TWO DEGREES OF FREEDOM SYSTEM WITH AN IMPACT DAMPER

STEADY STATE RESPONSE AND STABILITY ANALYSIS OF THE TWO DEGREES OF FREEDOM SYSTEM WITH AN IMPACT DAMPER

By

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SCOPE AND CONTENTS:

Steady State response of a system of two degrees of freedom with impact damper and their asymptotic stability criterian are derived analytically. Stability regions are also determined for a wide range of parameters of impact damper by using a digital computer.

An experimental study is also made to verify the assumptions taken in the analytical solution and to obtain general a response of the system for a wide range parameters of the impact damper.

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NOMENCLATURE

A_1 = displacement amplitude of primary mass 1 with out impact damper

A_2 = displacement amplitude of primary mass 2 with out impact damper

C = damping coeficient of primary masses 1 and 2

d = clearance in which the free mass is free to oscillate in primary masses 1 and 2

e = cofficient of restitution

F_o = maximum force of excitation

K_1 = stiffness of leaf springs

K_2 = stiffness of helical springs

M = mass of primary masses 1 and 2

m = mass of free mass

P = perturbation matrix

R = remainder matrix

r_1 = ratio, forcing frequency/first natural frequency

r_2 = ratio, forcing frequency/second natural frequency

t = time

t_1 = time difference in consecutive impacts in primary mass 1, and 2

V_1 = absolute velocity of free mass in primary mass 1

V_2 = absolute velocity of free mass in primary mass 2

x_1 = displacement of primary mass 1

x_2 = displacement of primary mass 2

x_a = displacement of primary mass 1 just before impact

x_b = displacement of primary mass 1 just after impact

x_c' = displacement of primary mass 2,at impact in mass 1

x_a' = displacement of primary mass 2 just after impact in it

x_b' = displacement of primary mass 2 just before impact in it

x_c = displacement of primary mass 1 at impact in mass 2

Y_1 = displacement of free mass in mass 1

Y_2 = displacement of free mass in mass 2

y_1 = relative displacement of free mass with respect to mass 1

y_2 = relative displacement of free mass with respect to mass 2

α = initially unknown phase angle

δ_1 = ratio of critical damping for mass 1

δ_2 = ratio of critical damping for mass 2

μ = mass ratio,m/M

$\bar{\xi}$ = perturbation vector

ψ_1 = phase angle of primary mass 1 (due to damping)

ψ_2 = phase angle of primary mass 2 (due to damping)

τ = phase angle of impact

ω_1 = first natural frequency

ω_2 = second natural frequency

Ω = forcing frequency

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ABSTRACT

Steady state response of a system of two degrees of freedom with impact damper and its asymptotic stability criterion are derived analytically.Stability regions are determined for wide range parameters of the impact damper by using digital computer.

Experimental study is also made to verify the assumptions taken in analytical solution and to obtain general response of the system for wide range parameters of the impact damper.

As a result of analytical and experimental studies,it is observed that the system,which is stable for an impact damper in one primary mass,may not be stable when the impact damper is transferred to other, even though the parameters of the impact damper are the same.

Stability boundaries are found to be a complicated function of the impact damper parameters.It has also been found that impact damper in both primary masses is not always more effective than that in a single primary mass.

1. INTRODUCTION

1.1 Review of Impact Damper in Single Degree of Freedom System

The impact damper is a device for reducing the vibration amplitude of a mechanical system through the mechanism of momentum transfer by collision and conversion of mechanical energy into heat.

Paget[(1)*] was the pioneer in making qualitative study of this damper. Lieber and Jenson[(2)] followed with a steady state solution for the case of zero coefficient of restitution and undamped single degree of freedom system. For their solution they assumed a simple harmonic motion of constant amplitude. As a result, they determined that for maximum energy dissipation per cycle, the clearance for the particle should be π times the amplitude of response. Grubin[(3)] determined the steady state solution for viscously damped single degree of freedom system. He assumed two symmetric impacts per cycle.

Arnold[(4)] also investigated a similiar system experimentally and theoretically. He represented the force that acts during impact by Fourier series. He also, assumed two symmetric impacts per cycle.

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* Number in parentheses designate refrences at the end.

Warburton[5] suggested a simpler method by which steady state solution can be found by considering only two consecutive impacts.

Masri[6] in his work obtained steady state solution and its asymptotically stable region,for two symmetric impacts per cycle motion of the system.

Sadek[7] obtained steady statesolution,assuming two unsymmetric impacts per cycle. The impact force-time curve is assumed to be of rectangular shape and of infinitesimal duration.He used Fourier series to represent impact cycle.

A number of experimental studies has also been made to this effect to establish the practical feasibility of impact damping.McGoldrick[8] investigated its effects on ship hulls;Lieber and Tripp[9] investigated its effects on cantilever beam;Sankey[10] studied its effects on single degree of freedom systems and Duckwald[11] studied its effects in reducing the vibrations of turbine buckets.

1.2 Objective and Scope

To the author's knowledge,no work has been published on impact damper in the system with two degree of freedom. In the system of the two degrees of freedom,three cases arises;

i Impact damper in primary mass 1

ii Impact damper in primary mass 2

iii Impact damper in both primary mass 1 and 2

In order to know which case is most efficient, it is necessary to know the general responses of the above mentioned cases.

The objective of the present study is to determine for the three cases;

a) the steady state solution for the symmetric two impacts per cycle motion of the impact damper

b) the stability criterion and stability region for steady state solution

c) the general responses of the impact damper to a wide range of its parameters

The exact solution for symmetric two impacts per cycle motion of the impact damper is derived analytically in chapter 2.The stability criterion for steady state solution is determined analytically in chapter 3.The experimental set up is described in chapter 4.The experimental results as well as theoretical results are discussed in chapter 5.The conclusion drawn from this research work and recommendations for future work are also stated in chapter 5.

The digital computer program for determing steady state solution ad stability region are given in appendix C

2. STEADY STATE SOLUTIONS WITH IMPACT DAMPER

2.1 Introduction

Here periodic solutions are determined with the assumption that, during a period of the sinusoidal forcing function, two impacts occur at equal time interval and at opposite sides of the container of the primary mass with impact damper. Experimentally it has been found that this motion of impact damper predominates.

Here three possible cases in two degrees of freedom system with impact damper are discussed:

i Impact damper in primary mass 1

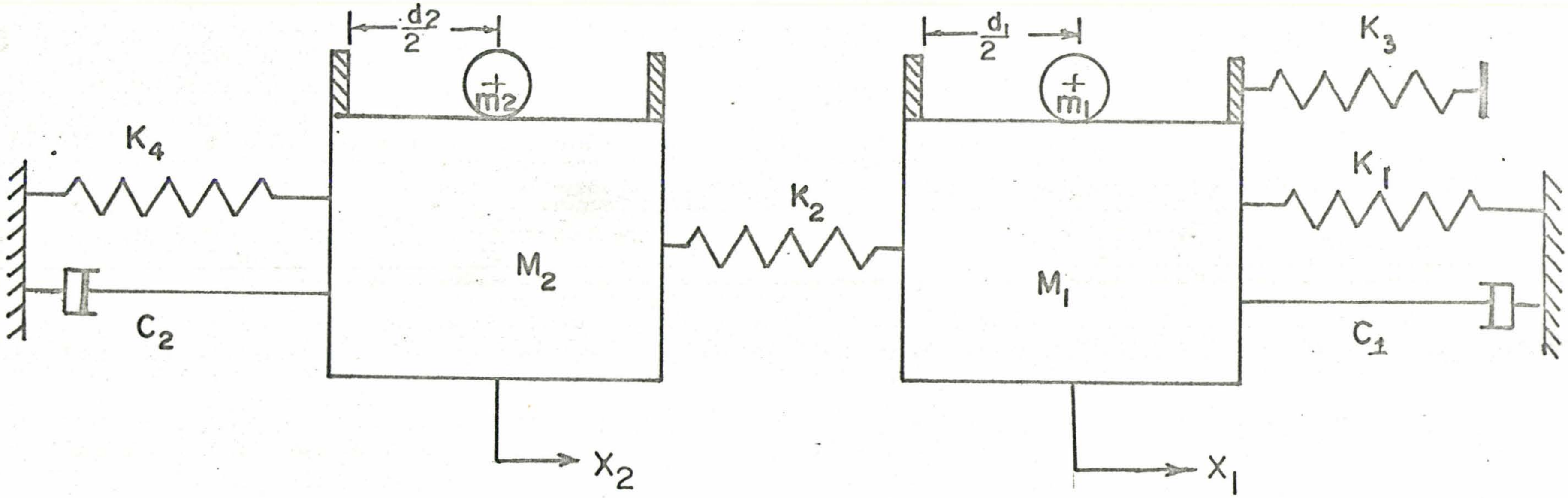
ii Impact damper in primary mass 2

iii Impact damper in both primary masses

2.2 Model of the System

The model of the system under discussion is shown in Fig. 1 . The free mass m_1 and m_2 are essentially a frictionless solid particles constrained to oscillate with clearance d_1 and d_2 respectively in containers attached to primary masses M_1 and M_2 respectively. Here without losing any generality, we assume that M_1, C_1, K_1, K_2, d_1 & m_1 are equal to M_2, C_2, K_4, K_3, d_2, and m_2 respectively.

Following the method suggested by Warburton[5] assume the disturbing force to be $F_0 \sin(\Omega t+\alpha)$



MODEL OF SYSTEM

Fig. 1

Where

$$F_0 = K_3 x$$

The equations of motion of primary mass 1 and mass 2 between consecutive impacts are :

$$M\ddot{x}_1 + C\dot{x}_1 + (K_1 + 2K_2)x_1 - K_2 x_2 = F_0 \sin(\Omega t + \alpha)$$
$$M\ddot{x}_2 + C\dot{x}_2 + (K_1 + K_2)x_2 - K_2 x_1 = 0 \qquad (2.1)$$

The complete solution of Equation(2.1) is

$$x_1 = (e^{-\delta_1\omega_1 t}(B_1 \sin\eta_1\omega_1 t + B_2\cos\eta_1\omega_1 t)) + (e^{-\delta_2\omega_2 t}(C_1\sin\eta_2\omega_2 t + C_2\cos\eta_2\omega_2 t)) + A_1\sin(\Omega t + \tau) \qquad (a)$$

$$x_2 = D_1(e^{-\delta_1\omega_1 t}(B_1\sin\eta_1\omega_1 t + B_2\cos\eta_1\omega_1 t)) + D_2(e^{-\delta_2\omega_2 t}(C_1\sin\eta_2\omega_2 t + C_2\cos\eta_2\omega_2 t)) + A_2\sin(\Omega t + \tau_1) \qquad (b) \quad (2.2)$$

Where

$$\omega_1^2 = \frac{3K_2 + 2K_1 - \sqrt{5}\,K_2}{2M} \qquad \tau = \alpha - \psi_1$$

$$\omega_2^2 = \frac{3K_2 + 2K_1 + \sqrt{5}\,K_2}{2M} \qquad \tau_1 = \tau + \psi$$

$$\delta_1 = \frac{C}{C_{r1}} \qquad \psi = -\psi_2 + \psi_1$$

$$\delta_2 = \frac{C}{C_{r2}} \qquad \psi_1 = \tan^{-1}\frac{(D\cdot A - C\Omega B)}{AB + C\Omega D}$$

$$C_{r1} = 2M\omega_1 \qquad \psi_2 = \tan^{-1}\left(\frac{D}{B}\right)$$

$$C_{r2} = 2M\omega_2 \qquad A_1 = \frac{F_0\sqrt{(AB + C\Omega D)^2 + (C\Omega B - DA)^2}}{B^2 + D^2}$$

$$\eta_1 = \sqrt{1-\delta_1^2} \qquad A_2 = \frac{K_2 F_0}{\sqrt{B^2 + D^2}}$$

$$\eta_2 = \sqrt{1-\delta_2^2} \qquad A = K_1 + K_2 - M\Omega^2$$

$$D_1 = \frac{1+\sqrt{5}}{2} \qquad D = C((2K_1 + 3K_2)\Omega - 2\Omega^3 M)$$

$$D_2 = \frac{1-\sqrt{5}}{2} \qquad B = (K_1 + K_2)(K_1 + 2K_2) - \Omega^2(C^2 + M(2K_1 + 3K_2)) + M^2\Omega^4 - K_2^2$$

2.3 Impact Damper in both Masses

Let us assume that a impact occurs in primary mass 1 at t=0 and next impact occurs in primary mass 2 at $t=t_1$

.As duration of the impact is small, it is reasonable to assume that at the time of impact, displacements of primary and free masses remain the same, while the velocities of primary and free masses which are under impact change discontinuously.

To summarize, the system should satisfy the following conditions:

At

$$t=0_-,\ x_1=x_b,\ y_1=\frac{d}{2},\ x_2=x_c',\ \dot{x}_2=\dot{x}_c',\ \dot{x}_1=\dot{x}_b,\ \dot{Y}_1=V_1 \quad (a)$$

$$t=0_+,\ x_1=x_b,\ y_1=\frac{d}{2},\ x_2=x_c',\ \dot{x}_2=\dot{x}_c',\ \dot{x}_1=\dot{x}_a,\ \dot{Y}_1=-V_1 \quad (b)$$

$$t=(t_1)_-\ x_1=x_c,\ y_2=\frac{d}{2},\ x_2=x_b',\ \dot{x}_2=\dot{x}_b',\ \dot{x}_1=\dot{x}_c,\ \dot{Y}_2=V_2 \quad (c) \qquad (2.3)$$

$$t=(t_1)_+\ x_1=x_c,\ y_2=\frac{d}{2},\ x_2=x_b',\ \dot{x}_2=\dot{x}_a',\ \dot{x}_1=\dot{x}_c,\ \dot{Y}_2=-V_2 \quad (d)$$

$$t=(\tfrac{\pi}{\Omega})_-\ x_1=-x_b,\ y_1=-\frac{d}{2},\ x_2=-x_c',\ \dot{x}_2=-\dot{x}_c',\ \dot{x}_1=-\dot{x}_b,\ \dot{Y}_1=-V_1 \quad (e)$$

Equation(2.2) describe the motion of primary masses 1 and 2 from $t=0_+$ to $t=(t_1)_-$

During impact, system should satisfy momentum and cofficient of restitution equations. So

$$M\dot{x}_{i-}+m\dot{y}_{i-} = M\dot{x}_{i+}+m\dot{y}_{i+} \qquad (2.4)$$

$$\dot{x}_{i+} - \dot{y}_{i+} = -e(\dot{x}_{i-}-\dot{y}_{i-}) \qquad (2.5)$$

Where $i = 1,2$ for impact in primary mass 1 and 2

In steady state motion, the absolute speed for frictionless free mass should be constant and equal to

$$V_1 = (d+2x_b)\frac{\Omega}{\pi} \qquad (2.6)$$

$$V_2 = (d+2x_b')\frac{\Omega}{\pi} \qquad (2.7)$$

By using Equations(2.4), (2.5), (2.6) and (2.7), we get

$$\dot{x}_{i+} = \left(\frac{1-\mu e}{1+\mu}\right)\dot{x}_{i-} + \mu\left(\frac{1+e}{1+\mu}\right)\dot{Y}_{i-} \tag{2.8}$$

$$\dot{Y}_{i+} = \left(\frac{1+e}{1-\mu}\right)\dot{x}_{i-} + \frac{(\mu-e)}{1+\mu}\dot{Y}_{i-} \tag{2.9}$$

$$x_b + \frac{\pi}{2\Omega}\left(\frac{1+e}{1-e+2\mu}\right)\dot{x}_b = -\frac{d}{2} \tag{2.10}$$

$$x_b + \frac{\pi}{2\Omega}\left(\frac{1+e}{1-e-2\mu e}\right)\dot{x}_a = -\frac{d}{2} \tag{2.11}$$

$$x'_b + \frac{\pi}{2\Omega}\left(\frac{1+e}{1-e+2\mu}\right)\dot{x}'_b = -\frac{d}{2} \tag{2.12}$$

$$x'_b + \frac{\pi}{2\Omega}\left(\frac{1+e}{1-e-2\mu e}\right)\dot{x}'_a = -\frac{d}{2} \tag{2.13}$$

An expression for velocities of primary mass 1 and 2 is

$$\dot{x}_1 = (-\delta_1\omega_1 e^{-\delta_1\omega_1 t}(B_1\sin\eta_1\omega_1 t + B_2\cos\eta_1\omega_1 t) + e^{-\delta_1\omega_1 t}(B_1\eta_1\omega_1\cos\eta_1\omega_1 t - B_2\eta_1\omega_1\sin\eta_1\omega_1 t)) + (-\delta_2\omega_2 e^{-\delta_2\omega_2 t}(C_1\sin\eta_2\omega_2 t + C_2\cos\eta_2\omega_2 t) + e^{-\delta_2\omega_2 t}(C_1\eta_2\omega_2\cos\eta_2\omega_2 t - C_2\eta_2\omega_2\sin\eta_2\omega_2 t)) + A_1\Omega\cos(\Omega t+\tau) \tag{2.14}$$

$$\dot{x}_2 = D_1(-\delta_1\omega_1 e^{-\delta_1\omega_1 t}(B_1\sin\eta_1\omega_1 t + B_2\cos\eta_1\omega_1 t) + e^{-\delta_1\omega_1 t}(B_1\eta_1\omega_1\cos\eta_1\omega_1 t - B_2\eta_1\omega_1\sin\eta_1\omega_1 t)) + D_2(-\delta_2\omega_2 e^{-\delta_2\omega_2 t}(C_1\sin\eta_2\omega_2 t + C_2\cos\eta_2\omega_2 t) + e^{-\delta_2\omega_2 t}(C_1\eta_2\omega_2\cos\eta_2\omega_2 t - C_2\eta_2\omega_2\sin\eta_2\omega_2 t)) + A_2\Omega\cos(\Omega t+\tau_1) \tag{2.15}$$

From Equations(2.2),(2.3-b) and (2.3-c)

$$x_1(0_+) = x_b = B_2 + C_2 + A_1\sin(\tau) \tag{2.16}$$

$$x_2(0_+) = x'_c = D_1B_2 + D_2C_2 + A_2\sin(\tau_1) \tag{2.17}$$

$$x_1(t_1)_- = x_c = (e^{-\delta_1\omega_1 t_1}(B_1\sin\eta_1\omega_1 t_1 + B_2\cos\eta_1\omega_1 t)) + (e^{-\delta_2\omega_2 t_1}(C_1\sin\eta_2\omega_2 t_1 + C_2\cos\eta_2\omega_2 t_1)) + A_1\sin(\Omega t_1+\tau) \tag{2.18}$$

$$x_2(t_1)_- = x'_b = D_1(e^{-\delta_1\omega_1 t_1}(B_1\sin\eta_1\omega_1 t_1 + B_2\cos\eta_1\omega_1 t)) + D_2(e^{-\delta_2\omega_2 t_1}(C_1\sin\eta_2\omega_2 t_1 + C_2\cos\eta_2\omega_2 t_1)) + A_2\sin(\Omega t+\tau_1) \tag{2.19}$$

Similiarly, from Equations (2.14)and(2.15),(2.4-b) and (2.4-c)

$$\dot{x}_1(0_+) = \dot{x}_a = -\delta_1\omega_1 B_2 + \eta_1\omega_1 B_1 - \delta_2\omega_2 C_2 + \eta_2\omega_2 C_1 + A_1\Omega\cos\tau \quad (2.20)$$

$$\dot{x}_2(0_+) = \dot{x}'_c = D_1(-\delta_1\omega_1 B_2 + \eta_1\omega_1 B_1) + D_2(-\delta_2\omega_2 C_2 + \eta_2\omega_2 C_1) + A_2\Omega\cos(\tau_1)$$

$$\dot{x}_1(t_1)_- = \dot{x}_c = (-\delta_1\omega_1 e^{-\delta_1\omega_1 t_1}(B_1\sin\eta_1\omega_1 t_1 + B_2\cos\eta_1\omega_1 t_1) + e^{-\delta_1\omega_1 t_1} \quad (2.21)$$
$$(B_1\eta_1\omega_1\cos\eta_1\omega_1 t_1 - B_2\eta_1\omega_1\sin\eta_1\omega_1 t_1)) + (-\delta_2\omega_2 e^{-\delta_2\omega_2 t_1}$$
$$(C_1\sin\eta_2\omega_2 t_1 + C_2\cos\eta_2\omega_2 t_1) + e^{-\delta_2\omega_2 t_1}(C_1\eta_2\omega_2\cos\eta_2\omega_2 t_1$$
$$-C_2\eta_2\omega_2\sin\eta_2\omega_2 t_1)) + \Omega A_1\cos(\Omega t_1 + \tau)$$

$$\dot{x}_2(t_1) = \dot{x}'_b = D_1(-\delta_1\omega_1 e^{-\delta_1\omega_1 t_1}(B_1\sin\eta_1\omega_1 t_1 + B_2\cos\eta_1\omega_1 t_1) + e^{-\delta_1\omega_1 t_1} \quad (2.22)$$
$$(B_1\eta_1\omega_1\cos\eta_1\omega_1 t_1 - B_2\eta_1\omega_1\sin\eta_1\omega_1 t_1)) + D_2(-\delta_2\omega_2 e^{-\delta_2\omega_2 t_1}$$
$$(C_1\sin\eta_2\omega_2 t_1 + C_2\cos\eta_2\omega_2 t_1) + e^{-\delta_2\omega_2 t_1}(C_1\eta_2\omega_2\cos\eta_2\omega_2 t_1$$
$$-C_2\eta_2\omega_2\sin\eta_2\omega_2 t_1)) + \Omega A_2\cos(\Omega t_1 + \tau_1) \quad (2.23)$$

Equations of motion of primary masses 1 and 2 for $t=t_{1+}$ to $t=\frac{\pi}{\Omega}-$

$$x_1 = (e^{-\delta_1\omega_1(t-t_1)}(B'_1\sin\eta_1\omega_1(t-t_1) + B'_2\cos\eta_1\omega_1(t-t_1)) + (e^{-\delta_2\omega_2(t-t_1)}$$
$$(C'_1\sin\eta_2\omega_2(t-t_1) + C'_2\cos\eta_2\omega_2(t-t_1)) + A_1\sin(\Omega t + \tau) \quad (2.24)$$

$$x_2 = D_1(e^{-\delta_1\omega_1(t-t_1)}(B'_1\sin\eta_1\omega_1(t-t_1) + B'_2\cos\eta_1\omega_1(t-t_1)) + D_2(e^{-\delta_2\omega_2(t-t_1)}$$
$$(C'_1\sin\eta_2\omega_2(t-t_1) + C'_2\cos\eta_2\omega_2(t-t_1)) + A_2\sin(\Omega t + \tau_1) \quad (2.25)$$

$$\dot{x}_1 = (-\delta_1\omega_1 e^{-\delta_1\omega_1(t-t_1)}(B'_1\sin\eta_1\omega_1(t-t_1) + B'_2\cos\eta_1\omega_1(t-t_1)) + e^{-\delta_1\omega_1(t-t_1)}$$
$$(B'_1\eta_1\omega_1\cos\eta_1\omega_1(t-t_1) - B'_2\eta_1\omega_1\sin\eta_1\omega_1(t-t_1))) + (-\delta_2\omega_2 e^{-\delta_2\omega_2(t-t_1)}$$
$$(C'_1\sin\eta_2\omega_2(t-t_1) + C'_2\cos\eta_2\omega_2(t-t_1)) + e^{-\delta_2\omega_2(t-t_1)}(C'_1\eta_2\omega_2$$
$$\cos\eta_2\omega_2(t-t_1) - C'_2\eta_2\omega_2\sin\eta_2\omega_2(t-t_1)) + A_1\Omega\cos(\Omega t + \tau) \quad (2.26)$$

$$\dot{x}_2 = D_1(-\delta_1\omega_1 e^{-\delta_1\omega_1(t-t_1)}(B'_1\sin\eta_1\omega_1(t-t_1) + B'_2\cos\eta_1\omega_1(t-t_1)) + e^{-\delta_1\omega_1(t-t_1)}$$
$$(B'_1\eta_1\omega_1\cos\eta_1\omega_1(t-t_1) - B'_2\eta_1\omega_1\sin\eta_1\omega_1(t-t_1))) + D_2(-\delta_2\omega_2 e^{-\delta_2\omega_2(t-t_1)}$$
$$(C'_1\sin\eta_2\omega_2(t-t_1) + C'_2\cos\eta_2\omega_2(t-t_1)) + e^{-\delta_2\omega_2(t-t_1)}(C'_1\eta_2\omega_2$$
$$\cos\eta_2\omega_2(t-t_1) - C'_2\eta_2\omega_2(t-t_1))) + A_2\Omega\cos(\Omega t + \tau_1) \quad (2.27)$$

From Equations (2.24), (2.25), (2.3-d) and (2.3-e)

$$x_1(t_1)_+ = x_c = B'_2 + C'_2 + A_1\sin(\Omega t_1 + \tau) \quad (2.28)$$

$$x_2(t_1)_+ = x'_b = D_1 B'_2 + D_2 C'_2 + A_2\sin(\Omega t_1 + \tau_1) \quad (2.29)$$

$$x_1(\tfrac{\pi}{\Omega}) = -x_b = e^{-\delta_1\omega_1(\frac{\pi}{\Omega}-t_1)}(B_1' \sin\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1) + B_2'\cos\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1))$$
$$+ e^{-\delta_2\omega_2(\frac{\pi}{\Omega}-t_1)}(C_1'\sin\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1) + C_2'\cos\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1))$$
$$-A_1\sin(\tau) \qquad (2.30)$$

$$x_2(\tfrac{\pi}{\Omega}) = -x_c' = D_1(e^{-\delta_1\omega_1(\frac{\pi}{\Omega}-t_1)}(B_1'\sin\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1) + B_2'\cos\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1)))$$
$$+ D_2(e^{-\delta_2\omega_2(\frac{\pi}{\Omega}-t_1)}(C_1'\sin\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1) + C_2'\cos\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1)))$$
$$-A_2\sin(\tau_1) \qquad (2.31)$$

Similiarly from Equations(2.26),(2.27),(2.3-d) and (2.3-e)

$$\dot{x}_1(t_1)_+ = \dot{x}_c = -\delta_1\omega_1 B_2' + \eta_1\omega_1 B_1' - \delta_2\omega_2 C_2' + \eta_2\omega_2 C_1' + A_1\Omega\cos(\Omega t_1+\tau) \qquad (2.32)$$

$$\dot{x}_2(t_1)_+ = \dot{x}_a' = D_1(-\delta_1\omega_2 B_2' + \eta_1\omega_1 B_1') + D_2(\eta_2\omega_2 C_1' - \delta_2\omega_2 C_2')$$
$$+ A_2\Omega\cos(\Omega t_1 + \tau_1) \qquad (2.33)$$

$$\dot{x}_1(\tfrac{\pi}{\Omega}) = -\dot{x}_b = -\delta_1\omega_1 e^{-\delta_1\omega_1(\frac{\pi}{\Omega}-t_1)}(B_1'\sin\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1) + B_2'\cos\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1))$$
$$+ e^{-\delta_1\omega_1(\frac{\pi}{\Omega}-t_1)}(B_1'\eta_1\omega_1\cos\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1) - B_2'\eta_1\omega_1$$
$$\sin\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1)) + (-\delta_2\omega_2 e^{-\delta_2\omega_2(\frac{\pi}{\Omega}-t_1)}(C_1'\sin\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1)$$
$$+ C_2'\cos\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1)) + e^{-\delta_2\omega_2(\frac{\pi}{\Omega}-t_1)}(C_1'\eta_2\omega_2\cos\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1)$$
$$-C_2'\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1))) - A_1\Omega\cos(\tau) \qquad (2.34)$$

$$\dot{x}_2(\tfrac{\pi}{\Omega}) = -\dot{x}_c' = D_1(-\delta_1\omega_1 e^{-\delta_1\omega_1(\frac{\pi}{\Omega}-t_1)}(B_1'\sin\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1) + B_2'\cos\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1))$$
$$+ e^{-\delta_1\omega_1(\frac{\pi}{\Omega}-t_1)}(B_1'\eta_1\omega_1\cos\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1) - B_2'\eta_1\omega_1\sin\eta_1\omega_1(\tfrac{\pi}{\Omega}-t_1)))$$
$$+ D_2(-\delta_2\omega_2 e^{-\delta_2\omega_2(\frac{\pi}{\Omega}-t_1)}(C_1'\sin\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1) + C_2'\cos\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1)) +$$
$$e^{-\delta_2\omega_2(\frac{\pi}{\Omega}-t_1)}(C_1'\eta_2\omega_2\cos\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1) - C_2'\eta_2\omega_2$$
$$\sin\eta_2\omega_2(\tfrac{\pi}{\Omega}-t_1))) - A_2\Omega\cos(\tau_1) \qquad (2.35)$$

By using the following new variables,

$$S_1 = \sin(\tau)$$
$$S_2 = \sin(\tau_1)$$
$$S_3 = \sin(\Omega t_1 + \tau)$$

$$S_4 = \sin(\Omega t_1 + \tau_1)$$

$$E_1 = \Omega \cos(\tau)$$

$$E_4 = \Omega \cos(\Omega t_1 + \tau_1)$$

$$E_3 = \Omega \cos(\Omega t_1 + \tau)$$

$$E_2 = \Omega \cos(\tau_1)$$

$$h_1 = \delta_1 \omega_1$$

$$h_2 = \delta_2 \omega_2$$

$$h_3 = \eta_1 \omega_1$$

$$h_4 = \eta_2 \omega_2$$

$$h_5 = e^{-\delta_1 \omega_1 t_1} \sin \eta_1 \omega_1 t_1$$

$$h_6 = e^{-\delta_1 \omega_1 t_1} \cos \eta_1 \omega_1 t_1$$

$$h_7 = e^{-\delta_2 \omega_2 t_1} \sin \eta_2 \omega_2 t_1$$

$$h_8 = e^{-\delta_2 \omega_2 t_1} \cos \eta_2 \omega_2 t_1$$

$$h_9 = e^{-\delta_1 \omega_1 (\frac{\pi}{\Omega} - t_1)} \cdot \sin \eta_1 \omega_1 (\frac{\pi}{\Omega} - t_1)$$

$$h_{10} = e^{-\delta_1 \omega_1 (\frac{\pi}{\Omega} - t_1)} \cos \eta_1 \omega_1 (\frac{\pi}{\Omega} - t_1)$$

$$h_{11} = e^{-\delta_2 \omega_2 (\frac{\pi}{\Omega} - t_1)} \cdot \sin \eta_2 \omega_2 (\frac{\pi}{\Omega} - t_1)$$

$$h_{12} = e^{-\delta_2 \omega_2 (\frac{\pi}{\Omega} - t_1)} \cdot \cos \eta_2 \omega_2 (\frac{\pi}{\Omega} - t_1)$$

$$\theta_1 = e^{-\delta_1 \omega_1 t_1} (-\delta_1 \omega_1 \sin \eta_1 \omega_1 t_1 + \eta_1 \omega_1 \cos \eta_1 \omega_1 t_1)$$

$$\theta_2 = e^{-\delta_1 \omega_1 t_1} (-\delta_1 \omega_1 \cos \eta_1 \omega_1 t_1 - \eta_1 \omega_1 \sin \eta_1 \omega_1 t_1)$$

$$\theta_3 = e^{-\delta_2 \omega_2 t_1} (-\delta_2 \omega_2 \sin \eta_2 \omega_2 t_1 + \eta_2 \omega_2 \cos \eta_2 \omega_2 t_1)$$

$$\theta_4 = e^{-\delta_2 \omega_2 t_1} (-\delta_2 \omega_2 \cos \eta_2 \omega_2 t_1 - \eta_2 \omega_2 \sin \eta_2 \omega_2 t_1)$$

$$\theta_5 = e^{-\delta_1 \omega_1 (\frac{\pi}{\Omega} - t_1)} (-\delta_1 \omega_1 \sin \eta_1 \omega_1 (\frac{\pi}{\Omega} - t_1) + \eta_1 \omega_1 \cos \eta_1 \omega_1 (\frac{\pi}{\Omega} - t_1))$$

$$\theta_6 = e^{-\delta_1 \omega_1 (\frac{\pi}{\Omega} - t_1)} (-\delta_1 \omega_1 \cos \eta_1 \omega_1 (\frac{\pi}{\Omega} - t_1) - \eta_1 \omega_1 \sin \eta_1 \omega_1 (\frac{\pi}{\Omega} - t_1))$$

$$\theta_7 = e^{-\delta_2 \omega_2 (\frac{\pi}{\Omega} - t_1)} (-\delta_2 \omega_2 \sin \eta_2 \omega_2 (\frac{\pi}{\Omega} - t_1) + \eta_2 \omega_2 \cos \eta_2 \omega_2 (\frac{\pi}{\Omega} - t_1))$$

$$\theta_8 = e^{-\delta_2 \omega_2 (\frac{\pi}{\Omega} - t_1)} (-\delta_2 \omega_2 \cos \eta_2 \omega_2 (\frac{\pi}{\Omega} - t_1) - \eta_2 \omega_2 \sin \eta_2 \omega_2 (\frac{\pi}{\Omega} - t_1))$$

$$\sigma_1' = \frac{\pi}{2\Omega} \frac{1 + e}{1 - e + 2\mu}$$

$$\sigma_2' = \frac{\pi}{2\Omega} \frac{1 + e}{1 - e - 2\mu e}$$

Equations(2.9) to (2.12),(2.16) to (2.23) and (2.28) to (2.35) can be put into matrix form

$$
\begin{bmatrix}
1 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & \sigma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \sigma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & -S_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -D_1 & 0 & -D_2 & 0 & 0 & 0 & 0 & 0 & -S_2 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -h_5 & -h_6 & -h_7 & -h_8 & 0 & 0 & 0 & 0 & -S_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -D_1h_5 & -D_1h_6 & -D_2h_7 & -D_2h_8 & 0 & 0 & 0 & 0 & 0 & -S_4 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -h_3 & h_1 & -h_4 & h_2 & 0 & 0 & 0 & 0 & -E_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -D_1h_3 & D_1h_1 & -D_2h_4 & D_2h_2 & 0 & 0 & 0 & 0 & 0 & -E_2 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\theta_1 & -\theta_2 & -\theta_3 & -\theta_4 & 0 & 0 & 0 & 0 & -E_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -D_1\theta_1 & -D_1\theta_2 & -D_2\theta_3 & -D_2\theta_4 & 0 & 0 & 0 & 0 & 0 & -E_4 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -S_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -D_1 & 0 & -D_2 & 0 & -S_4 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_9 & h_{10} & h_{11} & h_{12} & -S_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_1h_9 & D_1h_{10} & D_2h_{11} & D_2h_{12} & 0 & -S_2 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -h_3 & h_1 & -h_4 & h_2 & -E_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -D_1h_3 & D_1h_1 & -D_2h_4 & D_2h_2 & 0 & -E_4 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_5 & \theta_6 & \theta_7 & \theta_8 & -E_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & D_1\theta_5 & D_1\theta_6 & D_2\theta_7 & D_2\theta_8 & 0 & -E_2
\end{bmatrix}
\begin{Bmatrix}
x_b \\ x_c \\ \dot{x}_b \\ \dot{x}_a \\ \dot{x}_c \\ x'_b \\ x'_c \\ \dot{x}'_b \\ \dot{x}'_a \\ \dot{x}'_c \\ B_1 \\ B_2 \\ C_1 \\ C_2 \\ B'_1 \\ B'_2 \\ C'_1 \\ C'_2 \\ A_1 \\ A_2
\end{Bmatrix}
=
\begin{Bmatrix}
-\frac{d}{2} \\ -\frac{d}{2} \\ -\frac{d}{2} \\ -\frac{d}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{Bmatrix}
\qquad (2.36)
$$

Equation (2.36) furnishes 20 relations among 20 unknowns. Here A_1 and A_2 are found in terms of τ and t_1. As A_1 and A_2 are known, we get two equations among two unknown τ and t_1.

Thus

$$A_1 = \frac{N(A_1)}{\Delta} \tag{2.37}$$

$$A_2 = \frac{N(A_2)}{\Delta} \tag{2.38}$$

Where ,

$$N(A_1) = \sigma_1^2 \sigma_2^2 h_5 h_7 (D_2 - D_1)^2 M_{23} M_{59} M_{62} M_{74} \times (M_{81} M_{10} - M_{82} M_9)$$

$$N(A_2) = \sigma_1^2 \sigma_2^2 h_5 h_7 (D_2 - D_1)^2 M_{23} M_{59} M_{62} M_{74} \times (M_{76} M_{82} - M_{77} M_{81})$$

$$\Delta = \sigma_1^2 \sigma_2^2 h_5 h_7 (D_2 - D_1)^2 M_{23} M_{59} M_{62} M_{74} \times (M_{76} M_{10} - M_{77} M_9)$$

By relation of Equation (2.37) and (2.38), we get

$$\tau = \tan\left(\frac{M_{84}}{M_{85}}\right) \tag{2.39}$$

Knowning τ in terms of t_1, unknown τ and t_1 can be found by either (2.37) or (2.38). New variables M's are given in appendix A .

2.4 Impact Damper in Mass 1 only

Assuming that one impact occurs at $t=0_+$, the next impact will occur at $t=\frac{\pi}{\Omega}$

The system should satisfy the following conditions; at

$$t=0_- \quad x_1=x_b \quad y_1=\frac{d}{2} \quad x_2=x_c' \quad \dot{x}_2=\dot{x}_c' \quad \dot{x}_1=\dot{x}_b \quad \dot{Y}_1=V_1 \qquad (a)$$

$$t=0_+ \quad x_1=x_b \quad y_1=\frac{d}{2} \quad x_2=x_c' \quad \dot{x}_2=\dot{x}_c' \quad \dot{x}_1=\dot{x}_a \quad \dot{Y}_1=-V_1 \qquad (b) \quad (2.40)$$

$$t=(\tfrac{\pi}{\Omega})_- \quad x_1=-x_b \quad y_1=-\frac{d}{2} \quad x_2=-x_c' \quad \dot{x}_2=-\dot{x}_c' \quad \dot{x}_1=-\dot{x}_b \quad \dot{Y}_1=-V_1 \qquad (c)$$

Equations(2.2),(2.14) and (2.15) describe the motion of primary mass 1 and primary mass 2 from $t=0_+$ to $t=(\frac{\pi}{\Omega})_-$

By using Equations (2.2),(2.14),(2.15),(2.4),(2.5),(2.6),(2.40-a) and(2.40-b),we get the Equations(2.10),(2.11),(2.16),(2.17),(2.20) and (2.21).

From Equations(2.2),(2.14),(2.15) and(2.40-c)

$$x_1(\tfrac{\pi}{\Omega})_- = -x_b = \left(e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\left(B_1 \sin\eta_1\omega_1\tfrac{\pi}{\Omega} + B_2\cos\eta_1\omega_1\tfrac{\pi}{\Omega}\right)\right) + \left(e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\left(C_1\sin\eta_2\omega_2\tfrac{\pi}{\Omega} + C_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega}\right)\right) - A_1\sin(\tau) \qquad (2.41)$$

$$x_2(\tfrac{\pi}{\Omega})_- = -x_c' = D_1\left(e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\left(B_1\sin\eta_1\omega_1\tfrac{\pi}{\Omega} + B_2\cos\eta_1\omega_1\tfrac{\pi}{\Omega}\right)\right) + D_2\left(e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\left(C_1\sin\eta_2\omega_2\tfrac{\pi}{\Omega} + C_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega}\right)\right) - A_2\sin(\tau_1) \qquad (2.42)$$

$$\dot{x}_1(\tfrac{\pi}{\Omega})_- = -\dot{x}_b = \left(-\delta_1\omega_1 e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\left(B_1\sin\eta_1\omega_1\tfrac{\pi}{\Omega} + B_2\cos\eta_1\omega_1\tfrac{\pi}{\Omega}\right) + e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\left(B_1\eta_1\omega_1\cos\eta_1\omega_1\tfrac{\pi}{\Omega} - B_2\eta_1\omega_1\sin\eta_1\omega_1\tfrac{\pi}{\Omega}\right)\right) + \left(-\delta_2\omega_2 e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\left(C_1\sin\eta_2\omega_2\tfrac{\pi}{\Omega} + C_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega}\right) + e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\left(C_1\eta_2\omega_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega} - C_2\eta_2\omega_2\sin\eta_2\omega_2\tfrac{\pi}{\Omega}\right)\right) - A_1\Omega\cos(\tau) \qquad (2.43)$$

$$\dot{x}_2\left(\tfrac{\pi}{\Omega}\right)_- = -\dot{x}_c' = D_1\Big(-\delta_1\omega_1 e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\big(B_1\sin\eta_1\omega_1\tfrac{\pi}{\Omega} + B_2\cos\eta_1\omega_1\tfrac{\pi}{\Omega}\big) +$$
$$e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\big(B_1\eta_1\omega_1\cos\eta_1\omega_1\tfrac{\pi}{\Omega} - B_2\eta_1\omega_1\sin\eta_1\omega_1\tfrac{\pi}{\Omega}\big)\Big) +$$
$$D_2\Big(-\delta_2\omega_2 e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\big(C_1\sin\eta_2\omega_2\tfrac{\pi}{\Omega} + C_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega}\big) +$$
$$e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\big(C_1\eta_2\omega_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega} - C_2\eta_2\omega_2\sin\eta_2\omega_2\tfrac{\pi}{\Omega}\big)\Big)$$
$$- A_2\Omega\cos(\tau_1) \tag{2.44}$$

By using the following new variables

$$h_{13} = e^{-\delta_1\omega_1\frac{\pi}{\Omega}} \cdot \sin\eta_1\omega_1\tfrac{\pi}{\Omega}$$
$$h_{14} = e^{-\delta_1\omega_1\frac{\pi}{\Omega}} \cdot \cos\eta_1\omega_1\tfrac{\pi}{\Omega}$$
$$h_{15} = e^{-\delta_2\omega_2\frac{\pi}{\Omega}} \cdot \sin\eta_2\omega_2\tfrac{\pi}{\Omega}$$
$$h_{16} = e^{-\delta_2\omega_2\frac{\pi}{\Omega}} \cdot \cos\eta_2\omega_2\tfrac{\pi}{\Omega}$$
$$\theta_9 = e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\left(-\delta_1\omega_1\cdot\sin\eta_1\omega_1\tfrac{\pi}{\Omega} + \eta_1\omega_1\cos\eta_1\omega_1\tfrac{\pi}{\Omega}\right)$$

$$\theta_{10} = e^{-\delta_1\omega_1\frac{\pi}{\Omega}}\left(-\delta_1\omega_1\cdot\cos\eta_1\omega_1\tfrac{\pi}{\Omega} - \eta_1\omega_1\sin\eta_1\omega_1\tfrac{\pi}{\Omega}\right)$$

$$\theta_{11} = e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\left(-\delta_2\omega_2\cdot\sin\eta_2\omega_2\tfrac{\pi}{\Omega} + \eta_2\omega_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega}\right)$$

$$\theta_{12} = e^{-\delta_2\omega_2\frac{\pi}{\Omega}}\left(-\delta_2\omega_2\cos\eta_2\omega_2\tfrac{\pi}{\Omega} - \eta_2\omega_2\sin\eta_2\omega_2\tfrac{\pi}{\Omega}\right)$$

Equations(2.10),(2.11),(2.16),(2.17),(2.41),(2.42), (2.43) and (2.44) can be written in following matrix form

$$\begin{bmatrix} 1 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & \sigma_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -S_1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -D_1 & 0 & -D_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -h_3 & h_1 & -h_4 & h_2 & -E_1 \\ 0 & 0 & 0 & 0 & 1 & -D_1h_3 & D_1h_1 & -D_2h_4 & D_2h_2 & 0 \\ 1 & 0 & 0 & 0 & 0 & h_{13} & h_{14} & h_{15} & h_{16} & -S_1 \\ 0 & 1 & 0 & 0 & 0 & D_1h_{13} & D_1h_{14} & D_2h_{15} & D_2h_{16} & 0 \\ 0 & 0 & 1 & 0 & 0 & \theta_9 & \theta_{10} & \theta_{11} & \theta_{12} & -E_1 \\ 0 & 0 & 0 & 0 & 1 & D_1\theta_9 & D_1\theta_{10} & D_2\theta_{11} & D_2\theta_{12} & 0 \end{bmatrix} \begin{Bmatrix} x_b \\ x'_c \\ \dot{x}_b \\ \dot{x}_a \\ \dot{x}'_c \\ B_1 \\ B_2 \\ C_1 \\ C_2 \\ A_1 \end{Bmatrix} = \begin{Bmatrix} -\frac{d}{2} \\ -\frac{d}{2} \\ 0 \\ A_2S_2 \\ 0 \\ A_2E_2 \\ 0 \\ A_2S_2 \\ 0 \\ A_2E_2 \end{Bmatrix} \qquad (2.45)$$

Equations(2.45) furnishes 10 relations among 10 unknowns $\dot{x}_b$, x_b, $\dot{x}_a$, x_c', $\dot{x}_c'$, B_1, B_2, C_1, C_2 and τ

From the solution of Equation(2.45), expression for A_1 is

$$A_1 = \frac{N(A_1)}{\Delta} \qquad (2.46)$$

where,

$$N(A_1) = \sigma_1 \sigma_2 h_{13} Z_{16} h_{15} (D_2 - D_1) \cdot \left(\frac{d}{2\sigma_2} Z_{27} - \frac{d}{2\sigma_1} \cdot Z_{26}\right)$$

$$\Delta = \sigma_1 \sigma_2 h_{13} Z_{16} h_{15} (D_2 - D_1) \cdot (Z_5 \cdot Z_{27} - Z_8 Z_{26})$$

Equation (2.46) can be written in the form

$$2 \sin\tau + H \cos\tau = -\rho \qquad (2.47)$$

where

$$H = \frac{2 \cdot (Z_{26} - Z_{27})}{\left(\frac{Z_{26}}{\sigma_1} - \frac{Z_{27}}{\sigma_2}\right)} \cdot \Omega$$

$$\rho = \frac{d}{A_1}$$

Solution of Equation(2.47) is

$$\sin\tau = \frac{-2\rho \pm H\sqrt{H^2 + 4 - \rho^2}}{H^2 + 4}$$

$$\cos\tau = \frac{-\rho H \mp 2\sqrt{H^2 + 4 - \rho^2}}{H^2 + 4}$$

$$\tau = \tan^{-1}\left(\frac{-2\rho \pm H\sqrt{H^2 + 4 - \rho^2}}{-\rho H \mp 2\sqrt{H^2 + 4 - \rho^2}}\right)$$

In order to have real values for $\sin\tau$ and $\cos\tau$, the clearance d can not be arbitrarily large; it should satisfy the relation $\rho^2 \leq H^2 + 4$. Physically system will not have two impacts per cycle steady state motion. The two sets of signs appearing in Equation(2.48) corresponds to distinct steady state solutions.

