NON-LINEAR OSCILLATIONS IN TUNNEL DIODE

STUDY OF NON-LINEAR OSCILLATIONS

USING

TUNNEL DIODE

By

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SCOPE AND CONTENTS: Applications of the tunnel diode have been studied mainly as a large-signal oscillator and as a switching device. A tunnel diode oscillator has been analysed and studied in the small-signal, large-signal and the relaxation mode of operation. It is shown that the oscillating voltage and the current of the tunnel diode oscillator satisfy Van der Pol and the Raleigh equation respectively and how with increasing damping factor α of the Van der Pol equation, the non-linear oscillation changes from almost sinusoidal to relaxation type. The large-signal oscillation is solved graphically by Lienard method of construction of the limit cycle. Relaxation oscillations have been studied in detail and by means of experimental results it is shown how a tunnel diode relaxation oscillator can be used as a square-wave and staircase wave generators. The impedance of the diode has been analysed and the stability criteria as well as the conditions necessary for various modes of operation has been established.

Switching applications of the diode have been discussed along with the experimental results obtained from monostable and bistable operation. Other useful switching applications of the diode, e.g. as an amplitude discriminater, as a square-wave generator from sine waves and as a substitute for Schmidt trigger circuit, have been suggested along with experimental evidence.

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I Introduction

Since its introduction into the field of Electronics by Leo Esaki in 1957, the tunnel diode has opened up new Engineering perspectives. It is a two-terminal active semi-conductor device that exhibits a voltage-controlled negative-resistance over a portion of its V-I characteristics and thereby showing the prospects of manifold applications. It has a remarkable history in the sense that within a very short time of its arrival, it has attracted the attentions of thousands of the circuit designers who have thoroughly studied, analysed and already applied it for many useful purposes, specially in the field of Computers. This is because tunnel diode possesses several unique advantages over other known negative-resistance devices. It has also got its disadvantages and limitations which have disappointed most of its admirers by presenting some seemingly insoluble problems, but it is hoped that with the advent of new techniques these short-comings could easily be overcome.

A notable characteristic of the tunnel diode, is its fast switching speed. The inherent junction capacitance along with the series resistance and inductance tend to slow the switching time.

It has a theoretical frequency limit of about 10^7 megacycle per second. Up till now, diode with a cut-off frequency of about 200 kMc has been developed.

The tunnel diode is also capable of operation over wide temperature ranges and can withstand relatively large doses of nuclear radiation. These characteristics are due to the heavy doping and the lack of dependence on minority carrier life-time.

The heavy doping also results in a tolerance of surface contamination.

Another desirable characteristic of the diode is its extreme mechanical simplicity, having only two terminals. This makes the device to be manufactured at a reasonably low cost. Its low power consumption and miniature size also make it more attractive.

The above factors account for the interest which the tunnel diode has excited since its introduction in the field of Computers.

Because of its non-linear characteristic, a tunnel diode is very well suited for use as a self-excited converter. Operation in the negative-resistance region leads to components such as oscillator and amplifier with a good noise figure. It can be used as a harmonic generator, amplitude discriminator, limiter and non-sinusoidal pulse generator.

In parallel with a suitable resistance it can be used as a current source.

Prominent problems associated with its incorporation into the practical circuits are (i) spurious oscillation, (ii) requires low-impedance source, (iii) low voltage levels, (iv) very small output power. The most important of all is that it being a two-terminal device, complete isolation between the input and the output poses a great problem.

The aim of this work is to study the behaviour of the tunnel diode mainly as a large-signal oscillator and as a switching device.

The theory of small-signal operation of tunnel diode oscillator has been dealt with in many literatures assuming a constant negative resistance of the diode. Obviously, none of these analyses are satisfactory because of the fact that non-linearity must be present in a negative-resistance oscillator to provide amplitude limiting and as a result both the positive and the negative-resistance regions are involved in such an oscillator. Hence, sinusoidal oscillations may never be possible in such a circuit and the analysis must deal with the large-signal vehaviour and the relaxation oscillation.

Unfortunately, the theory of large-signal and the relaxation oscillator is in a very unstaisfactory state of development. Analytical methods are valid only for small-signal operation. For large-signal and relaxation behaviour both the analytical as well as the topological method must be used simultaneously, because a complete solution of the non-linear oscillator, i.e. wave shape, amplitude and frequency cannot be arrived at by either of the methods alone.

For analytical methods, the non-linearity must be expressed as a power series to get the accurate information, but it is very difficult to evaluate the coefficients unless a Computer is used. This difficulty could be avoided if the V-I characteristic is symmetrical, so that the non-linearity could be approximated by a cubic polynomial. Although inaccurate, this assumption has been made in the present analysis. It is shown that the voltage output of such an oscillator satisfies Van der Pol equation, whereas the current satisfies Raleigh equation.

This thesis contains the result of the complete investigation of a tunnel diode oscillator, for small-signal, large-signal and relaxation operation, how with the increasing damping factor α of the Van der Pol equation, the oscillation changes from almost sinusoidal to relaxation type. The relaxation oscillation is solved graphically be Lienard method of construction of limit cycle. By means of experimental result, it is shown how a tunnel diode relaxation can be used as a square wave and stair-case wave generator The equivalent impedance of the diode has been analysed and the stability criteria as well as the necessary conditions for various modes of operation have been established. The effect of the circuit parameters on the stability of the singularities is also considered.

Finally, the switching application of the diode is discussed and the experimental results are incorporated both for monostable and bistable modes of operation. The possibility of cascading several tunnel diode binaries in a scaler is also discussed along with the experimental evidence. In the switching region, various other useful applications of the diode, e.g. as an amplitude discriminator, as a square wave generator from a sine-wave, as a sbustitute for Schmidt trigger circuit, have been suggested and the experimental results obtained are shown.

II Negative-Resistance Devices in General

The theory of negative-resistance is quite established and its concept is properly understood now-a-days. Many devices posses a region of negative slope, i.e. decreasing current for an increasing voltage, in their V-I characteristic and as such, they exhibit the property of dynamic negative resistance in that particular region.

A negative resistance is not actually the reverse of a positive resistance but it is a two-terminal device with an internal source[1] of energy which is controlled either by the current flowing through it or by the voltage across the terminals but not by both.

A current produces a voltage drop in a positive resistance and hence a voltage rise in a negative resistance. A positive resistance is characterised by dissipation of energy whereas a negative resi tance implies a source of energy. Since any physically realizable source can supply only a finite amount of energy, a realizable resistance can be negative only over a limited range - a property not associated with positive resistance. Hence, a device possessing a negative resistance must be non-linear in character since a linear negative resistance implies an infinite source of energy.

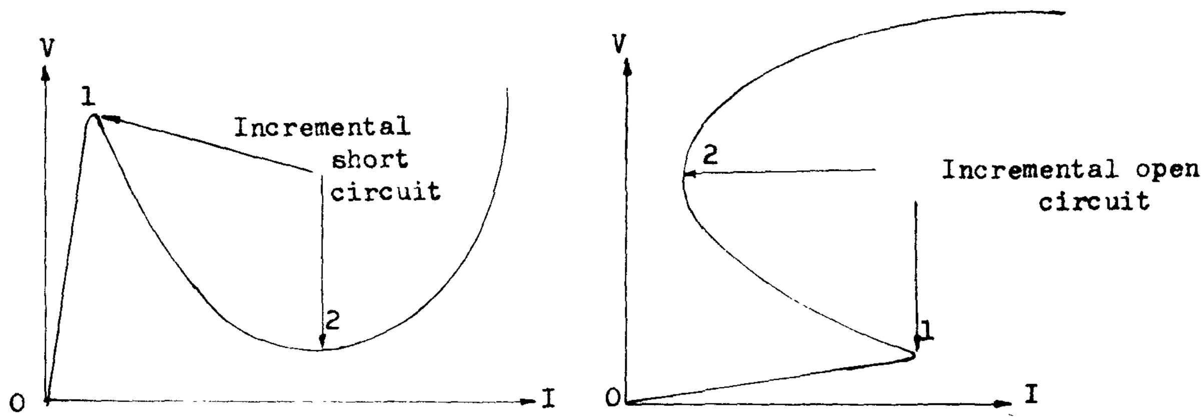
Accordingly, the negative resistances are classified into two classes:

(a) Current - controlled or N-type or open circuit stable;

(b) Voltage - controlled or S-type or short-circuit stable.

In a current-cohtrolled negative resistance, the voltage is a single-valued function of current, and not vice-versa. This is shown in Figure II.1A.

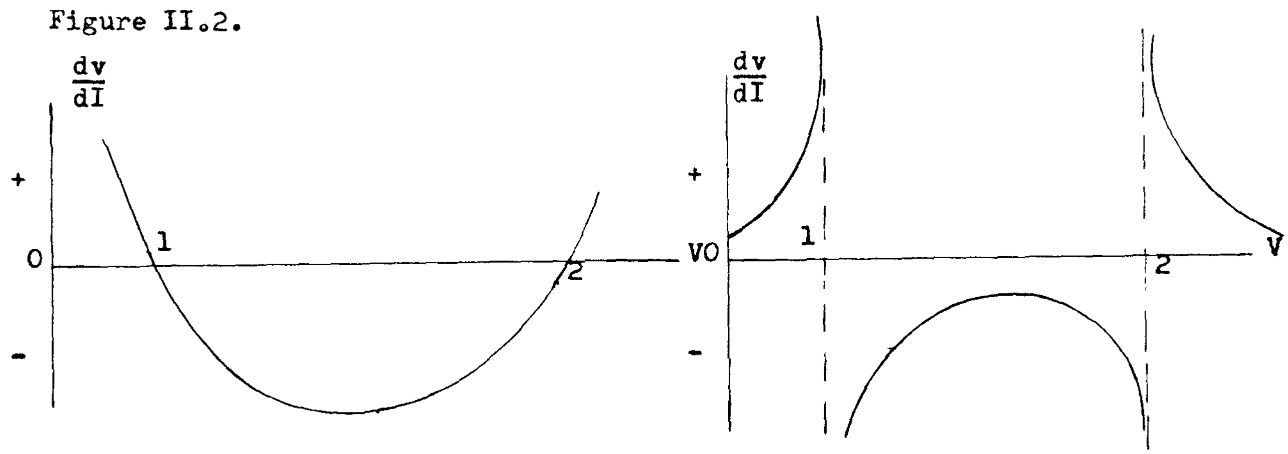
In a voltage-controlled type, the current is a unique function of voltage and not vice-versa, as shown in Figure II.1B.



A. Current-controlled or N-type B. Voltage-controlled or S-type

Figure II.1

In the case of current-controlled, the incremental dynamic resistance passes through zero before changing the sign. In voltage-controlled negative resistance, the incremental dynamic resistance passes through infinity before changing its sign. This is shown in Figure II.2.



A. Current-controlled or N-type. B. Voltage-controlled or S-type

Figure II.2

An S-type negative resi·tance is stable when the external positive resistance is less than the negative resistance, whereas an N-type negative-resistance is stable when the external positive resi·tance is greater than the negative resistance.

A voltage-controlled negative resistance is suitable for exciting oscillation in a parallel tuned circuit, but for an N-type negative resistance a series tuned circuit is needed.

If capacitance of a parallel tuned circuit is made negligible and the resistance of the inductance is very low, the connection to an S-type negative resistance with limited negative slope will result in oscillation whose period depends on a relaxation time.

Similarly, if the inductance of the series resonant circuit is made negligible, its connection with an N-type negative resistance will result in relaxation oscillation.

In a voltage-controlled resistance, the sense of rotation of the dynamic V-I characteristic loop is clockwise, and the effect of the circuit may be considered as the result of a negative resistance without lag shunted by a capacitance.

The corresponding loop in a current-controlled type is counter-clockwise so that the circuit effect is that of a negative resistance, without time lag in series with an inductance.

For an S-type the ratio of self-reactance to resistance at an angular frequency ω is $\omega C_v R_v$ which is a factor of poorness. Hence, its reciprocal $\frac{1}{\omega C_v R_v}$ is a figure of merit.

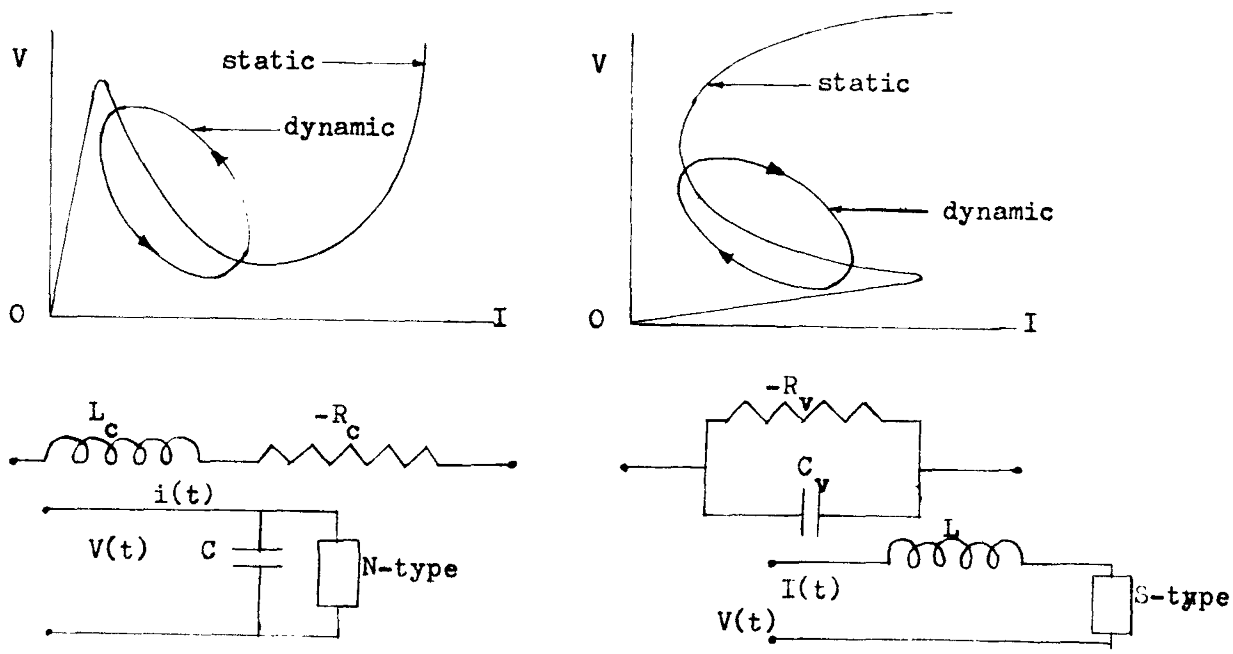
Similarly, for a current-controlled type, the figure of merit is $\frac{R_c}{\omega L_c}$.

In an N-type, since only current is unrestricted, it must have an energy-storage element allowing instantaneous current changes during the switching process. Hence, it must have a capacitor across it to allow for instantaneous current change during the oscillating cycle.

In a voltage-controlled negative resistance, an inductance in series with it is needed to permit rapid changes of voltage during the switching process.

Accordingly, an N-type negative resistance becomes unstable when the D.C. instability condition has been satisfied and the maintaining circuit is shunted by a capacitance.

And, an S-type negative resistance becomes unstable when the D.C. instability condition has been satisfied and the maintaining circuit is connected in series with an inductance. This is shown in Figure II.3.



A. Current-controlled or N-type. B. Voltage-controlled or S-type.

<u>Figure II.3</u>

Examples of practical devices having current-controlled negative resistance are

(a) Gaseous discharge tube

(b) Four-layer diodes and Controlled Rectifiers

(c) Unijunction and avalanche transistor

(d) Common base configuration of point-contact transistor using emitter-base terminals

(e) Using series triode circuit.

Devices showing voltage-controlled negative resistance are

(a) Tunnel diode

(b) Dynatron

(c) Transitron

(d) Point-contact transistor in common emitter configuration using base-emitter terminals.

The N-type and S-type negative resistances are duals of each other and the principle of duality is useful in transferring the knowledge of operation or design procedure from one type of device to another.

Circuit Duals

N-type	S-type
Voltage	Current
Resistance	Conductance
Inductance	Capacitance
Series	Parallel
Loop	Node

III The Mechanism of Tunneling Process

The tunnel diode utilizes the principle of quantum mechanical "tunneling" of carriers through a very thin P-N junction. The "tunnel effect" is the certain probability that an electron on one side of a potential barrier can leak through the barrier and appear instantaneously on the other side of the barrier even though it does not have enough energy to surmount this barrier. In order for the tunneling to take place the barrier must be very thin. In a P-N junction it is possible to have such a thin barrier and consequently a very narrow depletion region if the P-type and the N-type semi-conductors are both very heavily doped with impurities.[2]

As the impurity concentration or doping is increased in a P-N junction, the reverse breakdown voltage decreases and it can be brought to zero by increasing the doping.

In a tunnel diode the semi-conductors are very heavily doped, in the order of 10^{20} atoms/c.c. and the charge-free depletion region is made extremely narrow, of the order of about 100Å thick. In fact, the semi-conductors are so heavily doped that they become degenerate and represent alloys.

There is a second effect which the heavy doping has on the band structure. The donor level widens and overlaps the edge of the conduction band and unlike the moderately doped semi-conductor, the Fermi level moves up into the conduction band.

A similar phenomenon occurs in the P-type material where the Fermi level moves into the valence band.

Under this condition of zero breakdown voltage the diode conducts highly in the reverse direction for all back biasing. For small forward bias the current increases linearly with the voltage in the forward direction until it reaches a peak. With further increased forward bia , the current decreases and drops to a minimum valley, thereby manifesting a negative resistance. This current at low forward bias which disappears with increased bias is called the "Esaki Current" after L. Esaki who first observed this phenomenon in a heavily doped P-N junction.

At the valley voltage, the diffusion current is too small to account for the current actually observed, this value being in excess of that to be expected from the condition of "uncrossed bands". The origon of this excess current is not yet definitely known.

With still larger positive bias, the forward current increases as in an ordinary diode. This is shown in Figure III.1A. This I-V relation of the tunnel diode can be explained by means of the energy-band diagrams of a heavily doped P-N junction which are shown in Figure III.1B. The cause of the negative conductance will be obvious from these diagrams.

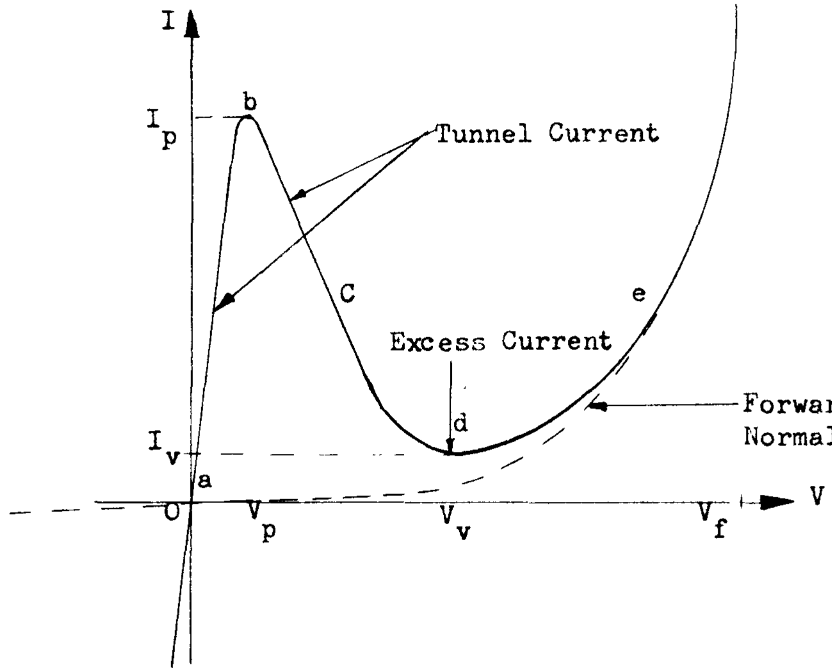
In (a) the net tunneling current is zero because there is just the same probability of electrons going from states in the conduction band on the N-side to states in the valence band on the P-side as in the opposite direction, because they are at the same energy level. This point is represented on the I-V characteristic at a.

With a small amount of forward bias as shown in (b) the height of the barrier is reduced and the electrons in the conduction band of the N-type are opposite the empty states in the valence band of the P-type at the same level of energy. Under this condition there is a finite probability that an electron originally on one side of the junction can appear on the other side at the same energy level by tunneling through the barrier at the speed of light. This tunneling probability can be made high enough to support large forward current. The valence electrons in the P-type, however, see forbidden levels to the right and cannot tunnel.

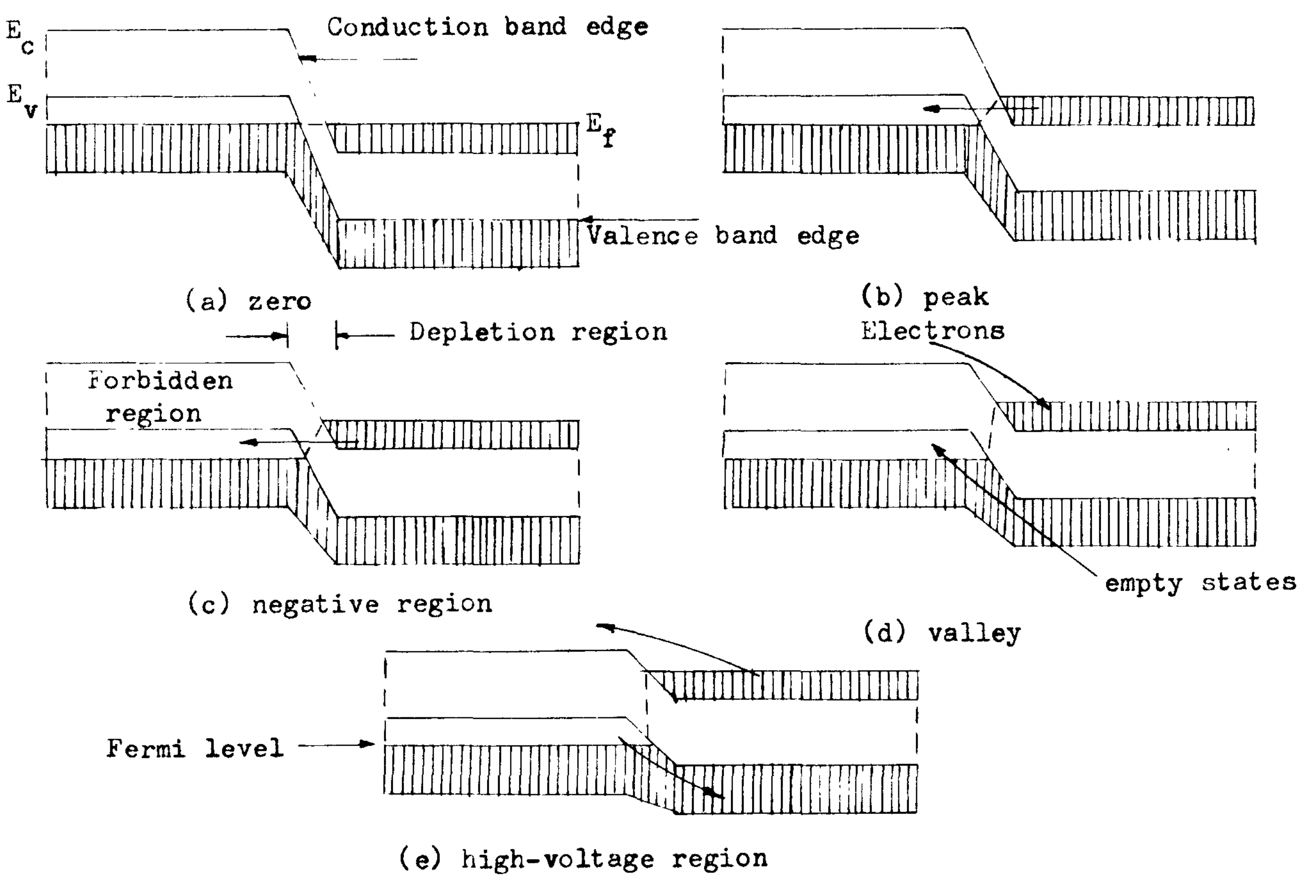
Hence a strong current flows from left to right. This situation corresponds to point b on the I-V characteristic.

With increased positive bias, the junction barrier is further reduced and some of the electrons in the conduction band of the N-type are opposite the forbidden levels of the P-type as shown in (c). Hence only part of the electrons see available energy levels across the barrier. Thus, the tunneling current is smaller than above. This phenomenon, the suppression of the tunneling makes the current decrease as the bias increases and thus causes the negative-resistance portion of the diode characteristics. This is represented by point c on the curve.

If the bias is further increased, the barrier height is still reduced, and all the electrons in the conduction band of the N-type are opposite the forbidden levels of the P-type. There are no available states to which electrons can tunnel. The current (theoretically zero) is very small, this being ordinary injection current. This is the possible minimum current. This situation is represented by point d on the curve.



A. I-V Characteristic of a heavily doped P-N junction or Tunnel diode.



B. Energy-band diagrams for tunnel diode at various points. Lettering corresponds to A.

Figure III.1

With further increase in bias the height of the barrier is sufficiently reduced and the electrons have sufficient energy to surmount this barrier and spill over.

At this bias current flows like that in an ordinary junction diode. Henceforth, the current increases linearly with voltage. This is shown at point e on the V-I characteristics.

