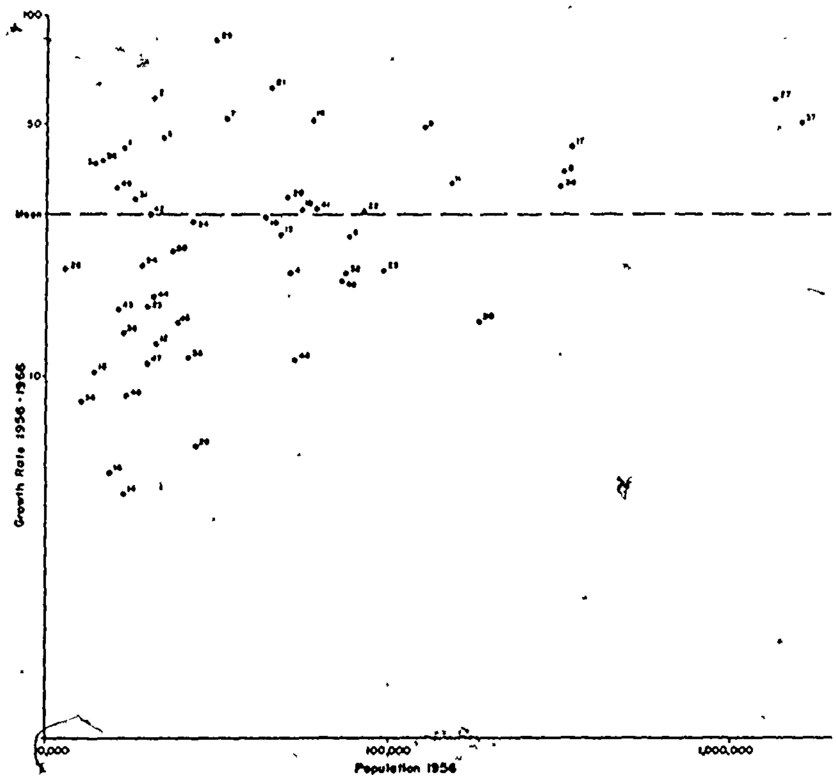
$$G_i = f(\bar{t}_i) \tag{1}$$

The form of this relationship could be estimated through regression, and variation in the structure of the regression linked to the parameters of the growth model, also through regression. The effects of further varying the form of Q and x could also be evaluated through simulation within a hypothetical system of cities. However, reliance upon an experimental approach would not be as suitable in this context as it was within the diffusion context. In the diffusion context, simulation within a hypothetical systems of cities was necessary because location is treated as a variable within the diffusion process. Within the growth context, such treatment of location would necessitate a prohibitive amount of experimentation and, anyway, a hypothetical system of cities of this type is unstable and therefore unsuitable for the long - run testing of a growth model. Thus, a third avenue for further research, the testing of the model within a real system of cities, appears both more feasible and more important than the second.

The distribution of growth rates by city size for Ontario and Quebec between 1956 and 1966 is graphed by Barber (1972) (Figure 7:1). This distribution is similar in form to that produced in several of the growth simulations (see Appendix 5), and it would therefore seem reasonable to attempt to duplicate this distribution through the application of the innovation - based growth model developed in this thesis. Given the location and size of each of the fifty cities and values for b and q (Simmons, 1972, and Davies, 1972, provide such values), the diffusion model could be used to calculate city adoption times. These adoption times would constitute a summary measure upon the diffusion potential of the system in 1956. However, there are no data from which to measure x and Q. To make even the simple growth model developed in Chapter 6 operational, values for N, the number of innovations available in a time period, for x, the period over which a city obtains a growth increment from adoption and for Q, the form of the growth increment, have to be determined exogenously. Although N can be derived from patent data, owing to the complexity of the growth process and the paucity of empirical evidence, similar data for x and Q are not available. Consequently, it is necessary to calculate x and Q from actual growth data. Using combinations of x and Q that produce reasonable distributions of growth rates within hypothetical systems of cities (Chapter 6) and the same iteration technique, growth rates for Ontario and Quebec cities could be estimated. Considerable

Figure 7:1

CENTRAL CANADA (1956 - 1966) : RATE OF GROWTH VERSUS INITIAL SIZE



City

1. Barrie
2 Belleville
3. Brockville
4. Brantford, M.U.A
5 Chatham
6 Thunder Bay, M U A
7 Guelph
8 Hamilton
9 Kitchener
10. Kingston, M U A
11 London
12 North Bay
13 Niagara Falls, M U A
14. Owen Sound
15 Orillia
16 Oshawa-Whitby, M.U A
17 Ottawa
18. Pembroke
19. Peterborough, M.U.A
20 Sarnia, M.U A
21. Sault Ste Marie, M.U A.
22 St. Catharines, M U A
23 Stratford
24. St. Thomas
25 Sudbury
26. Timmins
27 Toronto
28. Trenton
29 Welland-Port Colborne
30 Windsor
31. Woodstock
32. Lac St Jean, M U A
33. Drummondville
34. Granby
35. Joliette
36. Magog
37 Montreal
38 Quebec City
39. Rimouski
40. Rouyn
41. Sherbrooke, M.U.A
42. St. Jérôme
43. Sorel
44 St. Hyacinthe
45 St Jean
46 Shawinigan, M.U A
47 Thetford Mines
48 Trois Rivières, M.U.A.
49 Victoriaville
50. Valleyfield

Source: Barber (1972), p.149

differences between the levels of estimated and actual growth rates would indicate mis-specification of Q, whereas growth rates unduly biased in favour of one size class would indicate mis-specification of x. Thus, although such an analysis could not meaningfully predict growth rates, it would produce post - facto estimates of x and Q. The generality of these estimates could then be tested by similar analysis in other systems or for a different time period in Central Canada, and the possibility for predicting actual growth rates explored.

Two other aspects of the growth process could be investigated through analysis of Central Canada data. Firstly, the stability inherent in the system would permit long - run tests of the model. Secondly, and more importantly, the use of real data would permit meaningful city by city analysis of the growth process. The results produced by simulating the growth process within a hypothetical system of cities were discussed in terms of the distribution of growth rates by city size, or by size class, and not in terms of how individual cities performed or failed to perform. However, given that the growth rate of each city in the system is known, it would be possible to examine the city by city, as well as the overall, fit between actual and predicted growth rates. This constitutes the most important advantage associated with the use of real, as opposed to hypothetical, data.

Therefore, although the objective set at the beginning of the thesis is met, it is apparent from this discussion that this work is not an end in itself but a part of what should be on - going research. Of the avenues for further research available, the analysis of growth within a real system of cities appears to be the most promising, particularly since it would permit the post - facto estimation of the growth parameters, x and Q, and city by city comparison of actual and predicted growth.

BIBLIOGRAPHY

ARTLE, R. (1965), The Structure of the Stockholm Economy; Toward a Framework for Projecting Metropolitan Community Development, Ithica, New York: Cornell University Press.

BAILEY, N.T.J.(1957), The Mathematical Theory of Epidemics, New York: Hafner.

BANNISTER,G.(1975), "Population Change in Southern Ontario", Annals of the Association of American Geographers, Vol. 65, pp.177-188.

BARBER, G.(1972),"Growth Determinants in the Central Canada Urban System", in Bourne and McKinnon (1972), pp. 147-162.

BECKMANN, M.J. and J.C.McPHERSON (1970), "City Size Distribution in a Central Place Hierarchy: An Alternative Approach", Journal of Regional Science, Vol. 10, pp. 25-33.

BELL, F.W. (1967), "An Economic Forecasting Model for a Region", Journal of Regional Science, Vol. 7, pp. 109-117.

BERRY, B.J.L. (1961), "City Size Distribution and Economic Development", Economic Development and Cultural Change, Vol. 9, pp. 573-587.

__________ (1972), "Hierarchical Diffusion: The Basis of Developmental Filtering and Spread in a System of Growth Centres; in N.M. Hansen, ed., Growth Centres in Regional and Economic Development.

__________ and W.L. GARRISON (1958), "Alternative Explanations of Urban Rank - size Relationships", Annals of the Association of American Geographers, Vol. 48, pp. 81-91.

BLUMENFELD, H. (1955), "The Economic Base of the Metropolis", Journal of the American Institute of Planners, Vol. 21, pp. 114-32.

BOGUE, D.J. (1949), The Structure of the Metropolitan Community; a Study of Dominance and Subdominance, Ann Arbor: University of Michigan.

BONUS, H. (1973), "Quasi - Engel Curves, Diffusion, and the Ownership of Major Consumer Durables", Journal of Political Economy, Vol. 81, pp. 655-672.

BORTS, G.H. (1960), Regional Cycles in Manufacturing Employment in the United States, 1914-1953, New York: National Bureau of Economic Research.

__________ and J.L. STEIN (1964), Economic Growth in a Free Market, New York: Columbia University Press.

BOURNE, L.S. AND R. McKINNON, eds. (1972), Urban Systems Development in Central Canada: Selected Papers, Toronto: University of Toronto Press.

__________ and G. GAD (1972), "Urbanisation and Urban Growth in Ontario and Quebec: An Overview", in Bourne and McKinnon (1972), pp. 7-34.

BOWDEN, L.W. (1965), Diffusion of the Decision to Irrigate, Department of Geography, University of Chicago, Research Paper No. 97.

BROWN, L.A. (1968), Diffusion Processes and Location: A Conceptual Framework and Bibliography, Philadelphia: The Regional Science Research Institute, Bibliography No. 4.

CAMERON, R. (1975), "The Diffusion of Technology as a Problem in Economic History", Economic Geography, Vol. 51, pp. 217-230.

CASSETTI, E., L.J. KING and D. JEFFREY (1971), "Structural Imbalance in the U.S. Urban Economic System, 1960-1965", Geographical Analysis, Vol. 3, pp. 239-255.

CLIFF, A.D. and J.K. ORD (1971), "Evaluating the Percentage Points of a Spatial Autocorrelation Ceofficient", Geographical Analysis, Vol. 3, pp. 51-62.

COHEN, Y.S. (1972), Diffusion of an Innovation in an Urban System: The Spread of Planned Regional Shopping Centres in the United States, 1949-1968, Department of Geography, University of Chicago, Research Paper No. 140.

COLEMAN, J.S., E. KATZ and H. MENZEL (1957), "The Diffusion of an Innovation Among Physicians", Sociometry, Vol. 20, pp.253-270.

CONOVER, W.J. (1971), Practical Nonparametric Statistics, New York: Wiley.

CZAMANSKI, S. (1964), "A Model of Urban Growth", Paper and Proceedings of the Regional Science Association, Vol. 13, pp. 177-200.

__________ (1972), Regional Science Techniques in Practice: The Case of Nova Scotia, Lexington, Mass.: Lexington Books.

__________ et. al. (1968), An Econometric Model of Nova Scotia, Halifax, N.S.: Dalhousie University Institute of Public Affairs.

DARWENT, D.F. (1969), "Growth Poles and Growth Centres in Regional Planning - A Review", Environment and Planning, Vol. 1, pp.5-32.

DAVIES, J.B. (1972), "Behaviour of the Ontario - Quebec Urban System by Size Distribution", in Bourne and McKinnon (1972), pp. 35-50.

DAY, R.H. (1970), "A Theoretical Note on the Diffusion of Something New", Geographical Analysis, Vol. 2, pp. 68-76.

DZIEWONSKI, K. (1972), "General Theory of Rank - size Distributions in Regional Settlement Systems: Reappraisal and Reformulation of the Rank - size Rule", Papers and Proceedings of the Regional Science Association, Vol. 29, pp. 73-86.

FANO, P. (1969), "Organisation, City Size Distributions and Central Places", Papers and Proceedings of the Regional Science Association, Vol. 22, pp. 29-38.

FARRAR, D.E. and R.R. GLAUBER (1967), "Multicollinearity in Regression Analysis, the Problem Revisited", The Review of Economics and Statistics, Vol. 49, pp. 92-107.

FRIEDMANN, J. (1966), Regional Development Policy: A Case Study of Venezuela, Cambridge, Mass.: M.I.T. Press.

GIST, R.R. (1968), Retailing, New York: Wiley.

GOLANT, S.M. (1972), "Regression Models of Urban Growth in Ontario and Quebec", in Bourne and McKinnon (1972), pp. 117-131.

GOULD, P.E. (1969), Spatial Diffusion, Washington, D.C.: Association of American Geographers, Resource Paper No. 4.

GRILICHES, Z. (1957), "Hybrid Corn: An Exploration in the Economics of Technological Change", Econometrica, Vol. 25, pp. 501-522.

__________ (1962), "Profitability Versus Interaction: Another False Dichotomy", Rural Sociology, Vol. 27, pp. 327-330.

HÄGERSTRAND, T. (1952), The Propagation of Innovation Waves, Lund: Gleerup, Lund Studies in Geography, Series B, No. 4.

__________ (1967) Innovation Diffusion as a Spatial Process, translated by A.R. Pred, Chicago: University of Chicago Press.

HAGGETT, P. (1972), Geography: A Modern Synthesis, New York: Harper and Row.

HALL, G.P. and R.E. JOHNSON (1970), "Transfers of United States Aerospace Technolgoy to Japan", in R. Vernon, ed., The Technological Factor in International Trade, New York: Columbia University Press.

HANSEN , N.M. (1971), Intermediate - size Cities as Growth Centers: Applications for Kentucky, the Piedmont Crescent, the Ozarks and Texas, New York: Praeger.

HARTWELL, R.M. (1971), The Industrial Revolution and Economic Growth, London: Methuen.

HERMANSEN, T. (1972), "Development Poles and Related Theories: A Synoptic Review", in N.M. Hansen, ed., Growth Centres in Regional Economic Development, New York: Free Press, pp. 160-203.

HIRSCHMANN, A.O. (1958), The Strategy of Economic Development, New Haven, Conn.: Yale University Press.

HODGE, G. (1972), "Regional and Structural Components of Urban Growth", in Bourne and McKinnon (1972), pp. 108-116.

HODGSON, M.J. (1972), "Variability in the Growth of Small Urban Areas", in Bourne and McKinnon (1972), pp. 132-146.

HOEL, (1962), Introduction to Mathematical Statistics, Third Edition, New York: Wiley.

HUDSON, J.C. (1969), "Diffusion in a Central Place System", Geographical Analysis, Vol. 1, pp. 45-58.

__________ (1972), Geographical Diffusion Theory, Evanston, Illinois: Northwestern University Studies in Geography, No. 19.

ISARD, W. (1960), Methods of Regional Analysis, Cambridge, Mass.: M.I.T. Press.

JEFFERSON, M. (1939), "The Law of the Primate City", Geographical Review, Vol. 29, pp. 226-232.

JEFFREY, D. (1974), "Regional Fluctuations in Employment within the U.S. Urban Economic System: A Study of the Spatial Impact of Short Term Economic Change", Economic Geography, Vol. 50, pp. 111-123.

JOHNSTON, J. (1963), Econometric Methods, New York: McGraw Hill.

KALDOR, N. (1966), "Marginal Productivity and the Macro-economic Theories of Distribution", Review of Economic Studies, Vol. 33, pp. 309-319.

__________ (1970), " The Case for Regional Policies", Scottish Journal of Political Economy, Vol. 17, pp. 337-347.

KARASKA, G. (1969), "Manufacturing Linkages in the Philadelphia Economy: Some Evidence of External Agglomeration Forces", Geographical Analysis, Vol. 1, pp. 354-369.

KING, L.J. (1967), "Discriminatory Analysis of Urban Growth Patterns in Ontario and Quebec, 1951-1961", Annals of the Association of American Geographers, Vol. 57, pp. 566-578.

__________ (1969), Statistical Analysis in Geography, Englewood Cliffs, New Jersey: Prentice - Hall.

__________ (1974), " Conceptual Limitations and Data Problems in the Fashioning of Growth Pole Strategies: The Case of Ontario, Canada", Geoforum, No. 17, pp. 61-67.

KOTTIS, G.C. (1972), "Problems of Defining an Urban Economy", Planning Outlook, Vol. 12, pp. 9-16.

LANE, T. (1966), "The Urban Base Multiplier: An Evaluation of the State of the Art", Land Economics, Vol. 42.

LALONDE, B.J. (1967), "The Logistics of Retail Location", in R. Gist, ed., Management Perspectives in Retailing, New York: Wiley.

LASUEN, J.R. (1969), "On Growth Poles", Urban Studies, Vol. 6, pp. 137-161.

__________ (1973), "Urbanisation and Development - the Temporal Interaction between Geographical and Sectoral Clusters", Urban Studies, Vol. 10, pp. 163-188.

MADDEN, C.H. (1956), "On Some Indications of Stability in the Growth of Cities in the United States", Economic Development and Cultural Change, Vol. 4, pp. 236-252.

MANSFIELD, E. (1961), " Technical Change and the Rate of Imitation", Econometrica, Vol. 29, pp. 741-766.

__________ (1968), The Economics of Technological Change, New York: W.W. Norton.

__________ (1972), "Contribution of R. and D. to Economic Growth in the United States", Science, 166, pp. 477-486.

MATHIAS, P. (1969), The First Industrial Nation: An Economic History of Britain, 1700-1914, New York: Charles Scribner's Sons.

MATTILLA, J.M. (1973), " A Metropolitan Income Distribution Model and the Estimation of Metropolitan Income Multipliers". Journal of Regional Science, Vol. 13, pp. 1-16.

McKEEVER, R.J. (1953), Shopping Centres: Principles and Policies, Washington, D.C.: Urban Land Institute.

MILLS, E.S. (1972), Urban Economics, Grenview, Illinois: Scott Foresman and Co.

MISRA, R.A. (1968), Diffusion of Agricultural Innovations, Mysore, India: University of Mysore Press.

MORRILL, R.L. (1968), "Waves of Spatial Diffusion", Journal of Regional Science, Vol. 8, pp. 1-18.

MYRDAL, G. (1957), Economic Theory and Underdeveloped Regions, London: Duckworth.

NICHOLS, V. (1969), Growth Poles: An Investigation of their Potential as a Tool for Regional Economic Development, Regional Science Research Institute. D.P. No. 30.

NORTH, D.C. (1955), "Location Theory and Regional Economic Growth", Journal of Political Economy, Vol. 63, pp. 243-258.

ONTARIO DEPARTMENT OF REGIONAL ECONOMICS: REGIONAL DEVELOPMENT BRANCH (1970), Presentation of Design for Development: Toronto Centred Region, Toronto: Government Mimeograph.

PAELINCK, J. (1970), "Dynamic Urban Growth Models", Papers and Proceedings of the Regional Science Association, Vol. 24, pp. 25-37.

PARR, J.B. (1970), "Models of City Size in an Urban System", Papers and Proceedings of the Regional Science Association, Vol. 25, pp. 221-253.

PEDERSEN, P.O. (1970), "Innovation Diffusion within and Between National Urban Systems", Geographical Analysis, Vol. 2, pp. 203-255.

PERROUX, F. (1955), "Notes sur la Notion de 'Pole de Croissance'", Economique Appliquee, Vol. 1, pp. 307-320.

__________ (1970), "Note on the Concept of Growth Poles", in D.L. McKee, R.D. Dean and W.H. Leahy, eds., Regional Economics, New York: Free Press, pp. 93-104.

PRED, A.R. (1966), Spatial Dynamics of U.S. Urban Industrial Growth, 1800-1914, Cambridge, Mass.: M.I.T. Press.

__________ (1971), "Large City Interdependence and the Preelectronic Diffusion of Innovations in the U.S.", Geographical Analysis, Vol. 3, pp. 165-182.

__________ (1973), " The Growth and Development of Systems of Cities in Advanced Economies", in A.R. Pred and G.E. Tornquist, Systems of Cities and Information Flows: Two Essays, Lund: Gleerup, Lund Studies in Geography, Series B, No. 38.

__________ (1975), "Diffusion, Organisational Spatial Structure and City - system Development", Economic Geography, Vol. 51, pp. 252-268.

PRESTON, V.A. (1976), Preliminary Tests of a Dynamic Model of Urban Growth, McMaster University, Department of Geography, Unpublished Master's Research Paper.

PYLE, G. (1969), "The Diffusion of Cholera in the U.S. in the Nineteenth Century", Geographical Analysis, Vol. 1, pp. 59-76.

QUANDT, R.E. (1964), "Statistical Discrimination Among Alternative Hypotheses and Some Economic Regularities", Journal of Regional Science, Vol. 5, pp. 1-23.

RICHARDSON, H.W. (1969), Regional Economics: Location Theory, Urban Structure and Regional Change, London: Weidenfeld and Nicholson.

__________ (1973a), The Economics of Urban Size, Lexington, Mass: Lexington Books.

__________ (1973b), Regional Growth Theory, London: McMillan.

__________ (1973c), "Theory of the Distribution of City Sizes: Review and Prospects", Regional Studies, Vol. 7, pp. 239-251.

RIDDELL, J.B. (1970), "On Structuring a Migration Model", Geographical Analysis, Vol. 2, pp. 403-409.

ROBSON, B.T. (1973), Urban Growth: An Approach, London: Methuen.

ROGERS, E.M. (1962), Diffusion of Innovations, New York: Free Press.

__________ and A.E. Havens (1961), "Adoption of Hybrid Corn: Profitability and the Interaction Effect", Rural Sociology, Vol. 26, pp. 409-416.

ROSENBERG, N., Ed. (1971), The Economics of Technological Change, London: Penguin Books.

ROSENSTEIN - RODAN, P.N. (1943), "Problems of Industrialisation of Eastern and South - eastern Europe", Economic Journal, Vol. 53, pp. 202-211.

SCHMOOKLER, J. (1962),"Economic Sources of Inventive Activity", Journal of Economic History, pp. 1-20.

__________ (1966), Invention and Economic Growth, Cambridge, Mass.: Harvard University Press.

SCHULTZ, T.W. (1953), The Economic Organization of Agriculture, New York.

SCHUMPETER, J. (1928), "The Instability of Capitalism", Economic Journal, pp. 361-336.

SCHON, D.A. (1967), Technology and Change, New York: Dell Publishing Company.

SHEPPARD, E.S. (1974), Diffusion Through the Urban System: The Construction of Shopping Centres in Canada, University of Toronto, Department of Geography, Unpublished Geographical Research Paper.

SIMMONS, J.W. (1972), "Interaction among the Cities of Ontario and Quebec", in Bourne and McKinnon (1972), pp. 198-220.

SIMON, H.A. (1955), "On a Class of Skew Distribution Functions", Biometrika, Vol. 42, pp. 425-440.

__________ and C.P. Bonini (1958), "The Size Distribution of Business Firms", American Economic Review, Vol. 48, pp. 607-617.

THOMAS, M.D. and R.B. LeHERON (1975), "Perspectives on Technological Change and the Process of Diffusion in the Manufacturing Sector", Economic Geography, Vol. 51, pp. 231-251.

THOMPSON, W.R. (1962)," Locational Differences in Invention Effort and their Determinants", in The Rate and Direction of Inventive Activity: Economic and Social Factors, Princeton: National Bureau of Economic Research.

__________ (1965), A Preface to Urban Economics, Baltimore: John Hopkins University Press.

__________ (1968)," Internal and External Factors in the Development of Urban Economies", in H.S. Perloff and L. Wingo Jr., eds. Issues in Urban Economics, Baltimore; John Hopkins University Press.

TIEBOUT, C.M. (1956), "Exports and Regional Economic Growth", Journal of Political Economy, Vol. 64, pp. 160-169.

ULLMAN, E. (1962), "The Nature of Cities Reconsidered", Papers and Proceedings of the Regional Science Association, Vol. 9, pp. 7-23.

WEBBER, M.J. (1972), Impact of Uncertainty on Location, Cambridge, Mass.: M.I.T. Press.

_______ (1974), Private Communication.

WENSLEY, A.J. and P.S. FLORENCE (1940), "Recent Industrial Concentration, Especially in the Midlands", Review of Economic Studies, Vol. 7, pp. 139-158.

YEATES, M.H. (1975), Main Street, Toronto: MacMillan of Canada.

ADDENDUM

NADER, G.A. (1975), Cities of Canada, Volume 1: Theoretical, Historical and Planning Perspectives, Toronto: MacMillan of Canada.

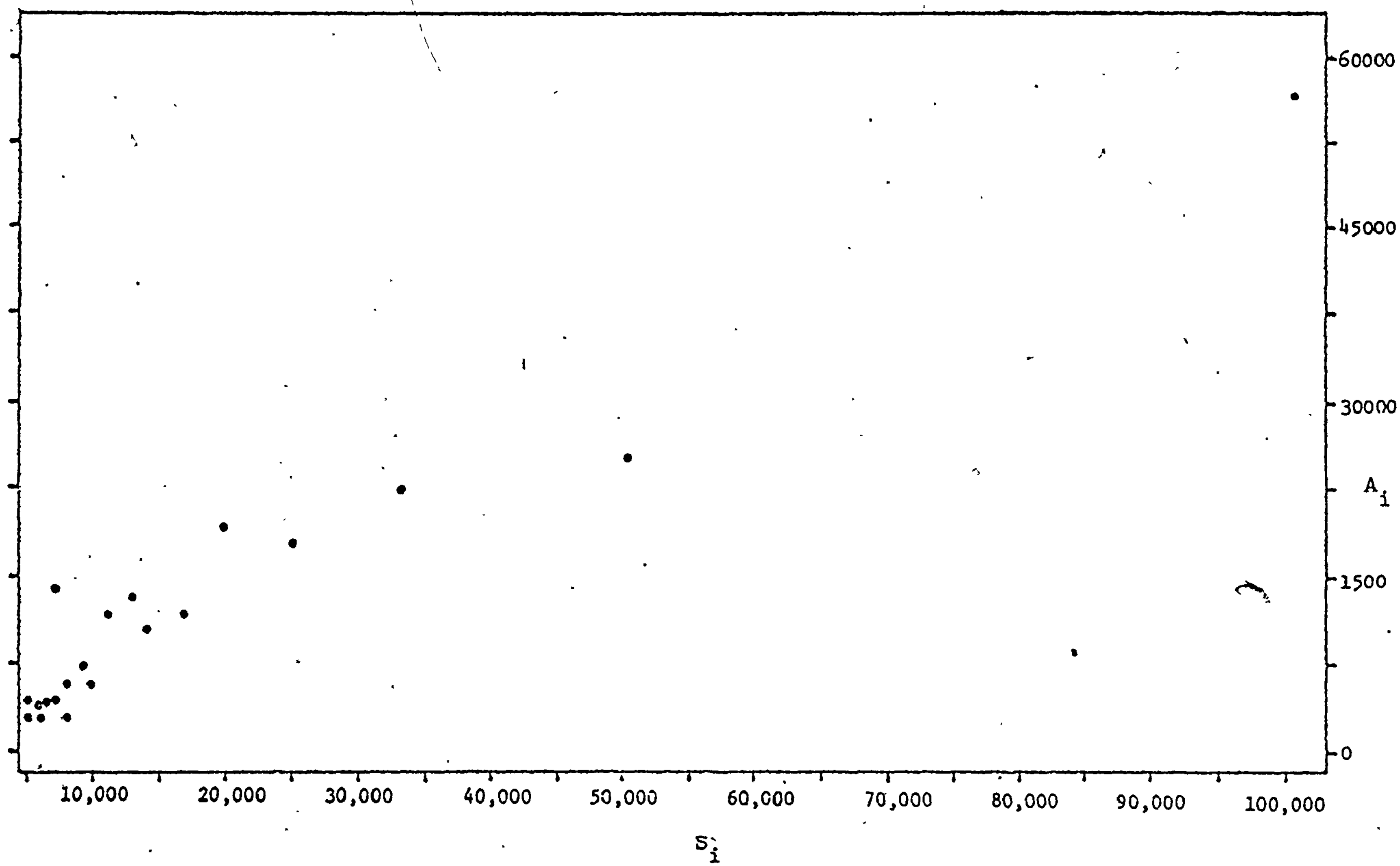
SHAW, R.P. (1975), Migration Theory and Fact: A Review and Bibliography of Current Literature, Philadelphia: The Regional Science Research Institute, Bibliography No. 5.

ZIPF, G.K. (1949), Human Behaviour and the Principle of Least Effort, Reading, Mass: Addison - Wesley.

