# SIMULATION AND OPTIMIZATION

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# OF

# WIRE DRAWING

By

ERROL MONTGOMERY ALEXANDER, B.Sc. (ENG)

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AUTHOR: Errol Montgomery Alexander, B.Sc. (Eng)
(University of the Witwatersrand)

SUPERVISOR: Professor M.C. de Malherbe.

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# TABLE OF CONTENTS

## LIST OF ILLUSTRATIONS

# ABSTRACT

A study of the production line process of wire manufacturing was made and a mathematical model of some aspects of the physical process then constructed. Using this model a computer program was written to simulate the process and hence predict the statistical distribution of wire strength. Additionally certain aspects of the wire drawing process are optimized so as to impart desirable metallurgical characteristics to the wire.

The simulation is achieved by two different techniques, in cases where the random variables are mutually independent the method of 'Transformation of Variables' is used, and in the more complex case of non-independent variables a 'Monte Carlo' simulation is performed.

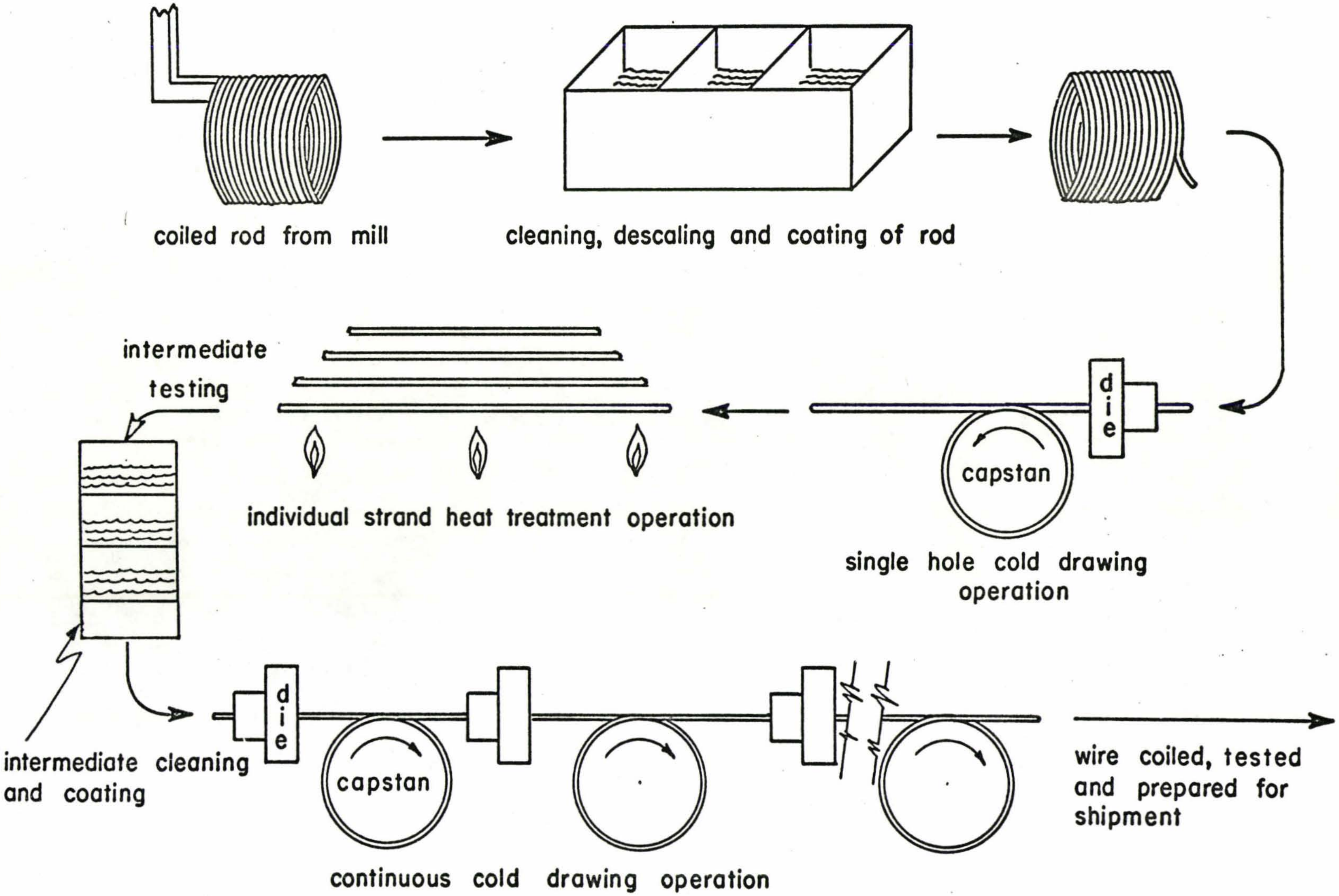
The area reduction of the wire due to reduction of successive die sizes, is optimized with respect to the ideal process when subjected to manufacturing machinery limitations. The optimization technique used is basically a 'Direct Search' modified to overcome the inherent inability of the method to move toward optimality in the face of certain constraints.

# INTRODUCTION

In the manufacture of steel rope wire there are a number of specifications that must be met, the most important being tensile strength and torsional standards. The tensile strength standard specifies both an upper and lower limit for wire fracture in either p.s.i. or lbs while the torsion standard specifies a minimum number of turns a set length of wire should undergo without fracture. As a result, the manufacturing process requires careful planning and control to ensure that the variation of the above properties and in particular tensile strength is not excessive.

The current production practice generally used by the wire industry is briefly described with reference to Figure 1.

The raw material of the process is steel rod, available in a number of discreet sizes and chemical compositions. The rod is produced by hot rolling from billets and as a result of this manufacturing process usually has a diameter variation of the order of $\pm$ .015 inches. In addition to this size variation there is also a random variation in tensile strength of the rods due to variation in chemical composition of the steel and non-uniformity of the heat treatment process.




Schematic Diagram of Rope Wire Production

Figure 1

The tensile strength of wire which is directly cold drawn from rod is predictable if the initial rod tensile stress and the gain in tensile stress due to work hardening are known.

The latter value can be determined from standard work hardening curves (Fig. 2) which provide a relationship between gain in tensile strength and percentage reduction in area from wire to rod.

WIRE TENSILE STRENGTH = ROD TENSILE + GAIN IN TENSILE DUE TO WORK HARDENING

If the tensile strength in pounds is required, the stress has to be multiplied by wire area, which is also a randomly distributed variable within specified tolerances.

Hence there are at least three random factors in the process that directly influence the final product properties and make accurate prediction difficult. There are, of course, other random effects, however, these are negligible compared to the above.

If the extremes of wire strength are calculated using the extremes of the three random variables above, it is found that the range of wire strength is far greater than the standard allowable variation. This means that even though all prior processes are within standard tolerances the resultant variation in wire strength cannot be guaranteed to be within limits; this in turn introduces an element of uncertainty and possible rejection of wire in the final product form.

From the above discussion it becomes apparent that meaningful and confident predictions of wire properties are not manually possible, so to avoid this uncertainty the traditional approach has been to reduce the spread of the component random variables by means of the intermediate processes.

Rod is selected on the basis of calculations using mean values, it is then drawn through a single hole, this acts as a sizing operation that virtually eliminates any spread in diameter of product. After this sizing operation the material is heat treated to achieve the required tensile stress. This treated material is then set up in the frame again and cold drawn to the required size. The sizing operation and heat treatment stage serve to eliminate random variables from the process and provide control of final product properties. However, it becomes apparent that much expense is incurred in set-up time, capital equipment and labour as a result.

Although it was known that the range of wire strength corresponding to the 3 Sigma limits of the input random variables would be outside the permissible limits as a rule, it was not known what percentage of product would be rejected as a result. If this percentage was sufficiently small it might be possible to cold draw the wire on one frame without interruption and the accompanying costs.

By simulating the direct cold drawing of rope wire from the rod and hence finding the resultant spread in tensile strength the factor of uncertainty would be eliminated from the prediction and the single hole sizing and heat treatment operations would become redundant, if a large enough proportion of the product was within allowable tolerances.

The resultant torsion for rope wire is more difficult to describe concisely by a mathematical relationship. Torsional properties are more sensitive to the drafting practice than to raw material input; the desired specification, therefore, can be achieved simply by observing certain empirical rules governing the drafting practice. By the term drafting practice the number of dies and size of each die is implied.

As the wire passes through each successive die the tensile strength increases so the work to deform the wire is increased. For this reason it is desirable in practice to decrease the strength gain through each successive die to prevent fracture and keep the temperature of the wire sufficiently low, this is known as taper drafting. Although an ideal taper drafting practice can be specified it is impossible to achieve this in practice due to certain machine limitations, so the practice must be designed as close to ideal as possible without violating machine constraints. This proves to be extremely tedious to do manually, as a result a feasible practice has been considered satisfactory, however an optimal solution is still the objective.

The above situation existed in the Wire Division of the Steel Company of Canada. From experience with other product lines less critical than rope wire, the desirability and possibility of direct drawing of rope wire was recognized. If therefore a reliable prediction of the statistical frequency distribution of rope wire tensile strength was possible and the prediction confirmed the direct drawing concept, this would permit simplification of the production process and also the associated realization of substantially reduced production costs.

In addition to this rather specific aim, some automatic means of designing drafting procedures for all Stelco wire products was considered. Many obsolete and inefficient practices are employed for low tonnage products simply due to the tedium of up-dating the approximately 20,000 existing wire practices with technological advances.

Hence the object was to develop a computerized method of predicting rope wire tensile strength and on this basis automatically select a suitable rod and design the production practice. In addition, this project was to test the concept of computerized design of all wire drawing practices with a view to eventual automation of the entire task.

## NEED FOR A COMPUTER PROGRAM

The need for additional electronically processed information arose mainly for the following reasons:

i) It is highly desirable from a productivity standpoint to cold draw rope wire directly from rod without any intermediate heat treatment stage, which has previously served as an important means of process control. The prior use of an intermediate heat treatment significantly reduced two inherent variables; size variations of the rod eliminated by a single hole sizing operation; and a reduction in tensile variation normally present in Stelmor rod. Furthermore since the operation was not continuous from rod to finished wire, an opportunity to test the semi-finished wire, was provided. This permitted re-application to other wire grades or products from the intermediate stage in the event that the test results were not satisfactory. However by simulating direct uninterrupted processes beforehand and selecting the most suitable practice, the need for this intermediate stage can be obviated and the associated economic advantages realized.

ii) An ideal drafting practice can be specified but it is impossible to achieve this in general, owing to machine limitations, so the practice most closely approximating this is considered optimal. The optimum draft is never attained through manual calculation and in fact more holes than necessary are often designed into the practice. Although only modest direct continuing manufacturing cost savings are provided by fewer reductions on continuous wire drawing, very significant capital investment savings are created since equipment with fewer holes is markedly less expensive. Therefore by drawing wire through an optimal process the above savings could be realized.

iii) Additional pertinent information is useful in monitoring the process and hence reducing quality control rejection.

iv) By having tedious calculations automatically executed, experienced personnel that are presently required to do these will be free to concentrate on more important tasks.

## PROBLEM SPECIFICATION

This section defines the problem and explains some of the constraints and empirical relationships used as a basis.

a) The following as output from the program was required:

i) The size and chemistry of the rod currently being simulated.

ii) The mean value of wire strength.

iii) Deviation of the expected mean strength from the required mean.

iv) The upper and lower limits of wire strength.

v) Percentage of the product that meets or falls below the maximum strength requirement.

vi) Percentage of the product that meets or exceeds the minimum strength requirement.

vii) Percentage simultaneously satisfying both the above requirements.

If the simulated result is acceptable the optimum drafting practice is also required. A 98% or greater product acceptance is the criterion for deciding whether or not to determine the drafting practice for a particular rod. Drafting practice is specified by:

| | |
|---|---|
| Tensile strength pick-up | lbs/sq inch. |
| Area reduction | % |
| Die size | inches |

for each hole of the draft.

b) The existing program covers seven different rod sizes and three chemical compositions which combine to 21 cases in all. The rod sizes are specified by an integer I, ranging in value from 1 to 7 from smallest to biggest diameter. The chemical compositions are specified by a second integer J, ranging from 1 to 3, from least to most carbon. The actual values represented by the above integer I may be found in the listing of the main program.
(Appendix III)

c) The relationship between reduction and strength gain was obtained from the experimental curve (Fig. 2) supplied by Stelco. This relationship was used in the program by storing some points from the curve and interpolating to find intermediate points.

d) A correction factor, which is a function of rate of drafting is also applied to the result obtained from the above relationship. An adjustment of 1.5% is applied to the strength gain for each hole less than the number required for a 25% drafting average, and similarly -1.5% for

STRENGTH GAIN VERSUS AREA REDUCTION

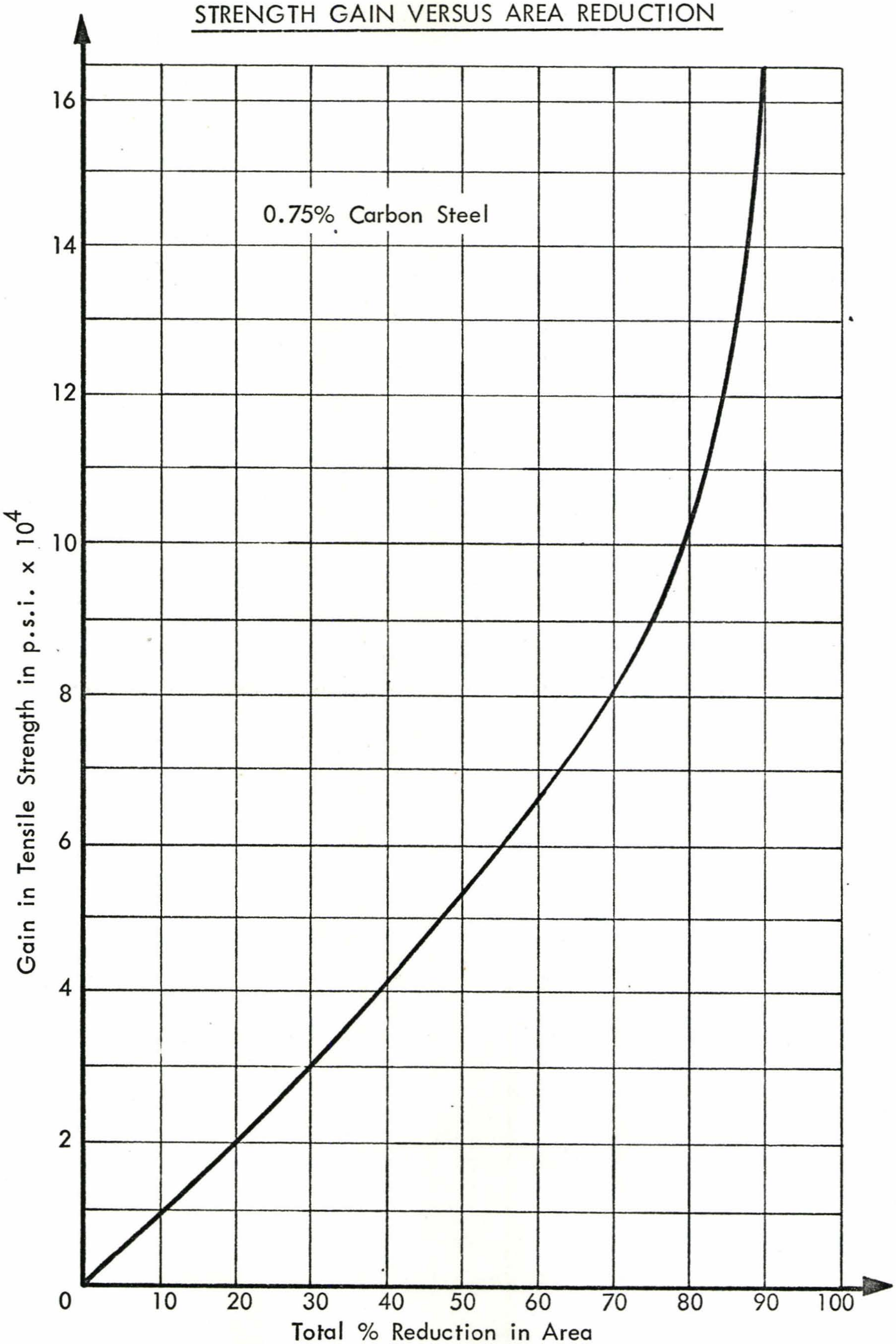



Figure 2.

each hole more than the above average.

Factor = .015 x (Number of holes for 25% drafting average - number of holes in optimum draft)

It appears that higher rates of drawing, i.e. fewer dies, produce an increase in wire temperature and as a result of strain aging, the wire tensile strength is increased (Ref. 1)

e) The pick-up reduction relationship must be used with total reduction only, never current hole reduction. i.e. If the draft of a current hole is 30% and the pick-up is required the total reduction before and after the hole must be found to apply the relationship. If the total reduction prior to the hole was 50% the total after the hole would be:

$$.50 + .30(1 - .50) = .65$$ (Appendix I for relationship)

To find wire tensile strength pick-up is added to rod initial tensile strength.

f) The maximum allowable total reduction must never exceed 90% the limit of the curve in Fig. 1.

g) The minimum allowable total reduction for rope wire must never be less than 70%. This is a rule of thumb used to obtain necessary torsional properties.

h) The ideal draft is specified as follows :

The reduction in the first hole is dependent on rod size being drawn and is a constant for each particular size. It is specified with due allowance for the variance of rod diameter.

The ideal decrease in pick-up of the second hole relative to the first is found using the empirical relationship;

Pick-up (2) = Pick-up (1) x X

where $X = (\text{overall reduction})^{.25} \times J \times .885$

and J is a factor dependent on the carbon content of the rod.

J ranges from .97 to 1.03.

The ideal decrease in pick-up of all other consecutive holes, except for the last, is similar to the above but the factor Y is used in place of X.

where $Y = (\text{overall reduction})^{.50} \times J \times .85$.

The last hole ideally has the same pick-up as the second last hole.

The above drafting specifications are maximum values not to be exceeded. The ideal drafting practice employs the minimum number of holes to achieve a particular reduction, without exceeding the maximum permissible drafts and with due observance of the relationship between successive drafts.

i) In determining the optimal drafting process an attempt is made to keep the difference between the actual strength pick-up of each hole and the ideal pick-up as small as possible. It is not acceptable for one difference to be very large and the others almost ideal so the following objective function was used in the optimization.

$$\text{Minimize } U = \sum (\text{Actual pick-up} - \text{Ideal pick-up})^2$$

over all holes.

j) Wire drawing frames have a series of capstans around which the wire is wound, these capstans are powered by variable speed motors. The motors speed ranges is such that the maximum difference in reduction from one hole to the next is approximately 3%. These machine limitations hence become inequality constraints in determining the optimum drafting practice.

$$\text{Absolute}\left[\text{Reduction } (n) - \text{Reduction } (n+1)\right] \leq .03$$

$(n = 2, 3, \ldots\ldots\ldots\ldots\ldots,$ number of holes $- 1)$

k) The last draft should be a reduction of 15% - 25% if possible to give the required torsional properties. When the last die is reached the wire is moving at a relatively high speed and the yield stress is almost at its maximum value by this stage of the process, the net result is that a lot of work is done here at a high rate which tends to reduce the wire's ductility. In order to meet the required torsional standards the wire must be able to twist sufficiently without fracture, in practice it has been found if the last draft is limited to 25% reduction of area this condition is usually satisfied. When drawing steel wire through a die if the area reduction is less than approximately 15% there is usually not sufficient 'bite' for a stable symmetric draft with the result that more deformation of the steel tends to take place at one side, resulting in non-uniform properties so a lower limit of 15% reduction in area is set on all drafts, but if the last draft is greater than 15% all others will be too owing to the inter-dependence of successive holes.

l) In the case of a seven hole practice the second hole reduction must always be 30% owing to machine construction.

m) The maximum number of holes allowable in any practice is twelve . As this already requires setting up in two seperate frames, if any more holes are required three set ups in different frames would be necessary., and this is not acceptable.

## GENERAL DESCRIPTION OF PROGRAM

The composite program consists of a main program served by 23 subprograms (Fig. 3), this subdivision facilitates debugging and addition to or alteration of the existing program at a later date. In order to provide a general overview of the package mechanics the subprogram flow sequence and basic function of each of these follows:

1. The particular problem is read in by the main program which also stores data on the random variable distribution parameters. The following data input specifies a problem.

   a) Nominal wire size in inches.

   b) Range of permissible yield stress in lb/sq inch or range of permissible wire breaking strength in lbs, whichever is required, the program automatically differentiates between the two.

   c) The sizes and chemistries of any unavailable rods coded as integer values.

The main program then calls subroutine START

2. START is a deterministic approach to the problem using mean values of the random variables to arrive at a solution. The aim of this is

## SUBPROGRAM ACCESS PATHS

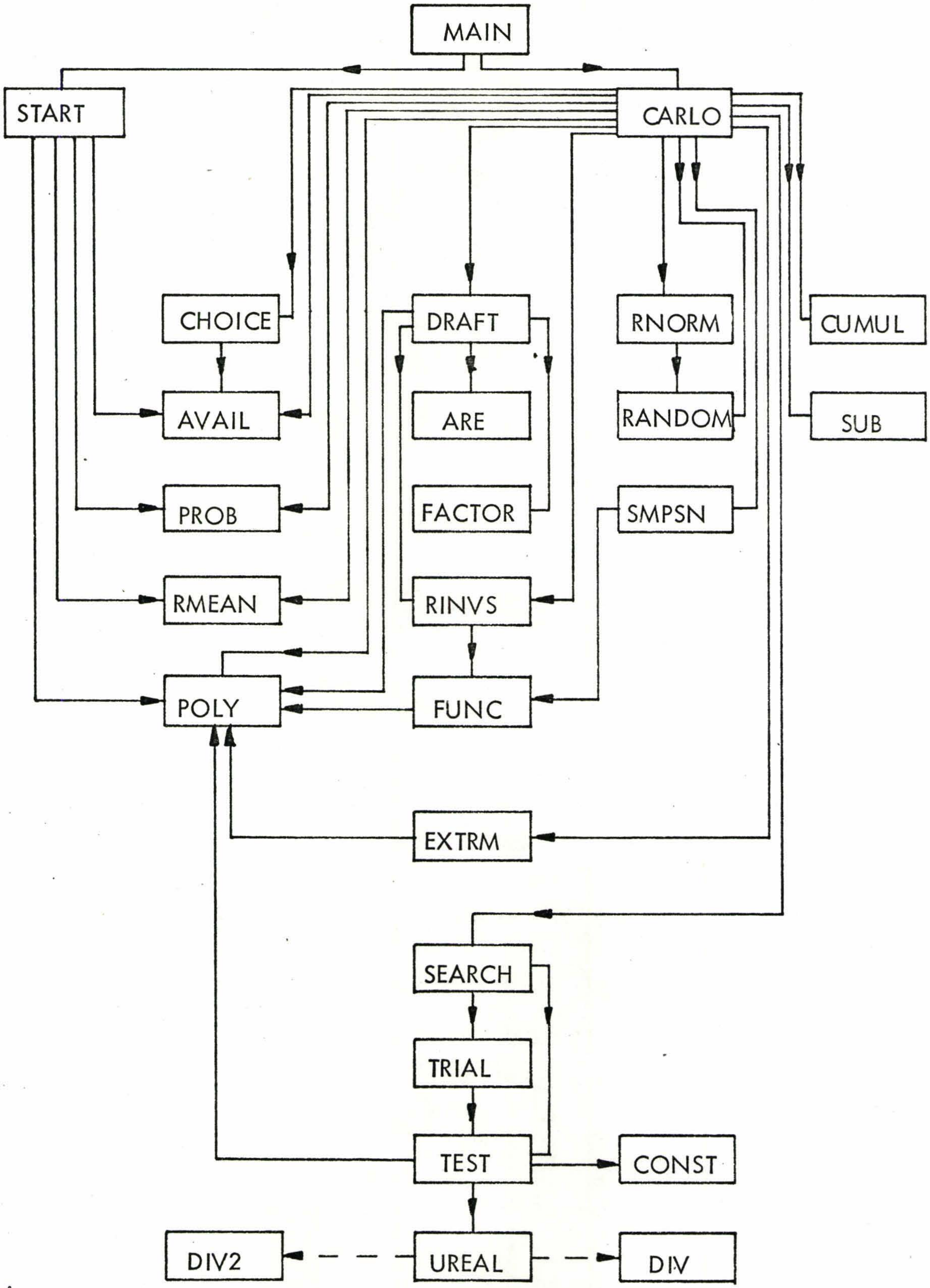



Figure 3.

to provide a realistic starting point for the time consuming probabilistic approach and so reduce computation time. Control is now returned to the main program.