IV Tunnel diode

In the previous chapter, we have found the physical phenomena giving rise to the non-linear I-V characteristic curve of the tunnel diode. In order to use tunnel diode as a circuit element and for proper understanding of its operational behaviour, the different regions of the characteristic of the device must be examined.

(a). Typical I-V characteristic of a GaAs type tunnel diode and its different regions.

In Figure IV.1, the I-V characteristic of a GaAs (TIXA650) type tunnel diode is shown. The points of great interest on the curve to a circuit designer are the following:

I_p = Peak current, for XA650 type shown, I_p = 10 ma.

V_p = Forward voltage at which peak current I_p occurs. The corresponging value for XA650 is V_p = 100 mv.

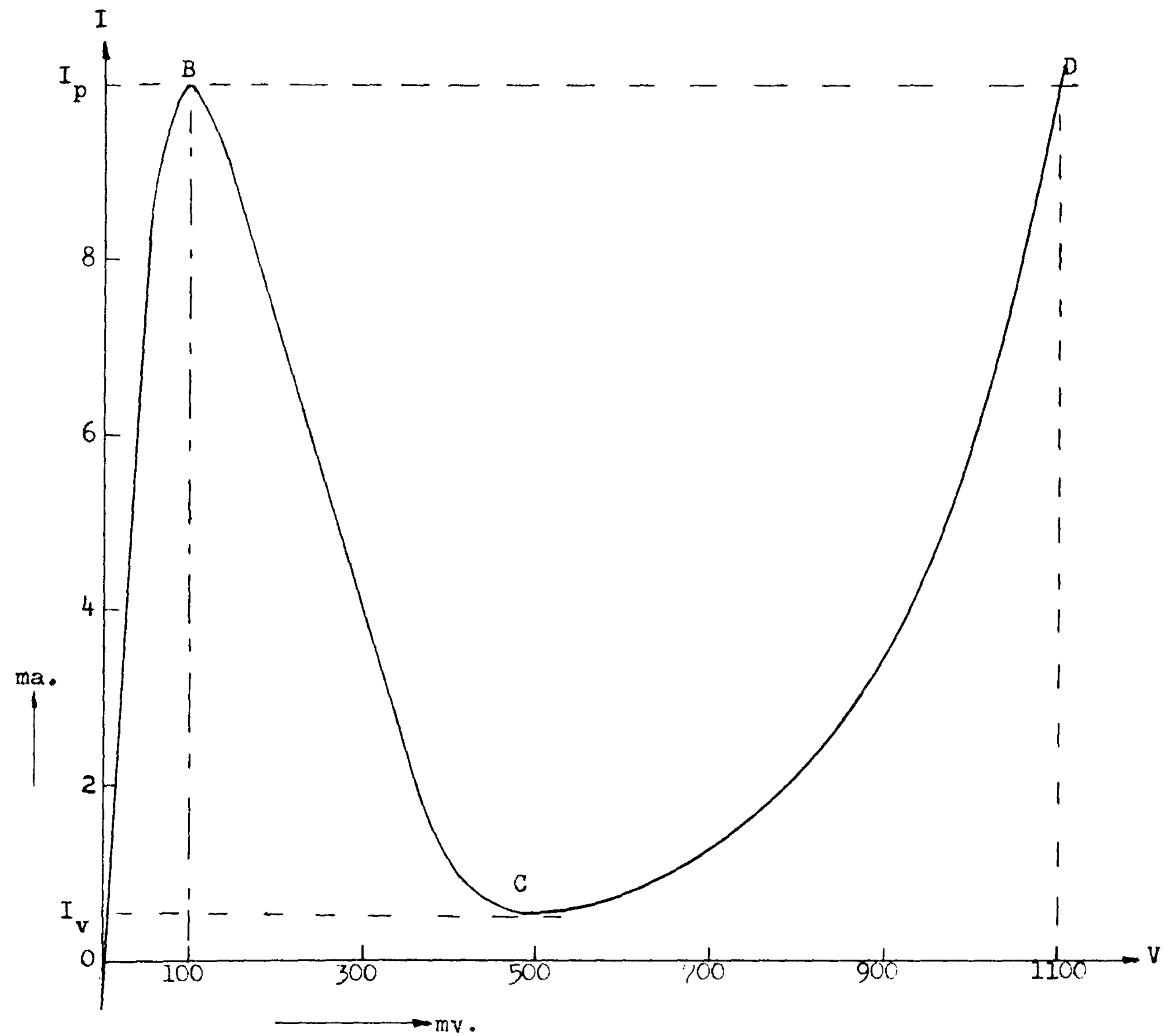
I_v = Valley or minimum current. For XA650, I_v = 0.5 ma.

V_v = Forward voltage at which I_v occurs. For XA650 V_v = 450 mv.

V_f = Forward voltage at which current through the diode again reaches the peak value. For XA650, V_f = 1100 mv.

Evidently, the characteristic curve has three distinct regions:-

(i) In the low-voltage region AB, where the current I increases from zero to its peak value I_p as the forward bias is increased from zero to V_p, the diode is characterized by a positive resistance given by



I-V Characteristic of a GaAs Type TIXA650 Tunnel Diode

Figure IV.1

the reciprocal of the slope of the segment AB.

(ii) The region BC is of greatest importance because here the curve has a negative slope exhibiting the so-called negative resistance. The average value of this negative r sistance is given by the reciprocal of the slope of the segment BC. It is this portion of the curve which is utilized in the manifold uses of the tunnel diode.

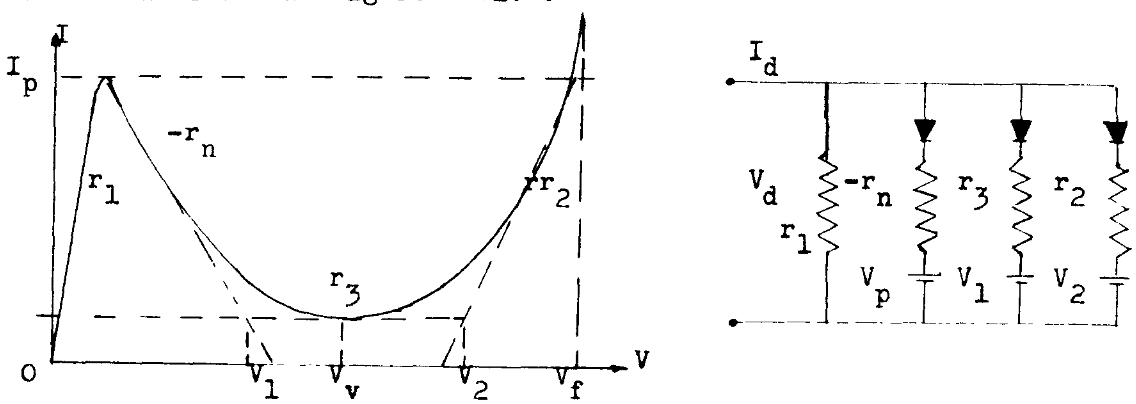
(iii) In the high-voltage region CD the slope is again positive and the tunnel diode is characterized by a positive resistance given by the reciprocal of the slope of CD.

(b). Piece-wise linearisation of the characteristic curve of the Tunnel Diode.

Since the I-V characteristic of the tunnel diode is non-linear, it is very difficult to establish the relation between the diode current and the diode voltage throughout its entire range of operating voltage. The difficulty can be overcome to a great extent by piece-wise linearization of the non-linear characteristic curve of the diode. In this method, the entire non-linear characteristic curve of the diode is approximated by a number of linear segments meeting at sharp corners, such that along each of these segments the current-voltage relationship is quite linear. Hence, in each of these regions of operation, the diode can be represented by an equivalent linear circuit. Based on this principle, the entire I-V characteristic of the tunnel diode can be approximated by four straight-line segments meeting at sharp corners, according to the four different voltage ranges as shown in Figure IV.2.A.

Each segment will be represented by a linear resistance either shunted by a current source I_d or in series with a voltage source V_d. The value of the resistances are given by the reciprocal of the slope of the respective segments and the magnitude of the voltage source, by the point of intersection of the segment with the voltage axis.

The slope of the second segment determines the negative resistance of the diode. The third section represents the region where the diode current is practically equal to the valley current I_v. Hence the entire Tunnel diode curve can be represented by an equivalent[4] circuit as shown in Figure IV.2.B.



A. Piece-wise linearization

B. Equivalent circuit representing the entire diode characteristic.

Figure IV.2

If the negative resistance of the diode is $-r_n$, then we have

$$V_1 = V_p + r_n (I_p - I_v) \text{ mv, neglecting } I_v r_n$$

$$V_2 = \frac{V_f + V_v}{2} \text{ mv, (good approximation)}$$

$$r_1 = \frac{V_p}{I_p} \text{ ohms}$$

$$r_3 = \infty$$

$$r_2 = \frac{V_f - V_2}{I_p - I_v} \text{ ohms.}$$

The corresponding values for the GaAs type XA650 tunnel diode are:-

V_p = 100 mv. $\qquad$ I_v = 0.5 ma.

I_p = 10 ma. $\qquad$ V_f = 1100 mv.

V_v = 450 mv. $\qquad$ r_n = -20 ohms.

$\therefore V_1 = 100 + 20 \times 9.5 = 300$ mv. $\qquad$ $r_1 = \frac{100}{10} = 10$ ohms.

$V_2 = \frac{1100 + 450}{2} = 775$ mv. $\qquad$ $r_2 = \frac{1100 - 775}{9.5} = 33$ ohms.

(c). Non-linear resistance of the diode

In a tunnel diode, since the current is a non-linear function of voltage, the I-V relation is denoted by $I = f(v)$. In this case, unlike a linear resistance, $\frac{dv}{dI}$ is not equal to $\frac{V}{I}$. Therefore, in such a non-linear device, we define two resistances:

A.C. or incremental or dynamic resistance = $\frac{dv}{dI}$

and D.C. absolute or static resistance = $\frac{V}{I}$.

In the case of the tunnel diode, the dynamic resistance is a function of voltage and is negative over the voltage range from V_p to V_v. From zero to V_p, it is positive . At V_p it becomes infinite and then changes its sign positive to negative. It remains negative up to V_v at which it again becomes infinite before changing its sign to positive.

The linearized average d.c. resistance evaluated before by piece-wise approximation of the characteristic are of little practical importance. It is the incremental resistance rather than d.c. resistance which is considered in the actual circuitary application of the diode.

(d). <u>Approximating the characteristic by a cubic polynomial.</u>

For analytical solution of the tunnel diode circuit, the non-linear I-V relationship must be expressed in terms of a mathematical expression.

With respect to any quiescent point in the negative resistance region of the diode, the characteristic curve can be approximated[5] by a cubic polynomial of the form $i = -av + bv^3$ where a and b are positive constants and can be evaulated from the curve. v and i are incremental voltage and current with respect to the quiescent point, i.e. $i = I - I_C$, and $v = V - V_0$. For a quiescent point (V_0, I_0) at the middle of the negative slope the constants can be evaulated as follows:-

Let $V_0 = \frac{1}{2}(V_v + V_p)$ and $I_0 = \frac{1}{2}(I_p + I_v)$

since, $\frac{di}{dv} = -a + 3bv^2$

$\therefore$ a is the negative conductance at the operation point.

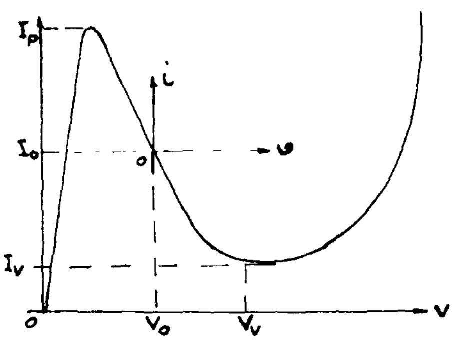
Since $\frac{di}{dv} = 0$ at $v = \frac{V_v - V_p}{2}$, we get from above $a = 3b\left(\frac{V_v - V_p}{2}\right)^2$

also when $i = -\left(\frac{I_p - I_v}{2}\right)$, $v = \frac{V_v - V_p}{2}$

$\therefore b = 2\frac{I_p - I_v}{(V_v - V_p)^3}$ and hence $a = \frac{3}{2}\frac{I_p - I_v}{V_v - V_p}$

for GaAs, XA650 diode I_p = 10 ma., V_p = 100 mv., I_v = 0.5 ma., V_v = 450 mv.

$\therefore$ a = 0.043 (given 0.05), $b = 0.44 \times 10^{-6}$



<u>Figure IV.3</u>

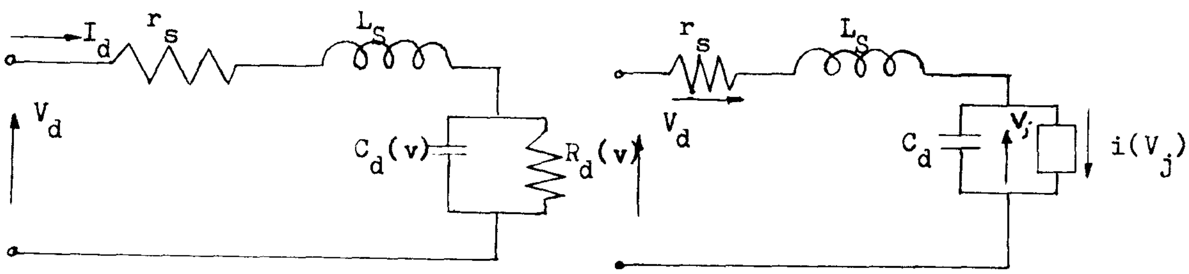
(e). <u>Tunnel diode equivalent circuit</u>.

The circuit analysis of tunnel diode falls into two groups according to its applications

(i) small-signal application

(ii) large-signal operation.

In (i) the operation of the diode is assumed to remain limited within a particular region, such that the diode can be approximately represented by a constant linear resistance (positive or negative) shunted by a junction capacitance as shown in Figure IV.5A.



A. Small-signal equivalent circuit. B. Large-signal equivalent circuit.

Figure VI.5

In A, $R_d(V)$ = Average resistance about the bias point and may be positive or negative depending on V.

= Reciprocal of the slope at the bias point.

$C_d(V)$ = Total junction capacitance at the voltage V.

L_s = Series inductance of the diode

r_s = Series resistance of the diode .

The D.C. bias voltage across the diode is given by

$V_d = I_d r_s + I_d (R_d(V)$ where I_d is the direct current through the diode. For most diodes r_s is very small and for the current range $0 < I_d < I_p$, therefore $V_d = I_d [R_d(V)]$.

The series inductance L_s and the junction capacitance $C_d(V)$ can be determined from A.C. impedance measurement. They are generally specified by the diode manufacturers.

Theoretically, $C_d(V)$ is a function of voltage but for all practical purposes C_d measured at V_v may be taken to be constant over the whole voltage range.

This small-signal equivalent circuit cannot be used for accurate explanation of the large signal applications in which entire range of the voltage is involved. Since the incremental resistance is a function of voltage and not constant as supposed before, the variation of this resistance over the entire positive and negative regions must be considered in its applications such as large-signal oscillator, monostable and bistable switches. The proposed large-signal equivalent[7] circuit consists of a non-linear current source in parallel with C_d and a series resistance r_s and a series inductance L_s as shown in Figure IV.5B. The large-signal operation is described by the equations

$$V_d = V_j + I_d r_s \quad \text{and} \quad I_d = C_d \frac{dv_j}{dt} + i(V_j).$$

Since r_s is very small, then for D.C. $V_d = V_j$ and

$$I_d = C_d \frac{dv_j}{dt} + i(V_j) \ .$$

As explained before $i(V_j)$ can be approximated to a cubic polynomial of the form $i(V_j) = -aV_j + bV_j^3$ where a and b are positive constants.

V Impedance Analysis of the Tunnel Diode Equivalent Circuit

From the small-signal lumped parameter equivalent circuit of the diode, biased in the neg tive resistance region, the a-c input impedance of the diode at an agular frequency ω, can be written as

$$Z = (r_s - \frac{R_n}{1 + \omega^2 C_d^2 R_n^2}) + j\omega(L - \frac{R_n^2 C_d}{1 + \omega^2 C_d^2 R_n^2})$$

$$= R(\omega) + j\,x(\omega) \qquad \ldots (V.1)$$

where $-R_n$ is the negative resistance at the bias point.

Both the real and imaginary part are functions of frequency. Hence there will be two critical frequencies which will determine the performance and stability of the diode. We define these two frequencies as follows:-

(i) Resistive cut-off frequency ω_R:- the frequency at which real part of Z becomes zero. Equating $R(\omega) = 0$, we get

$$\omega_R = \frac{1}{R_n C_d} (\frac{R_n}{r_s} - 1)^{1/2} \qquad \ldots (V.2)$$

(ii) Self-resonant frequency ω_X:- the frequency at which imaginary part of Z becomes zero. Equating $X(\omega) = 0$, we get

$$\omega_X = \frac{1}{R_n C_d} (\frac{C_d R_n^2}{L_s} - 1)^{1/2}. \qquad \ldots (V.3)$$

At zero frequency $R(\omega) = r_s - R_n$ which is negative since $R_n > r_s$. As the frequency increases the term $\frac{R_n}{1 + \omega^2 R_n^2 C_d^2}$ gradually decreases and approaches r_s.

At $\omega = \omega_R$, $r_s = \frac{R_n}{1 + \omega_R^2 R_n^2 C_d^2}$ and hence $R(\omega) = 0$.

Thus at $\omega = \omega_R$, the impedance of the diode is purely imaginary i.e. reactive, real part being zero. Its magnitude is given by

$$Z = j\left(\frac{R_n}{r_s} - 1\right)^{1/2} \left(\frac{L_s}{R_n C_d} - r_s\right) \qquad \ldots (V.4)$$

Below ω_R, $R(\omega)$ is negative and above ω_R it is positive. And as $\omega \rightarrow \infty$ $R(\omega)$ approaches r_s.

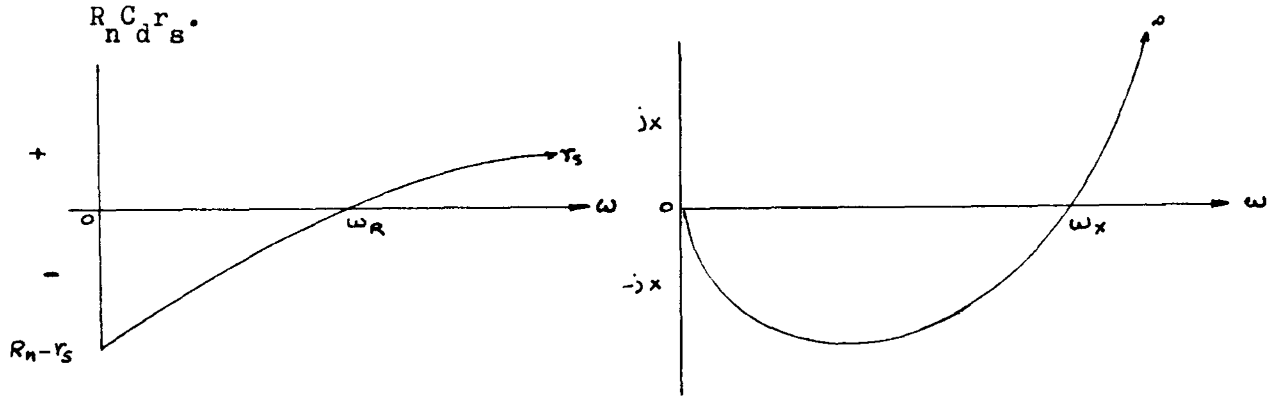
Since above $\omega = \omega_R$, the tunnel diode e xhibits no more negative resistance, it cannot oscillate at frequency more than ω_R which is the ultimate frequency limit of the diode. ω_R is maximum when $R_n = 2r_s$ and its maximum value is ω_R (maximum) $= \frac{1}{R_n C_d}$.

At zero frequency $X(\omega) = 0$ and as the frequency increases the term $\frac{R_n^2 C_d}{1 + \omega^2 r_n^2 C_d^2}$ gradually decreases and approaches L_s.

$$\text{At } \omega = \omega_X, \ \frac{R_n^2 C_d}{1 + \omega_X^2 C_d^2 R_n^2} = L_s \text{ and hence } X(\omega) = 0.$$

Thus at $\omega = \omega_X$ the impedance of the diode is real and resistive and is equal to $Z = r_s - \frac{L_s}{R_n C_d}$. $\qquad \ldots (V.5)$

Below ω_X, the reactance is capacitive and above ω_X it is inductive. ω_X can be increased by decreasing L_s and it approaches ω_R as L_s approaches $R_n C_d r_s$.



A. Real part as a function of ω. B. Imaginary part as a function of ω.

<u>Figure V.1</u>

(a). Criteria for oscillations, amplification and switching.

From above, we see real ω_R exists as long as $R_n > r_s$,

$\omega_R = 0$ when $R_n = r_s$ and ω_R is imaginary when $R_n < r_s$.

Similarly, real ω_X exists as long as $R_n^2 C_d > L_s$,

$\omega_X = 0$ when $R_n^2 C_d = L_s$ and ω_X is imaginary when $R_n^2 C_d < L_s$.

When $\omega_R = 0$, the diode can only be used as a switching device since $r_s = R_n$ and when $\omega_X = 0$, the diode can only operate as a non-linear relaxation oscillator at imaginary frequency, i.e. it acts as an astable multi-vibrator.

Provided ω_R and $\omega_X \neq 0$, three possible conditions may exist, in general.

(1) $\omega_R = \omega_X$

(2) $\omega_R > \omega_X$ . . . (V.6)

(3) $\omega_R < \omega_X$

We see that $\omega_R = \omega_X$ when $L_s = R_n r_s C_d$ and at this frequency both the real and the imaginary part of Z is zero and we get a sustained oscillation.

$\omega_R > \omega_X$ when $L_s > R_n C_d r_s$

$\omega_R < \omega_X$ when $L_s < R_n C_d r_s$

These three conditions can be plotted on the complex impedance plane and therefrom conditions of stability can be determined from the fact that the number of times the impedance plot in the complex Z-plane encircles the origin in the positive (counter clockwise) direction is equal to the difference of the number of poles and the number of zeros in the right half of complex frequency plane. These three conditions are plotted in the Z-plane and S-plane for comparison in Figure V.2.

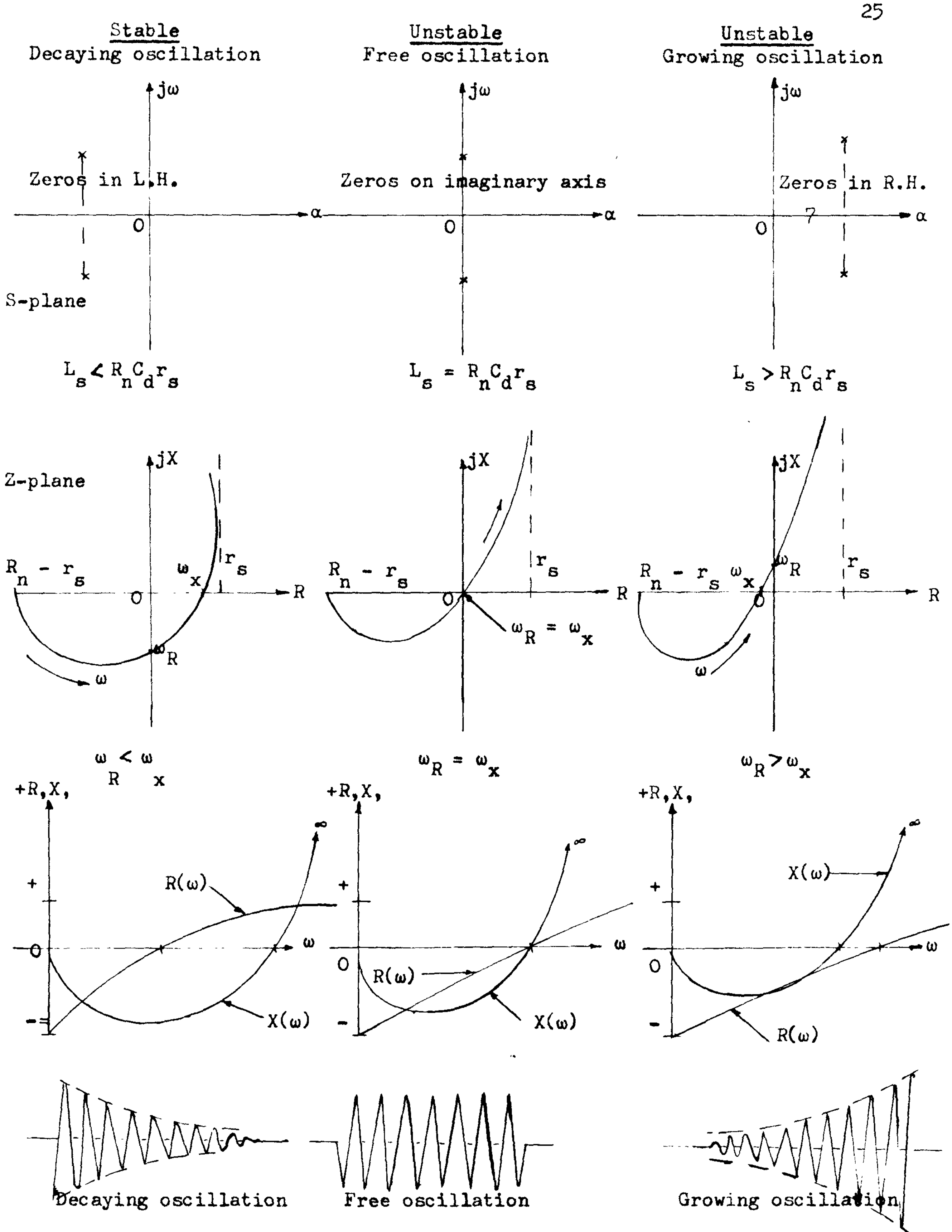

Stable
Decaying oscillation
Unstable
Free oscillation
Unstable
Growing oscillation
jω
Zeros in L.H.
Zeros on imaginary axis
Zeros in R.H.
α
0
S-plane
$L_s < R_n C_d r_s$
$L_s = R_n C_d r_s$
$L_s > R_n C_d r_s$
jX
Z-plane
$R_n - r_s$
r_s
ω_x
ω_R
R
ω
$\omega_R = \omega_x$
$\omega_R < \omega_x$
$\omega_R > \omega_x$
+R, X,
+
-
R(ω)
X(ω)
∞
Decaying oscillation
Free oscillation
Growing oscillation

Figure V.2

For a typical tunnel diode $\omega_R > \omega_X$. That is, the diode itself is an unstable device.