2.5 Impact Damper in Primary Mass 2 only

Assuming that a impact occurs at t=0 ,the next impact will occur at $t=\frac{\pi}{\Omega}$

The system should satisfy the following conditions; at

$$
\begin{aligned}
&t=0_- \quad x_1=x_c \quad x_2=x_b' \quad y_2=\tfrac{d}{2} \quad \dot{x}_1=\dot{x}_c \quad \dot{x}_2=\dot{x}_b' \quad \dot{Y}_2=V_2\\
&t=0_+ \quad x_1=x_c \quad x_2=x_b' \quad y_2=\tfrac{d}{2} \quad \dot{x}_1=\dot{x}_c \quad \dot{x}_2=\dot{x}_a' \quad \dot{Y}_2=-V_2\\
&t=(\tfrac{\pi}{\Omega})_- \quad x_1=-x_c \quad x_2=-x_b' \quad y_2=-\tfrac{d}{2} \quad \dot{x}_1=-\dot{x}_c \quad \dot{x}_2=-\dot{x}_b' \quad \dot{Y}_2=-V_2
\end{aligned}
\qquad (2.49)
$$

By proceeding as before ,we get following equations;

$$
\begin{aligned}
&x_b' + \sigma_1 \dot{x}_b' = -\tfrac{d}{2}\\
&x_b' + \sigma_2 \dot{x}_a' = -\tfrac{d}{2}\\
&x_c - B_2 - C_2 - A_1 S_1 = 0\\
&x_b' - D_1 B_2 - D_2 C_2 - A_2 S_2 = 0\\
&\dot{x}_c + h_1 B_2 - h_3 B_1 + h_2 C_2 - h_4 C_1 - A_1 E_1 = 0\\
&\dot{x}_a' + D_1 h_1 B_2 - D_1 h_3 B_1 + D_2 h_2 C_2 - D_2 h_4 C_1 - A_2 E_2 = 0\\
&x_c + h_{13} B_1 + h_{14} B_2 + h_{15} C_1 + h_{16} C_2 - A_1 S_1 = 0\\
&x_b' + D_1 h_{13} B_1 + D_1 h_{14} B_2 + D_2 h_{15} C_1 + D_2 h_{16} C_2 - A_2 S_2 = 0\\
&\dot{x}_c + \theta_9 B_1 + \theta_{10} B_2 + \theta_{11} C_1 + \theta_{12} C_2 - A_1 E_1 = 0\\
&\dot{x}_b' + D_1 \theta_9 B_1 + D_1 \theta_{10} B_2 + D_2 \theta_{11} C_1 + D_2 \theta_{12} C_2 - A_2 E_2 = 0
\end{aligned}
\qquad (2.50)
$$

Equations(2.50) furnishes 10 relations among 10 unknowns x_b ,$\dot{x}_b$,$\dot{x}_a'$,x_c' ,$\dot{x}_c'$,B_1 ,B_2 ,C_1 ,C_2 , τ

$$
\begin{bmatrix}
1 & 0 & \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & \sigma_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -D_1 & & -D_2 & -S_2 \\
0 & 0 & 0 & 0 & 1 & -h_3 & h_1 & -h_4 & h_2 & 0 \\
0 & 0 & 0 & 1 & 0 & -D_1 h_3 & D_1 h_1 & -D_2 h_4 & D_2 h_2 & -E_2 \\
0 & 1 & 0 & 0 & 0 & h_{13} & h_{14} & h_{15} & h_{16} & 0 \\
1 & 0 & 0 & 0 & 0 & D_1 h_{13} & D_1 h_{14} & D_2 h_{15} & D_2 h_{16} & -S_2 \\
0 & 0 & 0 & 0 & 1 & \theta_9 & \theta_{10} & \theta_{11} & \theta_{12} & 0 \\
0 & 0 & 1 & 0 & 0 & D_1 \theta_9 & D_1 \theta_{10} & D_2 \theta_{11} & D_2 \theta_{12} & -E_2
\end{bmatrix}
\begin{Bmatrix}
x'_b \\ x_c \\ \dot{x}'_b \\ \dot{x}'_a \\ \dot{x}_c \\ B_1 \\ B_2 \\ C_1 \\ C_2 \\ A_2
\end{Bmatrix}
=
\begin{Bmatrix}
-\frac{d}{2} \\ -\frac{d}{2} \\ A_1 S_1 \\ 0 \\ A_1 E_1 \\ 0 \\ A_1 S_1 \\ 0 \\ A_1 E_1 \\ 0
\end{Bmatrix}
\qquad (2.51)
$$

From the solution of Equation(2.51),expression for A_2 is

$$A_2 = \frac{N(A_2)}{\Delta} \tag{2.52}$$

where

$$N(A_2) = \sigma_1 \sigma_2 h_{13} U_{16} \cdot h_{15} (D_2 - D_1) \left(\frac{d}{2\sigma_2} U_{27} - \frac{d}{2\sigma_1} U_{26} \right)$$

$$\Delta = \sigma_1 \sigma_2 h_{13} U_{16} \cdot h_{15} (D_2 - D_1) (U_5 U_{27} - U_8 U_{26})$$

Equation(2.52) can be written in the form

$$2 \sin \tau_1 + H_1 \cos \tau_1 = -\rho_1 \tag{2.53}$$

where

$$H_1 = \frac{2(U_{26} - U_{27})}{\left(\frac{U_{26}}{\sigma_1} - \frac{U_{27}}{\sigma_2} \right)} \cdot \Omega$$

$$\rho_1 = \frac{d}{A_2}$$

Solution of Equation (2.53) for τ_1 results in

$$\sin \tau_1 = \frac{-2\rho_1 \pm H_1 \sqrt{H_1^2 + 4 - \rho_1^2}}{H_1^2 + 4}$$

$$\cos \tau_1 = \frac{-\rho_1 H_1 \mp 2\sqrt{H_1^2 + 4 - \rho_1^2}}{H_1^2 + 4}$$

In order to have real values of $\sin\tau_1$ and $\cos\tau_1$, the clearance d can not be arbitraily large;it should satisfy the relation $\rho_1^2 \leq H_1^2 + 4$ .Physically system will not have two impacts per cycle steady state motion.

$$\tau_1 = \tan^{-1} \left(\frac{-2\rho_1 \pm \sqrt{H_1^2 + 4 - \rho_1^2}}{-\rho_1 H_1 \mp 2\sqrt{H_1^2 + 4 - \rho_1^2}} \right)$$

The two sets of signs appearing in Equation (2.54) correspond to two distincit steady state solutions.

2.6 Special cases of Resonance and no Damping

For the system with forcing frequency approximately equal to the natural frequency and without viscous damping, the steady state solution with impact damper in action can be derived in much simpler way.

a) Impact Damper in Primary mass 1 at First Natural Frequency

Here $\delta_1 = \delta_2 = 0$

The Equations (2.2-a), (2.2-b), (2.14) and (2.15) are simplified as below;

$$x_1 = B_1 \sin\omega_1 t + B_2 \cos\omega_1 t + C_1 \sin\omega_2 t + C_2 \cos\omega_2 t + A_1 \sin(\Omega t + \tau) \qquad (2.55)$$

$$x_2 = D_1 B_1 \sin\omega_1 t + D_1 B_2 \cos\omega_1 t + D_2 C_1 \sin\omega_2 t + D_2 C_2 \cos\omega_2 t + A_2' \sin(\Omega t + \tau) \qquad (2.56)$$

$$\dot{x}_1 = \omega_1 B_1 \cos\omega_1 t - \omega_1 B_2 \sin\omega_1 t + \omega_2 C_1 \cos\omega_2 t - \omega_2 C_2 \sin\omega_2 t + A_1 \Omega \cos(\Omega t + \tau) \qquad (2.57)$$

$$\dot{x}_2 = \omega_1 D_1 B_1 \cos\omega_1 t - \omega_1 D_1 B_2 \sin\omega_1 t + \omega_2 D_2 C_1 \cos\omega_2 t - \omega_2 D_2 C_2 \sin\omega_2 t + A_2 \Omega \cos(\Omega t + \tau) \qquad (2.58)$$

For $\Omega \simeq \omega_1$, let $\Omega = \omega_1(1+\epsilon)$, $\epsilon \ll 1$ then

$$\sin(\Omega t + \tau) = \epsilon \omega_1 t \cos(\omega_1 t + \tau) + \sin\omega_1 t \cos\tau + \cos\omega_1 t \sin\tau \qquad (2.59)$$

$$\Omega \cos(\Omega t + \tau) = \omega_1 (\cos(\omega_1 t + \tau) - \epsilon(\omega_1 t \sin(\omega_1 t + \tau) - \cos(\omega_1 t + \tau))) \qquad (2.60)$$

Assuming that an impact occurs at t=0, the next impact will occur at $t = \frac{\pi}{\omega_1}$.

From Equations (2.40-b),(2.55) and (2.57) we get;

$$x_1(0) = x_b = B_2 + C_2 + A_1 \sin(\tau) \qquad (2.61)$$

$$\dot{x}_1(0) = \dot{x}_a = \omega_1 B_1 + \omega_2 C_1 + A_1 \omega_1 (1+\epsilon) \cos(\tau) \qquad (2.62)$$

Hence by substituting (2.61) and (2.62) in (2.55) and making use of (2.59), we get;

$$x_1 = x_b \cos\omega_1 t + \frac{\dot{x}_a}{\omega_1} \sin\omega_1 t + C_1(\sin\omega_2 t - \frac{\omega_2}{\omega_1}\sin\omega_1 t) + C_2(\cos\omega_2 t - \cos\omega_1 t) + A_1 \epsilon\, \omega_1 t \cos(\omega_1 t + \tau) - A_1 \epsilon \sin\omega_1 t \cos(\tau) \qquad (2.63)$$

since, $\lim_{\epsilon \to 0} A_1 \epsilon = \frac{-F_0'(\sqrt{5}-1)}{2\sqrt{5}(3K_2 + 2K_1 - K_2\sqrt{5})} = A_1'$ (2.64)

So

$$x_1 = x_b \cos\omega_1 t + \frac{\dot{x}_a}{\omega_1} \sin\omega_1 t + C_1(\sin\omega_2 t - \frac{\omega_2}{\omega_1}\sin\omega_1 t) + C_2(\cos\omega_2 t + \cos\omega_1 t) + A_1' \omega_1 t \cos(\omega_1 t + \tau) - A_1' \sin\omega_1 t \cos(\tau) \qquad (2.65)$$

From the condition that $\dot{x}_1(\frac{\pi}{\omega}) = -\dot{x}_b$ and by differenciating Equation (2.65) and substituting $\omega_1 t = \pi$ we obtain,

$$\dot{x}_a - \dot{x}_b = C_1(\omega_2 \cos\frac{\pi\omega_2}{\omega_1} + \omega_2) + C_2(-\omega_2 \sin\frac{\pi\omega_2}{\omega_1}) - A_1'\omega_1 \cos\tau + A_1' \pi \sin\tau + A_1' \omega \cos(\tau) \qquad (2.66)$$

By substituting Equation (2.10) from (2.11) and rearranging terms:

$$\dot{x}_a - \dot{x}_b = \frac{2u\omega_1(d + 2x_b)}{\pi} \qquad (2.67)$$

So Equation (2.66) becomes

$$\frac{2u\omega_1(d+2x_b)}{\pi} = C_1\omega_2(1 + \cos\frac{\pi\omega_2}{\omega_1}) + C_2(-\omega_2 \sin\frac{\pi\omega_2}{\omega_1}) - A_1'\omega_1\cos\tau + A_1'\pi\sin\tau + A_1'\omega_1\cos\tau \qquad (2.68)$$

Using the condition that $x_1(\frac{\pi}{\omega_1}) = -x_b$

$$-x_b = -x_b + C_1(\sin\frac{\pi\omega_2}{\omega_1}) + C_2(\cos\frac{\pi\omega_2}{\omega_1} + 1) - A_1'\pi\cos(\tau)$$

Or $$C_1(\sin\frac{\pi\omega_2}{\omega_1}) + C_2(\cos\frac{\pi\omega_2}{\omega_1} + 1) - A_1'\pi\cos(\tau) = 0 \quad (2.69)$$

By proceeding similiarly for primary mass 2, we get;

$$D_2C_1\sin(\frac{\pi\omega_2}{\omega_1}) + D_2C_2(\cos\frac{\pi\omega_2}{\omega_1} + 1) - A_2'\pi\cos\tau = 0 \quad (2.70)$$

where

$$A_2' = A_2\epsilon = \frac{F_0}{\sqrt{5}\cdot(3K_2 + 2K_1 - K_2\sqrt{5})} = D_1A_1' \quad (2.71)$$

$$D_2C_1\omega_2(1 + \cos\frac{\pi\omega_2}{\omega_1}) + D_2C_2(-\omega_2\sin\frac{\pi\omega_2}{\omega_1}) - A_2\omega_1'\cos(\tau) + A_2'\pi\sin(\tau) + A_2'\omega_1\cos(\tau) = 0 \quad (2.72)$$

By solving Equation(2.69) and (2.70)

$$(-D_2A_1' + A_2')\pi\cos\tau = 0 \quad (2.73)$$

The only solution is

$$\tau = -\pi/2 \quad (2.74)$$

From Equation(2.69)

$$C_1/C_2 = -(1 + \cos\frac{\pi\omega_2}{\omega_1})/\sin\frac{\pi\omega_2}{\omega_1} \quad (2.75)$$

From Equation (2.72) we obtain;

$$C_2 = -\frac{A_2'\pi\sin\frac{\pi\omega_2}{\omega_1}}{2(1 + \cos\frac{\pi\omega_2}{\omega_1})D_2\omega_2} \quad (2.76)$$

$$C_1 = \frac{A_2'\pi}{2D_2\omega_2} \quad (2.77)$$

From Equation(2.68)

$$x_b = \frac{\pi^2}{4\mu\omega_1}(\frac{A_2'}{D_2} - A_1') - \frac{d}{2}$$

$$= \frac{\pi^2}{4\mu\omega_1}A_1'(\frac{D_1}{D_2} - 1) - \frac{d}{2} \quad (2.78)$$

Making use of Equations(2.78) and (2.11),we get

$$\dot{x}_a = -\frac{(1-e-2\mu e)}{(1+e)} \frac{\pi A_1'}{2\mu} \left(\frac{D_1}{D_2} - 1\right) \qquad (2.79)$$

So the equations of the steady state are

$$x_1 = (x_b - C_2)\cos\omega_1 t + \left(\frac{\dot{x}_a}{\omega_1} - \frac{C_1\omega_2}{\omega_1}\right)\sin\omega_1 t + C_2\cos\omega_2 t + C_1\sin\omega_2 t + A_1'\,\omega_1 t\sin(\omega_1 t) \qquad (2.80)$$

$$x_2 = D_1(x_b - C_2)\cos\omega_1 t + D_1\left(\frac{\dot{x}_a}{\omega_1} - \frac{C_1\omega_2}{\omega_1}\right)\sin\omega_1 t + D_2 C_2\cos\omega_2 t + D_2 C_1\sin\omega_2 t + A_2'\,\omega_1 t\sin(\omega_1 t) \qquad (2.81)$$

b) <u>Impact Damper in Primary mass 1 and Second Natural Frequency</u>

Proceeding as before and taking $\Omega = \omega_2(1+\epsilon)$, the steady state solution will be;

$$x_1 = (x_b - B_2)\cos\omega_2 t + \left(\frac{\dot{x}_a}{\omega_2} - \frac{B_1\omega_1}{\omega_2}\right)\sin\omega_2 t + B_2\cos\omega_1 t + B_1\sin\omega_1 t + A_1''\,\omega_2 t\sin(\omega_2 t) \qquad (2.82)$$

$$x_2 = D_2(x_b - B_2)\cos\omega_2 t + D_2\left(\frac{\dot{x}_a}{\omega_2} - \frac{B_1\omega_1}{\omega_2}\right)\sin\omega_2 t + D_1 B_2\cos\omega_1 t + D_1 B_1\sin\omega_1 t + A_2''\,\omega_2 t\sin\omega_2 t \qquad (2.83)$$

where

$$B_2 = -\frac{A_2''\,\pi\sin\pi\frac{\omega_1}{\omega_2}}{2\left(1+\cos\frac{\pi\omega_1}{\omega_2}\right)D_1\omega_1}$$

$$B_1 = \frac{A_2''\,\pi}{2D_1\omega_1}$$

$$x_b = \frac{\pi^2 A_1''}{4\mu\omega_2}\left(\frac{D_2}{D_1} - 1\right) - \frac{d}{2}$$

$$\dot{x}_a = -\frac{(1-e-2\mu e)}{(1+e)}\frac{\pi A_2'}{4\mu}\left(\frac{D_2}{D_1} - 1\right)$$

$$A_1'' = -\frac{F_0(1+\sqrt{5})}{2\sqrt{5}\,(3K_2 + 2K_1 + \sqrt{5}K_2)}$$

$$A_2'' = \frac{F_0}{\sqrt{5}\,(3K_2 + 2K_1 + \sqrt{5}K_2)} = D_2 A_1''$$

c) <u>Impact Damper in Primary mass 2 at first natural frequency</u>

Using Equations(2,49),(2.55),(2.56),(2.57) and (2.58) proceeding as before;

$$x_2 = (x_b' - D_2C_2)\cos\omega_1 t + \left(\frac{\dot{x}_a'}{\omega_1} - \frac{D_2C_1\omega_2}{\omega_1}\right)\sin\omega_1 t + D_2C_2\cos\omega_2 t + D_2C_1\sin\omega_2 t + A_2'\omega_1 t\sin\omega_1 t \tag{2.84}$$

$$x_1 = \frac{1}{D_1}(x_b' - D_2C_2)\cos\omega_1 t + \frac{1}{D_1}\left(\frac{\dot{x}_a'}{\omega_1} - \frac{D_2C_1\omega_2}{\omega_1}\right)\sin\omega_1 t + C_2\cos\omega_2 t + C_1\sin\omega_2 t + A_1'\omega_1 t\sin\omega_1 t \tag{2.85}$$

Where

$$C_2 = \frac{A_1'\pi\sin\frac{\pi\omega_2}{\omega_1}}{2\left(1+\cos\frac{\pi\omega_2}{\omega_1}\right)\omega_2}$$

$$C_1 = \frac{A_1'\pi}{2\omega_2}$$

$$x_b' = -\frac{(1-e-2\mu e)}{(1+e)}\frac{\pi A_2'}{2\mu}\left(\frac{D_2}{D_1} - 1\right)$$

$$\dot{x}_a' = -\left(\frac{1-e-2\mu e}{1+e}\right)\frac{\pi A_2'}{2\mu}\left(\frac{D_2}{D_1}-1\right)$$

d) Impact Damper in Primary Mass 2 at Second Natural Frequency

Proceeding as before and taking $\Omega_2 = \omega_2(1+\epsilon)$, the steady state solution will be;

$$x_2 = (x_b' - D_1 B_2)\cos\omega_2 t + \left(\frac{\dot{x}_a'}{\omega_2} - \frac{D_1 B_1 \omega_1}{\omega_2}\right)\sin\omega_2 t +$$

$$D_1 B_2 \cos\omega_1 t + D_1 B_1 \sin\omega_1 t + A_2''\omega_2 t \sin\omega_2 t \qquad (2.86)$$

$$x_1 = \frac{1}{D_2}(x_b' - D_1 B_2)\cos\omega_2 t + \frac{1}{D_2}\left(\frac{\dot{x}_a'}{\omega_2} - \frac{D_1 B_1 \omega_1}{\omega_2}\right)\sin\omega_2 t +$$

$$B_2 \cos\omega_1 t + B_1 \sin\omega_1 t + A_1''\omega_2 t \sin\omega_2 t \qquad (2.87)$$

where

$$B_2 = \frac{A_1''\pi \sin\frac{\pi\omega_1}{\omega_2}}{2\left(1+\cos\frac{\pi\omega_1}{\omega_2}\right)\cdot\omega_1}$$

$$B_1 = \frac{A_1''\pi}{2\omega_2}$$

$$x_b' = \frac{\pi^2 A_2''}{4\mu\omega_2}\left(\frac{D_1}{D_2}-1\right) - \frac{d}{2}$$

$$\dot{x}_a' = -\frac{(1-e-2\mu e)}{(1+e)}\frac{\pi A_2''}{2\mu}\left(\frac{D_1}{D_2}-1\right)$$

2.7 Energy lost per Impact

From Equations (2.4) and (2.5), we can find that for impact in primary mass 1;

$$W_{imp_1} = 2\mu(1-e)\frac{(1+\mu)}{1+e}\left(\frac{\Omega}{\pi}(d+2x_b)\right)^2 M \tag{2.88}$$

For impact in primary mass 2

$$W_{imp_2} = 2\mu(1-e)\frac{(1+\mu)}{1+e}\left(\frac{\Omega}{\pi}(d+2x'_b)\right)^2 M \tag{2.89}$$

3. STABILITY ANALYSIS

3.1 Introduction

The steady state solutions,derived in chapter 2, are not necessary to be actual silutions as we have made certain assumptions.Thus the stability criterion for the three cases is obtained in this chapter.The stability analysis,as suggested by Masri[6],involves a perturbation of the phase space trajectory of the motion.The solution will be stable if the modulus of all the eigen values of a certain matrix is less than unity.The matrix continuously relates the perturbation immediately after each of c consecutive impact in one primary mass.

3.2 Theoretical consideration

Let the differential equation of our system be expressed in the form

$$\dot{\vec{Z}} = \vec{F}(Z_1, Z_2, \cdots, Z_8, t) \qquad (3.1)$$

and let the solution of Equation(3.1) be

$$\vec{Z} = \vec{S}(t) \quad .$$

If this solution is perturbed slightly,so that

$$\vec{Z}_p = \vec{S}(t) + \vec{\xi}(t) \quad ,$$

the solution is said to be asymptotically stable if

$$\lim_{t \to \infty} |\xi_i(t)| = 0 \quad ,\text{for } i=1,8 \ .$$

For present case,since steady state solution is

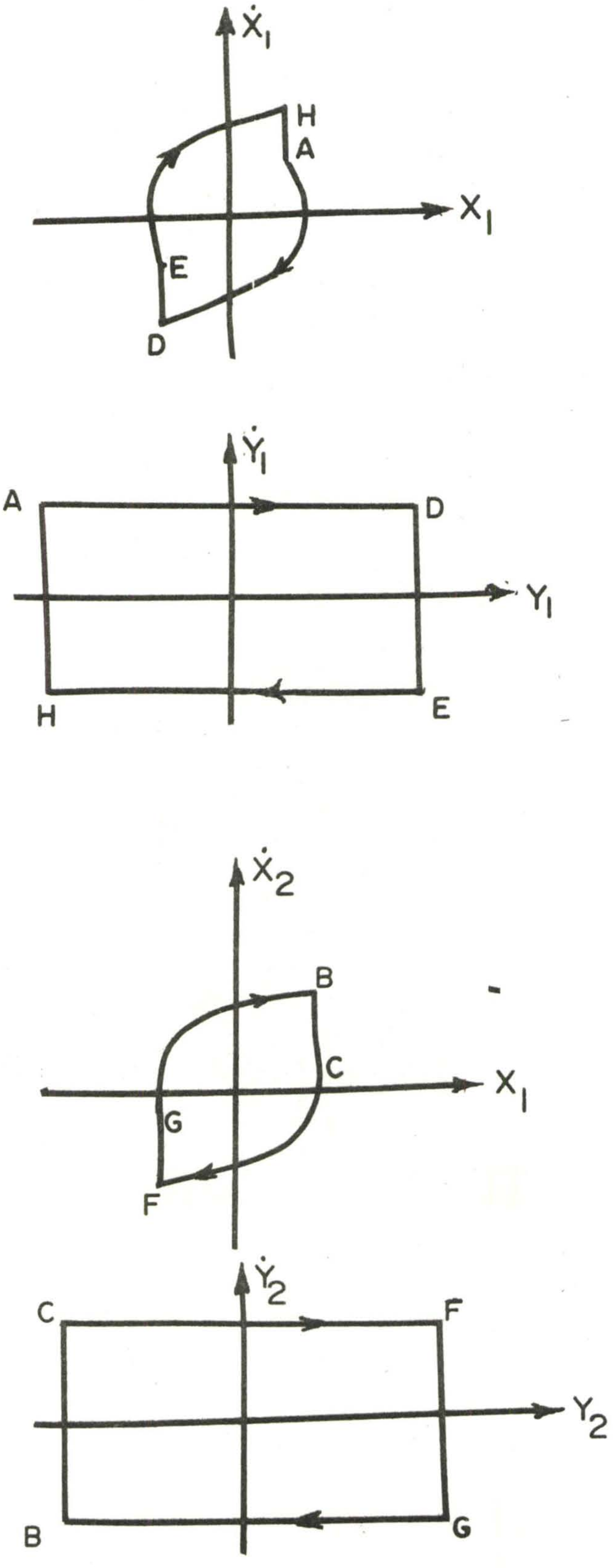


FIG 2 PHASE PLANE REPRESENTATION FOR IMPACT DAMPER IN BOTH MASSES

valid only between two consecutive impacts in the system, the stability or instability of the system is determined by whether or not the deviation from steady state decrease or increase as the number of the impacts is increased indefinitely(i.e. as $t \to \infty$). The differential equation of motion of our system between impacts is :

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x},t) \qquad (3.2)$$

where

$$\vec{x} = \begin{pmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{pmatrix} \qquad \vec{f}(\vec{x},t) = \begin{pmatrix} \dot{x}_1 \\ -\frac{(K_1+K_2)x_1}{M} + \frac{K_2 x_2}{M} - \frac{c\dot{x}_1}{M} + \frac{F_0 \sin(\Omega t+\alpha)}{M} \\ \dot{x}_2 \\ -\frac{(K_1+K_2)x_2}{M} + \frac{K_2 x_1}{M} - \frac{c\dot{x}_2}{M} \\ \dot{y}_1 \\ 0 \\ \dot{y}_2 \\ 0 \end{pmatrix} .$$

Now let the solution curve be perturbed slightly right after an impact i.e. at A.If at $\Omega t = (\Delta t_0)_+$ the steady state solution is perturbed by a small amount $\vec{\xi}_{(0)}$,then the time of the next impact, $\Omega t = \pi + \Delta t_0'$,is determined by a relation of the form

$$\Delta t_0' = g(\vec{\varsigma}, \vec{\xi}_{(0)}, \Delta t_0') \quad .$$

By making use of the theory of implicit function,at $\Omega t = (\pi + \Delta t_0')_+$ thedeviation of solution from steady state can be put in the form

$$\vec{\xi}_{(1)} = P\vec{\xi}_{(0)} + \vec{R}(\vec{\xi}_{(0)}) \qquad ,$$

where P is a constant matrix and $\vec{R}$ contains all the terms of ξ_i higher than the first power.Since the two-impact per cycle solution repeats itself after interval of $\Omega t = \pi$,the perturbation at $\Omega t = (2\pi + \Delta t_o'')_+$ will be

$$\vec{\xi}_{(2)} = P\vec{\xi}_{(1)} + \vec{R}(\vec{\xi}_{(1)}) \quad .$$

By following the perturbed solution from one impact to the next one,we obtain the continuous transformation

$$\vec{\xi}_{(n+1)} = P\vec{\xi}_{(n)} + \vec{R}(\vec{\xi}_{(n)}) \tag{3.3}$$

$$= P^{n+1}\vec{\xi}_{(0)} + \vec{R}(\vec{\xi}_{(0)}) \quad . \tag{3.4}$$

Consider the linear part of equation(3.4)

$$\vec{\xi}_{(n+1)} = P^{n+1}\xi_{(0)} \quad . \tag{3.5}$$

Equation(3.5) will be asymptotically stable if and only if

$$\lim_{n\to\infty} P^n = 0 \quad .$$

Now if the eigen values of P are distinct,it can be diagonalised by similiarty transformation,so that

$$P^n = S\Lambda^n S^{-1} \quad .$$

The requirement for stability,

$$\lim_{n\to\infty} P^n = 0$$

is satisfied if and only if all the eigen values of P have modulus less than unity,i.e.,if

$$|\lambda_i| < 1 \qquad (\ i=1,\ldots 8).$$

The theorem, derived in reference (6), states:

If 1) $\|\xi_{(0)}\|$ is sufficiently small

2) $\lim_{\|\vec{\xi}\| \to 0} \frac{\|\vec{R}(\xi)\|}{\|\vec{\xi}\|} = 0$

and if $\vec{\xi}_{(n+1)} = P^{n+1} \vec{\xi}_{(0)}$ is asymptotically stable

then also is $\vec{\xi}_{(n+1)} = P^{n+1} \vec{\xi}_{(0)} + \vec{R}'(\vec{\xi}_{(0)})$. Thus our problem is to determine P and to examine its eigen values.

3.3 Impact Damper in both Primary Masses

Since our system has four degrees of freedom, $\vec{\xi}$ should be eight component vector. Here the perturbation consist of small variation of steady state values of x_1, $\dot{x}_1$, x_2, $\dot{x}_2$, V_1, V_2, τ, t_1. Here o suscript refers to the unperturbed conditions.

The conditions that we have are:

Steady State	Perturbed
At $\Omega t = 0_+$	At $\Omega t = (0 + \Delta t_{10})_+$
$x_1 = x_{10}$	$x_1 = x_{10} + \Delta x_{10}$
$y_1 = \frac{d}{2}$	$y_1 = \frac{d}{2}$
$\dot{x}_1 = \dot{x}_{10}$	$\dot{x}_1 = \dot{x}_{10} + \Delta \dot{x}_{10}$
$\dot{Y}_1 = -V_{10}$	$\dot{Y}_1 = -(V_{10} + \Delta V_{10})$
$\tau = \tau_0$	$\tau = \tau_0 + \Delta \tau_0$
$x_2 = x_{2c}$	$x_2 = x_{2c} + \Delta x_{2c}$
$\dot{x}_2 = \dot{x}_{2c}$	$\dot{x}_2 = \dot{x}_{2c} + \Delta \dot{x}_{2c}$
$t_1 = t_{10}$	$t_1 = t_{10} + \Delta t_{10}$
$\dot{Y}_2 = V_{20}$	$\dot{Y}_2 = -V_{20} + \Delta V_{20}$

Steady State

At $\Omega t = t_{10+}$

$x_1 = x_{1c}$

$\dot{x}_1 = \dot{x}_{1c}$

$x_2 = x_{20}$

$\dot{x}_2 = \dot{x}_{20}$

$\dot{Y}_2 = -V_{20}$

$y_2 = \frac{d}{2}$

At $\Omega t = \pi_+$

$x_1 = -x_{10}$

$y_1 = -\frac{d}{2}$

$\dot{x}_1 = -\dot{x}_{10}$

$\dot{Y}_1 = V_{10}$

$\tau = \tau_0$

$x_2 = -x_{2c}$

$\dot{x}_2 = -\dot{x}_{2c}$

$t_1 = t_{10}$

$\dot{Y} = -V_{20}$

At $\Omega t = (\pi + t_{10})_-$

$x_2 = -x_{20}$

Perturbed

At $\Omega t = t_{10} + \Delta t_{10}$

$x_1 = x_{1c} + \Delta x_{1c}$

$\dot{x}_1 = \dot{x}_{1c} + \Delta \dot{x}_{1c}$

$x_2 = x_{20} + \Delta x_{20}$

$\dot{x}_2 = \dot{x}_{20} + \Delta \dot{x}_{20}$

$\dot{Y}_2 = -(V_{20} + \Delta V_{20}')$

$y_2 = \frac{d}{2}$

At $\Omega t = (\pi + \Delta t_0'')$

$x_1 = -(x_{10} + \Delta x_{10}')$

$y_1 = -\frac{d}{2}$

$\dot{x}_1 = -(\dot{x}_{10} + \Delta \dot{x}_{10}')$

$\dot{Y}_1 = (V_{10} + \Delta V_{10}')$

$\tau = \tau_0 + \Delta \tau_0'$

$x_2 = -(x_{2c} + \Delta x_{2c}')$

$\dot{x}_2 = -(\dot{x}_{2c} + \Delta \dot{x}_{2c})$

$t_1 = t_{10} + \Delta t_{10}'$

$\dot{Y}_2 = -(V_{20} + \Delta V_{20}')$

At $\Omega t = (\pi + t_{10} + \Delta t_0''')_-$

$x_2 = -(x_{20} + \Delta x_{20}')$

To simplify, l.t us put new variables

$\eta_1' = \eta_1 \omega_1$ $\qquad \delta_1' = \delta_1 \omega_1$

$\eta_2' = \eta_2 \omega_2$ $\qquad \delta_2' = \delta_2 \omega_2$

So Equations of motion are

$$x_1 = e^{-\delta_1' t}(B_1 \sin \eta_1' t + B_2 \cos \eta_1 t) + e^{-\delta_2' t}(C_1 \sin \eta_2' t + C_2 \cos \eta_2' t) + A_1 \sin(\Omega t + \tau) \quad (3.7)$$

$$x_2 = D_1 e^{-\delta_1' t}(B_1 \sin \eta_1' t + B_2 \cos \eta_1' t) + D_2 e^{-\delta_2' t}(C_1 \sin \eta_2' t + C_2 \cos \eta_2' t) + A_2 \sin(\Omega t + \tau + \psi) \quad (3.8)$$

$$\dot{x}_1 = e^{-\delta_1' t}(-\delta_1' \sin \eta_1' t + \eta_1' \cos \eta_1' t) B_1 + e^{-\delta_1' t}(-\delta_1' \cos \eta_1' t - \eta_1 \sin \eta_1' t) B_2 + e^{-\delta_2' t}(-\delta_2 \sin \eta_2' t + \eta_2 \cos \eta_2' t) C_1 + e^{-\delta_2' t}(-\delta_2 \cos \eta_2 t - \eta_2' \sin \eta_2 t) C_2 + \Omega A_1 \cos(\Omega t + \tau) \quad (3.9)$$

$$\dot{x}_2 = D_1 (e^{-\delta_1' t}(-\delta_1' \sin \eta_1' t + \eta_1' \cos \eta_1' t) B_1 + e^{-\delta_1' t}(-\delta_1' \cos \eta_1' t - \eta_1' \sin \eta_1' t) B_2) + D_2 (e^{-\delta_2' t}(-\delta_2' \sin \eta_2' t + \eta_2' \cos \eta_2' t) C_1 + e^{-\delta_2' t}(-\delta_2' \cos \eta_2' t - \eta_2' \sin \eta_2' t) C_2) + \Omega A_2 \cos(\Omega t + \tau + \psi) \quad (3.10)$$

Now

$$x_1(0)_+ = x_{10} = B_{20} + C_{20} + A_1 \sin(\tau_0) \quad (3.11)$$

$$\dot{x}_1(0)_+ = \dot{x}_{10} = \eta_1' B_{10} - \delta_1' B_{20} + \eta_2' C_{10} - \delta_2' C_{20} + \Omega A_1 \cos \tau_0 \quad (3.12)$$

$$x_2(0)_+ = x_{2c} = D_1 B_{20} + D_2 C_{20} + A_2 \sin(\tau_0 + \psi) \quad (3.13)$$

$$\dot{x}_2(0)_+ = \dot{x}_{2c} = D_1 \eta_1' B_{10} - D_1 \delta_1' B_{20} + D_2 \eta_2' C_{10} - D_2 \delta_2' C_{20} + \Omega A_2 \cos(\tau_0 + \psi) \quad (3.14)$$

From Equations(3.11),(3.12),(3.13) and (3.14)

$$B_{20} = \frac{D_2 x_{10} - x_{2c} - D_2 A_1 \sin \tau_0 + A_2 \sin(\tau_0 + \psi)}{D_2 - D_1} \quad (3.16)$$

$$C_{20} = \frac{D_1 x_{10} - x_{2c} - D_1 A_1 \sin \tau_0 + A_2 \sin(\tau_0 + \psi)}{D_1 - D_2}$$

$$B_{10} = (D_2(\dot{x}_{10} + \delta_1' x_{10}) - (\dot{x}_{2c} + \delta_1' x_{2c}) - D_2 A_1(\delta_1' \sin\tau_o + \Omega\cos\tau_o) + A_2(\delta_1' \sin(\tau_o+\psi) + \Omega\cos(\tau_o+\psi_o)))/(\eta_1'(D_2-D_1)) \tag{3.17}$$

$$C_{10} = (D_1(x_{10}' + \delta_2' x_{10}) - (\dot{x}_{2c} + \delta_2' x_{2c}) - D_1 A_1(\delta_2' \sin\tau_o + \Omega\cos\tau_o)) + A_2(\delta_2 \sin(\tau_o+\psi) + \Omega\cos(\tau_o+\psi)))/(\eta_2'(D_1-D_2)) \tag{3.18}$$

In finding out perturbed values of B_{10}, B_{20}, C_{10} and C_{20}, the quantities with subscript o in the above equations should be replaced by their perturbed values. Thus

$$B_{2(o+\Delta t_o)} = (D_2(x_{10}+\Delta x_{10}) - x_{2c} - \Delta x_{2c} - D_2 A_1 \sin(\tau_o+\Delta\tau_o) + A_2 \sin(\tau_o+\Delta\tau_o+\psi))/(D_2-D_1) \tag{3.19}$$

$$= B_{20} + (D_2\Delta x_{10} - \Delta x_{2c} - D_2\Delta\tau_o A_1 \cos\tau_o + A_2\Delta\tau_o\cos(\tau_o+\psi))/(D_2-D_1) \tag{3.20}$$

similiarly

$$\Delta B_{2o} = (D_2\Delta x_{10} - \Delta x_{2c} - D_2\Delta\tau_o A_1\cos\tau_o + A_2\Delta\tau_o\cos(\tau_o+\psi))/(D_2-D_1) \tag{3.21}$$

$$\Delta C_{2o} = (D_1\Delta x_{10} - \Delta x_{2c} - D_1\Delta\tau_o A_1\cos\tau_o + A_2\Delta\tau_o\cos(\tau_o+\psi))/(D_1-D_2) \tag{3.22}$$

$$\Delta B_{10} = (D_2(\Delta\dot{x}_{10} + \delta_1\Delta x_{10}) - (\Delta\dot{x}_{2c} + \delta_1'\Delta x_{2c}) - D_2 A_1(\delta_1\Delta\tau_o\cos\tau_o - \Omega\Delta\tau_o\sin\tau_o) + A_2(\delta_1'\Delta\tau_o\cos(\tau_o+\psi) - \Omega\Delta\tau_o\sin(\tau_o+\psi)))/(\eta_1'(D_2-D_1)) \tag{3.23}$$

$$\Delta C_{10} = (D_1(\Delta\dot{x}_{10} + \delta_2'\Delta x_{10}) - (\Delta\dot{x}_{2c} + \delta_2'\Delta x_{2c}) - D_1 A_1(\delta_2'\Delta\tau_0 \cos\tau_0 - \Omega\Delta\tau_0 \sin\tau_0) + A_2(\delta_2'\Delta\tau_0 \cos(\tau_0+\psi) - \Omega\Delta\tau_0 \sin(\tau_0+\psi)))/(\eta_2'(D_1 - D_2)) \qquad (3.24)$$

