DESIGN ANALYSIS OF

PERMANENT MAGNET MACHINES

DESIGN ANALYSIS OF PERMANENT MAGNET MACHINES WITH SPECIAL REFERENCE TO SYNCHRONOUS GENERATOR

By

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SCOPE AND CONTENTS:

A theoretical study has been made on the design aspects of permanent magnet machines. First, the design criterion for permanent magnets operating under dynamic conditions has been examined. Then, a method for optimum design of a synchronous generator has been described. This method is based on solution of simultaneous equations relating stator and rotor variables and solving them for minimum $D^2 l$ of the machine. Based on this analysis an example has been calculated to show how minimum length for a given diameter can be obtained.

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LIST OF SYMBOLS

A_m = cross-sectional area of the magnet

l_m = length of the magnet.

V_m = volume of the magnet

A_g = cross-sectional area of the air-gap.

l_g = radial length of the air-gap.

V_g = volume of the air-gap.

A = armature mmf (magnetomotive force).

A_d = demagnetizing armature mmf.

A_{sc} = steady-state short-circuit armature mmf.

E_g = electromative force generated by the air-gap flux.

ENL = excitation voltage, driving voltage in Blondel diagram.

F = mmf at magnet terminal.

F_o = F when $\emptyset_m$ = O, open-circuit mmf of equivalent magnetic source.

Q = F - F_1 , magnet terminal mmf over that for no-load on generator.

I = armature current.

I_d = direct-axis component of I.

I_q = quadrature-axis component of I.

I_{sc} = steady-state short-circuit I.

K_w = winding constant , product of pitch - factor and winding distribution factor.

K_g = generated emf per-unit air-gap flux = K_e N.

K_d = demagnetizing armature mmf per unit direct-axis current = K_a N.

K_D = number of armature slot per unit rotor diameter.

K_f = ratio of effective pole width to magnet width.

K_l = ratio of total permeance to that of path considered.

K_z = ratio of total leakage reactance to that of slot plus zig-zag reactance

$$= \frac{X_s + X_z}{X_l}$$

γ = per-unit voltage regulation.

n = per-unit short-circuit current

N = number of armature turns in series per phase.

Z = number of armature conductors in series per phase.

N_s = number of armature slots.

R = armature resistance.

K_R = resistance of direct current per unit length of conductor.

X_l = armature leakage reactance.

X_d = direct-axis synchronous reactance.

X_q = quadrature-axis synchronous reactance.

X_d' = direct-axis transient reactance.

X_d'' = direct-axis subtransient reactance.

Z_l = armature impedance, $Z_l^2 = R^2 + X_l^2$

P_g, R_g = permeance and reluctance of air-gap.

P_l, R_l = permeance and reluctance of pole-leakage.

P_o, R_o	=	internal permeance and reluctance of equivalent magnetic source.
P_t	=	$P_o + P_l + P_g$
Δ	=	$R_o R_l + R_l R_g + R_g R_o$
$\varnothing_g$	=	air-gap flux.
$\varnothing_l$	=	leakage flux.
$\varnothing_m$	=	magnet flux.
$\varnothing_o$	=	$\varnothing_m$ when $F = 0$, short-circuit flux of equivalent magnetic source.
B	=	magnet flux density.
B_m	=	B when the energy product (BH) is a maximum.
B_o	=	B when the stabilized magnet is short-circuited (by an infinite permeance).
B_r	=	B when the unstabilized magnet is short-circuited.
B_s	=	B at the point of stabilisation.
B_1	=	B for no-load at the generator.
f	=	frequency.
H	=	magnetizing force at magnet terminals.
H_c	=	H when the magnet faces an open circuit (coercivity).
D_{sc}	=	internal voltage drop due to short-circuit transformed to the B-H plane, a flux density.
Q_{sc}	=	armature mmf due to short-circuit current fransformed to the B-H plane, a demagnetizing force.
S	=	permeances transformed to the B-H plane.
μ_o	=	permeability of face space, $\mu_o = 4 \pi \times 10^{-7}$
m	=	number of phases.

CHAPTER I

INTRODUCTION

With the advent of high energy permanent magnet materials now available, these are finding more and more application in electric machines. Motors and generators with permanent magnet rotors have become more competitive to their electromagnetic version. In the past, the use of permanent magnet in electrical machine was limited to tachogenerators, synchronous time motors and hysteresis motors, etc. But in recent years, synchronous generators up to 75 KVa at 1714 rpm with permanent magnet rotor have been developed by U.S. signal corps division. Such machines, because of their inherent advantages over the electromagnet machines[5.1] are finding more and more application in mobile military applications. The most popular application to date for integral horse power permanent magnet dc motors has been used for round table drives for steel mill and aluminum mills. Thousands of these motors have been installed since 1962. They have had ratings between two and eight hp at speeds from 575 to 1750 rpm. Because of such increasing application of permanent magnet machines, study and research in this area has been growing in recent years.

The object of this work was to analyse the problems encountered in the design of permanent magnet machines. This study has been devoted mainly to synchronous generators and a method of designing an optimum permanent magnet machine with a given specification has been developed.

Before attempting the design of permanent magnet machines, a knowledge of design and operation of permanent magnets operating under static and dynamic conditions was essential. References(1) to (8) are worth mentioning on this subject. The author has tried to present in Chapter 3 the necessary information which is useful towards understanding the behaviour of permanent magnets operating under such conditions. Design relationship for magnets operating under static and dynamic conditions have been developed and the criteria for the most economic design of a permanent magnet has been established.

A comparison has been made between permanent magnets and electromagnets required to establish a certain air-gap energy in electrical machine. The ratio of magnet volume required to do the same job for both cases has been calculated and the result has been presented graphically. This analysis establishes the fact that for smaller sizes of machine permanent magnet machines should weigh less than the electromagnetic version. A direct cost comparison could not be made for it depends on so many other factors like, cost of materials, cost of fabrication, etc. which make this study very difficult.

Before attempting the design of permanent magnet generators, the theoretical analysis has been presented here as developed by Hanrahan and Toffolo(21). The subject of permanent magnet generator stabilisation is quite important and has been dealt here to explain why load - or short-circuit stabilisation is preferred for the economic design of permanent magnet generators.

References (12) to (18) on permanent magnet design indicate that the procedure of wound-field machine has been followed. But this method is not very satisfactory for the load stabilised permanent magnet generators. Hence, a new approach has been taken here to design an optimum machine for load or short-circuit stabilisation. This requires solution of simultaneous equations relating stator and rotor variables and solving these equations for a minimum D^2l for a given KVa. As there are more variables than the number of equations the design may be optimised. Based on the design equations derived in Chapter 6, the author has given a procedure for an actual design of machine. This has been done to obtain minimum length of the machine for a given diameter. Thus, by selecting different diameters, minimum length corresponding to each diameter can be calculated. The most optimum machine for a given KVa will be given by minimum D^2l product. A design example for a 2KVa generator has been given and the author has shown, how minimum length can be obtained for a given diameter.

The computer programmes used for calculations are given in Appendix. A chapter on field leakage permeance has also been included in Appendix.

CHAPTER II

HISTORY OF PERMANENT MAGNET SYNCHRONOUS MACHINE

A short history on the development of permanent magnet synchronous generator is presented in this chapter. Since the permanent magnet is the principal component in such a machine a brief account of permanent magnet material development is also given here.

Although magnets were known to mankind since the discovery of the loadstone - long before electricity - the use of permanent magnet material in electro-mechanical devices for the production of motive power is recent. This was mainly because permanent magnet materials of high energy contents were not available. There were two main reasons for it:

(a) Before Oersted's discovery in 1819 (of the magnetic properties of electric current) no knowledge existed of the relationship between electricity and magnetism. Hence magnetization by electromagnetic method was unknown.

(b) Metallurgists were not interested in magnetic properties of metal. Many steels subsequently employed as permanent magnets originated as tool steel.

A forward stride was made in 1917 due to discovery of COBALT MAGNET STEEL in Japan with four times more coercive force as compared to its predecessor Tungsten alloy. However, a tremendous advance in use and technology of permanent magnets started with the announcement in 1932 by MISHIMA of the excellent magnetic properties found in aluminum - nickle - iron alloy. Following the discovery of MISHIMA and perhaps of greater importance was the announcement of JONAS of a process to secure anisotropic magnetic properties in an ALNICO alloy containing higher percentage of Cobalt. Permanent magnets of this general type with maximum energy contents (BH max.) values of 5 mega-gauss-oersted or more are extensively manufactured as ALCOMAX II, III or IV, TICANOL C or G in Britain, ALNICO V or VI in the United States, ALNICO 500 in Germany, MAGNICO in Soviet Union and under somewhat similar names in a number of other countries. Today ALNICO alloys are the most widely used permanent magnet material in U.S.A. and account for about 85 to 90 percent of the total production of permanent magnets produced in that country. The second most widely used material is a CERAMIC MAGNET based on high crystal anisotropy of barium ferrite. TABLE 3.1 lists the electrical and magnetic properties of the most important permanent magnetic materials now in use. Because of the high energy (BH) content and high coercive force of ALNICO and low cost of Ceramic magnets, permanent magnets are now often selected over electromagnetic designs.

The use of permanent magnets in rotating electrical machinery such as motors and generators in preference to electromagnets is decided upon by the design requirements of particular equipment specifications. In fractional horsepower and other small motors and generators, it has been found that permanent magnets are more satisfactory than electromagnets. Considerable research is underway in the development of permanent magnets. As a result high energy permanent magnet should be available in the near future and permanent magnet machines may become technically and economically superior to the electromagnetic versions.

Possibly the first use of permanent magnets in synchronous machine was mentioned in a paper by Homes and Grundy in 1930's in connection of "Self-starting Synchronous Time Motors". Even at this early stage the demagnetization of permanent magnets and the maximum energy per unit volume were considered to be the critical quantities. After the introduction of iron-nickle-aluminum-cobalt alloys in the early thirties more and more use of permanent magnets was made in small"dynamo electric machines". Of course, a number of problems had to be solved before a satisfactory design could be evolved.

Around 1934 or 1935 attempts were made in the United States to make a small 4 inch diameter tachometer generator and permanent magnet motor with Alnico II. The energy content of Alnico II was very small and hence the output of the machine was considerably less than an electromagnetic version of similar size.

During the period 1935 to 1938 some alternators and d-c tachometers were made in U.S.A. In this period serious attention was given to the question of protection against demagnetization caused either by external or internal factors, and various remedies were suggested. In order to minimize the demagnetizing effect of armature reaction in the generator a heavy damper coil was put around the magnet. A patent of this type was given in United States sometime between 1937 and 1938. This type of damper winding was found to be fairly effective in case of alternators but it was very expensive. However, by this time the mechanical difficulties in construction of poles such as holding the pole face to the Alnico casting and of holding the pole to the rotor or stator core have been successfully overcome.

The use of permanent magnet in rotor offered the advantage of dispensing with slip rings and brush contacts usually found in electromagnetic alternators and thus improved the competitive position of permanent magnet synchronous machines. This stage was accomplished in the U.S.A. before the World War II. This type of rotor was provided with heavy amortisseur windings and the whole rotor was then die-cast in aluminum.

From 1940 to 1944 no further development could be traced in the application of permanent magnets to alternators or a.c. motors, although their use in d.c. motors was considerably extended. About 1945 a number of non-salient cylinderical shaped multipolar permanent magnet rotors were successfully constructed and used in alternators. These magnets were very difficult to heat-treat and had to be magnetized in a special multipole magnetizer. These magnets had very good output wave-form, but since the magnetic length of the magnets used in them was short they could be easily demagnetized. Further development in U.S.A. in permanent magnetic material during this time resulted new materials such as ALNICO V and VI which were extensively and profitably used in rotating machinery with permanent magnets.

Following the use of Alnico V and VI, considerable activities were reported during 1945 and 1955 in the field of dynamo-electric machinery using permanent magnets. At least twenty to twenty-five patents were issued in the United States patent office on the application and use of permanent magnets in rotating machinery. Technically they can be summarized in the following three groups:

1. Improved design to utilize magnets.
2. Protection of magnet against demagnetization, and
3. Improved mechanical features.

Some of the U.S. patents dealing with the above developments are listed in the Bibliography. As reported by Mr. M.W. Brainard in his paper presented in 1952 A.I.E.E., Summer General Meeting, permanent magnet generators in capacities from 0.1 KVa at 1200 rpm to 75 KVa at 1714 rpm are possible. The 75 KVa machine has been developed by U.S. Signal Corps Division. Because of the application of permanent magnet generators in mobile military units such as air craft and missiles, some research has been carried on by U.S. Naval Research laboratories and U.S. Signal Corps Division. However, very little information on the development and design of these machines has been available in the literature during the past few years.

CHAPTER III

PERMANENT MAGNETS PROPERTIES AND DESIGN RELATIONSHIPS

INTRODUCTION

Permanent magnets are classified as magnetically "hard" materials. Until recent years the proper utilization of such materials has been considered an art, cloaked in a bit of mystery, but as the modern magnets are receiving greater attention, it is becoming increasingly evident that the magnet performance can be predictable by physical laws. Extensive research is being carried on to explain the mechanism of magnetism for presently all the magnetic phenomena cannot be explained by physical laws. But as the object of this work is to make a study of design of permanent magnet machine rather than permanent magnet material itself no attempt has been made here to develop theories explaining the mechanism of magnetism. However, the adoption of permanent magnets in rotating machines would not have been possible without the present achievements in the metallurgy of permanent magnet materials. The future of permanent magnet machines depends to a large extent on the quality and cost of the future permanent magnets.

3.1 PROPERTIES OF PERMANENT MAGNET MATERIALS

Before attempting the design procedure of permanent magnets and the permanent magnet machine, an understanding of different magnetic properties of a permanent magnet material is essential. A brief account explaining the different magnetic, electrical and mechanical properties of permanent magnet materials is therefore presented here.

(A) MAGNETIC PROPERTIES

In order to define the suitability of a magnetic material for a particular use and to measure and specify its magnetic properties the following terms and definitions are generally used.

1. Remanence or B_r
2. Coercivity or H_c
3. Saturation flux density (B_{sat}) and
   Saturation magnetizing force (H_{sat})
4. Maximum energy or (BH) max
5. Permeability and minor hysteresis loop
6. Maximum available energy under recoil
7. Curie point temperature.

(B) MECHANICAL PROPERTIES

1. Hardness
2. Specific Gravity

(C) ELECTRICAL PROPERTIES

1. Resistivity

(D) MISCELLANEOUS PROPERTIES (STABILITY)

(A) MAGNETIC PROPERTIES

The well known static or d-c hysteresis loop is the basis of definitions of most of the magnetic material. In a permanent magnet the field strength measured at the magnet surface has a direction opposite to that of the induction inside the magnet, with positive induction the field strength is negative. The operating range of permanent magnet, therefore, will be a portion of the second quadrant of the major hysteresis loop called demagnetization curve and hence for permanent magnet design it is the demagnetization curve which is of the main concern. Fig. (3.1) is a typical hysteresis loop of the permanent magnet material. Fig. (3.2) shows the demagnetization characteristic and the energy curve is drawn by plotting the product of (B) and (-H) against B. These two curves show the magnetic quantities necessary to define the magnet.

1. Remanence or B_r

If a piece of magnetic material is magnetized to complete saturation, completely short-circuited magnetically and then the magnetizing force is withdrawn, then the maximum induction which is left is termed remanent magnetism or B_r. In a permanent magnet working under recoil conditions this term in itself is not very useful.

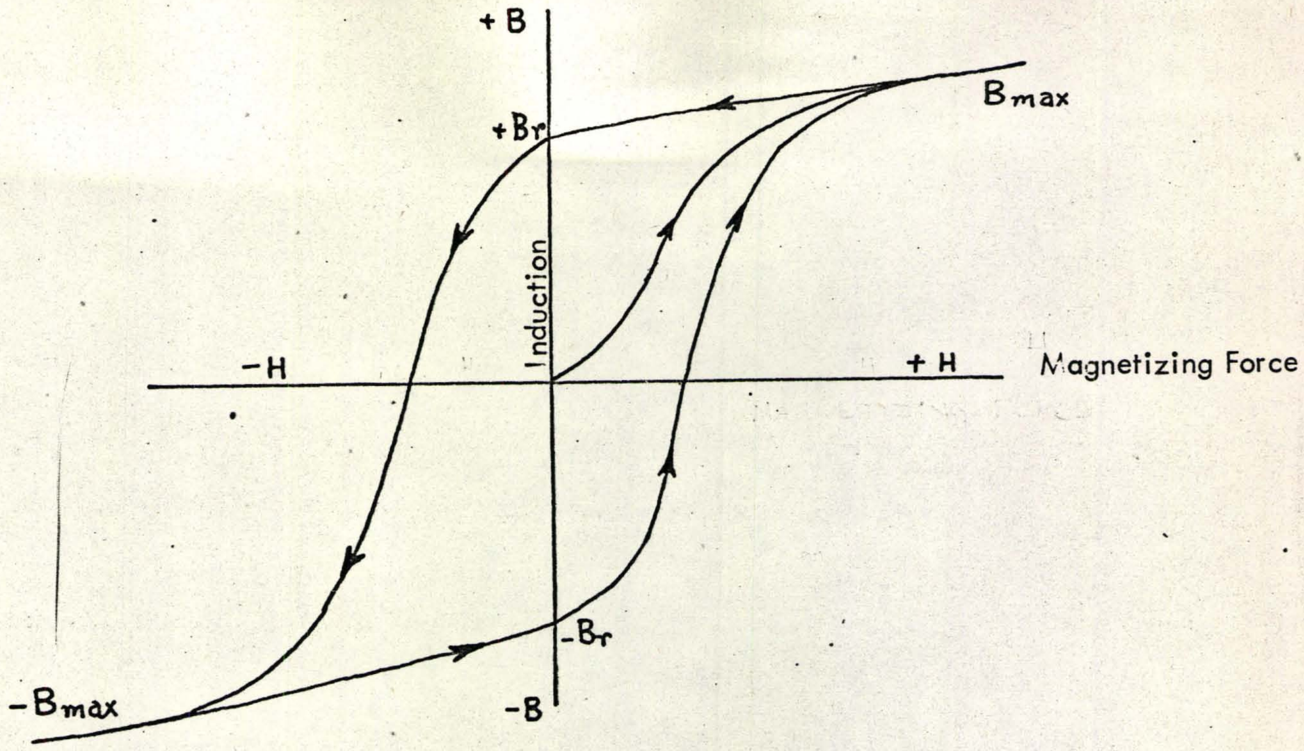


Fig. 3.1 Normal Magnetization Curve and Hysteresis Loop

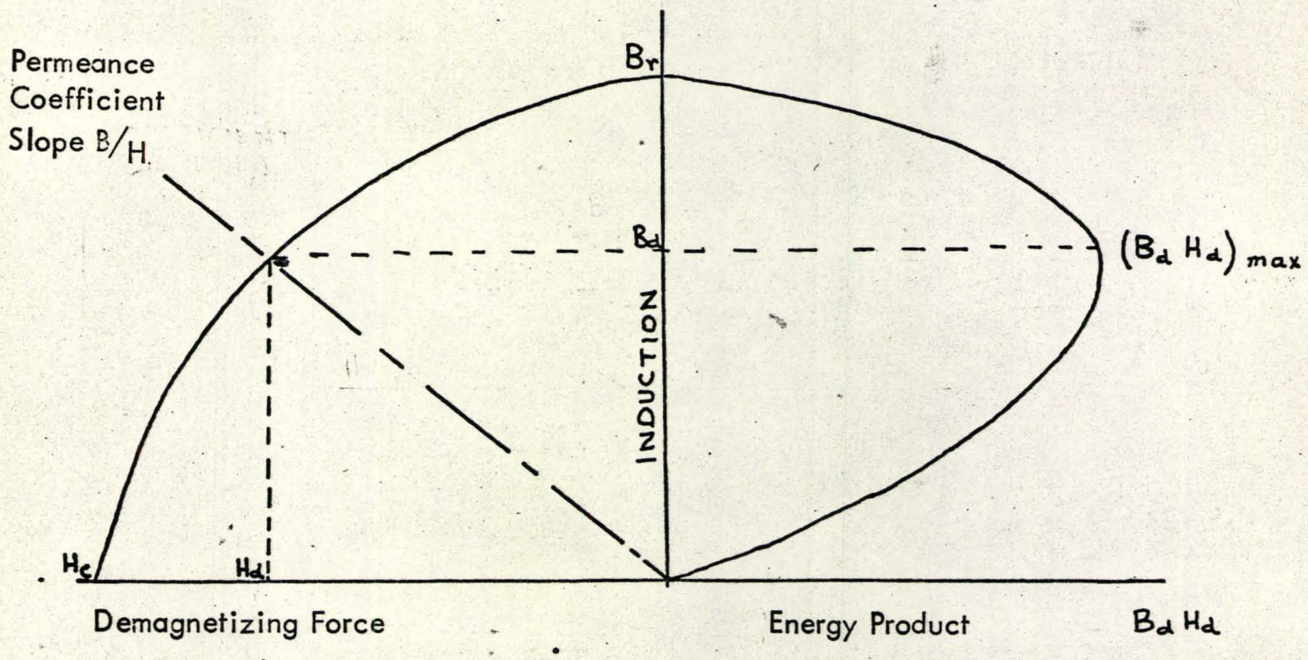


Fig. 3.2 Demagnetization and Energy Product Curve

2. Coercivity or H_c

This is the magnetomotive force which must be applied to a magnetic material to reduce the residual flux, in a colsed or short-circuited specimen after complete saturation, to zero. The maximum demagnetizing force equal to the corresponding coercivity would bring the magnetic induction to zero as long as the demagnetizing force is in operation; but if the intrinsic coercivity H_{ci} were applied then it would destroy the magnetism completely.

3. Saturation Flux Density (B_{sat}) and Saturation Magnetizing Force (H_{sat})

If a magnetic material is magnetized to such a flux density that no increase in intrinsic induction occurs when the magnetizing force is further increased, then the material may be called saturated and the corresponding flux density is called the saturation flux density or B_{sat}. The minimum magnetizing force to bring about this condition is called the saturation magnetizing force. In general, H_{sat} is approximately equal to 5 times H_{ci}

4. (BH) maximum is the maximum energy of the material per unit volume and is usually expressed in gauss-oersteds in C.G.S. units or joule per cubic meter in M.K.S. units. This could be directly obtained from the energy loop (Fig.(3.2) or the point at which it occurs on the demagnetization characteristic can be determined by simple geometric construction. The flux density and the coercive force corresponding to this maximum energy point are designated as B_d and H_d respectively. For static purposes, or where the reluctance of magnetic circuit does not vary during operation, the optimum design characteristics are given by

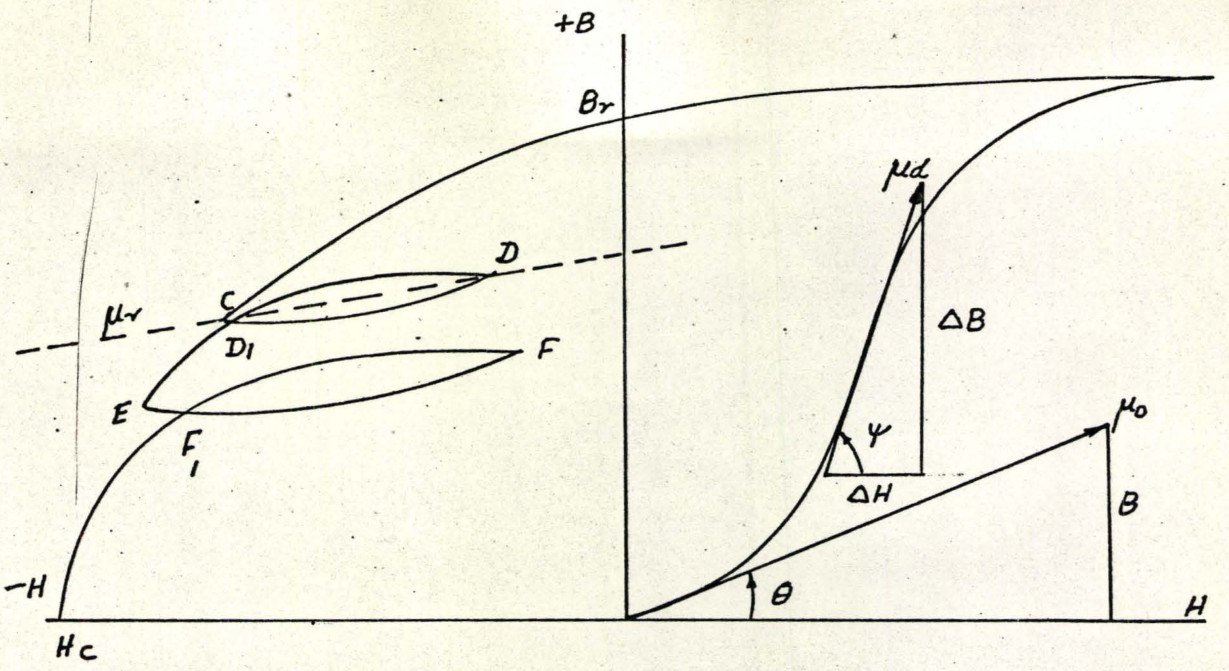


Fig. 3.3 Permeability in Permanent Magnet Material

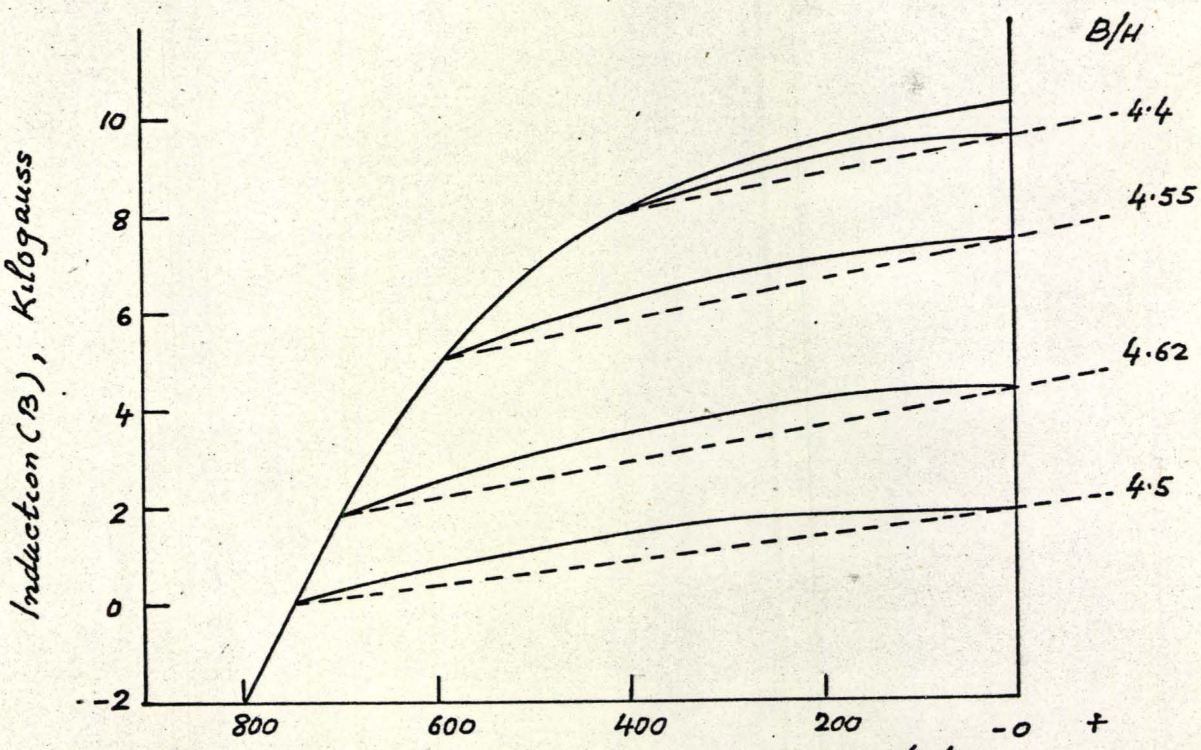


Fig. 3.4 The Variation of Recoil Permeability of Alnico 6 for Several Minor Loops (33)

these two quantities B_d and H_d. For dynamic working conditions such as in case of permanent magnet motors and generators, the phenomena of recoil must be taken into account. Working under the conditions of recoil a permanent magnet material gives less energy than its $(BH)_{max}$ and the actual conditions of recoil must be determined to find out the performance of the magnet. Hence, though the $(BH)_{max}$ has been assumed to be the sole figure of merit of a permanent magnet, the internal recoil permeability has great influence on the performance of permanent magnets.

5. Permeability

The ratio of induction to magnetizing force or permeability at various points on the magnetization curve and hysteresis loop is an important characteristic of a permanent magnet. Three such ratios tend to indicate qualitatively, if not quantitatively, the capabilities of permanent magnet. These are namely the initial permeability (μ_i), the maximum differential permeability $\mu_d(max)$ and the recoil permeability μ_r.

The initial permeability, μ_i as the name implies is the initial slope of the initial magnetization curve in the first quadrant as shown in Fig. (3.3). Thus $\mu_i = \tan \Theta$. The initial permeability of the magnetization curve is found to be of the same order as the recoil permeability in the second quadrant. Low values of initial permeability are characteristic of high coercive force and high anisotropic material.