For GaAs type XA650 tunnel diode, the inherent parameters are L_s = mμh, C_d = 40μμf, R_n = -20Ω and r_s = 1 ohm.

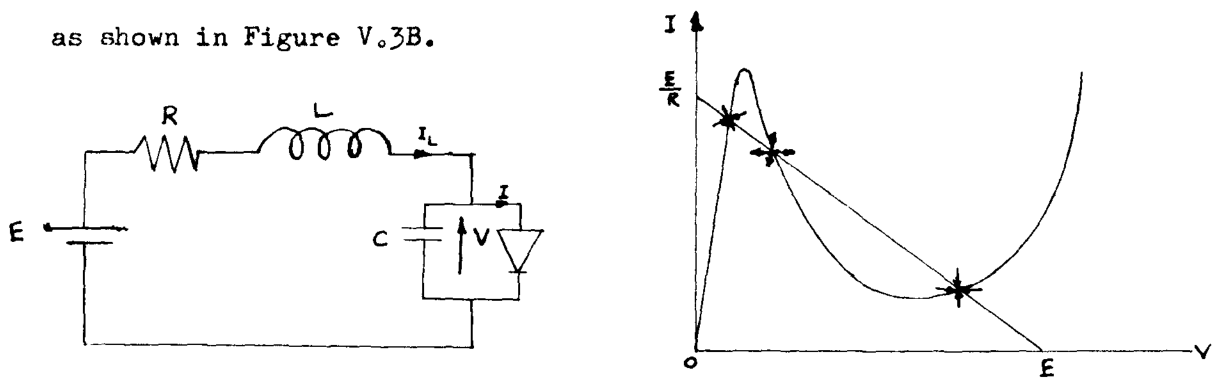
The calculated values of the two critical frequencies for this diode are

cut-off frequency ω_R = 5 KMC and

resonant frequency ω_X = 1.7 KMC.

(b). <u>Stability analysis of equilibrium points</u>.

A good deal of information pertaining to the behaviour of a system described by a pair of simultaneous first order equations can be obtained by studying the nature of its singularities or points of equilibrium. The stability or otherwise of the system depends on whether these points are stable or unstable. Let us consider the circuit shown in Figure V.3A, in which L and R are the total inductance and resistance respectively including the external ones. Let E and R be such that there are three equilbrium points or points of intersection of the load line with the char.cteristic, one in each region as shown in Figure V.3B.



A. Circuit

B. Load line giving three singularities, one in each region of the curve.

Figure V.3

Writing the voltage and current equations we get,

$$\frac{dI_\ell}{dt} = \frac{E - I_\ell R - V}{L} \quad , \frac{dV}{dt} = \frac{I_\ell - f(V)}{C} \qquad . . . (V.7)$$

where I_ℓ is the total current and $I = f(V)$ is the current through the diode.

Singularity is the point at which both $\frac{dI_\ell}{dt}$ and $\frac{dV}{dt}$ become simultaneously equal to zero. Hence, singularities are the points of intersection of the load line with the diode characteristic. Near a singularity (V_0, I_0) we can substitute $V = V_0 + v$ and $I = I_0 + i$ where v and i are small changes in voltage and current respectively and may be positive or negative.

By Taylor's expansion, we get

$$f(V) = f(V_0 + v) = f(V_0) + vf'(V_0) + - - - -$$

where $f'(V_0) = \frac{d}{dv}\left|f(V)\right|_{V = V_0}$ is the slope at the singularity and has the dimension of conductance. Let $f'(V_0) = g$.

Substituting for V and I in (V.7) we get,

$$\frac{di}{dt} = \frac{-iR - v}{L} \quad , \frac{dv}{dt} = \frac{i - vg}{C} \qquad . . . (V.8)$$

The characteristic equation of the system is then

$$S^2 + (\frac{R}{L} + \frac{g}{C})\ S + \frac{1}{LC}\ (1 + gR) = 0 \text{ and its roots are}$$

$$S_{1,2} = -\tfrac{1}{2}(\frac{R}{L} + \frac{g}{C}) \pm \tfrac{1}{2}\sqrt{(\frac{R}{L} + \frac{g}{C})^2 - \frac{4}{LC}(1+gR)} \qquad . . . (V.9)$$

These roots will determine the stability of the system. The parameters L, C and R can have only positive values whereas g can be either positive or negative depending on the location of the singularity on the characteristic.

In both the positive resistance regions of the diode, g is positive and, therefore, if $(\frac{R}{2L})^2 > \frac{1}{LC}$, S_1 and S_2 are real and negative. Hence, the singularity is a stable one (node). Moreover, if $(\frac{R}{2L})^2 < \frac{1}{LC}$, S_1 and S_2 are complex conjugate with negative real part. Therefore, the singularity is again stable (focus)[9]. Since along the negative slope of the diode $g < 0$ and also $g > \frac{1}{R}$, S_1 and S_2 are both real and opposite in sign. Therefore the singularity is unstable (saddle). Hence, we conclude that along the two positive slopes of the characteristic, the equilibrium points are always stable whereas on the negative slope of the characteristic the singularities are always unstable.

If the parameters are such that the circuit has a single equilibrium point on the negative slope, where g is negative then if $g < -\frac{CR}{L}$, S_1 and S_2 become complex conjugates with positive real part. The corresponding singularity is unstable (focus) and we get a growing oscillation at a real frequency.

On the other hand, if C = 0, the single root of the equation is $S_1 = - \frac{1}{Lg}(1 + Rg)$. Therefore, if $g < 0$ and $g < \frac{1}{R}$, the root is real and positive. The singularity then becomes unstable (node). This condition also gives a frowing solution but has no real frequency. This is the condition of aaperiodic or relaxation oscillation.

Putting $(\frac{R}{L} + \frac{g}{C}) = P$ and $\frac{1}{LC}(1 + gR) = q$, we can sum[10] up the various conditions as shown in Figure V.4.

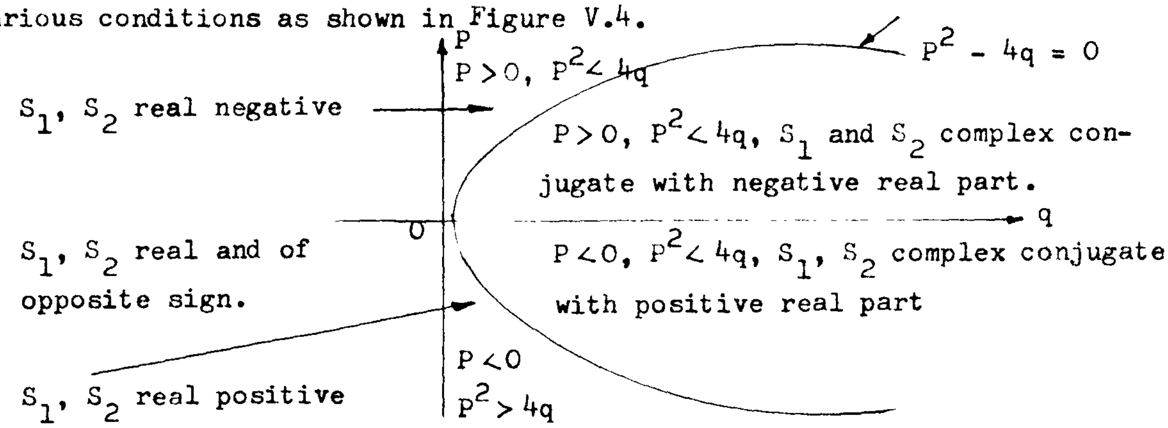


Figure V.4

VI Tunnel Diode Oscillator

A complete solution of a tunnel diode oscillator giving the output waveshape, amplitude and frequency has not yet been possible by any analytical or graphical method alone. These two methods are not independent but complimentary. For analytical solution, the non-linearity must be expressed as a power series and in the case of tunnel diode, a certain amount of qualitative information can be obtained if the diode characteristic is approximated by a simple cubic polynomial .

This analytical solution does not give the amplitude and waveshape directly but only as a Fourier series. It also shows the deviation of frequency of oscillation from its natural frequency.

On the other hand, topological method gives the peak to peak amplitude of oscillation quite rapidly but supplies almost no accurate information as to the frequency of operation. These methods are rather tedious when accurate solutions are needed. We shall consider the analytical method first. Since there are both inductance and capacitance inherent in a tunnel diode to excite oscillations in it, it is sufficient only to bias the diode in its negative resistance region.

However, to generalize the analysis of the oscillator, we shall consider the circuit shown in Figure VI.1 where the tunnel diode is represented by its large-signal equivalent circuit consisting of a non-linear current source shunted by the junction capacitance.

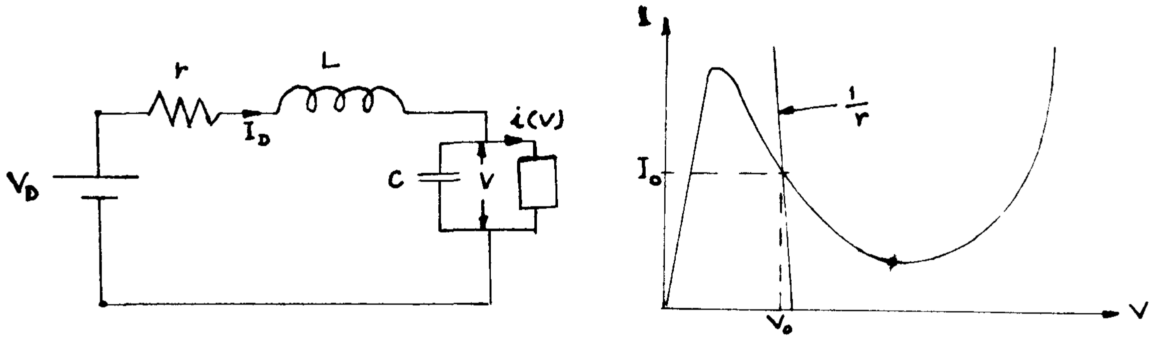


Figure VI.1

It is assumed that $L = L_{diode} + L_{ext.}$ and $r = r_{diode} + r_{source}$, $C = C_j + C_{ext.}$ and $i(V)$ = non-linear current source.

The d.c. bia voltage is such that the load line intersects the characteristic at a single point in its negative resistance portion. Since r is very small, the device voltage V_D and the junction voltage V are practically the same for D.C. and the d.c. junction current is $i(V_0) = I_0$.

Writing the loop voltage and current equations, we get

$$V_D = L\frac{dI_d}{dt} + I_d r + V, \quad I_D = C\frac{dv}{dt} + i(V) \qquad \ldots (VI.1)$$

Let $I_d = I_0 + i(t)$, and $V = V_0 + v(t)$. Since under D.C. condition $V_D = V_0 + I_0 r$, substituting these values in (VI.1) and separating A.C. and D.C. voltages we get

$$L\frac{di(t)}{dt} + i(t)r + v(t) = 0$$

$$i(t) = C\frac{dv(t)}{dt} + i\left[v(t)\right] \qquad \ldots (VI.2)$$

Let $i\left[v(t)\right] = -av(t) + bv^3(t)$ $\qquad \ldots$ (VI.3)

where a and b are positive constants. With this substitution, (VI.2) becomes

$$\frac{d^2v}{dt^2} + \left(\frac{r}{L} + \frac{3bv^2}{C} - \frac{a}{C}\right)\frac{dv}{dt} + v\left(\frac{1}{LC} - \frac{ar}{LC}\right) + \frac{br}{LC}v^3 = 0 \qquad \ldots (VI.4)$$

where $v = v(t)$.

(a). Small-signal analysis.

For small-signal operation, we can write by neglecting bv^3, $i = -av = -\frac{v}{R_n}$ where R_n is the magnitude of the negative resistance at the bias point. Therefore, for small-signal operation, neglecting v^2 and v^3 terms, (VI.4) becomes

$$\frac{d^2v}{dt^2} + \left(\frac{r}{L} - \frac{1}{R_nC}\right)\frac{dv}{dt} + \left(\frac{1}{LC} - \frac{r}{R_nLC}\right)v = 0 \qquad \text{. . . (VI.5)}$$

The roots of the characteristic equation are:

$$P_{1,2} = -\tfrac{1}{2}\left(\frac{r}{L} - \frac{1}{R_nC}\right) \pm j\sqrt{\frac{1}{LC}\left(1 - \frac{r}{R_n}\right) - \frac{1}{4}\left(\frac{r}{L} - \frac{1}{R_nC}\right)^2}$$

$$= \alpha \pm j\omega \qquad \text{. . . (VI.6)}$$

for a growing solution $\alpha = 0$. This condition is satisfied if either one or both of the following inequalities hold:

(i) $\frac{r}{L} - \frac{1}{R_nC} < 0$ or $L > R_n rC$

(ii) $\frac{r}{R_n} < 1$ or $R_n > r$

The second condition is that in order to have oscillation at a real frequency, ω must be real. This is satisfied if

$$\frac{1}{LC}\left(1 - \frac{r}{R_n}\right) > \frac{1}{4}\left(\frac{r}{L} - \frac{1}{R_nC}\right)^2$$

or $\frac{L}{R_nC} - \left(\frac{4L}{C}\right)^{1/2} \leq r \leq \frac{L}{R_nC} + \left(\frac{4l}{C}\right)^{1/2}$, that is for oscillations at real frequency, r should be within $\frac{L}{R_nC} \pm \left(\frac{4L}{C}\right)^{1/2}$.

If $r > \frac{L}{R_nC} + \left(\frac{4L}{C}\right)^{1/2}$ or $r < \frac{L}{R_nC} - \left(\frac{4L}{C}\right)^{1/2}$, ω is imaginary.

When $r = \frac{L}{R_nC}$, $\alpha = 0$ and $P_{1,2} = \pm j\omega = \pm j\sqrt{\frac{1}{LC}\left(1 - \frac{r}{R_n}\right)}$ therefore ω is real when $R_n > r$. In other words, when $L = R_n rC$ and $R_n > r$, the roots are purely imaginary lying on the vertical axis and it corresponds to free sinusoidal oscillation at a real frequency

$$\omega = \sqrt{\frac{1}{LC}\left(1 - \frac{r}{R_n}\right)}.$$

If $L < R_n rC$, $\alpha < 0$ and ω will be real when

$$\frac{L}{R_n C} < r < \frac{L}{R_n C} + \left(\frac{4L}{C}\right)^{1/2}$$

therefore within this range of r we will have decaying sinusoidal oscillation.

If $L < R_n rC$ and $r \geq \frac{L}{R_n C} + \left(\frac{4L}{C}\right)^{1/2}$, $\alpha < 0$ and ω is imaginary. Therefore this corresponds to decaying exponential.

On the other hand, if $L > R_n rC$ and $\frac{L}{R_n C} - \left(\frac{4L}{C}\right)^{1/2} < r < \frac{L}{R_n C}$

$\alpha > 0$ and ω is real. This corresponds to growing oscillation with real frequency. The growth being limited by the non-linearity of the circuit because large amplitude swings the operating point into the dissipative region of the diode where the condition $R_n > r$ does not hold.

When $L > R_n rC$ and $r \leq \frac{L}{R_n C} - \left(\frac{4L}{C}\right)^{1/2}$, $\alpha > 0$ and ω is imaginary. This corresponds to region of growing exponential. In this region we get oscillation at imaginary frequency" and this corresponds to astable multi-vibrator operation or the relaxation oscillation. Therefore, the conditions for oscillations are

(i) $R_n > r$

(ii) $L > R_n rC$.

As L is increased from $L = R_n rC$ to a very high value, the oscillation deviates from the sinusoidal type towards the relaxation type.

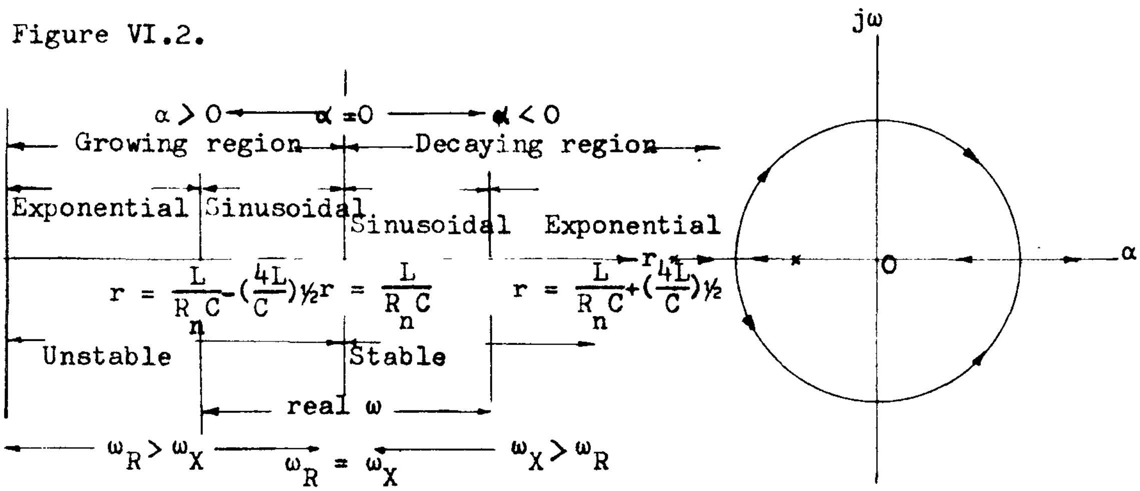
These oscillations are possible as long as $R_n \geq r$. For $R_n < r$, the diode can be used only as a switching device.

As r is reduced from a very high value to zero, the movements of the roots in the complex plane can be explained as follows:

Initially, $r \gg \frac{L}{R_n C}$ and the two roots lie on the real axis in the left hand half-plane. As r is reduced, the roots approach each other along the axis and then separate along the arcs of a circle. This occurs when $r = \frac{L}{R_n C} + (\frac{4L}{C})^{1/2}$. When $r = \frac{L}{R_n C}$, the two roots lie on the imaginary axis. This corresponds to free oscillation.

If $r < \frac{L}{R_n C}$, the roots move into the right hand half plane and we get non-linear sinusoidal oscillations.

As r is further reduced, the roots meet and again separate along the real axis. This occurs when $r < \frac{L}{R_n C} - (\frac{4L}{C})^{1/2}$. This corresponds to relaxation oscillation or astable operation. These are shown in Figure VI.2.



A. Various modes of operation with increasing r.

B. Movement of roots with decreasing r.

Figure VI.2

(b). Large-signal analysis.

For large-signal oscillation where the amplitude extends into the dissipative region of the characteristic, the non-linear term must also be considered. Hence, for large-signal analysis, we shall consider equation (VI.4).

$$\frac{d^2v}{dt^2} + \left(\frac{r}{L} + \frac{3bv^2}{C} - \frac{a}{C}\right)\frac{dv}{dt} + v\left(\frac{1}{LC} - \frac{ar}{LC}\right) + \frac{br}{LC}v^3 = 0$$

It is shown before, the minimum value of L for which a sustained oscillation is obtained is $L = R_n rC$, and the corresponding frequency of oscillation, provided $R_n > r$ is $\omega = \left(\frac{1 - r/R_n}{LC}\right)^{1/2}$, for value of L greater than this minimum $L = R_n rC$, oscillations are generally non-linear in character. For this non-linear oscillation analysis, we must consider the above equation.

Let us assume that r is very small and can be neglected later on. Hence, neglecting r, the above equation becomes

$$\frac{d^2v}{dt^2} - \frac{a}{c}\left(1 - \frac{3b}{a}v^2\right)\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad \text{. . . (VI.7)}$$

Let $\omega_0^2 = \frac{1}{LC}$ and $\frac{3b}{a} = \beta$, and changing the scale of time such that $\omega_0 t = T$, equation (VI.7) reduces to

$$\frac{d^2v}{dT^2} - \frac{a}{\omega_0 C}\left(1 - \beta v^2\right)\frac{dv}{dT} + v = 0 \qquad \text{. . . (VI.8)}$$

Finally changing the scale of v such that $\beta v^2 = V_1^2$, i.e. $v = \frac{V_1}{(\beta)^{1/2}}$,

$$\therefore \frac{dv}{dT} = \frac{1}{(\beta)^{1/2}}\frac{dV_1}{dT} \text{ and } \frac{d^2v}{dT^2} = \frac{1}{(\beta)^{1/2}}\frac{dV_1^2}{dT^2}.$$

Equation (VI.8) then reduces to the form

$$\frac{d^2V_1}{dT^2} - \alpha(1 - V_1^2)\frac{dV_1}{dT} + V_1 = 0 \qquad \text{. . . (VI.9)}$$

where $\alpha = \frac{a}{\omega_0 C}$, which is of the form of well-known Van der Pol equations.[12] The factor $-\alpha(1 - V_1^2)$ in (VI.9) represents a non-linear damping coefficient which alternates in sign during the steady oscillatory state. α being a positive constant, it is seen that for small voltage v_1 (such that $v_1^2 << 1$) the damping is negative supplying more energy to the system and helping the amplitude growth, but for large V_1 damping is positive dissipating energy from the system and thereby decreasing the amplitude. Hence this factor automatically brakes the oscillation such that a final stable and finite amplitude results.

Now, α is related to the Q of the circuit in the following manner. Since $Q = \pi, \frac{\text{maximum energy stored}}{\text{energy lost per half cycle}}$, if $v = V_{max.} \sin\omega_0 t$, then maximum stored energy $= \frac{1}{2}CV_{max.}^2$. Since there is no dissipative element in the circuit as assumed, the only change in the magnitude of this energy is due to the negative resistance. Therefore energy change per half-cycle, for small v, is $\frac{V^2_{max.}}{2} \frac{a\pi}{\omega_0}$. Therefore

$$Q = \pi \frac{\frac{1}{2}CV^2_{max.}}{\frac{V^2_{max.}}{2}\frac{a\pi}{\omega_0}} = \frac{C\omega_0}{a} = \frac{1}{\alpha}$$ i.e., the circuit Q is the reciprocal of the parameter α.

If $Q >> 1$ or $\alpha << 1$, any change in energy during a cycle is but a small fraction of the stored energy and hence the change in amplitude per cycle is very small.

On the other hand, if $\alpha >> 1$ or $Q << 1$, the change in energy per cycle is large compared with the stored or circulating energy and the amplitude may change abruptly during a cycle. Since only relatively slow changes can be considered by analytical methods, these analytical methods are limited to the case where $\alpha << 1$.

Since for $\alpha = 0$, $V_1 = A\cos T$ where A is an arbitrary constant, for $\alpha << 1$, we can assume a solution of the form $V_1 = A(T)\cos T$ where it assumed that percentage change in amplitude per cycle is very small, i.e., $\frac{1}{A}\frac{dA}{dT} \ll 1$. Substituting for V_1 and its derivatives in (VI.9) and neglecting the higher derivatives of A, as well as the higher harmonics, we get

$$(-A\cos T - 2\frac{dA}{dT}\sin T) - \alpha(-A\sin T + \frac{dA}{dT}\cos T + \frac{1}{4}A^3\sin T) + A\cos T = 0. \quad \ldots (VI.10)$$

The first and the last term cancel, which indicates that, to the present approximation the ffequency is equal to the natural frequency of the system.

Moreover, the term $\alpha\frac{dA}{dT}\cos T$ is negligible compared to the remaining terms. Multiplication by A and division by sinT reduces the above equation to

$$\frac{dA^2}{dT} - \alpha(A^2 - \frac{1}{4}A^4) = 0 \quad \ldots (VI.11)$$

which represents the variation of the amplitude A with respect to the time. Solving (VI.11) for A^2 we get

$$A^2 = \frac{4}{1 + e^{-\alpha(T - T_0)}}, \text{ therefore } V_1 = \pm\frac{2\cos T}{\sqrt{1 + e^{-\alpha(T - T_0)}}},$$

T_0 being a constant. Substituting for $V_1 = (\beta)^{1/2}v$ and $T = \omega_0 t$, we get

$$v(t) = \pm\frac{\frac{2}{(\beta)^{1/2}}\cos(\omega_0 t + \theta)}{\sqrt{1 + e^{-\alpha\omega_0 t + \alpha\omega_0 t_0}}} \quad \text{where } \theta \text{ is another constant.}$$

This equation describes the complete process of build-up and steady state oscillation.

