POWER SYSTEM STABILITY
INCLUDING SHAFT AND NETWORK DYNAMICS

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by

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SCOPE AND CONTENTS:

Dynamic stability of balanced multimachine power systems is considered. System models, which in addition to detailed electric machine, turbine governor and exciter models, incorporate dynamic representation of the mechanical shaft systems and of the electric network, are developed.

Eigenvalue methods are employed for the stability predictions. The relationships between modelling complexity and the aspects of stability reflected, are investigated.

A number of practical applications are considered - in particular situations involving insufficient synchronizing and damping torques, turbine generator and control equipment interactions, torsional subsynchronous resonance, turbine generator and asynchronous motor load interactions and shaft dynamic interactions in closely coupled generators,

ABSTRACT

This thesis describes dynamic stability modelling and analysis of balanced multimachine power systems where in addition to detailed generator, governor-turbine and excitation system simulation, the dynamics of the mechanical shaft system and electrical network are included. Eigenvalue and complementary eigenvalue sensitivity methods are used in the investigation of stability or lack thereof.

A unified structure is presented in the formulation of a state space model for the complete system in terms of subsystem models. A wide variety of subsystem types and complexities can be accommodated including all present industry standard models. In addition, general models describing the dynamics of the mechanical shaft system and electrical network can be accommodated. The formulation approach further preserves the identity of the various subsystems, thus allowing ease of system modification or update.

The overall modelling concepts are applied to a number of practical situations to demonstrate their applications. In particular, situations involving insufficient synchronizing torque, insufficient damping torque, interaction between turbine-generator and control equipment, interaction between turbine-generator and network dynamics and interaction between turbine-generator and asynchronous motor loads are examined.

The concepts developed are also applied in the analysis of the interaction between the shaft dynamics of closely coupled identical generators. A method is presented for

determining the shaft natural frequencies for an arbitrary number of identical units, in terms of two equivalents. The extent to which the shaft modes may be stimulated in feasible on-line experiments is investigated.

Results and comparisons for alternative dynamic models are presented. It is shown that a single high order 'Benchmark' model which embraces all important dynamic effects, can be systematically reduced to models of reduced complexity - the basis for the reduction being the approximations normally applied in practice.

The main contributions are:

1. A systematic state-space equation structure which can include shaft and/or network dynamics has been developed. The formulation structure, which includes facility for eigenvalue sensitivity evaluation, can be applied in general problems involving electrical machine and system dynamic stability.
2. A technique has been developed for deriving the multiplicity of normally considered low order power system models, from a single high order system model (Benchmark model). This permits the evaluation of a wide range of alternative models and also, the identification of sources of instability.
3. The fundamentals of interacting shaft dynamics in closely coupled turbine-generators, have been examined. It has been shown that only two equivalents are necessary for the prediction of natural frequencies

and mode shapes in a general N unit situation.

4. Insights have been presented into the interpretation of eigenvalues as they reflect the various aspects of power system stability.

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LIST OF PRINCIPAL SYMBOLS

OVERALL SYSTEM MODEL

$\underset{\sim}{f}\ \underset{\sim}{g}\ \underset{\sim}{h}\ \underset{\sim}{k}$	vector functions
A B C D	matrices for state space descriptions
n,m,r	number of states, inputs and outputs
$\underset{\sim}{x},\underset{\sim}{z},\underset{\sim}{u},\underset{\sim}{y}$	state, algebraic, input and output variable vectors
$\Phi(T),\theta(T)$	state transition and force distribution matrices
P,Q,R,S	matrices associated with P, Q, R method
$\lambda,\lambda_r,\lambda_i$	eigenvalue, real and imaginary components thereof
$\underset{\sim}{X}_i,\underset{\sim}{V}_i$	eigenvectors of coefficient matrix and its transpose
ξ,ω_n	damping coefficient and undamped natural frequency
I,O	identity and null matrices

SYSTEM OF GENERATING UNITS

a,b,γ,f,k,c,e,d	submatrices describing dynamics of a single isolated generator
$A_M\ B_M\ \Gamma_M\ F_M\ K_M$ $C_M\ E_M\ D_M$	matrices describing dynamics a group of isolated generators
$\underset{\sim}{i}_s$	vector of D, Q stator currents
$\underset{\sim}{e}_v$	vector of measured terminal voltages
$\underset{\sim}{X}_m$	vector of states of isolated generators
N	no. of electrical machines

NETWORK SYSTEM

R,L,C	diagonal matrices of resistive, inductive and capacitive elements
K_R,K_L,K_C	incidence matrices for resistive, inductive and capacitive elements

I_ℓ, I_c	fundamental loop and capacitive currents
n_ℓ, n_c	number of fundamental loops and capacitors
A_N, B_N, C_N, D_N, E_N	matrices describing network dynamics
$\underset{\sim}{X}_N$	vector of isolated network states
$\underset{\sim}{i}_s$	vector of D, Q stator currents
$\underset{\sim}{e}_s$	vector of D, Q stator voltages
NN	number of network states (two axes)

GENERATING UNIT SUBSYSTEMS

Synchronous Machines

Z	coefficient matrix
n_z	order of Z
Y	matrix representing tie to network voltage
$\underset{\sim}{z}_\delta, \underset{\sim}{z}_n, \underset{\sim}{z}_f$	vector representing ties to rotor angle and speed and field voltage
$\underset{\sim}{X}_z$	states associated with synchronous machine
$\underset{\sim}{\psi}_s$	stator flux linkage
$\underset{\sim}{\psi}_r$	rotor flux linkage
$\underset{\sim}{i}_a$	stator current (rotor frame)
$\underset{\sim}{i}_s$	stator current (synchronous frame)
$\underset{\sim}{e}_s$	stator voltage (synchronous frame)
P, Q, e_t, e_b	equilibrium real and reactive power, terminal voltage and bus voltage
T_e	electric torque
ω_0	rated angular frequency (377 rads/sec.)
δ_1	rotor angle (to synchronous frame)
n_1	rotor slip (to synchronous frame)
e_f	field voltage

$X_{ss}, X_{sr}, X_{rs}, X_{rr}$	reactance submatrices
T	transformation matrix = $\begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix}$
I_s	vector containing equilibrium currents $(-i_q, i_d)'$
$\underset{\sim}{\Psi}$	vector containing equilibrium fluxes $(-\psi_q, \psi_d)$
$\underset{\sim}{I}_a$	vector containing equilibrium currents $(-i_Q, i_D)$
$\underset{\sim}{E}_a$	vector containing equilibrium voltages $(-e_Q, e_D)$
subscript:	
s,r	stator, rotor
d,q	direct and quadrature axis quantities (rotor frame)

Excitation System

E	coefficient matrix
n_e	order of E
κ	matrix representing tie to network voltage
E_n	matrix representing tie to shaft system
$\underset{\sim}{x}_e$	states associated with exciter
e_f	field voltage
e_v	voltage sensor output
e_D, e_Q	D,Q components of voltage deviation
e_{D0}, e_{Q0}, e_{t0}	equilibrium values of D, Q and terminal voltages
τ_v	voltage sensor time constant

rotating exciter

τ_a, τ_e, τ_f	time constants associated with amplifier, exciter and stabilizing loop
K_a, K_f	exciter and stabilizing loop gains

e_r	amplifier output
e_s	stabilizing signal
q	$e_s - K_f/\tau_t \; e_f$

static-stabilizer

$\tau_e, \tau_q, \tau_a, \tau_x$	time constants associated with exciter, washout and lead lag circuit
K_e, K_Q	gains of exciter and of stabilizer loops
e_x, e_a	velocity and acceleration components of stabilizing signal
n	shaft speed
v	$e_x - K_Q n$
w	$e_a - \tau_a/\tau_x \; e_x$

Turbine-Governor

G	coefficient matrix
n_g	order of G
g_n	matrix representing tie to shaft system speed
X_g	states associated with turbine-governor

hydraulic turbine

τ_w, τ_G, τ_p	water start, gate servomotor and pilot valve time constants
δ, σ	transient and permanent droop settings
P	output power
h	gate position
g	pilot valve movement
r	transient droop signal
p	$P + 2h$
v	$r - \sigma h$

steam turbine

$\tau_{sr}, \tau_{sm}, \tau_{ch}, \tau_{co}$	time constants associated with speed relay, servomotor, steam chest, reheat and crossover piping
F_{HP}, F_{IP}, F_{LP}	power fractions of high, intermediate and low pressure turbine stages
K_g	governor gain
g	speed relay signal
h	valve movement
P_{HP}, P_{IP}, P_{LP}	high, intermediate and low pressure turbine powers

Mechanical Shaft Systems

S	coefficient matrix
n_s	order of S
S_p	matrix representing tie to turbine-governor system
S_z	matrix representing tie to generator equations
$\underset{\sim}{x}_s$	states associated with shaft system
M_i	inertia of i^{th} lumped mass element
S_{ij}	stiffness between i^{th} and j^{th} lumped masses
D_i	damping coefficient between i^{th} lumped mass and synchronous reference frame

Asynchronous Motor Loads

Y	coefficient matrix
n_y	order of Y
γ	matrix representing tie to network voltage
$\underset{\sim}{x}_y$	states associated with asynchronous motor load
L_s, L_r, L_{sr}	stator, rotor and mutual inductance
r_s, r_r	stator and rotor resistance
$i_{sD}, i_{sQ}, i_{rD}, i_{rQ}$	stator and rotor currents in D-Q axes

ψ_{rD}, ψ_{rQ}	rotor flux linkage in D-Q axes
e_D, e_Q	terminal voltage in D-Q axes
T_e	electric torque
n	slip
M	rotor and load inertia
D	equivalent damper representing load

MISCELLANEOUS

Δ	prescript denoting incremental change
$\cdot$	superscript denoting differentiation with respect to time
$\sim$	subscript denoting vector quantity
'	superscript denoting matrix transpose
-1	superscript denoting matrix inverse
0	subscript denoting equilibrium value
D,Q	subscript denoting direct and quadrature axis quantities (synchronous frame)
α	prescript denoting proportionality
s	Laplace operator
Units:	all time constants (including inertial time constants) in seconds, all angles in radians. Other quantities in per unit (pu)

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CHAPTER 1

INTRODUCTION

1.1 Power System Stability in Perspective

An interconnected power system represents, in the view of the author, an excellent example of a high order multivariable control system. System order may exceed 100, overall system dynamics include electrical, mechanical, thermal and hydraulic processes and response times range from milliseconds, for certain electrical effects, to many minutes, for certain thermal effects. The question of stability has traditionally been associated with the maintenance of synchronism between the individual synchronous machines in the system. Other important aspects of stability include the damping of mechanical oscillations and the stability of the load frequency control loop.

New aspects of stability are emerging in recent years as a result of increased system size and complexity, recent trends in equipment design and the policy of operating systems nearer their stability limits. Fortunately, concurrent with these trends, tremendous breakthroughs in computational facilities and analytical techniques render the stability problems surmountable.

Implied in the general topic of stability as applied in the power industry, is not only the binary question of whether the system is stable, but also is information on the dynamic properties which indicate the quality of stability. Furthermore, what is satisfactory dynamic performance will vary from one portion of the system to another and considerable experience is required in interpreting the results.

Dynamic problems of power systems may be broadly classified [1]:

(1) energy supply system dynamics

(2) load-frequency system dynamics

(3) electrical machine and system dynamics

Energy supply system dynamics (boiler effects) are usually analysed on an individual basis as the interaction of the rest of the system can be neglected over the time range of many minutes for this response [1]. For the analysis of load frequency dynamics the network and electrical transients of the machines can be neglected as in this case the prime mover, governing load frequency characteristics are the important considerations [1].

From the viewpoint of present industry requirements the most important class of dynamic problems are those involving electrical machine and system dynamics. Analysis of this class of problems is also the most involved since the interaction of a multiplicity of electrical machines, excitation systems, turbine-governors, etc. must be considered. Recently as a result of a number of incidents involving violent shaft oscillations, the importance of including mechanical shaft and electrical network dynamics has been shown [2].

This thesis is centred around the modelling and analysis of this latter aspect of power system dynamics with particular emphasis on mechanical shaft and electrical network dynamics.

1.2 Transient and Dynamic Stability

There are two main categories of power system stability and dynamic response, namely the transient (large disturbance) and 'dynamic' (small disturbance) aspects. The former implies system response to major disturbances such as tie-line faults, loss of excitation, etc. Immediate loss of synchronism is generally of concern and the differential equations describing the system are non linear due mainly to the sinusoidal nature of the torque-load angle relationships. Saturation in the exciter, prime mover response and magnetic saturation are also a significant factor in determining the non linearities. Stability or lack thereof is a property of the nature of the disturbance (location, magnitude and duration) as well as a property of the system.

Dynamic stability is associated with the system operating normally without major disturbances. It describes the dynamic properties of the system when subjected to 'small' disturbances. (It is unfortunate from the viewpoint of the nonspecialist, that 'transient' and 'dynamic' terms have restrictive connotation in power systems analysis. Henceforth, these terms will be used in their narrow power system context.) For sufficiently small disturbances linear differential equations may be used to describe the system's dynamics. These equations are derived by perturbing the non-linear equations of the system, about the equilibrium operating point.

Traditionally the stability problems in power systems were those of transient stability - it was generally true that a system which was transiently stable was dynamically stable [3]. However, in the last decade dynamic stability problems have emerged as major considerations in power system planning and operation [4]. The most significant factor is the inclusion of fast response, high ceiling static excitation schemes for synchronous generators. These have the capability of significantly improving the transient stability properties of the system. However, they have an adverse effect on dynamic stability in that they destroy the inherent damping in machine rotor oscillations. As a consequence, static exciters are usually supplied with power system stabilizers which, add at the exciter input, a signal which results in rotor oscillation damping [4]. The adverse effect of static exciters on stability and the design of power system stabilizers can be analysed using linear system models. For determination of overall performance, the results of dynamic stability predictions must be complemented by results showing the transient performance characteristics.

Another important mode of dynamic stability is that involving shaft torsional oscillations in turbine generators [2], [5]. The presence of static exciters (with stabilizers) and/or series compensated transmission lines may result in oscillatory instability at one or more of the shaft's natural frequencies. This phenomenon can also be analysed in terms of linear system theory.

It is important to appreciate the limitations in considering dynamic (small disturbance) stability. Some engineers, who have traditionally worked on transient (large disturbance) stability problems, scorn the use of linear system models, emphasising the inherent non-linearities in power systems. Others argue that as dynamic stability is concerned with small disturbances, the non-linearities will not play a significant role in response and in stability predictions. If the system is unstable dynamically, then of course, the response prediction (based on linear theory) will become inaccurate as the disturbance is amplified. However, the response characteristic by this time is generally not of fundamental importance and the important conclusion is that the system cannot operate with stability. (So-called limit cycles resulting when oscillations are limited due to saturation effects are usually considered intolerable).

The salient advantage of the whole concept of dynamic stability is that it admits of the use of linear system theory and many of the results of modern control theory are applicable.

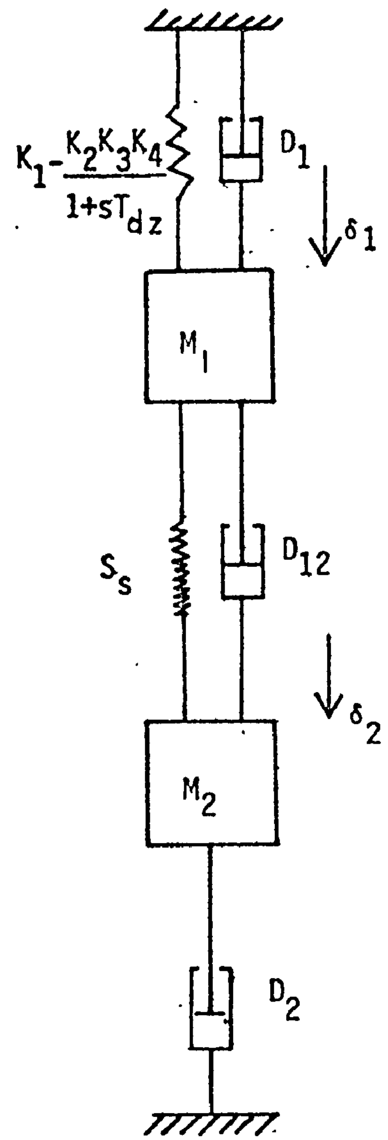
As the interactions involved in the stability problems described in this thesis are fundamentally linear, attention will be limited to dynamic stability analysis.

However it is very important to document that this stance is taken with a knowledge of the limitations of a linear analysis and the necessity of performing complementary transient stability studies for overall power system stability evaluation.

1.3 Important Aspects of Dynamic Stability

In this section an attempt is made to categorise the main aspects of power system stability associated with electrical machine and system dynamics. The fundamentals involved are described in terms of a simple fifth order model shown in Figure 1.1(a). A state space equation description is given in Figure 1.1(b). This model, which is an extension of that described in Figure 4 of de Mello and Concordia's paper [4], includes shaft dynamic effects. Masses M_1 and M_2 represent generator and turbine rotor inertias. Displacements between the equivalent masses and the stationary frame correspond to the angular displacement (δ_1, δ_2) between the actual turbine-generator inertias and a synchronous reference frame. The velocities (n_1, n_2) of the respective masses, represent the slip of the turbine generator inertias relative to the synchronously rotating reference frame.

The electric torque-rotor angle relationship is represented as a complex frequency dependent spring. K_1 - K_4 are the well known block diagram coefficients of de Mello and Concordia [4]. T_{dz} represents the effective field time constant on load.



(a) Mechanical System

$$\begin{bmatrix} \dot{T} \\ \dot{n}_1 \\ \dot{n}_2 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{T_{dz}} & & & \frac{-K_2K_3K_4}{T_{dz}} & \\ \frac{1}{M_1} & \frac{-(D_1+D_{12})}{M_1} & \frac{D_{12}}{M_1} & \frac{-(K_{tr}-S_s)}{M_1} & \frac{S_s}{M_1} \\ & \frac{D_{12}}{M_2} & -\frac{(D_2+D_{12})}{M_2} & \frac{S_s}{M2} & \frac{-S_s}{M_2} \\ & \omega_o & & & \\ & & \omega_o & & \end{bmatrix} \begin{bmatrix} T \\ n_1 \\ n_2 \\ \delta_1 \\ \delta_2 \end{bmatrix}$$

(b) State Space Equations

Fig. 1.1 Simple fifth order model illustrating main aspects of dynamic stability

$T_{\cdot}$ is a state associated with the electric torque component due to rotor demagnetization. K_{tr} (= K_1) is the transient synchronizing torque coefficient and K_{ss} (= K_1-K_2 K_3 K_4) is the steady-state synchronizing torque coefficient.

Spring S_s represents the torsional stiffness between the turbine and generator rotor inertias. D_{12}, D_1 and D_2 represent the viscous dampers between the two inertias and between each inertia and the synchronous reference frame.

Though a single machine power system is implied, the results may be extended to multi-machine situations if concepts of modal analysis are applied. In that case δ_1 will reflect the rotor angles of a group of machines swinging coherently.

The simplest possible representation of a synchronous machine in dynamic applications is that of two voltage sources (voltage behind synchronous reactance and bus voltage) separated by a reactance (synchronous and tie line reactance). From basic electric circuit principles it follows that maximum power transfer occurs when the phase angle between the machine and bus voltage is 90^{o}. An attempt to load the machine beyond this limit results in loss of synchronism.

Though the above criterion is derived for steady state conditions, it can be readily seen that it correctly predicts the dynamic stability limits of a synchronous machine in the absence of exciter action [4]. If exciter action is considered, operation with load angles in excess of 90^{o} is possible. This is referred to as 'dynamic operation' in the literature [4].

Figure 1.2(a) shows a stable reference response for a step disturbance in mechanical torque. Three modes are present in the response - a monotonic and two oscillatory modes. The low frequency mode represents the main system swing while the high frequency mode represents shaft oscillation.

In Figure 1.2(b) is the response stability and monotonic instability for different values of steady state synchronizing coefficient.

The effect of damping in the main system (rotor) swings is shown in Figure 1.2(c) for two cases of rotor damper coefficient.

Finally the response corresponding to unstable shaft oscillations is shown in Figure 1.2(d). The mechanisms resulting in a negative effective inter-mass damper will be discussed later.

The above model is a gross oversimplification in that it is of fifth order and cannot reflect the interaction in the overall high order system. (Certain aspects of stability are not primarily associated with the torque-load (rotor) angle loop - in particular the stability of the 'AVR' mode [3] and electrical subsynchronous resonance [6]). However, the model has significance in that it illustrates the main aspects of dynamic stability as they affect rotor angle response.

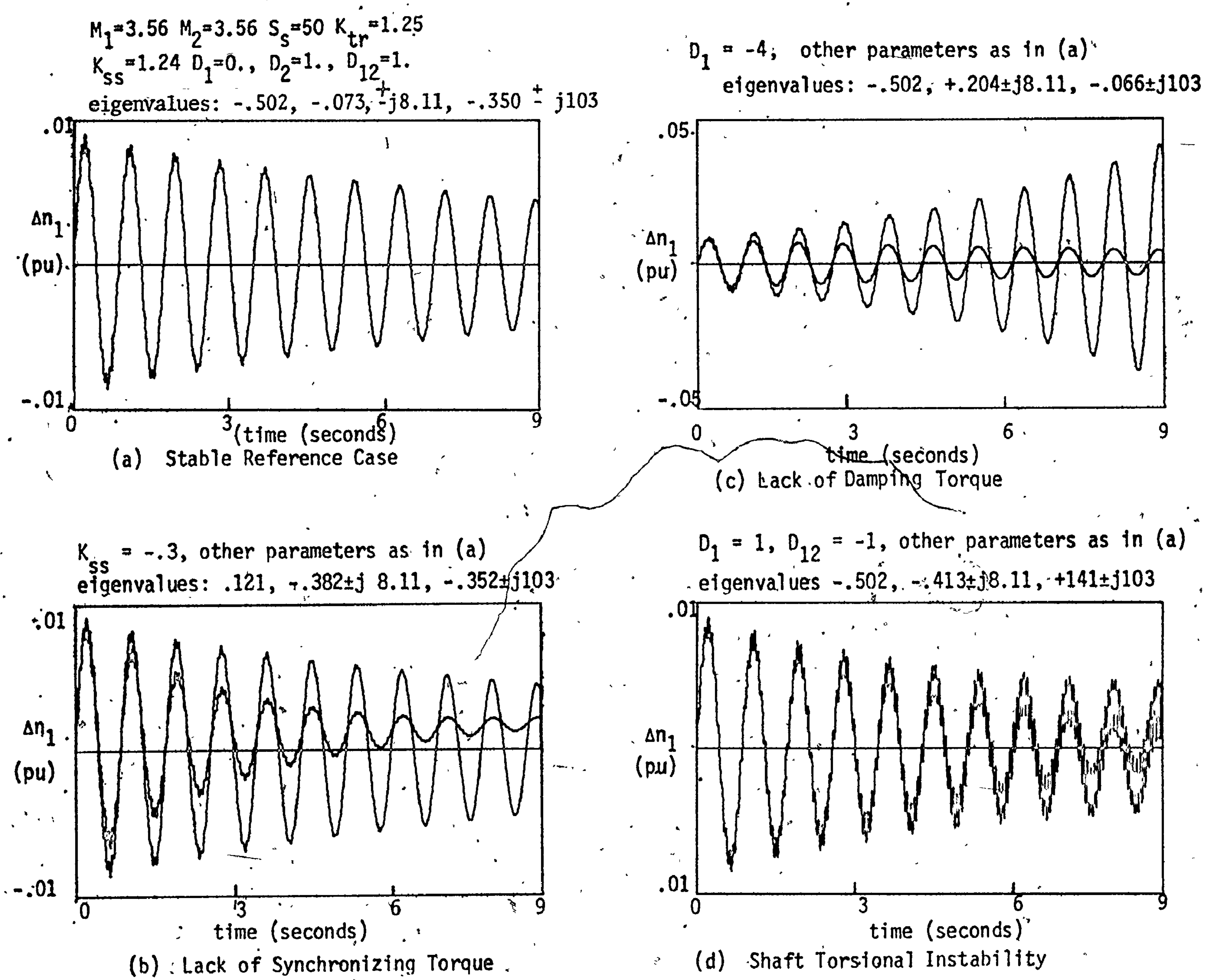


Fig. 1.2 Typical responses for unit step in mechanical torque

The usefulness of this model will be borne out repeatedly in this thesis.

The effect of excitation systems on dynamic stability has long been known [7] [8]. A recent 'classic' on the subject is by deMello and Concordia [4] where the general philosophy of power system stabilizers is also discussed. [9] is an extension of some of the ideas in [4] where a two machine situation is considered and the effects of different load types were considered. Pioneering work in the area of static excitation systems and power system stabilizers has been performed at Ontario Hydro [5], [10]. Other worksin this area which deserve mention are in [11] where stability at leading power factor loads is considered and [12] which is based on [4] and presents an analysis for a wide range of parameter values and loading conditions.

The two mechanisms which can potentially result in shaft instability are: (a) use of a speed sensitive power system stabilizer [5] and (b) use of series capacitor compensated transmission systems [2].

A general discussion on series capacitor compensation is [13]. An occurrence of instability resulting from the former cause was reported in [5] where the problem was discovered and diagnosed before shaft damage ensued. Incidents of the latter cause of instability are discussed in [2] - two shaft failures were recorded. This evidence of the potential danger prompted appraisal of other series compensated systems [14] and a special session on the subject at the 1975

Winter Power Meeting [15]. A number of analysis papers on the subject have recently appeared [16] to [20] and some novel approaches in the suppression of these unstable oscillations [21] have appeared. Experimental investigation recorded in [22] led to the awareness of coupling between the shaft modes of close generators which may have important significance.

There is another possibility of instability involving series capacitor compensated systems. This is a sustained oscillation involving only the stator currents and not involving the mechanical shaft system. Though such a situation has never been witnessed in North America, the phenomenon has been studied: [6], [13], [15].

The bibliography used on stability problems arising from interaction of controls will be included with references on system modelling and analysis which will be presented in Section 1.6.

1.4 Solution Methods for Stability

It will be assumed that the system is described by a set of first order differential equations [23] to [25].

$$\begin{aligned} \dot{\underset{\sim}{x}} &= \underset{\sim}{f}(\underset{\sim}{x}) + \underset{\sim}{g}(\underset{\sim}{u}) \\ \underset{\sim}{y} &= \underset{\sim}{h}(\underset{\sim}{x}) + \underset{\sim}{k}(\underset{\sim}{u}) \end{aligned} \qquad (1.1)$$

where $\underset{\sim}{x}$, $\underset{\sim}{u}$ and $\underset{\sim}{y}$ are vectors of state, input and output variables of order n, m and r respectively. f, g, h and k are vector functions, i.e.,

$$f(x) = \begin{matrix} f_1 (x_1, x_2 \ldots x_n) \\ \vdots \\ f_N (x_1, x_2 \ldots x_n) \end{matrix}$$

For small disturbances, a first order Taylor series yields:

$$\begin{aligned} \Delta \dot{\underset{\sim}{x}} &= A \, \Delta \underset{\sim}{x} + B \, \Delta \underset{\sim}{u} \\ \Delta \underset{\sim}{y} &= C \, \Delta \underset{\sim}{x} + D \, \Delta \underset{\sim}{u} \end{aligned} \tag{1.2}$$

which is the standard state space equation representation. A, B, C, D are real matrices which depend on the system parameter values and also on the equilibrium condition.

$$A = \left. \frac{\partial \underset{\sim}{f}}{\delta \underset{\sim}{x}} \right|_{\underset{\sim}{x}_o} = \begin{bmatrix} \frac{\delta f_1}{\delta x_1} & \frac{\delta f_1}{\delta x_n} \\ \frac{\delta f_n}{\delta x_1} & \frac{\delta f_n}{x_n} \end{bmatrix}_0 \tag{1.3}$$

The methods of stability determination may be classified as either direct or indirect. Indirect methods involve the numerical integration of the system's differential equations. For the transient stability case (non linear, Equation (1.1)) a variety of numerical integration schemes are available. Traditionally explicit integration methods (in particular , Runge Kutta methods) have been employed but recently implicit methods are receiving increased attention. As this thesis is concerned with dynamic stability the subject of numerical integration is outside our scope- a number of excellent references on the subject are [24], [26], [27].

For dynamic stability (linear equations - Equation (1.2) numerical integration is not necessary for linear systems. In that case the solution may be expressed in terms of the state transition matrix [28], [29]:

$$\underset{\sim}{X}_{NT} = \Phi(T)\, \underset{\sim}{X}_{(N-1)T} + \Theta(T)\, U_{(N-1)T}$$

where $\underset{\sim}{X}_{NT}$ and $\underset{\sim}{U}_{NT}$ are the values of $\underset{\sim}{X}$ and $\underset{\sim}{U}$ at $t = NT$. ($\underset{\sim}{U}$ is considered constant over the interval $(N-1)\,T \to NT$)

$$\Phi(T) \triangleq e^{AT} \quad \text{(state transition matrix)}$$

and $$\Theta = (\Phi - I)\, A^{-1}\, B \quad \text{(force distribution matrix)}$$

Determination of the state transition matrix is possible using a matrix power series expansion [29] or alternatively using matrix polynomial approximations [30], [31].

The best known direct method of transient stability determination is the so-called equal area method [32], [33]. Though this method gives good insight into the problem it has severe limitation in that it is restricted to a two machine system and employs only a second order system model (constant voltage behind transient reactance). The second method of Liapunov is the only practical direct method of determining transient stability [32], [34] to [37]. As is well known, the problem with this method is in the generation of suitable Liapunov functions.

The most widespread practical method of determining dynamic stability is the use of eigenvalue techniques [3], [11], [38] to [40]. Most modern computer libraries contain subroutines for the solution of the matrix eigenvalue problem [41] including routines for calculation of only the eigenvalues (as distinct from eigenvalues and eigenvectors). Methods for the calculation of only specific eigenvalues (eg., largest eigenvalue) also exist. The advantages of complementing eigenvalue calculations with those of eigenvalue sensitivity (with respect to variable parameters) has also been reported by a number of authors [3], [42] to [44]. Eigenvalue methods have also received practical application in transient stability studies where power system sections remote from the 'study area' are replaced by 'equivalents' [45].

The second method of Liapunov which has generally been applied in transient stability applications has also been used in dynamic stability calculations [46], [47]. By repeated evaluation of a performance index (which reflects the quality of response) for a variety of parameter values an optimum parameter setting may be determined [46]. The determination of optimal parameter settings for a number of variable parameters is possible using modern optimization techniques [48], [49].

The application of the so-called classical control techniques has been limited to small systems involving one or two machines. A disadvantage in the use of Routh Hurwitz [50], [51] and the D-partition method is that the characteristic polynomial must be first determined [25]. The most successful application of frequency response techniques is in 'synchronizing and damping torque component' analysis, [4], [52] to [54]. This has been most extensively applied in the explanation of the adverse effect of static excitation schemes on dynamic stability and in the design of power system stabilizers. Typical power system applications of Nyquist and Bode analysis are in [25], [55] and [56], respectively.

1.5 Approaches to System Stabilization

As emphasised in Section 1.2 satisfactory operation of a power system requires acceptable properties in both transient and dynamic aspects of stability. Proposed schemes for stabilization either through optimal parameter adjustment or through the incorporation of special stabilization loops have usually been applied for the enhancement of one of the two aspects. Consequently the approaches to system stabilization will be classified on the basis of the aspect of stability intended to be improved. It must be strongly emphasised that designs based on dynamic stability enhancement must be evaluated in transient stability studies and vice versa. The conflicting stabilization requirements on static exciters is well known [4]. It has also been shown that the incorporation of a power system stabilizer (which improves dynamic stability properties) can impair the transient stability properties [57].

For dynamic stability improvement, the application of synchronizing and damping torque concepts, have been used most successfully in designing power system stabilizers [4], [5], [10]. A successfully tested stabilizer based on 'Root Locus Design' has also been reported in the literature [58]. Some general considerations on power system stabilizers including the rational behind the choice of input variable (speed, frequency and power deviations are the candidates) are discussed in [10]. Other classical techniques have also been applied extensively, including; Nyquist Design [55], Routh Hurwitz [50] [51], D-partition [50], [59] and design based on time domain specification [51].

A variety of approaches based on modern control theory have also been considered for dynamic stability improvement. In particular, a state optimal controller (state regulator) has been suggested by a number of authors [60] to [63]. The advantages of using suboptimal controllers derived using low order system models has been demonstrated [64], [65]. Suboptimal controllers may also be designed using the state regulator with incomplete state feedback [66]. Some researchers have considered only feedback to the exciter input while others have considered the more general case of feedback to both exciter input and governor command input. (Consideration of the governor command as a control input is especially important nowadays due to the advances in fast valving.) An interesting alternative to the approach of a state regulator with feedback to the two input control is the application of state feedback decoupling. This results in two low order single input - single output systems - the poles of which can be optimally placed [67], [68]. The possibilities for stability improvement using a dual axis synchronous machine (two field windings) have also been demonstrated [69].

A major disadvantage with any state variable feedback scheme is that unmeasurable states may be required for feedback. Consequently, unmeasurable states have to be predicted which necessitates the inclusion of an observer system [70]. Though generally, single machine infinite bus configurations have been considered, some multimachine applications have been cited [71], [72]. The telemetering problems in implementing such a scheme are quite formidable.

The optimal output feedback has also been considered by a number of researchers. There are two types of output feedback - static output feedback and dynamic output feedback. In the former case a stabilizing signal is derived by passing the output signals through constant gains. In the latter case, the output signals are passed through dynamic compensators which introduce some desired dynamic properties. It is in the former (static) sense that output feedback is considered here (the power system stabilizer [4] is an example of a dynamic output feedback arrangement). Also it should be realized that there is no essential difference between static output feedback and a state regulator with incomplete state feedback.

Single machine infinite bus problems have generally been studied though multimachine examples have also been considered [73]. There are two approaches to the multi-machine problem, i.e., local and global control. In the local control strategy only the outputs of each machine are fed back to that machine. On the other hand, for global control, the outputs of every machine are fed back into each machine [73].

Though modern control theory approaches (state variable and global output feedback schemes) are not readily implementable, they should not be discounted too readily on the basis of impracticality. In the absence of actual implementation (and indeed some modern computer application in power systems, eg., state estimation, are comparably complex) analysis of the results of optimal type controllers is often extremely useful. Once the characteristics of the desired stabilizing signal are known, a good facsimile can often be generated using dynamic output feedback of key variables.

The parameters of stabilizing loops will in general vary with loading condition and changing system parameters. Implied in most approaches to system stabilization is that the parameters are adjusted as the system parameters change appreciably (a well known characteristic of feedback control systems is reduced sensitivity of dynamic performance to small system parameter changes). An alternative to adjustment of stabilizer parameters as system conditions change is to design a 'compromise' stabilizer which works over a range of system loading and parameter values [4].

For transient stability enhancement, the beneficial effects of fast response, high ceiling, static exciters is well known. It has long been realized that the so-called steady state stability limit of a 90^{o} torque angle can be extended right up to the transient stability limit of typically 140^{o} with such exciters. Pioneering work in the application of static exciters to enhance the stability of weakly coupled hydroelectric systems has been reported by Ontario Hydro [7].

For specific disturbances, time optimal control has been considered by a number of authors. The object here is to transfer the system from a pre to post fault equilibrium in minimum time. Bang-bang excitation control was proposed in [75]. In [76] and [77], the same problem was solved using intermittant duty reactor switching. A general discussion on capacitor switching and its effect on transient stability can be found in [78].

Driving the exciter into saturation and capacitor switching effectively causes step changes in electrical power. An alternative, (for thermal power systems) which is recently becoming feasible, is to 'fast valve', i.e., quickly close steam intake valves so as to reduce the turbine power upon fault inception, thereby reducing the accelerating torque [79]. General considerations of stabilization possibilities including fast valving are discussed in [80]. The feasibility of applying fast valving for system stabilization in a multi-machine situation has been demonstrated in [81].

So far in this section, approaches for the improvement of either transient or dynamic aspects of stability have been discussed. In some approaches, the conflict in requirements for both classes of stability is met 'head-on'. Such approaches consider 'dual-action' controllers in which a different strategy is used for small and large disturbances. In [57] control based on a speed sensitive power system stabilizer was considered. Logic circuitry was employed to disable the stabilizer signal during large angle swings. The fast response, high ceiling static exciter provides adequate stabilization in this mode. A similar approach was applied in [82] and the feasibility of the strategy on a multi-machine system model was demonstrated.

1.6 Formulation of Power System Models

A review of the main formulation approaches for a linear state-space equation system describing power system dynamics is given in this section. The object of the review is to attempt to place in perspective the contribution of this thesis and to explain the rational behind the choice in approach adopted.

The description of a power system involves large numbers of both differential and algebraic equations. For practical computation (either analog or digital) the equations must generally be reduced to standard state space form. For small single-machine-infinite bus problems it may be possible to reduce the equations (i.e., eliminate the unwanted algebraic equations) by hand. However, for large systems, it is mandatory to have systematic reduction techniques.

Enns et al. [83] described one such technique. The differential and algebraic equations are arranged in the following form:

$$P \begin{bmatrix} \dot{\underset{\sim}{x}} \\ \underset{\sim}{z} \end{bmatrix} = Q \underset{\sim}{x} + R \underset{\sim}{u}$$

where $\underset{\sim}{x}$ is an n dimensional state vector

$\underset{\sim}{z}$ is an r dimensional algebraic variable vector

$\underset{\sim}{u}$ is an m dimensional input control vector

P, Q, R are real matrices of compatible order with $\underset{\sim}{x}$, $\underset{\sim}{z}$, $\underset{\sim}{u}$. Upon reduction we obtain in general

$$\dot{\underset{\sim}{x}} = A \underset{\sim}{x} + B \underset{\sim}{u}$$

$$\underset{\sim}{z} = C \underset{\sim}{x} + D \underset{\sim}{u}$$

Enns et al. further classified three possibilities for the formulation of composite systems.

