

**INTEGER PROGRAMMING-BASED APPROACHES FOR
SUSTAINABLE COMMUNITY APPLICATIONS**

By Wendy Huang
B.A.Sc

Faculty of Engineering
Civil Engineering

A Thesis
Submitted to the School of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree

Masters of Applied Science

**INTEGER PROGRAMMING-BASED APPROACH FOR
SUSTAINABLE COMMUNITY APPLICATIONS**

Masters of Applied Science (2013)
(Department of Civil Engineering)

McMaster University
Hamilton, Ontario

TITLE: Integer Programming-Based Approaches for
Sustainable Community Applications

AUTHOR: Wendy Huang, B.A.Sc

SUPERVISOR: Dr. Brian W. Baetz, Dr. Saiedeh N. Razavi

**NUMBER OF
PAGES:** 85 pages (i-x, 11-85)

Abstract

Sustainability is an important consideration in municipalities, and decision-makers are often faced with making trade-offs in decisions on cost, location selection, and allocation. These decisions are often riddled with complexities and uncertainties, and a heuristic approach can become costly and inefficient. Integer programming is a branch of mathematical programming that can be applied to problems such as these and can be a helpful tool in the decision making process.

This thesis presents three papers that develop and apply integer programming-based methodologies and their applications to problems faced by many communities: urban noise, school location-allocation and municipal solid waste collection.

An interval-integer approach is applied to an urban noise problem to address the uncertainties in noise, variations in acceptable noise levels. The model determines the most optimal combination of common noise-control techniques to reduce noise in communities to acceptable levels based on the type of community.

Changing demographics and urban sprawl have resulted in a decrease in school enrolment in parts of many municipalities, resulting in schools being closed and students driving longer distances to get to school. A mixed-integer linear programming approach is applied to select the best school option based on minimizing vehicle distances.

Curbside solid waste collection, which is commonly used in North American municipalities, is a costly, labour intensive process. With fuel prices rising, it is becoming more expensive for municipalities to provide these services. This paper proposes a waste dropoff depot system for solid waste collection and provides a GIS-based integer programming model for site selection and bin sizing.

Acknowledgements

I would like to thank my supervisor, Dr. Brian W. Baetz for his guidance, support and leadership over the course of my studies, and challenging and encouraging me as a student and researcher. I am forever grateful for his guidance as I begin my journey into research. I would also like to thank Dr. S. Razavi for her co-supervision and valuable input in my research.

I would also like to thank my advisory committee, Dr. Y. Kim and Dr. Y.P. Guo for their interest, guidance and advice.

I gratefully acknowledge the funding that I received from the Natural Sciences and Engineering Research Council of Canada, Ontario Graduate Scholarship, and the Department of Civil Engineering at McMaster University.

Finally, I would like to thank my family for their endless support and encouragement over the years of my education.

Publication List

This thesis consists of the following papers:

Paper I

Huang, W., Dai, L.M., Baetz, B.W., Cao, M.F., Razavi, S., 2013. Interval Binary Programming Model for Noise Control Within an Urban Environment. *J. Environmental Informatics*, 21(2), 93-101.

Paper II

Huang, W., Razavi, S., Baetz, B.W., Minimizing Vehicular Travel Distances to Schools in Communities with Declining Enrolments Using Integer Programming. Submitted to *J. Urban Planning and Development (ASCE)*, August 2013.

Paper III

Huang, W., Razavi, S., Baetz, B.W., GIS-based Integer Programming for the Location of Solid Waste Collection Depots

Co-Authorship

This thesis has been prepared in accordance with the regulations for a ‘Sandwich’ thesis format or as a compilation of papers stipulated by the Faculty of Graduate Studies at McMaster University and has been co-authored.

Chapter 2: Interval Binary Programming Model for Noise Control Within an Urban Environment

The methodology, development, model formulation and solution of the model were completed by W. Huang with the assistance and consultation of Dr. B.W. Baetz, Dr. L.M. Dai and M.F. Cao. The paper was written by W.Huang and edited by Dr. B.W. Baetz and Dr. S. Razavi.

Chapter 3: Minimizing Vehicular Travel Distances to Schools in Communities with Declining Enrolments Using Integer Programming

The methodology, development, model formulation and solution of the model were completed by W. Huang with the consultation of Dr. B.W. Baetz. The paper was written by W. Huang and edited by Dr. B.W. Baetz and Dr. S. Razavi.

Chapter 4: GIS-based Integer Programming for the Location of Solid Waste Collection Depots

The methodology, development, model formulation and solution of the model were completed by W. Huang with the consultation of Dr. B.W. Baetz. The paper was written by W. Huang and edited by Dr. B.W. Baetz and Dr. S. Razavi.

Table of Contents

Abstract	iii
Acknowledgements	iv
Publication List	v
Co-Authorship	vi
List of Tables	ix
List of Figures	ix
1. Thesis Summary	11
1.1 Scope of Research	11
1.2 Background	12
1.2.1 Sustainable Communities	12
1.2.2 GIS applications	13
1.2.3 Urban noise pollution	13
1.2.4 Low enrolment schools	14
1.2.5 Solid Waste Management	15
1.3 Summary of Papers	17
1.4 References	19
2. Interval Binary Programming Model for Noise Control Within an Urban Environment	22
2.1 Introduction	22
2.2 Methodology	24
2.2.1 Interval Linear Programming (ILP)	24

2.2.2	Interval Binary Programming	26
2.3	Application	27
2.3.1	Overview of the system	27
2.4	Results and Discussion.....	35
2.5	Conclusions	42
2.6	References	43
3.	Minimizing Vehicular Travel Distances to Schools in Communities with Declining Enrolments Using Integer Programming	46
3.1	Introduction	46
3.2	Methodology	47
3.3	Hypothetical Problem.....	50
3.4	Results	53
3.5	Conclusions	55
3.6	References	56
4.	GIS-based Integer Programming Approach for the Location of Solid Waste Collection Depots.....	58
4.1	Introduction	58
4.2	Methodology	61
4.3	Case Study	62
4.4	Results	64
4.5	Conclusions	71
4.6	References	72

Appendix A – Technical Definitions for Paper I	73
Appendix B – Paper II Input Tables for Optimization.....	75
Appendix C – Paper III Maps and Input Tables for Optimization.....	81

List of Tables

Table 2.1 Noise control measures for Factory i	29
Table 2.2: Acceptable noise levels of considered communities	30
Table 2.3: Distance from the noise source i to community k	30
Table 2.4: Solutions from IBP model under different scenarios.....	36
Table 2.5: Results for Scenario 1	37
Table 2.6: Results for Scenario 2	38
Table 2.7: Results for Scenario 3.....	40
Table 3.1. Costs to the school board.....	51
Table 3.2. Number of students	52
Table 3.3. Summary of costs.....	54

List of Figures

Figure 2.1: Study System.....	28
Figure 2.2: Noise control measures under Scenario 1	37
Figure 2.3: Noise control measures under Scenario 2	39

Figure 2.4: Noise control measures under Scenario 3	40
Figure 2.5: Acceptable noise levels for Communities 1 and 2	41
Figure 2.6: Upper and lower bounds of cost for recommended noise reduction systems	41
Figure 3.1. Student distribution for Scenario 1	54
Figure 3.2. Student Distribution for Scenario 2	55
Figure 4.1: Study Area - Dundas, Ontario	63
Figure 4.2: 1 collection depot, 10080km travelled	65
Figure 4.3: 2 collection depots, 8148km travelled.....	66
Figure 4.4: 3 collection depots, 6384km travelled.....	67
Figure 4.5: Olde Dundas	68
Figure 4.6: 1 collection depot, 4600km travelled	69
Figure 4.7: 2 collection depots, 3200km travelled.....	69
Figure 4.8: 3 collection depots, 3130km travelled.....	70

1. Thesis Summary

1.1 Scope of Research

Since the inception of the concept in the 1980's, sustainability has become a key factor in current municipal decision making. Every decision made by a municipality should consider the three-legged stool of economy, society and environment, to make decisions that will benefit the municipality in the long run (World Commission on Environment and Development, 1987). The decision making process is not always simple, as there are trade-offs to be made between each leg of the stool, and optimization methods can aid in making a more optimal decision.

The goal of this thesis is to explore the application of integer programming methods to some problems that are not often addressed through these methods. These problems selected are related to issues that affect many North American municipalities today, and integer programming provides a tool that can assist decision makers in making decisions for a more sustainable community. Sustainability is not easily quantified as it is often difficult to place a dollar value on the the environmental and social benefits of sustainable development, so many decisions are made using heuristic approaches.

This thesis proposes three integer programming models for use in sustainable community development: an interval-binary programming method for decision making under uncertain parameters for a noise control context, a mixed-integer programming location-allocation model for school planning, and a GIS-based integer programming model for site selection for waste management depots.

The models developed are applied to hypothetical cases based on reasonable estimates using numbers realistic for present-day Ontario, Canada, and the Solver add-in in MS

Excel was the primary tool used to solve the developed models. ArcMap10 was used for GIS modeling and mapping.

1.2 Background

1.2.1 Sustainable Communities

Sustainability can be loosely defined as meeting today's needs without compromising the ability to meet the needs of the future. A sustainable community satisfies this, while maintaining a reasonable quality of life for its residents.

Quality of life in municipalities is affected by a number of elements. Available services, transportation, air quality, housing markets, safety, comfort, convenience, health care, education, and many more elements factor in to create an overall quality of life. It is up to the municipality to provide many of these services, which can be costly. It is important that these decisions be made optimally.

Municipalities across North America are planning for greener, more sustainable development for today and into the future. Optimization methods have been applied to many areas in municipal development for a greener future. Integer programming has been used to propose methods to improve the efficiency of public transportation through the location of public transit nodes such as bus and train modalities (Sohn, 2013) (Musso & Sciomachen, 1997). A number of variations of integer programming has been applied to waste management facility planning for location and allocation of facilities (Huang *et al.*, 1995) (Tanskanen, 2000) and transfer stations (Chatzouridis & Komilis, 2011), and overall management of municipal solid waste collection and disposal under uncertainty as a whole (Huang *et al.*, 2001) (Chang *et al.*, 1997). Integer programming has also been applied to the placement and maintenance of water and wastewater treatment and delivery infrastructure (Samani & Zanganeh, 2010) (Samani & Mottaghi, 2006).

When it comes to public facilities, in many instances municipalities are faced with the problem of what needs to be built, whether or not it should be built and where to build it. They are often faced with constraints such as geography, land use, surrounding properties, public opinion, cost, and many others. Some decisions can be made heuristically, but often the resulting solutions are not necessarily the most optimal solution.

This thesis looks at three separate cases for the application of integer programming as a decision-making tool for a municipality. These cases relate to issues that most city dwellers face on a day to day basis, but often go unnoticed due to their mundane nature. The applications are novel in that they have not been addressed from an integer programming approach in the past, and are considered to be viable cases for this approach.

1.2.2 GIS applications

Geographical Information Systems (GIS) tools are commonly used in conjunction with integer programming in planning in the public sector, particularly for planning tools involving persons or resource allocation and routing. Location of public facilities such as schools (Caro, Sirabe, Guignard, & Weintraub, 2004), recycling dropoff depots (Valeo *et al.*, 1998), and transit nodes (Sohn, 2013) have used GIS tools to aid in the decision-making process. GIS models have also been applied in the management and planning of solid wastes systems. GIS data from collection vehicles have been used to determine the fuel consumption and emissions for waste collection (Agar *et al.*, 2007).

1.2.3 Urban noise pollution

Noise is an issue that those who live in cities encounter daily. Noise has been shown to have detrimental effects on human health, and a correlation has been seen to problems such as hearing impairment, high blood pressure and some heart diseases (National Institute for Occupational Safety and Health, 1997). Noise affects the quality of life, and although is not generally considered when one addresses sustainability, it is an underlying

problem that exists in cities. Previous work has been done to manage noise at the source and at a micro scale, but little has been done to address noise management at a community scale. Common noise sources within a city are traffic, light industry, construction and maintenance, restaurants and public spaces. With the exception of traffic noise, many of these noises come from point sources. Traffic noise is a significant contributor to noise in cities, and most of this noise is generated from the impact of tire treads on pavement. Research has been done in the development of low-noise pavement using rubberized asphalt (Dai *et al.*, 2008), but it is not applicable to integer programming. Other noises within a city, such as light industry and construction, are point-sources.

Noise decreases over distance due to loss of sound energy from travelling through air and encountering obstacles. The noise level of point-sources received by the end receiver can be decreased through noise reduction at the source or the addition of obstacles such as sound barriers. Paper I uses an interval binary programming approach as a decision making tool for the selection of noise management options for point-source noises in a community.

1.2.4 Low enrolment schools

The current education system is also facing problems, one of which is the decline in enrolment in some urban public schools due to suburban sprawl. This, in combination with the increasing cost to operate older, less energy-efficient infrastructure, is making it increasingly more expensive per student to keep these low-enrolment schools open. For budgetary reasons many municipalities have opted to close down many smaller, low enrolment schools and replace them with one large centralized school. From a cost perspective, this may be efficient, as only one facility needs to be maintained. However, students are travelling greater and greater distances to attend school, resulting in greater distances travelled to arrive at school, thus shifting a portion of the system cost away from the school board and to the students' families. The cost reduction seen by the board

is instead diverted to the students and their families. This solution may not be the best solution for all communities, and integer programming is applied to the problem in Paper II to determine if the large school solution is the most optimal for a community.

1.2.5 Solid Waste Management

Municipal solid waste collection and disposal is becoming a problem for many cities today. Cities are faced with landfills approaching or even going over capacity and encountering resistance from residents for proposed new sites due to a “not in my backyard” stance on landfill locations. Waste generation from the public has been increasing over the last few decades, and although public awareness and recycling rates are improving, the rate of waste generation is still extremely high. Furthermore, rising fuel prices and suburban sprawl are causing the current model of weekly door-to-door collection seen in most North American cities to become increasingly more costly to maintain. Much modelling work has been put into making the current system more efficient, through the analysis of GIS data from collection vehicles (Wilson *et al.*, 2006), modelling of waste flows based on available facilities (Huang *et al.*, 1995) and modelling the costs of adding recycling, composting, and waste-to-energy facilities. However, given the current pattern of population growth, fuel price and waste generation, this waste collection model may be unsustainable.

In Paper III, a waste collection depot method is proposed, where depots are placed within the community and residents transport their own wastes to the depot. This change would be significant, however it accomplishes a number of goals: reduces distance travelled by collection trucks as they only need to travel from the depot to the landfill or transfer station, allows residents to notice their waste generation and potentially reduce waste generation, and encourage recycling to reduce waste disposal. The developed approach could also be used to temporarily locate depots in communities for short-term situations

where waste collection is suspended due to labour strikes, such as the incident in Toronto, Ontario in 2009.

1.3 Summary of Papers

Three technical manuscripts comprise the body of this thesis.