(i). Initial build-up process.

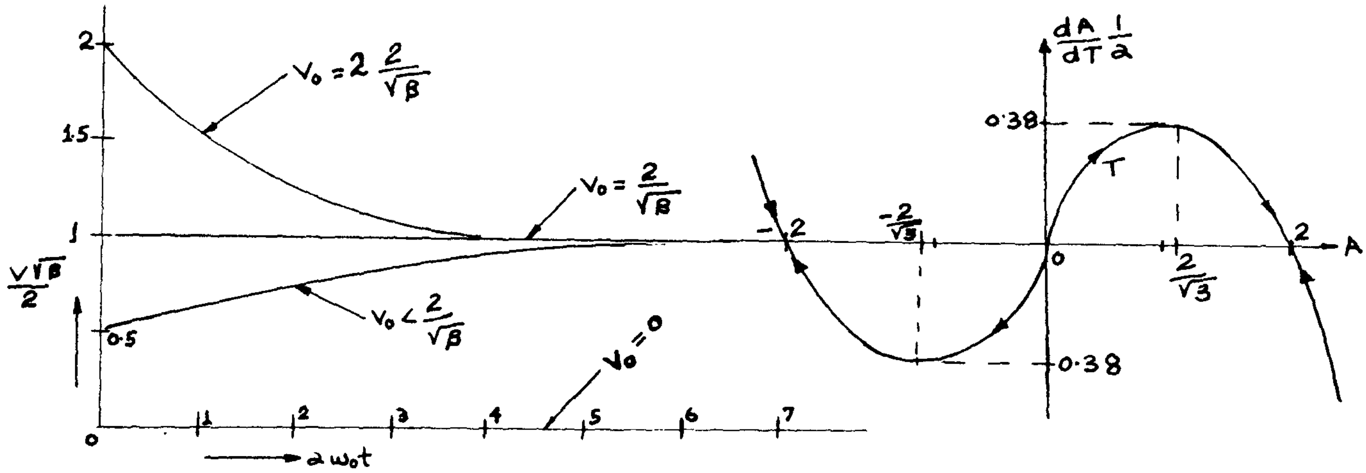
Let at $t = 0$, $v(t) = V_0$, therefore $V_0 = \pm\frac{\frac{2}{(\beta)^{1/2}}}{\sqrt{1 + e^{\alpha\omega_0 t_0}}}$

or, $e^{\alpha\omega_0 t_0} = \frac{4}{V_0^2\beta} - 1.$

Substituting this in equation for V(t) we get

$$v(t) = \pm \frac{\frac{2}{(\beta)^{1/2}}}{\left[1 - (1 - \frac{4}{\beta V_0^2})e^{-\alpha\omega_0 t}\right]^{1/2}} \cos\omega_0 t \qquad \ldots \text{(VI.12)}$$

From above, the variation of amplitude with time can be studied for different initial conditions i.e., for different values of V_0 as shown in Figure VI.3A.



A. Amplitude build-up for different V_0.

B. Phase-plane plot of amplitude

Figure VI.3

If $V_0 = 0$, amplitude v remains zero. If $V_0 \neq 0$ the amplitude ultimately assumes the final value $\frac{2}{(\beta)^{1/2}}$, growing if V_0 is initally small and decaying if V_0 is initially large.

In all cases V approaches $\frac{2}{(\beta)^{1/2}}$ monotonically with no overshot. This kind of oscillations which is self starting so long as there is any initial disturbance, is known as "soft"[5] oscillation.

(ii). Steady-state path of operation

Moreover, from (VI.10) we get, neglecting $-\alpha\frac{dA}{dT}\cos T$

$$\left[-2\frac{dA}{dT} + \alpha(A - \frac{A^3}{4})\right] \sin T = 0$$

or, $\frac{dA}{dT} = \frac{\alpha A}{2}(1 - \frac{A^2}{4})$. If this curve is plotted in the phase plane i.e., $\frac{dA}{dT}$ as a function of A as shown in Figure VI.3B we see there are three points where $\frac{dA}{dT} = 0$, these points being A = 0 and A = $\pm$ 2. Since A is the magnitude of $V_1 = v(\beta)^{1/2}$, $\frac{dv}{dT} = 0$ at $v = \pm \frac{2}{(\beta)^{1/2}}$ and at v = 0. These are the equilibrium points. The maximum value of $\frac{dv}{dT}$ occurs when $v = \pm \frac{2}{(3\beta)^{1/2}}$ and the corresponding maximum value of $\frac{dv}{dT} = \pm \frac{2\alpha}{3(3\beta)^{1/2}}$. Thus the origin is unstable, since a point on the curve tends to move away fromthe origin as time progresses.

Similarly, the other two equilibrium points are stable.

The steady-state voltage amplitude

$v = \frac{2}{(\beta)^{1/2}} = 2(\frac{a}{3b})^{1/2}$ is determined entirely by the coefficients a and b for the negative resistance element. This steady-state amplitude is easily related to the geometry of the characteristic of the diode as shown in Figure VI.4A.

Since $i = -av + bv^3$, $\frac{di}{dv} = 0$ at $v = \pm (\frac{a}{3b})^{1/2} = \pm \frac{1}{(\beta)^{1/2}}$. At these points, the current is $i = \pm \frac{2a/3}{(\beta)^{1/2}}$. If horizontal lines are drawn tangent to the characteristic, these lines intersect the portion of the char cteristic with positive slope at $v = \pm \frac{2}{(\beta)^{1/2}}$. This is due to the assumption of a symmetrical characteristic curve.

It is seen that for $\alpha = 0$, $\omega = \omega_0$. For $\alpha > 0$, $\omega = \frac{\omega_0}{1 + \frac{\alpha^2}{16}}$ which is less than ω_0. This is because for $\alpha > 0$, the voltage is not simple harmonic which acting with a non-linear resistance produces a current which is far from simple harmonic. One component of this current is at fundamental frequency. The frequency must be such that this fundamental component of current multiplied by the impedance of the

L-C circuit at this frequency gives the fundamental voltage. This condition requires that the fundamental frequency of oscillation be less than the resonance frequency of the L-C circuit.

(iii). Effect of r.

In the above analysis, r has been neglected. At the resonant frequency the impedance of the LrC circuit is a pure resistance $R' = \frac{L}{rC}$ which appears in parallel with the negative resistance and, therefore, has the effect of adding a positive conductance $\frac{1}{R'}$ to the negative conductance -a. Therefore

$$i = -(a - \frac{1}{R'})\ v + bv^3 = -a'v + bv^3.$$

Hence, in place of a, we can put a' to take r into consideration. Therefore $v = 2(\frac{a'}{3b})^{1/2}$ which will be zero and imaginary if a' becomes zero and negative. Therefore the condition of oscillation is that $\frac{1}{R'} = \frac{rC}{L} < a$.

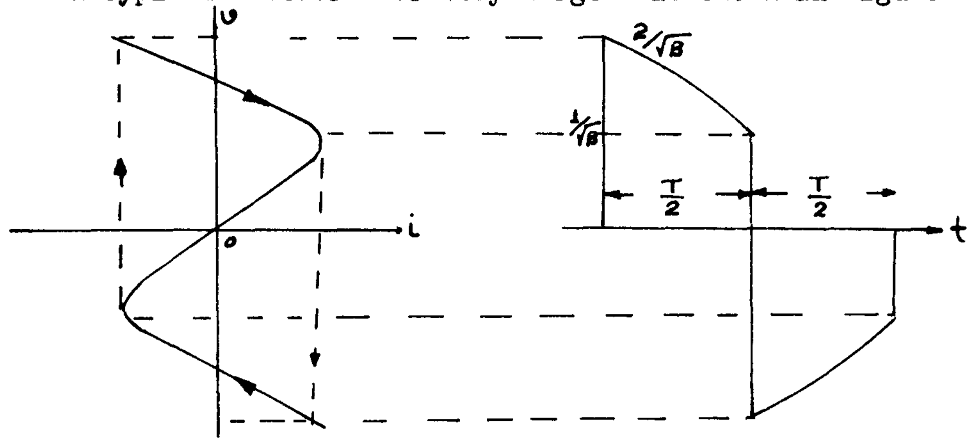
(iv). Large values of α: relaxation oscillation.

As α increases, the effect of the harmonics become more and more pronounced and the waveshape is greatly distorted. When $\alpha \gg 1$ the wave would have a flat top and sharp transition would occur at the ends. For large α the waveform is essentially composed of straight lines meeting in corners. This corresponds to oscillation with imaginary frequency and the circuit operates as an astable multivibrator. This mode of oscillation is called relaxation oscillation.

Since analytical methods are valid for $\alpha \ll 1$, this oscillation with $\alpha > 1$ can be solved only by topological or graphical methods to be described later on.

For large value of α the period of oscillations can be approximately calculated as follows:

A typical waveform for very large α is shown in Figure VI.4B.



A. Region of operation on the characteristic

B. Waveform for large α

Figure VI.4

Since the waveform is composed of straight lines, we can neglect $\frac{d^2V_1}{dT^2}$ everywhere except at corners. Therefore equation (VI.9) becomes $-\alpha(1 - v_1^2)\frac{dV_1}{dT} + v_1 = 0$. Sbustituting $V_1 = v(\beta)^{1/2}$ and $T = \omega_0 t$, we get $dt = \frac{\alpha}{\omega_0}(\frac{1}{v} - \beta v)dv$.

Integrating both sides

$$\int_0^t dt = \int_{2/\sqrt{\beta}}^{v} \frac{\alpha}{\omega_0}(\frac{1}{v} - \beta v)dv.$$

Assuming that at $t = 0$, $v = \frac{2}{(\beta)^{1/2}}$, its maximum value, and the instantaneous voltage v exists at time t.

Therefore $t = \frac{\alpha}{\omega_0}(\ell n \frac{v(\beta)^{1/2}}{2} - \frac{\beta v^2}{2} + 2)$. Let us assume that the voltage decreases to half of its peak value rather slowly and then suddenly jumps to the negative peak and that the negative half cycle is idéntical to the positive half cycle except for signs.

The interval during which v changes from $\frac{2}{(\beta)^{1/2}}$ to $\frac{1}{(\beta)^{1/2}}$ is half a period. This is because the transition is assumed to be

instantaneous requiring no time at all. Therefore when $t = \frac{T}{2}$, $v = \frac{1}{(\beta)^{1/2}}$.
Substituting in the above equations we get

$$\frac{T}{2} = \frac{\alpha}{\omega_0}\left(\ell n \tfrac{1}{2} + \frac{3}{2}\right) = 0.81\frac{\alpha}{\omega_0}\text{, therefore for } \alpha > 1.\ T = 1.62\frac{\alpha}{\omega_0}\text{,}$$

but $\alpha = \frac{a}{\omega_0 C}$ and $\omega_0 = \frac{1}{(LC)^{1/2}}$, therefore $T = 1.62\ aL$ second.

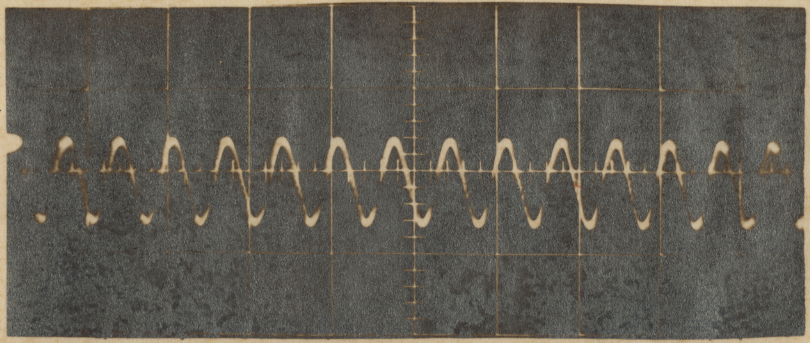
(c). <u>Experimental results</u>.

The tunnel diode used is GaAs type XA650 for which V_p = 100 mv., I_p = 10 ma., V_v = 950 mv., I_v = 0.5 ma., and V_f = 1100 mv. The average negative-resistance is -20 ohms.

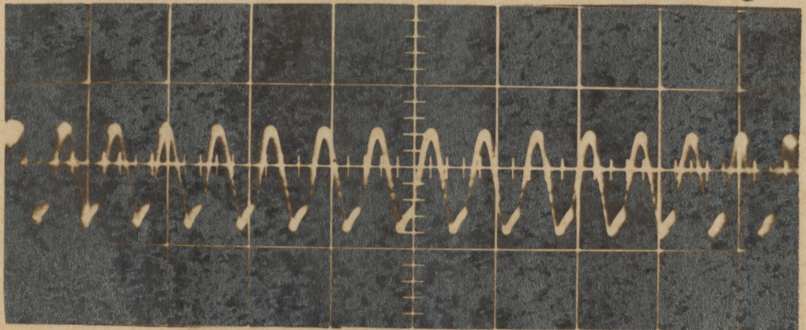
The diode is biased at the middle of the negative resistance region with a voltage source of 350 mv. having a source resistance of approximately 1 ohm. This bias voltage is held constant at 350 mv. whereas L and C are varied over a wide range, thereby varying α.
$\alpha = \frac{a}{\omega_0 C} = a(\frac{L}{C})^{1/2} = 0.05(\frac{L}{C})^{1/2}$, since a = 0.05.

The oscillation is observed by the voltage across the diode for values of α = 0.25, 1.25, 6.3 and 10. The oscillograms of the observed voltage waveforms for these values of α are shown in Figure VI.5. From the oscillograms it is seen how with increasing α, the oscillation changes from almost sinusoidal to relaxation type. For $\alpha < 1$, $\omega = \omega_0$ but $\alpha > 1$, $\omega = \frac{\omega_0}{1 + \frac{\alpha}{16}2}$. Both the calculated values of frequency and amplitude are in close agreement with the observed ones. Maximum amplitude is $V_{max.} = \frac{2}{(\beta)^{1/2}}$ = 350 mv. The following combination of L and C are used:

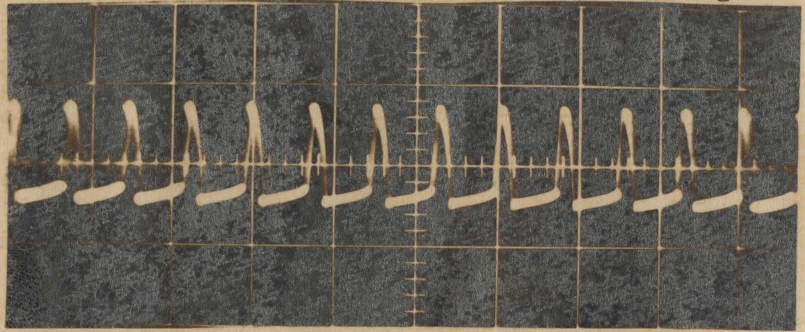
				Calculated	Observed
1.	L = 0.5μh	C = 0.02μf	α = 0.25	f = 1.6 mc/sec.	f = 1.4mc/sec.
2.	L = 2μh	C = 0.0032μf	α = 1.25	f = 2 mc/sec.	f = 1.7mc/sec.
3.	L = 40μh	C = 0.0025μf	α = 6.3	f = 0.5 mc/sec.	f = 0.3mc/sec.
4.	L = 40μh	C = 0.0006μf	α = 10	f = 1 mc/sec.	f = 0.6mc/sec.



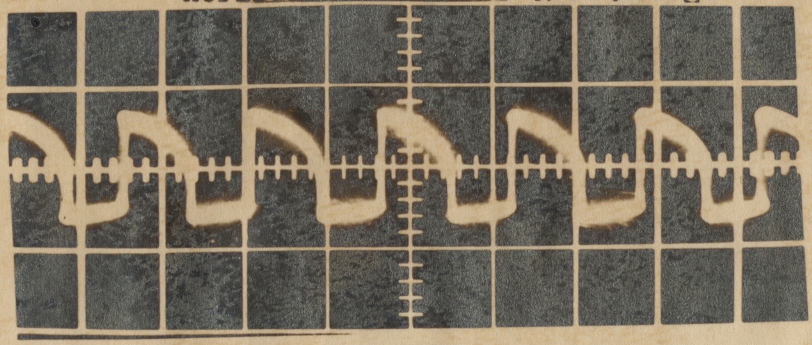
(a) $\alpha = 0.25$, $E = 350$ mv.
vertical scale is 0.5v/large division
Horizontal scale is 1μsec/large division



(b) $\alpha = 1.25$, $E = 350$ mv.
vertical scale is 0.5v/large division
horizontal scale is 1μsec/large division



(c) $\alpha = 6.3$, $E = 350$ mv.
vertical scale is 0.5v/large division
horizontal scale is 5μsec/large division



(d) $\alpha = 10$, $E = 350$ mv.
vertical scale is 0.5v/large division
horizontal scale is 1μsec/large division

VII Topological or Graphical Method of Solving Non-Linear Oscillations in Tunnel Diode.

The behaviour of a non-linear autonomous system described by a differential equation of the second-order can be studied by means of topological method without having to solve the actual differential equation for the time response. In this method, the periodic solution will appear as a closed contour, called the limit cycle, in a plane called the phase plane.

Since the analytical solution of the Van der Pol equation is valid only for $\alpha \not> 1$, for $\alpha > 1$, we must take recourse to graphical method. Graphical method for the solution of the above equation is the so-called method of isoclines.

Let us put $\frac{dy}{dT} = Z$, where $y = V_1$, therefore $\frac{d^2y}{dT^2} = \frac{dZ}{dT} = Z\frac{dZ}{dy}$.

Therefore equation (VI.9) reduces to $Z\frac{dZ}{dy} = \alpha(1-y^2)\,Z-y$, or

$$\frac{dZ}{dy} = \frac{\alpha(1-y^2)Z-y}{Z} \qquad \ldots \text{(VII.1)}$$

Hence in the phase plane $(Z = \frac{dy}{dT}, y)$, the above equation is the equation of the trajectory of y, and $\frac{dZ}{dy}$ is the slope of the tangent to the trajectory at a point. At an ordinary point in the phase plane, this slope has a definite value whereas at a singular point, this slope is indeterminate $(\frac{0}{0})$, both $\frac{dZ}{dT}$ and $\frac{dy}{dT}$ being zero at such a point. Therefore, in the phase plane a trajectory either converges to or diverges from a singular point depending whether it is stable or unstable respectively.

Through every ordinary point there passes one and only one phase trajectory.

A closed trajectory in the phase plane is called a limit cycle and it represents a periodic oscillation. In a non-linear system having variable damping, only the following three types of singularities[10] give rise to limit cycle in the phase plane and thereby indicate the presence of periodic oscillation.

(i) Vortex corresponding to imaginary roots.

(ii) Unstable node corresponding to positive real roots.

(iii) Unstable focus, corresponding to complex roots with positive real part.

If $\frac{dZ}{dy} = m$, then equation (VII.1) becomes $\frac{dy}{dT} = Z = \frac{y}{(\alpha - m) - ay^2}$

For a fixed value of α and for different values of m, a series of isoclines are drawn along each of which the slope of the trajectory is constant. The phase trajectories corresponding to $\alpha = 0$, 0.2, 1 and 10 are shown in the Figure VII.1. From the phase trajectory, the y(T) curve can be plotted by means of point-by-point integration method.

For small value of α, the limit cycles are only slightly different from a circle which represents a periodic harmonic oscillation. As α is increased, the limit cycles differ radically from circles and thus represents periodic motion that are distinctly non-harmonic. For $\alpha = 10$, the trajectory represents the relaxation where the motion is made up of sudden transitions between deflections of opposite signs.

As the isocline method is extremely tedious, we shall use a simpler graphical method devised by A. Lienard.[13] For this, we shall define

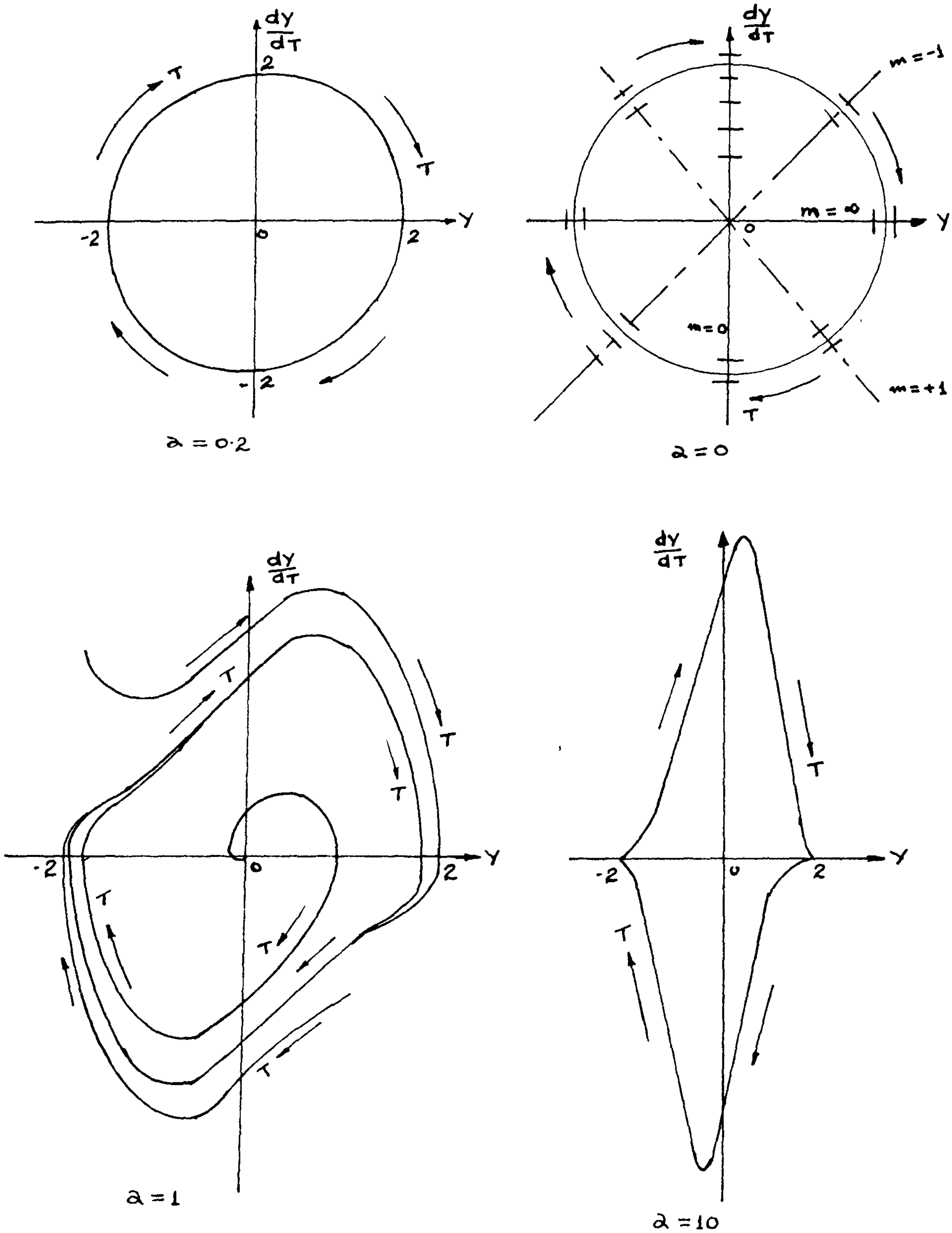


FIG. VII.1

$x(T) = \int y dT$, or $y = \frac{dx}{dT}$.

Substituting for y in the Van der Pol equation (VI.9) we get

$$\frac{d^3x}{dT^3} - \alpha\left[1 - (\frac{dx}{dT})^2\right] \frac{d^2x}{dT^2} + \frac{dx}{dT} = 0.$$

Integrating each term with respect to T and setting the constant of integration arbitrarily equal to zero, it reduces to

$$\frac{d^2x}{dT^2} - \alpha\left[\frac{dx}{dT} - \frac{1}{3}(\frac{dx}{dT})^3\right] + x = 0 \qquad \text{. . . (VII.2)}$$

which is of the form of Raleigh equation.

Since $I_L = \frac{1}{L}\int V\,dt$, $x(T)$ is the non-dimensional form of $I_L(t)$. Thus, we see that $y(T)$ which is the non-dimensional form of $v(t)$ satisfies Van der Pol equation, whereas $x(T)$ which is the non-dimensional form of $i_L(t)$ satisfies Raleigh equation.

To solve for $y(T)$ and $x(T)$ for different values of α, graphically we will construct Lienard diagram. Since $\frac{dx}{dT} = y$, and $\frac{d^2x}{dT^2} = \frac{dy}{dT} = y\frac{dy}{dx}$, then (VII.2) can be written as

$$\frac{dy}{dx} = \frac{\alpha(y - \frac{1}{3}y^3) - x}{y} \qquad \text{. . . (VII.3)}$$

In the phase plane ($y = \frac{dx}{dT}$, x), we plot the curve S(y) by the equation $S(y) = \alpha(y - \frac{y^3}{3})$ for a particular value of α as shown in Figure VII.2.

