EFFECT OF RANDOM-NOISE ON NON-LINEAR OSCILLATOR

EFFECT OF RANDOM-NOISE ON NON-LINEAR OSCILLATOR

By

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SCOPE AND CONTENTS: The thesis deals with the effect of external signals including random noise on nonlinear oscillator. By using a negative resistance across a L-C tuned circuit an oscillator representing van der Pol differential equation is obtained. A practical circuit of oscillator whose nonlinear characteristic closely approximates to that of the ideal van der Pol oscillator is described. The behaviour of this oscillator circuit is experimentally studied while the corresponding nonlinear differential equation is solved by means of a Digital Computer (IBM 7040). The study is divided into three important categories (1) system without disturbance (free-running oscillator) (2) system subjected to sinusoidal input (forced oscillator) (3) system subjected to random noise input (noise-perturbed oscillator).

In case of forced oscillations a necessary condition is established for locking which is useful from design point of view. For noise-perturbed oscillator the mean deviation of oscillator frequency from stable frequency of noiseless oscillator is found to depend on the Q of the tuned circuit and on the magnitude of injected noise. In all the above cases the results of numerical analysis by Digital Computer are in good agreement with the experimental results.

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INTRODUCTION

When a periodic force is applied to a system whose free oscillation is of the self excited type, a phenomenon known as frequency entrainment takes place. A typical case is the system governed by van der Pol's equation with an additional term for periodic excitation (1). The frequency of the self excited oscillation falls in synchronism with the driving frequency, provided these two frequencies are not far different. If their difference is large enough, the occurrence of a beat oscillation may be expected. However, a similar phenomenon of frequency entrainment still occurs when the ratio between the natural frequency of the self excited oscillation and the driving frequency is in the neighbourhood of an integer (other than unity) or a fraction.

The characteristics of the van der Pol oscillator subjected to a forcing signal have engaged the attention of several workers (2), (3), (4) because of its wide applications (5). The understanding of such a self-oscillator circuit is considerably complicated if the presence of random noise is taken into account as a forcing function. In this connection a variety of approaches have been taken, and results of undoubted value have been obtained. However, most of the publications (6), (7), (8) concern themselves with the response of a non-linear oscillator subjected to an injected synchronizing signal accompanied by noise or some interfering small amplitude signal.

The above approach is considerably different from that of the present work in which the forcing function is random noise of narrow band-width.

The response of a self-oscillator to random noise, is becoming of particular interest owing to the recent importance of monochromatic oscillators (9), (10), (11), (12) and the possibility of reducing oscillator noise by oscillator interaction (13), (14). In order to explore this problem further, an experimental system was used. This system included an oscillator circuit whose behaviour is very close to satisfying that of the "van der Pol negative resistance oscillator". Such an oscillator is simple to design and has stable characteristics. Over a long period of time the experimental oscillator was found to be very stable with respect to frequency.

To study the effect of noise on the nonlinear oscillator experimentally, an external source of random noise was used. By using the external noise source, the oscillator circuit was made so "noisy" that the effect of noise on the nonlinear oscillator could be recorded easily with the usual laboratory instruments. The choice of 240 kc/s frequency for the oscillator was also based on noise considerations. Thus, in the present work the low frequency noise which modulates the oscillator output is assumed to be negligible and only the effects of narrow band Gaussian noise (at or near the oscillator frequency) on the self-oscillator, are considered.

The first chapter of the thesis deals with the well known cases of the free running oscillator and then the forced oscillator with externally injected sinusoidal wave.

A great amount of work has been done regarding forced oscillations in a van der Pol type of nonlinear oscillator, yet no previous treatment seems to adequately describe the conditions of locking range in terms of useful circuit parameters. Van der Pol in his original article (15) and Van der Ziel (7) give similar conditions, but they do not seem to be sufficiently useful from a practical design point of view. A necessary condition for locking just to occur in forced oscillations is derived which eventually leads to an expression for the required amplitude of the injected voltage for boundary condition of locking. This gives a more useful relation of locking in terms of circuit parameters and is a simplification of a result previously obtained (16a).

A literature survey on the noise-perturbed oscillator is summarized in Chapter 2. The statistical effects of noise on oscillators are briefly described. A relation is developed for the number of zero crossings per unit time of the noise-perturbed oscillator output, a relation which must be understood in the probability sense.

It is to be emphasized that the above noise theory does not bear a direct relationship to the experimental and numerical analysis approach adopted for the problem. In fact, the theoretical approach to the problem of frequency deviation of noise-perturbed oscillator with respect to noiseless oscillator over a long time interval is quite complicated and no work has been done previously to predict this frequency deviation by theoretical analysis.

Chapter 3 describes the experimental system, beginning with the analysis and design considerations of the negative resistance oscillator circuit. This system was used to study the van der Pol nonlinear oscillator under various conditions of operation.

In Chapter 4 the results obtained for the experimental system are presented and discussed. A comparison is made with results obtained from a numerical analysis study of the van der Pol nonlinear differential equation. With the aid of a digital computer (IBM 7040), the equation was solved for the cases of: free oscillations, sinusoidal forcing function, and random noise forcing function. In both the experimental and computer studies the average number of zero crossings per unit time (twice the mean frequency) of the oscillator output voltage for the noise-perturbed oscillator and the noiseless unforced oscillator are compared and found to be in good agreement. Results are obtained for the deviation of the mean frequency of the oscillator as a function of the Q of the tuned circuit and the magnitude of the injected noise.

In summary, the thesis contains an investigation of oscillators whose behaviour is described by the van der Pol non-linear differential equation under the following operating conditions: (1) Free-running (No forcing function, (2) Sinusoidal forcing function (Locking condition) and (3) Random noise forcing function (Noise-perturbed oscillator).

One of the main considerations of the thesis is to find the deviation of the mean frequency of the noise-perturbed oscillator with respect to that of the "noiseless" free-running oscillator.

CHAPTER 1 - FREE AND FORCED OSCILLATIONS

1.1 DIFFERENTIAL EQUATIONS - OSCILLATORY MOTION

Consider an equation of the form

$$\ddot{x} + a\dot{x} + bx = f(t) \quad \text{where } \dot{x} = \frac{dx}{dt}$$

Three types of motion are represented by this equation. Free motion (associated with bx, which gives the restoring force), damped motion (associated with $a\dot{x}$, which gives the damping force) and forced motion (associated with f(t)). These facts are easily verified by considering representative physical systems and writing the differential equations. The left side is called the equation of motion of free oscillations, in which the natural frequencies of the undamped oscillation is obtained by putting a = o. For example, a coil spring with one end fixed and a weight attached to the other end would oscillate freely once it was started from equilibrium, However, the presence of air acts as a damping force which gradually diminishes the amplitude of oscillation. If the support of the spring were to move up and down according to some law, then the motion of the spring would be <u>forced</u>. If f(t) = o, the oscillations described by the equation are said to be <u>free</u>. The general equation of free oscillations is

$$\ddot{x} + w^2 x = g(x,\dot{x})$$

It includes both nonlinear damping and restoring forces. In many cases in which the nonlinearity occurs, deviations from the linearity are small, and the motion has approximately linear character. In a self-excited oscillation, as $t \to \infty$ every solution tends to a periodic solution corresponding to a limit cycle in phase plane.

1.2 THEORY OF NEGATIVE RESISTANCE OSCILLATOR

The principal parts of a negative resistance oscillator are (1) a negative resistance circuit or device, and (2) a connected load circuit. The nonlinear component is represented by an equivalent circuit consisting of a variable resistance R_N, which can vary both in magnitude and sign, in parallel with a current source i(t). This current source supplies the initial excitation necessary to produce the oscillation and can be disconnected when oscillation commences. In an actual circuit this current source might represent a current impulse caused by thermal agitation or some other circuit unbalance of transient character.

The integrodifferential equation for the circuit of Figure 1 is

$$C \frac{de}{dt} + \left[\frac{1}{R_L} + \frac{1}{R_N}\right] e + \frac{1}{L}\int e \, dt = i(t) \qquad (1)$$

If R_N is assumed to be linear for a short time during initiation of oscillation, then during this time R_N can be considered constant and thus the following analysis could be applied.

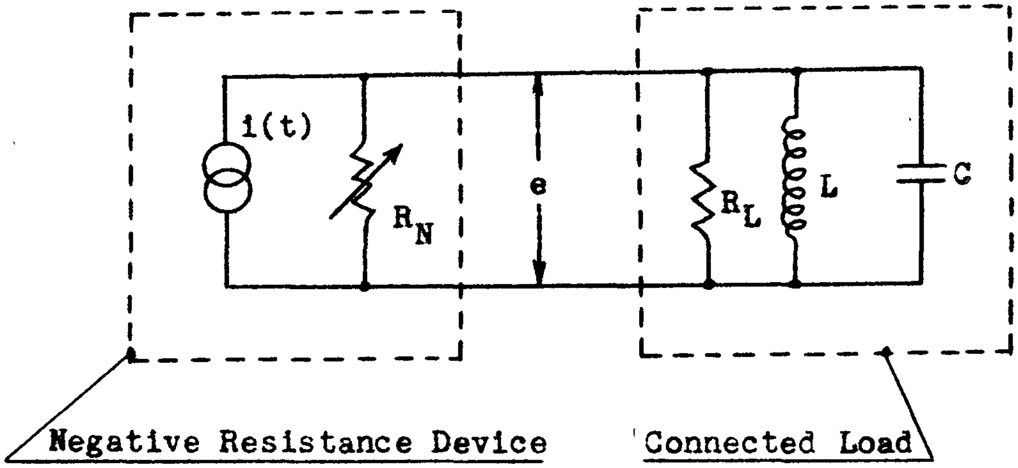


FIGURE 1 - ESSENTIAL COMPONENTS OF A NEGATIVE RESISTANCE OSCILLATOR

Assume that the capacitor is initially uncharged and that there is no initial current through the coil. Writing the Laplace transform of equation (1) and solving the result for the transform response voltage E(s) we have,

$$E(s) = \frac{I(s)}{C}\left[\frac{1}{(s^2 + (R_L + R_N)s/R_L R_N C + 1/LC)}\right] \tag{2}$$

So that the characteristic equation is

$$\left[s^2 + (R_L + R_N).s/R_L R_N.C + \frac{1}{LC}\right] = 0 \tag{3}$$

The roots of this equation are the poles of the response transform, and these roots are computed from the quadratic formulae to be,

$$S_{1,2} = \frac{-1}{2C}\,\frac{(R_L+R_N)}{R_L R_N} \pm j\sqrt{\frac{1}{LC} - \left[\frac{1}{2C}\right]^2 \left\{\frac{R_L+R_N}{R_L R_N}\right\}^2} \qquad (4)$$

$$= \alpha \pm j\omega$$

The poles of the response transform will be complex conjugates and the time response will be oscillatory only if the quantity under the radical is positive i.e. ω must be real. This is a necessary condition that must be fulfilled if oscillations of any type are to be produced.

There is another important condition. If the real part of the pole is positive, the amplitude of oscillation will increase as a function of time. If the real part is negative, the amplitude of oscillation would decay with time. A constant amplitude results when the real part of the pole is zero.

With the foregoing in mind we can see that the oscillation is produced only if

$$\left[\frac{1}{2C}\right]^2 \left[\frac{R_L+R_N}{R_L R_N}\right]^2 < \frac{1}{LC}$$

$$\text{or } \frac{4C}{L} > \left[\frac{1}{R_L} + \frac{1}{R_N}\right]^2 \qquad (5)$$

This inequality must be true at all times. The boundaries describing the inequality of equation (5) are two straight lines, which satisfy

$$\frac{-1}{R_L} + \sqrt{\frac{4C}{L}} \geq \frac{1}{R_N} \geq \frac{-1}{R_L} - \sqrt{\frac{4C}{L}} \qquad (6)$$

These regions are shown in Figure 2.

When the oscillation first starts out, it is small and must build up in amplitude. The real part of the pole must be positive so that,

$$-\left[\frac{R_L+R_N}{R_L R_N}\right] > 0$$

This condition can be achieved only if R_N is negative. That is, if $R_N = -|R_N|$, the inequality can be maintained if $R_L > |R_N|$. During the build up period the oscillator frequency

$$\omega_o = \sqrt{\left(\frac{1}{LC}\right) - \left[\frac{1}{2C}\right]^2 \left[\frac{R_L + R_N}{R_L\ R_N}\right]^2} \qquad (7)$$

will vary because R_N will change as a result of nonlinearity in the negative resistance circuit.

In Figure 2 we are interested in only that portion of the plane below the $\frac{1}{R_N} = \frac{-1}{R_L}$ line, because it is here that a self excited signal is generated, and we are primarily concerned with the narrow segment defining the sinusoidal oscillatory response.

To obtain an oscillation of constant amplitude the real part of the pole must vanish. This will occur only if $R_L = |R_N|$

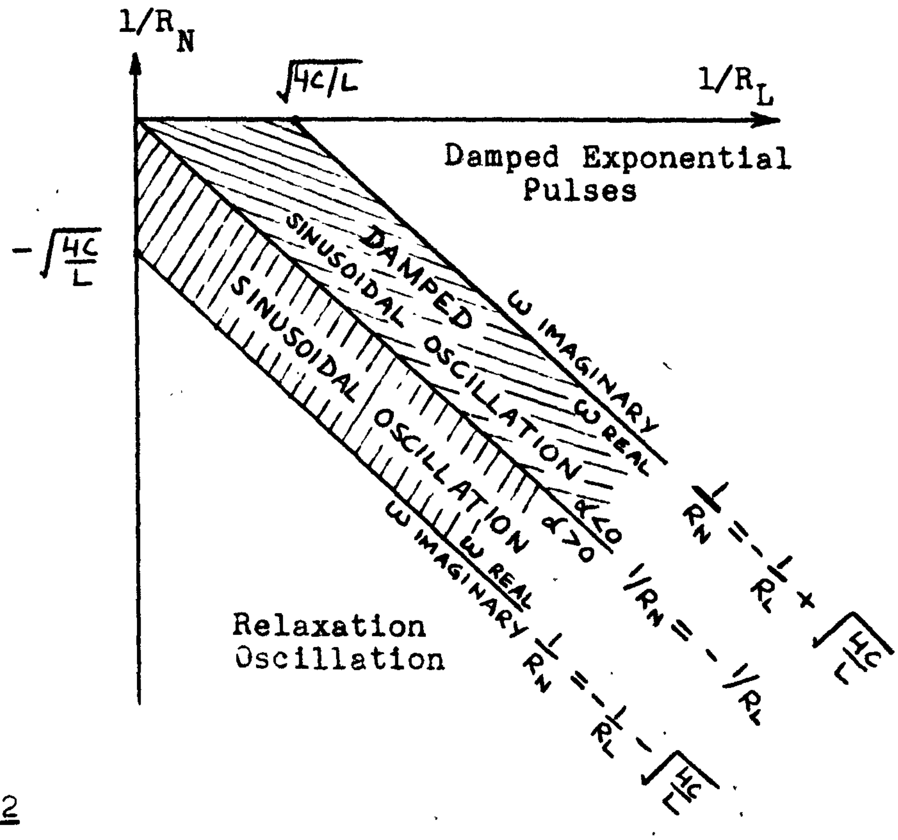


FIGURE 2

i.e. when the negative conductance of the active element exactly cancels the positive conductance of the tuned circuit. The purely imaginary poles are located at

$$S_{1,2} = \pm j \frac{1}{\sqrt{LC}}$$

When this transpires, the oscillator frequency becomes constant at $w_o = \sqrt{\frac{1}{LC}}$.

This value is governed by the constants of the load circuit if the negative resistance device or circuit is free from inductive and capacitance components.

1.3 NEGATIVE RESISTANCE CHARACTERISTIC

Consider a simple self-oscillatory circuit of Figure 3. L and C are the inductance and capacitance of a parallel tuned circuit, the total losses of which have been combined in the single parallel resistance R_L. Details of any steady voltage supply are omitted here.

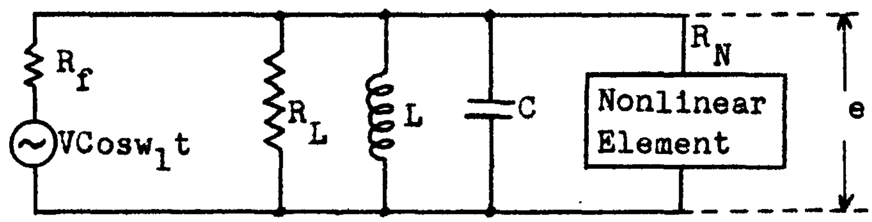


FIGURE 3 - CIRCUIT OF NEGATIVE RESISTANCE OSCILLATOR

Such a self-oscillating system has been studied in considerable detail originally by Van der Pol (1) and by many others. It has been shown that the waveform is essentially sinusoidal. A steady state is established corresponding to the appearance of a limit cycle with the amplitude of oscillation dependent solely upon the circuit parameters.

In parallel with the tuned circuit is a sinusoidal voltage generator in series with a resistance R_f. This external generator is the source of forcing voltage V to the self-oscillator. Also in parallel with the tuned circuit is R_N which represents the net effect of resistance in the system. This resistance is negative for small voltages, but becomes positive for large voltages as illustrated in Figure 4.

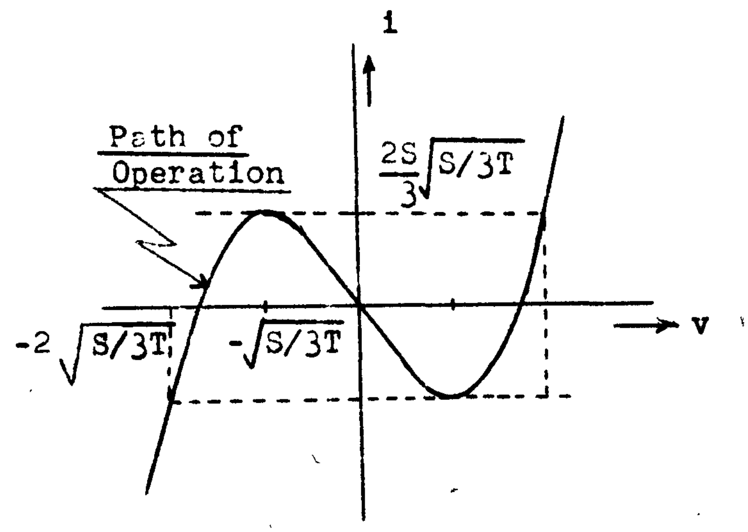


FIGURE 4 - PATH OF OPERATION ON NEGATIVE RESISTANCE CHARACTERISTIC

This negative resistance characteristic is described by the following relation.

$$i = -Se + Te^3 \qquad (8)$$

Here i = current through the element

e = voltage across the element

and S and T are positive constants.

These constants are determined by the negative resistance characteristic of Figure 4. The slope of the characteristic is zero,

$$\frac{di}{de} = 0 \quad \text{at } e_p = \pm\left(\frac{S}{3T}\right)^{\frac{1}{2}} \qquad (9)$$

At these points, from equation (8), the current is

$$i_p = \pm\frac{2S}{3}\sqrt{\frac{S}{3T}} \qquad (10)$$

Thus, from (9) and (10) the two constants S and T could be found out by knowing the geometry of the characteristic curve.

1.4 NON-LINEAR DIFFERENTIAL EQUATION OF OSCILLATOR

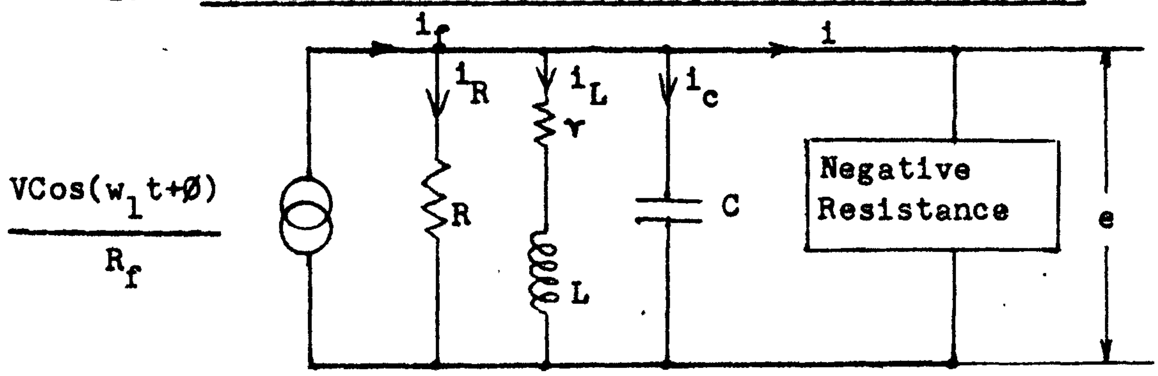


FIGURE 5 - EQUIVALENT CIRCUIT FOR NEGATIVE-RESISTANCE OSCILLATOR

(Refer Fig. 3 - Equivalent Circuit Obtained using Nortan's Theorem)

The differential equation for the circuit of Figure 5 is easily found from the condition

$$i_c + i + i_L + i_R = i_f \tag{11}$$

where, $i_c = C\frac{de}{dt}$, $\quad i = -Se + Te^3$, $\quad i_R = \frac{e}{R}$, $\quad R = \frac{R_f R_L}{(R_f + R_L)}$

$$\left[\frac{L di_L}{dt} + r i_L\right] = e, \qquad i_f = \frac{V}{R_f}\cos(w_1 t + \emptyset)$$

Here V/R_f is the amplitude of the driving current, its angular frequency is w_1, and angle $\emptyset$ is a phase angle inserted in the expression for the current as a matter of convenience.

After substitution in (11) we have

$$C\frac{de}{dt} - Se + Te^3 + \frac{1}{L}\int e\,dt + \frac{r}{L}\int (i_c + i + i_R - i_f)\,dt + \frac{e}{R} = \left[\frac{V}{R_f}\right]\cos(w_1 t + \emptyset)$$

Upon differentiating the above to remove the integral, the equation becomes

$$\ddot{e} - \frac{S}{C}\dot{e} + \frac{3T}{C}e^2\dot{e} + \frac{e}{LC} + \left[\frac{r}{LC}\right]\left[C\dot{e} + \frac{e}{R} - Se + Te^3 - \frac{V}{R_f}\operatorname{Cos}(w\ t + \emptyset)\right]$$

$$+ \frac{\dot{e}}{RC} = -\left[\frac{Vw_1}{CR_f}\right]\operatorname{Sin}(w_1 t + \emptyset)$$

Under most conditions of interest for an oscillator circuit, the resistance r is small enough so that the last term with the parentheses can be neglected and we have

$$\ddot{e} - \left[\frac{S}{C} - \frac{1}{RC}\right]\dot{e} + \frac{3T}{C}e^2\dot{e} + \frac{e}{LC} = -\left[\frac{Vw_1}{CR_f}\right]\operatorname{Sin}(w_1 t + \emptyset)$$

Thus, the nonlinear differential equation becomes,

$$\boxed{\ddot{e} - A(1 - Be^2)w_o\dot{e} + w_o^2 e = -\left[\frac{Iw_1}{C}\right]\operatorname{Sin}(w_1 t + \emptyset)} \qquad (12)$$

where $A = \frac{\left[S - \frac{1}{R}\right]}{Cw_o}$, $B = \frac{3T}{\left[S - \frac{1}{R}\right]}$, and $AB = \frac{3T}{Cw_o}$

$w_o^2 = \frac{1}{LC}$, and $R = \frac{R_f R_L}{(R_f + R_L)}$

1.5 <u>FREE OSCILLATIONS (FORCING CURRENT IS ZERO)</u>

The first order steady state solution of the non-linear differential equation (12) without forcing function is given by the perturbation method (16b) as follows,

$$e = E\operatorname{Cos}w_o t + \frac{AE}{8}(3\operatorname{Sin}w_o t - \operatorname{Sin}3w_o t) \qquad (13)$$

where, $A = \left[S - \frac{1}{R}\right]/Cw_o$

when $E = \frac{2}{\sqrt{B}}$ -------------- (14)

This particular condition has been dealt in detail by van der Pol for values of $1 \leq A < 1$. The period of sinusoidal oscillation existing for A<<1 is $T_o = 2\pi/w_o = 2\pi(LC)^{\frac{1}{2}}$. It depends upon the two reactive elements L and C and is independent of resistance.