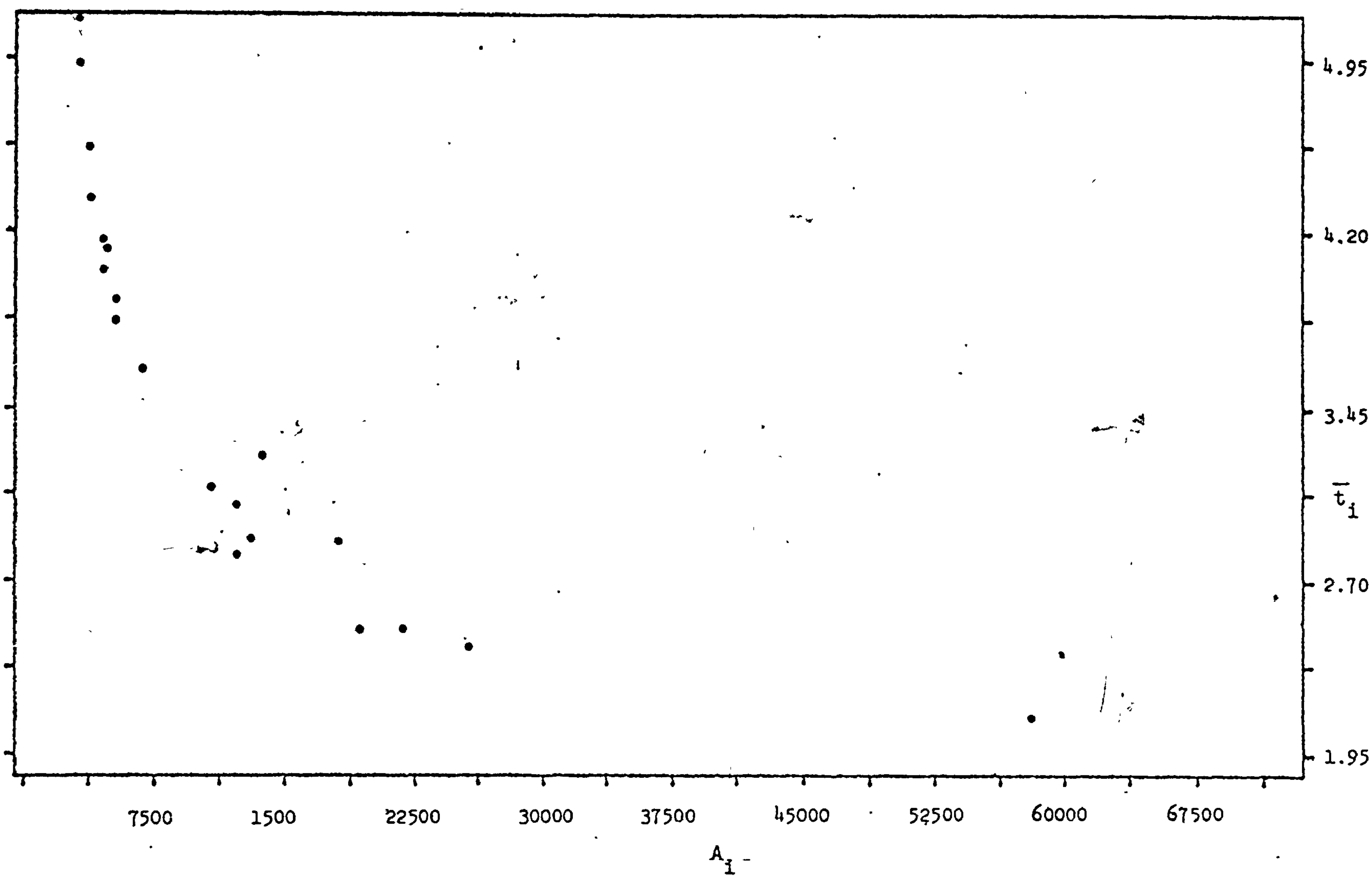
APPENDIX 1

SAMPLE GRAPHS FOR THE DIFFUSION PROCESS WITHIN DIFFERENT SYSTEMS OF CITIES

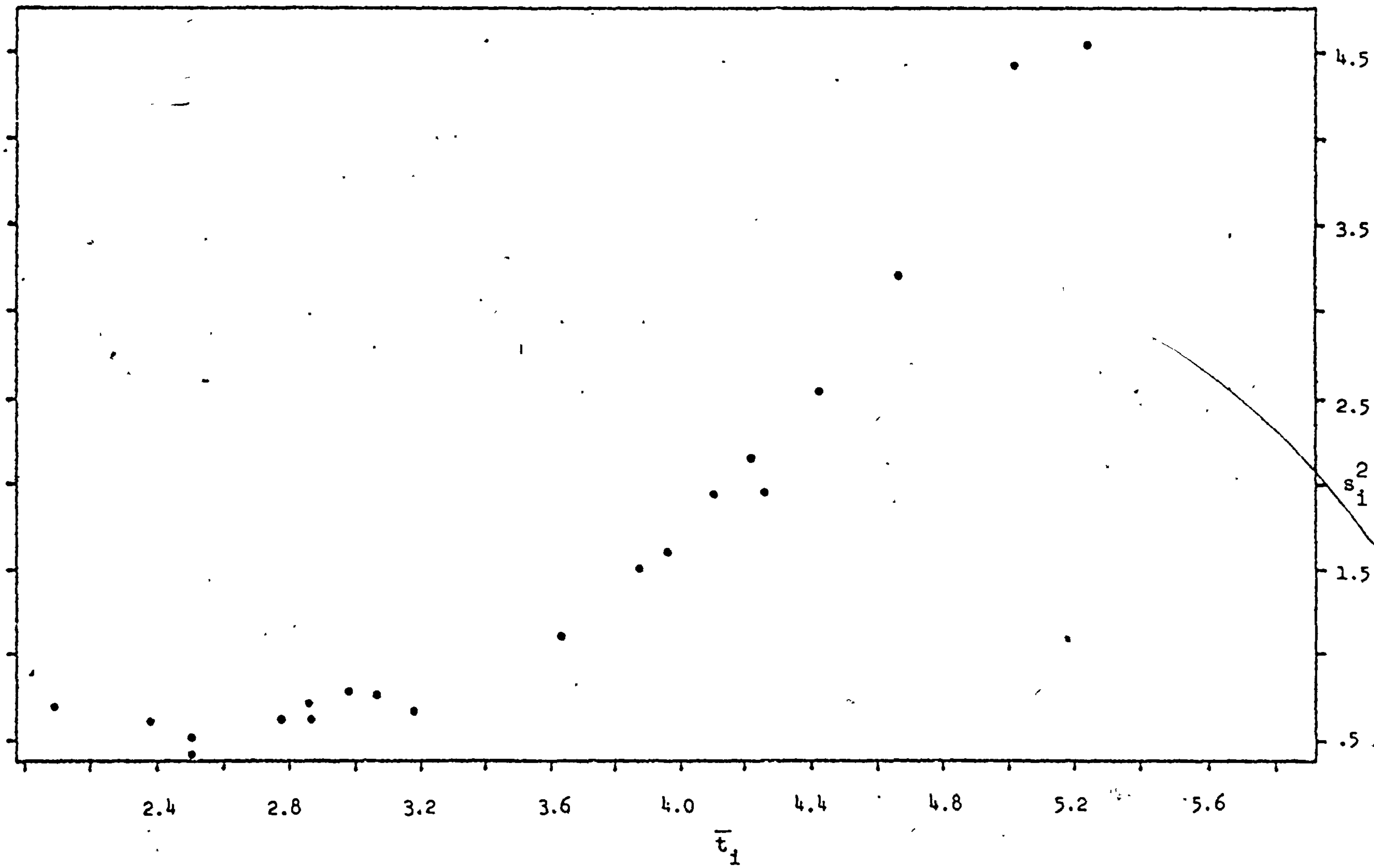
SYSTEM 2: CITY SIZE (S_i) AGAINST CITY INFORMATION POTENTIAL (A_i)



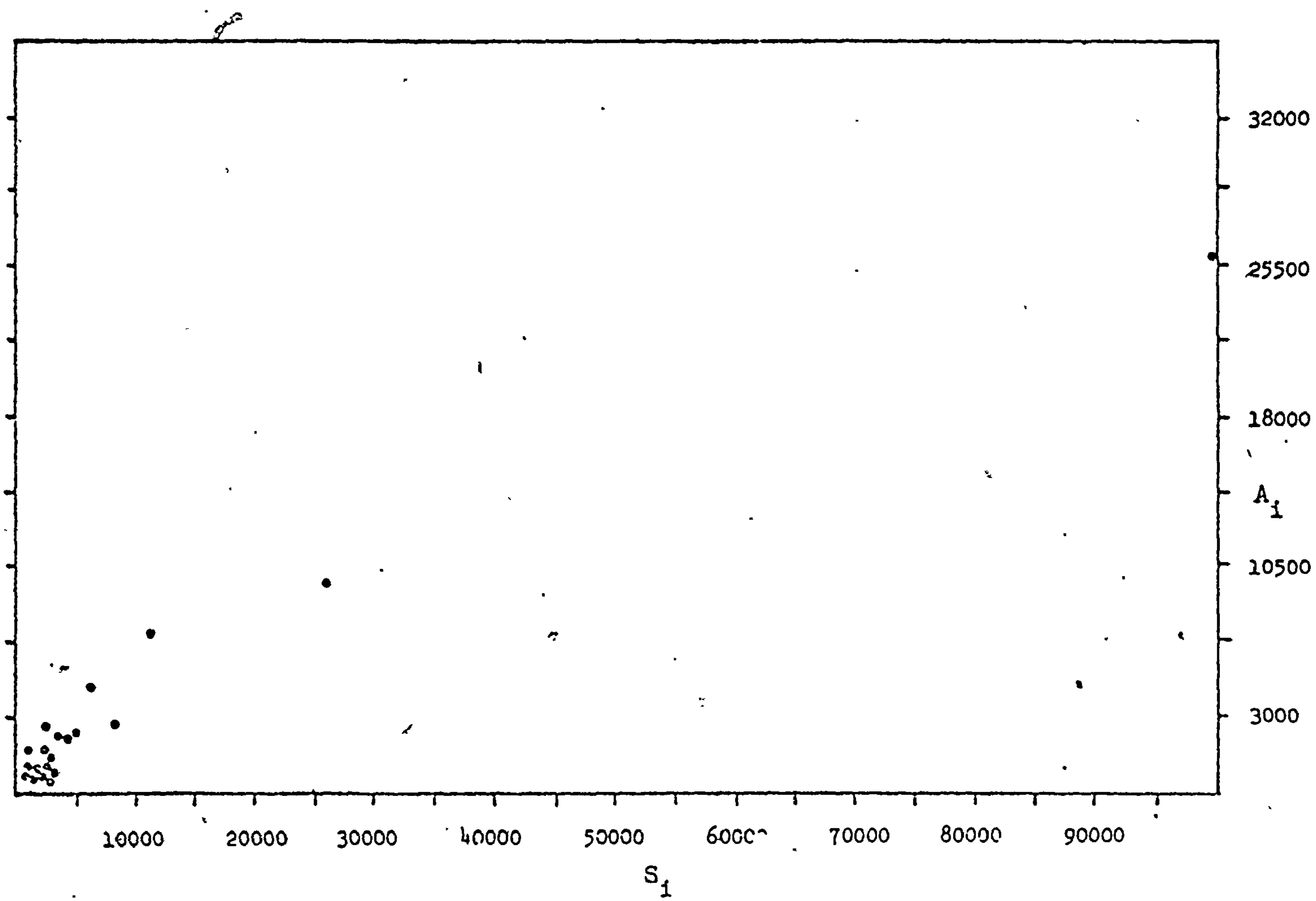
SYSTEM 2: CITY INFORMATION POTENTIAL (A_i) AGAINST CITY MEAN ADOPTION TIME ($\bar{t}_i$)



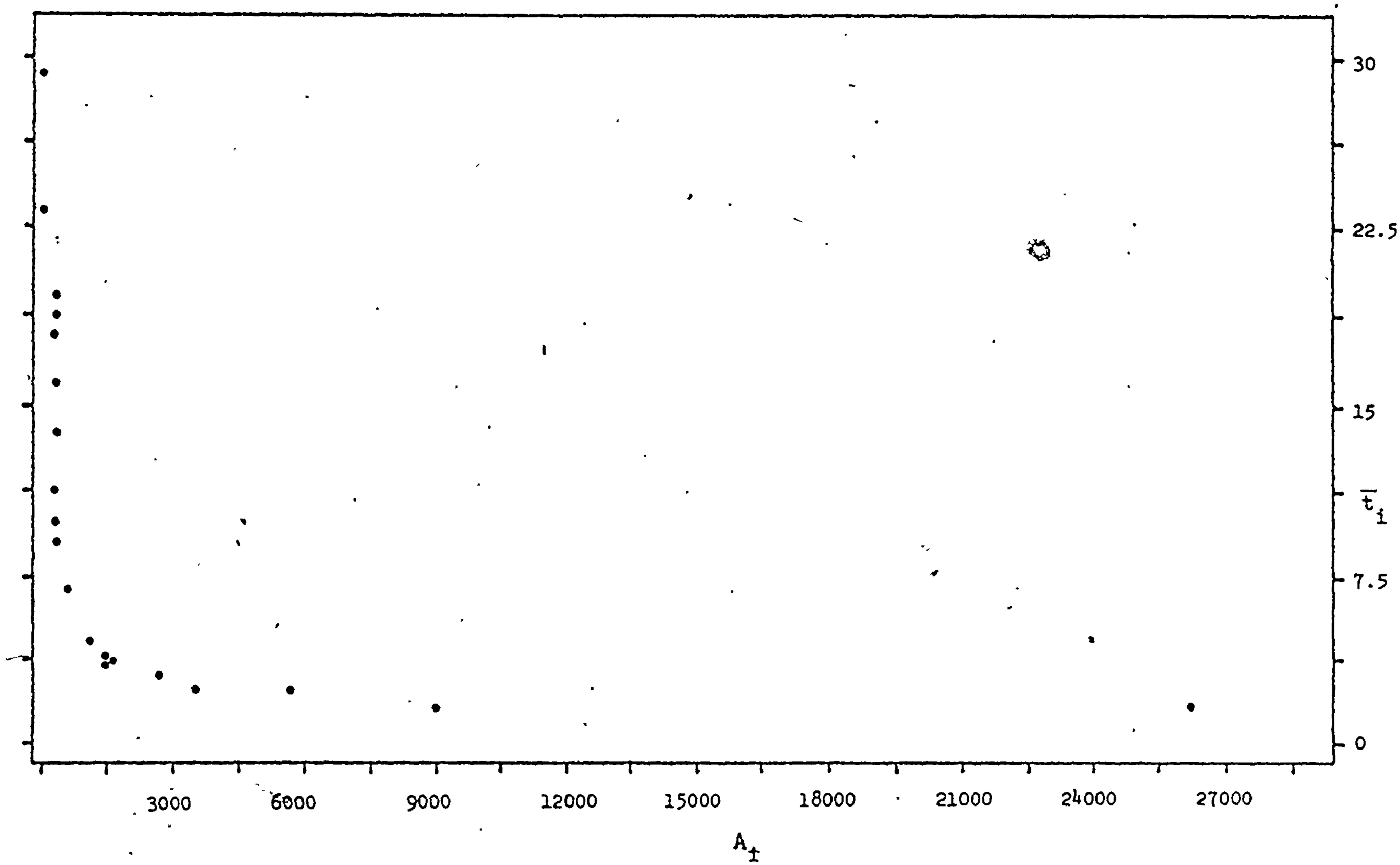
SYSTEM 2: MEAN ADOPTION TIME ($\bar{t}_i$) AGAINST VARIANCE OF ADOPTION TIME (s_i^2)



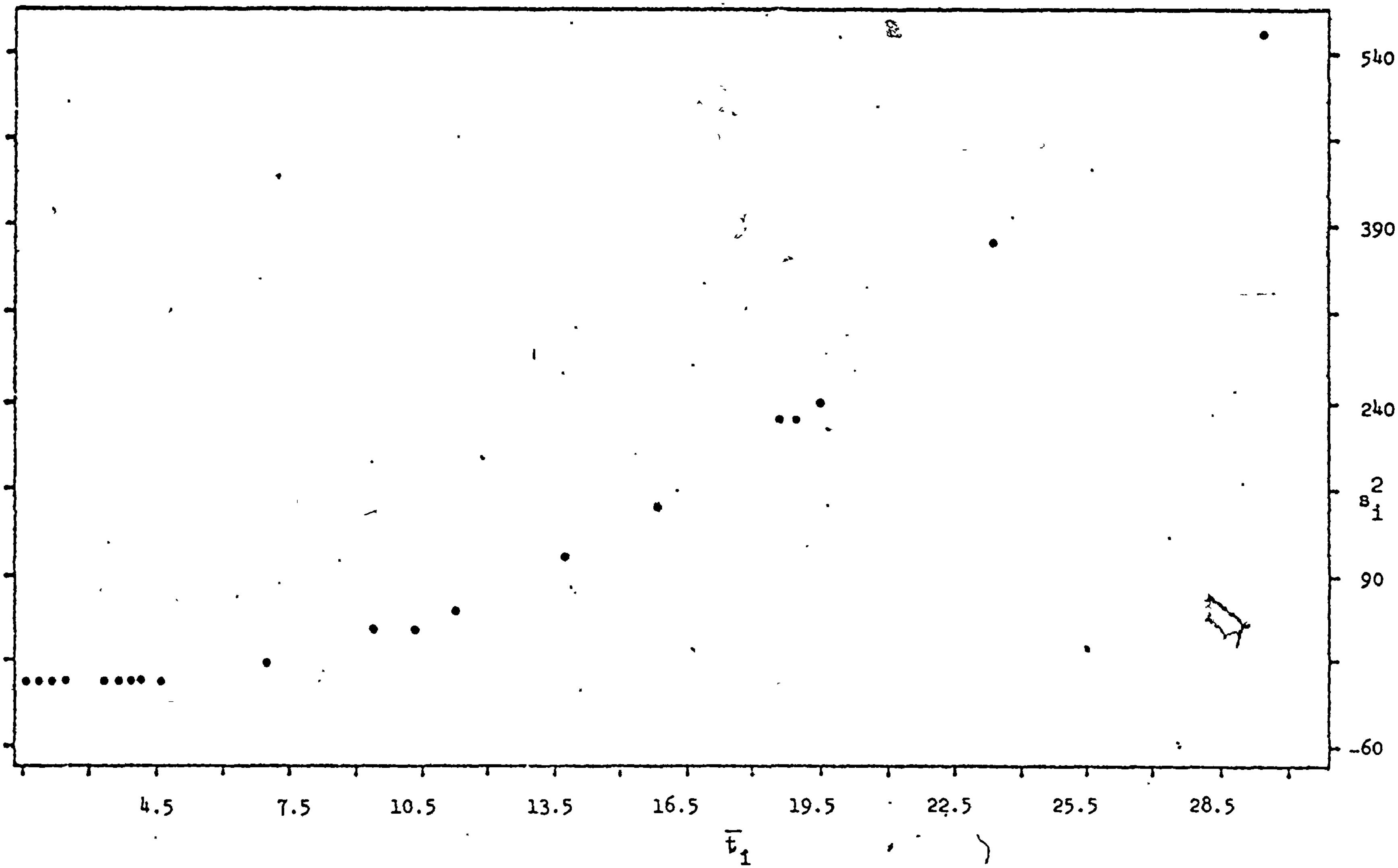
SYSTEM 3: CITY SIZE (S_i) AGAINST CITY INFORMATION POTENTIAL (A_i)



SYSTEM 3: CITY INFORMATION POTENTIAL (A_i) AGAINST CITY MEAN ADOPTION TIME ($\bar{t}_i$)



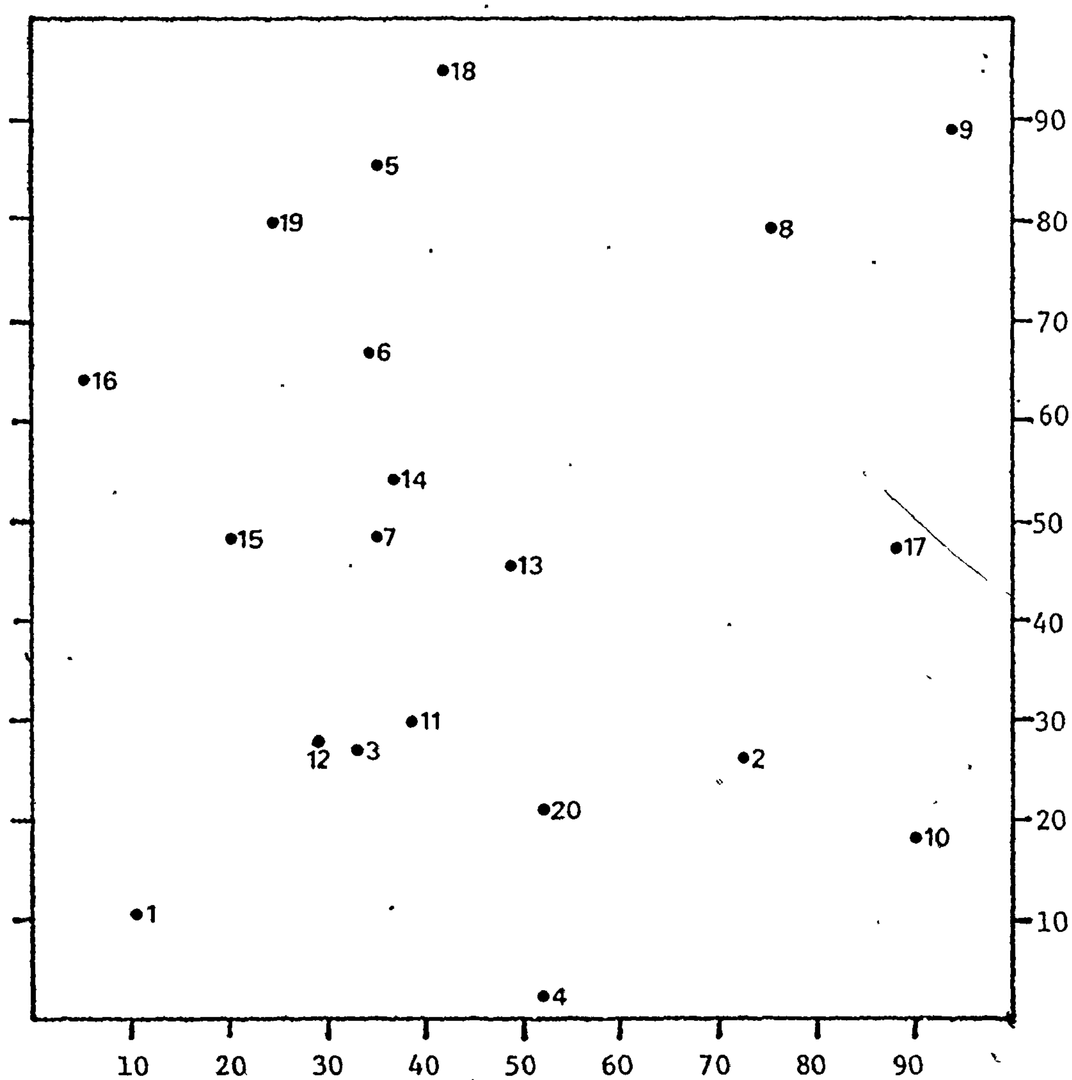
SYSTEM 3: MEAN ADOPTION TIME ($\overline{t}_i$) AGAINST VARIANCE OF ADOPTION TIME (s_i^2)