Differential permeability is the ratio of B to H for a small change in H at any point. As shown in Fig. (3.3) there is a point A on the initial magnetization

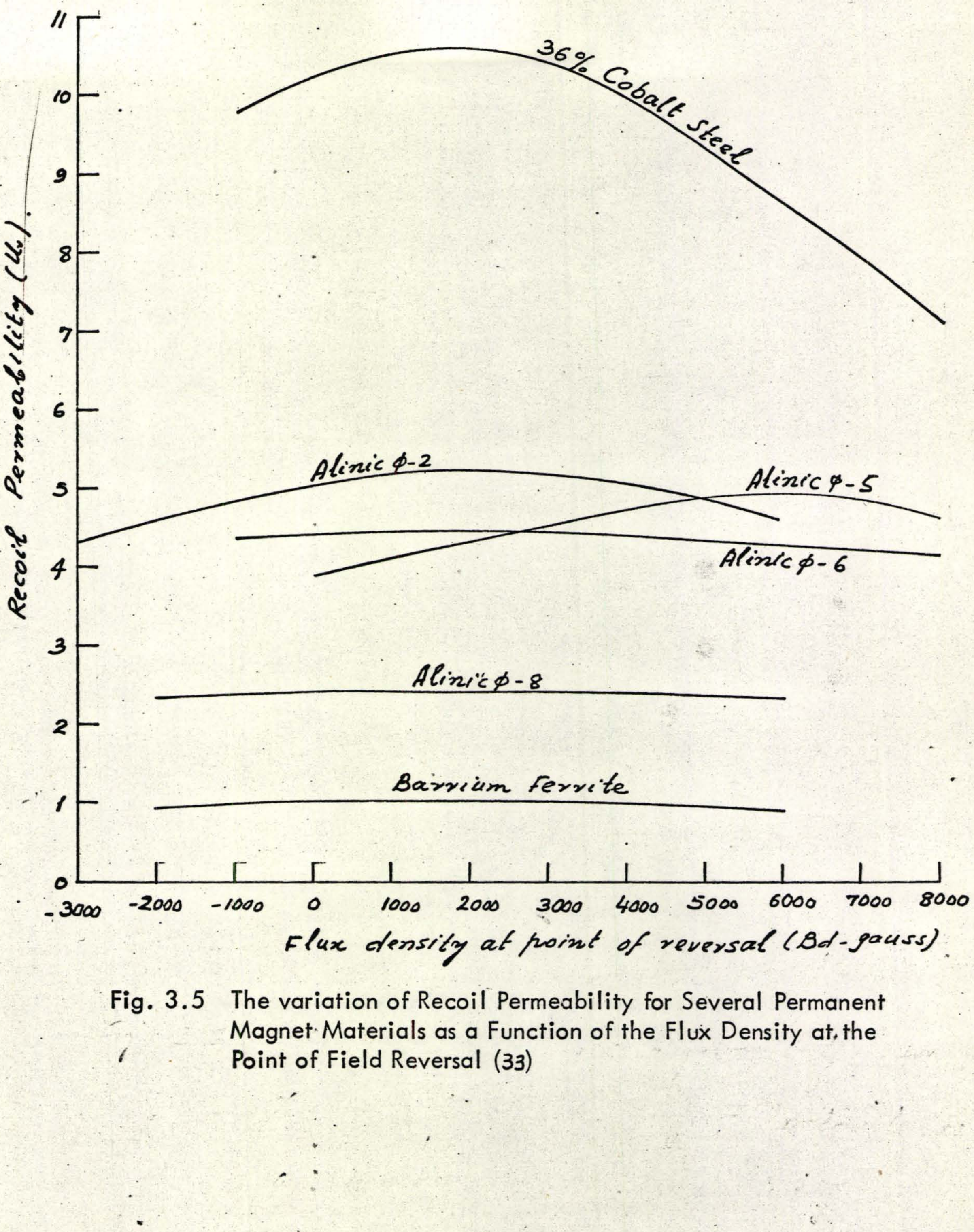


Fig. 3.5 The variation of Recoil Permeability for Several Permanent Magnet Materials as a Function of the Flux Density at the Point of Field Reversal (33)

curve where this ratio reaches a maximum. The ratio $\frac{\Delta B}{\Delta H}$ at this point is called the maximum differential permeability. This ratio is sometimes useful in estimating the properties of a permanent magnet material. Characteristics of different materials show that the H value of the point of maximum differential permeability is approximately equal to the intrinsic coercive force H_{ci}.

As a demagnetizing force is applied on a saturated permanent magnet the induction will decrease to a point C on a major hysteresis loop in the second quadrant Fig. (3.3). If now the demagnetizing force is removed the induction will not trace back the major loop but will follow a new path C D. Varying the demagnetizing field now at some intermediate point will cause the induction to trace a path DD_1. An infinite number of these loops can be traced with varying demagnetizing field. These loops are called minor hysteresis loops, their area being so small that for all practical purposes, the straight line through the tips may be used to represent the loops. The average slope of the minor loop is termed the recoil permeability (μ_r). Fig. (3.4) illustrates such a series of minor loops for Alinico - 6 with the average slope represented as a straight line. Measurements indicate that recoil permeability is nearly constant for materials with a permeability less than 6. Fig. (3.5) illustrates the variation of μ_r with flux densities at point of reversal for different magnetic materials. Recoil permeability is also sometimes called as increamental permeability.

6. Available Energy Under Recoil

Under conditions of recoil a permanent magnet cannot produce energy equivalent to (BH) product corresponding to its major loop as shown in Fig. (3.2). The

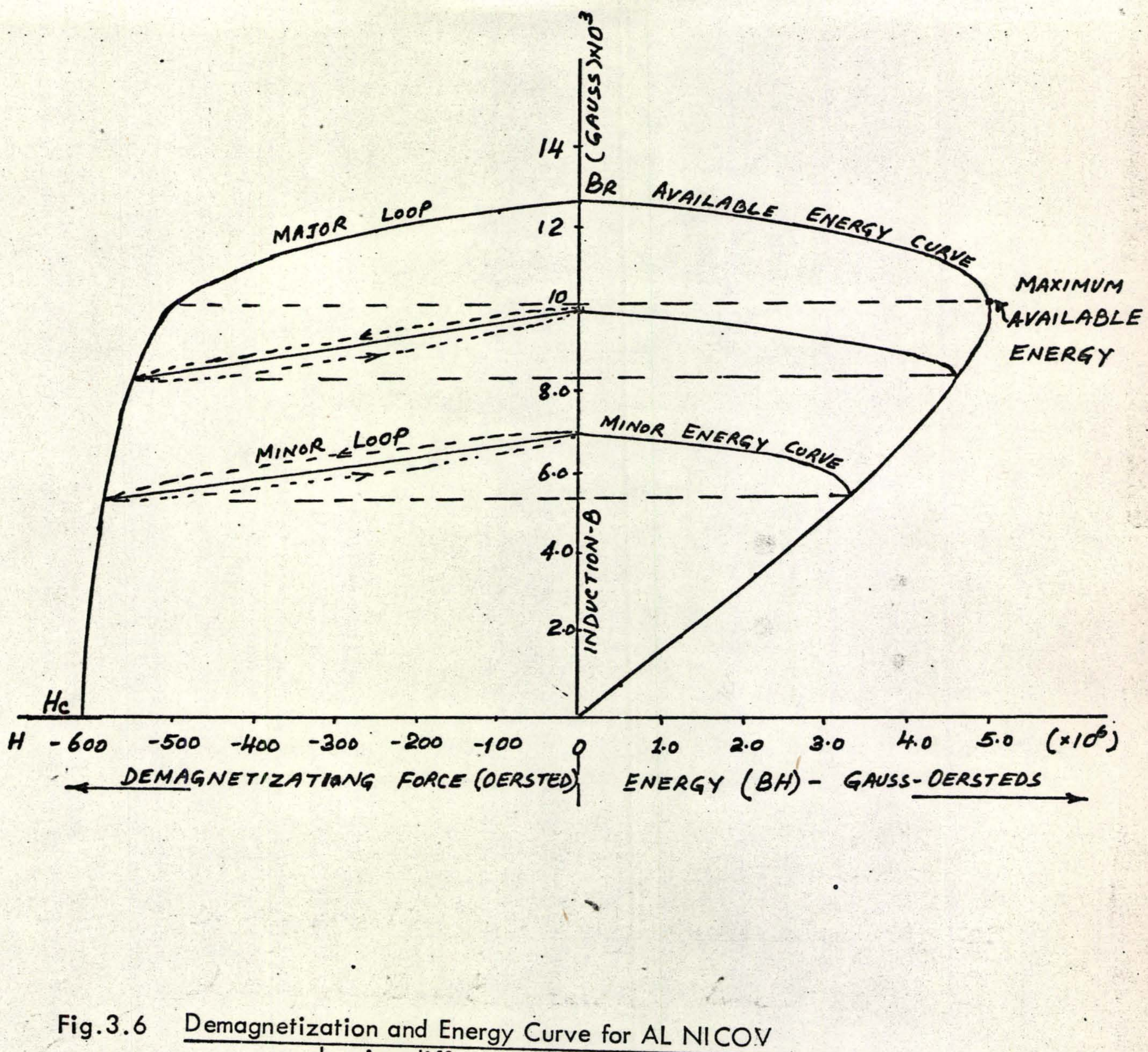


Fig.3.6 Demagnetization and Energy Curve for AL NICO.V showing different recoil line

energy under recoil is obtained for a particular recoil line by the product of H and B for different points on the line. A comparative curve of energy under recoil and energy for major loop is shown in Fig. (3.6). It shows the major loop and the minor loop lines originating at major loop and the corresponding energy (BH) curves plotted as a function of induction B.

7. Curie Point Temperature

The Curie point is defined as the temperature at which a "ferromagnetic" substance changes to a paramagnetic substance. The actual phenomena connected with this curie point is a favourite haunting ground for the physicists, but as far as an electrical engineer is concerned it may be regarded as a temperature at which a magnetic substance loses its magnetic properties permanently. Even before the Curie point of a material is reached, the effects of temperature are irreversible and a rapid decline in coercive force and remanence occurs. Hence, while designing a permanent magnet, care should be taken that the magnet is never heated up to the Curie point. Fortunately, the modern permanent magnets have a very high Curie temperature. For Alnico 5, this is as high as 1000°C. (See table 3.1) Hence in designing a permanent magnet for rotating machines, temperature effects would not be of much importance. The lowest temperature at which any observable change in permanent magnet occurs is about 500°C.

B. MECHANICAL PROPERTIES

1. Hardness

In the practical uses of a magnet, the mechanical properties are not negligible factors. Generally speaking, 'hard' magnetic materials i.e. materials having high values of H_c and B_r are also mechanically hard and brittle. This is principally due to the heat treatment carried on such materials to improve their magnetic properties. There is a relation between the hardness factor and the ultimate strength of a material though the coefficient of proportionality between hardness and ultimate strength is a function of the material itself, and of the factors associated with it and must be determined by test. Besides the question of ultimate strength, the hard and brittle nature of these magnetic materials limit the uses of different mechanical operations on them. For example, it is not practicable to drill them, grinding is the only practical method to finish them to size, hence, finishing is expensive. Because of these limitations, sintering has started as the alternative way of manufacture. Here finely powdered material is subjected to an extremely high pressure to produce magnets of various shapes. This method though more expensive than casting is particularly suited to the production of complicated shaped magnets. Metallurgical processes sometimes impose limitations on the shape and size of magnets, which can have large effects on the performance of the magnet. However, when magnet designers and users are not aware of these limitations and facts for a particular magnet material, size and shape of the magnet should not be finally decided upon without consultation with the manufacturers.

2. <u>Specific Gravity</u>

The specific gravity of a material is of considerable importance in all engineering design. Utilization per unit weight of a magnet is a useful factor, and is also a conventional basis of comparison in design. For ordinary metallic magnets the specific gravity lies between 6 and 8.5 while for ceramic magnets it is between 4.5 and 5.

C. ELECTRICAL PROPERTIES

The specific resistance of a magnet effects the eddy currents induced in it when subjected to an alternating flux. These eddy currents have two effects:

1. Production of heat and hence loss of electrical energy.

2. Production of magnetic field which according to LENZ'S law opposes the inducing field, thus tending to reduce the strength of the latter. The overall effect of the eddy current is thus to oppose any change of flux through the magnet. These eddy currents and their effects are functions of specific resistance of the material of the magnet, the section and depth of the magnet and the rate of flux linkage. Since usually the magnets are magnetized by direct current the effect of eddy current, which is present only during the switching on of the d.c., cannot permanently impede the magnetization. However, in the case of alternating flux which tends generally to demagnetize the magnet, the effect of eddy current is conspicuous. The distribution of eddy current field under such conditions in a magnet follows such a pattern that it causes the actual flux density inside the magnet to be non-uniform, the parasitic flux density generally diminishing from the perimeter to the centre. Modern anisotropic alloys have a high permeability and low specific resistance and hence the effect of eddy current is particularly noticeable in such materials. The actual amount of self-screening would depend obviously on (i) Specific resistance of the material (ii) Shape and section of the specimen, and (iii) Frequency and magnitude of the demagnetizing mmf.

(D) MISCELLANEOUS PROPERTIES (STABILITY)

Evershed [1] in one of the first classic papers written on permanent magnet considered this phenomena of magnetic stability. This term (magnetic stability) he applied to mean (i) resistance to stray field and (ii) resistance to vibration (resistance being used in general sense).

All practical applications utilizing magnets are subjected to influences which tend to alter the flux supplied by the magnet. In general, stability is measured by any change in magnetization detected as a change of remanence or air-gap flux. Factors causing such changes in a magnet are called demagnetizing influences. There are following important factors which influence magnet stability:

1. Structural stability
2. Temperature stability
3. Mechanical stress
4. Viscosity and relaxation effects
5. Radiation effects, and
6. Magnetic field effects.

Modern magnetic materials such as Alnico5 and 6 suffer no structural change for temperature below 550°C while Barium Ferrite is structurally stable below 400°C. They are also stable against relaxation and radiation effects and do not get easily demagnetized under such conditions. For a better understanding of above phenomenon (1 to 5) the reader is referred to the work of Gould, McCaig, Clegg, Tenzer and Kronenberg, for such a study is beyond the scope of this work. However, magnetic field effects on permanent magnets are quite important in electrical machinery and hence is explained here briefly.

Magnetic Field Effects

Magnetic field always disturbs the saturated permanent magnet. These fields may be due to external magnetic field such as field set up around high current carrying conductors or by the changing field of the magnet itself brought about by the change in environment referred to as reluctance change. The change on a magnet performance is similar in both cases. An ideal permanent magnet with a straight line demagnetization characteristic (Barium ferrite approaches this ideal condition.) will be unaffected by demagnetizing field and is relatively free of irreversible magnetization losses caused by external fields. However, in all widely used materials other than ferrites demagnetization due to field effects will be of major concern and must be taken into account in the design of the magnet.

A saturated permanent magnet material in a magnetic circuit with an air gap will exhibit a certain flux density given by the point A in Fig. (3.7), where line OA represents the effective permeance coefficient of the circuit. The application of an external demagnetizing force would cause the point of operation to move down the demagnetization curve to some point B. The magnitude of this demagnetizing force can be determined by constructing a line parallel to OA at a distance $H_1 = \dfrac{F_d}{l_m}$ where l_m is the length of magnet in centimeters. This line intersects the demagnetization curve at point B whereby ΔB, is the loss of air-gap flux supplied by the permanent magnet when the demagnetizing field is acting. In the case where the demagnetizing field H_a is known $H_1 = H_a(1 + \frac{1}{P})$ where P is the permeance coefficient.

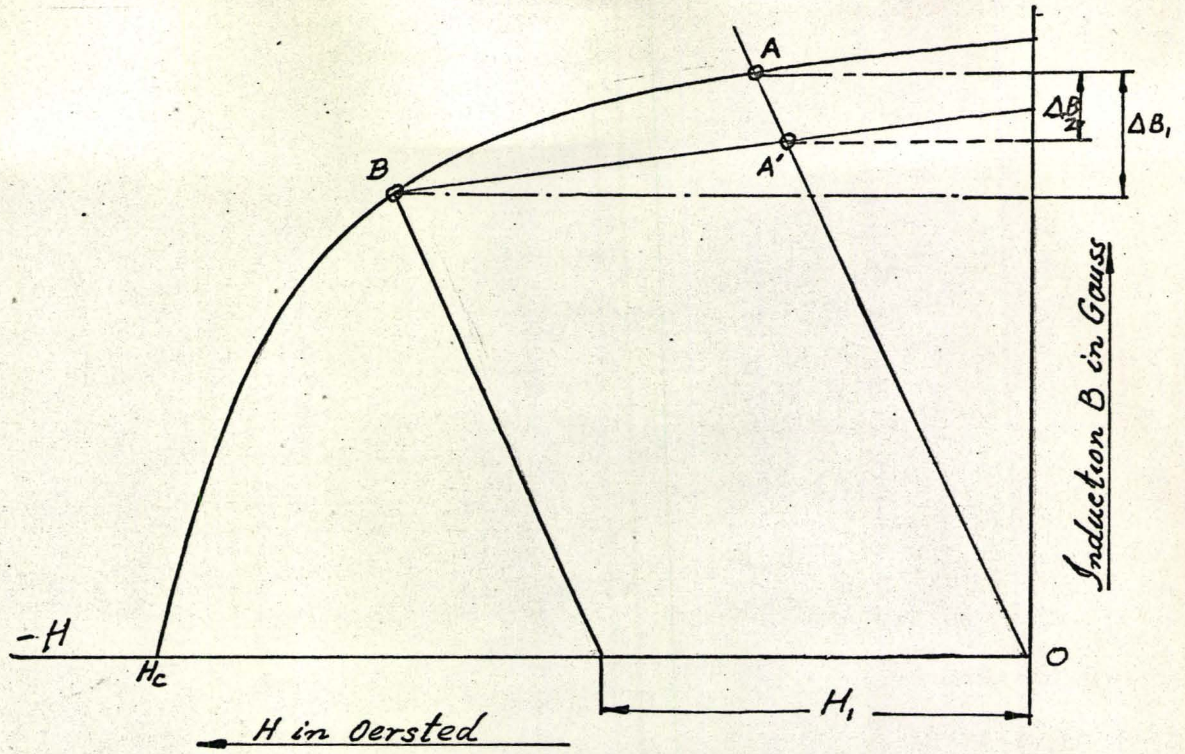
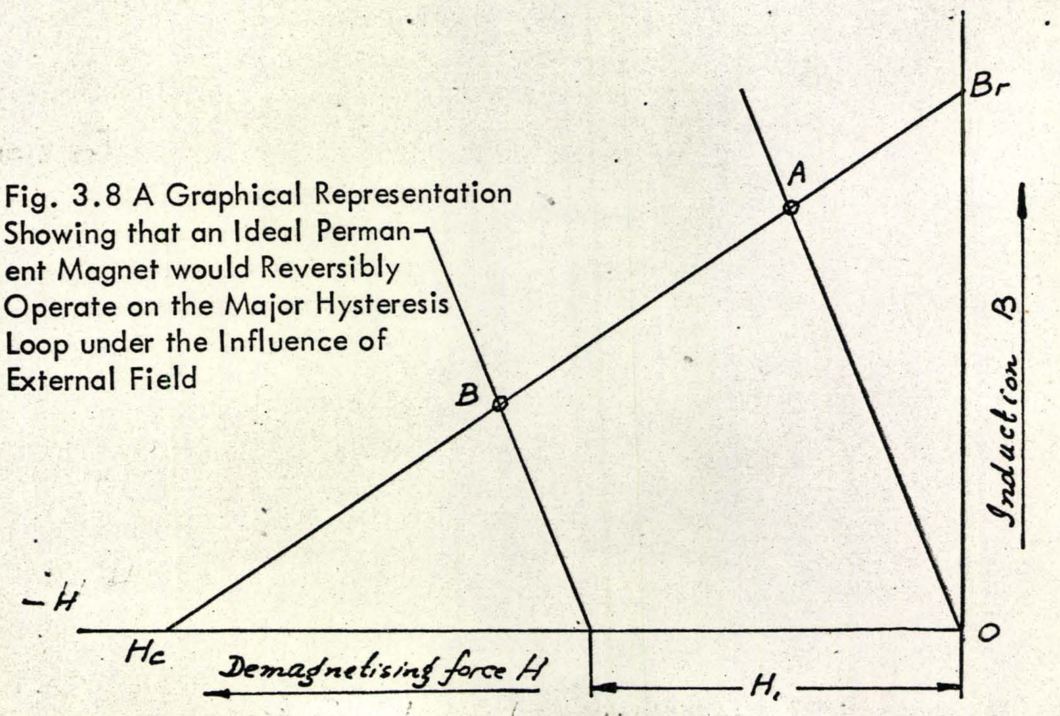


Fig. 3.7 Loss of External Induction Due to an External Demagnetizing Force on ALNICO

Fig. 3.8 A Graphical Representation Showing that an Ideal Permanent Magnet would Reversibly Operate on the Major Hysteresis Loop under the Influence of External Field



There are cases where both varying air gap and external demagnetizing forces exist. The one having the greater effect must be considered during design.

When the external field is removed the point of operation shifts to A' and the magnet suffers an irreversible loss of flux given by ΔB_2.

Fig. (3.8) illustrates that an ideal permanent magnet such as Barium ferrite whose demagnetization curve is given by a straight line will not suffer any irreversible loss due to any external demagnetizing field after the field is removed. This is because the magnet always operates on its major loop as shown in the Fig.(3.8).

3.2 ANISOTROPIC MAGNETS

Most of the high-energy modern magnetic alloys have directional properties. As a result, of special heat treatment accompanied by a magnetic field, anisotropy is developed in certain materials. In other words, $(BH)_{max}$, coercive force, induction and complete demagnetization characteristics of this material differ from one axis to another, generally showing a marked improvement in the direction of the originally applied field at the expense of the other axes.

3.3 COMMERCIALLY AVAILABLE MAGNETIC MATERIALS

A list of magnetic material commercially available in American markets has been included in table No. (3.1). The materials included are specially suited for use in rotating electrical machinery. This table has been compiled from manufacturer's (Indiana General Company, Valpariso, U.S.A.) published figures.

Here magnetic properties, important mechanical properties and general design data are included. The demagnetization curves for Al nico and Indox permanent magnet

Fig. 3.9

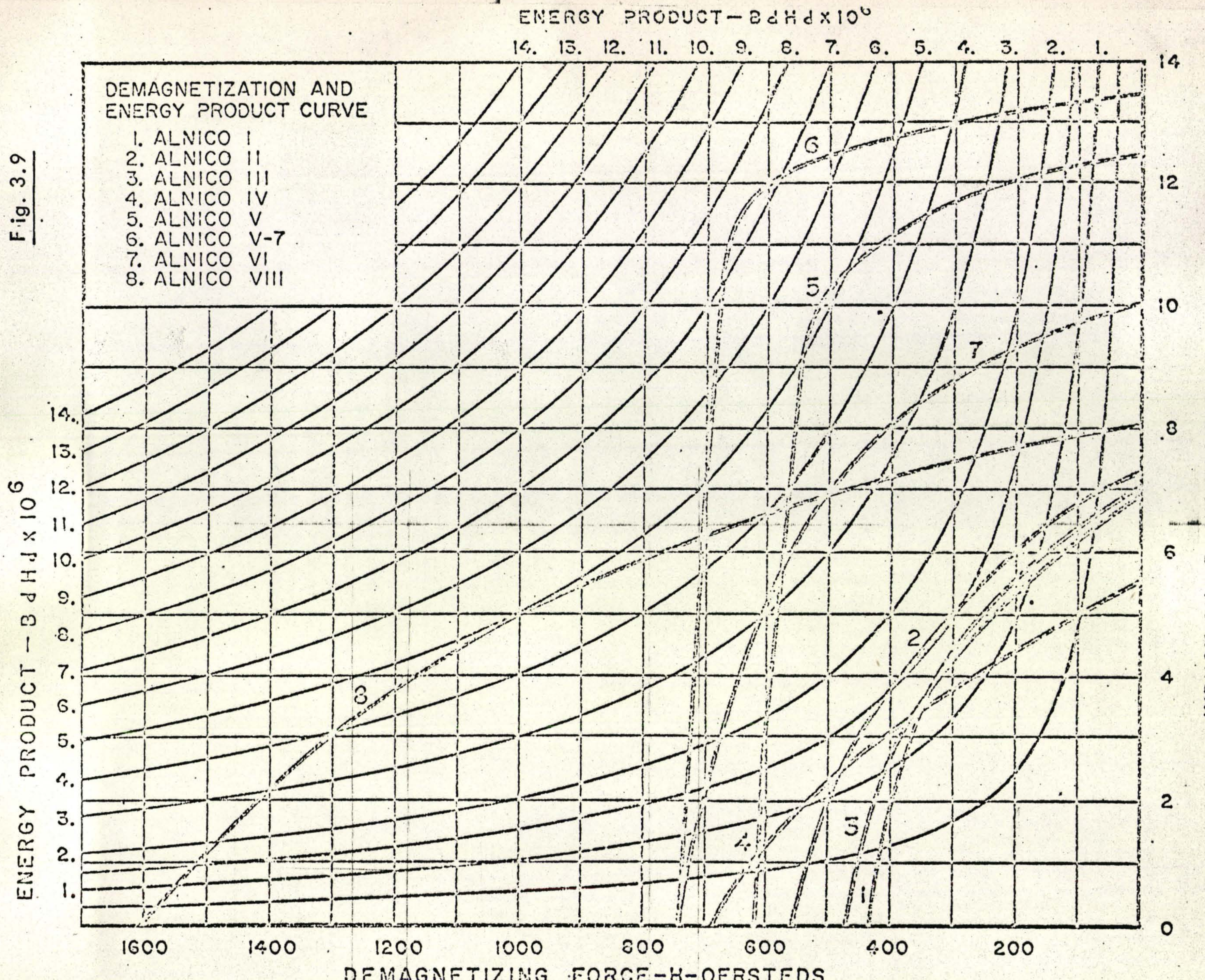


Fig. 3. Demagnetization and energy product curves for cast Alnico magnets.

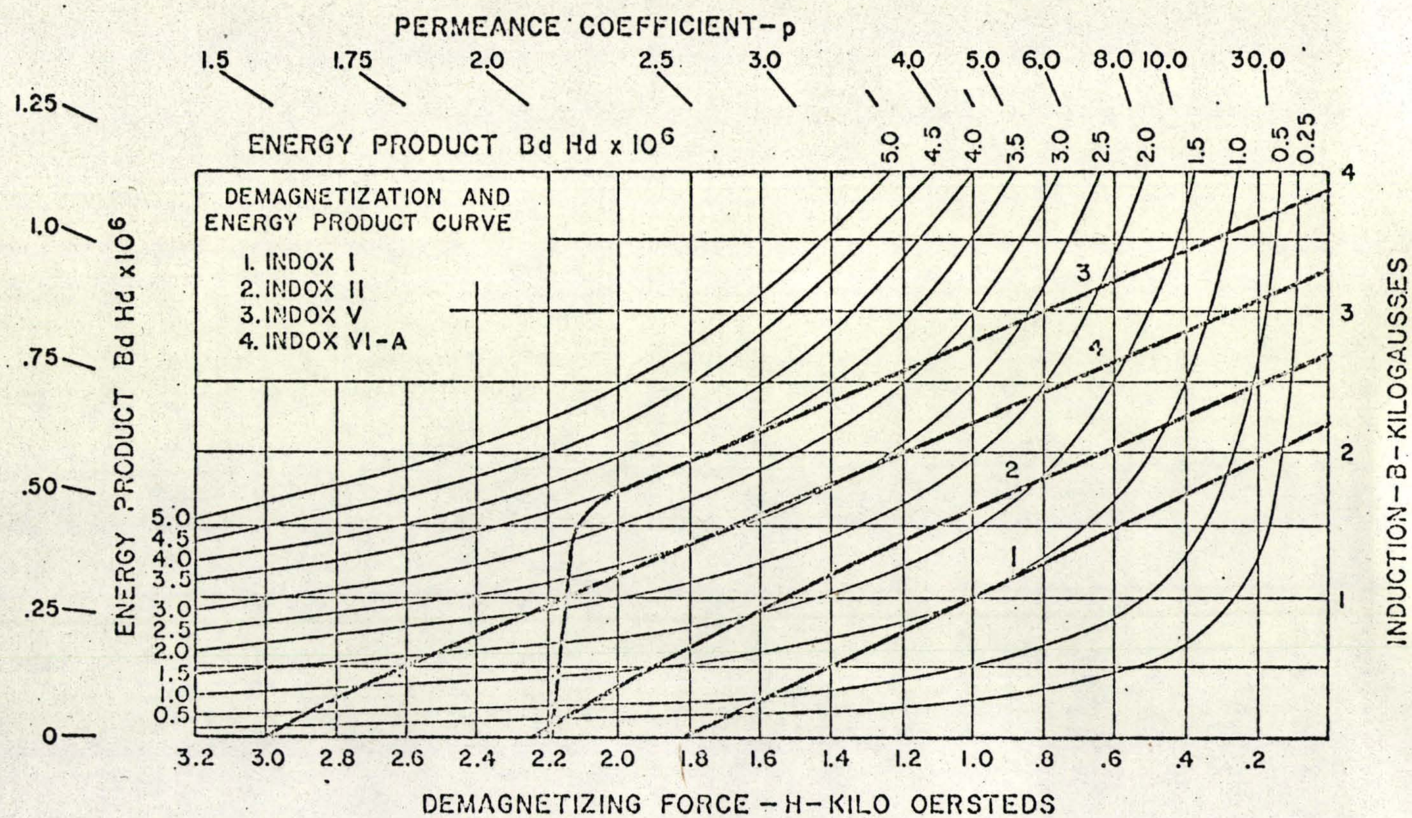


Fig. 4. Demagnetization and energy product curves for Indox ceramic magnets.