3. The next subroutine called is CARLO. This is the subprogram used to simulate the wire drawing process and predict the statistical distribution of product properties. Carlo firstly calls subprogram RMEAN.

4. RMEAN is used to find the mean value of total area reduction. The upper and lower limits of the range of reduction corresponding to the 3 Sigma limits of diameters are also determined by this subroutine. Control is returned to CARLO.

5. DRAFT is next called into action. This subroutine determines the ideal drafting practice without constraints for the particular rod and wire size. An iterative process is used to achieve the required total reduction in an integer number of holes, according to the specifications. This subroutine is served by two minor subprograms FACTOR and ARE.

6. FACTOR calculates an adjustment factor for rate of strength gain as laid down in the problem specification. ARE determines the reduction of the current hole knowing the total reduction before and after it.

7. Control returns to CARLO and the next step is to simulate the process.

If the wire strength is required a Monte Carlo method is used, but if the yield stress only is required the more rapid Transformation of Variables method is used. The Monte Carlo method will be considered first.

8. Subprogram EXTRM is called to find the range of wire strength corresponding to the 3 Sigma limits of wire and rod diameters.

9. RNORM in conjunction with RANDOM generates normally distributed random numbers. These subroutines are called to provide a random value for each of the three variables in turn, this done the numbers are manipulated as follows to find a value of rod strength; Firstly the total area reduction is calculated using the random diameters. Subroutine POLY is then called to find the strength gain corresponding to the reduction, the strength gain is adjusted by the necessary factor and hence,

Wire Strength = (Strength Gain due to work hardening + Rod Yield Strength x Wire Area)

for the particular sample. The wire strength is then tested against the permissible range of this variable and a record made as to whether or not the sample satisfied the test. This procedure is repeated until 4000 samples have been processed. From the information obtained the following are calculated:

Mean value of wire strength.

Percentage of cases satisfying individual upper and lower limits.

Percentage of cases simultaneously satisfying both limits.

10. Transformation of Variables method is now traced out: The limits of yield stress are firstly found. The probability density for various values of yield stress are found by integrating the function FUNC. The value of the integrand is calculated by subprogram FUNC, which is served by some minor subprograms. The integration is performed by subroutine SMPSN. The mean value of yield stress and all required percentages are determined by subroutines CUMUL and SUB.

11. If the percentage of cases satisfying the specified strength requirement is equal to or greater than 98% then an attempt is made to determine the optimum drafting practice. This is the function of subroutine SEARCH together with its peripheral subroutines.

12 SEARCH is basically a direct search method with exploratory moves and a pattern move. The problem to be optimized is rather a special case as there are only inequality constraints and the unconstrained optimum is known and used as a starting point for the problem. SEARCH calls TRIAL which performs the actual direct search. This subroutine is served by three others, TEST, CONST and UREAL.

13. CONST determines whether any constraints have been violated and if so, the amount by which the constraint is exceeded. UREAL

determines the current value of the objective function which is to be minimized. TEST is a pseudo-optimizing function set up to add penalties to the value found by UREAL in the case of violation of constraints.

14. TRIAL makes exploratory moves, then by calling TEST checks for any decrease in the pseudo-function, the search progresses in this way until no further improvement can be found. The step size is then reduced and the procedure repeated.

15. SEARCH then alters the value of each variable in turn by a fixed step, then treats it as a constant and recalls TRIAL to repeat the search. This way the search will not hang up on a 'fence'.

16. After determining the optimum, control is returned to CARLO which calls subroutine CHOICE to select another rod in a logical manner. The simulation is repeated for another rod until all logical trials have been made at which point the program stops.

## SIMULATION TECHNIQUE

In order to simulate the wire drawing process and hence predict wire properties an analytical model of the process was set up. Although the model is relatively simple it appears to be a good representation of the process judging from results obtained with it. The manufacturing process of wire and the problem specifications have been covered in previous chapters so the model will be stated without explanation.

| | | |
|---|---|---|
| Rod Area | = (Rod Diameter)$^2$ x 3.1416/4 . . . . . . | 1 |
| Wire Area | = (Wire Diameter)$^2$ x 3.1416/4 . . . . . . | 2 |
| Total Reduction | = 1. - Wire Area/Rod Area . . . . . . . | 3 |
| Gain in Tensile | = F(Total Reduction) . . . . . . . . . | 4 |
| Wire Tensile Strength | = Rod Tensile + Gain in Tensile due to Work Hardening . . . . . . . . . . . . | 5 |
| Wire Breaking Strength | = Wire Tensile Strength x Wire Area . . . . . | 6 |

In order to evaluate wire breaking strength, the values of the variables; wire diameter, rod diameter and rod tensile strength must be known, these are however continuous randomly distributed variables which complicates the above calculations somewhat. The problem therefore becomes one of evaluating functions of one or more random variables, so a survey of practical methods applicable to non-linear functions was made.

i) Transformation of Variables:

This is an exact technique applicable to finding the distributions of simple functions of independent random variables, however the method is powerless in the case of non-independent or correlated variables. This method will be discussed at greater depth later in the chapter.

ii) Generation of System (Function) Moments:

This approximate method sometimes referred to as statistical error propogation or the delta method is an attractive approach in situations where independence of component random variables exists. The complexity of the function of random variables is of no particular concern as long as a multivariate Taylor-series expansion about the mean exists and the resulting expression is not too cumbersome.
The method is essentially as follows :
The mean value of the function is calculated using the formula;

$$E(Z) = U\left[(E(v_1),\ E(v_2),\ \ldots\ldots,\ E(v_n)\right] + 1/2 \sum_{i=1}^{n} \frac{\partial^2 U}{\partial x_i{}^2} \quad Var.(v_i)$$

where Z is a function $U(v_1, v_2, \ldots\ldots, v_n)$ of random variables $v_i$ and $E(v_i)$ is the expected value of the $i^{th}$ variable. The partial derivatives must be evaluated for the mean values of the component variables and Var ($v_i$) is the variance of the variable $v_i$ .

Using formulae similar to the above derived in (Ref. 3) the second, third and fourth moments for the function are also found. Based upon these four estimates an empirical distribution such as a Pearson or Johnson distribution including many diverse shapes could be used to represent the result. None of the mathematical operations required would present much difficulty for this particular problem.

iii) Monte Carlo Simulation:

This is one of the most common methods of evaluating a function of random variables and is extremely simple to program for computers. This technique is also an approximation, however the accuracy is controllable and may be improved by conducting sufficient trials or samples.

From a comparison of these three techniques Monte Carlo simulation was selected to evaluate this particular problem. The function is too complex to handle by the method of Transformation of Variables, however this method was used for the simpler case of determining wire tensile stress.

Generation of System Moments although highly competitive in this situation, was rejected because there is no means of estimating the accuracy of the method. Also if the problem were altered slightly such as the addition of another random variable, the method would have to be completely redeveloped.

Alteration of the problem is quite conceivable as the program is still in the development stage. This method was used however to determine the mean value of area reduction as shown by application to subprogram RMEAN.

However a more important application of the method is in determining the expected or mean value of the objective function of the optimization problem. This is discussed in the next chapter.

The Monte Carlo method was selected mainly for its simplicity. If Stelco ever wished to extend or change the problem it is extremely simple to adapt this method. The fact that Monte Carlo simulation requires a large amount of central processor time is really unimportant when one looks at computer cost predictions.

"In 1965 it cost about 20 cents to provide internal storage capacity for one bit down from $2.61 in 1950 and 85 cents in 1960. The comparable cost in 1970 is estimated to be from 5 cents to 10 cents, while the 1975 figure is predicted to be 1/2 cent! The result of such cost reduction will be a further acceleration in the use of computers. Nor is the cost reduction limited to internal storage circuitry, an arithmetic logic unit which cost several dollars in 1955 and is now 50 cents, will go to 3 - 5 cents by 1975" (Ref. 9)

In addition to decrease in cost there is also an associated increase in computing speed of newer computers. Also when the cost of specialized software is considered the Monte Carlo method is very easily justified as being economical. No explanation of the method is necessary, except perhaps the determination of the number of trials. This may be determined from the formula, $n = p' (1-p') \times (z)^2 / E^2$ (Ref. 3).

This expression is based on the normal distribution approximation of the binomial distribution where;

n is the number of Monte Carlo trials

p' is an estimate of the proportion of the population between the specified limits

E is the maximum allowable error in p'

z is the conficence level percentage point of the standard normal distribution, in this case z is found for a one-sided bound.

We wish to determine the number of trials necessary to produce a result with 1/2 % maximum error when p' = .98 as this value of p' is the basis for differentiating between success or failure of a particular rod. The result is required with a 99% confidence level, therefore z = 2.33 from tables.

$$n = .98\ (1-.98) \times (2.33)^2 / (.005)^2$$
$$\approx 4000$$

The same accuracy would of course not be attained for p' less than .98

In practice it was found that more than 4000 trials did not significantly improve convergence of the results. In a check of the generator of normally distributed random numbers it was found that for 4000 trials the distribution of the random numbers approached the theoretical distribution very closely and more trials did not produce any improvement.

As stated in the problem specification it frequently occurs that wire tensile strength is required rather than breaking strength, this simplifies the evaluation of the functions of random variables sufficiently to allow the application of the method of 'Transformation of Variables'. This method is applicable to any type of distribution of variates or variables whether they be discrete or continuous. Univariate functions of one continuous random variable only are very easy to evaluate, also simple operations involving two or more independent variables although more complex can be handled by this method (Ref 3). The application of the method to equation 1 through 5 is as follows :

1. & 2. A general solution of functions of one variable can be obtained in terms of density functions provided it is a monatonically varying function and that the first derivative of the function with respect to the variable exists. Equation 1 satisfies these conditions as it is a continuously increasing function, therefore the following general result applies:

Let x be a continuous random variable with probability density function p(x) defined over some sample space.

Let $w = U(x)$ be a monatonic function of x, then the probability density function of w,

$$p(w) = f(x(w)) \left|\frac{dx}{dw}\right|$$

x(w) is used to denote the inverse of the function U(x). The term (dx/dw) in the expression is the Jacobian of the transformation hence gives a 1 to 1 mapping of the area.

It is seen that this general furmula can be easily applied to (1) and (2).

3. In this particular case where wire diameter and rod diameter are both normally distributed it is advantageous to apply certain other rules to evaluate Total Reduction. There are standard formulae available for determining the resulting distribution of certain simple mathematical operations on normally distributed independent variables. (Ref. 2). By applying these rules in conjunction with Transformation of Variables, Total Reduction is found as follows:

$$\text{Total Reduction} = 1 - (\text{Wire Diameter})^2 / (\text{Rod Diameter})^2$$
$$= 1 - (\text{Wire Diameter}/\text{Rod Diameter})^2$$

The distribution of the quotient in the above expression is normal and can easily be found from the following general formula for independent variables:

Let $q = x_1 / x_2$ , if $x_1$ and $x_2$ are normally distributed.

The expected value of q then is;

$$\mu = \mu_1 / \mu_2$$

And the standard deviation of q is :

$$\sigma = 1/\mu_2 \left[ \frac{\mu_2^2 \sigma_1^2 + \mu_1^2 \sigma_2^2}{\mu_2^2 + \sigma_2^2} \right]^{\frac{1}{2}}$$

$$\text{Total Reduction} = 1. - q^2$$

The result described under section 1 and 2 can now be used to find Total Reduction;

$$p(\text{Tot Red}) = f\left[(1 - \text{Tot Red})^{1/2}\right] \times \left| -\frac{1}{2}(1 - \text{Tot Red})^{1/2} \right|$$

$f[\quad]$ is the probability density distribution of q. Although this case was simplified by the distributions being normal the result could have been found for any distribution using Transformation of Variables.

4. Once again the equation for Gain in Tensile is a function of one variable only and so the result stated under section 1 is applicable, although no mathematical relationship exists between reduction and strength gain the method can be just as effectively applied by storing points from the curve (Fig. 2) and interpolating for intermediate points. This is the function of subprograms POLY and

RINVS, POLY determines the strength gain corresponding to a particular reduction and RINVS does the inverse of this. In fact this numerical method of interpolating is frequently much more powerful than a mathematical function, as it is very difficult if not impossible to find the inverse of some mathematical functions and numerical methods must be reverted to with very little loss in accuracy.

$$p(\text{Gain Tensile}) = f\left[(\text{RINVS}(\text{Gain})\right] \times \left|\frac{d\ \text{RINVS}}{d\ \text{GAIN}}\right|$$

$f\left[\quad\right]$ is the probability density of total reduction, and must be calculated for each value of Gain.

5. Wire tensile strength results from the summation of two independent variables, Gain in Tensile and Rod Tensile. This mathematical operation gives rise to a new random variable the distribution of which can be determined from the following general result :

$$\text{If } w = x + y$$

$$p(w) = \int_{\text{over sample space}} f(z)\, g(w-z)dz, \qquad \text{(Ref. 3)}$$

f(x) and g(y) are probability density functions of x and y respectively and z is an arbitrary function of x and y selected so as to facilitate the integration and bring the number of variables after the transformation to the same as that started with, $z = x-y$ in this case.

In applying this result to (5) the probability density of 'Rod Tensile' is normal and known, the probability density of Gain and Tensile however must be calculated via (1) to (4) as described, for each value of the variable required.

The sample space of z can be determined from the sample space of x and y. Although the sample space of a normal distribution is theoretically from $-\infty$ to $+\infty$ , for practical reasons it can be considered to range from $-3\sigma$ to $+3\sigma$ in this way 99.75 % of the area is accounted for and the problem can be handled numerically. The sample space of w is similarly found.

By generating probability densities for discrete values of w using the above method, the discrete points can be joined to give the statistical distribution.

If we were to try taking the method a step further and evaluate (6) it is seen that wire area is not completely independent of wire tensile strength but some correlation exists, hence the general results of Transformation of Variables are no longer applicable.

The simulation model can only be checked by running it in parallel with the production line and performing controlled tests. However, the mechanics of the simulation can be and have been checked in the following ways :

Monte Carlo simulation can be checked for convergence by increasing the sample sizes, this was done but no significant changes resulted, which is within keeping of the expected deviation. The method was further checked with different sets of random numbers and results once again corresponded very closely.

Finally the two methods, Monte Carlo Simulation and Transformation of Variables were checked against one another for determination of Wire Tensile Strength, this was made possible by slight modifications to the program. Results from the tests correlated very well.

The latter check is included and the results displayed and discussed under the section Results and Discussion.

## OPTIMIZATION TECHNIQUE

As previously stated an ideal drafting practice can be specified but this is rarely achieved in practice when machine limitations are imposed, as a result the objective is to approximate the ideal draft as closely as possible without violating machine constraints. The problem stated mathematically is as follows :

$$\text{Minimize } U = \sum_{i=2}^{n} (\text{Actual Tensile Gain of } i^{th} \text{ hole} - \text{Ideal Tensile Gain of } i^{th} \text{ hole})^2$$

$$\text{Subject to} \quad R_i - R_{i+1} \leq .03$$

$$R_{i+1} - R_i \leq .03 \quad (i = 1,2, \ldots ,n-1)$$

$$R_n \leq .25$$

$$R_n \geq .15$$

where n is the number of holes and $R_i$ is the reduction through the $i^{th}$ hole or die.

The above objective function, U, was chosen as it does not allow one term under the summation to be large while the others are almost zero or negative, as a linear function might for a particular value of U.

Die diameters form the problem variables of which Reduction and

Tensile Strength Gain are known functions. There are therefore, two less variables in the problem than there are holes in the draft as the first and last dies are necessarily of fixed dimensions. By making die diameters the variables instead of reductions $R_i$, the objective function and constraints become slightly more complex but the optimization problem is simplified substantially as equality constraints are avoided.

The following problem features formed the basis for the selection of an optimization technique:

a) The constraints and objective function are non-linear.

b) The unconstrained optimum is known and can be used as the starting point of the optimization.

c) There are only inequality constraints.

d) The constraints are extremely restrictive however and in some cases a feasible solution does not exist.

e) As many as ten variables may be encountered but mostly there will be about four variables.

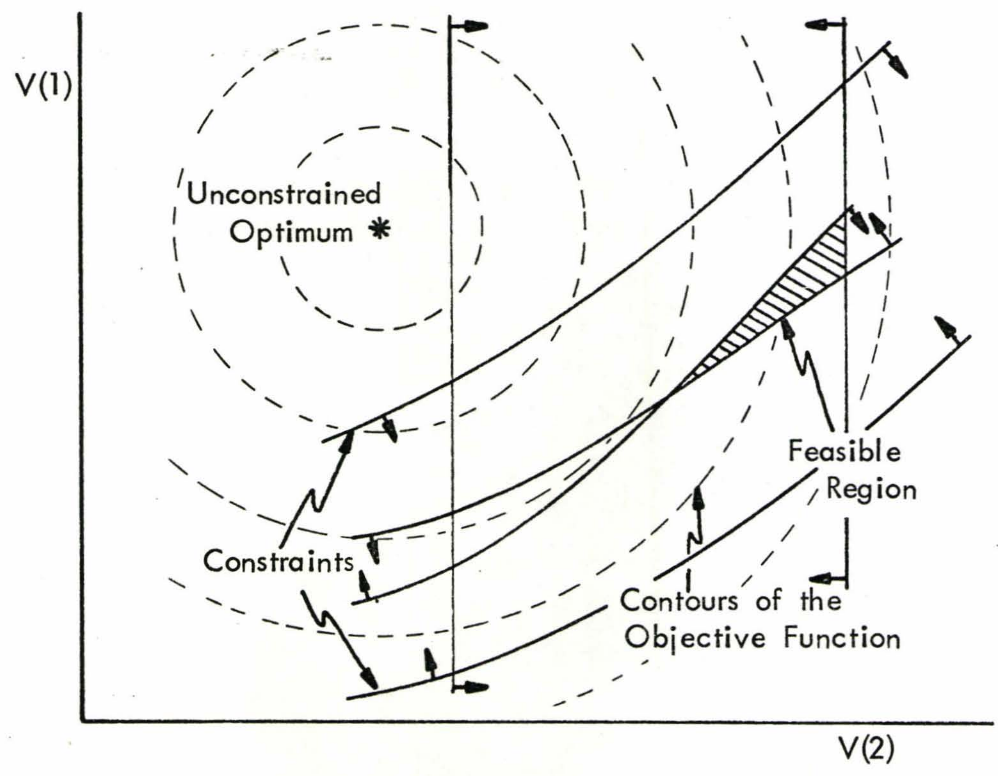
A simple direct search optimization technique was selected to solve the problem as this method is simple, easy to program and requires very little computer storage. The method was firstly shown to be adequate by testing sample problems on 'OPTIPAC' (Ref. 10).

Owing to the specialized nature of the problem the basic direct search technique was modified to speed it up and avoid 'hang ups'. This modified search technique has never failed in all test problems run and appears to be completely satisfactory. When compared with six different techniques in 'OPTIPAC' it repeatedly produced a better optimum although was slightly more time consuming on the average. On this basis the optimization technique was accepted as satisfactory.

Modifications to the basic direct search method:

1. Penalty Functions:

The search always starts in an infeasible region at the unconstrained optimum, the constrained optimum being the nearest feasible region to this point, as illustrated by the graphical representation of a typical two variable problem:




Graphical Representation of a Typical Two Variable Problem.

By providing quadratic penalty functions of the form,

$$\text{Penalty} = (\text{PHI} \times \text{Weighting factor})^2 + \text{PHI} \times \text{Weighting factor}$$

the search moves toward a feasible region along a direct route as an attempt is made to equalize and decrease violations simultaneously. A constant penalty is also added if one or more violations occur, this prevents the solution from being slightly infeasible.

2. Weighting Factor:

A weighting factor was also applied to these penalties as it was found that a change in one variable produces unequal changes in two adjacent reductions and hence in the value of the constraints. This phenomenon may cause a 'hang up' or slow the search down substantially when a violated constraint occurs at the first or last die. Consider the following hypothetical example in which one constraint PHI (3) is violated:

Die 1 Reduction (1) = .30

Die 2 = Variable (1), Reduction (2) = .26

PHI (1) = 0

Die 3 = Variable (2), Reduction (3) = .23

PHI (2) = 0

Die 4 = Variable (3), Reduction (4) = .20

PHI (3) = .1

Die 5 Reduction (5) = .16

In an attempt to satisfy this constraint V(3) can be increased or V(2) decreased but in either case some other constraint will be violated, this is acceptable as long as the associated penalty does not exceed the existing penalty for violating PHI (3). For this reason a weighting factor is applied to ensure that the penalty will be less, and so prevent the search from hanging up. This applies at either extreme so two weighting factors are used, one less than 1.0 and the other greater.

3. Repeated Search:

Having performed the initial search each of the variables in turn is changed by $\pm$ .00025", held constant and the search repeated. Convergence criteria for the repeated search are modified so as to reduce computation time. The reason for selecting a change of $\pm$ .00025" in each of the variables is that this is the smallest meaningful change in diameter of the dies.

This repeated search serves as a check on the initial search.

## Random Variables

The problem has been treated as a straight forward optimization so far, however the variables to be optimized are continuously distributed random variates. The optimization criteria is therefore to minimize the expected value of the objective function. As the objective function is non-linear in this case the expected value cannot simply be found by evaluation of the function for the means of the component variables. (Ref. 8)

There are various methods by which the expected value could be found and for this particular problem, Monte Carlo Simulation, Transformation of Variables or Moment Generating Functions would all be feasible. However for this application the method of Moment Generating Functions is far superior to the others.