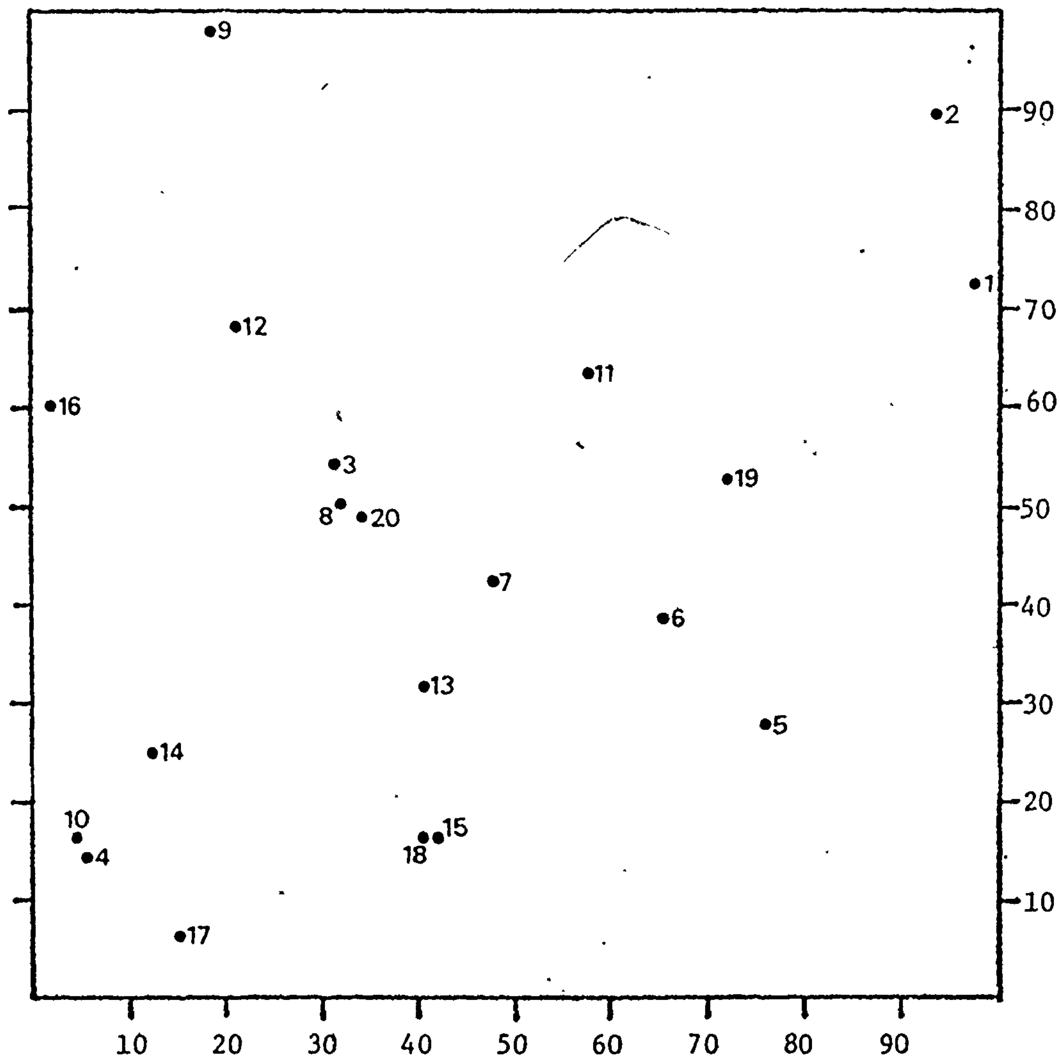
APPENDIX 2

EIGHTEEN LOCATION PATTERNS

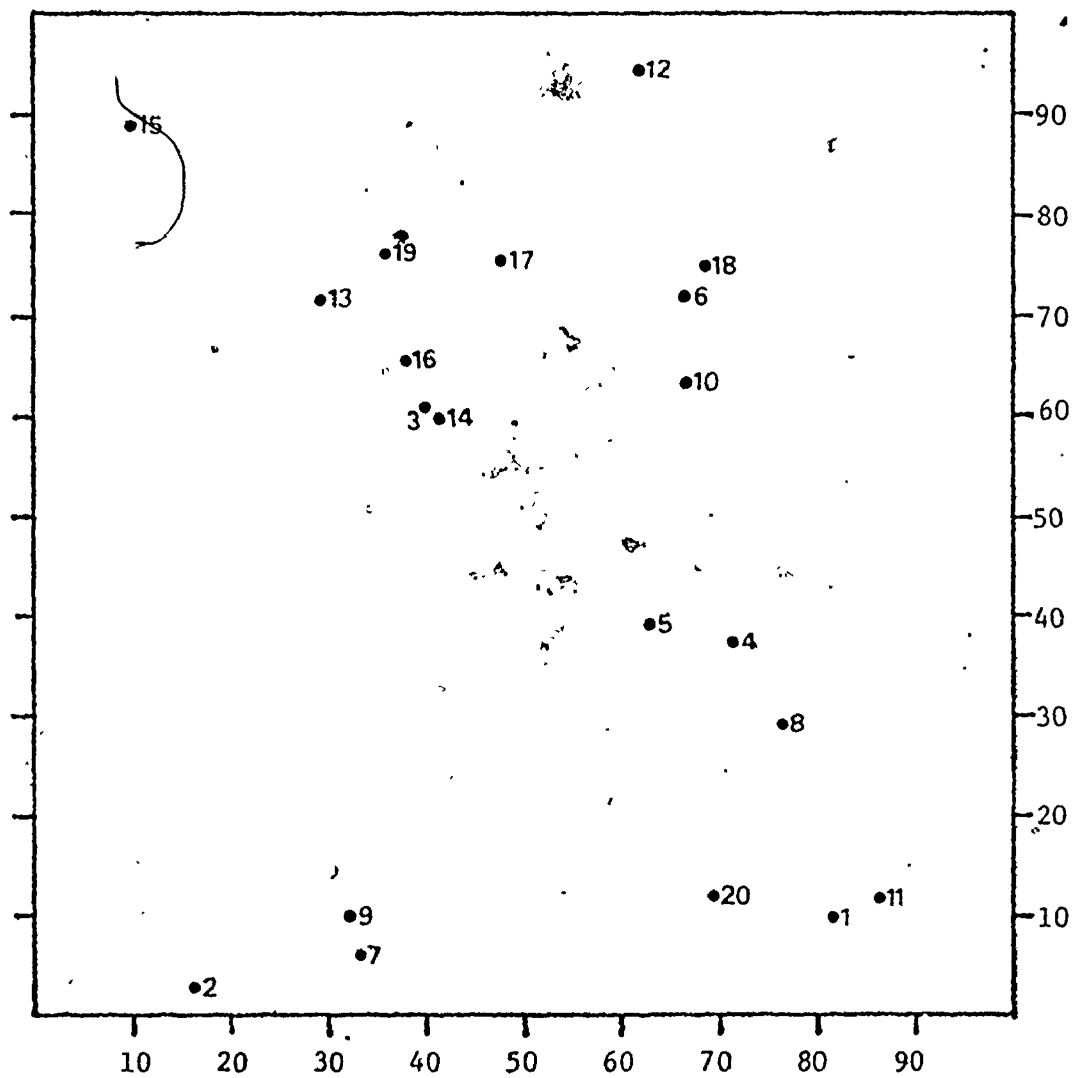
LOCATION PATTERN NO. 3



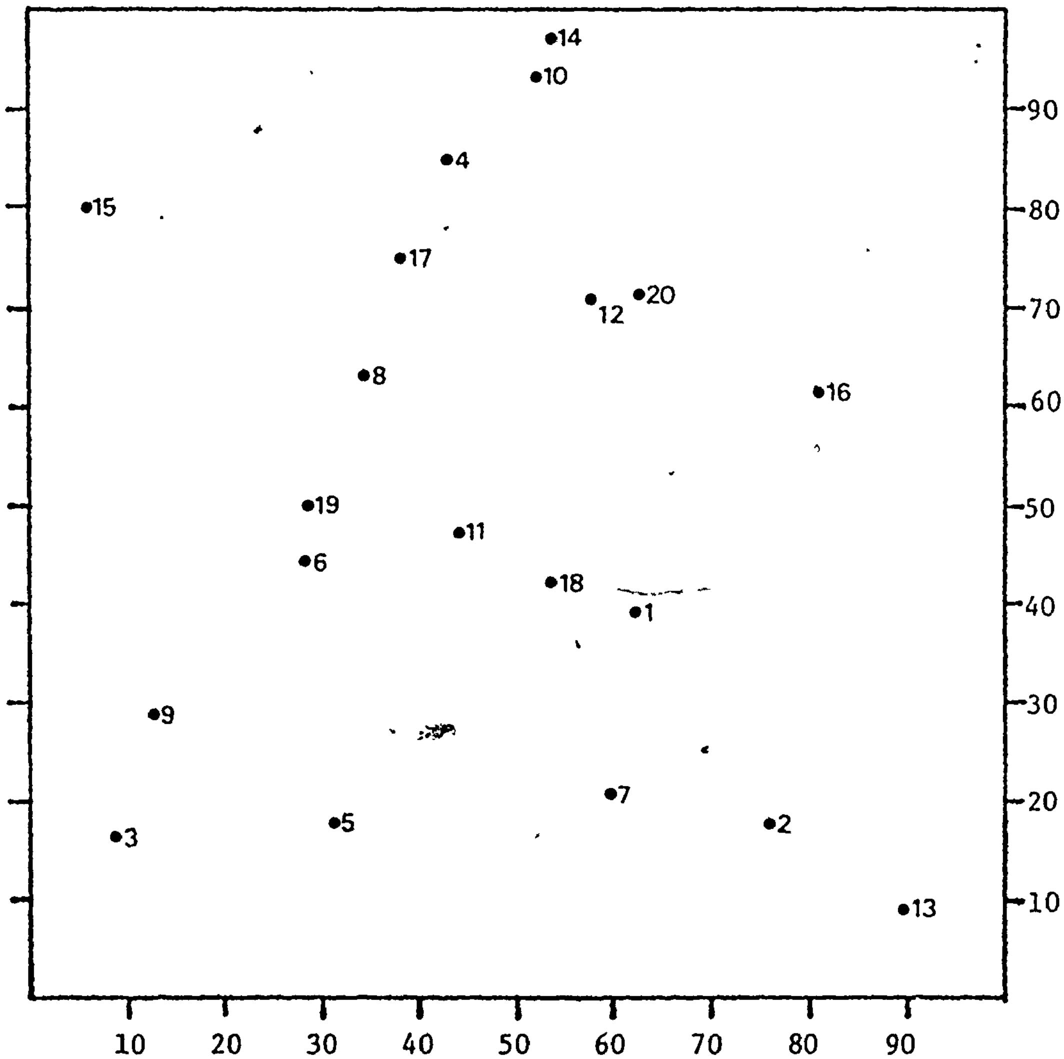
LOCATION PATTERN NO. 4



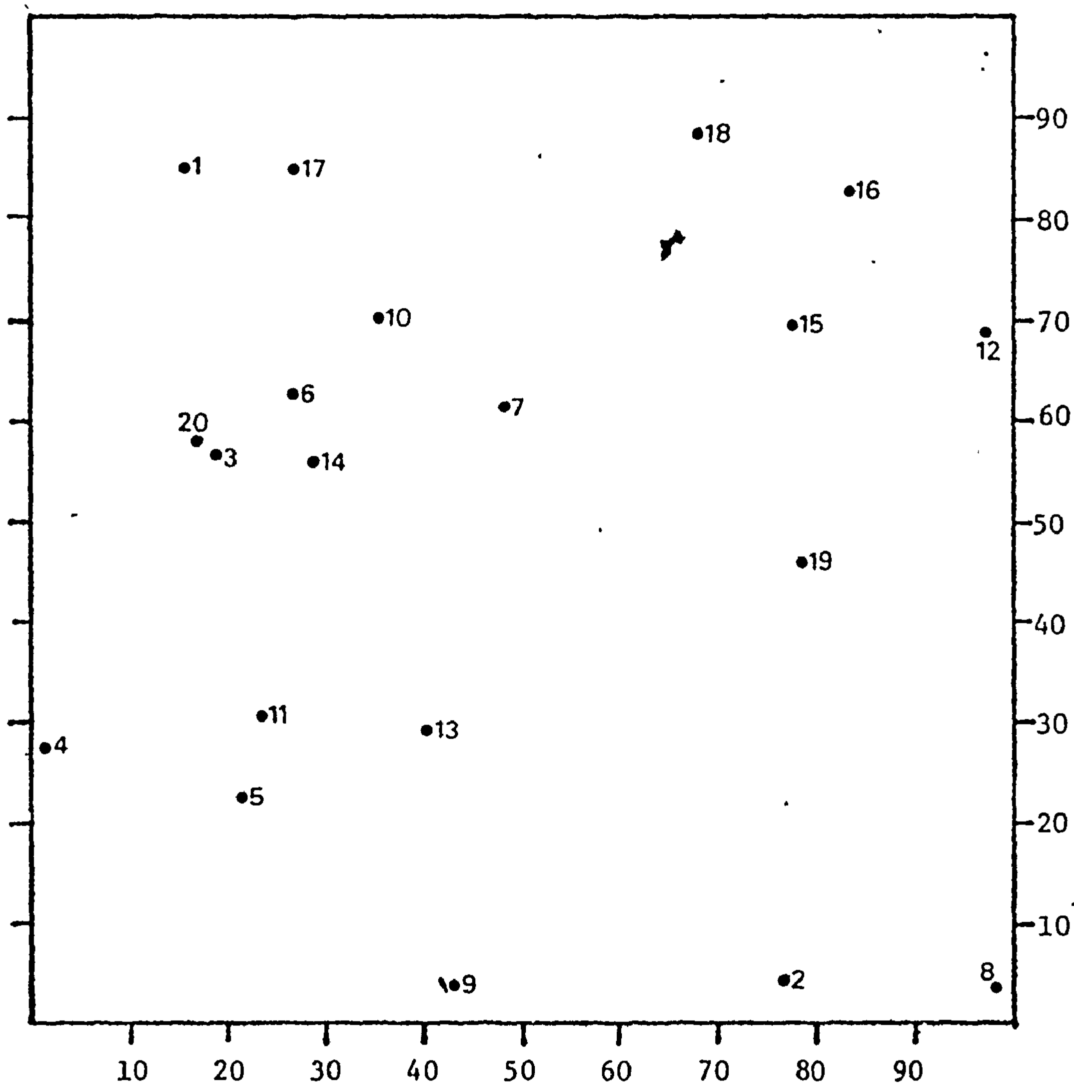
LOCATION PATTERN NO. 5



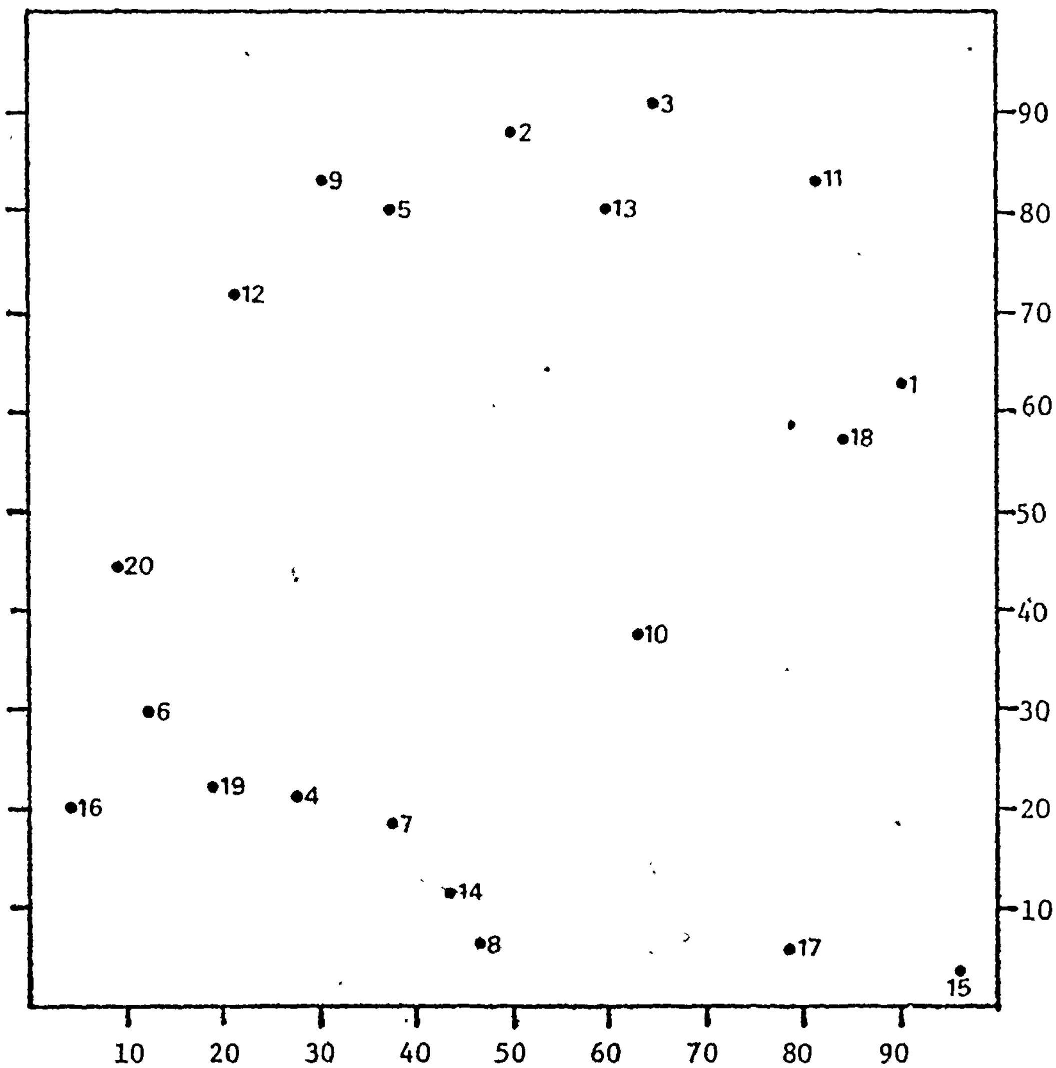
LOCATION PATTERN NO. 6



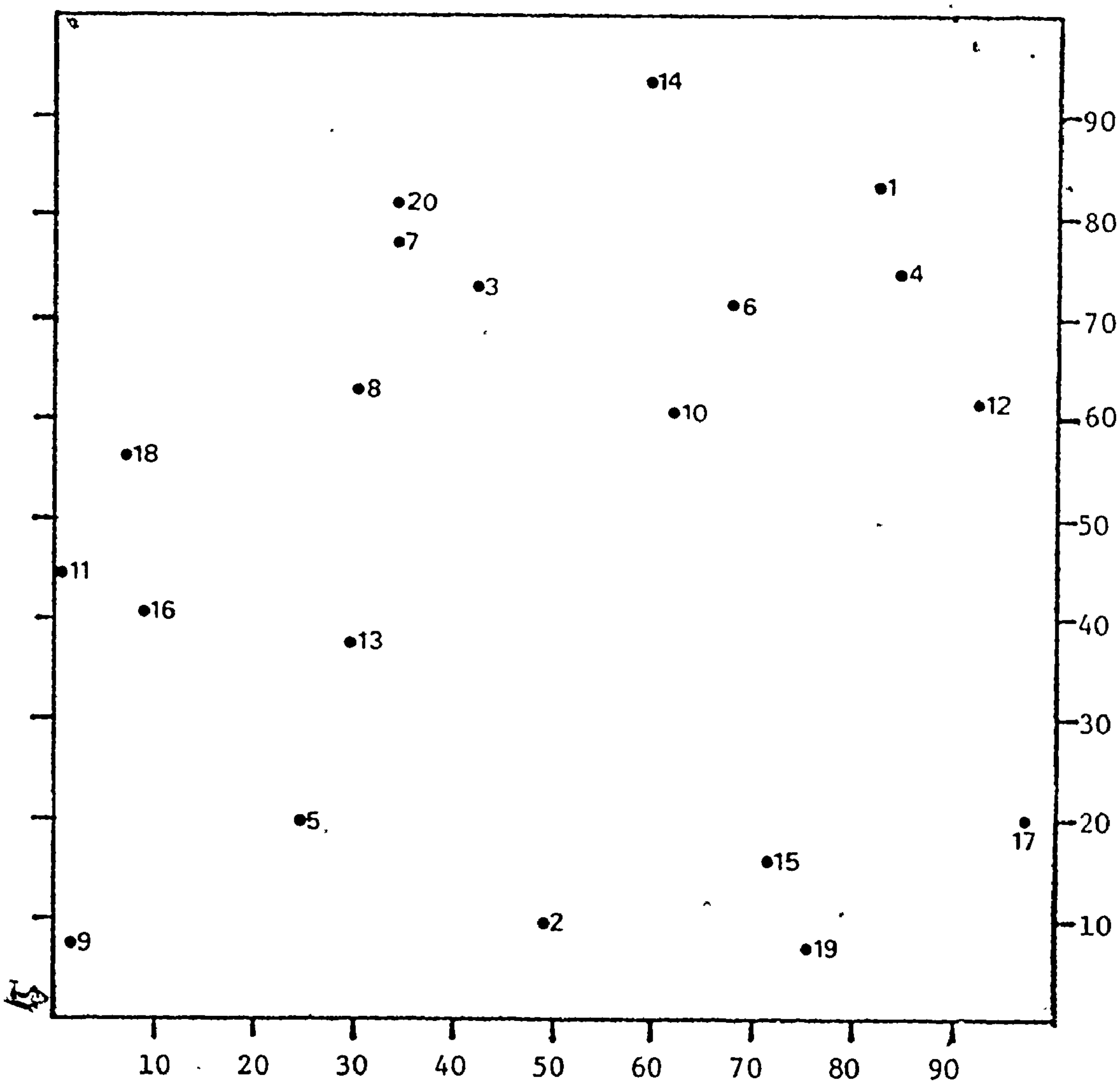
LOCATION PATTERN NO. 7



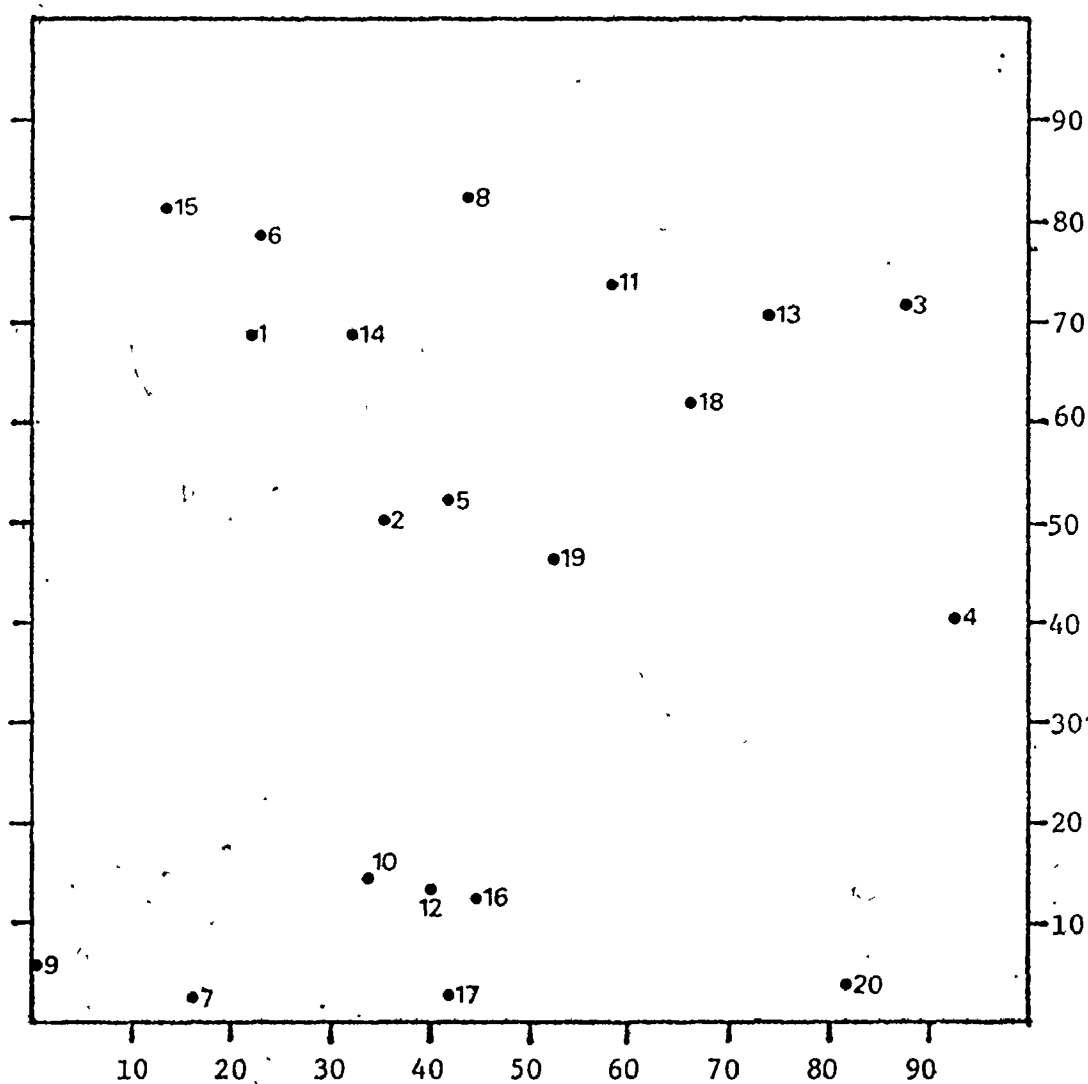
LOCATION PATTERN NO. 8



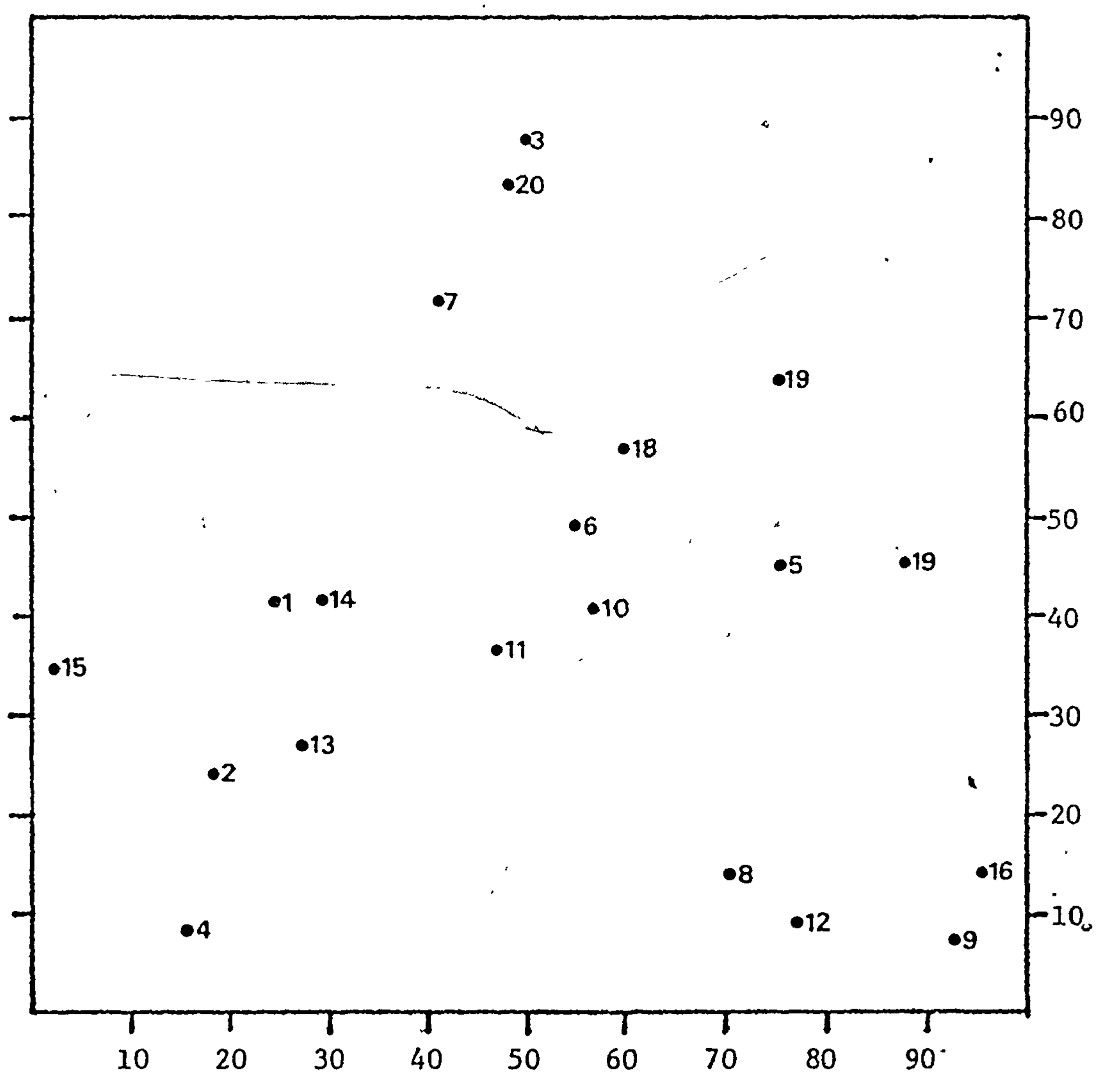
LOCATION PATTERN NO. 9



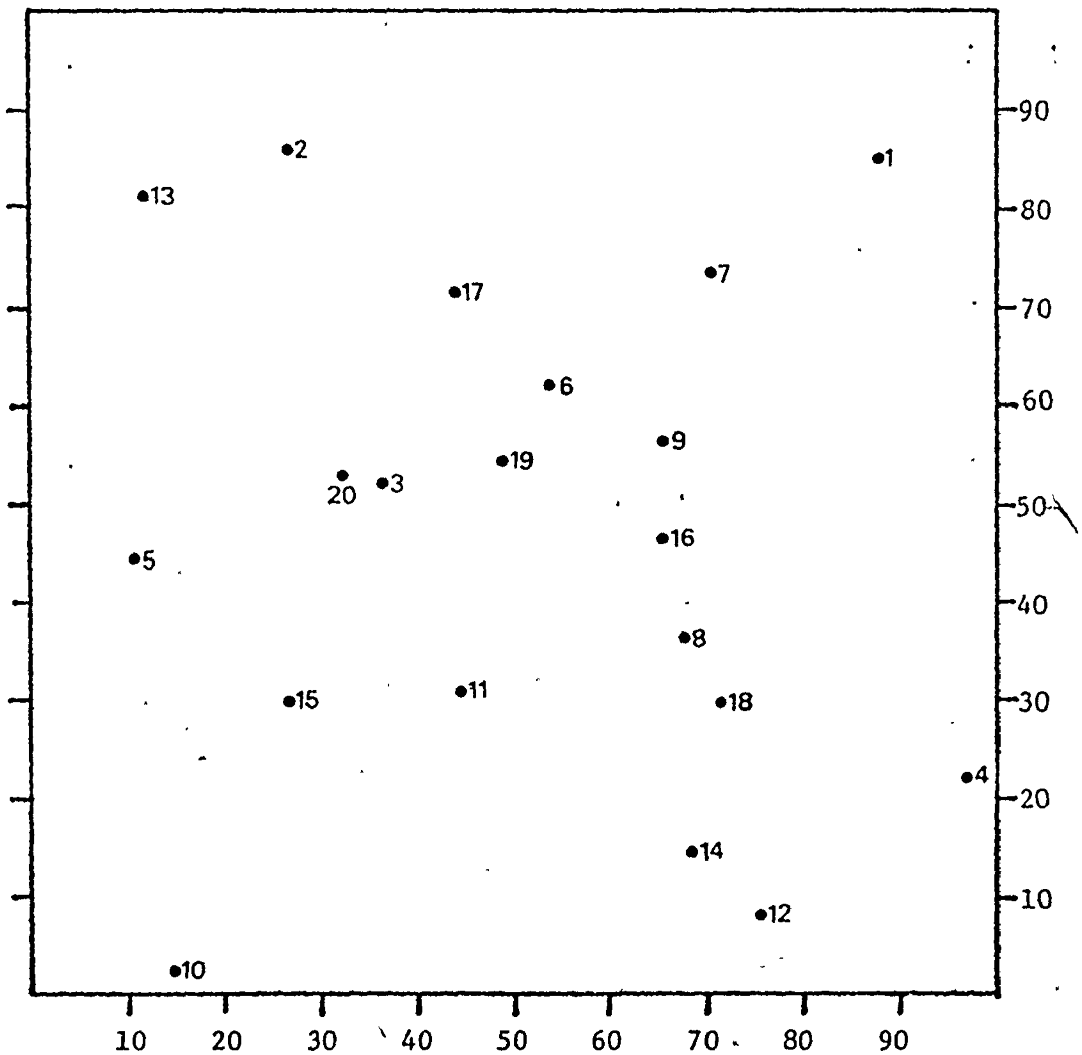
LOCATION PATTERN NO. 10



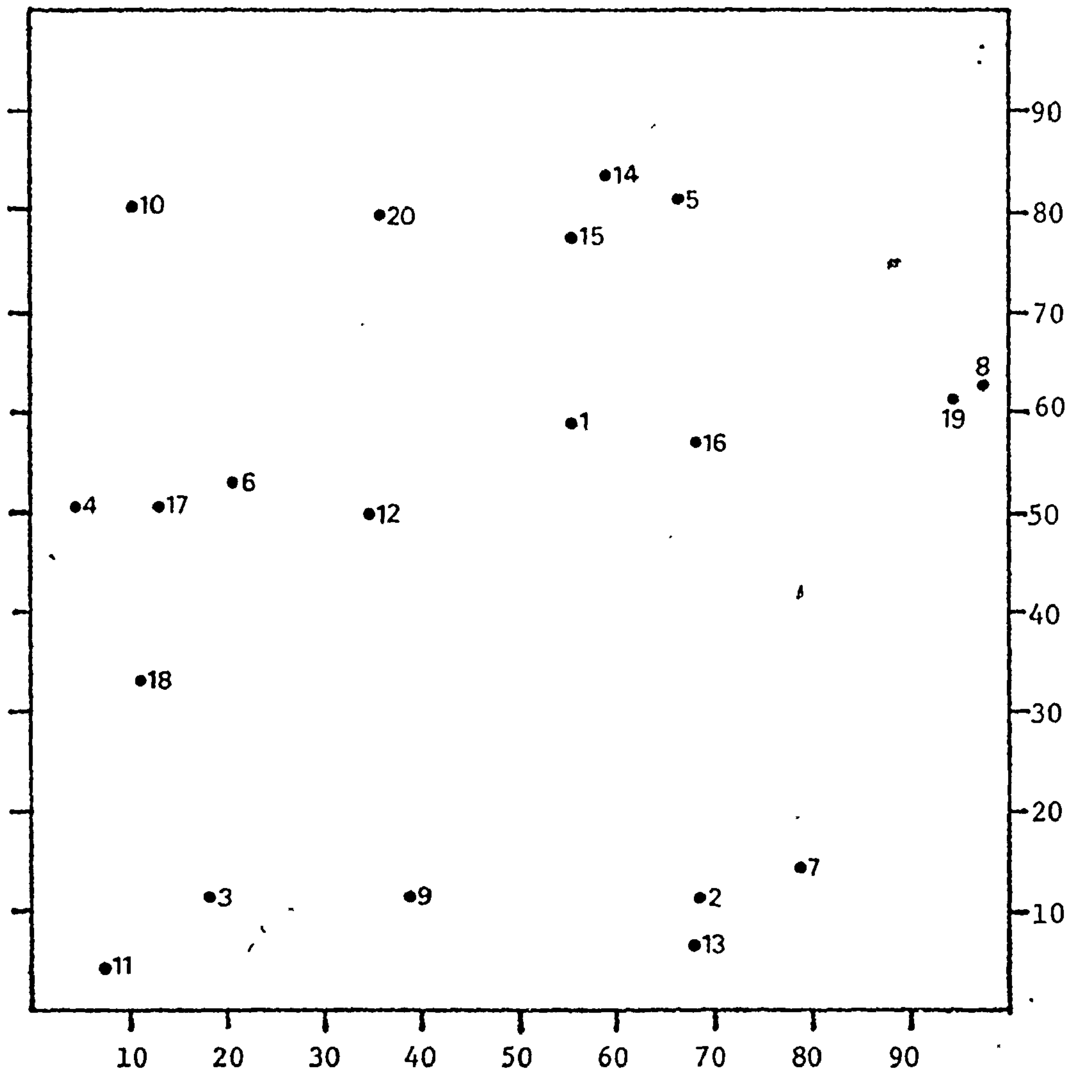
LOCATION PATTERN NO. 11



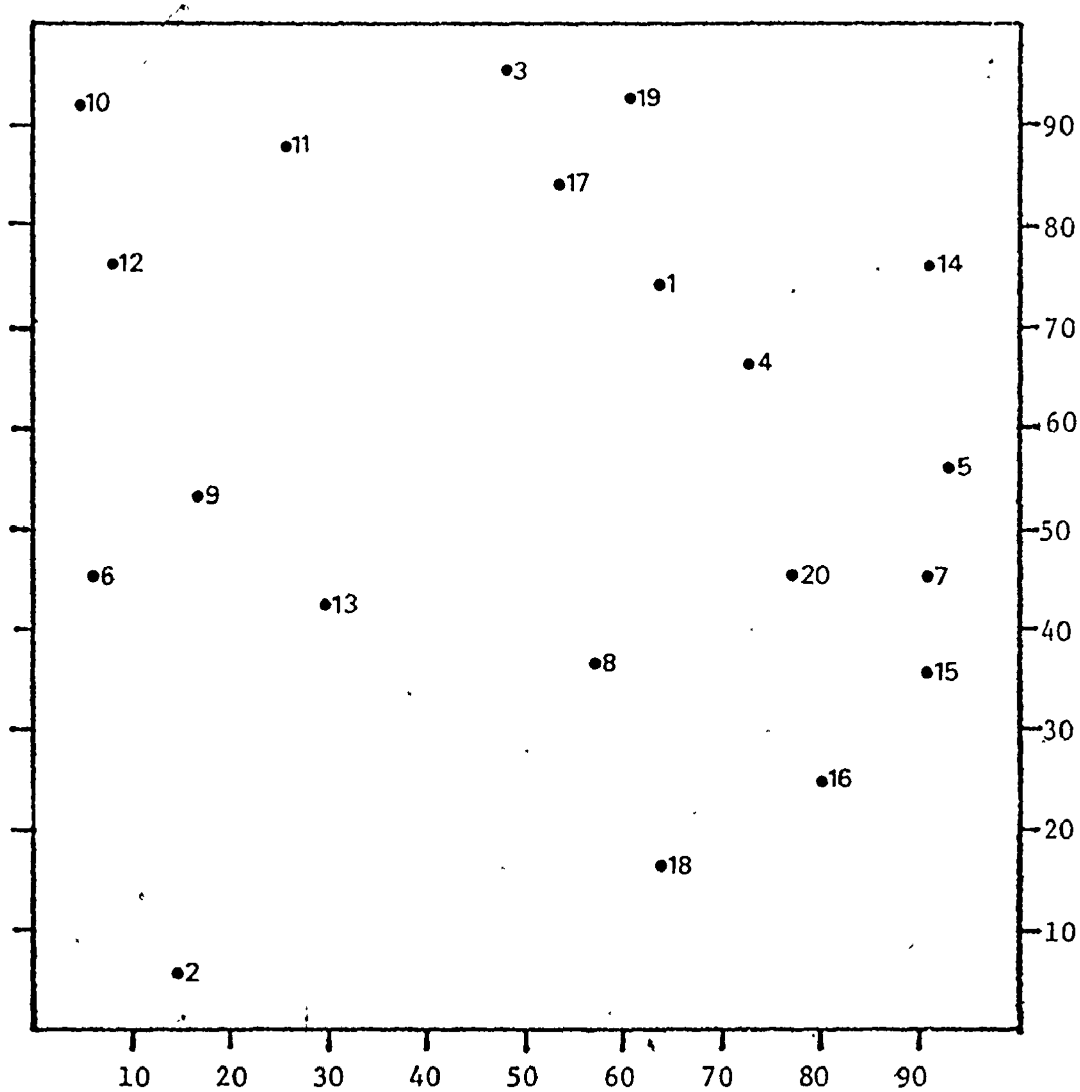
LOCATION PATTERN NO. 12



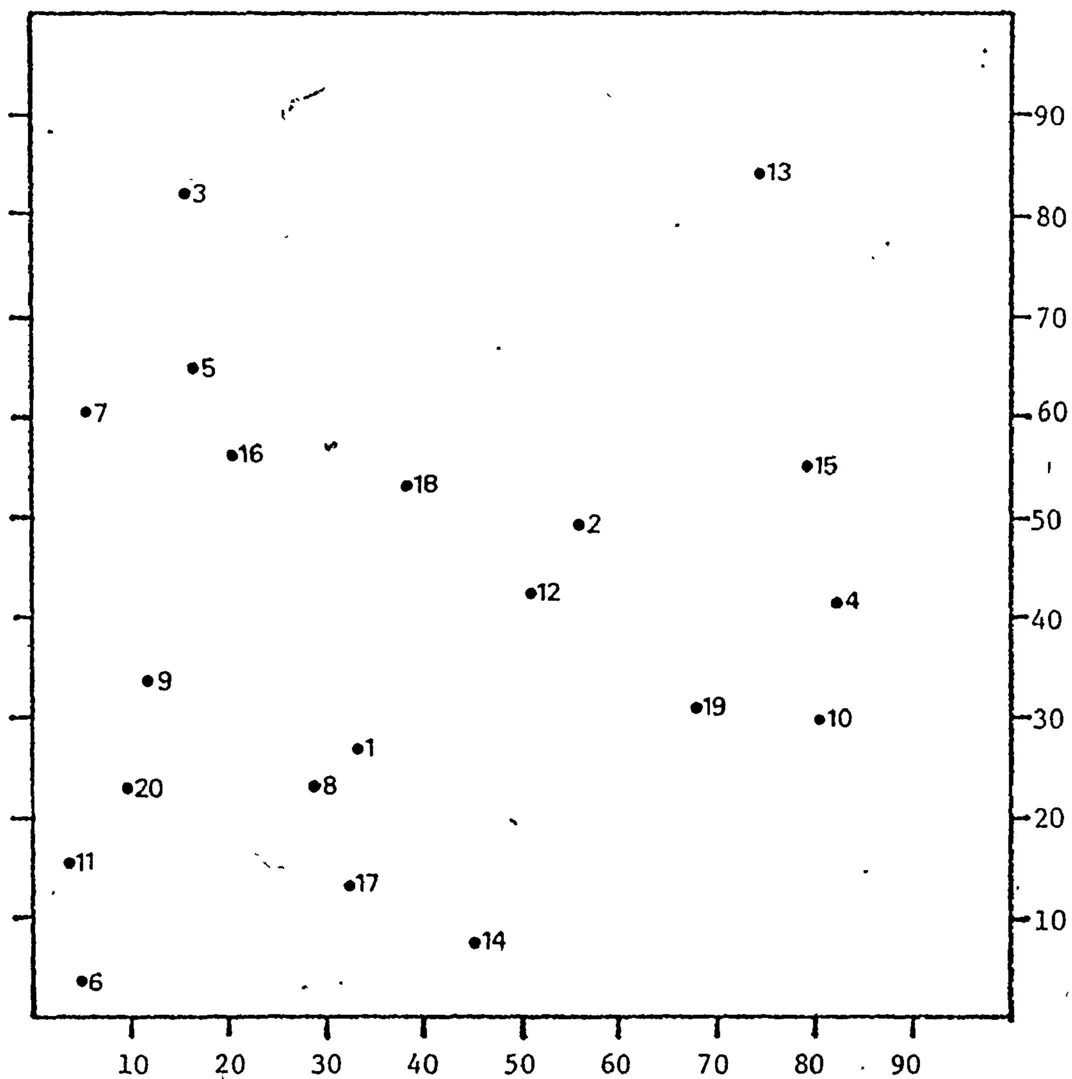
LOCATION PATTERN NO. 13



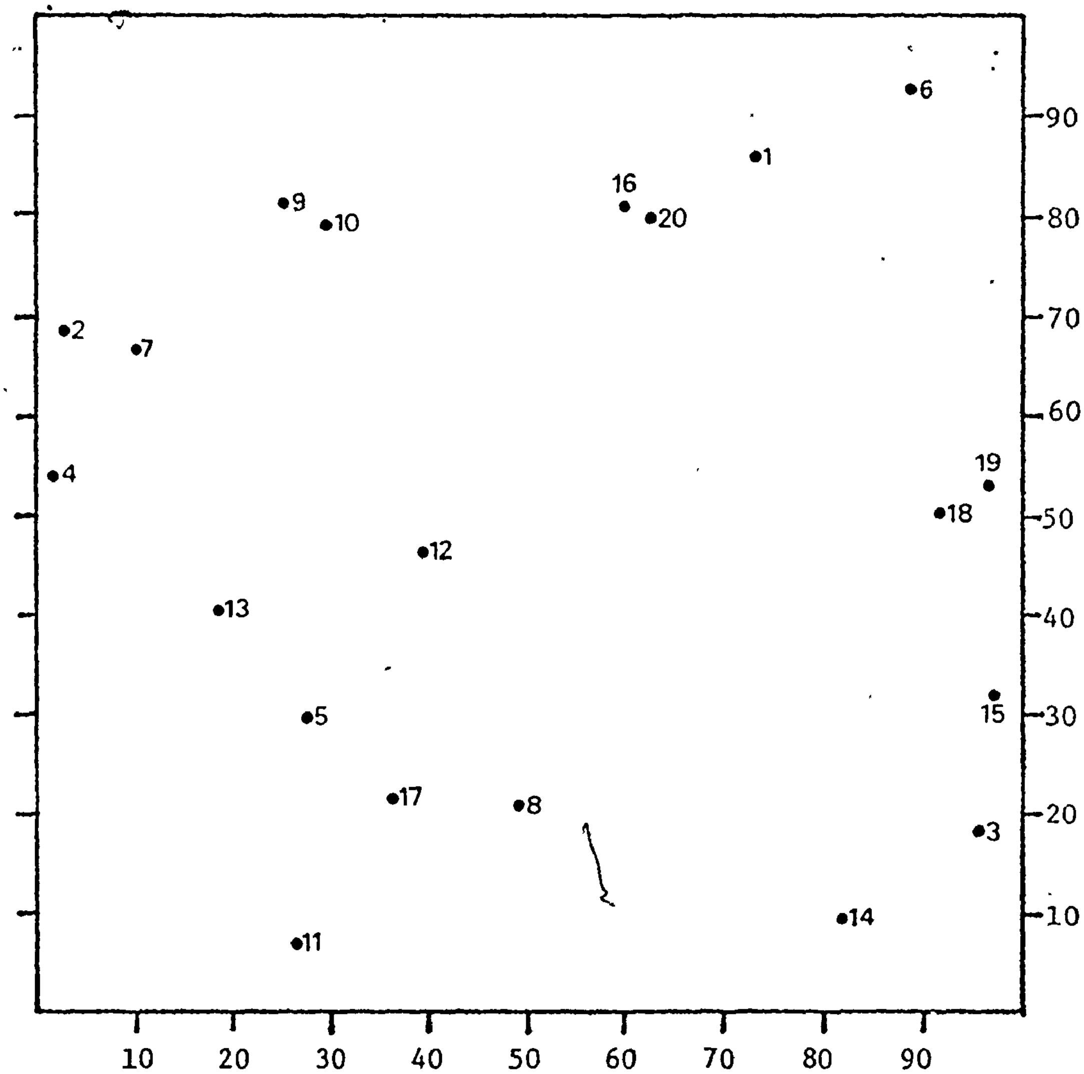
LOCATION PATTERN NO. 14



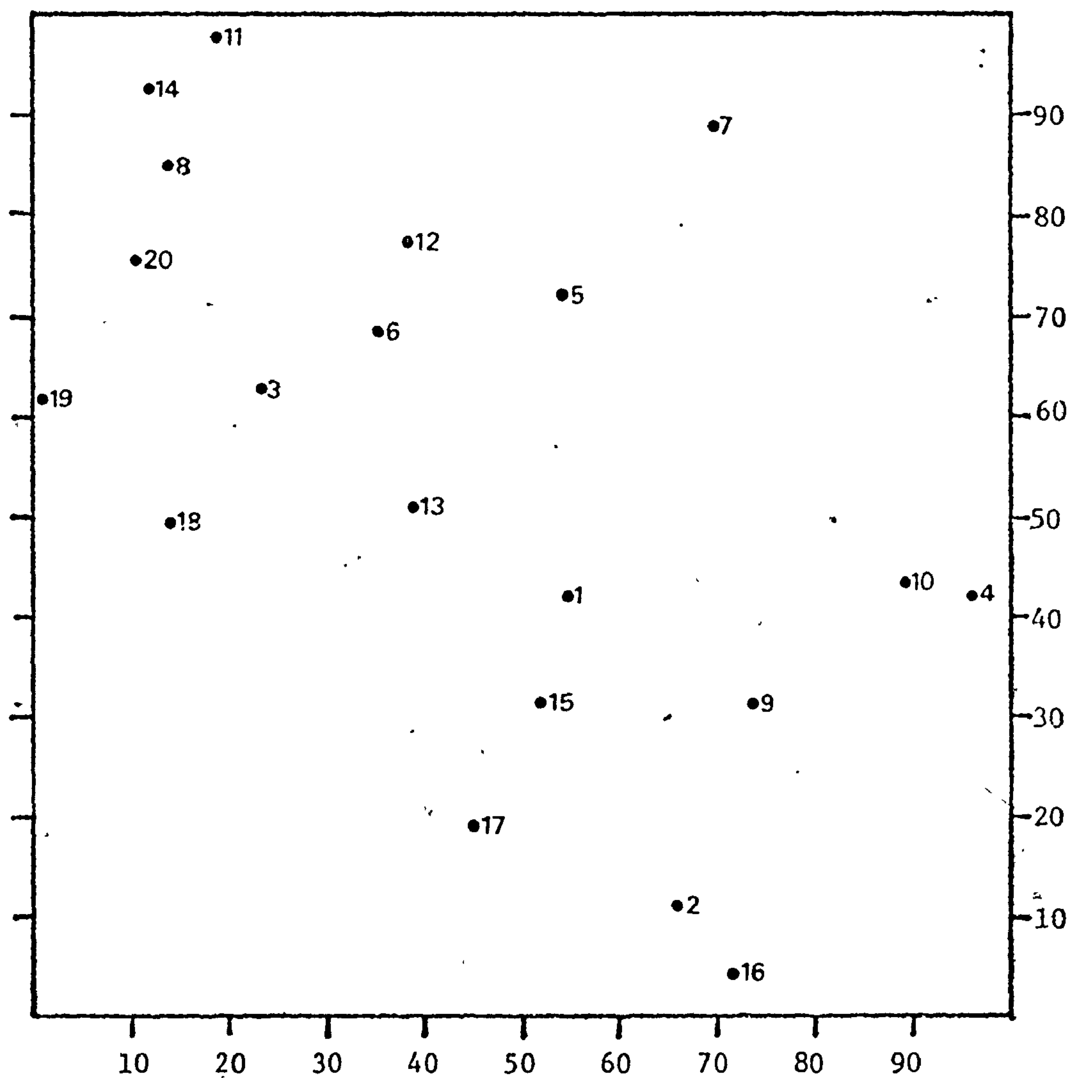
LOCATION PATTERN NO. 15



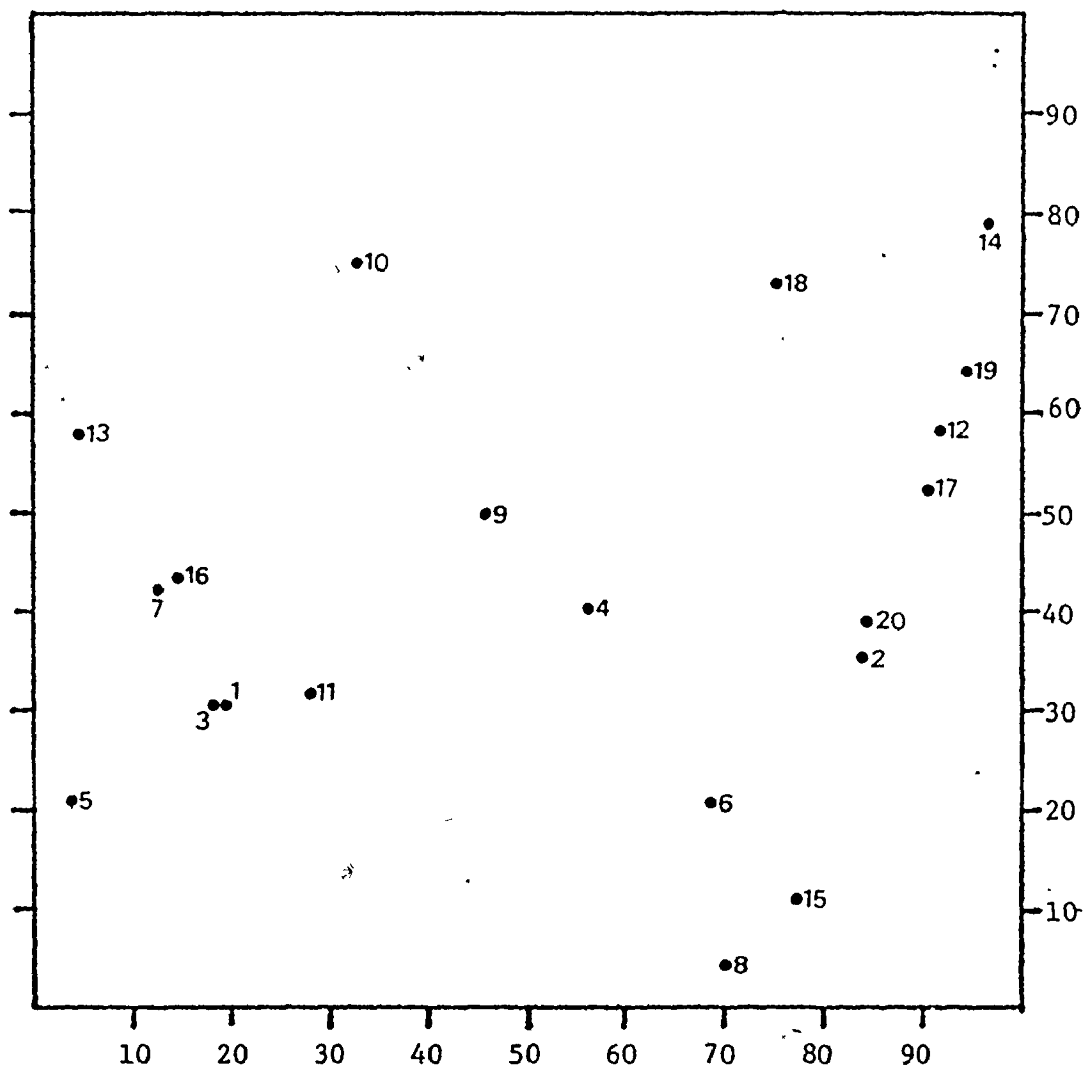
LOCATION PATTERN NO. 16



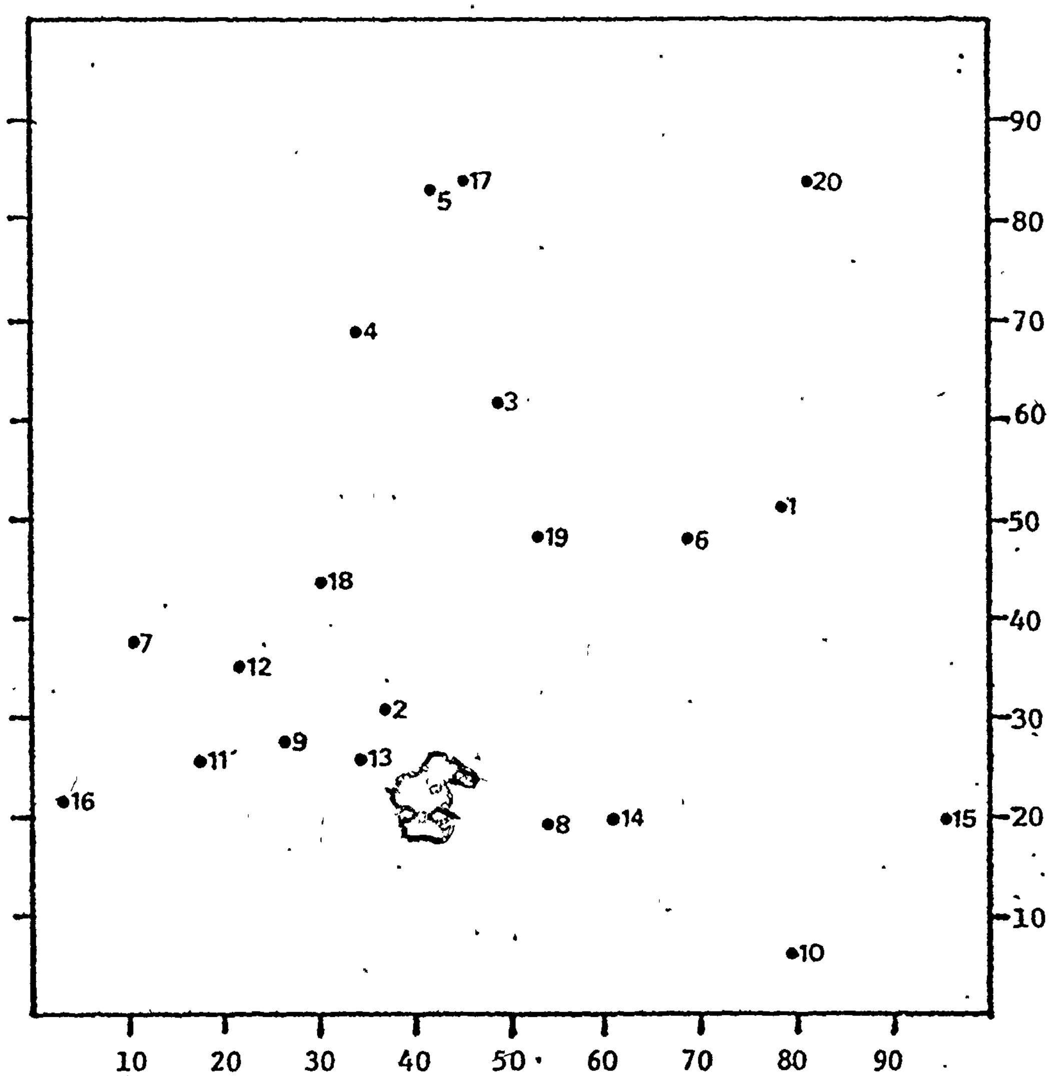
LOCATION PATTERN NO. 17



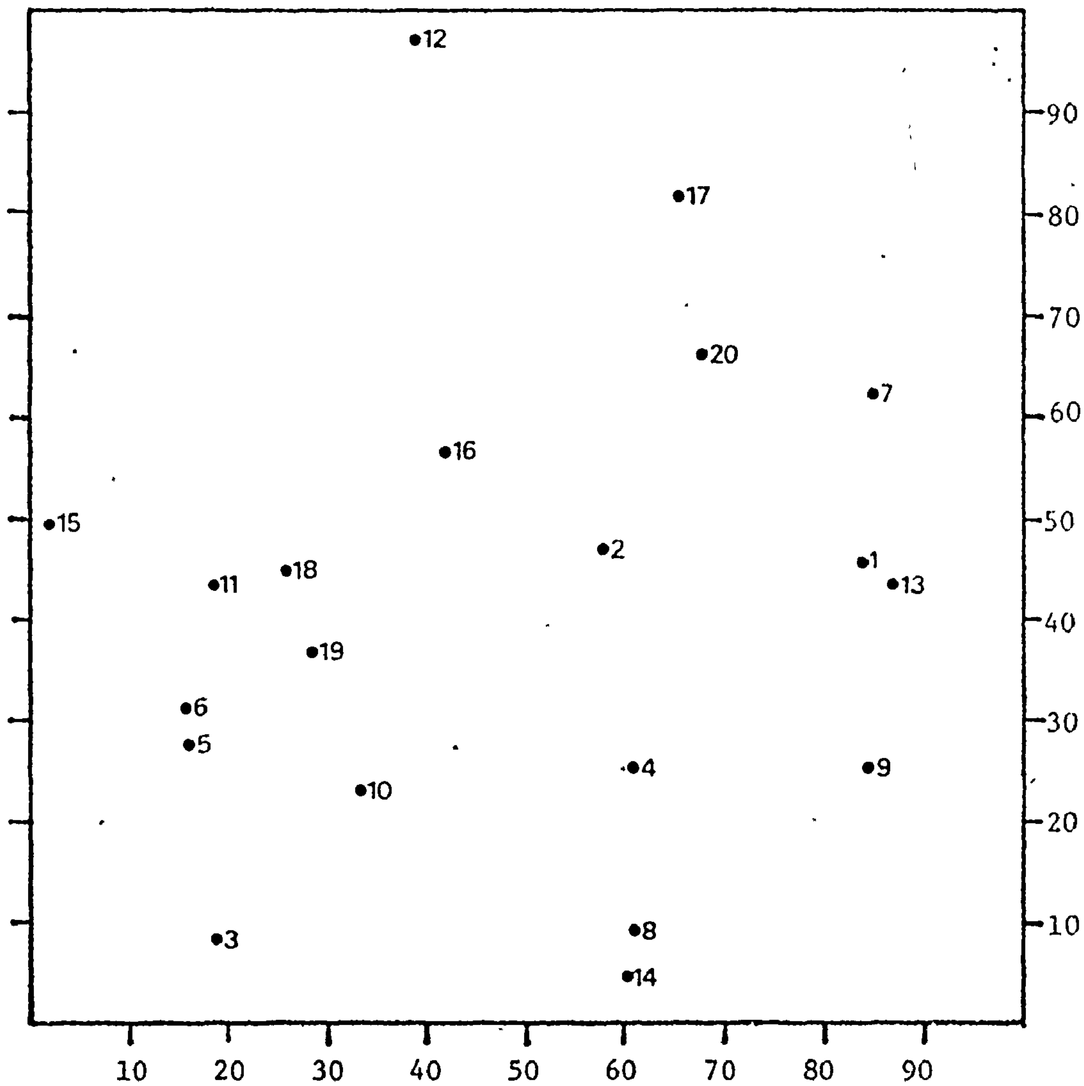
LOCATION PATTERN NO. 18



LOCATION PATTERN NO. 19



LOCATION PATTERN NO. 20



APPENDIX 3

SUMMARY RESULTS FOR LOCATION PATTERNS 2 TO 20

Key to column headings:

1. = Simulation number
2. = Mean A_i
3. = Variance of A_i (10^3)
4. = Coefficient of variation of $A_i (10^3)$
5. = Mean B_i
6. = Variance of $B_i (10^3)$
7. = Coefficient of variation of $B_i (10^3)$
8. = Mean $\bar{t}_i$
9. = Variance of $\bar{t}_i$
10. = α
11. = β_1
12. = β_2
13. = R^2
14. = Explanation added by R_i

LOCATION PATTERN NO. 2

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	33910.	358193.	11.59	1894.	1155.	.61	3.39	.23	4.0656	-.28003	-.17136	.9949	.0144
2	29160.	361224.	12.39	1799.	1248.	.69	3.53	.39	4.3131	-.30342	-.19707	.9918	.0198
3	27276.	357328.	13.10	1701.	1346.	.79	3.73	.69	4.5792	-.32627	-.23397	.9939	.0263
4	25367.	351683.	13.66	1605.	1467.	.91	3.99	1.27	4.8639	-.35717	-.27067	.9850	.0310
5	23538.	356672.	15.15	1514.	1618.	1.07	4.34	2.48	5.2067	-.38613	-.26376	.9792	.0253
6	12884.	234666.	18.21	1392.	2017.	1.45	3.49	.93	4.0898	-.31639	-.22027	.9773	.0322
7	11663.	204148.	17.21	1391.	1940.	1.49	3.78	1.56	4.4386	-.35460	-.23603	.9732	.0375
8	10766.	172653.	15.62	1214.	1632.	1.51	4.19	2.98	4.8197	-.39437	-.24305	.9712	.0372
9	9735.	139597.	14.34	1136.	1728.	1.52	4.76	6.16	5.2144	-.43604	-.24516	.9680	.0337
10	6762.	114679.	13.06	1068.	1657.	1.55	5.59	13.87	5.6166	-.47940	-.25767	.9642	.0312
11	6218.	119778.	19.26	1185.	3247.	2.74	4.62	5.63	4.7584	-.42275	-.20996	.9543	.0148
12	5629.	99067.	17.60	1091.	2981.	2.73	5.52	10.66	5.0975	-.46173	-.23090	.9537	.0176
13	5018.	78158.	15.56	1036.	2695.	2.66	6.56	21.51	5.4663	-.50509	-.23562	.9534	.0177
14	4435.	59631.	13.45	930.	2411.	2.59	7.95	41.29	5.6062	-.54619	-.24519	.9630	.0173
15	3918.	44756.	11.42	864.	2148.	2.49	9.61	63.21	6.0066	-.57639	-.27960	.9724	.0207
16	3369.	54812.	16.17	944.	3600.	3.61	10.32	62.27	5.6678	-.55312	-.25946	.9572	.0109
17	3045.	44492.	14.61	862.	3193.	3.70	11.90	64.95	5.6334	-.56997	-.24615	.9652	.0107
18	2691.	34449.	12.80	769.	2807.	3.56	13.54	105.90	5.8593	-.57724	-.24163	.9694	.0108

LOCATION PATTERN NO. 3

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	29547.	139019.	4.71	1742.	368.	.21	3.43	.11	3.7960	-.25150	-.12663	.9852	.0065
2	27317.	132060.	4.83	1586.	344.	.22	3.57	.16	3.9613	-.26591	-.09640	.9822	.0045
3	24923.	129403.	5.19	1429.	331.	.23	3.77	.25	4.1357	-.28126	-.11665	.9797	.0057
4	22537.	129195.	5.73	1274.	326.	.26	4.01	.43	4.4157	-.30710	-.13905	.9736	.0097
5	23297.	130349.	6.42	1132.	326.	.29	4.33	.77	4.6927	-.33271	-.15952	.9838	.0103
6	11264.	68941.	7.90	1043.	496.	.46	3.56	.56	4.0686	-.31168	-.13951	.9919	.0103
7	9609.	62020.	6.32	863.	323.	.37	3.89	.80	4.3113	-.33437	-.14261	.9870	.0119
8	8361.	42665.	5.13	703.	206.	.29	4.32	1.26	4.5933	-.36143	-.14416	.9649	.0129