For the oscillation to occur it is necessary that A be positive, which leads to the condition that $R \geq \frac{1}{S}$ (this has been established earlier also). For a parallel resonant circuit if we define $Q = \frac{R}{w_o L} = Rw_o C$ (15)

Then the condition for oscillation becomes

$$\frac{1}{S} \leq R \text{ or, } \frac{1}{S} \leq Q\sqrt{\frac{L}{C}}$$

Therefore $Q \geq \frac{1}{S}\sqrt{\frac{C}{L}}$

1.6 FORCED OSCILLATIONS (FORCING CURRENT IS NOT ZERO)

A somewhat different situation exists if the same oscillator is driven from an external source. Free running oscillators which are capable of being synchronized to an external signal are, of course, well known and extensively used. The synchronized oscillator is realized when the oscillation of desired frequency is injected into the free oscillator. The amplitude and frequency of the external driving voltage must be such as to quench the free oscillations, the quenching action being obtained through a zero memory type nonlinearity which attenuates the weaker signal more than the stronger. The analytical solution of such a system gives a second order nonlinear differential equation having a forcing term on the right side. Under certain conditions both free and forced oscillations exist simultaneously. When parameter A is small compared with unity, free and forced oscillations would be sinusoidal in nature. Analysis uses the principal of harmonic balance (16c). An approximate solution of the equation (12) may be expected to have the form

$$e = E\,\mathrm{Cos}wt + E_1\mathrm{Cos}w_1t \tag{16}$$

Where E is the peak amplitude output of free oscillation of frequency w and E_1 that of the forced oscillation at the driving frequency w_1.

The Following four relations result in accordance with the principle of harmonic balance. All other frequencies except the two fundamental frequencies w and w_1 are neglected and also it is assumed that $w \neq w_1$ (Appendix II)

$$E(w_o^2 - w^2) = 0 \tag{17}$$

$$Aww_oE(1 - \frac{B}{4}(E^2 + 2E_1^2)) = 0 \tag{18}$$

$$E_1(w_o^2 - w_1^2) = -(\frac{Iw_1}{C})\,\mathrm{Sin}\phi \tag{19}$$

$$Aw_1w_oE_1(1 - \frac{B}{4}(E_1^2 + 2E^2)) = -(\frac{Iw_1}{C})\,\mathrm{Cos}\phi \tag{20}$$

From (17) it is evident that the frequency of free oscillation is same as w_o. It could also be shown easily from (18) that if there are no forced oscillations, $E_1 = 0$ and $E = \frac{2}{\sqrt{B}}$ which is the result found in (14) for free oscillation case. The same equation could be rewritten $E^2 = \frac{4}{B} - 2E_1^2$. It is, thus, obvious that the output amplitude of forced oscillations would be given by

$$E_1 = \sqrt{\frac{2}{B}} \qquad (21)$$

subject to the condition that free oscillation amplitude is completely quenched i. e. $E = 0$.

Free and forced oscillations exist simultaneously if the output amplitude of the forced oscillation is within the limits set by $0 \leq E_1^2 \leq \frac{2}{B}$. Also when forced oscillation exists of amplitude large enough so that $E_1 \geq \sqrt{\frac{2}{B}}$, free oscillations are completely supressed and $E = 0$. It should be pointed out that the imaginary value of E (with large value of E_1) is interpreted to mean as if there is no free oscillation. Therefore it could be verified that under certain given conditions (locking) a relation

$$\boxed{E = \sqrt{2} \times \left[E_1 \right]} \qquad (22)$$

would exist between free and forced oscillations.

Detailed normalized response curves for the separate and simultaneous existance of both free and forced oscillations are given in Figure 7.30 pp 218 of Cunningham (16d).

1.7 BOUNDARY CONDITION FOR LOCKING

From the design point of view an engineer is interested in finding out some simplified relation between the injected voltage V with that of the circuit parameters for boundary condition of locking. This necessary condition for locking or pulling just to occur could be determined as follows.

For small amount of variation of w_1 from w_o

(i.e. $w_1 - w_o \ll w_o$)

$$w_o^2 - w_1^2 \cong (w_o - w_1)\, 2w_o \tag{23}$$

Also on the boundary condition of locking, $E_1 = \sqrt{2/B}$ and E = 0. Squaring and adding equations (19) and (20) results in

$$\left[(w_o^2 - w_1^2)^2 + (Aw_1w_o)^2\left[1 - \frac{BE_1^2}{4}\right]^2\right] = \left(\frac{Iw_1}{C}\right)^2 \frac{1}{E_1^2}$$

or

$$\left[(w_o^2 - w_1^2)^2 + A^2w_1^2w_o^2 . \frac{1}{4}\right] = \left(\frac{Iw_1}{C}\right)^2 \times \frac{B}{2}$$

or

$$\left[\frac{(w_o - w_1)^2}{w_o^2} \times \frac{(w_o + w_1)^2}{w_1^2} + \frac{A^2}{4}\right] = \frac{V^2}{R_f^2} \times \frac{1}{C^2} \times \frac{B}{2w_o^2}$$

This reduces to boundary condition for locking just to occur as follows

$$\left[4D^2 + \frac{A^2}{4}\right] = \frac{V^2}{R_f^{\,2}} \times \frac{B}{2C/L}$$

where $D = \frac{w_o - w_1}{w_o}$ = fractional variation in frequency

$$\boxed{\left[D^2 + \frac{A^2}{16}\right] = \left[\frac{BV^2}{8} \cdot \frac{L/C}{R_f^{\,2}}\right]} \qquad (24)$$

If Q of the tuned circuit is high then R_f could be made much less than R_L. In such cases $R_f \simeq R$ and $\frac{L/C}{R_f^{\,2}} = \frac{1}{Q^2}$ and (24) could be reduced to

$$\left[D^2 + \frac{A^2}{16}\right] = \left[\frac{BV^2}{8} \times \frac{1}{Q^2}\right]$$

This gives the input amplitude of forcing voltage required for locking just to occur as

$$\boxed{V = \left[\frac{8Q^2}{B} \left(D^2 + \frac{A^2}{16}\right)\right]^{\frac{1}{2}}} \qquad (25)$$

From (25) it is obvious that all other constants being same an increase of Q (or R_f) necessitates an increase in the input amplitude of the forcing voltage if locking is to be maintained. And the above relation is, indeed, the necessary [illegible] boundary condition for locking just to occur in the case of forced oscillations.

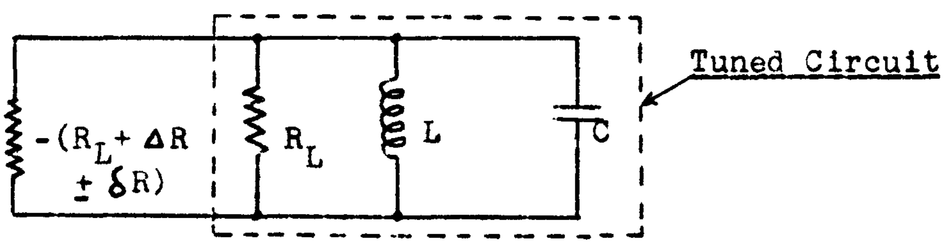
CHAPTER 2 - NOISE PERTURBED OSCILLATOR

2.1 STABILITY OF THE NEGATIVE RESISTANCE OSCILLATOR

No oscillator generates a pure Sine Wave. Noise present in the circuit always introduces random phase changes, so that the output power is spread over a narrow frequency band, and not concentrated at a single frequency. Consequently all oscillators will have orderly noise spectra. Extremely stable oscillators will have extremely narrow noise power spectrums which, for highly stable frequency standards, may be only a few hundreths of a cycle wide.

The stability of a negative resistance oscillator or the precise pole positions with steady state oscillation established but with noise still present, is examined by circuit analysis as follows.

FIGURE 6(a) - EQUIVALENT CIRCUIT OF NEGATIVE RESISTANCE OSCILLATOR WITH NOISE PRESENT



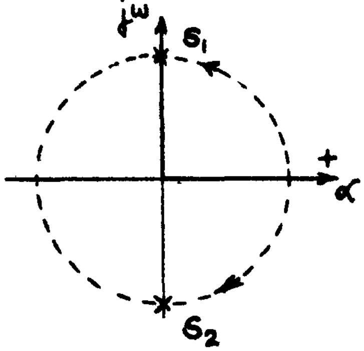


FIGURE 6(b) - POLE POSITIONS FOR STEADY STATE OSCILLATIONS

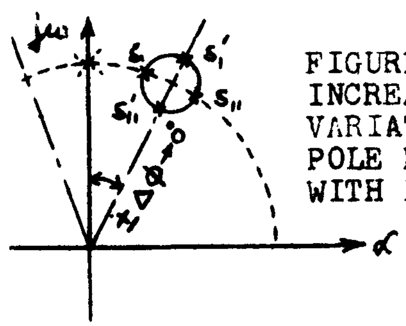


FIGURE 6(c) - INCREAMENTAL VARIATION IN POLE POSITION WITH NOISE IN OSCILLATOR

In Figure 6(a) a negative resistance ($R_L + \Delta R \pm \delta R$) is introduced that is fractionally larger than R_L, the sum of the positive resistances in the tuned circuit, to ensure the continuous oscillations. In this negative resistance the noise is represented by random variation $\pm \delta R$. The poles of the response transform for the negative resistance oscillator could now be expressed as follows.

$$S_{1,2} = -\frac{1}{2C}\frac{(\Delta R \pm \delta R)}{R_L^2} \pm j\sqrt{\frac{1}{LC} - \left(\frac{1}{2C}\right)^2 \left[\frac{\Delta R \pm \delta R}{R_L}\right]^2}$$

To meet the Barkhausen criterion, ($\Delta R \pm \delta R$) must become zero, making the oscillator frequency $\omega_0 = \sqrt{\frac{1}{LC}}$. However, the presence of the noise component $\pm \delta R$, which includes all circuit perturbations, requires that a small negative resistance component ΔR be present at all times to ensure sustained oscillations. As the equation indicates, perturbations $\pm \delta R$ in ΔR create both frequency and amplitude jitter with a corresponding jitter in S_1 and S_2.

The oscillator thus settles in a position of unbalanced equilibrium, as shown by the pole positions S_1, S_{11} S_1', S_{11}' in Figure 6(c). The angle between the $j\omega$ axis and the mean pole position $\Delta\phi \rightarrow 0^\circ$, and the jitter is represented to a greatly enlarged scale. Corresponding pole positions representing the amplitude and frequency jitter will also appear at S_2. Like the cyclist who must continually apply correction signals to the handle bars to maintain the balance, the oscillator must also continually apply correction signals to maintain equilibrium. It can easily be shown (11) that the above analysis is equally good for feedback type of oscillators as the amplifier and the feedback network can be transformed into equivalent negative resistance oscillator. (App. I)

2.2 REVIEW OF THE LITERATURE

Of the many publications related with noise (17, 18, 19) a considerable number (20, 21, 9, 10, 11, 22) concern themselves with the more special problem of noise in oscillators. The case of a self oscillator with van der Pol nonlinearity has been solved with many interesting applications by Blaquiáre (23). Rytov (8) extending the initial work of Berstein (24) has used van der Pol's technique of expressing oscillator output as an angle and amplitude modulated Sine Wave.

Edson (12) in an exhaustive paper described the noise effects on the behaviour of oscillator both during initiation of oscillator and also during sustained oscillations. During the starting of oscillation, noise constitutes the initial starting voltage and therefore affects the time required for the wave to reach pre-established amplitude. Noise also creates undesired perturbations both in amplitude and the phase of the wave during sustained oscillations. The amplitude perturbations produce a continuous spectrum which in typical situations is quite weak and broader than the bandwidth of the resonator. The phase perturbations disperse the nominal frequency into a continuous distribution which is of the same form but much stronger and narrower than for the amplitude perturbations.

An alternate method has been proposed which separates the output noise into linear and nonlinear parts but does not calculate any correlation between them (25).

Mullen in his paper has included resistive nonlinearity such that all the quantities which appear are measurable on a "black box" oscillator, at the desired operating point, and by obtaining the spectrum of the instantaneous output of the oscillator. It is shown that the noise output from noise bands around the oscillating frequency is composed of an additive noise of the shape of the oscillator resonant circuit and a very small FM broadening of the oscillator line. He further extends (26, 27) the previous treatment (12, 20, 21) of background noise in oscillators to include nonlinearities in which the frequency of oscillation is a function of amplitude of oscillation i.e. r.f. form of pushing. The inclusion of r.f form of pushing complicates the analysis considerably and includes correlation between AM noise and FM noise which not only broadens the spectrums but also produces an asymmetric spectrum.

The oscillator spectrum starting from a two port model is determined (28). Procedure is given for the evaluation of the output spectrum in the case of zero memory nonlinearity with an arbitrary narrow band filter.

In a recent work (29) on Spälti's Theory (30) of noise produced in oscillator, the power spectrum of a self excited oscillator consisting of linear elements is determined by calculation and experiments. It has been shown that at the output there appears not a single spectral line with superposed white noise, but a continuous spectrum, symmetrical with respect to the proper oscillating frequency. An oscillator for producing electrical vibrations can be imagined as a closed loop consisting of an active four pole (amplification factor A) whose output voltage is fed back to its input terminal by way of correctly phased passive four pole network (voltage transfer funtion B). If U_A is the output and U_E is the input voltage the overall amplification factor is $\frac{U_A}{U_E} = \frac{A}{(1-AB)}$.

According to the general theory of self oscillations an oscillator can build up when AB = 1. Thus U_E will be neglected, and the oscillation, once it has been started by random fluctuations is maintained. However, according to Spälti's theory, the input voltage, which consists of noise, may not be neglected. It is therefore assumed that in a small but finite frequency region Δf, (1-AB) is very small but not zero. A very large voltage amplification factor is thus achieved. In fact (1-AB) factor continually adjusts itself to the magnitude of noise voltage and also preserves the condition of stability.

It is well known that the sensitivity of any receiving system is determined ultimately by its band width and effective noise temperature. By narrowing the band width and transmitting at correspondingly low information rates the receiver sensitivity can be greatly increased. However, the minimum band width that can be achieved by this means depends upon the stability of the transmitter and receiver local oscillators. All sinusoidal oscillators have a finite spectral width determined by their phase stability. The spectrum is random in nature and the corresponding phase jitter is defined as the phase noise of the oscillator. The importance of highly phase stable oscillators for deep-space communication needs no emphasis here. Relationship between the phase noise, the spectrum, the short term stability and the Q for the oscillator is established by Malling (11).

In their recent (Nov. 1963) article Grivet and Blaquière (10) give a theoretical analysis of the effects of random noise in various types of electronic clocks. Leaving aside, the amplitude noise problems, the study concentrates on "line width" problems, and emphasizes the nonlinear theory of the thermal noise along a line proposed by Berstein (24), simplified and generalized by Blaquière (10). To eliminate the shot noise effect the tank circuit is to be connected to the grid of the oscillator. It has also been pointed out that the present study could be applied to maser clocks because the differential equation of these devices is also of the same type.

Golay (9) in his more recent (Nov. 1964) article shows that the use of the phasor concept and normalization of all the parameters, lead to relatively simple nonlinear equations of the regenerative oscillator. These equations are studied analytically and by means of an electronic computer for the three cases of free running oscillator, noise perturbed oscillator and continuous wave oscillator.

The problem of the effect of random noise on self-oscillator is, in general, quite complicated mathematically because of the random noise involved as a forcing function in the nonlinear differential equation. The nonlinearity that is responsible for the self-excited oscillations also complicates the analysis since it mixes various elements of the input noise. Although important results have been obtained (31), no complete solution of this differential equation exists at present. Partial solutions, however, do exist, (7) involving use of simplified representation of the noise in which mixing among the noise frequencies does not occur. But in the above analysis the noise voltage is represented as a single Sine Wave which in fact is not an appropriate model because, in the noise case, the continually fluctuating amplitude and phase give effects which are neglected entirely in the Sine Wave picture.

A similar problem, which is also of considerable interest in the fields of automatic feedback control system, has as yet no precise solution although results have been obtained to a considerable extent (32, 33). They involve linearization technique i.e. the replacement of nonlinear element by a linear element with an equivalent gain determined by the magnitude of the input signal and the nonlinear characteristic.

2.3 NARROW BAND NOISE

In this treatment only white noise of thermal or electronic origin is considered. The shot noise plays only a negligible role as long as one deals with thermal high frequency noise and if one chooses the right structure for the oscillator (10) (when the tank circuit of high Q is connected to the grid of the oscillator). For Q's larger than 100, the tube noise is less than 1/1000th of the tank noise and the tube may be considered as noiseless. Microphonics, element drift, low frequency abnormalities such as flicker effect are assumed to have also almost negligible effect in the present treatment.

Oscillators are characterized by relatively selective circuits and the fact that all the effects of interest are concentrated in a relatively narrow band of frequencies. Therefore, we may restrict our attention to noise within fractionally narrow band widths. The voltage which results when white noise is passed through any relatively narrow band filter may be described by

$$v(t) = x_1(t) \text{ Cos } w_o t + x_2(t) \text{ Sin } w_o t \tag{26}$$

Where w_o is the midband angular frequency and $x_1(t)$ and $x_2(t)$ are uncorrelated functions of time which vary slowly and randomly about zero in a manner described by the normal or Gaussian probability function.

2.4 PULSE ANALYSIS OF THE NOISE EXCITATION

The van der Pol differential equation (12) with noise excitation function En(t) could be written as

$$\frac{d^2e}{dt^2} - A(1 - Be^2)w_o \frac{de}{dt} + w_o^2 e = w_o^2 \, En(t) \qquad (27)$$

The noise e.m.f. En(t) consists of a great many small impulses, occuring randomly in time. No loss of generality results if pulses which occur within some finite period are collected or grouped together (34).

The average number of pulses per second N and the individual strength q of each pulse are related by Nyquist's law through the relation

$$Nq^2 = 2KTr \qquad (28)$$

Where r is the resistance of the tank circuit source of the noise, K is the ordinary Boltzmann constant, and T is the absolute temperature.

In order to derive (28) we proceed as follows: The mean square value of the noise e.m.f. is for this representation of noise (Figure 7).

$$\langle E^2(t) \rangle = N \int_0^{\tau} E_o^2 \, dt = NE_o^2 \tau = \frac{Nq^2}{\tau} \qquad (29)$$

where, $\tau = q/E_o$

On the other hand, in Nyquist's original phrasing it reads

$$\langle E^2(t) \rangle = 4KTr.\Delta f \qquad (30)$$

Δf being the noise bandwidth of the noise signal; this is no other than that of the individual pulses

$$\Delta f = 1/2\tau \tag{31}$$

and from (29), (30), (31), (28) results.

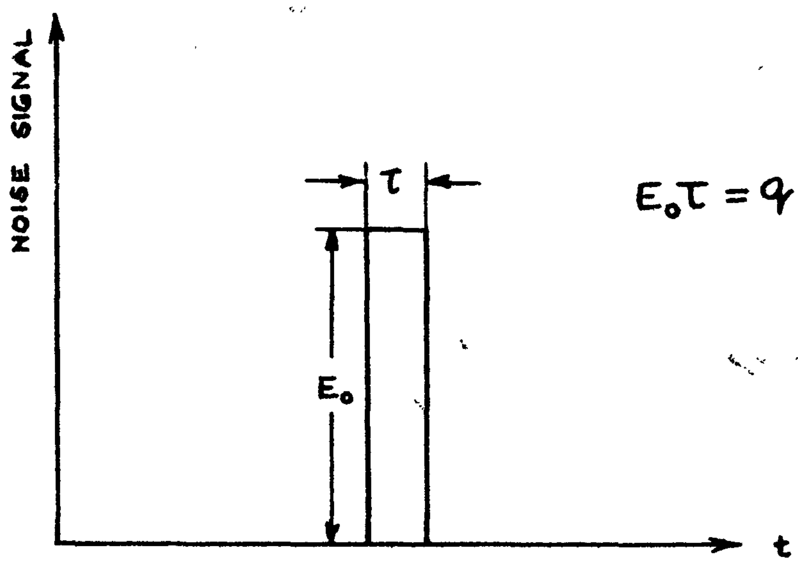


FIGURE 7 - THE STANDARD ELEMENTRY PULSE USED IN THE SYNTHESIS OF THE NYQUIST NOISE

The mechanism whereby random noise affects the self-oscillators, has been discussed in a number of sources. The following summary is based on the results of Blaquière (10) and Edson (12). A detailed account of statistical effects of noise on the oscillator is given later.

The noise impulses injected into the tank circuit of self-oscillator may be divided into two groups occuring at alternate quarter cycle of the oscillator cosine wave. Depending upon whether it occurs during the odd or even quarter cycle, an impulse produces a sinusoid which is in phase or in quadrature respectively with the original oscillation. The inphase component produces amplitude modulation, the quadrature component produces a step inphase. If no further disturbance occurs, the oscillator continues to operate at the new phase. But for large values of noise impulses, the accumulated phase error is summed and is a random walk phenomena.

The steady voltage, which one would observe in the absence of noise, would be

$$e = E \, Sin(w_o t + \emptyset) \tag{32}$$

Using this expression one can very simply calculate the effect of an elementary pulse; as is well known from the elementary theory of the ballistic galvanometer, one pulse causes a change in de/dt but no change in e. We then get $\delta \emptyset$ and δa by expanding the right side of (32) and imposing the conditions $\delta e = 0$, $\delta(de/dt) = qw_o^2$. One then gets the change δa in amplitude and $\delta \emptyset$ in phase produced by one pulse occuring at time t_j (Appendix IV)

$$\delta a = qw_o \, Cos(w_o t_j) \tag{33}$$

$$\delta \emptyset = -q \frac{w_o}{E} \, Sin(w_o t_j) \tag{34}$$

Of special interest are inphase and 90° out-of-phase pulses occuring respectively at times $t_j = 0$ and $t_j = T_o/4$ (multiples of $T_o/2$, T_o being the period $T_o = 2\pi/w_o$) Figure 8). For simplicity we call them (a) pulses and ($\emptyset$) pulses, because of their respective effects. (Refer Table No. 1).

An (a) pulse produces a change of amplitude only, and

(Ø) pulse a change in phase only. (a) and (Ø) pulses help to express the effect of general pulse in a more efficient way. Thus a general pulse occuring at time t_j is equivalent in its effect to a set of two (a) and (Ø) pulses components occuring at time 0 and $T_o/4$ and of intensity $q_Ø$ and q_a given by the right triangle of Figure 9. Going to statistical average values one has $\langle q_a^2\rangle = \langle q_Ø^2\rangle = q^2/2$

FIGURE 8 - TWO BASIC TYPE OF PULSES (a) AND (Ø)

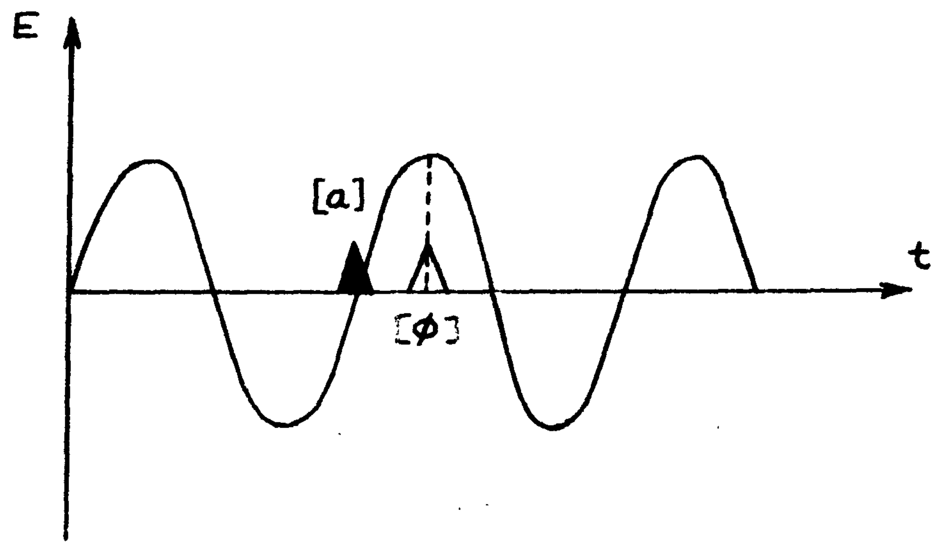


FIGURE 9 - THE USE OF THE (a) and (Ø) PULSES AS VECTOR COMPONENTS

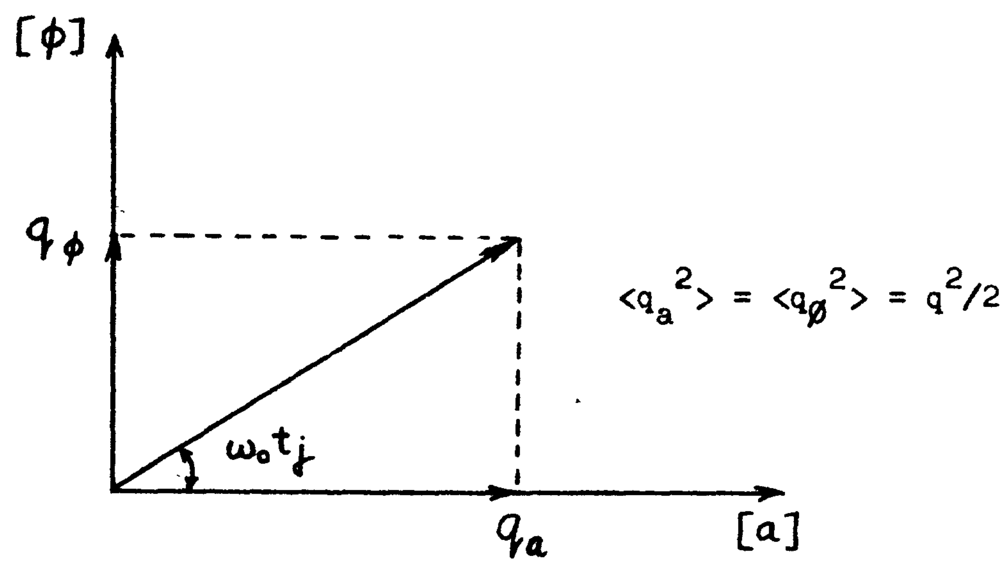


TABLE NO. 1

Time	Name	First Order Effect
$t_j = 0$ (in phase)	(a) pulse	$\begin{cases} \delta a = \epsilon q w_o \\ \delta \phi = 0 \end{cases}$
$t_j = T_o/4$ (90° out of phase i.e. quadrature)	(ϕ) pulse	$\begin{cases} \delta a = 0 \\ \delta \phi = -\epsilon q \frac{w_o}{a_o} \end{cases}$

As regards the average effect on the oscillator, we may replace the pulses of strength q occuring at random times t_j and in average number N per second by:

(1) (a) pulses occuring at time $t_a = 0 + \frac{T_o}{2}$, their individual intensity is q as for the original pulses, but their average number per second is N/2 only.