The method consists of expanding the objective function $U(v_1, v_2, \ldots\ldots v_n)$ about $(E(v_1), E(v_2), \ldots\ldots\ldots, E(v_n))$ the points at which the component variates take on their expected values, by a multivariable Taylor series expansion. By taking the expected value of this series and retaining terms up to second order the following expression is obtained:

$$E(U) = U(E(v_1), E(v_2), \ldots\ldots\ldots, E(v_n))$$

$$+ 1/2 \sum_{i=1}^{n} \frac{\partial^2 \bar{U}}{\partial v_i^2} \, Var(v_i)$$

where $\partial^2 \bar{U} / \partial v_i^2$ denotes the second partial derivative evaluated at the expected values of the variates. This expression is derived in Appendix II.

The expansion could have been taken about any point other than $E(v_i)$ but certain advantages accrue from using this as a reference point, viz. the second term of the expansion falls away as do parts of the third term, hence we have a much simpler final result.

The specified dies and hence the variables of the objective function are selected by measurement and so are subject to the associated random errors of this process. The distribution of these variables was assumed normal with an estimated standard deviation of .0003".

Using this data an equation for E(U) was found by applying the above expression to the objective function;

$$U = \sum_{i=2}^{n} \left[ F\left(1 - \frac{v_i^2}{v^2}\right) - F\left(1 - \frac{v_{i-1}^2}{v^2}\right) - \text{Ideal Tensile Gain of } i^{th} \text{ hole} \right]^2$$

where v is the input rod diameter and $v_i$ is the size of the $i^{th}$ die.

The constraints were not treated probabilistically as a margin of safety had originally been built into them, the maximum allowable difference of 3% in area reduction between holes is a conservative figure.

A problem was then tested using E(U) as the objective function in the search and this solution was compared with results from a program in which the mean values of the variates were simply substituted into the objective function. These results appear under the section on Results and Discussion. However it may be stated now that there was no significant difference in the solutions obtained by the methods so the simpler objective function was accepted as adequate.

The probable reasons for this outcome are that variances are relatively small and also the objective function is highly sensitive to changes in the variables and so conversely the variables are rather insensitive to changes in the objective function so in this case the solutions are effectively the same.

Two additional subprograms were added to the program in the test run, these subprograms DIV and DIV2 were used to calculate the 1st and 2nd derivatives of strength gain with respect to reduction. Listings appear under Appendix II.

## RESULTS AND DISCUSSION

Test problems were run on the program and these solutions compared with known experimental results. In this way arbitrary constraints and empirical formulae were checked and adjusted to give satisfactory results. Another batch of test problems, supplied by Stelco, were then run through as a check. Examples from these problems are used to discuss some of the more significant features. Reproductions of the computer printouts appear at the end of the chapter.

Example 1:

This problem is one of the simpler type in which the wire breaking stress is required so the simulation is performed by the method of transformation of variables. Firstly the problem specifications are printed out.

The starting point was found by the deterministic routine, this a .281 inch diameter rod 75/79 carbon. When simulated the result is that only 57.1 % of the product manufactured from this rod would be satisfactory from a strength requirement standpoint, so no further time is spent determining a drafting practice for this rod.

As seen from the output the resultant product would be too weak, so another rod must be chosen that would increase this property. The next rod is therefore automatically chosen with a higher carbon content, 79/83, with the result that 100% of the product satisfies the strength requirements, so a drafting practice is also required.

The drafting practice consists of seven holes or dies, for each of these the area reduction, tensile strength gain and die diameter is supplied for both the ideal and constrained case. Both cases are printed out as the machine limitations do not apply to some drafting frames. The area reduction of the second hole of a seven hole practice must always be 30% in accordance with the specifications hence the second die in this case is treated as a constant along with the first and last dies, so there are only four variables in the optimization.

The program next selects a .297 inch diameter rod for simulation, however the area reduction is too great for rope wire so the attempt is terminated. When a smaller size is attempted results are not acceptable so the solution is complete and no further attempts are made as these too would logically fail.

Example 2:

This example serves to check the simulation methods. The same problem as in example 1 is run however the program was modified slightly so that the simulation would now be performed by the Monte Carlo method.

For the .281 inch 75/75 carbon rod the mean value of wire tensile strength is found to be about 65 pounds greater than previously, and the percentage of product satisfying the minimum strength requirement is now 57.7% , an increase of 0.6% over the previous result.

The simulation result for the .281 inch diameter 79/83 carbon rod are identical to the results obtained by transformation of variables in example 2.

On comparison of results for the .263 inch diameter rod with 79/83 carbon it is seen that the mean value of wire tensile strength has decreased by 53 pounds over the previous results and there has been a corresponding decrease of 1.3% in percentage product satisfying the minimum strength requirement.

From the above comparison it is seen that the two independent methods check out reasonably well, furthermore it is seen that the variation between methods is of a random nature and not consistantly high or low, suggesting that the error is a result of random factors such as finite difference approximations and random number generation.

Another comparative point of interest is the central processor time used, the Monte Carlo simulation requires substantially more time, in fact if compilation and optimization times were subtracted it would be seen that Monte Carlo simulation requires approximately ten fold more time than Transformation of Variables.

Example 3:

Once again this is the same problem as in the two previous examples. The purpose of this example is to illustrate the effect of optimizing the expected value of the objective function, rather than just substituting mean values of the variables into the objective function. The simulation is performed by transformation of variables and the mean value of the objective function is found by generation of system moments, incorporated in a modified subroutine UREAL.

As seen from the computer printout there is no significant difference in the two optimized drafting practices. Owing to the increased computation of finding the expected value of the objective function it is seen that the central processor time has approximately doubled in this case.

Example 4:

This example is one of the more complex type in which wire breaking strength is required, hence Monte Carlo simulation is employed.

For the first rod tried, viz. .281" diameter 71/75 carbon, the simulation predicts 100% product acceptance, hence the drafting practice is determined. As seen from this example the range of strength, as determined from the 3 sigma limits of the normally distributed input variables, is outside of the strength requirements yet the product acceptance is 100%, this is due to the very low probability of an event at the tails of the distribution occurring.

The drafting practice for this rod has four holes, however the last hole has a reduction in excess of 25%, this solution is therefore infeasible and no feasible solution exists in fact. No error message is printed out as 25% is an arbitrary value and the final decision is left to the user.

The next rod tried is the same diameter but has a greater carbon content, the simulation is successful and another drafting practice is determined. This is a five hole practice with a feasible solution. The reason for different drafting practices for two rods of the same size is that a slightly softer draft is employed with the higher carbon steels, this must have been a borderline case, hence the different drafting practices.

Various other rods are then simulated and results given. The final choice of rod and practice is as yet still a manual one. The choice would be between :

i) .281" 71/75 carbon

ii) .281" 75/79 carbon

iii) .297" 71/75 carbon

all with 100% product acceptance. The criteria now used for deciding between these rods is deviation of the simulated mean value of strength from that required, and also rod size, the bigger rods having lower cost per pound. However as the drafting practice of the first rod is dubious the choice would be between the second and third. Deviation from the required mean is almost identical for these so the larger rod is chosen.

This decision could be quite easily built into the program and a dollar value assigned to alternatives, however it was not deemed necessary at this stage of development.

Numerous other problems were run with satisfactory results.

COMPUTER PRINTOUT

**EXAMPLE 1**

```
                    SELECTION OF WIRE DRAWING PRACTICES
                    ***********************************

REQUIRED NOMINAL WIRE DIAMETER =  .0980

MEAN WIRE DIAMETER             =  .0982

STANDARD DEVIATION OF DIAMETER =  .0004

REQUIRED  RANGE  OF  STRESS    =  300000.0   TO  345000.0


          * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          * USING A  .281   NOMINAL DIAMETER ROD AND 75/79 CARBON *
          * * * * * * * * * * * * * * * * * * * * * * * * * * * *

THE MEAN VALUE OF WIRE STRENGTH =  300721.8
 DEVIATION FROM REQUIRED MEAN   =  -21778.2
THE RANGE OF STRENGTH IS FROM  278879.5  TO  328455.0
  97.1  PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT
 100.0  PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT
  97.1  PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS


          * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          * USING A  .281   NOMINAL DIAMETER ROD AND 79/83 CARBON *
          * * * * * * * * * * * * * * * * * * * * * * * * * * * *

THE MEAN VALUE OF WIRE STRENGTH =  317221.8
 DEVIATION FROM REQUIRED MEAN   =   -5278.2
THE RANGE OF STRENGTH IS FROM  295379.5  TO  338955.9
 100.0  PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT
 100.0  PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT
 100.0  PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS
                                  IDEAL         OPTIMIZED


FOR DRAFT  NUMBER   1
REDUCTION IS                       29.8            29.4       PERCENT
STRENGTH GAIN IS                29437.0         29837.0       POUNDS/SQ. INCH
DIE DIAMETER IS                   .2405           .2405       INCHES

FOR DRAFT  NUMBER   2
REDUCTION IS                       30.0            30.0       PERCENT
STRENGTH GAIN IS                23654.2         23654.2       POUNDS/SQ. INCH
DIE DIAMETER IS                   .2012           .2012       INCHES

FOR DRAFT  NUMBER   3
REDUCTION IS                       33.7            30.6       PERCENT
STRENGTH GAIN IS                21444.2         19953.5       POUNDS/SQ. INCH
DIE DIAMETER IS                   .1639           .1676       INCHES

FOR DRAFT  NUMBER   4
REDUCTION IS                       30.5            27.8       PERCENT
STRENGTH GAIN IS                20246.6         19160.5       POUNDS/SQ. INCH
DIE DIAMETER IS                   .1306           .1424       INCHES

FOR DRAFT  NUMBER   5
REDUCTION IS                       28.9            24.9       PERCENT
STRENGTH GAIN IS                18731.5         16565.9       POUNDS/SQ. INCH
DIE DIAMETER IS                   .1104           .1234       INCHES

FOR DRAFT  NUMBER   6
REDUCTION IS                       19.7            21.0       PERCENT
STRENGTH GAIN IS                17329.8         13294.0       POUNDS/SQ. INCH
DIE DIAMETER IS                   .1001           .1141       INCHES

FOR DRAFT  NUMBER   7
REDUCTION IS                       14.3            18.9       PERCENT
STRENGTH GAIN IS                17351.6         21566.8       POUNDS/SQ. INCH
DIE DIAMETER IS                   .0983           .09[illegible]       INCHES


.297  NOMINAL DIAMETER ROD IS NOT ACCEPTABLE AS REDUCTION EXCEEDS 90 PERCENT


          * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          * USING A  .263   NOMINAL DIAMETER ROD AND 79/83 CARBON *
          * * * * * * * * * * * * * * * * * * * * * * * * * * * *

THE MEAN VALUE OF WIRE STRENGTH =  302992.7
 DEVIATION FROM REQUIRED MEAN   =  -19507.3

THE RANGE OF STRENGTH IS FROM  284957.3  TO  323[illegible]
  76.9  PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT
 100.0  PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT
  76.9  PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

  CENTRAL PROCESSOR TIME USED =  20.9  SECONDS
```

**EXAMPLE 2**

SELECTION OF WIRE DRAWING PRACTICES

REQUIRED NOMINAL WIRE DIAMETER = .0980

MEAN WIRE DIAMETER = .0982

STANDARD DEVIATION OF DIAMETER = .0004

REQUIRED RANGE OF STRESS = 300000.0 TO 345000.0

• USING A .281 NOMINAL DIAMETER ROD AND 75/79 CARBON •

THE MEAN VALUE OF WIRE STRENGTH = 300787.1

DEVIATION FROM REQUIRED MEAN = -21712.9

THE RANGE OF STRENGTH IS FROM 278879.5 TO 322155.9

57.6 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

57.6 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

• USING A .281 NOMINAL DIAMETER ROD AND 79/83 CARBON •

THE MEAN VALUE OF WIRE STRENGTH = 317387.9

DEVIATION FROM REQUIRED MEAN = -5112.1

THE RANGE OF STRENGTH IS FROM 295379.5 TO 336955.9

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

| | IDEAL | OPTIMIZED | |
|---|---|---|---|
| FOR DRAFT NUMBER 1 | | | |
| REDUCTION IS | 28.8 | 28.8 | PERCENT |
| STRENGTH GAIN IS | 24837.0 | 24837.0 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2405 | .2405 | INCHES |
| FOR DRAFT NUMBER 2 | | | |
| REDUCTION IS | 30.0 | 30.0 | PERCENT |
| STRENGTH GAIN IS | 23654.2 | 23654.2 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2012 | .2012 | INCHES |
| FOR DRAFT NUMBER 3 | | | |
| REDUCTION IS | 33.7 | 30.6 | PERCENT |
| STRENGTH GAIN IS | 21884.2 | 19953.5 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1639 | .1676 | INCHES |
| FOR DRAFT NUMBER 4 | | | |
| REDUCTION IS | 30.5 | 27.8 | PERCENT |
| STRENGTH GAIN IS | 20246.6 | 18160.5 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1366 | .1424 | INCHES |
| FOR DRAFT NUMBER 5 | | | |
| REDUCTION IS | 24.9 | 24.9 | PERCENT |
| STRENGTH GAIN IS | 18731.5 | 16565.9 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1184 | .1234 | INCHES |
| FOR DRAFT NUMBER 6 | | | |
| REDUCTION IS | 19.7 | 21.4 | PERCENT |
| STRENGTH GAIN IS | 17329.4 | 19244.0 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1061 | .1091 | INCHES |
| FOR DRAFT NUMBER 7 | | | |
| REDUCTION IS | 14.3 | 18.0 | PERCENT |
| STRENGTH GAIN IS | 17351.6 | 21564.4 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .0983 | .0983 | INCHES |

.297 NOMINAL DIAMETER ROD IS NOT ACCEPTABLE AS REDUCTION EXCEEDS 40 PERCENT

• USING A .263 NOMINAL DIAMETER ROD AND 79/83 CARBON •

THE MEAN VALUE OF WIRE STRENGTH = 302940.0

DEVIATION FROM REQUIRED MEAN = -19560.0

THE RANGE OF STRENGTH IS FROM 284957.3 TO 323054.4

75.6 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

75.6 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

CENTRAL PROCESSOR TIME USED = 56.3 SECONDS

EXAMPLE 3

SELECTION OF WIRE DRAWING PRACTICES

REQUIRED NOMINAL WIRE DIAMETER = .0980

MEAN WIRE DIAMETER = .0982

STANDARD DEVIATION OF DIAMETER = .0004

REQUIRED RANGE OF STRESS = 300000.0 TO 345000.0

* USING A .281 NOMINAL DIAMETER ROD AND 75/79 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 300721.8

DEVIATION FROM REQUIRED MEAN = -21778.2

THE RANGE OF STRENGTH IS FROM 278879.5 TO 320455.9

57.1 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

57.1 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

* USING A .281 NOMINAL DIAMETER ROD AND 79/83 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 317221.8

DEVIATION FROM REQUIRED MEAN = -5278.2

THE RANGE OF STRENGTH IS FROM 295379.5 TO 336955.9

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

| | IDEAL | OPTIMIZED | |
|---|---|---|---|
| FOR DRAFT NUMBER 1 | | | |
| REDUCTION IS | 29.8 | 28.8 | PERCENT |
| STRENGTH GAIN IS | 29837.0 | 29837.0 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2405 | .2405 | INCHES |
| FOR DRAFT NUMBER 2 | | | |
| REDUCTION IS | 30.0 | 30.0 | PERCENT |
| STRENGTH GAIN IS | 23654.2 | 23654.2 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2012 | .2012 | INCHES |
| FOR DRAFT NUMBER 3 | | | |
| REDUCTION IS | 33.7 | 30.6 | PERCENT |
| STRENGTH GAIN IS | 21884.2 | 19953.5 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1639 | .1676 | INCHES |
| FOR DRAFT NUMBER 4 | | | |
| REDUCTION IS | 30.5 | 27.8 | PERCENT |
| STRENGTH GAIN IS | 20246.6 | 18160.5 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1366 | .1424 | INCHES |
| FOR DRAFT NUMBER 5 | | | |
| REDUCTION IS | 24.9 | 24.9 | PERCENT |
| STRENGTH GAIN IS | 16731.5 | 16565.9 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1184 | .1234 | INCHES |
| FOR DRAFT NUMBER 6 | | | |
| REDUCTION IS | 19.7 | 21.9 | PERCENT |
| STRENGTH GAIN IS | 17329.8 | 19299.0 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1061 | .1091 | INCHES |
| FOR DRAFT NUMBER 7 | | | |
| REDUCTION IS | 14.3 | 18.9 | PERCENT |
| STRENGTH GAIN IS | 17351.6 | 21564.8 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .0983 | .0983 | INCHES |

.297 NOMINAL DIAMETER ROD IS NOT ACCEPTABLE AS REDUCTION EXCEEDS 90 PERCENT

* USING A .263 NOMINAL DIAMETER ROD AND 79/83 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 302492.7

DEVIATION FROM REQUIRED MEAN = -19507.3

THE RANGE OF STRENGTH IS FROM 284957.3 TO 323054.4

76.9 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

76.9 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

CENTRAL PROCESSOR TIME USED = 39.6 SECONDS

**EXAMPLE 4**

SELECTION OF WIRE DRAWING PRACTICES

REQUIRED NOMINAL WIRE DIAMETER = .1440

MEAN WIRE DIAMETER = .1440

STANDARD DEVIATION OF DIAMETER = .0005

REQUIRED RANGE OF STRENGTH = 3648.0 TO 4153.0

.240 NOMINAL DIAMETER ROD NOT ACCEPTABLE AS REDUCTION IS LESS THAN 70 PERCENT
.263 NOMINAL DIAMETER ROD NOT ACCEPTABLE AS REDUCTION IS LESS THAN 70 PERCENT

* USING A .281 NOMINAL DIAMETER ROD AND 71/75 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 3872.8

DEVIATION FROM REQUIRED MEAN = -27.7

THE RANGE OF STRENGTH IS FROM 3581.9 TO 4163.1

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

| | IDEAL | OPTIMIZED | |
|---|---|---|---|
| FOR DRAFT NUMBER 1 | | | |
| REDUCTION IS | 28.8 | 28.8 | PERCENT |
| STRENGTH GAIN IS | 29989.2 | 29989.2 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2405 | .2405 | INCHES |
| FOR DRAFT NUMBER 2 | | | |
| REDUCTION IS | 31.9 | 31.9 | PERCENT |
| STRENGTH GAIN IS | 25393.8 | 25380.2 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1984 | .1985 | INCHES |
| FOR DRAFT NUMBER 3 | | | |
| REDUCTION IS | 27.6 | 28.9 | PERCENT |
| STRENGTH GAIN IS | 17775.6 | 18606.7 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1689 | .1673 | INCHES |
| FOR DRAFT NUMBER 4 | | | |
| REDUCTION IS | 27.3 | 25.9 | PERCENT |
| STRENGTH GAIN IS | 17745.8 | 16928.2 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1440 | .1440 | INCHES |

* USING A .281 NOMINAL DIAMETER ROD AND 75/79 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 3933.9

DEVIATION FROM REQUIRED MEAN = 33.4

THE RANGE OF STRENGTH IS FROM 3640.9 TO 4223.2

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

| | IDEAL | OPTIMIZED | |
|---|---|---|---|
| FOR DRAFT NUMBER 1 | | | |
| REDUCTION IS | 28.8 | 28.3 | PERCENT |
| STRENGTH GAIN IS | 29544.4 | 29544.4 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2405 | .2405 | INCHES |
| FOR DRAFT NUMBER 2 | | | |
| REDUCTION IS | 31.0 | 27.0 | PERCENT |
| STRENGTH GAIN IS | 24288.4 | 21102.3 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1997 | .2054 | INCHES |
| FOR DRAFT NUMBER 3 | | | |
| REDUCTION IS | 24.2 | 24.[illegible] | PERCENT |
| STRENGTH GAIN IS | 15625.5 | 15804.5 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1738 | .1791 | INCHES |
| FOR DRAFT NUMBER 4 | | | |
| REDUCTION IS | 19.1 | 21.[illegible] | PERCENT |
| STRENGTH GAIN IS | 10052.4 | 11309.9 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1564 | .1591 | INCHES |
| FOR DRAFT NUMBER 5 | | | |
| REDUCTION IS | 15.2 | 18.[illegible] | PERCENT |
| STRENGTH GAIN IS | 10045.3 | 11794.8 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1440 | .1440 | INCHES |

* USING A .281 NOMINAL DIAMETER ROD AND 79/83 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 4200.7

DEVIATION FROM REQUIRED MEAN = 300.2

THE RANGE OF STRENGTH IS FROM 3904.0 TO 4497.5

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

22.1 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

22.1 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

* USING A .297 NOMINAL DIAMETER ROD AND 71/75 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 3933.4

DEVIATION FROM REQUIRED MEAN = 32.9

THE RANGE OF STRENGTH IS FROM 3655.6 TO 4222.7

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

100.0 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

| | IDEAL | OPTIMIZED | |
|---|---|---|---|
| FOR DRAFT NUMBER 1 | | | |
| REDUCTION IS | 28.5 | 28.5 | PERCENT |
| STRENGTH GAIN IS | 29360.7 | 29360.7 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2546 | .2546 | INCHES |
| FOR DRAFT NUMBER 2 | | | |
| REDUCTION IS | 31.7 | 29.2 | PERCENT |
| STRENGTH GAIN IS | 25079.4 | 23021.5 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2104 | .2142 | INCHES |
| FOR DRAFT NUMBER 3 | | | |
| REDUCTION IS | 26.8 | 26.2 | PERCENT |
| STRENGTH GAIN IS | 17267.2 | 17241.7 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1799 | .1840 | INCHES |
| FOR DRAFT NUMBER 4 | | | |
| REDUCTION IS | 20.9 | 23.2 | PERCENT |
| STRENGTH GAIN IS | 11888.6 | 13116.9 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1601 | .1613 | INCHES |
| FOR DRAFT NUMBER 5 | | | |
| REDUCTION IS | 19.0 | 20.2 | PERCENT |
| STRENGTH GAIN IS | 11842.3 | 12697.3 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1440 | .1440 | INCHES |

* USING A .297 NOMINAL DIAMETER ROD AND 75/79 CARBON *

THE MEAN VALUE OF WIRE STRENGTH = 4015.6

DEVIATION FROM REQUIRED MEAN = 115.1

THE RANGE OF STRENGTH IS FROM 3735.4 TO 4305.8

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

98.8 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

98.8 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

THE DRAFTING PRACTICE FOR THIS ROD SIZE HAS ALREADY BEEN DETERMINED

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

* USING A .312 NOMINAL DIAMETER ROD AND 71/75 CARBON *

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

THE MEAN VALUE OF WIRE STRENGTH = 4017.0

DEVIATION FROM REQUIRED MEAN = 116.5

THE RANGE OF STRENGTH IS FROM 3731.4 TO 4314.4

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

98.8 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

98.8 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

| | IDEAL | OPTIMIZED | |
|---|---|---|---|
| FOR DRAFT NUMBER 1 | | | |
| REDUCTION IS | 28.1 | 28.1 | PERCENT |
| STRENGTH GAIN IS | 29110.6 | 29110.6 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2679 | .2679 | INCHES |
| FOR DRAFT NUMBER 2 | | | |
| REDUCTION IS | 31.4 | 30.8 | PERCENT |
| STRENGTH GAIN IS | 25035.2 | 24568.1 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .2220 | .2229 | INCHES |
| FOR DRAFT NUMBER 3 | | | |
| REDUCTION IS | 29.1 | 27.9 | PERCENT |
| STRENGTH GAIN IS | 18860.7 | 19250.8 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1869 | .1892 | INCHES |
| FOR DRAFT NUMBER 4 | | | |
| REDUCTION IS | 23.3 | 24.0 | PERCENT |
| STRENGTH GAIN IS | 14209.0 | 15119.8 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1638 | .1640 | INCHES |
| FOR DRAFT NUMBER 5 | | | |
| REDUCTION IS | 22.6 | 22.8 | PERCENT |
| STRENGTH GAIN IS | 14278.2 | 14444.4 | POUNDS/SQ. INCH |
| DIE DIAMETER IS | .1440 | .1440 | INCHES |

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

* USING A .312 NOMINAL DIAMETER ROD AND 75/79 CARBON *

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

THE MEAN VALUE OF WIRE STRENGTH = 4072.8

DEVIATION FROM REQUIRED MEAN = 172.3

THE RANGE OF STRENGTH IS FROM 3787.6 TO 4371.7

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

90.9 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

90.9 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

* USING A .328 NOMINAL DIAMETER ROD AND 71/75 CARBON *

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

THE MEAN VALUE OF WIRE STRENGTH = 4089.0

DEVIATION FROM REQUIRED MEAN = 188.5

THE RANGE OF STRENGTH IS FROM 3804.4 TO 4378.9

100.0 PERCENT OF THE PRODUCT WILL MEET OR EXCEED THE MINIMUM REQUIREMENT

84.3 PERCENT OF THE PRODUCT WILL MEET OR FALL BELOW THE MAXIMUM REQUIREMENT

84.3 PERCENT OF THE PRODUCT FALLS BETWEEN BOTH LIMITS

CENTRAL PROCESSOR TIME USED = 123.9 SECONDS

# CONCLUSION

In any production process or action that involves uncertainty, adequate information to predict the result is necessary to reduce wastage and improve productivity. In many cases human experience and judgement is the best if not the only means of predicting the outcome, however the electronic computer is invaluable in assisting and complementing human judgement with thoroughness and speed.