Paper I: Interval Binary Programming Model for Noise Control Within an Urban Environment

(Published in the Journal of Environmental Informatics, July 2013)

Noise is a problem in modern cities, and has been shown to have negative effects on human health and quality of life. This paper introduces an interval binary programming model for the selection of noise mitigation technologies to implement based on their cost and effectiveness at noise reduction. Interval binary programming addresses uncertainties in noise levels by expressing parameters in the model as intervals, and the results are expressed as an optimistic and conservative model. This model was applied to a hypothetical residential community to compare the costs for several different scenarios, each with a different acceptable noise level. The results showed that through a combination of a number of noise reduction tools such as padding at the source and the addition of noise barriers and more, significant reductions in noise levels can be achieved.

Paper II: Minimizing Vehicular Travel Distances to Schools in Communities with Declining Enrolments Using Integer Programming

(Submitted to ASCE Journal of Urban Planning and Development, August 2013)

This study looks at the distance travelled by students in a community based on school availability. When enrolments decline in schools, school boards often consider closing down schools and replacing them with larger schools. This causes some students to travel longer distances to schools, and thus the total travel distance for the community as a whole increases. This study uses a mixed integer linear programming approach to optimize how many schools, which type of schools, the location of schools out of

predetermined options and allocation of students to the chosen schools, and calculates a cost for the system.

For this study, a grid system was used to represent a community. Each grid block consists of a random number of students, and three predetermined potential locations were available for schools. Schools currently exist at these locations, but the total number of students in the community is low and all schools are underutilized, making it necessary to downsize. The study uses integer programming to select schools to fulfil minimum enrolment and minimize the total distance travelled by all students.

Paper III: GIS-based Integer Programming for the Location of Solid Waste Collection Depots

Municipal solid waste disposal is a cost and labour intensive service provided by the city. Currently, most if not all major North American cities utilize a weekly door-to-door collection service. Millions of dollars are spent annually on just the collection of waste, and this practice is extremely energy intensive due to high fuel consumption. This study proposes the use of collection depots distributed around a community for people to bring their garbage to. The bins would be emptied with a higher frequency than door-to-door collection to prevent odor, pests, and overflow of wastes. This system would not only help to reduce the cost and energy consumption of solid waste collection, but would also make residents more accountable and aware of their garbage, and contribute to overall reduction in waste production as a whole. A GIS-based integer programming approach is used to select the number of depots and potential depot sites based on distance travelled from points in the community.

1.4 References

- Agar, B. J., Baetz, B. W., & Wilson, B. G. (2007). Fuel consumption, emissions estimation, and emissions costs estimates using global positioning data. *Journal of the Air and Waste Management Association*, 348-354.
- Caro, F., Sirabe, T., Guignard, M., & Weintraub, A. (2004, August). School redistricting: embedding GIS tools with integer programming. *Journal of the Operational Research Society*, 55(8), 836-849.
- Chang, N. B., Chen, Y. L., & Wang, S. F. (1997). A fuzzy interval multiobjective mixed integer linear programming approach for the optimal planning of solid waste management systems. *Fuzzy Set and Systems*, 35-60.
- Chatzouridis, C., & Komilis, D. (2011). A methodology to optimally site and design municipal solid waste transfer stations using binary programming. *Resources, Conservation and Recycling*, 89-98.
- Church, R. L., & Murray, A. T. (1993). Modeling School Utilization and Consolidation. *Journal of Urban Planning and Development*.
- Dai, L. M., Lou, Z., & Widger, A. (2008). A Study on the Performance of ARC Pavement for Traffic Noise Reduction. *Journal of Environmental Informatics*, 21-30.
- Diamond, J. T., & Wright, J. R. (1987, May). Multiobjective Analysis of Public School Consolidation. *Journal of Urban Planning and Development*, 113(1).
- Ericsson, E., Larsson, H., & Brundell-Frejj, K. (2006). Optimizing route choice for lowest fuel consumption. *Transportation Research*, 369-383.

- Huang, G. H., Baetz, B. W., & Patry, G. G. (1995). Grey integer programming: An application to waste management planning under uncertainty. *European Journal of Operational Research*, 594-620.
- Huang, G. H., Sae-Lim, N., Liu, L., & Chen, Z. (2001). An Interval-parameter fuzzy-stochastic programming approach for municipal solid waste management and planning. *Environmental Modeling & Assessment*, 271-283.
- Juha-Heikki, T. (1999). Strategic planning of municipal solid waste management. *Resources, Conservation and Recycling*, 111-133.
- Musso, E., & Sciomachen, A. (1997). Optimal location of bus depots in an urban area. *3rd International Conference on Urban Transport and the Environment for the 21st Century*, (pp. 93-103). Wessex.
- National Institute for Occupational Safety and Health. (1997). *Health Hazard Evaluations: Noise and Hearing Loss*. Cincinnati: DHHS.
- Samani, H. M., & Mottaghi, A. (2006). Optimization of Water Distribution Networks Using Integer Linear Programming. *Journal of Hydraulic Engineering*, 501-509.
- Samani, H. M., & Zanganeh, A. (2010). Optimisation of water networks using linear programming. *Proceedings of the Institution of Civil Engineers - Water Management*, (pp. 475-485).
- Sohn, K. (2013). Optimizing Train-Stop Positions Along a Platform to Distribute the Passenger Load More Evenly Across Individual Cars. *IEEE Transactions on Intelligent Transportation Systems*, 994-1002.

- Tanskanen, J. H. (2000). Strategic planning of municipal solid waste management. *Resources, Conservation and Recycling*, 111-133.
- Valeo, C., Baetz, B. W., & Tsanis, I. K. (1998). Location of Recycling Depots with GIS. *Journal of Urban Planning and Development*, 93-99.
- Wilson, B. B., Agar, B. J., Baetz, B. W., & Winning, A. (2006). Practical Applications for GPS Data from Solid Waste Collection Vehicles. *Canadian Journal for Civil Engineering*, 678-681.
- World Commission on Environment and Development. (1987). *Our Common Future*. Oxford: Oxford University Press.
- Zamorano, M., Molero, E., Grindlay, A., Rodriguez, M. L., Hurtado, A., & Calvo, F. J. (n.d.). A planning scenario for the application of geographical information systems in municipal waste collection: a case of Churriana de la Vega.

2. Interval Binary Programming Model for Noise Control Within an Urban Environment

By W. Huang, L.M. Dai, M.F. Cao, S. Razavi and B.W. Baetz

Abstract:

This paper introduces an interval binary programming (IBP) method to the selection of control measures for noise reduction under uncertainty, by incorporating the concepts of interval numbers and interval mathematical programming into a binary programming optimization framework. As an extension of the binary programming method, IBP can explicitly address complexities and uncertainties in a noise control system. Parameters in the IBP model can be expressed as intervals, and uncertainties are effectively incorporated within the model solution process. The modelling approach is applied to a representative control measure selection problem for noise reduction in an urban environment. Results of the application indicate that useful solutions for noise control practices can be generated. A number of decision alternatives have been obtained and analyzed under different acceptable noise levels for two communities, and they reflect complex tradeoffs between environmental and economic considerations.

Keywords: Interval Binary Programming, Uncertainty Analysis, Noise Control, System Optimization

2.1 Introduction

The problems associated with acoustic noise continues to be a major challenge for urban communities throughout the world due industrialisation and urbanisation. The noise generated from plants and factories can pose significant threats to the health of workers

and residents in nearby communities (King and Davis, 2003). Noise-induced hearing loss is one of the most common occupational diseases and the second most self-reported occupational illness or injury in the United States (Murray-Johnson et al., 2004). According to the National Institute for Occupational Safety and Health (NIOSH), approximately 30 million U.S. workers are currently exposed to noise hazards on the job and an additional 9 million U.S. workers are at-risk for developing hearing loss (NIOSH, 1998). Long-term exposure to excessive noise levels is recognized as the major cause of hearing loss. Since hearing loss is difficult to cure, appropriate engineering controls are strongly recommended to minimize noise and diminish the noise effect on workers and nearby residences. However, engineering controls differ in cost and noise reduction capability; more effective noise control measures usually require greater investment, while less effective measures may have lower costs. Therefore, optimization models are desired for helping decision makers make tradeoffs between system cost and noise control efficiency.

In the past decades, significant efforts have been made in developing optimization models for noise control systems. For example, Yeh *et al.* (2004) developed an optimization model for noise reduction in a multiple noise system by using a genetic algorithm. Asawarungsaengkul and Nanthavanij (2006) proposed six optimization models for identifying the optimal noise hazard control strategy, including two models for engineering controls, two for job rotation and two for the use of hearing protection devices. They then applied an algorithmic approach to the selection of engineering controls for optimal noise reduction (Asawarungsaengkul and Nanthavanij, 2007). Zachary et al. (2010) developed a multi-impact optimization model to reduce aviation noise and emissions at Luxembourg's Findel Airport. Prats et al. (2011) proposed a multi-objective optimization model for designing aircraft noise abatement strategies. Also, there are a number of other optimization models for identifying optimal noise control strategies (Waly and Sarker, 1998; King and Davis, 2003; Mun and Cho, 2009; Tokmechi, 2011).

In a practical noise control system, many parameters such as noise-reduction effects of different control measures, the unit cost of each measure, and acceptable noise levels for receptors may have some levels of uncertainty. However, previous optimization models are deterministic and only deal with parameters with crisp values. Therefore, in this paper, an interval binary programming method will be developed and applied to a representative noise control system for selecting optimal noise reduction measures. Interval solutions for binary variables will be analyzed and interpreted to provide useful decision alternatives for controlling noise from different sources and thus demonstrate the potential applicability of the developed method.

2.2 Methodology

2.2.1 Interval Linear Programming (ILP)

In ILP, interval values are allowed to be communicated into the optimization process. All parameters and decision variables in a linear programming model can be intervals (Huang et al., 1992).

Specifically, an ILP model can be defined as follows:

$$\text{Min/Max } f^\pm = \sum_{j=1}^n c_j^\pm x_j^\pm \quad (1a)$$

Subject to:

$$\sum_{j=1}^n a_{ij}^\pm x_j^\pm \leq b_i^\pm, \quad i = 1, 2, \dots, m \quad (1b)$$

$$x_j^\pm \geq 0, \quad j = 1, 2, \dots, n \quad (1c)$$

where a_{ij}^\pm, b_i^\pm , and $c_j^\pm \in R^\pm$ (R^\pm denotes a set of intervals). Definitions of interval, the related properties, and operation principals are shown in Appendix 1.

According to Huang et al. (1992, 1995), an interactive solution algorithm named the two-step-method (TSM) was proposed to solve the above problem. Interval solutions can be obtained based on the analysis of detailed interrelationships between the parameters and variables and between the objective function and constraints. The main idea of the TSM is to convert the original ILP model into two LP submodels corresponding to the lower and upper bounds of the objective-function value, respectively. In detail, when the objective function of model (1) is to be maximized, the first submodel would correspond to the upper bound of equation (1a). It can be formulated as follows (assume that $b_i^\pm > 0$ and $f^\pm > 0$):

$$\text{Max } f^+ = \sum_{j=1}^k c_j^+ x_j^+ + \sum_{j=k+1}^n c_j^+ x_j^- \quad (2a)$$

Subject to:

$$\sum_{j=1}^k |a_{ij}^\pm|^- \text{Sign}(a_{ij}^\pm) x_j^+ + \sum_{j=k+1}^n |a_{ij}^\pm|^+ \text{Sign}(a_{ij}^\pm) x_j^- \leq b_i^+, \quad i = 1, 2, \dots, m. \quad (2b)$$

$$x_j^+ \geq 0, \quad j = 1, 2, \dots, k \quad (2c)$$

$$x_j^- \geq 0, \quad j = k+1, k+2, \dots, n \quad (2d)$$

where for n interval coefficients c_j^\pm ($j=1, 2, \dots, n$) in the objective function of the model, we assume the former k coefficients of them are positive, and the latter $(n - k)$ coefficients are negative, i.e. $c_j^\pm \leq 0$ ($j = k+1, \dots, n$). Solutions of x_{jopt}^+ ($j = 1, 2, \dots, k$) and x_{jopt}^- ($j = k+1, \dots, n$) can be obtained through solving submodel (2). Based on solution for submodel (2), the submodel corresponding to the lower bound of equation (1a) can be formulated as follows:

$$\text{Max } f^- = \sum_{j=1}^k c_j^- x_j^- + \sum_{j=k+1}^n c_j^- x_j^+, \quad (3a)$$

subject to:

$$\sum_{j=1}^k |a_{ij}^\pm|^+ \text{Sign}(a_{ij}^\pm) x_j^- + \sum_{j=k+1}^n |a_{ij}^\pm|^- \text{Sign}(a_{ij}^\pm) x_j^+ \leq b_i^-, \quad i = 1, 2, \dots, m. \quad (3b)$$

$$0 \leq x_j^- \leq x_{j_{opt}}^+, j = 1, 2, \dots, k. \quad (3c)$$

$$x_j^+ \geq x_{j_{opt}}^-, j = k + 1, k + 2, \dots, n. \quad (3d)$$

From submodel (3), Solutions of $x_{j_{opt}}^- (j = 1, 2, \dots, k)$ and $x_{j_{opt}}^+ (j = k + 1, \dots, n)$ can be obtained.

Thus, the final solution of $f_{opt}^\pm = [f_{opt}^-, f_{opt}^+]$ and $x_{j_{opt}}^\pm = [x_{j_{opt}}^-, x_{j_{opt}}^+]$ can be obtained for model (1).

2.2.2 Interval Binary Programming

In integer programming, all the decision variables are integers. An integer programming model can be formulated as follows:

$$\text{Min/Max } f = \sum_{j=1}^n c_j y_j \quad (4a)$$

Subject to:

$$\sum_{j=1}^n a_{ij} y_j \leq b_i, i = 1, 2, \dots, m \quad (4b)$$

$$y_j \geq 0 \text{ and } y_j = \text{integer variable}, j = 1, \dots, n \quad (4c)$$

where a_{ij}, b_i , and $c_j \in R$.

A well-known approach to solve integer programming problems is the branch-and-bound algorithm (Garfinkel RS and Nemhauser GL, 1972; Blair CE and Jeroslow RG, 1982; Achterberg, 2007). Mixed integer linear programming (MILP) has been widely used in many engineering fields since it can deal with capacity-expansion issues (Baetz, 1990; Rajagopalan et al., 1998; Chen et al., 2002). However, it may not be effective when uncertain parameters exist (Jenkins, 1982). Therefore, Huang et al. (1995) introduced an interval mixed integer linear programming (IMILP) method, where input uncertainties

can be expressed as integer-intervals and/or intervals. In the IMILP model, the integer-intervals are interval binary variables, indicating capacity-expansion options. Therefore, as an extension of the IMILP model, an interval binary programming (IBP) method can be formulated as follows:

$$\text{Min/Max } f^\pm = \sum_{j=1}^n c_j^\pm x_j^\pm \quad (5a)$$

Subject to:

$$\sum_{j=1}^n a_{ij}^\pm x_j^\pm \leq b_i^\pm, \quad i = 1, 2, \dots, m \quad (5b)$$

$$x_j^\pm \geq 0, \quad j = 1, 2, \dots, n \quad (5c)$$

$$x_j^\pm = 0 \text{ or } 1, \quad j = 1, 2, \dots, n \quad (5d)$$

The solutions of the interval binary variables have four possible presentations ($[0, 0]$, $[1, 1]$, $[0, 1]$, and $[1, 0]$). They can represent the related interval decisions (for optimal scenarios) that can be interpreted to provide decision alternatives reflecting system condition variation caused by input uncertainties (Huang et al., 1995).