(2) (ϕ) pulses, occuring at times $t_\phi = \frac{T_o}{4} + \frac{T_o}{2}$, their strength is also q but their density is N/2 only.

2.5 STATISTICAL EFFECTS OF NOISE ON THE OSCILLATOR

2.5.1 Negligible role of (a) pulses - Campbell's theorem makes it easy to go from the individual action of one pulse to their statistical effect. As the elementary effect changes sign with the (a) pulse, the mean value of the disturbance is zero and the oscillator shows the same average amplitude in the presence of noise, or in its absence. The mean quadratic amplitude of the fluctuation is of no interest here, because it does not bear on the accurancy of frequency definition. We measure the frequency by counting the zeros of the time function: near a zero, the amplitude is very small and the fluctuation in amplitude negligible.

2.5.2 <u>Importance of (Ø) pulses</u> - In contrast to amplitude perturbations, which are counteracted by the inherent limiting action, no mechanism exists to counteract phase perturbations. The situation corresponds to a well regulated clock; if once set forward it continues to read fast until it is reset. The sine-phase noise impulses constitute a series of random clock settings, which produce much the same effect as a random deviation of rate.

The stochastic summation of the elementry phase angles is a random walk problem and after many periods the quadratic mean value $<\phi^2>$ is proportional to t (Figure 10).

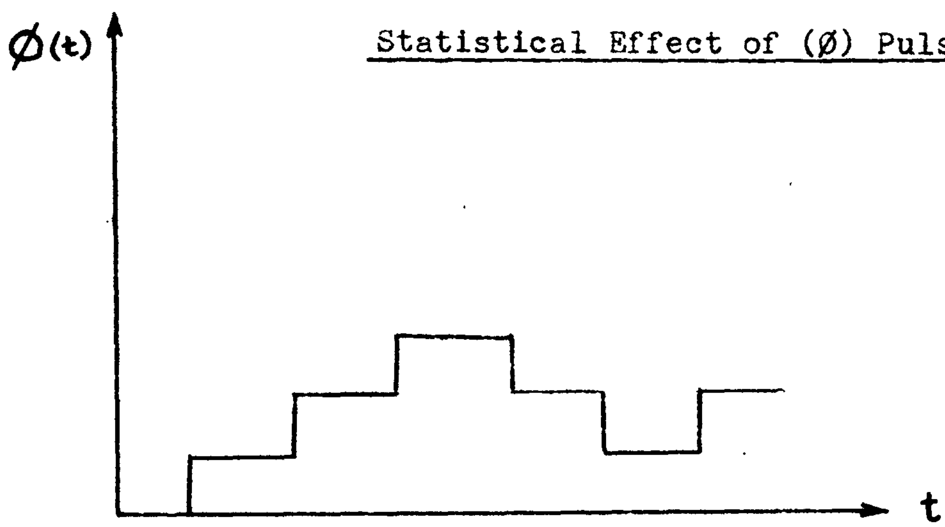


FIGURE 10 - THE RANDOM WALK PROBLEM ASSOCIATED WITH (Ø) PULSES

This results also from the Campbell's second theorem

$$\delta\phi = -\frac{qw_o}{E}$$

Therefore $$\langle\Delta\phi^2\rangle = \frac{N}{2}\int_o^t \frac{q^2 w_o^2}{E^2}\,dt = \frac{Nq^2 w_o^2}{2E^2}t \qquad (35)$$

and using Nyquists Law from (28) one gets

$$\langle\Delta\phi^2\rangle = \frac{KTrw_o^2}{E^2}\,t = Dt \qquad (36)$$

where, D is a "diffusion coefficient" for phase. It should be noted that the above relation is same as that obtained by Edson (12) in his equation (50).

2.5.3 <u>Correlation Time</u> - Choosing as time $t = 0$, that of a zero of the signal, we look at the mean value of signal, a long time t later and (Figure 11) we disregard the amplitude fluctuations. The procedure is permissible because frequency may be defined as the number of zeros of the function in one second, when observing the function near the value zero only, amplitude fluctuations are of no importance.

The expression e(t) at time t is,

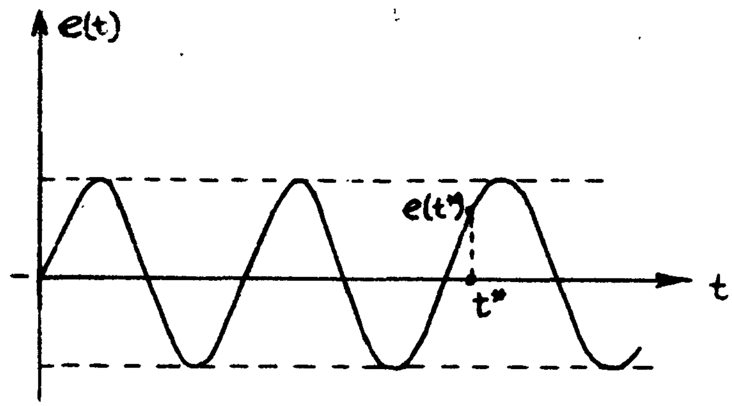
$$e(t^*) = E\,\mathrm{Sin}(w_o t^* + \Delta\phi) = E\,\mathrm{Cos}\Delta\phi\,\mathrm{Sin}\,wt^* + E\,\mathrm{Sin}\Delta\phi\,\mathrm{Cos}\,w_o t^*$$

and taking the mean value we find

$$\langle e(t^*)\rangle = E\,\langle\mathrm{Cos}\Delta\phi\rangle\,\mathrm{Sin}\,w_o t^* + E\,\langle\mathrm{Sin}\Delta\phi\rangle\,\mathrm{Cos}\,w_o t^*$$

$\langle\mathrm{Sin}\Delta\phi\rangle$ vanishes as $\langle\Delta\phi\rangle$ does. We write the first term in a more convenient form. Taking use of the approximation $\langle \cos\Delta\phi \rangle \simeq e^{-\frac{\Delta\phi^2}{2}}$ we find

$$\langle e(t^*) \rangle = E\, e^{-\frac{\Delta\phi^2}{2}} \sin w_o t^* \tag{37}$$



2.5.4 <u>Spectral Width</u> - One can represent the mean statistical effects of phase fluctuations by replacing the steady sine wave by a damped oscillation. Using (36) we have for the apparent damping factor

$$\langle e(t^*) \rangle = E\, e^{-\frac{D}{2}t^*} \sin w_o t^* \tag{38}$$

Obviously, the perturbed oscillation, when considered in the mean, behaves like a damped oscillation. This in turn results in the broadening of noise band of oscillator.

The correlation time τ_c is then

$$\tau_c = \frac{2}{D} = \frac{2E^2}{KTrw_o^2} \tag{39}$$

The total spectral width $2\delta w$ of such a damped sinusoid is well known and is expressed as

$$2\delta w = \frac{2}{\tau_c} = D$$

and we get $\delta w = KT \dfrac{\pi w_o^2}{E^2}$ (40)

It is well known that the most accurate frequency measurements are made by observing the time elapsed during a given number of zero crossings of the oscillator output voltage. Also any amplitude modulation of the output voltage due to noise will not, at least in the first order, cause any error in the observation of these zero crossings. A relation is developed to give the number of zero crossings per unit of time for a noise perturbed oscillator.

2.6 NUMBER OF ZERO CROSSINGS PER UNIT TIME OF THE OSCILLATOR OUTPUT

If the output of oscillator is a random function given by

$$e(t) = E(t) \text{ Cos } \left[w_o t + \phi(t)\right] \tag{41}$$

where, $E(t)$ and $\phi(t)$ are the envelope and the phase of the process respectively (they are random functions slowly varying in time compared with $\text{Cos } w_o t$). Since $E(t) \geq 0$ and the probability of the equality $E(t) = 0$ is small, the problem is of finding the number of zero crossings of function $\text{Cos }\left[w_o t + \phi(t)\right]$. In fact, if $\text{Cos }\left[w_o t + \phi(t)\right]$ crosses the zero level successively at instants t_1 and t_2 then,

$$w_o(t_2 - t_1) + \phi(t_2) - \phi(t_1) = \pi$$

Since $\emptyset(t)$ changes slowly compared with Cos $w_o t$, we shall replace $\emptyset(t_2) - \emptyset(t_1)$ by $(t_2 - t_1)\bar{\phi}$

$$\left[\text{Since } \frac{\partial \emptyset}{\partial t} \Delta t = \emptyset(t_2) - \emptyset(t_1) \right]$$

and then $t_2 - t_1 = \pi / (w_o + \bar{\phi})$

The number of zero crossings/unit interval of time dt is equal to

$$dN = \frac{dt}{t_2 - t_1} = \frac{(w_o + \bar{\phi})}{\pi} dt \quad .$$

After integrating the above expression for all values of time in the interval t, we obtain

$$N = \frac{w_o}{\pi} + \frac{1}{2\pi t} \int_{-t/2}^{t/2} \bar{\phi}(t)dt \tag{42}$$

where, N is the average number of crossings/unit time, $\bar{\phi}(t)$ is the derivative of phase $\emptyset(t)$ with respect to time.

Substituting from (35) in (42) we have

N = No. of Zero crossings/unit time

$$= 2f_o + \sqrt{\frac{Nq^2 w_o^2}{2\pi^2 E^2 t}} \tag{43}$$

where, t is the finite time interval

q individual strength of each pulse

E amplitude of noiseless oscillations

w_o angular frequency of free oscillations

The expression (43) must be understood in the probability sence. Also the narrower the noise bandwidth (slow variation of $\emptyset(t)$) the higher the probability that (43) is satisfied.

CHAPTER 3 - ANALYSIS AND DESIGN CONSIDERATIONS OF EXPERIMENTAL CIRCUIT

3.1 REALIZATION OF A PRACTICAL OSCILLATOR

Extensive mathematical treatment and the theoretical performance of van der Pol's nonlinear oscillating system is available in literature (36, 37, 38). In studying such a system, especially from an engineering point of view, it is desirable to have actual physical devices for demonstrating the phenomena involved. In order to realize the close approximation to cubic nonlinear characteristic of equation (8) a practical circuit of cathode coupled negative resistance oscillator is developed. Such a circuit is given in Figure 15 and is based on the circuits (39, 40, 41, 42) given earlier, so as to meet our requirements.

3.2. BASIC CIRCUIT

Negative transconductance provided by two stage amplifier gives the phase reversal necessary for circuit oscillation. Only two points in the circuit are needed for connection of a simple tuned circuit to provide oscillations of desired frequency.

Figure 12 shows the fundamental circuit. A twin triode is connected with one triode acting as a cathode follower driving the cathode of the second triode through coupling effected by the common cathode resistor R_k.

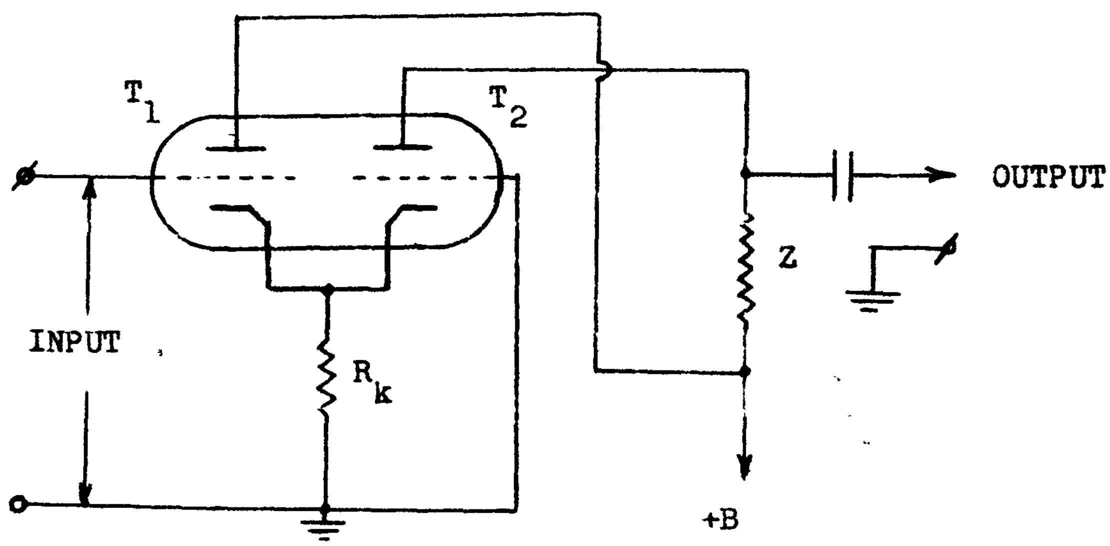


FIGURE 12 - BASIC NEGATIVE RESISTANCE OSCILLATOR CIRCUIT

A positive voltage applied to the grid of T_1 causes more current to flow in the plate circuit of T_1 and consequently more current through the common cathode resistor. The increased current through the cathode resistor raises the potential of cathodes in the positive direction.

An increased positive potential on the cathode of T_2 is equivalent to an increased negative potential on the grid of T_2. Thus a positive voltage applied to the grid of T_1 is converted, by the coupling system, to an equivalent negative potential on the grid of T_2. This phase reversal, together with the phase reversal effected between the grid and plate of T_2, forms a complete 360 degree phase rotation, so that the input is in phase with the output. Thus, for oscillations to take place a tuned circuit is required in the output plate circuit which is coupled back to the input of grid circuit of Tube T_1.

3.3 ANALYSIS OF CATHODE COUPLED NEGATIVE-RESISTANCE CIRCUIT

Consider the circuit of Figure 13(a). The experimental frequency of 240 kc/s is taken as medium frequency and is defined as that frequency at which the reactances of all capacitances (C_{gk}, C_{pk} and C_c) are neglected. Then the equivalent circuit of Figure 13(b) could be drawn.

When a voltage E is applied to the input of an amplifier whose voltage gain is $A\underline{/\theta}$, where θ is the phase angle of A, its output will be $EA\underline{/\theta}$. If the output is now connected back to the input, a single loop circuit results. It is apparent that the current i in this loop will be

$$i = \frac{E - EA\underline{/\theta}}{Z_i}$$

where Z_i is the internal impedance of amplifier in the absence of feedback.

The impedance Z seen by the source E will be

$$Z = \frac{e}{i} = \frac{Z_i}{1 - A\underline{/\theta}}$$

At medium frequencies Z_i is a pure resistance and A may have a phase angle of 0 or 180 degrees so that $A\underline{/\theta} = A\underline{/0} = A$. Z will then be the negative resistance

$$R = \frac{R_i}{1 - A\underline{/\theta}} \qquad (44)$$

where R_i is Z_i at medium frequencies and when $A > 1$, which is the case of interest here.

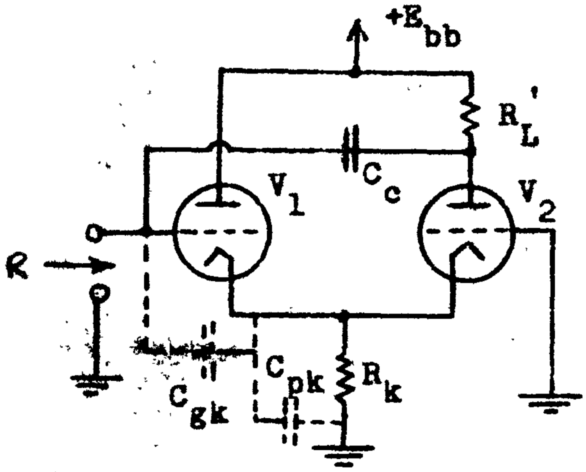


FIGURE 13(a) - NEGATIVE RESISTANCE CIRCUIT

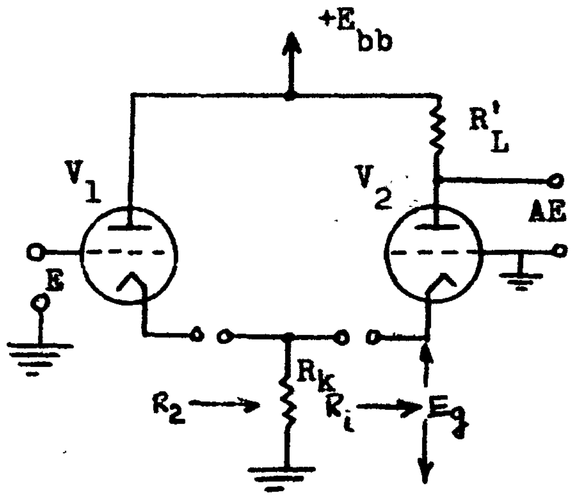


FIGURE 13(b) - EQUIVALENT CIRCUIT

The amplifier of Figure 13 (a) may be considered as a cathode follower V_1 driving a grounded grid stage V_2 through a coupling resistor R_K. The two tubes are taken as identical each having an amplification factor μ and a plate resistance r_p. The equivalent circuit may be drawn as follows.

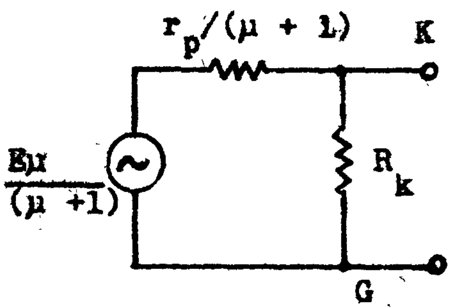


FIGURE 14(a) - EQUIVALENT CIRCUIT OF CATHODE FOLLOWER V_1

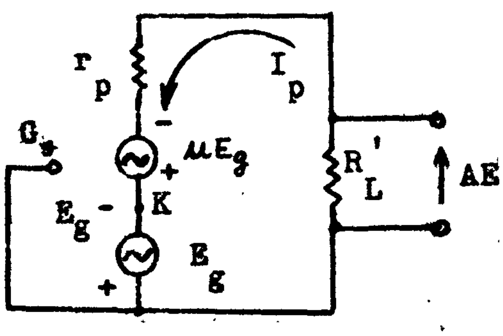


FIGURE 14(b) - EQUIVALENT CIRCUIT OF GROUNDED GRID STAGE V_2

Considering Figure 14(b), the equivalent circuit of stage V_2,

$$\mu E_g + E_g = I_p(r_p + R_L')$$

or $$I_p = E_g(\mu + 1)/(r_p + R_L')$$

where E_g is the ac voltage between grid and cathode of V_2, I_p is the ac plate current, and R_L' is the load resistance. If R_1 is the resistance seen looking into the cathode circuit of V_2 we have thus,

$$R_1 = E_g/I_p = (r_p + R_L')/(\mu + 1)$$

When R_2 is the equivalent Cathode Resistance (R_1 in parallel with R_K) of Cathode Follower V_1, the gain is (of cathode follower)

$$A_{CF} = \frac{\mu R_2}{(1+\mu)R_2 + r_p} = \frac{1}{\left[\left(\frac{\mu + 1}{\mu}\right)\left\{1 + \frac{r_p}{(r_p + R_L)}\right\} + \frac{r_p}{\mu R_K}\right]}$$

Similarly the gain of the grounded grid stage V_2 is

$$A_{GG} = (\mu + 1)/\left(1 + \frac{r_p}{R_L'}\right)$$

Thus the overall gain

$$A = A_{CF} \times A_{GG} = \frac{1}{\left[\frac{\mu + 1}{\mu}\left[1 + \frac{r_p}{r_p + R_L'}\right] + \frac{r_p}{\mu R_K}\right]} \times \frac{(\mu + 1)}{\left(1 + \frac{r_p}{R_L'}\right)}$$

which simplifies to give

$$A = \frac{\mu R_L'}{r_p\left[2 + \frac{(r_p + R_L')}{R_K(1 + \mu)}\right] + R_L'} \qquad (45)$$

This value of gain A could now be substituted in Equ. (44) to determine the magnitude of the negative resistance appearing between grid of V_1 and ground.

Now the value of R_i is found out with the help of Thevenin's Theorem. R_i will be the resistance looking back into the amplifier with all voltage sources shorted out. And this is R_L' in parallel with the effective plate resistance of V_2. The effective plate resistance R_e of V_2 is higher than r_p because of the impedance inserted in the cathode circuit. From equivalent circuit of V_1 and V_2 of Figure 14 we can write

$$E_g = I_p\left[\frac{R_K r_p}{(\mu + 1)}\right] \Big/ \left(R_K + \frac{r_p}{\mu + 1}\right)$$

and $R_e I_p = E_g + \mu E_g + I_p r_p$

or $R_e = \left[E_g(1 + \mu) + I_p r_p\right] \Big/ I_p$

Therefore $R_e = r_p + \dfrac{R_K r_p(\mu + 1)}{R_K(\mu + 1) + r_p}$

R_1 will be this resistance in parallel with R_L'

$$R_1 = \frac{r_p^2 R_L' + 2 r_p R_K R_L' (1 + \mu)}{\left[r_p (r_p + R_L') + (1 + \mu)(2 r_p R_K + R_K R_L') \right]} \qquad (46)$$

Substituting (45), (46) in (44) gives,

$$R' = \frac{R_1}{1-A} = \frac{R_L' \left[\frac{r_p^2}{R_K} + 2(1 + \mu)\, r_p \right]}{R_L' \left(\frac{r_p}{R_K} + 1 - \mu^2 \right) + \frac{r_p^2}{R_K} + 2(1 + \mu) r_p} \qquad (47)$$

For 6SN7-GT Dual-Triode used in the experimental set up for which r_p ~~= 7700 ohms~~ ≃ 30 kΩ, $\mu = 20$ and when $R_K = 5$K ohms and $R_L' = 10$K ohms we would have $R' \cong$ ~~925 ohms~~. 60 kΩ

3.4 PRINCIPLE OF MEASUREMENT

In view of the low noise voltages occuring in practice in the oscillator circuit, it is difficult to measure the effect of noise voltage on oscillator with existing techniques. The investigation of the random noise effect on oscillator carried out here, therefore, used a method in which the circuit was so noisy that the deviation of frequency could be recorded with the usual laboratory instruments. The noise voltage was injected in the circuit from a random noise generator (General Radio Company, Type 1390-B). This, in fact, is equivalent to a thermal noise produced across a resistance which is heated to a very high temperature. The whole measuring process is clear from block diagram of Figure 17. The experimental set-up for determining the number of zero crossings per unit time of oscillator output, is given in Figure 18.