9	7014.	30229.	4.31	567.	134.	.24	4.92	2.18	4.9325	-.39494	-.15070	.9607	.0139
10	5638.	22373.	3.63	456.	92.	.20	5.71	4.08	5.3024	-.43279	-.14405	.9767	.0120
11	4774.	44078.	9.23	697.	570.	.82	4.91	4.22	4.6518	-.42945	-.22762	.9680	.0150
12	3910.	26996.	6.90	527.	304.	.58	5.68	6.41	5.1730	-.45420	-.19960	.9809	.0125
13	3136.	14628.	4.71	393.	157.	.40	6.61	10.54	5.5330	-.50433	-.17962	.9874	.0136
14	2404.	7824.	3.26	290.	79.	.27	8.45	20.63	5.8787	-.54295	-.16464	.9637	.0095
15	1833.	4239.	2.31	214.	40.	.19	10.61	40.76	6.2233	-.58312	-.15395	.9639	.0085
16	2264.	19876.	6.70	457.	479.	1.05	10.60	57.51	5.9567	-.59364	-.30311	.9655	.0129
17	1772.	10814.	6.10	327.	233.	.71	12.97	80.53	6.1564	-.61753	-.21112	.9915	.0073
18	1319.	5231.	3.97	230.	109.	.47	15.65	107.76	6.2582	-.63353	-.13635	.9933	.0037

LOCATION PATTERN NO. 4

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	33373.	139709.	4.19	2009.	422.	.21	3.25	.08	3.5720	-.23163	-.18681	.9775	.0944
2	32958.	138201.	4.19	1975.	415.	.21	3.29	.12	3.6351	-.23716	-.18747	.9785	.1065
3	32616.	144531.	4.43	1946.	437.	.23	3.33	.18	3.7319	-.24567	-.18905	.9736	.1069
4	32334.	154023.	4.76	1922.	481.	.25	3.39	.33	3.9472	-.26002	-.19618	.9730	.0993
5	32103.	163471.	5.09	1912.	540.	.28	3.46	.52	4.2172	-.29146	-.20124	.9789	.0826
6	13735.	129188.	9.41	1439.	1426.	.99	3.19	.43	3.9216	-.29944	-.21099	.9691	.1062
7	13271.	101957.	7.62	1372.	1291.	.94	3.27	.56	4.1022	-.31717	-.21760	.9838	.1418
8	12860.	79379.	6.17	1314.	1172.	.69	3.37	.78	4.3097	-.33775	-.22432	.9809	.1049
9	12500.	63549.	5.08	1233.	1077.	.65	3.48	1.17	4.5440	-.36142	-.22756	.9771	.1093
10	12186.	52509.	4.31	1221.	1013.	.63	3.62	1.87	4.8066	-.38605	-.23244	.9729	.1044
11	6477.	89325.	13.79	1239.	2978.	2.46	3.95	2.65	4.5527	-.40021	-.20871	.9749	.0407
12	6077.	68755.	11.31	1121.	2656.	2.37	4.10	3.25	4.7215	-.41755	-.21601	.9753	.0603
13	5794.	51698.	9.06	1042.	2339.	2.25	4.29	4.34	4.9297	-.43972	-.22344	.9739	.0786
14	5305.	38488.	7.17	973.	2045.	2.10	4.55	6.41	5.1848	-.46767	-.22903	.9630	.0909
15	5063.	28692.	5.67	914.	1784.	1.95	4.87	10.05	5.4633	-.49876	-.23432	.9606	.0466
16	3474.	49832.	14.34	959.	3607.	3.76	7.86	31.88	5.5050	-.53467	-.23190	.9593	.0179
17	3150.	38986.	12.26	876.	3160.	3.61	8.12	36.02	5.6228	-.54614	-.21733	.9662	.0232
18	2892.	29351.	10.15	801.	2741.	3.42	8.45	41.40	5.7396	-.56118	-.21401	.9732	.0304

LOCATION PATTERN NO. 5

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	34279.	283687.	8.28	2107.	1326.	.63	3.31	.26	4.3699	-.30212	-.22905	.9855	.1363
2	34329.	333546.	9.72	2120.	1602.	.85	3.41	.44	4.5440	-.32340	-.24740	.9820	.1417
3	34448.	396024.	11.50	2138.	2366.	1.11	3.51	.70	4.7263	-.33989	-.26339	.9679	.1360
4	34577.	461488.	13.35	2156.	2940.	1.36	3.63	1.00	4.8564	-.35114	-.27162	.9547	.1244
5	34682.	522914.	15.05	2170.	3465.	1.60	3.77	1.32	4.9755	-.36129	-.27047	.9468	.1067
6	14793.	185637.	12.54	1677.	2759.	1.65	3.42	1.17	4.6856	-.37753	-.20916	.9693	.0946
7	14656.	172937.	11.64	1723.	3479.	2.02	3.65	1.93	5.0754	-.41501	-.21619	.9861	.1030
8	14997.	173719.	11.58	1782.	4580.	2.57	3.94	3.16	5.4053	-.44609	-.22760	.9634	.1066
9	15170.	186077.	12.27	1644.	6028.	3.27	4.27	5.11	5.6368	-.46681	-.23395	.9695	.1063
10	15330.	205458.	13.40	1501.	7693.	4.05	4.64	7.83	5.6061	-.46098	-.24190	.9579	.1060
11	7386.	109368.	14.81	1552.	5273.	3.40	4.47	6.57	5.1403	-.46360	-.21403	.9664	.0383
12	7427.	93025.	12.53	1621.	5849.	3.61	4.93	10.52	5.5015	-.49962	-.20428	.9635	.0306
13	7572.	83416.	11.02	1724.	6675.	3.99	5.57	17.64	5.7917	-.52584	-.19303	.9679	.0362
14	7600.	82238.	10.54	1852.	8677.	4.69	6.45	32.85	6.0594	-.54985	-.19141	.9618	.0373
15	8071.	89481.	11.09	1991.	11520.	5.79	7.64	63.51	6.3022	-.57177	-.19064	.9553	.0390
16	4111.	62231.	15.14	1296.	7405.	5.71	9.24	70.02	5.8649	-.57791	-.22024	.9466	.0153
17	4101.	53691.	13.09	1353.	7695.	5.64	9.10	92.59	6.0464	-.59617	-.20400	.9560	.0164
18	4164.	46328.	11.25	1446.	8458.	5.65	10.79	115.13	6.1132	-.59687	-.19841	.9660	.0190