These methodologies can be used to optimize the cost and effectiveness in the selection of noise-reducing technologies to mitigate the noise pollution problem outlined above. Given costs and the noise reduction potentials of each type of technology, IBP can be used to select the optimal combination of technologies to most efficiently achieve a desired level of noise reduction.

2.3 Application

2.3.1 Overview of the system

A representative problem has been developed to illustrate the IBP modelling approach, based upon cost and technical data extracted from the noise control system literature.

Noise sources considered in the studied system are factories and the receivers are adjoining communities. Amounts of sources and receivers are defined as I and K , respectively. The original generated noise levels in the factories and the acceptable noise levels in the communities are shown as ON_i (B) and AN_k (B), respectively. The distance between factory i and community k is denoted as D_{ik} (km), and a barrier could be established to mitigate noise. Figure 1 presents the hypothetical noise control system. Three noise sources and two affected communities are considered in this application. There are several alternatives that could be used to control noise from the original sources. In this case, four external noise control measures, as well as equipment updates are considered as the potential options for controlling noise, as presented in Table 1. These four external control measures can also be combined with each other to enhance the effect of noise-reduction.

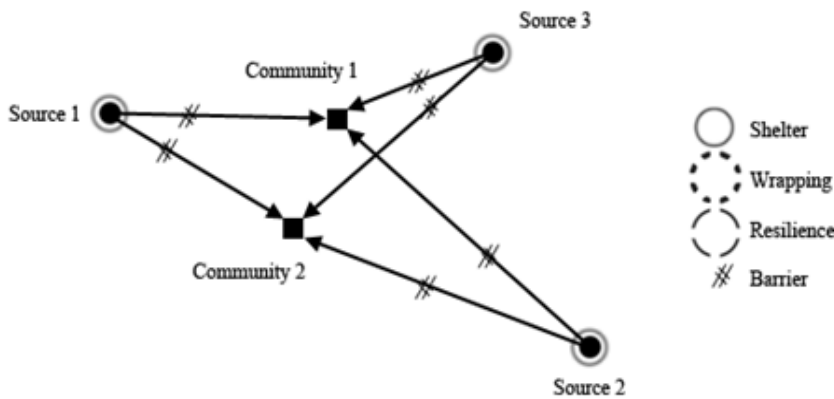


Figure 2.1: Study System

Eleven noise control options could be generated, as shown in Table 1. The unit cost for each combination of noise control is denoted as RE_j (B). As for the noise-reduction effect, it is related to the noise sources and noise control measures, thus C_{ij} (\$/B) is employed to represent this effect.

Table 1 shows the potential noise control measures and their combinations, and their related noise-reduction effect and unit cost for each measure. In this study, eleven control measures are considered for reducing excessive noise from different sources. These control measures include sheltering, wrapping, resilience features, barriers and combinations of the above methods. Equipment update is also one potential measure to control noise. The sheltering method reduces emitted noise by placing the noise sources inside a shelter; for example, housing noise-generating pieces of equipment inside a building. Wrapping is similar to sheltering, but instead of placing the sources inside a shelter, each source is separated from the environment by wrapping a noise absorbing material around it. Resilience features are noise control measures that reduce noise generated by placing padding and noise reducing materials, thus reducing noise generated from moving parts in the equipment. Barriers separate the source and the receiver through placement of a physical obstruction between them. The obstruction can absorb or redirect sound pressure, effectively reducing the amount of noise that is heard by the receiver.

Table 2.1 Noise control measures for Factory *i*

Options (<i>j</i>)	Noise control measures	Noise-reduction effect $RE_j(\text{dB})$	Cost for each scenario
1	Shelter	[9, 10]	[190, 210]
2	Wrapping	[7.5, 8]	[100, 110]
3	Resilience	[5.6, 6]	[55, 60]
4	Barrier	[11, 12]	[240, 260]
5	Equipment update	[23, 25]	[600, 650]
6	Shelter + resilience	[13.8, 14.5]	[260, 280]
7	Shelter + wrapping	[15, 16]	[320, 350]
8	Shelter + Barrier	[20.5, 22]	[520, 550]
9	Wrapping + Resilience	[10, 11]	[200, 220]
10	Wrapping + Barrier	[18, 20]	[400, 435]

11	Resilience + Barrier	[16, 18]	[350, 370]
----	----------------------	----------	------------

Different control measures produce different noise-reduction effects and also have different costs associated to them. The original noise levels of noise sources 1, 2 and 3 are [90, 92], [95, 97] and [100, 102] dB, respectively. For the 2 communities considered here, the acceptable noise levels are [60, 62] and [55, 57] dB, respectively. For each community, 3 scenarios are considered, as shown in Table 2. Table 3 presents the distance from Noise Source i to Community k .

Table 2.2: Acceptable noise levels of considered communities

	Strict level (dB)	Normal level (dB)	Loose level (dB)
Community 1 ($k = 1$)	60	61	62
Community 2 ($k = 2$)	55	56	57

Table 2.3: Distance from the noise source i to community k

Distance (m)	Noise source 1 ($i = 1$)	Noise source 2 ($i = 2$)	Noise source 3 ($i = 3$)
Community 1 ($k = 1$)	150	200	120
Community 2 ($k = 2$)	140	180	170

3.2. Interval Binary Programming (IBP) Model for Noise Control System

The problem under consideration is that excessive noise at emission sources would be reduced by different control measures over a given planning horizon. Binary variables

(X_{ij}) are introduced to denote whether or not the noise-control measure is selected. If X_{ij} is equal to one, the j^{th} noise control option would be implemented for factory i . Conversely, if the value of X_{ij} is zero, the corresponding options would not be implemented. In the case of interval binary solutions, $[0, 1]$ indicates a conservative model and $[1, 0]$ indicates an optimistic model. In addition, for factory i , no more than one of the J options can be implemented. Since this problem has several configurations of options – acceptable noise levels, combinations of mitigation methods – this paper will use a series of scenarios to describe sets of conditions. There are three noise scenarios: strict, normal, and relaxed. The objective is to minimize the total noise-reduction cost (TC) for identifying effective noise-control measures while the noise received in the considered communities is no higher than acceptable levels. Thus, the following optimization model can be formulated:

$$\text{Min TC} = \sum_{i=1}^I \sum_{j=1}^J C_{ij} X_{ij} \quad (6a)$$

Subject to:

(1) The noise received in community k should be no higher than the accepted level (AN_k). Noise reduction in the atmosphere is affected by a number of factors, including temperature and air pressure. At standard atmospheric conditions, noise reduction is 158.8 dB per kilometer, giving us:

$$\log \left\{ \sum_{i=1}^I 10^{\left[\left(ON_i - \frac{158.5D_{ik}}{1000} \sum_{j=1}^J X_{ij} RE_j \right) / 10 \right]} \right\} \leq AN_k, \quad k = 1, 2, \dots, K \quad (6b)$$

Noise levels are indicated by decibels, which is on a logarithmic scale. The source noise (ON_i) is reduced by distance and the selected noise control measure ($X_{ij}RE_j$).

(2) The longer the distance, the less the noise effect: For each community, we sort the distances between all the noise sources ($I = 1, 2, \dots, I$) to community k in a descending order, which are denoted as $D_{(i)k}$ ($i = 1, 2, \dots, i1, i2, \dots, I$), such that $D_{(i1)k} \geq D_{(i2)k}$. Thus, we have:

$$ON_{(i1)} - \frac{158.8D_{(i1)k}}{1000} - \sum_{j=1}^J X_{(i1)jk} RE_j \leq ON_{(i2)} - \frac{158.8D_{(i2)k}}{1000} - \sum_{j=1}^J X_{(i2)jk} RE_j, \quad (i1), (i2) = 1, 2, \dots, I; k = 1, 2, \dots, K \quad (6c)$$

(3) No more than one of the J combinations of noise control can be implemented for the combination of factory i and community k :

$$\sum_{j=1}^J X_{ij} \leq 1, \quad i = 1, 2, \dots, I; \quad (6d)$$

(4) Technical constraints:

$$X_{ij} = 0 \text{ or } 1, \quad i = 1, 2, \dots, I; j = 1, 2, 3, \dots, J. \quad (6e)$$

A few assumptions are made in this model in terms of noise transmission. Sound pressure decreases over distance, but this decrease is dependent on air pressure, temperature and humidity. Standard atmospheric conditions are assumed. Secondly, sound waves can be absorbed and deflected by obstacles between the transmitter and receiver. For the purposes of this model, obstacles are ignored and it is assumed that noise travels unhindered between the sources and the communities. In practical problems, many system parameters related to noise control systems such as unit costs, noise-reduction effects of different control measures, and noise levels from different sources may not be determined as crisp values. Most of them may present some levels of uncertainty. Moreover, the quality of information that can be obtained for these uncertainties is

generally not good enough to be presented as probability information. For example, the original noise levels for source 1 may vary with [90, 92] dB, which means that the lowest level of the original noise from source 1 would be 90 dB and the highest level would be 92 dB. Based on these considerations, interval parameters are introduced into the noise control optimization model framework to communicate uncertainties in C_{ij} , ON_i , and RE_j into the optimization process. This leads to an interval binary noise control optimization model as follows:

$$\text{Min TC}^\pm = \sum_{i=1}^I \sum_{j=1}^J C_{ij}^\pm X_{ij}^\pm \quad (7a)$$

Subject to:

$$\log \left\{ \sum_{i=1}^I 10^{\left[\left(ON_i^\pm - \frac{158.5D_{ik}}{1000} \sum_{j=1}^J X_{ij}^\pm RE_j^\pm \right) / 10 \right]} \right\} \leq AN_k^{(p)}, \quad k = 1, 2, \dots, K \quad (7b)$$

$$ON_{(i1)}^\pm - \frac{158.8D_{(i1)k}}{1000} - \sum_{j=1}^J X_{(i1)j}^\pm RE_j^\pm \leq ON_{(i2)}^\pm - \frac{158.8D_{(i2)k}}{1000} - \sum_{j=1}^J X_{(i2)j}^\pm RE_j^\pm, \quad (i1), (i2) = 1, 2, \dots, I; \quad k = 1, 2, \dots, K \quad (7c)$$

$$\sum_{j=1}^J X_{ij}^\pm \leq 1, \quad i = 1, 2, \dots, I. \quad (7d)$$

$$X_{ij}^\pm = [0, 1], [0, 0], [1, 0] \text{ or } [1, 1], \quad i = 1, 2, \dots, I, \text{ and } j = 1, 2, \dots, J. \quad (7e)$$

According to Huang *et al.*, 1995, model (7) can be transformed to the following two sub-models.

Sub-model 1

$$\text{Min TC}^- = \sum_{i=1}^I \sum_{j=1}^J C_{ij}^- X_{ij}^- \quad (8a)$$

Subject to

$$\log \left\{ \sum_{i=1}^I 10^{\left[\left(ON_i^- - \frac{158.5D_{ik}}{1000} \sum_{j=1}^J X_{ij}^- RE_j^+ \right) / 10 \right]} \right\} \leq AN_k^{(p)}, \quad k = 1, 2, \dots, K \quad (8b)$$

$$ON_{(i1)}^- - \frac{158.8D_{(i1)k}}{1000} - \sum_{j=1}^J X_{(i1)j}^- RE_j^+ \leq ON_{(i2)}^- - \frac{158.8D_{(i2)k}}{1000} - \sum_{j=1}^J X_{(i2)j}^- RE_j^+, \quad (i1), (i2) = 1, 2, \dots, I; \quad k = 1, 2, \dots, K \quad (8c)$$

$$\sum_{j=1}^J X_{ij}^- \leq 1, \quad i = 1, 2, \dots, I. \quad (8d)$$

$$X_{ij}^- = 0 \text{ or } 1, \quad i = 1, 2, \dots, I, \text{ and } j = 1, 2, \dots, J. \quad (8e)$$

Sub-model 2

$$\text{Min TC}^+ = \sum_{i=1}^I \sum_{j=1}^J C_{ij}^+ X_{ij}^+ \quad (9a)$$

Subject to

$$\log \left\{ \sum_{i=1}^I 10^{\left[\left(ON_i^+ - \frac{158.5D_{ik}}{1000} \sum_{j=1}^J X_{ij}^+ RE_j^- \right) / 10 \right]} \right\} \leq AN_k^{(p)}, \quad k = 1, 2, \dots, K \quad (9b)$$

$$ON_{(i1)}^+ - \frac{158.8D_{(i1)k}}{1000} - \sum_{j=1}^J X_{(i1)j}^+ RE_j^- \leq ON_{(i2)}^+ - \frac{158.8D_{(i2)k}}{1000} - \sum_{j=1}^J X_{(i2)j}^+ RE_j^-, i1, i2 = 1, 2, \dots, I; k = 1, 2, \dots, K \quad (9c)$$

$$\sum_{j=1}^J X_{ij}^+ \leq 1, i = 1, 2, \dots, I. \quad (9d)$$

$$X_{ij}^+ = 0 \text{ or } 1, i = 1, 2, \dots, I, \text{ and } j = 1, 2, \dots, J. \quad (9e)$$

Models 8 and 9 above can be solved by the solution proposed by Fan *et al.* (2012).

2.4 Results and Discussion

Table 4 presents the solutions obtained from the IBP model for noise abatement under uncertainty. The results show that different measures would be applied to different noise sources to mitigate the noise effect on nearby residences. Moreover, for one factory, the preferred noise control measure may be different under different acceptable noise levels.

Three Scenarios have been considered for this problem. Scenario 1 is the strictest, with the lowest acceptable noise levels, Scenario 2 is the middle range and Scenario 3 has the highest accepted noise levels. The type of scenario applicable to the problem would depend on the type of communities nearby – a residential community with retirement homes, for example, should have lower acceptable noise levels than a commercial area.