3.5 EXPERIMENTAL CIRCUIT OF OSCILLATOR

Figure 15 is a schematic diagram of the experimental circuit of nonlinear oscillator under consideration. It consists of a pair of triodes coupled so as to produce a negative resistance between terminals I and IV. For larger voltages the current voltage characteristic of triode does not change as fast as the theoratical Negative Resistance characteristic of Figure 4. Thus a pair of diodes (6AL5) is added to the circuit with provision for biasing and loading. At higher voltages these diodes conduct and add extra resistance to the circuit. An L-C resonant circuit, in series with a 100 ohm resistance, is connected at the terminals where the negative resistance appears. This small resistance is used for current measurement and is connected to the vertical amplifier of the oscilloscope. The horizontal amplifier is connected to terminal I while the ground of the oscilloscope is connected to terminal III. The 100K ohm variable resistance in the diode load controls the slope of the operating path after conduction has started. The 10K ohm resistance in the biasing circuit controls the level at which the diodes conduct, and 1K ohm potentiometer controls the symmetry of their conduction point. Proper adjustment of the above resistances would make the ideal nonlinear cubic characteristic to be matched closely.

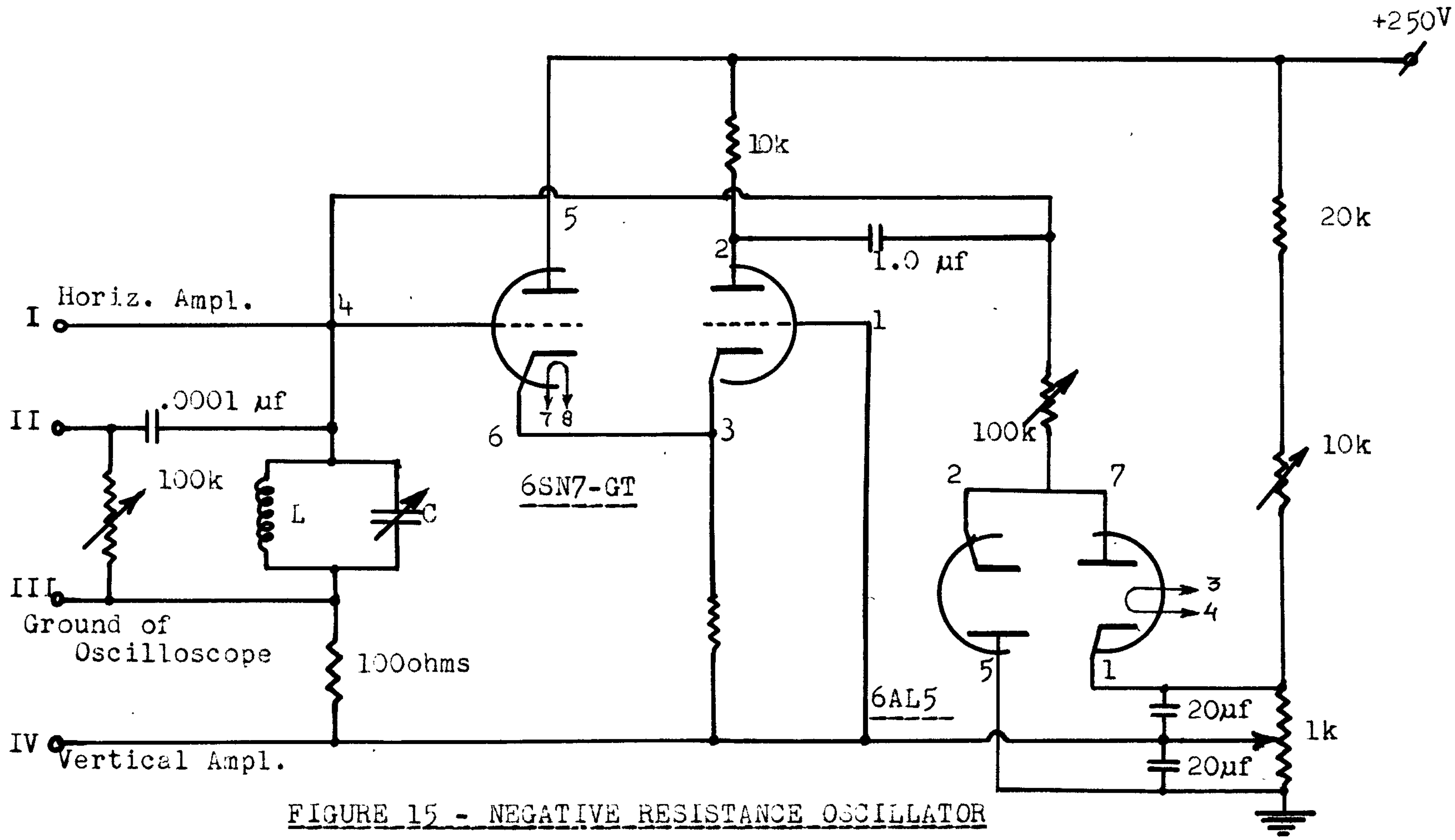


FIGURE 15 - NEGATIVE RESISTANCE OSCILLATOR

NOTE: (a) Negative Resistance Characteristic between I & IV
(b) Diode is used for adjusting loading & biasing. This diode conducts at larger voltages and add extra resistance to the circuit, thus providing for proper shaping of nonlinear characteristic.

Coefficients S and T for the current, voltage characteristic can be determined by making use of the geometrical properties of Figure 4 given by equation (9) and (10). The circuit of Figure 15 was used with $R_L^{'}$ = 10K ohms, R_K = 5K ohms and C_c= 1.0 uF.

This gave the result of

$S = 1.115 \times 10^{-4}$ ohms^{-1}

$T = 2.00 \times 10^{-6}$ ohm^{-1} volt^{-2}

Measurements were carried out at 240 KC/S (C adjustable) with L = 660 μ Henry and C = 660 p.f with an unloaded Q of 160.

A reasonable figure of merit for the oscillator is to be $Q\sqrt{C/L}$. This means that the best compromise is to have "high C" tuned circuit for stability (L/C is minimized) and also a high value of Q.

The cubic nonlinear characteristic ($i = -Se + Te^3$) was displayed on the C.R.O. tube using conventional technique described earlier. In order to realize the negative resistance characteristic, as close as possible to that of the ideal van der Pol oscillator, an extra diode circuit for adjusting proper loading and biasing was used. When the characteristic displayed on the C.R.O. is not symmetrical, indicating the presence of even order harmonics, it was found convenient to adjust diode circuitry to ensure symmetry and the desired cubic characteristic.

It is well known that with the decrease of resistance R across the tuned circuit $\left[R = R_L R_f/(R_L + R_f)\right.$, indicating total resistance across the tuned circuit$\left.\right]$ the Q of the circuit (Chapter 1) is lowered. Thus, during the experiment the reduction of R_f while keeping R_L = 100 K ohms, is analogous to reducing the Q of the circuit (Appendix V). Table No. 2 (Chapter 4) gives different values of Q ($Q = R/w_oL$) for various values of resistance R_f.

The output of the random noise generator was read using a VTVM which indicated the mean rectified voltage and which was calibrated in terms of r.m.s. values of sine waves. These readings were appropriately corrected by a factor of $2/\sqrt{\pi}$ in order to obtain the true r.m.s. noise voltage.

Figure 19 illustrates the oscillograms of noise-perturbed oscillator.

3.6 SCHMITT TRIGGER - Schmitt Trigger (43), a cathode coupled bistable multi-vibrator, is used as a voltage discriminator. Noise-perturbed oscillations after being amplified are reshaped by the voltage discriminating action of the Schmitt Trigger. This provides an appropriately small threshold for counting the number of zero crossings per unit time of the noise-perturbed oscillator with the help of a decade counter (Beckman, model 7370).

The Schmitt Trigger is capable of discriminating very low voltage levels. Hysterises or the backlash effect is minimized by adjusting the loop gain of the amplifier stage. A well regulated supply voltage is used and care is taken to avoid tube aging effects.

A typical circuit is shown in Figure 16. The direct coupling from plate of V_1 to the grid of V_2 is the same as for conventional Bistable multivibrator, but the plate to grid connection from V_2 to V_1 is eliminated. Instead, the common cathode resistor R_K provides the other necessary coupling for regeneration between stages. When the input signal is below a preset value, one tube conducts and the other is cut off. The moment the voltage exceeds the preset value there is a rapid transition of states.

The triggering level can be set by potentiometer R_1, which determines the grid voltage E_g, of tube V_1. With E_g sufficiently low, V_1 is cut off and the attenuator ratio $a = R_2/(R_{C1} + R_2)$ is selected by adjusting R_2 so that V_2 is conducting (e.g. grid bias of about -1 volt on V_2). The plate current I_2 of V_2 causes a voltage drop E_K across the common cathode resistor R_K. The difference between the value of E_{g1} set by potentiometer R_1 and the cathode voltage is the grid bias $(E_{g1} - E_K)$ of V_1. Since the transition of states occur the moment the cut off voltage E_{CO} is exceeded, it is the voltage difference E_t between E_{CO} and grid bias that determines the triggering level i.e.

$$E_t = E_{CO} - (E_{g1} - E_K)$$

For example if $E_{CO} = -5$ volts, $E_K = 68$ volts and $E_{g1} = 62.5$ volts then the triggering level

$$E_t = -5 - (62.5 - 68) = 0.5 \text{ volts}$$

Thus a minimum signal of 0.5 volts would be required to make V_1 conduct from cut off. As a result the plate voltage of V_1 and the grid of V_2 decrease, causing a decrease in plate current of V_2. The resulting drop of the cathode voltage E_K increases the plate current of V_1, and the regeneration process continues until V_2 is off and V_1 is on. The output from the plate of V_2 jumps to the plate supply value because of this transition.

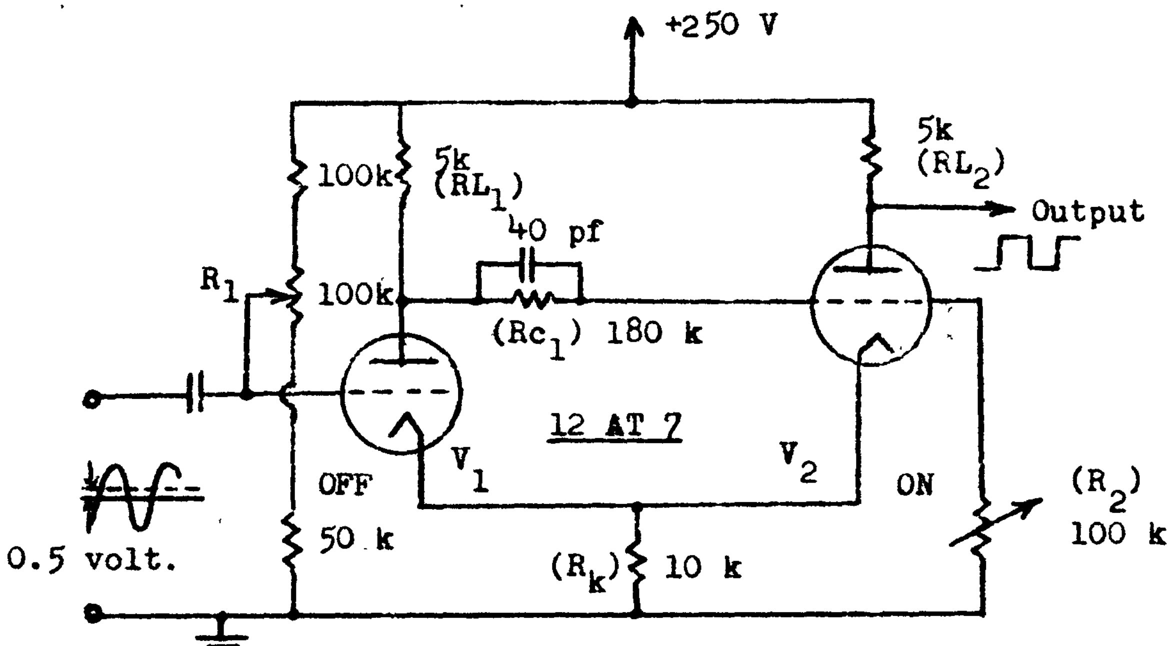


FIGURE 16 - SCHMITT TRIGGER

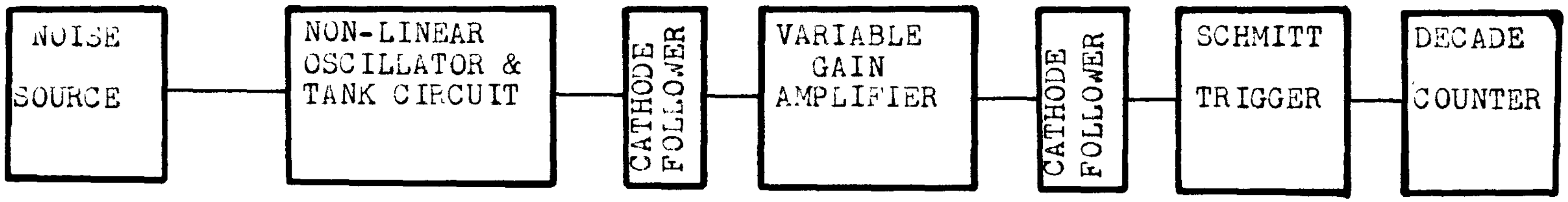
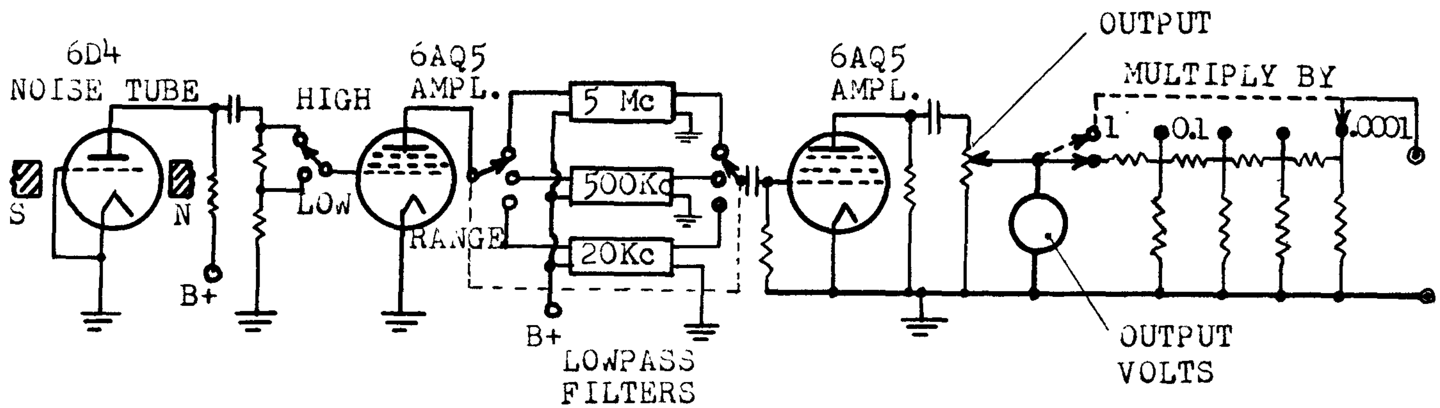


FIGURE 17 - BLOCK DIAGRAM OF EXPERIMENTAL CIRCUIT



SCHEMATIC OF RANDOM NOISE GENERATOR

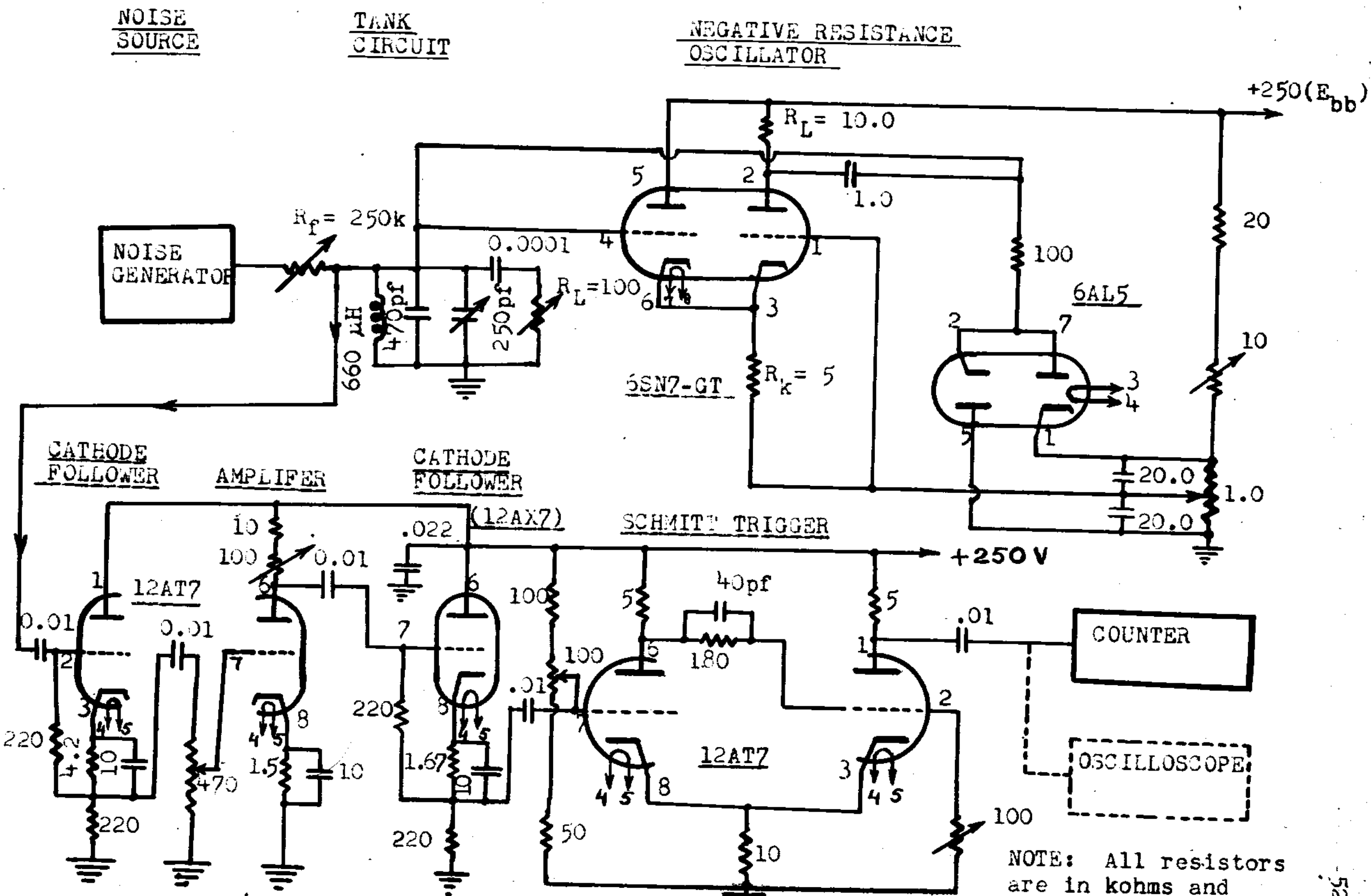
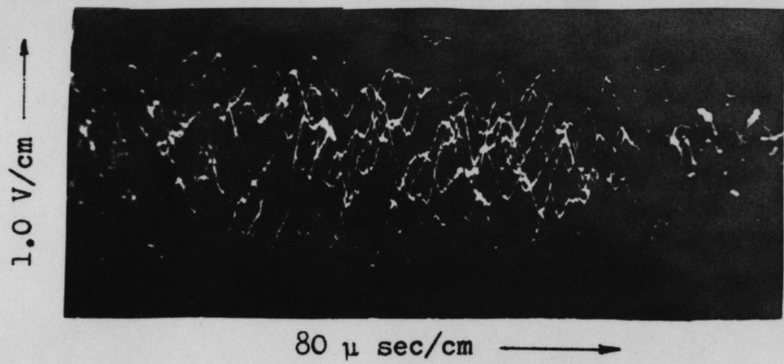


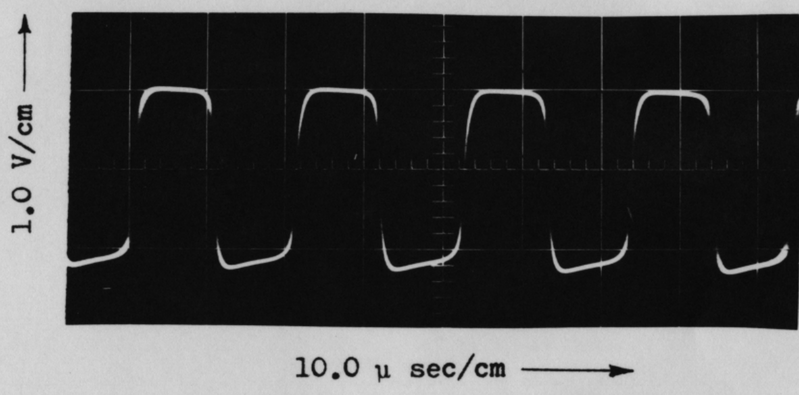
FIGURE 13 - EXPERIMENTAL CIRCUIT

NOTE: All resistors are in kohms and capacitors in μ farads unless otherwise specified.

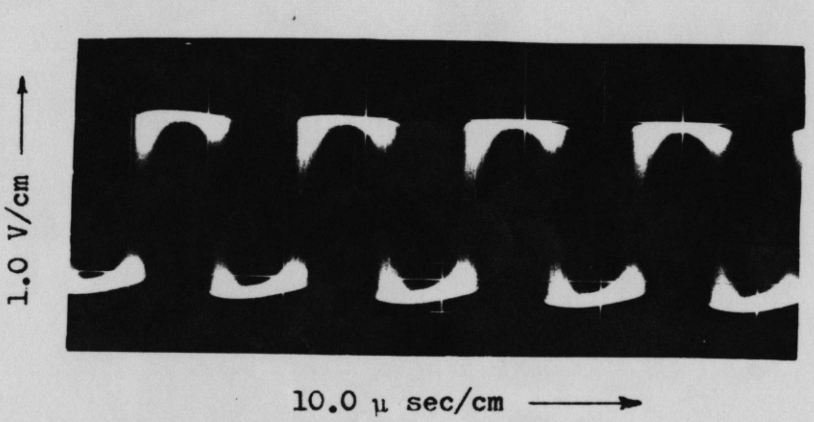
FIGURE 19 OSCILLOGRAMS OF NOISE PERTURBED OSCILLATOR



(1) Output From Random Noise Generator (500 Kc/s)



(2) Output of Schmitt-Trigger Without Noise



(3) Output of Schmitt-Trigger With Noise

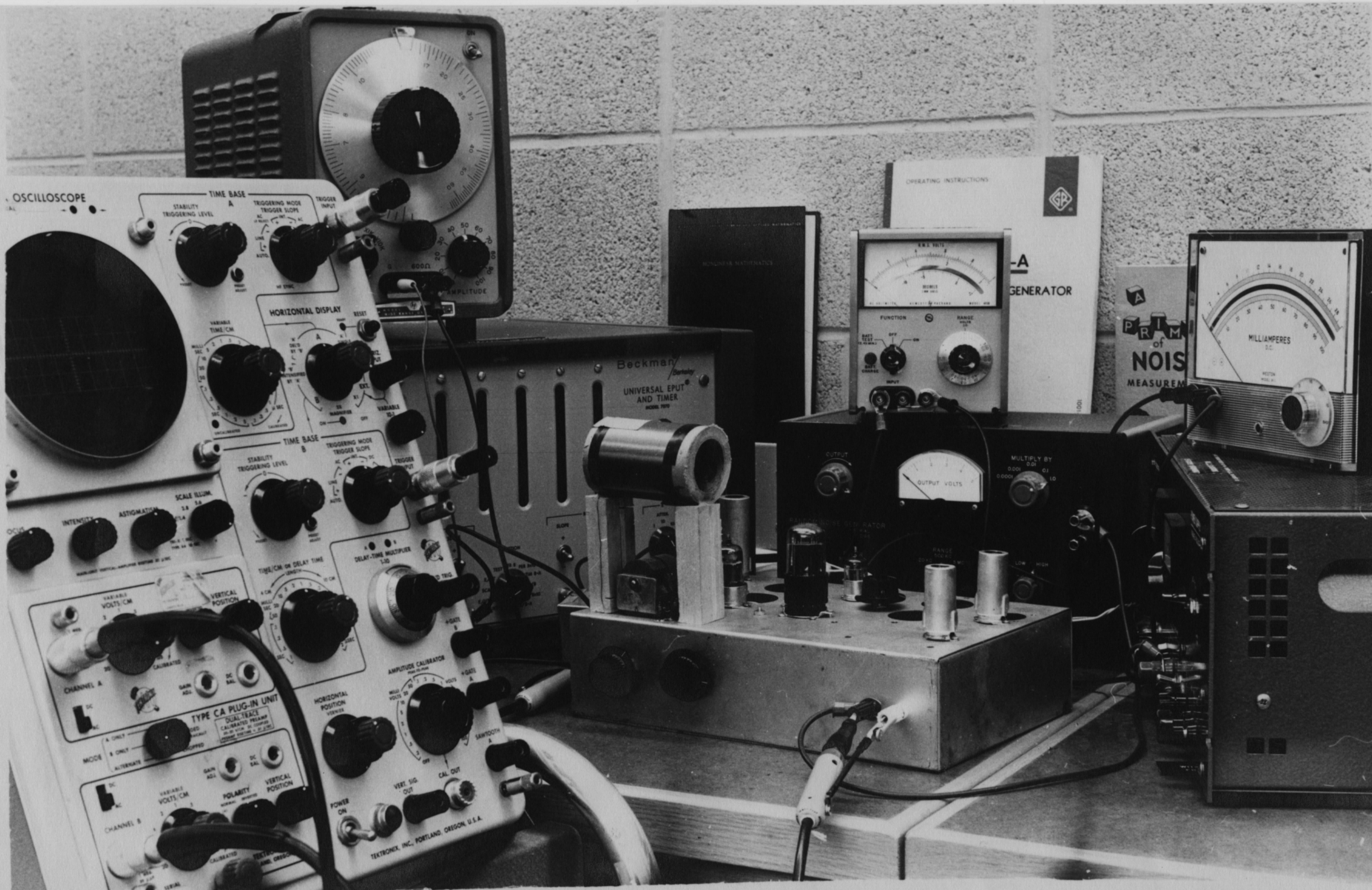


FIGURE 20 EXPERIMENTAL SET-UP

CHAPTER 4 - RESULTS OF EXPERIMENTAL AND COMPUTER STUDY

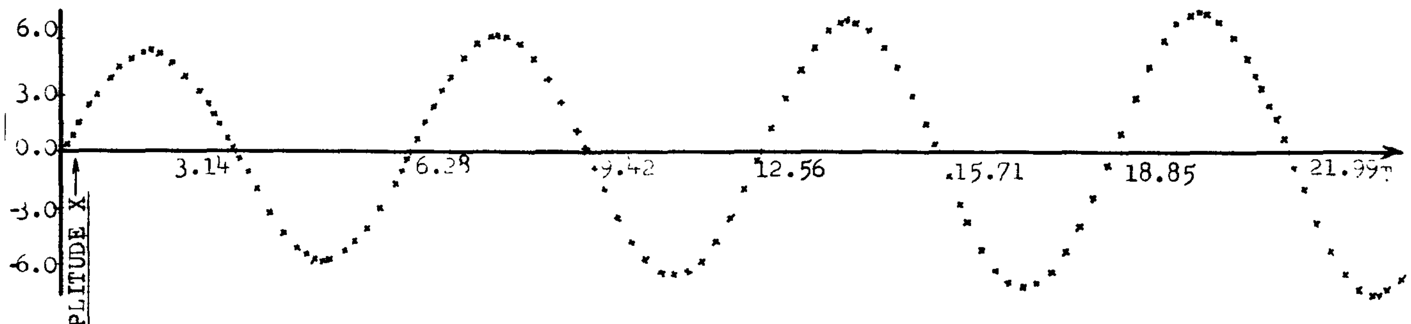
4.1 INTRODUCTION

Initially a comparative study was made of the well known results of free and forced oscillations in oscillators described by the van der Pol non-linear differential equation. These results were obtained both by numerical analysis using a digital computer and by experiments. The close agreement of these results enables one to proceed further, with confidence, to the case of the random noise forcing signal applied to the oscillator. The solution of the nonlinear differential equation with a random forcing function (representing the corresponding injected noise voltage in the experimental set-up), gave the deviation of the mean frequency of the noise-perturbed oscillator with respect to that of the "noiseless" oscillator.