Equation(3.7) describe the motion of primary mass 1 from $t = \frac{0+\Delta t_0}{\Omega}$ to immediately prior to $t = \frac{t_{10}+\Delta t_{10}'}{\Omega}$. So Equation (3.7) is applicable for $\Omega t = t_{10} + \Delta t_{10}$, where $\Delta t_{10} = \Delta t_0' - \Delta t_0$ .From the condition that;

$$x_1(t_{10}+\Delta t_0') = x_{1c} + \Delta x_{1c}$$

$$x_{1c} + \Delta x_{1c} = e^{-\delta_1'\frac{(t_{10}+\Delta t_{10})}{\Omega}}((B_{10}+\Delta B_{10})\sin\eta_1'\frac{(t_{10}+\Delta t_{10}')}{\Omega} + (B_{20}+\Delta B_{20})\cos\eta_1'\frac{(t_{10}+\Delta t_{10})}{\Omega}) + e^{-\delta_2'\frac{(t_{10}+\Delta t_{10})}{\Omega}}((C_{10}+\Delta C_{10})\sin\eta_2'\frac{(t_{10}+\Delta t_{10})}{\Omega} + (C_{20}+\Delta C_{20})\cdot\cos\eta_2'\frac{(t_{10}+\Delta t_{10})}{\Omega}) + A_1\sin(t_{10}+\Delta t_{10}+\tau_0+\Delta\tau_0) \qquad (3.25)$$

$$= x_{1c} + e^{-\frac{\delta_1' t_{10}}{\Omega}}(\frac{\Delta t_{10}}{\Omega}\eta_1' B_{10}\cos\frac{\eta_1' t_{10}}{\Omega} + \Delta B_{10}\sin\frac{\eta_1' t_{10}}{\Omega} + \Delta B_{20}\cos\frac{\eta_1' t_{10}}{\Omega} - \frac{\Delta t_{10}}{\Omega}\cdot\eta_1' B_{20}\cdot\sin\frac{\eta_1' t_{10}}{\Omega}) + e^{-\frac{\delta_2' t_{10}}{\Omega}}(\frac{\Delta t_{10}}{\Omega}\cdot\eta_2' C_{10}\cdot\cos\frac{\eta_2' t_{10}}{\Omega} + \Delta C_{10}\cdot\sin\frac{\eta_2' t_{10}}{\Omega} + \Delta C_{20}\cos\frac{\eta_2' t_{10}}{\Omega} - \frac{\Delta t_{10}}{\Omega}\eta_2' C_{20}\sin\frac{\eta_2' t_{10}}{\Omega}) + A_1(\Delta t_{10}+\Delta\tau_0)\cos(t_{10}+\tau_0)$$

$$\Delta x_{1c} = e^{-\frac{\delta_1' t_{10}}{\Omega}}(\frac{\Delta t_{10}}{\Omega}\cdot\eta_1' B_{10}\cos\frac{\eta_1' t_{10}}{\Omega} + \Delta B_{10}\sin\frac{\eta_1' t_{10}}{\Omega} + \Delta B_{20}\cos\frac{\eta_1' t_{10}}{\Omega} - \frac{\Delta t_{10}}{\Omega}\cdot\eta_1' B_{20}\sin\frac{\eta_1' t_{10}}{\Omega}) + e^{-\frac{\delta_2' t_{10}}{\Omega}}(\frac{\Delta t_{10}}{\Omega}\cdot\eta_2' C_{10}\cos\frac{\eta_2' t_{10}}{\Omega} + \Delta C_{10}\cdot\sin\frac{\eta_2' t_{10}}{\Omega} + \Delta C_{20}\cos\frac{\eta_2' t_{10}}{\Omega} - \frac{\Delta t_{10}}{\Omega}\cdot\eta_2' C_{20}\sin\frac{\eta_2' t_{10}}{\Omega}) + A_1(\Delta t_{10}+\Delta\tau_0)\cos(t_{10}+\tau_0) \qquad (3.26)$$

similiarly

$$\Delta x_{2c} = D_1 e^{-\frac{\delta_1' t_{10}}{\Omega}} \Big(\frac{\Delta t_{10}}{\Omega} \eta_1' B_{10} \cos \frac{\eta_1' t_{10}}{\Omega} + \Delta B_{10} \sin \frac{\eta_1' t_{10}}{\Omega} + \Delta B_{20} \cos \frac{\eta_1' t_{10}}{\Omega} - \frac{\Delta t_{10}}{\Omega} \eta_1' B_{20} \sin \frac{\eta_1' t_{10}}{\Omega} \Big) + D_2 e^{-\frac{\delta_2' t_{10}}{\Omega}} \Big(\frac{\Delta t_{10}}{\Omega} \eta_2' C_{10} \cos \frac{\eta_2' t_{10}}{\Omega} + \Delta C_{10} \sin \frac{\eta_2' t_{10}}{\Omega} + \Delta C_{20} \cos \frac{\eta_2' t_{10}}{\Omega} - \frac{\Delta t_{10}}{\Omega} \eta_2' C_{20} \sin \frac{\eta_2' t_{10}}{\Omega} \Big) + A_2 (\Delta t_{10} + \Delta \tau_0) \cos(t_{10} + \tau_0 + \psi) \qquad (3.27)$$

$$\Delta \dot{x}_{1c} = e^{-\frac{\delta_1' t_{10}}{\Omega}} \Big(\frac{\Delta t_{10}}{\Omega} \cdot \eta_1' \Big(-B_{10} \big(\delta_1' \cos \frac{\eta_1' t_{10}}{\Omega} + \eta_1' \sin \frac{\eta_1' t_{10}}{\Omega} \big) + B_{20} \big(\delta_1' \sin \frac{\eta_1' t_{10}}{\Omega} - \eta_1' \cos \frac{\eta_1' t_{10}}{\Omega} \big) \Big) + \Delta B_{10} \big(-\delta_1' \sin \frac{\eta_1' t_{10}}{\Omega} + \eta_1' \cos \frac{\eta_1' t_{10}}{\Omega} \big) \Big) + \Delta B_{20} \Big(-\delta_1' \cos \frac{\eta_1' t_{10}}{\Omega} - \eta_1' \sin \frac{\eta_1' t_{10}}{\Omega} \Big) \Big) + e^{-\frac{\delta_2' t_{10}}{\Omega}} \Big(\frac{\Delta t_{10}}{\Omega} \cdot \eta_2' \Big(-C_{10} \big(\delta_2' \cos \frac{\eta_2' t_{10}}{\Omega} + \eta_2' \sin \frac{\eta_2' t_{10}}{\Omega} \big) + C_{20} \big(\delta_2' \sin \frac{\eta_2' t_{10}}{\Omega} - \eta_2' \cos \frac{\eta_2' t_{10}}{\Omega} \big) \Big) + \Delta C_{10} \big(-\delta_2' \sin \frac{\eta_2' t_{10}}{\Omega} + \eta_2' \cos \frac{\eta_2' t_{10}}{\Omega} \big) + \Delta C_{20} \big(-\delta_2' \cos \frac{\eta_2' t_{10}}{\Omega} - \eta_2' \sin \frac{\eta_2' t_{10}}{\Omega} \big) - \Omega A_1 (\Delta t_{10} + \Delta \tau_0) \sin(t_{10} + \tau_0) \qquad (3.28)$$

similiarly if

$$\dot{x}_2 (t_{10} + \Delta t_0') = \dot{x}_{20b} + \Delta \dot{x}_{20b} \quad \text{, where}$$

$$x_{20b} = D_1 e^{-\frac{\delta_1' t_{10}}{\Omega}} \Big(\frac{\Delta t_{10}}{\Omega} \cdot \eta_1' \Big(-B_{10} \big(\delta_1' \cos \frac{\eta_1' t_{10}}{\Omega} + \eta_1' \sin \frac{\eta_1' t_{10}}{\Omega} \big) + B_{20} \big(\delta_1' \sin \frac{\eta_1' t_{10}}{\Omega} - \eta_1' \cos \frac{\eta_1' t_{10}}{\Omega} \big) \Big) + \Delta B_{10} \big(-\delta_1' \sin \frac{\eta_1' t_{10}}{\Omega} + \eta_1' \cos \frac{\eta_1' t_{10}}{\Omega} \big) + \Delta B_{20} \big(-\delta_1' \cos \frac{\eta_1' t_{10}}{\Omega} - \eta_1' \sin \frac{\eta_1' t_{10}}{\Omega} \big) \Big) + D_2 e^{-\frac{\delta_2' t_{10}}{\Omega}} \Big(\frac{\Delta t_{10}}{\Omega} \cdot \eta_2' \Big(-C_{10} \big(\delta_2' \cos \frac{\eta_2' t_{10}}{\Omega} + \eta_2' \sin \frac{\eta_2' t_{10}}{\Omega} \big) + C_{20} \big(\delta_2' \sin \frac{\eta_2' t_{10}}{\Omega} - \eta_2' \cos \frac{\eta_2' t_{10}}{\Omega} \big) \Big) + \Delta C_{10} \big(-\delta_2' \sin \frac{\eta_2' t_{10}}{\Omega} + \eta_2' \cos \frac{\eta_2' t_{10}}{\Omega} \big) + \Delta C_{20} \big(-\delta_2' \cos \frac{\eta_2' t_{10}}{\Omega} - \eta_2' \sin \frac{\eta_2' t_{10}}{\Omega} \big) - \Omega A_2 (\Delta t_{10} + \Delta \tau_0) \sin(t_{10} + \tau_0 + \psi) \qquad (3.29)$$

Using Equations(2.8) and (2.9) with new variables:

$$K_1 = (1 - \mu e)/(1 + \mu)$$

$$K_2 = \mu(1 + e)/(1 + \mu)$$

$$K_3 = (1+e)/(1+\mu)$$

$$K_4 = (\mu-e)/(1+\mu)$$

$$\dot{x}_{20} + \Delta\dot{x}_{20} = K_1(\dot{x}_{20b} + \Delta\dot{x}_{20b}) + K_2(V_{20} + \Delta V_{20}) \quad (3.30)$$

so

$$\Delta\dot{x}_{20} = K_1\Delta\dot{x}_{20b} + K_2\Delta V_{20} \quad (3.31)$$

$$-(V_{20} + \Delta V'_{20}) = K_3(\dot{x}_{20b} + \Delta\dot{x}_{20b}) + K_4(V_{20} + \Delta V_{20}) \quad (3.32)$$

$$-\Delta V'_{20} = K_3\Delta\dot{x}_{20b} + K_4\Delta V_{20} \quad (3.33)$$

After impact, B_1, B_2, C_1, C_2, t_1, τ will have perturbed values in Equations (2.24), (2.25), (2.26) and (2.27). So

$$\Delta B'_{20} = (D_2\Delta x_{1c} - \Delta x_{20} - D_2(\Delta\tau_0 + \Delta t_{10})A_1\cos(\tau_0+t_{10}) + A_2(\Delta\tau_0 + \Delta t_{10})\cos(\tau_0+t_{10}+\psi))/(D_2-D_1) \quad (3.34)$$

$$\Delta C'_{20} = (D_1\Delta x_{1c} - \Delta x_{20} - D_1(\Delta\tau_0 + \Delta t_{10})A_1\cos(\tau_0+t_{10}) + A_2(\Delta\tau_0 + \Delta t_{10})\cos(\tau_0+t_{10}+\psi))/(D_1-D_2) \quad (3.35)$$

$$\Delta B'_{10} = (D_2(\Delta\dot{x}_{1c} + \delta_1\Delta x_{1c}) - (\Delta\dot{x}'_{20} + \delta'_1\Delta x_{20}) - D_2A_1(\delta'_1\cdot(\Delta\tau_0 + \Delta t_{10})\cdot\cos(\tau_0+t_{10}) - \Omega(\Delta\tau_0 + \Delta t_{10})\sin(\tau_0+t_{10})) + A_2(\delta'_1(\Delta\tau_0 + \Delta t_{10})\cos(\tau_0+t_{10}+\psi) - \Omega(\Delta\tau_0 + \Delta t_{10})\cdot\sin(\tau_0+t_{10}+\psi))) /(\eta'_1(D_2-D_1)) \quad (3.36)$$

$$\Delta C'_{10} = D_1(\Delta\dot{x}_{1c} + \delta_2\Delta x_{1c}) - (\Delta\dot{x}_{20} + \delta_2\Delta x_{20}) - D_1A_1(\delta_2(\Delta\tau_0 + \Delta t_{10})\cos(\tau_0+t_{10}) - \Omega(\Delta\tau_0 + \Delta t_{10})\sin(\tau_0+t_{10})) + A_2(\delta'_2(\Delta\tau_0 + \Delta t_{10})\cdot\cos(\tau_0+t_{10}+\psi) - \Omega(\Delta\tau_0 + \Delta t_{10})\sin(\tau_0+t_{10}+\psi)))/(\eta'_2(D_1-D_2)) \quad (3.37)$$

Let $\Delta T_0 = \Delta t_0'' - \Delta t_0'$

So

$$-\Delta x'_{10} = e^{-\delta_1'\frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta_1' B'_{10} \cos \eta_1' \frac{(\pi - t_{10})}{\Omega} + \Delta B'_{10} \sin \eta_1' \frac{(\pi - t_{10})}{\Omega} + \Delta B'_{20} \cos \eta_1' \frac{(\pi - t_{10})}{\Omega} - \frac{\Delta T_0}{\Omega} \eta_1' B'_{20} \sin \eta_1' \frac{(\pi - t_{10})}{\Omega} \Big) + e^{-\delta_2'\frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta_2' C'_{10} \cos \eta_2' \frac{(\pi - t_{10})}{\Omega} + \Delta C'_{10} \sin \eta_2' \frac{(\pi - t_{10})}{\Omega} + \Delta C'_{20} \cos \eta_2' \frac{(\pi - t_{10})}{\Omega} - \frac{\Delta T_0}{\Omega} \eta_2' C'_{20} \sin \eta_2' \frac{(\pi - t_{10})}{\Omega} \Big) - A_1 (\Delta t_0'' - \Delta t_0 + \Delta \tau_0) \cos(\tau_0) \tag{3.38}$$

$$-\Delta x'_{2c} = D_1 e^{-\delta_1'\frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta_1' B'_{10} \cos \eta_1' \frac{(\pi - t_{10})}{\Omega} + \Delta B'_{10} \sin \eta_1' \frac{(\pi - t_{10})}{\Omega} + \Delta B'_{20} \cos \eta_1' \frac{(\pi - t_{10})}{\Omega} - \frac{\Delta T_0}{\Omega} \eta_1' B'_{20} \sin \eta_1' \frac{(\pi - t_{10})}{\Omega} \Big) + D_2 e^{-\delta_2'\frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta_2' C'_{10} \cos \eta_2' \frac{(\pi - t_{10})}{\Omega} + \Delta C'_{10} \sin \eta_2' \frac{(\pi - t_{10})}{\Omega} + \Delta C'_{20} \cos \eta_2' \frac{(\pi - t_{10})}{\Omega} - \frac{\Delta T_0}{\Omega} \eta_2' C'_{20} \sin \eta_2' \frac{(\pi - t_{10})}{\Omega} \Big) - A_1 (\Delta t_0'' - \Delta t_0 + \Delta \tau_0) \cos \tau_0 \tag{3.39}$$

$$\Delta \dot{x}'_{10b} = e^{-\delta_1'\frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta_1' \Big(-B'_{10} \Big(\delta_1' \cos \eta_1' \frac{(\pi - t_{10})}{\Omega} + \eta_1' \sin \eta_1' \frac{(\pi - t_{10})}{\Omega}\Big) + B'_{20} \Big(\delta_1' \sin \eta_1' \frac{(\pi - t_{10})}{\Omega} - \eta_1' \cos \eta_1' \frac{(\pi - t_{10})}{\Omega}\Big)\Big) + \Delta B'_{10} \Big(-\delta_1' \sin \eta_1' \frac{(\pi - t_{10})}{\Omega} + \eta_1' \cos \eta_1' \frac{(\pi - t_{10})}{\Omega}\Big) + \Delta B'_{20} \Big(-\delta_1' \cos \eta_1' \frac{(\pi - t_{10})}{\Omega} - \eta_1' \sin \eta_1' \frac{(\pi - t_{10})}{\Omega}\Big)\Big) + e^{-\delta_2'\frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta_2' \Big(-C_{10} \Big(\delta_2' \cos \eta_2' \frac{(\pi - t_{10})}{\Omega} + \eta_2' \sin \eta_2' \frac{(\pi - t_{10})}{\Omega}\Big) + C_{20} \Big(\delta_2' \sin \eta_2' \frac{(\pi - t_{10})}{\Omega} - \eta_2' \cos \eta_2' \frac{(\pi - t_{10})}{\Omega}\Big)\Big) + \Delta C_{10} \Big(-\delta_2' \sin \eta_2' \frac{(\pi - t_{10})}{\Omega} + \eta_2' \cos \eta_2' \frac{(\pi - t_{10})}{\Omega}\Big) + \Delta C_{20} \Big(-\delta_2' \cos \eta_2' \frac{(\pi - t_{10})}{\Omega} - \eta_2' \sin \eta_2' \frac{(\pi - t_{10})}{\Omega}\Big) \Big) + \Omega A_1 (\Delta t_0'' - \Delta t_0 + \Delta \tau_0) \sin \tau_0 \tag{3.40}$$

similiarly

$$-\Delta \dot{x}'_{2c} = D_1 e^{-\delta'_1 \frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta_1 \big(-B'_{10} (\delta'_1 \cos \eta'_1 \frac{(\pi - t_{10})}{\Omega} + \eta'_1 \sin \eta'_1 \frac{(\pi - t_{10})}{\Omega}) + B'_{20} (\delta'_1 \sin \eta'_1 \frac{(\pi - t_{10})}{\Omega} - \eta'_1 \cos \eta'_1 \frac{(\pi - t_{10})}{\Omega})\big) + \Delta B'_{10} (- \delta'_1 \sin \eta'_1 \frac{(\pi - t_{10})}{\Omega} + \eta'_1 \cos \eta'_1 \frac{(\pi - t_{10})}{\Omega}) + \Delta B'_{20} (- \delta'_1 \cos \eta'_1 \frac{(\pi - t_{10})}{\Omega} - \eta'_1 \sin \eta'_1 \frac{(\pi - t_{10})}{\Omega})\Big) + D_2 e^{-\delta'_2 \frac{(\pi - t_{10})}{\Omega}} \Big(\frac{\Delta T_0}{\Omega} \eta'_2 \big(-C_{10} (\delta'_2 \cos \eta'_2 \frac{(\pi - t_{10})}{\Omega} + \eta'_2 \sin \eta'_2 \frac{(\pi - t_{10})}{\Omega}) + C_{20} (\delta'_2 \sin \eta'_2 \frac{(\pi - t_{10})}{\Omega} - \eta'_2 \cos \eta'_2 \frac{(\pi - t_{10})}{\Omega})\big) + \Delta C_{10} (- \delta'_2 \sin \eta'_2 \frac{(\pi - t_{10})}{\Omega} + \eta'_2 \cos \eta'_2 \frac{(\pi - t_{10})}{\Omega}) + \Delta C_{20} (- \delta'_2 \cos \eta'_2 \frac{(\pi - t_{10})}{\Omega} - \eta'_2 \sin \eta'_2 \frac{(\pi - t_{10})}{\Omega}) + \Omega A_2 (\Delta t''_0 - \Delta t_0 + \Delta T_0) \sin (T_0 + \psi) \qquad (3.41)$$

Using Equations(2.8) and (2.9) with new variables:

$$-(\dot{x}_{10} + \Delta \dot{x}'_{10}) = K_1 (\dot{x}_{10b} + \Delta \dot{x}_{10b}) - K_2 (V_{10} + \Delta V_{10})$$

so

$$-\Delta \dot{x}'_{10} = K_1 \Delta \dot{x}_{10b} - K_2 \Delta V_1 \qquad (3.42)$$

$$V_{10} + \Delta V'_{10} = K_3 (\dot{x}_{10b} + \Delta \dot{x}_{10b}) - K_4 (V_{10} + \Delta V_{10})$$

or

$$\Delta V'_{10} = K_3 \Delta \dot{x}_{10b} - K_4 \Delta V_{10} \qquad (3.43)$$

The time required by particle to travel from one side of the container to other is equal to absolute distance travelled, divided by absolute velocity. So

$$\frac{(\pi + \Delta t''_0) - \Delta t_0}{\Omega} = -\big((y_1(\pi + \Delta t''_0) + x_1(\pi + \Delta t''_0)) - (y_1(\Delta t_0) + x_1(\Delta t_0))\big) / (V_{10} + \Delta V_{10}) \qquad (3.44)$$

Taking $\Delta T = \Delta t''_0 - \Delta t_0$,

$$\Delta T = \frac{\Omega}{V_{10}} \Delta x'_{10} + \frac{\Omega}{V_{10}} \Delta x_{10} - \frac{\pi}{V_{10}} \Delta V_{10} \qquad (3.45)$$

$$\Delta T_0 = \Delta T - \Delta t_{10} \qquad (3.46)$$

By using Equations (3.45) and (3.46), we get

$$-x'_{10} = \frac{1}{k}\Big(\Big(\frac{\Delta x_{10}}{V_{10}} - \frac{\pi}{\Omega}\frac{\Delta V_{10}}{V_{10}} - \frac{\Delta t_{10}}{\Omega}\Big)\Big(\eta'_1 e^{-\delta'_1\frac{(\pi-t_{10})}{\Omega}}$$
$$(B'_{10}\cos\eta'_1\frac{(\pi-t_{10})}{\Omega} - B'_{20}\sin\eta'_1\frac{(\pi-t_{10})}{\Omega}) +$$
$$\eta'_2 e^{-\delta'_2\frac{(\pi-t_{10})}{\Omega}}(C'_{10}\cos\eta'_2\frac{(\pi-t_{10})}{\Omega} +$$
$$-C'_{20}\sin\eta'_2\frac{(\pi-t_{10})}{\Omega})\Big) + e^{-\delta'_1\frac{(\pi-t_{10})}{\Omega}}(\Delta B'_{10}\sin\eta'_1\frac{(\pi-t_{10})}{\Omega} +$$
$$\Delta B'_{20}\cos\eta'_1\frac{(\pi-t_{10})}{\Omega}) + e^{-\delta'_2\frac{(\pi-t_{10})}{\Omega}}(\Delta C'_{10}\sin\eta'_2\frac{(\pi-t_{10})}{\Omega} +$$
$$\Delta C'_{20}\cos\eta'_2\frac{(\pi-t_{10})}{\Omega}) - A_1\Big(\frac{\Omega}{V_{10}}\Delta x_{10} - \frac{\pi}{V_{10}}\Delta V_{10} + \Delta\tau_0\Big)$$
$$\cos\tau_0\Big) \qquad (3.47)$$

So $\Delta\tau'_0 = \Delta\tau_0 + \Delta T$ (3.48)

For finding $\Delta t'_{10}$, let us consider time interval of impacts in primary mass 2

$$((\pi + t_{10} + \Delta t'''_0) - (t_{10} + \Delta t'_0))/\Omega = -((y_2(\pi + t_{10} + \Delta t'''_0) + x_2(\pi + t_{10} + \Delta t'''_0))$$
$$- (y_2(t_{10} + \Delta t'_0) + x_2(t_{10} + \Delta t'_0)))/$$
$$(V_{20} + \Delta V'_{20})$$

So $$\Delta t'''_0 - \Delta t'_0 = \frac{\Omega}{V_{20}}\Delta x'_{20} + \frac{\Omega}{V_{20}}\Delta x_{20} - \frac{\pi}{V_{20}}\Delta V'_{20} \qquad (3.49)$$

or $$\Delta t'_{10} + \Delta T - \Delta t_{10} = \frac{\Omega}{V_{20}}\Delta x'_{20} + \frac{\Omega}{V_{20}}\Delta x_{20} - \frac{\pi}{V_{20}}\Delta V'_{20} \qquad (3.50)$$

Here all terms are known except $\Delta x'_{20}$. By procee-ding as before, we get:

$$\Delta x'_{20} = D_1 e^{-\frac{\delta'_1 t_{10}}{\Omega}}\Big(\frac{\Delta t'_{10}}{\Omega}\eta'_1 B_{10}\cos\frac{\eta'_1 t_{10}}{\Omega} + \Delta B''_{10}\sin\frac{\eta'_1 t_{10}}{\Omega} +$$
$$\Delta B''_{20}\cos\frac{\eta'_1 t_{10}}{\Omega} - \frac{\Delta t'_{10}}{\Omega}\eta'_1 B_{20}\sin\frac{\eta'_1 t_{10}}{\Omega}\Big) + D_2 e^{-\frac{\delta'_2 t_{10}}{\Omega}}$$
$$\Big(\frac{\Delta t'_{10}}{\Omega}\eta'_2 C_{10}\cos\frac{\eta'_2 t_{10}}{\Omega} + \Delta C''_{10}\sin\frac{\eta'_2 t_{10}}{\Omega} +$$
$$\Delta C''_{20}\cos\frac{\eta'_2 t_{10}}{\Omega} - \frac{\Delta t'_{10}}{\Omega}\eta'_2 C''_{20}\sin\frac{\eta'_2 t_{10}}{\Omega}\Big) +$$
$$A_2(\Delta t'_{10} + \Delta\tau'_0)\cos(t_{10} + \tau_0 + \psi) \qquad (3.51)$$

where

$$\Delta B''_{20} = (D_2\Delta x'_{10} - \Delta x'_{20} - D_2\Delta\tau'_0 A_1\cos\tau_0 + A_2\Delta\tau'_0\cos(\tau_0+\psi))$$
$$/(D_2 - D_1) \qquad (3.52)$$

$$\Delta C''_{20} = (D_1 \Delta x'_{10} - \Delta x'_{2c} - D_1 \Delta \tau'_0 A_1 \cos \tau_0 + A_2 \Delta \tau'_0 \cos(\tau_0 + \psi))/(D_1 - D_2) \quad (3.53)$$

$$\Delta B''_{10} = (D_2(\Delta \dot{x}'_{10} + \delta_1 \Delta x'_{10}) - (\Delta \dot{x}'_{2c} + \delta_1 \Delta x'_{2c}) - D_2 A_1(\delta_1 \Delta \tau_0 \cos \tau_0 - \Omega \Delta \tau'_0 \sin \tau_0) + A_2(\delta_1 \Delta \tau'_0 \cos(\tau_0+\psi) - \Omega \Delta \tau'_0 \sin(\tau_0 + \psi)))/(\eta'_1(D_2 - D_1)) \quad (3.54)$$

$$\Delta C''_{10} = D_1(\Delta \dot{x}'_{10} + \delta_2 \Delta x'_{10}) - (\Delta \dot{x}'_{2c} + \delta_2 \Delta \dot{x}'_{2c}) - D_1 A_1(\delta_2 \Delta \tau'_0 \cos \tau_0 - \Omega \Delta \tau'_0 \sin(\tau_0 + \psi))/(\eta'_2(D_1 - D_2)) \quad (3.55)$$

The value of K is

$$K = \left(1 + \frac{1}{V_{10}}\left(\eta'_1 e^{-\delta'_1 \frac{(\pi - t_{10})}{\Omega}} \left(B'_{10} \cos \eta'_1 \frac{(\pi - t_{10})}{\Omega} - B'_{20} \sin \eta'_1 \frac{(\pi - t_{10})}{\Omega}\right) + \eta'_2 e^{-\delta'_2 \frac{(\pi - t_{10})}{\Omega}} \left(C'_{10} \cos \eta'_2 \frac{(\pi - t_{10})}{\Omega} - C'_{20} \sin \eta'_2 \frac{(\pi - t_{10})}{\Omega}\right)\right) - \frac{A_1 \Omega}{V_{10}}\right) \quad (3.56)$$

With new variables listed in appendix B, we get simplified Equations as:

$$\Delta B_{20} = G_1 \Delta x_{10} - G_5 \Delta x_{2c} + G_3 \Delta \tau_0 \quad (3.57)$$

$$\Delta C_{20} = G_2 \Delta x_{10} + G_5 \Delta x_{2c} + G_4 \Delta \tau_0 \quad (3.58)$$

$$\Delta B_{10} = \frac{G_1}{\eta'_1} \Delta \dot{x}_{10} + \frac{\delta'_1}{\eta'_1} G_1 \Delta x_{10} - \frac{G_5}{\eta'_1} \Delta \dot{x}_{2c} - \frac{\delta'_1}{\eta'_1} G_5 \Delta x_{2c} - G_6 \tau_0 \quad (3.59)$$

$$\Delta C_{10} = \frac{G_2}{\eta'_2} \Delta \dot{x}_{10} + \frac{\delta'_2}{\eta'_2} G_2 \Delta x_{10} + \frac{G_5}{\eta'_1} \Delta \dot{x}_{2c} + \frac{\delta'_2}{\eta'_2} G_5 \Delta x_{2c} - G_7 \tau_0 \quad (3.60)$$

$$\Delta x_{1c} = G_8 \Delta t_{10} + G_{14} \Delta \dot{x}_{10} + G_{15} \Delta x_{10} + G_{16} \Delta \dot{x}_{2c} + N_{17} \Delta x_{2c} + N_{18} \Delta \tau_0 \quad (3.61)$$

$$\Delta x_{20} = G_{17} \Delta t_{10} + G_{19} \Delta \dot{x}_{10} + G_{20} \Delta x_{10} + G_{21} \Delta \dot{x}_{2c} + G_{22} \Delta x_{2c} + G_{23} \Delta \tau_0 \quad (3.62)$$

$$\Delta\dot{x}_{1c} = G_{24}\Delta t_{10} + G_{23}\Delta\dot{x}_{10} + G_{31}\Delta x_{10} + G_{32}\Delta\dot{x}_{2c} + G_{33}\Delta x_{2c} + G_{34}\Delta\tau_o \quad (3.63)$$

$$\Delta\dot{x}_{20} = G_{42}\Delta t_{10} + G_{37}\Delta\dot{x}_{10} + G_{38}\Delta x_{10} + G_{39}\Delta\dot{x}_{20} + G_{40}\Delta x_{20} + G_{41}\Delta\tau_o + K_2\Delta V_{20} \quad (3.64)$$

$$\Delta\dot{B}'_{20} = G_{43}\Delta t_{10} + G_{44}\Delta\tau_o + G_{45}\Delta\dot{x}_{10} + G_{46}\Delta x_{10} + G_{47}\Delta\dot{x}_{2c} + G_{48}\Delta x_{2c} \quad (3.65)$$

$$\Delta C'_{20} = G_{49}\Delta t_{10} + G_{50}\Delta\tau_o + G_{51}\Delta\dot{x}_{10} + G_{52}\Delta x_{10} + G_{53}\Delta x'_{2c} + G_{54}\Delta x_{2c} \quad (3.66)$$

$$\Delta B'_{10} = G_{55}\Delta t_{10} + G_{56}\Delta\tau_o + G_{57}\Delta\dot{x}_{10} + G_{58}\Delta x_{10} + G_{59}\Delta\dot{x}_{2c} + G_{60}\Delta x_{2c} + G_{61}\Delta V_{20} \quad (3.67)$$

$$\Delta C'_{10} = G_{62}\Delta t_{10} + G_{63}\Delta\tau_o + G_{64}\Delta\dot{x}_{10} + G_{65}\Delta x_{10} + G_{66}\Delta\dot{x}_{2c} + G_{68}\Delta V_{20} + G_{67}\Delta x_{2c} \quad (3.68)$$

$$\Delta x'_{10} = G_{74}\Delta t_{10} + G_{75}\Delta\tau_o + G_{76}\Delta\dot{x}_{10} + G_{77}\Delta x_{10} + G_{78}\Delta\dot{x}_{2c} + G_{79}\Delta x_{2c} + G_{80}\Delta V_{10} + G_{81}\Delta V_{20} \quad (3.69)$$

$$\Delta T = G_{82}G_{74}\Delta t_{10} + G_{82}\cdot G_{75}\cdot\Delta\tau_o + G_{82}G_{76}\Delta\dot{x}_{10} + G_{82}\cdot(G_{77}+1.)\Delta x_{10} + G_{82}G_{78}\Delta\dot{x}_{2c} + G_{82}G_{79}\Delta x_{2c} + \left(G_{82}\cdot G_{80} - \frac{\pi}{V_{10}}\right)\Delta V_{10} + G_{82}G_{81}\Delta V_{20} \quad (3.70)$$

$$\Delta\tau_o = \Delta T - \Delta t_{10}$$

$$\Delta x'_{2c} = G_{84}\Delta t_{10} + G_{85}\Delta\tau_o + G_{86}\Delta\dot{x}_{10} + G_{87}\Delta x_{10} + G_{88}\Delta\dot{x}_{2c} + G_{89}\Delta x_{2c} + G_{90}\Delta V_{10} + G_{91}\Delta V_{20} \quad (3.71)$$

$$\Delta V'_{10} = K_3G_{97}\Delta t_{10} + K_3G_{98}\Delta\tau_o + K_3G_{99}\Delta\dot{x}_{10} + K_3G_{100}\Delta x_{10} + K_3G_{101}\Delta\dot{x}_{2c} + K_3G_{102}\Delta x_{2c} + (K_3G_{103} - K_4)\Delta V_{10} + K_3G_{104}\Delta V_{20} \quad (3.72)$$

$$\Delta\dot{x}_{10} = K_1 G_{97}\Delta t_{10} + K_1 G_{98}\Delta\tau_0 + K_1 G_{99}\Delta\dot{x}_{10} + K_1 G_{100}\Delta x_{10} + K_1 G_{101}\Delta\dot{x}_{2c} + K_1 G_{102}\Delta x_{2c} + (K_1 G_{103} - K_2)\Delta V_{10} + K_1 G_{104}\Delta V_{20} \tag{3.73}$$

$$\Delta\tau_0' = G_{82}G_{74}\Delta t_{10} + (1+G_{82}G_{75})\Delta\tau_0 + G_{82}G_{76}\Delta\dot{x}_{10} + G_{82}(1+G_{77})\Delta x_{10} + G_{82}G_{78}\Delta\dot{x}_{2c} + G_{82}G_{79}\Delta x_{2c} + \left(G_{82}G_{80} - \frac{\pi}{V_{10}}\right)\Delta V_{10} + G_{82}G_{81}\Delta V_{20} \tag{3.74}$$

$$\Delta V_{20}' = -\frac{K_3}{K_1}G_{42}\Delta t_{10} - \frac{K_3}{K_1}G\;\Delta\dot{x}_{10} - \frac{K_3}{K_1}G_{38}\Delta x_{10} - \frac{K_3}{K_1}G_{39}\Delta\dot{x}_{2c} - \frac{K_3}{K_1}G_{40}\Delta x_{2c} - \frac{K_3}{K_1}G_{41}\Delta\tau_0 - K_4\Delta V_{20} \tag{3.75}$$

$$\dot{x}_{2c}' = -G_{106}\Delta t_{10} - G_{107}\Delta\tau_0 - G_{108}\Delta x_{10} - G_{110}\Delta\dot{x}_{2c} - G_{109}\Delta\dot{x}_{10} - G_{111}\Delta x_{2c} - G_{112}\Delta V_{10} - G_{113}\Delta V_{20} \tag{3.76}$$

$$\Delta B_{20}' = G_{114}\Delta t_{10} + G_{115}\Delta\tau_0 + G_{116}\Delta\dot{x}_{10} + G_{117}\Delta x_{10} + G_{118}\Delta\dot{x}_{2c} + G_{119}\Delta x_{2c} + G_{120}\Delta V_{10} + G_{121}\Delta V_{20} \tag{3.77}$$

$$\Delta C_{20}' = G_{122}\Delta t_{10} + G_{123}\Delta\tau_0 + G_{124}\Delta\dot{x}_{10} + G_{125}\Delta x_{10} + G_{126}\Delta\dot{x}_{2c} + G_{127}\Delta x_{2c} + G_{128}\Delta V_{10} + G_{129}\Delta V_{20} \tag{3.78}$$

$$\Delta B_{10}'' = G_{130}\Delta t_{10} + G_{131}\Delta\tau_0 + G_{132}\Delta\dot{x}_{10} + G_{133}\Delta x_{10} + G_{134}\Delta\dot{x}_{2c} + G_{135}\Delta x_{2c} + G_{136}\Delta V_{10} + G_{137}\Delta V_{20} \tag{3.79}$$

$$\Delta C_{10}'' = G_{138}\Delta t_{10} + G_{139}\Delta\tau_0 + G_{140}\Delta\dot{x}_{10} + G_{141}\Delta x_{10} + G_{142}\Delta\dot{x}_{2c} + G_{143}\Delta x_{2c} + G_{144}\Delta V_{10} + G_{145}\Delta V_{20} \tag{3.80}$$

$$\Delta t_{10}' = G_{147}\Delta t_{10} + G_{148}\Delta\tau_0 + G_{149}\Delta\dot{x}_{10} + G_{150}\Delta x_{10} + G_{151}\Delta\dot{x}_{2c} + G_{152}\Delta x_{2c} + G_{153}\Delta V_{10} + G_{154}\Delta V_{20} \tag{3.81}$$

Equations(3.72),(3.69),(3.76),(3.71),(3.73),(3.74),(3.75) and(3.81) respectively can be written in the matrix form.The steady state solution is stable if modulus of all eigen values of the matrix are less than 1

$$
\begin{Bmatrix} \Delta \dot{x}'_{10} \\ \Delta x'_{10} \\ \Delta \dot{x}'_{2c} \\ \Delta x'_{2c} \\ \Delta V'_{10} \\ \Delta \tau'_{0} \\ \Delta V'_{20} \\ \Delta t'_{10} \end{Bmatrix}
=
\begin{bmatrix}
-K_1G_{99} & -K_1G_{100} & -K_1G_{101} & -K_1G_{102} & -K_1G_{103}+K_2 & -K_1G_{98} & -K_1G_{104} & -K_1G_{97} \\
G_{76} & G_{77} & G_{78} & G_{79} & G_{80} & G_{75} & G_{81} & G_{74} \\
-G_{108} & -G_{109} & -G_{110} & -G_{111} & -G_{112} & -G_{107} & -G_{113} & -G_{106} \\
-G_{86} & -G_{87} & -G_{88} & -G_{89} & -G_{90} & -G_{85} & -G_{91} & -G_{84} \\
K_3G_{99} & K_3G_{100} & K_3G_{101} & K_3G_{102} & K_3G_{103}-K_4 & K_3G_{98} & K_3G_{104} & K_3G_{97} \\
G_{82}G_{76} & G_{82}(1+G_{77}) & G_{82}G_{78} & G_{82}G_{79} & G_{82}G_{80}-\pi/V_{10} & 1+G_{82}G_{75} & G_{82}G_{81} & G_{82}G_{74} \\
\frac{K_3}{K_1} & \frac{K_3}{K_1} & \frac{K_3}{K_1} & \frac{K_3}{K_1} & 0 & \frac{K_3}{K_1} & -K_4 & \frac{K_3}{K_1}G_{42} \\
G_{149} & G_{150} & G_{151} & G_{152} & G_{153} & G_{148} & G_{154} & G_{147}
\end{bmatrix}
\begin{Bmatrix} \Delta \dot{x}_{10} \\ \Delta x_{10} \\ \Delta \dot{x}_{2c} \\ \Delta x_{2c} \\ \Delta V_{10} \\ \Delta \tau_{0} \\ \Delta V_{20} \\ \Delta t_{10} \end{Bmatrix}
$$

3.4 Impact Damper in Primary mass 1

Here our system consists of three degrees of freedom, $\vec{\xi}$ should be six component vector. Here perturbation consist of small variation of steady state values of x_2, $\dot{x}_2$, x_1, $\dot{x}_1$, V_1 and τ . Here o subscript refers to the unperturbed conditions.

The conditions that we have are

Steady State	Perturbed
At $\Omega t = 0_+$	At $\Omega t = (0+\Delta t_o)_+$
$x_1 = x_{10}$	$x_1 = x_{10} + \Delta x_{10}$
$y_1 = \frac{d}{2}$	$y_1 = \frac{d}{2}$
$\dot{x}_1 = \dot{x}_{10}$	$\dot{x}_1 = \dot{x}_{10} + \Delta \dot{x}_{10}$
$\dot{Y}_1 = -V_{10}$	$\dot{Y}_1 = -(V_{10} + \Delta V_{10})$
$\tau = \tau_o$	$\tau = \tau_o + \Delta\tau_o$
$x_2 = x_{2c}$	$x_2 = x_{2c} + \Delta x_{2c}$
$\dot{x}_2 = \dot{x}_{2c}$	$\dot{x}_2 = \dot{x}_{2c} + \Delta \dot{x}_{2c}$
At $\Omega t = \pi_+$	At $\Omega t = (\pi + \Delta t_o')_+$
$x_1 = -x_{10}$	$x_1 = -(x_{10} + \Delta x_{10}')$
$y_1 = -\frac{d}{2}$	$y_1 = -\frac{d}{2}$
$\dot{x}_1 = -\dot{x}_{10}$	$\dot{x}_1 = -(\dot{x}_{10} + \Delta \dot{x}_{10}')$
$\dot{Y}_1 = V_{10}$	$\dot{Y}_1 = V_{10} + \Delta V_{10}'$
$\tau = \tau_o$	$\tau = \tau_o + \Delta\tau_o'$
$x_2 = -x_{2c}$	$x_2 = -(x_{2c} + \Delta x_{2c}')$
$\dot{x}_2 = -\dot{x}_{2c}$	$\dot{x}_2 = -(\dot{x}_{2c} + \Delta \dot{x}_{2c}')$

Here Equations(3.21),(3.22),(3.23),(3.24) are the same here. Let

$$\Delta T = (\Delta t_0' - \Delta t_0), \tag{3.83}$$

So

$$-\Delta x_{10}' = e^{-\delta_1' \frac{\pi}{\Omega}} \left(\frac{\Delta T}{\Omega} \eta_1' B_{10} \cos \frac{\eta_1' \pi}{\Omega} + \Delta B_{10} \sin \frac{\eta_1' \pi}{\Omega} + \Delta B_{20} \cos \frac{\eta_1' \pi}{\Omega} - \frac{\Delta T}{\Omega} \eta_1' B_{20} \sin \frac{\eta_1' \pi}{\Omega} \right) + e^{-\delta_2' \frac{\pi}{\Omega}} \left(\frac{\Delta T}{\Omega} \eta_2' C_{10} \cos \frac{\eta_2' \pi}{\Omega} + \Delta C_{10} \sin \eta_2' \frac{\pi}{\Omega} + \Delta C_{20} \cos \eta_2' \frac{\pi}{\Omega} - \frac{\Delta T}{\Omega} \eta_2' C_{20} \sin \eta_2' \frac{\pi}{\Omega} \right) - A_1 (\Delta T + \Delta \tau_0) \cos \tau_0 \tag{3.84}$$

$$-\Delta x_{2c}' = D_1 e^{-\delta_1' \frac{\pi}{\Omega}} \left(\frac{\Delta T}{\Omega} \eta_1' B_{10} \cos \frac{\eta_1' \pi}{\Omega} + \Delta B_{10} \sin \eta_1' \frac{\pi}{\Omega} + \Delta B_{20} \cos \frac{\eta_1' \pi}{\Omega} - \frac{\Delta T}{\Omega} \eta_1' B_{20} \sin \frac{\eta_1' \pi}{\Omega} \right) + D_2 e^{-\frac{\delta_2' \pi}{\Omega}} \left(\frac{\Delta T}{\Omega} \cdot \eta_2' C_{10} \cos \frac{\eta_2' \pi}{\Omega} + \Delta C_{10} \sin \eta_2' \frac{\pi}{\Omega} + \Delta C_{20} \cos \eta_2' \frac{\pi}{\Omega} - \frac{\Delta T}{\Omega} \cdot \eta_2' C_{20} \sin \eta_2' \frac{\pi}{\Omega} \right) - A_2 (\Delta T + \Delta \tau_0) \cos (\tau_0 + \psi) \tag{3.85}$$

$$\Delta \dot{x}_1 \frac{(\pi + \Delta t_0')}{\Omega} = e^{-\frac{\delta_1' \pi}{\Omega}} \Big(\frac{\Delta T}{\Omega} \eta_1' \Big(-B_{10} \big(\delta_1' \cos \frac{\eta_1' \pi}{\Omega} + \eta_1' \sin \frac{\eta_1' \pi}{\Omega} \big) + B_{20} \big(\delta_1' \sin \frac{\eta_1' \pi}{\Omega} - \eta_1' \cos \eta_1' \frac{\pi}{\Omega} \big) \Big) + \Delta B_{10} \big(-\delta_1' \sin \frac{\eta_1' \pi}{\Omega} + \eta_1' \cos \frac{\eta_1' \pi}{\Omega} \big) + \Delta B_{20} \big(-\delta_1' \cos \frac{\eta_1' \pi}{\Omega} - \eta_1' \sin \frac{\eta_1' \pi}{\Omega} \big) \Big) + e^{-\delta_2' \frac{\pi}{\Omega}} \Big(\frac{\Delta T}{\Omega} \cdot \eta_2' \Big(-C_{10} \big(\delta_2' \cos \eta_2' \frac{\pi}{\Omega} + \eta_2' \sin \eta_2' \frac{\pi}{\Omega} \big) + C_{20} \big(\delta_2' \sin \eta_2' \frac{\pi}{\Omega} + \eta_2' \cos \eta_2' \frac{\pi}{\Omega} \big) \Big) + \Delta C_{20} \big(-\delta_2 \cos \frac{\eta_2' \pi}{\Omega} - \eta_2 \sin \eta_2' \frac{\pi}{\Omega} \big) \Big) + \Omega A_1 (\Delta T + \Delta \tau_0) \sin \tau_0 \tag{3.86}$$

$$-\Delta\dot{x}'_{2c} = D_1 e^{-\frac{\delta_1'\pi}{\Omega}} \left(\frac{\Delta T}{\Omega}\cdot\eta_1'(-B_{10}(\delta_1'\cos\eta_1'\frac{\pi}{\Omega}+\eta_1'\frac{\pi}{\Omega})+B_{20}\right.$$
$$(\delta_1'\sin\eta_1'\frac{\pi}{\Omega}-\eta_1'\cos\eta_1'\frac{\pi}{\Omega}))+\Delta B_{10}(-\delta_1'\sin\frac{\eta_1'\pi}{\Omega}+$$
$$\eta_1'\cos\eta_1'\frac{\pi}{\Omega})+\Delta B_{20}(-\delta_1'\cos\eta_1'\frac{\pi}{\Omega}-\eta_1'\sin\frac{\eta_1'\pi}{\Omega}))+$$
$$D_2 e^{\frac{\delta_2'\pi}{\Omega}}\left(\frac{\Delta T}{\Omega}\cdot\eta_2'(-C_{10}(\delta_2'\cos\eta_2'\frac{\pi}{\Omega}+\eta_2'\sin\eta_2'\frac{\pi}{\Omega})\right.$$
$$+C_{20}(\delta_2'\sin\eta_2'\frac{\pi}{\Omega}-\eta_2'\cos\eta_2'\frac{\pi}{\Omega}))+\Delta C_{10}(-\delta_2'\sin\eta_2'\frac{\pi}{\Omega}$$
$$+\eta_2'\cos\eta_2'\frac{\pi}{\Omega})+\Delta C_{20}(-\delta_2'\cos\eta_2'\frac{\pi}{\Omega}-\eta_2'\sin\eta_2'\frac{\pi}{\Omega}))+$$
$$\Omega A_2(\Delta T+\Delta T_0)\sin(T_0+\psi) \qquad (3.87)$$

Using Equations(2.8) and (2.9)

$$-(\dot{x}_{10}+\Delta\dot{x}'_{10}) = K_1(\dot{x}_1(\frac{\pi+\Delta t_0'}{\Omega})+\Delta\dot{x}_1(\frac{\pi+\Delta t_0'}{\Omega}))-K_2(V_{10}+\Delta V_{10})$$

$$-\Delta\dot{x}'_{10} = K_1\Delta\dot{x}_1(\frac{\pi+\Delta t_0'}{\Omega}) - K_2\Delta V_{10} \qquad (3.88)$$

$$V_{10}+\Delta V'_{10} = K_3(\dot{x}_1(\frac{\pi+\Delta t_0'}{\Omega})+\Delta\dot{x}_1(\frac{\pi+\Delta t_0'}{\Omega})) - K_4(V_{10}+\Delta V_{10})$$

$$\Delta V'_{10} = K_3\Delta\dot{x}_1(\frac{\pi+\Delta t_0'}{\Omega}) - K_4\Delta V_{10} \qquad (3.89)$$

The time required by particle to travel from one side of the container to the other side equals the absolute distance travelled,divided by the absolute velocity.Hence

$$((\pi+\Delta t_0')-\Delta t_0)/\Omega = -((y_1(\pi+\Delta t_0')+x_1(\pi+\Delta t_0'))-(y_1(\Delta t_0)+x_1(\Delta t_0)))/(V_{10}+\Delta V_{10}) \qquad (3.90)$$

Substituting the values of the quantities on right hand side :

$$\Delta T = \frac{\Omega}{V_{10}}\Delta x'_{10} + \frac{\Omega}{V_{10}}\Delta x_{10} - \frac{\pi}{V_{10}}\Delta V_{10} \qquad (3.91)$$

Using new variables listed in appendix B , and by further simplifying,we shall get:

$$\Delta x'_{10} = \frac{G_{163}}{G_{168}} \Delta \dot{x}_{10} + \frac{G_{164}}{G_{168}} \Delta x_{10} + \frac{G_{165}}{G_{168}} \Delta \dot{x}_{2c} + \frac{G_{166}}{G_{168}} \Delta x_{2c} +$$

$$\frac{G_{167}}{G_{168}} \Delta \tau_o - \frac{G_{155}}{G_{168}} \frac{\pi}{V_{10}} \Delta V_{10} \qquad (3.92)$$

$$\Delta T = G_{169} \Delta \dot{x}_{10} + G_{170} \Delta x_{10} + G_{171} \Delta \dot{x}_{2c} + G_{172} \Delta x_{2c} +$$

$$G_{173} \Delta \tau_o + G_{174} \Delta V_{10} \qquad (3.93)$$

$$\Delta \tau'_o = G_{169} \Delta \dot{x}_{10} + G_{170} \Delta x_{10} + G_{171} \Delta \dot{x}_{2c} + G_{172} \Delta x_{2c} +$$

$$(1 + G_{173}) \Delta \tau_o + G_{174} \Delta V_{10} \qquad (3.94)$$

$$\Delta \dot{x}'_{2c} = G_{188} \Delta \dot{x}_{10} + G_{189} \Delta x_{10} + G_{190} \Delta \dot{x}_{2c} + G_{191} \Delta x_{2c} +$$

$$G_{192} \Delta \tau_o + G_{193} \Delta V_{10} \qquad (3.95)$$

$$\Delta \dot{x}'_{10} = G_{195} \Delta \dot{x}_{10} + G_{196} \Delta x_{10} + G_{197} \Delta \dot{x}_{2c} + G_{198} \Delta x_{2c} +$$

$$G_{199} \Delta \tau_o + G_{200} \Delta V_{10} \qquad (3.96)$$

$$\Delta V'_{10} = -\frac{K_3}{K_1} G_{195} \Delta \dot{x}_{10} - \frac{K_3}{K_1} G_{196} \Delta x_{10} - \frac{K_3}{K_1} G_{197} \Delta \dot{x}_{2c} -$$

$$\frac{K_3}{K_1} G_{198} \Delta x_{2c} - \frac{K_3}{K_1} G_{199} \Delta \tau_o + G_{201} \Delta V_{10} \qquad (3.97)$$

$$\Delta x'_{2c} = G_{175} \Delta \dot{x}_{10} + G_{176} \Delta x_{10} + G_{177} \Delta \dot{x}_{2c} + G_{178} \Delta x_{2c} +$$

$$G_{179} \Delta \tau_o + G_{180} \Delta V_{10} \qquad (3.98)$$

Equations (3.96),(3.92),(3.95),(3.98),(3.94) and (3.97) can be put into the matrix form.The steady state solution is stable if modulus of all the eigen values of the matrix are less than unity.

$$
\begin{Bmatrix} \Delta \dot{x}'_{10} \\ \Delta x'_{10} \\ \Delta \dot{x}'_{2c} \\ \Delta x'_{2c} \\ \Delta \tau'_{0} \\ \Delta V'_{10} \end{Bmatrix}
=
\begin{bmatrix}
G_{195} & G_{196} & G_{197} & G_{198} & G_{199} & G_{200} \\
\frac{G_{163}}{G_{168}} & \frac{G_{164}}{G_{168}} & \frac{G_{165}}{G_{168}} & \frac{G_{166}}{G_{168}} & \frac{G_{167}}{G_{168}} & -\frac{G_{155}\,\pi}{G_{168}\,V_{10}} \\
G_{188} & G_{189} & G_{190} & G_{191} & G_{192} & G_{193} \\
G_{175} & G_{176} & G_{177} & G_{178} & G_{179} & G_{180} \\
G_{169} & G_{170} & G_{171} & G_{172} & (1+G_{173}) & G_{174} \\
-\frac{K_3}{K_1}G_{195} & -\frac{K_3}{K_1}G_{196} & -\frac{K_3}{K_1}G_{197} & -\frac{K_3}{K_1}G_{198} & -\frac{K_3}{K_1}G_{199} & G_{201}
\end{bmatrix}
\begin{Bmatrix} \Delta \dot{x}_{10} \\ \Delta x_{10} \\ \Delta \dot{x}_{2c} \\ \Delta x_{2c} \\ \Delta \tau_{0} \\ \Delta V_{10} \end{Bmatrix}
$$

3.5 Impact Damper in Primary Mass 2

Here our system consists of three degrees of freedom,   should be six component vector.Here ξ perturbation consist of small variation of steady state value of x_1 ,x_2 ,x_1 ,x_2 ,V_2 ,τ . Hence subscript refers to the unperturbed condition.