Table 2.4: Solutions from IBP model under different scenarios

X_{ij}	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	TC
Scenario 1												
$i = 1$	0	0	0	0	0	[1, 0]	[0, 1]	0	0	0	0	[1100, 1435]
$i = 2$	0	0	0	0	0	0	[1, 0]	0	0	[0, 1]	0	
$i = 3$	0	0	0	0	[0, 1]	0	0	[1, 0]	0	0	0	
Scenario 2												
$i = 1$	0	0	0	[1, 0]	0	[0, 1]	0	0	0	0	0	[900, 1365]
$i = 2$	0	0	0	0	0	[1, 0]	0	0	0	[0, 1]	0	
$i = 3$	0	0	0	0	[0, 1]	0	0	0	0	[1, 0]	0	
Scenario 3												
$i = 1$	0	0	0	0	0	[0, 1]	0	0	[1, 0]	0	0	[860, 1180]
$i = 2$	0	0	0	0	0	[1, 0]	[0, 1]	0	0	0	0	
$i = 3$	0	0	0	0	0	0	0	[0, 1]	0	[1, 0]	0	

Scenario 1 corresponds to the strictest standard, where both Communities 1 and 2 are exposed to the lowest acceptable noise levels. In this scenario, option 6 (i.e. Shelter and Resilience) would be applied to control noise for factory 1 under the optimistic condition, which corresponds to the lower bound of the objective-function value, while option 7 would be considered under the conservative condition, as shown in Table 5.

Table 2.5: Results for Scenario 1

	Optimistic		Conservative	
	Option	Cost	Option	Cost
Source 1	6	260	7	350
Source 2	7	320	10	435
Source 3	8	520	5	650
System Cost	1100		1435	

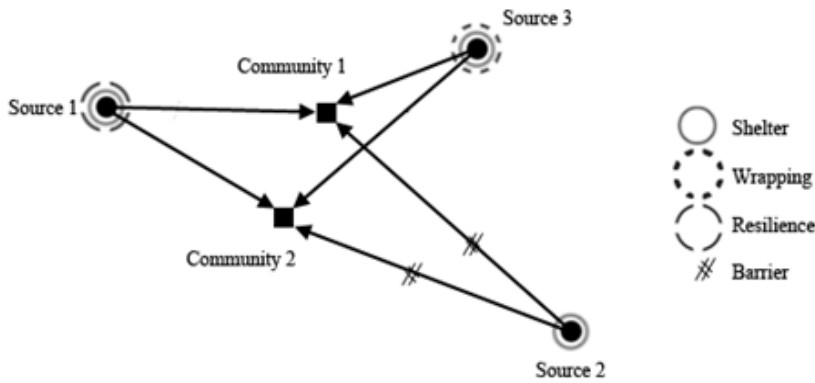


Figure 2.2: Noise control measures under Scenario 1

This is because the optimistic condition would predominately focus on reducing system cost while the conservative condition mainly considers the effects of noise reduction. Under the optimistic condition, the noise-reduction effect is considered to achieve its upper bound (e.g. 14.5 dB for option 6) and the cost of each option goes to its lower bound (e.g. \$260 for option 6), as presented in Table 1. Consequently, the noise from factory 1 (i.e. noise source 1) can just be reduced through option 6 to satisfy the acceptable noise levels for these two communities.

Conversely, the conservative condition primarily wants to guarantee the noise control effect, which regards all control measures as achieving their lower bounds (e.g. 15 dB for

option 7) and the cost of each control measure as achieving its upper bound (e.g. \$350 for option 7), as shown in Table 1. Therefore, option 7 should be applied to control noise from source 1 to satisfy the standards of the two communities. For noise source 2, the noise control options would be similar to noise source 1. A less expensive option (i.e. option 7) is to be applied under the optimistic conditions while a more expensive option (option 10) is to be used under conservative conditions. For noise source 3, option 8 (i.e. Shelter and Barrier), which will cost less and has a lower efficiency, would be used to control noise under optimistic conditions while option 5 (i.e. Equipment Update), which is more expensive but more effective, would be applied under conservative conditions.

Scenario 2 would allow normal standards for both communities, indicating acceptable noise levels to be 61 and 56 dB for Communities 1 and 2, respectively (as shown in Table 2). In this scenario, the solution of $X_{14}^{\pm} = [1, 0]$ means that option 4 (i.e. Barrier) is used to control the noise from factory 1 (i.e. noise source 1) under optimistic conditions, as shown in Table 6.

Table 2.6: Results for Scenario 2

	Optimistic		Conservative	
	Option	Cost	Option	Cost
Source 1	4	240	6	280
Source 2	6	260	10	435
Source 3	10	400	5	650
System Cost	900		1365	

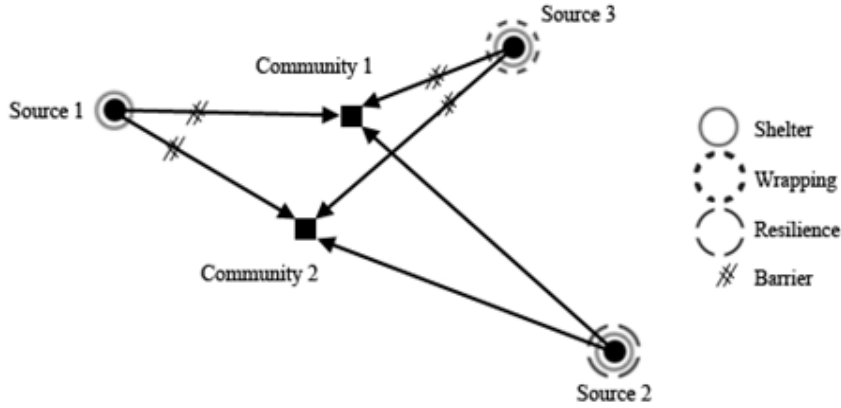


Figure 2.3: Noise control measures under Scenario 2

The solution of $X_{16}^{\pm} = [0, 1]$ indicates that option 6 is applied to control the noise from source 1 under conservative conditions. Compared with the options for source 1 in scenario 1, the control measures in scenario 2 would be less effective but also less expensive due to the loosening of the acceptable noise level constraints for the two communities. For noise source 2, option 6 ($X_{26}^{\pm} = [1, 0]$) would be applied to control its noise under less stringent conditions, while the same option as scenario 1 (i.e. option 10) would be used under more stringent noise control requirements. For noise source 3, the main difference for control measures between scenarios 1 and 2 is that a less effective option ($X_{310}^{\pm} = [1, 0]$) is desirable under advantageous conditions. Under demanding conditions, the noise control measure would be the same (i.e. option 5) for these two scenarios.

In scenario 3, which implements the loosest noise control standards in two communities, the main difference for noise control measures for three sources under optimistic conditions is that source 1 requires a less effective and less expensive measure (i.e. $X_{19}^{\pm} = [1, 0]$ with a cost of \$[200, 220] and a noise-reduction effect of [10, 11] dB). Under conservative conditions, both sources 2 and 3 would change to some less effective noise

control measures due to the relaxation of the standards. Options 7 and 8 (i.e. $X_{27}^{\pm} = [0, 1]$ and $X_{38}^{\pm} = [0, 1]$) would be applied to noise sources 2 and 3, respectively.

Table 2.7: Results for Scenario 3

	Optimistic		Conservative	
	Option	Cost	Option	Cost
Source 1	9	200	6	280
Source 2	6	260	7	350
Source 3	10	400	8	550
System Cost	860		1180	

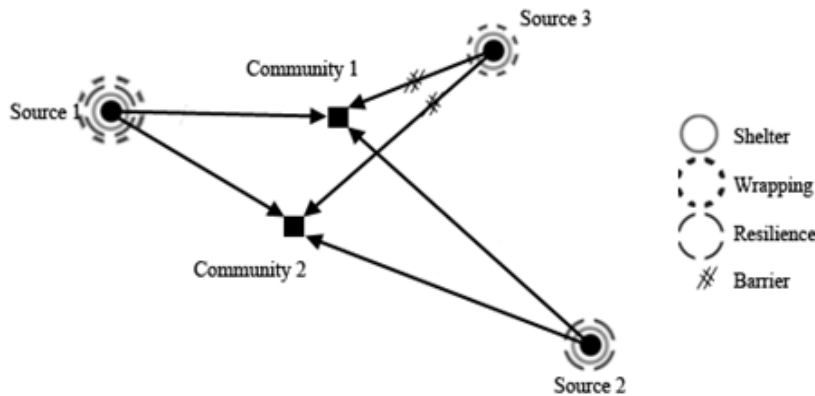


Figure 2.4: Noise control measures under Scenario 3

Generally, the above results indicate that through the proposed modeling approach, uncertainties presented as intervals in parameters can be communicated into the IBP model process. Table 4 provides the total system cost from the IBP model under different scenarios. The results suggest that different acceptable noise levels for the two communities would lead to varied objective function values. The system cost would have an opposite tendency to that of the acceptable noise levels for the two communities. As shown in Figures 5 and 6, both the lower and upper bounds of the system cost will

decrease as acceptable noise levels increase. This is because as higher acceptable noise levels are implemented, lower effective noise control measures are required, and thus system costs are lowered.

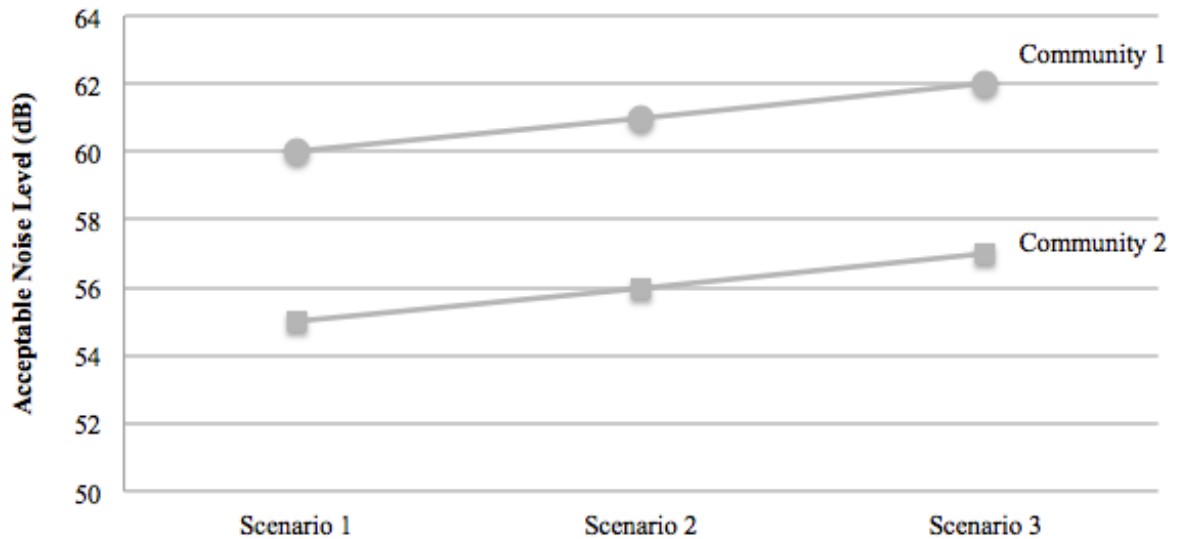


Figure 2.5: Acceptable noise levels for Communities 1 and 2

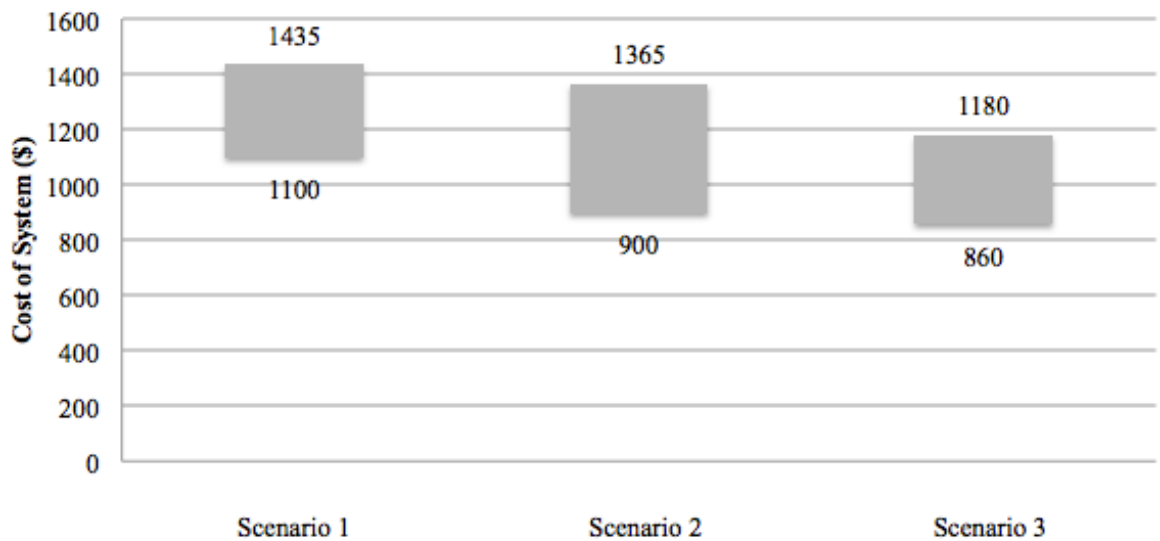


Figure 2.6: Upper and lower bounds of cost for recommended noise reduction systems

Under each scenario, the lower-bound cost corresponds to optimistic conditions where noise control measures are assumed to be most effective, while the upper-bound cost is

associated with more demanding conditions where the same measures are assumed to be less effective. For example, the system cost would be \$[1100, 1435] under scenario 1, indicating that the system cost would be \$1100 under advantageous conditions and \$1435 under demanding conditions. Moreover, the system cost would fluctuate within \$1100 and \$1435 as the model parameters vary within their lower and upper bounds. The system costs under the other two scenarios would have similar characteristics as that under scenario 1. Furthermore, the lower bound of the system cost is obtained under such consideration that each noise control measure would achieve the upper bound for the noise-reduction effect. Therefore, this may generate the highest risk of violating the acceptable noise levels of two communities. Conversely, the upper bound of the system cost is obtained with the most conservative noise-reduction effect of each control measure to be considered. This would definitely guarantee acceptable noise levels to be satisfied but may lead to excessive costs. Therefore, decision makers can make tradeoffs between system costs and the violation risk of acceptable noise levels, based on the solutions from the IBP model.

2.5 Conclusions

An interval binary programming (IBP) method has been proposed and applied to a representative noise control problem. As an extension of the binary programming method, IBP can explicitly address complexities and uncertainties in a noise control system. Parameters in the IBP model can be expressed as intervals, and also such uncertainties can be effectively incorporated within the model solution process. Two submodels corresponding to the lower and upper bounds of the objective-function value would be obtained based on an interactive algorithm, and interval solutions are then generated by solving the two submodels sequentially. Results of the model application indicate that useful solutions for noise control practices can be generated. A number of decision alternatives have been obtained and analyzed under different acceptable noise levels for the two communities. They reflect complex tradeoffs between environmental and

economic considerations. A willingness to pay higher operating costs will guarantee meeting the noise control standards; however, a desire to reduce the costs will run into the risk of potentially violating acceptable noise levels.

Although this study is a new application for the IBP methodology, the results suggest that this approach is applicable to practical noise control problems that are associated with highly complex and uncertain information.