4.2 FREE AND FORCED OSCILLATIONS

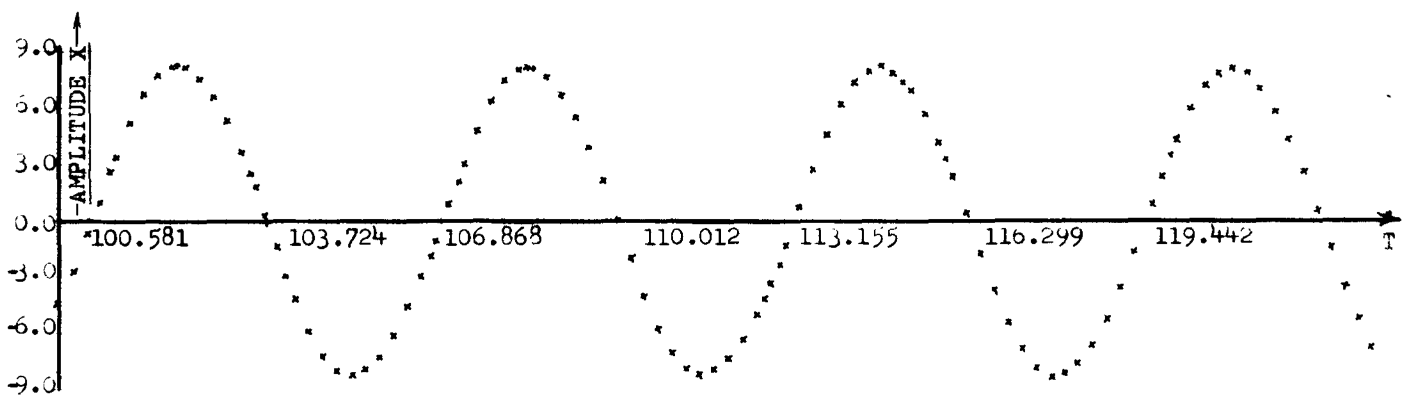
Graph 1 and Graph 2 give the waveforms of the oscillator having same value of Q, obtained by solving the nonlinear differential equation (12) for both the cases of free-running and forced oscillations respectively (see appendix 3, program No. 4, 5 and 6).

A study of the predictions quoted earlier in Chapter 1 (particularly relation (22) and (24)) was made on the practical circuit of the negative resistance oscillator and good agreement was obtained. Measurements were carried out to record the



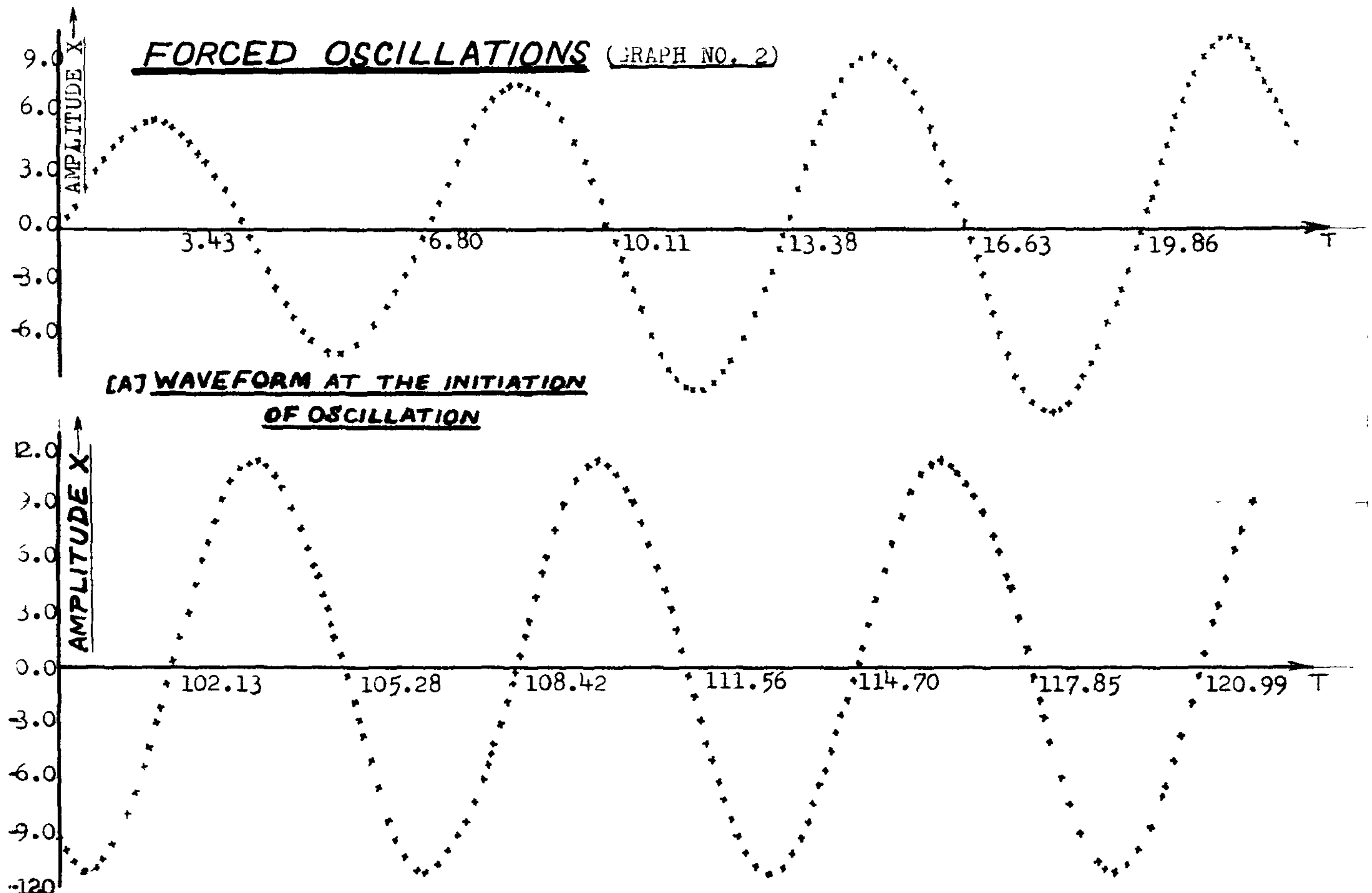
[A] WAVEFORM AT THE INITIATION OF OSCILLATIONS

FREE-RUNNING OSCILLATOR (GRAPH NO. 1)



[B] WAVEFORM AT THE STEADY STATE OSCILLATIONS

FORCED OSCILLATIONS (GRAPH NO. 2)



[A] WAVEFORM AT THE INITIATION OF OSCILLATION

[B] WAVEFORM AT THE STEADY STATE OSCILLATIONS amplitude of free-running oscillator output (E) as a function of Q. The computed results were found to be in very good agreement with the practical results (Graph 3). Graph 4 gives the injected voltage amplitude (V) at the locking condition (equation (24)), for various values of R_f (or Q).

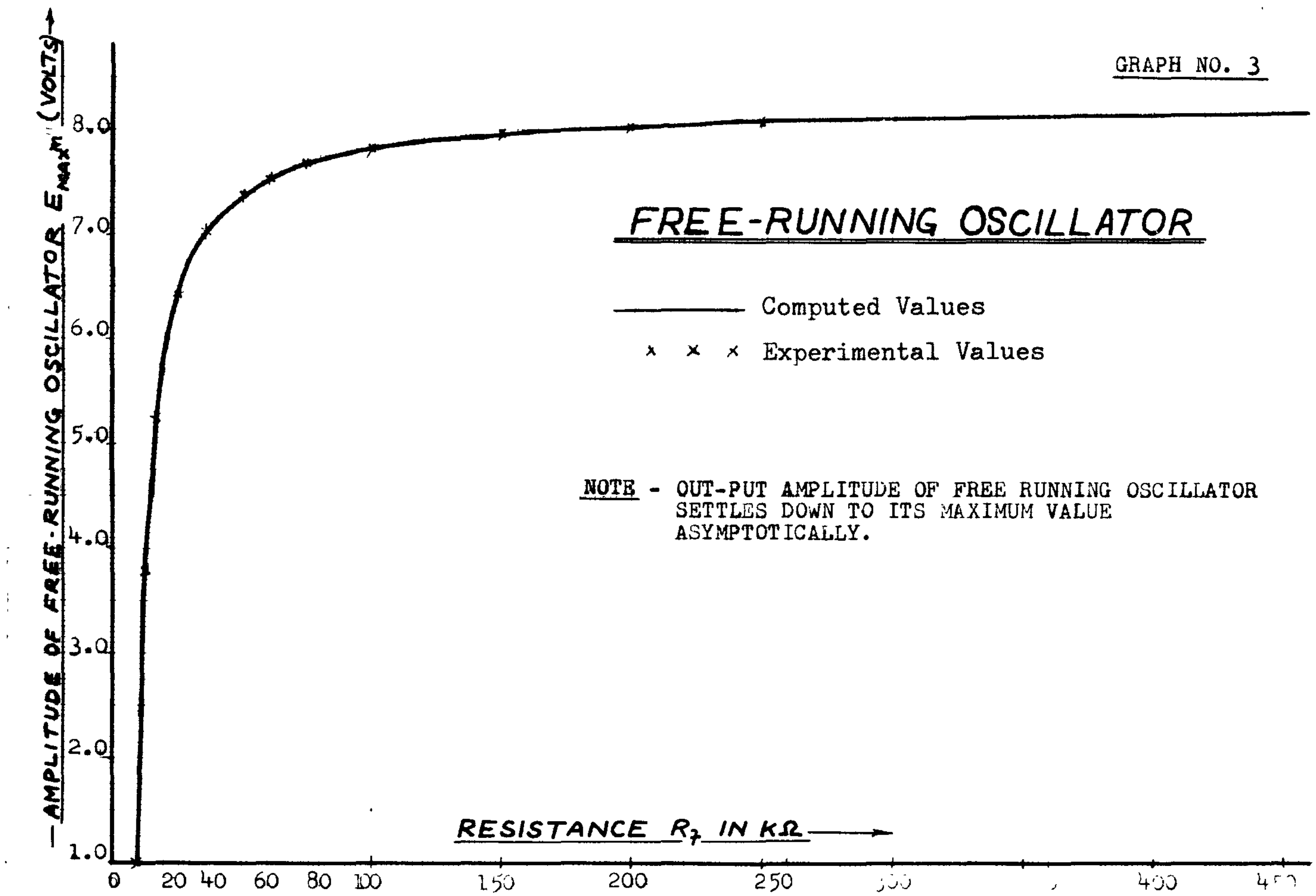
The results of the different measurements made may be summarized as follows.

<u>Amplitude of the free-running and forced oscillator</u>

(a) With no injected voltage i.e. V = 0, the free-running amplitude is $E = 2/\sqrt{B}$, where B is a function of R_f $\left[B = 3T \Big/ \left[S - \frac{1}{R}\right]\right.$ where, $\left. R = R_L R_f \Big/ (R_L + R_f)\right]$. A curve is plotted of free-running amplitude E with R_f (or Q, see Appendix 5). As R_f increases (for a fixed value of R_L = 100 k), B decreases and approaches the value 3T/S asymptotically. Hence the output E of the free-running oscillator increases with increase of R_f (or Q) as shown in Graph 3.

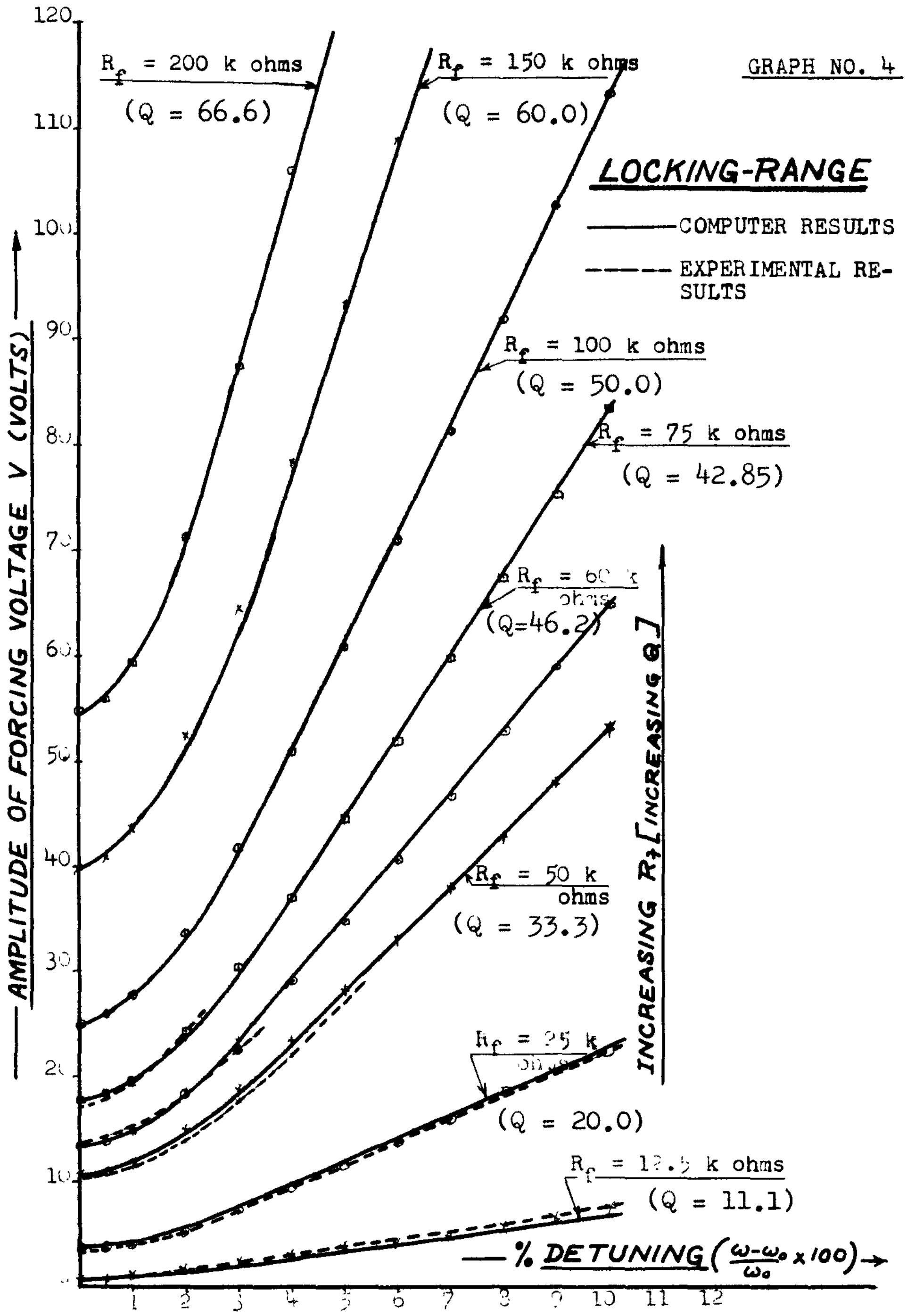
(b) When E = 0 and the injected voltage V is just sufficient for locking, it was found experimentally that the output amplitude of the forced oscillation was given by the relation $E_1 = \sqrt{2/B}$.

(c) From the results of (a) and (b) it is observed that for the experimental measurements made, the relation $E = \sqrt{2}\, E_1$ was satisfied. However, this relation is true only for locking condition.



$$Q = Q_o / \left[R_L/R_f + 1 \right]$$

where, Q_o = 100 and R_L = 100 k ohms


GRAPH NO. 4
LOCKING-RANGE
COMPUTER RESULTS
EXPERIMENTAL RESULTS
R_f = 200 k ohms (Q = 66.6)
R_f = 150 k ohms (Q = 60.0)
R_f = 100 k ohms (Q = 50.0)
R_f = 75 k ohms (Q = 42.85)
(Q=46.2)
R_f = 50 k ohms (Q = 33.3)
(Q = 20.0)
(Q = 11.1)
AMPLITUDE OF FORCING VOLTAGE V (VOLTS)
INCREASING R_f [INCREASING Q]
% DETUNING (ω-ω₀/ω₀ x 100)
120
110
90
40
1 2 3 4 5 6 7 8 9 10 11 12

Frequency of the free-running oscillator

(a) When the Q of the tuned circuit is decreased, the mean frequency of the oscillator increases from its value when the LC tuned circuit has its highest Q. (See Graph No. 5).

(b) The free-running oscillator over a long time interval (several hours) was found to be very stable. The oscillator frequency was periodically checked with a frequency measuring device (Beckman Counter) and was found to be stable within ±5 parts in 10^6.

The free-running oscillator frequency was also determined by solving the nonlinear differential equation with no forcing function, and the results were found to agree with the experimental results.

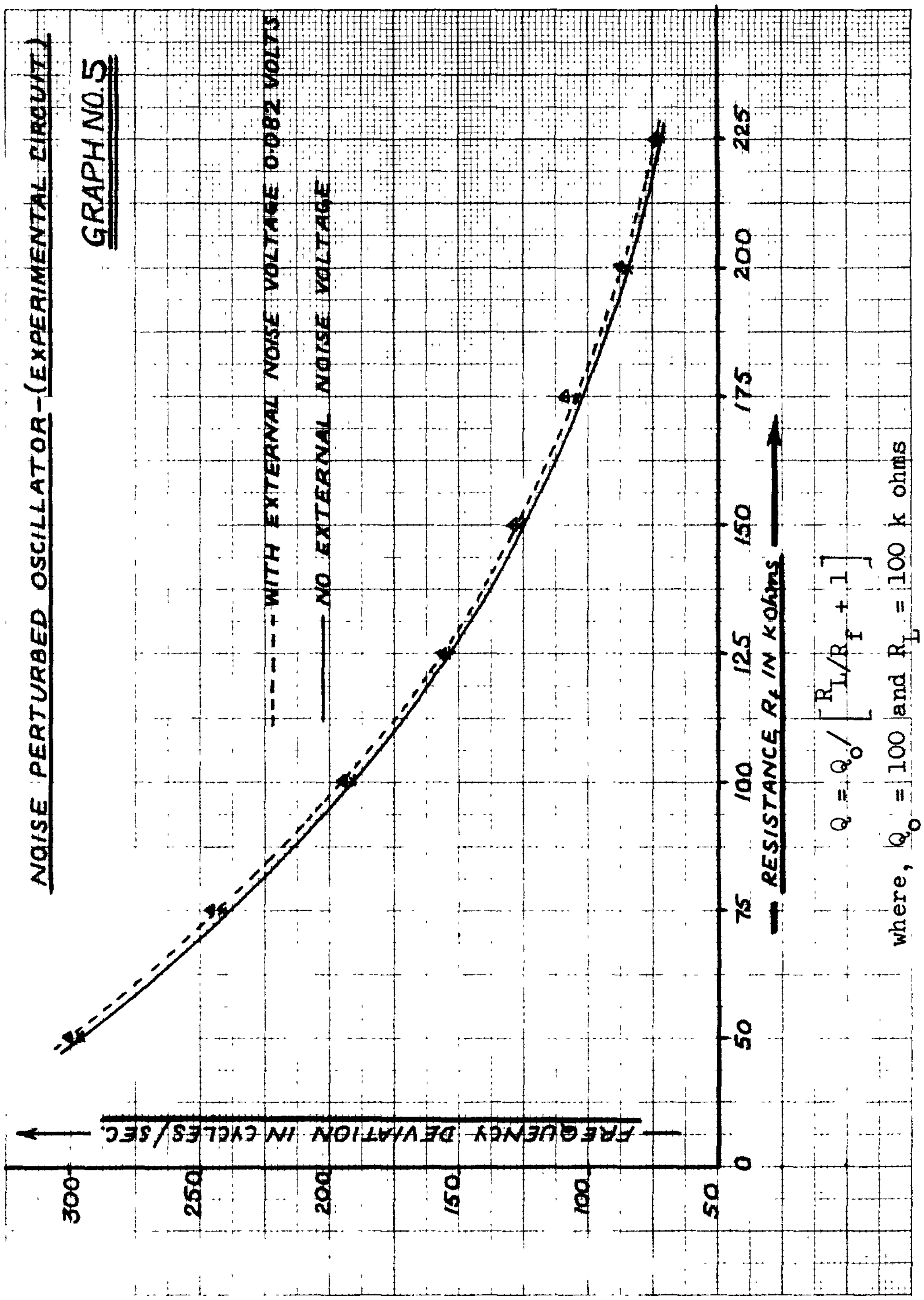
Locking Range

Curves of injected voltage V vs. percentage frequency detuning are plotted with R_f (or Q) as parameter according to relation (24). Graph 4 shows that the results both from computation and experiments are in good agreement over the limited range of experimental measurements. This graph can be used to determine the required amplitude of injected voltage V for a particular amount of detuning, with Q (or R_f) as parameter.

Furthermore, the study of the results indicates that an increase of Q requires an increase in the amplitude of V if locking is to be maintained.

