Minimum Variance Linear Receiver for Interference Rejection in Space-Time Block Code Communications

MINIMUM VARIANCE LINEAR RECEIVER FOR INTERFERENCE REJECTION IN SPACE-TIME BLOCK CODE COMMUNICATIONS

BY

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NOVEMBER 2004

A THESIS

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

AND THE SCHOOL OF GRADUATE STUDIES

OF MCMASTER UNIVERSITY

IN PARTIAL FULFILMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

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Master of Applied Science (2004) McMaster University

(Electrical & Computer Engineering) Hamilton, Ontario

TITLE:	Minimum Variance Linear Receiver for Interference Rejection in Space-Time Block Code Communications
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NUMBER OF PAGES:	xii, 68

Dedication

To my parents and family
for their guidance, support and love
without them this work was impossible
and to the memory of my beloved niece Sadaf

Acknowledgement

I like to express my thanks to my supervisor, Prof. Kon Max Wong for his precious time and constructive comments on my thesis and my research, and I would like to express my appreciation to Dr. Shahram Shahbazpanahi and Dr. Mohammad Gharavi Alkhansari for their guidance and support, without them this work was impossible. I am thankful to Prof. Alex Gershman for giving me the directions and motivation to work on my course project on which this thesis is based. I would like to thank Prof. Tim Davidson for his comments which encouraged me to study more in this area.

I express my thanks to our secretaries, specially Cheryl Gies and Helen Jachna for their efforts and patience in answering my questions and requests.

Abstract

In this thesis, we consider a wireless communication system which has several transmitters (users), and one receiver (base station). The goal is to extract the data from the desired transmitter, using received data . To extract data from the desired transmitter, we have to combat noise and interferences from the other transmitters (users). In order to decrease the probability of error we use space diversity; therefore, the desired transmitter (user), and the receiver (base station) are equipped with arrays of antennas.

We use Orthogonal Space-Time Block Codes (OSTBC) because they reach full diversity gain. Furthermore, these codes enable us to introduce a new linear receiver, which suppresses the interference and decodes the received data simultaneously.

The proposed receiver is designed by enforcing an *underline operator* on the vector of the received data. We minimize the output power subject to a constraint which preserves a unity gain for all symbols in the desired transmitter's block of data, at the same time, the intra block interference (self-interference) for the desired transmitter (user) is completely suppressed.

Simulation results show that the proposed method has a substantially lower symbol error rate compared to the matched filter receiver and another linear receiver presented by Naguib and Seshadri. Note that in the absence of interference, our receiver simplifies to the matched filter, which is the Maximum Likelihood receiver for a single user MIMO system using Orthogonal Space-Time Block Codes.

Notations and Abbreviations

Symbol	Definition
x	scalar
$\mathbf{x}$	vector
$\mathbf{X}$	matrix

Abbreviations

CSI	Channel State Information
DL	Diagonal Loading
IC	Interference Canceller
i.i.d.	independent identically distributed
INR	Interference to Noise Ratio
ITU	International Telecommunication Union
LDC	Linear Dispersion Code
LOS	Line Of Sight
MAI	Multi-Access Interference
MF	Matched Filter
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MOE	Minimum Output Energy
MUD	Multi-User Detection
MV	Minimum Variance
NLOS	Non Line Of Sight
OSTBC	Orthogonal Space Time Block Code

SER	Symbol Error Rate
SIMO	Single Input Multiple Output
SINR	Signal to Interference plus Noise Ratio
SMI	Sample Matrix Inverse
SNR	Signal to Noise Ratio
STBC	Space Time Block Code
STC	Space Time Code
STTC	Space Time Trellis Code
V-BLAST	Vertical Bell Laboratories Layered Space-Time
ZF	Zero Forcing

Notations

$E(\cdot)$	Statistical Expectation
$\sim N(\mathbf{a},\mathbf{A})$	A vector with Normal distribution, expectation $\mathbf{a}$ and covariance $\mathbf{A}$
dB	Decibel

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Chapter 1

Introduction

In the first section of this chapter, wireless channel modeling in the presence of fading is discussed. In the second section, single input single output systems and their reliability in the presence of fading are introduced. Section 1.3 mentions MIMO systems and their advantages compared to the single input single output systems. Different transmission schemes and space-time codes are mentioned in Section 1.4 and Section 1.5, respectively. Section 1.6 gives a brief introduction to the applications of MIMO systems, and some groups doing research in this area.

1.1 Wireless Channel

In a wireless communication system, due to the time varying behavior of the channel, the amplitude of the received signal can undergo severe fluctuations. These fluctuations can be either the result of the existence of multiple paths between transmitter and receiver or the result of the time varying behavior of the channel. In the communication literature, the fluctuation caused by the existence of multiple paths is called *multi-path fading* phenomenon, and the fluctuation resulted from the time variations of the channel is called *time variation fading.*

The multi-path fading is characterized by *maximum excess delay time.* The maximum excess delay time is defined as the time difference between the first and the last copies of

a transmitted impulse when they are received at the receiver [4]. A channel is said to be flat fading if the symbol period is greater than the maximum excess delay time [4]. In this thesis, it is assumed that this condition is satisfied, and the flat fading channel is considered.

Time variations of the channel may happen due to the movement of the transmitter and the receiver. Depending on how fast the receiver and the transmitter move, a channel is called fast or slow fading. To distinguish fast and slow fading, one has to compare the symbol period with the *coherence time of the channel* [4]. The coherence time of a channel is the period during which the channel can be assumed to be constant. If this channel coherence time is much larger than the symbol period then the channel is called slow fading, otherwise it is referred to as fast fading [4]. The case of slow fading is considered throughout this thesis.

When the channel is flat fading, it can be represented by a complex scalar gain (or loss). This coefficient can be modeled as random variable due to the random behavior of the channel. The Rayleigh and Rician distributions are two most popular distributions that are considered as the distribution of flat fading channel coefficient. A complex random variable with real and imaginary parts shown as X and Y has Rayleigh distribution if X and Y are two independent Gaussian random variables with zero mean and variance σ^2. Defining r and ϕ, respectively, as the magnitude and the phase of a Rayleigh complex random variable, probability distribution function (pdf) in the polar coordinate is given by

$$p(r,\phi) = \frac{r}{2\pi\sigma^2}\exp(-\frac{r^2}{2\sigma^2}) \tag{1.1}$$

which is independent of ϕ. It can be seen that the phase is independent of the magnitude and it is uniformly distributed over the interval $[0, 2\pi)$.

The Rician probability distribution function for a complex random variable with magnitude r and phase ϕ is given by

$$p(r,\phi) = \frac{r}{2\pi\sigma^2}\exp(-\frac{r^2 + A^2 - 2A\cdot r\cos(\phi)}{2\sigma^2}) \tag{1.2}$$

In a wireless communication channel, the constant A represents the channel path gain on the direct path from the transmitter to the receiver. It can be seen that if the direct path

Figure 1.1: SISO wireless communication system

does not exist then the Rician pdf simplifies to the Rayleigh pdf. The component that is received via the direct path between the transmitter and the receiver is called Line-Of-Sight (LOS) component, and the components received via other paths are called Non-Line-Of-Sight (NLOS) components. To compare the power of these two components, *K factor* is defined as the ratio of the power of the LOS component to the mean power of the NLOS component [2]. If the K factor is zero or almost zero the Rayleigh distribution can be chosen as the distribution of the channel coefficients, otherwise the Rician distribution will be used. In the simulation part of the Chapter 3, it is assumed that the channel is flat Rayleigh fading. This assumption is used only for simulating purposes, and the new linear receiver presented in this thesis is applicable for any flat fading channel.

1.2 SISO Communication Systems

A Single Input Single Output (SISO) communication system is shown in the Fig.1.1.
In a SISO system the communication link is not reliable because if the transmitted signal experiences a deep fade then it may not be possible to recover it at the receiver.

Figure 1.2 shows the effect of fading. This figure shows the fluctuations in the channel gain due to the fading phenomenon: In deep fades the Signal-to-Noise Ratio (SNR) can be very low and it may not be possible to recover the signal.

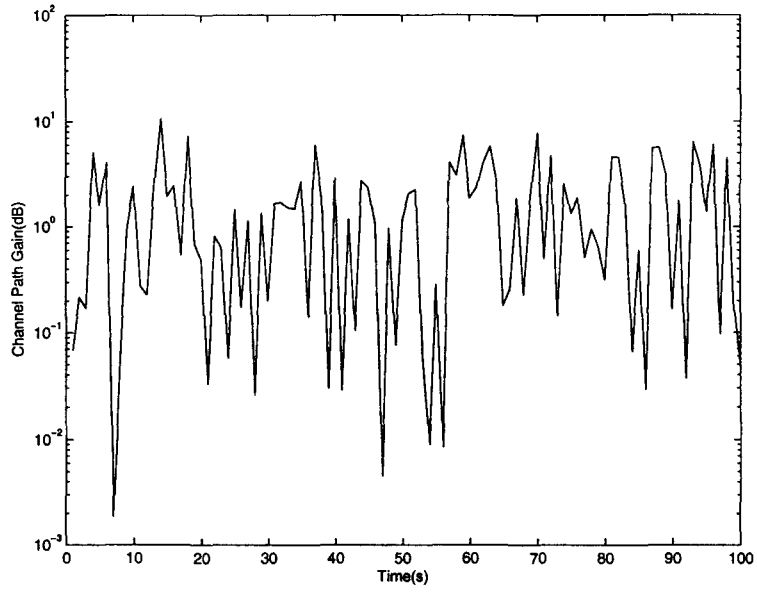

Figure 1.2: Channel gain for a SISO system in the presence of fading.

1.3 MIMO Communication Systems

Multiple Input Multiple Output communication (MIMO) systems deploy multiple antennas both at the transmitter and at the receiver. Figure 1.3 shows a typical MIMO system. As shown in this figure, the transmitter performs coding and modulation to obtain the transmitter signals for multiple antennas, and transmits the so-obtained signals over multiple antennas. As it is the case in any other communication system, the transmitted signals are affected by the channel, are corrupted by the noise, and are then received by the receiver. The receiver demodulates the received signal and then decodes the data. The goal of the whole process is to recover the transmitted information bit stream with the lowest possible error level.

In a MIMO system the transmitted signals reach the receiver through different paths. It is less likely that all these paths suffer from a deep fade at the same time. This fact has been shown in Fig.1.4 and Fig.1.5. These figures show the channel path gains for two separate paths, $|h_1|^2$ and $|h_2|^2$ in dB, and $|h_1|^2 + |h_1|^2$, it can be seen that there is less

Figure 1.3: MIMO wireless transmission system (This Figure is based on Fig.1. in [2])

fluctuation in $|h_1|^2 + |h_1|^2$.

To quantitatively study the advantages of MIMO system compared to SISO system, one can use the elements of information theory. Foschini in [5] and Telatar in [6] have shown that for a flat Rayleigh fading channel, the ergodic capacity of a MIMO channel increases linearly with the minimum of the number of transmitter and receiver antennas. This linear growth of the capacity is an important property of MIMO systems which has stimulated an enormous amount of research efforts. The capacity increase for MIMO systems is achieved without any need for using more frequency bandwidth, or without increasing the transmitted power. There is however a price to be paid: more complicated hardware and more complex coding, modulation, and decoding processes are needed.

MIMO systems combine two different diversity techniques: Receive and Transmit diversity. Receive diversity based techniques use multiple antennas at the receiver [7]. In these techniques, the goal is to enhance the quality of the signal by properly combining the received signals by different antennas.

On the other hand, transmit diversity based techniques use multiple antennas at the transmitter side. Contrary to receive diversity which has been exploited for quit a long time [7], transmit diversity is a rather new topic in wireless communications. Transmit diversity techniques aim at transmitting signals by different transmitter antennas (and possibly at different time slots) to combat space-time dependent fading. MIMO systems combine transmit and receive diversity to combat fading more efficiently.

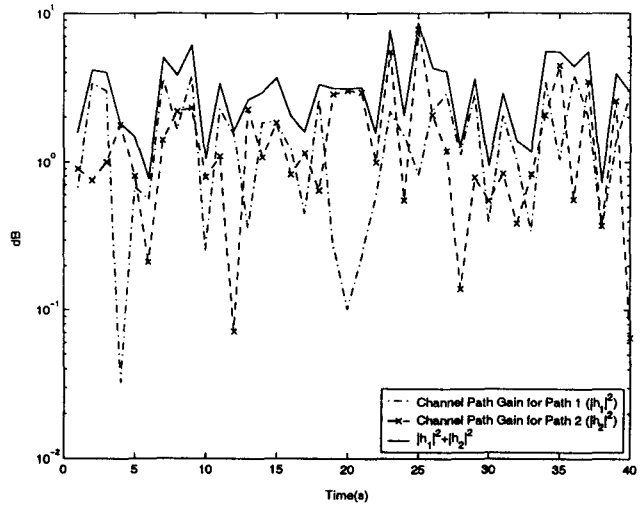

Figure 1.4: Channel Gains for MIMO and SISO systems for Rayleigh Channel

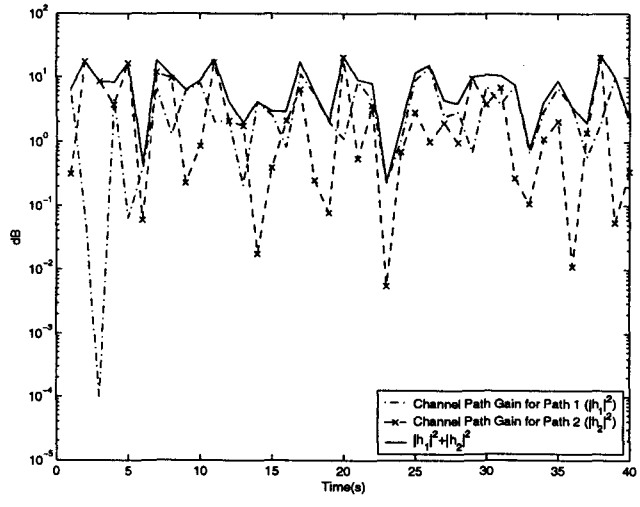

Figure 1.5: Channel Gains for MIMO and SISO systems for Rician Channel

1.4 Transmission Schemes for MIMO Communication Systems

Transmission schemes proposed in the literature for a MIMO system can be divided into two major categories, *data rate maximization transmission schemes* and *diversity maximization transmission schemes.*

1.4.1 Data Rate Maximization Transmission Schemes

In data rate maximization schemes, the goal is to increase the transmission data rate for a fixed level of error probability. An example of these schemes is Vertical Bell Laboratories Layered Space-Time (VBLAST) technique [8], which is shown in Fig.1.6.

In VBLAST, a stream of symbols is demultiplexed among the different transmitter antennas; therefore, the transmitted symbols are independent of each other in time and space. Using the Channel State Information (CSI) knowledge, the receiver detects the transmitted symbols through a nullifying and cancelling process. The receiver first nullifies the effect of the channel for the strongest symbol and then knowing the channel coefficients for that symbol, it removes the contribution of that symbol from the received data vector. This nullifying and cancelling process continues until all of the transmitted symbols are detected.

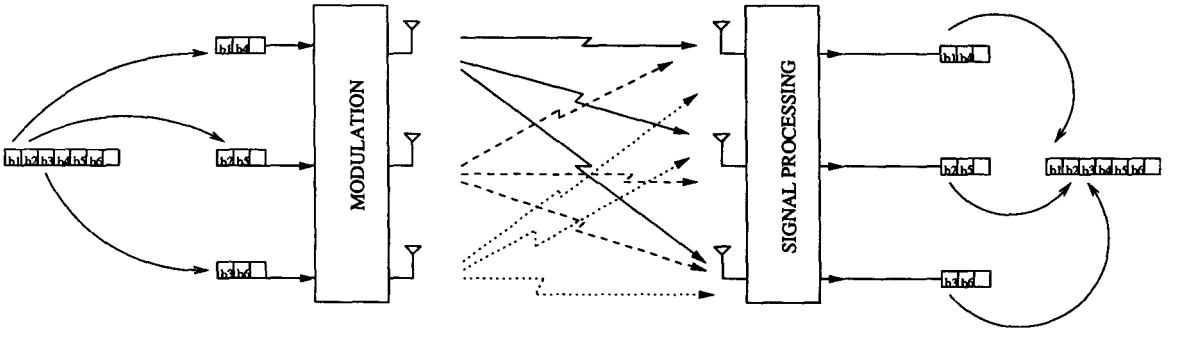

Figure 1.6: VBLAST, This figure is based on Fig.2 in [2]

VBLAST has two major drawbacks: first, in the process of nullifying and cancelling if a wrong decision is made for one symbol, then the error propagates to the next step, and the remaining symbols will be incorrectly detected. Second, if one of the transmitted symbols goes through a deep fade then because there is not any redundancy (diversity) in the transmission that symbol will be lost.

1.4.2 Diversity Maximization Transmission Schemes

These schemes improve the error performance for a constant transmission data rate. Their approaches are based on introducing redundancy in the transmitted symbols in time and space. This redundancy provides differently faded versions of the transmitted data at different time slots for the receiver. This in turn helps the receiver to combat fading and to reduce the error probability.

As is evident from their name, these schemes maximize the diversity gain. Diversity gain for a MIMO system is defined based on the reduction rate of probability of error when SNR is increased. More specifically, if the probability of error for a system is proportional to SNR^{-d} then d is called the diversity gain. For a MIMO system with N transmitter and M receiver antennas the maximum diversity gain is MN. One example of the diversity maximization schemes is Orthogonal Space Time Block Code (OSTBC). By sacrificing transmission rate, OSTBCs achieve maximum diversity gain.