This project is an illustration of the use of the digital computer in decision making under risk. Even in a 'black art' such as wire making where experience and intuition have been heavily relied upon in the design of drafting practices and choice of raw materials, partial or complete decisions can be made automatically, reliably and quickly.

Owing to the very simple nature of input to the program anyone can use it without a wire drawing or a computer programming background. Preliminary results obtained from the program have been very promising and it has been proposed by Stelco that the program be extended to cover more cases and perform more functions, such as automatic rod selection.

In addition to the direct economic advantages expected from elimination of the intermediate processes, numerous secondary advantages are expected as a result of this project. Although difficult to put a dollar value on these, substantial savings are predicted.

## GLOSSARY

| | | |
|---|---|---|
| 1) | DRAFT | The process of drawing wire through a die in order to reduce its cross sectional area. |
| 2) | HOLE | The wire die. |
| 3) | FRAME | A machine for wire drawing consisting of one or more sets of dies and capstans. |
| 4) | ROD | The raw material used as input to the drafting process. |
| 5) | PICK-UP OR STRENGTH GAIN | The tensile strength gain due to drawing the wire. |
| 6) | REDUCTION | Reduction of area sometimes expressed as a percentage. |
| 7) | TOTAL REDUCTION | The reduction in area from rod to wire. |
| 8) | HOLE REDUCTION | The reduction of area through a single die or hole. |
| 9) | FENCE | A constraining function of an optimization problem. |
| 10) | FEASIBLE SOLUTION | A solution to the optimization problem for which no constraints are violated. |

# APPENDICES

## APPENDIX I

a) Total Reduction

The total reduction is frequently required as a function of hole reductions. If $R_i$ denotes the reduction through the $i^{th}$ die and $A_i$ and $A_{i+1}$ denote cross sectional areas of the wire before and after the die then by definition;

$$R_i = 1 - \frac{A_{i+1}}{A_i}$$

where $i = 1, 2, \ldots\ldots n$ for an n hole practice.

$$R_1 = 1 - \frac{A_2}{A_1} \qquad \text{and} \qquad A_2 = (1-R_1)\,A_1$$

$$R_2 = 1 - \frac{A_3}{A_2} \qquad \text{and} \qquad A_3 = (1-R_2)\,A_2 = (1-R_1)\,(1-R_2)\,A_1$$

$$\begin{matrix} \cdot & \qquad\qquad & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \end{matrix}$$

$$R_n = 1 - \frac{A_{n+1}}{A_n} \qquad \text{and} \qquad A_{n+1} = (1-R_1)\,(1-R_2)\ldots\ldots(1-R_n)\,A_1$$

Now total reduction $RT = 1 - \dfrac{A_{n+1}}{A_1}$ by definition,

$$\therefore \quad RT = 1 - \frac{(1-R_1)\ (1-R_2)\ldots\ldots(1-R_n)\ A_1}{A_1}$$

$$\therefore \quad RT = 1 - (1-R_1)\ (1-R_2)\ldots\ldots(1-R_n)$$

for an n hole practice.

b) Hole Reduction:

Conversely the hole reduction is frequently required by the program, when total reduction at all dies is known.

If $RT_i$ denotes total reduction after the $i^{th}$ die and $R_i$ denotes reduction through the $i^{th}$ die.

$$RT_{i-1} = 1 - (1-R_1)\ (1-R_2)\ldots\ldots(1 - R_{i-1})$$

$$RT_i = 1 - (1-R_1)\ (1-R_2)\ldots\ldots(1-R_i)$$

$$RT_{i+1} \times (1-R_i) = (1-R_i) - (1-R_1)\ (1-R_2)\ldots\ldots(1 - R_{i-1})\ (1-R_i)$$

$$= 1 - (1-R_1)\ (1-R_2)\ldots\ldots(1-R_i) - R_i$$

$$\therefore \quad RT_i = RT_{i-1} \times (1-R_i) + R_i$$

$$= R_i\ (1 - RT_{i-1}) + RT_{i-1}$$

$$\therefore \quad R_i = \frac{RT_i - RT_{i-1}}{(1 - RT_{i-1})} \qquad \text{for } i \geq 2$$

## APPENDIX II

Derivation of expression for the expected value of a function of random variables ;

$$U = U(v_1, v_2, \ldots\ldots, v_n)$$

then for uncorrelated component variables, if the function is expanded in a multivariable Taylor series expansion about the expected value of each of the variables, $E(v_i)$ then;

$$U = U\left[E(v_1), E(v_2), \ldots\ldots, E(v_n)\right]$$

$$+ \sum_{i=1}^{n} \frac{\partial U}{\partial v_i}\left[v_i - E(v_i)\right] + \frac{1}{2} \sum_{i=1}^{n} \frac{\partial^2 U}{\partial v_i^2}\left[v_i - E(v_i)\right]^2$$

$$+ 2 \sum_{\substack{i\ j=1 \\ i<j}}^{n} \frac{\partial^2 U}{\partial v_i \partial v_j}\left[v_i - E(v_i)\right]\left[v_j - E(v_j)\right]$$

where all derivatives are evaluated at their expected values.

Now taking expected values of both sides of the equation

$$E(U) = E\left\{U\left[E(v_1), E(v_2), \ldots\ldots\ldots, E(v_n)\right]\right\}$$

$$+ E\left\{\sum_{i=1}^{n} \frac{\partial U}{\partial v_i}\left[v_i - E(v_i)\right]\right\}$$

$$+ E\ \frac{1}{2}\left\{\sum_{i=1}^{n} \frac{\partial^2 U}{\partial v_i^2}\left[v_i - E(v_i)\right]^2\right\}$$

$$+ E\left\{\sum_{i} \sum_{\substack{j=1 \\ i<j}}^{n} \frac{\partial^2 U}{\partial v_i \partial v_j}\left[v_i - E(v_i)\right]\left[v_j - E(v_j)\right]\right\}$$

Following the laws of probability:

$$E(U) = U\left[E(v_1), E(v_2), \ldots\ldots, E(v_n)\right] \quad \text{(as } E(\text{const}) = \text{const)}$$

$$+ 0 \quad \text{(as } E\left[v_i - E(v_i)\right] = E(v_i) - E(v_i)$$

$$+ \frac{1}{2}\sum_{i=1}^{n} \frac{\partial^2 U}{\partial v_i^2} \text{Var.}(v_i) \quad \text{(by definition } E\left[v_i - E(v_i)\right]^2 = \text{Var.}(v_i))$$

$$+ 0 \quad \text{(as } E\left[v_i - E(v_i)\right] \times E\left[v_j - E(v_j)\right] = 0\text{ )}$$

Therefore:

$$E(U) = U\left[E(v_1), E(v_2), \ldots\ldots, E(v_n)\right]$$

$$+ \frac{1}{2}\sum_{i=1}^{n} \frac{\partial^2 U}{\partial v_i^2} \text{Var.}(v_i)$$

Now applying this expression to the objective function,

$$U = \sum_{i=2}^{n} \left[ F(1-\frac{v_i^2}{v^2}) - F(1-\frac{v_{i-1}^2}{v^2}) - s_i \right]^2$$

where $v$ = input rod diameter

$v_i$ = diameter of the $i^{th}$ hole

$s_i$ = ideal tensile gain for the $i^{th}$ hole

The function F ( ) is of course the strength gain reduction relationship. Hence for each term under the summation;

$$U_i = \left[ f(1-\frac{v_i^2}{v^2}) - f(1-\frac{v_{i-1}^2}{v^2}) - s_i \right]^2$$

$$\frac{\partial^2 U_i}{\partial v_i^2} = 2\left[ f'(1-\frac{v_i^2}{v^2})(-2\frac{v_i}{v^2}) \right]^2$$

$$+ 2\left[ f(1-\frac{v_i^2}{v^2}) - f(1-\frac{v_{i-1}^2}{v^2}) - s_i \right] \left[ f''(1-\frac{v_i^2}{v^2})(4\frac{v_i^2}{v^4}) \right.$$

$$\left. + f'(1-\frac{v_i^2}{v^2})(-\frac{2}{v^2}) \right]$$

$$\frac{\partial^2 U_i}{\partial v_{i-1}^2} = 2\left[ f'(1-\frac{v_{i-1}^2}{v^2})(2\frac{v_{i-1}}{v^2}) \right]^2$$

$$+ 2\left[f(1-\frac{v_i^2}{v^2}) - f(1-\frac{v_{i-1}^2}{v^2}) - s_i\right]$$

$$x\left[-f''(1-\frac{v_{i-1}^2}{v^2})\ (\frac{4v_{i-1}^2}{v^4}) + f'(1-\frac{v_{i-1}^2}{v^2})\ (\frac{2}{v^2})\right]$$

$$\frac{\partial^2 U_i}{\partial v} = 2\left[f'(1-\frac{v_i^2}{v^2})\ \frac{2v_i^2}{v^3} - f'(1-\frac{v_{i-1}^2}{v^2})\ \frac{2v_{i-1}^2}{v^3}\right]$$

$$+ 2\left[f(1-\frac{v_i^2}{v^2}) - f(1-\frac{v_{i-1}^2}{v^2}) - s_i\right]$$

$$x\left[f''(1-\frac{v_i^2}{v^2})(\frac{4v_i^4}{v^6}) - f'(1-\frac{v_i^2}{v^2})(\frac{6v_i^2}{v^4})\right.$$

$$\left. - f''(1-\frac{v_{i-1}^2}{v^2})(\frac{4v_{i-1}^4}{v^6}) + f'(1-\frac{v_{i-1}^2}{v^2})\ (\frac{6v_{i-1}^2}{v^4})\right]$$

$$\therefore E(U) = \sum_{i=2}^{n}\left\{\left[(f(1-\frac{E(v_i)^2}{E(v)^2}) - f(1-\frac{Ev_{i-1})^2}{E(v)^2} - s_i\right]^2\right.$$

$$+ \frac{1}{2}\left[\frac{\partial^2 U_i}{\partial v_i^2}\ Var(v_i) + \frac{\partial^2 U_i}{\partial v_{i-1}^2}\ Var(v_{i-1})\right.$$

$$\left.\left. + \frac{\partial^2 U_i}{\partial v^2}\ Var(v)\right]\right\}$$

This expression was programmed in fortran and substituted for U in subroutine UREAL. The subroutine listing follows, along with DIV and DIV2 which are function subprograms used to evaluate $f'(R)$ and $f''(R)$.

```
      SUBROUTINE UREAL (U,A,X,RTOT,I,DIV,DIV2 )
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      COMMON/B/NP,STEP,RDT(15),DIAM(15),     RDTMU,S(15),FF,IND
      DIMENSION P(15),A(1),X(1),RTOT(1),R( 15),V(15),VAR(15)
C   THE FUNCTION OF THIS SUBPROGRAM IS TO CALCULATE THE CURRENT VALUE OF
C   THE OBJECTIVE FUNCTION
      U=0.
      R(1)=RDT(1)
      V(1)=DIAM(1)
      DO 2 M=1,NP
      MP=M+1
      VAR(M)=(.001/3.)**2
      R(MP)=RTOT(M)
    2 V(MP)=X(M)
      NPP=NP+1
      VAR(NPP)=WSIG**2
      DO 1 M=1,NP
      MP=M+1
      ONE=(DIV(R(M))*(2.*V(M)/SIZMU(I)**2))**2+(A(M)-S(MP))*
     1(-1.*DIV2(R(M))* 4.*V(M)**2/SIZMU(I)**4+DIV(R(M))*2./SIZMU(I)**2)
      TWO=(DIV(R(MP))*(-2.*V(MP)/SIZMU(I)**2))**2+(A(M)-S(MP))
     1*(DIV2(R(MP))*4.*V(MP)**2/SIZMU(I)**4+DIV(R(MP))*(-2./SIZMU(I)**2)
     2)
      THREE=(DIV(R(MP))*2.*V(MP)**2/SIZMU(I)**3-DIV(R(M))*2.*V(M)**2/
     1SIZMU(I)**3)**2+(A(M)-S(MP))*(DIV2(R(MP))*4.*V(MP)**4/SIZMU(I)**6
     3-DIV(R(MP))*6.*V(MP)**2/SIZMU(I)**4-DIV2(R(M))*4.*V(M)**4/SIZMU(I)
     4**6+DIV(R(M))*6.*V(M)**2/SIZMU(I)**4)
      P(M)=(A(M)-S(MP))**2+ONE*VAR(M)+TWO*VAR(MP)+THREE*SIZSIG**2
    1 U=U+P(M)
      U=U/1000000.
      RETURN
      END
```

```
      FUNCTION DIV(X)
C   THE FIRST DERIVATIVE OF STRENGTH GAIN WITH RESPECT TO REDUCTION IS
C   FOUND BY DIFFERENTIATING  THE QUADRATIC INTERPOLANT
      COMMON /D/  S(23),R(23),DELS,DELR
      DO 5 K=3,23,2
      KM=K-1
      KMM=K-2
      IF(X.GT.R(K)) GO TO 5
      DIV= (2.*X-R(KM)-R(K))/((R(KMM)-R(KM))*(R(KMM)-R(K)))*S(KMM)
     1+(2.*X-R(KMM)-R(K))/((R(KM)-R(KMM))*(R(KM)-R(K)))*S(KM)
     2+(2.*X-R(KMM)-R(KM))/((R(K)-R(KMM))*(R(K)-R(KM)))*S(K)
      RETURN
    5 CONTINUE
      DIV=1.E+06
      RETURN
      END


      FUNCTION DIV2(X)
C   THE SECOND DERIVATIVE IS FOUND BY DIFFERENTIATING THE QUADRATIC
C   INTERPOLANT
      COMMON /D/  S(23),R(23),DELS,DELR
      DO 5 K=3,23,2
      KM=K-1
      KMM=K-2
      IF(X.GT.R(K)) GO TO 5
      DIV2=2./((R(KMM)-R(KM))*(R(KMM)-R(K)))*S(KMM)
     1 +2./((R(KM)--(KMM))*(R(KM)-R(K)))*S(KM)
     2+2./((R(K)-R(KMM))*(R(K)-R(KM)))*S(K)
      RETURN
    5 CONTINUE
      DIV2=0.
      RETURN
       END
```