(a) subsystems reduced - composite system reduced

(b) subsystems unreduced - composite system unreduced

(c) subsystems reduced - composite system unreduced

It is further observed that (a) directly yields (without further reduction) the final (state space) equations, but is restrictive in the type of subsystem connection. (b) offers no restriction on the connections between subsystems but is very wasteful of computer storage. (c) has practical importance in that while it does not restrict connection possibilities, unwanted algebraic variables may be eliminated.

An alternative approach to the general problem of reducing large sets of differential and algebraic equations to state space form, is that of Van Ness [84]. In the approach, state variables were grouped according to the process type described, i.e., the states associated with all pure integrators were grouped together. This can be compared with the approach of Enns et al. [83] where states are grouped on the basis of physical subsystems.

In comparing alternative formulations for power systems, the state grouping approach is a key differentiating feature. The main possibilities are:

(1) 'Type' grouping, i.e., all states associated with the same process in each machine are grouped together, eg., rotor angles of all machines grouped together.

(2) 'Generator' grouping, i.e., all states associated with a particular generator are grouped together.

The overall coefficient matrix structure for the two state grouping possibilities is shown in Figure 1.3(a) and (b). It can be seen that the second scheme is simpler than that of the first for the general case when the generating unit models are different. Also note that the identity of each is maintained - this allowing sections of the system to be analysed independently.

A review of the main formulation approaches is listed in Table 1.1. Each column describes the modelling approach

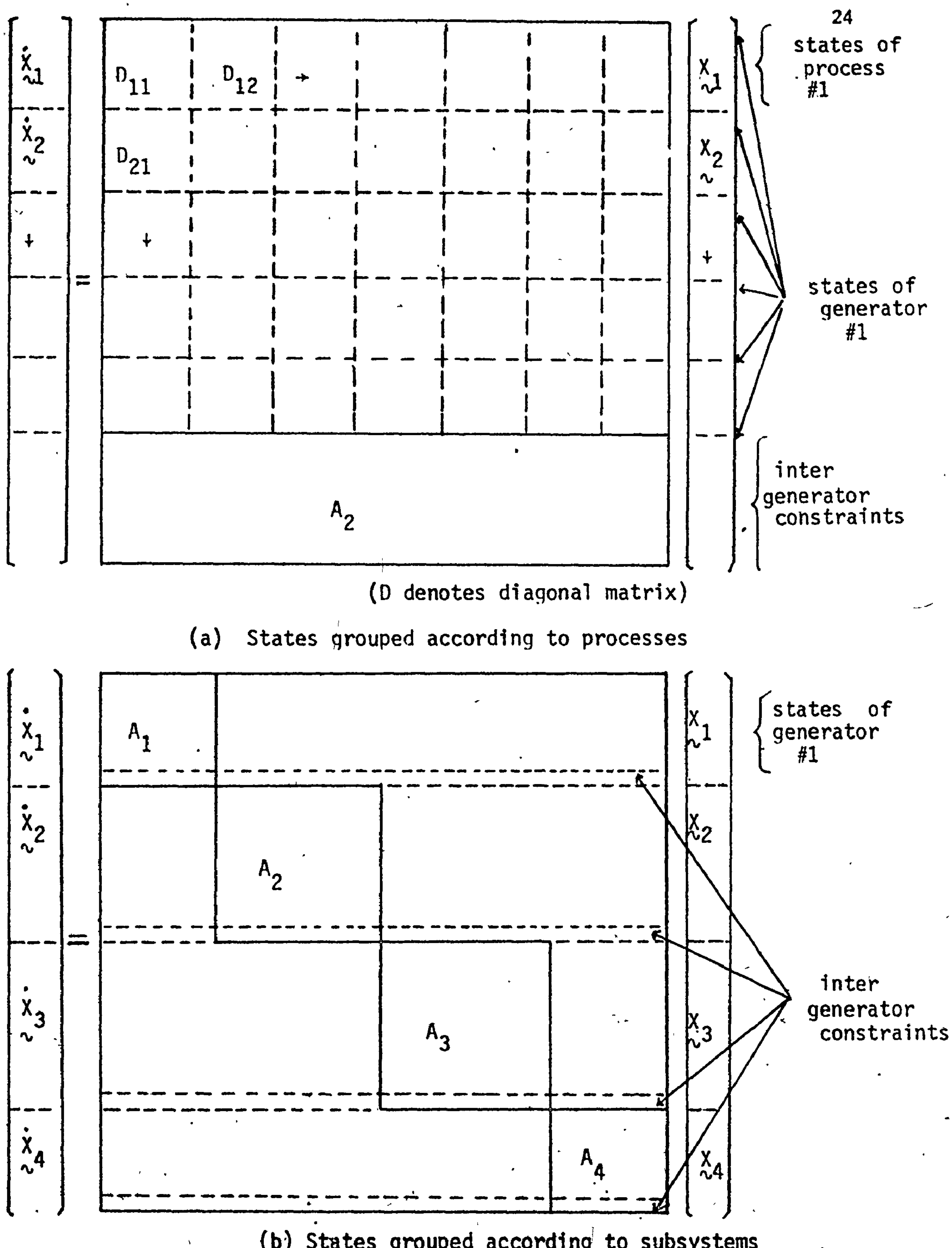


Fig. 1.3 Structure of coefficient matrix for alternative formulations

Author	damper windings	network stator transients	shaft dynamics	asynchronous motor loads	eigenvalue sensitivity	state grouping	subsystem	composite system
Laughton[86] (1966)	-	-	-	-	-	T	U	U
Undrill [87] (1967)	✓	-	-	-	-	T	U	R
Prabhashanker and Janischewskyj[24] (1968)	✓	-	-	-	-	G	R	U
Anderson et al[88] (1973)	✓	-	-	-	-	G	R	U
Smith et al [89] (1974)	✓	✓	-	-	-	T	U	U
Nolan et al [90] (1974)	✓	-	-	-	✓	G	U	U
Alden and Zein el Din[44] (1975)	✓	-	-	-	✓	G	U	U
Kundur and Dandeno [3] (1975)	✓	-	✓	-	✓	T	R	R
"Proposed Method" (1975)	✓	✓	✓	✓	✓	G	R	U

T : states grouped according to type
G : states grouped on generator basis
U : unreduced . R : reduced

Table 1.1 Main formulation approaches for dynamic stability taken and the effects included. It should be pointed out that the absence of a check mark does not necessarily imply that the formulation approach could not be modified to accommodate the appropriate effects. Only those effects specifically outlined in the pertinent reference are included in Table 1.1. The check marks in the row describing the 'proposed method' reflect the requirements of the formulation presented in this thesis.

An important comment should be made about the representation of stator-network transients. Network equations in stability studies are usually written as algebraic rather than differential. A full version of Parks Equations [85] includes stator flux linkage as states. It is inconsistent to combine a full version of Park's Equations with algebraic network equations. This inconsistency (which results in an erroneous prediction for the 60 hertz frequency modes) has been overlooked by a number of researchers. It is well known that elimination of $p\psi_d$, $p\psi_q$ terms does not seriously affect accuracy in most dynamic stability studies [109]. The error introduced can be approximately counter-balanced if the effect of speed deviation is neglected in the voltage equations [109]. If $p\psi_d$, $p\psi_q$ terms are included, then differential equations must be used to describe the network. This will be required in system modelling for subsynchronous resonance studies.

The choice of reference frame is an important question [87]. For systems which include a tie to an infinite bus, a reference frame synchronized to the infinite bus is the obvious choice. For multimachine studies without an infinite bus, a common frame synchronized to a large machine can be considered [87], [88]. In a discussion on [87] it was pointed out that a common frame rotating at any known speed can be used. It was also emphasised in the discussion that effective changes in network admittance are accounted for if a common synchronous frame is chosen. This idea is used in the thesis in that the network equations with 'transformer' voltages included, are referred to a synchronously rotating reference frame. If the frame of a particular machine is adopted, the effect of the oscillation of this frame during transients would have to be included in the network equations.

For successful modelling of an overall power system it is important to understand well the modelling and dynamic behaviour of its individual subsystems. The development of these individual subsystem models and subsequent analysis is in itself a formidable task and fortunately is available in the technical literature.

The subsystems considered in this thesis are described in Appendix A and the modelling concepts and dynamic properties are described in the following references:

1.	Synchronous Machine	[24], [25], [92]-[96] [114]
2.	Excitation Systems	[56], [98]
3.	Turbine-Governors, Boilers	[79]-[81], [99]-[101]
4.	Mechanical Shaft System	[15]-[22], [95]
5.	Transformers, Transmission Network and Static Loads	[102]-[105], [116]

6. Asynchronous Motor Loads [106]-[108]

General References [72], [96], [97], [109], [110], [117]

1.7 Objects of this Thesis

1. To develop a general state space equation formulation suitable for analysis of dynamic stability properties of a balanced multimachine power system. In addition, to detailed generator, governor-turbine and exciter representation, the dynamics of the mechanical shaft system and electrical network are to be included. Special emphasis is to be placed on modelling flexibility and on the relationship between alternative models and the aspects of stability that they reflect.

2. To apply the modelling concepts developed in the study of the interaction of shaft dynamics in closely coupled turbine-generators. The usual practice in power system analysis has been to 'lump' into a single equivalent, all nominally identical generators within a given power station. Recently, as a result of tests [22], close generators were found to have interacting shaft dynamics which resulted in a shift in shaft natural frequencies. This shift may have very serious practical implications for thermal units using speed sensitive power system stabilizers. Consequently, the task of analysing the interaction in shaft dynamics of a general number of electrically close generators is to be undertaken.

1.8 Arrangement of the Material

Chapter 2 is devoted to the development of a general state space model for the system. The overall model is characterised by the particular choice of representing the overall system in terms of two interacting systems, namely a network system and a 'generating units' system. The network equations are differential (as distinct to algebraic) and are developed using network topological principles. The 'generating units' system is built up using subsystem models for excitation systems, turbine governors, etc. (In Appendix A, the models of the individual subsystems are presented.) The overall systems' equations are presented in terms of the interaction of the two systems. Finally, an overall solution sequence which includes the computation of eigenvalue sensitivity is discussed.

The application of the modelling concepts (described in Chapter 2) to practical power system problems is demonstrated in Chapter 3. An overall solution strategy is discussed and subsequently typical results for sample systems are presented. In particular, situations involving stability problems due to lack of synchronizing and damping torques, interaction of controls and subsynchronous resonance, are illustrated. The facility for the study of power systems where asynchronous load effects may be significant, is also illustrated. Finally, as a multimachine example, a sample four machine hydro-electric system is modelled and analysed using eigenvalue and eigenvalue sensitivity methods.

The topic of shaft dynamics in closely coupled identical turbine-generators is discussed in Chapter 4. The 'double resonant peak' phenomenon is first explained in terms of a 2 generator, 2 mass system. It is then shown that a general N generator, M mass system may be analysed in terms of two equivalents. The computed response showing the stimulation of shaft transients is presented. It is hoped that such simulation results will provide a basis for future experimental testing. Finally, in Chapter 4 the application of eigenvalue sensitivities in the study of shaft effects is outlined.

In Chapter 5 the general problem of evaluating alternative models is considered. A technique for developing alternative low order models from a single high order system model, is presented. The basis for the derivation of the low order models is the assumptions normally applied in practice in the industry. Sample systems are taken from Chapter 3 and the effect of modelling complexity on stability predictions is discussed.

In Chapter 6 the main conclusions of the thesis are summarized and the specific contributions of the research and suggestions for future work, are outlined.

CHAPTER 2

FORMULATION OF POWER SYSTEM DYNAMIC STABILITY MODELS

2.1 Introduction

This chapter documents the development of a state space model, for digital computer applications, of an integrated power system comprising a general number of generator and load units. Unlike previous formulations [86] to [90] the dynamics of the electrical network and mechanical shaft system can be included. Provision is made for including a wide variety of machine and control equipment model types and complexities. The overall formulation structure was designed to allow ease of system modification and update and evaluation of the co-efficient matrix sensitivity. Consequently the model and formulation structure is suitable for stability analysis using eigenvalue and eigenvalue sensitivity techniques. Furthermore, the structure of the overall system description was chosen so as to retain (as far as possible) the identity of individual subsystem descriptions. This facilitates the analysis of chosen portions of the overall system thus providing insight into the overall system dynamics.

The development of the system model in terms of multi level dynamic subsystems is described in [111] and eigenvalue sensitivity determination and applications in [112].

In Section 2.2 the overall system is described in terms of two interacting systems. The first of these systems is described in Section 2.3 in terms of a series of mutually uncoupled generating (or load) systems each of which is represented as a group of interacting subsystems such as synchronous machines, turbine governors, etc. The modelling of these individual subsystems is well documented and is considered outside the scope of this thesis. An understanding of the dynamic properties of the individual subsystems (either isolated or with appropriate boundary conditions) is a desirable prerequisite for a satisfactory appreciation of overall system analysis. Consequently a list of appropriate references has been presented in Section 1.6 which can be used in interpreting the subsystem models documented in Appendix A.

For Section 2.4, the second major system, the Network System, is described. Topological concepts are applied in the development and the state space equations are arranged in a format in which they can be readily merged with the equations of the system of generating units.

The state space equations of the two interacting systems are merged in Section 2.5, thus yielding the overall system state space equations. Finally in Section 2.6, the evaluation of eigenvalue sensitivities is discussed.

The important aspects of this chapter are summarized in Section 2.7

2.2 Overall System Representation

The formulation described here is characterized by a particular choice of overall system description in which two major systems interact (Figure 2.1). The first major system comprises mutually uncoupled generating (and rotating load) units. The second major system comprises the electrical network which includes transmission and static load representation. This choice of representation has inherent advantages over alternative schemes (Section 1.6) in that the identity of individual generator (together with their control equipment) is maintained. This facilitates the analysis of individual portions of the overall system including 'single machine-infinite bus' configurations for the individual generators. The choice of representation also simplifies the task of integrating generator models of different types and complexities. This is cumbersome to implement using schemes where states are grouped according to the type of variables they represent (i.e., rotor angles of all machines grouped together) rather than grouping states of individual machines together (Section 1.6).

2.3 System of Generating Units

The dynamics of individual generating (or load) units may be described in terms of the block diagram of Figure 2.2. This diagram shows the interaction of generator, shaft, exciter and governor subsystems. Figure 2.2 is drawn specifically for generator description. Synchronous

Fig. 2.1

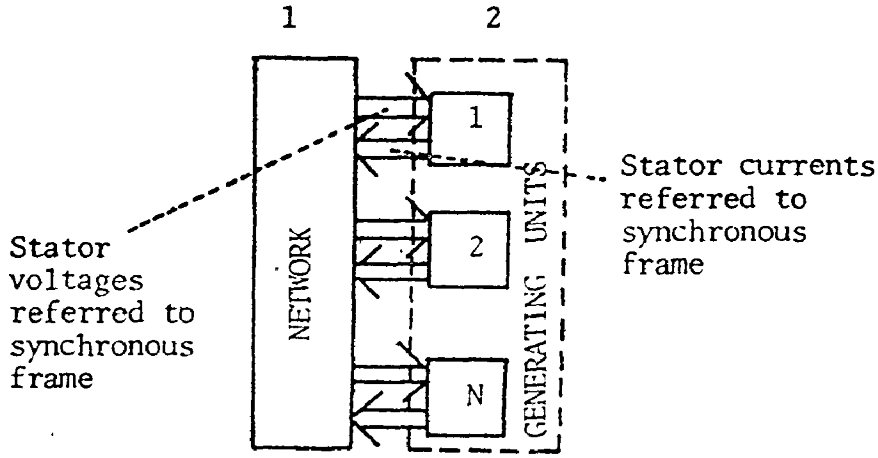


Fig. 2.1 Choice of overall power system description.

Fig. 2.2

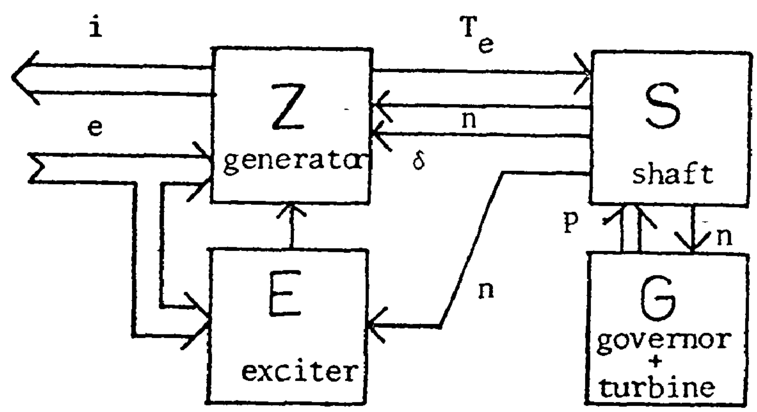


Fig. 2.2 Generating unit system structure.

motor loads are represented similarly except that the governor block is removed and the shaft block is modified to represent load dynamics. Asynchronous motor loads are represented by replacing the synchronous machine model by that of an asynchronous machine and removal of the exciter and governor blocks. By appropriate choice of generator, shaft, exciter and turbine-governor subsystems, a wide variety of model types and complexitities may be considered. Typical subsystems (which have been considered during the course of this thesis preparation) are shown in Figure 2.3 with appropriate description in Table 2.1. These subsystems are described in Appendix A.

An essential requirement in the development of the subsystem state space equations is that the input-output variables of subsystem normally interacting be compatible, i.e., output variables for one subsystem are the input variables for the next.

The state space equations of an isolated generating (or load) system are determined by combining the state space equations of the ZSGE (generator, shaft, turbine-governor and exciter) subsystems. The structure of the overall state space matrix is shown in Figure 2.4 where the origin of the off-block diagonal terms is explained.

It will be apparent later that it is expedient to partition the states into three groups: -

$\underset{\sim}{i}'_s$ = (i_D, i_Q) = stator currents in D-Q frame

e_v = output of terminal voltage sensor

$\underset{\sim}{x}'_m$ = all state variables of isolated generating system

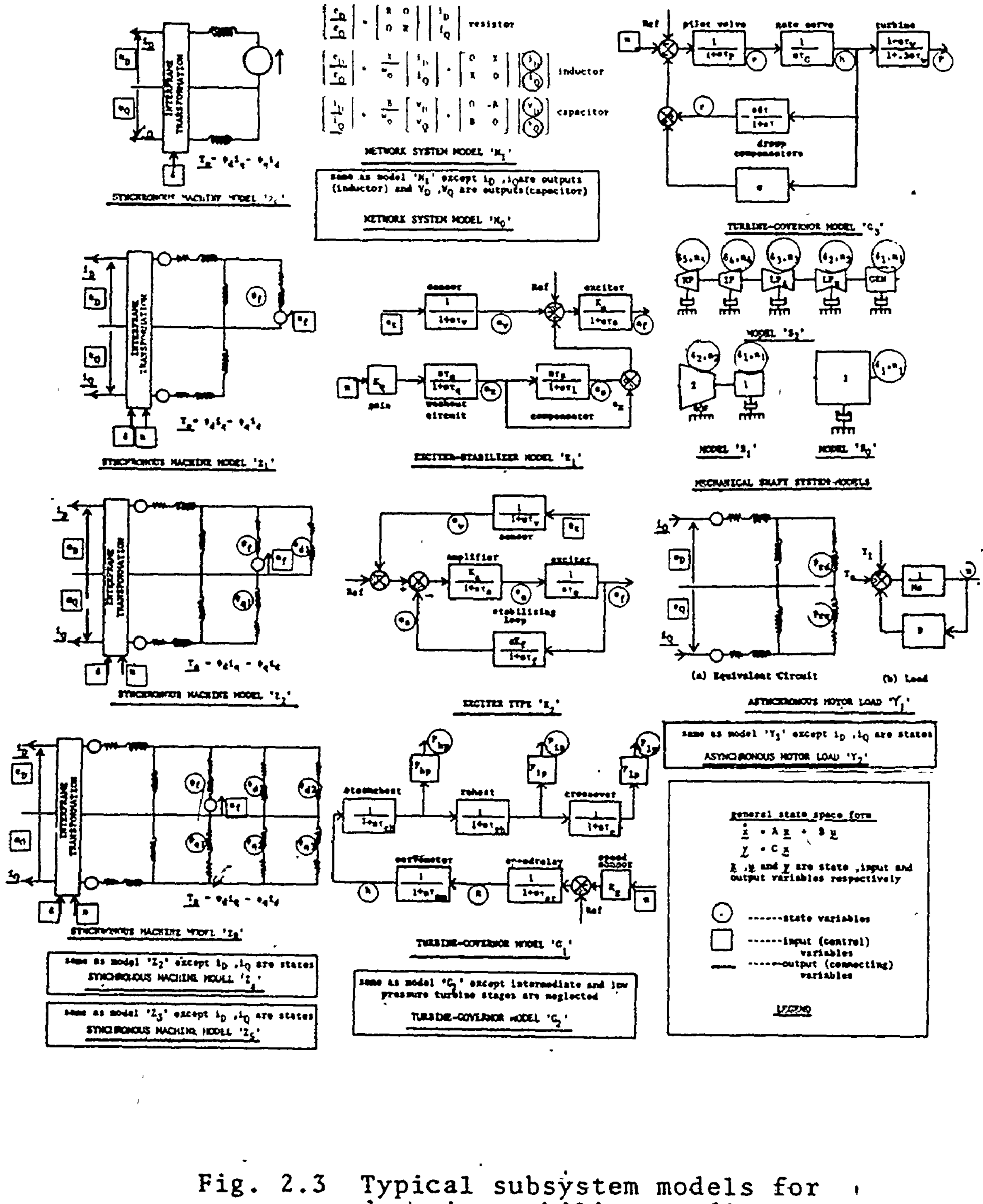


Fig. 2.3 Typical subsystem models for dynamic stability studies.

ELEMENT	SYMBOL		CLASSIFICATION	STATOR TRANSIENTS	ORDER n m r	INPUT VARIABLES	OUTPUT VARIABLES	STATE VARIABLES
SYNCHRONOUS MACHINES	Z	0	'classical model'	x	0 3 3	D, Q Voltages, rotor angle	D-Q stator currents electric torque	---
		1	1 rotor circuit	x	1 5 3	D, Q Voltages, rotor angle and speed, field voltage	"	field flux linkage
		2	3 "	x	3 5 3	"	"	rotor flux linkages
		3	6 "	x	6 5 3	"	"	"
		4	3 "	✓	5 5 1	"	electric torque	rotor flux linkages and D, Q stator currents
		5	6 "	✓	8 5 1	"	"	"
TRANSMISSION NETWORK STATIC LOADS	N	0	constant impedance	x	0 2 2	D, Q Voltages	D, Q Currents	---
		1	linear circuit element	✓	* 2 0	"	---	D, Q inductor currents and D, Q capacitor voltages
EXCITATION SYSTEM	E	1	static-stabilizer	-	4 3 0	D, Q voltages + speed signal	---	field voltage + states associated with exciter and stabilizer
		2	rotational	-	4 2 0	D, Q voltages	---	field voltage + states associated with exciter
TURBINE-GOVERNOR	G	1	steam (including reheat dynamics)	-	5 1 3	speed signal	---	turbine powers + States associated with steam storage and regulator equipment
		2	steam (neglecting reheat dynamics)	-	3 1 1	"	---	turbine powers + states associated with regulator equipment
		3	hydraulic	-	4 1 1	"	---	turbine power + states associated with regulator and compensators
MECHANICAL SHAFT	S	0	lumped mass	-	2 2 0	electrical + turbine powers	---	rotor angle and speed
		1	2 mass model	-	4 2 0	"	---	angles and speeds of 'equivalent' inertias
		2	5 mass model	-	10 4 0	"	---	"
ASYNCHRONOUS MOTOR LOAD	Y	1	neglecting stator transients	x	3 2 0	D, Q voltages	D,Q stator currents	rotor fluxes and speed
		2	including stator transients	✓	5 2 0	"	---	rotor fluxes, speed, D-Q stator current

*varies depending on network

Table 2.1 Classification of typical subsystem models

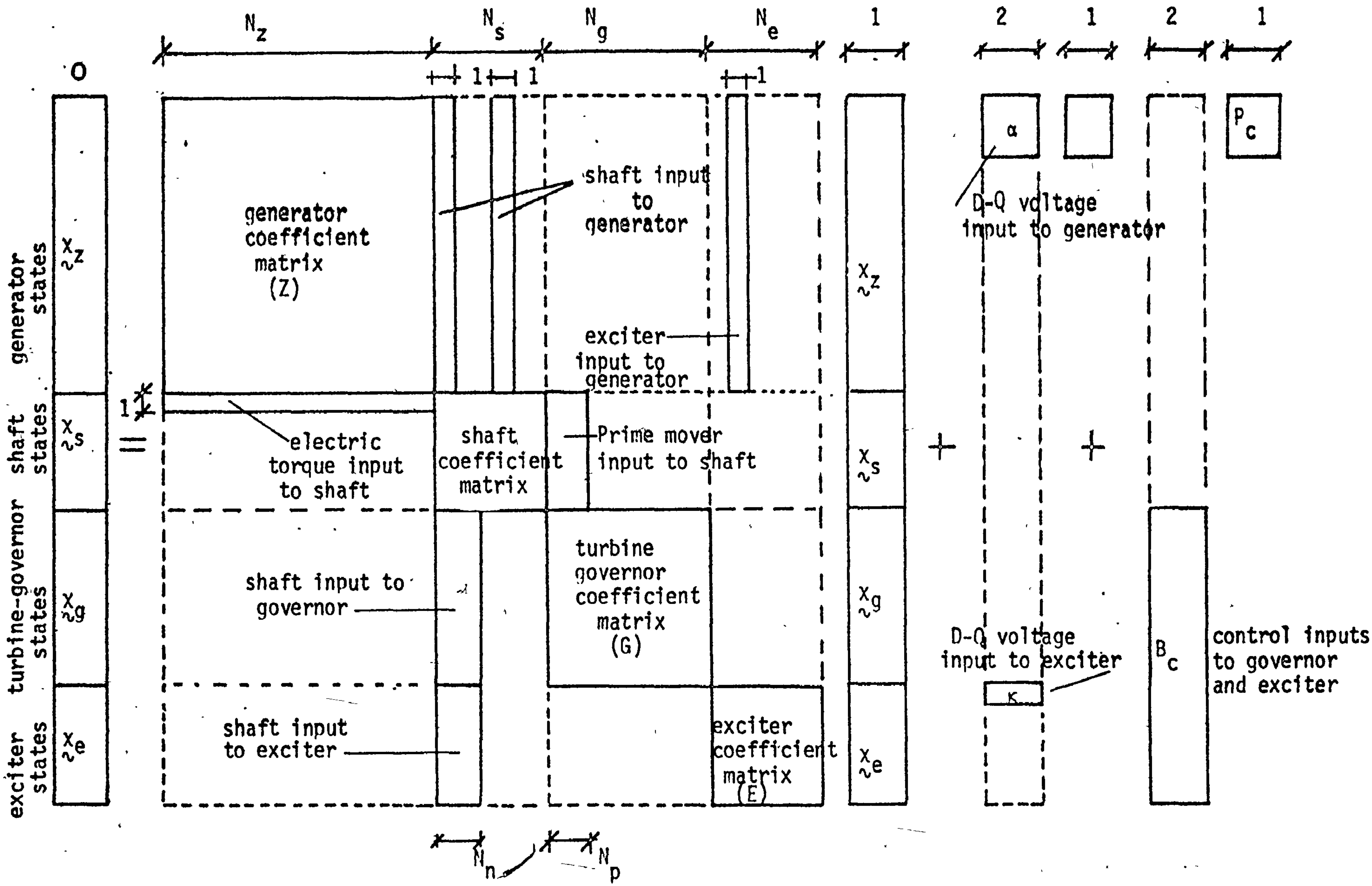


Fig. 2.4 Structure of overall coefficient matrix for isolated generator

other than i_D, i_Q, e_V

$\underset{\sim}{i}_s$ are 'connecting' states in that they are common to both the network and generator systems (Figure 2.1).

The equations of an isolated single generator unit can be expressed in terms of the partitions of the state variables:

$$\begin{aligned} \dot{\underset{\sim}{i}}_s &= a\,\underset{\sim}{i}_s + b\,\underset{\sim}{x}_m + \gamma\,\underset{\sim}{e}_s \\ \dot{e}_V &= f\,e_V + \kappa\,\underset{\sim}{e}_s \\ \dot{\underset{\sim}{x}}_m &= c\,\underset{\sim}{i}_s + e\,e_V + d\,\underset{\sim}{x}_m \end{aligned} \qquad (2.1)$$

Notice that if e_s is set to zero the equations of a 'single' machine - infinite bus are described. The infinite bus in this case is incident to the stator's terminals and, of course, there is no exciter action as the terminal voltage is maintained constant.

The effect of a variable length transmission line to an infinite bus can be easily accounted for by substituting $Z\,i_s + L\,\dot{i}_s$ for e_s (Subsystem N_1) and then reducing to state space form.

The state space equations for a multiplicity of such generating (or load) units can be expressed in terms of block diagonal matrices of the constituent a, b, c, d, e, f, κ, γ submatrices: -

$$\dot{\underset{\sim}{i}}_s = A_M \underset{\sim}{i}_s \qquad B_M \underset{\sim}{X}_M + \Gamma_M \underset{\sim}{e}_s$$

$$\dot{\underset{\sim}{e}}_v = \qquad F_M \underset{\sim}{e}_v \qquad + \kappa_M \underset{\sim}{e}_s \qquad (2.2)$$

$$\dot{\underset{\sim}{x}}_M \quad C_M \underset{\sim}{i}_s + E_M \underset{\sim}{e}_v + D_M \underset{\sim}{X}_M$$

with A_M = diag $\{a_1, a_2 \ldots a_N\}$

and $\underset{\sim}{i}_s = \{\underset{\sim}{i}'_{s1}, \underset{\sim}{i}'_{s2} \ldots \underset{\sim}{i}'_{s\ N}\}$ etc.

It can be seen that the states in the above have been ordered so that the stator currents of the individual synchronous machines and asynchronous motors appear consecutively as also do the measured terminal voltages. The remaining states are grouped according to the unit which they represent, eg., the states of generator #1 (with all its control equipment) appear together. This tends to preserve the identity of each portion of the representation within the overall equation structure.

The grouping of the stator currents and measured terminal voltages separately (from the other states) will be seen to facilitate the merging of the network equations with those of the generating units. This merging results in a coupling between the dynamics of the individual uncoupled generating units.

2.4 Network System

As mentioned previously (Section 1.6) in most previous formulations, static network equations have been used except in special single machine - infinite bus situations.

One of the tasks of the research reported in this thesis was to include dynamic network equations in a general multi-machine formulation. The difficulty of so doing is mainly in imposing the correct constraints on the equations to account for the interface between the network and generators. Actual development of state space equations for a general balanced three phase network is quite straightforward using modern topological concepts.

It has been shown in [116] that the equations of a balanced three phase network can be expressed in terms of an equivalent uncoupled balanced two phase system. Since the two phase system is balanced it is obviously expedient to set up the network in terms of a single phase system. Consequently we proceed in three steps:

(1) A single phase representation for the network. (Such single phase systems are universally employed in the power industry for network description-excepting unbalanced system representation.) The state space equations describing this network are developed using topological concepts.

(2) The single phase equations are extended to those of a balanced uncoupled two phase system and subsequently the equations are transformed from a stationary to a synchronously rotating frame. This makes the network equations compatible with those of the rotating machines.

(3) Finally, the state space equations describing the network system are partitioned so as to separate those network currents which form fundamental loops with the stators of the electrical machines from the remaining network currents and capacitor voltages which comprise the states of the network system.

2.4.1 Single Phase Network Equations

The topological concepts presented here are very well developed and adequately documented in modern textbooks on network theory. Such concepts have been applied extensively in setting up the static network equations for power flow studies, but have not, to the knowledge of the author, been employed in formulating state space equations for stability studies.

By applying directly such concepts we obtain the following matrix equation for the network:

$$\underset{N_N:\ N_\ell,\ N_C}{}\left[\begin{array}{c|c} K_L \ L \ K_L^1 & 0 \\ \hline 0 & C \end{array}\right]\left[\begin{array}{c} \dot{I}_\ell \\ \dot{\underset{\sim}{V}}_C \end{array}\right] = \left[\begin{array}{c|c} -K_R \ R \ K_R^1 & -K_C^1 \\ \hline K_C & 0 \end{array}\right]\left[\begin{array}{c} \underset{\sim}{I}_\ell \\ \hline \underset{\sim}{V}_C \end{array}\right] + \left[\begin{array}{c} \underset{\sim}{V}_S \\ \hline \underset{\sim}{0} \end{array}\right] \updownarrow N \qquad (2.3)$$

where:

K_L, K_R and K_e are incidence matrices corresponding to inductive, resistive and capacitive circuit elements, respectively.

L, R and C are diagonal matrices of inductance, resistance and capacitance values. I_ℓ and V_C are vectors of fundamental loop currents and capacitor voltages and constitute the states of the network system. $\underset{\sim}{V}_S$ is vector of generator voltages and the ordering of the fundamental loops is such that the first N loops (N = number of machines) are incident to generator unit terminals. Furthermore the reference direction of these N loops is consistent with the normal sense of current flow for the machines.

Typical examples of incidence matrices are given in Figures 2.5(a) and (b).

2.4.2 Two Phase Network Equations

For the previous subsection, the equations of one phase of an uncoupled two phase system were developed. The equations for the second phase are an exact duplicate of those for the first.

Consequently if the reduced equations for the single phase network are:

$$\dot{\underset{\sim}{x}}_1 = A_1 \underset{\sim}{x}_1 + B_1 \underset{\sim}{e}_1 \tag{2.4}$$

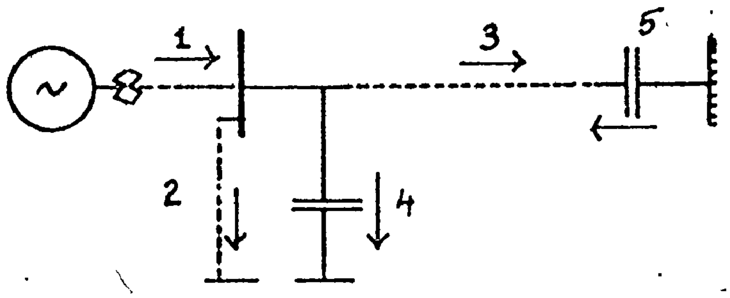
the equations for an uncoupled two phase system are:

$$\dot{\underset{\sim}{x}}_2 = A_2 \underset{\sim}{x}_2 + B_2 \underset{\sim}{e}_2 \tag{2.5}$$

where $\underset{\sim}{x}_2' = (\underset{\sim}{x}_d', \underset{\sim}{x}_q')$

$\underset{\sim}{e}_2' = (\underset{\sim}{e}_d' \ \underset{\sim}{e}_q')$

$A_2 = \text{diag}\ \{A_1, A_1\}$



inductive + resistive elements

	1	2	3
1	1		
2		1	
3			1

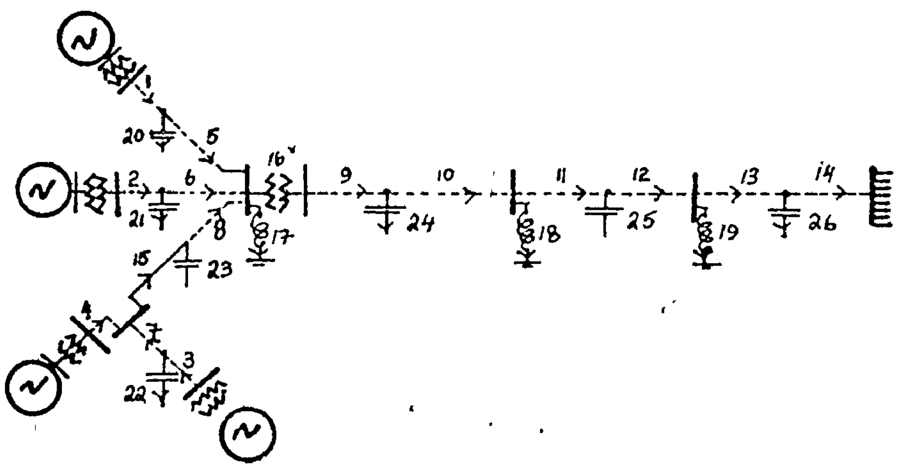
K_L and K_R

capacitive elements

	4	5
1	1	
2	-1	
3	-1	-1

K_C

(a) Single machine system



inductive + resistive elements

inductive links	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1																		
2		1																	
3			1																
4				1											1				
5					1												1		
6						1											1		
7							1								1				
8								1									1		
9									1							1	-1		
10										1								1	
11											1							-1	
12												1							1
13													1						-1
14														1					

K_L and K_R

capacitive elements

	20	21	22	23	24	25	26
1	1						
2		1					
3			1				
4				1			
5	-1						
6		-1					
7			-1	1			
8				-1			
9					1		
10					-1		
11						1	
12						-1	
13							1
14							-1

K_C

(b) Four machine system

Fig. 2.5 Examples of typical incidence matrices.

and $\quad B_2 = \text{diag}\{B_1, B_1\}$

and d and q designate the phases of the two phase system.

The above equations are in terms of state and input variables referred to a stationary reference frame. To have compatibility with the equations for the system of generating units the equations must be transformed to a synchronously rotating (D-Q) frame.

Using Krons transformation [118] we obtain:

$$\underset{\sim}{x}_2 = T\,\underset{\sim}{\hat{x}}_2 \tag{2.6}$$

with
$$T = \left[\begin{array}{c|c} \cos\omega_o t\, I & -\sin\omega_o t\, I \\ \hline \sin\omega_o t\, I & \cos\omega_o t\, I \end{array}\right]$$

$$\hat{x}_2' = (X_D', X_Q')$$

and I is an identity matrix of order NN.