Fig. 3.10

TABLE 3.1

Permanent Magnet Materials

CAST ALNICO

	ALNICO I	ALNICO II	ALNICO III	ALNICO IV	ALNICO V	ALNICO V-7	ALNICO VI
MAGNETIC CHARACTERISTICS							
Peak energy product, $(B_dH_d)_{max} \times 10^6$	1.4	1.6	1.35	1.3	5.25	7.50	3.5
Residual Induction, B_r (kilogauss)	7.0	7.2	6.9	5.5	12.5	13.40	10.1
Coercive force H_c (oersted)	440	560	470	700	600	730	750
Permeance coefficient B/H at $(B_dH_d)_{max}$	14.4	13.4	13.7	6.83	20	18.5	12.5
Induction B_d at $(B_dH_d)_{max}$ (kilogauss)	4.5	4.7	4.3	3.0	10.2	11.8	7.0
Mmf per unit length, H_d at $(B_dH_d)_{max}$ (oersteds)	310	350	315	430	512	635	543
Incremental permeability (Note 1)	6.8	6.4	6.5	4.1	3.8	1.9	5.0
Peak magnetizing force required H_p (oersteds)	2000	2000	2000	3000	3000	3000	3000
APPLICATION AND DESIGN FACTORS							
Ability to withstand demagnetizing fields (Note 2)	G	G	G	E	G	G	E
Approx. temp. permanently affecting material (deg F) (Note 3)	1000	1000	900	1100	1000	1000	1000
Preferred magnetic orientation (Note 4)	No	No	No	No	YH	YH	YH
Importance of working at $(B_dH_d)_{max}$ (Note 5)	B	B	B	C	A	A	B
MATERIAL CHARACTERISTICS							
Weight (lb per cu in.)	0.249	0.256	0.249	0.253	0.265	0.265	0.268
Mechanical properties (Note 6)	HB	HB	HB	HB	HB	HB	HB
Critical materials present	Co,Ni	Co,Ni	Ni	Co, Ni	Co, Ni	Co, Ni	Co, Ni
Relative material cost per unit of energy (Note 7)	B	B	A	B	A	B	B
Relative processing cost (Note 7)	B	B	B	B	C	D	C
Electrical resistivity at 25 C (microohm cm)	75	65	60	75	47	47	50
Tensile Strength (psi)	4100	3000	12,000	9100	5450	5000	23,000
Transverse modulus of rupture (psi)	13,900	7700	22,500	24,000	10,500	...	45,000
Coefficient of thermal expansion (per deg C $\times 10^{-6}$)	12.6	12.4	13.0	13.1	11.3	...	11.4
MANUFACTURING METHODS AND LIMITATIONS							
Method (Note 8)	C	C	C	C	CM	CM	C
Shape limitations (Note 9)	C	C	C	C	C	CB	C
Machinability (Note 10)	G	G	G	G	G	G	G

TABLE 3.1

Permanent Magnet Materials

		CAST ALNICO			SINTERED ALNICO		
		ALNICO VII-5 Oriented	ALNICO VII-5 Unoriented	ALNICO VIII	ALNICO II	ALNICO IV	ALNICO V
MAGNETIC CHARACTERISTICS							
Peak energy product, $(B_dH_d)_{max} \times 10^6$		3.7	2.5	5.0	1.45	1.2	3.5
Residual Induction, B_r (kilogauss)		8.57	7.54	8.0	6.9	5.2	10.5
Coercive force H_c (oersted)		1040	890	1600	520	700	600
Permeance coefficient, B/H at $(B_dH_d)_{max}$		8.0	8.5	5.25	12.7	7.5	17.4
Induction, B_d at $(B_dH_d)_{max}$ (Kilogauss)		5.46	4.5	5.12	4.3	3.0	7.8
Mmf per unit length, H_d at $(B_dH_d)_{max}$ (oersteds)		680	555	980	340	400	450
Incremental permeability	(Note 1)	...	...	2.1	6.4	4.1	4.0
Peak magnetizing force required H_p (oersteds)		3500	3500	3000	2000	3000	3000
APPLICATION AND DESIGN FACTORS							
Ability to withstand demagnetizing fields	(Note 2)	E	E	Ex	G	E	G
Approx. temp. permanently affecting material (deg F)'	(Note 3)	1000	1000	1000	900	1100	1000
Preferred magnetic orientation	(Note 4)	YH	No	YH	No	No	YH
Importance of working at $(B_dH_d)_{max}$	(Note 5)	B	B	...	B	C	A
MATERIAL CHARACTERISTICS							
Weight (lb per cu in.)	(Note 6)	0.265	0.265	0.265	0.243	0.250	0.241
Mechanical properties		HB	HB	HB	H	H	H
Critical materials present		Co,Ni	Co,Ni	Co,Ni	Co,Ni	Co,Ni	Co,Ni
Relative material cost per unit of energy	(Note 7)	D	D	D	C	C	B
Relative processing cost	(Note7)	C	C	D	B	B	C
Electrical resistivity at 25 C (microohm cm)		...	...	...	68	68	50
Tensile Strength (psi)		...	...	...	65,000	60,000	50,000
Transverse modulus of rupture (psi)		...	...	...	70,000	85,000	...
Coefficient of thermal expansion (per deg C $\times 10^{-6}$		...	...	...	12.4	13.1	11.3
MANUFACTURING METHODS AND LIMITATIONS							
Method	(Note 8)	CM	C	C	S	S	SM
Shape limitations	(Note 9)	C	C	CB	P	P	P
Machinability	(Note 10)	G	G	G	G	G	G

TABLE 3.1

Permanent Magnet Materials

	INDOX				DUCTILE
	INDOX I	INDOX II	INDOX V	INDOX VI-A	CUNIFE I
MAGNETIC CHARACTERISTICS					
Peak energy product, $(B_dH_d)_{max} \times 10^6$	1.0	1.65	3.5	2.6	1.3
Residual Induction, B_r (kilogauss)	2.2	2.7	3.84	3.3	5.4
Coercive force H_c (oersted)	1825	2250	2200	3000	500
Permeance coefficient, B/H at $(B_dH_d)_{max}$	1.2	1.1	1.05	1.06	12.3
Induction, B_d at $(B_dH_d)_{max}$ (kilogauss)	1.1	1.35	1.92	1650	4.0
Mmf per unit length, H_d at $(B_dH_d)_{max}$ (oersteds)	900	1220	1820	1575	325
Incremental permeability (Note 1)	1.1	1.15	1.05	1.06	1.7
Peak magnetizing force required H_p (oersteds)	10,000	10,000	10,000	10,000	2400
APPLICATION AND DESIGN FACTORS					
Ability to withstand demagnetizing fields (Note 2)	S	S		S	G
Approx. Temp. permanently affecting material (deg F) (Note 3)	1800	1800	1800	1800	900
Preferred magnetic orientation (Note 4)	YP	Yes	Yes	Yes	YR
Importance of working at $(B_dH_d)_{max}$ (Note 5)	B	C	C	C	B
MATERIAL CHARACTERISTICS					
Weight (lb per cu in.)	0.167	.162	0.18	0.162	0.311
Mechanical properties (Note 6)	HB	HB	HB	HB	D
Critical materials present	None	None	None	None	Ni
Relative material cost per unit of energy (Note 7)	A	A	A	A	C
Relative processing cost (Note 7)	B	B	E	E	D
Electrical resistivity at 25 C (microohm cm)	10^{12}	...	10^{10}	...	18
Tensile Strength (psi)	Low	Low	Low	Low	100,000
Transverse modulus of rupture (psi)	...	...	...	...	...
Coefficient of thermal expansion (per deg C $\times 10^{-6}$)	10	10.3	10	10.3	14.0
MANUFACTURING METHODS AND LIMITATIONS					
Method (Note 8)	S	S	S	S	CR
Shape limitations (Note 9)	P	P	P	P	Fa
Machinability (Note 10)	G	G	G	G	Yes

NOTES

1. Approximate value.

2. F, fair; G, good; E, excellent; Ex, exceptional; S, superior.

3. See section on Thermal Properties.

4. YH, oriented in heat treatment; YR, in cold reduction; YP, in pressed direction (slightly).

5. A, most important; D, least important.

6. H, hard; B, brittle; D, ductile.

7. A indicates lowest; D indicates highest.

8. C, cast, heat treated; CR, cold reduced; M, cooled in magnetic field; S, pressed from powder and sintered.

9. C, cast; P, pressed; F, formed from rolled stock; a, maximum cross-section is 0.1 sq. in.

10. G, grind only; holes cored at time of casting; A, annealed for machining.

materials as given by Indiana General Company are also given in Figures (3.9) and (3.10).

The list of magnetic materials given here is by no means exhaustive. It is included here just to give an idea of the present range and qualities of high-energy magnetic materials. The future prospects of permanent magnet machines obviously depend to a large extent on the development of these materials.

3.4 OUTPUT AND DESIGN OF PERMANENT MAGNETS SUBJECTED TO DEMAGNETIZING FORCES

In general the permanent magnet design problem is to specify a permanent magnet material with defined unit properties and to arrive at a given volume and configuration which will establish the required energy in space most efficiently. The problem in this rigid analysis is quite complicated for one faces the solution of difficult field problems and determination of actual flux in space and its distribution. However, some simplifying assumptions based on experience is to be made while dealing uniquely any design problem.

The essential pre-requisites in determining the principal dimensions of permanent magnet which should be known to a designer are:

(a) knowledge of the conditions under which the permanent magnet is supposed to work

(b) the demagnetization characteristic and any other special property of the magnetic materials to be used.

and (c) the limit of variation if the magnet is subjected to variable reluctances during operation.

A systematic attempt on the design of permanent magnet operating under variable demagnetizing force has been presented by Hornfeek and Edgar[4]. These methods are based on major and minor hysteresis loop data and are essentially extensions of Evershed's[2] and Watson's[3] method.

An attempt will be made here to present briefly the design relationships of magnet volume and its output when operating under different conditions.

Firstly, the design for magnet to fulfill a static function is considered. By static function is implied the fact that the reluctance of the magnetic circuit is constant, in other words the magnet is used to establish field energy in an air-gap of fixed dimension and operating under no demagnetizing field influence. Secondly, magnets operating under dynamic conditions will be considered. This includes magnets operating under variable reluctance of magnetic circuit or under variable demagnetizing forces. Such magnets will operate under minor hysteresis loop with recoil.

The design relationships given here are based on the following important assumptions:

1. The magnet is uniform in cross-section and its properties and magnetic states are uniform throughout. In other words, every particle of the magnet works on the same B/H curve.
2. The magnet has been initially magnetized to saturation.
3. The minor hysteresis loops have their origin on the major loop and they can be replaced by recoil lines.

I. DEMAGNETIZATION CURVE

The equation to the demagnetizing curve of a permanent magnet has been considered by Watson using a modified form of Lamont's law. This has been improved and presented by Desmond[7] in one of the most classic papers written on this subject after Evershed (1), (2). For the purpose of design it is only the B-H curve which is considered (and not the intrinsic magnetization curve) as the magnet can only maintain the B value in an external field. For convenience the negative sign H is omitted although it is obvious that the demagnetization curve (second quadrant of B-H curve) is being considered.

The equation to the B-H curve is (Refer to Appendix AI)

$$B = \frac{B_r (H - H_c)}{(a H - H_c)} \tag{3.1}$$

where

$$a = \frac{2\sqrt{r} - 1}{r}$$

and

$$r = \frac{(BH)_{max}}{B_r H_c}$$

3.5 MAGNET OPERATING UNDER STATIC CONDITION

1. Case 1 - Without Leakage

Considering that a magnet after being magnetized to saturation is subjected to an insertion of an air gap in the magnetic circuit, then some point A on the de-

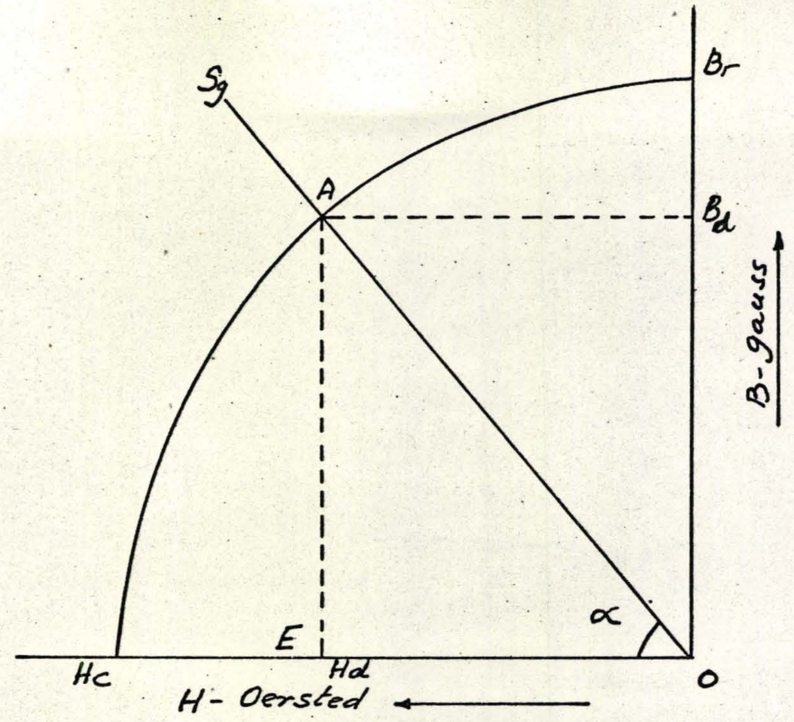


Fig. 3.11 Constant Reluctance without Leakage

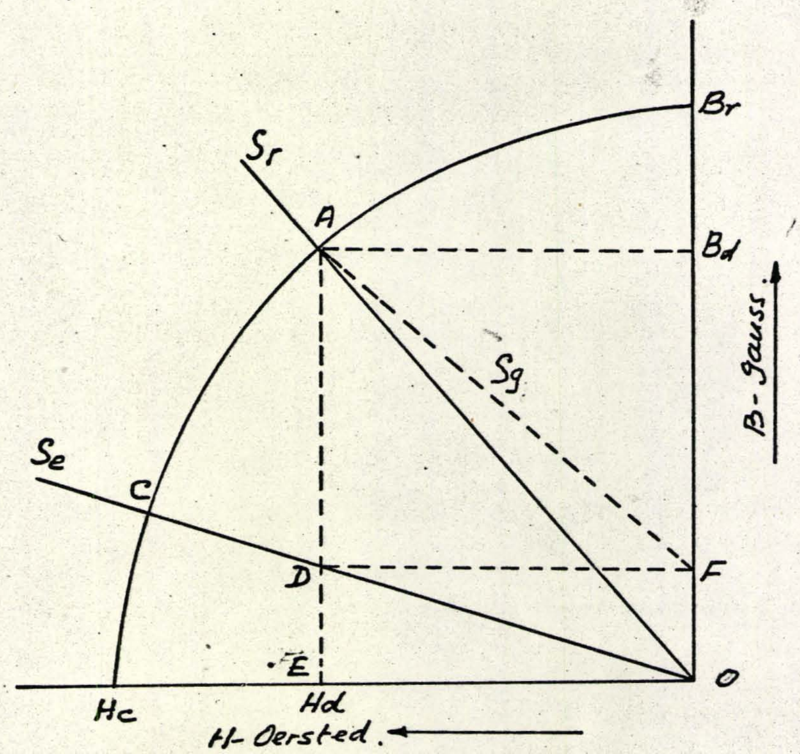


Fig. 3.12 Constant Reluctance with Leakage

magnetization curve (Fig. 3.II) will represent the magnetizing force H_d which the magnet is applying to the air-gap and the induction B_d existing in the magnet. The location of the point A will depend on the relative dimension of the magnet and the air gap. Let H_g be the air-gap potential and B_g the flux density in the air gap.

Since the magnetomotive force applied to the circuit is zero

$$H_d l_m = H_g l_g = \frac{B_g l_g}{\mu_o} \tag{3.2}$$

assuming, of course, that the total mmf is required for the air-gap only

Also $B_d A_m = B_g A_g$ (3.3)

From equation (3,2) and (3.3)

$$V_m = A_m l_m = \frac{B_g^2 A_g l_g}{\mu_o (B_d H_d)} \tag{3.4}$$

The slope of the line OA is

$$S_g = \tan \alpha = \frac{B_d}{H_d}$$

Substituting for B_d and H_d from equation (3.2) and (3.3)

$$S_g = \frac{B_d}{H_d} = \frac{l_m}{A_m} \left(\frac{\mu_o A_g}{l_g}\right) = \frac{l_m}{A_m} P_g \tag{3.5}$$

S_g is called as unit air gap permeance and is related to the air-gap permeance (P_g) by magnet dimensions.

From equation (3.4) it is obvious that the volume of the magnet will degrease with increasing ($B_d \times H_d$). Hence for the minimum magnet volume, the slope of the line OA should be such that A lies where the $(B_d \times H_d)$ product of the magnet is maximum.

Considering a complex magnetic circuit, consisting of an air-gap and various sections of ferrous metal in series with each other,

$$H_d l_m + \frac{B_g l_g}{\mu_o} + \frac{B_1 l_1}{\mu_1} + \frac{B_2 l_2}{\mu_2} + \text{etc.} = 0 \quad (3.6)$$

$$B_d \times A_m = B_g A_g = B_1 A_1 = B_2 A_2 \text{ etc.} \quad (3.7)$$

Where l_1, l_2 and A_1, A_2 are the respective lengths and areas of different magnetic paths and μ_1 and μ_2 are their permeabilities at flux density B_1 and B_2.

From equation (3.5) and (3.6),

$$H_d = \frac{B_d A_m}{l_m} \left(\frac{l_g}{\mu_o A_g} + \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \text{etc.} \right) \quad (3.8)$$

The value of B and H in the magnet may be determined from the intersection of the curve given by equation (3.7) and the demagnetization curve.

The total volume of the magnet required is

$$V_m = \frac{1}{(B_d H_d)} \left(\frac{B_g^2 l_g A_g}{\mu_o} + \frac{B_1^2 A_1 l_1}{\mu_1} + \frac{B_2^2 A_2 l_2}{\mu_2} \right) \quad (3.9)$$

Thus in this case also for a given magnetic circuit, the volume of the magnet required is again a minimum when $(B_d \times H_d)$ is a maximum.

From Appendix (A I) at the point of $(B_d H_d)_{max}$.

$$\frac{B_d}{H_d} = \frac{B_r}{H_c} \quad (3.10)$$

Hence, if the magnet is working at the $(B_dH_d)_{max}$ point, we shall get from equation (3.4), (3.5), the dimensions of the magnet as ,

$$A_m = B_gA_g \sqrt{\frac{H_c}{B_r (B_dH_d)_{max}}} \quad (3.11)$$

$$l_m = B_gl_g \sqrt{\frac{B_r}{H_c (B_d H_d)_{max}}} \quad (3.12)$$

And in the case of complex magnetic circuit,

$$A_m = B_gA_g \sqrt{\frac{H_c}{B_r (B_dH_d)_{max}}} \quad (3.13)$$

$$l_m = (B_gl_g + B_1 l_1 + B_2 l_2 + elc) \sqrt{\frac{B_r}{H_c (B_d H_d)_{max}}} \quad (3.14)$$

2. Case 2 - With Leakage

The above equations neglect the leakage flux which may be quite a considerable part of the total flux depending on the length and area of cross-section of the magnet.

Assuming that the leakage flux is a terminal factor and is given by

$$K_1 = \frac{P_g + P_l}{P_g} \quad (3.15)$$

where P_l is the permeance of all the leakage paths in parallel with the air gap

Referring to Fig. (3.12), $K_1 = \dfrac{AE}{AD} = \dfrac{S_g + S_l}{S_g}$

Thus, considering leakage, the unit permeance

$$(S_g + S_l) = \frac{l_m}{A_m}(P_g + P_l) = \frac{l_m}{A_m} P_g K_1$$

$$(S_g + S_l) = \frac{l_m}{A_m}\left(\frac{\mu_o A_g}{l_g}\right) K_1 \qquad (3.16)$$

and the volume of the magnet is given by

$$V_m = \frac{B_g^2 A_g l_g}{(B_d H_d)} K_1 \qquad (3.17)$$

If the point A lies where $(B_d H_d)$ is max. considering complex magnetic circuit equation (3.13) and (3.14) are modified as

$$A_m = B_g A_g K_1 \sqrt{\frac{H_c}{B_r (B_d H_d)_{max}}} \qquad (3.18)$$

$$l_m = (B_g l_g + B_1 l_1 + B_2 l_2 + etc) \sqrt{\frac{B_r}{H_c (B_d H_d)_{max}}} \qquad (3.19)$$

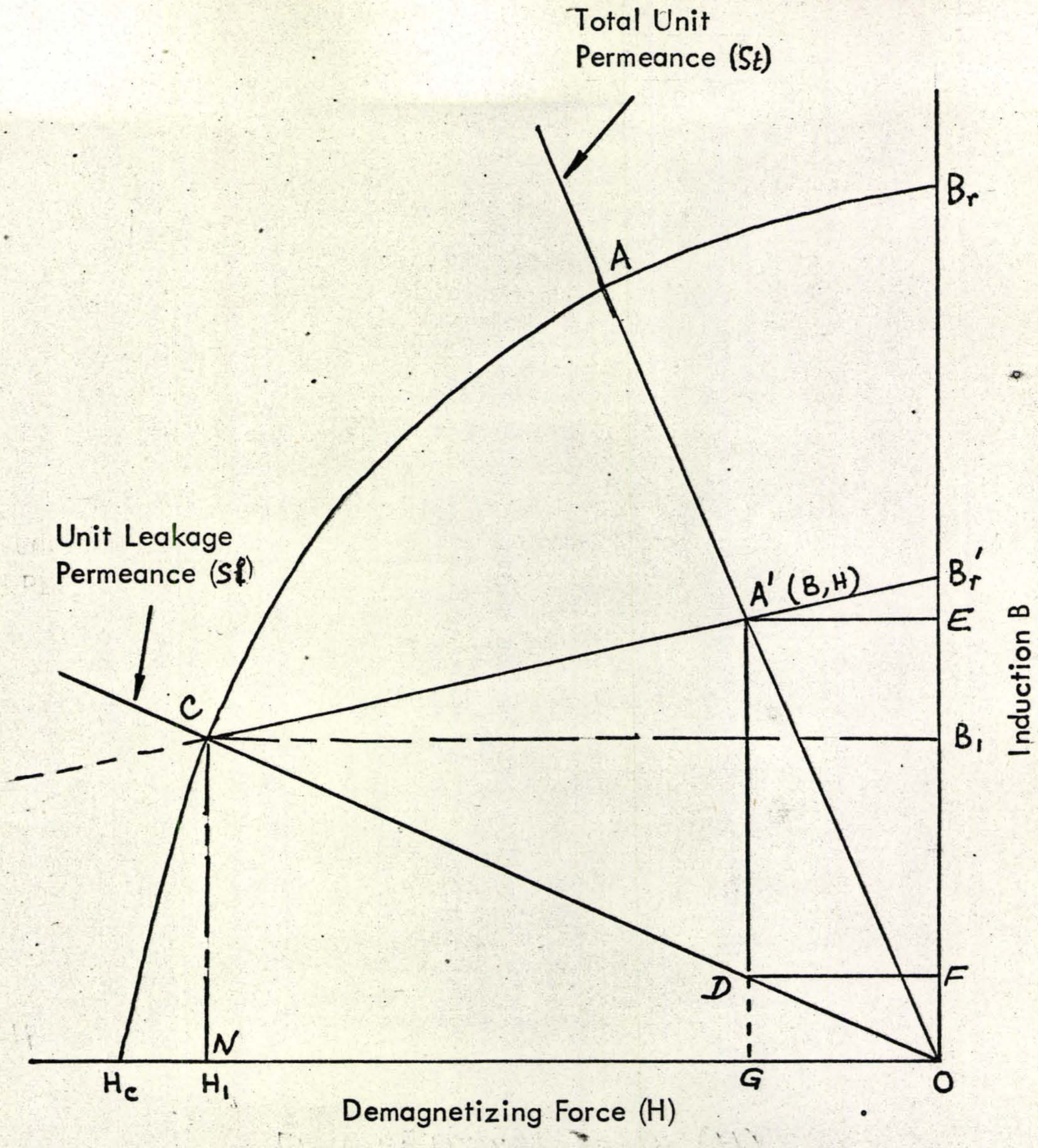


Fig. 3.13 Graphical Representation of Permanent Magnet Operating Under Dynamic Load Condition

3.6 MAGNET OPERATING UNDER DYNAMIC CONDITION

In the constant reluctance problem, the total permeance is controlled such that the total available energy supplied by the magnet is maximum . For this to happen the leakage permeance and the useful air-gap permeance are fixed with respect to each other. But for the permanent magnet operating under dynamic condition the useful and leakage energies change corresponding to the changing conditions.

Consider a magnet initially fully magnetized and connected to a circuit of which OA, Fig. (3.13) represents the maximum unit permeance equal to sum of unit leakage and unit air gap permeance. In the case of rotating machine represents the total unit permeance of the flux path when the rotor is in the stator. Now let the circuit reluctance be increased until the unit permeance is OC and the flux exists in leakage paths only. This would correspond to the condition in the machine when the rotor is out of stator. Let S_1 be the unit permeance corresponding to this condition. The working point moves down now from A to C. Now when the reluctance is decreased again, say, by putting the rotor back into stator the unit permeance is again given by OA. During this process the magnet operates along the recoil line CB_r' called minor loop and the working point corresponding to total permeance is given by A' and not A.

The minor loop CB_r' on which the magnet operates depends on the slope of OC and the slope of major loop at B_r. The magnet energy available now is given by (BH) at A' and not (BH) at point A on major loop.

If the leakage permeance line remains constant, the useful flux A'D exists, the useful energy per unit volume of magnet is given by area A'EFD while the leakage energy by area DFOG. Now for any fixed position C the useful area A'EFD will depend on the position of A'. To maximize area A'EFD for greatest design economy, the total permeance line should be such that $A'B_r' = \frac{1}{2} CB_r'$ and $EF = \frac{1}{2} OB_r'$, and the leakage permeance line should be such that OCB_r' is maximum. This will uniquely determine the point C and A'.

The particular position of C for the optimum performance will occur when the product of

$H (B + \mu_r H)$ is a maximum

i.e. $BH + \mu_r H^2$ is a maximum

And for this to be true

$$\frac{d\,(BH)}{dH} + 2\mu_r H = 0$$

assuming, of course, that μ_r is constant which is quite true in the region of maximum (BH) point.

Now at (BH) max, $\frac{d(BH)}{dH}$ is zero but $2\mu_r H$ is positive which shows that for area OCB_r' to be maximum C lies below the point $(BH)_{max}$ on the demagnetizing curve. However, the actual co-ordinate of H can be determined by substituting the value of B and $\frac{dB}{dH}$ from equation (3.1) in euqation (3.20) which gives

$$\frac{B_r (H - H_c)}{aH - H_c} + H \frac{H_c B_r (a-1)}{(aH - H_c)^2} + 2\mu_r H = 0$$

Simplifying the above equation

$$H^3(2\mu_r a^2) + H^2(aB_r - 4a\mu_r) + 2H(\mu_r H_c^2 - H_c B_r) + B_r H_c^2 = 0 \quad (3.21)$$

Thus, from the above equation the values of H can be calculated one of which will determine the position of C for the maximum area of OCB_r'. And for most economical design the magnet dimensions can be fixed such that A' lies midway of CB_r'. For design purposes one can draw the variation of energy (BH) supplied by the magnet for magnet operating along different minor loops and can then determine the magnet dimensions for most optimum design. We shall now consider briefly the design procedure for permanent magnets subjected to varying applied magnetomotive forces.

3.7 PERMANENT MAGNET SUBJECTED TO VARYING APPLIED MAGNETOMOTIVE FORCES - DESIGN PROCEDURE

In some cases the permanent magnet may be subjected to a variable applied magnetomotive force as well as variable external reluctances. Assuming that the maximum demagnetizing force is not unmanageable and its value could be determined, then the following method as suggested by Hornfeck and Edgar [4] could be used to estimate optimum magnet dimensions and predict its maximum energy. The operation of the permanent magnet subjected to demagnetizing forces is illustrated in Fig. (3.14).

Let R_1 = maximum external reluctance.

R_2 = minimum external reluctance.

H_{d_1} = maximum demagnetizing force on the magnet.

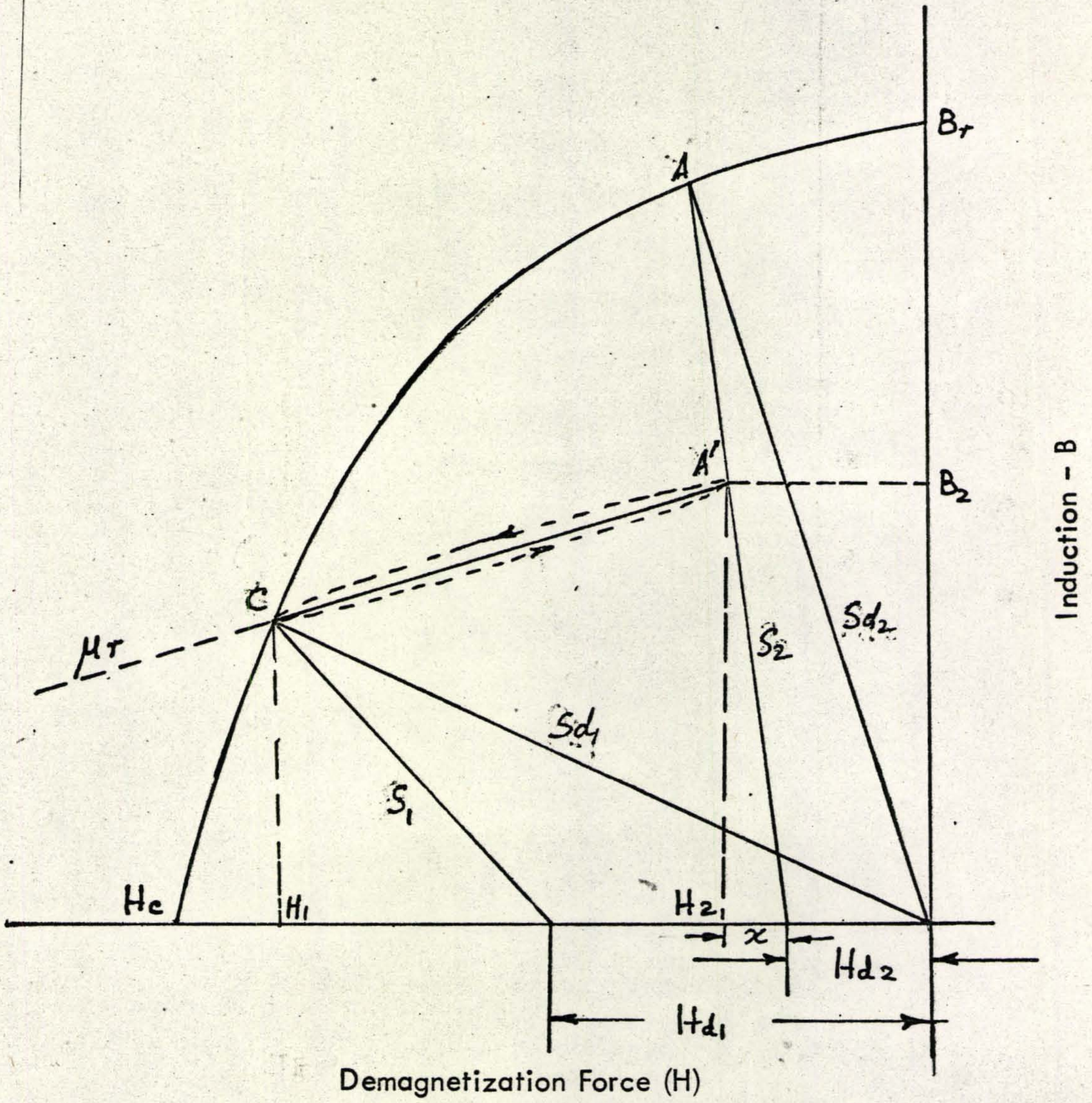


Fig. 3.14 Demagnetization Curve and minor hysteresis loop for a condition of combined variation in external reluctance and demagnetizing force.