LOCATION PATTERN NO. 6

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	30371.	191345.	6.30	1628.	546.	.30	3.36	.11	3.9103	-.26383	-.17143	.9904	.0331
2	28500.	163642.	5.74	1709.	490.	.29	3.49	.16	4.0097	-.27249	-.20519	.9891	.0626
3	25543.	137192.	5.17	1588.	442.	.28	3.61	.23	4.0622	-.27800	-.22125	.9931	.0861
4	24602.	115343.	4.69	1469.	414.	.28	3.77	.37	4.2672	-.29465	-.22292	.9907	.0068
5	22770.	99951.	4.39	1357.	411.	.30	3.97	.65	4.5118	-.31772	-.21656	.9683	.0697
6	12116.	164995.	13.62	1245.	1098.	.88	3.44	.57	4.0364	-.31113	-.20762	.9950	.0416
7	10858.	126435.	11.64	1136.	696.	.81	3.67	.81	4.3131	-.33868	-.21490	.9922	.0567
8	9557.	90819.	9.50	972.	716.	.74	3.95	1.23	4.6489	-.37353	-.22645	.9915	.0727
9	8291.	61775.	7.45	650.	574.	.68	4.33	2.06	5.0526	-.41622	-.22924	.9904	.0785
10	7130.	49479.	5.65	742.	473.	.64	4.62	3.80	5.4544	-.45933	-.23394	.9694	.0614
11	5530.	90525.	16.37	975.	1761.	1.81	4.64	4.26	4.8075	-.42951	-.25260	.9767	.0255
12	4753.	65842.	13.65	631.	1349.	1.62	5.20	6.36	5.1192	-.46460	-.25462	.9791	.0311
13	3973.	44137.	11.11	731.	1008.	1.44	5.95	10.45	5.4942	-.50663	-.26100	.9815	.0351
14	3241.	27543.	8.50	590.	743.	1.26	6.99	19.07	5.6625	-.55566	-.24511	.9626	.0356
15	2598.	16307.	6.28	469.	545.	1.16	8.35	34.63	6.2551	-.60264	-.24258	.9666	.0363
16	2881.	42024.	14.59	726.	1931.	2.66	10.11	54.64	5.7977	-.57408	-.33301	.9605	.0171
17	2413.	29715.	12.32	605.	1427.	2.36	11.70	76.24	5.9946	-.59944	-.30708	.9755	.0165
18	1956.	19176.	9.80	590.	1037.	2.07	13.44	97.78	6.1361	-.61660	-.27720	.9631	.0161

LOCATION PATTERN NO. 7

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	29600.	178923.	6.05	1765.	532.	.30	3.45	.17	4.1488	-.28587	-.16704	.9932	.0244
2	27417.	174186.	6.35	1627.	574.	.35	3.63	.26	4.4797	-.31635	-.16667	.9914	.0280
3	25137.	177132.	7.05	1489.	655.	.44	3.87	.47	4.6402	-.35011	-.16929	.9857	.0275
4	22692.	186476.	8.15	1358.	766.	.56	4.17	.80	5.2114	-.38537	-.17199	.9799	.0252
5	20803.	119197.	5.73	1239.	693.	.72	4.56	1.40	5.5181	-.41418	-.16615	.9734	.0206
6	11395.	105998.	9.30	1102.	611.	.55	3.62	.83	4.2755	-.33406	-.19562	.9839	.0267
7	9999.	75152.	7.52	944.	473.	.59	3.97	1.44	4.7487	-.38111	-.20678	.9651	.0292
8	6036.	52905.	6.13	806.	386.	.48	4.50	2.67	5.2206	-.43240	-.20539	.9945	.0297
9	7406.	39748.	5.37	691.	346.	.50	5.27	5.28	5.7102	-.48330	-.20573	.9831	.0263
10	6378.	34131.	5.35	631.	345.	.57	6.37	11.08	6.1592	-.53139	-.19922	.9786	.0205
11	4551.	51306.	10.58	756.	655.	.87	4.98	6.36	5.0222	-.45215	-.23552	.9692	.0189
12	4018.	32229.	8.02	601.	431.	.72	5.93	12.13	5.4081	-.50596	-.22193	.9693	.0163
13	3251.	16772.	5.77	478.	290.	.61	7.32	25.17	6.0102	-.56909	-.21243	.9743	.0140
14	2599.	10561.	4.06	365.	201.	.52	9.54	56.72	6.4847	-.62724	-.20216	.9799	.0114
15	2092.	6287.	3.01	315.	147.	.47	12.51	113.47	6.7371	-.65667	-.18794	.9846	.0091
16	2363.	21882.	9.50	492.	469.	.99	10.76	71.55	5.9749	-.60014	-.27632	.9795	.0136
17	1817.	12953.	7.13	371.	293.	.79	12.73	104.82	6.1112	-.61035	-.20178	.9850	.0062
18	1369.	6953.	5.01	261.	182.	.65	14.50	126.84	6.0625	-.61167	-.12243	.9873	.0037

LOCATION PATTERN NO. 8

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	29954.	174514.	5.63	1793.	564.	.32	3.42	.17	3.9753	-.26915	-.17968	.9934	.0477
2	28042.	154929.	5.53	1675.	556.	.33	3.55	.26	4.2063	-.29067	-.18248	.9869	.0540
3	26108.	139468.	5.34	1561.	581.	.37	3.72	.44	4.4744	-.31603	-.19196	.9602	.0560
4	24261.	128961.	5.32	1457.	637.	.44	3.92	.75	4.7750	-.34460	-.18573	.9715	.0445
5	22584.	123303.	5.46	1367.	727.	.53	4.16	1.34	5.0640	-.37457	-.18567	.9620	.0355
6	11751.	116091.	9.88	1142.	690.	.78	3.62	.88	4.2237	-.32831	-.18750	.9601	.0417
7	10491.	84017.	8.01	996.	707.	.71	3.92	1.44	4.6318	-.37644	-.19635	.9825	.0488
8	9242.	59285.	6.42	868.	581.	.67	4.33	2.53	5.0255	-.41120	-.20246	.9775	.0510
9	8078.	41790.	5.17	760.	511.	.67	4.92	4.76	5.4376	-.45758	-.20576	.9758	.0478
10	7055.	30198.	4.28	673.	492.	.73	5.68	9.58	5.8300	-.49626	-.20488	.9766	.0408
11	5167.	57101.	11.05	809.	1051.	1.30	5.10	6.83	5.0188	-.45003	-.21694	.9710	.0209
12	4382.	37091.	8.46	656.	695.	1.06	5.93	11.51	5.3714	-.48665	-.20366	.9716	.0217
13	3631.	22791.	6.28	531.	458.	.86	7.13	22.24	5.6093	-.53998	-.19543	.9715	.0221
14	2959.	13628.	4.61	433.	309.	.71	8.89	46.73	6.2027	-.59535	-.19411	.9727	.0208
15	2395.	8188.	3.42	357.	227.	.64	11.03	63.69	6.4862	-.62238	-.20364	.9782	.0233
16	2567.	25507.	9.94	554.	665.	1.60	11.33	61.67	5.9258	-.58726	-.25498	.9760	.0109
17	2086.	15554.	7.46	426.	524.	1.23	13.05	107.55	6.0530	-.60314	-.21333	.9870	.0100
18	1640.	8662.	5.29	327.	305.	.93	14.76	124.02	6.0457	-.60109	-.20019	.9683	.0117

LOCATION PATTERN NO. 9

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	23872.	219487.	7.35	1800.	871.	.48	3.43	.19	4.1364	-.26582	-.19139	.9847	.0294
2	27816.	209676.	7.54	1676.	986.	.59	3.57	.27	4.3576	-.30624	-.21464	.9617	.0408
3	25658.	203526.	7.93	1557.	1131.	.73	3.76	.42	4.6012	-.32683	-.25005	.9785	.0542
4	23552.	202909.	8.62	1442.	1307.	.91	3.99	.69	4.8743	-.35461	-.27474	.9714	.0565
5	21579.	208325.	9.65	1338.	1505.	1.13	4.29	1.14	5.1098	-.37706	-.30595	.9646	.0626
6	11926.	152448.	12.78	1215.	1565.	1.31	3.66	.91	4.2796	-.33499	-.16517	.9920	.0303
7	10738.	126073.	11.77	1097.	1816.	1.65	4.01	1.42	4.6060	-.36745	-.19628	.9857	.0370
8	9490.	104814.	11.05	996.	2112.	2.12	4.48	2.39	4.9927	-.40726	-.20670	.9663	.0406
9	3355.	89957.	10.77	916.	2477.	2.70	5.13	4.32	5.3926	-.44926	-.21715	.9825	.0414
10	7371.	81165.	11.01	855.	2692.	3.38	6.02	8.24	5.7512	-.48676	-.22335	.9704	.0395
11	5342.	70490.	13.20	926.	1768.	1.91	5.27	6.85	5.0260	-.45011	-.21580	.9794	.0194
12	4646.	53011.	11.41	817.	1959.	2.40	6.22	11.61	5.3549	-.45544	-.19034	.9732	.0167
13	3994.	40131.	10.05	737.	2267.	3.08	7.57	21.46	5.7060	-.52489	-.17116	.9759	.0141
14	2427.	31930.	13.14	682.	2660.	3.90	9.57	43.58	6.0719	-.56717	-.15912	.9730	.0119
15	2973.	27473.	9.24	649.	3117.	4.80	12.21	60.57	6.3287	-.59724	-.16523	.9732	.0125
16	2634.	27977.	10.62	649.	1160.	1.62	11.82	81.16	5.9995	-.59544	-.27201	.9624	.0141
17	2224.	19370.	8.71	560.	1223.	2.18	14.35	110.28	6.1134	-.61100	-.21215	.9878	.0131
18	1665.	13491.	7.23	501.	1378.	2.75	16.40	135.80	6.1090	-.60654	-.15661	.9054	.0068

LOCATION PATTERN NO. 10

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	30448.	265469.	8.72	1848.	960.	.52	3.40	.19	4.1686	-.28926	-.17560	.9839	.0313
2	28646.	256001.	8.94	1743.	1030.	.59	3.53	.29	4.3499	-.30595	-.23203	.9654	.0468
3	26788.	246916.	9.22	1638.	1105.	.66	3.69	.46	4.5261	-.32209	-.22298	.9633	.0577
4	24965.	240906.	9.64	1540.	1192.	.77	3.90	.81	4.7679	-.34488	-.23063	.9777	.0554
5	23327.	239131.	10.25	1453.	1295.	.89	4.17	1.49	5.0461	-.37142	-.23663	.9637	.0486
6	12379.	191470.	15.47	1306.	1635.	1.25	3.58	.94	4.2466	-.33279	-.21258	.9869	.0462
7	11259.	158315.	14.06	1199.	1576.	1.31	3.89	1.54	4.5943	-.36806	-.22176	.9845	.0566
8	10128.	127736.	12.61	1103.	1538.	1.39	4.31	2.71	4.9657	-.40612	-.23126	.9628	.0635
9	9074.	103236.	11.38	1022.	1538.	1.51	4.88	5.22	5.3530	-.44650	-.23764	.9802	.0636
10	8157.	85596.	10.49	956.	1579.	1.65	5.66	10.88	5.7317	-.48081	-.24170	.9770	.0590
11	5769.	95422.	16.54	1056.	2293.	2.17	5.10	7.89	4.9870	-.44784	-.29167	.9707	.0316
12	5095.	72767.	14.28	945.	1969.	2.08	5.94	14.07	5.3531	-.48892	-.29563	.9746	.0358
13	4433.	53189.	11.99	851.	1705.	2.00	7.14	27.57	5.7511	-.53511	-.29332	.9779	.0366
14	3648.	38411.	9.98	774.	1513.	1.96	8.59	44.60	6.0443	-.56953	-.29815	.9676	.0394
15	3357.	28315.	8.44	714.	1390.	1.95	10.31	56.65	6.1905	-.58541	-.31193	.9917	.0443
16	3037.	42790.	14.09	798.	2405.	3.01	11.07	76.60	5.6094	-.57305	-.35026	.9723	.0190
17	2626.	31290.	11.92	700.	1920.	2.74	12.61	96.07	5.9265	-.58673	-.32210	.9749	.0176
18	2235.	21661.	9.69	618.	1532.	2.48	14.67	113.19	5.9432	-.58719	-.29995	.9612	.0172

LOCATION PATTERN NO. 11

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	30840.	234670.	7.61	1873.	834.	.45	3.36	.15	4.1589	-.28615	-.18170	.9651	.0519
2	29264.	218719.	7.47	1785.	942.	.53	3.46	.20	4.3708	-.30604	-.18925	.9736	.0065
3	27734.	206563.	7.45	1703.	1141.	.67	3.59	.28	4.5602	-.32552	-.20433	.9741	.0051
4	26268.	202702.	7.72	1630.	1461.	.90	3.73	.39	4.6936	-.33750	-.21553	.9680	.0539
5	24939.	209122.	8.39	1570.	1908.	1.22	3.90	.54	4.8112	-.34776	-.21532	.9504	.0073
6	12541.	189671.	15.12	1333.	1697.	1.27	3.46	.73	4.2305	-.33205	-.23261	.9940	.0013
7	11491.	153949.	13.40	1236.	1671.	1.35	3.70	1.05	4.5639	-.36600	-.23950	.9918	.0745
8	10435.	121297.	11.62	1153.	1746.	1.51	4.01	1.54	4.9231	-.40249	-.24564	.9907	.0023
9	9445.	96308.	10.20	1069.	2001.	1.84	4.40	2.39	5.3253	-.44435	-.25548	.9078	.0644
10	8579.	81099.	9.45	1045.	2512.	2.40	3.91	3.79	5.0949	-.46338	-.26941	.9838	.0050
11	5895.	106358.	18.04	1096.	2647.	2.60	4.77	5.51	4.8911	-.43826	-.30445	.9705	.0377
12	5236.	82476.	15.75	991.	2534.	2.56	5.36	8.30	5.2373	-.47755	-.30637	.9736	.0418
13	4577.	60570.	13.23	902.	2262.	2.51	6.18	13.31	5.6272	-.52243	-.30794	.9767	.0430
14	3969.	43264.	10.90	834.	2060.	2.49	7.33	22.80	6.0259	-.56971	-.31207	.9796	.0417
15	3454.	31292.	9.06	756.	2040.	2.60	8.98	41.97	6.4183	-.61770	-.32261	.9819	.0395
16	3169.	51078.	16.12	652.	3275.	3.64	10.56	71.87	5.7918	-.57160	-.38157	.9620	.0241
17	2772.	39221.	14.15	757.	2610.	3.71	12.08	94.78	5.9610	-.59263	-.34615	.9595	.0225
18	2374.	28253.	11.90	677.	2390.	3.53	13.60	119.33	6.0778	-.60765	-.30831	.9740	.0196

LOCATION PATTERN NO. 12

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	29574.	113324.	3.83	1740.	322.	.19	3.42	.09	3.8497	-.26597	-.16234	.9833	.0208
2	27327.	102396.	3.75	1566.	294.	.19	3.56	.13	4.0656	-.27792	-.17335	.9730	.0313
3	24956.	98392.	3.94	1428.	290.	.20	3.76	.21	4.3038	-.29714	-.16710	.9743	.0320
4	22602.	99482.	4.40	1274.	309.	.24	4.01	.37	4.5604	-.32052	-.16752	.9748	.0306
5	20388.	104000.	5.10	1134.	348.	.31	4.32	.68	4.8377	-.34595	-.15334	.9744	.0225
6	11193.	86963.	7.77	1047.	531.	.51	3.52	.48	4.1592	-.32164	-.13661	.9917	.0118
7	9661.	57012.	5.09	866.	347.	.40	3.34	.68	4.4937	-.35462	-.15449	.9622	.0173
8	8179.	35737.	4.37	703.	224.	.32	4.28	1.00	4.8616	-.39097	-.18357	.9626	.0262
9	6790.	22312.	3.29	566.	151.	.27	4.88	1.93	5.2964	-.43554	-.20269	.9790	.0348
10	5591.	14677.	2.66	455.	113.	.25	5.72	3.82	5.7123	-.47650	-.20162	.9753	.0336
11	4745.	47375.	9.98	711.	614.	.66	4.78	3.40	4.8476	-.42981	-.23030	.9892	.0174
12	3836.	27924.	7.26	541.	336.	.63	5.51	5.08	5.2397	-.47362	-.20399	.9679	.0142
13	2985.	14829.	4.97	404.	181.	.45	6.56	8.76	5.6943	-.52073	-.18372	.9854	.0118
14	2251.	7229.	3.21	299.	96.	.33	8.21	17.01	6.1562	-.56158	-.17577	.9644	.0114
15	1667.	3366.	2.02	222.	54.	.24	13.67	37.56	5.5975	-.63574	-.17910	.9831	.0111
16	2288.	21785.	9.52	472.	509.	-1.08	14.50	43.73	5.9607	-.59532	-.32633	.9673	.0173
17	1753.	12067.	6.88	340.	253.	.74	12.70	67.17	6.2518	-.63262	-.24642	.9667	.0139
18	1277.	5828.	4.56	242.	121.	.50	15.69	105.33	6.5213	-.67065	-.15656	.9916	.0346

LOCATION PATTERN NO. 13

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	30157.	179775.	5.96	1613.	381.	.21	3.37	.09	3.7488	-.24789	-.17175	.9942	.0289
2	28375.	149990.	5.29	1701.	335.	.20	3.47	.11	3.8376	-.25534	-.16682	.9668	.0432
3	26616.	129226.	4.86	1592.	342.	.22	3.61	.16	3.9509	-.26469	-.16591	.9734	.0542
4	24955.	120245.	4.82	1492.	403.	.27	3.76	.26	4.1608	-.28381	-.15756	.9789	.0498
5	23444.	122307.	5.20	1403.	507.	.36	3.93	.43	4.3599	-.30216	-.14960	.9620	.0413
6	11619.	134898.	11.41	1160.	535.	.45	3.41	.42	3.6468	-.29023	-.16086	.9926	.0281
7	13561.	95793.	9.07	1033.	356.	.35	3.61	.56	4.1199	-.31766	-.18460	.9935	.0367
8	9330.	64022.	6.86	901.	253.	.28	3.87	.79	4.3937	-.34482	-.19005	.9906	.0514
9	8201.	41968.	5.12	787.	214.	.27	4.24	1.21	4.6866	-.37320	-.19806	.9903	.0631
10	7223.	28980.	4.01	694.	223.	.32	4.70	2.07	5.0477	-.41019	-.19491	.9845	.0609
11	5150.	66476.	12.91	842.	700.	.83	4.41	3.30	4.6504	-.41180	-.24866	.9734	.0266
12	4341.	43550.	10.03	683.	402.	.59	4.89	4.43	4.9167	-.44100	-.22970	.9772	.0274
13	3576.	25922.	7.25	551.	225.	.41	5.51	6.44	5.2642	-.48056	-.21089	.9815	.0272
14	2902.	14255.	4.91	446.	129.	.29	6.38	10.27	5.6334	-.52334	-.23299	.9644	.0265
15	2347.	7565.	3.22	364.	63.	.23	7.57	17.80	6.0075	-.56728	-.19641	.9659	.0292
16	2517.	28475.	11.31	567.	665.	-1.17	9.12	44.68	5.5622	-.54325	-.20750	.9747	.0184
17	2018.	17574.	8.71	435.	358.	.82	10.42	59.31	5.9496	-.59979	-.23546	.9771	.0135
18	1563.	9655.	6.18	331.	187.	.57	11.92	75.18	6.1801	-.63145	-.16316	.9650	.0095

LOCATION PATTERN NO. 14

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	29479.	232552.	7.69	1771.	724.	.41	3.41	.12	4.0701	-.27935	-.14661	.9607	.0147
2	27148.	214771.	7.91	1628.	744.	.46	3.55	.17	4.2514	-.29587	-.17265	.9651	.0256
3	24660.	194261.	7.88	1480.	760.	.51	3.73	.24	4.5094	-.32008	-.19708	.9629	.0352
4	22158.	174091.	7.86	1336.	777.	.58	3.97	.38	4.7391	-.34125	-.21909	.9759	.0425
5	19772.	156269.	7.90	1203.	796.	.66	4.28	.92	5.0450	-.37039	-.23712	.9723	.0430
6	11664.	179650.	15.40	1194.	1216.	1.02	3.55	.59	4.0660	-.31333	-.18503	.9932	.0304
7	10276.	146054.	14.21	1052.	1197.	1.14	3.86	.87	4.3761	-.34453	-.21057	.9919	.0428
8	6865.	113713.	12.63	924.	1202.	1.30	4.29	1.39	4.7565	-.38355	-.21900	.9699	.0477
9	7528.	86488.	11.49	814.	1228.	1.51	4.86	2.37	5.1809	-.42977	-.23238	.9676	.0500
10	6341.	65694.	10.39	724.	1269.	1.75	5.65	4.34	5.6372	-.47925	-.23970	.9850	.0462
11	5233.	85087.	16.26	912.	1355.	1.49	4.90	4.45	4.8556	-.43352	-.25648	.9652	.0293
12	4457.	63604.	14.27	760.	1259.	1.61	5.70	7.12	5.1733	-.45545	-.25770	.9670	.0313
13	3712.	44963.	12.11	674.	1226.	1.82	6.69	12.48	5.5498	-.51156	-.25103	.9672	.0303
14	3042.	30829.	10.13	591.	1232.	2.09	8.48	23.79	5.9507	-.56016	-.25366	.9602	.0271
15	2486.	21355.	8.59	527.	1259.	2.39	10.84	47.51	6.3253	-.60778	-.24414	.9856	.0226
16	2605.	33094.	12.70	640.	957.	1.50	10.62	54.71	5.8906	-.56643	-.33659	.9652	.0221
17	2165.	23205.	10.72	535.	618.	1.53	12.75	77.39	6.0778	-.61606	-.29219	.9910	.0161
18	1765.	15325.	8.68	455.	757.	1.06	15.30	102.26	6.1926	-.62525	-.23920	.9914	.0132

LOCATION PATTERN NO. 15

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	30603.	280914.	9.16	1656.	789.	.42	3.36	.13	3.6796	-.26155	-.12511	.9916	.0122
2	28943.	277605.	9.59	1767.	937.	.53	3.47	.18	4.0243	-.27476	-.15612	.9912	.0236
3	27122.	273409.	10.08	1669.	1177.	.71	3.60	.27	4.2465	-.29558	-.15722	.9887	.0249
4	25314.	271861.	10.74	1576.	1524.	.97	3.78	.44	4.4798	-.31741	-.17651	.9636	.0295
5	23601.	275593.	11.68	1494.	1575.	1.32	3.99	.71	4.6760	-.33585	-.15910	.9793	.0219
6	12408.	185232.	14.93	1309.	1153.	.89	3.40	.43	3.8364	-.29633	-.17442	.9933	.0286
7	11302.	153370.	13.57	1236.	1330.	1.10	3.61	.69	4.0928	-.31602	-.17558	.9932	.0323
8	10183.	124935.	12.27	1119.	1750.	1.56	3.91	1.06	4.3962	-.34726	-.18166	.9966	.0348
9	9125.	104624.	11.47	1105.	2464.	2.23	4.31	1.78	4.7371	-.36176	-.17764	.9694	.0318
10	8190.	94795.	11.57	1009.	3500.	3.47	4.65	3.17	5.0357	-.41166	-.14727	.9841	.0205
11	5638.	86225.	15.29	1015.	1415.	1.39	4.36	3.17	4.6209	-.40963	-.24476	.9837	.0255
12	4941.	63732.	12.90	902.	1290.	1.43	4.84	4.53	4.8709	-.43526	-.23076	.9620	.0243
13	4269.	45027.	10.55	815.	1419.	1.74	5.53	7.86	5.1626	-.46696	-.16669	.9615	.0166
14	3669.	32163.	8.77	757.	1835.	2.42	6.45	12.20	5.4435	-.50111	-.13618	.9791	.0091
15	3180.	25586.	8.05	729.	2574.	3.53	7.74	23.47	5.7172	-.53201	-.09606	.9748	.0042
16	2859.	36704.	12.67	721.	1349.	1.67	8.69	36.49	5.6790	-.56194	-.33363	.9728	.0208
17	2417.	25307.	10.47	616.	962.	1.59	10.26	52.17	5.9122	-.59301	-.29567	.9731	.0172
18	2002.	16097.	8.04	536.	806.	1.50	11.92	73.23	6.1160	-.62177	-.23684	.9692	.0118

LOCATION PATTERN NO. 16

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	29565.	142232.	4.81	1764.	397.	.23	3.41	.11	4.0576	-.27721	-.17280	.9921	.0399
2	27563.	117725.	4.27	1637.	395.	.24	3.54	.15	4.3526	-.30461	-.16607	.9934	.0477
3	25598.	101963.	3.98	1516.	425.	.28	3.70	.23	4.6745	-.33479	-.18266	.9782	.0625
4	23776.	96665.	4.07	1466.	485.	.35	3.69	.35	4.9663	-.36233	-.18560	.9714	.0636
5	22163.	100555.	4.54	1312.	565.	.43	4.11	.55	5.2174	-.36568	-.19126	.9632	.0616
6	11371.	106652.	9.40	1094.	518.	.47	3.56	.57	4.1065	-.31557	-.16296	.9936	.0341
7	10046.	75836.	7.55	944.	415.	.44	3.65	.65	4.4964	-.35550	-.16563	.9913	.0409
8	8777.	52684.	6.00	616.	365.	.45	4.24	1.34	4.9594	-.40321	-.18454	.9666	.0528
9	7639.	37420.	4.90	711.	350.	.49	4.75	2.25	5.4466	-.45452	-.18548	.9651	.0505
10	6679.	28543.	4.27	629.	352.	.56	5.40	3.89	5.9723	-.51134	-.17772	.9600	.0397
11	4616.	48471.	10.07	731.	505.	.69	4.84	4.13	4.6551	-.43099	-.21495	.9636	.0238
12	4014.	30317.	7.55	560.	310.	.53	5.59	6.61	5.2301	-.47251	-.21225	.9603	.0269
13	3268.	18094.	5.50	465.	212.	.46	6.62	11.26	5.6713	-.52346	-.20209	.9600	.0262
14	2673.	19740.	4.02	379.	164.	.43	8.11	20.52	6.1192	-.57646	-.19967	.9851	.0244
15	2164.	6696.	3.07	317.	140.	.44	10.22	39.72	6.5756	-.63300	-.16566	.9640	.0200
16	2270.	19608.	6.64	462.	387.	.84	10.31	52.23	5.9499	-.59517	-.20515	.9613	.0141
17	1788.	10948.	6.12	342.	193.	.56	12.36	76.74	6.1650	-.62490	-.23023	.9698	.0124
~~18~~	~~1374.~~	5657.	4.12	256.	104.	.41	14.97	109.44	6.3505	-.64567	-.19766	.9941	.0108

LOCATION PATTERN NO. 17

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	30382.	207870.	6.64	1831.	536.	.29	3.36	.11	3.8350	-.25666	-.18186	.9907	.0408
2	28532.	178516.	6.26	1716.	467.	.27	3.46	.14	3.6870	-.26665	-.19456	.9936	.0663
3	26594.	149743.	5.63	1597.	408.	.26	3.58	.18	3.9550	-.27613	-.18347	.9641	.0717
4	24656.	125074.	5.07	1482.	372.	.25	3.72	.26	4.1718	-.28614	-.18477	.9837	.0767
5	22802.	106217.	4.66	1373.	360.	.26	3.90	.37	4.3414	-.30149	-.18206	.9772	.0728
6	12123.	160952.	13.28	1238.	896.	.72	3.43	.60	3.9943	-.30713	-.21732	.9946	.0481
7	10671.	121582.	11.18	1095.	669.	.61	3.65	.81	4.2513	-.33276	-.23156	.9936	.0691
8	9573.	85938.	8.96	958.	485.	.51	3.92	1.17	4.5016	-.36462	-.23202	.9941	.0837
9	8305.	57446.	6.92	833.	352.	.42	4.27	1.76	4.8992	-.40006	-.23367	.9937	.0660
10	7132.	36926.	5.18	722.	267.	.37	4.73	2.77	5.2488	-.43063	-.23993	.9932	.0936
11	5493.	85404.	15.55	951.	1288.	1.35	4.64	4.95	4.6380	-.43434	-.32747	.9831	.0406
12	4734.	63488.	12.86	798.	880.	1.10	5.18	7.27	5.1460	-.46959	-.31353	.9624	.0439
13	3913.	39038.	9.98	661.	578.	.87	5.92	11.31	5.4839	-.50861	-.29964	.9640	.0405
14	3169.	23125.	7.30	544.	371.	.68	6.92	16.69	5.6905	-.55669	-.26688	.9867	.0457
15	2516.	12775.	5.08	446.	237.	.53	8.30	33.55	6.3640	-.61213	-.27029	.9859	.0413
16	2823.	39031.	13.63	691.	1311.	1.90	10.08	63.38	5.6084	-.57782	-.35306	.9706	.0252
17	2343.	26783.	11.43	561.	856.	1.53	11.48	62.74	5.9918	-.60302	-.36595	.9603	.0249
18	1972.	16439.	8.34	450.	544.	1.21	13.07	103.05	6.1393	-.62479	-.32428	.9905	.0227

LOCATION PATTERN NO. 18

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	32189.	754598.	23.44	2022.	5745.	2.64	3.60	.35	4.3701	-.30569	-.13729	.9936	.0269
2	31125.	861852.	27.09	1967.	7569.	3.81	3.83	.55	4.6107	-.32007	-.19024	.9930	.0247
3	29960.	910125.	30.38	1945.	8763.	4.51	4.10	.93	4.8789	-.35066	-.10962	.9945	.0162
4	28763.	920995.	32.02	1900.	9434.	4.97	4.42	1.59	5.1202	-.37237	-.13060	.9946	.0082
5	27603.	916304.	33.20	1854.	9798.	5.29	4.83	2.87	5.3891	-.39719	-.06360	.9943	.0026
6	14458.	526459.	36.55	1776.	18746.	10.56	4.16	1.50	4.3521	-.33332	-.13421	.9655	.0089
7	14027.	588010.	41.92	1820.	25744.	14.15	4.76	2.46	4.6509	-.35809	-.08352	.9672	.0055
8	13459.	612370.	45.53	1841.	30056.	16.65	5.48	4.21	4.9618	-.38539	-.05936	.9727	.0025
9	12632.	611835.	47.68	1644.	33582.	16.21	6.35	7.72	5.2853	-.41592	-.03004	.9702	.0005
10	12233.	600273.	49.05	1640.	35173.	19.12	7.51	15.19	5.5962	-.44034	-.00000	.9706	.0000
11	7985.	315157.	39.47	1979.	41024.	21.13	6.30	10.99	4.8699	-.40749	-.07979	.9159	.0029
12	6040.	387755.	48.23	2164.	61450.	26.40	7.86	19.60	5.0925	-.42642	-.05730	.9240	.0016
13	7946.	440359.	55.42	2295.	76940.	33.53	9.98	30.86	5.4082	-.45416	-.02670	.9309	.0003
14	7744.	470323.	60.73	2373.	87140.	36.72	12.85	73.25	5.7244	-.48230	-.01167	.9530	.0001
15	7512.	484317.	64.47	2415.	93169.	38.58	15.92	116.65	5.8794	-.49385	-.03064	.9609	.0004
16	5378.	182847.	36.61	2041.	55708.	27.29	13.33	98.23	5.5050	-.50606	-.08247	.9132	.0017
17	5467.	264262.	48.34	2412.	91397.	37.89	16.22	124.24	5.5090	-.49452	-.01257	.9293	.0001
18	5730.	340471.	59.42	2710.	98143.	45.99	18.95	139.12	5.4145	-.40046	-.02193	.9467	.0002