D and Q designate a synchronously rotating reference frame. Differentiating Equation (2.6) yields

$$\dot{x}_2 = T\,\dot{\hat{x}}_2 - U\,T\,\hat{x}_2 \tag{2.7}$$

where
$$U = \left[\begin{array}{c|c} 0 & \omega_o I \\ \hline -\omega_o I & 0 \end{array}\right]$$

Substituting in Equation (2.5) we obtain:

$$\dot{\hat{x}}_2 = (T^{-1}A_2T + T^{-1}UT)\,\hat{x}_2 + T^{-1}\,B_2\,T\,\hat{e}_2 \tag{2.8}$$

Because of the orthogonal property of the transformation, the above reduces to

$$\dot{\hat{X}}_2 = (A_2+U)\,\hat{X}_2 + B_2\,\hat{e}_2 \qquad (2.9)$$

It is interesting to note the effect on eigenvalues due to the change from a single phase representation (stationary frame) to a two phase representation (rotating frame).

We can easily see (by subsititution) that if λ_i and X_i are an eigenvalue and eigenvector, respectively of A_1, then $(\lambda_i \pm j\,\omega_o)$ and $(\underset{\sim}{X}_i', \pm j\underset{\sim}{X}_i')'$ are an eigenvalue and eigenvector of $(A_2 + U)$. This is consistent with the fact that stator transients are reflected as fundamental frequency oscillations in the rotor (synchronously rotating).

2.4.3 Partitioning of Network Equations

The results of the previous subsections allow us to write the network system equations in state space form with fundamental loop currents and capacitor voltages as states. The loops are ordered so as to coincide with the order in which the electrical machines are ordered. Moreover, the reference frame of the equation system is synchronously rotating, thereby ensuring compatibility between the network equations and those of the electrical machines.

We now partition the network state space equations so that the stator currents are separated from the remaining loop currents. As a preliminary step the network states and stator voltages must be reordered so that the D and Q components of each variable are grouped together:

$$\hat{\underset{\sim}{x}}_2' = (x_{D1},\ x_{D2}, \ldots X_{DNN},\ x_{Q1},\ x_{Q2} \ldots x_{QNN})$$

$$\hat{\underset{\sim}{e}}_2' = e_{D1},\ e_{D2} \ldots e_{DNN}\ e_{Q1},\ e_{Q1} \ldots e_{QN}$$

are rewritten

$$\underset{\sim}{X}_N' = (X_{D1},\ X_{Q1},\ X_{D2},\ X_{Q2} \ldots\ldots\ldots X_{DNN},\ X_{QNN})$$

$$\underset{\sim}{e}_S' = (e_{D1},\ e_{Q1},\ e_{D2},\ e_{Q2} \ldots\ldots e_{DNN},\ e_{QNN}$$

Equation (2.9) is then rewritten

$$\begin{aligned} \dot{\underset{\sim}{X}}_N &= A_N \underset{\sim}{X}_N + B_N \underset{\sim}{i}_S \\ \dot{\underset{\sim}{i}}_S &= C_N \underset{\sim}{X}_N + D_N \underset{\sim}{i}_S + E_N \underset{\sim}{e}_S \end{aligned} \qquad (2.10)$$

2.5 Complete System Model

The overall matrix equation describing the interaction of the network and generating units systems is shown in Figure 2.6. For convenience a description of the constituent vectors is summarized here:

- $\underset{\sim}{X}_N$ - a vector of network states comprising all fundamental loop currents (other than stator currents) and capacitor voltages.
- $\underset{\sim}{i}_S$ - a vector of D-Q stator currents
- $\underset{\sim}{e}_v$ - a vector of measured terminal voltages.

$$\overset{\displaystyle n_N \quad\; 2N \quad\; n \quad\; n_M \quad\; 2N}{\begin{pmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & -E_N \\ 0 & 0 & I & 0 & -K_M \\ 0 & 0 & 0 & I & 0 \\ 0 & I & 0 & 0 & -\Gamma_M \end{pmatrix}} \begin{pmatrix} \dot{\underset{\sim}{x}}_N \\ \dot{\underset{\sim}{i}}_S \\ \dot{\underset{\sim}{e}}_V \\ \dot{\underset{\sim}{x}}_M \\ \underset{\sim}{e}_S \end{pmatrix} = \begin{pmatrix} A_N & B_N & 0 & 0 \\ C_N & D_N & 0 & 0 \\ 0 & 0 & F_M & 0 \\ 0 & C_M & E_M & D_M \\ 0 & A_M & 0 & B_M \end{pmatrix} \begin{pmatrix} \underset{\sim}{x}_N \\ \underset{\sim}{i}_S \\ \underset{\sim}{e}_V \\ \underset{\sim}{x}_M \end{pmatrix}$$

Fig. 2.6 Unreduced equation for overall system

$$\begin{pmatrix} \dot{\underset{\sim}{x}}_N \\ \dot{\underset{\sim}{i}}_S \\ \dot{\underset{\sim}{e}}_V \\ \dot{\underset{\sim}{x}}_M \end{pmatrix} = \begin{pmatrix} A_N & B_N & 0 & 0 \\ (I-E_N\S)C_N & D_N+E_N\S(A_M-D_N) & 0 & E_N \; \S \; B_M \\ K_M \; \S \; C_N & K_M \; \S \; (A_M-D_N) & F_M & -K_M \; \S \; B_M \\ 0 & C_M & E_M & D_M \end{pmatrix} \begin{pmatrix} \underset{\sim}{x}_N \\ \underset{\sim}{i}_S \\ \underset{\sim}{e}_V \\ \underset{\sim}{x}_M \end{pmatrix}$$

$$[\; \S = (E_N - \Gamma_M)^{-1} \;]$$

Fig. 2.7 State space equation for overall system

$\underset{\sim}{X}_M$ - a vector comprising all states of generating units (other than stator currents of machines)

$\underset{\sim}{e}_S$ - a vector of D-Q stator voltages

The equations described in Figure 2.6 can be reduced to standard state space form by the inversion of a 2N matrix. The reduced equation is shown in Figure 2.7 and the overall structure is depicted in Figure 2.8. It is important to note that A_N, B_N, C_M, E_M and D_M are unaffected by the reduction process - this property is important in system updating or eigenvalue sensitivity evaluation.

2.6 Eigenvalue Sensitivity Evaluation

The expression for eigenvalue sensitivity is taken directly from Van Ness et al. [42].

$$\frac{\delta \lambda_i}{\delta \xi} = \frac{\{\frac{\delta A}{\delta \xi} \underset{\sim}{X}_i\} \underset{\sim}{V}_i}{\{\underset{\sim}{X}_i \; V_i'\}} \tag{2.11}$$

where:

λ_i = i^{th} eigenvalue

$\underset{\sim}{X}_i$ = eigenvector corresponding to i^{th} eigenvalue

$\underset{\sim}{V}_i$ = eigenvector of transposed matrix corresponding to i^{th} eigenvalue

The method for obtaining the 'A' matrix and its derivative described in [84] is general purpose and consequently fails to take advantage of specific properties encountered in power

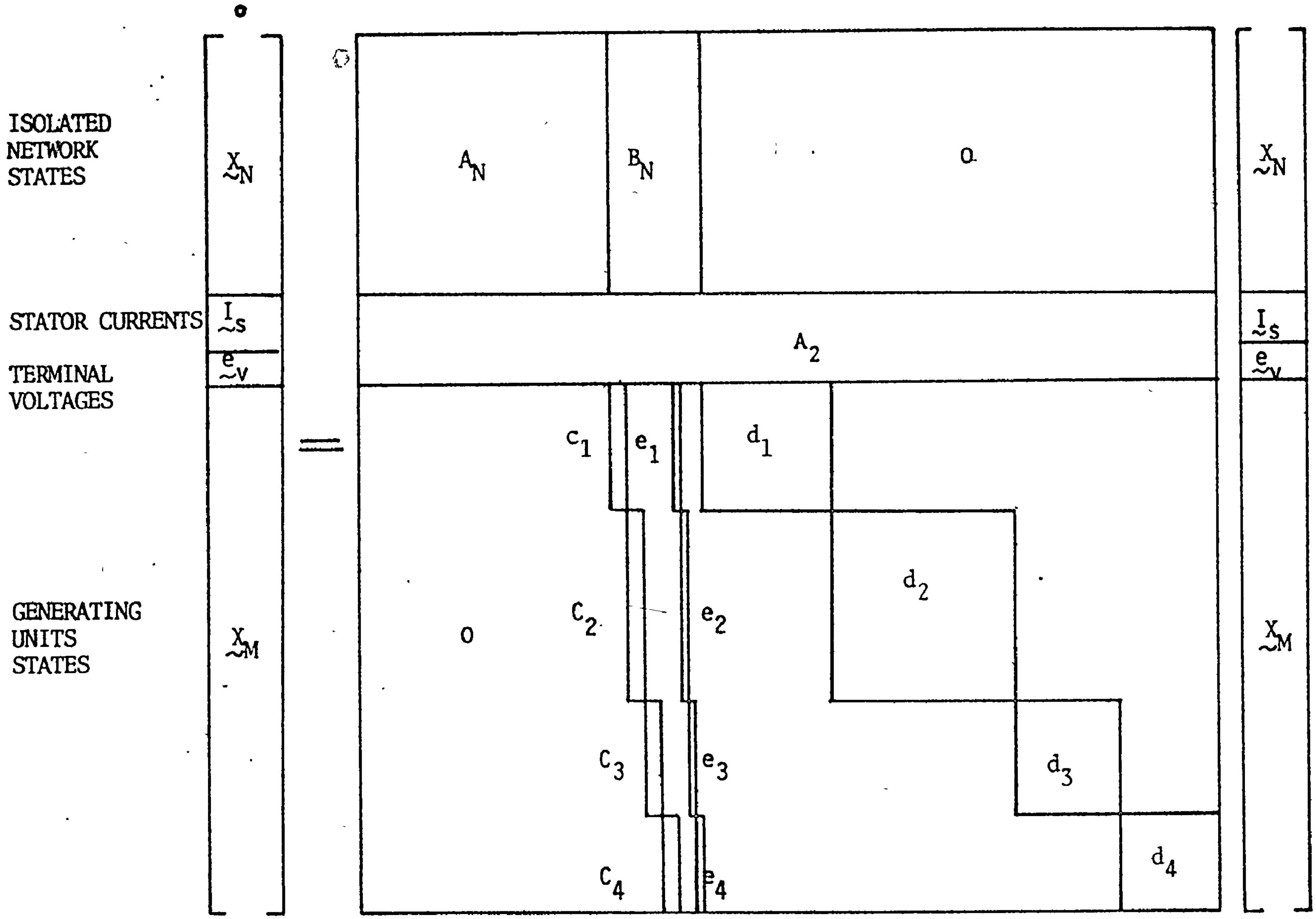


Fig. 2.8 Structure of state space equation for overall system

systems. Such properties include choice of state variables, choice of connecting variables, choice of system partitioning, etc. The formulation of the equation structure as described in the previous sections has evolved as the author has gained experience in power system dynamic analysis and offers substantial computational saving over general purpose methods.

The unreduced system equation (Figure 2.6) has the general form

$$P \begin{bmatrix} \dot{\underset{\sim}{x}} \\ \\ \underset{\sim}{z} \end{bmatrix} = Q \underset{\sim}{x} \tag{2.12}$$

which upon reduction is

$$\begin{bmatrix} \dot{\underset{\sim}{x}} \\ \\ \underset{\sim}{z} \end{bmatrix} = S \underset{\sim}{x} \tag{2.13}$$

where

$$S = \begin{bmatrix} A \\ \hline C \end{bmatrix}$$

A is the state space matrix and C is the output matrix expressing the relation between terminal voltage and the system states. It can be shown that

$$\frac{\delta A}{\delta \xi} = [I \;\; 0] \; P^{-1} \; [\frac{\delta Q}{\delta \xi} - \frac{\delta P}{\delta \xi} \; S] \tag{2.14}$$

Consequently the eigenvalue sensitivities can be expressed in terms of the sensitivities of the two major systems describing the overall system. It should be recalled that P^{-1} is largely the identity matrix and $\frac{\delta P}{\delta \xi}$ is zero for most variable parameters.

Consequently for most variable parameters and specifically for all control parameters (control gains, time constants, etc.)

$$\frac{\delta A}{\delta \xi} = \frac{\delta Q_1}{\delta \xi} \tag{2.15}$$

where $Q_1 = [I \; 0] \; Q.$

The same simplification occurs at the individual generating unit subsystem level. In this case the d matrix (D_M = diag (d_1, $d_2 \ldots d_N$)) will be found to contain all the control parameters.

Consequently, though the system matrix is developed using a two level subsystem approach (level 1: subsystems comprising each individual generating unit, level 2: system of generating units and network system) the identity of the individual dynamic subsystems representing control action is preserved. Appropriate control parameters may be readily identified thus allowing the sensitivity of the 'A' matrix to be determined without any matrix manipulations. Also the A matrix may be updated by simply changing the appropriate parameters without matrix operations. Similar comments as were made for control parameters can be made for the shaft system parameters and network parameters (except those forming fundamental loops with the machines).

For electrical machine parameters and parameters of the network elements forming fundamental loops with the stator windings, the full version of Equation (2.14) will generally have to be applied.

Even in this case the partitioned form of Equation (2.14) offers significant saving over general purpose techniques.

Only first order sensitivity evaluation has been considered in this thesis. First order sensitivities have been found adequate in applications considered (including the determination of stability limits). However use of second and higher order sensitivities may be especially useful as an alternative to repeated eigenvalue calculations and has not been applied in the power industry. In addition to the normal and transposed eigenvector and $\frac{\delta A}{\delta \xi}$ (Equation (2.11)), $\frac{\delta_2 A}{\delta \rho^2}$ is also required for second order sensitivities ([120], Sec. 9.2). The development of the 'A' matrix in terms of submatrices which largely retain the identity of the individual dynamic subsystem matrices is very desirable. This will avoid application of the chain rule for differentiation to Equation (2.14).

Two comments are in order concerning computation. Firstly, an implicit assumption in the use of Equation (2.11) is that there are no repeated eigenvalues. It was decided to postpone an investigation of this ill conditioning until the problem arose in practice. During the course of the study reported, the problem did not arise - successful calculation of sensitivities for a system with eigenvalues at $-.3001 \pm j\,276.47$ and $-.3004 \pm j\,276.47$ is reported in Section 4.6. Secondly, since $\frac{\delta A}{\delta \xi}$ is extremely sparse (<.1% full for large systems (50 states)) sparse matrix methods are employed in the sensitivity calculation.

2.7 Summary

A structured, efficient approach for the development of a state space model for a power system has been presented. A two level hierarchical approach has been adopted. At the first level the individual subsystems comprising synchronous machines, shaft systems, exciters and turbine governors are interconnected to yield a system of generating units (including any asynchronous motor loads). This has inherent conceptual and implementational advantages which stem from the property that the equations of a single machine - infinite bus representation for each machine appear explicitly. This facilitates the checking of the dynamics of each machine separately thus giving a basis for comparison for the more complex interactions in the overall system.

Also at the first level in the modelling approach the state space equations describing the transmission system and static loads are developed using modern circuit theory topological concepts.

At the second level the equations describing the interacting system of generating units and network system are combined to yield the overall system model. The structure of the overall system model is shown to preserve the identity of certain individual dynamic subsystems (exciter, shaft turbine-governor). This greatly simplifies the task of system update and differentiation of the 'A' matrix which is required in eigenvalue sensitivity evaluation.

CHAPTER 3

APPLICATION TO PRACTICAL SYSTEMS

3.1 Introduction

In the previous chapter a state space model for an interconnected power system comprising a general number of generators and loads was presented. Provision was made for the inclusion of mechanical shaft system and electrical network dynamics. The model is suited to general analysis and control applications for the major category of power system problems involving electrical machine dynamics.

In this chapter the application of the developed model and formulation structure is outlined as applied to the study of the following aspects of power system stability:

(a) monotonic instability associated with synchronization problems.

(b) oscillatory instability associated with poor rotor damping.

(c) interaction between turbine-generators and excitation, governing and load systems.

(d) subsynchronous torsional instability in series compensated transmission systems.

In Section 3.2 the overall strategy for problem solution is discussed with emphasis on the selection of modelling detail and analysis approach. Five examples involving single machine - infinite bus situations are discussed in

Section 3.3. The rational behind the choice of modelling detail and analysis approach is discussed and typical computed results are presented.

A sample four machine hydroelectric system is analysed in Section 3.4. Because the modelling of this situation draws heavily on the material developed in the previous chapter, all the main stages in the development are documented. This will enable other workers to apply the models developed in problems of comparable complexity. Results for the system including and excluding stator-network dynamics are presented. The application of eigenvalue sensitivities is also discussed.

General concluding comments are made in Section 3.5.

3.2 Overall Solution Strategy

Two distinct uses are envisaged for the model developed. The first is in a diagnostic use where a particular power system exhibits poor stability properties and the cause is to be determined. Unless a clear precedent is known, a full system model encompassing all known effects should be employed and the cause of instability determined by examining the dynamic properties, their sensitivities to parameter variation and subsystem inclusion. The second use is in the exploratory studies required in system planning. This will generally involve a very large number of stability predictions corresponding to a variety of system parameters and loading conditions. Evaluations involving alternative system configurations (alternative line status) will in general be required. Also comparative evaluation of alternative control equipment (eg., rotating versus static exciters) may be required. In this case the choice of modelling detail will be reasonably well defined as the aspect of stability will generally be understood. Here, efficiency both of stability prediction and system update will be of paramount importance due to the large number of predictions.

Table 3.1 shows the relationship between the dynamic subsystems modelled and the various aspects of power system dynamic stability predicted.

This table has been compiled from a review of the modelling approaches adopted by other authors where individual aspects of stability were studied (Section 1.3). The validity of not including all dynamic subsystem models is discussed in Chapter 5.

The structure of formulation described in the previous chapter satisfies both the requirement of modelling detail and efficiency of formulation.

As stated in Section 1.4, eigenvalue analysis is the most practical method for determining the dynamic stability properties of a power system. As is well known, negativity of all real components of the eigenvalues indicates absolute stability. A positive real eigenvalue corresponds to monotonic instability and a positive real component of a complex conjugate eigenvalue pair indicates oscillatory instability. Useful yardsticks in elementary servomechanism theory are damping ratio, ξ, and undamped natural frequency, ω_n. These may be readily determined from the individual eigenvalues:

subsystem models → / study ↓	SYNCHRONOUS MACHINES				SHAFT		EXCITER		TURBINE		NETWORK		LOADS			
	steady state	field effects	damper windings	stator transients	sinlge lumped mass	shaft torsional effects	voltage regulation	stabilizer (if present)	mechanical hydraulic	electro hydraulic	static eqtns.	dynamic eqtns.	static representation	voltage dependency	frequency dependency	dynamic model
monotonic stability (synchronization)		✓	✓				✓	✓			✓		✓			
rotor oscillations		✓	✓		✓		✓	✓		✓	✓		✓			
exciter effects		✓	✓		✓		✓	✓			✓		✓			
subsynchronous resonance (electrical)		✓	✓	✓			✓					✓		A	A	A
subsynchronous resonance (torsional)		✓	✓	✓		✓	✓	✓		✓		✓		A	A	A
load-frequency	✓				✓				✓	✓	✓			A	A	A
interaction between controls		✓	✓		✓		✓	✓	✓	✓	✓					✓

A denotes choice between alternatives depending on specific application.

Table 3.1 Relationship between dynamic subsystems modelled and aspects of stability reflected.

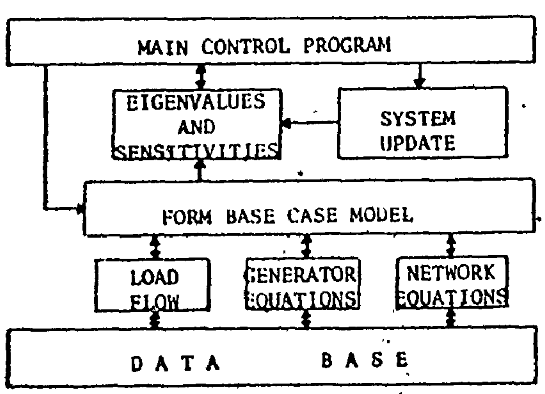


Fig. 3.1 Basic stages in overall formulation and anaylsis.

say $\lambda = \lambda_r \pm j \lambda_i$

$= \xi \omega_n \pm j \omega_n \sqrt{1-\xi^2}$

hence $\omega_n = \sqrt{\lambda_r^2 + \lambda_i^2}$

and $\xi = \lambda_r / \omega_n$

The high system order and multiplicity of parameter variation possibilities generally prohibit repeated eigenvalue evaluations over large ranges of parameter variation because of the computation cost.

Eigenvalue sensitivity evaluations render eigenvalue analysis far more economical for high order systems. This will in general allow:

(a) Mode identification

By obtaining the sensitivities of particular eigenvalues to system parameters, the interactions involved may be ascertained. This allows identification of potential sources of instability.

(b) Basis for selection of optimal parameter setting.

By optimal parameter setting is meant a setting which satisfies some specific design criterion. This criterion may often be expressed in terms of the systems' eigenvalues. Furthermore the allowable limits of parameter variation may be obtained by determining the stability limits in a Newton-Raphson type process.

(c) Basis for the determination of modelling precision.

By knowing the sensitivities of the critical eigenvalues with respect to system parameters the precision to which these parameters must be estimated can be determined.

The basic stages in the overall formulation and analysis are depicted in Figure 3.1.

3.3 Single Machine - Infinite Bus Problem

Though the model development of the previous chapter was aimed at general multimachine situations, a single machine problem will be analysed here as it can be used to illustrate the various aspects of stability. Because of the relatively low order, the results are relatively easy to interpret. Also due to the relatively small data base, general conclusions regarding the stability and allowable parameter variations can be made. This can be contrasted with multimachine situations where due to the multiplicty of parameter variation possibilities, no general conclusions can be drawn and problems must be solved on an individual basis.

Though only strictly valid in situations which actually approximate single machine - infinite bus behaviour, the results of such analysis yields valuable insights into the more general multimachine problems which as stated before, must be analysed on an individual basis.

3.3.1 Stabilization of Torque-load Angle Loop

Stabilization of the torque-load angle loop is the most common dynamic stability problem presently confronting power system engineers. As mentioned previously, this is largely due to the introduction of high gain static excitation systems which help with transient stability but destroy inherent machine damping. Excellent insights into the effects of static excitation schemes on dynamic stability can be obtained from a study of [4]. An approach for the design of power system stabilizers has been also outlined in [4].

In this subsection the application of the detailed model developed to this problem will be outlined. Application of the results of [4] though mandatory (in the opinion of the author) for a sound understanding of the processes involved can lead to sub-optimal designs since a very simplified machine model (in particular no damper representation) was assumed.

A line diagram for the system studied is shown in Figure 3.2 and the system data is listed in Table 3.2. Notice in particular that the rotor angle relative to the infinite bus is in excess of 90^o ($\delta = 105^o$). The eigenvalues for the system corresponding to the case of zero stabilizer and different values of exciter gain are shown in Table 3.3. Time responses for δ (rotor angle) for a .01 p.u. disturbance in shaft torque are shown in Figure 3.3 for three values of exciter gain. For zero exciter gain the system exhibits a monotonic instability - real positive eigenvalue at .046. Increasing the exciter gain results in stability - mode at .046 is replaced by a mode at - .471, for $K_e = 5.$ but is seen to impair the damping in the oscillatory mode at 8 rads/sec. As can be seen from Figure 3.3, this mode is dominant in the rotor response and becomes unstable for $K_e = 25$.

Attention is now given to the effect of a speed sensitive power stabilizer. In this application the data again

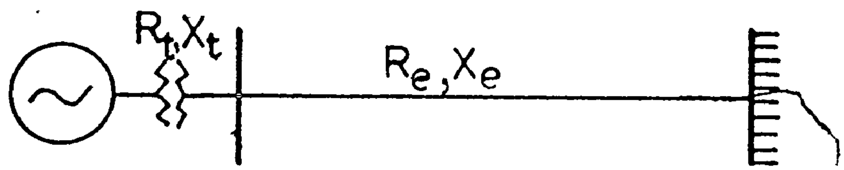


Fig. 3.2 Line diagram for single machine-infinite bus system.

generator	(Z_4)	$X_d = 1.297$, $X_q = 1.197$, $X_f = 1.216$
		$X_{kd1} = 1.226$, $X_{kq1} = 1.265$, $X_l = .159$
		$r_a = .0015$, $r_f = .00075$, $r_{kd1} = .0081$, $r_{kd2} = .0025$
exciter	(E_1)	K_e, K_q - varied
		$T_e = .002$, $Tv = .01$, $Tq = 1.41$, $T_a = 121$, $T_x = .033$
shaft	(So)	$M = 7.112$, $D = 1$
network	(N1)	$R_t = .005$, $X_t = .133$, $R_e = .03$, $X_e = .54$
loading		$P = .8$, $Q = -.6$, $e_t = 1.$, $\delta = 2.0$

(all time constants in seconds, other quantities in p.u. on machine base)

Table 3.2 Data for single machine-infinite bus system

N	K_e = 0.	K_e = 5.	K_e = 10.	K_e = 15.	K_e = 20.	K_e = .25
1	- .616 + j 377	- .616 + j 377	- .616 + j 377	- .616 + j 377	- .616 + j 377	- .616 + j 377
2	- .616 - j 377	- .616 - j 377	- ,616 - j 377	- .616 - j 377	- .616 - j 377	- .616 - j 377
3	- 100.	- 100.	- 100.	- ,100	- 99.9	- 99.9
4	- 20.9	- 20.8	- 20.7	- 20.5	- 20.4	- 20.3
5	- .407 + j 8.19	- .313 + j 8.14	- .214 + j 8.10	- .111 + j 8.07	- .005 + j 8.05	+ .101 + j 8.04
6	- .407 - j 8.19	- .313 - j 8.14	- .214 - j 8.10	- .111 - j 8.07	- .005 - j 8.05	+ .101 - j 8.04
7	- 1.37	- 1.29	- 1.11 + j .271	- 1.34 + j .460	- 1.58 + j .490	- 1.82 + j .389
8	+ .046	- .471	- 1.11 - j .271	- 1.34 - j .460	- 1.48 - j .490	- 1.82 - j .389
9	- 500.	- 500.	- 500.	- 500.	- 500.	- 499.
10	- 30.3	- 30.3	- 30.3	- 30.3	- 30.3	- 30.3
11	- .709	- .709	- .709	- .709	- .709	- .709

Table 3.3 Variation of eigenvalues with exciter gain

N	K_q = 0.	K_q = 5.	K_q = 10.	K_q = 15.	K_q = 20.	K_q = 25.
1	- .786 + j 377	- .786 + j 377	- .787 + j 377	- .787 + j 377	- .787 + j 377	- .787 + j 377
2	- .786 - j 377	- .786 - j 377	- .787 - j 377	- .787 - j 377	- .787 - j 377	- .787 - j 377
3	- 500.	- 500.	- 501	- 510	- 5 01	- 501
4	- 85.4	- 85.7	- 86.0	- 86.3	- 86.6	- 86.8
5	- 18.4 + j 17.7	- 16.8 + j 20.6	- 15.6 + j 23.4	- 14.7 + j 26.0	- 14.7 + j 28.4	- 13.5 + j 30.6
6	- 18.4 - j 17.7	- 16.8 - j 20.6	- 15.6 - j 23.4	- 14.7 - j 26.0	- 14.1 - j 28.4	- 13.5 - j 30.6
7	+ .147 + j 8.30	- .786 + j 7.76	- 1.47 + j 7.12	- 1.88 + j 6.49	- 2.16 + j 5.92	- 2.35 + j 5.43
8	+ .147 - j 8.30	- .786 - j 7.76	- 1.47 - j 7.12	- 1.88 - j 6.49	- 2.16 - j 5.92	- 2.35 - j 5.43
9	- 1.24	- 1.23	- 1.21	- 1.19	- 1.17	- 1.14
10	- 30.3	- 31.1	- 31.7	- 32.0	- 32.3	- 32.6
11	- .709	- .725	- .742	- .761	- .784	- .812

Table 3.4 Variation of eigenvalues with stabilizer gain

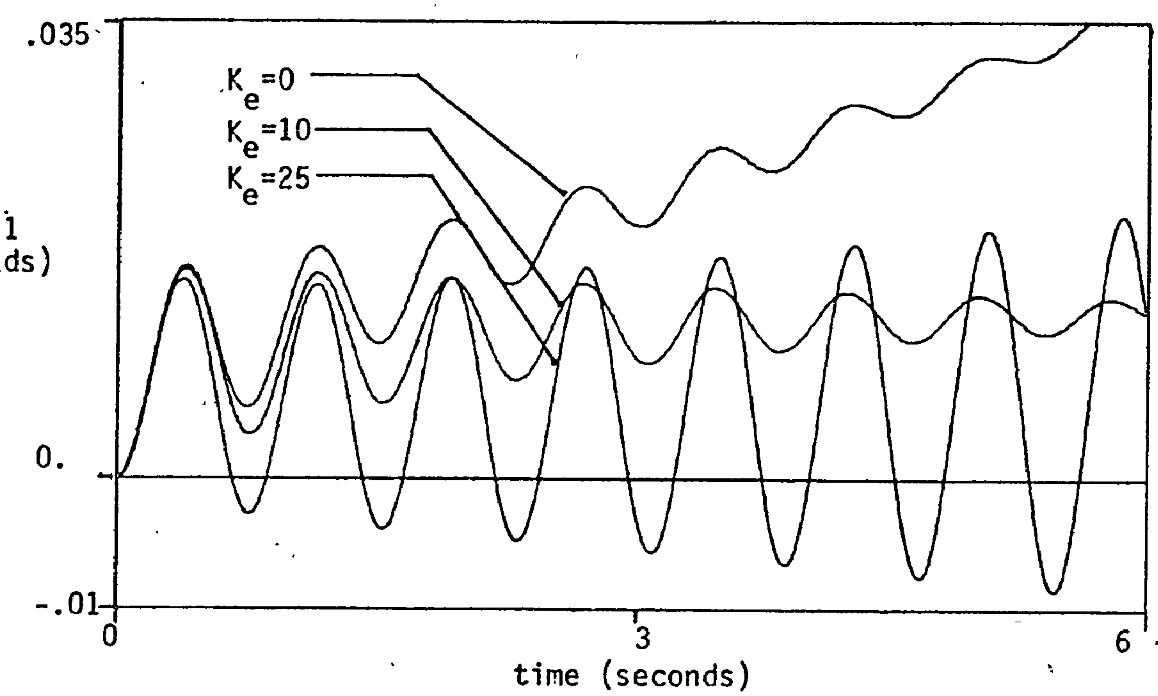
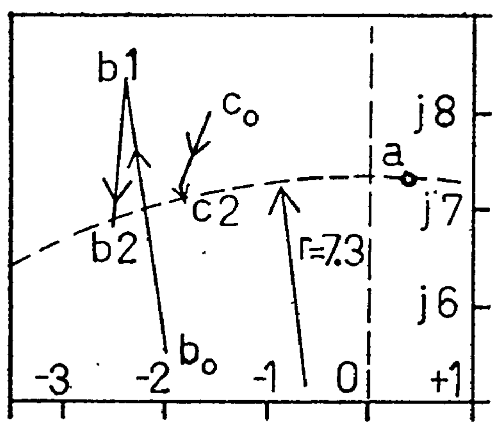


Fig. 3.3 Response of rotor following .01 pu step in shaft torque for different exciter gains



(a) K_q=0 used to define inherent natural frequency
(b) K_q=20 initial T_a= .15 final T_a = .06
(c) K_q=15 initial T_a= .01 final T_a = .06

Fig. 3.4 Use of eigenvalue sensitivities in stabilizer design

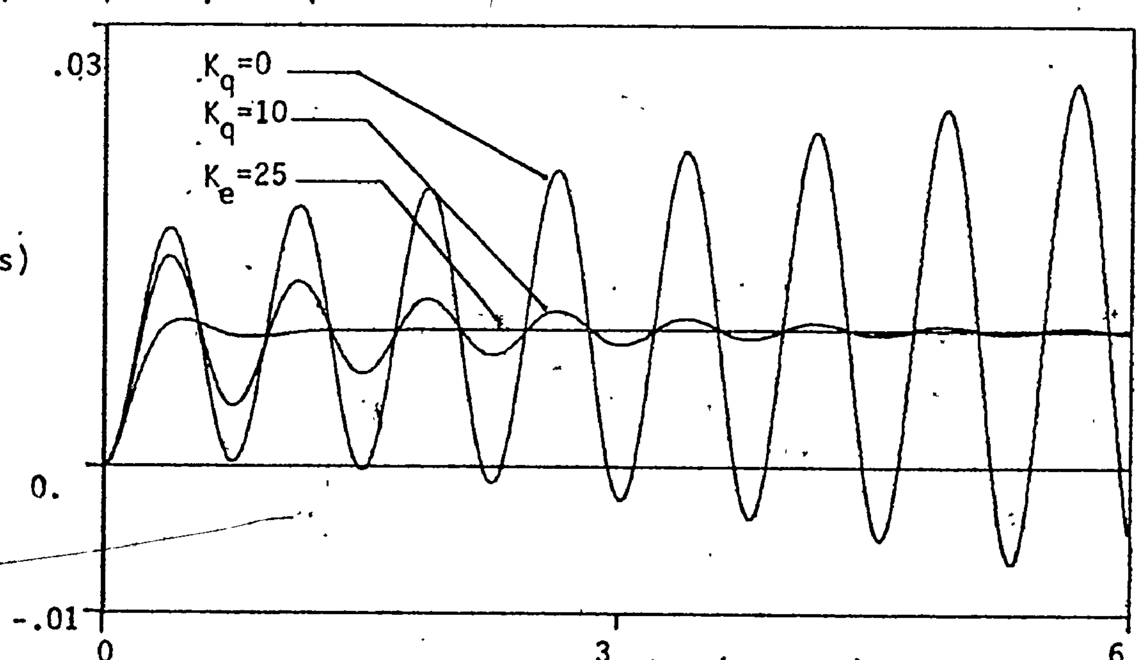


Fig. 3.5 Response of rotor following .01 pu steps in shaft torque for different stabilizer

as in Table 3.2, except rated loading at .9 pf lag, is considered. The purpose of a stabilizer is to introduce damping torque in the torque-load angle loop. The approach, suggested in [4], is to use a phase lead lag circuit in series with the exciter - the lead in which cancels the lag in the reference voltage - electric torque, transfer function. Consequently the input to the lead-lag circuit will be in phase with the electric torque produced and if a speed signal is chosen as the input, a pure damping torque results.

In Figure 3.4, a novel use of eigenvalue sensitivities is demonstrated - in which they are used to determine the lead parameter of the lead-lag circuit which just results in the correct amount of lead required to cancel the lag in the reference voltage - electric torque transfer function. An initial guess of τ_a is made and then using eigenvalue sensitivity calculations the value is adjusted until the natural frequency is that of an unstabilized system - this indicating the introduction of pure damping torque as distinct to damping and synchronizing torque. By adjusting the stabilizer gain any desired damping coefficient may be introduced.

The eigenvalue listings corresponding to a range of stabilizer gains are shown in Table 3.4. It can be noted that there are two oscillatory modes - which are referred to as the rotor and exciter (AVR) modes, respectively. The former is associated mainly with the torque - load angle loop and is the mode which has been traditionally studied. The latter mode (typically, 20 rads/sec) is associated primarily with the reference voltage - terminal voltage loop - however, as is evident in the eigenvalue listings in Table 3.4, it is sensitive to stabilizer gain. This charateristic has been observed previously [3], the authors stating that the design of a good stabilizer involved introducing sufficient damping in the rotor mode without sacrificing the stability of the exciter mode.

It can be seen from Table 3.4 that as K_q is increased from zero to 25 p.u., the damping in the rotor mode is increased from being negative to a value with coefficient of .4. It can be seen that the exciter mode becomes less damped with increased stabilizer gain. Responses of rotor angle for a step disturbance in shaft torque for three values of stabilizer gain are shown in Figure 3.5.

3.3.2 Mechanical Shaft Instability

During the commissioning of a speed sensitive power system stabilizer on a 555 MVA turbine generator at Ontario Hydro, sustained 16 hertz oscillations in the stabilizer output were noticed [5]. The cause was diagnosed as that of excitment of a shaft natural frequency by the stabilizer. The problem was solved by placing the stabilizer's speed sensor near a node of the critical shaft modes. As an added measure, a notch filter was incorporated to attenuate shaft oscillation signal in the feedback loop.

In this subsection, the application of the models developed will be outlined, in this application. Typical data for a large (500 → 700 MVA) tandem compound, single reheat turbine-generator mechanical shaft system is given in Table 3.5. As governor (mechanical-hydraulic) action did not significantly affect the stability, governor representation was omitted in the case presented (113). This is not generally the case however - El Serafi pointed out in a discussion on(113) that the action of electro-hydraulic governor with acceleration feedforward may be significant in studies involving shaft torsional stability.

Eigenvalue listings corresponding to four cases of parameter variation are shown in Table 3.6. The eigenvalues corresponding to the case of zero stabilizer gain appear in column 1. The main rotor mode at 8.2 rads/sec. is unstable while the shaft modes (easily identified as occurring at 103, 152, 193 and 276 rads/sec. - Appendix A5) are stable. In column 2, the stabilizing effect of introducing stabilizer gain is shown. (Speed pick-up is located midway between the two low pressure turbine stages). The effect of moving the location of the speed sensor to the rotor end of the shaft is shown in column 3. It can be seen that this result is a shaft instability at 193 rads/sec. The predicted response of rotor speed deviation following a line switching (resulting in an equilibrium rotor angle of .1 rad) is shown in Figure 3.6. Speed deviation was chosen rather than rotor angle, as it contains a greater proportion of shaft modes.