H_{d2} = minimum demagnetizing force on the magnet.

S_1 = slope of maximum external reluctance on the magnet.

S_2 = slope of minimum external reluctance on the magnet.

S_{d_1} = slope of maximum unit permeance line.

S_{d2} = slope of minimum unit permeance line.

H_o = a fictitious coercive force obtained by extending the working minor loop line to intersect the abscissa - called internal mmf/unit length.

μ_r = recoil permeability, given by slope of minor hysteresis loop.

Under such conditions the internal magnetomotive force of the magnet is reduced as can be seen from the Fig. (3.14) and the magnet output as well as the effective permeance are given by the following equations.

$$B_2 = \frac{\mu_r (H_o - H_{d2}) S_2}{\mu_r + S_2} \tag{3.22}$$

$$H_2 = \frac{\mu_r (H_o - H_{d2})}{\mu_r + S_2} \tag{3.23}$$

$$S_{d_1} = \frac{[\mu_r (H_o - H_{d1}) - H_{d1} (\mu_r + S_1)] S_1}{\mu_r (H_o - H_{d1})} \tag{3.24}$$

$$S_{d2} = \frac{[\mu_r (H_o - H_{d2}) - H_{d2} (\mu_r + S_2)] S_2}{\mu_r (H_o - H_{d2})} \tag{3.25}$$

Available energy

$$B_2H_2 = \frac{\mu r^2}{(\mu_r + S_2)^2} (H_o - H_{d2})^2 S_2 \qquad (3.26)$$

$$\text{And} \quad \Delta R' = \frac{100 (S_{d1} - S_{d2})}{S_{d2}} \qquad (3.27)$$

The next step is to find the optimum value of S_{d1} for a particular material if $\Delta R'$ is fixed. To accomplish this the value of B_2H_2 is estimated for different values of S_{d1} and a curve is drawn with (B_2H_2) as the ordinate and S_{d1} as abscissa. It is quite apparent from Fig. (3.14) that the value of H_o depends on the position of C and hence upon the slope of S_{d1}. μ_r also varies somewhat for different values of S_{d1}. These two quantities are plotted as functions of S_{d1} in Fig. (3.15). For a particular $\Delta R'$, S_{d1} and S_{d2} are determined. (BH) is then calculated from equation (3.26) and plotted for different values of S_{d1} as shown in Fig. (3.16). The value of S_{d1} corresponding to maximum (BH) is the optimum value of S_{d1}. Now for this optimum value of S_{d1}, from equation (3.24) corresponding value of S_1 can be calculated. Hence, the optimum magnet dimensions are obtained for

$$S_1 = \frac{l_m}{A_m} \frac{1}{R_1}$$

For this optimum value of S_{d1}, $(BH)_{opt}$ is obtained from Fig. (3.16) and the optimum volume of the magnet

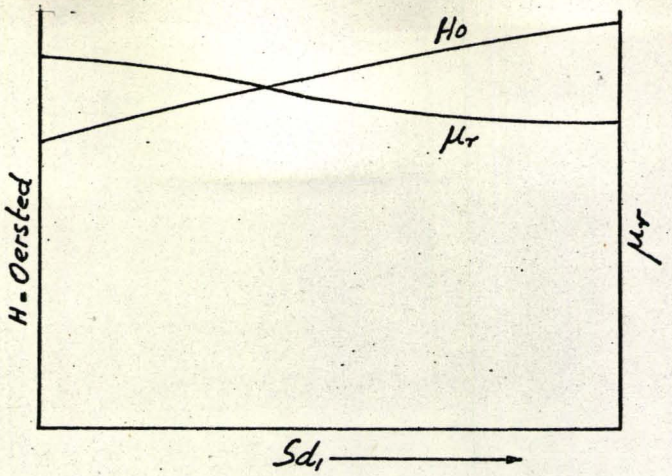


Fig. 3.15 Curves showing values of H_o and $_r$ for assumed Straight-Line Minor Hysteresis Loops

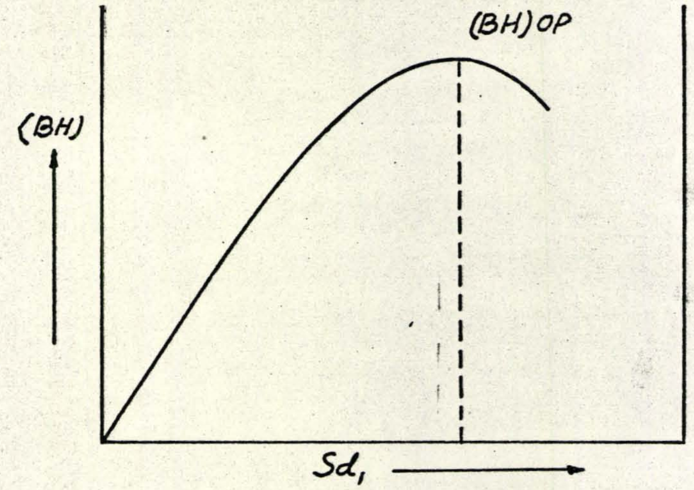


Fig. 3.16 Curve showing values of (BH) which can be obtained with a single set of Sd_1 and Sd_2

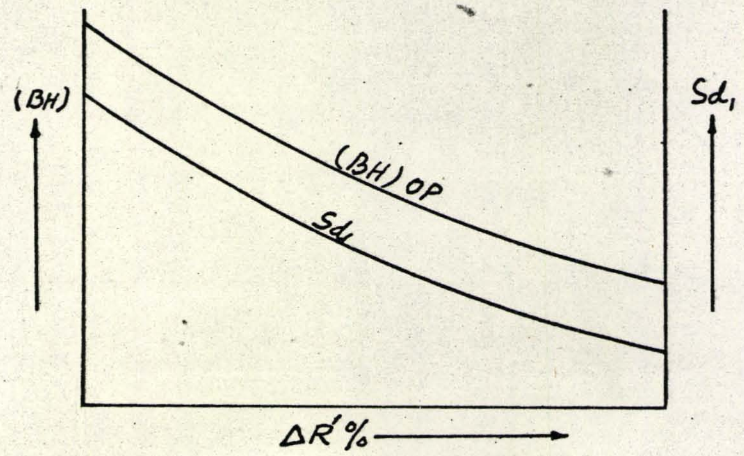


Fig. 3.17 Curves showing the Optimum value of (Sd_1) and the Optimum value of (BH) as function of R'

$$V_m = \frac{\text{Maximum energy required}}{(BH)_{opt}}$$

If the choice of $\Delta R'$ is possible then similar curves as in Fig. (3.15) and (3.16) are plotted for each value of $\Delta R'$ and the $(BH)_{opt}$ and corresponding S_{d1} determined. From these values curves as shown in Fig. (3.17) can be drawn which give $(BH)_{opt}$ and corresponding S_{d1} for any value of $\Delta R'$.

The utilization of permanent magnet material

$$U = \frac{(B_2 H_2)}{(BH)_{opt}} \tag{3.28}$$

which is simply a ratio of total available energy of the actual design to the maximum available energy $(BH)_{opt}$.

The overall efficiency of design is given by

$$E_{ff} = \frac{B_2 H_2}{(BH)_{opt} K_1} \tag{3.29}$$

3.8 LIMITATIONS IN USING THE DESIGN EQUATIONS

Obviously there are limitations to this method of design because of certain assumptions made. In magnet configurations having appreciable limb leakage the assumption of each unit volume operating at a particular point on the demagnetization curve is invalid. The exact field theory to refine these methods will be too cumbersome to apply. However, the method of permanent magnet design which has been outlined here has been found in practice to be sufficiently accurate for a wide variety of problems.

CHAPTER IV

COMPARISON OF PERMANENT MAGNET WITH ELECTROMAGNET

Since permanent magnet materials of high energy contents are now available they are finding more and more application in electrical machines (motors and generators). Experience has shown that permanent magnets are preferable for small devices requiring a constant magnetic field but electromagnets are preferable for large sizes. It is worthwhile to investigate more specifically the magnetic criteria which would more or less give a rough idea whether permanent magnets or electromagnets may be used in any application. However, other design factors or cost may also play an important factor, but these considerations are beyond the scope of this analysis. This comparison is based on three factors:

1. Energy per unit volume of the magnetic field which is to be established.
2. Energy per unit available for permanent magnet material, and
3. Volume of copper and iron required for an electromagnet performing the same task.

The following simplifying assumptions are made:

(a) The flux density is low enough for the reluctance of the iron in the return path to be a minor correction. In other words, all the

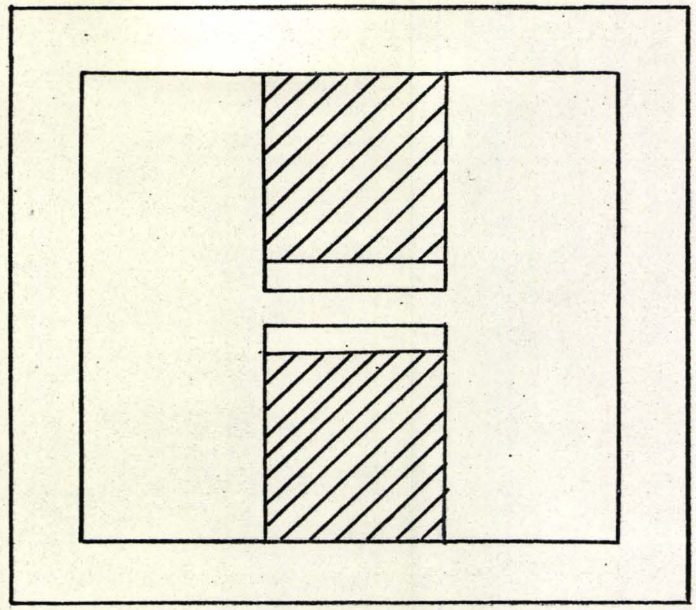


Fig. 4.1 Permanent magnet with pole-pieces and yoke

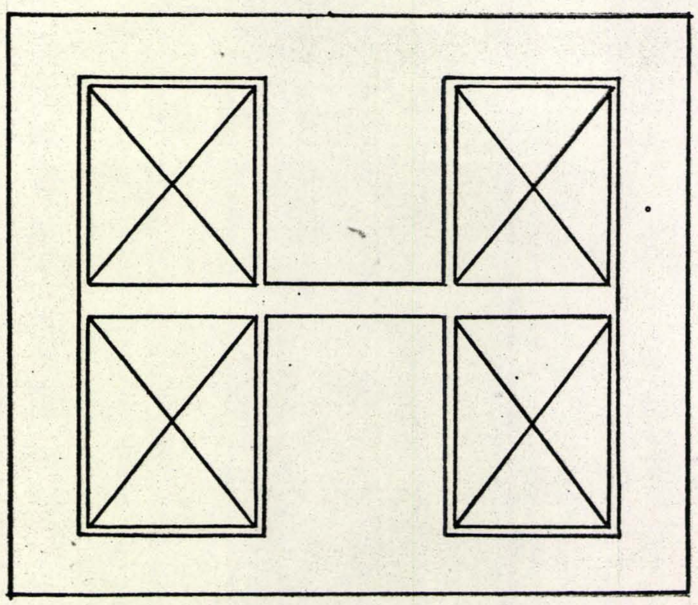


Fig. 4.2 Electromagnet with yoke

mmf produced by the field winding is used in the air gap.

(b) The geometry of air gap is such that the fringing is also a minor correction and hence neglected.

Let us assume that the magnet is used to produce a uniform flux density B_g in an air-gap of cross-section A_g and radial length l_g. The basic models for the system with permanent magnets and electromagnets are then as shown in Fig. (4.1) and Fig. (4.2) respectively.

4.1 PERMANENT MAGNET

The volume of the magnet as derived before from equation (3.4) is

$$V_m \doteq \frac{B_g^2 V_g}{\mu_o \text{(BH) mag}} \qquad (4.1)$$

Thus the volume of the magnet will directly depend on the (BH) product of the material used. Magnet materials with maximum energy content of up to 6×10^6 gauss-oersted are available. However, in electrical machines because of armature reaction, there is always a demagnetizing force acting against the magnet, and the magnet operates on minor hysteresis loop as explained in the previous chapter. Consider the case of the magnet being stabilized under the condition when the rotor is absent (air-stabilized). This represents the condition of stabilization at point C in Fig. (3.13) while the magnet operates in the minor-loop (CA' B_r'). Under this condition, the maximum available energy (BH) for Alnico V is of the order of 1.5×10^6 gauss-oersted.[33] Assuming B_g to be 15000 Gauss, the volume of permanent magnet then must be $\frac{(15000)^2}{1.5 \times 10^6} = 150$ times the volume of the gap.

4.2 ELECTROMAGNET

Considering the case of electromagnets, we have the relation

$$\frac{4\pi NI}{10} = H_g l_g = \frac{B_g l_g}{\mu_o} = B_g l_g$$

(Where B_g is in Gauss and l_g in Centimeter)

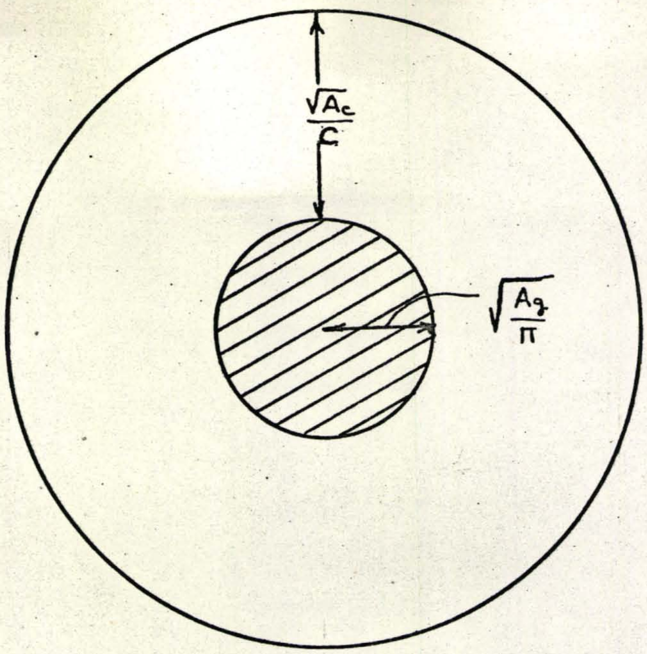
Thus, from above relationship we see that $(\frac{4\pi}{10})$ times the current circulating round the pole pieces equals $(B_g l_g)$ and assuming the current density to be J, the cross-section of copper

$$A_c = \frac{10 B_g l_g}{4\pi J} \quad (cm^2) \qquad (4.2)$$

By making J less than the net value in copper by the packing factor of the winding the cross-section A_c can be taken to represent the gross size of the winding cross-section.

Let us assume that the basic shape of coil section is square and departure from the shape is represented by C by which the length of the pole is multiplied and the radial width of the winding is divided. The pole length is then $C\sqrt{A_c}$ and if the area of core is the same as A_g then the core volume is $A_g C\sqrt{A_c}$.

Let us now assume a circular pole face as shown in Fig. (4.3). This is most efficient having least perimeter and least length of the winding, and for any other shape of the same area, the perimeter will be allowed to increase by a factor, say, f_1.



(A) Section PP - Pole Section Circular

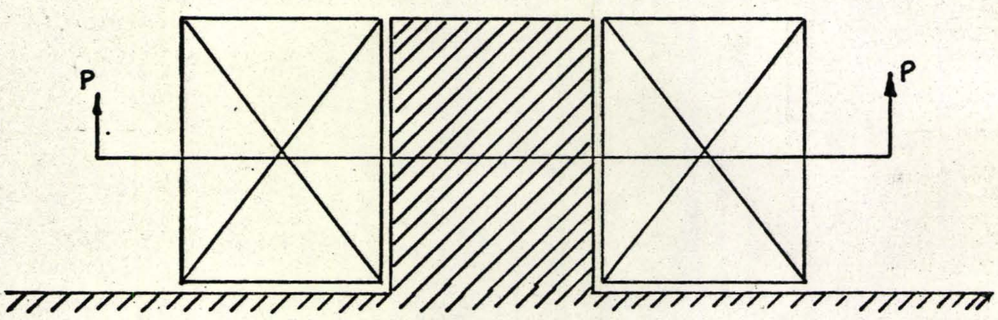
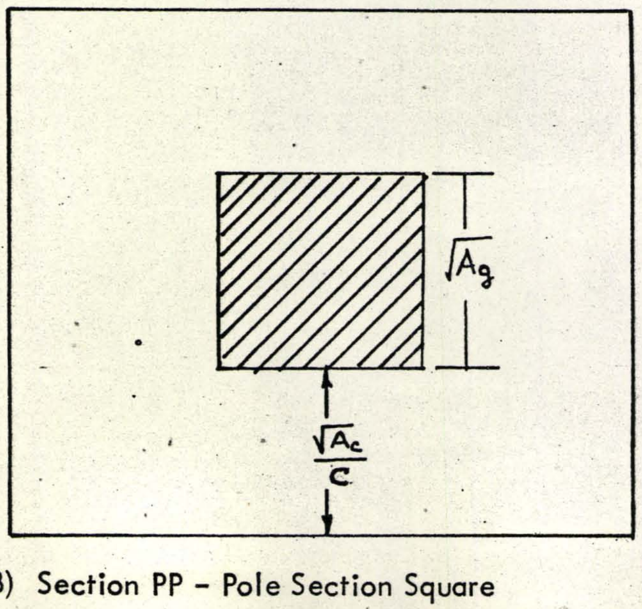


Fig. 4.3



(B) Section PP - Pole Section Square

e.g: ("f_1" for square-section = $\frac{4\sqrt{A_g}}{2\sqrt{\pi A_g}} = \frac{2}{\sqrt{\pi}} = 1.13$)

Now let S be the cross-section of copper in the plane PP as shown in Fig. (4.3A).

For Circular Section (See Fig. 4.3A)

$$S = \pi\left[\left(\sqrt{\frac{A_g}{\pi}} + \frac{\sqrt{A_c}}{C}\right)^2 - \left(\sqrt{\frac{A_g}{\pi}}\right)^2\right]$$

$$= \pi\left[\frac{A_c}{C^2} + \frac{2}{C}\sqrt{\frac{A_g A_c}{\pi}}\right] \tag{4.3}$$

For Square Section (See Fig. 4.3B)

$$S = 4\left[\left(\frac{\sqrt{A_c}}{C}\right)^2 + \frac{\sqrt{A_c A_g}}{C}\right]$$

$$= 4\left[\frac{A_c}{C^2} + \frac{\sqrt{A_c A_g}}{C}\right] \tag{4.4}$$

The volume of copper is then

$$V_c = SC\sqrt{A_c} \tag{4.5}$$

Taking the case of circular pole face by combining equation (4.3) and (4.5) the volume of copper

$$V_c = \left[\frac{\pi A_c C\sqrt{A_c}}{C^2} + \frac{\pi\, 2}{C}\sqrt{\frac{A_g A_c}{\pi}}\, C\sqrt{A_c}\right]$$

$$V_c = A_c\left[\frac{\pi\sqrt{A_c}}{C} + 2\sqrt{A_g \pi}\right] \tag{4.6}$$

Now from equation (4.2)

$$A_c = M I_g \quad \text{where } M = \frac{10\, H_g}{4\pi J}$$

Hence, substituting this value of A_c in equation (4.6)

$$V_c = \left[\frac{\pi}{C} M^{3/2} I_g^{3/2} + 2 M I_g \sqrt{A_g \pi}\right] \tag{4.7}$$

M is proportional to "magnetic loading" to "electrical loading" since H_g depends on the desired flux density in the gap and J is the effective current density in the winding. It is also seen that M has a dimension of length $\frac{\text{(amp-turns/cm)}}{\text{amp/cm}^2}$. Thus, the copper volume is expressible only in terms including l_g and A_g and so depends on gap shape as well as gap volume.

The total volume of copper and core

$$V_c + V_i = V_c + A_g C\sqrt{A_c} = V_c + A_g C\sqrt{M l_g} \tag{4.8}$$

$$V_g = A_g l_g \tag{4.9}$$

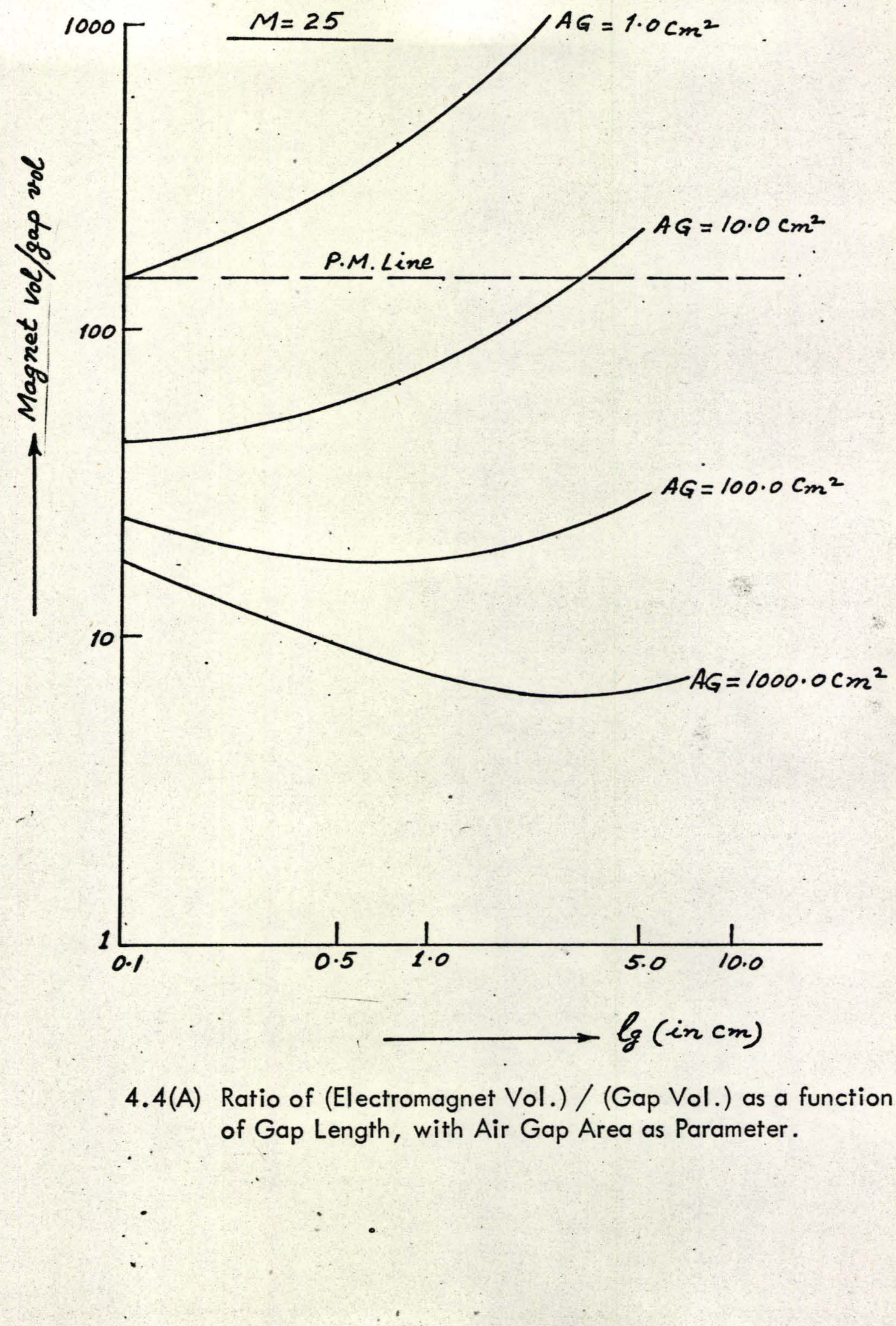
Hence

$$\frac{V_c + V_i}{V_g} = \left[\frac{\pi M^{3/2}}{C} \left(\frac{l_g^{1/2}}{A_g} \right) + 2M\sqrt{\frac{\pi}{A_g}} + C\sqrt{\frac{M}{l_g}} \right] \tag{4.10}$$

Thus, the dependance of the ratio of magnet volume to gap volume on gap length and gap area is a function of the magnetic to electric loading ratio M.

Hence, a plot of magnet volume/gap volume for different values of A_g with varying gap length can be calculated for different values of M. Typical range of values of M lies between 25 to 100, which corresponds to H_g between 10000 oersteds to 20000 oersteds and between 300 A/cm^2 to 200 A/cm^2.

Fig. (4.4(A) to Fig. (4.4(D) show the four set of curves of the ratio of magnetic volume to gap volume for electromagnet as a function of gap length in cm and gap area in cm^2, for M = 25, 50, 75 and 100. The dotted line marked PM shows this ratio of 150 for permanent magnet as calculated before.



4.4(A) Ratio of (Electromagnet Vol.) / (Gap Vol.) as a function of Gap Length, with Air Gap Area as Parameter.

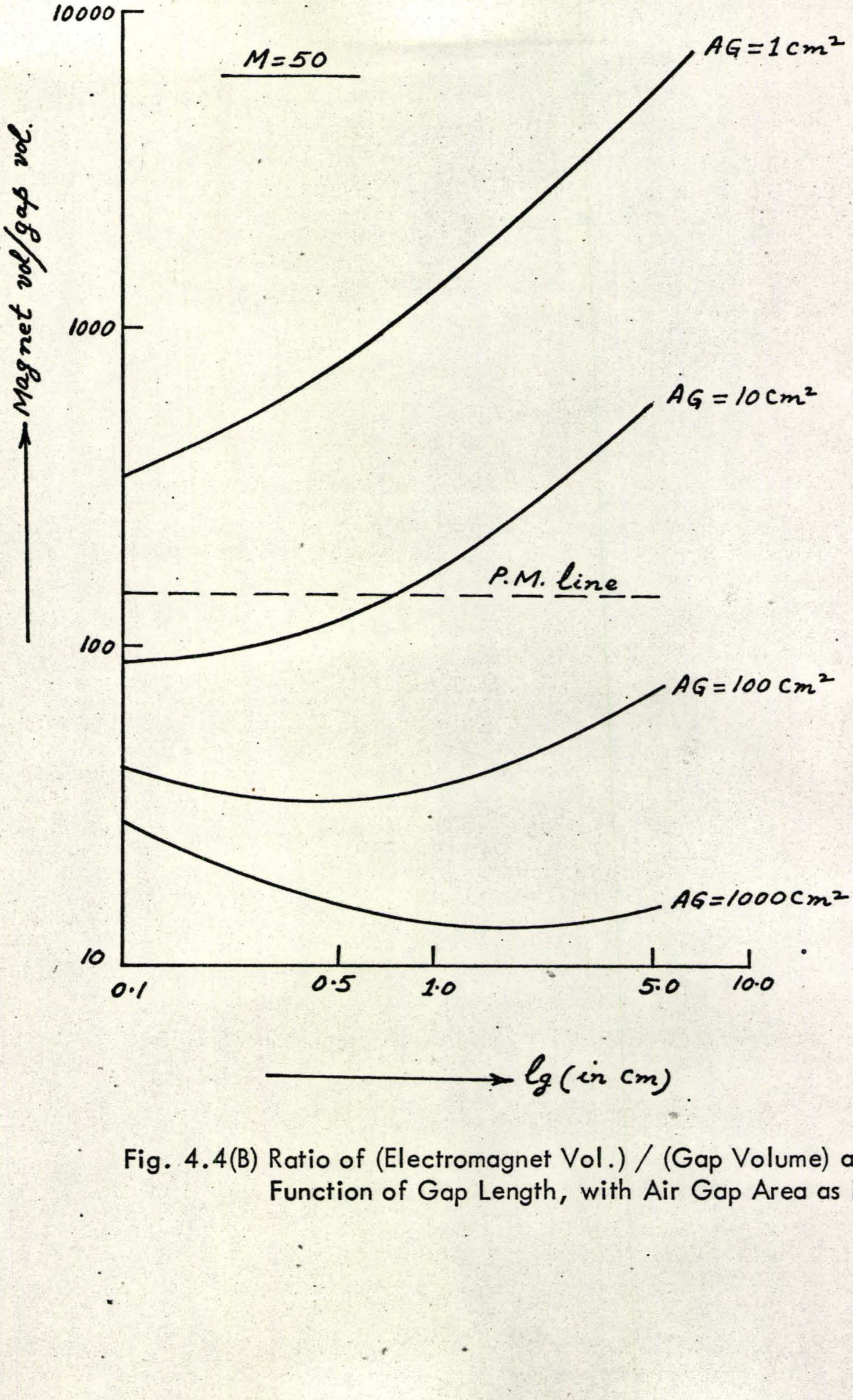


Fig. 4.4(B) Ratio of (Electromagnet Vol.) / (Gap Volume) as a Function of Gap Length, with Air Gap Area as Parameter.