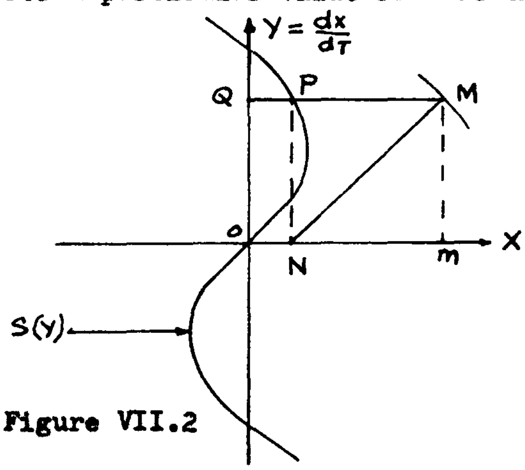


Figure VII.2

If MN be the normal to the trajectory satisfying (VII.3) and passing through any point M(x, y) in the x-y plane, then the projection Nm of NM on ox is $Nm = -y\frac{dy}{dx}$. Hence, (VII.3) can be written as Nm = QM - QP since $QP = \alpha(y - \frac{y^3}{3})$ and QM = x. This means that given any point M in the plane, all we have to do in order to obtain the tangent to the trajectory passing through it, is to draw MPQ horizontal and PN vertical and joint NM. The required tangent is perpendicular to NM.

Starting from any initial point and repeating the above process of construction in a clockwise direction, the entire phase trajectory can be drawn. For a sustained oscillation, a limit cycle is ultimately obtained regardless of the initial starting point. If the initial point is within the limit cycle, the ensuing trajectory spirals outward whereas if the initial point is outside the limit cycle the ensuing trajectory spirals inward. At steady-state, the spirals near the origin as well as those far from the origin tend to a single closed curve, the limit cycle. The path leading to the limit cycle from any starting point is the transient portion of the solution. From the limit cycle $\dot{x}(T)$ can be plotted.

The shape of the limit cycle depends on the value of α. As α increased from a very small value to a very large value, the shape of the limit cycle changes from a circle to an almost rectangle and the corresponding oscillation changes from almost sinusoidal to relaxation type. Since the two planes are quite different, the geometrical shape of the trajectory for the Van der Pol equation and that for the Raleigh equation are necessarily different.

Results

Using Lienard method, the limit cycle and the corresponding waveforms for α = 0.2, 1 and 5 are shown in Figures VII.4A, B and C. Moreover, if we put $Z = \frac{dy}{dT} = \frac{d^2x}{dT^2}$, then to a scale, if y(T) represents V(t), then x(T) represents $i_L(t)$ and Z(T) represents $I_C(t)$. These functions are shown in Figure VII.3 for α = 10

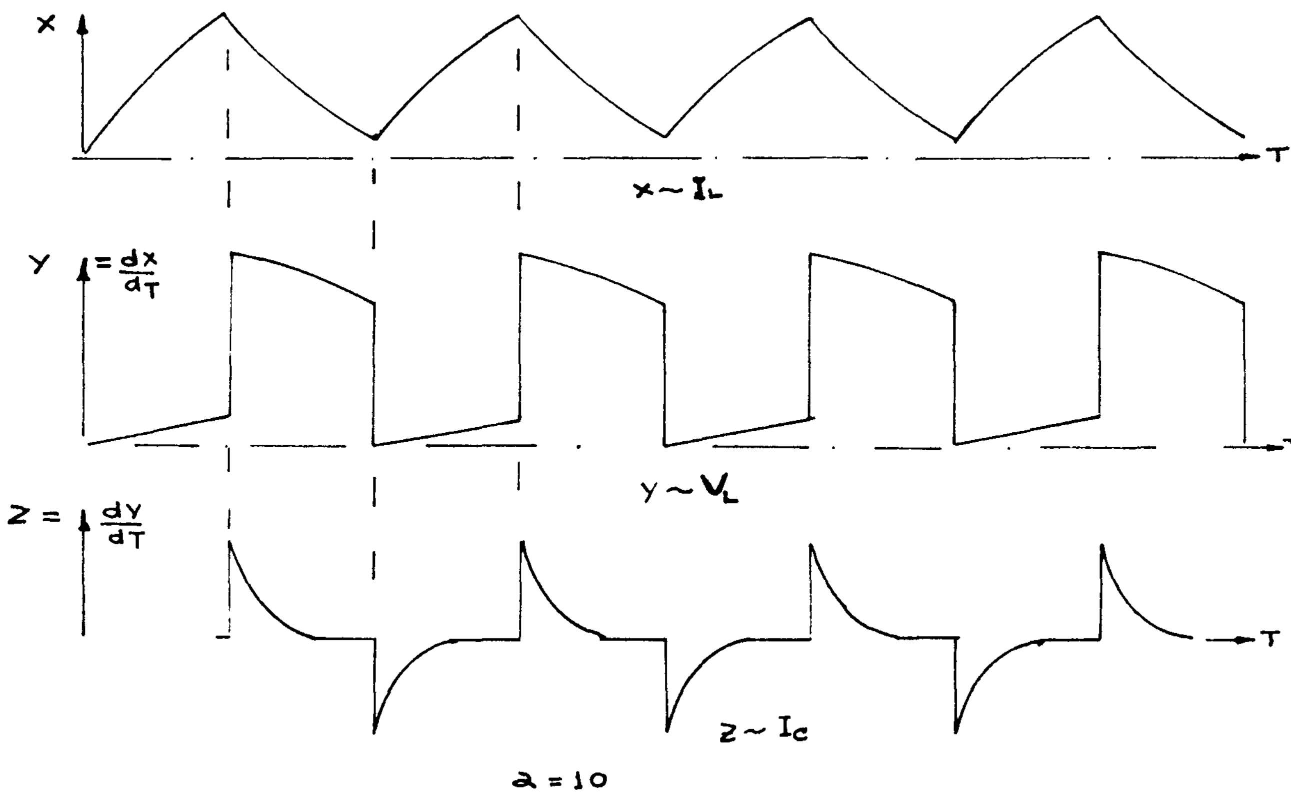
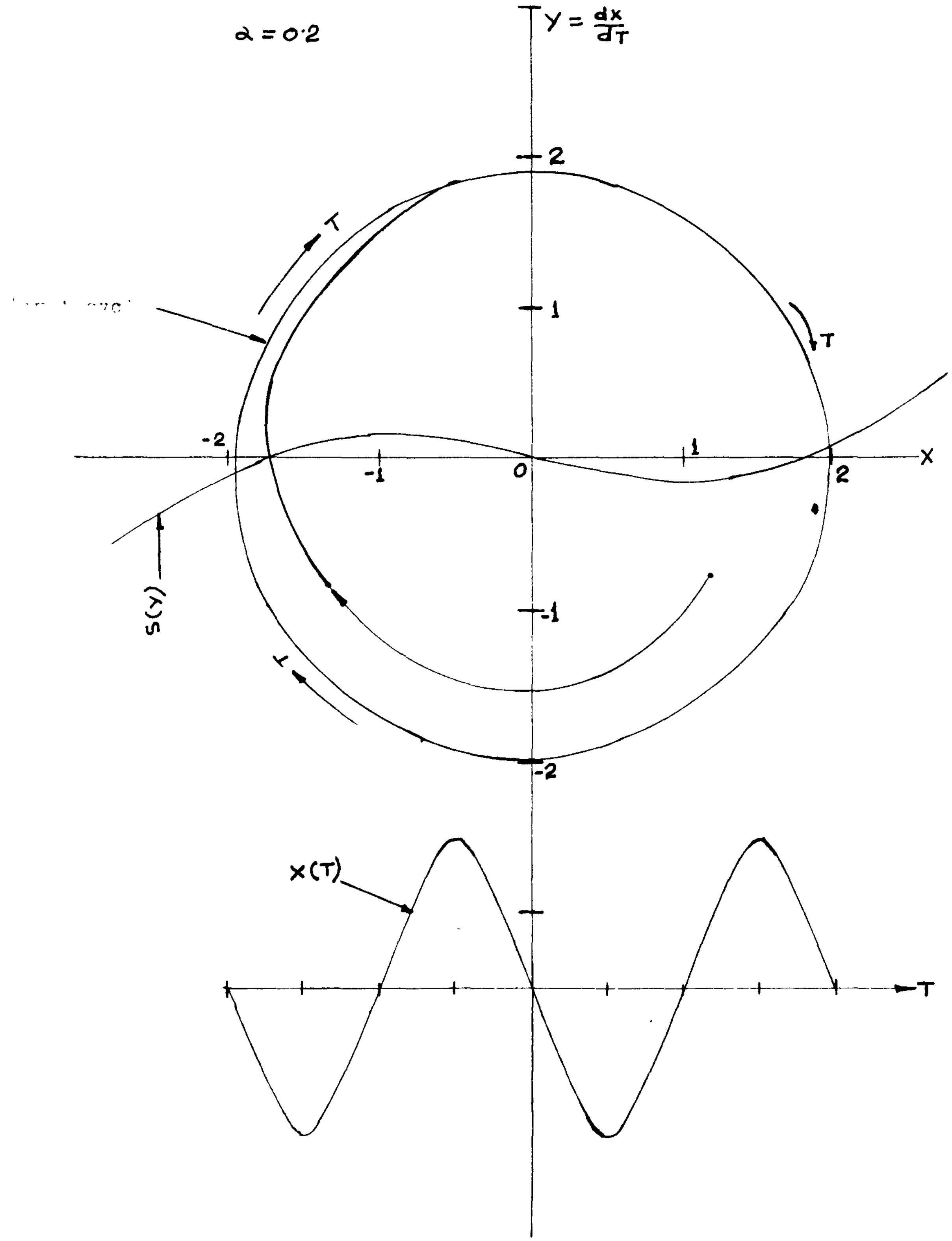
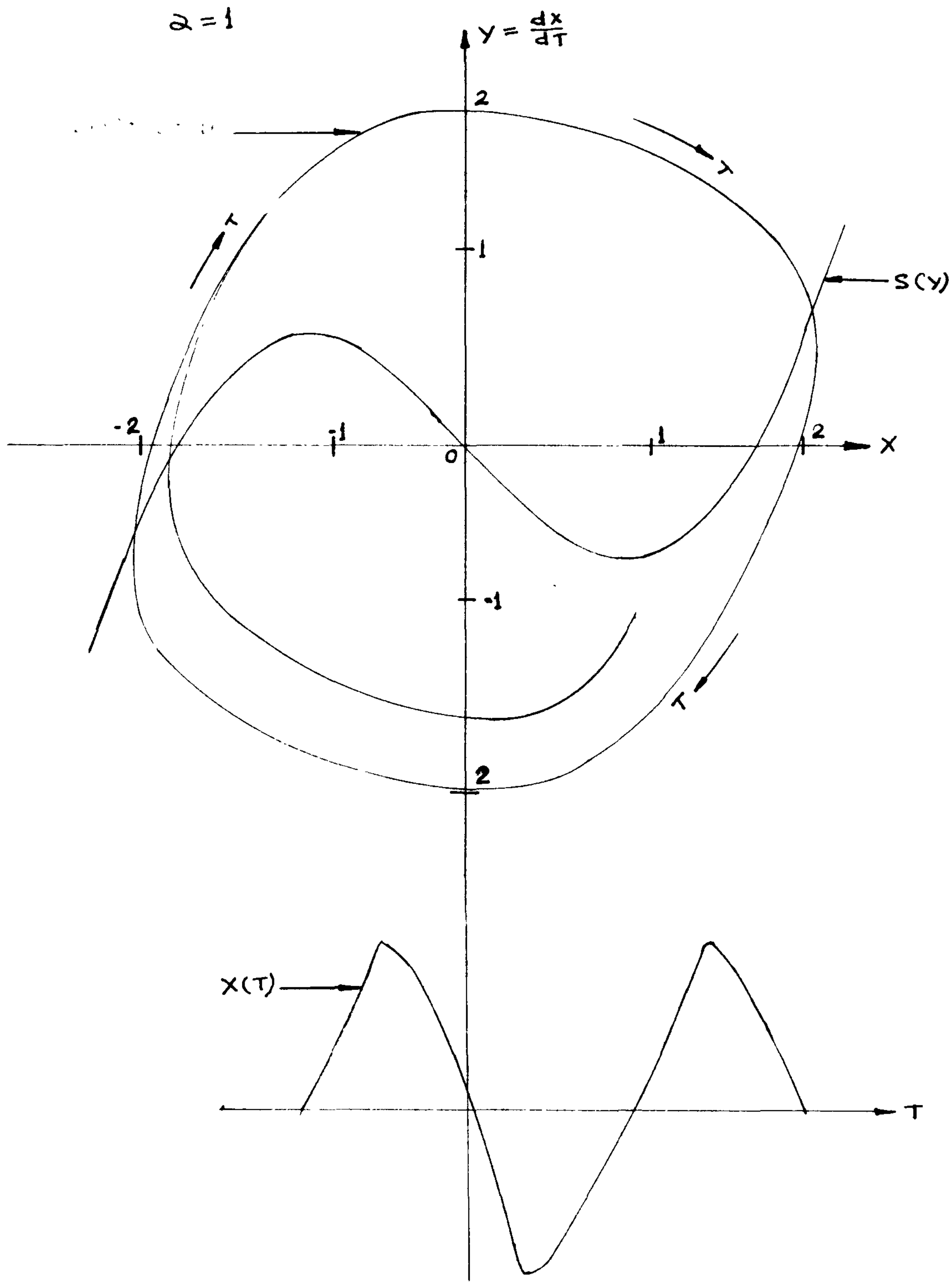


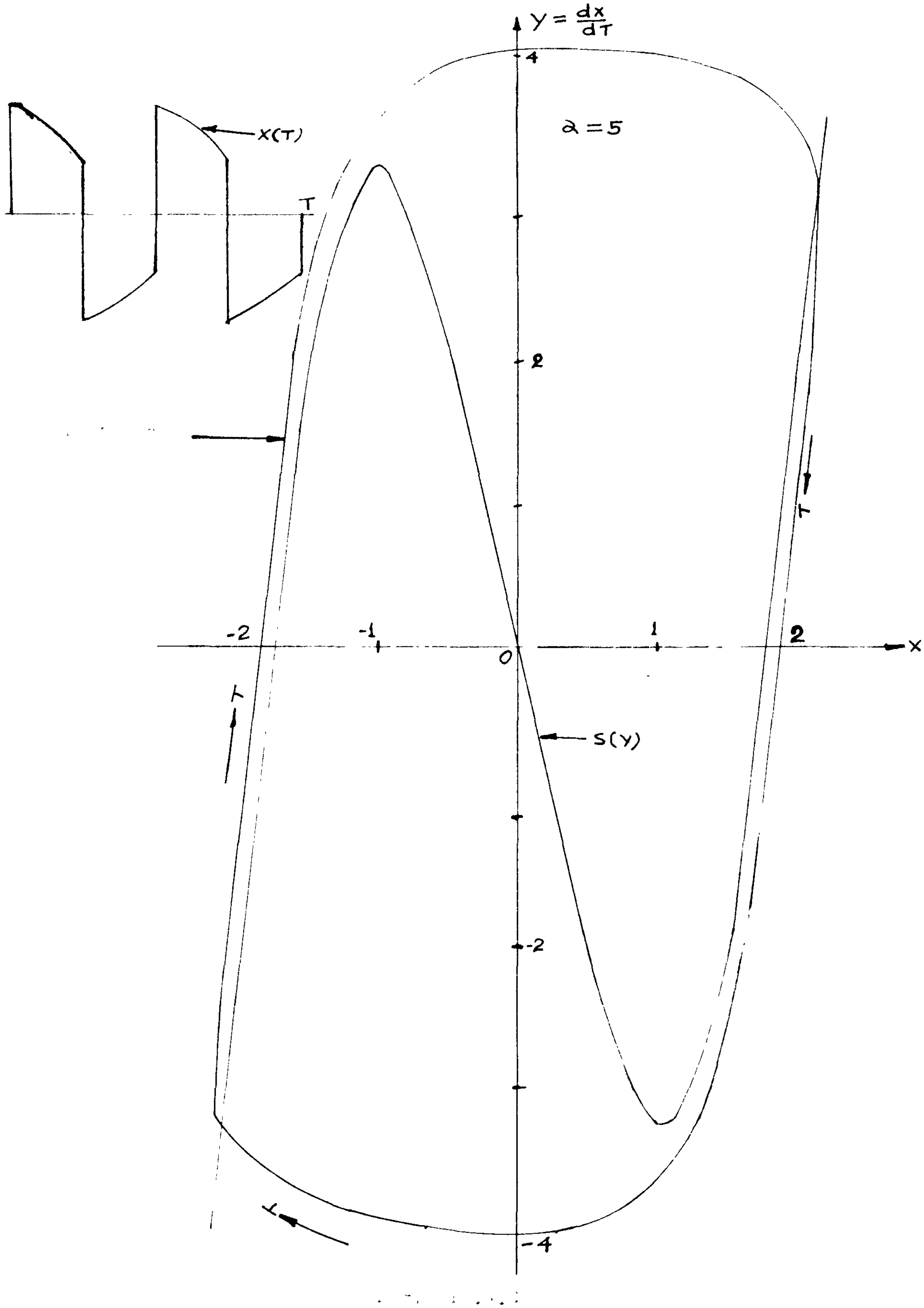
FIG. VII.3



α = 0.2
Y = dx/dT
X
T
S(Y)
X(T)



α = 1
y = dx/dT
2
1
-1
2
T
T
T
S(y)
-2
-1
0
1
2
X
X(T)
T



$y = \frac{dx}{dT}$
4
2
-2
-4
$\alpha = 5$
x(T)
T
-2
-1
0
1
2
x
S(y)
T
T
T

VIII Relaxation Oscillation

As shown before, the conditions of Relaxation oscillation are:

(i) $R_n > r$, i.e., the load line should intersect the characteristic at a single point in its negative slope.

(ii) $L \gg rR_nC$ or C should be very small as shown in Figure VIII.1A.

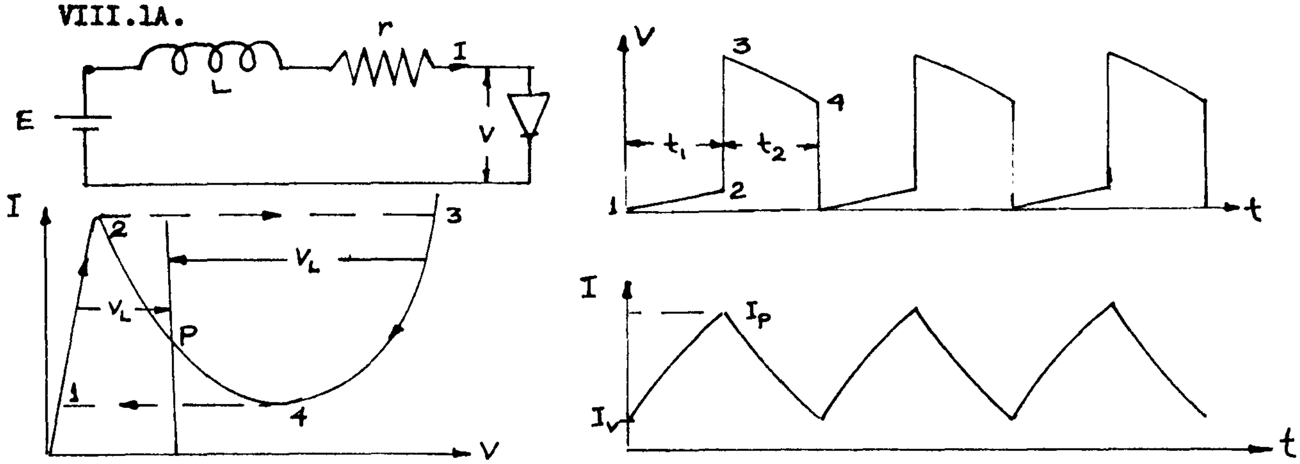


Figure VIII.1

In order for any oscillation to be self-starting, the origin must be a point of unstable equilibrium. As discussed before, if $\frac{L}{R_nC} - (\frac{4L}{C})^{1/2} < r < \frac{L}{R_nC}$, the point P in Figure VIII.1B above is an unstable focus corresponding to complex roots with positive real part. This gives oscillation at a real frequency. But if $r < \frac{L}{R_nC} - (\frac{4L}{C})^{1/2}$, the point P is an unstable node corresponding to positive real root and we get relaxation oscillation. Since $\alpha = a(\frac{L}{C})^{1/2}$, this also corresponds to a very large value of α.

Whatever be the initial conditions, once the oscillation builds up, it attains a final steady-state amplitude which is independent of the source voltage or any other condition. During each cycle of oscillation, the current and the voltage relation of the diode traces the path 1-2-3-4 along the diode characteristic as shown in Figure VIII.1B. The corresponding current and voltage waveforms are shown in Figure VIII.1C. This can be explained in the following manner:-

Writing the loop equation, we get,

$$E = L\frac{dI}{dt} + rI + V$$

$$\text{or} \quad L\frac{dI}{dt} = (E - Ir) - V = V_L (\text{say}) \qquad \ldots (\text{VIII.1})$$

But $(E - Ir)$ is the equation of the D.C. load-line. Therefore V_L is the distance from any point on the V-I characteristic to the load line and it can be treated as a vector. But V_L is proportional to the time rate of change of current through the inductance L. Only at the point of intersection of the two curves $V_L = 0$, i.e. $\frac{dI}{dt} = 0$ and this point is the steady-state operating point which is unstable in this case.

Now, as there was no initial current through the inductance after closing the switch, the current starts to increase from zero along the line 1-2. Now V_L drawn from any point along this line to the load line is positive indicating that the current through the inductance must increase, i.e. $\frac{dI}{dt}$ is positive. Hence current increases from zero to I_p and the operating point moves to the position 2.

At 2, V_L is still positive, i.e. $\frac{dI}{dt}$ is positive but current cannot increase beyond I_p. The operating point cannot travel down the negative slope because that would mean a decreasing current or negative $\frac{dI}{dt}$. Moreover, the movement of the operating point must remain restricted along the characteristic curve of the diode. This condition can be satisfied if the operating point makes a jump from point 2 to point 3. The current through the inductance cannot change instantaneously but the voltage can and it does. As soon as the operating point reaches the peak point 2 it jumps at constant current I_p to the point 3. At 3, V_L is negative. Therefore, current through the inductance must decrease. As the current decreases from I_p to I_v, the operating point moves down from 3 to 4 along the curve 3-4.

At 4, V_L is still negative. Therefore, current must decrease. But it cannot decrease beyong I_v. It cannot also travel up along the path 4-2 because that would mean increasing current. Hence, the operating point makes another jump from point 4 to 1, again at constant current I_v. At 1, $\frac{dI}{dt}$ is again positive. Therefore, the current increases again and the cycle repeats itself. Hence we find that the operating point never reaches the steady-state position p, but it oscillates around p indefinitely.

(a) Period of oscillation.

If the shunt capacitance of the diode is very small the time taken by the operating point to jump from point 2 to point 3 and also from point 4 to 1, is quite negligible. This time of instantaneous transition from one state to the other is called the switching time of the tunnel diode and is given by[14]

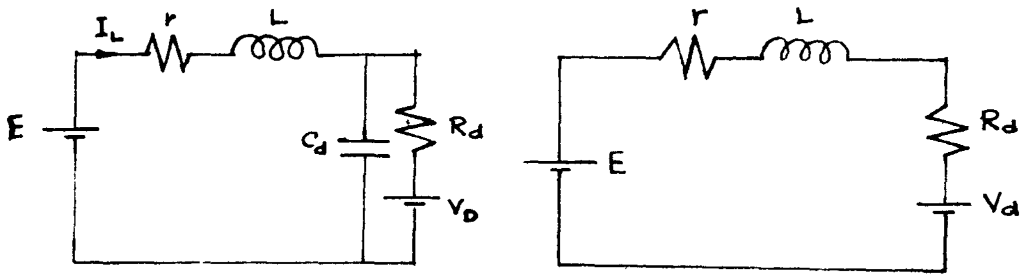
$$t_S = C_v \frac{V_f - V_p}{I_p - I_v} \text{ for points 2-3, and}$$

$$t_S = C_v \frac{V_v - V_1}{I_p - I_v} \text{ for points 4-1.}$$

For typical tunnel diode, this switching time is of the order of neno-seconds (10^{-9} seconds).

For XA650, GaAs tunnel diode $t_s = 2$ to 3×10^{-9} seconds. Neglecting this switching time, the time taken by the operating point to traverse the distance from 3-4 in the high-voltage state and from 1-2 in the low-voltage state, can be calculated as follows. Let t_1 and t_2 be the time taken respectively by the current to increase from I_v to I_p along 1-2, and to decrease from I_p to I_v along 3-4.

In order to calculate this time, the tunnel diode is replaced by its equivalent voltage source V_d, series resistance R_d and the shunt capacitance C_d as shown in Figure VIII.2A



<u>Figure VIII.2</u>

The current through the inductance is of the form

$$I_L(t) = K_0 + K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t} \qquad \text{(VIII.2)}$$

where α_1 and α_2 arethe roots of the characteristic equation and are given by

$$\alpha_1, \alpha_2 = \frac{r}{2L} + \frac{1}{2R_d C_d} \pm \sqrt{\left(\frac{r}{2L} + \frac{1}{2R_d C_d}\right)^2 - \frac{1}{LC_d}\left(1 + \frac{r}{R_d}\right)} \qquad \text{(VIII.3)}$$

For typical circuit values

$$\frac{1}{LC_d}\left(1 + \frac{r}{R_d}\right) < \left(\frac{r}{2L} + \frac{1}{2R_d C_d}\right)^2 \quad \text{and} \quad \frac{r}{L} < \frac{1}{R_d C_d}$$

Therefore α_1 and α_2 may be approximated by $\alpha_1 \cong \frac{1}{R_d C_d}$ and $\alpha_2 \cong \frac{r + R_d}{L}$.