The sensitivity of the results to change in stabilizer design can be seen from the results in column 4 where it is seen that the instability occurs in the 103 rads/sec (16 hertz) shaft mode. This data for this case is as in Table 3.5 with the generator

loading: $P = .9$, $Q = .436$, $e_t = 1.$

Synchronous Machine (Z_4) $X_d = 1.297$, $X_q = 1.197$, $X_f = 1.216$, $X_{kd} = 1.226$

$X_{kq} = 1.265$, $X_e = .159$, $r_a = .0015$, $r_f = .00075$, $r_{kd} = .0081$, $r_{kq} = .0025$

Mechanical Shaft $M_1 = 1.71$, $M_2 = 2.28$, $M_3 = 2.310$, $M_4 = .464$, $M_5 = .248$

$D_1 = 0.$, $D_2 = .25$, $D_3 = .26$, $D_4 = .26$, $D_5 = .24$

$S_{12} = 62.3$, $S_{23} = 75.6$, $S_{34} = 48.4$, $S_{45} = 21.8$

Exciter (E_1) $K_e = 212$, $T_e = .002$, $K_q = 9.6$, $T_q = 1.41$, $T_a = .121$, $T_x = .033$

(All time constants in seconds, other quantities in per unit on machine base)

Table. 3.5 Data for system exhibiting stabilizer introduced shaft torsional instability

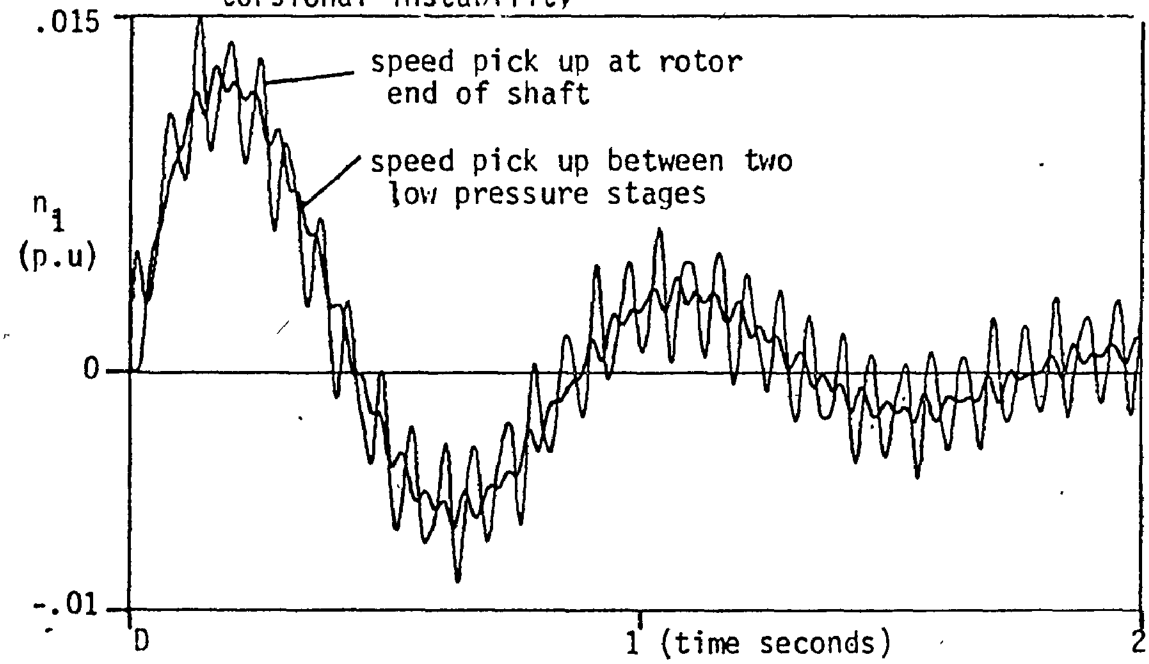


Fig. 3.6 Rotor response following line switching-stabilizer introduced shaft torsional instability

	Column 1	Column 2	Column 3	Column 4
1	- 500.	- 500.	- 502	- 501
2	- 18.8 + j 377	- 18.8 + j 377	- 18.8 + j 377	- 18.8 + j 377
3	- 18.8 - j 377	- 18.8 - j 377	- 18.8 - j 377	- 18.8 - j 377
4	- .301 + j 275	- .301 + j 276	-	- .302 + j 277
5	- .301 - j 275	- .301 - j 276	- .301 - j 276	- .302 - j 277
6	- .085 + j 193	- .116 + j 193	+ .022 + j 193	- .129 + j 193
7	- .086 - J 193	- .116 - j 193	+ .022 - j 193	- .129 - j 193
8	- .267 + j 152	- .351 + j 152	- .146 + j 152	- .245 + j 152
9	- .267 - j 152	- .351 - j 152	- .146 - j 152	- .245 - j 152
10	- .144 + j 103	- .149 + j 103	- .071 + j 105	+ .034 + j 164
11	- .144 - j 103	- .149 - j 103	- .071 - j 105	+ .034 - j 104
12	- 84.9	- 85.3	- 86.1	- 86.9
13	- 18.6 + j 18.1	- 16.0 + j 18.1	- 15.0 + j 23.2	- 12.7 + j 21.2
14	- 18.6 - j 18.1	- 16.0 - j 24.0	- 15.0 - j 23.2	- 12.7 - j 21.2
15	+ .097 + j 8.20	- 1.45 + j 7.00	- 1.44 + j 7.03	- 4.10 + j 7.38
16	+ .097 - j 8.20	- 1.45 - j 7.00	- 1.44 - j 7.03	- 4.10 - j 7.38
17	- 1.24	- 1.21	- 1.21	- 1.09
18	- 30.3	- 31.6	- 31.8	- 31.4
19	- .709	- .742	- .742	- .833

exception that $K_q = 23.6$, $\tau_a = .141$.

3.3.3 Torsional Subsynchronous Resonance

As mentioned in Section 1.4, there are two possible ways in which subsynchronous resonance may manifest itself. The first case is subsynchronous resonance which exists in the absence of shaft torsional interaction. The resonance in this case involves large amplitude oscillations in only electrical variables. In this case, the subsynchronous currents (at frequency f_N) entering the generator terminals, produce subsynchronous terminal voltage components which sustain these currents. The self excitation is a result of the effective negative resistance due to induction-generator action [6].

Self excitation may also exist even when the subsynchronous terminal voltage components due to the subsynchronous currents, cannot sustain these currents. In this case sufficient additional subsynchronous voltage components may be induced due to rotor oscillation. The amplitude of these oscillations can have sufficient magnitude when a shaft natural frequency is sufficiently close to the subsynchronous forcing torque. (The subsynchronous forcing torque will have a frequency $60 - f_N$ due to the interaction of the rotor flux (travelling at approximately synchronous speed) with the mmf due to subsynchronous currents). Subsynchronous resonance in this case involves large amplitude oscillations in both electrical and mechanical variables.

The modelling concepts developed in the previous chapter can be applied in the study of both aspects of sub-synchronous resonance. As the problem of torsional sub-synchronous resonance is far more common and certainly more serious in consequence (Section 1.4), results of typical stability prediction for this case will be presented.

The line diagram is again as shown in Figure 3.2 except the long transmission line is compensated by a series capacitor. The parameters of the system are shown in Table 3.7. Note that exciter, stabilizer and governor representation has been omitted in the interests of analysing the fundamental interactions. Results for cases where these effects are considered can be found in Section 4.6 and Section 5.4.3.

The transmission line parameters were chosen so that the effective reactance of the transmission line is .47 p.u. and the resonant frequency of the compensated line is 250 rads/sec. As can be seen from Table 3.8, the 103 rad/sec. shaft mode is unstable. This instability can be attributed to interaction between subsynchronous currents at 124 rads/sec. and the mechanical shaft system at its 103 rad/sec. natural frequency. Electrical subsynchronous resonance, on the other hand, would result in an instability in the electrical mode at 124 rads/sec. and does not involve appreciable change in the damping of the mechanical shaft modes.

Predicted time responses for rotor angle and speed deviation following a line switching (resulting in a .1 rad change in equilibrium rotor angle) are shown in Figure 3.7.

<u>generator</u>

loading $P = .9,\ Q = .436,\ e_t = 1$

Synchronous Machine (Z_4) $X_d = 1.297,\ X_q = 1.197\ x_f = 1.126,\ X_{kd} = 1.226$

$X_{kd} = 1.265,\ \ x_e = .159$

Mechanical Shaft (S_2) $M_1 = 1.71,\ M_2 = 2.38,\ M_3 = 231,\ M_4 = .46,\ M_5 = .25$

$D_1 = 0,\ D_2 = .25,\ D_3 = 26,\ D_4 = .26,\ D_5 = .24$

$S_{12} = 62,\ S_{23} = 75,\ S_{34} = 48,\ S_{45} = 22$

Exciter and Turbine Governor: Representation neglected

Network: $X_e = 1.10,\ X_c = .63,\ R_e = .11$

Time constants in seconds, Angles in radians, other quantities in pu on machine base

Table 3.7 Data for study of tornsional subsynchronous resonance

1	+ .894 ± j 103
2	- .107 ± j 190
3	- .1 ± j 151
4	- .228 ± j 7.46
5	- .302 ± j 276
6	- 16.6 ± j 124
7	- 16.9 ± j 634
8	- .263
9	- 21.2
10	- 1301.

Table 3.8 Eigenvalues for system exhibiting torsional subsychronous resonance

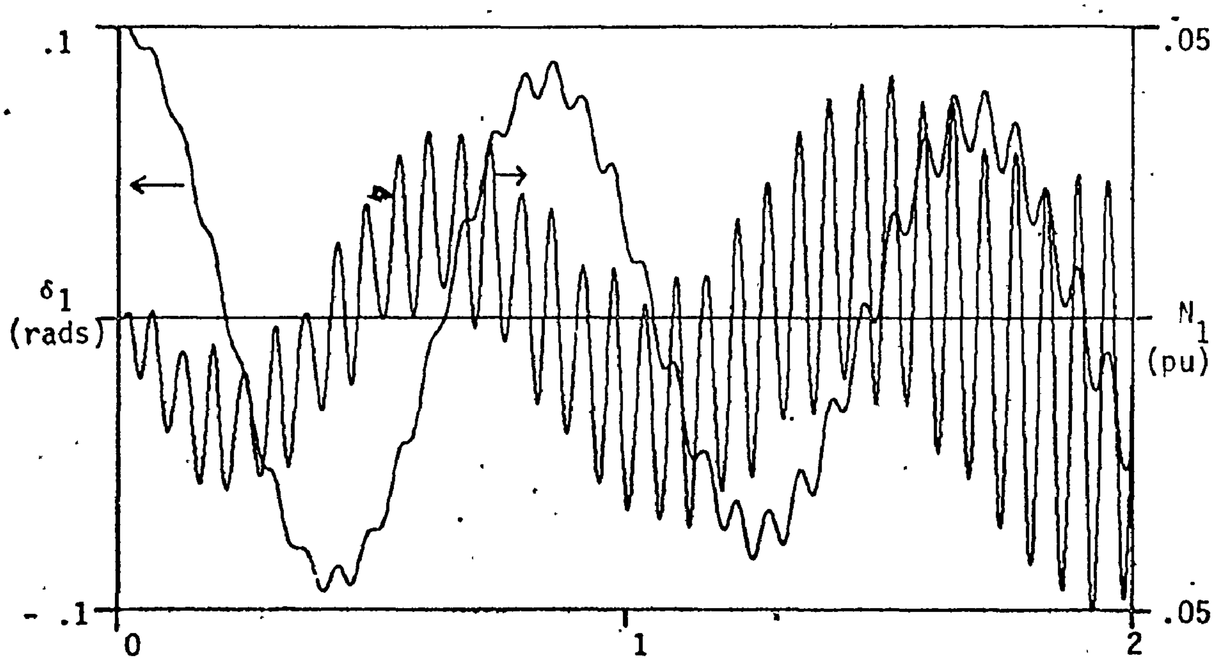


Fig. 3.7 Rotor response following line switching-torsional

3.3.4 Effect of Asynchronous Motor Load on Stability

In this subsection, the application of the models of Chapter 2 in the analysis of a power system comprising a significant proportion of asynchronous motor load is considered. The configuration analysed is shown in Figure 3.8 and has been considered previously in [9].

In this study a more detailed representation is considered: specifically stator - network transients are considered and results for cases including and excluding exciter action, are presented.

The local load is assumed to total 600 MW. In case (i) 500 MW of the load is assumed asynchronous and 100 MW resistive. In case (ii), the propertions are interchanged. A centrifugal pump type load characteristic (torque α speed^2) was chosen for the motor - of course under the assumption of only 'small' disturbances such a load characteristic will be reflected as an 'equivalent' damper. The system data is shown in Table 3.9.

The eigenvalue listings corresponding to the case of zero exciter gain are shown in Table 3.10(a) and predicted rotor responses for .01 p.u. step inputs in shaft torque are shown in Figure 3.9(a). As expected, the responses are characterized by slow monotonic rises indicating poor steady-state synchronizing coefficients in the absence of exciter gain. The results also show that the asynchronous load has a stabilizing effect on rotor oscillations. This result is in agreement with the findings of de Mello et al. [9].

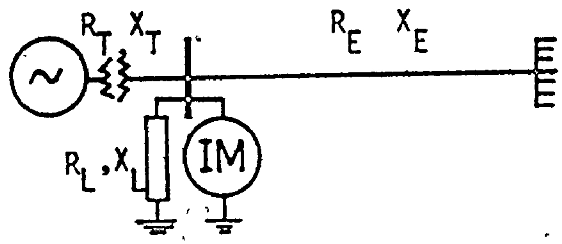


Fig. 3.8 Line diagram of single generator with local asynchronous motor load system

<u>generating station</u> (2 x 555 MVA)

loading P = .9, Q =.436, $e_t = 1$

synchronous machine (Z_4) $X_d = 1.297$, $X_q = 1.197$, $x_f = 1.216$, $X_{kd} = 1.226$

$X_{kq} = 1.265$, $X_e = .159$, $r_a = .00075$, $r_{kd} = .0081$, $r_{kq} = .0025$

Mechanical Shaft (So) $M_1 = 7.116$, $D_1 = 1$

Exciter (E_i) $K_e = 212$, $T_e = .002$, $K_q = 0.$

<u>Synchronous Motor Load</u> (500 MW)

Asynchronous Motor (A_1) $X_s = 2.32$, $X_r = 2.33$, $X_m = 2.20$

Mechanical load M = 1.5 secs, D = 1. pu

<u>Network</u> $R_T = .01$, $R_E = .05$, $X_E = .4$, load PF = .9

Time constants in seconds, other quantities in per unit on generator base

Table 3.9 Data for single generator with local asynchronous motor load system

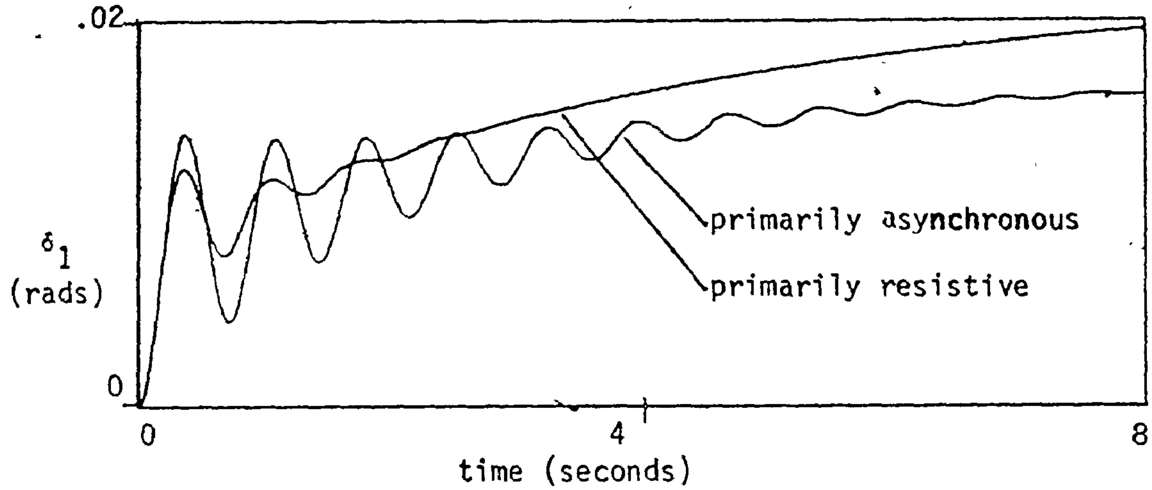
n	500 MW (asych) 100 MW (Resistive)	100 MW (asynch) 500 MW (Resistive
1	-.213	- .250
2	-.709	- .709
3	- 1.39	- 1.44
4	- 8.49	- 17.4
5	- 21.4	- 21.3
6	- 100.	- 100.
7	- 500.	- 500.
8	- 1.57±j8.24	- .461 ±j7.73
9	- 6.50±j22.4	- 9.56 ±j27.6
10	- 18.6±j377	- 18.6 ±j376
11	- 40.3±j377	- 32.1 ±j377

(a) Zero exciter gain

n	500 MW (Asynch) 100 MW (Resistive)	100 MW (Asynch) 100 MW (Resistive
1	-.709	-.709
2	-1.24	-1.24
3	-8.60	-17.4
4	-30.3	-30.3
5	-87.4	-83.2
6	-500.	-502
7	-.776±j8.85	-.235 ±j7.85
8	-4.65±j23.8	-8.96 ±j27.8
9	-19.6±j15.6	-19.3 ±j18.8
10	-18.6±j377	-18.6 ±j376
11	-40.3±j377	-32.2 ±j377

(b) Exciter gain = 212 pu

Table 3.10 Eigenvalues for single generator with local asynchronous motor load system



(a) Without exciter action

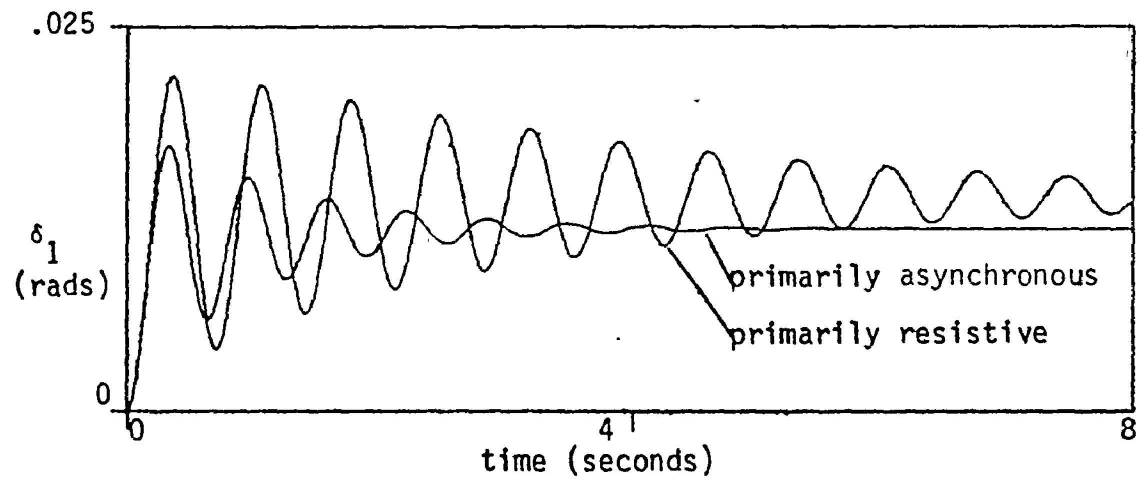


Fig. 3.9 Rotor response following .01 step in shaft torque for different local load characteristics

Eigenvalue listings for the case with excitation is shown in Table 3.10(b) and the rotor angle time response again following a .01 p.u. step disturbance in shaft torque is shown in Figure 3.9(b). Improved damping for the case of primarily asynchronous motor load over primarily resistive is observed.

Caution must be observed in attempting to generalize these results on the effects of asynchronous motor loads on system stability. As pointed out in [9] the action of the exciter is highly sensitive to load and parameter value variation and can dominate the results. The effect of any power system stabilizers must also, of course, be included. The models developed in Chapter 2 can be used in more exhaustive studies and consequently lead to general guidelines and better insight into this important question.

3.4 Four Machine Hydroelectric System

In this section the modelling concepts of the previous chapter are applied to a sample multimachine system. The system chosen is the Moose River section of Ontario Hydro's system. This system which comprises four generating stations with a total capacity of 500 MW has been analysed using a 43rd order model by Leffen [72]. In the study presented here, all machines within each station will be assumed to be equally loaded and to have identical excitation and governing schemes. This allows 'lumping' of the generators within each station, into a single 'equivalent' - a usual practice in such an analysis. For parameter values pertaining to the shaft damper windings, and more detailed governor representation (not usually considered) which were not available in [72], typical values were assumed [115].

3.4.1 Development of the System Model

A line diagram showing the system appears in Figure 3.10 and the loading and parameter values are listed in Table 3.11.

For convenience the system model development is divided into four steps:

(a) Load-flow: The first step in the development is the solution of the load flow equations to determine the initial bus voltages and load angles which subsequently lead to values of flux linkage, rotor angles, etc. Though Gauss-Seidel methods have been largely superceded by more efficient and reliable Newton-Raphson and more recently, decoupled load flow methods [121], it was used in the present study because of its simplicity.

The network data is listed in Table 3.12 and the bus incidence matrix appears in Figure 3.11. The results of the load flow are listed in Table 3.13.

(b) Equations for Generating Units: The equations for each generating unit in isolation are developed by combining the subsystem models for the synchronous machine, exciter, governor and shaft representation. As a specific illustration, the structure of the equations for generating unit #1 is shown in Figure 3.12 and the eigenvalues for this unit with

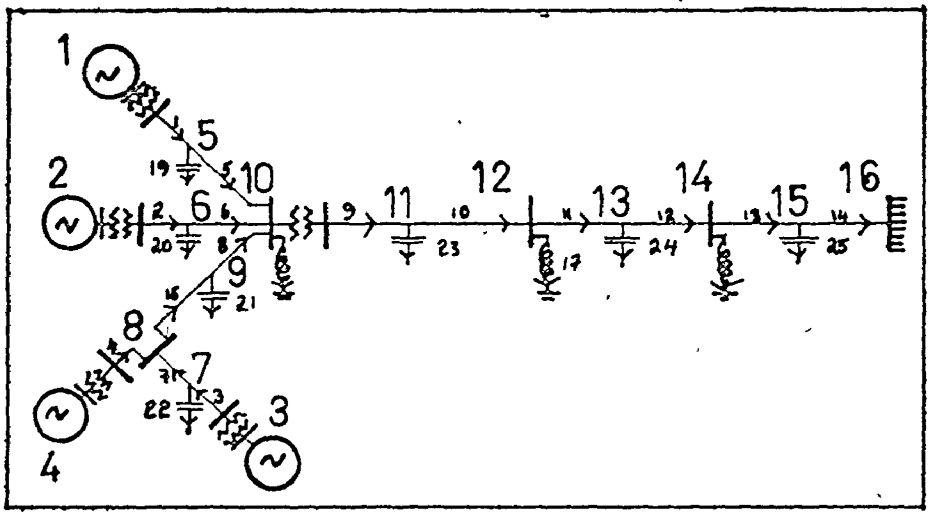


Fig. 3.10 Line diagram of four machine hydroelectric system

Link	from bus	to bus
1	1	5
2	2	6
3	3	7
4	4	8
5	5	10
6	6	10
7	7	8
8	9	10
9	10	11
10	11	12
11	12	13
12	13	14
13	14	15
14	15	16
15	8	9
16	10	11
17	12	12
18	14	14
19	5	5
20	6	6
21	7	7
22	9	9
23	11	11
24	13	13
25	15	15

Fig. 3.11 Bus incidence matrix for four machine

Link	60 hertz impedance (13.8kv,100MVA base)
1	.0033+j.0511
2	.0057+j.1078
3	.0039+j.0909
4	.0033+j.0844
5	.0016+j.0167
6	.0024+j.0235
7	.0007+j.0066
8	.0019+j.0189
9	.0007+j.0097
10	.0007+j.0097
11	.0011+j.0141
12	.0011+j.0141
13	.0017+j.0219
14	.0019+j.0395
15	.0019+j.0189
16	.0026+j.9709
17	.0500+j2.000
18	.0162+j1.600
19	-j14.48
20	-j9.261
21	-j33.90
22	-j11.50
23	-j5.565
24	-j3.220
25	-j2.200

Table 3.12 Network data for four machine system

Bus	voltage (pu)	angle (rads)
1	1.014	0.
2	1.015	.807
3	.996	.860
4	1.004	.926
5	1.002	.897
6	.954	.723
7	.946	.724
8	.968	.804
9	.965	.795
10	.949	.745
11	.934	.694
12	.912	.558
13	.906	.496
14	.909	.405
15	.886	.315
16	.983	.187

(voltage base = 13.8kV

Table 3.13 Results of load flow for four machine system.

Station #1 (4 x 46 MVA)

loading: $P = 1.67$, $Q = 1.16$; $e_t = 1.01$, $\delta_b = .807$

Synchronous Machine (Z_1) $X_d = .370$, $X_q = .239$, $X_f = .425$, $X_{d1} = .379$
$X = .237$ $X_e = .038$

Mechanical Shaft (S_1) $M_1 = 6.11$, $M_2 = 9.16$, $D_1 = 0.$, $D_2 = 1$, $S_{12} = 40.$

Exciter (E_1) $K_e = 300.$, $\tau_e = .005$, $K_q = 10.2$, $\tau_q = 1.16$, $\tau_v = .01$

Turbine-Governor (G_3) $\tau_\omega = 1.5$, $\tau_g = .2$, $\tau_p = .04$, $\tau_r = 6.0$
$\delta = .45$, $\sigma = .03$

Station # 2 (2 x 68 MVA)

Loading $P = 1.210$, $Q = .469$, $e_t = .996$, $\delta_b = .860$

Synchronous Machine (Z_1) $X_d = .542$, $X_q = .357$, $X_f = .611$, $X_{d1} = .559$
$X_{q1} = .454$, $X_e = .067$

Mechanical Shaft (S_1) $M_1 = 3.07$, $M_2 = 4.60$, $D_1 = 0$, $D_2 = 1$, $S_{12} = 20$

Exciter (E_1) $K_e = 165$, $\tau_e = .005$, $K_q = 23.9$, $\tau_q = 1.6$, $\tau_v = .01$

Turbine-Governor (G_3) $\tau_\omega = 1.75$, $\tau_g = .2$, $\tau_p = .04$, $\tau_r = 7.0$
$\delta = .45$, $\sigma = .03$

Station #3 (s x 66 MVA)

loading: $P = 1.316$, $Q = .424$, $e_t = 1.00$, $\delta_b = .926$

Synchronous Machine (Z_1) $X_d = .522$, $X_q = .343$, $X_f = .592$, $X_{d1} = .576$
$X_{q1} = 414$, $X_e = .093$

Mechanical Shaft (S_1) $M_1 = 3.16$, $M_2 = 4.74$, $D_1 = 0$, $D_2 = 1$, $S_{12} = 20.$

Exciter (E_1) $K_e = 240.$, $\tau_e = .055$, $K_q = 23.0$, $\tau_q = 1.6$, $\tau_v = .01$

Turbine-Governor (G_3) $\tau_\omega = 1.6$, $\tau_g = .2$, $\tau_p = .04$, $\tau_r = 6.4$

$\delta = .45$ $\sigma = .03$

Station #4 (2 x 64 MVA)

loading: $P = 1.19$, $Q = .451$, $e_t = 1.00$, $\delta_b = .897$

Synchronous Machine (Z_1) $X_d = .497$, $X_q = .314$, $X_f = .504$, $X_{d1} = .532$

Mechanical Shaft (S_1) $M_1 = 2.93$, $M_2 = 4.40$, $D_1 = 0.$, $D_2 = 1$, $S_{12} = 20$

Exciter (E_1) $K_e = 117$, $\tau_e = .005$, $q = 19.3$, $\tau_q = 1.16$, $\tau_v = .01$

Turbine-Governor (G_3) $\tau_\omega = 1.6$, $\tau_g = .2$, $\tau_p = .04$, $\tau_r = 6.4$

$\delta = .50$ $\sigma = .03$

All time constants in seconds, angles in rads, other quantities in per unit on 13.8 kv, 100 M V A base

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Table 3.11 Data for four machine hydrolectric system

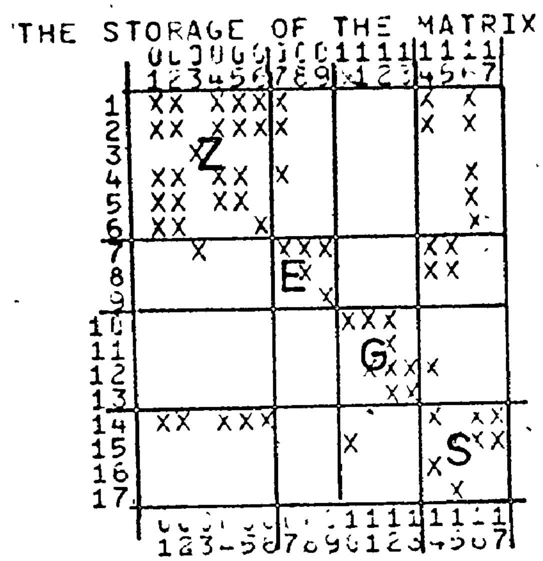


Fig. 3.12 Structure of coefficient matrix for generator #1

1	- 468.
2	- 2.75 ± j 377
3	- 208.
4	- 116.
5	- .093 ± j 64.6
6	- 10.9 ± j 30.7
7	- 1.60 ± j 9.60
8	- 22.3
9	- 2.93
10	- 1.32
11	- .638
12	- .010

Table 3.14 Eigenvalues for generator #1 connected to an infinite bus through reactance of .1 pu

a tie line reactance of .4 p.u. are listed in Table 3.14 for reference.

The equations for the system of generating units are developed in terms of the constituent matrices: A_M, B_M, Γ_M, K_M, C_M, E_M and D_M - the order of which are shown in Figure 3.13(a).

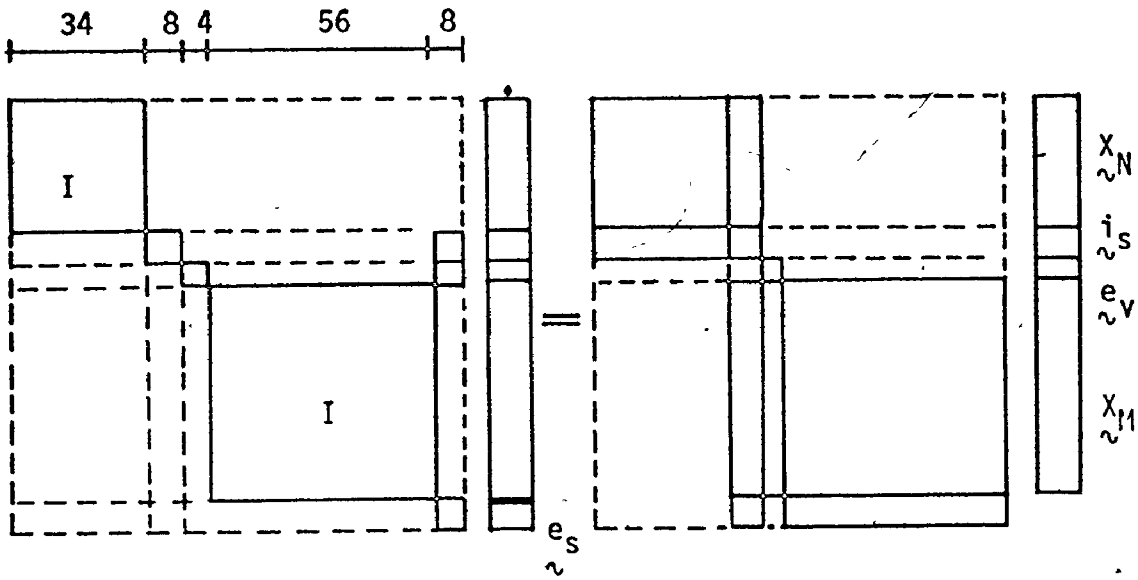
(c) Network Equations: The dynamic network equations are set up as outlined in Section 2.4 . The incidence matrices and tree graph for the network have already been presented in Section 2.4. For the network we have $N = 4$ (machines), $N_\ell = 14$ (# fundamental loops) and $N_c = 7$.

As outlined in Section 2.4, the single phase network equations are first set up and subsequently the equations are extended to that of a two phase system. Finally, the states are reordered and partitioned to yield the network matrices A_N, B_N, C_N, D_N, E_N - the order of which are again shown in Figure 3.13(a).

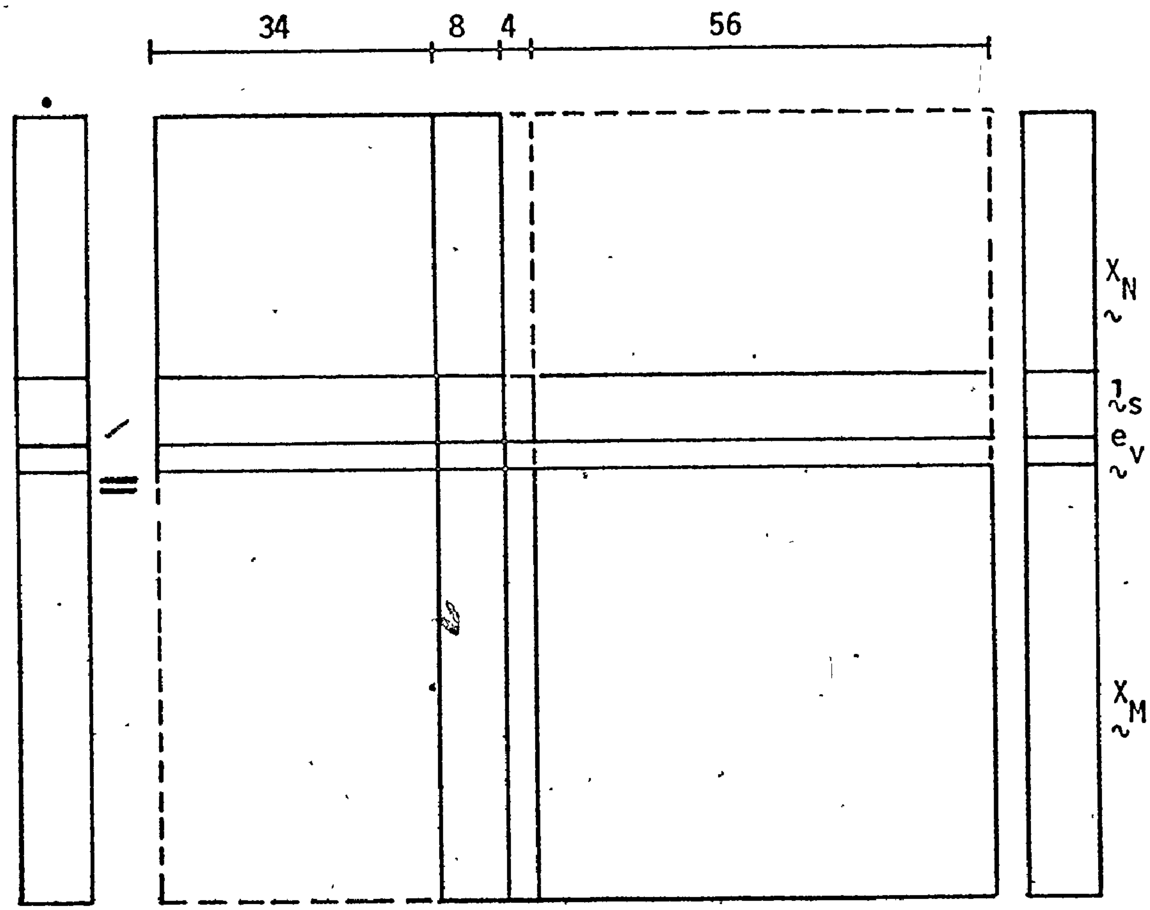
(d) Overall System Equations: The equations describing the interaction of the system of generating units and network systems is shown in Figure 3.13(a). As described in Section 2.5, this can be reduced to standard state space form by the inversion of an 8 x 8 (2N x 2N) matrix. The structure of the overall matrix is shown in Figure 3.13(b) and a photograph of a computer printout showing the matrix structure and distribution of elements within the submatrices, is shown in Figure 3.14.

3.4.2 Eigenvalue Analysis

The eigenvalues for the system are listed in Table 3.15.