LOCATION PATTERN NO. 19

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	31099.	215639.	6.93	1876.	670.	.36	3.36	.16	3.7852	-.25206	-.09708	.9934	.0070
2	29600.	202925.	6.66	1764.	713.	.40	3.47	.25	3.9736	-.26923	-.11456	.9929	.0114
3	28367.	194817.	6.94	1694.	803.	.47	3.61	.41	4.2090	-.29129	-.12628	.9909	.0139
4	26572.	192863.	7.26	1611.	944.	.59	3.77	.69	4.4579	-.31468	-.11622	.9660	.0113
5	25177.	196844.	7.82	1536.	1131.	.74	3.98	1.92	4.7255	-.34013	-.09599	.9836	.0071
6	12519.	142531.	11.39	1268.	1091.	.86	3.46	.69	4.0153	-.30823	-.12644	.9069	.0126
7	11494.	111693.	9.74	1152.	1057.	.92	3.68	1.05	4.3136	-.33651	-.12138	.9971	.0126
8	10450.	85979.	8.22	1048.	1104.	1.05	3.98	1.71	4.6406	-.37200	-.11656	.9871	.0117
9	9471.	66619.	7.03	960.	1231.	1.28	4.38	2.93	4.9469	-.40317	-.11669	.9552	.0118
10	8533.	54173.	6.31	890.	1426.	1.60	4.91	5.29	5.3018	-.44063	-.11158	.9830	.0094
11	5636.	69303.	12.30	955.	1254.	1.31	4.62	4.64	4.7739	-.42445	-.12606	.9689	.0063
12	4961.	49436.	9.97	624.	1028.	1.25	5.19	7.27	5.0726	-.45733	-.10053	.9705	.0043
13	4297.	33531.	7.80	715.	932.	1.30	5.93	12.20	5.4322	-.49929	-.08295	.9766	.0030
14	3682.	22213.	6.03	629.	936.	1.49	6.99	21.63	5.7634	-.54023	-.07341	.9627	.0024
15	3152.	15138.	4.80	564.	1014.	1.80	8.44	40.62	6.1066	-.57666	-.06126	.9876	.0016
16	2619.	30457.	10.30	671.	1012.	1.51	9.75	52.06	5.8045	-.57617	-.20465	.9716	.0074
17	2365.	20362.	8.55	555.	694.	1.25	11.11	70.78	5.9678	-.60024	-.14551	.9797	.0044
18	1971.	12634.	6.41	462.	520.	1.13	12.64	90.23	6.1095	-.61656	-.10397	.9876	.0026

LOCATION PATTERN NO. 20

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	31102.	270688.	8.71	1894.	1103.	.58	3.39	.20	4.0059	-.27326	-.17766	.9803	.0395
2	29706.	301696.	10.16	1623.	1709.	.94	3.55	.34	4.2690	-.29729	-.17826	.9675	.0348
3	28301.	352181.	12.44	1758.	2631.	1.50	3.75	.58	4.5032	-.31864	-.15781	.9885	.0223
4	26910.	414049.	15.39	1699.	3741.	2.20	4.03	1.02	4.7581	-.34162	-.15196	.9872	.0161
5	25557.	475477.	18.61	1642.	4848.	2.95	4.40	1.87	5.0847	-.37167	-.14416	.9872	.0109
6	12648.	159086.	12.58	1334.	1584.	1.19	3.53	.83	4.1346	-.32026	-.16362	.9915	.0431
7	11732.	136839.	11.61	1271.	2296.	1.81	3.64	1.38	4.4787	-.35479	-.19395	.9930	.0400
8	10331.	129099.	11.76	1240.	3854.	3.11	4.28	2.43	4.7875	-.38464	-.16865	.9636	.0320
9	10291.	140262.	13.63	1241.	6466.	5.21	4.91	4.61	5.0450	-.40639	-.14253	.9746	.0209
10	9738.	170084.	17.47	1268.	10065.	7.95	5.78	9.44	5.3295	-.43530	-.13063	.9632	.0150
11	5771.	77336.	13.40	1039.	1831.	1.76	4.85	5.63	4.9368	-.44203	-.21267	.9802	.0231
12	5098.	57173.	11.22	956.	1620.	1.90	5.57	9.33	5.2763	-.48007	-.20611	.9633	.0237
13	4635.	43257.	9.21	920.	2407.	2.62	6.67	17.08	5.5784	-.50913	-.17371	.9796	.0179
14	4308.	38174.	8.66	931.	3980.	4.28	8.27	34.37	5.8247	-.53631	-.14718	.9694	.0131
15	4075.	44096.	10.82	993.	7075.	7.13	10.48	69.12	6.0157	-.55486	-.13716	.9606	.0112
16	2946.	35857.	12.17	749.	4823.	2.43	10.65	66.14	5.9277	-.58937	-.27064	.9704	.0141
17	2556.	24819.	9.71	660.	1387.	2.10	12.71	100.65	6.1501	-.61737	-.23167	.9804	.0125
18	2208.	16351.	7.41	603.	1188.	1.97	14.73	135.85	6.2047	-.62407	-.21569	.9931	.0133

APPENDIX 4

AUTOCORRELATION OF $\bar{t}_i$ AND REGRESSION OF $\bar{t}_i$ ON s_i^2 FOR LOCATION PATTERNS 2 TO 20

Key to column headings:

1. = Simulation number
2. = First nearest neighbour autocorrelation statistic for $\bar{t}_i$
3. = γ
4. = δ
5. = r^2

LOCATION PATTERN NO. [illegible]

1.	2.	3.	4.	5.
1	.5035	-2.6229	2.1065	.8894
2	.5715	-2.9755	2.4282	.9166
3	.5917	-3.2312	2.0693	.9301
4	.5745	-3.4526	2.8629	.9463
5	.5339	-3.5197	2.9321	.9062
6	.3867	-3.1276	2.6828	.8792
7	.4555	-3.3035	2.8651	.9035
8	.4666	-3.3169	2.9170	.9368
9	.4505	-3.2656	2.9092	.9593
10	.4131	-3.1748	2.8602	.9724
11	.1949	-3.3583	3.1390	.9722
12	.2574	-3.2166	3.0378	.973-
13	.2856	-3.1030	2.9494	.9720
14	.3093	-3.0121	2.8750	.9671
15	.3828	-2.9543	2.8297	.9672
16	.0313	-2.8829	2.8691	.9759
17	.1004	-2.7213	2.7776	.9760
18	.2091	-2.6228	2.7243	.9762

LOCATION PATTERN NO. 3

1.	2.	3.	4.	5.
1	.0956	-1.4770	1.2026	.5264
2	.2221	-1.9734	1.6607	.7276
3	.3877	-2.5104	2.1321	.7740
4	.5255	-2.9902	2.5260	.8629
5	.6262	-3.3052	2.7808	.9179
6	-.0939	-2.4985	2.1581	.6615
7	.0634	-2.5056	2.2533	.6593
8	.1785	-2.6445	2.4224	.8734
9	.3040	-2.7205	2.5322	.8967
10	.4242	-2.8216	2.6290	.9171
11	-.0216	-3.0750	2.9312	.9504
12	.0234	-2.8543	2.8015	.9614
13	.0967	-2.6367	2.6828	.9636
14	.1374	-2.3520	2.5576	.9644
15	.2899	-2.1742	2.4639	.9636
16	-.1416	-2.6017	2.7327	.9819
17	.0262	-2.3135	2.5698	.9864
18	.0963	-2.0333	2.4674	.9872

LOCATION PATTERN NO. 4

1.	2.	3.	4.	5.
1	-.1328	-1.2652	.9298	.3058
2	-.0016	-1.9258	1.5157	.4997
3	.1386	-2.5097	2.0971	.6570
4	.2142	-3.1207	2.5542	.9869
5	.2260	-3.4330	2.0756	.8551
6	.0997	-2.2900	1.8253	.7490
7	.2044	-2.4890	2.0089	.7167
8	.2896	-2.7249	2.2264	.7621
9	.3415	-2.9514	2.4508	.8236
10	.3666	-3.1153	2.5792	.8609
11	.0465	-3.4370	3.0696	.9409
12	.1051	-3.2579	2.9212	.9337
13	.1814	-3.1467	2.8233	.9234
14	.2541	-3.0892	2.7645	.9294
15	.3022	-3.0558	2.7308	.9322
16	.0354	-2.9731	2.9107	.9691
17	.0241	-2.6626	2.8348	.9060
18	.0506	-2.6877	2.7331	.9670

LOCATION ... RUN NO. 5

1.	2.	3.	4.	5.
1	.8954	-2.5479	2.0365	.8438
2	.9169	-2.8654	2.3457	.9060
3	.9187	-3.0061	2.5019	.9295
4	.9027	-3.1440	2.5830	.9432
5	.8751	-3.1653	2.6302	.9123
6	.9120	-3.0766	2.5265	.8877
7	.9444	-3.1502	2.6032	.9140
8	.9591	-3.3539	2.7433	.8940
9	.9642	-3.2997	2.7095	.9105
10	.9650	-3.2692	2.6657	.9203
11	.8717	-3.7050	3.1658	.9002
12	.9221	-3.8784	3.1843	.9391
13	.9521	-3.6770	3.0145	.9399
14	.9703	-3.5623	2.8951	.9412
15	.9605	-3.6469	2.6720	.9290
16	.8089	-3.0601	2.6501	.9800
17	.8679	-3.3183	2.8956	.9543
18	.8965	-3.3561	2.6625	.9491

... RUN NO. 6

1.	2.	3.	4.	5.
1	.1165	-1.6364	1.3400	.7383
2	.2224	-2.0374	1.6713	.7631
3	.3176	-2.4426	2.0514	.7865
4	.3613	-2.6693	2.4311	.8430
5	.3960	-3.2756	2.7437	.8667
6	.1828	-2.5745	2.2136	.8456
7	.2625	-2.7111	2.3731	.8712
8	.3765	-2.8115	2.4955	.8947
9	.4422	-2.8600	2.5607	.9110
10	.4610	-2.9000	2.6347	.9248
11	.1157	-3.3492	3.1116	.9604
12	.2145	-3.1095	2.9564	.9646
13	.3025	-2.8304	2.0236	.9684
14	.3722	-2.5765	2.6554	.9769
15	.4252	-2.3560	2.5475	.9800
16	.0416	-2.6401	2.7053	.9832
17	.1053	-2.4033	2.6475	.9827
18	.1596	-2.2273	2.5645	.9812

LOCATION PATTERN NO. 7

1.	2.	3.	4.	5.
1	.6898	-2.0792	1.6914	.7495
2	.8259	-2.4646	2.0652	.6312
3	.8804	-2.7550	2.3349	.9208
4	.9106	-2.9803	2.5349	.9460
5	.9259	-3.0926	2.6408	.9615
6	.7090	-2.9353	2.5020	.8509
7	.3223	-3.1022	2.6952	.9107
8	.8924	-3.0539	2.7123	.9328
9	.9299	-3.0170	2.7134	.9516
10	.9514	-2.9297	2.6841	.9652
11	.7278	-3.0447	2.9114	.9621
12	.8316	-2.8521	2.7603	.9716
13	.8689	-2.6822	2.6838	.9764
14	.9219	-2.4739	2.5752	.9754
15	.9569	-2.3113	2.4943	.9755
16	.7208	-2.4928	2.6873	.9790
17	.8368	-2.2946	2.5517	.9835
18	.3629	-1.9978	2.4777	.9891

LOCATION PATTERN NO.

1.	2.	3.	4.	5.
1	.4964	-1.8343	1.4583	.7261
2	.6742	-2.2691	1.9005	.6091
3	.7647	-2.6627	2.2510	.8834
4	.8055	-2.9772	2.5006	.9314
5	.8296	-3.1956	2.6917	.9622
6	.4170	-2.8692	2.4775	.8590
7	.5940	-3.0649	2.6485	.9393
8	.7188	-3.1308	2.7309	.9334
9	.7981	-3.0610	2.7186	.9433
10	.8484	-3.0124	2.7076	.9573
11	.2659	-3.0663	2.9241	.9629
12	.4734	-2.6446	2.7733	.9709
13	.6242	-2.7272	2.6919	.9771
14	.7308	-2.5797	2.6056	.9737
15	.7566	-2.3597	2.4957	.9838
16	.1157	-2.5041	2.7016	.9763
17	.2352	-2.3464	2.5911	.9832
18	.2676	-2.2476	2.5473	.9820

LOCATION PATTERN NO. 9

1.	2.	3.	4.	5.
1	.6663	-2.0527	1.6502	.8558
2	.7105	-2.3537	1.9372	.6912
3	.7355	-2.7203	2.2725	.9206
4	.7452	-3.0350	2.5391	.9458
5	.7495	-3.2489	2.7219	.9617
6	.5913	-2.9123	2.5198	.9012
7	.6438	-2.9973	2.6378	.9225
8	.6835	-3.1956	2.8028	.9336
9	.7137	-3.3505	2.9097	.9386
10	.7338	-3.2679	2.8644	.9461
11	.5222	-3.3475	3.0922	.9491
12	.5914	-3.1473	2.9547	.9479
13	.6485	-2.9796	2.8405	.9578
14	.6916	-2.8032	2.7310	.9609
15	.7161	-2.6495	2.6402	.9624
16	.4410	-2.8159	2.7969	.9666
17	.5001	-2.6176	2.6917	.9666
18	.5281	-2.5061	2.6299	.9662

LOCATION PATTERN NO. 10

1.	2.	3.	4.	5.
1	.5383	-2.2047	1.6174	.8864
2	.6160	-2.5440	2.0906	.8943
3	.6754	-2.8183	2.3470	.9036
4	.7096	-3.0625	2.5649	.9258
5	.7342	-3.2296	2.7178	.9511
6	.4424	-2.9121	2.5125	.8945
7	.4729	-2.9330	2.6238	.9265
8	.4677	-3.0264	2.6911	.9478
9	.4907	-3.0332	2.7265	.9615
10	.4690	-3.0169	2.7336	.9700
11	.3034	-3.2199	3.0263	.9693
12	.3049	-3.0575	2.9164	.9712
13	.2910	-2.9272	2.8244	.9705
14	.3296	-2.8539	2.7645	.9669
15	.4305	-2.8157	2.7321	.9690
16	.3138	-2.7844	2.8017	.9762
17	.3954	-2.6631	2.7365	.9755
18	.5015	-2.6396	2.7062	.9715

LOCATION PATTERN NO. 11

1.	2.	3.	4.	5.
1	.4435	-1.9096	1.5599	.8760
2	.5934	-2.2290	1.8001	.9123
3	.6756	-2.5101	2.0699	.9214
4	.6996	-2.6218	2.3433	.9239
5	.6956	-2.9679	2.4945	.9491
6	.3658	-2.6753	2.4331	.8678
7	.4992	-2.6934	2.4936	.9016
8	.5977	-2.6821	2.5332	.9250
9	.6634	-2.9424	2.6072	.9401
10	.7049	-3.1783	2.7636	.9305
11	.1711	-3.3082	3.0052	.9704
12	.3138	-3.1552	2.9536	.9666
13	.4233	-2.9394	2.6436	.9686
14	.5036	-2.8530	2.7535	.9686
15	.5626	-2.9098	2.7522	.9541
16	.0302	-2.6617	2.7322	.9612
17	.1598	-2.4030	2.6234	.9818
18	.2975	-2.2816	2.6666	.9815

[illegible]

1.	2.	3.	4.	5.
1	.2211	-1.4697	1.2163	.4609
2	.4127	-2.0328	1.6832	.6407
3	.5640	-2.4504	2.0651	.7185
4	.6961	-2.9194	2.4734	.8339
5	.7590	-3.2589	2.7404	.8903
6	.0419	-2.2626	1.9513	.8514
7	.2354	-2.2605	2.0304	.8543
8	.4041	-2.3068	2.1590	.8574
9	.5412	-2.3636	2.2802	.8832
10	.6375	-2.4097	2.3603	.9048
11	[illegible]	-2.9011	2.6540	.9523
12	.0443	-2.6995	2.5931	.9659
13	.2195	-2.4659	2.5453	.9721
14	.3704	-2.1011	2.4056	.9718
15	.4678	-1.8209	2.2975	.9695
16	[illegible]	-2.5790	2.7286	.9835
17	[illegible]	-2.2089	2.5505	.9896
18	.0207	-1.5509	2.3972	.9926

LOCATION PATTERN NO. 13

1.	2.	3.	4.	5.
1	.1650	-1.3290	1.0535	.6092
2	.3055	-1.6116	1.3371	.7000
3	.4881	-1.9953	1.7097	.7755
4	.6002	-2.5009	2.1416	.8087
5	.6711	-2.9124	2.4802	.9054
6	.1446	-2.2379	1.9775	.8181
7	.2891	-2.5092	2.1977	.8503
8	.4247	-2.6022	2.3323	.8644
9	.5425	-2.6312	2.4232	.8095
10	.6263	-2.7947	2.5733	.9024
11	.0802	-3.0854	2.9522	.9465
12	.1935	-2.8049	2.8163	.9522
13	.3036	-2.6094	2.7057	.9566
14	.3993	-2.4547	2.5941	.9570
15	.4802	-2.2715	2.5048	.9575
16	.0103	-2.6500	2.7596	.9769
17	.0896	-2.2645	2.6174	.9853
18	.1723	-2.0033	2.4902	.9096

[illegible] NO. 14

1.	2.	3.	4.	5.
1	.6646	-1.9150	1.5359	.5218
2	.7197	-2.0975	1.7357	.7930
3	.7606	-2.3026	1.9496	.8587
4	.7898	-2.5406	2.1900	.8917
5	.8021	-2.8020	2.4235	.9253
6	.5035	-2.5604	2.2351	.8812
7	.5652	-2.6036	2.3916	.9192
8	.6127	-2.7217	2.5372	.9414
9	.6491	-2.9009	2.6171	.9561
10	.6719	-2.8402	2.6704	.9607
11	.3421	-3.3194	3.1161	.9420
12	.3936	-3.1350	2.9947	.9459
13	.4407	-2.9301	2.8735	.9493
14	.4794	-2.7712	2.77-5	.9526
15	.5157	-2.6319	2.6925	.9563
16	.2499	-2.9123	2.8700	.9017
17	.2872	-2.7060	2.7617	.9639
18	.3222	-2.5366	2.6702	.9010

LOCATION PATTERN NO. 15

1.	2.	3.	4.	5.
1	.5230	-1.9481	1.5473	.8301
2	.6099	-2.3662	1.9320	.8641
3	.6665	-2.7965	2.3045	.8823
4	.6844	-3.1003	2.6120	.9225
5	.6756	-3.3356	2.7735	.9526
6	.3835	-2.5777	2.2198	.8444
7	.4665	-2.8158	2.4696	.8657
8	.5333	-3.1304	2.7332	.9049
9	.5782	-3.5112	3.1109	.8879
10	.5967	-3.6117	3.1167	.9145
11	.2625	-3.4425	3.2042	.9532
12	.3304	-3.3277	3.1273	.9474
13	.3940	-3.3345	3.1130	.9258
14	.4455	-3.1893	3.0149	.9305
15	.4649	-3.0263	2.9120	.9340
16	.1821	-2.8330	2.8734	.9784
17	.2385	-2.7195	2.8039	.9755
18	.2908	-2.6707	2.7622	.9694

[illegible] NO. 16

1.	2.	3.	4.	5.
1	.4536	-1.5106	1.2134	.6068
2	.6399	-1.8311	1.5346	.6763
3	.7510	-2.2467	1.9057	.8806
4	.8111	-2.5415	2.1712	.8729
5	.8513	-2.8053	2.4025	.9034
6	.3829	-2.4716	2.1515	.8603
7	.5147	-2.6042	2.3118	.9120
8	.6301	-2.6103	2.3627	.9255
9	.7232	-2.6563	2.4535	.9352
10	.7097	-2.6028	2.5013	.9550
11	.3006	-3.0357	2.5131	.9463
12	.4045	-2.8459	2.7940	.9512
13	.5022	-2.8439	2.6701	.9669
14	.5652	-2.4230	2.5579	.9707
15	.6540	-2.2531	2.4725	.9755
16	.2495	-2.3070	2.7244	.9317
17	.2809	-2.2301	2.5852	.9843
18	.2949	-2.0326	2.4675	.9833

[illegible] NO. 17

1.	2.	3.	4.	5.
1	.1648	-1.5562	1.2276	.6831
2	.2832	-1.7232	1.4090	.7353
3	.3771	-1.9277	1.6251	.7959
4	.4594	-2.2464	1.9210	.8526
5	.5373	-2.4004	2.1493	.8772
6	.2290	-2.6093	2.2797	.8499
7	.3569	-2.7237	2.3803	.8805
8	.4652	-2.7702	2.4651	.8990
9	.5478	-2.8395	2.5415	.9189
10	.6143	-2.7582	2.5531	.9263
11	.2820	-3.2537	3.0756	.9016
12	.3620	-3.0001	2.6909	.9679
13	.4761	-2.7123	2.7356	.9752
14	.5585	-2.5115	2.6293	.9606
15	.6277	-2.3940	2.5300	.9848
16	.2803	-2.6474	2.7070	.9786
17	.3136	-2.4452	2.6655	.9793
18	.3424	-2.2610	2.5933	.9796

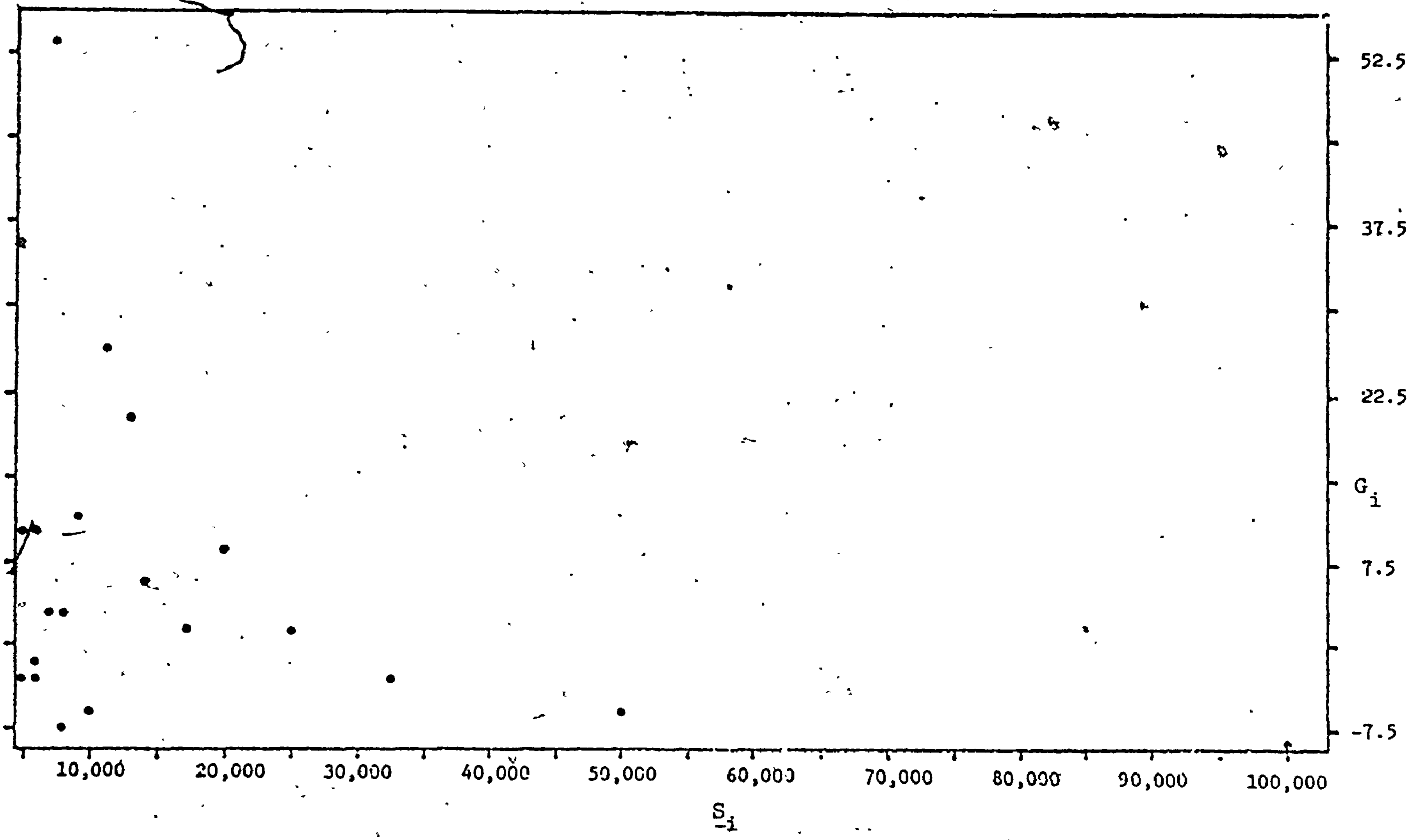
[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	5.
1	.7013	-2.4776	2.0596	.9398
2	.6577	-2.6462	2.3752	.9538
3	.5728	-3.1255	2.6079	.9642
4	.4088	-3.3231	2.7718	.9727
5	.4071	-3.4595	2.8615	.9831
6	.7145	-3.1216	2.7000	.9221
7	.7190	-3.0879	2.6999	.9334
8	.6827	-3.0562	2.7056	.9445
9	.6236	-3.1221	2.7254	.9569
10	.5545	-3.1467	2.7355	.9670
11	.6629	-2.9580	2.6536	.9672
12	.7059	-2.7805	2.7054	.9650
13	.7134	-2.6902	2.6250	.9659
14	.6561	-2.6499	2.5734	.9699
15	.6611	-2.6420	2.5435	.9726
16	.5609	-2.4923	2.6503	.9723
17	.5523	-2.3502	2.5793	.9703
18	.5537	-2.4329	2.5674	.9656

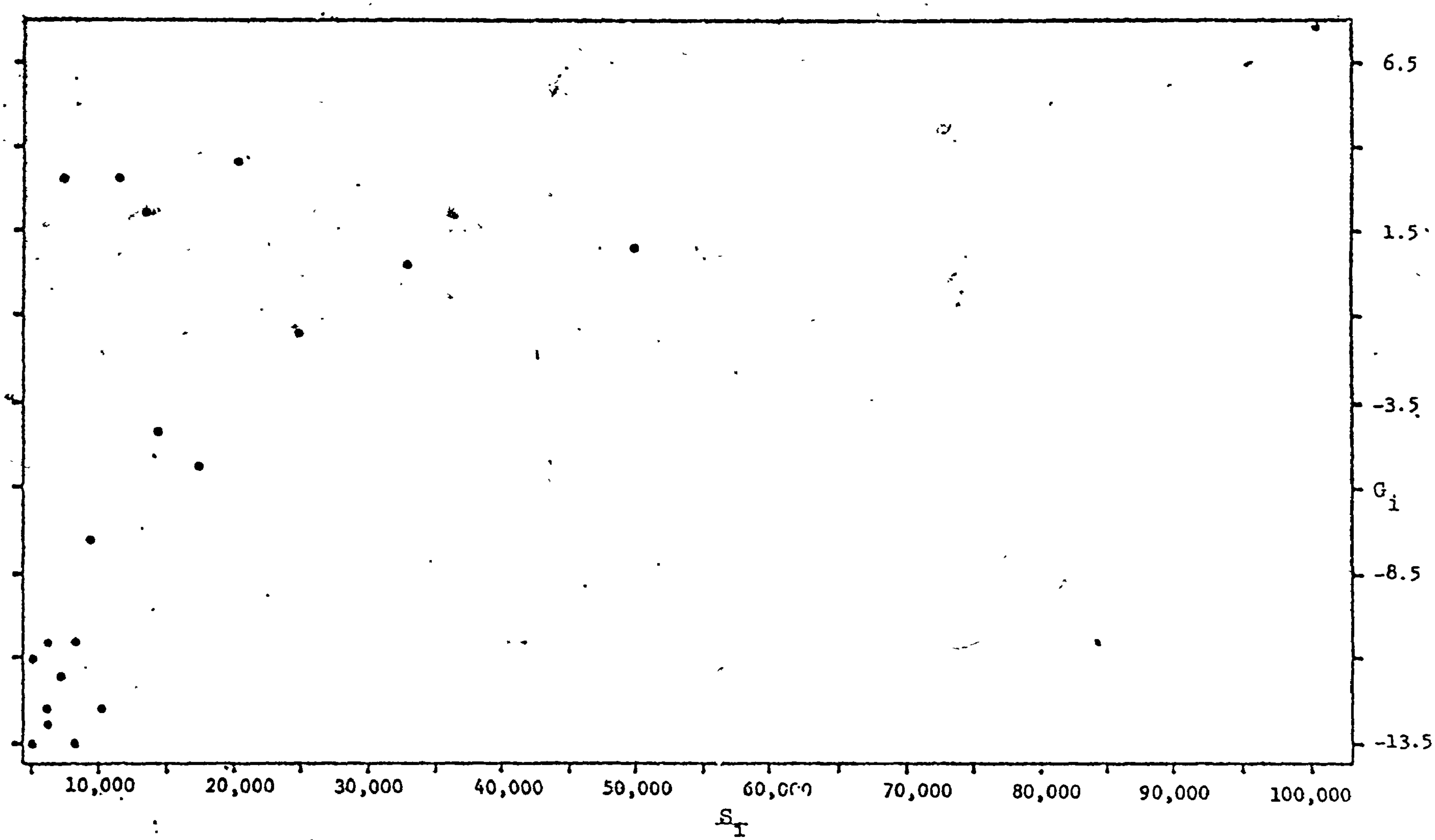
APPENDIX 5

GRAPHS OF CITY SIZE AGAINST GROWTH RATE FOR LONG - TERM AND SHORT - TERM GROWTH [illegible]

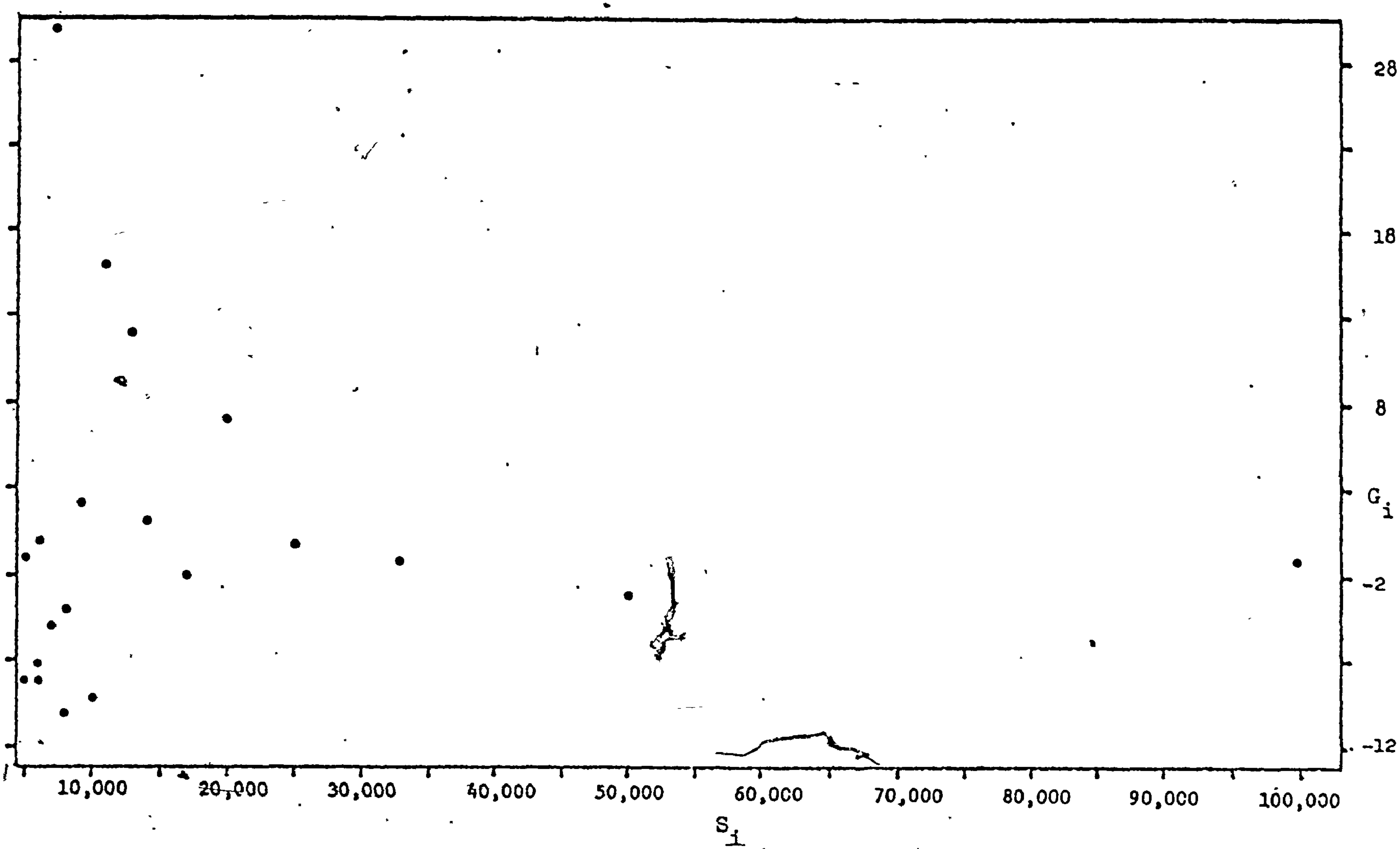
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=4, Q_1$)