2.6 References

- Achterberg, T. (2007) Conflict analysis in mixed integer programming. *Discrete Optimization*, 4(1): 1-20.
- Asawarungsaengkul, K. and Nanthavanij, S. (2006) Design of Optimal Noise Hazard Control Strategy with Budget Constraint. *International Journal of Occupational Safety and Ergonomics*, 12(4): 355-367.
- Asawarungsaengkul, K. and Nanthavanij, S. (2007) A Genetic Algorithm Approach to the Selection of Engineering Controls for Optimal Noise Reduction. *ScienceAsia*, 33: 89-101
- Baetz, B.W, (1990). Optimization/simulation modelling for waste management capacity planning. *Journal of Urban Planning and Development-ASCE*, 116(2): 59-79.
- Blair, C.E. and Jeroslow, R.G. (1982) The value function of an integer program. *Mathematical Programming*, 23: 237-273.

- Chen, Z.L., Li, S.L., Tirupati, D., (2002). A scenario-based stochastic programming approach for technology and capacity planning. *Computer & Operations Research*, 29: 781-806.
- Fan, Y.R. and Huang, G.H. (2012) A robust two-step method for solving linear programming problems within an environmental management context. *Journal of Environmental Informatics*, 19(1): 1-9.
- Garfinkel, R.S. and Nemhauser, G.L. (1972) *Integer programming*. John Wiley and Sons, New York.
- Huang, G.H., Baetz, B.W., and Patry, G.G. (1992) A grey linear programming approach for municipal solid waste management planning under uncertainty. *Civil Engineering Systems*, 10: 123-149.
- Huang, G.H., Baetz, B.W., and Patry, G.G. (1995) Grey integer programming: An application to waste management planning under uncertainty. *European Journal of Operational Research*, 83: 594-620.
- Jenkins, L., (1982). Parametric Mixed-Integer Programming: An Application to Solid Waste Management. *Management Science*, 28: 1270-1284.
- King, R.P. and Davis, J.R. (2003) Community noise: Health effects and management. *International Journal of Hygiene and Environmental Health*, 206: 123-131.
- Mun, S. and Cho, Y.H. (2009) Noise barrier optimization using a simulated annealing algorithm. *Applied Acoustic*, 70: 1094-1098.

- Murray-Johnson L, Witte K, Patel D, Orrego V, Zuckerman C, Maxfield AM, et al, (2004). Using the Extended Parallel Process Model to prevent noise-induced hearing loss among coal miners in Appalachia. *Health Education & Behavior*, 31:741-55.
- National Institute for Occupational Safety and Health (1998). *Health Hazard Evaluations: Noise and Hearing Loss, 1986-1997*. DHHS Publication NO. 98-126, Cincinnati, Ohio
- Prats, X., Puig, Vicenc, Quevedo, J., (2011) A multi-objective optimization strategy for designing aircraft noise abatement procedures. Case study at Girona airport. *Transportation Research Part D*, 16:31-41.
- Rajagopalan, S., Singh, M., Morton, T.E., (1998). Capacity Expansion and Replacement with Uncertain Technological Breakthroughs. *Management Science*, 44(1): 12-30
- Tokmechi, Z. (2011) Uses of Monte Carlo Method for Noise Pollution Optimization. *Advances in Environmental Biology*, 5(9): 2937-2943.
- Waly, S.M. and Sarker, B.R. (1998) Noise Reduction Using Nonlinear Optimization Modeling. *Computers and Industrial Engineering*, 35(1): 327 – 330.
- Yeh, L.J., Chang, Y.C., Chiu, M.C., (2004) Optimization of allocation and noise reduction on multi-system by using genetic algorithm, *Noise & Vibration Worldwide*, 35(4): 11-18.
- Zachary, D.S., Gervais, J., Leopold, U. (2010) Multi-impact optimization to reduce aviation noise and emissions. *Transportation Research Part D*, 15:82-93.

3. Minimizing Vehicular Travel Distances to Schools in Communities with Declining Enrolments Using Integer Programming

By W. Huang, S. Razavi, and B.W. Baetz

Abstract: Due to suburban sprawl in our cities, many elementary schools in urban core areas are seeing declines in student enrolment. This often results in the closure of smaller under-utilized schools and the replacement of these with a fewer number of larger schools, resulting in students commuting greater distances to and from school each day. This work proposes an integer programming-based method to aid in the decision-making process of which schools to close and whether or not to construct a large school to replace several small schools, by minimizing travel distances between home and school for subsections of a hypothetical community.

3.1 Introduction

In any community, a public school is an important piece of infrastructure that can significantly affect entire neighborhoods. Often, public schools serve more than just students. Schools have been used as a venue for elections, community events, extracurricular activities, and much more.

Ageing buildings in need of renovation and changing demographics have led public school boards across North America to re-evaluate their facility planning strategies. With suburban development and family sizes becoming smaller, many school boards are seeing enrolment declines in urban core schools. At the same time, due to the outdated and inefficient infrastructure of older buildings, the cost to run these schools is ever increasing, leading school boards to shut down smaller schools and relocate students to larger, more centralized schools.

Currently, these decisions are made primarily from a budgetary standpoint. The decision-making processes often do not fully account for the overall sustainability of the community as a whole. Up until the early 1990's, some research was done to determine optimal student allocation to schools based on capacity, utilization, diversity, age distribution, and other demographic characteristics (Church & Murray, 1993). Diamond *et al.* (1987) looked at distributing students to schools based on zoning and safety, which determined a student distribution method that minimizes the potential risks encountered by a student while travelling from home to school. In 2004, Caro *et al.* developed a GIS and integer programming-based school district redistribution model to re-draw school districts to allocate students more effectively for the city of Philadelphia, Pennsylvania. This work addressed redistributing the student population based on age, gender, and location to existing schools in the school system, but did not address the option of building new schools.

This technical note will introduce the concept of using integer programming for school location/allocation planning from a sustainable community standpoint. The costs will not only consider monetary costs for running the schools, but also the costs to the community in the form of energy consumption due to travel distances. Integer programming methods are applied to a hypothetical community to illustrate the proposed school selection and student assignment methods.

3.2 Methodology

Integer programming is proposed for this problem where decisions need to be made as to which schools are available for use and the allocation of students to these schools. The costs are minimized based on enrolment, school capacity; operating, construction and decommissioning costs; and transportation costs. Students in the community are assigned to selected schools, based on minimizing vehicular travel distance. For this concept,

students are assumed to either utilize a non-energy consuming form of transportation (walking, cycling, skateboarding, etc.) or an energy consuming form of transportation (driving, carpooling, busing). Although some forms of energy-consuming transportation are less costly from an energy standpoint than others, for the purposes of this study they will be considered the same. Following this, the transportation costs for the blocks adjacent to the school blocks are set to zero, since for the most part, these students will be assumed to choose non-vehicular methods of transportation due to their close proximity to the school.

The objective function represents the goal to minimize the overall system cost over a planning horizon of K years. The costs c_i , c_{build} , and c_{close} represent the cost to operate a school, cost to construct a school, and cost to close a school, respectively. These costs are each multiplied by the integer variable y_i (or conversely $1-y_i$) to determine if a particular school option is to be included in the optimal solution. The unit transportation cost represented by $trans$ is multiplied by the student population from a given sub-community and the distance travelled to a possible school option, represented by $x_{ij}d_{ij}$, where x_{ij} is the student population in sub-community j that will attend school at location i and d_{ij} is the distance between sub-community j and school i . These yearly costs are summed with the one-time construction and closing costs in the objective function.

For the model proposed, I is the number of schools, J is the number of sub-communities in the overall community, and K is the number of years in the planning period. S is the total number of students in the overall community.

(1) Objective Function

$$\min z = \left[\sum_{k=1}^K year_k \left(\sum_{i=1}^I c_i y_i + trans \sum_{i=1}^I \sum_{j=1}^J x_{ij} d_{ij} \right) \right] + \sum_{i=1}^I c_{build,i} y_i + \sum_{i=1}^I c_{close,i} (1 - y_i)$$

Subject to the following constraints:

(2) A school must be present in order for a student to attend it.

$$\sum_{j=1}^J x_{ij} \leq S y_i, \text{ for all } i$$

All students are assigned to a school.

$$\sum_{i=1}^I x_{ij} = s_j, \text{ for all } j$$

(4) Maximum capacity must not be exceeded.

$$\sum_{j=1}^J x_{ij} < cap_i, \text{ for all } i$$

(5) Minimum enrolment must be satisfied.

$$\sum_{j=1}^J x_{ij} \geq enrol_i, \text{ for all } i$$

(6) If the large school is selected, the small schools will not be selected.

$$y_I + y_i \leq 1, \text{ for all } i$$

Where y_I denotes the large school and $i = 1, 2, 3, \dots, I-1$ denote the smaller schools.

Technical constraints

$$x_{ij} \geq 0$$

$$y_i \in \text{binary}$$

Constraint (2) specifies that a school must be present in order for students to be allocated to it. The binary variable, y_i , will be equal to 1 if school i is present and 0 if school i is not present. Constraint (3) states that all students from section j must be accounted for across all i schools. Each school will have a maximum capacity and a minimum enrolment. These constraints are reflected in (4) and (5). Lastly, if the large centralized school that can house all students from the community is selected, it will not be cost efficient to

select the smaller schools as well, thus constraint (6) is implemented. The optimization was done using the Solver add-in in Microsoft Excel, which was appropriate for a reasonably sized problem.

3.3 Hypothetical Problem

To illustrate the use of this methodology, a simplified hypothetical problem is proposed. A 1 km by 1 km community is delineated with a 5 by 5 grid, with a random number of students assigned to each grid block. This grid size was selected because each grid block would be 200m across, and thus the maximum distance from the west-most end of one cell and the east-most end of the cell directly to the east of the first cell is 400m, which is considered a typical comfortable walking distance (Walker, 1962). Three schools currently exist in the community, however, due to low enrolment in all schools, the school board must make a decision on whether to maintain the existing schools or to construct a new one. The integer programming methodology was applied to assist in this decision-making process, using the Solver add-in for MS Excel to solve the equations. Student movement is simplified in this model. Instead of determining the actual distance between a point in a block and the location of a school, a simple counting method was applied. Each cell a student encounters from his or her home cell to the school cell is 200m. To illustrate, a student from Block 1 who attends the school at Block 7 will travel through Blocks 1, 2, 7 to reach the school, thus the distance travelled is estimated as 400m.

Table 1 shows the costs for constructing, operating, and decommissioning/repurposing each school. The costs were determined from approximations based on the Hamilton-Wentworth School Board 2012 budget (Hamilton-Wentworth District School Board, 2013).

Table 3.1. Costs to the school board

School Number	1	2	3	4
Annual cost to run school n	5	8	7	10
One time cost to open school n	0	0	0	15
One-time cost to close school n	0.25	0.25	0.25	0
School Capacity	300	500	400	1000
Minimum Enrolled	150	250	200	500

* cost in \$M

Table 2 summarizes the number of students living in each sub-community 1 through 25 and the distance from each sub-community to each school option.

Table 3.2. Number of students

Locations	Number of Students	Distance to School (m)			
		1	2	3	4
1	20	1000	600	1600	600
2	19	800	400	1400	400
3	24	600	600	1200	600
4	13	400	800	1000	800
5	35	200	1000	1200	1000
6	21	1200	400	1400	400
7	30	1000	200	1200	200
8	32	800	400	1000	400
9	24	600	600	800	600
10	19	400	800	1000	800
11	26	1400	600	1200	600
12	12	1200	400	1000	400
13	16	1000	600	800	600
14	18	800	800	600	800
15	26	600	1000	800	1000
16	11	1600	800	1000	800
17	18	1400	600	800	600
18	25	1200	800	600	800
19	16	1000	1000	400	1000
20	31	800	1200	600	1200
21	17	1800	1000	800	1000
22	15	1600	800	600	800
23	14	1400	1000	400	1000
24	9	1200	1200	200	1200
25	28	1000	1400	400	1400
Total	519				

For the purposes of minimizing the distance travelled in the system, a cost of \$5/km was assigned to the distance travelled per student, and a total of 200 school days per year is assumed. This cost is the cost per kilometer travelled and was estimated, assuming two students per vehicle, travelling to and from school, in a vehicle with 12 MPG fuel

efficiency. After a school is selected, the costs for student distance travelled is further refined using a heuristic approach. Students that live within one grid cell distance from their school have a transportation cost of zero, while those that exceed this limit have a transportation cost that follows the estimate outlined above. It should be noted that there are alternative forms of vehicle transportation that are less costly to the environment, such as school buses and all forms of public transportation. However, use of such forms of transportation varies dramatically depending on the community, season, and other societal factors. Thus, the above assumptions for determining travel costs were assumed to be reasonable for this analysis.

3.4 Results

The results of the method as applied to a hypothetical problem show that in the short term, it is better to keep two of the existing schools, close one school, and reallocate students from the closed school to the two remaining schools. This satisfies the minimum enrolment requirements of the two schools. For a twelve year planning horizon, the overall system cost for operating these two smaller schools becomes higher than that to construct one larger school. The high cost for construction makes it economically unattractive to construct new schools in a community when a planning horizon shorter than 12 years is considered.

Figure 1 below illustrates the short-term optimal model. This model is for planning horizons shorter than 12 years. The darker cells indicate schools selected and the lighter cells indicate which students are allocated to each selected school, as determined by the integer programming model. According to this model, students in Blocks 4, 5, 9 and 10 will walk (or bicycle, rollerblade, skateboard, etc.) to the school at Block 5, and students from Blocks 18, 19, 20, 23, 24, 25 will walk to the school at Block 24. Thus, their transportation cost will be zero. The system cost for this scenario is shown in Table 2. As

seen in Table 3, there are two possible scenarios. Scenario 1 has two open schools, 1 and 3. Scenario 2 has one large school and zero small schools.

Table 3.3. Summary of costs

Years	Schools Open	Board	Vehicular		Total Cost	Total Cost/Year
		Operating Cost	Distance Travelled	Travel Cost		
1	1, 3	12.25	52160	0.26	12.51	12.51
5	1, 3	62.25	260800	1.30	61.55	12.31
10	1, 3	122.25	521600	2.61	122.86	12.29
12	4	145.75	722880	3.61	149.36	12.45
15	4	175.75	903600	4.52	180.27	12.02
20	4	225.75	1204800	6.02	231.77	11.59

*costs in \$M

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure 3.1. Student distribution for Scenario 1

Under Scenario 1, the vehicular distance travelled annually is 52,160 km. Since economic growth was excluded in this analysis, this number is assumed to increase linearly each year.

Figure 2 shows the long-term optimal model; according to this model, for this particular problem, at 12 years it becomes optimal to construct a large school at Block 7. With this option, all students in all 25 blocks will attend the same school. Students in Blocks 1, 2, 3,

6, 7, 8, 11, 12, 13 will travel via non-vehicular methods and have a transportation cost of zero. A summary of these costs can also be found in Table 2 above.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure 3.2. Student Distribution for Scenario 2

Under this scenario, the vehicular distance travelled annually is 60,240 km. This option becomes increasingly optimal over a longer duration because the construction and demolition costs are spread out over a longer period.