(a) Unreduced equations



(b) Reduced equations

Fig. 3.13 Structure of unreduced and reduced state space equations for four machine hydroelectric system

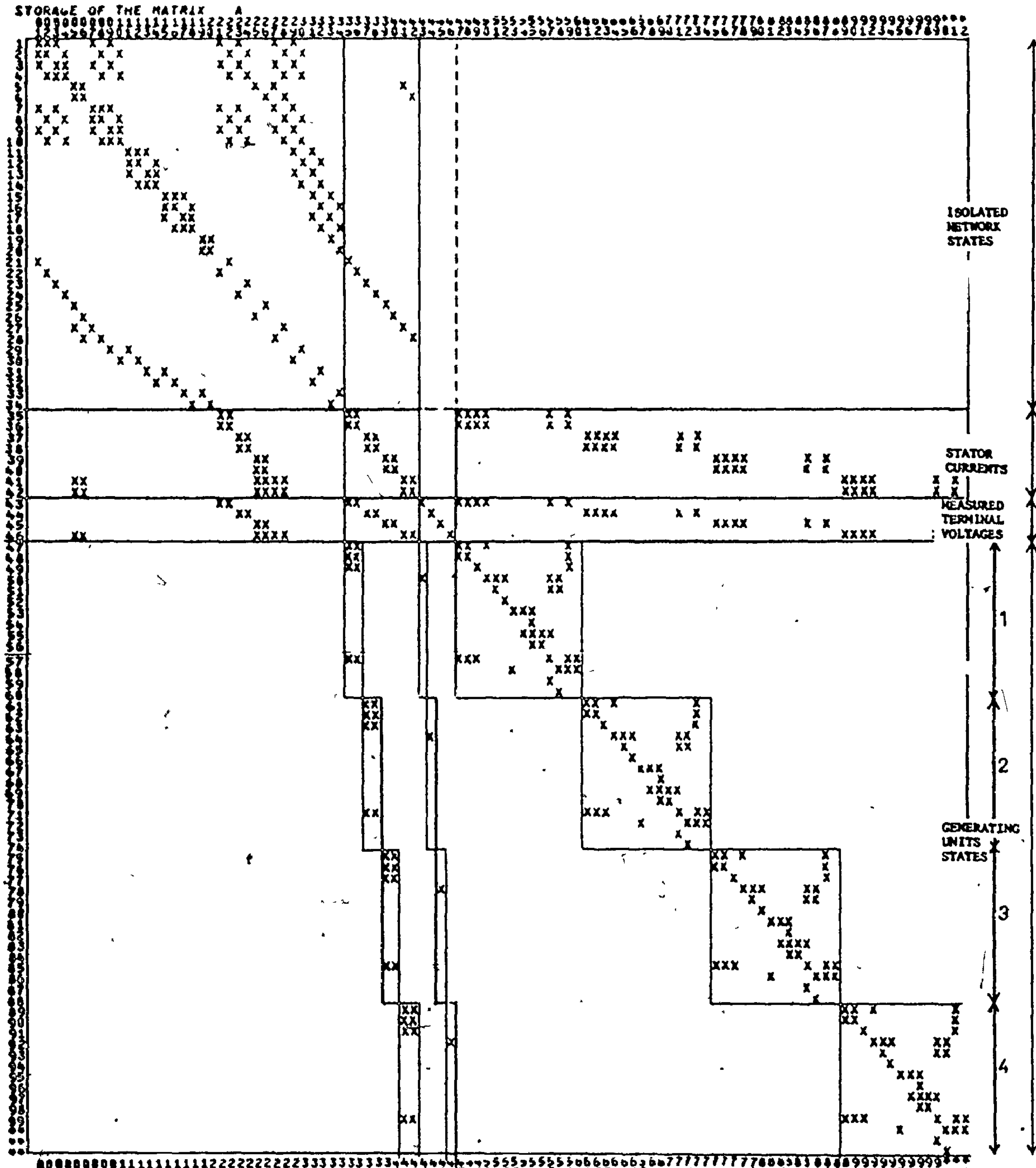


Fig. 3.14 Computer print out showing matrix structure for four machine system.

1	- .008	18	- 1.64±j 377	35	-12.8±j 3640	52	-22.2
2	- .008	19	- 2.37±j 15.1	36	-13.0±j 5960	53	-22.3
3	- .009	20	- 2.93	37	-13.1±j 5210	54	-23.2
4	- .010	21	- 2.97	38	-13.2±j 9290	55	-35.6
5	- .135±j 64.2	22	- 3.05	39	-14.3±j 8540	56	-42.4
6	.264±j 65.3	23	- 3.34	40	-14.3±j 377	57	-44.9
7	- .288±j 67.3	24	- 4.25±j 377	41	-16.0±j 377	58	-50.5
8	- .395±j 65.9	25	- 6.56	42	-16.1±j 32.8	59	-76.1
9	- .642	26	- 6.85±j 18.3	43	-16.9±j 8520	60	-84.3
10	- .660	27	- 7.19±j 10.5	44	-17.5±j 11300	61	-87.5
11	- .672	28	- 8.67±j 377	45	-17.6±j 10500	62	-90.5
12	- .675	29	- 9.65±j 377	46	-18.0±j 7770	63	-205
13	- 1.12	30	- 10.7±j 2510	47	-18.3±j 17900	64	-208
14	- 1.22	31	- 10.9±j 18.9	48	-18.3±j 17200	65	-209
15	- 1.23	32	- 11.9±j 1760	49	-20.9±j 377	66	-213
16	- 1.32	33	- 12.5±j 4390	50	-22.0		
17	- 1.47±j 6.44	34	- 12.8±j 38.6	51	-22.1		

Table 3.15 Eigenvalues for four machine system

N	base case	zero stabilizer gain	zero exciter gain
1	- .642	- .625	- .625
2	- .660	- .625	- .625
3	- .672	- .625	- .625
4	- .674	- .625	- .625
5	- 1.52±j 6.60	+ .105 + j6.55	- .223+j5-83
6	- 1.52-j 6.60	+ .105 - j6.55	- .223-j5.83
7	- 2.14 j 15.7	- .243 + j11.9	- .588+j11.9
8	- 2.14-j 15.7	- .243 - j11.9	- .558-j11.9
9	- 6.16 j 19.3	- .272 + j11.0	- .655+j10.8
10	- 6.16 j 19.3	- .272 - j11.0	- .655-j10.8
11	- 8.48 j 10.6	- .466 + j10.6	- .690+j10.4
12	- 8.48-j 10.6	- .446 - j10.6	- .690-j10.4
13	- 8.85 j 18.9	- 17.0 + j4.46	- 25.2
14	- 8.85-j 18.9	- 17.0 - j4.46	- 27.6
15	- 13.0 j 2.88	- 17.3 + j32.4	- 29.3
16	- 13.0-j 2.88	- 17.3 - j32.4	- 31.6
17	- 15.7 j 32.4	- 19.0 + j16.4	- 35.5
18	- 15.7-j 32.4	- 19.0 - j16.4	- 42.5
19	- 6.56	- 19.4 + j32.4	- 45.1
20	- 23.7	- 19.4 - j32.4	- 50.6
21	- 35.8	- 35.7	- .202
22	- 42.5	- 42.5	- .421
23	- 45.0	- 45.0	- .433
24	- 51.0	- 51.1	- .480
25	- 76.8	- 75.9	- 100.
26	- 85.0	- 84.5	- 100.
27	- 88,5	- 88.3	- 100,
28	- 90.9	- 90.8	- 200.
29	- 205.	- 205	- 200.
30	- 208	- 207	- 200.
31	- 210	- 209	- 200.
32	- 213.	- 212	- 200.

Table 3.16 Eigenvalues for four machine system neglecting governor, shaft and network-stator

The eigenvalues associated with stator-network transients and shaft modes have been grouped. Those corresponding to stator-network transients are easily identified as they appear approximately either as 60 hertz or $\pm\ |60 \pm f_N|$ (where f_N is a natural frequency of the network - Section 2.4.1). The modes associated with the mechanical shaft system are readily recognized since the frequency can be approximately calculated:

$$w_n = \sqrt{\frac{w_o\ S_s(M_1+M_2)}{M_1 M_2}}$$

Consequently, using the data in Table 3.11, the shaft natural frequencies are determined to be in the range 60 → 70 rads/sec.

The overall system is predicted to be stable - this is reassuring since we are analysing an actual operating system. There is no instability in the mechanical shaft or stator-network modes. The possibility of shaft instability introduction by the power stabilizers has been eliminated by choosing the node of the shaft mode shape as the location of the speed sensor. Torsional subsynchronous resonance is unlikely in such a system because of the absence of series compensation. Electrical subsynchronous resonance have been reported in long uncompensated transmission systems [89]. As line charging capacitance representation in this study, such instability would be evident in the results, if present.

It is common practice in stability studies on electrical machine dynamics to omit governor action. The validity of this practice and also the validity of neglecting shaft and stator-network transients can be seen from the results in Table 3.16 where it is seen that there is little loss in accuracy in the remaining modes (specifically the oscillatory modes reflecting rotor oscillation). The mode at 6.4 rads/sec. represents main system swing against the infinite bus and the three higher frequency modes reflect inter-generating unit swings (Section 3.4.3). For comparative purposes, the eigenvalue listings corresponding to the cases of no stabilizer and no exciter gains respectively, are shown in columns 2 and 3 of Table 3.16. Two important charcteristics which have already been observed in single machine studies, are evident. The first is the poor damping associated with static excitation systems. The second is the large synchronizing torque coefficient (as reflected by the small magnitude of the real eigenvalues in column 3). Predicted time responses for a step disturbance in prime mover torque in machine #1 are shown in Figure 3.15 to complement the information of the eigenvalue listings.

3.4.3 Eigenvalue Sensitivities

As there is no evidence of subsynchronous resonance in the results of the complete system model - see Table 3.15, it was decided to omit stator-network transients from the representation. In the first column of Table 3.17 are listed the 56 eigenvalues for the reduced system. It can be seen that there is little loss of accuracy in the remaining modes.

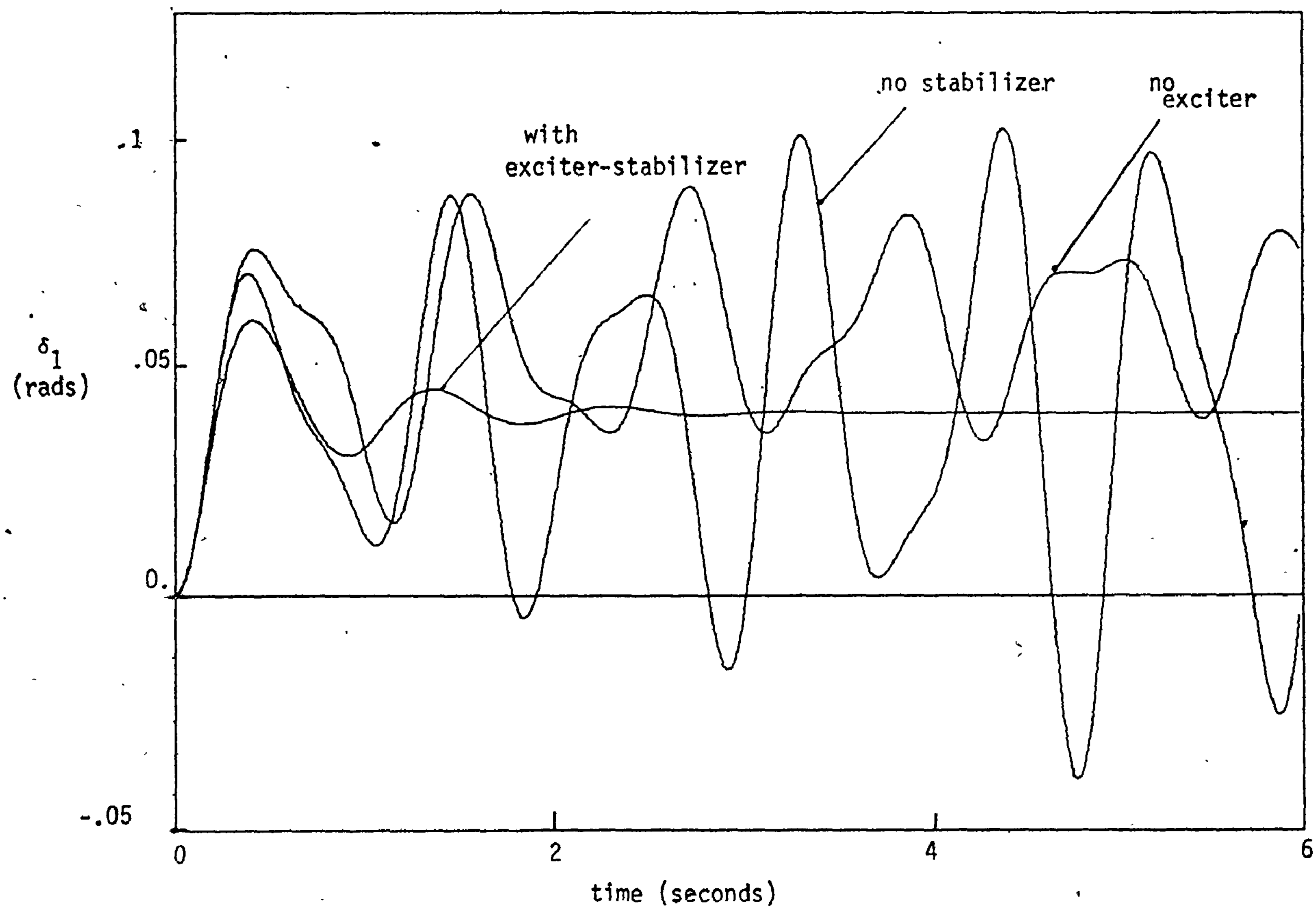


Fig. 3.15 Rotor angle response following 1 pu step in mechanical power to generator #1

Eigenvalue Sensitivities for Four Machine Hydro System								
n	Eigenvalue (1)	M (2)	K_q (3)	z (4)	K_e (5)	τ_e (6)	τ_r (7)	n
1	-.008						+.008	1
2	-.008						+.008	2
3	-.009						+.008	3
4	-.010						+.009	4
5	-.163 ± j64.2	+.138 ∓ j32.2	+.022 ± j.035	+.824 ± j1.15	-.007 ∓ j.068	+.003 ± j.043		5
6	-.289 ± j65.3	+.148 ∓ j32.8	+.032 ± j.069	+.790 ± j1.13	+.001 ± j.029	+.042 ± j.066		6
7	-.312 ± j67.3	+.167 ∓ j33.8	+.025 ± j.034	+.483 ± j.515	+.014 ± j.018	+.029 ± j.028		7
8	-.427 ± j65.9	+.212 ∓ j33.1	+.032 ± j.056	+.773 ± j.866	+.004 ± j.026	+.011 ± j.052		8
9	-.642		-.018					9
10	-.660		-.040		+.001	-.002		10
11	-.672		-.055		+.001	-.004	+.001	11
12	-.675		-.050		+.001	-.004	+.001	12
13	-1.12	-.001	+.009			+.004	-.036	13
14	-1.22	-.001	+.009			+.004	-.045	14
15	-1.23	-.001	+.005			+.003	-.025	15
16	-1.32	-.001	+.002			+.001	-.029	16
17	-1.48 ± j6.46	+1.57 ∓ j3.36	-1.61 ∓ j.011	+.017 ± j.008	-.116 ∓ j.225	-1.59 ± j.141	+.037 ± j.008	17
18	-2.13 ± j15.3	-.480 ∓ j8.10	-.455 ± j3.64	+.086 ∓ j.194	-2.51 ± j3.62	-.038 ± j3.85	+.002 ∓ j.007	18
19	-2.94	+.008	-.020	+.001	+.002	-.015	-2.94	19
20	-2.97	+.016	-.091	+.003	+.011	-.071	-3.02	20
21	-3.05	+.042	-.128	+.002	+.015	-.102	-3.04	21
22	-3.34	+.034	-.104	+.003	+.026	-.084	-3.48	22
23	-6.26 ± j18.7	-3.78 ∓ j12.2	+.937 ± j9.27	+.563 ∓ j.790	-8.43 ± j7.81	+1.78 ± j10.3	-.005 ∓ j.002	23
24	-6.46	-1.19	+5.21	+.039	-5.06	+4.54	+.188	24
25	-7.79 ± j10.6	+17.2 ± j1.32	-13.4 ∓ j10.8	-.281 ± j.686	+11.8 ± j3.91	-13.2 ± j10.8	+.029 ∓ j.045	25
26	-9.67 ± j18.6	-19.5 ∓ j4.77	+16.1 ± j7.92	-.167 ∓ j2.09	-16.5 ± j24.7	+17.9 ± j10.1	-.001 ± j.008	26
27	-12.9 ± j3.08	+5.30 ∓ j22.1	-1.68 ± j34.9	-.672 ± j1.20	+6.67 ± j26.7	-1.06 ± j31.6		27
28	-15.7 ± j32.5	-1.58 ± j.134	+1.69 ± j.027	-.240 ± j.595	-1.49 ∓ j22.6	+5.79 ∓ j1.41		28
29	-22.0	-.179	+.195	+.035	-.444	+1.34	+3.40	29
30	-22.1	-.526	+.413	+.012	-.245	+.371	+3.07	30
31	-22.3	-.061	+.026	+.002	-.045	+.020	+2.93	31
32	-22.3	-.342	+.556	+.104	-1.533	+.348	+3.47	32
33	-23.3	+3.23	-4.34	-.552	+7.92	-3.18	-.585	33
34	-35.7	-.140	-.023	-.005	-.051	-.033		34
35	-42.2	-.740	-.009	-.003	-.001	-.009		35
36	-44.7	-.774	+.029	+.010	+.097	+.057		36
37	-50.4	-1.23	+.012	+.005	-.108	-.025		37
38	-76.6	+.523	-.831	-.495	+20.3	+11.8		38
39	-84.9	+.203	-.431	-.276	+13.4	+9.47		39
40	-88.4	-.031	-.186	-.122	+9.79	+7.56		40
41	-90.9	+.029	-.188	-.124	+9.75	+7.10		41
42	-205.	+.754	-.717	-.651	-4.94	+191.		42
43	-208.	+.969	-.907	-.828	-7.30	+187.		43
44	-210.	+.985	-.917	-.840	-8.49	+185.		44
45	-213.	+.783	-.726	-.668	-10.9	+181.		45

Table 3.17 Eigenvalue sensitivities of four machine system

Columns 2 → 7 show the sensitivities to simultaneous change of corresponding parameters in each generating unit. The sensitivities are normalised for ease of interpretation so that the numerical results reflect changes in eigenvalues for a one per unit increase in the parameter. Note that since we are considering a first order sensitivity - the change in eigenvalue is only strictly valid for an incremental parameter change. As mentioned in Section 2.6, higher order derivatives and a Taylor series expansion must be employed in general for updating eigenvalues from a base case solution.

In column 2 are shown the sensitivities to the inertial time constants. If, as a first approximation, the system stiffness is assumed frequency independent then it follows that the natural frequencies α $M^{-1/2}$. Consequently, it follows that for a change of 1 p.u. in M (inertial time constants), the corresponding change in w (undamped natural freqeuncies) is -.5 p.u. Using this knowledge we can infer that modes 5, 6, 7, 8, 17, 18, 23 are associated with mechanical oscillation. Mode 26, the eigenvector of which contains a large component states corresponding to rotor angles can also be associated with mechanical oscillation. Obviously, because of their relatively high frequency, the first four modes reflect shaft oscillation.

In column 3 are shown the sensitivities to stabilizer gains (all stabilizer gains adjusted simultaneously). It is very interesting to note that if the stabilizer gains are reduced by 1 p.u., the oscillatory mode at 6.4 rads/sec. loses stability. This mode (lowest frequency mechanical mode) can be interpreted as that reflecting main system swing against the infinite bus. This can be verified by examining the eigenvector for this mode which shows that all rotors swing in phase . Consequently, in the absence of stabilizing circuits, stable operation is impossible at this load condition. Also it should be noted that there is very little change in frequency of oscillation with stabilizer gain - this implying a satisfactory stabilizer design in that only damping torque is introduced. This can be contrasted with the frequency changes (with stabilizer gain) in the other three mechanical oscillatory modes which indicates the introduction of synchronizing torque components. This emphasizes one important characteristic of standard stabilizers : pure damping torque is introduced at only one frequency of oscillation [4].

Another very interesting result is shown in column 4 where the variable is z, the point along the shaft at which the speed signal is taken. The nominal position (for which the modes of column 1 pertain) is midway between the turbine and rotor masses. As would be expected, the location has little effect on the operation of the stabilizer on the main rotor oscillations. However, moving the speed sensor towards the rotor end of the shaft results in shaft oscillatory instability (mutual oscillation between the turbine and rotor masses). This form of instability originates from the action of the electric torque component derived from the speed signal and has been discussed in [5].

In columns 5 and 6, the results are presented for exciter gain and time constant variations, respectively. In particular it can be seen that an increase in exciter gain tends to cause instability in the oscillatory modes, 25 and 27. Oscillatory mode #28 and real modes 24, 33 also show relatively high sensitivity to exciter gain and time constant. These modes (normally oscillatory and having relatively large field voltage component in eigenvectors are referred to as 'exciter or AVR' modes [3]. (It was verified that at reduced exciter gain the real modes #24, 33 appear as an oscillatory as usual). Finally, in column 7, the sensitivities to all transient drop settings are shown. As expected, the first four very slow monotonic modes are sensitive to this parameter and can be identified with governor operation. Also the damping in the main system oscillatory mode is not especially sensitive to this parameter. The sensitivities to additional governor parameters have also been determined (not shown) and the results indicate that governor action does not significantly affect the rotor modes.

3.5 Summary

The application of the modelling concepts developed in Chapter 2 have been applied in practical power system problems. Rather than obtaining definitive results for specific problems, the emphasis was directed towards showing the application of the model to a variety of situations. In particular, situations illustrating the main aspects of dynamic stability listed in

Section 1.4, have been discussed.

Overall solution strategy in dynamic stability problems and the relationship between the dynamic subsystems modelled and the aspects of stability reflected, have been discussed. The general applications of eigenvalue sensitivities have been considered, and the specific application in mode identification in a complex system has been considered in detail.

CHAPTER 4

SHAFT DYNAMICS IN CLOSELY COUPLED IDENTICAL TURBINE-GENERATORS

4.1 Introduction

Analysis of shaft dynamics is currently a subject of intense interest to power system planners and operators. [15]. This interest has been initiated by a number of incidents involving violent shaft oscillations in turbine-generators in two cases resulting in shaft failure [2]. There are two aspects to the problem of shaft torsional oscillations, namely the dynamic and transient stability aspects.

If the mechanical shaft system is dynamically unstable, then, following any small disturbance, oscillations of growing amplitude will result which will eventually cause fatigue failure or at least unit shut down if the situation is recognized in time. Even if the mechanical shaft system is dynamically stable, following major incidents, transient vibrating torques are impressed on the shaft system and cause fatigue damage. This is the transient stability aspect and currently techniques are being developed to assess the fatigue damage in typical incidents and record the expected loss of shaft life [122]. This chapter is restricted to the dynamic stability aspect of the problem.

Shaft torsional oscillations have, of course, always been present in power systems. However, though there has never been very significant damping in these oscillations, it was not until this decade that a mechanism capable of introducing negative damping in these oscillations was introduced in practice. The inclusion of series capacitor compensation in long transmission lines [13] and the incorporation of speed derived stabilizing signals [5] result in two such mechanisms which have the potential of introducing negative damping in the shaft oscillatory modes.

From a review of the bibliography on Subsynchronous Resonance [15] it is seen that the analysis is generally carried out on the basis of a single machine representation for units operating in parallel. Such practice of 'lumping' identical generators feeding into the same bus is universally followed in power system analysis except in special cases [115]. Recently, however, a test on two parallel units conducted at the Mohave Generating Station, Nevada [22] resulted in the discovery of a double resonant peak near one of the natural frequences of the shaft system. This led to the awareness of cross coupling between the shaft's natural modes as a result of the electrical coupling between the generators.

In the paper reporting the discovery of the 'double resonant peak' [22] it was stated that, in general, if there are N nominally identical units feeding into the same bus, N closely separated modes will result due to the electrical cross coupling between the N units.

If this prediction were correct, it could possibly have serious implications for thermal power stations where a number of units each with a static exciter-stabilizer are connected to a common bus. As has been shown in Section 3.3 there is a possibility of shaft instability if the speed pick up point is not properly chosen. The choice of such pick up point is normally made using a single generator representation for all units. Obviously, a substantial shift in natural frequency and the corresponding change in mode shape could result in instability. Corresponding comments can be made on the detrimental effects of a large shift in shaft natural frequency in series compensated systems. Obviously it is very important to quantify the electrical cross coupling between identical generator units operating in parallel, and determine the shifts in natural frequency that result therefrom.

In this chapter an attempt is made to answer some of the important questions relating to shaft dynamics in closely coupled identical generators. In Section 4.2 we qualitatively illustrate the cross coupling between adjacent shaft systems in terms of a 2 unit, 2 mass system. A general N unit, M mass mechanical system is analysed in Section 4.3 and it is shown that in contradiction to the prediction made in [22], that we obtain only two closely separated modes corresponding to each of the M-1 shaft modes, the mode of higher frequency having multiplicity N-1. Reference [22] states that N closely separated (though unequal) modes would result. As a consequence it is shown that the shaft natural frequencies and mode shapes of an arbitrary number of identical units can be analysed in terms of a single M mass system.

In Section 4.4 the conclusions of the mechanical system are extended to the 'full' system model comprising N generators, each with M equivalent masses. It is seen that the natural frequencies and mode shapes for an arbitrary number of generators can be obtained in terms of two evaluations of a single generator situation which are designated 'system' and 'inter-unit' cases, respectively. Typical results are shown for the 'system' and 'inter-unit' case and as a general illustration, a set of mode shapes is drawn for a three generator situation.

In Section 4.5 the extent to which the shaft modes may be stimulated in feasible on-line experiments is investigated. This will hopefully provide a basis for future experimental tests to investigate the phenomenon.

In Section 4.6 the results of an eigenvalue sensitivity analysis are presented for a two turbine-generator system. This system employs static exciter-stabilizers and series compensation. The conclusion which can be drawn from the sensitivities complement and amplify some of the main conclusions of the previous sections.

Finally, in Section 4.7 the main conclusions of this chapter are summarized.

4.2 Fundamental Interactions

In Figure 4.1(a) is depicted a two generator-infinite bus situation which is the simplest configuration exhibiting the 'double resonant peak' phenomenon. In Figure 4.1(b) is shown the mechanical system representation which can be used to explain the frequency shift. Each generator unit is represented by two equivalent masses, one representing the turbine mass and the second representing the rotor mass. Springs of coefficient S_s represent the torsional stiffness of the shafts separating the turbine and rotor masses. S_o represents the stiffness corresponding to the synchronizing coefficient of the rotors against the infinite bus. S_m represents synchronizing coefficient between the units. Such a representation is, of course, in most cases inadequate. If complete system models including exciter and turbine-governor modelling were included then S_s and S_m would be functions of the Laplace operator s and would imply both frequency dependency of the spring coefficients and a damping component. (There is a direct correspondence between the electrical spring coefficients and de Mello and Concordia's synchronizing coefficient(4).) However, such inclusion would tend to obscure the fundamental interaction resulting in the phenomenon and will be omitted in this section.

The mechanical system of Figure 4.1(b) can be represented by the following matrix equation

$$M \ddot{\underset{\sim}{\delta}} + K \underset{\sim}{\delta} = \underset{\sim}{0} \qquad (4.1)$$

where M and K are inertia and stiffness matrices respectively.

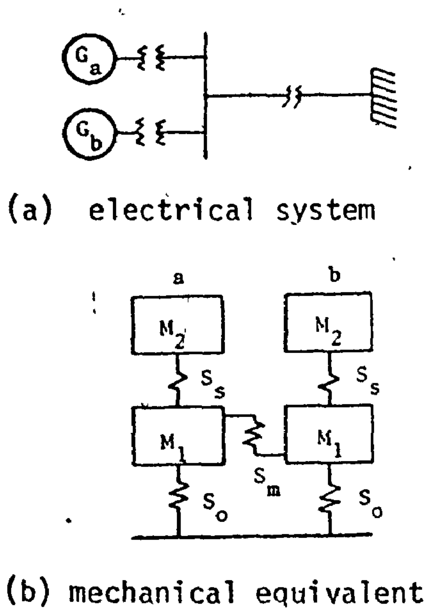


Fig. 4.1 Simplest configuration exhibiting double peak phenomenon

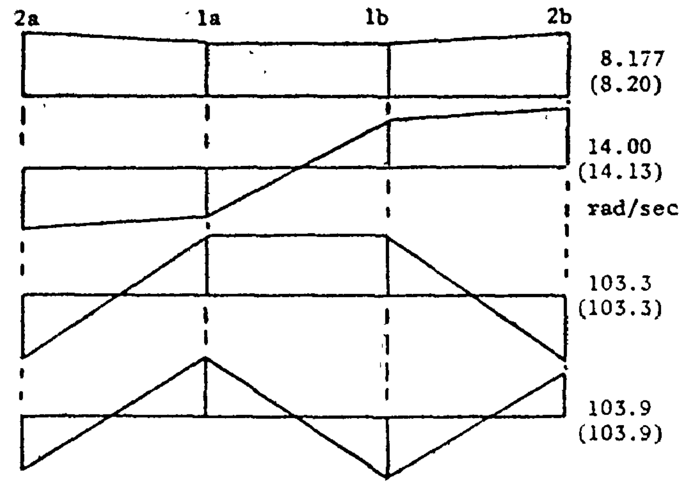


Fig. 4.2 Mode shapes and natural frequencies for 2 generator, 2 mass system

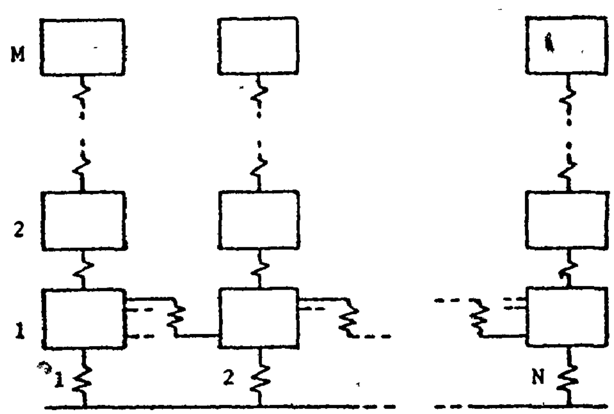


Fig. 4.3. N unit, M mass mechanical system

$M = \text{diag } (M_2, M_1, M_1, M_2)$

$$K = \omega_o \begin{bmatrix} S_s & -S_s & 0 & 0 \\ -S_s & S_o+S_m+S_s & -S_m & 0 \\ 0 & -S_m & S_o+S_m+S_s & -S_s \\ 0 & 0 & -S_s & S_s \end{bmatrix} \tag{4.2}$$

$\omega_o = 2\pi f_o$ where f_o is the electrical frequency in hertz and its incorporation results from the per-unit (pu) equations representation.

and

$\underset{\sim}{\delta} = (\delta_{2a}, \delta_{1a}, \delta_{1b}, \delta_{2b})$ is a vector representing the displacements in radians relative to the reference frame. The natural frequencies and mode shapes can be obtained by solving the real eigenvalue problem where $\underset{\sim}{\delta}$ is a vector of sinusoidal functions.

$$\omega^2 M \underset{\sim}{\delta} = K \underset{\sim}{\delta} \tag{4.3}$$

Using the following data:

$M_1 = M_2 = 3.558$ (secs), $S_o = 1.27$, $S_m = 1.25$, $S_s = 50$ (pu) (The value of total mass which corresponds to the thermal unit analysed in Section 3.3 was assumed divided into two equal portions for the purposes of this section and S_s was chosen to yield a typical shaft natural frequency. The values of S_o and S_m are also typical for the thermal unit of Section 3.3) we obtain the results shown in Figure 4.2.

The natural frequencies fall into two groups - a low frequency group (8.177 and 14.00 rad/sec.) and a closely separated high frequency group (103.3 and 103.9 rads/sec). The very close separation in the high frequency group is the so-called double resonant peak phenomenon.

The low frequency modes do not involve appreciable shaft strain ($\delta_{1a} \doteqdot \delta_{2a}$ and $\delta_{1b} \cong \delta_{2b}$). The lowest mode represents the oscillation of all four masses against the reference frame and is the mode usually analysed in stability studies. The next mode (14.0 rads/sec) represents inter unit oscillations: masses M_1 and M_2 of one unit oscillate as a group against the group of the second units masses.

The third and fourth modes represent oscillations between the individual masses in each group. In mode 3 corresponding masses (M_{1a} and M_{1b}, M_{2a} and M_{2b}) oscillate in phase while in mode 4 corresponding masses oscillate in anti phase.

Because of the symmetry of the arrangement it is possible to avoid solving a fourth order eigenvalue problem. If 'system' oscillation ($\delta_{1a} = \delta_{1b}$, $\delta_{2a} = \delta_{2b}$) is assumed M_{1a} and M_{1b} may be considered joined by a rigid coupling as also may M_{2a} and M_{2b} and the analysis conducted on a two mass equivalent. On the other hand for 'inter-unit' oscillation ($\delta_{1a} = -\delta_{1b}$, $\delta_{2a} = -\delta_{2b}$ the mid point of spring S_m is a node and the analysis may also be conducted on a two mass basis.

Consequently for both in-phase and anti phase modes we have:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \ddot{\delta}_1 \\ \ddot{\delta}_2 \end{bmatrix} = - \omega_o \begin{bmatrix} S_s + \hat{S}_o & -S_s \\ - S_s & S_s \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \qquad (4.4)$$

where:

$\hat{S}_o = S_o$: in-phase modes

$\hat{S}_o = S_o + 2 S_m$: anti-phase modes

Equation (4.4) leads to simple quadratic equations for the solution of the natural frequencies.

If we assume $M_1 = M_2 = M$ and use the property that the shaft stiffness, S_s, and electrical stiffnesses, S_o and S_m, are well separated in magnitude, it can be easily seen:

$$\omega^2 = \frac{\omega_o}{2M} \{\hat{S}_o + 2 S_s \pm 2 S_s\} \qquad (4.5)$$

This leads to 4 solutions of ω^2 and hence ω

$$\begin{aligned} \omega_1^2 &= \omega_o S_o / 2M \\ \omega_2^2 &= \omega_o (S_o + 2 S_m)/2M \\ \omega_3^2 &= \omega_o (S_o + 4 S_s)/2M \\ \omega_4^2 &= \omega_o (S_o + 2 S_m + 4 S_s)/2M \end{aligned} \qquad (4.6)$$

The expressions for ω corresponding to the shaft modes are simplified further:

$$\begin{aligned} \omega_3 &= \omega_s (1 + S_o/8 S_s) \\ \omega_4 &= \omega_s \{1 + (S_o + 2 S_m)/8 S_s\} \\ \omega_s^2 &= \frac{2\omega_o S_s}{M} \end{aligned} \qquad (4.7)$$

ω_s is the shaft natural frequency in the absence of any electrical coupling.

The close separation can be expressed:

$$\omega_4 - \omega_3 = \omega_s \; S_m/4 \; S_s \tag{4.8}$$

In Figure 4.2 the frequencies corresponding to the full model are followed by the approximate values which are shown as bracketed quantities

$$\omega_s = 102.9 \text{ rads/sec}$$
$$\omega_4 - \omega_3 = .64 \text{ rads/sec}$$

The above analysis may be easily extended to a general M mass system. In the above case, a two mass system gave rise to a single pair of close modes. For the M mass case, M-1 close modes result. This can be readily seen by repeated application of the 2 unit, 2 mass analysis with appropriate value of S_s for each shaft frequency prediction.

4.3 N Unit, M Mass Mechanical System

Consider the case of N nominally identical generators each with M equivalent masses feeding into a single bus. The corresponding mechanical system representation (again subject to the same limitation of frequency independence of parameters as outlined in the previous section) is shown in Figure 4.3.

It will be recalled from the previous section that the stiffness matrix for N = 2 case is block symmetric, i.e., it has the following form

$$K = \begin{bmatrix} K_d & -K_m \\ -K_m & K_d \end{bmatrix} \tag{4.9}$$

This can be rewritten:

$$K = \begin{bmatrix} K_o & 0 \\ 0 & K_o \end{bmatrix} + \begin{bmatrix} K_m & -K_m \\ -K_m & K_m \end{bmatrix} \tag{4.10}$$

K_o matrices represent the coupling between the individual masses in each unit and between each unit and the reference frame. K_m matrices represent the coupling between the units and will contain only one element as only one mass (corresponding to the rotor) is spring coupled between the units.

It can be argued that the general N Unit, M Mass case has the following stiffness matrix

$$\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & N \end{array} \\ \begin{bmatrix} K_d & -K_m & -K_m & -K_m \\ -K_m & K_d & & \\ -K_m & & K_d & \\ -K_m & & & K_d \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ N \end{array} \end{array} \tag{4.11}$$

where K_d and K_m are square matrices of order M and $K_d = K_o + (N-1)\,K_m$.

The inertia matrix for the N Unit case will have the general form:

$$M = \operatorname{diag}\{\overset{1}{M_u}, \overset{2}{M_u} \ldots \overset{N}{M_u}\} \tag{4.12}$$

where:

$$M_u = \operatorname{diag}\{M_m, M_{n-1} \ldots M_1\} \tag{4.13}$$

It can be verified that the following constitute the eigenvalues and eigenvectors for the system.

eigenvalue:	ω_+^2	ω_-^2	(Multiplicity, N-1)
eigenvector:	$\underset{\sim}{x}_+$	$r_1\underset{\sim}{x}_-$	1
	$\underset{\sim}{x}_+$	$r_2\underset{\sim}{x}_-$	2
	$\underset{\sim}{x}_+$	$r_3\underset{\sim}{x}_-$	3
	$\vdots$	$\vdots$	$\vdots$
	$\underset{\sim}{x}_+$	$r_N\underset{\sim}{x}_-$	N

(4.14)

where ω_+, x_+, ω_- and x_- are the M solutions of the following eigenvalue problems:

$$\omega_+ M_u \underset{\sim}{x}_+ = K_o \underset{\sim}{x}_+$$
$$\omega_- M_u \underset{\sim}{x}_- = (K_d + K_m) \underset{\sim}{x}_- \tag{4.15}$$

and where $\sum_{r=1}^{N} r_i = 0$ and $r_i \neq 0$; all i.

It can be shown that by careful choice of the 'r' coefficients, distinct eigenvectors can be obtained. Consider the matrix:

$$\begin{bmatrix} 1 & 1 \quad 1 \quad 1 \\ S & A_{22} \end{bmatrix} \tag{4.16}$$

All the rows of this matrix are linearly independent provided

$$\rho = A_{22} - [\underset{\sim}{S}^{1},\underset{\sim}{S}^{2},\ldots S^{N-1}] \tag{4.17}$$

can be inverted. This results from the formula for matrix inversion by parts([41], Sec. 4.9). By choosing any non singular ρ, the elements of the i^{th} row may be obtained in terms of the elements of S and ρ and subsequently the elements of S obtained using the constraint:

$$\sum_{j=1}^{N} A(i.j) = 0, \quad i = 2 \rightarrow N \tag{4.18}$$

The above represents a general method of obtaining distinct eigenvectors - an alternative approach is to assume values of r (Equation (4.15)) and check for independence.