1.4.3 Fundamental Trade-off between Diversity and Multiplexing Gain

In a MIMO system, if the data rate is kept constant, then for higher SNRs, the gap between the data rate and the capacity grows, because the capacity increases as the SNR increases. In order to reduce this gap, one can increase the number of bits per symbol for higher SNRs, and consequently can increase the rate as SNR increases. In such a scheme the rate of transmission is not constant any more and changes with SNR. If the data rate is shown as R, then the *multiplexing gain* is defined as, [3]

$$r = \lim_{\mathrm{SNR}\to\infty} \left(\frac{R}{\log \mathrm{SNR}}\right). \tag{1.3}$$

Based on this definition, the multiplexing gain for a scheme which has a fixed data rate is equal to zero. While diversity gain can be seen as a measure of the error performance, the

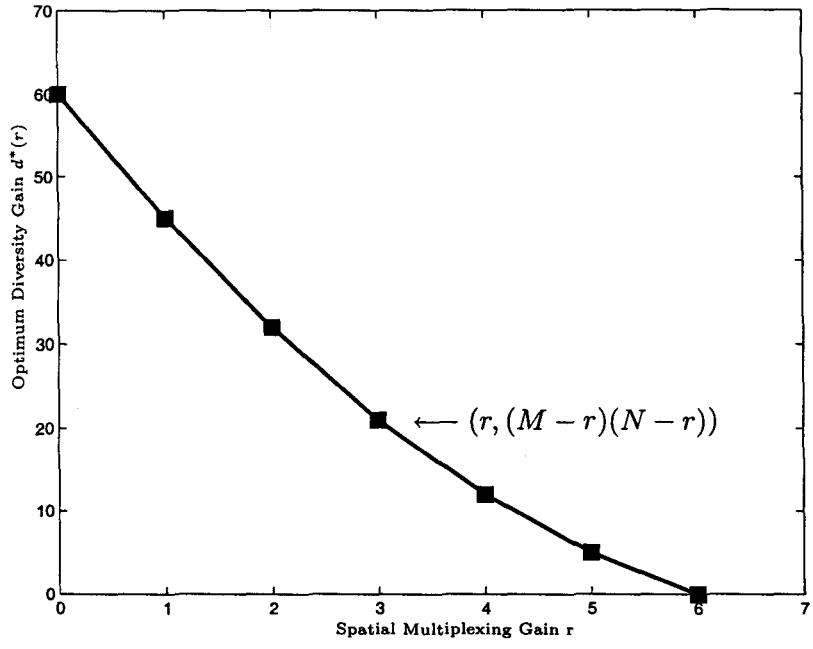

Figure 1.7: Optimal Diversity-Multiplexing tradeoff, $d^*(r)$ for $M = 10$ and $N = 6$ and $T \geq M + N - 1$, using Formula (1.3). This figure is based on Fig.1 in [3].

multiplexing gain can be viewed as a measure of the data rate. The relationship between the maximum achievable diversity gain and the multiplexing gain is shown by the optimal trade-off curve [3] shown in the Fig.1.7.

Zheng and Tse proved that for an i.i.d flat Rayleigh fading channel which remains constant for T time slots while $T \geq M + N - 1$, the relationship between the maximum achievable diversity gain and the multiplexing gain is simple [3]:

$$d^*(r) = (M - r)(N - r) \tag{1.4}$$

where $d^*(r)$ is the maximum achievable diversity gain. Using (1.4) for a MIMO system with $M = 10$ receiver and $N = 6$ transmitter antennas, the optimal trade-off curve is shown in Fig.1.7. This figure shows that there is a trade-off between the multiplexing gain and the

diversity gain.

1.5 Space-Time Codes

Space-Time Coding (STC) is an efficient tool to provide diversity. The first attempt to design a STC for MIMO system resulted in trellis coding [9]. Unfortunately, the decoding of trellis codes requires a multidimensional Viterbi algorithm whose complexity increases exponentially with the number of the states in the trellis.

1.5.1 Space Time Block Codes

To provide a coding scheme with affordable decoding computational complexity, Space-Time Block Codes (STBC) were presented in [10], [11]. In space-time block coding a vector of information symbols is mapped to a code matrix with number of columns equal to the number of transmitter antennas, and number of rows equal to the length of the STBC. Each column of the code matrix will be transmitted by the transmitter antennas during the block period [12]. If the vector of information symbols has K entries, and the block length is T, then the code rate is defined as K/T. The set of all possible code matrices is called a *code book*. To decode STBCs, a Maximum Likelihood (ML) receiver searches through the code book to find the code which maximizes the likelihood function.

1.5.2 Orthogonal Space Time Block Codes

The set of Orthogonal Space Time Block Codes (OSTBCs) is a sub-class of STBCs in which the information symbol vector $\mathbf{s} = [s_1, s_2, ..., s_K]$ is mapped to a $T \times N$ code matrix $\mathbf{X}(\mathbf{s})$ which has the following properties:

- all elements of $\mathbf{X}(\mathbf{s})$ are linear functions of the K complex variables $s_1, s_2, ..., s_K$ and their complex conjugates.
- for any arbitrary $\mathbf{s}$, the relation $\mathbf{X}^H(\mathbf{s})\mathbf{X}(\mathbf{s}) = \|\mathbf{s}\|^2\mathbf{I}_N$ is satisfied, where $\|\cdot\|$ is the l_2 norm, and $\mathbf{I}_N$ is the $N \times N$ identity matrix.

while $\{\cdot\}^H$ represents hermitian transpose. As an example for OSTBCs the Alamouti's code is shown here [11].

$$\mathbf{X(s)} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \tag{1.5}$$

where * represents complex conjugate. This code is the simplest OSTBC and it is also the only full rate complex OSTBC. Another example of OSTBCs is the 3/4 rate code [12].

$$\mathbf{X(s)} = \begin{bmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(-s_1-s_1^*+s_2-s_2^*)}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{(s_2+s_2^*+s_1-s_1^*)}{2} \end{bmatrix} \tag{1.6}$$

For a point to point MIMO system with N transmitter and M receiver antennas, and flat fading channel, the relation between the transmitted data vector and the received data vector at time slot t, is given by [3], [13], [14].

$$\mathbf{y}(t) = \mathbf{x}(t)\mathbf{H} + \mathbf{v}(t) \tag{1.7}$$

where $\mathbf{x}(t)$ and $\mathbf{y}(t)$ are the transmitted data vector and the received data vector at the time slot t, respectively, $\mathbf{H}$ is the channel matrix, and $\mathbf{v}(t)$ is the vector of complex i.i.d. white Gaussian noise. The vectors $\mathbf{x}(t)$, $\mathbf{y}(t)$ and $\mathbf{v}(t)$ are shown here.

$$\mathbf{y}(t) = [y_1(t)\ y_2(t)\ \ldots y_M(t)] \tag{1.8}$$

$$\mathbf{x}(t) = [x_1(t)\ x_2(t)\ \ldots x_N(t)] \tag{1.9}$$

$$\mathbf{v}(t) = [v_1(t)\ v_2(t)\ \ldots v_M(t)] \tag{1.10}$$

Over period of T consecutive time slots for which the channel is constant, the received and transmitted data vectors can be stacked in matrices $\mathbf{Y}$ and $\mathbf{X}$, respectively. Doing so, the relationship between the received data matrix and the transmitted data matrix can be shown in matrix form as, [3], [13], and [15]:

$$\mathbf{Y} = \mathbf{XH} + \mathbf{V} \tag{1.11}$$

where

$$\mathbf{Y} \triangleq \begin{bmatrix} \mathbf{y}(1) \\ \mathbf{y}(2) \\ \vdots \\ \mathbf{y}(T) \end{bmatrix}, \quad \mathbf{X} \triangleq \begin{bmatrix} \mathbf{x}(1) \\ \mathbf{x}(2) \\ \vdots \\ \mathbf{x}(T) \end{bmatrix}, \quad \mathbf{V} \triangleq \begin{bmatrix} \mathbf{v}(1) \\ \mathbf{v}(2) \\ \vdots \\ \mathbf{v}(T) \end{bmatrix} \tag{1.12}$$

Assuming that $\mathbf{X}$ is an OSTBC, we hereafter denote it as $\mathbf{X}(\mathbf{s})$. It is assumed that $\mathbf{s} \in \mathcal{S}$, where $\mathcal{S} = \{\mathbf{s}^{(1)}\ \mathbf{s}^{(2)}\ \ldots \mathbf{s}^{(\mathrm{L})}\}$ is the set of all possible symbol vectors, and L is the cardinality of this set.

An OSTBC can be expanded using its *dispersion matrices*, this can be shown as, [16], [17]:

$$\mathbf{X}(\mathbf{s}) = \sum_{k=1}^{K} (\mathbf{C}_k \mathrm{Re}\{s_k\} + \mathbf{D}_k \mathrm{Im}\{s_k\}) \tag{1.13}$$

where, $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ stand for real and imaginary parts, and s_k is the kth symbol in the information symbol vector, K is the length of the information symbol vector, $\mathbf{C}_k$ and $\mathbf{D}_k$ are dispersion matrices, defined as:

$$\mathbf{C}_k \triangleq \mathbf{X}(\mathbf{e}_k) \tag{1.14}$$

$$\mathbf{D}_k \triangleq \mathbf{X}(j\mathbf{e}_k) \tag{1.15}$$

where, $j = \sqrt{-1}$, and $\mathbf{e}_k$ is a $K \times 1$ vector having a "1" in the kth position and zeros elsewhere. Substituting (1.13) back in (1.11) one can write,

$$\mathbf{Y} = (\sum_{k=1}^{K} (\mathbf{C}_k \mathrm{Re}\{s_k\} + \mathbf{D}_k \mathrm{Im}\{s_k\})) \cdot \mathbf{H} + \mathbf{V} \tag{1.16}$$

Using (1.16), one can rewrite (1.11) in the following way [17], [18]:

$$\underline{\mathbf{Y}} = \mathbf{A}\underline{\mathbf{s}} + \underline{\mathbf{V}} \tag{1.17}$$

where the "underline" operator for any matrix $\mathbf{P}$ is defined as

$$\underline{\mathbf{P}} = \begin{bmatrix} \mathrm{vec}\{\mathrm{Re}(\mathbf{P})\} \\ \mathrm{vec}\{\mathrm{Im}(\mathbf{P})\} \end{bmatrix} \tag{1.18}$$

and vec$\{\cdot\}$ is the vectorization operator stacking all columns of a matrix on top of each other. Here the $2MT \times 2K$ real matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = [\underline{\mathbf{C}_1\mathbf{H}} \ \ldots \ \underline{\mathbf{C}_K\mathbf{H}} \ \underline{\mathbf{D}_1\mathbf{H}} \ \ldots \ \underline{\mathbf{D}_K\mathbf{H}}] \tag{1.19}$$

This matrix has an important property that its columns have the same norm and are orthogonal to each other [14],[17], [18]:

$$\mathbf{A}^T\mathbf{A} = \|\mathbf{H}\|_F^2\mathbf{I}_{2K} \tag{1.20}$$

where T denotes the transpose and, $\|\cdot\|_F$ denotes the Ferobenius norm.

The optimal (ML) space-time decoder is based on the nearest neighbor decoding principle. It uses CSI to find the closest point to the received signal in the noise-free observation space $\mathcal{Y} = \{\mathbf{Y}^{(1)}, \mathbf{Y}^{(2)}, \ldots, \mathbf{Y}^{(L)}\}$, i.e., it obtains

$$l_{\text{opt}} = \underset{l\in\{1,\ldots,L\}}{\arg\min} \|\mathbf{Y} - \mathbf{Y}^{(l)}\|_F \tag{1.21}$$

and then uses this index to decode the transmitted bits [10]. Here $\mathbf{Y}^{(l)}$ is the noise-free received signal matrix that corresponds to the vector of information symbols $\mathbf{s}^{(l)}$.

The ML receiver can also be viewed as a Matched Filter (MF) [19]. Matched filter is a linear receiver which maximizes SNR at the receiver. It has been shown in [14] that (1.21) is equivalent to the MF linear receiver which computes the following estimate of $\underline{\mathbf{s}}$:

$$\hat{\underline{\mathbf{s}}} = \frac{1}{\|\mathbf{H}\|_F^2}\mathbf{A}^T\underline{\mathbf{Y}} \tag{1.22}$$

and builds the estimate of the vector $\mathbf{s}$ as $\hat{\mathbf{s}} = [\mathbf{I}_K \ j\mathbf{I}_K]\hat{\underline{\mathbf{s}}}$. The kth element of $\hat{\mathbf{s}}$ is then compared with all points in the constellation. The closest point is accepted as an estimate of kth entry of $\mathbf{s}$. This procedure is repeated for all $k = 1, \ldots, K$, that is, the decoding is done symbol-by-symbol. Therefore, the maximum likelihood detection for a MIMO system using OSTBCs is composed of a linear processing followed by a symbol by symbol detection. Achieving maximum diversity gain and having simple coding and decoding processes make the OSTBCs very attractive. These good properties are provided by trading off the rate of transmission which is the price one has to pay for using OSTBCs.

1.6 Applications of MIMO Systems

MIMO systems are currently used in indoor communication [20], for example in the mobile audio and video devices in homes, and in WLAN (Wireless Local Area Network) [20]. One example of the application of MIMO in WLAN is Airgo's AGN100 chipset. This chipset can support data rates up to 108 Mbps, which is twice as much as those of its predecessor. It can cover larger areas, and has a reasonable cost. The Airgo has already developed the router and notebook network card based on this chip. This chip can work at 2.4 GHz and 5 GHz bands and is compatible with 802.11/a/b/g standards [21]. Currently 3GPP (Third Generation Partnership Project), Airgo, Lucent, Agere are some of the companies that are working on the development of new MIMO techniques and devices, see [20]-[22].

Chapter 2

Some Existing Methods for Interference Suppression in OSTBC Communication Systems

In this chapter, we consider multiple-access MIMO communications systems, where several transmitters equipped with multiple-antennas are communicating with a receiver which is also equipped with multi-antennas. For such systems, several multi-access reception schemes that have been presented in the literature will be reviewed.

2.1 Data Model

A multi-access MIMO wireless communication system for P transmitters is shown in the Figure 2.1. Assuming that each transmitter is using OSTBCs to encode its data, the received data matrix is the superposition of the received data matrices of each user, i.e.

$$\mathbf{Y} = \sum_{p=1}^{P} \mathbf{X}_p(\mathbf{s}_p)\mathbf{H}_p + \mathbf{V} \tag{2.1}$$

while $\mathbf{X}_p(\mathbf{s}_p)$ is the $T \times N$ code matrix of pth transmitter, $\mathbf{H}_p$ is the $N \times M$ channel matrix for pth transmitter, and $\mathbf{s}_p$ is the information symbol vector of the pth transmitter. It is

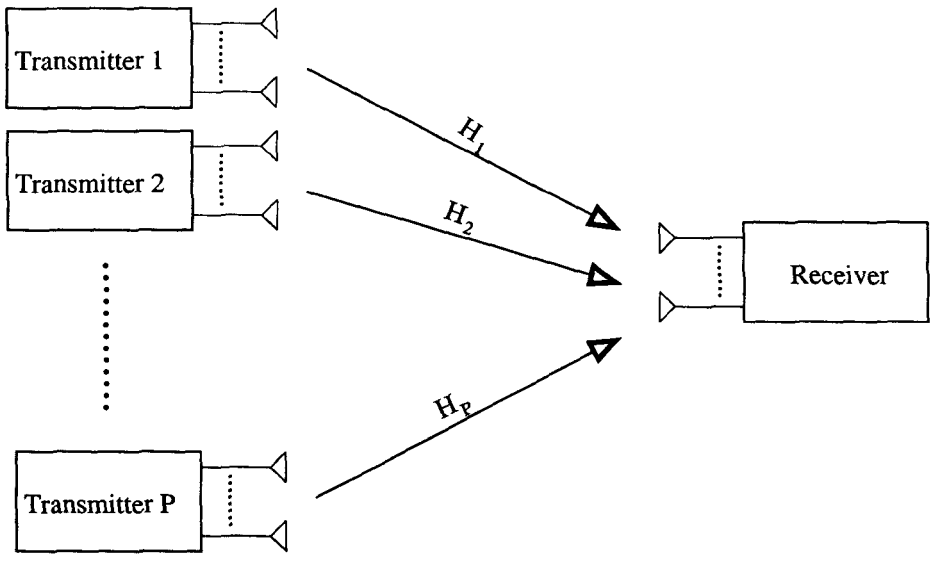

Figure 2.1: Diagram of a Multi-Access MIMO System.

assumed that following conditions are satisfied:

$$\mathrm{E}\{\underline{\mathbf{s}_p}\,\underline{\mathbf{s}_p}^H\} = \frac{\sigma_p^2}{2}\mathbf{I}_{2K} \tag{2.2}$$

$$\mathrm{E}\{\underline{\mathbf{s}_p}\,\underline{\mathbf{s}_k}^H\} = \mathbf{0} \quad \text{for } p \neq k \tag{2.3}$$

where $\mathbf{0}$ is $2K \times 2K$ zero matrix, $\mathrm{E}\{\cdot\}$ stands for statistical expectation, and σ_p^2 is the power of the symbols in the constellation of the pth transmitter. Furthermore the information symbol vectors are independent of noise for all transmitters and all time slots. Employing the underline operator defined in (1.18) on (2.1), it can be shown as

$$\underline{\mathbf{Y}} = \sum_{p=1}^{P} \mathbf{A}_p \underline{\mathbf{s}_p} + \underline{\mathbf{V}} \tag{2.4}$$

where $\mathbf{A}_p \triangleq [\underline{\mathbf{C}_{1,p}\mathbf{H}_p} \ldots \underline{\mathbf{C}_{K,p}\mathbf{H}_p}\ \underline{\mathbf{D}_{1,p}\mathbf{H}_p} \ldots \underline{\mathbf{D}_{K,p}\mathbf{H}_p}]$, while $\mathbf{C}_{k,p}$ and $\mathbf{D}_{k,p}$ are dispersion matrices for the code matrix of the pth transmitter defined as

$$\mathbf{C}_{k,p} \triangleq \mathbf{X}_p(\mathbf{e}_k) \tag{2.5}$$

$$\mathbf{D}_{k,p} \triangleq \mathbf{X}_p(j\mathbf{e}_k) \tag{2.6}$$

and $\mathbf{e}_k$ is a $K \times 1$ vector having a "1" in the kth position and zeros elsewhere.

2.2 ML Multi-User Detection

For the above-mentioned data model, with the assumption of white Gaussian noise, ML detection is equal to the solution of the following optimization problem:

$$\min_{\{\mathbf{s}_p\}_{p=1}^{P}} \|\mathbf{Y} - \sum_{p=1}^{P} \mathbf{X}_p(\mathbf{s}_p)\mathbf{H}_p\|_F^2 \tag{2.7}$$

or, equivalently

$$\min_{\{\mathbf{s}_p\}_{p=1}^{P}} \|\underline{\mathbf{Y}} - \sum_{p=1}^{P} \mathbf{A}_p \underline{\mathbf{s}_p}\|^2 \tag{2.8}$$

To solve these optimization problems, the receiver needs to know the CSI for all transmitters. Moreover, the computational complexity of this ML multi-user receiver grows exponentially with the number of transmitters, number of symbols per block K.