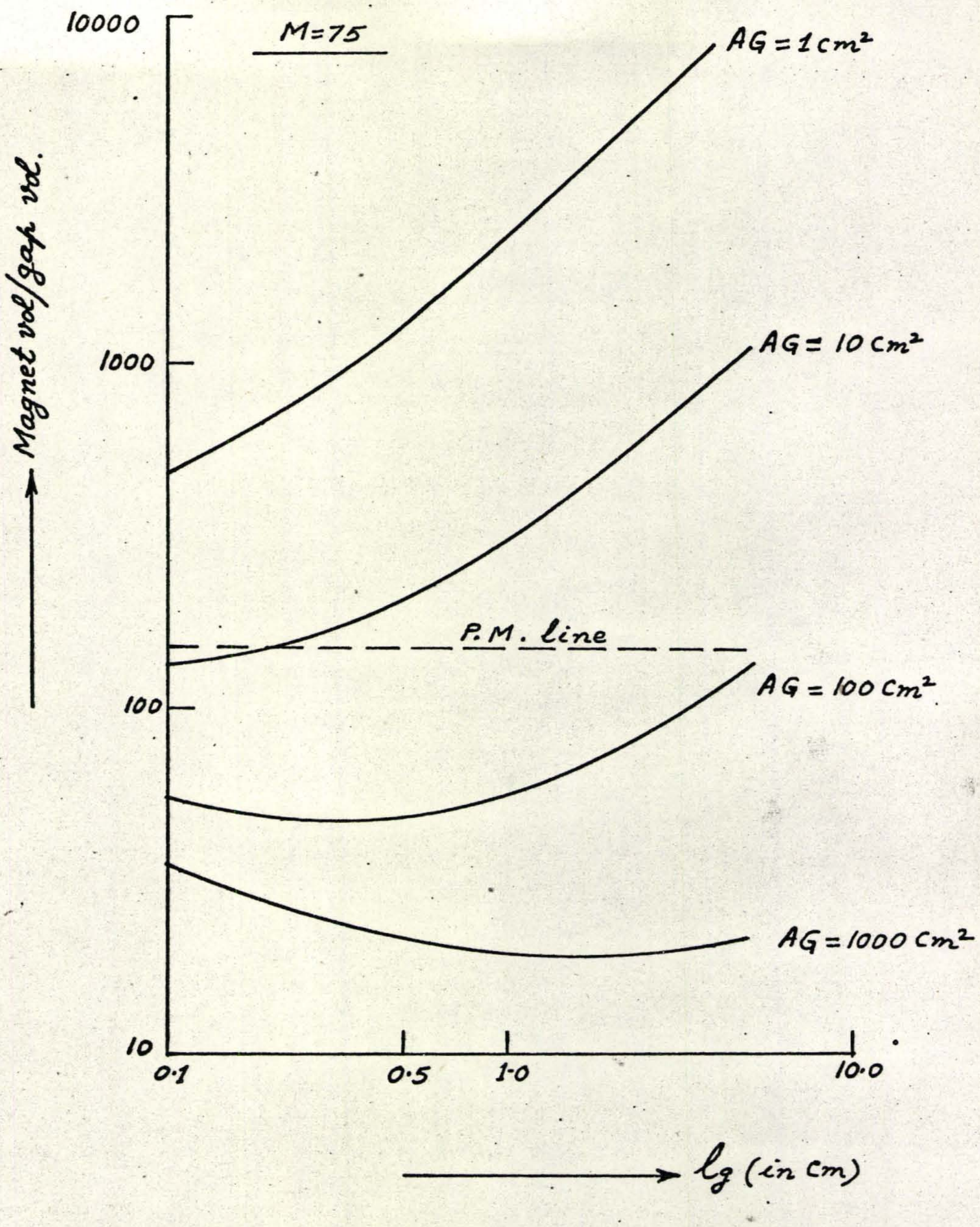


Fig. 4.4(C) Ratio of (Electromagnet Vol.) / (Gap Volume) as a Function of Gap Lenght, with Air Gap Area as Parameter.

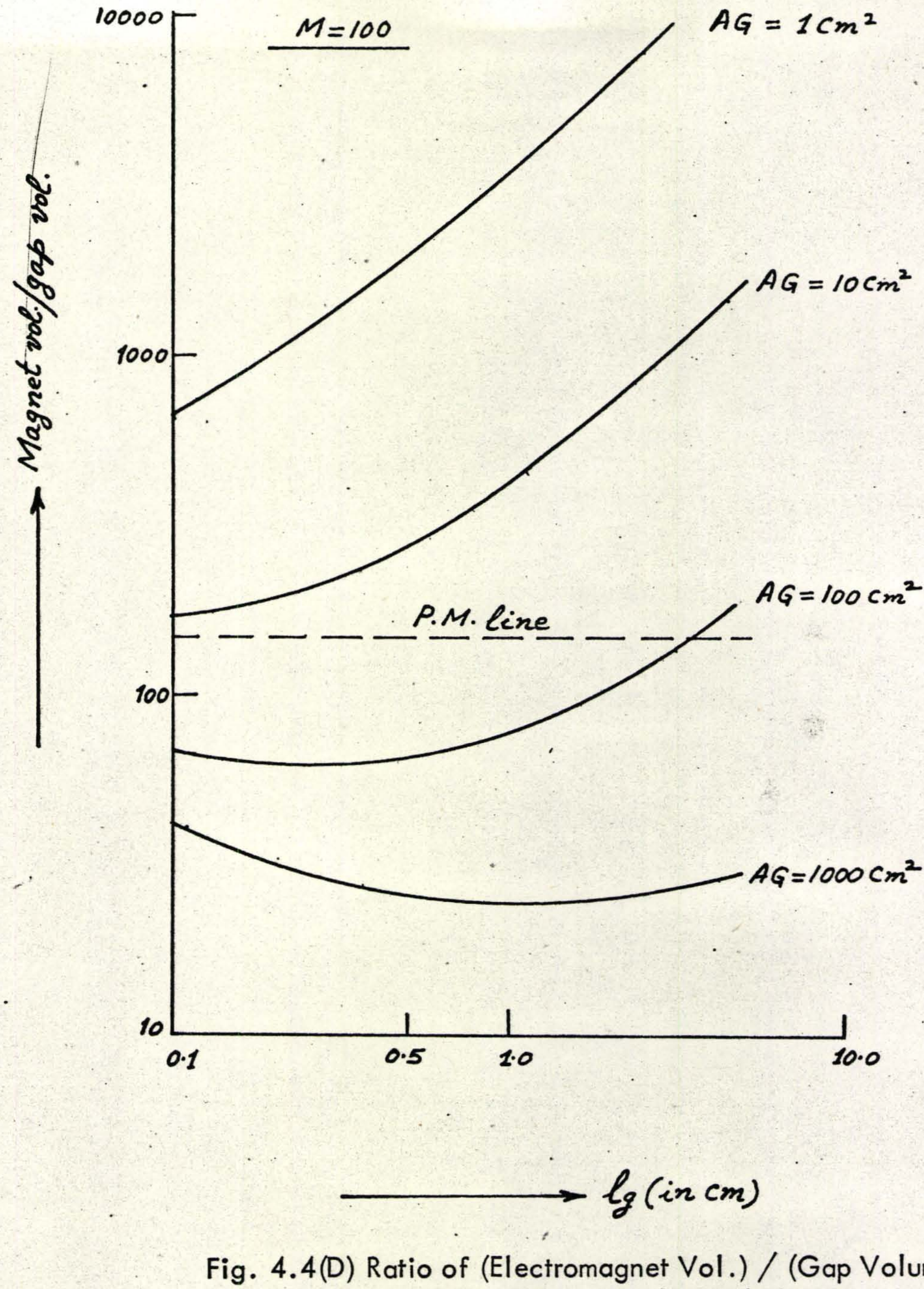


Fig. 4.4(D) Ratio of (Electromagnet Vol.) / (Gap Volume) as a Function of Gap Length, with Air Gap Area as Parameter.

From these curves it can be seen that the electromagnet occupies less volume than the permanent magnet whenever it is presented below the permanent magnet (dotted) line. However, the curves are only a rough guide since there are various shape coefficients and fringing factors to be taken into account in an actual design. These volumes are the net volumes and so are the rough guide to the weight, though copper is slightly heavier (density 9.8) than iron (density 7.8). C. Gould in his thesis (25) has shown that Power $\propto$ (Weight)$^{3/2}$.

Applying this analysis to electric machine, it is very difficult to say up to what power level the permanent magnet is economical to electromagnet for their costs depend on so many factors which make it almost impossible to come up with some concrete values. However, it is obvious that for smaller machines the permanent magnet would be preferable. According to some manufacturer's remark, a comparison of overall costs often show permanent magnet motors to be more economical than wound field motors up to 2" to 3" overall diameter. In case of generators the overall reduction in weight and cost is affected because of the elimination of the exciter. There are many other factors besides weight which make the permanent magnet machine more attractive than an electromagnet one. This will be dealt with in the next chapter while dealing with permanent magnet synchronous generators. However, for fractional horsepower, permanent magnet machines definitely tend to weigh less and cost less than the electromagnetic version.

CHAPTER V

PERMANENT MAGNET SYNCHRONOUS GENERATOR

THEORETICAL ANALYSIS

INTRODUCTION

The advent of anisotropic permanent magnet alloys has made possible considerable progress in the development of permanent magnet generators. The characterstics and physical properties of permanent magnet materials and the design criteria have been discussed in Chapter 3. In this chapter the magnetic characteristic of permanent magnet generators and methods of stabilization will be discussed. Equivalent magnetic circuit on a per-pole basis for no-load as well as load conditions will be presented. Based on the circuit analogy, relationship for excitation voltage, voltage regulation, short-circuit current and the demagnetization effect due to steady-state, short-circuit current, as derived by Hanrahan and Toffolo [21] has been presented. Demagnetization effect of short-circuit transient and consequently X_d' and X_d'' has not been considered for in designing the machine the point of stabilization is assumed to be given by the steady-state short-circuit current.

5.1 ADVANTAGES OF PERMANENT MAGNET GENERATOR

The armature of a permanent-magnet generator may be of any conventional type and winding but the field structure is entirely different since the necessary magnetic energy is obtained from the blocks of permenant magnet material in the field structure instead of from the coils energized by the exciter. As it has no field winding it requires no external excitation, no slip rings and no brushes and so forth. It has no arcing contacts to cause radio interference and is inherently as explosion-proof as squirrel cage induction motor.

1. Size and Weight

Permanent magnet generators inherently are smaller and lighter due mainly to the elimination of the exciter and slip rings. This is especially marked in very small high-speed units and in high-frequency multipole designs where the diameter is large compared to the axial length.

2. Cost

Permanent-magnet rotors are inherently expensive due to the high price of Alnico. Elimination of the exciter brings the cost of the smaller permanent magnet generators below that of a conventional unit complete with exciter. Precise data as to the cost comparison of permanent magnet rotors to the electromagnet types are not readily available. According to some manufacturer's remark, a comparison of overall costs, however, show permanent magnet machines of overall diameter of 2" to 3" to be more economical. However, as the permanent magnet generator finds its application greatly in missile and aircraft industires, it is usually the weight which is the deciding factor instead of the cost.

3. Maintenance

Maintenance is low as compared to electromagnet ones. Rotor is practically indestructible with a solid mass of metal with no insulation material. Modern permanent magnets are also pretty stable against vibration and temperature. (No apparent change in magnetization takes place below 500°C for Alnico metals).

4. Rating - Heating

Correctly designed permanent magnet generators have almost no heat generated in the rotor and usually have less than conventional stator losses. For this reason, the maximum capacity of a given unit is usually determined by inherent voltage regulation limitations rather than by heating. Very small high speed units may be operated completely sealed without exceeding class A temperature. However, for larger generators operating on high power-factor loads heating may become the limiting factor.

5. Efficiency

Due to elimination of all excitation losses it has a higher over-all efficiency as compared to its counterpart (electromagnet machine). Efficiency of permanent magnet generators ranges from 75% for a 0.1 KVa, 1200 rpm generator with high windage and friction losses up to approximately 93% for the 1800 rpm units rated at 10 KVa or larger.

6. Voltage Regulation

The inherent voltage regulation of a permanent magnet generator is somewhat similar to that of a highly saturated electromagnet machine (with full saturation maintained at all speeds.) In other words it varies from fairly close regu-

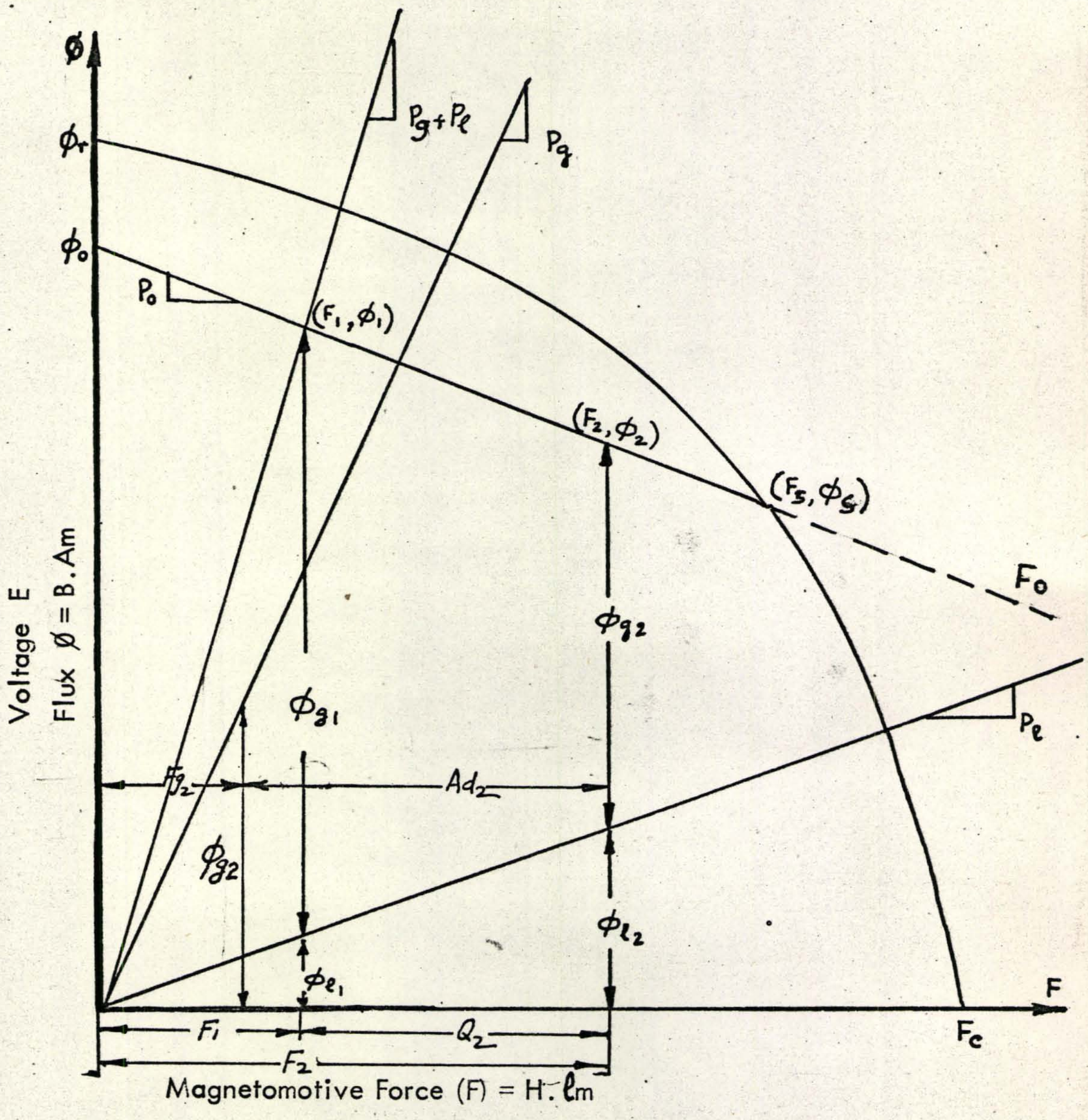


Fig. 5.1 Magnetic Characteristic of a Permanent Magnet Alternator

lation for unity power factor loads to poor regulation for highly lagging power factor loads. And this is because the mmf supplied by the magnet increases with the load as will be explained later while dealing with the operation of permanent magnet generators.

7. Improved Voltage Control

The actual voltage regulations of permanent-magnet generator obviously appears to be impossible except for such complicated methods as mechanically moving the rotor axially in relation to the stator and thus varying the flux interlinkages.

A simple method of obtaining a small range (up to 10%) of effective and accurate control of the generated voltage is by means of a special control winding on the stator called "Toroidal Winding". However, the subject of voltage control of permanent magnet generators is a full development project in itself and is beyond the scope of this work to deal with.

The advantages of permanent magnet generators mentioned before, make them specially attractive for mobile military applications e.g., air craft, missiles and engine generator sets for shipboard. To be competitive, permanent magnet generator together with voltage regulator (if used) must compare favourably in weight, total cost, installation and operation with conventional machines.

5.2 EQUIVALENT MAGNETIC CIRCUIT

1. Magnetic Characteristic of Permanent Magnet Generator

Fig. (5.1) represents the typical demagnetization curve with flux and magnetmotive force (ϕ_r to F_c) and represents the second quadrant in the major hysteresis loop with the abscissa reversed. This is related to the B-H curve of the material by the magnet dimensions. ϕ_r and F_c are values per pole. If the magnet is saturated and then demagnetized either by separating the rotor or by electrically short-circuiting the generator, the point on the demagnetization curve moves down to some point (F_s, ϕ_s). If now the short-circuit is removed the hysteresis loop will not trace its path along the major loop rather along the minor loop joining ϕ_s and ϕ_o where the minor loop is for all practical purposes a straight line the slope of which is equal to the slope of major hysteresis loop at point ϕ_r. So long as the demagnetization does not increase beyond the point (F_s, ϕ_s), the magnet will always operate along this line and the flux level is somewhere in the band ϕ_s to ϕ_o depending on the generator load. The line of return gives the magnet 'terminal' flux and mmf for any condition of stabilized operation, hence it represents the volt-ampere characterstic of the permanent magnet generator. ϕ_o is the magnetic short-circuit flux and the open-circuit potential (F_o) is given by the intersection of this line with F - axis, although operation outside the major loop is impossible.

Point (F_1, ϕ_1) is the no-load operating point. When the load is applied the operating point shifts to (F_2, ϕ_2) as given by the demagnetization caused by the armature-reaction due to load. Thus, it is seen from

Fig. (5.1) that the magnetomative force supplied by the rotor increases as the demagnetizing load is applied, and hence the permanent magnet actually acts as a self-regulating source of excitation. The regulation is naturally not perfect due to the slope of the minor hysteresis loop and the armature impedance. Thus, as compared to separately excited electromagnetic generator, this machine has a better inherent voltage regulation. Where the permanent magnet machine inherently cannot meet regulation limits the "Toroidal Winding" method of control may be used.

2. Permanent Magnet Stabilization

While designing a permanent magnet as used in electromagnetic machinery, it is common practice to expose the permanent magnet to the maximum demagnetization that the magnet will encounter. The magnet is then said to be stabilized. There are three methods of stabilization in common use.

(a) Air Stabilization

(b) Short-Circuit Stabilization, and

(c) Load Stabilization.

Since the methods of stabilization can have considerable influence on the volume of permanent magnet required, each method and operation of machine under this condition is described here briefly. Fig. (5.1) represents the magnetic characteristic of the machine and can be drawn only after the magnet has been dimensioned. For design analysis we must, therefore, return to the B-H curve and the unit permeances. This is related to the $\emptyset - I$ curve by the magnet dimension. Hence, the relationship of S_t to P_t is $S_t = P_t \left(\frac{l_m}{A_m} \right)$ as explained in Chapter 3. Also

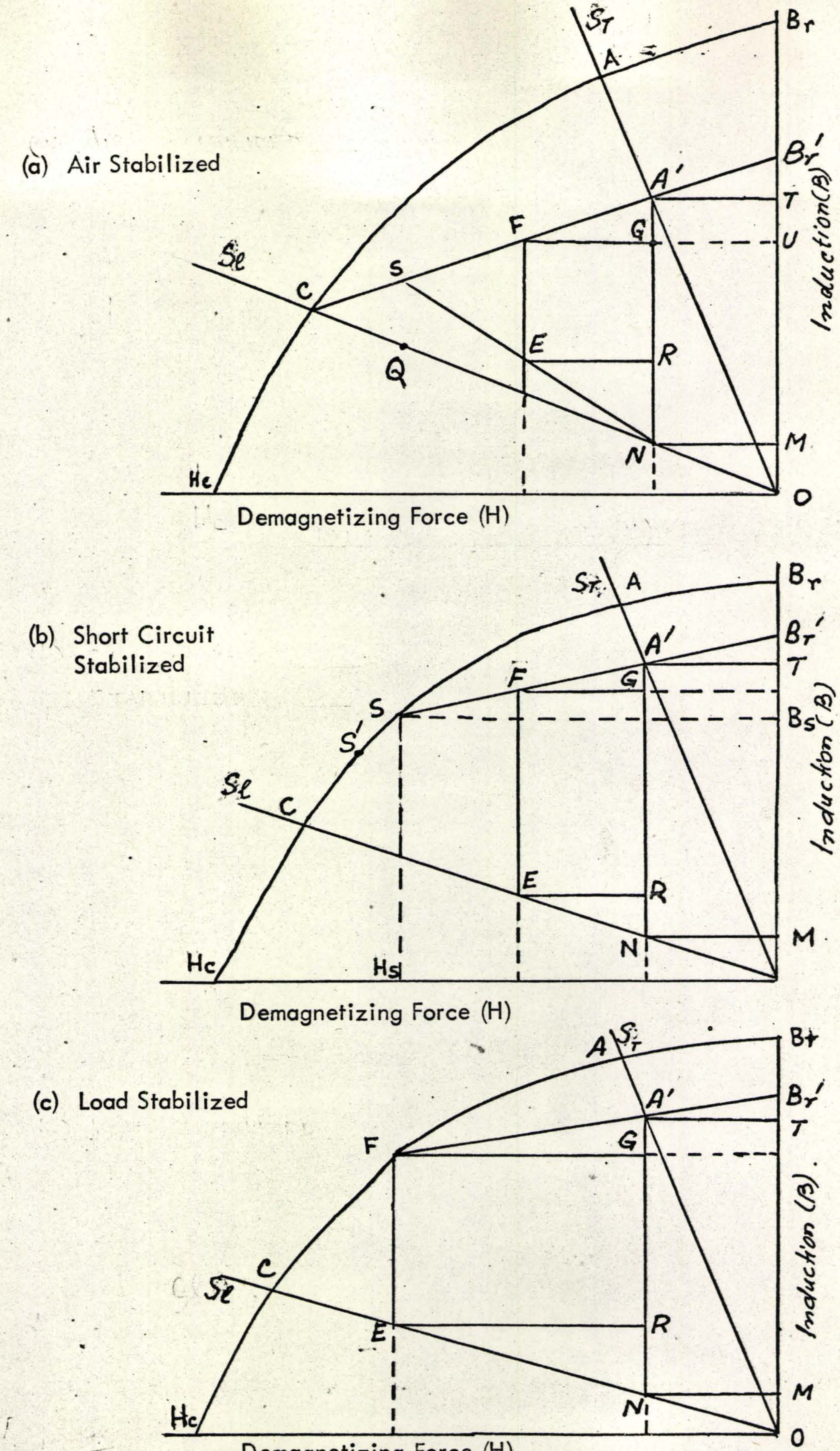


Fig. 5.2 Stabilizing Practices in Permanent Magnet Generators

the slope of the minor loop is related to the slope of $F_o \phi_o$ by the same conversion.

(a) Air Stabilization

Considering Fig. (5.2(a), OC represents the unit permeance open circuit load line or the unit leakage permeance line when the rotor is out of stator and OA represents the total assembly unit permeance (gap plus leakage). After magnetization and exposure to the air path, point C represents the point of stabilization and the rotor is said to be air stabilized. When the rotor is now inserted into the stator the permanent magnet will recover along the recoil line CB_r' and under no-load conditions the generator will operate at point A' in terms of unit magnetic properties. Loading the machine cause the operating point S where S represents the short-circuit condition. SQ then becomes the flux required to drive the short-circuit current through the armature. CS depends on armature impedance drop. Line SN then becomes the equivalent open-circuit leakage line of the magnet. The flux density below SN represents that required for internal armature voltage drop and that above SN represents the flux density available for generating useful output voltage. If F represents the load point, then FE will be proportional to generated voltage and FG proportional to load current. Consequently, area FGRE represents electrical energy output. For this to be maximum FE = $\frac{1}{2}$ A' N and E lies at mid-point of SN. Physically this can be interpretted as saying that maximum load energy occurs when the load current is half of the value of short-circuit current, and at this condition the load voltage becomes half of the open-circuit voltage.

(b) Short-Circuit Stabilization

Fig. (5.2(b) illustrates the case of a generator stabilized by short-circuit. Here again OC and OA represent the unit permeance load line for leakage and gap plus leakage permeance. In this case, the rotor is magnetized in the stator and no-load operation would be at point A. As the machine is short-circuited armature reaction moves the operating point down to S. Upon removal of short-circuit the magnet would recover along the recoil line through S to A' which would now represent the stable no-load operating condition. Area FGRE now represent the electrical energy output.

In this analysis the point S must be the condition resulting from steady state short-circuit. However, in alternators the transients associated with short-circuit momentarily produce a field which can cause further demagnetization on the magnet. To reduce the effect of transients a damper winding can be added to the rotor. A single low resistance turns around each pole is interconnected between poles. In the short-circuit stabilized machine, point S in Fig. (5.2(b) must be the worst condition the magnet will use. If conditions are such as to have transient adding to state , then S must be located in from the major loop by the amount the transient exceeds the steady state short-circuit demagnetizing magnetomotive force, this is given by point S'. In practice a carefully designed damper can limit the transient magnetomotive force so that combined effect only exceeds steady state demagnetization by small percentage and then point S may be located only due to steady state short-circuit current.

(c) Load Stabilization

It is similar to the short-circuit case except that the stabilized point F in Fig. (5.2(c) is reached by applying only the maximum load current value instead of short-circuit current. In load stabilization a generator will for a given weight produce more output than any other method of stabilization and consequently load stabilized generators find applications wher e emphasis is placed on eliminating weight and where the consequences of short circuits can be tolerated. Short-circuit stabilization is used for missile and air-craft generators while for small generators where one wants to avoid the necessity of remagnetization, air-stabilization is more commonly used. However, as compared to a generator stabilized by short-circuit the volume of permanent magnet and consequently the weight and size of the air stabilized machine will be of the order of 25 to 50 percent greater.

In permanent magnet generators it should be appreciated that the volume of the magnet required will be the sum of two requirements: (1) The magnetic energy necessary to combat armature reaction and (2) the amount of energy stored in the air gap of the machine. In practice invariably more energy is stored in the gap than is used to balance armature reaction. For example in Fig. (5.2(a) the area FGRE is proportional to electrical energy output and A' TMN represents the energy stored in the air gap. By far, the greatest portion of the magnet volume is unavoidably necessitated by the presence of the air gap.

3. Magnetic Circuit

Because the line of return in Fig. (5.1) is straight the stabilized magnet may be represented by a constant - mmf (F_o) source in series with a constant reluctance (R_o, the negative reciprocal of the slope of the line) or alternatively as a constant - flux (ϕ_o) source shunted by the same reluctance. On these equivalent source representation of the stabilized magnet, the equivalent magnetic circuit on per pole basis when there is no load on the generator is given in Fig. (5.3). Fig. (5.3(A) shows a developed view of the machine where the air-gaps are such that the shaded portion of the circuit represents the soft iron parts of negligible reluctance. Considering just one pole the magnetic circuit can be given as in Fig. (5.3(B), here the neutral planes are represented by a magnetic ground common to stator yoke and rotor core. The electric analogies of Fig.(5.3(B) are given in Fig. (5.3(C), (D) and (E). Fig. (5.3(D) uses the potential-source representation while Fig. (5.3(E) uses the flux-source representation.

R_l in the circuit is the reluctance of the equivalent single-leakage path and R_g is the reluctance of the equivalent air-gap. Also, since soft-iron saturation is neglected and R_o is constant the circuits are linear. Permeances shown in Fig. (5.3(E) are the reciprocal of reluctances and are the slope of the lines through the origin except for P_o which is the negative of the slope of line of return.

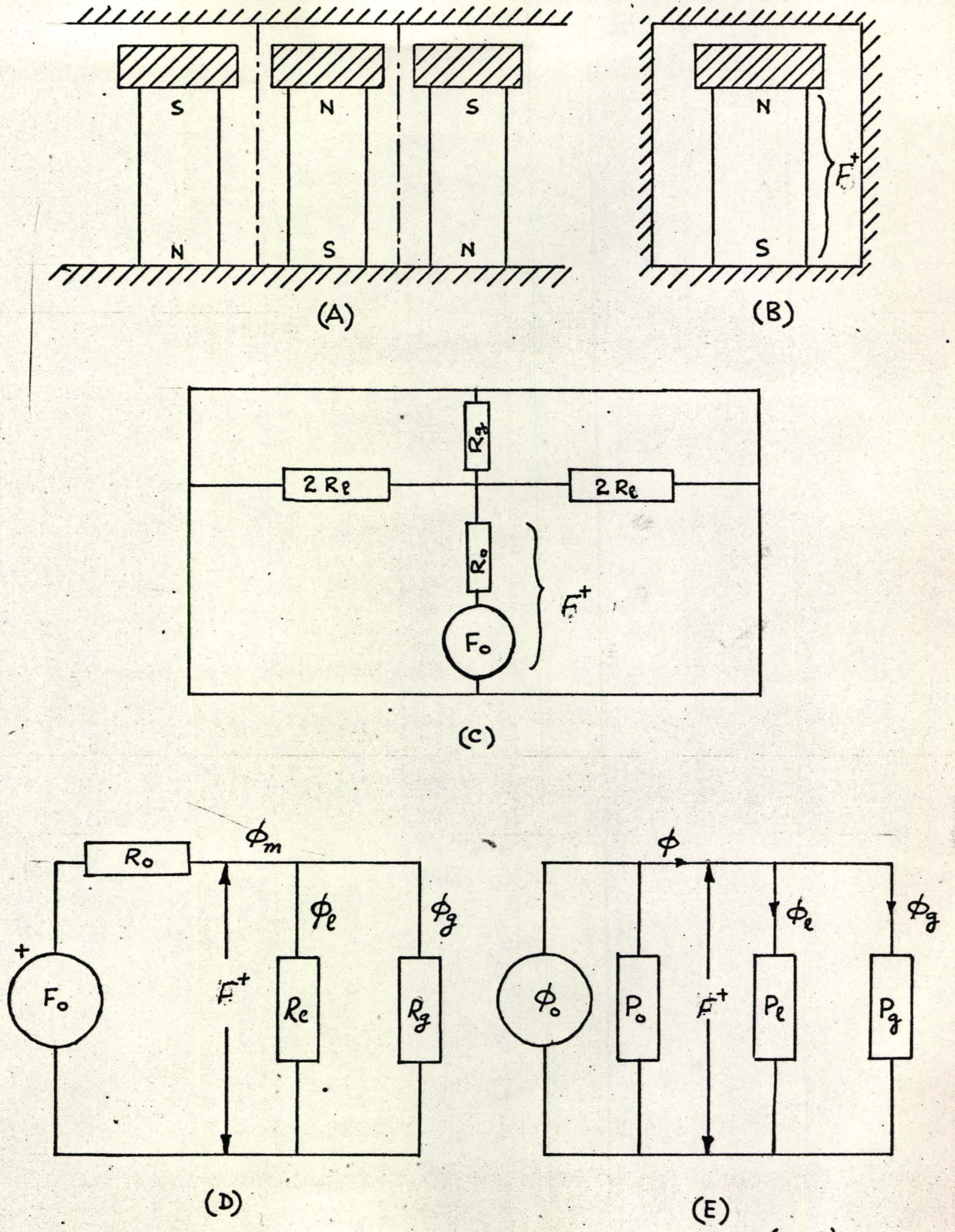


Fig. 5.3 Equivalent Magnetic Circuits on Per-Pole Basis (without load)

4. No-Load Operation

With no-load on the generator the point of operation is given by the intersection of $(P_g + P_l)$ line with the line of return as indicated here as $(F_1, \emptyset_1)$.