A number of variables were, however, not considered in the analysis. The time value of money, changes in gasoline prices, and changes in available transportation technology were excluded from the analysis, as these aspects of economics can become very involved and were considered outside the scope of this study. Fluctuation in community demographics was also not considered.

3.5 Conclusions

The school location-allocation problem is a concern that many North American communities are currently facing. The proposed methodology can aid decision makers in determining systematically which schools to keep, which schools to build and when to build them, and where to distribute students. This model not only takes into consideration the costs to the public school board for running the system, but also the transportation

costs to the broader community. Schools are a key part of any community. They shape the demographics around them, and have a considerable effect on housing prices. Proper location-allocation of schools and students will have long-term benefits to the overall health and sustainability of a community.

Notation

The following symbols are used in this note

$c_{build,i}$ = cost to build school i

$c_{close,i}$ = cost to close school i

c_i = annual operation cost of school i

cap_i = school student capacity of school i

d = distance

$enrol$ = enrolment

s = number of students

$trans$ = unit cost of transportation

3.6 References

Caro, F., Sirabe, T., Guignard, M., & Weintraub, A. (2004, August). School redistricting: embedding GIS tools with integer programming. *Journal of the Operational Research Society*, 55(8), 836-849.

Church, R. L., & Murray, A. T. (1993). Modeling School Utilization and Consolidation. *Journal of Urban Planning and Development*.

Diamond, J. T., & Wright, J. R. (1987, May). Multiobjective Analysis of Public School Consolidation. *Journal of Urban Planning and Development*, 113(1).

Hamilton-Wentworth District School Board. (2013). Retrieved January 27, 2013, from
Hamilton-Wentworth District School Board:
<http://www.hwdsb.on.ca/aboutus/budget/>

Walker, J. (1962). *Human Transit - How Clearer Thinking About Public Transit Can
Enrich Our Communities and Our Lives*. Washington, DC: Island Press.

4. GIS-based Integer Programming Approach for the Location of Solid Waste Collection Depots

By W. Huang, B.W. Baetz and S. Razavi

4.1 Introduction

Municipal solid waste collection is an issue that every community has to deal with, and many citizens operate on a day-to-day basis without giving it much thought. Only when waste collection is disrupted do we see how important it is. For example, in 2009, the City of Toronto's worker strike caused a disruption in waste collection, which resulted in citizens dumping wastes around the city.

The current popular model of solid waste collection in North America is door-to-door garbage pickup. Residents place their wastes, compostables and recyclables in bags and bins on the curb, and a scheduled collection truck comes by, picks up the waste materials, and transports them to a materials recycling facility, composting facility, transfer station or directly to a landfill. A collection truck must drive to every household in the community, stop, pick up the wastes, drive to the next house, stop, pick up wastes, and so on. Stop-and-go driving consumes even larger amounts of fuel than driving without stops. Furthermore, suburban development patterns where houses are sprawled out and where houses sit on large properties results in longer distances being driven in order to collect every household's wastes.

In the past, garbage was dealt with differently. Prior to door-to-door pickup, towns often had the "town dump" where residents would take their own wastes to the dump. Although a minor inconvenience, this method of waste management makes people more aware of their own garbage and more accountable for it. Many jurisdictions in the world use a centralized bin or depot system where citizens transport their wastes to the depot and it is handled centrally from there.

In addition, households are generating larger and larger volumes of wastes. In 1990, average waste generation was 550kg per capita annually. This has steadily increased and the average waste generation in 2008 was 777kg per capita annually (The Conference Board of Canada, 2008). This can be attributed to today's consumerist culture and a lack of accountability for one's own garbage. Once waste is placed on the curb, it is considered out of the generator's hands.

This work looks at an alternate method to deal with waste disposal in municipalities by re-introducing the concept of taking one's own garbage to a centralized collection point. This change in policy could be prompted by operating budget constraints, where a cash-strapped municipality in the future explores options for reducing municipal services. However, the collection points would not be the final disposal place for the materials. Rather, they would serve as a depot for waste collection trucks to pick up garbage without needing to stop at each door. Not only does this reduce collection truck driving distances, but it also makes people more accountable for their own garbage. This work aims to select the most optimal (or convenient) sites for depot placement by minimizing the distance which residents must travel in order to dispose of their wastes.

GIS and integer programming models were used in this study to determine potential site locations for such a depot system in the town of Dundas, Ontario, Canada. The integer programming model was applied to determine optimal and near-optimal locations for depots based on travel distance from households to depots.

For the purposes of this study, waste generation volumes and depot capacities were not considered. It has been assumed that each depot will be large enough to collect all of the waste generated by the sub-community that will use it. As seen in the labour strike situation in Toronto, people will dispose of their waste regardless of the situation. This can also be observed in apartment dumpsters where people will put their garbage in an

already full dumpster or beside the dumpster, donation bins where people will place their donations beside the bins when full, and even in towns with recycling depots where people will overflow the bins. Thus, instead of optimizing for the capacity of the bins at depots, a municipality in this scenario would develop a bin-clearing frequency to minimize overflow occurrence. People generally would not turn away and find another depot just because the depot nearest them is full.

Integer and binary programming have been used to determine optimal solutions to various problems related to solid waste management. Mathematical optimization has been widely used to efficiently allocate resources for solid waste management (Juha-Heikki, 1999; Zamorano *et al*, 2009). Transfer station locations are a topic of interest in mathematical optimization programming, as the proper optimal placement of a transfer station can have effects on the cost and efficiency of waste management systems. A study was done for a Greek community to locate transfer stations optimally using binary programming methods (Chatzouridis & Komilis, 2011)(Chatzouridis *et al*, 2011), however, this study does not solve the problem of collection of wastes initially. Trucks are still needed to haul wastes from households to the transfer stations, which accounts for a considerable travel distance associated with solid waste management systems. The issue with the energy consumption required for municipal solid waste management is a known issue, and studies have been performed to analyse and optimize trip routes, vehicle speeds, vehicle types, and vehicle loads in an effort to mitigate this problem (Ericsson *et al*, 2006). In 1998, a study looked at the sizing and placement of recycling depots based on GIS models (Valeo *et al*, 1998). This study used the idea of letting residents take care of transporting the recyclable portion of their wastes to depots. The study used radial distances, which can be significantly different than the actual transport distances. This present work incorporates actual travel distances, and the full waste stream generated by households under the projected scenario of no individual household waste collection.

4.2 Methodology

Integer programming was used to as a tool to assist in the decision-making process to determine the optimal locations to site a set number of waste bins. GIS software was used to determine potential locations for a waste management depot. Potential depot sites were selected based on land-use type and proximity to sensitive areas, such as schools. Commercial sites may be desirable because they are typically high-traffic and encourage trip chaining (a person can dispose of their wastes while on their way to a store, for example). Public green spaces may be selected because they are city-owned, and encouraging more people to come and go from these areas can enhance the safety of the area. Potential depot sites should be high traffic, in easily accessible locations, safe and away from children. Frequent collection would be needed to prevent the depot area from becoming odorous. Bins would need to be designed with tight-closing covers to keep out scavenging wildlife.

For the purposes of this study, travel distances along roads will be used, as all potential users were assumed to drive or cycle along roads. Road-travel distances between the community points and each potential depot point were determined using the distance-measuring function in ArcMap 10 and MS Excel was used to perform the optimization with the data obtained from ArcMap. Integer programming methods were applied to select depot sites that minimize the travel distance of the community as a whole, given a set number of sub-communities.

Each variable i represents a depot location, j represents a subsection of the community, and d_{ij} represents the distance between depot point i and subcommunity j . y_i is an integer variable representing whether or not a depot will exist at location i , and x_{ij} indicates whether or not residents of subcommunity j drop their wastes off at depot location i .

The objective function for this problem is the following:

(1)

$$\min z = \sum_{i=1}^I \sum_{j=1}^J x_{ij} d_{ij}$$

Subject to the following constraints:

(2) Each sub-community is assigned to a depot

$$\sum_{i=1}^I x_{ij} = 1, \text{ for all } j$$

(3) A depot must be present at the location in order for it to receive waste from subcommunities

$$Ny_i - \sum_{j=1}^J x_{ij} \geq 0, \text{ for all } i$$

Where N is the number of sub-communities in the community.

(4) Total number of depots

$$\sum_{i=1}^I y_i = T$$

Where T is the total number of depot sites needed for the community, based on waste generation.

(5) Technical constraints

$$y_i \in \text{binary}$$

$$x_{ij} \in \text{binary}$$

The Solver add-in in Microsoft Excel was used to solve this problem.

4.3 Case Study

This methodology was applied to the community of Dundas, Ontario. Dundas is a community consisting of North Dundas (Olde Dundas) and South Dundas, with a combined population of approximately 25,000 people or 8000 households (Statistics

Canada, 2011). The community is divided down the middle by a road running east to west. The northern half has a grid-like configuration while the southern half has a more sprawling suburban pattern.



Figure 4.1: Study Area - Dundas, Ontario

Figure 1 shows the study area with potential depot sites (letters) and subcommunities (numbers). Due to the software limitations of Microsoft Excel's Solver add-in, a total of 5 potential depot sites and 40 subcommunities were selected. All depot sites are located either in a commercial parking lot or adjacent to public green spaces. Sub-communities were selected based on representative points. For example, most of the Pleasant Valley area in the south will always follow the same path to leave the area – by following the arterial road eastwards. Thus, point #44 is a point that the entire community will pass through and is used to represent that subcommunity.

The Olde Dundas area of the town has a grid network and more commercial areas, and thus has more potential depot sites. A second optimization scenario was run for that part

of Dundas specifically, since it is more detailed. Figure 2 shows the points selected for analysis for this second scenario.

For the purposes of this study, population distribution was not considered. It will be assumed that all subcommunities have an equal number of households, and thus their distance travelled will each be weighted the same. Thus, the distances calculated are not true distances. Realistically, slight variations in population density within the larger community may sway the decision to place depots closer to higher density areas. However, since Dundas is a mostly residential town, the population densities are considered to be reasonably equal.

4.4 Results

Figures 2 through 5 show the results for having 1, 2, and 3 collection depots. As seen in the figures, when only one depot is allowed, it takes a more centralized location. When 3 depots are allowed, they are spread out to be closer to clusters of subcommunities.

8000 households are assumed to be evenly distributed among the selected subcommunity points, thus there are 420 households per subcommunity.



Figure 4.2: 1 collection depot, 10080km travelled



Figure 4.3: 2 collection depots, 8148km travelled



Figure 4.4: 3 collection depots, 6384km travelled



Figure 4.5: Olde Dundas

For the Olde Dundas area, it can be seen that there is a more grid-like network of roads, and thus there are more potential paths that users can take. Figure 5 above shows the points selected for analysis in the Olde Dundas area. Figures 6 through 8 summarize the results of the optimization for the Olde Dundas area, with 1 to 3 depots selected for the communities. Since this is a sub-set of the entire town, and the subcommunities are divided more finely, it will be assumed that there are 100 households per subcommunity.