# APPENDIX III

FORTRAN PROGRAM LISTING.

```
C    MAIN PROGRAM
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      DIMENSION CARBON(3)
      DATA CARBON(1),CARBON(2),CARBON(3)/5H71/75,5H75/79,5H79/83/
      EXTERNAL FACTOR,FUNC      ,DIV,DIV2
C   PROGRAM TO DETERMINE A SUITABLE WIRE DRAWING PRACTICE
C   ROD SIZES ARE NO-MALLY DISTRIBUTED WITHIN WORKING LIMITS
C    ROD STRENGTH IS NORMALLY DISTRIBUTED WITHIN THE WORKING LIMITS OF
C        A PARTICULA- CHEMISTRY
C   SIZMU IS THE MEAN VALUE OF A PARTICULAR ROD SIZE CLASSIFIED BY THE
C   SUBSCRIPT
C   RNOM IS THE NOMINAL ROD DIAMETER CORRESPONDING TO THE PARTICULAR
C   SUBSCRIPT
C   SIZSIG IS THE STANDARD DEVIATION OF ROD DIAMETER, ALL BEING EQUAL
C   BRKMU(I,J) IS THE MEAN BREAKING STRESS OF A ROD
C   THE FIRST SUBSCR+PT INDICATES ROD SIZE AND THE SECOND ROD CHEMISTRY
C   BRKSIG IS THE STANDARD DEVIATION OF ROD BREAKING STRESS
C    IPSI IS AN INDICATOR USED TO DIFFERENTIATE BETWEEN WIRE STRENGTH OR
C   STRESS   IF STRESS IS REQUIRED THEN IPSI =1
      IPSI=0
C   ASSIGNING VALUES TO THE PARAMETERS
      BRKSIG=10000./3.
      SIZSIG=.01/3.
      BRKMU(1,1)=152000.
      BRKMU(1,2)=15 000.
      BRKMU(1,3)=175000.
      BRKMU(2,1)=150000.
      BRKMU(2,2)=155000.
      BRKMU(2,3)=172500.
      BRKMU(3,1)=148000.
      BRKMU(3,2)=153000.
      BRKMU(3,3)=170000.
      BRKMU(4,1)=147000.
      BRKMU(4,2)=152000.
      BRKMU(4,3)=168500.
      BRKMU(5,1)=146000.
      BRKMU(5,2)=151000.
      BRKMU(5,3)=167500.
      BRKMU(6,1)=145000.
      BRKMU(6,2)=150000.
      BRKMU(6,3)=166000.
      BRKMU(7,1)=144000.
      BRKMU(7,2)=14 000.
      BRKMU(7,3)=165000.
      RNOM(7)=.328
      RNOM(6)=.312
      RNOM(5)=.297
      RNOM(4)=.281
      RNOM(3)=.263
      RNOM(2)=.240
```

```
      RNOM(1)=.218
      SIZMU(7)=.332
      SIZMU(6)=.316
      SIZMU(5)=.301
      SIZMU(4)=.285
      SIZMU(3)=.267
      SIZMU(2)=.244
      SIZMU(1)=.222
C   INFEAS IS AN IND+CATOR IDENTIFYING NON FEASIBLE ATTEMPTS
C   INFEAS  IS INITIALLY SET AT 0
      INFEAS=0
C   ISTOP AND IEXIT ARE INDICATORS USED TO STOP THE PROGRAM UNCER CERTAIN
C   CONDITIONS
      IEXIT=0
      ISTOP=0
      WRITE (6,100)
  100 FORMAT(1H1,40X,35HSELECTION OF WIRE DRAWING PRACTICES,/41X,35H----
     1-------------------------------,///)
C   INPUT TO THE PROGRAM
      READ(5,200) WSIZE,WBREAK,WHI
  200 FORMAT(F6.4,2F12.1)
C   TEST TO CHECK WHETHER WIRE SIZE IS WITHIN PERMISSABLE LIMITS
      IF(WSIZE.GE..010.OR.WSIZE.LE..200) GO TO 1
      WRITE(6,101)
  101 FORMAT(53H     REQUIRED WIRE SIZE IS OUTSIDE PERMISSABLE LIMITS//)
      STOP
C    ASSIGN A VALUE TO IPSI
    1 IF(WBREAK.GT.100000.) IPSI=1
C   WSIZE IS THE REQUIRED NOMINAL WIRE SIZE
C   WBREAK IS THE MINIMUM BREAKING STRENGTH OF THE WIRE
C   WHI IS THE MAXIMUM BREAKING STRENGTH OF THE WIRE
C   WMU IS THE MEAN VALUE OF WIRE SIZE
C   WSIG IS THE STANDARD DEVIATION OF WIRE DIAMETER
C   THE WIRE MUST NOW BE CLASSIFIED BY SIZE IN ORDER TO DETERMINE THE
C   DISTRIBUTION PARAMETERS
    2 IF(WSIZE.GE..025) GO TO 3
      WMU=WSIZE+.0002
      WSIG=.0001/6.
      GO TO 7
    3 IF(WSIZE.GE..060) GO TO 4
      WMU=WSIZE+.00025
      WSIG=.00025
      GO TO 7
    4 IF(WSIZE.GE..093) GO TO 5
      WMU=WSIZE
      WSIG=.001/3.
      GO TO 7
    5 IF(WSIZE.GE..142) GO TO 6
      WMU=WSIZE+.00025
      WSIG=.0025/6.
```

```
      GO TO 7
    6 WMU=WSIZE
      WSIG=.0005
C  IF ANY STOCK RODS ARE UNAVAILABLE THIS INFORMATION IS NOW READ IN
C  NASIG(I) IS THE +NTEGER CLASSIFICATION OF ROD SIZES UNAVAILABLE
C E.G. IF NASIZ(I)=1 THEN THE SMALLEST ROD NAMELY RNOM(1)=.218
C  IS UNAVAILABLE AND MUST BE EXCLUDED
C  NACHEM(I) IS THE INTEGER CLASSIFICATION OF ROD CHEMISTRY NOT
C  AVAILABLE  E.G. +F NACHEM(I)=2 THEN THE 75/79 CHEMISTRY OF A SIZE
C  DEFINED BY NASIZ IS NOT AVAILABLE
    7 READ(5,201) (NASIZ(I),NACHEM(I),I=1,20)
  201 FORMAT(40I2)
      WRITE(6,104)  WSIZE
      WRITE(6,105)  WMU
      WRITE(6,106)  WSIG
      IF (IPSI.EQ.0) WRITE(6,107) WBREAK,WHI
      IF (IPSI.EQ.1) WRITE(6,207) WBREAK,WHI
  104 FORMAT(37H       REQUIRED NOMINAL WIRE DIAMETER =,F7.4//)
  105 FORMAT(37H       MEAN WIRE DIAMETER             =,F7.4//)
  106 FORMAT(37H       STANDARD DEVIATION OF DIAMETER =,F7.4//)
  107 FORMAT(37H       REQUIRED  RANGE  OF  STRENGTH  =,F8.1,5H   TO,F8.1/
     1/)
  207 FORMAT(37H       REQUIRED  RANGE  OF  STRESS    =,F10.1,5H   TO,F10.
     11//)
C  TESTING TO FIND WHICH RODS ARE UNAVAILABLE
      DO 8 I=1,20
      IF(NASIZ(I).EQ.0) GO TO 8
      NN=NASIZ(I)
      NNN=NACHEM(I)
      WRITE(6,109) -NOM(NN)  ,CARBON(NNN)
    8 CONTINUE
  109 FORMAT(46H       THE FOLLOWING STOCK ROD IS NOT AVAILABLE ,F6.3,23H
     1 NOMINAL DIAMETER AND ,A5,7H CARBON//)
C  THE WIRE IS NOW CLASSIFIED BY THE PARAMETERS OF ITS DISTRIBUTION
C  RATHER THAN ITS NOMINAL SIZE
C  THE DETERMINISTIC SUBROUTINE START IS CALLED NOW,TO MAKE A ROUGH
C  ESTIMATE OF THE NECESSARY ROD SIZE SO AS TO REDUCE WORK DONE BY THE
C        MORE TIME CONSUMING STOCHASTIC SUBROUTINE
      CALL START(  +SUB1,ISUB2)
      IF(IEXIT.EQ.1) STOP
C  HENCE THE STARTING POINT FOR THE STOCHASTIC SUBROUTINE IS
C  ROD(ISUB1,ISUB2)
   15 CALL CARLO(ISUB1,ISUB2,FACTOR,DIV,FUNC,DIV2)
      CALL SECOND(T+ME)
      WRITE(6,300) TIME
  300 FORMAT(/1H ,*      CENTRAL PROCESSOR TIME USED =*,F6.1,*  SECONDS*)
      STOP
      END
```

```
      SUBROUTINE START(I,J)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      COMMON/D/ S(23),R(23),DELS,DELR
C   THIS SUBROUTINE SERVES TO SELECT A ROD IN A DETERMINISTIC APPROACH
C   USING MEAN VALUES FOR THE RANDOMLY DISTRIBUTED VARIABLES)
C   THIS PROVIDES A STARTING POINT IN THE VICINITY OF THE SOLUTION
C   AND SO SERVES TO SAVE TIME
C   IFLAG IS AN INDICATOR TO SHOW WHETHER THE LAST TRIAL VALUE OF WIRE
C   STRENGTH WAS LESS THAN WBREAK,IFLAG=0, OR GREATER THAN WBREAK,
C   IFLAG=1.   IFLAG IS INITIALLY SET =2
      IFLAG=2
C   IMAX AND IMIN ARE THE BOUNDS OF A FEASIBLE ROD SIZE
C   INITIALLY ALL RODS ARE CONSIDERED FEASIBLE
      IMAX=7
      IMIN=1
C   ARBITRALLY SELECTING A ROD AT THE MIDPOINT OF THE RANGE (4,2)
      I=4
      J=2
C   KOUNT IS USED IN CHECKING FOR SIZES NOT AVAILABLE
      KOUNT=0
C   FINDING THE REDUCTION KNOWING THE SIZES
    1 CALL RMEAN (RDT   ,RDTLO,RDTHI,I)
C   BEFORE CALLING POLY DATA IS ASSIGNED
      DATA S(1 ),S(2 ),S(3 ),S(4 )/20000.,31000.,42000.,53000./
      DATA S(5 ),S(6 ),S(7 ),S(8 )/60000.,66000.,70000.,75000./
      DATA S(9 ),S(10),S(11),S(12)/80000.,85000.,90000.,95000./
      DATA S(13),S(14),S(15),S(16)/100000. ,103500.,110000.,115000./
      DATA S(17),S(18),S(19),S(20)/120000.,125000.,130000.,140000./
      DATA S(21),S(22),S(23)/150000.,160000.,165000./
      DATA R(1 ),R(2 ),R(3 ),R(4 ),R(5 )/.20,.30,.40,.50,.56/
      DATA R(6 ),R(7 ),R(8 ),R(9 ),R(10)/.60,.63,.67,.70,.725/
      DATA R(11),R(12),R(13),R(14),R(15)/.745,.77,.79,.80,.82/
      DATA R(16),R(17),R(18),R(19),R(20)/.83,.84,.85,.86,.875/
      DATA R(21),R(22),R(23)/.8825,.895,.90/
C   NOW  USING THE SUBROUTINE TO FIND THE STRENGTH GAIN
      CALL POLY(RDT,GAIN,I)
C A CHECK IS MADE AND ACTION IS TAKEN IF THE REDUCTION IS INFEASIBLE
      IF(INFEAS.EQ.1) GO TO 5
      IF(INFEAS.EQ.2) GO TO 6
      GO TO 2
    5 I=I+1
      IF(I.GT.7) GO TO 7
      GO TO 1
    6 I=I-1
      IF (I.LT.1) GO TO 7
      GO TO 1
    2 STRGTH=GAIN+B-KMU(I,J)
      IF(IPSI.EQ.1) TRIAL=STRGTH
      IF(IPSI.EQ.0)TRIAL=STRGTH*WMU*WMU*3.1416/4.
C   FINDING A ROD THAT SATISFIES THE REQUIREMENTS
```

```
      IF(TRIAL.GT.WBREAK) GO TO 3
      IF(I.EQ.IMAX) GO TO 44
      I=I+1
      IF(IFLAG.EQ.1) GO TO 4
      IFLAG=0
      GO TO 1
    3 IF(IFLAG.EQ.0)  GO TO 4
      IF(I.EQ.IMIN) GO TO 45
      I=I-1
      IFLAG=1
      GO TO 1
    7 WRITE(6,106)
  106 FORMAT(47H     THE REQUIRED CONFIGURATION IS NOT FEASIBLE//)
      IEXIT=1
      RETURN
C   THIS PART CHECKS IF SELECTED ROD IS AVAILABLE IF NOT A CHANGE IS MADE
   44 J=3
      GO TO 4
   45 J=1
    4 CALL AVAIL(I,J,KO)
      IF(KO.EQ.1) GO TO 10
C   THIS IS THE ESTIMATED ROD SIZE TO BE USED AS A STARTING POINT
   10 IF(J.NE.1.OR.J.NE.3) GO TO 11
C   THE CLOSEST POINT IS J = 2
      J=2
      CALL AVAIL (I,J,KO)
      IF(KO.NE.1) RETURN
C   J=2 NOW AND I LIES BETWEEN IMAX AND IMIN
   11 IF(I.EQ.IMAX) GO TO 15
   12 CALL AVAIL(I,3,KO)
      IF(KO.NE.1) RETURN
  13  IF(I.EQ.IMAX) GO TO 15
      I=I+1
      DO 14 J=1,3
      CALL AVAIL (I,J,KO)
      IF(KO.NE.1) RETURN
   14 CONTINUE
      GO TO 13
   15 DO 16 JJ=1,3
      J=4-JJ
      CALL AVAIL (I,J,KO)
      IF(KO.NE.1) RETURN
   16 CONTINUE
      IF(I.EQ.IMIN) GO TO 17
      I=I-1
      GO TO 15
   17 WRITE(6,107)
  107 FORMAT(57H    THE REQUIREMENT CANNOT BE MET WITH THE AVAILABLE ROD
     1S//)
      IEXIT=1
      RETURN
      END
```

```
      SUBROUTINE AVAIL(I,J,KO)
C  THIS SUBROUTINE +S USED TO INDICATE WHETHER OR NOT TO SIMULATE FOR
C  A PARTICULAR ROD
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
C  NASIZ IS AN ARRAY STORING CODE NAMES OF ROD SIZES THAT ARE UNAVAILABLE
C  NACHEM STORES THE CORRESPONDING CHEMISTRY TO DEFINE THE ROD
C  IDONE AND JDONE ARE ARRAYS THAT DEFINE RODS ALREADY TRIED
C   ILO IS AN ARRAY STORING THE CODE SIZE, FOR EACH CHEMISTRY OF ROD
C  BELOW WHICH A SIMULATION WILL LOGICALLY GIVE UNACCEPTABLY HIGH
C  PRODUCT REJECTION
C  IHI IS A SIMILAR ARRAY DEFINING THE LIMITS OF MAXIMUM ROD SIZES,
C  KO IS AN INDICATOR SET = 0 IF ROD IS AVAILABLE,WORTH SIMULATING,
C  AND HAS NOT YET BEEN TRIED
      KO=0
C  THIS TEST DETERM+NES WHETHER OR NOT THE ROD IS AVAILABLE AND ALSO
C  IF THE PARTICULA- ROD SIZE HAS ALREADY BEEN TRIED AS INDICATED BY
C  IDONE AND JDONE
      DO 8 K=1,20
      IF(NASIZ(K).EQ.I.AND.NACHEM(K).EQ.J) KO=1
      IF(IDONE(K).EQ.I.AND.JDONE(K).EQ.J) KO=2
    8 CONTINUE
C  THIS TEST SERVES TO PREVENT ATTEMPTS THAT WILL LOGICALLY BE WORSE
C  PREVIOUS UNACCEPTABLE RESULTS
      IF(I.LT.ILO(J).OR.I.GT.IHI(J)) KO=3
      RETURN



      END
      SUBROUTINE PROB (I,RDT,P,KK)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
C  THE FUNCTION OF THIS SUBPROGRAM IS TO PROVIDE THE PROBABILITY DENSITY
C  OF REDUCTION FOR ANY VALUE OF THAT VARIABLE
C  KK IS AN INDICATOR USED TO INITIALIZE THE VALUES OF FMU AND FSIG .
C  ON THE FIRST CALL KK = 2 AND THEREAFTER KK = 0
      IF(KK.EQ.0) GO TO 1
C  F=WIRE DIAM/ROD DIAM
C   NOW CALCULATING THE PARAMETERS OF  THE NORMAL DISTRIBUTION FOR
C  THE QUOTIENT
      FMU=WMU/SIZMU(I)
      FSIG=1./SIZMU(I)*SQRT(((SIZMU(I)**2)*(WSIG**2)+(WMU**2)*(SIZSIG**2
     1))/((SIZMU(I)**2)+(SIZSIG**2)))
      IF(KK.EQ.2) GO TO 2
C  NOW USING THE TRANSFORMATION OF VARIABLES METHOD TO DETERMINE
C   PROBABILITY DENSITY  OF A FUNCTION OF ONE RANDOM VARIABLE
    1 X=SQRT(1.-RDTD
      P=1./(FSIG*SQ-T(2.*3.1416))*EXP(-1.*(X-FMU)**2/(2.*FSIG**2))*ABS
     1(-1./(2.*X))
    2 RETURN
      END
```

```
      SUBROUTINE RMEAN (RDTMU,RDTLO,RDTHI,I)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
C  THE FUNCTION OF THIS SUBROUTINE IS TO CALCULATE THE MEAN VALUE OF
C  REDUCTION AS WELL AS THE LIMITS CORRESPONDING TO THE 3 SIGMA LIMITS
C  OF THE COMPONENT VARIABLES
      RDTHI=1.-(WMU-3.*WSIG)**2/(SIZMU(I)+3.*SIZSIG)**2
      RDTLO=1.-(WMU+3.*WSIG)**2/(SIZMU(I)-3.*SIZSIG)**2
C  APPLYING THE GENERAL RESULT TO FIND THE MEAN USING A TAYLOR SERIES
C  EXPANSION
      RDTMU=1.-WMU**2/SIZMU(I)**2-WSIG**2/SIZMU(I)**2-3.*WMU**2*SIZSIG**
     12/SIZMU(I)**4
      RETURN
      END


      SUBROUTINE RNORM(R)
C  THIS SUBPROGRAM GENERATES NORMALLY DISTRIBUTED RANDOM NUMBERS IN THE
C  RANGE 0-1 BETWEEN THE 3 SIGMA LIMITS.
C  THIS ACCOUNTS FO- 99.75 PERCENT OF THE AREA  SO IS SUFFICIENTLY
C  ACCURATE FOR THE PURPOSE
      DIMENSIONA(2)
    1 CALL RANDOM (A,2,0)
C  THE FREQUENCY IS NORMALISED  (LIES BETWEEN 0 AND 1)
      FREQ=EXP(-(A(1)-.5)**2*18.)
      IF(A(2).GT.FREQ) GO TO 1
C  THE NUMBER LIES WITHIN THE DESIRED AREA
      R=A(1)
      RETURN
      END


      SUBROUTINE    -ANDOM(A,N,M)
C  RANDOM NUMBER GENERATOR (MODIFIED IBM ROUTINE)
      DIMENSION A(1)
C  B IS A MACHINE DEPENDANT CONSTANT...........B=2.**(I/2+1)+3.
C  I ABOVE IS THE NUMBER OF BITS IN AN INTEGER WORD
      B=64.**3+3.
      X=M
      X=X/.8719467
   20 IF(X.NE.0.)  Y=AMOD(ABS(X),3.18967)
       DO  10  K=1,N
       DO  11  J=1,2
   11 Y=AMOD(B*Y,1.)
       A(K)  =  Y
   10 IF(Y.EQ.0..OR.Y.EQ.1.) Y=.182818285
      RETURN
      END
```

```
      SUBROUTINE CA-LO(I,J,FACTOR,DIV,FUNC,DIV2)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIG(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      COMMON /B/NP,STEP,R(15),DIAM(15),RDTMU,S(15),FF   ,IEX,WATE
      COMMON/C/SHAPE,SCALE,W,ICOM,JCOM ,FFF,RDTSIG,RDTM
      COMMON/E/ZS(15),ZR(15),ZD(15)
      DIMENSION     P(21),G(21),CUM(21),IHOLD(21)   ,CARBON(3),NNHOLD(21)
      REAL MOMENT
      DATA CARBON(1),CARBON(2),CARBON(3)/5H71/75,5H75/79,5H79/83/
C   THE PURPOSE OF THIS SUBROUTINE IS TO FIND THE WIRE STRENGTH USING
C   A PROBABILISTIC APPROACH ,VIZ. MONTE CARLO IF IPSI = 0 OTHERWISE BY
C   TRANSFORMATION OF VARIABLES WHEN IPSI = 1
C   THE FOLLOWING VA-IABLES ARE RANDOM AND NORMALLY DISTRIBUTED
C   1 WIRE SIZE REPRESENTED BY WMU AND WSIG
C   2 ROD SIZE REPRESENTED BY SIZMU AND SIZSIG
C   3 ROD STRENGTH REPRESENTED BY BRKMU AND BRKSIG
C   FIRSTLY REDUCTION IS CALCULATED RDT=1-WIRE AREA/ROD AREA
C   N IS THE SAMPLE SIZE OF THE SIMULATION
      N=4000
C   BEFORE USING THE RANDOM NUMBER GENERATOR IT MUST BE CALLED
C   WITH A POSITIVE +NTEGER VALUE FOR THE THIRD ARGUMENT
      CALL RANDOM(ZT1,13)
C   INITIALLY ZEROING ARRAY THAT RECORDS WHICH RODS HAVE BEEN TRIED
      DO 53 L=1,21
      NNHOLD(L)=0
      IHOLD(L)=0
      IDONE(L)=0
   53 JDONE(L)=0
C    THE ARRAY STORING THE LIMITS OF THE RANGE OF INTEREST ARE INITIALLY
C   ZEROED
      DO 54 L=1,3
      ILO(L)=IMIN
   54 IHI(L)=IMAX
C   A COUNTER ,LOT, +S INITIALLY SET
      LOT=1
C   RDTMU IS THE MEAN REDUCTION
    1 CALL PROB (I,-DT,P,2)
      CALL RMEAN(RDTMU,RDTLO,RDTHI,I)
      IF(RDTLO.LT..70) IMIN=I+1
      IF(RDTLO.LT..70) WRITE(6,204) RNOM(I)
  204 FORMAT(1H ,F5.3,  74H  NOMINAL DIAMETER ROD NOT ACCEPTABLE AS REDU
     1CTION IS LESS THAN 70 PERCENT)
      IF(RDTLO.LT..70) GO TO 98
C   THE NEXT STEP IS TO DETERMINE THE UPPER AND LOWER LIMITS OF STRENGTH
C   GAIN CORRESPONDING TO THE 3 SIGMA LIMITS OF DIAMETERS
C   LOWER LIMIT
   19 KATE=0
      CALL POLY(RDTLO,GAINLO,I)
      IF(INFEAS.NE.1) GO TO 2
   98 KATE=1
```

```
      IF(I.EQ.7) RETURN
      PRCNT=0.
      DO 56 L=1,3
      IF(ILO(L).GT.+MIN) GO TO 56
      ILO(L)=IMIN
   56 CONTINUE
      GO TO 93
C UPPER LIMIT
    2 CALL POLY (RDTHI,GAINHI,I)
      IF(INFEAS.NE.2) GO TO 3
      IF(I.EQ.1) RETURN
      KATE=1
      PRCNT=100.
      DO 57 L=1,3
      IF( IHI(L).LT.IMAX) GO TO 57
      IHI(L)=IMAX
   57 CONTINUE
      GO TO 93
    3 CONTINUE
C   THE SUBROUTINE D-AFT IS CALLED TO DETERMINE THE NUMBER OF HOLES
      CALL DRAFT (RDTMU,H,I,J,FF,NN,R1,D1,S1)
      FFF=FF
   33 WRITE (6,107)
      WRITE(6,100) -NOM(I) ,CARBON(J)
  107 FORMAT(////1H ,18X,58H * * * * * * * * * * * * * * * * * * * * * * * *
     1 * * * * * * *)
  108 FORMAT(     1H ,18X,58H * * * * * * * * * * * * * * * * * * * * * * * *
     1 * * * * * * * //)
  100 FORMAT(1H ,18X,11H * USING A , F5.3,28H   NOMINAL DIAMETER ROD AND
     1 ,A5,9H CARBON *)
  4   WRITE(6,108)
      IF(IPSI.EQ.1) GO TO 5
C   THE NEXT STEP IS TO DETERMINE THE MIN AND MAX VALUES OF STRENGTH
C   THE SUBROUTINE EXTRM   IS CALLED FOR THIS PURPOSE
      CALL EXTRM (STRMIN,STRMAX,I,J,FF)
      GO TO 44
C   THE MIN AND MAX VALUES OF STRENGTH
    5 GAINLO=FF*GAINLO
      GAINHI=FF*GAINHI
      STRMIN=(GAINLO+BRKMU(I,J)-3.*BRKSIG)
      STRMAX=(GAINH++BRKMU(I,J)+3.*BRKSIG)
C   TEST IF WBREAK IS WITHIN THE RANGE OF HTE DISTRIBUTION
   44 IF(WBREAK.GT.STRMIN.AND.WBREAK.LT.STRMAX) GO TO 55
      IF(WHI.GT.STRMIN.AND.WHI.LT.STRMAX) GO TO 55
      IF(WBREAK.LE.STRMIN) PRCNT=100.
      IF(WBREAK.GE.STRMAX) PRCNT=0.
      IF(WHI.GE.STRMAX) PHI=100.
      IF(WHI.LE.STRMIN) PHI=0.
      GO TO 7
   55 IF(IPSI.EQ.1) GO TO 21
C   NOW READY TO STA-T THE MONTE CARLO SIMULATION OF (GAIN+ROD STRENGTH)
C   *WIRE AREA
```

```
      SUM=0.
      PRCNT=0.
      PHI=0.
      KOUNT=0
C   NOW A VALUE OF ROD STRENGTH ,BRK,IS FOUND
    6 CALL RNORM(RR)
      BRK=RR    *6.*BRKSIG+BRKMU(I,J)-3.*BRKSIG
C   FINDING A RANDOM VALUE OF ROD AREA
      CALL RNORM (R-)
      RAREA=(RR*6.*SIZSIG+SIZMU(I)-3.*SIZSIG)**2*3.1416/4.
C   SIMILARILY FINDING A VALUE OF WIRE AREA
      CALL RNORM(RR)
      WAREA=3.1416*(RR    *6.*WSIG+WMU-3.*WSIG)**2/4.
C   NOW FINDING RDT
      RDT=1.-WAREA/-AREA
C   FINDING THE CORRESPONDING STRENGTH GAIN
      CALL POLY(RDT,GAIN,I)
      GAIN= GAIN*FF
      STRGTH=(GAIN+BRK)*WAREA
      KOUNT=KOUNT+1
C   FINDING SUM OF STRENGTH
      SUM=SUM+STRGTH
C FINDING HOW MANY OF THE CASES SATISFY THE MAXIMUM RESTRICTION
      IF(STRGTH.GT.WHI) GO TO 14
      PHI=PHI+1
   14 CONTINUE
C   FINDING HOW MANY OF THE CASES SATISFY THE MINIMUM REQUIREMENT
      IF(STRGTH.LT.WBREAK) GO TO 9
      PRCNT=PRCNT+1.
    9 IF(KOUNT.LT.N) GO TO 6
C   NOW FINDING THE MEAN STRENGTH
      STRMU=SUM/FLOAT(N)
C   PERCENTAGE OF TIME THE REQUIREMENT IS SATISFIED
      PHI=PHI*100./FLOAT(N)
      PRCNT=PRCNT*100./FLOAT(N)
      GO TO 13
C    THIS SECTION OF THE PROGRAM CALCULATES WIRE TENSILE STRESS BY
C   THE METHOD OF TRANS FORMATION OF VARIABLES
   21 ICOM=I
      JCOM=J
      A=GAINLO
      B=GAINHI
C   THE LIMITS OF THE VARIBLE W=GAIN+BRK ARE STRMIN AND STRMAX
      DELTA=(STRMAX-STRMIN)/20.
      W=STRMIN
C   INTEGRATING THE FUNCTION FUNC FOR VARIOUS VALUES OF W
      DO 26 K=1,21
      CALL SMPSN(A,B,FUNC,SUM)
      P(K)=SUM
      W=W+DELTA
   26 CONTINUE
C   FINDING THE CUMULATIVE DENSITIES AT VARIOUS POINTS
```

```
      CALL CUMUL (P,DELTA,19,CUM)
      DO 30 L=1,10
      LL=1 2-L
      LM=LL-1
   30 CUM(LL)=CUM(LM)
      CUM(1)=0.
C  FINDING PERCENTAGE OF CASES THAT MEET OR EXCEED REQUIREMENT
      IF(WBREAK.GT.STRMIN) GO TO 15
      PRCNT=100.
      GO TO 16
   15 CALL SUB (WBREAK,DELTA,CUM,AREA,STRMIN,P)
      PRCNT=(1.-AREA/CUM(11))*100.
   16 IF(WHI.LT.STRMAX) GO TO 17
      PHI=100.
      GO TO 18
   17 CALL SUB (WHI    ,DELTA,CUM,AREA,STRMIN,P)
      PHI=AREA/CUM(11)*100.
   18 CONTINUE
C  NOW CALCULATING THE MEAN VALUE OF TENSILE STRESS
      SUM=0.
      DO 29 K=1,20
      KP=K+1
      MOMENT=(P(K)+P(KP))/2.*DELTA*(FLOAT(K)-.5)*DELTA
   29 SUM=SUM+MOMENT
      STRMU=STRMIN+SUM/CUM(11)
C   OUTPUT
   13 WRITE(6,103) STRMU
  103 FORMAT(37H     THE MEAN VALUE OF WIRE STRENGTH =,F10.1/)
      DEV=STRMU-(WH++WBREAK)/2.
      WRITE(6,203) DEV
  203 FORMAT(37H      DEVIATION FROM REQUIRED MEAN    =,F10.1/)
    7 WRITE(6,104) STRMIN,STRMAX
  104 FORMAT(33H     THE RANGE OF STRENGTH IS FROM,F10.1,4H  TO,F10.1/)
      WRITE(6,105) PRCNT
      WRITE(6,101) PHI
  105 FORMAT(4H    ,F6.1,68H  PERCENT OF THE PRODUCT WILL MEET OR EXCEED
     1 THE MINIMUM -EQUIREMENT/)
  101 FORMAT(4H    ,F6.1,72H  PERCENT OF THE PRODUCT WILL MEET OR FALL B
     1ELOW THE MAXIMUM REQUIREMENT/)
      PTOT=100.-(100.-PHI)-(100.-PRCNT)
      IF(PTOT.LT.0.) PTOT=0.
      WRITE(6,102) PTOT
  102 FORMAT(4H    ,F6.1,50H  PERCENT OF THE PRODUCT FALLS BETWEEN BOTH
     1LIMITS/)
C  THE NAME OF THE LAST ROD TRIED IS RECORDED
      IDONE(LOT)=I
      JDONE(LOT)=J
      LOT=LOT+1
      JACK=J
      LAST=I
      IF(PTOT.LT.98.) GO TO 93
C  CHECKING WHETHER DRAFTING PROCEDURE HAS BEEN DETERMINED FOR THIS SIZE
C  AND NUMBER OF HOLES
```

```
      LOOK=1
      IF(NN.LE.2) GO TO 91
      DO 95 M=1,21
      IF(IHOLD(M).EQ.I.AND.NNHOLD(M).EQ.NN) GO TO 94
   95 CONTINUE
      IEX=IEXIT
      NM2=NN-2
   20 CALL SEARCH (NM2,I,INFEAS,DIV,DIV2)
      IF(INFEAS.LT.4) GO TO 90
      LOOK=LOOK+1
      IF(LOOK.EQ.4) GO TO 90
      CALL DRAFT (RDTMU,H,I,J,FF,NN,R1,D1,S1)
      GO TO 20
   90 CONTINUE
      IHOLD(LOT)=I
      NNHOLD(LOT)=NN
C   OUTPUT
      IF(INFEAS.EQ.4) WRITE(6,201)
  201 FORMAT(25H    SOLUTION IS INFEASIBLE)
      WRITE(6,205)
  205 FORMAT(1H ,35X,5HIDEAL,10X,9HOPTIMIZED/)
      IF(IEXIT.NE.7) GO TO 91
      M=1
      R1=R1*100.
      WRITE(6,200) M,R1,R1,S1,S1,D1,D1
   91 DO 92 M=1,NN
      MP=M
      IF(IEXIT.EQ.7) MP=M+1
      IF(IEXIT.EQ.7) R(1)=(R(1)-R1/100.)/(1.-R1/100.)
      IF(IEXIT.EQ.7) S(1)=S(1)-S1
      R(M)=R(M)*100.
      ZR(M)=ZR(M)*100.
   92 WRITE(6,200) MP,ZR(M),R(M),ZS(M),S(M),ZD(M),DIAM(M)
  200 FORMAT(1H ,4X,19HFOR DRAFT  NUMBER  ,I2,/17H      REDUCTION IS,17X,
     1F6.1,12X,F6.1,14H        PERCENT,/21H      STRENGTH GAIN IS,8X,F12.1
     2,6X,F12.1,5X  ,16H POUNDS/SQ. INCH,/20H     DIE DIAMETER IS, 14X,
     3F7.4,11X,F7.4,12H      INCHES,//)
      GO TO 93
   94 WRITE(6,106)
  106 FORMAT(//72H      THE DRAFTING PRACTICE FOR THIS ROD SIZE HAS ALREA
     1DY BEEN DETERMINED)
   93 IEXIT=0
C   CHOICE IS CALLED TO SELECT ANOTHER ROD
      CALL CHOICE(P-CNT,J,I,PHI)
      IF(ISTOP.EQ.3D RETURN
      GO TO 1
      END
```

```
      SUBROUTINE CHOICE (PRCNT,J,I,PHI)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
C    THIS SUBROUTINE SELCTS RODS FOR SIMULATION IN A LOGICAL MANNER SO
C    THAT AS FEW NON-FEASIBLE ATTEMPTS AS POSSIBLE WILL BE MADE
C    THIS PART OF THE PROGRAM DEALS WITH CASES WHERE PRCNT IS LESS THAN 98
C    PHI IS THE PERCENTAGE WITHIN THE UPPER BOUND
C    PRCNT IS THE PERCENTAGE WITHIN THE LOWER BOUND
      IF(PRCNT.GE.98.)GO TO 17
      ILO(J)=I+1
C    THIS LOGIC PREVENTS WASTEFUL CALCULATIONS  BY EXAMINING THE LIMITS
      IF(ILO(2).LT.+LO(3)) ILO(2)=ILO(3)
      IF(ILO(1).LT.+LO(2)) ILO(1)=ILO(2)
      IF(IHI(2).GT.+HI(1)) IHI(2)=IHI(1)
      IF(IHI(3).GT.+HI(2)) IHI(3)=IHI(2)
      IF(J-2)14,15,16
   14 J=2
      CALL AVAIL(I,J,KO)
C    KO IS AN INDICATOR SET BY AVAIL, IF KO = 0 SELECTION IS ACCEPTABLE
C    ISTOP IS AN INDICATOR SET = 3 WHEN ALL POSIBILITIES HAVE BEEN TRIED
       IF(KO.EQ.0) -ETURN
   15 J=3
      CALL AVAIL(I,J,KO)
       IF(KO.EQ.0) -ETURN
   16 IF(I.EQ.IMAX) GO TO 26
      J=1
      I=I+1
      CALL AVAIL (I,J,KO)
       IF(KO.EQ.0) -ETURN
      GO TO 14
C    THIS PART OF THE PROGRAM DEALS WITH CASES WHERE PHI LESS THAN 98
   17 IF(PHI.GE.98.) GO TO 21
      IHI(J)=I-1
C    THIS LOGIC PREVENTS WASTEFUL CALCULATIONS
      IF(ILO(2).LT.+LO(3)) ILO(2)=ILO(3)