If the shunt-capacitance is neglected, then $\alpha_1 = 0$, $\alpha_2 = \frac{r + R_d}{L}$ and the equivalent circuit becomes as shown in Figure VIII.2B. Therefore $I_L(t) = K_0 + K_2 e^{-\alpha_2 t}$. The values of R_d and V_d to be substituted depend on the region where the diode operates. This is shown in the piece-wise equivalent circuit of the diode.

t_1:- As soon as the diode enters the low-voltage state at point 1, it can be replaced by a series resistance r_1, and a voltage source $V_d = 0$. The equivalent circuit then becomes as shown in Figure VIII.3A.

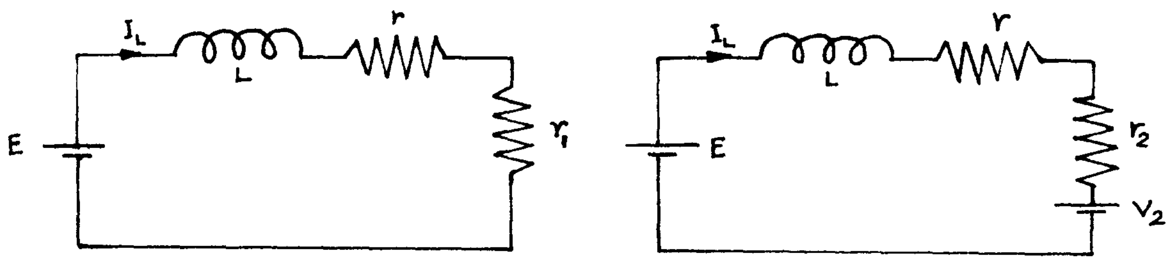


Figure VIII.3

The conditions are at $t = 0$, $I_L = I_v$ and $t = t_1$, $I_L = I_p$. Solving for the current I_L, we get

$$I_L = \frac{E}{R_1}(1 - e^{-R_1 t/L}) + I_v\, e^{-R_1 t/L}, \text{ where } R_1 = r + r_1$$

$$= I_a(1 - e^{-R_1 t/L}) + I_v\, e^{-R_1 t/L}, \text{ where } I_a = \frac{E}{R_1}.$$

When $t = t_1$, $I_L = I_p$. Therefore $t_1 = \frac{L}{R_1} \ell n \frac{I_a - I_v}{I_a - I_p}$

$$= \frac{L}{R_1} \ell n \frac{E - I_v R_1}{E - I_p R_1}$$

If r is negligible, $t_1 = \frac{L}{r_1} \ell n \frac{E - E_1}{E - E_2}$ (VIII.4)

where $E_1 = I_v r_1$ and $E_2 = V_p$.

t_2:- When the diode voltage jumps from point 2 to point 3, the equivalent circuit becomes as shown in Figure VIII.3B, where r_2 is given by the reciprocal of the slope of the characteristic segment between the point 3 and 4 and V_2 is the corresponding voltage source. As shown before, we will take $V_2 = \frac{V_v + V_f}{2}$ and $r_2 = \frac{V_f - V_2}{I_p - I_v}$.

In this case the conditions are when $t = 0$, $I_L = I_p$ and when $t = t_2$, $I_L = I_v$. Solving the loop equations for $I_L(t)$, we get, after substituting the initial condition

$$I_L(t) = \frac{E - V_2}{R_2}(1 - e^{-R_2 t/L}) + I_p e^{-R_2 t/L}, \text{ where}$$

$R_2 = r + r_2$. When $t = t_2$, $I_L = I_v$, therefore

$$t_2 = \frac{L}{R_2} \ln \frac{I_p R_2 + V_2 - E}{I_v R_2 + V_2 - E}, \text{ since } V_2 + I_p r_2 = V_f$$

therefore, $t_2 = \frac{L}{R_2} \ln \frac{V_f - E + I_p r}{V_2 + I_v R_2 - E}$. If r is negligible[17]

$$t_2 = \frac{L}{R_2} \ln \frac{V_f - E}{V_2 - E} \qquad \text{(VIII.5)}$$

Therefore, the total period of a cycle is $T = t_1 + t_2$ and hence the frequency $f = \frac{1}{T}$.

As shown before, the period T is a function of negative conductance a at the bias point and the series inductance $L(T = 1.62\ aL)$.

Since the negative conductance a is a function of bias voltage, the period T varies with the bias voltage. Hence, there is a large variation of frequency with thesupply voltage.

(b) <u>Experimental results</u>.

The tunnel diode used is a GaAs type XA650 having a switching time $t_s = C_v \frac{V_f - V_p}{I_p - I_v} = 4 \times 10^{-9}$ seconds. The diode is connected in series with an inductance $L = 40\mu h$ and is biased in the middle of its negative slope region with a voltage of $E = 300$ mv., having a series resistance of approximately $r = 2$ ohms. The voltage and the current waveforms obtained are shown in Figure VIII.5.

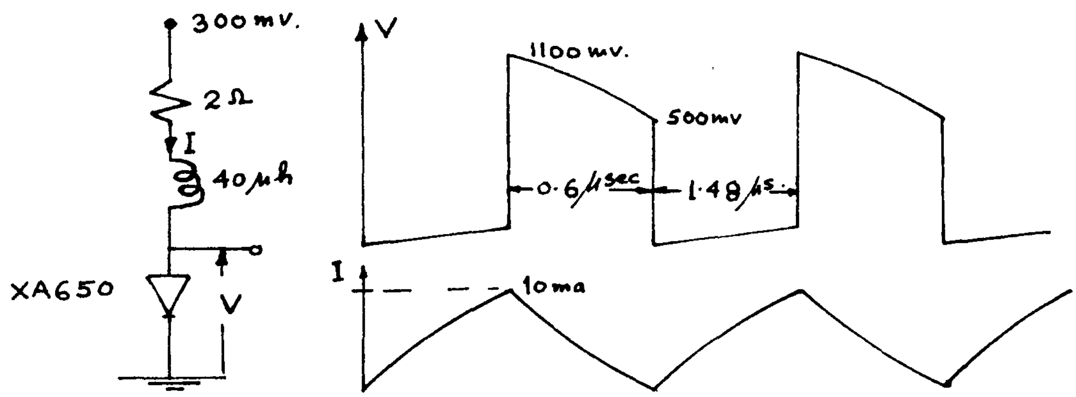


Figure VIII.5

The period is found to be $T = t_1 + t_2 = 2.06$ μsec., and the corresponding frequency $f = \frac{1}{T} = 0.5$ mc/sec.

Since r is negligible, using the equations (VIII.4) and (VIII.5) established before, t_1 and t_2 can be calculated as

$$t_1 = \frac{L}{r_1} \ell n \frac{E - E_1}{E - E_2} = 1.5 \text{ μsec., and}$$

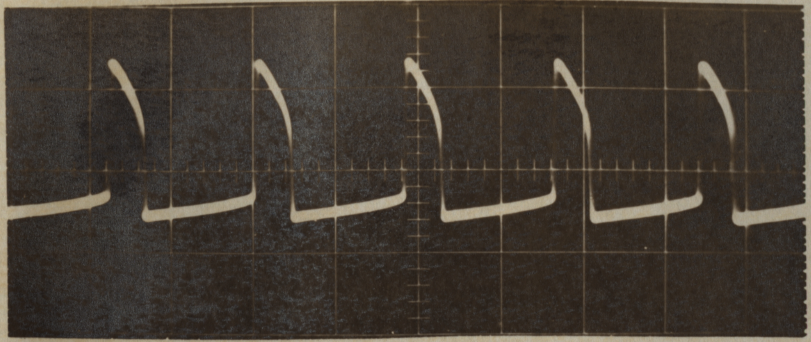
$$t_2 = \frac{L}{r_2} \ell n \frac{V_f - E}{V_2 - E} = 0.5 \text{ μsec.}$$

Therefore, the period $T = t_1 + t_2 = 2.1$ μsec. The observed values are $t_1 = 1.48$ μsec. and $t_2 = 0.6$ μsec. Hence, we see that the calculated values are in close agreement with the experimental ones. An oscillogram of the voltage waveform is shown in Figure VIII.6a. In VIII.6b, the voltage and the current waveforms are superimposed for comparison.

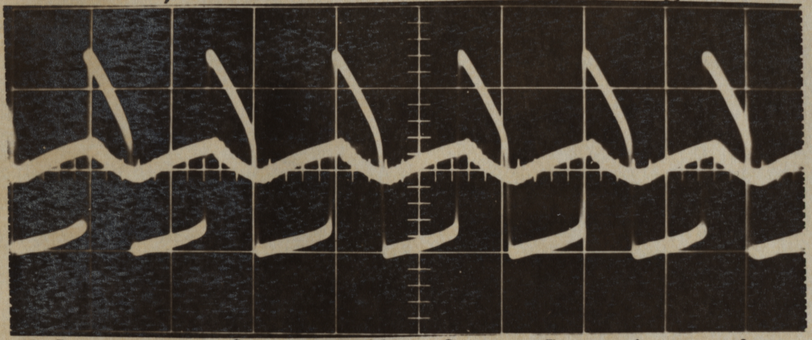
(c) Variation of t_1 and t_2 with bias voltage.

If the bias voltage is varied over the entire range of the negative slope of the diode characteristics, the amplitudes remain uneffected by this variation of the bias voltage but t_1 and t_2 vary over a wide range as shown in the following table.

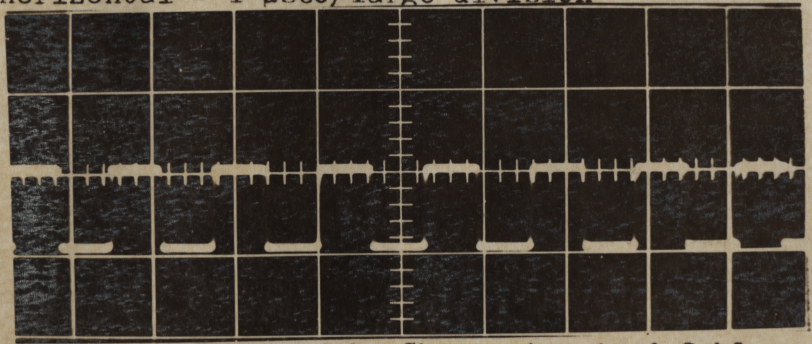
These values of t_1 and t_2 are plotted against bias voltage in Figure VIII.7. The frequency is maximum and the waveform is quite



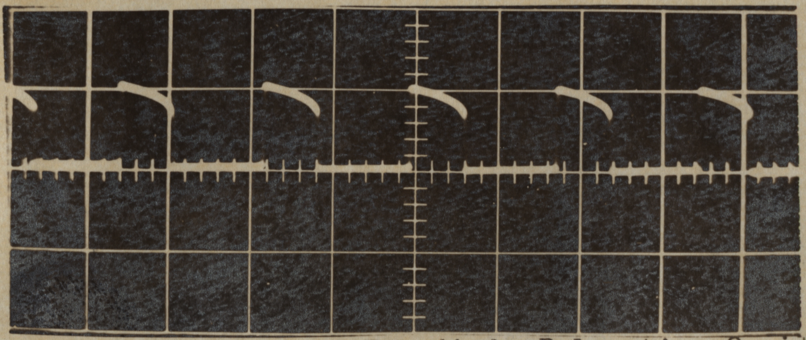
(a) Voltage Waveform of Tunnel Diode Relaxation Oscillator
Bias voltage 350 mv., vertical scale is 0.5 volt/large division; horizontal scale 1 μsec/large division



(b) Voltage and Current Waveforms Superimposed on Each Other
Bias voltage = 400 mv., vertical = 0.5v/large division; horizontal = 1 μsec/large division



(c) Inductance Replaced by Short-circuited Cable generates Square Waves. Bias voltage = 400 mv. 1 volt/large division (vertical); 1 μsec/large division (horizontal)



(d) Voltage Waveform of Two-diode Relaxation Oscillator
Bias voltage = 400 mv. Vertical Scale - 1 v/large division; horizontal scale = 1 μsec/large division

symmetrical near the valley voltage.

E(mv)	Calculated values t_1(μsec)	t_2(μsec)	T(μsec)	Observed values t_1(μsec)	t_2(μsec)	T(μsec)
150	4	0.48	4.48	3.8	0.45	4.25
200	2.56	0.56	3.12	2.5	0.54	3.04
250	1.88	0.64	2.52	1.9	0.55	2.45
300	1.6	0.70	2.30	1.48	0.60	2.08
350	1.32	0.77	2.09	1.25	0.64	1.89
400	1.04	0.84	1.88	1.00	0.75	1.75
450	1	0.96	1.96	0.90	0.90	1.8
480	1	1	2	0.80	1.00	1.80
500	0.88	1.06	1.94	0.78	1.10	1.88

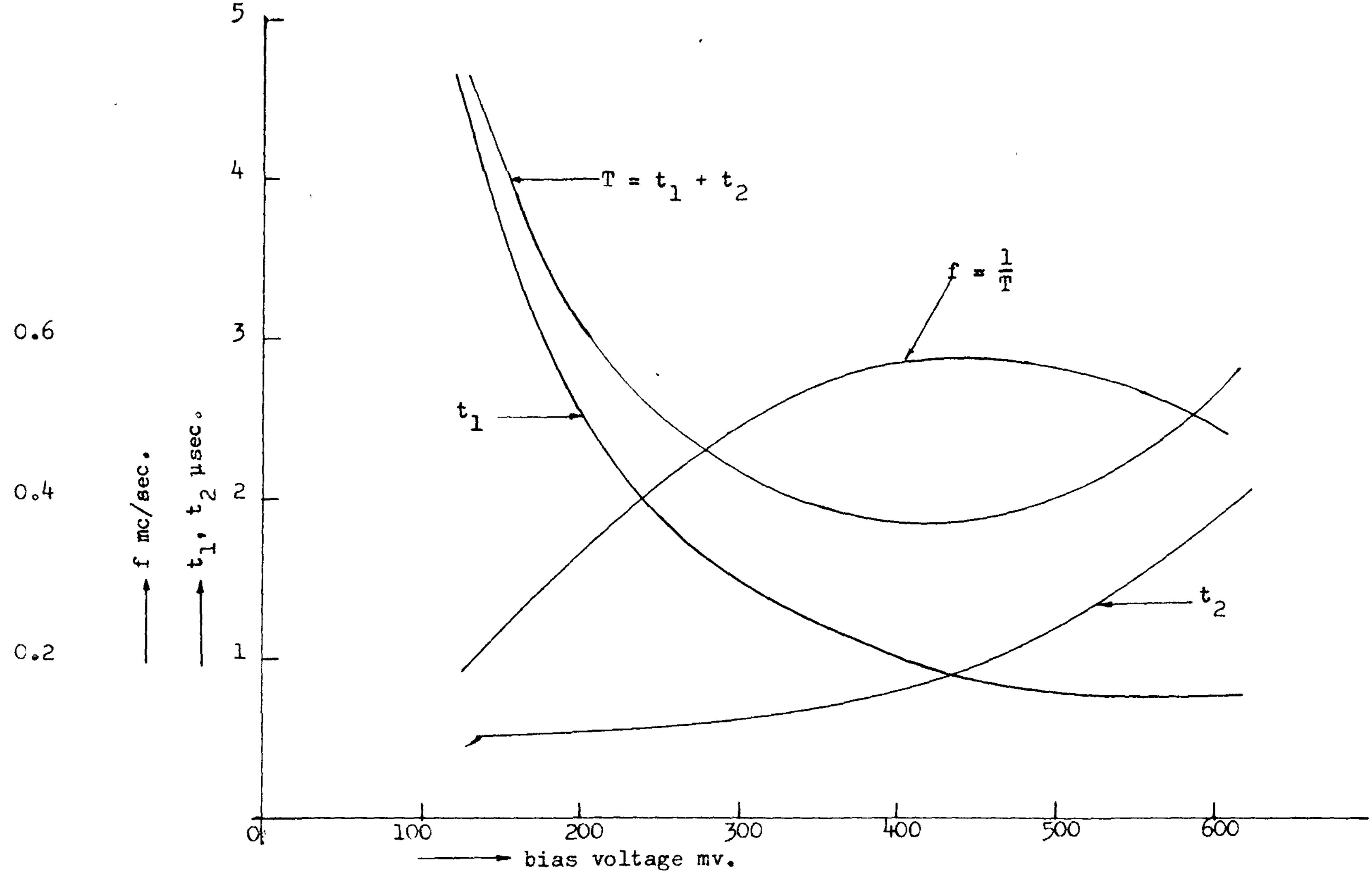
(d) Wave shaping: square wave generation.

(1) The waveform of the relaxation oscillator is neither sinusoidal nor rectangular. By using another tunnel diode in series with a suitable resistance across the output terminals, the bottom rising portion of the wave can be smoothed out. The top drooping portion can be eliminated by means of clipper action. The resulting waveform then becomes a square one.

An oscillogram of the output voltage waveform without clipping is shown in Figure VIII.6d.

(2) If a short-circuited co-axial cable is used instead of the series inductance, a good square wave is generated with excellent frequency stability over a considerable bias range.

The period of the square wave is four times the period needed for the electro-magnetic wave to travel the length of the cable. The



Variation of t_1 and t_2 with bias

Figure VIII.7

short-circuited cable can be used either in series or in parallel with the diode. In the latter case, the cable is connected to the diode through a blocking condenser.

Experimental Results

The diode is biased in the negative-resistance region and connected in series with a short-circuited co-axiable cable of type RG-8A/U, having a characteristic impedance Z = 50 ohms, as shown in Figure VIII.8A. The output voltage waveform across the diode is shown in Figure VIII.8B.

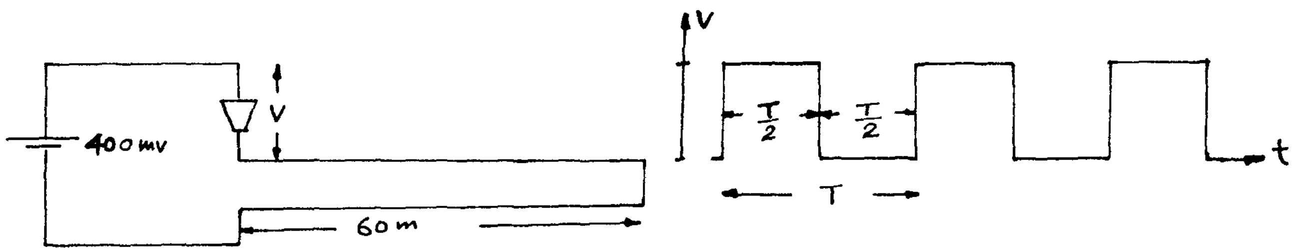


Figure VIII.8

Length of the cable is $\ell = 60$ meters, and the velocity of propagation of the electro-magnetic waves along the cable is approximately $V = 2 \times 10^8$ meters/second. Therefore, time of one way travel down the line is $t = \frac{\ell}{V} = 0.3$ µsec. The observed value of the period is 1.3 µsec., which is approximately equal to 4t, t being the time of one way travel along the length of the cable. An oscillogram of the observed square wave is shown at (c) in Figure VIII.6. The period remains constant over a considerable range of bias voltage.

When the cable is connected in parallel with the diode, a high inductance choke of 100 µh is placed between the diode and the source voltage in order to isolate the source path a.c. wise. The d.c. is blocked through the cable by a condenser of 0.1 µf. The amplitude and the period of the square wave are not affected by the parallel connection of the cable.

(3) On the other hand, if an open-circuited co-axial cable is connected across the tunnel diode in the above relaxation oscillator circuit, an excellent rectangular wave is generated. But this time the period of the rectangular wave is twice the period needed by the electromagnetic wave to travel the length of the cable. This is shown in Figure VIII.9

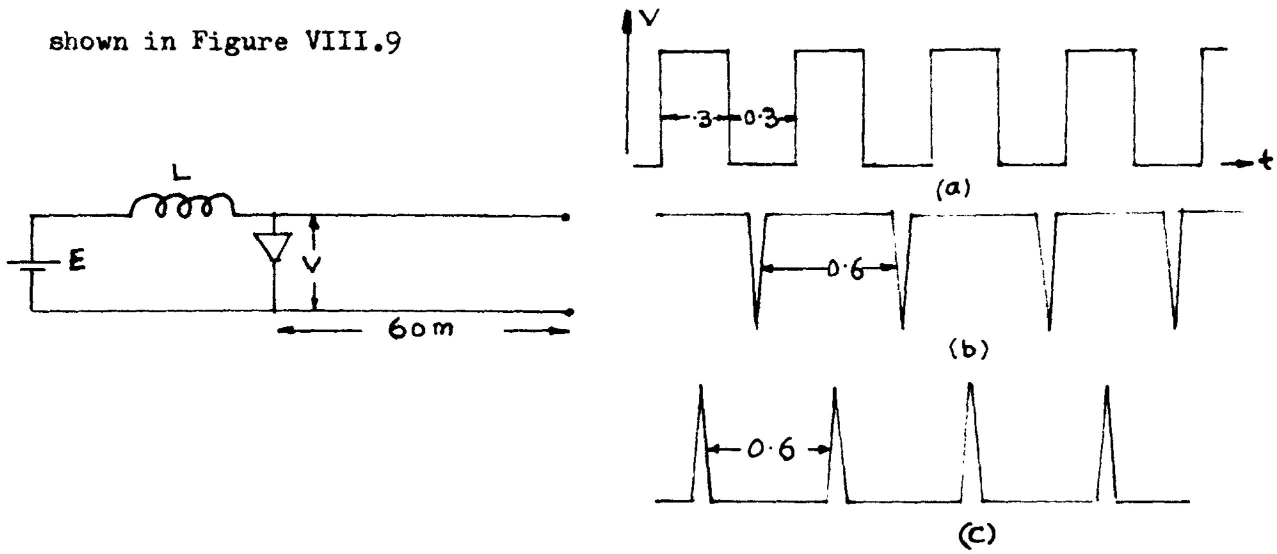


Figure VIII.9

The length of the cable is L = 60 m. and the velocity of the propagation along the cable is $V = 2 \times 10^8$ m/sec. Therefore, the time of one way travel is $t = \frac{\ell}{v} = 0.3$ µsec. The period of the rectangular wave is found to be T = 0.65 µsec., which is equal to 2t. At a bias, E = 400 mv. corresponding to the middle of the negative slope of the diode characteristic, the wave is symmetrical as shown at (a) in Figure VIII.9B, and the half periods $t_1 = t_2 = 0.3$ µsec.

As the bias is increased gradually to E = 500 mv., corresponding to the valley of the characteristic, t_2 gradually decreases from 0.3 µsec. to about zero and t_1 increases from 0.3 µsec. to 0.6 µsec.

The waveform then becomes a series of negative pulses as shown at (b).

On the other hand, when the bias is decreased from 400 mv. to about 200 mv, t_1 decreases from 0.3 to almost zero and t_2 increases from 0.3 to 0.6 μsec., and the waveform becomes a series of positive pulses as shown at (c).

IX Self-Oscillations in a Transmission Line with a Tunnel Diode Generate Staircase and Square Wave Forms

In the previous chapter we observed that a tunnel diode generates square-wave when connected either in series or in parallel with a short-circuited transmission line, the diode being biased in the negative resistance region.

This phenomenon of self-oscillation in a transmission line with a tunnel diode has been explained with appropriate theoretical evidence in the reference 15. There it has been shown how with increasing bias the waveform changes from staircase to a square type. This method is graphical rather than analytical. We shall give a brief account of the theory given therein.

In a loss-less line, both the voltage and the current satisfies the wave equation

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} \qquad \ldots \text{(IX.1)}$$

$$\frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 i}{\partial t^2}$$

where v(x, t) and i(x, t) are the instantaneous voltage and the current at a point x as shown in Figure IX.1

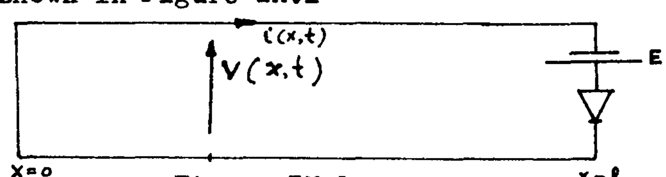


Figure IX.1

L and C are the series inductance and the shunt capacitance per unit length of the line. D'Alemberts' solution of (IX.1) is of the form

$$v(x,\ t) = \phi_1(t - \frac{x}{V}) + \phi_2(t + \frac{x}{V}) \qquad \ldots (IX.2)$$

where $V = \frac{1}{(LC)^{1/2}}$ is the velocity of propagation of the wave along the line.

Since $\frac{\partial v}{\partial x} = -L\,\frac{\partial i}{\partial t}$, and $\frac{\partial i}{\partial x} = -C\,\frac{\partial v}{\partial t}$, from (IX.2) we get

$$i(x,\ t) = \frac{1}{Z}\left[\phi_1(t - \frac{x}{V}) - \phi_2(t + \frac{x}{V})\right] \qquad \ldots (IX.3)$$

where $Z = (\frac{L}{C})^{1/2}$ is the characteristic impedance of the line.