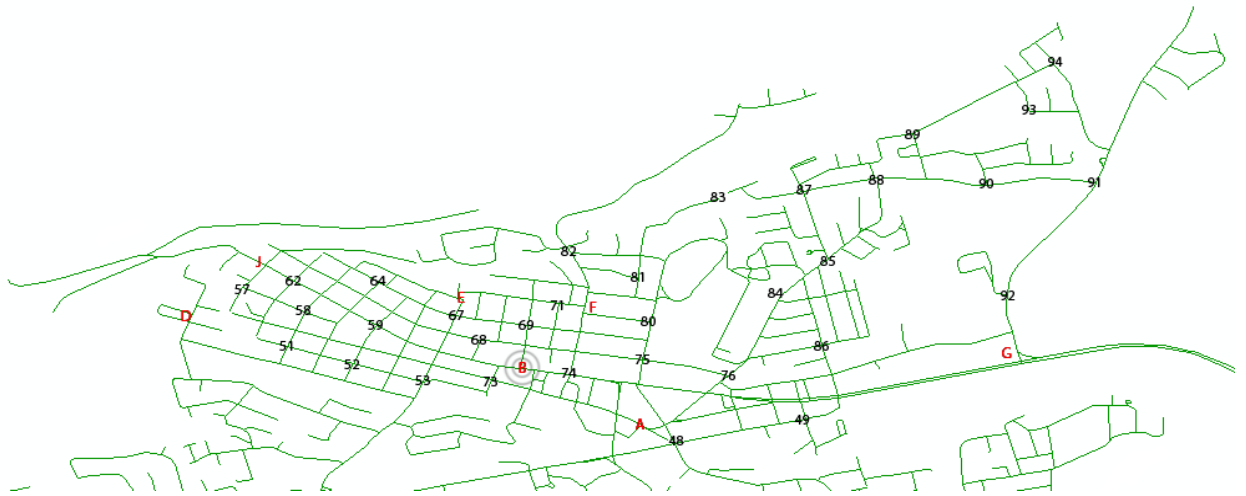


Figure 4.6: 1 collection depot, 4600km travelled

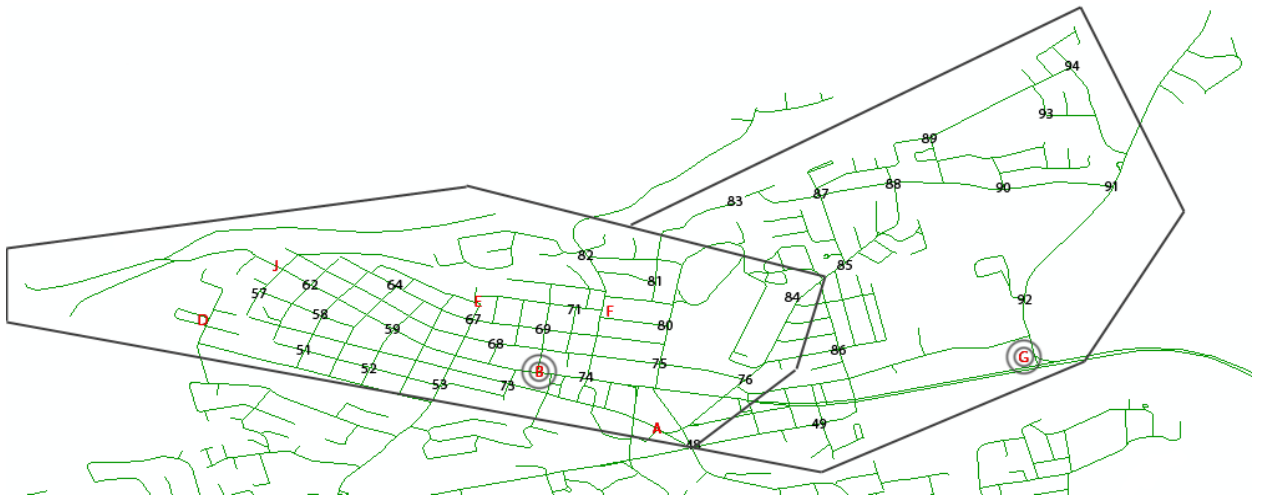


Figure 4.7: 2 collection depots, 3200km travelled

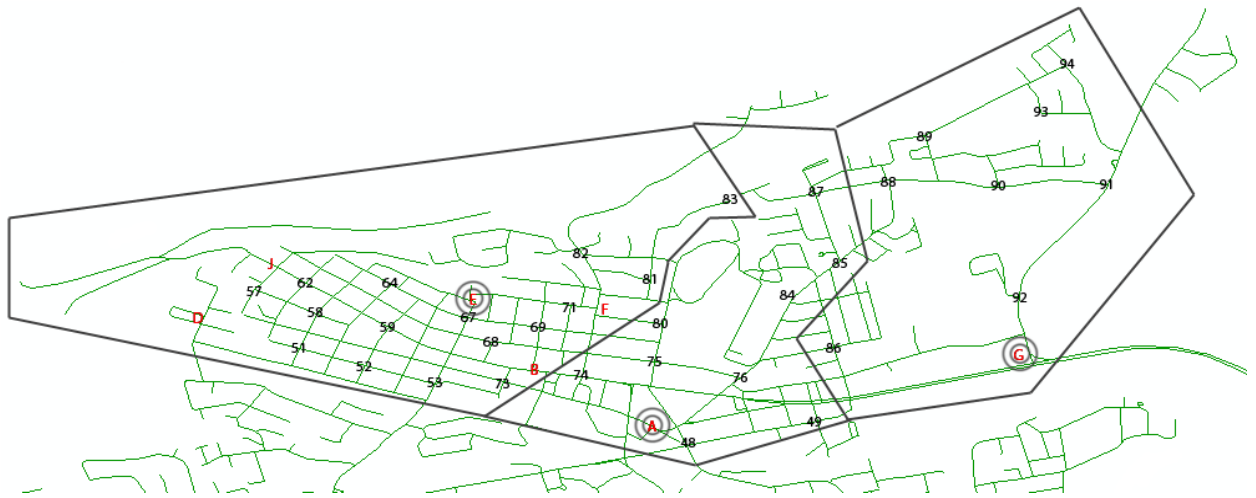


Figure 4.8: 3 collection depots, 3130km travelled

From the analyses above, it can be seen that Site B is the most frequently selected depot site based on an optimized decision. This is due to Site B being in a centralized location geographically. Furthermore, Site B is located on a commercial parking lot, and is a very suitable site for the placement of a collection depot. It can be seen that there is a large decrease in travel distance when the number of depot locations increases from 1 to 2, but the difference from 2 to 3 depot locations is much smaller.

The average amount of waste generated annually by Canadians is 777kg per capita (The Conference Board of Canada, 2008), or 15kg per week assuming no waste diversion. With a population of approximately 25,000 people, we can assume that the waste generation is approximately 375,000kg per week. Assuming a bulk density of 160kg per cubic meter, the total weekly volume of wastes is approximately 2350 cubic meters of waste. If a 50% diversion rate by volume is assumed, this would result in 1175 cubic meters of waste. Thus, if 3 depots are placed in the community and emptied weekly, they would have to have a capacity of approximately 400 cubic metres, which is rather large. Assuming bins with 40 cubic meter capacities are used, this would require 10 bins for each depot location. Thus, there would need to be more bins in more locations, more bins per location, or more frequent collection. However, one must note that solid wastes can become odorous very quickly, especially in warmer months, and more frequent collection

may be beneficial for several purposes: reduction of odour and potential to attract vermin and wildlife, smaller and less noticeable bin sizes, less likelihood of overflowing, and less likelihood for vandalism.

The approximate cost for municipal waste collection assuming door-to-door collection ranges from \$150 - \$200 per household (Kelleher, Robins, & Dixie, 2005). Dundas has approximately 8000 households so following this assumption, the cost for waste collection would total to \$1.2 – 1.6 million annually. These costs assume weekly curbside pickup.

In either a curbside scenario or a waste depot scenario, wastes will have to be transported downstream to waste management facilities (such as recycling facilities, composting facilities, landfill). The depot scenario would shift the burden to the individual household to a large degree, and for a community population of 25,000 people would realize an annual cost savings in excess of a million dollars.

4.5 Conclusions

This work proposes an alternative method for the collection of municipal solid waste. The current door-to-door curbside collection system is energy and cost-intensive and unsustainable, and it is important to consider alternative methods. This work proposes the idea of the use of waste drop off depots in urban communities, and have members of the community bring their waste materials to depots located around the community. This work provides an integer programming optimization method for selecting optimal locations in the case study community, using GIS to select potential sites and measure distances and Excel to perform the optimization to determine the best locations based on minimizing travel distances.

4.6 References

- Chatzouridis, C., & Komilis, D. (2011). A methodology to optimally site and design municipal solid waste transfer stations using binary programming. *Resources, Conservation and Recycling*, 89-98.
- Juha-Heikki, T. (1999). Strategic planning of municipal solid waste management. *Resources, Conservation and Recycling*, 111-133.
- Kelleher, M., Robins, J., & Dixie, J. (2005). *Taking out the trash: how to allocate costs fairly*. C.D. Howe Institute.
- Statistics Canada. (2011). *Population and dwelling counts, for Canada, provinces and territories, and census subdivisions (municipalities), 2011 and 2006 censuses*. Retrieved from Statistics Canada: <http://www12.statcan.gc.ca/census-recensement/2011/dp-pd/hltfst/pd-pl/Table-Tableau.cfm?LANG=Eng&T=302&SR=326&S=51&O=A&RPP=25&PR=35&CMA=0>
- Tanskanen, J. H. (2000). Strategic planning of municipal solid waste management. *Resources, Conservation and Recycling*, 111-133.
- Valeo, C., Baetz, B. W., & Tsanis, I. K. (1998). Location of Recycling Depots with GIS. *Journal of Urban Planning and Development*, 93-99.
- Zamorano, M., Molero, E., Grindlay, A., Rodriguez, M. L., Hurtado, A., & Calvo, F. J. (n.d.). A planning scenario for the application of geographical information systems in municipal waste collection: a case of Churriana de la Vega.

Appendix A – Technical Definitions for Paper I

Appendix 1.

Definition 1. Let x denote a closed and bounded set of real number. An interval number x^\pm is defined as an interval with known lower and upper bounds but unknown distribution information for x (Huang et al., 1992):

$$x^\pm = [x^-, x^+] = \{t \in x / x^- \leq t \leq x^+\}$$

where x^- and x^+ are the lower and upper bounds of x^\pm , respectively. When $x^- = x^+$, x^\pm becomes a deterministic number, i.e. $x^\pm = x^- = x^+$.

Definition 2. Let R' denote a set of real integer numbers. An interval integer is an interval number with integer lower and upper bounds, and all of its elements are integers (Huang et al., 1995):

$$y^\pm = [y^-, y^+]$$

$$y^- \in R', y^+ \in R'$$

$$\text{any } y \in y^\pm, y \in R'$$

Definition 3. An interval binary number is an interval integer with its two bounds being 0 and 1, and its elements can only be 0 or 1 (Huang et al., 1992).

Definition 4. Let $*$ $\in \{+, -, \times, \div\}$ be a binary operation on interval numbers. For interval numbers x^\pm and y^\pm , we have (Huang et al., 1995):

$$x^\pm * y^\pm = [\min(x*y), \max(x*y)], x^- \leq x \leq x^+, y^- \leq y \leq y^+$$

In the case of division, it is assumed that $\forall y \neq 0$. Hence we have:

$$x^\pm + y^\pm = [x^- + y^-, x^+ + y^+]$$

$$x^\pm - y^\pm = [x^- - y^+, x^+ - y^-]$$

$$x^\pm \times y^\pm = [\min(x \times y), \max(x \times y)], x^- \leq x \leq x^+, y^- \leq y \leq y^+$$

$$x^\pm \div y^\pm = [\min(x \div y), \max(x \div y)], x^- \leq x \leq x^+, y^- \leq y \leq y^+$$

Definition 5. For an interval number, we have (Huang et al., 1995):

$$x^\pm \geq 0 \text{ iff } x^+ \geq 0 \text{ and } x^- \geq 0$$

$$x^\pm \leq 0 \text{ iff } x^+ \leq 0 \text{ and } x^- \leq 0$$

Definition 6. For $x^\pm = [x^-, x^+]$ and $y^\pm = [y^-, y^+]$, we have their order relations as follows (Huang et al., 1995):

$$x^\pm \leq y^\pm \text{ iff } x^- \leq y^- \text{ and } x^+ \leq y^+,$$

$$x^\pm < y^\pm \text{ iff } x^- < y^- \text{ and } x^+ < y^+.$$

Definition 7. For an interval number x^\pm , we define $\text{Sign}(x^\pm)$ as follows (Huang et al., 1995):

$$\text{Sign}(x^\pm) = \begin{cases} 1 & \text{if } x^\pm \geq 0 \\ -1 & \text{if } x^\pm < 0 \end{cases}$$

Definition 8. For an interval number x^\pm , we define its grey absolute value $|x/^\pm$ as follows (Huang et al., 1995):

$$|x/^\pm = \begin{cases} |x/^\pm & \text{if } x^\pm \geq 0 \\ -|x/^\pm & \text{if } x^\pm < 0 \end{cases}$$

Thus we have

$$|x/^\pm = \begin{cases} |x/^\pm & \text{if } x^\pm \geq 0 \\ -|x/^\pm & \text{if } x^\pm < 0 \end{cases}$$

and

$$|x/^\pm = \begin{cases} |x/^\pm & \text{if } x^\pm \geq 0 \\ -|x/^\pm & \text{if } x^\pm < 0 \end{cases}$$

Appendix B – Paper II Input Tables for Optimization

Years	1	Initial Cost:	12.25
Transportation	million\$/km/perso	Total Cost:	12.25
Cost	5.00E-06 n	Distance:	1768
min z =	2.60E+0	Vehicular Distance	52160
	1	Travelled	2.61E-01
		Travel Cost	01

School n=	1	2	3	4
yn	1	0	1	0
x1n	20	0	0	0
x2n	19	0	0	0
x3n	24	0	0	0
x4n	13	0	0	0
x5n	35	0	0	0
x6n	21	0	0	0
x7n	30	0	0	0
x8n	32	0	0	0
x9n	24	0	0	0
x10n	19	0	0	0
x11n	0	0	26	0
x12n	0	0	12	0
x13n	0	0	16	0
x14n	0	0	18	0
x15n	26	0	0	0
x16n	0	0	11	0
x17n	0	0	18	0
x18n	0	0	25	0
x19n	0	0	16	0
x20n	0	0	31	0
x21n	0	0	17	0
x22n	0	0	15	0
x23n	0	0	14	0
x24n	0	0	9	0
x25n	0	0	28	0

Years	5			
Transportation		million\$/km/perso		
Cost	5.00E-06	n	Cost:	60.25
	8.11E+0			
min z =	1		Distance:	1768
			Vehicular Distance	
			Travelled	260800
				1.30E+0
			Travel Cost	0
School n=	1	2	3	4
yn	1	0	1	0
x1n	20	0	0	0
x2n	19	0	0	0
x3n	24	0	0	0
x4n	13	0	0	0
x5n	35	0	0	0
x6n	21	0	0	0
x7n	30	0	0	0
x8n	32	0	0	0
x9n	24	0	0	0
x10n	19	0	0	0
x11n	0	0	26	0
x12n	0	0	12	0
x13n	0	0	16	0
x14n	0	0	18	0
x15n	26	0	0	0
x16n	0	0	11	0
x17n	0	0	18	0
x18n	0	0	25	0
x19n	0	0	16	0
x20n	0	0	31	0
x21n	0	0	17	0
x22n	0	0	15	0
x23n	0	0	14	0
x24n	0	0	9	0
x25n	0	0	28	0

Years	10			
Transportation		million\$/km/perso		
Cost	5.00E-06	n	Cost:	120.25
	1.50E+0			
min z =	2		Distance:	1768
			Vehicular Distance	
			Travelled	521600
				2.61E+0
			Travel Cost	0

School n=	1	2	3	4
yn	1	0	1	0
x1n	20	0	0	0
x2n	19	0	0	0
x3n	24	0	0	0
x4n	13	0	0	0
x5n	35	0	0	0
x6n	21	0	0	0
x7n	30	0	0	0
x8n	32	0	0	0
x9n	24	0	0	0
x10n	19	0	0	0
x11n	0	0	26	0
x12n	0	0	12	0
x13n	0	0	16	0
x14n	0	0	18	0
x15n	26	0	0	0
x16n	0	0	11	0
x17n	0	0	18	0
x18n	0	0	25	0
x19n	0	0	16	0
x20n	0	0	31	0
x21n	0	0	17	0
x22n	0	0	15	0
x23n	0	0	14	0
x24n	0	0	9	0
x25n	0	0	28	0

Years	12			
Transportation Cost	5.00E-06	million\$/km/person	Cost:	145.75
min z =	1.79E+02		Distance:	1962
			Vehicular Distance Travelled	722880
			Travel Cost	3.6144

School n=	1	2	3	4
yn	0	0	0	1
x1n	0	0	0	20
x2n	0	0	0	19
x3n	0	0	0	24
x4n	0	0	0	13
x5n	0	0	0	35
x6n	0	0	0	21
x7n	0	0	0	30
x8n	0	0	0	32
x9n	0	0	0	24
x10n	0	0	0	19
x11n	0	0	0	26
x12n	0	0	0	12
x13n	0	0	0	16
x14n	0	0	0	18
x15n	0	0	0	26
x16n	0	0	0	11
x17n	0	0	0	18
x18n	0	0	0	25
x19n	0	0	0	16
x20n	0	0	0	31
x21n	0	0	0	17
x22n	0	0	0	15
x23n	0	0	0	14
x24n	0	0	0	9
x25n	0	0	0	28

Years	15			
Transportation Cost	5.00E-06	million\$/km/person	Cost:	175.75
min z =	2.15E+02		Distance:	1962
			Vehicular Distance Travelled	903600
			Travel Cost	4.518