Sphere decoding is one method presented to decrease the computational complexity of the ML detection. To maximize the likelihood function at the receiver, one has to decode the transmitted symbols $\{\mathbf{s}_p\}_{p=1}^{P}$ by searching through all points in the constellation, such an exhaustive search can be quite time consuming even for low to moderate number of users. Sphere decoding decreases the number of points in the constellation to be checked, by checking only those points which lie in a sphere around the observation point. To apply this technique to the data model presented here, the receiver has to know the CSI for all transmitters.

2.3 ML Detection for a Point-to-Point MIMO System Using OSTBCs in the Presence of Interference

In this method, presented in [13], a point-to-point MIMO system is considered. It is assumed that L information symbol vectors of K symbols each, are transmitted by the transmitter. Each symbol in the above-mentioned information symbol vectors comes from a complex constellation $\mathcal{X}$. The kth symbol in the lth information symbol vector is shown as $s_{k,l}$, and the set of transmitted symbols are $\{s_{k,l}\}_{k=1,l=1}^{K,L}$. It is assumed that the constellation $\mathcal{X}$ is

of unit modulus, i.e., $|s_{k,l}|=1$ for all l and k. The number of antennas at the transmitter is equal to N, and there are M antennas at the receiver. Assuming a flat fading channel, the channel matrix $\mathbf{H}$ is shown as an $N \times M$ matrix [13]. The (i,j) element of $\mathbf{H}$, $(h_{i,j})$ is the complex gain between the ith transmitter antenna, and the jth receiver antenna. It is assumed that the lth information symbol vector is mapped to a $T \times N$ OSTBC code matrix $\mathbf{X}_l$ for $l = 1, ..., L$. The transmitted code matrix thus can be shown as

$$\mathbf{X} \triangleq [\mathbf{X}_1^T \ \ldots \ \mathbf{X}_L^T]^T \tag{2.9}$$

which takes LT time slots to be transmitted [13].

Using the expansion of OSTBCs based on their dispersion matrices, the code matrix $\mathbf{X}_l$ can be shown as

$$\mathbf{X}_l = \sum_{k=1}^{K} (\mathrm{Re}\{s_{k,l}\}\mathbf{C}_{k,l} + \mathrm{Im}\{s_{k,l}\}\mathbf{D}_{k,l}) \tag{2.10}$$

where $\mathbf{C}_{k,l}$ and $\mathbf{D}_{k,l}$ are $T \times N$ dispersion matrices [13]. For OSTBCs, the dispersion matrices $\mathbf{C}_{k,l}$ and $\mathbf{D}_{k,l}$ satisfy the following conditions, [1], [12], [19]:

$$\begin{aligned} &\mathbf{C}_{i,l}^H \mathbf{C}_{i,l} = \mathbf{I}, && \mathbf{D}_{i,l}^H \mathbf{D}_{i,l} = \mathbf{I} \\ &\mathbf{C}_{i,l}^H \mathbf{C}_{j,l} = -\mathbf{C}_{j,l}^H \mathbf{C}_{i,l}, && \mathbf{D}_{i,l}^H \mathbf{D}_{j,l} = -\mathbf{D}_{j,l}^H \mathbf{D}_{i,l}, \quad i \neq j \\ &\mathbf{C}_{i,l}^H \mathbf{D}_{j,l} = -\mathbf{D}_{j,l}^H \mathbf{C}_{i,l}. \end{aligned} \tag{2.11}$$

In the communication system mentioned above, the received data matrix has the following relationship with the transmitted data matrix [13]:

$$\mathbf{Y} = [\mathbf{Y}_1^T \ \ldots \ \mathbf{Y}_L^T]^T = \mathbf{XH} + \mathbf{\Phi} \tag{2.12}$$

where

$$\mathbf{\Phi} = [\mathbf{\Phi}_1^T \ \ldots \ \mathbf{\Phi}_L^T]^T = [\phi_1^T \ \ldots \ \phi_{LT}^T]^T \tag{2.13}$$

is an $(LT) \times N$ matrix representing the contribution of noise and interferers in the received data matrix. The ϕ_is are modeled as temporarily white zero mean Gaussian random vectors with covariance matrix $\mathbf{Q}$. This can be shown as:

$$\mathrm{E}[\phi_i^H \phi_i] = \mathbf{Q} \tag{2.14}$$

$$\mathrm{E}[\phi_i^H \phi_j] = 0 \ \ for \ \ i \neq j. \tag{2.15}$$

The logarithm of the likelihood function for this scenario is given as:

$$\begin{aligned} L(\mathbf{Y}|\mathbf{H},\mathbf{Q},\mathbf{X}) &= L(\mathbf{Y}|\mathbf{H},\mathbf{Q},\mathcal{X}) \\ &= -(LT)\text{logdet}(\mathbf{Q}) \\ &\quad -\text{Tr}\{\mathbf{Q}^{-1}(\mathbf{Y}-\mathbf{XH})^H(\mathbf{Y}-\mathbf{XH})\} \end{aligned} \tag{2.16}$$

where logdet$(\cdot)$ represents logarithm of the determinant, and Tr$\{\cdot\}$ stands for trace. The ML detector finds the point in $\mathcal{X}$ which maximizes the above likelihood function, or equivalently the point which is the solution for the following minimization

$$\min_{\mathcal{X}} \text{Tr}\{\mathbf{Q}^{-1}(\mathbf{Y}-\mathbf{XH})^H(\mathbf{Y}-\mathbf{XH})\} \tag{2.17}$$

which can be simplified to

$$\max_{\mathcal{X}} \text{Re}\{\text{Tr}\{\mathbf{Q}^{-1}\mathbf{H}^H\mathbf{X}^H\mathbf{Y}\}\}. \tag{2.18}$$

Substituting the expansion from (2.10) for each $\mathbf{X}_l$ in $\mathbf{X}$, the maximization problem can be shown as

$$\max_{\mathcal{X}}(\sum_{1}^{L}\sum_{1}^{K}(\text{Re}\{\text{Tr}\{\mathbf{Q}^{-1}\mathbf{H}^H(\text{Re}\{s_{k,l}\}\mathbf{C}_{k,l}+\text{Im}\{s_{k,l}\}\mathbf{D}_{k,l})^H\mathbf{Y}_l\}\})). \tag{2.19}$$

The resulted detector based on this maximization is called Coherent ML (COHML), [13], or *Clairvoyant* detector, [23].

2.3.1 Unstructured Interference and Unknown Channel

If $\mathbf{H}$ and $\mathbf{Q}$ are not known by the receiver, an approximation to the above-mentioned exact ML detector has been presented in [13]. This method estimates $\mathbf{H}$ and $\mathbf{Q}$ using a training code matrix shown as $\mathbf{X}_t$, which is an $T_t \times N$ OSTBC code. In this case the likelihood function in (2.16) is shown as $L(\mathbf{Y}_t|\mathbf{H},\mathbf{Q},\mathcal{X}_t)$, while $\mathbf{Y}_t$ is the received data matrix associated with $\mathbf{X}_t$, $\mathcal{X}_t$ represents set of constellation points for the training data. To maximize this likelihood function over $\mathbf{Q}$, one has to take the derivative of this likelihood function with respect to $\mathbf{Q}$ and equate it to zero, this is shown as

$$\frac{\partial L(\mathbf{Y}_t|\mathbf{H},\mathbf{Q},\mathcal{X}_t)}{\partial \mathbf{Q}_{ij}} = 0 \tag{2.20}$$

and $\frac{\partial}{\partial x} \triangleq (\frac{\partial}{\partial \mathrm{Re(x)}}) + i(\frac{\partial}{\partial \mathrm{Im(x)}})$. Solving (2.20) for $\mathbf{Q}$ results in $\hat{\mathbf{Q}}$, given as

$$\hat{\mathbf{Q}} = \frac{1}{T_t}(\mathbf{Y}_t - \mathbf{X}_t\mathbf{H})^H(\mathbf{Y}_t - \mathbf{X}_t\mathbf{H}) \tag{2.21}$$

which depends on $\mathbf{H}$. Substituting this $\hat{\mathbf{Q}}$ back in the likelihood function $L(\mathbf{Y}_t|\mathbf{H}, \mathbf{Q}, \mathcal{X}_t)$, and maximizing the resulted likelihood function in terms of $\mathbf{H}$, gives the estimate for $\mathbf{H}$ as

$$\hat{\mathbf{H}} = (\mathbf{X}_t^H\mathbf{X}_t)^{-1}\mathbf{X}_t^H\mathbf{Y}_t \tag{2.22}$$

which only depends on the received and transmitted training data matrices. In the process of estimating $\mathbf{Q}$ and $\mathbf{H}$ it is assumed that $T_t \geqslant M$. Substituting (2.22) back in (2.21), $\hat{\mathbf{Q}}$ can be shown as

$$\hat{\mathbf{Q}} = \frac{1}{T_t}\mathbf{Y}_t^H\Pi_{\mathbf{X}}\mathbf{Y}_t \tag{2.23}$$

where $\Pi_{\mathbf{X}} \triangleq (\mathbf{I} - \mathbf{X}_t(\mathbf{X}_t^H\mathbf{X}_t)^{-1}\mathbf{X}_t^H)$.

Plugging (2.22) and (2.23) back in the coherent ML receiver (2.19), an approximation to the exact ML can be reached. This method is called training based ML, [13].

2.4 Matched Filter

Due to the high computational complexity of ML detection, suboptimal but simple methods are of great interest, the simplest one is the matched filter. As shown in the Chapter 1, the matched filter for a point-to-point MIMO system which uses OSTBC to encode its information symbols is a linear receiver, and can be easily implemented. To recover the data transmitted by the transmitter of interest in the above-mentioned data model, one can use the matched filter.

Although for a multi-access MIMO system shown in Fig.2.1, the matched filter can be implemented easily, but its performance is far from being optimal in any sense. This non-optimality is due to the fact that matched filter ignores the structure of Multi-Access Interference (MAI) treating it as noise . Matched filter maximizes SNR while Signal-to-Interference-plus-Noise Ratio (SINR) is the proper criterion that has to be used.

2.5 Naguib-Seshadri Method

An interference suppression method has been presented for a MIMO communication system which consists of two transmitters by Naguib and Seshadri in [1]. Each transmitter uses Alamouti's scheme to encode the information symbols; therefore, each transmitter has two antennas, the base station also has two antennas. For such a system a multiuser receiver is proposed. It is assumed that the CSI for both transmitters is known by the receiver. Figure 2.2 shows the diagram of the system.

In the Naguib-Seshadri method, it is assumed that the coherence time of the channel is much larger than two time slots, [1]. This implies that

$$h_{ij}^{p}(nT_s) = h_{ij}^{p}((n+1)T_s), \qquad i,j,p = 1,2 \tag{2.24}$$

where h_{ij}^{p} is the (i,j)th entry of the Matrix $\mathbf{H}_p$, and T_s is the period of one time slot.

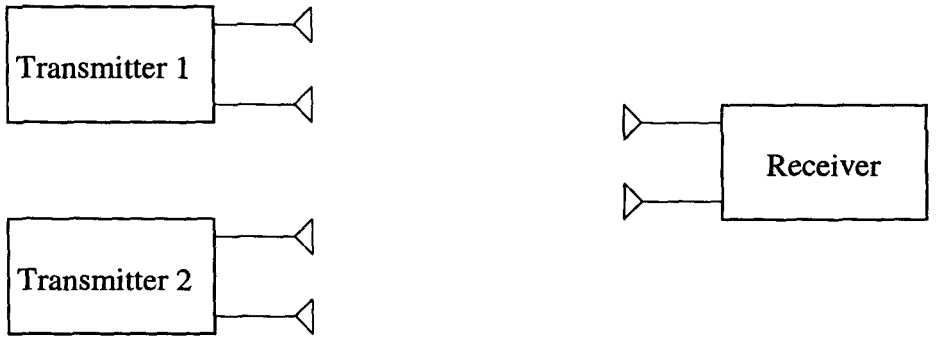

Figure 2.2: Diagram of the system for Naguib-Seshadri method.

y_{ij} and v_{ij} are the entries of the matrices $\mathbf{Y}$ and $\mathbf{V}$, respectively. The information data symbol vector for the pth transmitter is represented by

$$\mathbf{s}_p = [s_1^p \quad s_2^p]^T \quad \text{for} \quad p = 1,2 \quad \text{and} \quad \mathbf{s}_p \in \mathcal{S}_p. \tag{2.25}$$

where $\mathcal{S}_p$ is the set of constellation points for $\mathbf{s}_p$. Since the Alamouti code is used for both transmitters, the first transmitter sends s_1^1 and s_2^1 from the first and the second antenna, respectively during the first time slot. The second transmitter sends s_1^2 and s_2^2 from the first and the second antenna, respectively during the same time. During the second time slot,

the first transmitter sends $-s_2^{1*}$ and s_1^{1*} from its first and second antenna, respectively, and the second transmitter sends $-s_2^{2*}$ and s_1^{2*} from its first and second antenna, respectively.

At the receiver, the signals received by the first receiver antenna over two consecutive time slots are shown as y_{11} and y_{21}:

$$y_{11} = h_{11}^1 s_1^1 + h_{21}^1 s_2^1 + h_{11}^2 s_1^2 + h_{21}^2 s_2^2 + v_{11} \tag{2.26}$$

$$y_{21} = -h_{11}^1 s_2^{1*} + h_{21}^1 s_1^{1*} - h_{11}^2 s_2^{2*} + h_{21}^2 s_1^{2*} + v_{21}. \tag{2.27}$$

Using the following definitions

$$\mathbf{y}_1 \triangleq [y_{11}\ y_{21}^*]^T \tag{2.28}$$

$$\mathbf{v}_1 \triangleq [v_{11}\ v_{21}^*]^T \tag{2.29}$$

one can combine (2.26) and (2.27) as:

$$\mathbf{y}_1 = \mathbb{A}_1^1 \cdot \mathbf{s}_1 + \mathbb{A}_2^1 \cdot \mathbf{s}_2 + \mathbf{v}_1 \tag{2.30}$$

where the matrices $\mathbb{A}_1^1$ and $\mathbb{A}_2^1$, are defined as [1]:

$$\mathbb{A}_1^1 \triangleq \begin{bmatrix} h_{11}^1 & h_{21}^1 \\ h_{21}^{1\,*} & -h_{11}^{1\,*} \end{bmatrix}, \quad \mathbb{A}_2^1 \triangleq \begin{bmatrix} h_{11}^2 & h_{21}^2 \\ h_{21}^{2\,*} & -h_{11}^{2\,*} \end{bmatrix}. \tag{2.31}$$

The elements of the noise vectors in this method are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and a variance of $\sigma_n^2/2$ per its real or imaginary dimensions, [1]. In the same way, the signals received by the second receiver antenna over two consecutive time slots can be written in this form [1]:

$$\mathbf{y}_2 = \mathbb{A}_1^2 \cdot \mathbf{s}_1 + \mathbb{A}_2^2 \cdot \mathbf{s}_2 + \mathbf{v}_2 \tag{2.32}$$

where

$$\mathbf{y}_2 \triangleq [y_{12}\ y_{22}^*]^T \tag{2.33}$$

$$\mathbf{v}_2 \triangleq [v_{12}\ v_{22}^*]^T \tag{2.34}$$

$$\mathbb{A}_1^2 \triangleq \begin{bmatrix} h_{12}^1 & h_{22}^1 \\ h_{22}^{1\,*} & -h_{12}^{1\,*} \end{bmatrix}, \quad \mathbb{A}_2^2 \triangleq \begin{bmatrix} h_{12}^2 & h_{22}^2 \\ h_{22}^{2\,*} & -h_{12}^{2\,*} \end{bmatrix}. \tag{2.35}$$

Combining (2.30) and (2.32) yields:

$$\mathbf{y} \triangleq \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbb{A}_1^1 & \mathbb{A}_2^1 \\ \mathbb{A}_1^2 & \mathbb{A}_2^2 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}. \tag{2.36}$$

It is assumed that the first transmitter is the desired user, and the Minimum Mean Square Error (MMSE) technique is used to design a linear receiver to suppress noise and interference of the second transmitter. To estimate each symbol in the desired user's information symbol vector using MMSE criterion, a vector of four coefficients has to be found, [1]. This vector of coefficients for s_1^1 is shown as $\boldsymbol{\alpha}_1^1$, and can be found by solving the following minimization problem, [1]:

$$\boldsymbol{\alpha}_1^1 = \underset{\boldsymbol{\alpha}}{\operatorname{argmin}}\ \mathrm{E}\{|\boldsymbol{\alpha}^H \mathbf{y} - s_1^1|^2\} \tag{2.37}$$

and $|\cdot|$ stands for absolute value. The solution of (2.37) is given as:

$$\boldsymbol{\alpha}_1^1 = \mathbf{M}^{-1}\mathbf{a}_{tot1}. \tag{2.38}$$

where $\mathbf{a}_{tot1}$ is the first column of the matrix $\mathbf{A}_{tot}$ defined as

$$\mathbf{A}_{tot} \triangleq \begin{bmatrix} \mathbb{A}_1^1 & \mathbb{A}_2^1 \\ \mathbb{A}_1^2 & \mathbb{A}_2^2 \end{bmatrix} \tag{2.39}$$

and $\boldsymbol{\alpha}_1^1 = [\alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4]^T$, $\mathbf{M}$ is the covariance matrix of the received data vector $\mathbf{y}$, and $\mathbf{a}_{tot1} \triangleq \left[h_{11}^1 \ \ h_{21}^{1\,*} \ \ h_{12}^1 \ \ h_{22}^{1\,*}\right]^T$ which is the first column of $\mathbf{A}_{tot}$, [1]. Since it is assumed that the conditions (2.2) and (2.3) are satisfied for information symbol vectors, the covariance matrix of received data vector $\mathbf{y}$ can be shown as

$$\mathbf{M} = \mathbf{A}_{tot}\mathbf{A}_{tot}^H + \sigma_n^2 \mathbf{I}_4. \tag{2.40}$$

The vector of coefficients for the second symbol of the first user s_2^1, can be calculated in the same way as:

$$\boldsymbol{\alpha}_2^1 = \mathbf{M}^{-1}\mathbf{a}_{tot2} \tag{2.41}$$

where $\mathbf{a}_{tot2}$ is the second column of $\mathbf{A}_{tot}$, [1].

It can be seen that in this method, using the MMSE criterion, there are two sets of coefficients for the first user. Each vector of the coefficients mentioned above is for one symbol of the desired user.