4.3 NOISE PERTURBED OSCILLATOR

Results of Experimental Set-Up — The average frequency of the


NOISE PERTURBED OSCILLATOR-(EXPERIMENTAL CIRCUIT)
GRAPH NO.5
WITH EXTERNAL NOISE VOLTAGE 0·082 VOLTS
NO EXTERNAL NOISE VOLTAGE
FREQUENCY DEVIATION IN CYCLES/SEC.
300
250
200
150
100
50
0
50
75
100
125
150
175
200
225
RESISTANCE R_f IN K ohms
$Q = Q_o / \left[R_L/R_f + 1 \right]$
where, Q_o = 100 and R_L = 100 k ohms

noise-perturbed oscillator is determined by the experimental measuring system described earlier (Figure 17 and Figure 18). The experiments indicate unambiguously that the oscillator output does not consist of single frequency sinusoidal oscillations. When the magnitude of the external noise voltage is increased, a definite frequency deviation from the noiseless oscillator case is observed. This mean frequency deviation was recorded for various values of noise voltages. Considerable frequency deviation of the oscillator frequency takes place when the resistance R_f and thus Q of the tuned circuit is decreased. (Graph No. 5).

<u>Computer Study</u> - The nonlinear differential equation having random forcing function, representing the van der Pol type of negative resistance oscillator, was solved with the help of a Digital Computer. The programmes are based on the Runge-Kutta method of solving the nonlinear differential equation. Different details of the programmes including the library function used to simulate random noise having Gaussian distribution, and the programme for finding the r.m.s. value of the random noise voltage, are given in the Appendix III.

From equation (12) of Chapter 1

$$\frac{d^2e}{dt^2} - A(1 - Be^2)\frac{de}{dt} + w^2e = F(t) \tag{48}$$

where, e is the voltage across the tuned circuit and nonlinear element.

A and B are constants of nonlinearity

$w = 1/\sqrt{LC}$

and F(t) is the forcing function.

Relation (48) is transformed to a system of two first order equations, in two new variables.

And so,

let $x = e$ and $y = \frac{1}{w}\frac{de}{dt}$

This gives,

$$\frac{dx}{dt} = wy \tag{49}$$

and
$$\frac{dy}{dt} = - wx - A(1 - Bx^2)y - F(t) \tag{50}$$

where, the forcing function F(t) could be,

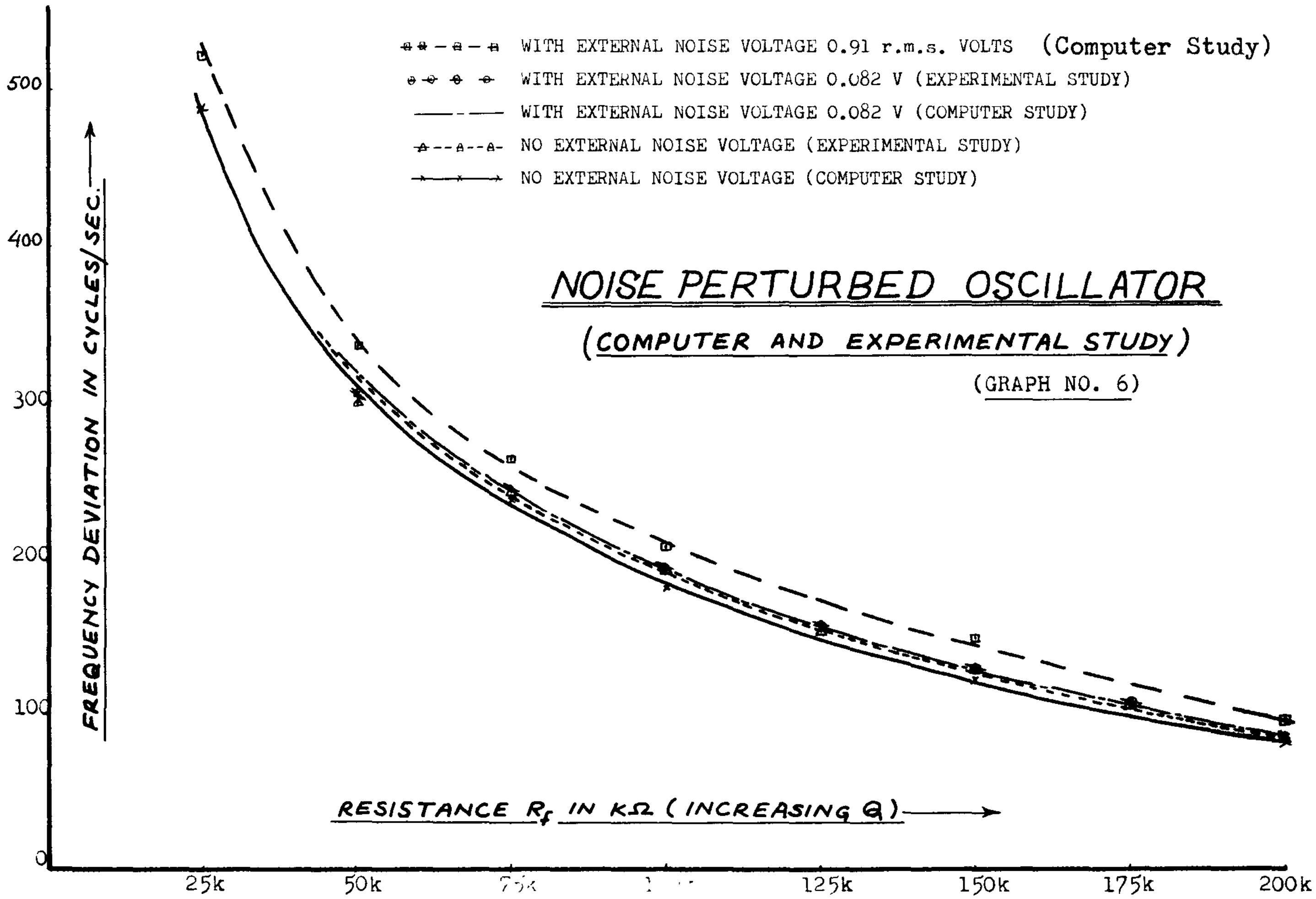
(1) 0 i.e. No forcing function

(2) Sinusoidal Signal

(3) Random Number (having Gaussian distribution)

The computer study of normalized equations (49) and (50) for all the above three cases were made by programming an IBM-7040 Digital Computer to calculate step by step, the T (time), x and y values. An analog computer used initially was promptly discarded for a more "deterministic" digital computer, disturbed only by the round off noise of its twelfth place of decimal. Results of computation using analog computer were found to be unsatisfactory even for free and forced oscillation cases. This seems to be due to the stringent requirement in simulating the nonlinear characteristic.

After the initial transient state, when the oscillator settles down to steady state oscillations, the number of zero crossings in a given time should give the average frequency of



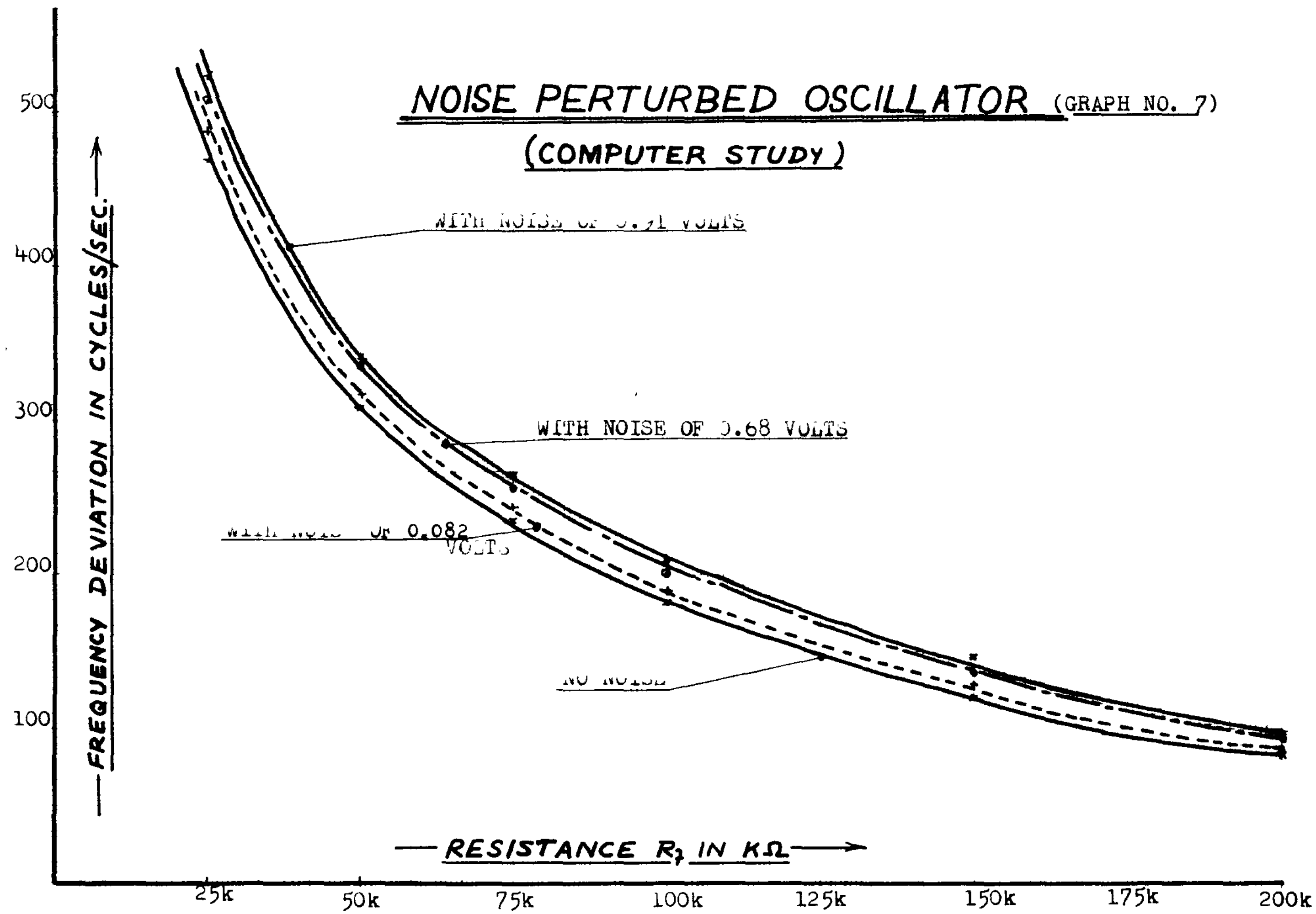
$$Q = Q_o / \left[R_L / R_f + 1 \right]$$

where, Q_o = 100 and R_L = 100 k ohms

the oscillator. From the results it was found, as expected, that the frequency of the oscillator, as defined by the intervals between zero crossings of the output voltage, remains "constant" (free oscillator frequency) for the case of no forcing function. Synchronization or pulling takes place resulting in forced oscillations when the forcing function is a sinusoidal signal of sufficient amplitude and of frequency w_1, where w_1 is nearly same as w but $w \neq w_1$. These results agreed with the experimental results.

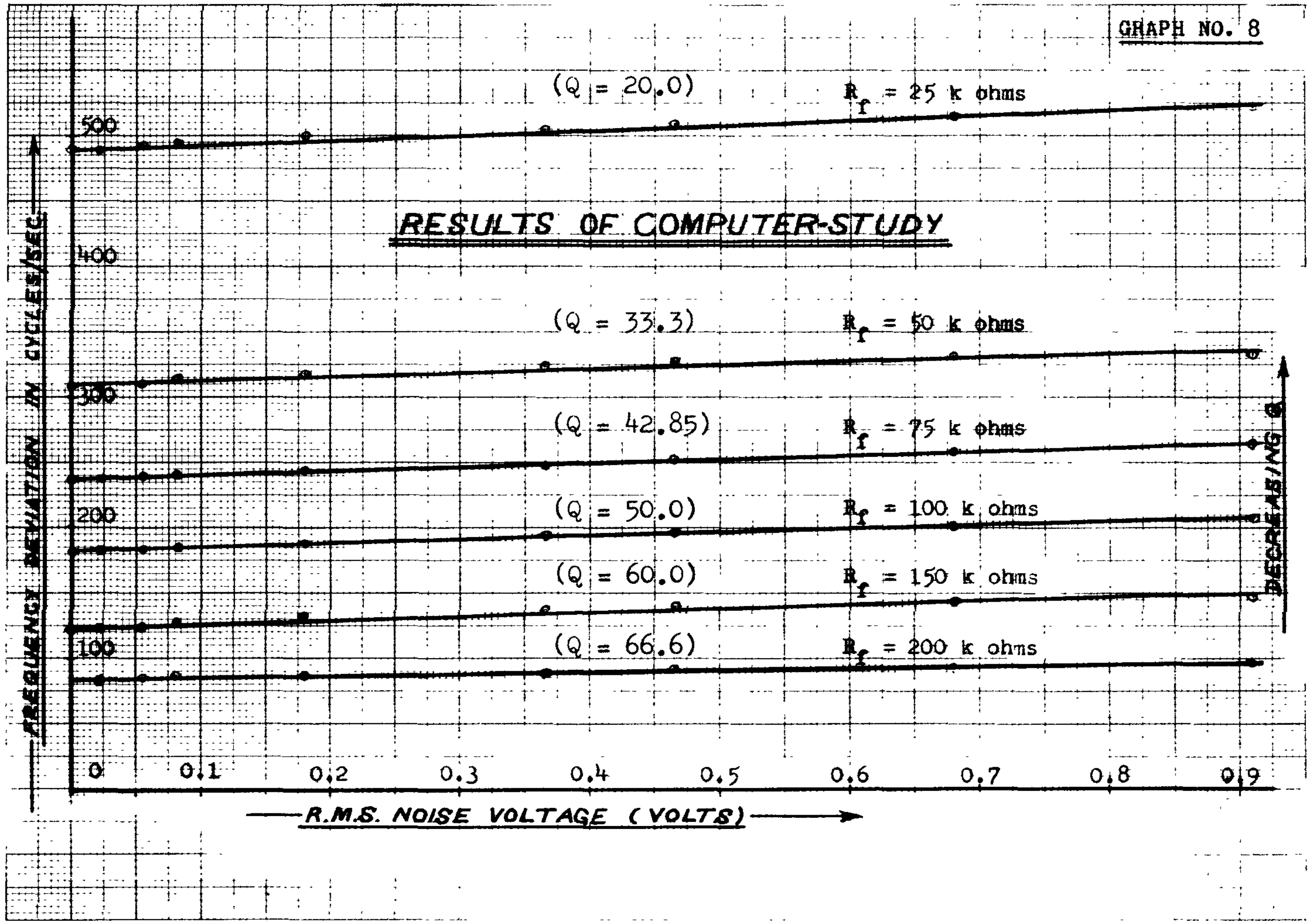
When the forcing function is random noise, the computer results showed that the mean frequency of the oscillator increased from that of the "noiseless" oscillator case. This deviation of the mean oscillator frequency from the stable frequency of the noiseless oscillator was determined by finding the average number of zero crossings per unit time and comparing this with the corresponding results for the noiseless oscillator. The mean frequency so obtained for various values of resistance R_f(or Q) and r.m.s. noise voltages are shown in Graph 6, Graph 7, Graph 8 and Graph 9. Graph 6 shows the results of both the computer and experimental studies for several values of the applied external noise voltage. Graph 7 shows the deviation of mean frequency for various values of noise voltage obtained with the aid of the digital computer.

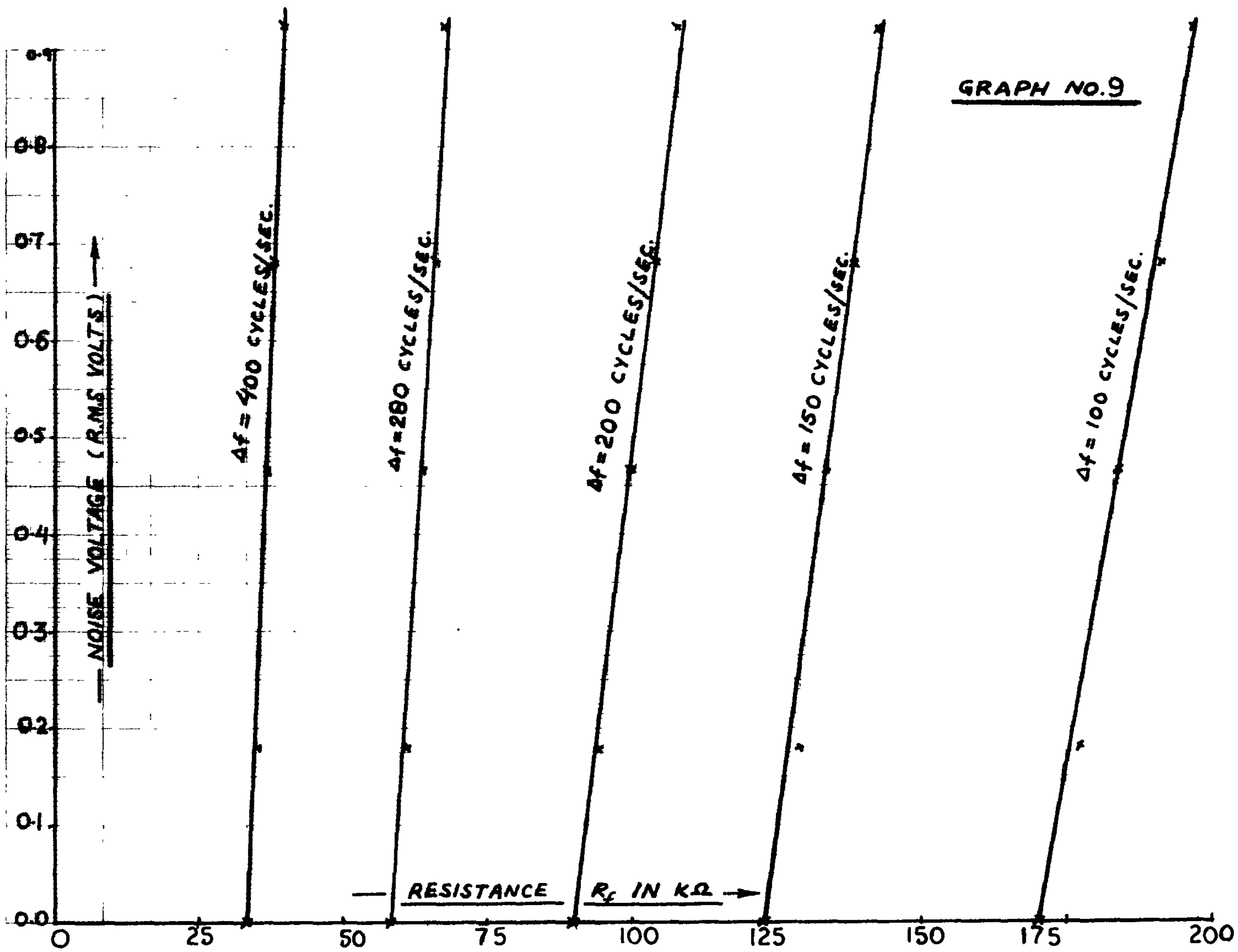
Graph 8 shows the deviation of the mean frequency as a function of injected noise voltage with the Q of the circuit (or resistance R_f) as a parameter. The results indicate that



$$Q = Q_o / \left[R_L/R_f + 1 \right]$$

where, Q_o = 100 and R_L = 100 k ohms


GRAPH NO. 8
RESULTS OF COMPUTER-STUDY
(Q = 20.0)
Rf = 25 k ohms
(Q = 33.3)
Rf = 50 k ohms
(Q = 42.85)
Rf = 75 k ohms
(Q = 50.0)
Rf = 100 k ohms
(Q = 60.0)
Rf = 150 k ohms
(Q = 66.6)
Rf = 200 k ohms
FREQUENCY DEVIATION IN CYCLES/SEC.
DECREASING Q
R.M.S. NOISE VOLTAGE (VOLTS)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
100
200
300
400
500



$$Q = Q_o / \left[R_L/R_f + 1 \right]$$

where, Q_o = 100 and R_L = 100 k ohms

for higher values of Q, the percentage change in the mean frequency with noise from that of the noiseless oscillator is less than that for low values of Q. In all cases the effect of noise is to increase the mean frequency. For a given value of noise voltage, a low Q circuit gives higher mean frequency of the oscillator as compared to the high Q tuned-circuit. Graph 7, 8 and 9 are obtained from the same results.

4.4 ENSURING SIMILARITY BETWEEN EXPERIMENTAL AND COMPUTER STUDY

It is to be noted that the computer study is made for corresponding values of R_f chosen in experimental cases. For both the experimental and computer studies R_L is kept constant at a value of 100 K ohms. The reduction of R_f decreases the equivalent resistance R $\left[R = R_L R_f / (R_L + R_f) \right]$ thereby reducing the Q of the tuned circuit (Appendix 5). As the constants A and B of the non-linear differential equation (48) are related to the coefficients S and T of the cubic characteristic (Chapter 1), various values of R_f give varying A and B. Before analyzing the noise-perturbed oscillator case, care was taken to verify that, for both the free and forced oscillation cases, various values of R_f give identical results from both the experimental and computer studies. Table No. 2 gives different values of constants A and B for various resistances R_f.

Table No. 2

R_f	$R = Qw_oL$ $= R_LR_f/(R_L + R_f)$	Q	$A = (S-\frac{1}{R})\sqrt{\frac{L}{C}}$	$B = 3T/(S-\frac{1}{R})$
OPEN CIRCUIT	100.0 K	100	0.1015	0.0592
200 K	66.6 K	66.6	0.0965	0.062
150 K	60.0 K	60.0	0.0948	0.063
125 K	55.6 K	55.6	0.0935	0.064
100 K	50.0 K	50.0	0.0915	0.066
75 K	42.85 K	42.85	0.0882	0.068
50 K	33.3 K	33.3	0.0815	0.074
25 K	20.0 K	20.0	0.0615	0.098
12.5 K	11.1 K	11.1	0.0215	0.279

CHAPTER 5 - SUMMARY

Some effects of interfering external signals, including random noise, on nonlinear oscillations have been investigated. A simple and practical negative resistance oscillator whose behaviour closely approximates that of the van der Pol oscillator, is used for experimental purposes. For all the three important cases of (a) free-running oscillator (b) forced oscillator and (c) noise-perturbed oscillator, the experimental results are found to be in good agreement with the results obtained by numerical analysis using a Digital Computer.

A locking phenomenon occurs when a sinusoidal input is injected into the free-running oscillator. A necessary condition is established for locking which is useful from the design point of view.

Statistical effects of noise on oscillators are discussed and a relation is developed for the average number of zero crossings per unit time in terms of the phase error for a noise-perturbed oscillator. This expression gives the instantaneous frequency of the oscillator and is to be understood in the probability sense. The expression also indicates that the mean frequency of a noise-perturbed oscillator over a long time interval should be more than that of the frequency of a "noiseless" free-running oscillator.

Although no satisfactory theoretical analysis could be obtained to predict the deviation of the mean frequency of a noise-perturbed oscillator, the experimental and numerical analysis results given in the thesis are likely to help in any further investigation related to this problem.

Important conclusions from the experimental and computer studies are summarized as follows:

5.1 FREE AND FORCED OSCILLATIONS

(a) The output voltage of the free-running oscillator increases with increase of Q (Graph 3). The output amplitude of the free-running oscillator (E) is related to the output amplitude of the forced oscillator (E_1) by the expression $E = \sqrt{2}\, E_1$. However, this condition is true only when the oscillator is just locked.

(b) For the free-running oscillator, when the Q of the tuned circuit is decreased, the mean frequency (half the number of zero crossings per unit time) increases from its value when the LC tuned circuit has its highest Q (Graph 5 and Graph 6).

(c) Locking range - when the Q of the tuned circuit is increased (by increasing R_f), the injected voltage has to be increased correspondingly if the condition of locking is to be maintained (Graph 4). Also for a given Q, if the percentage detuning of the injected signal frequency from the free-running oscillation frequency is increased, the injected voltage has to be increased to maintain the locking condition. The results (Graph 4) also help in predicting, for a given resistance R across the tuned circuit (and hence Q of the tuned circuit), the required amplitude of forcing voltage when the injected signal frequency (or percentage detuning) is known.

5.2 NOISE-PERTURBED OSCILLATOR

(a) A comparison of the experimentally obtained values and those computed from the nonlinear differential equation, shows unambiguously that with an increase of noise voltage the oscillator mean frequency increases from its normal "constant" value of the free-running oscillator. Thus, the deviation of the mean frequency of the oscillator over a long time interval, depends upon the magnitude of the input noise. This deviation in mean frequency increases with an increase of noise magnitude (Graph 5).

The results (Graph 8) show that for lower values of Q, the effect of noise is greater. For a given value of noise voltage the percentage deviation in mean frequency of the noise-perturbed oscillator from that of the "noiseless" free oscillator increases with decreasing values of Q.