The conditions that we have are:

	Steady State	Perturbed
At	$\Omega t = 0_+$	$\Omega t = (0+\Delta t_o)_+$
	$x_1 = x_{1c}$	$x_1 = x_{1c} + \Delta x_{1c}$
	$\dot{x}_1 = \dot{x}_{1c}$	$\dot{x}_1 = \dot{x}_{1c} + \Delta \dot{x}_{1c}$
	$x_2 = x_{20}$	$x_2 = x_{20} + \Delta x_{20}$
	$\dot{x}_2 = \dot{x}_{20}$	$\dot{x}_2 = \dot{x}_{20} + \Delta \dot{x}_{20}$
	$y_2 = \frac{d}{2}$	$y_2 = \frac{d}{2}$
	$\tau = \tau_o$	$\tau = \tau_o + \Delta \tau_o$
	$\dot{Y}_2 = -V_{20}$	$\dot{Y}_2 = -(V_{20} + \Delta V_{20})$
At	$\Omega t = \pi_+$	$\Omega t = (\pi + \Delta t_o')_+$
	$x_1 = -x_{1c}$	$x_1 = -(x_{1c} + \Delta x_{1c}')$
	$\dot{x}_1 = -\dot{x}_{1c}$	$\dot{x}_1 = -(\dot{x}_{1c} + \Delta \dot{x}_{1c})$
	$x_2 = -x_{20}$	$x_2 = -(x_{20} + \Delta x_{20}')$
	$y_2 = -\frac{d}{2}$	$y_2 = -\frac{d}{2}$
	$\tau = \tau_o$	$\tau = \tau_o + \Delta \tau_o'$
	$\dot{Y}_2 = V_{20}$	$\dot{Y}_2 = V_{20} + \Delta V_{20}'$
	$\dot{x}_2 = -\dot{x}_{20}$	$\dot{x}_2 = -(\dot{x}_{20} + \Delta \dot{x}_{20}')$

Here Equations (3.21),(3.22),(3.23) and (3.24) are same ~~here~~.

Let $\Delta T = (\Delta t'_0 - \Delta\tau)$ (3.100)

Proceeding as before

$$\begin{aligned}
-\Delta x'_{1c} = {} & e^{-\delta_1\frac{\pi}{\Omega}}\Big(\tfrac{\Delta T}{\Omega}\eta'_1 B_{10}\cos\eta'_1\tfrac{\pi}{\Omega} + \Delta B_{10}\sin\eta'_1\tfrac{\pi}{\Omega} + \\
& \Delta B_{20}\cos\eta'_1\tfrac{\pi}{\Omega} - \tfrac{\Delta T}{\Omega}\eta'_1 B_{20}\sin\eta'_1\tfrac{\pi}{\Omega}\Big) + e^{-\delta'_2\frac{\pi}{\Omega}}\Big(\\
& \tfrac{\Delta T}{\Omega}\eta'_2 C_{10}\cos\eta'_2\tfrac{\pi}{\Omega} + \Delta C_{10}\sin\eta'_2\tfrac{\pi}{\Omega} + \Delta C_{20}\cos\eta'_2\tfrac{\pi}{\Omega} - \\
& \tfrac{\Delta T}{\Omega}\eta'_2 C_{20}\sin\eta'_2\tfrac{\pi}{\Omega}\Big) - A_1(\Delta T + \Delta\tau_0)\cos\tau_0
\end{aligned} \qquad (3.101)$$

$$\begin{aligned}
-\Delta x'_{20} = {} & D_1 e^{-\delta_1\frac{\pi}{\Omega}}\Big(\tfrac{\Delta T}{\Omega}\eta'_1 B_{10}\cos\eta'_1\tfrac{\pi}{\Omega} + \Delta B_{10}\sin\eta'_1\tfrac{\pi}{\Omega} + \\
& \Delta B_{20}\cos\eta'_1\tfrac{\pi}{\Omega} - \tfrac{\Delta T}{\Omega}\eta'_1 B_{20}\sin\eta'_1\tfrac{\pi}{\Omega}\Big) + D_2 e^{-\delta'_2\frac{\pi}{\Omega}}\Big(\\
& \tfrac{\Delta T}{\Omega}\eta'_2 C_{10}\cos\eta'_2\tfrac{\pi}{\Omega} + \Delta C_{10}\sin\eta'_2\tfrac{\pi}{\Omega} + \Delta C_{20}\cos\eta'_2\tfrac{\pi}{\Omega} - \\
& \tfrac{\Delta T}{\Omega}\eta'_2 C_{20}\sin\eta'_2\tfrac{\pi}{\Omega}\Big) - A_1(\Delta T + \Delta\tau_0)\cos(\tau_0 + \psi)
\end{aligned} \qquad (3.102)$$

$$\begin{aligned}
-\Delta\dot{x}'_{1c} = {} & e^{-\delta_1\frac{\pi}{\Omega}}\Big(\tfrac{\Delta T}{\Omega}\eta'_1\big(-B_{10}(\delta'_1\cos\eta'_1\tfrac{\pi}{\Omega} + \eta'_1\sin\eta'_1\tfrac{\pi}{\Omega}) + \\
& B_{20}(\delta'_1\sin\eta'_1\tfrac{\pi}{\Omega} - \eta'_1\cos\eta'_1\tfrac{\pi}{\Omega})\big) + \Delta B_{10}(-\delta'_1\sin\eta'_1\tfrac{\pi}{\Omega} \\
& + \eta'_1\cos\eta'_1\tfrac{\pi}{\Omega}) + \Delta B_{20}(-\delta'_1\cos\eta'_1\tfrac{\pi}{\Omega} - \eta'_1\sin\eta'_1\tfrac{\pi}{\Omega})\Big) + \\
& e^{-\delta'_2\frac{\pi}{\Omega}}\Big(\tfrac{\Delta T}{\Omega}\eta'_2\big(-C_{10}(\delta'_2\cos\eta'_2\tfrac{\pi}{\Omega} + \eta'_2\sin\eta'_2\tfrac{\pi}{\Omega}) + \\
& C_{20}(\delta'_2\sin\eta'_2\tfrac{\pi}{\Omega} - \eta'_2\cos\eta'_2\tfrac{\pi}{\Omega})\big) + \Delta C_{10}(-\delta'_2\sin\eta'_2\tfrac{\pi}{\Omega} + \\
& \eta'_2\cos\eta'_2\tfrac{\pi}{\Omega}) + \Delta C_{20}(-\delta'_2\cos\eta'_2\tfrac{\pi}{\Omega} - \eta'_2\sin\eta'_2\tfrac{\pi}{\Omega})\Big) + \\
& \Omega A_1(\Delta T + \Delta\tau_0)\sin\tau_0
\end{aligned} \qquad (3.103)$$

$$\begin{aligned}
\Delta\dot{x}_{2(\frac{\pi+\Delta t}{\Omega})} = {} & D_1 e^{-\delta'_1\frac{\pi}{\Omega}}\Big(\tfrac{\Delta T}{\Omega}\eta'_1\big(-B_{10}(\delta'_1\cos\eta'_1\tfrac{\pi}{\Omega} + \eta'_1\sin\eta'_1\tfrac{\pi}{\Omega}) + \\
& B_{20}(\delta'_1\sin\eta'_1\tfrac{\pi}{\Omega} - \eta'_1\cos\eta'_1\tfrac{\pi}{\Omega})\big) + \Delta B_{10}(-\delta'_1\sin\eta'_1\tfrac{\pi}{\Omega} + \\
& \eta'_1\cos\eta'_1\tfrac{\pi}{\Omega})\Big) + D_2 e^{-\delta'_2\frac{\pi}{\Omega}}\Big(\tfrac{\Delta T}{\Omega}\eta'_2\big(-C_{10}(\delta'_2\cos\eta'_2\tfrac{\pi}{\Omega} + \\
& \eta'_2\sin\eta'_2\tfrac{\pi}{\Omega} + \eta'_2\sin\eta'_2\tfrac{\pi}{\Omega}) + C_{20}(\delta'_2\sin\eta'_2\tfrac{\pi}{\Omega} - \eta'_2\cos\eta'_2\tfrac{\pi}{\Omega})\big) \\
& + \Delta C_{10}(-\delta'_2\sin\eta'_2\tfrac{\pi}{\Omega} + \eta'_2\cos\eta'_2\tfrac{\pi}{\Omega}) + \Delta C_{20}(-\delta'_2\cos\eta'_2\tfrac{\pi}{\Omega} - \\
& \eta'_2\sin\eta'_2\tfrac{\pi}{\Omega})\Big) + \Omega A_2(\Delta T + \Delta\tau_0)\sin(\tau_0 + \psi)
\end{aligned}$$

(3.104)

Using Equations(2.8) and(2.9)

$$-\Delta x'_{20} = K_1\,\Delta \dot{x}_2\left(\frac{\pi+\Delta t'_o}{\Omega}\right) - K_2\,\Delta V_{20} \qquad (3.105)$$

$$\Delta V'_{20} = K_3\,\Delta \dot{x}_2\left(\frac{\pi+\Delta t'_o}{\Omega}\right) - K_4\,\Delta V_{20} \qquad (3.106)$$

The time required by particle to travel from one side of container to the other side equals to the absolute distance travelled,divided by the absolute velocity.

Hence

$$((\pi+\Delta t'_o)-\Delta t_o)/\Omega = -((y_2(\pi+\Delta t'_o) + x_2(\pi+\Delta t'_o)) - (y_2(\Delta t_o) + x_2(\Delta t_o))/(V_{20}+\Delta V_{20}) \qquad (3.107)$$

Substituting the values of the quantities

$$\Delta T = \frac{\Omega}{V_{20}}\Delta x'_{20} + \frac{\Omega}{V_{20}}\Delta x_{20} - \frac{\pi}{V_{20}}\Delta V_{20} \qquad (3.108)$$

$$\Delta x'_{20} = \frac{G_{202}}{G_{207}}\Delta \dot{x}_{1C} + \frac{G_{203}}{G_{207}}\Delta x_{1C} + \frac{G_{204}}{G_{207}}\Delta \dot{x}_{20} + \frac{G_{205}}{G_{207}}\Delta x_{20} + \frac{G_{206}}{G_{207}}\Delta \tau_o - \frac{G_{161}\,\pi}{V_{20}\,G_{207}}\Delta V_{20} \qquad (3.109)$$

$$\Delta T = G_{208}\Delta \dot{x}_{1C} + G_{209}\Delta x_{1C} + G_{210}\Delta \dot{x}_{20} + G_{211}\Delta x_{20} + G_{212}\Delta \tau_o + G_{213}\Delta V_{20} \qquad (3.110)$$

$$\Delta \tau'_o = G_{208}\Delta \dot{x}_{1C} + G_{209}\Delta x_{1C} + G_{210}\Delta \dot{x}_{20} + G_{211}\Delta x_{20} + (1+G_{212})\Delta \tau_o + G_{213}\Delta V_{20} \qquad (3.111)$$

$$\Delta x'_{1C} = G_{214}\Delta \dot{x}_{1C} + G_{215}\Delta x_{1C} + G_{216}\Delta \dot{x}_{20} + G_{217}\Delta x_{20} + G_{218}\Delta \tau_o + G_{219}\Delta V_{20} \qquad (3.112)$$

$$\Delta \dot{x}'_{1C} = G_{220}\Delta \dot{x}_{1C} + G_{221}\Delta x_{1C} + G_{222}\Delta \dot{x}_{20} + G_{223}\Delta x_{20} + G_{224}\Delta \tau_o + G_{225}\Delta V_{20} \qquad (3.113)$$

$$\Delta \dot{x}'_{20} = G_{226}\Delta \dot{x}_{1C} + G_{227}\Delta x_{1C} + G_{228}\Delta \dot{x}_{20} + G_{229}\Delta x_{20} + G_{230}\Delta \tau_o + G_{231}\Delta V_{20} \qquad (3.114)$$

$$\Delta \dot{V}_{20} = -\frac{K_3}{K_1} G_{226}\, \Delta \dot{x}_{1c} - \frac{K_3}{K_1} G_{227}\, \Delta x_{1c} - \frac{K_3}{K_1} G_{228}\, \Delta \dot{x}_{20} - $$

$$- \frac{K_3}{K_1} G_{229}\, \Delta \dot{x}_{20} - \frac{K_3}{K_1} G_{230}\, \Delta T_0 + G_{232}\, \Delta V_{20} \tag{3.115}$$

$$
\begin{Bmatrix} \Delta \dot{x}'_{1c} \\ \Delta x'_{1c} \\ \Delta \dot{x}'_{20} \\ \Delta x'_{20} \\ \Delta \tau'_{0} \\ \Delta V'_{20} \end{Bmatrix}
=
\begin{bmatrix}
G_{220} & G_{221} & G_{222} & G_{223} & G_{224} & G_{225} \\
G_{214} & G_{215} & G_{216} & G_{217} & G_{218} & G_{219} \\
G_{226} & G_{227} & G_{228} & G_{229} & G_{230} & G_{231} \\
\frac{G_{202}}{G_{207}} & \frac{G_{203}}{G_{207}} & \frac{G_{204}}{G_{207}} & \frac{G_{205}}{G_{207}} & \frac{G_{206}}{G_{207}} & -\frac{G_{161}\pi}{G_{207}V_{20}} \\
G_{208} & G_{209} & G_{210} & G_{211} & 1+G_{212} & G_{213} \\
-\frac{K_3}{K_1}G_{226} & -\frac{K_3}{K_1}G_{227} & -\frac{K_3}{K_1}G_{228} & -\frac{K_3}{K_1}G_{229} & -\frac{K_3}{K_1}G_{230} & G_{232}
\end{bmatrix}
\begin{Bmatrix} \Delta \dot{x}_{1c} \\ \Delta x_{1c} \\ \Delta \dot{x}_{20} \\ \Delta x_{20} \\ \Delta \tau_{0} \\ \Delta V_{20} \end{Bmatrix}
$$

4. EXPERIMENTAL STUDIES

4.1 Introduction

The objectives of the present experimental study are :

a) To study the general response of the system for wide range parameters of the impact damper.

b) To verify the assumptions made in determining steady state solutions.

c) To study the motions, other than two symmetric impact per cycle.

4.2 Experimental Model

A schematic diagram of the experimental model is shown in Figure 1 .The photographs of the test rig and actual model are shown in Figures 3 and 4 respectively.The primary mass 1 and primary mass 2 are simple rectangular boxes with rigid stops at the ends to constrain the movement of the frictionless solid particle m_1 and m_2 a horizontal oscillation with in the clearance d_1 and d_2 respectively.The particles are hardened steel balls.Leaf springs K_1 and K_4 ,which support the primary mass 1 and primary mass 2 respectively also produce restoring forces.The springs K_2 and K_3 are helical springs.

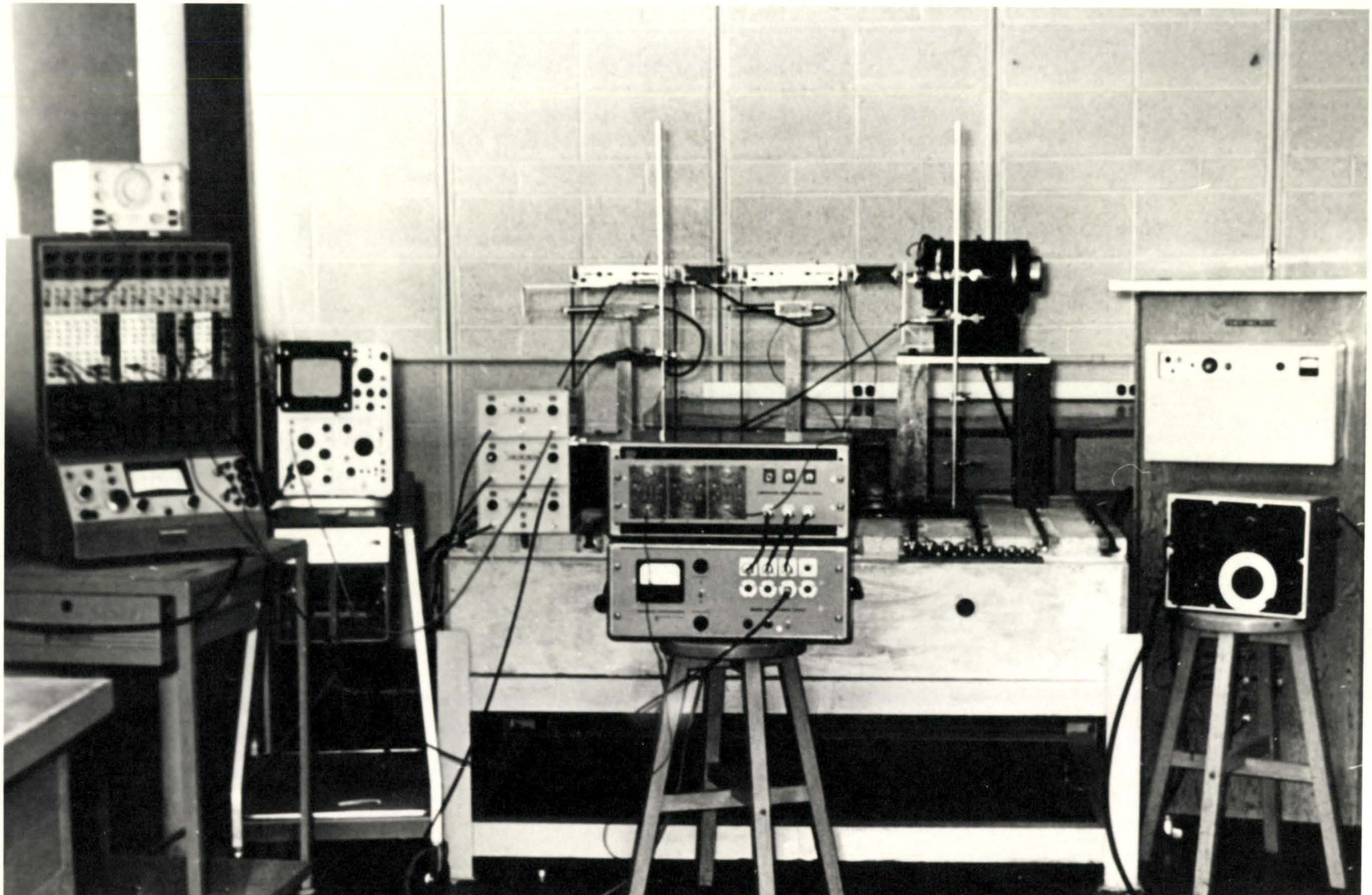


FIG. 3. GENERAL VIEW OF EXPERIMENTAL SET UP

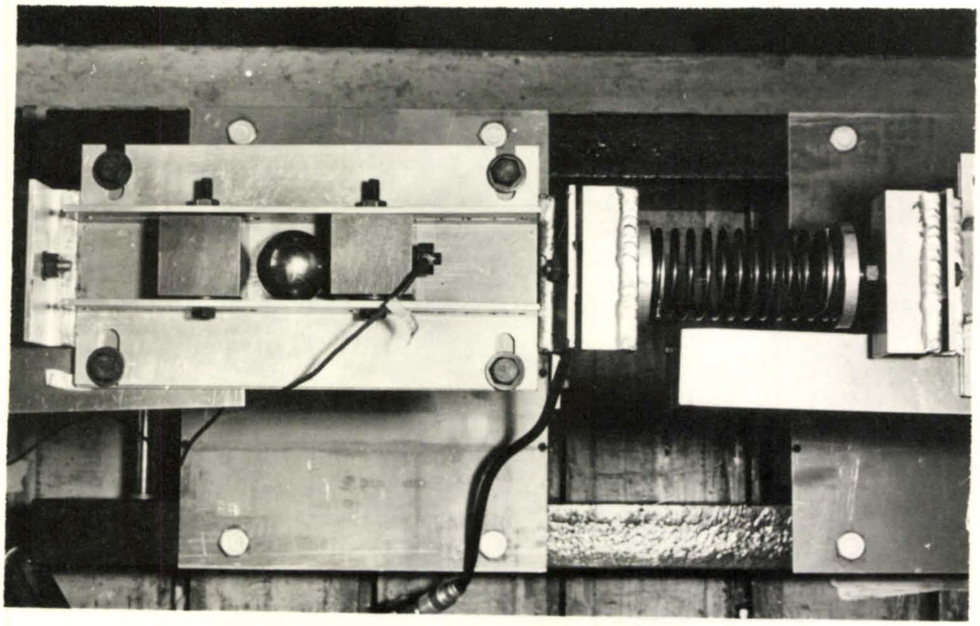


FIG. 4(a) TOP VIEW OF THE MODEL

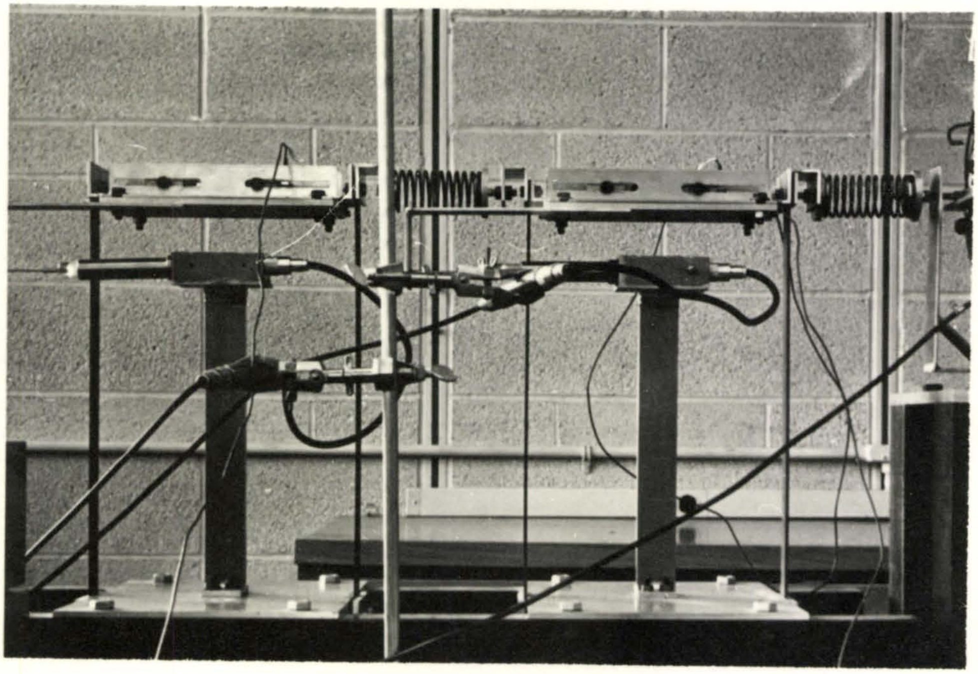
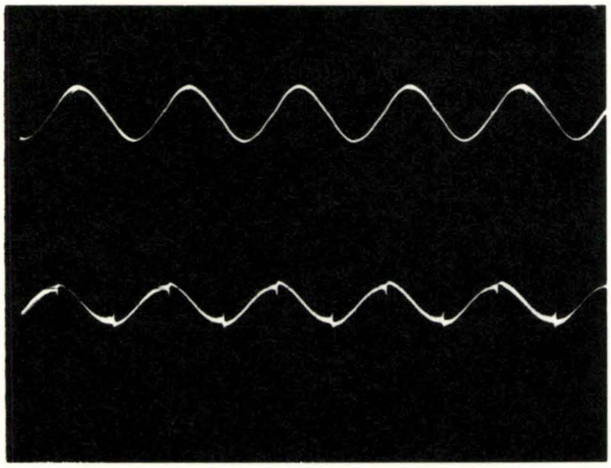
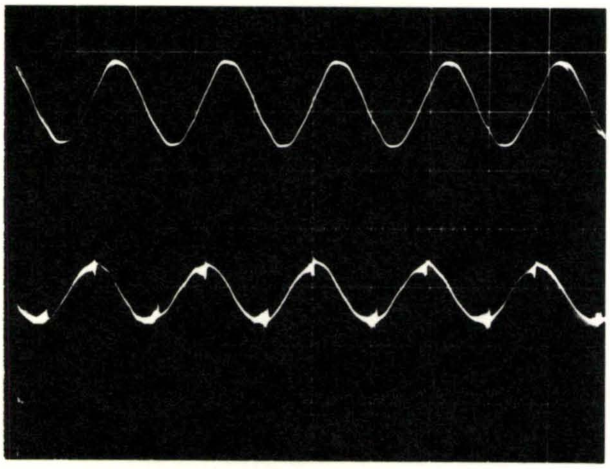


FIG. 4(b) FRONT VIEW OF THE MODEL



(a)



(b)

FIGURE 5 DISPLACEMENT AND VELOCITY CURVE AT SECOND NATURAL FREQUENCY

(a) Uppertrace X_1, lowertrace $\dot{X}_1$ for impact damper in mass 1.

(b) Uppertrace X_2, lowertrace $\dot{X}_2$ for impact damper in mass 2.

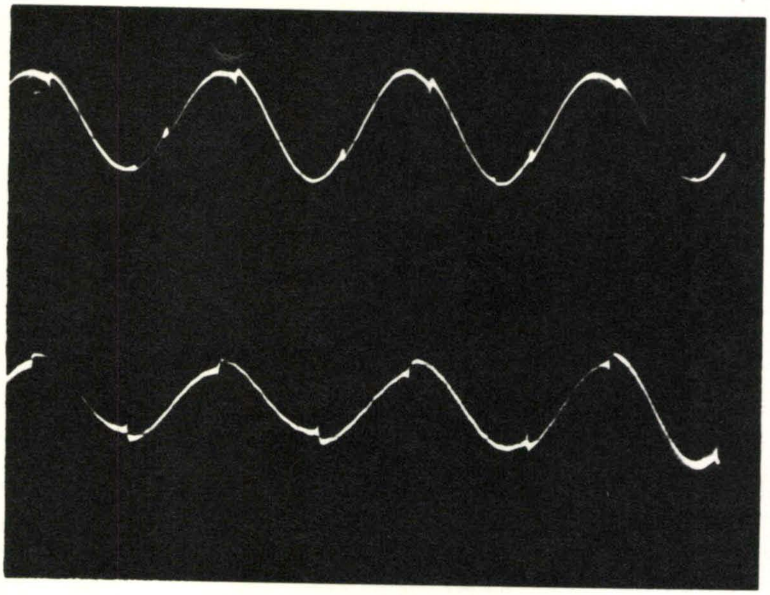


FIGURE 6 VELOCITY CURVE FOR IMPACT DAMPER IN BOTH MASSES AT 2nd NATURAL FREQUENCY

UPPER TRACE $\dot{X}_1$, LOWER TRACE $\dot{X}_2$

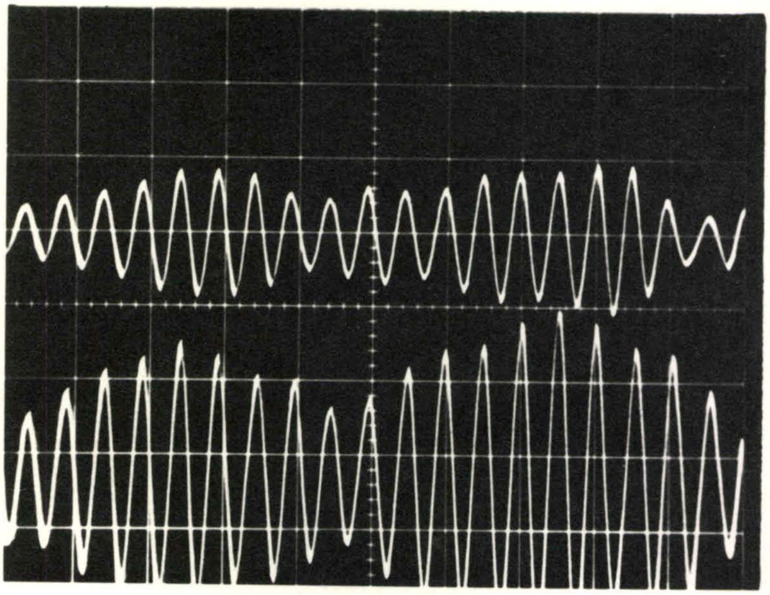
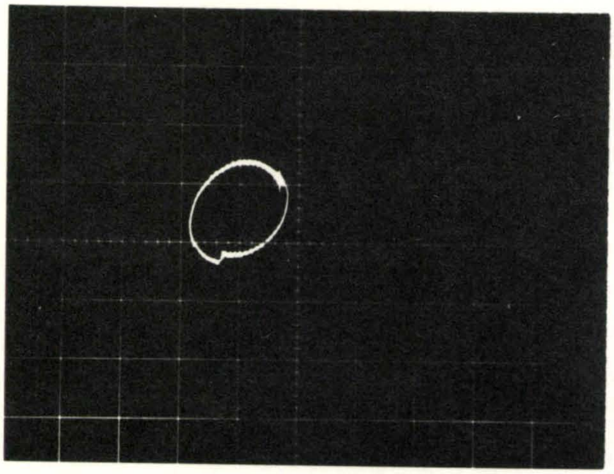
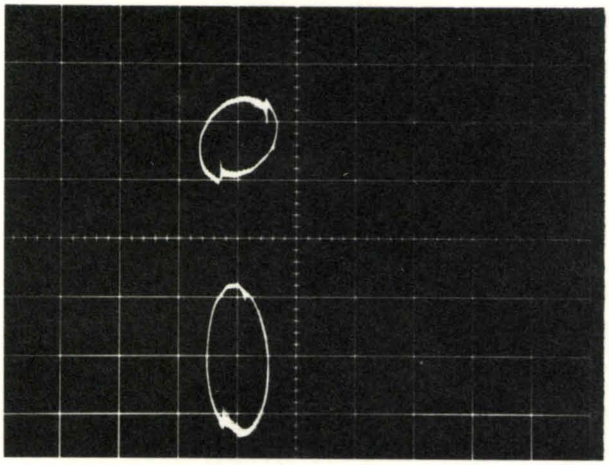


FIGURE 7 BEATING PHENOMENON

UPPER TRACE X_1 AND LOWER TRACE X_2



(a)



(b)

FIGURE 8 LIMIT CYCLES OF IMPACT DAMPER AT FIRST NATURAL FREQUENCY

(a) X_1, $\dot{X}_1$ for impact damper in primary mass 1.

(b) uppertrace X_1, $\dot{X}_1$, lowertrace X_1, $\dot{X}_2$ for impact damper in both masses.

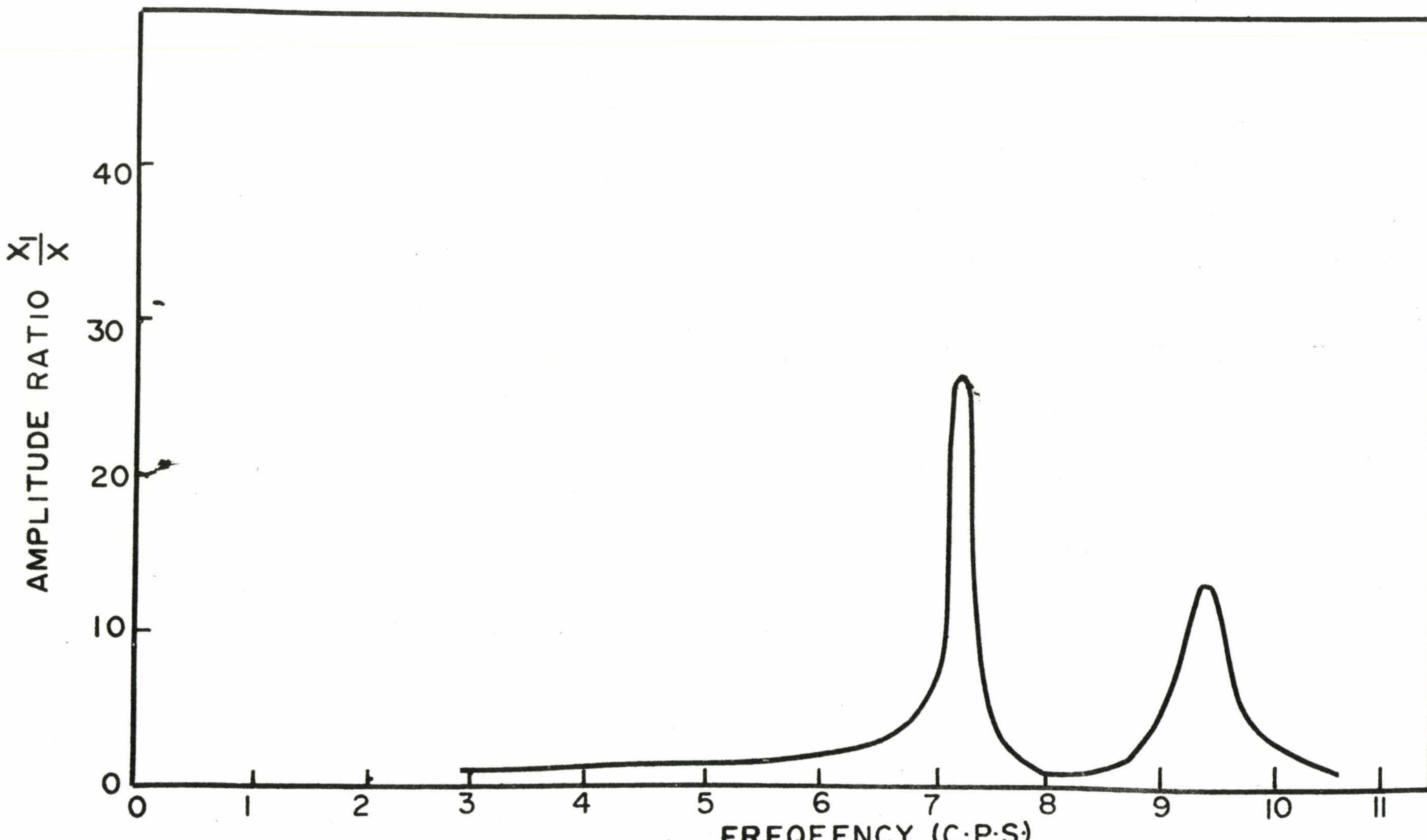


FIG 9 STEADY STATE RESPONSE OF PRIMARY MASS I
(WITH OUT IMPACT DAMPER)

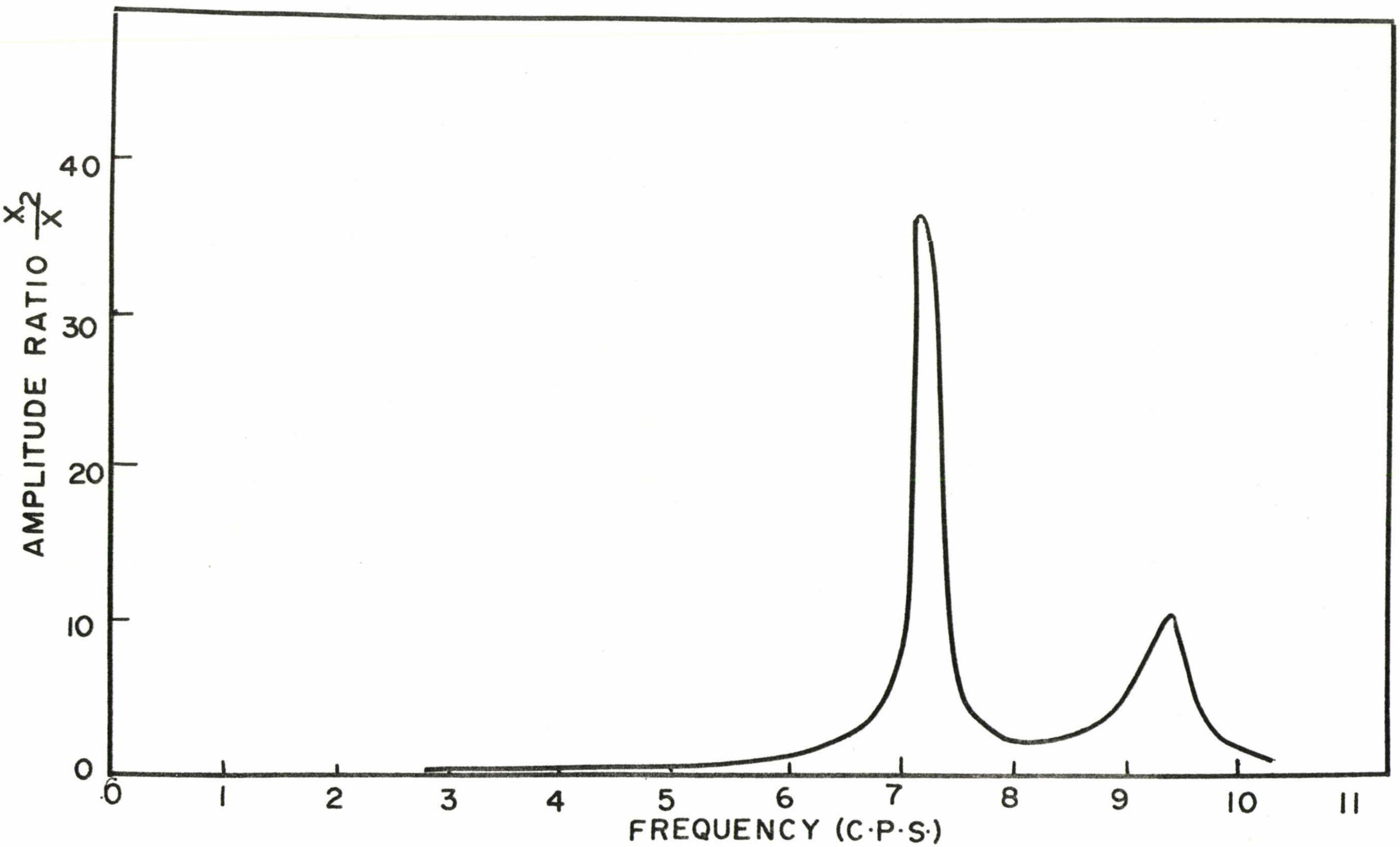


FIG 10 STEADY STATE RESPONSE OF PRIMARY MASS 2
(WITH OUT IMPACT DAMPER)

The system is excited by an electro-magnetic shaker through spring K_3.

4.3 Electronic Measurement

A capacitance pick-up,with associated electronic equipment,is used to find the displacement of the primary mass 1,primary mass 2 and also exciting displacement. Velocities of primary mass 1 and primary mass 2 are obtained by integrating with respect to time the outputs of Piezoelectric accelerometers attached to the respective masses.This integration is accomplished by using an integrating network in conjunction with an operation amplifier.