The first M modes represent system oscillations where the corresponding masses in each unit oscillate in unison. These modes would be stimulated by symmetric disturbances on each unit. The remaining (N-1) M modes appear as M groups (one group corresponding to each equivalent mass) of N-1 repeated eigenvalues and reflect inter-unit dynamics. It should be emphasized that the choice of eigenvectors for the 'inter-unit' case is not unique. Consequently a variety of eigenvectors may be considered. As a specific case consider a 4 unit situation. In incidents involving pairs of units, a convenient mode shape is that in which one pair of units swings in unison against the remaining pair. For other disturbances, involving only one unit, the most convenient mode shape is that with one unit swinging against the remaining three with three times the amplitude. In both of these cases, the units considered swinging in unison can be equivalenced and then the problem solved as that of a single unit.

Consequently the following important conclusions can be drawn for the mechanical system of Figure 4.3.

(a) There is a group of N-1 repeated eigenvalues closely separated for each of the shafts M-1 modes. Furthermore, there corresponds a distinct eigenvector for each repeated eigenvalue.

(b) The natural frequencies and mode shapes (eigenvalues and eigenvectors) can be predicted using a single unit M Mass system using two different values of equivalent spring between the unit and reference frame.

4.4 N Unit, M Mass Generator System

The simple mechanical representation of the previous subsections is helpful in providing insight into the 'system' and 'inter-unit' dynamics. The conclusions regarding the repetition of eigenvalues and the number of units to be analyzed may be duplicated if the general state space equation set (including representation of all processes of importance -

eg. exciter effects) is used to replace Equation (4.3).

In subsection 4.2 it was noted that the mid point of spring S_m is a node for the inter-unit situation. Correspondingly, the synchronizing bus voltage magnitude and angle deviations are zero for anti phase or inter-unit oscillations.

The two separate equivalents which can be employed to analyze the in-phase and anti phase modes are shown in Figure 4.4.

Though the development which has been presented is for N generators feeding into an infinite bus through a transmission line, identical conclusions (regarding repetition of eigenvalues) can be drawn for the general situation shown in Figure 4.5. Taking N = 3 as an example, it can be argued that the state space matrix has the following structure:

$$\begin{bmatrix} \dot{\underset{\sim}{x}}_{g1} \\ \dot{\underset{\sim}{x}}_{g2} \\ \dot{\underset{\sim}{x}}_{g3} \\ \dot{\underset{\sim}{x}}_{n} \end{bmatrix} = \begin{bmatrix} A_d & A_m & A_m & B \\ A_m & A_d & A_m & B \\ A_m & A_n & A_d & B \\ C & C & C & A_n \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_{g1} \\ \underset{\sim}{x}_{g2} \\ \underset{\sim}{x}_{g3} \\ \underset{\sim}{x}_{n} \end{bmatrix} \qquad (4.19)$$

$\underset{\sim}{x}_{g1}$, $\underset{\sim}{x}_{g2}$, $\underset{\sim}{x}_{g3}$ and $\underset{\sim}{x}_{n}$ are the states associated with generators 1, 2, 3 and the external system. It can be verified by substitution that the following constitute the eigenvalues and a choice of eigenvectors for the system.

	1	2	3
eigenvalue	λ_+	λ_-	λ_-
	$\underset{\sim}{x}_{g+}$	$\underset{\sim}{x}_{g-}$	$-.5\underset{\sim}{x}_{g-}$
	$\underset{\sim}{x}_{g+}$	$-.5\underset{\sim}{x}_{g-}$	$\underset{\sim}{x}_{g-}$
eigenvector	$\underset{\sim}{x}_{g+}$	$-.5\underset{\sim}{x}_{g-}$	$-.5\underset{\sim}{x}_{g-}$
	$\underset{\sim}{x}_{n+}$	$\underset{\sim}{0}$	$\underset{\sim}{0}$

(4.20)

where λ_+, $\underset{\sim}{x}_{g+}$ and $\underset{\sim}{x}_{n+}$ satisfy the following eigenvalue equation:

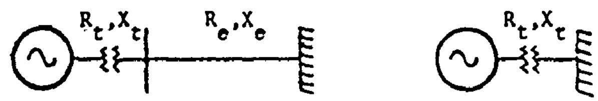
$$\lambda_+ \begin{bmatrix} x_{g+} \\ x_{n+} \end{bmatrix} = \begin{bmatrix} A_d + 2A_m & B \\ 3C & A_n \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_{g+} \\ \underset{\sim}{x}_{n+} \end{bmatrix} \qquad (4.21)$$

and λ_-, x_{g-} satisfy the equation:

$$\lambda_- \underset{\sim}{x}_{g-} = (A_d + A_m)\, \underset{\sim}{x}_{g-}$$

Predicted results for a system with the data shown in Table 4.1 is presented in Table 4.2.

Because of their frequency separation and limited amount of interaction, certain modes may be associated with specific portions of the system. Those relating to the exciter, the rotor angle, and the shaft torsional modes are listed in Table 4.3. The corresponding modes without a stabilizer are presented for reference. The mode shapes corresponding to the shaft system are shown in Figure 4.6. The double peak phenomenon is clearly evident in the results.



(a) system mode (b) inter unit modes

Fig. 4.4 Equivalents for study of shaft dynamics in general number of generators

Generator: $X_d = 1.30$, $X_q = 1.20$, $X_f = 1.22$, $X_{kd1} = 1.27$
$X_{kq1} = 1.27$, $X_\ell = .159$,
$r_a = .0015$, $r_f = .00075$, $r_{kd1} = .0081$, $r_{kq1} = .0025$

Exciter: $K_e = 212$, $\tau_e = .002$, $\tau_v = .01$

Stablizer: $K_Q = 9.6$, $\tau_Q = 1.41$, $\tau_a = .121$, $\tau_x = .033$

Network: $R_t = .005$ $X_t = .133$ $R_e = .03$ $X_e = .34$

Mechanical Shaft $M_1 = 1.71$, $M_2 = 2.38$, $M_3 = 2.31$
$M_4 = .464$, $M_5 = .248$, $D_1 = 0$, $D_2 = .249$, $D_3 = .259$,
$D_4 = .255$, $D_5 = .237$, $S_{12} = 62.3$, $S_{23} = 75.6$,
$S_{34} = 48.4$, $S_{45} = 21.8$

Governor and turbine: representation omitted in simulation presented

Loading: MVA = 1., PF = .9 (lag)

Table 4.1 System data for analysis of shaft dynamics effect

	system	inter unit
1	-500.3	-500.2
2	-18.79 ± j377.0	-6.303 ± j377.0
3	-.3005 ± j276.5	-.3002 ± j276.5
4	-.1158 ± j192.6	-.1316 ± j192.7
5	-.3509 ± j151.4	-.3946 ± j151.5
6	-.1480 ± j102.2	-.1778 ± j102.8
7	-85.34	-92.04
8	-31.62	-32.09
9	-15.98 ± j24.05	-10.54 ± j22.95
10	-1.449 ± j7.008	-4.149 ± j7.563
11	-1.210	-1.071
12	-.7418	-.8088

Table 4.2 Eigenvalues for general N generator infinite bus system.

MODE	system				inter unit			
	$K_Q = 0$		$K_Q = 10$		$K_Q = 0$		$K_Q = 10$	
	ω_n	ξ	ω_n	ξ	ω_n	ξ	ω_n	ξ
EXCITER	25.90	.7164	28.87	.5534	19.38	.8101	25.25	.4172
ROTOR	8.197	-.0119	7.156	.2025	11.46	.0193	8.626	.4810
16 Hz	102.6	.0014	102.2	.0015	103.3	.0016	102.8	.0017
24 Hz	151.8	.0018	151.4	.0023	152.0	.0018	151.5	.0026
30 Hz	192.7	.0004	192.6	.0006	192.8	.0005	192.7	.0007
44 Hz	276.5	.0011	276.5	.0011	276.5	.0011	276.5	.0011

Table 4.3 Natural frequencies and mode shapes for system.

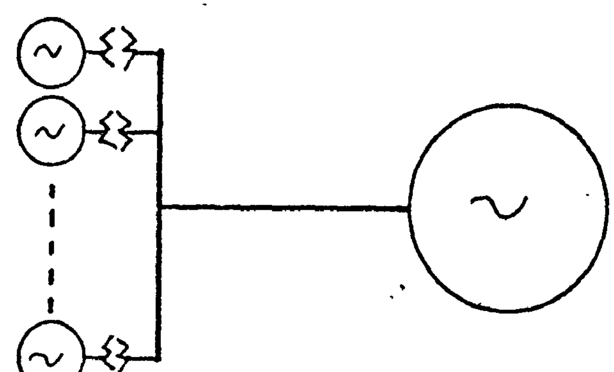


Fig. 4.5 General situation of N generators connected to external system

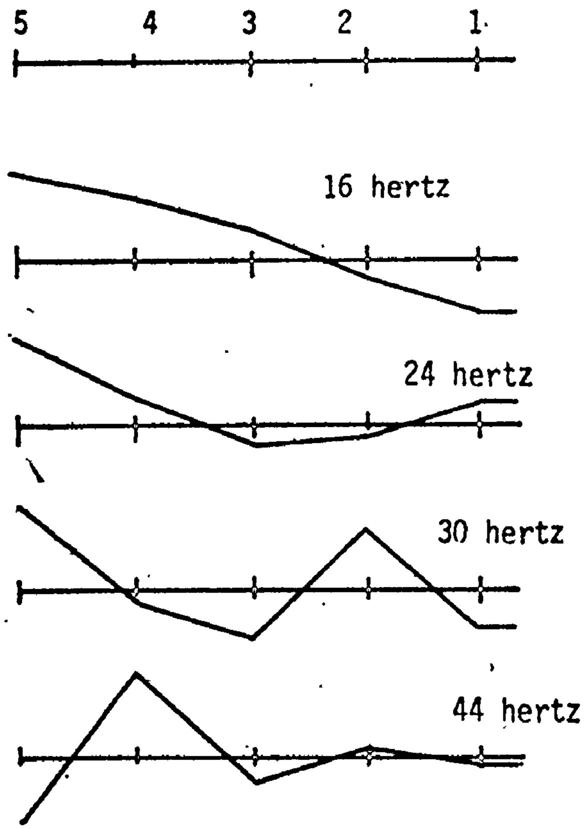


Fig. 4.6 Mode shapes for turbine-generator shaft system

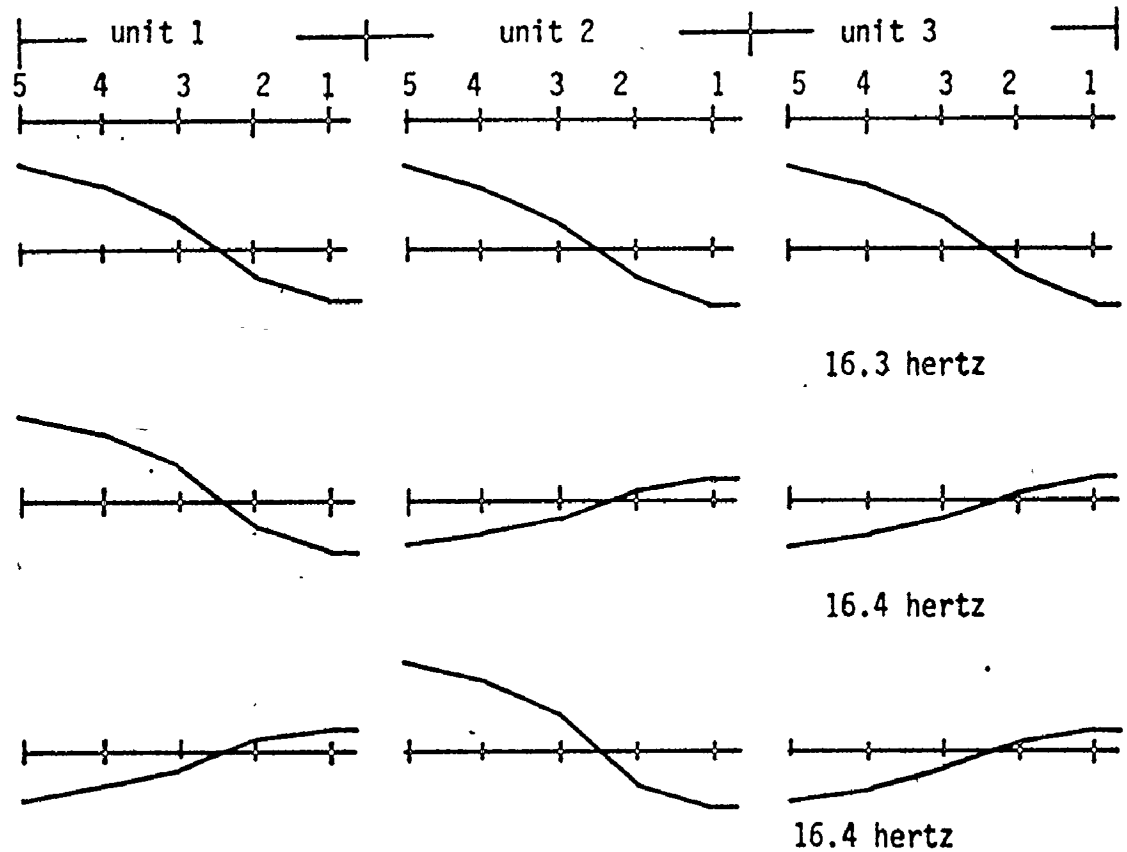


Fig. 4.7 Mode shapes of first natural frequency for three machine system

The separation in the 16 hertz mode is 0.6 radians/sec which is close to the value of .57 radians/sec measured in the Mohave test [22].

As a specific illustration of the generation of mode shapes for the general case, a set of 'inter-unit' mode shapes corresponding to the first shaft mode for a three unit system is shown in Figure 4.7. As mentioned previously, the main system mode shape is uniquely determined (to within a multiplicative constant) whereas the inter-unit mode shapes are not unique and a variety of solutions are possible.

4.5 Response Prediction

In this section we present results of predictions showing the extent to which the shaft natural modes may be excited in on-line experiments. The reason for the prediction is to provide a basis for the design of experiments to substantiate the theoretical results obtained.

Both time and frequency response experiments were considered. For the former case, two tests suggested themselves as being suitable. Firstly, a step change in reference voltage results in a rapid disturbance in electrical torques which may cause shaft transients. Secondly, by tie line switching the electrical torques are instantaneously changed, thereby transmitting a step input in torque to the mechanical system.

The most convenient frequency response test is the introduction of a sinusoidal perturbation in the reference voltage. This was the technique employed in the test at Mohave which resulted in the discovery of the double-resonant peak phenomenon.

4.5.1 Time Response Prediction

Figure 4.8(a) shows the computed rotor angle (δ_1), rotor speed (n_1) and the speed of the coupling (n_s) between the two low pressure stages in response to a step input of the same polarity applied in the reference voltage of each of the four machines. Figure 4.8(b) shows the response when the inputs for two machines are of one polarity and the inputs for the remaining two machines are of the opposite polarity.

The dominance of the rotor mode with a slight contribution of shaft oscillation can be seen. By filtering the response through a band pass filter broadly tuned to the various shaft frequencies, an accurate measurement of the frequency and damping may be made.

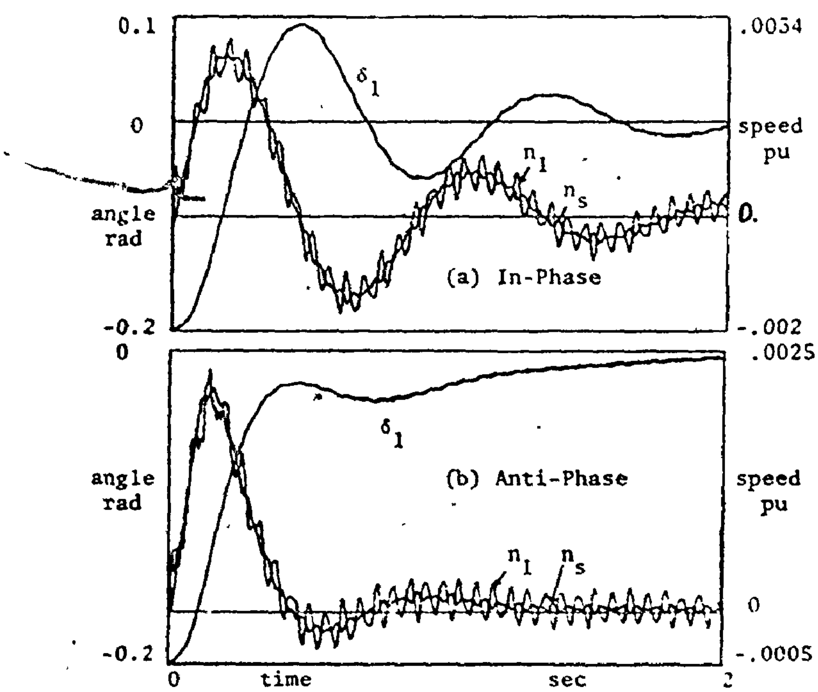
Figure 4.9 shows the computed response following line switchings resulting in rotor angle changes of .2 radians. Figure 4.9(a) represents the 'main-system' case and results from a line switching on a common transmission line. For the 'inter-unit' case (4.9(b)) the station bus is split and initially each pair of generators independently feeds into a strong system through a separate transmission line. The generators are assumed to be initially operating with rotor angles at $\pm$ .2 radians relative to the final equilibrium angle when the bus is closed.

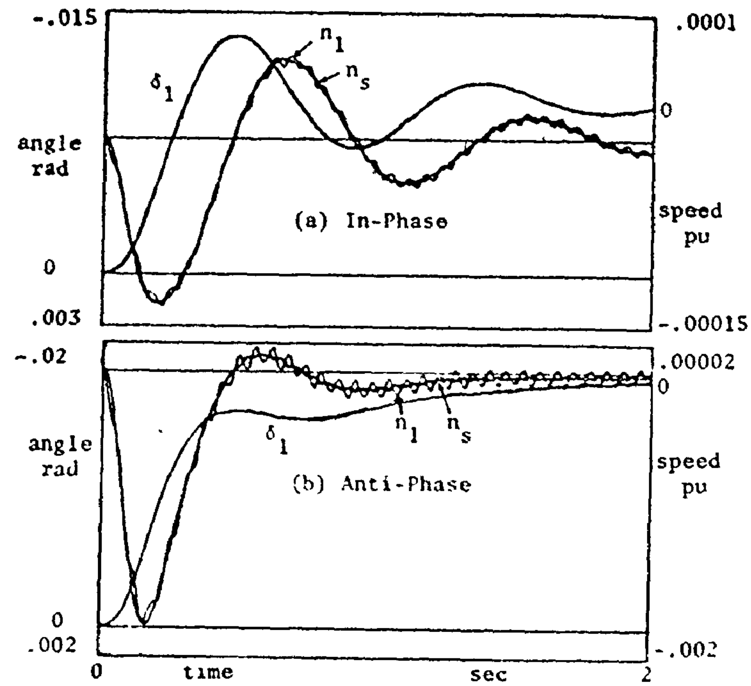
In both cases there is a significant contribution of the 16 hertz shaft oscillation in the rotor speed (n_I). This 16 hertz component is absent from the speed (n_s) signal

(a)
same polarity

Fig 4.8 Response following .05 pu step in reference voltage

(b)
opposite polarity





(a)
same polarity

Fig. 4.9 Response following line switching resulting in equilibrium rotor single change of .2 rads

(b)
opposite polarity

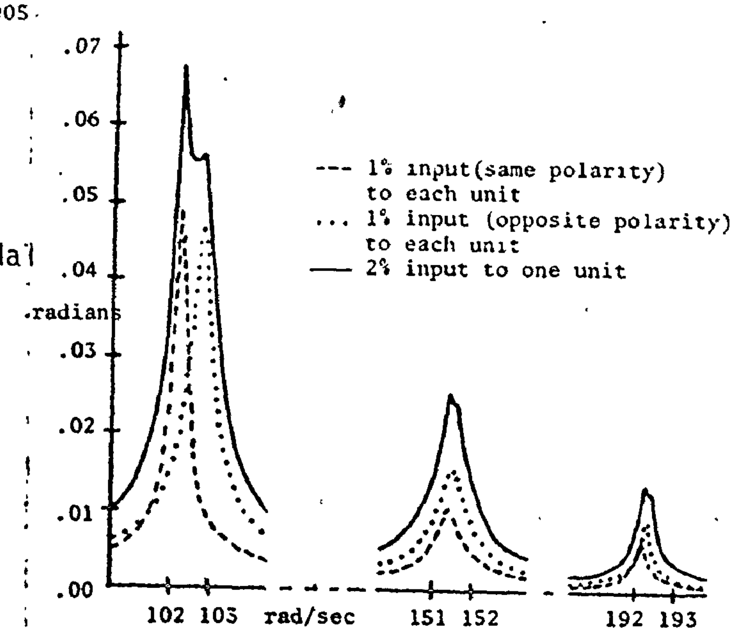


Fig. 4.10 Rotor frequency response to sinusoidal perturbation in reference voltage

fed into the stabilizer. This point represents a node in the 16 hertz mode (Figure 4.6).

From the point of view of correlating theoretical predictions with experimentation, it can be noted that closed form solutions for the response of any variable for any initial condition or input, can be written:

$$y_i = \sum_{i=1}^{N} \alpha_{ij} e^{\lambda_i t} \tag{4.22}$$

where i refers to the state or output number and j refers to the eigenvalue. α_{ij} is a constant which is determined from the initial condition or inputs.

The modal contents in the state variable responses for the 'main system' line switching disturbance is shown in Table 4.4 Specifically we see: that

$$N_1 \cong (-.0003 \pm j.0016)e^{-18.8 \pm j377} + (.0001 \pm j.0017)e^{.1 \pm j102.2}$$

$$= (.0009 \pm j.0098)e^{-.14 \pm j7.00} \tag{4.23}$$

The rotor response contains only three significant modal contributions. By examining the content in each state and output variable amenable to measurement, a variety of measurement possibilities can be considered.

4.5.2 Frequency Response Prediction

Figure 4.10 shows the computed frequency responses of δ_5 for a sinusoidal input in reference voltage. Case (a) shows results for in-phase excitation to all 4 units. Case (b)

	eigenvalues																			
	-500.7 0.0	-16.8 377.0	-16.8 -377.0	-.3 270.3	-.3 -270.3	-.1 192.0	-.1 -192.0	-.4 151.4	-.4 -151.4	-.1 102.2	-.1 -102.2	-55.3 0.0	-31.0 0.0	-15.0 24.0	-15.0 -24.0	-1.4 7.0	-1.4 -7.0	-1.2 0.0	-.7 0.0	
	modal content of state response																			
STATE NO. 1	-.0021 -.0000	.1392 .5405	.1392 -.5405	.0000 -.0000	.0000 .0000	-.0009 .0004	-.0009 -.0004	-.0037 .0013	-.0037 -.0013	-.0002 .0006	-.0002 -.0006	.0312 -.0000	.0331 -.0000	.0679 -.1543	.0679 .1543	-.2400 .1772	.2400 -.1772	-.0039 .0000	.0425 .0000	i_D
STATE NO. 2	-.0015 -.0000	-.7020 .2731	-.7020 -.2731	.0000 -.0000	.0000 .0000	.0006 .0001	.0006 -.0001	-.0006 .0007	-.0006 -.0007	.0039 -.0033	.0059 .0003	.0233 -.0000	.0220 -.0000	.0360 -.1343	.0360 .1343	.7093 .0717	.7093 -.0717	-.0014 -.0000	-.0220 -.0000	i_Q
STATE NO. 3	.0000 .0000	.0070 .0213	.0070 -.0213	.0000 -.0000	.0000 .0000	.0000 .0002	.0000 -.0002	-.0004 .0010	-.0004 -.0010	-.0007 .0019	-.0007 -.0019	.0007 -.0000	.0220 -.0000	.0169 -.0922	.0169 .0922	-.0073 .0990	-.0073 -.0990	-.0010 .0000	.0194 .0000	e_v
STATE NO. 4	-.0033 -.0000	-.0015 .0016	-.0015 -.0016	.0000 -.0000	.0000 .0000	-.0012 .0002	-.0012 -.0002	-.0057 .0013	-.0057 -.0013	-.0071 -.0000	-.0071 .0000	.0067 -.0000	-.2645 .0000	.1747 -.1390	.1747 .1390	-.1009 .1407	-.1009 -.1407	-.0129 .0000	.0303 .0000	ψ_f
STATE NO. 5	.0031 .0000	-.0031 -.0011	-.0031 .0011	-.0000 -.0000	-.0000 .0000	.0000 .0001	.0000 -.0001	.0001 .0007	.0001 -.0007	-.0002 .0012	-.0002 -.0012	-.0168 .0000	.2909 -.0000	-.0742 -.1340	-.0742 .1340	-.0710 .1009	-.0710 -.1009	-.0130 .0000	.0374 .0000	ψ_{d1}
STATE NO. 6	-.0000 -.0000	.0005 -.0012	.0005 .0012	.0000 .0000	.0000 -.0000	.0000 .0000	.0000 -.0000	.0000 .0000	.0000 -.0000	.0000 .0000	.0000 -.0000	.0000 -.0000	.0030 -.0000	.0003 -.0002	.0003 .0002	.0021 .0235	.0021 -.0235	-.0208 .0000	.0229 .0000	ψ_{q1}
STATE NO. 7	.0044 .0000	-.0019 -.0012	-.0019 .0012	.0000 .0000	.0000 -.0000	-.0003 -.0006	-.0003 .0006	-.0006 -.0023	-.0006 .0023	-.0000 -.0017	-.0000 .0017	-.0143 .0000	.0143 -.0000	.0026 .0103	.0026 -.0103	-.0024 -.0024	-.0024 .0024	.0000 -.0000	-.0031 -.0000	e_f
STATE NO. 8	-.0000 -.0000	-.0000 .0000	-.0000 -.0030	-.0000 .0000	-.0000 -.0000	.0000 .0000	.0000 .0000	.0000 -.0000	.0000 .0000	.0000 -.0030	.0000 .0000	.0000 -.0000	.0000 -.0000	.0002 .0002	.0002 -.0002	-.0097 .0001	-.0097 -.0001	-.0010 .0000	.0199 .0000	e_{s1}
STATE NO. 9	-.0000 -.0000	-.0002 .0000	-.0002 -.0000	-.0000 .0000	-.0000 -.0000	.0006 .0000	.0006 -.0000	.0026 .0002	.0026 -.0002	.0026 .0006	.0026 -.0006	.0006 -.0003	.1105 -.0000	-.0124 .0339	-.0124 -.0339	-.0102 .3045	-.0102 .3040	.0007 -.0000	-.0704 -.0000	e_{s2}
STATE NO. 10	-.0000 -.0000	-.0003 .0010	-.0003 .0010	.0000 .0000	.0000 -.0000	.0001 .0004	.0001 -.0004	.0001 .0005	.0001 -.0005	.0001 .0017	.0001 -.0017	.0001 -.0000	.0002 -.0000	.0006 -.0004	.0006 .0004	-.0009 .0090	-.0009 -.0090	-.0001 .0000	.0001 .0000	n_1
STATE NO. 11	-.0000 -.0000	.0000 .0001	.0000 -.0001	-.0000 -.0000	-.0000 .0000	-.0001 -.0006	-.0001 .0006	-.0000 -.0003	-.0000 .0003	.0000 .0004	.0000 -.0004	.0000 -.0000	.0001 -.0000	.0009 -.0002	.0009 .0002	-.0009 .0093	-.0009 -.0093	-.0001 .0000	.0001 .0000	n_2
STATE NO. 12	-.0000 -.0000	-.0000 -.0000	.0000 .0000	.0000 -.0000	.0000 -.0000	.0001 .0004	.0001 -.0004	-.0001 -.0004	-.0001 .0004	-.0001 -.0010	-.0001 .0010	.0000 -.0000	.0001 -.0000	.0009 -.0002	.0009 .0002	-.0010 .0100	-.0010 -.0100	-.0001 .0000	.0001 .0000	n_3
STATE NO. 13	-.0000 -.0000	.0000 .0000	.0000 -.0000	-.0000 -.0001	-.0000 .0001	.0003 .0001	.0000 -.0001	.0001 .0007	.0001 -.0007	-.0001 -.0017	-.0001 .0017	.0000 -.0000	.0001 -.0000	.0010 -.0001	.0010 .0001	-.0010 .0100	-.0010 -.0100	-.0001 .0000	.0001 .0000	n_4
STATE NO. 14	-.0000 -.0000	-.0000 -.0000	-.0000 .0000	.0000 .0001	.0000 -.0001	-.0001 -.0000	-.0001 .0005	.0005 .0022	.0005 -.0022	-.0022 -.0027	-.0022 .0027	.0000 -.0000	.0001 -.0000	.0010 -.0001	.0010 .0001	-.0010 .0100	-.0010 -.0100	-.0001 .0000	.0001 .0000	n_5
STATE NO. 15	.0000 .0000	-.0010 .0005	-.0010 -.0005	.0000 -.0000	.0000 .0000	.0007 -.0001	.0007 .0001	.0012 -.0001	.0012 .0001	.0002 -.0002	.0002 .0002	-.0005 .0000	-.0021 .0000	-.0090 -.0029	-.0090 .0029	.5170 -.0592	.5170 .0592	.0234 -.0000	-.0465 -.0000	δ_1
STATE NO. 16	.0000 .0000	.0001 -.0001	.0001 .0001	-.0000 .0000	-.0000 -.0000	-.0012 .0002	-.0012 -.0002	-.0003 .0001	-.0003 -.0001	.0015 -.0001	.0015 .0001	-.0004 .0000	-.0010 .0000	-.0005 -.0001	-.0005 .0001	.5229 -.0570	.5229 .0570	.0234 -.0000	-.0464 -.0000	δ_2
STATE NO. 17	.0000 .0000	-.0000 .0000	-.0000 -.0000	.0000 -.0000	.0000 .0000	.0000 -.0001	.0006 .0001	-.0010 .0001	-.0010 -.0001	-.0030 .0003	-.0030 -.0003	-.0001 .0000	-.0010 .0000	-.0005 -.0071	-.0005 .0071	.5250 -.0500	.5250 .0500	.0234 -.0000	-.0464 -.0000	δ_3
STATE NO. 18	.0000 .0000	.0000 -.0000	.0000 .0000	-.0001 .0000	-.0001 -.0000	.0002 .0000	.0002 .0000	.0017 -.0002	.0017 .0002	-.0000 .0000	-.0000 -.0000	-.0001 .0000	-.0015 .0000	-.0083 -.0074	-.0083 .0074	.5267 -.0500	.5267 .0500	.0234 -.0000	-.0464 -.0000	δ_4
STATE NO. 19	.0000 .0000	-.0000 .0000	-.0000 -.0000	.0001 -.0000	.0001 .0000	-.0010 .0002	-.0010 -.0002	.0055 -.0007	.0055 .0007	-.0100 .0009	-.0100 -.0009	-.0001 .0000	-.0015 .0000	-.0062 -.0027	-.0062 .0027	.5275 -.0505	.5275 .0505	.0234 -.0000	-.0464 -.0000	δ_5

Table 4.4 Modal content of state response following line switching

shows results for one pair of units being excited with opposite phase signal to the other pair. δ_5 was chosen as this state exhibits a large relative amplitude in all modes.

Only 3 of the 4 shaft modes have detectable amplitudes, the fourth mode having a predicted amplitude of 1.2×10^{-5} radians. This result can be appreciated by examining the element's canonical input distribution matrix [29] which shows a negligible component for the 44 hertz shaft mode demonstrating the ineffectiveness of the exciter in stimulating this mode.

It was assumed in the computation of the frequency response that over the band width of response around 16, 24, 30 and 44 hertz (the shaft natural frequencies) only the corresponding modes contribute significantly to the overall response. This was verified by computing the total response (all modes considered) at sample points over the frequency range. The difference was not detectable over the frequency range (16-40 hertz). (At lower frequencies the rotor and exciter modes have appreciable contributions.)

Once the response of a state in which all modes are observable, the response of every other state and output can be determined, if the eigenvectors of the system are known. This will allow a variety of measurement possibilities to be compared, and variables which are amenable to measurement and reflect the modes of interest, chosen.

4.6 Eigenvalue Sensitivity Analysis

In this section results of an eigenvalue sensitivity study on a two thermal machine power system (Figure 4.11) are given. The long transmission line is series capacitor compensated. Both units (which are identical) are equipped with static exciter stabilizers.

The material presented in this section was not part of the main development for this chapter and continuity will not be sacrificed if this section is bypassed. The results presented here were obtained using a full system model evaluation (as distinct to separate evaluations for the 'main system' and inter-unit cases) and the interpretation of the results complements and amplifies the conclusions of the previous sections.

Table 4.5 shows the eigenvalue sensitivities for X_c, X_e, Re, K_q and z (series capacitance, tie line reactance and resistance, stabilizer gain and speed sensor location (z = 0 when midway between the low pressure turbine stages)).

X_c, X_e and R_e are 'main system' parameters (they relate to the common tie line). It can be seen from Table 4.5 that the shaft modes occur in pairs (102, 150, 193, 276 rads/sec corresponding to 16, 24, 30, and 44 Hz.) and that only one mode of each pair is sensitive to these 'main system' parameters.

K_q and z, on the other hand are individual machine properties and as such will influence inter-unit dynamics. This is borne out by the sensitivity of all shaft modes to these parameters.

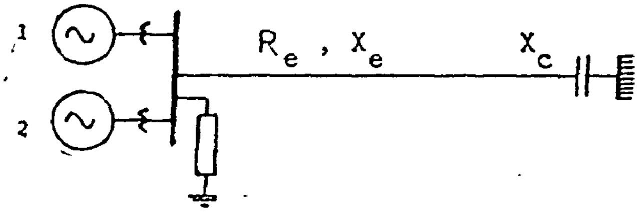


Fig. 4.11 Two-thermal machine power system

n	Eigenvalue $K_q = 0$ (1)	Eigenvalue $K_q = 10$ (2)	Eigenvalue sensitivity ($K_q = 10$) X_c (3)	X_e (4)	R_e (5)	K_q (6)	z (7)
1	+.035 ± j103	+.198 ± j102.	+1.22 ± j5.62	-1.12 ∓ j.707	+.058 ∓ j.205	+.178 ∓ j.499	-.377 ± j1.24
2	-.086 ± j193	-.132 ± j193				-.045 ∓ j.138	-.217 ∓ j.668
3	-.074 ± j192	-.173 ± j192	+.043 ± j.147	+.013 ∓ j.109	+.010 ∓ j.008	-.099 ∓ j.015	-.486 ∓ j.076
4	-.157 ± j103	-.176 ± j102				-.020 ∓ j.536	+.065 ± j1.39
5	-.301 ± j276	-.300 ± j276				+.001 ± j.002	-.002 ∓ j.004
6	-.301 ± j277	-.300 ± j277				+.001 ± j.001	-.002 ∓ j.001
7	+.176 ± j152	-.400 ± j151				-.135 ∓ j.612	+.009 ± j.046
8	-.709	-.741	-.010	+.019	-.004	-.036	
9	-.267 ± j152	-.768 ± j151	-7.82 ∓ j2.90	+5.69 ± j2.11	+.345 ∓ j.908	-.990 ± j1.18	-.014 ∓ j.046
10	-.709	-.813				-.178	
11	-1.226	-1.07				+.240	
12	+.160 ± j7.10	-1.11 ± j6.12	-.856 ± j1.89	+1.78 ∓ j3.66	-.036 ∓ j.382	-.897 ∓ j1.04	+.001 ± j.003
13	-1.24	-1.22	+.039	-.076	-.001	+.023	
14	-.301 ± j11.4	-4.16 ± j7.42				-.146 ∓ j3.44	-.011 ± j.008
15	-15.7 ± j11.4	-10.5 ± j23.3				+.994 ± j10.0	+.249 ∓ j.254
16	-11.2 ± j377	-11.2 ± j377				+.003 ± j.001	+.004 ± j.001
17	-15.3 ± j150	-14.1 ± j150	+8.91 ∓ j117.	+3.93 ± j82.9	-14.1 ∓ j.186	+1.20 ± j.701	+.721 ∓ j.798
18	-15.8 ± j604	-15.8 ± j604	-.383 ± j114.	+11.5 ∓ j80.1	-13.7 ∓ j1.00	+0.00 ∓ j.001	+0.00 ∓ j.001
19	-19.4 ± j19.0	-17.6 ± j24.6	+.755 ∓ j.839	-2.84 ± j.289	-.183 ± j1.23	+1.22 ± j5.25	+.232 ∓ j.128
20	-30.3	-31.5	-.233	+.629	-.107	-.846	-.042
21	-30.3	-32.1				-.995	-.051
22	-55.3 ± j3710	-55.3 ± j3710	+.104 ± j288	-1.63 ∓ j445.	-5.14 ± j.102		
23	-59.8 ± j2850	-59.8 ± j2850	+.164 ± j3.51	-3.14 ∓ j475.	-5.85 ± j.155		
24	-85.0	-85.1	-5.36	+8.27	+.606	-.350	-.093
25	-91.6	-92.0				-.319	-.094
26	-500.	-500.	+.326	-.361	-.054	-.020	-.018
27	-500.	-500.				-.054	-.049
28	-1790 ± j377	-1790 ± j377.	+.466 ± j.004	+20.5 ± j.040	-.444 ∓ j.002		

Table 4.5 Eigenvalue sensitivities for two thermal machine power system

A more detailed discussion of the results is given in [112].

A variety of other predictions using eigenvalue sensitivities suggest themselves. In particular, the effect of slight unbalance (either in system parameters or in loading) on the 'double resonant peak' phenomenon should be investigated.

4.7 Summary

The fundamentals of torsional oscillations in closely coupled identical generating units have been demonstrated using the simplest possible four mass equivalent. This can be extended to explain the double resonant peaks occurring in multi-unit generating stations.