      IF(ILO(1).LT.+LO(2)) ILO(1)=ILO(2)
      IF(IHI(2).GT.+HI(1)) IHI(2)=IHI(1)
      IF(IHI(3).GT.+HI(2)) IHI(3)=IHI(2)
 1717 IF (J-2) 18,1 ,20
   18 IF(I.EQ.IMIN) GO TO 25
      I=I-1
      J=3
      CALL AVAIL(I,J,KO)
       IF(KO.EQ.0) -ETURN
      GO TO 20
   19 J=1
      CALL AVAIL(I,J,KO)
       IF(KO.EQ.0) -ETURN
      GO TO18
```

```
   20 J=2
      CALL AVAIL(I,J,KO)
       IF(KO.EQ.0) -ETURN
      GO TO 19
C  THIS LOGIC PREVENTS WASTEFUL CALCULATIONS
   21 IF(ILO(2).LT.+LO(3)) ILO(2)=ILO(3)
      IF(ILO(1).LT.+LO(2)) ILO(1)=ILO(2)
      IF(IHI(2).GT.+HI(1)) IHI(2)=IHI(1)
      IF(IHI(3).GT.+HI(2)) IHI(3)=IHI(2)
C  THIS SECTION OF THE PRAGRAM NOW SELECTS A ROD FROM THE AREA OF INTEREST
      IF(J-2) 22,23,24
   22 J=2
      CALL AVAIL(I,J,KO)
       IF(KO.EQ.0) -ETURN
   23 J=3
      CALL AVAIL(I,J,KO)
      IF(KO.EQ.0) RETURN
   24 IF(I.GE.IMAX) GO TO  26
       I=I+1
      J=1
      CALL AVAIL (I,J,KO)
      IF(KO.EQ.0) RETURN
      GO TO 22
C  CHECK THAT ALL POSIBILITIES HAVE BEEN TRIED
   25 ISTOP=2
      GO TO 21
   26 IF(ISTOP.EQ.2) GO TO 27
      GO TO 1717
   27 ISTOP=3
      RETURN
      END
```

```
      SUBROUTINE POLY( X ,GAIN,I)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      COMMON/D/ S(23),R(23),DELS,DELR
C   THIS SUBROUTINE +S USED TO CALCULATE THE GAIN IN TENSILE STRENGTH FOR
C   A PARTICULAR REDUCTION IN AREA
C   TEST TO SEE IF DESIRED REDUCTION IS IN FACT FEASIBLE
      INFEAS=0
      IF (X.LT..20) GO TO 1
      IF(X.GT..90) GO TO 2
C   FINDING THE LINEAR INTERPOLANT OF X
      DO 5 K=2,23
      KM=K-1
      IF(X.GT.R(K)) GO TO 5
      DELR=R(K)-R(KM)
      DELS=S(K)-S(KM)
      GAIN=S(K)-DELS*(R(K)-X)/DELR
      GO TO 4
    5 CONTINUE
    1 INFEAS=1
      WRITE(6,109) -NOM(I)
  109 FORMAT(//1H ,F6.3,73H  NOMINAL DIAMETER ROD IS NOT ACCEPTABLE AS R
     1EDUCTION IS BELOW 20 PERCENT//)
      IMIN=I+1
      GO TO 4
    2 INFEAS=2
      WRITE(6,110) -NOM(I)
  110 FORMAT(//1H ,F6.3,72H  NOMINAL DIAMETER ROD IS NOT ACCEPTABLE AS R
     1EDUCTION EXCEEDS 90 PERCENT//)
      IMAX=I-1
    4 RETURN
      END


      FUNCTION FACTOR(H,RDTMU)
C   THIS SUBPROGRAM +S USED TO  CALCULATE AN ADJUSTMENT FACTOR FOR THE
C  STRENGTH GAIN OF THE WIRE
C   THE ADJUSTMENT IS DEPENDANT ON THE NUMBER OF HOLES
C   THE ADJUSTMENT IS A 1.5 PERCENT INCREASE IN STRENGTH GAIN FOR EVERY
C   HOLE LESS THAN THE NUMBER  REQUIRED  FOR A 25 PERCENT DRAFTING AVERAGE
C   AND 1.5 PERCENT DECREASE  FOR EVERY HOLE IN EXESS OF THE ABOVE
C   H25= NUMBER OF HOLES TO ACHIEVE REDUCTION WITH 25 PERCENT AVERAGE
      H25=ALOG(1.-RDTMU)/ALOG(.75)
C   H IS THE NUMBER OF HOLES AS OPTIMISED BY SUB DRAFT
      FACTOR=1.+(H25-H)*.015
      RETURN
      END
```

```
      SUBROUTINE RINVS (RDT,X)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
C   THIS SUBROUTINE +S ONE OF THE INVERSE FUNCTION OF THAT IN SUB POLY
      COMMON/D/ S(23),R(23),DELS,DELR
C   TEST IF THE DESI-ED REDUCTION IS FEASIBLE
      IF(X.LT.20000C) GO TO 1
      IF(X.GT.165000.) GO TO 2
C   FINDING THE LINEAR INTERPOLANT OF X
      DO 5 K=2,23
      KM=K-1
      IF(X.GT.S(K)) GO TO 5
      DELR=R(K)-R(KM)
      DELS=S(K)-S(KM)
      RDT=R(K)-DELR*(S(K)-X)/DELS
      GO TO 4
    5 CONTINUE
    1 INFEAS=0
      GO TO 3
    2 INFEAS=1
    3 WRITE(6,101) X
  101 FORMAT(16H      PICKUP OF   ,F12.1,18H  IS NOT FEASIBLE //)
    4 RETURN
      END



      SUBROUTINE EXTRM (A,B,I,J,FF)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
C   THIS SUBROUTINE DETERMINES THE UPPER AND LOWER LIMITS OF WIRE STRENGTH
C   FIRSTLY FINDING THE MINIMUM VALUE OF WIRE STRENGTH,A
C   THIS OCCURS FOR MINIMUM WIRE DIAMETER AND MINIMUM ROD DIAMETER AND
C   MINIMUM ROD STRENGTH,I.E. -3 SIGMA LIMITS OF ALL THE VARIABLES
      W=WMU-3.*WSIG
      ROD=SIZMU(I)-3.*SIZSIG
    1 RDT=1.-W**2/ROD**2
      CALL POLY (RDT,GAIN,I)
      GAIN=GAIN*FF
      A=(GAIN+BRKMU(I,J)-3.*BRKSIG)*W**2*3.1416/4.
C   FINDING TH E MAX+MUM VALUE OF WIRE STRENGTH ,B
C   THIS OCCURS FOR MAXIMUM VALUES OF ALL THE VARIABLES
      W=WMU+3.*WSIG
      ROD=SIZMU(I)+3.*SIZSIG
    5 RDT=1.-W**2/ROD**2
      CALL POLY (RDT,GAIN,I)
      GAIN=GAIN*FF
      B=(GAIN+BRKMU(I,J)+3.*BRKSIG)*W**2*3.1416/4.
      RETURN
      END
```

```
      SUBROUTINE DRAFT (RDTMU,H,I,J,FF,NN,R1,D1,S1)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      COMMON /B/NP,STEP,R(15),DIAM(15),RDTM ,S(15),FFF ,IEX,WATE
      COMMON/E/ZS(15),ZR(15),ZD(15)
C   THE FUNCTION OF THIS SUBROUTINE IS TO DETERMINE THE NUMBER OF HOLES
C   AND IDEAL PICKUP OF EACH HOLE
C   S  IS STRENGTH GAIN AND R IS REDUCTION OF EACH HOLE
C    X IS THE FRACTIONAL REDUCTION OF STRENGTH GAIN FOR CONSECUTIVE HOLES
C   RT(I) IS THE TOTAL REDUCTION UP TO THE ITH HOLE
C   Y IS FRACTIONAL -EDUCTION OF STRENGTH PICKUP FOR THE SECOND HOLE
      DIMENSION FIRST(7),RT(15),C(15),ARRAY(3)
      IND=INFEAS
      RDTM=RDTMU
      ISTRT=1
C   ARRAY STORES CONSTANTS USED IN THE DETERMINATION OF X AND Y AND IS
C   DEPENDANT ON THE ROD CHEMISTRY
      ARRAY(2)=1.0
      ARRAY(1)=1.03
      ARRAY(3)=.97
C   X AND Y ARE CALCULATED EMPIRICALLY  AND ARE THE TAPER FACTORS
      X=SQRT(RDTMU)*ARRAY(J)*.85
      Y=(RDTMU**.25*ARRAY(J)*.885) +1.
      EX=X
      ISTOP=0
C   IND IS AN INDICATOR USED TO IDENTIFY 3 HOLE PRACTICES THAT ARE TOO
C   HEAVEY AND HENCE ARE INFEASIBLE
C    IND = 4 INDICATES THE PREVIOUS ATTEMPT WITH 3 HOLES WAS INFEASIBLE
      IF(IND.EQ.4) +STRT=2
C   ZEROING ARRAYS IF IND = 4 THE FIRST HOLE IS NOT ZEROED
      DO 1 K=ISTRT,15
      S(K)=0.
    1 R(K)=0.
C   FINDING THE OVERALL STRENGTH GAIN CORRESPONDING TO RDTMU
      CALL POLY(RDTMU,GAINMU,I)
C   ASSIGNING THE EMPIRICAL VALUE FOR FIRST REDUCTION WHERE THE SUBSCRIPT
C   INDICATES ROD SIOE
      DATA FIRST(1),FIRST(2),FIRST(3),FIRST(4),FIRST(5),FIRST(6),
     1FIRST(7)/.3103,.3045,.2955,.2878,.2846,.2810,.2768/
C   IF A 3 HOLE PRACTICE IS INFEASIBLE THE REDUCTION OF THE FIRST HOLE IS
C   REDUCED
      IF(IND.EQ.4) -(1)=R(1)*.9
      IF(IND.EQ.4) GO TO 11
      R(1)=FIRST(I)
   11 RT(1)=R(1)
      IF(RDTMU.GT.R(1)) GO TO 2
C   CASE OF ONE HOLE ONLY
      R(1)=RDTMU
C   NN IS THE TOTAL NUMBER OF HOLES IN THE PRACTICE
      NN=1
      GO TO 20
```

```
C    FINDING STRENGTH GAIN CORRESPONDING TO FIRST REDUCTION
   2 IF(IEXIT.EQ.7) GO TO 5
     CALL POLY(RT(1),S(1),I)
     S(2)=Y*S(1)
     IF(S(2).LT.GA+NMU) GO TO 4
C  ONLY 2 HOLES REQUIRED
     S(2)=GAINMU
C  FINDING THE NECESSARY OVERALL REDUCTION
     CALL RINVS(RT(2),S(2))
C  THE FUNCTION 'ARE' IS USED TO FIND THE DRAFT OF THE CURRENT HOLE
   3 CALL ARE (RT(1),RT(2),R(2))
     NN=2
     IF(ABS(R(1)-R( 2)).LE..03) GO TO 20
     IF(R(1)-R(2).GT..03) R(1)=R(1)-.005
     IF(R(1)-R(2).LT.-.03) R(1)=R(1)+.005
     RT(1)=R(1)
     CALL POLY(RT(1),S(1),I)
     GO TO 3
C  IN THE CASE OF SEVEN HOLES THE SECOND DRAFT IS SET AT .30
   5 R(2)=.30
     RT(2)=1.-(1.--(1))*(1.-R(2))
     CALL POLY(RT(2),S(2),I)
     GO TO 6
   4 CALL RINVS(RT(2),S(2))
     CALL ARE (RT(1),RT(2),R(2))
C  3 OR MORE HOLES -EQUIRED
   6 S(3)=S(1)+(S(2)-S(1))*2.
     IF(S(3).LT.GA+NMU) GO TO 7
C  3 HOLES ONLY NEEDED
     S(3)=GAINMU
     NN=3
     CALL RINVS(RT(3),S(3))
     CALL ARE (RT(2),RT(3),R(3))
     GO TO 20
C  4 OR MORE HOLES NEEDED
   7 DO 8 K=1,12
     K2=K+1
     K3=K+2
     K4=K+3
     S(K3)=X*(S(K2)-S(K))+S(K2)
     S(K4)=X*(S(K2)-S(K))+S(K3)
     CALL RINVS(RT(K3),S(K3))
 777 CALL ARE(RT(K2),RT(K3),R(K3))
     IF(ISTOP.EQ.1.AND.K4.EQ.NN) GOTO 20
     IF(K4.EQ.7.AND.IEXIT.EQ.7) GO TO 9
     IF(S(K4).GT.GAINMU) GO TO 9
   8 CONTINUE
   9 NN=K4
     EPS1=S(K4)-GA+NMU
C  NOW FINDING A VALUE OF X SUCH THAT EXACTLY I4 HOLES ARE NEEDED
C  INITIALLY ASSIGN+NG VALUES TO THE COEFFICIENTS
     INDEX=NN-3
```

```
      IF(INDEX.EQ.1) GO TO 111
      DO 10 M=1,INDEX
   10 C(M)=1.
  111 DO 12 M=INDEX,15
   12 C(M)=0.
      XP=X
      X=.70
      IF(XP.EQ..70) X=.71
C  CALCULATING A TR+AL VALUE OF GAINMU-S(2) FOR A NEW X VALUE
C  THESE COEFFICIENTS ENSURE THAT THE CORRECT NUMBER OF HOLES ARE
C  CONSIDERED AND THE LAST HOLE WILL HAVE THE SAME STRENGTH GAIN AS THE
C  SECOND LAST OWING TO THE COEFFICIENT 1/X
   13 C(INDEX)=1./X
      TRY=(S(2)-S(1))*X*(1.+C(1)*X+C(2)*X**2+C(3)*X**3+C(4)*X**4+C(5)*X*
     1*5+C(6)*X**6+C(7)*X**7+C(8)*X**8+C(9)*X**9+C(10)*X**10+C(11)*X**11
     2+C(12)*X**12)
      EPS2=TRY+S(2)-GAINMU
C   NOW ITTERATING
      IF(ABS( EPS2).LE.200.) GO TO 14
      TEMP=X
      X=X-(X-XP)*EPS2/(EPS2-EPS1)
      EPS1=EPS2
      XP=TEMP
      GO TO 13
   14 ISTOP=1
      GO TO 7
C  OUTPUT
   20 H=NN
C  NO MORE THAN 12 HOLES ARE PERMISSABLE, IF THIS OCCURS THEN THE
C  VALUE OF X AND Y ARE INCREASED
      IF(NN.GT.12) GO TO 29
      IF(NN.EQ.7.AND.IEXIT.NE.7) GO TO 24
C  THE ADJUSTMENT FACTOR FOR NUMBER OF HOLES MUST BE APPLIED TO GAIN
      FF=FACTOR(H,RDTMU)
      FFF=FF
      JJ=NN-1
      RT(NN)=RDTMU
      S(NN)=GAINMU
      CALL ARE (RT(JJ),RT(NN),R(NN))
      IF(R(NN).GT..12) GO TO 28
C  IF THE IDEAL REDUCTION ON THE LAST HOLE IS LESS THAN .12 THEN X IS
C  INCREASED
   29 X=EX+.01
      Y=X+1.
      EX=X
      IF(X.GE.1.01) GO TO 28
      GO TO 11
C  DIE DIAMETERS,PICKUP AND REDUCTION ARE CALCULATED FOR EACH DRAFT
   28 DO 23 M=1,NN
      MM=M-1
      S(M)=S(M)*FF
      IF(M.NE.1) GO TO 21
      R(1)=RT(1)
```

```
      DIAM(1)=SQRT(U1.-R(1))*SIZMU(I)**2)
      SL=S(1)
      ZD(1)=DIAM(1)
      ZS(1)=S(1)
      ZR(1)=R(1)
      GO TO 22
   21 TEMP=S(M)
      S(M)=S(M)-SL
      ZS(M)=S(M)
      ZR(M)=R(M)
      SL=TEMP
      DIAM(M)=SQRT((1.-R(M))*DIAM(MM)**2)
      ZD(M)=DIAM(M)
   22 CONTINUE
   23 CONTINUE
      GO TO 25
C   IN THE CASE OF 7 HOLES CERTAIN INDICATORS ARE SET
   24 IEXIT=7
      ISTOP=0
      GO TO 2
C   IN THE CASE OF 7 HOLES THE LAST 6 ARE TREATED AS THOUGH A 6 HOLE
C   DRAFT
   25 IF(IEXIT.NE.7) GO TO 27
      NN=NN-1
      R1=R(1)
      D1=DIAM(1)
      S1=S(1)
      DO 26 M=1,NN
      MP=M+1
      R(M)=R(MP)
      ZR(M)=R(M)
      DIAM(M)=DIAM(MP)
      ZD(M)=DIAM(M)
      ZS(M)=S(MP)
   26 S(M)=S(MP)
      S(1)=S(1)+S1
      R(1)=R1+R(1)*(1.-R1)
   27 IND=0
       RETURN
      END


      SUBROUTINE ARE (X,Y,R)
C   THIS SUBPROGRAM +S USED TOCALCULATE THE HOLE REDUCTION KNOWING THE
C   TOTAL REDUCTION BEFORE AND AFTER THE HOLE IN QUESTION
       R =(Y-X)/(1.-X)
      RETURN
      END
```

```
      FUNCTION FUNC (Z)
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIG(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      COMMON/C/SHAPE,SCALE,W,I,J,FF,RDTSIG,RDTM
      COMMON/D/ S(23),R(23),DELS,DELR
C   THIS FUNCTION IS USED TO CALCULATE THE VALUE OF THE INTEGRAND FOR
C   SUB SIMPSN   USING THE METHOD OF TRANSFORMATION OF VARIABLES
      WM=W-Z
C   THE ADJUSTMENT IS NECESSARY AS THE SUBROUTINE FOR REDUCTION IS TO BE
C   USED
      X=Z/FF
C   THE REDUCTION CO-RESPONDING TO THIS GAIN IS NOW NEEDED
      CALL RINVS(RDT,X)
C   THIS   SECTION  +S USED TO FIND THE DERIVATIVE OF REDUCTION WITH
C   RESPECT TO GAIN
      DRDS=DELR/(DELS*FF)
C   FINDING THE PROBABILITY DENSITY FOR THE PARTICULAR VALUE OF RDT
      CALL PROB(I,RDT,FREQ,0)
C   NOW CALCULATING PROBABILITY DENSITY FOR STRENGTH GAIN
      P=ABS(DRDS)*F-EQ
      FUNC=P*(1./(B-KSIG*SQRT(2.*3.1416)))*EXP(-1.*(WM-BRKMU(I,J))*(WM-B
     1RKMU(I,J))/(2.*BRKSIG**2))
      RETURN
      END


      SUBROUTINE CUMUL (P,H,N,CUM)
C   THE FUNCTION OF THIS SUBPROGRAM IS TO FIND THE CUMULATIVE DENSITIES
C   AT VARIOUS POINTS FROM THE PROBABILITY DENSITIES AT DOUBLE THIS
C   NUMBER OF POINTS
C   INTEGRATION IS CARRIED OUT USING SIMPSON'S RULE
C   P IS THE PROBABILITY DENSITY
C   CUM IS THE CUMULATIVE DENSITY
C   H IS THE STEP LENGTH
C   N IS THE TOTAL NUMBER OF POINTS, MUST BE AN ODD INTEGER
      DIMENSION P(1),CUM(1)
      KOUNT=0
      HOLD=0.
      DO 1 K=1,N,2
      K1=K+1
      K2=K+2
C   APPLYING SIMPSON'S RULE
      G=H/3.*(P(K)+4.*P(K1)+P(K2))
      KOUNT=KOUNT+1
      CUM(KOUNT)=HOLD+G
      HOLD=CUM(KOUNT)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SUB (W,DELTA,CUM,AREA,STRMIN,P)
      DIMENSION CUM(1),P(1)
C   THIS SUBROUTINE +S USED TO CALCULATE ARES WITHIN THE UPPER AND LOWER
C   BOUNDS OF STRESS  BY LINEAR INTERPOLATION
C   CUM IS THE CUMULATIVE PROBABILITY AT VARIOUS STATIONS
C   P IS THE PROBABILITY DENSITY AT THESE STATIONS
C   DELTA IS THE STEP LENGTH BETWEEN STATIONS
      DEC=(W     -STRMIN)/DELTA+1.
      INT=IFIX(DEC)
      IPLUS=INT+1
      REEL= FLOAT(INT)
      RPLUS=REEL+1.
      IND=IFIX((REEL+1.)/2.+.6)
      DIFF=(DEC-FLOAT(IND)*2.+1.)
C   NOW CALCULATING THE AREA
      IF(DIFF.LT.0.) GO TO 27
      AREA= CUM(IND)+(2.*P(INT)+(P(IPLUS)-P(INT))*DIFF)*DIFF*DELTA/2.
      GO TO 28
   27 AREA=CUM(IND)+(P(INT)+(P(IPLUS)-P(INT))*(1.+DIFF)+P(IPLUS))*DIFF*
     1DELTA/2.
   28 RETURN
      END


      SUBROUTINE SMPSN (A,B,FUNC,SUM)
C   THE FUNCTION OF THIS SUBPROGRAM IS TO INTEGRATE ANY FUNCTION,FUNC,
C   BETWEEN THE LIMITS A AND B USING SIMPSONS RULE
C   THE INITIAL STEP LENGTH H IS SET
      H=(B-A)/20.
C   KOUNT RECORDS THE NUMBER OF TIMES THE STEP IS HALVED
      KOUNT=0
    1 SUM=0.
      AA=A
C   APPLYING SIMPSON'S RULE
    2 G=H/3.*(FUNC(AA)+4.*FUNC(AA+H)+FUNC(AA+2.*H))
      AA=AA+2.*H
      SUM=SUM+G
      DIFF=B-AA
      TRY=(B-A)/200000.
      IF(DIFF.GT.TRY) GO TO 2
      KOUNT=KOUNT+1
      IF(KOUNT.EQ.1) GO TO 3
      DIFF=ABS(SUM--ECORD)
C   TWO PERCENT ERRO- IS ACCEPTIBLE
      TEST=ABS(.02*SUM)
      IF( KOUNT.GE.20) GO TO 4
      IF(DIFF.LT.TEST.OR.DIFF.LT..0001) RETURN
    3 RECORD=SUM
      H=H/2.
      GO TO 1
    4 WRITE(6,100)
  100 FORMAT(52H    INTEGRATION FAILED TO CONVERGE IN 20 ITTERATIONS)
      RETURN
      END
```

```
      SUBROUTINE SEARCH (NN,I,INFEAS,DIV,DIV2)
      COMMON /B/NP,STEP,RDT(15),DIAM(15),RDTMU,S(15),FF   ,IEX,WATE
      DIMENSION VS(15),STORE(15),A(15),V(15),COMP(15)
C  A DIRECT SEARCH TECHNIQUE  IS USED TO OPTIMISE THE VARIABLES V
C T IS A TEMPORARY VALUE OF V
C   WATE IS A WEIGHTING FACTOR APPLIED TO THE CONSTRAINTS INITIALLY=.8
C  ARRAYS VS AND BASE ARE SEARCH AND BASE POINTS RESPECTIVELY
C ASSIGN A STARTING VALUE TO THE VARIABLE V
C  WATE IS THE WEIGHTING FACTOR INITIALLY = .8 THEN CHANGED TO 1.2
C   NN NOW IS THE TOTAL NUMBER OF VARIABLES ,I.E. NUMBER OF HOLES -2
      WATE=.8
      NP=NN+1
      DO 12 M=1,NP
      MP=M+1
   12 V(M)=DIAM(MP)
      ISET=0
C  THIS SECTION CALLS THE DIRECT SEARCH INTO ACTION TO OPTIMISE THE
C  PROBLEM INITIALLY
      CALL        TR+AL( NN,I,INFEAS,VS,IWIN,USTORE,A,ISET,V,DIV,DIV2)
      NPP=NP+1
      VS(NP)=DIAM(NPP)
      DO1 M=1,NP
      V(M)=VS(M)
      COMP(M)=VS(M)
    1 STORE(M)=VS(M)
    4 IMPRV=0
C THIS SECTION IMPOSES A CHANGE OF +.00025 ON THE VALUE OF EACH VARIABLE
C  IN TURN, HOLDS THE VARIABLE TEMPORARILY CONSTANT AND CHECKS FOR
C  IMPROVEMENT IN THE OBJECTIVE FUNCTION
      DO20 L=1,NN
      LP=L+1
      HOLD=STORE(L)
C  IF THE SOLUTION +S FEASIBLE NO ATTEMPT IS MADE THAT WILL TAKE THE
C  SOLUTION ANY FURTHER FROM THE UNCONSTRAINED OPTIMUM
      IF(INFEAS.EQ.3.AND.STORE(L).GT.DIAM(LP)) GO TO 3
      V(L)=STORE(L)+.00025
      ISET=L
      VS(L)=V(L)
      CALL        TR+AL( NN,I,INFEAS,VS,IWIN,UOLD  ,A,ISET,V,DIV,DIV2)
      IF(UOLD.GT.USTORE) GO TO 3
      USTORE=UOLD
      IMPRV=1
C THIS SECTION IMPOSES A CHANGE OF -.00025 ON THE VALUE OF EACH VARIABLE
C  IN TURN, HOLDS THE VARIABLE TEMPORARILY CONSTANT AND CHECKS FOR
C  IMPROVEMENT IN THE OBJECTIVE FUNCTION
      DO 33 M=1,NN
      V(M)=VS(M)
   33 STORE(M)=VS(M)
    3 CONTINUE
      IF(INFEAS.EQ.3.AND.STORE(L).LT.DIAM(LP)) GO TO 20
      DO 8 M=1,NN
```

```
    8 V(M)=STORE(M)
      V(L)=HOLD-.00025
      VS(L)=V(L)
      CALL        TR+AL( NN,I,INFEAS,VS,IWIN,UOLD  ,A,ISET,V,DIV,DIV2)
      IF(UOLD.GT.USTORE) GO TO 2
      USTORE=UOLD
      IMPRV=1
      DO 22 M=1,NN
      V(M)=VS(M)
   22 STORE(M)=VS(M)
    2 CONTINUE
      DO 9 M=1,NN
    9 V(M)=STORE(M)
   20 CONTINUE
C  IF NO MORE IMPROVEMENT IS FOUND AT THIS STAGE THE SEARCH IS STOPPED
      IF (IMPRV.EQ.0) GO TO 6
C  IF IMPROVEMENT IS FOUND A PATTERN MOVE IS EXECUTED TO SPEED THE SEARCH
   11 DO 10 M=1,NN
   10 V(M)=STORE(M)-COMP(M)+STORE(M)
      CALL TEST(UOLD,V,NN,I,A,DIV,DIV2)
      IF(UOLD.GT.USTORE) GO TO 15
       USTORE=UOLD
      DO 16 M=1,NN
      COMP(M)=STORE(M)
   16 STORE(M)=V(M)
      GO TO 4
C  IF NO ONE DIE SIOE IS CHANGED BY MORE THAN .00025 THE SOLUTION IS
C  CONSIDERED OPTIMAL
   15 DO 19 M=1,NN
      V(M)=STORE(M)
   19 IF(ABS(COMP(M)-STORE(M)).GT..00026) GO TO 4
    6 CALL TEST (UF+NAL,STORE,NN,I,A,DIV,DIV2)
      DO 13 M=1,NP
      MP=M+1
   13 S(MP)=A(M)
      DO 7 M=2,NP
      MM=M-1
      DIAM(M)=STORE(MM)
    7 CONTINUE
       DO 5 M=2,NPP
      MM=M-1
    5 RDT(M)=1.-DIAM(M)**2/DIAM(MM)**2
C  VARIABLES ARE NOW OPTIMAL OR SEARCH HAS FAILED TO CONVERGE
      IF(UFINAL.LT.1.E+09) GO TO 14
      IF(NN.EQ.1) INFEAS=4
      IF (INFEAS.NE.4) WRITE(6,101)
  101 FORMAT(/40H      SOLUTION IS INFEASIBLE..............,/)
   14 RETURN
      END
```

```
      SUBROUTINE TR+AL( NN,I,INFEAS,VS,IWIN,UOLD  ,A,ISET,V,DIV,DIV2)
      COMMON /B/NP,STEP,RDT(15),DIAM(15),RDTMU,S(15),FF  ,IEX,WATE
      DIMENSION T(15),V(1 ),BASE(15),VS(1 ),A(1 )
C   INITIALISE THE STEP LENGTH AND COUNTER
      K10=8+2*NN
      KOUNT=0
   14 STEP=.002
C AFTER THE INITIAL SOLUTION HAS BEEN FOUND A REDUCED SEARCH IS
C   PERFORMED, THAT +S A SMALLER STEP RANGE IS USED AND FEWER ITTERATIONS
      IF(ISET.GE.1) STEP=.000125
      T(NP)=V(NP)
C    INITIALISE THE ARTIFICIAL OBJECTIVE FUNCTION
   66 CALL TEST (UOLD,V,NN,I,A,DIV,DIV2)
C INTIALISING THE NECESSARY ARRAYS
   44 DO 6 L=1,NN
      T(L)=V(L)
    6 BASE(L)=V(L)
   11 IWIN=0
C   CHANGING THE VALUES OF ALL THE VARIABLES IN TURN AND TESTING FOR
C   DECREASE IN THE VALUE OF THE ARTIFICIAL OBJECTIVE FUNCTION
    4 KOUNT=KOUNT+1
C   A TERMINATION CR+TERION OF THE REDUCED SEARCH
      IF( KOUNT.GE.K10.AND.ISET.GE.1) GO TO 12
      IF(KOUNT.GE.1000) GO TO 10
      DO 22 L=1,NN
      IF(ISET .EQ.L) GO TO 22
      T(L)=V(L)+STEP
      CALL TEST (UA-T,T,NN,I,A,DIV,DIV2)
      IF(UART.GT.UOLD) GO TO 2
      UOLD=UART
      IWIN=1
      VS(L)=T(L)
      GO TO 22
    2 VS(L)=V(L)
      T(L)=VS(L)
   22 CONTINUE
C   REPEAT FOR A DEC-EASE IN THE VARIABLE VALUE
      DO 1 L=1,NN
      IF(ISET .EQ.L) GO TO 1
      T(L)=V(L)-STEP
      CALL TEST (UA-T,T,NN,I,A,DIV,DIV2)
      IF(UART.GT.UOLD) GO TO 111
      UOLD=UART
      IWIN=1
      VS(L)=T(L)
      GO TO 1
  111 T(L)=VS(L)
    1 CONTINUE
C   IF THE ATTEMPT WAS NOT SUCCESSFUL DECREASE THE STEP SIZE
      IF(IWIN.EQ.1) GO TO 3
      STEP=STEP/2.
```

```
      IF(STEP.LE..000001.AND.WATE.EQ.0.8) GO TO 13
      IF(STEP.LE..000001) GO TO 10
      IF(ISET.GE.1.AND.STEP.LT..00001) GO TO 12
      GO TO 44
C   AFTER THE INITIAL SEARCH THE WEIGHTING FACTOR IS CHANGED AND THE
C   SEARCH REPEATED
   13 WATE=1.2
      GO TO 14
C   A SEARCH POINT IS ESTABLISHED NOW TRY REPEATING THE LAST STEP
    3 DO 5 L=1,NN
      T(L)=VS(L)-BASE(L)+VS(L)
    5 CONTINUE
      CALL TEST (UA-T,T,NN,I,A,DIV,DIV2)
      IF(UART.LE.UOLD) GO TO 8
      DO 7 L=1,NN
      V(L)=VS(L)
    7 CONTINUE
      GO TO 44
C   SUCCESSFUL ATTEMPT RECORDED
    8 DO 9 L=1,NN
      VS(L)=T(L)
      V(L)=T(L)
    9 BASE(L)=VS(L)
      UOLD=UART
      GO TO 11
   12 CONTINUE
   10 IF(UOLD.LT.1.E+09) INFEAS=3
      RETURN
      END
```

```
      SUBROUTINE CONST (R,PHI,RIN)
      DIMENSION PHI(1),R(1)
      COMMON /B/NP,STEP,RDT(15),DIAM(15),RDTMU,S(15),FF   ,IEX,WATE
C   THE FUNCTION OF THIS SUBPROGRAM IS TO CALCULATE ANY VIOLATION OF
C   THE DRAFTING CONSTRAINTS
C    ANY POSITIVE PH+ INDICATES A VIOLATED CONSTRAINT
      DO 1 L=2,NP
      LM=L-1
    1 PHI(L)=-.03+ABS(R(L)-R(LM))
      RETURN
      END
```

```
      SUBROUTINE TEST(UART,X,NN,I,A,DIV,DIV2)
      DIMENSION PHI(15),R(15),RTOT(15),SG(15),A(1),X(1)
      COMMON /B/NP,STEP,RDT(15),DIAM(15),RDTMU,S(15),FF   ,IEX,WATE
C  THIS SUBROUTINE CALCULATES AN ARTIFICIAL VALUE OF THE OBJECTIVE
C  FUNCTION
C  UART=U+PENAL
C  FIRSTLY REDUCTIONS ARE CALCULATED KNOWING DIAMETERS
C  THE REDUCTION AND DIAMETER OF THE FIRST DRAFT ARE CONSTANTS
      RIN=RDT(1)
      DIN=DIAM(1)
C  NVIOL IS AN INDICATOR TO SHOW IF ANY CONSTRAINTS HAVE BEEN VIOLATED
      NVIOL=0
      R(1)=1.-X(1)**2/DIN**2
      DO 1 M=2,NP
      MM=M-1
    1 R(M)=1.-X(M)**2/X(MM)**2
C  CALCULATING THE TOTAL REDUCTION AT EACH HOLE
      RTOT(1)=RIN+R(1)*(1.-RIN)
      DO 2 M=2,NN
      MM=M-1
    2 RTOT(M)=RTOT(MM)+R    (M)*(1.-RTOT(MM))
      RTOT(NP)=RDTMU
C  CALCULATING STRENGTH GAIN AT EACH HOLE
      DO 3 M=1,NP
      CALL POLY (RTOT(M),GAIN,I)
    3 SG(M)=GAIN*FF
      A(1)=SG(1)-S(1)
      DO 4 M=2,NP
      MM=M-1
    4 A(M)=SG(M)-SG(MM)
      CALL UREAL (U,A,X,RTOT,I,DIV,DIV2)
      CALL CONST(R,PHI,RIN)
      PENAL=0.
C  IF ANY CONSTRAINTS ARE VIOLATED A PENALTY IS APPLIED
      DO 5 M=2,NP
      IF(PHI(M).LE.0.) GO TO 5
      NVIOL=1
      IPOW=(NP+1)-M
      X1=WATE**IPOW*100.
      PENAL = PENAL+((PHI(M)*X1)**2)*1.E+08 + PHI(M)*X1*1.E+08
    5 CONTINUE
C  IF ANY CONSTRAINTS WERE VIOLATED A STEP PENATY IS USED TO KEEP THE
C  SOLUTION FEASIBLE
      IF(NVIOL.NE.1) GO TO 6
      PENAL=PENAL+1.E+09
    6 CONTINUE
C  IF THE REDUCTION OF THE LAST HOLE EXCEEDS .25 A PENALTY IS APPLIED
      TORS=R(NP)-.25
      IF(TORS.LE.0.) GO TO 9
      PENAL=PENAL+TORS*100000.
    9 CONTINUE
```

```
C   IF THE REDUCTION OF THE LAST HOLE IS LESS THAN .15 A PENALTY IS
C   APPLIED
      DO 10 M=2,NP
       DMIN=.15-R(M)
      IF(DMIN.LE.0.) GO TO 10
      PENAL=PENAL+DMIN*100000.
   10 CONTINUE
      UART=U+PENAL
      RETURN
      END
```

```
      SUBROUTINE UREAL (U,A,X,RTOT,I,DIV,DIV2 )
      COMMON SIZMU(7),BRKMU(7,3),WMU,WBREAK,RNOM(7),NASIZ(20),NACHEM(20)
     1,INFEAS,ISTOP,IEXIT,IMAX,IMIN,IDONE(21),JDONE(21),IHI(3),ILO(3)
     2,WSIG,SIZSIG,BRKSIG,IPSI,WHI
      COMMON /B/NP,STEP,RDT(15),DIAM(15),RDTMU,S(15),FF   ,IEX,WATE
      DIMENSION P(15),A(1),X(1),RTOT(1),R( 15),V(15),VAR(15)
C   THE FUNCTION OF THIS SUBPROGRAM IS TO CALCULATE THE CURRENT VALUE OF
C   THE OBJECTIVE FUNCTION
      U=0.
      DO 1 M=1,NP
      MP=M+1
C    THE OBJECTIVE FUNCTION IS DIVIDED THROUGH BY 1.E+06 SO THAT IT'S
C   EFFECT WILL NOT OVERIDE THE CONSTRAINTS
      P(M)=(A(M)-S(MP))**2/1000000.
    1 U=U+P(M)
      RETURN
      END
```