4.4 Experimental Results

a) Characteristics of the system without Impact Damper

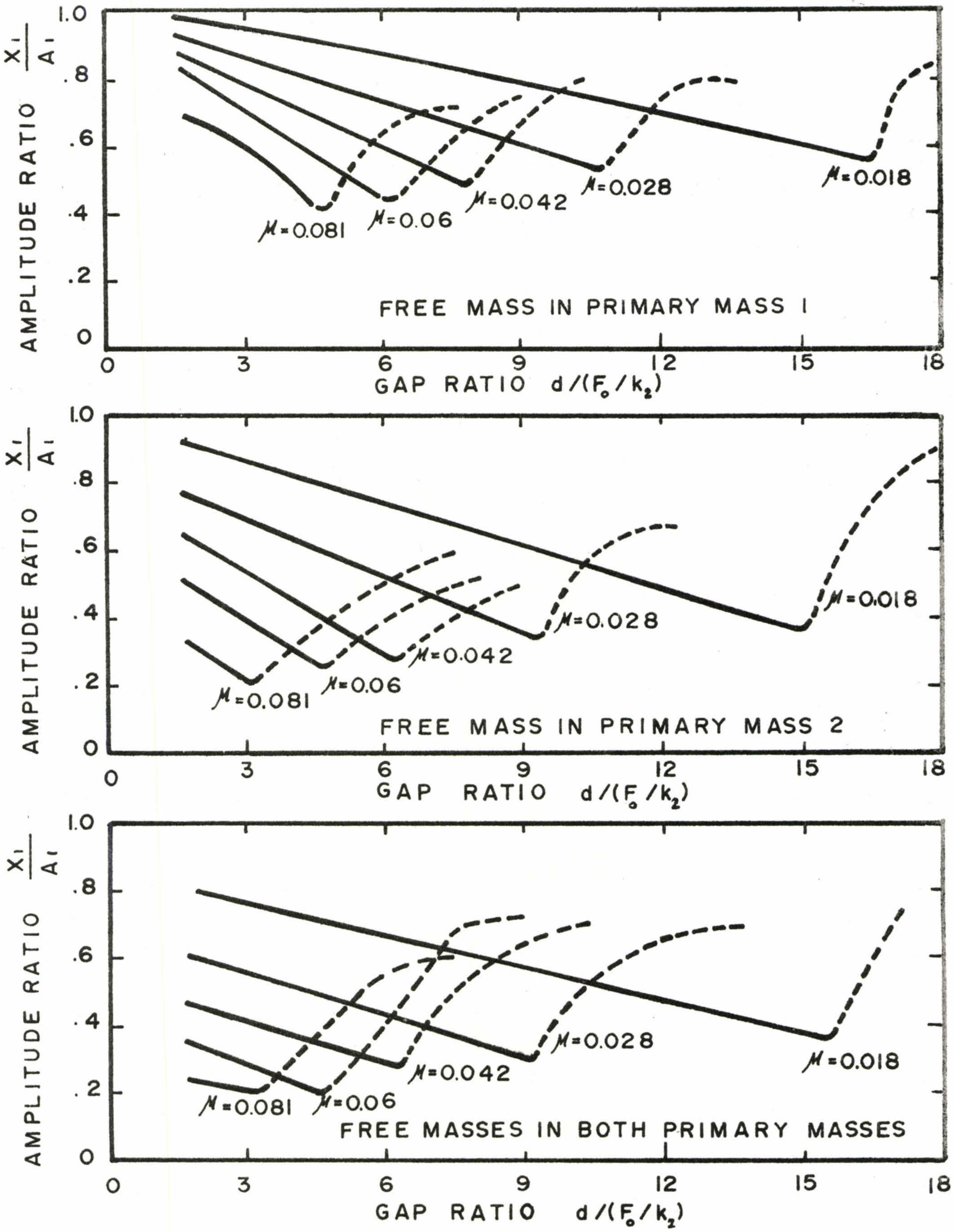
The equivalent characteristic constants of the system are;

$\omega_1 = 7.15$ $M_1 = M_2 = 4.4$ lbs. $K_2 = K_3 = 8$ lbs/in.

$\omega_2 = 9.35$ $K_1 = K_4 = 19.5$ lbs/ inch

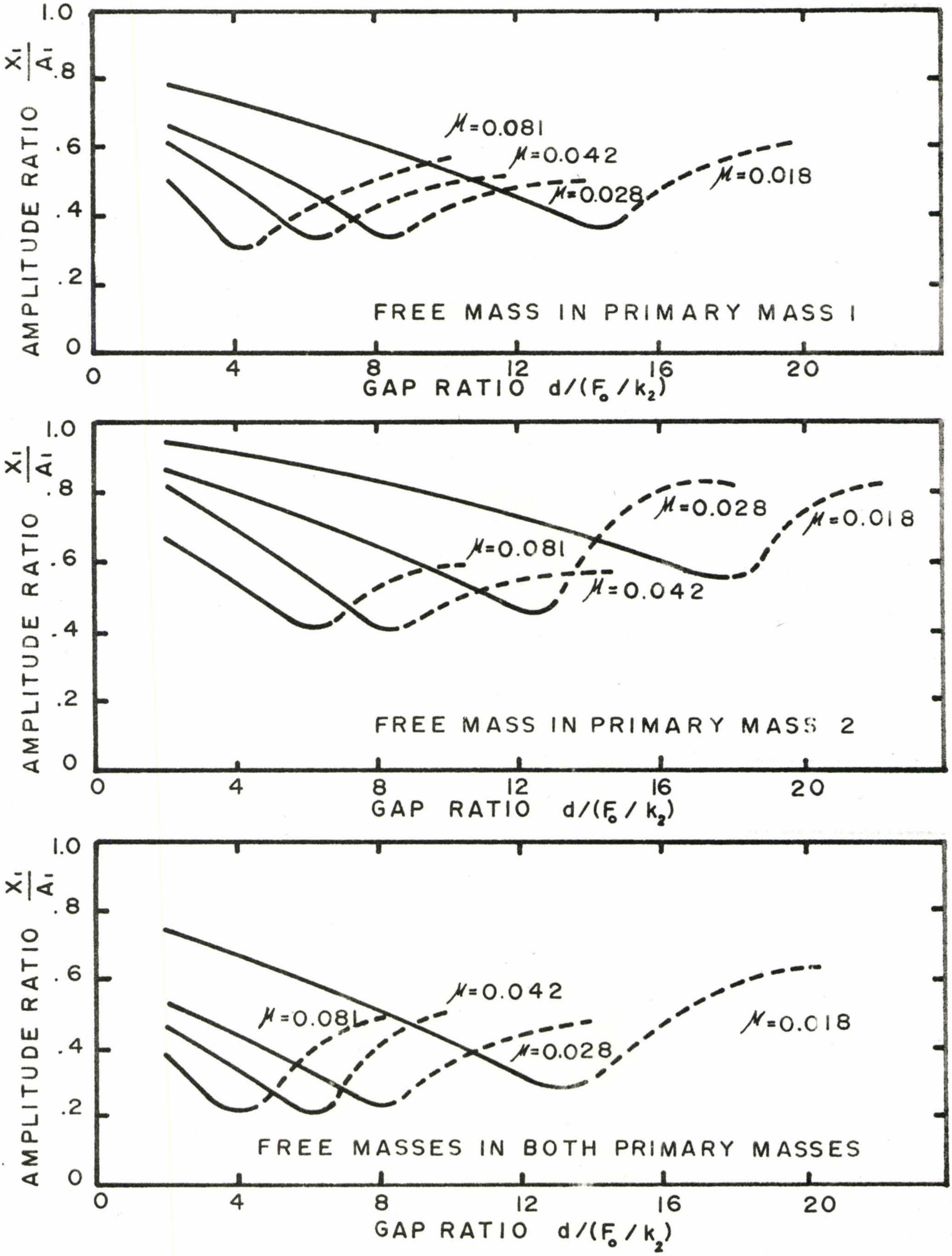
b) Characteristics of the system with Impact Damper in action

The effect of various parameters of impact damper namely mass ratio μ and gap ratio on the system responses for all the three cases,previously cited,is investigated.



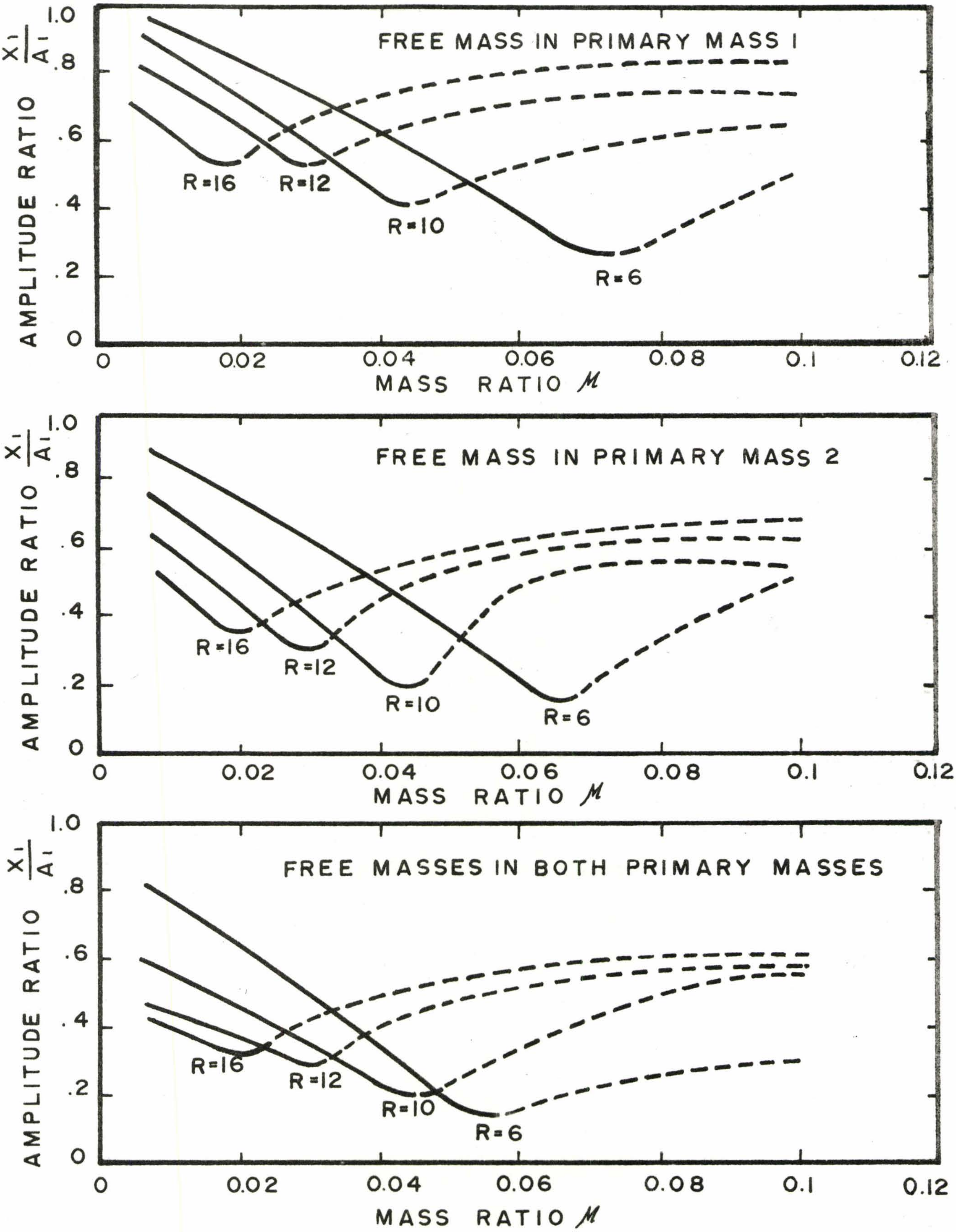
FIRST NATURAL FREQUENCY

FIG 11

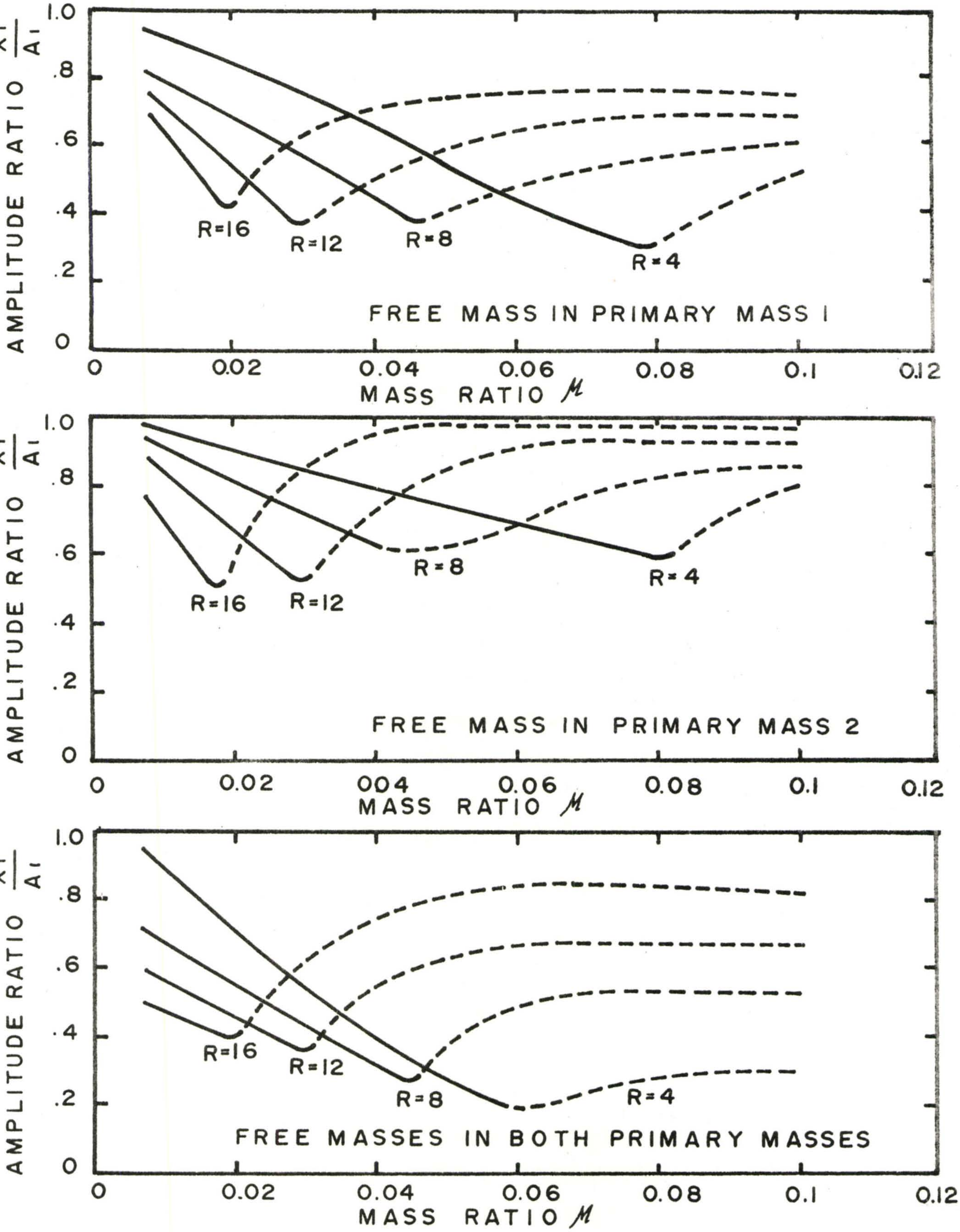


SECOND NATURAL FREQUENCY

FIG 12

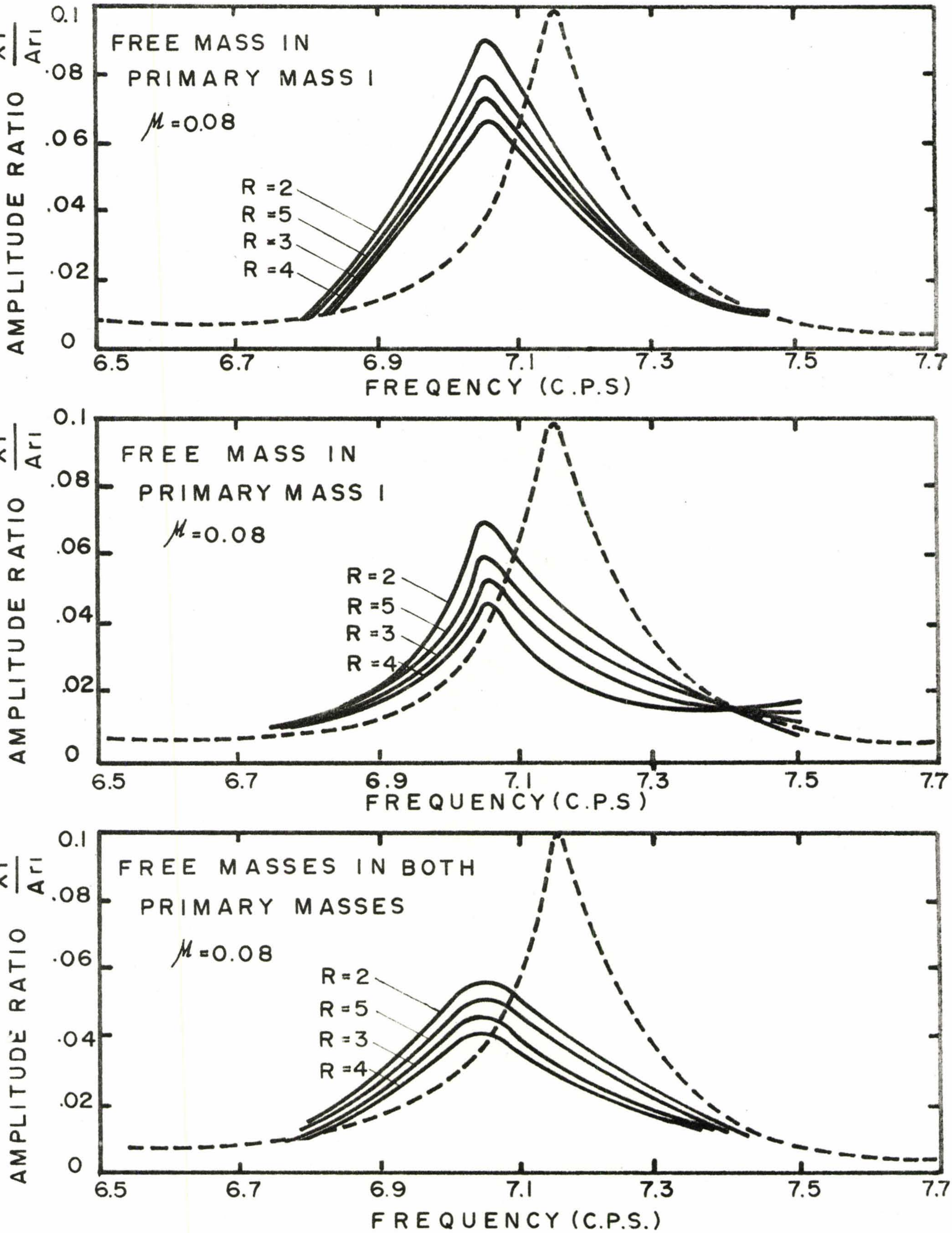


FIRST NATURAL FREQUENCY
FIG 13

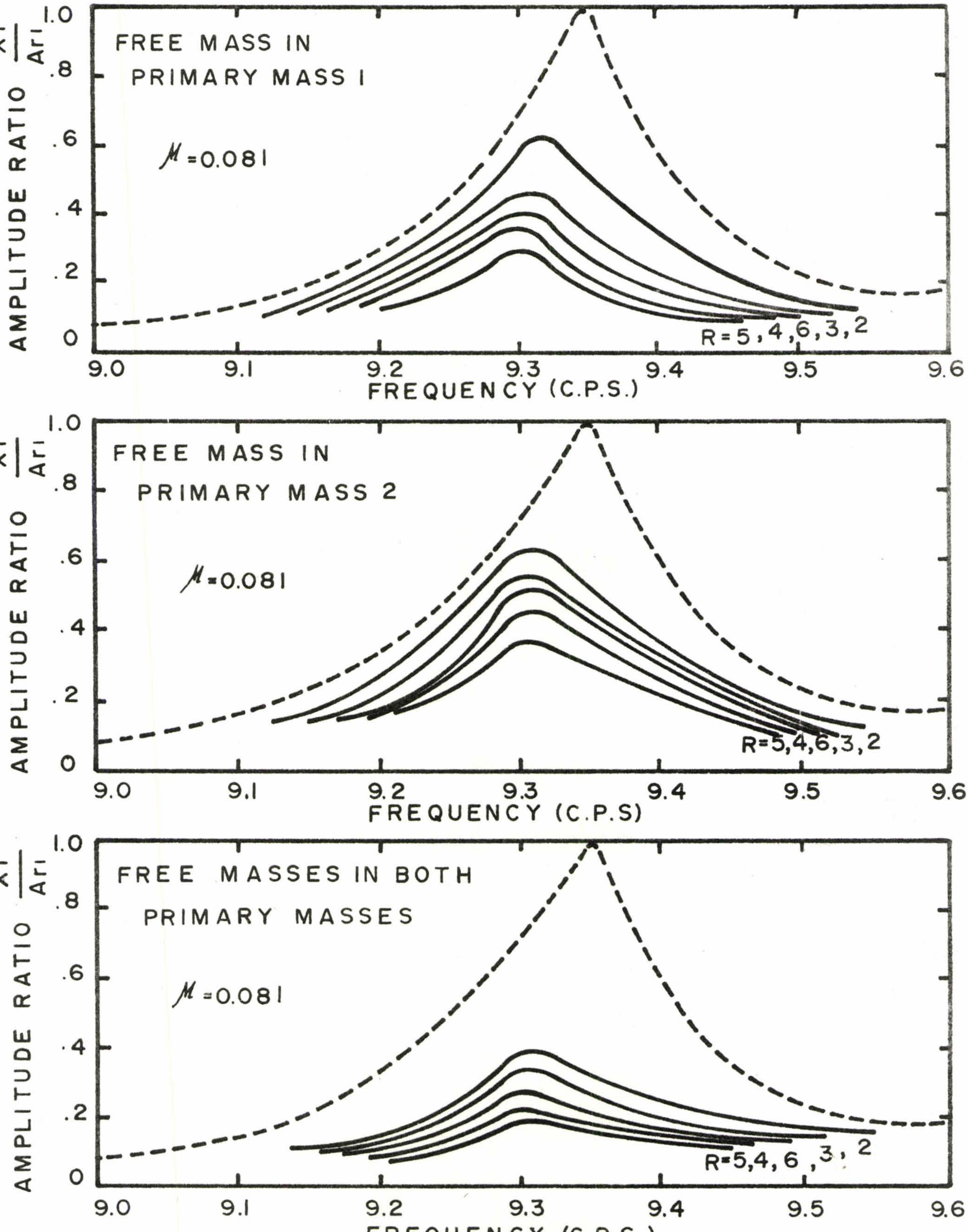


SECOND NATURAL FREQUENCY

FIG 14

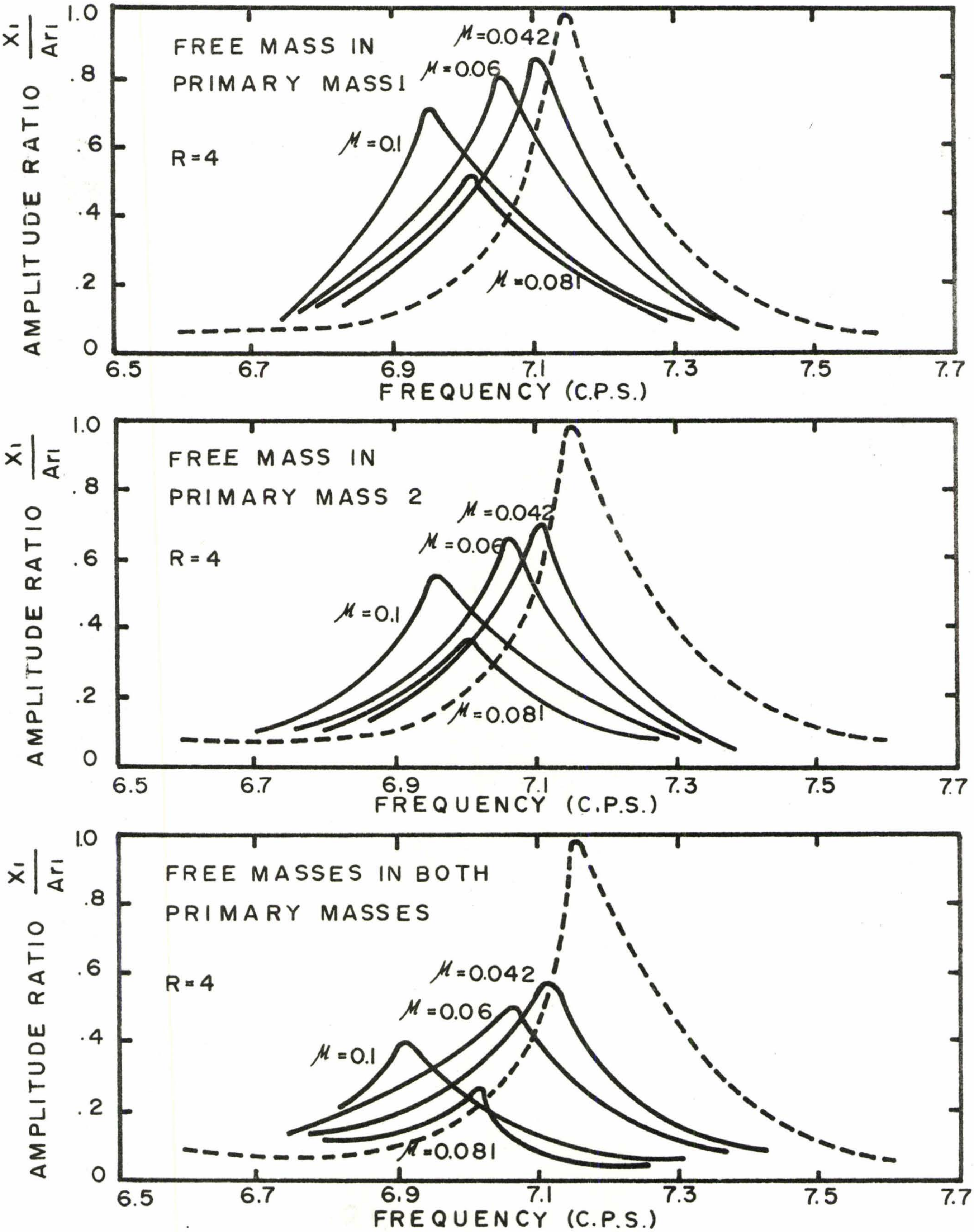


FIRST NATURAL FREQUENCY
FIG 15



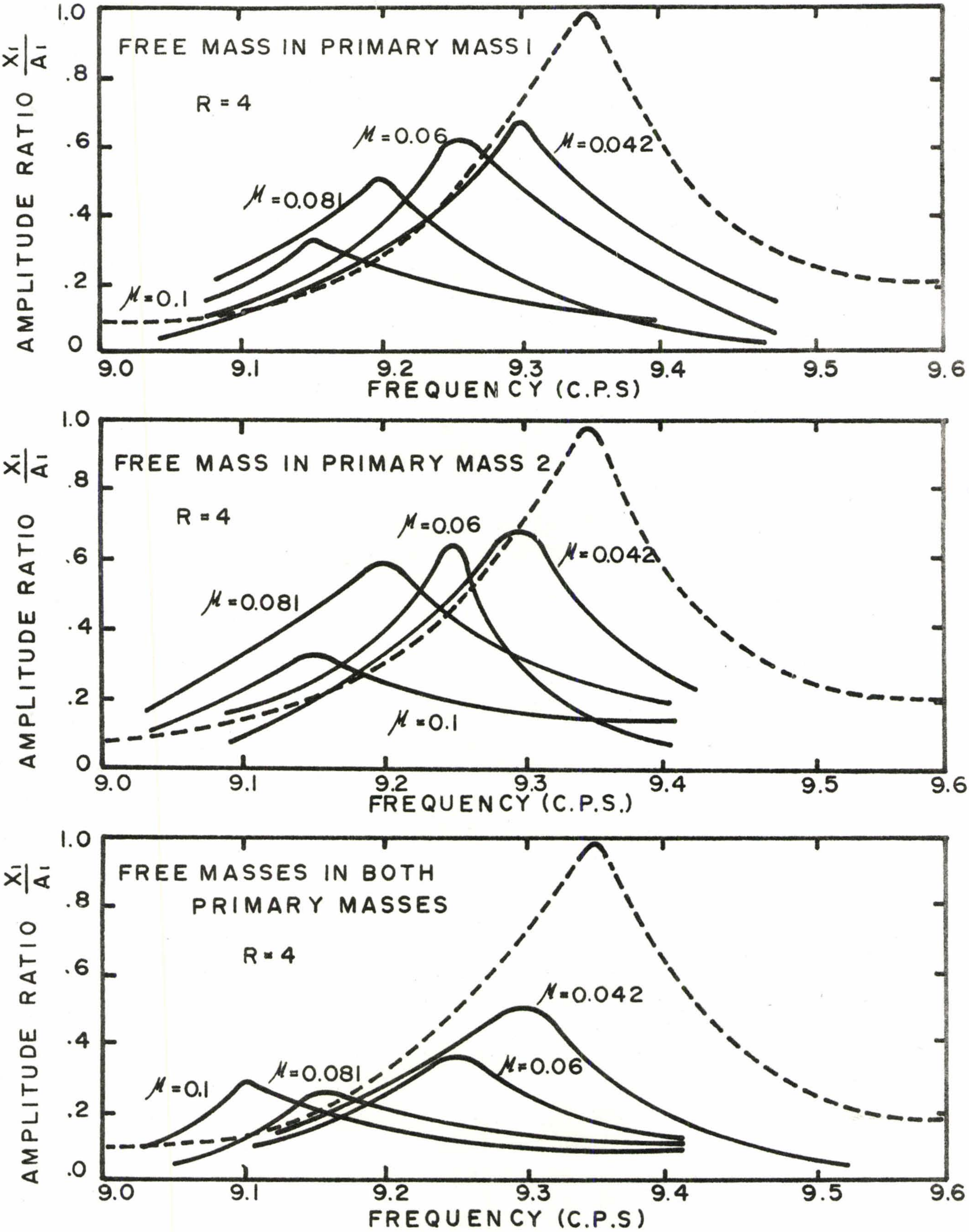
SECOND NATURAL FREQUENCY

FIG 16



FIRST NATURAL FREQUENCY

FIG 17



SECOND NATURAL FREQUENCY

FIG 18

Eight free mass ratios ranging from μ=0.005 to μ=0.1 are used.The free mass stroke are varied from d=0.005 to d=0.75 inch.

For the three cases of the system and for both natural frequencies,the ratio of resonant amplitude X_r (with impact damper in action) to A_r (with impact damper removed) is plotted versus d.The results are shown in Figures 11 to 14

For all the three cases as shown in Figures 15,16,17&18, $\frac{X_r}{A_r}$ is plotted versus frequency in the neighbourhood of first and second natural frequency by varying d and μ .

The velocities and displacements of the primary masses,with impact damper in action are shown in Figures 5 (a),(b)

The displacement of primary mass 1 versus its velocity,with impact damper in it,is shown in Figure 8(a).Figures 8(b) show a plot of displacement of primary mass 1 against its velocity and against the velocity ofprimary mass 2 respectively for the case of impact damper in both primary mass 1 and primary mass 2.

5. DISCUSSION OF RESULTS AND CONCLUSION

5.1 Discussion of Results

a) Experimental Results

From Figure 13 & 14, it is observed that for a system with fixed gap ratio, the increase in mass ratio results in a increase of damping effeciency. There exist however an optimum mass ratio, after which erratic behaviour of the damper starts and effeciency decreases. This erratic behaviour can be attributed to the fact that energy imparted to the free mass is inadequate to force the free mass to the opposite side of the container. Thus the free mass starts to oscillate, and the amplitude of the vibrating system builds up and subsequently impact occurs between container end and free mass. Due to the impact the vibrational amplitude decreases, resulting in a vibration wave form that resembles that of the beating phenomena, shown in Figure 7 . The beating zone is shown by dotted lines. The same effect is observed when the gap ratio, for the fixed mass ratio (as shown in Figure 11 & 12) is increased.

As shown in Figures 11-14, it will be observed that at the first natural frequency the system has more damping effeciency (for the same parameters of the impact damper),if impact damper is used in primary mass 2,instead of primary mass 1.This may be due to the fact that at the first natural frequency,the amplitude of displacement of primary mass 2 is higher than that of primary mass 1, so from Equation (2.88) it is obvious that for the same parameters of the impact damper,primary mass 2 will dissipate more energy on impact.Hence the impact damper will be more effecient in primary mass 2 than in primary mass 1.A similiar explanation can be given for second natural frequency.

It is observed that an impact damper in both primary masses is not always more effective than in a single primary mass.The reason is that the free mass starts its movement in the primary which has the higher amplitude of the displacement and is thus able to impart more energy.Sometime the amplitude of displacement of the other primary mass becomes so small that the energy imparted to the free mass will be inadequate to force the free mass to travel to the opposite end of the damper container.The free mass starts to oscillate and strikes the ends.So the effeciency remains same as for an impact damper in single primary mass.Effeciency can some times decrease due to beating.

As shown in Figure 11 ,it is found that for the same mass ratio,an impact damper in the primary mass, having higher amplitude of displacement,has a smaller optimum gap than when it is in the other primary mass. This can be attributed to the fact that a primary mass having higher amplitude of displacement,will have to dissipate more energy,for the same parameters of the impact damper.Therefore the maximum gap in which it can dissipate sufficient energy for the free mass to traverse will be less than the other.

From the Figure 17 it is clear that if no compensation is made for the increase in the primary mass due to the addition of the free mass (as in our system) ,then the natural frequency decreases with impact in action.

It is also evident that the impact damper is most efficient at natural frquencies.

Figures 5 & 6 & 8 show that in most cases,there are two symmetric impacts per cycle of motion.So the assumption of 2 symmetric impacts per cycle is justified.

From Figures 5 & 6 it is clear that the velocity of the primary mass which is under impact,changes at impact discontinuously,while the velocity of the other remains same.The displacement of the primary masses however do not change due to impact.

b) Theoretical Results

Figure 19 ,shows two distinct curves marked as τ_2 and τ_3 ,and correspond to two theoretical solutions of the system in the case of the impact damper in primary mass 1.Here two solutions are obtained by using the value of τ in one case

$$\tau = \tau_2 = \tan^{-1}\left(\frac{-2\rho + H\sqrt{H^2+4-\rho^2}}{-\rho H - 2\sqrt{H^2+4-\rho^2}}\right)$$

and in other

$$\tau = \tau_3 = \tan^{-1}\left(\frac{-2\rho - H\sqrt{H^2+4-\rho^2}}{-\rho H - 2\sqrt{H^2+4-\rho^2}}\right)$$

The two solutions concide at the extreme d=0 for which

$$\tau_2 = \tau_3 = \tan^{-1}\left(-\frac{H_2}{2}\right)$$

and at gap ratio,where, $H^2+4-\rho^2$ =0;Here

$$\tau_2 = \tau_3 = \tan^{-1}\left(\frac{2}{H}\right)$$

By further increasing the gap ratio, τ becomes complex;consequently our 2 impact per cycle solutions do not exist.

The stability analysis indicates that the curve τ_3 is entirely unstable,while τ_2 curve is only partly stable.

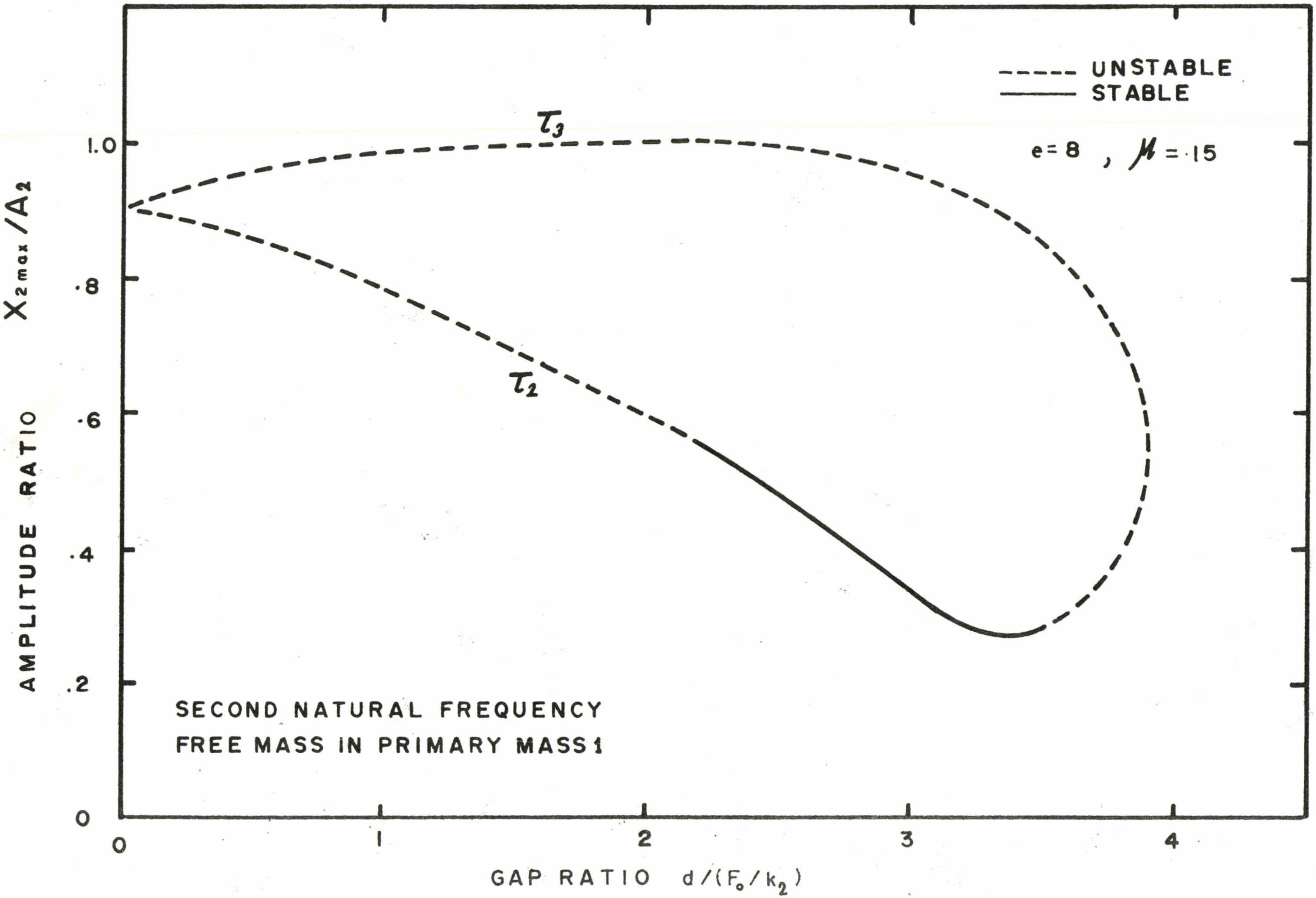
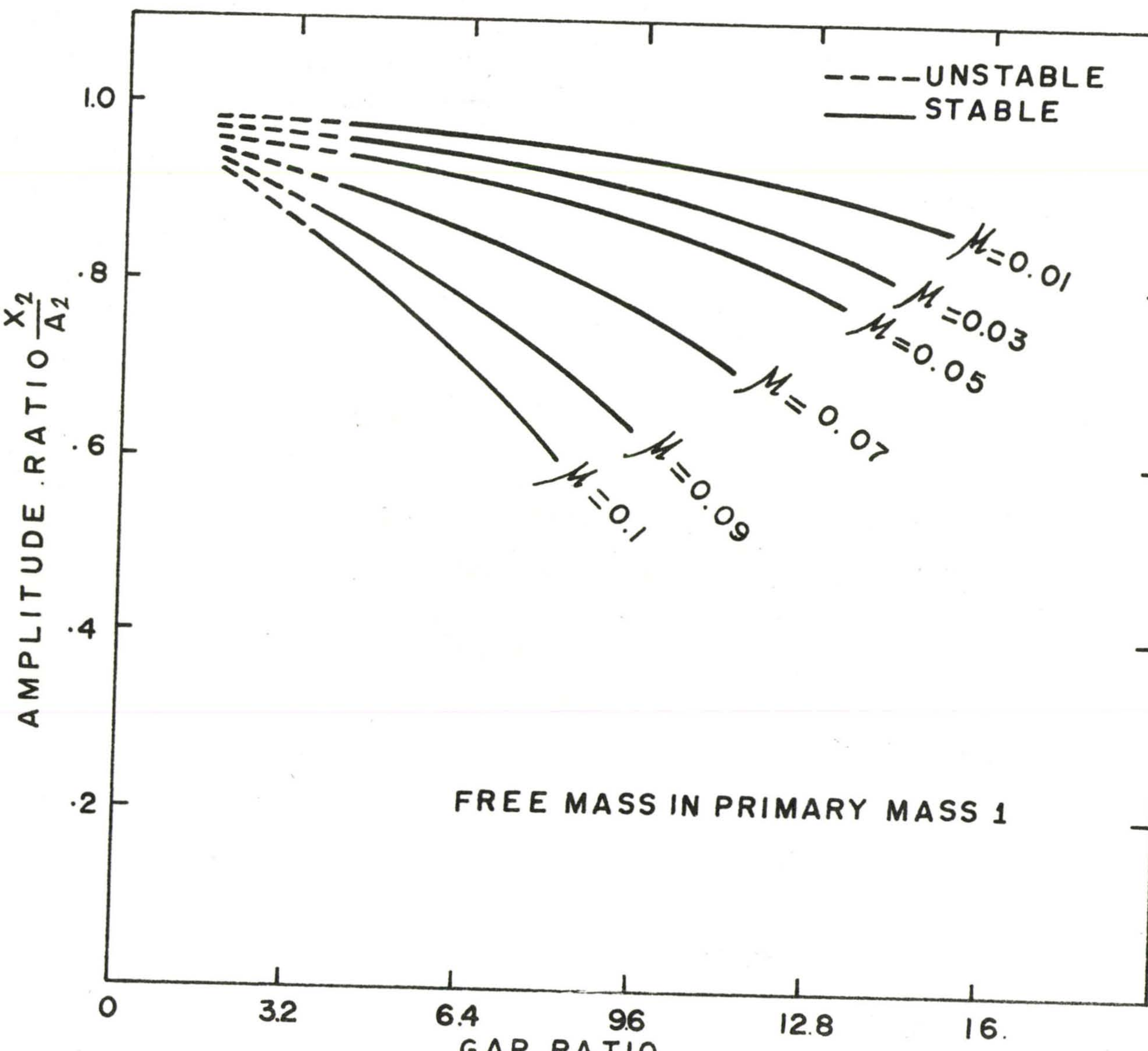
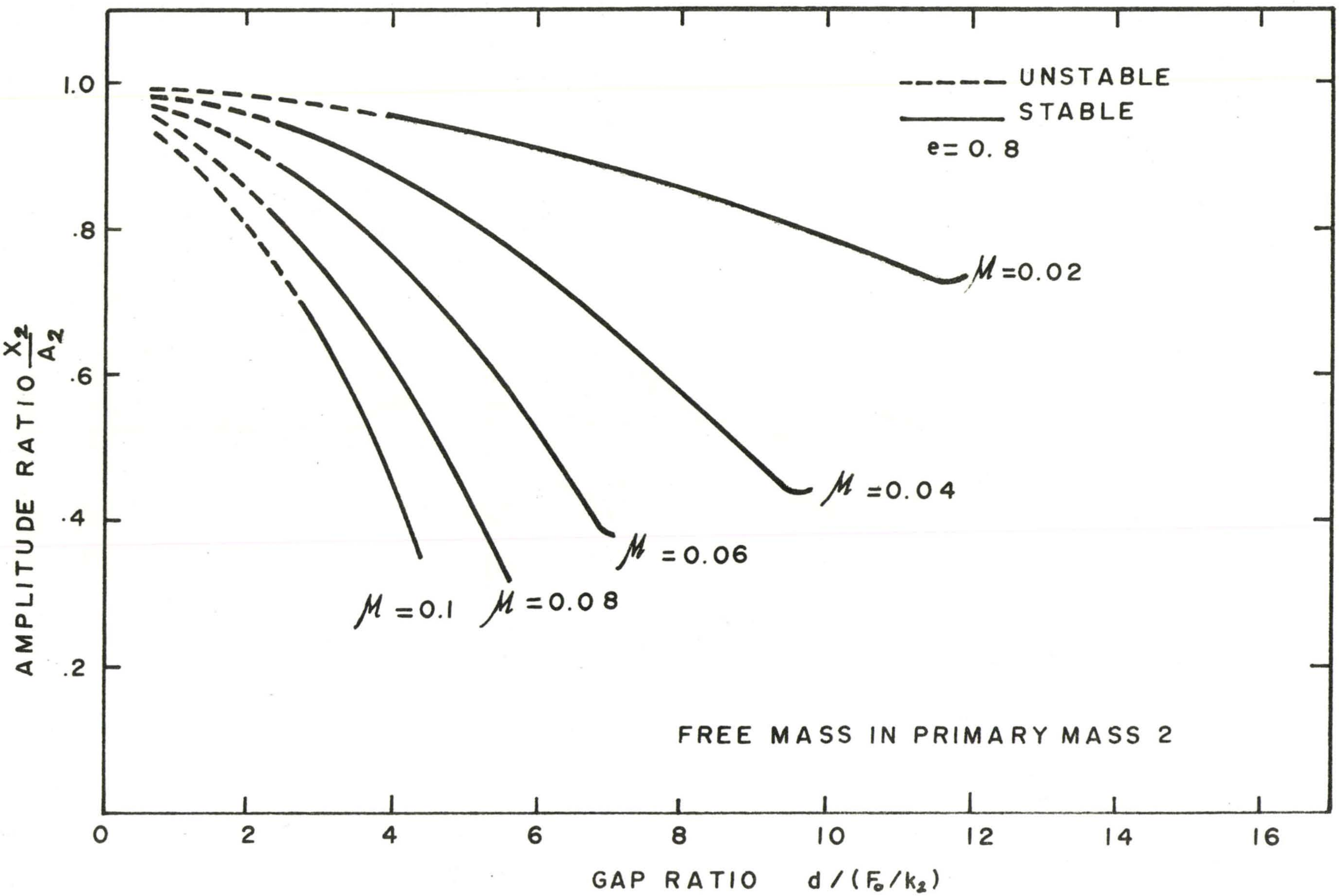