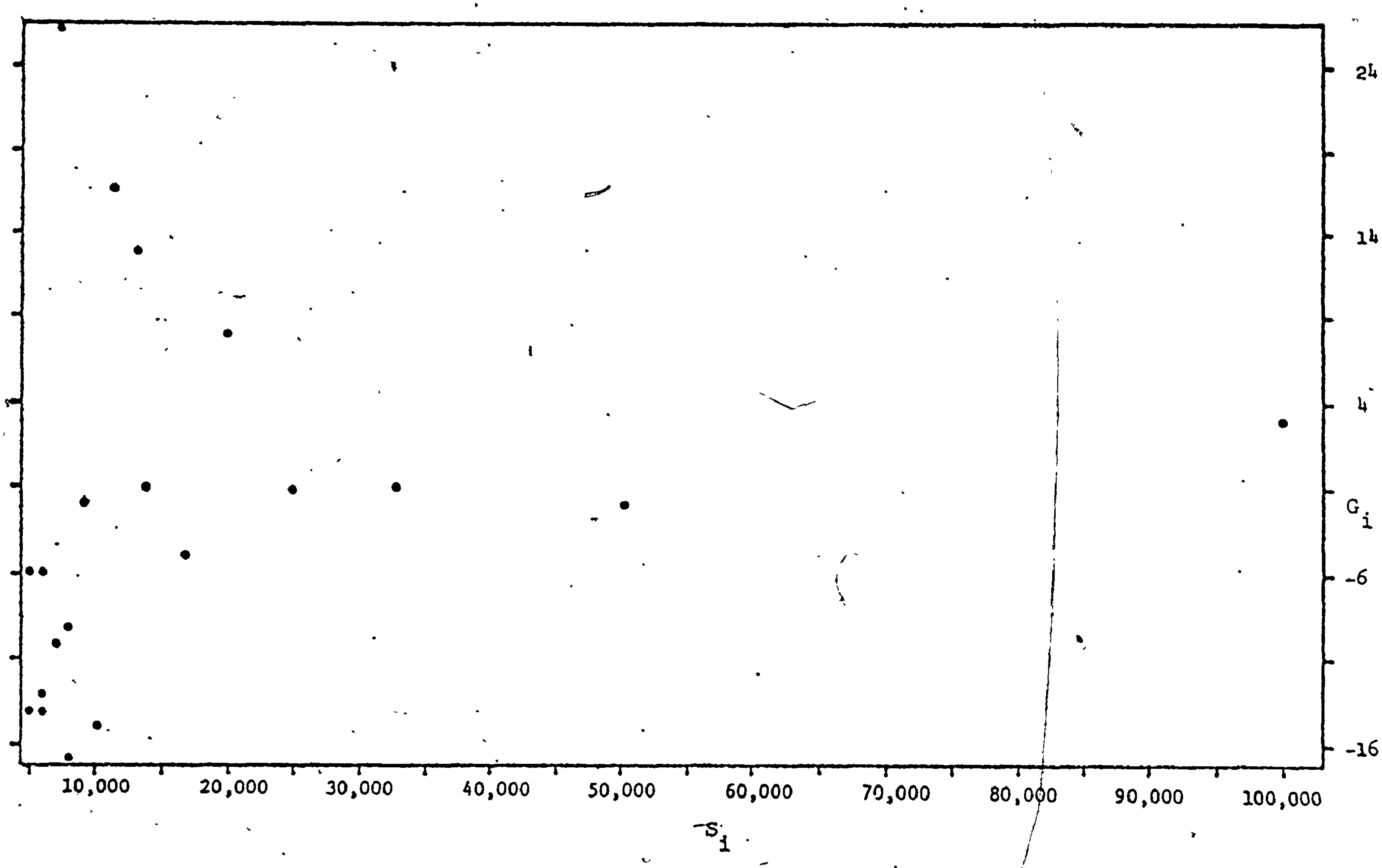
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=4,Q_2$)



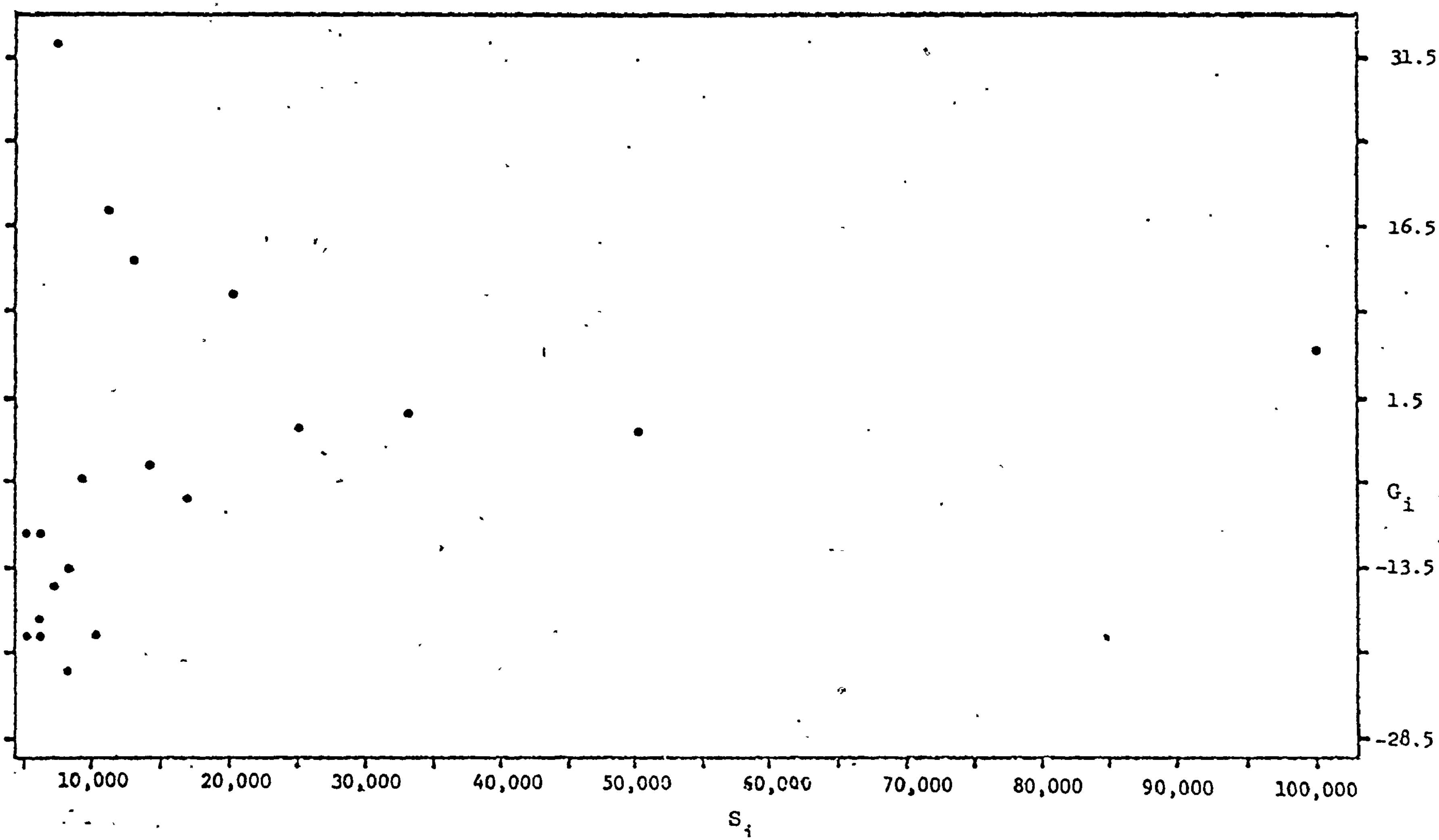
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=4, Q_3$)



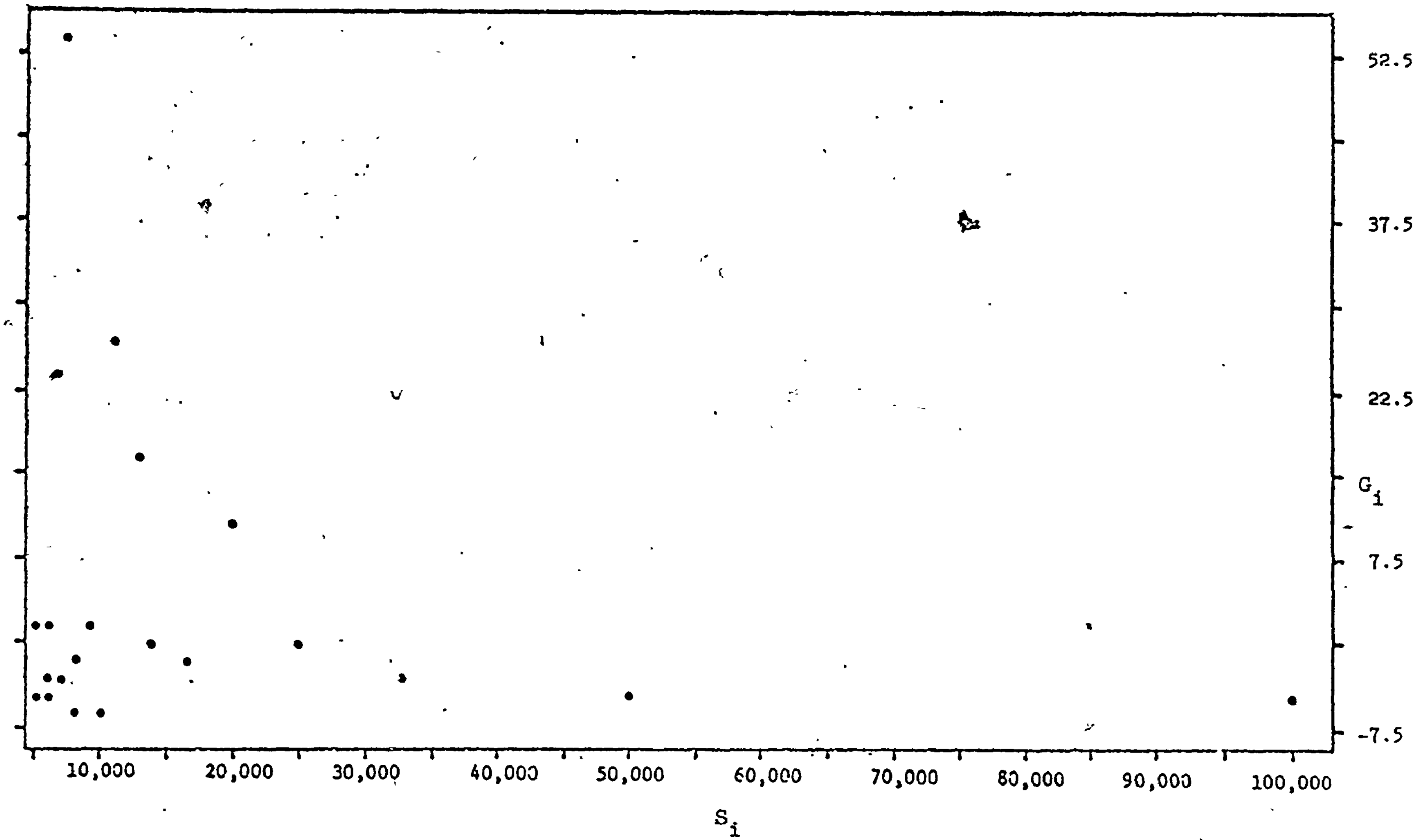
SHORT – TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=4, Q_4$)



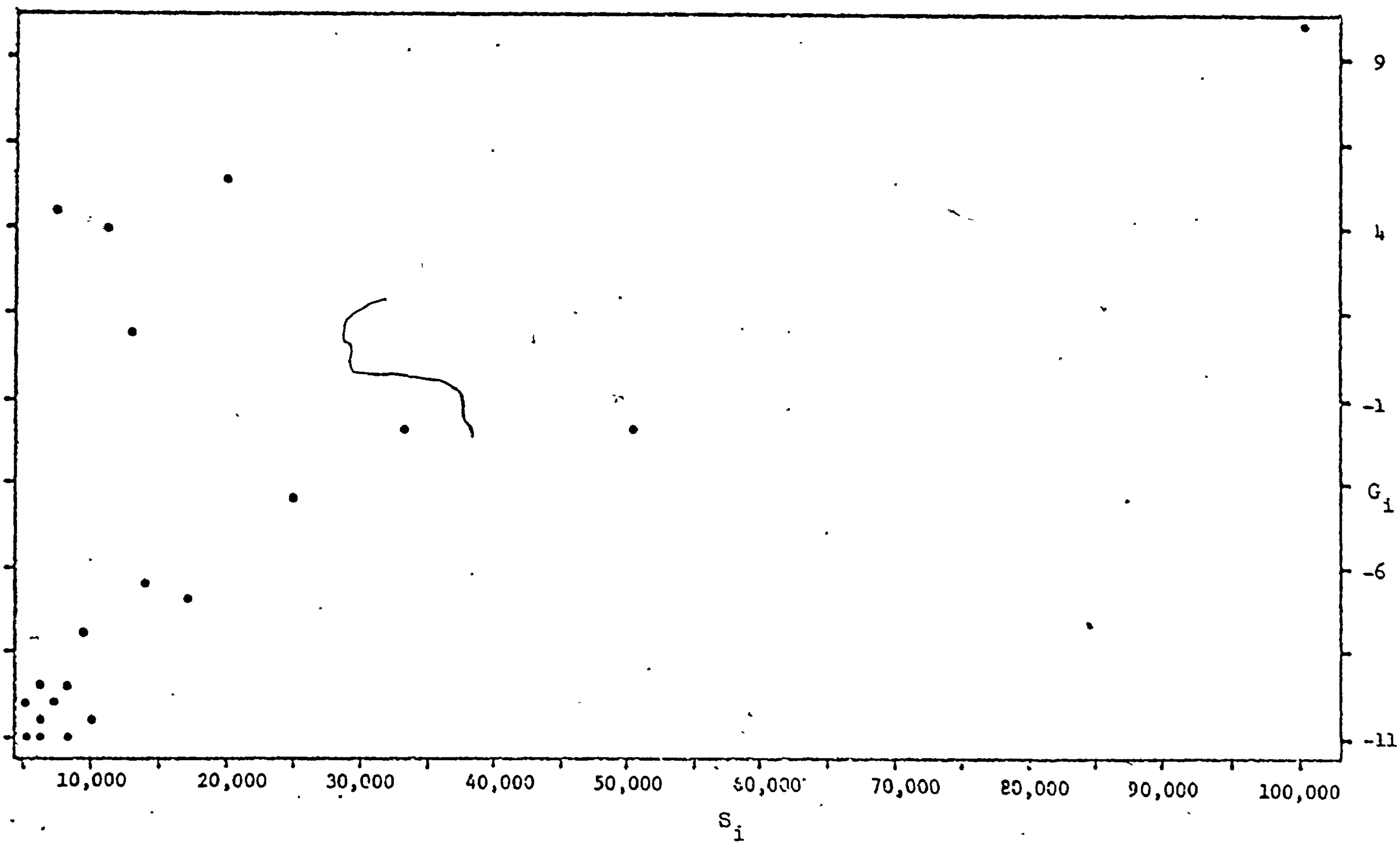
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=4,Q_5$)



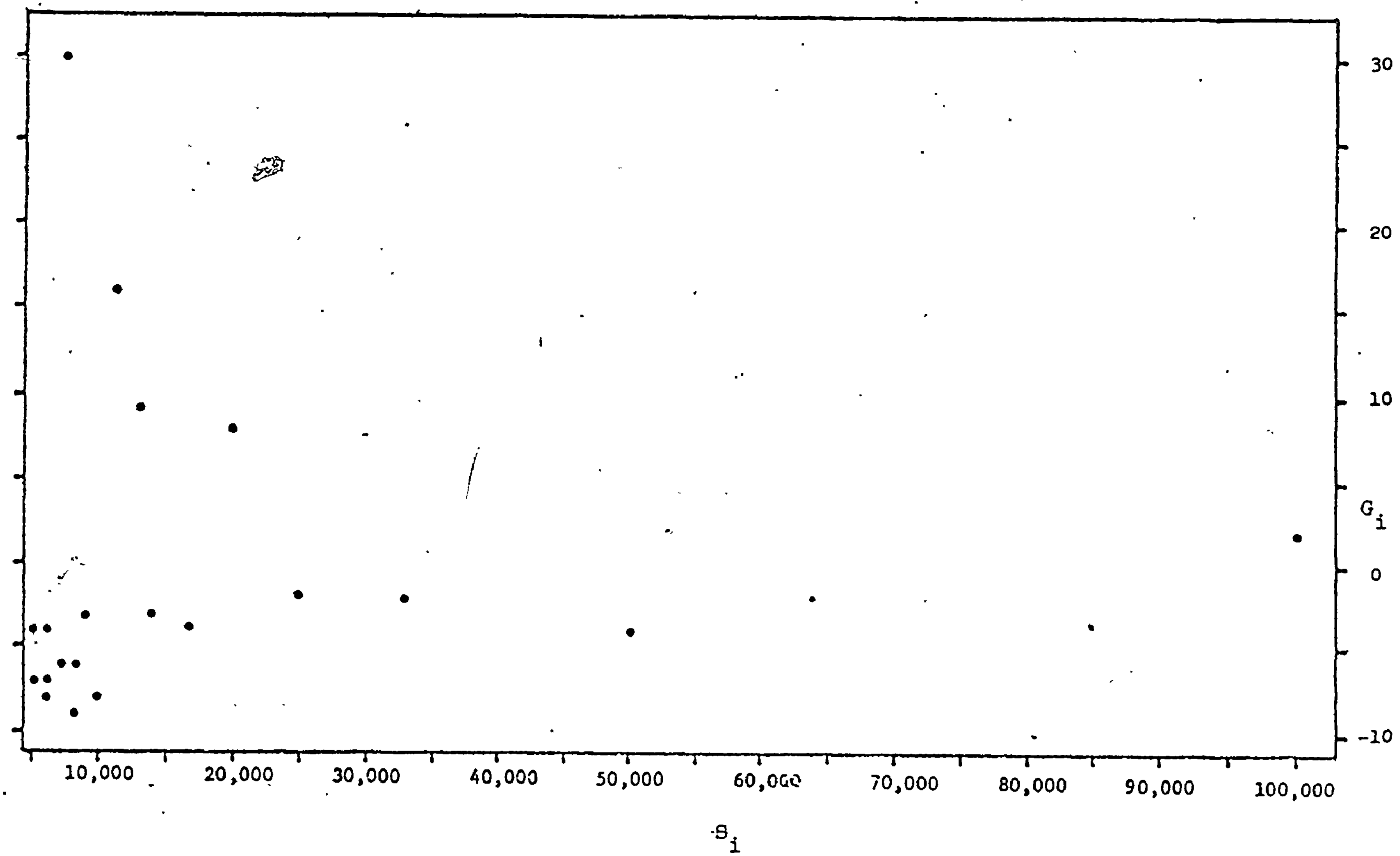
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=3,Q_1$)



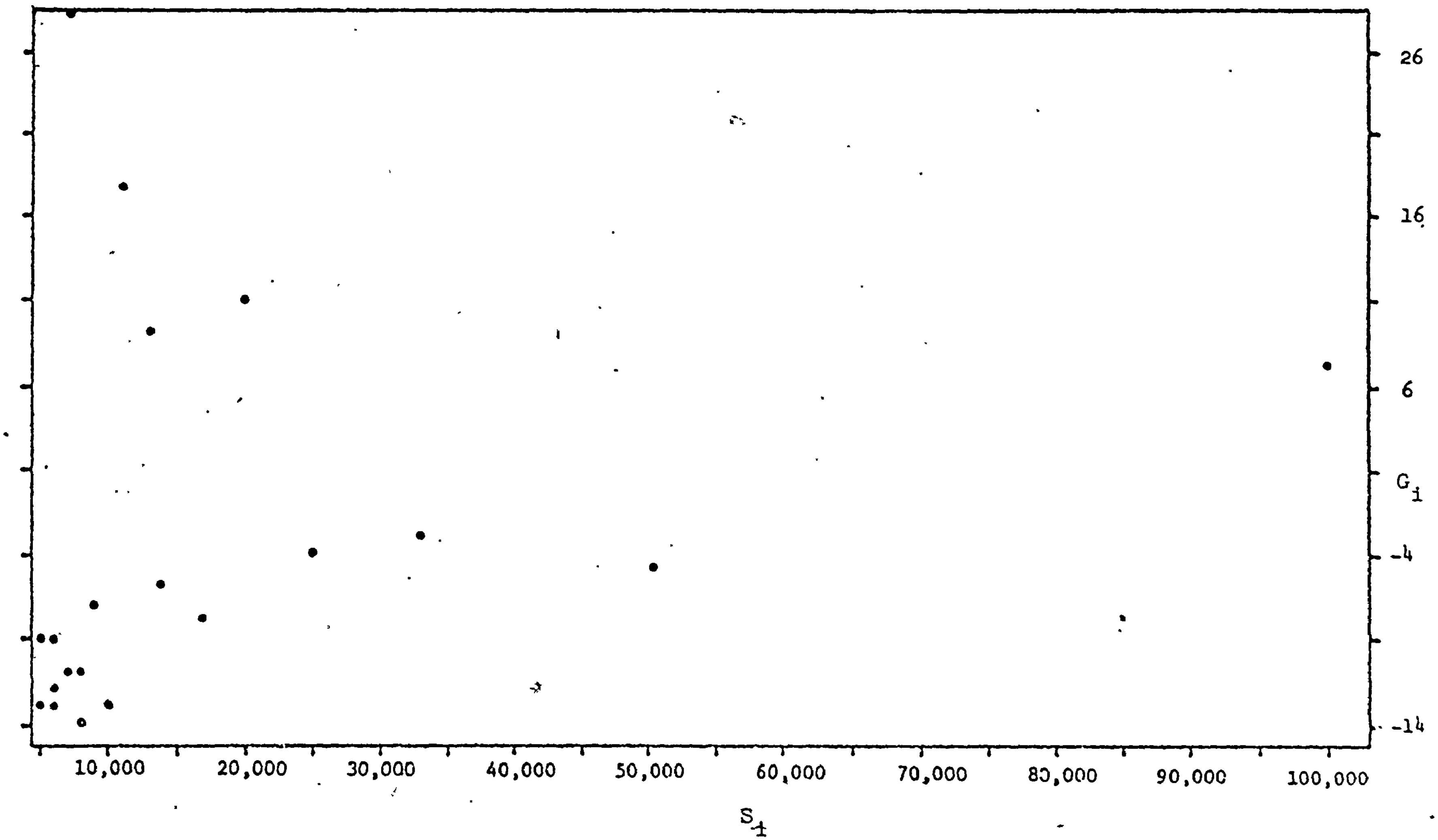
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=3,Q_2$)



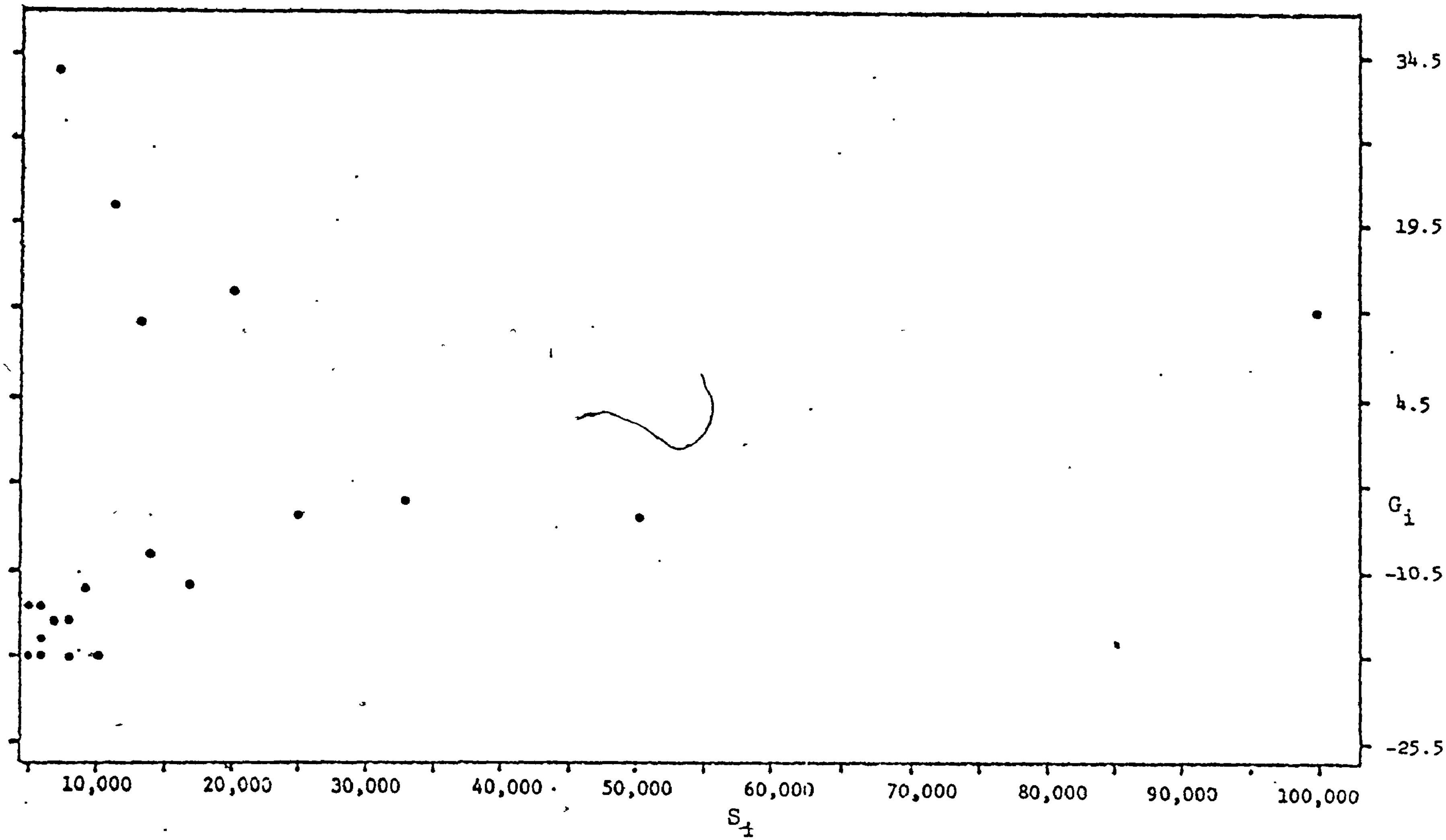
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=3,Q_3$)



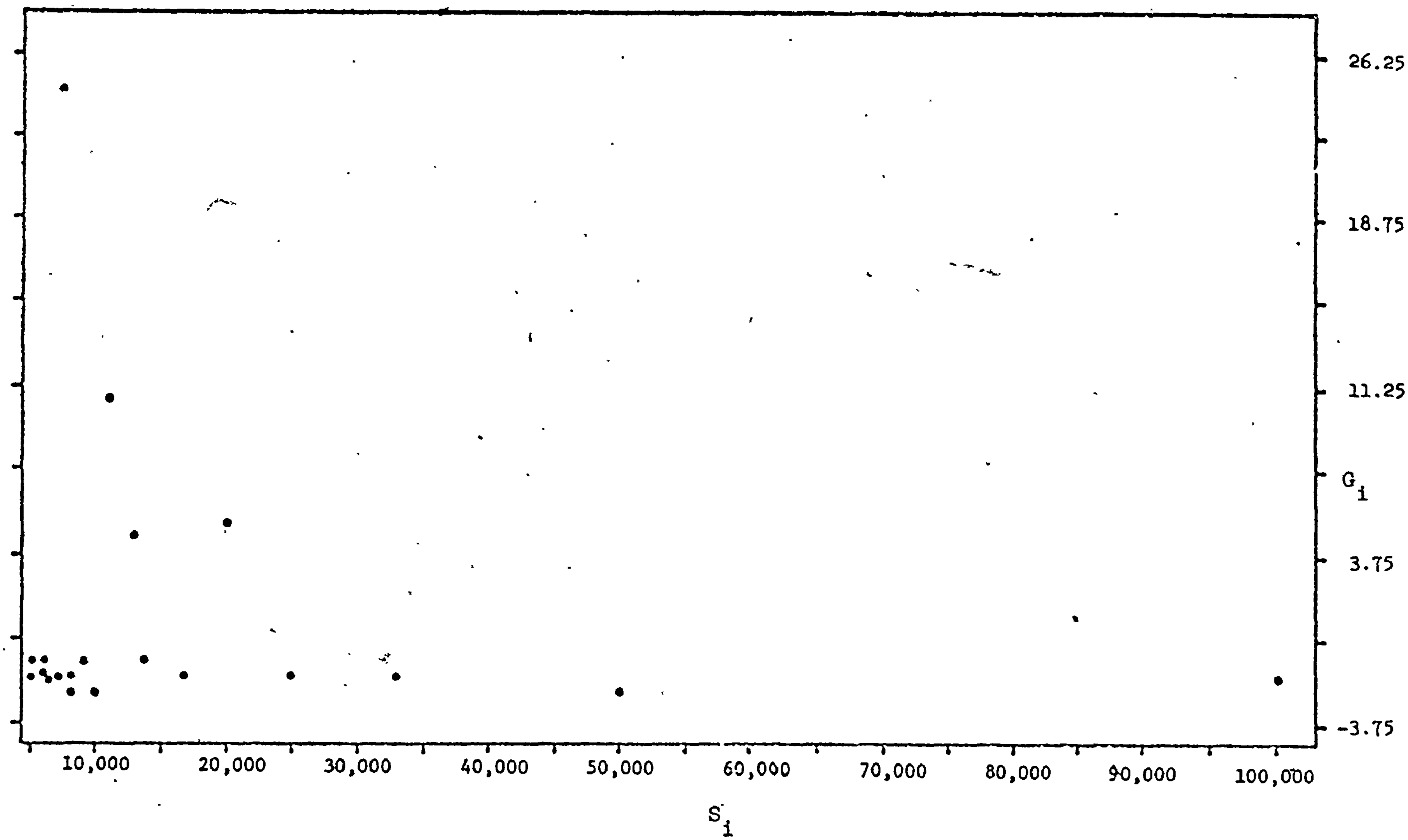
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=3$, Q_4)