## APPENDIX IV

### Program Documentation

Program documentation provides a brief general description of each subprogram's purpose and logic and is intended to be used in conjunction with the program listing which includes explanatory comments. Some of the simpler subprograms are self explanatory however, so have not been included in this documentation.

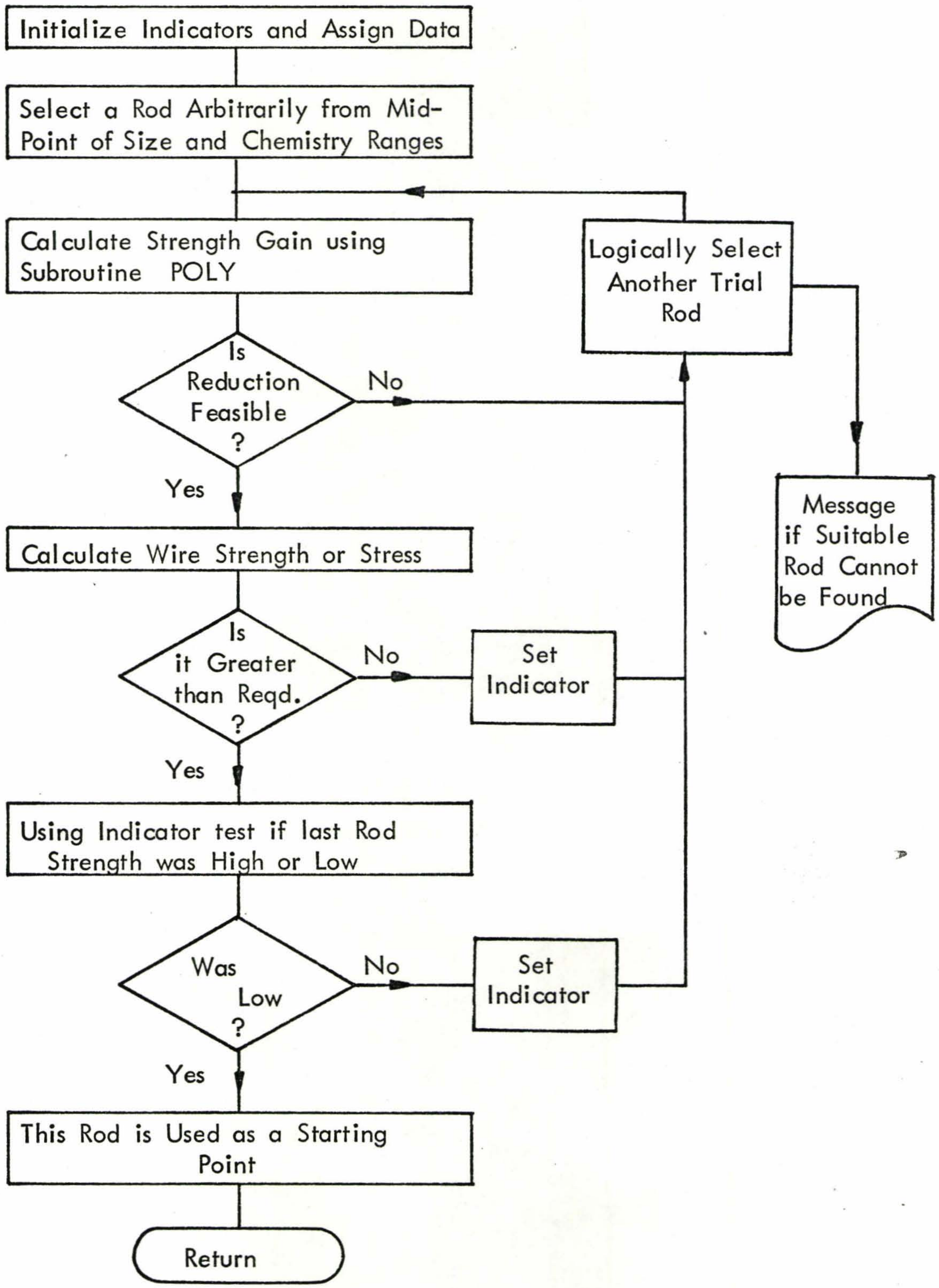
a) MAIN Program

This program contains very little logic and primarily is used to assign values to parameters and initialize indicators. Some output is produced by the program and it calls subroutines START and CARLO into action.

b) Subroutine START

This subroutine produces a deterministic solution to the problem using mean values of all random variables. This solution is then used as a starting point by the time consuming probabilistic subroutine CARLO. Data is assigned and some indicators are also initialized within the subprogram.