FIG 19 STEADY STATE SOLUTION



FIRST NATURAL FREQENCY
FIG 20



FIRST NATURAL FREQENCY
FIG 21

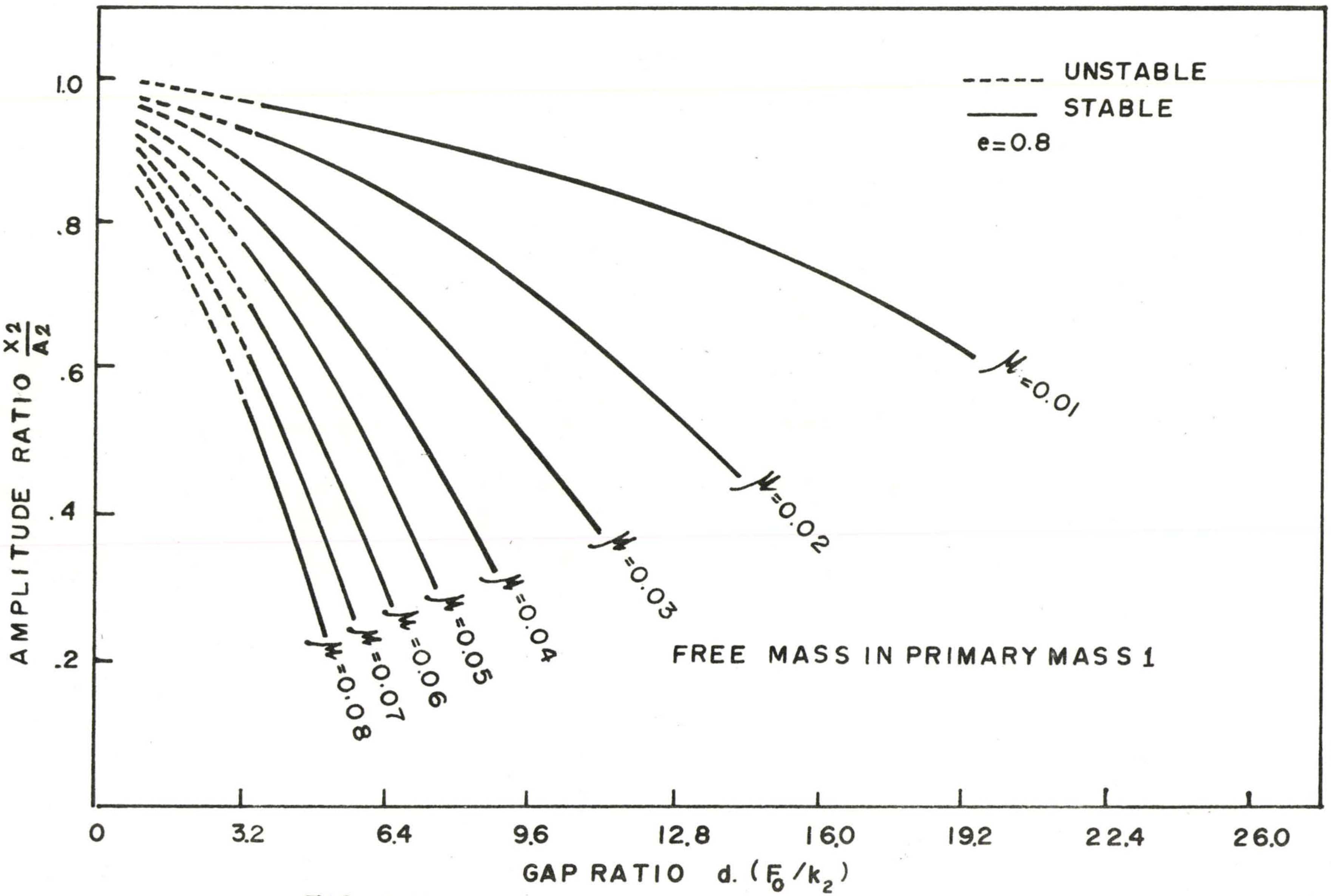
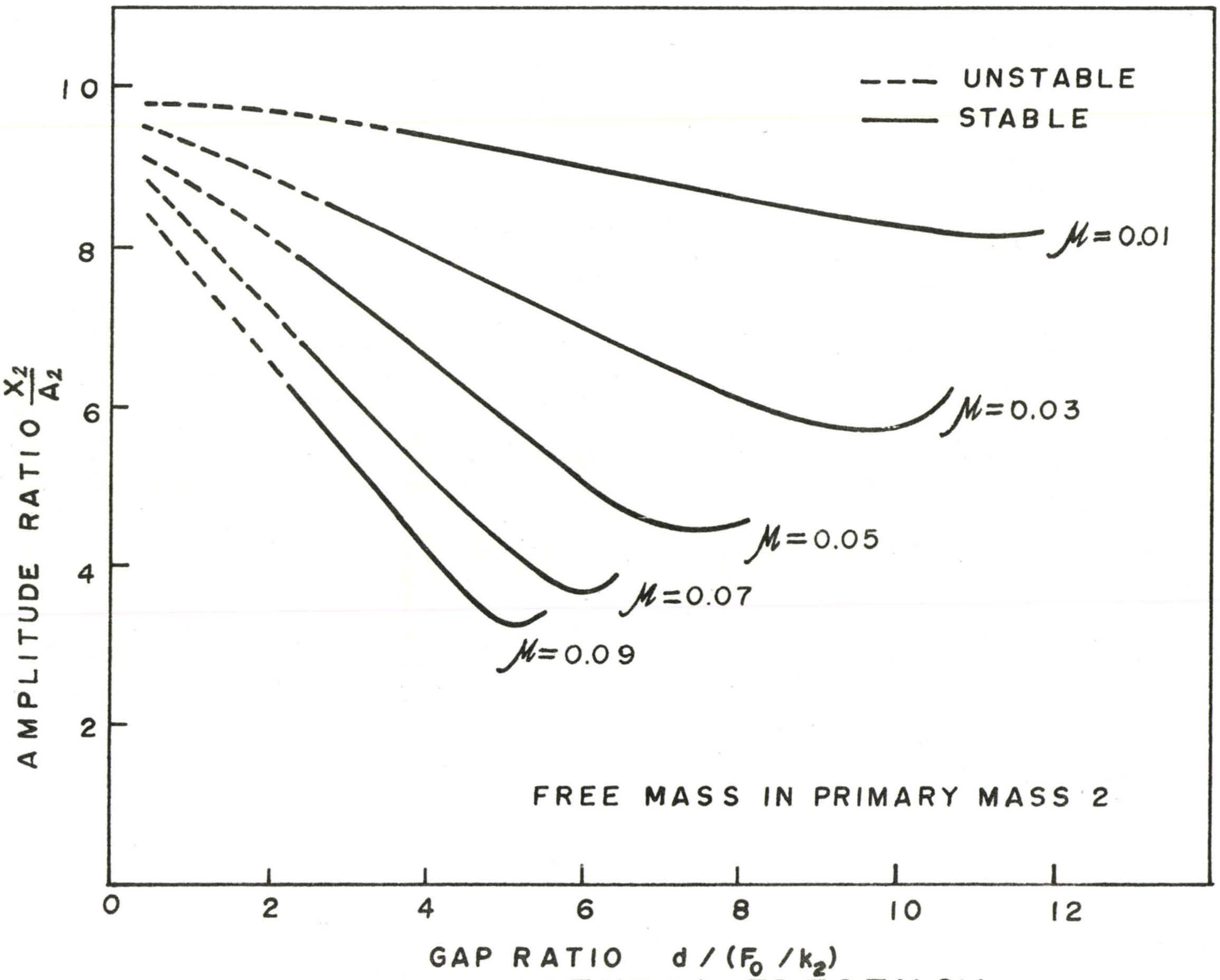
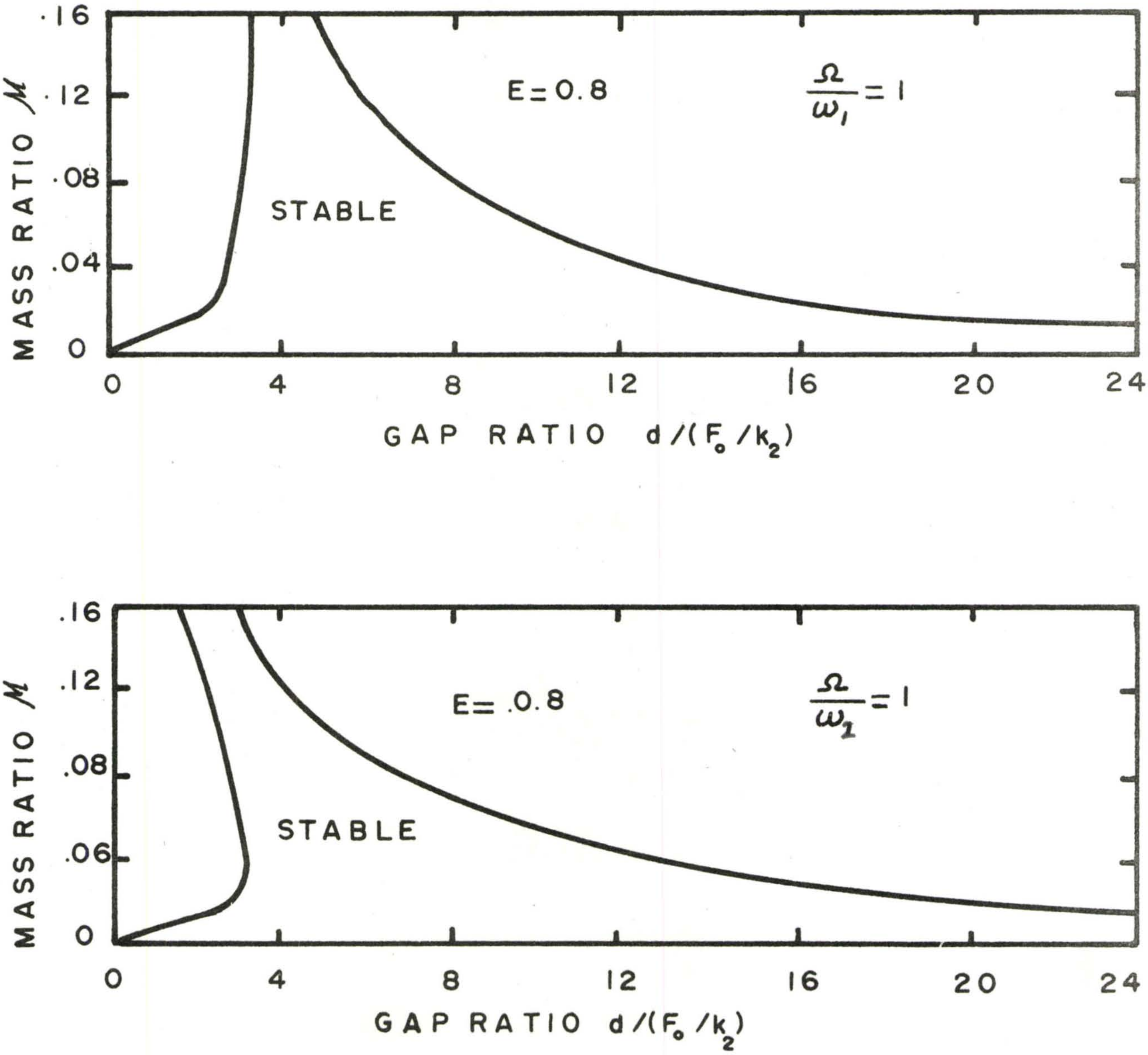


FIG 22 SECOND NATURAL FREQENCY



SECOND NATURAL FREQENCY

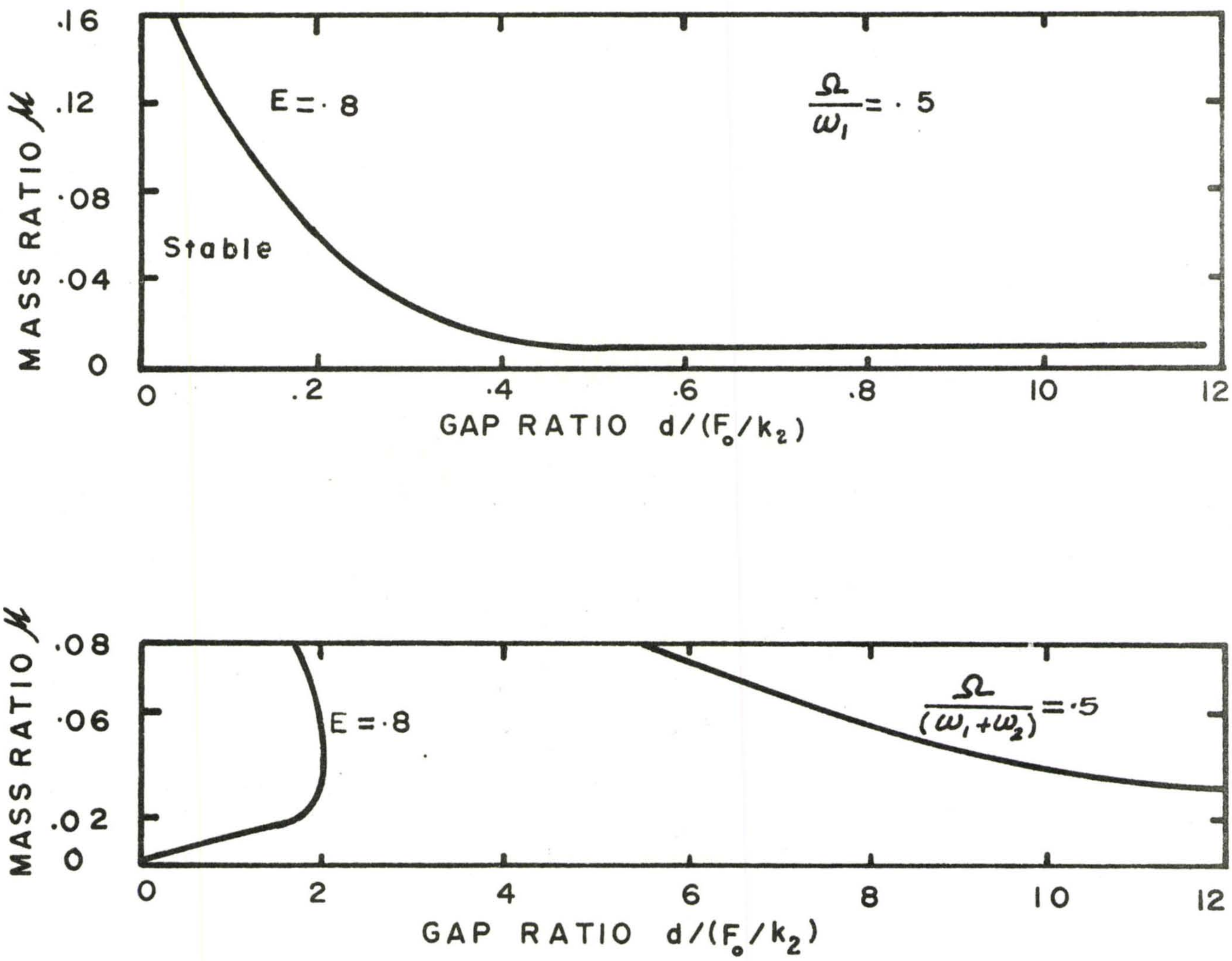
FIG23



FREE MASS IN PRIMARY MASS I

STABILITY BOUNDARIES ($e=\cdot 8$ $\delta_1=\cdot 1$)

FIG 24

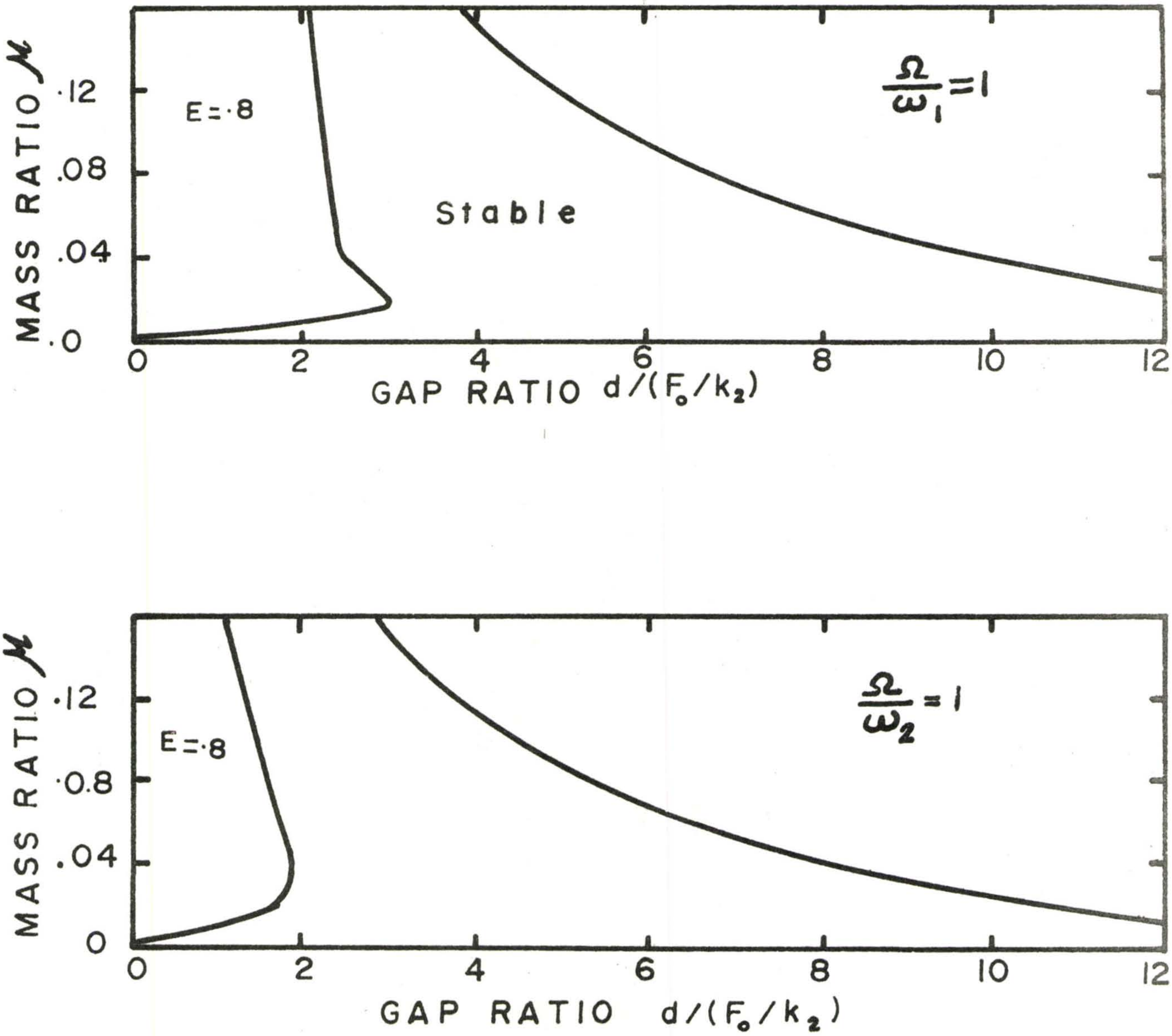


FREE MASS IN PRIMARY MASS 2

STABILITY BOUNDARIES

($e = .8$ $\delta = .1$)

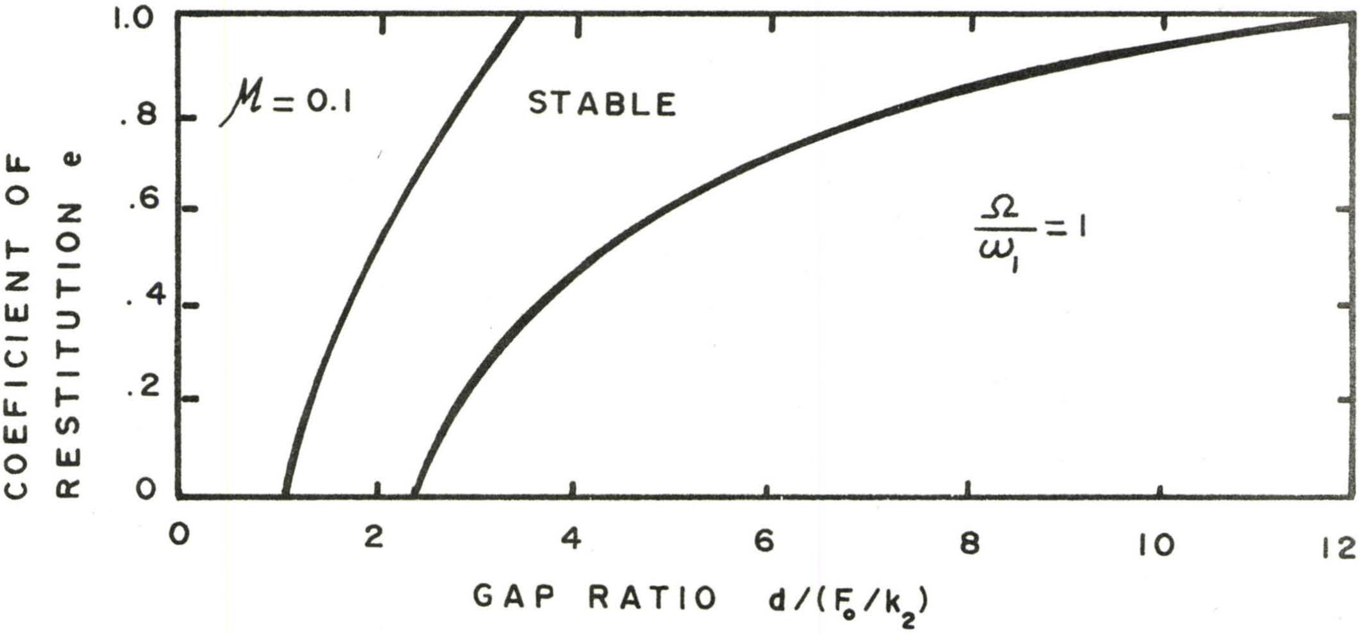
FIG 25

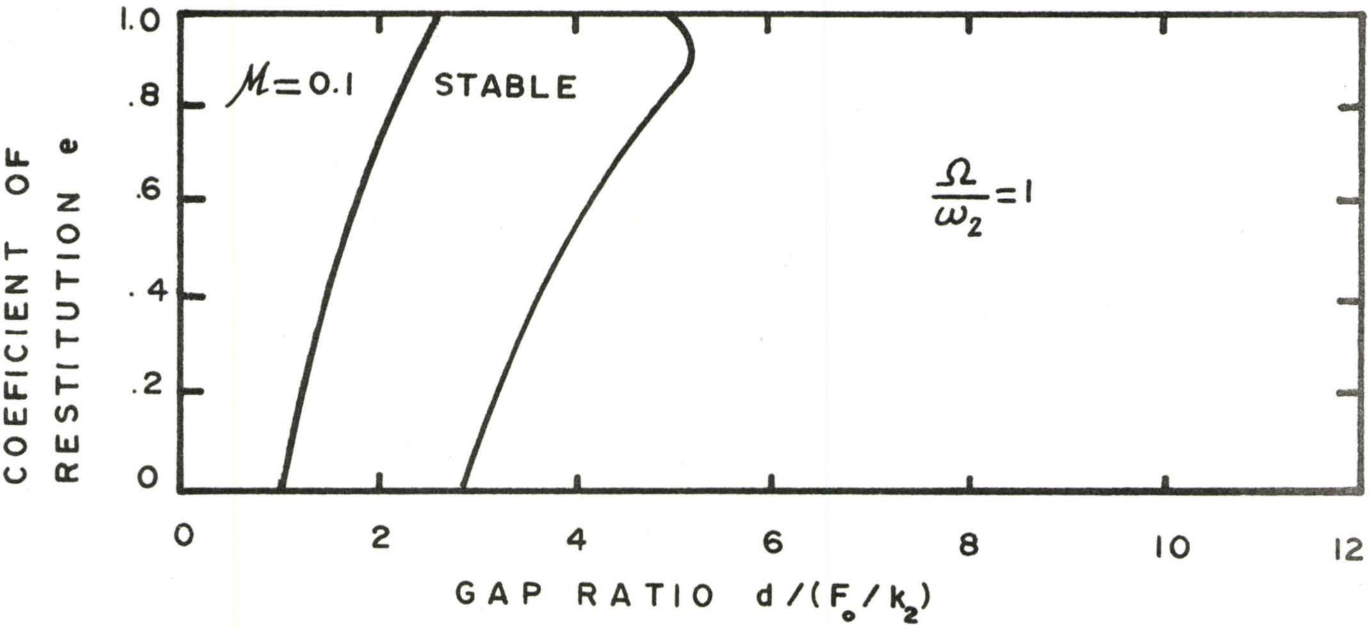


FREE MASS IN PRIMARY MASS 2

STABILITY BOUNDARIES
($e = .8$ $\delta = .1$)

FIG 26

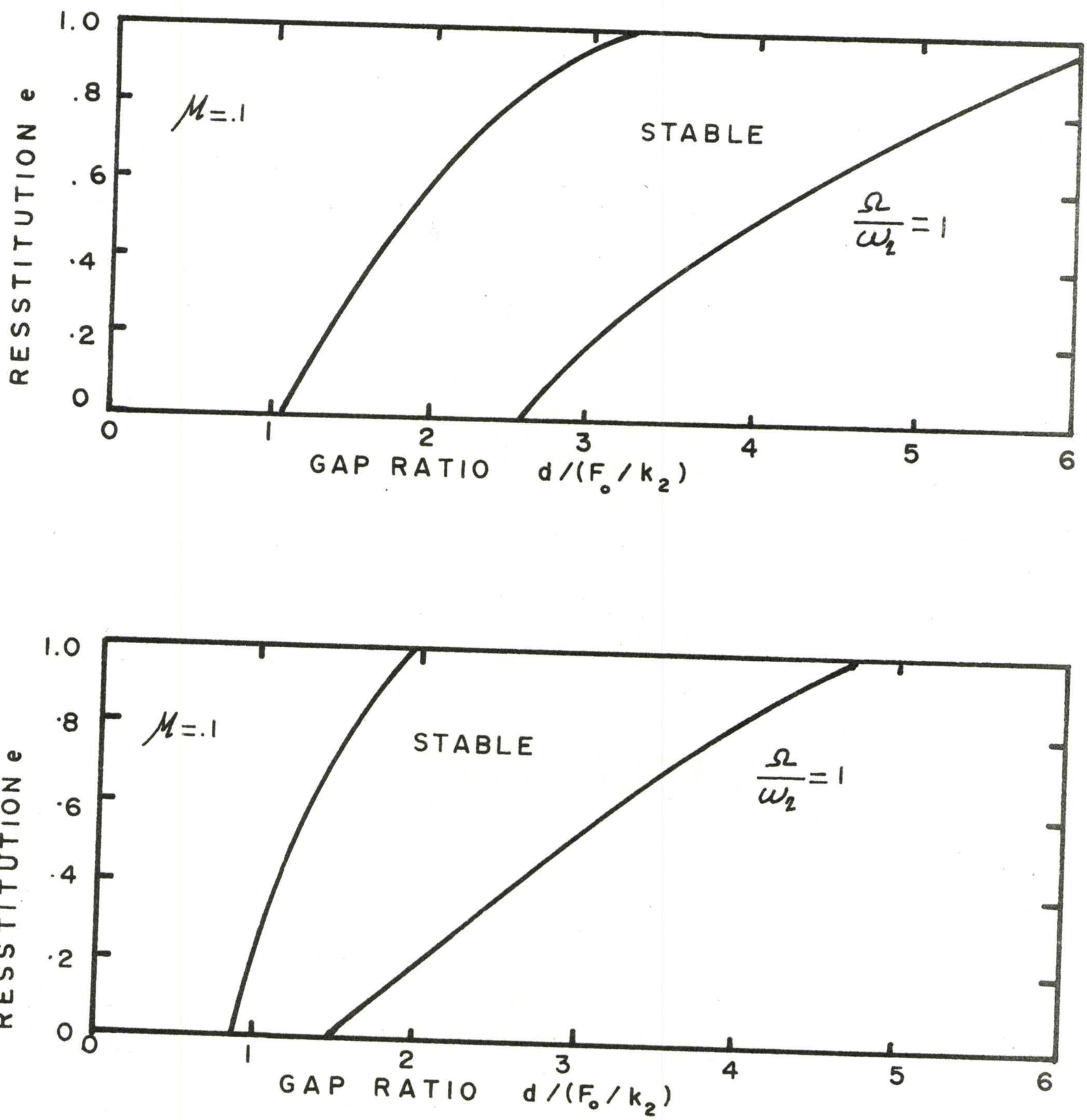




FREE MASS IN PRIMARY MASS 1

STABILITY BOUNDARIES ($\delta_1=.1$, $\mathcal{M}=.1$)

FIG 27



FREE MASS IN PRIMARY MASS 2

STABILITY BOUNDARIES (δ_1=0.1, $\mathcal{M}$=0.1)

FIG 28

For impact damper in primary mass 2, two solutions are obtained by using the value of τ_1 in one case;

$$\tau_1 = \tau_2' = \tan^{-1}\left(\frac{-2\rho_1 + H_1\sqrt{H_1^2+4-\rho_1^2}}{-\rho_1 H_1 - 2\sqrt{H_1^2+4-\rho_1^2}}\right)$$

and in other case

$$\tau_1 = \tau_3' = \tan^{-1}\left(\frac{-2\rho_1 - H_1\sqrt{H_1^2+4-\rho_1^2}}{-\rho_1 H_1 + 2\sqrt{H_1^2+4-\rho_1^2}}\right)$$

The two solution coincides at the extreme d=0 ,for which

$$\tau_2' = \tau_3' = \tan^{-1}\left(-\frac{H_1}{2}\right)$$

and at gap ratio, where, $H_1^2+4-\rho_1^2=0$, Here

$$\tau_2' = \tau_3' = \tan^{-1}\left(\frac{2}{H_1}\right)$$

By further increasing the gap ratio, τ_1 becomes complex, consequently our 2 impacts per cycle solutions do not exist. The stability analysis indicates that the curve τ_3' is entirely unstable, while τ_2' curve is only partly stable.

From Figures 20 & 21, it is obvious that a system, which is stable with the impact damper in one primary mass, may not be stable when the impact damper is transferred to the other primary mass, even though the parameter of the impact damper is the same.

From Figure 28 ,it is clear that generally by increasing the coefficent of restitution, the stability increases

5.2 Conclusion

On the basis of the present investigation,we can conclude that;

1) The impact damper will be more effective in the system,if it is used in the primary mass that has the higher amplitude of displacement.

2) A system,which is stable with the impact damper in one primary mass,may not be stable when the impact damper is transferred to the other primary mass,even though the parameter of the impact damper is the same.

3) Stability boundaries of the steady state solutions are the complicated function of the parameters of the impact damper and the system.

4) For a system with fixed gap ratio,an increase in mass ratio results in an increase in the damping efficiency of the system.However,there exists an optimum mass ratio after which the damping effeciency decreases. Beating also starts at a point slightly higher than the optimum mass ratio.The same effect is observed by increasing the gap ratio for fixed mass ratio.

5) An impact damper in both primary masses is not always more effective than one impact damper in single primary mass.

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APPENDIX A

$$M_1 = h_1 - \frac{1}{\sigma_2}$$

$$M_2 = h_2 - \frac{1}{\sigma_2}$$

$$M_3 = -E_1 - \frac{S_1}{\sigma_2}$$

$$M_4 = -E_1 - \frac{S_1}{\sigma_1}$$

$$M_5 = -D_1\left(\theta_1 + \frac{h_5}{\sigma_1}\right)$$

$$M_6 = -D_2\left(\theta_2 + \frac{h_6}{\sigma_1}\right)$$

$$M_7 = -D_2\left(\theta_3 + \frac{h_7}{\sigma_1}\right)$$

$$M_8 = -D_2\left(\theta_4 + \frac{h_8}{\sigma_1}\right)$$

$$M_9 = -E_4 - \frac{S_4}{\sigma_1}$$

$$M_{10} = -E_4 - \frac{S_4}{\sigma_2}$$

$$M_{11} = M_1 + \frac{h_3}{h_5} h_6$$

$$M_{12} = -\frac{h_3}{h_5}$$

$$M_{13} = M_6 - \frac{M_5}{h_5} h_6$$

$$M_{14} = \frac{M_5}{h_5}$$

$$M_{15} = \theta_2 - \frac{\theta_1}{h_5} h_6$$

$$M_{16} = h_1 + \frac{\theta_1}{h_5}$$

$$M_{17} = -D_1 h_1 - \frac{D_1 h_3}{h_5} h_6$$

$$M_{18} = D_1 \theta_6 + \frac{D_1 h_3}{h_5}$$

$$M_{19} = -M_{11} h_9$$

$$M_{20} = M_{12} - M_{11} h_{10}$$

$$M_{21} = -M_{13} h_9$$

$$M_{22} = M_{14} - M_{13} h_{10}$$

$$M_{23} = -h_3 - M_{15} h_9$$

$$M_{24} = M_{16} - M_{15} h_{10}$$

$$M_{25} = \theta_5 + \frac{h_9}{\sigma_1}$$

$$M_{26} = \theta_6 + \frac{h_{10}}{\sigma_1}$$

$$M_{27} = D_1 \theta_5 - M_{17} h_9$$

$$M_{28} = M_{18} - M_{17} h_{10}$$

$$M_{29} = M_2 + \frac{h_4}{h_7} h_8$$

$$M_{30} = -\frac{h_4}{h_7}$$

$$M_{31} = M_8 - \frac{M_7}{h_7} h_8$$

$$M_{32} = \frac{M_7}{h_7}$$

$$M_{33} = \theta_4 - \frac{\theta_3}{h_7} h_8$$

$$M_{34} = h_2 + \frac{\theta_3}{h_7}$$

$$M_{35} = -D_2 h_2 - \frac{D_2 h_4}{h_7} h_8$$

$$M_{36} = D_2 \theta_8 + \frac{D_2 h_4}{h_7}$$

$$M_{37} = -h_{11} M_{29}$$

$$M_{38} = M_{30} - h_{12} M_{29}$$

$$M_{39} = -h_{11} M_{31}$$

$$M_{40} = M_{32} - h_{12} M_{31}$$

$$M_{41} = -h_4 - M_{33} h_{11}$$

$$M_{42} = M_{34} - M_{33} h_{12}$$

$$M_{43} = \theta_7 + \frac{h_{11}}{\sigma_1}$$

$$M_{44} = \theta_8 + \frac{h_{12}}{\sigma_1}$$

$$M_{45} = D_2 \theta_7 - M_{35} h_{11}$$

$$M_{46} = M_{36} - M_{35} h_{12}$$

$$M_{47} = M_{20} - \frac{M_{19}}{M_{23}} M_{24}$$

$$M_{48} = M_{37} - \frac{M_{19}}{M_{23}} M_{41}$$

$$M_{49} = M_{38} - \frac{M_{21}}{M_{23}} M_{42}$$

$$M_{50} = M_{22} - \frac{M_{21}}{M_{23}} M_{24}$$

$$M_{51} = M_{39} - \frac{M_{21}}{M_{23}} M_{41}$$

$$M_{52} = M_{40} - \frac{M_{21}}{M_{23}} M_{42}$$

$$M_{53} = D_1 M_1 + \frac{D_1 h_3}{M_{23}} M_{24}$$

$$M_{54} = -D_2 h_4 + \frac{D_1 h_3}{M_{23}} M_{41}$$

$$M_{55} = D_2 M_2 + \frac{D_1 h_3}{M_{23}} M_{42}$$

$$M_{56} = M_{26} - \frac{M_{25}}{M_{23}} M_{24}$$

$$M_{57} = M_{43} - \frac{M_{25}}{M_{23}} M_{41}$$

$$M_{58} = M_{44} - \frac{M_{25}}{M_{23}} M_{42}$$

$$M_{59} = M_{28} - \frac{M_{27}}{M_{23}} M_{24}$$

$$M_{60} = M_{45} - \frac{M_{27}}{M_{23}} M_{41}$$

$$M_{61} = M_{46} - \frac{M_{27}}{M_{23}} M_{42}$$

$$M_{62} = M_{48} - \frac{M_{47}}{M_{59}} M_{60}$$

$$M_{63} = M_{49} - \frac{M_{47}}{M_{59}} M_{61}$$

$$M_{64} = M_{51} - \frac{M_{50}}{M_{59}} M_{60}$$

$$M_{65} = M_{52} - \frac{M_{50}}{M_{59}} M_{61}$$

$$M_{66} = M_{54} - \frac{M_{53}}{M_{59}} M_{60}$$

$$M_{67} = M_{55} - \frac{M_{53}}{M_{59}} M_{61}$$

$$M_{68} = M_{57} - \frac{M_{56}}{M_{59}} M_{60}$$

$$M_{69} = M_{58} - \frac{M_{56}}{M_{59}} M_{61}$$

$$M_{70} = M_{65} - \frac{M_{64}}{M_{62}} M_{63}$$

$$M_{71} = -\frac{M_{64}}{M_{62}} M_3$$

$$M_{72} = M_{67} - \frac{M_{66}}{M_{62}} M_{63}$$

$$M_{73} = -\frac{M_{66}}{M_{62}} M_3$$

$$M_{74} = M_{64} - \frac{M_{68}}{M_{62}} M_{63}$$

$$M_{75} = M_4 - \frac{M_{68}}{M_{62}} M_3$$

$$M_{76} = M_{71} - \frac{M_{70}}{M_{74}} M_{75}$$

$$M_{77} = M_{73} - \frac{M_{72}}{M_{74}} M_{75}$$

$$M_{78} = \frac{d}{2\sigma_1} - \frac{M_{64}}{M_{62}} \frac{d}{2\sigma_2}$$

$$M_{79} = \frac{d}{2\sigma_2} - \frac{M_{66}}{M_{62}} \frac{d}{2\sigma_2}$$

$$M_{80} = \frac{d}{2\sigma_1} - \frac{M_{68}}{M_{62}} \frac{d}{2\sigma_2}$$

$$M_{81} = M_{78} - \frac{M_{70}}{M_{74}} M_{80}$$

$$M_{82} = M_{79} - \frac{M_{72}}{M_{74}} M_{80}$$

$$M_{83} = \frac{M_{82} M_{70} M_{68}}{M_{74} M_{62}} - \frac{M_{64} M_{82}}{M_{62}} + \frac{M_{81} M_{64}}{M_{62}}$$

$$M_{84} = \omega \left(-M_{83} + \frac{M_{82} M_{70}}{M_{74}} - M_{81} \frac{A_2}{A_1} \left(\cos(\omega t_1 + \psi) + \frac{\sin(\omega t_1 + \psi)}{\sigma_1} \right) \right) - M_{81} \frac{A_2}{A_1} \left(\cos(\omega t_1 + \psi) + \frac{\sin(\omega t_1 + \psi)}{\sigma_2} \right)$$

$$M_{85} = -\left(-\frac{M_{83}}{\sigma_2} + \frac{M_{82} M_{70}}{M_{74}\, \sigma_1} + \frac{M_{82} A_2}{A_1} \left(\omega \sin(\omega t_1 + \psi) - \frac{\cos(\omega t_1 + \psi)}{\sigma_1} \right) + \frac{M_{81} A_2}{A_1} \left(\omega \sin(\omega t_1 + \psi) - \frac{\cos(\omega t_1 + \psi)}{\sigma_2} \right) \right)$$

$$Z_1 = (1 + h_{14})$$

$$Z_2 = (1 + h_{16})$$

$$Z_3 = (h_1 - \frac{1}{\sigma_2})$$

$$Z_4 = (h_2 - \frac{1}{\sigma_2})$$

$$Z_5 = (-E_1 - \frac{S_1}{\sigma_2})$$

$$Z_6 = (\theta_{10} - \frac{1}{\sigma_1})$$

$$Z_7 = (\theta_{12} - \frac{1}{\sigma_2})$$

$$Z_8 = (-E_1 - \frac{S_1}{\sigma_1})$$

$$Z_9 = -D_1(h_3 + \theta_9)$$

$$Z_{10} = D_1(h_1 - \theta_{10})$$

$$Z_{11} = -D_2(h_4 + \theta_{11})$$

$$Z_{12} = D_2(h_2 - \theta_{12})$$

$$Z_{13} = Z_3 + \frac{h_3}{h_{13}} Z_1$$

$$Z_{14} = -h_4 + \frac{h_3}{h_{13}} h_{15}$$

$$Z_{15} = Z_4 + \frac{h_3}{h_{13}} Z_2$$

$$Z_{16} = Z_{10} - \frac{Z_9}{h_{13}} Z_1$$

$$Z_{17} = Z_{11} - \frac{Z_9 Z_1}{h_{13}}$$

$$Z_{18} = Z_{12} - \frac{Z_9 Z_2}{h_{13}}$$

$$Z_{19} = Z_6 - \frac{\theta_9}{h_{13}} Z_1$$

$$Z_{20} = \theta_{11} - \frac{\theta_9}{h_{13}} h_{15}$$

$$Z_{21} = Z_7 - \frac{\theta_9}{h_{13}} Z_2$$

$$Z_{22} = Z_{14} - \frac{Z_{13}}{Z_{16}} Z_{17}$$

$$Z_{23} = Z_{15} - \frac{Z_{13}}{Z_{16}} Z_{18}$$

$$Z_{24} = Z_{20} - \frac{Z_{19}}{Z_{16}} Z_{17}$$

$$Z_{25} = Z_{21} - \frac{Z_{19}}{Z_{16}} Z_{18}$$

$$Z_{26} = Z_{23} - \frac{Z_{22}}{h_{16}} Z_2$$

$$Z_{27} = Z_{25} - \frac{Z_{24} Z_2}{h_{15}}$$

$$U_1 = (1 + h_{14})$$

$$U_2 = (1 + h_{16})$$

$$U_3 = D_1 \left(h_1 - \frac{1}{\sigma_2}\right)$$

$$U_{14} = D_2 \left(h_2 - \frac{1}{\sigma_2}\right)$$

$$U_5 = \left(-E_2 - \frac{S_2}{\sigma_2}\right)$$

$$U_6 = D_1 \left(\theta_{10} - \frac{1}{\sigma_1}\right)$$

$$U_7 = D_2 \left(\theta_{12} - \frac{1}{\sigma_1}\right)$$

$$U_8 = \left(-E_2 - \frac{S_2}{\sigma_1}\right)$$

$$U_9 = -(h_3 + \theta_9)$$

$$U_{10} = (h_1 - \theta_{10})$$

$$U_{11} = -(h_4 + \theta_{11})$$

$$U_{12} = (h_2 - \theta_{12})$$

$$U_{13} = U_3 + \frac{D_1 h_3}{h_{13}} U_1$$

$$U_{14} = -D_2 h_4 + \frac{D_1 h_3}{h_{13}} h_{15}$$

$$U_{15} = U_4 + \frac{D_1 h_3}{h_{13}} U_2$$

$$U_{16} = U_{10} - \frac{U_9 U_1}{h_{13}}$$

$$U_{17} = U_{11} - \frac{U_9 h_{15}}{h_{13}}$$

$$U_{18} = U_{12} - \frac{U_9}{h_{13}} U_2$$

$$U_{19} = U_6 - \frac{D_1 \theta_9}{h_{13}} U_1$$

$$U_{20} = D_2 \theta_{11} - \frac{D_1 \theta_9}{h_{13}} h_{15}$$

$$U_{21} = U_7 - \frac{D_1 \theta_9}{h_{13}} U_2$$

$$U_{22} = U_{14} - \frac{U_{15}}{U_{16}} U_{17}$$

$$U_{23} = U_{15} - \frac{U_{13}}{U_{16}} U_{18}$$

$$U_{24} = U_{20} - \frac{U_{19}}{U_{16}} U_{17}$$

$$U_{25} = U_{21} - \frac{U_{19}}{U_{16}} U_{18}$$

$$U_{26} = U_{23} - \frac{U_{22}}{h_{15}} U_2$$

$$U_{27} = U_{25} - \frac{U_{24}}{h_{15}} U_2$$

APPENDIX B

$$G_1 = D_2/(D_2 - D_1)$$

$$G_2 = D_1/(D_1 - D_2)$$

$$G_3 = (-D_2 A_1 \cos(\tau_0) + A_2 \cos(\tau_0 + \psi))/(D_1 - D_2)$$

$$G_5 = 1/(D_2 - D_1)$$

$$G_4 = (-D_1 A_1 \cos(\tau_0) + A_2 \cos(\tau_0 + \psi))/(D_1 - D_2)$$

$$G_6 = (-D_2 A_1(\delta_1 \cos\tau_0 - \Omega \sin\tau_0) + A_2(\delta_1 \cos(\tau_0 + \psi) - \Omega \sin(\tau_0 + \psi)))/(\eta'_1(D_1 - D_2))$$