Table 2.1 shows a summary of the Naguib-Seshadri method. The parameters $\mathbf{y}_1, \mathbf{y}_2, \mathbf{A}_{tot}$ and σ_n^2 have to be passed to the algorithm in the Table 2.1. The output of this algorithm is the MMSE estimate of the desired user's information symbol vector (e.g. $\mathbf{s}_1$), and the error value Δ which is defined in the Table 2.1 and shows the estimation error; therefore, the algorithm in the Table 2.1 can be shown as

$$(\hat{\mathbf{s}}_1, \Delta_{\hat{\mathbf{s}}_1}) = \text{MMSE.DEC}(\mathbf{y}_1, \mathbf{y}_2, \mathbf{A}_{tot}, \sigma_n^2)$$

If the desired transmitter is the second one, then the algorithm in Table 2.1 can be rewritten as

$$(\hat{\mathbf{s}}_2, \Delta_{\hat{\mathbf{s}}_2}) = \text{MMSE.DEC}(\mathbf{y}_2, \mathbf{y}_1, \mathbf{A}_{tot}, \sigma_n^2)$$

which is shown in Table 2.2.

Using Tables 2.1 and 2.2, an estimate of the desired user's information symbol vector can be found. The algorithm in Table 2.3 extends the algorithms in Tables 2.1 and 2.2 to a multi-user detection algorithm.

The algorithm in the Table 2.3, has the same inputs as the algorithm in the Table 2.1, outputs of this algorithm are the estimates of $\mathbf{s}_1$ and $\mathbf{s}_2$ denoted as $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$, and it is called two step interference cancellation algorithm and is shown as

$$(\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2) = \text{IIC.DEC}\ (\mathbf{y}_1, \mathbf{y}_2, \mathbf{A}_{tot}, \sigma_n^2)$$

This two step interference cancellation algorithm works in the following way: Using the algorithm in the Table 2.1, the information symbol vector for the first user is estimated as $\hat{\mathbf{s}}_1'$, then the contribution of the estimate of the first user's information symbol vector is removed from the received data vectors $\mathbf{y}_1$ and $\mathbf{y}_2$, the resulted vectors are shown as $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Defining $\mathcal{F}(\mathbf{s}_2)$ as shown in the table, the receiver searches through the constellation of the second transmitter to find the point which minimizes $\mathcal{F}(\mathbf{s}_2)$, this point

is shown as $\hat{s}_2'$. If $\hat{s}_1' = s_1$ then minimizing $\mathcal{F}(s_2)$ gives us the ML estimate for s_2. Here the summation of the errors in estimating s_1 and s_2 is shown as $\Delta_{s_1'} + \Delta_{s_2'}$.

Then the above-mentioned process will be repeated by first estimating s_2 using Table 2.2, and then minimizing $\mathcal{G}(s_1)$, defined in the Table 2.3 to estimate s_1. This time the estimates of s_1 and s_2 are shown as $\hat{s}_1''$ and $\hat{s}_2''$. Here the summation of the errors in estimating s_1 and s_2 is shown as $\Delta_{s_1''} + \Delta_{s_2''}$.

Finally by comparing the errors resulted from this two repetition of the above-mentioned process, receiver decides in favor of the pair with lower summation of errors as shown in the Table 2.3.

The algorithm in Table (2.1) can be used as an interference canceller even for the case that only the CSI of the desired user is known. In this case one can use the algorithm in the Table 2.1 with a minor change. This change happens in the calculation of the matrix $\mathbf{M}$. Here instead of using the channels of two users and the noise covariance matrix to find the covariance matrix $\mathbf{M}$, the sample covariance matrix $\hat{\mathbf{M}}$ has to be used.

$$\hat{\mathbf{M}} = \frac{1}{Q_{\mathbf{y}}} \sum_{q=1}^{Q_{\mathbf{y}}} \mathbf{y}_q \mathbf{y}_q^H. \tag{2.42}$$

while $Q_{\mathbf{y}}$ is the number of sample vectors used to estimate $\hat{\mathbf{M}}$.

$$
\begin{array}{|rcl|}
\hline
(\hat{\mathbf{s}}_1, \Delta_{\hat{\mathbf{s}}_1}) & = & \text{MMSE.DEC}\,(\mathbf{y}_2, \mathbf{y}_1, \mathbf{A}_{tot}, \sigma_n^2) \\
\hline
\mathbf{A}_{tot} & \triangleq & \begin{bmatrix} \mathbb{A}_1^1 & \mathbb{A}_2^1 \\ \mathbb{A}_1^2 & \mathbb{A}_2^2 \end{bmatrix} \\
\mathbf{M} & = & \mathbf{A}_{tot}\mathbf{A}_{tot}^H + \sigma_n^2 \mathbf{I}_4 \\
\mathbf{a}_{tot1} & \triangleq & \left[h_{11}^1 \quad {h_{21}^1}^* \quad h_{12}^1 \quad {h_{22}^1}^*\right]^T \\
 & = & \text{first column of } \mathbf{A}_{tot} \\
\mathbf{a}_{tot2} & \triangleq & \left[h_{21}^1 \quad -{h_{11}^1}^* \quad h_{22}^1 \quad -{h_{12}^1}^*\right]^T \\
 & = & \text{second column of } \mathbf{A}_{tot} \\
\boldsymbol{\alpha}_1^1 & = & \mathbf{M}^{-1}\mathbf{a}_{tot1} \\
\boldsymbol{\alpha}_2^1 & = & \mathbf{M}^{-1}\mathbf{a}_{tot2} \\
\hat{\mathbf{s}}_1 & = & \underset{\mathbf{s}_1 \in \mathcal{S}_1}{\text{argmin}} \; \{|{\boldsymbol{\alpha}_1^1}^H \mathbf{y} - s_1^1|^2 + |{\boldsymbol{\alpha}_2^1}^H \mathbf{y} - s_2^1|^2\} \\
\Delta_{\hat{\mathbf{s}}_1} & \triangleq & |{\boldsymbol{\alpha}_1^1}^H \mathbf{y} - \hat{s}_1^1|^2 + |{\boldsymbol{\alpha}_2^1}^H \mathbf{y} - \hat{s}_2^1|^2 \\
\hline
\end{array}
$$

Table 2.1: MMSE interference cancellation algorithm for space-time block codes when the desired user is the first one [1].

$$
\begin{aligned}
(\hat{\mathbf{s}}_2, \Delta_{\hat{\mathbf{s}}_2}) &= \text{MMSE.DEC}\,(\mathbf{y}_1, \mathbf{y}_2, \mathbf{A}_{tot}, \sigma_n^2) \\
\mathbf{A}_{tot} &\triangleq \begin{bmatrix} \mathbb{A}_1^1 & \mathbb{A}_2^1 \\ \mathbb{A}_1^2 & \mathbb{A}_2^2 \end{bmatrix} \\
\mathbf{M} &= \mathbf{A}_{tot}\mathbf{A}_{tot}^H + \sigma_n^2 \mathbf{I}_4 \\
\mathbf{a}_{tot3} &\triangleq \left[h_{11}^2 \quad h_{21}^{2\,*} \quad h_{12}^2 \quad h_{22}^{2\,*}\right]^T \\
&= \text{third column of } \mathbf{A}_{tot} \\
\mathbf{a}_{tot4} &\triangleq \left[h_{21}^2 \quad -h_{11}^{2\,*} \quad h_{22}^2 \quad -h_{12}^{2\,*}\right]^T \\
&= \text{fourth column of } \mathbf{A}_{tot} \\
\boldsymbol{\alpha}_1^2 &= \mathbf{M}^{-1}\mathbf{a}_{tot3} \\
\boldsymbol{\alpha}_2^2 &= \mathbf{M}^{-1}\mathbf{a}_{tot4} \\
\hat{\mathbf{s}}_2 &= \underset{\mathbf{s}_2 \in \mathcal{S}_2}{\text{argmin}} \; \{|\boldsymbol{\alpha}_1^{2H}\mathbf{y} - s_1^2|^2 + |\boldsymbol{\alpha}_2^{2H}\mathbf{y} - s_2^2|^2\} \\
\Delta_{\hat{\mathbf{s}}_2} &\triangleq |\boldsymbol{\alpha}_1^{2H}\mathbf{y} - \hat{s}_1^1|^2 + |\boldsymbol{\alpha}_2^{1H}\mathbf{y} - \hat{s}_2^2|^2
\end{aligned}
$$

Table 2.2: MMSE interference cancellation algorithm for space-time block codes when the desire user is the second one [1].

$(\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2)$	$=$	IIC.DEC $(\mathbf{y}_1, \mathbf{y}_2, \mathbf{A}_{tot}, \sigma_n^2)$
$(\hat{\mathbf{s}}_1^{'}, \Delta_{\mathbf{s}_1^{'}})$	$=$	MMSE.DEC $(\mathbf{y}_1, \mathbf{y}_2, \mathbf{A}_{tot}, \sigma_n^2)$
$\mathbf{z}_1$	$\triangleq$	$\mathbf{y}_1 - \mathbb{A}_1^1 \cdot \hat{\mathbf{s}}_1^{'}$, $\mathbf{z}_2 \triangleq \mathbf{y}_2 - \mathbb{A}_1^2 \cdot \hat{\mathbf{s}}_1^{'}$
$\mathcal{F}(\mathbf{s}_2)$	$\triangleq$	$\|\mathbf{z}_1 - \mathbb{A}_2^1 \cdot \mathbf{s}_2\|^2 + \|\mathbf{z}_2 - \mathbb{A}_2^2 \cdot \mathbf{s}_2\|^2$
$\hat{\mathbf{s}}_2^{'}$	$=$	$\underset{\mathbf{s}_2^{'} \in \mathcal{S}_2}{\text{argmin}}\ \mathcal{F}(\mathbf{s}_2^{'})$, $\Delta_{\mathbf{s}_2^{'}} \triangleq \mathcal{F}(\hat{\mathbf{s}}_2^{'})$
$(\hat{\mathbf{s}}_2^{''}, \Delta_{\mathbf{s}_2^{''}})$	$=$	MMSE.DEC $(\mathbf{y}_1, \mathbf{y}_2, \mathbf{A}_{tot}, \sigma_n^2)$
$\mathbf{u}_1$	$\triangleq$	$\mathbf{y}_1 - \mathbb{A}_2^1 \cdot \hat{\mathbf{s}}_2^{''}$, $\mathbf{u}_2 \triangleq \mathbf{y}_2 - \mathbb{A}_2^2 \cdot \hat{\mathbf{s}}_2^{''}$
$\mathcal{G}(\mathbf{s}_1)$	$\triangleq$	$\|\mathbf{u}_1 - \mathbb{A}_1^1 \cdot \mathbf{s}_1\|^2 + \|\mathbf{u}_2 - \mathbb{A}_1^2 \cdot \mathbf{s}_1\|^2$
$\hat{\mathbf{s}}_1^{''}$	$=$	$\underset{\mathbf{s}_1 \in \mathcal{S}_1}{\text{argmin}}\ \mathcal{G}(\mathbf{s}_1)$, $\Delta_{\mathbf{s}_1^{''}} \triangleq \mathcal{G}(\hat{\mathbf{s}}_1^{''})$
if		$(\Delta_{\mathbf{s}_1^{'}} + \Delta_{\mathbf{s}_2^{'}}) < (\Delta_{\mathbf{s}_1^{''}} + \Delta_{\mathbf{s}_2^{''}})$
		$(\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2) = (\hat{\mathbf{s}}_1^{'}, \hat{\mathbf{s}}_2^{'})$
else		
		$(\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2) = (\hat{\mathbf{s}}_1^{''}, \hat{\mathbf{s}}_2^{''})$

Table 2.3: Two-step interference cancellation for space-time block codes [1].

2.6 Interference Rejection Using Decorrelator for Systems Using OSTBCs

In this section, another sub-optimal method for interference suppression in STBC communication systems is briefly reviewed [24]. The multi-user scenario considered in [24] is the same as the one considered in Section 2.4, i.e. two transmitters each with two transmitter antennas and a receiver with two antennas are considered.

Each transmitter is using Alamouti code to encode the information symbols; therefore, they are equipped with two transmitter antennas. Using the data model from the previous section,

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbb{A}_1^1 & \mathbb{A}_2^1 \\ \mathbb{A}_1^2 & \mathbb{A}_2^2 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} \tag{2.43}$$

and applying the following decorrelator matrix $\mathbf{W}_d$, presented in [24]

$$\mathbf{W}_d = \begin{bmatrix} \mathbf{I}_2 & -\mathbb{A}_2^1 \mathbb{A}_2^{2-1} \\ -\mathbb{A}_1^2 \mathbb{A}_1^{1-1} & \mathbf{I}_2 \end{bmatrix} \tag{2.44}$$

to $\mathbf{y}$ in (2.43) results in:

$$\tilde{\mathbf{y}} \triangleq \mathbf{W}_d \mathbf{y} = \begin{bmatrix} \tilde{\mathbf{H}}_1 & 0 \\ 0 & \tilde{\mathbf{H}}_2 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{v}}_1 \\ \tilde{\mathbf{v}}_2 \end{bmatrix} \tag{2.45}$$

where matrices $\tilde{\mathbf{H}}_1$ and $\tilde{\mathbf{H}}_2$ are defined as [24]:

$$\tilde{\mathbf{H}}_1 \triangleq \mathbb{A}_1^1 - \mathbb{A}_2^1 \mathbb{A}_2^{2-1} \mathbb{A}_1^2 \tag{2.46}$$

$$\tilde{\mathbf{H}}_2 \triangleq \mathbb{A}_2^2 - \mathbb{A}_1^2 \mathbb{A}_1^{1-1} \mathbb{A}_2^1 \tag{2.47}$$

Matrices $\mathbb{A}_1^1, \mathbb{A}_1^2, \mathbb{A}_2^1$ and $\mathbb{A}_2^2$ are orthogonal and invertible; therefore, $\tilde{\mathbf{H}}_1$ and $\tilde{\mathbf{H}}_2$ exist. It has been shown in [24] that $\tilde{\mathbf{H}}_1$ and $\tilde{\mathbf{H}}_2$ are invertible.

In this method, effect of noise is not considered; therefore, noise enhancement may happen. To avoid noise enhancement this linear receiver may be used for the case that the SNR is high. This receiver also needs to know the CSI of both users to decorrelate their data.

2.7 Conclusion

In this chapter, for the data model presented in Section 2.1, six different existing interference suppression methods were presented. The optimum method is the ML multi-user receiver, but its computational complexity increases exponentially with the number of transmitters, number of symbols per block K, and number of constellation points, and the CSI of all transmitters should be known by the receiver. Sphere decoding decreases the computational complexity of ML detector by confining the set of points to be checked in the likelihood function to a sphere centered at the observation point. In this method, the receiver must know the CSI of all transmitters; furthermore, the radius of sphere should be chosen properly to decrease the computational complexity.

Another ML receiver for the case that only the CSI of the desired transmitter is known is presented in Section 2.3, in which the receiver has to know the structure of interference. The computational complexity of this method is also high, specially if the structure of interference has to be estimated from the training data.

To avoid the computational complexity of the ML method, suboptimal methods were introduced in the literature, three of them considered in Sections 2.4-2.6. The simplest suboptimal receiver is the matched filter. As shown in the Section 1.6, the matched filter is a linear receiver for the scenario considered in this thesis, and one only needs to know the CSI and code of the desired transmitter to implement it. Although it is easy to implement, its performance for the data model presented in the Section 2.1 is far from optimal, because it ignores MAI, treating it as noise.

The Naguib-seshadri method presented in Section 2.5 is a linear receiver designed based on an MMSE criterion. This method can even be used for the case where the CSI of the second transmitter is not known by the receiver. Naguib-Seshadri method is only presented for a system using Alamouti's code and does not show a general closed form for applying it to systems using other OSTBCs.

The decorrelator receiver presented in Section 2.6, decorrelates the information symbol vectors of two transmitters, but it does not consider the effect of noise, and there may

be noise enhancement. This receiver should know the CSI of both transmitters, and it is designed for a system with two transmitters using Alamouti code; furthermore, the number of antennas at the receiver should be equal to the number of antennas at each transmitter.

Chapter 3

Minimum Variance Linear Receiver for Interference Rejection in OSTBC Communications

In the first section of this chapter, a new linear receiver is introduced, then two applications for this receiver are mentioned in Section 3.2. In Section 3.3, simulation results for this new linear receiver are shown. Section 3.4 compares the computational complexity and degrees of freedom of our method with Naguib-Seshadri method. The last section suggests work to be done in future.

3.1 New Linear Receiver

In the multi-access MIMO scenario presented in the Chapter 2, the MF receiver of (1.22) will become highly non-optimal because it ignores the effect of Interference treating it as noise. In this case, the receiver performance is determined by the signal-to-interference-plus-noise ratio (SINR) rather than the SNR and some cancellation of co-channel interference is required. Motivated by this fact, in this section we design linear receivers which maximize the SINR and achieve much better multi-access performance than the MF receiver.

Using the vectorized model (2.4) and assuming without any loss of generality that the first transmitter is the transmitter-of-interest, we can express the output vector of a linear receiver as

$$\underline{\hat{\mathbf{s}}_1} = \mathbf{W}^T \underline{\mathbf{Y}} \tag{3.1}$$

where $\mathbf{W} = [\mathbf{w}_1\, \mathbf{w}_2 \cdots \mathbf{w}_{2K}]$ is the $2MT \times 2K$ real matrix of the receiver coefficients. Note that for each transmitter-of-interest, a different matrix $\mathbf{W}$ should be formed. However, for the sake of notational simplicity we consider only one transmitter-of-interest and omit the corresponding index in $\mathbf{W}$. The $\underline{\hat{\mathbf{s}}_1}$ can be viewed as an estimate of the vector $\underline{\mathbf{s}_1}$. The vector $\mathbf{w}_k$ can be interpreted as the receiver weight vector for the kth entry of $\underline{\mathbf{s}_1}$.

Given the matrix $\mathbf{W}$, the estimate of the vector of information symbols for the transmitter of interest can be computed as

$$\hat{\mathbf{s}}_1 = [\mathbf{I}_K\ j\mathbf{I}_K]\, \underline{\hat{\mathbf{s}}_1} \tag{3.2}$$

Using the linear estimate (3.2), the kth information symbol can be detected as a point in the constellation which is the nearest neighbor to the kth entry of $\hat{\mathbf{s}}_1$.

Our goal is to design the matrix $\mathbf{W}$ which suppresses Interference as much as possible while preserving a distortionless response to the transmitter of interest. The key idea of our receiver is inspired by the popular Minimum Variance (MV) and Minimum Output Energy (MOE) approaches used in adaptive beamforming and multiuser detection, respectively.