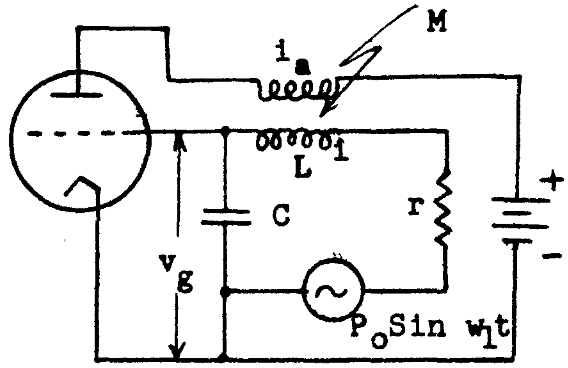
(c) From the above two results it is concluded that in-cre-asing the injected noise magnitude has a similar effect on the mean frequency of the oscillator as decreasing the Q of the tuned circuit. However, no specific relation between the Q and the noise voltage for a given circuit of oscillator could be established.

(d) The above results suggest that the shift in the mean frequency of the oscillator with noise is due to the asymmetry of the resonance curve. Since the noise power has a peak at the maximum impedance frequency, the output frequency of the oscillator is pulled towards this maximum impedance frequency. The latter is on the higher side and thus, this results in the mean frequency of the noise-perturbed oscillator to be more than that of the "noiseless" oscillator frequency.

APPENDICES

APPENDIX I - FORCED OSCILLATIONS IN FEED BACK OSCILLATOR

The purpose of this note is to show that a feedback-type of oscillator can be represented by van der Pol nonlinear differential equation. Thus, in the case of feedback type of oscillator the approach adopted in this work would be equally good for all the three cases of free-running oscillations, forced-oscillations and noise-perturbed oscillations.



The oscillator circuit consists of an LCr circuit in in the grid lead and a magnetic feedback from the output; M is the mutual inductance of the coupling. The grid and plate circuits are inductively coupled. In addition a source $P_o \text{Sin } w_1 t$ of alternating voltage (forcing signal) is present in the grid circuit, as indicated in the figure. The differential equations for the system in terms of the current i in the grid circuit, the current i_a in the anode circuit, and the grid potential v_g are readily derived; they are

$$\left.\begin{aligned} L\frac{di}{dt} + ri + v_g - M\frac{di_a}{dt} &= P_o \text{Sin } w_1 t \\ C\frac{dv_g}{dt} &= i \end{aligned}\right\} \qquad (1)$$

In deriving the above equation the current in grid itself is ignored. Assuming that the anode current i_a depends only upon the potential v_g between the grid and cathode, then the relation between these two quantities is as follows:

$$i_a = H\, v_g \left[1 - \frac{v_g^{\,2}}{3N^2} \right] \qquad (2)$$

where, H and N are positive constants. The quantity H is sometimes called the steepness of the characteristic and N is called the saturation potential. Both i and i_a in (1) can be replaced in terms of v_g through use of (2). For convenience the following new quantities are introduced.

$$e = v_g/N, \quad A' = (MH - rC)/LC, \quad B' = MH/3LC$$

$$F = P_o/N \quad \text{and} \quad w_o^{\,2} = 1/LC$$

Equation (1) can now be written as

$$\frac{d^2e}{dt^2} - A'\frac{de}{dt} + B'\frac{d}{dt}(e^3) + w_o^{\,2}e = Fw_o^{\,2}\,\text{Sin}\, w_1 t$$

The differential equation represented by equation (3) is the van der Pol equation and has been the basis of discussion throughout the thesis.

APPENDIX II - SOLUTION OF NON-LINEAR DIFFERENTIAL EQUATION

The nonlinear differential equation representing non-linear negative resistance oscillator, driven by an injected simple-harmonic current is given (Equation (12)) as follows.

$$\ddot{e} - A(1 - Be^2)\,\dot{e} + w_o^2 e = -\left(\frac{Iw_1}{C}\right)\text{Sin}(w_1 t + \phi)$$

The meanings of the different terms have already been explained in Chapter 1. In general, it might be expected that a solution of the above equation would involve both a free oscillations and a forced oscillations produced by the driving current, If parameter A is small compared with unity, the free oscillations have been found to be essentially sinusoidal. It is not unreasonable to expect that the forced oscillations will also be essentially sinusoidal for this condition. Suppose that the approximate solution of the above equation has the form

$$e = E\ \text{Cos}\ wt + E_1\ \text{Cos}\ w_1 t$$

where, the first term represents the free oscillations at a frequency w and second term represents the forced oscillations at the driving frequency w_1. The four unknown quantities in the solution are the two amplitudes E and E_1, the frequency w of the free oscillations, and the phase angle ϕ. Four simultaneous equations for determining these unknowns can be set up by substituting the assumed solution into the circuit nonlinear differential equation. Thus,

$$\dot{e} = -wE \sin wt - w_1E_1 \sin w_1t$$

$$\ddot{e} = -w^2E \cos wt - w_1^2E_1 \cos w_1t$$

$$e^2\dot{e} = -\frac{wE^3}{4}(\sin wt + \sin 3wt)$$

$$-\frac{wE^2E_1}{2}\left[\sin(2w + w_1)t + \sin(2w - w_1)t\right]$$

$$-\frac{wEE_1^2}{4}\left[2 \sin wt + \sin(w + 2w_1)t + \sin(w - 2w_1)t\right]$$

$$-\frac{wE_1E^2}{4}\left[2 \sin w_1t + \sin(w_1 + 2w)t + \sin(w_1 - 2w)t\right]$$

$$-\frac{w_1EE_1^2}{2}\left[\sin(2w_1 + w)t + \sin(2w_1 - w)t\right]$$

$$-\frac{w_1E_1^3}{4}\left[(\sin w_1t + \sin 3w_1t)\right]$$

According to the principle of harmonic balance, only terms containing fundamental frequencies (w and w_1) of the free and forced oscillations, are considered in the following calculations. All other terms are neglected. Also, it is assumed that the two frequencies are different i.e. $w \neq w_1$. On substituting the above relations in the nonlinear differential equation and collecting terms, the following four relations are obtained.

For Cos wt: $E(w_o^2 - w^2) = 0$

For Sin wt: $Aw w_o \left[1 - \frac{B}{4}(E^2 + 2E_1^2)\right] = 0$

For Cos w_1t: $E_1(w_o^2 - w_1^2) = -\frac{Iw_1}{C} \operatorname{Sin} \phi$

For Sin w_1t: $Aw_1 w_o E_1 \left[1 - \frac{B}{4}(E_1^2 + 2E^2)\right] = -\frac{Iw_1}{C} \operatorname{Cos} \phi$

From the above relations it is evident that conditions could easily be obtained (Chapter 2) which satisfy the assumed solution of differential equation (i.e. conditions which give either free, forced, or free and forced oscillations).

APPENDIX III - IBM 7040 COMPUTER PROGRAMMES

The program for solving nonlinear differential equation having a forcing function is based on Runge-Kutta process. The numerical values of T, x and y are calculated step by step. A graph plot can be made for amplitude of oscillation (x) with respect to time (T). The number of zero crossings per unit time obtained from these results give the frequency of oscillations for either free oscillations, forced oscillations or noise perturbed oscillations depending upon the program used.

The second order nonlinear differential equation can be reduced to a system of two first order equations and is written in general form as follows,

$$\frac{dx}{dt} = f_1(T, x, y)$$

$$\frac{dy}{dt} = f\ (T, x, y)$$

Starting at T_o, x_o, y_o, the increments in x_o and y_o for the first increments in T are computed by means of the formulas

$$K_1 = f_1(T_o, x_o, y_o)\ DT$$

$$K_2 = f_1(T_o + \frac{DT}{2}, x_o + \frac{K_1}{2}, y_o + \frac{L_1}{2})\ DT$$

$$K_3 = f_1(T_o + \frac{DT}{2}, x_o + \frac{K_2}{2}, y_o + \frac{L_2}{2})\ DT$$

$$K_4 = f_1(T_o + DT,\ x_o + K_3,\ y_o + L_3)\ DT$$

$$DX = (K_1 + 2K_2 + 2K_3 + K_4)/6$$

$$L_1 = f_2(T_o,\ x_o,\ y_o)\ DT$$

$$L_2 = f_2(T_o + \frac{DT}{2},\ x_o + \frac{K_1}{2},\ y_o + \frac{L_1}{2})\ DT$$

$$L_3 = f_2(T_o + \frac{DT}{2},\ x_o + \frac{K_2}{2},\ y_o + \frac{L_2}{2})\ DT$$

$$L_4 = f_2(T_o + DT,\ x_o + K_3,\ y_o + L_3)\ DT$$

$$Dy = (L_1 + 2L_2 + 2L_3 + L_4)/6$$

To compute next increment, it is necessary only to replace T_o, x_o, y_o in the above formulas by T_1, x_1, y_1.

Small increments to the value of DT should be used while computing the x and y values as the changes in the slopes of the curves are quite steep. It is preferable that for every new system, the approximate values for initial guesses i.e. the starting points be obtained from the program first by having the values printed out after a few iterations. This is advisable in order to reduce the computer time and to speed up convergence to a great extent - and arrangements have been made in the program for this and to make it as general as possible.

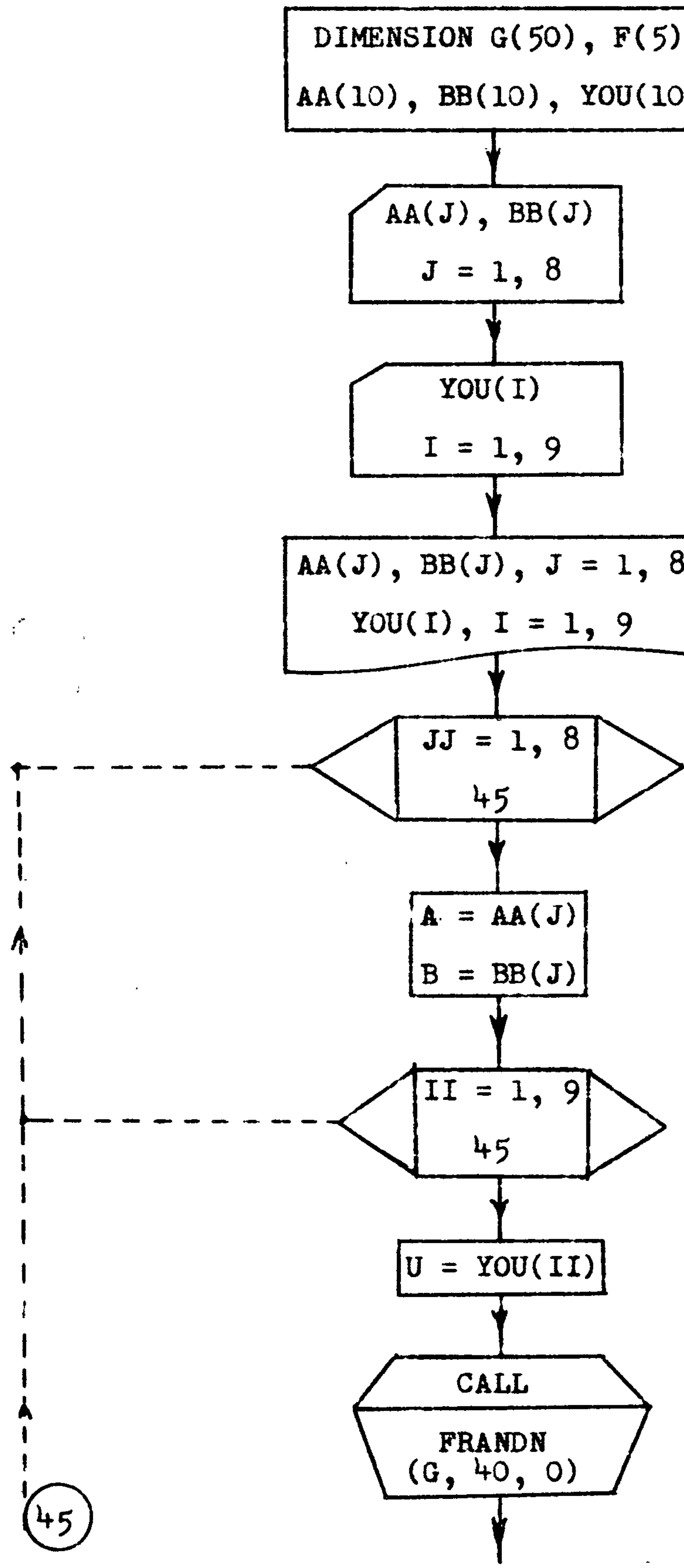
For varying Q of the tuned circuit the nonlinear constants A and B are changed while the amplitude of random number (noise) is controlled by the factor U.

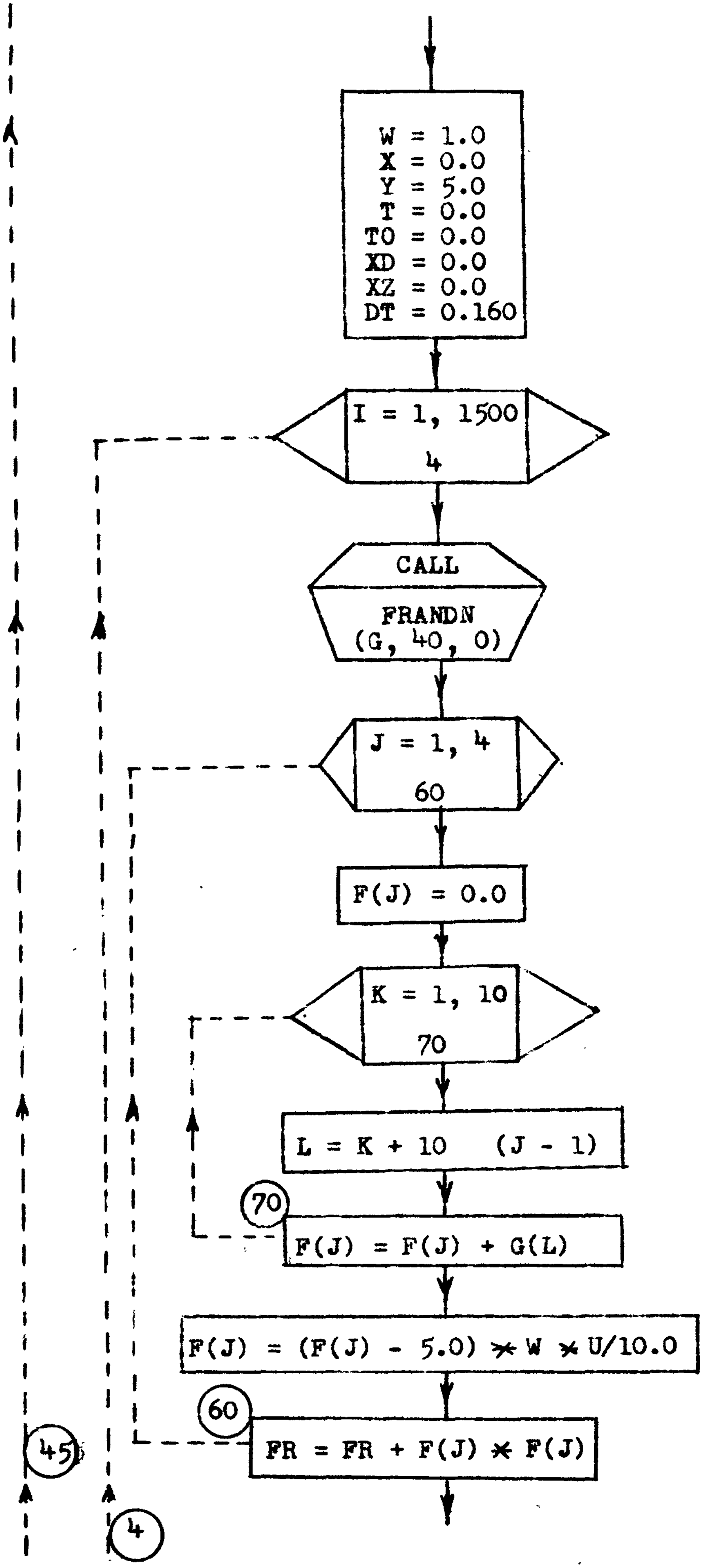
Call FRANDN (A, N, M) causes N pseudo-random numbers to be placed in the first N elements of the array A. A non zero M defines the starting point, a zero M continues from where the previous call left-off, or from a given starting point if this is the first call. If M is small compared with 10^{10} it may take 5 or 6 points before we get away from very small numbers, but otherwise the numbers are uniformly distributed over (0,1).

In order to get random forcing function having Gaussian distribution, the above subroutine was used. In our case 10 numbers are summed up from $-\frac{1}{2}$ to $+\frac{1}{2}$. This arrangement gives random number (noise) having Gaussian distribution and zero mean value. The amplitude of random number could also be varied by giving different values of U. (Programme No. 3)

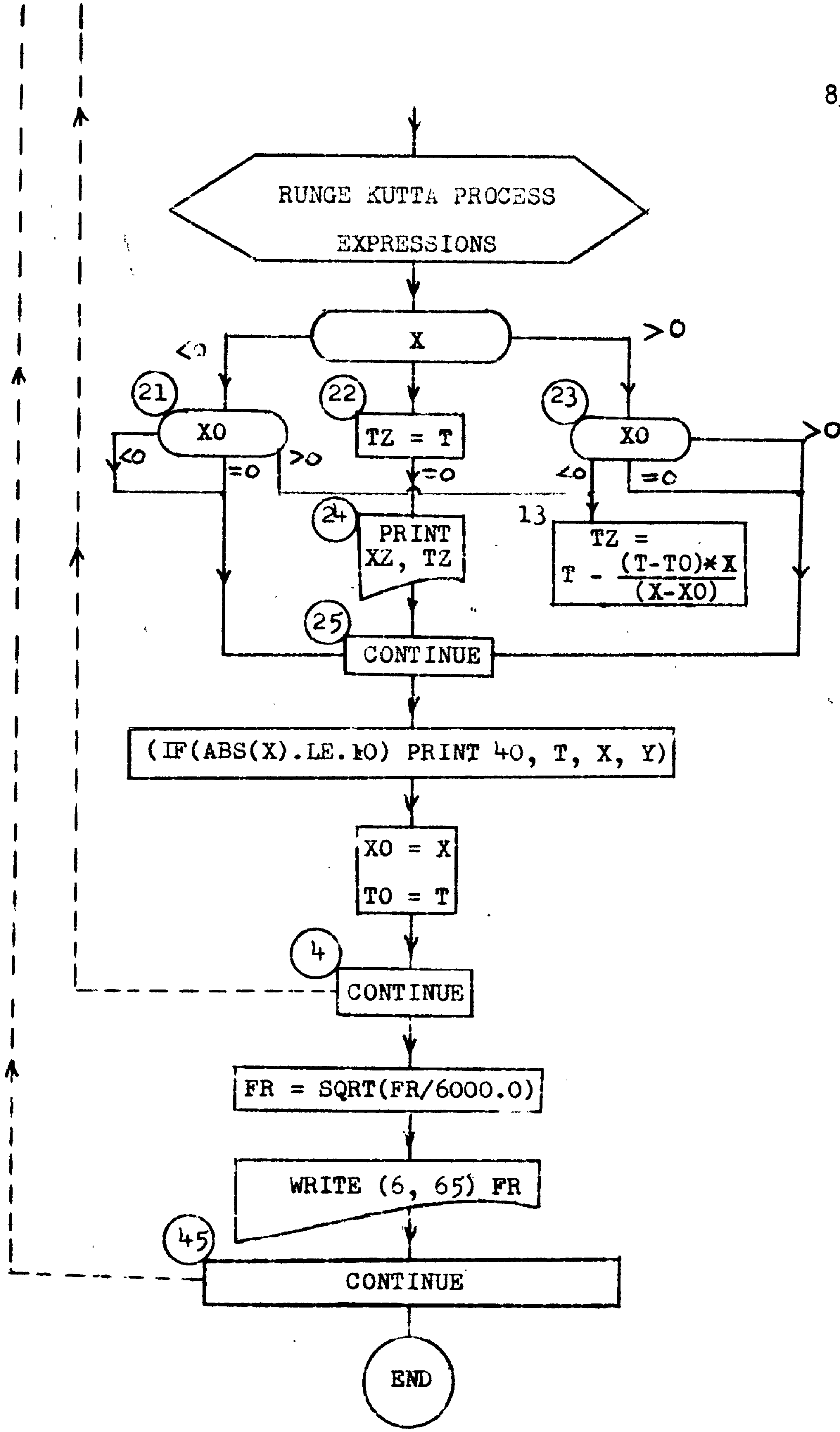
The following flow diagram illustrates the general steps in the calculation procedure of the first program. All other programs are similar and therefore self expla-natory.

ALGORITHM OF THE PROGRAM





W = 1.0
X = 0.0
Y = 5.0
T = 0.0
TO = 0.0
XD = 0.0
XZ = 0.0
DT = 0.160
I = 1, 1500
4
CALL
FRANDN
(G, 40, 0)
J = 1, 4
60
F(J) = 0.0
K = 1, 10
70
L = K + 10 (J - 1)
70
F(J) = F(J) + G(L)
F(J) = (F(J) - 5.0) * W * U/10.0
60
FR = FR + F(J) * F(J)
45
4