$$G_7 = (-D_1 A_1(\delta'_2 \cos\tau_0 - \Omega \sin\tau_0) + A_2(\delta'_2 \cos(\tau_0 + \psi) - \Omega \sin(\tau_0 + \psi)))/(\eta'_2(D_2 - D_1))$$

$$G_8 = e^{-\frac{\delta'_1 t_{10}}{\Omega}}\left(\frac{\eta'_1 B_{10}}{\Omega} \cos\frac{\eta'_1 t_{10}}{\Omega} - \frac{\eta'_1 B_{20}}{\Omega} \sin\frac{\eta'_1 t_{10}}{\Omega}\right) + e^{-\frac{\delta'_2 t_{10}}{\Omega}}\left(\frac{\eta'_2 C_{10}}{\Omega} \cos\frac{\eta'_2 t_{10}}{\Omega} - \frac{\eta'_2 C_{20}}{\Omega} \sin\frac{\eta'_2 t_{10}}{\Omega}\right) + A_1 \cos(t_{10} + \tau_0)$$

$$G_9 = e^{-\frac{\delta'_1 t_{10}}{\Omega}} \cdot \sin\frac{\eta'_1 t_{10}}{\Omega}$$

$$G_{10} = e^{-\frac{\delta'_1 t_{10}}{\Omega}} \cdot \cos\frac{\eta'_1 t_{10}}{\Omega}$$

$$G_{11} = e^{-\frac{\delta'_2 t_{10}}{\Omega}} \cdot \sin\frac{\eta'_2 t_{10}}{\Omega}$$

$$G_{12} = e^{-\frac{\delta'_2 t_{10}}{\Omega}} \cos\frac{\eta'_2 t_{10}}{\Omega}$$

$$G_{13} = A_1 \cos(t_{10} + \tau_0)$$

$$G_{14} = \frac{A_1 G_9}{\eta'_1} + \frac{G_{11} G_2}{\eta'_2}$$

$$G_{15} = \frac{G_9 G_1 \delta_1}{\eta'_1} + G_1 G_{10} + \frac{G_{11} G_2 \delta_2}{\eta'_2} + G_{12} G_2$$

$$G_{16} = -\frac{G_9 G_5}{\eta'_1} + \frac{G_{11} G_5}{\eta'_2}$$

$$N_{17} = -\frac{G_9 G_5 \delta'_1}{\eta'_1} + \frac{G_{11} G_5}{\eta'_2}$$

$$N_{18} = -G_9 G_6 + G_{10} G_3 - G_7 G_{11} + G_{12} G_4 + G_{13}$$

$$G_{17} = D_1 e^{-\frac{\delta'_1 t_{10}}{\Omega}}\left(\frac{\eta'_1 B_{10}}{\Omega} \cos\frac{\eta'_1 t_{10}}{\Omega} - \frac{\eta'_1 B_{20}}{\Omega} \sin\frac{\eta'_1 t_{10}}{\Omega}\right) + D_2 e^{-\frac{\delta'_2 t_{10}}{\Omega}}\left(\frac{\eta'_2 C_{10}}{\Omega} \cos\frac{\eta'_2 t_{10}}{\Omega} - \frac{\eta'_2 C_{20}}{\Omega} \sin\frac{\eta'_2 t_{10}}{\Omega}\right) + A_2 \cos(t_{10} + \tau_0 + \psi)$$

$$G_{18} = A_2 \cos(t_{10} + \tau_0 + \psi)$$

$$G_{19} = \frac{G_1 G_9 D_1}{\eta_1'} + \frac{D_2 G_{11} G_2}{\eta_2'}$$

$$G_{20} = \frac{D_1 G_9 G_1 \delta_1'}{\eta_1'} + D_1 G_1 G_{10} + \frac{D_2 G_{11} G_2 \delta_2'}{\eta_2'} + D_2 G_{12} G_2$$

$$G_{21} = -\frac{D_1 G_9 G_1 G_5}{\eta_1'} + \frac{D_2 G_{11} G_5}{\eta_2'}$$

$$G_{22} = -\frac{D_1 G_9 G_5 \delta_1'}{\eta_1'} - D_1 G_{10} G_5 + \frac{D_2 G_{11} \delta_2' G_5}{\eta_2'} + D_2 G_{15} G_5$$

$$G_{23} = -D_1 G_9 G_6 + D_1 G_{10} G_3 - D_2 G_7 G_{11} + D_2 G_{12} G_4 + G_{18}$$

$$G_{24} = \frac{e^{-\frac{\delta_1' t_{10}}{\Omega}} \cdot \eta_1}{\Omega} \left(-B_{10}\left(\delta_1' \cos\frac{\eta_1' t_{10}}{\Omega} + \eta_1' \sin\frac{\eta_1' t_{10}}{\Omega}\right) + B_{20}\left(\delta_1' \sin\frac{\eta_1' t_{10}}{\Omega} - \eta_1' \cos\frac{\eta_1' t_{10}}{\Omega}\right)\right) + \frac{e^{-\frac{\delta_2' t_{10}}{\Omega}}}{\Omega} \cdot \eta_2' \left(-C_{10}\left(\delta_2' \cos\frac{\eta_2' t_{10}}{\Omega} + \eta_2' \sin\frac{\eta_2' t_{10}}{\Omega}\right) + C_{20}\left(\delta_2' \cdot \sin\frac{\eta_2' t_{10}}{\Omega} - \eta_2' \cos\frac{\eta_2' t_{10}}{\Omega}\right)\right) - \Omega A_1 \sin(t_{10} + \tau_0)$$

$$G_{25} = e^{-\frac{\delta_1' t_{10}}{\Omega}} \left(-\delta_1' \sin\frac{\eta_1' t_{10}}{\Omega} + \eta_1' \cos\frac{\eta_1' t_{10}}{\Omega}\right)$$

$$G_{26} = e^{-\frac{\delta_1' t_{10}}{\Omega}} \left(-\delta_1' \cos\frac{\eta_1' t_{10}}{\Omega} - \eta_1' \sin\frac{\eta_1' t_{10}}{\Omega}\right)$$

$$G_{27} = e^{-\frac{\delta_2' t_{10}}{\Omega}} \left(-\delta_2' \sin\frac{\eta_2' t_{10}}{\Omega} + \eta_2' \cos\frac{\eta_2' t_{10}}{\Omega}\right)$$

$$G_{28} = e^{-\frac{\delta_2' t_{10}}{\Omega}} \left(-\delta_2' \cos\frac{\eta_2' t_{10}}{\Omega} - \eta_2' \sin\frac{\eta_2' t_{10}}{\Omega}\right)$$

$$G_{29} = -\Omega A_1 \sin(t_{10} + \tau_0)$$

$$G_{30} = \frac{G_1 G_{25}}{\eta_1'} + \frac{G_2 G_{27}}{\eta_2'}$$

$$G_{31} = \frac{G_{25} G_1 \delta_1'}{\eta_1'} + G_1 G_{26} + \frac{G_{27} G_2 \delta_2'}{\eta_2'} + G_{28} G_2$$

$$G_{32} = \frac{-G_{25} G_5}{\eta_1'} + \frac{G_{27} G_5}{\eta_2'}$$

$$G_{33} = -\frac{G_{25} G_5 \delta_1'}{\eta_1'} - G_{26} G_5 + \frac{G_{27} \delta_2' G_5}{\eta_2'} + G_{28} G_5$$

$$G_{34} = -G_{25} G_6 + G_{26} G_3 - G_{27} G_7 + G_{28} G_4 + G_{29}$$

$$G_{35} = D_1 \frac{e^{-\frac{\delta_1' t_{10}}{\Omega}}}{\Omega} \eta_1' \left(-B_{10}\left(\delta_1' \cos\frac{\eta_1' t_{10}}{\Omega} + \eta_1' \sin\frac{\eta_1' t_{10}}{\Omega}\right) + B_{20}\left(\delta_1' \sin\frac{\eta_1' t_{10}}{\Omega} - \eta_1' \cos\frac{\eta_1' t_{10}}{\Omega}\right)\right) + D_2 \frac{e^{-\frac{\delta_2' t_{10}}{\Omega}}}{\Omega} \cdot \eta_2' \left(-C_{10}\left(\delta_2' \cos\frac{\eta_2' t_{10}}{\Omega} + \eta_2' \sin\frac{\eta_2' t_{10}}{\Omega}\right) + C_{20}\left(\delta_2' \sin\frac{\eta_2' t_{10}}{\Omega} - \eta_2' \cos\frac{\eta_2' t_{10}}{\Omega}\right)\right) - \Omega A_2 \sin(t_{10} + \tau_0 + \psi)$$

$$G_{36} = -\Omega A_2 \sin(t_{10} + \tau_0 + \psi)$$

$$G_{37} = K_1 \left(\frac{D_1 G_1 G_{25}}{\eta'_1} + \frac{D_2 G_2 G_{27}}{\eta'_2} \right)$$

$$G_{38} = K_1 \left(\frac{D_1 G_1 G_{25} \delta'_1}{\eta'_1} + D_1 G_{26} G_1 + \frac{D_2 G_{27} G_2 \delta'_2}{\eta'_2} + D_2 G_{28} G_2 \right)$$

$$G_{39} = K_1 \left(-\frac{D_1 G_{25} G_5}{\eta'_1} + \frac{D_2 G_{27} G_5}{\eta'_2} \right)$$

$$G_{40} = K_1 \left(-\frac{D_1 G_{25} G_6 \delta'_1}{\eta'_1} - D_1 G_{26} G_5 + \frac{D_2 G_{27} \delta'_2 G_5}{\eta'_2} + D_2 G_{28} G_5 \right)$$

$$G_{41} = K_1 (-D_2 G_{25} G_6 + D_1 G_{26} G_3 - D_2 G_{27} G_7 + D_2 G_{28} G_4 + G_{36})$$

$$G_{42} = K_1 G_{35}$$

$$G_{43} = (D_2 G_8 - G_{17} + A_2 \cos(\tau_0 + t_{10} + \psi) - D_2 A_1 \cos(\tau_0 + t_{10})) / (D_2 - D_1)$$

$$G_{44} = (D_2 N_{18} - G_{23} + A_2 \cos(\tau_0 + t_{10} + \psi) - D_2 A_1 \cos(\tau_0 + t_{10})) / (D_2 - D_1)$$

$$G_{45} = (D_2 G_{14} - G_{19}) / (D_2 - D_1)$$

$$G_{46} = (D_2 G_{15} - G_{20}) / (D_2 - D_1)$$

$$G_{47} = (D_2 G_{16} - G_{21}) / (D_2 - D_1)$$

$$G_{48} = (D_2 N_{17} - G_{22}) / (D_2 - D_1)$$

$$G_{49} = (D_1 G_8 - G_{17} + A_2 \cos(\tau_0 + t_{10} + \psi) - D_1 A_1 \cos(\tau_0 + t_{10})) / (D_1 - D_2)$$

$$G_{50} = (D_1 N_{18} - G_{23} + A_2 \cos(\tau_0 + t_{10} + \psi) - D_1 A_1 \cos(\tau_0 + t_{10})) / (D_1 - D_2)$$

$$G_{51} = (D_1 G_{14} - G_{19}) / (D_1 - D_2)$$

$$G_{52} = (D_1 G_{15} - G_{20}) / (D_1 - D_2)$$

$$G_{53} = (D_1 G_{16} - G_{21}) / (D_1 - D_2)$$

$$G_{54} = (D_1 N_{17} - G_{22}) / (D_1 - D_2)$$

$$G_{55} = (D_2 G_{24} + D_2 \delta_1' G_8 - G_{42} - \delta_1' G_{17} - D_2 A_1(\delta_1 \cos(\tau_0+t_{10}) - \Omega \sin(\tau_0+t_{10})) + A_2(\delta_1' \cos(\tau_0+t_{10}+\psi) - \Omega \sin(\tau_0+t_{10})))/(\eta_1'(D_2-D_1))$$

$$G_{56} = (D_2 G_{34} + D_2 \delta_1' N_{18} - G_{41} - \delta_1' G_{23} - D_2 A_1(\delta_1' \cos(\tau_0+t_{10}) - \Omega \sin(\tau_0+t_{10})) + A_2(\delta_1' \cos(\tau_0+t_{10}+\psi) - \Omega \sin(\tau_0+t_{10})))/(\eta_1'(D_2-D_1))$$

$$G_{57} = (D_2 G_{30} + D_2 \delta_1' G_{14} - G_{37} - \delta_1' G_{19})/(\eta_1'(D_2-D_1))$$

$$G_{58} = (D_2 G_{31} + D_2 \delta_1' G_{15} - G_{38} - \delta_1' G_{20})/(\eta_1'(D_2-D_1))$$

$$G_{59} = (D_2 G_{32} + D_2 \delta_1' G_{16} - G_{39} - \delta_1' G_{21})/(\eta_1'(D_2-D_1))$$

$$G_{60} = (D_2 G_{33} + D_2 \delta_1' N_{17} - G_{40} - \delta_1' G_{22})/(\eta_1'(D_2-D_1))$$

$$G_{61} = -K_2/(\eta_1'(D_2-D_1))$$

$$G_{62} = (D_1 G_{24} + D_1 \delta_2' G_8 - G_{42} - \delta_2' G_{17} - D_1 A_1(\delta_2' \cos(\tau_0+t_{10}) - \Omega \sin(\tau_0+t_{10})) + A_2(\delta_2' \cos(\tau_0+t_{10}+\psi) - \Omega \sin(\tau_0+t_{10})))/(\eta_2'(D_1-D_2))$$

$$G_{63} = (D_1 G_{34} + D_1 \delta_2' N_{18} - G_{41} - \delta_2' G_{43} - D_1 A_1(\delta_2' \cos(\tau_0+t_{10}) - \Omega \sin(\tau_0+t_{10})) + A_2(\delta_2' \cos(\tau_0+t_{10}+\psi) - \Omega \sin(\tau_0+t_{10})))/(\eta_2'(D_1-D_2))$$

$$G_{64} = (D_1 G_{30} + D_1 \delta_2' G_{14} - G_{37} - \delta_2' G_{19})/(\eta_2'(D_1-D_2))$$

$$G_{65} = (D_1 G_{31} + D_1 \delta_2' G_{15} - G_{38} - \delta_2' G_{20})/(\eta_2'(D_1-D_2))$$

$$G_{66} = (D_1 G_{32} + D_1 \delta_2' G_{16} - G_{39} - \delta_2' G_{21}) / (\eta_2' (D_1 - D_2))$$

$$G_{67} = (D_1 G_{33} + D_1 \delta_2' N_{17} - G_{40} - \delta_2' G_{22}) / (\eta_2' (D_1 - D_2))$$

$$G_{68} = -K_2 / (\eta_2' (D_1 - D_2))$$

$$G_{69} = \eta_1' \, e^{-\delta_1' \frac{(\pi - t_{10})}{\Omega}} \left(B_{10}' \cos \eta_1' \left(\frac{\pi - t_{10}}{\Omega}\right) - B_{20}' \sin \eta_1' \left(\frac{\pi - t_{10}}{\Omega}\right)\right) + \eta_2' \, e^{-\delta_2' \frac{(\pi - t_{10})}{\Omega}} \left(C_{10}' \cos \eta_2' \left(\frac{\pi - t_{10}}{\Omega}\right) - C_{20}' \sin \eta_2' \left(\frac{\pi - t_{10}}{\Omega}\right)\right)$$

$$G_{70} = e^{-\delta_1' \left(\frac{\pi - t_{10}}{\Omega}\right)} \sin \eta_1' \left(\frac{\pi - t_{10}}{\Omega}\right)$$

$$G_{71} = e^{-\delta_1' \left(\frac{\pi - t_{10}}{\Omega}\right)} \cdot \cos \eta_1' \left(\frac{\pi - t_{10}}{\Omega}\right)$$

$$G_{72} = e^{-\delta_2' \left(\frac{\pi - t_{10}}{\Omega}\right)} \sin \eta_2' \left(\frac{\pi - t_{10}}{\Omega}\right)$$

$$G_{73} = e^{-\delta_2' \left(\frac{\pi - t_{10}}{\Omega}\right)} \cos \eta_2' \left(\frac{\pi - t_{10}}{\Omega}\right)$$

$$G_{74} = -\frac{1}{K}\left(-\frac{G_{69}}{\Omega} + G_{70} G_{55} + G_{71} G_{43} + G_{72} G_{62} + G_{73} G_{49}\right)$$

$$G_{75} = -\frac{1}{K}\left(-A_1 \cos \tau_0 + G_{70} G_{56} + G_{71} G_{44} + G_{72} G_{63} + G_{73} G_{50}\right)$$

$$G_{76} = -\frac{1}{K}\left(G_{70} G_{57} + G_{71} G_{45} + G_{72} G_{64} + G_{73} G_{57}\right)$$

$$G_{77} = -\frac{1}{K}\left(\frac{G_{69}}{V_{10}} - \frac{A_1 \cos(\tau_0)\,\Omega}{V_{10}} + G_{70} G_{58} + G_{71} G_{46} + G_{72} G_{65} + G_{73} G_{52}\right)$$

$$G_{78} = -\frac{1}{K}\left(G_{70} G_{59} + G_{71} G_{47} + G_{72} G_{66} + G_{73} G_{53}\right)$$

$$G_{79} = -\frac{1}{K}\left(G_{70} G_{60} + G_{71} G_{48} + G_{72} G_{67} + G_{73} G_{54}\right)$$

$$G_{80} = -\frac{1}{K}\left(-\frac{\pi}{\Omega V_{10}} G_{69} + \frac{\pi}{V_{10}} A_1\right)$$

$$G_{81} = -\frac{1}{K}\left(G_{70} G_{61} + G_{72} G_{68}\right)$$

$$G_{82} = \frac{\Omega}{V_{10}}$$

$$G_{83} = \frac{D_1 \eta_1'}{\Omega} e^{-\delta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)} \left(B_{10}' \cos \eta_1' \left(\frac{\pi - t_{10}}{\Omega}\right) - B_{20}' \sin \eta_1' \left(\frac{\pi - t_{10}}{\Omega}\right)\right)$$
$$+ \frac{D_2 \eta_2'}{\Omega} e^{-\delta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)} \left(C_{10}' \cos \eta_2' \left(\frac{\pi - t_{10}}{\Omega}\right) -\right.$$
$$\left. C_{20}' \sin \eta_2' \left(\frac{\pi - t_{10}}{\Omega}\right)\right)$$

$$G_{84} = G_{83}(G_{82} G_{14} - 1) + D_1 G_{70} G_{55} + D_1 G_{71} G_{43} +$$
$$D_2 G_{72} G_{62} + D_2 G_{73} G_{49} - A_2 \cos(\tau_0 + \psi) G_{82} G_{74}$$

$$G_{85} = G_{83} G_{82} G_{75} + D_1 G_{70} G_{56} + D_1 G_{71} G_{44} + D_2 G_{72} G_{63} +$$
$$D_2 G_{73} G_{50} - A_2 \cos(\tau_0 + \psi)(G_{82} G_{75} + 1)$$

$$G_{86} = G_{83} G_{82} G_{76} + D_1 G_{70} G_{57} + D_1 G_{71} G_{45} +$$
$$D_2 G_{72} G_{64} + D_2 G_{73} G_{51} - A_2 \cos(\tau_0 + \psi) G_{82} G_{76}$$

$$G_{87} = G_{83} G_{82} (1 + G_{77}) + D_1 G_{70} G_{58} + D_1 G_{71} G_{46} +$$
$$D_2 G_{72} G_{65} + D_2 G_{73} G_{52} - A_2 \cos(\tau_0 + \psi) \cdot$$
$$G_{82}(1 + G_{77})$$

$$G_{88} = G_{83} G_{82} G_{78} + D_1 G_{70} G_{59} + D_1 G_{71} G_{47} +$$
$$D_2 G_{72} G_{66} + D_2 G_{73} G_{53} - A_2 \cos(\tau_0 + \psi) G_{82} G_{78}$$

$$G_{89} = G_{83} G_{82} G_{79} + D_1 G_{70} G_{60} + D_1 G_{71} G_{48} + D_2 G_{72} G_{67} +$$
$$D_2 G_{73} G_{54} - A_2 \cos(\tau_0 + \psi) G_{82} G_{79}$$

$$G_{90} = \left(G_{82} G_{80} - \frac{\pi}{V_{10}}\right)(-A_2 \cos(\tau_0 + \psi) + G_{83})$$

$$G_{91} = G_{83} G_{82} G_{81} + D_1 G_{70} G_{61} + D_2 G_{72} G_{8} -$$
$$A_2 \cos(\tau_0 + \psi) G_{82} G_{81}$$

$$G_{92} = \frac{e^{-\delta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)}}{\Omega} \cdot \eta_1' \left(-B_{10}'\left(\delta_1' \cos \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right) + \eta_1' \sin \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)\right.$$
$$\left. + B_{20}'\left(\delta_1' \sin \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right) - \eta_1' \cos \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)\right) +$$
$$e^{-\delta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)} \cdot \eta_2' \left(-C_{10}'\left(\delta_2' \cos \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right) + \eta_2' \sin \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)\right.$$
$$\left. + C_{20}\left(\delta_2' \sin \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right) - \eta_2' \cos \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)\right)$$

$$G_{93} = e^{-\delta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)} \left(-\delta_1' \sin \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right) + \eta_1' \cos \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)$$

$$G_{94} = e^{-\delta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)} \left(-\delta_1' \cos \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right) - \eta_1' \sin \eta_1'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)$$

$$G_{95} = e^{-\delta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)} \left(-\delta_2' \sin \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right) + \eta_2' \cos \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)$$

$$G_{96} = e^{-\delta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)} \left(-\delta_2' \cos \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right) - \eta_2' \sin \eta_2'\left(\frac{\pi - t_{10}}{\Omega}\right)\right)$$

$$G_{97} = G_{92}\left(G_{82} G_{74} + G_{93} G_{55} + G_{94} G_{43} + G_{95} G_{62} + G_{96} G_{49} + \Omega A_1 \sin(\tau_0)\, G_{82} G_{74}\right)$$

$$G_{98} = G_{92} G_{82} G_{75} + G_{93} G_{56} + G_{94} G_{44} + G_{95} G_{63} + G_{96} G_{50} + \Omega A_1 \sin(\tau_0)\left(1 + G_{82} G_{75}\right)$$

$$G_{99} = G_{92} G_{82} G_{76} + G_{93} G_{57} + G_{94} G_{45} + G_{95} G_{64} + G_{96} G_{51} + \Omega A_1 \sin(\tau_0)\, G_{82} G_{76}$$

$$G_{100} = G_{92} G_{82}\left(1 + G_{77}\right) + G_{93} G_{58} + G_{94} G_{46} + G_{95} G_{65} + G_{96} G_{52} + \Omega A_1 \sin(\tau_0)\, G_{82}\left(1 + G_{77}\right)$$

$$G_{101} = G_{92} G_{82} G_{78} + G_{93} G_{59} + G_{94} G_{47} + G_{95} G_{66} + G_{96} G_{53} + \Omega A_1 \sin(\tau_0)\, G_{82} G_{78}$$

$$G_{102} = G_{92} G_{82} G_{79} + G_{93} G_{60} + G_{94} G_{48} + G_{95} G_{67} + G_{96} G_{54} + \Omega A_1 \sin(\tau_0)\, G_{82} G_{79}$$

$$G_{103} = \left(G_{82} G_{80} - \frac{\pi}{v_{10}}\right)\left(\Omega A_1 \sin(\tau_0) + G_{92}\right)$$

$$G_{104} = G_{92} \cdot G_{82} G_{81} + \Omega A_1 \sin(\tau_0)\, G_{82} G_{81} + G_{83} G_{61} + G_{95} G_{68}$$

$$G_{104} = D_1 \frac{e^{-\delta_1' \frac{(\pi - t_{10})}{\Omega}}}{\Omega} \eta_1' \left(-B_{10}' \left(\delta_1' \cos \eta_1' \frac{(\pi - t_{10})}{\Omega} + \eta_1' \sin \eta_1' \frac{(\pi - t_{10})}{\Omega}\right) + B_{20}' \left(\delta_1' \sin \eta_1' \frac{(\pi - t_{10})}{\Omega} - \eta_1' \cos \eta_1' \frac{(\pi - t_{10})}{\Omega}\right)\right) + D_2 \frac{e^{-\delta_2' (\pi - t_{10})}}{\Omega} \eta_2' \left(-C_{10}' \left(\delta_2' \cos \eta_2' \frac{(\pi - t_{10})}{\Omega} + \eta_2' \sin \eta_2' \frac{(\pi - t_{10})}{\Omega}\right) + C_{20} \left(\delta_2' \sin \eta_2' \frac{(\pi - t_{10})}{\Omega} - \eta_2' \cos \eta_2' \frac{(\pi - t_{10})}{\Omega}\right)\right)$$

$$G_{106} = G_{105} (G_{82} G_{75} - 1) + G_{93} G_{55} D_1 + D_1 G_{94} G_{43} + D_2 G_{95} G_{62} + D_2 G_{96} G_{49} + \Omega A_2 \sin(\tau_0 + \psi)\, G_{82} G_{74}$$

$$G_{107} = G_{105} G_{82} G_{75} + D_1 G_{93} G_{56} + D_1 G_{94} G_{44} + D_2 G_{95} G_{63} + D_2 G_{96} G_{50} + \Omega A_2 \sin(\tau_0 + \psi)(1 + G_{82} G_{75})$$

$$G_{108} = G_{105} G_{82} G_{76} + D_1 G_{93} G_{57} + D_1 G_{94} G_{45} + D_2 G_{95} G_{64} + D_2 G_{96} G_{51} + \Omega A_2 \sin(\tau_0 + \psi)\, G_{82} G_{76}$$

$$G_{109} = G_{105} G_{82} (1 + G_{77}) + D_1 G_{93} G_{58} + D_1 G_{94} G_{46} + D_2 G_{95} G_{65} + D_2 G_{96} G_{52} + \Omega A_2 \sin(\tau_0 + \psi) \cdot G_{82} (1 + G_{77})$$

$$G_{110} = G_{105} G_{82} G_{78} + D_1 G_{93} G_{59} + D_1 G_{94} G_{47} + D_2 G_{95} G_{66} + D_2 G_{96} G_{53} + \Omega A_2 \sin(\tau_0 + \psi)\, G_{82} G_{78}$$

$$G_{111} = G_{105} G_{82} G_{79} + D_1 G_{93} G_{60} + D_1 G_{94} G_{48} + D_2 G_{95} G_{67} + D_2 G_{94} G_{54} + \Omega A_2 \sin(\tau_0 + \psi) G_{82} G_{79}$$

$$G_{112} = \left(G_{82} G_{80} - \frac{\pi}{V_{10}} \right) \left(\Omega A_2 \sin(\tau_0 + \psi) + G_{105} \right)$$

$$G_{113} = \left(G_{105} G_{82} G_{81} + \Omega A_2 \sin(\tau_0 + \psi) G_{82} G_{81} + D_1 G_{93} G_{61} + D_2 G_{95} G_{68} \right)$$

$$G_{114} = \left(G_1 G_{74} + G_5 G_{84} + G_3 G_{82} G_{74} \right)$$

$$G_{115} = \left(G_1 G_{75} + G_5 G_{85} + G_3 (1 + G_{82} G_{75}) \right)$$

$$G_{116} = \left(G_1 G_{76} + G_5 G_{86} + G_3 G_{82} G_{76} \right)$$

$$G_{117} = \left(G_1 G_{77} + G_5 G_{87} + G_3 G_{82} (1 + G_{77}) \right)$$

$$G_{118} = \left(G_1 G_{78} + G_5 G_{88} + G_3 G_{82} G_{78} \right)$$

$$G_{119} = \left(G_1 G_{79} + G_5 G_{89} + G_3 G_{82} G_{79} \right)$$

$$G_{120} = \left(G_1 G_{80} + G_5 G_{90} + G_3 \left(G_{82} G_{80} - \frac{\pi}{V_{10}} \right) \right)$$

$$G_{121} = \left(G_1 G_{81} + G_5 G_{91} + G_3 G_{82} G_{81} \right)$$

$$G_{122} = \left(G_2 G_{74} - G_5 G_{84} + G_4 G_{82} G_{74} \right)$$

$$G_{123} = \left(G_2 G_{75} - G_5 G_{85} + G_4 (G_{82} G_{75} + 1) \right)$$

$$G_{124} = (G_2 G_{76} - G_5 G_{86} + G_4 G_{82} G_{76})$$

$$G_{125} = (G_2 G_{77} - G_5 G_{87} + G_4 G_{82}(1 + G_{77}))$$

$$G_{126} = (G_2 G_{78} - G_5 G_{88} + G_4 G_{82} G_{78})$$

$$G_{127} = (G_2 G_{79} - G_5 G_{89} + G_4 G_{82} G_{79})$$

$$G_{128} = (G_2 G_{80} - G_5 G_{90} + G_4 (G_{82} G_{80} - \frac{\pi}{V_{10}}))$$

$$G_{129} = (G_2 G_{81} - G_5 G_{91} + G_4 G_{82} G_{81})$$

$$G_{130} = (-\frac{K_1 G_{97} G_1}{\eta_1'} + \frac{\delta_1' G_1 G_{74}}{\eta_1'} + \frac{G_5 G_{106}}{\eta_1'} + \frac{\delta_1' G_5 G_{84}}{\eta_1'} - G_6 G_{82} G_{74})$$

$$G_{131} = (-\frac{K_1 G_{98} G_1}{\eta_1'} + \frac{\delta_1 G_1' G_{75}}{\eta_1'} + \frac{G_5 G_{107}}{\eta_1'} + \frac{\delta_1' G_5 G_{85}}{\eta_1'} - G_6 (1 + G_{82} G_{75}))$$

$$G_{132} = (-\frac{K_1 G_{99} G_1}{\eta_1'} + \frac{\delta_1' G_1 G_{76}}{\eta_1'} + \frac{G_5 G_{108}}{\eta_1'} + \frac{\delta_1' G_5 G_{86}}{\eta_1'} - G_6 G_{82} G_{76})$$

$$G_{133} = (-\frac{K_1 G_{100} G_1}{\eta_1'} + \frac{\delta_1' G_1 G_{77}}{\eta_1'} + \frac{G_5 G_{109}}{\eta_1'} + \frac{\delta_1' G_5 G_{87}}{\eta_1'} - G_6 G_{82}(1 + G_{77}))$$

$$G_{134} = (-\frac{K_1 G_{101} G_1}{\eta_1'} + \frac{\delta_1' G_1 G_{78}}{\eta_1'} + \frac{G_5 G_{110}}{\eta_1'} + \frac{\delta_1' G_5 G_{88}}{\eta_1'} - G_6 G_{82} G_{78})$$

$$G_{135} = (-\frac{K_1 G_{102} G_1}{\eta_1'} + \frac{\delta_1' G_1 G_{79}}{\eta_1'} + \frac{G_5 G_{111}}{\eta_1'} + \frac{\delta_1' G_5 G_{89}}{\eta_1'} - G_6 G_{82} G_{79})$$

$$G_{136} = (-\frac{(K_1 G_{103} - K_2) G_1}{\eta_1'} + \frac{\delta_1' G_1 G_{80}}{\eta_1'} + \frac{G_5 G_{112}}{\eta_1'} + \frac{\delta_1' G_5 G_{90}}{\eta_1'} - G_6 (G_{82} G_{80} - \frac{\pi}{V_{10}}))$$

$$G_{137} = (-\frac{K_1 G_{104} G_1}{\eta_1'} + \frac{\delta_1' G_1 G_{81}}{\eta_1'} + \frac{G_5 G_{113}}{\eta_1'} + \frac{\delta_1' G_5 G_{91}}{\eta_1'} - G_6 G_{82} G_{81})$$

$$G_{138} = -(+\frac{K_1 G_{98} G_2}{\eta_2'} - \frac{\delta_2' G_2 G_{74}}{\eta_2'} + \frac{G_5 G_{108}}{\eta_2'} + \frac{\delta_2' G_5 G_{84}}{\eta_2'} + G_7 G_{82} G_{74})$$

$$G_{139} = -\left(\frac{K_1 G_{98} G_2}{\eta'_2} - \frac{\delta'_2 G_2 G_{75}}{\eta'_2} + \frac{G_5 G_{107}}{\eta'_2} + \frac{\delta'_2 G_5 G_{85}}{\eta'_2} + G_7 (1 + G_{82} G_{75}) \right)$$

$$G_{140} = -\left(\frac{K_1 G_{99} G_2}{\eta'_2} - \frac{\delta'_2 G_2 G_{76}}{\eta'_2} + \frac{G_5 G_{108}}{\eta'_2} + \frac{\delta'_2 G_5 G_{86}}{\eta'_2} + G_7 G_{82} G_{76} \right)$$

$$G_{141} = -\left(\frac{K_1 G_{100} G_2}{\eta'_2} - \frac{\delta'_2 G_2 G_{77}}{\eta'_2} + \frac{G_5 G_{109}}{\eta'_2} + \frac{\delta'_2 G_5 G_{87}}{\eta'_2} + G_7 \cdot G_{82} (1 + G_{77}) \right)$$

$$G_{142} = -\left(\frac{K_1 G_{101} G_2}{\eta'_2} - \frac{\delta'_2 G_2 G_{78}}{\eta'_2} + \frac{G_5 G_{110}}{\eta'_2} + \frac{\delta'_2 G_5 G_{88}}{\eta'_2} + G_7 G_{82} G_{78} \right)$$

$$G_{143} = -\left(\frac{K_1 G_{102} G_2}{\eta'_2} - \frac{\delta'_2 G_2 G_{79}}{\eta'_2} + \frac{G_5 G_{111}}{\eta'_2} + \frac{\delta'_2 G_5 G_{89}}{\eta'_2} + G_7 G_{82} G_{79} \right)$$

$$G_{144} = -\left(\frac{(K_1 G_{103} - K_2) G_2}{\eta'_2} - \frac{\delta'_2 G_2 G_{80}}{\eta'_2} + \frac{G_5 G_{112}}{\eta'_2} + \frac{\delta'_2 G_5 G_{90}}{\eta'_2} + G_7 \left(G_{82} G_{80} - \frac{\pi}{V_{10}} \right) \right)$$

$$G_{145} = -\left(\frac{K_1 G_{104} G_2}{\eta'_2} - \frac{\delta'_2 G_2 G_{81}}{\eta'_2} + \frac{G_5 G_{113}}{\eta'_2} + \frac{\delta'_2 G_5 G_{91}}{\eta'_2} + G_7 G_{82} G_{81} \right)$$

$$G_{146} = 1 / \left(\left(\frac{V_{20}}{\Omega} - G_{11} - A_2 \cos(t_{10} + \tau_0 + \psi) \right) \right)$$

$$G_{147} = \left(-\frac{V_{20} G_{82} G_{74}}{\Omega} + G_{17} + \frac{\pi K_3 G_{42}}{\Omega K_1} + D_1 G_9 G_{130} + D_1 G_{10} G_{114} + D_2 G_{11} G_{138} + D_2 G_{12} G_{122} + A_2 \cos(t_{10} + \tau_0 + \psi) G_{82} G_{74} + \frac{V_{20}}{\Omega} \right)$$

$$G_{148} = \left(-\frac{V_{20} G_{82} G_{75}}{\Omega} + G_{23} + \frac{\pi}{\Omega} \frac{K_3}{K_1} G_{41} + D_1 G_9 G_{131} + D_1 G_{10} G_{115} + D_2 G_{11} G_{139} + D_2 G_{12} G_{123} + A_2 \cos(t_{10} + \tau_0 + \psi) \cdot (1 + G_{82} G_{75}) \right)$$

$$G_{149} = \left(-\frac{V_{20} G_{82} G_{76}}{\Omega} + G_{19} + \frac{\pi}{\Omega} \frac{K_3}{K_1} G_{37} + D_1 G_9 G_{132} + D_1 G_{10} G_{116} + D_2 G_{11} G_{140} + D_2 G_{12} G_{124} + A_2 \cos(t_{10} + \tau_0 + \psi) G_{82} G_{76} \right)$$

$$G_{150} = \left(-\frac{V_{20} G_{82} (1 + G_{77})}{\Omega} + G_{20} + \frac{\pi}{\Omega} \frac{K_3}{K_1} G_{38} + D_1 G_9 G_{133} + D_1 G_{10} G_{117} + D_2 G_{11} G_{141} + D_2 G_{13} G_{125} + A_2 \cos(t_{10} + \tau_0 + \psi) G_{82} (1 + G_{77}) \right)$$

$$G_{151} = \left(-\frac{V_{20} G_{82} G_{78}}{\Omega} + G_{21} + \frac{\pi}{\Omega}\frac{K_3}{K_1} G_{39} + D_1 G_9 G_{134} + D_1 G_{10} G_{118} + D_2 G_{11} G_{142} + D_2 G_{13} G_{126} + A_2 \cos(t_{10} + \tau_0 + \psi) G_{82} G_{78}\right)$$

$$G_{152} = \left(-\frac{V_{20} G_{82} G_{79}}{\Omega} + G_{22} + \frac{\pi}{\Omega}\frac{K_3}{K_1} G_{40} + D_1 G_9 G_{135} + D_1 G_{10} G_{119} + D_2 G_{11} G_{143} + D_2 G_{13} G_{127} + A_2 \cos(t_{10} + \psi + \tau_0) G_{82} G_{79}\right)$$

$$G_{153} = \left(-\frac{V_{20}\left(G_{82} G_{80} - \frac{\pi}{V_{10}}\right)}{\Omega} + D_1 G_9 G_{136} + D_1 G_{10} G_{120} + D_2 G_{11} G_{143} + D_2 G_{12} G_{128} + A_2 \cos(t_{10} + \tau_0 + \psi) \cdot \left(G_{82} G_{80} - \frac{\pi}{V_{10}}\right)\right)$$

$$G_{154} = \left(-V_{20}\frac{G_{82} G_{81}}{\Omega} + \frac{\pi K_4}{\Omega} + D_1 G_9 G_{137} + D_1 G_{10} G_{121} + D_2 G_{11} G_{145} + D_2 G_{12} G_{129} + A_2 \cos(t_{10} + \tau_0 + \psi) \cdot G_{82} G_{81}\right)$$

$$G_{155} = e^{-\frac{\delta_1' \pi}{\Omega}}\left(\frac{\eta_1' B_{10}}{\Omega}\cos\frac{\eta_1'\pi}{\Omega} - \frac{\eta_1' B_{20}}{\Omega}\sin\frac{\eta_1'\pi}{\Omega}\right) + e^{-\delta_2'\frac{\pi}{\Omega}}\left(\frac{\eta_2' C_{10}}{\Omega}\cos\frac{\eta_2'\pi}{\Omega} - \frac{\eta_2' C_{20}}{\Omega}\sin\frac{\eta_2'\pi}{\Omega}\right) - A_1\cos(\tau_0)$$

$$G_{156} = e^{-\frac{\delta_1'\pi}{\Omega}} \sin\frac{\eta_1'\pi}{\Omega}$$

$$G_{157} = e^{-\frac{\delta_1'\pi}{\Omega}} \cos\frac{\eta_1'\pi}{\Omega}$$

$$G_{158} = e^{-\frac{\delta_2'\pi}{\Omega}} \sin\frac{\eta_2'\pi}{\Omega}$$

$$G_{159} = e^{-\frac{\delta_2'\pi}{\Omega}} \cos\frac{\eta_2'\pi}{\Omega}$$

$$G_{160} = -A_1 \cos(\tau_0)$$

$$G_{161} = D_1 e^{-\frac{\delta_1'\pi}{\Omega}}\left(\eta_1'\frac{B_{10}}{\Omega}\cos\frac{\eta_1'\pi}{\Omega} - \eta_1'\frac{B_{20}}{\Omega}\sin\frac{\eta_1'\pi}{\Omega}\right) + D_2 e^{-\frac{\delta_2'\pi}{\Omega}}\left(\eta_2'\frac{C_{10}}{\Omega}\cos\frac{\eta_2'\pi}{\Omega} - \eta_2'\frac{C_{20}}{\Omega}\sin\frac{\eta_2'\pi}{\Omega}\right) - A_2\cos(\tau_0 + \psi)$$

$$G_{162} = -A_2 \cos(\tau_0 + \psi)$$

$$G_{163} = \frac{G_{156} G_1}{\eta_1'} + \frac{G_{158} G_2}{\eta_2'}$$

$$G_{164} = \frac{G_{156} G_1 \delta_1'}{\eta_1'} + G_1 G_{157} + \frac{G_{158} G_2 \delta_2'}{\eta_2'} + G_{159} G_2 + G_{155}\frac{\Omega}{V_{10}}$$

$$G_{165} = -\frac{G_5 G_{156}}{\eta'_1} - \frac{G_{158} G_5}{\eta'_2}$$

$$G_{166} = -\frac{G_{156} G_5 \delta'_1}{\eta'_1} - G_{157} G_5 + \frac{G_{158} G_5 \delta'_2}{\eta'_2} + G_{159} G_5$$

$$G_{167} = -G_{156} G_6 + G_{157} G_3 - G_{158} G_7 + G_{159} G_4 + G_{160}$$

$$G_{168} = -\left(1 + G_{155} \frac{\Omega}{V_{10}}\right)$$

$$G_{169} = \frac{\Omega}{V_{10}} \frac{G_{163}}{G_{168}}$$

$$G_{170} = \frac{\Omega}{V_{10}} \left(\frac{G_{164}}{G_{168}} + 1\right)$$

$$G_{171} = \frac{\Omega}{V_{10}} \frac{G_{165}}{G_{168}}$$

$$G_{172} = \frac{\Omega}{V_{10}} \frac{G_{166}}{G_{168}}$$

$$G_{173} = \frac{\Omega}{V_{10}} \frac{G_{167}}{G_{168}}$$

$$G_{174} = -\frac{\pi}{V_{10}} \left(\frac{\Omega}{V_{10}} \frac{G_{155}}{G_{168}} + 1\right)$$

$$G_{175} = -\left(G_{161} G_{169} + \frac{D_1 G_{156} G_1}{\eta'_1} + \frac{D_2 G_{158} G_2}{\eta'_2}\right)$$

$$G_{176} = -\left(G_{161} G_{170} + \frac{D_1 G_{156} G_1 \delta'_1}{\eta'_1} + D_1 G_1 G_{157} + \frac{D_2 G_{158} G_2 \delta'_2}{\eta'_2} + D_2 G_{159} G_2\right)$$

$$G_{177} = -\left(G_{161} G_{171} - \frac{D_1 G_{156} G_1 \delta'_1}{\eta'_1} + \frac{D_2 G_{158} G_5}{\eta'_2}\right)$$

$$G_{178} = -\left(G_{161} G_{172} - \frac{D_1 G_{158} \delta'_1 G_5}{\eta'_1} - D_1 G_{157} G_5 + \frac{D_2 G_{158} \delta'_2 G_5}{\eta'_2} + D_2 G_{159} G_5\right)$$

$$G_{179} = -\left(G_{161} G_{173} - D_1 G_{156} G_6 + D_1 G_{157} G_3 - D_2 G_{158} G_7 + D_2 G_{159} G_4 + G_{162}\right)$$

$$G_{180} = -G_{161} G_{174}$$

$$G_{181} = \frac{e^{-\delta'_1 \frac{\pi}{\Omega}} \eta_1}{\Omega} \left(-B_{10}\left(\delta'_1 \cos\frac{\eta'_1 \pi}{\Omega} + \eta'_1 \sin\frac{\eta'_1 \pi}{\Omega}\right) + B_{20}\left(\delta'_1 \sin\frac{\eta'_1 \pi}{\Omega} - \eta'_1 \cos\frac{\eta'_1 \pi}{\Omega}\right)\right)$$

$$G_{182} = \frac{e^{-\delta'_2 \frac{\pi}{\Omega}} \cdot \eta'_2}{\Omega} \left(-C_{10}\left(\delta'_2 \cos\frac{\eta'_2 \pi}{\Omega} + \eta'_2 \sin\frac{\eta'_2 \pi}{\Omega}\right) + C_{20}\left(\delta'_2 \sin\frac{\eta'_2 \pi}{\Omega} - \eta'_2 \cos\eta'_2 \frac{\pi}{\Omega}\right)\right)$$