For each entry of the vector $\underline{\mathbf{s}_1}$, let us minimize the receiver output power while preserving a unity gain for this particular entry of $\underline{\mathbf{s}_1}$. This is equivalent to solving the following optimization problem:

$$\min_{\mathbf{w}_k} \mathbf{w}_k^T \mathbf{R} \mathbf{w}_k \quad \text{s.t.} \quad \mathbf{a}_{1,k}^T \mathbf{w}_k = 1 \quad \text{for all} \quad k = 1, \ldots, 2K \tag{3.3}$$

where $\mathbf{R} \triangleq \mathrm{E}\{\underline{\mathbf{Y}}\ \underline{\mathbf{Y}}^T\}$ is the $2MT \times 2MT$ covariance matrix of the vectorized data $\underline{\mathbf{Y}}$ and $\mathrm{E}\{\cdot\}$ denotes the statistical expectation.

The problem (3.3) can be solved independently for each k, we obtain that the solution to (3.3) is given by [25] as

$$\mathbf{w}_k = \frac{1}{\mathbf{a}_{1,k}^T \mathbf{R}^{-1} \mathbf{a}_{1,k}} \mathbf{R}^{-1} \mathbf{a}_{1,k}\,, \quad k = 1, \ldots, 2K \tag{3.4}$$

Although the receiver (3.4) is able to reject the interference, it does not completely cancel *self-interference* [26] which, for each $\mathbf{w}_k$, is caused by other entries of $\underline{\mathbf{s}_1}$ than the kth one. It is important to emphasize that the *complete cancellation* of self-interference is a strongly desirable feature because, otherwise, the symbol-by-symbol detector becomes non-optimal. Indeed, this detector is based on the assumption that the output of each linear receiver corresponding to any particular symbol is independent of the other symbols. This assumption is violated in the presence of even small amount of non-cancelled self-interference.

To incorporate the complete self-interference cancellation feature into (3.3), let us add the additional zero-forcing constraints to this problem:

$$\mathbf{a}_{1,l}^T \mathbf{w}_k = 0 \quad \text{for all} \quad l \neq k \tag{3.5}$$

These additional constraints guarantee that self-interference is *completely* rejected.

It can be readily verified that the problem (3.3) with the additional constraints (3.5) is equivalent to the following optimization problem:

$$\min_{\mathbf{W}} \ \mathrm{Tr}\{\mathbf{W}^T \mathbf{R} \mathbf{W}\} \quad \text{s.t.} \quad \mathbf{A}_1^T \mathbf{W} = \mathbf{I}_{2K}. \tag{3.6}$$

To solve the optimization problem in (3.6), we construct the Lagrangian function [17]:

$$L(\mathbf{W}, \mathbf{\Lambda}) = \mathrm{Tr}\{\mathbf{W}^T \mathbf{R} \mathbf{W}\} - \mathrm{Tr}\{\mathbf{\Lambda}^T (\mathbf{A}_1^T \mathbf{W} - \mathbf{I}_{2K})\} \tag{3.7}$$

here $\mathbf{\Lambda}$ is a $2K \times 2K$ matrix of Lagrange multipliers. We differentiate the Lagrangian function with respect to the $\mathbf{W}$, and let it equal to zero, which gives us:

$$2\mathbf{R}\mathbf{W} = \mathbf{A}_1 \mathbf{\Lambda}. \tag{3.8}$$

The optimum $\mathbf{W}$ based on $\mathbf{\Lambda}$ can be found as:

$$\mathbf{W}_{opt} = \frac{1}{2} \mathbf{R}^{-1} \mathbf{A}_1 \mathbf{\Lambda}. \tag{3.9}$$

Substituting this $\mathbf{W}$ back in the constraint $\mathbf{A}_1^T \mathbf{W} = \mathbf{I}_{2K}$, gives us $\mathbf{\Lambda}$ as:

$$\mathbf{\Lambda} = 2(\mathbf{A}_1^T \mathbf{R}^{-1} \mathbf{A}_1)^{-1} \tag{3.10}$$

therefore, the optimum receiver matrix is:

$$\mathbf{W}_{opt} = \mathbf{R}^{-1}\mathbf{A}_1(\mathbf{A}_1^T\mathbf{R}^{-1}\mathbf{A}_1)^{-1}. \tag{3.11}$$

This $\mathbf{W}$ will be substituted in (3.1) to find $\underline{\hat{\mathbf{s}}_1}$, and subsequently $\hat{\mathbf{s}}_1$.

Assuming that the effect of noise and interference is suppressed by the linear receiver, the information symbol vector of the desired transmitter can be detected by finding the closest point to the $\hat{\mathbf{s}}_1$ in the constellation, in this way we avoid the complexity of an ML detection.

Because of the additional zero-forcing constraints used, the obtained receiver is different from the traditional MV and MOE receivers used in adaptive beamforming and multiuser detection, respectively. To clarify this, note that the traditional MV and MOE receivers are solutions for a single weight vector, while our receiver (3.11) represents the expression for the *weight matrix* whose columns represent different weight vectors that must maintain a specific relationship to each other (which is dictated by the zero-forcing constraints in (3.6)).

Since we assume that (2.2) and (2.3) are satisfied for the data model presented in the Chapter 2, the true covariance matrix can be shown as

$$\mathbf{R} = \frac{\sigma_1^2}{2}\mathbf{A}_1\mathbf{A}_1^T + \frac{\sigma_2^2}{2}\mathbf{A}_2\mathbf{A}_2^T + ... + \frac{\sigma_P^2}{2}\mathbf{A}_P\mathbf{A}_P^T + \frac{\sigma_n^2}{2}\mathbf{I}_{2MT}. \tag{3.12}$$

Hence if the channel matrices $\mathbf{H}_p(p = 1, \ldots P)$ and the noise variance are known at the receiver, the true covariance matrix can be computed directly. However, in practice not all channel matrices may be available at the receiver. In such cases, the true covariance matrix $\mathbf{R}$ can be replaced by its sample estimate $\hat{\mathbf{R}}$, given as

$$\hat{\mathbf{R}} = \frac{1}{Q}\sum_{q=1}^{Q}\underline{\mathbf{Y}_q}\,\underline{\mathbf{Y}_q}^T \tag{3.13}$$

where $\underline{\mathbf{Y}_q}$ is the qth received data block and Q is the number of data blocks we use to estimate the covariance matrix. Replacing the true covariance matrix by the sample one, we obtain the finite sample version of the MV receiver (3.11) can be written as

$$\mathbf{W}_{MV} = \hat{\mathbf{R}}^{-1}\mathbf{A}_1(\mathbf{A}_1^T\hat{\mathbf{R}}^{-1}\mathbf{A}_1)^{-1} \tag{3.14}$$

For our new linear receiver we do not have any restriction on the number of antennas on the both sides, or on the type of the OSTBC used by the desired transmitter; furthermore, there is not any restriction on the structure of the interference.

3.2 Applications

Two applications for our new linear receiver are presented

Jamming: Our new linear receiver can be viewed as an interference rejection technique which can be used to prevent jamming. Jamming happens when one or more transmitters generate signals which dramatically decrease the quality of a communication link intentionally or unintentionally. These interference rejection techniques can also be used to derive techniques to jam the signal of one transmitter. Jamming cancellation is mostly required in military communications [27].

Cellular Communication: The new linear receiver presented in this chapter, can also be used in the cellular communication systems to reject the interference from other cells, generated due to the use of the same frequency bands in the adjacent cells. In this scenario the base station does not know the CSI of the transmitters in the other cells.

3.3 Simulations

In the case of jamming, the jammer's signal can be stronger than the signal level at the receiver. In the case of rejecting interference from other cells in a cellular communication system, the level of interference is lower than the signal level at the receiver. Due to these facts, in the simulations two cases of stronger interference and weaker interference compared to the signal level are considered.

The data model of Section 2.1 is simulated in this section and all transmitters use OSTBCs to code their data. As mentioned before, there is not any restriction on the structure of interference for our new linear receiver; therefore, using OSTBC for interferers

is only for simulation purposes. In the simulations two codes are considered

$$\mathbf{X}_p(\mathbf{s}_p) = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1^p & s_2^p & s_3^p \\ -s_2^p & s_1^p & -s_4^p \\ -s_3^p & s_4^p & s_1^p \\ -s_4^p & -s_3^p & s_2^p \\ s_1^{p*} & s_2^{p*} & s_3^{p*} \\ -s_2^{p*} & s_1^{p*} & -s_4^{p*} \\ -s_3^{p*} & s_4^{p*} & s_1^{p*} \\ -s_4^{p*} & -s_3^{p*} & s_2^{p*} \end{bmatrix} \tag{3.15}$$

which is Tarokh's half rate code [12], and

$$\mathbf{X}_p(\mathbf{s}_p) = \begin{bmatrix} s_1^p & s_2^p \\ -s_2^{p*} & s_1^{p*} \end{bmatrix} \tag{3.16}$$

which is Alamouti code [12].

In the case of Alamouti code $N = 2, T = 2, K = 2$, and the error performance of our linear receiver, matched filter, and Naguib-Seshadri receiver are compared. In the case of Tarokh's half rate code $N = 3, T = 8, K = 4$, and error performance of our new linear receiver and matched filter are compared.

In each figure, the Symbol Error Rate (SER) is shown on the vertical axis. The SER is calculated by taking average over 200 different channel realizations. For each channel realization, 10000 information symbol vectors used to calculate SER. The symbols of each transmitter uniformly come from a Quadrature Phase Shift Keying (QPSK) constellation. The SER is shown versus SNR, Interference-to-Noise Ratio (INR), M and Q, for $P = 1, 3, 5$.

The SNR and INR are defined at the transmitter as $\text{SNR} = \frac{\sigma_1^2}{\sigma_n^2}$, and $\text{INR}_p = \frac{\sigma_p^2}{\sigma_n^2}, p = 2 \ldots P$, while the Frobenius norm of the channel matrices for all transmitters are kept equal to 1. The INR is assumed to be equal for all $P - 1$ interferers.

If the true covariance matrix for the case that there is only one transmitter is known by the receiver, as mentioned in Appendix A, the error performance of our method is the same as matched filter. However, in practice we have to calculate the error rate by taking the

average of error rate on finite number of symbol vectors. These symbol vectors are different for each method in our simulations. Therefore, there is a mismatch between the new linear receiver and the matched filter for the case that the true covariance matrix is known.

The matched filter followed by symbol by symbol detection is the ML receiver for the case of one transmitter. The ultimate goal of interference rejection is to reach the performance of the ML receiver for the case where there is no interference. Therefore, in our simulations we show the error performance of the matched filter for the case of one transmitter, as the lower bound on the achievable error performance.

3.3.1 Simulation Results for Strong Interference

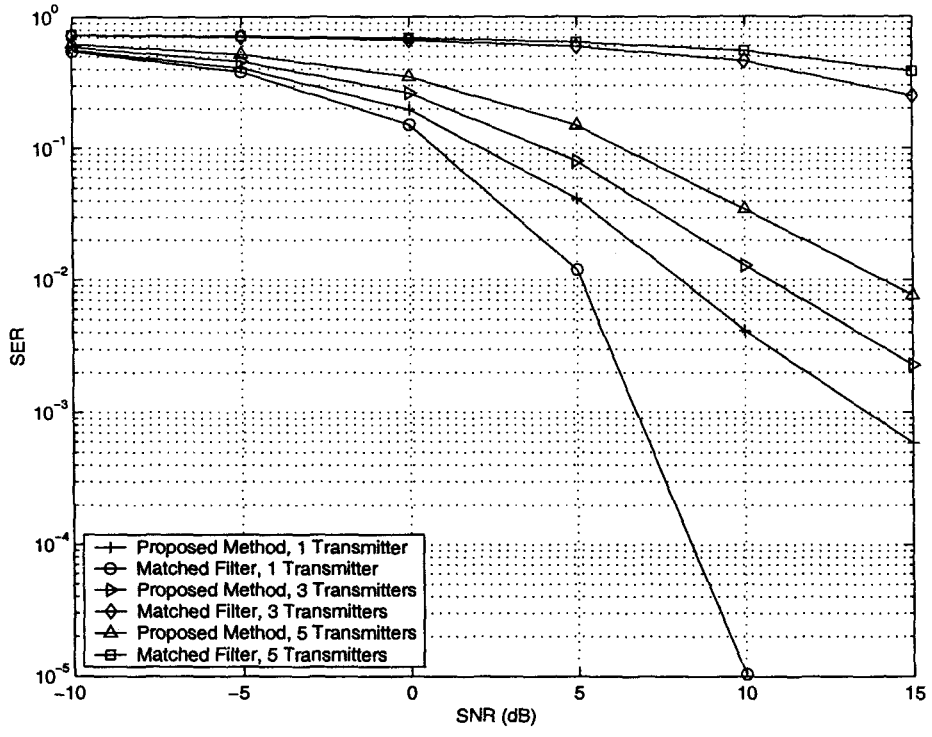

Figure 3.1: SER versus SNR, Tarokh's Code, INR = 20dB.

Code	SNR Range	INR	Q	P	M	N	T	K
Tarokh	(-10)dB-15dB	20 dB	500	1,3,5	4	3	8	4

Figure 3.1 shows the SER versus SNR for our receiver, and the matched filter, for one, three, and five transmitters, when we use Tarokh's code.

It can be seen from this figure that the proposed MV receiver outperforms the MF receiver, e.g. at SER of 10^{-1}, for three and five transmitters, the new linear receiver has more than 10 dB gain compared to the matched filter performance.

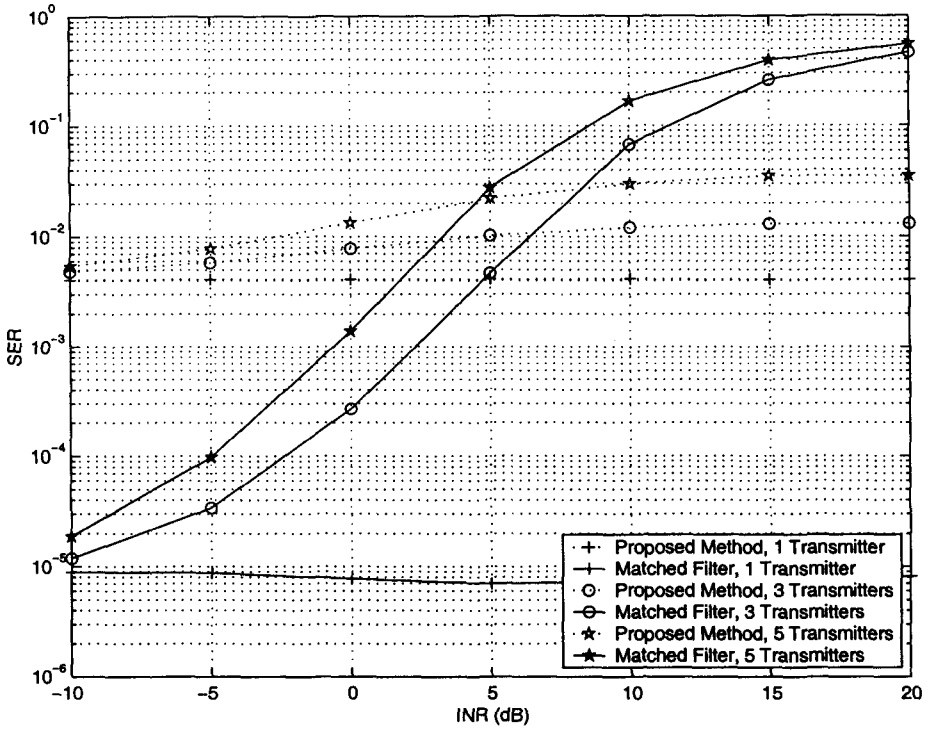

Figure 3.2: SER versus INR, Tarokh's code, SNR = 10dB

Code	SNR	INR Range	Q	P	M	N	T	K
Tarokh	10dB	(-10)dB-20dB	500	1,3,5	4	3	8	4

In Fig.3.2, the behavior of SER versus INR for our receiver, and the matched filter are compared when all interference signals arrive with the same power at the receiver, and we use the Tarokh's code.

We can see that by using 500 blocks of data to estimate $\mathbf{R}$, our receiver is performing better compared to the matched filter for higher INRs. In the three transmitters case, for INRs higher than 7 dB, and in the case of five transmitters, for INRs higher than 4 dB, our receiver is performing better than the matched filter.

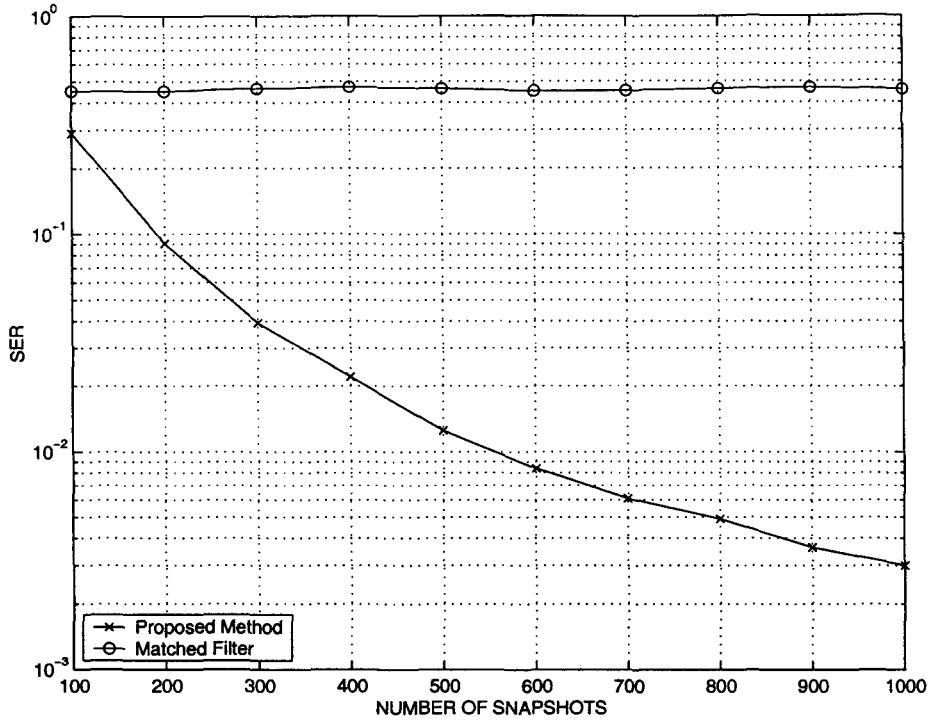

Figure 3.3: SER versus Q, Tarokh's code, SNR = 10dB, INR = 20dB.