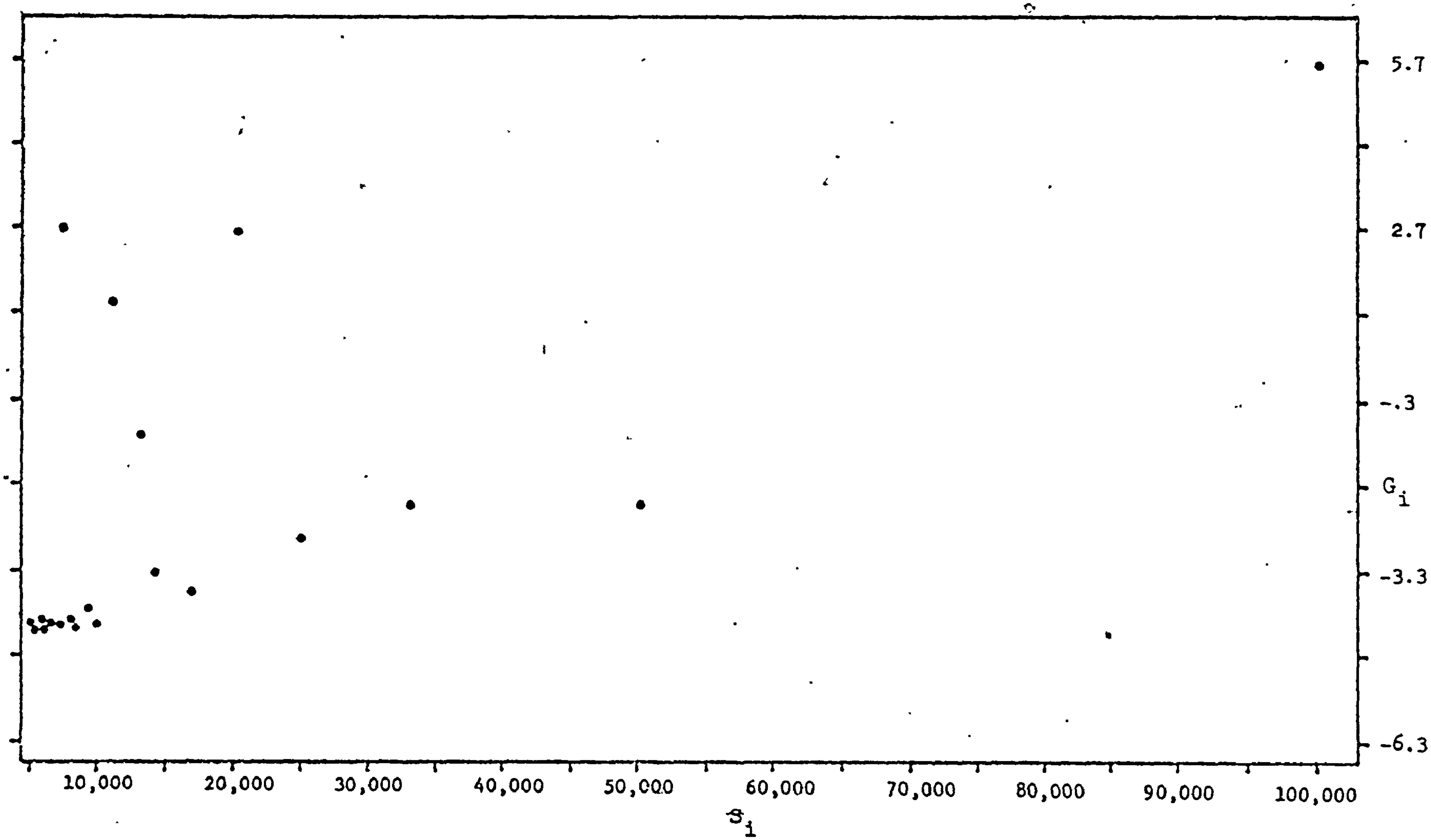
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=3,Q_5$)



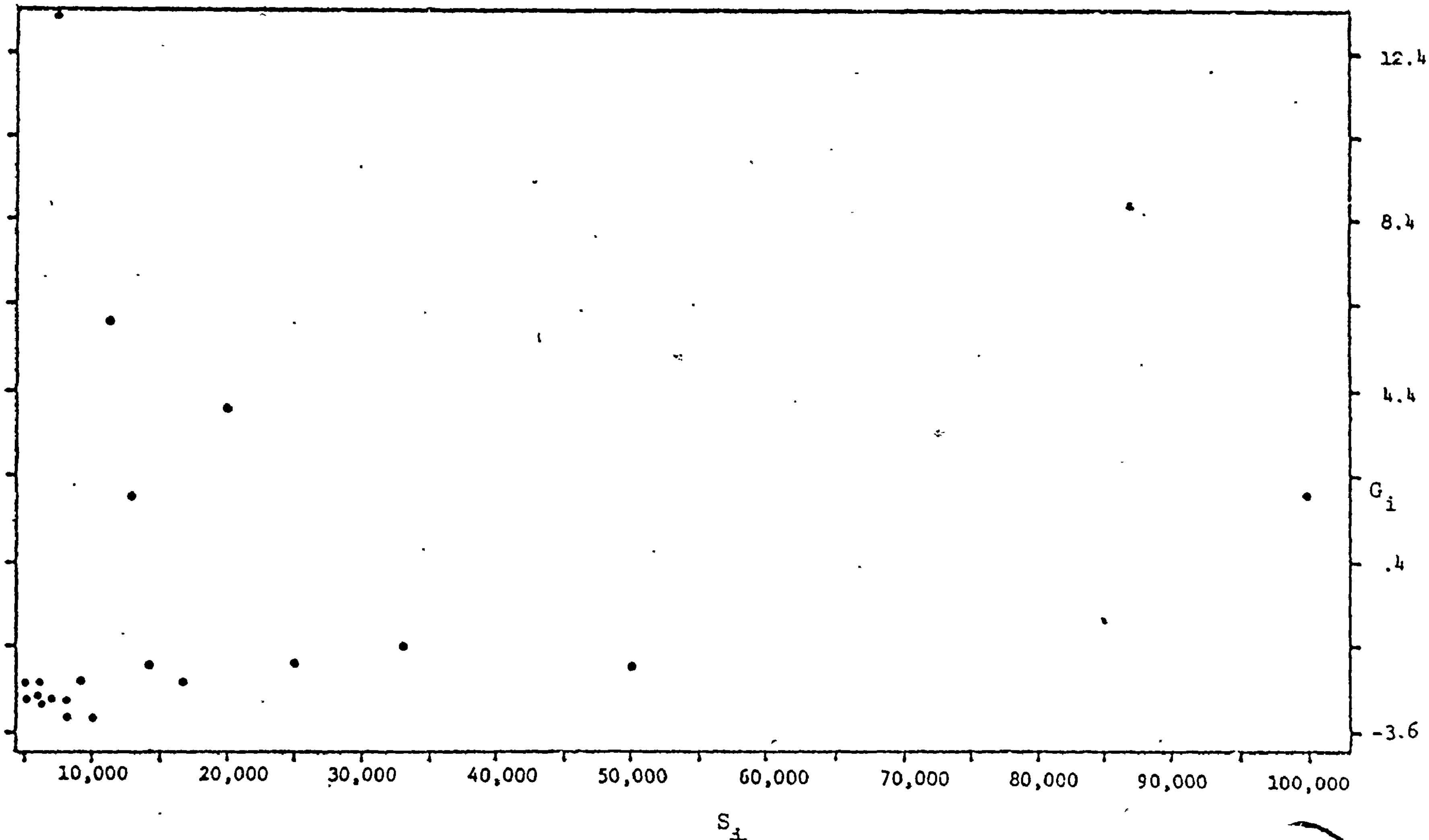
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=2,Q_1$)



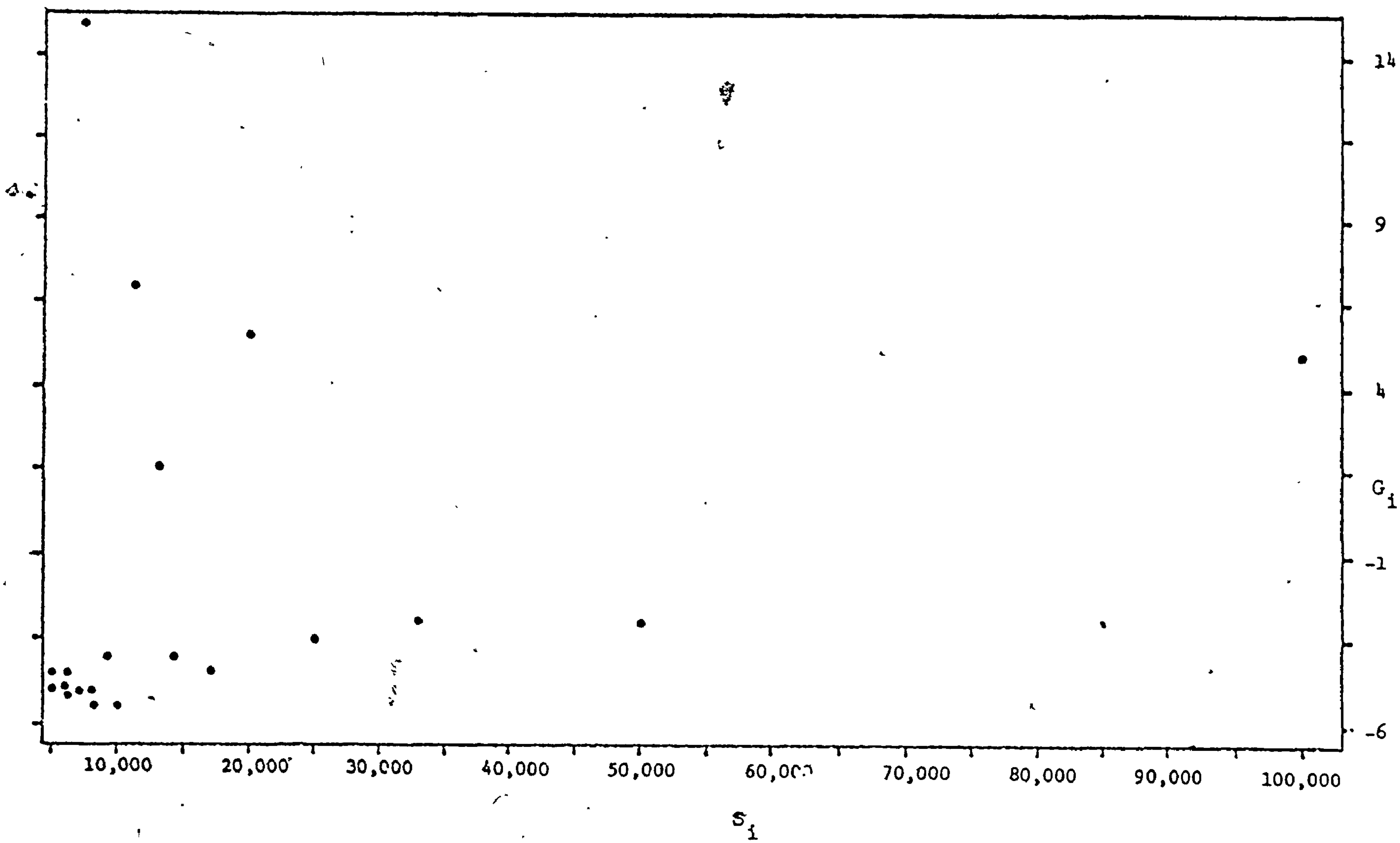
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=2,Q_2$)



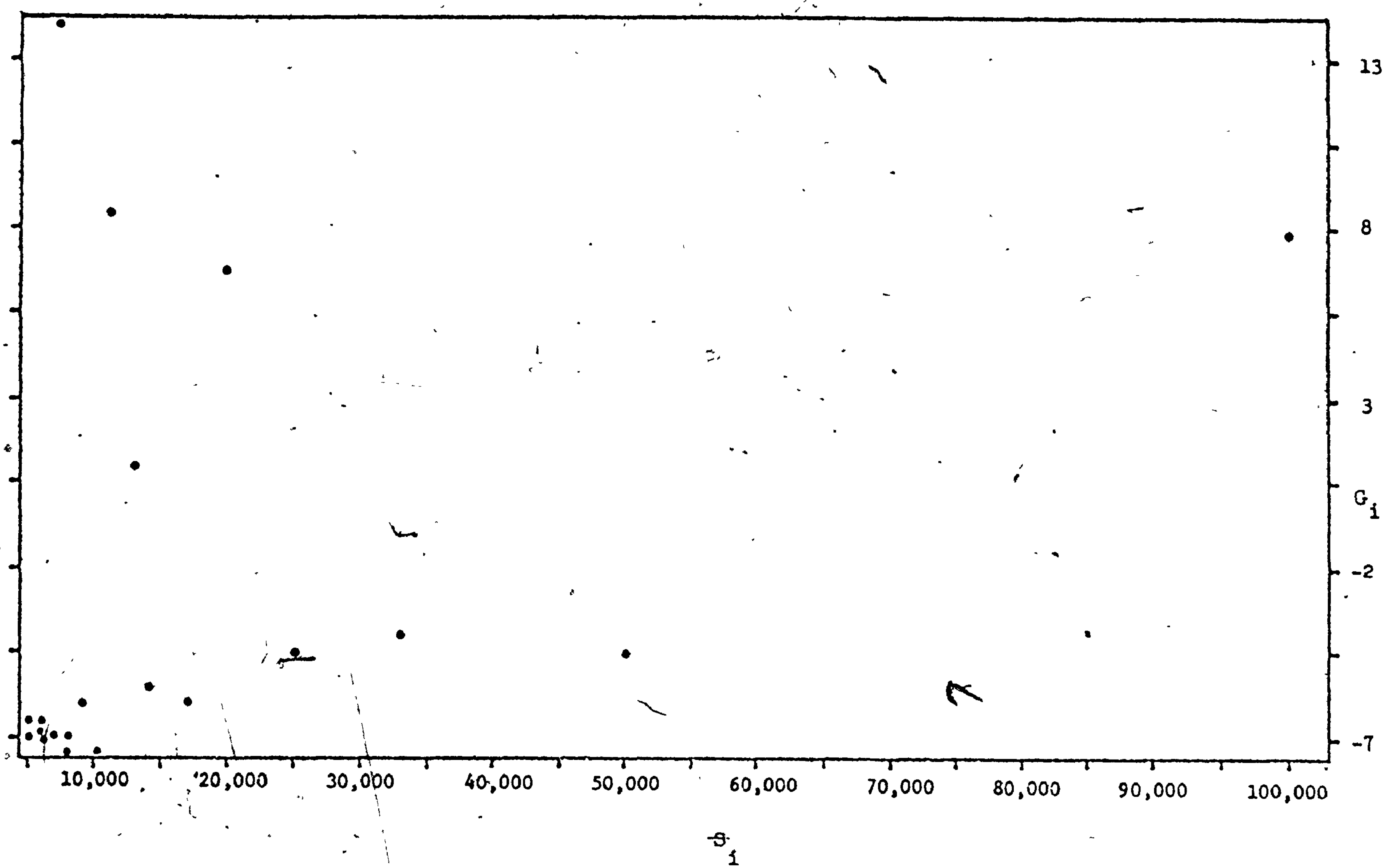
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=2,Q_3$)



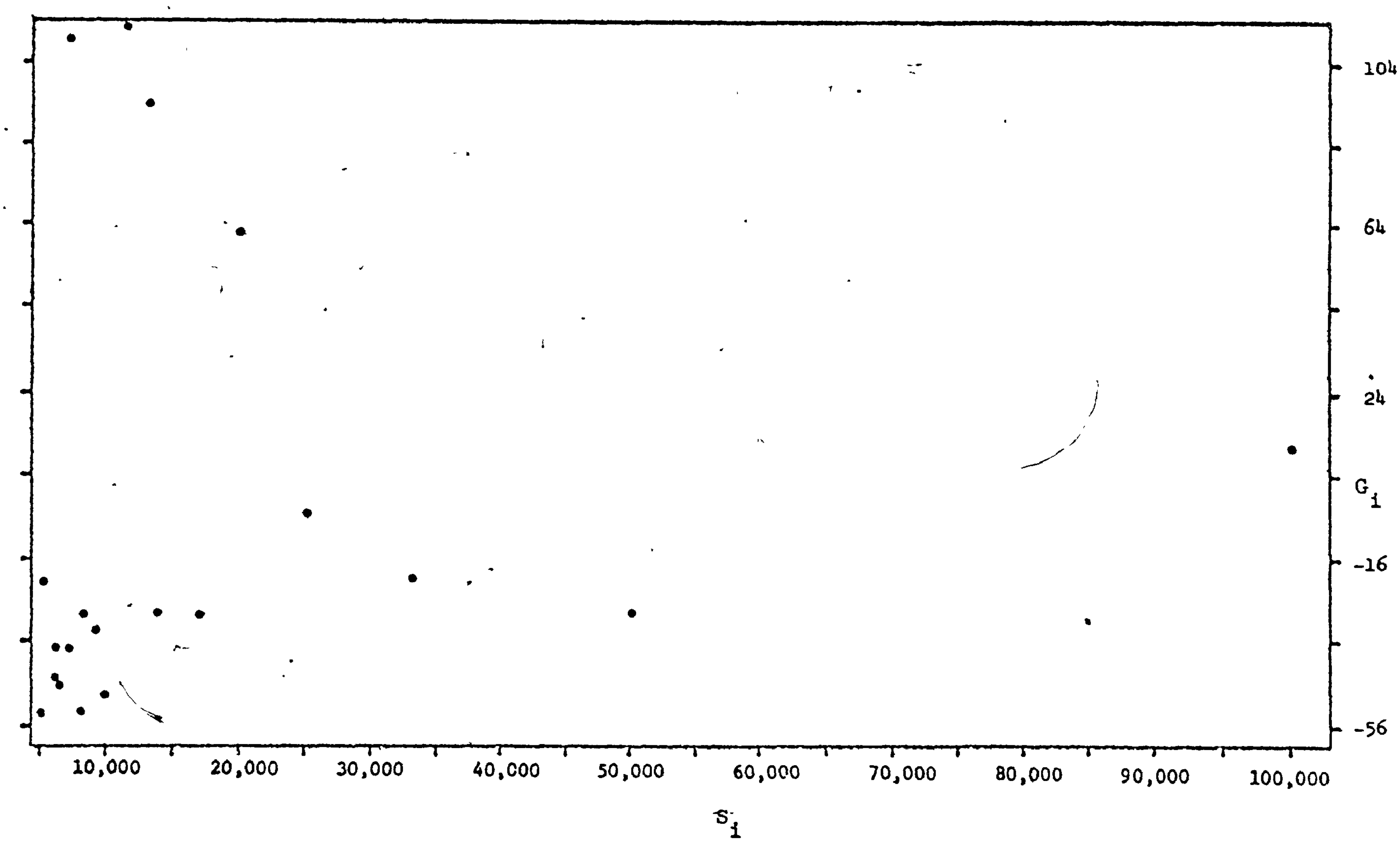
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=2, Q_4$)



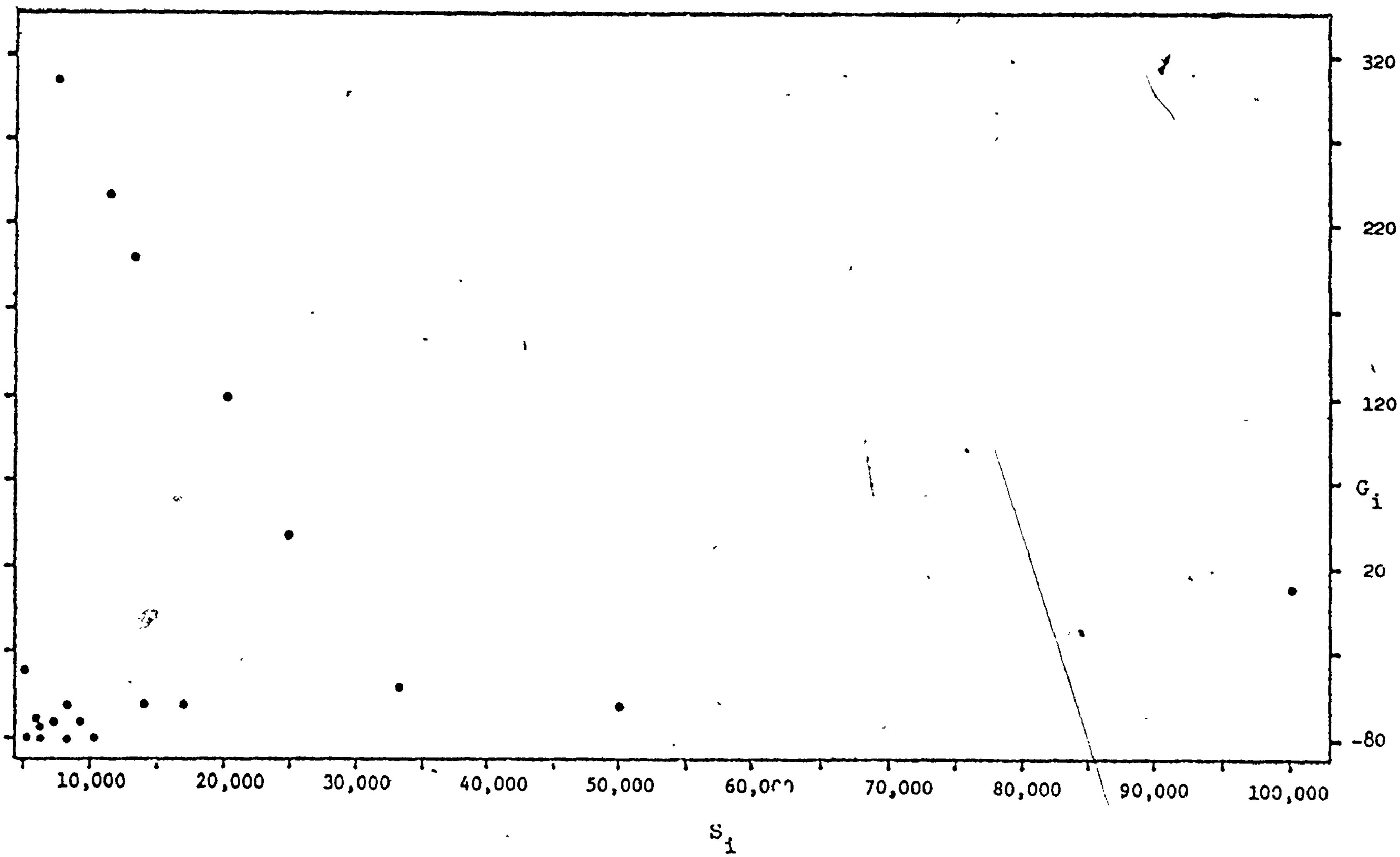
SHORT - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) ($x=2,Q_5$)



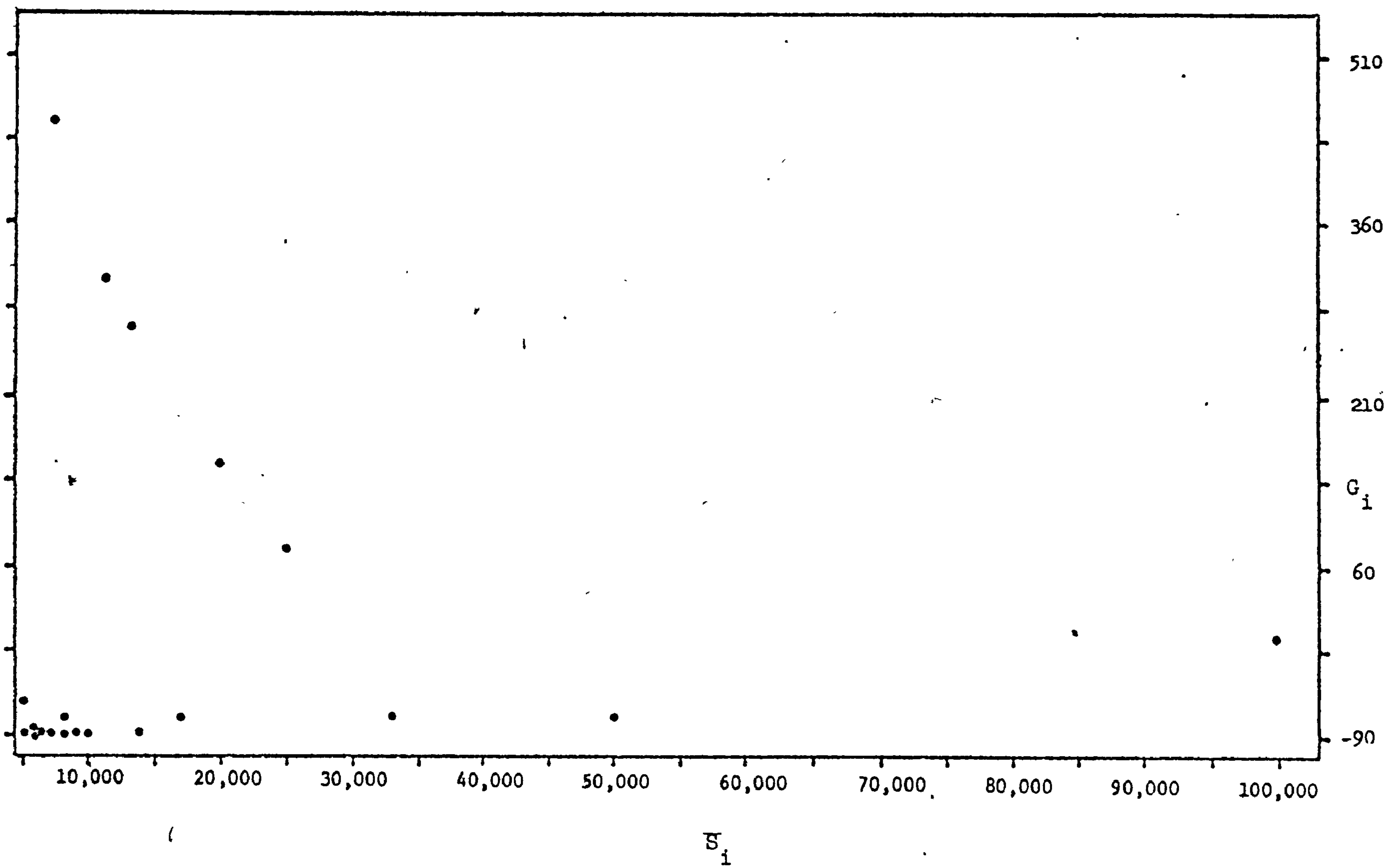
LONG - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) (5 GROWTH PERIODS)



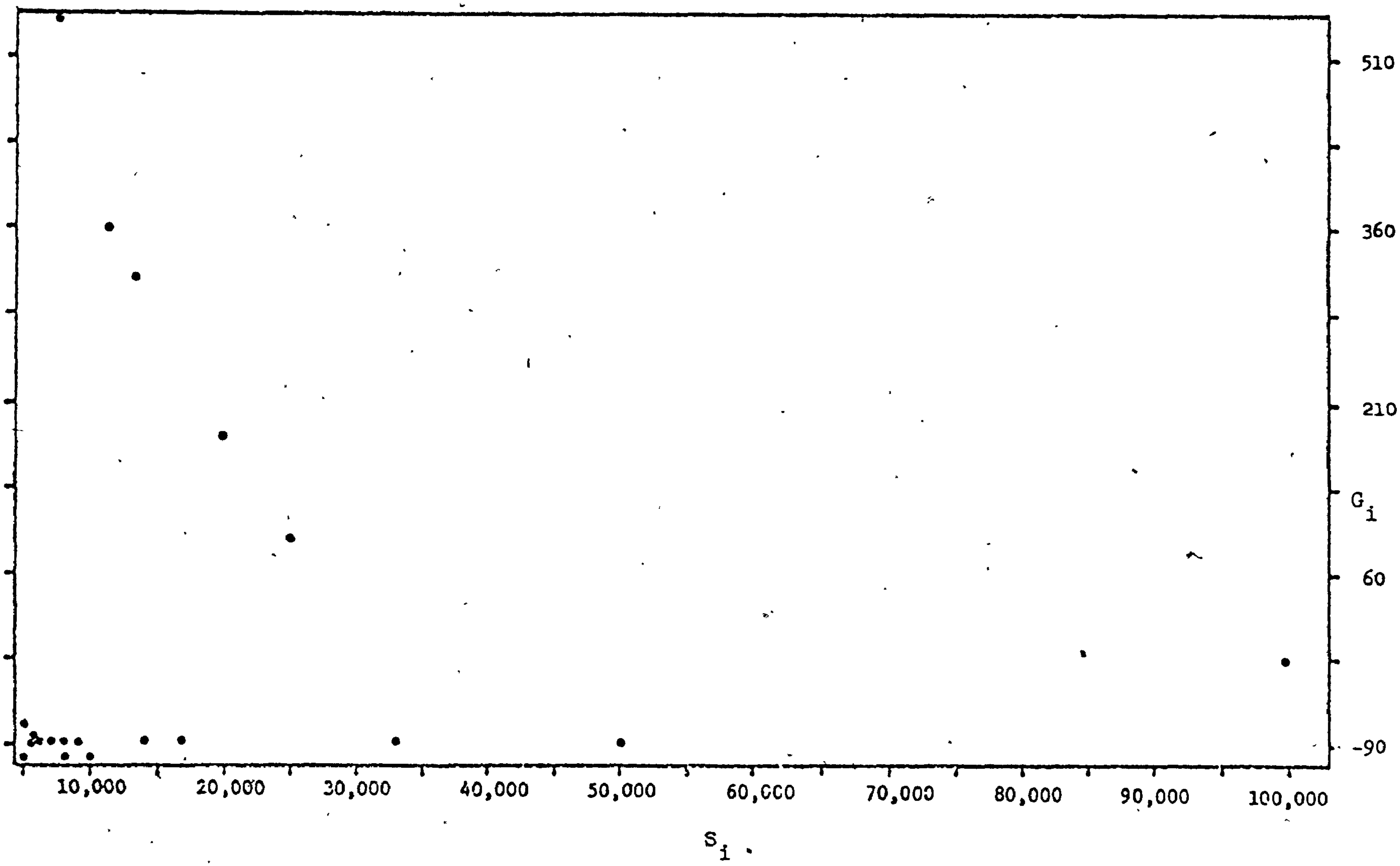
LONG - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) (10 GROWTH PERIODS)[1]



LONG - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY-GROWTH RATE (G_i) (15-GROWTH PERIODS)[1]



LONG - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) (20 GROWTH PERIODS)[1]



LONG - TERM SIMULATION OF GROWTH: INITIAL CITY SIZE (S_i) AGAINST CITY GROWTH RATE (G_i) (25 GROWTH PERIODS)[1]