The boundary conditions are

(i) $V(0,\ t) = 0$

(ii) $i(L,\ t) = f\left[v(L,\ t) + E\right]$

Sbustituting the boundary conditions (i) in (IX.2) we get

$$\phi_1(t) + \phi_2(t) = 0, \text{ or } \phi_1(t) = -\phi_2(t).$$

Substituting this into (IX.2) and (IX.3) and applying the boundary condition (ii), we get

$$\phi_1(t - \frac{\ell}{V}) + \phi_1(t + \frac{\ell}{V}) = Zf\left[\phi_1(t - \frac{\ell}{V}) - \phi_1(t + \frac{\ell}{V}) + E\right]$$

Let $\frac{\ell}{V} = \frac{T}{2}$, the time taken by the wave to travel the length of the line.

$$\phi_1(t - \frac{T}{2}) + \phi_1(t + \frac{T}{2}) = Zf\left[\phi_1(t - \frac{T}{2}) - \phi_1(t + \frac{T}{2}) + E\right] \qquad \ldots (IX.4)$$

The above equation can be written as[15]

$$\phi_1(t + \frac{T}{2}) = g\left[\phi_1(t - \frac{T}{2})\right] \qquad (IX.5)$$

which is a difference equation with the difference T. If

$\phi_1(t - \frac{T}{2}) = y(t)$, then (IX.5) becomes $y(t + T) = g\left[y(t)\right]$.

It is assumed that the initial function is of the form $y(t) = y_0(t)$.

Let t_0 be a fixed time, then we can define a sequence $\{y_n\}$, such that

$$y_0 = y_0(t_0)$$

$$y_1 = y_0(t_0 + T) = g\left[y_0(t_0)\right]$$

$$y_2 = y_0(t_0 + 2T) = gg\ [y_0(t_0)]$$

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$$y_n = y_0(t_0 + nT) = gg\ .\ .\ .\ (n)\ g\ [y_0(t_0)]$$

As n approaches ∞, this sequence has two possible steady states.

(1) There exists a certain value y_∞ such that as n approaches ∞, this sequence converges asymptotically to y_∞ and hence no oscillation can occur.

(2) The steady-state may take ■ alternative different values cyclically.

It can be shown that[15] if

(i) $\left|\frac{dg}{dy}\right| < 1$, then the sequence $\{y_n\}$ converges to y_∞ and no oscillation can occur. On the other hand if,

(ii) $\left|\frac{dg}{dy}\right| > 1$, then the sequence alternates between steady-state cyclically and hence we get oscillation.

Now defining new variables η and φ such that

$$\eta = \frac{1}{(2)^{1/2}}\left[y(t) + y(t + T)\right] \text{ and}$$

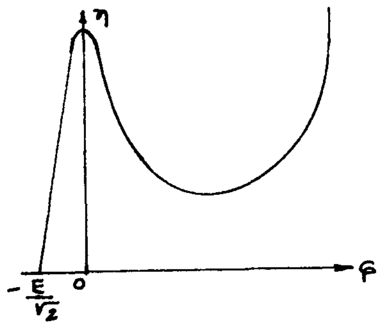
$$\varphi = \frac{1}{(2)^{1/2}}\left[y(t) - y(t + T)\right]$$, equation (IX.4) can be written as

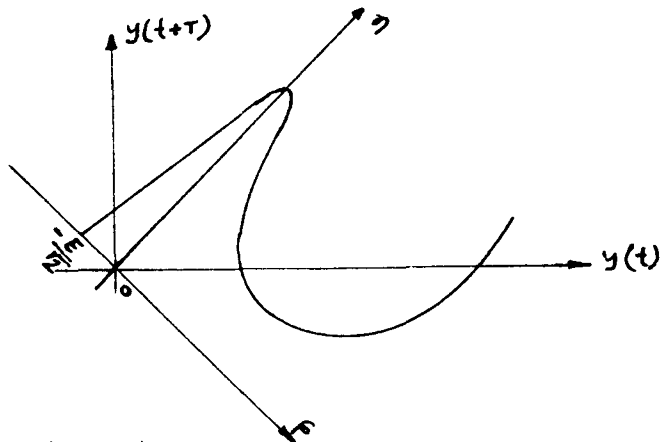
$$\eta = \frac{Z}{(2)^{1/2}} f\left[(2)^{1/2}\varphi + E\right]$$, but $v(\ell t) = y(t) - y(t + T)$ and

$i(\ell,t) = \frac{1}{Z}\left[y(t) + y(t + T)\right]$ Therefore, η and φ are related to $v(\ell,t)$ and $i(\ell,t)$ by $\varphi = \frac{1}{(2)^{1/2}} v(\ell,t)$, and $\eta = \frac{Z}{(2)^{1/2}} i(\ell,t)$.

Hence the characteristic of the tunnel diode can be plotted on the η-φ plane by simply changing the scale of $i(\ell,t)$ and $v(\ell,t)$ such that $\eta = \frac{Z}{(2)^{1/2}} i(\ell,t)$, and $\varphi = \frac{1}{(2)^{1/2}} v(\ell,t)$. This is shown in

Figure IX.2A.





The relation between y(t) and y(t + T) can be obtained by rotating the above curve 45° to the right as shown in Figure IX.2B.

If $E < V_p$ or $E > V_v$, the curve g intersects the η-axis at a point in the positive resistance region. At this point $\left|\frac{dg}{dy}\right| < 1$ and hence this point is stable and no oscillation is possible. If $V_p < E < V_v$, $\left|\frac{dg}{dy}\right|$ is greater than unity at the point of intersection of the g curve with the η-axis. Hence, this point is unstable and various types of self-oscillation can occur in this region.

If this oscillation is observed by the voltage across the tunnel diode, the oscillation generally has a staircase waveform.

If E is set near the middle point between V_p and V_v, the number of steady operating points in the low-voltage region (m) is nearly equal to that o n the high-voltage region (n). When Z is small we get a square wave corresponding to the mode m = 1 and n = 1 and the current is constant. When E is set near to V_p, m is greater than n, and when E is set near to V_v, m is smaller than n. In any case, the period is (m + n)T where $T = \frac{2\ell}{V}$.

Experimental Results

The tunnel diode used is GaAs type XA650. For transmission line, RG-8A/U type co-axial cable is used. The characteristic impedance of the cable is about $Z = 50\Omega$. The length of the cable is $\ell = 60$ m., velocity of propagation of wave along the cable is $V = \frac{1}{(LC)^{1/2}} = 2x10^8$ m/sec.

Therefore $T = \frac{2\ell}{V} = 0.6$ µsec., where T is the time needed for the wave to travel the length of the cable both ways.

By varying the bias voltage E within the negative resistance portion of the diode V_p E V_v, the following three modes of oscillation are observed.

(i) m = 3, n = 1, period = (m + n)T = 4T = 2.4 µsec.

(ii) m = 2, n = 1, period = (m + n)T = 3T = 1.8 µsec.

(iii) m = 1, n = 1, period = 2T = 1.2 µsec.

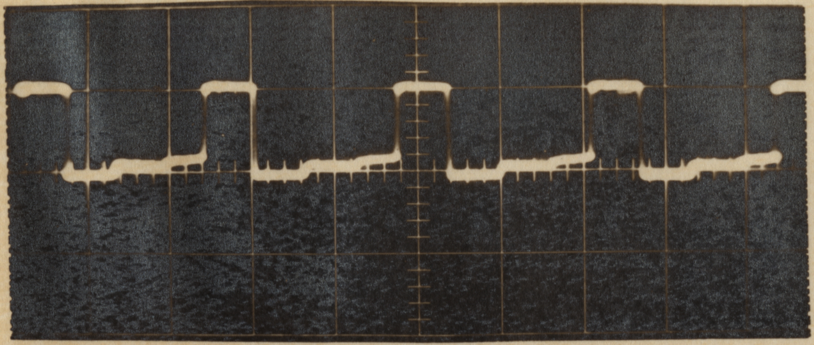
These results are shown in the oscillograms of Figure IX.3. These are the voltage waveforms across the diode for different bias voltages. The observed periods are in close agreement with the calculated ones, the slight difference being due to the assumed approximate value for the propagation velocity V along the cable. In Figure IX.3,

(a) corresponds to the case m = 3, n = 1. The bias is set very near to the peak voltage of the diode. E = 200 mv., period = 2.6 µsec.

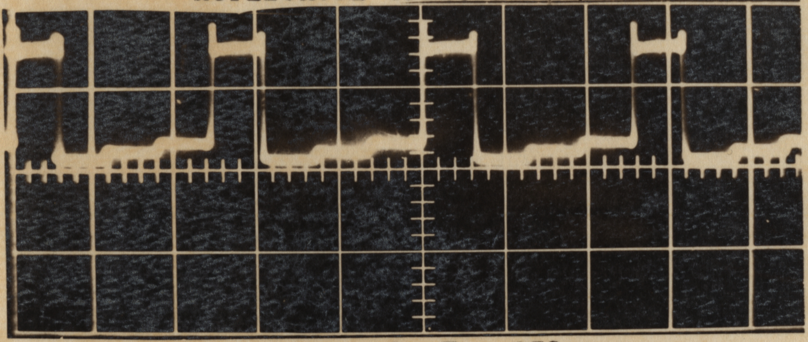
(b) also shows m = 3, n = 1, but the bias is slightly greater than that in (a). E = 250 mv., period = 2.4 µsec.

(c) corresponds to the case m = 2, n = 1, E = 350 mv., period = 1.9 µsec.

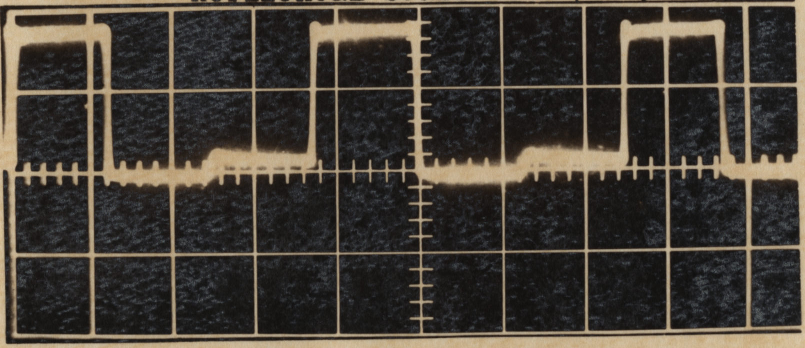
(d) corresponds to the mode m = 1, n = 1. The wave form is a square one. The bias voltage E = 400 mv., period = 1.25 µsec.



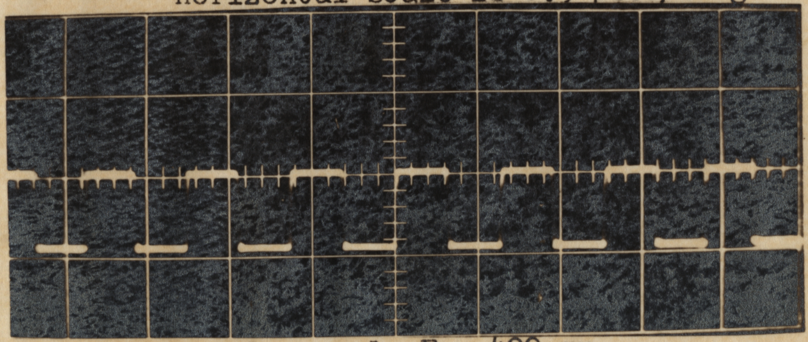
(a) m = 3, n = 1, E = 200 mv.
vertical scale is 1 v/large division
horizontal scale is 1 μsec/large division



(b) m = 3, n = 1, E = 250 mv.
vertical scale is 0.5 v/large division
horizontal scale is 1 μsec/large division



(c) m = 2, n = 1, E = 350 mv.
vertical scale is 0.5 v/large division
horizontal scale is 0.5 μsec/large division



(d) m = 1, n = 1, E = 400 mv.
vertical scale is 1 v/large division
horizontal scale is 1 μsec/large division

X Tunnel Diode Switching Circuit

Any two-terminal active device that exhibits a negative resistance possesses three distinct regions in its characteristic curve; two stable regions separated by an unstable region. This is because no physical device can have unlimited range of negative-resistance as that would mean the device is a source of infinite energy. Such a device makes an ideal switching circuit which needs two stable or quasi-stable states separated by a single transition zone.

As shown before, to be active or regenerative, a two-terminal device must have an energy-storage element in it and hence as a voltage-controlled negative-resistance a tunnel diode needs an inductance in series with it, in order to allow instantaneous voltage changes during the switching process.

By proper choice of the load resistance R and the battery voltage E, the load-line can be made to intersect the characteristic of the tunnel diode either at a single point in its negative slope or at a single point in one of its positive slopes or it can intersect at three points in the three regions of the curve and thereby signifying that the diode can be operated in the astable, monostable or bistable mode of multivibrator respectively. The steady-state operating point of the circuit which is given by the intersection of the load-line with the characteristic, is always stable if it is on one of its positive slopes and unstable if it is on its negative slope. The intersection of the load line with the negative slope is always unstable since here a change in current in either direction changes the diode voltage in such a direction so as to cause an additional current change in the same direction.

(a) Monostable operation.

As the name suggests, the circuit has a single, stable, steady-state operating point. This is achieved by choosing the load line such that it intersects the characteristic curve of the diode at a single point in one of its positive-resistance regions.

(i) Let the bias voltage E and the load resistance R be such that the load line intersects the characteristic in its low-voltage positive slope at a single point P as shown in Figure X.1B.

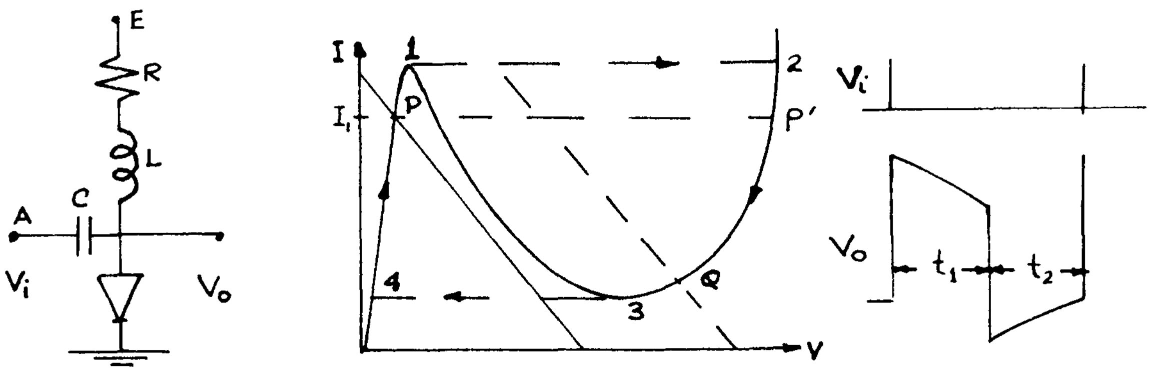


Figure X.1

The steady-state operating point which is given by the intersection of the load-line with the characteristic is stable when and only when any momentary perturbation will create conditions forching a return to the original position. Thus, the point P is stable and a quiescent current I_1 flows corresponding to a voltage V_1 across the diode.

In order to trigger the diode, either a positive current pulse of magnitude greater than $(I_p - I_1)$ or a positive voltage pulse of magnitude greater than $(V_p - V_1)$ must be applied at A, such that the stable operating point P is momentarily shifted beyond the peak of the curve. Since a triggering pulse shifts the load-line upward and parallel to its quiescent slope, for efficient and guaranteed triggering, the slope of the load line and the magnitude of the triggering pulse should be such that the temporary load line establishes a new stable operating point Q in the high-voltage positive slope of the characteristic during the time the pulse is present. Moreover, the nearer the quiescent point P to the peak of the curve, the smaller is the magnitude of the tirggering pulse needed.

During the presence of the triggering pulse, the operating point quickly reaches the peak point of the characteristic and then jumps to point 2 along a constant current line and then proceeds towards the point Q. The time needed for the operating point to jump from the peak to the point 2 is called the switching time of the diode. This time given by $t_S = C_V \frac{V_f - V_p}{I_p - I_v}$ determines the rise time of the pulse.

At the termination of the triggering pulse, the load line returns to its original position and the diode draws its quiescent current I_1, since the series inductance does not permit instantaneous change in current. Hence, with the termination of the triggering pulse, the operating point falls to P' from 2. At P', the diode voltage being greater than the source voltage the current through L decreases and reaches the minimum at point 3. The current cannot decrease beyond this minimum and hence the operating point jumps to the point 4 again at constant current. From 4 the current increases again and finally reaches its steady-state value at P. In order to repeat the cycle another triggering pulse can be injected as soon as the operating point reaches its stable position P.

Hence, for each small positive triggering pulse, an output pulse is obtained as shown in Figure X.1C.

The rise and fall time of the output pulse depends on the switching time of the diode from the point 1 to 2 and from 3 to 4 respectively. These are of the order of 1 - 4 nanoseconds (10^{-9}sec.) and can be neglected.

The width of the output pulse depends on the time t_1 needed for the current to decrease from point P' to point 3, i.e. from I_1 to I_v along the high-voltage slope. The maximum repetition rate is limited by the time t_2 required for the current to increase from I_v to I_1 along the low-voltage slope. This is because the diode will not respond to the triggering pulse from any position of the operating point other than P. This recovery time can be varied by varying L and R, and hence this circuit will operate as a frequency divider if the recovery time is made larger than the time between the successive triggering pulses.

(ii) If the bias voltage E is now increased beyond V_v, such that the steady-state stable operating point is at S, as shown in Figure X.2A, a negative pulse (voltage or current) is needed for triggering the diode.

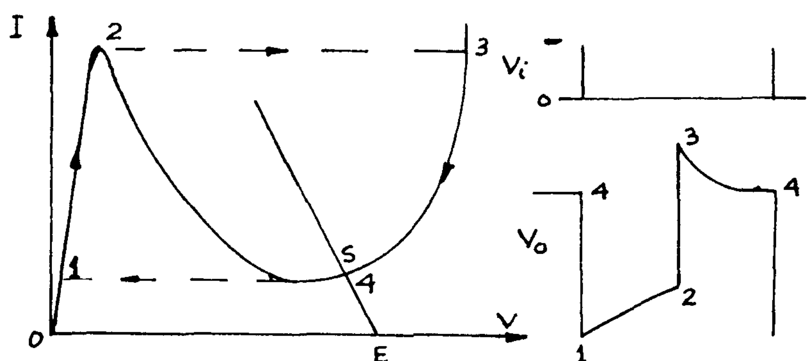
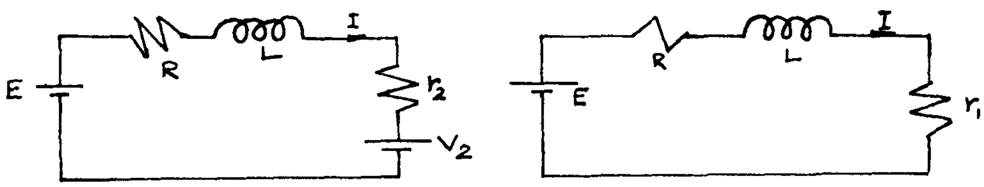


Figure X.2

The magnitude of the negative triggering current pulse should be greater than $(I_2 - I_v)$, and that of the voltage pulse should be greater than $(V_2 - V_v)$. The path of operation is shown by the dotted arrow line. The output voltage waveform is shown at Figure X.2B.

(b) <u>Calculation of output pulse duration time t_1 and recovery time t_2.</u>

In (i) the output pulse duration time t_1 is the time required by the current to decrease from its quiescent value I_1 to I_v along the high-voltage slope of the diode, i.e. from point P' to 3. In this region the diode can be replaced by a resistance r_2 in series with a voltage source V_2, r_2 being the reciprocal of the slope of the characteristic in this region. This is given by $r_2 = \frac{V_f - V_2}{I_p - I_v}$ where $V_2 = \frac{V_f + V_v}{2}$ as discussed before. Hence, the equivalent circuit becomes as shown in Figure X.3A.



<u>Figure X.3</u>

The conditions are at $t = 0$, $I = I_1$ and at $t = t_1$, $I = I_v$.

Solving for I(t) with the above initial condition, one gets

$$I(t) = \frac{E - V_2}{R_2}(1 - e^{-R_2 t/L}) + I_1 e^{-R_2 t/L}$$

where $R_2 = R + r_2$. At $t = t_1$, $I = I_v$; therefore substituting this, we get

$$t_1 = \frac{L}{R_2} \ell n \frac{I_1 R_2 + V_2 - E}{I_v R_2 + V_2 - E} .$$

Similarly, the recovery time t_2 can be calculated by replacing the diode with a resistance r_1 given by the reciprocal of the slope of the characteristic between point 4 and 1 in series with a zero voltage source as shown at B in Figure X.3. The conditions are at $t = 0$, $I = I_v$, and at $t = t_2$, $I = I_1$. Solving for I(t) and substituting these conditions, one gets $t_2 = \frac{L}{R_1} \ln \frac{E - I_v R_1}{E - I_1 R_1}$.

Similarly for (ii) the time t_1 and t_2 can be calculated as above and are given by

output pulse duration time $t_1 = \frac{L}{R_1} \ln \frac{E_2 - I_2 R_1}{E_2 - I_p R_1}$ and

recovery time $t_2 = \frac{L}{R_2} \ln \frac{I_p R_2 + V_2 - E_2}{I_2 R_2 + V_2 - E_2}$.

(c) Experimental Results.

(i) Positive triggering:- The circuit used is shown at A andthe output pulses are shown in B in Figure X.4.

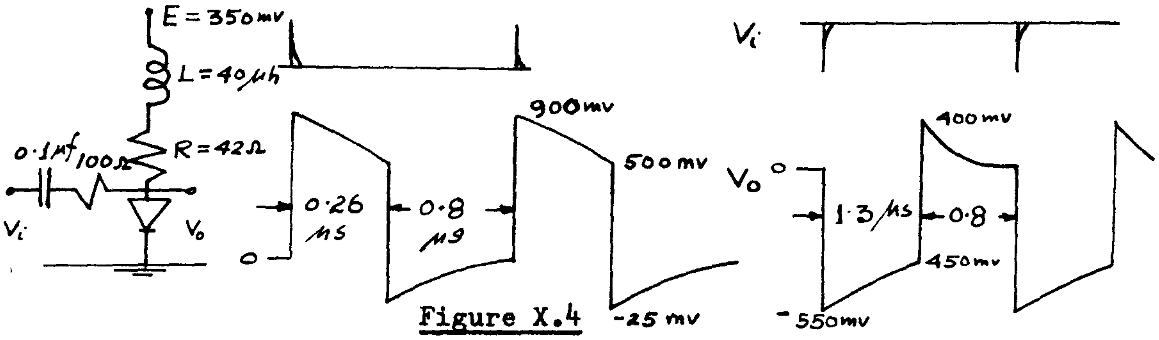


Figure X.4

The tunnel diode used is GaAs type XA650 for which V_2 = 775 mv., r_1 = 10Ω and r_2 = 33 ohms. The positive triggering pulses at 100 x 10^3 pulses/sec. are obtained from a unit pulser (General Radio Co.) type 1217-A., having an output impedance of 200 ohms. The observed values of t_1 and t_2 are 0.26 μsec. and 0.8 μsec. respectively and the calculated values are

$$t_1 = \frac{L}{R_2} \ln \frac{I_1 R_2 + V_2 - E}{I_v R_2 + V_2 - E} = 0.27 \text{ μsec., and}$$

$$t_2 = \frac{L}{R_1} \ln \frac{E - I_v R_1}{E - I_1 R_1} = 0.77 \text{ μsec.}$$

Thus, calculated and observed values are in close agreement. An oscillogram of the output pulse is shown at (a) in Figure X.5.

(ii) Negative triggering:- The bias is increased to 650 mv. and negative pulses at 100×10^3 pulses/sec. from a unit pulser are used for triggering the diode. The other circuit parameters are kept the same as in (i). The output pulses are shown at C in Figure X.4.

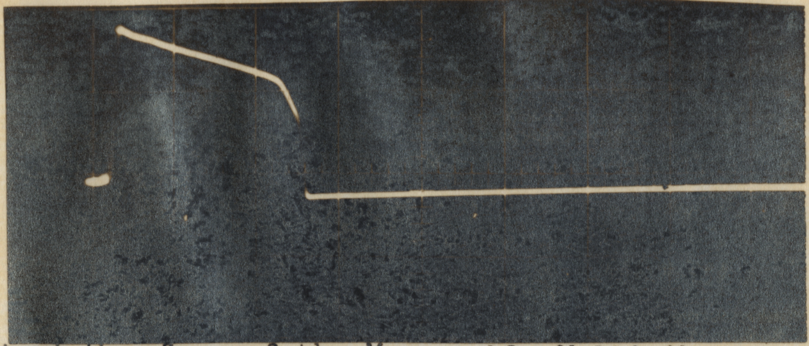
The observed values of t_1 and t_2 are 1.3 µsec. and 0.8 µsec. respectively whereas the calculated values are $t_1 = 1.2$ µsec. and $t_2 = 0.77$ µsec. An oscillogram of the output pulse is shown at (b) in Figure X.5.