It has been shown that for an N unit station all the natural frequencies and mode shapes can be predicted using two analyses of a single generating unit system. This result disproves an earlier prediction which stated N different modes corresponding to each frequency of the shaft system would result.

Analysis of the frequency and time responses presented indicate that significant experimental data may be obtained using relatively simple station tests. The results will help in the selection of variables for measurement and in choosing the amplitude of the test disturbance.

Finally, some results showing eigenvalue sensitivity predictions have been presented, which emphasise the 'main system' and 'inter-unit' dynamic properties of the overall system. A condensed version of the material in this chapter can be found in Reference (119).

CHAPTER 5

EVALUATING ALTERNATIVE MODELS

5.1 Introduction

In this chapter the evaluation of alternative dynamic stability models is considered. The basis for the evaluation is the accuracy of stability prediction as reflected by critical eigenvalues. The computational advantages of using reduced order models can be readily appreciated - the computational cost of an eigenvalue evaluation $\alpha\ N^3$, where N is the system order. Apart from cost consideration, the use of reduced order models may be mandatory due to computer storage limitations. Also as applied to eigenvalue analysis - the interpretation and tractability of results is extremely difficult for high system order.

The obvious disadvantage in using a lower order model is that erroneous predictions, on the stability of the system, may be made if all dynamic processes contributing significantly to critical modes of stability are not included.

Though a number of authors have considered the problem of setting up overall system models (Section 1.6) and have allowed for flexibility in representation, they do not justify the degree of complexity chosen. On the other hand, for transient stability studies a number of attempts have been made to evaluate alternative models [114] [128]. The absence of comparisons of alternative dynamic stability models was the motivation for the work described in this chapter.

In this chapter a 'Benchmark' model is described which embraces all dynamical effects normally considered of importance including mechanical shaft and electrical network dynamics. This model can be used to predict all the usual aspects of power system dynamic stability (Section 1.3). By systematically reducing the 'Benchmark' model, a family of reduced order models which span the range of models currently employed in the industry, is developed.

The effects of using alternative reduced order models is studied by comparing their stability prediction with that of the Benchmark model. This will yield a basis for the selection of modelling complexity and also will be shown to allow identification sources of instability.

In Section 5.2, a general discussion is given on the development of alternative models. The basis for the derivation of the reduced order models considered in this chapter, and the method of reduction is explained in Section 5.3. Practical examples, illustrating applications in a single machine-infinite bus and multimachine problems, are presented in Sections 5.4 and 5.5. A summary of the chapter is given in Section 5.6.

An expanded version of the material presented in this chapter is available in Reference (113).

5.2 Development of Alternative Models

Within the context of the multilevel system description adopted in this thesis, there are four basic possibilities in the development of alternative models for the entire system.

1. full subsystem model - full system model (Benchmark)
2. full subsystem model - reduced system model
3. reduced subsystem model - full system model
4. reduced subsystem model - reduced system model

By full subsystem model is meant a model in which all significant dynamic properties are represented. Full system model (in the context here) implies a model which is a union of all subsystem models - irrespective of whether they are 'full' or 'reduced'.

The first possibility is a luxury which is only possible for relatively small systems (< 10 machines) and would be evaluated rarely due to the prohibitive computation cost. A more economical possibility is the second listed where even though full subsystem models are employed, a reduced system model (which approximates the full model in some sense) is used in subsequent analysis and control studies. The third possibility is the most widely used in the industry - the basis for the derivation of the reduced subsystem models being the neglect of certain physical processes (eg. exciter action, governor action). The fourth possibility in the development of alternative system models is significant in control applications for very large systems.

Obviously only methods 1 and 2 will guarantee a correct stability prediction. Once reduced sybsystem models are employed, then either an explicit (#3) or an implicit (#2) evaluation, may fail to reveal any unstable interaction between the individually stable subsystems. Considerable engineering judgement and experience is necessary before confident use of approaches 3 and 4 can be expected.

The main techniques for obtaining reduced order models may be classified according to the sense in which the reduced order model approximates the original, [123], [124].

(a) state variable grouping technique

(b) eigenvalue grouping technique

(c) matching reduced order and original model responses

  (i) frequency response

  (ii) time response

(d) transfer function approximations

The state variable grouping technique is implicit in the derivation of the classical time constants of synchronous machines and in the derivation of the 'constant voltage behind transient reactance' model for synchronous machines.

The use of eigenvalue grouping techniques has received most successful application in the derivation of electromechanical equivalents for sections of power systems [45]. A number of authors have also proposed this method in the derivation of low order models for control applications [64], [65].

The most widespread examples of frequency response matched low order models is the use of synchronizing and damping torque coefficients (4). Though these coefficients have been traditionally used in analysis, recently they have been widely used in defining 2nd order models which are employed in the design of power system stabilizers. Another example of frequency response matching is in the pioneering work at Ontario Hydro fitting low order rotor circuit models to experimental data [93].

Well known examples of transfer function approximation in the industry are the development of approximate expressions for operational impedances in terms of open circuit and short circuit time constants [125].

5.3 Basis for the Derivation of Reduced Order Models

The general methods of model reduction summarized in the previous section were cited with the intent of placing in perspective the approach adopted in this chapter. As mentioned previously, the object here is to evaluate the alternative models normally employed in practice. These alternative models could have been individually programmed but for greater flexibility it shall be shown that they may be obtained by systematically reducing the benchmark model subject to the assumption normally applied in practice. Such assumptions are the neglect of:

(1) Torsional oscillation effects (shaft dynamics)
(2) Stator and network dynamics
(3) Individual turbine stages
(4) Individual rotor circuits
(5) Stabilizer action
(6) Exciter action

(7) Field demagnetization and all damper representation

Application of the above assumptions results in a family of models which span the whole range of modelling complexity from the benchmark model to the simplest possible case of a 2nd order representation per machine with constant field flux linkage.

Consider the general state space equation in terms of partitioned state variables:

$$\begin{bmatrix} \dot{\underset{\sim}{x}}_1 \\ \dot{\underset{\sim}{x}}_2 \\ \dot{\underset{\sim}{x}}_3 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_1 \\ \underset{\sim}{x}_2 \\ \underset{\sim}{x}_3 \end{bmatrix} \qquad (5.1)$$

Say $\dot{\underset{\sim}{x}}_1 = \underset{\sim}{0}$ and $\underset{\sim}{x}_3 = \underset{\sim}{0}$, then we obtain the reduced representation:

$$\underset{\sim}{x}_2 = \hat{A}_{22}\, x_2 \qquad (5.2)$$

with

$$\hat{A}_2 = A_{22} - A_{21} A_{11}^{-1} A_{12}$$

It can be seen that if a suitable choice of state variables is chosen, all and any combination of the above assumptions may be expressed mathematically as $\dot{\underset{\sim}{x}}_1 = 0$ and $\underset{\sim}{x}_3 = \underset{\sim}{0}$.

As a specific example, if the derivative of the network states are set to zero, the differential equations describing the network become algebraic and network dynamics are neglected. If the differential angle and speed between each turbine mass and the synchronous machine rotor are chosen as states and set to zero, a flat mode shape will be imposed on the mechanical shaft system, and shaft dynamics will be neglected. By setting the derivative of specific rotor circuit flux linkage to zero the current in that circuit is set to zero and the circuit is effectively removed.

Table 5.1 shows the assumptions considered with the relevant state derivatives and states to be equated to zero. The saving in system order per machine is also shown.

It is important to emphasise the extremeness of the reduction approach. The appropriate processes are completely removed and no attempt is made to modify the remaining system parameters so as to compensate (in some sense) for the removed system portion. (This can be contrasted with techniques which retain dominant eigenvalues) However this reduction process does yield models with physically identifiable, as distinct from mathematical, assumptions.

5.4 Practical Application - Single Machine System

A single machine-infinite bus configuration is considered in this section. This system has been chosen to depict three aspects of power system stability - specifically two aspects which involve shaft and network dynamics and one aspect which involves the effect of a speed sensitive power stabilizer on stability.

A line diagram of the system is shown in Figure 5.1. The long transmission line is compensated by a series capacitor, X_C, and capacitor X_S represents line charging capacitance. R_L, X_L represent a local impedance load and the generator

#	effect neglected	states or state derivatives set to zero	typical order reduction machine
1	torsional oscillation	displacements and slip (relative to rotor) of equivalent inertias	8
2	stator and dynamics	derivative of network states and stator currents	6
3	individual turbine stages	turbine power and states associated with reheater and crossover piping delays	4
4	individual rotor circuits	derivative of rotor circuit flux linkage	4
5	stabilizer action	components of stabilizing signal	2
6	exciter action	voltage sensor output and field voltage	2
7	field demagnetization and all damper representation	field flux linkage and derivative of all remaining rotor circuit flux linkages	6

Table 5.1 State constraints for reduced order models

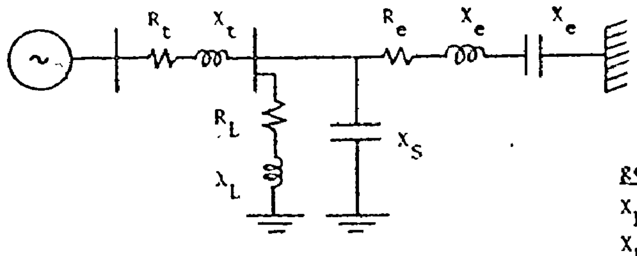


Fig. 5.1 Line diagram for single machine-infinite bus system

subsystem	model classification
generator	Z_5
shaft	S_2
turbine-governor	G_1
exciter	E_1
network	N_1

Table 5.2 Subsystem description for "Benchmark model"

<u>generator</u>. X_d = 1.82, X_{md} = 1.65, X_f = 1.92, X_{kd1} = 1.85, X_{kd2} = 1.76, X_q = 1.73, X_{mq} = 1.59, X_{kq1} = 3.76, X_{kq2} = 2.05, X_{kq3} = 1.71, r_a = .0015, r_f = .000666, r_{kd1} = .00430, r_{kd2} = .120, r_{kq1} = .00891, r_{kq2} = .0224, r_{kq3} = .117

<u>exciter</u> K_e = 212, τ_e = .002, τ_v = .01

<u>stabilizer</u> K_Q = 9.6, τ_Q = 1.41, τ_a = .121, τ_x = .033

<u>network</u> R_t = 01, X_t = .1, R_L = 5., X_l = 1., R_e = .02, X_e = 3, X_s = 20., X_c = 1

<u>governor and turbine</u> K_g = 25., F_{hp} = [illegible], F_{ip} = .3, F_{lp} = .15, τ_{ch} = .3, τ_{co} = .2, τ_{sr} = .2, τ_{sm} = .3, τ_{rh} = 7.

<u>mechanical system</u> M_1 = 1.71, M_2 = 2.38, M_3 = 2.31, M_4 = .464, M_5 = .248, D_1 = 0, D_2 = .249, D_3 = .259, D_4 = .255, D_5 = .25[illegible], S_{12} = 62.3, S_{23} = 75.6, S_{34} = 48.4, S_{45} = 21.6

Table 5.3 Base case parameters for single-infinite bus system

Assumptions	None	1	1,2	1,2,3a	1,2,3	1,2,3,4a	1,2,3,4b	1→3,4b,5
Shaft	-.301±j276 -.184±j191 -.423±j151 -.367±j103							
Stator/ Network	-9960±j377 -750 ±j4920 -752 ±j4160 -14.1±j527 -3.55±j228	-9960±j377 -750 ±j4910 -752 ±j4160 -14.2±j527 -3.66±j229						
Machine/ Exciter-stabilizer	-501. -227. -149. -101. -26.1 -10.9 -3.06±j3.08 -2.63±j11.8 -3.68 -.778	-501. -228. -149. -101. -25.8 -10.9 -3.07±j3.07 -2.63±j11.9 -3.68 -.778	-501. -223. -158. -97.2 -25.7 -10.9 -3.04±j3.06 -2.54±j12.0 -3.70 -.779	-501. -223. -157. -97.2 -25.7 -10.9 -3.05±j3.06 -2.54±j12.0 -3.70 -.779	-501. -223. -158. -97.2 -25.6 -10.9 -3.00±j3.02 -2.55±j11.9 -3.63 -.778	-501. -97.8 -25.9 -11.2 -3.05±j3.02 -2.62±j11.9 -3.63 -.778	-501. -95.6 -13.0 -9.62 -5.42±j9.65 -3.69 -.777	-501. -95.4 -7.08 +1.33±j8.89 -3.66
Turbine-Governor	-.143 -2.64 -4.52 ±.642 -4.86	-.143 -2.64 -4.51±j.639 -4.86	-.143 -2.63 -4.52±j.676 -4.86	-2.58 -4.47±j.762				
Order	34	27	17	15	12	10	8	6

(3a: all but HP turbine stage neglected
4a and 4b: 1 and 2 rotor circuits neglected, respectively)

Table 5.4 Eigenvalues of reduced order models for system with inherent negative synchronizing torque.

transformer and tie line is represented by R_t, X_t. The 'Benchmark model' for this system has subsystems defined in Table 5.2 and the base case parameter values are listed in Table 5.3.

5.4.1 Insufficient Synchronizing Torque

This case represents a system which is unstable in the absence of automatic control. The equilbrium load angle (δ) is greater than 90° and consequently the system fails to meet the classical steady state stability criterion.

The first column of Table 5.4 shows that the overall system is stable.

The frequency and damping characteristics of individual portions of the representation are known (to a good approximation) from previous experience involving eigenvalue sensitivity and eigenvector interpretation. Consequently the eigenvalues may be classified according to the portion of the system to which they primarily correspond. (The subsequent reduction to the lower order models, also provides a basis for the initial classification.) When the model is modified so that the shaft states are eliminated, the oscillatory modes at 103, 151, 191 and 276 rads/sec. (16, 24, 30 and 44 hertz) are removed with little effect on the other modes - specifically, the mode at 11.8 rads/sec (1.9 hertz) which represents oscillation in the main torque-load angle loop. Elimination of the network and stator transients removes the next five oscillatory modes. The heavily damped 377 rad/sec mode represents flux decay in the circuit comprising the stator and local load. The remaining two modes reflect oscillations (as seen from a synchronously rotating frame) due to the capacitor representation. Removal of the stator and network transients produces negligible loss of accuracy on the remaining modes.

In columns 4 and 5 are shown the effects of neglecting the contribution of all but the HP turbine stage and all governing actions, respectively. Again little loss of accuracy in the remaining modes, results.

The effect of rotor representation simplification can be seen from columns 6 and 7. It is seen that in the presence of the static exciter-stabilizer, the neglect of additional rotor circuits results in increased damping in the main torque-load angle loop. The reason for this seems to be the result of change in phase lag in the torque-reference voltage transfer function which will affect stabilizer operation. Finally, column 8 shows the effect of removal of the stabilizer. The damping in the torque-load angle loop is negative indicating oscillatory instability.

If we proceed further and eliminate excitation effects (constant field voltage) we obtain the following eigenvalues:

$$- .364 \pm j\, 8.05, \; - .365, \; + .020$$

The positive eigenvalue indicates negative steady-state synchronizing coefficient corresponding to a load angle greater than 90^{o}. The above eigenvalue can be compared with those in column 8 where the smallest real eigenvalue is -3.66 indicating a strong synchronizing coefficient introduced by the exciter.

Finally, if the field flux linkage is constrained to be constant and all q-axis damper effects are neglected we obtain as eigenvalues:

$$-.154 \pm j\,8.05$$

This results in the classical 'constant voltage behind transient reactance' model. The above numerical result is in agreement with that obtained using equation 1 of deMello and Concordia'a paper [4].

5.4.2 Subsynchronous Resonance

The data is as given in Table 5.3 with the exceptions: $X_c = .625$ and $X_e = 1.0$ (pu). The series capacitive compensation was chosen to cause resonance at 123 rads/sec (20 hertz) which strongly interacts with the shaft mode at 103 rads/sec (16 hertz). By comparing column 1 from Table 5.5 with the corresponding column of Table 5.4 we can see that the interaction results in two modes - one of which is unstable and the other highly damped. This behaviour has been previously reported in [16]. If the shaft states are eliminated, a stable situation is predicted. Also if the stator-network transients are neglected, a stable system results and the damping in the shaft modes is approximately the same as in the case analysed in Subsection 5.4.2.

5.4.3 Stabilizer Introduced Shaft Instability

The system data is again as listed in Table 5.3 with

Assumptions	none	1	2
Shaft	-.301 ± j276		-.301 ± j276
	-.130 ± j190		-.165 ± j191
	-.366 ± j151		-.425 ± j151
	+.078 ± j103		-.358 ± 102
Stator/ Network	-9.74 ± j122	-9.39 ± j123	
	-19.8 ± j365	-19.8 ± j635	
	-810 ± j3240	-810 ± j3240	
	-806 ± j3990	-806 ± j3990	
	-9860 ± j377	-9860 ± j377	
Machine/ exciter-stabilizer	-501.	-501.	-501.
	-217.	-217.	-217.
	-130.	-130.	-146.
	-104.	-104.	-95.1
	-26.7	-26.5	-26.3
	-9.75	-9.75	-9.68
	-3.10 ± j4.32	-3.11 ± j4.31	-3.05 ± j4.25
	-2.76 ± j10.1	-2.75 ± j10.2	-2.61 ± j10.5
	-3.66	-3.66	-3.67
	-.756	-.757	-.757
governor	-.142	-.142	-.142
	-2.53	-2.53	-2.51
	-4.62 ± j.881	-4.62 ± j.882	-4.64 ± j.927
	-4.87	-4.87	-4.87
Order	35	27	25

Table 5.5 Eigenvalues of reduced order models for system exhibiting torsional subsynchronous resonance

Assumptions	none	5	1
Shaft	-.302 ± j276	-.302 ± j276	
	+.050 ± j192	-.160 ± j191	
	-.248 ± j152	-.354 ± j152	
	-.239 ± j104	-.349 ± j103	
Stator/ Network	-9960 ± j377	-9960 ± j377	-9960 ± j377
	-750 ± j4920	-750 ± j4920	-750 ± j4920
	-752 ± j160	-752 ± j4160	-752 ± j4160
	-14.2 ± j527	-14.1 ± j527	-142 ± j527
	-3.66 ± j227	-3.56 ± j228	-3.66 ± j229
Machine/ exciter-stabilizer	-501.	-501.	-501.
	-229.	-227.	-228.
	-149.	-149.	-129.
	-101.	-101.	-101.
	-25.1		-25.6
	-10.9	-10.9	-10.9
	-3.04 ± j3.03	-4.20 ± j3.92	-3.04 ± j3.03
	-2.58 ± j11.8	+.398 ± j8.81	-2.74 ± j12.0
	-3.69	-3.69	-3.69
	-.782		-.782
governor	-.143	-.143	-.143
	-2.64	-2.74	-2.64
	-4.52 ± j.641	-4.44 ± j.514	-4.52 ± j.644
	-4.86	-4.85	-4.86
Order	35	33	27

Table 5.6 Eigenvalues of reduced order models for system exhibiting stabilizer introduced shaft instability

the exception that the speed signal for the stabilizer is taken from the rotor end of the shaft (as distinct from midway between the two low pressure turbine stages). The results in Table 5.6 show an instability in the 30 hertz shaft mode. This can be attributed to stabilizer action as when the equations are constrained so that the stabilizer effect is removed, the shaft system is stable (Column 2).

It can be seen that though the stabilizer introduces significant damping in the unstable torque-load angle (rotor angle) oscillation at 8.8 rads/sec (1.4 hertz) it causes a negative damping in the shaft natural frequency at 191 rads/sec (30 hertz). If the shaft dynamics were neglected, as is common, there is little loss in accuracy in the remaining modes but, of course, the instability would be overlooked.

5.5 Practical Application - Multimachine System

In this section the methods of Section 5.3 are applied to the hydroelectric power system which has been analysed in Section 3.4. The 'Benchmark model' for this system is defined and the parameter values listed in Table 3.11.

Two examples are considered - one with uniform reduction (i.e., all generating unit models reduced to some extent) and one with non-uniform reduction.

Table 5.7 shows the results for eight alternative models. The results corresponding to the 'Benchmark model' are listed in column 1 and are grouped according to the portion

		1	2	3	4	5	6	7	8
Assumptions: None			2	1, 2, 3	1, 2, 3, 6a	1→4 , 6a	1→5 , 6a	1→6	1→7
n		98	56	32	28	20	16	12	8
ROTOR		-1.47±j6.44	-1.46±j6.46	-1.52±j6.60	-1.52±j6.60	-1.4.±j6.62	+.159±j6.53	-.127±j5.78	-.052±j5.79
		-2.37±j15.1	-2.13±j15.3	-2.14±j15.7	-2.40±j15.5	-2.37±j15.1	+.220±j11.8	-.097±j11.7	-.071±j11.7
		-6.85±j18.3	-6.27±j18.7	-6.16±j19.3	-6.54±j18.3	-4.95±j12.0	+.156±j10.5	-.088±j10.3	-.053±j10.3
		-11.0 ±j18.9	-9.67±j18.6	-8.85±j18.9	-9.94±j15.4	-7.13±j15.7	+.204±j10.9	-.116±j10.7	-.072±j10.7
MACHINE/EXCITER-STABILIZER		-.642	-.642	-.642	-.642	-.642		-.205	
		-.660	-.660	-.660	-.660	-.660		-.424	
		-.672	-.672	-.672	-.672	-.671		-.436	
		-.675	-.675	-.674	-.674	-.674		-.484	
		-6.56	-6.46	-6.56	-6.40	-6.38	-12.1		
		-7.19±j10.5	-7.79±j10.6	-8.48±j10.6	-12.2	-11.0	-31.7		
		-12.8±j3.85	-12.9±j3.08	-13.0±j2.88	-9.10±j12.0	-61.7	-55.5		
		-16.1±j32.8	-15.7±j32.5	-15.7±j32.4	-19.1±j30.1	-81.9	-81.3		
		-23.2	-23.3	-23.7	-24.3	-35.0±j48.8	-36.7±j49.3		
		-35.6	-35.7	-35.8	-35.8	-34.9±j10.1	-41.9±j22.2		
		-42.4	-42.4	-42.5	-42.5				
		-44.9	-44.7	-45.0	-45.0				
		-50.5	-50.4	-51.0	-51.0				
		-76.1	-76.6	-76.8	-86.0				
		-84.3	-84.9	-85.0	-91.4				
		-87.5	-88.4	-88.5	-93.4				
		-90.5	-90.9	-90.9	-95.1				
		-205.	-205.	-205.	-15.6				
		-208.	-208.	-208.					
		-209.	-210	-210					
		-212.	-213	-213.					
TURBINE-GOVERNOR		-.008	-.008						
		-.008	-.008						
		-.009	-.009						
		-.010	-.010						
		-.642	-.642						
		-.660	-.660						
		-.672	-.672						
		-.675	-.675						
		-2.93	-2.94						
		-2.97	-2.97						
		-3.05	-3.05						
		-3.34	-3.34						
		-22.0	-22.0						
		-22.1	-22.1						
		-22.3	-22.3						
		-22.3	-22.3						
SHAFT		-.135±j64.2	-.163±j64.2						
		-.264±j65.3	-.289±j65.3						
		-.288±j67.3	-.312±j67.3						
		-.395±j65.9	-.427±j65.9						
NETWORK-STATOR		-1.64±j377							
		-4.25±j377							
		-8.67±j377							
		-9.65±j377							
		-10.7±j2510							
		-11.9±j1760							
		-12.5±j4390							
		-12.8±j3640							
		-13.0±j5960							
		-13.1±j5210							
		-13.2±j9290							
		-14.2±j8540							
		-14.3±j377							
		-16.0±j377							
		-16.9±j8520							
		-17.5±j11300							
		-17.6±j10520							
		-18.0±j7760							
		-18.3±j17900							
		-18.3±j17200							
		-21.0±j377							

Table 5.7 Eigenvalues of reduced order model for four machine hydroelectric system

of the representation to which they primarily correspond. The eigenvalue sensitivity study of Section 3.4.2 helps in the grouping. In column 2 are shown the eigenvalues when network-stator transients are neglected. It can be seen that neglect of these transients results in a slightly conservative estimate of the damping in the intergenerating unit oscillations. Shaft dynamics, governor action and stator network transients, effects are removed in column 3 - again with little loss in accuracy in the remaining modes.

The additional assumption of neglecting τ_e (exciter time constants) is considered in column 4. It can be seen that this results in little loss in accuracy in the two lower frequency rotor modes. The erroneous prediction in the higher frequency rotor modes seems to be caused by the larger error in phase lag in the reference voltage-electric torque transfer function at higher frequencies.

In column 5, the effects of damper representation omission in addition to the previous assumptions, can be seen. The results show that the higher frequency rotor modes are significantly different from those in column 1. This again seems to be due to the sensitivity of stabilizer action to the phase lag in the 'reference voltage-electric torque' transfer function - a result which is in agreement with the predictions for the single machine case (Section 5.4.1).

The stabilizer gain has been set to zero for the case shown in column 6. The striking result is that all rotor modes are unstable. This is a well known property (destruction of inherent machine damping) of high gain, fast response exciters.

In column 7, the additional constraint of constant field voltage is considered. The frequencies of the rotor modes are not appreciably different from those in column 6, but the instability is removed. Also notice the presence of the four relatively long time constants for this case. These represent modes associated with rotor demagnetization and their small value is indicative of poor synchronization.

Finally, in column 8 is shown the results of the so-called constant voltage behind the transient reactance model. It is gratifying to note that the frequencies of rotor oscillation in the absence of the stabilizers (column 6) is approximately predicted by this simple model (column 8). The reason for the differences between these predictions and those of the 'Benchmark model' (column 7) is the large component of synchronizing torque introduced by the stabilizers at the higher frequencies of the intergenerating unit rotor oscillations.

In the previous example, all generating units were reduced to the same extent. In practice, one is generally concerned with analysing the stability of a specific portion of the system. In such cases it may be desirable (and indeed often mandatory) to replace generating units outside the 'study area' by simplified models while retaining a relatively detailed model for the 'study area'. Traditionally distant machines have been replaced by 'constant voltage behind transient reactance' models. Less distant machines are modelled in intermediate detail and local generating units are modelled in detail.

The reduction scheme of Section 5.3 lends itself very well to this application. Let us suppose that we wish to study the effect on the stability of generating unit #1, of equipping it with a stabilizer. Generating units 2, 3 and 4 are equipped with static exciters but do not have stabilizers. The traditional approach to this problem would be to model these machines as constant voltages behind transient reactance. This can be readily achieved using the model reduction scheme presented.

The results are shown in Table 5.8. In column 1 is shown the reference case where excitation, damper windings, and field effects for all machines, were considered. In column 2 are shown the results when the model of column 3 of Table 5.7 is reduced by eliminating exciter, damper winding and field states in generating units 2, 3 and 4. It can be seen that the frequency of all rotor modes and the damping in the mode of primary interest at 10.7 rad/sec (machine #1 swinging against the remaining three) are quite close. The erroneous predictions in the 11.1 and 12.0 rad/sec rotor modes are not of serious consequence since we are not primarily interested in these modes.

Rather than selling the merits of the chosen approximation in the above application, the motivation for the inclusion of the example was to demonstrate the application of the developed reduction technique in deriving a multiplicity of low order models from a single high order system where corresponding portions of the representation are reduced to

	Column 1	Column 2
	-212.	
	-209.	-210.
	-207.	
	-205.	
	-90.8	
	-88.4	-87.1
	-84.5	
	-75.9	
	-51.1	-47.3
	-45.0	
	-42.5	
	-35.8	
exciter modes	-16.2 ± j 32.0	-15.2 ±j30.3
exciter modes	-18.7 ± j 20.3	
exciter modes	-18.9 ± j 16.3	
exciter modes	-16.9 ± j 4.32	
rotor modes	-.338 ± j 6.67	-.456 ±J6.33 ← system mode
rotor modes	-1.78 ± j 10.7	-1.49 ±j10.3 ← machine #1
rotor modes	-.242 ± j 12.0	-.091 ±j11.7 against group
rotor modes	-.268 ± j 11.1	-.082 ±j10.7
	-.642	-.642

Table 5.8 Eigenvalues for four machine system representing three machines as constant voltages behind transient reactances.

different extents.

5.6 Summary

It has been shown that the multiplicty of power system models associated with identifiable physical assumptions regarding the presence or absence of certain effects can be systematically obtained from a single high order model (Benchmark model).

This facilitates the choice of modelling complexity in any particular study because the cost of using any model can be weighed against the accuracy of the resulting prediction. It has been shown that sources of instability can be readily identified by systematically isolating certain effects.

The results presented emphasize that the complexity of modelling required is highly problem dependent. For example, the inclusion of shaft dynamics may have anything from an insignificant effect, to a completely erroneous effect on the stability prediction depending on the characteristics of any series compensation and/or speed sensitive power stabilizers. However, once the 'Benchmark model's' characteristics are known, effects not contributing significantly to the modes of interest can be eliminated and more economical reduced order models can be used in simulations over ranges of parameter variation.

Rather than to obtain definitive results for any specific system, the emphasis in this chapter has been on presenting a method by which families of low order models, each with normally applied assumptions, can be easily determined from the 'Benchmark model'. It is hoped that this method will form the basis of future studies in which a wide range of parameter variation and loading conditions will be considered.

CHAPTER 6

CONCLUSIONS

The main aspects of power system dynamic stability including aspects involving the interactions of the mechanical shaft and electrical network dynamic subsystems have been discussed. State-space equation formulations for digital computer applications have been reviewed. A formulation has been presented which includes representation of mechanical shaft and electric network dynamics. A key feature of the development is the choice of stator current referred to a synchronously rotating frame and rotor flux linkages, as states. This results in compatibility for the inclusion of stator-network transients, between the network and synchronous and asynchronous machine equations.

The overall system model is structured so that a wide variety of subsystem model types and complexities may be included. An important requirement in the model development was the facility for ease of differentiation of the overall coefficient matrix - this leads to the determination of eigenvalue sensitivities with respect to system parameters.

The approach presented is conceptually simple in that the equations for each generating unit are developed separately. Besides allowing the dynamics of individual generating units to be independently checked, this simplifies the task of system update and modification.

Sample systems have been analysed using the formulation presented and for those aspects of dynamic stability predictable by alternative formulations, consistent results are obtained. For those aspects of stability (involving shaft and network dynamic effects), not predictable by alternative models, results in agreement with operating experience are obtained.

The methods developed have been applied in the analysis of interacting shaft dynamics in a general number of closely coupled turbine-generators. It has been shown that, in contradiction to a previous prediction, two sets of closely coupled modes corresponding to each natural frequency of the shaft system, result in the general case. The determination of natural frequencies and mode shapes in terms of two equivalent turbine generators is explained. The shift in shaft natural frequencies which results from the interaction of close turbine-generators which has been predicted, is in agreement with test results. Predicted responses showing the stimulation of shaft oscillations are presented which hopefully will provide the basis of future experimentation.

A technique has been presented for the reduction of a high order system model (Benchmark model) which embraces all effects usually considered of importance, to the family of models currently used in the industry. This provides a basis for the choice of modelling complexity since a variety of models (each with identifiable assumptions and corresponding to industry standard models) may be readily developed from a single model and subsequently compared. The method has also been shown to be very useful in analysing the dynamics of complex systems in that, by systematically isolating specific effects, sources of instability may be identified.

The specific contributions are considered to be:

(1.) Development of systematic formulation for the construction of a state-space model for power system dynamic stability where network-stator and mechanical shaft dynamics are included.

(2.) Presentation of a technique for the development of a multiplicity of low order models, each with identifiable physical assumptions, from a single high order power system model.

(3.) Determination of the dynamic properties of closely coupled identical turbine-generators.

(4.) Insights into the interpretation of eigenvalues as they reflect various aspects of dynamic stability, results from the analyses presented.

The system models developed in this thesis will permit other investigators to study a wide variety of multi-machine power system dynamic problems. In particular, situations involving subsynchronous resonance and/or asynchronous motor load interactions may be analysed as well as all currently investigated problems involving electric machine and system dynamics.

Specific topics which seem especially worthwhile are:

(a) In the thesis, a hierarchical modelling approach has been employed in defining the overall system model. A variety of techniques are currently available for obtaining low order equivalents for higher order systems (123), (124). It would be interesting to investigate how the predicted dynamics of the overall system vary as alternative low order models are used to describe the individual subsystems comprising the overall system.

(b) It has been shown that the choice of D, Q stator currents of electrical machines facilitates the inclusion of network-stator transients. Furthermore, it has been shown (Chapter 5) that setting the derivative of the stator currents to zero yields acceptable accuracy in eliminating network stator transient effects. A comparison of this accuracy with that obtained using the traditional approach (114) in which the derivatives of d-q stator flux linkages are set to zero, is desirable.

(c) Shaft dynamics in closely coupled identical generators has been investigated. In practice there may be small design and/or loading differences between such closely coupled generators. Investigation of the interacting shaft dynamics in such situations is an important task. Eigenvalue sensitivity methods seem to be a logical approach in this application.

(d) The modelling and formulation structure presented in the thesis, has been based on the premise of 'dynamic' or small signal behaviour. It would be useful to determine how the hierarchial modelling approach can be accommodated within a transient stability algorithm. Specifically, such a development would permit analysis of the 'transient' aspects of torsional subsynchronous oscillations and subsequently lead to estimates of shaft life expenditure following severe faults.

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APPENDIX A

SUBSYSTEM MODELS

In this Appendix, state variable equations for the individual subsystems comprising a power system will be presented. As mentioned in Section 1.6, the development of the individual models is beyond the scope of this thesis. In Section 1.6, appropriate references for the modelling and analyses of these subsystems, were cited. The block diagram and/or equation descriptions have been taken from these references. The main reason for the inclusion of the models here (apart from the sake of completeness) is that the equations here have been all arranged in state-space form. Furthermore, the states, input and output variables have been chosen so that the overall model can be efficiently (minimum number of algebraic equations) built up in terms of subsystem models.

A1 Synchronous Machines

The development of state space representation for synchronous machines has been considered by a number of authors - in particular by Anderson [25]. In previous studies one of the two approaches has been adopted regarding the representation of network-stator transients. The former is to eliminate these effects by equating the derivatives of stator flux linkages to zero and subsequently to use algebraic network equations [109]. The second approach which has been previously applied only in simple single machine problems is to chose stator current (referred to a rotor frame) as states and to subsequently include the effects of an oscillating frame in the network differential equation [25]. Retention of stator flux linkages as states and the use of algebraic network equations leads to erroneous predictions of network-stator transients.

In this appendix, a choice of synchronous machine equation system , is described which is compatible (i.e., can be readily joined) with network differential equations (referred to a common synchronously rotating frame). This facilitates the inclusion of network-stator transients in general multi-machine situations.

Humpage [126] summarized the practical reference frames for electrical machines including, in particular, a synchronously rotating frame for stator current and a rotor frame for rotor quantities. Stator currents were considered as algebraic variables as algebraic network equations were employed.

In the approach presented here, stator currents referred to a common synchronously rotating frame and the rotor fluxes referred to the machine rotor reference frame are chosen as states. The choice of stator currents (in the common frame) facilitates the combination of the electric machine and network equations. The choice of rotor fluxes (besides having clearer physical significance over currents in stability studies) simplifies the task of combining machine and network equations.

The equations describing the synchronous machine have been taken from [24]. The equations are assumed to be in incremental form - a variety of approaches may be considered in representation of machine saturation in determining equilibrium values of flux linkage and reactances. The method which is most commonly used is to assume only mutual reactance is affected by saturation, which permits the determination of saturation factor from open circuit characterisic [24].

The equations are arranged in matrix form in Figure A1. Notice that besides developing the basic state-space equations, the state transformations are included in one matrix equation. This form of equation development is consistent with the so-called P, Q R technique [83] and is used throughout the thesis. Upon reduction, the equations have the general form shown in Figure A2.

The above equations have been developed for a machine model with three rotor circuits per axis. This model is designated 'Synchronous Machine Model 'Z_5' in the text. The development of state space equations for machines with alternative rotor representation can be written by inspection using the above as a reference.