The flux $\emptyset 1 = \emptyset_{l_1} + \emptyset_{g_1}$

The generated voltage

$$ENL = K_g \emptyset_g$$

Using the flux-source equivalent circuit Fig. (5.3(E)

$$ENL = K_g \left(\frac{P_g}{P_t} \emptyset_o \right)$$

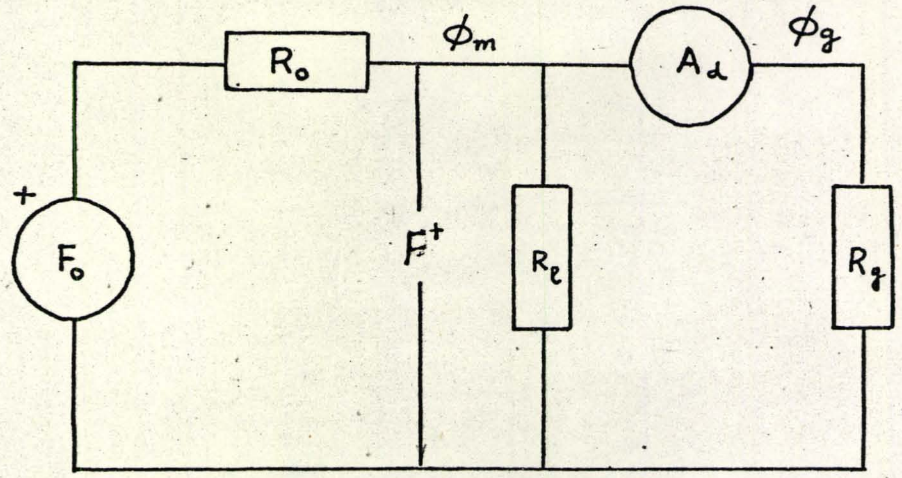
While using the potential-source equivalence

$$\emptyset_m = \frac{F_o \ (R_l + R_g)}{R_o R_l + R_o R_g + R_l R_g} = \frac{F_o}{\Delta} (R_l + R_g)$$

Which gives

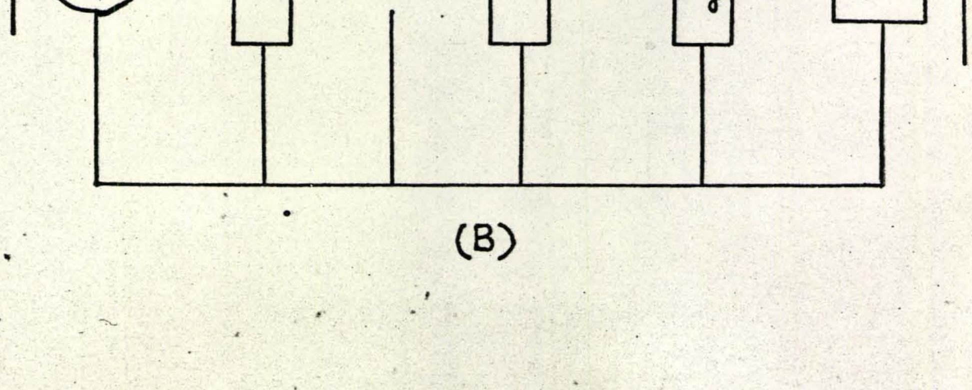
$$\emptyset_g = \frac{F_o}{\Delta} R_l$$

Hence $$ENL = K_g \frac{P_g}{P_t} \emptyset_o = K_g \frac{F_o}{\Delta} R_l \qquad (5.1)$$



(A)

Fig. 5.4 Equivalent Magnetic Circuit for a Demagnetizing Load



(B)

5.3 STEDY-STATE OPERATION

I. Operating Point due to Demagnetizing Load

When the generator is operating on load the armature current gives rise to an mmf which can be either in a magnetizing or demagnetizing direction depending on load power factor. As in most cases when the generator load is inductive, the armature reaction always has a demagnetizing effect.

The equivalent magnetic circuits for a demagnetizing load can be represented as in Fig. (5.4(A) and (B). With the help of these circuits and generator characteristic the operating point for a given armature demagnetizing mmf can be found as follows:

Let the operating point because of the load be at $(F_2, \emptyset_2)$. The leakage flux at this point is $\emptyset_{l_2}$ and the air-gap flux $\emptyset_{g_2} = \emptyset_2 - \emptyset_{l_2}$. This value of air-gap flux requires a certain mmf drop across the gap $= F_{g_2} = R_g \emptyset_{g_2}$.

Substracting this value from F_2 will give the demagnetizing mmf (A_{d_2}) causing operation at $(F_2, \emptyset_2)$ i.e. $A_{d_2} = F_2 - F_{g_2}$.

Hence to find the operating point for a given (A_d), let us define Q such that

$$F = F_g + A_d = F_1 + Q \qquad (5.2)$$

then from Fig. (5.4(B),

$$(P_o + P_l + P_g) F = \emptyset_o + P_g A_d$$

$$P_t F = \emptyset_o + P_g A_d$$

$$P_t Q = P_t F - P_t F_1$$

$$= \emptyset_o + P_g A_d - \emptyset_o$$

Therefore $$Q = \frac{P_g}{P_t} A_d \qquad (5.3)$$

Thus the point of operation (F_2, $\emptyset_2$) is found simply be adding Q_2 to the no-load mmf F_1.

2. Excitation of Permanent Magnet Generators

In the circuit analogy the excitation of the generator is represented by F, in other words, F would be the effective ampere turns of the field winding in case of an electromagnet machine. In an electromagnet machine the excitation F is constant for a particular field current and the machine exhibits a large voltage regulation. But in the case of a permanent magnet generator the demagnetizing mmf is added to a permanent magnet i.e. the mmf increases with increase in load. Hence, the permanent magnet generator acts as a self-regulating machine. The regulation is, of course, imperfect because of armature impedance, the slope of the line of return and slope of leakage permeance. However, inherent regulation of permanent magnet generators is much less than that of an electromagnet machine.

3. Machine Reactances

Fig. (5.5) shows the Blondel diagram with resistance neglected. In the case of wound-field machine, the excitation voltage E_o is the same as ENL for the same excitation. However, as explained before, for permanent magnet generator E_o is variable. For operation at (F_2, $\emptyset_2$), E_o will be given by the ordinate to the P_g line at F_2.

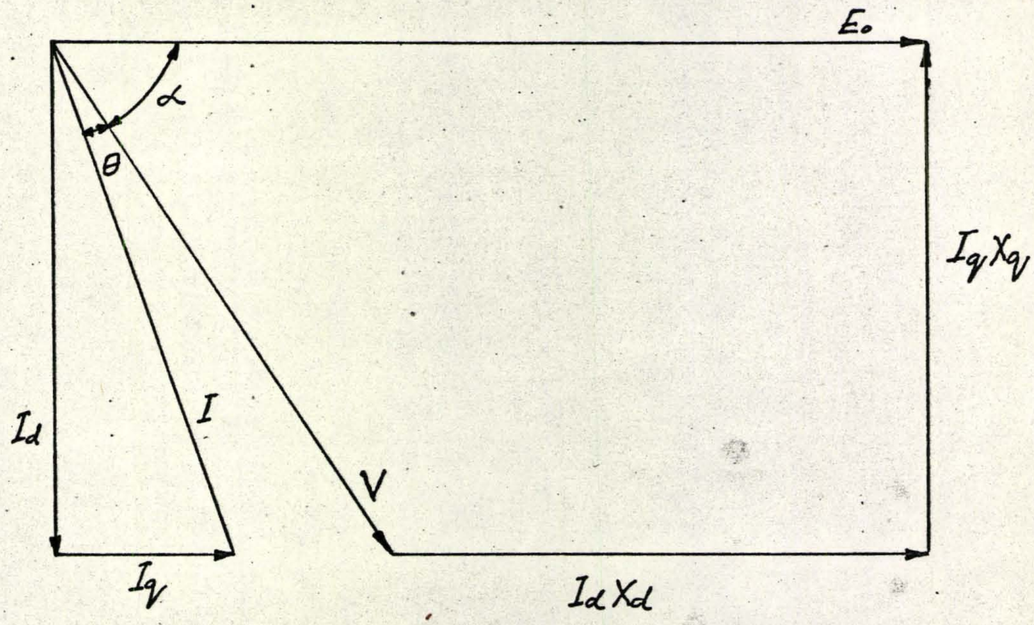


Fig. 5.5 Blondel Diagram (Armature Resistance Neglected)

$$E_o = K_g P_g F$$

Hence from equation (5.2) and (5.3)

$$E_o = K_g P_g \left(F_1 + \frac{P_g}{P_t} A_d\right)$$

$$= ENL + K_g \frac{P_g^{\,2}}{P_t} A_d \qquad (5.4)$$

Thus, excitation increases with the load, i.e. E_o is a function of load.

This peculiarity of permanent magnet generator is taken into account by defining the reactance of armature reaction as that of an electromagnet machine and E_o is the true excitation voltage, the no-load voltage given by equation (5.4).

The other steady-state reactances are defined and determined as for the wound-field machine.

4. <u>Voltage Regulation</u>

From Blondel diagram

$$\sin \alpha = \frac{\cos \theta}{\sqrt{1 + \frac{2Z}{X_q} \sin \theta + \frac{Z^2}{X_q^{\,2}}}}, \text{ and}$$

$$\text{Cos}\,\alpha = \frac{\text{Sin}\,\Theta + \dfrac{Z}{X_q}}{\sqrt{1 + \dfrac{2Z}{X_q}\,\text{Sin}\,\Theta + \dfrac{Z^2}{X_q^{\,2}}}}$$

The excitation voltage

$$E_o = V\cos\alpha + I\,\text{Sin}\,(\alpha + \Theta)\,X_d$$

Substituting the value of Sinα and Cosα as obtained and using equation (5.4), the terminal voltage as derived in Appendix (B1) is shown to be

$$V = \frac{ENL\,P_t\,Z\sqrt{Z^2 + 2ZX_q\,\text{Sin}\,\Theta + X_q^{\,2}}}{[Z^2 + Z(X_d + X_q)\,\text{Sin}\,\Theta + X_dX_q]\,P_t - K_gK_dP_g^{\,2}X_q\left(1 + \dfrac{Z}{X_q}\text{Sin}\,\Theta\right)} \qquad (5.6)$$

5. <u>Short-circuit Current</u>

Substituting ZI for V in equation (5.6) and then setting Z = O

$$I_{sc} = \frac{P_t\ ENL\,X_q}{X_d\,X_q\,P_t - K_gK_dX_qP_g^{\,2}}$$

$$= \frac{ENL}{X_d - K_gK_d\,\dfrac{P_g^{\,2}}{P_t}} \qquad (5.7)$$

6. Direct - Axis Reactance of Armature Reaction (X_{ad})

The direct-axis reactance of armature reaction is derived as follows:

$$I_{sc} = \frac{(Eg)_{sc}}{X_l} = \frac{K_g(\varnothing_g)_{sc}}{X_l}$$

From Fig. (5.3(D)

$$\varnothing_g \Delta = R_l F_o$$

and

$$\varnothing_l \Delta = R_g F_o$$

And from the equivalent circuit of Fig. (5.4) the flux under load

$$\varnothing_g \Delta = R_l F_o - (R_o + R_l) A_d$$

Under short-circuit condition

$$(\varnothing_g)_{sc} = \frac{R_l F_o - (R_o + R_l) A_{sc}}{\Delta}$$

Therefore

$$I_{sc} = \frac{K_g [R_l F_o - (R_o + R_l) A_{sc}]}{X_l \Delta} \qquad (5.8)$$

Substituting $A_{sc} = K_d I_{sc}$ in equation (5.8) and simplifying

$$I_{sc} = \frac{ENL \ \Delta}{X_l \Delta + K_g K_d (R_o + R_l)} \qquad (5.9)$$

Thus, combining equation (5.7) and (5.9) gives

$$X_d - K_g K_d \frac{P_g^2}{P_t} = X_l + K_g K_d \left(\frac{R_o + R_l}{\Delta} \right)$$

Simplifying the above equation gives

$$X_d = X_l + K_g K_d P_g$$

Hence

$$X_{ad} = K_g K_d P_g \qquad (5.10)$$

Based on this theoretical analysis the method of designing a permanent magnet generator is considered in the following chapter.

CHAPTER VI

PERMANENT MAGNET GENERATOR

OPTIMUM DESIGN

6.1 INTRODUCTION

Progress reported in the literatures on the design of permanent magnet generators has not been satisfactory. Papers (12), (13), (14), (15), (16), (17), and (18) on permanent magnet design indicate that the procedure of wound-field machines has been followed i.e. an armature is chosen and then the rotor is calculated accordingly. This method could perhaps be satisfactorily employed for the design of air-stabilized machine. But as mentioned in Chapter (5.2.2) this is accompanied by the weight penalty and is consequently not economical because of the high cost of permanent magnet materials. However, the permanent magnet generator finds it application mostly in mobile military units such as air-craft and missile generators. Minimum weight for such applications dictates the necessity of short-circuit or even full-load stabilization. Short-circuit stabilization offers the advantage of retaining the magnetic characterstic of the permanent magnet unchanged in case of short-circuit and hence does not need to be remagnetized. This advantage over load-stabilization is associated with some weight penalty on the machine. However, air-craft and missile generators are preferably designed for short-circuit stabilization.

This requires solution of simultaneous equations relating machine parameters and the conditions obtained from the geometry of the curve. Because there are more variables than relations to be satisfied, the design may be optimized. Here the design is attempted to find the minimum length of the machine for a given diameter and this gives the minimum D^2l of the machine for that diameter. Thus, by repetitive calculations for different diameters the most optimum machine having minimum D^2l can be calculated. This criteria of minimum D^2l for a given KVa in case of generators gives naturally the best machine as far as weight is concerned.

Hanrahan and Toffolo (22) first suggested such an approach, although they made no attempt to give a practical design of a machine. The author has tried to give an actual design procedure which makes the calculation of an optimum machine possible. Simplifying assumptions have been made which can be easily modified according to the actual design problem and yet the same design relationship can be utilized to achieve the designer's goal.

6.2 DESIGN EQUATIONS

All the design equations are derived in m.k.s. units. The magnetic quantities are on per pole basis and the electric quantities on per phase basis.

I. Machine Specifications

Typical specifications for a 3-phase a-c generator are: Kilovolt - ampere (KVa), frequency, speed, voltage and power factor. Thus, the number of poles are known. Inherent voltage regulation or short-circuit current may also be specified. However, a relation between short-circuit current and voltage regulation does exist.

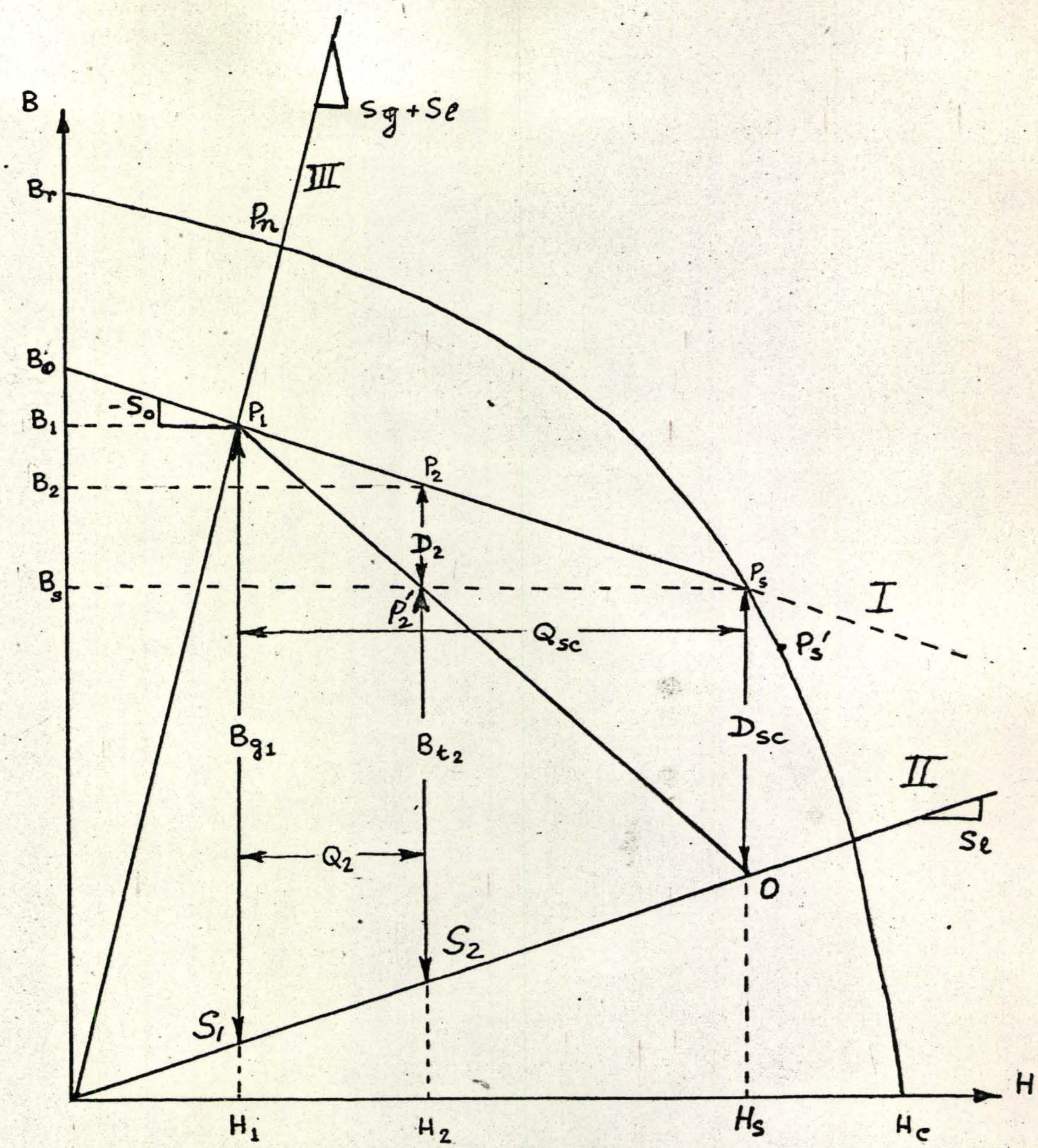


Fig. 6.1 The B-H Plane

2. THE B-H PLANE

The B-H curve of the magnet material is shown in Fig. (6.1), Line I, II and III determine the magnet operation. Suppose the magnet is stabilized by bringing the generator up to rated speed under short-circuit. The operating point moves down to point P_s as explained in paragraph 5.2. Slope S_l, S_g and S_o are the unit permeances of the machine corresponding to P_l, P_g and P_o. The normal operation would occur now along line I, and the machine is said to be stabilized. For no-load on the generator the condition is given by the intersection of line I and III at point P_1. Point P_2 represent the operating point under full-load conditions.

A simple geometry of the machine considered is shown in Fig. (6.2), where b, h and l are the magnet dimensions, and l_g is the radial air-gap length of the machine. Permeances P_g and P_l can now be transformed to the B-H plane as follows:

Air-Gap Permeance Transformed to B-H Plane

From Fig. (6.2)

$$P_g = \mu_o \frac{l\,b'}{l_g}$$

Where b' is the effective pole width of the machine. This can be written as

$$b' = b\,k_f$$

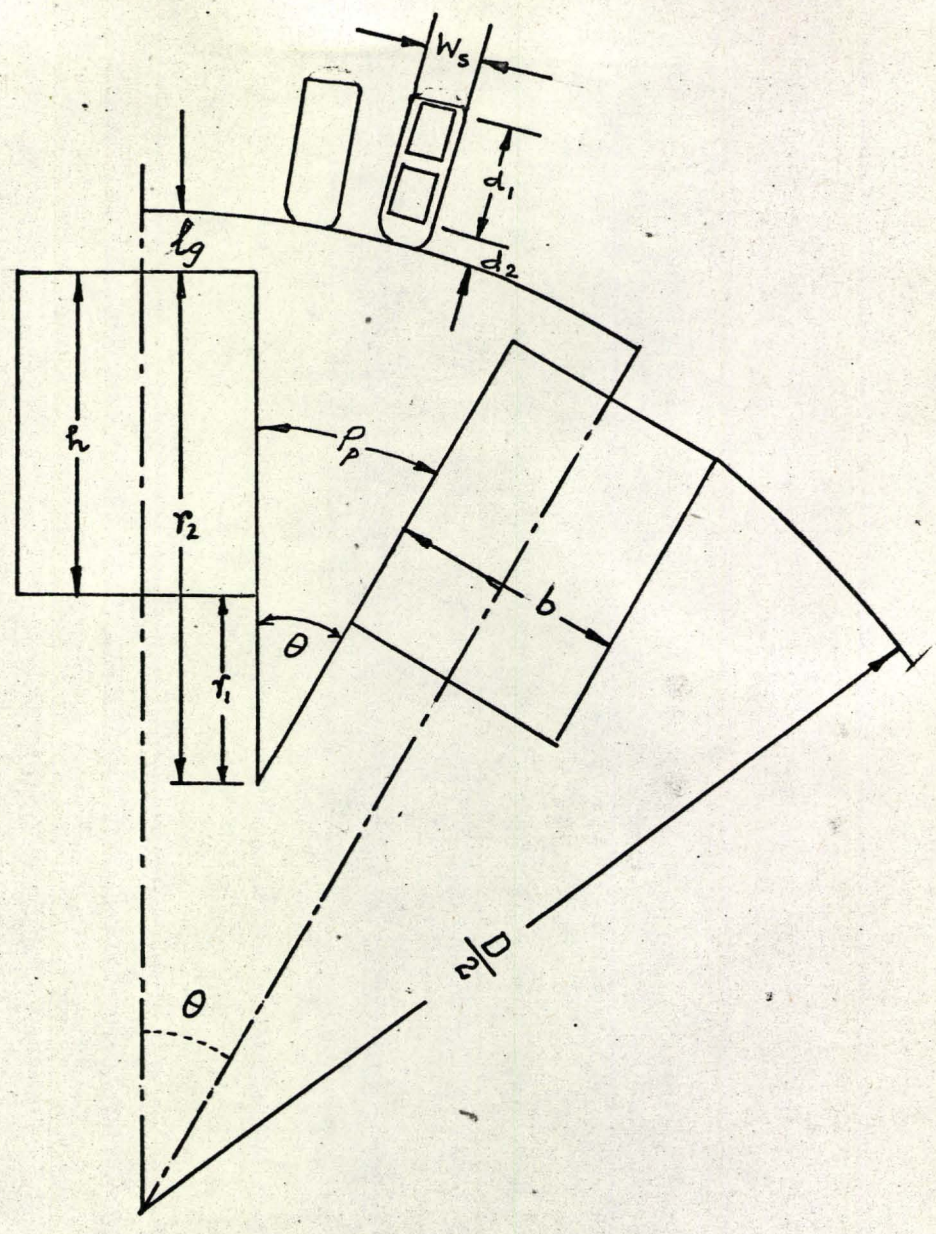


Fig. 6.2 Machine Geometry

Therefore

$$P_g = \mu_o \frac{lbk_f}{l_g}$$

and

$$S_g = \frac{h}{bl} P_g = \mu_o k_f \frac{h}{l_g} \qquad (6.1)$$

Pole-Leakage Permeance Transformed to B-H Plane

There are a number of paths contributing to P_l. Calculation of leakage permeances of the machine is dealt separately in the Appendix (C). As maximum leakage takes place between the adjacent pole forces, a good first approximation can be made by using this largest path and then relating this permeance to the total by a factor K_l

From Appendix (C1)

$$P_p = \frac{\mu_o Pl}{2\pi}\left(1 - \frac{r_1}{h} l_n \frac{r_2}{r_1}\right)$$

Now $r_2 = (\pi D - Pb)/2\pi$ and $r_1 = r_2 - h$

Thus

$$P_p = \frac{\mu_o Pl}{2\pi}\left(1 - \frac{\pi D - Pb - 2\pi h}{2\pi h} \ln \frac{\pi D - Pb}{\pi D - Pb - 2\pi h}\right)$$

But as is seen from the equivalent magnetic circuit Fig. (5. 3C), the pole-to-pole permeance should be multiplied by 4 to convert it to P_l.

Consequently

$$S_1 = \frac{h}{bl} P_1 = K_1' \frac{h}{b} \left[1 - \frac{\pi D - Pb - 2\pi h}{2\pi h} \ln \frac{\pi D - Pb}{\pi D - Pb - 2\pi h} \right] \quad (6.2)$$

where $K_1' = \dfrac{2\mu_o PK_1}{\pi}$

3. NO-LOAD VOLTAGE

At no-load the magnet operates at point P_1 in Fig. (6.1) at which point

$$ENL = K_e N \phi_{g1}$$

where $K_e = 4.44 \, f \, kw$

But $\phi_{g1} = b l B_{g1}$, and

$B_{g1} = S_g H_1$

Thus $ENL = K_e N (bl) S_g H_1$

Using equation (6.1) for the value of S_{g_1}

$$ENL = \mu_o K_f K_e \frac{(bhl) NH_1}{l_g} \quad (6.3)$$

4. SHORT-CIRCUIT CURRENT

For the steady-state short-circuit the generator will operate at point P_s. At the short-circuit point all the voltage is utilized as the impedance drop. Now the line joining O to P_1 divides any air-gap voltage into impedance drop and terminal voltage. At the short-circuit the generated voltage, equal to the armature drop is represented by the flux density D_{sc} in the B-H plane, and the demagnetizing mmf because of the short-circuit current as Q_{sc}.

From the geometry of the B-H curve

$$Q_{sc} = nQ_2$$

where n is the per unit short-circuit current (n >1).

Also $$B_{g1} = B_{t_2} + \gamma B_{t_2}$$

where γ is per unit voltage - regulation

From similar triangles $P_1 S_1 O$ and $P_2' S_2 O$

$$\frac{B_{g_1}}{B_{t_2}} = \frac{Q_{sc}}{Q_{sc} - Q_2} = \frac{n}{n - 1}$$

And using the relationship of B_{g_1} to B_{t_2}

$$n = \frac{1 + \gamma}{\gamma} \qquad (6.4)$$

5. SHORT-CIRCUIT ARMATURE REACTION

It is shown in equation (5.3) that the operating point in $\emptyset$ - F plane under load moves to the right of the no-load point by $\left(\frac{P_g}{P_t} A\right)$ by a demagnetizing mmf A.

Transforming to the B-H plane the effect of short-circuit demagnetization can be written as

$$Q_{sc} = \frac{S_g \; Asc}{S_t \; h}$$

Since short-circuit current is almost completely direct-axis current

$$A_{sc} = K_a N I_{sc}$$

where K_a is given by (Reference 39, Page 187)

$$K_a = \frac{3}{2} \frac{4}{\pi} \sqrt{2} \frac{kw}{p}$$

Thus

$$Q_{sc} = \frac{S_g K_a N I_{sc}}{S_t h}$$

Using equation (6.1)

$$Q_{sc} = K_q \frac{N}{l_g S_t} \tag{6.5}$$

where

$$K_q = \mu_o K_f K_a I_{sc}$$

6. SHORT-CIRCUIT VOLTAGE DROP

The air-gap voltage on the short-circuit is consumed as an internal impedance drop. This voltage drop is represented by line OP_s in the B-H plane in Fig. (6.1).

From Appendix (B.2)

$$D_{sc} = \frac{1}{b} \sqrt{\left(\frac{4K_R I_{sc}}{K_e}\right)^2 + \left(\frac{K_x I_{sc}}{K_e}\right)^2 \cdot \frac{N^2}{D^2}} \tag{6.6}$$

Which can be reduced to

$$D_{sc}^{\ 2} = R_r' + \left(\frac{K_x'}{\lambda^2}\right) Q_{sc}^{\ 2} \tag{6.7}$$

Where

$$R_r' = \left(\frac{4K_R I_{sc}}{K_e b}\right)^2 \quad (6.7\ (a))$$

$$K_x' = \left(\frac{K_x I_{sc}}{K_e b D}\right)^2 \quad (6.7\ (b))$$

and

$$\lambda = \frac{Q_{sc}}{N} \quad (6.7\ (c))$$

7. GEOMETRY IN THE B-H PLANE

The first geometrical relation is imposed at point P_s where line I cuts the B-H axis.

$$D_{sc} = B_s - S_l H_s \quad (6.8)$$

and

$$Q_{sc} = H_s - H_l \quad (6.9)$$

Also

$$Q_{sc}(S_g + S_l) = (S_g + S_l) H_s - S_l H_s - (D_{sc} + S_o Q_{sc})$$

which can be reduced to

$$Q_{sc} S_t = S_g H_s - D_{sc} \quad (6.10)$$

The relationship

$$B_s = B(H_s)$$

can be given by the equation (3. 1) as derived by Desmond [7]. However, the equation to the demagnetization curve can also be written in the form of a polynomial as in Appendix (D 2).