Code	SNR	INR	Q Range	P	M	N	T	K
Tarokh	10dB	20dB	100-1000	3	4	3	8	4

Figure 3.3 shows SER versus the number of blocks (Q), used to estimate $\mathbf{R}$, for the case where we use Tarokh's code, and we have three transmitters. It can be seen from this figure that the performance of our MV receiver improves as Q increases, while the performance of the MF receiver does not depend on Q. For $Q = 1000$, SER for MV receiver is around 10^{-3}, while it is more than 10^{-1} for the MF receiver.

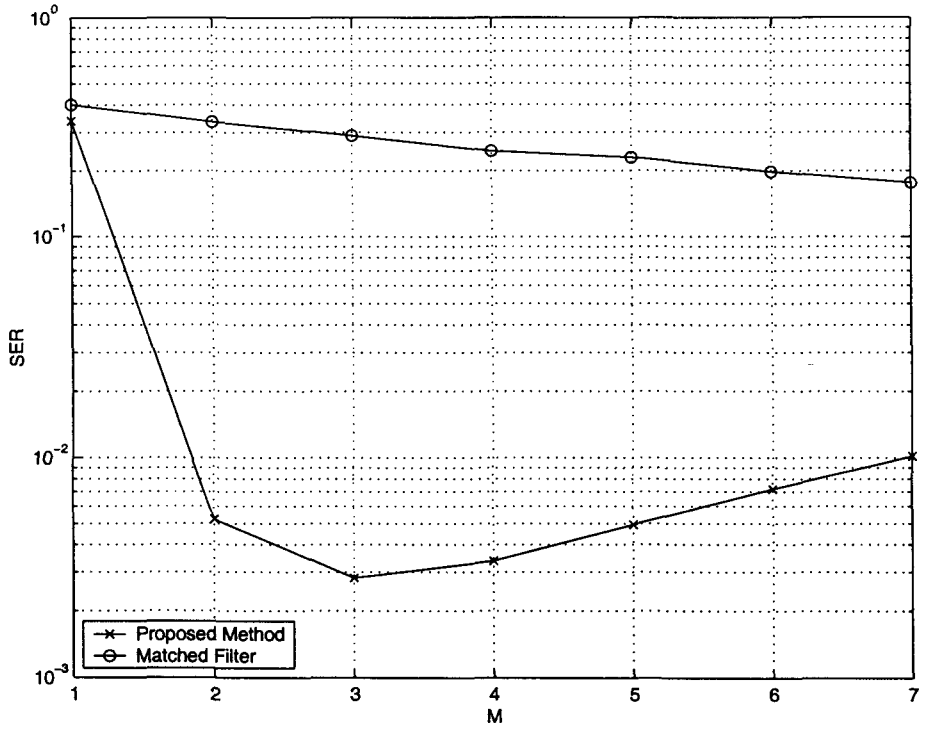

Figure 3.4: SER versus M for Tarokh's code, 3 transmitters.

Code	SNR	INR	Q	P	M Range	N	T	K
Tarokh	10dB	20dB	500	3	1-7	3	8	4

To show the effect of the number of antennas at the receiver, we consider the behavior of SER when the number of antennas in the receiver increases. Fig.3.4, shows the SER versus M (number of antennas in the receiver) for the case that we are using the Tarokh's half rate code, and we have three transmitters. In this figure, Q is kept constant as M increases. Considering the simulation results in Fig.3.4, it seems that an optimum number of antenna, $M = 3$ exists.

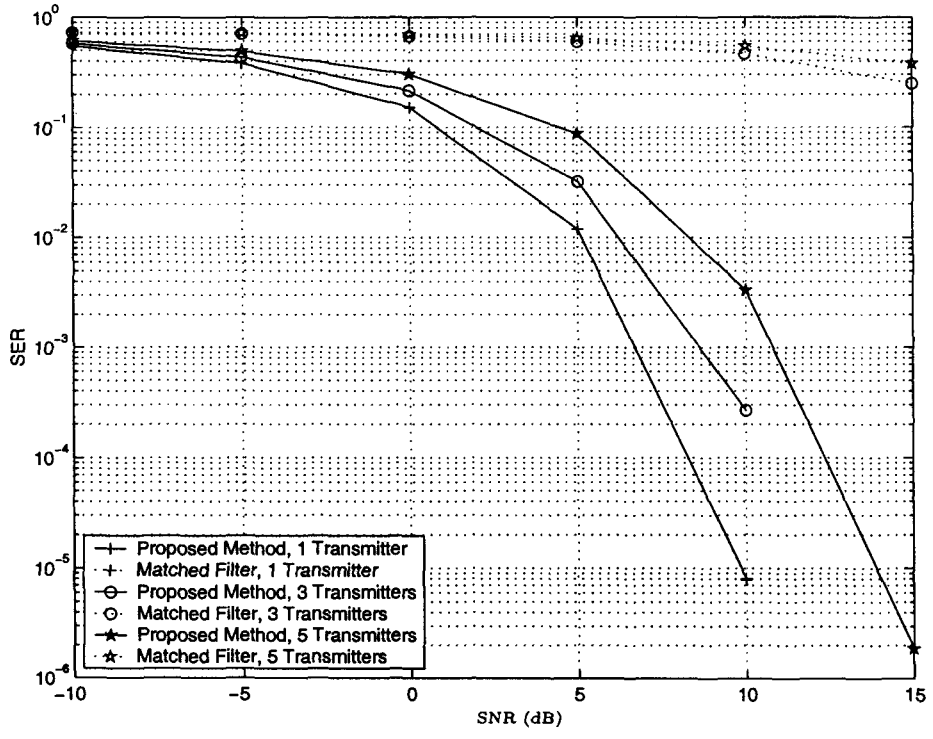

Figure 3.5: SER versus SNR, Tarokh's code, using the true covariance matrix, INR = 20dB.

Code	SNR Range	INR	Q	P	M	N	T	K
Tarokh	(-10)dB-15dB	20dB	-	1,3,5	4	3	8	4

Figure 3.5 shows the behavior of SER vs SNR for the matched filter and our receiver when we use Tarokh's code, and we know the true covariance matrix. The comparison of Fig.3.5 and Fig.3.1 shows that the estimate of $\mathbf{R}$ converges slowly; therefore, if the time coherence of the channel and symbol rate permit us, we have to increase Q to have better results. In Fig.3.5, at SNR = 10 dB, SER for our receiver is near 10^{-4} for the three transmitters case, while it is worse than 10^{-1} for the matched filter.

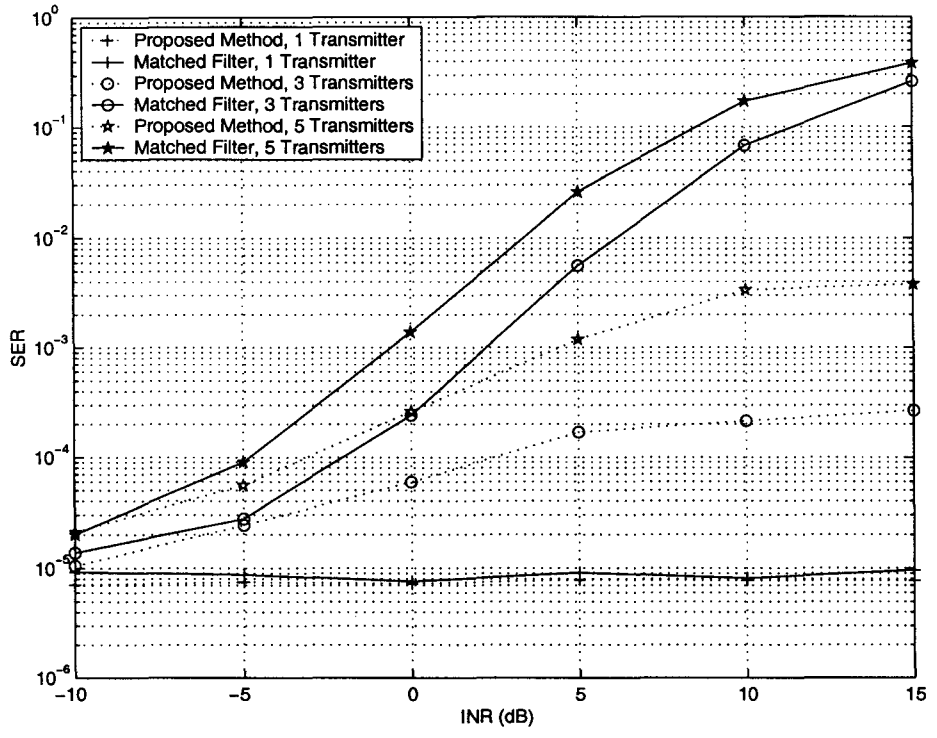

Figure 3.6: SER versus INR, Tarokh's code, using the true covariance matrix, SNR = 10dB.

Code	SNR	INR Range	Q	P	M	N	T	K
Tarokh	10dB	(-10)dB-15dB	-	1,3,5	4	3	8	4

Figure 3.6 shows the SER performance of our receiver, and the matched filter, versus INR, when we use Tarokh's code and we know the true covariance matrix. This figure shows that knowing the true covariance matrix, SER performance of our receiver is superior to the matched filter for any INR, and for low INRs their performance is nearly the same as each other. At INR = 15 dB for the three transmitters case, SER is near 10^{-4} for our new receiver, while SER is worse than 10^{-1} for the matched filter.

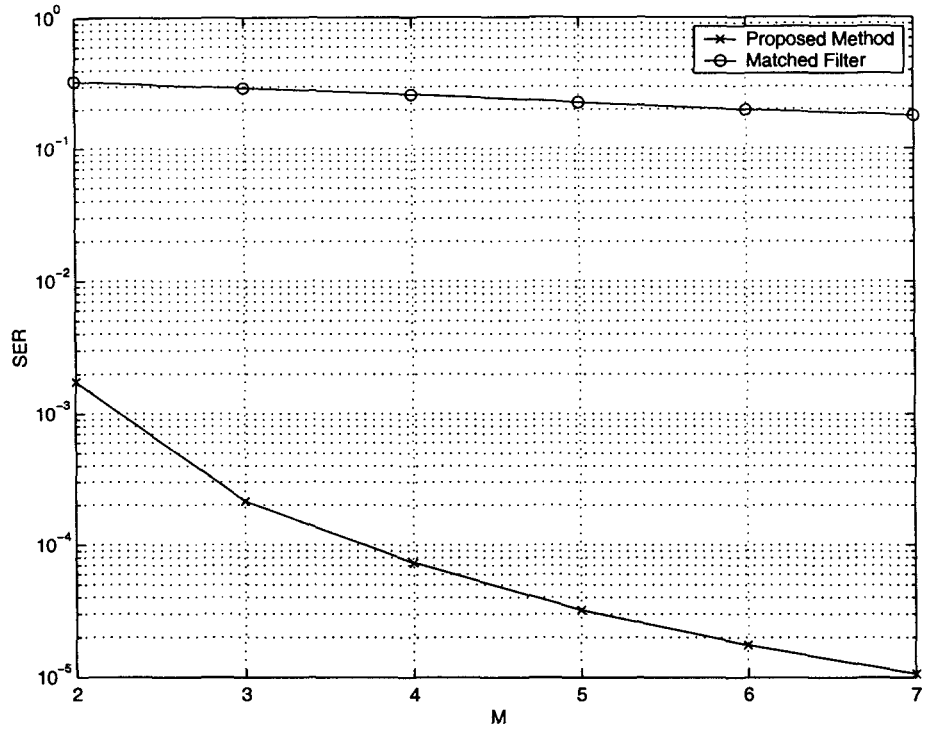

Figure 3.7: SER versus M, Tarokh's code, using the true covariance matrix, SNR = 10dB, INR = 20dB.

Code	SNR	INR	Q	P	M Range	N	T	K
Tarokh	10dB	20dB	-	3	2-7	3	8	4

Figure 3.7 shows the SER performance of our receiver, and the matched filter versus the number of antennas at the receiver for the case that we use Tarokh's code, and we know the true covariance matrix. If we compare this figure with the Figure 3.4 , we see that in the Fig.3.7, we do not have an optimum number of antennas, because in the Fig.3.4, when we increase M, we keep the Q constant. This gives us an underestimated $\mathbf{R}$, while M increases; therefore, the optimum point in Fig.3.4 is the result of the poor estimate of $\mathbf{R}$.

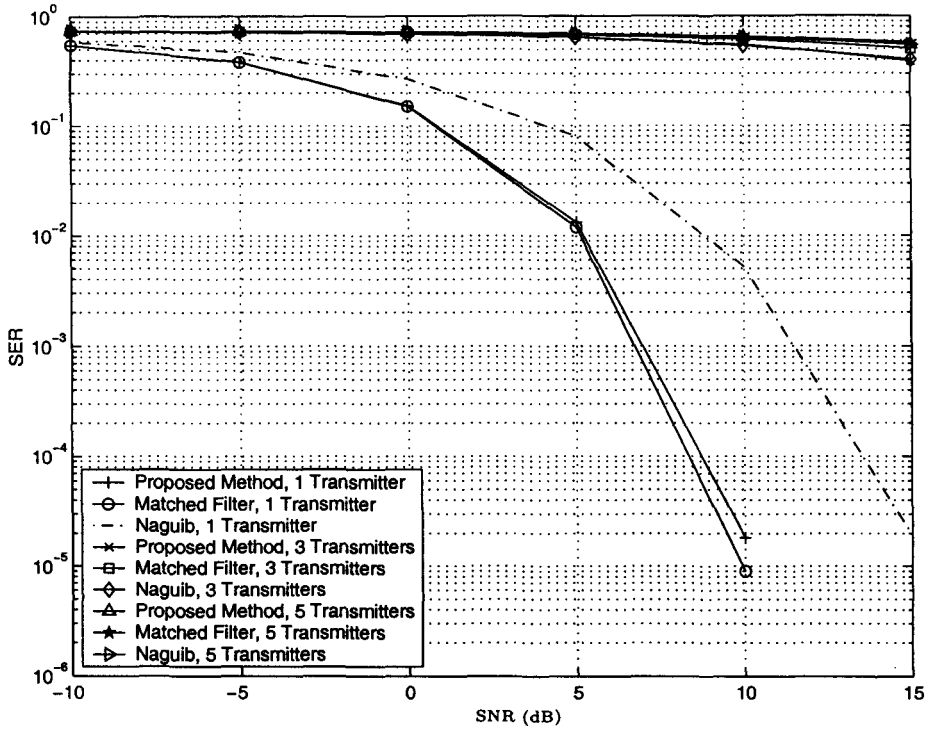

Figure 3.8: SER versus SNR, Alamouti code.

Code	SNR Range	INR	Q	P	M	N	T	K
Alamouti	(-10)dB-15dB	20dB	500	1,3,5	2	2	2	2

Figure 3.10 shows the SER performance of our receiver, Naguib-Seshadri receiver, and the matched filter versus SNR when we use Alamouti code. This figure shows that estimating $\mathbf{R}$ using 500 data matrices, none of the three methods can suppress the interference for the case that there are three or more transmitters. For the case of one transmitter our method behaves almost the same as matched filter, and at SER $= 10^{-4}$ they have 5 dB gain over Naguib-Seshadri method

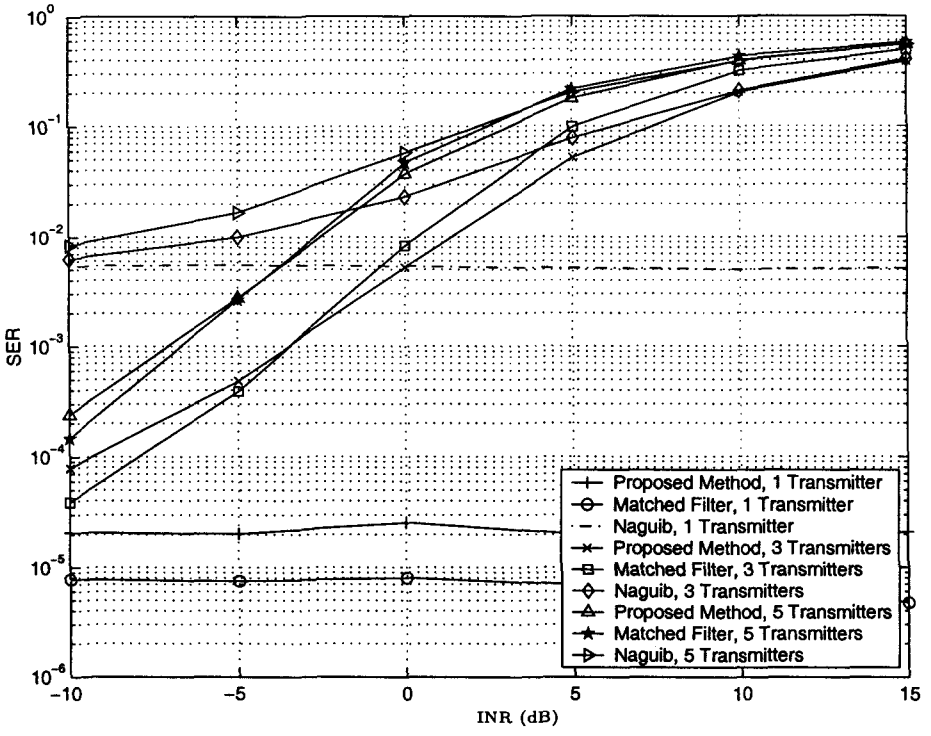

Figure 3.9: SER versus INR, Alamouti code, SNR = 10 dB.

Code	SNR	INR Range	Q	P	M	N	T	K
Alamouti	10dB	(-10)dB-15dB	500	1,3,5	2	2	2	2

This figure shows SER performance of our receiver, Naguib-Seshadri receiver, and the matched filter versus INR for the case that we use Alamouti code. Here we see that for all INRs investigated, our method is performing better than Naguib-Seshadri method, but compared to the matched filter (due to the fact that we do not know the true covariance matrix) our receiver performs better than the matched filter only for INRs higher than -4 dB in the two transmitters case, and for INRs higher than -5 dB for five transmitters.