A2 Network Elements

Transmission system transformers and non-rotating electric loads are modelled as linear networks in this thesis. Transmission lines are modelled in terms of nominal π or T sections developed under the assumption that line sections are

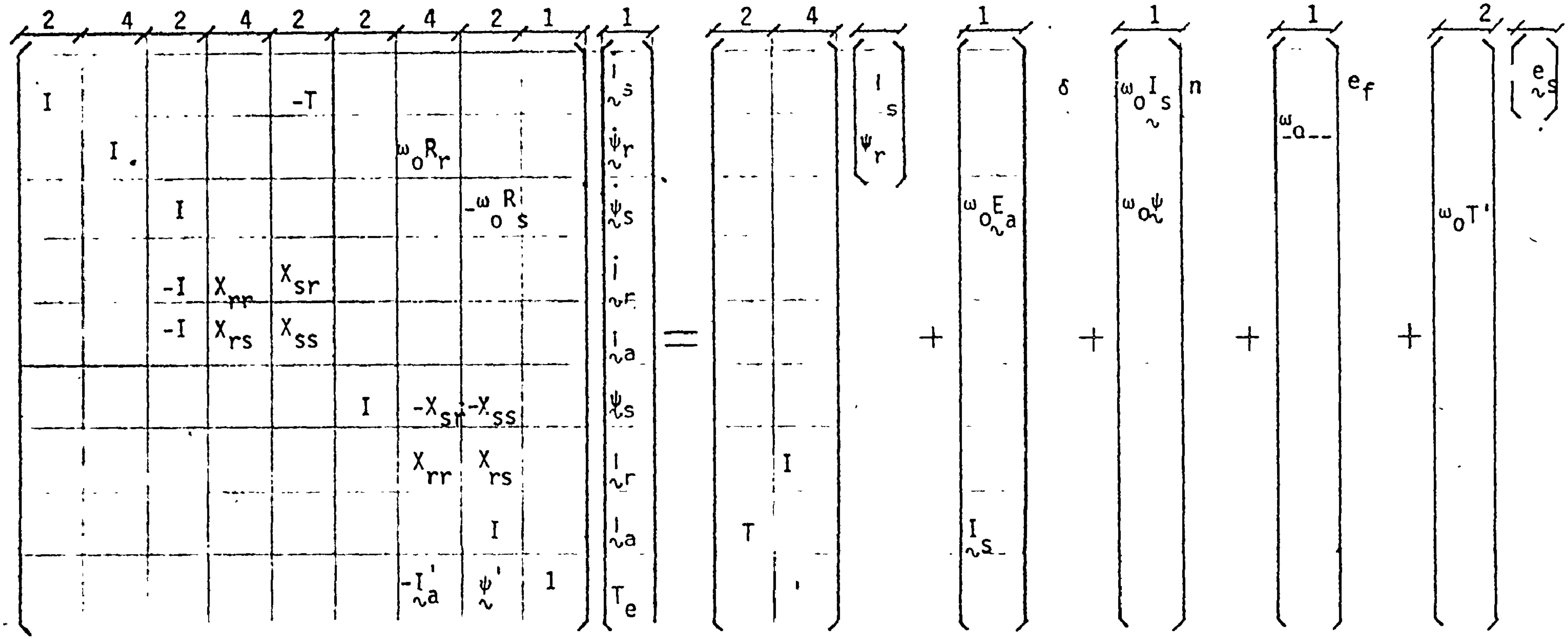


Fig. A1 Unreduced state space equations for synchronous machine

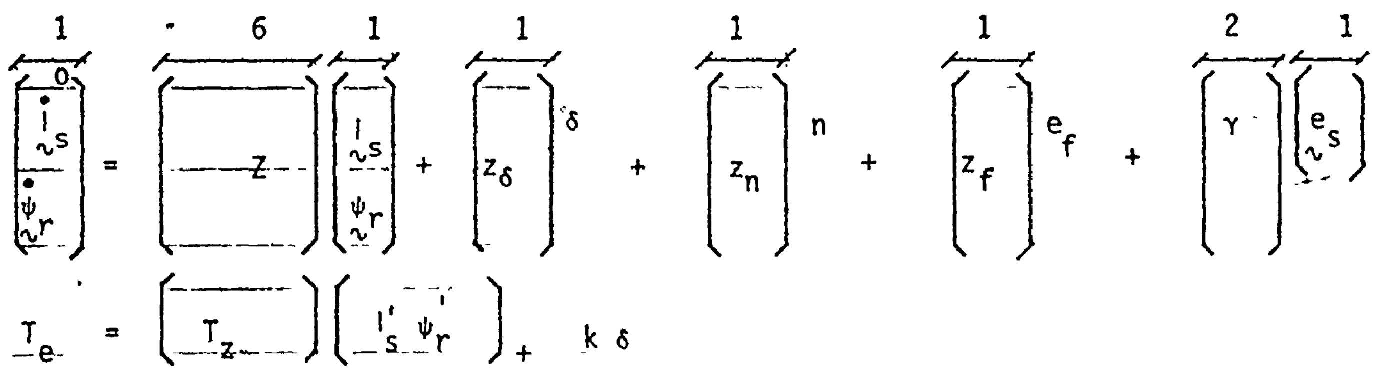


Fig. A2 Reduced state space equations for synchronous machine 'electrically short'. Since we are primarily interested in low frequency network effects (subsynchronous electrical resonance) the use of lumped equivalents which demonstrate good accuracy at 60 hertz are suitable. For 60 hertz, a line of 100 miles may be considered short [32]. The inductance, resistance and capacitance values are obtained by multiplying the value per metre by the appropriate line length.

As dynamic stability is limited to balanced conditions, an equivalent balanced two-phase system comprising independent two-axis equivalents can be obtained for the balanced three-phase system [116]:

resistive elements: $$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = R \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \qquad \text{(A1)}$$

inductive elements: $$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = \frac{X}{w_o} \begin{bmatrix} \dot{i}_\alpha \\ i_\beta \end{bmatrix} \qquad \text{(A2)}$$

capacitive elements: $$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{B}{w_o} \begin{bmatrix} \dot{e}_\alpha \\ e_\beta \end{bmatrix} \qquad \text{(A3)}$$

where α, β designate the uncoupled two-phase system.

Upon transforming the equation from a stationary reference frame to a synchronously rotating frame (Section 2.4.2) the equations for the two-phase system become interacting:

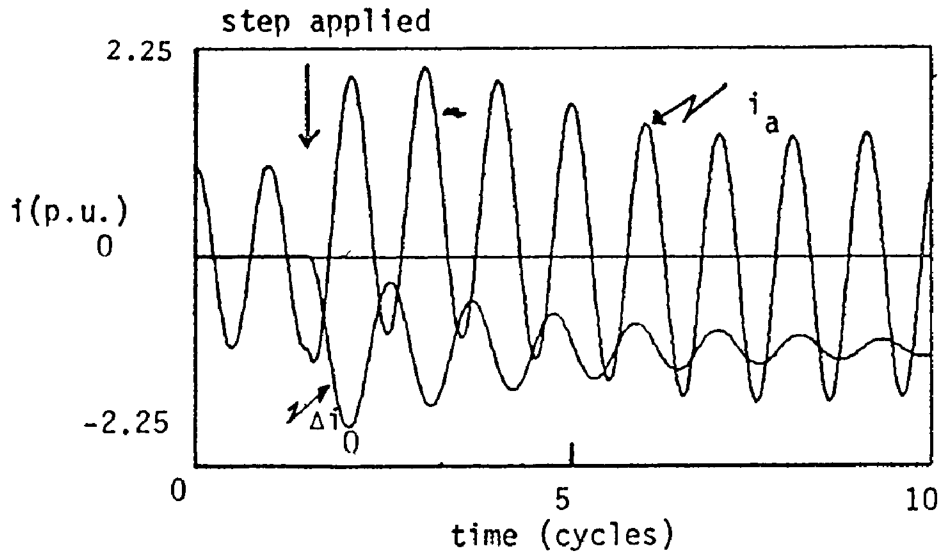
resistive elements: $$\begin{bmatrix} e_D \\ e_Q \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} i_D \\ i_Q \end{bmatrix} \qquad \text{(A4)}$$

inductive elements: $$\begin{bmatrix} e_D \\ e_Q \end{bmatrix} = \begin{bmatrix} 0 & -X \\ X & 0 \end{bmatrix} \begin{bmatrix} i_D \\ i_Q \end{bmatrix} + \frac{X}{w_o} \begin{bmatrix} \dot{i}_D \\ i_Q \end{bmatrix} \quad \text{(A5)}$$

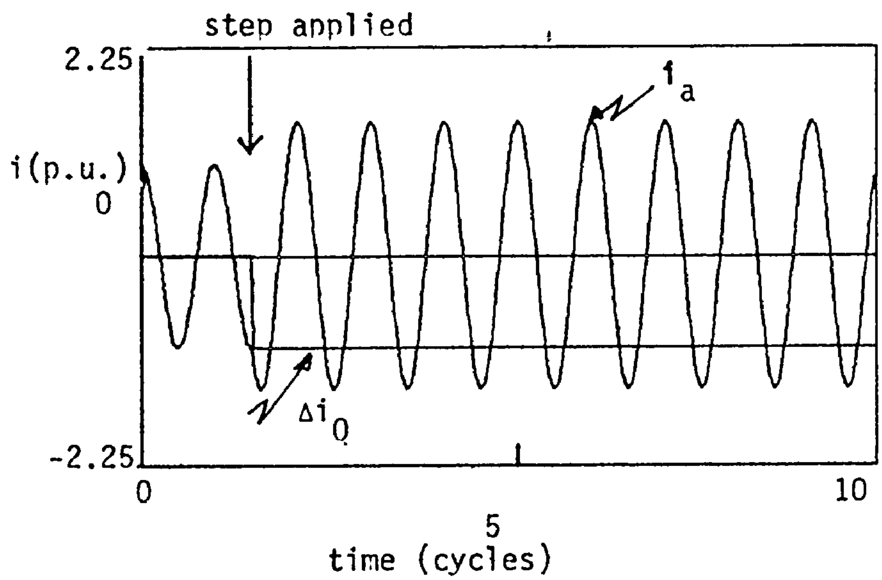
capacitive elements: $$\begin{bmatrix} i_D \\ i_Q \end{bmatrix} = \begin{bmatrix} 0 & -B \\ B & 0 \end{bmatrix} \begin{bmatrix} e_D \\ e_Q \end{bmatrix} + \frac{B}{w_o} \begin{bmatrix} \dot{e}_D \\ e_Q \end{bmatrix} \quad \text{(A6)}$$

Traditionally, the derivative terms in the above equations have been neglected. To get a feel for how this approximation manifests itself, a input disturbance in e_D was considered on a single RL circuit. The response is shown in Figure A3 where it can be seen that inclusion of the derivative terms results in a fundamental frequency (60 hertz) component in i_D, i_Q and a dc offset in the phase currents. The main reason for excluding the derivative terms in transient stability studies is the need to choose very small integration step lengths - typically .001 as distinct to .02 secs. if these effects are neglected.

Transformers may be modelled as coupled circuits or simply as a series RL circuits [89]. Electrical loads, which are recently being represented in transient stability studies using general nonlinear functional relationships between power and voltage and between reactive power and voltage [102], may be represented as equivalent constant circuit elements in dynamic stability studies. The values of these equivalent circuit elements can be obtained by perturbing the original nonlinear load relationships.



(a) Derivative terms included



(b) Derivative terms neglected

Fig. A3 Response of line current to voltage step including and excluding derivative terms

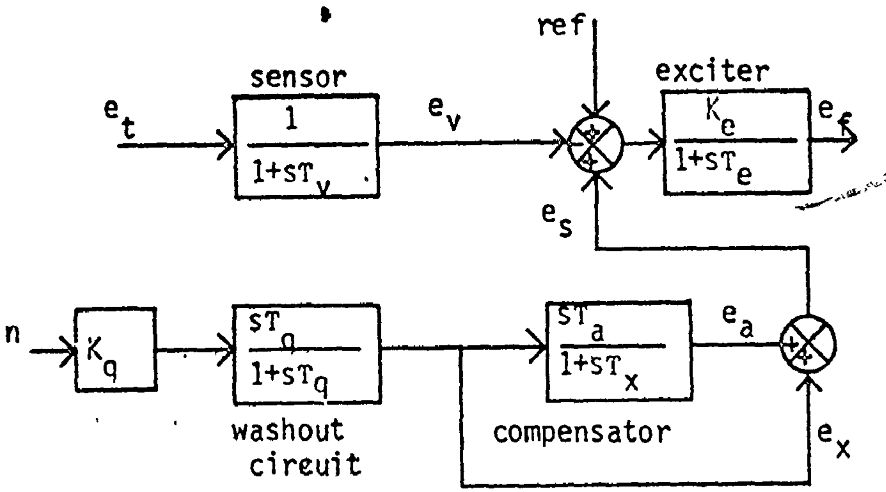
A3 Excitation System Models

In this section models are presented for modern static exciters with speed sensitive stabilizing circuits and for a commonly used rotating exciter. Block diagram descriptions are taken from [5] and [98] respectively.

In Figure A4 (a), a block diagram description for the static exciter-stabilizer used by Ontario Hydro, is shown. The exciter is modelled as a single time constant transfer function, the inputs being the stabilizing signal and the difference between the reference voltage and measured terminal voltage. At the front end of the stabilizing circuit is a washout function - the purpose of which is to eliminate steady state offset in exciter input with speed. The washout time constant is typically 2 seconds. Phase lead compensation is affected through the addition of an acceleration as well as a velocity component of signal. Damping torque is achieved by having phase cancellation between the lead in compensation and the total lags in the washout and reference voltage - electric torque transfer functions.

State space equations are presented in Figure A4 (b). Note that if e_x and e_a are chosen as states - only an unreduced equation set can be obtained. However by choosing the states as shown, a reduced equation set results - this simplifies the task of combining the equations of the exciter with those of other subsystems.

A simplified model for IEEE Type 1 rotating exciter is shown in Figure A5 (a). Saturation is omitted in the



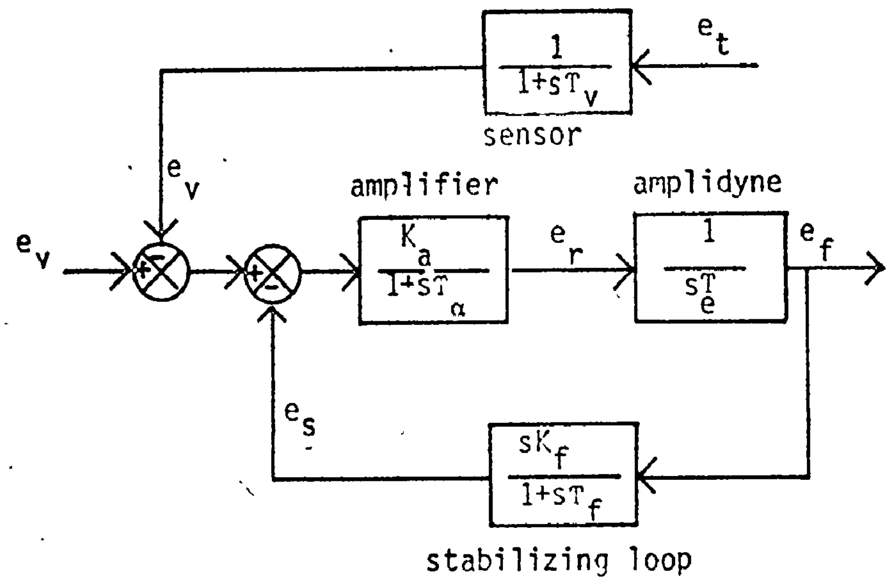
$$\dot{\begin{pmatrix} e_v \\ e_f \\ v \\ \omega \end{pmatrix}} = \begin{pmatrix} \frac{-1}{T_v} & & & \\ \frac{-K_e}{T_e} & \frac{-1}{T_e} & \frac{(1+T_d)K_e}{T_x T_e} & \frac{K_e}{T_e} \\ & & \frac{1}{T_Q} & \\ & & \frac{-T_a}{T_x^2} & \frac{-1}{T_x} \end{pmatrix} \begin{pmatrix} e_v \\ e_f \\ v \\ \omega \end{pmatrix} + \begin{pmatrix} \frac{e_{Do}}{e_{to}} & \frac{e_{Qo}}{e_{to}} \\ & \\ & \\ & \end{pmatrix} \begin{pmatrix} e_D \\ e_Q \end{pmatrix} + \begin{pmatrix} \\ \frac{1+T_a K_Q K_e}{T_e \; T_e} \\ \frac{-K\alpha}{T_Q} \\ \frac{-T_a K_\alpha}{T_x^2} \end{pmatrix} n$$

$$\{v \triangleq e_x - K_Q n,$$
$$\omega \triangleq e_a - \frac{T_a}{T_x} e_x\}$$

(a) Block diagram description

(a) State space equations

Fig. A4 Static exciter stabilizer Model "E_1"



(a) Block diagram description

$$\frac{d}{dt}\begin{pmatrix} e_v \\ e_f \\ e_r \\ q \end{pmatrix} = \begin{bmatrix} -\frac{1}{T_v} & 0 & 0 & 0 \\ -\frac{K_a}{T_a} & 0 & \frac{-1}{T_e} & 0 \\ 0 & \frac{-K_a K_f}{T_a T_a} & \frac{-1}{T_a} & \frac{-K_a}{T_a} \\ 0 & \frac{-K_f}{T_f^2} & 0 & 0 \end{bmatrix} \begin{pmatrix} e_v \\ e_f \\ e_r \\ q \end{pmatrix} + \begin{bmatrix} \frac{e_{Do}}{e_{to}} & \frac{e_{Qo}}{e_{to}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{\gamma} \begin{pmatrix} e_D \\ e_Q \end{pmatrix}$$

$$q \triangleq e_s - \frac{K_f}{T_f} e_f$$

$$q \triangleq e_s - \frac{K_f}{T_f} e_f$$

(b) State space equations

Fig. A_5 Continuously acting rotating exciter Model "E_2"

representation as this effect is neglected in dynamic stability studies since ceiling values are generally not reached. The state space equations are given in Figure A5 (b).

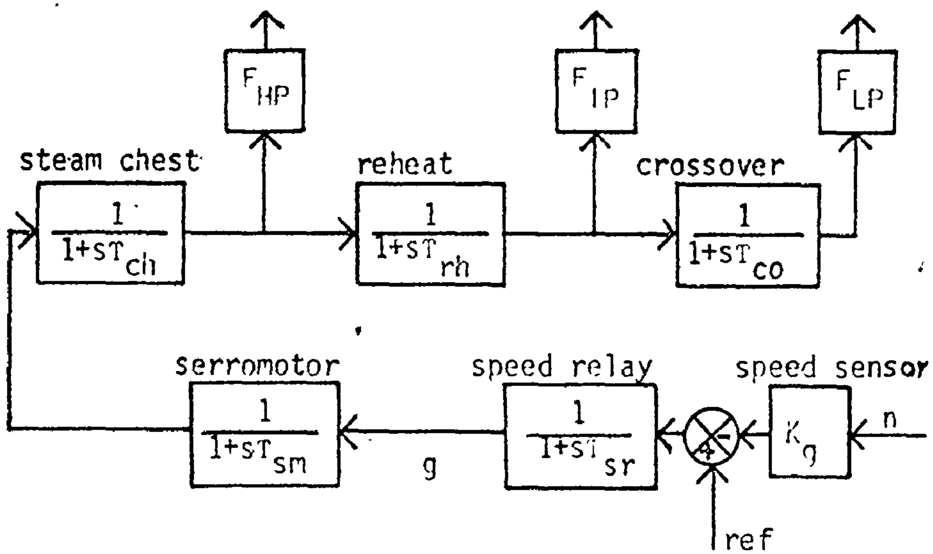
Though only two exciter types have been described, the general format of the equations in Figure A5 (b) can be used to describe all commonly used exciters including those listed in [98].

A4 Turbine-Governor Model

A block diagram description for a tandem compound, single reheat steam turbine and mechanical hydraulic governor is shown in Figure A6(a) and the corresponding state space equations are given in Figure A6(b). This representation has been taken from (115). The same block diagram may be employed in describing similar turbines with electro-hydraulic governing schemes, by appropriately adjusting the parameters. If acceleration feedforward is included, [115] then a signal comprising a combination of speed signal and speed relay output can be used to circumvent the requirement to use unreduced equations rather than the reduced form.

Boiler representation has been neglected as is standard in such studies. Boiler controls regulate governor valve pressure but the response is not sufficiently fast to compensate for pressure variation due to movement of the governor valves. However, in dynamic stability studies, the pressure variation is assumed to be negligible during the small valve movements.

In many studies such a detailed turbine representation may not be necessary. An appropriate approximation in such

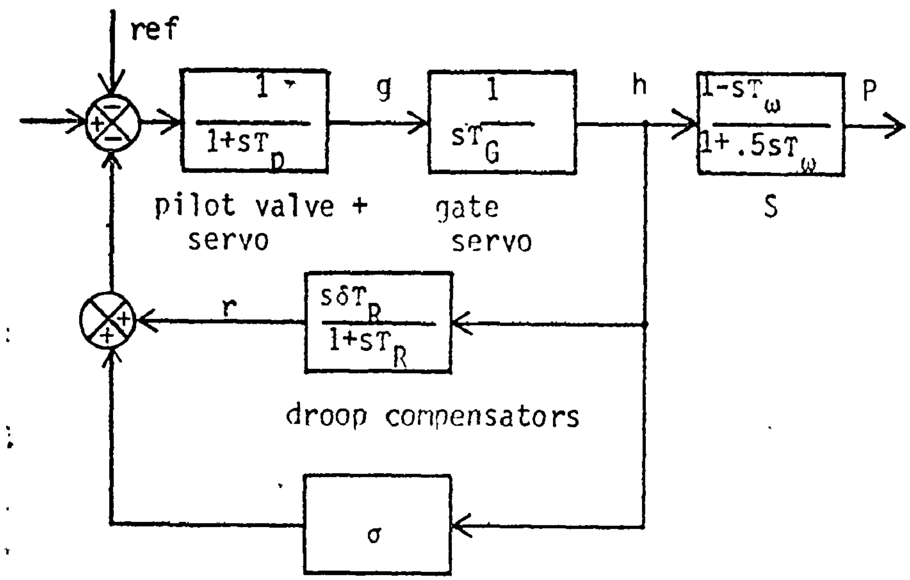


(a) Block diagram description

$$\begin{bmatrix} \dot{g} \\ \dot{h} \\ \dot{P}_{HP} \\ \dot{P}_{IP} \\ \dot{P}_{LP} \end{bmatrix} = \begin{bmatrix} \frac{-1}{T_{sr}} & & & & \\ \frac{-1}{T_{sm}} & \frac{-1}{T_{sm}} & & & \\ & \frac{F_{HP}}{T_{ch}} & \frac{-1}{T_{ch}} & & \\ & & \frac{F_{LP}}{F_{HP}T_{rh}} & \frac{-1}{T_{rh}} & \\ & & \frac{F_{LP}}{F_{IP}T_{co}} & \frac{-1}{T_{co}} & \end{bmatrix} \begin{bmatrix} g \\ h \\ P_{HP} \\ P_{IP} \\ P_{LP} \end{bmatrix} + \begin{bmatrix} \frac{-K_g}{T_{sr}} \\ \\ \\ \\ \\ \end{bmatrix} n$$

(b) State space equations

Fig A 6 Steam turbine and governor Model "G_1"



(a) Block diagram description

$$\dot{\begin{bmatrix} p \\ h \\ g \\ v \end{bmatrix}} = \begin{bmatrix} \frac{-2}{T_\omega} & \frac{-6}{T_\omega} & & \\ & & \frac{-1}{T_G} & \\ & \frac{-(\delta+\sigma)}{T_p} & \frac{-1}{T_p} & \frac{-1}{T_p} \\ & \frac{-\delta}{T_r} & & \frac{-1}{T_r} \end{bmatrix} \begin{bmatrix} p \\ h \\ g \\ v \end{bmatrix} + \begin{bmatrix} \\ \\ \frac{-1}{T_p} \\ \\ \end{bmatrix} n$$

$$v \triangleq r - \sigma h$$
$$p \triangleq P + 2h$$

(b) State space equations

Fig. A 7 Hydraulic turbine and governor Model "G_3"

cases is to neglect all but the H.P. turbine stages because of the relatively long reheat time constant (typically 7 seconds). This model is designated 'Turbine-governor model 'G2'' in this thesis.

A block diagram description for a hydraulic turbine and mechanical-hydraulic speed governor is Figure A7(a) and the state space equations are given in Figure A7(b). The same model can be used to describe electro-hydraulic governors by appropriate choice of parameters.

A5 Mechanical Shaft System

For the analysis of shaft torsional effects in power system stability studies, the approach adopted is to approximate the actual system by a number of concentrated rotating masses connected by weightless springs. Data for such representation is available (though not too readily) from turbine manufacturers.

For a thermal generating unit a simple representation would be to have one equivalent rotating mass corresponding to each turbine stage and one equivalent mass representing the generator rotor (Figure A8). Such a representation proves sufficiently accurate for the prediction of the lower shaft natural frequencies (below 60 hertz) at which torsional sub-synchronous resonance occurs.

Quantifying the damping in such a system constitutes a very 'grey area' at present. The main contributions are:

(a) Steam damping on turbine buckets and rotor windage damping.

(b) Shaft material hysteresis damping

(c) Bearing oil film damping

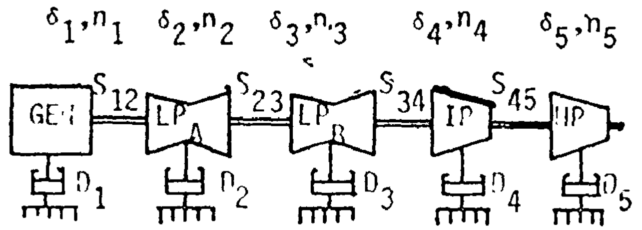
(d) Generator electrical damping

Damping effects due to governor action should also be included in the above list. Included in generator electrical damping are damping effects due to subsynchronous resonance interactions and stabilizer introduced effects.

As effects (b) and (c) are generally considered of minor importance in relation to (a) and (d), and also are not readily amenable to calculation, they are omitted. The usual approximation for steam damping is to set the damping coefficient of a simple dashpot equal to the per unit turbine power [113].

Electrical damping cannot be represented as a simple dashpot because of the frequency dependency of the electric torque-rotor slip transfer function. The spring coefficient (synchronizing torque coefficient) introduced by the generator, of course, must be included in the representation. The spring coefficient is also frequency dependent but accurate predictions of shaft natural frequencies can be obtained if a constant coefficient is used.

The natural frequencies and mode shapes for the system shown in Figure A8 are listed in Figure A9. For comparative purposes, the lowest frequency is calculated for the case where all masses and dampers are lumped into a single equivalent. Such representation has traditionally been

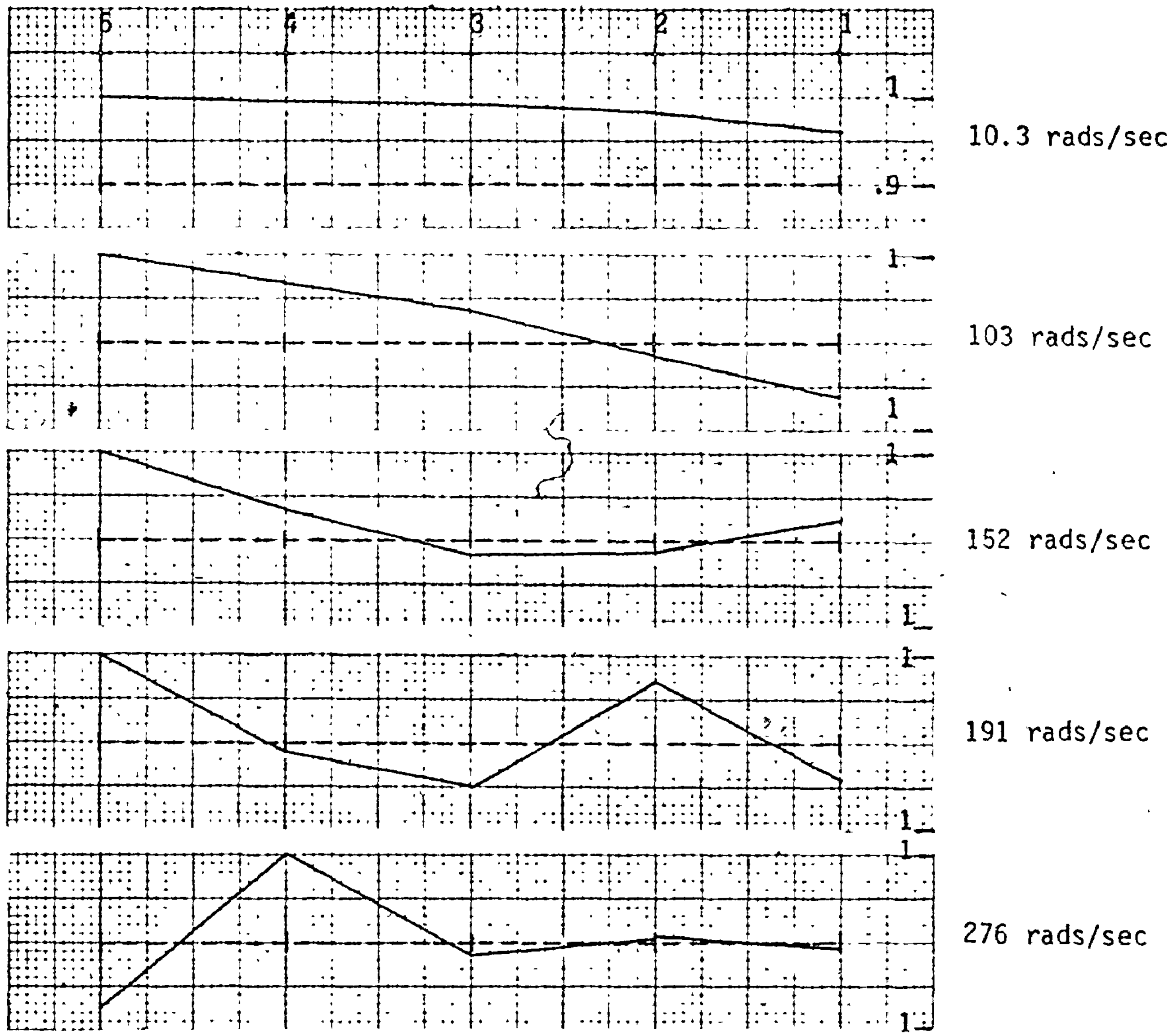


(a) Equivalent mechanical system

$$
\begin{bmatrix} \dot{n}_1 \\ \dot{n}_2 \\ \dot{n}_3 \\ \dot{n}_4 \\ \dot{n}_5 \\ \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \\ \dot{\delta}_4 \\ \dot{\delta}_5 \end{bmatrix}
=
\begin{bmatrix}
\frac{-D_1}{M_1} & & & & & \frac{-S_{12}}{M_1} & \frac{S_{12}}{M_1} & & & \\
& \frac{-D_2}{M_2} & & & & \frac{S_{12}}{M_2} & \frac{-S_{12}-S_{23}}{M_2} & \frac{S_{23}}{M_2} & & \\
& & \frac{-D_3}{M_3} & & & & \frac{S_{23}}{M_3} & \frac{-S_{23}-S_{34}}{M_3} & \frac{S_{34}}{M_3} & \\
& & & \frac{-D_4}{M_4} & & & & \frac{S_{34}}{M_4} & \frac{-S_{34}-S_{45}}{M_4} & \frac{S_{45}}{M_5} \\
& & & & \frac{-D_5}{M_5} & & & & \frac{S_{45}}{M_5} & \frac{-S_{45}}{M_5} \\
\omega_o & & & & & & & & & \\
& \omega_o & & & & & & & & \\
& & \omega_o & & & & & & & \\
& & & \omega_o & & & & & & \\
& & & & \omega_o & & & & &
\end{bmatrix}
\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \end{bmatrix}
+
\begin{bmatrix} \frac{1}{M_1} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} T_e
+
\begin{bmatrix}
 & & \\
 & & \frac{1}{M_2} \\
 & & \frac{1}{M_3} \\
 & \frac{1}{M_4} & \\
\frac{1}{M_5} & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & &
\end{bmatrix}
\begin{bmatrix} P_{HP} \\ P_{IP} \\ P_{LP} \end{bmatrix}
$$

(b) State space equations

Fig. A 8 Turbine-generator mechanical shaft system "S_2"



Typical Data:

$M_1 = 1.71$, $M_2 = 2.38$, $M_3 = 2.31$, $M_4 = .464$, $M_5 = .248$

$D_2 = .25$, D_3, $= .26$, $D_4 = .26$, D_5, $.24$

$S_{12} = 62.3$, $S_{23} = 75.6$, $S_{34} = 48.4$, $S_{45} = 21.8$

Synchronization represented as "equivalent" spring with coefficinet $S_o = 2.04$

Lowest frequency when shaft dynamics neglected

$$\sqrt{\frac{\omega_o S_o}{\Sigma M}} = 10.4 \text{ rads/sec}$$

Fig. A 9 Mode shapes for mechanical shaft system "S_2"

used in the industry and this single lumped mass equivalent is designated 'Mechanical Shaft System 'S_o''.

For hydraulic turbines, a two mass equivalent is considered adequate - one corresponding to the rotor inertia and the other representing the turbine inertia. The equations can be written by inspection using the 5 mass system as a reference. Dashpot D_2, represents water damping on the turbine rotor.

In applications involving many stability predictions in which the stability of a specific shaft mode is in question, a single mass equivalent may be employed. This single mass equivalent is defined in terms of modal mass, spring and damping coefficient values [20].

A6 Asynchronous Motor Loads

The rational behind the choice of state variables for asynchronous (induction) motors is along the same vein as that used for synchronous machines. Stator currents and rotor fluxes referred to a synchronously rotating frame are chosen. This particular choice results in compatibility between machine and network equations and facilitates the inclusion of network-stator transients.

The overall equations, taken from [107] have been expressed in incremental form and arranged in unreduced and reduced state space form in Figures A10(a) and (b) respectively. One rotor circuit has been considered in the equations shown - the extension to include more rotor circuits (eg., to represent 'deep bar' effects) is straightforward.

The mechanical load has been represented as a simple inertia and dashpot combination. If more detailed representation is required (eg., if load dynamic effects are important), any appropriate load model may be included within the same equation structure.

$$\begin{bmatrix} L_s & & L_{sr} & & & & & L_{sr} & & \\ & L_s & & L_{sr} & & & -L_{sr} & & & \\ L_{sr} & & L_r & & & & -r_r & -n_{Lr} & t_1 & \\ & L_{sr} & & L_r & & & -n_{Lr} & -r_r & t_2 & \\ & & & & & & -L_{sr} & & & \\ & & & & & & & -L_{sr} & & \\ & & & & & & -L_r & & & \\ & & & & & & & L_r & & \\ & & & & & & & & M & \\ & & & & & & & & & \frac{1}{L_{sr}} \end{bmatrix} \begin{bmatrix} \dot{i}_{SD} \\ \dot{i}_{SQ} \\ i_{rD} \\ i_{rQ} \\ \dot{\psi}_{rD} \\ \dot{\psi}_{rQ} \\ i_{rD} \\ i_{rQ} \\ \dot{n} \\ T_e \end{bmatrix} = \begin{bmatrix} r_s & -L_s & & & \\ L_s & r_s & & & \\ & nL_{sr} & & & \\ nL_{sr} & & & & \\ L_r & & & & \\ & L_r & & & \\ -L_{sr} & & 1 & & \\ & -L_{sr} & & 1 & \\ & & & & -D \\ i_{rQ} & -i_{rD} & -i_{SQ} & i_{SD} & \end{bmatrix} \begin{bmatrix} i_{SD} \\ i_{SQ} \\ \psi_{rD} \\ \psi_{rQ} \\ n \end{bmatrix} + \begin{bmatrix} 1 & \\ & 1 \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \end{bmatrix} \begin{bmatrix} e_D \\ e_Q \end{bmatrix}$$

$t_1 = -L_{sr} i_{SD} - L_r i_{rQ}, \; t_2 = L_{sr} i_{SQ} + L_r i_{rD}$

(a) Unreduced equations

$$\begin{bmatrix} \dot{i}_{SD} \\ \dot{i}_{SQ} \\ \dot{\psi}_{rD} \\ \dot{\psi}_{rQ} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & Y & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} i_{SD} \\ i_{SQ} \\ \psi_{rD} \\ \psi_{rQ} \\ n \end{bmatrix} + \begin{bmatrix} Y \\ \hline 0 \\ \\ 1 \end{bmatrix} \begin{bmatrix} e_D \\ e_Q \end{bmatrix}$$

(b) Reduced equations

Fig. A10 State space equations for asynchronous motor loads

APPENDIX B

SYSTEM EQUATIONS NEGLECTING NETWORK DYNAMICS

In this appendix the formulation of the overall power system state space equations when network-stator transients are neglected, is outlined. The general development presented in Chapter 2 has been based on the premise that network-stator transient effects are to be included. It has subsequently been shown (Chapter 5) how these effects may be eliminated from the final state space equation set.

In many applications it will be known from the outset that network-stator transients are not significant and consequently may be neglected. It is more efficient (and perhaps often mandatory due to storage constraints) to omit the appropriate dynamic subsystem during formulation rather than eliminate it from the final equation set.

The equations describing synchronous and asynchronous machines are developed as in Appendix A except the derivative of stator flux linkages are equated to zero. The equations are arranged as in Section 2.2 with the differential equation describing stator current (Equation (2.2)):

$$\dot{\underset{\sim}{i}}_s = A_M \underset{\sim}{i}_s + B_M \underset{\sim}{X}_M + \Gamma_M \underset{\sim}{e}_s$$

being replaced by the algebraic equation:

$$\underset{\sim}{i}_s = G_M e_s + H_M \underset{\sim}{X}_M$$

Similarly the differential equations describing network dynamics (Equation (2.10)):

$$\dot{\underset{\sim}{X}}_N = A_N \underset{\sim}{X}_N + B_N \underset{\sim}{i}_s$$

$$\underset{\sim}{i}_s = C_N X_N + D_N \underset{\sim}{i}_s + E_N \underset{\sim}{e}_s$$

are replaced by the algebraic relationship

$$\underset{\sim}{e}_s = Z \underset{\sim}{i}_s \quad \text{or} \quad \underset{\sim}{i}_s = Y \underset{\sim}{e}_s$$

(Methods of constructing the 'Z' matrix are well documented [116]).

The unreduced and reduced state space equations are shown in Figure A11.

$$\begin{bmatrix} I & & & -K_M \\ & I & & \\ & & I & -G_M \\ & & -Z & I \end{bmatrix} \begin{bmatrix} \dot{\underset{\sim}{e}}_v \\ \dot{\underset{\sim}{X}}_M \\ \underset{\sim}{i}_s \\ \underset{\sim}{e}_s \end{bmatrix} = \begin{bmatrix} F_M & \\ E_M & D_M \\ & H_M \\ & \end{bmatrix} \begin{bmatrix} \underset{\sim}{e}_v \\ \underset{\sim}{X}_M \end{bmatrix}$$

(a) Unreduced equations

$$\begin{bmatrix} \dot{\underset{\sim}{e}}_v \\ \dot{\underset{\sim}{X}}_M \end{bmatrix} = \begin{bmatrix} F_M & K_M(Y-G_M)^{-1} H_M \\ E_M & D_M \end{bmatrix} \begin{bmatrix} \underset{\sim}{e}_v \\ \underset{\sim}{X}_M \end{bmatrix}$$

(b) Reduced equations

Fig. A11 State space equations for system neglecting network-stator transients