Considering the geometry at point P_1

$$B_o - S_o H_1 = (S_l + S_g) H_1 \tag{6.11}$$

and

$$B_o = S_l H_s + D_{sc} \quad S_o H_s \tag{6.12}$$

Subtracting equation (6.11) from (6.12)

$$S_o H_1 = H_s S_l + H_s S_o + D_{sc} - S_l H_1 - S_g H_1 \tag{6.13}$$

Which gives

$$S_g H_1 = D_{sc} + (H_s - H_1)(S_o + S_l) \tag{6.14}$$

and using equation (6.9)

$$S_g H_1 = D_{sc} + (S_o + S_l) Q_{sc} \tag{6.15}$$

Equations (6.8), (6.9), (6.10) and (6.15) are the four independent equations defining the curve geometry completely.

6.3 DESIGN PROCEDURE

From the above design equations it is obvious that two different sets of independent equations exist. Equations (6.8), (6.9), (6.10) and (6.15) come from the geometry of the curve only, while equations (6.3), (6.4), (6.5) and (6.6) depend on the machine parameters and the electrical quantities like no-load voltage, short-circuit current, armature reaction and voltage drop due to short-circuit current of the machine.

The quantities Q_{sc}, D_{sc} obtained from the machine parameters fix the point P_s which must lie on the curve to satisfy all equations. In other words, the two sets of equations are solved for a Q-D fit. A simple approach to this problem is given below -

Select a reasonable diameter of the machine. For this diameter assume a value of b and h. S_g and S_l can then be calculated from equations (6.1) and (6.2), while S_o is known for any material. Q_{sc} and D_{sc} can then be easily obtained from equations (6.8), (6.9) and (6.10). Thus, variation of Q_{sc}^2 and D_{sc}^2 can be plotted which is shown in Fig. (6.4). The corresponding B-H curve of the material is shown in Fig. (6.3) and the computer programme is given in Appendix (D3).

Now D_{sc} can also be obtained from equation (6.6) which depends on the stator variables of the machine. Equation (6.6) can be reduced to equation (6.7) as explained in Appendix (B2). This is obviously an equation to a straight line in Q_{sc}^2 - D_{sc}^2. The gradient of this straight line and the constant R_r' can then be calculated. The computer programme for this is given in Appendix (D4). Thus, the intersection of this straight line with the curve gives one point which satisfies all equations and hence gives the required Q - D fit.

Thus, H_s, B_s being known, H_l can be easily calculated from equation (6.9) and the number of turns is given by equation (6.7C). The load voltage and the regulation being known no-load voltage is readily obtained and hence the length of the machine can be calculated from equation (6.3). Hence, for a particular diameter and breadth of the pole,variation of length of the machine with different values of h are calculated and plotted as shown in Fig. (6.5). Thus, the optimum length for each diameter of the machine isknown from the curve of Fig. (6.5). The same procedure can be repeated for different diameters and the optimum machine for minimum

D^2l can be obtained. To illustrate the above method an example is given below.

6.4 DESIGN EXAMPLE

A 2KVa generator has been calculated. Calculation for minimum length has been done only for one diameter. By repetitive calculation, minimum length for different values of D can be calculated, and hence the best machine for minimum D^2l can be obtained.

The calculation of stator variables, that is, number of slots, number of conductors per slot and slot dimensions is done exactly in the same way as for conventional machine and hence is not described here. It may be mentioned that for a given machine the ratio of slot height to slot width should not vary very much even if the winding is different, for the resistance and leakage reactance per phase are machine constants. However, no attempt has been made here to calculate the machine completely for the purpose is just to demonstrate how an optimum machine with minimum D^2l can be calculated.

Machine Specification

$2KV_a$, 3200 c/s, 115 volts / Δ - connected

24000 rpm generator.

Calculations

$$f = 3200; \quad n = 24000 \text{ rpm}$$

$$\text{gives } P = \frac{120 . 3200}{24000} = 16$$

Number of pole = 16

$$I\ phase = \frac{2 \times 10^3}{3 \times 115} = \underline{5.65\ amps}$$

Assume diameter = 2.5 inches

gives pole pitch = 0.49 inches.

Assuming ratio of length to pole pitch to be approximately 2.5, gives

l = 1.23 = 1.25 inches (say)

Now taking the pole arc to be approximately 75% of the pole pitch

Pole shoe = 0.37 inches

Now if K_f = 1.2 to 1.6, then

approximate breadth of the pole = 0.23 to 0.31 inches

b = 0.2 to 0.3 inches

The probable height of the pole can be easily obtained from simple geometric construction of Fig. (6.2).

Let h vary from 0.25 to 0.55 inches

Slot dimensions

Inherent voltage regulations is assumed to be 25% gives

$$ENL = (115 + \frac{115}{4}) = 144\ volts$$

Assuming sinusoidal wave-form of air-gap flux, the form factor equals 1.11.

$$\text{Hence}\ ENL = 4.44\ f\ K_w\ N\ \phi_{g_1} 10^{-8}$$

$$\phi_g = B_{g_1}\ b\ l$$

B_{g1} lies between 40.0 to 55.0 kilo-lines per square inch.

Let B_{g1} = 45 kilo-lines / in^2

Hence N = 72.5 for b = 0.25 inch

Let $\underline{N = 80}$

Total number of conductors = 480

1 slot per pole per phase is selected

Hence N_s = 48

and conductors per slot = 10

Full pitch winding gives K_w = 1.0

S.W.G. number 20 wire selected

S.C.C. wire diameter = 0.037 inches

Full pitch single layer winding is used

Now $d_1 = (0.037.5 + .005) = 0.190$ inches

Let $d_2 = 0.002$ inches (20 mils)

$W_s = (0.037 + 5)^2 = 0.087$ inches

Thus, the slot dimensions are known for our calculations of X_1

It may be mentioned here that the ratio height of the slot to width of the slot is fairly constant for a particular machine and does not vary very much if the winding is changed.

K_R for N_o 20 S W G = 12.3 Ω per 1000 ft. at 75°C.

<u>Short-Circuit Current</u>

If n is the per unit short-circuit current then from equation (6.4)

$$n = \frac{1 + r}{r} = \frac{1 + 0.25}{0.25} = 5$$

Hence, $I_{sc} = 5 \times 5.65 = \underline{28.2 \text{ amps}}$

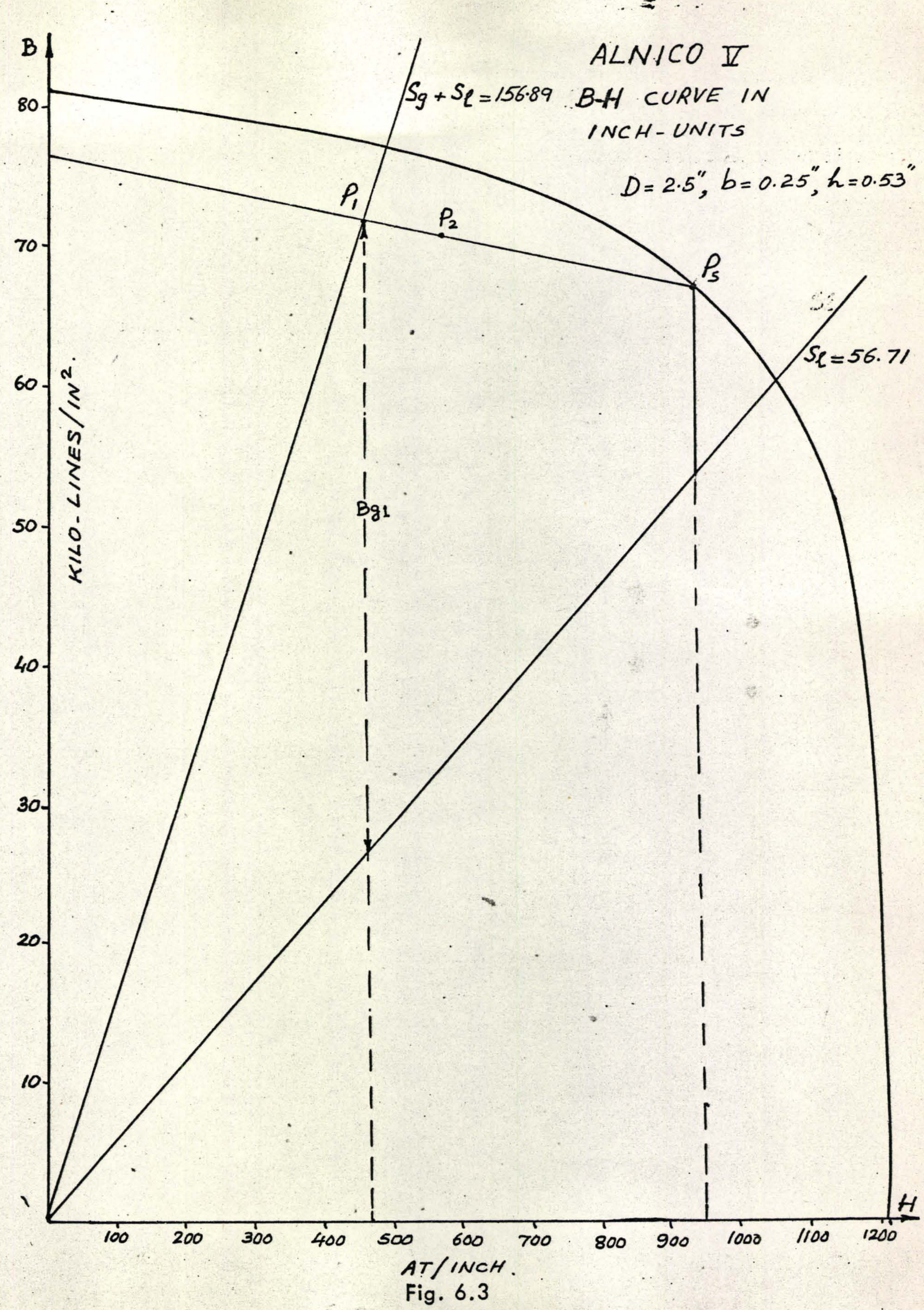

ALNICO V
B-H CURVE IN
INCH-UNITS
D = 2.5", b = 0.25", h = 0.53"
Sg + Sl = 156.89
Sl = 56.71
P1
P2
Ps
Bg1
B
80
70
60
50
40
30
20
10
KILO-LINES/IN².
H
100
200
300
400
500
600
700
800
900
1000
1100
1200
AT/INCH.

Fig. 6.3

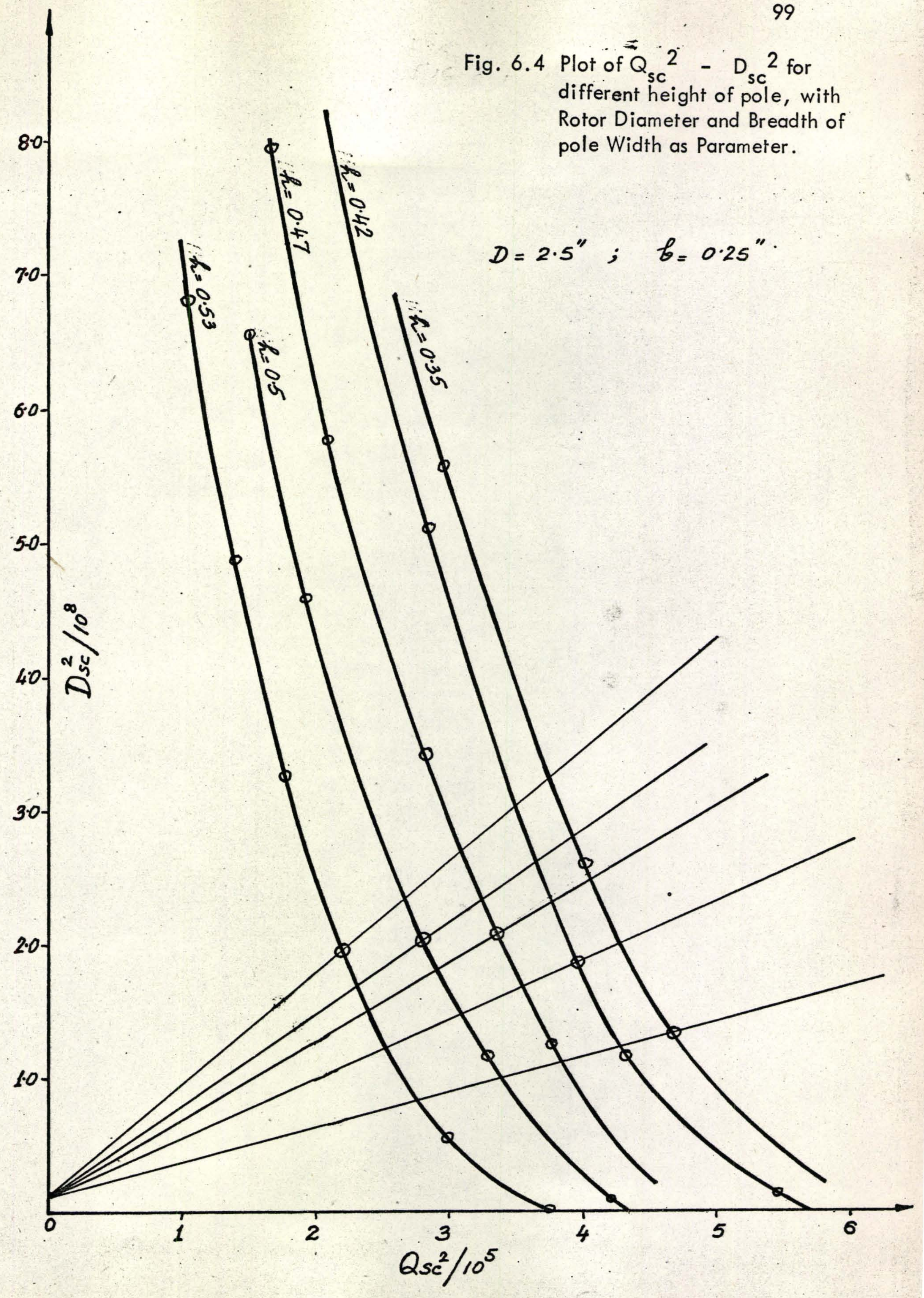


Fig. 6.4 Plot of Q_{sc}^2 - D_{sc}^2 for different height of pole, with Rotor Diameter and Breadth of pole Width as Parameter.

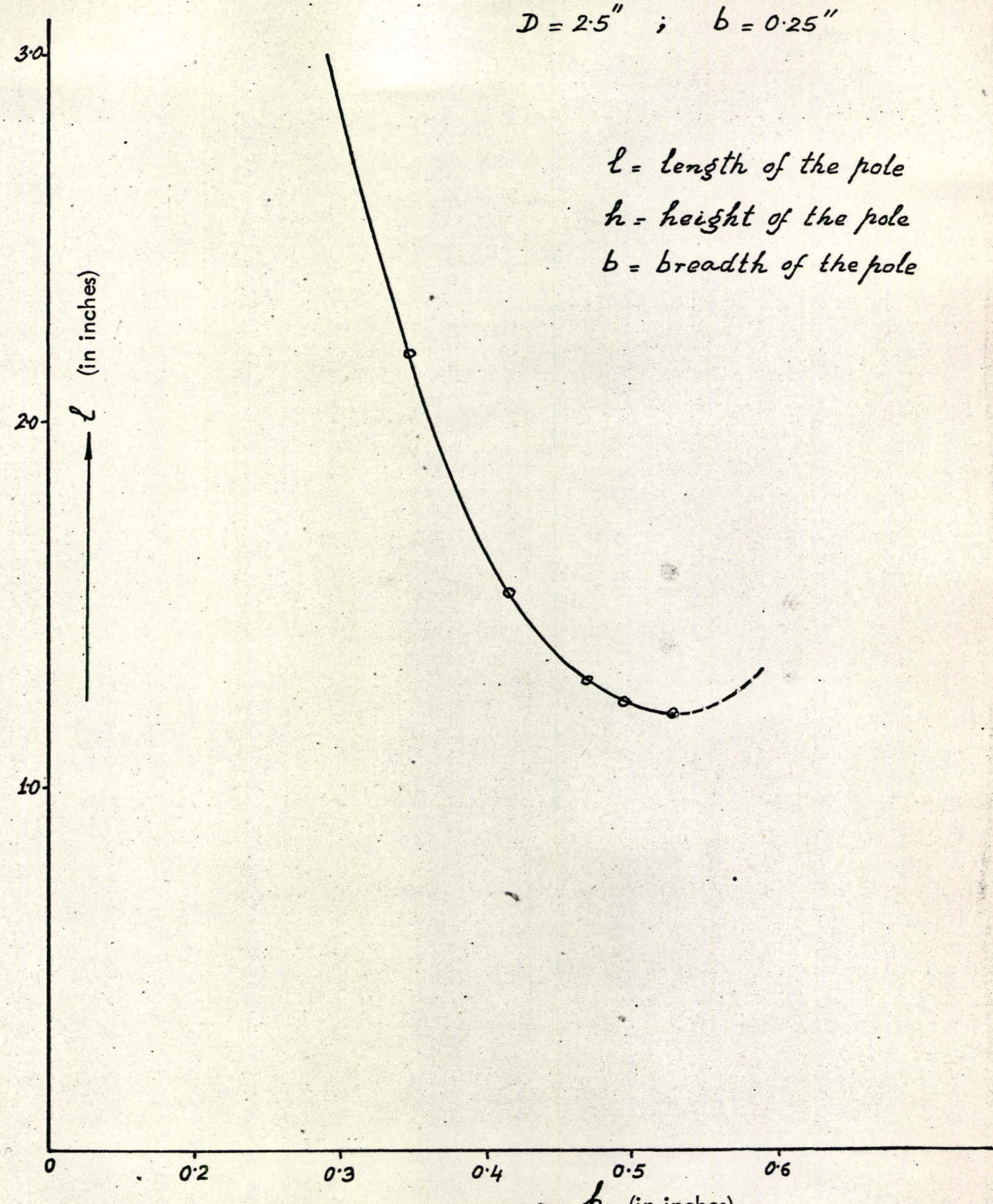


Fig. 6.5 Variation of Machine Length (l) with Height of Pole (h) Varying, Rotor Diameter and Breadth of Pole as Parameter.

From these values, the minimum length has been calculated as explained before and the result is plotted in Fig. (6.5). The minimum length is 1.2 inches for D = 2.5 inches.

CHAPTER VII

CONCLUSIONS

Design analysis of permanent magnets operating under static conditions indicates that for the most economical design of a permanent magnet its operating line must intersect the major loop at $(BH)_{max}$ point. For dynamic conditions however the maximum energy (BH) available will vary according to the operation on different minor loops and depend on unit leakage permeance line and air-gap permeance line. Criteria for the most economical design of permanent magnets operating under dynamic conditions have been developed in Chapter 3 of this project. Papers on permanant magnet generators indicate that the procedure of wound-field machine has been followed. That is, a stator is chosen and then the most economical rotor is designed based on the principles of optimum magnet design. This method is satisfactory for air-stabilised machine but minimum weight requirements for permanent magnet generators dictate load or short-circuit stabilisation. For such a machine to be economical a new approach to the design of permanent magnet generator was essential. Hanrahan and Toffolo [21], [22] first suggested such an approach. Optimum design based on this approach has been thoroughly dealt with in Chapter 6.

The design example calculated indicate that the values of $Q_{sc}^2 - D_{sc}^2$ from the curve geometry are quite sensitive to the geometry of the B-H curve of the material. Hence, the choice of material is an important factor. Alnico V and VI are most suitable permanent magnets for machines of integral hp capacity. For smaller machines ceramic magnets are preferred because of their low cost, but they have much lower values of residual induction as compared to Alnico's.

While calculating Q_{sc} and D_{sc} from the geometry of B-H curve, many more points for B_s and H_s at smaller intervals should be taken in the region the point of stabilisation is expected to lie on the curve.

The solution for a Q - D fit is obtained graphically and is subject to inaccuracy in result. Calculation of the slope of straight line in $Q_{sc}^2 - D_{sc}^2$ as obtained from short-circuit voltage drop depends on the leakage reactance. Hence, a more rigorous calculation of leakage reactance would increase the accuracy in result.

The calculation of no-load voltage is based here on the inherent voltage regulation. Permanent magnet generators suffer due to this disadvantage for voltage regulation in permanent magnet is still a problem. No effective method has been developed and this area is still open to further research.

The example calculated here for the given generators specification demonstrate the method of optimum design for minimum D^2 l. Values of the length of machine for a particular diameter have been plotted as a function of the height of the pole diameter of pole and breadth of pole as parameters. Thus, for the same diameter, a set of such curves can be obtained with different values of breadth and the absolute minimum length can be calculated. Thus, by repetitive calculations minimum length for each diameter can be calculated and, hence, an optimum machine with minimum $D^2 l$ can be designed. However, while calculating length, number of turns is also a parameter and sometimes might be the deciding factor for the choice of length. However, this calculation could not be completed because of lack of time.

The choice of different machine constants like K_l, K_f, K_w, etc. also effect the calculation and should be chosen carefully. However, these values may be obtained from previous design experience.

The results obtained for S_l and S_g and Q_{sc} etc. for the machine are plotted on the B-H curve as in Fig. (6.3), satisfy the conditions of no-load at point P_1.

APPENDICES

APPENDIX (A)

I. DEMAGNETIZATION CURVE

In deriving the equation to the demagnetization curve the negative sign of H is omitted for convenience.

A general equation connecting B and H is then

$$aBH + bBH_c + CB_rH + B_rH_c = 0 \qquad (A.I.I)$$

where a, b and c are numberical constants when $B = B_r$, $H = 0$ and when $H = H_c$, $B = 0$ gives $b = c = -I$

giving the simplest equation of B-H curve as

$$aBH + B_rH_c = BH_c + B_rH \qquad (A.I.2)$$

$$\text{or} \quad B = \frac{B_r\,(H - H_c)}{(aH - H_c)} \qquad (A.I.3)$$

Now as $H \longrightarrow -\infty$, $B \longrightarrow B_s$

Therefore $aB_s = B_r$

$$\text{or} \quad a = \frac{B_r}{B_s}$$

which is called the "remanance factor"

This equation is identical to that obtained by Watson.

Assuming that the $(BH)_{max}$ point occurs at $B = B_m$ and $H = H_m$ at the magnetization curve. At this point $\partial (BH) = 0$

Therefore, $H_m \partial B + B_m \partial H = 0$ (A.1.4)

and differentiating equation (A.2) gives

$$H_c \partial B + B_r \partial H = a \partial (BH) = 0 \quad (A.1.5)$$

Hence, by division

$$H_m/H_c = B_m/B_r \quad (A.1.6)$$

Now substituting B_m and H_m for B and H in equation (A.1.2) and dividing by $B_r H_c$ gives

$$\frac{a B_m H_m}{B_r H_c} + 1 = \frac{B_m}{B_r} + \frac{H_m}{H_c} \quad (A.1.7)$$

Using equation (A.1.6) gives

$$a r + 1 = 2\sqrt{r}$$

whence $a = \left(\frac{2\sqrt{r} - 1}{r} \right)$. (A.1.8)

where $r = \frac{B_m H_m}{B_r H_c}$ and is called "curve factor".

2. DERIVATION OF THE RELATIONSHIP GIVEN IN EQUATION (3.22)

From the geometry of the curve (Fig. (3.14)

$$B_2 = \mu_r H_o - \mu_r (x + H_{d2})$$

where

$$B_2 = x S_2$$

Hence

$$B_2 = \mu_r H_o - \mu_r \left(\frac{B_2}{S_2} + H_{d2} \right)$$

or $$B_2 \left(1 + \frac{\mu_r}{S_2}\right) = \mu_r H_o - \mu_r H_{d_2}$$

therefore

$$B_2 = \frac{\mu_r (H_o - H_{d2})}{(S_2 + \mu_r)} S_2$$

Similarly

$$H_2 = \frac{\mu_r (H_o - H_{d2})}{(\mu_r + S_2)}$$

Similarly other relationships given by Hornfeck and Edgar in equation (3.23) to (3.25) can be easily derived.

APPENDIX (B)

I. VOLTAGE REGULATION OF PERMANENT MAGNET GENERATOR

E_o in the Blondel diagram is the true excitation voltage, no-load voltage corresponding to main field mmf and is given by equation (5.4).

From Blondel diagram

$$\text{Sin} \quad = \quad \frac{I_q X_q}{V} \qquad \frac{I \cos(\alpha + \Theta) X_q}{V}$$

$$\text{or} \quad \text{Sin} \quad = \quad \frac{\cos\alpha \cos\Theta \; - \; \sin\alpha \sin\Theta}{\frac{V}{I} X_q}$$

$$\text{or} \quad \text{Sin} \quad \left(\frac{Z}{X_q} + \sin\Theta \right) \quad = \quad \cos\Theta \sqrt{1 - \sin^2\alpha}$$

Squaring both sides and simplifying

$$\sin\alpha \quad = \quad \frac{\cos\Theta}{\sqrt{1 + \frac{2Z}{X_q}\sin\Theta + \frac{Z^2}{X_q^2}}} \qquad \text{(B.I.I)}$$

Hence

$$\text{Cos}\,\alpha = \sqrt{1 - \text{Sin}^2\alpha}$$

$$= \frac{\text{Sin}\,\Theta + \frac{Z}{X_q}}{\sqrt{1 + \frac{2Z}{X_q}\,\text{Sin}\,\Theta + \frac{Z^2}{X_q^{\,2}}}} \qquad \text{(B.1.2)}$$

From Blondel diagram

$$E_o = V\,\text{Cos}\,\alpha + I_d X_d$$

$$= V\,\text{Cos}\,\alpha + I\,\text{Sin}\,(\alpha + \Theta)\,X_d \qquad \text{(B.1.3)}$$

Substituting the value of Sinα and Cosα from equation(B.1.1) and (B.1.2) in the equation (B.1.3), and simplifying gives

$$V = E_o \frac{Z\sqrt{Z^2 + 2ZX_q\,\text{Sin}\,\Theta + X_q^{\,2}}}{[Z^2 + Z\,(X_d + X_q)\,\text{Sin}\,\Theta + X_dX_q]} \qquad \text{(B.1.4)}$$

Using equation (5.4) in equation (B.1.6) gives

$$V = [\,ENL + \frac{K_gP_g^{\,2}}{P_t}A_d]\;\frac{Z\sqrt{Z^2 + 2ZX_q\,\text{Sin}\,\Theta + X_q^{\,2}}}{[Z^2 + Z\,(X_d + X_q)\,\text{Sin}\,\Theta + X_dX_q]} \qquad \text{(B.1.5)}$$

where

$$A_d = K_dI_d = K_d\,I\,\text{Sin}\,(\alpha + \Theta)$$

Substituting the value of Sin and Cos

$$A_d = K_d\frac{V}{Z}\left(\frac{1 + \frac{Z}{X_q}\,\text{Sin}\,\Theta}{\sqrt{1 + \frac{2Z}{X_q}\,\text{Sin}\,\Theta + \frac{Z^2}{X_q^{\,2}}}}\right) \qquad \text{(B.1.6)}$$

Substituting equation (B.1.6) and simplifying gives

$$V = \frac{ENL\, P_t\, Z \sqrt{Z^2 + 2ZX_q \,\mathrm{Sin}\,\Theta + X_q^2}}{[Z^2 + Z(X_d + X_q)\,\mathrm{Sin}\,\Theta + X_d X_q]\, P_t - K_g K_d P_g^2 X_q \left(1 + \frac{Z}{X_q}\,\mathrm{Sin}\,\Theta\right)} \qquad (B.1.7)$$

2. VOLTAGE DROP TRANSFORMED TO B-H PLANE

The internal voltage drop on short-circuit is equal to the air gap voltage represented as D_{sc} in the B-H plane Fig. (6.1), and is given by

$$D_{sc} = \frac{(E_g)sc}{K_e\, N\, (bl)}$$

$$= \frac{I_{sc}\, Z_l}{K_e\, N\, (bl)} \qquad (B.2.1)$$

where

$$Z_l^2 = R^2 + X_l^2$$

The d.c. resistance for the active length of the winding

$$R_{dc} = K_R\, N\, 2\, l \qquad (B.2.2)$$

Where K_R is the resistance per unit length, multiplying K_R by factor 2 to take into account increase in resistance because of end-turns and alternating current, the resistnace of the winding per phase becomes

$$R = 4\, K_R\, N\, l \qquad (B.2.3)$$

The leakage reactance in the machine can be broadly classified as (1) slot reactance, (2) zig-zag reactance (3) belt leakage reactance and (4) end-leakage reactance. For a detailed study of this subject the reader is referred to reference (23) and (40).

The maximum leakage reactance is due to slot-reactance and zig-zag reactance and is given by (40).

$$X_s + X_z = \frac{0.79\ flmZ^2}{N_s\ 10^7}\ [K_s\ (\frac{d_1}{3W_s} + \frac{d_2}{W_s}) + \frac{0.266\ DKa^2}{l_g\ N_s}] \quad (B.2.4)$$

where dimensions are in centimeters. The factor 0.79 is replaced by 2.0 where dimensions are in inches.