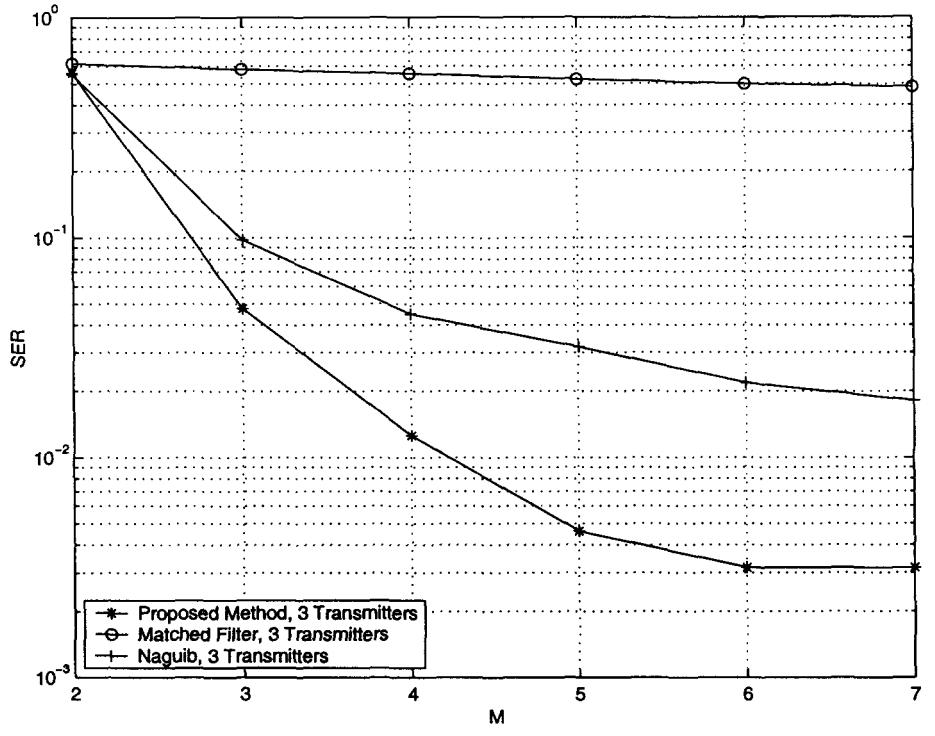

Figure 3.10: SER versus M, Alamouti code, SNR = 10dB, INR = 20dB.

Code	SNR	INR	Q	P	M Range	N	T	K
Alamouti	10dB	20dB	500	1,3,5	2-7	2	2	2

This figure shows the SER performance of our receiver, Naguib-Seshadri receiver, and the matched filter versus M, when we use Alamouti code. This figure does not show some optimum value for M because 500 data matrices is enough to have an exact estimate for $\mathbf{R}$. This figure shows that if we add to the number of antennas at the receiver, our method will be applicable for the cases of three and five transmitters.

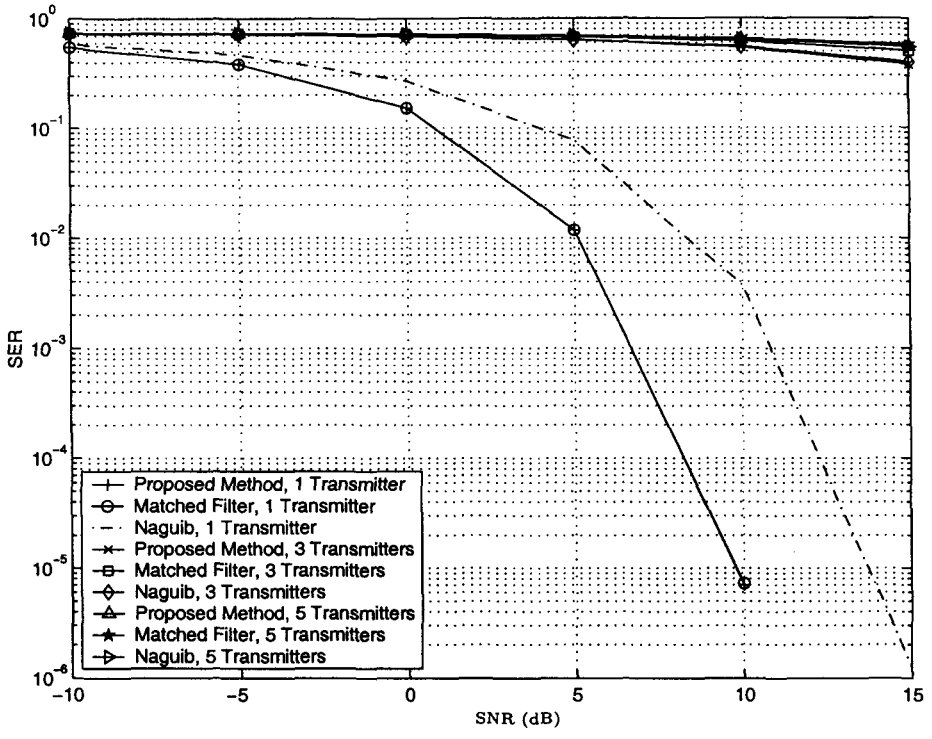

Figure 3.11: SER versus SNR, Alamouti code, using the true covariance matrix, INR = 20dB.

Code	SNR Range	INR	Q	P	M	N	T	K
Alamouti	(-10)dB-15dB	20dB	-	1,3,5	2	2	2	2

This figure shows the SER performance of our receiver, Naguib-Seshadri receiver, and the matched filter versus SNR, for the case that we use Alamouti code, and we know the true covariance matrix. It shows that even knowing the true covariance matrix for the case that $M = 2$, and there are three or more transmitters in the system, these methods are unable to suppress the interference.

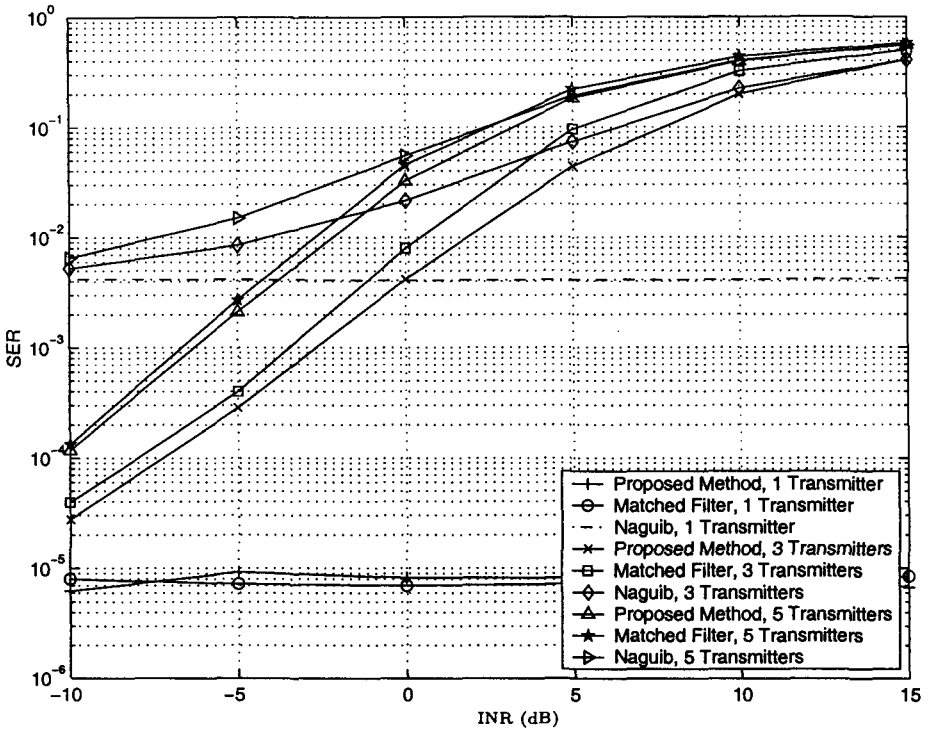

Figure 3.12: SER versus INR, Alamouti code, using the true covariance matrix, SNR = 10dB.

Code	SNR	INR Range	Q	P	M	N	T	K
Alamouti	10dB	(-10)dB-15dB	-	1,3,5	2	2	2	2

This figure shows the SER performance of our receiver, Naguib-Seshadri receiver, and the matched filter versus INR for the case that we use Alamouti code, and we know the true covariance matrix. This figure shows that for any INR investigated, the SER performance of our receiver is better than the other two methods compared here.

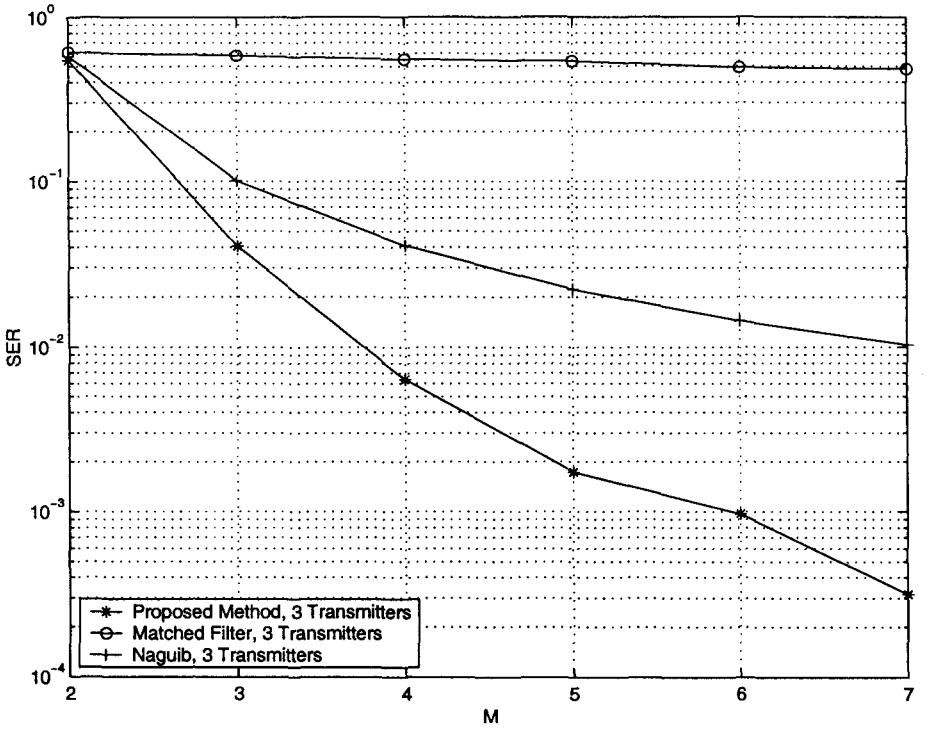

Figure 3.13: SER versus M, Alamouti code, using the true covariance matrix, SNR = 10dB, INR = 20dB.

Code	SNR	INR	Q	P	M Range	N	T	K
Alamouti	10dB	20dB	-	3	2-7	2	2	2

This figure shows the SER performance of all three methods versus M, for the case that we use Alamouti code, and we know the true covariance matrix.

3.3.2 Simulation Results for Weak Interference

As mentioned in Section 3.2, our new receiver can be used in a cellular mobile communication system to remove the interference from the other cells due to the frequency reuse. In this case the level of the interferer signals are the same as, or weaker than the desired transmitter's signal; therefore, here we investigate the case that the interferers are weaker than the desired transmitter.

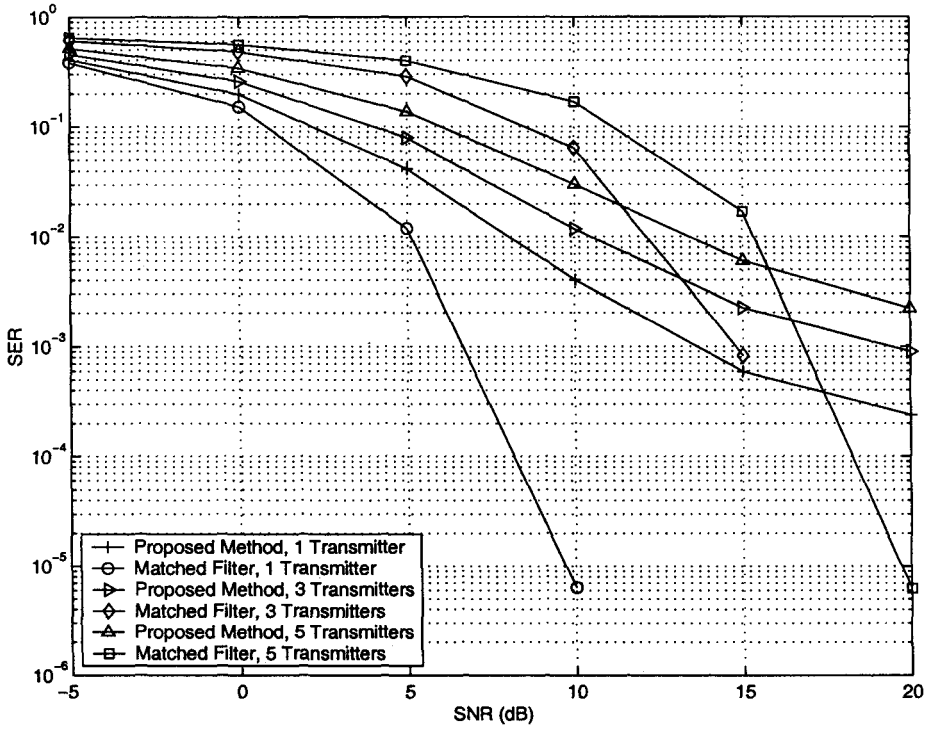

Figure 3.14: SER versus SNR, Tarokh's code, INR = 10dB.

Code	SNR Range	INR	Q	P	M	N	T	K
Tarokh	(-5)dB-20dB	10dB	500	1,3,5	4	3	8	4

This figure shows that the error performance of our receiver is not good for higher SNR. It will be shown that this is due to underestimating $\mathbf{R}$, and by improving the condition number for $\hat{\mathbf{R}}$, our method performs better.

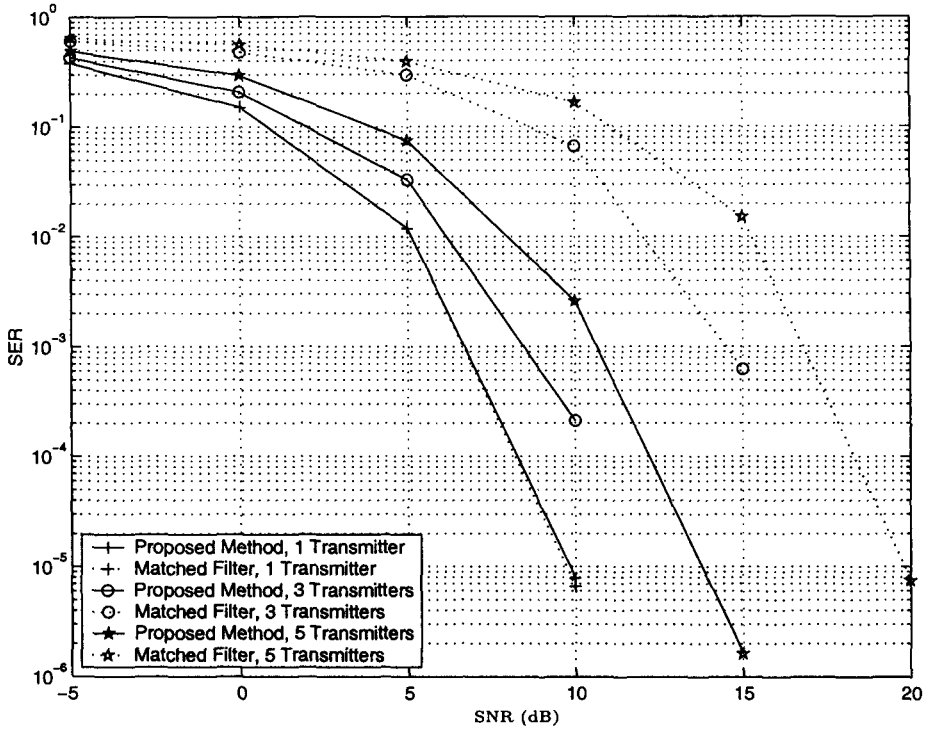

Figure 3.15: SER versus SNR, Tarokh's code, using the true covariance matrix, INR = 10dB.

Code	SNR Range	INR	Q	P	M	N	T	K
Tarokh	(-5)dB-20dB	10dB	-	1,3,5	4	3	8	4

This figure shows that the ultimate performance of the new linear receiver is much better than what shown in Fig.3.14, and it performs better than the matched filter for any SNR. For example at SER = 10^{-4}, for the case of five transmitters, our method has a 5dB gain over matched filter.

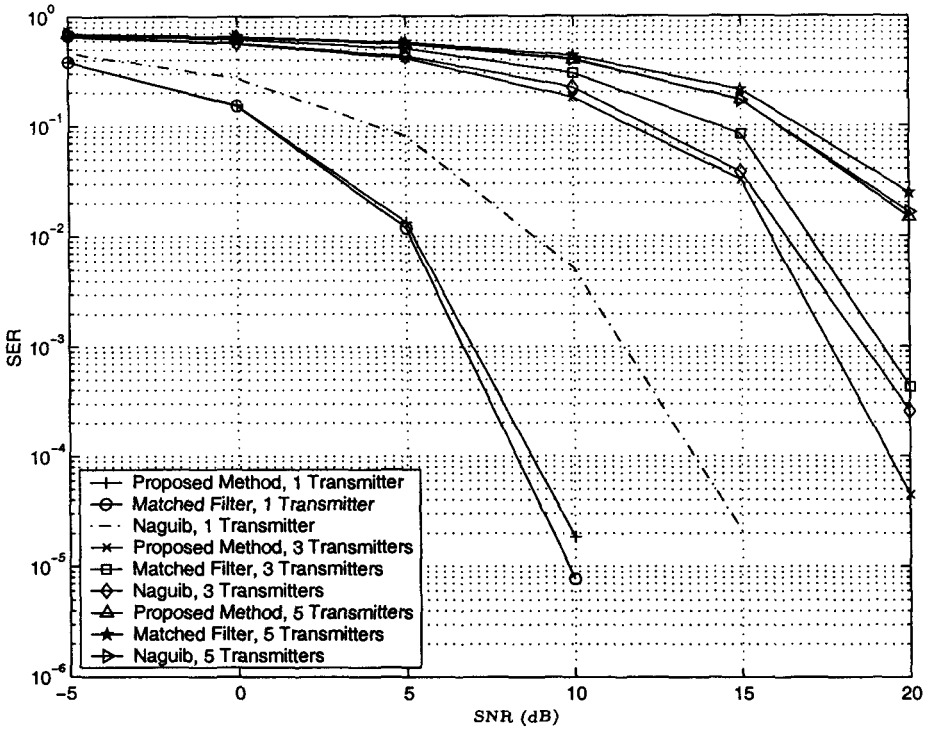

Figure 3.16: SER versus SNR, Alamouti code, INR = 10dB.