(d) Bistable Operation

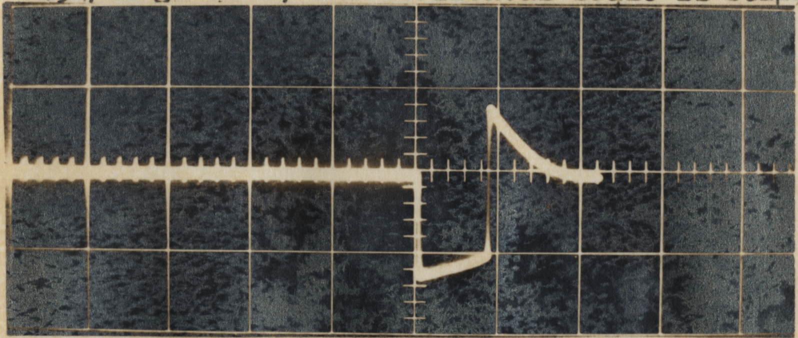
For bistable operation, as the name suggests, there must be two stable steady-state operating points and the diode may rest in any one of these stable positions. Transition from one state to the other is achieved by means of external triggering. The bias voltage E and series resistance R are such that the load line intersects the characteristic of the diode at three points as shown in Figure X.6B. Of the three equilibrium points 1 and 2 are stable whereas 3 is unstable.

The diode can be triggered from point 1 to the point 2 by means of a positive pulse of such magnitude that the operating point is forced out beyond the peak of the curve. Once triggered, the diode stays at point 2 until it is triggered back to point 1 by means of a negative pulse.

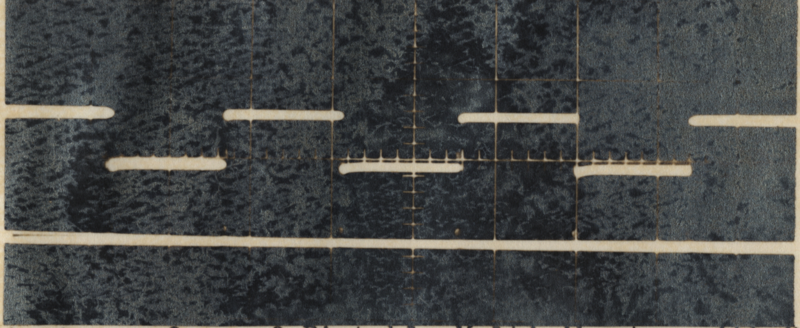
If triggered with alternate positive and negative pulses the path of operation is along the arrows and the output pulse is as shown



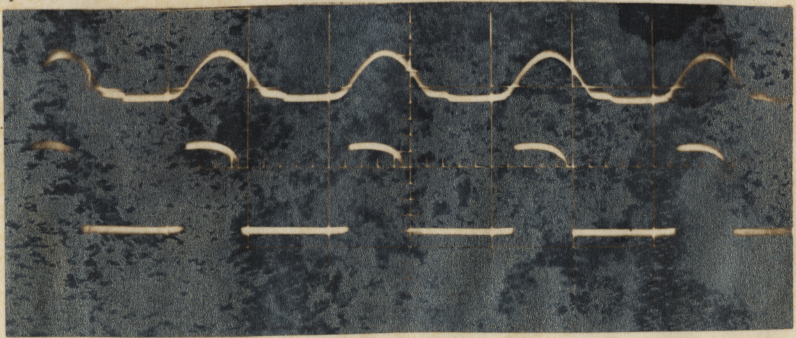
(a) Output Waveform of the Monostable Multivibrator with Positive Triggering Pulses. Bias voltage = 350 mv. Vertical scale is 0.5v/large div., and Horizontal scale is 0.1μsec./large div.



(b) Output waveform of the Monostable Multivibrator with Negative Triggering Pulses. Bias voltage = 650 mv. Vertical scale is 0.5v/large div., and Horizontal scale is 1μsec/ large div.



(c) Output waveform of Bistable Multivibrator with Only Positive Triggering Pulses. Bias voltage = 600 mv. Vertical scale is 1v/div. and Horizontal scale is 0.5μsec/Div.



(d) Tunnel diode generates square waves from Sine waves. Peak value of half sine wave = 6.5 volts. Amplitude of square wave = 500 mv.

at C in Figure X.6.

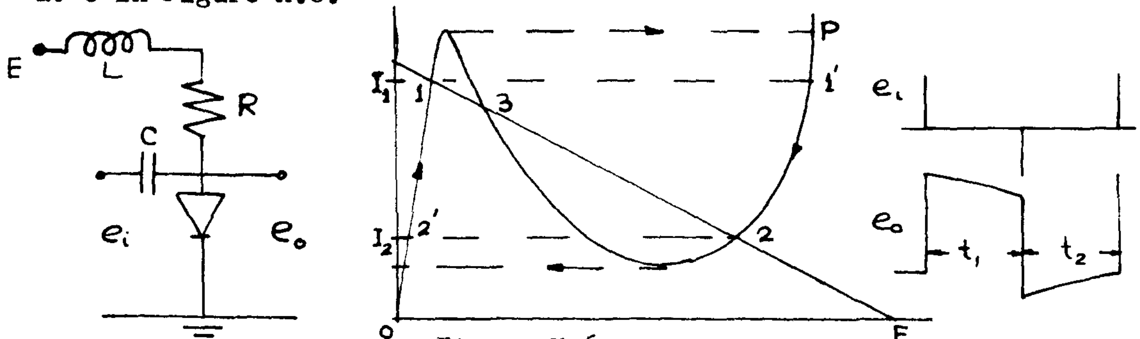


Figure X.6

Suppose, initially the diode is at point 1, and current through the diode and the inductance is I_1. When triggered by a positive pulse, then during the presence of the pulse the operating point jumps to P. But as soon as the external pulse is over, the diode draws its quiescent current I_1, since current through L cannot change instantaneously, and hence the operating point falls to 1' with the termination of the triggering pulse. From 1', it proceeds to the stable position 2, with a time constant depending on L and the total resistance in this region.

The operating point stays at 2 until the negative pulse arrives, when it is triggered to point 2' again at constant current. From 2' it rises towards 1, with a different time constant.

The diode will not respond to the positive pulses unless the operating point is at 1, and it will not be triggered by the negative pulses from any position other than 2. Hence, if the time t_1 and t_2 are greater than the interval between the positive and negative pulses, then this circuit will act as a frequency divider.

The time t_1 and t_2 can be calculated as shown before.

$$t_1 = \frac{L}{R_1} \ln \frac{E - I_2 R_1}{E - I_1 R_1}$$

$$t_2 = \frac{L}{R_2} \ln \frac{I_1 R_2 + V_2 - E}{I_2 R_2 + V_2 - E}$$

Triggering with positive pulses only.

In order to trigger the diode alternately between the two stable states, triggering pulses of alternating polarity are needed. This causes a great limitation on the practical applications of the diode such as a counter. This is because the output pulses of any practical particle detector are always of the same polarity, either positive or negative.

By using a catching diode for steering the input pulses of the same polarity alternately to the anode and the cathode of the diode, a bistable operation of the tunnel diode can be achieved(16) with triggering pulses of one polarity. This is shown in Figure X.7.

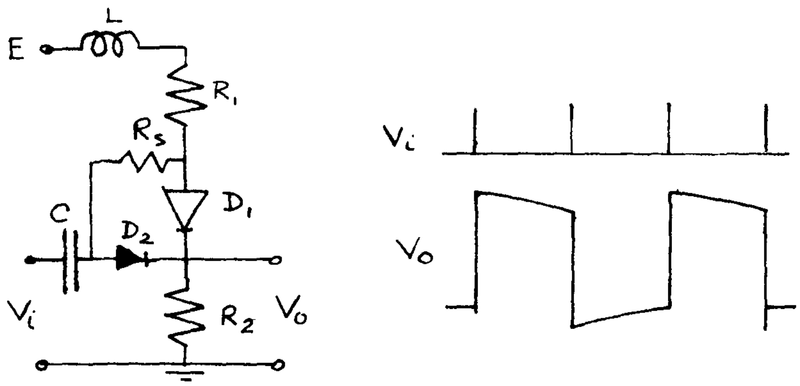


Figure X.7

In the above circuit D_2 is an ordinary diode. Suppose initially the diode is at 1 in the low-voltage state. The current is I_1 and the voltage across the diode is very low. Hence, the diode D_2 will not conduct. The first positive pulse is thus applied to the anode of the tunnel diode and it switches to the high-voltage low-current state 2. The voltage across the tunnel diode is very large and the current is very low. Hence, D_2 conducts now. When the next pulse arrives, it is applied to the cathode of the tunnel diode through the catching diode D_2. Therefore, the tunnel diode is again switched back to the low-voltage state. The second pulse is also applied to the anode but the diode being in high-voltage state, this positive pulse has no effect on it. Thus we see, in a binary for every two input pulses, there is one output pulse. If this output pulse of the first stage is fed into a second stage, then for two such output pulses, the second stage will give a single output pulse and this corresponds to four input pulses. In this way, by cascading several of these binary stages, the input pulses per second can be scaled down to a very low value when it can be counted with a mechanical counter.

Such a scaler of several tunnel diodes will have the following advantages over other available scalers.

(a) Economical

(b) Miniature in size

(c) Low power consumption

(d) High rate of input pulses

(e) Insensitive to nuclear radiation

(f) Simpler construction.

Experimental Results

(i) Triggering with alternating pulses:- The circuit used and the output pulses obtained are shown in Figure X.8.

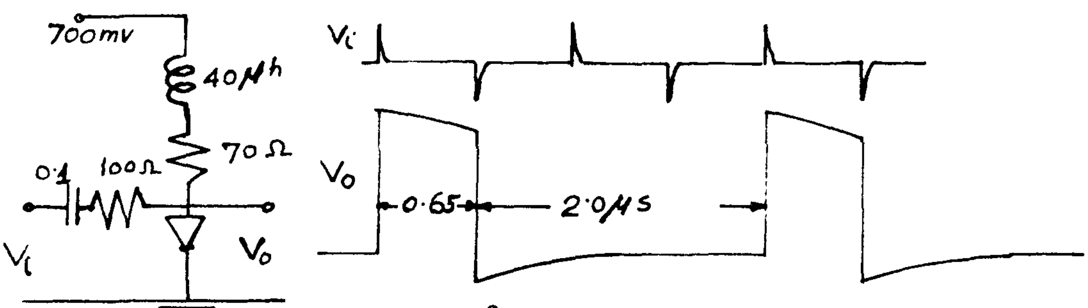


Figure X.8

The alternate positive and negative triggering pulses are obtained by differentiating square waves generated by another tunnel diode connected in series with a short-circuited co-axial cable. The internal T_1 and T_2 between pulses of the opposite polarity is the same and equal to $T_1 = T_2 = 0.65$ µsec. The pulse durations t_1 and t_2 are found to be t_1=0.65 µsec. and t_2=2 µsec. This is because the time constants of the circuit are different in the two regions of the diode characteristic and the recovery time in the low-voltage state is greater than T_2, the interval between a negative and the following positive pulse. Hence, a frequency division of 4 : 1 is attained, since the diode responds to every third positive pulse.

(ii) Triggering with positive pulses:- The circuit arrangement and the output waveform are shown below. The triggering pulses are obtained from a unit pulser.

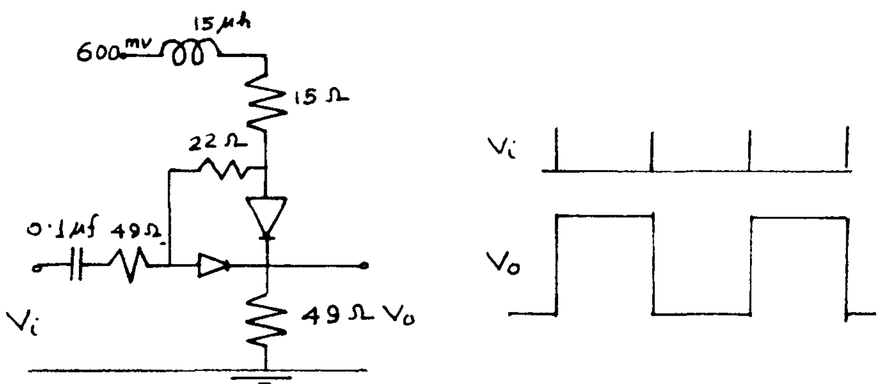


Figure X.9

An oscillogram of the input triggering pulses and the output pulse is shown at (c) in Figure X.5.

XI Other Switching Applications of Tunnel Diode

For any input voltage, the output voltage across a tunnel diode with a finite load resistance is a non-linear fraction of the input voltage. Except for no-load condition ($R_L = 0$) these two voltages do not bear any definite linear relationship. This is because current through the diode is a non-linear function of the voltage, the relation being given the V-I characteristic curve of the diode. Hence, in Figure XI.1, V_{out} is given by the intersection of the load-line with the characteristic of the diode, for different values of V_{in}.

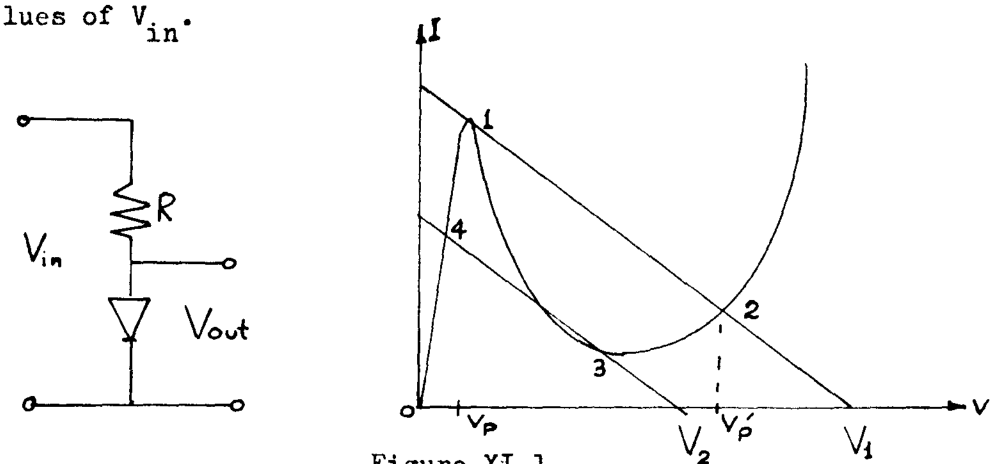


Figure XI.1

If R is constant, the slope of the load-line is the same for all values of V_{in}. As the input voltage is increased from zero to V_1, the current through the diode increases from zero to I_p and the output voltage increases from zero to V_p and the operating point moves to the peak point 1.

Now, if V_{in} is slightly increased beyond V_1, the operating point is shifted beyond the peak and it sees a stable position at 2, which is the intersection of the new load-line with the characteristic. Hence, the operating point is switched from 1 to 2.

If R is very large, the switching takes place almost at constant current as shown by the dotted lines. Thus, for a slight increase in V_{in} from V_1, the output voltage increases from V_p to V_p'. If V_{in} is further increased, V_{out} also increases along the high-voltage slope of the characteristic.

Hence, for a particular value of R, we have a fixed value of V_{in} above which the tunnel diode switches from the low-voltage to the high-voltage state. We shall call this the <u>upper threshold voltage</u> of the diode for a particular load. This voltage is given by the equation $V_1 = V_p + I_pR$. Since V_p and I_p are fixed for a diode, V_1 is proportional to R.

Now, if after the diode voltage is switched from V_p to V_p', the input voltage is gradually decreased from V_1 to V_2, then the output voltage will decrease from V_p' to V_v and the operating point will be at point 3. If V_{in} is slightly decreased beyond V_2, the operating point will be shifted beyond the valley and it will see a stable position at point 4 given by the intersection of the new load-line with the characteristic. Hence the diode voltage will switch back from point 3 in the high-voltage state to the point 4 in the low-voltage state.

Thus, V_2 is another threshold voltage below which the diode switches from the high-voltage to the low-voltage state. We shall call this the <u>lower threshold voltage</u> which is given by the equation $V_2 = V_v + I_vR$. Since V_v and I_v are constants, V_2 is proportional to R.

If R is large, switching occurs at constant current. The switching time in the two cases is given by $t_1 = C_v \frac{V_f - V_p}{I_p - I_v}$ and $t_2 = C_v \frac{V_v}{I_p}$ which are of the order of several nanoseconds (10^{-9} seconds).

Thus, V_{out} is not the same for the increasing the decreasing V_{in} as shown in Figure XI.2.

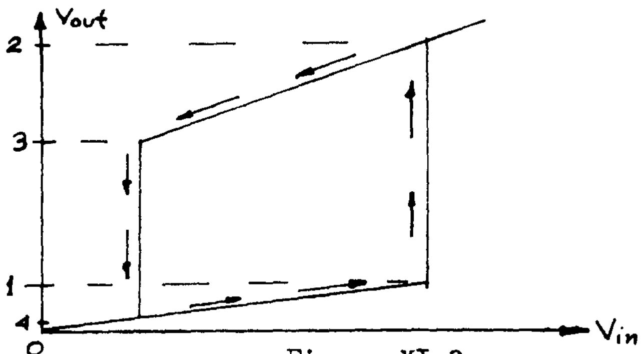


Figure XI.2

We find that there is a "hysteresis" loop which is equal to $(V_1 - V_2)$. This is due to the non-linear characteristic of the diode. This hysteresis width depends on the value of R and increases with R.

For stable, switching, both the threshold voltages V_1 and V_2 should be greater than V_v, the valley voltage of the diode. The load resistance R should be such that the slope of the load-line should be equal to or less than the negative slope of the characteristic curve.

From above, it is found that with a particular load resistance R, the tunnel diode circuit has two definite threshold levels of input voltage at which switching occurs.

(a) An upper threshold limit above which it switches from the low-voltage to the high-voltage state and,

(b) a lower threshold limit below which the diode switches back to the low-voltage state from the high-voltage one.

This switching property of the diode can be applied to a number of useful applications.

(i) It can be used as an amplitude discriminator.

(ii) As a substitute for Schmidt trigger circuit,

(iii) Generation of rectangular pulses from sine wave.

(i) As an amplitude discriminator or detector.

Suppose we have an irregular stream of pulses having different amplitudes. All pulses having an amplitude greater than a predetermined level V_T can be detected by feeding the stream into the diode circuit. The diode will not be triggered by those pulses having amplitude less than V_T. V_T can be adjusted at any level by choosing the value of R. In any case, V_T should be greater than the valley voltage of the diode, which is in the case of GaAs type about 450 mv.

Thus we can separate or discriminate pulses of different amplitudes by using a number of diodes having different threshold voltages as shown in Figure XI.3.

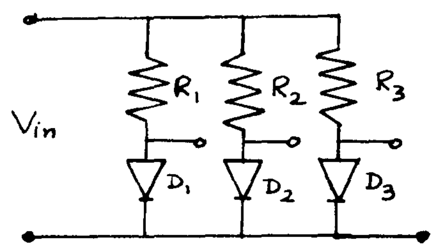


Figure XI.3

D_1 will be triggered by all the pulses having amplitude greater than V_1. D_2 will detect only those pulses having amplitude greater than V_2 whereas D_3 will only be actuated by those having amplitude greater than V_3.

Moreover, suppose we have a single input having amplitude greater than V_3. As soon as the amplitude exceeds V_1, D_1 will be triggered and when it exceeds V_2, D_2 will be triggered. Finally when the input voltage exceeds V_3, D_3 will switch to its high-voltage state. During the decreasing portion, D_3 will switch back to its low-voltage first, and then D_2 and finally D_1. Now, if the output of the three diodes are combined together to give a single output, then from the above arrangement, it is evident that for a sine function input, the output will be a staircase type function.

(ii) As a substitute for Schmidt trigger circuit.

A Schmidt trigger circuit is most commonly used to deliver a stepped output pulse from a continuously varying input pulse when the input pulse passes certain set levels.

The output from a particle detector employing the ionizing property of the particle as it passes through a gas, is in the form of a pulse of varying amplitude and duration. Moreover, associated with the ionization pulses there are spurious noise pulses which are to be eliminated. These ionization pulses must be converted to rectangular or stepped pulses before they are fed to the counter. These are accomplished by setting the two trigger levels of the Schmidt circuit.

The simple tunnel diode circuit shown in Figure XI.4 perform the same thing.

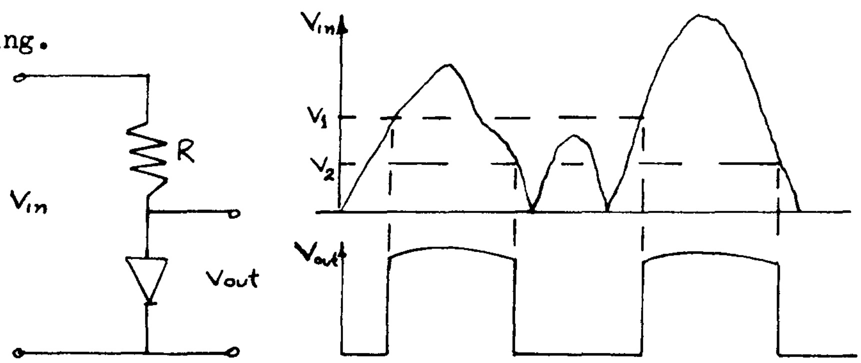


Figure XI.4

The diode will trigger only for those pulses having amplitude greater than V_1, where $V_1 = V_p + I_pR$. Hence, it will eliminate all the unwanted noise pulses. When the amplitude decreases below V_2, it will again trigger back to its low-voltage state, where $V_2 = V_v + I_vR$. The diode will remain in the high-voltage state as long as the input amplitude is greater than V_2, and it will remain in the low-voltage state as long as V_{in} is less than V_1. Thus, for a continuously varying input, the output is a rectangular pulse.

(iii) Experimental results.

The tunnel diode is used in generating rectangular pulses from sine wave. The circuit arrangement as well as the input and the output pulses are shown in Figure XI.5. Since there is no bias on the diode, only positive half-cycle of the wave is used. If a full-wave rectification arrangement is used, two pulses for each cycle are obtained.

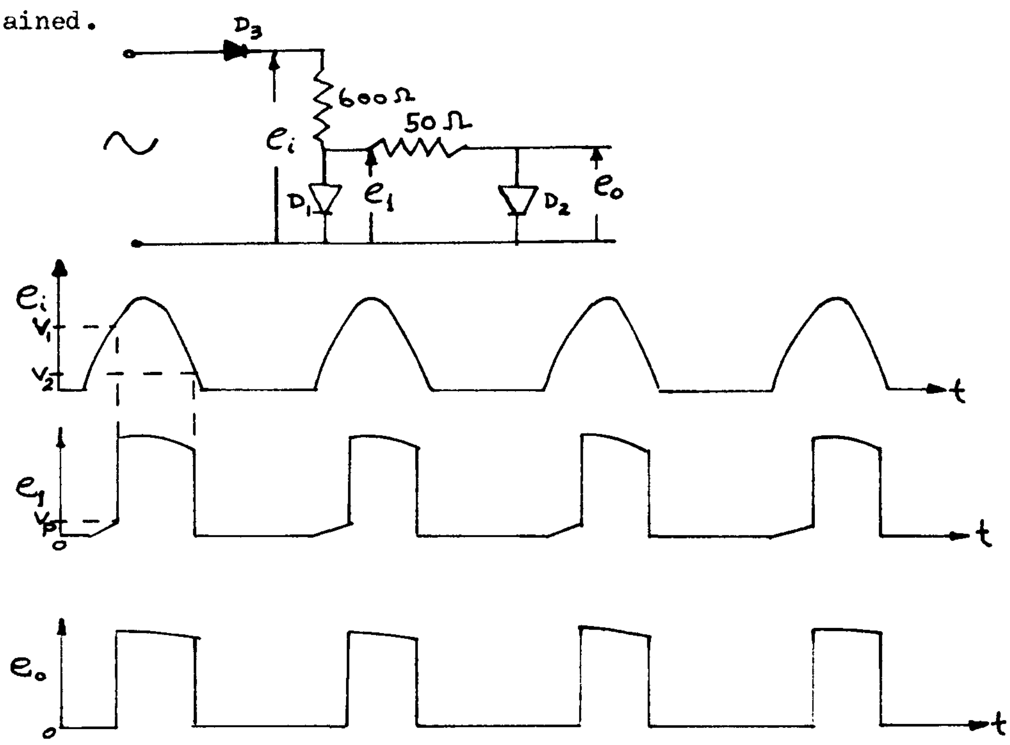


Figure XI.5

D_3 is an ordinary diode used to block the negative half-cycle. D_1

and D_2 are GaAs type tunnel diode having $V_p = 100$ mv., $V_v = 450$ mv., $I_p = 10$ ma., and $I_v = 0.5$ ma.

Upper threshold voltage $V_1 = V_p + I_pR = 6$ volts,

and Lower threshold voltage $V_2 = V_v + I_vR = 0.75$ volts.

Therefore peak value of the input voltage must be greater than 6 volts for effective triggering. As soon as the input voltage is greater than 6 volts, the diode voltage jumps from V_p to the high-voltage state, and when the input voltage decreases below 0.75 volts, the diode voltage again jumps from V_v to the low-voltage state.

R_2 and D_2 are used to smooth out the initial rising portion of the output pulse, from 0 to V_p.

An oscillogram of the waveform is shown at (d) in Figure X.5.

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