In m.k.s. unit

$$X_s + X_z = 2\pi\mu_o \frac{flmZ^2}{N_s} [\ K_s\ (\frac{d_1}{3W_s} + \frac{d_2}{W_s}) + \frac{0.266\ DK_w^2}{l_g\ N_s}]$$

$$[\ \frac{0.79}{10^7}\ (cm) = \frac{0.8\pi}{10^8}\ (cm) = \frac{2\pi}{10^2}\ \frac{4\ \pi}{10^7}\ (cm) = 2\pi\mu_o\ (meter)\ ]$$

Let the diameter and N_s be related by the relation $N_s = K_D D$, then

$$X_s + X_z = \frac{8\pi\mu_o flm\ N^2}{K_D\ D}\ [K_s(\frac{d_1}{3W_s} + \frac{d_2}{W_s}) + \frac{0.266\ K_w^2}{l_g K_D}]$$

Let K_2 be a factor by which slot and zig-zag reactance is related to the total leakage reactance, then

$$X_l = \frac{8\pi\mu_o\ flm\ N^2}{K_D\ D\ K_z}\ [K_s(\frac{d_1}{3W_s} + \frac{d_2}{W_s}) + \frac{0.266K_w^2}{l_g\ K_D}] \quad (B.2.5)$$

$$= K_x(\frac{lN^2}{D})$$

Where

$$K_x = \frac{8\pi\mu_o fm}{K_D K_z} \left[K_s \left(\frac{d_1}{3W_s} + \frac{d_2}{W_s} \right) + \frac{0.266 K_w^2}{l_g K_D} \right] \quad (B.2.6)$$

Thus

$$Z_1 = \sqrt{R^2 + X_1^2}$$

$$= \sqrt{(4K_R NI)^2 + K_x^2 \left(\frac{I^2 N^4}{D^2} \right)}$$

$$= NI \sqrt{(4K_R)^2 + K_x^2 \frac{N^2}{D^2}} \quad (B.2.7)$$

Using equation (B.2.7) in equation (B.2.1) gives

$$D_{sc} = \frac{I_{sc}(NI)}{K_1 b(NI)} \sqrt{(4K_R)^2 + K_x^2 \frac{N^2}{D^2}}$$

$$= \sqrt{\left(\frac{4K_R I_{sc}}{K_1 b} \right)^2 + \left(\frac{K_x I_{sc}}{K_1 bD} \right)^2 N^2}$$

Hence

$$D_{sc}^2 = K_R' + K_x' N^2 \quad (B.2.8)$$

Where

$$K_R' = \left(\frac{4K_R I_{sc}}{K_1 b} \right)^2 \quad \text{and} \quad K_x' = \left(\frac{K_x I_{sc}}{K_1 bD} \right)^2$$

Using equation (6.5) in equation (B.2.8)

$$D_{sc}^2 = K_R' + \frac{K_x'}{\lambda^2} Q_{sc}^2 \quad (B.2.9)$$

Where

$$\lambda = \frac{Q_{sc}}{N} = \frac{K_q}{I_g S_t} = \frac{K_a \mu_o K_f}{I_g S_t} \qquad (B.2.10)$$

APPENDIX (C)

I. FIELD LEAKAGE PERMEANCES

In any practical magnet application leakage flux is a considerable part of the total flux, hence leakage calculation is important. Graphical field mapping, as it is generally used in design of electrical machinery gives comparatively good results. But this procedure is tedious and requires too much labour for practical use. The most convenient method seems to be that of "estimating permeance of probable flux paths". This consists of arbitrarily dividing the total magnetic flux through the air into suitable partial fluxes, the permeance of which are estimated by appropriate geometrical assumptions.

Fig. (C.I) shows a common mechanical construction showing the various flux leakages divided into individual components and is the same as presented by Ginsberg[20]. The 3-dimensional leakage calculations are an adaption of flux path permeances as contained in the book of Herbert C. Roters [37].

Maximum leakage takes place along path 4 and the equation to the leakage permeance for this path is given here. For calculation of permeances of other leakage paths the reader is referred to [20]. However, the leakage permeance P_p for path 4 is derived below.

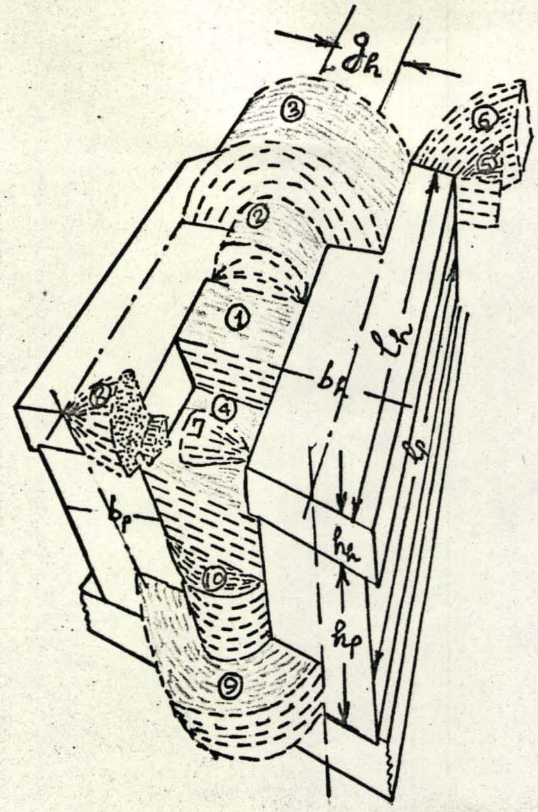


Fig. C(l) A Common Mechanical Construction showing the Various Flux Leakages.

Paths 1 and 4 - Non parallel Planes (Normal to Flow of Flux)

Paths 2 and 3 - Semicircular Cylinder (between Edges Axial to Flow)

Paths 5 and 10 - Semicircular Cylinder (between Edges Normal to Flow)

Paths 6 and 9 - Half-Annulus (between Parallel Planes)

Path 7 - Spherical Quadrant (between Corners)

Path 8 - Quadrant of Sperical Shell (between Edges)

$$F = 2F_o \frac{r}{a}$$

and $$d\phi = F\,dP$$

$$dP = \frac{l\,dr}{r\Theta}$$

Permeance P_p of Path 4

Path 4 represents distributed potential problem for the potential varies along the pole face from zero in the centre to maximum at the pole head. The pole faces are usually non-parallel planes.

The potential distribution can be represented in circuit analogy by Fig. (C.2C).

Consider a smallstrip dr as in Fig. (C.2b) and let dP be the permeance of this path

Now $$F = 2F_o \frac{r}{a}$$

and $$d\phi = F\,dP$$

$$dP = \frac{\mu_o\, l\, dr}{r\,\Theta}$$

Thus the total leakage flux

$$\phi_l = \int_o^a d\phi = \frac{2\,F_o}{a}\;\frac{\mu_o\, l}{\Theta}\int_o^a dr = \frac{2\mu_o l\, F_o}{O}$$

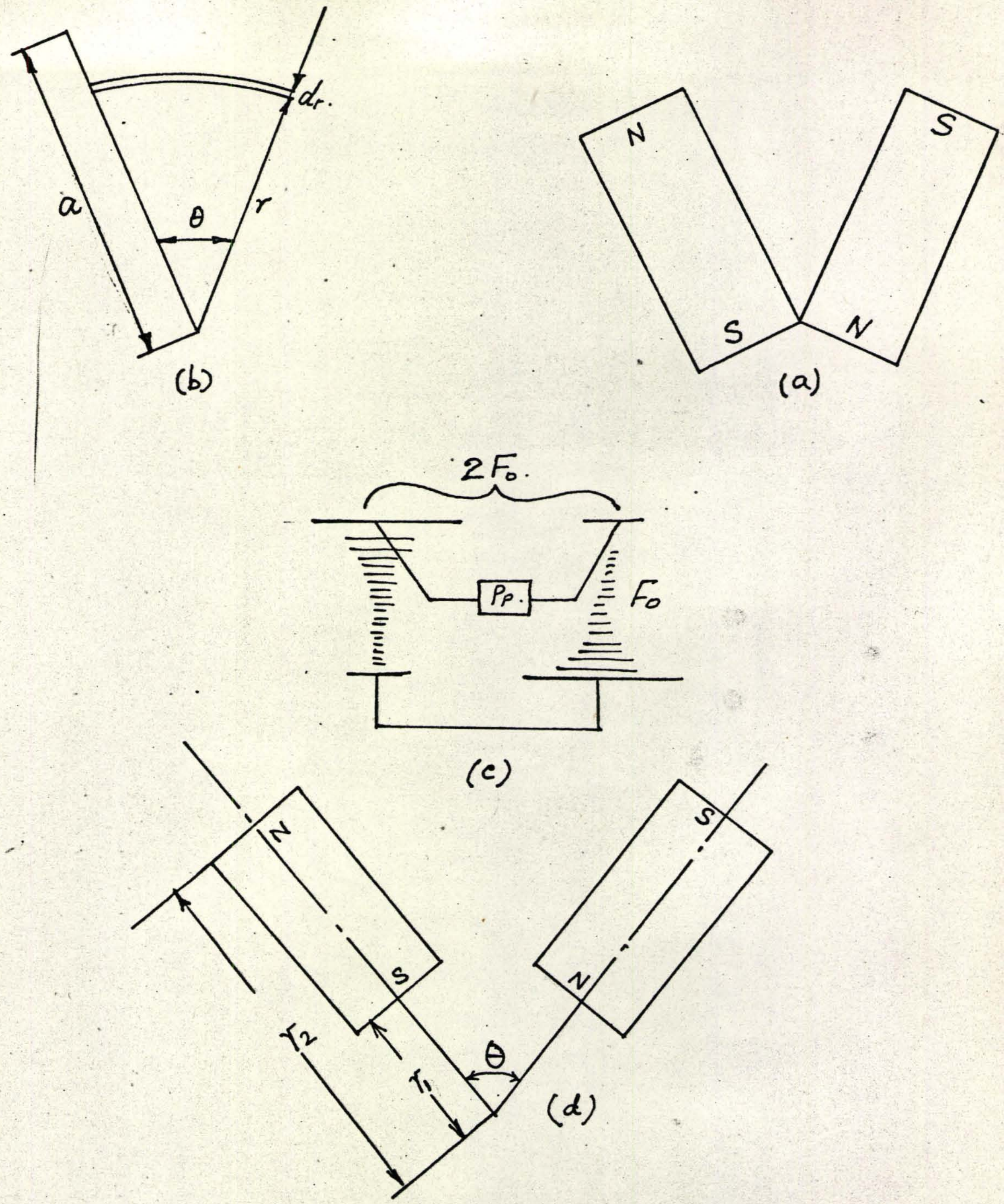


Fig.C(2)

Hence, if P_p is the leakage permeance of path Fig. (C.2A), then

$$P_p = \frac{\phi_l}{2F_o} = \frac{2\mu_o l F_o}{\Theta\, 2F_o} = \frac{\mu_o l}{\Theta} \qquad \text{(C.l)}$$

But for the configuration of Fig. (C.2d) which is that of our machine under consideration in Chapter 6

$$F = 2F_o \frac{r - r_1}{r_2 - r_1}$$

$$\text{Hence } d\phi = F\,dP = \frac{\mu_o l\, d_r}{r\Theta} 2F_o \frac{r - r_1}{r_2 - r_1}$$

Therefore the total leakage flux along this path

$$\phi_l = \frac{2\mu_o l F_o}{\Theta\,(r_2 - r_1)} \int_{r_1}^{r_2} \left(\frac{r - r_1}{r}\right) dr$$

$$= \frac{2\mu_o l F_o}{\Theta\,(r_2 - r_1)} \left[(r_2 - r_1) - r_1 l_n \frac{r_2}{r_1} \right]$$

$$= \frac{2\mu_o l F_o}{O} \left[1 - \frac{r_1}{r_2 - r_1} l_n \frac{r_2}{r_1} \right]$$

Now $\phi_l = P_p\, 2F_o$

Therefore

$$P_p = \frac{\mu_o l}{\Theta} \left[1 - \frac{r_1}{h} l_n \frac{r_2}{r_1} \right] \qquad \text{(C.2)}$$

APPENDIX D

Computer Programmes

```
$JOB            003516 HESAMUDDIN        100    010    030
$IBJOB          NODECK
$IBFTC
C      CALCULATION OF VM/VG FOR ELECTROMAGNET
C      RE=VM/VG,VM=VOLUME OF COPPER+VOL. OF IRON
C      M IS THE RATIO OF MAGNETIC LOADING TO ELECTRIC LOADING
C      LG AND AG ARE LENGTH AND AREA OF AIR-GAP IN CENTIMETERS
C      RATIO RE IS CALCULATED FOR M=25,50,75,100
       DATA M,C,PI/0.0,1.0,3.14159265/
       REAL M,LG
       DO 10 I=1,4
       WRITE(6,1)
       AG=0.1
       M=M+25.
       DO 10 J=1,4
       LG=0.
       AG=AG*10.
       WRITE(6,4)
       DO 10 L=1,10
       LG=LG+0.1
       IF(LG.GE.0.5) LG=LG+0.4
       RE=(PI*SQRT(M*M*M)*LG)/(C*AG)+2.*M*SQRT(PI/AG)+C*SQRT(M/LG)
    10 WRITE(6,2) M,AG,LG,RE
     1 FORMAT(1H1,14X,1HM,19X,2HAG,18X,2HLG,18X,2HRE/1H0)
     2 FORMAT(1H+,4F20.5)
     4 FORMAT(1H-)
       STOP
       END
$ENTRY
$IBSYS
```

CD TOT   0030

Appendix D 1

```
$JOB            003516 HESAMUDDIN        100    010    030
$IBJOB          NODECK
$IBFTC
C      TO FIND THE LEAST SQUARE FIT OF A SET OF POINTS(X,Y) TO ACURVE
C      Y= A POLYNOMIAL OF A GIVEN DEGREE IN X
C      Y=B1+B2*X+B3*X**2+B4*X**3+B5*X**4+B6*X**5
C      N IS TNHE NUMBER OF POINTS(X(I),Y(I)),I=1,) TO BE FITTED
C      M IS THE DEGREE OF POLYNOMIAL TO BE FITTED
C      B IS AN ARRAY OF M+1 ELEMENTS,IN WHICH THE M+1CO-EFFICIENTS ARE
C      RETURNED,STARTING WITH THE CONSTANT TERMS IN B(1) AND FINISHING
C      WITH THE CO-EFFICIENT OF X*M IN B(M+1).
C      A IS A WORKING ARRAY OF AT LEAST((M+2)**2/2) ELEMENTS
C      Y REPRESENTS THE MAGNETOMOTIVE FORCE (H) IN AMPERE-TURNS PER IN
C      X REPRESENTS THE FLUX DENSITY(B) IN KILO-LINES PER INCH-SQUARE
       DIMENSION X(19),Y(19),B(6),A(25)
       N=19
       READ(5,10)  (X(I),Y(I),I=1,N)
10     FORMAT (2F20.5)
       WRITE(6,1)
1      FORMAT(1H1,5X,39HDEMAGNETIZATION-CURVE DATA OF ALINICO-5,/)
       WRITE(6,5)
5      FORMAT(15X,1HH,20X,1HB,//)
       WRITE(6,15) (X(I),Y(I),I=1,N)
15     FORMAT(2F20.5)
       M=5
       CALL LESQ(A,B,X,Y,M,N)
       WRITE(6,6)
6      FORMAT(1H-,10X,38HTHE COEFFICIENTS OF THE POLYNOMIAL ARE,)
       WRITE(6,20) (B(I),I=1,6)
20     FORMAT(1H-,20X,1PE12.5)
       STOP
       END
$ENTRY
0.0                    81.
200.                   79.4
400.                   77.8
600.                   75.0
750.                   72.5
800.                   71.54
850.                   70.4
900.                   69.0
950.                   67.2
1000.                  64.9
1050.                  61.6
1100.                  56.4
1150.                  46.5
1170.                  39.5
1180.                  34.55
1190.                  28.1
1200.                  19.4
1210.                  10.
1215.                  0.0
$IBSYS
```

Appendix D2

DEMAGNETIZATIØN-CURVE DATA ØF ALINICØ-5

H	B
0.00000	81.00000
200.00000	79.40000
400.00000	77.80000
600.00000	75.00000
750.00000	72.50000
800.00000	71.54000
850.00000	70.40000
900.00000	69.00000
950.00000	67.20000
1000.00000	64.90000
1050.00000	61.60000
1100.00000	56.40000
1150.00000	46.50000
1170.00000	39.50000
1180.00000	34.55000
1190.00000	28.10000
1200.00000	19.40000
1210.00000	10.00000
1215.00000	0.00000

THE CØEFFICIENTS ØF THE PØLYNØMIAL ARE

8.16331E 01

-1.86131E-01

1.36325E-03

-3.44425E-06

3.52194E-09

-1.26756E-12

03516 HESAMUDDIN 100 000MIN 47SEC CØST$003.54 REM. TIME 0097MIN

000MI 49SEC00900=

A4231. Appendix D3 HES

```
A4231.
RUN(S,,,,,,5000)
LGO.
'         6400 END RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     CALCULATION OF QSC-DSC FIT FROM DEMAGNETIZATION CURVE
C     QSC IS THE DEMAGNETIZATION-MMF BECAUSE OF ARMATUE-REACTION(AT/INC
C     DSC IS THE DROP IN FLUX-DENSITY DUE TO QSC(MAXWELL/SQ.INCH)
C     BP AND HP ARE THE BREADTH AND HEIGHT OF THE PERMANENT-MAGNET-POLE
C     DR=ROTOR DIAMATEROF THE MACHINE
C     PN=NUMBER OF POLES,DELTA=AIR-GAP OF MACHINE
C     DELTA IS AIR-GAP,CF IS THE CARTERSCHER FACTOR
C     RF IS RELUCTANCEFACTOR,FF=RATIO OF POLEWIDTHTO POLESHOE
      DIMENSION DR(13),BP(13),HP(13),HS(13),BS(13)
      COMMON/QDFIT/PI,PN,U00,FF,CF,RF,DELTA
      DATA PI,U00,FF,CF,RF,DELTA/3.14159265,3.19,1.6,1.,1.2,0.027/
      DATA PN,FR,PH,EFL/16.,3200.,3.,120./
      READ(5,1) DR,BP,HP,HS,BS
    1 FORMAT(13F6.0)
      CALL QDFIT(DR,BP,HP,HS,BS)
      STOP
      END
$IBFTC  QDFIT
      SUBROUTINE QDFIT(DR,BP,HP,HS,BS)
      DIMENSION DR(13),BP(13),HP(13),HS(13),BS(13)
      COMMON/QDFIT/PI,PN,U00,FF,CF,RF,DELTA
      REAL MATPY
      DO 199 K=1,10
C     SELECT DIFFERENT DIAMATER
      IF(DR(K).EQ.0.) GO TO 199
      DO 200 I=1,10
      IF(BP(I).EQ.0.) GO TO 200
      WRITE(6,10) DR(K),BP(I)
   10 FORMAT(1H1,25X,9HDIAMATER=,F10.2/,25X,11HPOLE WIDTH=,F10.2/1H-)
      DO 201 J=1,10
C     CALCULATION OF UNIT AIR-GAP AND LEAKAGE PERMEANCES
C     SO IS THE SLOPE OF RECOIL LINE
C     SG=UNIT AIR-GAP PERMEANCE AND SL=UNIT LEAKAGE PERMEANCE
C     THE UNIT OF SG,SLAND ST ARE(MAXWELL/AT-INCH)
      IF(HP(J).EQ.0.) GO TO 201
      R2=(PI*DR(K)-PN*BP(I))/(2.*PI)
      IF(R2.LE.HP(J)) GO TO 201
      R1=(PI*DR(K)-PN*BP(I)-2.*PI*HP(J))/(2.*PI)
      SO=10.7
      SG=U00*FF*HP(J)/(DELTA*CF)
      SL=2.*RF*U00*HP(J)*PN/(BP(I)*PI)*(1.-R1/HP(J)*ALOG(R2/R1))
      ST=SG+SL+SO
      WRITE(6,4) HP(J),SG,SL,ST
    4 FORMAT(1H-,9X,3HHP=,F10.3,2X,3HSG=,F10.5,2X,3HSL=,F10.5,
     C2X,3HST=,F10.5/)
      WRITE(6,6)
    6 FORMAT(9X,2HHS,9X,2HBS,10X,3HQSC,10X,3HDSC,10X,5HQSCSQ,10X,
     C5HDSCSQ)
      DO 202 L=1,10
C     QSC AND DSC MUST BE GREATER THAN ZERO
      IF(HS(L).EQ.0.) GO TO 202
      MATPY=SL*HS(L)
      IF(MATPY.GE.BS(L)) GO TO 202
```

```
      DSC=BS(L)-MATPY
      PROD=SG*HS(L)-DSC
      IF(PROD.LE.0.) GO TO 202
      QSC=PROD/ST
      DSCSQ=DSC*DSC
      QSCSQ=QSC*QSC
      WRITE(6,5) HS(L),BS(L),QSC,DSC,QSCSQ,DSCSQ
    5 FORMAT(5X,2F10.2,2X,1P4E13.4)
  202 CONTINUE
  201 CONTINUE
  200 CONTINUE
  199 CONTINUE
      RETURN
      END
'        6400 END RECORD
2.5   2.4
0.167 0.20  0.22  0.25  0.275
0.25  0.35  0.40  0.42  0.43  0.47  0.50  0.51  0.53
750.  800.  850.  900.  950.  1000. 1050. 1100. 1150. 1170. 1180. 1190.
72500.71540.70400.69000.67200.64900.61600.56400.46500.39500.34550.28100
'        6400 END FILE

                              CD TOT    0079
```

Appendix D4

```
A4231.
RUN(S,,,,,,5000)
LGO.
'        6400 END RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     CALCULATION FOR THE SLOPE OF STRAIGHT LINE
C     REACTANCE  IS CALCULATED WHEN THE DIMENSION IS IN INCHES
      DIMENSION ST(10),LUMDA(10),SLOPE(10),ANGLE(10)
      REAL KD,KW,KF,KFF,KE,KS,KA,NS,KQ,KX1,KX2,MUO,LG,LUMDA,ISC
      DATA KW,KF,KS,D1,D2,WS,LG/1.0,1.6,1.0,0.019,0.002,0.009,0.02/
      DATA PI,NS,FR,ISC,D,BP/3.14159265,48.,3200.,28.2,2.5,0.25/
      READ(5,3) ST
    3 FORMAT(10F7.0)
      MUO=3.192
      KD=19.7
      RO=12.3*1.E-03/12.
      KE=4.44*FR*KW*1.E-08
      RR=(4.*RO*ISC/(KE*BP))**2.
      KX1=24.*FR/(KD*1.E07)*(KS*(D1/(3.*WS)+D2/WS)
     C+0.266*KW**2./(LG*KD))
      KX2=(KX1*ISC/(KE*BP*D))**2.
      KFF=MUO*KF
      KA=6.*1.414/(PI*16.)*KW
      KQ=KA*KFF
      WRITE(6,2)
      DO 10 J=1,10
      LUMDA(J)=KQ*ISC/(LG*ST(J))
      SLOPE(J)=KX2/LUMDA(J)**2.
      SLOPE(J)=SLOPE(J)*1.E-03
      ANGLE(J)=ATAN(SLOPE(J))
      ANGLE(J)=ANGLE(J)/PI*180.
   10 WRITE(6,1) RR,KX1,KX2,LUMDA(J),SLOPE(J),ANGLE(J)
    1 FORMAT(10X,6E20.5)
    2 FORMAT(1H0,20X,2HRR,18X,3HKX1,17X,3HKX2,15X,5HLUMDA,
     C15X,5HSLOPE,15X,5HANGLE//)
      STOP
      END
'        6400 END RECORD
67.27   96.72   120.78 140.348153.347167.004
'        6400 END FILE

                 CD TOT    0040
```

```
A4231.
RUN(S,,,,,,5000)
LGO.
'        6400 END RECORD.
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     CALCULATION OF OPTIMUM LENGTH OF MACHINE
C     FOR A PARTICULAR DIAMATER OF MACHINE MINIMUM LENGTH OF MACHINE FOR
C     PARTICULAR POLE-WIDTH IS CALCULATED
C     HS WHICH SATISFIES THE Q-D FIT IS GIVEN BY THE INTERSECTION
C     OF THE STRAIGHT LINE WITH THE QSCSQ-DSCSQ CURVE
      DIMENSION BP(10),HP(10),SG(10),ST(10),LUMDA(10),DSCSQ(10),
     CHS(10),DSC(10),QSC(10),H1(10),N(10),LP(10),DR(10),QSCSQ(10)
      REAL KF,KW,MUO,LUMDA,KE,LP,N
      DATA KF,MUO,FR,KW/1.6,3.192,3200.,1.0/
      ENL=144.
      READ(5,1) DR,BP,HP,SG,ST,LUMDA,QSCSQ,DSCSQ
    1 FORMAT(10F7.)
      DO 5 K=1,10
      IF(DR(K).EQ.0.) GO TO 5
      DO 10 I=1,10
      IF(BP(I).EQ.0.) GO TO 10
      WRITE(6,2) DR(K),BP(I)
    2 FORMAT(1H1,30X,9HDIAMETER=,F10.3/36X,3HBP=,F10.3//)
      WRITE(6,4)
    4 FORMAT(10X,2HHP,11X,2HSG,11X,2HST,11X,5HLUMDA,8X,2HHS,10X,
     C3HDSC,10X,3HQSC,10X,1HN,12X,2HLP//)
      DO 100 J=1,10
      KE=4.44*FR*KW*1.0E-08
      QSC(J)=SQRT(QSCSQ(J))
      DSC(J)=SQRT(DSCSQ(J))
      HS(J)=(QSC(J)*ST(J)+DSC(J))/SG(J)
      H1(J)=HS(J)-QSC(J)
      N(J)=QSC(J)/LUMDA(J)
      LP(J)=ENL/(KE*N(J)*SG(J)*H1(J)*BP(I))
      WRITE(6,3) HP(J),SG(J),ST(J),LUMDA(J),HS(J),DSC(J),QSC(J),N(J),
     CLP(J)
    3 FORMAT(1H ,9F13.3)
  100 CONTINUE
   10 CONTINUE
    5 CONTINUE
      STOP
      END
'        6400 END RECORD
2.5
0.25
0.35   0.42   0.47   0.50   0.53
66.16  79.39  88.84  94.51  100.18
96.72  120.78 140.34 153.34 167.60
12.03  9.42   8.04   7.40   6.73
4.65E053.95E053.35E052.80E052.40E05 0.      0.      0.      0.      0.
1.37E081.90E082.10E082.10E082.00E08
'        6400 END FILE

                         CD TOT    0052
```

REFERENCES

REFERENCES

1. Evershed, S., "Theory and Practice of Permanent Magnets", Proc. I.E.E., Vol. 58, (1920).

2. Evershed, S., "Theory and Practice of Permanent Magnets", Proc. I.E.E., Vol. 63, (1925).

3. Watson, E.A., "The economic Utilization of Permanent Magnets in Electrical Apparatus". Proc. I.E.E., Vol. 61, (1923).

4. Hornfeck and Edgar, "The Output and Optimum Design of Permanent Magnets Subjected to Demagnetizing Forces", A.I.E.E. Transactions, Vol. 59, (1940).

5. Underhill, Earl M., "Permanent Magnet", Electronics, December, (1943).

6. Underhill, Earl M., "Designing and Stabilizing Permanent Magnets", Electronics, January, (1944).

7. Desmond, D.J., "The Economic Utilization of Modern Permanent Magnets", Journal, I.E.E., Vol. 92, (1945).

8. Tyrrel, A.J., "The Designing and Application of Modern Permanent Magnets", Journal, British Institute of Radio Engineers, Vol. 6, (1946).

9. Gould, J.E., "Progress in Permanent Magnet Materials", I.E.E., December (1959).

10. Scott, K.L., "Magnet Steels and Permanent Magnets - Relationship among their Magnetic Properties", A.I.E.E. Journal, June (1932).

11. Parker, R.J., "Understanding and Predicting Permanent Magnet Performance by Electrical Analog Methods", Journal of Applied Physics, (1958).

12. Thomas and Ireland, "The Design and Performance of Permanent - Magnet Rotors Electrical Manufacturing", August, (1947).

13. Saunders and Weekly, "Design of Permanent Magnet Alternators", A.I.E.E. Transactions, Vol. 70, (1951).

14. Ginsberg, David, "Design Calculations for A.C. Generators", A.I.E.E. Transaction, Vol. 69, (1950).

15. Strauss, Fritz, "Synchronous Machines with Rotating Permanent - Magnet Fields", A.I.E.E. Transaction, Vol. 71, October, (1952).

16. Brainard, M.W., "Synchronous Machines with Rotating Permanent - Magnet Fields", A.I.E.E. Transaction, Vol. 71, August, (1952).

17. Hershberger, D.D., "Design Considerations of Fractional Horsepower Size Permanent - Magnet Motors and Generators", A.I.E.E., Vol. 72, June, (1953).

18. Puder and Strauss, "Salient - Pole Permanent Magnet Alternators for High Speed Drive", A.I.E E. Transaction, Vol. 78, November, (1957).

19. Merril, F.W., "Permanent Magnet Synchronous Motors", A.I.E.E., Trans. Vol.74.

20. Ginsberg and Miesenheimer, "Design Calculations for Permanent Magnet Generators", A.I.E.E., April, (1953).

21. Hanrahan and Toffolo, "Permanent Magnet Generators Part I - Theory", A.I.E.E., Vol. 78, June, (1957).

22. Hanrahan and Toffolo, "Permanent Magnet Generators Optimum Design", A.I.E.E., Vol. 58, April, (1953).

23. Alger, P.L., "Calculation of Synchronous Machine Constants", A.I.E.E., June, (1931).

24. Kilgore, L.A., "Calculation of Synchronous Machine Constants", A.I.E.E., June, (1931).

25. Gould, C., M.Sc. Thesis, University of Birmingham, (1953).

26. Bgchi, C., M.Sc. Thesis, University of London, (1958).

27. Saunders, R.M.,"Digital Computer as an Aid in Electrical Machine Design", A.I.E.E., May (1954).

28. U.S. Patent - No. 2643350.

29. U.S. Patent - No. 2525455.

30. U.S. Patent - No. 2719931.

31. U.S. Patent - No. 2632123.

33. Parker and Studder, "Permanent Magnets and their Application", Book, (1962).

34. Hadfield, D., "Permanent Magnets and Magnetism", Book, (1962).

35. Ireland, James R., "Ceramic Permanent Magnet Motors", Book, (1968).

36. Brailsford, F., "Magnetic Materials", Book, (1960).

37. Roters, Herbert C., "Electromagnet Devices", Book, (1955).

38. Spreadbury, "Permanent Magnets", Book, (1949).

39. Puchstein and Lloyd, "Alternating Current Machinery", Book, (1964).

40 Kuhlman, "Design of Electrical Apparatus", Book, (1950).