## Logic Flow of Subroutine Start



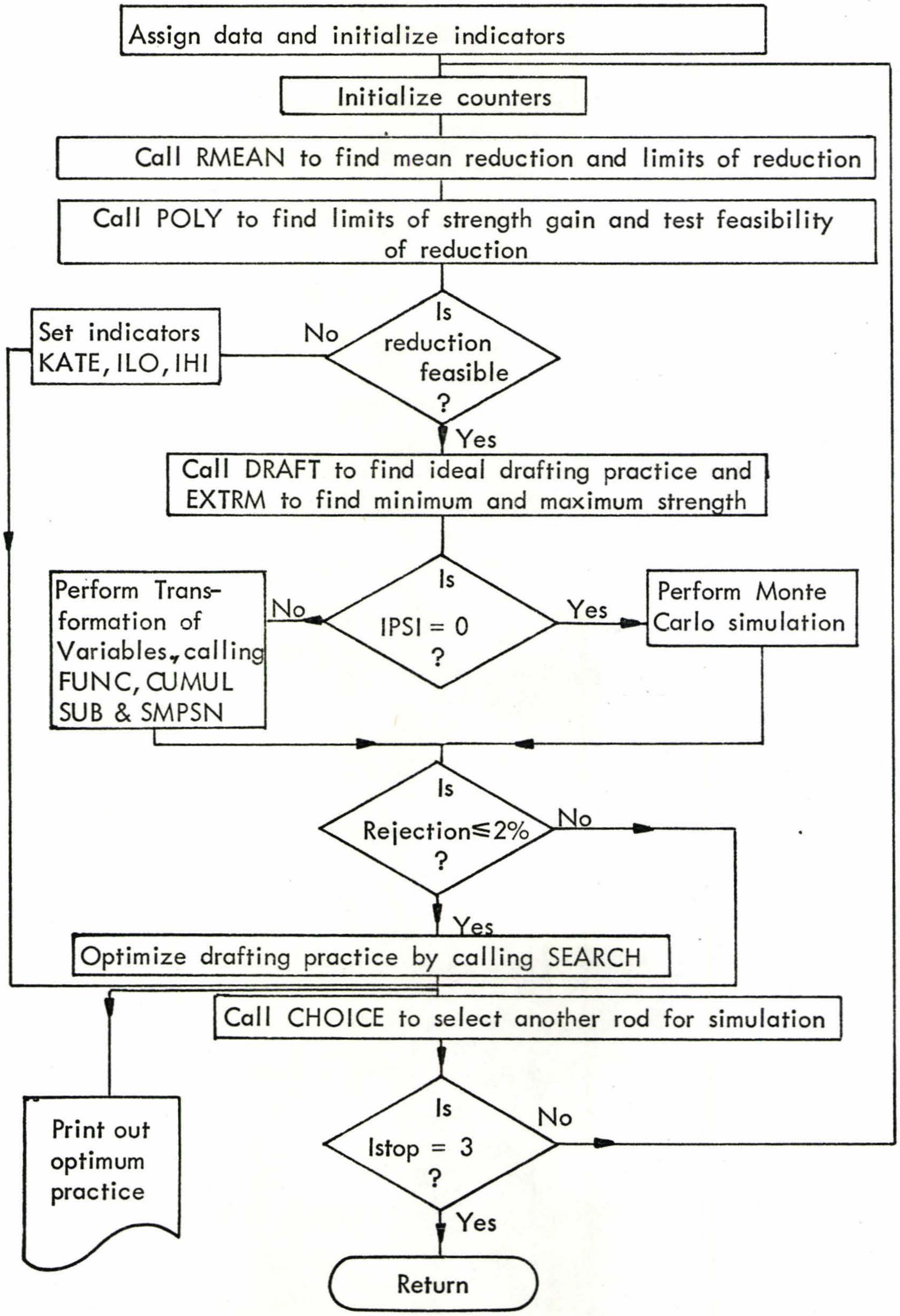
c) Subroutine CARLO

This subroutine performs the simulation and also provides most of the program control. If the simulation suggests a 98% product acceptance the optimization routine SEARCH is called from CARLO to optimize the drafting practice with respect to the ideal practice as determined by subroutine DRAFT.

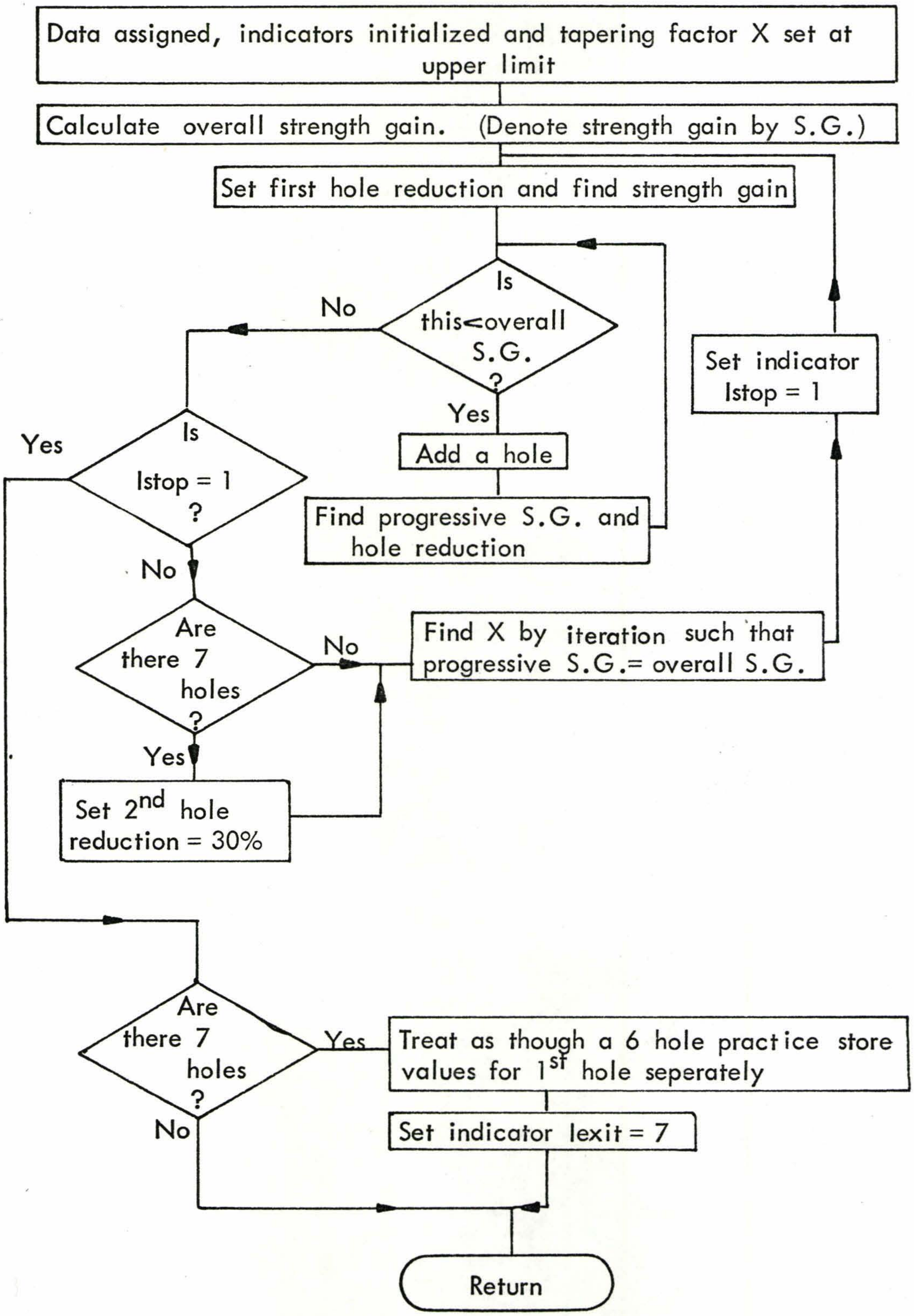
d) Subroutine DRAFT

This subprogram calculates the ideal taper drafting practice neglecting constraints The fractional reduction, the factor by which successive strength gain is reduced is initially set at the upper limit. The number of holes is established by adding on one hole at a time until the progressive strength gain and reduction exceed the overall value required. The draft is then softened, i.e. the value of the strength gain reduction factor is reduced until the progressive strength gain and reduction at the last hole exactly equal the overall values. In the case of 7 holes the second hole reduction is set at 30% and the practice is then treated as though it were a six hole practice, this facilitates handling by the optimization subroutines.

Logic Flow of Subroutine CARLO



Logic Flow of Subroutine DRAFT



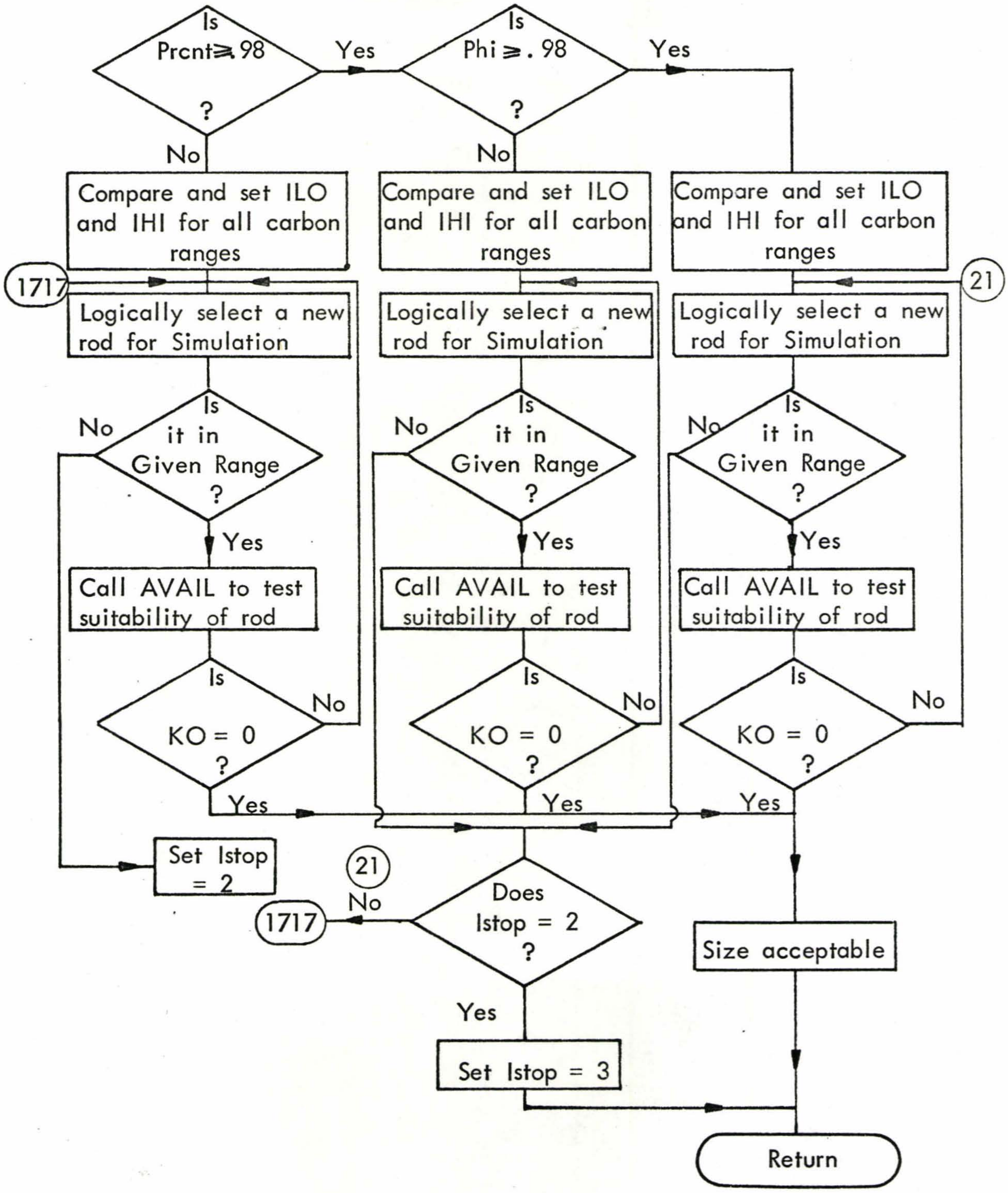
e) Subroutine CHOICE

This subroutine chooses the rods for simulation by considering previous results All possibilities with an acceptable product rejection level are chosen and by logical testing, as few unacceptable rods as possible are chosen, however some unacceptable rods must of necessity be tried in order to determine the limits in each of the carbon ranges.

f) Subroutine AVAIL

This subprogram called by CHOICE serves to check if a particular rod is not available, i.e. not in stock, if a simulation has been performed for this rod in the present computer run, and if this rod will logically be a bad choice. An indicator KO is set in the event of any of the above occurring, this then prevents selection of the rod for simulation.

Logic Flow of Subroutine CHOICE



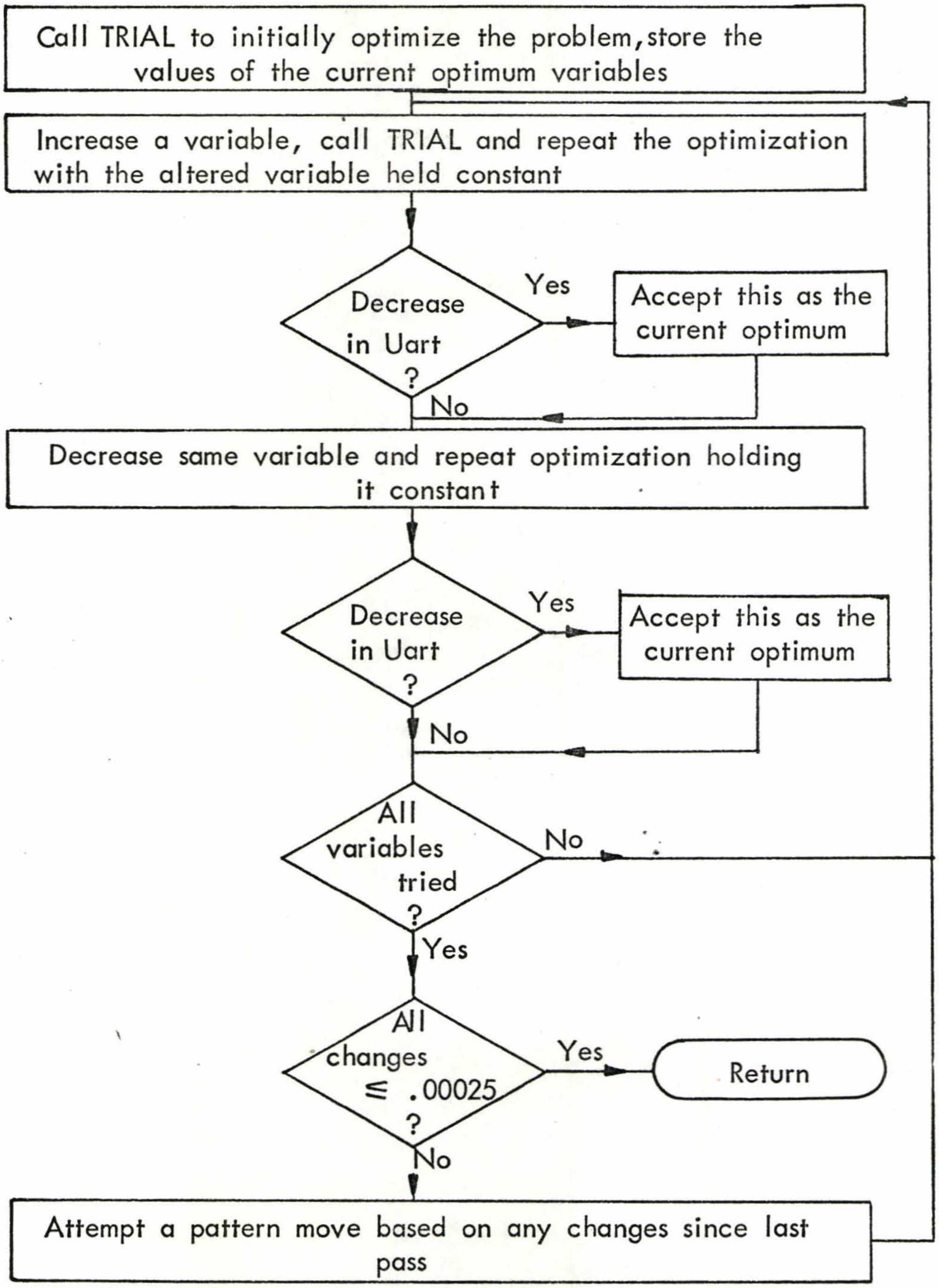
g) Subroutine SEARCH

The function of this subprogram is to control the optimization subroutine TRIAL so as to ensure the search does not 'hang up'. As soon as the initial search has been completed each of the variables in the optimization is changed by a small amount, both positive and negative, held constant and the optimization performed using the remaining variables. In this way the search may enter an infeasible region initially but then work back into the feasible region closer to the optimum. A pattern move is then made in an attempt to accelerate the process.

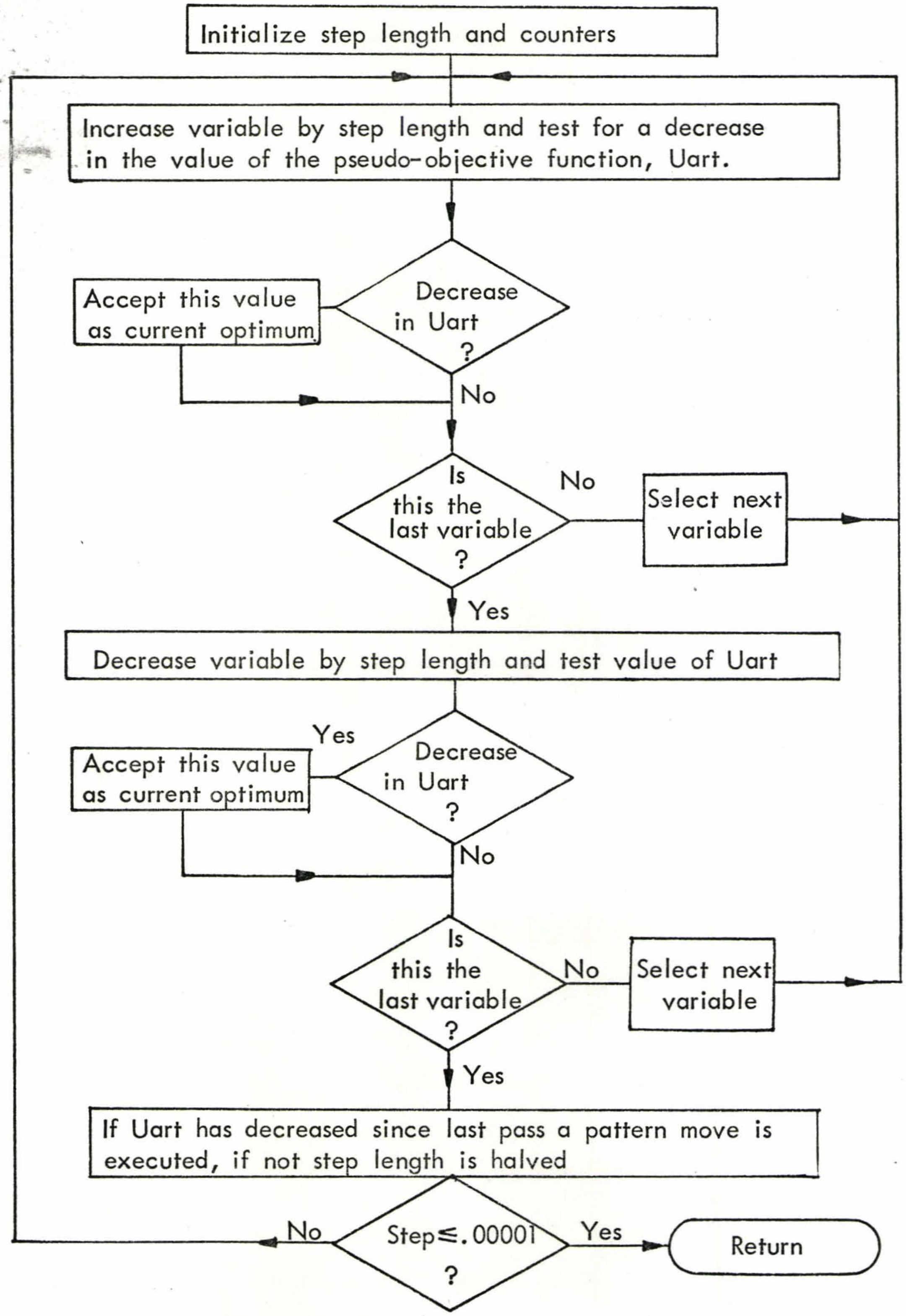
h) Subroutine TRIAL

This subroutine executes the direct search pattern, changing each variable in turn and testing for improvement in the value of the objective function. Owing to the constraint imposed a pseudo-optimization function is used in place of the actual objective function in order to direct the search to a feasible solution. The pseudo-optimization function is evaluated by subroutine TEST.

Logic Flow of Subroutine SEARCH



Logic Flow of Subroutine TRIAL



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