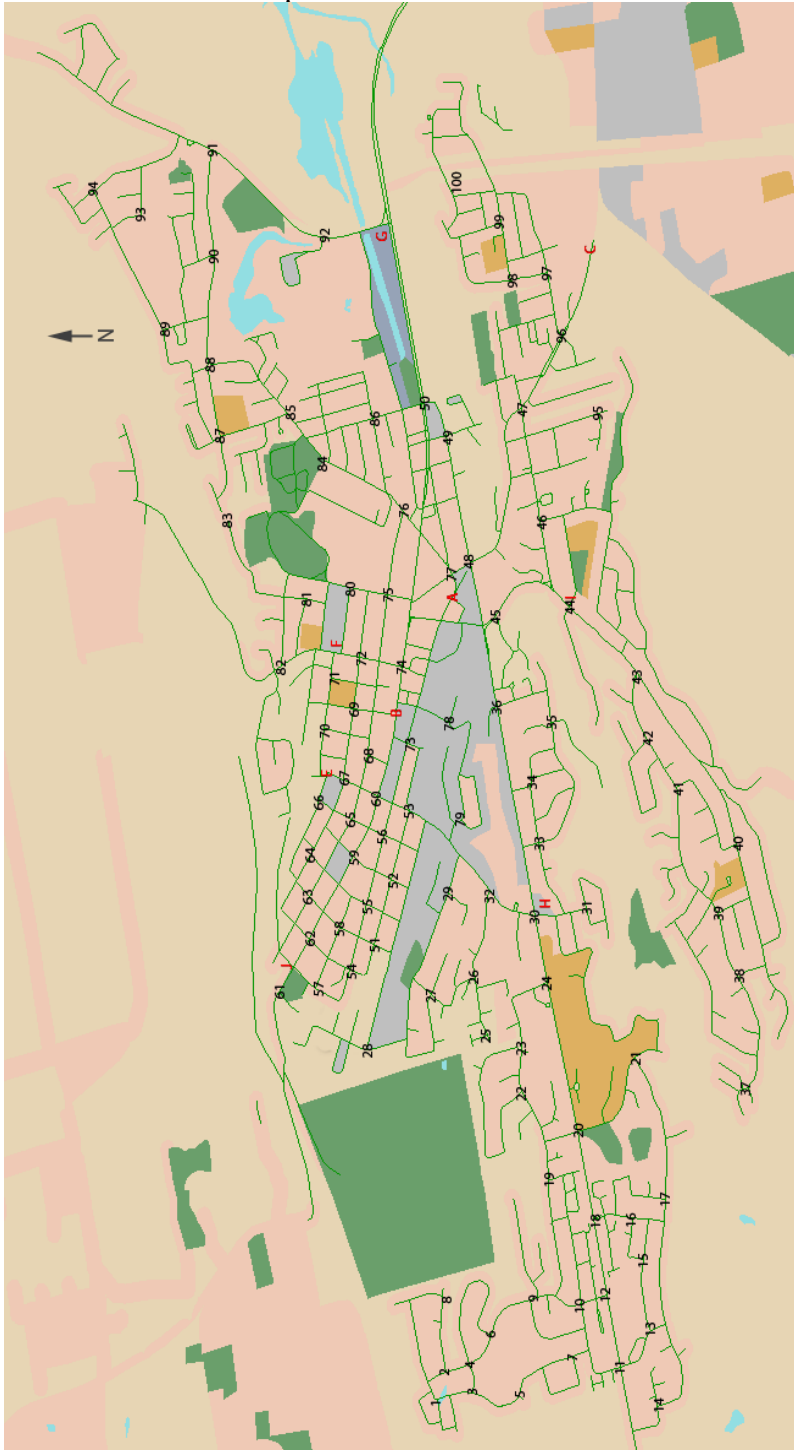
School n=	1	2	3	4
yn	0	0	0	1
x1n	0	0	0	20
x2n	0	0	0	19
x3n	0	0	0	24
x4n	0	0	0	13
x5n	0	0	0	35
x6n	0	0	0	21
x7n	0	0	0	30
x8n	0	0	0	32
x9n	0	0	0	24
x10n	0	0	0	19
x11n	0	0	0	26
x12n	0	0	0	12
x13n	0	0	0	16
x14n	0	0	0	18
x15n	0	0	0	26
x16n	0	0	0	11
x17n	0	0	0	18
x18n	0	0	0	25
x19n	0	0	0	16
x20n	0	0	0	31
x21n	0	0	0	17
x22n	0	0	0	15
x23n	0	0	0	14
x24n	0	0	0	9
x25n	0	0	0	28

Years	20			
Transportation		million\$/km/perso		
Cost	5.00E-06	n	Cost:	225.75
	2.75E+0			
min z =	2		Distance:	1962
			Vehicular Distance	120480
			Travelled	0
			Travel Cost	6.024

School n=	1	2	3	4
yn	0	0	0	1
x1n	0	0	0	20
x2n	0	0	0	19
x3n	0	0	0	24
x4n	0	0	0	13
x5n	0	0	0	35
x6n	0	0	0	21
x7n	0	0	0	30
x8n	0	0	0	32
x9n	0	0	0	24
x10n	0	0	0	19
x11n	0	0	0	26
x12n	0	0	0	12
x13n	0	0	0	16
x14n	0	0	0	18
x15n	0	0	0	26
x16n	0	0	0	11
x17n	0	0	0	18
x18n	0	0	0	25
x19n	0	0	0	16
x20n	0	0	0	31
x21n	0	0	0	17
x22n	0	0	0	15
x23n	0	0	0	14
x24n	0	0	0	9
x25n	0	0	0	28

Appendix C – Paper III maps and input tables for optimization

Dundas Land Use Map



Distances between sub-communities and depots in km

Community	Depot									
	A	B	C	D	E	F	G	H	I	J
1	4.22	4.11	5.68	4.59	4.07	4.72	5.66	2.84	4.50	4.57
2	3.80	3.52	5.09	4.12	3.66	4.32	5.24	2.44	4.09	4.16
3	4.00	3.92	5.49	4.05	3.88	4.54	5.44	2.65	4.31	4.38
4	3.66	3.39	4.96	4.00	3.53	4.19	5.10	2.31	3.97	4.04
5	3.74	3.67	5.24	4.15	3.63	4.28	5.18	2.40	4.06	4.13
6	3.51	3.23	4.80	3.84	3.38	4.04	4.95	2.15	3.81	3.88
7	3.24	3.33	4.90	3.81	3.29	3.94	4.68	2.06	3.72	3.79
8	4.10	3.81	5.38	4.44	3.97	4.62	5.54	2.74	4.40	4.47
9	3.47	2.94	4.51	3.65	3.12	3.77	4.91	1.89	3.55	3.62
10	2.97	2.70	4.27	3.31	2.85	3.51	4.41	1.62	3.28	3.35
11	3.11	3.03	4.60	3.45	2.99	3.92	4.55	1.77	3.42	3.49
12	2.84	2.75	4.32	3.17	2.71	3.65	4.28	1.49	3.14	3.21
13	3.34	3.16	4.61	3.62	3.18	4.15	4.78	1.95	3.61	3.68
14	3.58	3.65	5.08	3.90	3.10	4.39	5.02	1.87	3.53	3.60
15	2.98	2.88	4.33	3.93	2.60	3.79	4.42	1.37	3.03	3.10
16	2.73	2.65	4.09	3.07	2.36	3.54	4.17	1.13	2.79	2.86
17	3.22	2.46	4.34	3.31	3.10	4.03	4.66	1.88	3.54	3.61
18	2.54	2.46	4.03	2.88	2.42	3.35	3.98	1.19	2.85	2.92
19	2.51	2.25	3.82	2.86	2.39	3.05	3.95	1.16	2.82	2.89
20	2.20	2.12	3.69	2.53	2.08	2.73	3.64	0.85	2.51	2.58
21	2.69	2.39	4.01	3.01	2.53	3.18	4.13	1.30	2.96	3.03
22	2.17	1.89	3.46	2.88	2.09	2.75	3.61	0.86	2.52	2.59
23	2.02	1.74	3.30	2.34	1.94	2.59	3.46	0.71	2.37	2.44
24	1.63	1.53	3.10	1.95	1.49	2.44	3.07	0.26	1.92	1.99
25	2.18	1.40	2.97	1.16	1.54	2.19	3.62	0.82	2.48	2.04
26	1.95	1.17	2.74	0.94	1.32	1.97	3.39	0.64	2.30	1.82
27	2.45	1.44	3.01	0.65	1.40	2.09	3.89	1.10	2.76	1.90
28	2.97	1.50	3.07	0.09	1.47	2.12	4.41	1.61	3.27	1.04
29	2.00	0.96	2.52	0.91	0.92	1.51	3.44	0.62	2.28	1.42
30	1.36	1.27	2.84	1.68	1.23	2.17	2.80	0.00	1.66	1.73
31	1.62	0.54	2.97	1.97	1.50	2.43	3.06	0.27	1.93	2.00
32	1.61	1.01	2.58	1.43	0.97	1.63	3.05	0.26	1.91	1.47
33	1.19	1.61	2.52	2.03	1.58	2.00	2.63	0.46	1.49	2.08
34	0.99	1.85	2.29	2.29	1.81	1.80	2.43	0.68	1.28	2.31
35	0.94	2.15	2.28	2.66	2.43	1.75	2.38	1.06	1.24	2.95
36	0.55	1.06	2.63	2.51	1.68	1.36	1.99	0.81	0.84	2.54
37	2.99	3.59	3.68	5.07	4.19	3.80	4.43	4.23	2.23	4.62

38	2.54	3.10	3.19	4.62	3.70	3.35	3.98	3.74	1.77	4.16
39	2.27	2.83	2.92	4.35	3.43	3.08	3.71	3.46	1.50	3.89
40	2.05	2.57	2.70	4.13	3.17	2.86	3.49	3.08	1.40	3.79
41	1.71	2.27	2.36	3.79	2.87	2.52	3.15	2.62	0.95	3.34
42	1.47	2.03	2.13	3.55	2.63	2.28	2.91	2.38	0.71	3.10
43	1.20	1.76	1.92	3.28	2.36	2.01	2.64	2.11	0.44	2.83
44	0.76	1.33	1.42	2.84	1.93	1.57	2.20	1.47	0.00	2.39
45	0.29	0.83	1.65	2.37	1.43	1.10	1.73	1.19	0.56	2.06
46	0.91	1.41	1.04	2.99	2.01	1.72	2.35	1.81	0.37	2.68
47	0.87	1.70	0.60	2.96	2.30	1.68	2.31	2.25	0.81	2.64
48	0.14	0.98	2.55	2.22	1.58	0.95	1.40	1.37	0.78	1.86
49	0.66	1.51	3.08	2.74	1.86	1.47	0.89	1.87	1.29	2.26
50	0.85	1.67	3.24	2.93	2.05	1.44	0.70	2.06	1.48	2.30
51	1.49	1.03	2.60	0.58	0.99	1.46	2.93	1.31	2.13	0.55
52	1.23	0.77	2.34	0.83	0.73	1.20	2.67	1.06	1.87	0.84
53	0.95	0.49	2.06	1.11	0.43	0.90	2.38	0.78	1.60	0.37
54	1.53	1.06	2.63	0.80	1.03	1.50	2.96	1.52	2.17	0.38
55	1.27	0.80	2.37	0.79	0.76	1.24	2.70	1.28	1.91	0.55
56	0.99	0.53	2.10	1.07	0.49	0.96	2.43	1.00	1.63	0.91
57	1.79	1.19	2.75	1.03	1.17	1.65	3.08	1.78	2.43	0.10
58	1.68	0.95	2.51	0.77	0.83	1.30	2.84	1.50	2.32	0.40
59	1.20	0.58	2.15	1.05	0.55	1.02	2.48	1.21	1.85	0.52
60	0.94	0.32	1.89	1.27	0.28	0.76	2.22	0.95	1.58	0.79
61	1.85	1.22	2.79	1.25	1.03	1.51	3.12	2.04	2.49	0.13
62	1.59	0.97	2.54	1.03	0.78	1.25	2.87	1.60	2.23	0.03
63	1.88	0.94	2.50	0.97	0.56	1.03	2.83	0.54	2.52	0.35
64	1.46	0.81	2.38	1.19	0.33	0.81	2.71	1.44	2.11	0.57
65	1.17	0.54	2.11	1.35	0.31	0.79	2.44	1.18	1.81	0.72
66	1.35	0.69	2.26	1.48	0.09	0.56	2.58	1.32	1.99	0.80
67	1.13	0.51	2.08	1.47	0.09	0.57	2.41	1.14	1.77	0.97
68	0.91	0.29	1.86	1.52	0.31	0.66	2.19	1.18	1.55	1.02
69	0.98	0.22	1.79	1.82	0.37	0.33	2.12	1.41	1.63	1.34
70	1.13	0.34	1.90	1.80	0.41	0.30	2.23	1.64	1.78	1.31
71	0.90	0.46	2.03	2.01	0.60	0.10	2.17	1.65	1.54	1.48
72	0.70	0.42	1.99	2.00	0.56	0.10	1.94	1.68	1.34	1.51
73	0.62	0.16	1.73	1.44	0.60	0.67	2.06	1.11	1.27	1.28
74	0.48	0.19	1.76	1.78	0.79	0.43	1.71	1.50	1.12	1.29
75	0.43	0.57	2.14	2.17	0.92	0.59	1.60	1.89	1.07	1.66
76	0.30	0.90	2.47	2.48	1.25	0.92	1.32	2.22	0.95	1.99
77	0.07	0.68	2.25	2.17	1.29	1.02	1.37	1.48	0.72	1.77

78	0.65	0.33	1.90	1.78	0.93	0.88	2.13	1.86	1.30	1.43
79	1.12	0.83	2.40	1.26	0.79	1.44	2.63	0.64	1.77	1.27
80	0.66	0.76	2.32	2.37	0.73	0.26	1.77	2.04	1.30	1.87
81	0.93	0.90	2.47	2.66	0.82	0.39	2.06	2.20	1.57	2.00
82	1.30	0.83	2.40	2.97	0.76	0.32	2.38	2.51	1.94	1.94
83	1.51	1.44	3.01	3.23	1.40	0.86	2.53	2.80	2.16	2.46
84	1.86	1.25	2.82	2.85	1.86	1.50	1.45	2.83	2.50	2.51
85	1.14	1.54	3.11	3.70	2.82	1.78	1.46	2.86	1.78	2.79
86	1.02	1.22	2.79	3.21	2.33	1.47	0.97	2.34	1.66	2.58
87	1.50	1.90	3.47	4.06	3.18	2.17	1.81	2.98	2.14	3.00
88	1.79	1.90	3.47	4.35	3.47	2.20	2.08	3.32	2.43	3.29
89	2.20	2.60	4.17	4.81	3.88	2.61	2.52	3.37	2.84	3.70
90	2.19	2.59	4.16	4.55	3.19	2.60	1.44	3.49	2.83	3.69
91	2.60	2.93	4.50	4.96	3.51	3.40	1.03	3.80	3.24	4.10
92	1.79	2.25	3.82	3.87	2.84	2.51	0.36	3.13	2.43	3.37
93	3.15	3.53	5.09	5.56	4.20	3.78	1.70	4.13	3.80	4.70
94	3.14	4.89	6.45	5.57	4.52	4.10	1.72	4.38	3.79	4.79
95	1.18	1.83	3.39	3.27	2.43	2.10	2.50	2.42	1.09	2.95
96	1.25	2.04	0.34	3.34	2.64	2.02	2.66	2.59	1.16	3.02
97	1.46	2.30	0.60	3.54	2.90	2.28	2.91	2.84	1.42	3.23
98	1.65	2.49	0.79	3.74	3.09	2.74	3.10	3.03	1.61	3.42
99	1.88	2.70	1.02	3.96	3.30	2.70	3.32	3.26	1.83	3.65
100	2.22	3.06	1.36	4.30	3.66	3.04	3.66	3.60	2.18	3.99