$$G_{183} = D_1 G_{181} + D_2 G_{182} + \Omega A_2 \sin(\tau_0 + \psi)$$

$$G_{184} = e^{-\frac{\delta_1'\pi}{\Omega}} \left(-\delta_1' \sin\frac{\eta_1'\pi}{\Omega} + \eta_1' \cos\frac{\eta_1'\pi}{\Omega}\right)$$

$$G_{185} = e^{-\frac{\delta_1'\pi}{\Omega}} \left(-\delta_1' \cos\frac{\eta_1'\pi}{\Omega} - \eta_1' \sin\frac{\eta_1'\pi}{\Omega}\right)$$

$$G_{186} = e^{-\frac{\delta_2'\pi}{\Omega}} \left(-\delta_2' \cos\frac{\eta_2'\pi}{\Omega} + \eta_2' \cos\frac{\eta_2'\pi}{\Omega}\right)$$

$$G_{187} = e^{-\frac{\delta_2'\pi}{\Omega}} \left(-\delta_2' \cos\frac{\eta_2'\pi}{\Omega} - \eta_2' \sin\frac{\eta_2'\pi}{\Omega}\right)$$

$$G_{188} = -\left(G_{183} G_{169} + \frac{D_1 G_{184} G_1}{\eta_1'} + \frac{D_2 G_{186} G_2}{\eta_2'}\right)$$

$$G_{189} = -\left(G_{183} G_{170} + \frac{D_1 G_{184} G_1 \delta_1'}{\eta_1'} + D_1 G_{185} G_1 + \frac{D_2 G_{186} G_2 \delta_2'}{\eta_2'} + D_2 G_{187} G_2\right)$$

$$G_{190} = -\left(G_{183} G_{171} - \frac{D_1 G_{184} G_5}{\eta_1'} + \frac{D_2 G_{186} G_5}{\eta_2'}\right)$$

$$G_{191} = -\left(G_{183} G_{172} - \frac{D_1 G_{184} \delta_1' G_5}{\eta_1'} - D_1 G_{185} G_5 + \frac{D_2 G_{186} \delta_2' G_5}{\eta_2'} + D_2 G_{187} G_5\right)$$

$$G_{192} = -\left(G_{183} G_{173} - D_1 G_{184} G_6 + D_1 G_{185} G_3 - D_2 G_{186} G_7 + D_2 G_{187} G_4 + \Omega A_2 \sin(\tau_0 + \psi)\right)$$

$$G_{193} = -G_{183} G_{174}$$

$$G_{194} = G_{181} + G_{182} + \Omega A_1 \sin\tau_0$$

$$G_{195} = -K_1\left(G_{194} G_{169} + G_{184}\frac{G_1}{\eta_1'} + G_{186}\frac{G_2}{\eta_2'}\right)$$

$$G_{196} = -K_1\left(G_{194} G_{170} + G_{184}\frac{G_1 \delta_1'}{\eta_1'} + G_{185} G_1 + G_{186}\frac{G_2 \delta_2'}{\eta_2'} + G_{187} G_2\right)$$

$$G_{197} = -K_1\left(G_{194} G_{171} - \frac{G_{184} G_5}{\eta_1'} + \frac{G_{186} G_5}{\eta_2'}\right)$$

$$G_{198} = -K_1\left(G_{194} G_{172} - \frac{G_{184} \delta_1' G_5}{\eta_1'} - G_5 G_{185} + \frac{G_{188} \delta_2' G_5}{\eta_2'} + G_{187} G_5\right)$$

$$G_{199} = -K_1\left(G_{194} G_{173} - G_{186} G_6 + G_{185} G_3 - G_{186} G_7 + G_{187} G_4 + \Omega A_1 \sin(\tau_0)\right)$$

$$G_{200} = -(K_1 G_{194} G_{174} - K_2)$$

$$G_{201} = -(K_3 G_{194} G_{174} - K_4)$$

$$G_{202} = \frac{D_1 G_{156} G_1}{\eta_1'} + \frac{D_2 G_{158} G_2}{\eta_2'}$$

$$G_{203} = \frac{D_1 G_{156} G_1 \delta_1'}{\eta_1'} + D_1 G_{157} G_1 + \frac{D_2 G_{158} G_2 \delta_2'}{\eta_2'} + D_2 G_{159} G_2$$

$$G_{204} = -\frac{D_1 G_{156} G_5}{\eta'_1} + \frac{D_2 G_{158} G_5}{\eta'_2}$$

$$G_{205} = \frac{G_{161}\Omega}{V_{20}} - \frac{D_1 G_{156}\delta'_1 G_5}{\eta'_1} - D_1 G_{157} G_5 + \frac{D_2 G_{158}\delta'_2 G_5}{\eta'_2} + D_2 G_{159} G_5$$

$$G_{206} = -D_1 G_{156} G_6 + D_1 G_{157} G_3 - D_2 G_{158} G_7 + D_2 G_{159} G_4 + G_{162}$$

$$G_{207} = -\left(1 + \frac{G_{161}\Omega}{V_{20}}\right)$$

$$G_{208} = \frac{\Omega}{V_{20}} \frac{G_{202}}{G_{207}}$$

$$G_{209} = \frac{\Omega}{V_{20}} \frac{G_{203}}{G_{207}}$$

$$G_{210} = \frac{\Omega}{V_{20}} \frac{G_{204}}{G_{207}}$$

$$G_{211} = \frac{\Omega}{V_{20}} \left(\frac{G_{205}}{G_{207}} + 1\right)$$

$$G_{212} = \frac{\Omega}{V_{20}} \frac{G_{206}}{G_{207}}$$

$$G_{213} = -\frac{\pi}{V_{20}} \left(\frac{\Omega G_{161}}{V_{20} G_{207}} + 1\right)$$

$$G_{214} = -\left(G_{155} G_{208} + \frac{G_{156} G_1}{\eta'_1} + \frac{G_{158} G_2}{\eta'_2}\right)$$

$$G_{215} = -\left(G_{155} G_{209} + \frac{G_{156} G_1 \delta'_1}{\eta'_1} + G_{157} G_1 + \frac{G_{158} G_2 \delta'_2}{\eta'_2} + G_{159} G_2\right)$$

$$G_{216} = -\left(G_{155} G_{210} - \frac{G_{156} G_5}{\eta'_1} + \frac{G_{158} G_5}{\eta'_2}\right)$$

$$G_{217} = -\left(G_{155} G_{211} - \frac{G_{156} G_5 \delta'_1}{\eta'_1} - G_{157} G_5 + \frac{G_{158} G_5 \delta'_2}{\eta'_2} + G_{159} G_5\right)$$

$$G_{218} = -\left(G_{155} G_{212} - G_{156} G_6 + G_{157} G_3 - G_{158} G_7 + G_{159} G_4 + G_{160}\right)$$

$$G_{219} = -G_{155} G_{213}$$

$$G_{220} = -\left(G_{194} G_{208} + \frac{G_{184} G_1}{\eta'_1} + \frac{G_{186} G_2}{\eta'_2}\right)$$

$$G_{221} = -\left(G_{194} G_{209} + \frac{G_{184} G_1 \delta'_1}{\eta'_1} + G_{185} G_1 + \frac{G_{186} G_2 \delta'_2}{\eta'_2} + G_{187} G_2\right)$$

$$G_{222} = -\left(G_{194} G_{210} - \frac{G_{184} G_5}{\eta'_1} + \frac{G_{186} G_5}{\eta'_2}\right)$$

$$G_{223} = -\left(G_{194}G_{211} - \frac{G_{184}G_6\delta_1'}{\eta_1'} - G_{185}G_5 + \frac{G_{186}\delta_2'G_5}{\eta_2'} + G_{187}G_5\right)$$

$$G_{224} = -\left(G_{194}G_{212} - G_{184}G_6 + G_{185}G_3 - G_{186}G_7 + G_{187}G_4 + \Omega A_1 \sin(\tau_0)\right)$$

$$G_{225} = -G_{194}G_{213}$$

$$G_{226} = -K_1\left(G_{183}G_{208} + \frac{D_1G_{184}G_1}{\eta_1'} + \frac{D_2G_{186}G_2}{\eta_2'}\right)$$

$$G_{227} = -K_1\left(G_{183}G_{209} + \frac{D_1G_{184}G_1\delta_1'}{\eta_1'} + D_1G_{185}G_1 + \frac{D_2G_{186}G_2\delta_2'}{\eta_2'} + D_2G_{187}G_2\right)$$

$$G_{228} = -K_1\left(G_{183}G_{210} - \frac{D_1G_{184}G_5}{\eta_1'} + \frac{D_2G_{186}G_5}{\eta_2'}\right)$$

$$G_{229} = -K_1\left(G_{183}G_{211} - \frac{D_1G_{184}\delta_1'G_5}{\eta_1'} - D_1G_{185}G_5 + \frac{D_2G_{186}\delta_2'G_5}{\eta_2'} + D_2G_{187}G_5\right)$$

$$G_{230} = -K_1\left(G_{183}G_{212} - D_1G_{184}G_6 + D_1G_{185}G_3 - D_2G_{185}G_7 + D_2G_{186}G_7 + D_2G_{187}G_4 + \Omega A_2 \sin(\tau_0 + \psi)\right)$$

$$G_{231} = -K_1G_{183}G_{213} + K_2$$

$$G_{232} = K_3G_{183}G_{213} - K_4$$

APPENDIX C

```
$JOB            003721 BN AGRAWAL        100   010   030
$IBJOB          NODECK
$IBFTC
C
C       IMPACT DAMPER IN PRIMARY MASS 1.HERE WE DETERMINE THE STABILITY
C       REGION AT FIRST NATURAL FREQUENCY.MASS RATIO AND GAP RATIO ARE
C      .VARIED.COEFFICIENT OF RESTITUTION IS CONSTANT.
C
C
C       DETERMINE THE ELEMENTS OF MATRIX OF STEADY STATE SOLUTION.
C
      DIMENSION P(20,20),Q(20,20),X(10),A(20,20),Z(10),Y(10)
      READ(5,1) AK1,AK2,AM,G,D,W,F
      AM=AM/386.4
      F=1.
      EMU=0.15
      E=0.8
      R=AK1/AK2
      W2=SQRT(AK2*(3.+2.*R+SQRT(5.))/(2.*AM))
      W1=SQRT(AK2*(3.+2.*R-SQRT(5.))/(2.*AM))
      CR1=2.*W1*AM
      CR2=2.*W2*AM
      D1=0.1
      D=D1*CR1
      D2=D/CR2
      AN1=SQRT(1.-D1*D1)
      AN2=SQRT(1.-D2*D2)
      W11=AN1*W1
      W22=AN2*W2
      C1=(1.+SQRT(5.))/2.
      C2=(1.-SQRT(5.))/2.
      W=W22
215   C=3.143/W
      K2=0
      DO 210 K7=1,2
      K2=K2+1
      CONS=0.
      DO 105 I7=1,50
      CONS=CONS+0.2
      G=CONS*F/AK2
      DO 30 I=1,11
      DO 30 J=1,10
30    P(I,J)=0.
      P(1,1)=1.
      P(1,2)=1.
      P(1,3)=1.
      P(1,7)=1.
      P(2,4)=1.
      P(2,8)=1.
      P(3,9)=1.
      P(4,5)=1.
```

```
 P(5,6)=1.
 P(5,10)=1.
 P(7,3)=-1.
 P(9,3)=-1.
 EQUIVALENCE(P(6,7),H13),(P(7,7),H14),(P(8,7),H15),(P(9,7),H16),
C(P(6,9),T9),(P(7,9),T10),(P(8,9),T11),(P(9,9),T12),(P(3,1),SG1),
C(P(4,2),SG2)
 E1=AK1+AK2-AM*W*W
 E2=(AK1+AK2)*(AK1+2.*AK2)-W*W*(D*D+AM*(2.*AK1+3.*AK2))+AM*AM*W**4
C-AK2*AK2
 E3=D*((2.*AK1+3.*AK2)*W-2.*W**3*AM)
 A1=F*SQRT((E1*E2+D*W*E3)**2+(D*W*E2-E3*E1)**2)/(E2*E2+E3*E3)
 PH1=ATAN((-D*W*E2+E3*E1)/(E1*E2+D*W*E3))
 A2=AK2*F/SQRT(E2*E2+E3*E3)
 PH2=ATAN(E3/E2)
 PH=PH1-PH2
 H1=D1*W1
 H2=D2*W2
 H13=EXP(-H1*C)*SIN(W11*C)
 H14=EXP(-H1*C)*COS(W11*C)
 H15= EXP(-H2*C)*SIN(W22*C)
 H16=EXP(-H2*C)*COS(W22*C)
 T9=EXP(-H1*C)*(-H1*SIN(W11*C)+W11*COS(W11*C))
 T10=EXP(-H1*C)*(-H1*COS(W11*C)-W11*SIN(W11*C))
 T11=EXP(-H2*C)*(-H2*SIN(W22*C)+W22*COS(W22*C))
 T12=EXP(-H2*C)*(-H2*COS(W22*C)-W22*SIN(W22*C))
 SG1=3.143*(1.+E)/(2.*W*(1.-E+2.*EMU   ))
 SG2=3.143*(1.+E)/(2.*W*(1.-E-2.*EMU*E))
 Z1=1.+H14
 Z2=1. +H16
 Z3=H1-1./SG2
 Z4=H2-1./SG2
 Z6=T10-1./SG1
 Z7=T12-1./SG1
 Z9=-C1*(W11+T9)
 Z10=C1*(H1-T10)
 Z11=-C2*(W22+T11)
 Z12=C2*(H2-T12)
 Z13=Z3+W11*Z1/H13
 Z14=-W22+W11*H15/H13
 Z15=Z4+W11*Z2/H13
 Z16=Z10-Z9*Z1/H13
 Z17=Z11-Z9*H15/H13
 Z18=Z12-Z9*Z2/H13
 Z19=Z6-T9*Z1/H13
 Z20=T11-T9*H15/H13
 Z21=Z7-T9*Z2/H13
 Z22=Z14-Z13*Z17/Z16
 Z23=Z15-Z13*Z18/Z16
 Z24=Z20-Z19*Z17/Z16
 Z25=Z21-Z19*Z18/Z16
 Z26=Z23-Z22*Z2/H15
 Z27=Z25-Z24*Z2/H15
 H=2.*(Z26-Z27)*W/(Z26/SG1-Z27/SG2)
 RO=G/A1
 CONS1=H*H+4.-RO*RO
 IF(CONS1.GE.0.0) GO TO 170
 WRITE(6,9)
```

```
      GO TO 210
170   T=ATAN((-2.*RO+H*SQRT(H*H+4.-RO*RO))/(-RO*H-2.*SQRT(H*H+4.-RO*RO
     C)))
      IF(K2.EQ.1) GO TO 240
      T=ATAN((-2.*RO-H*SQRT(H*H+4.-RO*RO))/(-RO*H+2.*SQRT(H*H+4.-RO*RO
     C)))
240   IF(T.GE.0.) GO TO 160
      P(10,3)=SIN(T)
      P(10,5)=W*COS(T)
      P(11,4)=-A2*SIN(T+PH)
      P(11,6)=-A2*W*COS(T+PH)
      P(11,10)=P(11,6)
      P(11,8)=P(11,4)
      P(10,7)=P(10,3)
      P(10,9)=P(10,5)
      T=22./7.+T
      GO TO 200
160   P(10,3)=-SIN(T)
      P(10,5)=-W*COS(T)
      P(11,4)=A2*SIN(T+PH)
      P(11,6)=A2*W*COS(T+PH)
      P(10,7)=P(10,3)
      P(10,9)=P(10,5)
      P(11,8)=P(11,4)
      P(11,10)=P(11,6)
200   P(7,4)=-C1
      P(9,4)=-C2
       P(6,5)=-W11
      P(7,5)=H1
      P(8,5)=-W22
      P(9,5)=H2
      P(6,6)=-C1*W11
      P(7,6)=C1*H1
      P(9,6)=C2*H2
      P(8,6)=-C2*W22
      P(6,8)=C1*H13
      P(7,8)=C1*H14
      P(8,8)=C2*H15
      P(9,8)=C2*H16
      P(6,10)=C1*T9
      P(7,10)=C1*T10
      P(8,10)=C2*T11
      P(9,10)=C2*T12
      P(11,1)=-G/2.
      P(11,2)=-G/2.
      DO 35 I=1,10
      DO 35 J=1,11
35    A(I,J)=P(J,I)
C
C      SOLUTION OF STEADY STATE EQUATIONS.
C
      N=10
      NM1=N-1
      NP1=N+1
      DO 50 I=1,NM1
      I1=I+1
      R=0.0
      DO 12 K=I,N
```

```
      IF(ABS(R).GE.ABS(A(K,I)))GO TO 12
      IT=K
      R=A(K,I)
   12 CONTINUE
C
C     SINGULARITY CHECK
C
      IF(R.EQ.0.0) GO TO 51
C
C     SET LARGEST ELEMENT ON THE DIOGNAL
C
      IF(IT.EQ.I) GO TO 16
      DO 17 L=I,NP1
      R=A(IT,L)
      A(IT,L)=A(I,L)
   17 A(I,L)=R
   16 DO 50 M=I1,N
      CM=A(M,I)/A(I,I)
      DO 45 J=I,NP1
   45 A(M,J)=A(M,J)-CM*A(I,J)
   50 CONTINUE
C     REQUIRED ROOTS
C
      X(N)=A(N,NP1)/A(N,N)
      I=N
      I1=I-1
      DO 80 K=1,I1
      I=I-1
      L=I+1
      SUM=0.0
      DO 70 M1=L,N
   70 SUM=SUM+A(I,M1)*X(M1)
      X(I)=(A(I,NP1)-SUM)/A(I,I)
   80 CONTINUE
C
      WRITE(6,4) A1,X(10)
      XB=X(1)
      V1=(G+2.*ABS(XB))*W/3.143
      B1=X(6)
      B2=X(7)
      E1=X(8)
      E2=X(9)
      A1=X(10)
      TPH=T+PH
      XMAX=0.
      TANG=-0.1
      DO  230 I=1,31
      TANG=TANG+0.1
      T1=TANG/W
      X2=C1*EXP(-H1*T1)*(B1*SIN(W11*T1)+B2*COS(W11*T1))+C2*EXP(-H2*T1)*
     C(E1*SIN(W22*T1)+E2*COS(W22*T1))+A2*SIN(W*T1+TPH)
      IF(ABS(X2).LE.XMAX) GO TO 230
      XMAX=ABS(X2)
230   CONTINUE
      RATIO=XMAX/A2
C
C      DETERMINE THE ELEMENT OF STABILITY MATRIX.
C
```

```
 G1=C2/(C2-C1)
 G2=C1/(C1-C2)
 G3=(-C2*A1*COS(T)+A2*COS(T+PH))/(C2-C1)
 G4=(-C1*A1*COS(T)+A2*COS(T+PH))/(C1-C2)
 G5=1./(C2-C1)
 G6=((-C2*A1*(H1*COS(T)-W*SIN(T))+A2*(H1*COS(T+PH)-W*SIN(T+PH)))/(W
C11*(C1-C2)))
 G7=(-C1*A1*(H2*COS(T)-W*SIN(T))+A2*(H2*COS(T+PH)-W*SIN(T+PH)))/(W2
C2*(C2-C1))
 G155=EXP(-H1*C)*(W11*B1*COS(W11*C)/W-W11*B2*SIN(W11*C)/W)+EXP(-H2
C*C)*(W22*E1*COS(W22*C)/W-W22*E2*SIN(W22*C)/W)-A1*COS(T)
 G156=EXP(-H1*C)*SIN(W11*C)
 G157=EXP(-H1*C)*COS(W11*C)
 G158=EXP(-H2*C)*SIN(W22*C)
 G159=EXP(-H2*C)*COS(W22*C)
 G160=-A1*COS(T)
 G161=C1*EXP(-H1*C)*(W11*B1*COS(W11*C)/W-W11*B2*SIN(W11*C)/W)+C2*
CEXP(-H2*C)*(W22*E1*COS(W22*C)/W-W22*E2*SIN(W22*C)/W)-A2*COS(T+PH)
 G162=-A2*COS(T+PH)
 G163=G156*G1/W11+G158*G2/W22
 G164=G156*G1*H1/W11+G1*G157+G158*G2*H2/W22+G159*G2+G155*W/V1
 G165=-G5*G156/W11+G158*G5/W22
 G166=-G156*G5*H1/W11-G157*G5+G158*G5*H2/W22+G159*G5
 G167=-G156*G6+G157*G3-G158*G7+G159*G4+G160
 G168=-(1.+G155*W/V1)
 G169=W*G163/(V1*G168)
 G170=W*(G164/G168+1.)/V1
 G171=W*G165/(V1*G168)
 G172=W*G166/(V1*G168)
 G173=W*G167/(V1*G168)
 G174=-3.143*(W*G155/(V1*G168)+1.)/V1
 G175=-(G161*G169+C1*G156*G1/W11+C2*G158*G2/W22)
 G176=-(G161*G170+C1*G156*G1*H1/W11+C1*G1*G157+C2*G158*G2*H2/W22
C+C2*G159*G2)
 G177=-(G161*G171-C1*G156*G5/W11+C2*G158*G5/W22)
 G178=-(G161*G172-C1*G156*H1*G5/W11-C1*G157*G5+C2*G158*H2*G5/W22+C2
C*G159*G5)
 G179=-(G161*G173-C1*G156*G6+C1*G157*G3-C2*G158*G7+C2*G159*G4+G162)
 G180=-G161*G174
 G181=EXP(-H1*C)*W11*(-B1*(H1*COS(W11*C)+W11*SIN(W11*C))+B2*(H1*SIN
C(W11*C)-W11*COS(W11*C)))/W
 G182=EXP(-H2*C)*W22*(-E1*(H2*COS(W22*C)+W22*SIN(W22*C))+E2*(H2*SIN
C(W22*C)-W22*COS(W22*C)))/W
 G183=C1*G181+C2*G182+W*A2*SIN(T+PH)
 G184=EXP(-H1*C)*(-H1*SIN(W11*C)+W11*COS(W11*C))
 G185=EXP(-H1*C)*(-H1*COS(W11*C)-W11*SIN(W11*C))
 G186=EXP(-H2*C)*(-H2*SIN(W22*C)+W22*COS(W22*C))
 G187=EXP(-H2*C)*(-H2*COS(W22*C)-W22*SIN(W22*C))
 G188=-(G183*G169+C1*G184*G1/W11+C2*G186*G2/W22)
 G189=-(G183*G170+C1*G184*G1*H1/W11+C1*G185*G1+C2*G186*G2*H2/W22
C+C2*G187*G2)
 G190=-(G183*G171-C1*G184*G5/W11+C2*G186*G5/W22)
 G191=-(G183*G172-C1*G184*H1*G5/W11-C1*G185*G5+C2*G186*H2*G5/W22
C+C2*G187*G5)
 G192=-(G183*G173-C1*G184*G6+C1*G185*G3-C2*G186*G7+C2*G187*G4
C+W*A2*SIN(T+PH))
 G193=-G183*G174
 G194=G181+G182+W*A1*SIN(T)
```

```
      BK1=(1.-EMU*E)/(1.+EMU)
      BK2=EMU*(1.+E)/(1.+EMU)
      BK3=(1.+E)/(1.+EMU)
      BK4=(EMU-E)/(1.+EMU)
      G195=-BK1*(G194*G169+G184*G1/W11+G186*G2/W22)
      G196=-BK1*(G194*G170+G184*G1*H1/W11+G185*G1+G186*G2*H2/W22+G187*
     CG2)
      G197=-BK1*(G194*G171-G184*G5/W11+G186*G5/W22)
      G198=-BK1*(G194* G172-G184*H1*G5/W11-G5*G185+G186*H2*G5/W22+G187*G
     C5)
      G199=-BK1*(G194*G173-G184*G6+G185*G3-G186*G7+G187*G4+W*A1*SIN(T))
      G200=-(BK1*G194*G174-BK2)
      G201=(BK3*G194*G174-BK4)
      EQUIVALENCE(Q(1,1),G195),(Q(1,2),G196),(Q(1,3),G197),
     C(Q(1,5),G199),(Q(1,6),G200),(Q(3,1),G188),(Q(3,2),G189),(Q(3,3),G1
     C90),(Q(3,4),G191),(Q(3,5),G192),(Q(3,6),G193),(Q(4,1),G175),(Q(4,2
     C),G176),(Q(4,3),G177),(Q(4,4),G178),(Q(4,5),G179),(Q(4,6),G180),
     C(Q(5,1),G169),(Q(5,2),G170),(Q(5,3),G171),(Q(5,4),G172),(Q(5,6),G1
     C74),(Q(6,6),G201),(Q(1,4),G198)
      Q(2,1)=G163/G168
      Q(2,2)=G164/G168
      Q(2,3)=G165/G168
      Q(2,4)=G166/G168
      Q(2,5)=G167/G168
      Q(2,6)=-G155*3.143/(G168*V1)
      Q(5,5)=1.+G173
      Q(6,1)=-BK3*G195/BK1
      Q(6,2)=-BK3*G196/BK1
      Q(6,3)=-BK3*G197/BK1
      Q(6,4)=-BK3*G198/BK1
      Q(6,5)=-BK3*G199/BK1
C
C      DETERMINE THE EIGEN VALUES BY THE SUBROUTINE RUTI
C
      CALL RUTI(Q,6,20,Z,Y,0.001)
C
C      CHECK IF ALL THE EIGEN VALUES OF STABILITY MATRIX ARE LESS THAN
C      UNITY
C
      DO 75 L=1,6
      AMO=SQRT(Z(L)**2+Y(L)**2)
      IF(AMO.GT.1.) GO TO 220
 75   CONTINUE
      WRITE(6,3) RATIO,CONS
      GO TO 105
220   WRITE(6,2) RATIO,CONS
      GO TO 105
   51 WRITE(6,8)
      GO TO 110
105     CONTINUE
210   CONTINUE
C
C      PRINT THE RESULTS
C
1     FORMAT(7F10.6)
2     FORMAT(5X,7HRATIO =,F5.2,6HCONS =,F5.2,8HUNSTABLE    //)
3     FORMAT(5X,7HRATIO =,F5.2,6HCONS =,F5.2,6HSTABLE  //)
4     FORMAT(1X,11F10.6)
```

```
    8 FORMAT(10X,18HMATRIX IS SINGULAR   //)
9     FORMAT(10X,15HUNSTEADY MOTION  //)
110   CALL EXIT
      END
$ENTRY
20.        10.        5.        0.5        0.36        50.        3.0
$IBSYS
```

```
$JOB            003721 BN AGRAWAL        100    010    030
$IBJOB          NODECK
$IBFTC
C
C      IMPACT DAMPER IN PRIMARY MASS 2.HERE STEADY STATE AMPLITUDES ARE
C      DETERMINED FOR BOTH PRIMARY MASSES AT BOTH NATURAL FREQUENCIES.
C      HERE MASS RATIO IS KEPT CONSTANT AND GAP RATIO IS VARYING.
C      STABILITY IS ALSO CHECKED.
C
C
C      DETERMINE THE ELEMENTS OF MATRIX OF STEADY STATE SOLUTION.
C
      DIMENSION P(20,20),Q(20,20),X(10),A(20,20),Z(10),Y(10)
      READ(5,1) AK1,AK2,AM,G,D,W,F
      AM=AM/386.4
      EMU=0.1
      E=0.8
      R=AK1/AK2
      W2=SQRT(AK2*(3.+2.*R+SQRT(5.))/(2.*AM))
      W1=SQRT(AK2*(3.+2.*R-SQRT(5.))/(2.*AM))
      CR1=2.*W1*AM
      CR2=2.*W2*AM
      D1=0.1
      D=D1*CR1
      D2=D/CR2
      AN1=SQRT(1.-D1*D1)
      AN2=SQRT(1.-D2*D2)
      W11=AN1*W1
      W11=AN1*W1
      W22=AN2*W2
      C1=(1.+SQRT(5.))/2.
      C2=(1.-SQRT(5.))/2.
      WRITE(6,6)
      M2=0
      DO 210 J7=1,2
      M2=M2+1
      W=W22
      IF(M2.EQ.1) GO TO 215
      W=W11
      WRITE(6,5)
215   EMU=0.02
      DO 210 L7=1,8
      EMU=EMU+0.01
      WRITE(6,7) EMU
      C=3.143/W
      CONS=-0.4
      DO 110 I7=1,30
      CONS=CONS+0.4
      G=CONS*F/AK2
      DO 160 L=1,10
      DO 160 M=1,11
160   A(L,M)=0.
      A(1,1)=1.
```

```
      A(2,1)=1.
      A(4,1)=1.
      A(8,1)=1.
      A(3,2)=1.
      A(7,2)=1.
      A(10,3)=1.
      A(6,4)=1.
      A(5,5)=1.
      A(9,5)=1.
      A(3,7)=-1.
      A(3,9)=-1.
      EQUIVALENCE(A(7,6),H13),(A(7,7),H14),(A(7,8),H15),(A(7,9),H16),(A(
     C9,6),T9),(A(9,7),T10),(A(9,8),T11),(A(9,9),T12),(A(1,3),SG1),
     C(A(2,4),SG2)
      E1=AK1+AK2-AM*W*W
      E2=(AK1+AK2)*(AK1+2.*AK2)-W*W*(D*D+AM*(2.*AK1+3.*AK2))+AM*AM*W**4
     C-AK2*AK2
      E3=D*((2.*AK1+3.*AK2)*W-2.*W**3*AM)
      A1=F*SQRT((E1*E2+D*W*E3)**2+(D*W*E2-E3*E1)**2)/(E2*E2+E3*E3)
      PH1=ATAN((-D*W*E2+E3*E1)/(E1*E2+D*W*E3))
      A2=AK2*F/SQRT(E2*E2+E3*E3)
      PH2=ATAN(E3/E2)
      PH=PH1-PH2
      H1=D1*W1
      H2=D2*W2
      H3=W11
      H4=W22
      T9=EXP(-H1*C)*(-H1*SIN(W11*C)+W11*COS(W11*C))
      T10=EXP(-H1*C)*(-H1*COS(W11*C)-W11*SIN(W11*C))
      T11=EXP(-H2*C)*(-H2*SIN(W22*C)+W22*COS(W22*C))
      T12=EXP(-H2*C)*(-H2*COS(W22*C)-W22*SIN(W22*C))
      H13=EXP(-H1*C)*SIN(W11*C)
      H14=EXP(-H1*C)*COS(W11*C)
      H15= EXP(-H2*C)*SIN(W22*C)
      H16=EXP(-H2*C)*COS(W22*C)
      SG1=3.143*(1.+E)/(2.*W*(1.-E+2.*EMU   ))
      SG2=3.143*(1.+E)/(2.*W*(1.-E-2.*EMU*E))
      U1=(1.+H14)
      U2=(1.+H16)
      U3=C1*(H1-1./SG2)
      U4=C2*(H2-1./SG2)
      U6=C1*(T10-1./SG1)
      U7=C2*(T12-1./SG1)
      U9=-(W11+T9)
      U10=(H1-T10)
      U11=-(W22+T11)
      U12=(H2-T12)
      U13=U3+C1*H3*U1/H13
      U14=-C2*H4+C1*H3*H15/H13
      U15=U4+C1*H3*U2/H13
      U16=U10-U9*U1/H13
      U17=U11-U9*H15/H13
      U18=U12-U9*U2/H13
      U19=U6-C1*T9*U1/H13
      U20=C2*T11-C1*T9*H15/H13
      U21=U7-C1*T9*U2/H13
      U22=U14-U13*U17/U16
      U23=U15-U13*U18/U16
```

```
      U24=U20-U19*U17/U16
      U25=U21-U19*U18/U16
      U26=U23-U22*U2/H15
      U27=U25-U24*U2/H15
      H=2.*(U26-U27)*W/(U26/SG1-U27/SG2)
      RO=G/A2
      CONS1=H*H+4.-RO*RO
      IF(CONS1.GE.0.0) GO TO 180
      WRITE(6,9)
      GO TO 210
180   TPH=ATAN((-2.*RO+H*SQRT(H*H+4.-RO*RO))/(-RO*H-2.*SQRT(H*H+4.-RO*
     CRO)))
240   IF(TPH.GE.0.) GO TO 200
      A(4,10)=SIN(TPH)
      A(8,10)=SIN(TPH)
      A(6,10)=W*COS(TPH)
      A(10,10)=W*COS(TPH)
      A(3,11)=-A1*SIN(TPH-PH)
      A(7,11)=-A1*SIN(TPH-PH)
      A(9,11)=-A1*W*COS(TPH-PH)
      A(5,11)=A(9,11)
      TPH=22./7.+TPH
      T=TPH-PH
      GO TO 170
200   A(4,10)=-SIN(T+PH)
      A(8,10)=-SIN(T+PH)
      A(5,11)=A1*W*COS(T)
      A(7,11)=A1*SIN(T)
      A(9,11)=A1*W*COS(T)
      A(3,11)=A1*SIN(T)
      A(10,10)=-W*COS(T+PH)
      A(6,10)=-W*COS(T+PH)
170   A(4,7)=-C1
      A(4,9)=-C2
      A(5,6)=-W11
      A(5,7)=H1
      A(5,8)=-W22
      A(5,9)=H2
      A(6,6)=-C1*W11
      A(6,7)=C1*H1
      A(6,8)=-C2*W22
      A(6,9)=C2*H2
      A(8,6)=C1*H13
      A(8,7)=C1*H14
      A(8,8)=C2*H15
      A(8,9)=C2*H16
      A(10,6)=C1*T9
      A(10,7)=C1*T10
      A(10,8)=C2*T11
      A(10,9)=C2*T12
      A(1,11)=-G/2.
      A(2,11)=-G/2.
C
C      SOLUTION OF STEADY STATE EQUATIONS
C
      N=10
      NM1=N-1
      NP1=N+1
```

```
      DO 50 I=1,NM1
      I1=I+1
      R=0.0
      DO 12 K=I,N
      IF(ABS(R).GE.ABS(A(K,I)))GO TO 12
      IT=K
      R=A(K,I)
   12 CONTINUE
C
C     SINGULARITY CHECK
C
      IF(R.EQ.0.0) GO TO 51
C
C     SET LARGEST ELEMENT ON THE DIOGNAL
C
      IF(IT.EQ.I) GO TO 16
      DO 17 L=I,NP1
      R=A(IT,L)
      A(IT,L)=A(I,L)
   17 A(I,L)=R
   16 DO 50 M=I1,N
      CM=A(M,I)/A(I,I)
      DO 45 J=I,NP1
   45 A(M,J)=A(M,J)-CM*A(I,J)
   50 CONTINUE
      X(N)=A(N,NP1)/A(N,N)
      I=N
      I1=I-1
      DO 80 K=1,I1
      I=I-1
      L=I+1
      SUM=0.0
      DO 70 M1=L,N
   70 SUM=SUM+A(I,M1)*X(M1)
      X(I)=(A(I,NP1)-SUM)/A(I,I)
   80 CONTINUE
C
      WRITE(6,4) A2,X(10)
      XB1=X(1)
      IF(XB1.LT.0.0) GO TO 130
      V20=(G+2.*XB1)*W/3.143
      GO TO 140
  130 V20=(2.*XB1-G)*W/3.143
  140 B1=X(6)
      B2=X(7)
      E1=X(8)
      E2=X(9)
      A2=X(10)
C
C      DETERMINE THE AMPLITUDE OF DISPLACEMENT OF PRIMARY MASS 1 AND 2
C
      X1MX=0.
      X2MX=0.
      TANG=-0.1
      DO 230 I=1,32
      TANG=TANG+0.1
      T1=TANG/W
      X1=EXP(-H1*T1)*(B1*SIN(W11*T1)+B2*COS(W11*T1))+EXP(-H2*T1)*
```

```
     C(E1*SIN(W22*T1)+E2*COS(W22*T1))+A1*SIN(W*T1+T)
      X2=C1*EXP(-H1*T1)*(B1*SIN(W11*T1)+B2*COS(W11*T1))+C2*EXP(-H2*T1)*
     C(E1*SIN(W22*T1)+E2*COS(W22*T1))+A2*SIN(W*T1+TPH)
      IF(ABS(X1).LE.X1MX) GO TO 260
      X1MX=ABS(X1)
260   IF(ABS(X2).LE.X2MX) GO TO 230
      X2MX=ABS(X2)
230   CONTINUE
      AMPR1=X1MX/A1
      AMPR2=X2MX/A2
      BK1=(1.-EMU*E)/(1.+EMU)
      BK2=EMU*(1.+E)/(1.+EMU)
      BK3=(1.+E)/(1.+EMU)
      BK4=(EMU-E)/(1.+EMU)
C
C      DETERMINE THE ELEMENTS OF STABILITY MATRIX.
C
      G1=C2/(C2-C1)
      G2=C1/(C1-C2)
      G3=(-C2*A1*COS(T)+A2*COS(T+PH))/(C2-C1)
      G4=(-C1*A1*COS(T)+A2*COS(T+PH))/(C1-C2)
      G5=1./(C2-C1)
      G6=((-C2*A1*(H1*COS(T)-W*SIN(T))+A2*(H1*COS(T+PH)-W*SIN(T+PH)))/(W
     C11*(C1-C2)))
      G7=(-C1*A1*(H2*COS(T)-W*SIN(T))+A2*(H2*COS(T+PH)-W*SIN(T+PH)))/(W2
     C2*(C2-C1))
      G155=EXP(-H1*C)*(W11*B1*COS(W11*C)/W-W11*B2*SIN(W11*C)/W)+EXP(-H2
     C*C)*(W22*E1*COS(W22*C)/W-W22*E2*SIN(W22*C)/W)-A1*COS(T)
      G156=EXP(-H1*C)*SIN(W11*C)
      G157=EXP(-H1*C)*COS(W11*C)
      G158=EXP(-H2*C)*SIN(W22*C)
      G159=EXP(-H2*C)*COS(W22*C)
      G160=-A1*COS(T)
      G161=C1*EXP(-H1*C)*(W11*B1*COS(W11*C)/W-W11*B2*SIN(W11*C)/W)+C2*
     CEXP(-H2*C)*(W22*E1*COS(W22*C)/W-W22*E2*SIN(W22*C)/W)-A2*COS(T+PH)
      G162=-A2*COS(T+PH)
      G181=EXP(-H1*C)*W11*(-B1*(H1*COS(W11*C)+W11*SIN(W11*C))+B2*(H1*SIN
     C(W11*C)-W11*COS(W11*C)))/W
      G182=EXP(-H2*C)*W22*(-E1*(H2*COS(W22*C)+W22*SIN(W22*C))+E2*(H2*SIN
     C(W22*C)-W22*COS(W22*C)))/W
      G183=C1*G181+C2*G182+W*A2*SIN(T+PH)
      G184=EXP(-H1*C)*(-H1*SIN(W11*C)+W11*COS(W11*C))
      G185=EXP(-H1*C)*(-H1*COS(W11*C)-W11*SIN(W11*C))
      G186=EXP(-H2*C)*(-H2*SIN(W22*C)+W22*COS(W22*C))
      G187=EXP(-H2*C)*(-H2*COS(W22*C)-W22*SIN(W22*C))
      G194=G181+G182+W*A1*SIN(T)
      G202=C1*G156*G1/W11+C2*G158*G2/W22
      G203=C1*G156*G1*H1/W11+C1*G157*G1+C2*G158*G2*H2/W22+C2*G159*G2
      G204=-C1*G156*G5/W11+C2*G158*G5/W22
      G205=G161*W/V20-C1*G156*H1*G5/W11-C1*G157*G5+C2*G158*H2*G5/W22
     C+C2*G159*G5
      G206=-C1*G156*G6+C1*G157*G3-C2*G158*G7+C2*G159*G4+G162
      G207=-(1.+G161*W/V20)
      G208=W*G202/(V20*G207)
      G209=W*G203/(V20*G207)
      G210=W*G204/(V20*G207)
      G211=W*(G205/G207+1.)/V20
      G212=W*G206/(V20*G207)
```

```
 G213=-3.143*(W*G161/(V20*G207)+1.)/V20
 G214=-(G155*G208+G156*G1/W11+G158*G2/W22)
 G215=  -(G155*G209+G156*G1*H1/W11+G157*G1+G158*G2*H2/W22+G159*G2)
 G216=-(G155*G210-G156*G5/W11+G158*G5/W22)
 G217=-(G155*G211-G156*G5*H1/W11-G157*G5+G158*G5*H2/W22+G159*G5)
 G218=-(G155*G212-G156*G6+G157*G3-G18*G7+G159*G4+G160)
 G219=-G155*G213
 G220=-(G194*G208+G184*G1/W11+G186*G2/W22)
 G221=-(G194*G209+G184*G1*H1/W11+G185*G1+G186*G2*H2/W22+G187*G2)
 G222=-(G194*G210-G184*G5/W11+G186*G5/W22)
 G223=-(G194*G211-G184*G5*H1/W11-G185*G5+G186*H2*G5/W22+G187*G5)
 G224=-(G194*G212-G184*G6+G185*G3-G186*G7+G187*G4+W*A1*SIN(T))
 G225=-G194*G213
 G226=-BK1*(G183*G208+C1*G184*G1/W11+C2*G186*G2/W22)
 G227=-BK1*(G183*G209+C1*G184*G1*H1/W11+C1*G185*G1+C2*G186*G2*H2/W2
C2+C2*G187*G2)
 G228=-BK1*(G183*G210-C1*G184*G5/W11+C2*G186*G5/W22)
 G229=-BK1*(G183*G211-C1*G184*H1*G5/W11-C1*G185*G5+C2*G186*H2*G5/W2
C2+C2*G187*G5)
 G230=-BK1*(G183*G212-C1*G184*G6+C1*G185*G3-C2*G186*G7+C2*G187*G4
C+W*A2*SIN(T+RH))
 G231=-BK1*G183*G213+BK2
 G232=BK3*G183*G213-BK4
 EQUIVALENCE(Q(1,1),G220),(Q(1,2),G221),(Q(1,3),G222),(Q(1,4),G223)
C,(Q(1,5),G224),(Q(1,6),G225),(Q(2,1),G214),(Q(2,2),G215),
C     (Q(2,4),G217),(Q(2,5),G218),(Q(2,6),G219),(Q(3,1),G226),
C(Q(3,2),G227),(Q(3,3),G228),(Q(3,4),G229),(Q(3,5),G230),(Q(3,6),
CG231),(Q(5,1),G208),(Q(5,2),G209),(Q(5,3),G210),(Q(5,4),G211)
C,(Q(5,6),G213),(Q(2,3),G216)
 Q(5,5)=1.+G212
 Q(4,1)=G202/G207
 Q(4,2)=G203/G207
 Q(4,3)=G204/G207
 Q(4,4)=G205/G207
 Q(4,5)=G206/G207
 Q(4,6)=-G161*3.143/(V20*G207)
 Q(6,1)=-BK3*G226/BK1
 Q(6,2)=-BK3*G227/BK1
 Q(6,3)=-BK3*G228/BK1
 Q(6,4)=-BK3*G229/BK1
 Q(6,5)=-BK3*G230/BK1
 Q(6,6)=G232

  DETERMINE THE EIGEN VALUES BY THE SUBROUTINE RUTI

 CALL RUTI(Q,6,20,Z,Y,0.0001)

  CHECK IF ALL THE EIGEN VALUES OF STABILITY MATRIX ARE LESS THAN
  UNITY

 DO 75 L=1,6
 AMO=SQRT(Z(L)**2+Y(L)**2)
 IF(AMO.GT.1.) GO TO 220
 CONTINUE
 WRITE(6,3) CONS,AMPR1,AMPR2
 GO TO 110
0 WRITE(6,2) CONS,AMPR1,AMPR2
 GO TO 110
```

```
51     WRITE(6,8)
       GO TO 110
110    CONTINUE
210    CONTINUE
C
C        PRINT THE RESULTS
C
1      FORMAT(7F10.6)
2      FORMAT(5X,12HGAP  RATIO =,F5.2,19HAMPLITUDE RATIO 1 =,F5.2,
     C19HAMPLITUDE RATIO 2 =,F5.2,8HUNSTABLE  //)
3      FORMAT(5X,12HGAP  RATIO =,F5.2,19HAMPLITUDE RATIO 1 =,F5.2,
     C19HAMPLITUDE RATIO 2 =,F5.2,6HSTABLE //)
4      FORMAT(1X,11F10.6)
5      FORMAT(10X,23HFIRST NATURAL FREQUENCY  //)
6      FORMAT(10X,24HSECOND NATURAL FREQUENCY  //)
7      FORMAT(10X,12HMASS RATIO =,F5.2 //)
     8 FORMAT(10X,18HMATRIX IS SINGULAR    //)
9      FORMAT(10X,15HUNSTEADY MOTION  //)
190    CALL EXIT
       END
$ENTRY
20.       10.       5.        0.05      0.36      50.       1.
```