Code	SNR Range	INR	Q	P	M	N	T	K
Alamouti	(-5)dB-20dB	10dB	500	1,3,5	2	2	2	2

This figure shows the SER versus SNR for the case that $Q = 500$ and INR = 10dB. Contrary to Fig.3.8, we see that having two antennas at the receiver, all methods can suppress interference, because the level of interference is lower, but their performance is almost the same. However, the performance of our receiver becomes better compared to the two other methods as M increases.

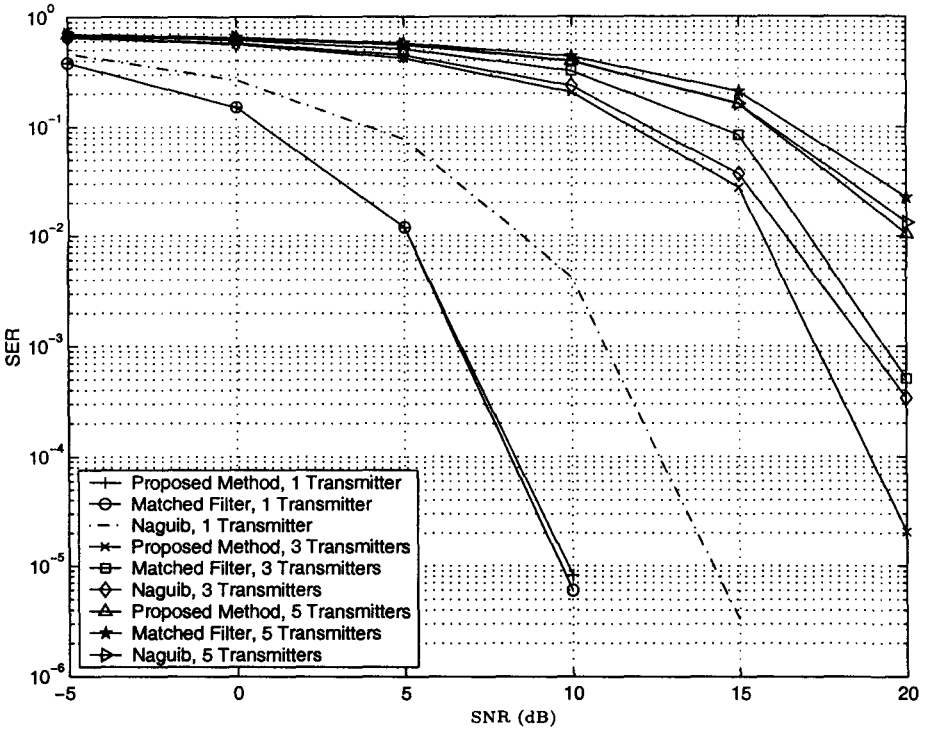

Figure 3.17: SER versus SNR, Alamouti code, using the true covariance matrix, INR = 10dB.

Code	SNR Range	INR	Q	P	M	N	T	K
Alamouti	(-5)dB-20dB	10dB	-	1,3,5	2	2	2	2

Figure 3.17 shows the SER versus SNR when the true covariance matrix is known by the receiver and $M = 2$. At SER $= 10^{-3}$ for the case of three transmitters, our receiver has a 2dB gain over the Naguib-Seshadri method, and a 2.5 dB gain over the matched filter.

3.3.3 Convergence of Sample Covariance Matrix

One important variable which may specify the feasibility of implementing our method is Q. If the coherence time of the channel is not large enough, there is a limit on Q. Theoretically we need to use infinite number of data matrices to find the true covariance matrix.

For a problem of SNR maximization using an array of antenna at the receiver while the signal has the steering vector $\mathbf{a}_s$, the optimum coefficients for the array of antennas is given as $\mathbf{R}_n^{-1}\mathbf{a}_s$, while the $\mathbf{R}_n$ is the noise covariance matrix, [28]. The number of sample vectors needed to reach in within 3dB of the maximum SNR achievable for this problem is shown to be two times of the length of the noise sample vectors [28]. This result is obtained by finding the distribution of SNR based on the distribution of sample vectors which are Gaussian noise vectors here and gives us a reasonable choice for Q instead of infinity.

In our receiver we do not know the probability distribution of the interferers; furthermore, our receiver is more complicated than a Sample Matrix Inversion (SMI) problem; therefore, it is very hard to find a threshold value (acceptable) Q using our receiver. We can choose the Q based on the coherence time of the channel. For example we can dedicate 10% or less of the time over which we can assume that the channel is constant to estimate $\mathbf{R}$.

The Diagonal Loading technique can be used to improve the condition number of the sample covariance matrix, [25] . In this technique a term $\gamma\mathbf{I}$ will be added to $\hat{\mathbf{R}}$, in which γ is equal to few times of the noise power, e.g. $4\sigma_n^2$.

$$\hat{\mathbf{R}}_{DL} = \frac{1}{Q}\sum_{q=1}^{Q}\underline{\mathbf{Y}_q}\,\underline{\mathbf{Y}_q}^T + \gamma\mathbf{I}. \tag{3.17}$$

Using $\hat{\mathbf{R}}_{DL}$ instead of $\hat{\mathbf{R}}$, our simulation results improve dramatically, this is shown in Fig.3.18 and Fig.3.19.

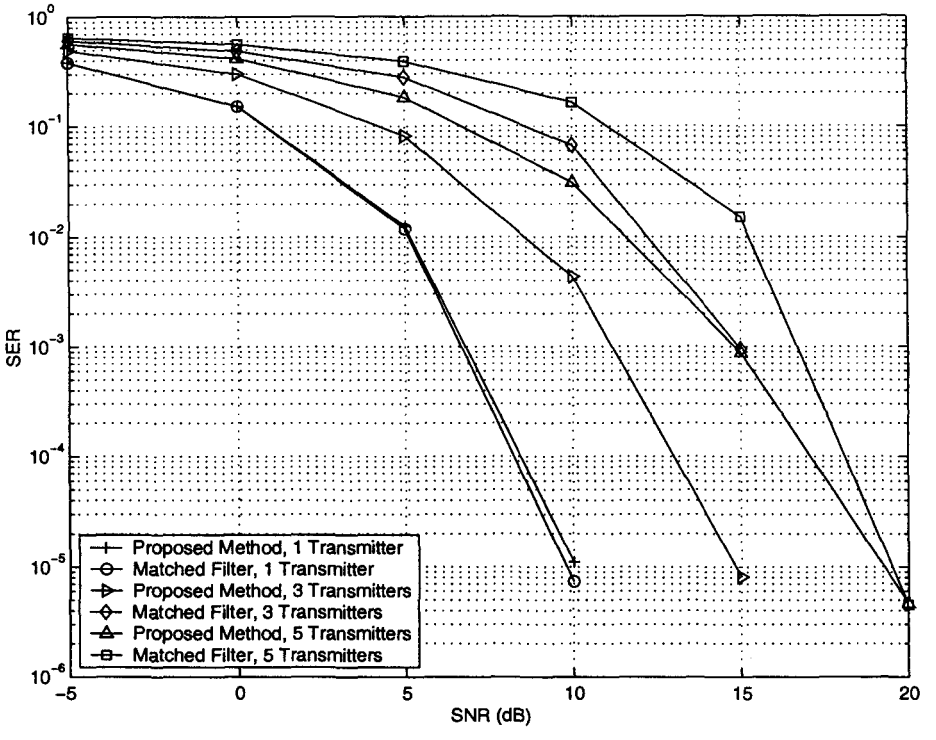

Figure 3.18: SER versus SNR, Tarokh's code, using DL technique, INR = 10dB.

Code	SNR Range	INR	Q	P	M	N	T	K	γ
Tarokh	(-5)dB-20dB	10dB	500	1,3,5	4	3	8	4	4

This figure shows SER versus SNR for the case that Tarokh's code is used, INR = 10dB, and diagonal loading is used to estimate the covariance matrix. It can be seen that using the same Q, the results are much better compared to Fig.3.14. For example for the case of three transmitters and using diagonal loading, at SER = 10^{-3}, the error performance has 9 dB gain over its equivalent in Fig.3.14. By comparing Fig.3.18 and Fig.3.15, It can be seen that using diagonal loading with $Q = 500$ we have reached in to 3 dB of the best performance of our receiver, for the case of 3 Transmitters and SER of 10^{-3}.

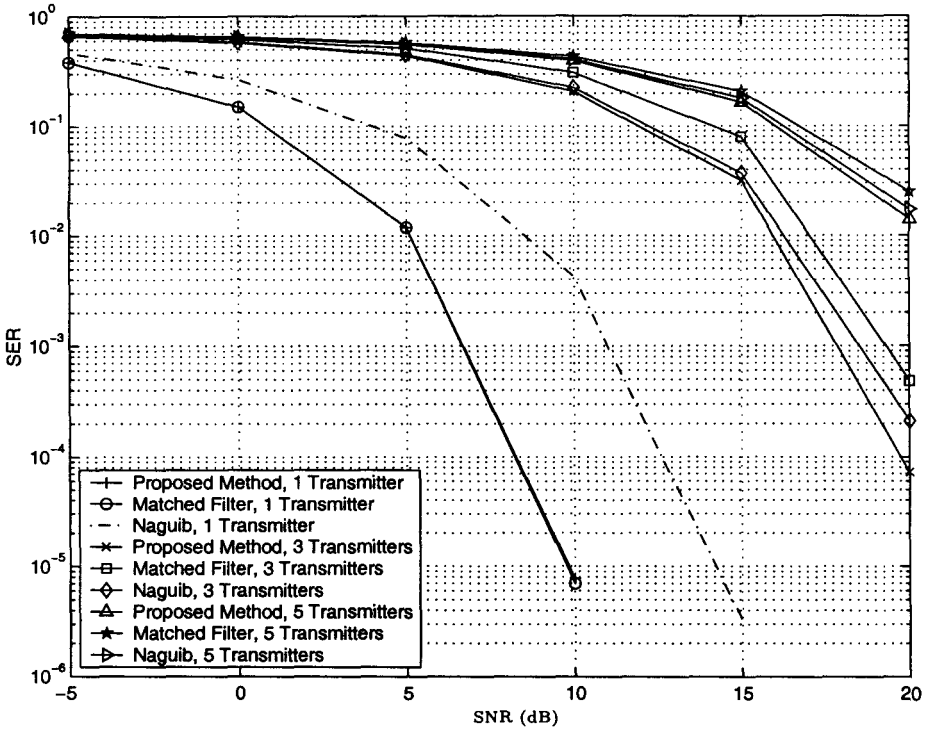

Figure 3.19: SER versus SNR, Alamouti code, using DL technique.

Code	SNR Range	INR	Q	P	M	N	T	K	γ
Alamouti	(-5)dB-20dB	10dB	500	1,3,5	2	2	2	2	4

This figure shows SER versus SNR for the case that Alamouti code is used, INR = 10dB, and DL technique is used to estimate $\mathbf{R}$. Using DL technique does not make much difference here, because Q is large enough to have a $\hat{\mathbf{R}}$ close to the $\mathbf{R}$ without using DL technique.

3.4 Discussion and Comparing

In this thesis it is assumed that the channel is flat fading. The assumption that the channel is not frequency selective is valid if the transmitted signals are narrow-band. The assumption of a slow fading channel is also valid if the symbol generation rate in the transmitter, compared to the speed of change in the channel is high. This is a realistic assumption for carrier frequency in the scale of GHz, a relative speed of around 100 Km/h, and transmission rates more than 10^4 symbols per second, [4].

We use OSTBCs for our desired transmitter for the following two reasons: In this case the matrix $\mathbf{A}$, defined in (1.19) is orthogonal regardless of the channel, and as mentioned in Chapter 1, OSTBCs reach the maximum diversity gain possible.

In the Naguib-Seshadri method, for each symbol, by using MMSE criterion a set of coefficients was presented [1]. Here we take a look at the number of coefficients that one can design in the Naguib and Seshadri method. If there are M antennas at the base station, the code takes T time slots to be transmitted, and the block length is K, then we have MT coefficients to design for each symbol in this method. Therefore, for all K symbols, there are MTK complex coefficients to design for one transmitter. But in our new method, $\mathbf{W}$ has dimension equal to $2MT \times 2K$ and we estimate the block of symbols jointly. Hence, we have $4MTK$ real coefficients to design. This means that there is two times more degree of freedom available, compared to the method in the Section 2.5. Furthermore, in our method we add another condition, which omits the self-interference.

In comparing our method with the Naguib-Seshadri method another important factor is the computational complexity, or the number of flops needed. In the simulations part, the command *flops* is used to compare the computational complexity. It is assumed that the desired transmitter's code is Alamouti code, and we know the channel matrix of the desired transmitter. For the case of two transmitters, and for seven different SNRs, and one-hundred different channel realizations, with ten thousand information symbol vectors for each channel realization, the flops needed are compared for the MV method and the Naguib-Seshadri method. The flops for the MV method, contain the calculation of the

matrix $\mathbf{A}_1$, the estimation of $\mathbf{R}$, the calculation of $\mathbf{W}$, and the detection part. The flops for the Naguib-Seshadri method contain the estimation of $\mathbf{M}$, using the received data, the calculation of the coefficients vector $\boldsymbol{\alpha}$, and finally the detection part. The simulation results show that the MV method needs slightly more flops. The ratio of the flops needed for the MV receiver to the flops needed for the Naguib-Seshadri receiver is 1.23.

3.5 Future Work

The linear receiver, presented in this thesis is mainly designed for interference rejection, but it can be extended to the case of a multi-access system. If the channels of all the transmitters and the type of their OSTBCs are known by the receiver then we can use this linear method to reject the Multi Access Interference (MAI). In this case there is no need to estimate the covariance matrix any more. There are many Multi User Detection (MUD) techniques available. In future work our receiver can be compared to these methods in terms of SER performance, complexity, and robustness.

In the new linear receiver, the desired transmitter is using OSTBCs, which are one subclass of a larger class of codes called LDCs (Linear Dispersion Codes). One possible future research is to use our method for some LDCs. The condition for using our method for LDCs is to choose those codes for which the matrix $\mathbf{A}$ in (1.19), is full rank regardless of the channel. Therefore, one has to derive the constraints on the dispersion matrices to satisfy this condition.

This new linear receiver can also be extended to the case where the channel of the desired transmitter should be estimated and it is unknown by the receiver.

Chapter 4

Conclusion

In this thesis, the problem of joint space-time decoding and interference rejection in a MIMO wireless communication system has been considered. The general case has been addressed when both the receiver and interferers are equipped with multiple antennas, and when the orthogonal space-time block codes are used to send the data simultaneously from the desired user, and interferers to the receiver. A new linear receiver scheme has been proposed to decode the data sent from the transmitter of interest while rejecting interference and noise, and eliminating self-interference. The proposed receiver has been designed by minimizing the output power, subject to a matrix constraint which preserves a unity gain for all symbols of the transmitter of interest, and guarantees that self-interference is cancelled. The resulting receiver has a computationally attractive closed-form solution. Simulation results have demonstrated that in this MIMO scenario, the proposed receiver has a substantially lower symbol error rate compared to the traditional matched filter receiver. Comparison with another existing method presented by Naguib and Seshadri also shows that our method is superior in performance. In our method we have more degrees of freedom in designing the receiver, compared to the Naguib-Seshadri method. The better SER performance here is achieved by trading off between SER and implementation complexity. Our method is 1.23 times more complex than the Naguib-Seshadri method.

This thesis shows that if we know the true covariance matrix, the SER performance of

our receiver is always better than the other two existing linear receivers,

Appendix A

Matched Filter as a Special Case of New Linear Receiver

In the case that there is not any interference, using (3.12) the true correlation matrix of the $\underline{\mathbf{Y}}$ is given as

$$\mathbf{R} = \mathbf{A}_1\mathbf{A}_1^T + \mathbf{I}. \tag{A.1}$$

Using the matrix inversion lemma [29], and defining $\mu \triangleq \|\mathbf{H}_1\|_F$, the $\mathbf{R}^{-1}$ can be expanded in the following way

$$\begin{aligned} \mathbf{R}^{-1} &= (\mathbf{I} + \mathbf{A}_1\mathbf{A}_1^T)^{-1} && \text{(A.2)} \\ &= \mathbf{I} - (\mathbf{A}_1(\mathbf{I} + \mathbf{A}_1^T\mathbf{A}_1)^{-1}\mathbf{A}_1^T). && \text{(A.3)} \end{aligned}$$

Considering (1.20), it can be simplified as

$$\begin{aligned} \mathbf{R}^{-1} &= \mathbf{I} - (\mathbf{A}_1(\mathbf{I} + \mu^2\mathbf{I})^{-1}\mathbf{A}_1^T) && \text{(A.4)} \\ &= \mathbf{I} - \frac{\mathbf{A}_1\mathbf{A}_1^T}{1+\mu^2}. && \text{(A.5)} \end{aligned}$$

Substituting it back in (3.14), the new linear receiver for this scenario is shown as $\mathbf{W}_{opt1}$, and given as

$$\mathbf{W}_{opt1} = (\mathbf{I} - \frac{\mathbf{A}_1\mathbf{A}_1^T}{1+\mu^2})\mathbf{A}_1[\mathbf{A}_1^T(\mathbf{I} - \frac{\mathbf{A}_1\mathbf{A}_1^T}{1+\mu^2})\mathbf{A}_1]^{-1} \quad (A.6)$$

$$= (\mathbf{A}_1 - \frac{\mu^2\mathbf{A}_1}{\mu^2+1})[\mu^2\mathbf{I} - \frac{\mu^4}{\mu^2+1}\mathbf{I}]^{-1} \quad (A.7)$$

$$= \frac{\mathbf{A}_1}{\mu^2+1} \cdot \frac{\mu^2+1}{\mu^2} \quad (A.8)$$

$$= \frac{\mathbf{A}_1}{\mu^2} \quad (A.9)$$

This shows that matched filter is a special case of our new linear receiver, if the estimate of the correlation matrix is exact, and there is not any interference. This is what we should expect, because maximizing SINR for this case is equal to maximizing SNR, and the condition in (3.6) is satisfied by matched filter.

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