RUNGE KUTTA PROCESS
EXPRESSIONS
X
<0
>0
21
XO
<0
=0
>0
22
TZ = T
=0
23
XO
>0
<0
=0
13
24
PRINT
XZ, TZ
TZ =
T - (T-TO)*X / (X-XO)
25
CONTINUE
(IF(ABS(X).LE.10) PRINT 40, T, X, Y)
XO = X
TO = T
4
CONTINUE
FR = SQRT(FR/6000.0)
WRITE (6, 65) FR
45
CONTINUE
END

```
$JOB            003504V K AGARWAL        100
$IBJOB          NODECK
$IBFTC
C      PROGRAM NO.1, (RUNGE KUTTA PROCESS)
C      NON LINEAR DIFFERENTIAL EQUATION WITH RANDOM FORCING FUNCTION.
C      NOISE PERTURBED OSCILLATOR FREQUENCY DEVIATION W.R.TO NOISELESS-
C      OSCILLATOR,FOR VARYING NOISE VOLTAGE AND VARIOUS RF(Q)VALUES.
       DIMENSION G(50),F(5),AA(10),BB(10),YOU(10)
       READ(5,80)(AA(J),BB(J),J=1,8)
       READ(5,29)(YOU(I),I=1,9)
       WRITE(6,90)(AA(J),BB(J),J=1,8),(YOU(I),I=1,9)
       DO 45 JJ=1,8
       A=AA(JJ)
       B=BB(JJ)
       DO 45 II=1,9
       U=YOU(II)
       CALL FRANDN (G,42,1)
C      INITIAL CONDITIONS TO START OSCILLATIONS.
       W=1.0
       X=0.0
       Y=5.0
       T=0.0
       TO=0.0
       XO=0.0
       XZ=0.00
       DT=0.160
       DO 4 I=1,1500
C      CALL LIBRARY FUNCTION-TO GIVE RANDOM NUMBER (NOISE) HAVING-
C      GAUSSIAN DISTRIBUTION.
       CALL FRANDN (G,40,0)
       DO 60 J=1,4
       F(J)=0.0
       DO 70 K=1,10
       L=K+10*(J-1)
    70 F(J)=F(J)+G(L)
       F(J)=(F(J)-5.0)*W*U/10.0
    60 FR=FR+F(J)*F(J)
C      RUNGE-KUTTA EXPRESSIONS.
       RK1=W*Y*DT
       RL1=-(W*X-A*(1.0-B*X*X)*Y-F(1))*DT
       RK2=W*(Y+RL1/2.0)*DT
       RL2=-(W*(X+RK1/2.0)-A*(1.0-B*(X+RK1/2.0)**2)*(Y+RL1/2.0)-F(2))*DT
       RK3=W*(Y+RL2/2.0)*DT
       RL3=-(W*(X+RK2/2.0)-A*(1.0-B*(X+RK2/2.0)**2)*(Y+RL2/2.0)-F(3))*DT
       RK4=W*(Y+RL3)*DT
       RL4=-(W*(X+RK3)-A*(1.0-B*(X+RK3)**2)*(Y+RL3)-F(4))*DT
       DX=(RK1+2.0*RK2+2.0*RK3+RK4)/6.0
       DY=(RL1+2.0*RL2+2.0*RL3+RL4)/6.0
       T=T+DT
       X=X+DX
       Y=Y+DY
```

```
C     EXTRAPOLATION-TO FIND TIME OF ZERO CROSSINGS.
   21 IF(XO) 25,25,13
   23 IF(XO) 13,25,25
   13 TZ=T-(T-TO)*(X)/(X-XO)
      GO TO 24
   22 TZ=T
   24 PRINT 30,XZ,TZ
   25 CONTINUE
C     IF(ABS(X).LE.1.0) PRINT40,T,X,Y
      XO=X
      TO=T
    4 CONTINUE
C     R.M.S.AMPLITUDE OF RANDOM NUMBER(NOISE).
      FR=SQRT(FR/6000.0)
      WRITE(6,65) FR
   45 CONTINUE
   29 FORMAT(9F8.5)
   30 FORMAT(1H+,90X,2F20.10/)
   40 FORMAT(1H ,3F30.10)
   65 FORMAT(F20.10)
   80 FORMAT(2F10.6)
   90 FORMAT(11HOINPUT DATA,9X,8F9.4/14HNOISE VOLTAGE,9F7.4//)
C     VARIOUS VALUES OF RESISTANCE RF(Q) USED FOR CALCULATION
C     OPEN CIRCUIT,200K,150K,100K,75K,50K,25K,12.5K
C     VARIOUS VALUES OF R.M.S.  NOISE VOLTAGES USED
C     0.0(0),0.023(0.25),0.055(0.55),0.082(0.90),0.18(2.0),0.365(4.0)
C     0.465(5.0),0.68(7.5),0.90(10.0)
      STOP
      END
$ENTRY
  0.1015     0.0592
  0.0965     0.062
  0.09483    0.063
  0.0915     0.066
  0.0882     0.068
  0.0815     0.074
  0.0615     0.098
  0.0215     0.279
 0.00    0.25     .55    0.90    2.0     4.0     5.0     7.5    10.0
$IBSYS

                                          CD TOT   0099
```

```
$JOB            003504V K AGARWAL       100   010
$IBJOB          NODECK
$IBFTC
C      PROGRAM NO.2-ALTERNATE METHOD
C      NON LINEAR  DIFF. EQUATION. (RUNGE-KUTTA METHOD),EFFECT OF RANDOM-
C      NOISE ON NON-LINEAR OSCILLATIONS,LOW Q(RF=50K),NOISE=0.18 VOLTS.
       DIMENSION G(50),F(5),Z(5)
       CALL FRANDN(G,48,1)
       W=1.0
       A=0.0815
       B=0.0740
       X=0.0
       Y=5.0
       T=0.0
       TO=0.0
       XO=0.0
       XZ=0.00
       DT=0.160
       DO 4 I=1,1500
       CALL FRANDN (G,40,0)
       DO 60 J=1,4
       F(J)=0.0
       DO 70 K=1,10
       L=K+10*(J-1)
    70 F(J)=F(J)+G(L)
    60 F(J)=(F(J)-5.0)*W*2.0/10.0
       CALL FRANDN (Z,4,0)
       RK1=W*Y*DT
       RL1=-(W*X-A*(1.0-B*X*X)*Y-(1.0+F(1))*(SIN(T+DT*(Z(1)-0.5)))*1.0)*D
      1T
       RK2=W*(Y+RL1/2.0)*DT
       RL2=-(W*(X+RK1/2.0)-A*(1.0-B*(X+RK1/2.0)**2)*(Y+RL1/2.0)-(1.0+F(2)
      1)*(SIN(T+DT*(Z(2)-0.5)/2.0))*1.0)*DT
       RK3=W*(Y+RL2/2.0)*DT
       RL3=-(W*(X+RK2/2.0)-A*(1.0-B*(X+RK2/2.0)**2)*(Y+RL2/2.0)-(1.0+F(3)
      1)*(SIN(T+DT*(Z(3)-0.5)/2.0))*1.0)*DT
       RK4=W*(Y+RL3)*DT
       RL4=-(W*(X+RK3)-A*(1.0-B*(X+RK3)**2)*(Y+RL3)-(1.0+F(4))*(SIN(T+DT*
      1(Z(4)-0.5)))*1.0)*DT
       DX=(RK1+2.0*RK2+2.0*RK3+RK4)/6.0
       DY=(RL1+2.0*RL2+2.0*RL3+RL4)/6.0
       T=T+DT
       X=X+DX
       Y=Y+DY
       IF(X) 21,22,23
    21 IF(XO) 25,25,13
    23 IF(XO) 13,25,25
    13 TZ=T-(T-TO)*(X)/(X-XO)
       GO TO 24
    22 TZ=T
    24 PRINT 30,XZ,TZ
```

```
   25 PRINT 40,T,X,Y
      XO=X
      TO=T
    4 CONTINUE
   30 FORMAT(1H+,90X,2F20.10/)
   40 FORMAT(1H ,3F30.10)
      STOP
      END
$ENTRY
$IBSYS
```

CD TOT 0069

```
$JOB            003504V K AGARWAL        100    010
$IBJOB          NODECK
$IBFTC
C      PROGRAM NO.3-USED AS A SUB-ROUTINE IN MAIN PROGRAM.
C      R.M.S.VOLTAGE AMPLITUDE OF RANDOM NOISE.
       DIMENSION G(50),F(5)
       CALL FRANDN (G,46,1)
   10  READ80,U
       W=1.0
       FR=0.0
       DO 4 I=1,1500
       CALL FRANDN (G,40,0)
       DO 60 J=1,4
       F(J)=0.0
       DO 70 K=1,10
       L=K+10*(J-1)
   70  F(J)=F(J)+G(L)
       F(J)=(F(J)-5.0)*W*U/10.0
   60  FR=FR+F(J)*F(J)
    4  CONTINUE
       FR=SQRT(FR/6000.0)
       WRITE(6,65) FR
       GO TO 10
   65  FORMAT (F20.10)
   80  FORMAT(F6.3)
$ENTRY
 0.25
 0.55
 0.9
 2.0
 4.0
 5.0
 7.5
10.0
$IBSYS

                                  CD TOT   0035
```

```
$JOB            003504V K AGARWAL        100   010
$IBJOB          NODECK
$IBFTC
C      PROGRAM NO.4-FORCED OSCILLATIONS.
C      NON LINEAR  DIFF  EQUATION. (RUNGE-KUTTA METHOD)
C      WITH SIN FORCING FUNCTION.
       W=1.0
       A=0.1015
       B=0.0592
       X=0.0
       Y=5.0
       T=0.0
       TO=0.0
       XO=0.0
       XZ=0.00
       DT=0.160
       DO 4I=1,500
       RK1=W*Y*DT
       RL1=-(W*X-A*(1.0-B*X*X)*Y-SIN((T+DT)*W))*DT
       RK2=W*(Y+RL1/2.0)*DT
       RL2=-(W*(X+RK1/2.0)-A*(1.0-B*(X+RK1/2.0)**2)*(Y+RL1/2.0)-SIN((T+DT
      1/2.0)*W))*DT
       RK3=W*(Y+RL2/2.0)*DT
       RL3=-(W*(X+RK2/2.0)-A*(1.0-B*(X+RK2/2.0)**2)*(Y+RL2/2.0)-SIN((T+DT
      1/2.0)*W))*DT
       RK4=W*(Y+RL3)*DT
       RL4=-(W*(X+RK3)-A*(1.0-B*(X+RK3)**2)*(Y+RL3)-SIN((T+DT)*W))*DT
       DX=(RK1+2.0*RK2+2.0*RK3+RK4)/6.0
       DY=(RL1+2.0*RL2+2.0*RL3+RL4)/6.0
       T=T+DT
       Y=Y+DY
       X=X+DX
       IF(X) 21,22,23
    21 IF(XO) 25,25,13
    23 IF(XO) 13,25,25
    13 TZ=T-(T-TO)*(X)/(X-XO)
       GO TO 24
    22 TZ=T
    24 PRINT 30,XZ,TZ
    25 PRINT 40,T,X,Y
       XO=X
       TO=T
     4 CONTINUE
    30 FORMAT(1H+,90X,2F20.10/)
    40 FORMAT(1H ,3F30.10)
       STOP
       END
$ENTRY
$IBSYS

                                  CD TOT   0049
```

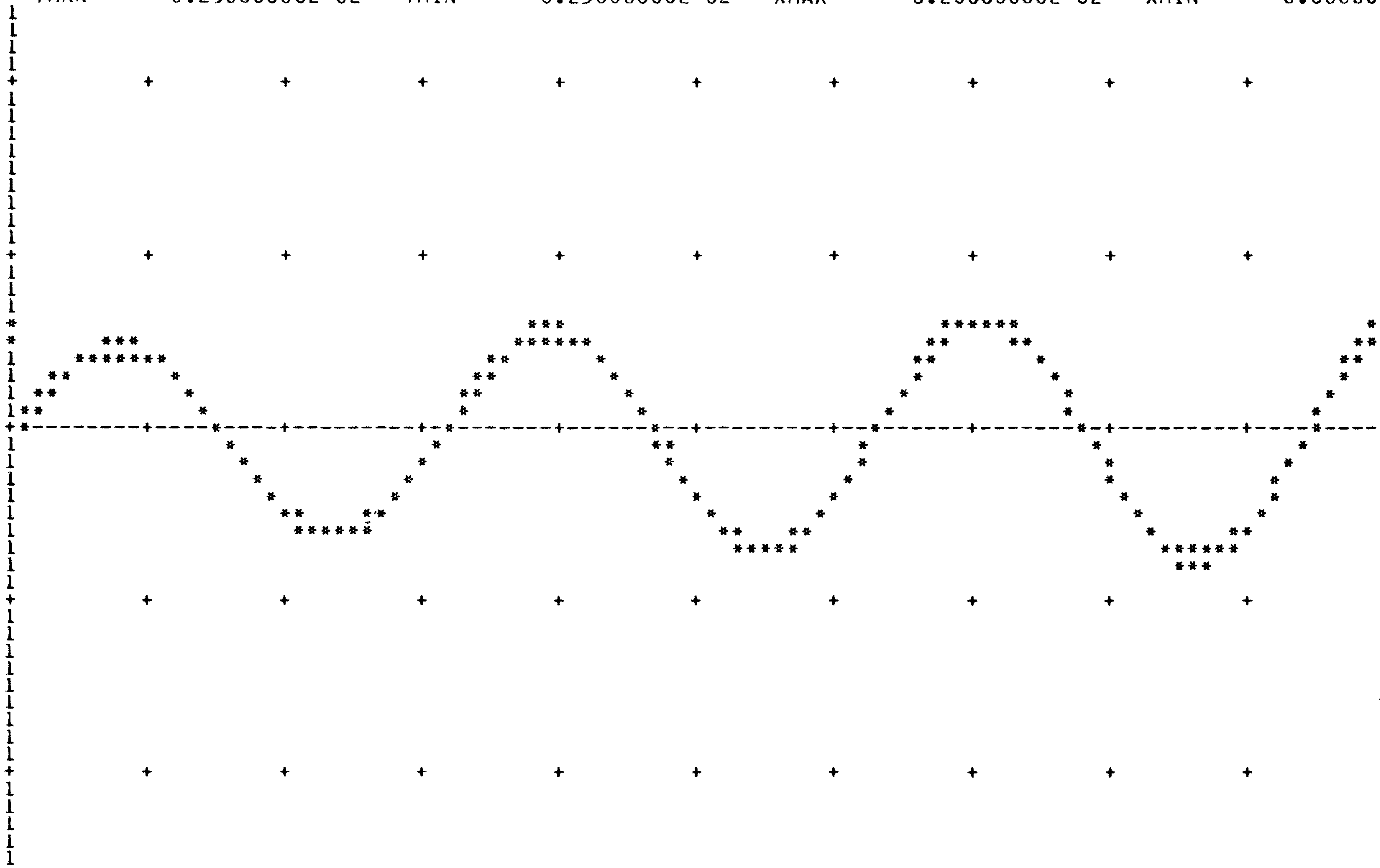
```
$JOB            003504V K AGARWAL        100    010
$IBJOB          NODECK
$IBFTC
C      PROGRAM NO.5-FREE OSCILLATIONS.
C      NON LINEAR  DIFF  EQUATION. (RUNGE-KUTTA METHOD)
C      WITH NO FORCING FUNCTION.
       W=1.0
       A=0.1015
       B=0.0592
       X=0.0
       Y=5.0
       T=0.0
       TO=0.0
       XO=0.0
       XZ=0.00
       DT=0.160
       DO 4 I=1,1500
       RK1=W*Y*DT
       RL1=-(W*X-A*(1.0-B*X*X)*Y)*DT
       RK2=W*(Y+RL1/2.0)*DT
       RL2=-(W*(X+RK1/2.0)-A*(1.0-B*(X+RK1/2.0)**2)*(Y+RL1/2.0))*DT
       RK3=W*(Y+RL2/2.0)*DT
       RL3=-(W*(X+RK2/2.0)-A*(1.0-B*(X+RK2/2.0)**2)*(Y+RL2/2.0))*DT
       RK4=W*(Y+RL3)*DT
       RL4=-(W*(X+RK3)-A*(1.0-B*(X+RK3)**2)*(Y+RL3))*DT
       DX=(RK1+2.0*RK2+2.0*RK3+RK4)/6.0
       DY=(RL1+2.0*RL2+2.0*RL3+RL4)/6.0
       T=T+DT
       X=X+DX
       Y=Y+DY
       IF(X) 21,22,23
    21 IF(XO) 25,25,13
    23 IF(XO) 13,25,25
    13 TZ=T-(T-TO)*(X)/(X-XO)
       GO TO 24
    22 TZ=T
    24 PRINT 30,XZ,TZ
    25 PRINT 40,T,X,Y
       XO=X
       TO=T
     4 CONTINUE
    30 FORMAT(1H+,90X,2F20.10/)
    40 FORMAT(1H ,3F30.10)
       STOP
       END
$ENTRY
$IBSYS

                                     CD TOT    0047
```

```
JOB           003504V K AGARWAL        100    010
IBJOB         NODECK
IBFTC
      PROGRAM NO.6-FREE OSCILLATIONS.
      PLOTTING OF GRAPH,(NO FORCING FUNCTION)
      NON LINEAR  DIFF  EQUATION. (RUNGE-KUTTA METHOD)
      DIMENSION T(100),X(100)
      W=1.0
      A=0.1015
      B=0.0592
      X(1)=0.0
      T(1)=0.0
      Y=5.0
      DT=0.20
      DO 4 I=1,100
      X=X(I)
      T=T(I)
      RK1=W*Y*DT
      RL1=-(W*X-A*(1.0-B*X*X)*Y)*DT
      RK2=W*(Y+RL1/2.0)*DT
      RL2=-(W*(X+RK1/2.0)-A*(1.0-B*(X+RK1/2.0)**2)*(Y+RL1/2.0))*DT
      RK3=W*(Y+RL2/2.0)*DT
      RL3=-(W*(X+RK2/2.0)-A*(1.0-B*(X+RK2/2.0)**2)*(Y+RL2/2.0))*DT
      RK4=W*(Y+RL3)*DT
      RL4=-(W*(X+RK3)-A*(1.0-B*(X+RK3)**2)*(Y+RL3))*DT
      DX=(RK1+2.0*RK2+2.0*RK3+RK4)/6.0
      DY=(RL1+2.0*RL2+2.0*RL3+RL4)/6.0
      X(I+1)=X(I)+DX
      T(I+1)=T(I)+DT
      Y=Y+DY
      PRINT 40,T(I),X(I),Y
    4 CONTINUE
      CALL PLOT1(X,25.0,-25.0,20.0,0.0,50,100,10)
   40 FORMAT(1H ,3F30.10)
      STOP
      END
ENTRY
IBSYS

                          CD TOT   0038
```

YMAX = 0.25000000E 02 YMIN = -0.25000000E 02 XMAX = 0.20000000E 02 XMIN = 0.00000000E



03504V K AGARWAL 100 001MIN 21SEC COST$004.88 REM. TIME 0023MIN 33SEC

```
$JOB           003504V K AGARWAL       100   010
$IBFTC
$IBJOB         NODECK
C       PROGRAM NO.7-FREE AND FORCED OSCILLATIONS.
C       GRAPH PLOT OF FORCING VOLTAGE W.R.TO PERCENTAGE FREQUENCY DEVIAT.
C       NECESSARY AND SUFFICIENT CONDITION FOR LOCKING TO OCCOUR.
        DIMENSION RF(15),D(15)
        READ 20,(RF(I),I=1,10)
        READ 30,(D(J),J=1,12)
        RL=100000.0
        S=1.115E-04
        T=2.000E-06
        EL1=660.0E-06
        C1=660.0E-12
        DO 10 I=1,10
        DO 10 J=1,12
        R=RL*RF(I)/(RL+RF(I))
        A=(S-1.0/R)*(EL1/C1)**0.5
        B=3.0*T/(S-1.0/R)
        E=2.0/B**0.5
        V=(RF(I)*RF(I)*8.0*C1/(B*EL1)*(D(J)*D(J)+A*A/16.0))**0.5
     10 PRINT40,A,B,R,RF(I),D(J),V,E
     20 FORMAT(7F10.2/3F10.2)
     30 FORMAT(12F6.3)
     40 FORMAT(1H ,F15.7,5X,F6.3,5X,F8.1,5X,F8.1,5X,F8.3,5X,F15.5,5X,F8.3)
        STOP
        END
$ENTRY
1000000.0  200000.0  150000.0  100000.0   75000.0   60000.0   50000.0
  25000.0   12500.0   10000.0
 0.00  0.005 0.01  0.02  0.03  0.04  0.05  0.06  0.07  0.08  0.09  0.10
$IBSYS

                              CD TOT   0032
```

APPENDIX IV - NOISE EXCITATION EFFECT OF AN ELEMENTARY PULSE

$$e = E \operatorname{Sin}(w_o t + \phi) \qquad (1)$$

It has been explained on Page 30 that one pulse causes a change in $\frac{de}{dt}$ while e remains constant. Thus,

$$\delta(e) = 0 \qquad (2)$$

$$\text{and } \delta\left(\frac{de}{dt}\right) = \frac{\delta}{dt}\left(\frac{de}{dt}\right)\cdot\Delta t$$

$$= -Ew_o^{\,2} \operatorname{Sin}(w_o t + \phi)\cdot\Delta t$$

$$\delta\left(\frac{de}{dt}\right) = qw_o^{\,2} \qquad (3)$$

For simplicity it is assumed throughout the analysis that $\delta E = \delta a$.

$$e = E\left[\operatorname{Sin} w_o t \operatorname{Cos} \phi + \operatorname{Cos} w_o t \operatorname{Sin} \phi\right]$$

$$\delta(e) = \delta E\left[\operatorname{Sin} w_o t \operatorname{Cos} \phi + \operatorname{Cos} w_o t \operatorname{Sin} \phi\right]$$

$$+ \delta\phi\left[E \operatorname{Cos} w_o t \operatorname{Cos} \phi - E \operatorname{Sin} w_o t \operatorname{Sin} \phi\right]$$

$$= \delta E\left[\operatorname{Sin}(w_o t + \phi)\right] + E\cdot\delta\phi\left[\operatorname{Cos}(w_o t + \phi)\right]$$

From (2), $\delta(e) = 0$. Replacing δE by δa we have,

$$\delta a\left[\operatorname{Sin}(w_o t + \phi)\right] + E\cdot\delta\phi\left[\operatorname{Cos}(w_o t + \phi)\right] = 0 \qquad \ldots\ldots(4)$$

Again from (1),

$$\frac{de}{dt} = Ew_o \left[\text{Cos } w_o t \text{ Cos } \phi - \text{Sin } w_o t \text{ Sin } \phi \right]$$

$$\delta\left[\frac{de}{dt}\right] = \delta E . w_o \left[\text{Cos } (w_o t + \phi) \quad - E\,\delta\phi . w_o \left[\text{Sin } (w_o t + \phi) \right]\right.$$

From (3)

$$qw_o{}^2 = \delta a . w_o \left[\text{Cos } (w_o t + \phi) \right] - E\,\delta\phi \cdot w_o \left[\text{Sin } (w_o t + \phi) \right] \quad ......(5)$$

From (4) and (5) we have,

$$qw_o^2 = - w_o E \cdot \delta\phi \frac{\text{Cos}^2 (w_o t + \phi)}{\text{Sin } (w_o t + \phi)} - Ew_o \cdot \delta\phi \left[\text{Sin } (w_o t + \phi) \right]$$

$$= - \frac{w_o E \cdot \delta\phi}{\text{Sin } (w_o t + \phi)} \left[\text{Cos}^2 (w_o t + \phi) + \text{Sin}^2 (w_o t + \phi) \right]$$

Therefore,

$$\delta\phi = - \frac{qw_o{}^2}{w_o E} \text{ Sin } (w_o t + \phi)$$

Similarly from relation (4)

$$\delta a = qw_o \text{ Cos } (w_o t + \phi)$$

Thus, the change δa in amplitude and $\delta\phi$ in phase produced by one pulse occuring at time t_j is given by,

$$\delta a = qw_o \text{ Cos } (w_o t_j)$$

$$\delta\phi = - \frac{qw_o}{E} \text{ Sin } (w_o t_j)$$

APPENDIX V - RELATION OF RESISTANCE R_f TO Q OF TUNED CIRCUIT

Refer figure 3 of Chapter 1

$$R = R_f R_L / (R_f + R_L) \tag{1}$$

For a parallel tuned circuit

$$Q = R/w_o L \tag{2}$$

From equations (1) and (2)

$$R_f = Qw_o L \cdot R_L / (R_L - w_o L)$$

For the unloaded circuit, let $Q_o = R_L/w_o L$ (3)

Thus, $R_f = R_L\left[Q/(Q_o - Q)\right]$

$$\text{or } Q = Q_o \Big/ \left[\frac{R_L}{R_f} + 1\right] \tag{4}$$

In the experimental circuit,

For R_L = 100 K ohms, frequency of oscillation (f_o) = 242 kc/s

and L = 660 μ henry

$Q_o = 100$ - [from relation (3)]

Various values of Q are calculated for corresponding values of R_f and are tabulated in Table No. 2 (Chapter 4)

APPENDIX VI - RESULTS OF COMPUTER AND EXPERIMENTAL STUDY

COMPUTER STUDY - DEVIATION OF THE MEAN FREQUENCY FOR NOISE-PERTURBED OSCILLATOR

NOISE VOLTAGE (r.m.s volts) / R_f (KΩ) or (Q)	MEAN FREQUENCY DEVIATION (C/S) W.R. TO FREQ. OF NOISELESS OSCILLATOR (With Q=100)							
	0.00	0.055	0.082	0.18	0.365	0.465	0.68	0.91
200 (66.6)	83.6	84.5	85.75	87.0	89.1	90.75	94.1	96.5
150 (60.0)	122.0	125.0	128.1	132.0	137.0	140.0	143.5	148.75
100 (50.0)	181.0	183.9	185.5	188.5	193.0	195.5	201.0	206.5
75 (42.85)	237.5	239.0	240.75	244.5	247.5	252.0	258.0	264.0
50 (33.3)	307.0	310.0	314.0	318.0	324.0	326.5	331.0	337.0
25 (20.0)	488.0	491.5	494.5	499.75	505.0	508.5	515.0	522.0

EXPERIMENTAL RESULTS

(A) NO EXTERNAL NOISE

Resistance R_f (K ohms)	Mean frequency in cycles/sec.	Deviation in frequency c/s.
R_f = Open Circuit	242030	0
225	242102	72
200	242115	85
175	242136	106
150	242157	127
125	242183	153
100	242221	191
75	242272	242
50	242326	296
25	242270	240

(B) R. M. S. NOISE-VOLTAGE = 0.082 volts

Resistance R_f (K ohms)	Mean frequency in cycles/sec.	Deviation in frequency c/s
Open Circuit	242030.5	0
225	242104.0	73.5
200	242118.0	87.5
175	242139.5	109.0
150	242160.5	130.0
125	242186.5	156.0
100	242225.0	194.5
75	242276.5	246.0
50	242331.0	300.5
25	242276.0	245.5

(C) R.M.S. NOISE VOLTAGE = 0.37 VOLTS

Resistance R_f (K ohms)	Mean frequency in cycles/sec.	Deviation in frequency c/s
Open Circuit	242031.0	0
225	242112.0	81.0
200	242119.5	88.5
175	242146.5	115.5
150	242167.0	136.0
125	242194.5	163.5
100	242225.0	194.0
75	242276.0	245.0
50	242347.5	316.5

(D) R. M. S. NOISE VOLTAGE = 0.91 VOLTS

Resistance R_f (K ohms)	Mean frequency in cycles/sec.	Deviation in frequency c/s
Open circuit	242031.0	0
200	242128.0	97.0
175	242154.0	123.0
150	242181.5	150.5
125	242204.0	173.0
100	242238.0	207.0
75	242293.0	262.0
50	242358.5	327.5

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